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NELSON WA MATHS
for the Australian Curriculum
Revised 1st Edition

Stephen Corcoran | Ross Brodie | Stephen Swift | Sue Garner

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NELSON
CENGAGE Learning™

Contents

About this book

At the beginning of each chapter

- Clear outline of chapter contents
- Strong links to the curriculum
- Prior learning provides a quick check of understanding for the individualisation of instructional sequence

Contents
6.1 Patterns and symbols
6.2 Number rules
6.3 Algebraic expressions
6.4 Solving equations
Chapter summary
Chapter review

Prior learning
Chapter 6

Parent guide
Chapter 6

Curriculum guide
Chapter 6

9780170361910

Australian Curriculum statements

Patterns and algebra
Introduce the concept of variables as a way of representing numbers using letters. (ACMNA175)
Create algebraic expressions and evaluate them by substituting a given value for each variable. (ACMNA176)
Extend and apply the laws and properties of arithmetic to algebraic terms and expressions. (ACMNA177)
Linear and non-linear relationships
Solve simple linear equations. (ACMNA179)

231

6.1 Patterns and symbols

Investigate: Restaurant tables

Many restaurants use square tables that can seat four people. When they get a booking for more than four people, they put tables together so that more people can sit at them. Some examples are shown below. Your teacher might have some cut-outs for you to use in this investigation. Work in small groups.

4   8   10

- How many tables do you need to put together for 6 people?
- How many people can sit at 6 tables put together?
- How many tables will need to be put together for a big birthday party of 20 people?
- Write a rule for the number of people that can sit at any number of tables.

- Parent guide provides vital information for parents
- Curriculum guide provides a rationale in words that students will understand
- Clear sectioning consistent with the exercises
- Many sections begin with a concrete investigation, with teacher support materials provided
- Intriguing aids for students provided

In each chapter

- Examples are clearly set out with explanations on the left and expected student answers in bold on the right
- Colour coded examples for ease of teacher directions to students
  Green – nearly all students will master
  Gold – most students will master
  Blue – some students should master
- Examples are sequenced in a natural learning progression that promotes chunking and effective use of working memory
- Clear examples enable parents to assist their student with confidence that they are 'doing it the way the teacher explained it'
- Important boxes highlight key terms and concepts

Example 2

The table below shows input and output numbers for a rule. Write a rule for the output numbers.

Input	5	6	7	8	9	10
Output	15	18	21	24	27	30

Solution

Look for a pattern.   15 is 3 times 5
                             18 is 3 times 6
Write the rule.   **The output number is three times the input number.**

Example 4

For the number pattern 5, 10, 15, 20, …, use a variable to:
a write a rule for the pattern   b work out the 9th term.

Example 6

Write a rule in symbols for the following table of inputs and outputs.

Input p	2	3	4	5
Output v	19	16	13	10

Important!

Variables and number patterns

A **variable** (**pronumeral** or **unknown**) is a letter or symbol that stands for a number.
We often use the symbol n for the term number of a number pattern. The symbol a is often used for the term itself, so a_4 is the fourth term of a number pattern.

- Initial exercise questions focus on understanding and refer directly back to examples
- Questions are categorised according to the four proficiency strands - understanding, fluency, problem solving and reasoning
- Questions and parts are colour coded to be consistent with the examples

- Colour is used pedagogically to enhance learning

- There is an abundance of carefully designed investigations proven to aid deep understanding

At the end of each chapter

- A chapter summary that re-states all important concepts

- A comprehensive review set out with example references and consistent with exercise layout
- Answers to all questions are given at the back of the book

Other books in the series

- There are five books in this Years 7 to 10 series, which has been written specifically for the Australian Mathematics Curriculum, including two books for Year 10 (see back cover).
- There is also a printed Teacher's Edition of each textbook with additional guidance and advice on implementing the Australian Mathematics Curriculum in your classroom.
- Accompanying each printed textbook is a digital textbook called the NelsonNetBook (see opposite page).
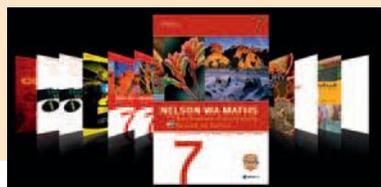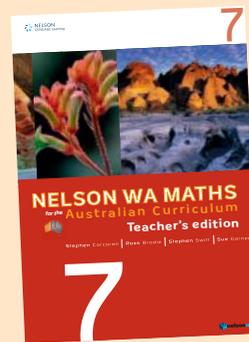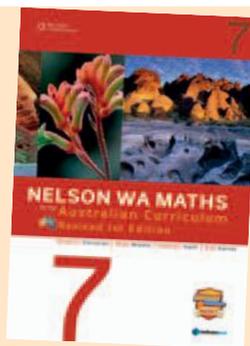
Icons in the textbook

- In the outside margins of your textbook you will find numerous icons for resources that can be accessed through the NelsonNet website or through the NelsonNetBook.

Icon	Description	Icon	Description
Alternative method / Moving the decimal	Explains different ways of solving a maths problem	Puzzle sheet / Find the number	Makes learning maths methods and skills fun
Animated example / Stem-and-leaf plots	Shows how to tackle difficult problems	Quiz / Expressions	Tests and marks your knowledge of a topic
Calculator skills / Length and area	Develops your scientific calculator skills	Teacher notes / Restaurant tables	Gives guidance for your teacher
Curriculum guide / Chapter 6	Explains what you need to know and be able to do	Technology / Excel	Provides a spreadsheet or Geogebra activity
Extra questions / Exercise 6.1	Allows you to do additional practice and drill	Learning object / Rectangle division (L3704)	Provides interactive ways of learning about maths*
Maths clip / Statistics	Gives the 'big picture' view of your maths topic	Video tutorial / Algebraic expressions	Fully explains maths methods and skills
Maths dictionary	Provides an illustrated explanations of all terms	Weblink / Algebra masterclass	Makes maths relevant to everyday life
Parent guide / Chapter 6	Provides guidance for your parent to help you	Worked solutions / Exercise 6.1	Gives solutions steps for selected problems
Prior learning / Chapter 6	Finds out what you already know and can do	Worksheet / Writing a rule	Supplies tasks to be done digitally or by using a pen

* Your teacher will explain how to access The Learning Federation learning objects.

NelsonNet Website

- Go to www.nelsonnet.com.au to log in or find out more.
- The first screen you will see on the student website is called 'Chapter resources'.
- Click on a chapter and a list of different resources types will appear.
- Click on a resource type and a list of specific resources will pop up.
- Clicking on one of those resources will open a pdf file, start a video, or some special maths software.
- Use the blue tabs on the left of the screen to access calculator resources and the maths dictionary.

NelsonNetBook

- This is a web-based ebook that you can customise to meet your own learning needs.
- The icons with the blue NelsonNet logo are 'hotspots'. Click on an icon to open that resource.
- The tools on the vertical toolbar allow you to personalise pages in a variety of ways, including voice recordings, drawings and links to favourite websites. They also enable you to zoom in and out.
- The tools on the horizontal toolbar allow you to navigate around your ebook and change settings.
- Administration tools allow your teacher to manage classes and share annotations with you.
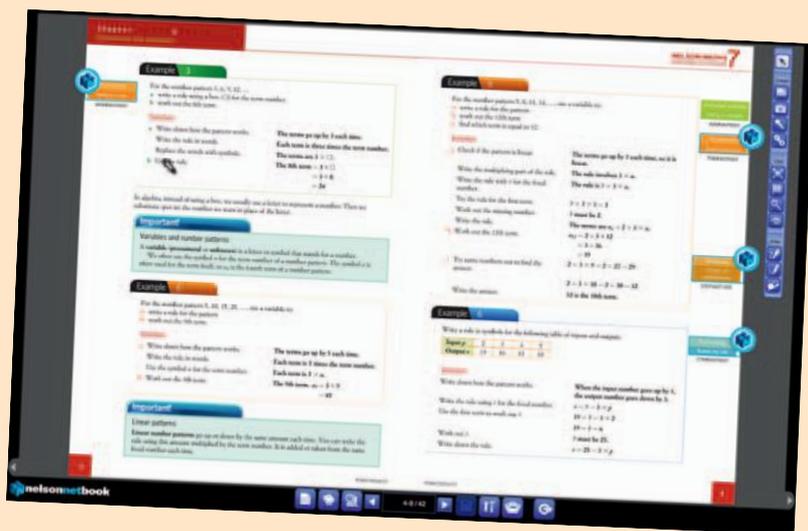
Please note that complimentary access to NelsonNet and the NelsonNetBook is only available to teachers who use the accompanying student textbook as a core educational resource in their classroom. Contact your sales representative for information about access codes and conditions.

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$\therefore$	therefore		QED	*quod erat demonstrandum*
$=$	is equal to		$\triangle ABC$	triangle *ABC*
$\neq$	is not equal to		$\angle ABC$	angle *ABC*
$\approx, \sim, \doteq$	is approximately equal to		$\equiv, \cong$	is congruent to
$>$	is greater than		$\||, \sim$	is similar to
$\ngtr$	is not greater than		$\|$	is parallel to
$\geq$	is greater than or equal to		$\perp$	is perpendicular to
$<$	is less than		$\%$	percentage
$\nless$	is not less than		$\frac{a}{b}$	fraction, division $a \div b$ or ratio $a : b$
$\leq$	is less than or equal to		$\pm$	plus or minus
$\propto$	is proportional to		$\sqrt{}$	square root
$\in$	is a member of		$\sqrt[3]{}$	cube root
$\notin$	is not a member of		$\sqrt[n]{}$	*n*th root
$\llcorner$	right angle		$45°$	45 degrees
BC	before Christ		π	pi $\approx 3.1415926\ldots$
BCE	before the common era		$\{\}, \varnothing$	null set, empty set
AD	Anno Domini		$\subseteq$	subset of
CE	common era		$\subset$	proper subset of

Metric system

Symbols

millimetre — mm
centimetre — cm
metre — m
kilometre — km
hectare — ha
millilitre — mL
litre — L
kilolitre — kL
megalitre — ML
milligram — mg
gram — g
kilogram — kg
tonne — t

Length

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

Area

$100 \text{ mm}^2 = 1 \text{ cm}^2$
$10\,000 \text{ cm}^2 = 1 \text{ m}^2$
$10\,000 \text{ m}^2 = 1 \text{ ha}$
$100 \text{ ha} = 1 \text{ km}^2$

Capacity

1000 mL = 1 L
1000 L = 1 kL
1000 kL = 1 ML

Mass

1000 mg = 1 g
1000 g = 1 kg
1000 kg = 1 t

Greek alphabet

A, α alpha
B, β beta
Γ, γ gamma
Δ, δ delta
E, ε epsilon
Z, ζ zeta
H, η eta
Θ, θ theta
I, ι iota

K, κ kappa
Λ, λ lambda
M, μ mu
N, ν nu
Ξ, ξ xi
O, o omicron
Π, π pi
P, ρ rho

Σ, σ sigma
T, τ tau
Y, υ upsilon
Φ, ϕ phi
X, χ chi
Ψ, ψ psi
Ω, ω omega

Statistics and probability

1

Statistics

Contents

Prior learning
Chapter 1

Parent guide
Chapter 1

Curriculum guide
Chapter 1

Australian Curriculum statements

Data representation and interpretation

Identify and investigate issues involving numerical data collected from primary and secondary sources. (ACMSP169)

Construct and compare a range of data displays including stem-and-leaf plots and dot plots. (ACMSP170)

Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data. (ACMSP171)

Describe and interpret data displays using median, mean and range. (ACMSP172) AC

Herman Hollerith was the founder of IBM, one of the largest computer companies in the world today. He was also responsible for inventing a tabulating machine that was used to count the 1890 United States census. Statistics still relies heavily on computers for calculations. However, statistics goes far beyond just counting. It is the mathematics of the collection, organisation and analysis of information.

Mathematical literacy

Maths dictionary

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

average	data	line graph	sample
axis	dependent	mean	scale
bar chart	discrete	median	score
bias	divided bar chart	mode	spread
bimodal	dot plot	numerical	statistical
categorical	fair	outlier	stem-and-leaf plot
cell	frequency	picture graph	survey
central tendency	frequency	pie graph	tabulate
circle graph	distribution	population	tally mark
column	graph	range	
continuous	histogram	relationship	
cumulative frequency	independent	row	

1.1   Data and surveys

Weblink

ABS: CensusAtSchool

Weblink

Earth's population chain letter

Weblink

Worldometer

Weblink

Nation master: Country comparisons

The information that you would collect about a car could include the make, the number of cylinders, the power of the engine, the colour, the type of engine, the weight, the legal number of occupants, fuel economy, gear ratios, whether it is front-wheel drive, rear-wheel drive or four-wheel drive, and so on. Some items of information have number values and some are categories (names). The number of cylinders can only be a whole number, but the weight could be given to any degree of accuracy.

Normally, information such as that above is collected about many cars, not just one. Before starting, you need to decide what information to gather.

Important!

Statistical data

The information that you collect about related things (or people) is called **statistical data**.
Categorical data places the information in categories such as colour, sex and nationality.
Numerical data consists of numbers. There are two kinds of numerical data.
Continuous data can take any value between the smallest and largest values, so we may need to show how accurate it is.
Discrete data can only have particular values. In most cases, the values are whole numbers.

Example 1

Classify the type of data for each of the following.
a The numbers of children in the families of students in a Grade 7 class.
b The heights of students in a Grade 7 class.
c The body type (ectomorph, mesomorph, endomorph) of each student in a Grade 7 class.
d The favourite TV channel (2, 7, 9, 10) of each student in a Grade 7 class.
e The shoe size (. . ., 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$, . . .) of each student in a Grade 7 class.

Solution

a The number of children must be a whole number.   **It is discrete.**

b Exact height can be any number. For example,
a student can be 147.28 cm tall.   **It is continuous.**

c Body type is a category.   **It is categorical.**

d TV channel is really a name, not a number.   **It is categorical.**

e The shoe size can only be certain numbers.   **It is discrete.**

When you think of your age, it could be any type of data, depending on what it is used for.
• If you mean the time since you were born, then the data would be continuous.
• If you mean the number of birthdays you've had, then the data would be discrete.
• If you mean your age group for a sport, then it would be categorical.
The 'number of birthdays' is the most common meaning, so the data is discrete.
Suppose you wanted to find the funniest cartoon show on Queensland TV. Not every person in Queensland actually watches cartoon shows. It would be very expensive and take a long time to ask every person. If you just asked children at a childcare centre, you might not get the same results as if you asked Grade 7 students.

Important!

Samples, surveys and bias

When you collect information, the **population** is the complete group that the information would relate to. A **census** obtains data from the whole population.
A **sample** is part of the population. A **survey** obtains data from a sample.
A **fair sample** represents the whole population. A **biased sample** is not typical of the population.
Using poor methods or questions can lead to **bias** in a sample.

Example 2

Is a sample or the population used in each of the following?
a Grade 7 students at Marysville High School are asked for their favourite cartoon show.
b The fastest runner is chosen to represent a school.
c Every thousandth person on the electoral roll is asked who they would vote for at the next election.
d A TV show asks parents to phone in to vote Yes or No to outlaw smacking children.

Solution

a Not all children are asked.   **It is a sample.**

b Every student at the school would have a chance.   **The population is used.**

c Not every person is asked.   **It is a sample.**

d Only some parents would ring.   **It is a sample.**

Example 3

State whether each of the following is fair or biased.

a Three students from each class are asked if they prefer 'Pop, Jazz, Hip-Hop or Disco' music to find the most popular.

b People leaving a supermarket on a Friday morning are asked which washing powder they use.

c Every 200th person in the phone book is asked if they are happy with their phone service.

d A market survey company rings people at random and asks 'Do you prefer fresh Queensland fish or fish from other states?'

e Every 1000th packet of chips coming from a factory is weighed.

Solution

a The question leaves out some kinds of music. **The question is biased.**

b Only the people leaving one supermarket at one time in one town are questioned. **The sample is biased.**

c Mobiles would not be included. **This is a fair sample for fixed line services.**

d Using the word 'fresh' for one kind is unfair. **The question is biased towards Queensland.**

e The sampling method is good for this case. **The method is fair.**

Investigate: Questionnaire design

Work in groups to find what is wrong with each of the following survey questions. One person in each group should take notes.

1 What is your name?
2 How old are you?
3 Do you go to bed early or late?
4 Do you play a lot of sport?
5 What kind of music do you like?
6 What do you like about your work/school?
7 What should the government do about unemployment?
8 What do you think of dole bludgers?
9 How much fast food do you eat?

When you have finished, your teacher may want each group to report back to the whole class.

Investigate: Data collection

In this investigation, you will use a data collection sheet to obtain the responses to a simple survey about TV watching habits.

The questions are as follows.

1 What is your age group?
   Under 6, 6–11, 12–18, 19–24, 25–34, 35–44, 45–64, 65 or over
2 What is your sex: M, F?
3 How much TV did you watch yesterday before 9 a.m.?
   0 hours, up to 1 hour, up to 2 hours, more than 2 hours
4 How much TV did you watch yesterday between 9 a.m. and 3 p.m.?
   0 hours, up to 1 hour, up to 2 hours, more than 2 hours
5 How much TV did you watch yesterday between 3 p.m. and 7 p.m.?
   0 hours, up to 1 hour, up to 2 hours, more than 2 hours
6 How much TV did you watch yesterday between 7 p.m. and 10 p.m.?
   0 hours, up to 1 hour, up to 2 hours, more than 2 hours
7 How much TV did you watch yesterday after 10 p.m.?
   0 hours, up to 1 hour, up to 2 hours, more than 2 hours
8 How much TV did you watch last weekend?
   0 hours, about 1, 2, 3, 4, 5, 6, . . . hours

The data collection sheet you will use is shown below. You will need to make a copy of this sheet to use.

TV Survey Data Collection sheet

Instructions:
    Write the age group for Q1: U6, 6–11, 12–18, 19–24, 25–34, 35–44, 45–64, 65+
    Write M or F for Q2
    Write 0, 1, 2 or 2+ for Q3–7
    Write 0 or the number of hours for Q8

Person	1 Age	2 Sex	3	4	5	6	7	8
1								
2								
3								
4								
5								

Collect data for different groups of, say, 10 people. What do you find? Is the information the same for each group?

Example 4

A mobile phone company wants to know how frequently mobiles are used for sending pictures and SMSs, making voice calls, internet and games. What is the population for the survey?

Solution

The owners of mobile phones are the people who determine how they are used.

The population is all the people who own mobile phones.

Investigate: Year 7 countries of birth and languages

In this investigation, you will use a large collection of data about Year 7 students to investigate languages spoken by Australian school students. The data collection is on the textbook website. Your teacher may have already downloaded it for you to use.

Year 7 countries of birth and languages

Gender ▾	State ▾	Country of birth ▾
Male	WA	(All)
Male	VIC	(Top 10…)
Male	SA	(Custom…)
Male	TAS	Australia
Female	WA	Austria
Female	VIC	Cambodia
Male	SA	Canada
Male	WA	Chile
Male	QLD	China (excluding Hong
Female	QLD	England
Male	SA	Fiji
Female	SA	Hong Kong (SAR of Cl
Female	SA	India
Female	TAS	Indonesia
Male	QLD	Iran
Male	VIC	Iraq
Male	TAS	Ireland
		Italy
		Japan
		Korea; Republic of (Sc
		Singapore
		Australia

- Find how many different countries this group was born in. Click on the little arrow next to the blue heading **Country of birth**. A box will pop up showing the different countries. Count how many there are.
- Scroll down until you find New Zealand and click on this. Count how many students are from New Zealand. Notice that the arrow has changed to a blue colour.
- Click on the blue arrow next to **Country of birth** and scroll back up to (All). Click on this to restore the list of students. Notice that the arrow has changed back to black.
- Find out how many students speak the language Tagalog at home. Find out what country uses Tagalog.
- Many students in Australia speak Chinese at home. They speak two different Chinese languages, although they are both written the same way. What are they called? How many students speak Chinese?

- To find how many girls were born in England, click on **Gender** and choose Female. Then click on **Country of birth** and choose England.
- Count the number of students from NSW. Count the number of students from NSW who speak Tagalog at home. What percentage of NSW students speak Tagalog? What percentage of WA students speak Tagalog? What does this suggest?
- Use the spreadsheet to find whether students in Tasmania speak more or fewer languages than students in other states.

Exercise 1.1   Data and surveys

Understanding

1 For each of the following, state whether the data is categorical, discrete or continuous.
  a The masses of Year 7 students.
  b The first initials of Year 7 students.
  c The shoe sizes of Year 7 students.
  d The distances students have to travel home.
  e The ages of Year 7 students.
  f The kinds of pets each student has.

Extra questions
Exercise 1.1
MAT07SPEQ00005
See Example 1

2 For each of the following, state whether the population or a sample has been used.
  a To find out what the most popular TV program was the previous night in Brisbane, the pupils in a Year 3 class at a state school in Brisbane were asked to write down the programs they watched the previous night.
  ⓑ To find out what size the spaces in a new car park should be, a supermarket measured the sizes of cars parked in the old car park one Saturday morning.
  c To find out whether pupils at a school wanted to have an evening or daytime prize-giving ceremony, all pupils were asked 'Do you want a daytime prize-giving ceremony?'

See Example 2

Worked solutions
Exercise 1.1
MAT07SPWS00005

3 For each part of question **2**, state whether the method is fair or biased.

See Example 3

4 For each of these surveys, state whether a sample or the population should be used.
  ⓐ The staff of the school tuckshop want to find out how many pies and pasties they should have on their regular order.
  b The factory canteen wants to know the number of rolls and sandwiches to make for the workers' picnic.
  c A publisher of pocket diaries for Australia wants to know whether or not to put a vinyl cover on the diary. The vinyl cover adds 50c to the price.
  d The manager of a hardware shop in Gympie wants to find out whether to stock plastic or brass plumbing materials. There isn't enough space for a full range of both.

Fluency

Worked solutions
Exercise 1.1
MAT07SPWS00005

5 State the population for each part of question **4**.

6 Each of the following surveys has a problem with the method. Indicate a possible problem and how you could improve the method.
  a To find out whether a new freeway should be built through Townsville, the people who lived in affected suburbs were asked 'Would you like a freeway to make it easier to get around and through Townsville?'

Problem solving

See Example 4

b A firm that sells pool products Australia-wide sent a free sample of a new pool cleaner to pool owners in Sydney, asking them to try the product and send back a prepaid card with answers to the questions on the back. The questions were:
- 'Did you find the cleaner effective? YES/NO'
- 'If it cost $4.20 per kg, would you buy it? YES/NO'

c To find out which party was most likely to win the next election, a current affairs program asked people watching to ring up and register a vote.

Reasoning

7 Would you expect the same result if you did a survey of two different classes that had the same number of students in each class? Why/Why not?

8 What happens to the results of surveys as the number in the samples is increased?

1.2 Tabulating data

When data is collected, it is normally put into a table. This is called **tabulating**. The simplest way to tabulate data is by counting.

The boxes in a table are called **cells**. Each line of cells going across the table is a **row**, while a line going down the table is called a **column**. The row at the top of a table often contains a **heading** to describe the data in each column.

Data can be tabulated in different ways. You need to be able to read a table to get information from it.

Example 5

The table below shows some of the results from a trial of different moisturisers. Each moisturiser was repackaged in plain containers and rated by 50 different people who used the moisturiser on their faces for three days.

Brand	Overall rating %	Texture %	Fragrance %	Absorption %	Price $/50 mL
Estee Lauder	54	61	50	69	69.00
L'Oreal	61	69	65	84	13.10
Lancome	73	78	71	85	59.00
Home Brand	67	67	62	89	0.32
Olay	63	71	57	80	6.48
Dior	57	60	62	76	57.00
Nutrimetics	61	68	62	74	16.40
Priceline	43	58	38	88	2.00

Source: Choice magazine, April 2003

a Which moisturiser had the highest overall rating?
b Which moisturiser had the highest absorption rating?
c Which moisturiser had the lowest rating for fragrance?
d Which moisturiser was the best value?

Solution

a Use the second column.

Lancome had the highest overall rating, 73%.

b Use the fifth column.

Home Brand had the highest absorption rating, 89%.

c Use the fourth column.

Priceline had the lowest fragrance rating, 38%.

d Compare the costs of the best overall rating moisturisers.

Lancome is the highest rating, but costs $59 for 50 mL. Home Brand is the second best, but costs only $0.32 for 50 mL.

Give your answer.

Home Brand is the best value of these moisturisers.

Important!

Frequency tables

A **score** is a thing that you count. It can be a number like height, an object like a car or a property such as the colour of a person's eyes.

Scores are often counted using vertical strokes called **tally marks**. Every fifth one is crossed through the last 4 to make it easier to count.

The **frequency** of a score is the number of times it occurs. It is the count shown by the tally.

A **frequency table** is a table of scores and their frequencies. Other columns can be included as well. It is sometimes called a **frequency distribution** or a **one-way table**.

Data is often grouped into convenient intervals to construct frequency tables, such as counts of 51–60, 61–70, etc. (discrete), or heights of 130–139 cm, 140–149 cm, etc. (continuous).

Example 6

TLF Learning object

Dice duels (L2641)

The data below represents the heights of a group of Year 8 students. The heights have been taken to the nearest 5 cm.

145	140	160	135	135	145	140	145	135
150	145	140	140	145	125	140	140	150
145	140	155	145	145	135	155		

Make a frequency table for this data.

Solution

Draw up a table with the headings Score, Tally and Frequency.

Score	Tally	Frequency

Find the lowest and highest scores.

Write the lowest score at the top of the score column. Fill down the column in order with all the other scores. The highest score is at the bottom.

Work through the data; each time an item occurs put a stroke in the tally column. Cross off the item (in pencil) so that you don't lose track.

The lowest score is 125 and the highest is 160.

Score	Tally	Frequency
125		
130		
135	||	
140	|	
145	|	
150		
155		
160	|	

For every fifth tally mark, put a stroke through the four marks before it to help keep a count on the score.

Leave a space after each group of five marks.

When you have tallied all the data, write the frequency next to each group of tally marks.

Add up the frequencies to make sure that you haven't missed any.

Score	Tally	Frequency
125	|	1
130		0
135	||||	4
140	ⅣⅡⅡ	7
145	ⅣⅢⅢ	8
150	||	2
155	||	2
160	|	1
Total		25

Example 7

Use groups of 20–24, 25–29, 30–34, 35–39, . . . to make a frequency table for the following information about the times (in seconds) Year 8 students take to read a short paragraph.

45.9	26.7	38.1	22.5	42	36	33.4	23.1	30.1
26.1	28.8	29.1	27.6	29.8	29.4	41.1	29.5	48.9
31.2	47.1	38.1	39	45	29.1	21.6	34.2	21.9
27	27	33.5	27.6	39				

Solution

Draw up the table, with the headings of Time (score in seconds), Tally and Frequency.

Put the score groups in the table, and insert the tally marks. Scores of 29.1 and 29.4 go in the 25–29 group but scores of 29.5 and 29.8 go in the 30–34 group.

Complete the frequency column.

Time (score in seconds)	Tally	Frequency
20–24	||||	4
25–29	ⅢⅢ ⅢⅢ	10
30–34	ⅢⅢⅡ	7
35–39	ⅢⅢ	5
40–44	||	2
45–49	||||	4
Total		32

Exercise 1.2   Tabulating data

1 The following timetable shows the Brisbane City Council Route 120 bus service for Saturday.

Garden City to City												
Saturday												
Route number	120	120	120	120		120	120	120	120	120	120	120
	a.m.	a.m.	a.m.	a.m.		p.m.	p.m.	p.m.	p.m.	p.m.	p.m.	p.m.
Garden City Interchange 'D'	7:24	7:54	8:24	8:54		5:54	6:24	6:54	7:24	8:24	9:24	10:24
Hibiscus	7:28	7:58	8:28	8:58		5:58	6:28	6:58	7:28	8:28	9:28	10:28
Griffith University A	7:34	8:04	8:34	9:04		6:04	6:34	7:04	7:34	8:34	9:34	10:34
QEII Hospital	7:37	8:07	8:37	9:08		6:08	6:37	7:07	7:37	8:37	9:37	10:37
Salisbury East	7:42	8:12	8:42	9:14	Route 120 services operate every 30 minutes until:	6:14	6:41	7:11	7:41	8:41	9:41	10:41
Tarragindi	7:47	8:17	8:47	9:20		6:20	6:46	7:16	7:46	8:46	9:46	10:46
Ekibin	7:51	8:21	8:51	9:25		6:25	6:50	7:20	7:50	8:50	9:50	10:50
Buranda Busway Station	7:56	8:26	8:56	9:30		6:30	6:55	7:25	7:55	8:55	9:55	10:55
Mater Hill Busway Station	7:58	8:28	8:58	9:32		6:32	6:57	7:27	7:59	8:59	9:59	10:59
South Bank Busway Station	8:00	8:30	9:00	9:34		6:34	6:59	7:29	8:01	9:01	10:01	11:01
Cultural Centre Busway station	8:02	8:32	9:02	9:36		6:36	7:01	7:31	8:03	9:03	10:03	11:03
Queen St Bus Station	8:05	8:35	9:05	9:39		6:39	7:04	7:34	8:06	9:06	10:06	11:06

a What time does the first bus leave Garden City on Saturday?

b What time does the last bus from Garden City arrive at Queen St Bus Station on Saturday?

c What is the earliest time after 9 a.m. on Saturday that you can catch a bus at Tarragindi?

d Paul reached the Tarragindi bus stop at 7 p.m. How long does he have to wait for the next 120 bus?

e How long does the first bus after 8 a.m. take to travel from QEII Hospital to Ekibin?

2 The table below shows the football results for a local competition.

Team	Played	Won	Drawn	Lost	Points for	Points against	Premiership points
Norths	9	7	1	1	183	126	15
Tigers	9	7	0	2	262	130	14
Bulldogs	9	6	0	3	207	109	12
Falcons	9	6	0	3	205	135	12
Souths	9	4	1	4	170	172	9
Dragons	8	4	0	4	138	118	8
Bulls	9	4	0	5	214	143	8
Easts	9	1	0	8	88	310	2
Wests	9	0	0	9	67	291	0

a How many matches did the Bulldogs lose?
b How many columns are in the table?
c How many matches did the Bulldogs draw?
d Which team scored the highest number of points?
e Which team scored the lowest number of points?
f Which team had the highest number of points scored against it?

3 The table on the next page shows the quantity of paint needed for different-sized rooms.
Use the table to work out how much paint would be needed for the walls of a room with:
a perimeter 12.3 m, wall height 2.2 m
b perimeter 15 m, wall height 3.1 m
c perimeter 16.5 m, wall height 2.5 m
d How much paint would be needed for the ceiling of a room with a perimeter of:
   i 11 m   **ii** 16 m?

Quantity calculator									
Room size (perimeter in metres)	Walls: required in litres for single coat (average cover: 16 m²/L)								Ceilings: requirement in litres of 'Ceiling White' is for single coat (average cover: 14 m²/L)
	Height of walls (metres)								
	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.3	
9.8	1.5	1.5	2	2	2.5	2.5	2.5	2	0.5
11.1	1.5	2	2	2.5	2.5	3	3	3	1
12.3	2	2	2.5	2.5	3	3	3.5	3.5	1
13.5	2	2.5	2.5	3	3	3.5	3.5	4	1
14.8	2	2.5	3	3	3.5	3.5	4	4	1
15.4	2	2.5	3	3	3.5	4	4	4.5	1.5
16.0	2.5	2.5	3	3.5	3.5	4	4	4.5	1.5
16.6	2.5	3	3	3.5	3.5	4	4.5	4.5	1.5
17.8	2.5	3	3.5	3.5	4	4.5	4.5	5	1.5
18.5	2.5	3	3.5	4	4	4.5	5	5	1.5
19.1	3	3	3.5	4	4	4.5	5	5.5	2
20.3	3	3.5	3.5	4	4.5	5	5.5	5.5	2
20.9	3	3.5	4	4	4.5	5	5.5	6	2

4 Make a frequency table for the following information showing the Maths test scores from a Year 8 class.

See Example 6

7	8	7	7	2	8	7	7	5	8	5	6	2	7	4
5	6	6	5	8	4	7	9	5	8	8	6	4	3	0

5 Make a frequency table for the following information showing the number of children in the families of some Year 8 students.

Worked solutions

Exercise 1.2

1	2	3	6	3	6	2	4	1	6	1	3	3	4	2
7	2	2	2	1	2	2	2	3	2	2	3			

MAT07SPWS00050

6 Make a frequency table for the following results of a survey asking 'Do you agree with the amount of money that the government provides for schools?' The data is coded as SA = Strongly agree, A = Agree, I = Don't know/No opinion, D = Disagree, SD = Strongly disagree.

SA	A	I	SA	SA	A	A	D	D	I
SD	SD	SD	D	D	SA	A	I	I	I
D	D	D	D	A	A	SA	SA	SA	SD
SD	D	D	D	A	D	SD	SA	SD	SD

7 Use the BCC 120 timetable for the bus service for Saturday in question **1** to answer these questions.

Fluency

  a How long does the first bus take to travel from Garden City to Queen St Bus Station?

  b How long do buses in the middle of the day take to travel from Garden City to Queen St Bus Station?

  c If you arrived at the Hibiscus bus stop at 11:05 a.m., how long would you have to wait for a 120 bus?

8 Use the table of football results from question **2** to answer the following.
   a Which team did the Norths draw with?
   b Which team did the Easts beat?
   c Have any teams played each other twice?
   d Is it possible to work out which team beat the Norths?

See Example 7

9 Use groups of 0–4, 5–9, 10–14, 15–19 and 20–24 to make a frequency table for the following lengths (in cm) of watermelon seedlings.

17.8	9	14.3	20.9	20.4	14.1	15	12.7	23.4
18.2	9.5	13	13.9	4.5	7.7	19.3	17.8	12.1
13	11.3	9	3.8	10.7	17.3	10.3		

10 Use groups of 80–99, 100–119, 120–139, 140–159, 160–179 to make a frequency table for the following masses of avocados, where the masses are in grams.

85	130	150	137	95	85	142	113	98
103	128	128	105	118	174	113	132	150
137	123	113	137	98	140	115	125	130
162	123	140						

11 The rooms in a house all have a height of 2.4 m. You have 15 L of paint to do the walls and must apply two coats of paint. The rooms are 3 m by 4 m, 2.5 m by 3 m, 2.5 m by 3.5 m, and 3.5 m by 4 m. Use the information in question **3** to find which rooms you can finish.

Reasoning

12 Complete the cross-table below, showing the results of a round-robin tennis match in which every contestant plays every other contestant.

	T.S.	S.A.	W.C.	J.L.	N.S.	P.G.	Games W	Games L	Sets W	Sets L
Toni Scarpelli	***	6–3	6–2	3–6	6–4	6–1	27	16	4	1
Sonja Alexopolis	3–6	***		3–6		6–4				
Wendy Cohen	2–6	6–4	***	4–6	1–6	6–2				
Jacqui Lee		6–3		***					17	4	1
Nicole Sharma		6–1		6–3	***					
Patty Green						***	13	31	0	

1.3 Data display

There are many different ways to display data. The most appropriate way will depend on the type and amount of data and its purpose. Graphs are a very useful way of displaying information. The easiest graphs to draw are those that have only one scale. Graphs that use areas of colour or numbers of pictures or dots are especially useful when you want your information to give an immediate impression.

Important!

Types of graphs

A **dot plot** is constructed by putting a dot for each data item above a scale. They are most useful for small sets of numerical data, particularly discrete data.

Shoes owned

Number of pairs
1 2 3 4 5 6 7
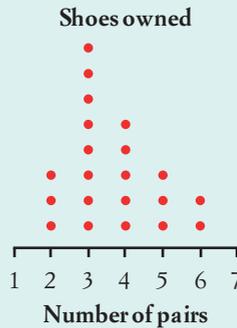
A **picture graph** (**pictogram** or **pictograph**) shows different quantities using different-sized pictures or different numbers of pictures. For example, the picture on the right of Cairns tourist arrivals shows that the greatest number of tourists arrive by road. This is followed by those arriving by air. Only a small proportion of tourists arrive by rail or sea.

Cairns tourist arrivals
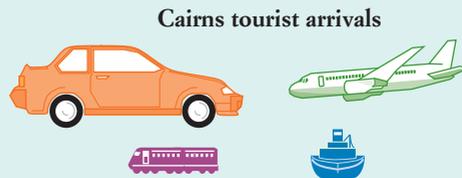
A **bar chart** or **bar graph** displays the different data values as bars. It can be drawn so that the bars are vertical **or** horizontal. Bar graphs are most often used to show the relative sizes of related measurements.

Cairns tourist arrivals

Rail
Air
Sea
Road
Number
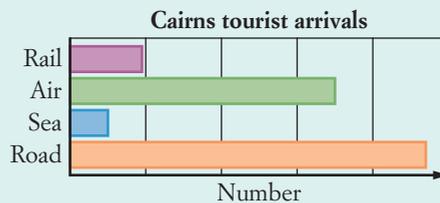
A **histogram** is a column graph showing the frequencies of continuous data. It has no spaces between the columns except for frequencies of zero.

A **circle graph** (**pie graph** or **sector graph**) is a circle that has been divided into sectors to show the parts of a total amount. The sector angles are proportional to the amounts.
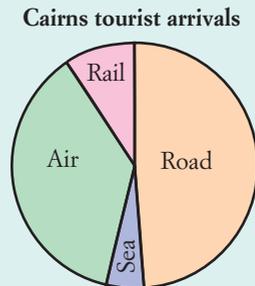
Cairns tourist arrivals

Rail
Air
Road
Sea

A **divided bar chart** is a long bar that has been divided into sections to show the parts of a total amount. The lengths of the sections are proportional to the amounts.

Cairns tourist arrivals

Road Sea Air Rail
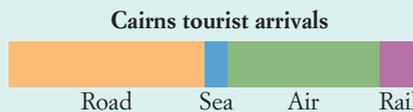
All charts and graphs should have a **title** describing what they are about.
An **axis** (plural is **axes**) of a graph or chart is a line that shows the measurements.
The **scale** of a graph or chart (usually marked on the axes) shows the size of the measurements.

Dot plots are particularly easy to draw, and can be used to collect data in addition to displaying information.

Example 8

The following shows the minimum daily temperatures in Perth during July 2010.

3, 5, 0, 1, 0, 2, 1, 5, 9, 8, 10, 11, 8, 5, 8, 5, 6, 4, 7, 4, 5, 3, 3, 4, 4, 11, 12, 12, 13, 4, 9

Draw a dot plot to show the information.

Solution

Draw a horizontal scale from the lowest temperature to the highest temperature. Leave room above the scale for the dots.

Put dots above each number on the scale, starting from the first temperature and working through to the end of the data. Fill in the dots as you go.

Keep going until all the data is used.
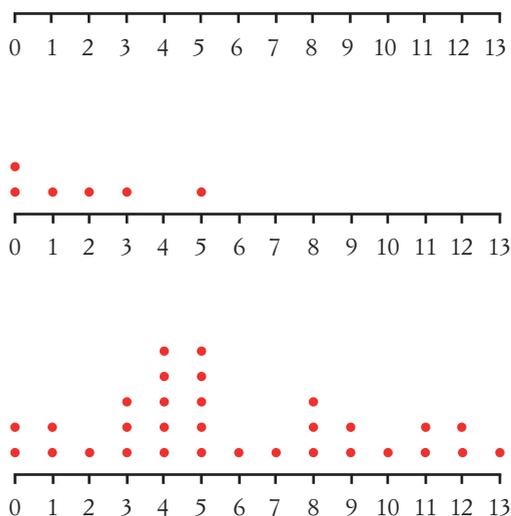
You should choose the scale of a picture graph so that the biggest number is represented by about 10 pictures.

You should choose the scale of a bar graph to fill the space, but round off to an easy number like 2, 4, 5, 10, 20 . . . per centimetre. When you work out the width of the bars, allow for any spaces between them.

Example 9

The table on the right shows the time spent on different activities during a week.
a Work out the scale that should be used for a picture graph.
b Draw the picture graph using clock faces.

Activity	Hours
Eating	15
Sleeping	56
TV	21
School	30
Washing	9
Travel	12
Play	15
Other	10

Solution

a Looking at the Hours column, the largest number is **56**.

Divide by 10 to find the scale. **56 ÷ 10 = 5.6**

We round the result 5.6 down to 5 because 5 is easier to work with than 6.

The scale should be 5 hours/picture.

b Show each activity on a line.

15 ÷ 5 = 3, so eating needs 3 pictures.

56 ÷ 5 = 11.2, so sleeping needs 11-and-a-bit pictures. Continue with the rest of the activities.

Include the scale in a convenient place.

Put a title at the top.

Activities in a week

Eating	✓ ✓ ✓
Sleeping	✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ (
TV	✓ ✓ ✓ ✓ (
School	✓ ✓ ✓ ✓ ✓ ✓
Washing	✓ ✓
Travel	✓ ✓ (
Play	✓ ✓ ✓
Other	✓ ✓

✓ = 5 hours
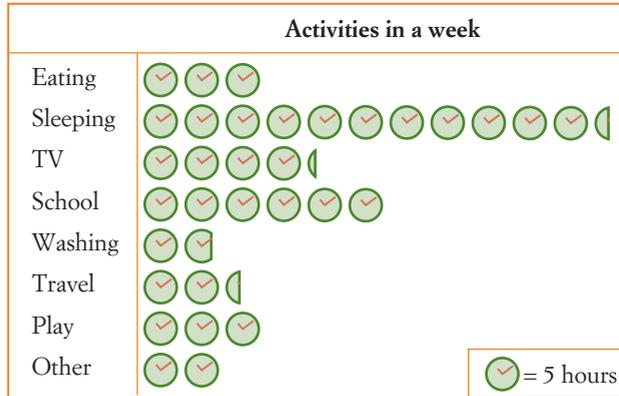
Example 10

The data on the right shows the lengths of students' pointer fingers, correct to the nearest cm. Draw a bar chart using a scale of 2 students/cm.

Length (cm)	Frequency
5	2
6	0
7	5
8	9
9	3
10	1
Total	20

Solution

The shortest pointer finger length is 5 cm and the longest is 10 cm.

You could write 5.5, 6.5, 7.5, . . . under the lines to show the lengths before rounding. In this case it is easier to write the rounded lengths under the columns instead.

Show the frequencies on the vertical scale.

Leave no spaces between the columns.

Include axis labels and a title for the graph.

Pointer finger lengths

Technology Electronic circle graphs

You can use a spreadsheet such as Excel to draw a circle graph.

Type the following into cells A1 to B5 in your spreadsheet.

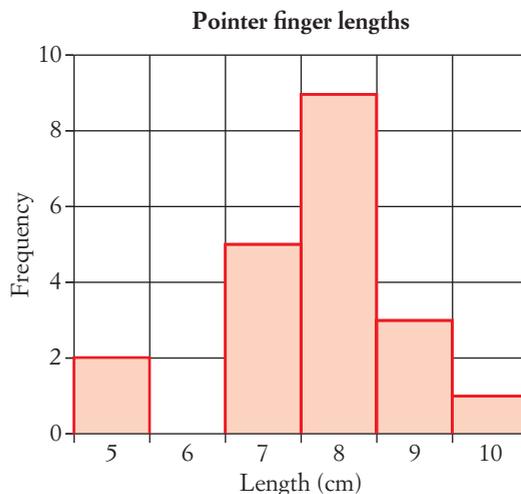| Carbs	200
Protein	70
Fat	50
Roughage	300
Other	30

Highlight (block) these cells and click on the Chart Wizard.

Follow the instructions to get a pie graph.
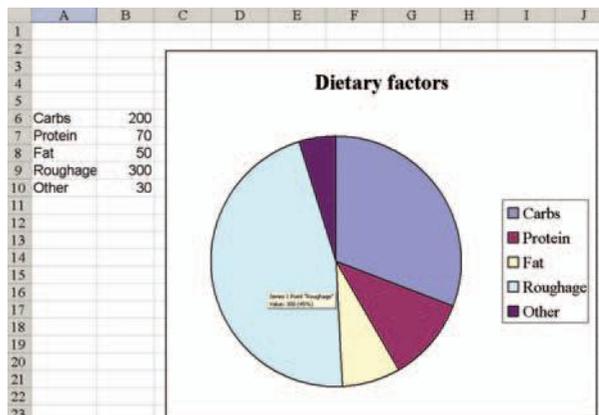
Example 11

The following table shows the number of students in each year of a local high school.

Year	7	8	9	10	11	12
Number	210	220	240	200	180	150

a Calculate the angles and draw a circle graph.
b Make a divided bar chart about 10 cm long.

Solution

a Redraw the table with an extra column for the angle.

Include an extra row at the bottom for the totals.

Year	Number	Angle
7	210	
8	220	
9	240	
10	200	
11	180	
12	150	
Total	**1200**	

Use your calculator to find the scale. The scale is 360° divided by the total number of students.

Enter as 360 [÷] 1200 [STO] [A].

Scale = 360° ÷ 1200 = 0.3°/student

```
360÷1200          A
```

Store the exact answer in the memory and multiply by each number to find the angles.

Year 7: Angle = 210 × 0.3 = 63°
Year 8: Angle = 220 × 0.3 = 66°
Year 9: Angle = 240 × 0.3 = 72°
Year 10: Angle = 200 × 0.3 = 60°
Year 11: Angle = 180 × 0.3 = 54°
Year 12: Angle = 150 × 0.3 = 45°

Enter as 210 ⊗ [RCL] [A] [=] .

```
210×A                63
```

Add the numbers down the second and third columns to find the total and to check your calculations.

Year	Number	Angle
7	210	63°
8	220	66°
9	240	72°
10	200	60°
11	180	54°
12	150	45°
Total	**1200**	**360°**

Draw a circle and mark its centre. Draw a line from the centre to the circumference. Progressively mark each angle with your protractor, starting from the top. Label and colour or shade each sector. Write the total amount in the title.

School composition (1200 students)
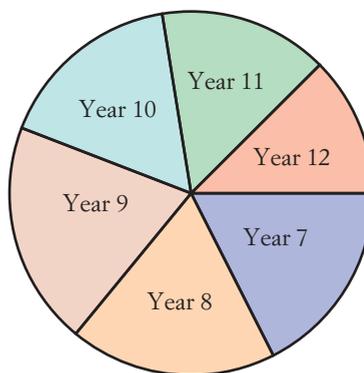
b Put an extra column for the bar length in the table, and an extra row at the bottom for the total.

Year	Number	Length (cm)
7	210	
8	220	
9	240	
10	200	
11	180	
12	150	
Total		

Use your calculator to find the scale.

The scale is the total length divided by the number of students.

Scale = 10 cm ÷ 1200
        = 0.00933 . . . cm/student

It will be easier to use a scale of 0.01 cm/ student.

Scale = 0.01 cm/student

Multiply the scale by each number to find the lengths.

Year 7: Length = 210 × 0.01 = 2.1 cm
Year 8: Length = 220 × 0.01 = 2.2 cm
Year 9: Length = 240 × 0.01 = 2.4 cm
Year 10: Length = 200 × 0.01 = 2.0 cm
Year 11: Length = 180 × 0.01 = 1.8 cm
Year 12: Length = 150 × 0.01 = 1.5 cm

Add the lengths to check your calculations.

Year	Number	Length (cm)
7	210	2.1
8	220	2.2
9	240	2.4
10	200	2.0
11	180	1.8
12	150	1.5
Total	**1200**	**12.0**

Draw the bar and mark each length. Label and colour or shade each section of the bar. Write the total amount in the title.

Students at a School, total number = 1200

Year 7	Year 8	Year 9	Year 10	Year 11	Year 12
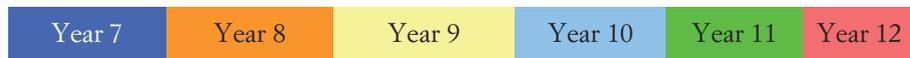
Example 12

Draw a histogram in a space about 6 cm high and 12 cm wide to show the times taken by some Year 8 students to read a short paragraph.

Time (seconds)	Frequency
20–24	4
25–29	10
30–34	7
35–39	5
40–44	2
45–49	4

Solution

Calculate the vertical scale using the largest frequency.

**Vertical scale = 10 ÷ 6 cm = 1.66**

Round *up* to an easy number.   **Make the scale 2/cm.**

Draw the histogram. The divisions between the groups are actually halfway between 24 and 25, 29 and 30, and so on. Show that the axis does not start from 0 by using a zigzag on the axis.

It would have taken more space if we had used a horizontal scale of 4/cm instead of 5/cm, but it would have been harder to work out where the divisions were.

Reading times
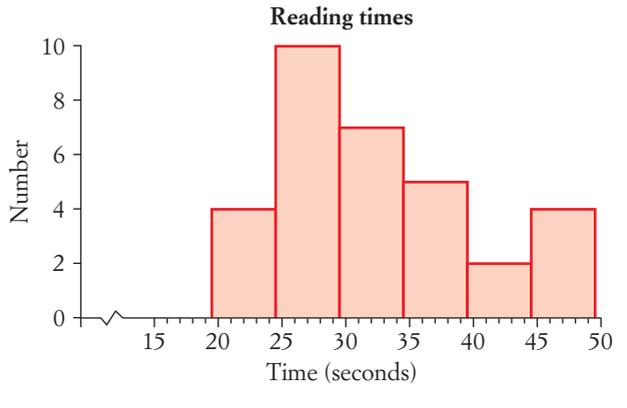
A dot plot is tedious to draw for data that is spread over a large number of scores. You can draw a stem-and-leaf plot instead of a dot plot to display the information quickly and easily.

Important!

Stem-and-leaf plot

A **stem-and-leaf plot** (**stemplot**) can be made for 2-digit or 3-digit scores. The first one or two digits are written vertically to make the **stem**. The remaining digits are written in a row across from the first digit(s) to make the **leaves**. You usually separate the stem from the leaves with a line. The lengths of the rows make a 'graph' of the data.

Video tutorial
Stem-and-leaf plots
MAT07PSVT10022

Example   13

Make a stem-and-leaf plot of the following masses (in grams) of avocados.

85	130	150	137	95	85	142	113	98	103
128	128	105	118	174	113	132	150	137	123
113	137	98	140	115	125	130	162	123	140

Animated example
Stem-and-leaf plots
MAT07SPAE00001

TLF Learning object
Stem-and-leaf-plots
(L5911)

Solution

Use stems from 80–170 to get a reasonable number of groups.

Write the 5 from 85.
Write the 0 from 130.
Write the 0 from 150.
Write the 7 from 137.

Continue until the whole list is done.

Stem	Leaf
8	5
9	
10	
11	
12	
13	0 7
14	
15	0
16	
17	

Stem	Leaf
8	5 5
9	5 8 8
10	3 5
11	3 8 3 3 5
12	8 8 3 5 3
13	0 7 2 7 7 0
14	2 0 0
15	0 0
16	2
17	4

Redo the plot with the 'leaves' in order. Use the same space between each 'leaf'. Put a title and key on the stemplot.

Avocado masses (grams)
Key: 12|6 = 126 g

| 8	5 5
9	5 8 8
10	3 5
11	3 3 3 5 8
12	3 3 5 8 8
13	0 2 7 7 7
14	0 0 2
15	0 0
16	2
17	4

Important!

Line graphs

Line graphs have points connected by lines. They are most commonly used for time-series, continuous data, such as temperature and concentration.

A **dependent variable** is one that is controlled in some way by another quantity. An **independent variable** is not controlled by another variable. If there is one, the independent variable is shown horizontally in line graphs. If one of the variables is time, it is usually shown horizontally.
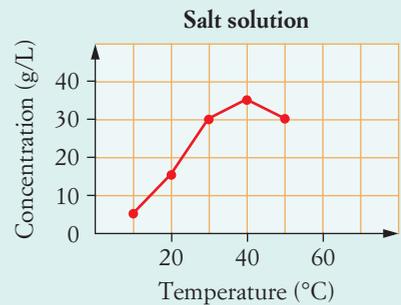
Example 14

The following graph shows the average monthly rainfall in Brisbane and Melbourne.
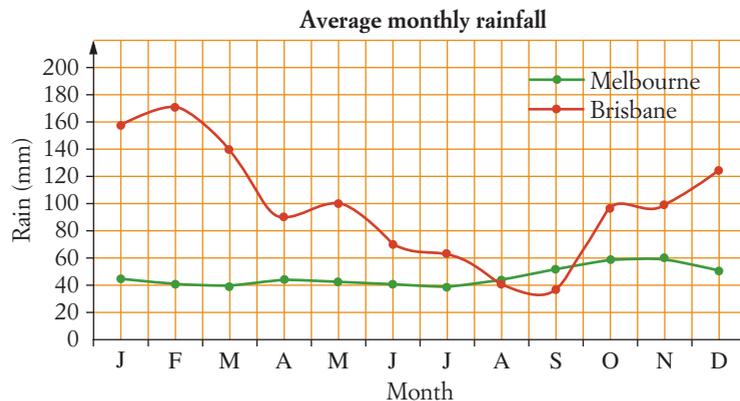
a In which months does more rain fall in Melbourne than in Brisbane?
b In which season does it rain the most in each capital?

Solution

a The Melbourne line is higher in August and September.

More rain falls in Melbourne than in Brisbane in August and September.

b The highest part is J-F-M for Brisbane and S-O-N for Melbourne.

It rains the most in Brisbane during summer and the most in Melbourne during spring.

Investigate: Favourite flavours

As a class activity, everyone writes their favourite ice-cream flavour on the board. You have to choose *one*!

Make a table of the ice-cream flavour data and choose a graph type to display the information.

Your teacher may be able to obtain information from the other Year 7 classes. In this case, make new tables and graphs for all Year 7 classes.

Compare the percentage of people who prefer chocolate in your class with the percentage in the other Year 7 classes. Are they the same?

Exercise 1.3 Data display

Your teacher may ask you to draw some of the graphs in this exercise using spreadsheets, as well as by hand.

1 Draw dot plots for each of the following data sets.

a 5, 9, 7, 9, 2, 9, 8, 10, 4, 10, 6, 9, 5, 8, 7, 8, 4, 9, 2, 9, 8, 9, 6, 8, 7

b 19, 25, 23, 22, 21, 19, 23, 27, 19, 20, 23, 23, 18, 19, 15, 22, 23, 16, 15, 20

c 15, 16, 15, 18, 11, 14, 15, 17, 11, 15, 12, 17, 19, 14, 20, 15, 15, 13, 13, 14, 17, 15, 19, 16, 18, 15, 12, 18, 15, 13

See Example 9

**2** The graph below shows the numbers of native frogs thought to have been in a Brisbane creek over a number of years.

Adult frogs in Mimosa Creek	
1900	🐸 🐸 🐸 🐸 🐸 🐸 🐸 🐸
1920	🐸 🐸 🐸 🐸 🐸 🐸 🐸 🐸
1950	🐸 🐸 🐸 🐸
1980	🐸 🐸 🐸
2000	🐸 🐸 🐸 = 500 frogs
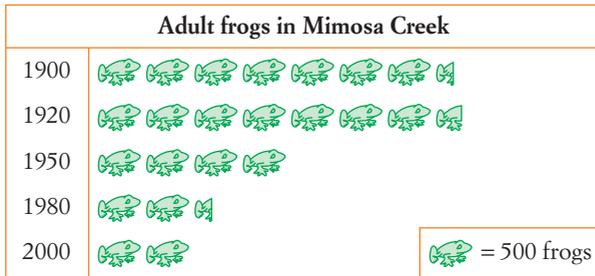
a What is the scale of the graph?
b How many frogs were thought to have been in the creek in 1950?
c How many frogs were thought to have been there in 2000?

See Example 10

3 The graph on the right shows the number of pieces of fruit that some people said they ate.

Fruit eaten in one day

Number of people (y-axis: 0, 2, 4, 6, 8)
Number of pieces (x-axis: 0, 1, 2, 3, 4, 5, 6)
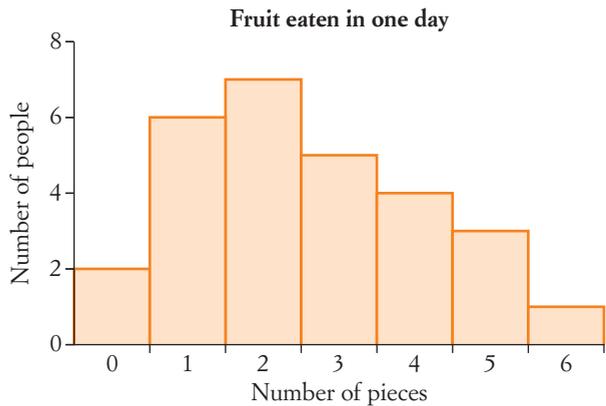
a What is the scale of the graph?
b What was the largest number of pieces of fruit eaten?
c How many people ate three pieces of fruit?
d How many people ate five pieces of fruit?

See Examples 9, 10

4 The use of coastal land in Queensland is shown below.
  a What scale should be used for a picture graph?
  b Use a picture such as a fenced area to draw a picture graph.
  c Draw a horizontal bar graph using a scale of 1000 km²/cm.

Use	**Area (km^2)**
Conservation	2750
Military	900
Grazing	5100
Urban/mining	350
Aboriginal reserves	4200
Agriculture	500
Forestry	500
Unused	12 900

5 The sales of different kinds of doughnuts from a stall in a shopping centre are shown on the right.

Type of doughnut	Number sold
Plain	250
Cinnamon	185
Iced	278
Cream-filled	110
Chocolate	140
Fancy	87

  a What scale should be used for a picture graph?
  b Use a drawing of a doughnut to draw a picture graph.
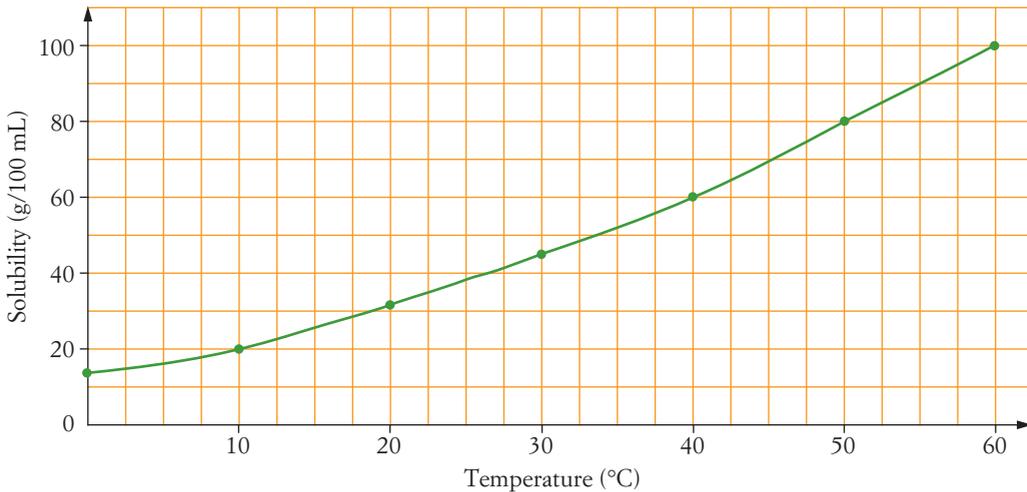
  c Use a scale of 50 doughnuts/cm to draw a vertical bar graph.

6 This graph shows the amount of potassium nitrate that will dissolve in 100 mL of water.

See Example 14

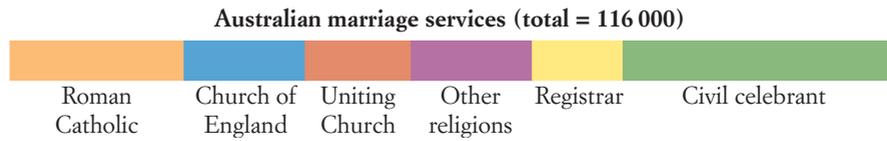**Solubility of potassium nitrate at different temperatures**

  a What are the scales?
  b How much potassium nitrate will dissolve in 100 mL of water:

  i at 20°C?   **ii** at 40°C?   **iii** in the freezing cold?
  iv in hot tap water?   **v** at normal room temperature?

  c How much would dissolve in 500 mL of water at body temperature?
  d How much water would be needed to dissolve 1 kg of potassium nitrate at 15°C?

7 The number of pets owned by families is shown below.

See Example 10

Pets	0	1	2	3	4	5	6
Frequency	2	5	14	2	4	1	2

Use a scale of 2 pets/cm to draw a column graph.

8   The graph below shows the number of marriage services in one year in Australia.

Australian marriage services (total = 116 000)

| Roman Catholic | Church of England | Uniting Church | Other religions | Registrar | Civil celebrant |

a   What is the scale?

b   How many marriages were performed in:

   i   Roman Catholic churches?   ii   registry offices?   iii   Uniting Church churches?

See Example 13

9   The numbers of words in 17 paragraphs from a children's book were counted. The results were as follows:

| 19 | 37 | 8 | 20 | 42 | 17 | 34 | 20 | 43 | 24 | 31 | 13 |
| 25 | 22 | 30 | 26 | 48 |

Draw a stem-and-leaf plot to represent this data.

10   The numbers of DVDs owned by the families of some Year 7 students were as follows:

1	10	34	18	0	65	22	78	35	43	66	76
55	65	42	34	39	98	43	29	54	38	3	12
40	38	34	72	60	51	18	9	41

Draw a stem-and-leaf plot to represent this data.

See Example 12

11   The amount of pocket money that Year 7 students say they get is shown below.

Amount	$5–$9	$10–$14	$15–$19	$20–$24	$25–$29
Frequency	6	9	7	3	2

a   What scales should be used to draw a histogram 6 cm high and 12 cm wide?

b   Draw the histogram.

Fluency

See Example 11

12   Some students were surveyed to find their preferred September holiday.

Holiday	Gold Coast	Sunshine Coast	Other Queensland	At home	Interstate	Overseas
Number	52	32	45	60	30	21

Draw a circle graph to show this information.

See Example 14

13   The density of battery acid is as shown in the table.

Percentage of acid	0	10	20	30	40	50	60	70	80	90	100
Density (g/mL)	1.00	1.07	1.14	1.22	1.30	1.40	1.50	1.61	1.73	1.81	1.83

Show this information as a line graph.

14   The original Australian Parliament, established in 1901, had the following numbers of members for each state in the House of Representatives. Draw a divided bar graph to show the representation of states.
House of Representatives, 1901

State	NSW	Vic.	Qld.	SA	WA	Tas.
Members	26	23	9	7	5	5

15 Refer to the graph for question **2**.
   a What has happened to the population of frogs from 1900 to 2000?
   b What could be some reasons for the change in population?

16 Refer to the graph for question **3**.
   a How many people were interviewed?
   b How many pieces of fruit were eaten altogether by the people who ate three pieces of fruit?
   c How many pieces of fruit were eaten altogether by all the people interviewed?

17 The distances (to the nearest km) that some children live from school are shown below.

2	1	1	8	1	1	6	3	1	5	3	3	6	1
2	2	0	3	2	3	5	1	5	1	2	4		

   a Make a frequency table.
   b Use a suitable scale to draw a histogram.
   c What is the most common distance from school?

18 This graph shows how a student spent his class time in one 40-minute lesson.
   a What is the scale?
   b How long did he spend ruling up, etc.?
   c How long did he spend listening to the teacher?
   d How long did he spend writing?
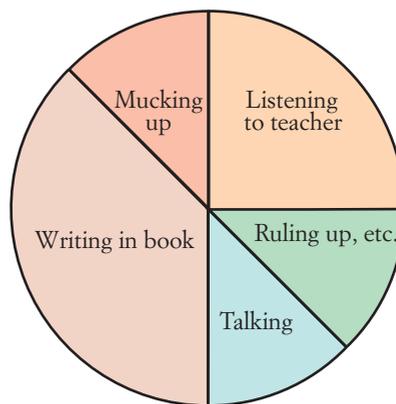
19 The results of the positive blood-alcohol tests of people involved in accidents for a fortnight are recorded below. The results are in grams of alcohol/100 mL of blood.
0.09, 0.13, 0.04, 0.01, 0.02, 0.09, 0.12, 0.01, 0.07, 0.02, 0.04, 0.12, 0.07, 0.06, 0.15, 0.04, 0.11, 0.07, 0.02, 0.06, 0.17, 0.01, 0.03, 0.01, 0.03, 0.11, 0.23, 0.09, 0.18, 0.14, 0.03, 0.14, 0.09, 0.08, 0.02, 0.05, 0.08, 0.01, 0.05, 0.08, 0.19, 0.19, 0. 07, 0.02, 0.02, 0.02, 0.11, 0.13, 0.03, 0.02, 0.19, 0.02, 0.01, 0.10, 0.10, 0.07, 0.12, 0.16, 0.09, 0.07
   a Is blood-alcohol level discrete or continuous?
   b Make a frequency table of the results using intervals of 0.05 g/100 mL.
   c Draw a histogram for the results.

20 This table shows the temperature of some sugar and water as it is heated in a saucepan.

Time (min)	0	1	2	3	4	5	6	7	8	9	10
Temperature (°C)	25	40	53	65	75	83	90	96	100	103	104

   a Draw a line graph of the temperature against time.
   b Does the temperature rise evenly?

21 The table below shows the average maximum monthly temperature at Davis Base in Antarctica.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Max. temp. (°C)	3	0	−6	−11	−13	−12	−13	−14	−13	−9	−3	2

   a Draw a line graph of the temperature against time.

   b Estimate the number of days in the year when the temperature is above zero.

22 The graph of mine production on the right was used to show how much mine production rose from 1998 to 2002.

   a What is wrong with the way that the graph is drawn?

   b Why do you think it was drawn in this way?

Mine production
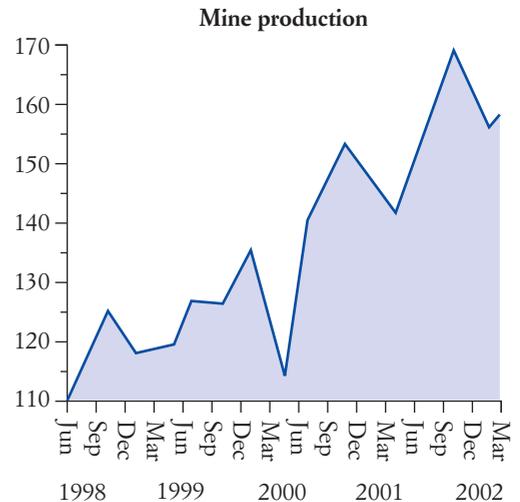
1.4 Analysing data

After you have collected data about something, you may want to summarise the information. In addition to tables and graphs, you can use a single number to make a statement.

Important!

Mode, median, mean and average

The **mode** is the score with the highest frequency. It is the most common score, so it occurs more often than any other score. In cases where there are two scores with equal highest frequencies, we say that there are two modes and that the distribution is **bimodal**.

The **median** is the *middle* score. To find the median, all scores are arranged in order from smallest to largest and the middle score is chosen. If there is an even number of scores, then the average of the middle two scores is used. **Cumulative frequency** is the progressive total of frequencies and can be used to work out the median.

The **mean** is the **average** score. It is calculated by adding all the scores and dividing by the number of scores. The symbol $\bar{x}$, pronounced as 'x-bar', is often used for the mean. In most cases, the mean is considered to be the typical score.

The mean, median and mode are called measures of **central tendency** and are usually very similar. The mean is generally considered typical as it summarises all the scores. The relationship between the mean and the median can be changed by a few very high or very low scores. If there are a few very high or very low scores that would distort the mean, you should use the median. The mode should be used in cases where the most common score is needed.

The **range** is the difference between the highest and lowest scores and measures the **spread** of the data.

Example 15

The following are the results of a spelling test out of 20.

9	14	11	17	16	16	13	14	16	13
13	13	14	16	12	16	18	16	16	13
15	12	17	15	16					

a Find the mode and median of the spelling test results.
b What is the most likely spelling score?

Solution

a Find the most common score.   **There are more 16s than any other score.**

   Write the result.   **The mode is 16.**

   Arrange all 25 scores in order.

9 11 12 12 13 13 13 13 13 14 14 14 **15** 15 16 16 16 16 16 16 16 17 17 18

   12 scores on the left.   **12 scores on the right.**

   Choose the middle score.   **The median is 15.**

b The mode is the most common score.   **The most likely score is 16.**

Example 16

The dot plot below shows the number of hours of sunshine in Essendon in February 2010. Use the information to find the mode and median.
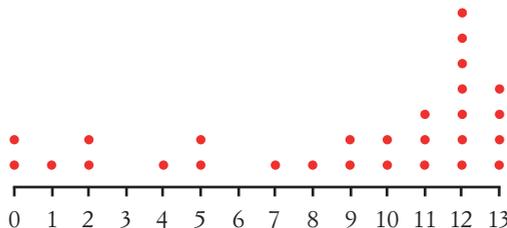
Solution

Since the mode is the most common score, it is the highest column.

The mode is 12 hours of sunshine.

To find the median, first count all the dots.

There are 28 scores.

There are an even number of dots. Determine the middle two scores. There will be 13 before the 14th score and 13 after the 15th score.

The middle two scores are the 14th and 15th ones.

Find the 14th and 15th scores.

The middle two scores are 10 and 11.

Find the average of the middle two scores.

The median is 10.5.

Example 17

Find the mean and range for the ages of the mothers of some Year 7 students.

35	40	30	42	34	37	36	36	31	40	36	43
35	45	33	41	33	39	34	36				

Solution

The mean is calculated by dividing the total of all the scores by the number of scores.

Total of scores

We first need to add up all the scores.

$$= 35 + 40 + 30 + 42 + 34 + 37 + 36$$
$$+ 36 + 31 + 40 + 36 + 43 + 35$$
$$+ 45 + 33 + 41 + 33 + 39 + 34 + 36$$
$$= 736$$

Count the scores to find the total number.

Number of scores $= 20$

Find the average.

Mean $= \dfrac{736}{20} = 36.8$

Write the answer, rounding appropriately.

The average age is about 37.

Find the lowest and highest ages.

Lowest score $= 30$
Highest score $= 45$

Find the range.

Range $=$ highest $-$ lowest score
$$= 45 - 30$$
$$= 15$$

Write the answers, rounding the mean appropriately.

The average age is about 37 and the range is 15 years.

You could use the statistics mode on your scientific calculator to find the mean, but for small data sets it is just as easy to add them up and divide. You may learn how to do this later.

Example 18

Scientific calculator exercise

Statistics

The Science marks for a Year 9 class are shown below.

Mark	2	3	4	5	6	7	8
Frequency	2	3	8	6	5	4	3

a Extend the table to include score × frequency and cumulative frequency.
b Use the table to find the mean and median.
c What is the typical score?

Solution

a Rearrange the table in columns. Include columns for score × frequency (xf) and cumulative frequency.

Work out each value of xf. For example, when $x = 4$, $xf = 4 \times 8 = 32$

Work out the cumulative frequency as you move down the Marks column by adding the corresponding frequency value to the cumulative frequency of the mark above it. For example,
Mark $x = 6$: $5 + 19 = 24$

Add the xf column.

The total frequency can simply be read from the cumulative frequency column.

Mark x	Frequency f	xf	Cumulative frequency
2	2	4	2
3	3	9	5
4	8	32	13
5	6	30	19
6	5	30	24
7	4	28	28
8	3	24	31
Total	**31**	**157**	

Total of scores
= 4 + 9 + 32 + 30 + 30 + 28 + 24
= 157

b Divide the totals to get the mean.

$$\textbf{Mean} = \frac{\textbf{total of scores}}{\textbf{number of scores}}$$

$$= \frac{\textbf{157}}{\textbf{31}} \approx \textbf{5.1}$$

The median is the 16th score, as there is an odd number of scores and $31 \div 2 = 15.5$. There are 13 scores up to 4, and the next 6 scores, including the 16th, are all 5s.

Write the median.

Median = 5

c The median and mean are both 5.

The typical score is 5, although the most likely score (mode) is 4.

By comparing the data from different groups in a population, we can show the relationships between different aspects of the data. For example, there is often a difference between boys and girls. A **side-by-side** column graph can show differences of this kind.

Example 19

The information below shows the heights of some 200 Year 7 boys and girls.

Height (cm)	130–139	140–149	150–159	160–169	170–179	180–180
Boys	6	31	106	41	13	3
Girls	1	35	89	64	11	0

a Draw a side-by-side column graph
b Comment on the results.

Solution

a Make the girls red and the boys green. The graph has to go up to 110.

Year 7 heights
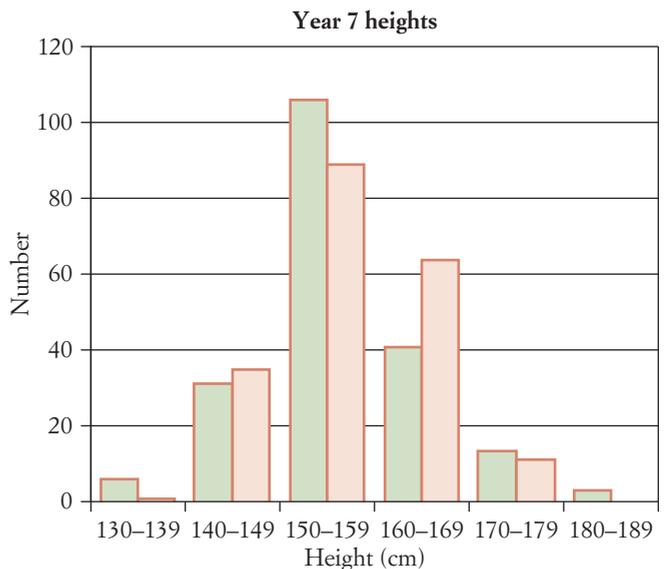
b Notice most are in the middle.

Most students in Year 7 are 150–169 cm tall, with more 150–159 than 160–169.

Notice the imbalance of boys and girls.

The boys outnumber the girls from 150–159, but the girls outnumber the boys from 160–169.

Look at the ends.

There are more very short boys than girls, and there are no very tall girls, but there are some boys.

Summarise your comments.

Most girls are a little taller than most boys at Year 7, but there are some very tall and very short boys, but not many very tall or very short girls. The range of heights is smaller for girls than boys.

In some cases, a set of data may have a value that is a long way outside the range of the rest of the data. For example, if you happened to have the Australian boys' age champion runner in your class, then his running times would be much lower than the times of other Year 7 students at your school. A value that is very different from the other values is called an **outlier**.

Example 20

The ages of people waiting to be served at the school tuckshop were as follows:

| 12 | 14 | 11 | 13 | 14 | 12 | 11 | 13 | 16 | 12 | 14 | 53 |
| 16 | 13 | 14 | 17 |

a Find the mean, median, mode and range of the ages.
b Suggest the reason for an outlier.
c Find the mean, median and mode without the outlier and comment on the results.

Solution

a Arrange the ages in order and find the mode and median. There are 16 pieces of data, so the median is the average of 7th and 8th scores. Add the scores and find the mean.

**11 11 12 12 12 13 13 13 14 14 14
14 16 16 17 53**
The mode is 14 and the median is 13.

$$\textbf{Total} = \textbf{11} + \textbf{11} + \textbf{12} + \textbf{12} + \textbf{12} + \textbf{13}$$
$$+ \textbf{13} + \textbf{13} + \textbf{14} + \textbf{14} + \textbf{14} + \textbf{16}$$
$$+ \textbf{16} + \textbf{17} + \textbf{53}$$
$$= \textbf{255}$$

Number of people = 16.
Mean = 255 ÷ 16 ≈ 15.9

Find the range.

Range = 53 − 11 = 42

b The outlier is 53.

The 53-year-old could be a teacher and the others students.

c Find the median and mode again.

**11 11 12 12 12 13 13 13 14 14 14
14 16 16 17**
The mode is still 14 and the median 13.

Find the mean and range again.

$$\textbf{Total} = \textbf{11} + \textbf{11} + \textbf{12} + \textbf{12} + \textbf{12} + \textbf{13}$$
$$+ \textbf{13} + \textbf{13} + \textbf{14} + \textbf{14} + \textbf{14} + \textbf{16}$$
$$+ \textbf{16} + \textbf{17} = \textbf{202}$$

Number of people = 15
Mean = 202 ÷ 15 ≈ 13.5
Range = 17 − 11 = 6

Comment on the results.

The outlier makes a big difference to the mean and range, but no difference to the mode and median.

The result in Example **20** shows that outliers can have a huge effect on the mean and range. If there are outliers, we should use the median as the typical score, rather than the mean. In some cases, an outlier could even indicate an error in data collection.

As can be seen in Example **20** the **relationship** between the mean and median is affected by outliers. With no outliers, the mean and median are (almost) the same. With high outlier(s) the mean is generally higher than the median. Similarly, with low outliers, the mean is generally lower than the median.

Investigate: Global health and resources

In this investigation, you will use the internet site gapminder.org.

Go to this site and click on the tab 'Gapminder World' at the top of the page. You should then load up the 'Health and Wealth' Chart.

- Click on Afghanistan in the window on the right. What does the chart show about Afghanistan?
- Click on Australia. What does the chart show about Australia?
- Your teacher might want groups to investigate particular aspects of health, education and wealth.
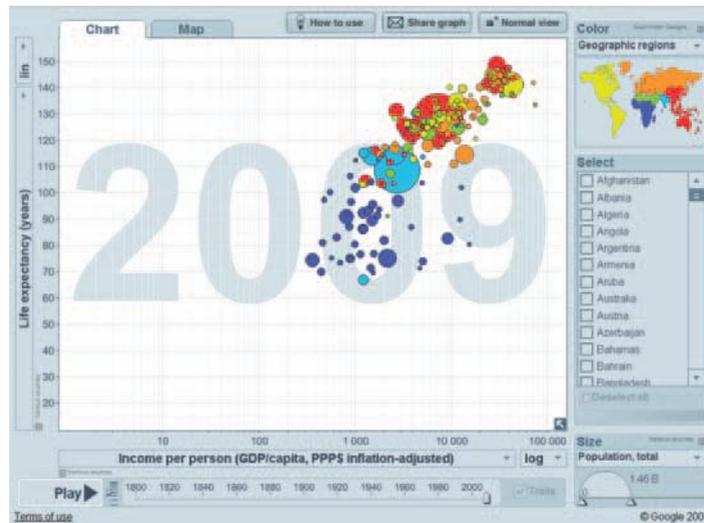
- Now choose the 'Data tab'
- Type 'Oil' into the search box.
- Compare the oil consumption and oil production of different countries around the world.
- Look at the oil reserves of different countries. Compare oil use and oil reserves for different countries.
- Look for other sites that show reserves, production and consumption of non-renewable resources. You might find the site 'www.mapsofworld.com' useful. You can search for maps of different resources such as iron ore, gold, etc.
- Your teacher might want you to investigate mismatches between reserves and consumption around the world.

Exercise 1.4 Analysing data

1 Find the median and mode for each of the following sets of scores.
  a 5, 7, 6, 7, 3, 8, 5, 4, 8, 4, 6, 5, 4, 4, 4, 4, 5, 4, 5, 6, 5, 6, 6
  b 3, 4, 7, 6, 1, 8, 6, 4, 7, 6, 7, 7, 8, 9, 6, 2, 3, 3, 7, 8, 7, 6, 7, 3, 6, 7, 7, 6, 4, 8
  c 24, 29, 27, 25, 24, 26, 25, 28, 28, 27, 27, 24, 28, 26, 22, 28, 23, 26, 27, 29, 28, 23, 25, 26, 26, 27, 26, 26, 27
  d 22, 24, 20, 24, 27, 26, 22, 24, 24, 23, 25, 21, 27, 27, 25, 24, 23, 22, 20, 22, 17, 20, 22, 25, 25, 24, 26, 22
  e 48, 54, 46, 58, 46, 55, 55, 53, 51, 60, 54, 46, 56, 49, 55, 54, 49, 57, 57, 49, 46, 59, 55, 55, 51, 42, 48, 55, 44, 51, 57, 54

2 Find the mean and range of each set of scores in question **1**.

3 Find the median and mode of the sets of data in each of the following dot plots and stem-and-leaf plots.

a
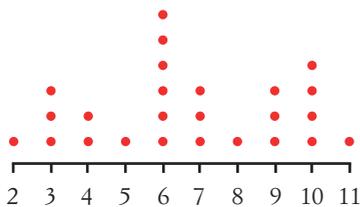
b Key:12|3 = 123

```
10 | 7
11 |
12 | 0  0  3  7  8  9
13 | 1  2  5  7  7
14 | 4  4  5  7  8
15 | 1  3  5  6
16 | 1  2
17 |
18 | 5
```

c
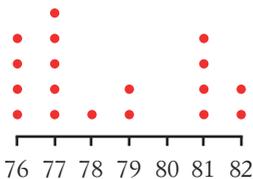
d Key:2|7 = 27

```
3 | 2  5  6  7  9
4 | 1  1  3  4  4  4  9
5 | 2  4  4  6
6 | 2  2  8  9  9
7 |
8 | 2
9 | 1
```

4 The following table shows the times taken by Year 7 boys and girls to complete a questionnaire. Draw a side-by-side column graph to show the results.

Time in seconds	1–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40
Number of Boys	0	24	45	16	21	9	1	0
Number of Girls	2	15	37	23	21	15	2	1

See Example 18

5 Rearrange each of the following tables into columns, then extend the table to include score ×
frequency and cumulative frequency columns. Hence find the mean, median, mode and range.

a

x	3	4	5	6	7	8	9	10
f	2	4	5	6	7	5	4	2

b

x	12	13	14	15	16	17	18	19	20	21	22
f	6	7	8	5	3	4	2	5	3	2	1

c

x	7	8	9	10	11	12	13	14	15
f	2	3	5	7	9	11	8	6	5

d

x	42	43	44	45	46	47	48	49	50	51
f	2	4	5	7	8	9	7	4	3	1

e

x	22	23	24	25	26	27	28
f	2	5	9	11	6	4	6

Problem solving

6 The ages of the grandparents of a class were collected. They are shown below.
76, 70, 74, 58, 62, 66, 62, 64, 78, 64, 72, 58, 68, 70, 58, 70, 78, 60, 68, 62, 62, 60, 62,
58, 62, 66, 64, 62, 62, 59, 64, 62, 70, 60, 60, 72, 62

a Calculate the mean, median and mode of the ages.

b What is the typical age?

7 The prices of the houses and units sold
by a real estate firm in the first
week in March were:

$420 000	$740 000	$380 000
$780 000	$420 000	$760 000
$460 000	$960 000	$1 400 000
$460 000	$700 000	$700 000
$440 000	$460 000	$400 000

a Calculate the mean, median and range of the prices.

b What was the typical price?

c In general, the mean is not used to measure real estate prices. Why not?

d The firm charges an average of 2.5% commission. What is its likely total in commissions for
the year?

8 a Work out the median and mode of the following shoe sizes for some members of a class.
   10, 7, 6, 6, 3, 7, 9, 7, 8, 9, 8, 10, 10, 6, 6

   b During the year, the student with the smallest feet left the class. Find the effect on the median and mode.

   c Calculate the mean before and after the student left and comment.

See Example 20

9 a Work out the median and mode of the following waist measurements (in cm) of a group of people starting a fitness class.
   100, 104, 106, 108, 102, 122, 134, 102, 100, 92, 100, 96, 106, 100, 92, 78

   b After the fitness instructor told the group that they would be showing their waist measurements on a chart during the course, the two largest people left the group. Calculate the effect on the median and mode.

   c Calculate the effect on the mean.

Worked solutions

Exercise 14

MAT07SPWS00052

10 The mean age of a group of 50 people is 32, and their median age is 38. A married couple both aged 25 join the group.

   a What is the new mean?

   b Explain why the new median cannot be exactly predicted.

   c Estimate the new median.

   d Explain the effect on the mean and the median when the married couple join the group.

Reasoning

11 Wendy wants to pass Business Studies, and has scored 53, 44 and 56 on the first three tests. She needs an average of 45 for all four tests to pass. What does she need to score on the last test to pass?

12 The ages of a big firm's apprentices are shown below:
   17, 23, 17, 21, 18, 16, 21, 21, 18, 17, 18, 17, 15, 17, 20, 17, 17, 19, 18, 20, 18, 18, 17, 20, 18, 22, 23, 17, 21, 17, 19, 17, 18, 19, 15, 17, 18, 17, 22, 22

   a Find the mean, median and mode of the ages.

   b What is the typical age of these apprentices?

Worked solutions

Exercise 1.4

MAT07SPWS00052

13 The weekly wages (to the nearest $50) of 50 people (including casuals and part-timers) from Gladstone are given below:
   850, 550, 200, 500, 250, 500, 350, 400, 500, 700, 500, 900, 750, 250, 500, 350, 350, 250, 300, 300, 350, 600, 150, 450, 150, 200, 500, 300, 850, 750, 200, 450, 150, 250, 200, 450, 500, 200, 300, 300, 300, 400, 650, 250, 300, 350, 950, 300, 450, 900

   a Calculate the mean, median, mode and range.

   b What is the typical amount earned?

   c What would be the total earnings of a group of five workers?

   d Explain the relationship between the mean and the median.

14 Comment on the results for question **4**.

15 The side-by-side histogram below shows the Australian population. Comment on the results.

Australian population 2010
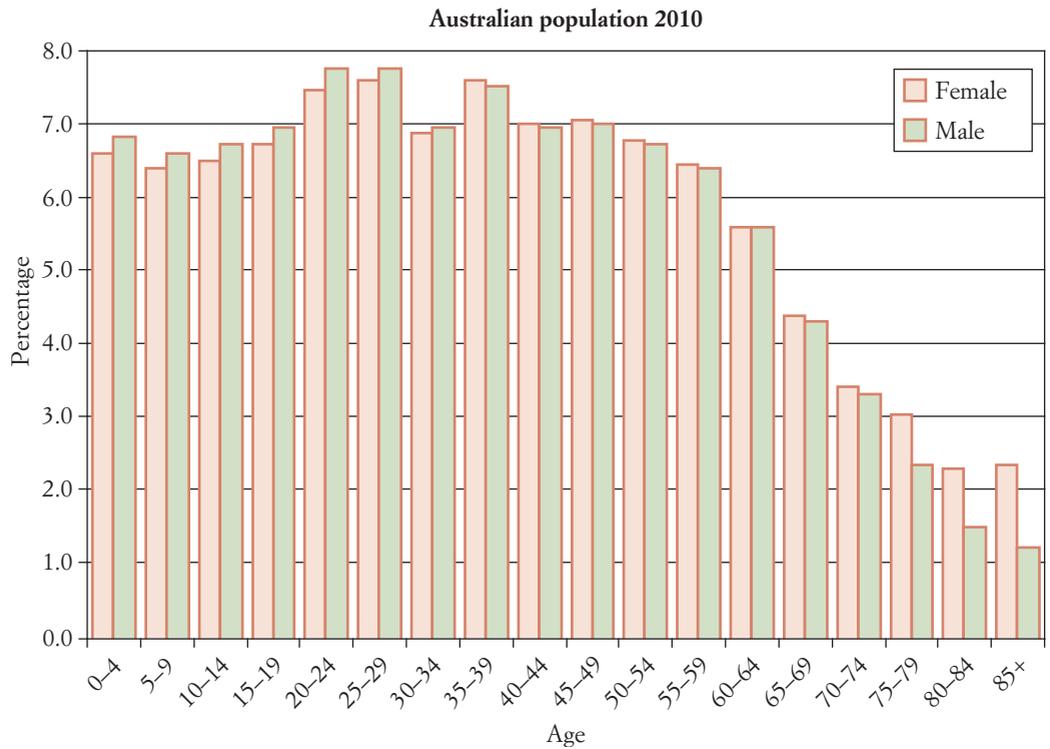
16 The average height of a group of people is 150 cm. When two people of height 180 cm join the group, the average increases to 155 cm. How many people were in the original group?

- The exact information you collect about many similar things (or people) is called **statistical data**.
- **Categorical** data places the information in categories such as colour, sex and nationality.
- **Numerical** data consists of numbers. There are two kinds of numerical data.
- **Continuous** data can have any values between the smallest and largest, so we may need to show how accurate it is.
- **Discrete** data can have only particular values, which are most often whole numbers. A **dot plot** is constructed by putting a dot for each data item above a scale. They are most useful for small sets of numerical data, particularly discrete data.
- In a statistical survey, the same information (**data**) is collected from each person. The **population** for a survey is the group of all possible sources. A **sample** is part of a population. **Census** data is collected from the whole population.
- A **fair** sample is like the population. A **biased** sample is not like the population.
- A **frequency table** (**frequency distribution**) is a table of scores and their frequencies. The **score** is the thing that is counted. The **frequency** of a score is the number of times it occurs. **Tally marks** are strokes grouped in fives to help counting.
- Continuous data is often **grouped** into convenient intervals to construct frequency tables. Discrete data may also be grouped.
- Graphs can be constructed using pictures (a **picture graph**, **pictogram** or **pictograph**), bars (**bar chart** or **column graph**), or points and lines (**line graph**). A **histogram** is a column graph of frequencies of continuous data. It has no gaps between the columns.
- All charts and graphs should have a **title**. The **axes** of a graph show the measurements and usually incorporate a **scale** showing the measurements.
- A **circle graph** (**pie graph** or **sector graph**) is a circle that has been divided into sectors to show the parts of a total amount. The angles of the sectors show the amounts.
- A **divided bar chart** is a long bar that has been divided into sections to show the parts of a total amount. The lengths of the sections show the amounts.
- **Line graphs** have points connected by lines. They are most commonly used for time-series data or bivariate data.
- A **dependent variable** is one that is controlled in some way by another quantity. An **independent variable** is not controlled by another variable.
- A **stem-and-leaf plot** (**stemplot**) is drawn with the first one or two digits written vertically to make the **stem**. The remaining digits are written in rows across from the first digit(s) to make the **leaves**.
- The **mean** $\overline{x}$ is the **average** score. It is the total divided by the number of scores.
- The **median** is the middle score. It is found by arranging all scores in order to choose the middle score. **Cumulative frequency** is the progressive total of frequencies and can be used to work out the median.
- The **mode** is the score with the highest frequency. A **bimodal** distribution has two scores with equal highest frequencies.
- The mean, median and mode are called **measures of central tendency**.

Quiz

Statistics

MAT07SPQZ00006

- The mean is most often used as the typical score. However, if there are a few high or low scores that would distort the mean, you should use the median. The mode should be used in cases where the most common score is needed.

- The **range** is the difference between the highest and lowest scores and measures the **spread** of the data.

- A **side-by-side** display shows the data for two different groups next to each other on a single graph. It is often used to compare the data from two groups (such as males and females).

- An **outlier** is a score that has a very different value to the rest of a set of data. The **relationship** between the mean and median can be affected by outliers. With no outliers, the mean and median are (almost) the same. With high outlier(s) the mean is generally higher than the median. With low outliers, the mean is generally lower than the median.

1 Classify the type of data for the pets of the families of a Year 7 class in each of the following.

   a The weights of the pets.
   b The names of the pets.
   c The numbers of pets owned by each family.
   d The amount of food eaten by the pets of each family.

2 For each of the following, state whether a sample or the population has been used.

   a People coming out of a particular bank were asked 'Do banks do a good job?'
   b The students in Mt Isa were asked 'Can you swim?' to find out if students in Mt Isa needed a special swimming program.

3 The numbers of steps at the back doors of some highset houses are shown below.

   15, 10, 14, 14, 13, 13, 15, 13, 9, 12, 14, 15, 12, 14, 16, 12, 11, 13, 15, 14, 14, 13, 10, 11.
   a Make a frequency table.
   b Draw a column graph of the numbers of back door steps.

4 The table below shows the numbers of different breeds of cattle on a particular property.

Breed	Angus	Hereford	Brahman	Simmental	Charolais
Number	120	350	200	180	130

   a How many Brahman cattle are there?
   b What is the mode?
   c What scale should be used for a picture graph of this data?
   d Draw a picture graph using a steer head symbol like .

5 Make a stem-and-leaf plot to show the following data:

   32, 47, 28, 37, 36, 42, 51, 64, 53, 47, 35, 17, 29, 22, 36, 34, 45, 37, 32, 24, 19, 61, 55, 38, 32

6 **a** Find the median and mode of the following scores:

     45, 48, 52, 41, 43, 47, 48, 49, 55, 56, 51, 51, 49, 51, 45
   b Find the mean and range of the scores.

7 For each part of question **2**, state if the method was fair or biased.

8 Draw a dot plot for the following data set:

   5, 7, 4, 3, 7, 9, 10, 2, 6, 8, 8, 8, 6, 9, 7, 8, 9

9 The following information shows the preferences of students for drinks on a school Music trip. Draw a circle graph to show the information.

Drink	Soft drink	Milk	Juice	Water
Number	25	8	17	10

10 For each of these surveys, state whether a sample or the population should be used.

   a The Education Department wants to find out how parents of students in Queensland schools feel about school uniforms.
   b The principal of a high school wants to know what types of takeaway foods to order for teachers' meals at a parent-teacher meeting.

11 State the population for each part of question **10**.

12 a Use groups of 15–19, 20–24, . . . to make a frequency table for these tyre pressures.
25.8, 24.9, 20.6, 36.1, 32.8, 26.7, 22, 19.6, 32.2, 28.5, 34.2, 41.1, 35.6, 30.3, 24, 22.2, 21.7, 38.2, 25.8, 21.3, 30, 35.7, 35.1, 32.7, 36.3, 41.7, 32.1, 32, 29.4, 18.6
b Draw a histogram of the tyre pressures.
c How are the mean and median related?

13 Rearrange the table into columns, then extend the table to include score × frequency and cumulative frequency columns. Hence find the mean, median, mode and range.

x	6	7	8	9	10	11	12	13
f	1	3	6	5	6	4	3	2

Problem solving

14 This graph shows the marks of Year 9 students for their English essay assignment.
a How many students scored 8 marks?
b What was the highest mark?
c What mark did the greatest number of students get?
d How many students' marks are shown?

English essay marks (out of 20)
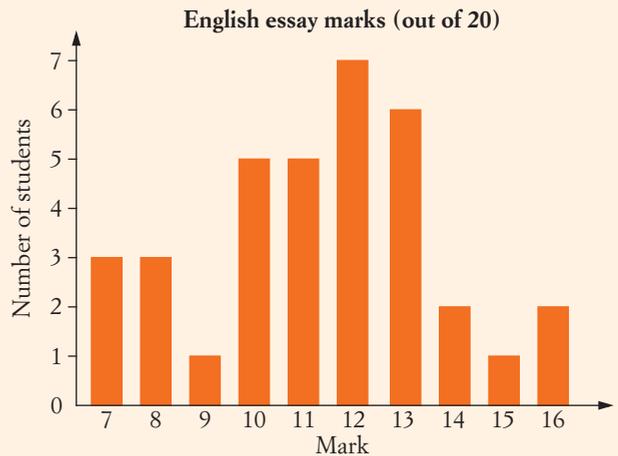
15 The percentages of people who die of cancer at different ages is shown on the right.
a What percentage of 50-year-old males die from cancer?
b At what ages does the percentage of female cancer deaths exceed that of males?

Cancer deaths
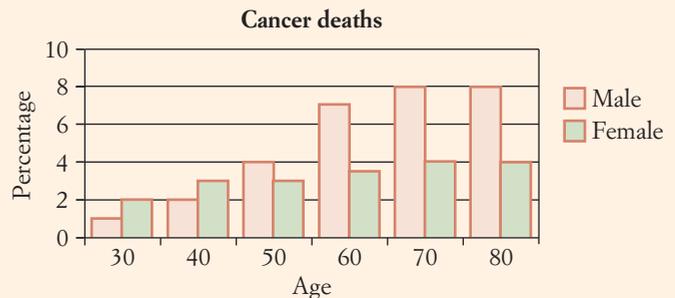
16 The graph below shows the solubility of oxygen in water at different temperatures.

Oxygen solubility
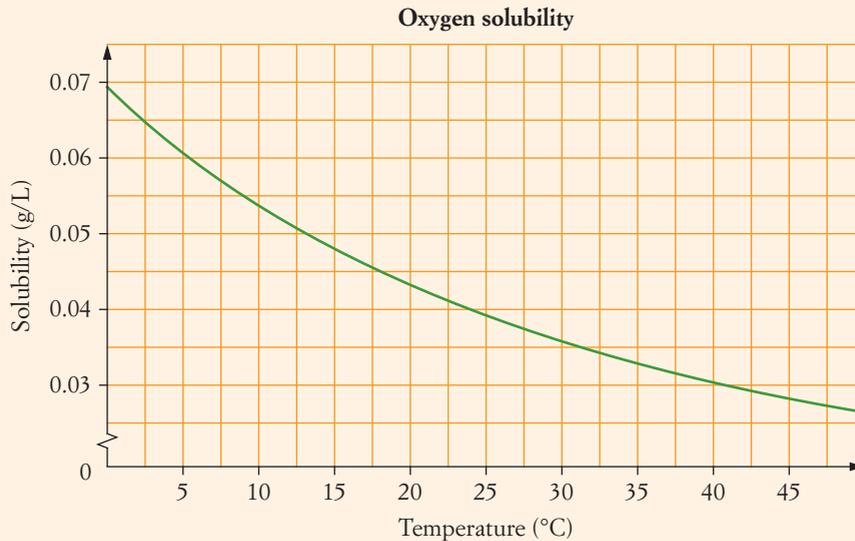
a What are the scales?
b What is the solubility of oxygen at 10°C?
c At what temperature is the solubility 0.05 g/L?
d A tropical fish needs oxygen at a minimum level of 0.035 g/L, but will die if the level exceeds 0.04 g/L. What is the temperature range of the water that it can live in?

17 The ages of people riding a roller coaster on the Gold Coast were as follows:
32, 16, 16, 20, 17, 15, 16, 15, 28, 21, 21, 22, 16, 16, 16, 27, 17, 16, 24, 25, 17, 14, 22
a What is the typical age? Explain your answer.
b Why were there no 10-year-olds?
c Which is more likely: under 20 or over 20?

18 a Draw a line graph about 6 cm high to illustrate these records of a patient's temperature.

Reasoning

Time	5 a.m.	8 a.m.	11 a.m.	2 p.m.	7 p.m.	10 p.m.
Temperature (°C)	38.7	39.0	38.6	38.2	37.9	37.5

b Estimate the patient's temperature at 12 noon.

19 The dot plot below shows the scores out of 20 obtained by a class in a vocabulary test.

    a Use the plot to find the mode and median of the scores.

    b Are there any outliers?

    c What effect would omitting the outlier(s) have on the mean?

    d Without calculation, comment on the relationship between the mean and median and explain your reasoning.
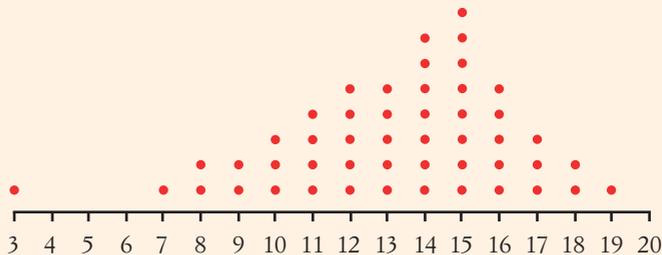
20 The side-by-side column graph below shows the Australian weekly earnings of males and females, rounded to the nearest $100 and 1%. Comment on the results.

Full-time weekly earnings
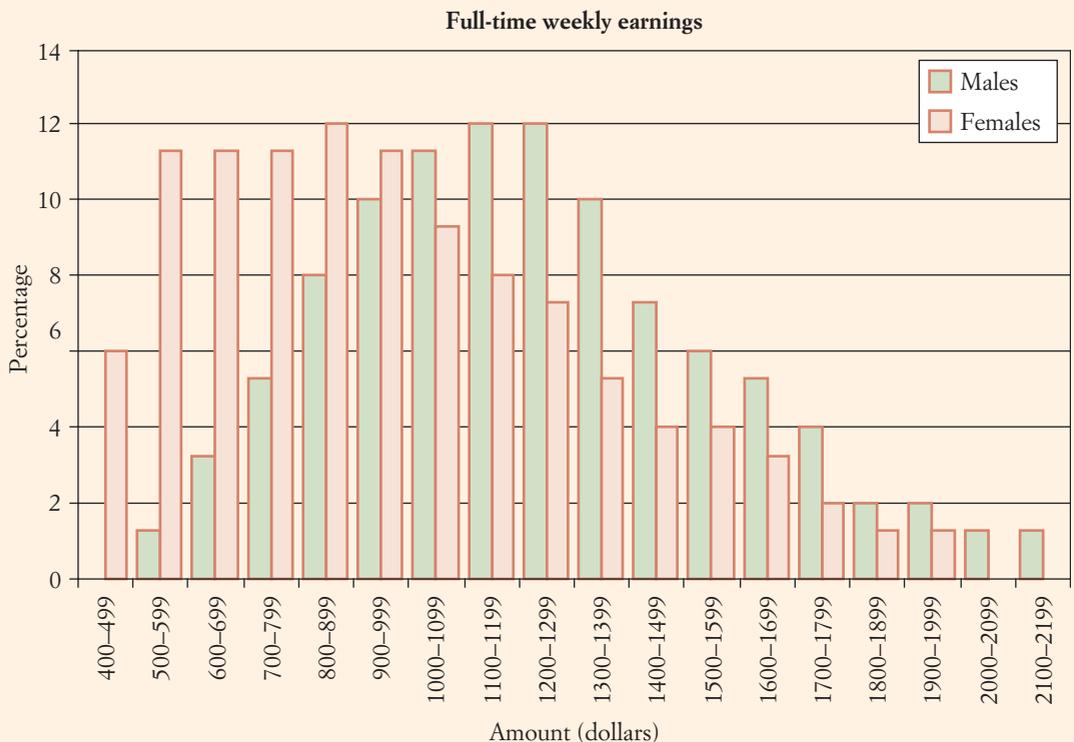
21 What things are wrong with the following survey? People on the last page of the White Pages were phoned and asked 'Do you think the discrimination in Australia's immigration policies should be changed?'

22 The side-by-side column graph below shows the monthly average daily maximum and
    minimum temperatures for Brisbane and Roma. Comment on the results.

Monthly average daily maximum and minimum temperatures
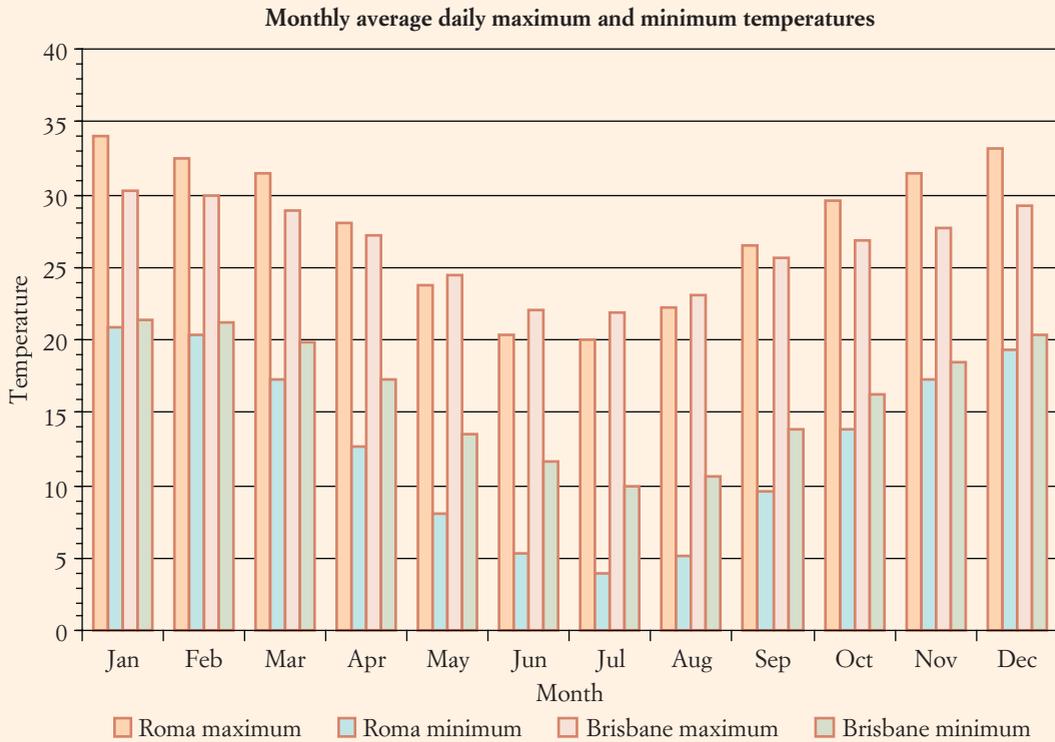
Number and algebra

2

Whole numbers

Contents

| Prior learning |
| Chapter 2 |

| Parent guide |
| Chapter 2 |

| Curriculum guide |
| Chapter 2 |

◼ Australian Curriculum statements

Number and place value

Investigate index notation and represent whole numbers as products of powers of prime numbers. (ACMNA149)

Investigate and use square roots of perfect square numbers. (ACMNA150)

Apply the associative, commutative and distributive laws to aid mental and written computation. (ACMNA151)

Compare, order, add and subtract integers. (ACMNA280) **AC**

Whole numbers are simply the numbers 0, 1, 2, 3, 4, 5, … (and so on). The first numbers that you learnt at school were probably the **counting numbers**. If you remove the zero from the set of whole numbers you end up with the counting numbers.

Depending on the context, counting numbers and whole numbers can also be called **natural numbers**. Whole numbers can also be called positive integers. **Integers** can be positive or negative and zero is included. Integers = { …, ⁻5, ⁻4, ⁻3, ⁻2, ⁻1, 0, 1, 2, 3, 4, 5, … }

While all mathematicians agree on what an integer is, some say that whole numbers can also be negative, so that would make them the same as integers. Some people also say that zero is not a whole number. In this chapter we will consider zero as a whole number but we will not include negative numbers.

■ Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

addition	directed numbers	multiplication	prime decomposition
ascending	division	natural numbers	product
base	estimate	negative	round
composite number	expanded form	number line	square
counting numbers	factor	place value	square root
cube	factor tree	positive	subtraction
cube root	index	power	sum
descending	integer	prime number	whole numbers

2.1 Using whole numbers

Over thousands of years people have used many different number systems. For example, one of the earliest number systems was used about 5000 years ago by the ancient Egyptians. Pictures called hieroglyphs represented words or sounds. They were written on papyrus (a type of paper made from reeds) or painted on walls.

The ancient kingdom of Babylon existed from about 3000 to 200 BCE (or BC, see page 214) where Iraq is today. Babylonian writing used wedge shapes called cuneiform. The wedges were stamped into clay tablets which were then baked.

Roman numerals were introduced during the time of the Roman Empire over 2000 years ago and they remain in use today. You may see them on an old clock face, on page numbers in a book or in the credits of a film to indicate the year of release.

1	2	3	4	5
I	II	III	IV	V
6	7	8	9	10
VI	VII	VIII	IX	X
50	100	500	1000	
L	C	D	M	
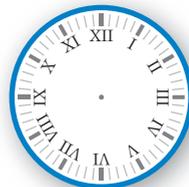
Aboriginal people did not have symbols for numbers. Different regions had their own names for numbers. The Belyando River people of central Queensland used only two words in their counting system:

1 = wogin

2 = booleroo

3 = booleroo wogin

4 = booleroo booleroo

The Kamilaroi people lived in northern New South Wales. They used three words in their counting system:

1 = mal

2 = bularr

3 = guliba

4 = bularr bularr

5 = bularr guliba

6 = guliba guliba

We use the Hindu-Arabic system that uses the following ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The system was first adopted from India into Europe, then into the Middle East/Arabia.
We use a **place value** system for bigger numbers.

Important!

Place value system

The place in which you write a digit affects its value. The larger place values are on the left. Each place is 10 times as big as the one before it.
When there are more than four digits, the modern practice is to separate each group of three places by a small gap. Some people use a comma between the groups of three instead of a small space.

Trillions			Billions			Millions			Thousands					
H	T	U	H	T	U	H	T	U	H	T	U			

Hundreds | Tens | Units
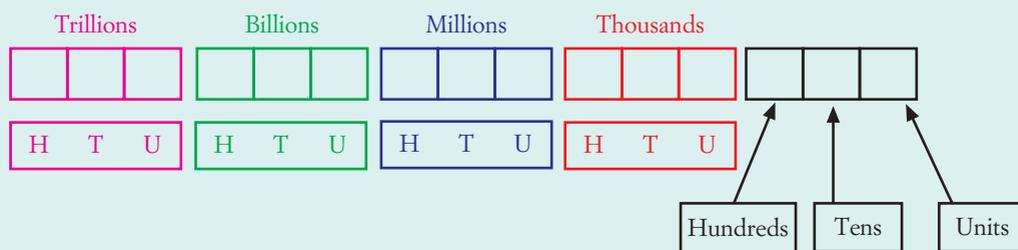
When you write numbers in words, you should use a hyphen in the numbers from twenty-one to ninety-nine.
In Australia, we now use the system where a billion is a thousand million, a trillion is a thousand billion, and so on. Some people still use a system where each new 'illion' is a million times bigger than the previous one.
You can use quadrillion, quintillion, sextillion, septillion and so on for larger numbers but this is uncommon.

You can think of each group of three numbers as a 'house'. Inside each house the numbers go from one to nine hundred and ninety-nine. The lowest black 'house' does not have a separate name, but all the other houses have names so you know the size of the numbers. The first big house is the 'thousands' house, then the 'millions' house, the 'billions' house and so on.

Example 1

Write the value of each of the digits in 5316.

Solution

Look at the place value of each digit in 5316.

6 has a value of 6 or 6 × 1

1 has a value of 10 or 1 × 10

3 has a value of 300 or 3 × 100

5 has a value of 5000 or 5 × 1000

You can write a number in **expanded form** (**extended form**) by separating each digit with its value. You can write 100 as 10^2 and 1000 as 10^3.

Example 2

Write each of these numbers in extended form.

a 247   **b** 3587

Solution

a In 247, the 2 = 200, 4 = 40 and 7 = 7. $247 = (2 × 100) + (4 × 10) + (7 × 1)$

$= 2 × 10^2 + 4 × 10 + 7 × 1$

b Separate 3587 using the place value of each digit.

$3587 = (3 × 1000) + (5 × 100) + (8 × 10) + (7 × 1)$

Change the place values to powers of 10.

$= 3 × 10^3 + 5 × 10^2 + 8 × 10 + 7 × 1$

Investigate: Double-digit dice game

This is a game for two or more players using one die.

Instructions

Step 1: Copy the scoresheet shown on the right.

Step 2: Each player rolls the die seven times and, for each roll, can choose to write the number in either the tens column or the units column of his or her scoresheet.

Step 3: Each player finds the total of his or her seven numbers. The winner is the person with a total closest to 99.

Step 4: Play the game again and work out a strategy to improve your score.

Scoresheet		
Roll	**Tens**	**Units**
1st		
2nd		
3rd		
4th		
5th		
6th		
7th		
Total		

Example 3

A long-distance runner has a training schedule that calls for her to run 130 km every week. On Monday, Tuesday and Wednesday she ran 14, 15 and 16 km respectively. On Thursday she got up late so could only go 10 km, but she compensated by running 15 km in the morning on Friday and another 15 km after work. On Saturday she ran 24 km.

a How much did she run from Monday to Saturday?

b How far should she run on Sunday?
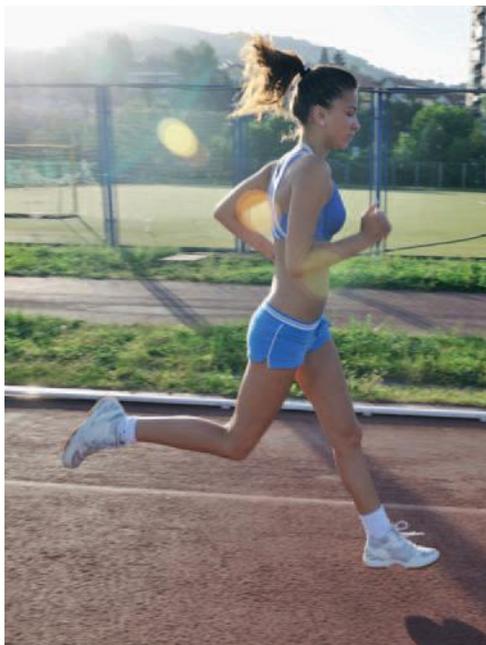
Solution

a You need to add all the distances to get the total.

$14 + 15 + 16 + 10 + 15 + 15 + 24 = 109$

Write the answer in a sentence.

She ran 109 km from Monday to Saturday.

b You need to subtract the distance that she ran from 130.

$130 - 109 = 21$

Write the answer in a sentence.

She should run 21 km on Sunday.

Example 4

A hardware store has 200 m of rope on a reel. It is to be cut into 7 m lengths, which will be sold for $4 each.

a How many lengths will there be?

b How much money will they get altogether for the rope?

Solution

a Divide the total by 7 to get the number of 7 m lengths of rope.

$$\text{Number} = 200 \div 7$$
$$= 28 \text{ with remainder } 4$$

Write the answer in a sentence.

There will be 28 full lengths, with 4 m left over.

b Multiply the number of lengths by the price.

$$\text{Total amount} = \$4 \times 28$$
$$= \$112$$

Write the answer in a sentence.

They will get $112 altogether.

Exercise 2.1 Using whole numbers

1 Write the value of each digit in the following numbers, then write each number in words.

  a 609   **b** 1039   **c** 70 104   **d** 504 860   **e** 9 134 671

  f 5 837 000   **g** 4001   **h** 205 689   **i** 34 000 036

2 Write each of the following using numerals.

  a eight thousand, seven hundred and ninety-six

  b three million and eighty-eight

  c two thousand, three hundred and eighty-five

  d six thousand, nine hundred and seven

  e four hundred and twenty thousand, eight hundred and thirty

  f three hundred and nine thousand, two hundred and eleven

  g one million, two hundred and eighty thousand, four hundred and sixty

  h twelve million, nine hundred and one

3 Write each of these numbers in extended form.

  a 56   **b** 3562   **c** 416   **d** 502   **e** 1001

  f 10 253   **g** 38 002   **h** 59 644   **i** 3809   **j** 120 435

4 Write each of these as a single number.

  a $(5 \times 100) + (2 \times 10) + (4 \times 1)$

  b $(6 \times 1000) + (5 \times 100) + (3 \times 10) + (7 \times 1)$

  c $(4 \times 10^2) + (2 \times 10) + (9 \times 1)$

  d $(6 \times 10^3) + (4 \times 10^2) + (7 \times 10) + (3 \times 1)$

  e $8 \times 10^4 + 2 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 3 \times 1$

  f $3 \times 10^3 + 0 \times 10^2 + 5 \times 10 + 7 \times 1$

  g $7 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 0 \times 10 + 1 \times 1$

  h $1 \times 10^4 + 0 \times 10^3 + 9 \times 10^2 + 9 \times 10 + 9 \times 1$

  i $3 \times 10^5 + 4 \times 10^4 + 4 \times 10^3 + 2 \times 10^2 + 2 \times 10 + 0 \times 1$

  j $9 \times 10^5 + 0 \times 10^4 + 0 \times 10^3 + 9 \times 10^2 + 9 \times 10 + 9 \times 1$

Fluency

5 Calculate the answers to these.

  a Find the total of 472 and 3621.

  b By how much does 487 exceed 299?

  c Find the sum of 88, 67, 132 and 431.

  d How much more than 312 is 1876?

  e Find the sum of 381, 2174 and 82.

6 Arrange the numbers in each of these sets in order, from smallest to largest.

  a 321, 17, 8000   **b** 17, 707, 27, 63

  c 246, 3596, 5369, 432, 16, 6125   **d** 123, 321, 132, 231, 213

  e 1045, 450, 145, 82   **f** 721, 243, 43, 4372, 722

  g 380 211, 308 022, 300 806, 392 084   **h** 4 856 231, 4 766 372, 1 429 950, 3 006 853

7 How many times is the first 3 bigger than the second 3 in each of these numbers?

   a 1433   **b** 1343   **c** 3143   **d** 2 352 312

8 Answer the following, using the correct Aboriginal words.

   a wogin + booleroo wogin   **b** guliba × bularr   **c** bularr + bularr + mal

   d booleroo × booleroo   **e** guliba guliba − guliba   **f** bularr bularr − mal

9 At Big Town High there are 404 students in Year 7, 376 in Year 8, 417 in Year 9, 283 in Year 10, 216 in Year 11 and 209 in Year 12. How many students are there at the school?

10 A certain whole number was trebled and then reduced by 4 to give a result of 14. What was the number?

11 Elena the truckie travels around Queensland in her 'Big Mack' delivering smallgoods. Use the map to calculate the distances of the trips below.

   a Brisbane to Cairns

   b Brisbane to Mt Isa via Charleville

   c Rockhampton to Mt Isa via Longreach and return via Townsville

   d Brisbane direct to Rockhampton and return via Longreach and Charleville

   e Brisbane direct to Cairns and return via Mt Isa and Charleville

Distances shown in km

12 Rianna bought 18 golf balls.

   a What was the total cost?

   b How much did she save by buying them at the special price?

GOLF BALLS
$3 each

TOP SHOT

(Normally $3.75)

13 Joshua wants to count the number of words in his three-page essay. He knows that he writes about 12 words per line. He has written his essay on pages that fit 28 lines per page. How many words are in Joshua's essay?

14 A science experiment started with 183 217 live bacteria cultures. It finished with 64 128 live cultures. How many died?

15 The daily attendances at an extreme sports exhibition were:

Mon	Tues	Wed	Thur	Fri	Sat	Sun
3267	3025	4162	4275	4891	8213	7921

   a What was the total attendance at the exhibition over the week?

   b How many more people went on Monday to Friday than on the weekend?

   c If all people were charged $3 entry, how much money was collected from attendances?
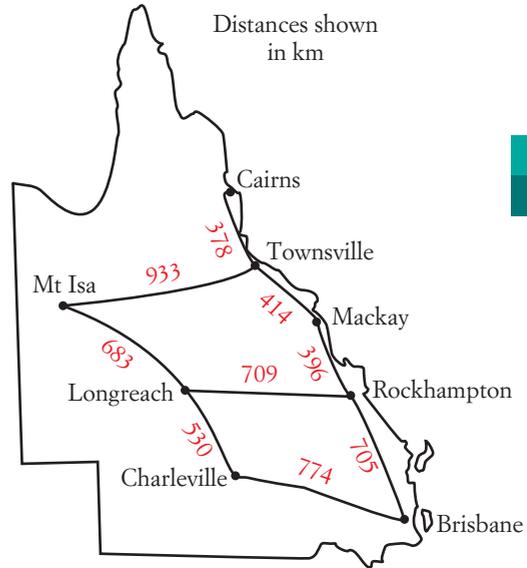
16 An arcade game had a record of 756 251 points. Jomar set a new record of 874 275 points. What was the increase in the record?

17 Sandra organised tickets for a group of people to go to a rock concert. The tickets cost $72 each. How much did Sandra need to collect if 46 people wanted to go to the concert?

18 Kristina is paid $738 per fortnight. She pays $95 per week rent and it costs her $125 per week for food and entertainment. Her other expenses amount to $105 per week.
  a How much could she save each week?
  b How much could she save in a year?

19

BUY NOW AND SAVE!

You pay only $256 per week for 48 months

DRIVE-AWAY PRICES

  a How much would be repaid each year?
  b How much would the car cost over the full 4 years?
  c How much more was paid for the car if its cash price was $40 000?

20 Each morning, 504 people use a lift to enter an office building. The lift carries a maximum of 14 people. What would be the minimum number of trips that the lift would need to make to take the workers up to their offices?

21 A plane travels 1045 km on a regular daily trip delivering mail in outback Victoria.
  a How many kilometres are travelled in a normal five-day week?
  b An average of 786 letters and parcels are delivered each trip. How many would be delivered in a four-week period?

22 During one year, Jessica and Stefan saved $2041 and $1892 respectively. They wanted to pool their savings to buy a car worth $10 701.

  a How much more did Jessica save than Stefan?
  b How much did they need to borrow?
  c They decided to borrow the extra money and repay it over three years. The interest charges were $3600. How much did they actually pay for the car?
  d How much did they repay on the loan?
  e How much were the monthly repayments?

23 The total quantity of diamonds mined in Western Australia over a period of six years is illustrated in the table. (Diamonds are measured in carats.)
  a What was the total quantity of diamonds mined in Western Australia from 2002 to 2007 (inclusive)?

b How many more carats were produced in 2002 than in 2007?

c If the diamonds sold at an average of $17 per carat, how much did the Western Australian diamond industry gross for 2005?

d If the diamonds sold for an average of $12 per carat, how much did the Western Australian diamond industry gross for 2003?

Year	Diamonds (Carats)
2002	35 486 604
2003	34 367 807
2004	22 842 000
2005	30 845 157
2006	29 300 000
2007	20 467 000

24 What would be the weekly payment for a mobile phone that costs $468 and is paid off in two years?

25 Olly the orchardist has 240 apple trees. If his total production of apples was 115 200, how many apples did each tree produce on average?

26 A special double-decker bus carries 85 people. How many of these buses would be required to transport 361 760 people?

27 You are to spend exactly $100 on items costing $3, $5 and $13 each. If you must buy at least one of each, what are the largest and smallest numbers of items you can buy?

Full price $468
Paid over 2 years

2.2 Order of operations

What do you think is the answer to the question $3 + 4 \times 7$? Is it 49 or 31? The correct answer is 31. Order of operations is a mathematical convention that tells you the order that operations should be performed in. In the above example you are supposed to evaluate the multiplication part of the problem before you do the addition.

Important!

Order of operations

First: Work out the value within any grouping symbols, starting with the innermost grouping symbols: parentheses or round brackets ( ), square brackets [ ] and braces { }.
Second: Work out multiplication or division as you come to it, going from left to right.
Third: Work out addition or subtraction as you come to it, going from left to right.

Example 5

Find the value of $(5 + 13) \div 2$.

Solution

Write the expression.	$(5 + 13) \div 2$
Calculate the brackets first.	$= 18 \div 2$
Now do the division.	$= 9$

Example 6

a Find the value of $15 \div 5 \times 8$.
b Find the value of $5 + 6 \times 2 - 7$.

Solution

a	Write the expression.	$15 \div 5 \times 8$
	Work from left to right — do the division first.	$= 3 \times 8$
	Now do the multiplication.	$= 24$
b	Write the expression.	$5 + 6 \times 2 - 7$
	Do the multiplication first.	$= 5 + 12 - 7$
	Work from left to right — do the addition next.	$= 17 - 7$
	Now do the subtraction.	$= 10$

Modern scientific calculators perform calculations in the correct order of operations.

Example 7

Find the value of $25 - [7 \times (5 - 3) + 4]$.

Solution

Write the expression.	$25 - [7 \times (5 - 3) + 4]$
Do the operation in the innermost brackets first.	$= 25 - [7 \times 2 + 4]$
Now do the multiplication in the brackets.	$= 25 - [14 + 4]$
Brackets come first, so do the addition.	$= 25 - 18$
Now do the subtraction.	$= 7$

To check, enter as
25 [−] [(] 7 [×] [(] 5 [−] 3 [)] [+] 4 [)] [=].

25−(7×(5−3)+4)	7

Investigate: Brain bender

Various forms of 'brain benders' are common in daily newspapers and magazines. Here is one for you. Copy the grids and fill in the six gaps to complete each of the lines, using the remaining digits from 1 to 9 only once. Be sure to use the 'order of operations' rules. The aim is to make the sum of the answers for the three lines total 45.

5	+		×		=	
	×	3	−		=	
	−		+	4	=	
						45

Investigate: Number puzzles

Various number puzzles appear in newspapers and magazines, such as Sudoku and the one shown below. To complete the puzzle, every shaded square must be filled in with a number from 1–9. The operations shown in black give the answers at the end of the lines and the operations shown in red give the answers at the bottom of the columns, using the correct order of operations.

	÷		−		=	1
÷		+		×		
	×		−		=	7
+		−		×		
	+		÷		=	10
=		=		=		
11		1		15		

Exercise 2.2 Order of operations

1 Evaluate (find the value of) each of the following.

   a $12 \times (3 + 5)$ **b** $(16 - 3) \times 2$ **c** $(60 + 12) \div 6$

   d $(3 - 2) \times 5$ **e** $(2 + 5) \times 6$ **f** $(12 - 4) \div 4$

   g $7 \times (25 - 12)$ **h** $36 \div (14 - 10)$ **i** $(5 \times 7) - 16$

   j $120 \div (34 - 24)$ **k** $5 + 6 \times (50 - 10)$ **l** $(77 \div 11) - 7$

2 Evaluate the following.

   a $3 + 5 \times 2$ **b** $20 - 2 \times 5$ **c** $5 + 3 \times 2 - 7$

   d $19 - 4 \times 4 - 1$ **e** $24 - 5 \div 5 + 7$ **f** $17 + 8 - 3 \times 2$

   g $2 \times 10 - 9 + 28$ **h** $42 \div 7 - 5$ **i** $9 + 28 - 12$

   j $4 \times 8 - 3 \times 3$ **k** $109 + 36 \div 4$ **l** $60 - 8 \times 4 + 20$

3 $12 \div 4 + 8 \times 5 = ?$ Select **A**, **B**, **C** or **D**.

   A 5   **B** 16   **C** 43   **D** 55

4 Find the answer to each of the following.

   a $(24 - 4) \div 5 + 7$ **b** $2 \times (10 - 9) + 28$

   c $(8 + 2) \times (17 - 7)$ **d** $7 + 7 + (11 - 8)$

   e $(16 - 5 + 8) \times 9$ **f** $(8 + 8 - 5) \times (7 + 4)$

   g $9 + 3 \times (15 - 4) - 5 \times 6$ **h** $16 \times 3 - 4 \times (15 - 6 \times 2) + 7$

   i $(5 + 8) \times 2 - (25 \div 5)$ **j** $4 \times [(5 + 11) \div 2] - (15 \times 2)$

   k $100 - [12 + (3 \times 5) \div 3]$ **l** $120 \div \{16 + [(2 \times 5) + 4]\}$

   m $\{15 - [3 \times (12 - 9) + 1]\} - [(44 \times 2) + 12] \div 50$

   n $[(16 - 4) \times 10] \div [(45 \div 3) + 25]$ **o** $86 + [(15 \div 3) + (65 \div 5)] \times 2$

   p $[20 \div (5 - 4) \times 2] - \{[(4 + 5) \times 3] \div [15 - (30 \div 5)]\}$

Fluency

5 Put grouping symbols where necessary to make each of the following statements true. The first one has been done for you.

   a $5 - 2 \times 4 = 12$ becomes $(5 - 2) \times 4 = 12$ **b** $3 + 8 - 7 = 4$ **c** $15 - 3 \times 5 = 60$

   d $15 - 3 \times 5 = 0$ **e** $8 + 4 - 3 \times 2 = 10$ **f** $8 + 4 - 3 \times 2 = 6$

   g $8 + 4 - 3 \times 2 = 18$ **h** $6 + 4 \times 0 = 6$ **i** $6 + 4 \times 0 = 0$

   j $100 \div 10 + 10 = 5$ **k** $100 \div 10 + 10 = 20$

6 Put grouping symbols where necessary to make each of the answers correct.

   a $84 \div 3 + 9 \times 15 - 11 = 152$ **b** $84 \div 3 + 9 \times 15 - 11 = 64$

   c $84 \div 3 + 9 \times 15 - 11 = 94$

Problem solving

7 Use the four numbers in each set only once (in any order), with the operations $+$, $-$, $\times$, $\div$ or grouping symbols, to make an equation that equals the number in the red box.

   a $2, 7, 8, 9$ 12   **b** $1, 2, 3, 5$ 18   **c** $3, 4, 6, 8$ 41

   d $2, 6, 8, 11$ 21   **e** $2, 4, 6, 8$ 10   **f** $2, 5, 8, 10$ 44

   g $3, 5, 7, 9$ 2   **h** $4, 5, 7, 9$ 8   **i** $2, 5, 7, 10$ 60

2.3 Index notation

The following table shows some of the most common symbols used in mathematics.

> **Important!**
>
> Mathematical symbols
>
> | Symbol | Meaning | Symbol | Meaning |
> |:---:|:---|:---:|:---|
> | $+$ | plus, add, sum | $\sqrt{}$ | square root ($\sqrt{25} = 5$) |
> | $-$ | minus, subtract, difference | $\sqrt[3]{}$ | cube root ($\sqrt[3]{8} = 2$) |
> | $\times$ | multiply, times, product | $\therefore$ | therefore |
> | $\div$ | divided by, quotient | $\approx$ | approximately equal to |
> | $=$ | equal to | 3^2 | squared (3×3) |
> | $\neq$ | not equal to | 5^3 | cubed ($5 \times 5 \times 5$) |
> | $<$ | less than | $(\ )$ | parentheses or brackets |
> | $\leq$ | less than or equal to | $[\ ]$ | square brackets |
> | $>$ | greater than | $\{\ \}$ | braces |
> | $\geq$ | greater than or equal to | | |

> **Important!**
>
> Powers and roots
>
> A repeated multiplication is written as a **power**.
> The number that is multiplied, the **base**, is written in normal size.
> The number of times it is in the product, the **index**, is written as a small number at the top right of the base. The index is also called the **exponent**.

$$\text{Power form } \underbrace{4^{\overbrace{5}^{\text{Index or exponent}}}}_{\text{Base}} = \underbrace{4 \times 4 \times 4 \times 4 \times 4}_{\text{Extended form}}$$

> The power may be written in **extended form** by writing out the product in full.
> The power shown is said as 'four to the fifth' or 'the fifth power of four'.
> A number raised to the power of 2 is called a **square**. 6^2 is said as 'six squared'.
> A number raised to the power of 3 is called a **cube**. 8^3 is said as 'eight cubed'.
> The **square root** of a given number is the value which if squared will give that number.
> The **cube root** of a number is the value which if cubed will give the number.

Example 8

Write the meaning of each of the following.

a $5 > 2$ **b** $3 \leq 7$ **c** $5 \geq 5$

Solution

a $>$ means 'greater than'.   **5 is greater than 2.**

b $\leq$ means 'less than or equal to'.   **3 is less than or equal to 7.**

c $\geq$ means greater than or equal to.   **5 is greater than or equal to 5.**

Example 9

Find the answer for each of the following.

a 7^2 **b** 5^3 **c** $\sqrt{9}$ **d** $\sqrt[3]{8}$

Solution

a 7^2 means '7 squared'. $7^2 = 7 \times 7$

Evaluate. $= 49$

b 5^3 means '5 cubed'. $5^3 = 5 \times 5 \times 5$

Evaluate. $= 125$

c If you factorise 9, you can see that $3 \times 3 = 9$. So when 3
is squared, we get 9. $\sqrt{9} = 3$

You may also remember that $3^2 = 3 \times 3 = 9$.

d If you factorise 8, you can see that $8 = 2 \times 2 \times 2 = 2^3$. $\sqrt[3]{8} = 2$

Example 10

Use your calculator to determine each of the following.

a 3^5 **b** 5^4 **c** $\sqrt[3]{343}$ **d** $\sqrt{600}$ (correct to 2 decimal places)

Solution

a Enter as 3 $\boxed{a^b}$ 5 $\boxed{=}$.

3^5 243

b Enter as 5 $\boxed{a^b}$ 4 $\boxed{=}$.

5^4 625

c Enter as $\boxed{\sqrt[3]{}}$ 343 $\boxed{=}$.

$\sqrt[3]{343}$ 7

d Enter as $\boxed{\sqrt{}}$ 600 $\boxed{=}$.

$\sqrt{600}$ 24.49489743

Round to 2 decimal places and write the answer. $\sqrt{600} \approx 24.49$

Investigate: Locating square roots

How can you find the approximate value of $\sqrt{60}$ without using a calculator?
60 is between 49 and 64, so $\sqrt{60}$ must be between $\sqrt{49} = 7$ and $\sqrt{64} = 8$.
You can write this as $7 < \sqrt{60} < 8$ and say it is between 7 and 8.
Since 60 is closer to 64 than 49, try working out 7.7^2, 7.8^2 and 7.9^2.
$7.7^2 = 59.2$; $7.8^2 = 60.84$; and $6.9^2 = 62.41$ so it is clear that $\sqrt{60} \approx 7.8$.
Try finding which whole numbers the following lie between. $\sqrt{20}$, $\sqrt{40}$, $\sqrt{90}$, $\sqrt{110}$ and $\sqrt{127}$.
Now see if you can find approximations (correct to one decimal place).

You can write the place values of the decimal system as powers of 10, so $100 = 10^2$, $1000 = 10^3$, $10\,000 = 10^4$ and so on. Instead of writing decompositions like $4372 = 4000 + 300 + 70 + 2$, you can write them as place value decompositions like $4372 = 4 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 2$.

Example 11

Write 72 065 as a place value decomposition using powers.

Solution

Write the decomposition.	$72\,065 = 70\,000 + 2000 + 60 + 5$
Separate the place values.	$= 7 \times 10\,000 + 2 \times 1000 + 6 \times 10 + 5$
Write the place values as powers of 10.	$= 7 \times 10^4 + 2 \times 10^3 + 6 \times 10^1 + 5$
Include all the place values.	$= 7 \times 10^4 + 2 \times 10^3 + 0 \times 10^2 + 6 \times 10^1 + 5$

Example 12

Work out the following and write in index notation.
a $4^2 \times 4$ **b** $2^3 \times 2^4$ **c** $3^5 \div 3^2$

Solution

a Work out 4^2.	$4^2 = 16$
Replace 4^2 in the problem.	$4^2 \times 4 = 16 \times 4$
Work out the answer.	$= 64$
Write in expanded form.	$= 4 \times 4 \times 4$
Write the answer as a power.	$= 4^3$
b Work out 2^3 and 2^4.	$2^3 = 8, 2^4 = 16$
Replace 2^3 and 2^4 in the problem.	$2^3 \times 2^4 = 8 \times 16$
Work out the answer.	$= 128$

Write in extended form.

$$= 64 \times 2$$
$$= 32 \times 2 \times 2$$
$$= \ldots$$
$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

Write the answer as a power. $= 2^7$

c Work out 3^5 and 3^2. $\quad 3^5 = 243, \; 3^2 = 9$

Replace 3^5 and 3^2 in the problem. $\quad 3^5 \div 3^2 = 243 \div 9$
Work out the answer. $\qquad = 27$
Write in expanded form. $\qquad = 9 \times 3$
$\qquad = 3 \times 3 \times 3$
Write the answer as a power. $\qquad = 3^3$

Exercise 2.3 Index notation

1 Here is a list of words that relate to the four basic operations $+, -, \times$ and $\div$.

plus	*minus*	*times*	*multiply*	*and*	*divide*
subtract	*share*	*decrease*	*product*	*difference*	*less*
increase	*total*	*groups of*	*quotient*	*take away*	*more than*

Draw a table with column headings as shown below in your notebook, and write each of the given words in the appropriate column.

+	−	×	÷

2 Rewrite these questions using mathematical symbols.

a 15 minus 6   **b** 48 plus 12   **c** 12 is greater than 5
d 5 is not equal to 3 plus 6   **e** the product of 7 and 8   **f** square root of 16
g 36 divided by 4   **h** 5 squared   **i** 8 more than 12
j 6 less than 13   **k** increase 3 by 13   **l** quotient of 39 and 3
m the difference between 25 and 8   **n** the cube root of 125
o 13 is not equal to 3   **p** 999 is approximately equal to 1000

3 Write down how you would say each of the following.

a 6^2 **b** 7^3 **c** 6^7 **d** 2^{10} **e** 8^4 **f** 24^4

4 Work out the following by hand.

a 5^2 **b** 6^3 **c** 7^2 **d** 9^2 **e** 2^7 **f** 4^5
g 3^4 **h** 9^3 **i** 5^3 **j** 8^1

5 Use a calculator to find each of the following.

a 2^{12} **b** 7^4 **c** 6^5 **d** 14^4 **e** 4^6 **f** 15^3
g 3^{10} **h** 9^5 **i** 6^4 **j** $2^4 \times 3^4$

6 Use a calculator to find each of the following correct to 2 decimal places.

a $\sqrt{26}$ **b** $\sqrt{24}$ **c** $\sqrt{120}$ **d** $\sqrt{54}$ **e** $\sqrt{13}$ **f** $\sqrt{150}$
g $\sqrt{600}$ **h** $\sqrt{75}$ **i** $\sqrt{276}$ **j** $\sqrt{164}$

7 Work out each of the following by hand.

a $\sqrt{25}$ b $\sqrt{196}$ c $\sqrt{81}$ d $\sqrt{64}$ e $\sqrt{576}$ f $\sqrt{121}$

g $\sqrt{16}$ h $\sqrt{49}$ i $\sqrt{225}$ j $\sqrt{900}$

8 Write the answer to each of the following.

a the number 6 less than 18   b the sum of 26 and 14   c the total of 6, 8 and 22

d 9 times 8   e 7 squared   f the quotient of 36 and 4

g the number 14 more than 8   h decrease 33 by 11   i increase 83 by 27

j 7 lots of 13   k the cube root of 64   l the difference between 135 and 29

9 Which of these statements is true? Select from **A**, **B**, **C** or **D**.

A $\sqrt{36} = 18$ **B** $18 \div 2 \neq 9$ **C** $6 \times 4 \leq 15$ **D** $7^2 > 12$

10 Write whether each of the following is true (T) or false (F).

a $16 > 2$ b $4^2 = 8$ c $300 > 5 \times 100$

d $3602 = 3600$ e $\sqrt{25} = 5$ f $8 \times 201 \geq 8 \times 200$

g $2 \neq \sqrt[3]{27}$ h product of 2 and 15 = 17   i $63 \div 3 < 60 \div 5$

j $3^3 = 27$ k $5^2 - 3 = 7$ l $7^2 < 7^3$

m $16 \times 0 \neq 7 \times 0$ n $(30 - 6) \times 5 \neq 12 \times 10$ o $\sqrt{36} = 6$

p $\sqrt[3]{1} = 1$ q $5^3 = 15$ r $\sqrt{2^4} \neq 4$

11 Complete the blank with $>$ or $<$ to make each statement true.

a 7130 ___ 860   b 2001 ___ 2010   c 352 140 ___ 4 082 716   d 2651 ___ 2561

e 3602 ___ 3206   f 13 253 ___ 1353   g 8079 ___ 8097   h 1432 ___ 1483

12 Write each of the following as place value decompositions using powers.   *See* Example 11

a 5896   b 314 528   c 503   d 60 307   e 930 200

13 Without using a calculator, work out each of the following and write in index notation.   *See* Example 12

a $2^2 \times 2$ b $2^4 \times 2^3$ c $2^2 \times 2^5$ d $2^{12} \div 2^{10}$ e $2^{10} \div 2^7$

14 Without using a calculator, work out each of the following and write in index notation.

a $3^2 \times 3$ b $3^4 \div 3^3$ c $3^8 \div 3^8$ d $3^6 \times 3^7 \div 3^{10}$

15 For each of the following statements, select all the numbers from this list of seven numbers that make the statement true: 2, 3, 7, 8, 11, 36, 41.

a $w > 13$ b $w \leq 5$ c $w \neq 8$ d $w \approx 42$

e $w^3 = 8$ f $w \geq 11$ g $\sqrt[3]{w} = 2$ h $5 + w \geq 8$

16 a Examine the powers of 2, 4 and 8. Make a conclusion.

b Examine the powers of 3, 9 and 27. Make a conclusion.

c What is likely to be true of the powers of 5, 25 and 125?

Can you make a general rule?

17 Draw a square of dots as shown below.

a How many dots are in a square with a side length of 3 dots?

b Try some other squares and make a general conclusion.

18 Explain how to work out $5^4 \times 5^6$ and write in index notation without calculating 5^4 and 5^6 first.

19 Explain how to work out $7^8 \div 7^2$ and write in index notation without calculating 7^8 and 7^2 first.

2.4 Integers on the number line

Positive integers are all the whole numbers greater than zero {1, 2, 3, 4, 5, … }.
Negative integers are all the opposites of those whole numbers: {$^-1, ^-2, ^-3, ^-4, ^-5$, …}.
We do not consider zero to be a positive or negative number.
$^-5$ is the opposite of 5 and $^-11$ is the opposite of 11. If an integer is greater than zero, we say that its sign is positive ($^+5$).
If an integer is less than zero, we say that its sign is negative, e.g. $^-5$.
Positive numbers and negative numbers together are called **directed numbers**. Directed numbers show both direction and magnitude (size). For example, 7 or $^+7$ means 7 units in the positive direction, while $^-3$ means 3 units in the negative direction. Negative numbers are in the opposite direction to positive numbers. Together, they are called **integers**.
Integers are useful in indicating a direction associated with certain events. If I take three steps forwards this could be viewed as a positive 3. If I take 4 steps backwards, we might write this as $^-4$.
Temperature is another way negative numbers are used. On a cold day, the temperature might be 2 degrees below zero Celsius, or $-2°C$.
The **number line** is used to represent integers.
The number line is labelled with the integers. It increases from left to right and extends in both directions. This is shown by arrows on both ends of the line.
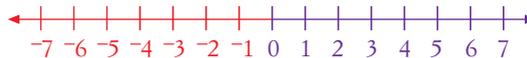
For any two different places on the number line, the integer on the right is greater than the integer on the left.
3 is greater (or more) than 1.
This can be written as 3 > 1.
Think of the > and < signs as being the wings of hunting eagles. They always open towards the *bigger* number and point to the *smaller* number (prey).
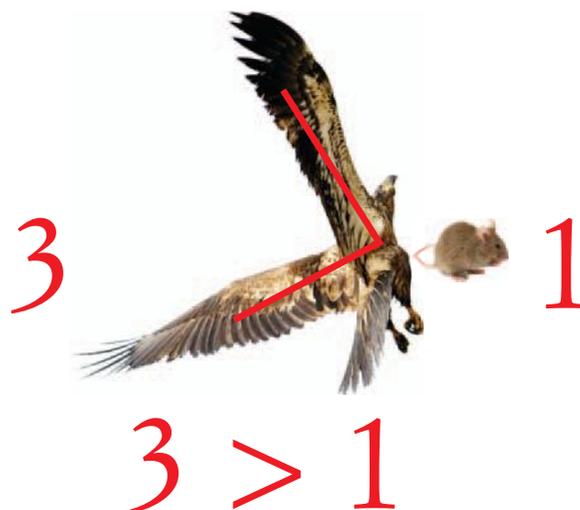
⁻4 is less than 3.
This is written as ⁻4 < 3.
It follows that:

   1 is greater than ⁻4, or 1 > ⁻4
   ⁻5 is less than ⁻3, or ⁻5 < ⁻3
   ⁻6 is more than ⁻7, or ⁻6 > ⁻7.

$$-4$$ $$3$$

$$-4 < 3$$
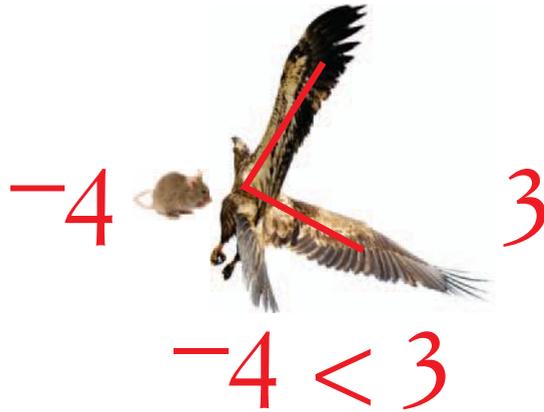
Example 13

Here is a business account showing money coming in, and money to be paid to others
(withdrawals have a minus sign). Use a calculator to find the *final balance* at the end of the day.

Balance at start: $721
                                      −$49
                                       $69
                                       −$1
                                     −$101
                                       $24
                                     −$261
Balance at the end of the day: $

Solution

We find the balance at the end of the day by adding all the money
that comes in and subtracting the money that is paid to others.

The opening balance is $721.	**$721**
The first transaction recorded is ⁻$49. This is a withdrawal since there is a minus sign in front of it. So we subtract $49.	**−$49**
The second transaction is $69. This does not have a minus sign in front of it, so it is money coming in. We add the $69.	**+$69**
The third transaction is a withdrawal of $1. This is most likely a bank fee. We subtract $1.	**−$1**
We continue adding and subtracting until we reach the end.	**−$101**
	+$24
	−$261
Evaluate.	**= $402**
Write the result	**At the end of the day, the business has $402 in the account.**

Example 14

Here are some numbers: 1, $^-2$, 3, $^-4$, 5, $^-6$, 7, $^-8$.

a Which is the biggest number? **b** Which is the smallest number?

c Rewrite the numbers in order, from smallest to biggest.

Solution

Place the numbers on a number line.
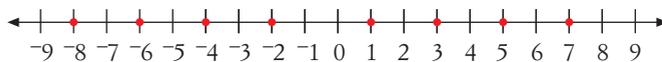
a 7 is furthest to the *right*. **7 is the biggest number.**

b $^-8$ is furthest to the *left*. **$^-$8 is the smallest number.**

c Start from left and write in order.

The numbers in order from smallest to biggest are $^-$8, $^-$6, $^-$4, $^-$2, 1, 3, 5, 7.

You can use a number line to help you add directed numbers. You can use the counting method you used when you first learned to add. For adding positive numbers, you count to the right. For adding negative numbers, you count to the left.

Example 15

Calculate each of the following:

a $^-6 + 7$ **b** $8 + {}^-6$ **c** $2 + {}^-5$

Solution

a Start from $^-6$.
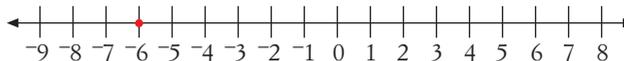
Move 7 steps to the *right* to add *positive* 7.
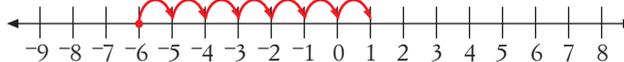
Write the answer. $^-6 + 7 = 1$

b Start from 8.
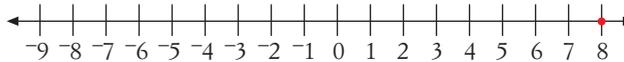
Move 6 steps to the *left* to add *negative* 6.
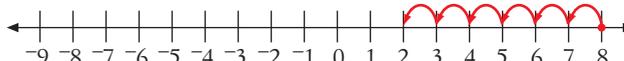
Write the answer. $8 + {}^-6 = 2$ **or** $8 - 6 = 2$

c Start from 2 and move 5 steps to the *left* to add *negative* 5.
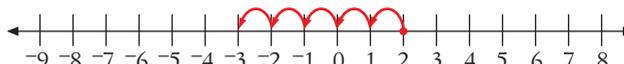
Write the answer. $2 + {}^-5 = {}^-3$ **or** $2 - 5 = {}^-3$

When you learnt to subtract, you counted back from the starting number. You went the opposite way to adding. You can do this on the number line when you subtract integers.

Example 16

Work out the following:

a $3 - 5$ **b** $^-4 - 2$ **c** $1 - ^-6$ **d** $^-7 - ^-4$

Solution

a Start from 3.
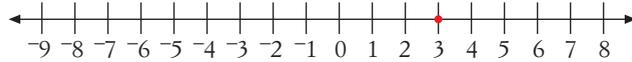
Instead of going to the right to add positive 5, you go the opposite way.
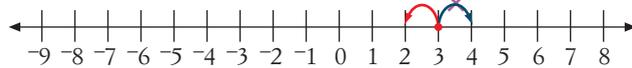
Count five steps to the *left*.
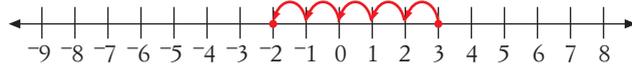
Write the answer. $3 + ^-5 = ^-2$ **or** $3 - 5 = ^-2$

b Start from $^-4$.
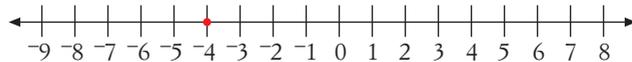
Instead of going to the right to add positive 2, you go the opposite way.
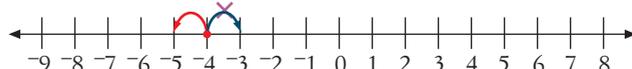
Count two steps to the left.
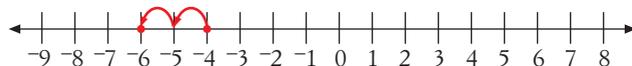
Write the answer. $^-4 - 2 = ^-6$

c Start from 1.
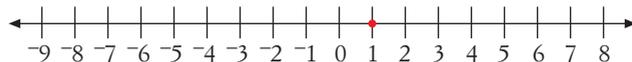
$^-6$ is how far you move in the opposite direction. You end up moving 6 to the *right*.
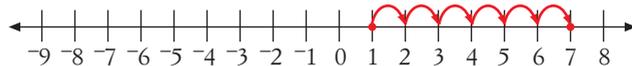
Write the answer. $1 - (^-6) = 7$ **or** $1 + 6 = 7$

d Start from $^-7$.
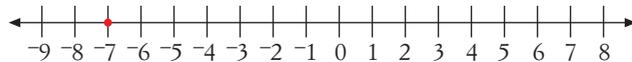
$^-4$ is how far you move in the opposite direction. You end up moving 4 to the *right*.
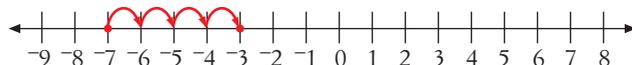
Write the answer. $^-7 - ^-4 = ^-3$ **or** $^-7 + 4 = ^-3$

Note: Negative numbers can be entered into a calculator using the sign change key +/− or (−).

Investigate: Adding and subtracting integers

You should work in pairs for this investigation.

You will need some strips of 1 cm wide paper of lengths 1–20 cm, coloured on one side and labelled with the lengths on both sides. A different coloured working sheet makes the modelling easier.

The coloured sides show negative numbers and the white sides are positive.

Use your strips to show $5 + 7 = 12$, as shown below.
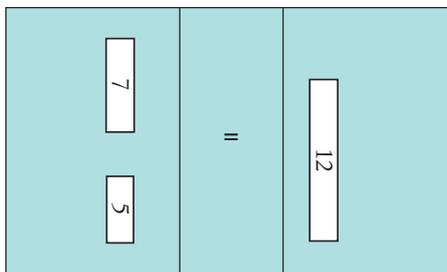
Turn your strips over.
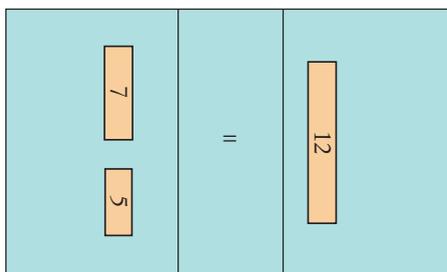
What does this show?

Now get an 8 and a $^-8$ strip. Put them on the left side.

What will they make altogether?

Follow your teacher's instructions for the rest of this investigation.

Exercise 2.4 — Integers on the number line

1 Copy these number lines, using a ruler to mark the positions evenly. Fill in the missing number for each mark.

a

b

c

d

e

9780170361910
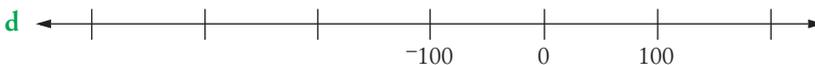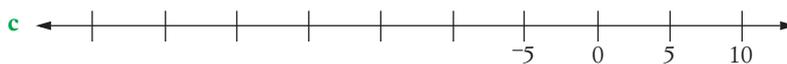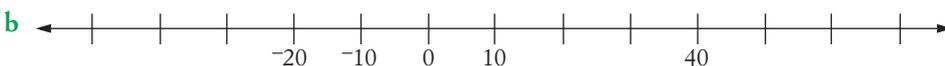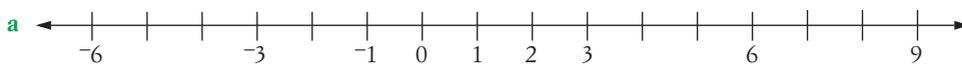
2 Copy each pair of numbers and use > or < to make true statements.

a 4 ___ 1	**b** 0 ___ 9	**c** 7 ___ ⁻7	**d** ⁻3 ___ 0
e 5 ___ ⁻11	**f** ⁻1 ___ ⁻6	**g** ⁻12 ___ 8	**h** ⁻10 ___ ⁻2
i ⁻6 ___ 5	**j** ⁻6 ___ ⁻2	**k** ⁻2 ___ ⁻6	**l** 24 ___ ⁻24
m ⁻136 ___ 36	**n** ⁻872 ___ 5	**o** ⁻120 ___ 120	**p** ⁻12 ___ ⁻78
q 17 ___ 23	**r** 8 ___ ⁻47		

3 What directed number is the opposite of:

a 6?　　**b** 1?　　**c** ⁻10?　　**d** 11?　　**e** 0?　　**f** ⁻2?

4 Write a word that is the opposite of:

a up	**b** more	**c** down	**d** left	**e** ascending
f south	**g** withdraw	**h** decrease	**i** west	

5 Rewrite these numbers in ascending order (smallest to largest).　　　*See* Example 14

a ⁻3, 2, ⁻1, 3　　　　　　**b** 5, ⁻5, 2, ⁻8, ⁻3　　　　　**c** ⁻4, ⁻6, ⁻3, ⁻10, 0
d 6, ⁻3, 4, ⁻2, ⁻5　　　　**e** ⁻48, 36, ⁻24, 8, 0, ⁻11　　**f** 15, 12, ⁻10, ⁻26, 3, ⁻2

6 Rewrite these numbers in descending order (largest to smallest).

a 4, 3, ⁻1, 5, ⁻2　　　　**(b)** ⁻4, 8, ⁻7, ⁻2, 0　　　　**c** 1, ⁻1, 4, ⁻5, ⁻11, ⁻3
d 8, ⁻4, ⁻18, 3, ⁻2　　　**e** ⁻6, ⁻15, ⁻48, ⁻3, 1　　　**f** 33, 1, ⁻100, ⁻58, ⁻36

7 The distance between ⁻3 and 1 on a number line is 4.
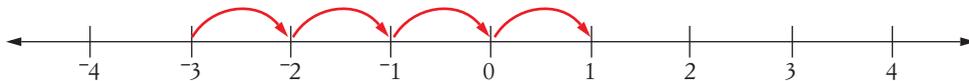
Find the distance between these pairs of numbers.

a 2 and 4	**b** ⁻2 and ⁻4	**c** 0 and 4	**d** 0 and ⁻3
e ⁻3 and 3	**f** ⁻10 and 10	**g** ⁻6 and 1	**h** ⁻1 and 6

8 Find the answers to these additions.　　　*See* Example 15

a 6 + 9	**b** 8 + ⁻9	**c** 8 + ⁻5	**d** 8 + ⁻15
e ⁻2 + 2	**f** ⁻2 + 5	**g** ⁻4 + ⁻7	**h** ⁻2 + ⁻2
i ⁻8 + ⁻18	**j** 8 + 12	**k** ⁻11 + 3	**l** ⁻10 + 10
m 7 + ⁻9	**n** ⁻13 + 4	**o** ⁻16 + 15	**p** ⁻7 + 13

9 Find the answers to these subtractions.　　　*See* Example 16

a 4 − 2	**b** 13 − 12	**c** 2 − 13	**d** 5 − ⁻1
e 7 − ⁻5	**f** ⁻3 − ⁻2	**g** ⁻5 − ⁻5	**h** ⁻5 − 5
i ⁻6 − 4	**(j)** ⁻3 − ⁻8	**k** 5 − 11	**l** 4 − ⁻2
m ⁻6 − 7	**n** 12 − 18	**o** 9 − ⁻3	**p** ⁻2 − 7

10 Copy each question and work out the answer. Use the number line if you are unsure.

a 6 + ⁻6	**b** ⁻2 + 2	**c** ⁻5 + 12	**d** ⁻11 + ⁻9
e 3 + ⁻8	**f** ⁻7 + ⁻7	**g** 0 + 70	**h** ⁻27 + 6
i ⁻4 + ⁻15	**j** 9 + ⁻1	**k** ⁻32 + 0	**l** ⁻13 + 21

11 Find the value of:

a $20 - 9$ **b** $9 - 20$ **c** $20 - {}^-9$ **d** ${}^-9 - 20$

e ${}^-4 - 7$ **f** ${}^-4 - {}^-7$ **g** $13 - {}^-3$ **h** ${}^-13 - {}^-3$

i $0 - 8$ **j** $0 - {}^-8$ **k** $5 - {}^-5$ **l** ${}^-5 - 5$

12 Use your calculator to find the value of:

a $6 - 13$ **b** $4 - {}^-2$ **c** ${}^-7 + 3$

d ${}^-5 - {}^-6$ **e** $3 + {}^-8$ **f** $8 - {}^-23$

g $11 - {}^-15$ **h** ${}^-83 + {}^-95$ **i** $12 - {}^-108$

Fluency

13 Simplify the following.

a ${}^-3 + {}^-3 + {}^-4$ **b** ${}^-3 + 3 + {}^-4$ **c** ${}^-3 + {}^-3 + 4$

d $5 + {}^-5 + {}^-7 + 7$ **e** $5 + {}^-5 + {}^-7 + {}^-7$ **f** ${}^-5 + {}^-5 + {}^-7 + 7$

g $9 + {}^-2 + 2 + {}^-8 + {}^-9$ **h** $6 - 8 + {}^-3 + 1$

14 Simplify the following.

a $1 - 9 + 9 - 10 - 1$ **b** ${}^-4 - 7 - 6 + 7 - 4$ **c** $4 - 7 - 6 - 7 - 4$

d ${}^-1 - 2 - 3 - 4 - 5 - 6$ **e** $6 - 9 + 4 - 11$ **f** $7 - 10 + 6 - 5$

g ${}^-13 + 20 - 4 + 10 - 6$ **h** ${}^-6 + 10 - 1 + 3$

15 Find the missing number in each of the following.

a $7 + \underline{\ \ \ } = 0$ **b** ${}^-3 + \underline{\ \ \ } = 0$ **c** ${}^-3 + \underline{\ \ \ } = {}^-6$

d ${}^-8 + 1 = \underline{\ \ \ }$ **e** $5 + \underline{\ \ \ } = 12$ **f** ${}^-5 + \underline{\ \ \ } = {}^-12$

g $10 + \underline{\ \ \ } = 4$ **h** ${}^-10 + \underline{\ \ \ } = {}^-4$ **i** $\underline{\ \ \ } + {}^-2 = 18$

j $\underline{\ \ \ } + {}^-2 = {}^-18$ **k** $\underline{\ \ \ } + 7 = {}^-2$ **l** $\underline{\ \ \ } + {}^-7 = {}^-2$

Problem solving

See Example 13

16 Karina had $74 in her bank account yesterday but was allowed to withdraw $121. Today, she deposited $40 into the account. What is her account balance now?

17 The heights above (and below) sea level of some well-known places are:

The Dead Sea (Jordan Valley)	${}^-397$ metres
Death Valley (California, USA)	${}^-86$ metres
Mt Kosciuszko (Australia)	2230 metres
Mt Everest (Nepal)	8840 metres

a How much higher is Mt Everest than Mt Kosciuszko?

b How much higher is Mt Kosciuszko than Death Valley?

c How much lower is the Dead Sea than Death Valley?

d The highest point on Earth is Mt Everest and the lowest point on land is the Dead Sea. What is the difference in altitude between these two points?

18 The temperature on a day in winter reached a maximum of 11°C. It dropped to a minimum overnight of ${}^-2$°C. How many degrees did it drop?

19 A man walked 16 kilometres east and then 20 kilometres west. How far is he from his starting point?

20 Anita left home and walked three kilometres west to her friend's home. Together, they then walked west for another 4 kilometres. How far from Anita's home were they then?

21 Mt Etna is an active volcano in Italy. It is 3200 m high. The lava starts 652 m below the Earth's surface and shoots to a height of 750 metres above Etna. How far does the lava travel?

22 The submarine Nemesis descends from the surface to a depth of 955 metres to inspect a shipwreck. It then ascends 800 metres to send a message. How far below the surface is it when it sends the message?

23 The graph on the right shows the profit made by Moneta Pty Ltd over 6 years.
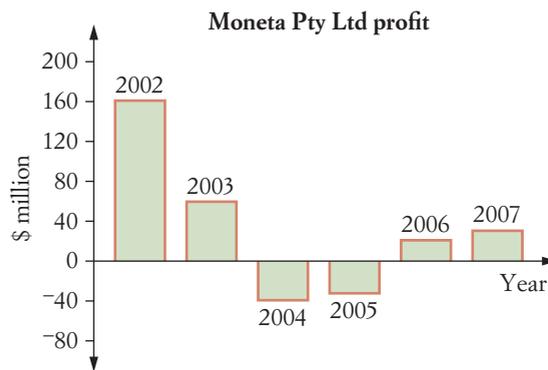
   a In which years did Moneta make a profit?

   b In which years did Moneta make a loss?

   c What was the overall change in profit between 2003 and 2004?

   d Did the company make an overall profit or loss during the six years shown? How much profit or loss did they make?

Moneta Pty Ltd profit

24 A scuba diver was swimming at a depth of 18 metres. Her diving friend was at a depth of 7 metres. What was the vertical distance between them?

2.5   Operating with integers

Investigate: Multiplying integers

1 Copy the table below onto a large piece of paper and, as a group:
   a complete the shaded section
   b complete the pattern for the first row, that is 25, 20, 15, 10, 5, 0, ⁻5, …
   c complete the pattern for the next row, that is 20, 16, 12, 8, 4, 0, ⁻4, …
   d complete the next four rows
   e complete the pattern for each column.

×	5	4	3	2	1	0	⁻1	⁻2	⁻3	⁻4	⁻5
5											
4											
3											
2											
1											
0											
⁻1											
⁻2											
⁻3											
⁻4											
⁻5											

How do the signs (positive or negative) of the numbers in the question affect the sign of the answer? Look for a pattern.

Use your completed table to help you answer the following.

a $4 \times {}^{-}3 =$ **b** ${}^{-}3 \times 5 =$ **c** ${}^{-}2 \times {}^{-}2 =$ **d** ${}^{-}5 \times {}^{-}1 =$

e $2 \times 5 =$ **f** ${}^{-}3 \times 3 =$ **g** ${}^{-}1 \times 4 =$ **h** ${}^{-}5 \times 5 =$

Important!

The rules for multiplying and dividing directed numbers are:

positive × positive = positive	positive ÷ positive = positive
positive × negative = negative	positive ÷ negative = negative
negative × positive = negative	negative ÷ positive = negative
negative × negative = positive	negative ÷ negative = positive

÷/×	+	−
+	+	−
−	−	+

When multiplying or dividing two numbers that have the same sign, the answer is positive.
When multiplying or dividing two numbers that have different signs, the answer is negative.

Division is the reverse operation of multiplication. This is why the rules for multiplying and dividing directed numbers or integers are the same.

Example 17

Find the answers for these, taking care to get the signs correct.

a $^-4 \times 2$ **b** $^-5 \times\ ^-8$ **c** $3 \times\ ^-3$

Solution

a Negative × positive = negative. $^-4 \times 2 = \ ^-8$
(Unlike signs ⟶ negative.)

b Negative × negative = positive. $^-5 \times (^-8) = 40$
(Like signs ⟶ positive.)

c Positive × negative = negative. $3 \times (^-3) = \ ^-9$
(Unlike signs ⟶ negative.)

Example 18

Find answers for these, taking care to get the signs correct.

a $^-18 \div 3$ **b** $15 \div\ ^-5$ **c** $^-20 \div\ ^-4$ **d** $\dfrac{16}{^-8}$

Solution

a Negative ÷ positive = negative. $^-18 \div 3 = \ ^-6$
(Unlike signs ⟶ negative.)

b Positive ÷ negative = negative. $15 \div (^-5) = \ ^-3$
(Unlike signs ⟶ negative.)

c Negative ÷ negative = positive. $^-20 \div (^-4) = 5$
(Like signs ⟶ positive.)

d Positive ÷ negative = negative. $\dfrac{16}{^-8} = 16 \div (^-8) = \ ^-2$
(Unlike signs ⟶ negative.)

The order of operations rules apply to integers in the same way.

First: Work out the value within any grouping symbols, starting with the innermost grouping symbols. Grouping symbols include parentheses or round brackets ( ), square brackets [ ], and braces { }.

Second: Work out multiplication or division as you come to it, going from left to right.

Third: Work out addition or subtraction as you come to it, going from left to right.

Example   19

a Find the value of $^-5 \times (5 - 7)$.

b Find the value of $^-5 + 16 \div 2 - 7$.

Solution

a Write the expression.

$^-\mathbf{5} \times (\mathbf{5} - \mathbf{7})$

Do the brackets first.

$= {}^-\mathbf{5} \times {}^-\mathbf{2}$

Negative $\times$ negative = positive.

$= \mathbf{10}$

To check, enter as (−) 5 × (5 − 7) = .

| -5×(5−7) | 10 |

b Write the expression.

$^-\mathbf{5} + \mathbf{16} \div \mathbf{2} - \mathbf{7}$

Do division before addition or subtraction.

$= {}^-\mathbf{5} + (\mathbf{16} \div \mathbf{2}) - \mathbf{7}$

$= {}^-\mathbf{5} + \mathbf{8} - \mathbf{7}$

Add then subtract from left to right.

$= {}^-\mathbf{4}$

To check, enter as (−) 5 + 16 ÷ 2 − 7 = .

| -5+16÷2−7 | -4 |

Exercise 2.5   Operating with integers

1 Find the answer for each of these.

a $^-3 \times 6$ **b** $6 \times {}^-4$ **c** $^-3 \times {}^-7$ **d** $4 \times {}^-8$

e $^-7 \times {}^-9$ **f** $^-9 \times 5$ **g** 7×6 **h** $^-7 \times {}^-6$

i $3 \times {}^-8$ **j** $^-9 \times 1$ **k** $^-1 \times {}^-1$ **l** $^-9 \times 6$

m $5 \times {}^-2$ **n** $^-7 \times 6$ **o** $^-11 \times {}^-4$ **p** $^-9 \times 9$

q $^-12 \times {}^-5$ **r** $20 \times {}^-3$ **s** $^-10 \times {}^-10$ **t** $^-7 \times {}^-8$

2 Find the answers for the following divisions.

a $15 \div {}^-5$ **b** $^-8 \div 4$ **c** $^-12 \div {}^-3$ **d** $6 \div {}^-2$

e $27 \div {}^-3$ **f** $^-18 \div {}^-6$ **g** $24 \div {}^-6$ **h** $100 \div {}^-2$

i $^-20 \div 5$ **j** $^-51 \div {}^-3$ **k** $45 \div {}^-5$ **l** $^-72 \div 6$

m $^-80 \div {}^-5$ **n** $^-10 \div 2$ **o** $98 \div {}^-7$ **p** $48 \div {}^-8$

q $^-27 \div 9$ **r** $^-80 \div {}^-4$

3 Find the answers for these divisions.

a $\dfrac{12}{^-2}$ **b** $\dfrac{100}{^-50}$ **c** $\dfrac{^-18}{^-3}$ **d** $\dfrac{^-66}{^-2}$

e $\dfrac{^-126}{6}$ **f** $\dfrac{45}{^-5}$ **g** $\dfrac{100}{5}$ **h** $\dfrac{^-81}{^-9}$

4 Find the answer for each of these.

a $^-6 \times {}^-4$ **b** 2×15 **c** $^-9 \times {}^-9$ **d** $11 \times {}^-3$

e $16 \times {}^-1$ **f** $^-3 \times 81$ **g** 63×300 **h** $^-104 \times {}^-40$

i $^-99 \times 1000$ **j** $^-56 \times {}^-100$ **k** $^-1 \times {}^-1 \times {}^-1$ **l** $13 \times {}^-4$

m $^-300 \times 100$ **n** $^-4 \times 4$ **o** $^-9 \times {}^-4$ **p** $^-6 \times 3 \times {}^-5$

q $8 \times {}^-70 \times {}^-200$ **r** $^-13 \times 100$ **s** 0×4 **t** $12 \times {}^-3 \times {}^-10$

5 Find the value of:

a 4^2 **b** $(^-3)^2$ **c** $(^-7)^2$
d $(^-8)^2 \times (^-2)$ **e** 5^3 **f** $(^-2)^3$
g $^-4 \times (^-1)^3$ **h** $(^-2)^3 \times (^-5)^2$ **i** $(^-3)^3 \times (^-2)^3$

6 Use your calculator to find the answer for each of the following.

a $^-8 \times 6$ **b** $^-11 \times ^-4$ **c** $^-15 \times ^-16$
d $13 \times ^-5$ **e** $38 \times ^-11$ **f** $^-25 \times ^-12$
g $(^-5)^2$ **h** $(^-8)^3$ **i** $^-15 \times (^-7)^2$

7 This question involves both division and multiplication. Remember to work from left to right, and find the answers.

a $5 \times ^-4$ **b** $6 \times ^-3 \div 9$ **c** $7 \times 6 \div ^-2$
d $^-3 \div ^-1 \times 1$ **e** $24 \div ^-6 \times 50$ **f** $^-56 \div ^-8 \times 4$
g $^-100 \div ^-20 \div ^-5 \times 23$ **h** $^-27 \times ^-9 \div 9$ **i** $3 \times 8 \div ^-4 \div ^-2$
j $^-14 \div 2 \times 3 \times ^-2$ **k** $^-16 \times 5 \div 4 \div ^-5$ **l** $13 \times 2 \times ^-5 \div ^-10$
m $500 \div ^-10 \times 4$ **n** $^-14 \times ^-5 \div ^-7 \div ^-2$

8 Use your calculator to answer the following.

a $45 \div ^-9 \times 2$ **b** $^-56 \times ^-4 \div 2$ **c** $^-200 \div ^-100 \div 2$
d $32 \times ^-3 \times ^-15$ **e** $^-40 \times ^-3 \div ^-20$ **f** $78 \times ^-2 \times ^-5 \div ^-39$
g $^-544 \times ^-15 \times ^-6$ **h** $48 \div 3 \times ^-12$

9 Find the answers to the following.

a $6 - 8$ **b** $^-3 + 5$ **c** $^-8 - 4$ **d** $^-7 + 9$
e $^-2 - 12$ **f** $10 - 35$ **g** $^-8 - 13$ **h** $19 - 22$
i $4 - ^-7$ **j** $^-3 - ^-11$ **k** $^-15 + 9$ **l** $^-12 - ^-3$
m $9 - 12$ **n** $^-13 - 10 - ^-1$ **o** $4 - 6 + 2 + ^-1$ **p** $9 - 12 + 7 - 9$

10 Find the answers to the following.

a $^-3 \times 6$ **b** $^-2 \times ^-6$ **c** $18 \div 2$
d $^-16 \div 8$ **e** $^-25 \times ^-2$ **f** $^-30 \div ^-6$
g $12 \times ^-4$ **h** $48 \div ^-6$ **i** $^-100 \div ^-10$
j $21 \div ^-3 \div ^-7$ **k** $^-2 \times 5 \times ^-3$ **l** $15 \div ^-3 \times ^-2$
m $^-32 \div ^-4 \times ^-2$ **n** $6 \div ^-3 \times ^-4$ **o** $^-24 \times 2 \div ^-3 \div ^-4$

11 Find the value of each of the following.

a $^-5 \times 2 + 3$ **b** $^-5 \times (2 + 3)$ **c** $3 \times 6 \times ^-2$ **d** $12 - 8 \div 4$
e $12 - 8 \div ^-4$ **f** $^-3 \times (9 - 10)$ **g** $21 \div ^-7 + 8$ **h** $[^-6 - ^-4] \times ^-4$
i $(^-6 - 2) \div ^-4$ **j** $^-3 \div 3 + 9$ **k** $12 \times (6 - 7) + 1$ **l** $^-4 \times 7 \div (^-7 - 7)$

12 Find the value of:

a $^-5 \times 6 - 8$ **b** $^-5 \times (6 - 8)$ **c** $[18 - ^-3] \times 4$
d $5 + ^-8 \times ^-7 + 72$ **e** $81 - ^-3 \times 12$ **f** $102 \times (^-5 + 3)$
g $6 \times ^-2 + ^-4$ **h** $^-7 - 5 + 9$ **i** $^-19 + 4 \times 6$
j $[^-6 - ^-9] \times 2$ **k** $(^-6 - 3) \div ^-3$ **l** $25 \div ^-5 + 6$
m $^-8 + ^-9 \times ^-10 \div ^-5$ **n** $6 - 9 + 5 - 11$ **o** $^-15 + 19 \times 2 - 8$
p $6 - 13 + 14 - 3 \times ^-5$ **q** $^-3 - 10 \div 2 + 19$ **r** $3 - 10 \times ^-2 - ^-19$

13 Use your calculator to find the value of:

a $^-2 \times 15 + 11$ **b** $6 \times (13 - 27)$ **c** $^-14 - 6 \times ^-8$
d $25 \div ^-5 \div ^-5$ **e** $[12 - ^-10] \times ^-3 + 6$ **f** $^-3 \times ^-8 \div ^-4 + ^-11$
g $9 - 19 + 36 - 8 \times ^-6$ **h** $^-12 - 30 \div ^-2 + 58$ **i** $6 + 4 \times ^-5 \div ^-2$

14 Follow the arrows along all the stepping stones and multiply by the next number as you go. Is your answer positive or negative at the finish?
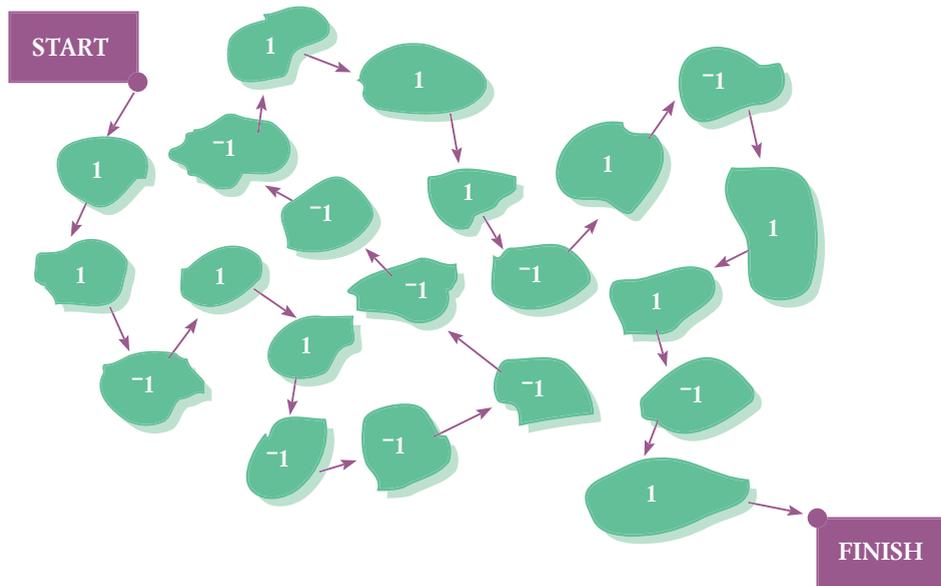
15 A Year 7 student completed these 10 questions. Mark them, and correct the mistakes.

a $^-3 \times 5 = ^-15$ **b** $6 - 8 = 2$ **c** $^-6 - 8 = ^-2$

d $^-11 + 3 = ^-8$ **e** $12 \div ^-6 = ^-2$ **f** $(^-2 + 6) \times 2 = ^-8$

g $^-7 - 6 + 10 = 3$ **h** $(^-12 - 3) \div ^-3 = 3$ **i** $12 - 3 \times 5 = 45$

j $2 - 9 \times ^-3 - ^-18 = 39$

2.6   Special numbers

In this chapter we have already explored counting numbers, whole numbers and integers. In mathematics there are many other fascinating number types.

Investigate: Number types

Figurate numbers

Numbers formed from geometric shapes, such as triangular or square numbers, are called *figurate numbers*. There are many figurate number patterns.
Investigate the pentagonal numbers.
Investigate the hexagonal numbers.
What are the names of the other types of figurate numbers?

More types of numbers

Investigate one or more of the following types of numbers and find out the relationships and patterns in them. You may find the Internet useful. Prepare a short talk for the class on your topic.

- Amicable numbers
- Perfect numbers
- The golden ratio/rectangle
- Irrational numbers
- Pythagorean triads
- Binary, octal and hexadecimal numbers
- Factorial numbers, for example the meaning of 5!

Weblinks
Binary numbers

Weblink
The golden ratio

Weblink
Number spirals

Important!

The **factors** of a number are those whole numbers that divide exactly into it.
The **highest common factor** (**HCF**) of two or more numbers is the largest factor that is common to all those numbers.

Technology
Excel spreadsheet:
Developing number patterns

MAT07NACT00016

Example 20

What are the factors of 18?

Solution

Think of all the possible of ways of multiplying to get 18.

$$1 \times 18 = 18$$
$$2 \times 9 = 18$$
$$3 \times 6 = 18$$

List the factors of 18. **1, 2, 3, 6, 9 and 18**

Note: 1 is a factor of every number.

Worksheet
Whole numbers 9
MAT07NAWK00012

Worksheet
Whole numbers 10
MAT07NAWK00013

Example 21

Find the highest common factor (HCF) of 16 and 40.

Solution

Find the factors of 16.

$$1 \times 16 = 16$$
$$2 \times 8 = 16$$
$$4 \times 4 = 16$$

List the factors of 16. **1, 2, 4, 8 and 16.**

Find the factors of 40.

$$1 \times 40 = 40$$
$$2 \times 20 = 40$$
$$4 \times 10 = 40$$
$$5 \times 8 = 40$$

List the factors of 40. **1, 2, 4, 5, 8, 10, 20 and 40.**

Consider the common factors of 16 and 40. **1, 2, 4 and 8.**

The largest of these is the HCF. **The highest common factor is 8.**

Investigate: Tests for divisibility

It is useful to know if a number is divisible by another number.

Is 75 divisible by 3? Does 3 divide into 75 with no remainder?

You already know that even numbers are divisible by 2.

You should also know that numbers are divisible by 3 if the sum of their digits is divisible by 3.

Hence 75 is divisible by 3 because $7 + 5 = 12$ and $12 \div 3 = 4$. ($75 \div 3 = 25$ with no remainder)

A number is divisible by 4 if its last two digits are divisible by 4.

Research and explore divisibility of a number by 5, 6, 8, 9 and 10.

Investigate the divisibility tests for a number that is divided by 7 or 11.

Important!

A **prime number** has only two factors: 1 and itself.

The prime numbers are {2, 3, 5, 7, 11, 13, 17, 19, 23, …}

A **composite number** has more than two factors.

The composite numbers are {4, 6, 8, 9, 10, 12, 14, 15, 16, …}

1 is neither prime nor composite (it has only one factor).

A **factor tree** is a diagram that lists the prime factors of a number. All composite numbers can be written as a product of its prime factors. This is called prime decomposition.

Example   22

Write the prime decomposition of 24.

Solution

3 and 8 are factors of 24

2 and 4 are factors of 8
3 is prime

2 is a factor of 4
2 is prime — stop

Write 24 as a product of prime factors. $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$$
\begin{array}{c}
24 \\
3 \times 8 \\
3 \times 2 \times 4 \\
3 \times 2 \times 2 \times 2
\end{array}
$$
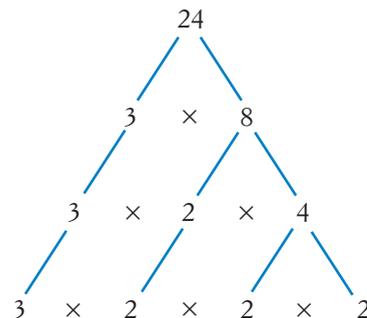
Example 23

Use a factor tree to find the highest common factor (HCF) of 1960 and 2000.

Solution

10 and 196 are factors of 1960.
2 and 5 are factors of 10 and both are prime — stop.
196 is divisible by 4. Dividing 196 by 4, we get 49.
So 4 and 49 are factors of 196.
2 is a factor of 4
2 is prime — stop
7 is a factor of 49
7 is prime — stop

Write 1960 as a product of all its prime factors. $1960 = 2 \times 2 \times 2 \times 5 \times 7 \times 7$
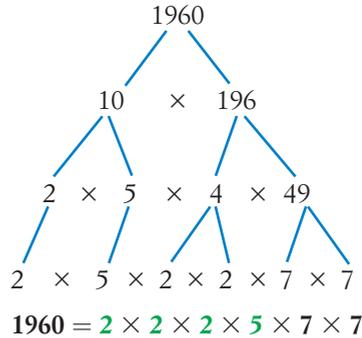
2 and 1000 are factors of 2000.
2 is already prime so we only need to find the factors of 1000.
100 and 10 are factors of 1000.

100 is 10 multiplied by itself.

2 and 5 are factors of 10 and both are prime — stop.

Write 2000 as a product of all its prime factors. $2000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

Both numbers contain $2 \times 2 \times 2 \times 5$.   **The HCF is $2 \times 2 \times 2 \times 5 = 40$.**
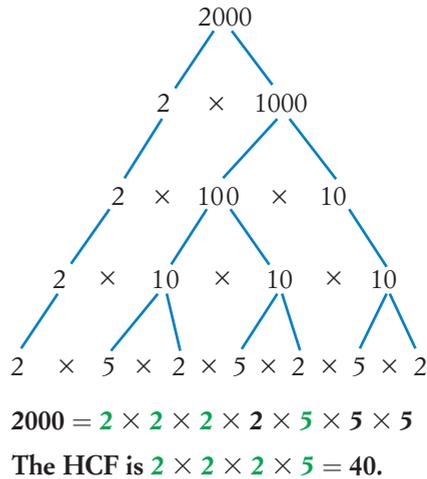
Example 24

Write 648 as a product of its prime factors, using **index notation** (powers).

Solution

2 and 324 are factors of 648.
324 is divisible by 4.
Dividing 324 by 4, we get 81. So 4 and 81 are factors of 324.

4 is 2 multiplied by itself.
81 is 9 multiplied by itself.

9 is 3 squared.
3 is prime — stop

Write 648 as a product of all its prime factors. $648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^3 \times 3^4$
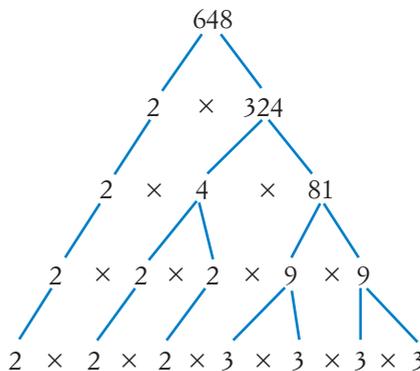
Exercise 2.6 Special numbers

1 In each of these pairs, is the smaller number a factor of the larger number?

   a 8, 24   **b** 3, 39   **c** 4, 42   **d** 9, 45   **e** 8, 54

   f 7, 91   **g** 7, 133   **h** 6, 48   **i** 5, 57   **j** 11, 143

See Example 20

2 List all the factors of:

Extra questions

Exercise 2.6

MAT07NAEQ00014

   a 16   **b** 21   **c** 24   **d** 36   **e** 35

   f 48   **g** 52   **h** 80   **i** 112   **j** 144

   k 28   **l** 100   **m** 45   **n** 200   **o** 363

3 Find the common factors for each of these pairs of numbers.

   a 2, 4   **b** 9, 6   **c** 6, 14   **d** 8, 12

   e 50, 150 **f** 46, 69   **g** 10, 15   **h** 12, 16

   i 30, 20   **j** 18, 24   **k** 60, 90   **l** 39, 26

   m 45, 15   **n** 36, 39   **o** 27, 64   **p** 350, 210

Worked solutions

Exercise 2.6

MAT07NAWS00011

4 Find the common factors for each of these sets of numbers.

   a 2, 4, 6   **b** 10, 50, 60   **c** 22, 33, 121

   d 24, 36, 144   **e** 6, 9, 12   **f** 16, 24, 40, 56

   g 28, 70, 42, 98   **h** 30, 90, 75, 135   **i** 50, 60, 90, 120

See Example 21

5 Find the highest common factor for each of these pairs of numbers.

   a 12 and 60   **b** 33 and 22   **c** 132 and 60

   d 9 and 21   **e** 45 and 78   **f** 64 and 144

   g 16 and 12   **h** 8 and 14   **i** 50 and 150

   j 18 and 24   **k** 48 and 72   **l** 15 and 25

   m 35 and 21   **n** 45 and 18   **o** 75 and 125

6 a List the prime numbers between 36 and 50.

   b List the composite numbers between 65 and 80.

   c List the prime numbers less than 20.

   d List the composite numbers larger than 30 but less than 47.

See Example 22

7 Use factor trees to express each of these numbers as a product of its prime factors.

   a 8   **b** 63   **c** 45   **d** 36   **e** 51

   f 49   **g** 90   **h** 27   **i** 130   **j** 200

   k 275   **l** 342   **m** 1250   **n** 1020   **o** 837

See Example 23

8 Use factor trees to find the highest common factor of each of these pairs of numbers.

   a 324 and 486   **b** 6000 and 1260   **c** 2475 and 3375

   d 4900 and 1960   **e** 4950 and 1530   **f** 1404 and 900

See Example 24

9 Use factor trees to write prime decompositions of each of the following.

   a 18   **b** 20   **c** 45

   d 72   **e** 98   **f** 196

10 Triangular numbers are shown in the diagram below.

| 1 | 3 | 6 | 10 |

a Why are they called 'triangular numbers'?

b Work out all the triangular numbers less than 100.

c Complete four more lines of this pattern:

$1 = 1$

$1 + 2 = 3$

$1 + 2 + 3 = 6$

$1 + 2 + 3 + 4 = 10$

d Describe how the pattern in part **c** works.

e Use what you have worked out to help you find the 100th triangular number.

(*Hint*: Do you know a quick way to add up all the numbers from 1 to 100?)

11 Square numbers are shown in the diagram below.

| 1 | 4 | 9 | 16 |

a Why are these called 'square numbers'?

b Work out all the square numbers up to 100.

c Complete four more lines of this pattern:

$1 = 1$

$1 + 3 = 4$

$1 + 3 + 5 = 9$

$1 + 3 + 5 + 7 = 16$
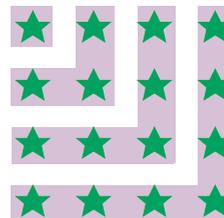
d Describe how the pattern works.

e Work out another pattern to help you find the square numbers. What is the 50th square number?

f Complete four more lines of these patterns:

i
$$1 = 1^2$$
$$1 + 2 + 1 = 2^2$$
$$1 + 2 + 3 + 2 + 1 = 3^2$$

ii
$$2^2 = 1^2 + (1 + 2)$$
$$3^2 = 2^2 + (2 + 3)$$
$$4^2 = 3^2 + (3 + 4)$$

g Each square number is said to be the sum of two consecutive triangular numbers.

$$3 \qquad + \qquad 6 \qquad = \qquad 9$$

Show that this is true for the square numbers up to 100.

h Find two numbers that are both triangular numbers and square numbers.

12 Use a factor tree to find the value of.

a $\sqrt{196}$ **b** $\sqrt{784}$ **c** $\sqrt{3136}$

13 Use a factor tree to find the value of.

a $\sqrt[3]{10\,648}$ **b** $\sqrt[3]{4913}$ **c** $\sqrt[3]{9261}$

Reasoning

14 'Every whole number has at least two factors.' Is this true or false? Why?

15 Look up other meanings for the word 'composite'. Suggest why this word is used the way it is in mathematics.

16 Leonardo **Fibonacci** was an Italian mathematician who lived in the early 13th century. He discovered this pattern when studying the breeding habits of rabbits:
1, 1, 2, 3, 5, 8, 13, 21, …
The diagram below illustrates this. The vertical arrows labelled 'Birth' indicate the new offspring of a pair of rabbits every two months. The unlabelled arrows indicate the same pair of rabbits. After each month, the number of pairs is a term in Fibonacci's pattern.

a How is the Fibonacci pattern formed?

b Add five more lines to this pattern:

1

1

$1 + 1 = 2$

$1 + 2 = 3$

$2 + 3 = 5$

$3 + 5 = 8$

c Write the first 20 Fibonacci numbers.

  i Write every *third* Fibonacci number, beginning with 2. What number divides evenly into all these numbers?

  ii Write every *fourth* Fibonacci number, beginning with 3. What number divides evenly into all these numbers?

  iii Write every *fifth* Fibonacci number, beginning with 5. What number divides evenly into all these numbers?

d **i** Find any triangular numbers in the Fibonacci numbers up to 100.

  ii Find any square numbers in the Fibonacci numbers up to 100.

e Pairs of Fibonacci numbers are found by counting along the spirals on pine cones. Investigate how and where else Fibonacci numbers occur in nature.

17 Blaise Pascal, a French mathematician who lived in the 17th century, studied a triangle of numbers known to the Chinese as the Yanghui triangle. Each row of the triangle is created using the numbers in the row above it. The triangle is known as **Pascal's triangle**. The first seven rows are shown on the right.

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
1   5  10  10   5   1
1  6  15  20  15  6   1
```
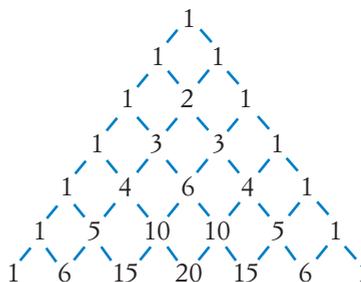
Weblink

Pascal's triangle

TLF Learning object

Pascal's triangle
(L3538)

a Complete the next four rows of Pascal's triangle.

b Describe how the pattern works.

c Add each row in Pascal's triangle. What do you notice?

d The diagonals in Pascal's triangle produce some interesting patterns. Write the triangular numbers using Pascal's triangle.

e We can even find Fibonacci numbers in this pattern. Rewrite the triangle above as a right-angled triangle.

```
1
1   1
1   2   1
1   3   3   1
1   4   6   4   1
1   5  10  10   5   1
```
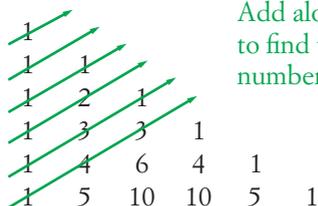
Add along the arrows to find the Fibonacci numbers.

18 A **palindrome** is a word, number or sentence that reads the same forward and backward. The following word, number and sentence are all palindromes:

noon   151   *Able was I ere I saw Elba* (Napoleon Bonaparte)

a Select the palindromes from these numbers.

447   373   656   281   37   22   899   191   797   516

b Find the numbers between 1000 and 2000 that are palindromes.

c The following steps change any number into a palindrome:

Choose any number to start with $\longrightarrow$ 64
Reverse the digits and add   +46
     110
Reverse the digits and add   +011
Repeat until you get a palindrome.   121

Find out how many steps it takes to form a palindrome from each of these numbers.

  i 26   **ii** 28   **iii** 47   **iv** 75   **v** 149
vi 273   **vii** 1756   **viii** 2379   **ix** 4021

d List some other words and place names that are palindromes.

- **Whole numbers** are the numbers {0, 1, 2, 3, 4, 5, …}.
- Counting numbers or natural numbers are the numbers {1, 2, 3, 4, 5, …}.
- Integers are the numbers {…, ⁻5, ⁻4, ⁻3, ⁻2, ⁻1, 0, 1, 2, 3, 4, 5, …}.
- Our decimal system is a **place value** system of numbers. The value of each digit depends on its place. In writing numbers and giving their names, the digits are grouped in threes. Each group of three digits has hundreds, tens and units. The lowest three place values have no other name, but then the groups go up as thousands, millions, billions, trillions, quadrillions and so on.
- The order of mixed operations is:
    1. Work out brackets first, from innermost outwards and from left to right.
    2. Work out multiplications and divisions next, from left to right.
    3. Work out addition and subtraction last, from left to right.
- A repeated multiplication may be written as a **power**. The number multiplied is written once as the **base**. The number of times it is in the multiplication is written as the small **index** (**exponent**) at the top right. The **extended form** of a power has the multiplication written out in full. For example, $4^3 = 4 \times 4 \times 4$.
- A **square** is a power with index 2, while a **cube** has index 3.
- The **square root** of a number is written like $\sqrt{49}$. It is the new number that must be squared to give the original number. $\sqrt{49} = 7$ because $7^2 = 7 \times 7 = 49$.
- The **cube root** of a number is written like $\sqrt[3]{216}$. It is the new number that must be cubed to give the original number. $\sqrt[3]{216} = 6$ because $6^3 = 6 \times 6 \times 6 = 216$.
- Positive numbers and negative numbers together are called **directed numbers**. Directed numbers show both direction and magnitude (size). For example, 8 or $^+8$ means 8 units in the positive direction, while ($^-4$) means 4 units in the negative direction.
- A **number line** can be used to represent integers. The number line is labelled with the integers in increasing order from left to right. The line extends in both directions and this is shown by the arrows on both of its ends.

  The rules for multiplying and dividing directed numbers are:

positive × positive = positive	positive ÷ positive = positive
positive × negative = negative	positive ÷ negative = negative
negative × positive = negative	negative ÷ positive = negative
negative × negative = positive	negative ÷ negative = positive

- A **factor** is a value that divides evenly into a given number, for example, 3 is a factor of 15. A **factor tree** is a diagram that lists the prime factors of a number.
- A **prime number** is a number with only two factors, 1 and the number itself. A **composite number** is a number with more than two factors. For example 7 is a prime number with factors 1 and 7. (1 × 7 = 7). 12 is a composite number with factors 1, 2, 3, 4, 6 and 12. (1 × 12 = 12), (2 × 6 = 12) and (3 × 4 = 12).
- The prime decomposition of a number shows it as a product of (powers of) prime numbers.

Chapter 2 review

1 Write each of the following using numerals.

  a six hundred and twelve

  b nine hundred and forty-three

  c five thousand, four hundred and ninety-nine

  d six thousand and two

  e nine million, seven hundred and fifty thousand and seventy-six

See Example 1

2 What is the place value of the digit 4 in:

  a 47?   **b** 3024?

  c 8412?   **d** 146 235?

See Example 2

3 Write each of these in extended form.

  a 19   **b** 283

  c 665   **d** 42 891

See Examples 5-7

4 Find the value of each of these.

  a $16 - (5 \times 3)$ **b** $6 + 5 \times 3$ **c** $30 - 10 \div 2$

  d $(16 \div 2) + (18 - 11)$ **e** $(320 - 120) \times 12$ **f** $35 \times (19 - 17) \times 20$

  g $(36 - 14) \times 2 \div 4$ **h** $36 - (28 - 13) + (20 - 3 \times 5)$

  i $(256 - 120) \div 17$ **j** $[394 + (30 \div 5)] \div (440 \div 11)$

  k $36 - (4 \times 3) \div (35 - 23)$ **l** $2\,000\,000 - [(300 \times 100) + 1]$

See Example 9

5 Work out each of the following without using a calculator.

  a 2^5 **b** 9^2 **c** 5^4 **d** $\sqrt{49}$ **e** $\sqrt{400}$ **f** $\sqrt{256}$

6 Copy and complete these number patterns.

  a $^-2, \, ^-1, \, 0,$ ___, ___, ___   **b** $1, \, 0, \, ^-1,$ ___, ___, ___

  c $^-6, \, ^-8,$ ___, ___, ___   **d** $2, \, 0, \, ^-2,$ ___, ___, ___

  e $1, \, ^-2, \, 3,$ ___, ___, ___   **f** $^-1, \, ^-3, \, ^-5,$ ___, ___, ___

See Example 14

7 Answer true (T) or false (F) for each of these.

  a $^-4 > 2$ **b** $^-2 < \, ^-3$ **c** $^-1 > 1$ **d** $2 > \, ^-1$

  e $^-11 < \, ^-10$ **f** $^-3 > \, ^-4$ **g** $^-8 < 11$ **h** $^-4 > \, ^-3$

  i $5 < \, ^-4$ **j** $^-16 < \, ^-17$ **k** $1 > \, ^-5$ **l** $^-9 > \, ^-2$

8 Find the opposite of each of these integers.

  a $^-2$ **b** $^-23$ **c** 56 **d** $^-10$ **e** 8 **f** 0

See Examples 15, 16

9 Evaluate the following.

  a $3 + \, ^-4$ **b** $3 - \, ^-4$ **c** $^-8 + \, ^-2$

  d $10 + \, ^-13$ **e** $^-5 - \, ^-9$ **f** $7 + 11$

  g $^-4 - 8$ **h** $^-5 - \, ^-5$ **i** $^-7 - 8$

  j $^-3 + 6$ **k** $^-5 + \, ^-4 + \, ^-2$ **l** $^-8 + 5 - 4$

  m $8 - 6 - \, ^-4$ **n** $1 - 3 + 0$ **o** $^-3 - 14 - 5$

  p $^-7 + 6 + \, ^-7$ **q** $4 + 6 + 7$ **r** $3 - 8 + \, ^-6$

10 Evaluate the following. *See* Example 17

   a $6 \times {}^-3$ **b** ${}^-8 \times {}^-6$ **c** ${}^-2 \times 5$

   d ${}^-100 \times 4$ **e** $7 \times {}^-9$ **f** 6×8

   g ${}^-6 \times {}^-4 \times {}^-1$ **h** ${}^-2 \times 3 \times 7$ **i** ${}^-4 \times 2 \times {}^-5$

11 Evaluate the following. *See* Example 18

   a ${}^-12 \div {}^-4$ **b** $15 \div {}^-5$ **c** ${}^-6 \div {}^-6$

   d $16 \div {}^-4$ **e** ${}^-24 \div 12$ **f** ${}^-1000 \div 10$

   g $24 \div {}^-6 \div {}^-2$ **h** ${}^-36 \div 3 \div 4$ **i** ${}^-64 \div 4 \div 2$

12 Write the next three numbers in each of these patterns.

   a $1, 3, 5, 7, \ldots$ **b** $2, 4, 6, 8, \ldots$ **c** $1, 3, 6, 10, \ldots$

   d $1, 4, 9, 16, \ldots$ **e** $1, 1, 2, 3, 5, 8, \ldots$ **f** $60, 55, 50, 45, \ldots$

13 In question **12**, which set of numbers are the:

   a square numbers?   **b** triangular numbers?   **c** Fibonacci numbers?

14 **a** Find the common factors of 42 and 60. *See* Example 21

   b Find the highest common factor of 20 and 48.

   c Find the highest common factor of 36 and 84.

15 Find the prime numbers from:

   27   93   6   29   19   39   96   31   57   2   51

   100   65   37   17   13   1   67   73   83   89   27

16 Use a calculator to determine each of the following, rounding to 2 decimal places if necessary. *See* Example 10

   a 13^4 **b** 126^3 **c** $\sqrt{130}$ **d** $\sqrt[3]{1742}$

Fluency

17 Arrange the numbers in each of these sets in order, from largest to smallest.

   a $16, 21, 38, 19, 14$

   b $89, 36, 101, 98, 88$

   c $2356, 2534, 2635, 2300, 2533$

   d $12\,391, 12\,913, 11\,990, 11\,391, 12\,300$

18 Write whether each of these is true (T) or false (F).

   a $5 < 8$ **b** $7 > 2 + 4$

   c $52 \neq 10$ **d** $6 \times 7 \leq 43$

   e $23 \geq 5 + 1$ **f** $\sqrt{36} = 6$

19 Write 76 725 as a place value decomposition using powers. *See* Example 11

20 Evaluate the following. *See* Example 19

   a ${}^-6 + ({}^-3) \times 2$ **b** $4 - 3 \times 6$

   c ${}^-12 \div 4 - 10$ **d** ${}^-3 \times 5 + 6 \div ({}^-2)$

   e $30 \div ({}^-6) + 5$ **f** ${}^-2 \times 7 + 15 \div ({}^-3)$

   g ${}^-12 + ({}^-3) \times ({}^-7) - 2$ **h** $14 + 7 \times ({}^-3) + 5$

21 Write the following.

    a a triangular number between 10 and 20   **b** the highest Fibonacci number below 40
    c the next palindrome after 2002   **d** the next prime number after 29
    e the first five composite numbers   **f** the square number between 40 and 50

See Example 22   **22** Use factor trees to write the following as products of their prime factors.

    a 24   **b** 60   **c** 27   **d** 200
    e 36   **f** 45   **g** 72   **h** 144

See Example 24   **23** Use the factor trees you drew for question **22** to write the following as prime number decompositions in index notation.

    a 24   **b** 60   **c** 27   **d** 200
    e 36   **f** 45   **g** 72   **h** 144

24 Write the next three lines of Pascal's triangle as shown on the right.

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
```
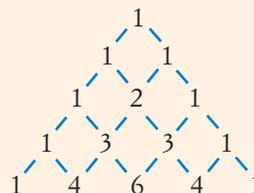
See Example 14   **25** Show on separate number lines.

    a $^-6, 2, {}^-3, 3, {}^-1$ **b** Whole numbers n so $^-3 \le n < 4$

26 Use grouping symbols and operations signs $(+, -, \times, \div)$ to make each of these true.

    a $7 \, ? \, 3 \, ? \, 1 = 9$ **b** $10 \, ? \, 5 \, ? \, 5 = 10$
    c $8 \, ? \, 3 \, ? \, 6 \, ? \, 2 = 8$ **d** $28 \, ? \, 4 \, ? \, 7 = 49$
    e $6 \, ? \, 4 \, ? \, 3 \, ? \, 5 = 40$ **f** $19 \, ? \, 1 \, ? \, 5 \, ? \, 3 \, ? \, 1 = 0$

27 The heights of several places are given as $^-120$ m, 85 m and 39 m. Arrange them in order from the lowest to the highest points.

28 Between which two consecutive whole numbers does $\sqrt{55}$ lie?

Problem solving

29 **a** Mary has \$45, Joan has \$63 and Maria has \$52 to spend at a theme park. How much do the three friends have altogether?

See Example 3

    b At the end of the day they are left with \$7, \$9 and \$16 respectively. Altogether, how much did they spend?

See Example 13   **30** The minimum overnight temperature in Goulburn was $^-5$°C. During the day, it rose to 11°C. By how much did the temperature rise?

31 The match fee for a game of Rugby in the under-14s is \$8. Hassan played 14 games in the season. How much did he pay during the season?

32 It takes about 17 minutes for an officer in the Australian Tax Office to process a handwritten tax return. How much time would it take him to process 347 tax returns?

See Example 4   **33** A book is 14 mm thick. How many books would be needed to make a stack 120 cm high?

34 Use factor trees to find:

    a $\sqrt{225}$ **b** $\sqrt{256}$ **c** $\sqrt{1764}$ **d** $\sqrt[3]{5832}$

Reasoning

35 A shark cruising at a depth of 1.5 m spotted a large cod below it. The shark dived 4.2 m and the cod tried to avoid the shark by rising 0.8 m, but the shark caught it. What depth was the cod originally at?

36 A landscaping company estimates that about 12 paving bricks are wasted from every pallet of 250 bricks. Roughly how many bricks will be wasted for a large paving job that is estimated to need 7780 bricks?

37 To find your body mass index, you square your mass in kg and divide by your height in cm. What is the body mass index of a 78 kg man of height 170 cm?

38 Here is the test paper done by a Year 7 student. Mark the test out of 10 and correct any incorrect answers.

    a $^-5 + {}^-4 = {}^-9$ **b** $^-5 + 4 = {}^-9$

    c $^-2 \times 3 = 6$ **d** $^-2 \times {}^-3 = 6$

    e $5 \times {}^-4 = -20$ **f** $0 \times {}^-7 = {}^-7$

    g $8 + {}^-7 + {}^-2 = {}^-7$ **h** $^-4 - {}^-6 = 2$

    i $[8 \times {}^-9] + (6 - 7) = {}^-71$ **j** $^-1 \times 2 \times {}^-3 \times 4 = 24$

Measurement and geometry

3

Flat shapes

Contents

Prior learning
Chapter 3

Parent guide
Chapter 3

Curriculum guide
Chapter 3

Australian Curriculum statements

Location and transformation
Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries. (ACMMG181)

Geometric reasoning
Identify corresponding, alternate and co-interior angles when two parallel straight lines are crossed by a transversal. (ACMMG163)

Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning. (ACMMG164)

Classify triangles according to their side and angle properties and describe quadrilaterals. (ACMMG165)

Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral. (ACMMG166) AC

Weblink

GeoGebra

The word **geometry** comes from two ancient Greek words, *geo* meaning earth, and *metria* meaning measure. It is a field of mathematics that studies the size, shape and relative position of figures. Many people think that the study of geometry began because in ancient times it was important to be able to measure distances and land areas accurately. So, geometry began as a very practical science. As time went by, mathematicians developed formulas for lengths and areas to help do calculations. Another example of the practical nature of geometry is shown by the fact that ancient builders used geometrical techniques to draw and construct right angles. This skill remains a fundamental part of the building and construction industry.

■ Mathematical literacy

Maths dictionary

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

acute	corresponding	order	scale factor
acute-angled	decagon	parallel	scalene
adjacent	degree	parallelogram	set square
alternate	diagonal	perpendicular	side
angle	draw	polygon	similar
arc	equilateral	protractor	similarity
arm	geometry	quadrilateral	square
axis of symmetry	grads	rectangle	straight angle
bisect	intersecting line	reflection	supplementary
co-interior	interval	reflex	symmetry
compasses	isosceles	regular	translation
complementary	kite	revolution	transversal
congruence	line	rhombus	trapezium
congruent	line symmetry	right angle	triangle
construct	magnification	rotation	vertex
construction	obtuse-angled	rotational symmetry	vertically opposite
convex	octagon	scale	

3.1   Points, lines and angles

A **point** is a position represented by a dot which is labelled by a capital letter. A **line** is a straight edge that continues infinitely (forever) in both directions, so it is usually drawn with arrowheads on both ends. The name of a line passing through two different points is determined by the names of those points.

The line in this diagram is labelled *LM*.

An **interval** is a *part* of a line. It has a starting point, an end point and a definite length. The interval in this diagram is *RS*.
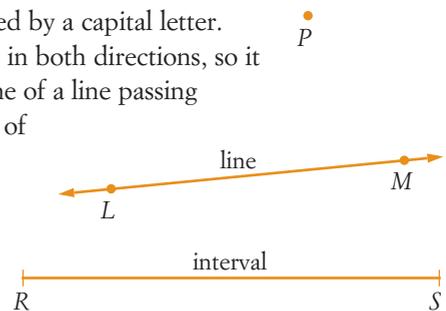
An angle is a description of the size of a turn or rotation.

It is drawn with two arms which meet at a **vertex**. Angles are normally marked with a curved line called an **arc**. This shows the size of the turn. The angle marked in this diagram can be written as:
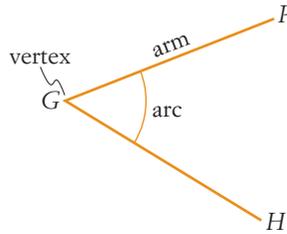
∠*G* or *Ĝ*

∠*PGH* or ∠*HGP* } The middle letter is always the letter
PĜH or *HĜP*   that labels the vertex of the angle.

Example 1

Name the angle marked with • in each of these diagrams.

a

Y

X   *Z*

b

P

Q   *S*

R
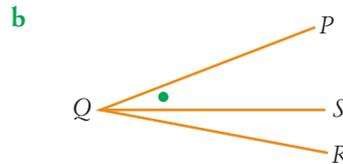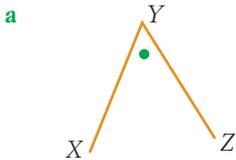
Solution

a The angle marked • is at the vertex labelled *Y*.

• **is called**
∠*Y* or ∠*XYZ* or ∠*ZYX*.

b The angle marked • is at the vertex labelled *Q* but there are other angles with *Q* as the vertex.

The arms end in *P* and *S*.

• **is called**
∠*SQP* or ∠*PQS*.

Circles are measured in **degrees** (°). As shown in the diagram below, a complete turn or revolution is divided into 360 degrees or 360°.

0°

270°   90°

180°
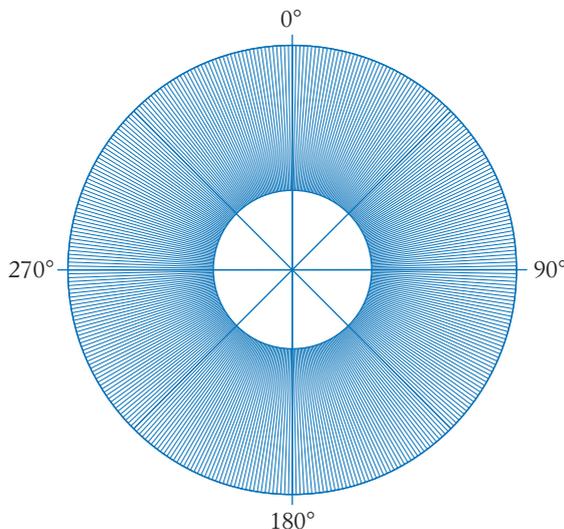
Investigate: Why 360 degrees?

Why are there 90° in a right angle and 360° in a revolution? Why do we use such strange numbers instead of more conventional numbers like 10 and 100?

The reason is that in 2000 BCE, the ancient Babylonians used a base 60 system of numbers. They used a base 60 number system because:

• 60 is a rounder, more convenient number that has more factors than 10. You can divide 60 by 2, 3, 4, 5, 6, 10, 12, 15, 20 and 30.

• $6 \times 60 = 360$, which was the Babylonian approximation of the number of days in a year. They defined a revolution as being 360° so that, each day, the Earth would travel 1° around the Sun. A right angle, being a quarter-revolution, thus became $360° \div 4 = 90°$.

Find out more about the calendars used by the ancient Babylonians and Persians.
Some people who prefer a base 10 system of measurement use **grads** instead of degrees to measure angles. With this system, a right angle is 100 grads and a revolution is 400 grads. Find out more information about grads, including the exact relationship between degrees and grads.
Find out more about the smaller divisions (subdivisions) into which a degree may be divided.

A protractor is an instrument used to measure angles. Basic protractors were used in ancient times for measurements during building construction. Today, they can be used by architects and builders when designing and constructing buildings, and in navigation.

Example 2

Measure $\angle TEX$.
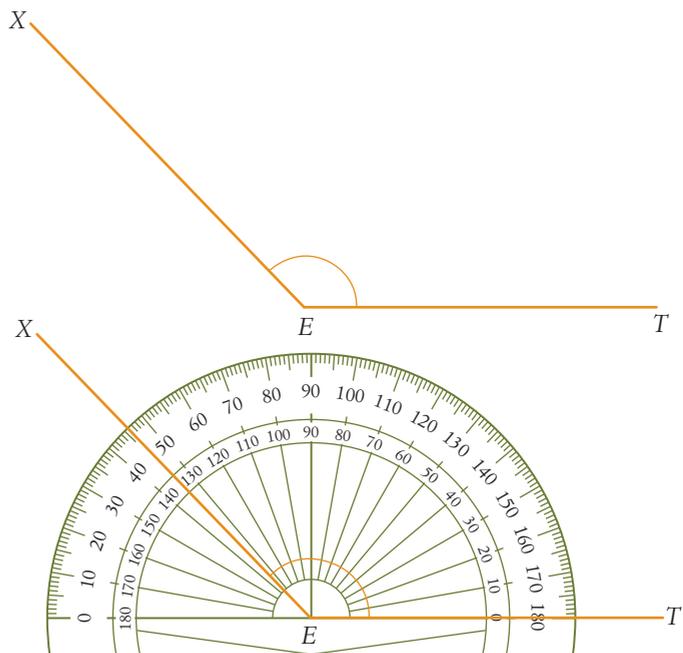
Solution

Line up TE with the base line of the protractor.

Place the centre mark over the vertex E.
$\angle TEX$ is bigger than 90°.

Use the inside scale.

Write the result.

$\angle \textbf{\textit{TEX}} = \textbf{134°}$

Example 3

Measure the reflex angle ∠GHK.
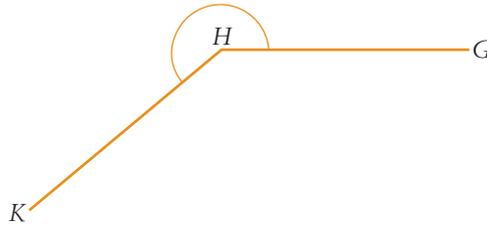
Solution

The angle *H* is a reflex angle (greater than 180°) so we cannot just read its size from a protractor. Instead, we must first measure the obtuse angle, ∠GHK. It is 140°.

We then subtract 140° from 360° to find the size of reflex angle *H*.

We then subtract 140° from 360° to find the size of reflex angle *H*. **360° – 140° = 220°**
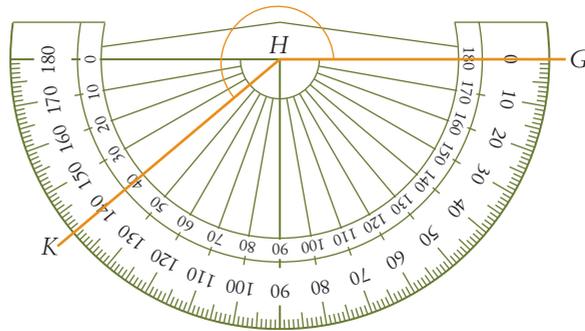
Write the result. **Reflex ∠GHK = 220°**

You can use your protractor to draw angles.

Example 4

Use a protractor to draw ∠KPM that measures 76°.

Solution

Draw a line with endpoints *P* and *M*.

Line up the base line of the protractor over *PM*.
Place the centre mark over *P*.
Follow the inside scale around on the protractor, from 0° to 76°.
Mark this point.

Mark 76°
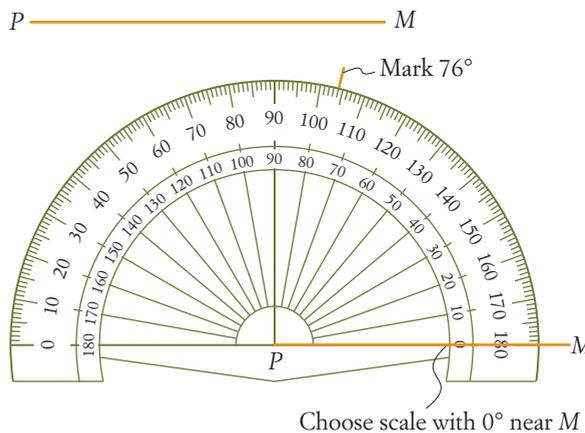
Choose scale with 0° near *M*

Draw a line from *P* through this mark. Label the end of this line *K*. You have now drawn ∠*KPM*, measuring 76°.

K

line ruled
from *P* through
mark at 76°

P ⌐ *M*
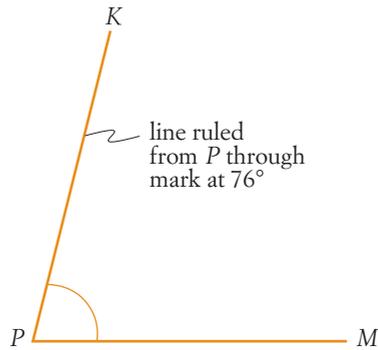
Angles may be classified according to their size as shown below.

Angle	Type	Description
	acute	less than 90°
	right	90° (quarter turn) Note that a right angle is marked with a box symbol.
	obtuse	greater than 90° but less than 180°
	straight	180° (half turn)
	reflex	greater than 180° but less than 360°
	revolution	360° (complete turn)

Example 5

Classify these angle sizes

a 200°   **b** 84°   **c** 145°   **d** 180°

Solution

a 200° is between 180° and 360°.   **200° is a reflex angle.**

b 84° is less than 90°.   **84° is an acute angle.**

c 145° is between 90° and 180°. **145° is an obtuse angle.**

d 180° is half a revolution. **180° is a straight angle.**

Example 6

Classify and estimate the size of these angles.

a

b

c

d
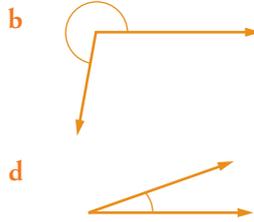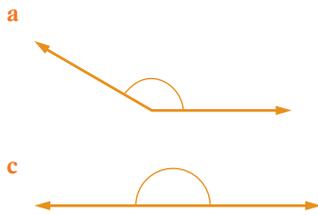
Solution

a The angle is just over halfway from 90° to 180°. **The angle is an obtuse angle about 150°.**

b The angle is just before halfway from 180° to 360°. **The angle is a reflex angle about 260°.**

c The angle is 180°. **The angle is a straight angle, 180°.**

d The angle is small, between 0° and 90°. **The angle is an acute angle, about 20°.**

Exercise 3.1 Points, lines and angles

1 Write down two different names for each of the angles below.
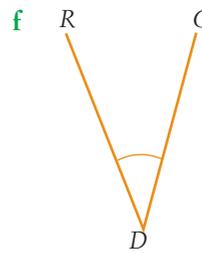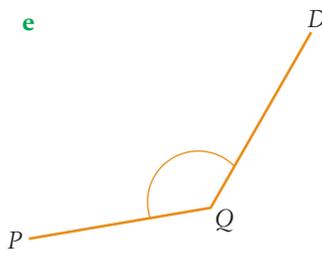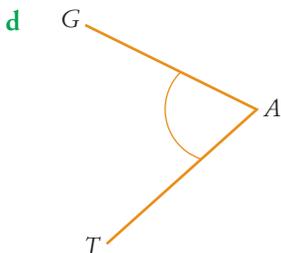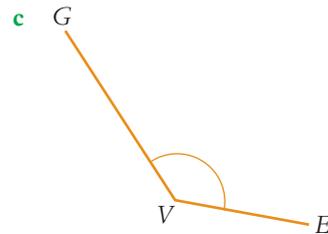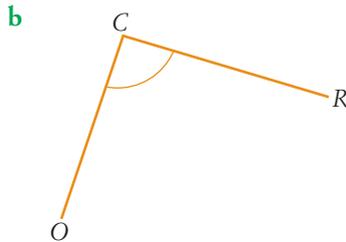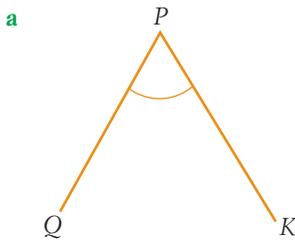
a *P* *Q* *K*

b *C* *R* *O*

c *G* *V* *E*

d *G* *A* *T*

e *D* *P* *Q*

f *R* *C* *D*

2 The name of the angle marked • is which of the following? Select **A**, **B**, **C** or **D**.

A $\angle ABD$ **B** $\angle CBD$

C $\angle ABC$ **D** $\angle BCA$
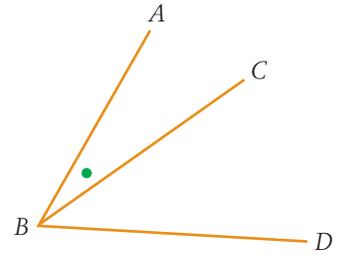
3 Write the name of the angle marked with • in each of these diagrams.

a

b

c

d

e

f
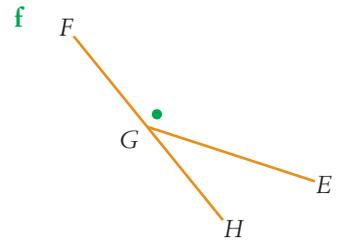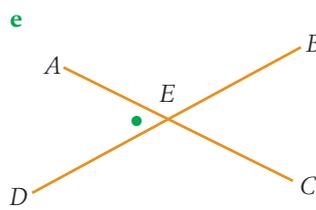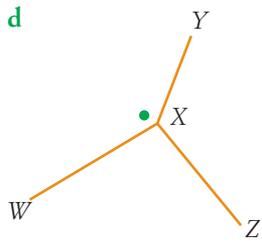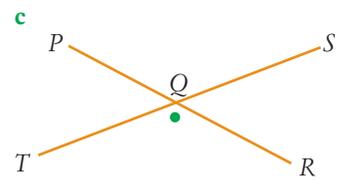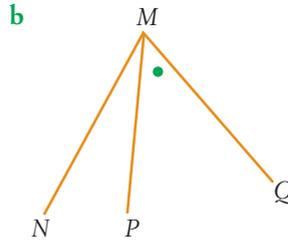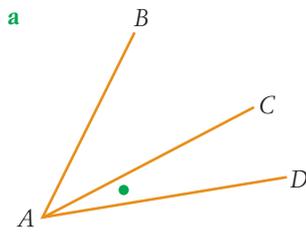
4 Draw each of these angles, labelling them correctly.

   a $\angle POT$ **b** $\angle TAF$ **c** $\angle AFE$ **d** $\angle H$

See Example 2

5 Find the size of each of these angles.

a

b

c

d

e

f

g

h

6 Estimate the size of each of these angles. Write the name of each angle and use a protractor to measure the angles accurately.

a

b

c

d

e

f

Z · Y · X

g M · A · L

D · F · N

h G

B

j Z · M · P · E

i D

k C · B · A · G

l F

7 Estimate the size of each of these angles. Write the name of each angle and use a protractor to measure the angles accurately.

a B · A · C

b X · Y · Z

c M · N · L

d G · H · K

8 Accurately draw these angles, using your protractor. *See* Example 4

   a 35°   **b** 115°   **c** 150°   **d** 40°

   e 15°   **f** 170°   **g** 117°   **h** 200°

9 Use your protractor to accurately draw and label these angles.

   a $\angle DRE = 65°$ **b** $\angle BGH = 145°$ **c** $\angle GRT = 32°$

   d $\angle ABC = 45°$ **e** $\angle SAQ = 110°$ **f** $\angle NMH = 265°$

   g $\angle KLY = 28°$ **h** $\angle LMN = 180°$ **i** $\angle LKY = 90°$

10 **a** There are 13 different angles inside the diagram on the right. Write down all their names.

   b What type of angle is $\angle NCY$?
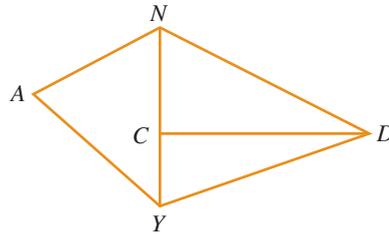
11 Write the name of the angles marked • and × in each of the following diagrams.

12 Angles *AMP* and *PMN* share a common arm, *PM*. They also share a common vertex, *M*. Angles that are next to each other in this way are called adjacent angles.

Name a pair of adjacent angles for each diagram in question **11**.

13 The diagram shows Daniel shooting for goal in a game of football. His shooting angle is shown on the diagram. Estimate the size of this angle. Select **A**, **B**, **C** or **D**.

A 60°   **B** 120°
C 150°   **D** 240°

14 Measure the angles marked with a • and a × on each of these diagrams.

ⓐ

b

c

d

e

f

15 Draw two different examples of:

    a an acute angle   **b** an obtuse angle   **c** a right angle

    d a reflex angle   **e** a straight angle   **f** a revolution

16 Classify each of the following angles.

    a 37°   **b** 107°   **c** 252°

    d 195°   **e** 79°   **f** 180°

    g 163°   **h** 179°   **i** 360°

    j 5°   **k** 345°   **l** 91°

    m 14°   **n** 299°   **o** 90°

    p 205°   **q** 126°   **r** 44°

See Example 5
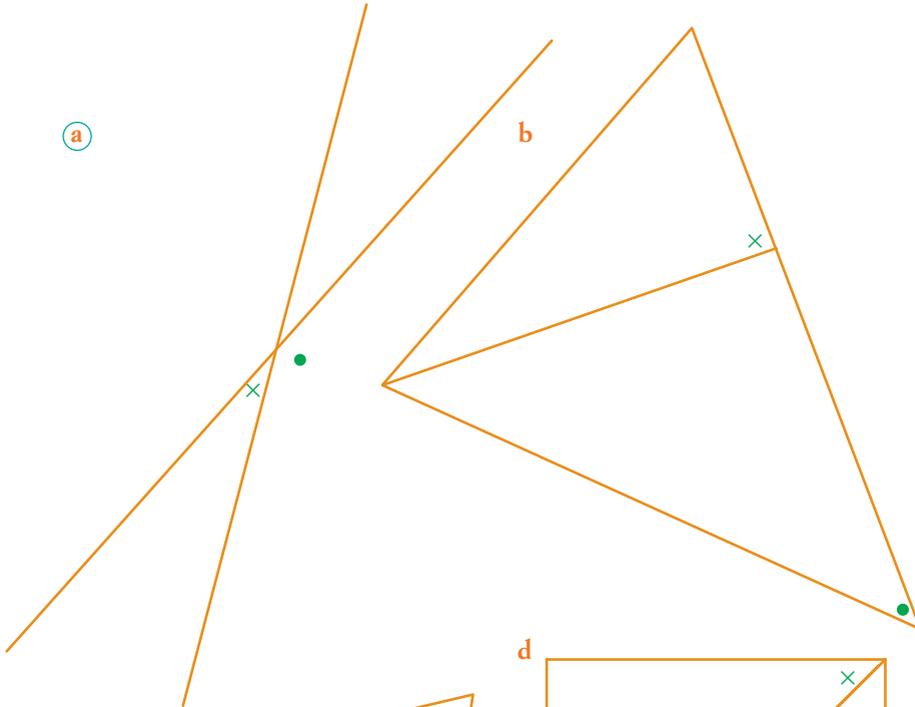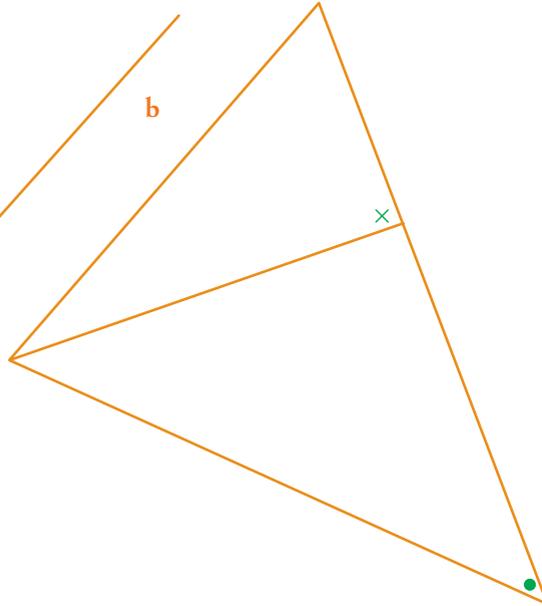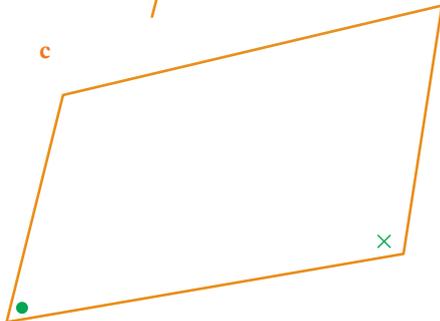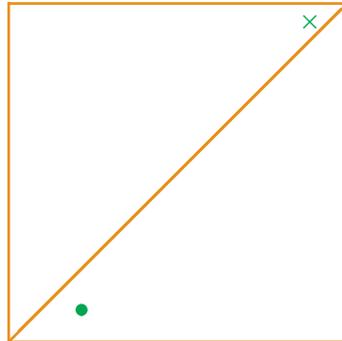
Worked solutions

Exercise 3.1

MAT07MGWS00012

17 Decide whether each of these angles is acute, obtuse or reflex.

    a

    b

    c

    d

18 Select **A**, **B**, **C** or **D**. Angles m and n are respectively:

A obtuse and reflex

B obtuse and a revolution

C acute and a revolution

D acute and reflex
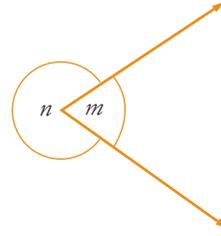
Problem solving

19 List the following angles from smallest to largest.
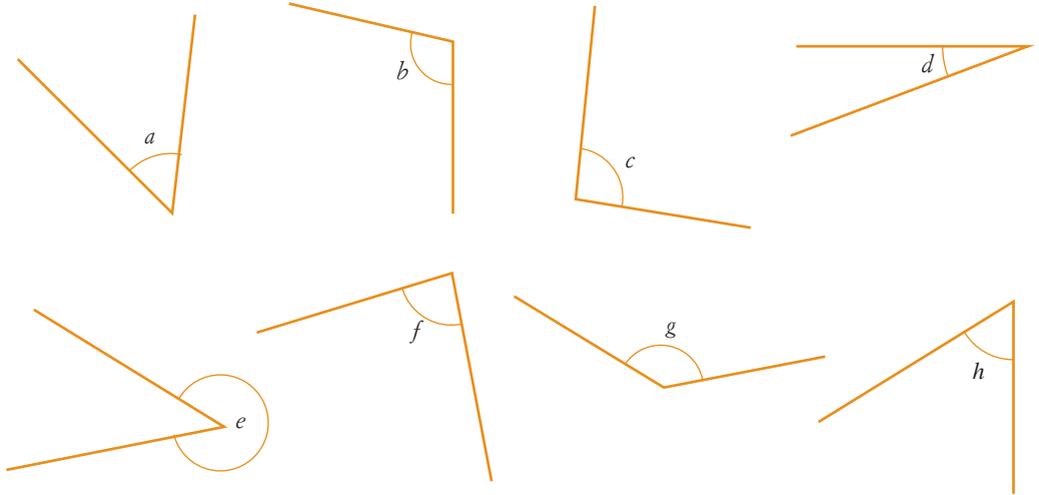
20 List the angles below in order, from smallest to largest, without using a protractor.

a   **b**   **c**

d   **e**   **f**

g   **h**   **i**   **j**
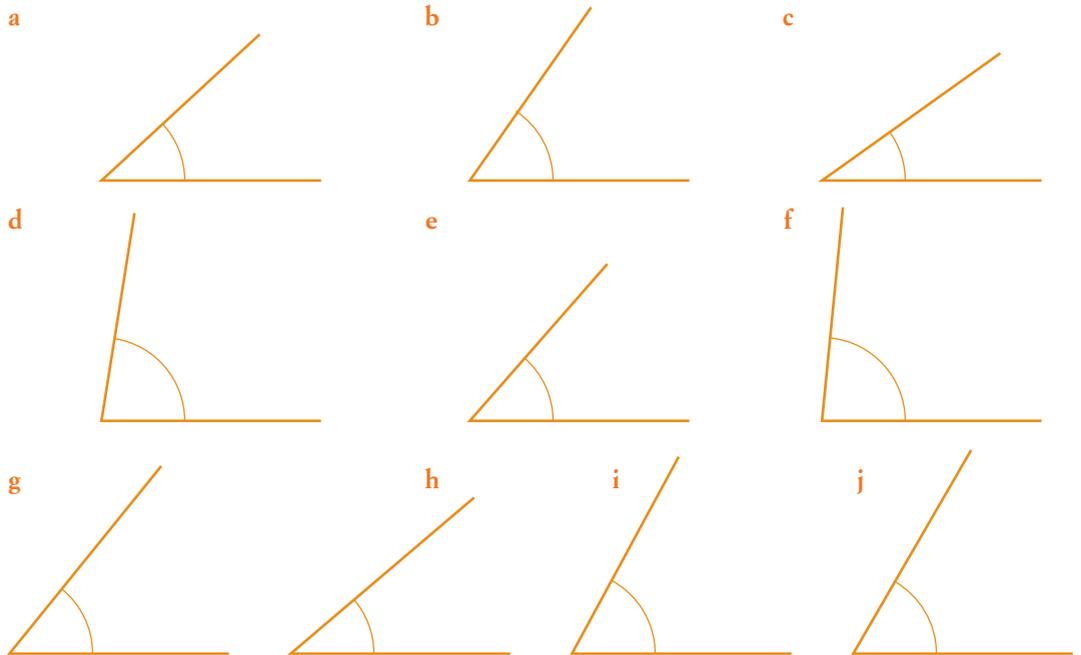
3.2 Angle relationships

You can describe angles according to their sizes. You can also describe how they relate to each other. In the following investigation you will discover some of these relationships.

Investigate: Relationships between angles

1 Copy and complete the information below each of these diagrams. Use your protractor to measure the angles, extending arms as necessary.
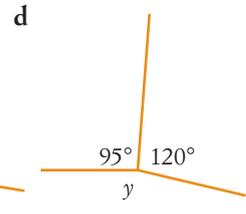
a

$\angle ABD = \underline{\qquad}$
$\angle CBD = \underline{\qquad}$
$\angle ABD + \angle CBD = \underline{\qquad}$

b

$\angle XYZ = \underline{\qquad}$
$\angle XZY = \underline{\qquad}$
$\angle XYZ + \angle XZY = \underline{\qquad}$
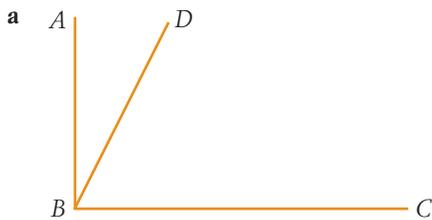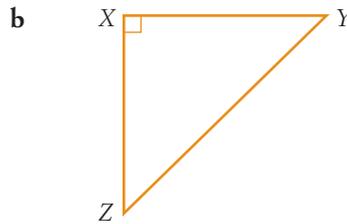
The pairs of angles you measured are called **complementary angles**. They complement each other to form 90°.

2 Look up 'complement' in a dictionary. Write one non-mathematical meaning you find.

3 What is the complement of:

a 30°?	**b** 70°?	**c** 25°?	**d** 38°?	**e** 89°?	**f** 57°?
g 42°?	**h** 66°?	**i** 11°?	**j** 74°?	**k** 1°?	**l** 12°?

4 Copy and complete the information below each of these diagrams. Use your protractor to measure the angles, extending arms as necessary.

a

$\angle ABD = \underline{\qquad}$
$\angle CBD = \underline{\qquad}$
$\angle ABD + \angle CBD = \underline{\qquad}$

b

$\angle PQR = \underline{\qquad}$
$\angle RQS = \underline{\qquad}$
$\angle PQR + \angle RQS = \underline{\qquad}$
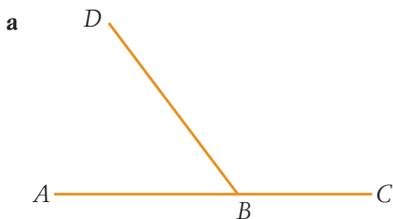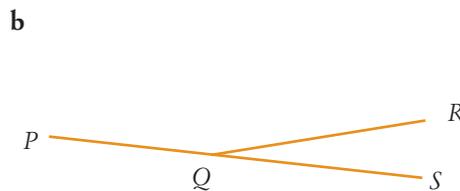
The pairs of angles you measured are called **supplementary angles**. They supplement each other, together forming 180°.

5 Look up 'supplement' in your dictionary. Write a non-mathematical meaning for it.

6 What is the supplement of:

a 18°?	**b** 150°?	**c** 35°?	**d** 125°?	**e** 62°?	**f** 87°?
g 111°?	**h** 173°?	**i** 54°?	**j** 132°?	**k** 8°?	**l** 91°?

7 a How many degrees are there in a complete turn or revolution?
  b Copy and measure the angles and complete the statements below each of these diagrams, extending the arms if necessary.

i

ii
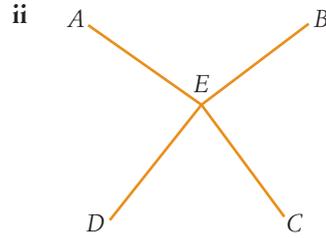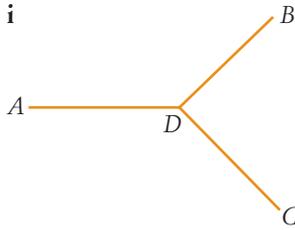
$$\angle ADB= \underline{\qquad}$$
$$\angle ADC= \underline{\qquad}$$
$$\angle BDC= \underline{\qquad}$$
$$\angle ADB + \angle ADC + \angle BDC= \underline{\qquad}$$
(These angles all meet at a point.)

$$\angle AEB = \underline{\qquad}$$
$$\angle BEC = \underline{\qquad}$$
$$\angle CED = \underline{\qquad}$$
$$\angle DEA = \underline{\qquad}$$
$$\angle AEB + \angle BEC + \angle CED + \angle DEA = \underline{\qquad}$$

In each case, the angles you measured all meet at a point. They make up one revolution.

8 Use the given information to find the size of the angle shown by the letter each time.

a 150° 160° m

b 70° q 170°

c 62° 87° x

d 95° 120° y

e 102° a 25° 135°

f 71° 116° d 22°

g 105° 110° 55° w

h 132° 123° 48° f

i n 30° 220°

j 152° k

k 118° t 47° 15°

l 303° h
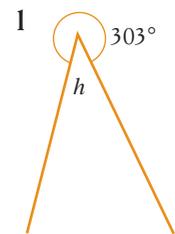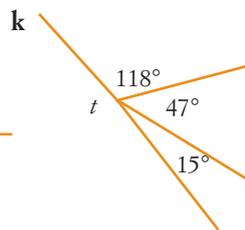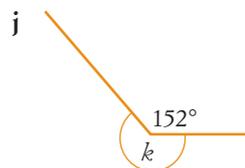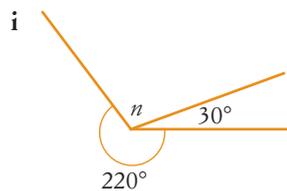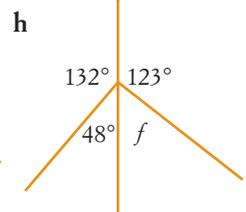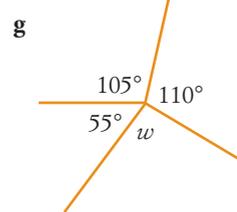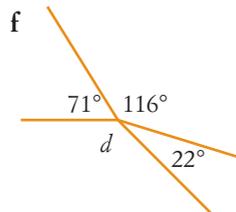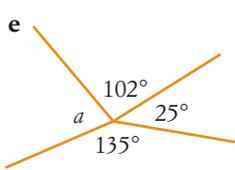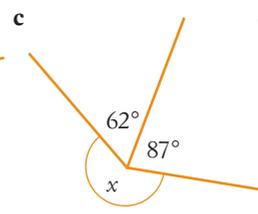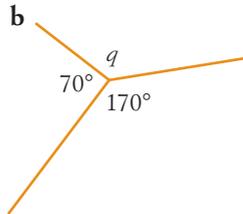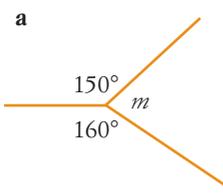
9 Find the value of d. Select **A**, **B**, **C** or **D**.

A 122 **B** 61
C 142 **D** 81

38°
d d
160°
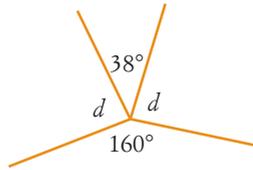
When two lines cross, four angles are created.
Angles that are opposite each other when lines cross
in this way are called **vertically opposite angles**.
You can see that a and c are vertically opposite angles.
Can you name another pair of vertically opposite angles
in this diagram?
What is true about angles a and c, b and d?

a
d b
c
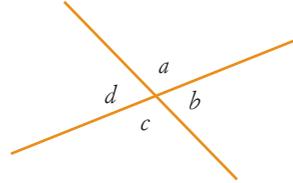
Important!

Complementary angles add to 90°.
Supplementary angles add to 180°.
Angles at a point (in a revolution) add to 360°.
Vertically opposite angles are equal.

Example 7

What angle is vertically opposite $\angle XKY$?

W X

K

Z Y
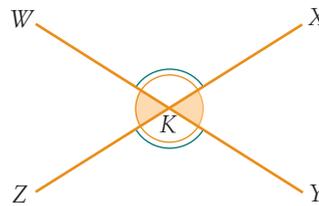
Solution

Identify the angle that is opposite $\angle XKY$. $\quad$ **$\angle WKZ$ is vertically opposite $\angle XKY$.**

You can see from Example **7** that angles that are equal in size are marked on diagrams with the
same type of arc or symbol.

Example 8

Find the size of the angles shown by the
letters in this diagram.

130°
50° m
k
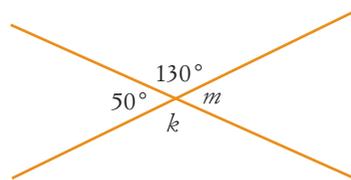
Solution

k is vertically opposite 130° so it must be equal to it.	$\boldsymbol{k = 130°}$
m is vertically opposite 50° so it must be equal to it.	$\boldsymbol{m = 50°}$

The various angle relationships covered so far in this chapter are summarised below.

Type of angle	Relationship	Diagram
Adjacent angles	Angles that share a common arm and a common vertex. ($\angle ABD$ and $\angle DBC$ are adjacent angles)	
Complementary angles	Two angles that add to 90°. ($a + b = 90°$)	
Supplementary angles	Two angles that add to 180°. ($m + n = 180°$)	
Angles at a point	Form a revolution and add to 360°. ($a + b + c = 360°$)	
Vertically opposite angles	Formed when two straight lines cross. Vertically opposite angles are equal. ($a = c, b = d$)	

Example 9

Calculate the size of the angle shown by the letters in these diagrams.

a

130° x

b

60°

y
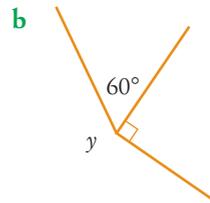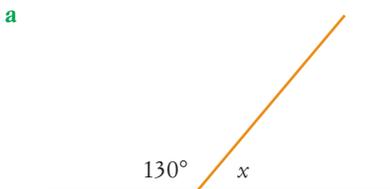
Solution

a 130° and x are supplementary angles.
This means that they must add up to 180°. $x + 130° = 180°$

Subtract 130° from both sides of the equation.	$x = 180° - 130°$
Simplify the right-hand side.	$= 50°$
State the result.	$x = 50°$

b 60°, 90° and y are angles at a point. This means that they form a revolution and add up to 360°.

	$y + 60° + 90° = 360°$
Subtract 60° and 90° from both sides of the equation.	$y = 360° - 60° - 90°$
Simplify the right-hand side.	$= 210°$
State the result.	$y = 210°$

Important!

The name of a line passing through two different points is determined by the names of those points.

For example, this is the line *AB*.

When two lines cross, we say that they **intersect**. Two lines intersect at a point.

For example, in this diagram, the line *DE* intersects the line *FG* at point *H*.

Lines that intersect at right angles are called **perpendicular lines**.

For example, in this diagram, *PQ* is perpendicular to *XY*. This is written as '*PQ* ⊥ *XY*', where the ⊥ symbol stands for 'is perpendicular to'.

Lines that point in the same direction are called **parallel** lines. Parallel lines are marked with identical arrowheads. In normal geometry they do not intersect because they are always the same distance apart.

For example, in this diagram, *MN* is parallel to *RS*. This is written as *MN* ∥ *RS*, where the symbol ∥ stands for 'is parallel to'.

indicates these lines are parallel

A line that crosses two or more other lines is called a **transversal**. Transverse means 'crossing'.
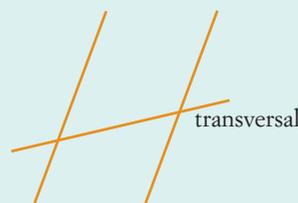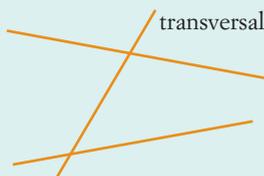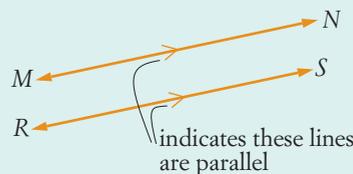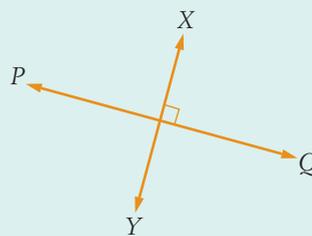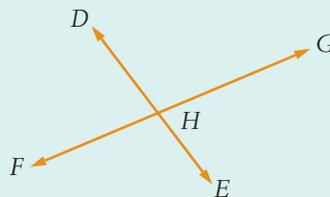
transversal

transversal

Alternate angles are between two lines and on opposite sides of a transversal crossing the lines. Alternate angles on *parallel lines* are equal.

On this diagram the alternate angles are marked with dots. 'Alternate' means 'going back and forth'. Draw a pair of parallel lines and mark the alternate angles as shown. Draw in the dotted line and cut along it.

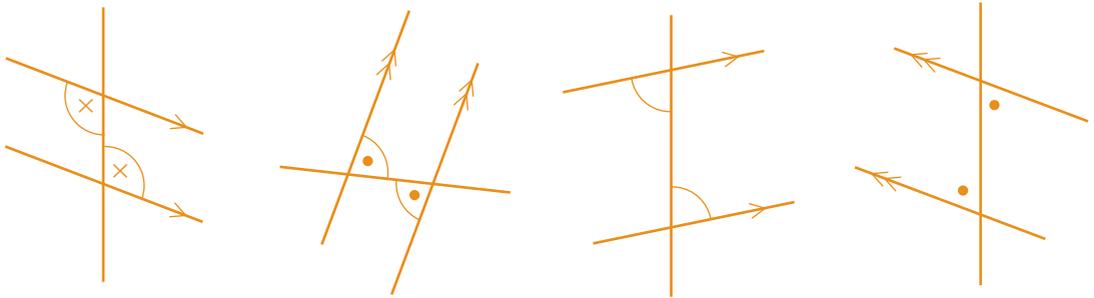Rotate the two alternate angles and place them on top of each other. You should see that they are the same.

transversal
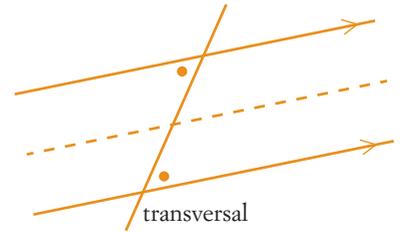
Important!

Alternate angles
Alternate angles on parallel lines are equal.

Alternate angles on parallel lines

The marked pairs of angles are alternate. Measure them and check that alternate angles are equal. (*Remember*: Equal angles are marked by the same symbol.)

Corresponding angles are on the same side of the transversal and are both either above or below the other two lines. 'Corresponding' means 'matching'.

Important!

Corresponding angles
Corresponding angles on parallel lines are equal.

Corresponding angles on parallel lines
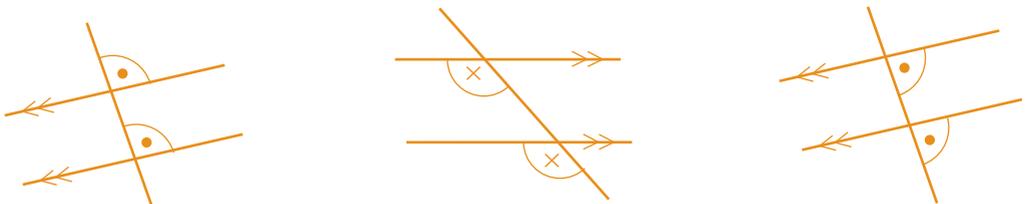
The marked pairs of angles are corresponding. Measure them and check that they are equal.
We can prove that corresponding angles on parallel lines are equal.

$a = b$ (They are vertically opposite angles.)
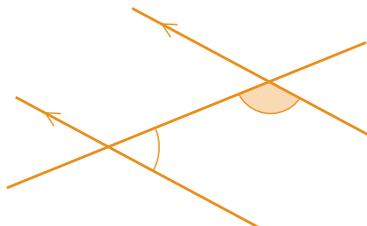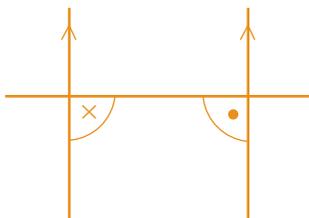$b = c$ (They are alternate angles.)
So $a = c$.
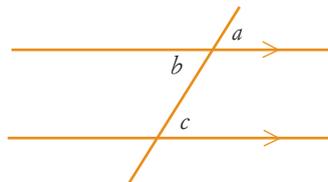
Co-interior angles (**allied angles**) are on the same side of the transversal but *between* the other two lines. 'Co-interior' means 'together inside'.

Important!

Co-interior angles (allied angles)
Co-interior angles on parallel lines are supplementary. They add to 180°.

Co-interior angles on parallel lines

Measure the following pairs of angles and see if they really are supplementary.
We can also show that co-interior angles on parallel lines add to 180° using the following method.

$a + b = 180°$ (They are angles on a straight line.)
$a = c$ (They are alternate angles.)
So $c + b = 180°$
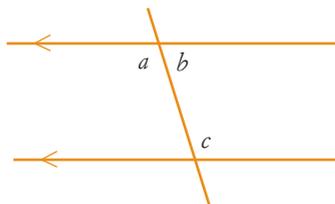
Example 10

Find the size of the angle marked a in this diagram.

Solution

The angles marked a and 80° are co-interior angles on parallel lines. So they must add up to 180°.

$a + 80° = 180°$

Subtract 80° from both sides of the equation. $a = 180° - 80°$

Simplify the right-hand side of the equation. $= 100°$

State the result. $a = 100°$
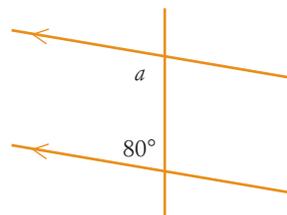
Example 11

Find the size of the angle marked *m*.
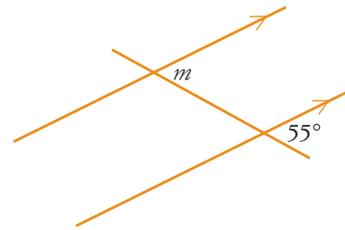
Solution

Angles marked *m* and 55° are corresponding angles on parallel lines. So they are equal.

$m = 55°$

Below is a summary of all that we have found out about the angles formed on parallel lines. When parallel lines are crossed by a transversal:

- alternate angles are equal
- corresponding angles are equal
- co-interior angles are supplementary (add to 180°).

Type of angle	Relationship	Diagram
Alternate angles on parallel lines	Alternate angles are between two lines and on opposite sides of a transversal. Alternate angles on parallel lines are equal. ($a = b$)	
Corresponding angles on parallel lines	Corresponding angles are on the same side of the transversal and are both either above or below parallel lines. Corresponding angles on parallel lines are equal. ($m = n$)	
Co-interior angles on parallel lines	Co-interior angles are on the same side of the transversal and between parallel lines. Co-interior angles on parallel lines are supplementary. ($x + y = 180°$)	

Example 12

Is *AB* parallel to *CD* in the diagram on the right?
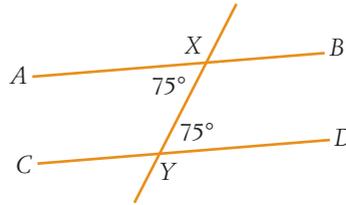
Solution

Look at the positions of the angles marked 75°.	**∠AXY and ∠DYX are a pair of alternate angles.**
Write the relationship between ∠AXY and ∠DYX.	**∠AXY = ∠DYX**
Alternate angles on parallel lines are equal.	**AB ∥ CD**

Example 13

Is *MN* parallel to *PQ* in the diagram on the right?
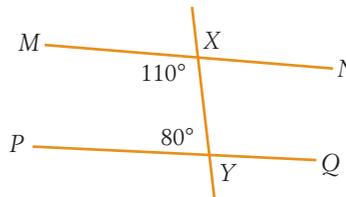
Solution

|	
Look at the positions of the 80° and 110°angles.	**∠MXY and ∠PYX are a pair of co-interior angles.**
Write the relationship between ∠MXY and ∠PYX.	**∠MXY + ∠PYX = 190°**
Co-interior angles on parallel lines are supplementary and add up to 180°.	**MN is not parallel to PQ.**

Investigate: Angles in triangles and quadrilaterals

Measure the angles in the triangles below.
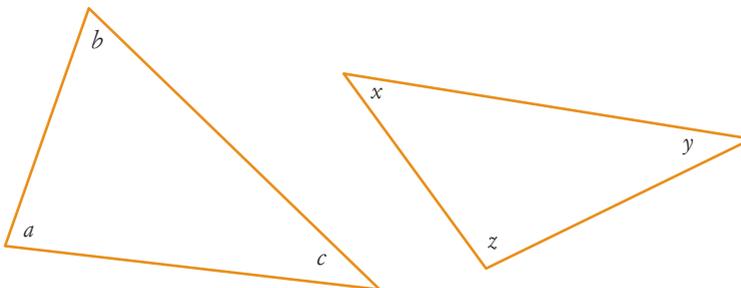
Work out $a + b + c$. Work out $x + y + z$. What do you find?

Draw some triangles of your own and measure the angles in the triangles.
What do you find about the sum of the angles?
Complete the following.
The sum of the angles in a triangle is always _____ degrees.
Now consider the quadrilaterals below.
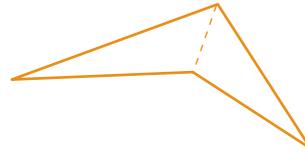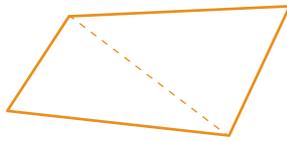
How many triangles does it take to make a quadrilateral?
What is the total of the angles in the triangles that make a quadrilateral?
What is the total of the angles in a quadrilateral?
Complete the following.
The sum of the angles in a quadrilateral is always _____ degrees.

The relationship of angles in triangles is very important in geometry as it allows you to find out and prove many other relationships.
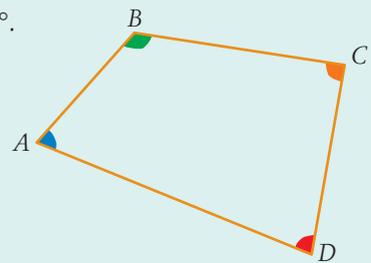
Important!

Angle sums

The sum of the angles inside a triangle is always 180°.
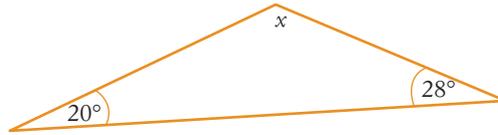For any triangle $\triangle ABC$, $\angle A + \angle B + \angle C = 180°$
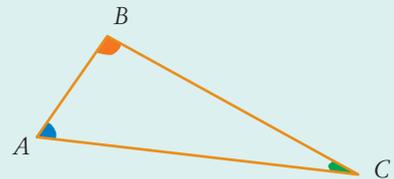
The sum of the angles inside a quadrilateral is always 360°.

For any quadrilateral $ABCD$, $\angle A + \angle B + \angle C + \angle D = 360°$

Example 14

Find x.

Solution

Look at the positions of the angles.	x, 20° and 28° are the angles of a triangle.
Write the relationship.	$x + 20° + 28° = 180°$
Simplify.	$x + 48° = 180°$
Solve for x.	$x = 132°$

Example 15

Find e.
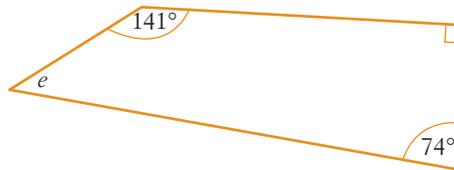
Solution

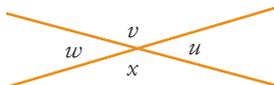|	
Look at the positions of the angles.	e, 141°, 90° and 74° are the angles of a quadrilateral.
Write the relationship.	$e + 141° + 90° + 74° = 360°$
Simplify.	$e + 305° = 360°$
Solve for e.	$e = 55°$

Exercise 3.2 Angle relationships

1 What angle is vertically opposite to:
  a the angle marked a?
  b the angle marked w?
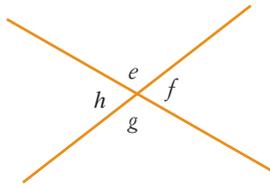 **c** the angle marked c?
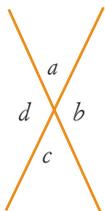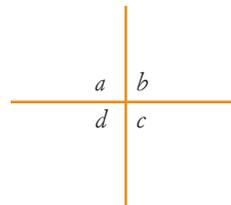
d the angle marked *h*?   **e** the angle marked *k*?   **f** the angle marked *m*?
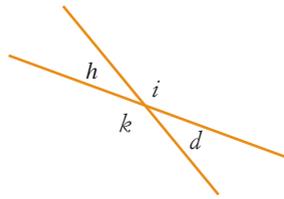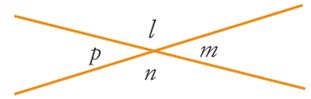
See Example 9

2 **a** If ∠*TAF* = 42°, what is the size of its complementary angle?
   b If ∠*ZAB* = 127°, what is the size of its supplementary angle?

3 Refer to the diagram shown on the right.
   a Which angle is vertically opposite to ∠*NDP*?
   b Which angle is equal to ∠*MDQ*?
   c Name two straight angles in the diagram.
   d Name two different pairs of supplementary angles
      in the diagram.
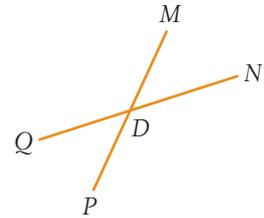
4 Which of the following is an angle adjacent to ∠*AXB*? Select **A**, **B**, **C** or **D**.
   A ∠*BXC*
   B ∠*DXE*
   C ∠*DXC*
   D ∠*CXE*
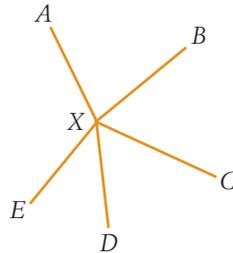
5 Refer to the diagram shown on the right.
   a Name a pair of adjacent angles.
   b Name a pair of complementary angles.
   c How do you know that the angles you named
      are complementary?
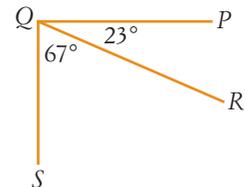
6 In this diagram, name two lines that:
   a are perpendicular
   b are parallel
   c intersect.
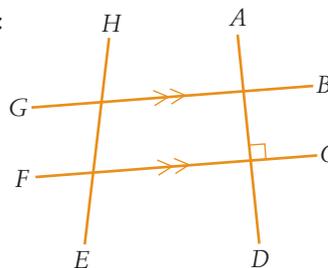
7 Rewrite your answers to question **6** parts **a** and **b** using the symbols for 'is perpendicular to' and 'is parallel to'.

8 Draw and label the following correctly,
   a line *FG*   **b** line *AB* intersecting line *CD* at point *E*
   c line *PQ* parallel to line *YZ*   **d** line *JK* perpendicular to line *LM*.

9 Which angle is alternate to the marked angle each time?

a

b

c
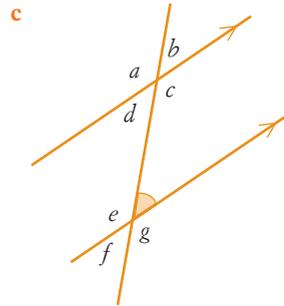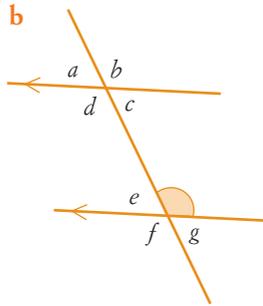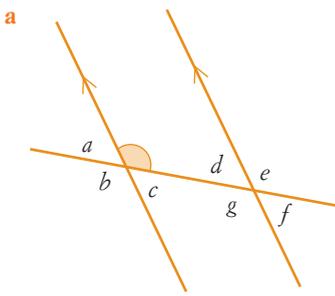
10 Copy these diagrams and mark in a pair of alternate angles on each one.

a

b

c
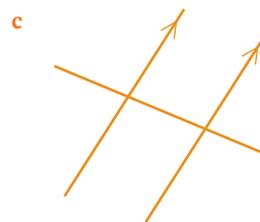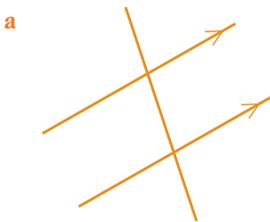
11 Which angle corresponds to the marked angle in each of the diagrams below?

See Example 11

a

b

c
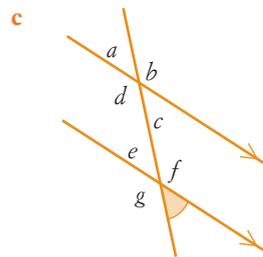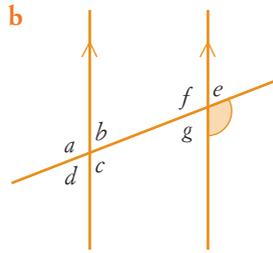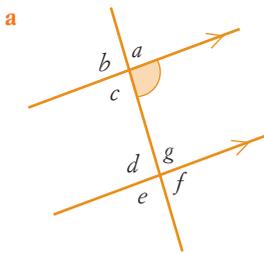
12 Which angle is co-interior with the marked angle in each of the diagrams below?

a

b

c
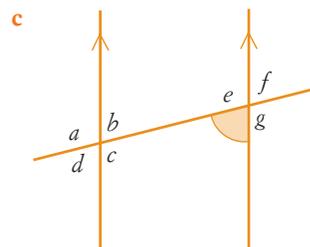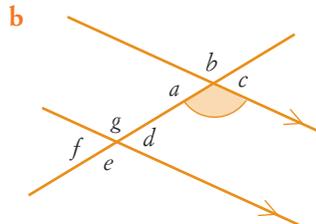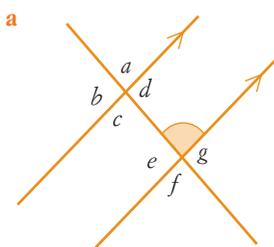
13 In the diagram on the right, name the angle that is:

  a corresponding to ∠*VWA*

  b alternate to ∠*QXW*

  c co-interior with ∠*PWX*

  d supplementary with ∠*AWX*

  e alternate to ∠*SXV*

  f corresponding to ∠*ZXS*.
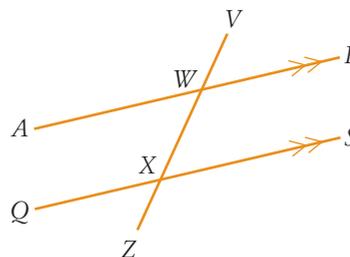
14 Without measuring, find the size of the angle shown by the letter each time.

a

70°

a

b

110°

m

c

k

85°

d

90°

m

e

135° *x*

f

25°   *f*

g

w 133°

h

29°

n

i

q   62°

j

t

163°

k

h   160°

g   20°

l

90°

q   *s*

r
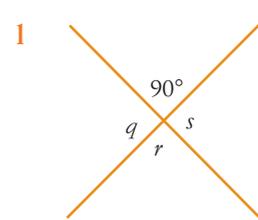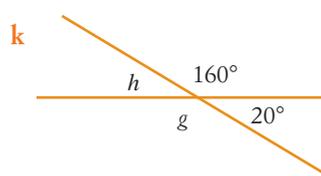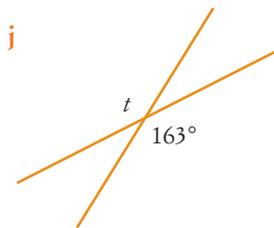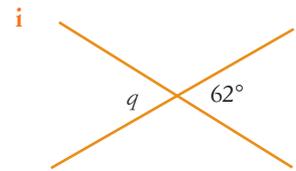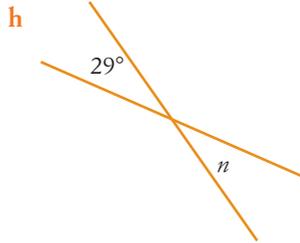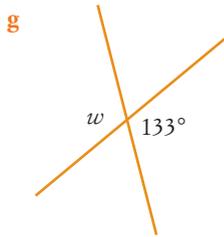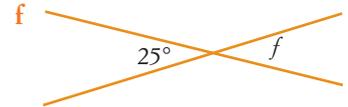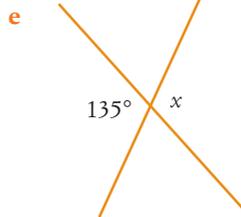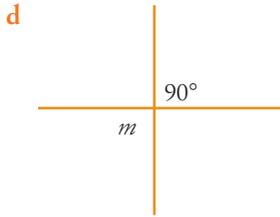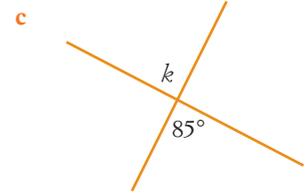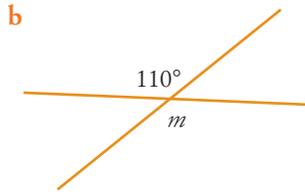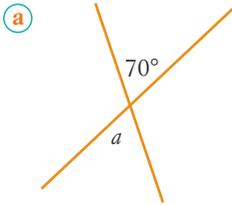
15 Determine the types of angles in each of the following diagrams and then calculate the size of the angle shown by the letter.

a

a 120°

b

a

70°

c

100°

y

d

m
45°

e

p
150°

f

100°
100°   40°
a

g

m
19°

h

41°
x

i

15°
f

j

h
170°

k

a
a

l

t t

m

32°
b
a

n

135°
y 82°

o

d
d

p

20°
x
y

q

e 112°
e f

r

j 48°
k l

s

118° x
y 75°

t

p
85°   155°
p

u

e e
e
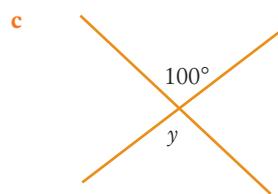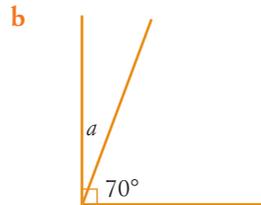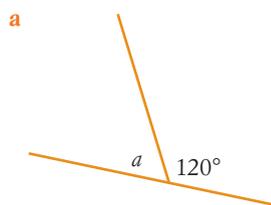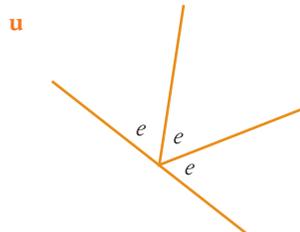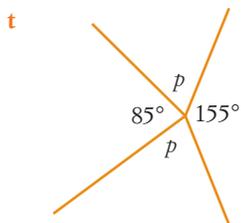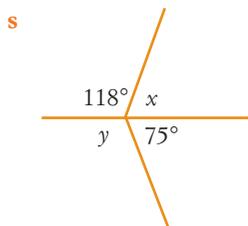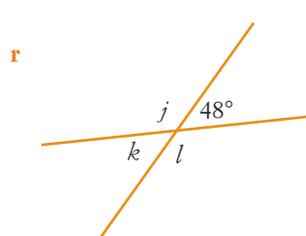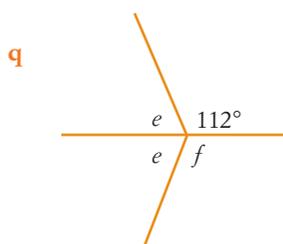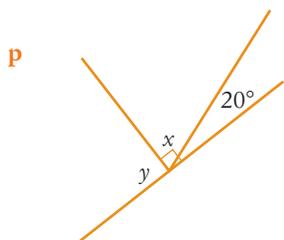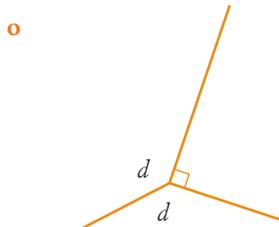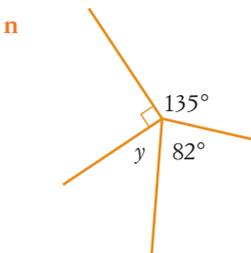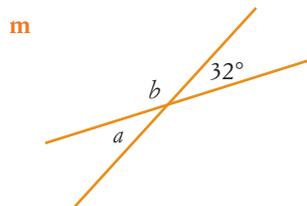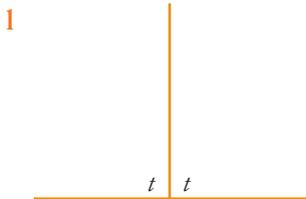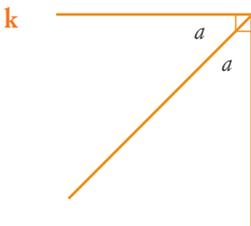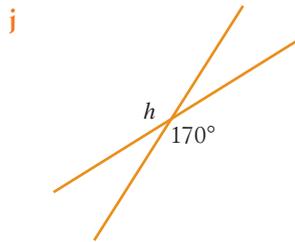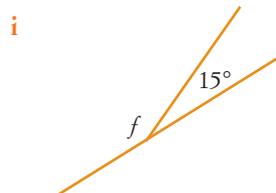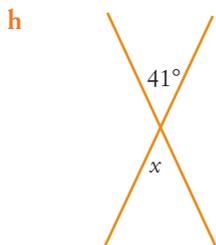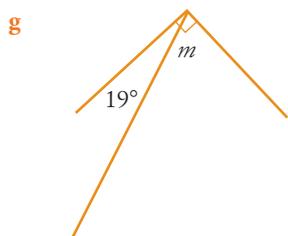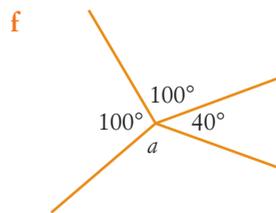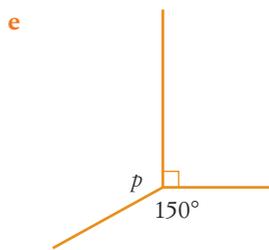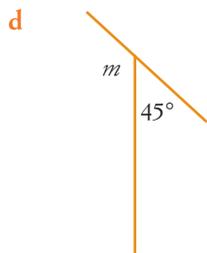
16 Write the size of each angle shown by a letter.

a

110°

m

b

a 50°

c

n

80°

d

122°

b

e

20°

h

f

n

m

50°

p

17 Write the size of each angle shown by a letter.

a

120°

a

b

y

28°

c

m

63°

d

108°

c

e

t

74°

f

50°

a b

60°
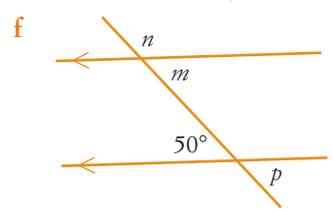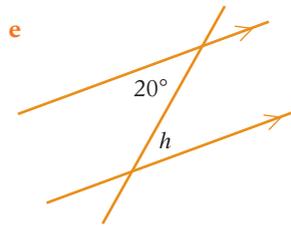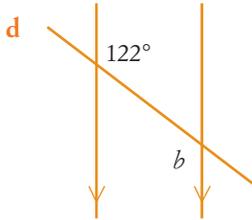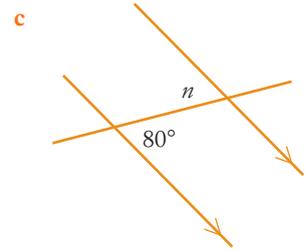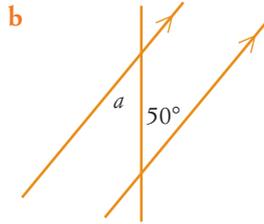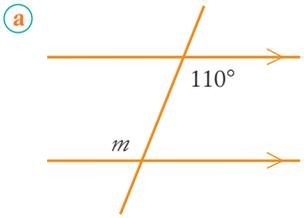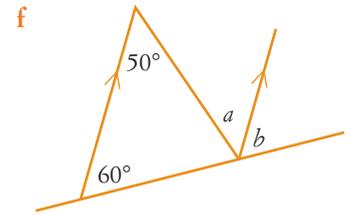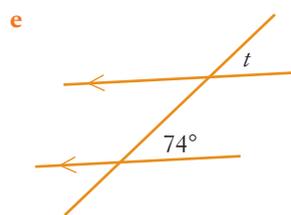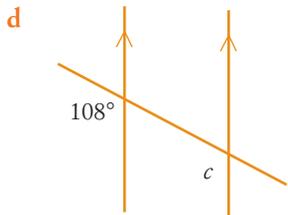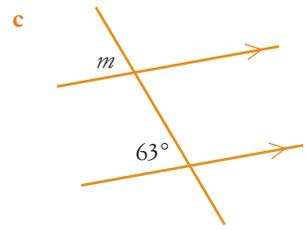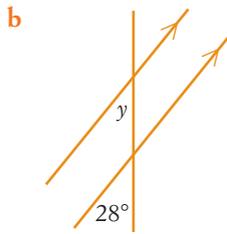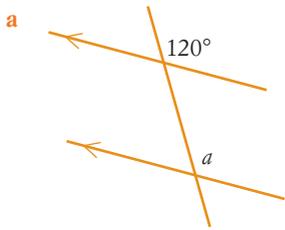
18 Without measuring, find the size of the other seven angles in this diagram.
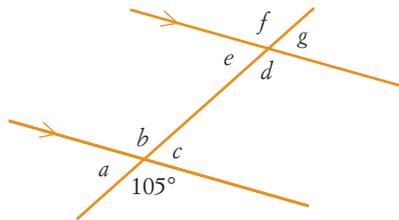
f g
e d

b c
a 105°

19 Without the use of instruments, find the size of the angles shown by the letters.
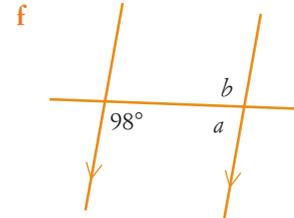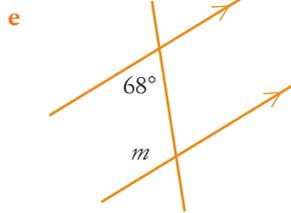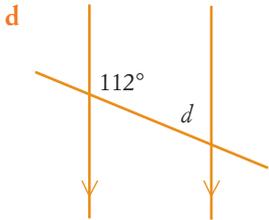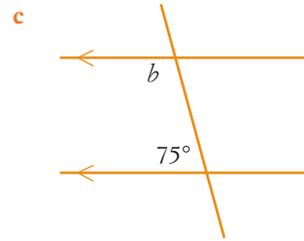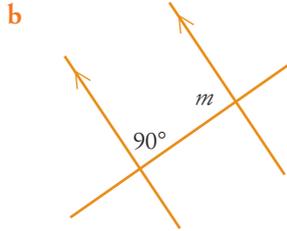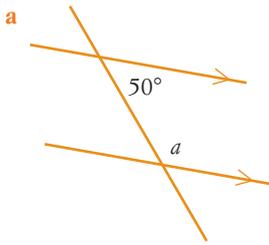
a

50°

a

b

m

90°

c

b

75°

d

112°

d

e

68°

m

f

b

98°   a

20 Without the use of instruments, find the size of each angle shown by a letter.

a

p

115°

b

71°

k

c

105°

t

d

m

120°

e

70°

n

f

132°

a

g

28°

x

h

72°   s

i

85°

k
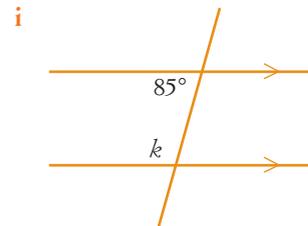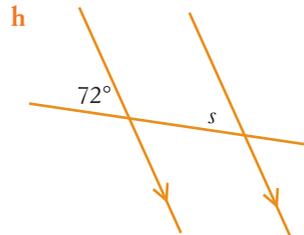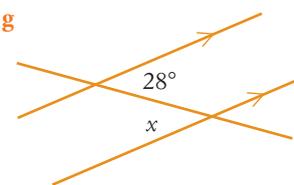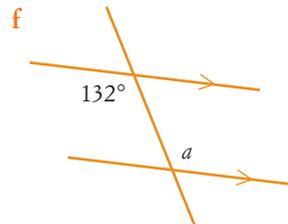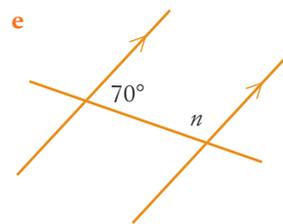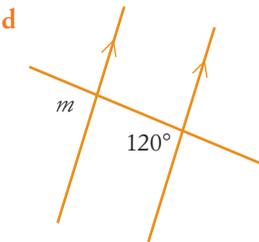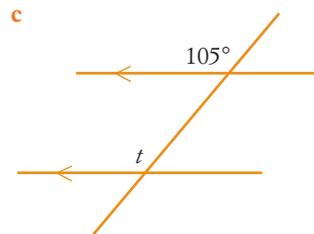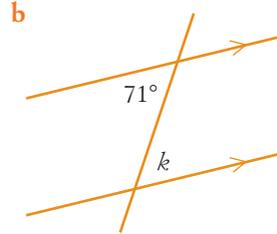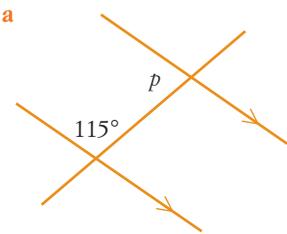
21 Without measuring, find the size of all the angles labelled with letters in these diagrams.

a

b

c

d

e

f

g

h

i
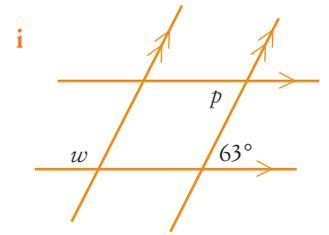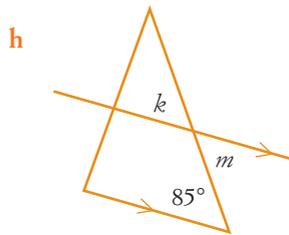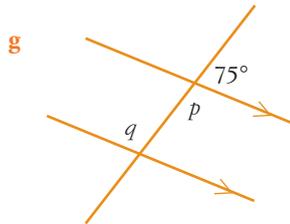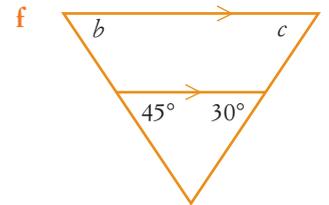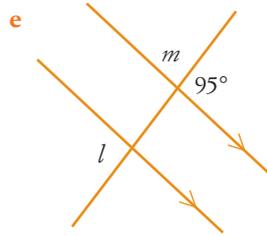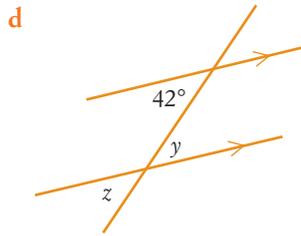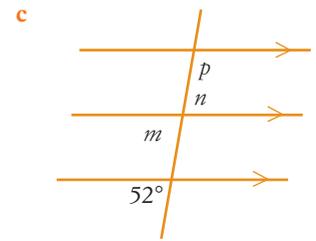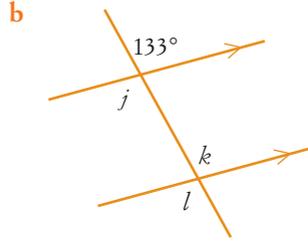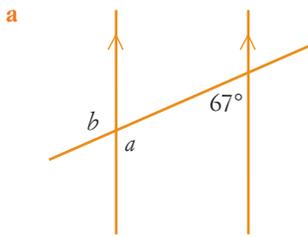
See Examples 14, 15 **22** Find the sizes of the lettered angles in the following diagrams without measuring.

a

b

c

d

e

f

g

h

i
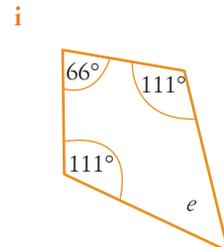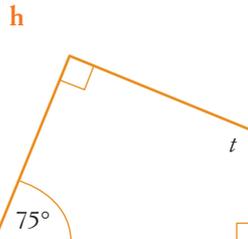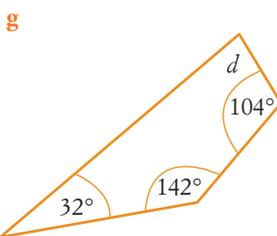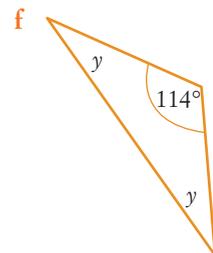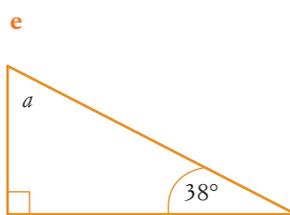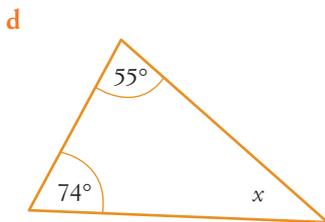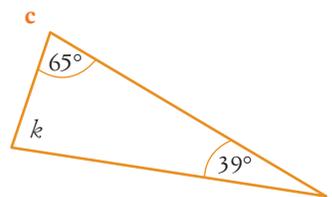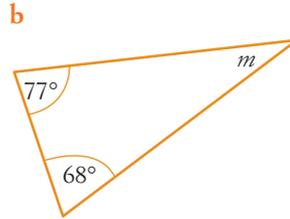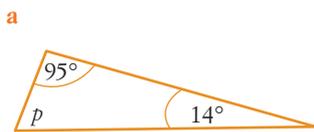
23 Describe all the examples of parallel lines, perpendicular lines and intersecting lines you can find in the photograph below.

24 Which of the following does *y* equal? Select **A**, **B**, **C** or **D**.
   A 28°   **B** 47°   **C** 77°   **D** 152°

y

28°

105°
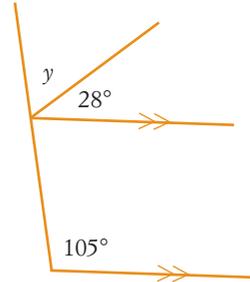
25 In each diagram below, name a pair of alternate angles and use them to decide if *AB* is parallel to *CD*.

a

E

A ———— *B*

64°

64° *F*

C ———— *D*

b

A

C

100°

G

H

100°

B

D

c

B

D

E 32°

35° *F*

A

C
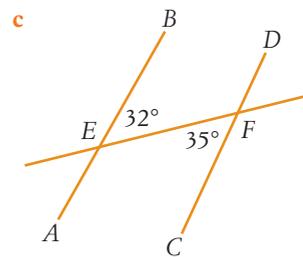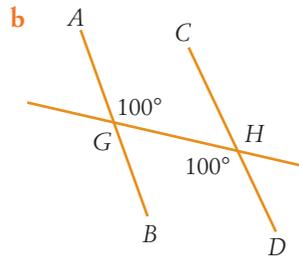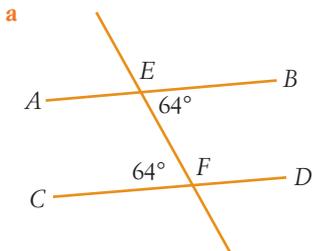
26 In each diagram below, name a pair of corresponding angles and use them to decide if *AB* is parallel to *CD*.

a

b

c
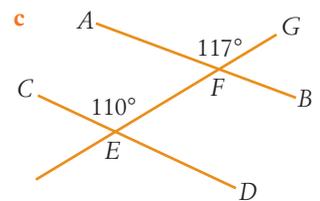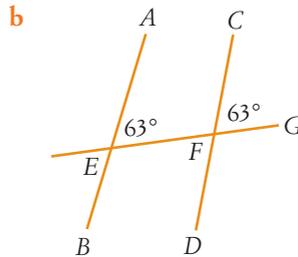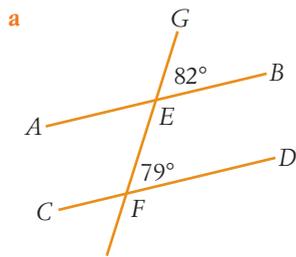
27 In each diagram below, name a pair of co-interior angles and use them to decide if *AB* is parallel to *CD*.

(a)

b

c
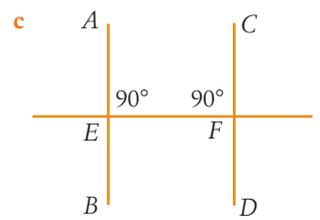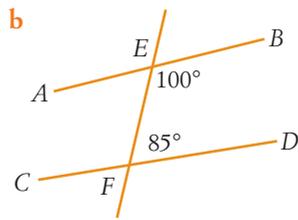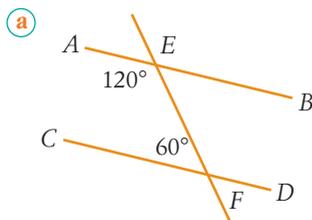
28 For each diagram, determine if the line *PQ* is parallel to the line *MN*. Explain your reason.

a

b

c

d
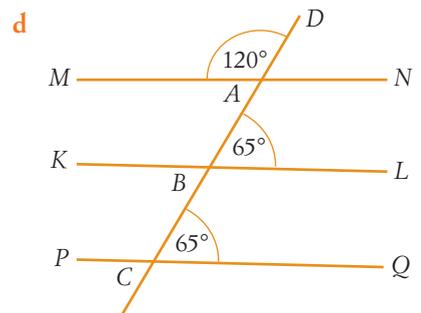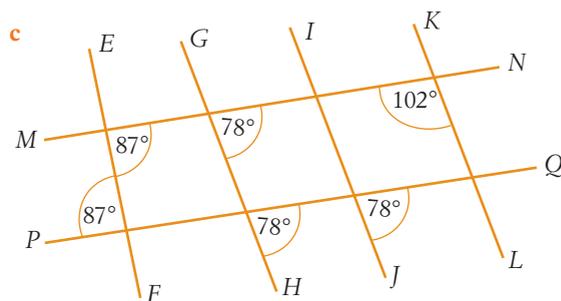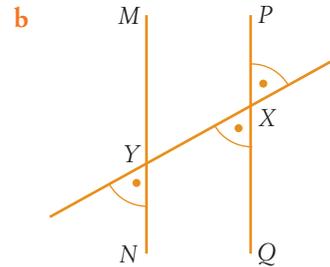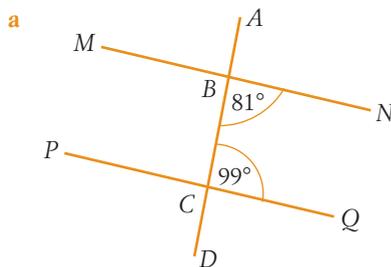
29 What reason can be used to prove that *GC* ‖ *HE*?
Select **A**, **B**, **C** or **D**.
A ∠*ABC* = ∠*HDF* (corresponding angles)
B ∠*CBD* = ∠*BDH* (alternate angles)
C ∠*ADE* = 91° (corresponding angles)
D ∠*BDE* = ∠*FDH* (vertically opposite angles)
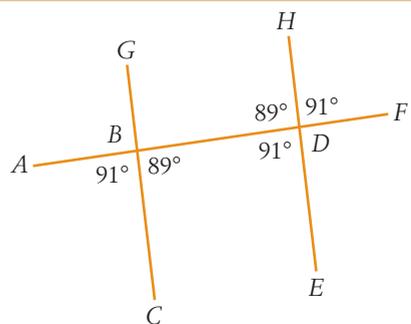
3.3 Naming flat shapes

You have seen how to name points, lines and angles. It is also important to be able to name and label flat shapes in a consistent way.

Triangles

A **triangle** is identified by the capital letters that label its angles. The triangle in this diagram can be labelled $\triangle ABC$ or $\triangle BCA$ or $\triangle CAB$.

The **sides** of a triangle can be described in two ways:

- by two capital letters labelling their endpoints
- by a small letter that matches the capital letter naming its opposite angle.

This diagram shows angles labelled A, B, C. The sides are labelled a, b, c, where side a is opposite $\angle A$, side b is opposite $\angle B$, and side c is opposite $\angle C$.
The purple side can be called CB, BC or a.
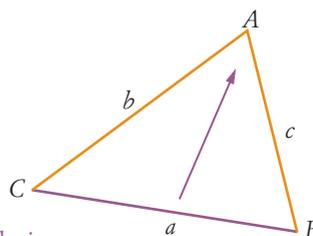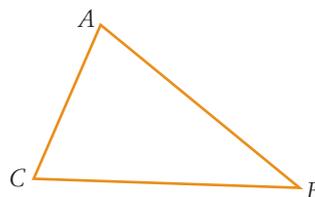
The side labelled a is opposite the angle labelled A.

Quadrilaterals

A **quadrilateral** is any plane shape with four sides and is identified by the capital letters that label its angles.
The quadrilateral in this diagram is labelled $PQRS$ or $QRSP$ or $RSPQ$ or $SPQR$.
The **sides** of a quadrilateral can be identified by the two capital letters labelling their endpoints. The purple side can be labelled PS or SP.
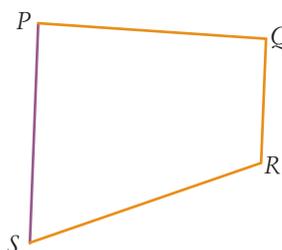
Equal angles and intervals

In geometric diagrams, **equal angles** are marked by identical symbols, while **equal intervals** are marked by identical dashes. In this diagram, DF, DE and EG are all the same length while $\angle F$ and $\angle DEF$ are the same size.
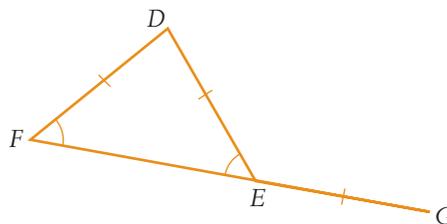
Parallel and perpendicular intervals

Parallel intervals point in the same direction and do not intersect. In rectangle $WXYZ$ shown on the right, WZ is parallel to XY, which is written $WZ \parallel XY$.
Perpendicular intervals meet at right angles (90°). In the rectangle $WXYZ$, WX is perpendicular to XY, which is written as $WX \perp XY$.
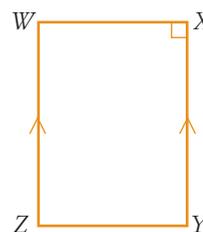
Example 16

An isosceles triangle has two equal sides. $\triangle JKL$ is an isosceles triangle. The interval JM divides $\triangle JKL$ into two smaller triangles.
a Name the two smaller triangles.
b What can be said about sides JL and JK?
c Name the two equal angles in $\triangle JKL$.
d Explain the meaning of this sentence:
   'If $JM \perp LK$, then $LM = MK$'.
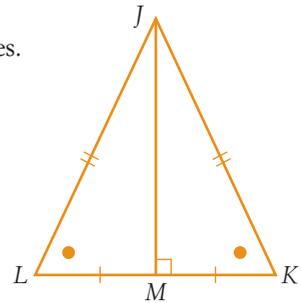
Solution

a Use the letters naming the angles.   **The smaller triangles are $\triangle JML$ and $\triangle JKM$.**

b JL and JK have identical symbols.   **JL and JK are of equal length.**

c Look for angles with
   identical symbols.   **$\angle JLM = \angle JKM$ or $\angle L = \angle K$**

d Replace the symbols with words.   **If side JM is perpendicular to side LK, then the lengths of intervals LM and MK are equal.**

Example 17

The figure shown here is called a parallelogram.
a Name the parallelogram.
b Name both pairs of parallel sides.
c Name any pairs of equal sides.
d Name any pairs of equal angles.
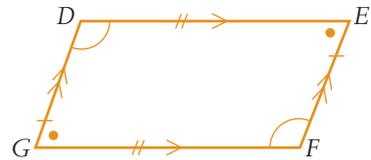
Solution

a Use the angle labels to name the parallelogram.   **The parallelogram is $DEFG$.**

b Parallel intervals are marked with arrowheads.   **$DE \parallel GF$ and $DG \parallel EF$.**

c Equal intervals are marked with identical symbols. **$DE = GF$ and $DG = EF$.**

d Equal angles are marked with identical symbols.   **$\angle GDE = \angle GFE$ or $\angle D = \angle F$ and $\angle DGF = \angle DEF$ or $\angle G = \angle E$.**

A **polygon** is a plane shape made up of straight sides. The word 'polygon' means 'many angles'. These shapes are all polygons.
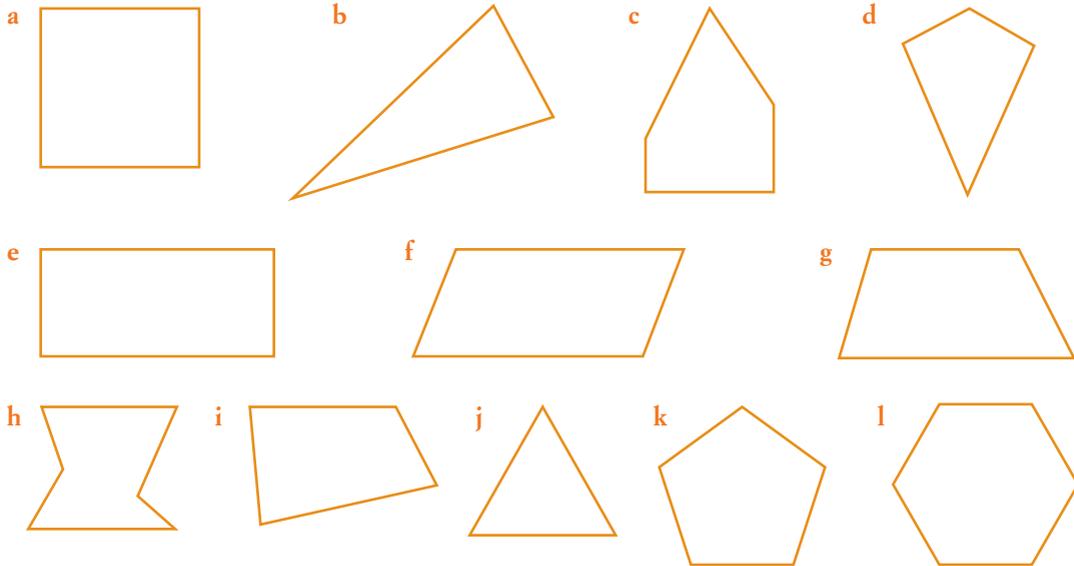
a

b

c

d

e

f

g

h   **i**   **j**   **k**   **l**

A polygon's name is determined by the number of sides that it has.

Name	Number of sides
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10
Undecagon	11
Dodecagon	12

Convex and non-convex polygons

Convex polygons have vertices that *all* point *outwards* while **concave** (or **non-convex**) polygons have *some* vertices that point or cave *inwards*.

Convex polygon

Concave polygon

A simple test to determine whether a polygon is convex or concave is to draw diagonals joining two vertices of the polygon.

If all diagonals lie completely *inside* the polygon, then it is convex.
If all or part of any diagonal lies *outside* of the polygon, then it is concave.
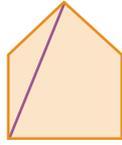
Investigate: Logos and designs

Find examples of company logos.
Draw them in your book. Discuss
the shapes used to make them.
Research some Islamic or Grecian
art in the library or on the Internet.
Bring some pictures to class.

A triangle, having three sides, is the simplest type of polygon. It is an important shape that has
been used by many civilisations throughout history for building, construction and packaging. It
has even been used as a cultural or religious symbol.

Triangles can be classified in two ways:

• by their **sides** (equilateral, isosceles or scalene)
• by their **angles** (acute-angled, right-angled, obtuse-angled).

Technology

Geogebra:
Classifying triangles

MAT07MGCT00003

Sides			
	Equilateral Three equal sides	**Isosceles** Two equal sides	**Scalene** No equal sides
Angles	**Acute-angled** All three angles acute	**Right-angled** One right angle	**Obtuse-angled** One obtuse angle

Example 18

Classify this triangle using sides and angles.
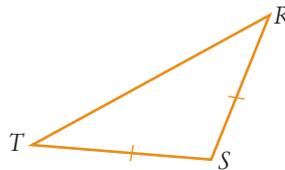
Solution

The triangle has two equal sides.	$\triangle RST$ **is isosceles.**
$\angle RST$ is an obtuse angle.	$\triangle RST$ **is obtuse-angled.**
State the result.	$\triangle RST$ **is an obtuse-angled isosceles triangle.**

Investigate: Building with shapes

1 When you look at the shapes of buildings and other constructions, you will notice that some shapes are more common than others. Write the names of the most commonly used shapes.

2 When building any structure, strength is important. Which is the strongest shape of those you have listed?

3 **a** Use ice block sticks or geo-sticks to make a triangle, a square and a pentagon as shown on the right.
  b Stand each shape up and push one corner. What happens?
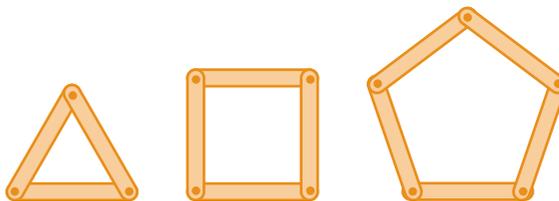
4 You saw in task **3** that a triangular framework is very strong or rigid, which is why that shape is used in many types of constructions. How can you make the other shapes in task **3** stronger?

5 Find as many pictures as you can of triangular frameworks in everyday use. The Anzac Bridge in Sydney is a good example.

A quadrilateral is any shape with four sides, but there are six special quadrilaterals that you need to know. These are listed in the table below.

Name	A quadrilateral with:	Diagrams
Trapezium	one pair of opposite sides parallel	
Parallelogram	two pairs of opposite sides parallel	
Rhombus (or diamond)	four equal sides	
Rectangle	four right angles	
Square	four equal sides and four right angles	
Kite	two pairs of adjacent sides equal	

Example 19

PQRS is a parallelogram, as shown on the right.
a Measure the lengths of its sides. Are opposite sides equal?
b Measure the size of its angles. Are opposite angles equal?
c Draw the diagonals *PR* and *QS* and measure them. Are the diagonals equal?

Solution

a Measure the sides.

State the result.

$PQ = SR = $ **4.2 cm**
$PS = QR = $ **2 cm**
Opposite sides are equal.

b Measure the angles.

$\angle PSR = \angle PQR = 80°$
$\angle QPS = \angle QRS = 100°$

State the result.

Opposite angles are equal.

c $PR = 4.2$ cm and $QS = 5$ cm.

The diagonals of the parallelogram are not equal.

Example 20

This diagram illustrates the properties of the diagonals of a rectangle.

a Are the diagonals equal?
b Do the diagonals bisect each other?
c Do the diagonals intersect at right angles?
d Do the diagonals bisect the angles of the rectangle?
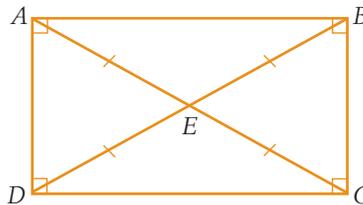
Solution

a Look at the markings on the diagonals.

$AE = DE$ **and** $BE = CE$.

State the result.

Diagonals AC **and** BD **have equal length.**

b Look at the markings on the diagonals.

$DE = EB$ **and** $AE = EC$.

State the result.

The diagonals bisect each other.

c Measure the angles at E.

$\angle AED = \angle BEC \neq 90°$
$\angle AEB = \angle DEC \neq 90°$

State the result.

The diagonals do not intersect at right angles.

d Measure the angles at the corners.

$\angle EAB \neq \angle EAD$, $\angle ABE \neq \angle EBC$
$\angle ADE \neq \angle EDC$, $\angle ECD \neq \angle ECB$

State the result.

The diagonals do not bisect the angles of the rectangle.

Investigate: Shape puzzles

All of the shapes used in this investigation can be printed using the Puzzle sheet material.

1 a How many squares can you find in this shape?
(The answer is not 16!)
b How many rectangles can you find?

2 Can you trace this shape without going over any line twice and without lifting your pencil off the paper?
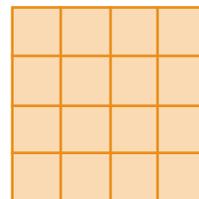
Puzzle sheet
Shapes puzzle 1
MAT07MGPS00019

Weblink
Tangrams

3 There are 12 different ways to join five squares edge to edge. These shapes are called pentominoes. Here are five of them. Draw the other seven.
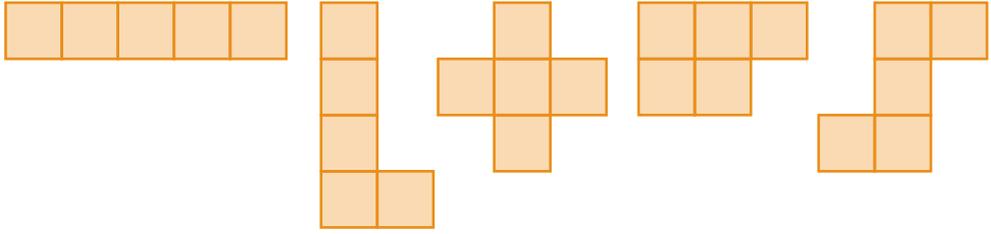
4 Copy this equilateral triangle, cut out the four pieces and rearrange them into a square.
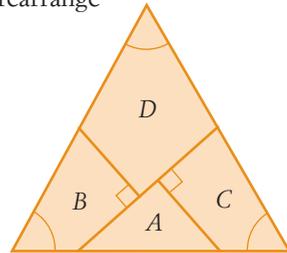
5 How many triangles can you find in each of these shapes?

   a   **b**   **c**   **d**
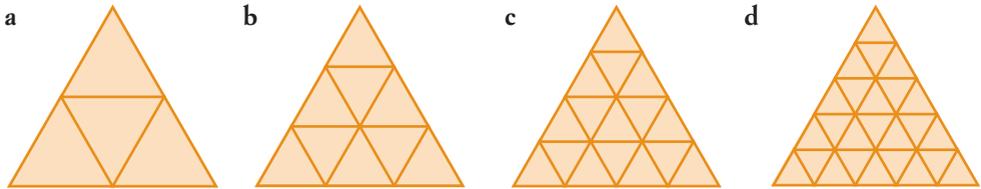
6 Copy the hexagon twice onto a piece of paper and then:
   a cut the first hexagon into two pieces and rearrange them to make a parallelogram
   b cut the second hexagon into three pieces and rearrange them to make a rhombus.
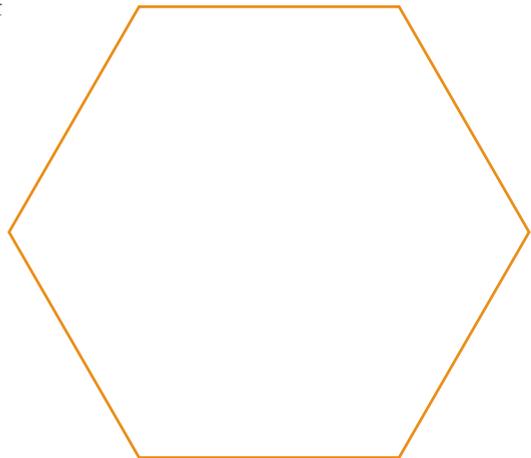
Exercise 3.3 Naming flat shapes

1 In this diagram, name a pair of intervals that are:
   a equal
   b perpendicular
   c parallel.
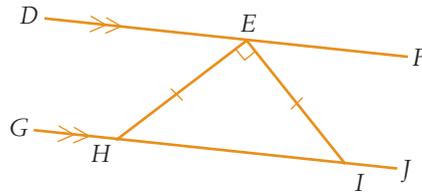
2 a Draw two lines, *PQ* and *RS*, intersecting at *T*.
   b Mark the equal angles ∠*PTR* and ∠*STQ*.

   c ∠*PTR* = ∠*STQ*. Why?

3 Copy the triangle *KLM* shown on the right, and correctly label its sides *k*, *l*, and *m*.
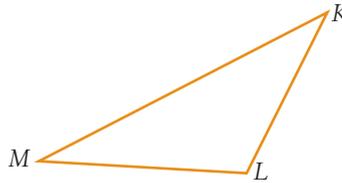
4 What is the difference between the **line** *EF* and the **interval** *EF*?

5 *CDEF* is a kite.
   a Copy the diagram and mark in the equal sides.
   b What side is equal to *DE*?
   c Mark the equal angles ∠*F* and ∠*D*.
   d Draw the two diagonals *FD* and *CE*.
   e Show on your diagram that *FD* ⊥ *CE*.
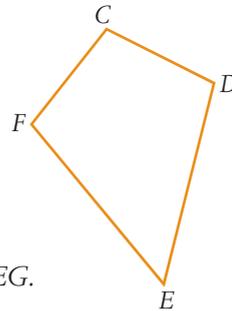
6 a Draw an isosceles triangle △*EFG* where *EF* = *EG*.
   b Label the sides of the triangle *e*, *f*, and *g*.
   c What is another name for the side *EF*?
   d Mark on the triangle the equal angles ∠*F* and ∠*G*.

7 a Draw parallel lines *AB* and *CD*.
   b Draw a transversal *EF* crossing both lines *AB* and *CD*, where *EF* ⊥ *AB*.
   c *CD* ⊥ *EF*. True or false?

8 a Draw a square, *WXYZ*, and mark all equal sides and angles.
   b Name the point where side *XY* meets side *ZY*.
   c Name a pair of parallel sides and mark them.
   d Name a pair of perpendicular sides.
   e Explain the meaning of:
      i *WX* ⊥ *XY*
      ii *WX* = *XY*

9 How many sides has:

a a hexagon?   **b** a quadrilateral?   **c** a nonagon?

d a decagon?   **e** a heptagon?   **f** a pentagon?

g a dodecagon?   **h** an octagon?   **i** a undecagon?

10 Which shape is a regular convex octagon? Select **A**, **B**, **C** or **D**.

A   **B**   **C**   **D**

11 Draw the following shapes.

a a regular hexagon   **b** a non-regular hexagon   **c** a regular triangle

d a non-regular heptagon   **e** a convex pentagon   **f** a non-convex dodecagon

Fluency

12 **a** Name the two triangles in the diagram on the right.

b Name the interval that is equal to:

i *QR*   **ii** *PT*.

c If *TQRS* is a trapezium, name the parallel sides.

d Copy the diagram and mark the parallel sides.

e Mark the angles $\angle PTQ$ and $\angle TSR$.

f $\angle PTQ = \angle TSR$. Why?

13 **a** $VS = ST$. True or false?

b $VS \parallel ST$. True or false?

c $\angle SVT = \angle VTU$. True or false?

d Name the marked pair of equal intervals.

14 Name each of the polygons (*a* to *l*) on page 129, and state which one is non-convex.

15 Regular polygons have all sides equal and all angles equal. Which of the polygons from question **10** are regular?

16 Which of the following shapes are not polygons?

A a trapezium   **B** an ellipse   **C** a square

D a diamond   **E** a prism   **F** a circle

Worked solutions

Exercise 3.3

MAT07MGWS00014

See Example 18

17 How many diagonals has a:

a kite?   **b** pentagon?   **c** hexagon?

18 Rule up a table with these headings.

Acute-angled	Obtuse-angled	Right-angled	Equilateral	Isosceles	Scalene

Place the letter of each of the following triangles under the headings that match. (The same triangle may appear under more than one heading.)

a

b

c
30°
88°
62°

d

e
20°
140°
20°

f

g
3 cm
3 cm
3 cm

h

i

j
60°
60°
60°

k
12 cm
12 cm

l
15 cm
12 cm
6 cm
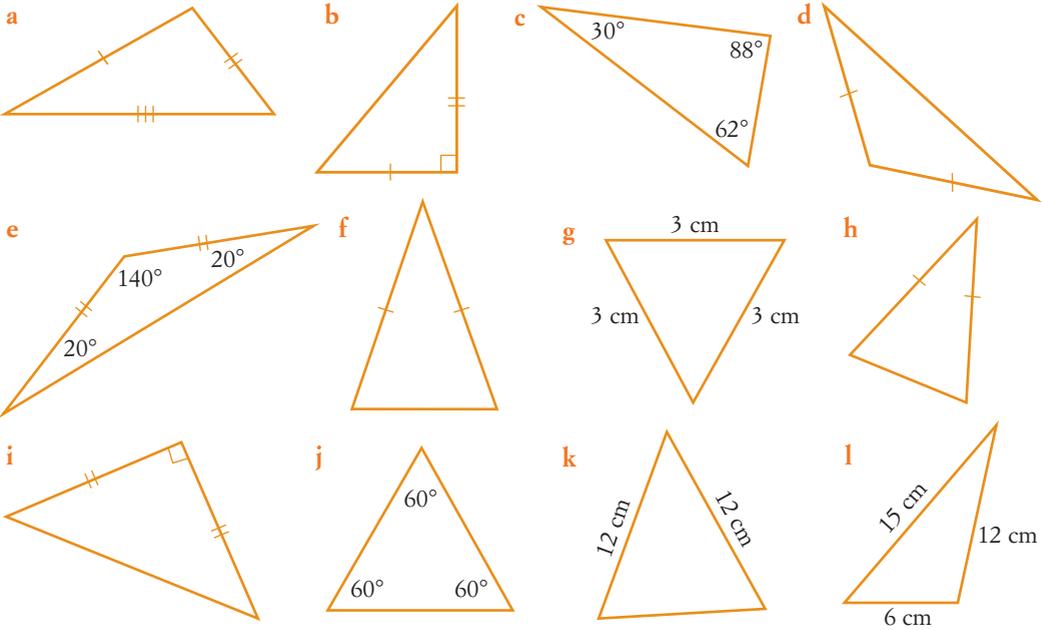
19 Draw the following triangles.

See Example 16

   a a scalene triangle
   b a right-angled scalene triangle
   c an isosceles triangle
   d an equilateral triangle
   e a right-angled isosceles triangle
   f an acute scalene triangle

20 Measure these triangles and determine their side names.

a

b

c

d

e

f
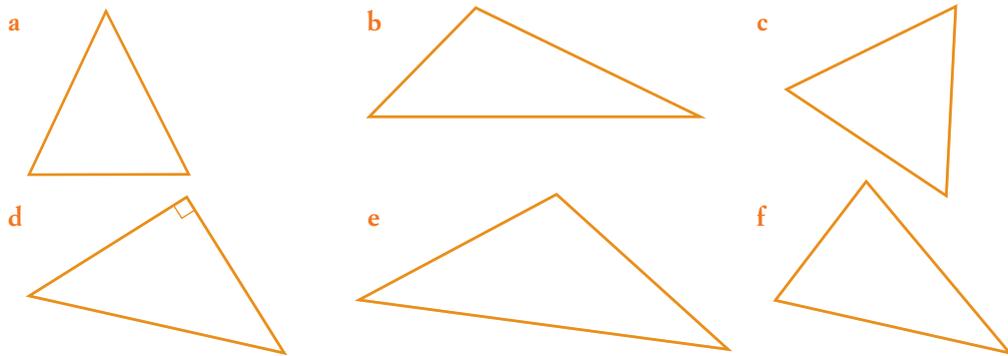
21 Which triangle is an obtuse-angled scalene triangle? Select **A**, **B**, **C** or **D**.

A

B

C

D
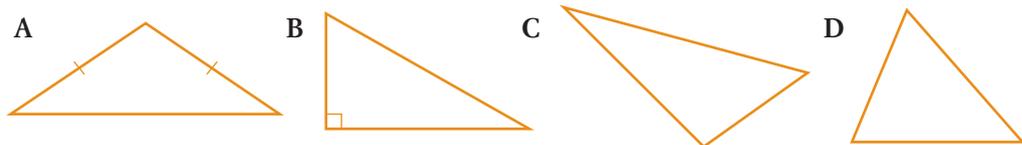
22 Find out what the prefix 'quad' means. List other words beginning with 'quad'.

23 Label each of these quadrilaterals as convex or concave (non-convex).

a

b

c

d

e

f
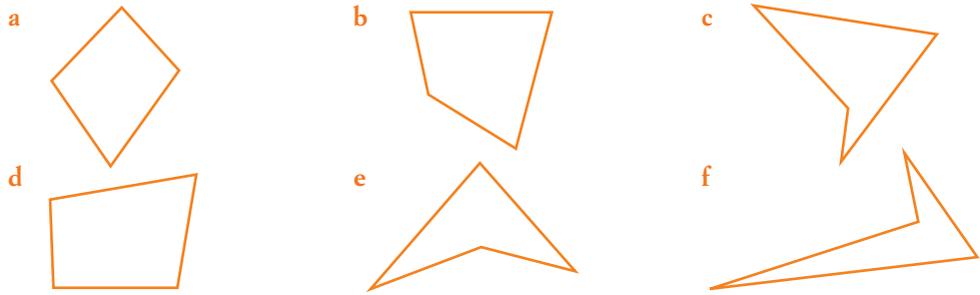
24 Classify each quadrilateral in this diagram.
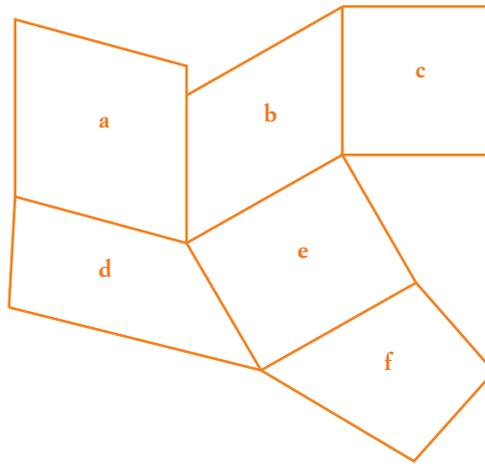
See Examples 17,18   **25 a** Cut out an example of each quadrilateral in question **24**. Copy this table.

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Opposite sides are equal						
Opposite sides are parallel						
Opposite angles are equal						
All angles are equal						
Diagonals are equal						

   b Test the properties of each quadrilateral listed in the table by folding and measuring them with a ruler, protractor and set square. If the listed property is true, then place a tick in the appropriate space in the table.

   c Check your results with your teacher.

   d You should have noticed that there are no ticks for the kite. Write two properties of the kite (that is two things that are always true about its sides, angles or diagonals).

26 Use a ruler and protractor with the quadrilaterals you cut out in question **25** to discover the properties of the diagonals of each one, as listed in the table below. Copy or print out this table. Place ticks in the appropriate spaces.

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Diagonals are equal						
Diagonals bisect each other						
Diagonals intersect at right angles						
Diagonals bisect angles of quadrilateral						

27 Copy this table of properties of quadrilaterals into your book and complete it.

Shape	Properties
Trapezium	• One pair of _____ sides
Kite	• _____ pairs of adjacent sides are equal • One pair of opposite angles are _____ • Diagonals intersect at _____ angles
Parallelogram	• _____ sides are equal and parallel • Opposite angles are _____ • Diagonals _____ each other
Rhombus	• All four sides are _____ • _____ sides are parallel • Opposite angles are _____ • _____ bisect at right angles • Diagonals bisect the _____ of the rhombus
Rectangle	• All four angles measure _____ • Opposite sides are _____ and _____ • Diagonals are _____ • _____ bisect each other
Square	• All four sides are _____ • All four angles measure _____ • Diagonals are equal and _____ each other at right angles • _____ bisect the angles of the square
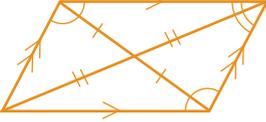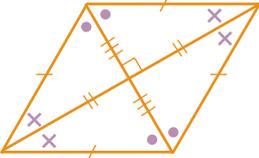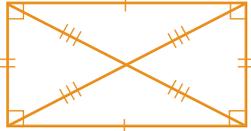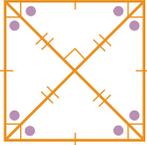
Problem solving

28 What minimum number of shapes have been put together to form each of these composite shapes?

a

b

c

d

e
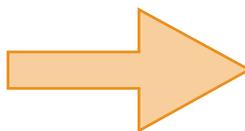
f

29 Copy these composite shapes into your book and divide them into the shapes listed.

a
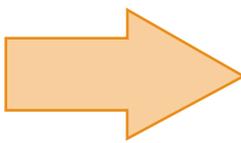
Two pentagons

b
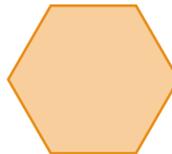
Two triangles and one rectangle

c
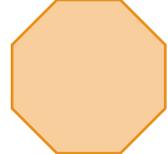
One trapezium and one hexagon

d

Two trapeziums

e
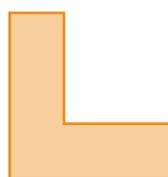
Four triangles

f

One triangle and one trapezium

g

One square and one heptagon

30 Which quadrilateral am I? (There may be more than one answer.)
  a My diagonals are equal.
  b All my sides are equal.
  c My opposite sides are equal.
  d My diagonals bisect each other.
  e I have four right angles.
  f I have two pairs of parallel opposite sides.
  g My diagonals bisect each other at right angles.

Reasoning

31 Is it possible to draw an equilateral right-angled triangle? Why?

32 Is it possible to draw a triangle with two obtuse angles? Why?

33 a Does a square have all the properties of a rectangle? Why?
  b Does a rhombus have all the properties of a parallelogram? Why?

3.4 Sketching and constructing flat shapes

The set square

A set square is made in the shape of a right-angled triangle. It is used for measuring and drawing right angles, perpendicular and parallel lines. There are two types of set squares, named according to their angle sizes.

The 60°–30° set square

The 45° set square
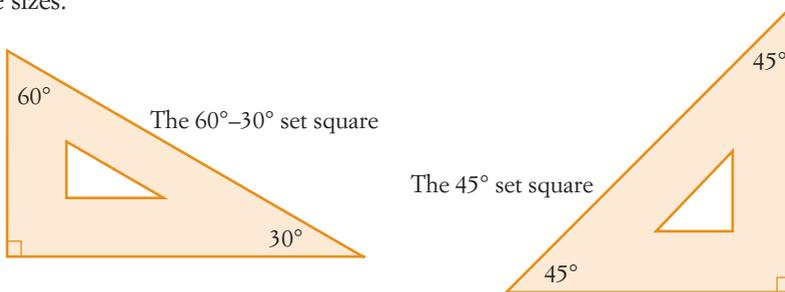
Perpendicular lines and parallel lines can be drawn using a set square or a protractor.

Example 21

Use a set square to draw a line perpendicular to *XY* through point *W*.

Solution

Place the set square on *XY* with the right angle at *W*.

Draw a perpendicular line through *W* using the side of the set square.
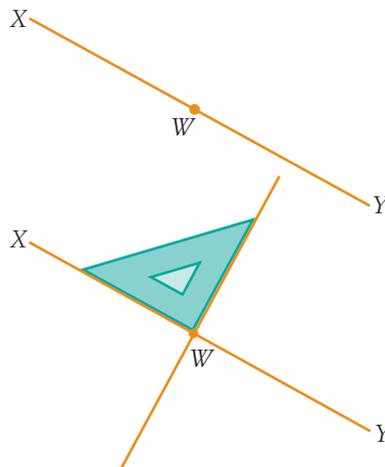
In the previous example, the perpendicular line could also have been drawn by using a protractor to measure a 90° angle.

In Geometry, the words sketch, draw and construct have different meanings.

Important!

Sketching, drawing and constructing

In mathematics, **sketch** means to make a freehand drawing, without the use of a ruler or other instruments. **Draw** means to make an accurate (or scale) drawing, using whatever instruments are available. **Construct** means to make an accurate drawing using only a straight-edge (ruler without markings) and a pair of compasses. Construction is sometimes called **Euclidean construction**, after the ancient Greek mathematician Euclid (about 300 BCE).

It is also possible to construct a line perpendicular to another line using a compass.

1
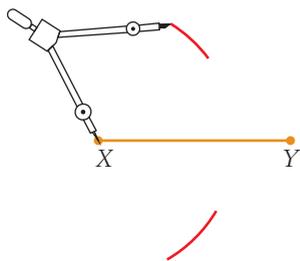
Begin with the line and mark points X and Y on it.
Set the compasses to more than half XY.
From X, draw arcs above and below XY.

2
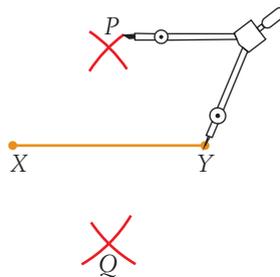
Keep the same compass setting. From Y, draw arcs above and below XY.
Mark intersections P and Q.

3
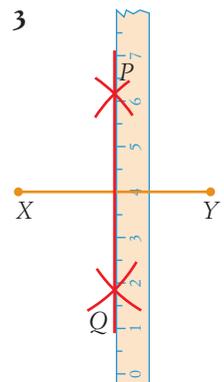
Draw the line PQ.
PQ bisects XY at right angles.

Example   22

Construct a line perpendicular to AB through the point C.

Solution

Copy the diagram and place the compass point on C and draw arcs through AB, cutting through the line at X and Y.
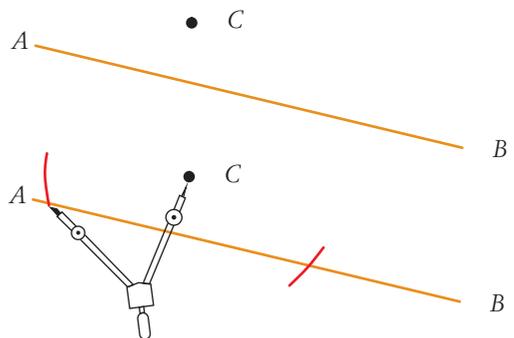
Now bisect the interval *XY* to obtain a perpendicular to *AB* passing through *C*.
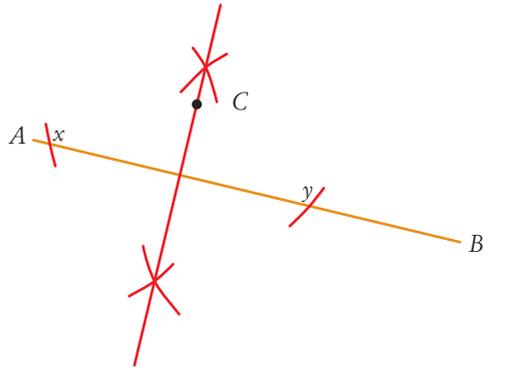
Example   23

Use a set square to draw a line parallel to *AB* through point *P*.

Solution

Step 1: Place the set square on the line *AB*.

Step 2: Place the ruler next to the set square, at right angles to *AB* so that the edge of the ruler passes through point *P*.

Step 3: Hold the ruler firmly and slide the set square until its edge passes through point *P*.

Step 4: Rule the parallel line.
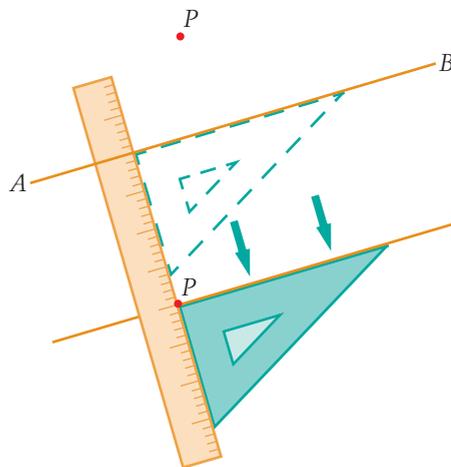
To construct a triangle we need to know the length of its sides and the size of its angles. We also need a ruler, a protractor and a compass.

The following examples will show you how to construct triangles. *Hint*: Draw a rough sketch before beginning the construction.

Example 24

Draw a triangle with sides 3 cm, 5 cm and 4 cm.

3 cm   4 cm

Rough sketch

5 cm
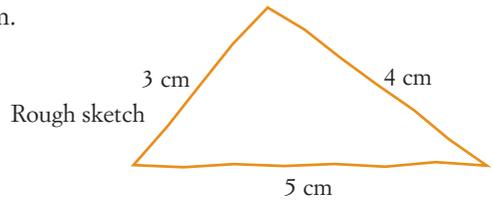
Solution

Step 1: Draw an interval 5 cm long. (It is easier to start with the longest side.)

5 cm

Step 2: Open a compass to a 3 cm radius and draw an arc from one end of the interval.

(Every point on this arc is 3 cm from the end of the interval.)

3 cm

5 cm
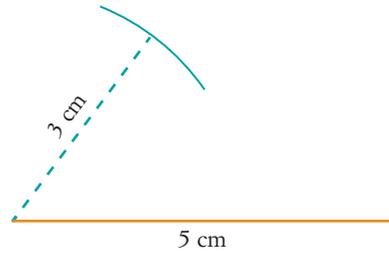
Step 3: Open the compass to 4 cm and draw an arc from the other end of the interval. (Every point on this second arc is 4 cm from the other end of the interval.)

4 cm

5 cm
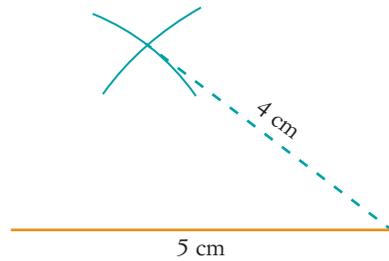
Step 4: Complete the triangle by joining the intersecting point of the arcs to the ends of the interval.

3 cm   4 cm

5 cm
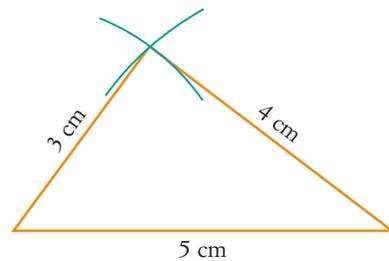
Example 25

Draw $\triangle ABC$ where $a = 5$ cm, $\angle C = 30°$ and $\angle B = 70°$.

A

b

Rough sketch

c

C 30°

70°

B

$a = 5$ cm
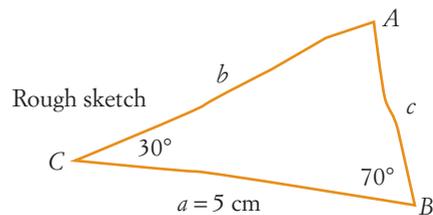
Solution

Step 1: Draw an interval 5 cm long.

Step 2: Draw a 70° angle at *B*.
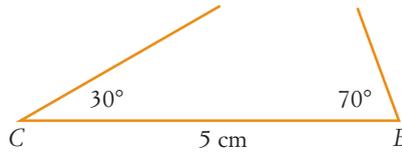
Step 3: At *C*, draw a 30° angle.

Step 4: Join the arms to complete the triangle.
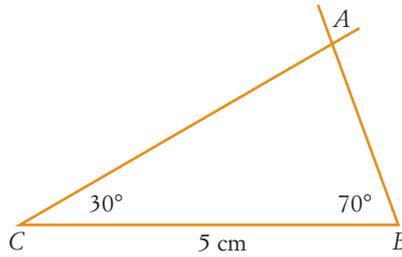
Example 26

Draw a triangle with one side measuring 6 cm, another side measuring 4 cm and an angle between them of 35°.

Rough sketch
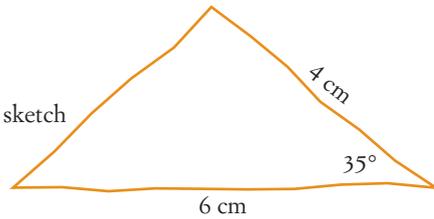
Solution

Step 1: Draw an interval 6 cm long.

Step 2: Draw an angle of 35° at one end.
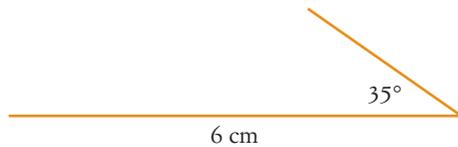
Step 3: Measure an interval of 4 cm on the new arm.
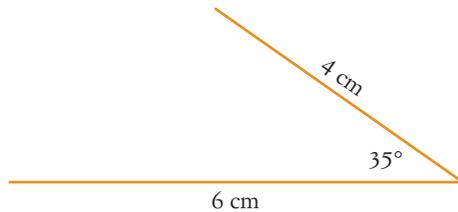
Step 4: Complete the triangle.
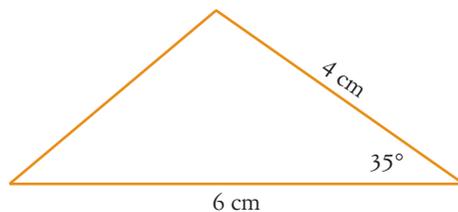
Note: 35° is called the **included angle** because it is between the two sides.

You can use similar techniques to construct quadrilaterals.

Example 27

Draw a square, *FGHI*, of side length 4 cm.

Rough sketch
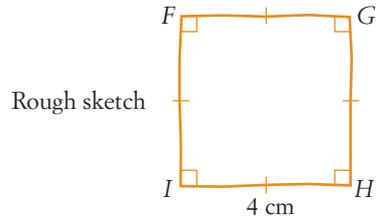
Solution

Step 1: Construct the base, *IH*, of length 4 cm.

Step 2: Use a set square to construct the perpendiculars, *FI* and *GH*, of length 4 cm.

Step 3: Join *FG*.
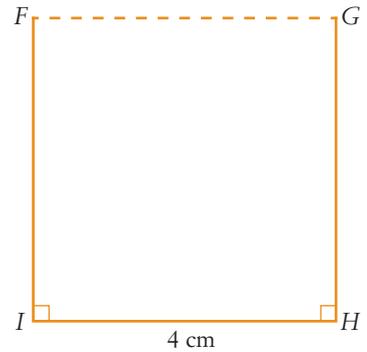
Example 28

Draw this kite, *PQRS*.
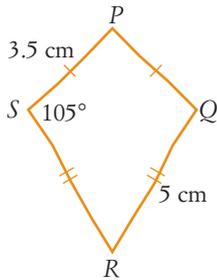
Solution

Step 1: Draw *PS* of length 3.5 cm.

Step 2: Measure 105° at ∠*S*.

Step 3: Draw *SR* of length 5 cm.

Step 4: At *R*, use a compass to draw an arc of radius 5 cm.

Step 5: At *P*, use a compass to draw an arc of radius 3.5 cm.

Step 6: The arcs cross at *Q*, the fourth vertex of the kite. Join *P* and *R* to *Q*.

Exercise 3.4 Sketching and constructing flat shapes

1 What is the name of the set square that is:
  a tall and thin?
  b short and wide, and half of a square?
  c an isosceles, right-angled triangle?
  d a scalene, right-angled triangle?

2 Why do you think the word 'square' in 'set square' is used to describe a right angle? What other types of squares are used to draw or measure right angles?

3 Draw a line and mark a point L on it.
Draw a perpendicular line through L.

4 Draw a line and mark a point X below it.
Construct a perpendicular line to the line through X.

5 Draw a line and mark a point T above it.
Draw a parallel line through T.

6 Draw a line and mark a point P above it.
Draw a line through P perpendicular to the line.

7 **a** Draw an interval, AB, 4 cm long. This will be the base of a triangle.
  b Mark X, the midpoint of AB, and draw a perpendicular interval XC of length 5 cm at X.
  c Join C to A and then to B to make a triangle, $\triangle CBA$.
  d What special type of triangle is $\triangle CBA$?

8 Draw each of the following triangles. Do a rough sketch first.
  a $\triangle ABC$ with $a = 4$ cm, $b = 3$ cm, and $\angle C = 50°$.
  b $\triangle RST$ with $r = 5$ cm, $s = 3$ cm, and $t = 3$ cm.
  c $\triangle PQR$ with $\angle P = 60°$, $\angle Q = 60°$, and $PQ = 4$ cm.
  d $\triangle LMN$ with $LN = 5$ cm, $ML = 4$ cm, and $\angle NLM = 25°$.

9 **a** Which triangle in question **8** is isosceles?
  b Measure its angles. What do you notice?

10 Copy this table of triangle properties into your book and complete it.

Shape	Properties
Equilateral triangle	• All three sides are_____ • All three angles measure _____

Shape	Properties
Isosceles triangle	• Two sides are _____ • Two _____ equal (opposite the equal sides)
Scalene triangle	• No _____ or angles are equal
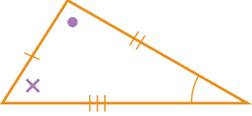
Problem solving

11 a Draw *BA* measuring 6 cm.
  b Draw a perpendicular, *BC*, 3 cm long.
  c Complete the rectangle, *ABCD*.

See Example 27

12 Draw a square, *KLMN*, of side length 5 cm.

13 Draw the trapezium shown at right.
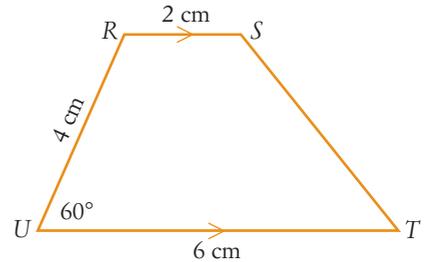
14 Draw a parallelogram with sides of 6 cm and 4 cm and an included angle of 65°.

15 a Draw two parallel intervals 4 cm long and 3 cm apart.
  b Join the ends to make a quadrilateral.
  c Measure the lengths of the two new sides.
  d Are these sides both equal and parallel?
  e What type of quadrilateral is it?
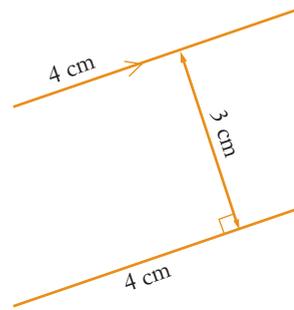
16 a Draw two joined intervals of the same length and use your instruments to complete these shapes.

  i

  ii
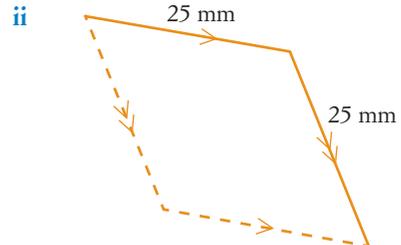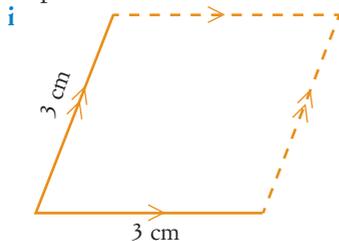
  b What is each shape called?

17 A trapezium with two equal (non-parallel) sides
is called an **isosceles trapezium**.
   a Name the equal sides in the isosceles trapezium,
PQRS, shown on the right.
   b Draw the isosceles trapezium *PQRS*.
   c Measure all four angles of the trapezium.
   d Name all pairs of equal angles.
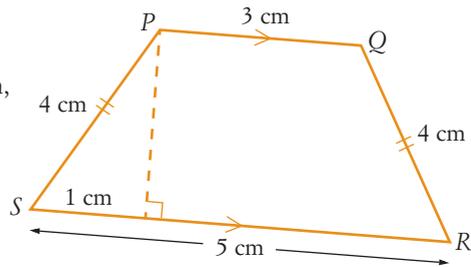
18 Draw a rhombus with sides of 6 cm and an included angle of 50°.

19 **a** Draw the quadrilateral shown at right.
   b What type of quadrilateral is *JKLM*?
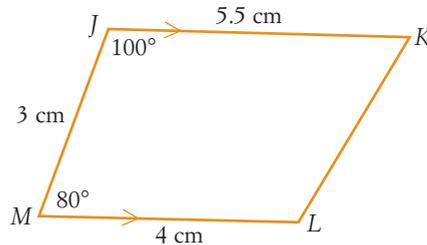
20 Draw the kite shown below.
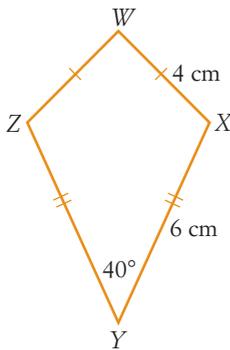
See Example 28

21 Draw the quadrilateral *ABCD*, where $AB \perp BC$, $AB = 7$ cm, $BC = 3$ cm, $DC = 5$ cm and $AD = 4.5$ cm.

22 Draw the trapezium *DEFG* where $DE \parallel GF$, $DE = 6$ cm, $EF = GF = 3$ cm and $\angle F = 135°$.

Reasoning

23 Are parallel lines always the same distance apart?
   a Draw a pair of parallel lines and mark the points *D* and *E*
on one of them.
   b Draw perpendiculars from *D* and *E* to the other line.
Where the lines intersect, mark the points *F* and *G*.
   c Measure the lengths of *DG* and *EF*. What do you notice?
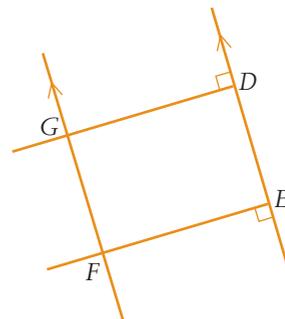
24 Draw an interval and mark its midpoint. Draw another interval of a different length through
this midpoint, perpendicular to the first interval. Join the ends of both intervals to make a
quadrilateral. What type of quadrilateral have you drawn?

25 What is the difference in meaning between 'perpendicular' and 'vertical'?

26 Draw each of these triangles accurately.

a

b

c

d

e

f
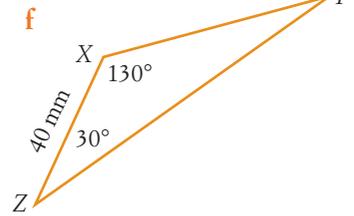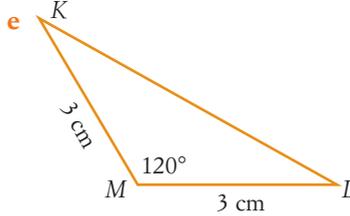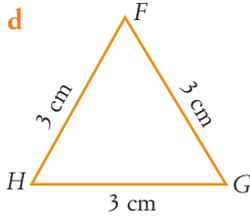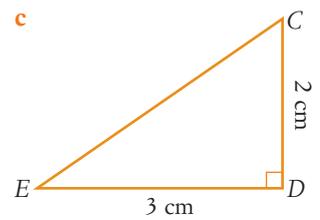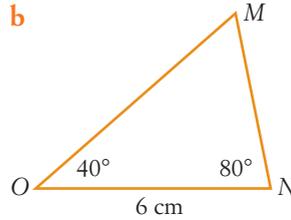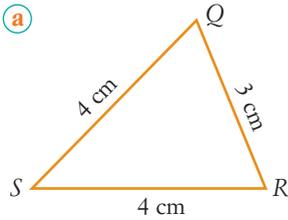
27 For each of the triangles drawn in question **26**:
  i name the largest angle and the longest side
  ii name the smallest angle and the shortest side.

28 Copy and complete this sentence:
In any triangle, the longest side is always _____ the largest angle, while the shortest side is always _____ the _____ angle.

29 a Which triangle in question **26** is equilateral?
  b Measure its angles. What do you notice?

30 a On a sheet of paper, draw an equilateral triangle of side length 5 cm, and cut it out.
  b By folding along each of its axes of symmetry, what do you observe about the sizes of the triangle's angles?
  c Measure the angles. What do you notice?
  d Copy and complete:

An equilateral triangle has three _____ sides, and three _____ angles each of size _____ .

31 The **triangle inequality rule** says that 'if you add any two sides of a triangle, the combined length is always greater than the length of the third side'. (This inequality can be written as $a + b > c$.)
  a Test that this inequality is true for all of the triangles you drew in question **8**.
  b Why is it impossible to draw a triangle with sides of length 7 cm, 15 cm and 5 cm?

3.5 Similarity, congruence and transformations

In mathematics, the terms similar and congruent have special meanings.

> ## Important!
>
> ### Similarity and congruence
> **Similar** figures have exactly the same shape but not the same size.
> **Congruent** figures have exactly the same shape and size.

Similar figures are magnifications of each other.
Two circles are always similar.

Two squares are always similar.
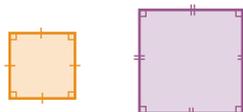
Not all rectangles are similar.

It is often helpful to be able to refer to similar and congruent figures in a more precise way. Look at the figures shown below. The two larger figures have the same shape and size, so they are congruent. We name congruent shapes so that the vertices in each name correspond. The symbol '≡' means 'congruent'.

From the diagram you can see that A corresponds to V ($A \leftrightarrow V$), $E \leftrightarrow Z$, $B \leftrightarrow Y$, $D \leftrightarrow X$ and $C \leftrightarrow W$.

So we can write $AEBDC \equiv VZYXW$.

It is also true to say that
$AE \leftrightarrow VZ$, $BD \leftrightarrow XY$,
$\angle A \leftrightarrow \angle V$, and so on.

All of the figures shown below have the same shape. This means that each one is similar to the others. The symbol '|||' means 'similar'.

So, $QSMRP \;|||\; AEBDC$,
$QSMRP \;|||\; VZYXW$ and
$AEBDC \;|||\; VZYXW$.

Many people use ≅ instead of ≡ for congruence or : instead of ||| for similarity.
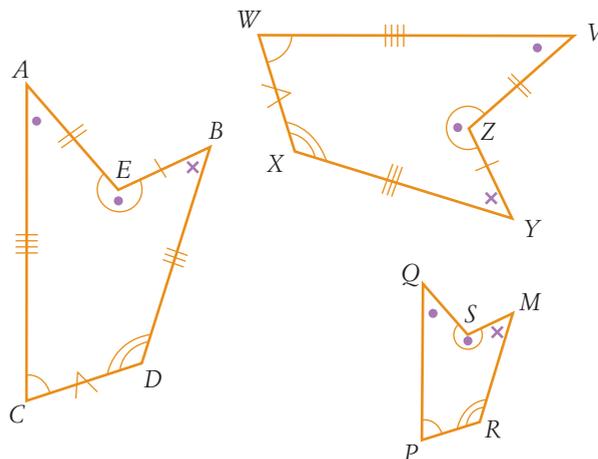
Example 29

a Why are the triangles shown here similar?
b Complete the following: $\triangle ABC \;|||\; \triangle\ldots$
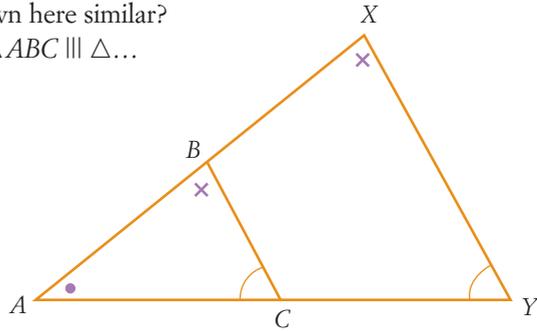
Solution

a From the diagrams, we can see that $\angle A$ is
the same for both triangles,
$\angle B = \angle X$ and $\angle C = \angle Y$.

The triangles are similar because they have the same shape. Their angles are the same.

b From the diagrams, we can see that A is
the same *for both triangles*,
$B \leftrightarrow X$ and $C \leftrightarrow Y$.

$\triangle \boldsymbol{ABC} \;|||\; \triangle \boldsymbol{AXY}$

In mathematics, a **transformation** is a change that produces an image of a figure (or shape).
Some transformations result in an image that is congruent to the initial figure.

Important!

Translation

A **translation** is a sliding movement. All parts of the figure move the same distance and
direction. The figure and its image are congruent.

Translation

9780170361910
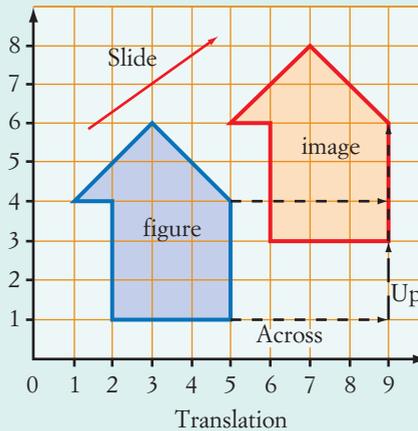
Any translation can be described as a combination of movements up (or down) and across. The translation shown on the previous page is 4 units to the right and 2 units up.

Important!

Reflection

When a figure is **reflected** (or **flipped**) all points swap to an equal distance on the other side of the line of reflection. The figure and its image are congruent.

Reflection
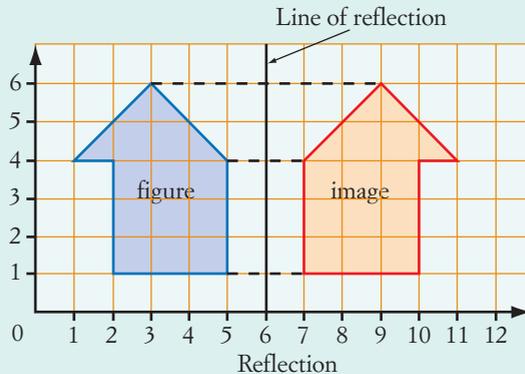
Important!

Rotation

A **rotation** is a turn. All points remain the same distance from the **centre of rotation**. The angle of the turn is either clockwise or anticlockwise. The figure and its image are congruent.
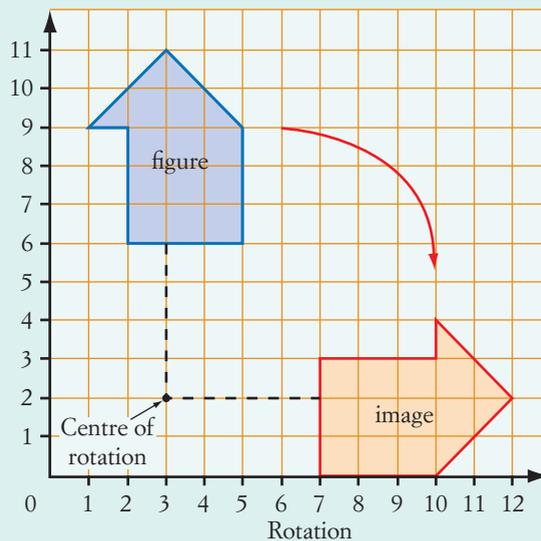
Rotation

The rotation shown here is 90° clockwise.

Some transformations result in an image that is similar to the initial figure.

Important!

Enlargement and reduction

Enlargements and **reductions** produce an image that is a different size from the original. The image is the same shape as the original, so they are similar. The amount by which the shape is enlarged or reduced is called the **magnification** or **scale factor**.
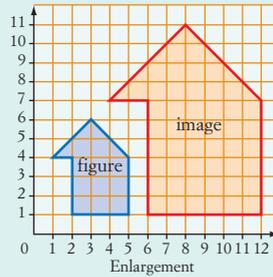
Enlargement

In the enlargement shown here, the magnification is 2.

You can use coordinates to describe transformations.

Investigate: Combining transformations

1 Draw an *x*- and *y*-axis on a piece of centimetre grid paper and draw a square as shown here. Label the square *A*.

  a Translate the square four units to the right. Label this shape *A'*.

  b Now rotate *A'* 90° clockwise about the origin. Label this shape *A''*.

  c Reflect *A''* in the *y*-axis and label this shape *A'''*.

  d Describe the single transformation that could have moved *A* to *A'''*.

  e Find two reflections that give the same result as a translation.

2 Look at the diagram on the right.

  a Find combinations of transformations of the shape on the left that will make the pattern on the right.

  b Make your own pattern from one or more basic shapes using transformations.
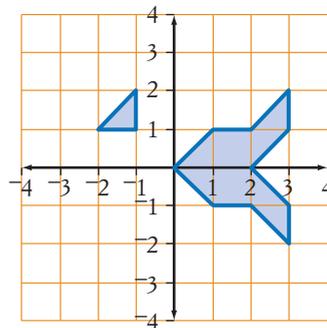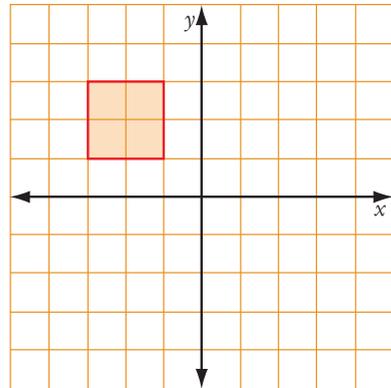
3 Draw an *x*- and *y*-axis on another piece of centimetre grid paper and draw a triangle as shown here. Label the triangle *B*.
  a Reflect *B* in the *y*-axis. Label this shape *B′*.
  b Now rotate *B′* 180° clockwise about the origin. Label this shape *B″*.
  c Describe the single transformation that could have moved *B* to *B″*.
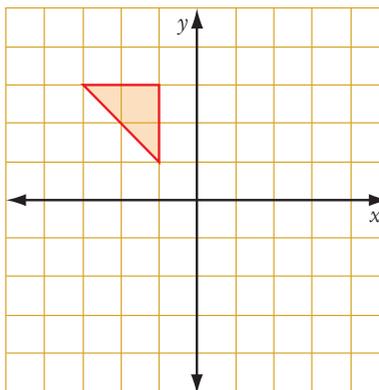
Example   30

$\triangle ABC$ shown below on the left has undergone three different transformations to produce the images $\triangle PRQ$, $\triangle JHI$ and $\triangle KLM$ shown in the diagram on the right.
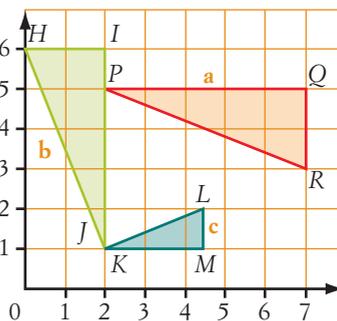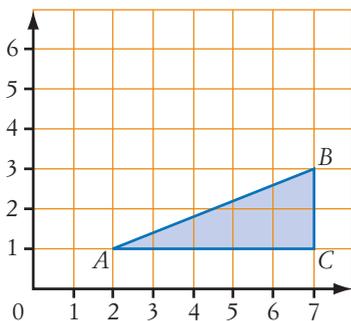
Describe each of the transformations.

Solution

a $\triangle ABC \equiv \triangle PRQ$ so the transformation is either a rotation, reflection or a translation. $\triangle ABC$ has been flipped to form $\triangle PRQ$.

The distance from points in $\triangle ABC$ and its image to the horizontal line passing through *B* is the same.

The transformation is a reflection. The line of reflection is the horizontal line passing through *B*.

b $\triangle ABC \equiv \triangle JHI$ so the transformation is either a rotation, reflection or a translation.

The image has *not* resulted from a flip or a slide.

The transformation is a rotation.

The centre of rotation is *A*.

The figure has been rotated 90° anticlockwise or 270° clockwise.

c $\triangle ABC \,|||\, \triangle KLM$ so the transformation is an enlargement or reduction.

The image is smaller than the original. **The transformation is a reduction.**

Calculate the magnification. $\dfrac{LM}{BC} = \dfrac{1}{2}$

State the result. **The transformation is a reduction with a magnification of $\frac{1}{2}$.**

The symmetry of a shape can be used to help identify it. Shapes can have either line or rotational symmetry.

A plane shape has **line symmetry** if it can be folded so that one half fits exactly on top of the other half. In other words, if one half is the mirror image of the other, then the shape has line symmetry. The diagram at right shows that a rectangle has line symmetry.

axis of symmetry ⟶

The dotted line or fold is called the **axis of symmetry**. Some shapes have more than one axis of symmetry. The plural of 'axis' is '**axes**'. For example, a rectangle actually has two axes of symmetry, shown at right, because it can be folded in half two different ways.

Here are some more examples of shapes and their axes of symmetry.
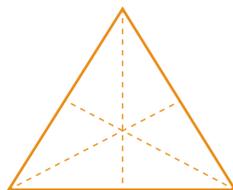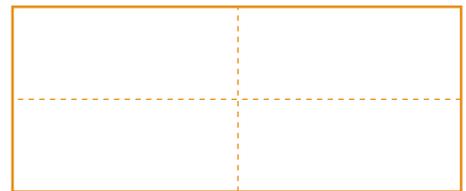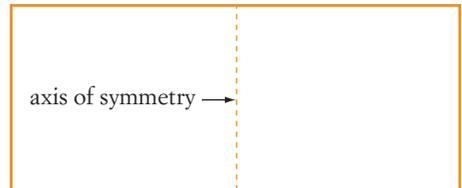
Equilateral triangle
Three axes of symmetry
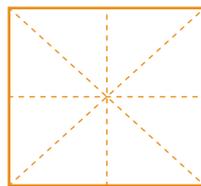
Square
Four axes of symmetry

Parallelogram
No axes of symmetry

A plane shape has **rotational symmetry** if it can be rotated (spun) around its centre so that it fits onto itself again before making one full rotation. The diagram below shows that a regular pentagon has rotational symmetry.
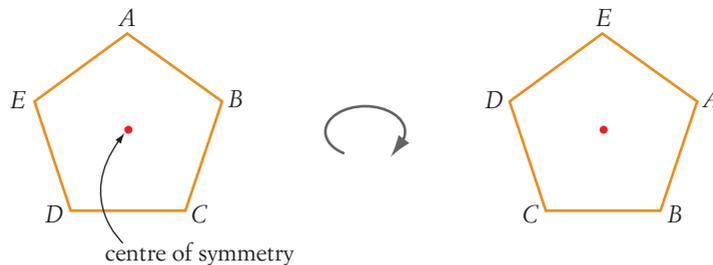
centre of symmetry

The centre point is called the **centre of symmetry**. Because a regular pentagon can fit onto itself five times during one complete rotation, we say that it has order 5 rotational symmetry.
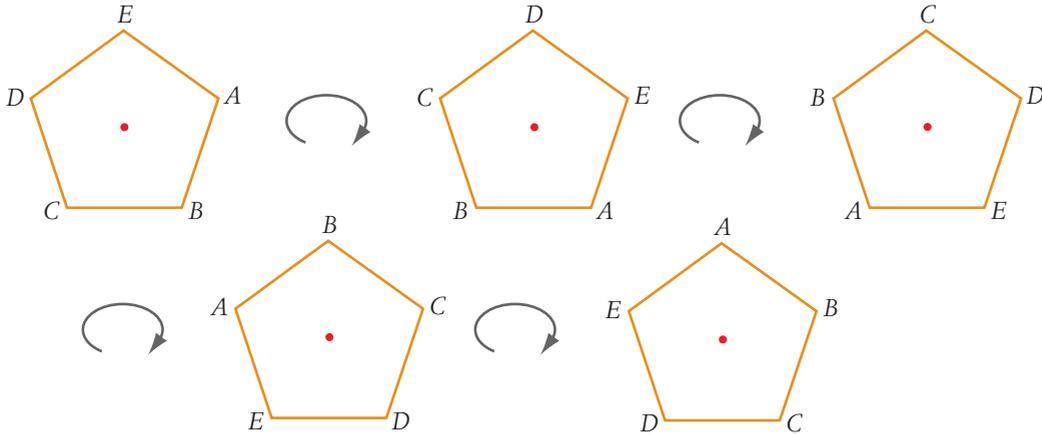
Here are some examples of shapes with or without rotational symmetry.

Parallelogram
Has rotational symmetry
of order 2

Isosceles triangle
Does not have rotational
symmetry

Regular octagon
Has rotational symmetry
of order 8
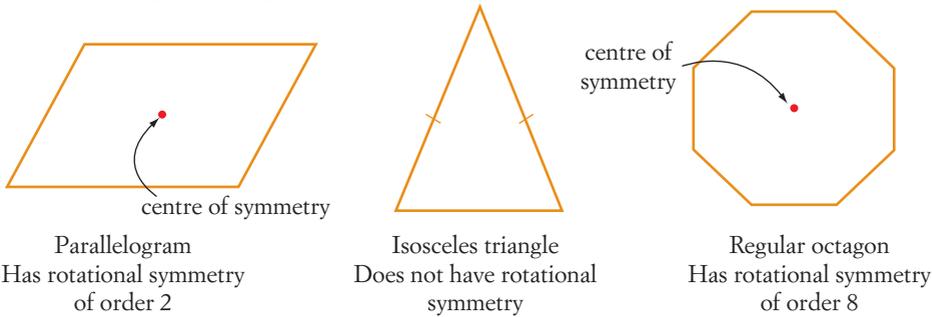
You can test your understanding of the symmetry of flat shapes by completing the questions in the Symmetry of flat shapes worksheet.

Example   31

PQRS is a parallelogram, as shown below.

a Does a parallelogram have line symmetry? If so, draw its axes of symmetry.

b Does a parallelogram have rotational symmetry? If so, state the order.

Solution

a It is not possible to fold *PQRS* in half.

The parallelogram does not have an axis of symmetry.

b You can rotate *PQRS* 180° so that it maps onto itself. Mark the centre of symmetry as *O*.
There is no other rotation that will make *PQRS* fit on itself.

State the result.
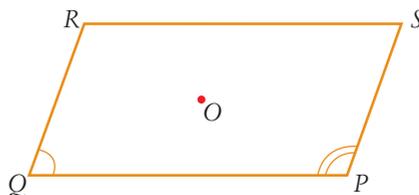
It has rotational symmetry of order 2.

Exercise 3.5   Similarity, congruence and transformations

1 These two triangles are congruent.
   a Identify the corresponding vertices.
   b Complete the statement
      $\triangle NLM \equiv \triangle\dots$
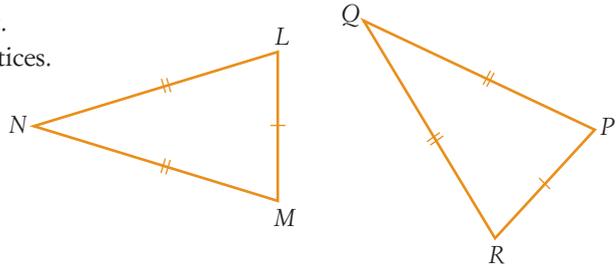
2 These two shapes are similar.
   a Identify the corresponding vertices.
   b Complete the statement
      $ACBD \parallel\!\parallel\!\parallel \dots$
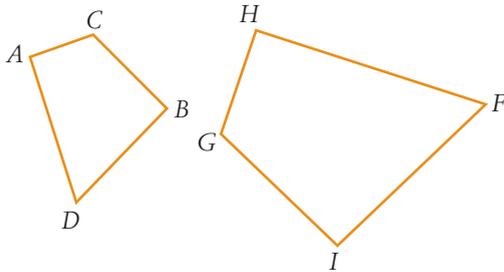
3 Which of the following shapes are congruent?

   a

   b

   c   5 cm

   7.5 cm

   d

   e
   2 cm
   (Circle)

   f

   g

   h   15 cm

   10 cm

   i

   3 cm

   (Circle)

   k

   l

   5 cm

   7.5 cm

   m
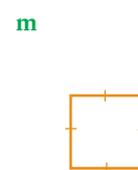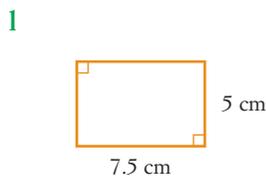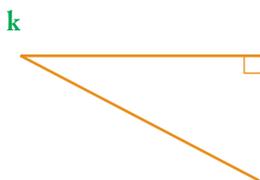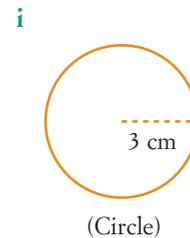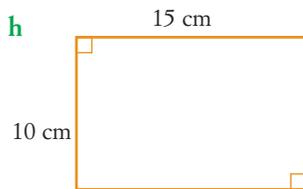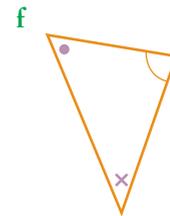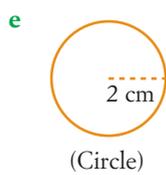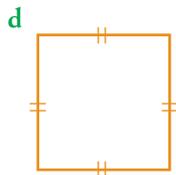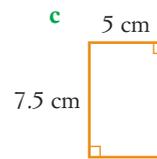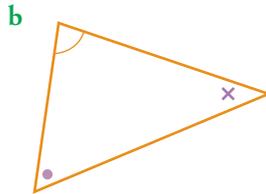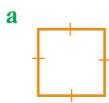
4 Which of the shapes shown in question **3** that are not congruent are similar?

5 a Show that the shapes below are similar.
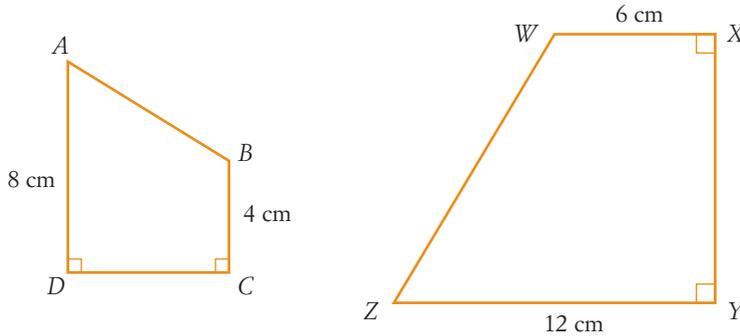
b Calculate the scale factor.

6 Shape *X* has been translated to each of the images *A*, *B*, *C*, *D* and *E*.

See Example 30
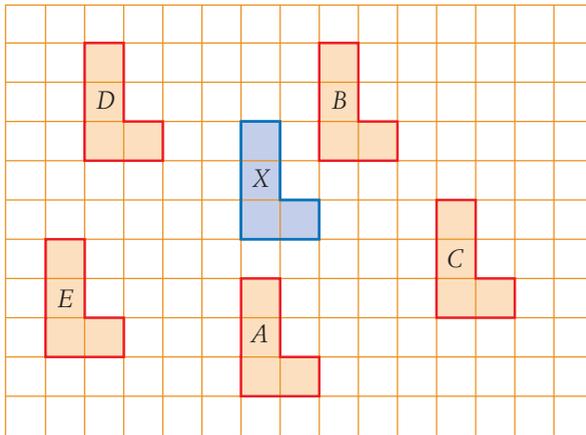
a Describe the translation used to produce each of the images *A*, *B*, *C*, *D* and *E* from the original shape *X*.

b Describe the translation needed to slide:

i shape *D* to shape *C* **ii** shape *C* to shape *A*
iii shape *E* to shape *B* **iv** shape *A* to shape *D*
v shape *D* to shape *X*

7 The blue shape shown below can be rotated around any of the points *P*, *Q*, *R*, *S*, *T* or *U*. The results of some rotations are shown here. Describe the rotation of the blue rectangle to form the image shown in red. In each case, give the size and direction of the rotation.
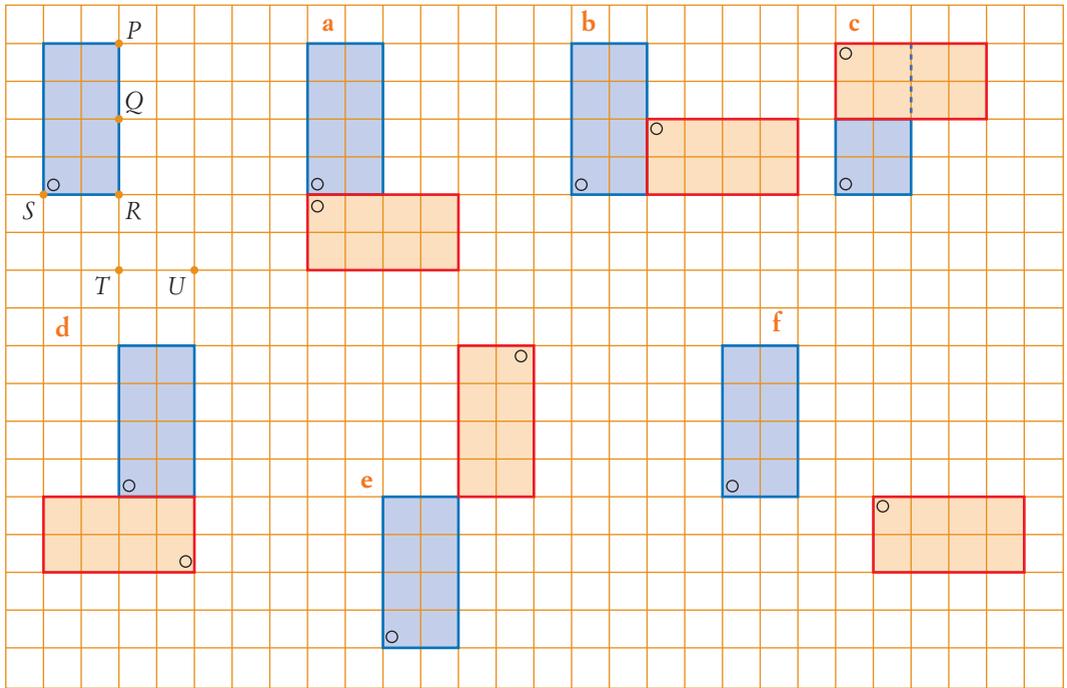
8 Copy each of these diagrams onto grid paper. In each case, draw the reflected image of the triangle in the line of reflection *m*.
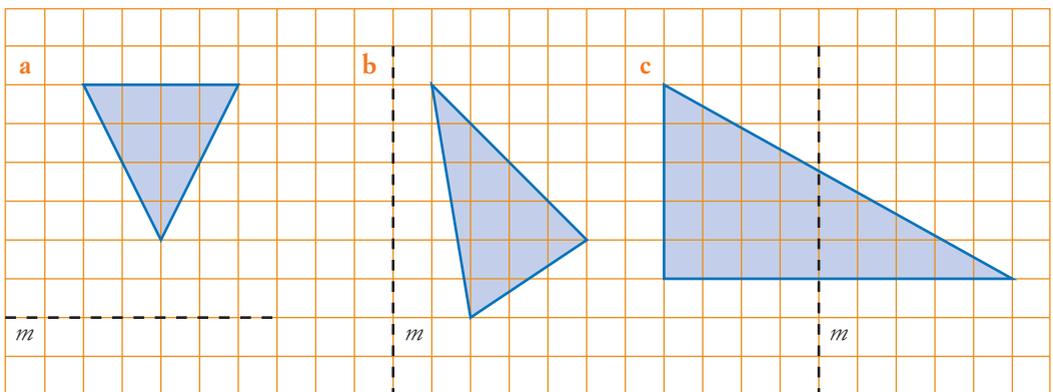
9 Copy this shape onto grid paper. Enlarge the shape by a factor of 3.
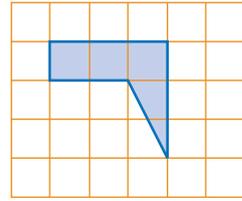
10 Copy this shape onto grid paper. Reduce the shape by a factor of one half.
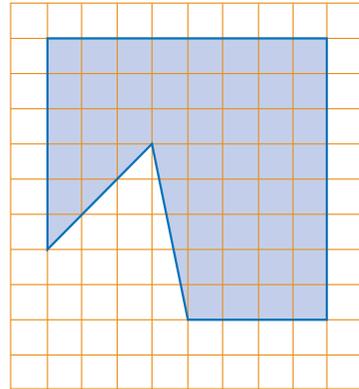
11 Use the coordinates of A, B, A' and B' to show the transformations $A \to A'$ and $B \to B'$ of each shape.

See Example 31
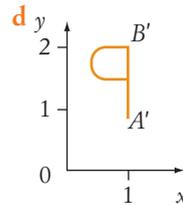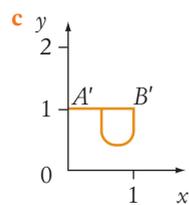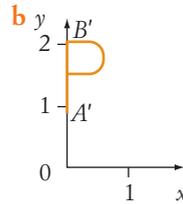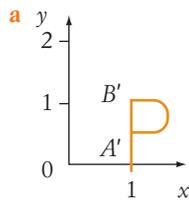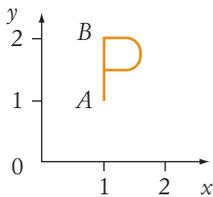
12 a Draw a pentagon with one axis of symmetry.
   b Draw a quadrilateral with four axes of symmetry.
   c Draw a hexagon with six axes of symmetry.
   d Draw a decagon with two axes of symmetry.

13 Draw each of the following quadrilaterals and mark all axes of symmetry.
   a rectangle   b square   c parallelogram
   d rhombus   e trapezium   f kite

14 List the quadrilaterals in question **13** that have rotational symmetry and state the order of rotational symmetry for each.

Problem solving

15 Using measurement, show that the shapes on the right are similar.
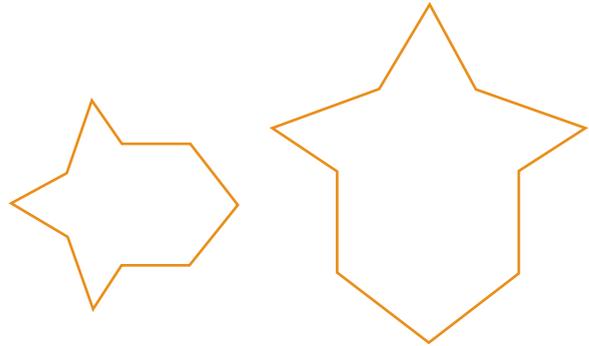
16 I am a quadrilateral with equal opposite sides that are parallel. My diagonals are equal and I have four axes of symmetry. Which quadrilateral am I? Select **A**, **B**, **C** or **D**.

A rectangle   **B** square   **C** parallelogram   **D** rhombus

Reasoning

17 I have rotational symmetry, but no axes of symmetry. Which quadrilateral am I?

Worked solution

Exercise 3.5

MAT07MGWS00016

18 Consider shape *A* shown here.
  a Rotate *A* 90° clockwise about the origin. Label the resulting shape *A′*.
  b Reflect *A′* in the *x*-axis and label the resulting shape *A″*.
  c Translate *A″* 5 units to the left. Label the resulting shape *A‴*.
  d What single transformation would produce *A‴* from *A′*?
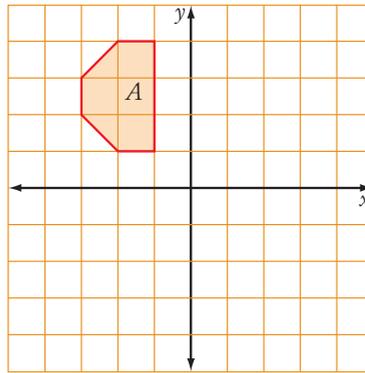
19 Do any triangles have rotational symmetry? Give examples to support your answer.

- A **point** is a position and it is represented by a dot labelled with a capital letter.
- A **line** is a straight edge that continues infinitely in both directions. An **interval** is part of a line. Equal intervals are marked with identical dashes.
- An **angle** consists of two lines or **arms** with a common **vertex**. Angles are marked with an **arc** to show the size of the turn for one arm to meet the other.
  The angle shown here has arms QP and QR and vertex Q. This angle can be written as $\angle PQR$, $\angle RQP$, a, $\angle Q$, $R\hat{Q}P$, $P\hat{Q}R$ or $\tilde{Q}$.
- There are 360° in a complete **turn** or **full rotation**. This means that a **half-turn** (**straight angle**) is 180° and a **quarter turn** (**right angle**) is 90°.
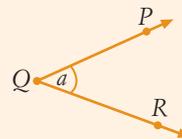
Full rotation

Straight angle

Right angle

- Angles can be classified according to their size. An **acute angle** is less than 90°. An **obtuse angle** is larger than 90° but less than 180°. A **reflex angle** is larger than 180° but less than 360°.
- You can use a **protractor** to measure angles. Protractors have clockwise and anticlockwise scales. Protractors can also be used to draw angles of a given size.
- Angles that share a common arm are called **adjacent angles**. Angles that are opposite each other when lines cross are called **vertically opposite angles**. Vertically opposite angles are equal.
- **Complementary angles** add to 90° and **supplementary angles** add to 180°. **Angles at a point** add to 360°.
- If two lines cross we say that they **intersect**. Lines that intersect at right angles are called **perpendicular lines**. We write this as $AB \perp CD$. Lines that never intersect are called **parallel lines**. We write this as $WX \parallel ZY$.
- A line that crosses two or more other lines is called a **transversal**.
- **Alternate angles** are between two lines and on opposite sides of the transversal. Alternate angles on parallel lines are equal, e.g. $a = b$.
- **Corresponding angles** are on the same side of the transversal and are either both above or below parallel lines. Corresponding angles on parallel lines are equal, e.g. $a = c$.
- **Co-interior angles (allied angles)** are on the same side of the transversal and between parallel lines. Co-interior angles on parallel lines are supplementary, e.g., $a + d = 180°$.
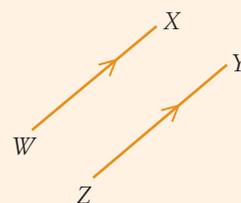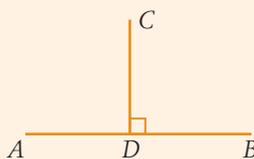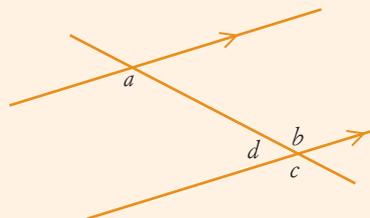
- A **triangle** is a plane shape with three sides and is named using its vertices. A **quadrilateral** is a plane shape with four sides. Other plane shapes with straight sides are called polygons with names that identify the number of sides as follows:

Name	No. of sides
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10
Undecagon	11
Dodecagon	12

- **Convex polygons** have vertices that all point outwards. **Concave** (**non-convex**) **polygons** have at least one inward pointing vertex.
- Triangles can be classified using their side lengths or their angles as follows:

Sides			
	Equilateral Three equal sides	**Isosceles** Two equal sides	**Scalene** No equal sides
Angles	**Acute-angled** All three angles acute	**Right-angled** One right angle	**Obtuse-angled** One obtuse angle

- There are six special types of convex quadrilaterals: the **trapezium** or **trapezoid**, the **parallelogram**, the **rhombus**, the **kite**, the **rectangle** and the **square**.
- The sum of the angles in a triangle is 180°.

$\angle A + \angle B + \angle C = 180°$

- The sum of the angles in a quadrilateral is 360°.
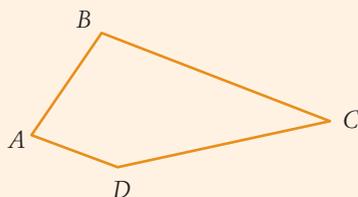
$\angle A + \angle B + \angle C + \angle D = 180°$

- A **transformation** changes a shape or its position to form an **image**.
- Three types of transformation produce an image that is congruent with the original shape.

  A **translation** is a sliding movement where all parts of the shape move the same distance and direction.

  A **rotation** is a clockwise or anticlockwise turn where all points remain the same distance from the centre of the turn (**centre of rotation**).

  A **reflection** is a flip, with all points swapped to an equal distance on the other side of the **line of reflection**.

- An **enlargement** or a **reduction** is a transformation that produces an image that is similar to the original shape. The amount by which the shape is enlarged or reduced is called the **magnification**.

- Coordinates can be used to show transformations.

- A plane shape has **line symmetry** if it can be folded so that one half is a mirror image of the other. The fold is called the **axis of symmetry**. A plane shape has **rotational symmetry** if it can be rotated (spun) around a point called the **centre of symmetry** so that it fits onto itself before making a complete revolution.

1 Draw labelled diagrams of each of these angles.

a ∠*BKT* **b** ∠*FPR* **c** ∠*MZQ*

2 Use a protractor to measure each angle you drew in question **1**. Name the smallest angle and the largest angle.

3 Write the name of each of these angles. Then label each one as acute, obtuse, right, reflex or straight.

a **b** **c**

d **e** **f**

g **h** **i**
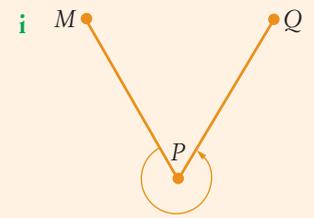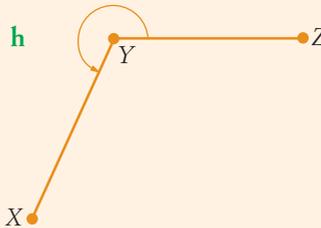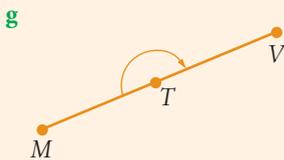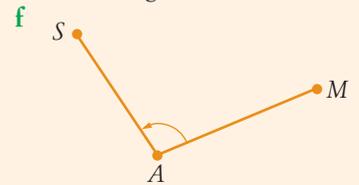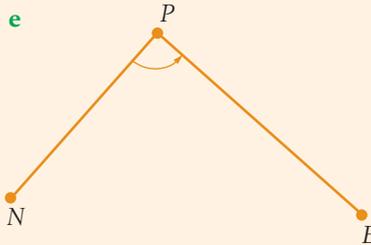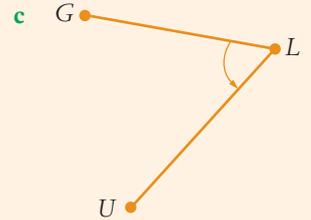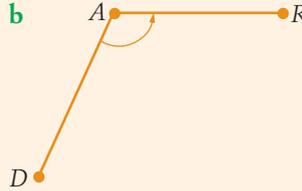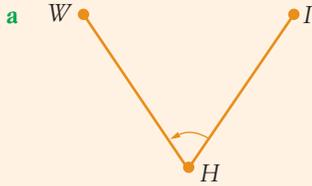
4 **a** What is the complement of each of these angles?

    i 35° **ii** 78° **iii** 4°

  b What is the supplement of each of these angles?

    i 45° **ii** 100° **iii** 178°

  c Without measuring, find the size of the angle shown by each letter.

i 70° *a* 70°

ii 25° *m*

iii 35° *y*
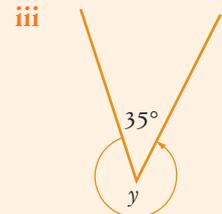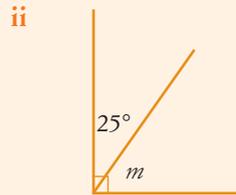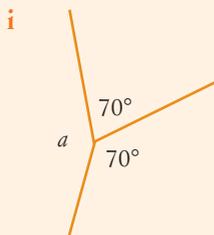
5 In this diagram, name two lines that:

  a are parallel   **b** are perpendicular   **c** intersect.
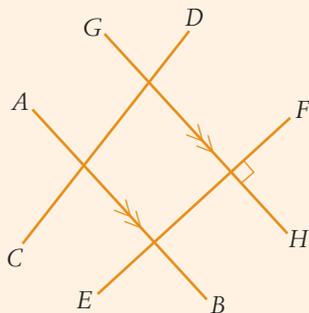
6 **a** Copy each diagram and mark in the alternate angle to the one shown.

    i   **ii**
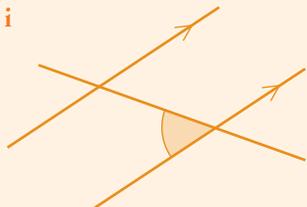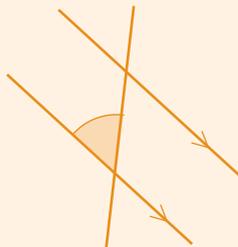
  b Without the use of instruments, find the size of each angle shown by a letter.
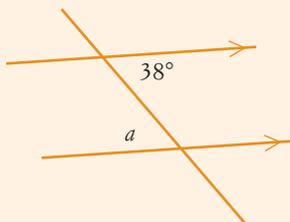
    i   **ii**   **iii**
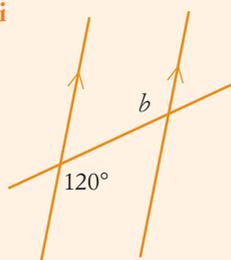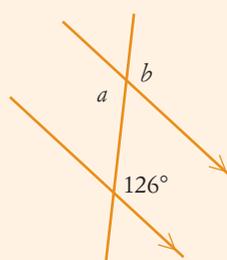
7 **a** Copy each diagram and mark in the corresponding angle to the one shown.
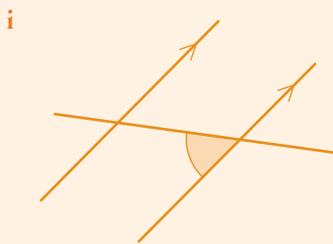
i

ii

b Without the use of instruments, find the size of each angle shown by a letter.

i

112°

a

ii

p

117°

iii

n

p

150°
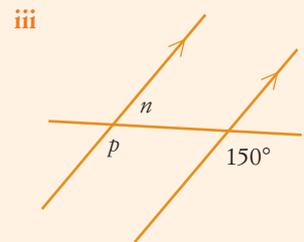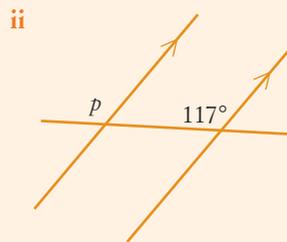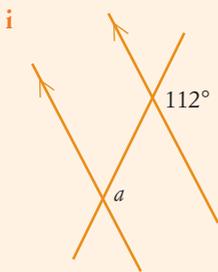
See Example 10

8 **a** Copy each diagram and mark in the co-interior angle to the one shown.

i

ii
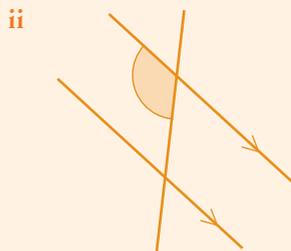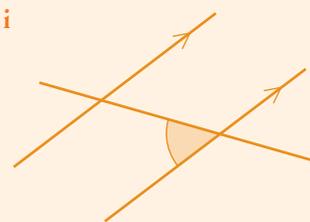
b Find the size of the angle shown by each letter.

i

k

72°

ii

112°

x

iii

82°

y

x
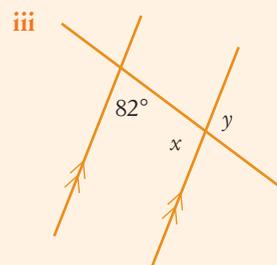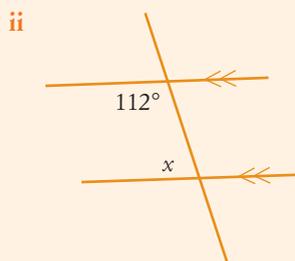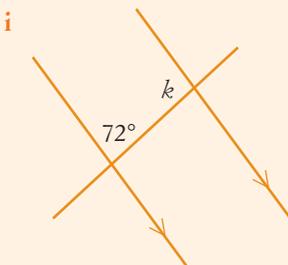
9 Label the marked pairs of angles as alternate, co-interior or corresponding.

a

b

c

d

e

f
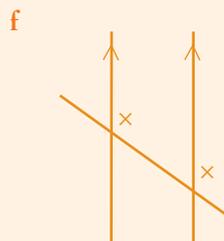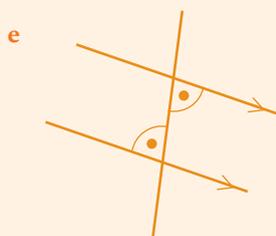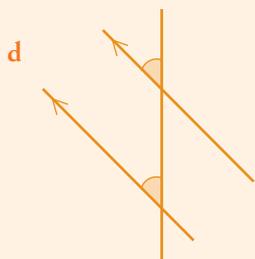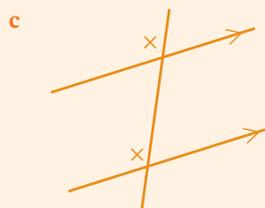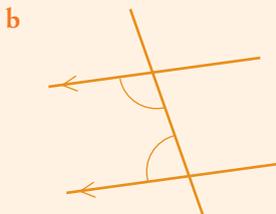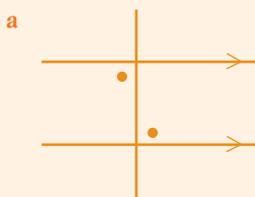
10 **a** Name a pair of parallel sides in this figure.
b Name a pair of perpendicular sides.
c Name a pair of equal angles.
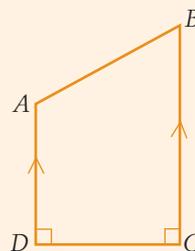
See Example 15

11 What type of polygon has 10 sides?

12 Name a shape that is not a polygon.

13 Draw the following shapes.
a a regular pentagon   **b** a non-regular pentagon
c a convex quadrilateral   **d** a non-convex quadrilateral

14 Classify these triangles, by sides and angles.

See Example 16

a

b

c

d

e

f
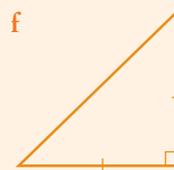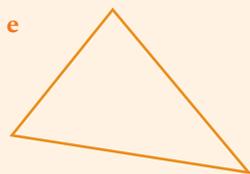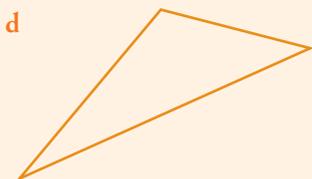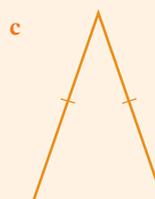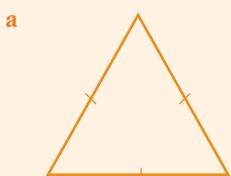
15 Name each of the following quadrilaterals.

a

b

c

d

e

f
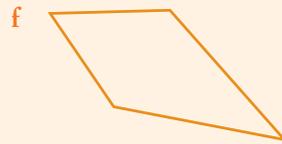
16 What is the definition of a rhombus?

17 Write two properties of a parallelogram.

18 Which of the following shapes are similar and which are congruent?

A

B

C

D

E

F

G

H

I
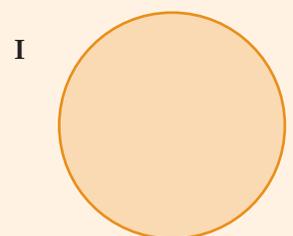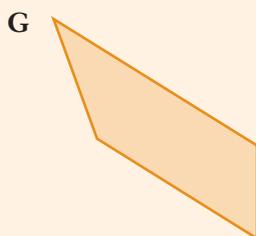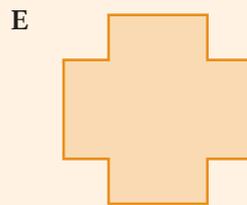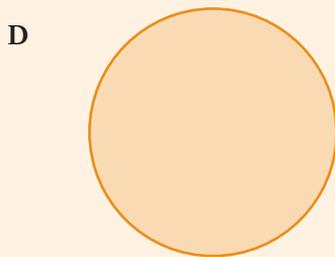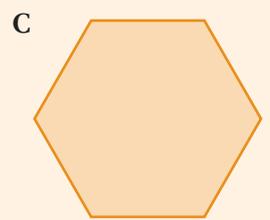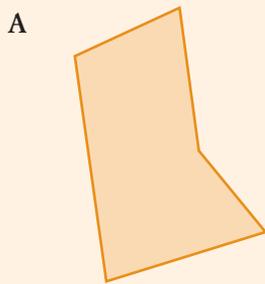
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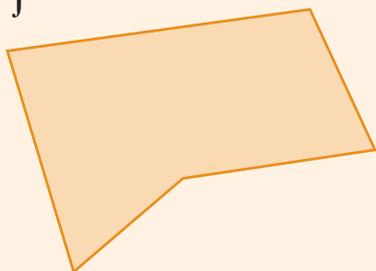
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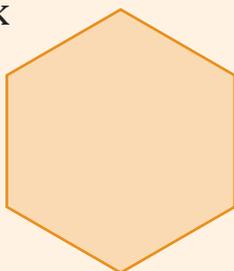
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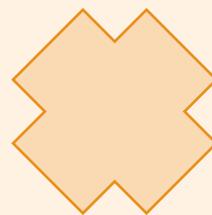
19 Use a protractor to draw these angles.

   a $\angle JUG = 84°$ **b** $\angle QRA = 117°$ **c** $\angle POT = 41°$

   d $\angle DGE = 150°$ **e** $\angle SAR = 96°$ **f** $\angle XDW = 210°$

   g $\angle MNB = 195°$ **h** $\angle PLO = 270°$ **i** $\angle AMP = 300°$

20 Find the size of each angle shown by a letter. Do not use a protractor to measure the angle.
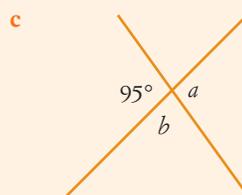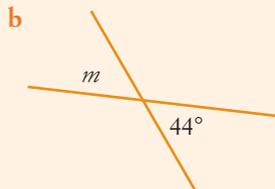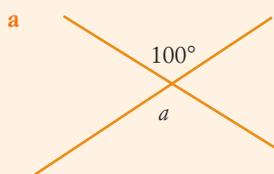
   a   **b**   **c**

      100° m 95° a

         a 44° b

21 Without measuring, find the size of each angle shown by a letter.

   a   28°   **b** k **c** x

     m 47° y 122°

   d   **e**   **f**

     140°   110°

       75° f

     p 48°

                         x

   g   **h** r q **i**

   105° 82° p 25°

     t x x

                                             x
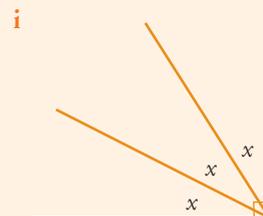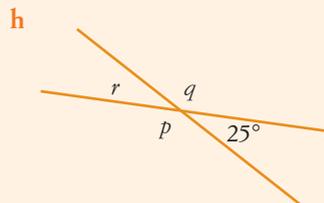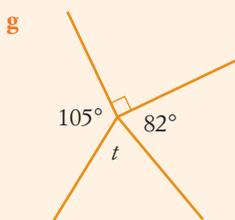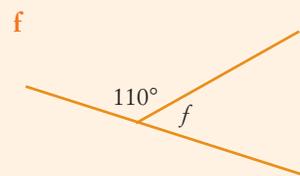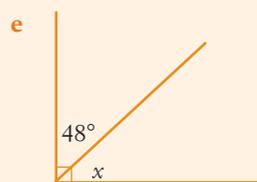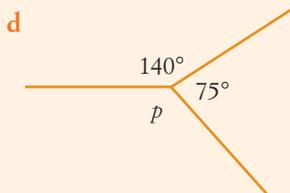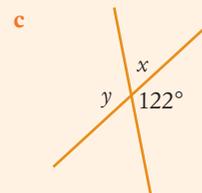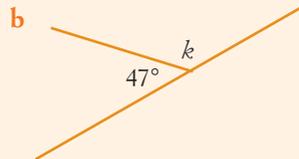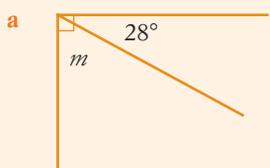
See Examples 10–13

22 Find the size of each angle shown with a letter.

a

115°

a

b

35°

m

c

65°

k

d

125°

d

e

x

130°

f

q

62°

g

37°

y

x

h

z

62°

m *p*

i

112°

t

d *a*

23 Find the size of each angle shown with a letter.

a

130°

x

y

b

64°

x

m

c

c

z

a

70° 38°

d

38°

a

c

e

x *y*

57°

z

145°
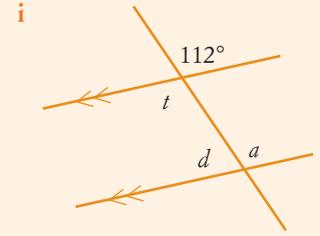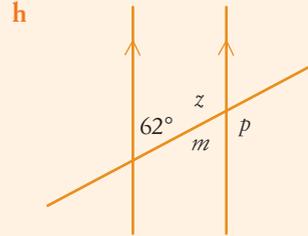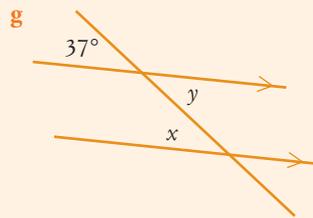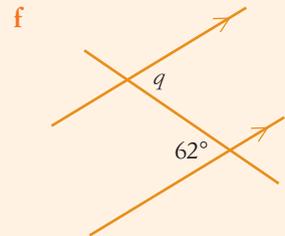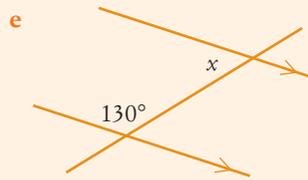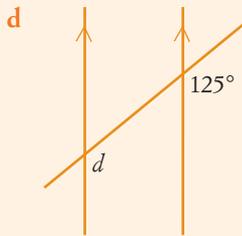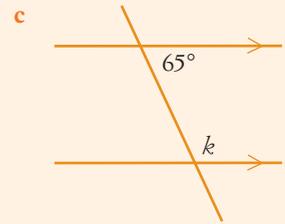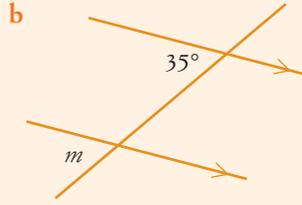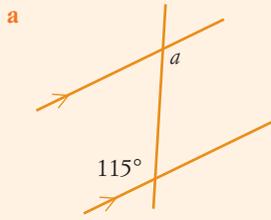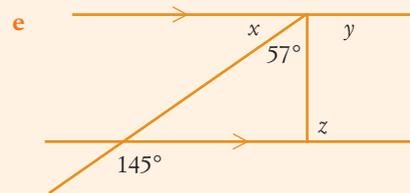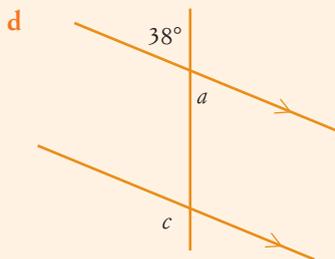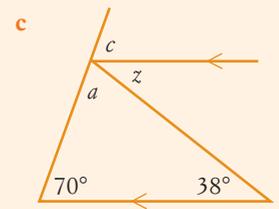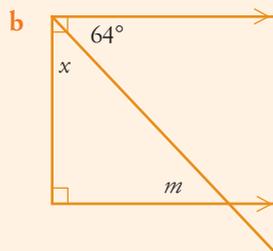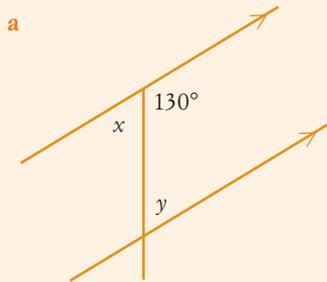
24 Find the sizes of the lettered angles in the following diagrams without measuring.   *See* Examples 14, 15

a

b
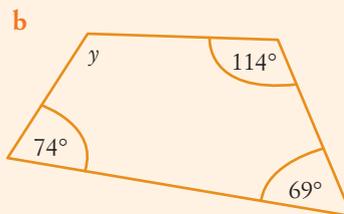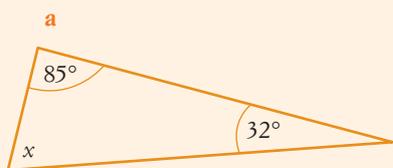
25 Draw a neat diagram to illustrate each of the following.
  a an acute angle   **b** supplementary angles
  c a straight angle   **d** vertically opposite angles
  e alternate angles   **f** an obtuse angle
  g corresponding angles   **h** a reflex angle

26 a Classify $\triangle FGH$ by sides and angles.
  b Which angles in *FGH* are equal?
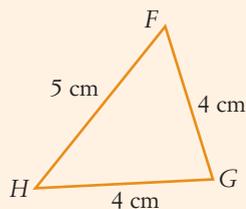
See Examples 24–26

27 Draw the following triangles.
  a

  Not to scale
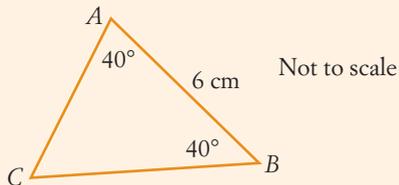
  b $\triangle PQR$ with $\angle P = 20°$, $PR = 3$ cm and $PQ = 4$ cm.
  c $\triangle MNO$ with $MN = 4$ cm, $NO = 5$ cm and $OM = 6$ cm.

28 a Draw an obtuse-angled triangle, $\triangle XYZ$, and label its sides *x*, *y* and *z*.
  b What is the relationship between the triangle's longest side and its largest angle?

29 Copy this diagram and: *See* Examples 22, 23
  a construct a perpendicular through *P*, leaving construction lines
  b draw a line parallel to *QR*.
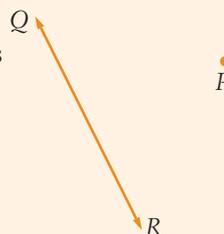
30 Copy this shape onto grid paper.
- **a** Translate it 2 units to the right and 3 units up.
- **b** Rotate it 90° clockwise around *A*.
- **c** Reflect it in *m*.
- **d** Enlarge it with a magnification of 2, keeping *B* the same.
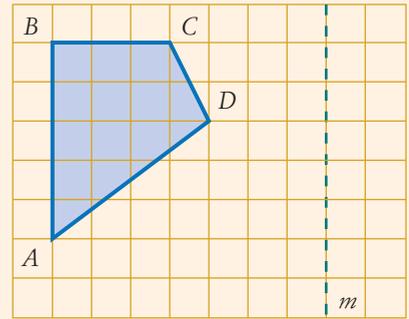
31 Copy these figures and draw in any lines of symmetry.

a

b
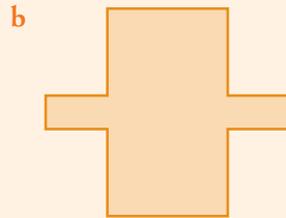
32 **a** Copy each shape in question **15** and mark all the axes of symmetry.
- **b** List the shapes in question **15** that have rotational symmetry, and state the order of rotational symmetry of each one.

33 In each diagram below, is *AB* parallel to *CD*? Give a reason for your answer each time.

a

b

c
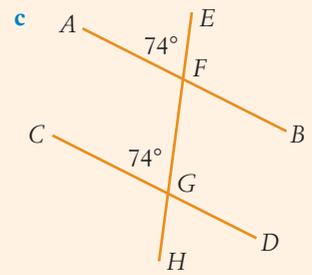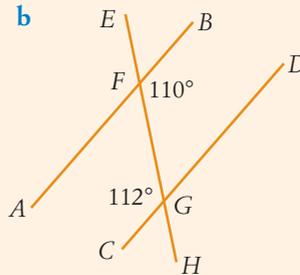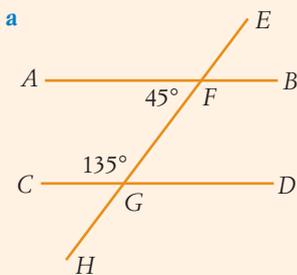
34 Draw this parallelogram.

4 cm

6 cm

80°
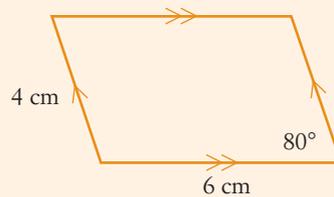
35 Draw this quadrilateral.
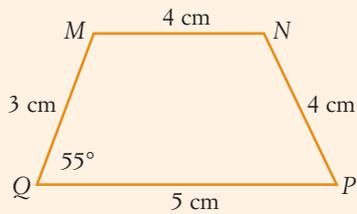
36 What polygon am I? (There may be more than one answer.)

   a I have three sides and all of my angles are equal.
   b I am a quadrilateral with parallel opposite sides.
   c I have five sides.
   d I have four sides and my diagonals bisect each other.
   e I am a quadrilateral with one pair of parallel sides.
   f I have three sides. My angles are 60°, 80° and 40°.

37 Are all equilateral triangles similar? Why?

Number and algebra

4 Decimals

Contents

Prior learning
Chapter 4

Parent guide
Chapter 4

Curriculum guide
Chapter 4

Australian Curriculum statements

Real numbers

Multiply and divide fractions and decimals using efficient written strategies and digital technologies. (ACMNA154)

Round decimals to a specified number of decimal places. (ACMNA156) **AC**

Whole numbers are most useful for counting. However, measurement often requires you to use partial numbers (fractions). Decimal fractions extend our whole numbers to cover parts of numbers. A shelf that is 25.4 cm wide is between 25 cm and 26 cm wide, but closer to 25 cm than 26 cm. Just as with whole numbers, the fact that we have ten fingers and ten toes makes it easier for us to think in tens than any other number.

■ Mathematical literacy

Maths dictionary

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

decimal places	extended form	round	thousandth
decimal point	hundredth	tenth	
estimate	recurring decimal	terminating decimal	

4.1   Naming and comparing decimals

In our number system, each place value is ten times as big as the one before it. Thus 100 is ten lots of 10, and ten 100s make 1000.

In the same way, you can divide 1 into 10 parts, and each is part is called a **tenth**, which is written as $\frac{1}{10}$ or 0.1. The **decimal point** is the dot that separates the whole number part from the fractional part of the number.

You can divide 0.1 into 10 parts to make a **hundredth**, which is written as $\frac{1}{100}$ or 0.01.

A tenth is bigger than a hundredth, so 0.4 is bigger than 0.08, or 0.4 > 0.08.

You can think of the place value sizes and names as a staircase, where each step going up is ten times bigger than the one below it. This also means that each step going down is ten times smaller than the one above it.
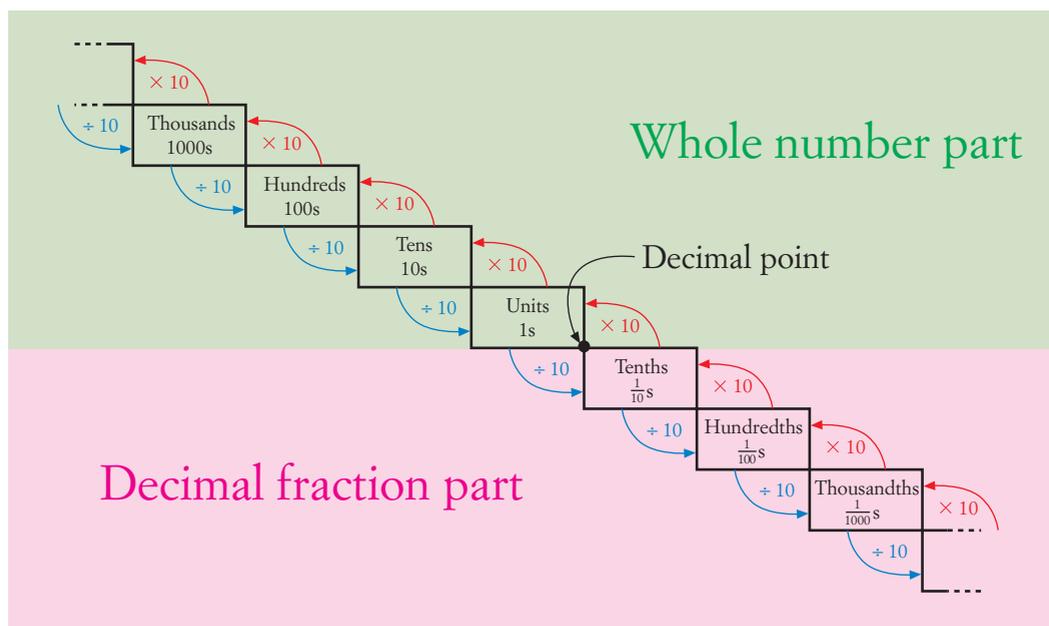
Investigate: Decimal distances

Equipment: A tape measure and some coloured chalk.

Step 1: Form a group of three or four students and find a flat area such as a path, basketball court or the classroom floor.

Step 2: Mark a starting point on your flat area.

Step 3: Choose a distance between 1 and 4 metres, for example 3.25 m.

Step 4: Using chalk, take turns to mark your estimate of 3.25 m from the starting point. Measure the exact length and mark it with an X.

Step 5: Give points to each person. Award a score of 5 points for the best estimate, 3 points for the second best, 2 points for the third best and 1 point for the fourth best. Take turns to choose distances, and play the game until someone reaches 20 points and wins.

Sample score sheet for one person.

Trial	Distance	Guess	Points
1	3.25	2.57	
2	4.50	3.16	
3	3.50	3.41	
4	1.75	1.82	
5	2.20	2.38	
		Total	

A number such as 346.78 has a whole number part before the decimal point and a decimal fraction part after the decimal point. The whole number part is 346, which we say as 'three hundred and forty-six'. The decimal part is .78, which we say as 'point seven eight'. It is really seven tenths ($\frac{7}{10}$) and eight hundredths ($\frac{8}{100}$). We can write decimals in **extended form** in a similar way to whole numbers. The number of digits after the decimal point tells us the number of **decimal places** in the decimal number. The number 5.632 has three decimal places and the number 12.1005 has four decimal places.

Notice that 'seventy-eight' is a whole number. *Figures after the decimal point are always said individually.* So 5.632 is said as 'five point six three two'.

Example 1

a Write 703.25 in extended form.
b What is the value of the digit 4 in the number 312.47?
c Write $5 + \frac{1}{10} + 3 \times \frac{1}{1000}$ in decimal form.

Solution

a In 703.25, the 7 is in the hundreds column, the 3 is in the units column, the 2 is in the tenths column and the 5 is in the hundredths column.

So $7 = 700$, $3 = 3$, $2 = \frac{2}{10}$ and $5 = \frac{5}{100}$

b In 312.47, the **4** is in the tenths column.

$$703.25 = 7 \times 100 + 0 \times 10 + 3 \times 1$$
$$+ 2 \times \frac{1}{10} + 5 \times \frac{1}{100}$$

$$4 \times \frac{1}{10} = \frac{4}{10} \text{ or 4 tenths}$$

c In $5 + \frac{1}{10} + 3 \times \frac{1}{1000}$, the $5 = 5$, $\frac{1}{10} = 0.1$ (one tenth) and $3 \times \frac{1}{1000} = 0.003$. (3 thousandths) Note that there are 0 hundredths.

$$5 + \frac{1}{10} + 3 \times \frac{1}{1000} = 5.103$$

When you compare the values of decimals, you work in the same way as for whole numbers by comparing the highest place values first.

You can put extra zeros at the end of a decimal fraction so that it has the same number of decimal places as another number. You can always write 304.6 as 304.600 because you are not changing the place values of the 4 or 6 with the extra zeros. You are just emphasising the fact that there are no hundredths or thousandths in this number.

Example 2

Arrange these numbers in ascending order (smallest to largest).

| 51.82 | 51.28 | 5.182 | 51.08 | 51.8 |

Solution

To make things easier to see, insert zeros at the end to give all the numbers the same number of decimal places. In this example the number with the most decimal places is 5.182. It has three decimal places.

51.820
51.280
5.182
51.080
51.800

Place in ascending order, smallest to largest.

5.182 is the smallest because it is the only one with no value in the tens column.

5.182

The whole number part in all the remaining numbers is 51. So we have to compare the decimal parts. The lowest part is 080, followed by 280, then 800 and **820**.

51.080, 51.280, 51.800, 51.820

Write all the numbers in order.

5.182, 51.080, 51.280, 51.800, 51.820

Remove the extra zeros that were added.

5.182, 51.08, 51.28, 51.8, 51.82

Exercise 4.1 Naming and comparing decimals

1 Copy this place-value table. Put the 12 numbers given below in the table, with their digits in the correct columns.

Hundreds	Tens	Units	·	tenths	hundredths	thousandths

| **a** 14.82 | **b** 6.014 | **c** 931.02 | **d** 70.8 | **e** 297.86 | **f** 11.14 |
| **g** 503.92 | **h** 8.3 | **i** 0.375 | **j** 200.047 | **k** 4.025 | **l** 0.81 |

2 Write each of the following as a decimal.

  a five and four-tenths
  b six and fifteen-hundredths
  c eight and three tenths
  d eleven and thirty-eight hundredths
  e fourteen and six-hundredths
  f four hundred and two and three-thousandths
  g 6 + 2 tenths + 3 hundredths
  h 3 + 7 tenths
  i nineteen and nine-tenths
  j 14 + 3 tenths + 9 hundredths
  k 2 tens + 4 + 0 tenths + 9 hundredths
  l 2 + 6 hundredths
  m 8 + 7 tenths + 5 hundredths
  n seventy-three hundredths
  o nine-thousandths
  p 8 tenths + 5 hundredths + 9 thousandths

3 Write the following in expanded notation.   *See* Example 1

  a 1.234 **b** 102.34 **c** 30.12 **d** 0.751 **e** 2.09
  f 12.71 **g** 8.003 **h** 4.509 **i** 0.04

4 What is the value of the digit **4** in each of these numbers?

  a 431.70 **b** 31.047 **c** 761.04 **d** 114.37 **e** 3.734 **f** 907.431
  g 0.064 75 **h** 42 376 **i** 72.314 **j** 26.74 **k** 100.0407 **l** 94.071

5 What is the value of the digit **7** in each number listed in question **4**?

6 Write the following in decimal notation.

  a $4 + \frac{1}{10}$
  b $3 + \frac{8}{10}$
  c $4 + \frac{3}{10} + \frac{5}{100}$
  d $12 + \frac{9}{10} + \frac{3}{100}$
  e $5 + \frac{2}{10} + \frac{7}{100} + \frac{9}{1000}$
  f $0 + \frac{4}{100} + \frac{6}{1000} + \frac{1}{10\,000}$
  g $19 + \frac{9}{1000}$
  h $2 + \frac{7}{10} + \frac{3}{1000}$
  i $11 + \frac{6}{10} + \frac{2}{10\,000}$
  j $3 \times 1 + 2 \times \frac{1}{10} + 7 \times \frac{1}{100}$
  k $2 \times 100 + 7 \times 10 + 6 \times \frac{1}{10} + 1 \times \frac{1}{1000}$
  l $7 \times 10 + 6 \times 1 + 5 \times \frac{1}{100}$
  m $3 \times \frac{1}{10} + 4 \times \frac{1}{100} + 1 \times \frac{1}{1000}$
  n $9 \times \frac{1}{10} + 7 \times \frac{1}{1000}$
  o $2 \times 10 + 7 + 2 \times \frac{1}{10} + 3 \times \frac{1}{100}$
  p $4 \times 10 + 9 \times \frac{1}{100}$
  q $9 + 4 \times \frac{1}{100} + 1 \times \frac{1}{1000}$
  r $5 \times 100 + 4 \times \frac{1}{10} + 3 \times \frac{1}{1000}$

7 How many decimal places does each of these numbers have?

  a 1.65 **b** 3.881 **c** 15.3062 **d** 0.005 **e** 7.045 73 **f** 814.3
  g 9.100 001 **h** 203.222 22 **i** 0.040 400 4

8 Arrange these numbers in ascending order (smallest first).   *See* Example 2

  a 43.89, 56.324, 9.998, 80.879, 400, 23.89, 56.314
  b 0.568, 0.684, 0.099, 1.002, 0.586, 5.608, 0.0586
  c 1.23, 0.891, 1.814, 0.222, 7.007, 0.89
  d 0.5, 0.05, 0.005
  e 3.441, 3.404, 3.4, 3.44, 3.004, 3.044
  f 0.2, 0.202, 0.22, 0.022
  g 1.01, 1.002, 1.012, 1.21
  h 0.07, 0.67, 0.71, 0.007, 7

Worksheet
Decimals 4
MAT07NAWK00042

9 Arrange these numbers in descending order (largest first).

a 570.25, 125.63, 0.9899, 4000.99, 1256.3, 400.099

b 5.37, 6.539, 5.639, 5.367, 3.659, 3.66, 5.369

c 1.6, 1.61, 1.599, 1.601, 1.509 **d** 6, 0.06, 0.6, 6.6

e 0.7, 0.07, 0.707, 0.77, 0.007, 7.07 **f** 0.4004, 0.044, 0.404, 0.44

g 0.1, 0.08, 0.65, 0.029 **h** 0.92, 0.921, 0.09, 0.099

10 Insert < or > to make each of these true.

a 0.2 ___ 0.25 **b** 0.731 ___ 0.73

c 0.035 ___ 0.305 **d** 0.007 ___ 0.070

e 1.59 ___ 1.059 **f** 0.099 ___ 0.99

g 44.44 ___ 4.444 **h** 0.7932 ___ 0.7239

11 Copy each of these number lines carefully and fill in the values of the points marked with dots.
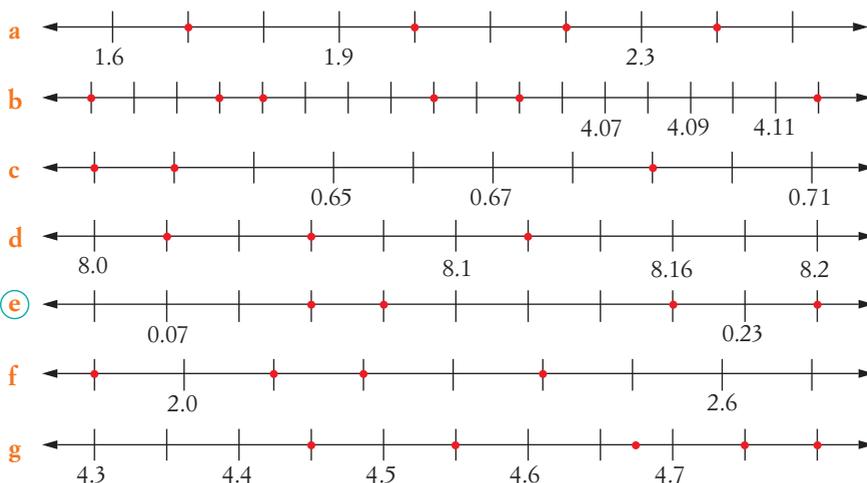
12 Read the following story carefully. List the numbers appearing as **a** to **r** and place a decimal point in each so that the story makes sense.

Maria filled her car, which was nearly empty with **a** 434 litres of petrol at **b** 1399 cents per litre and handed the attendant a **c** $5000 note and a **d** $2000 note. After she received change of **e** $930 she picked up her friend Fred, a tall man of **f** 188 metres in height. Into the car jumped Charlie the dog, weighing **g** 144 kilograms, happily wagging his **h** 150 centimetres of tail. Maria and Fred had each packed a **i** 200 kg backpack. They stopped for a snack at McDougall's and bought two beefburgers each, so that they could have their daily intake of **j** 250 kg of meat.

After they had driven for several hours, averaging **k** 850 kilometres per hour, they were within sight of Mt Kosciuszko, the highest mountain in Australia, about **l** 23 000 m above sea level. They enjoyed seeing the small eucalypt trees about **m** 105 m high. They also saw a kangaroo that jumped about **n** 80 metres, which was **o** 110 times the Olympic record for the long jump (set in Mexico City). After a hike of **p** 134 km they arrived at their campsite. After hiking back to the car the next day, they drove the **q** 2500 kilometres home in about **r** 300 hours.

13 Look at the following diagram.
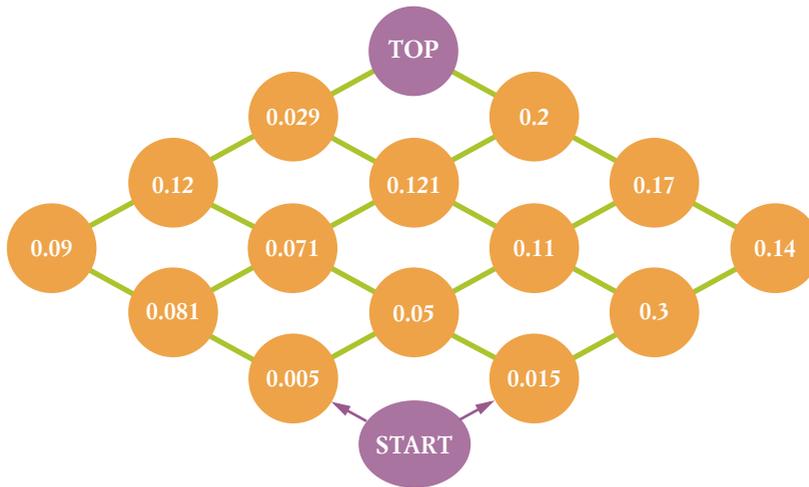
a Find a path from the START circle to the TOP circle. You can make your first move in either direction, but then you can only move to a circle with a larger number.

b Now write out the number sequence for:
   i the longest path
   ii a path requiring seven moves
   iii a route requiring nine moves

c There are several paths that use the smallest number of moves.
   i Find and write the number sequence for all the shortest paths you can find.
   ii How many moves are in the shortest path?

4.2 Decimals, fractions and rounding

This shape has been divided into 10 equal parts. Nine out of the 10 parts are coloured green.
Writing this as a fraction, $\frac{9}{10}$ or nine-tenths of the shape is coloured.
Writing it as a decimal, 0.9 or 'zero point nine' of the shape is coloured.
A decimal number has a **decimal point** to mark where the whole part is separated from the fraction part of the number.
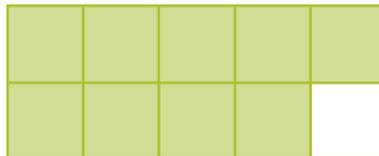
Important!

Decimal places
- tenths have one decimal place ($\frac{1}{10}$ has 1 zero)
- hundredths have two decimal places ($\frac{1}{100}$ has 2 zeros)
- thousandths have three decimal places ($\frac{1}{1000}$ has 3 zeros)
- ten thousandths have four decimal places ($\frac{1}{10\,000}$ has 4 zeros)

Example 3

Convert each fraction to a decimal.

a $\frac{22}{100}$ **b** $3\frac{7}{10}$ **c** eighty-five thousandths

Solution

a $\frac{1}{100}$ has 2 zeros which means 2 decimal places. $\frac{22}{100} = 0.22$

b $\frac{1}{10}$ has 1 zero which means 1 decimal place. The 3 is a whole number. $3\frac{7}{10} = 3.7$

c Thousandths $\left(\frac{1}{1000}\right)$ has 3 zeros which means 3 decimal places.   eighty five thousandths $= \frac{85}{1000} = 0.085$

Example 4

Convert each decimal to a fraction.

a 0.09 **b** 0.273 **c** 8.1

Solution

a 2 decimal places means hundredths $\left(\frac{}{100}\right)$. $0.09 = \frac{9}{100}$

b 3 decimal places means thousandths $\left(\frac{}{1000}\right)$. $0.273 = \frac{273}{1000}$

c 1 decimal place means tenths $\left(\frac{}{10}\right)$ 8 is a whole number. $8.1 = 8 + \frac{1}{10} = 8\frac{1}{10}$

Whole numbers can be rounded to the nearest ten, hundred, thousand and so on. Decimals can be rounded in the same way to the nearest tenth, hundredth and so on. However, we normally call this rounding **to 1 decimal place**, **to 2 decimal places** and so on.

We assume that a number written as 5.43 is accurate to 2 decimal places, as the smallest place value is the hundredths. In the same way, 67.4 is assumed to be accurate to 1 decimal place. If we wanted to show that 67.4 was actually accurate to 2 decimal places, we would write it as 67.40.

This shows that we know that there were no hundredths.

Important!

Rounding steps

1 Underline the digit to be rounded.
2 Circle the next digit to (the right of) the underlined digit.
3 If the circled digit is less than 5, leave the underlined digit as it is.
4 If the circled digit is 5 or more, round the underlined digit up to the next digit.
5 Leave off the digits after the rounded digit.

Example 5

Round each of the following to the stated degree of accuracy.
a 2.68 (1 decimal place)
b 16.5749 (2 decimal places)
c 19.9545 (1 decimal place)

Solution

a Underline the number in the tenths position and circle the next digit. $\text{2.6}\underline{6}\,\text{⑧}$

The circled digit is 8, so round up the underlined digit. $2.68 \approx 2.7$

Change 6 to 7 and leave off the 8.

b Underline the number in the hundredths position and circle the next digit. $16.5\underline{7}\,④\,9$

The circled digit is 4, so the 7 stays. $16.5749 \approx 16.57$

Keep the 7 and leave off the other digits.

c Underline the number in the tenths position and circle the next digit. $19.\underline{9}\,⑤\,45$

The circled digit is 5, so round up. This 'carries' across to the units and then the tens digit. Change the 19 to 20 and make the tenths zero. $19.9545 \approx 20.0$

In Example **5c**, the .0 in 20.0 means that the number is accurate to 1 decimal place.

Important!

Combining measurements

When measurements of different accuracy are combined, the answer is rounded to the least accurate of the original figures.

Investigate: Rounding prices

Since the use of 1c coins and 2c coins stopped in 1990, prices have been rounded to the nearest 5 cents. Find out:
a when amounts are rounded
b how amounts are rounded (check the major stores in your area)
c what happens with bills such as electricity, phone, etc.
d who decides how the rounding will work.

Exercise 4.2 Decimals, fractions and rounding

1 What part of each of these shapes has been shaded? (Give your answers as decimals.)

a

b   **c**

d
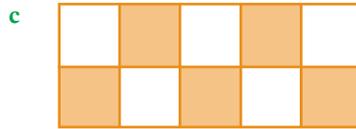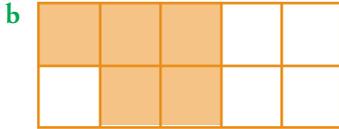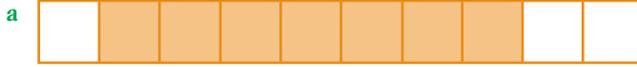
See Example 3 **2** Convert each fraction to a decimal.

a $\frac{9}{10}$ **b** $\frac{15}{100}$ **c** $\frac{79}{100}$ **d** $\frac{60}{100}$

e four tenths   **f** $\frac{23}{100}$ **g** $\frac{6}{10}$ **h** $\frac{411}{1000}$

i $\frac{704}{1000}$ **j** eighty-seven hundredths **k** $\frac{7}{100}$ **l** $\frac{14}{100}$

m $\frac{8}{10}$ **n** $\frac{235}{1000}$ **o** $\frac{247}{1000}$ **p** $\frac{17}{100}$

q $\frac{368}{1000}$ **r** $\frac{9345}{10\,000}$ **s** $\frac{493}{1000}$

3 Write each of these as a decimal.

a $\frac{67}{100}$ **b** $\frac{3}{10}$ **c** $\frac{4}{100}$

d $\frac{11}{1000}$ **e** $\frac{8}{100}$ **f** $\frac{6}{1000}$

g $\frac{1}{100}$ **h** thirty-two hundredths   **i** five thousandths

j $\frac{17}{10\,000}$ **k** $\frac{2}{100}$ **l** $\frac{57}{1000}$

m $\frac{33}{10\,000}$ **n** $\frac{46}{100\,000}$ **o** seven millionths

4 Write each of these as a decimal.

a $1\frac{67}{100}$ **b** $4\frac{47}{100}$ **c** $23\frac{9}{10}$ **d** $6\frac{8}{10}$

e forty-five and twenty-three thousandths   **f** $2\frac{3}{10}$ **g** $6\frac{11}{1000}$

h $89\frac{143}{1000}$ **i** $42\frac{0}{10}$ **j** $5\frac{0}{100}$ **k** $4\frac{6}{1000}$

See Example 4 **5** Convert each decimal to a proper fraction.

a 0.7   **b** 0.4   **c** 0.39   **d** 0.572
e 0.003   **f** 0.05   **g** 0.11   **h** 0.309
i 0.9   **j** 0.999   **k** 0.013   **l** 0.0004
m 0.0471   **n** 0.3333   **o** 0.5001   **p** 0.91
q 0.087   **r** 1.9   **s** 27.33   **t** 2.007
u 10.349   **v** 7.41   **w** 101.3   **x** 6.0102

See Example 5 **6** Write each of the following correct to one decimal place.

a 0.35   **b** 0.47   **c** 0.81   **d** 0.69
e 2.55   **f** 0.32   **g** 0.90   **h** 2.88

7 Round each of the following to two decimal places.

a 0.481	**b** 0.736	**c** 0.069	**d** 0.293
e 0.309	**f** 0.655	**g** 2.096	**h** 3.995

8 Write each of the following correct to two decimal places.

a 25.3456	**b** 341.7675	**c** 321.3333
d 734.6541	**e** 27.757 575	**f** 1314.123 45

9 Copy this table into your book. Use a calculator to help you complete it.

Question	Calculator display	Rounded to 1 decimal place	Rounded to 2 decimal places
12.19 ÷ 3	4.0633333333	4.1	4.06
12.32 ÷ 6			
19.82 ÷ 9			
56.85 ÷ 11			
17.13 ÷ 4			
12.65 ÷ 12			
4.875 ÷ 21			
27.45 ÷ 8			
17 ÷ 12			
254.678 ÷ 32			

10 Use a calculator to find the value of each of the following divisions. Give your answers to the nearest hundredths.

a 2.89 ÷ 3	**b** 8.57 ÷ 6	**c** 0.812 ÷ 9	**d** 7 ÷ 11
e 5.12 ÷ 6	**f** 11.71 ÷ 7	**g** 8 ÷ 6	**h** 12 ÷ 7
i 18.87 ÷ 11	**j** 12.62 ÷ 13	**k** 7 ÷ 12	**l** 9.38 ÷ 15

11 Round each of these numbers to four decimal places.

a 10.333 74	**b** 431.543 27	**c** 1.444 95
d 3217.654 061	**e** 4.670 89	**f** 0.888 88

12 The answers to the following are whole numbers but, for particular reasons, some need to be rounded up and some need to be rounded down. Find the answers.

a A box of 33 chocolates is shared among a family of five people. How many chocolates does each person receive?

b A new bathroom requires 32 square metres of tiles. The tiles come in boxes containing 1.5 square metres. How many boxes are needed to tile the bathroom?

c A team of four golfers wins 27 new golf balls. How many does each person receive?

d Some timber comes in 1.8 m lengths. How many lengths are needed to build a chicken coop needing 23 m of timber?

e Each blouse requires 1.3 m of material. How many blouses can be made from a 5 m length of material?

13 Give a reason for:

a rounding up

b rounding down

4.3 Adding and subtracting decimals

To add or subtract decimals, we use methods similar to those for whole numbers. We use mental arithmetic for simple cases, especially those where the place values are the same.

Before you add or subtract decimals it is a good idea to try and estimate your answer.

Important!

Written method of addition and subtraction of decimals

1 Put a decimal point at the right end of whole numbers.
2 Set out vertically with decimal points under each other.
3 Put zeros on the right end if needed to make all the decimal parts the same length.
4 Add or subtract as if the algorithm was of whole numbers.
5 Put the decimal point in the answer in line with those in the problem.

Example 6

Add 16.27, 10.92, 4.03, 0.89, 32, 0.6.

Solution

Estimate your answer. $16 + 11 + 4 + 1 + 32 + 1 = 65$

The answer should be about 65.

Set the numbers out vertically with the
decimal points under each other. Add extra
zeros so that all the decimals are the same length.
Add in the usual way, ignoring the decimal point.

$$
\begin{array}{r}
{\scriptstyle 1\ 2\ 2}\\
16.27\\
10.92\\
4.03\\
0.89\\
32.00\\
+\ 0.60\\
\hline
64.71
\end{array}
$$

Example 7

a Subtract 8.914 from 46.029.   **b** Find the answer to 4.31 − 2.183.

Solution

a Estimate your answer. $46 - 9 = 37$

The answer should be about 37.

Set the numbers out vertically with the
decimal points under each other.
The decimals are the same length so there is
no need to add extra zeros. Subtract in the usual way,
ignoring the decimal point.

$$
\begin{array}{r}
{\scriptstyle 3\ \ 5\ \ 1}\\
4\ 6.029\\
-\ 8.914\\
\hline
37.115
\end{array}
$$

b Estimate your answer.

$$4 - 2 = 2$$

The answer should be about 2.

Set the numbers out vertically with the decimal points under each other. Add extra zeros so all the decimals are the same length. Subtract in the usual way, ignoring the decimal point.

$$
\begin{array}{r}
4.3\,1\,0 \\
-\ 2.1\,8\,3 \\
\hline
2.1\,2\,7
\end{array}
$$

Investigate: Decimal clues

1 Use the clues below to find each girl's name and height.

- Mandy is taller than Sarah.
- Sarah is shorter than Yin.
- Kelly is taller than Sarah but shorter than Mandy.

- Mandy is not the tallest.
- The heights of the girls are 168.5 cm, 166.3 cm, 164.2 cm and 160.7 cm.

2 Use the clues below to find each boy's name and height.

- Steve is 164.7 cm tall.
- Mike is 14.7 cm taller than Milof.

- Steve is 3.9 cm shorter than Milof.
- Ganesh is 1.6 cm taller than Mike.
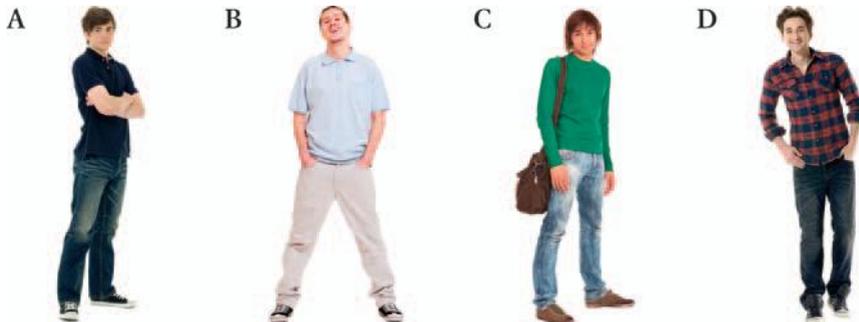
3 Use the clues below to find each student's name and height.

- Yoko is 15.1 cm taller than Peter.
- Jade is 13.7 cm shorter than Yoko.
- Karl is 20.6 cm taller than Jade.
- Yoko is 6.9 cm shorter than Karl.
- Peter is 163 cm tall.

A   B   C   D

4 Devise your own problem using four to six class members. Estimate their heights and height differences. Write a set of clues. (Don't forget to change their names!)

Exercise 4.3   Adding and subtracting decimals

Understanding

1 Copy and complete these addition calculations.

a   5.3
     6.2
  + 0.5

b   4.723
     0.01
  + 12.2

c   43.5
     116.29
     7.3
  + 0.227

d   0.0076
     1.23
  + 0.9

e   37
     3.45
     1.98
  + 548.7

2 Copy and complete these subtraction calculations.

a   57.703
  − 16.21

b   6.1
  − 0.2

c   23.57
  − 16.88

d   22.6
  − 13.54

e   48.6
  − 9.951

See Examples 6, 7

3 Find the answers to the following.

a $103.67 + 9.81 + 0.24 + 3.7$
b $4.6 + 2.9 + 15 + 0.16 + 32.32$
c $98.64 − 41.09$
d $7.41 − 3.95$
e $38.624 + 1.109 + 23.7 + 0.65$
f $58.94 − 2.31 − 46.13$

Problem solving

4 An electrician needed these lengths of cable to complete a wiring job:
12.3 m, 4.8 m, 18.7 m, 7.98 m, 13.65 m and 23.6 m.
  a How many metres of cable did the electrician use?
  b If the full spool of cable was 100 m long, how many metres of cable were left in the spool after the electrician completed the job?

5 Add each set of prices. Calculate the exact change from the amount shown in brackets.
  a $7.01 $0.34 $2.19   ($10)
  b $0.85 $4.34 $1.17 $8.79   ($20)
  c $11.34 $9.15 $3.95 $7.92 $2.36   ($50)

6 To keep fit, Angela runs each day. Last week she ran 3.8 km, 4.1 km, 2.3 km, 2.6 km, 3.0 km, 0.9 km and 1.8 km. How far did she run last week?

7 A truck carrying sand had a total mass of 13 248 kg. If the truck alone had a mass of 5210.8 kg, what is the mass of the sand?

8 Five runners in the school's 100 m race recorded the following times: 13.5 s, 13.81 s, 12.7 s, 14.62 s and 12.45 s.
  a Place these times in order, from fastest to slowest.
  b What is the time difference between the first runner to finish and the last?
  c If the runner in second place had run 0.3 seconds faster, would she have won the race? Explain your answer.

9 Krysten's expenses for one week are shown in the table below.
  a How much did she spend?
  b How much did she have left out of her salary of $620.80?

Item	Cost ($)
Food	128.80
Clothing	88.45
Car	58.35
Rent	185.00
Entertainment	78.95
Savings	66.00

10 A block of wood 11.27 cm thick has 0.34 cm shaved off one side and 0.55 cm shaved off the other side. How thick is the block of wood now?

11 Wendy was making teddy bears. She needed these amounts of material for five bears: 2.6 m, 0.8 m, 1.2 m, 0.75 m and 0.88 m. How much material did she need altogether?

12 The odometer in a car measures the total distance that the car has travelled. The odometer below reads 21 456.9 km. The red digit shows tenths of a kilometre.

2	1	4	5	6	9

Find the distance travelled during a holiday if the odometers below give the readings at the start of the holiday and at the end of the holiday.

2	1	5	7	6	4

2	2	3	1	5	3

4.4 Multiplying and dividing decimals

You can mentally multiply or divide decimals by whole numbers such as 2, 4 and 5 using the same methods as for whole numbers.

You can use place value to multiply or divide by 10, 100, 1000 and so on in your head. This can be used with whole numbers or decimals and can give a whole number or decimal result. The value of each digit will increase by one place value for every multiplication by 10. Similarly, the value will decrease by one place for every division by 10. If multiply or divide by 100, then each digit's place value will increase or decrease by two places. This is because $100 = 10 \times 10$. For instance, $4.8 \times 100 = 480$.

Another way to look at multiplication by 10, 100, 1000, etc. is the following:

- When multiplying a number by 10, move the decimal point one place to the right.
- When multiplying a number by 100, move the decimal point two places to the right.
- When multiplying a number by 1000, move the decimal point three places to the right.
- When dividing a number by 10, 100 or 1000, simply move the decimal point the same number of places to the left.

Multiplication or division by 0.1, 0.01, etc. is actually multiplication or division by $\frac{1}{10}$, $\frac{1}{100}$, etc., so must be the opposite of multiplication or division by 10, 100, etc. This means that multiplication by 0.1 decreases the place values by one place, while division by 0.1 increases the place values by one place. For example, $23.5 \times 0.1 = 2.35$.

Example 8

Work out the following in your head.

a 5.86×10 **b** $76.9 \div 10$ **c** $\frac{7.84}{2}$ **d** $\frac{19.3}{1000}$ **e** $6.05 \times 10\ 000$

f 6.78×0.01 **g** $\frac{8.24}{0.001}$ **h** 5×21.79 **i** 4.5×12 **j** $0.42 \div 400$

Solution

a When multiplying by 10, move the decimal point one place to the right. $5.86 \times 10 = 58.6$

b When dividing by 10, move the decimal point one place to the left. $76.9 \div 10 = 7.69$

c Halve the number. $\frac{7.84}{2} = 3.92$

d When dividing by 1000, move the decimal point three places to the left. $\frac{19.3}{1000} = 0.0193$

e When multiplying by 10 000, move the decimal point four places to the right. $6.05 \times 10\ 000 = 60\ 500$

f Multiplying by 0.01 is the same as dividing by 100. So move the decimal point two places to the left. $6.78 \times 0.01 = 0.0678$

g Dividing by 0.001 is the same as multiplying by 1000. So move the decimal point three places to the right. $\frac{8.24}{0.001} = 8240$

h Use $5 = \frac{10}{2}$, so increase values by one place and halve the answer.

$$5 \times 21.79 = \frac{217.9}{2}$$
$$= 108.95$$

i Treat 12 as 2×6.

$$4.5 \times 12 = 4.5 \times 2 \times 6$$
$$= 9 \times 6$$
$$= 54$$

j Treat 400 as 4×100.

$$0.42 \div 400 = 0.42 \div (100 \times 4)$$

We first divide by 100, so move the decimal point 2 places to the left.

$$= 0.0042 \div 4$$

We then divide the result by 4.

$$= 0.001\,05$$

Consider the problem 6.5×4.63. This could be rewritten as $\frac{65}{10} \times \frac{463}{100}$ because $65 \div 10 = 6.5$ and $463 \div 100 = 4.63$. This means we could rewrite the whole problem as $65 \times 463 \div 10 \div 100$, which is the same as $65 \times 463 \div 1000$. Now $65 \times 463 = 30\,095$, and $30\,095 \div 1000 = 30.095$, so altogether $6.5 \times 4.63 = 30.095$. Consideration of similar problems leads to the general rule for multiplication of decimals.

Important!

Written multiplication of decimals

1 Ignore the decimal points and multiply as whole numbers.
2 Count the total number of decimal places altogether in the numbers.
3 Insert the decimal point to give this total number of decimal places in the answer.

Video tutorial

Multiplying decimals

MAT07NAVT10015

Investigate: Multiplication maze

Can you work out how to get through the maze below? Start with 1 in your calculator display and as you travel along each path multiply the number in the display by each number you pass. You may travel along each path only once but it can be in any direction. You must finish with a 5 in the display.
How many steps are needed to complete the maze?
Is there only one solution to this maze?
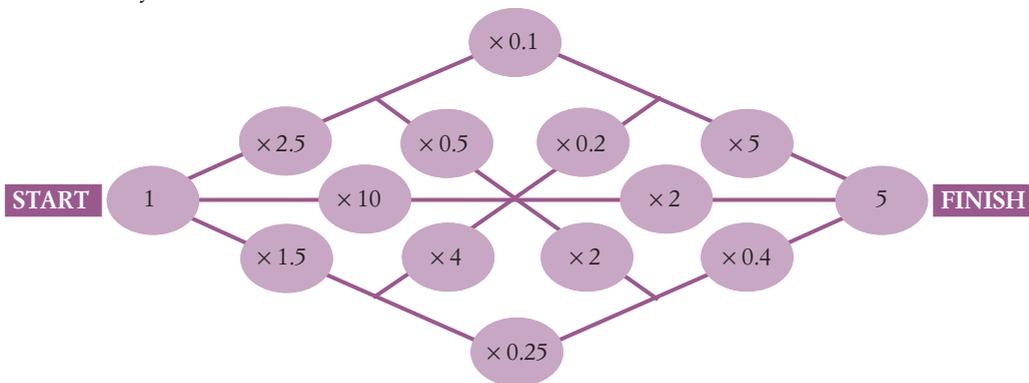
Example 9

Use written methods to complete the following.

a 6.5×0.7 **b** 4.56×0.024 **c** $9.467 \times 0.007\,52$

Solution

a Ignore the decimal and set out like 65×7, so put the 5 over the 7.
Multiply out.
Put in the decimal point with 2 places.

$$
\begin{array}{r}
6.5 \quad \leftarrow \textbf{1 place} \\
\times\, 0.7 \quad \leftarrow \textbf{1 place} \\
\hline
4.55 \qquad \textbf{2 places}
\end{array}
\Big\}\ \textbf{Add}
$$

b Ignore the decimal and set out like 456×24, so put the 6 over the 4.
Multiply out.

Put in the decimal point with 5 places.

$$
\begin{array}{r}
4.56 \quad \leftarrow \textbf{2 places} \\
\times\, 0.024 \quad \leftarrow \textbf{3 places} \\
\hline
1824 \qquad \textbf{5 places} \\
9120 \\
\hline
0.10944
\end{array}
\Big\}\ \textbf{Add}
$$

c Ignore the decimal and set out like 9467×752, so put the 7 over the 2.
Multiply out.

Put in the decimal point with 8 places.

$$
\begin{array}{r}
9.467 \quad \leftarrow \textbf{3 places} \\
\times\, 0.00752 \quad \leftarrow \textbf{5 places} \\
\hline
18934 \qquad \textbf{8 places} \\
473350 \\
6626900 \\
\hline
0.07119184
\end{array}
\Big\}\ \textbf{Add}
$$

From basic number facts, you know that $\frac{24}{6} = 4$. It is also clear that this means that $\frac{240}{60} = 4$, $\frac{2400}{600} = 4$ and so on. In other words, if we change *both* of the numbers in a division problem by the same number of place values, it makes no difference to the answer. Applying this to decimals, it means that $\frac{2.4}{0.6} = 4$ and $\frac{0.24}{0.06} = 4$.

The answer to $31.34 \div 4$ must be between 7 and 8 because $4 \times 7 = 28$ and $4 \times 8 = 32$. We can perform the calculation in the usual way by regarding the decimal point as just another separator of place values. Since $31.34 = 31.340 = 31.3400$ and so on, we can add as many zeros on the end as we like to obtain the decimal answer.

In some cases, when we perform a division, the division keeps going. In this case, there is a pattern of repeating digits in the decimal part of the answer. Division by any whole number always terminates or gives a repeating pattern.

We get the following rules for dividing by decimals.

Important!

Written division of decimals

1 Change *both* numbers by the same number of place values to make the divisor a whole number.
2 Perform the new division, adding zeros as needed to complete the calculation.
3 If the answer does not terminate, it will have repeating digits. We show that the digits are repeating by putting a bar over the repeating digits. This can also be shown by putting dots over the repeating digits or over the first and last repeating digits.

Example 10

Use written methods to complete the following.

a $61.36 \div 5$ **b** $24.71 \div 0.4$ **c** $\frac{5.1}{0.06}$

d $\frac{4.85}{0.6}$ **e** $17\,041 \div 0\,07$

MAT07NAWK00039

Solution

a Division of decimal numbers is the same as division for whole numbers.
Set out as usual, but putting the decimal point for the answer above the one in the question.
Divide through as usual for whole numbers.
Add a zero to finish the division.

$$
\begin{array}{r}
12.272 \\
5 \overline{)6\,1.\,^{1}3\,^{1}6\,^{3}0\,^{1}}
\end{array}
$$

Write the answer.

$61.36 \div 5 = 12.272$

b Change both numbers by the same number of places to divide by a whole number.

$24.71 \div 0.4 = 247.1 \div 4$

Now set out as usual, but putting the decimal point for the answer above the one in the question.
Divide through as usual for whole numbers.
Add zeros as necessary to finish the division.

$$
\begin{array}{r}
61.775 \\
4 \overline{)247.1\,^{3}0\,^{3}0\,^{2}}
\end{array}
$$

Write the answer.

$24.71 \div 0.4 = 61.775$

c Change both numbers by the same number of places to divide by a whole number.

$\dfrac{5.1}{0.06} = \dfrac{510}{6}$

Now set out as usual, but putting the decimal point for the answer above the one in the question.
Divide through as usual for whole numbers.
In this case the result is a whole number.

$$
\begin{array}{r}
85. \\
6 \overline{)5\,^{3}1\,0.}
\end{array}
$$

Write the answer.

$\dfrac{5.1}{0.06} = 85$

d Change both numbers by the same number of places to divide by a whole number.

$\dfrac{4.85}{0.6} = \dfrac{48.5}{6}$

Now set out as usual, but putting the decimal point for the answer above the one in the question.
Divide through as usual for whole numbers.
Add zeros as necessary to finish the division.
In this case, the division keeps repeating.

$$
\begin{array}{r}
8.08333 \\
6 \overline{)48.5\,^{5}0\,^{2}0\,^{2}0\,^{2}0\,^{2}0\,^{2}0}
\end{array}
$$

Write the answer using a dot over the repeating digit.

$\dfrac{4.85}{0.6} = 8.08\overline{3}$

e Change both numbers by the same number of places to divide by a whole number.

$$17.041 \div 0.07 = 1704.1 \div 7$$

Now set out as usual, but putting the decimal point for the answer above the one in the question.
Divide through as usual for whole numbers.
Add zeros as necessary to finish the division.
In this case, there are six digits that repeat.

$$\begin{array}{r} 243.4428571428 \\[2pt] {}^{3\,2\ \ 3\,3\,2\,6\,4\,5\,1\,3\,2\,6} \\[-2pt] 7\overline{)1704.1000000000} \end{array}$$

Write the answer using a dot over the first and last numbers in the sequence of repeating digits.

$$17.041 \div 0.07 = 243.4\overline{428571}$$

Investigate: Back-to-front problems

The cards for this set of questions have been printed without any decimal points. Insert the decimal points so that the numbers on the cards fit the clues.

1 The difference between these two numbers is 53.7. The sum of the numbers is 58.5.

| 561 | 24 |

2 The difference between the numbers is 178.8. When you divide the greater number by the smaller number, the quotient is between 43 and 44.

| 183 | 42 |

3 The sum of the three numbers is 5.36. The product of the numbers is 1.2096.

| 8 | 42 | 36 |

4 The sum of the three numbers is 4.61. The product of the numbers is 0.9135.

| 15 | 29 | 21 |

5 The sum of the four numbers is 2.55. The product of two of the numbers is 0.196. The product of the other two numbers is 0.217.

| 7 | 31 | 14 |

| 14 |

Exercise 4.4 Multiplying and dividing decimals

Understanding

1 Copy and complete each of these statements.
  a To multiply by 10, move the decimal point _____ place to the _____.
  b To multiply by 100, move the decimal point _____ places to the _____.
  c To multiply by 1000, move the decimal point _____ places to the _____.
  d To divide by 10, move the decimal point _____ places to the _____.
  e To divide by 100, move the decimal point _____ places to the _____.
  f To divide by 1000, move the decimal point _____ places to the _____.

2 Copy and complete the following.

a $46.3 \div 10 =$ **b** $507 \div 100 =$ **c** $1203 \div 1000 =$
d $36.4 \div 100 =$ **e** $705 \div 1000 =$ **f** $3102 \div 10 =$
g $6.43 \div 1000 =$ **h** $64.3 \div 1000 =$ **i** $4.28 \div 1000 =$
j $66 \div 100 =$ **k** $0.31 \div 1000 =$ **l** $0.02 \div 10 =$
m $24.9 \times 100 =$ **n** $0.81 \times 10 =$ **o** $37.42 \times 1000 =$
p $0.416 \times 100 =$ **q** $0.81 \times 100 =$ **r** $2.192 \times 1000 =$
s $60451 \times 100 =$ **t** $6.02 \times 100 =$ **u** $0.031 \times 10 =$

3 Evaluate the following, using the rules that you have found.   *See* Example 8

a 45.213×100 **b** 10.64×1000 **c** $6304 \div 100$
d $5.98 \div 1000$ **e** 847.612×100 **f** 0.0592×10
g $36.28 \div 100$ **h** 519.4×1000 **i** $40.075 \div 10$
j $81.348 \div 1000$ **k** 502×100 **l** $0.61 \div 100$
m $0.4 \div 1000$ **n** 17.01×100 **o** 12.3×10000
p $66 \div 10000$ **q** $5.2 \times 10 \div 100$ **r** $14.71 \div 100 \times 10$

4 How many decimal places will the answers to the following have?

a 0.25×11 **b** 10.2×4 **c** 5×10
d 7×2.193 **e** 0.9×0.75 **f** 8.06×4.1
g 0.11×1.01 **h** 6.3×0.04 **i** 2.95×5.13
j 0.237×1.2 **k** 0.023×0.042 **l** 321.2×8.1

5 Evaluate the following.

a $469\ 187 \div 100\ 000$ **b** $437.421 \div 1000$
c $27.43 \div 1$ million   **d** $1\ 200\ 000 \div 1\ 000\ 000$
e $235\ 000\ 137 \div 10\ 000$ **f** $137.429 \times 1000 \div 10\ 000$

6 Given that $63 \times 34 = 2142$, find:

a 630×340 **b** 0.63×3.4 **c** 0.63×3400
d 3.4×630 **e** $6.3 \times 34\ 000$ **f** 63×340

7 Given that $1.7 \times 1.2 = 2.04$, find:

a 1.7×12 **b** 17×12 **c** 0.17×1.2
d 120×170 **e** 0.12×17 **f** $17\ 000 \times 0.12$

8 Given that $7.2 \times 3.4 = 24.48$, find:

a 7.2×34 **b** 72×3.4 **c** 0.72×3.4
d 72×34 **e** 7.2×0.34 **f** 720×340

9 Given that $1.26 \times 6 = 7.56$, find:

a 12.6×6 **b** 126×6 **c** 1.26×0.6
d 0.126×6 **e** 0.126×0.6 **f** $126\ 000 \times 600$

10 Use the fact that $0.3 \times 0.24 = 0.072$ to find:

a 3×0.24 **b** 0.3×2.4 **c** 0.3×24
d 30×0.24 **e** 300×2.4 **f** 0.03×0.24

11 Calculate the following.

a 3.05 × 4   **b** 1.02 × 7   **c** 2.001 × 9   **d** 17.1 × 2
e 10 × 2.25   **f** 3 × 4.20   **g** 6.95 × 5   **h** 1.0004 × 8
i 0.18 × 5   **j** 0.4 × 12   **k** 6 × 0.002   **l** 3 × 4.2
m 10.941 × 3   **n** 492 × 0.12   **o** 0.11 × 365

See Example 9

12 Calculate the following.

a 0.4 × 0.8   **b** 3.9 × 0.5   **c** 0.8 × 0.6   **d** 0.3 × 0.24
e 0.08 × 0.04   **f** 3.1 × 0.4   **g** 12.6 × 0.06   **h** 0.28 × 3
i 0.39 × 9   **j** 2.93 × 0.3   **k** 6.80 × 4   **l** 0.54 × 20

13 Evaluate the following.

a 47.9 × 23   **b** 6.43 × 7.2   **c** 83.4 × 6.3
d 94.6 × 5.1   **e** 9.2 × 7.9   **f** 0.7521 × 3.6

See Example 10

14 Evaluate each of the following.

a 4.8 ÷ 2   **b** 18.6 ÷ 3   **c** 20.8 ÷ 5   **d** 32.8 ÷ 8
e 29.3 ÷ 2   **f** 8.79 ÷ 4   **g** 0.056 ÷ 7   **h** 10.71 ÷ 4
i 195.6 ÷ 8   **j** 7.35 ÷ 2   **k** 4.15 ÷ 8   **l** 0.318 ÷ 3

15 Evaluate each of the following.

a 12 ÷ 5   **b** 13.56 ÷ 12   **c** 23 ÷ 4
d 88.88 ÷ 11   **e** 107.1 ÷ 9   **f** 82.5 ÷ 6
g 177 ÷ 12   **h** 2075.6 ÷ 8   **i** 0.732 ÷ 6

16 Evaluate each of the following.

a 0.651 ÷ 3   **b** 37 ÷ 8   **c** 0.078 ÷ 6   **d** 675 ÷ 12
e 89.341 ÷ 7   **f** 4.275 ÷ 5   **g** 2.75 ÷ 4   **h** 0.0913 ÷ 11
i 117.09 ÷ 9   **j** 0.471 ÷ 3   **k** 256.84 ÷ 4   **l** 0.696 ÷ 12

17 Rewrite each of the following divisions so that the second decimal is a whole number.

a 508.8 ÷ 1.2   **b** 17.82 ÷ 0.11   **c** 333 ÷ 4.5
d 1.725 ÷ 2.5   **e** 129.2 ÷ 0.38   **f** 49.5 ÷ 1.5
g 168 ÷ 0.75   **h** 14.823 ÷ 0.61   **i** 0.66 ÷ 0.022

Fluency

18 Evaluate each of these, and check that your answers seem reasonable by estimating.

a 3.48 ÷ 0.4   **b** 7.32 ÷ 0.2   **c** 2.94 ÷ 0.6
d 16.28 ÷ 2.2   **e** 27 ÷ 0.25   **f** 10.08 ÷ 2.4
g 10.4 ÷ 0.05   **h** 5.6 ÷ 0.07   **i** 1.71 ÷ 0.3
j 40.82 ÷ 5.2   **k** 532 ÷ 3.5   **l** 78.12 ÷ 6.2
m 0.272 ÷ 0.08   **n** 98.4 ÷ 0.08   **o** 465 ÷ 7.5

Problem solving

19 Fly screen netting costs $2.75/metre. How much will it cost for 30 metres to screen the windows of a house?

Worked solutions

Exercise 4.4

MAT07NAWS00020

20 Xin-ging has to cut a length of rope 7.8 m long into five equal pieces. How long will each piece be?

21 Cartons of cereal are stacked on pallets for shipment to supermarkets. Each carton is 1.2 m wide and 1.45 m long. A stack in the warehouse is 6 cartons wide by 4 cartons long. What is the length and width of the stack?

22 Sarah needs 5.25 metres of 150 cm wide material for her formal gown. The material costs $42.45/metre. How much will the material for the gown cost?

23 Earthenware drainage pipe is available in lengths of 0.3 m. How many lengths does Nazeh need to make a drain 12.6 m long?

Worked solutions

Exercise 4.4

MAT07NAWS00020

24 Potatoes cost $1.40/kg. What mass of potatoes can Jomar buy for $12.50?

25 Curtain material is available in widths of 0.9, 1.2 and 1.5 metres. The cost per metre of each is $8.95, $11.95 and $15.95 respectively. For all the materials, the pattern repeats every 1.2 metres and drops of 2.1 metres are needed. The large window to be curtained requires a total width of 4.5 metres, including allowance for pleats. Which is the cheapest material to buy, taking the waste into account?

26 Tasha wants to tile a hallway 5.7 m long by 0.95 m wide. She is going to use square tiles, but has a choice of 0.09 m, 0.13 m or 0.2 m tiles. In any case, allowance of 0.004 m between each tile must be made for grout. Based on the amount of waste generated, which would be the best choice for the job?

Reasoning

27 a 18 ÷ 0.5 means 'how many times does 0.5 go into 18?'. Is the answer more or less than 18? Why?

   b Estimate the answer to 18 ÷ 0.5.

   c Use the method from Example **10** to evaluate 18 ÷ 0.5.

28 a 20.4 ÷ 0.3 means 'how many times does 0.3 go into 20.4?'. Is the answer more or less than 20.4?

   b Estimate the answer to 20.4 ÷ 0.3.

   c Find the exact answer to 20.4 ÷ 0.3.

29 What happens when you divide by a number less than 1? Is the answer more or less than the number?

Worked solutions

Exercise 4.4

MAT07NAWS00020

4.5 Applying decimals

It is useful to recognise fractions in their decimal form. It is important to memorise common fractions and their decimal equivalent.

A fraction can be thought of as a division calculation. $\frac{3}{4}$ has the same meaning as $3 \div 4 = 0.75$.

Example 11

Write each of these as a decimal.

a $\frac{3}{5}$ **b** $\frac{5}{8}$

Solution

a $\frac{3}{5}$ means $3 \div 5$

$$5\overline{)3.0}^{\,0.6}$$

To check, enter as: 3 ÷ 5 = .

3÷5	0.6

b $\frac{5}{8}$ means $5 \div 8$

$$8\overline{)5.000}^{\,0.625}$$

To check, enter as: 5 ÷ 8 = .

5÷8	0.625

Remember to add zeros, if necessary, to complete the division.

Decimals such as 0.625 are called **terminating decimals**. 'Terminate' means 'to stop'. Sometimes when a common fraction is converted into a decimal, we get a **repeating** or **recurring decimal**. One or more of the digits in the decimal repeat forever. For example, $\frac{1}{3} = 0.333\ldots$ To show that the pattern goes on forever, we use dots or a line to identify the repeating section: for example, $0.259\,259\,259\ldots = 0.2\dot{5}\dot{9}$ or $0.\overline{259}$.

We only indicate a repeating or recurring pattern for digits after the decimal point. For example, $100\,000 \div 3 = 33\,333.\dot{3}$.

Example 12

Change each of the following to a decimal.

a $\frac{5}{6}$ **b** $\frac{2}{11}$

Solution

a $\frac{5}{6}$ means $5 \div 6$.

$$6\overline{)5.0000\ldots}^{\,0.833\ldots}$$

To check, enter as: 5 ÷ 6 = .

5÷6	0.8333333333

Write the answer as recurring.

$$\frac{5}{6} = 0.8\dot{3}$$

b $\frac{2}{11}$ means $2 \div 11$.

$$11\overline{)2.00000\ldots}^{\,0.18181\ldots}$$

To check, enter as: 2 ÷ 11 = .

2÷11	0.1818181818

Write the answer as recurring.

$$\frac{2}{11} = 0.\dot{1}\dot{8}$$

Investigate: Decimal currency and time

Currency
a Find out about the history of the decimal money system in Australia.
   i When was it introduced?
   ii What system did it replace?
   iii Why was the change made?
   iv What coins and notes were introduced?
   v What changes have occurred since?
b Design a set of notes ($5, $10, $20, $50, $100) that blind and sighted people could both use.

Time
a Investigate the idea of decimal time. How would you measure time in this system?
b Design a decimal time calendar.
c How would decimal time affect your birthday and age?
d Do you think decimal time is possible? Explain.
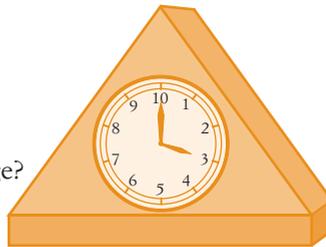
Exercise 4.5 Applying decimals

1 Copy and complete this table. Use a calculator if you need to.

	Common fraction	Meaning as division	Decimal
a	$\frac{3}{5}$	$3 \div 5$	0.6
b	$\frac{1}{2}$	$1 \div 2$	
c	$\frac{1}{4}$		0.25
d	$\frac{4}{5}$		
e	$\frac{2}{5}$		
f		$3 \div 4$	0.75
g		$1 \div 5$	
h		$1 \div 8$	

2 Use your completed table from question 1 to help you find decimals equal to the following fractions.
a $\frac{2}{5}$ b $\frac{3}{8}$ c $\frac{3}{4}$ d $\frac{2}{2}$
e $\frac{2}{4}$ f $\frac{6}{8}$ g $\frac{3}{5}$ h $\frac{2}{8}$
i $\frac{5}{8}$ j $\frac{7}{8}$ k $\frac{4}{8}$ l $\frac{5}{5}$

3 Change these fractions to repeating decimals.   *See* Example 12
a $\frac{1}{9}$ b $\frac{1}{3}$ c $\frac{1}{6}$ d $\frac{1}{7}$ e $\frac{2}{3}$
f $\frac{2}{7}$ g $\frac{2}{9}$ h $\frac{3}{7}$ i $\frac{4}{7}$ j $\frac{4}{9}$
k $\frac{5}{7}$ l $\frac{5}{9}$ m $\frac{6}{7}$ n $\frac{7}{9}$ o $\frac{8}{9}$

4 Copy and complete the following pattern:

Fraction:	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$
Decimal:	$0.\dot{1}$	$0.\dot{2}$						

5 Perform these calculations and write the answers as recurring decimals.

a $11 \div 3$ **b** $11.3 \div 7$ **c** $58.43 \div 9$
d $1.9 \div 6$ **e** $76 \div 9$ **f** $0.67 \div 6$
g $2 \div 7$ **h** $13.4 \div 6$ **i** $149 \div 7$

Fluency

6 Write the following as decimals.

a $4\frac{8}{10}$ **b** $23\frac{3}{4}$ **c** $12\frac{5}{8}$ **d** $6\frac{3}{5}$
e $57\frac{2}{5}$ **f** $19\frac{1}{8}$ **g** $110\frac{7}{8}$ **h** $80\frac{1}{4}$

Problem solving

7 Find the cost in dollars and cents of 352 units of electricity at 12.3 cents per unit.

8 A farmer wants to fence a rectangular paddock. The paddock is 35.6 metres long and 20.85 metres wide. How many metres of fencing will be needed?

9 Mark buys golf balls for $4.85 each and sells them for $5.15 each. How much money does he make if he sells 30 golf balls?

10 A drink bottle holds 0.75 litres. How many drink bottles can be filled from a tub that holds 4.5 litres?

11 A car travels 220.6 kilometres on 14 litres of petrol. How many kilometres would the car travel on one litre of petrol? (Give the answer correct to one decimal place.)

12 Pamela runs 3.8 kilometres each day of the week. How far does she run in one week?

13 A long distance train is made up of a diesel engine, two dining cars and 15 passenger carriages. The engine has a mass of 20.2 tonnes, each dining car has a mass of 14.35 tonnes and each passenger carriage has a mass of 13.96 tonnes. How heavy is the entire train?

14 George is cutting shelves from a board which is 4.6 metres long. Each shelf needs to be 1.25 metres long. How many shelves can be cut?

15 Samantha ran 100 metres in 15.21 seconds. How long would Samantha take to run 400 metres at this pace?

16 Henry has a faulty calculator; it does not show the decimal point. For each of the calculations in the table shown on the right, write the correct answer.

Calculation	Answer
3.42 × 12	4104
4.145 × 0.2	829
37.3 × 8.8	32824
0.03 × 157.64	47292
8.3902 × 0.3	251706

17 Mick walks to work and back each day. He works six days a week and, in one week, walks 16.8 kilometres. How far is Mick's apartment from work?

18 The following are calculator displays for amounts in dollars and cents. Rewrite each amount in dollars and cents, to the nearest cent.

a `15.236`

b `6.8412`

c `487.759318`

d `1234.047126`

19 A sheet of paper is 0.01 cm thick. How many sheets would be in a stack that is 4 cm high?

20 In the Perth FM radio and television guide you will find a list of FM stations and their allocated frequencies measured in megahertz (MHz).

Station	Frequency (MHz)
RTR-FM	92.1
All New 92.9	92.9
NOVA 93.7	93.7
Mix 94.5 FM	94.5
6EBA - FM	95.3
96.1 FM Perth	96.1
SBS National	96.9
Coast FM	97.3
ABC Classic FM	97.7
Sonshine FM	98.5
Triple JJJ	99.3
100 FM	100.1
101 FM	100.7

  a Locate the stations on a number line.

      90   100   110

  b What is the frequency difference in megahertz between Coast FM and 96.1 FM Perth?

  c Find the smallest frequency difference between adjacent stations. ('Adjacent' means side-by-side.)

  d What is the largest difference in frequency between adjacent stations?

21 Explain why some of the fractions in question **2** have the same decimal value.

22 A long time ago, the warrior Wolf was trying to break a code that would open some dungeon doors. He had to free the prisoners before midnight so that they would not be turned into frogs by an evil spell. Each castle door was operated by a combination lock.

Combinations	Rooms
8.262	Queen's chamber
9.24	armoury
9.96	throne room
8.07	banquet hall
8.16	kitchen
8.79	dungeon

  a Use the following clues to match the various combinations with the doors to the different rooms in the castle.

    Clues
- Combination 9.24 opens a door to a room that deals with food.
- The combination to the armoury has a 6 in the hundredths place.
- The combinations of the throne room and the banquet hall add to 18.03.
- The Queen's chamber has a combination that is bigger than 3.78 × 2.1 but smaller than 25.11 + 3.1. (*Hint*: The doors in the dungeon and the throne room remained locked when Wolf tried 9.96 and 8.262.)
- The kitchen combination is one of the three largest combination numbers.

  b Invent another story that sets a code-breaking task involving decimals and ask your classmates to solve it.

23 Calculate the following.

    a 0.02×0.02 **b** $(0.02)^3$ **c** $(1.1)^3$

    d $\sqrt{0.04}$ **e** $\sqrt{0.36}$ **f** $\sqrt[3]{0.027}$

24 Decide where the decimal point should be so that the numbers in the ovals fit the clues.

    a   The product of the two numbers is 91.02. The sum of the numbers is 28.3.

    b   The sum of the three numbers is 14.
        The product of the three numbers is 78.66.

    c   The sum of the four numbers is 28.32.
        The product of two of the numbers is 3.672.
        The product of the other two is 81.

37   246

23   57   6

51   72

45   18
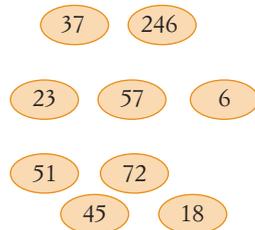
25 The assistant at a pet motel had to feed some unusual pets. Each pet had to be fed a precise amount of food at various times in the week.

The Smiths' pet ate 0.3 kg at a time,

the Nguyens' pet ate 0.7 kg at a time,

the Schulzes' ate 2.9 kg at a time,

the Perezes' ate 3.8 kg at a time

and the Zoghbis' ate 4.4 kg at a time.

After a week, the assistant had made 71 feedings and carried 177.3 kg of food. How many times did each pet get fed? Whose pet ate the most altogether?

- A **decimal point** is used to separate the whole number from the fractional part of a number. The first decimal place is **tenths** (0.1 or $\frac{1}{10}$), the next is hundredths (0.01 or $\frac{1}{100}$) and so on. Each extra decimal place is ten times smaller than the one above it. Decimal fractions can be written in **extended form** by writing out the values of each digit separately.
- The relative sizes of decimal numbers are most easily compared by writing in extra zeros to make the fractional parts the same length.
- An **estimate** is an educated guess concerning the answer to a calculation.
- A **recurring decimal** is a decimal that has one or more digits that repeat forever. We show that digits are repeating by putting a bar (or dots) over the repeating digits. A **terminating decimal** is a decimal that is not recurring, but comes to an end.
- Decimals are added and subtracted mentally by the same methods as for whole numbers. When decimals are added or subtracted on paper:
  – Put a decimal point at the right-most end of whole numbers.
  – Set out vertically with decimal points under each other.
  – Put zeros on the right-most end if needed to make all the decimal parts the same length.
  – Add or subtract as if the algorithm was of whole numbers.
  – Put the decimal point in the answer in line with those in the problem.
- Decimals are multiplied or divided by 10, 100 and so on by changing the place values by one place for every 10. Multiplication or division by 0.1, 0.01 and so on works in the opposite direction to 10, 100 and so on. For other numbers, you use the same methods as for whole numbers.
- When decimals are multiplied on paper:
  – Ignore the decimal points and multiply as whole numbers.
  – Count the total number of decimal places altogether in the numbers.
  – Insert the decimal point to give the total number of decimal places in the answer.
- When decimals are divided on paper:
  – Change *both* numbers by the same number of place values to make the divisor a whole.
  – Perform the new division, adding zeros as needed to complete the calculation.
  – If the answer does not terminate, it will have repeating digits.
  – Show that digits are repeating by putting a bar (or dots) over the repeating digits.
- Decimals can be rounded off to **1 decimal place**, to **2 decimal places** and so on. You can use common sense to find the nearest or follow these steps:
  – Underline the digit to be rounded.
  – Circle the next digit (to the right) to the underlined digit.
  – If the circled digit is less than 5, leave the underlined digit as it is.
  – If the circled digit is 5 or more, round the underlined digit up to the next digit.
  – Leave off the digits after the rounded digit.
- When measurements of different accuracy are combined, the answer is rounded to the least accurate of the original figures.

Quiz
Decimals
MAT07NAQZ00004

1 Write each of the following in decimal form.

   a $4 \times 10 + 3 \times \frac{1}{10} + 6 \times \frac{1}{100}$ **b** $5 \times 100 + 2 \times 10 + 7 \times \frac{1}{10} + 3 \times \frac{1}{1000}$

   c twelve thousandths   **d** $8 + 6 \times \frac{1}{100} + 9 \times \frac{1}{1000}$

2 Evaluate the following.

   a 7.54×10 **b** 7.54×100 **c** 7.54×1000

   d $13.9 \div 10$ **e** $13.9 \div 100$ **f** $13.9 \div 1000$

3 Find the value of each of the following.

   a $12.35 + 4.53 + 0.56 + 3.125 + 24.7 + 20.09$

   b $214.33 - 109.84$

   c $0.568 + 23 + 4.027 - 16.28 + 0.65$

   d $1600.8 - 562.9$

   e $1453.6 + 1287.31 - 2344.4$

   f $204.9 + 23.24 + 0.015$

   g $9.23 - 6.851$

   h $16.51 + 9.003 + 125 + 0.9$

4 Draw a decimal number line for each of these sets of decimals and mark them in their correct locations.

   a 1.6   1.7   1.9   2.0   2.2   2.3   2.4

   b 0.63   0.64   0.67   0.69   0.71

5 Arrange each group of numbers in descending order.

   a 34.98   56.86   3.998   50.141   340   34.89   3.099

   b 0.136   0.86   0.652   0.662   0.23   1.006   0.086   8.6

   c 1.015   1.293   1.1015   1.239

6 Convert each fraction to a decimal.

   a $\frac{4}{10}$ **b** $\frac{13}{100}$ **c** $\frac{7}{100}$ **d** $\frac{11}{1000}$ **e** $\frac{45}{1000}$

7 Convert each decimal to a fraction.

   a 0.5   **b** 0.89   **c** 0.09   **d** 0.444   **e** 4.051

8 Round each of these numbers to the place value shown.

   a 456.8 to the nearest ten   **b** 125.84 to the nearest whole number

   c 2345.876 to two decimal places   **d** 3.8967 to the nearest tenth

   e 78 654.056 to the nearest thousand   **f** 678.439 to one decimal place

   g 102.007 to the nearest hundred   **h** 102.007 to the nearest hundredth

9 Given that $42 \times 76 = 3192$, find:

   a 4.2×76 **b** 4.2×7.6 **c** 0.42×760

   d 420×76 **e** 42×7.6 **f** 420×760

10 Find the answer to each of the following.

See Example 9

   a 2.75 × 6   **b** 0.5 × 1.2   **c** 12.23 × 4

   d 6.1 × 1.2   **e** 0.92 × 10   **f** 100 × 6.7

   g 3.25 × 0.41   **h** 0.05 × 0.02   **i** 4.67 × 1.1

11 Find:

See Example 10

   a 762.4 ÷ 2   **b** 97.6 ÷ 8   **c** 2.75 ÷ 4   **d** 195.6 ÷ 12

12 Find:

   a 12.5 ÷ 0.5   **b** 12.72 ÷ 0.4   **c** 6.9 ÷ 0.03   **d** 27.94 ÷ 1.1

| Puzzle sheet |
| Decimals puzzle 2 |
| MAT07NAPS00022 |

13 Change the following to decimals.

See Examples 11, 12

   a $\frac{4}{5}$ **b** $\frac{3}{8}$ **c** $\frac{5}{9}$ **d** $6\frac{2}{3}$

Problem solving

14 Read the story below and write the number in each part with a decimal point so that the story makes sense.

   a Angelo and Loren decided to go to the motorbike races. They caught the train at 12:45 p.m. after walking 123 km to the station.

   b They gave the station master a $2000 note

   c and received $460 change.

   d The train trip took 15 hours.

   e Angelo said that each motorbike weighed 1400 kg.

   f There were 4 500 000 people at the race meeting

   g who paid a total of $50 850 000 in entrance costs.

   h The average price of a ticket was $113.

   i A mechanic told Loren that the price of race fuel was 1349 cents per litre.

15 The Liverpool Women's Cricket Club is having a pizza night. They order 16 Super Supreme pizzas at $13.70 each and 10 Hawaiian pizzas at $12.10 each. How much will the club need to spend?

16 Maria saved $90 to go to a rock concert. Her return fare cost $5.60, her concert ticket cost $48.95, the program cost $11.00 and food cost $8.70. She did not have enough to buy the band's latest compact disc (priced $24.00) after the concert. How much did she need to borrow from her friend Sam to buy the disc?

17 Rae bought 1200 bricks for $465.70. How much did one brick cost?

5 Time

Contents

Prior learning
Chapter 5

Parent guide
Chapter 5

Curriculum guide
Chapter 5

Australian Curriculum statements

Using units of measurement

Interpret and use timetables. (Year 6) (ACMMG139)

Solve problems involving duration, including using 12-hour and 24-hour time within a single timezone. (Year 8) (ACMMG199) AC

It seems that people never have enough time to do things. Have you ever heard someone say 'so much to do and so little time' or 'there are only so many hours in a day'? It is common to refer to today's society as 'time-poor'. Now, more than ever, it is important to use your time effectively. An important part of using your time well is being able to measure and calculate time periods accurately.

■ Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

12-hour time	hour	noon	week
24-hour time	midday	second	year
century	midnight	time zone	
day	minute	time line	
decade	month	timetable	

5.1   12-hour and 24-hour time

Being able to tell the time is an important life skill. The middle of the day is referred to as **noon** (or **midday**) while the middle of the night is known as **midnight**. Other times are written or said in two common forms.

Important!

12-hour time

The time from midnight to noon is written as **a.m.** from the Latin phrase *ante meridiem* which means 'before midday'. The time from noon to midnight is written as **p.m.** from the Latin phrase *post meridiem* which means 'after midday'.

The hours, minutes and seconds are written in order and separated by a colon (:) – for example 7:30:25 a.m. means 7 hours 30 minutes and 25 seconds after midnight.

Times up to the half-hour like 5:21 are normally said as minutes after the hour – for example 5:21 is 'five twenty-one'. Times after the half-hour like 5:40 are normally said as minutes before the hour (until the hour is reached) – for example 5:40 'twenty to six'. Times such as 8:15, 10:30 and 7:45 are often said as 'a quarter past eight', 'half past ten' and 'a quarter to eight' respectively.

Example 1

Write down how you would say the following times.

a

```
3:54 AM
```
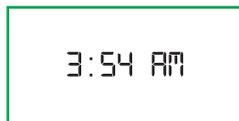
b
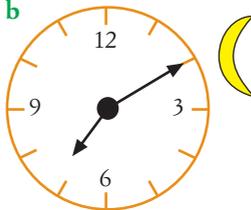
Solution

a It is before noon, so it is in the morning.

It is after the half hour, so work out how long it is to the next hour. Some people would prefer not to do this.

'Six minutes to four in the morning' or 'Three fifty-four in the morning'.

b The moon indicates a p.m. (evening) time. It is before the half hour.

'Ten past seven in the evening' or 'Seven ten in the evening'.

There can be confusion between a.m. and p.m. times when using 12-hour time. You can use 24-hour time to avoid misunderstandings. The armed forces, airlines and many programmable devices such as DVD recorders use 24-hour time.

Important!

24-hour time

The time of day is given as four digits in 24-hour time. The first two digits show the number of hours after midnight and the last two digits show the minutes after the hour. The time is stated as 'hours'. For example 7:32 a.m. is given as 0732 hours (said 'oh seven thirty-two hours') while 7:32 p.m. is given as 1932 hours.

The 24-hour times and their 12-hour equivalents are shown below.

24-hour time	12-hour time	24-hour time	12-hour time
0000 hours	12:00 midnight	1200 hours	12:00 midday
0100 hours	1:00 a.m.	1300 hours	1:00 p.m.
0200 hours	2:00 a.m.	1400 hours	2:00 p.m.
0300 hours	3:00 a.m.	1500 hours	3:00 p.m.
0400 hours	4:00 a.m.	1600 hours	4:00 p.m.
0500 hours	5:00 a.m.	1700 hours	5:00 p.m.
0600 hours	6:00 a.m.	1800 hours	6:00 p.m.
0700 hours	7:00 a.m.	1900 hours	7:00 p.m.
0800 hours	8:00 a.m.	2000 hours	8:00 p.m.
0900 hours	9:00 a.m.	2100 hours	9:00 p.m.
1000 hours	10:00 a.m.	2200 hours	10:00 p.m.
1100 hours	11:00 a.m.	2300 hours	11:00 p.m.

To convert from 24-hour time to 12-hour time, look at the first two digits.
• If they are 00, then it is just after 12:00 midnight.
• If they are less than 12, then it is 'a.m.' (morning) time. Write 'a.m.'.
• If they are 12 or more, it is 'p.m.' (evening) time. Subtract 12 and write 'p.m.'.

Then insert a colon before the last two digits.

Example 2

Convert:
a 6:24 p.m. to 24-hour time
b 1009 h to 12-hour time

Solution

a It is after noon, so remove the colon and add 1200 h.

b The first two digits are less than 12, so it is a.m.

$$6{:}24 \text{ p.m.} = 624 + 1200 \text{ h}$$
$$= 1824 \text{ h}$$

$$1009 \text{ h} = 10{:}09 \text{ a.m.}$$

Exercise 5.1 | 12-hour and 24-hour time

1 Write down how you would say the following times.

a

```
3:23 PM
```

b

c

d

e

```
11:37 AM
```

f

g

```
6:15 AM
```

h

i

```
12:45 PM
```
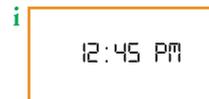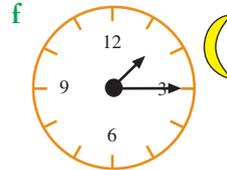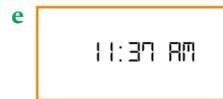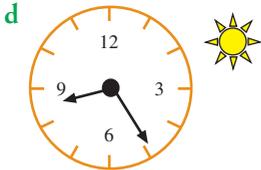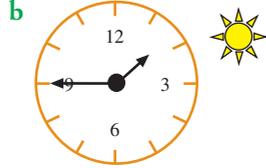
2 Write each of the following in 12-hour time using numbers.

  a twenty to one in the day   **b** sixteen after four in the morning

  c a quarter to four in the day   **d** twenty to five in the morning

  e twenty-six after midnight   **f** a quarter past midday

3 Write each of the times in question **2** in 24-hour time.

4 Change each of the following to 24-hour time.

  a 1 p.m.   **b** 1 a.m.   **c** 10:30 a.m.   **d** 11:10 p.m.   **e** 5:17 p.m.

  f 8:08 p.m.   **g** 12 noon   **h** 11:50 p.m.   **i** 9:09 a.m.

5 Change each of the following to 12-hour time.

  a 0600 hours   **b** 1800 hours   **c** 0330 hours   **d** 1340 hours   **e** 0945 hours

  f 2150 hours   **g** 1220 hours   **h** 2340 hours   **i** 0005 hours

6 Convert these times to 12-hour time.

  a 1440 hours   **b** 0320 hours   **c** 1655 hours   **d** 2331 hours

  e 0108 hours   **f** 1018 hours   **g** 2000 hours   **h** 0643 hours

7 Convert each of these times to 24-hour time.

  a 9:08 a.m.   **b** 9:50 p.m.   **c** 12:42 a.m.   **d** 7:39 a.m.

  e 1:59 a.m.   **f** 10:18 p.m.   **g** 10:46 a.m.   **h** 5:43 p.m.

8 Mandy wants to record a 3 h 26 min TV program on her DVD recorder. The program will begin at 9:35 p.m. The DVD has a 24-hour clock. What times should she set to start and finish recording?

5.2 Using time lines

Time lines record events in the order in which they happen, along a regular scale. A time line for a puppy's first 32 weeks could look like this:
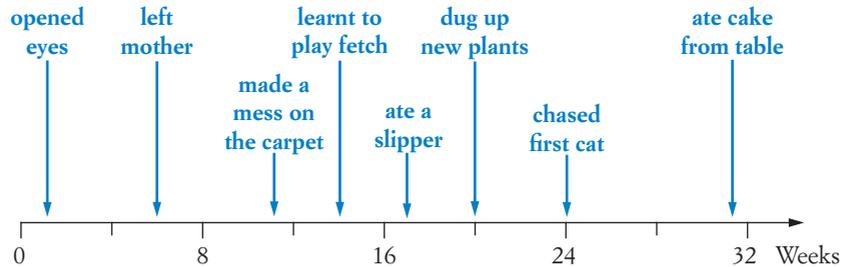
You need to work out the **scale** used on the time line before you can get information from it. On this time line there are eight major divisions between 0 and 32, so each interval (unit) represents 4 weeks.
From the time line, you can see that, at 24 weeks, the puppy chased its first cat. It left its mother at about 6 weeks and at 20 weeks it started digging up the garden.

Important!

Century abbreviations

In the Gregorian calendar, years are counted backward and forward from the birth of Christ. The Gregorian calendar is now used internationally for business, so years in this calendar are abbreviated AD or CE for years after the birth of Christ and BC or BCE for years before the birth of Christ. These stand for Anno Domini (the year of our lord), Common Era, Before Christ and Before Common Era respectively. There is no year 0, so 1 BC (or 1 BCE) is followed by 1 AD (or 1 CE). It is 2250 years from 250 BCE to 2000 CE inclusive.

Example 3

Pete decided to go shopping while on holidays. The following events show some of the things he did during the day.

7:10 a.m. Get up	2:20 p.m. Watch movie	7:30 a.m. Breakfast
9:40 a.m. Start shopping	12:50 p.m. Lunch	9:20 p.m. Bed
6:30 p.m. Dinner	8:15 a.m. Catch bus	

Construct a time line to show these events.

Solution

The period of time covers approximately
15 hours. Choose a scale. **1 cm = 1 hour will fit on a page.**

Draw the line and mark in the hours.

Show the events.
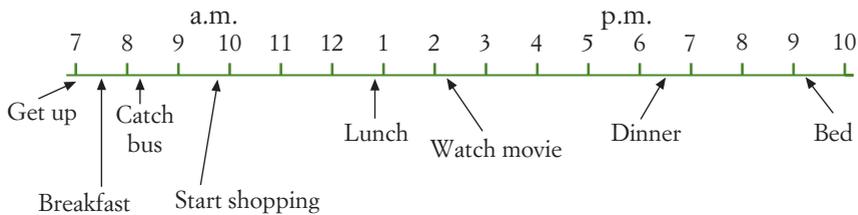
Investigate: Time line display

By yourself, or with a partner, create a time line for one of the following:

- Major disasters of the world
- Historical events of another country
- Achievements in science
- Achievements in sport
- Wars of the last 150 years
- Women in history
- Prime Ministers of Australia
- A topic approved by your teacher

Exercise 5.2 Using time lines

1 Copy this time line.
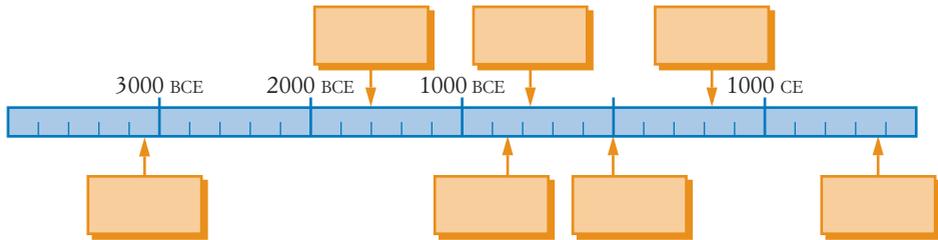
a How many years does each interval (unit) on the time line represent? (This is called the scale of the time line.)

b Write the following dates on the time line in the correct boxes.

- 1 CE   The birth of Christ
- 753 BCE   The founding of the city of Rome
- About 1600 BCE   Introduction of the current Chinese year system
- 3111 BCE   Start of the Mayan 'Long Count'
- 544 BCE   Date recorded as the birth of Buddha
- 1792 CE   Declaration of the 1st French Republic
- 622 CE   Traditional date for the flight of Muhammad

2 This time line shows events from the first 200 years of white settlement in Australia.
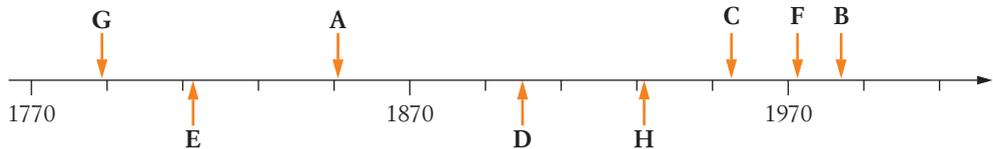
a What is the scale of this time line?

b Match the letters on the time line with these events.

- 1851   Gold was discovered at Warrandyte, Victoria
- 1932   Sydney Harbour Bridge was opened
- 1974   Darwin was devastated by Cyclone Tracy
- 1956   Melbourne hosted the Olympic Games
- 1813   The explorers Blaxland, Wentworth and
                  Lawson crossed the Blue Mountains
- 1788   The First Fleet arrived in Port Jackson
- 1982   Brisbane hosted the Commonwealth Games
- 1901   The Federation of the Australian States to form
                  the Commonwealth of Australia

3 The table below shows the names of Australia's Governors-General and the year they each took office, from 1960 to 2003.

	Name	Year
A	Viscount Dunrossil	1960
B	Lord Casey	1965

	Name	**Year**
C	Sir Zelman Cowen	1977
D	Viscount De L'Isle	1961
E	Right Reverend Dr Peter Hollingsworth	2001
F	Sir William Deane	1996
G	Sir Paul Hasluck	1969
H	Sir John Kerr	1974
I	William Hayden	1989
J	Sir Ninian Stephen	1982
K	Major-General Michael Jeffery	2003

a Copy the time line below and complete it by writing in the letters to indicate when each Governor-General took office. (Two have been done for you.)
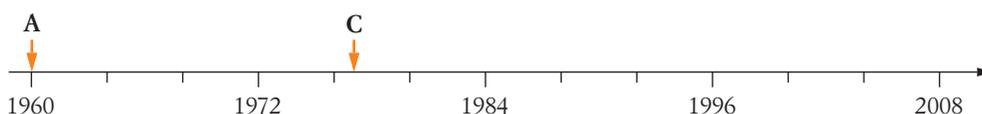
b What is the scale of this time line?

c Which Governor-General was in office for the longest period of time?

d Which Governor-General was in office for the shortest time?

4 Draw a time line to show these events for the period between 1945 and now.

- 1969   Astronauts first walked on the moon
- 1945   World War II ended
- 1989   Wayne Gardner won his first Australian 500 cc Motorcycle Grand Prix
- 1985   The Aboriginal people were granted land rights to Uluru (Ayers Rock)
- 1964   The Beatles toured Australia
- 1983   *Australia II* won the America's Cup
- 1956   The first television transmission in Australia occurred
- 2000   The Olympic Games were held in Sydney
- 2002   Steve Fossett flew solo around the world in a balloon
- 2006   The Crocodile Hunter, Steve Irwin, died
- 20__   (Enter your own important event.)

5 Draw a time line to display these famous Australian inventions and discoveries.

- 1906   The surf-lifesaving reel for use at Bondi Beach was invented
- 1919   The preferential voting system was first used for the House of Representatives
- 1922   Vegemite was developed by Dr Cyril Callister
- 1930   The world's first mechanised letter-sorter was installed in the Sydney GPO, built by A. B. Corbett
- 1945   The Hills rotary clothes line was invented by Lance Hill
- 1952   The Victa rotary lawnmower was developed by Mervyn Victor Richardson
- 1979   Race-cam was first used by Channel Seven at the Bathurst 1000 car races
- 1983   The 'Bionic ear' cochlear implant came on the market
- 1988   Plastic banknotes, developed by the CSIRO, were first released

6 a Work with a partner or in a small group to write a list of important events that have occurred in your lifetime. Try to make a personal list.

    b Draw a time line to show these events

5.3 Converting units of time

Important!

Time units and conversions

Unit	Abbreviation	Conversion
second	s	
minute	min	1 min = 60 s
hour	hr or h	1 hr = 60 min = 3600 s
day	day	1 day = 24 h

A **year** is the time taken for the Earth to go around the Sun.
1 year ≈ 365.2422 days ≈ 12 **months** ≈ 52 **weeks**
In our calendar, 1 year = 365 days and 1 leap year = 366 days.
1 **decade** = 10 years
1 **century** = 100 years

The following diagram may be of help when converting between units of time.
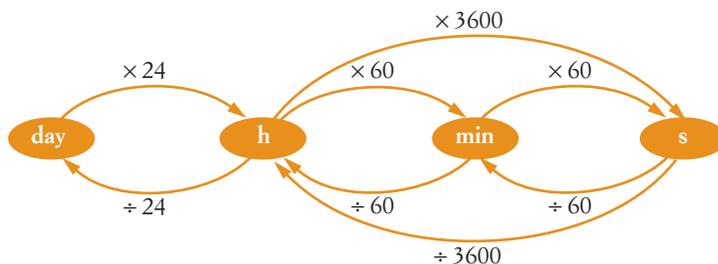
When rounding hours and minutes to the nearest hour, we use 30 minutes as the halfway mark because there are 60 minutes in an hour.
When rounding minutes and seconds to the nearest minute, we use 30 seconds as the halfway mark because there are 60 seconds in a minute.

Example 4

Round each of these amounts of time to the nearest hour.
a 7.83 hours   **b** 12 hours 19 minutes   **c** 2 hours 43 minutes 6 seconds

Solution

a Round off as with normal decimals.
Round up because 0.83 > 0.5   **7.83 h ≈ 8 hours**

b Round down because 19 min < 30 min.   **12 h 19 min ≈ 12 h**

c Round up because 43 min > 30 min.   **2 h 43 min 6 sec ≈ 3 h**

Example 5

Round each of these amounts of time to the nearest minute.

a 11.4 minutes **b** 25 minutes 37 seconds **c** 3 hours 6 minutes 30 seconds

Solution

a Round off as with normal decimals.
Round down because 0.4 < 0.5 **11.4 min ≈ 11 min**

b Round up because 37 s > 30 s. **25 min 37 s ≈ 26 min**

c Just as we round 0.5 up with decimals,
we round 30 s up. **3 h 6 min 30 sec ≈ 3 h 7 min**

Example 6

Convert:
a 7 minutes into seconds. **b** 91 days into weeks.

Solution

a There are 60 s in a minute. **7 min = 7×60 s**
= 420 s

b There are 7 days in a week. **91 days = 91 ÷ 7 weeks**
= 13 weeks

Example 7

Convert 275 minutes into hours and minutes.

Solution

There are 60 min in 1 hour. **275 ÷ 60 = 4 remainder 35**
The remainder is the minutes left over. **275 min = 4 h 35 min**

Most scientific calculators have a degrees-minutes-seconds key ○'" or DMS, that is useful for calculations involving minutes and seconds (base 60). This key can be used to convert decimal answers for time to hours-and-minutes or minutes-and-seconds. Calculating the answer to Example **7** in this way:

$$275 \text{ minutes} = 275 \div 60 \text{ h}$$
$$= 4.583\ 333\ 3\ldots \text{ h}$$

Press ○'" to get 4° 35′ 0″ on the calculator display which means 4 h 35 min.

Technology Predicting gold medal times

The table below shows the Olympic gold medal times for the men's 400 m track event for the games from 1972 to 2008.

Year	1972	1976	1980	1984	1988	1992	1996	2000	2004	2008
Time (s)	44.66	44.26	44.60	44.27	43.87	43.51	43.49	43.84	44.01	43.75

Enter this data into a spreadsheet with the years in row 1 and the times in row 2.

Highlight the data (A1 to K2) and under the **Insert** tab, choose **Scatter** ( ).

Then click on the option ( ) that shows points joined with lines and markers.
Give the graph an appropriate title and label the axes ('Year' for the *x*-axis and 'Time (s)' for the *y*-axis).
Move the cursor over a data point on the graph but don't click on it. What do you see?
Describe the general trend in gold medal-winning times over the period shown in the graph.
Right-click on the line on the graph and select the **Add Trendline** option.
Select the **Linear** trendline ( ). What does the trendline show?
Use the graph to predict the 400 m gold medal winning time for the 2016 Olympics.

Exercise 5.3 Converting units of time

1 State which unit of time (seconds, hours, minutes, or days) would be used to measure each event.

  a snapping your fingers five times, as fast as possible   **b** a day-night cricket match

  c running once around the school oval   **d** building a house

  e flying from Sydney to Broken Hill   **f** watching a DVD from beginning to end

  g the life span of a grasshopper

2 Write these times correct to the nearest hour.

  a 4 h 14 min   **b** 11.5 h   **c** 6 h 27 min

  d 7 h 48 min 19 s   **e** 3.42 h   **f** 2 h 30 min

3 Write these times correct to the nearest minute.

  a 17 min 51 s   **b** 8.8 min   **c** 4 min 7 s

  d 4 h 20 min 19 s   **e** 12.31 min   **f** 1 h 28 min 40 s

4 Convert:

  a 6 hours to minutes   **b** 15 minutes to seconds

  c 9 weeks to days   **d** 2.5 years to weeks

  e 3 days to hours   **f** 2 years to days

  g 2 weeks to hours   **h** 4.25 hours to minutes

  i 8.5 days to hours   **j** $10\frac{1}{2}$ minutes to seconds

  k 7.2 centuries to years   **l** 3 fortnights to days

5 Convert: *See* Example 7

a 480 seconds to minutes   **b** 70 days to weeks
c 96 hours to days   **d** 200 minutes to hours and minutes
e 468 weeks to years   **f** 560 seconds to minutes and seconds
g 60 hours to days   **h** 126 days to weeks
i 330 seconds to minutes and seconds   **j** 24 weeks to fortnights
k 135 minutes to hours and minutes   **l** 470 years to centuries
m 405 minutes to hours and minutes   **n** 167 minutes to hours and minutes

6 Find the number of seconds in:

a 1 hour   **b** 1 day   **c** 1 year

7 Are you over a million seconds old? Find your age in seconds to answer this question.

<div style="background:#eee">**Problem solving**</div>

<div>Worked solutions</div>
<div>Exercise 5.3</div>
MAT07MGWS00024

Investigate: Minutes and seconds

In Chapter 3, you learnt that there are 360° in a revolution because the ancient Babylonians used a base 60 number system and believed that a year lasted 360 days. (How many days is a year actually?) The Babylonians, who lived where Iraq is today, in 2000 BCE, invented the units for measuring angles and time. That is why there are 60 minutes in an hour and 60 seconds in a minute.

The word 'minute' has another meaning. When pronounced '*my-newt*', it means tiny but this meaning is also related to the minute as a unit of time. A minute is a tiny fraction of an hour, and comes from the Latin '*pars minuta prima*', meaning the first division (or part) of an hour. The word 'second' also means coming after first, and this meaning is also related to the second as a unit of time. Find out how.

5.4   Time calculations and timetables

Diagrams can help to do some time calculations.

Example 8

What is the time 7 hours 40 minutes after 11:52 p.m.?

Solution

Draw a line and mark in 11:52 p.m. at the start.

7 hours after 11:52 p.m. is 6:52 a.m.

It's easier to break up the 40 minutes to 8 min + 32 min.

40 minutes after 6:52 a.m. is 7:32 a.m.
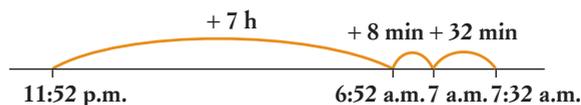
State the result.   **7 h 40 minutes after 11:52 p.m. is 7:32 a.m.**

<div>Worksheet</div>
<div>Tide chart</div>
MAT07MGWK00092

<div>Puzzle sheet</div>
<div>Time calculations</div>
MAT07MGPS00026

Example 9

What is the difference in time between 8:35 a.m. and 3:10 p.m.?

Solution

Draw a line and mark in 8:35 a.m. at the start.

First move up to the next hour.

From 8:35 a.m. to 9:00 a.m. = 25 min.

From 9:00 a.m. to 3:00 p.m. = 6 h.

From 3:00 p.m. to 3:10 p.m. = 10 min.

Use the diagram to work out the difference.
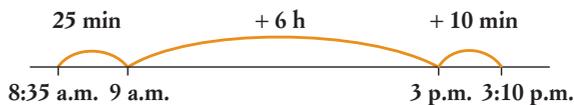
Total time difference = 25 min + 6 h + 10 min
$$= \textbf{6 h 35 min}$$

There is another way to do time calculations. First convert times to 24-hour time. Then use the calculator's ⟨ o , " ⟩ or ⟨ DMS ⟩ key to enter hours and minutes, and subtract the times.

Example 10

Find 7 h 5 min – 3 h 24 min.

Solution

Method 1

You can't subtract 24 min from 5 min.

7 h 5 min – 3 h 24 min

Change 7 h 5 min to 6 h 65 min.

= 6 h 65 min – 3 h 24 min

Subtract the hours and minutes separately.

= (6 – 3) h + (65 – 24) min

Evaluate.

= 3 h 41 min

Method 2

Enter as 7 ⟨DMS⟩ 5 ⟨DMS⟩ –3 ⟨DMS⟩ 24 ⟨DMS⟩ ⟨=⟩ .

7°5'–3°24'   3°41'0"

State the results.

7 h 5 min – 3 h 24 min = 3 h 41 min

Most forms of transport publish timetables so that people can plan their journeys. You will need to do time calculations when you want to find the duration of journeys, the time between trips and so on.

Technology | Timetables

Timetables for many trains, buses and airlines are published on the Internet. This makes it easy for transport companies to keep timetables up to date. It also makes it convenient for travellers to access information.
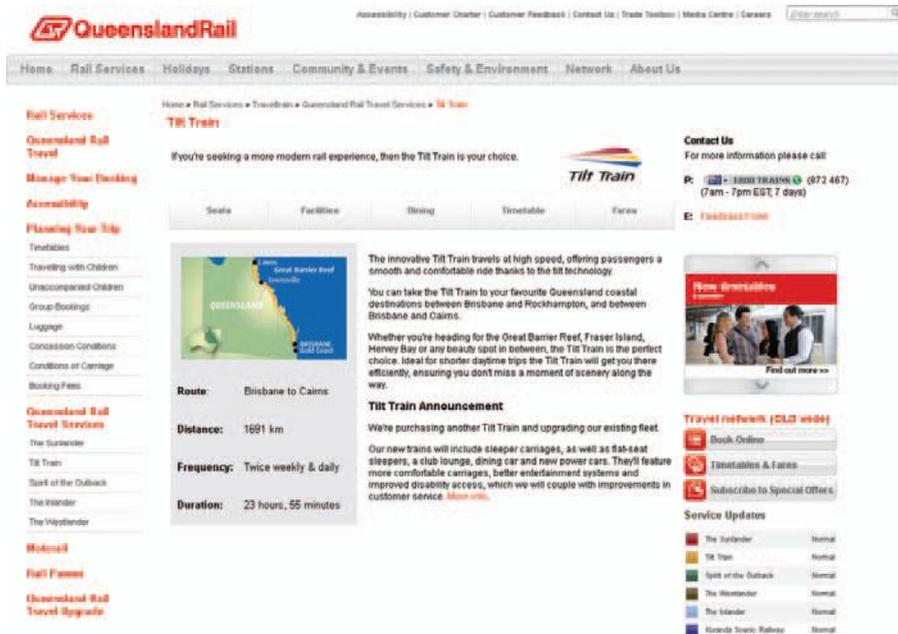
Work with your partner under the direction of your teacher to examine some timetables on the Internet. Compare the timetables using the following:

Is the timetable up-to-date?

Is it easy to find the departure and destination locations?

Are times stated in 12-hour or 24-hour time?

Are trip durations given?

Is the timetable in a static format like PDF or is it interactive?

Exercise 5.4 | Time calculations and timetables

1 What time will it be:

   a 5 hours after 3:00 p.m.?   **b** 8 hours after 11:00 a.m.?

   c 28 minutes after 7:15 p.m.?   **d** 3 hours 32 minutes after 9:45 a.m.?

   e 3 hours 19 minutes after 10:49 p.m.?   **f** 4 hours after 9:32 a.m.?

   g 9 hours after 5:14 p.m.?   **h** 45 minutes after 3:30 p.m.?

   i $2\frac{1}{4}$ hours after 4:02 a.m.?   **j** 12 hours 40 minutes after 2:45 a.m.?

2 What is the difference in time between 10:42 a.m. and 2:13 p.m.? Select **A**, **B**, **C** or **D**.

   A 3 h 31 min   **B** 4 h 55 min   **C** 8 h 29 min   **D** 12 h 55 min

3 What is the difference in time between:

  a 7:15 p.m. and 8:20 p.m.?   **b** 1016 h and 1206 h?
  c 4:09 a.m. and 9:53 a.m.?   **d** 11:15 p.m. and 3:08 a.m.?
  e 0727 h and 1312 h?   **f** 9:36 p.m. and 9:14 a.m.?
  g 7:45 p.m. and 10:10 p.m.?   **h** 2:24 a.m. and 3:07 a.m.?
  i 4:15 p.m. and 6:02 p.m.?   **j** 10:25 a.m. and 1433 h?
  k 8:40 a.m. and 4:19 p.m.?   **l** 0645 h and 2010 h?

4 Find:

  a 2 h 15 min + 4 h 32 min   **b** 3 h 25 min + 8 h 27 min
  c 7 h 12 min + 5 h 18 min   **d** 1 h 42 min + 6 h 27 min
  e 9 h 37 min + 2 h 52 min   **f** 4 h 49 min + 7 h 18 min

See Example 10

5 Find:

  a 6 h 42 min – 3 h 13 min   **b** 12 h 37 min – 5 h 6 min
  c 15 h 57 min – 9 h 48 min   **d** 6 h 2 min – 4 h 17 min
  e 8 h 18 min – 3 h 27 min   **f** 5 h 31 min – 3 h 48 min

Fluency

6 A marathon began at 10:20 a.m. Here are some of the competitors and the times they ran. Write the runners in their order of finishing and the time each crossed the finishing line.

  Mike 3:11 (3 h 11 min)   Joe 2:23
  Anna 2:54   Pathena 3:01
  Ken 2:59   Gail 3:42

Problem solving

7 Daniel and his volleyball team need to fly from Brisbane to Melbourne for a championship tournament. Daniel logged onto the Internet site for Thomson Airways and found the following flight schedule for 12 October.

Flight number	Brisbane departure time	Melbourne arrival time
TH503	0905	1140
TH511	0935	1210
TH038	1005	1240
TH114	1040	1400
TH514	1105	1340
TH051	1135	1410

  a How long does flight TH503 take to fly from Brisbane to Melbourne?

  b The team plans to meet in Brisbane airport at 10:45 a.m. How long will they need to wait for the next available flight?

  c The team needs to be at the hotel in Melbourne by 2:00 p.m. If it takes 30 minutes to drive from the airport to the hotel, what is the latest flight that the team can catch from Brisbane?

  d What is the flight number of the flight that takes longer to reach Melbourne than the others? Give one reason why it might take longer.

8 The table below shows a bus service timetable between Brisbane and Toowoomba.

Forward: Brisbane to Toowoomba		Return: Toowoomba to Brisbane	
Brisbane	1730	Toowoomba	1800
Goodna	1755	Withcott	1810
Blacksoil	1810	Helidon Spa	1815
Haigslea	1815	Gatton College	1830
Minden Crossroads	1823	Plainland	1837
Plainland	1830	Minden Crossroads	1847
Gatton College	1838	Haigslea	1855
Helidon Spa	1855	Blacksoil	1900
Withcott	1857	Brisbane	1935
Toowoomba	1915		

a How long does the trip from Brisbane to Toowoomba take?

b How long does the return trip from Toowoomba to Brisbane take?

c Ali joins the bus at Blacksoil and gets off at Withcott. How long was his trip?

d Rada joins the bus at Helidon Spa and gets off at Blacksoil. How long was her trip?

e Steph catches the bus from Brisbane to Toowoomba to visit her father and then returns to Brisbane on the bus the following day. How long must she wait in Toowoomba between trips?

9 The table below shows a train service timetable

■ City to Ipswich\Rosewood Outbound

Station	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
Bowen Hills pform	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3
Bowen Hills	3:43	3:58	4:01	4:13	4:26	4:43	4:48	4:58	5:01	5:05	5:13	5:19	5:27	5:35	5:40	5:58	6:12
Fortitude Valley pform	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3	#3
Fortitude Valley	3:46	4:01	4:04	4:16	4:29	4:46	4:51	5:01	5:04	5:08	5:16	5:22	5:29	5:37	5:43	6:01	6:15
Central arrive	**3:49**	**4:03**	**4:07**	**4:18**	**4:32**	**4:48**	**4:53**	**5:04**	**5:07**	**5:11**	**5:19**	**5:25**	**5:32**	**5:40**	**5:46**	**6:03**	**6:18**
Central pform	#5	#5	#5	#5	#5	#5	#5	#5	#5	#5	#5	#5	#5	#5	#5	#5	#5
Central depart	3:51	4:05	4:09	4:20	4:35	4:50	4:55	5:06	5:09	5:13	5:21	5:27	5:34	5:42	5:48	6:05	6:20
Roma Street pform	#8	#8	#8	#8	#8	#8	#8	#8	#8	#8	#8	#8	#8	#8	#8	#8	#8
Roma Street	3:53	4:07	4:11	4:22	4:37	4:52	4:57	5:11	5:15	5:23	5:29	5:36	5:44	5:50	6:07	6:22	
Milton	3:56	4:10	4:14	4:25	4:40	4:55	5:00	5:11	5:15	5:19	exp	5:32	5:39	5:47	5:53	6:10	6:26
Auchenflower	3:58	4:12	4:16	4:27	4:42	4:57	5:02	5:13	exp	5:21	exp	5:34	5:41	5:49	5:55	6:12	6:27
Toowong	4:00	4:14	4:18	4:29	4:44	4:59	5:04	5:15	exp	5:23	exp	5:36	5:43	5:51	5:57	6:14	6:29
Taringa	4:02	4:16	4:20	4:31	4:46	5:01	5:06	5:17	exp	5:25	exp	5:39	5:45	5:53	5:59	6:16	6:31
Indooroopilly	4:05	4:18	4:23	4:33	4:48	5:03	5:09	5:20	5:22	5:27	exp	5:42	5:48	5:56	6:02	6:18	6:33
Chelmar	4:07	4:20	4:25	4:35	4:50	5:05	5:11	exp	5:24	5:29	exp	5:43	exp	5:58	6:04	6:20	6:35
Graceville	4:09	4:22	4:27	4:37	4:52	5:07	5:13	exp	5:26	5:31	exp	5:45	exp	6:00	6:06	6:22	6:37
Sharwood	4:11	4:24	4:29	4:39	4:54	5:09	5:15	exp	5:28	5:33	exp	5:47	exp	6:02	6:08	6:24	6:39
Corinda	4:13	4:26	4:32	4:41	4:56	5:11	5:17	5:25	5:30	5:35	exp	5:48	5:53	6:04	6:10	6:26	6:41
Oxley	4:16	4:29	4:35	4:44	4:59	5:14	5:20	5:28	5:33	5:38	exp	5:51	5:56	6:07	6:13	6:29	6:44
Darra	4:19	4:32	4:38	4:47	5:02	5:17	5:23	5:31	5:37	5:41	5:43	5:55	5:59	6:10	6:16	6:32	6:47
Wacol	4:23	4:35	4:42	4:50	5:05	5:20	5:27	exp	5:40	....	exp	5:58	6:03	6:13	6:19	6:35	6:50
Gailes	4:25	4:37	4:44	4:52	5:07	5:22	5:29	exp	5:42	.	exp	6:00	6:05	6:15	6:21	6:37	6:52
Goodna	4:27	4:39	4:46	4:54	5:09	5:24	5:31	5:37	5:44	.	exp	6:02	6:07	6:17	6:23	6:39	6:54
Redbank	4:30	4:43	4:49	4:58	5:13	5:28	5:35	5:40	5:48	.	5:54	6:05	6:11	6:22	6:27	6:43	6:58
Riverview	....	4:45	....	5:00	5:15	5:30	5:37	5:42	5:51	.	5:57	6:07	6:14	6:24	6:29	6:45	7:00
Dinmore	.	4:47	.	5:02	5:17	5:32	5:39	5:44	5:53	.	5:59	6:09	6:16	6:27	6:32	6:47	7:02
Ebbw Vale	.	4:49	.	5:04	5:19	5:34	5:41	5:46	5:55	.	6:01	6:11	6:18	6:29	6:34	6:49	7:04
Bundamba	.	4:51	.	5:06	5:21	5:36	5:43	5:48	5:57	.	6:03	6:13	6:20	6:31	6:36	6:51	7:06
Booval	.	4:53	.	5:08	5:23	5:38	5:45	5:50	5:59	.	6:05	6:15	6:22	6:33	6:38	6:53	7:08
East Ipswich	.	4:55	.	5:10	5:25	5:40	5:47	5:52	6:01	.	6:07	6:17	6:24	6:35	6:40	6:55	7:10
Ipswich	.	**4:58**	.	**5:13**	**5:28**	**5:43**	**5:50**	**5:55**	**6:05**	.	**6:11**	**6:20**	**6:28**	**6:38**	**6:43**	**6:58**	**7:13**

a Michael has a meeting in Ipswich at 6:00 p.m. His meeting is located 5 minutes walk from Ipswich station. At what time must he catch the train from Central?

b What is the difference in time taken to travel from Central to Ipswich on the 4:35 p.m. train and the 5:21 p.m. train?

c Georgina travels from Milton, arriving at Redbank at 5:28 p.m. How long did the trip take?

d What is the difference in time taken to travel from Roma Street to Ebbw Vale on the 4:22 p.m. train and the 5:36 p.m. train?

10 An Explorer Bus operates in most capital cities. It takes tourists on a tour of the city and allows them to visit places of interest. Below is the winter timetable for an Explorer Bus in a particular capital city.

Depart									
Explorer depot	10:00	10:25	10:50	11:15	11:45	12:00	12:25	12:50	1:15
City cathedral	10:08	10:33	10:58	11:23	11:53	12:08	12:33	12:58	1:23
Railway station	10:15	10:40	11:05	11:30	12:00	12:15	12:40	1:05	1:30
Parliament	10:24	10:49	11:14	11:39	12:09	12:24	12:49	1:14	1:39
Museum	10:35	11:00	11:25	11:50	12:20	12:35	1:00	1:25	1:50
City square	10:45	11:10	11:35	12:00	12:30	12:45	1:10	1:35	2:00
Zoo	11:00	11:25	11:50	12:15	12:45	1:00	1:25	1:50	2:15
Dockland shops	11:12	11:37	12:02	12:27	12:57	1:12	1:37	2:02	2:27
Arts centre	11:19	11:44	12:09	12:34	1:04	1:19	1:44	2:09	2:34
Water gardens	11:30	11:55	12:20	12:45	1:15	1:30	1:55	2:20	2:45
Hall of fame	11:38	12:03	12:28	12:53	1:23	1:38	2:03	2:28	2:53
Arrive									
Explorer depot	11:50	12:15	12:40	1:05	1:35	1:50	2:15	2:40	3:05

a How many buses are needed to meet the winter Explorer Bus timetable? Explain how you arrived at your answer.

b Vo, Binh and Vicki came to the city by train, arriving at the station at 11:42 a.m. They caught the Explorer Bus to the zoo. What is the earliest time they could expect to arrive at the zoo? Explain your answer.

c Manuel and Sofia are dropped off by car at the 'City cathedral' at 10:25 a.m. They arrange to meet their hosts at the 'Hall of fame' at 2:45 p.m. They want to spend at least half an hour at the museum, photograph the 'City square' and do some souvenir shopping at the Dockland shops. Plan a list of times for them to catch the Explorer Bus to do these things and meet their hosts on time.

d In summer, extra Explorer tours leave the depot at 11:30 a.m., 1:30 p.m. and 2:30 p.m. Make a list of departure times that appear in the timetable for each of these tours.

- The middle of the day is referred to as **noon** (or **midday**) while the middle of the night is known as **midnight**.

- For **12-hour time**, the time from midnight to noon is written as **a.m.** (ante meridiem) and the time from noon to midnight is written as **p.m.** (post meridiem). The hours, minutes and seconds are written in order and separated by a colon (:), e.g. 4:38 p.m.

- For **24-hour time** the time of day is given as four digits. The first two digits show the number of hours after midnight and the last two digits show the minutes after the hour. The time is stated as 'hours', e.g. 1638 h.

- A **time line** is a record of events in the order in which they happen.

- Common units of time are the **second** (**s**), **minute** (**min**), **hour** (**h** or **hr**) and **day**. You can convert between units of time using the following:

$$1 \text{ min} = 60 \text{ s}; \quad 1 \text{ h} = 60 \text{ min} = 3600 \text{ s}; \quad 1 \text{ day} = 24 \text{ h}.$$

- A **year** is the time taken for the Earth to go around the Sun.

$$1 \text{ year} \approx 365.2422 \text{ days} \approx 12 \text{ months} \approx 52 \text{ weeks}.$$

- In our calendar, 1 year = 365 days and 1 leap year = 366 days.

- 1 **decade** = 10 years and 1 **century** = 100 years

- You can use the degrees-minutes-seconds key on your calculator ○' '' or DMS to convert between units of time.

See Example 1

1 Write each of the following times in words.

a

6:34 PM

b

c

2 Write twenty past nine in the morning in 12-hour time.

See Example 6

3 **a** Change 4 minutes to seconds.   **b** Change $3\frac{1}{2}$ hours to minutes.

See Example 2

4 Change each of the following to 24-hour time.

a 7:30 p.m.   **b** 3:35 a.m.

5 Change each of the following to 12-hour time.

a 0400 h   **b** 1952 h

See Example 3

6 Make up a time line for your life from age 0 to 12.

See Example 4

7 Write each of these amounts of time correct to the nearest hour.

a 9 h 50 min   **b** 3.2 h   **c** 4 h 12 min 49 s

See Example 5

8 Write each of these amounts of time correct to the nearest minute.

a 2 min 36 s   **b** 10.5 min   **c** 3 h 23 min 40 s

See Example 6

9 Copy and complete the following.

a 56 days = _____ weeks   **b** 4 h = _____ min
c 960 s = _____ min   **d** 5 years = _____ weeks
e 7 days = _____ h   **f** 750 min = _____ h

See Example 8

10 What is the time:

a 5 hours after 10:42 p.m.?   **b** 2 hours 28 minutes after 5:23 a.m.?
c 55 minutes before 7:15 p.m.?   **d** 7 hours 36 minutes before 1:19 a.m.?
e 15 hours 34 minutes after 7:00 a.m.?   **f** $3\frac{1}{4}$ hours after 3:40 p.m.?

See Example 9

11 What is the difference in time between:

a 5.26 a.m. and 9:45 a.m.?   **b** 11:56 p.m. and 7:30 a.m.?
c 1316 hours and 2003 hours?   **d** 0750 hours and 1425 hours?
e 2347 hours and 0006 hours?   **f** 1529 hours and 3:28 p.m.?

See Example 10

12 Find:

a 6 h 45 min + 3 h 20 min   **b** 3 h 16 min – 1 h 26 min
c 4 h 33 min + 2 h 24 min   **d** 4 h 19 min – 2 h 50 min

13 Use the Cairns to Townsville bus timetable shown below to answer the following questions.

		GX410	GX411
TOWN		**DAILY**	**DAILY**
Cairns	**Dep**	**12:25A**	**7:00A**
Edmonton		12:40A	7:15A
Gordonvale		12:55A	7:25A
Babinda		-	7:50A
Innisfail		1:40A	8:20A
Silkwood		2:00A	8:40A
El Arish		2:05A	8:45A
Wongaling – Mission Beach Central		-	9:15A
Tully		2:20A	9:40A
Cardwell	**Arr**	**2:55A**	**10:10A**
	Dep	**3:35A**	**10:45A**
Ingham		4:15A	11:30A
Rollingstone		4:40A	12:01P
Townsville	**Arr**	**5:50A**	**1:00P**

a How often do the services travel from Cairns to Townsville?

b How long does it take service GX410 to travel from Cairns to Townsville?

c How long does it take service GX411 to make the same journey?

d Which service would you catch if you wanted to visit someone in Babinda?

e What is the difference in the time taken by each service to travel between Cairns and Innisfail?

14 A catering firm is staging a large breakfast function. Draw a time line to show the events in order.

7:30 a.m. Begin serving food   10:30 a.m. Pack van and leave
9:30 a.m. Clean up   4:40 a.m. Arrive at venue
11:00 a.m. Arrive back at depot   10:00 a.m. Finish washing up
5:30 a.m. Start cooking

15 A movie lasting 122 minutes starts at 7:48 p.m. At what time does it finish?

16 A school readathon starts at 8:30 a.m. on Wednesday and the last reader in the relay finishes at 4:38 p.m. on Friday. How long did the readathon last?

Number and algebra

6

Expressions and equations

Contents

Prior learning
Chapter 6

Parent guide
Chapter 6

Curriculum guide
Chapter 6

▪ Australian Curriculum statements

Patterns and algebra
Introduce the concept of variables as a way of representing numbers using letters. (ACMNA175)
Create algebraic expressions and evaluate them by substituting a given value for each variable. (ACMNA176)
Extend and apply the laws and properties of arithmetic to algebraic terms and expressions. (ACMNA177)
Linear and non-linear relationships
Solve simple linear equations. (ACMNA179) **AC**

Algebra is like a written language and it is used to write down ideas in mathematics. To fully understand mathematics, you need to learn to read and write in algebraic ways, just like you learnt to read and write in primary school.

Learning algebra is like learning the alphabet. You need to know the basics before you can start using it properly.

When you work out the area of a rectangle by multiplying the length and width, you are already using algebra. The formula $A = l \times w$ is an example of algebra.

◼ Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

associative	flow chart	right-hand side
coefficient	input	simplify
commutative	inverse operation	solution
constant	law	solve
distributive	left-hand side	substitution
equation	like terms	term
evaluate	linear	unknown
expression	output	variable

6.1 Patterns and symbols

Investigate: Restaurant tables

Many restaurants use square tables that can seat four people. When they get a booking for more than four people, they put tables together so that more people can sit at them. Some examples are shown below. Your teacher might have some cut-outs for you to use in this investigation. Work in small groups.

- How many tables do you need to put together for 6 people?
- How many people can sit at 6 tables put together?
- How many tables will need to be put together for a big birthday party of 20 people?
- Write a rule for the number of people that can sit at any number of tables.

You can see that for 1, 2, 3, 4, … tables, the number of people that can be seated in the investigation above are 4, 6, 8, 10, … If you just look at the number pattern 4, 6, 8, 10, …, the 4 is called the **first term**, the 6 is the **second term**, the 8 is the **third term**, and so on. Each number in the pattern is called a **term** and the **term number** tells you which one it is.

If you can write a **rule** for the number pattern, it gives you a way to work out other numbers in the pattern. If you can connect the term number to the value of the term, it is easy to work out any term without having to write out the pattern from the start.

Example 1

For the number pattern 3, 4, 5, 6, 7, …
a write a rule for the pattern
b work out the 9th term.

Solution

Method 1

a The numbers go up by 1 each time.   **Start from 3 and add 1 each time to get the next number.**

b Write out the pattern up to 9 terms.   **3, 4, 5, 6, 7, 8, 9, 10, 11, …**

Write the answer.   **The 9th term is 11.**

Method 2

a The 1st term is 3, the 2nd term is 4, the 3rd term is 5, the 4th term is 6, etc., so each term is 2 more than the term number.   **Add 2 to the term number.**

b Work out the 9th term using the rule.   **The 9th term is $9 + 2 = 11$.**

In Example **1** we could also write the rule using Method 2 as 'the $\square$th term is $\square + 2$'. For any term, we just fill in the box with the term number. You can also write the numbers in a table. We call the top row of numbers the **input** numbers and the bottom row the **output** numbers, as shown below.

Input	1	2	3	4	5		
Output	3	4	5	6	7		

Example 2

The table below shows input and output numbers for a rule. Write a rule for the output numbers.

Input	5	6	7	8	9	10
Output	15	18	21	24	27	30

Solution

Look for a pattern.   **15 is 3 times 5**

18 is 3 times 6

Write the rule.   **The output number is three times the input number.**

Example 3

For the number pattern 3, 6, 9, 12, …
a write a rule using a box (□) for the term number
b work out the 8th term.

Solution

a Write down how the pattern works.   **The terms go up by 3 each time.**

   Write the rule in words.   **Each term is three times the term number.**

   Replace the words with symbols.   **The terms are $3 \times \square$.**

b Use the rule.   **The 8th term $= 3 \times \square$**

   $$= 3 \times 8$$
   $$= 24$$

In algebra, instead of using a box, we usually use a letter to represent a number. Then we substitute (put in) the number we want in place of the letter.

Important!

Variables and number patterns

A **variable** (**pronumeral** or **unknown**) is a letter or symbol that stands for a number.
We often use the symbol n for the term number of a number pattern. The symbol a is often used for the term itself, so a_4 is the fourth term of a number pattern.

Example 4

For the number pattern 5, 10, 15, 20, …, use a variable to:
a write a rule for the pattern   b work out the 9th term.

Solution

a Write down how the pattern works.   **The terms go up by 5 each time.**

   Write the rule in words.   **Each term is 5 times the term number.**

   Use the symbol n for the term number. **Each term is $5 \times n$.**

b Work out the 9th term.   **The 9th term, $a_9 = 5 \times 9$**

   $$= 45$$

Important!

Linear patterns

Linear number patterns go up or down by the same amount each time. You can write the rule using this amount multiplied by the term number. It is added or taken from the same fixed number each time.

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Example 5

For the number pattern 5, 8, 11, 14, …, use a variable to:
a write a rule for the pattern
b work out the 12th term
c find which term is equal to 32.

Solution

a Check if the pattern is linear.   **The terms go up by 3 each time, so it is linear.**

Write the multiplying part of the rule.   **The rule involves $3 \times n$.**

Write the rule with ? for the fixed number.   **The rule is $? + 3 \times n$.**

Try the rule for the first term. $? + 3 \times 1 = 5$

Work out the missing number.   **? must be 2.**

Write the rule.   **The terms are $a_n = 2 + 3 \times n$.**

b Work out the 12th term.
$$a_{12} = 2 + 3 \times 12$$
$$= 2 + 36$$
$$= 38$$

c Try some numbers out to find the answer. $2 + 3 \times 9 = 2 + 27 = 29$

Write the answer. $2 + 3 \times 10 = 2 + 30 = 32$
32 is the 10th term.

Example 6

Write a rule in symbols for the following table of inputs and outputs.

Input p	2	3	4	5
Output v	19	16	13	10

Solution

Write down how the pattern works.   **When the input number goes up by 1, the output number goes down by 3.**

Write the rule using ? for the fixed number. $v = ? - 3 \times p$

Use the first term to work out ?. $19 = ? - 3 \times 2$
$$19 = ? - 6$$

Work out ?.   **? must be 25.**

Write down the rule. $v = 25 - 3 \times p$

Example 7

A large barbecue costs $30 for 1 day's hire and $10 for every extra day after that.

a Write a rule for the cost of hire.

b Find the cost to hire for 3 days.

Solution

a Change the rule so the first day includes $10.

Now write down the rule.

The first day costs $20 plus $10.

It costs $20 to hire plus $10 for each day of hire.

Write the rule for d days.

Hire cost = $20 + $10 × d

b Work out the cost for 3 days.

Cost for 3 days = $20 + $10 × 3

= $50

Enter as: 20 `+` 10 `×` 3 `=` .

| 20+10×3 | 50 |

Exercise 6.1 Patterns and symbols

1 A caterer uses portable trapezoidal tables for seating guests at functions. Tables are joined end-to-end to seat the guests as shown here.
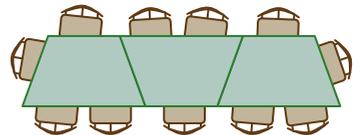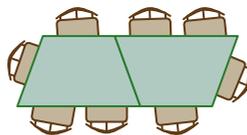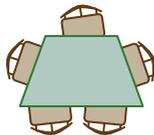
a Extend the pattern to complete this table.

Number of tables (t)	1	2	3	4	5	6
Number of guests (g)	5	8				

b Use the information to make a rule in words for the number of guests that can be seated.

c Write the rule using □ for the number of tables

d How many people could be seated at 6 tables?

e How many tables would need to be joined in this arrangement to seat 38 people?

See Example 1

2 Write a rule in words for each of the following number patterns and work out the indicated term:

a 6, 7, 8, 9, 10, …; 15th term

b 4, 8, 12, 16, 20, …; 21st term

c 7, 9, 11, 13, 15, …; 9th term

d 20, 19, 18, 17, …; 11th term

e 12, 15, 18, 21, 24, …; 12th term

f 11, 8, 5, 2, ⁻1, …; 10th term

See Example 2

3 Write a rule in words for each of the following number patterns.

a

Input	1	2	3	4	5
Output	5	10	15	20	25

b

Input	6	7	8	9	10
Output	4	5	6	7	8

c

Input	2	3	4	9	10
Output	8	11	14	29	32

d

Input	3	6	9	12	15
Output	7	13	19	25	31

e

Input	4	7	8	9	12
Output	5	11	13	15	21

f

Input	3	8	10	9	7
Output	13	33	41	37	29

4 For the number pattern 3, 6, 9, 12, …:   *See* Example 3
  a write a rule using a box (□) for the term number   **b** work out the 8th term.

5 For the number pattern 2, 7, 12, 17, …:
  a write a rule using a box (□) for the term number   **b** work out the 6th term.

6 For the number pattern 7, 11, 15, 19, …:
  a write a rule using a box (□) for the term number   **b** work out the 10th term.

Fluency

7 For each of the following number patterns, use the variable *n* to write the rule and find the indicated term.

See Examples 4, 5

  a 17, 20, 23, 26, 29, …; 10th term   **b** 1, 3, 5, 7, 9, …; 8th term
  c 1, 5, 9, 13, 17, …; 12th term   **d** 18, 16, 14, 12, 10, …; 20th term
  e ⁻3, 1, 5, 9, 13, …; 14th term   **f** 4, 1, ⁻2, ⁻5, ⁻8, …; 9th term

Worked solutions

Exercise 6.1

MAT07NAWS00001

8 Write rules in symbols for the following tables of inputs and outputs.

a

See Example 6

f	1	2	3	4	5	6
h	1	4	7	10	13	16

b

m	1	2	3	4	5	6
p	2	7	12	17	22	27

c

m	0	1	2	3	4	5
b	3	6	9	12	15	18

d

h	3	4	5	6	7	8
k	8	10	12	14	16	18

e

r	0	1	2	3	4	5
s	1	4	7	10	13	16

f

a	2	3	4	5	6	7
b	2	4	6	8	10	12

9 Doreen is saying some numbers and someone hears her say '11, 14, 17, 20, 23'. What will be the 15th number she says?

10 '*One elephant went out to play, upon a spider's web one day*
He found it such tremendous fun, he called three more elephants to come
Four elephants went out to play, upon a spider's web one day
They found it such tremendous fun, they called three more elephants to come
Seven elephants went …

…
They all fell down, and the poem is done!'
The spider's web will not hold more than 20 elephants. How many lines does the poem have?

See Example 7

11 A silo has 2050 kg of cattle feed. Each day, 70 kg is used to supplement the feed of a herd of cattle. How long is it before the silo needs refilling?

Worked solutions
Exercise 6.1
MAT07NAWS00001

12 As a child in a family gets older, they get more pocket money. Peter is 6 and gets 80 cents a week. Pat is 8 and gets $2 a week. Sue is 11 and gets $3.80 a week. Cherie is 16. How much does she get each week?

6.2 Number rules

You already know that you can use 'turn-arounds' for multiplying and adding. The product 8×3 has the same answer as 3×8, and $4 + 5$ has the same answer as $5 + 4$. You can always turn around addition and multiplication.
When you multiply three numbers, for example $7 \times 4 \times 5$, it doesn't matter if you do 7×4 first or 4×5 first. You would probably calculate 4×5 first to get $7 \times (4 \times 5) = 7 \times 20 = 140$ because it is easier than doing $(7 \times 4) \times 5 = 28 \times 5 = 140$. When you add three numbers you can always choose whether to start with the front numbers or the back numbers. For $123 + 27 + 48$ you would add the front numbers first to get $150 + 48$, but for $24 + 38 + 52$ you would add the back numbers first to get $24 + 90$.
Mathematicians have special names for turn-around and front-or-back-first rules.

Important!

Commutative and associative laws

Addition and multiplication are both **commutative**.
The order in which the operation is done does not matter, so $a + b = b + a$ and $a \times b = b \times a$ for any numbers.
Addition and multiplication are both **associative**.
When the operation is performed on three numbers, it does not matter which pair is done first, so $a + b + c = (a + b) + c = a + (b + c)$ and $a \times b \times c = (a \times b) \times c = a \times (b \times c)$.
These rules are called **laws** because they work for all numbers.

Although $5 - 5$ can be turned around to $5 - 5$, you cannot turn $5 - 3$ around, so subtraction is not commutative. To be commutative, you must be able to change the order for *any* numbers. Division is not commutative because $6 \div 3 \neq 3 \div 6$.

Example 8

Calculate $18 \times 25 \times 40$ in your head.

Solution

Calculate 25×40 first as it is easiest.	**Think 25×4 tens $= 100$ tens $= 1000$**
Now calculate 18×1000.	**Think $18 \times 1000 = 18$ thousand**
Write the answer.	**$18 \times 25 \times 40 = 18\ 000$**

When you have many numbers to add together, you can add them in any order because addition is both commutative and associative. It makes sense to choose the order to make tens.

Example 9

Work out $3 + 24 + 22 + 9 + 11 + 17 + 38$.

Solution

Put the units together to make tens.	$3 + 24 + 22 + 9 + 11 + 17 + 38$
Write down what the units add up to.	**The units make 3 tens and there are 4 units left over**
Add the rest of the tens in your head.	**3 and 2 and 2 and 1 and 1 and 3 makes 12 tens**
Write the answer.	**$3 + 9 + 22 + 17 + 24 + 11 + 38 = 124$**

You can calculate 36×24 in your head by multiplying 36 by 20 and then adding 36×4. These are both easy to do using 'double and put on a zero' and 'double double' to give $720 + 144$, which is 864. You can do 47×19 by calculating 47×20 and subtracting 47×1 to get $940 - 47$, which is 893. Any multiplication can be broken into two parts like these, but you can also do the opposite when two numbers are multiplied by the same number and then added or subtracted.

$28 \times 34 + 28 \times 16$ can be calculated more easily as 28×50. $28 \times 34 = 952$, $28 \times 16 = 448$ and $28 \times 50 = 1400$, which is indeed $952 + 448$. This works for any case, so if you have to add or subtract the product of the same number with some others, you can always perform the addition or subtraction first. Sometimes this is easier, and sometimes it's easier to do it the other way around. Mathematicians also have a special name for this rule.

Important!

Distributive law

The product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each one separately. This can be written in symbols as:

$$a \times (b + c) = a \times b + a \times c$$
$$\text{and} \quad a \times (b - c) = a \times b - a \times c.$$

The addition and subtraction cases of the **distributive law** can be summarised as:

$$a \times (b \pm c) = a \times b \pm a \times c, \text{ using the symbol } \pm \text{ for 'plus or minus'.}$$

Example 10

Find a short cut to work out 102×128.

Solution

Look for a break-up.	**Think 102 is 100 plus 2.**
Work out the parts.	**Think 100×128 is 12 800 and 2×128 is 256.**
Add them together.	**Think $12\ 800 + 256$ is 13 056.**
Write the answer.	**$102 \times 128 = 13\ 056$**

The distributive law can be used with variables to get rid of brackets. This is called **expanding brackets**. This is what you are really doing when you use a short cut to work out 47×19.

Example 11

Show the short cut to work out 47×19 using the distributive law.

Solution

Write down the problem.	**47×19**
Show the break-up of 19 in brackets.	**$= 47 \times (20 - 1)$**
Show the use of the distributive law.	**$= 47 \times 20 - 47 \times 1$**
Show the answers to the multiplications.	**$= 940 - 47$**
Write the final answer.	**$= 893$**

You should write the equals signs underneath each other so that the reasoning for each step is shown clearly, just like it is in Example **11**.

You can expand the brackets of expressions with variables to work out part of the answer, even if you don't know what the variable stands for.

Example 12

Expand the brackets and simplify $5 \times (m + 8)$.

Solution

Write down the problem.	**$5 \times (m + 8)$**
Use the distributive law.	**$= 5 \times m + 5 \times 8$**
Simplify (work out) 5×8.	**$= 5 \times m + 40$**

In algebra, we often leave out the multiply sign between symbols, so $5 \times (m + 8)$ can be written as $5(m + 8)$ and $5 \times m$ can be written as $5m$. Try using this convention to write out the expansion of $5(m + 8)$ without using multiply signs between symbols.

Example   13

Expand the brackets and simplify $6(3p - 7)$.

Solution

Write down the problem.	$6(3p - 7)$
Put in the multiply sign in your head.	Think $6 \times (3p - 7)$
Use the distributive law.	$= 6 \times 3p - 6 \times 7$
Simplify $6 \times 3p$ and 6×7.	$= 18p - 42$

Investigate: Modelling expansion

You can use a simple model to show expansion of brackets. You will need some paperclips, small squares and strips of paper coloured on one side to model numbers and variables. You can make the squares and strips as follows:

- Draw lines 1 cm apart on a blank piece of paper in the pattern shown on the right.
- It doesn't matter how long the strips are, as long as they are not an exact centimetre length. Label the strips of equal length m, p and x.
- Use a highlighter or coloured pencil to shade the other side of the paper.
- Cut out the squares and strips and label the reverse sides of the strips.
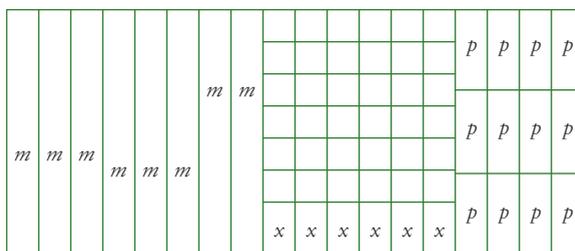
The white side represents a positive unit and the coloured side represents a negative unit. Use your strips and squares to model $(2x - 3)$ by putting 2 x-strips and 3 coloured-side squares together with a paperclip. Make another 3 models of $(2x - 3)$. Your models should look like this:
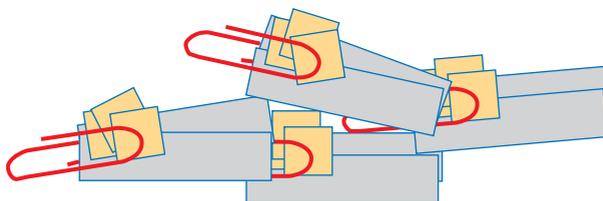
Unclip your 4 models to expand $4(2x - 3)$ and collect the x-strips and squares to work out the result of the expansion.
Use your models to work out $3(2 - x)$, $2(4m - 3p)$ and $3(m + 4p - 2x + 5)$.

You can model the expansion of $^-3(x - 2)$ by making 3 models of $(x - 2)$ and turning them over to change from 3 times to $^-3$ times. What do you get as the answer?
Now use your models to work out $^-2(5 - 2p)$, $-(2m - 3p + x)$ and $^-4(4 - 3p + 2m - x)$. Write down your results and discuss them in groups of 3 people.

Exercise 6.2 Number rules

Understanding

See Examples 8, 9

1 Work out each of the following in your head and explain how you did it. Your teacher may ask you to explain it to your neighbour.

a $20 \times 154 \times 500$
b $80 \times 136 \times 12.5$
c $75 \times 234 \times 40$
d $68 + 90 + 13 + 43 + 73 + 7 + 37$
e $88 + 52 + 25 + 59 + 24 + 46 + 64$
f $3 + 27 + 74 + 86 + 50 + 92 + 22$
g $79 + 32 + 61 + 22 + 38 + 28 + 20 + 67$
h $88 + 83 + 16 + 77 + 17 + 84 + 20 + 34$
i $72 + 45 + 68 + 11 + 40 + 45 + 67 + 35$

Fluency

See Examples 10,11

See Example 12

See Example 13

2 Use the distributive law to find short cuts for working out each of the following.

a 101×143
b 152×99
c 327×102
d 999×68
e 302×35
f 49×247
g 19×28
h 21×422
i 99×1002

3 Expand the brackets and simplify each of the following.

a $3(a + 2)$
b $2(h + 2)$
c $2(m + 3)$
d $4(x + 6)$
e $4(x - 2)$
f $3(m - 7)$
g $8(k - 3)$
h $5(y - 5)$

4 Expand the brackets and simplify each of the following.

a $4(2m + 3)$
b $7(a + b)$
c $12(2p + 5)$
d $5(a + 2)$
e $6(3x + 4)$
f $12(2m + n)$
g $10(4p + 2q)$
h $3(2a + 4b)$

Problem solving

5 Peter was paid $12 an hour to pack. He worked 3 hours on Monday, 5 hours on Tuesday, 4 hours on Wednesday and 3 hours on Friday. Explain how to use a mathematical law to work out how much he should be paid.

6 Expand the brackets and simplify each of the following.

a $^-3(x + 2)$
b $^-7(p + 1)$
c $^-2(m - 3)$
d $^-5(k - 4)$
e $^-4(6 - y)$
f $^-9(a + 4)$
g $^-(k + 3)$
h $^-(m - 2)$
i $^-(6 - 2x)$
j $^-3(4m + 5)$
k $^-5(3y - 6)$
l $^-(4 - 7x)$

Reasoning

7 Write down short cuts using the distributive law to work out each of the following. Your teacher may ask you to explain it to your neighbour.

a 103×49
b 999×104
c 205×51
d 1002×458
e 11×734
f 111×75

8 Show how to expand the brackets in each of the following, explaining every step.

a $2(m - p)$
b $7(3f - 2g)$
c $4(3m - 5)$
d $5(3m - 3)$
e $6(3x - 4)$
f $3(1 - k)$
g $6(2 - 3p)$
h $10(2 - 2m)$

9 Work out a short cut for multiplying by 11 in your head and explain the rule.

10 Work out a rule for multiplying a two-digit number by 99 and explain the rule.

6.3   Algebraic expressions

You have already seen that symbols and letters can be used to represent unknown numbers and that $4 \times a$ is usually written as $4a$. We also usually write $a \times 4$ as $4a$. It follows that $5x + 2p - 6$ means 'multiply 5 by some number, add twice another number and take 6 from the total'. There are special names for the algebraic way of writing this.

Important!

Expressions

An **expression** has variables and/or numbers connected by arithmetic operations like $+$, $\div$ and powers. An expression with numbers only is an **arithmetic expression**, while one with variables is an **algebraic expression**. The numbers are called constants. The numbers that are multiplied by the variables are called **coefficients**, the parts separated from the rest by $+$ or $-$ are called **terms** and a number on its own is called a **constant term**.

Terms

Constant term

$$\overbrace{5x} \, \overbrace{+ \, 2p} \, \overbrace{- \, 6}$$

Coefficients
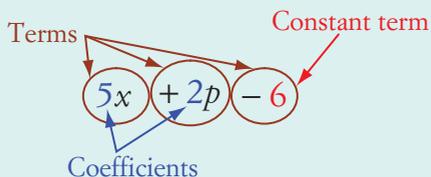
To **evaluate** an algebraic expression we put in values for the variables (**substitute** values) and work out the answer.

The sign in front of the term is usually included as part of the term. In the expression above, the terms can be considered as $+5x$, $+2p$ and -6 (that is: ^+5x, ^+2p and $^-6$).

Example   14

What are the coefficients of each of the following?

a $7e$ **b** ^-10f **c** v **d** $-w$

Solution

a	Write the number multiplied by the variable.	**The coefficient is 7.**
b	Include the sign.	**The coefficient is $^-10$.**
c	There is one v, so $v = 1v$.	**The coefficient is 1.**
d	Include the sign.	**The coefficient is $^-1$.**

> **Example** 15
>
> How many terms are there in each of the following expressions?
>
> **a** $g + 8$ **b** $8 - m + 3pq$ **c** $2(v - 2p)$ **d** $6m + 3(m + 2)$
>
> ### Solution
>
> **a** g and 8 are separated by $+$.   **There are 2 terms.**
>
> **b** The $-$ and $+$ separate the parts.   **There are 3 terms.**
>
> **c** The '$v - 2p$' is not separated from the 2.   **There is 1 term.**
>
> **d** $3(m + 2)$ is one term.   **There are 2 terms.**

The English language has many words that mean the same thing, so you need to be able to interpret all the words that describe arithmetic operations.

> ## Important!
>
> ### Key words for operations
>
> The symbol n is used for a number in the table below. It doesn't make any difference if a different symbol is used for the variable.
>
> | Word | Example | Meaning |
> |---|---|---|
> | **Sum** | The sum of a number and 6. | $n + 6$ |
> | **Quotient** | The quotient of 20 and a number. | $20 \div n$ or $\dfrac{20}{n}$ |
> | **Product** | The product of a number and negative 8. | ^-8n or $-8n$ |
> | **Difference** | The difference between 30 and a number. | $30 - n$ |
> | **Double** | A number doubled. | $2n$ |
> | **Quarter** | A quarter of a number. | $n \div 4$ or $\dfrac{n}{4}$ |
> | **Increase** | A number is increased by 5. | $n + 5$ |
> | **Less** | 8 less than a number. | $n - 8$ |
> | **Decrease** | 11 is decreased by a number. | $11 - n$ |

> **Example** 16

> Write each of the following as an algebraic expression, using p and q as variables.
> **a** The difference between a number and 18.
> **b** 21 more than triple a number.
> **c** The product of 8 and the sum of two numbers.
> **d** Substitute the values $p = 3$ and $q = 4$ to evaluate the expression in part **c**.
>
> ### Solution
>
> **a** Difference means subtract. $\boldsymbol{p - 18}$
>
> **b** Triple means 3 times. $\boldsymbol{3p + 21}$

c The total is multiplied by 8. $8(p + q)$

d Write the expression. $8(p + q)$

Substitute values. $= 8(3 + 4)$

Work out the brackets. $= 8 \times 7$

Multiply. $= 56$

To check, enter as: 8 **(** 3 **+** 4 **)** **=** .

| 8(3+4) | 56 |

You could substitute $m = 3$ in the **formula** $c = 100m$ to work out the number of centimetres in 3 metres.

One reason for writing algebraic expressions is to make it clear when they can be simplified. The distributive law means that if you have $5p + 8p$ then you can add the 5 and 8 before multiplying by the number.

Important!

Collecting like terms

If an algebraic expression contains terms with exactly the same variables, we call these **like terms**. Like terms can be added or subtracted to **simplify** the expression.

Example 17

Simplify each of the following by collecting like terms.
a $7m + 8m$
b $9p - 3p$

Solution

a The terms are alike, so you can add them. $7m + 8m$

$7 + 8 = 15$ $= 15m$

b The terms are alike, so you can take one away from the other. $9p - 3p$

$9 - 3 = 6$ $= 6p$

Example 18

Simplify each of the following by collecting like terms.
a $5x + 3y - 7x + 6y$
b $g^2 + 7gh + 2g - gh$

Solution

a Identify the like terms. $5x + 3y - 7x + 6y$

Simplify $5x - 7x$ and $3y + 6y$. $= {}^-2x + 9y$

We write a positive term first if we can. $= {}^+9y - 2x$ or $9y - 2x$

b Identify the like terms (g^2 and g are different). $\qquad$ $g^2 \underset{\smile}{\boxed{+ 7gh}} + 2g \underset{\smile}{\boxed{- gh}}$

Simplify $7gh - gh$. $\qquad\qquad\qquad\qquad$ $= g^2 + 6gh + 2g$

When expressions are multiplied or divided, it is usual to simplify them if possible by multiplying or dividing the coefficients.

Example 19

Simplify each of the following.

a $3f \times 5d$ $\qquad$ **b** $18p \div 6$ $\qquad$ **c** $6x \times 5x^2 \times 2y$ $\qquad$ **d** $\dfrac{15mg^3}{9m^2g}$

Solution

a Rearrange the order using the associative law and put the variables in alphabetical order. $\qquad$ $3f \times 5d$

Multiply the coefficients. $\qquad\qquad\qquad\qquad$ $= 3 \times 5 \times d \times f$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $= 15df$

b Divide the coefficients. $\qquad\qquad\qquad\qquad$ $18p \div 6$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $= 3p$

c Multiply the coefficients. $\qquad\qquad\qquad\qquad$ $6x \times 5x^2 \times 2y$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $= 60x \times x^2 \times y$

Use $x \times x^2 = x^3$. $\qquad\qquad\qquad\qquad$ $= 60x^3y$

d Simplify the 15 and 9 by dividing each by 3, write g^3 as $g \times g^2$ and m^2 as $m \times m$. $\qquad$ $\dfrac{\overset{5}{\cancel{15}}mg^3}{\underset{3}{\cancel{9}}m^2g}$

Cancel the m and the g. $\qquad\qquad\qquad\qquad$ $= \dfrac{5\cancel{m}\cancel{g} \times g^2}{3\cancel{m} \times m\cancel{g}}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $= \dfrac{5g^2}{3m}$

Exercise 6.3 Algebraic expressions

1 What are the coefficients of each of the following expressions?

a $5h$ $\qquad\qquad$ **b** ^-7x $\qquad\qquad$ **c** $6g$

d $2y$ $\qquad\qquad$ **e** ^-12ab $\qquad\qquad$ **f** p

g $-gh$ $\qquad\qquad$ **h** ^-5t $\qquad\qquad$ **i** $16g^2$

j e^3 $\qquad\qquad$ **k** $^-892x^3yz$ $\qquad\qquad$ **l** $412m$

m $6(3x - 5)$ $\qquad\qquad$ **n** $-(7x - 8y)$ $\qquad\qquad$ **o** $7x^2(y + 5z)$

p $(3c - d)^2$ $\qquad\qquad$ **q** $\dfrac{2q}{3}$ $\qquad\qquad$ **r** $\dfrac{3y}{4}$

2 How many terms are there in each of the following expressions?

a $2k + 1$ $\qquad\qquad\qquad\qquad$ **b** $m + 3p + n$

c $4x$ $\qquad\qquad\qquad\qquad\qquad$ **d** $2w + 7y + v$

e $4 + 7n$

g 9

i $x + y + z$

k $6k - 3km + 2m - m^2$

m $6(3x - 5)$

o $7(3m + n) - 4(m - 2n)$

q $\frac{1}{2}(2x + 4) + 1 - 3(x + 1)$

f $4m - 2k - 7$

h $3p + 2q$

j $4pq + 6q + 7p$

l $6ef + 4e$

n $3x + 2(x + 2y)$

p $\dfrac{x - 3}{2}$

r $\dfrac{3(k^2 - 3m)}{5(k - 3)} + 2k - k^2 m$

3 Find the like terms in each of these sets.

a $2my, 3x, 6am, 16x, 4mb$

c $8k, 3x, 2w, 12g, 23w$

e $x, 3x^2, y, x^2$

g $x^2y, 2x, 3y, 4x^2y$

b $2mw, 3km, 4w, 5mw, 6m, 7aw$

d $2p, 5mq, 5p, 7q, 7m$

f $4mn, 3m, 4, 2nm, mn, 2n$

h $7, 2a, 4b, 5p, 9, d, 2$

4 Simplify each of these expressions by adding like terms.

a $2m + 5m$

d $5mn + 2nm$

b $4k + 7k$

e $xy + xy$

c $2ab + 5ab$

f $3abc + 4abc + 2bac$

See Example 17

5 Simplify each of these expressions by subtracting like terms.

a $5m - 3m$

d $45abc - 12abc$

b $8d - 3d$

e $45fg - 13fg$

c $12mk - 7mk$

f $48mn - 29mn$

6 Simplify each of these by adding or subtracting like terms.

a $5m + 2m - 3m$

d $12f + 15f - 18f$

g $3pq - 2pq + pq$

b $12a - 4a - 5a$

e $8s^2 + 7s^2 - 6s^2$

h $11abc + 3abc - 5abc$

c $3x + 5x + 7x$

f $5mn + 6mn - 4nm$

i $8p - 3p - 5p$

7 Simplify the following.

a $3x + 4 + 5x + 6$

c $2mn + 3f + 5mn + 7f$

e $23xy + 23ab + 17xy - 17ab$

b $4m - 6 + 4m + 10$

d $10k - 4 - 6k + 12$

f $15r + 15 - 15r - r$

8 Simplify the following.

a $6m + 3m$

d $2d + 9 + 3d$

g $11p - p$

b $k + 2 + 3k$

e $2f + 3g + 4f + 7g$

h $7 + 5q - 3$

c $12mx + 4xm$

f $4y + 3x + 2x + 8y$

i $8abc - 4abc - abc$

9 Simplify each of the following.

a $3x + 8y - 8x + 2y$

c $-x^2y + 4xy^2 - 6x + 7x$

e $5abc + 3ab - 2bc - 9abc$

b $u - 2w - 5u + 7w$

d $2p - 8p - 8p + 2q$

f $-d + 4d^3 + 6d + 2d^2$

See Example 18

10 Simplify each of the following.

a $3m \times 2m$

d $6g \times 3 \times 4h$

g $4x \times 7x \times 3$

j $18yz \div 6z$

b $5k \times 11k$

e $10 \times 2m \times 6n$

h $5m \times 2 \times 3m$

k $\dfrac{21p^2q}{7pq}$

c $12f \times 9f$

f $4a \times 3 \times 7a$

i $20x^2 \div 5$

l $\dfrac{9mn}{18m^2}$

See Example 19

m $24pq^3 \div {}^-6q^2$

n ${}^-7ef \times {}^-9ef^3$

o $\dfrac{{}^-8kp}{24kp}$

p $\dfrac{28t^3s}{{}^-12t^2s^3}$

q ${}^-15m^2 \times {}^-8m^4$

r $dgh \times {}^-4ah^2$

s $\dfrac{{}^-7t^2m}{{}^-35t^3m^3}$

t $25gk \div 15km$

Problem solving

11 Write an expression for each of the following. Use N to represent any number.

See Example 16

  a double the number

  b half the number

  c triple the number

  d one-quarter of the number

  e one-tenth of the number

  f the next consecutive whole number

  g 5 times the number

  h the sum of the number with 21

  i the difference between the number and 10

  j 2 more than the number

  k the number increased by 3

  l the number times itself

  m the square root of the number

12 If A, B and C represent any three numbers, write an expression for:

  a the sum of A and B

  b the sum of all three numbers A, B and C

  c the difference between B and C, where B is greater than C

  d the product of A and C

  e the product of all three numbers A, B and C

  f the quotient of A and B

  g the sum of A and B, divided by C

  h the quotient of C and B.

13 Evaluate each expression in question **12** by substituting $A = 12$, $B = 4$ and $C = 2$.

14 Write an expression for:

  a the sum of 3 and A

  b 3 less than B

  c 5 added to C

  d 8 increased by D

  e 3 taken away from E

  f X decreased by F

  g the sum of A, B and W

  h m divided by m

  i R to the power of 2

  j the sum of A and B, divided by 2.

15 Write an expression for:

  a the number of students in a class if there are B boys and G girls

  b the number of pies needed at a party if there are N children and each child can eat two pies

  c the number of children remaining in class if X leave for the library out of a total group of T

  d the amount of money earned by selling N cakes at the school fete, where each cake is priced at \$2

  e the cost of each film ticket where the total cost is \$$M$ and there are three people going to the film

  f the total cost of buying A cans of lemonade and B ice-creams, where each can costs \$1 and each ice-cream costs \$2.

16 Imagine that you must repeat question **11** using the pronumeral A, instead of N. What difference would this make to your answers? Does it matter which letter of the alphabet you choose to use?

17 Write an expression for the cost of buying c children's tickets at $21 each and a adult tickets at $27 each. Select **A**, **B**, **C** or **D**.

    A $21a + 27c$ **B** $a + c$
    C $27a + 21c$ **D** $a + c + 21 + 27$

18 Write formulas for each of the following situations.

    a Change M metres to m millimetres.

    b Change h hours to d days.

    c A curtain has a cut length c, 15 cm more than the finished length l.

19 Substitute the values $M = 1.4$, $h = 72$ and $l = 900$ to evaluate the formulas in question **18**.

20 In the USA, temperature is measured in degrees Fahrenheit. To change degrees Fahrenheit to Celsius, you need to subtract 32, then multiply by 5 and finally divide by 9.

    a Write a formula to change f degrees Fahrenheit to c degrees Celsius.

    b One mild December afternoon in Miami it was 50°F. Find the temperature in Celsius.

    c One summer's day in New York it was 95°F. What was this in Celsius?

6.4   Solving equations

The equals sign, $=$, is used in different ways in mathematics. $5 + 4 = ?$ means that we have to work out the answer to the sum $5 + 4$. But $3 \times 4 = 5 + 7$ is a mathematical sentence that says that the product of 3 and 4 is the same as the result of adding 5 and 7. The equation $7 - 3 = 2 \times 3$ is another mathematical sentence. One of these mathematical sentences is true and the other one is false. Which one is true and which one is false?

Important!

Equations

An **equation** is a mathematical sentence. It has an equals sign with expressions on both sides. The expressions can be arithmetic or algebraic.

Example   20

Write equations to show each of the following mathematical sentences and state whether or not the equation is true.

    a Five times four is the same as six times three.

    b The sum of eleven and eight is the same as the sum of twelve and seven.

    c Five times a number is the same as thirty.

Solution

a Write the expressions with
an equals sign between them.

$5 \times 4 = 6 \times 3$

Work out each side.

Left-hand side $= 5 \times 4 = 20$

Right-hand side $= 6 \times 3 = 18$

The LHS and RHS are not the same.

The equation $5 \times 4 = 6 \times 3$ is not true.

b Write the expressions with
an equals sign between them.

$11 + 8 = 12 + 7$

Work out each side.

Left-hand side $= 11 + 8 = 19$

Right-hand side $= 12 + 7 = 19$

The LHS and RHS are the same.

The equation $11 + 8 = 12 + 7$ is true.

c Write the expression, using x
as the 'number' and 30 on the RHS.

$5x = 30$

You don't know what number x is, so you
can't say whether or not the equation is true.

**The equation might be true, but it cannot be
stated without knowing the number.**

In Example **20**, to work out whether or not the first two equations were true, you worked out the answer to each side. This is called **evaluating** the expressions. You should be able to see that the last equation is true for the number 6, but is not true for any other number. The equation $x^2 - 6x + 8 = 0$ is true for 2 or 4. Some algebraic equations are not true for any numbers, and some are true for more than one number.

An equation can be true, false, or true for some number(s) and false for others.

Important!

Solutions of an equation

A **solution** of an algebraic equation is a value of the variable that makes the equation true.
A solution makes the left-hand side (**LHS**) equal to the right-hand side (**RHS**).
The process of finding the solution to an equation is called **solving** an equation.

Balances like the one below were used for centuries by merchants and traders. If the mass in the left-hand pan is the same as the mass in the right-hand pan, the balance is level. If not, the balance tilts at an angle.

A simple diagram of the balance is shown below.

Left-hand side   Right-hand side

Example 21

Find the unknown mass if the balance is level.

Solution

The left-hand side and the right-hand side are equal.

Think what must be added to 2 to obtain 7.

$$\boxed{?} + 2 = 7$$

$$\boxed{?} \text{ must be 5.}$$

The balance problem below shows two equal masses on the left-hand side.

The balance problem can be written as the following equation.

$$\boxed{?} + \boxed{?} = 24$$

We can also write this equation using a variable as follows.

$$m + m = 24$$
$$\text{or } 2m = 24$$
$$\text{So } m = 12$$

Example 22

State whether the number in the brackets is a solution to the equation.

a $d - 3 = 10$ (7)   **b** $f + 8 = 12$ (4)

Solution

a Write down the left-hand side of the equation.   **LHS** $= d - 3$

Replace d with 7 and work out the result. $= 7 - 3 = 4$

Compare the result with the right-hand side.   **LHS** $= 4 \neq 10 =$ **RHS**

The left-hand side $\neq$ right-hand side.   **7 is not a solution to** $d - 3 = 10$**.**

b Write down the left-hand side of the equation.   **LHS** $= f + 8$

Replace f with 4 and work out the result. $= 4 + 8 = 12$

Compare the result with the right-hand side.   **LHS** $= 12 =$ **RHS**

The left-hand side $=$ right-hand side.   **4 is a solution to** $f + 8 = 12$**.**

One way of solving equations is to guess a solution and check the guess by inserting it into the equation to see if it makes the left-hand side equal the right-hand side.

Example   23

Find a solution to each of the following equations.
a $10 - t = 8$
b $3d = 15$

Solution

a	Write down the left-hand side of the equation.	**LHS = 10 − t**
	Try $t = 4$ as a solution.	$= 10 - 4 = 6$
	$6 \neq 8$.	**4 is not a solution.**
	Try $t = 2$ as a solution.	**LHS = 10 − 2 = 8**
	State the result.	$t = 2$ **is a solution to** $10 - t = 8$.
b	Write down the left-hand side of the equation.	**LHS = 3d**
	Try $d = 7$ as a solution.	$= 3 \times 7 = 21$
	$21 \neq 15$.	$d = 7$ **is not a solution.**
	Try $d = 5$ as a solution.	$3 \times 5 = 15$
	State the result.	$d = 5$ **is a solution to** $3d = 15$.

The process used in Example **23** is called the '**guess and check and improve**' or '**trial and error**' method of solving equations.

Investigate: Modelling equations

Work in pairs for this investigation.
You can use the modelling method from Section 6.2 to solve equations. For this investigation, you will need the strips and squares from that investigation, and a piece of paper with an equals sign in the middle. Use your strips and squares to model the equation $x - 5 = 3$ as shown below.

To get the x on its own, you need to get rid of the 5 negative (red) squares.
Add 5 positive (white) squares to both sides so that it looks like this:

9780170361910

Now put red and white squares in pairs together on the LHS, so that it looks like this:

But each pair means $(1 - 1)$, so each pair of red and white squares comes to nothing. Take them away so the model looks like this:

Counting the squares on the right-hand side gives the answer $x = 8$!

Try using your model to solve the following equations. Remember, to get rid of a number on the left-hand side, add the opposite number to both sides.

$$x + 3 = 7$$
$$p - 5 = 3$$
$$m + 8 = 6$$

Set each other some problems to solve.

There is another way of informally solving equations as well as the balance model, guess and check, and the model in the investigation above. This method uses a **flow chart** that shows the steps involved on the left-hand side of an equation using arrows between a line of boxes. The variable is shown in the first box. The answer from the right-hand side of the equation is shown in the last box. The boxes in-between are for the numbers that result from each operation. The operations are shown above the arrows between the boxes.

Example 24

Write the equation $3d + 5 = 20$ as a flow chart.

Solution

Write the equation.	$3d + 5 = 20$
Make all operations obvious.	$3 \times d + 5 = 20$
Show the amounts as boxes and operations as arrows.	$\boxed{d} \xrightarrow{\times 3} \boxed{} \xrightarrow{+5} \boxed{20}$

A flow chart is used to solve an equation by moving backwards along the boxes (**backtracking**) from the right-hand side to the beginning. At each step, the operation is 'undone' by doing the opposite operation. We call the opposite operation the **inverse operation**.

Example 25

Solve the equation $3d + 5 = 20$ using backtracking on a flow chart.

Solution

Draw the flow chart.

$$d \xrightarrow{\times 3} \square \xrightarrow{+5} 20$$

Put in the inverse operations.

$$d \xrightarrow[\div 3]{\times 3} \square \xrightarrow[-5]{+5} 20$$

Apply the inverse operations, one step at a time.

$$\square \xleftarrow[\div 3]{} 15 \xleftarrow[-5]{} 20$$

Go all the way to the start.

$$5 \xleftarrow[\div 3]{} 15 \xleftarrow[-5]{} 20$$

State the result.

$d = 5$ is a solution to $3d + 5 = 20$.

When you find what you think is a solution to an equation, you should always check to see if it works. Do this by putting the value back into the equation. If the LHS = RHS, the value is a solution. In Example **25**, if you substitute 5 in for d on the LHS, you have:

$$3d + 5 = 3 \times 5 + 5$$
$$= 20$$

This means that $d = 5$ is a solution to the equation $3d + 5 = 20$.
Sometimes you will need to swap the left- and right-hand sides and/or make the operations in the equation more obvious before you begin backtracking.

Example 26

Use backtracking to solve the following equations.
a $19 = 4y - 17$
b $30 = 5(2e - 4)$
c $\dfrac{3r - 8}{2} = 11$

Solution

a Swap the LHS and RHS of the equation to
get the variable on the LHS.

$4y - 17 = 19$

Make all operations obvious.

$4 \times y - 17 = 19$

Write the equation as a flow chart.

$$y \xrightarrow{\times 4} \square \xrightarrow{-17} 19$$

Apply the inverse operations.

$$y \xrightarrow[\div 4]{\times 4} \square \xrightarrow[+17]{-17} 19$$

Begin at the final result and backtrack.

$$9 \xleftarrow[\div 4]{} 36 \xleftarrow[+17]{} 19$$

State the result.

$y = 9$

b Swap the LHS and RHS of the equation to
get the variable on the LHS.

$5(2e - 4) = 30$

Show all operations.

$5 \times (2 \times e - 4) = 30$

Express the equation as a flow chart.

Apply the inverse operations.

Begin at the result and backtrack.

State the result.

$e = 5$

c

$$\frac{3r - 8}{2} = 11$$

Show all operations.

$(3 \times r - 8) \div 2 = 11$

Express the rule as a flow chart.

Apply the inverse operations.

Begin at the result and backtrack.

State the result.

$r = 10$

The modelling method using paper coloured on both sides and the backtracking method both use inverse operations to find the solution to an equation. This is the formal method of solving equations where the left-hand side is a **linear rule** (like $x + 3$ or $6x + 5$, which produce linear number patterns if we input $x = 1, 2, 3, 4, \ldots$).

Important!

Rules for solving equations using inverse operations

1. What is done to one side must be done to the other.
2. To 'undo' or cancel an operation, apply its inverse.
3. Keep the equals signs underneath each other.

Example 27

Solve each of these equations using inverse operations.
a $x + 3 = 12$
b $y - 4 = 8$

Solution

a Write down the equation.

$x + 3 = 12$

Undo '+ 3' by applying '− 3'.

$x + 3 - 3 = 12 - 3$

Evaluate both sides.

$x + 0 = 9$

State the result.

$x = 9$

b Write down the equation. $\qquad\qquad y - 4 = 8$

Undo '− 4' by applying '+ 4'. $\qquad y - 4 + 4 = 8 + 4$

Evaluate both sides. $\qquad\qquad\qquad y + 0 = 12$

State the result. $\qquad\qquad\qquad\qquad y = 12$

Example 28

Solve each of these equations using inverse operations.

a $2a = 14$

b $\dfrac{b}{3} = 5$

Solution

a Write down the equation. $\qquad\qquad\qquad 2a = 14$

$2a$ means $2 \times a$. We need to undo 'multiplication by 2'.

Undo '× 2' by applying '÷ 2'. $\qquad\qquad 2a \div 2 = 14 \div 2$

Rewrite using fractions. $\qquad\qquad\qquad \dfrac{2a}{2} = \dfrac{14}{2}$

Evaluate both sides and state the result. $\qquad a = 7$

b Write down the equation. $\qquad\qquad\qquad \dfrac{b}{3} = 5$

$\dfrac{b}{3}$ means $b \div 3$. We need to undo 'division by 3'.

Undo '÷ 3' by applying '× 3'. $\qquad\qquad \dfrac{b}{3} \times 3 = 5 \times 3$

Evaluate both sides and state the result. $\qquad b = 15$

If there is more than one operation, you should do the inverse operations in the opposite order to the normal order of operations.

Example 29

Solve the following equations using inverse operations.

a $6m + 5 = 47$ $\qquad\qquad$ **b** $16 = 4(5y - 11)$ $\qquad\qquad$ **c** $\dfrac{7h - 14}{3} = 14$

Solution

a Write down the equation showing all operations. $\qquad 6 \times m + 5 = 47$

Undo the '+ 5' by subtracting 5 from both sides. $\qquad 6 \times m + 5 - 5 = 47 - 5$

Simplify. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad 6 \times m = 42$

Undo the '× 6' by dividing both sides by 6. $\qquad 6 \times m \div 6 = 42 \div 6$

Simplify. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad m = 7$

To check, enter as: 6 $\boxed{\times}$ 7 $\boxed{+}$ 5 $\boxed{=}$.

$\boxed{6\times7+5 \qquad\qquad\qquad 47}$

b Rearrange the equation. $$4(5y - 11) = 16$$

Show all operations. $$4 \times (5 \times y - 11) = 16$$

Undo the '× 4' by dividing both sides by 4. $$\frac{\cancel{4} \times (5 \times y - 11)}{\cancel{4}} = \frac{16}{4}$$

Simplify. $$5 \times y - 11 = 4$$

Undo the '− 11' by adding 11 to both sides. $$5 \times y - 11 + 11 = 4 + 11$$

Simplify. $$5 \times y = 15$$

Undo the '× 5' by dividing both sides by 5. $$\frac{5 \times y}{5} = \frac{15}{5}$$

Simplify. $$y = 3$$

To check, enter as: 4 **(** 5 **×** 3 **−** 11 **)** **=** .

$$\boxed{4(5\times3-11) \qquad\qquad 16}$$

c Write down the equation showing all operations. $$\frac{7 \times h - 14}{3} = 14$$

Undo the '÷ 3' by multiplying both sides by 3. $$\frac{7 \times h - 14}{3} \times 3 = 14 \times 3$$

Simplify. $$7 \times h - 14 = 42$$

Undo the '− 14' by adding 14 to both sides. $$7 \times h - 14 + 14 = 42 + 14$$

Simplify. $$7 \times h = 56$$

Undo the '× 7' by dividing both sides by 7. $$7 \times h \div 7 = 56 \div 7$$

Simplify. $$h = 8$$

To check, enter as:
(7 **×** 8 **−** 14 **)** **÷** 3 **=** .

$$\boxed{(7\times8-14)\div3 \qquad\qquad 14}$$

When equations involve brackets, it is often easier to expand the brackets using the distributive law before solving the equation.

Example 30

Solve the following equations.
a $3(f - 7) = 24$
b $5(2g + 1) + 8 = 33$

Solution

a Write down the equation. $$3(f - 7) = 24$$

Expand the brackets. $$3f - 21 = 24$$

Undo the '− 21' by adding 21 to both sides. $$3f - 21 + 21 = 24 + 21$$

Simplify. $$3f = 45$$

Undo the '× 3' by dividing both sides by 3. $$3f \div 3 = 45 \div 3$$

Simplify. $$f = 15$$

To check, enter as: 3 **(** 15 **−** 7 **)** **=** .

$$\boxed{3(15-7) \qquad\qquad 24}$$

b Write down the equation. $5(2g + 1) + 8 = 33$

Expand the brackets. $10g + 5 + 8 = 33$

Simplify. $10g + 13 = 33$

Undo the '13' by subtracting 13 from both sides. $10g + 13 - 13 = 33 - 13$

Simplify. $10g = 20$

Undo the '× 10' by dividing both sides by 10. $10g \div 10 = 20 \div 10$

Simplify. $g = 2$

To check, enter as:
5 **(** 2 **×** 2 **+** 1 **)** **+** 8 **=** .

| 5(2×2+1)+8 | 33 |

Exercise 6.4 Solving equations

For questions **1** to **6**, the masses shown on the balances are in kilograms. All parcels marked as '?' in any one question are of equal mass. Find the masses of these parcels.

1 a

b

2 a

b

3 a

b

4 a

b

5 a

b

6 a

b

For questions **7** to **9**, the level balance has been replaced by an equals sign. Find the masses of the parcels marked '?'.

7 a

b

8 a

b

9 a

b

10 Find the value of the variable that makes the following equations true.

See Example 23

a $d + 1 = 7$ b $3 + m = 14$ c $a - 3 = 10$ d $6 - c = 2$

e $3e = 15$ f $\dfrac{12}{f} = 4$ g $s + 2s = 9$ h $15 - j = 10$

i $8q = 16$ j $g - 6 = 8$ k $5 + i = 12$ l $3k = 15$

m $5x - 2x = 18$ n $a + 4a = 25$ o $\dfrac{p}{3} = 9$ p $3z - z = 8$

11 Is the number in brackets a solution to the equation (yes or no)?

See Example 22

a $a + 4 = 7$ (3)   b $m - 6 = 3$ (9)   c $15 - n = 7$ (5)

d $f + 4 = 11$ (6)   e $10 - g = 6$ (4)   f $2h + 3 = 17$ (6)

g $3d - 4 = 11$ (5)   h $2b + 3 = 17$ (7)   i $8 - 4z = 16$ (2)

j $11 + c = 7$ (4)

12 Write the following equations as flow charts, rearranging if necessary.

See Example 24

a $3h + 4 = 19$ b $5y - 7 = 13$ c $27 = 7u + 6$

d $5 = 6j - 13$ e $\dfrac{k + 6}{5} = 4$ f $\dfrac{p - 8}{3} = 11$

13 Solve each of the equations in question **12** by backtracking the flow charts you formed.

See Example 25

14 Put in the inverse operations for each of the following flow charts.

a $h \xrightarrow{\times 3} \square \xrightarrow{-8} \boxed{1}$

b $S \xrightarrow{\times 5} \square \xrightarrow{+12} \boxed{22}$

c $b \xrightarrow{\div 2} \square \xrightarrow{-9} \boxed{5}$

d $k \xrightarrow{\div 4} \square \xrightarrow{+11} \boxed{15}$

e $M \xrightarrow{\times 4} \square \xrightarrow{-16} \square \xrightarrow{\div 5} \boxed{4}$

f $f \xrightarrow{\div 3} \square \xrightarrow{+7} \square \xrightarrow{\times 2} \boxed{22}$

15 Solve each of the flow charts in question **14** by backtracking.

See Example 26

16 Use flow charts and backtracking to solve the following equations.

a $2d + 6 = 22$

ⓑ $3k - 17 = 13$

c $\dfrac{r}{3} + 5 = 12$

d $2(m + 3) = 16$

e $3(x - 8) = 27$

f $\dfrac{g + 4}{7} = 3$

g $7n + 11 = 60$

h $48 = 6e - 12$

i $\dfrac{4f - 9}{3} = 13$

17 Give the inverses of the following operations.

a *Walk* in the door.
b *Pick up* a book.
c *Add* 3.
d *Subtract* 2.
e Look *up*.
f *Multiply* by 7.
g *Divide* by 3.
h Fall *down*.
i *Take off* from the airstrip.
j *Put on* a hat.

18 Copy and complete the following.

a $7 - \square = 0$
b $\square - 4 = 0$
c $8 \div \square = 1$
d $\dfrac{1}{4} \times \square = 1$

e $12 - \square = 0$
f $11 \times \square = 1$
g $2 \div \square = 1$
h $a - \square = 0$

i $\square - d = 0$
j $m \times \square = 1$

19 What must be done to each expression to get b by itself?

a $b - 6$
b $8b$
c $b + 5$
d $b \div 3$

e $b \times 10$
f $7 + b$
g $\dfrac{b}{4}$
h $b - 12$

i $b + a$
j $b \times c$

20 Copy and complete the following.

a $d + 3 - \underline{\quad} = d$
b $d \times 4 \div \underline{\quad} = d$
c $d - 4 \underline{\quad} = d$

d $3d \underline{\quad} = d$
e $4 + d \underline{\quad} = d$
f $d \div 7 \underline{\quad} = d$

g $\dfrac{d}{13} \underline{\quad} = d$
h $d \times \dfrac{1}{2} \underline{\quad} = d$
i $d + a \underline{\quad} = d$

j $\dfrac{d}{m} \underline{\quad} = d$

21 Solve these equations by using the instructions given.

a $x + 14 = 20$ (Subtract 14 from both sides.)
b $y - 7 = 4$ (Add 7 to both sides.)

c $3w = 12$ (Divide both sides by 3.)
d $\dfrac{m}{12} = 3$ (Multiply both sides by 12.)

e $9 + d = 17$ (Subtract 9 from both sides.)
f $q \div 12 = 6$ (Multiply both sides by 12.)

22 Copy and complete the following.

a
$$a + 5 = 14$$
$$a + 5 - \boxed{} = 14 - \boxed{}$$
$$a = \boxed{}$$

b $6b = 18$
$$\frac{6b}{} = \frac{18}{}$$
$$\boxed{} \quad \boxed{}$$
$$b = \boxed{}$$

c
$$d - 12 = 21$$
$$d - 12 + \boxed{} = 21 + \boxed{}$$
$$d = \boxed{}$$

d
$$\frac{f}{7} = 13$$
$$\frac{f}{7} \times \boxed{} = 13 \times \boxed{}$$
$$f = \boxed{}$$

23 Solve these equations by using inverse operations. *See* Examples 27, 28

a $5p = 35$ **b** $x - 10 = 8$ **c** $\dfrac{z}{8} = 3$ **d** $s + 4 = 5$

e $d - 5 = 6$ **f** $8f = 32$ **g** $y - 13 = 7$ **h** $b \div 4 = 12$

i $p \times 7 = 49$ **j** $36 \times g = 72$ **k** $a - 14 = 20$ **l** $\dfrac{h}{12} = 7$

m $9i = 81$ **n** $9 + f = 17$ **o** $\dfrac{u}{8} = 8$

For questions **24** and **25** below, $\boxed{}$ indicates that one or more numbers and/or operations need to be completed.

24 Use inverse operations to complete the following steps for solving each equation.

a
$$3h + 4 = 22$$
$$3h + 4 \boxed{} = 22 \boxed{}$$
$$\frac{3h}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$
$$h = \boxed{}$$

b
$$5m + 3 = 28$$
$$5m + 3 \boxed{} = 28 \boxed{}$$
$$\frac{5m}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$
$$m = \boxed{}$$

c
$$2p - 4 = 12$$
$$2p - 4 \boxed{} = 12 \boxed{}$$
$$\frac{2p}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

25 Use inverse operations to complete the following steps for solving each equation.

a
$$\frac{c + 4}{5} = 3$$
$$\frac{c + 4}{5} \boxed{} = 3 \boxed{}$$
$$c + 4 = \boxed{}$$
$$c + 4 \boxed{} = \boxed{}$$
$$c = \boxed{}$$

b
$$\frac{k + 2}{7} = 5$$
$$\frac{k + 2}{7} \boxed{} = 5 \boxed{}$$
$$k + 2 = \boxed{}$$
$$k + 2 \boxed{} = \boxed{}$$
$$k = \boxed{}$$

c
$$\frac{r - 5}{3} = 8$$
$$\frac{r - 5}{3} \boxed{} = 8 \boxed{}$$
$$r - 5 = \boxed{}$$
$$r - 5 \boxed{} = \boxed{}$$
$$r = \boxed{}$$

26 Solve these equations. *See* Example 29

a $7a + 9 = 30$ **b** $3s + 6 = 18$ **c** $3y - 1 = 8$
d $2x + 3 = 15$ **e** $3e - 12 = 6$ **f** $10p - 2 = 98$
g $4 = 3m - 2$ **h** $5w - 8 = 2$ **i** $5y + 14 = 89$
j $2d - 7 = 1$ **k** $44 = 9 + 7x$ **l** $10 + 4m = 34$
m $5n - 14 = 16$ **n** $5b + 7 = 42$ **o** $3c - 8 = 16$

See Example 30

27 Solve these equations by first using the distributive law.

   a $2(a + 1) = 6$ **b** $4(b + 3) = 12$ **c** $3(c - 4) = 27$
   d $3(f - 10) = 18$ **e** $4(2g + 3) = 36$ **f** $42 = 6(3h - 2)$
   g $5(3k + 2) = 25$ **h** $8(2j - 1) = 64$ **i** $2(m + 3) - 7 = 19$

Problem solving

28 Find the value of the variable in each of the following.

   a x is the number of eggs in a dozen.
   b y is the number of wheels on a car.
   c p is the number of runs in a century in cricket.
   d m is the number of days in a week.
   e r is the number of letters in the English alphabet.
   f q is the number of points for a goal in netball.
   g t is the number of cards in a pack without jokers.

See Example 30

29 For each of the following, write an equation that shows the information.

   a When 6 is added to a number, the answer is 11.
   b When a number is reduced by 7, the result is 8.
   c Three times a number is 12.
   d Half a number is 9.
   e When a number is doubled, the result is 16.
   f Multiply a number by 3 and add 1. The result is 10.
   g Multiply a number by 4 and subtract 5. The result is 19.

30 Work out the number in each case in question **29**.

Reasoning

31 Write each of the following as an equation and solve using inverse operations.

   a The sum of a number and eight is 6.
   b The product of a number and 7 is 42.
   c The difference between a number and ten is 3.
   d The quotient of a number and 5 is 35.
   e The sum of double a number and 7 is 29.

Worked solutions

Exercise 6.4

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32 Write each of the following as an equation and solve using inverse operations.

   a Three times the sum of a number and 5 is 36.
   b The difference between seven times a number and 5 is 51.
   c The quotient of eight added to a number and nine is 4.
   d The product of 4 and the sum of twice a number and 3 is 96.
   e The sum of a third of a number and 20 is 2.

33 The following are the solutions of equations by a student. In each case there is an error. State the line on which the first error occurred and the correct answer for each one.

a $3x - 6 = 18$

$$3x = 18 - 6 \quad \text{A}$$
$$3x = 12 \quad \text{B}$$
$$x = 4 \quad \text{C}$$

b $4x + 8 = 32$

$$x + 2 = 8 \quad \text{A}$$
$$x = 8 + 2 \quad \text{B}$$
$$x = 10 \quad \text{C}$$

c $2 - 7x = 30$

$$^{-}7x = 30 - 2 \quad \text{A}$$
$$^{-}7x = 28 \quad \text{B}$$
$$x = 4 \quad \text{C}$$

d $2x + 3 = 31$

$$2x = 31 + 3 \quad \text{A}$$
$$2x = 34 \quad \text{B}$$
$$x = 17 \quad \text{C}$$

e $3(x + 2) = 24$

$$3x + 6 = 24 \quad \text{A}$$
$$3x = 24 - 6 \quad \text{B}$$
$$x = 2 \quad \text{C}$$

f $4(x + 8) = 16$

$$4x + 8 = 16 \quad \text{A}$$
$$4x = 8 \quad \text{B}$$
$$x = 2 \quad \text{C}$$

Chapter 6 summary

- A number pattern consists of **terms**. The **term number** shows which term it is; the second, third and fifth terms of 4, 7, 10, 13, 16, 19, 25, … are 7, 10 and 16 respectively.

- The **rule** for a number pattern is written with the term number as the **input** and the term as the **output** number. A number pattern can be written as a table with the input numbers in the first row and the output numbers in the second row.

- A **variable** (**pronumeral** or **unknown**) is a letter or symbol that stands for a number. The rule for a number pattern can be written with a variable for the term number.

- A **linear** number pattern goes up or down by the same amount from one term to the next.

- Addition and multiplication are both **commutative**: when two numbers are added or subtracted, the order makes no difference, so $a + b = b + a$ and $a \times b = b \times a$ for any numbers.

- Addition and multiplication are both **associative**. When the operation is performed on three numbers, it does not matter which pair is done first, so $a + b + c = (a + b) + c = a + (b + c)$ and $a \times b \times c = (a \times b) \times c = a \times (b \times c)$ for any three numbers.

- The commutative and associative properties are called **laws** because they work for all numbers.

- The **distributive law** states that the product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each one separately. This can be written in symbols as:

$$a \times (b + c) = a \times b + a \times c \quad \text{and} \quad a \times (b - c) = a \times b - a \times c.$$

- The removal of brackets using the distributive law is called **expanding brackets**.

- An **arithmetic expression** has numbers connected by arithmetic operations such as $-$ and $\times$.

- An **algebraic expression** includes variables. Replacement of the variables by values is called **substitution**.

- A **constant** is a number in an algebraic expression. The numbers that are multiplied by the variables are called **coefficients**, the parts *separated* from the rest by $+$ or $-$ are called **terms**, and a number on its own is called a **constant term**.

- If an algebraic expression contains terms with exactly the same variables, we call these **like terms**. Like terms can be added or subtracted to **simplify** the expression.

- An **equation** is a mathematical sentence. It has an equals sign with expressions on both sides. The expressions can be arithmetic or algebraic.

- Calculation of the answer to an expression is called **evaluation** of the expression.

- A **solution** of an algebraic equation is a value of the variable that makes the equation true. A solution makes the left-hand side (**LHS**) equal to the right-hand side (**RHS**).

- The process of finding the solution to an equation is called **solving** an equation.

- The '**guess and check**' (**trial and error**) method of solving equations involves the evaluation of both sides of the equation with different values of the variable until a solution is found.

- An equation with only a number on the right-hand side can be shown as a **flow chart**. The operations are shown in the correct order on arrows between boxes that contain the values at each step. The variable is shown in the first box and the right-hand side value is in the last box.

- The **backtracking** method solves an equation by working backwards on a flow chart.

- **Inverse operations** are opposites. The inverse of $\times$ is $\div$ and the inverse of $-$ is $+$. They can be used to solve an equation by isolating the variable on the left-hand side.

1 Write a rule in words for each of the following number patterns and work out the indicated term.

   a 3, 8, 13, 18, 23, …, 12th term   **b** 7, 11, 15, 19, …, 20th term

See Example 1

2 Write a rule in words for each of the following number patterns.

See Example 2

   a

Input	1	2	3	4	5
Output	5	9	13	17	21

   b

Input	3	4	5	6	7
Output	7	10	13	16	19

3 For the number pattern 3, 9, 15, 21, 27, …

   a write a rule using a box ($\square$) for the term number

   b work out the 11th term.

See Example 3

4 What is the coefficient of $^-3x^4z$?

See Example 14

5 How many terms are in the expression $^-3x - 5y + 8 + z$?

See Example 15

6 For the number pattern 4, 10, 16, 22, 28, … use the variable n to write the rule and find the 15th term.

See Examples 4, 5

7 Work out these problems in your head.

   a $20 \times 123 \times 5$ **b** $16 + 28 + 12 + 34$

See Examples 8, 9

8 How many terms are there in the expression $3(2x + 5y) - 3z + 2$?

See Example 15

9 What are the like terms in the set $3p, 3q, 5pq, \,^-2p, 6q$?

10 Simplify each of the following.

   a $7e + 5e$ **b** $3k - 2m + 6m + 4k - 1$

See Examples 17, 18

11 For this question, the masses shown on the balances are in kilograms. For each part, all masses marked with ? are the same. Find the masses of the parcels marked.

See Example 21

   a

   b

   c

   d

12 Is the number in brackets a solution to the equation (yes or no)?

See Example 22

   a $k + 8 = 15$ (7)   **b** $6h = 18$ (4)

See Example 23

13 Find the values of the variables that make each of the following equations true.

    a $t + 14 = 27$ **b** $4u = 36$

See Example 24

14 Write the following equations as flow charts.

    a $4p + 8 = 27$ **b** $\dfrac{k - 11}{2} = 7$

See Example 25

15 Insert inverse operations in each of the following flow charts.

    a $\boxed{y} \xrightarrow{\times 4} \boxed{} \xrightarrow{-12} \boxed{20}$ **b** $\boxed{f} \xrightarrow{\div 6} \boxed{} \xrightarrow{+11} \boxed{17}$

See Example 26

16 Use flow charts and backtracking to solve the following equations.

    a $2h + 5 = 17$ **b** $4m - 14 = 18$ **c** $\dfrac{k}{5} + 3 = 7$ **d** $\dfrac{3r - 5}{2} = 8$

Fluency

17 For the number pattern 44, 41, 38, 35, 33, … use the variable n to write the rule and find the 20th term.

18 Write rules in symbols for the following tables of inputs and outputs.

    a

f	1	2	3	4	5	6
h	2	9	16	23	30	37

    b

m	1	2	3	4	5	6
p	99	95	91	87	83	79

See Examples 12, 13

19 Expand the brackets and simplify each of the following.

    a $5(d + 8)$ **b** $6(2h - 3p)$

20 Simplify each of the following.

    a $7e^2 - 5e + e^2 - 2e$ **b** $6x + 5y - 3 + y + 8 - 7y - 9z$

21 Simplify each of the following.

    a $3x \times {}^- 5y$ **b** $24pqr^3 \div 6pr$ **c** $\dfrac{18vj^2}{24jv^2}$

See Examples 27–29

22 Solve these equations using inverse operations.

    a $9g + 5 = 41$ **b** $\dfrac{8p}{3} - 5 = 11$

See Example 30

23 Solve these equations by first using the distributive law.

    a $3(m + 2) = 15$ **b** $2(3w - 11) = 20$

Problem solving

24 Peter says "5, 8, 11, 14, 17". What would be the 10th number he says?

25 A life raft has 75 litres of water on board. Each person on the life raft drinks 1.8 litres of water each day. There are 4 people on the life raft. How much water is left after 4 days? How long will the water last?

26 Write down and explain a short cut for working out 171×99.

27 David was paid \$34 for every ten books he sold at a bookshop. On Tuesday he sold 180, on Wednesday 60 and on Thursday 120. Explain how to use a mathematical law to work out how much he was paid.

28 Write expressions for:
- **a** the difference between 9 and g
- **b** 40 less the product of f and 7
- **c** the number of men and women at a wedding.

29 Write each of the following as an equation and solve to find the number.
- **a** The sum of a number and 7 is 20.
- **b** The difference between triple a number and 8 is 7.

30 Write a formula to find the total number of seconds, t, in m minutes and s seconds. Use your formula to find the total number of seconds in 3 minutes 21 seconds.

Reasoning

31 The daily hiring cost of a pneumatic drill is a linear rule of the number of days. It costs \$59 to hire for 2 days and \$95 to hire for 5 days. How much would it cost for 11 days?

32 Expand and simplify $-(6x - 5)$.

33 Work out a rule for multiplying a two-digit number by 201 and explain the rule.

34 Write each of the following as an equation and solve using inverse operations.
- **a** The quotient of a number and 5 is $^{-}3$.
- **b** The product of a number and 3 is 5 less than 29.

7

Fractions and ratios

Contents

Prior learning
Chapter 7

Parent guide
Chapter 7

Curriculum guide
Chapter 7

Australian Curriculum statements

Real numbers

Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line. (ACMNA152)

Solve problems involving addition and subtraction of fractions, including those with unrelated denominators. (ACMNA153)

Multiply and divide fractions and decimals using efficient written strategies and digital technologies. (ACMNA154)

Express one quantity as a fraction of another, with and without the use of digital technologies. (ACMNA155)

Connect fractions, decimals and percentages and carry out simple conversions. (ACMNA157)

Recognise and solve problems involving simple ratios. (ACMNA173) 🔵AC

If you divide a pizza to share equally among four people, you should then all get the same amount of pizza. In this case, each share is called a quarter. This is usually written in symbols as $\frac{1}{4}$.

You can think of this as the result of dividing 1 whole amount into 4 equal shares. In a similar way, you can divide whole amounts into other fractions by dividing into different numbers of equal shares.

Imagine that you divide a bag of lollies into quarters and give one of your friends three of the shares, keeping only one share for yourself.

In this case, your friend would have three-quarters of the lollies.

This is usually written as $\frac{3}{4}$ or 3/4 and means three out of four equal shares of the whole amount.

If the bag contained 48 lollies, then three-quarters would amount to 36 lollies.

Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

common factor	mixed numeral	ratio	vulgar fractions
denominator	numerator	reciprocal	
equivalent fraction	proper fraction	simplest form	
improper fraction	rate	vinculum	

7.1   Naming and comparing common fractions

A fraction is written as two numbers separated by a line.

The top part is called the **numerator**.

$$\frac{4}{9}$$

The line between the top and bottom is called the **vinculum**.

The bottom part is called the **denominator**.

Fractions written in this way are called **common fractions** (or **vulgar fractions**).
We have already learnt about decimal fractions in Chapter 4.

- **Proper fractions** have a numerator that is less than the denominator.
  For example $\frac{4}{9}, \frac{4}{13}, \frac{37}{60}, \frac{261}{365}, \frac{80}{100}$.
- **Improper fractions** have a numerator that is more than the denominator.
  For example $\frac{9}{4}, \frac{17}{3}, \frac{81}{60}, \frac{121}{100}$.
- **Mixed numerals** have a whole part and a fraction part.
  For example $4\frac{1}{2}, 5\frac{2}{3}, 3\frac{1}{7}, 4\frac{8}{31}$.

Example 1

a Write $1\frac{3}{4}$ as an improper fraction.

b Write $5\frac{3}{7}$ as an improper fraction.

Solution

a $1\frac{3}{4}$ is made up of 1 whole and 3 quarters.

$$1\frac{3}{4} = 1 + \frac{3}{4}$$

1 whole is 4 quarters. 4 quarters + 3 quarters makes 7 quarters.

$$\frac{4}{4} + \frac{3}{4} = \frac{7}{4}$$

b $5\frac{3}{7}$ is made up of 5 wholes and 3 sevenths.

$$5\frac{3}{7} = 5 + \frac{3}{7}$$

5 wholes is 35 sevenths.
$5 \times 7 = 35$
35 sevenths + 3 sevenths makes 38 sevenths.

$$\frac{5 \times 7}{7} + \frac{3}{7} = \frac{38}{7}$$

A quicker way is to calculate 5×7 for the whole part and then add 3. Divide all this by 7.

$$5\frac{3}{7} = \frac{(5 \times 7) + 3}{7} = \frac{38}{7}$$

Example 2

a Write $\frac{38}{3}$ as a mixed numeral.

b Write $\frac{27}{4}$ as a mixed numeral.

Solution

To change an improper fraction into a mixed numeral, divide the numerator by the denominator and write the remainder as a proper fraction.

a $38 \div 3 = 12$ remainder 2.

$$\frac{38}{3} = 12\frac{2}{3}$$

b $27 \div 4 = 6$ remainder 3.

$$\frac{27}{4} = 6\frac{3}{4}$$

In Chapter **2** you learnt about finding the highest common factor (HCF). When working with fractions you will need to be able to determine the highest common factor and the lowest common multiple (LCM) of two or more numbers.

> **Important!**
>
> Highest common factors and lowest common multiples
> The **highest common factor** (**HCF**) of two or more numbers is the largest factor that divides evenly into all of those numbers.
> The **lowest common multiple** (**LCM**) of two or more numbers is the smallest number that is a multiple of all of those numbers.

Example 3

Find the highest common factor (HCF) of 27 and 36.

Solution

Find the factors of 27.	**$(1 \times 27 = 27)$ and $(3 \times 9 = 27)$.**
List the factors of 27.	**1, 3, 9 and 27**
Find the factors of 36.	**$(1 \times 36 = 36)$, $(2 \times 18 = 36)$, $(3 \times 12 = 36)$, $(4 \times 9 = 36)$ and $(6 \times 6 = 36)$**
List the factors of 36.	**1, 2, 3, 4, 6, 9, 12, 18 and 36**
Consider the common factors of 27 and 36.	**1, 3 and 9**
The largest of these is the HCF.	**The highest common factor is 9**

Example 4

Find the lowest common multiple (LCM) of:

a 8 and 10   **b** 3 and 6

Solution

a	List the multiples of 8.	**8, 16, 24, 32, 40, 48, . . .**
	List the multiples of 10.	**10, 20, 30, 40, 50, 60, . . .**
	The smallest number to appear in both lists is the lowest common multiple.	**LCM = 40**
b	List the multiples of 3.	**3, 6, 9, 12, 15, 18, . . .**
	List the multiples of 6.	**6, 12, 18, 24, 30, 36 . . .**
	The smallest number to appear in both lists is the lowest common multiple.	**LCM = 6**

There is more than one way to show the same number. For example, a quarter of a cake will be the same amount of cake as two-eighths of the same cake.

We say that $\frac{1}{4} = \frac{2}{8}$ and that the fractions are **equivalent**.

When you multiply the denominator and the numerator of $\frac{4}{5}$ by the same number you create equivalent or equal fractions.

$$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} = \frac{28}{35}$$

The 'cross-products' of equivalent fractions are equal. The cross-products of two fractions are found by multiplying the denominator of each fraction by the numerator of the other.

$$\frac{4}{5} = \frac{8}{10}$$

$$\frac{4}{5} \times \frac{8}{10}$$

$4 \times 10 = 40$ and $5 \times 8 = 40$

Important!

Equivalent fractions

Different fractions that represent the same amount are called **equivalent fractions**.

The **simplest form** of a fraction is the equivalent fraction (or mixed number) with the lowest possible denominator. This is also called the **lowest form**. The simplest form of the two fractions shown on the right is 1/3.

Improper fractions are not considered to be in simplest form.

A number that divides both the numerator and the denominator exactly is called a **common factor**.

The simplest form of a fraction has no common factors in its numerator and denominator (apart from 1), and is obtained by dividing both the top and bottom by common factors.

$\frac{2}{6} = \frac{1}{3}$

Simplest form $= \frac{1}{3}$

Example 5

Find the missing term in each of these pairs of equivalent fractions.

a $\frac{3}{4} = \frac{?}{20}$ **b** $\frac{5}{9} = \frac{35}{?}$ **c** $\frac{16}{24} = \frac{?}{6}$

Solution

a To find the missing numerator, we look at the two given denominators, 4 and 20.

   4 is multiplied by 5 to give 20, so we do the same thing to the numerator 3.

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

b To find the missing denominator, we look at 5 and 35.

   5 is multiplied by 7 to give 35, so we do the same to the denominator 9.

$$\frac{5}{9} = \frac{5 \times 7}{9 \times 7} = \frac{35}{63}$$

c To find the missing numerator, we look at 24 and 6.

   24 is divided by 4 to give 6, so we do the same to the numerator 16.

$$\frac{16}{24} = \frac{16 \div 4}{24 \div 4} = \frac{4}{6}$$

Example 6

Determine whether $\frac{3}{8}$ and $\frac{5}{8}$ are equivalent fractions.

Solution

The cross-products of two fractions are found by multiplying the denominator of each fraction by the numerator of the other.

$$\frac{3}{8} \diagdown\!\!\!\!\diagup \frac{5}{8}$$

The cross-products are different.

$$3 \times 8 \neq 8 \times 5$$

The cross-products of the two fractions are not the same, so $\frac{3}{8}$ and $\frac{5}{8}$ are not equivalent fractions.

Example 7

Reduce each of the following fractions to its simplest form.

a $\frac{20}{30}$ **b** $\frac{16}{48}$

Solution

a Factors of 20 are 1, 2, 4, 5, **10** and 20.   **The HCF of 20 and 30 is 10.**

   Factors of 30 are 1, 2, 3, 5, 6, **10**, 15 and 30.

   Divide the numerator and denominator by the HCF.

$$\frac{20}{30} = \frac{20 \div 10}{30 \div 10} = \frac{2}{3}$$

   To check, enter as 20 $\boxed{a^{b}/_{c}}$ 30 $\boxed{=}$.

$$\boxed{\frac{20}{30} \qquad\qquad\qquad\qquad \frac{2}{3}}$$

b Factors of 16 are 1, 2, 4, 8 and **16**.

Factors of 48 are 1, 2, 3, 4, 6, 8, 12, **16**, 24 and 48.

The HCF of 16 and 48 is 16.

Divide the numerator and denominator by the HCF.

$$\frac{16}{48} = \frac{16 \div 16}{48 \div 16} = \frac{1}{3}$$

To check, enter as 16 $a^{b}/_{c}$ 48 $=$.

$\frac{16}{48}$	$\frac{1}{3}$

You can find the simplest form of a fraction in steps. In part **a** above, you could change $\frac{20}{30}$ to $\frac{4}{6}$ by dividing by 5 and then change to $\frac{2}{3}$ by dividing by 2. Some people show this by crossing out and writing the new numbers like this: $\frac{^4 20}{_6 30} = \frac{^2 4}{_3 6} = \frac{2}{3}$.

Exercise 7.1 Naming and comparing common fractions

1 Classify the fractions below. Make three lists: proper fractions, improper fractions and mixed numerals.

a $1\frac{1}{2}$ **b** $\frac{3}{100}$ **c** $3\frac{2}{5}$ **d** $20\frac{1}{4}$

e $\frac{2}{3}$ **f** $\frac{100}{45}$ **g** $\frac{150}{250}$ **h** $\frac{86}{87}$

i $\frac{3}{2}$ **j** $\frac{3}{5}$ **k** $4\frac{1}{20}$ **l** $\frac{87}{86}$

m $\frac{3}{12}$ **n** $\frac{25}{1}$ **o** $\frac{1000}{25}$ **p** $\frac{301}{300}$

2 What mixed numerals are shaded orange in the following diagrams?

a

1 + 1 + $\frac{1}{3}$

b

+ + +

c

+ +

d

+ + +

e

+ + +

3 Draw rectangles to show the following mixed numerals.

   a $1\frac{2}{3}$ **b** $1\frac{5}{8}$ **c** 2 and three-quarters

   d $3\frac{1}{4}$ **e** $1\frac{1}{6}$ **f** $2\frac{3}{8}$

See Example 1

4 Write each of these mixed numerals as an improper fraction.

   a $1\frac{1}{2}$ **b** $1\frac{5}{8}$ **c** $1\frac{2}{7}$ **d** $3\frac{1}{3}$

   e $4\frac{1}{2}$ **f** $3\frac{7}{10}$ **g** $6\frac{1}{6}$ **h** $1\frac{11}{12}$

   i $7\frac{3}{5}$ **j** $9\frac{7}{15}$ **k** $11\frac{5}{11}$ **l** $7\frac{6}{7}$

See Example 2

5 Write each of these improper fractions as a mixed numeral.

   a $\frac{13}{3}$ **b** $\frac{13}{4}$ **c** $\frac{13}{10}$ **d** $\frac{21}{5}$

   e $\frac{38}{35}$ **f** $\frac{35}{8}$ **g** $\frac{41}{6}$ **h** $\frac{36}{5}$

   i $\frac{38}{7}$ **j** $\frac{120}{11}$ **k** $\frac{72}{19}$ **l** $\frac{101}{10}$

See Example 3

6 Find the HCF of each of the following.

   a 6 and 16   **b** 40 and 50   **c** 16 and 48   **d** 24 and 32
   e 28 and 12   **f** 42 and 18   **g** 5 and 30   **h** 35 and 50
   i 12 and 32   **j** 8 and 24   **k** 60, 12 and 30   **l** 20, 75 and 15

See Example 4

7 Find the LCM for each of the following.

   a 3 and 5   **b** 6 and 7   **c** 4 and 6   **d** 15 and 10
   e 5 and 8   **f** 4 and 10   **g** 10 and 5   **h** 2 and 8
   i 9 and 6   **j** 3 and 7   **k** 2, 3 and 4   **l** 20, 12 and 15

8 Write four equivalent fractions for each of the following.

   a $\frac{3}{5}$ **b** $\frac{2}{3}$ **c** $\frac{3}{10}$ **d** $\frac{1}{8}$ **e** $\frac{2}{7}$ **f** $\frac{5}{6}$

9 Copy and complete these diagrams. Start with the middle fraction. Multiply its numerator and denominator by the numbers along the arrows to make equivalent fractions in the circles.

a

b

10 Complete the equivalent fractions.

   a $\frac{1}{4} = \frac{2}{8} = \frac{3}{?} = \frac{4}{?} = \frac{?}{20} = \frac{6}{?} = \frac{?}{28}$

   b $\frac{5}{6} = \frac{10}{?} = \frac{?}{18} = \frac{?}{30} = \frac{30}{?} = \frac{60}{?} = \frac{100}{?}$

   c $\frac{2}{3} = \frac{4}{?} = \frac{?}{9} = \frac{?}{12} = \frac{10}{?} = \frac{?}{30} = \frac{200}{?}$

   d $\frac{7}{10} = \frac{?}{20} = \frac{21}{?} = \frac{?}{50} = \frac{?}{80} = \frac{70}{?} = \frac{?}{150}$

   e $\frac{7}{8} = \frac{14}{?} = \frac{?}{24} = \frac{?}{40} = \frac{63}{?} = \frac{77}{?} = \frac{?}{400}$

   f $\frac{3}{4} = \frac{?}{8} = \frac{9}{?} = \frac{?}{20} = \frac{24}{?} = \frac{36}{?} = \frac{?}{100}$

11 Find the missing number to complete each number sentence.

a $\dfrac{7}{12} = \dfrac{?}{36}$ **b** $\dfrac{3}{?} = \dfrac{9}{24}$ **c** $\dfrac{4}{5} = \dfrac{28}{?}$ **d** $\dfrac{3}{5} = \dfrac{?}{20}$

e $\dfrac{?}{15} = \dfrac{18}{45}$ **f** $\dfrac{14}{21} = \dfrac{?}{3}$ **g** $\dfrac{1}{3} = \dfrac{?}{12}$ **h** $\dfrac{25}{50} = \dfrac{1}{?}$

i $\dfrac{600}{1000} = \dfrac{?}{5}$ **j** $\dfrac{7}{8} = \dfrac{49}{?}$ **k** $\dfrac{4}{7} = \dfrac{16}{?}$ **l** $\dfrac{28}{40} = \dfrac{?}{10}$

m $\dfrac{5}{6} = \dfrac{45}{?}$ **n** $\dfrac{9}{36} = \dfrac{1}{?}$ **o** $\dfrac{65}{100} = \dfrac{?}{20}$

12 Write four equivalent fractions for each of the following.

a $\dfrac{1}{10}$ **b** $\dfrac{2}{5}$ **c** $\dfrac{3}{8}$ **d** $\dfrac{7}{100}$

13 For each pair of fractions, write $=$ between those that are equivalent, and $\neq$ between those that are not.

a $\dfrac{1}{2} \underline{\quad} \dfrac{10}{20}$ **b** $\dfrac{1}{3} \underline{\quad} \dfrac{4}{12}$ **c** $\dfrac{1}{4} \underline{\quad} \dfrac{4}{12}$ **d** $\dfrac{3}{4} \underline{\quad} \dfrac{5}{7}$

e $\dfrac{14}{21} \underline{\quad} \dfrac{2}{3}$ **f** $\dfrac{6}{12} \underline{\quad} \dfrac{7}{13}$ **g** $\dfrac{5}{9} \underline{\quad} \dfrac{4}{7}$ **h** $\dfrac{3}{12} \underline{\quad} \dfrac{9}{35}$

14 Reduce each of these fractions to its simplest equivalent form.

a $\dfrac{6}{16}$ **b** $\dfrac{6}{24}$ **c** $\dfrac{40}{50}$ **d** $\dfrac{24}{32}$ **e** $\dfrac{56}{104}$

f $\dfrac{18}{42}$ **g** $\dfrac{60}{75}$ **h** $\dfrac{5}{20}$ **i** $\dfrac{14}{21}$ **j** $\dfrac{20}{75}$

15 Write these in simplest form, changing them to mixed numerals.

a $\dfrac{48}{36}$ **b** $\dfrac{32}{24}$ **c** $\dfrac{50}{25}$ **d** $\dfrac{90}{54}$ **e** $\dfrac{35}{14}$

f $\dfrac{28}{12}$ **g** $\dfrac{75}{60}$ **h** $\dfrac{192}{128}$ **i** $\dfrac{104}{56}$ **j** $\dfrac{252}{120}$

16 Write these in simplest form, leaving them as mixed numerals.

a $1\dfrac{4}{8}$ **b** $3\dfrac{5}{30}$ **c** $4\dfrac{5}{20}$ **d** $6\dfrac{30}{40}$ **e** $100\dfrac{4}{6}$

f $17\dfrac{35}{100}$ **g** $100\dfrac{44}{64}$ **h** $1\dfrac{54}{100}$ **i** $5\dfrac{4}{12}$ **j** $15\dfrac{16}{20}$

17 What fraction of the months of the year start with the letter 'F'?

18 What fraction of the days of the week contain the letter 'i'?

19 What fraction of the value of a dollar coin are each of the following coins?

   a 50c coin   **c** 5c coin

   b 20c coin   **d** 10c coin

20 Sharon and Ben decided to share a chocolate bar, but Sharon finished up eating twice as much as Ben. What fraction of the bar did each of them eat?

21 What fraction of a *minute* is each of the following?

   a 20 seconds   **b** 50 seconds

   c 24 seconds   **d** 42 seconds

7.2   Ordering fractions

To compare fractions with different denominators, we must change them to the same denominator. A common denominator can be found by multiplying the denominators of both fractions together or by using the lowest common multiple of the denominators.
$\frac{4}{8}$ is larger than $\frac{3}{8}$. Since the fractions have the same denominator, we compare the numerators: 4 is larger than 3.

Example   8

Which fraction is larger, $\frac{4}{10}$ or $\frac{3}{8}$?

Solution

Method 1

Multiply the denominators to give a common denominator.

$10 \times 8 = 80$

Determine equivalent fractions that have a common denominator of 80.

$$\frac{4}{10} = \frac{4 \times 8}{10 \times 8} = \frac{32}{80}$$

$$\text{and } \frac{3}{8} = \frac{3 \times 10}{8 \times 10} = \frac{30}{80}$$

Compare the numerators.

$$\frac{32}{80} > \frac{30}{80}$$

$$\therefore \frac{4}{10} \text{ is larger than } \frac{3}{8}.$$

Method 2

Use the lowest common multiple as a common denominator.

Multiples of 10 are 10, 20, 30, 40, 50, . . .

Multiples of 8 are 8, 16, 24, 32, 40, 48, . . .

LCM = 40

Determine equivalent fractions that have a common denominator of 40.

$$\frac{4}{10} = \frac{4 \times 4}{10 \times 4} = \frac{16}{40}$$

$$\text{and } \frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$$

Compare the numerators.

$$\frac{16}{40} > \frac{15}{40}$$

$$\therefore \frac{4}{10} \text{ is larger than } \frac{3}{8}.$$

You can use your calculator to subtract fractions to find which is bigger.
On some calculators the fraction key looks like this   .

Example 9

Which fraction is smaller, $1\frac{5}{9}$ or $1\frac{2}{3}$?

Solution

Since both whole numbers are 1, we only need to compare the fraction parts.

Method 1

Multiply the denominators to give a common denominator.

$$3 \times 9 = 27$$

Determine equivalent fractions that have a common denominator of 27.

$$\frac{5}{9} = \frac{5 \times 3}{9 \times 3} = \frac{15}{27} \text{ and } \frac{2}{3} = \frac{2 \times 9}{3 \times 9} = \frac{18}{27}$$

Compare the numerators.

$$\frac{15}{27} < \frac{18}{27}$$

$$\therefore 1\frac{5}{9} \text{ is smaller than } 1\frac{2}{3}.$$

Method 2

Use the lowest common multiple as a common denominator.

LCM of 3 and 9 is 9.

Determine equivalent fractions that have a common denominator of 9.

$$\frac{5}{9} = \frac{5}{9} \text{ and } \frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

Compare the numerators.

$$\frac{5}{9} < \frac{6}{9}$$

$$\therefore 1\frac{5}{9} \text{ is smaller than } 1\frac{2}{3}.$$

To check, enter as:

1 $a^{b}/_{c}$ 5 $a^{b}/_{c}$ 9 $-$ 1 $a^{b}/_{c}$ 2 $a^{b}/_{c}$ 3 $=$.

$1\frac{5}{9} - 1\frac{2}{3}$	$-\frac{1}{9}$

Answer is negative so $1\frac{2}{3}$ is bigger.

$$1\frac{5}{9} \text{ is smaller than } 1\frac{2}{3}.$$

Example 10

Draw a number line and mark in the fractions $\frac{1}{6}, 2\frac{1}{7}$ and $2\frac{5}{7}$.

Solution

Show the number line from 0 to 4.

Divide the space between 0 and 1 into six equal parts, and the space between 2 and 3 into seven equal parts to find the positions for these fractions.

Mark the numbers with large dots.

Investigate: Shaded fractions

What fraction of this diagram is shaded? Describe how you were able to answer the question.

Exercise 7.2 Ordering fractions

1 Write the smaller fraction from each of these pairs.

a $\frac{7}{10}$, $\frac{3}{10}$ **b** $\frac{5}{3}$, $\frac{2}{3}$ **c** $\frac{3}{6}$, $\frac{8}{6}$

d $\frac{3}{4}$, $\frac{7}{8}$ **e** $\frac{3}{5}$, $\frac{7}{10}$ **f** $\frac{3}{5}$, $\frac{5}{8}$

2 Copy and complete each of these, using $<$ or $>$ to make a true statement.

a $\frac{1}{2}$____$\frac{1}{3}$ **b** $\frac{2}{3}$____$\frac{3}{4}$ **c** $\frac{3}{8}$____$\frac{1}{2}$ **d** $\frac{7}{8}$____$\frac{3}{4}$

e $\frac{1}{4}$____$\frac{1}{3}$ **f** $\frac{3}{5}$____$\frac{7}{10}$ **g** $\frac{5}{6}$____$\frac{1}{2}$ **h** $\frac{1}{6}$____$\frac{1}{4}$

i $\frac{5}{12}$____$\frac{1}{3}$ **j** $\frac{17}{100}$____$\frac{2}{5}$ **k** $\frac{7}{10}$____$\frac{4}{5}$ **l** $\frac{17}{20}$____$\frac{6}{10}$

m $\frac{11}{15}$____$\frac{2}{3}$ **n** $\frac{1}{3}$____$\frac{2}{5}$ **o** $\frac{63}{100}$____$\frac{4}{5}$ **(p)** $\frac{2}{5}$____$\frac{3}{7}$

q $3\frac{3}{4}$____$3\frac{5}{8}$ **r** $2\frac{3}{7}$____$5\frac{2}{5}$ **s** $1\frac{5}{7}$____$1\frac{2}{3}$ **t** $2\frac{1}{3}$____$2\frac{2}{5}$

3 Write each of these groups of fractions in ascending order.

a $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{2}$ **b** $\frac{3}{5}$, $\frac{3}{7}$, $\frac{2}{5}$, $\frac{5}{7}$ **c** $\frac{1}{8}$, $\frac{8}{9}$, $\frac{1}{2}$

d $\frac{3}{8}$, $\frac{2}{5}$, $\frac{4}{7}$, $\frac{3}{10}$, $\frac{3}{5}$, $\frac{2}{9}$ **e** 0.9, $\frac{2}{7}$, 0.5, $\frac{4}{5}$ **f** $\frac{2}{3}$, $\frac{3}{5}$, 0.7, $\frac{1}{2}$

4 Write each of these sets of numbers in descending order.

a $\frac{6}{7}$, $\frac{6}{9}$, $\frac{6}{8}$ **(b)** $\frac{5}{6}$, $\frac{5}{9}$, $\frac{5}{4}$ **c** 1, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{1}{2}$ **d** $\frac{12}{5}$, $\frac{3}{4}$, $\frac{40}{50}$, $1\frac{1}{5}$, 3

5 Write the fractions marked by a dot on each of these number lines.

6 Show each of these sets of fractions on a separate number line.

    a $\frac{2}{3}$, $\frac{3}{3}$, $1\frac{1}{3}$, $\frac{7}{3}$ **b** $\frac{1}{5}$, $1\frac{3}{5}$, $\frac{5}{5}$, $2\frac{2}{5}$, $\frac{3}{5}$, $\frac{9}{5}$

    c $\frac{1}{4}$, $\frac{3}{4}$, $1\frac{1}{4}$, $2\frac{3}{4}$, $\frac{7}{4}$ **d** $\frac{5}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $1\frac{1}{6}$

    e $\frac{3}{8}$, $\frac{1}{4}$, $\frac{5}{8}$, $1\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{8}$ **f** $\frac{7}{10}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{9}{10}$

Problem solving

7 Liliane saves $\frac{5}{9}$ of her pocket money and Ali saves $\frac{5}{7}$ of his. Who saves the greater fraction of his or her money?

8 Theo and Jenny are test pilots. Theo's plane can fly at $\frac{4}{5}$ the speed of sound while Jenny's can fly at $\frac{7}{8}$ the speed of sound. Whose plane is faster?

9 What fractions of an hour are equal to each of the following?

    a a whole number of minutes   **b** a whole number of seconds

10 Which cake has had the larger slice cut out?

A   B

$\frac{5}{6}$ $\frac{9}{11}$

Reasoning

11 A chocolate block can be divided easily into quarters or eighths. There are 3 blocks of chocolate. Among how many people can these be easily divided evenly?

12 Some mandarins have been segmented and given to the children in a kindergarten class. Each child has three segments and the teacher has two segments. There were 10 segments in each mandarin and there was a whole number of mandarins to start with. How many children could be in the class?

7.3   Adding and subtracting fractions

Fractions with the same denominator have 'pieces' that are the same size. This means that you can add or subtract fractions with the same denominator. In some cases, you can do this in your head. You will also be able to do this for fractions that can easily be made into fractions having the same denominator.

Example 11

Find:

a $\frac{1}{5} + \frac{2}{5}$

b $\frac{7}{10} - \frac{5}{10}$

Solution

a

$\frac{1}{5}$ + $\frac{2}{5}$ = $\frac{3}{5}$

b

$\frac{7}{10}$ − $\frac{5}{10}$ = $\frac{2}{10}$ = $\frac{1}{5}$

Consider adding $\frac{2}{3} + \frac{4}{7}$. The denominators are 3 and 7.

Step 1: Draw a 3×7 grid. This gives 21 squares.

Step 2: There are three rows. Two rows will be $\frac{2}{3}$ of the grid. This gives 14 squares, or $\frac{14}{21}$.

Step 3: There are seven columns. Four columns will be $\frac{4}{7}$ of the grid. This gives 12 squares, or $\frac{12}{21}$.

Step 4: Adding $\frac{2}{3}$ and $\frac{4}{7}$:

14 squares + 12 squares = 26 squares, or $\frac{26}{21}$

This gives one complete grid plus 5 extra squares, or $1\frac{5}{21}$.

Important!

Adding and subtracting fractions

To add or subtract fractions with different denominators, we must first change them into equivalent fractions with the same denominator.

Example 12

Find:

a $\frac{1}{4} + \frac{5}{6}$

b $\frac{2}{5} - \frac{3}{20}$

Solution

a Method 1

Multiply the denominators to get a common denominator.

$$4 \times 6 = 24$$

Determine equivalent fractions that have a common denominator of 24.

$$\frac{1}{4} = \frac{1 \times 6}{4 \times 6} = \frac{6}{24}, \ \frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

Add the fractions together.

$$\frac{1}{4} + \frac{5}{6} = \frac{6}{24} + \frac{20}{24}$$
$$= \frac{26}{24}$$

Change to the simplest form.

$$= \frac{13}{12}$$
$$= 1\frac{1}{12}$$

Method 2

Use the lowest common multiple as a common denominator.

LCM of 4 and 6 is 12

Determine equivalent fractions that have a common denominator of 12.

$$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}, \ \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

Add the fractions together.

$$\frac{1}{4} + \frac{5}{6} = \frac{3}{12} + \frac{10}{12}$$
$$= \frac{13}{12}$$
$$= 1\frac{1}{12}$$

b Method 1

Multiply the denominators to get a common denominator.

$$5 \times 20 = 100$$

Determine equivalent fractions that have a common denominator of 100 and then subtract.

$$\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100}, \ \frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100}$$
$$\frac{2}{5} - \frac{3}{20} = \frac{40}{100} - \frac{15}{100}$$

Give your answer in simplest form.

$$= \frac{25}{100} = \frac{1}{4}$$

Method 2

Use the lowest common multiple as a common denominator.

LCM of 5 and 20 = 20

Determine equivalent fractions that have a common denominator of 20 and then subtract.

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}, \ \frac{3}{20} = \frac{3}{20}$$
$$\frac{2}{5} - \frac{3}{20} = \frac{8}{20} - \frac{3}{20}$$

Give your answer in simplest form.

$$= \frac{5}{20} = \frac{1}{4}$$

Important!

Adding and subtracting mixed numerals

When adding or subtracting mixed numerals, add or subtract the whole numbers first and then add or subtract the fractions.

Example   13

Find:

a $6\frac{1}{3} + 4\frac{1}{2}$

b $6\frac{2}{3} - 4\frac{1}{2}$

Solution

a Add the whole numbers first.

$$6\frac{1}{3} + 4\frac{1}{2} = 6 + 4 + \frac{1}{3} + \frac{1}{2}$$

Find a common denominator for the fractional parts and add them together.

$$= 10 + \frac{2}{6} + \frac{3}{6}$$

$$= 10\frac{5}{6}$$

b Subtract the whole numbers first.

$$6\frac{2}{3} - 4\frac{1}{2} = 6 - 4 + \frac{2}{3} - \frac{1}{2}$$

Find a common denominator for the fractional parts and subtract.

$$= 2 + \frac{4}{6} - \frac{3}{6}$$

$$= 2\frac{1}{6}$$

Example   14

Find $9\frac{2}{3} - 3\frac{3}{4}$.

Solution

Subtract the whole numbers first.

$$9\frac{2}{3} - 3\frac{3}{4} = 9 - 3 + \frac{2}{3} - \frac{3}{4}$$

$$= 6 + \frac{2}{3} - \frac{3}{4}$$

Find a common denominator for the fractional parts and subtract.

$$= 6 + \frac{8}{12} - \frac{9}{12}$$

$$= 6 - \frac{1}{12}$$

You will need to change 1 whole to $\frac{12}{12}$.

$$= 5 + \frac{12}{12} - \frac{1}{12}$$

Complete the subtraction.

$$= 5\frac{11}{12}$$

To check, enter as:

9 $\boxed{a^{b}/_{c}}$ 2 $\boxed{a^{b}/_{c}}$ 3 $\boxed{-}$ 3 $\boxed{a^{b}/_{c}}$ 3 $\boxed{a^{b}/_{c}}$ 4 $\boxed{=}$.

$$\boxed{9\frac{2}{3} - 3\frac{3}{4} \qquad\qquad \frac{71}{12}}$$

Exercise 7.3 Adding and subtracting fractions

1 Evaluate the following.

a $\frac{1}{2} + \frac{1}{2}$ **b** $\frac{7}{10} - \frac{1}{10}$ **c** $\frac{2}{3} + \frac{1}{3}$ **d** $\frac{2}{3} + \frac{2}{3}$

e $\frac{3}{5} - \frac{1}{5}$ **f** $\frac{1}{4} + \frac{1}{4} + \frac{2}{4}$ **g** $\frac{17}{10} - \frac{14}{10}$ **h** $\frac{7}{12} - \frac{3}{12}$

i $\frac{5}{6} + \frac{3}{6}$ **j** $\frac{9}{15} - \frac{5}{15} - \frac{4}{15}$ **k** $\frac{14}{6} + \frac{7}{6}$ **l** $\frac{5}{8} + \frac{5}{8} - \frac{1}{8}$

2 Evaluate the following.

a $\frac{1}{2} + \frac{1}{3}$ **b** $\frac{2}{3} + \frac{1}{5}$ **c** $\frac{3}{4} - \frac{1}{2}$ **d** $\frac{5}{9} - \frac{1}{2}$ **e** $\frac{7}{8} + \frac{1}{3}$

f $\frac{4}{5} - \frac{3}{4}$ **g** $\frac{3}{7} - \frac{1}{9}$ **h** $\frac{7}{12} - \frac{1}{5}$ **i** $\frac{5}{9} + \frac{2}{5}$ **j** $\frac{9}{10} - \frac{3}{5}$

k $\frac{7}{9} - \frac{1}{5}$ **l** $\frac{5}{6} + \frac{3}{8}$ **m** $\frac{18}{27} - \frac{2}{3}$ **n** $\frac{3}{4} + \frac{4}{5}$ **o** $\frac{5}{8} - \frac{1}{3}$

p $\frac{6}{7} - \frac{1}{5}$ **q** $\frac{15}{21} + \frac{1}{2}$ **r** $\frac{1}{4} + \frac{1}{5}$ **s** $\frac{3}{4} + \frac{2}{5}$ **t** $\frac{5}{8} - \frac{1}{3}$

u $\frac{7}{8} - \frac{1}{3}$ **v** $\frac{2}{3} - \frac{5}{8}$ **w** $\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$ **x** $\frac{5}{6} - \frac{1}{2} - \frac{1}{3}$ **y** $\frac{2}{3} + \frac{1}{4} - \frac{3}{5}$

3 Find the sum of $\frac{1}{4}$ and $\frac{2}{3}$.

4 Find the difference between $\frac{2}{3}$ and $\frac{1}{2}$.

5 Evaluate the following expressions.

a $4 + \frac{1}{2}$ **b** $3\frac{2}{3} - 3$ **c** $4\frac{1}{6} - \frac{1}{6}$ **d** $5\frac{3}{4} - \frac{1}{4}$

e $2\frac{3}{5} + \frac{4}{5}$ **f** $6\frac{5}{8} - \frac{2}{8}$ **g** $\frac{5}{6} + 1\frac{5}{6}$ **h** $3\frac{4}{5} + 2\frac{1}{5}$

i $7\frac{3}{4} + 2\frac{1}{4}$ **j** $8\frac{7}{8} - \frac{3}{8}$ **k** $6\frac{2}{7} + \frac{3}{7}$ **l** $5\frac{3}{5} - 5$

6 Simplify the following.

a $1\frac{1}{2} + 1\frac{1}{2}$ **b** $2\frac{1}{2} + 1\frac{3}{4}$ **c** $5\frac{1}{3} - 2\frac{1}{5}$ **d** $4\frac{3}{8} - 1\frac{1}{4}$

e $1\frac{1}{3} + 2\frac{3}{4}$ **f** $2\frac{3}{5} + 1\frac{1}{2}$ **g** $4\frac{5}{6} - 2\frac{1}{3}$ **h** $1\frac{1}{2} + 2\frac{1}{3}$

i $3\frac{3}{8} - 1\frac{1}{4}$ **j** $2\frac{3}{4} + 1\frac{2}{5}$ **k** $3\frac{1}{3} - 1\frac{1}{4}$ **l** $3\frac{1}{7} + 2\frac{1}{3}$

m $2\frac{9}{10} - 1\frac{4}{5}$ **n** $6\frac{1}{4} - 4\frac{7}{8}$ **o** $2\frac{1}{2} + 2\frac{3}{8}$ **p** $8\frac{5}{12} + 3\frac{4}{5}$

7 Simplify the following.

a $1\frac{1}{2} + 2\frac{1}{4}$ **b** $3\frac{3}{4} - 2\frac{1}{2}$ **c** $2\frac{3}{4} + 3\frac{3}{4}$ **d** $3\frac{7}{8} + 1\frac{3}{8}$

e $3\frac{3}{10} + 2\frac{1}{2}$ **f** $1\frac{9}{10} + 3\frac{1}{2}$ **g** $3\frac{7}{8} - 1\frac{1}{2}$ **h** $2\frac{3}{5} + 1\frac{2}{3}$

i $3\frac{2}{3} + 2\frac{2}{5}$ **j** $4\frac{3}{4} - 1\frac{1}{5}$ **k** $2\frac{3}{4} + 1\frac{1}{2}$ **l** $3\frac{7}{8} - 1\frac{2}{5}$

8 Simplify the following.

a $3 - \frac{1}{2}$ **b** $2 - \frac{3}{5}$ **c** $1 - \frac{4}{7}$ **d** $4 - \frac{2}{3}$

e $2 - \frac{1}{4}$ **f** $3 - \frac{5}{12}$ **g** $3\frac{1}{5} - 1\frac{1}{2}$ **h** $3\frac{1}{4} - 2\frac{1}{2}$

i $6\frac{2}{5} - 4\frac{7}{8}$ **j** $4\frac{3}{4} - 2\frac{4}{5}$ **k** $2\frac{1}{3} - 1\frac{1}{2}$ **l** $4\frac{3}{5} - 2\frac{9}{10}$

m $3\frac{1}{6} - 1\frac{2}{3}$ **n** $5\frac{3}{10} - 2\frac{1}{2}$ **o** $5\frac{2}{3} - 3\frac{3}{4}$ **p** $4\frac{5}{12} - 3\frac{2}{3}$

⑨ James used a leftover piece of material to make some shorts. The torn pieces were trimmed so that one-tenth of the original material was lost from each end. What fraction of the original remained for James to work with?

10 When Penny walks at her quickest rate to school she takes half an hour. At her normal pace she takes an extra one-third of an hour. What fraction of an hour does Penny take walking at her normal pace?

11 Which of these fractions is closest to $\frac{3}{4}$: $\frac{1}{2}$, $\frac{7}{8}$ or $\frac{13}{16}$?

12 Two hundred sausages were required for a barbecue. One-tenth of them were donated by a parent, 25 had been left in the freezer from a previous occasion, and a supplier gave one-quarter of the total free. The rest had to be bought at wholesale price. What fraction were bought?

13 Simplify the following.

a $5 + \frac{5}{8} - 2\frac{2}{3}$

b $4\frac{1}{9} - \left(2\frac{1}{6} + \frac{2}{3}\right)$

c $1\frac{5}{6} - \left(\frac{2}{3} - 2\frac{2}{5}\right)$

7.4   Multiplying fractions

What is $\frac{2}{3} \times 12$? Here are 12 lollies:

If these lollies were placed into three equal piles, each pile would be one-third of the total.

$\frac{1}{3}$ of $12 = 4$ $\frac{1}{3}$ of $12 = 4$ $\frac{1}{3}$ of $12 = 4$

$\frac{2}{3}$ of $12 = 8$

Note that $\frac{2}{3}$ of $12 = 2 \times \frac{1}{3}$ of 12.

Example 15

Find:

a $\frac{1}{4}$ of 28

b $\frac{2}{3}$ of 30

Solution

a Divide 28 by 4.

$$\frac{1}{4} \text{ of } 28 = 28 \div 4$$
$$= 7$$

b Divide 30 by 3 and then multiply this result by 2.

$$\frac{2}{3} \text{ of } 30 = \left(\frac{1}{3} \times 30\right) \times 2$$
$$= 10 \times 2$$
$$= 20$$

Example 16

Find:

a $\frac{1}{10}$ of 1 km (in metres)

b $\frac{3}{5}$ of 1 minute

Solution

a Divide 1000 m by 10.
(Change 1 km to the smaller unit of length, 1000 m.)

$$\frac{1}{10} \text{ of } 1 \text{ km} = \frac{1}{10} \times 1000 \text{ m}$$
$$= 100 \text{ m}$$

b Divide 60 by 5 and then multiply your answer by 3.
(Change 1 min to 60 seconds.)

$$\frac{3}{5} \text{ of } 1 \text{ minute} = \left(\frac{1}{5} \times 60 \text{ sec}\right) \times 3$$
$$= 12 \times 3$$
$$= 36 \text{ sec}$$

Investigate: Netball

Read the following fraction story carefully.
Elvira and Christina play netball for the Jets. In netball, each goal is worth one point. Last Saturday, Elvira and Christina scored all their team's goals in the first quarter. Elvira shot $\frac{1}{4}$ of all the goals. Christina shot 12 goals. How many points did the Jets team score? In the second quarter of the game, Christina scored a fifth of the team's 20 points while again Elvira scored the rest. How many points did Elvira score?
The following may help you to solve the problems.
First quarter

1 Explain how to work out $\frac{1}{4}$ of an amount.
2 If Elvira shot $\frac{1}{4}$, what fraction did Christina shoot?
3 How many points did the Jets score?

Second quarter

4 How many goals made up $\frac{1}{5}$ of the points scored?

5 What fraction of the points did Elvira score?

6 How many points did Elvira score?

What happened in the last two quarters? See if you can finish off the story of the netball game by writing two more stories involving fractions.

You have probably discovered that the word 'of' usually means 'to multiply'. $\frac{2}{5}$ of $\frac{3}{5}$ can be thought of as $\frac{2}{5} \times \frac{3}{5}$.

You can use diagrams to show other multiplications, for example $\frac{2}{5} \times \frac{3}{5}$:

$$\frac{2}{5} \qquad \times \qquad \frac{3}{5} \qquad = \qquad \frac{6}{25}$$

If you look carefully at the diagram above, you might see the pattern for multiplication.

Important!

Multiplying fractions

To multiply fractions, multiply the numbers in the numerator and separately multiply the numbers in the denominator.

Example 17

Find:

a $\frac{3}{5}$ of $\frac{7}{8}$ 　　　　**b** $\frac{2}{5} \times \frac{5}{7}$ 　　　　**c** $\frac{21}{100} \times \frac{8}{15}$ 　　　　**d** $3\frac{2}{5} \times 1\frac{2}{3}$

Solution

a Multiply the numbers in the numerator. Multiply the numbers in the denominator.

$$\frac{3}{5} \times \frac{7}{8} = \frac{3 \times 7}{5 \times 8}$$
$$= \frac{21}{40}$$

b **Method 1**

Multiply the numbers in the numerator and the denominator.

Simplify your answer by dividing by 5.

$$\frac{2}{5} \times \frac{5}{7} = \frac{2 \times 5}{5 \times 7}$$
$$= \frac{\cancel{10}^2}{\cancel{35}_7}$$
$$= \frac{2}{7}$$

Method 2

Cancel by 5 at the beginning.

$$\frac{2}{5} \times \frac{5}{7} = \frac{2 \times \overset{1}{\cancel{5}}}{\cancel{5} \times 7}$$

$$= \frac{2}{7}$$

c Multiply the numbers in the numerator and the denominator. Cancel out first.

$$\frac{21}{100} \times \frac{8}{15} = \frac{\overset{7}{\cancel{21}}}{\underset{25}{\cancel{100}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{15}}}$$

Simplify your answer.

$$= \frac{7 \times 2}{25 \times 5} = \frac{14}{125}$$

d First change the mixed numerals to improper fractions.

$$3\frac{2}{5} \times 1\frac{2}{3} = \frac{17}{\cancel{5}} \times \frac{\overset{1}{\cancel{5}}}{3}$$

$$= \frac{17}{3} = 5\frac{2}{3}$$

To check, enter as:

3 [ab/c] 2 [ab/c] 5 [×] 1 [ab/c] 2 [ab/c] 3 [=] .

$3\frac{2}{5} \times 1\frac{2}{3}$	$\frac{17}{3}$

In parts **c** and **d** above, top and bottom numbers have been cancelled *before* multiplication to make the calculations easier. Check that you get the same answers if you change to simplest form after multiplying (so in **c**, you get $\frac{168}{1500}$ first).

Exercise 7.4 Multiplying fractions

1 Find the following amounts.

a $\frac{1}{4}$ of 20   **b** $\frac{1}{5}$ of 20   **c** $\frac{2}{5}$ of 20   **d** $\frac{1}{2}$ of 12

e $\frac{5}{12}$ of 12   **f** $\frac{2}{3}$ of 27   **g** $\frac{1}{9}$ of 27   **h** $\frac{6}{9}$ of 27

i $\frac{3}{10}$ of 50   **j** $\frac{5}{6}$ of 600   **k** $\frac{3}{4}$ of 1000   **l** $\frac{2}{7}$ of 91

2 Calculate each of the following.

a $\frac{1}{2}$ of \$2   **b** $\frac{2}{3}$ of \$12   **c** $\frac{1}{3}$ of \$60   **d** $\frac{1}{3}$ of 60c

e $\frac{1}{4}$ of \$1.20   **f** $\frac{1}{4}$ of \$120   **g** $\frac{3}{8}$ of \$120   **h** $\frac{2}{5}$ of 90c

i $\frac{5}{6}$ of \$3   **j** $\frac{3}{4}$ of \$3   **k** $\frac{4}{5}$ of \$250   **l** $\frac{6}{11}$ of \$1430

3 You will need to use time conversions to find these fractions. Give the answer in the unit shown in brackets.

a $\frac{1}{4}$ of 2 hours (in minutes)   **b** $\frac{3}{4}$ of 2 hours (in minutes)

c $\frac{3}{4}$ of 12 hours (in hours)   **d** $\frac{3}{5}$ of half an hour (in minutes)

e $\frac{1}{5}$ of 1 year (in days)   **f** $\frac{1}{4}$ of 3 minutes (in seconds)

g $\frac{2}{3}$ of 5 minutes (in seconds)   **h** $\frac{3}{10}$ of 12 minutes (in seconds)

i $\frac{2}{3}$ of 2 days (in hours)   **j** $\frac{1}{5}$ of 2 hours (in minutes)

k $\frac{1}{4}$ of 3 days (in hours)   **l** $\frac{4}{5}$ of 7 minutes (in seconds)

m $\frac{1}{3}$ of 2 years (in months)   **n** $\frac{7}{10}$ of 50 minutes (in minutes)

4 Find:

a $\frac{1}{4}$ of 1 km (in m)   **b** $\frac{3}{4}$ of 2 km (in m)   **c** $\frac{5}{8}$ of 1 km (in m)

d $\frac{3}{4}$ of 1 km (in m)   **e** $\frac{1}{4}$ of 3000 m   **f** $\frac{7}{10}$ of 5000 m

g $\frac{1}{5}$ of 16 cm   **h** $\frac{3}{10}$ of 1 cm (in mm)   **i** $\frac{3}{8}$ of 72 mm

j $\frac{5}{8}$ of 7.2 cm   **k** $\frac{3}{5}$ of 85 mm   **l** $\frac{1}{20}$ of 3 m (in cm)

m $\frac{3}{4}$ of 6 cm (in mm)   **n** $\frac{2}{3}$ of 60 km   **o** $\frac{5}{12}$ of 84 m

5 **a** Copy the diagram on the right.
b On your diagram, dot in $\frac{1}{3}$ of the block.
c Now shade $\frac{1}{4}$ of the dotted $\frac{1}{3}$ bit.
d You have now shaded $\frac{1}{4}$ of $\frac{1}{3}$. What fraction is this?

6 Copy and complete each of these statements.

a $\frac{1}{4}+\frac{1}{4}+\frac{1}{4} = 3 \times \frac{1}{4}$ **b** $\frac{3}{5}+\frac{3}{5} = 2 \times \frac{3}{5}$ **c** $\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8} = 5 \times$

$=$ $=$ $=$

$=$ $=$

d $5 \times \frac{1}{7} =$ **e** $7 \times \frac{3}{20} =$ **f** $8 \times \frac{3}{5} =$

$=$ $=$

7 Find the value of:

a $\frac{1}{4}$ of 12   **b** $\frac{3}{4}$ of 20   **c** $\frac{2}{5}$ of 14   **d** $\frac{1}{3}$ of 7   **e** $\frac{2}{3}$ of 20

f $\frac{5}{6}$ of 27   **g** $\frac{7}{10}$ of 7   **h** $\frac{8}{5}$ of 12   **i** $\frac{1}{2}$ of $\frac{1}{2}$ **j** $\frac{3}{2}$ of 5

See Example 17

8 Multiply these fractions. Simplify your answer.

a $\frac{1}{3} \times \frac{2}{5}$ **b** $\frac{3}{4} \times \frac{3}{5}$ **c** $\frac{5}{6} \times \frac{7}{8}$ **d** $\frac{1}{8} \times \frac{9}{10}$ **e** $\frac{3}{8} \times \frac{5}{6}$

f $\frac{3}{8} \times \frac{3}{8}$ **g** $\frac{4}{9} \times \frac{5}{7}$ **h** $\frac{7}{8} \times \frac{3}{20}$ **i** $\frac{2}{3} \times \frac{2}{5}$ **j** $\frac{2}{3} \times \frac{3}{10}$

9 Multiply these fractions. Simplify your answer.

a $\frac{1}{5} \times \frac{5}{7}$ **b** $\frac{2}{5} \times \frac{5}{7}$ **c** $\frac{1}{8} \times \frac{4}{9}$ **d** $\frac{3}{8} \times \frac{4}{9}$ **e** $\frac{3}{8} \times \frac{5}{6}$

f $\frac{3}{7} \times \frac{4}{9}$ **g** $\frac{2}{5} \times \frac{3}{8}$ **h** $\frac{2}{5} \times \frac{5}{8}$ **i** $\frac{3}{10} \times \frac{2}{5}$ **j** $\frac{8}{5} \times \frac{7}{10}$

k $\frac{8}{5} \times \frac{10}{7}$ **l** $\frac{1}{6} \times \frac{12}{20}$ **m** $\frac{5}{6} \times \frac{11}{20}$ **n** $\frac{60}{100} \times \frac{10}{15}$ **o** $\frac{2}{3} \times \frac{9}{10}$

p $\frac{15}{20} \times \frac{12}{5}$ **q** $\frac{3}{7} \times \frac{7}{3}$ **r** $\frac{3}{7} \times \frac{7}{10}$ **s** $\frac{8}{3} \times \frac{12}{5}$ **t** $\frac{9}{5} \times \frac{25}{3}$

10 Convert these mixed numerals to improper fractions before multiplying.

See Example 17

Worked solutions

Exercise 7.4

MAT07NAWS00029

a $1\frac{2}{3} \times \frac{3}{10}$ **b** $4\frac{2}{3} \times \frac{2}{7}$ **c** $4\frac{2}{3} \times \frac{3}{7}$ **d** $3\frac{3}{5} \times 1\frac{2}{3}$ **e** $1\frac{2}{3} \times 1\frac{2}{3}$

f $3\frac{1}{5} \times \frac{5}{8}$ **g** $3\frac{1}{5} \times 1\frac{5}{8}$ **ⓗ** $4\frac{2}{5} \times 2\frac{1}{7}$ **i** $5\frac{1}{3} \times 1\frac{1}{8}$

11 Calculate the answer for each of these.

a $\left(\frac{2}{5}\right)^2$ **b** $\left(\frac{1}{3}\right)^3$ **c** $\left(\frac{1}{2}\right)^2$ **d** $\left(\frac{2}{3}\right)^2$ **e** $\left(\frac{3}{4}\right)^3$

f $\sqrt{\frac{4}{9}}$ **g** $\sqrt{\frac{25}{49}}$ **h** $\sqrt{\frac{81}{100}}$ **i** $\sqrt{\frac{9}{16}}$ **j** $\sqrt[3]{\frac{27}{64}}$

12 In a class of 24 students, one-sixth were a day late handing in assignments. How many were late?

13 In a swimming squad of 45 people, 3 out of 5 were female. How many are female? How many are male?

14 Melissa bought six books while shopping. The orange juice leaked and ruined two-thirds of them. How many could she still read?

15 A class of students discovered that 1 out of 8 cars passing their school was white. If 2160 cars pass the school in a day, how many would you expect to be white?

16 Three-quarters of the Year 7 students at a school are boys. There are 24 students in 7A, 20 in 7B, 28 in 7C and 16 in 7D. If the boys and girls are spread as evenly as possible through all four classes, how many boys and girls are in each class?

17 An echidna has a mass of $1\frac{1}{2}$ kg. A field mouse has a mass of one-twentieth that of the echidna, but 5000 ants are needed to balance the mass of one field mouse. Find the mass of one ant.

18 Use this diagram to help you fill in the correct fractions below.
 a Height of fir = _____ height of wattle
 b Height of fir = _____ height of eucalypt
 c Height of grevillea = _____ height of banksia
 d Height of banksia = _____ height of eucalypt
 e Height of grevillea = _____ height of fir
 f Height of banksia = _____ height of fir
 g Height of wattle = _____ height of eucalypt

19 Mrs Rizzo's patio measures 5 m × 3 m. How many paving stones does it contain if there are $8\frac{1}{2}$ stones per m²?

20 A cake recipe needs 1 egg, $2\frac{1}{2}$ cups of flour and $\frac{1}{4}$ cup of sugar. What quantities of eggs, flour and sugar will you need to make eight cakes?

21 The school hall has 840 seats. If it is two-thirds full, how many seats are vacant?

7.5 Dividing fractions

You can draw the problem $\frac{3}{4} \div \frac{2}{5}$ as

You can think of $\frac{3}{4} \div \frac{2}{5}$ as asking how many lots of $\frac{2}{5}$ will fit into $\frac{3}{4}$? How many pink rectangles

will fit into the orange rectangle ?

Divide the boxes both ways.

The big pink rectangle has 8 little rectangles the same size as the 15 little orange rectangles in the big orange rectangle. These make one big pink rectangle, with seven (out of 8) left over. This means that $\frac{3}{4} \div \frac{2}{5} = 1\frac{7}{8}$.

The opposite (**reciprocal**) of the fraction $\frac{2}{5}$ is $\frac{5}{2}$, which is upside down. The opposite of $\div$ is $\times$. Two opposites cancel each other out, so dividing by a fraction has the same answer as multiplying by its reciprocal. $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$, the same answer as above.

Important!

Dividing fractions

To divide by a fraction $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$. The reciprocal of a whole number is the unit fraction for that number, so the reciprocal of 5 is $\frac{1}{5}$.

Consider $2 \div \frac{1}{3}$. This can be thought of as 'How many times does $\frac{1}{3}$ go into 2?'.
2 wholes contain 6 thirds.

$$2 \div \frac{1}{3} = 2 \times \frac{3}{1}$$
$$= 6$$

Example 18

Find:

a $\frac{3}{4} \div \frac{1}{2}$ **b** $\frac{7}{3} \div \frac{5}{6}$ **c** $\frac{5}{7} \div 5$ **d** $7\frac{1}{2} \div 5\frac{5}{6}$

Solution

a Flip over the second fraction and then multiply.
$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1}$$

Simplify your answer.
$$= \frac{6}{4} = \frac{3}{2}$$

Write as a mixed numeral.
$$= 1\frac{1}{2}$$

You can do this in your head.

How many $\frac{1}{2}$ s in $\frac{3}{4}$?

b Multiply by the reciprocal of the 2nd fraction. Cancel out where possible.
$$\frac{7}{3} \div \frac{5}{6} = \frac{7}{{}_1\cancel{3}} \times \frac{\cancel{6}^2}{5}$$

Simplify your answer.
$$= \frac{14}{5}$$

Write as a mixed numeral.
$$= 2\frac{4}{5}$$

c Multiply by the reciprocal of the 2nd fraction. The reciprocal of 5 is $\frac{1}{5}$. Cancel out where possible.

$$\frac{5}{7} \div 5 = \frac{{}^{1}\cancel{5}}{7} \times \frac{1}{{}_{1}\cancel{5}}$$

Simplify your answer.

$$= \frac{1}{7}$$

d As with multiplication, mixed numerals must be changed to improper fractions before you can divide.

$$7\frac{1}{2} \div 5\frac{5}{6} = \frac{15}{2} \div \frac{35}{6}$$

Multiply by the reciprocal of the 2nd fraction. Cancel out where possible.

$$= \frac{{}^{3}\cancel{15}}{{}^{1}\cancel{2}} \times \frac{{}^{3}\cancel{6}}{{}^{7}\cancel{35}}$$

Simplify your answer.

$$= \frac{9}{7}$$

Write as a mixed numeral.

$$= 1\frac{2}{7}$$

To check, enter as:

7 $a^{b}/_{c}$ 1 $a^{b}/_{c}$ 2 ÷ 5 $a^{b}/_{c}$ 5 $a^{b}/_{c}$ 6 = .

$7\frac{1}{2} \div 5\frac{5}{6}$	$\frac{9}{7}$

Investigate: Thanh's gorgeous garden

In this activity you will be using fractions to make a decorative pattern.
In Thanh's garden there are places for 120 bulbs. Thanh wants to plant them in the following proportions:

$\frac{1}{3}$ golden daffodils $\qquad$ $\frac{3}{10}$ red tulips

$\frac{1}{6}$ white snowdrops $\qquad$ $\frac{1}{5}$ blue hyacinths

1 How many of each type of flower should Thanh plant?
2 Copy the plan below or use one provided by your teacher. Colour in squares for each flower. Make up your own pattern.

3 Make up different fractions for the same garden plot. Make a list of flowers and the fractions of their proportions and colour them on a fresh plan (or colour the plan first and list the fractions afterwards).

Investigate: Fraction puzzles

Concentrate on finding strategies to solve these problems. Write down the thinking you used to work them out.

1 The pencil box

Ruth has a pencil box which is half full. She puts four extra pencils in and the box is now two-thirds full. How many pencils will fit into her pencil box?

2 Magic squares

In a magic square, each row, column and diagonal must add to the same total.

i

4	$1\frac{1}{2}$	
$\frac{1}{2}$		$4\frac{1}{2}$
3		

ii

$\frac{1}{6}$		
	$\frac{2}{3}$	
	$\frac{1}{3}$	$1\frac{1}{6}$

a Find the missing numbers to complete these magic squares.

b Make up one or two magic squares of your own.

3 Friendly mints

Samantha has a bag of mints to share. Ali takes $\frac{1}{4}$ of them. Duyen takes $\frac{1}{2}$ of what is left. Sharelle takes $\frac{2}{3}$ of the remainder. Samantha has 7 mints left. How many mints were in the bag to start with?

4 Fit fractions

Find a fraction which fits between:

a $\frac{1}{2}$ and $\frac{5}{8}$ **b** $\frac{3}{4}$ and $\frac{4}{5}$ **c** $\frac{5}{8}$ and $\frac{2}{3}$

5 The race

- Jane finished half a minute behind Bill.
- Bill took less than half the time that Joel took.
- Joel's time was 5 minutes 23 seconds.
- Dimitra took about $3\frac{1}{4}$ minutes.
- Esther finished about $1\frac{1}{2}$ minutes after Bill.
- David finished three-quarters of a minute in front of Joel.

What is the finishing order of the six competitors?

6 Sweet tooth

Some packets of lollies are emptied onto a table. Colleen takes away half of the pile. Gino then takes one-third of what was left. Heather takes two and eats them. Half of those left are red and there are eight red lollies. How many lollies were there originally?

7 Fractions in history

When asked her age, a lady gave this reply:

If to my age there added be
One-half, one-third (of it) and 3 times 3,
Six score and 10 the sum you would see.
Now pray tell me what age I be.

How old is she?

8 How much is unshaded?
Each rectangle is half of the one before it.
The whole square is 1. How much is unshaded?

Exercise 7.5 Dividing fractions

1 How many thirds in 1 whole?

2 How many fifths in 1 whole?

3 How many sixths are there:
  a in 1 whole?   **b** in 2 wholes?   **c** in 3 wholes?

4 a How many quarters are there in 5?   **b** How many quarters are there in 10?

5 Find answers to the following.
  a $4 \div \frac{1}{3}$ **b** $5 \div \frac{1}{3}$ **c** $7 \div \frac{1}{3}$ **d** $3 \div \frac{1}{5}$

  e $7 \div \frac{1}{5}$ **f** $8 \div \frac{1}{5}$ **g** $4 \div \frac{1}{4}$ **h** $1 \div \frac{1}{4}$

  i $8 \div \frac{1}{4}$ **j** $1 \div \frac{1}{3}$ **k** $1 \div \frac{1}{8}$ **l** $1 \div \frac{1}{10}$

6 Find each of the following. Simplify your answers.

See Example 18

  a $\frac{1}{4} \div \frac{2}{3}$ **b** $\frac{8}{7} \div \frac{2}{7}$ **c** $\frac{3}{5} \div \frac{2}{5}$ **d** $\frac{5}{7} \div \frac{2}{5}$

  e $\frac{5}{7} \div \frac{5}{6}$ **f** $\frac{3}{5} \div \frac{5}{8}$ **g** $\frac{3}{5} \div \frac{9}{10}$ **h** $\frac{5}{12} \div \frac{5}{6}$

  i $\frac{5}{6} \div \frac{5}{12}$ **j** $\frac{1}{12} \div \frac{1}{6}$ **k** $\frac{1}{5} \div \frac{1}{2}$ **l** $\frac{1}{2} \div \frac{1}{5}$

  m $\frac{9}{10} \div \frac{6}{25}$ **n** $\frac{9}{10} \div \frac{6}{100}$ **o** $\frac{7}{10} \div \frac{5}{100}$ **p** $\frac{1}{3} \div \frac{2}{5}$

  q $\frac{3}{4} \div \frac{3}{5}$ **r** $\frac{5}{6} \div \frac{7}{8}$ **s** $\frac{1}{8} \div \frac{9}{10}$ **t** $\frac{3}{8} \div \frac{3}{8}$

  u $\frac{2}{3} \div \frac{2}{5}$ **v** $\frac{2}{3} \div \frac{4}{7}$ **w** $\frac{3}{7} \div \frac{4}{9}$ **x** $\frac{60}{100} \div \frac{10}{15}$

7 Find each of the following. Simplify your answers.

See Example 18

  a $\frac{5}{7} \div 5$ **b** $\frac{5}{7} \div 7$ **c** $\frac{1}{4} \div 3$ **d** $\frac{7}{8} \div 5$

  e $\frac{3}{4} \div 3$ **f** $\frac{8}{9} \div 4$ **g** $\frac{4}{5} \div 6$ **h** $\frac{2}{3} \div 4$

  i $\frac{1}{2} \div 5$ **j** $\frac{9}{10} \div 9$ **k** $\frac{3}{5} \div 9$ **l** $\frac{5}{6} \div 2$

See Example 18

8 Find answers to each of the following.
  a $3\frac{1}{8} \div \frac{5}{6}$ **b** $3\frac{1}{2} \div 1\frac{1}{4}$ **c** $2\frac{3}{4} \div 3\frac{1}{7}$ **d** $2\frac{1}{2} \div 1\frac{7}{8}$

  e $1\frac{7}{8} \div 2\frac{1}{2}$ **f** $6\frac{2}{5} \div 4\frac{4}{5}$ **g** $12 \div 1\frac{1}{5}$ **h** $1\frac{1}{4} \div 1\frac{1}{3}$

  i $6 \div 1\frac{4}{5}$ **j** $2\frac{1}{2} \div \frac{4}{5}$ **(k)** $4\frac{2}{3} \div 1\frac{1}{2}$ **l** $5 \div 3\frac{1}{3}$

9 If a farm of 78 hectares is subdivided into lots of $4\frac{1}{2}$ hectares, how many lots are there?

10 A bag holds five-eighths of a kilogram of oranges. How many bags can $12\frac{1}{2}$ kg of oranges be packed into?

11 A car needs $52\frac{1}{2}$ litres to fill its tank. If a tanker delivers 5040 litres of petrol, how many times could this car's tank be filled?

12 How many speakers were there if each speaker was allowed $3\frac{1}{4}$ minutes and the debate went for about 26 minutes?

13 Water is dripping from a tank at the rate of five-eighths of a litre per hour. How long will it take for a 24 litre tank to empty?

14 Write a question for each operation ($+$, $-$, $\times$, and $\div$) that gives the answer:

   a $\frac{5}{8}$ **b** $2\frac{2}{3}$

15 Give three fractions between:

   a $\frac{1}{3}$ and $\frac{3}{4}$ **ⓑ** $\frac{1}{2}$ and $\frac{1}{3}$

16 Rukaya sold all of her eggs. She first sold half her eggs plus half an egg to her neighbour, without having to break an egg. Then she sold half her remaining eggs plus half an egg to her uncle, again without breaking an egg. Finally she sold half her remaining eggs plus half an egg to her friend, without breaking an egg. How many eggs did Rukaya begin with?

17 Find answers for these questions using the correct order of operations.

   a $5 + \frac{5}{8} \times 2\frac{2}{3}$ **b** $4\frac{1}{3} - \left(2\frac{1}{6} - \frac{2}{3}\right)$ **c** $\frac{5}{6} \times \left(\frac{2}{5} + 2 \times \frac{2}{5}\right)$

   d $\frac{7}{8} + 5 \times \frac{3}{4}$ **e** $4\frac{7}{8} - \left(1\frac{1}{2} + 2\frac{3}{4}\right)$ **f** $\left(1\frac{1}{5} + 1\frac{1}{4}\right) \div \left(1\frac{3}{4} + \frac{3}{5}\right)$

7.6   Rates and ratios

A fruit and vegetable shop shows the price of tomatoes as a rate. The rate tells you how much you pay for each kilogram. If you paid $5.40 for 3 kg, you could divide the price by the mass to work out the price rate as $1.80/kg.

Important!

Rates

A **rate** is obtained by dividing one quantity by a different, related quantity. It is sometimes useful to express a rate as a fraction.

In general, a rate compares related measurements. The measurements that are compared within a rate are usually referred to as 'quantities'.

The measurement units for the quantities in a rate are separated by the word 'per' or the symbol /, with the second one being the denominator.

Prices, pay rates and speeds are the most common rates.

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}}$$

Example 19

Sally earns $62.80 working for 4 hours. Work out her rate of pay.

Solution

Write the division.	$\text{Rate of pay} = \dfrac{\$62.80}{4 \text{ hours}}$
Do the division and write as a rate.	$= \$15.70/\text{hour}$

Example 20

A car travels 180 km in 2 hours. What is its average speed?

Solution

Write the formula.	$\text{Average speed} = \dfrac{\text{Distance}}{\text{Time}}$
Write the division.	$= \dfrac{180 \text{ km}}{2 \text{ h}}$
Do the division and write the speed.	$= 90 \text{ km/h}$

Rates can be used to calculate the first quantity from the second. In this case, we use multiplication to reverse the division.

Example 21

Katherine paid $11.25 for 3 kg of bananas. What was the unit price of the bananas?

Solution

Write the division.	$\text{Unit price} = \dfrac{\$11.25}{3 \text{ kg}}$
Do the division and write as a rate.	$= \$3.75/\text{kg}$

Example 22

Potatoes cost $2.49/kg. How much will 3.4 kg cost?

Solution

In this case, you are given the second quantity of the rate, so use multiplication.

Write the multiplication.	$\text{Cost} = \$2.49/\text{kg} \times 3.4 \text{ kg}$
Use your calculator.	$= \$8.466$
Round off to the nearest cent.	$\approx \$8.47$
Write the answer.	**The potatoes will cost $8.47.**

Important!

Ratios

A ratio compares quantities of the *same kind* in a definite order.

When you compare quantities of the same kind, you obtain a ratio.

In order to simplify a ratio, we must write the quantities in the *same units*.

A ratio is stated without units using

- the word '**to**', as in 5 to 3
- the symbol **:**, as in 5 : 3

or in **fraction** notation with the second quantity as the denominator, as in $\frac{5}{3}$.

We usually write ratios in simplest form using whole numbers, in the same way that we write fractions in simplest form.

Example 23

Write each of the following as ratios in simplest form.

a 15 to 20   **b** 4 days : 2 weeks   **c** 2.01 : 0.3   **d** $2\frac{1}{4} : \frac{3}{10}$

Solution

a

	15 to 20
Write in fraction format.	$= \dfrac{15}{20}$
Divide both numerator and denominator by 5.	$= \dfrac{15 \div 5}{20 \div 5}$
	$= \dfrac{3}{4}$
Write in ratio format.	**3 : 4**

b

	4 days : 2 weeks
Write in the same units.	**= 4 days : 14 days**
Divide both sides by 2.	**= 4 ÷ 2 : 14 ÷ 2**
	= 2 : 7

c

	2.01 : 0.3
Multiply both sides by 100 to make wholes.	**= 201 : 30**
Divide both sides by 3.	**= 201 ÷ 3 : 30 ÷ 3**
	= 67 : 10

d

	$2\dfrac{1}{4} : \dfrac{3}{10}$
Write both as improper fractions.	$\dfrac{9}{4} : \dfrac{3}{10}$
Multiply both by 20 to make them wholes.	$\dfrac{9}{4} \times 20 : \dfrac{3}{10} \times 20$
	= 45 : 6
Divide both sides by 3.	**= 15 : 2**

Rates and ratios are closely related. For example, a speed of 5 m/s means that the object travels 20 m in 4 s or 30 m in 6 s. The ratio of the distances, 20 : 30, is the same as the ratio of the times, 4 : 6, because they both simplify to give 2 : 3.

You can use the relationship between ratios and rates to solve some kinds of problems.

Example 24

A snail crawls at 15 cm/min. How long will it take to move 117 cm?

Solution

Write as a ratio problem with the same quantities together in each ratio, writing the new time and distance as the numerators.

$$\frac{117}{15} = \frac{\text{New time}}{1} \quad \text{—— Rate}$$

Now write the ratio as a division problem.

New time = 117 ÷ 15

Use your calculator.

= 7.8 minutes

Change 0.8 minutes to seconds.

0.8 minutes = 0.8 × 60 = 48 seconds

Write the answer.

The snail will take 7.8 minutes, which is 7 minutes 48 seconds.

Exercise 7.6 Rates and ratios

1 For the following rates, identify the quantities being compared. Select from: mass, time, money, distance, volume and area.

a $15/h	**b** 25 kg/L	**c** $3.2/m^2
d $3/kg	**e** 12 kg/m	**f** 14 km/h
g 38 kg/L	**h** 2000 L/km^2	**i** 21 cm/min

2 Express each of the following rates in simplest form.

See Example 19

a 120 km in 6 h	**b** 40 kg in 20 L	**c** 500 rev. in 25 min
d 38 words in 2 min	**e** 28 L in 7 s	**f** 36°C in 6 min
g $300 for 20 people	**h** $2.40 for 15 m	**i** 3 kg for 40 L

3 Calculate the average speed in each of the following cases.

See Example 20

a 10 m in 1 s	**b** 300 m in 6 s	**c** 25 km in 1 h
d 60 km in 2 h	**e** 100 km in 5 min	**f** 1000 km in 10 h
g 2000 m in 40 s	**h** 1000 km in 80 h	**i** 400 m in 64 s

4 Express the following ratios in simplest form.

See Example 23

a 30 : 18	**b** 45 is to 60	**c** 28 : 224
d 84 to 112	**e** $\frac{54}{81}$	**f** 35 s : 14 s
g 32 girls : 24 girls	**h** 20 L : 12 L	**i** $1.20 : 30c

5 Simplify these ratios.

a 30c : $1.50	**b** 1 m : 20 cm	**c** 500 g : 2 kg
d 15 mm : 2.5 cm	**e** 18 s : 2 min	**f** 3.2 km : 2400 m
g 8 months : 1 year	**h** $4.50 : 120c	**i** 3 weeks : 12 days

6 Simplify each of the following ratios.

   a $1 : \frac{1}{2}$ **b** $1\frac{2}{3} : 2$ **c** $2 : 2\frac{1}{3}$

   d $3\frac{1}{3} : 1\frac{1}{2}$ **e** $5\frac{1}{7} : 2\frac{1}{2}$ **f** $3\frac{3}{4} : 2\frac{1}{5}$

7 Simplify each of the following ratios.

   a   8 : 1.2   **b**   0.9 : 12   **c**   1.8 : 15

   d   22 : 3.3   **e**   2.1 : 2.4   (**f**)   0.16 : 2.4

   g   2.5 : 0.75   **h**   24 : 7.2   **i**   72 : 3.6

8 Change the following ratios so that the second part is 1, rounding to 2 decimal places if necessary.

   a   3 : 2   **b**   4 : 5   **c**   12 : 5

   d   7 : 4   **e**   18 : 8   **f**   20 : 6

   g   14 : 52   **h**   62 : 17   **i**   83 : 111

9 Peter and Josephine delivered some advertising material in their suburb. The ratio of the amounts they delivered was 3 : 4, with Josephine distributing more. They divided the $28 payment they received in the ratio 3 : 4.

   a   How many parts altogether were there?

   b   How much would 1 part be worth?

   c   How much did they each get?

10 Divide each quantity in the given ratio.

   a $15 in the ratio 2 : 1   **b**   10 L in the ratio 3 : 2   **c**   36c in the ratio 1 : 3

   d   77 kg in the ratio 3 : 4   **e** $35 in the ratio 5 : 2   **f**   200 mL in the ratio 3 : 5

   g   100 cm in the ratio 7 : 3   **h**   72 g in the ratio 1 : 5   **i**   81 s in the ratio 4 : 5

Problem solving

11 Calculate the rate in each of the following.

   a   The cost per kilogram if 5 kg of apples costs $16.25.

   b   The cost per m^3 of soil if 12 m^3 costs $168.

   c   The flow rate if 224 L passed through in 16 min.

   d   The cost per m^2 of land if $240 000 was paid for 800 m^2.

12 A large tank of water has a hole that lets water leak out at the rate of 24 L/min.

   a   How much water leaks out after 3 min?

   b   What volume of water passes through the hole after $7\frac{1}{2}$ min?

13 Electricity is charged at the rate of 11.8c/unit. How much would be charged for these amounts?

   a   1200 units

   b   850 units

   c   420 units

14 A car averages 80 km/h on a journey. How long will it take to travel 320 km?

15 An oil tank has a hole that lets oil leak out at the rate of 12 L/min.

   a   How long will it take for 600 L to leak out?

   b   How long will it be before the tank loses 780 L?

16 Copy and complete the table below for the ratios of the values of Australian coins.

a b c

d e f

Pair	a, e	d, f		f, b		d, c	f, c		b, e	c, e
Values			20 : 50		10 : 200			100 : 10		
Simplest ratio										

17 The average distance from the Sun to various planets is shown in millions of kilometres.

Planet	Pluto	Uranus	Jupiter	Earth	Mercury
Distance (million km)	5865.6	2852.8	772.8	148.8	57.6

Find the following ratios of average distances from the Sun.
a Earth : Uranus (round to 2 decimal places)
b Pluto : Mercury (round to the nearest whole number)
c Jupiter : Uranus (round to 1 decimal place)
d Earth : Pluto (round to 2 decimal places)

18 Sally made some punch for her party. The punch had three parts pineapple juice for every two parts of lemonade. She made up 25 litres of punch altogether.
What volume of each ingredient was required?

19 Eddie is saving to buy a home computer worth $2640. His parents promise to match every $3 saved by him with $5 of their own.
a How much will Eddie contribute?
b How much will be left for his parents to contribute?

20 Pure gold is called 24-carat gold; 18-carat gold contains 18 parts gold for every 6 parts alloy.
a What mass of gold would be required to make an 18-carat chain weighing 144 g?
b How much gold would be required to make the chain if 22-carat gold were used?

Reasoning

21 During an electrical storm, lightning strikes the ground and a clap of thunder results.
Take the speed of sound to be 340 m/s. Light would travel right around the Earth in less than 0.1 seconds.
a How long will it take for people in a town 10 km away to hear the thunder? (Answer to the nearest second.)
b People in another town heard the thunder 15 s after the lightning struck. How far were they from the lightning?

22 A mixture of olive oil and vinegar contains 2 parts vinegar to 3 parts olive oil. In what proportion should this mixture be made up with additional vinegar to produce equal amounts of olive oil and vinegar?

- A **common fraction** (**vulgar fraction**) shows part of a whole number.

- The **denominator** is written underneath and shows the number of equal shares in one whole number. The **numerator** is written on top and shows the number of these equal shares in the fraction. The bar that separates the numerator and denominator is called a **vinculum**. Fractions may also be written horizontally with the numerator and denominator separated by a slash.

- Fractions are named by giving the numerator first, then the denominator with the ending 'ths'. Fractions with denominators of 2, 3 and 4 have the special names **half**, **third** and **quarter**.

- A **proper fraction** is less than 1. Fractions greater than 1, with the numerator larger than the denominator, are called **improper fractions**.

- **Mixed fractions**, or **mixed numerals**, have a whole number part and a proper fraction. Improper fractions can be changed to mixed fractions by division. Mixed fractions are changed to improper fractions by multiplying the whole number by the denominator and adding to the numerator.

- **Equivalent fractions** show the same amount using different denominators. The **simplest form** of a fraction is the equivalent fraction with the lowest possible denominator. Fractions can be changed to simplest form by dividing both the numerator and the denominator by common factors. This is called **cancelling** (**down**).

- To compare fractions with different denominators, we must change them to the same denominator. A common denominator can be found by multiplying the denominators of both fractions together or by using the lowest common multiple of the denominators.

- Fractions with the *same denominator* may be *added* or *subtracted*. Fractions with different denominators must be changed to equivalent fractions with the same denominator before they can be added or subtracted. Some calculators have a fraction key that allows adding and subtracting.

- To multiply fractions, multiply the numbers in the numerator and multiply the numbers in the denominator.

- To divide by a fraction $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$.

- A **rate** is obtained by dividing one quantity by another. The units of measurement of a rate are separated by the word 'per' or the symbol /. The second quantity is the denominator. Average speed may be measured in metres per second or kilometres per hour.

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}}$$

- A **ratio** compares two quantities of the same kind in a definite order. It is stated without units using the word 'to' or the symbol : between the parts.

- Ratios may be shown in fraction notation. This may help in simplifying a ratio.

1 Write these mixed numerals as improper fractions.

a $1\frac{3}{4}$ **b** $1\frac{4}{9}$ **c** $2\frac{1}{4}$ **d** $3\frac{1}{7}$

e $5\frac{4}{10}$ **f** $3\frac{5}{8}$ **g** $7\frac{3}{5}$ **h** $10\frac{2}{7}$

2 Write these improper fractions as mixed numerals.

a $\frac{7}{4}$ **b** $\frac{12}{11}$ **c** $\frac{23}{7}$ **d** $\frac{36}{5}$

e $\frac{45}{8}$ **f** $\frac{26}{3}$ **g** $\frac{36}{24}$ **h** $\frac{121}{12}$

3 Find the highest common factor of each pair.

a 14 and 10   **b** 25 and 20   **c** 16 and 48

4 Find the lowest common multiple of each pair.

a 8 and 3   **b** 4 and 6   **c** 20 and 15

5 Copy and complete these pairs of equivalent fractions.

a $\frac{1}{3} = \frac{?}{6}$ **b** $\frac{3}{4} = \frac{12}{?}$ **c** $\frac{4}{5} = \frac{?}{20}$

d $\frac{5}{12} = \frac{15}{?}$ **e** $\frac{1}{2} = \frac{?}{50}$ **f** $\frac{3}{20} = \frac{?}{100}$

6 Reduce each of these fractions to simplest form.

a $\frac{25}{30}$ **b** $\frac{30}{40}$ **c** $\frac{15}{20}$

d $\frac{20}{15}$ **e** $\frac{24}{48}$ **f** $\frac{24}{42}$

7 Order these sets of numbers, from smallest to largest.

a $\frac{4}{5}, \frac{3}{10}, \frac{2}{5}, \frac{7}{10}$ **b** $\frac{1}{2}, \frac{1}{3}, \frac{7}{12}, \frac{11}{12}$

c $1\frac{1}{4}, \frac{7}{8}, \frac{7}{4}, \frac{3}{8}$ **d** $3, 2\frac{1}{4}, \frac{5}{8}, \frac{8}{5}, \frac{8}{12}, \frac{3}{5}$

8 Find answers for each of the following.

a $\frac{3}{4} + \frac{1}{4}$ **b** $\frac{1}{6} + \frac{4}{6} + \frac{5}{6}$ **c** $8 - \frac{4}{7}$

d $\frac{2}{5} + \frac{3}{10}$ **e** $\frac{1}{2} + \frac{1}{3}$ **f** $2\frac{1}{4} + 3\frac{1}{5}$

g $\frac{1}{2} - \frac{3}{8}$ **h** $\frac{3}{4} - \frac{1}{5}$ **i** $\frac{5}{10} + \frac{1}{3}$

9 Find the answer to each of the following.

a $10 - \frac{1}{3}$ **b** $2\frac{5}{6} + 1\frac{1}{2}$ **c** $3\frac{2}{3} - 2\frac{5}{6}$

d $3 + 2\frac{1}{2} - 1\frac{1}{5}$ **e** $6 - 4\frac{2}{3}$ **f** $2\frac{5}{6} + 4\frac{1}{2} - 3\frac{2}{3}$

10 a Find the answer to each of the questions. (The letters are not in order.)

A $\frac{3}{7}$ of 21   **B** $\frac{2}{5}$ of 20   **R** $\frac{4}{9}$ of 27

O $\frac{1}{4}$ of 28   **T** $\frac{2}{3}$ of 30   **H** $\frac{1}{10}$ of 35

     b Order the six answers, beginning with the smallest. Replace each answer by its letter. What word do you spell?

11 Find the value of:

a $\frac{2}{5}$ of 45   **b** $\frac{2}{7}$ of 14   **c** $\frac{1}{3}$ of 15

d $\frac{3}{8}$ of 32   **e** $\frac{2}{5}$ of 15   **f** $\frac{4}{9}$ of 18

g $\frac{2}{3}$ of 1 day   **h** $\frac{4}{5}$ of 2 km (in metres)   **i** $\frac{3}{4}$ of 3 hours

j $\frac{7}{10}$ of 15 cm (in mm)   **k** $\frac{1}{6}$ of 5 minutes   **l** $\frac{5}{8}$ of 2 m (in cm)

12 Find answers to the following.

a $\frac{2}{3} \times \frac{1}{5}$ **b** $\frac{3}{5} \times 10$ **c** $\frac{4}{7} \times \frac{5}{8}$

d $\frac{1}{3} \times \frac{1}{7}$ **e** $1\frac{1}{2} \times \frac{1}{2}$ **f** $\frac{1}{5} \times 2\frac{1}{2}$

g $1\frac{1}{2} \times 1\frac{1}{2}$ **h** $2\frac{2}{7} \times 3\frac{1}{2}$ **i** $\frac{3}{4} \times 8 \times \frac{1}{3}$

j $4\frac{1}{2} \times \frac{3}{4}$ **k** $1\frac{2}{3} \times 2\frac{1}{4}$ **l** $1\frac{1}{3} \times 1\frac{1}{5} \times \frac{1}{2}$

13 How many fifths are in 3?

14 Find answers to the following.

a $10 \div \frac{1}{5}$ **b** $\frac{1}{2} \div 3$ **c** $\frac{1}{2} \div \frac{1}{3}$

d $\frac{3}{4} \div \frac{3}{8}$ **e** $\frac{3}{5} \div \frac{1}{10}$ **f** $\frac{4}{7} \div \frac{5}{8}$

g $1\frac{1}{2} \div \frac{1}{2}$ **h** $2\frac{1}{4} \div 1\frac{1}{8}$ **i** $1\frac{7}{8} \div 1\frac{1}{4}$

15 Express the following ratios in simplest form.

a 24 : 36   **b** 50 to 35   **c** 6 weeks : 3 days

d $6\frac{1}{4} : 2\frac{1}{2}$ **e** 0.5 : 1.45   **f** 8 months : 1 year

g $4.50 : 120c   **h** 3 weeks : 12 days

16 Divide 630 mL in the ratio 5 : 4.

Fluency

17 Calculate each of the following.

a $\frac{5}{7} - \frac{2}{3}$ **b** $2\frac{5}{9} + 3\frac{4}{15}$ **c** $5\frac{7}{12} - 3\frac{5}{7}$

d $5\frac{3}{5} \div \frac{2}{3}$ **e** $3\frac{1}{6} \times 3\frac{3}{5}$ **f** $7\frac{1}{2} \div 1\frac{3}{7}$

18 Solve each of the following.

a $\frac{7}{12} - \frac{4}{15}$ **b** $3\frac{5}{12} + 6\frac{9}{17}$ **c** $5\frac{8}{15} - 2\frac{6}{7}$

d $\frac{7}{9} \div \frac{4}{17}$ **e** $2\frac{1}{12} \times 4\frac{7}{12}$ **f** $6\frac{5}{16} \div 7\frac{1}{8}$

Problem solving

19 In a small music shop, these types of CDs were in stock:

male vocal	63	specials	17
jazz	28	female vocal	37
classical	56	country	17
rock groups	82		

a How many CDs were in stock?

b What fraction was vocal (male or female)?

c What fraction was specials?

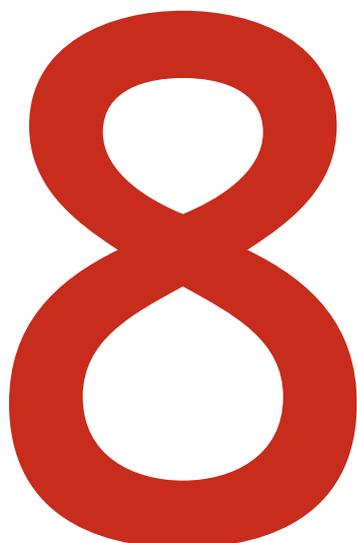**d** What type of CDs made up $\frac{14}{75}$ of the stock?

e What type of CDs made up more than $\frac{1}{5}$ of the stock?

20 The netball team scored 26 goals. Jessica scored 8 goals. What fraction of her team's score did Jessica make?

21 In a cross-country relay, the first person runs 3 km, the next runs 4 km and the last runner runs 2 km. What fraction of the event does the middle person run?

22 The recipe for a fruit punch is 2 litres of ginger ale, $1\frac{1}{2}$ litres of orange juice, $2\frac{3}{4}$ litres of lemonade and $\frac{1}{3}$ of a litre of raspberry juice. How many litres of punch does this recipe make?

23 A farmer owns 24 goats. One-quarter of the goats are black, two-thirds are white, and the rest are divided equally between brown and spotted. How many of the goats are spotted?

24 A bottle is two-thirds full. One quarter of the liquid is then poured out. What fraction remains in the bottle?

25 The instructions for mixing a liquid insecticide said to use 3 parts of the insecticide to 22 parts of water. What volumes of insecticide and water should be used to make 10 L for spraying on plants?

26 A drawing is at a scale of 1 : 250.
   a What size on the drawing is used to show a length of 2.4 m?
   b What is the true size of something shown on the drawing as 4.2 cm?

Reasoning

27 Julio was shipwrecked and had only a limited supply of water. On the first day he drank $\frac{1}{2}$ of his supply and on the second day he drank $\frac{2}{3}$ of the rest. What fraction of his original water did he have left?

28 Explain why division can be done as multiplication by the reciprocal.

29 Explain how cancelling before multiplication works.

30 Work out the following:
   a $2\frac{2}{5} \times \left(\frac{2}{5} - \frac{3}{8}\right) - 0.3$
                                              b $2\frac{3}{4} \times 2\frac{1}{22} + 4\frac{3}{4} \div \frac{2}{9}$

Measurement and geometry

8

Length and area

Contents

Prior learning
Chapter 8

Parent guide
Chapter 8

Curriculum guide
Chapter 8

Australian Curriculum statements

Using units of measurement

Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving. (ACMMG159) AC

The ability to measure lengths and areas has always been important. In ancient times, measuring length was based on things such as a hand span or a stride. These forms of measure were not very accurate and so eventually standard units of measure and instruments were developed. The metric system is now used across most of the world to measure length and area as well as other quantities. The French developed the metric system in the late 1700s. It is a decimal system and it became widely used because it is relatively simple.

■ Mathematical literacy

Maths dictionary

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

area	length	radius
centimetre	mass	rectangle
circle	measurement	scale
circumference	metre	square
composite shape	metric system	square metre
diameter	millimetre	triangle
hectare	perimeter	
kilometre	pi	

8.1 The metric system

No one really knows how the earliest civilisations measured lengths. However, we do know that the ancient Egyptians could measure with great accuracy using units based on parts of the body.

digit   palm   span

cubit   foot   pace

Investigate: Measurement systems

Work in a group of four or five to complete this investigation. Your group will need a ruler and tape measure for this investigation.

Early civilisations developed distance units for short distances based on the dimensions of the human body.

Inch

- An inch was based on the width of an adult's thumb.
- Measure the widths (in cm) of the thumbs of each member of the group and record your results.
- Find the difference between the largest and smallest 'inch' in your group.
- Calculate the length in cm of the average 'inch' for your group.
- Use the Internet or some other method to complete: 1 inch ≈ … cm.
- Compare the length of an inch with the average 'inch' for your group and comment on your result.

Span

- A span is the distance between the tips of the thumb and little finger of an outstretched hand.
- Complete the steps outlined above for a span.

Foot

- The foot (12 inches) was originally the length of a human foot.
- Complete the steps outlined above for a foot.

Yard

- The yard is the distance from the tip of the nose to the end of the middle finger of the outstretched hand.
- Complete the steps outlined above for a yard.

Fathom

- The fathom is the distance from one fingertip to the other if you stretch your arms out to the sides as far as possible.
- Complete the steps outlined above for a fathom.

Other non-metric units of length include the rod, furlong and mile. Research the rod, furlong and mile to determine:

- the equivalent measurements in the metric system
- instances where these measures are still used.

In Australia we measure using the **metric system**. This system began in France in the 1790s after the French revolution and, because it is based on powers of 10, it is logical and easy to use. The word 'metric' comes from the Greek word *metron* meaning 'to measure'.

The metre was originally defined as one ten-millionth of the distance from the North Pole to the Equator (along the meridian through Paris).

In 1970, the Metric Conversion Board was established in Australia and Australia started the change to metric units.

The following metric units are most commonly used.

Quantity	Name of unit	Abbreviation
Length	millimetre	mm
	centimetre	cm
	metre	m
	kilometre	km
Mass	milligram	mg
	gram	g
	kilogram	kg
	tonne	t
Area	square centimetre	cm^2
	square metre	m^2
	hectare	ha
	square kilometre	km^2
Volume	cubic centimetre	cm^3
	cubic metre	m^3
Capacity	millilitre	mL
	litre	L
	kilolitre	kL
Temperature	degree Celsius	°C

Investigate: Measuring the height of horses

Horses and ponies are measured in a non-metric unit called the hand.

- Find out what a hand is as a unit of measurement.
- Complete: 1 hand ≈ … cm
- Why do you think that the hand was originally used to measure horses?
- The height of a horse or pony is measured from the ground to the withers – the point where the animal's neck meets its back.
- Find out the height of a typical horse.
- How many hands is a typical pony?
- What is your height in hands?

The basic unit of length is the metre.
The metric system uses the prefixes **milli**, **centi** and **kilo** to combine units of measure.

Prefix	Meaning	Example
milli	one thousandth	1 millimetre $= \frac{1}{1000}$ of 1 metre $= 0.001$ m
centi	one hundredth	1 centimetre $= \frac{1}{100}$ of 1 metre $= 0.01$ m
kilo	one thousand	1 kilometre $= 1000$ metres

Note: 1 mm is about the thickness of a fingernail.
    1 cm is about the width of a fingernail.
    1 m is about the height of a typical door handle.
    1 km is about the distance covered in a 20 minute walk
    around the block.

Important!

Metric units of length
    1 kilometre (km) = 1000 metres (m)
    1 metre (m) = 100 centimetres (cm)
    1 centimetre (cm) = 10 millimetres (mm)

To convert from a *larger* unit to a *smaller* unit, *multiply* by 10, 100 or 1000.
To convert from a *smaller* unit to a *larger* unit, *divide* by 10, 100 or 1000.

Example 1

Convert the following lengths as indicated.
a   2.4 km to m   b   36 m to cm   c   65 mm to cm

Solution

a   We are converting from kilometres to metres. Metres is a
    smaller unit than kilometres, so we multiply.

    Kilo = 1000, so multiply by 1000.   **2.4 km = (2.4 × 1000) m**

                                                       = 2400 m

b   We are converting from metres to centimetres. Centimetres
    is a smaller unit than metres, so we multiply.

    1 m = 100 cm, so multiply by 100.   **36 m = (36 × 100) cm**

                                                       = 3600 cm

c We are converting from millimetres to centimetres.
Centimetres is a larger unit than millimetres, so we divide.

1 cm = 10 mm, so divide by 10.

$$65 \text{ mm} = (65 \div 10) \text{ cm}$$
$$= 6.5 \text{ cm}$$

Sometimes an accurate measure is not important because an estimate is sufficient. For this reason it is often useful to be able to estimate distances.

Investigate: Estimating short lengths

Work in a group of four or five to complete this investigation. Your group will need a ruler or tape measure for this investigation. You can record your results on the data sheet in the web material.

- Mark a point on the ground and take two normal paces. Make a mark where your second pace finishes.
- Measure the distance between the two points, divide it by 2 and round it off to the nearest 10 cm. This is your pace length.
- Each member of the group should do this.
- Record the pace length of each group member for future use.
- Draw a line a metre long on the blackboard or whiteboard.
- Now estimate the width of the classroom in metres.
- Once you have recorded your estimate, record how many paces you need to walk across the width of the room.
- Multiply the number of paces by your pace length and compare it with your estimate.
- Now measure the width of the room and compare this with your original estimate and the distance calculated by pacing.
- Comment on the results.
- Try the same activity with lengths that are smaller and larger than the width of the classroom.

Maps present a type of scale drawing of an area and are a convenient way of representing large distances. Map scales are often expressed as ratios. A scale of 1 cm : 1 km means that 1 cm on the map represents an actual distance of 1 km.

Example 2

Write this scale as a ratio in simplest form.

Metres

0	2	4	6	8	10

Solution

Measure the length of the scale.	**Scale length = 5 cm**
Write the scale as a ratio.	**5 cm : 10 m**
Convert each measure to the same unit.	**= 5 cm : 10 × 100 cm**
Evaluate.	**= 5 cm : 1000 cm**
Simplify by dividing both numbers by 5 and remove the units.	**= 1 : 200**
State the result.	**The scale is 1 : 200.**

You can use a scale to find actual distances by taking a measurement on a map.

Example 3

This map is drawn using a scale of 1 : 10 000. Use the map to calculate the actual distance from the Rotunda in New Farm Park to the Rivercity church on Sydney Street.

Solution

Measure the distance on the map.	**Map distance = 5 cm**
Use the scale.	**Actual distance = 5 × 10 000 cm**
Evaluate.	**= 50 000 cm**

Convert to metres. $= 50\ 000 \div 100$ m

$= 500$ m

State the result. **The Rotunda is 500 m from the Rivercity church.**

Exercise 8.1 The metric system

1 Convert these lengths to centimetres (cm).

a 2 m	**b** 15 m	**c** 0.5 m	**d** 3.5 m
e 0.25 m	**f** 1.25 m	**g** 15.48 m	**h** 12.6 m
i 3 km	**j** 25 km	**k** 12.5 km	**l** 20.25 km
m 420 mm	**n** 65 mm	**o** 7 mm	**p** 125 mm

2 Convert to millimetres (mm).

a 3 cm	**b** 8 m	**c** 40 cm	**d** 25 m
e 11.5 cm	**f** 4.2 m	**g** 3.25 cm	**h** 15.75 m
i 5 km	**j** 12 km	**k** 8.4 km	**l** 32.75 km
m 78 cm	**n** 2 m	**o** 125 cm	**p** 0.7 m

3 Convert to metres (m).

a 200 cm	**b** 3000 mm	**c** 850 cm	**d** 9800 mm
e 1250 cm	**f** 2750 mm	**g** 80 cm	**h** 250 mm
i 75 cm	**j** 325 mm	**k** 6 km	**l** 18 km
m 23.6 km	**n** 48.25 km	**o** 1380 cm	**p** 0.8 km

4 Convert to kilometres (km).

a 2000 m	**b** 1385 m	**c** 140 000 cm	**d** 293 870 cm
e 375 m	**f** 98 m	**g** 23 500 cm	**h** 6480 cm
i 500 000 mm	**j** 2 700 000 mm	**k** 3200 m	**l** 607 000 cm

5 Which length is the longest?

a 8 m or 1200 cm	**b** 2800 cm or 5700 mm
c 10 000 mm or 20 m	**d** 3500 m or 24 km
e 1.5 km or 25 000 cm	**f** 0.65 km or 78 m
g 230 000 mm or 2.3 km	**h** 6 m or 580 cm or 4000 mm
i 250 cm or 1.5 m or 1800 mm	**j** 0.2 km or 120 m or 18 000 cm

6 Place each of these sets of lengths in order, from smallest to largest.

a 200 cm, 3 m, 2500 mm	**b** 3200 mm, 240 cm, 4 m
c 900 cm, 0.8 m, 700 mm	**d** 0.045 km, 450 m, 4800 cm
e 650 mm, 60 cm, 0.69 m	**f** 1.5 km, 150 m, 1500 cm
g 1750 mm, 0.18 m, 180 cm	**h** 120 cm, 1.3 m, 0.011 m

7 Which length is the shortest? Select **A**, **B**, **C** or **D**.

A 0.3 m   **B** 29 cm   **C** 300 mm   **D** 160 mm

8 Copy and complete the following.

a the prefix meaning 1000 is _____

b the prefix meaning $\frac{1}{100}$ is _____

c the prefix meaning $\frac{1}{1000}$ is _____

9 For each map scale given as a diagram in parts **a** to **f** below, write the scale in simplest form.   *See* Example 2
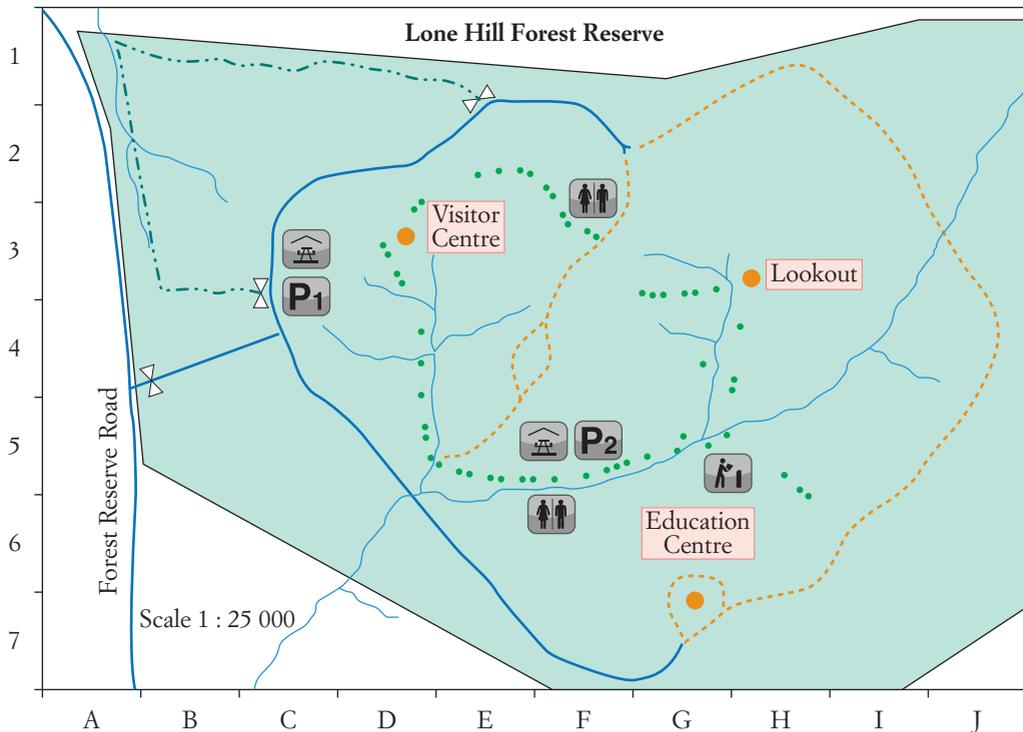
a
```
0    2    4    6    8   10
```
metres

b
```
0         5        10                  20
```
km

c
```
0  5 10   20   30   40
```
km

d
```
0       150    300    450    600
```
metres

e
```
0           500        1000       1500       2000
```
metres

f
```
0    50   100      200      300      400      500
```
metres

10 The map below shows part of a forest reserve. Use it to identify the feature located at:

Problem solving

a H3   **b** B4   **c** D3   **d** F6

Lone Hill Forest Reserve

Forest Reserve Road

Visitor Centre

P₁

Lookout

P₂

Education Centre

Scale 1 : 25 000

A   B   C   D   E   F   G   H   I   J

Forest reserve	Multiple use track	● Place of interest	**P** Parking area
Minor road (sealed)	••• Walking track	Toilets	Information point
Management road	Gate	Sheltered table	

See Example 3

11 Use the map in question **10** to calculate the actual distances between the following pairs of features.
   a the Visitor Centre and the Education Centre
   b the gate near Forest Reserve Road and the Lookout
   c the information point and the sheltered table near parking area 1
   d parking area 1 and parking area 2
   e parking area 1 and the visitor centre
   f parking area 2 and the visitor centre

12 Use the map in Example **3** to calculate:
   a the width of the river at Norris Point
   b the distance between the New Farm neighbourhood centre on Brunswick Street and the Energex substation near Powerhouse Park
   c the distance from the intersection of Sydney and Brunswick Streets to the bank of the river near the end of Brunswick Street
   d the distance from the public telephone (☎) on Sydney Street to the Rotunda in New Farm Park
   e the length of Fuljames Lane
   f the distance from the Jehovah's Witnesses hall on Welsby Street to Norris Point

Worked solutions

Exercise 8.1

MAT07MGWS00032

13 Look carefully at each of these photographs. Then make up a measurement question for each picture. Ask what could be measured and choose the best unit for measuring it.

a

b

c

d

e

f

g

h

i

14 Compare your questions and units from question **13** with those of others in your group.

15 For each of the questions below, write down what is being measured each time and the unit of measurement that could be used.

  a How long is your favourite song?

  b How hot was it in the town in Western Australia that recorded the hottest day?

  c How thick is the ozone layer?

  d How big was the 'Welcome Stranger' gold nugget mined in Ballarat in 1859?

  e When is the next low tide?

  f How long is the highway between Brisbane and Melbourne?

  g How long is your school day?

  h If a plane flew the 7000 km from London to New York in 5.5 hours, how fast was it travelling?

  i How much soft drink will a bottle hold?

  j How big is the school playground?

16 State what each of these newspaper headlines is measuring and suggest a suitable unit.

   a FISHERMAN MAKES RECORD CATCH

   b RECORD-BREAKING WINDS LASH QUEENSLAND COAST

   c HIGH JUMP RECORD BROKEN

   d STATE SIZZLES – FIRE ALERT

   e OLYMPIC STADIUM LARGEST IN WORLD

17 Make up five newspaper headlines similar to those in question **16**. Explain what is being measured and suggest a suitable unit.

18 Karla and Cassie measured these objects in the classroom.

length of pencil	12 cm
width of computer screen	33 cm
width of door	80 cm
width of ruler	3.5 cm
length of room	740 cm

   a Karla wrote her measurements in millimetres. What did she write?

   b Cassie wrote her measurements in metres. What did she write?

   c What unit would you use (cm, mm or m)? Would you use different units for different objects? If so, what would they be?

19 Find other prefixes that are used in the metric system (for example 'micro' and 'mega'). Write their meanings.

Refer to the diagrams below when answering the next question.

face circumference   palm span   hand span

head circumference   arm span

20 a Use a tape measure to measure yourself. Copy and complete this table.

	Body part	Measurement (to nearest cm)
i	Wrist	
ii	Neck	
iii	Waist	
iv	Head circumference (distance around head)	
v	Face circumference (distance around face)	
vi	Height	
vii	Hand span	
viii	Arm span (fingertip to fingertip)	

	Body part	Measurement (to nearest cm)
ix	Palm span	
x	Middle finger (length)	
xi	Foot (length)	

b Compare your neck and wrist measurements. Compare them with those of some classmates. Write a conclusion.

c Compare your arm span and your height. Compare them with those of some classmates. Write a conclusion.

d Write a conclusion about the head and face circumference for the human race.

21 Look at the line segments below. Answer the following questions without measuring the segments.

a Which segment is closest in length to 1 cm?

b Which segment is closest in length to 10 cm?

c Which segment is closest in length to 5 cm?

d Which segment is closest in length to 3 cm?

Reasoning

22 a Estimate the length of segment *GH* in question **21**.

b Measure *GH* and calculate the error in your estimate.

c Compare the accuracy of your estimate with those of others in your class.

23 a Repeat the steps in question **22** for segment *EF*.

b Was your estimate of the length of *EF* more or less accurate than your estimate of the length of *GH*?

24 a Estimate the length of a basketball court.

b In pairs, measure the length of your school basketball court by counting paces. How close were you to your estimate? Explain your results by looking at the length of your paces.

c Use a trundle wheel or tape measure to measure the basketball court. Which unit of length do you think is more appropriate?

25 Refer to the map of the Lone Hill Forest Reserve in question **10**. Estimate the total length of the walking track on the reserve.

26 Select an area with which you are familiar such as your backyard, the school parade area or the school oval. Draw a scaled map of the area.

8.2 Calculating perimeter

Important!

Perimeter

The distance around the edges forming the boundary of a shape is called the **perimeter**. The perimeter of a polygon is the sum of the lengths of the sides of the shape.

Example 4

Find the perimeter of this shape.

35 mm

26 mm

26 mm

80 mm

14 mm

14 mm

61 mm

Solution

Find the sum of the sides.

Perimeter $= 80 + 35 + 26 + 26 + 14 + 14 + 61$

Evaluate.

$= 256$ **mm**

Example 5

Find the perimeter of this shape.

Solution

First we need to find the lengths of the unknown sides a and b.

The sum of the vertical sides must equal a.

$a = 2 + 11 = 13$ **cm**

The sum of b and 5 cm must equal 12 cm.

$b + 5 = 12$

$b = 12 - 5$

$= 7$ **cm**

Now add up all the sides.

Perimeter $= 12 + 13 + 5 + 11 + 7 + 2$

Evaluate.

$= 50$ **cm**

12 cm

2 cm

b

11 cm

a cm

5 cm

Important!

Rules for calculating perimeter

For some shapes it is quicker to use a rule or formula to calculate the perimeter. A rectangle is one such shape.

$$\begin{aligned}\text{Perimeter} &= l + w + l + w \\ &= (l + w) + (l + w) \\ &= 2 \text{ lots of } (l + w) \\ &= 2(l + w)\end{aligned}$$

length = l

width = w

A square is really a special rectangle that has all its sides equal.

$$\begin{aligned}\text{Perimeter} &= 2(l + w) \\ &= 2(w + w) \\ &= 2 \times 2w \\ &= 4w\end{aligned}$$

length = w

width = w

Example 6

Calculate the perimeters of:

a   12 cm

7 m

b   8.2 m

Solution

a Use the rule for the perimeter of a rectangle. $P = 2(l + w)$

Replace l with 12 cm and w with 7 cm. $= 2(12 + 7) \text{ cm}$

Evaluate the brackets. $= 2 \times 19 \text{ cm}$

Evaluate. $= 38 \text{ cm}$

b Use the rule for the perimeter of a square. $P = 4w$

Replace w with 8.2 m. $= 4 \times 8.2 \text{ m}$

Evaluate. $= 32.8 \text{ m}$

Alternative method

Use the rule for the perimeter of a rectangle. $P = 2(l + w)$

Replace l with 8.2 m and w with 8.2 m. $= 2(8.2 + 8.2) \text{ m}$

                                                        $= 2 \times 16.4 \text{ m}$

Evaluate. $= 32.8 \text{ m}$

It is also possible to calculate the perimeter of some curved shapes such as circles.

The distance across a circle through its centre is called the **diameter** (**D**). The **radius** (**r**) is the distance from the centre to the edge of the circle and is half the diameter.

The perimeter of a circle has a special name – **circumference**.

The circumference of a circle can be measured directly in some cases but it can be calculated accurately using a special number called **pi** (**π**).

The value of π cannot be written exactly, so an approximation for π is used when calculating the circumference of a circle.

Investigate: Estimating a value for pi (π)

Work in groups of four or five for this investigation. You will need an object with a circular section such as a dinner plate, a drinking glass, a soccer ball or a tennis ball. You will also need some twine, a ruler and a flexible measuring tape.

- Use the ruler to measure the diameter of the object. Do this more than once and calculate the average diameter (*D*). Record your result.
- Now we will measure the circumference of the object. Wrap some twine (or thread) around the circumference of the object.
- Use a whiteboard marker to mark the twine where it overlaps.
- Lay out the twine and measure the distance between the marks made when the twine was overlapped.
- Have others in the group do the same thing and work out an average for the circumference (*C*). Record your result.
- You can improve the accuracy of your measurements if you follow the same procedure but wrap the twine around the object several times. The circumference is then found by measuring the twine and dividing by the number of times the twine was wrapped around the object.
- Find an estimate for pi by completing:

$$\pi \approx C \div D$$
$$= \ldots \div \ldots$$
$$= \ldots$$

Another way of measuring the circumference is to mark a point on the circumference of the object and roll the object on a flat surface in a straight line until the mark completes one full revolution. The distance travelled by the mark is the circumference.

- Use the method described above to calculate the circumference of your object and calculate an estimate for π using the new value of *C*.
- You can improve the accuracy of your measurements if you follow the same procedure but complete several revolutions of the object.

Around 1900 BCE, the ancient Babylonians and Egyptians estimated the value of pi to be 25/8 and 256/81 respectively.

Much later, Archimedes of Syracuse (287–212 BCE) obtained the approximation $223/71 < \pi < 22/7$.

• Find out more about the history of pi.
• How accurately is the value of pi now known?

You can use the $\boxed{\pi}$ button on your calculator but this is still an approximate value.

Important!

Approximation for π

$\pi \approx 3.14$ is sufficiently accurate for most calculations involving π.

From the previous investigation you saw that $\pi = C \div D$. This can be used to find a rule for calculating the circumference of a circle.

Important!

Rule for circumference of a circle

Circumference $= \pi \times$ Diameter

$$C = \pi D$$

or $\qquad C = 2\pi r \qquad (r = \text{radius})$

Example 7

Find the circumference of a circle with a diameter of 10 cm, giving your answer correct to 1 decimal place.

Solution

Write the rule for a circumference, involving D.

$$C = \pi D$$

Replace D with 10 cm.

$$= \pi \times \textbf{10 cm}$$

Enter as: $\boxed{\pi}$ $\boxed{\times}$ 10 $\boxed{=}$.

π×10	31.41592654

Write the result correct to 1 decimal place.

$$C \approx \textbf{31.4 cm}$$

Exercise 8.2 Calculating perimeter

1 Find the perimeter of each shape.

(a) 15 mm, 8 mm, 8 mm, 15 mm

b 7 cm, 9 cm, 12 cm, 16 cm

c 22 m, 40 m, 17 m, 18 m, 21 m

d 12 cm, 12 cm, 15 cm

e 41 cm

f 300 mm, 700 mm

g 7 km, 2.9 km, 12 km

h 5.2 cm, 7.8 cm

i 53 mm, 75 mm, 40 mm

j 15 mm

k 8 m, 4 m, 12 m

2 Measure the sides of each shape in mm and find the perimeter.

a

b

c

d

Fluency

3 Find the missing lengths and then calculate the perimeter of each figure.

a

3 m

10 m

3 m

12 m

b

8 cm

5 cm

2 cm

2 cm

c

12 mm

5 mm

5 mm

5 mm

7 mm

d

9 m

4 m

3 m

3 m

2 m

2 m

e

4 km

5 km

5 km

2 km

3 km

10 km

f

7 m

2 m

3 m

3 m

2 m

3 m

5 m

2 m

4 Use a rule to find the perimeter of each of the following.

Worked solutions

Exercise 8.2

MAT07MGWS00033

a

5 m

11 m

b

9 mm

c

49 mm

7.8 cm

d

14.8 cm

6.3 cm

e

88 cm

2.4 m

f

3.7 km

5 Calculate the perimeters of the rectangles whose lengths and widths are given below.

   a 12 cm, 5 cm   **b** 3.2 m, 4 m   **c** 15 cm, 15 cm

   d 1.6 m, 80 cm   **e** 17 cm, 22 mm   **f** 2.2 km, 900 m

See Example 7

6 Calculate the circumference of each of the following circles.

   a   **b**   **c**   **d**

7 Calculate the circumference of a circle with:

   a diameter = 30 cm   **b** diameter = 15 m   **c** radius = 4.8 km

   d diameter = 80 cm   **e** radius = 7 mm   **f** diameter = 1.25 m

Problem solving

8 **a** A square has a perimeter of 16 cm. Find the length of one side.

   b A rectangle has a perimeter of 40 cm. If its length is 12 cm, find its width.

   c A square has a perimeter of 56 cm. Find the length of one side.

   d A rectangle has a perimeter of 62 cm. If its width is 13 cm, find its length.

   e A square has a perimeter of 38 cm. Find the length of one side.

   f A rectangle has a perimeter of 45 cm. If its length is 15 cm, find its width.

Worked solutions

Exercise 8.2

MAT07NAWS00033

9 A rectangle has a perimeter of 30 cm. Find a possible length and a possible width.

Reasoning

10 The figure below is made up of a rectangle and an equilateral triangle. Find its perimeter.

11 A small crop farmer wishes to fence a rectangular area measuring 110 m by 65 m for planting lettuce. If fencing costs $38.50 a metre to supply and install, how much will the job cost?

12 Jiang has a pace length of 90 cm. He wants to fence off a rectangular area at the back of his house so his children have somewhere to play. The area he wants to fence is 28 paces long and 15 paces wide. What total length of fencing will Jiang need to complete the job?

13 A bicycle wheel has a circumference of 2.5 m. What is the diameter of the wheel?

8.3   Area and its measurement

A person estimating the number of tiles needed to cover the floor of a room needs to know the size of a tile and the size of the room. In this case, the **area of the shape** gives its 'size'.

Important!

Area of a shape

The **area** of a shape is the amount of surface enclosed by the boundary of the shape.
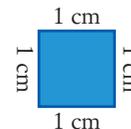The area of the shape on the right has been coloured orange.

Metric units of area

This is 1 **square millimetre** (mm^2): $1 \text{ mm}^2 = 1 \text{ mm} \times 1 \text{ mm}$

This is 1 **square centimetre** (cm^2): $1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm}$

1 square metre has sides measuring 1 m: $1 \text{ m}^2 = 1 \text{ m} \times 1 \text{ m}$

1 cm
1 cm · 1 cm
1 cm

You can see from the diagrams above that mm^2 or cm^2 are useful for measuring small areas.
A square centimetre is about the area of a fingernail. Larger areas such as the floor area of a house or the area of a block of ground are measured in m^2. The square metre or m^2 is approximately the size of the floor of a large shower recess.

Investigate: Estimating area

As a group, mark out a 1 m² area in a suitable place (e.g. in the playground).

- Estimate how many people could stand in the square without any body parts outside the 1 m² area.
- Check your estimate using the people in your class.
- Based on this, how many students could you fit in your classroom?
- What is the greatest population density of your classroom?

Example 8

Find the area of each shape shown on the 1 cm dot paper below.

a

Shape A

b

Shape B

Solution

The area bounded by each set of four dots like this $\vdots\ \vdots$ represents 1 cm².

a Count the number of cm² in the shape.   **Area of Shape A $= 4$ cm²**

b Count the full and part cm² in the shape.   **Area of Shape B $= 3$ cm² $+ \frac{1}{2}$ cm² $+ \frac{1}{2}$ cm²**
$$= 4 \text{ cm}^2$$

To compare areas they need to be in the same unit. For this reason it is sometimes necessary to convert from one area unit to another.

1 cm $= 10$ mm
1 cm² $= (10 \times 10)$ mm²
      $= 100$ mm²
(double the number of zeros)

10 mm

1 cm²   10 mm

1 m $= 100$ cm
1 m² $= (100 \times 100)$ cm²
      $= 10\ 000$ cm²
(double the number of zeros)

100 cm

1 m²   100 cm

1 m = 1000 mm
$1 \text{ m}^2 = (1000 \times 1000) \text{ mm}^2$
$\qquad = 1\,000\,000 \text{ mm}^2$
(double the number of zeros)

1000 mm

1 m^2

1000 mm

Important!

Metric area units

$1 \text{ cm}^2 = 100 \text{ mm}^2$ $\qquad$ $1 \text{ m}^2 = 10\,000 \text{ cm}^2$ $\qquad$ $1 \text{ m}^2 = 1\,000\,000 \text{ mm}^2$

This diagram shows how to convert between different units of area.

$\times 1\,000\,000$

$\times 10\,000$ $\qquad$ $\times 100$

m² $\qquad$ **cm²** $\qquad$ **mm²**

$\div 10\,000$ $\qquad$ $\div 100$

$\div 1\,000\,000$

Example 9

Convert
a 3 cm^2 to mm² $\qquad\qquad$ **b** 4000 mm^2 to m²

Solution

a Converting to a smaller unit, so multiply.

$1 \text{ cm}^2 = 100 \text{ mm}^2$. $\qquad\qquad$ $\mathbf{3 \text{ cm}^2 = (3 \times 100) \text{ mm}^2}$

Evaluate. $\qquad\qquad\qquad\qquad\qquad$ $\mathbf{= 300 \text{ mm}^2}$

b Converting to a larger unit, so divide.

Convert to cm² first. $\qquad\qquad$ $\mathbf{4000 \text{ mm}^2 = (4000 \div 100) \text{ cm}^2}$

$1 \text{ cm}^2 = 100 \text{ mm}^2$

Evaluate. $\qquad\qquad\qquad\qquad\qquad$ $\mathbf{= 40 \text{ cm}^2}$

Now convert to m². $\qquad\qquad\qquad$ $\mathbf{40 \text{ cm}^2 = (40 \div 10\,000) \text{ m}^2}$

$1 \text{ m}^2 = 10\,000 \text{ cm}^2$.

Evaluate. $\qquad\qquad\qquad\qquad\qquad$ $\mathbf{= 0.004 \text{ m}^2}$

Square metres are fine for measuring the area of floors and gardens, but we need a larger unit of measure for areas such as farms and fields. One such unit is the **hectare** (ha). 1 ha has the same area as a square with 100 m sides. It is about the same size as two football fields.

An even larger unit of area is the **square kilometre** (km^2).

1 km^2 has the same area as a square with 1 km sides. It is about the same size as a large theme park, such as Dreamworld on Queensland's Gold Coast.

How many hectares are there in 1 km^2?

$$1 \text{ km}^2 = (1000 \times 1000) \text{ m}^2$$
$$= 1\ 000\ 000 \text{ m}^2$$
$$= (1\ 000\ 000 \div 10\ 000) \text{ ha}$$
$$= 100 \text{ ha}$$

Important!

Metric area units for large areas

$$1 \text{ hectare (ha)} = (100 \times 100) \text{ m}^2 \qquad 1 \text{ km}^2 = (1000 \times 1000) \text{ m}^2$$
$$= 10\ 000 \text{ m}^2 \qquad\qquad\qquad = 1\ 000\ 000 \text{ m}^2 = 100 \text{ ha}$$

This diagram shows how to convert between large units of area.

Example 10

A nature reserve has an area of 9 577 000 000 m^2.

a What is its area in hectares? **b** What is its area in square kilometres?

Solution

a Converting to a larger unit, so divide. **Area of reserve = 9 577 000 000 m²**

1 ha = 10 000 m^2, so divide by 10 000. **= (9 577 000 000 ÷ 10 000) ha**

Evaluate. **= 957 700 ha**

Write the answer. **The area of the reserve is 957 700 hectares.**

b Converting to a larger unit, so divide. **Area of reserve = 9 577 000 000 m²**

1 km^2 = 1 000 000 m^2, so divide by 1 000 000. **= (9 577 000 000 ÷ 1 000 000) km²**

Evaluate. **= 9577 km²**

Write the answer. **The area of the reserve is 957 700 km².**

Investigate: Local areas

1 a Use a map or street directory to find a recreation area (park, garden or reserve) near where you live.
  b Find the scale for measuring distance used on your map or street directory.
  c Estimate the area of the park, garden or reserve.
2 a Rule lines on a sheet of tracing paper to make a grid of squares scaled like your map or street directory. Trace your park, garden or reserve from task **1** onto the paper.
  b Approximate the area (in m^2 and ha) of your park, garden or reserve. Use the method of counting squares. (It may be more accurate to subdivide each square on the grid into smaller shapes.)

Investigate: Population density

You may have heard of the term population density. For humans, this usually means the number of people per unit of area – usually per km^2. Population density is commonly calculated for a country, region, city or some other large area.

The world population is approximately 6.8 billion, and Earth's land area is about 510 000 000 km^2.

- Use this information to calculate the population density of the Earth.

By comparison, Australia has a land area of approximately 7 692 000 km^2 and a population of about 22.2 million people.

- What is Australia's population density?
- Undertake some research to compare Australia's population density with other countries including the USA, England, India, Singapore and Indonesia.
- Comment on your results.

Now let's look at your classroom.

- Calculate the area of the classroom.
- Calculate the population density of your classroom when everyone is present.

Exercise 8.3 Area and its measurement

1 Use square dot paper to draw four different shapes which have the same area as each of these shapes.

a

b

2 Which shape has the greatest area? Select **A**, **B**, **C** or **D**.

Shape 1

Shape 2

Shape 3

A Shape 1   **B** Shape 2   **C** Shape 3   **D** All the same

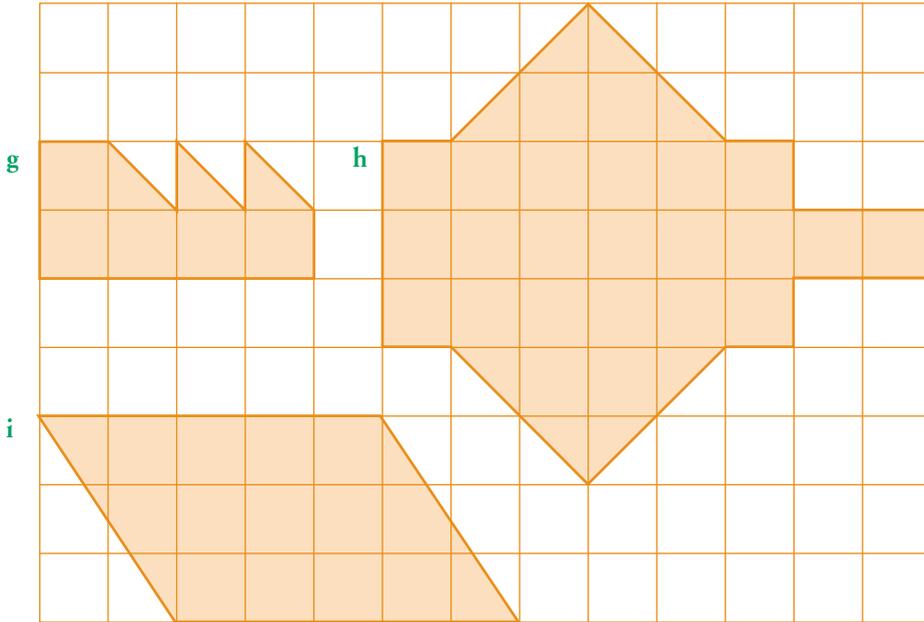**3** Use triangular dot paper to draw three different shapes which have the same area as each of these shapes.

a

b

4 These shapes are drawn on a grid of 1 cm squares. Find the area of each shape.

a

b

c

d

e

f

5 Copy and complete each of the following conversions. *See* Example 9

  a $15 \text{ cm}^2 = $ _____ mm^2 **b** $1\,500\,000 \text{ mm}^2 = $ _____ m^2

  c $690 \text{ mm}^2 = $ _____ cm^2 **d** $6.5 \text{ m}^2 = $ _____ cm^2

  e $0.5 \text{ m}^2 = $ _____ mm^2 **f** $12\,200 \text{ cm}^2 = $ _____ m^2

  g $1250 \text{ mm}^2 = $ _____ cm^2 **h** $7.9 \text{ cm}^2 = $ _____ mm^2

  i $0.75 \text{ m}^2 = $ _____ mm^2 **j** $865\,000 \text{ cm}^2 = $ _____ m^2

  k $690\,000 \text{ mm}^2 = $ _____ m^2 **l** $0.47 \text{ m}^2 = $ _____ cm^2

6 Write each of these areas in hectares. *See* Example 10

  a $2\,450\,000 \text{ m}^2$ **b** $34\,452\,000 \text{ m}^2$ **c** $12\,750\,200\,000 \text{ m}^2$

  d 1500 m^2 **e** 854 m^2 **f** 2000 m^2

7 Write the areas in question **6**, parts **a**, **b** and **c** in km^2.

Problem solving

8 Finding the exact area of a shape that is not made of straight lines is difficult. It is easier to estimate the area by placing a grid over the shape. Count a square only if more than half of it is included in the shape. Find the approximate area of each of these shapes in square units.

Weblink

Pick's rule

  a   **b**   **c**

d   **e**   **f**

g   **h**

9 A square tile has an area of 1024 cm^2. What is this in:
  a mm^2?   **b** m^2?

10 A square garden has an area of 25 m^2. Express this area first in cm^2 and then in mm^2.

11 A desk top has an area of 10 800 mm^2. Express this area first in cm^2 and then in m^2.

12 Measure each of the following shapes and redraw them on 1 cm grid paper. Estimate the area of each first in cm^2, then in mm^2.

a   **b**   **c**

d

13 A large cattle station in Queensland has an area of 30 028 km^2. What is this in hectares?

14 The Cradle Mountain–St Clair National Park in Tasmania has an area of 134 805 ha. What is this in km^2?

15 Sydney Airport has an area of 881 hectares. Sydney Harbour has an area of about 5500 hectares. How many 'Sydney Airports' would fit into Sydney Harbour?

16 The area of Australia is 7 691 951 km^2. Switzerland's area is 41 290 km^2. How many times bigger than Switzerland is Australia?

17 The area of the USA is 9 629 091 km^2. The area of Italy is 301 230 km^2. About how many 'Italys' would make the USA?

18 Western Australia has an area of 2 526 000 km². Approximately what fraction of Australia is Western Australia? (See question **16** for the total area of Australia.)

19 The United Kingdom is made up of about 240 000 km² of land and 3000 km² of water. What fraction of the United Kingdom is water?

20 In Britain, a town with an area of at least 2.5 square kilometres is defined as a centre of business or population. What is the smallest area of a town in hectares?
Select **A**, **B**, **C** or **D**.

A 2500 **B** 0. 025 **C** 250 **D** 2 500 000

8.4 Calculating area

You have seen how areas of shapes can be calculated by counting squares on a grid. It is also possible to calculate areas using special rules.

Investigate: Area rules

1 Copy the following rectangles onto cm grid paper.

2 Use the rectangles you have drawn to complete the following table.

Rectangle	Number of columns (length)	Number of rows (width)	Number of squares (area)
a			
b			
c			
d			
e			
f			
g			

3 Use the information in your table to complete the following.

Area of rectangle $= \cdots \times \cdots$

4 Copy the following rectangles onto some grid paper.

5 Draw in the triangles and shade the areas as shown above.

6 Cut out each rectangle.

7 For each rectangle: cut out the lighter triangles and use them to exactly cover the darker triangle – you may need to flip pieces to make them fit.

8 Complete the following.

Area of triangle $= \frac{1}{2}$ area of

$\qquad = \frac{1}{2} \cdots \times \cdots$

For the triangle below, a rectangle has been drawn to enclose it.

The base (b) of the triangle is the same as the length (l) of the rectangle.

The perpendicular height (h) of the triangle is the same as the width (w) of the rectangle.

9 Complete the following rule for the area of a triangle using b and h.

Area of triangle $= \frac{1}{2} \cdots \times \cdots$

The rule for the area of a parallelogram can be found by cutting and moving one end of the parallelogram to the other to make a rectangle, as shown in the diagram below.

From the diagram, you can see that the area of the parallelogram is the same as that of the rectangle. The rectangle is the same *height* and *length* as the parallelogram. The height is different from the length of the diagonal side.

Investigate: Area of a circle

For this part of the investigation you will need some circles cut from coloured paper. Your teacher may have some circles photocopied on coloured paper. Circles of 20 cm in diameter will work well. Each circle should have the centre marked.

1 Use a protractor to divide the first circle into eight parts, each with an angle at the centre of 45°.

2 Now cut out your parts and stick them onto some white paper in the following arrangement.

The shape looks a bit like a rectangle that has been pushed to one side.

3 Repeat the procedure, but divide the next circle into 15 pieces using angles at the centre of 24°.

4 What effect does this have on your new shape?

5 Repeat the procedure again, using angles of 10° to get 36 pieces.

6 The shape looks more like a rectangle as the number of pieces increases. Imagine that the circle was divided into 5000 pieces and the rectangle below is formed using these pieces.

Width

Length

7 The length of the rectangular shape must be half the circumference of the circle. The circumference of a circle is $2\pi r$. Complete the following for a rectangle formed from a circle of radius r:

$$\text{Length} = \frac{1}{2} \times \cdots = \cdots$$
$$\text{Width} = \cdots$$
$$\text{Area of rectangle} = \text{length} \times \text{width}$$
$$= \cdots \times \cdots$$
$$= \cdots$$

8 What can you say about the area of the rectangle and the area of the circle?

Important!

Area rules

Rectangle:

w

l

$$A = l \times w$$

Triangle:

h

b

$$A = \frac{1}{2} b \times h$$

Circle:

r

$$A = \pi r^2$$
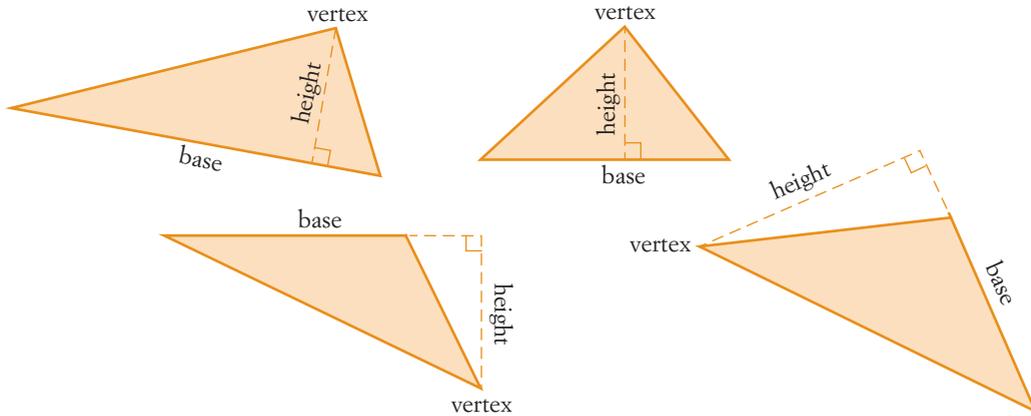
Parallelogram:

h

l

$$A = l \times h$$

When you use these rules, you should note the following points.

A square is a special rectangle. To find the area of a square, you should use the rule for the area of a rectangle, where $l = w$.

When working with a triangle we use special names for its dimensions.

Any side of a triangle can be called the **base**. The **height** is the distance from the base to the vertex opposite the base. This distance is measured at right angles to the base, so it is also called the **perpendicular height**.

The height of a parallelogram is always taken as the width perpendicular to the side we have called the length.

Example 11

Find the area of this triangle.

Solution

Write the rule for the area of a triangle.	$A = \frac{1}{2} b \times h$
Substitute the values for b and h.	$= \frac{1}{2} \times 8 \text{ m} \times 6 \text{ m}$
Evaluate.	$= 24 \text{ m}^2$

Note: The length of 7 m was not required to find this triangle's area.

Example 12

Use a rule to find the area of each of the following.

a

15 mm

24 mm

b

8 cm

12 cm

c

10 cm

9 cm

14 cm

d

20 m

Solution

a Write the rule for the area of a rectangle.　$A = l \times w$

Substitute the values for l and w.　$= 24 \text{ mm} \times 15 \text{ mm}$

Evaluate.　$= 360 \text{ mm}^2$

b Write the rule for the area of a triangle.　$A = \frac{1}{2} b \times h$

Substitute the values for b and h.　$= \frac{1}{2} \times 12 \text{ cm} \times 8 \text{ cm}$

Evaluate.　$= 48 \text{ cm}^2$

c Write the rule for the area of a parallelogram.　$A = l \times h$

Write the length and height.　$l = 14 \text{ cm}, h = 9 \text{ cm}$

Substitute the values for l and h.　$A = 14 \text{ cm} \times 9 \text{ cm}$

Evaluate.　$= 126 \text{ cm}^2$

d Write the rule for the area of a circle.　$A = \pi r^2$

Substitute the value for r.　$= \pi \times 20 \text{ m} \times 20 \text{ m}$

Enter as: $\boxed{\pi}$ $\boxed{\times}$ 20 $\boxed{x^2}$ $\boxed{=}$.　$\boxed{\pi \times 20^2 \qquad 1256.637061}$

Evaluate.　$\approx 1257 \text{ m}^2$

Investigate: Paper sizes

There is an international standard for paper size. Under this standard the base format is a sheet of paper measuring 1189 mm by 841 mm. This is known as A0 paper size.
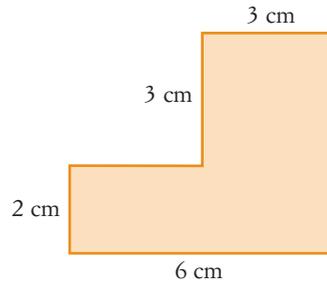

- Calculate the area of an A0 sheet of paper in m^2.
- Calculate the ratio of the sides of a sheet of A0 paper. This is called the aspect ratio.
- Successive paper sizes in the series are A1, A2, A3, and so on as shown in the diagram above (dimensions in mm).
- What are the dimensions of an A1 sheet?
- Calculate the aspect ratio for an A1 sheet.
- How does that compare with the aspect ratio for an A0 sheet?
- Calculate the ratio of corresponding sides of an A0 and A1 sheet. What do you notice?
- What is the area (m^2) of an A1 sheet?
- How does this compare with the area of an A0 sheet?
- Compare the areas of an A1 and A2 sheet.
- What are the advantages of having standard sizes for paper?

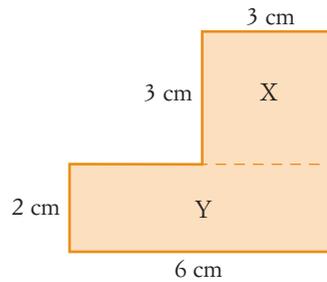A **composite shape** is made up of two or more shapes.

Example 13

Find the area of this shape.

3 cm

3 cm

2 cm

6 cm

Solution

Method 1

Separate the shape into a square (X) and a rectangle (Y).

3 cm

3 cm   X

2 cm   Y

6 cm

The shape is made up of X and Y.

Use the area rule $A = l \times w$.

Evaluate.

Area of shape = area of Y + area of X

$$= 6 \times 2 + 3 \times 3$$
$$= 12 + 9$$
$$= 21 \text{ cm}^2$$

Method 2

Another way to divide the shape into rectangles is like this:

3 cm

3 cm

3 cm   Q   5 cm

2 cm   P

6 cm

The shape is made up of P and Q.

Use the area rule $A = l \times w$.

Evaluate.

Area of shape = area of P + area of Q

$$= 3 \times 2 + 5 \times 3$$
$$= 6 + 15$$
$$= 21 \text{ cm}^2$$

Method 3

The area can also be found by subtracting areas.

Here we have a large rectangle, S, which is 6 cm by 5 cm and a square, R, which has a side of 3 cm.

3 cm   3 cm

3 cm   R

5 cm

2 cm   S

6 cm

The area of the shape is the difference between the areas of S and R.

Use the area rule $A = l \times w$.
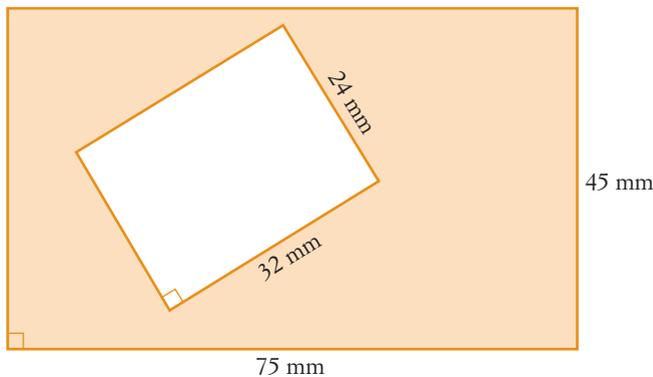
Evaluate.

Area of shape = area of S – area of R
$$= 6 \times 5 - 3 \times 3$$
$$= 30 - 9$$
$$= 21 \text{ cm}^2$$

Example 14

Find the shaded area.

Solution

The shaded area is the difference between the areas of the large rectangle and the small rectangle.

Use the area of a rectangle rule.

Evaluate.

Shaded area = A

$$A = 75 \times 45 - 32 \times 24$$
$$= 3375 - 768$$
$$= 2607 \text{ mm}^2$$
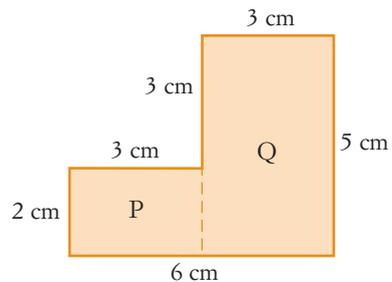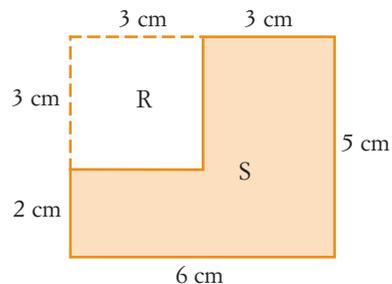
Investigate: Area puzzle

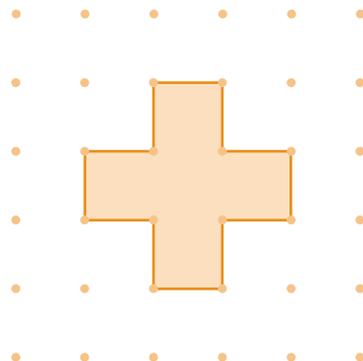The shape below has a perimeter of 12 units and an area of 5 square units.

- Use grid paper to draw a shape with a perimeter of 12 units and an area of 4 square units.
- Can you draw a shape with a perimeter of 12 units and an area of 3 square units?
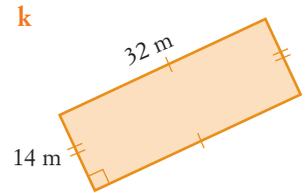
Exercise 8.4 Calculating area

1 Find the area of each of these rectangles and squares.
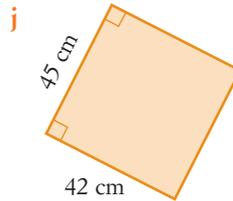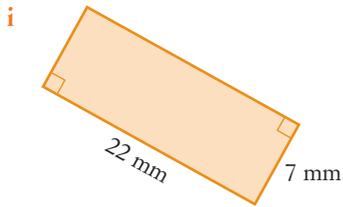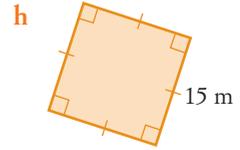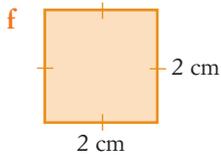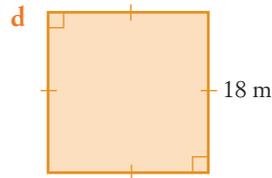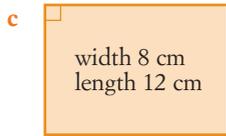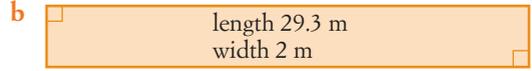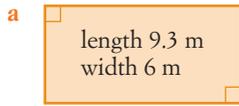
a
length 9.3 m
width 6 m

b
length 29.3 m
width 2 m

c
width 8 cm
length 12 cm

d
18 m

e
length 16 cm
width 0.09 cm

f
2 cm
2 cm

g
8 mm
12 mm

h
15 m

i
22 mm
7 mm

j
45 cm
42 cm

k
32 m
14 m

2 Find the area of each of these triangles.
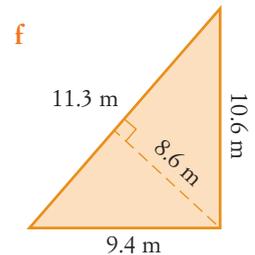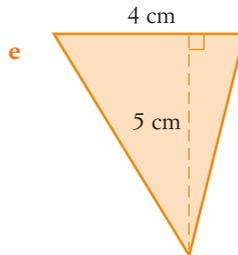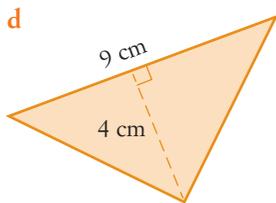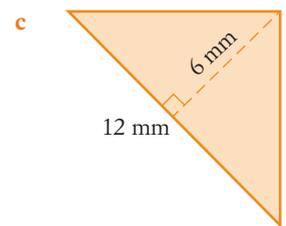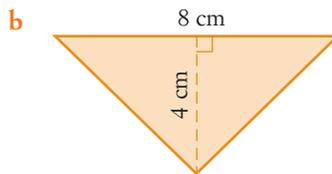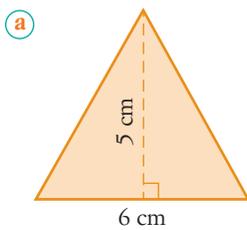
a
5 cm
6 cm

b
8 cm
4 cm

c
6 mm
12 mm

d
9 cm
4 cm

e
4 cm
5 cm

f
11.3 m
8.6 m
10.6 m
9.4 m

g

4.9 m
4.2 m
4.5 m
4.2 m

h

5.3 mm
3.4 mm

i

4.8 cm
7.5 cm
3.2 cm

j

9.1 cm
7.2 cm
10.8 cm
12.7 cm

k

6.8 m
8.9 m
12.8 m
6.4 m

l

50 mm
150 mm
120 mm
70 mm
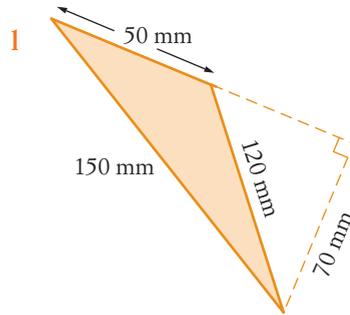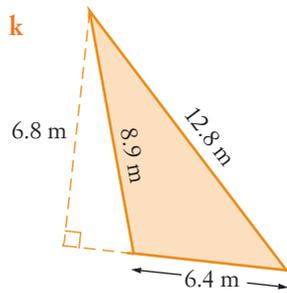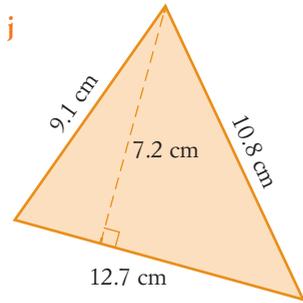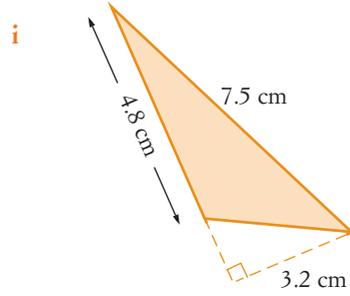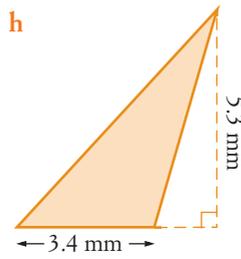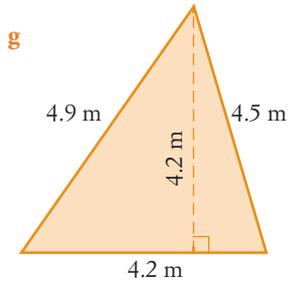
3 Find the area of each of the following parallelograms.

See Example 12

a

5.9 m
5.2 m

b

9 cm
14 cm
7.5 cm

c

3.2 cm
30 mm
28 mm

d

11 m
12 m
10 m

e

4.1 cm
8.4 cm
11.5 cm
7.9 cm

f

2.4 m
48 cm
43 cm
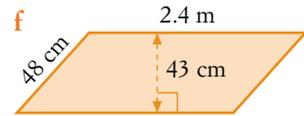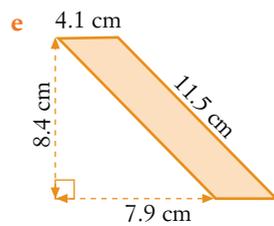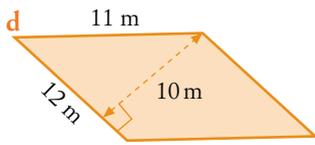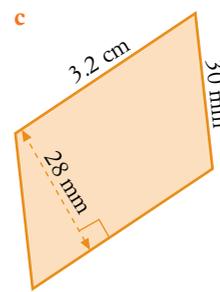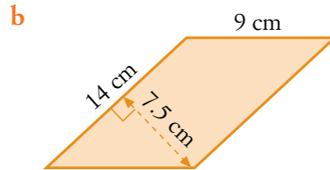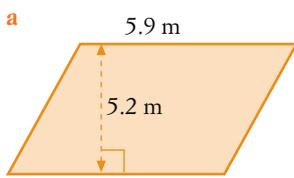
4 Find the area of each of these circles.

a

7 cm

(Correct to
1 decimal place)

b

9.3 m

c

11.7 cm
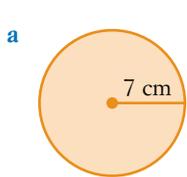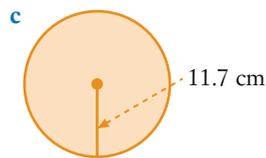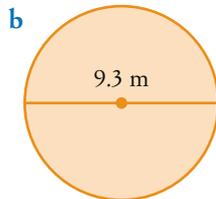
5 Find the area of each of these figures. Be careful with mixed units. Answer in the units shown in the figure.

a

35 cm · 50 mm · (mm²)

b

780 cm · 4 m · (m²)

c

(mm²) · 1.2 cm · 50 mm

d

60 mm · 8 cm · (cm²)

e

3 m · 5200 mm · (m²)

f

80 mm · (cm²)

(Correct to nearest cm²)
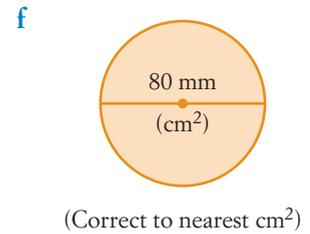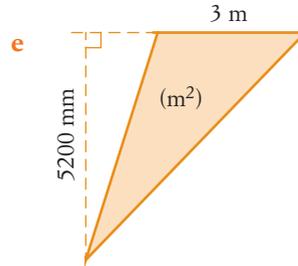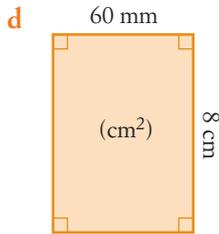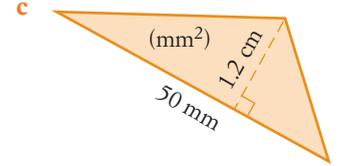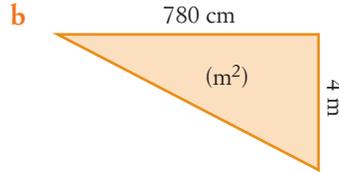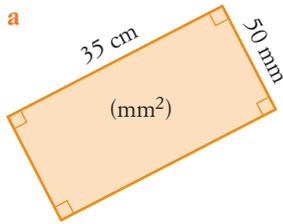
6 What is the area of this triangle? Select from **A**, **B**, **C** or **D**.

   A 360 cm²   **B** 432 cm²

   C 270 cm²   **D** 216 cm²

18 mm · 24 mm · 30 mm
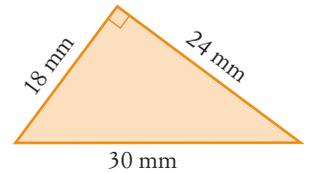
7 A window is 2 metres long and 90 centimetres wide. Find, in square metres, the area of glass in the window. Select **A**, **B**, **C** or **D**.

   A 180 cm²   **B** 1800 cm²

   C 1.8 m²   **D** 18 m²

90 cm · 2 m
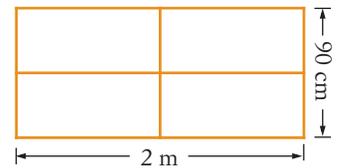
See Example 13

8 Find the area of each of these shapes. All measurements are in centimetres.

a

16 · 8 · 14 · 9

b

22 · 6 · 8 · 6 · 30 · 8

c

8

2 2

6

9

4

d

5

1

1

1

1

3

1
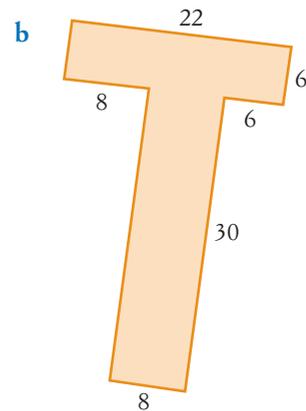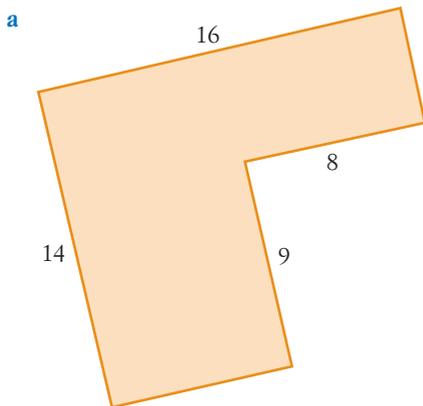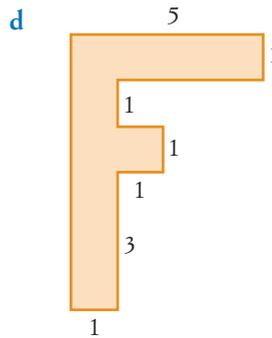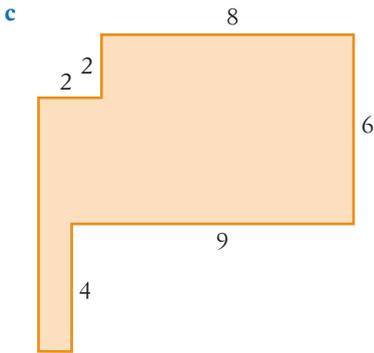
9 Find the area of each of these orange shapes.

a Outside dimensions: 230 cm × 55 cm. Hole dimensions: 40 cm × 35 cm

See Example 14

b Total height: 30 cm

Total width: 30 cm

Dimensions of cut-out: 20 cm × 20 cm

height

width
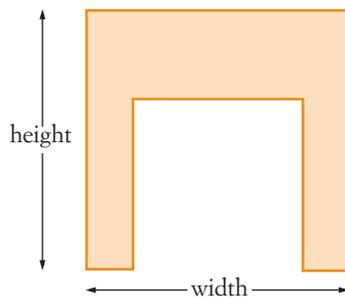
c This shape is formed from strips of metal 10 cm wide. The height of the long leg is 80 cm. The base is 50 cm wide. The height of the short leg is 20 cm.

height of long leg

width of strip

height of short leg

base
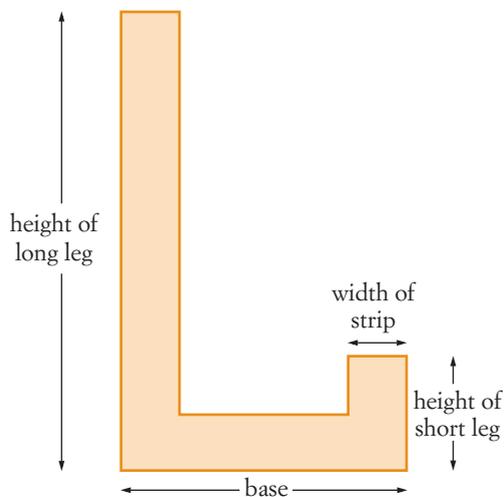
10 Find the area of each of these shapes in:

i square units (m^2 or km^2)   **ii** hectares

a

2 km   6 km

b

200 m

c

8 km

d

120 m   60 m

e

40 m   240 m

f

4 km

---
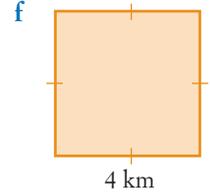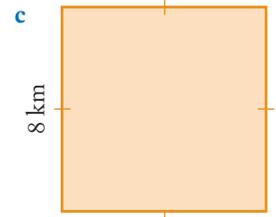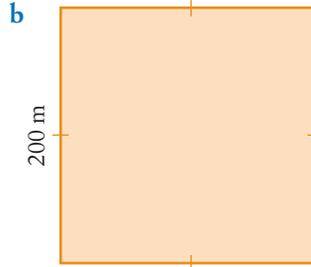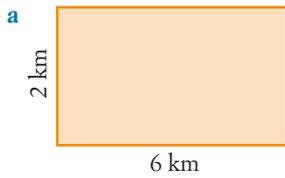
11 Find the area of turf needed to grass the front and back yards of a house. The front yard measures 20 m by 8 m and the back yard measures 35 m by 7.5 m.

12 A 4 m × 5 m room has a ceiling height of 3.2 m.
   a Ignoring windows and doors, find:
     i the area of the floor of the room
     ii the area of the ceiling
     iii the area of one of the smaller walls.

3.2 m   5 m   4 m

   b How many square carpet tiles measuring 50 cm on each side are needed to cover the floor of the room?
   c How many litres are needed to paint the walls of the room, if 1 L covers 16 m^2?
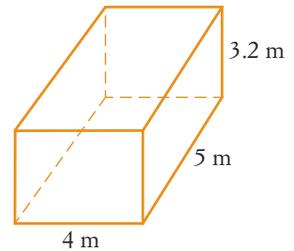
13 A rectangular lawn contains two square flower gardens, each measuring 3 m by 3 m. If the lawn measures 9.5 m by 6.4 m, find the area of the grass.

14 A garden measuring 5 m by 6 m has a 2 m wide strip of paving around its border. What is the area of the paving?

15 The diagram on the right shows a swimming pool in a holiday resort.
All angles are right angles. What is the area of the surface of the pool? Select **A**, **B**, **C** or **D**.
   A 156 m^2 **B** 180 m^2
   C 72 m^2 **D** 260 m^2

8 m   3 m   3 m   6 m   6 m   22 m

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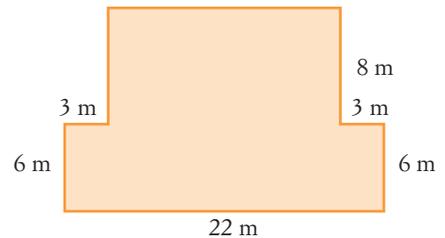
16 The area of a rectangle is 1 ha. Give its width in metres if the length is:
   a 1000 m   **b** 500 m   **c** 400 m
   d 2 km   **e** 1.25 km   **f** 5 km

17 The area of a rectangle is 1 km^2. Give its length in metres if the width is:
   a 100 m   **b** 160 m   **c** 250 m
   d 0.5 km   **e** 0.08 km   **f** 2 km

- The metric system of units has been used in Australia since the early 1970s. Some common metric units are shown in the following table.

Quantity	Metric unit	Larger metric unit	Smaller metric unit
Length	metre (m)	kilometre (km)	centimetre (cm)
Mass	gram (g)	kilogram (kg)	milligram (mg)
Capacity	litre (L)	kilolitre (kL)	millilitre (mL)
Area	square metre (m^2)	square kilometre (km^2)	square millimetre (mm^2)

- The metric system uses degrees Celsius (°C) to measure temperature.
- The metric units of length are:
    - 1 kilometre (km) = 1000 metres (m)
    - 1 metre (m) = 100 centimetres (cm)
    - 1 centimetre (cm) = 10 millimetres
- To convert from a *larger* to a *smaller* unit, *multiply* by the amount. To convert from a *smaller* to a *larger* unit, *divide* by the amount.
- The distance around the edges forming the boundary of a shape is called the **perimeter**. The perimeter of a polygon is the sum of the lengths of the sides of the shape.
- You can use rules to find the perimeters of rectangles and squares.
    - For a rectangle: $P = 2(l + w)$
    - For a square: $P = 4w$
- The perimeter of a circle is called its **circumference** (C).
  The circumference of a circle is about 3 times its diameter.
  The rule for the circumference of a circle is:
    - $C = \pi D$ or $C = 2\pi r$ (where $\pi \approx 3.14$)
- The **area** of a shape is the amount of surface enclosed by the boundary of the shape.
- The metric units of area are:
    - $1 cm^2 = 100 mm^2$
    - $1 m^2 = 10\ 000 cm^2 = 1\ 000\ 000 mm^2$
    - 1 hectare (ha) = $(100 \times 100) m^2 = 10\ 000 m^2$
    - $1 km^2 = (1000 \times 1000) m^2 = 1\ 000\ 000 m^2 = 100$ ha
- The rules of calculating the areas of common plane figures are:
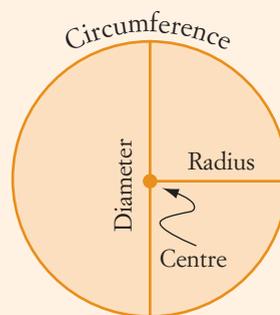
Rectangle
$A = l \times w$
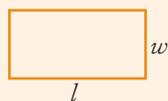
Triangle
$A = \frac{1}{2} b \times h$
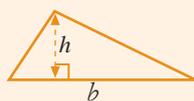
Parallelogram
$A = l \times h$
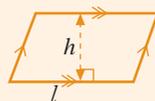
Circle
$A = \pi r^2$

- A **composite shape** is made up of other shapes. The area of a composite shape can be found by dividing the shape into other shapes whose areas may be found.

Understanding

1 Which unit of length (mm, cm, m or km) would be most appropriate to measure:
  a the width of your thumb?
  b the distance from your house to your school?
  c the length of your arm?
  d the length of the walls in your classroom?

2 Give an example of an item that is:
  a 1 cm long   **b** 1 m long   **c** 1 mm wide

See Example 1

3 How many:
  a centimetres in 3 m?   **b** millimetres in 8 m?
  c metres in 3000 km?   **d** kilometres in 6500 m?
  e centimetres in 750 mm?   **f** millimetres in 2.5 cm?
  g metres in 7800 mm?   **h** metres in 520 cm?

4 Which of the following is closest to the length of a house brick? Select **A**, **B**, **C** or **D**.
  A 23 mm   **B** 230 mm   **C** 230 cm   **D** 23 m

See Example 4

5 Measure the side lengths of each shape and find the perimeter in mm and then in cm.

  a   **b**   **c**
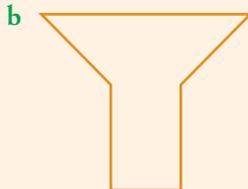
Fluency

6 Find the perimeter of each of these shapes.

  a

    13 m

  b

    5 mm

    13 mm

  c

    25 mm

  d

    16 mm

e

17 cm
4 cm
5 cm
6 cm
10 cm
22 cm

f

36 mm
12 mm
20 mm
20 mm
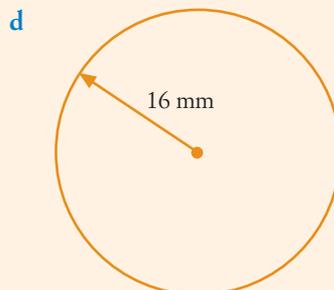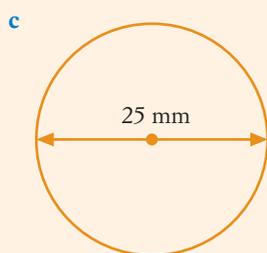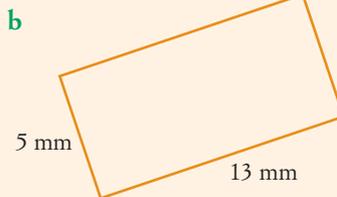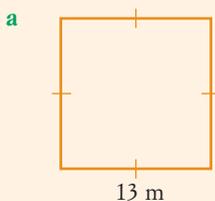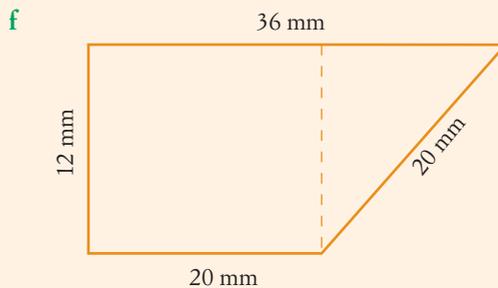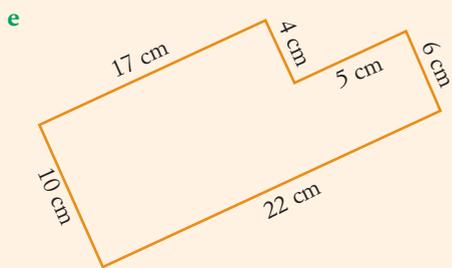
7 First find the missing lengths, then find the perimeter of each shape. (All lengths are in metres.)   *See* Example 5

a

6
2.5
9
5

b

14
15
12
14
6

c

12
5
3   3
1.4   1.2

d

2
2
2
2
2
11
2

e

8   8
12
15
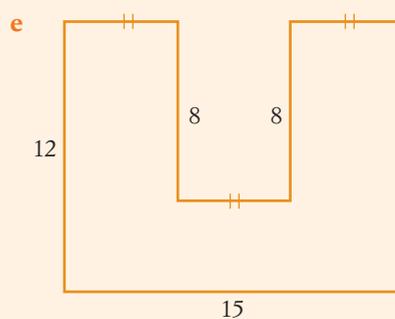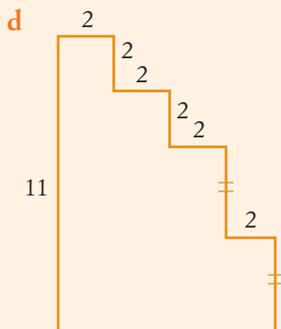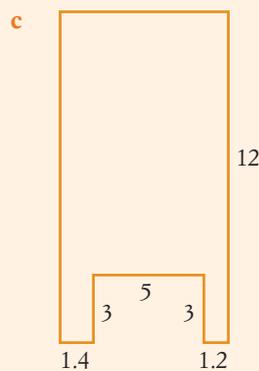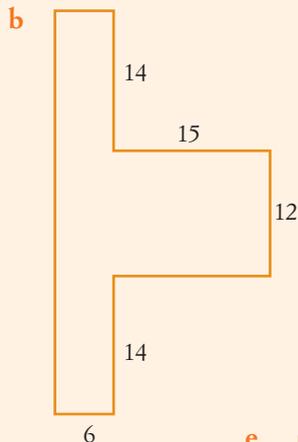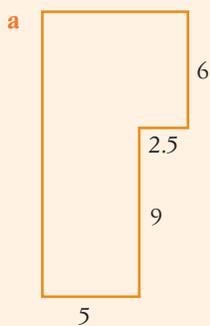
8 Find the area of each shape if the grid is made up of 1 cm squares.   *See* Example 8

a   **b**   **c**
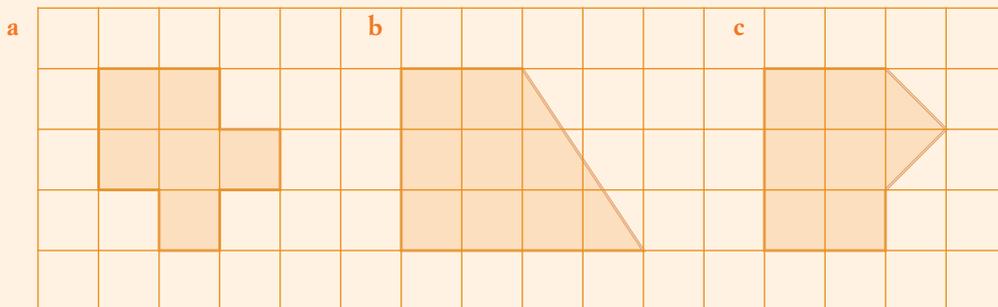
See Example 9

9 How many:

a cm^2 in 9.1 m^2?
b mm^2 in 2.5 cm^2?
c m^2 in 4 ha?
d m^2 in 1 km^2?
e m^2 in 38 000 cm^2?
f mm^2 in 8.6 m^2?
g cm^2 in 240 mm^2?
h m^2 in 175 000 cm^2?

See Example 12

10 Find the area of each of the following.

a square with sides 4 m, 4 m

b rectangle 6 cm by 4 cm

c triangle, base 30 mm, height 24 mm

d right triangle with 14 mm and 16 mm sides

e triangle base 4 cm, height 2.6 cm

f triangle base 34 km, height 11 km

g parallelogram base 18 m, height 9 m

h circle diameter 15 km

i circle radius 2.2 m
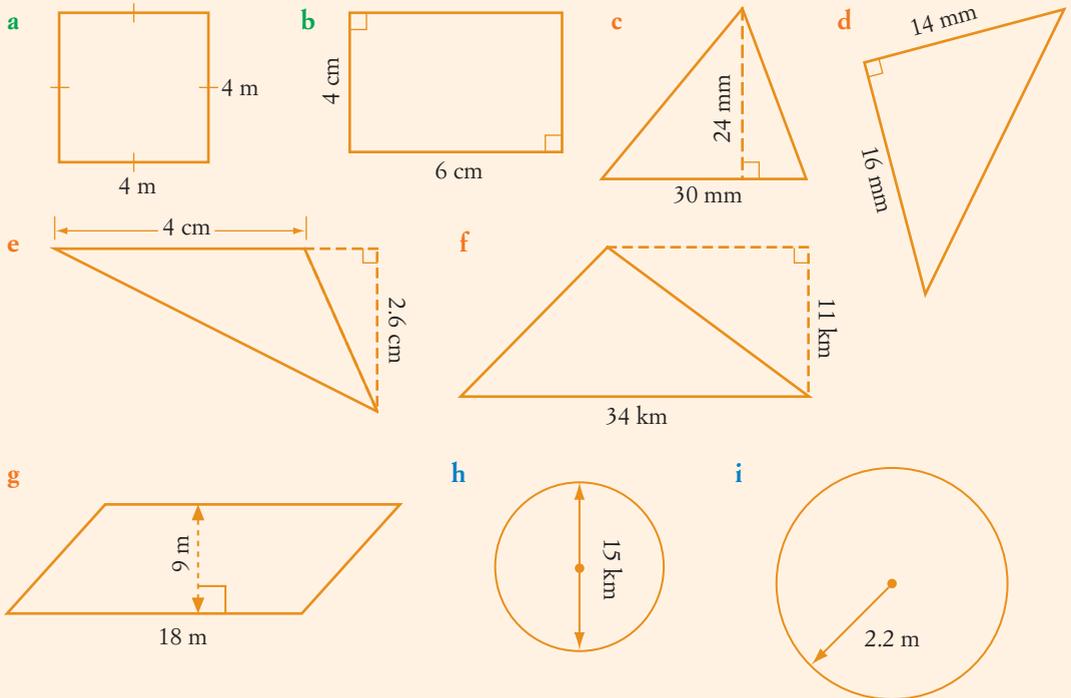
See Examples 13, 14

11 Find the area of each of the following.

a L-shape: 6 cm, 1 cm, 4 cm, 1 cm

b T-shape: 8 m, 3 m, 3 m, 3 m, 12 m
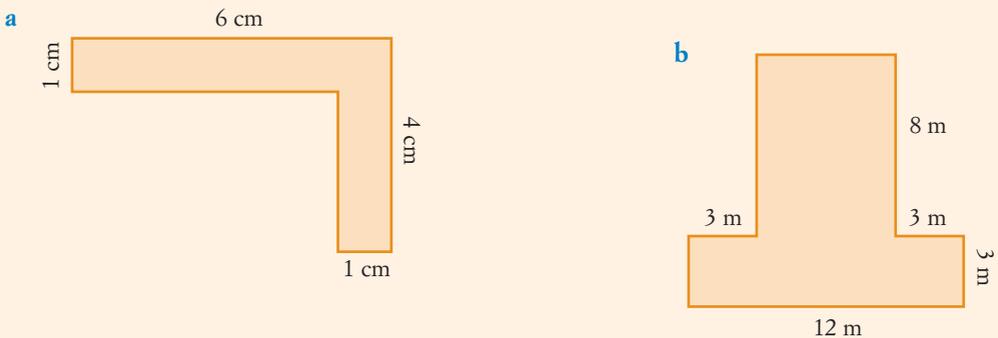
12 A large circular tray has a circumference of 88 cm.
What is the diameter of the tray?

13 Find the approximate area of this shape in square units.
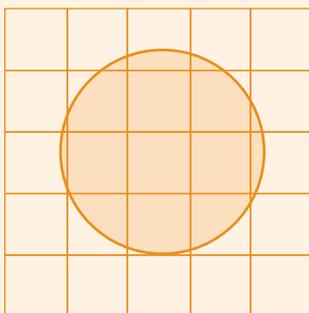
14 **a** Water covers about 391 000 ha of Zimbabwe. How many square kilometres is this?
   b Tasmania has an area of 68 000 km². The Australian mainland has an area of 7 682 300 km². How many Tasmanias would fit into mainland Australia?
   c What is the area of Tasmania in hectares?

15 The wall shown on the right needs painting. If one litre of paint covers 3 m² of wall, how many whole litres should be bought?
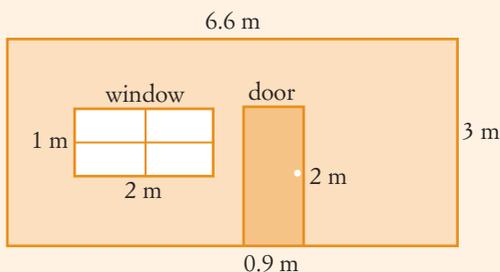
Reasoning

16 A rectangle has a perimeter of 30 cm.
   a List some possible dimensions for this rectangle.
   b For each pair of dimensions, calculate the area of the rectangle.
   c Which dimensions give the greatest area?

17 A right-angled triangle has an area of 18 cm².
   a List some possible dimensions for this triangle.
   b By drawing each possible triangle and measuring the third side, find the triangle with the shortest perimeter.

9 Probability

Contents

Prior learning
Chapter 9

Parent guide
Chapter 9

Curriculum guide
Chapter 9

Australian Curriculum statements

Chance

Construct sample spaces for single-step experiments with equally likely outcomes. (ACMSP167)

Assign probabilities to the outcomes of events and determine probabilities for events. (ACMSP168) AC

Probability is the mathematical word that describes chances. Mathematicians have made theories about the chances of things happening. They have worked out ways to measure the chances of particular things occurring. Casinos and insurance companies use probability to work out how to make a profit. Doctors use probability to work out what they should do for their patients. There are many uses for probability.

Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

certain	favourable outcome	probability	sample space
complementary	frequency	probable	simple outcome
element	impossible	random	theoretical probability
even chance	improbable	relative	trial
event	likely	frequency	unlikely
experimental probability	outcome	sample point	

9.1 Describing probability

Investigate: Dice game

Work in pairs or small groups for this investigation.

• Divide a sheet of paper up into different parts, choosing one of the following patterns. Your teacher may already have made some outlines for you.

A   B   C   D   E
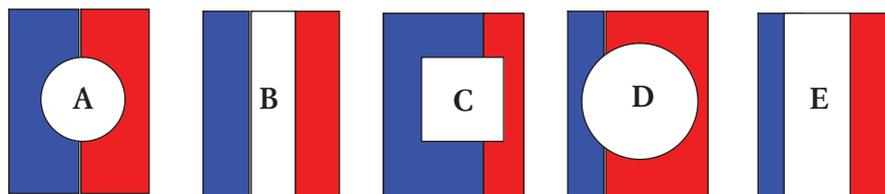
• You need one die. What is the plural?
• You will also need a recording sheet or a page of your book divided in half with headings.
• Write A, B, C, D or E at the top of your recording sheet to show which pattern you are using.
• Divide the top half of the recording sheet into three parts. You can use 'Left', 'Right' and 'Middle' or 'Red', 'Blue' and 'White' for your headings on this half.
• Divide the bottom half of your sheet into 6 parts, labelled 1, 2, 3, 4, 5, 6. Now do the experiment.
• Roll the die so it lands on the sheet. If it rolls off the sheet, do it again.

- Put a mark or a tick on the top half of your recording sheet to show where it lands.
- Put a mark or a tick on the bottom half to show what number it lands on.
- Keep doing this until you have done it as many times as your teacher says.
- Your recording sheet should end up looking something like the one on the right.
- Count how many marks you have in each column for Blue, Red and White.
  – Compare with other groups who had the same outline as you.
  – Can you explain the results?
  – What about the results for the numbers?
  – What happens to the results if you add all the results for the dice from the whole class?

C		
Blue	Red	White
x x x x x x x x x x x x x x x	x x x x x	x x x x x x x x x x

1	2	3	4	5	6
x x x x	x x	x x x x x x	x x x x	x x x x x x x x	x x x x

In the investigation above, there were different things that could happen. When you look at where the die lands, it could be on blue, red or white. These are called the **outcomes** for where the die could land. When you look at the number, the outcomes are 1, 2, 3, 4, 5, or 6.

From results for the whole class, it is obvious that some outcomes *usually* occur more often than others. We say that these outcomes are **more likely** than others.

Probability is the part of maths about situations like this. We can say what will *usually* happen, but not what will happen.

Important!

Probability terms

The **outcomes** of a probability situation are the things that *could* happen. They are also called **events**.

More likely outcomes *usually happen* more often than **less likely** outcomes.

Equally likely outcomes usually happen as often as each other.

An outcome with an **even chance** usually happens about half the time. It is also called a **fifty-fifty** or **equal chance**.

An **unlikely** outcome usually happens less than half the time. It is also called **improbable**.

A **likely** outcome usually happens more than half the time. It is also called **probable**.

An **impossible** outcome *never happens*.

A **certain** outcome always happens. It is **sure** to happen.

Example 1

A bag contains 6 beads. Three are glass and 3 are plastic. Two of the glass beads are red and so are 2 of the plastic beads. The others are blue. Without looking inside, one is taken out. Describe the chance that it is:

a Red   **b** Glass   **c** A bead

d Yellow   **e** Blue

Solution

a More than half the beads are red.   **It is likely to be red.**

b Half the beads are glass.   **There is an even chance it is glass.**

c Everything in the bag is a bead.   **It is certain to be a bead.**

d There is nothing yellow.   **It is impossible to be yellow.**

e Less than half the beads are blue.   **It is unlikely to be blue.**

In Example **1**, there are six **simple outcomes** because there are exactly six different objects that can be picked. It is often useful to list the simple outcomes. This makes it easier to work out chances. A list of all the simple outcomes is called the **event space** (or **sample space**). The sample space for Example **1** has 6 items. We usually write the sample space in curly brackets like {Red, Red, Red, Red, Blue, Blue}.

Example 2

Give examples of events that are impossible, improbable, fifty-fifty, probable and certain.

Solution

It is impossible that the winner of the Melbourne cup in 2015 will be a zebra.
It is unlikely that the temperature will be above 40°C in Hobart today.
There is an equal chance that the next child born is Sydney will be a girl.
It is likely that Sydney will be the biggest city in Australia in 2030.
Tossing a normal die is certain to give a number from 1 to 6.

Example 3

List the sample space for the kind of bead in Example **1**.

Solution

Write G for glass and P for plastic. **Sample space = {G, G, G, P, P, P}**

You choose a pen to write with in your book. You normally look at your pens and pick a blue or black pen for ordinary writing. Imagine picking one without looking, so that each pen in your case had an equal chance. This is called choosing **at random**. It means that each simple outcome has the same chance of happening.

Example 4

A card is taken at random from a normal pack. Which is more likely, an Ace or a King?

Solution

There are the same number of Aces as Kings in a normal pack of cards. **Aces and Kings are equally likely.**

Example 5

List the sample space for Example **1** if you are interested in both the colour and the type of bead.

Solution

Write RG for a red glass bead, BP for a blue plastic bead, etc. **Sample space = {RG, RG, BG, BP, RP, RP}**

Investigate: Rolling two dice

You can roll a red die and a blue die at the same time. Some of the outcomes of rolling the dice are shown in the following partial table.

- How many ways can you get a total of 3 from rolling the dice?
- How many ways can you get a total of 4 from rolling the dice?

Copy and complete the table.

- How many ways can you get a total of 9 from rolling the dice?
- What is the most likely total?

Blue die		Red die		
		1	2	3
	1	1, **1**	1, **2**	1, **3**
	2	2, **1**	2, **2**	2, **3**
	3	3, **1**	3, **2**	3, **3**

Investigate: Coin game

This is a game for two people. The 'board' is shown on the right. Draw it up on A4 paper, unless your teacher has printed copies ready.

The idea is to flick a coin so that it ends up in a shaded square.
You must start from behind the line and give it one flick with your finger or a ruler.
You are not allowed to slowly push it forward.
It has to be a clean hit.
You will need some coins to flick.
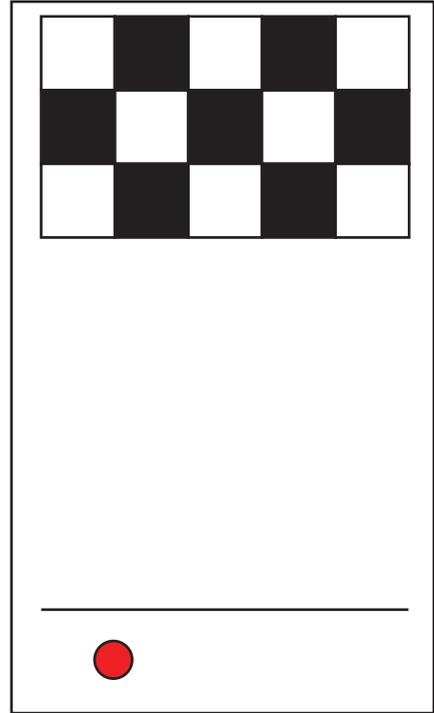
The coin has to be completely inside the shaded square to count as a win.

✓   ✗   ✗   ✗
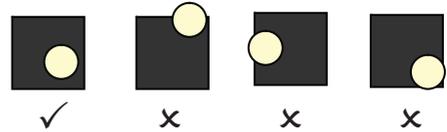
Before you start, write down what you think about the likelihood of winning.
- Is it unlikely, fifty-fifty, or likely?
- Can you estimate a probability from 0 to 1?

Now play the game. Each player gets 10 goes. Write down how many wins you get from 10 tries.
- You should both have a go with a 5-cent coin first.
- Then try a 10-cent coin.
- Do a 20-cent coin last.

Were your predictions right? Why/Why not?
Which coin is easiest to win with? Why?
Your teacher might want the whole class to pool the results together.

Exercise 9.1 — Describing probability

1 The following words and phrases are used to describe the chances of something happening. Arrange them in order from the lowest chance to the highest chance. Some of the words can show events that could be equally likely.

certain	fifty-fifty	very unlikely
probable	likely	highly probable
sure	even	improbable
impossible	almost certain	highly improbable
very likely	almost impossible	unlikely
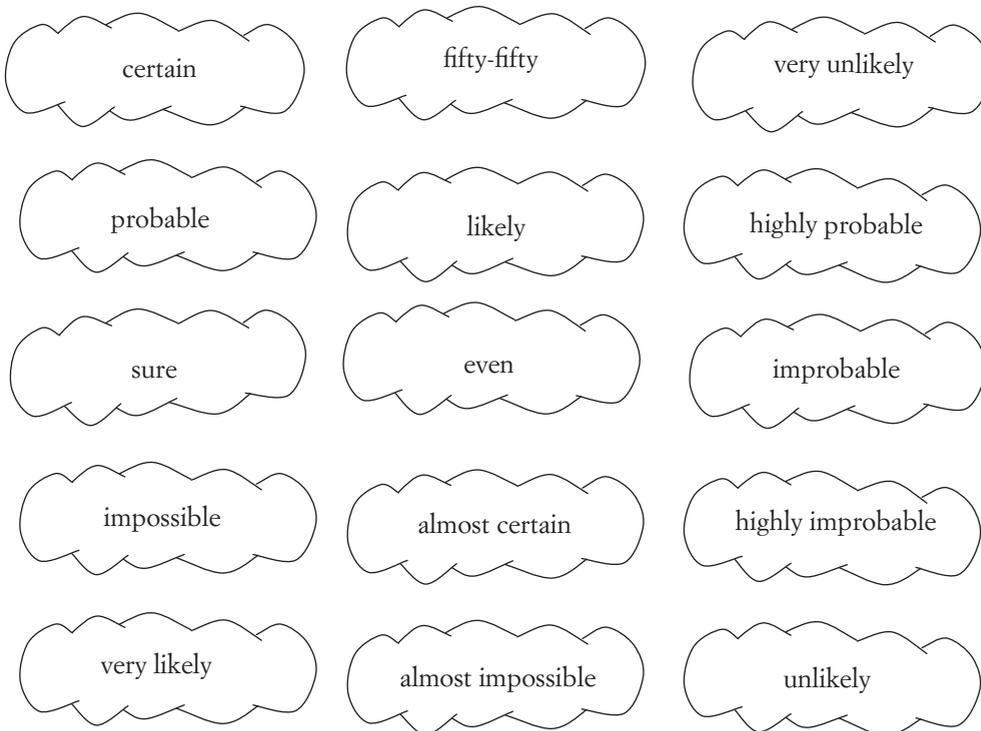
2 A normal die is rolled and the outcome is noted. Describe the chance that the number is:

  a 6   **b** An even number   **c** Bigger than 2   **d** 0.

See Example 1

3 A marble is taken at random from a bag with 3 red, 1 green and 5 blue marbles. Describe the chance that the marble is:

  a red   **b** green   **c** blue   **d** coloured.

4 A 4-sided die is rolled and the outcome is noted. Describe the chance that the number is:

  a 6   **b** an odd number   **c** bigger than 1   **d** 2.

5 A biro is taken at random from a case with 4 red, 2 black and 6 blue ones and nothing else in the case. Describe the chance that the biro is:

  a red   **b** black   **c** blue   **d** a biro.

6 Describe the chance that someone chosen at random has their birthday this year on a:

  a Saturday

  b day with an S in it

  c day with a D in it

  d day with a K in it

  e weekday

  f weekend.

See Examples 3, 4

7 a List the sample space for picking a DVD from a shelf with 5 comedies, 6 action movies, 3 romances and 2 westerns.

  b Which is more likely, a western or a comedy?

  c Which is the least likely to be chosen?

  d Which is the most likely to be chosen?

  e Andrew only likes actions and westerns, while Bella only likes comedies and romances. David picks the movie. Is it more likely to suit Andrew or Bella?

8 A card is taken from a normal pack of cards at random. Which is more likely from the following pairs of possibilities?

  a A Queen or a Jack

  b A number card or a picture card

  c The 7 of spades or an ace

  d A number card below 5 or a number card above 5

  e A number card below 6 or a number card above 6

  f A club or a red card

  g A heart or a diamond

9 Describe the likelihood of each of the following events, choosing from impossible, very unlikely, unlikely, even, likely, very likely or certain.

  a The sun will rise tomorrow

  b A shark will walk along St Kilda Rd

  c Mixing red and yellow paints will make orange

  d It will take more than 15 minutes to drive from Fremantle to Perth (20 km on city roads)

  e A bus trip from anywhere in the suburbs to the CBD will cost less than 50 cents.

10 A normal die is rolled. Which is more likely from the following pairs of possibilities?

  a A 2 or an odd number

  b A 6 or a 1

  c A 9 or a 4

  d An even number or a number divisible by 3

11 A 10-cent coin and a 20-cent coin are tossed.

  a Write down the sample space, with the result for the 20-cent coin first.

  b Which is more likely, 2 heads or 2 tails?

  c Which is more likely, 2 heads or 1 head?

  d What is the likelihood of getting a head and a tail?

  e What is the likelihood of getting 2 tails?

12 A coin is tossed 3 times, and the results noted in order.
   a Write down the sample space with the results in order.
   b What is the likelihood of more heads than tails?
   c What is the likelihood of 3 heads?
   d What is the likelihood that there will be more of one than the other?
   e What is the likelihood of exactly 2 heads?

13 Give 3 examples of likely events, and carefully explain why you think they are likely.

14 The spinner on the right has the 3 colours shown.
Describe the chance of landing on each colour.
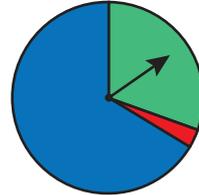
15 Lee, Ahmed and Jo are playing a game. They open a book with pages numbered from 1 to 53 to a random page. If the page number is divisible by 3, then Lee wins. If it is even and not divisible by 3, then Ahmed wins, otherwise Jo wins. Work out who is the most likely to win, who is the least likely to win and explain your reasons.

16 A family has three children. Write down the sample space of the genders of the children, with the eldest first. Work out the likelihood of exactly 2 girls and explain your reasoning.

9.2   Experimental probability

Lots of students would agree that when you want to use coloured pencils 'it's usually the one you want that needs sharpening'. Is this really true? Is it because you only notice when you want to use it that a pencil needs sharpening?

Investigate: Coloured pencils

Before starting this investigation:

- Swap your coloured pencils with a neighbour
- Check how many of the pencils need sharpening
- Write down the total number of pencils and the number of sharp ones
- Give the pencils back

For the main part of the investigation:
- Your teacher will call out something to colour
- Without looking, choose the colour you would use
- Find the colour and see if the pencil is sharp
- Write down 'sharp' or 'blunt'

When you have finished, write down what you think about the likelihood of the colour you want needing to be sharpened.

To predict the likelihood of future events, you really need to use accurate information. This can come from doing a careful experiment, or from using data that has already been collected.

Important!

Experimental probability

Experimental probability (empirical probability) uses real data to work out chances.

Each time we try an experiment it is called a **trial**.

The result that we are interested in is called the **favourable outcome** of a trial.

Outcomes are also called **events**.

More likely outcomes happen more often than less likely outcomes. They have *higher frequencies*.

The probability of an outcome can be measured using the formula:

$$\text{Probability} = \frac{\text{Frequency of event}}{\text{Total frequency}}$$
$$= \text{Number of favourable outcomes} \div \text{Number of trials}$$

Probability calculated in this way is often called **relative frequency**.

Example 6

A wooden ruler is dropped on the floor 20 times. It lands with the brand name upwards 8 times.

a Describe the likelihood of the ruler landing with the brand name showing.

b What is the probability of the ruler landing with the brand name showing?

Solution

a	The brand name lands upwards just less than half the time.	**The brand name is unlikely to show.**
b	Write the formula.	$\textbf{Probability} = \dfrac{\textbf{Frequency of event}}{\textbf{Total frequency}}$
	Put in the numbers.	$= \textbf{8} \div \textbf{20}$
	Work out the answer.	$= \textbf{0.4 or } \dfrac{\textbf{2}}{\textbf{5}} \textbf{ or 40\%}$
	Write the answer in a sentence.	**The probability of the brand name showing is 0.4.**

Example   7

A weighted die is rolled.
The results are shown below.

1	2	2	4	6	2	4	2	6	3	6	4
6	3	6	3	5	6	6	2	1	4	6	

a How many trials are there?
b Make a table to show the results.
c What is the most likely outcome?
d Find the probability of getting a 6.

Solution

a The die is rolled 24 times altogether.

There are 24 trials.

b Make a table to show the results.

Score	Tally	Frequency
1					3
2						5
3					3
4						4
5			1
6									8
	Total	24

c Six occurs the most times.

The most likely outcome is 6.

d Find the probability of 6.

$$\textbf{Probability} = \frac{\textbf{Frequency of event}}{\textbf{Total frequency}}$$

$$= \frac{8}{24}$$

$$= \frac{1}{3} \approx 0.33 \textbf{ or } 33\%$$

Write the answer in a sentence.

The probability of a 6 is about 0.33.

When an experiment is repeated, we expect the results to be similar unless the conditions are changed. We can work out the probability of an event from the first experiment and use it to predict what is likely to happen in other experiments.

Important!

Expected frequency

The **expected frequency** of an event is the number of times we expect it to occur from a particular number of trials. It can be calculated using this formula.

$$\text{Expected frequency} = \text{Probability} \times \text{Number of trials}$$

Example 8

The probability of being dealt a spade first from a shuffled pack of cards is 0.25. How many times would you expect to get a spade if this is done 36 times?

Solution

Write the number of trials.	**Number of trials = 36.**
Write the formula.	**Expected frequency = Probability $\times$ Number of trials**
Put in the numbers.	$= 0.25 \times 36$
Calculate.	$= 9$
Write the answer in a sentence.	**You would expect to get a spade 9 times.**

Example 9

A biased coin is tossed 20 times and there are only 6 heads. How many times would you expect to get tails from 50 tosses?

Solution

Calculate the probability.	$\textbf{Probability} = \dfrac{\textbf{Frequency of event}}{\textbf{Total frequency}}$
There were 14 tails.	$= \dfrac{14}{20}$
	$= 0.7$
Calculate the expected frequency.	**Expected frequency = Probability $\times$ Number of trials**
Put in the numbers.	$= 0.7 \times 50$
Calculate.	$= 35$
Write the answer in a sentence.	**There would probably be about 35 tails from 50 tosses.**

Investigate: Boys and girls

In the animal kingdom, the proportions of males and females being born varies with the species. It is even true that for some animals, all the newly born animals are one sex and the sex of some of them changes later. For people, it is generally assumed that there is an equal chance of a newly-born child being a boy or a girl. Similarly, we generally assume that a coin has equal chances of landing on heads or tails. In this investigation, you will simulate births, taking heads to mean girls and tails to mean boys. In Australia, more than half of high school students come from families with 3 or more children. You will simulate the number of boys and girls in families with 4 children.

Work in groups of two for this investigation.

- Toss a coin 4 times and note the number of heads as girls and the number of tails as boys.
- Repeat the experiment 4 times.
- For the first 5 families, how many have only boys?
- How many have only girls?
- How many have 2 boys and 2 girls?
- Do another 5 families: what do you find for the next 5 families?
- Keep going in lots of 5 families until you have done 100 families.
- Find the proportions of all-boy, all-girl and equal-sex families for the first 5, first 10, first 20, first 50 and all 100 families.
- What do you find happens to the proportions as the numbers of families sampled increases?
- Compare the results of the whole class.

In Australia, the number of boys born is actually more than half the number of births. What does this mean about using a coin to simulate the birth of boys and girls? Historically, it has been found that the proportion of boys and girls born in a country changes: the proportion of boys born in Australia *increased* for a few years after the First World War.

From the results of your experiment, do you think that using experimental probability always gives accurate results? Is it possible that some samples could be misleading about what happens with a large number of trials?

Exercise 9.2 Experimental probability

1 A coin is tossed and the side facing up is noted as shown below:

T, T, T, H, H, T, H, H, H, H, H, T, T, T, T, T, T, H, H, H, H, H, H, T, T, H, H, H, T, T, H

  a How many trials were there?
  b Draw a frequency table of the results.
  c Describe the likelihood of getting heads.
  d Compare the likelihood of getting heads to getting tails.
  e What is the probability of getting heads with this coin?

2 A coin is repeatedly tossed twice and the outcomes recorded in order as shown.
HH, TT, TH, HT, HH, TT, TT, HT, HT, HT, TH, HH, TT, HT, HT, TH, TT, TT, TT, HT
  a How many trials were there?
  b Draw a frequency table of the results.
  c Describe the likelihood of landing with the same side up in two tosses.
  d Compare the likelihood of getting two heads to getting two tails.
  e What is the probability of HH?
  f What is the probability of HT?

3 A normal die is tossed many times and the results recorded as shown below:
4, 4, 1, 2, 2, 1, 3, 3, 5, 3, 5, 6, 6, 5, 3, 3, 4, 2, 1, 2, 6, 6, 2, 3
  a How many trials were there?
  b Draw a frequency table of the results.
  c What appears to be the most likely outcome?
  d Compare the likelihood of getting a 4 or a 5.
  e What is the probability of getting a 6?
  f What is the probability of getting a 3?
  g What is the probability of getting the same number at least twice in a row?

Fluency

4 A spinner with sectors marked in red, blue and green is spun 30 times and lands on red 8
times, blue 12 times and green 10 times.
  a What is the experimental probability of landing on blue?
  b What is the experimental probability of landing on red?
  c Do the results prove that the blue sector is the largest?
  d If the spinner was used another 55 times, how many reds would you expect?

5 An 8 sided die is tossed 40 times and the following results obtained.
1, 3, 8, 1, 2, 8, 6, 5, 4, 8, 2, 7, 5, 1, 5, 8, 2, 5, 1, 5, 5, 6, 3, 1, 6, 8, 5, 8, 6, 4, 7, 7, 5, 4, 4, 4, 8, 8, 7, 3
  a What is the experimental probability of getting a 6?
  b What is the experimental probability of getting a 7?
  c What is the experimental probability of getting a 1?
  d If it was tossed 20 times, how many 3s would you expect?

6 A biased die is rolled 30 times and lands on six 8 times, on one 3 times, on four 5 times, on
five 6 times and on three and two 4 times. If it was tossed 80 times, how many of each result
would you expect?

7 A weighted coin is tossed 16 times and lands on heads 10 times and tails 6 times. If it was
tossed 45 times, how many tails would you expect?

Problem solving

8 The numbers of cherry tomatoes in 250 g punnets were counted to get the following results:
10, 14, 11, 12, 11, 11, 14, 13, 16, 10, 12, 13, 15, 11, 11, 15, 14, 12
  a How many trials were there?
  b Draw a frequency table of the results.
  c What appears to be the most likely outcome?

d Compare the likelihood of getting punnets of 14 or 15 tomatoes.

e What is the probability of getting a punnet with 10 tomatoes?

f What is the probability of getting a punnet with 12 tomatoes?

g There were 50 punnets on display in the fruit and vegetables. How many punnets would you expect to have 14 tomatoes?

h Another shop had 120 punnets from the same supplier. How many punnets in this shop would you expect to have 15 tomatoes?

9 The table below shows the brands of mobile phones owned by 49 students in two Year 7 classes.

Brand	Apple	LG	Nokia	Samsung	Sony
Number	2	3	10	8	2

a How many students altogether had mobile phones?

b What was the most likely kind of phone at this school?

c Find the probability of each brand.

d From another school with five Year 7 classes, with a total of 123 students, how many would you expect to have Samsung mobiles?

10 Longreach has an average of 23 clear days in September. What is the probability of Longreach having a cloudy or rainy day on 14/09/2016?

11 The probability that Year 7 students have access to the Internet at home is 0.93. How many from 210 would you expect NOT to have access?

12 After many trials, a student worked out that the probability of picking a red marble at random from a bag containing 40 marbles was 0.6. What is the most likely number of red marbles in the bag?

Reasoning

Worked solutions

Exercise 9.2

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See Example 8

9.3 Theoretical probability

You can often use your experience to work out probabilities. For example, it is common sense that a coin has equal chance of landing heads or tails. There are only two possibilities and we assume that they both have a probability of $1 \div 2 = 0.5$.

In **theoretical probability**, we assume that all of the outcomes in a probability situation are equally likely. This means that we can work out the chances by counting the number of outcomes.

Investigate: Equal chances

You should work in groups of 3 or 4 for this investigation.

- Make spinners using light cardboard circles coloured as shown below. Your teacher may have printed sheets with circles for you to cut out.
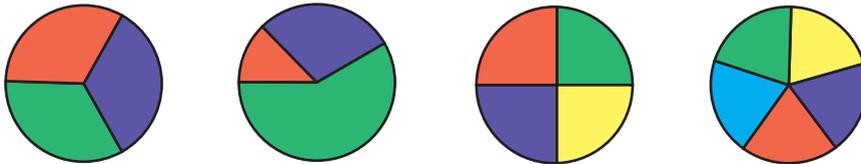
- For each spinner, write down whether the colours have equal chances.
- Work out an experiment to test your theory.
- Perform your experiment to see if you are right.
- Now work out what the chance of landing on red is for each spinner.
- Work out an experiment to test your theory.
- Do the experiment to see if you are right.
- Discuss your results as a class.

Example   10

A bag contains a red, a blue and a green marble. There are no other marbles in the bag. Peter picks a marble without looking. What is the chance that it is blue?

Solution

State the assumption.	**Each marble has an equal chance of being picked.**
Count the simple outcomes.	**There are three marbles.**
State the chance of each outcome.	**Each marble has a 1 in 3 chance of being picked.**
Write the answer.	**The chance of getting a blue marble is $\frac{1}{3}$.**

Example 11

A bag contains 7 marbles. One is red, one is light green, one is dark green, one is light blue, one is sky blue, one is dark blue and the other one is yellow. Lee picks a marble without looking. What is the probability that it is green?

Solution

State the assumption.	**Each marble has an equal chance of being picked.**
Count the simple outcomes.	**There are 7 marbles.**
State the chance of the desired outcome.	**Green marbles have a 2 in 7 chance of being picked.**
Write the answer.	**The probability of a green marble is $\frac{2}{7}$.**

Important!

Theoretical probability

A **simple outcome** (or **atomic outcome**) is an outcome that has only one way of happening. A list of all the simple outcomes of a probability situation is called the **sample space** (or **event space**). The outcomes in a sample space are also called **sample points** or **elements** of the sample space.

An **event** is part of a sample space. It can have any number of the sample points.

The **chance** of an event is the ratio of the number of elements in the event to the number in the sample space, such as 2 in 7.

The **probability** of an event is calculated using the formula

$$\text{Probability} = \frac{\text{Number of elements in the event}}{\text{Number of elements in the sample space}}$$

A probability can be written as a fraction, decimal or percentage.

In Example **11**, it doesn't make any difference to the result if both the green marbles are the same shade of green. Similarly, it won't make any difference if all three blue marbles are the same shade of blue.

Example 12

A spinner has five coloured parts of equal size.
Two of the parts are green, one is red, one is blue and the other is purple.
What is the chance of landing on green?
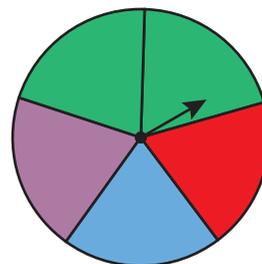
Solution

State the assumption.	**Each part has an equal chance of being picked.**
Count the simple outcomes.	**There are five parts.**
State the chance of the desired outcome.	**Green parts have a 2 in 5 chance of being picked.**
Calculate the probability.	**Probability of a green part $= \dfrac{2}{5} = 0.4 = 40\%$.**

Example 13

A bag contains 8 marbles: 2 red, 3 green, 2 blue and 1 orange. One is selected at random and then replaced.
a What is the probability of getting a green marble?
b How many times would you expect to get a blue marble from 33 such attempts?

Solution

a Write the number of elements in the event.

Number of green marbles $= 3$

Write the number in the sample space.

Number in sample space $= 8$

Write the formula.

$$\text{Probability} = \frac{\text{No. in the event}}{\text{No. in the sample space}}$$

Substitute in the numbers.

$$= \frac{3}{8}$$

Do the division.

$$= 0.375 \text{ (or } 37.5\%)$$

Write the answer.

The probability of a green marble is 0.375.

b Work out the probability of blue.

Write the formula.

$$\text{Probability} = \frac{\text{No. of blue marbles}}{\text{No. of marbles altogether}}$$

Substitute in the numbers.

$$= \frac{2}{8}$$

Do the division.

$$= 0.25 \left(= \frac{1}{4} \text{ or } 25\% \right)$$

Write the formula for expected frequency.

Expected frequency $=$ Probability $\times$ Number of trials

Put in the numbers.

$$= 0.25 \times 33$$

Calculate.

$$= 8.25$$

Round and state the answer.

You would expect about 8 blue marbles.

Example 14

A simple card game for three players has the following rules:

- Each player, in turn from the dealer, chooses 'evens', 'odds' or 'pictures'.
- Aces count as 1.
- The pack is shuffled and cards are turned over one at a time.
- The 'picture' player wins if the card turned over is a king, queen or jack.
- The 'evens' player wins if the card turned over is an even number card.
- The 'odds' player wins if the card turned over is an odd number card.

Calculate the chances of each player winning.
Would you choose to be the dealer, who picks last?

Solution

Write down the number of cards.	**There are 52 cards in the pack.**
Write down the number of each card.	**There are 12 picture, 20 even and 20 odd cards, counting the aces as odd.**
Work out the probabilities.	$P(\text{picture}) = 12 \div 52 \approx 0.23$
	$P(\text{even}) = 20 \div 52 \approx 0.38$
	$P(\text{odd}) = 20 \div 52 \approx 0.38$
To check, enter as: 20 $\boxed{\div}$ 52 $\boxed{=}$.	`20÷52          0.3846153846`
Comment on the chances for the dealer.	**The dealer will probably be stuck with 'pictures' so it's better not to be the dealer.**

Example 15

A spinner has red, blue and green parts. The probability of red is 1 in 3 and the probability of blue is 0.25. What is the probability of green?

Solution

Write 0.25 as a fraction.	$0.25 = \dfrac{1}{4} = 1 \text{ in } 4.$
Consider 1 in 4 and 1 in 3.	**They must both be out of the same total.**
Write them as larger ratios.	1 in 4 = 2 in 8 = 3 in 12 = 4 in 16 = . . .
	1 in 3 = 2 in 6 = 3 in 9 = 4 in 12 = 5 in 15 = . . .
Write the smallest possible total.	**There must be 12 parts.**
Work out the number of green parts.	**Green parts = 12 – 3 – 4 = 5**
Write the probability of green.	$\text{Probability} = \dfrac{\text{No. of green parts}}{\text{No. of parts altogether}}$
Substitute in the numbers.	$= \dfrac{5}{12}$
To check, enter as: 5 $\boxed{\div}$ 12 $\boxed{=}$.	`5÷12          0.4166666667`
Do the division.	$= 0.416\,666 \ldots$
Round and write the answer.	**The probability of green is about 0.417 or 41.7%.**

Sometimes you have information about two aspects of the same subject. You may find it useful to display the information as a cross-table.

Example 16

There are 14 girls in a class and 8 of them have brown eyes. Of the 12 boys in the class, 3 have blue eyes, one has grey eyes and the others have brown eyes. Altogether, there are 7 students with blue eyes. What is the probability that a student chosen at random will be a blue-eyed girl?

Solution

Make a cross-table with gender in the first column and eye colour in the top row.

Fill in the information given.

		Eye colour			
Sex		**Blue**	**Brown**	**Other**	**Total**
	Female		8		14
	Male	3		1	12
	Total	7			

Use the first column to work out the number of blue-eyed girls.
$7 - 3 = 4$

Use the last column to work out the total number of students.
$14 + 12 = 26$

Use the first row to work out the number of girls with 'Other' colours.
$14 - (8 + 4) = 2$

Work out the rest of the missing numbers in a similar way.

		Eye colour			
Sex		**Blue**	**Brown**	**Other**	**Total**
	Female	4	8	2	14
	Male	3	8	1	12
	Total	7	16	3	26

There are 4 blue-eyed girls out of 26 students.

$$\text{Required probability} = \frac{4}{26}$$
$$= \frac{2}{13} \approx 0.1536 \text{ or } 15.38\%$$

To check, enter as: 4 [÷] 26 [=].

4÷26	0.1538461538

Write the answer.

The chance of picking a blue-eyed girl is about 15%.

A table like the one in Example **16** is also called a **two-way table**.

Technology Coin tossing simulation

There is a coin tossing spreadsheet on the NelsonNet student website. You can use it to simulate the tossing of a coin many times. By simulating the tossing of a coin for different numbers of times you can see what happens to the ratio of heads to tails as the number of tosses is increased.
Try using the simulator for 10, 20, 50, 100, 200, 500, 1000 and 2000 tosses.
What happens to the ratio of heads to tails as the total is increased?
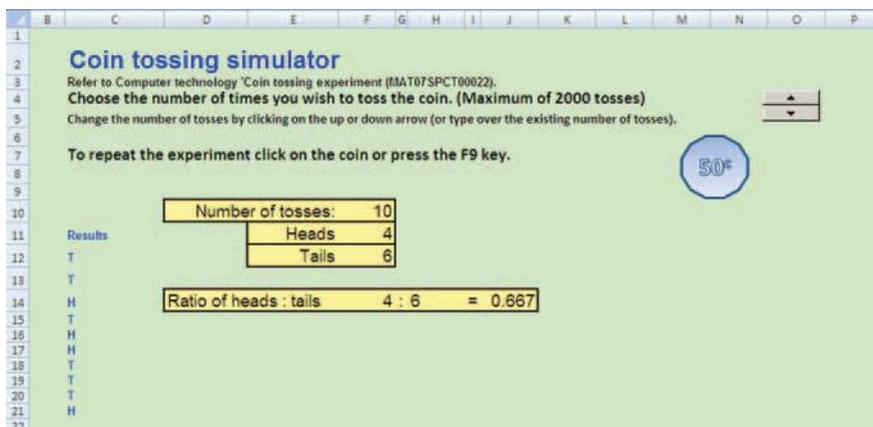
Example 17

The chance of getting a chocolate covered sweet from a bag of lollies is 4 in 15. What is the chance of getting one that is not chocolate covered?

Solution

Write the total number of parts.   **There are 15 parts.**

Work out how many are not chocolate.   **There are 15 – 4 = 11 that are not chocolate covered.**

Write the chance.   **There is an 11 in 15 chance of getting a lolly that is not chocolate covered.**

Important!

Complementary events

Two separate events are called **complementary** if, together, they make the whole sample space. They have no sample points in common and their probabilities add up to 1. One event is *not* the other.

Example 18

The probability of being left-handed is about 15%. What is the probability of being right-handed?

Solution

|	
Left- and right-handedness are complementary events.	**Probability (right-handed)**
	= 1 – Probability (left-handed)
	= 1 – 0.15
	= 0.85
Write the answer.	**The probability of being right-handed is 85%.**

It is obvious that the biggest advantage of theoretical probability is that the long-term probability can be worked out exactly. What are some disadvantages of theoretical probability? Your teacher may want you to discuss this as a class.

Exercise 9.3 Theoretical probability

Understanding

1 State if each of the spinners shown below has equal or unequal chances of each colour.
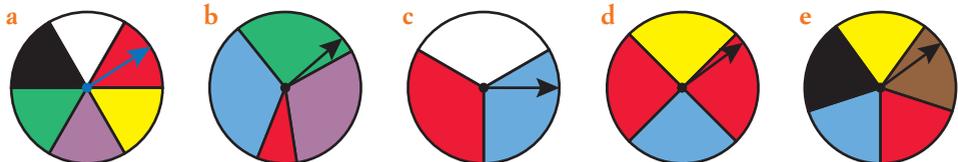
2 The marbles illustrated below are put into a bag and one is taken out at random from the bag.

    a What is the chance that it is red?
    b What is the chance that it is white?
    c What is the chance that it is black?
    d What is the chance that it is blue?

3 State the chance of landing on red for each of the spinners shown.
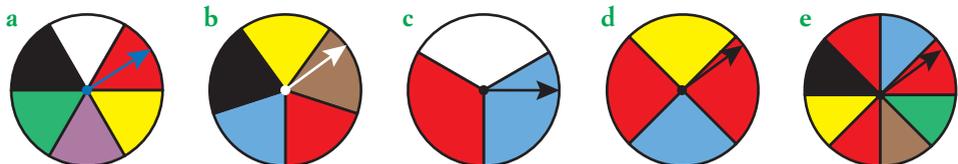
4 State the chance of landing on black for each of the spinners in question **3**.

5 State the chance of landing on blue for each of the spinners in question **3**.

6 Ten marbles are placed in a bag. Four are white, 3 are red, 2 are yellow and 1 is green. One marble is taken from the bag. What is the chance that it is:

 a White   **c** Yellow   **e** Black

 b Red   **d** Not white

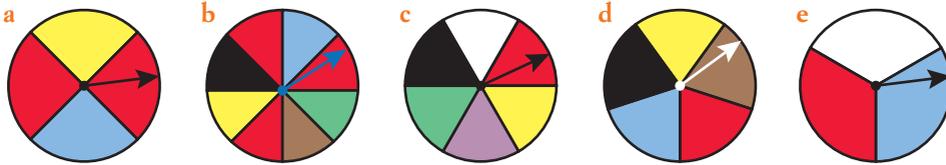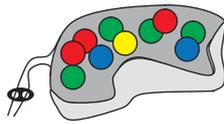**7** State the probability of landing on yellow for each of the spinners shown below.

8 The marbles bag shown below is closed and one marble is taken at random from the bag.

 a What is the probability that it is red?

 b What is the probability that it is yellow?

 c What is the probability that it is green?

 d What is the probability that it is blue?

 e What is the probability that it is not white?

 f What is the probability that it is white?

9 What is the probability of a normal die landing on 4?

10 What is the probability that the first card dealt from a normal pack of cards is an ace?

11 Repeat Question 8 with the bag of marbles shown here.
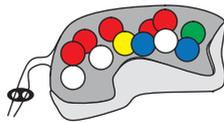
12 Each time a marble is taken out of the bag in question **8** it is put back before the experiment is repeated. This is done 55 times. How many times would you expect to get a green marble?

13 The spinners in question **7** are each spun 50 times. For each spinner, state how many times you would expect it to land on red.

14 The probability of having more than 1 mm of rain on a single day in August in Wangaratta is about 0.294. What is the probability of having less than 1 mm of rain on August 25 this year in Wangaratta?

15 The probability that the next child born in Australia is a boy is 51.3%. What is the probability that the next child born is a girl?

16 A potter's website had 2000 'hits' in a week, resulting in 40 sales of porcelain. What is the chance that someone visiting the website will buy something?

17 A collection of old DVDs has 5 that won't play properly. The probability of getting one that does play properly is 6 in 7. How many DVDs are there altogether?

18 A large school has a premier concert band with 25 students and a beginner's band with 40 students. Every student learning an instrument at the school plays in one of the bands. There are 20 students at the school learning the flute and there are 6 flutes in the premier band. Stacey plays the flute. What is the probability that she is in the premier band?

19 After playing chess for about 15 minutes, Sol has taken 3 black pawns and Ibrahim has captured a knight and a pawn from Sol. A normal chessboard was set up at the start as shown in the photo. What is the probability that the f5 square marked by the cross now has a pawn on it in their game?

20 A display board in a classroom of art work has 12 paintings, 4 pieces of pottery, 6 drawings and 3 time lines shown on it. The teacher has to take off one piece of work to put a new piece up. If she does this at random, what is the probability that it will be a painting?

See Example 14

21 A dice game is played between 2 players. The thrower is the person who tosses the die. If the die lands on an even number the thrower wins, otherwise the other player wins. What is the probability of winning?

22 A dice game played with two dice has the following rules. The total of the dice is used to work out the winner. If the thrower rolls a total of 7 or 11, the thrower wins. If a 2, 3, 4 or 12 is rolled the thrower loses. If any other total is rolled, the thrower rolls again. What is the probability that the thrower will win on any one roll? What is the probability that the thrower will lose on any one roll?

23 A simple card game is played by dealing out all the cards face down. If there are any cards left over they are placed face-down in the centre. Players then take it in turns to turn over the top card of their pile. If a player turns over a card that is the same suit or the same card as the previous one, they take all the cards that have been turned over (and the ones in the middle, if it is the first time). This player shuffles their cards and turns them face down. Play resumes with this player starting. A player is out if all their cards are gone. The winner is the last person left in.
For any play, what is the probability of winning the cards?

24 Acey-deucey is a simple card game where players bet against the 'pot' in the centre. At the start, all players put an agreed amount into the pot. Players take turns to play. The dealer gives a player two cards face-up, one at a time. If a card is an ace, the player must nominate it as high (above the king) or low (below the 2) when it is dealt. The player then can choose to bet or pass. The only bet is that the next card is strictly between the first two cards. The player may bet any amount up to the total amount in the centre. If the player wins, they take the

amount bet from the pot. If a player wins the whole pot, the deal passes to the next person. If the player loses, the bet is added to the pot. If the pack of cards is exhausted, the discards are shuffled and cut and play continues. The game can take a very long time and the pot can get very large and there are lots of variations.

a Can you lose if you are dealt two aces?

b Calculate the probability of winning from two aces.

c Calculate the probability of winning from a 3 and a 10.

d Calculate the probability of getting two aces.

25 The game of two-up is played by tossing two coins in the air in the centre of a large ring. Only two heads or two tails counts as a result. The spinner in the ring who tosses the coins must bet on heads, and other players on the outside of the ring on tails. If two tails are tossed, the spinner loses and a new spinner comes in. The tail-betters outside the ring bet by placing their money inside the ring before the toss. The total amount bet is usually limited to the amount the spinner has put up at the start. If the spinner gets three 'heads' in a row, they win and can choose to pass onto a new spinner.

a Work out the probability that a result occurs on any one toss.

b Work out the probability that the spinner loses on any one toss.

c Work out the probability that the spinner wins, with three 'heads' in three tosses.

26 A pacer pencil lead lasts for about 5 A4 pages of writing with about 20 lines to a page. Twelve students in a class use pacers for writing. How many students would you expect to need a new lead when they all do about 17 lines of writing?

27 The ABS classifies people aged 15–24 as young men and women. In Australia, the probability that a young person admitted to hospital is male is about 0.75. 1 in 3 of these are from transport accidents and of these more are from motorbikes than any other cause. Only a quarter of young women are admitted to hospital due to transport accidents. What is the probability that a young Australian is admitted to hospital as a result of a transport accident?

28 In March in Broome, there are an average of 10.6 sunny days and 6 cloudy days. What is the probability that it will rain on March 15 next year in Broome?

29 Hair colour can be classed as fair or dark (red usually counts as fair). A year 7 class has 8 dark-haired girls out of 14 girls and 10 dark-haired boys. There are 27 students in the class altogether. What is the probability that the first student leaving the classroom at lunchtime is a fair-haired girl?

30 A class has been making pottery. 10 students made coil pots and the others made little animals. In the kiln, 10 pieces of pottery cracked. Nine students out of the class of 25 were able to take home little pottery animals that had no cracks in them. What is the probability that a student had a cracked coil pot?

Chapter 9 summary

- **Probability** is the mathematics of chance, where more than one result may occur. The **outcomes** of a probability situation are the things that *could* happen.

- **More likely** outcomes *usually happen* more often than **less likely** outcomes. **Equally likely** outcomes usually happen as often as each other.

- An outcome with an **even chance** usually happens about half the time. It is also called a **fifty-fifty** or **equal chance**.

- An **unlikely** outcome usually happens less than half the time. It is also said to be **improbable**. A **likely** outcome usually happens more than half the time. It is also said to be **probable**.

- An **impossible** outcome *never happens*. A **certain** outcome always happens. It is **sure** to happen.

- Choosing at **random** means that each possible outcome could occur.

- **Experimental probability** uses real data to work out chances. Each time we try an experiment it is called a **trial**.

- The result that we are interested in is called the **favourable outcome** of a trial.

- The **probability** of an event is given by the formula

$$\text{Probability} = \frac{\text{Frequency of event}}{\text{Total frequency}}$$

- The **expected frequency** of an event is the number of times we expect it to occur from a particular number of trials. It can be worked out using the formula.

$$\text{Expected frequency} = \text{Probability} \times \text{Number of trials}$$

- A **simple outcome** (or **atomic outcome**) is an outcome that has only one way of happening.

- In **theoretical probability**, we assume that all the simple outcomes in a probability situation are equally likely. This means that the outcomes occur at **random**.

- A list of all the outcomes of a probability situation is called the **sample space** (or **event space**). The outcomes in a sample space are also called **sample points** or **elements** of the sample space.

- An **event** is part of a sample space. It can have any number of the sample points.

- The **theoretical probability** of an event is calculated using the formula

$$\text{Probability} = \frac{\text{Number of elements in the event}}{\text{Number of elements in the sample space}}$$

- It can be written as a fraction, decimal or percentage.

- A **two-way table** is a cross-table showing two aspects of the same subjects.

- **Complementary events** together make the whole sample space. They have no sample points in common and their probabilities add up to 1. One event is not the other.

1 Describe the chance that a card taken at random from a normal pack of cards is:

  a red   **b** green   **c** a spade   **d** a number card

See Example 1

2 A wallet contains four 2-dollar coins, ten 20-cent coins, a 50-cent coin, two dollar coins and a 5-cent coin. Describe the chance that a coin taken at random is:

  a 50c   **b** Less than 50c   **c** more than 50c   **d** 10c

3 A survey of Year 7 students' normal bedtimes gave the following:

10:00 p.m., 9:00 p.m., 8:30 p.m., 10:00 p.m., 8:00 p.m., 10:30 p.m., 8:00 p.m., 9:00 p.m., 9:00 p.m., 10:00 p.m., 10:00 p.m., 9:00 p.m., 9:30 p.m., 9:00 p.m., 10:30 p.m., 11:00 p.m., 9:00 p.m., 9:30 p.m., 9:00 p.m., 10:00 p.m., 8:30 p.m., 10:30 p.m., 10:00 p.m., 8:30 p.m., 8:00 p.m., 8:30 p.m.

See Example 7

  a How many trials were there?

  b Make a frequency table of the results

  c Compare the likelihood of a 9 p.m. or 10 p.m. bedtime.

  d Find the probability of an 8:30 bedtime.

  e If another 40 students were surveyed, how many would you expect to have a bedtime after 10 p.m.?

4 The spinners below are made by students.
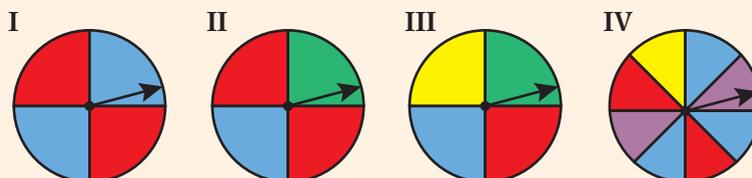
  a Which spinner(s) have equal chances of landing on each colour?

  b For each spinner, what is the chance of landing on blue?

  c What is the probability of landing on yellow for spinners II, III and IV respectively?

  d If spinner IV was spun 50 times, how many times would you expect it to land on yellow?

5 **a** List the sample space for choosing a card from a normal pack with the 2s, 3s, 4s, 5s and 6s removed.

See Example 3

  b Which is more likely, a court card or a number card?

  c Are you more likely to get a card higher than 10 or lower than 10? Ace is high.

6 **a** List the sample space for throwing a ten-sided die.

See Example 4

  b Are you less likely to get a number less than 7 or more than 7?

  c Which is more likely, a number less than 5 or a number bigger than 5?

7 A pack of 20 cards has drawings of animals on the front sides. After shuffling, one card is taken from the pack and the animal on the front is noted. The card is replaced and this is repeated 40 times. There were 12 monkeys, 10 donkeys, 8 horses, 6 dogs and 4 cats.

See Examples 7-9

  a What is the experimental probability of getting a horse?

  b What is the experimental probability of getting a monkey?

  c Can you be certain there are no cards with ducks on them?

  d If this was done another 55 times, how many dogs would you expect?

8   A bag of fruit nougat chews has 5 strawberry, 4 lime, 3 pineapple and 3 orange ones. Find the probability of choosing each type at random.

9   A card is taken from a normal pack and the suit is noted. The card is replaced and the pack shuffled before this is done again.
    a   Write the sample space for the 2 cards in order, using the letters S, H, D and C for the suits.
    b   Which is more likely, 2 cards of the same suit or different suits?
    c   What is the probability of at least one of the cards being a spade?

See Example 18

10 The probability that a person is ambidextrous is about 3%. What is the probability of being right-handed or left-handed?

Problem solving

See Example 12

11 Draw a spinner with red, green and blue sectors such that it is likely to land on red and very unlikely to land on blue.

See Example 2

12 Give an example each of an unlikely, likely and even-chance event.

13 A student takes a pen at random from his pencil case at the beginning of each non-practical lesson. The first pens taken over a few days are the following colours:

    Black, Black, Blue, Blue, Blue, Red, Blue, Red, Black

    a   How many trials were there?
    b   Draw up a frequency table of the results.
    c   What is the most likely outcome?
    d   What is the probability of getting a red pen?
    e   From 30 lessons next week, what is the most likely number of times the student would get a blue pen?

14 There are 8 winning numbers in a 'lucky dip' at a school fete. The probability of getting a winning envelope is 1 in 25. How many envelopes were in the lucky dip? Explain your reasoning.

15 Are you more likely to get a queue waiting for a bus at the main city stop or at a stop in the suburbs? Explain your answer.

16 Describe the likelihood of each of the following events:
    a   It will take less than 2 hours to legally drive from Melbourne to Adelaide.
    b   The Bombers will win their next game.
    c   A ferry berthing at Kangaroo Point will do so safely.
    d   A coin will land on heads.
    e   Friday the 13th will be considered unlucky by some people.

17 Ballarat airport has about 7 clear days in March. How many clear days would you expect in a fortnight in March?

18 Luigi has taken home his Maths text, English novel, Science text and three books from the Library for an assignment. On the way home his bag was jumbled about. What is the probability that his Maths book is on top?

19 There is an even chance of getting a red marble from a bag containing red, yellow and blue marbles and you are very unlikely to get a blue marble. Describe the likelihood of getting a yellow marble and explain your logic.

20 The probability of rain in Fremantle in August is about 0.25. How many rainy days would you expect in a week in August?

21 On average, a Year 7 student at Adam Road needs a new blue pen every fifteen weeks. Explain how many students in a class of 27 students would need a new pen during Term 2 (10 weeks).

22 Fifty students were asked about pets. Of the 26 girls, 7 had dogs and 5 had cats. 8 of the boys had cats and there were 30 students altogether that had either a dog or a cat. None of these students had both a dog and a cat. What is the probability that a particular student was a boy with a pet dog?

See Example 16

10

Money and percentages

Contents

Prior learning
Chapter 10

Parent guide
Chapter 10

Curriculum guide
Chapter 10

Australian Curriculum statements

Real numbers

Connect fractions, decimals and percentages and carry out simple conversions. (ACMNA157)

Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. (ACMNA158)

Money and financial mathematics

Investigate and calculate 'best buys', with and without digital technologies. (ACMNA174) AC

Money can't buy happiness, but poor money management can cause a lot of misery. It is very important to manage your money properly. You should be able to check a shopkeeper's bill and the change you get when you make purchases. You should also understand the differences between methods of payment and have a basic understanding of finances.

Percentages are traditionally used in many money calculations, so you also need to have a clear understanding of percentages.

■ Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

amount tendered   cash-out   Goods and Services Tax (GST)
best buy   change   percentage
bill   discount

10.1   Money transactions

When you buy things at a shop, the shopkeeper or checkout operator works out the total amount you have to pay. You can usually pay by cash, cheque or EFTPOS (Electronic Funds Transfer at Point of Sale). When you pay by cash in Australian currency, the amount must be rounded to the nearest 5 cents.

Important!

Bills and change

The total amount you have to pay for purchases is called the **bill**. If you pay by cash, the bill is adjusted to the nearest 5 cents. The amount you give in payment is called the **amount tendered**. The difference between the bill and the amount tendered, which you get back, is the **change**.

Example   1

a Work out the total bill and the change for the following purchases if a $50 note is tendered.

- 2 kg packet of flour @ $2.17/packet
- 5 iceblocks @ $1.15 each
- 2 L bottle of milk @ $3.13/bottle
- 1.2 kg of fat-free mince @ $8.57/kg

b In what appropriate combinations can the change be given?

Solution

a Set out the costs and totals in a table to make it easier to follow. You can use your calculator to multiply the cost by the number (or mass) to get each amount.

Add to get the total.

Item	Cost/unit	Units	Amount
Flour	$2.17	1	$2.17
Iceblock	$1.15	5	$5.75
Milk	$3.13	1	$3.13
Mince	$8.57	1.2	$10.28
		Total	$21.33

Round to the nearest 5 cents for cash.

Total amount ≈ $21.35

Subtract to get the change.

Change = $50 − $21.35
= $28.65

Write the answer.

The total bill is $21.35 and the change is $28.65.

b Write possible totals.

$28 = $20 + $5 + $2 + $1
= $10 + $10 + $2 + $2 + $2 + $2
65c = 50c + 10c + 5c
= 20c + 20c + 20c + 5c

Write some answers.

You would probably get the change as $20 and $5 notes, with $2, $1, 50c, 10c and 5c coins; or it could be given as two $10 notes, four $2 coins, three 20c coins and a 5c coin.

In Example **1**, the way the change is given would depend on the notes and coins that were in the till at the time. The answers could vary immensely.

Investigate: Counting change

When you are given change in most shops, the person at the till does not work out the change by subtraction. It is much more common to let the register carry out the calculation or to count out the change by addition.

Work in pairs to practise giving change by addition from a 'float' of money in a 'till' provided by your teacher.

If you pay by cheque or EFTPOS (debit or credit card), the shop may make an extra charge to cover its handling costs. Some large retail chains can do a cash-out with EFTPOS transactions so you can withdraw money. There is usually a minimum purchase required for cash-outs.

Worksheet

Calculating change

MAT07NAWK00067

Example 2

A hardware store charges 50 cents for EFTPOS transactions. Rianna bought seven tubes of Gap-filler at a price of $7.43 each and paid with her Visa card. How much was she charged?

Solution

Work out the cost of the Gap-filler.	$\textbf{Cost} = \textbf{\$7.43} \times \textbf{7}$
	$= \textbf{\$52.01}$
Add the charge.	$\textbf{Total cost} = \textbf{\$52.01} + \textbf{\$0.50}$
	$= \textbf{\$52.51}$
Write the answer.	**Rianna was charged \$52.51.**

Exercise 10.1 Money transactions

1 What will three 140 g tins of tomato paste cost at 63 cents each?

2 What is the cash price for four 165 mL cans of coconut milk at $1.16 each?

3 Smoked salmon costs $58.40/kg. What is the cash price for 128 g?

4 Greek-style yoghurt is $4.57 per container. What is the cash price for four containers?

5 Cumin seeds cost $1.59 per packet. What is the cash price for six packets?

6 A bill comes to $14.38. What change will you get when you tender a $20 note?

7 What change will you get when you tender a $50 note for a bill that comes to $42.63?

8 What change do you get when you pay a $26.22 bill with $30?

9 Work out the total bill and change from $50 for the following supermarket purchases.

- two tins of tomatoes @ $1.43 per tin
- two 1 kg packets of sugar @ $1.78 per packet
- 1.34 kg of cheese @ $7.63/kg
- three frozen dinners @ $6.49 each

10 Work out the total bill and change from $50 for the following purchases from the fruit shop.

- 3 kg potatoes @ $1.39/kg
- 3.4 kg bananas @ $2.45/kg
- six grapefruit @ $0.95 each
- half-case (9.5 kg) apples @ $2.46/kg

11 If you tender $30 at the newsagent for the following, what is the total bill and the change?

- two newspapers @ $1.20 each
- one paperback novel @ $12.95
- four manila folders @ 27 cents each
- three ballpoint pens @ 65 cents each
- two lined paper refills @ $2.37 each

12 List some appropriate combinations to give $6.85 in change.

13 List some appropriate combinations to give $12.70 in change.

14 What is the total amount paid if an EFTPOS charge of $1.50 is made at the drapery for you to put the following on your Mastercard?

- 2.5 metres cotton print @ $12.39/metre
- nine fancy buttons @ $1.20 for three
- three reels of cotton @ $5.90 per reel
- 4.2 metres of braid @ $4.63/metre

15 Harry the Handyman paid for the following at his local hardware store using his Visa card. The store charges $2.50 for the use of EFTPOS.

- twelve 2.7-metre lengths of 45 × 90 pine @ $4.45/metre
- five bags of cement @ $9.75 per bag
- 3 kg of 75-mm BH nails @ $5.75/kg
- 145 spadefuls of concrete blend @ 24 cents a spadeful

How much did Harry pay?

16 Filipa bought the following at her chemist.

- two prescriptions @ $23.40 each
- 675 g of barley sugar @ $2.47/kg
- three bottles of fluoride tablets @ $4.85/bottle
- two packets of sticking plaster @ $4.65/packet

She also got a cash-out for $50 and had to pay a $2 EFTPOS fee. How much was the total EFTPOS charge?

17 Simone bought petrol, milk and bread at the service station and paid by EFTPOS, with a fee of $2.50. She also got cash-out of $40. What was the total bill for the following?

- 43 litres of petrol at 94.6c/L
- two small bottles of milk at $1.87 each
- a loaf of bread for $1.43

10.2 Percentages, fractions and decimals

Important!

Percentages

Percentages are used to compare quantities that are 'out of' different amounts. Percentages are all 'out of' the same total – 100. In other words, percentages are really fractions with denominators of 100.

A **percentage** is a number written as a fraction with a denominator of 100, but the 100 is replaced by the % sign. For example, $\frac{9}{100} = 9\%$.

A fraction or decimal can be changed to a percentage by multiplying by 100%.

Example 3

Change the following to percentages.

a $\frac{1}{2}$ **b** 0.15 **c** $\frac{15}{37}$ **d** 0.045 **e** 37 out of 60

Solution

a Change the denominator to 100.

$$\frac{1}{2} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100}$$

Replace the denominator 100 by %. $= 50\%$

b Write in extended form.

$$0.15 = \frac{15}{100}$$

Replace the denominator 100 by %. $= 15\%$

c For more complicated cases, multiply by 100% to change to a %.

$$\frac{15}{37} = \frac{15}{37} \times 100\%$$

$$= \frac{1500}{37}\%$$

Enter as: 15 ÷ 37 × 100 = .

```
15÷37×100
          40.54054054
```

Round off. $\approx 40.5\%$

d Multiply by 100%.

$$0.045 = 0.045 \times 100\%$$

$$= 4.5\%$$

e Write as a fraction.

$$37 \text{ out of } 60 = \frac{37}{60}$$

Multiply by 100%.

$$= \frac{37}{60} \times 100\%$$

$$= \frac{3700}{60}\%$$

Enter as: 37 ÷ 60 × 100 = .

```
37÷60×100
          61.66666667
```

Round off. $\approx 61.7\%$

You should be familiar with the common percentages shown below.

Important!

Common percentages

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	0.01
5%	$\frac{1}{20}$	0.05
10%	$\frac{1}{10}$	0.1
20%	$\frac{1}{5}$	0.2

Percentage	Fraction	Decimal
25%	$\frac{1}{4}$	0.25
50%	$\frac{1}{2}$	0.5
75%	$\frac{3}{4}$	0.75
100%	1	1

Example 4

Sandra got 23 out of 35 for Maths and 27 out of 40 for Science. In which subject did she appear to do better?

Solution

Write the Maths mark as a fraction.

$$23 \text{ out of } 35 = \frac{23}{35}$$

Change the Maths mark to a percentage.

$$= \frac{23}{35} \times 100\%$$

Enter as: 23 $\div$ 35 $\times$ 100 $=$.

```
23÷35×100
              65.71428571
```

Write the Science mark as a fraction.

$$27 \text{ out of } 40 = \frac{27}{40}$$

Change the Science mark to a percentage.

$$= \frac{27}{40} \times 100\%$$

Enter as: 27 $\div$ 40 $\times$ 100 $=$.

```
27÷40×100        67.5
```

Write the answer.

Sandra did a little better in Science than in Maths.

To change a percentage to a common fraction or decimal, you only need to remember that a percentage has a denominator of 100. Fractions should be cancelled down to the simplest form.

Example 5

a Change 35% to a fraction.
b Change 47% to a decimal.
c Change 120% to a fraction.
d Change 7.6% to a decimal.

Solution

a Write in the denominator of 100.

$$35\% = \frac{35}{100}$$

Cancel down.

$$= \frac{35 \div 5}{100 \div 5}$$

$$= \frac{7}{20}$$

b Think in words as hundredths.

$$47\% = 47 \text{ hundredths}$$

Now write as a decimal.

$$= 0.47$$

c Write in the denominator of 100.

$$120\% = \frac{120}{100}$$

Cancel down.

$$= 1\frac{1}{5}$$

d Think in words as hundredths.

Change to thousandths.

Now write as a decimal.

7.6% = 7.6 hundredths

= 76 thousandths

= 0.076

Many people use the short cut 'move the decimal point two places' to change a percentage to or from a decimal. Make sure you understand any short cuts before you use them.
When you want to find an amount given as a percentage, you need to think of the percentage as a fraction or decimal to perform the calculation.

Example 6

Calculate the answers to the following in your head.

a 20% of $300

b 75% of $500

Solution

a 20% is the same as $\frac{1}{5}$.

Divide by 5 to find $\frac{1}{5}$.

Write the answer.

$$20\% \text{ of } 300 = \frac{1}{5} \times \$300$$

$$= \$60$$

20% of $300 is $60.

b 75% is the same as $\frac{3}{4}$.

Think of $\frac{3}{4}$ as $3 \times \frac{1}{4}$.

Divide by 4 to find $\frac{1}{4}$.

Write the answer.

$$75\% \text{ of } \$500 = \frac{3}{4} \times \$500$$

$$= 3 \times \frac{1}{4} \times \$500$$

$$= 3 \times \$125$$

$$= \$375$$

75% of $500 is $375.

More complicated percentage problems can be done using a calculator, or using written methods. In either case, you can choose to make the percentage a fraction or a decimal.

Example 7

Use a calculator to find 36% of $380.

Solution

Method 1

Write 36% as 36 hundredths.

36% of $380

$$= \frac{36}{100} \times \$380$$

Enter as: 36 [÷] 100 [×] 380 [=] .

```
36÷100×380        136.8
```

Method 2

	36% of \$380
Write 36% as a decimal.	$= \mathbf{0.36 \times \$380}$

Enter as: 0.36 ⊗ 380 ⊜ .

$$\boxed{0.36 \times 380 \qquad 136.8}$$

Answer from both methods

Write the amount to 2 decimal places.   **36% of \$380 is \$136.80.**

Example   8

Use written methods to find the following.

a   28% of 750 kg   **b**   4.7% of \$424.75

Solution

a   **Method 1**

	28% of 750 kg	750
Write 28% as a decimal.	$= \mathbf{0.28 \times 750\ kg}$	$\times\ 0.28$
		6 000
Do the multiplication.	$= \mathbf{210\ kg}$	**15 000**
		210.00

Method 2

	28% of 750 kg
Write 28% as 28 hundredths.	$= \dfrac{28}{100} \times 750\ \mathbf{kg}$
Multiply 28 by 750.	$= \dfrac{21\ 000}{100}\ \mathbf{kg}$
Divide by 100.	$= \mathbf{210\ kg}$

b   **Method 1**

	4.7% of \$424.75
Change to a decimal.	$= \mathbf{0.047 \times \$424.75}$
Do the multiplication.	$= \mathbf{\$19.96325}$

Method 2

	4.7% of \$424.75	424.75
Write 4.7% as $\frac{4.7}{100} = \frac{47}{1000}$.	$\dfrac{47}{1000} \times \$424.75$	$\times\ 47$
		297 325
Multiply 47 by 424.75.	$= \$\dfrac{19\ 963.25}{1000}$	**1 699 000**
	$= \mathbf{\$19.96325}$	**19 963.25**

Answer to both methods

Write the amount to two decimal places.   **4.7% of \$424.75 is \$19.96.**

Some people prefer to use the decimal method for all cases to avoid the change to a denominator of 1000 shown in Method 2 for part **b** of Example **8**.

Exercise 10.2 — Percentages, fractions and decimals

1 Change each of the following to a percentage.

   a $\frac{15}{20}$ **b** $\frac{15}{30}$ **c** $\frac{15}{40}$ **d** $\frac{17}{50}$ **e** $\frac{4}{5}$

   f $\frac{18}{45}$ **g** $\frac{25}{20}$ **h** $\frac{1}{4}$ **i** $\frac{37}{25}$

2 Change each of the following to a percentage.

   a 0.25   **b** 0.37   **c** 0.24   **d** 0.78   **e** 0.23

   f 0.03   **g** 0.2   **h** 0.7   **i** 0.06

3 Change each of the following to a fraction.

   a 45%   **b** 8%   **c** 16%
   d 20%   **e** 48%   **f** 76%
   g 12%   **h** 75%   **i** 30%

4 Change each of the following to a decimal.

   a 15%   **b** 28%   **c** 16%
   d 7%   **e** 40%   **f** 21%
   g 34%   **h** 85%   **i** 98%

5 Find the answers to the following in your head.

   a 50% of 40 kg   **b** 25% of 24 L   **c** 10% of $320
   d 5% of 120 g   **e** 5% of $600   **f** 20% of 82 kg
   g 100% of 50 mL   **h** 1% of 200 km   **i** 25% of 24 kg

6 Find the answers to the following using a calculator.

   a 60% of 500 kg   **b** 30% of 20 g   **c** 45% of $78
   d 34% of 240 mL   **e** 22% of 195 km   **f** 18% of $346
   g 45% of 92 L   **h** 7% of 400 g   **i** 17% of 96 m

7 Change each of the following to a percentage. Give the answer to 1 decimal place.

   a $\frac{56}{90}$ **b** $\frac{112}{120}$ **c** $\frac{59}{60}$

   d $\frac{3}{8}$ **e** $\frac{5}{8}$ **f** $\frac{7}{12}$

   g $1\frac{5}{6}$ **h** $2\frac{13}{40}$ **i** $\frac{18}{35}$

8 Change each of the following to a percentage.

   a 0.05   **b** 0.096   **c** 0.24
   d 1.24   **e** 3.85   **f** 1.6
   g 0.6   **h** 0.004   **i** 2

9 Change each of the following to a percentage. Give the answer to 1 decimal place if necessary.

   a 15 out of 20   **b** 24 out of 60   **c** 13 out of 25
   d 17 out of 27   **e** 58 out of 130   **f** 3 out of 7
   g 11 out of 12   **h** 27 out of 32   **i** 46 out of 340

10 Change each of the following to a fraction.

   a 245%   **b** 163%   **c** 145%

   d 0.8%   **e** 140%   **f** 360%

11 Change each of the following to a decimal.

   a 4.6%   **b** 13.8%   **c** 126%

   d 42.5%   **e** 18.4%   **f** 186%

   g 9.42%   **h** 109%   **i** 53.6%

12 Calculate the answers to the following in your head.

   a 75% of $180   **b** 60% of 35 kg   **c** 40% of 28 L

   d 80% of 240 g   **e** 15% of 320 km   **f** 3% of $600

   g 9% of 800 mL   **h** 45% of $800   **i** 75% of 690 m

13 Use written methods to find the answers to the following.   *See* Example 8

   a 38% of $42   **b** 12% of 500 g   **c** 18% of $4.50

   d 17% of 28 km   **e** 4.5% of 240 L   **f** 12.5% of 524 km

   g 120% of $340   **h** 7.6% of 4.9 L   **i** 46% of 59 m

   j 13% of $45.57   **k** 11% of $68.79   **l** 117% of $600

14 Find the answers to the following using a calculator.

   a 11.4% of 530 mL   **b** 128% of 465 kg   **c** 13.4% of $824

   d 6.3% of 19.8 mg   **e** 40% of 521.4 m   **f** 0.8% of $3025

   g 125.9% of 340 km   **h** 16.5% of 28 mL   **i** 37.5% of $88

Problem solving

15 A chemist analyses 28 g of an alloy and finds that it contains 7 g of tin, 14 g of lead and 7 g of other metals. Work out the percentage of lead and the percentage of tin in the alloy.

See Example 4

16 A science pad has 45 graph pages and 80 ruled pages.

   a How many pages does it have altogether?

   b What percentage are graph pages?

   c What percentage are ruled pages?

   d What is the total percentage?

17 The use of space in the City of Melbourne for offices, manufacturing, retail and residential use is about 7.2 km^2, 1.2 km^2, 1.3 km^2 and 7 km^2. The total area of the city of Melbourne is about 37.6 km^2.

   a Find the percentage of the total area devoted to each of these uses.

   b Would you expect the percentages to be similar in Kalgoorlie?

18 A vegetable grower picked 500 heads of lettuce, but found that 130 had too much snail damage and had to be composted. Of the ones sent to the market, only 300 were sold. By the next day the remainder were too wilted to sell.

   a What percentage of the lettuce was ruined by snails?

   b What percentage wilted?

   c What percentage was sold?

19 Lucy and Ciara are twin sisters who go to different schools. Lucy scored 58 on her Maths test and Ciara only got 47.

Lucy's test was out of 90 and Ciara's was out of 70. Use percentages to work out who performed better in Maths.

20 A fertiliser has 37% superphosphate. How much superphosphate is needed by the manufacturer to make 70 tonnes of fertiliser?

21 A taxpayer pays 35% income tax. How much tax does she pay on $31 000?

22 Hydrochloric acid is diluted to a concentration of 0.0045% in swimming pool water. How much acid should there be in a backyard pool that holds 42 000 L of water?

10.3 Money and percentages

When you purchase something from a shop you will probably have GST included in the price. The **Goods and Services Tax (GST)** is currently set at 10% in Australia. This means that whenever you buy goods or pay for a service, you must pay an extra 10%. This helps the government to pay for roads, schools and so on. The shop does not keep the GST – it must be sent to the Australian Tax Office. The amount of GST paid for an item is shown on the shop's receipt. Ex-GST refers to the price of goods before the GST has been added.

Example 9

Ellen was sold a silver Pandora bracelet that the shop priced at $80 (ex-GST).

a How much GST will need to be added to the price?

b What does Ellen pay for the bracelet?

Solution

a Write the calculation in words.

Do this in your head by thinking 10% is $\frac{1}{10}$

Write the answer.

$$\textbf{GST} = \textbf{10\% of \$80}$$
$$= \frac{\textbf{1}}{\textbf{10}} \times \textbf{\$80}$$
$$\textbf{GST} = \textbf{\$8.00}$$

b Add the GST to the ex-GST price.

Final price $=$ **$80 + GST**
$= $80 + 8
$= 88

Write the answer.

Ellen paid $88 for the bracelet.

For every $100 of the retailer's price, $10 extra has to be paid by the customer, so the customer actually pays $110, and $10 of this goes to the government. An item with an ex-GST price of $100 would have $10 GST added. The customer would pay $110, and $10 of this would be GST. An item with an ex-GST price of $40 would have $4 added, so $4 out of the final price of $44 would be GST. For something with an ex-GST price of $300, $30 GST would be paid. Out of the final price of $330, $30 would be GST. The fractions would be $10 out of $110, $4 out of $44 and $30 out of $330. All of these work out to be $\frac{1}{11}$ of the total.

Important!

Goods and Services Tax (GST)

The GST is currently set at 10% in Australia.

$$\text{Amount of GST} = \frac{\text{price (Excluding GST)}}{10}$$

$$\text{Amount of GST} = \frac{\text{price (Including GST)}}{11}$$

Example 10

Brian purchased an iPad for $1078 (including GST).
a How much GST had Brian paid?　　　**b** How much does the shopkeeper get?

Solution

a Write the calculation in words.

$$\textbf{Amount of GST paid} = \frac{\textbf{1}}{\textbf{11}} \times \frac{\textbf{Price}}{\textbf{1}}$$

Multiply the price by $\frac{1}{11}$.

$$= \frac{\textbf{1}}{\textbf{11}} \times \frac{\textbf{1078}}{\textbf{1}}$$

Simplify.

$$= \frac{\textbf{1}}{_1\textbf{11}} \times \frac{\textbf{1078}^{98}}{\textbf{1}}$$

$$= \$98$$

Write the answer.

Brian paid GST of $98.

b Subtract the amount for the government.

Shopkeeper's amount $=$ **Final price – GST**

Put in values.

$= $1078 – 98

Work out the answer.

$= 980

Write the answer.

The shopkeeper gets $980, and the other $98 is sent to the Tax Office.

When you buy something from a shop, you might pay less than the price marked on the item. To attract buyers shopkeepers frequently sell items for a **discount**. Discounts may be expressed as amounts or percentages.

Example 11

A shirt marked at $49 is sold at a sale for $37.
a What is the discount?
b What is the percentage discount?

Solution

a Identify the amounts.

Marked price = $49, Price paid = $37

Work out the discount.

$$\textbf{Discount} = \textbf{Marked price} - \textbf{Price paid}$$
$$= \textbf{\$49} - \textbf{\$37}$$
$$= \textbf{\$12}$$

b Identify the amounts.

Discount = $12, Original amount = $49

Calculate the percentage.

Percentage discount = $12 out of $49

Remember to multiply by 100%.

$$= \frac{\textbf{12}}{\textbf{49}} \times \textbf{100\%}$$

Enter as: 12 ÷ 49 × 100 = .

```
12÷49×100
           24.48979592
```

Round the answer.

The percentage discount is about 24.5%

You already know that 20% is the same as $\frac{1}{5}$. This means that to work out 20% of $420 you only need to divide by 5. So 20% of $420 is $420 ÷ 5 = $84.

Example 12

An office supplier is selling a computer desk for $1200 (Including GST).
Calculate the new price of the desk if it is discounted by
a 25%   **b** 20%   **c** 5%

Solution

a Write the percentage as a fraction.

$$\textbf{25\%} = \frac{\textbf{1}}{\textbf{4}}$$

Calculate the discount.

Discount = $1200 ÷ 4

Find the answer.

= $300

Work out the price.

New price = $1200 − $300

= $900

Write the answer.

The discounted price is $900.

b Write the percentage as a fraction. $\qquad$ $20\% = \dfrac{1}{5}$

Calculate the discount. $\qquad$ **Discount = \$1200 ÷ 5**

Find the answer. $\qquad$ **= \$240**

Work out the price. $\qquad$ **New price = \$1200 – \$240**

$\qquad$ **= \$960**

Write the answer. $\qquad$ **The discounted price is \$960.**

c Write the percentage as a fraction. $\qquad$ $5\% = \dfrac{1}{20}$

Calculate the discount. $\qquad$ **Discount = \$1200 ÷ 20**

Find the answer. $\qquad$ **= \$60**

Work out the price. $\qquad$ **New price = \$1200 – \$60**

$\qquad$ **= \$1140**

Write the answer. $\qquad$ **The discounted price is \$1140.**

Example 13

At a boat show, a catamaran that usually costs \$25 000 (Including GST) has 15% off. What is the boat show price?

Solution

Write the calculation in words. $\qquad$ **Discount = 15% of \$25 000**

Change to a mathematical calculation. $\qquad$ **= 0.15 × \$25 000**

= \$3750

Determine the discounted price. $\qquad$ **= \$25 000 − \$3750**

= \$21 250

Write the answer. $\qquad$ **The catamaran costs \$21 250 at the show.**

You can use simple ratios or rates to compare the value of different brands or package sizes of the same things. You usually work out the cost of the same amount of the items you want to compare. This is called a **unitary method**.

Important!

Unitary method and 'best buy'

To use the unitary method, a ratio or rate is used to work out the value for a fixed amount (usually 1) of another.
To compare prices to find the best buy, you usually work out what 1 kg or 100 g would cost. You can also use other easy amounts.

Example 14

You can buy vacuum-packed sliced ham in 150 g or 250 g packets. The 150 g packets have about 5 slices and cost $4.40. The 250 g packets cost $6.20. Which is the best buy?

Solution

In this case, you can see immediately that both packets have a simple multiple of 50 g in them.

Work out the cost of 50 g for the 150 g packet.	**Cost of 50 g for 1st packet $= \$4.40 \div 3$** **$\approx \$1.47/50\,g$**
Work out the cost of 50 g for the 250 g packet.	**Cost of 50 g for 250 g packet $= \$6.20 \div 5$** **$= \$1.24/50\,g$**
Compare the cost.	**The 250 g packet is cheaper for 50 g.**
Write the answer.	**The 250 g packet is the best buy, provided that the ham is not wasted.**

In Example **14**, it would be 'false economy' to buy the 250 g packet if you wanted to make lunch for only one person for a couple of days because the ham wouldn't all be used. If you were going to make lunches for a week, you would need to be sure that it wouldn't go off before you used it all.

Example 15

A supermarket sells cans of condensed milk in two sizes. A 375 ml can costs $2.28 and a 410 ml can is priced at $2.56. Which is the best buy?

Solution

To compare, work out the price per litre for each size of can.

Convert the 375 ml can to litres by dividing by 1 000.   **375 ml = 0.375 L**

Work out the cost for 1 L by division.   **= \$2.28/0.375 L**

= \$6.08/L

Convert the 410 ml can to litres by dividing by 1 000.   **410 ml = 0.410 L**

Work out the cost for 1 L by division.   **= \$2.56/0.410 L**

≈ \$6.24/L

Compare the cost.   **The 375 ml can is cheaper for 1 L.**

Write the answer.   **The 375 ml can is the best buy.**

Exercise 10.3 — Money and percentages

1 Work out the GST for each of the following items.

	Item	Price (Ex-GST) $
a	Notebook computer	500.00
b	LCD Television	1700.00
c	Vacuum cleaner	350.00
d	Micro Hi-Fi system	88.00
e	Chest Freezer	312.00
f	Microwave Oven	102.00
g	Cordless Telephone	27.50
h	Mattress	317.50
i	In-car GPS Navigator	119.50
j	Hair cut	22.50

2 An electrical store is selling a home theatre package for \$9900 (Including GST).
Calculate the new price of the home theatre package if it is discounted by

a	25%	**c**	20%	**e**	30%	**g**	15%
b	50%	**d**	10%	**f**	5%		

3 Work out the discount for each of the following items.

	Item	Marked price $	Discount Price $
a	Card table	25.00	12.00
b	LCD TV	500.00	451.00
c	Old-model cleaner	235.00	188.00
d	Toy castle set	25.00	18.00
e	Filing cabinet	175.00	147.00
f	Surfboard	860.00	652.00
g	Damaged book	45.00	27.00
h	Mattress	450.00	317.50
i	CD player	80.00	62.08
j	Large soft drink	3.20	2.87

Fluency

See Example 9

4 Calculate the price of the following items after GST has been included.

	Item	Price (Ex-GST) $
a	Aluminium ladder	100.00
b	Painting	1200.00
c	Bread maker	160.00
d	Binary clock	18.00
e	Outdoor barbeque	346.00
f	Radio controlled car	117.00
g	Wireless router	119.50
h	Mattress	517.50
i	Rechargeable batteries	18.50
j	Gardening services	92.50

See Example 13

5 Work out the discounted price for each of the following items.

	Item	Marked price $	Discount percentage off %
a	Outdoor table	225.00	50
b	Kayak	900.00	25
c	Diamond ring	2050.00	10
d	Coin collection	255.00	20
e	Kite surfer	1175.00	5
f	Floor rug	965.00	35

See Example 14

6 Which is the best buy?

a 300 g of toothpaste for $4.49 or 450 g of toothpaste at a cost of $5.90

b 650 g of peanut butter for $6.99 or 750 g of peanut butter priced at $7.85

c A box or 15 rewriteable DVDs for $4.99 or a spool of 100 rewriteable DVDs for $29.90

d 645 g of premium mince for $7.89 or 956 g of premium mince costing $10.16

e A pack of two 300 g tins of deodorant for $5.13 or one 750 g tin of deodorant priced at $6.87

See Example 10

7 Work out the GST for each of the following items.

	Item	Price (Including GST) $
a	Astronomy book	99.00
b	DVD box set	167.20
c	Sheet set	40.15
d	Outdoor Christmas lights	463.43
e	Ornament	495.55
f	Amplifier	530.09

Worked solution

Exercise 10.3

MAT07NAWS00041

Problem solving

See Example 13

8 A jewellery store is having a 12% off sale. Determine the sale price of a pair of sapphire earrings with a price tag of $299.

9 John found a sports store with a $23\frac{1}{2}$% off sale on all shoes. Work out the sale price David would have to pay for a pair of sports shoes originally priced at $319.

10 A pair of jeans have been discounted from $84.50 to $67.60. Calculate the discount as a percentage of the original price.

See Example 11

11 Determine the percentage discount for a pair of sunglasses originally priced at $109.95, reduced to $60.50.

12 A butcher sells a 560 g packet of premium beef mince for $5.89 and a 1.2 kg packet of premium mince priced at $12.98. Which is the best buy?

See Example 15

13 Which is the better buy? A 390 ml bottle of soft drink priced at 75c or 1.5 L of the same soft drink priced at $2.60.

Reasoning

14 To calculate the amount of GST added to an item you divide the Ex-GST cost of the item by 10. Remember 10% is $\frac{1}{10}$ as a fraction. If the GST was increased to $12\frac{1}{2}$%, what fraction of the Ex-GST cost would you use to determine the amount of GST?

15 A supermarket has a special on beauty products of 25% off and if you use a rewards card you get a further 20% off. Rita purchased $128 of beauty products.

Worked solutions

Exercise 10.3

MAT07NAWS00041

  a Calculate the discount price if Rita did not use her rewards card.
  b Calculate the discount price if Rita did use a rewards card.
  c Why would it be false advertising for the supermarket to display a sign saying 'Get a total of 45% off all beauty products if you use your rewards card'?

16 Graeme receives a 20% discount on a jacket for sale at a clothing store. Graeme is a VIP member of the store which entitles him to a further 20% off the already discounted jacket.

  a If the jacket was originally priced at $95, determine his overall discount.
  b Explain why his overall discount is not 40%.

17 Show that two discounts of 30% do not give a total discount of 60%.

18 After getting a discount of 10%, a customer paid only $270 for a new outfit.

  a What was the discount? Explain your method. Hint: If the price was $10, what would the discount be?
  b What is this as a fraction of the final price?

19 At a 20%-off sale, Jane bought a pair of designer jeans for only $144. How much did she save? Explain your reasoning.

20 Peter bought a second-hand desk for only $289 after getting a discount for cash of 15%. Work out the original price, and explain how you work it out.

21 Lloyd traded in his van for $420. He was offered a moped at a 25% discount for the amount he got for the van. He said 'No way man, I need more than $105 off'. Explain how he got $105 and what is wrong with his working out.

22 To calculate the amount of GST that has already been added to an item you divide the cost of the item by 11. If the GST was increased to $12\frac{1}{2}\%$, what would you divide the cost (Including GST) by to determine the amount of GST?

23 David wants to buy some mince to make Spaghetti Bolognese for his Mum's birthday. There are plenty of spices, tomato paste, tomatoes and spaghetti at home for the recipe. His sister is making the dessert, and their baby brother will only have a little bit of strained sauce. Dad is buying the birthday cake, so everyone in the family (except the baby) is doing something. At the supermarket after school, there are only a few packets of mince left. There are 3 trays of 250 g for $2.10, 3 trays of 500 g for $3.90, and 2 trays of 1.5 kg for $7.20. What should David buy? Explain your reasoning.

24 Suellen has persuaded her mother to let her paint her bedroom walls herself in whatever colour she wants. She has to pay for the paint out of her own money. Suellen has a normal-sized kid's bedroom (2.6 m by 2.5 m) and the ceilings are the standard 2.4 m height. She can buy the paint in 1 L tins for $15, 2 L tins for $29, 4 L tins for $56 and 10 L tins for $120. On the tins it says the coverage is 11 m²/L, but she has never painted a wall before so will probably waste some. Explain what she should buy.

- Purchases at a shop may be paid by cash, cheque or EFTPOS (debit or credit card). Cash payment is rounded to the nearest 5 cents. EFTPOS payment may incur an extra charge.
- The **bill** is the total amount to pay. When paying in cash, the **amount tendered** is the money you give to pay. The **change** is the difference between the bill and the amount tendered.
- **Cash-out** is extra money withdrawn from your account in cash at a checkout.
- A **percentage** is a number written as a fraction with a denominator of 100, but the 100 is replaced by the % sign.
- A fraction or decimal can be changed to a percentage by multiplying by 100%.
- Some common percentages are 1% ($\frac{1}{100}$), 5% ($\frac{1}{20}$), 10% ($\frac{1}{10}$), 20% ($\frac{1}{5}$), 25% ($\frac{1}{4}$), 50% ($\frac{1}{2}$), 75% ($\frac{3}{4}$) and 100% (1).
- To find a percentage of an amount, the percentage is changed to a fraction or decimal and multiplied by the amount.
- A **discount** is an amount taken off a normal price.
- The **Goods and Services Tax** (**GST**) is currently set at 10% in Australia and is paid to the government.
- The amount of GST can be found by using either of these two formulas.

$$\text{Amount of GST} = \frac{\text{Price (Excluding GST)}}{10}$$

$$\text{Amount of GST} = \frac{\text{Price (Including GST)}}{11}$$

- When doing money problems, it is often helpful to write the calculation in words before doing the mathematics.
- To compare prices to find the **best buy**, you usually work out what 1 kg or 100 g would cost. You can also use other easy amounts.

Chapter 10 review

Understanding

1 What will four tins of salmon cost at $2.47 each?

2 What will the cash price be for three chocolates costing $2.43 each?

3 What change will you get from a $50 note when you tender it for a bill that comes to $35.72?

4 Change each of the following to percentages.

   a $\frac{13}{20}$ **b** $\frac{4}{5}$ **c** 0.46   **d** 0.8

5 Change to fractions.

   a 55%   **b** 30%   **c** 6%

6 Change to decimals.

   a 68%   **b** 5%   **c** 40%

7 Calculate the answers to the following in your head.

   a 25% of $60   **b** 50% of 38 kg

8 Find the answers to the following using a calculator.

   a 12% of 450 L   **b** 28% of $75

9 Bridie went to a sale and bought a T-shirt with a sticker showing an original price of $18 reduced to $7. What discount did she get?

10 Work out the discount price for a $360 desk that is reduced by

   a 20%   **b** 5%

Fluency

11 Change each of the following to percentages.

   a $\frac{7}{16}$ **b** $2\frac{7}{8}$ **c** 1.45

   d 0.067   **e** 42 out of 60   **f** 14 out of 30

12 Change to fractions.

   a 205%   **b** 0.6%

13 Change to decimals.

   a 5.45%   **b** 125%

14 Calculate the answers to the following in your head.

   a 70% of 500 km   **b** 75% of 84 kg   **c** 3% of $5000

15 Find the answers to the following using written methods.

   a 32% of 820 mm   **b** 16.5% of $217

16 Find the answers to the following using a calculator.

   a 112% of $529.40   **b** 6.3% of 455 g

17 Jemima got a gold necklace at a 15% off sale. The normal price was $45. How much did she pay?

18 Which is the better buy? 600 ml of milk priced at $1.15 or 2 litres costing $3.46?

19 A pair of jeans with a marked price of $95 is sold at a discount of 15%. What is the selling price?

20 Kylie works on a commission of 8%. What does she get for selling goods worth $980?

See Example 13

21 A graphics calculator has a price of $140 pre-GST.

   a What is the GST?   **b** What is the final price?

Problem solving

22 Work out the total bill and change from $50 for the following purchases at a bakery.

See Example 1

- three loaves of bread @ $1.87 each
- six bread rolls @ $0.48 each
- four cream buns @ $1.26 each
- three meat pies costing $4.43 each

23 List some appropriate combinations to give $7.90 in change.

24 Xuan bought the following from a home entertainment shop.

- three CDs costing $29.98 each
- headphones costing $49.63
- an MP3-CD player for $87.65

The shop charged her $3.50 to use her debit card for payment. How much did it cost altogether?

25 If 19 people in a class of 28 have a computer at home, what percentage of the class has computers?

26 Tomasina got 14 out of 20 for English and 28 out of 35 for Maths. In which subject did she perform better?

27 11.9% of water is the element hydrogen and the rest is oxygen. What mass of oxygen is present in 15 kg of water?

28 Determine the percentage discount for a pair of bathers originally priced at $89.95, reduced to $62.95.

See Example 11

29 A baker sells a 680 g loaf of bread for $2.98 and a 1.1 kg loaf of the same type of bread priced at $5.98. Which is the best buy?

See Example 15

Reasoning

30 A customer got a loyalty discount of 10% and another discount of 20% at a sale. What was the overall discount? Hint: It is not 30%.

31 A store has discounted all items by 25% during a sale. During the sale, a customer pays $60 for an MP3 player. How much money did the customer save?

11

Solid shapes and measures

Contents

Prior learning
Chapter 11

Parent guide
Chapter 11

Curriculum guide
Chapter 11

Australian Curriculum statements

Using units of measurement
Establish the formulas for areas of rectangles, triangles and parallelograms, and use these in problem solving. (ACMMG159)

Calculate volumes of rectangular prisms. (ACMMG160)

Shape
Draw different views of prisms and solids formed from combinations of prisms. (ACMMG161)

The images in this book are two-dimensional (2D) but we live in a three-dimensional (3D) world. The objects around us are called 3D because they have length, width and thickness. Because you live in a 3D world it is important that you can estimate and calculate quantities such as volume, capacity and mass. This could be a simple estimation of how much milk you need to put on your cereal or something more complex such as calculating how much concrete is needed for the foundations of a house.

■ Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

base	cylinder	octahedron	solid
concave solid	dodecahedron	perpendicular	sphere
cone	edge	Platonic solid	surface area
congruent	face	polyhedron	tetrahedron
convex solid	hexahedron	prism	vertex
cross-section	icosahedron	pyramid	
cube	net	regular	

11.1   Solids, nets and surface area

Here are examples of some solid shapes that you should know.
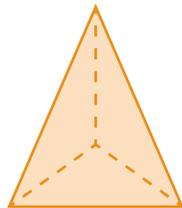
Triangular pyramid
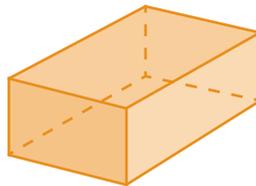
Rectangular prism
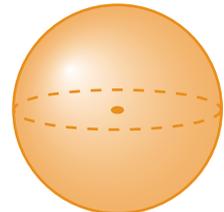
Sphere
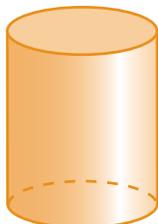
Cylinder
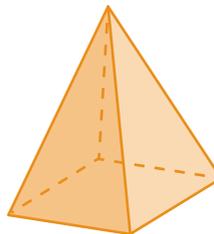
Square pyramid
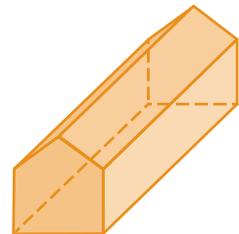
Pentagonal prism

An **edge** is a line of the solid, where two **faces** meet.
A **vertex** is a corner of the solid, where edges meet.
The solid shape in the diagram has six faces, eight vertices and 12 edges.

edge

vertex (plural: vertices)
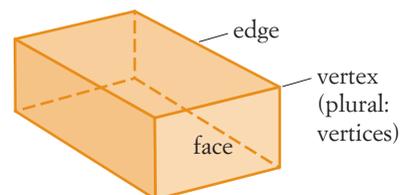
face

These solids are **convex**.
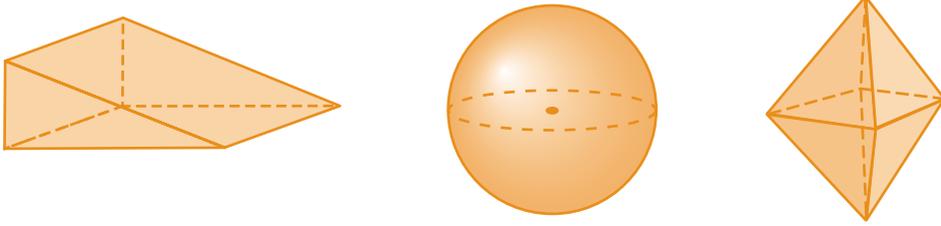
These solids are **non-convex** (or **concave**).
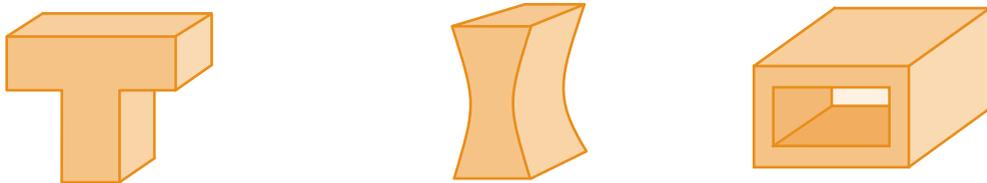
Convex solids have faces that all point, curve or bulge *outwards*. Non-convex solids have some faces that point, curve or cave *inwards*, having 'dents' or 'holes'. A simple test to determine whether a solid is convex or non-convex is to draw straight lines between any two faces on the solid.
If every line that you draw lies completely *inside* the solid, then the solid is convex.
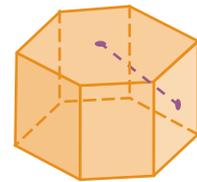
If all or part of any line that you draw lies *outside* the solid, then the solid is non-convex.
A **net** of a 3D shape is a 2D shape that can be folded up to form the 3D shape.
The diagram below shows the net for a box.

Net:
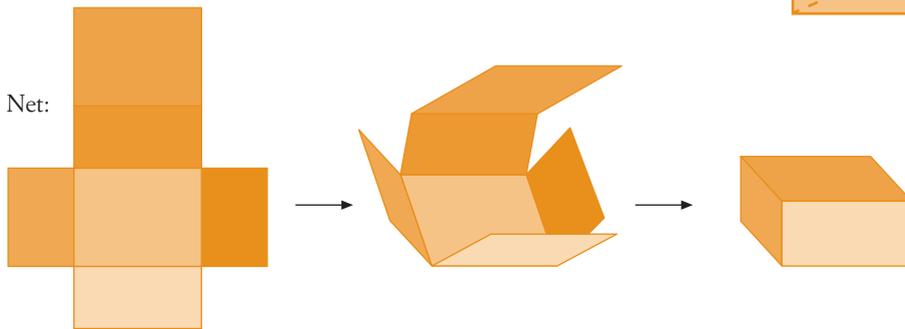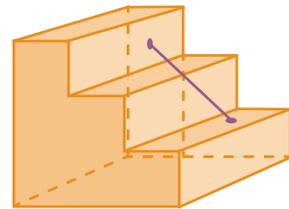
Example 1

Draw a net for this rectangular prism, including measurements.

8 cm

12 cm   4 cm
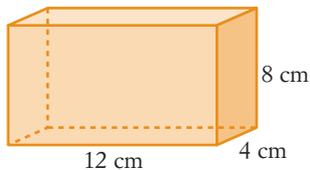
Solution

The rectangular prism has six faces, with the front and back the same (12 cm by 8 cm), top and bottom the same (4 cm by 12 cm), left and right sides the same (4 cm by 8 cm).

The top and back are connected along the 12 cm edge, the bottom is connected to the back along the 12 cm edge, and the front is connected to the bottom along the 12 cm edge.

The left and right faces are connected to the bottom along the 4 cm edge.

Draw each face and connect along the edges mentioned above to make the net.

Write the measurements on the net.

	Top	4 cm
8 cm	Back	8 cm
Left face	Bottom	Right face	4 cm
	Front	8 cm
	12 cm	
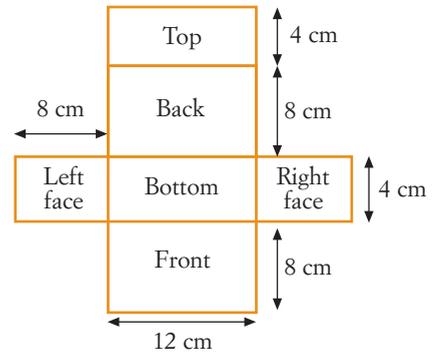
Example 2

Draw a net for this cylinder, including the measurements.

3 m

4 m
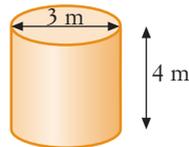
Solution

Imagine cutting the cylinder from the top to the bottom, and then unrolling the curved part. When the curved part is rolled out, it forms a rectangle. The length of the rectangle is the same as the circumference of the circle.

Find the circumference of the circle.

Write the measurements on the net.

$$C = \pi \times D = \pi \times 3 \approx 9.4 \text{ m}$$

3 m

9.4 m   4 m
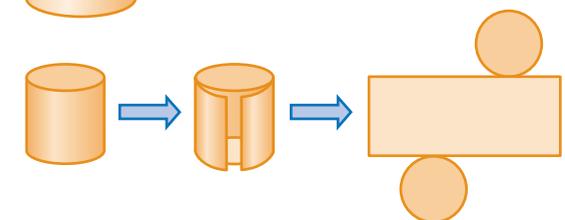
3 m
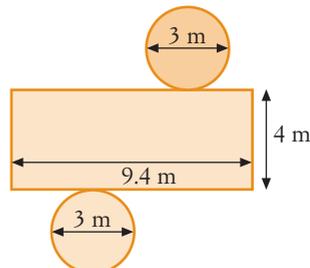
If you need to paint all the faces of a box, it would be useful to know the amount of surface area that is going to be painted.

Important!

Surface area

The **surface area** (**SA**) of a 3D shape is the total area of all the faces. It is calculated by adding the areas of the faces.

Example 3

Find the surface area of this shape.
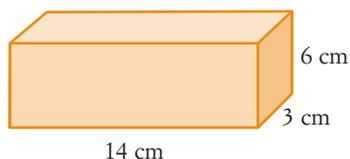
6 cm

3 cm

14 cm

Solution

Draw a net of the shape.

Write the measurements on the net.

	Left	3 cm		
14 cm	Front	Top	Back	Bottom
	6 cm	3 cm	Right	3 cm

6 cm

3 cm
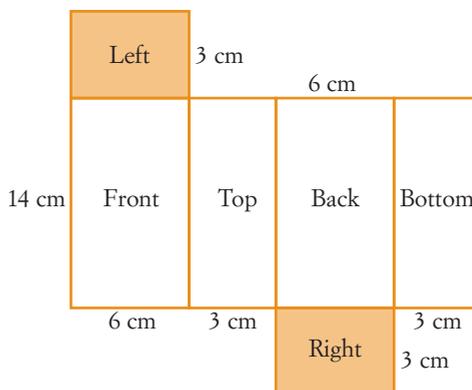
Work out the area of each face.

Left $= l \times w = 3 \times 6 = 18$ **cm^2**

Front $= l \times w = 14 \times 6 = 84$ **cm^2**

Top $= l \times w = 14 \times 3 = 42$ **cm^2**

Back $= l \times w = 14 \times 6 = 84$ **cm^2**

Bottom $= l \times w = 14 \times 3 = 42$ **cm^2**

Right $= l \times w = 3 \times 6 = 18$ **cm^2**

Find the total by adding all the areas.

SA $= 18 + 84 + 42 + 84 + 42 + 18$ **cm^2**

$= 288$ **cm^2**

Exercise 11.1 Solids, nets and surface area

1 **a** Copy the following nets, or print out the worksheet Nets of solids 1 pages from NelsonNet.

b Cut out each net along the solid lines.

c Fold each net along the dotted lines to make a solid, but do not paste the edges together.

d Write the name of each solid on one of the light green faces.

e State if each solid is convex or non-convex.

f Unfold each net and paste it in your workbook on its darker green face.

i

ii

iii

iv
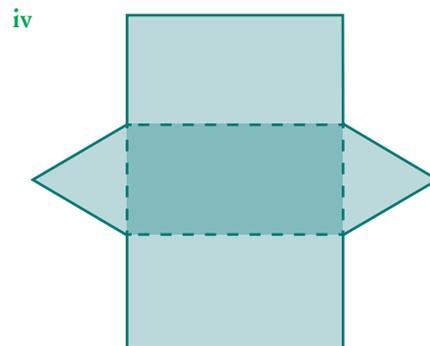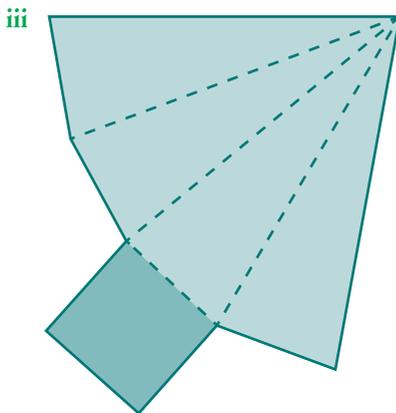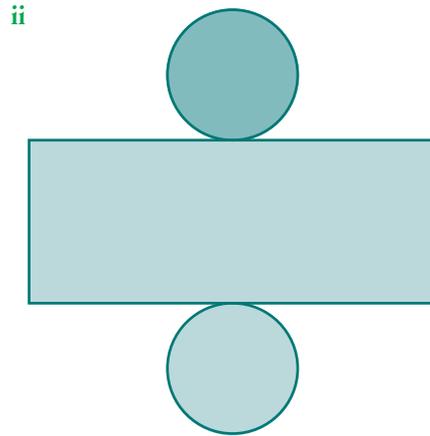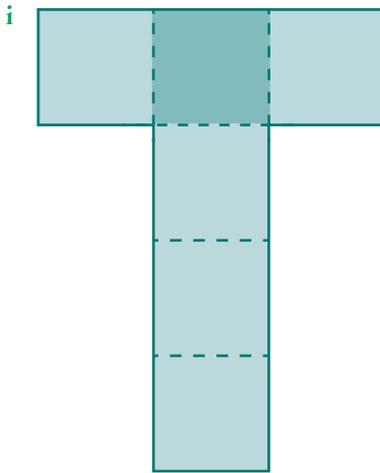
v
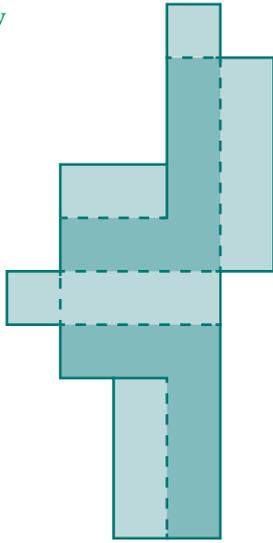
vi
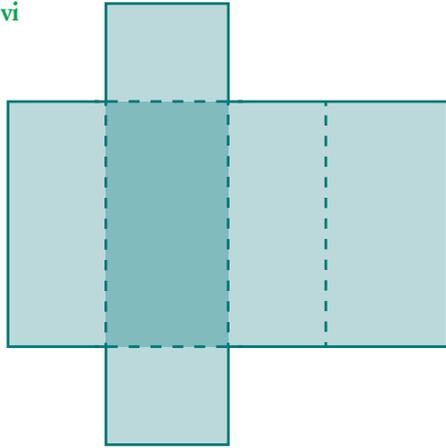
vii
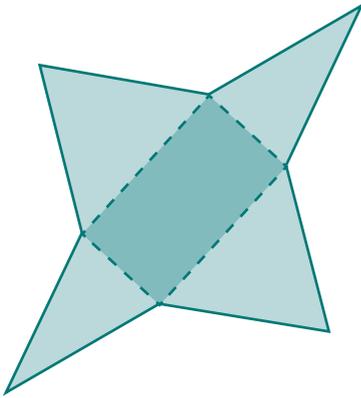
viii
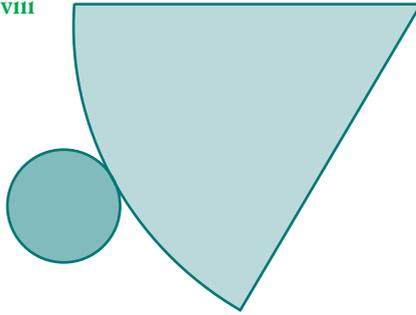
ix
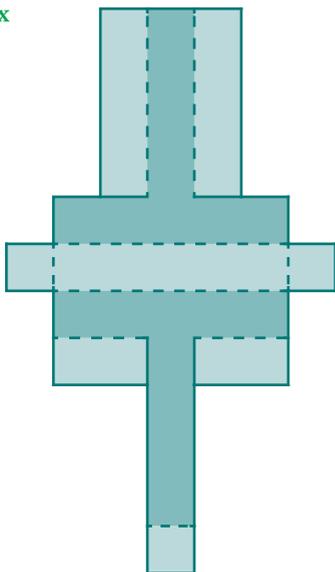
x
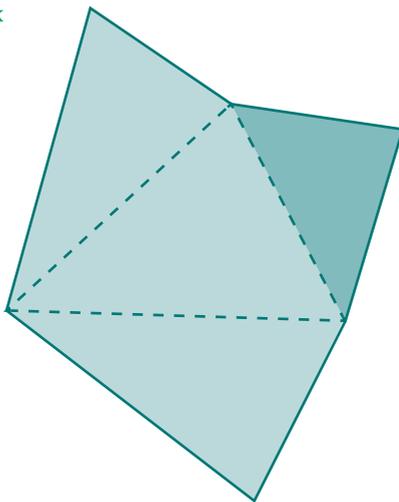
The nets in questions **2** and **3** have tabs for pasting or taping the edges of each solid together. For best results, copy them onto cardboard first.

2 Here is the net for a triangular prism. Using the measurements in the net, copy it to make a model of a triangular prism.

4 cm
4 cm

4 cm   4 cm   4 cm

4 cm   4 cm   4 cm

4 cm   4 cm   4 cm

4 cm   4 cm
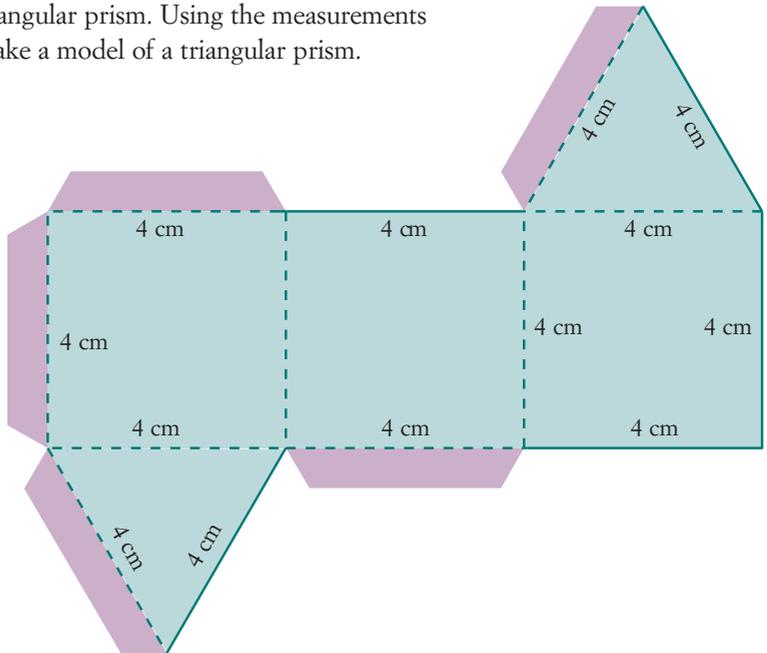
3 **a** Copy the nets below and on the next two pages, or print out the Nets of solids 2 pages.
   b Cut out each net along its boundary.
   c Fold each net along the dotted lines to make a solid.

   i

   tab   tab

   Triangular prism

   tab   tab

   tab

ii

Cube net with tabs

iii

Pentagonal pyramid net with tabs

iv

Triangular pyramid

tab

tab

tab

v

Trapezoidal prism

tab

tab

tab

tab

tab

tab

tab
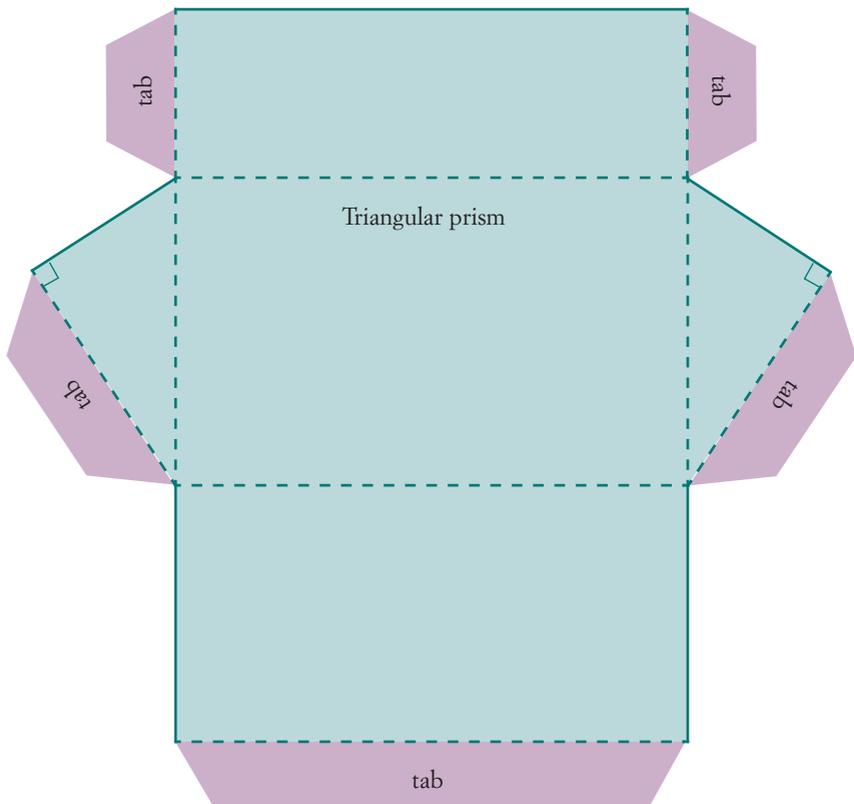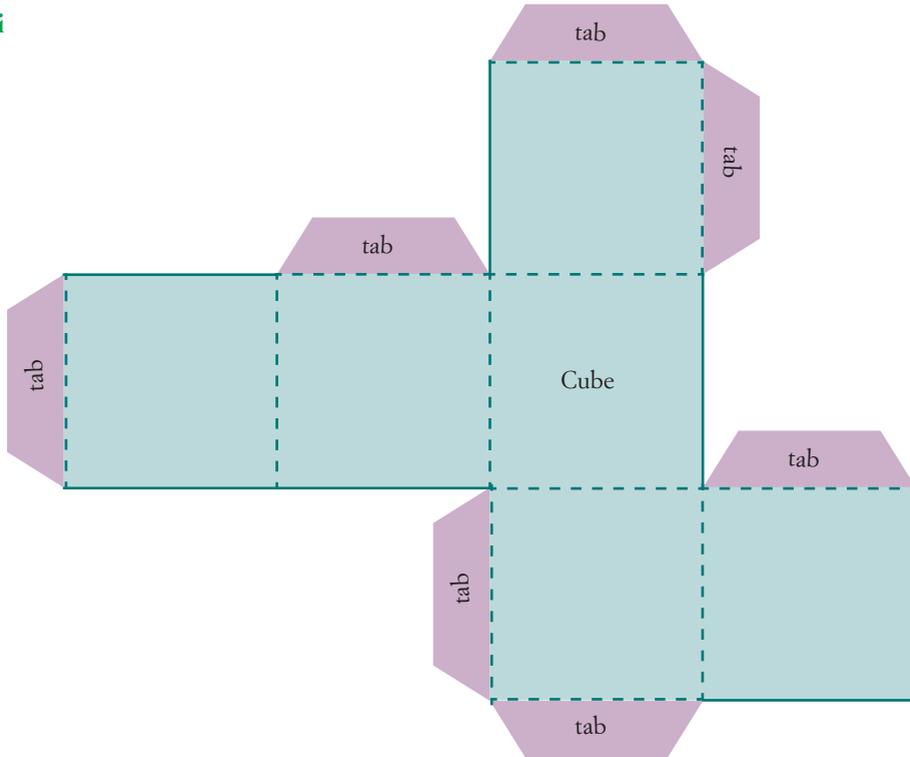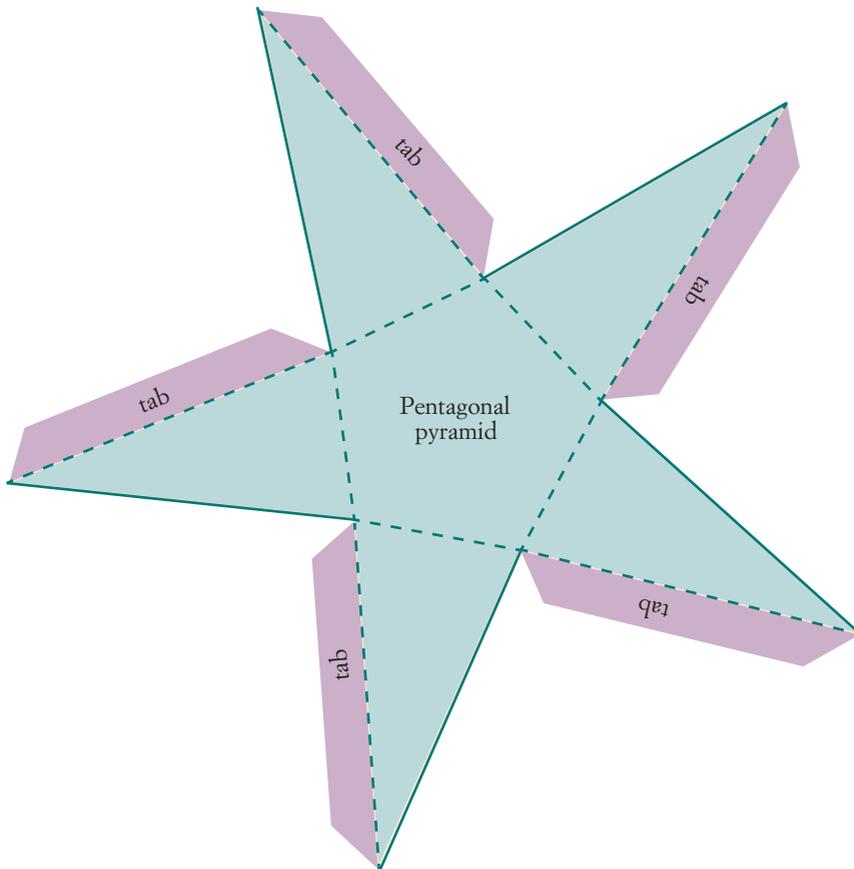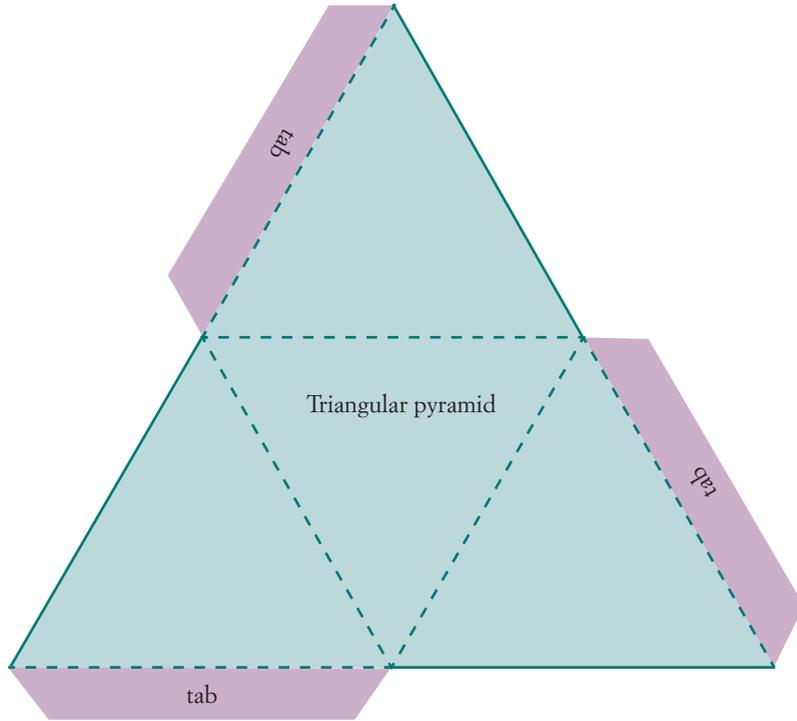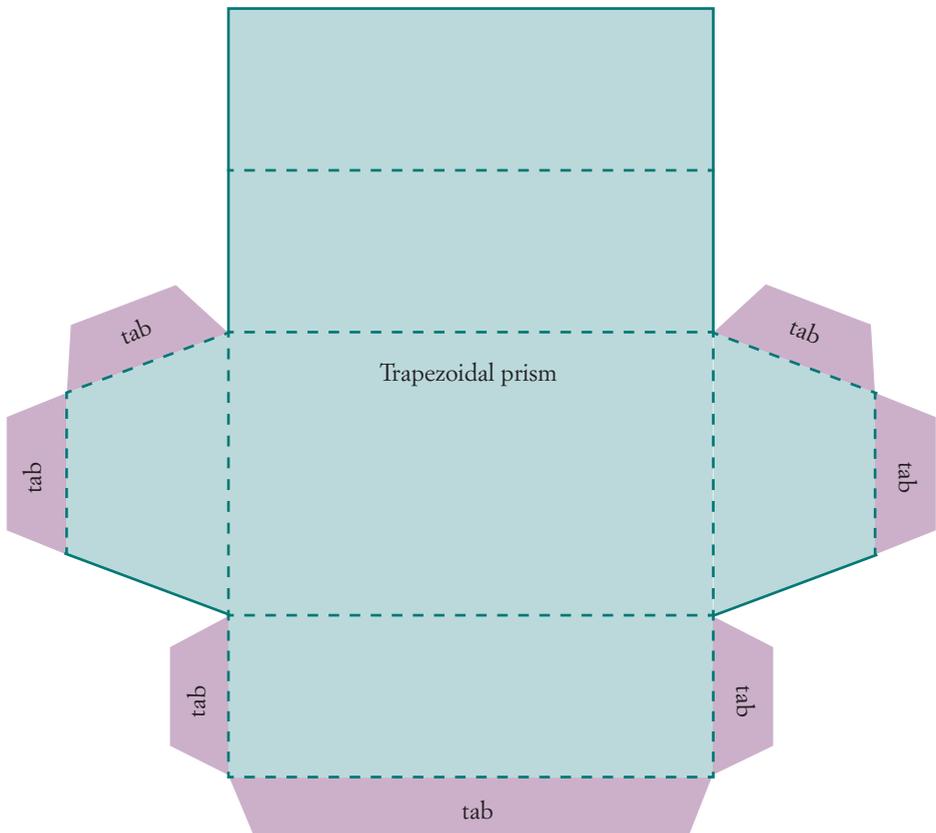
4 Draw nets for each of the following shapes, including measurements.

a

5 cm
3 cm
10 cm

b

120 mm
120 mm
120 mm

c

4.5 m
1.2 m
5.1 m

5 Test whether each of these solids is convex or non-convex.

a

b

c

d

e

f

g

h

i
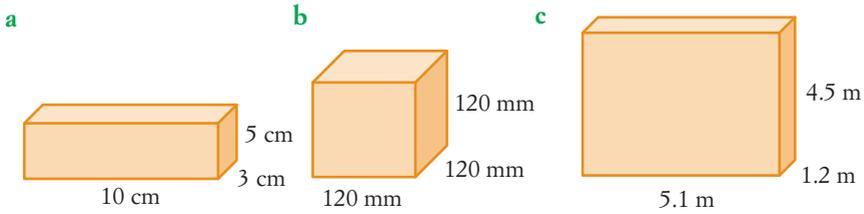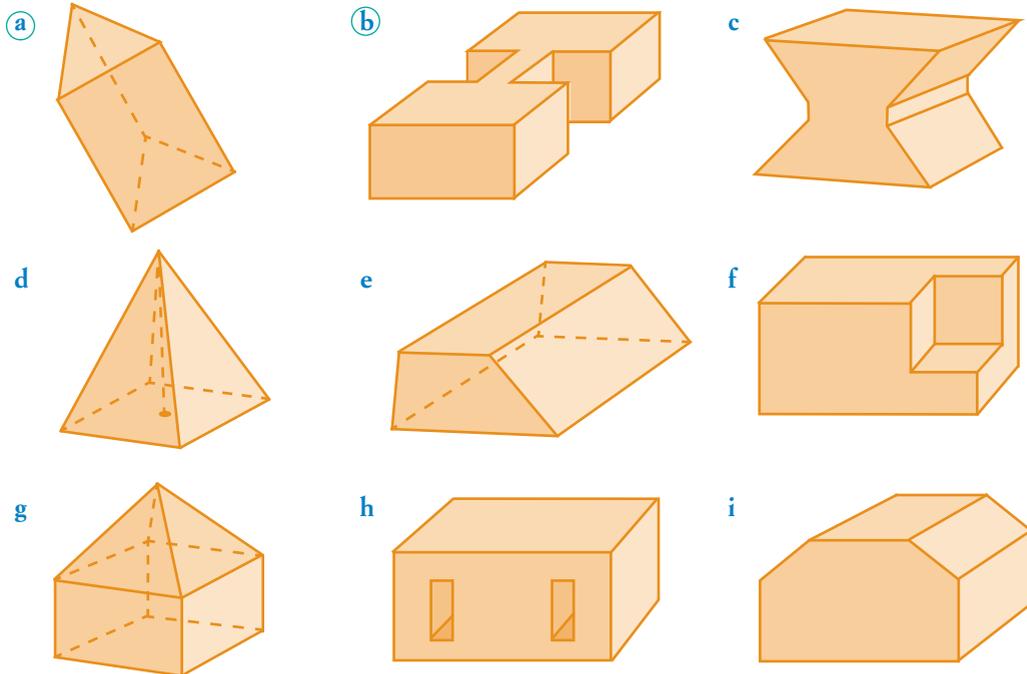
6 Draw nets for each of the following shapes, including measurements.

a

30 m
6 m

b

0.8 m
3 m

c

12 cm
12 cm
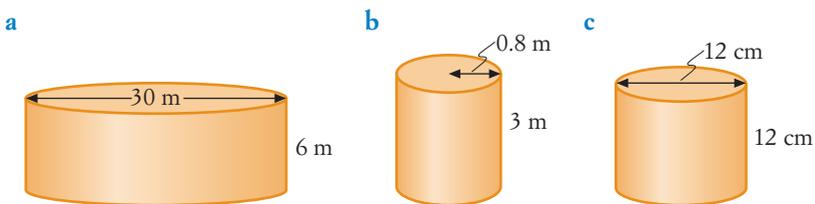
7 Match the correct shape name to each net (**a** to **h**). (Some have more than one net.)

triangular prism   cube   rectangular prism
square pyramid   cylinder   trapezoidal prism

a

b

c

d

e

f

g

h
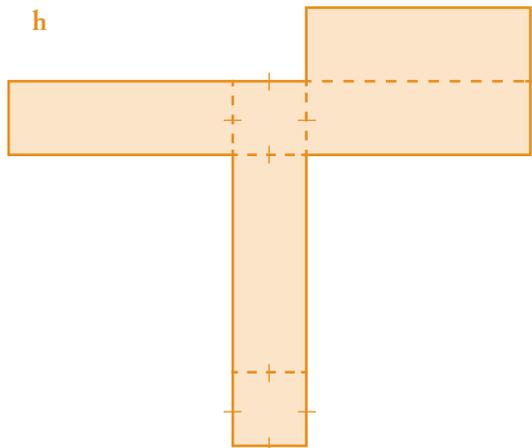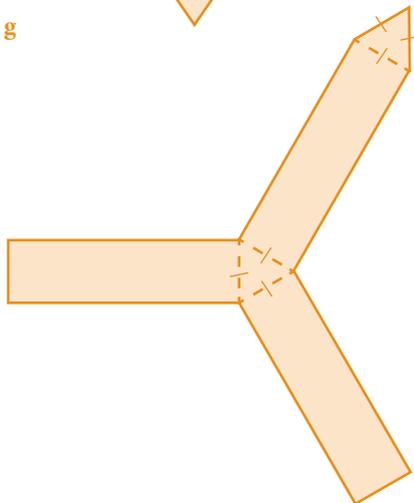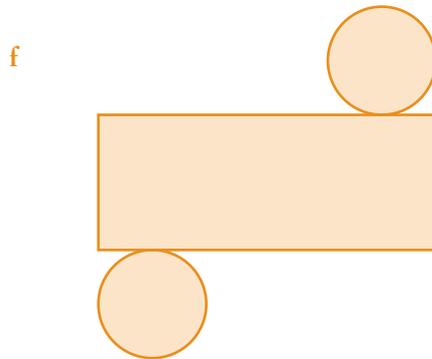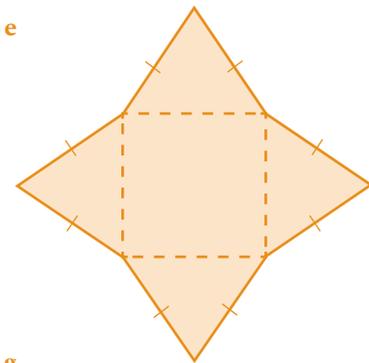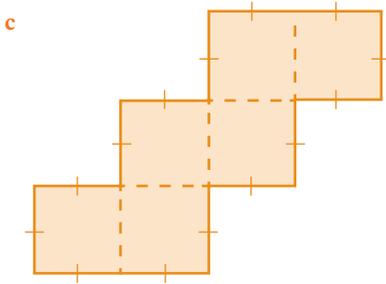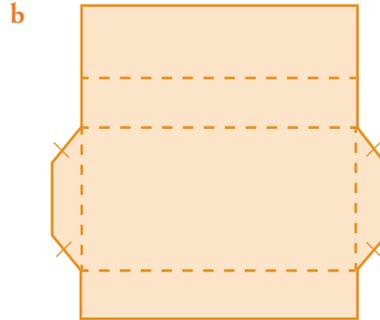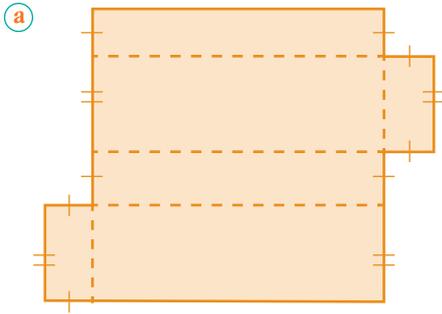
8 Which of the following *cannot* be folded to make a cube? Select **A**, **B**, **C** or **D**.

A   B   C   D
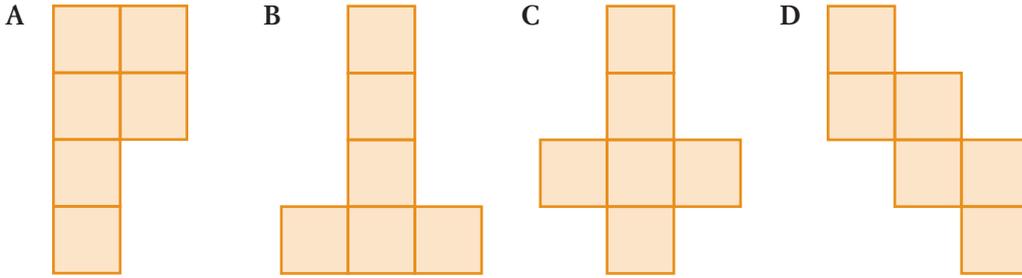
9 Find the surface area of these shapes.

See Example 3
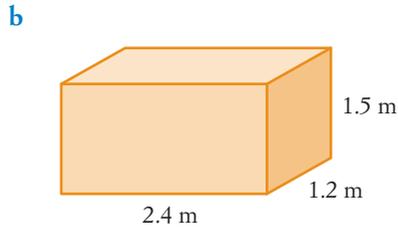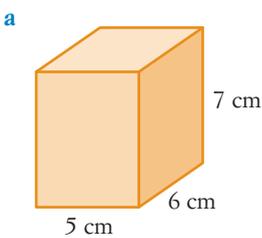
a

7 cm
6 cm
5 cm

b

1.5 m
1.2 m
2.4 m

10 Find the surface area of this shape.
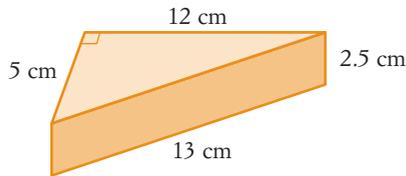
12 cm
5 cm
2.5 cm
13 cm

11.2 Naming and classifying solids

Solids have **flat faces** and **curved faces**.
This cylinder has two flat faces and one curved face.
The flat faces are circles, and the curved face is a rectangle (when flattened).
A hexagonal prism has eight faces. Two of the faces are hexagons and the other six faces are rectangles.
The solid, shown below right, whose faces are all flat is called a **polyhedron**. The plural of 'polyhedron' is '**polyhedra**' or '**polyhedrons**'. A hexagonal prism is a polyhedron. A cylinder, however, is not because it has a curved face.
Just as polygons such as pentagons, hexagons and octagons are named according to the number of sides they have, polyhedra are named according to the number of faces they have.

face
curved face

hexagon
rectangle
face
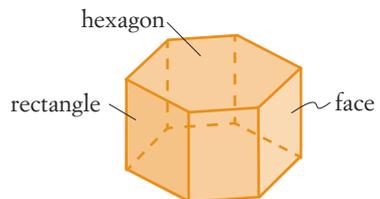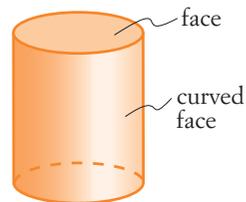
For example, a tetrahedron has four faces (tetra = 4), and a hexahedron has six faces (hexa = 6).
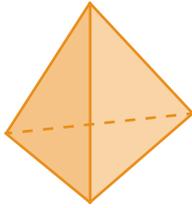
Tetrahedron (4 faces)
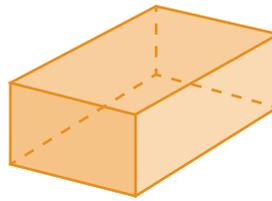
Hexahedron (6 faces)

This table lists the names of some polyhedra.

Polyhedron	Number of faces
Tetrahedron	4
Pentahedron	5
Hexahedron	6
Heptahedron	7
Octahedron	8

Polyhedron	Number of faces
Nonahedron	9
Decahedron	10
Undecahedron	11
Dodecahedron	12
Icosahedron	20

Example 4

What type of polyhedron is the shape shown here?

Solution

Count the faces.　　　　　　　**The shape has 6 faces.**

Refer to the table of polyhedra
names above.　　　　　　　**The shape is a hexahedron.**
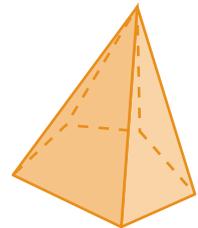
Investigate: Solids around you

Bricks are commonly used in buildings.

- What type of solid is a brick?
- What shape is your classroom?
- What shape are most books?

Imagine that you need to stack oranges in a box.

- Use some oranges or tennis balls or similar shapes to investigate the different ways that a layer of oranges can be arranged.
- Which arrangement seems to waste the least amount of space?

Imagine that you need to stack oranges in piles – this time without a box to provide support.

- Use some oranges or tennis balls or similar shapes to investigate the different ways to stack the oranges.
- Describe the method of stacking that seems to work best.

Visit your local supermarket and look at the different shapes that are stacked on the shelves.

- Which packaging shapes can be stacked on the shelf easily?
- Which packaging shapes are difficult to stack on the shelf?
- Which packaging shape can be stacked most efficiently, i.e., wastes the least space?
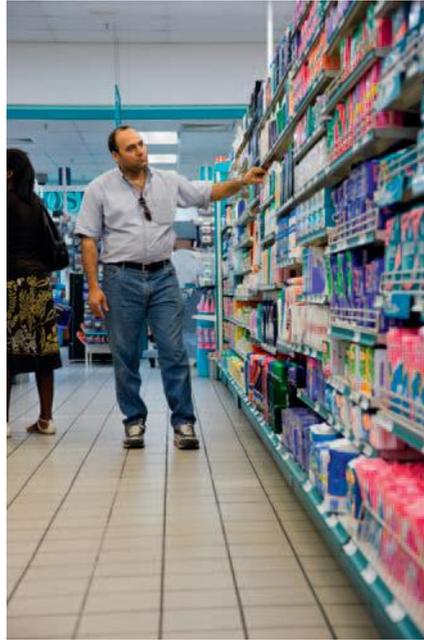
A **cross-section** of a solid is a 'slice' of the solid, cut *across* it, parallel to its end faces, rather than along it. These diagrams show cross-sections of a rectangular prism, a sphere and a square pyramid.
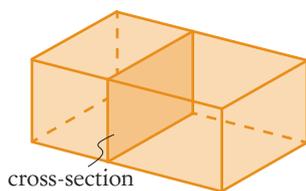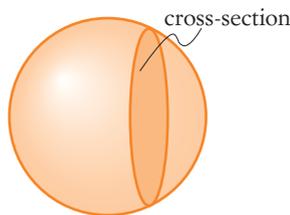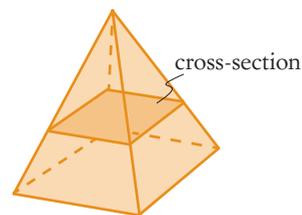
Rectangular prism   **Sphere**   **Square pyramid**

Note that the cross-sections of a rectangular prism are congruent (identical) rectangles, the cross-sections of a sphere are circles (but of different sizes), and the cross-sections of a square pyramid are squares (again, of different sizes).

Prisms and pyramids

Prisms and **pyramids** are special types of polyhedra. A **prism** has the same (uniform) cross-section along its length. Each cross-section is a polygon. Either of the end faces is called the prism's **base**. Prisms take their names from their base.
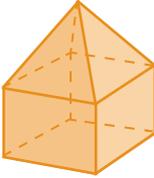
The prism shown below has trapezium-shaped cross-sections, identical and parallel to its base. This is a **trapezoidal prism**.
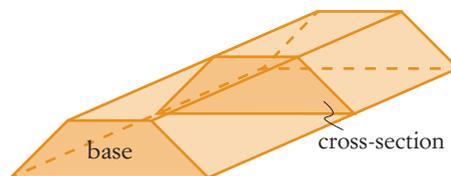
Trapezoidal prism

A **pyramid** has a pointed top called the **apex**. The face opposite the apex is a polygon and is called the pyramid's **base**. A pyramid's cross-sections have the same shape as the base but are not the same size. The pyramid on the right has a triangular base, so it is called a **triangular pyramid**.
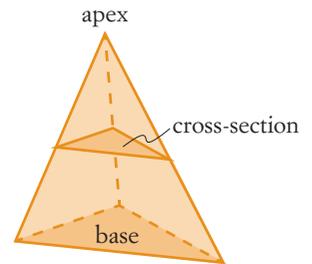
Triangular pyramid

Cylinders, cones and spheres

Cylinders, cones and spheres are not polyhedra because they have curved faces. Although it has a uniform cross-section, a **cylinder** is *not* a prism because its cross-sections are circles, which are not polygons.

Although it has an apex, a **cone** is *not* a pyramid because its base is a circle, which is not a polygon.
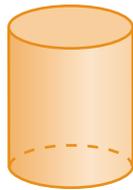
Cylinder   **Cone**

A **sphere** is a perfectly circular solid, the shape of a ball. All of the points on a sphere's surface are exactly the same distance from the centre of the sphere. In this diagram of a sphere, O is the centre and P and Q are points on the surface of the sphere. The distance OP (the radius) is the same as the distance OQ for any points on the surface.
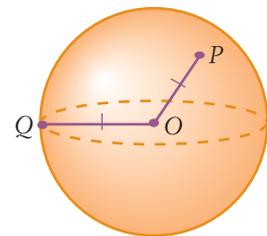
Example   5

What is the full name of this solid?

Solution

Draw a cross-section of the solid.

Name the base (or cross-section).   **The base is a hexagon.**

All cross-sections are identical hexagons.   **The solid is a hexagonal prism.**
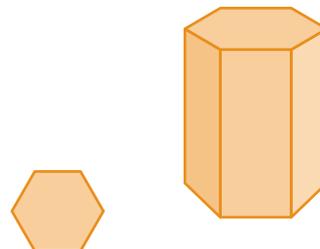
Throughout this chapter, we have seen that solids can be classified in different ways. A solid may belong to one or more of the categories shown below.

Category	Meaning	Example
Polyhedron	Any solid whose faces are all flat	
Prism	A polyhedron with a uniform polygonal cross-section	
Pyramid	A polyhedron with a polygonal base and an apex	
Cylinder	Any solid with identical circular ends and one curved surface.	
Cone	Any solid with a circular base and an apex	
Sphere	A ball shape that is completely round, with all points on its surface the same distance from the centre	
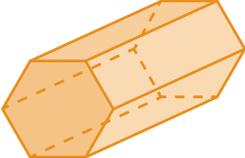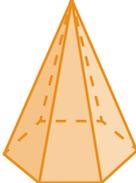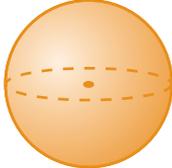
A **regular polygon** is a flat shape with equal sides. For example, a regular hexagon has six equal
sides.

A **regular polyhedron** or **Platonic solid** is a convex polyhedron with identical faces. Every face is
the same regular polygon. For example, a cube is a regular polyhedron because every face is a
square. Because a **cube** has six identical faces, it is also called a **regular hexahedron**.

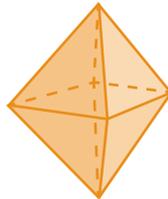There are only five Platonic solids, as listed in the table and drawn below.

Platonic solid	Number of equal faces
Regular tetrahedron	tetra = 4
Regular hexahedron	hexa = 6
Regular octahedron	octa = 8
Regular dodecahedron	dodeca = 12
Regular icosahedron	icosa = 20
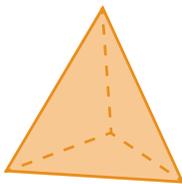
**Tetrahedron
(equilateral
triangular pyramid)** **Hexahedron
(cube)** **Octahedron** **Dodecahedron** **Icosahedron**
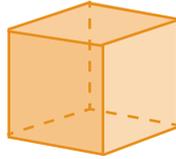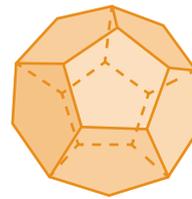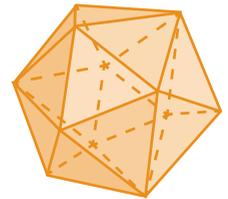
The Platonic solids were discovered by the Greeks in the 5th century BCE. However, ancient
artefacts displaying pictures of these solids have been found in Europe, Egypt, Africa and South
America. It was believed that these shapes had mystical properties. They are named after the
Greek philosopher Plato (427–327 BCE) who claimed that these solids were 'cosmic bodies'
representing the elements.

Investigate: Special solids

For the following investigation, work in pairs or small groups and use the Internet or
library to undertake any research required.

• Plato associated each of the Platonic solids with one of five fundamental elements. Match
each of the Platonic solids in Column A with an element from Column B.

Column A	Column B
Tetrahedron	Cosmos (divine force)
Cube	Water
Octahedron	Fire
Icosahedron	Earth
Dodecahedron	Air

Another Greek philosopher and mathematician named Archimedes studied Plato's work and
developed another set of solids based on the Platonic solids. These were named the
Archimedean solids (polyhedra) in his honour.

- How many Archimedean solids are there?
- Describe the features of Archimedean solids.
- Research the nets of Archimedean solids.

Many other types of solids have been studied in the field of mathematics called solid geometry. The geodesic dome is one such solid. A geodesic dome was first constructed in Germany in 1922.

- Describe a geodesic dome.
- How is a geodesic dome related to the Platonic solids?
- Find out where buildings based on geodesic domes have been constructed.

Exercise 11.2   Naming and classifying solids

1 What is a more common name for a tetrahedron?

2 Which of the following solids are polyhedra?

  A a rectangular prism   **B** a triangular prism   **C** a sphere
  D a square pyramid   **E** a cone   **F** a donut shape
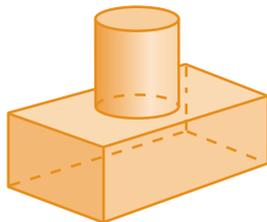
3 Two solids have been combined to make each of these composite figures. For each one:
  i   name the two solids that have been combined
  ii state whether the composite figure is a polyhedron
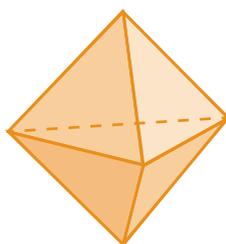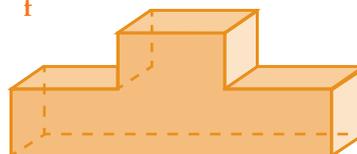 **iii** state whether the composite figure is convex.

a
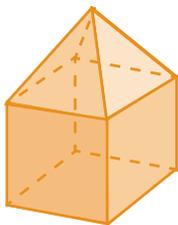
b

c

d

e

f

4 Copy the figures from question **3** parts **a**, **b** and **f** and colour one pair of parallel flat faces.

5 Copy and complete this table.

Solid	Number of faces	Shapes of faces	Number of identical faces	Number of pairs of parallel flat faces
Cube				
Cylinder				
Square pyramid				
Triangular prism				
Rectangular prism				
Rectangular pyramid				
Cone				
Triangular pyramid				

6 How many faces has:
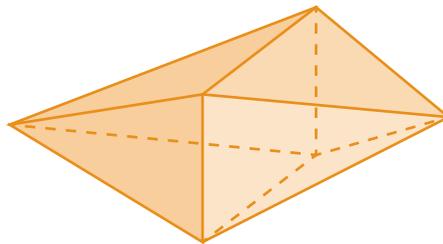   **a** a tetrahedron?   **b** an octahedron?
   c a hexahedron?   **d** a pentahedron?

7 Use the table of polyhedra names on page 422 to name these polyhedra.

a

b

c
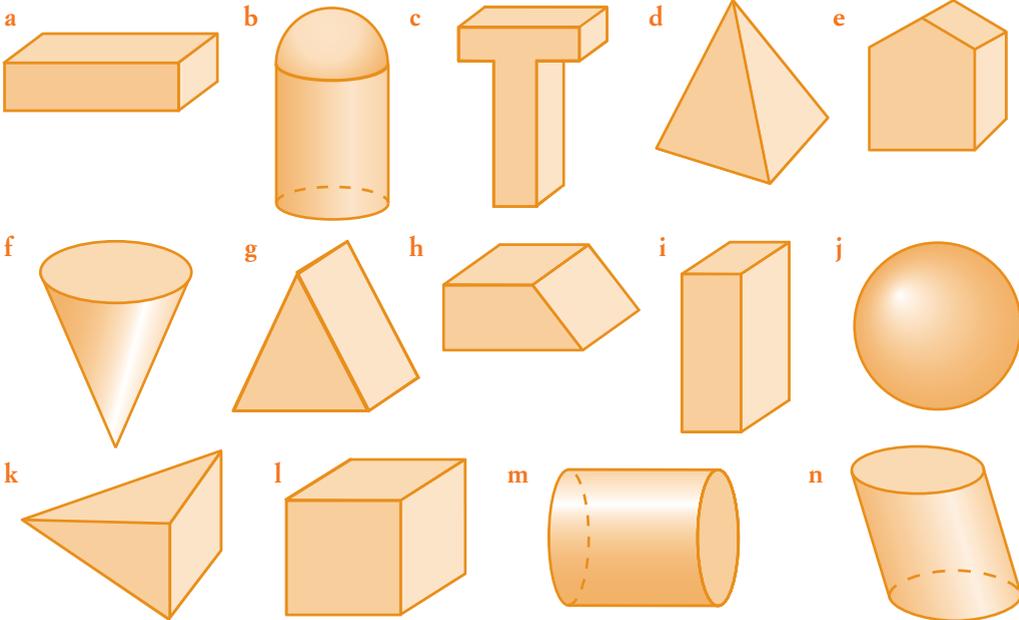
d

e

f
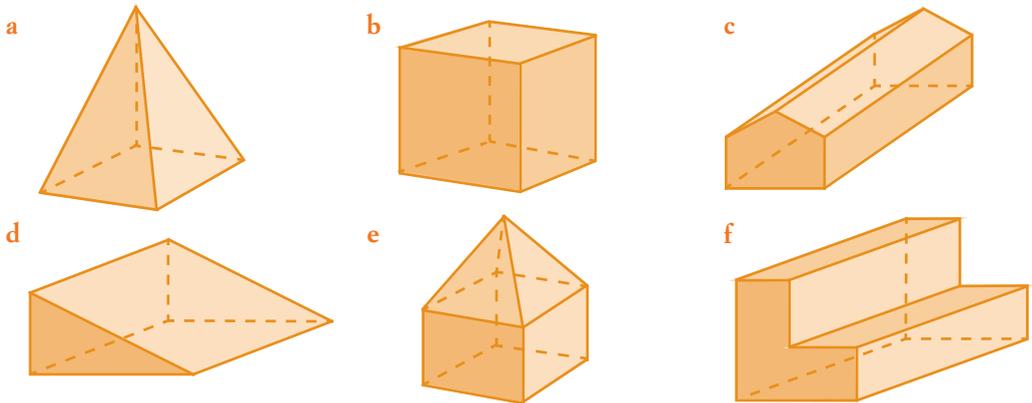
8 Complete this statement, selecting **A**, **B**, **C** or **D**. The name of this polyhedron is:
   A hexahedron   **B** decahedron   **C** pentahedron   **D** octahedron
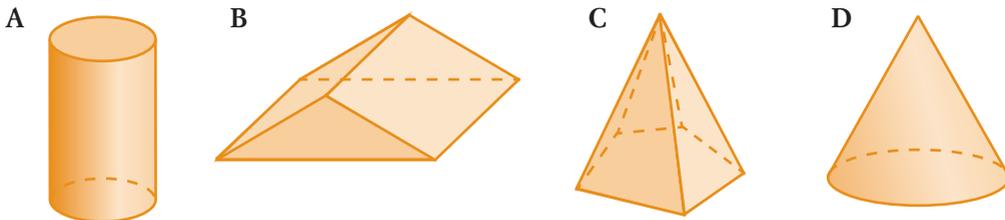
9 Draw a cross-section of each of these solids.

a　**b**　**c**　**d**　**e**

f　**g**　**h**　**i**　**j**

k　**l**　**m**　**n**

10 Which solids in question **9** are:
　a polyhedra?　　　　**b** prisms?
　c pyramids?　　　　**d** neither prisms nor pyramids?

11 Which solid shown is a prism? Select **A**, **B**, **C** or **D**.

A　**B**　**C**　**D**

12 Describe in your own words:
　a a prism　　　　　　　　　　　**b** a pyramid.

13 Using each of these shapes as a base, draw a prism and shade its base.
　a square　　　**b** isosceles triangle　　　**c** trapezium　　　**d** hexagon

14 Draw a pentagonal pyramid and shade its base.

15 For each of these prisms, state:　　　　　　　　　　　　　*See* Example 5
　i the shape of its cross-section　　　　　　**ii** its name

a　**b**　**c**

16 Name these pyramids.

a

b

c

d

e
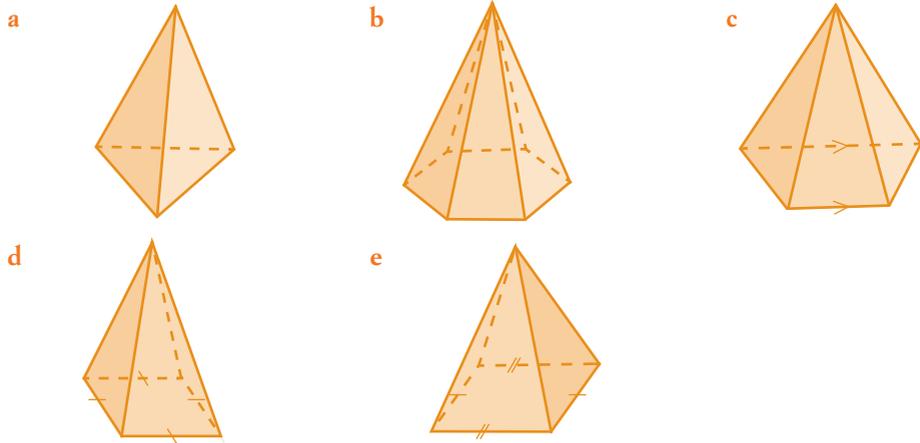
17 Write the name of each of these solids:

a

b

c

d
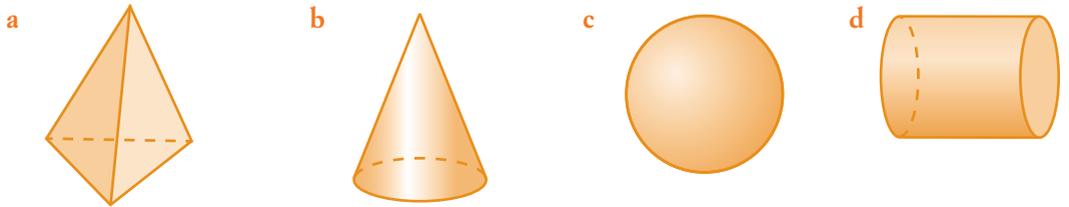
18 Answer these questions for the following shapes.

a Which are convex?   b Which are prisms?   c Which are pyramids?

A

B

C

D

E

F

G

H

I

J

K

L

M

N

O

P

Q

R

S

T
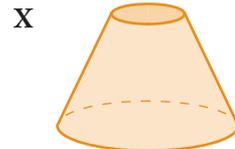
U

V

W

X

Y

Z
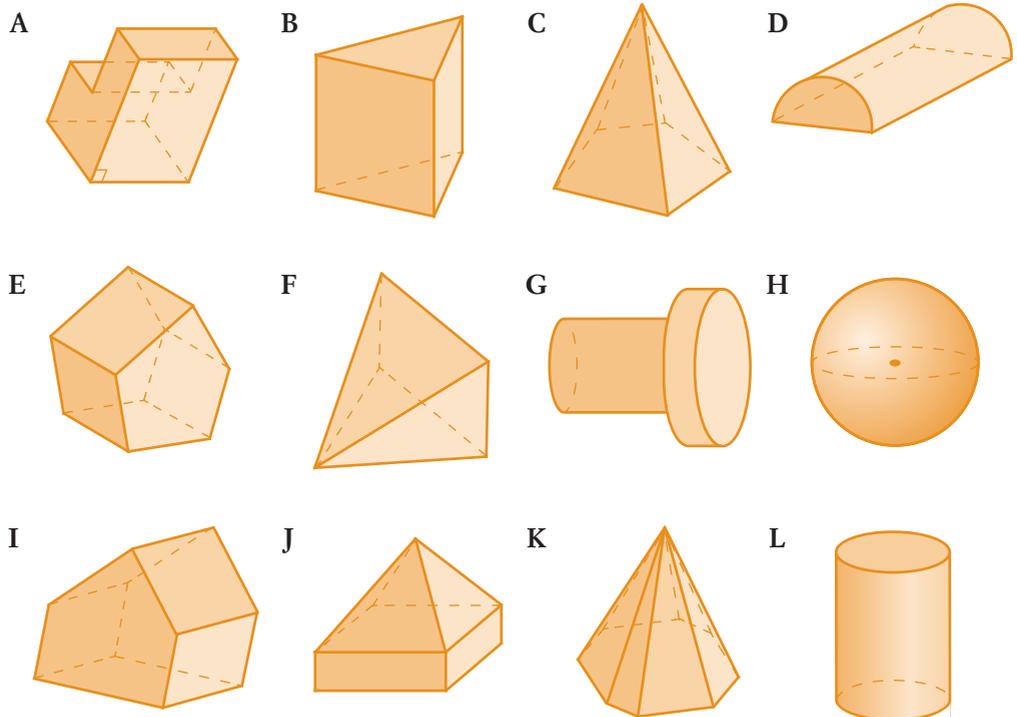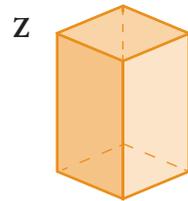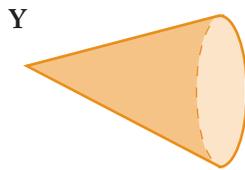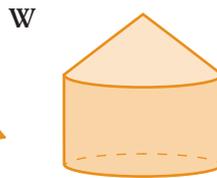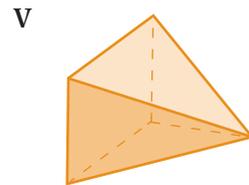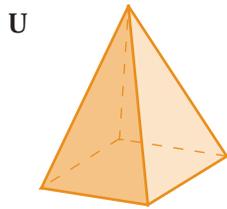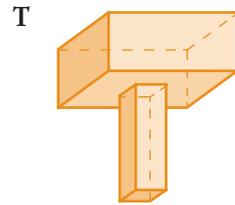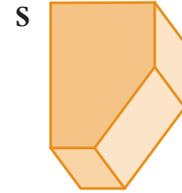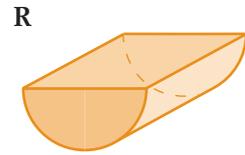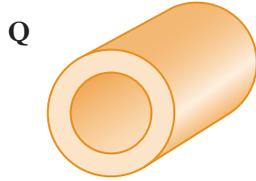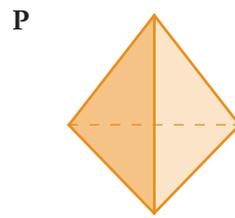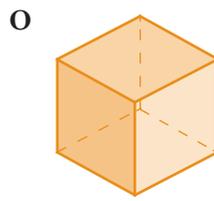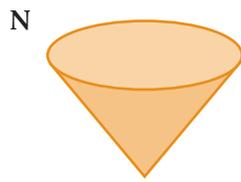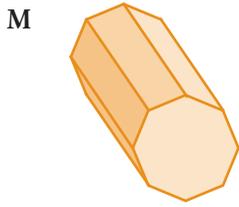
19 Which solids have been used to form the solid shown on the right? Select **A**, **B**, **C** or **D**.

A cylinder and rectangular prism

B half cylinder and trapezoidal prism

C cylinder and trapezoidal prism

D sphere and trapezium
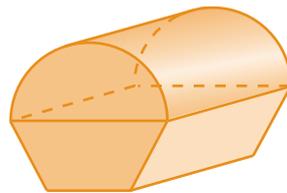
You can print out the Nets of Platonic solids pages for the nets used in this exercise.

20 A regular tetrahedron is shown on the right.

   a Draw and cut out the net of a regular tetrahedron with side lengths of 5 cm. For best results, draw it onto cardboard. Fold along the dotted lines and paste the tabs to make the tetrahedron.

   b How many faces has the regular tetrahedron?

   c What is the more common name for a tetrahedron?

   d What type of triangle is each face?

   e How many vertices has this tetrahedron?

   f How many edges has this tetrahedron?

   g Does this tetrahedron have any:

      i parallel edges?

      ii skew edges?

      iii intersecting edges?
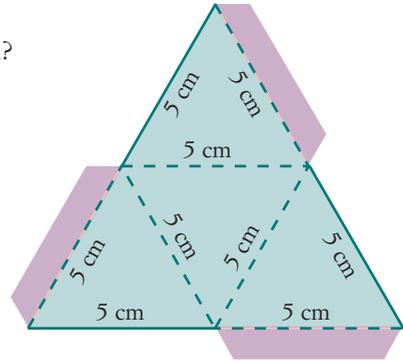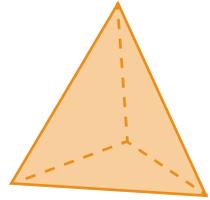
21 A regular octahedron is shown on the right.

   a Draw and cut out the net of a regular octahedron with side lengths of 5 cm. Fold and paste to make the octahedron.

   b How many faces does this regular octahedron have?

   c What is the shape of each face?

   d A regular octahedron is a composite solid. What two solids can be combined to make it?

   e How many edges does this regular octahedron have?

   f Does this octahedron have any:

      i parallel edges?

      ii skew edges?

      iii intersecting edges?
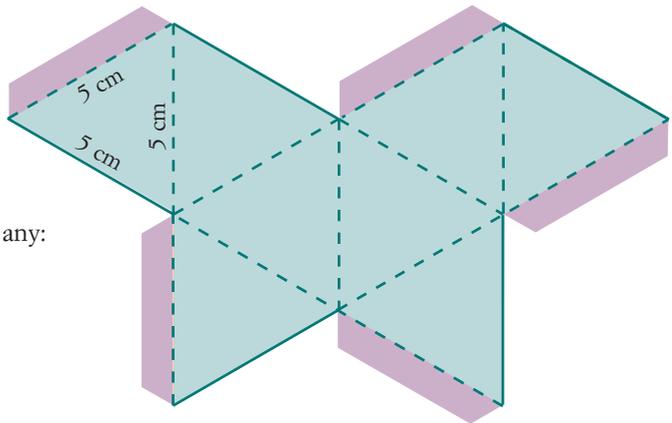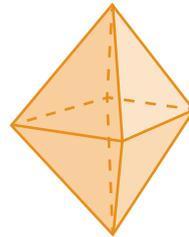
22 A regular icosahedron is shown on the right.

   a Draw and cut out the net of a regular icosahedron with side lengths of 4 cm as shown at the top of the next page. Then make the icosahedron.

   b How many faces has this regular icosahedron?

   c What is the shape of each face?

   d How many edges does it have?
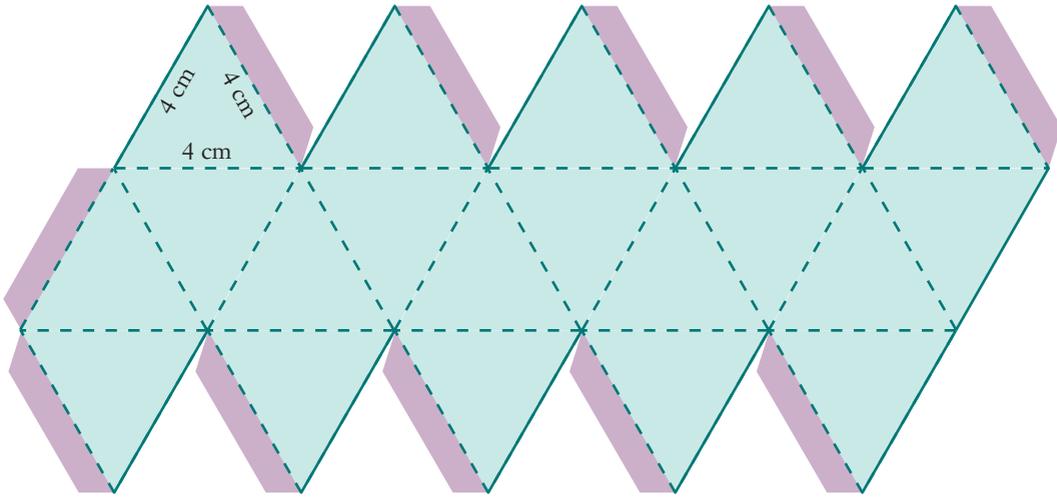
e How many vertices does it have?

f Does this icosahedron have any:

   i parallel edges?   **ii** skew edges?

23 A regular hexahedron is shown on the right and its net is shown below.

  a Draw and cut out this net of a regular hexahedron with side lengths of 5 cm. Then make the hexahedron.

  b How many faces does the regular hexahedron have?

  c What is the shape of each face?

  d What is the more common name for a regular hexahedron?
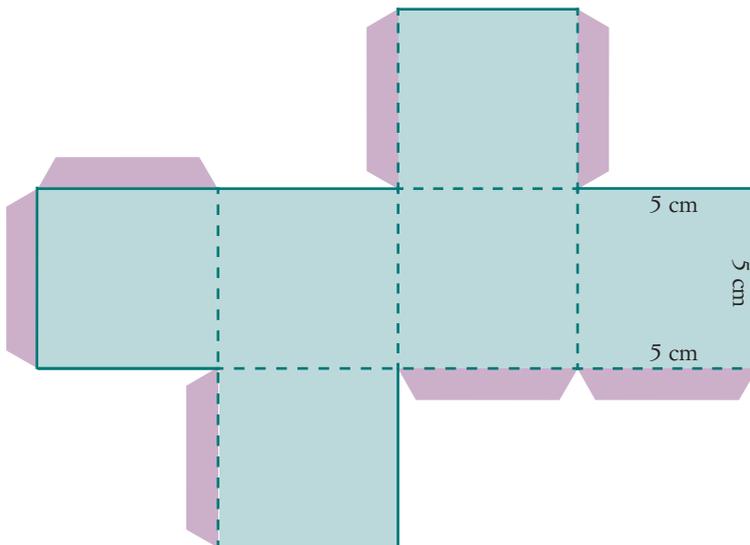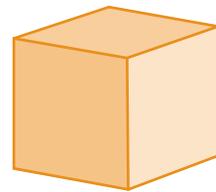
24 A regular dodecahedron is shown on the right and its net is shown below.

a Draw and cut out this net of a regular dodecahedron
with side lengths of 3 cm, then make it.

b How many faces does this the regular dodecahedron have?

c What is the shape of each face?

d How many edges does it have?

e How many vertices does it have?

f Does this dodecahedron have any:

   i parallel edges?   **ii** skew edges?
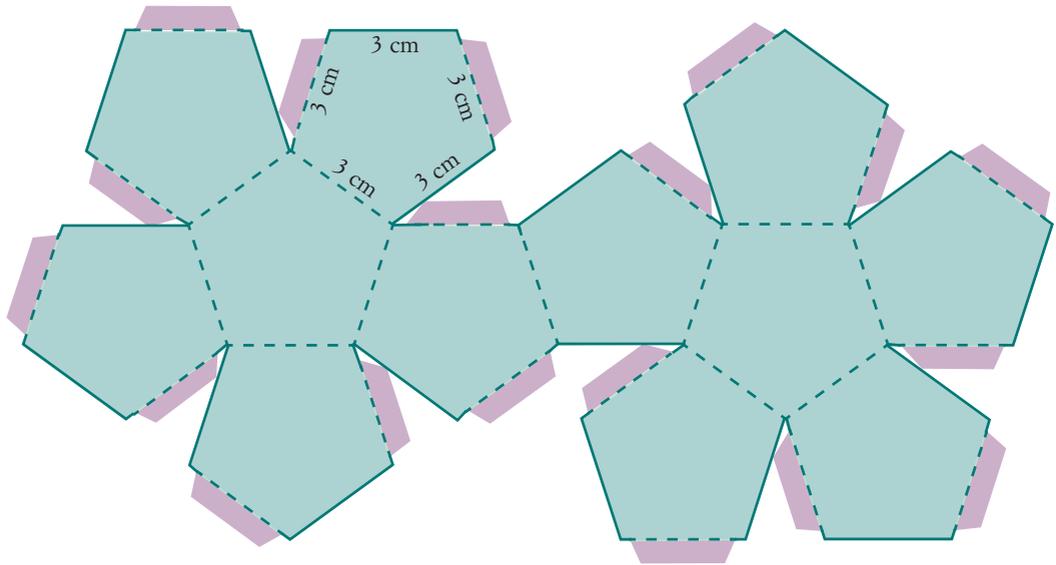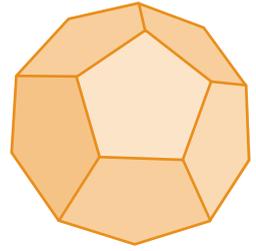
25 Which of the following is a Platonic solid? Select **A**, **B**, **C** or **D**.

A

B

C

D
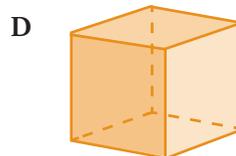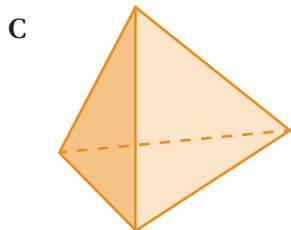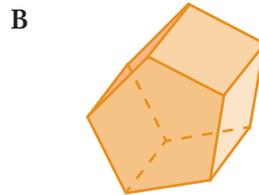
26 Copy and complete this table of Platonic solids.

Platonic solid	Description	Shape of each face	Number of faces
	Two square pyramids joined at bases		
	Similar to a soccer ball pattern	Regular pentagon	
Icosahedron			20
			4
		Square	

27 Which Platonic solid:

a has 12 faces? **b** is a pyramid? **c** has 12 edges?
d has 12 vertices? **e** is a prism? **f** has pentagonal faces?

28 A slice of a cone is cut vertically as shown in the diagram below. What shape is the slice? Select **A**, **B**, **C** or **D** from the diagrams below.

A **B** **C** **D**
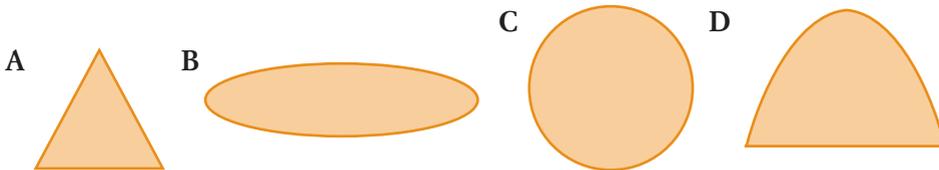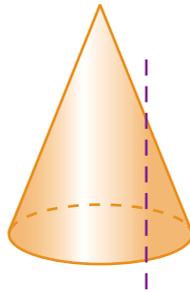
29 Cans with a diameter of 10 cm are to be packed in layers in a carton with a base measuring 50 cm by 40 cm.

a What is the greatest number of cans that could be packed in each layer in the carton?
b What percentage of the area of the base is covered by the cans?

Worked solutions

Exercise 11.2

MAT07MGWS00043

30 Find out what a tetrapak and the computer game Tetris are. How are they related to the meaning of 'tetra'?

31 What is the smallest number of faces a polyhedron can have?

32 Why can a cube, a rectangular prism and a pentagonal pyramid all be called hexahedrons?

33 What shape are the side faces of:

   a any right prism?   **b** any pyramid?

34 Explain why a cylinder is not a prism and why a cone is not a pyramid.

11.3   Drawing solids

Solids can be difficult to draw. Sometimes it is easier to use square dot paper or isometric dot paper to draw them.
The rectangular prisms shown below are drawn on **square dot paper**, so called because the dots sit on a grid of squares.

The rectangular prisms shown below are drawn on **isometric dot paper**, with the dots arranged in a triangular pattern. Horizontal edges are drawn at a slant on this paper. A 2D (flat) diagram of a 3D figure (solid) looks more natural when drawn on isometric paper.

When drawing on square paper, one of the solid's *faces* is shown at the front. When drawing on isometric paper, one of the solid's vertical *edges* is shown at the front.
House plans and other two dimensional drawings are used to help understand the shape of three dimensional objects. When you look at plans you need to imagine how the object will appear from various positions.

Example 6

For the solid on the right, assume that there are no hidden cubes. Draw:

a the front view
b the left view
c the top view.
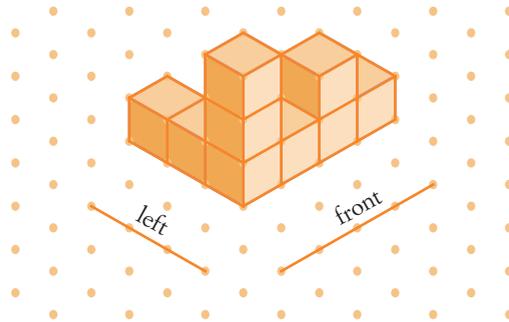
Solution

a Imagine that you are standing so that you are directly opposite the front face. You will see a vertical stack of 3 cubes on the very left. On its right will be 1 cube, a stack of 2 cubes and 1 cube.

Draw the faces of the cubes you would see.
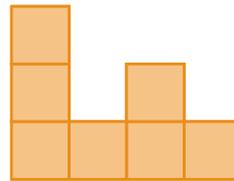
b Imagine that you are standing so that you are directly opposite the left face. You will see 1 cube on the very left. On its right will be 1 cube and a stack of 3 cubes.

Draw the faces of the cubes you would see.
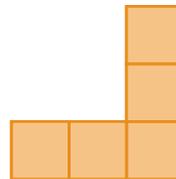
c Imagine that you are hovering directly above the shape. You will see the tops of the cubes forming an L shape. The L shape has 3 cubes along one arm and 4 cubes along the other.

Draw the faces of the cubes you would see.
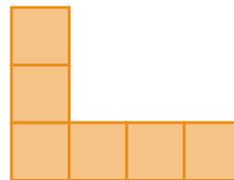
Example 7

For the solid shown on the right, draw:
a the front view
b the top view.
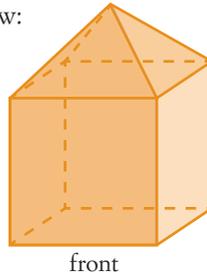
front

Solution

a Imagine that you are standing directly in front of the solid. The bottom of the shape looks like a square and the top looks like a triangle.

You will see a line between the square and the triangle.

Draw what you would see.

b Imagine that you are hovering directly above the solid. The outside edges form a square.

You will see the edges of the pyramid as lines meeting at the centre of the square.

Draw the faces of the cubes you would see.

Exercise 11.3   Drawing solids

For this exercise you will need square dot paper, isometric dot paper and construction cubes. Both types of dot paper can be printed out from NelsonNet. You can also draw shapes directly into a geometry program, using a square grid or an isometric grid.

1 Copy the rectangular prisms shown below onto isometric dot paper.
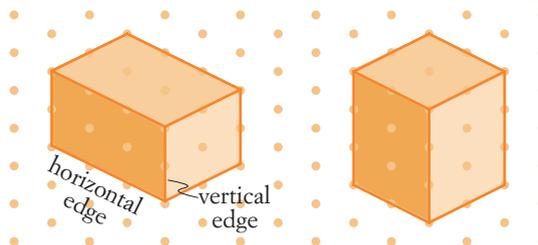
horizontal edge vertical edge

2 Copy the rectangular prisms shown below onto square dot paper.

face
horizontal
edge
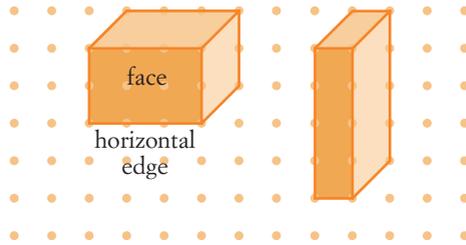
3 Use cubes to make each of the following solids, then draw them on isometric dot paper.

a   **b**   **c**   **d**

e   **f**   **g**
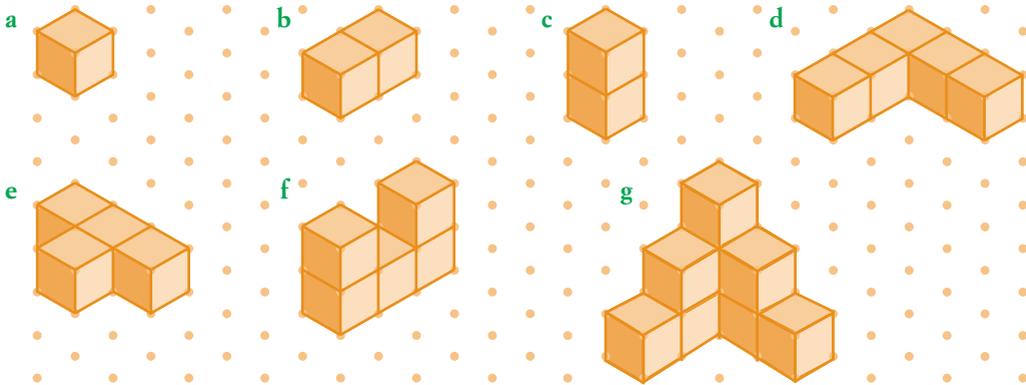
4 Draw this shape on isometric dot paper.
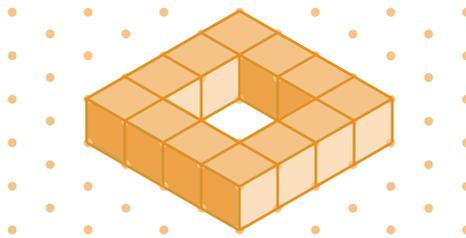
5 Make three different solids using five cubes. Then draw them on isometric dot paper.

6 Each of the following is the start of a drawing of a rectangular prism. Copy each drawing onto isometric dot paper and complete it.

a   **b**   **c**   **d**   **e**
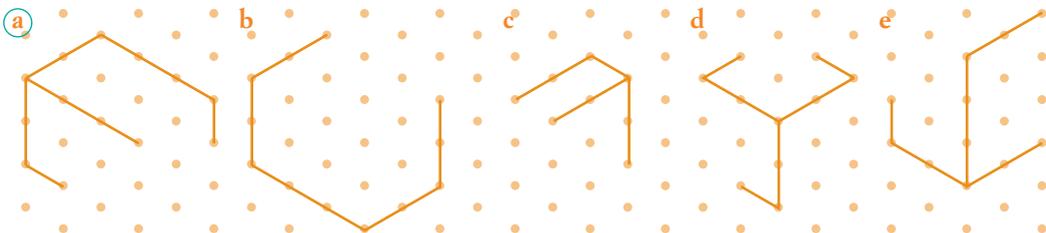
See Example 6

7 For each of these solids, draw the views listed underneath.

a
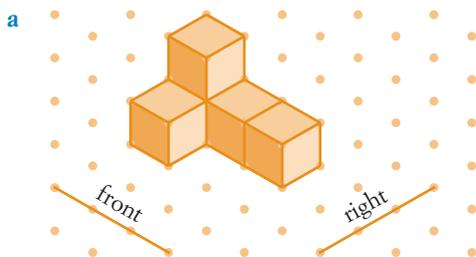
    i   top view
  ii   right view
 iii   front view

b
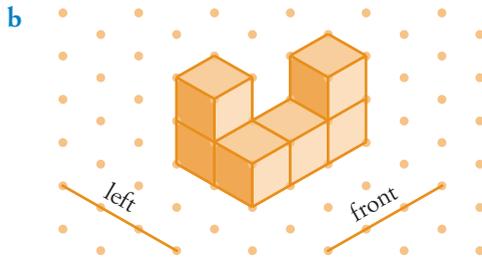
    i   left view
  ii   back view
 iii   top view

c
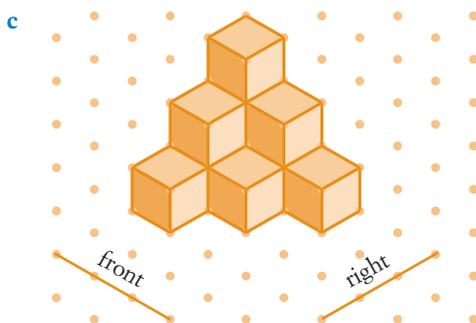
    i   back view
  ii   top view
 iii   right view

d

    i   right view
  ii   front view
 iii   top view

e
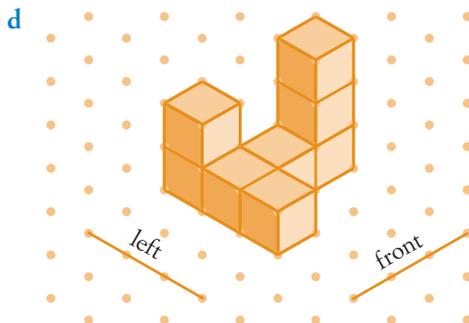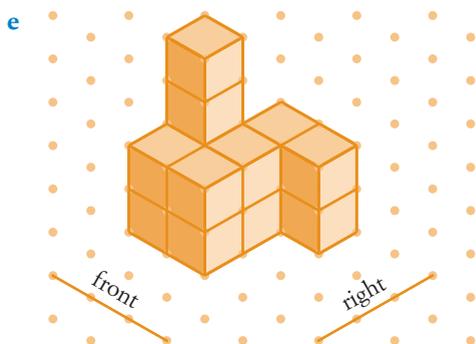
    i   front view
  ii   left view
 iii   top view

f
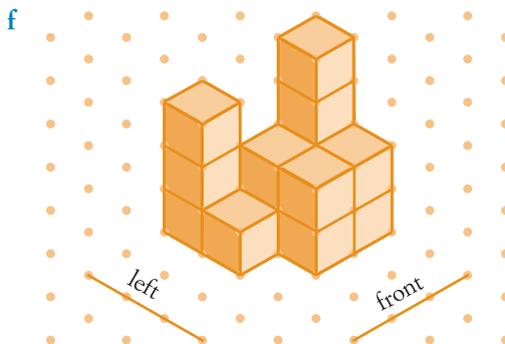
    i   back view
  ii   right view
 iii   top view

8 Imagine and then draw what would remain of this large block if the smaller block was taken away from it.
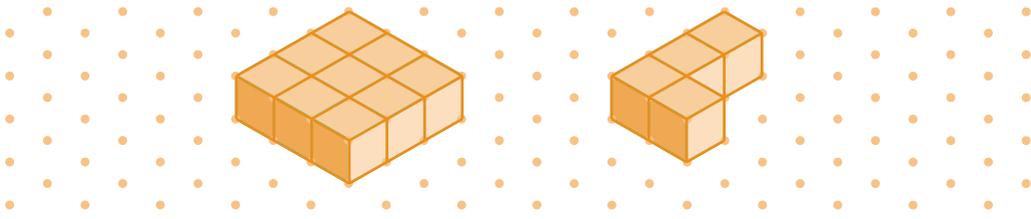
The use of cubes should help you complete questions **9– 13**.

9 Imagine and then draw what you would see of the following shape if you were looking from A, then from B, and then from C.
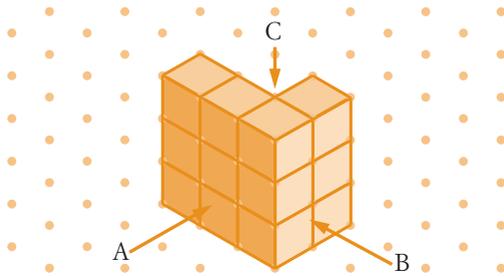
10 Copy each of these drawings of solids onto isometric dot paper exactly as shown.

a

4 cubes

b

5 cubes

c

6 cubes

d

7 cubes

e

10 cubes

f

18 cubes
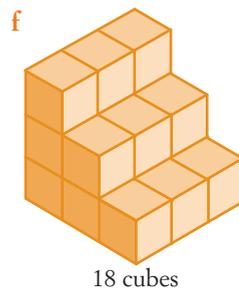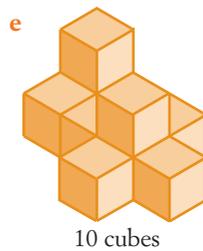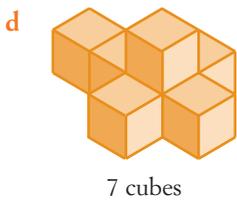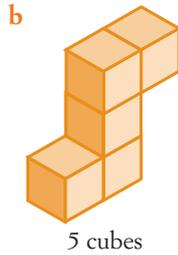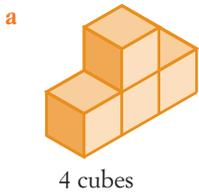
11 Here are four different shapes, called **A**, **B**, **C** and **D**.

A   **B**   **C**   **D**
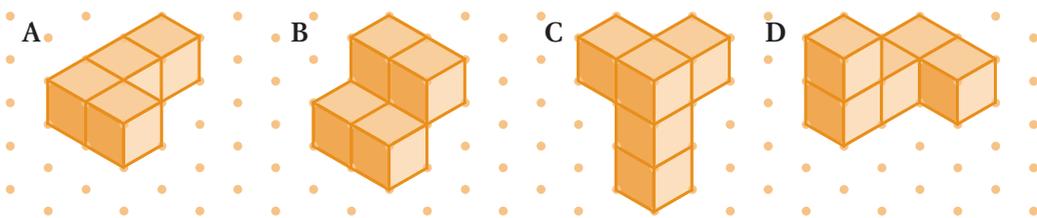
Which of the following shapes are the same as:

a A?   **b B**?   **c C**?   **d D**?

i   **ii**   **iii**

iv   **v**   **vi**   **vii**

viii   **ix**   **x**
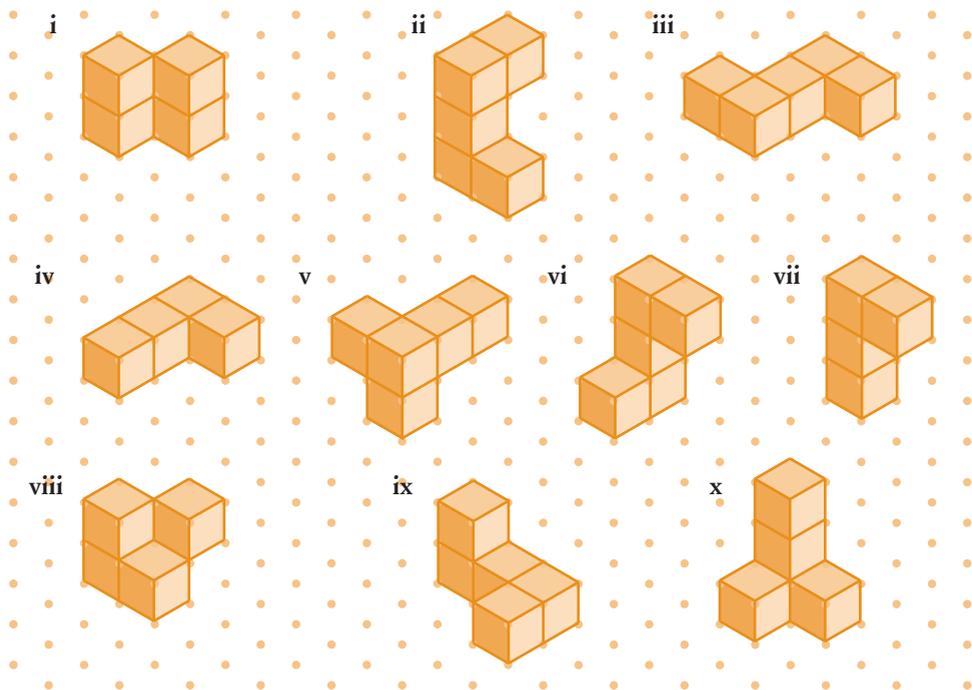
12 On isometric dot paper draw the different solids that can be made if four cubes are joined together. Two have already been drawn below for you. How many different solids are there?
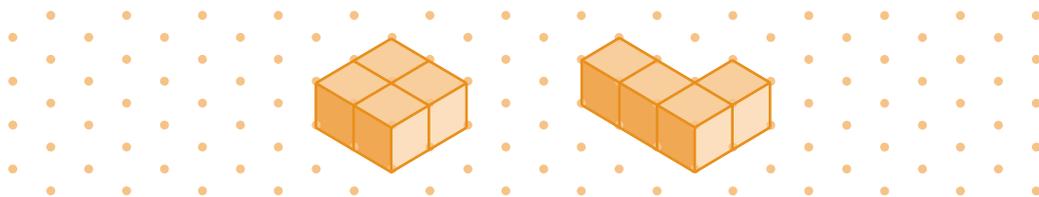
13 Which of the given solids is the same as the solid on the left? Select **A**, **B**, **C**, **D** or **E**.

A   **B**   **C**
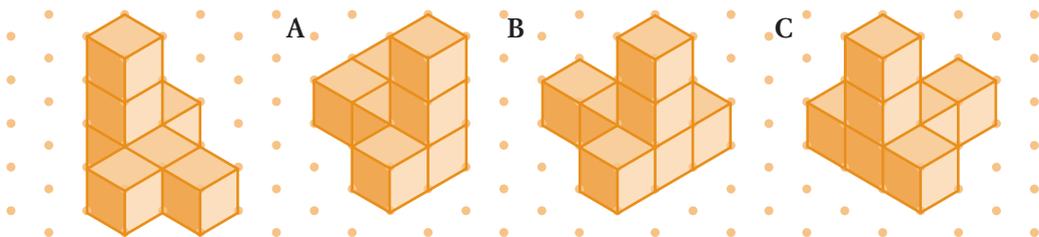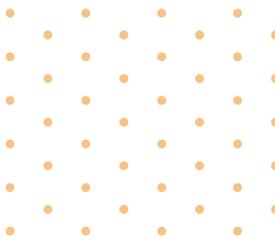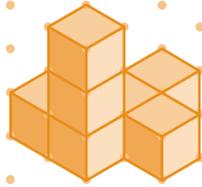
D   **E**
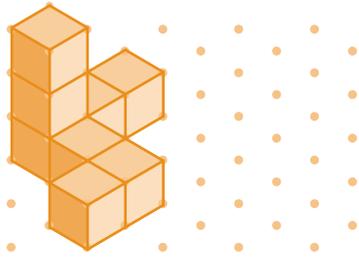
14 For each of these solids, draw the views requested.   *See* Example 7

a
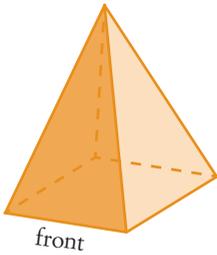
front

  i front view
  ii top view

b
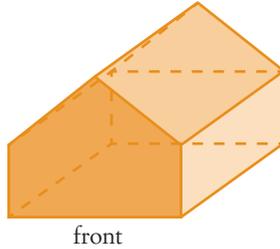
front

  i front view
  ii right view
  iii top view

c
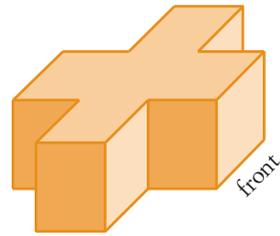
front

  i left view
  ii top view
  iii front view

d
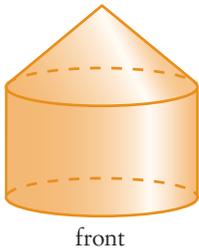
front

  i top view
  ii front view

e
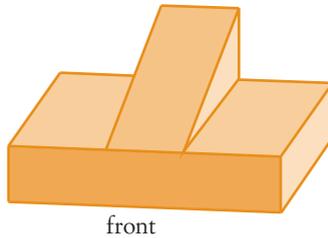
front

  i front view
  ii left view
  iii right view
  iv top view

f
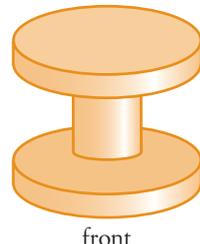
front

  i top view
  ii front view

15 Draw each of these solids from the top and front views.

a

b

c

d

16 Chuan flew over the top of a building shaped like the object on the right. Which view would he see when directly overhead? Select **A**, **B**, **C** or **D**.

A

B

C

D
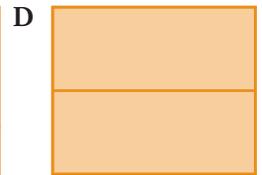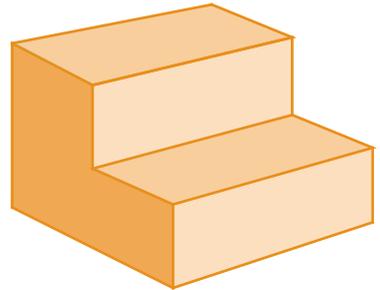
17 What would Diana see if she were looking at this solid from point E? Select **A**, **B**, **C** or **D**.

E

A

B

C

D
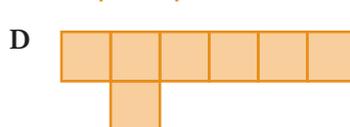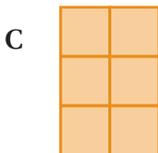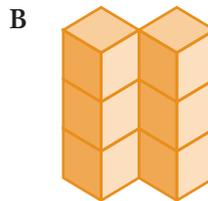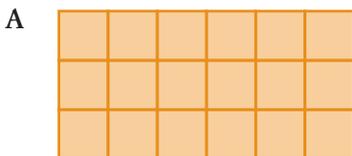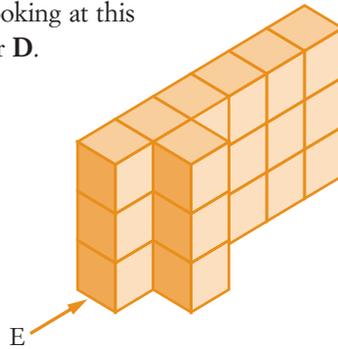
18 In each set of diagrams below, three views of a solid shape are shown, along with the number of cubes needed to make it. Make each of these solids, then draw them on isometric paper.

a 5 cubes

top   front   side

b 6 cubes

top   front   side

c 5 cubes

top   front   side

d 5 cubes

top   front   side
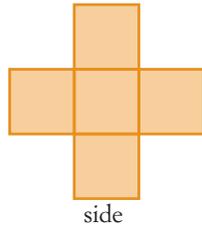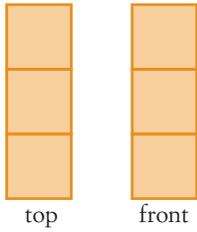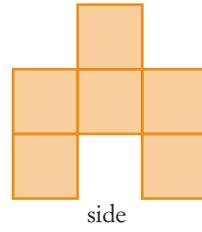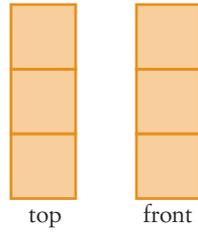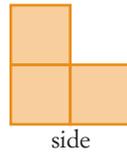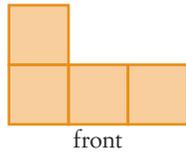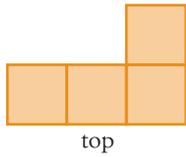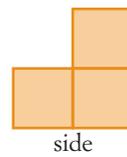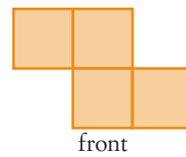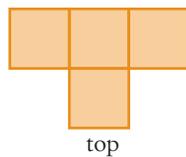
11.4 Volume and capacity

Important!

Volume

The **volume** of a solid is the amount of space occupied by the solid.

Often, informal (everyday) units are used to refer to volume. For example:

• a *cup* of flour
• a *tablespoon* of oil
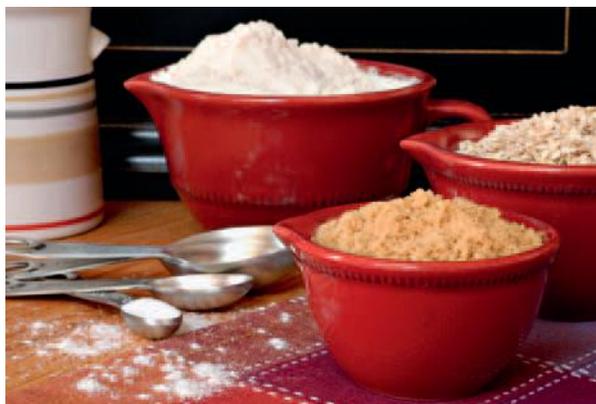
Standard units of volume

A **cubic centimetre** is the amount of space that a cube with each side measuring 1 cm would occupy. The volume of the cube is one cubic centimetre, or 1 cm^3.
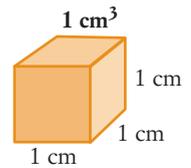
A **cubic millimetre** is the amount of space that a cube with each side measuring 1 mm would occupy. The volume of the tiny orange cube on the corner of the diagram below left is one cubic millimetre, or 1 mm^3. There are 1000 cubic millimetres in one cubic centimetre.

1 cm^3

1 cm
1 cm
1 cm

1 cubic millimetre

1 cubic centimetre

1 cm
1 cm
1 cm
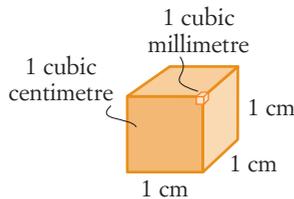
$1 \text{ cm}^3 = 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$
$= 1000 \text{ mm}^3$

1000 mm^3

10 mm
10 mm
10 mm
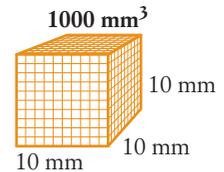
A **cubic metre** is the amount of space that a cube with each side 1 m would occupy, that is, 1 m^3. A washing machine is about half a cubic metre. There are 1 000 000 cubic centimetres in one cubic metre. The diagram below illustrates this.

100 cm (or 1 m)

100 cm (or 1 m)

100 cm (or 1 m)
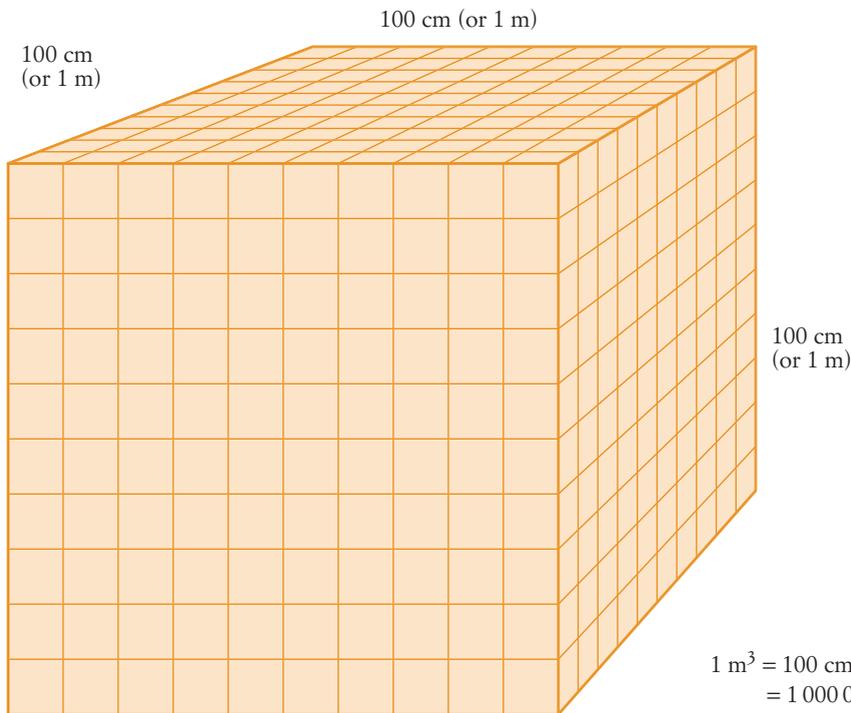
$1 \text{ m}^3 = 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$
$= 1\,000\,000 \text{ cm}^3$

Important!

Metric units of volume

Unit	Abbreviation	Conversion
cubic millimetre	mm^3	
cubic centimetre	cm^3	$1\ cm^3 = (10\ mm)^3 = 1000\ mm^3$
cubic metre	m^3	$1\ m^3 = (100\ cm)^3 = 1\ 000\ 000\ cm^3$

The diagram below will help you convert units of volume.

Converting to a smaller unit so multiply

$\times 1\ 000\ 000$ $\times 1000$

m^3 cm^3 mm^3

$\div 1\ 000\ 000$ $\div 1000$

Converting to a larger unit so divide
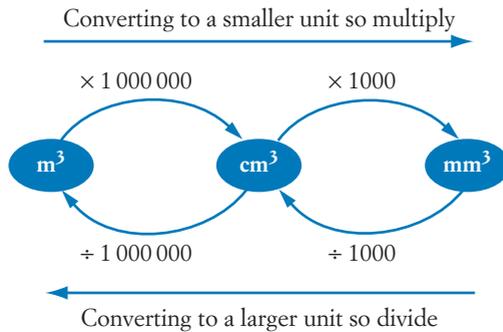
Example   8

a Convert 12 000 mm^3 into cm^3.
b Convert 48 m^3 into cm^3.

Solution

a Converting to a larger unit, so divide.
       $12\ 000\ mm^3 = (12\ 000 \div 1000)\ cm^3$
                    $= 12\ cm^3$

cm^3 mm^3
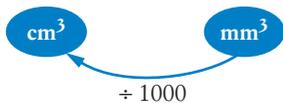
$\div 1000$

b Converting to a smaller unit, so multiply.
       $48\ m^3 = (48 \times 1\ 000\ 000)\ cm^3$
             $= 48\ 000\ 000\ cm^3$

$\times 1\ 000\ 000$

m^3 cm^3
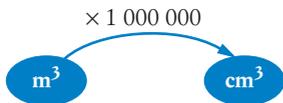
Investigate: Volume of a rectangular prism

The following rectangular prisms are made up of 1 cm cubes.

a
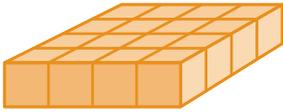
b
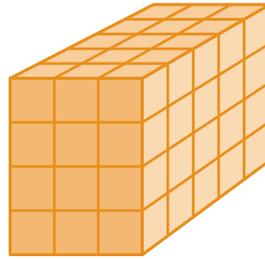
c
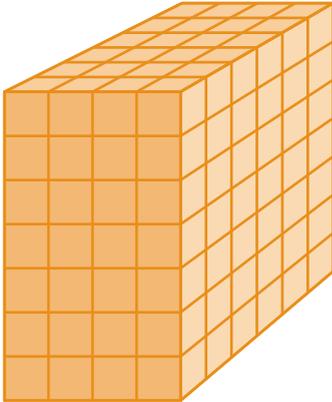
d
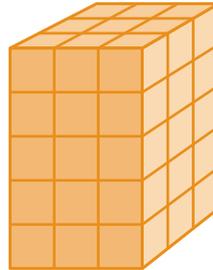
e
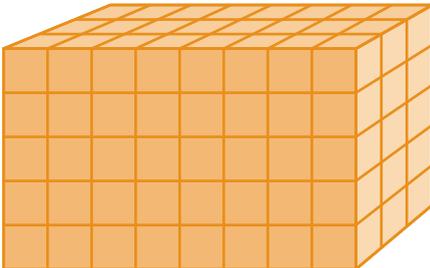
f
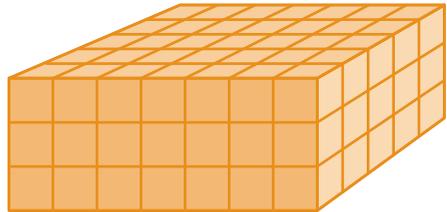
1 Use the rectangular prisms above to complete the table below.

Shape	Length (cm)	Width (cm)	Height (cm)	Volume (cm³)
a	4	4	1	16
b				
c				
d				
e				
f				

Use the information in the table to answer the following questions.

2 What is the relationship between length, width and height of a rectangular prism and its volume?

3 Write the relationship as a rule by completing the following:
Volume of a rectangular prism = _____ × _____ × _____

Important!

Volume of a rectangular prism

The volume of a rectangular prism is:

$$V = \text{length} \times \text{width} \times \text{height}$$
$$= l \times w \times h$$
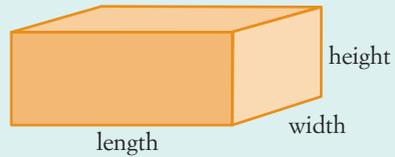
Example 9

Find the volume of the rectangular prism on the right.
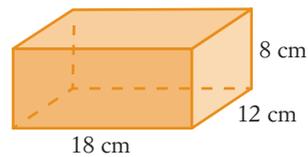
Solution

Write the rule for the volume of a rectangular prism.	$V = l \times w \times h$
Substitute the measurements from the diagram.	$= 18 \times 12 \times 8$
Evaluate.	$= 1728$
Write the answer including the unit.	**The volume is 1728 cm³.**

The volume of the rectangular prism is:

$$V = l \times w \times h$$

But the area of the base of the rectangular prism is:

$$A = l \times w$$

So, the volume of the rectangular prism could be written as:

$$V = A \times h$$

where A is the area of the base.
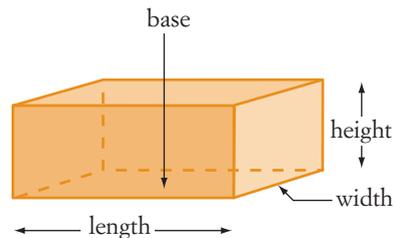
Important!

Volume of a rectangular prism

The volume of a rectangular prism is:

$$V = \text{area of base} \times \text{height}$$
$$= A \times h$$
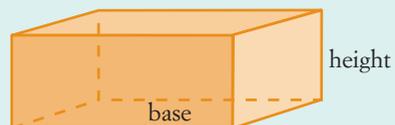
Example | 10

Find the volume of this rectangular prism.
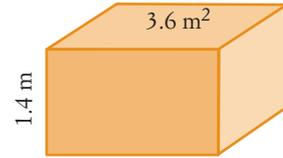
Solution

Write the rule for the volume of the prism.	$V = A \times h$
Substitute for the area of the base and the height.	$= 3.6 \times 1.4$
Evaluate.	$= 5.04$
State the result.	**The volume is 5.04 m^3.**

Investigate: Calculating your volume
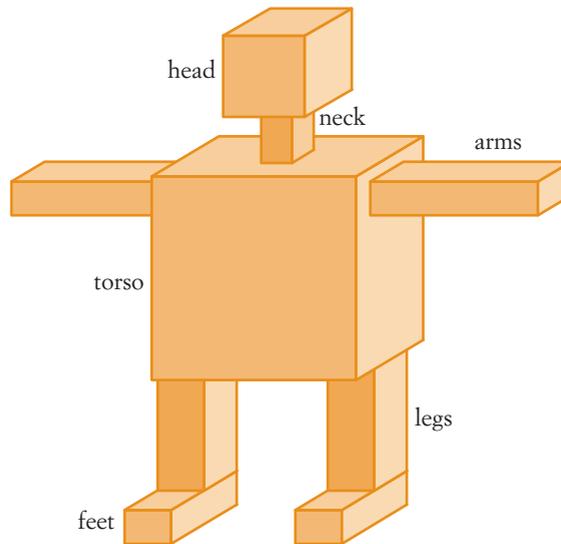
Imagine that you are made up of rectangular prisms.

1 With the help of a partner, make measurements of your body. Use them to find dimensions (to the nearest centimetre) for each of the 'prism' body parts as shown above.
2 Sketch each body part prism and label its dimensions.
3 Use the prisms to find your volume in cm^3.
4 How accurate is the value you calculated for the volume of your body?
5 How could you improve the accuracy of the calculation?

'What is the capacity of the water tank?'

Capacity is the amount of fluid (liquid or gas) in a container.

The standard units of capacity are the litre (L), the millilitre (mL) and the kilolitre (kL).

The same units are used to describe the volume of any liquid.

A teaspoon holds about 5 mL.

A tall standard carton of milk holds 1 L.

Important!

Metric units of capacity

Unit	Abbreviation	Conversion
millilitre	mL	
litre	L	1 L = 1000 mL
kilolitre	kL	1 kL = 1000 L

The diagram below will help you convert capacity units.

Converting to a smaller unit so multiply

× 1000   × 1000

1 kL   1 L   1 mL

÷ 1000   ÷ 1000

Converting to a larger unit so divide
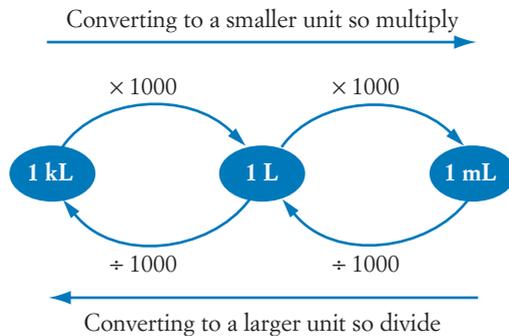
It is also useful to know the relationship between volume and capacity units.

Important!

Volume and capacity units

1 cm^3 contains 1 mL
1 m^3 contains 1 kL

This means that a cubic centimetre can hold 1 mL of liquid, while a cubic metre can hold 1000 L of liquid.
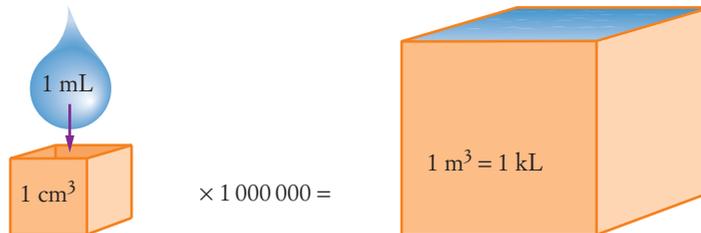
Investigate: How much water does your household use?

Water is a precious commodity and Australia is one of the driest countries in the world. This means that we all need to be careful about how we use water. The table below lists some common household activities and the water usage associated with each.

You may be interested in your household's daily water consumption. To calculate this, you will need to use the information in the table and knowledge about your household including the answers to following questions:

- How many people are in your household?
- How many showers or baths does each person take a day?
- How long do household members take to shower?
- How many loads of washing are done each week?
- How many times do household members use the toilet at home?
- How many times do household members brush their teeth?
- How often does your household water the garden or hose paths and driveways?
- How many times a week is a car washed?

Use (appliance)	Type	Average total water used
Shower	Normal shower rose	15 L/min
	Water saver rose	8.5 L/min
Bath		50–150 L/bath
Toilet	Single flush	11 L
	Dual flush	5 L
Washing machine	Front loading	80 L/wash
	Top loading	170 L/wash
Dishwasher		20–50 L/load

Use (appliance)	Type	Average total water used
Cooking, cleaning, drinking		150 L/day
Brushing teeth (water running)		5 L/brush
Brushing teeth (no water running)		1 L/brush
Outdoors		
Sprinkler or handheld hose		10–20 L/min
Drip system		4 L/hr
Hosing paths/driveways		20 L/min
Washing the car with a running hose		10–20 L/min
Filling a swimming pool		20 L/min

- Use this information to calculate your household's weekly water consumption.
- What is your household's daily water consumption?
- What is the daily water consumption per person in your household?
- Compare your findings with others in the class and comment on the results.

Technology   Dam levels

Both Brisbane and Melbourne rely on multiple dams for their water supply. The combined capacities of these dams are 1 759 000 ML for Brisbane and 1 773 000 ML for Melbourne. (Note that 1 ML = 1 000 000 L.)

The spreadsheet below shows the volumes for each of the dam systems over a 16 month period.

Technology
Calculating capacity
MAT07MGCT00025

	A	B	C	D	E	F	G	H	I
1	Brisbane		Capacity (ML) =	1759000		Melbourne		Capacity (ML) =	1773000
2									
3	Date	Year	Volume (ML)	% Capacity		Date	Year	Volume (ML)	% Capacity
4	8-Jan	2009	813713			8-Jan	2009	615231	
5	8-Feb	2009	799993			8-Feb	2009	579771	
6	8-Mar	2009	827434			8-Mar	2009	542538	
7	8-Apr	2009	828313			8-Apr	2009	514170	
8	8-May	2009	860679			8-May	2009	492894	
9	8-Jun	2009	1047309			8-Jun	2009	462753	
10	8-Jul	2009	1347922			8-Jul	2009	460980	
11	8-Aug	2009	1350384			8-Aug	2009	482256	
12	8-Sep	2009	1337192			8-Sep	2009	508851	
13	8-Oct	2009	1316612			8-Oct	2009	588636	
14	8-Nov	2009	1291634			8-Nov	2009	675513	
15	8-Dec	2009	1264545			8-Dec	2009	673740	
16	8-Jan	2010	1249594			8-Jan	2010	663102	
17	8-Feb	2010	1232883			8-Feb	2010	636507	
18	8-Mar	2010	1447657			8-Mar	2010	615231	
19	8-Apr	2010	1692158			8-Apr	2010	620550	

Sources: www.seqwater.com.au and www.melbournewater.com.au

- Open a new spreadsheet and enter the data shown in the screen shot above.

- Now we will calculate the '% Capacity' which is the volume of water as a percentage of the total for each dam. Click on cell D4 and enter '=C4/D1'.
- Format the cell as a percentage with no decimal places by selecting the Format menu, 'Cells', 'Percentage' from the pop-up menu, choosing 0 decimal places, then click on 'OK'.
- Copy this formula down the column to cell D19.
- Repeat these steps for the Melbourne data.
- Answer the following questions using the completed spreadsheet.
  - **a** In which month was Brisbane's water capacity at its highest level?
  - **b** In which month was Melbourne's water capacity at its lowest level?
  - **c** In which month did Brisbane's water capacity have the greatest increase compared with the previous month? How much was the increase?
  - **d** In which month did Brisbane's water capacity have the greatest decrease compared with the previous month? How much was the decrease?
  - **e** In which month did Melbourne's water capacity have the greatest increase compared with the previous month? How much was the increase?
  - **f** In which month did Melbourne's water capacity have the greatest decrease compared with the previous month? How much was the decrease?
  - **g** Use the Internet to find out if Brisbane and Melbourne experience wet and dry seasons.
  - **h** Compare the results of your research in part **g** with the results of your previous calculations and comment on your findings.

Exercise 11.4 Volume and capacity

Understanding

1 Write an example of the items that could be measured by each informal unit of volume.
- **a** cup(s)
- **b** box(es)
- **c** handful
- **d** pinch
- **e** bucket(s)
- **f** packet
- **g** capsule(s)
- **h** can(s)
- **i** teaspoon
- **j** wheelbarrow
- **k** carton
- **l** capful

2 What unit of volume would you use when measuring the volume of:
- **a** a textbook?
- **b** a backpack?
- **c** the carton for a large TV?
- **d** a large suitcase?
- **e** a match box?
- **f** a room?

3 Copy and complete the following.
- **a** $3 \text{ cm}^3 = $ _____ mm^3
- **b** $5 \text{ m}^3 = $ _____ cm^3
- **c** $2.6 \text{ cm}^3 = $ _____ mm^3
- **d** $4000 \text{ mm}^3 = $ _____ cm^3
- **e** $7.2 \text{ m}^3 = $ _____ cm^3
- **f** $66\,000 \text{ mm}^3 = $ _____ cm^3
- **g** $1 \text{ m}^3 = $ _____ mm^3
- **h** $2300 \text{ cm}^3 = $ _____ m^3
- **i** $126\,000\,000 \text{ cm}^3 = $ _____ m^3
- **j** $3450 \text{ mm}^3 = $ _____ cm^3
- **k** $25 \text{ m}^3 = $ _____ mm^3
- **l** $78\,000 \text{ mm}^3 = $ _____ m^3
- **m** $63\,000 \text{ cm}^3 = $ _____ m^3
- **n** $1.4 \text{ mm}^3 = $ _____ cm^3

4 Find the volume of each of these rectangular prisms.   *See* Example 9

a

9 cm
5 cm
5 cm

b

21 cm
3 cm
17 cm

c

4 cm
4 cm
36 cm

d

17 cm
15 cm
9 cm

e

3 cm
3 cm
3 cm

f

3 m
11 m
1.2 m

g

15 cm
15 cm
180 cm

h

2.4 m
1.8 m
33.5 m

5 Find the missing measurements for the rectangular prisms in the table.

Prism	Length	Breadth	Height	Volume
a	50 cm	50 cm	50 cm	
b	5 cm	10 cm	18 cm	
c	4 m	2.5 m	1.4 m	
d	24 mm	16 mm	11 mm	
e		10 cm	10 cm	2000 cm^3
f	5 mm	2 mm		100 mm^3
g		1.5 m	3 m	27 m^3
h	22 cm		5 cm	880 cm^3
i	70 mm	10 mm		70 000 mm^3
j	1.8 m		10 m	9 m^3

6 Find the capacity of:
  a a variety of milk containers
  b four different-sized soft drink bottles
  c a standard soft drink can
  d a standard cup
  e the petrol tanks of a variety of cars
  f your local swimming pool
  g a petrol tanker
  h a small fruit juice pack

7 State what unit of capacity you would use when measuring:

   a a glass of milk   **b** a dam   **c** a petrol tank

   d a bottle of medicine   **e** the amount of soft drink consumed in a week

8 Copy and complete the following.

   a 7000 mL = _____ L   **b** 2 L = _____ mL

   c $3\frac{1}{2}$ L = _____ mL   **d** 10 000 mL = _____ L

   e 2500 mL = _____ L   **f** 1.5 L = _____ mL

   g 4000 mL = _____ L   **h** 8.5 L = _____ mL

   i 6.2 L = _____ mL   **j** 1750 mL = _____ L

   k 5 kL = _____ L   **l** 9000 L = _____ kL

   m 25 000 kL = _____ L   **n** 520 mL = _____ L

   o 2.3 mL = _____ L   **p** 6 mL = _____ kL

Fluency

See Example 10

9 Find the volume of each of these rectangular prisms.

a

42 cm^2

6 cm

b

4 mm^2

12.5 mm

c

9.5 m^2

5.2 m

d

2.3 m^2

3.4 m

e

310 cm^2

24 cm

f

98 mm^2

28 mm

g

2.4 m^2

1.7 m

h

255 cm^2

17 cm

10 Find the volume of these rectangular prisms. (Hint: make sure that all the measurements you use are in the same units.)

a

5 cm
40 mm
10 cm

b

15 mm
2.5 cm
15 mm

c

0.5 m
10 cm   10 cm

d

2 m
50 cm
3 m

e

500 mm
80 cm
1.5 m
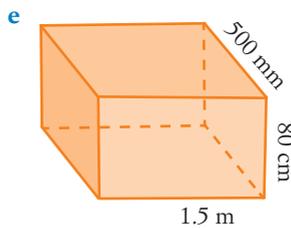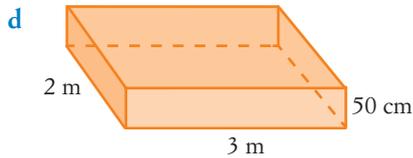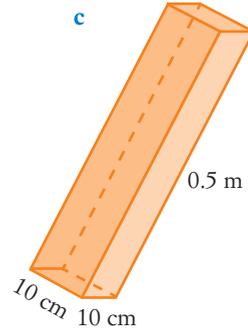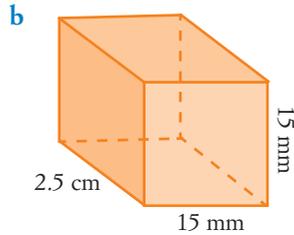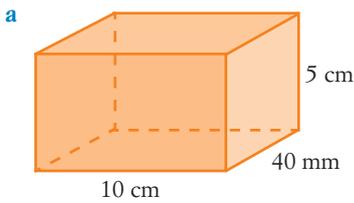
11 Select **A**, **B**, **C** or **D** to complete this statement.
The capacity of a bottle of cough medicine is approximately equal to:

    A 200 mL   **B** 500 mL   **C** 1500 mL   **D** 2000 mL

12 Match the correct capacity (**A** to **J**) with the items (**a** to **j**) listed.

a	car petrol tank	**A**	200 mL
b	liquid paper	**B**	23 kL
c	bath tub	**C**	5 mL
d	bucket of water	**D**	70 L
e	can of drink	**E**	1250 mL
f	glass of water	**F**	1875 kL
g	Olympic swimming pool	**G**	20 mL
h	bottle of lemonade	**H**	9 L
i	teaspoon	**I**	375 mL
j	water storage tank	**J**	180 L

Problem solving

13 Use any types of cubes to complete these constructions.

  a Build as many different solids as you can with a volume of 3 cubes (that is using 3 cubes). Sketch each one.

  b Build as many different solids as you can with a volume of 4 cubes (that is using 4 cubes). Sketch each one.

  c Build as many different solids as you can with a volume of 5 cubes. Sketch each one.

14 What is the approximate volume of a brick? Select from **A**, **B**, **C** or **D**.

    A 1000 cm^3 **B** 20 cm^3 **C** 1600 cm^3 **D** 2100 cm^3

15 Match the correct volume (**A** to **G**) with each of the items (**a** to **g**) listed.

a	a bottle of liquid paper	**A**	200 m³
b	a box of tissues	**B**	3890 m³
c	a glass of water	**C**	1250 cm³
d	a bottle of lemonade	**D**	5000 cm³
e	a classroom	**E**	20 000 mm³
f	a school hall	**F**	250 cm³
g	a cereal package	**G**	2200 cm³

16 Find the volume of each of these solids. (Hint: You will need to find the volume of two rectangular prisms each time.)

a

b

c

d

e

f

g
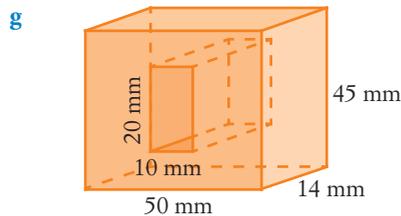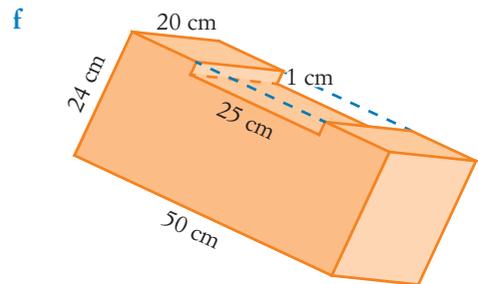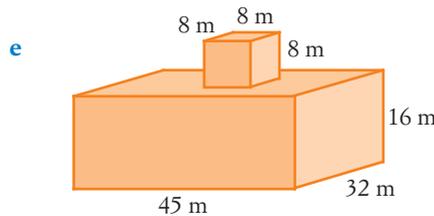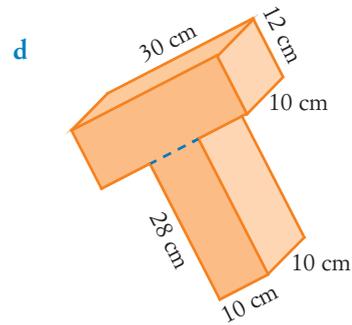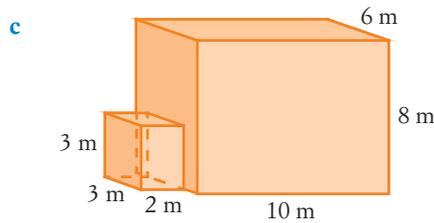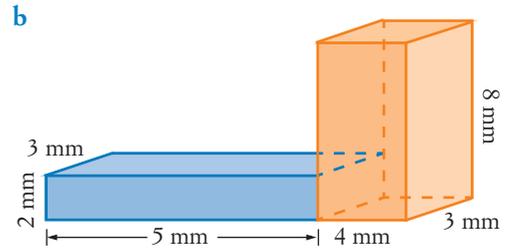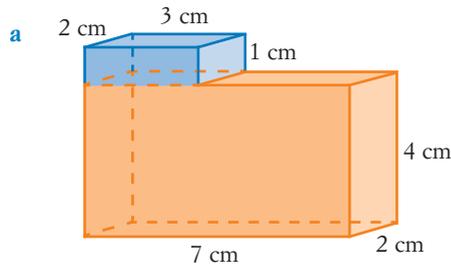
17 A jug holds 2 L of water. How many 250 mL glasses could be filled from it?

18 James is inviting 30 friends to a party. He calculates that each person will drink 1800 mL of soft drink.
   a How many litres of soft drink must he buy?
   b If James intends to buy large 2 L bottles of drink, how many bottles must he buy?

19 A bottle of medicine holds 100 mL. Tara was told to take 5 mL twice a day. For how many days can Tara take the medicine before it runs out?

20 A tap leaks 10 mL of water every 50 seconds. How much water will be lost in:

    a 1 second?   **b** 1 minute?   **c** 3 hours?   **d** 1 day?

21 Your skin releases moisture as a way of controlling body temperature. On average 200 mL is released per hour. If all this moisture was captured, how long would it take to fill a 1.25 L soft drink bottle?

22 A lunch box is made in the shape of a rectangular prism. Its dimensions are 20 cm, 15 cm and 9 cm.

    a Find the volume of the lunch box, in cm^3.

    b How many mL of water would fit in the lunch box?

> **Worked solutions**
>
> Exercise 11.4
>
> MAT07MGWS00045

23 Gina's swimming pool is a rectangular prism 8 m long, 4 m wide and 1.5 m deep.

    a Find the volume of the swimming pool.

    b How many litres of water would be needed to fill the pool? (Hint: 1 m^3 holds 1 kL)

24 A fish tank is a rectangular prism 60 cm long, 40 cm high and 30 cm wide.

    a Find the volume of the tank.

    b How many litres of water will it hold?

> Reasoning

25 Cans with a diameter of 8 cm and a height of 12 cm are to be packed in layers in a carton with a base measuring 60 cm by 35 cm and a height of 40 cm.

    a What is the greatest number of cans that could be packed in each layer in the carton? (Assume all cans are packed in equal rows.)

    b How many layers of cans could be packed in the carton?

    c If each can has a volume of 605 cm^3, what percentage of the volume of the carton is occupied by the cans?

11.5 Mass

You are asked to pick up:

- a cubic metre of feathers
- a cubic metre of concrete

You can lift the feathers but not the concrete!

The *volume* is the *same* but the *mass* is *different*. Even though they each take up the same amount of space, one is much heavier.

Mass is the amount of matter in an object. The standard unit of mass is one **kilogram** (kg).
Other units used are the **milligram** (mg), the **gram** (g) and the **tonne** (t).
A drawing pin has a mass of about 1 g.
An egg has a mass of about 60 g.
A litre of water has a mass of exactly 1 kg.
A medium-sized car has a mass of about 1.5 t.

Important!

Metric units of mass

Unit	Abbreviation	Conversion
milligram	mg	
gram	g	1 g = 1000 mg
kilogram	kg	1 kg = 1000 g
tonne	t	1 t = 1000 kg

The diagram below will help you convert mass units.

Converting to a smaller unit so multiply

$\times 1000 \qquad \times 1000 \qquad \times 1000$

t **kg** **g** **mg**

$\div 1000 \qquad \div 1000 \qquad \div 1000$

Converting to a larger unit so divide
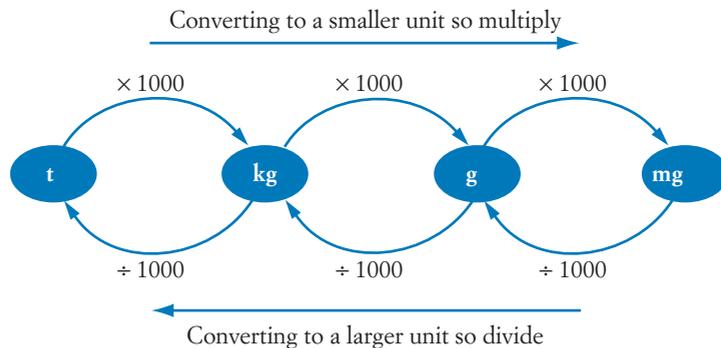
Example   11

Change each of the following to the unit indicated in brackets.

a 577 g (kg)   **b** 9.4 kg (g)   **c** 87 kg (t)

Solution

a Kilogram is the larger unit, so divide. $577\,\text{g} = \dfrac{577}{1000}\,\text{kg} = 0.577\,\text{kg}$

b Gram is the smaller unit, so multiply. $9.4\,\text{kg} = 9.4 \times 1000\,\text{g} = 9400\,\text{g}$

c Tonne is the larger unit, so divide. $87\,\text{kg} = \dfrac{87}{1000}\,\text{t} = 0.087\,\text{t}$

Exercise 11.5　Mass

Note: You will need a variety of weighing scales.

1 What unit of mass would you use when measuring:

   (a) a piece of fruit?　　**b** an elephant?　　**c** a schoolbag?

   d a car?　　**e** a television?

2 Measure the mass of:

   a this textbook　　**b** your lunchbox　　**c** your schoolbag

   d shoe　　**e** a pencil case　　**f** yourself

   g a jumper　　**h** a brick　　**i** an apple

3 Copy and complete the following.

   (a) 3000 g = _____ kg　　**b** 2 t = _____ kg　　**c** 4 kg = _____ g

   d 9000 kg = _____ t　　**e** 7.5 t = _____ kg　　**f** 10 000 mg = _____ g

   g 1.5 kg = _____ g　　**h** 3800 kg = _____ t　　**i** 3 g = _____ mg

4 Select **A**, **B**, **C** or **D** to complete the following statement.

   A tub of margarine weighing 500 g has a mass greater than:

   A 2.5 kg　　　　**B** 0.01 tonnes　　　　**C** 60 000 mg　　　　**D** 0.8 kg

5 Copy and complete the following, using a $<$, $>$ or $=$ sign.

   (a) 700 g _____ 0.6 kg　　**b** 0.8 g _____ 95 mg　　**c** 3500 kg _____ 3.5 t

   d 1.7 kg _____ 1700 g　　**e** 0.007 t _____ 7 kg　　**f** 640 mg _____ 0.7 g

   g 4000 mg _____ 0.04 kg　　**h** 0.03 kg _____ 3 g

6 Match the items given (**a** to **j**) with the masses listed (**A** to **J**).

   a an egg　　　　　　　　　　**A** 400 g

   b an elephant　　　　　　　　**B** 16 g

   c a house brick　　　　　　　**C** 25 kg

   d a medium-sized car　　　　**D** 80 kg

   e an adult　　　　　　　　　**E** 6 t

   f a can of soft drink　　　　　**F** 500 g

   g a 50c piece　　　　　　　　**G** 10 kg

   h a 7-year-old child　　　　　**H** 50 g

   i a tub of margarine　　　　　**I** 3 kg

   j a large watermelon　　　　　**J** 1 t

7 Measure the mass of 1 L of water. Write a report on how you did it.

- **Convex solids** have faces that all point, curve or bulge *outwards*, while **non-convex** (or **concave**) **solids** have some faces that point, curve or bulge *inwards*.
- The **net** of a 3D shape is a 2D shape that can be folded up to form the 3D shape.
- The surface area of a 3D shape is the total area of all the faces.
- A **polyhedron** is a solid whose faces are all flat. Polyhedra (or polyhedrons) are named according to the number of faces they possess: **tetrahedron** (4 faces), **pentahedron** (5 faces), **hexahedron** (6 faces), **heptahedron** (7 faces), **octahedron** (8 faces), **nonahedron** (9 faces) **decahedron** (10 faces), **undecahedron** (11 faces), **dodecahedron** (12 faces) and **icosahedron** (20 faces).
- An **edge** is the line where two faces of a solid meet.
- A **vertex** is where edges of a solid meet to form a corner.
- A **cross-section** of a solid is a slice of the solid cut parallel to its end faces.
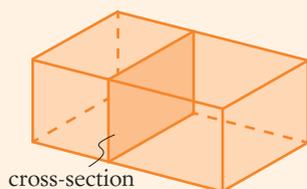
cross-section

Rectangular prism

- A **prism** is a polyhedron with a uniform cross-section. The cross-section is a polygon and the end face is called the **base**. Prisms are named according to the shape of the base.
- A **pyramid** is a polyhedron with a pointed top called an **apex**. The base of a pyramid is the face opposite its apex. The cross-sections of a pyramid are polygons with the same shape but not the same size. A pyramid is named according to the shape of its base.
- A **cylinder** is a solid with a uniform circular cross-section.
- A **cone** is a solid with circular cross-sections that vary in size.

apex

cross-section

base
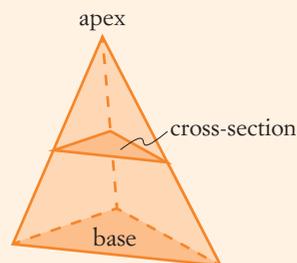
Triangular pyramid

Cylinder　　　**Cone**

- A **sphere** is a ball-shaped solid with all points on its surface the same distance from its centre.
- Cylinders, cones and spheres are not polyhedra because they have curved faces.

- A **regular polyhedron** or **Platonic solid** is a convex polyhedron with identical faces. There are only 5 Platonic solids: **regular tetrahedron** (4 equal faces), **regular hexahedron** (6 equal faces), **regular octahedron** (8 equal faces), **regular dodecahedron** (12 equal faces), **regular icosahedron** (20 equal faces).
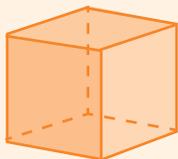
| **Tetrahedron** (equilateral triangular pyramid) | **Hexahedron** (cube) | **Octahedron** | **Dodecahedron** | **Icosahedron** |

- The **volume** of a solid is the amount of space occupied by the solid. The metric units of volume are **cubic millimetre** (mm^3), **cubic centimetre** (cm^3) and **cubic metre** (m^3).

    1 cm^3 = 1000 mm^3 and 1 m^3 = 1 000 000 cm^3

- The volume of a rectangular prism is:

    V = length × width × height

    $= l \times w \times h$

    or

    V = area of base × height

    $= A \times h$
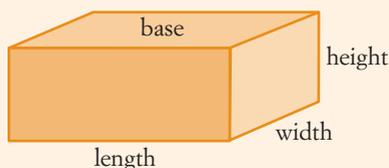
- The **capacity** of a shape is the amount of fluid held by the shape. The metric units of capacity are millilitre (mL), litre (L) and kilolitre (kL).

    1 L = 1000 mL and 1 kL = 1000 L

- Volume and capacity units are related as follows:

    1 cm^3 contains 1 mL

    1 m^3 contains 1 kL

- Mass is the amount of matter in an object. The metric units of mass are **milligram** (mg), **gram** (g), **kilogram** (kg) and **tonne** (t).

    1 g = 1000 mg

    1 kg = 1000 g

    1 t = 1000 kg

Understanding

See Example 1

1 Draw the net of this shape, including measurements.

5 cm

4 cm

6 cm
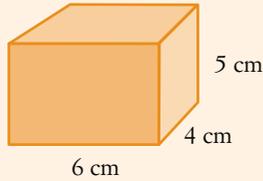
2 Count the cubes in this hollow solid to find its volume. Each cube equals 1 cm³.
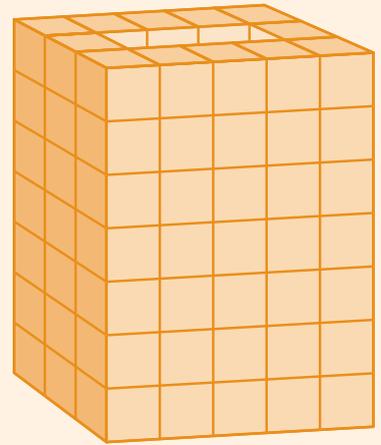
3 State whether each of these solids is convex or non-convex.

a

b
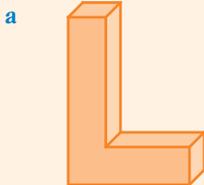
4 Copy and complete the table below for the three solids shown.

A   **B**   **C**

Features	Solid A	Solid B	Solid C
Number of faces			
Number of congruent faces			
Shapes of the faces			
Number of parallel flat faces			
Shape of a cross-section			
Are the cross-sections uniform?			
Is the solid a polyhedron?			

5 Draw each of the following solids and shade its base.

   a a hexagonal prism   **b** a cylinder   **c** a rectangular prism

6 Draw a net of this shape, including measurements. *See* Example 2

10 cm

12 cm
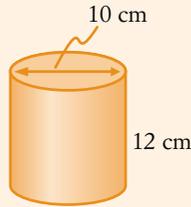
7 Find the surface area of this shape. *See* Example 3

8 cm

6 cm

20 cm
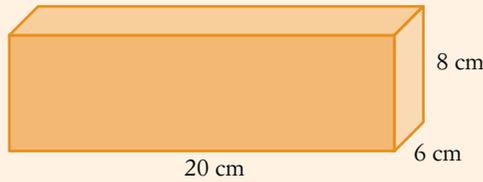
8 Which shape below is the net of a triangular prism? Select **A**, **B**, **C** or **D**.

A   **B**   **C**   **D**
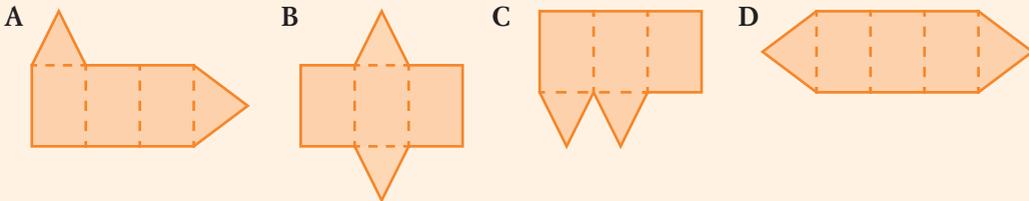
9 Draw a cross-section of each of these solids and state whether each one is a prism. *See* Example 4

  a   **b**   **c**

  d   **e**   **f**
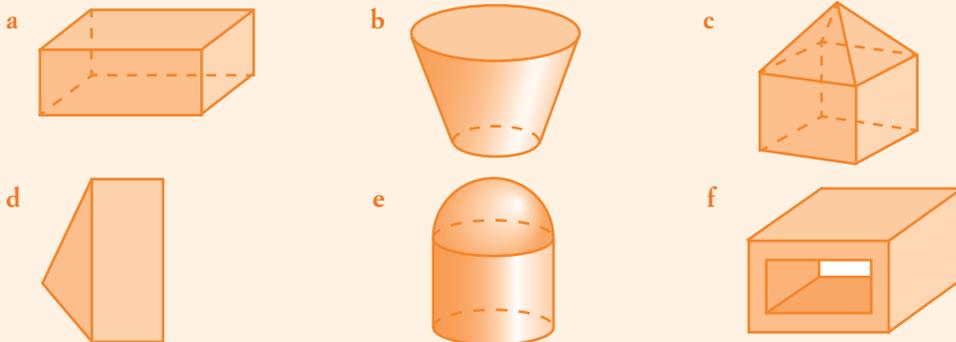
10 Name each of the following solids. *See* Example 5

  a   **b**   **c**
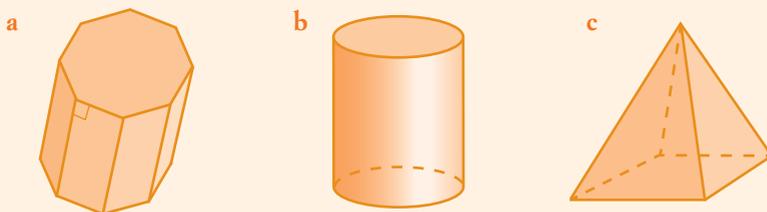
11 Which solids from questions **9** and **10** are concave (non-convex)?

12 Match each shape with its correct property.

Shape	Property
a octahedron	**A** another name for a cube
b pyramid	**B** a polyhedron with triangular faces and a square base
c sphere	**C** has all side faces meeting at a point called the apex
d octagonal prism	**D** a Platonic solid with eight equilateral triangles as faces
e regular hexahedron	**E** has a uniform cross-section but is not a prism
f oblique prism	**F** a polyhedron with 10 faces, 16 vertices and 24 edges
g square pyramid	**G** another name for a triangular pyramid
h cylinder	**H** a box shape that is slanted
i right pyramid	**I** a solid with no flat faces
j tetrahedron	**J** a pyramid that is upright

13 What is a Platonic solid? Give an example of one.

14 How many diagonals has:

   a a cube?   **b** a square pyramid?   **c** a triangular prism?

15 The solid on the right is a regular octahedron.
   a Which two solids can be combined to make this figure?
   b How is this octahedron regular?
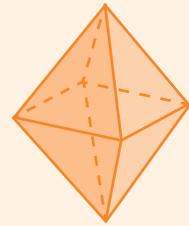
16 Copy and complete these rectangular prisms on dot paper.

   a   **b**
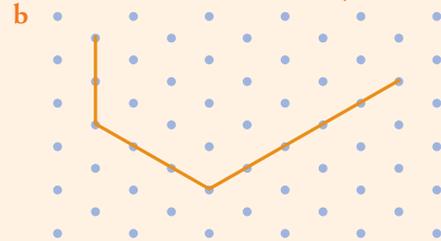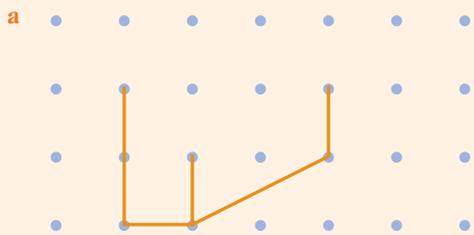
17 For each of these solids, sketch the views requested.

   a   **b**

              i front view   **i** top view
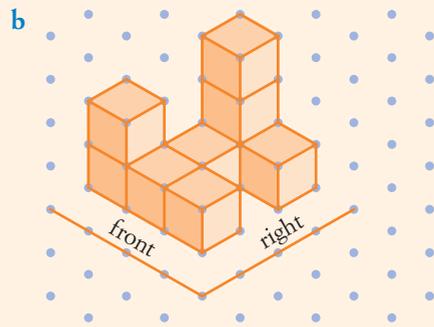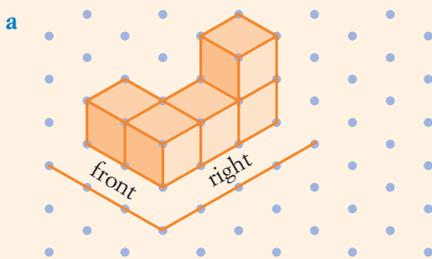
            ii left view   **ii** right view

          iii top view   **iii** front view

18 Copy and complete the following.   *See* Example 8

  a $20 \text{ cm}^3 = $ _____ mm^3 **b** $0.5 \text{ m}^3 = $ _____ cm^3
  c $7500 \text{ mm}^3 = $ _____ cm^3 **d** $230\ 000 \text{ mm}^3 = $ _____ m^3

19 Find the volume of each of these prisms.   *See* Example 9

  a

2 m
5 m
8 m

  b

4 cm
4 cm
20 cm

  c

7 mm
8 mm
12 mm

  d

10 m
6 m
6 m

  e

15 cm
20 cm
10 cm

  f

7 m
6 m
15 m
5 m
15 m
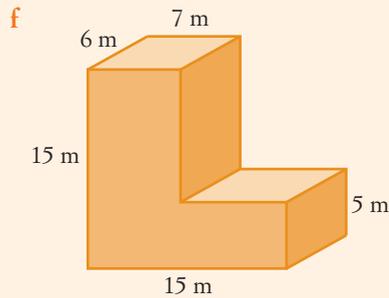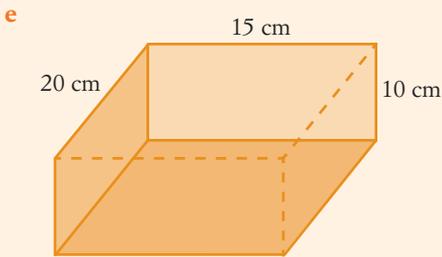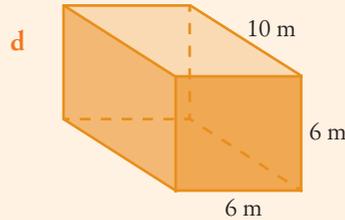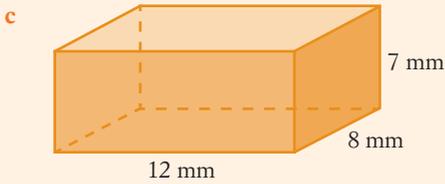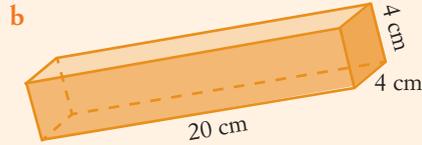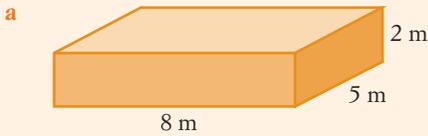
20 Copy and complete the following.

  a $2000 \text{ mL} = $ _____ L   **b** $3 \text{ kL} = $ _____ L
  c $7 \text{ L} = $ _____ mL   **d** $3300 \text{ L} = $ _____ kL
  e $1750 \text{ mL} = $ _____ L   **f** $2.5 \text{ L} = $ _____ mL

21 Copy and complete the following.   *See* Example 11

  a $5000 \text{ g} = $ _____ kg   **b** $2 \text{ g} = $ _____ mg
  c $1\frac{1}{2} \text{ t} = $ _____ kg   **d** $6500 \text{ kg} = $ _____ t
  e $4000 \text{ mg} = $ _____ g   **f** $1.5 \text{ kg} = $ _____ g

Problem solving

22 Calculate the surface area of the triangular prism shown here.

10 cm
24 cm
8 cm
26 cm
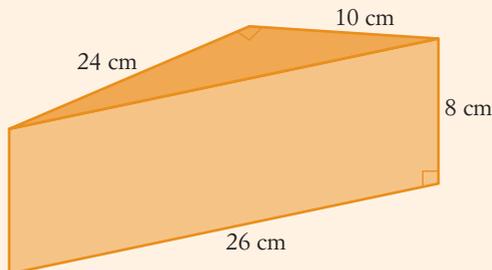
23 Draw each of the following.

  a a non-regular hexahedron   **b** a regular octahedron   **c** a pentahedron

24 For each of these solids, sketch the views listed.

a
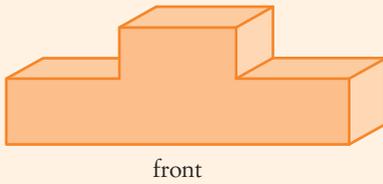
front

   i right view
   ii front view
   iii top view

b

   i right view
   ii front view
   iii top view

left
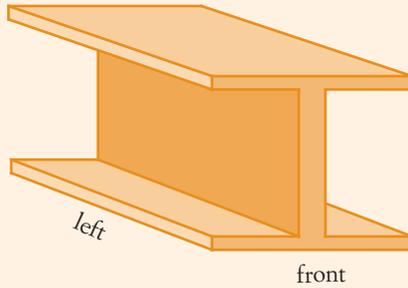
front

25 The diagram on the right shows a tank. The tank is half-filled with water. Find the amount of water in the tank.

14 cm

15 cm

30 cm
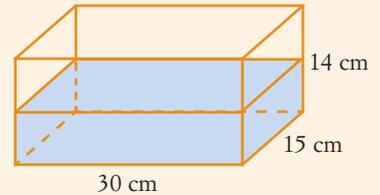
26 A children's pool is in the shape of a cross as shown on the right. Each side is 3 m long. The pool is filled with water to a depth of 300 mm.
   a Find the area of the pool's surface.
   b Calculate the volume of water, in cubic metres (m^3).
   c If water is charged for at $0.80 per kL, how much does it cost to fill the pool?

3 m

3 m

3 m

3 m
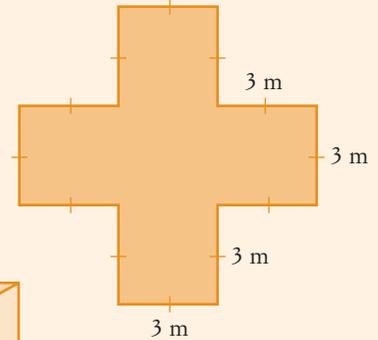
27 Calculate the volume of each solid below.

a

8 cm

16 cm

8 cm

16 cm

16 cm

16 cm

b

7 m

6 m

15 m

5 m

15 m
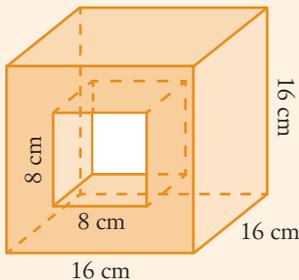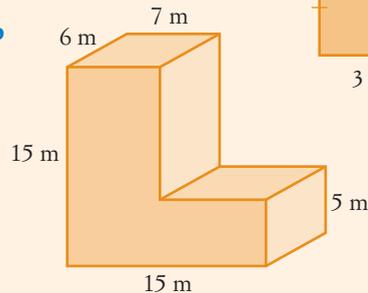
28 A rectangular box 40 cm long and 12 cm wide contains 2880 cm^3 of sugar. How deep is the sugar in the box if it is spread evenly?

29 Select **A**, **B**, **C** or **D** to complete the following. The mass of an egg is closest to:
   A 5 g   **B** 50 g   **C** 500 g   **D** 5 kg

30 Eighteen trucks, each carrying 12 000 kg of debris, were required to clear a building site. How many tonnes of debris were cleared altogether?

31 The biggest iceberg on record was called B9. It had the same volume as a rectangular prism with dimensions 160 km long, 50 km wide and 250 metres high. When B9 melted, how many litres of water was contained in it? (1 kL of water will occupy 1 m^3.)

32 A cube has a volume of 512 cm^3. Find the length of each side of the cube.

33 A doctor orders 5.2 litres of fluid each day to be given to a patient in drops. Each 1 mL of fluid is equivalent to 15 drops. How many drops of fluid per minute are needed for the patient to receive the required dose?

34 The diagram below shows a shipping container in the shape of a rectangular prism.
  a How many cubes of side length 60 cm could be stacked in the container?
  b If each cube has a mass of 25 kg, how many tonnes would the container carry?

3 m

12 m   3 m
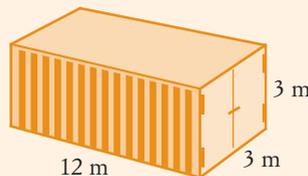
35 A rectangular prism is 5 m long, 3 m wide and 2 m high.
  a Calculate its volume.
  b Calculate the volume of a rectangular prism with dimensions twice as big.
  c Calculate the volume for dimensions 3 times as big.
  d Calculate the volume for dimensions 4 times as big.
  e Find the ratios of the volumes of each prism to the first one.
  f If the dimensions are multiplied by n, by what factor will the volume increase?
  g Consider the sauce bottle shown below. Explain what will happen if it was expanded so that its size increased by a factor of 5 in each direction. Hint: consider it enclosed in a rectangular prism.

Number and algebra

12

Number patterns

Contents

Prior learning
Chapter 12

Parent guide
Chapter 12

Curriculum guide
Chapter 12

Australian curriculum statements

Patterns and algebra

Introduce the concept of variables as a way of representing numbers using letters. (ACMNA175) Create algebraic expressions and evaluate them by substituting a given value for each variable. (ACMNA176)

Linear and non-linear relationships

Given coordinates, plot points on the Cartesian plane and find coordinates for a given point. (ACMNA178) Investigate, interpret and analyse graphs from authentic data. (ACMNA180) AC

You have seen how graphs are used in statistics to give a visual idea of information and relationships. In algebra, you can also use graphs to give a picture of relationships. In fact, the use of graphs in algebra makes up a whole field of study called coordinate geometry. It is a cross between algebra and geometry that is a powerful tool when it is applied to a wide range of real-world situations.

◼ Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

Cartesian plane	independent	output	x-axis
coordinates	input	quadrant	y-axis
dependent	linear graph	satisfies	
function	origin	variable	

12.1 Graphs of number patterns

An algebraic rule relates an input number and an output number. In Chapter 1 you have already examined line graphs. These often represent pairs of related numbers by points on the line on a graph. For example, a graph might show how the much of a compound will dissolve at different temperatures. Each point represents a temperature and a measure of solubility. The same kind of system is extended to the **Cartesian plane** to show algebraic rules. This system is named after its inventor, René Descartes (1596–1650).

Important!

The Cartesian plane

The **Cartesian plane** has two axes at right angles. It is also called a **number lattice**.
The **first axis** is horizontal and is called the **x-axis**. The **second axis** is vertical and is the **y-axis**. The axes cross at the **origin**.
The position of a point on the Cartesian plane is determined by its **coordinates**.
Coordinates are shown in parentheses (round brackets) with a comma between them. The origin is the point (0, 0). The **x-coordinate** is always shown *first* and the **y-coordinate** *last*.
The x-coordinate gives the horizontal position of the point from the origin, and the y-coordinate is the vertical position of the point from the origin.
Each quarter of the Cartesian plane is called a **quadrant**.
In the diagram, Point A is called (3, 2), B is (⁻2, 2), C is (⁻3, ⁻1) and D is (4, ⁻3).
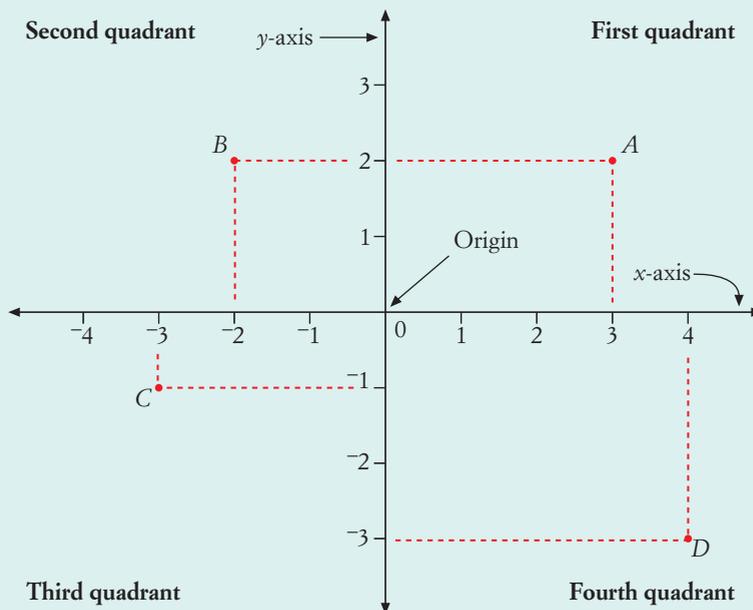
Example 1

For each of the following points, state its position (coordinates) and quadrant or axis.
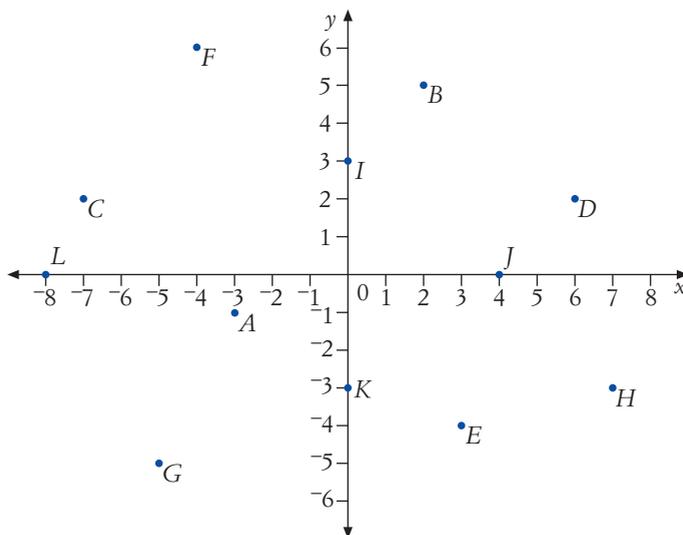
Solution

Find the number on the *x*-axis vertically above (or below) the letter *A*. This is the *x*-coordinate.

The *x*-coordinate of *A* is ⁻3.

Now find the number on the *y*-axis across to the right (or left) of the letter *A*. This is the *y*-coordinate.

The *y*-coordinate of *A* is ⁻1.

Think of the numbering of the quadrants and check which one *A* is in.

$$\frac{2 \quad | \quad 1}{A \quad 3 \quad | \quad 4}$$

A is in the third quadrant.

Write down the letter, its coordinates and quadrant. The *x*-coordinate must come first.

A(⁻3, ⁻1), third quadrant

Now do the point *B*. The *x*-axis is vertically above this point and the *y*-axis is across to the left.

B(2, 5), first quadrant

Keep going with the other points.

C(⁻7, 2), second quadrant

D(6, 2), first quadrant

E(3, ⁻4), fourth quadrant

F(⁻4, 6), second quadrant

G(⁻5, ⁻5), third quadrant

H(7, ⁻3), fourth quadrant

I is on the *y*-axis, so the *x*-coordinate is 0.

I(0, 3), *y*-axis

J is on the *x*-axis, so the *y*-coordinate is 0.

J(4, 0), *x*-axis

K(0, ⁻3), *y*-axis

L(⁻8, 0), *x*-axis

Example 2

Plot the following points on a Cartesian plane and join them in order. $A(2, 11)$, $B(^-3, ^-4)$, $C(10, 5)$, $D(^-6, 5)$, $E(7, ^-4)$, A – Twinkle!

Solution

Find the greatest and least values of the coordinates.

The least value on the x-axis is $^-6$, and the greatest value is 10. The least value on the y-axis is $^-4$ and the greatest value is 11.

Draw a set of axes.

Make the scales big enough to fit all the coordinates.

Plot the points in order.

Join up the points.

$A(2, 11)$ has an x-coordinate of 2, so is vertically opposite 2 on the x-axis. It has a y-coordinate of 11, so is across from 11 on the y-axis.
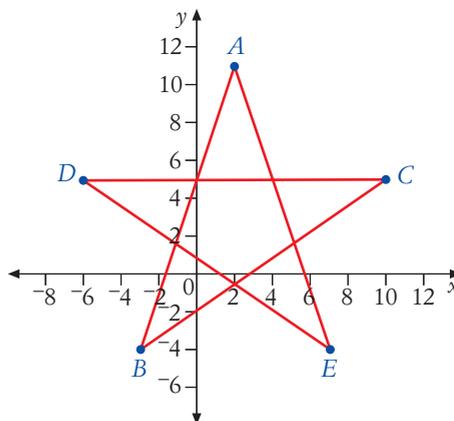
We use the Cartesian plane to draw graphs of number patterns.

Important!

Graphing number patterns

When graphing a number pattern, you do the following:

- draw the x-axis across the page and the y-axis vertically at right-angles
- make the axes big enough to clearly show the information
- mark each axis with a suitable scale
- the x-axis is used for the input number and the y-axis for the output number
- label each axis to show the actual variable names of the first and second numbers of the number pattern
- mark the points on the graph as (*input*, *output*)
- connect with a line if appropriate

In many cases it is also useful to name the graph.

Example 3

Look at this triangular pattern.

a Complete this table of values using the pattern.

Number of triangles (t)	1	2	3	4	5	6
Number of matches (m)	3	5				

b Draw a graph of the pattern using the horizontal axis for the number of triangles.
c Use the information in the table to find a rule for the pattern.

Solution

a Extend the pattern and complete the table.

t	1	2	3	4	5	6
m	3	5	7	9	11	13

b Use the values in the table to plot the points in the graph.

Join the points.
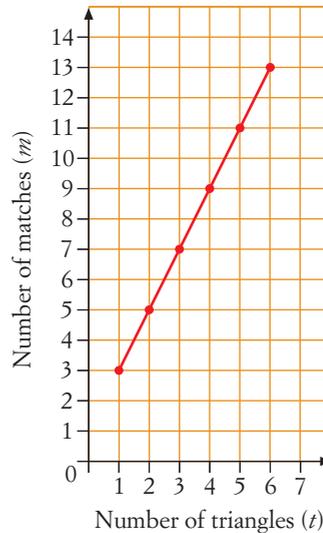
c The difference between successive terms is 2. Use this fact to form the rule. $m = 2 \times t + 1$

Check the rule using different values of t. $2 \times 1 + 1 = 3, 2 \times 2 + 1 = 5,$ $2 \times 3 + 1 = 7 \ldots,$ **so the rule works.**

Write the rule algebraically. $m = 2t + 1$

You can see that the graph in Example **3** is a straight line. This is also called a **linear graph**. If we extend the graph, it can be used to find the number of matches needed for other terms in the triangular pattern.

Example 4

Look at this pattern.

Term 1   Term 2   Term 3
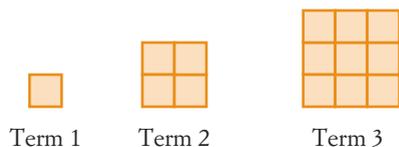
a Extend the pattern and count the squares to complete this table of values.

Term number (T)	1	2	3	4	5	6
Number of squares (S)	1	4	9			

b Draw a graph of this pattern using the horizontal axis for the term number.
c Is the graph linear?

Solution

a Extend the pattern and complete the table.

T	1	2	3	4	5	6
S	1	4	9	16	25	36

b Use the values in the table to plot the points in the graph.

Join the points.
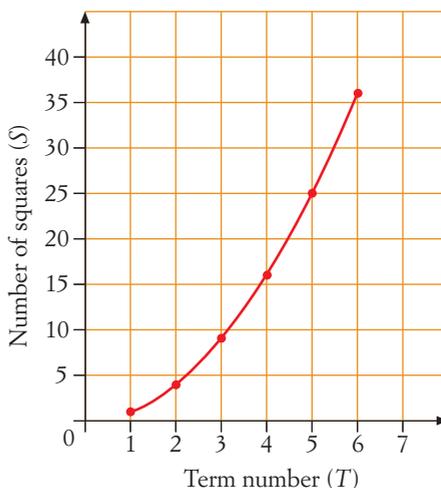
c Look at the shape of the graph. **The graph is not linear.**

The graph of the pattern in Example **4** is based on a rule (how *do* you work out the second number from the first one?), but it is not linear. Some patterns of numbers do not have rules at all. For example, during the day humidity is generally lowest just before dawn, and is highest in the early afternoon.

Example 5

The table below shows the humidity at 9 a.m. in September 2010 in Broome. Plot the points using the information in the table below and comment on the shape of the graph.

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Humidity %	80	69	33	28	22	24	56	64	45	37	35	55	47	61	68
Date	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Humidity %	60	63	40	41	28	22	21	33	27	27	28	16	33	47	26

Solution

Make time horizontal.

Humidity in Broome
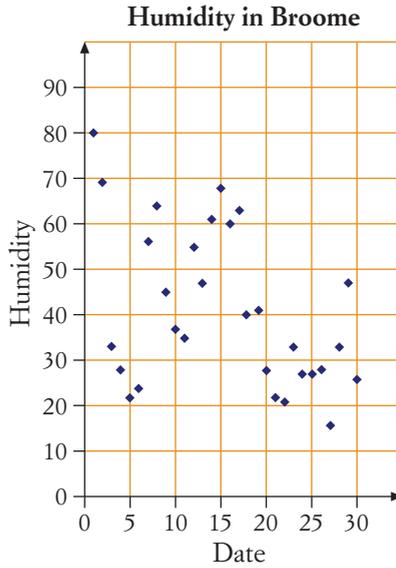
Comment.

The graph is not linear as a straight line cannot be drawn through the points.

Exercise 12.1 Graphs of number patterns

1 For each of the following points, state its position and quadrant or axis.
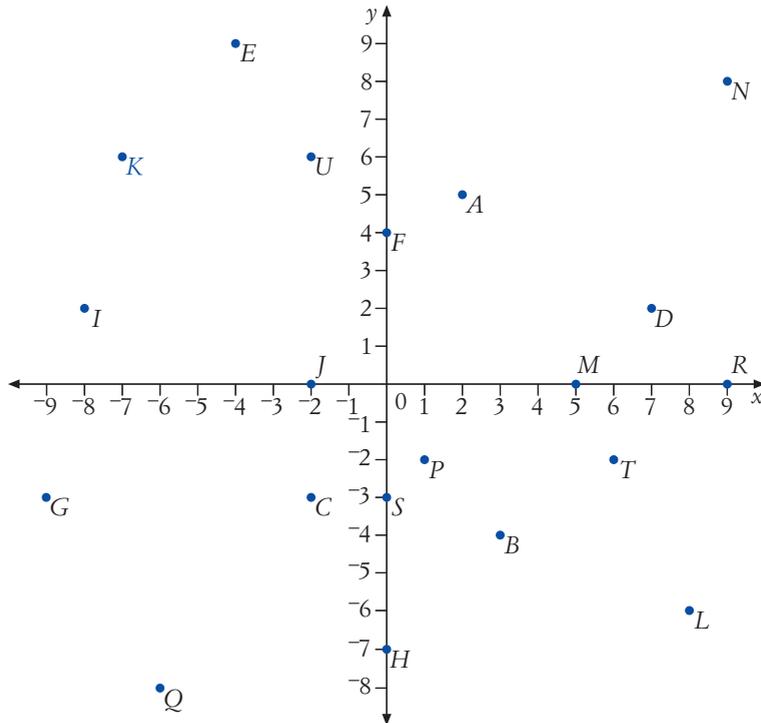
2 Plot each group of points on a grid and join them in order.

   a $A(1, 4)$, $B(2, 3)$, $C(3, 3)$, $D(3, 1)$, $E(1, 1)$, $F(1, 3)$, B, $G(3, 4)$ – What's on?

   b $A(1, 2)$, $B(1, 3)$, $C(2, 3)$, $D(2, 4)$, $E(3, 4)$, $F(3, 5)$, $G(4, 5)$, $H(4, 2)$, A – Stepping up!

   c $A(5, 4)$, $B(2, 4)$, $C(1, 2)$, $D(2, 1)$, $E(3, 2)$, $F(6, 2)$, $G(7, 1)$, $H(8, 2)$, $I(8, 3)$, $J(6, 3)$, A – Vrooom!

3 Plot each group of points on a grid and join them in order.

   a $A(^-2, 1)$, $B(2, ^-2)$, $C(^-5, ^-2)$, $D(^-5, 1)$, A, $E(1, 5)$, $F(5, 2)$, B, $G(2, ^-6)$, $H(^-2, ^-6)$, A – Greek!

   b $A(2, ^-4)$, $B(2, 0)$, $C(3, 0)$, $D(3, 6)$, $E(2, 6)$, $F(2, 7)$, $G(5, 7)$, $H(5, 6)$, $I(4, 6)$, $J(4, 0)$, $K(5, 0)$, $L(5, ^-4)$, $M(^-4, ^-4)$, $N(^-6, 2)$, $P(2, 2)$, $Q(0, ^-4)$ – By the sea!

4 Look at the pattern on the right.
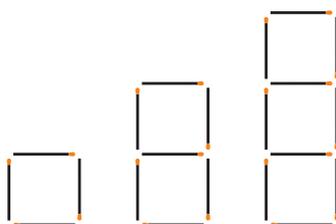
   a Extend the pattern and count the matchsticks to complete the following table.

Number of squares (s)	1	2	3	4	5	6
Number of matches (m)	4	7				

   b Draw a graph of the pattern using the horizontal axis to show the number of squares.

   c Use the information in the table to find a rule for the pattern.

5 Refer to the triangular pattern below.
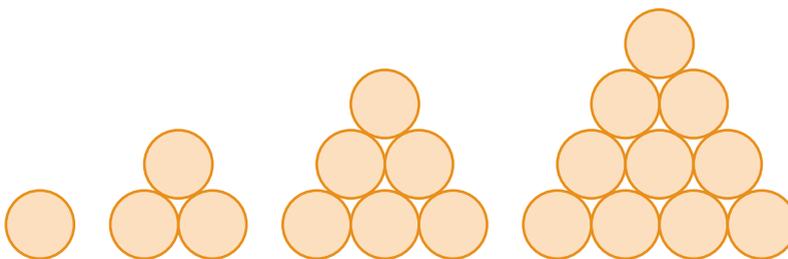
   a Extend the pattern and count the discs to complete this table of values.

Term number (T)	1	2	3	4	5	6
Number of discs (D)	1	3	6			

   b Draw a graph of this pattern using the horizontal axis for the term number.

   c Is the graph linear?

6 Look at the matchstick patterns.

Quadrilateral pattern:

Pentagonal pattern:

Hexagonal pattern:
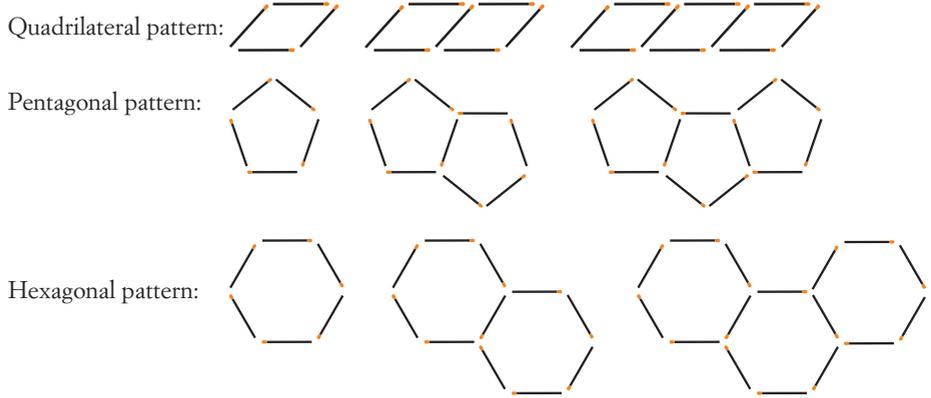
a For each pattern complete a table showing the number of matches required to make up to 6 terms.

b Use one set of axes to draw a graph of each pattern, showing the term number on the horizontal axis.

c Use the information in the table to find a rule for each pattern.

7 Look at this pattern.
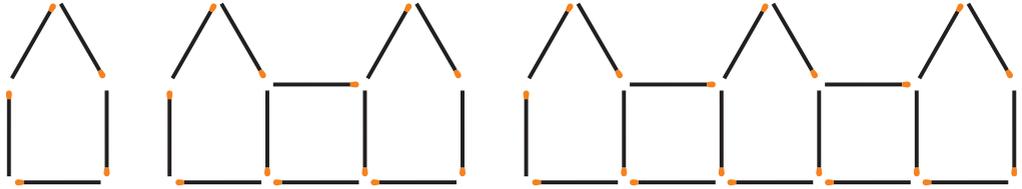
a Extend the pattern to complete this table.

Term number (t)	1	2	3	4	5	6
Number of matches (m)	5	12	19			

b Draw a graph of this pattern using the horizontal axis for the term number.

c Use your graph to find the number of matches in the 10th term.

d Use the information in the table to find a rule for the pattern.

8 The following shapes are formed using tiles. Each successive shape has one extra tile in each 'arm'.
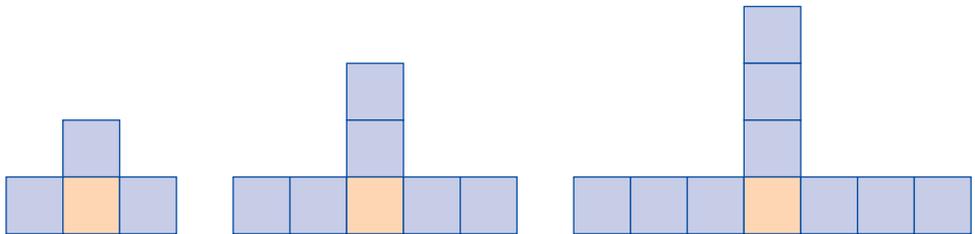
a Extend the pattern and complete this table.

Arm length (*a*)	1	2	3	4	5	6
Number of tiles (*t*)	4	7				

b Draw a graph of this pattern using the horizontal axis for the arm length.

c Use the information in the table to find a rule for the pattern.

9 A restaurant uses rectangular tables. A table can seat six diners but, when two tables are joined, only 10 people can be seated as shown below.
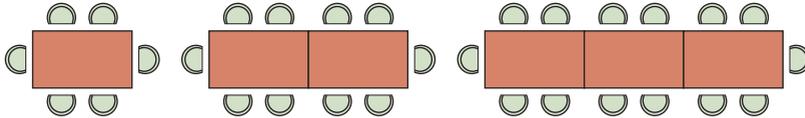
a Extend the pattern to complete this table.

Number of tables (*t*)	1	2	3	4	5	6
Number of people (*p*)	6	10				

b Draw a graph of this pattern using the horizontal axis for the number of tables.

c Extend your graph to find how many tables would need to be joined to seat 40 people.

d Use the information in the table to find a rule for the pattern.

10 Make up some grid drawings like the one in Example **2**. It is easiest to use grid paper to work out the points. Give your instructions to someone else to try out. Discuss the drawings with that person.

Problem solving

11 When people meet for the first time they usually shake hands. Consider the number of handshakes (*h*) which occur when a number of people (*p*) meet and everyone shakes the hand of every other person once.

a Copy and complete this table:

Number of people (*p*)	1	2	3	4	5	6
Number of handshakes (*h*)						

b Draw a graph showing the number of handshakes for different numbers of people.

c Use your graph to find out how many handshakes would occur if eight people meet.

12 The information below shows the number of rainy days each month in Cairns. Draw a graph of the information and comment on the shape of the graph.

See Example 5

Month	1	2	3	4	5	6	7	8	9	10	11	12
Rainy days	15	17	16	15	10	7	6	5	5	6	8	11

12.2 Number rules and functions

Input/output devices are common in computing. These devices accept an input and then produce an output as a result. A printer is an example of an input/output device. It accepts signals from a computer, processes these and produces a printed output.

Imagine a **number machine** that acts like an input/output device by accepting numbers, processing these numbers using a rule and then producing different numbers as outputs.

Number machine

Input
4

Output =
$2 \times$ Input + 1
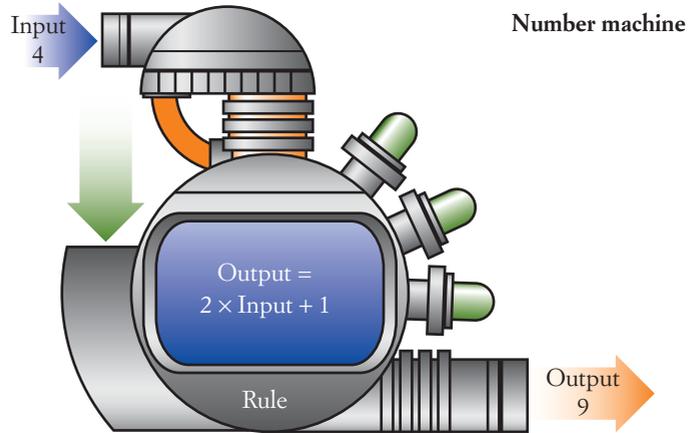
Rule

Output
9

Example 6

a The rule on a number machine is:

$$\text{output} = 4 \times \text{input} + 3$$

Find the outputs when the inputs are 1, 3 and 5 and show the results in a table.

b The rule for a number machine is:

$$\text{output} = 50 - 3 \times \text{input}$$

Are the output numbers in the following table correct for the corresponding input numbers?

Input	Output
2	44
5	35
8	26

Solution

a For this number machine we take the input, multiply it by 4 and then add 3 to find the output.

Write the number machine rule.	**output = 4 × input + 3**
Replace input with 1.	**= 4 × 1 + 3**
Calculate the output.	**= 4 + 3**
	= 7

When the input is 3, the rule stays the same.	**output = 4 × input + 3**
This time we replace input with 3.	**= 4 × 3 + 3**
Calculate the output.	**= 12 + 3**
	= 15

Now find the last output.	**output = 4 × input + 3**
Replace input with 5.	**= 4 × 5 + 3**
Calculate the output.	**= 20 + 3**
	= 23

Show the results in a table.

Input	Output
1	7
3	15
5	23

b For this number machine we take the input, multiply it by 3 and then subtract it from 50 to find the output. We need to check each input with the corresponding output.

Write the rule.	**output = 50 – 3 × input**
Replace input with 2.	**= 50 – 3 × 2**
Calculate the output.	**= 50 – 6**
	= 44

For the second input, the rule does not change.	**output = 50 – 3 × input**
Replace input with 5.	**= 50 – 3 × 5**
Calculate the output.	**= 50 – 15**
	= 35

For the third input, we use the same rule.	**output = 50 – 3 × input**
Replace input with 8.	**= 50 – 3 × 8**
Calculate the output.	**= 50 – 24**
	= 26

Compare with the values in the table.	**The results in the table are correct.**

Sometimes we won't know the rule for the number machine. However, if we have a table with the inputs and corresponding outputs, we may be able to find the number machine rule.

Example 7

The following tables of inputs and outputs were produced by different number machines. Find the rule for each machine. Write the rule in words and complete the table.

a

Input	Output
5	10
8	13
2	7
9	
15	
40	

b

Input	Output
7	43
3	47
20	30
18	
2	
30	

c

Input	Output
4	13
10	31
8	25
11	
23	
16	

Solution

a Compare each input with its output.

Each output is 5 more than its input.

Write the rule in words.

$output = input + 5$

Complete the table by adding 5 to each input.

Input	Output
5	10
8	13
2	7
9	14
15	20
40	45

$9 + 5 = 14$
$15 + 5 = 20$
$40 + 5 = 45$

b Compare inputs and outputs.

To get the output take the input away from 50.

Write the rule.

$output = 50 - input$

Complete the table by taking each input away from 50.

Input	Output
7	43
3	47
20	30
18	32
2	48
30	20

$50 - 18 = 32$
$50 - 2 = 48$
$50 - 30 = 20$

c Compare inputs and outputs.

Each output is 1 more than 3 times the input.

Write the rule.

output = 3 × input + 1

Complete the table by multiplying each input by 3 and then adding 1.

Input	Output
4	13
10	31
8	25
11	34
23	70
16	49

$3 \times 11 + 1 = 33 + 1 = 34$
$3 \times 23 + 1 = 69 + 1 = 70$
$3 \times 16 + 1 = 48 + 1 = 49$

You can work out the rule for linear rules by comparing inputs and outputs.

Input	Output
7	31
2	11
5	23
1	7

Arrange the inputs in order. Make sure you keep the corresponding inputs and outputs together.

Input	Output
1	7
2	11
5	23
7	31

Linear rules are of the form: output = □ × input + △ or output = □ × input − △.

It looks like each output is 4 more than the one before. Extend the table to include 0 as an input and include the missing inputs between 1 and 7.

Find missing outputs by adding 4 to the previous output.

You need to backtrack to find the output for the input of 0.

When the input goes up by 1, each output is 4 more than the previous one. So the multiplier in the number machine rule must be 4. This means that the rule will involve 4 × input.

For an input of 0, the output is 3. This means that + 3 will also be part of the rule.

Putting these together, the rule is:

output = 4 × input + 3

Input	Output	
0	3	+ 4
1	7	+ 4
2	11	+ 4
3	15	+ 4
4	19	+ 4
5	23	+ 4
6	27	+ 4
7	31	

A check of the values in the table shows that the rule works.

Example 8

Find the rule in symbols for the number machine that produced the values in this table.

Input	Output
5	26
6	31
1	6
8	41
3	16

Solution

Rewrite the table by arranging the inputs in ascending order, and including the missing input numbers.

Input	Output
0	
1	6
2	
3	16
4	
5	26
6	31
7	
8	41

Looking at inputs 5 and 6, the difference between successive outputs is 5. Since the output for 1 is 6, the output for 0 must be 1, the output for 2 is 11 and so on.

Include the missing outputs.

Input	Output
0	$6 - 5 = 1$
1	6
2	$6 + 5 = 11$
3	16
4	$16 + 5 = 21$
5	26
6	31
7	$31 + 5 = 36$
8	41

Write the rule.

output $= 5 \times$ input $+ 1$

Check the output values in the table using the rule.

The rule works.

Write the rule in symbols.

$b = 5a + 1$

Technology Guess my rule

Guess my rule is a mathematical game played in pairs. The aim of the game is to guess the rule made up by your partner.

Open a spreadsheet and copy the headings in rows 1 and 2 as shown below.

Enter a number into cell A3. Select a whole number less than 10 to begin with so that the game isn't too difficult. You may wish to change this for later games.

Enter a mathematical rule that uses A3 into cell B3 as shown below. You can use up to two of the operations +, −, × or ÷. For example, if A3 = 5, in cell B3 you might type '=3*A3+2', then hit 'Enter'. This will give you the answer but hide the formula. Remember when using a spreadsheet the * symbol means × and the / symbol means ÷.

	A	B	C	D
1	**Guess my rule**			
2	Input	Output	Rules	
3	5	= 3*A3+2		
4				
5				

Now give the keyboard to your partner, who must try to guess the rule you have entered. Your partner must enter rules in column C. There are many different rules that will give the same output for the input in A3. He or she has 2 minutes to enter as many rules as possible moving down column C.

When the time is up, check all the rules entered by your partner. Any rule that gives the correct output receives 2 points. If your partner actually guesses the rule you entered in B3, he or she receives 10 points.

The first player to reach 100 points wins the game.

In mathematics, we have a special name for rules that change one number into another.

Important!

Functions

A mathematical rule that changes numbers into new ones is called a **function**. The starting number is called the **input** or **independent variable**. The number produced is called the **output** or **dependent variable**. If you change the value of the independent variable, then the dependent variable works out to a new value.

The rule for a function is usually written with the dependent variable on the left-hand side of the equals sign, like $f = 4i + 2$. This function changes an input of $i = 7$ to an output of $f = 30$.

A number machine should really be called a **function machine**.

Example 9
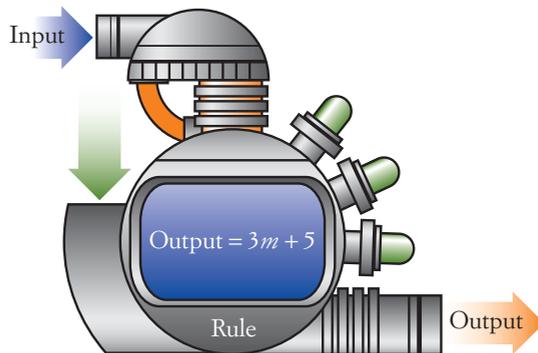
Find the output for each of the following inputs of the function $y = 3m + 5$.

a 4 **b** 7 **c** $^-2$

Solution

You can imagine the function as a function machine to help your thinking.
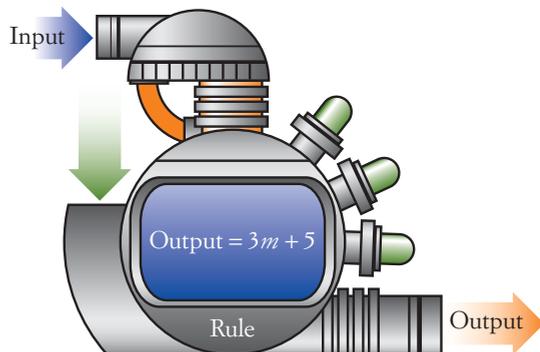
Write the function using a $\times$ sign. **Output** $= 3 \times m + 5$

a Replace m with the input value 4. $y = 3 \times 4 + 5$

Evaluate. $= 17$

b Put 7 in place of the variable. $y = 3 \times 7 + 5$

Work out the answer. $= 26$

c Put $m = {}^-2$. $y = 3 \times {}^-2 + 5$

Remember that
positive $\times$ negative = negative. $= {}^-6 + 5$

Write the answer. $y = {}^-1$

Exercise 12.2 — Number rules and functions

1 For each of the following tables, use the rule to complete the table.

a The rule is
output = 3 × input + 2

Input	Output
1	
2	
3	
4	
5	

b The rule is
output = 2 × input − 3.

Input	Output
2	
3	
4	
5	
6	

c The rule is
output = 5 × input + 3.

Input	Output
2	
3	
4	
6	
11	

d The rule is
output = 50 − 2 × input.

Input	Output
1	
2	
3	
8	
13	

e The rule is $B = 10A + 7$.

A	B
0	
1	
2	
6	
	87
12	

f The rule is $y = 6x - 5$.

x	y
3	
4	
5	
	43
11	
	115

2 Calculate the output for each of the
following input values.

a	4	**b**	3
c	2	**d**	7
e	12	**f**	10

Input

$4m - 3$

Rule   Output

3 Calculate the inputs that will result in the following output values.

  a 7   **b** 5
  c 13   **d** 25
  e 21   **f** 10

Input

$2d + 5$

Rule   Output
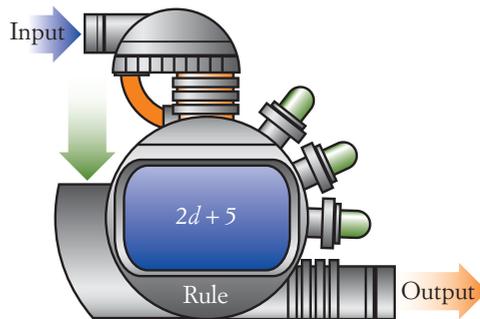
4 Which of the following rules was used to produce this table?

Input	Output
0	7
4	19

A output $= 2 \times$ input $+ 7$
B output $= 3 \times$ input $+ 1$
C output $= 3 \times$ input $+ 7$
D output $= 3 \times$ input $- 2$

5 Which of the following rules was used to produce this table?

A	B
4	11
9	31

A $B = 2 \times A + 3$
B $B = 5 \times A - 9$
C $B = 3 \times A + 4$
D $B = 4 \times A - 5$

Fluency

See Example 7

Worked solutions

Exercise 12.2

MAT07NAWS00048

6 Find the rule that relates output and input values in the following tables. In each case, use the rule to complete the table.

a

Input	Output
1	5
2	10
3	15
5	
7	
9	

b

Input	Output
4	15
7	18
5	
	20
	28
11	

c

Input	Output
2	11
6	15
14	23
7	
9	18
12	

d

Input	Output
2	6
11	33
6	18
9	
	54
	48

e

A	*B*
7	43
11	39
4	
22	28
16	
	16

f

m	*n*
24	3
56	7
112	
72	9
	11
64	

7 Complete the output tables for the following functions. *See* Example 9

a $t = 4b - 3$

b	1	2	3	4
t				

b $g = 5n + 2$

n	0	2	3	5
g				

c $y = 2(x + 1)$

x	1	3	5	7
y				

d $q = 14 - 3p$

p	0	1	3	4
q				

8 Name the dependent and independent variables for each function in question **7**.

9 Complete the output tables for the following functions.

a $h = {}^{-}2g + 7$

g	0	1	3	6
h				

b $k = 2(j + 4)$

j	$^{-}2$	$^{-}1$	0	1
k				

c $y = {}^{-}4(2x - 3)$

x	1	3	5	7
y				

d $d = 3e(e + 6)$

e	$^{-}2$	0	3	5
d				

10 Use the rule $a = 1 + 3b$ to find the value of *a* when:

    a $b = 4$ **b** $b = 7$ **c** $b = 0$

11 Use the rule $M = 2N - 5$ to find the value of *M* when:

    a $N = 4$ **b** $N = 7$ **c** $N = 12$

12 Name the independent variables in questions **10** and **11**.

Problem solving

13 Find the rule that relates output and input values in the following tables. In each case, use the rule to complete the table.

a

Input	Output
0	4
1	7
2	10
5	19
7	
10	

b

Input	Output
2	16
3	21
5	31
8	46
6	
11	

c

Input	Output
2	7
3	11
4	15
7	27
9	
	55

d

Input	Output
6	40
2	12
7	47
10	68
	54
15	

e

S	T
1	13
8	27
6	23
11	
9	29
	15

f

a	b
10	89
1	8
7	
9	80
	116
	44

See Example 8

14 Here are the inputs and corresponding outputs for different functions. Use the variables shown to write the function rule.

a

Input, n	1	3	5	8
Output, m	1	9	17	29

b

Input, m	1	2	3	4
Output, p	5	7	9	11

c

Input, c	1	2	4	6
Output, d	1	4	10	16

d

Input, d	1	2	5	7
Output, k	6	11	26	36

e

Input, x	1	2	3	5
Output, y	17	14	11	5

15 Look at these figures.

4 sides   5 sides   6 sides
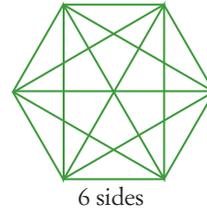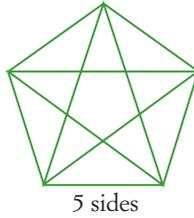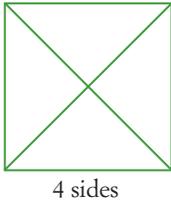
There are six lines in the first figure: the four sides and the two diagonals. How many lines are in the other two figures?

Now look at the design below. Build up a table using the information from the three figures above to calculate the number of lines there are in this design.
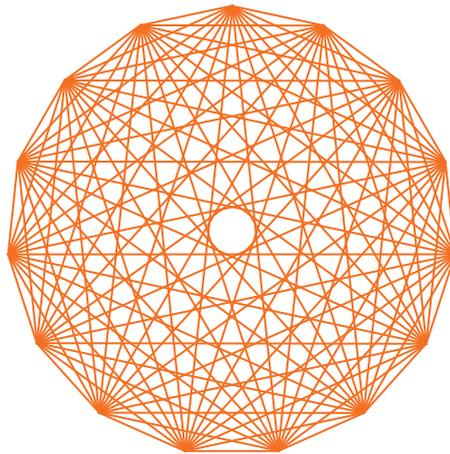
12.3   Functions and graphs

You have already seen that linear rules (**linear functions**) have graphs that are straight lines. This means that you really only have to plot two points to be able to draw the line. It is a good idea to plot three points so that the third point acts as a check. If the points don't line up, you will know that you need to check your working.
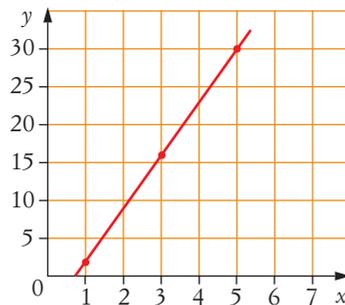
Example 10

Consider the function $y = 3x + 2$.
a Draw a graph of the function using $x = 0$, 2 and 4.
b Describe the graph and identify the dependent and independent variables.
c Use the graph to find the value of y when x is 3.
d Use the graph to find the value of x when $y = 20$.

Solution

a Find the output value for $x = 0$.　　$\boldsymbol{y = 3 \times 0 + 2}$

$\boldsymbol{= 0 + 2}$

$\boldsymbol{= 2}$

Find the output value for $x = 2$.　　$\boldsymbol{y = 3 \times 2 + 2}$

$\boldsymbol{= 8}$

Find the output value for $x = 4$.　　$\boldsymbol{y = 3 \times 4 + 2}$

$\boldsymbol{= 14}$

Complete the table of values.

x	0	2	4
y	2	8	14

Draw the x- and y-axes. Plot the points and join them. Extend your graph beyond the points that have been plotted.
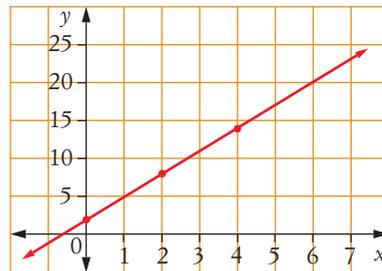
b The graph is a straight line.　　**The graph is linear.**

The value of y depends on the value of x.　　**y is the dependent variable.**

x is the independent variable.

c Locate 3 on the x-axis. Draw a line vertically up until it touches the graph.

Draw a line horizontally across until it touches the y-axis.
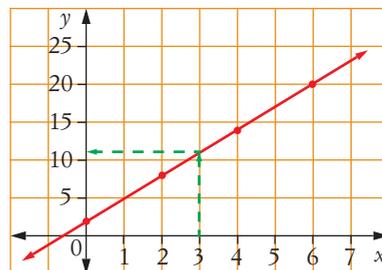
Read the value.　　$\boldsymbol{y = 11}$

State the result.　　**When $x = 3$, $y = 11$.**

d Repeat for $x = 6$.
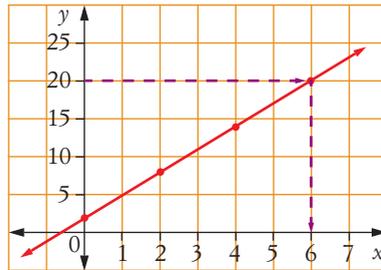
State the result.   **When $x = 6$, $y = 20$.**

In part **d** above, you found that, for the function $y = 3x + 2$, $x = 6$ when $y = 20$. We say that the point $(6, 20)$ **satisfies** the function $y = 3x + 2$.

Example   11

Mandy sells mobile phones. She is paid \$350 per week and an extra \$40 for each phone she sells. Some weeks she doesn't sell any phones but she has sold as many as 10 phones in a week.

a Write the function for Mandy's weekly wage (w) using the number of phones (p) sold.
b Draw a graph showing her weekly wage and phones sold.
c Describe the relationship between Mandy's wage and the number of phones she sells.

Solution

a Write the function in words.

Mandy's wage is \$350 plus \$40 for each phone sold.

Write the rule using symbols.

$w = 350 + 40 \times p$

Write without using the multiplication sign.

$w = 350 + 40p$

b Draw a table showing the number of phones sold and Mandy's wage. Any values of p between 0 and 10 can be used.

Phones sold (p)	0	2	4	7	10
Wages (w)	350	430	510	630	750

Draw the axes with the number of phones sold on the horizontal axis. Plot the points and join them.

Mandy's weekly wage
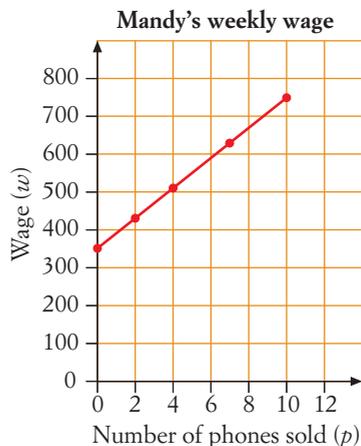
c Look at the graph of the relationship between Mandy's wage and the number of phones she sells.

The graph is a straight line.

Write the result.

There is a linear relationship between Mandy's wage and the number of phones she sells.

A **travel graph** shows the distance travelled by something over time. When the speed is constant, you get a straight line, as shown in parts of the following example. A travel graph can also tell you about what happened during a trip.

Example   12

This graph shows the travel of a girl to and from the corner shop to get some milk.

a How long did it take her to get to the shop?
b What does the flat part show?
c How long did she spend at the shop?
d How long did she take to get home?
e Find her speeds going to and from the shop. When did she travel fastest?

Going to the shop
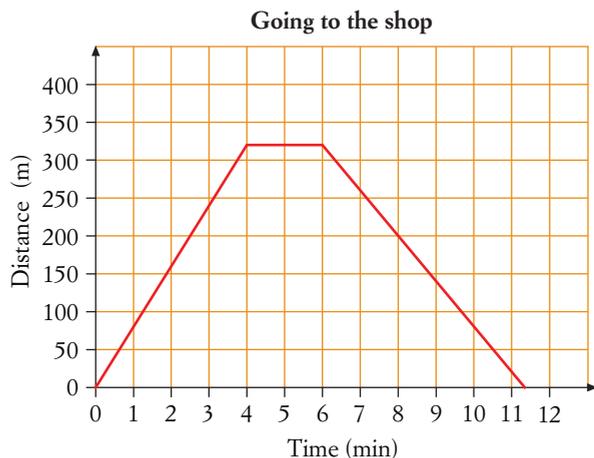
Solution

a The time will be up to when she was furthest from home.

Time to the shop = 4 minutes

b On the flat part, she does not change her distance from home.

It shows when she was at the shop.

c Check how long the flat part is.

She stayed 2 minutes at the shop.

d She took from 6 minutes to 11 minutes and 20 seconds.

She took 5 minutes and 20 seconds to get home.

e The distance is about 320 m.

$$\text{Speed to the shop} = \frac{320\,\text{m}}{4\,\text{min}}$$
$$= 80\,\text{m/minute}$$

The time to get home is about 5 min 20 s, or $5\frac{1}{3}$ minutes.

$$\text{Speed on the way home} = \frac{320\,\text{m}}{5\frac{1}{3}\,\text{min}}$$

Use your calculator.

$$= 60\,\text{m/min}$$

Write the answer.

She travelled fastest on the way to the shop.

In Example **12** you can see that the part of the trip on the way to the shop, where she travelled fastest, is steeper than on the way home. The steeper the graph, the faster the speed. When the graph is flat, the person is not moving.

Investigate: Evaporation rates

Evaporation and transpiration of water by plants are the main reasons that water is lost from the land. The rate of evaporation from a lake or dam depends on the surface area, the temperature, the humidity, the wind speed and the amount of cloud cover. The amount of transpiration depends on the type of plants, their density and the nature of the plant surfaces. Evaporation and transpiration taken together are known as *evapotranspiration*. The graphs below show the typical rates of evaporation in mm/day for grassland for different temperatures and wind speeds with relative humidity of 50%. The graph on the left shows what happens as the speed of the wind increases at a temperature of 25°C. The one on the right shows what happens as the temperature increases for a wind speed of 2 m/s (about 7 km/h).

Evaporation at RH 50%, T = 25°C   **Evaporation at RH 50%, windspeed 2 m/s**
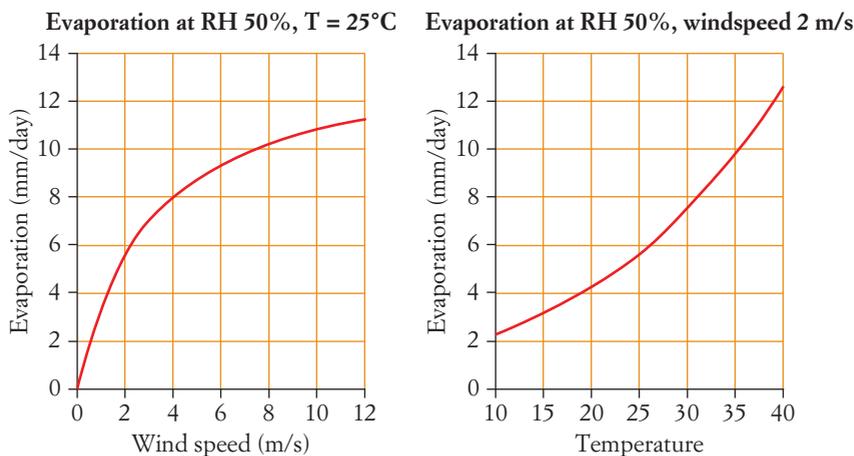
The rate of evaporation over open water storages like dams is similar to that in (°C) the graphs. As you can see from the graphs, it is quite possible for the amount of rainfall to be less than the amount of evaporation.

What does this mean for water storage for dry parts of Australia or places that have most of their rainfall in one part of the year?

To draw a linear function, you only need to plot two points. Remember that a linear function is of the form *output = □ × input ± △*. If a function is not in this form, it is probably not linear. If it is not linear, you will need to plot many points to draw it. If a function is given as a table of values anyway, you can just plot the points to see if it is linear.

Example 13

Draw a graph of the function $y = x(x + 2)$ for $x = 0$ to 4.

Solution

The function is *not* of the form
$□ × x ± △$.

The function is not linear.

Make a table of values.

$y = x(x + 2)$

For $x = 3$,
$y = 3 × (3 + 2) = 15.$

x	0	1	2	3	4
y	0	3	8	15	24

Plot the points and draw the graph.
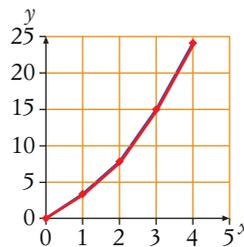
In Example **13**, straight lines between the points have been drawn in blue. It is preferable to draw a smooth curve, as shown in red, if the graph appears to be smoothly changing, like this one.

Exercise 12.3   Functions and graphs

1 For each of the following, complete the table and use the values to draw a graph of the rule.

a $t = 5s$

s	0	3	5
t			

b $P = 4K - 1$

K	1	3	6
P			

c $a = 15 - 3b$

b	0	2	4
a			

d $d = 2f + 7$

f	1	3	6
d			

2 Draw a graph of each of the following rules by first completing the table supplied.

a $y = x + 3$

x	0	3	5
y			

b $y = 3x + 5$

x	0	2	4
y			

c $y = 13 - 2x$

x	0	3	6
y			

d $y = 7x - 5$

x	1	3	5
y			

3 Draw up tables of values using x from 1 to 4 for the following rules and plot the graphs.

a $y = 2x + 3$ **b** $y = 3x + 1$
c $y = 4x - 3$ **d** $y = 13 - 2x$
e $y = 3x - 3$ **f** $y = 16 - 3x$
g $y = 5x - 4$ **h** $y = 9 - 2x$

4 In each of the following, state if the ordered pair satisfies the given function.

a $y = x + 3$ (2, 5)   **b** $y = x - 4$ (6, 2)
c $y = 2x + 5$ (3, 11)   **d** $y = 3x - 7$ (2, 1)
e $y = 4x - 8$ (3, 2)   **f** $y = 15 - 2x$ (4, 7)

5 For each of the following functions, complete the ordered pair that satisfies the given function.

a $y = x - 2$ (3, . . .)   **b** $y = x + 3$ (1, . . .)
c $y = 3x - 6$ (2, . . .)   **d** $y = 2x + 3$ (4, . . .)
e $y = 6x - 5$ (6, . . .)   **f** $y = 13 - 3x$ (3, . . .)

6 In each of the following, state if the ordered pair satisfies the given function.

a $y = 2 - 2x$ $(1, 0)$ b $y = 3x - 2$ $(^-1, ^-5)$
c $y = 4x + 7$ $(^-3, 5)$ d $y = 3 - 4x$ $(^-2, 11)$
e $y = 5x + 3$ $(^-2, 7)$ f $y = 9 - 3x$ $(4, 4)$

7 For each of these functions, complete the ordered pair that satisfies the given function.

a $y = 4x - 5$ $(^-3, \ldots)$ b $y = 6 - 3x$ $(\ldots, 0)$
c $y = 2x + 9$ $(\ldots, 5)$ d $y = 7 - 5x$ $(^-4, \ldots)$
e $y = 7x + 18$ $(\ldots, 4)$ f $y = 8 - 4x$ $(^-2, \ldots)$

8 A family drove along the Great Ocean Road from Geelong to Port Campbell. They left
Geelong at 9 a.m. The travel graph for their trip is shown below.
  a What time did they stop for an ice-cream and a break?
  b How long did they stop for?
  c When were they travelling the slowest?
  d At about what time did they get to Port Campbell?
  e About how far did they travel?

Geelong-Port Campbell trip
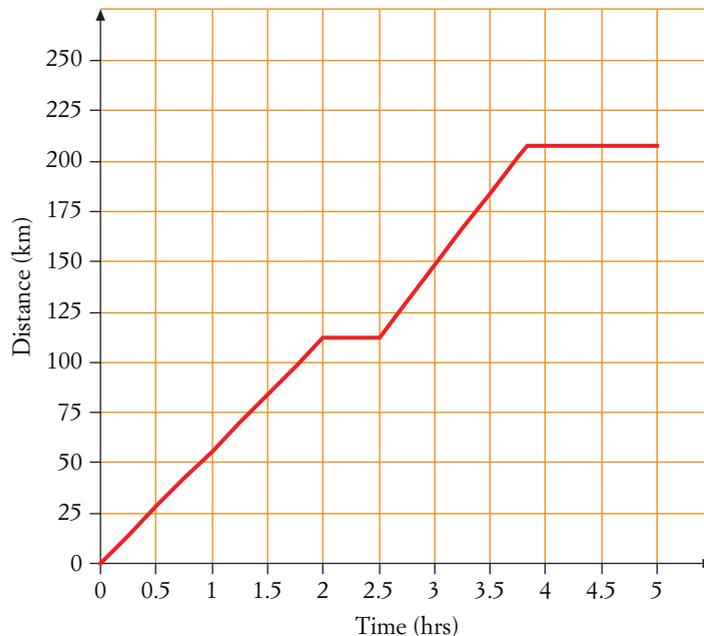
9 A boy walked 1.8 km to school at a speed of 5.4 km/h (90 metres/minute). Make a table of his
distance from home every two minutes and draw a travel graph.

10 Julia rode her bike to her friend Samantha's house and back one Saturday. She rode at a speed of 20 km/h on the way there but did only 10 km/h on the way back. It took her half an hour to ride there, and she stayed for 2 hours. Draw a travel graph to show the whole trip and explain the slopes on the graph.

11 A Year 8 class wants to stage a concert to raise money. The setup costs including the hire of the venue and equipment is $500. There are no other costs as all performers are donating their time. They decide to charge $4 per ticket.

   a Find the function for the profit (P) using the number of tickets sold (t).

   b Draw up a table showing the number of tickets sold (t) and the profit (P) using 200, 250, 300, 350 and 400 for values of t.

   c Use a graph to find the number of tickets that must be sold to make a profit of $400.

   d How many tickets must they sell to cover their costs?

See Example 11

Worked solutions

Exercise 12.3

MAT07NAWS00049

12 A catering business charges $50 for a meal for two people delivered to their home plus $10 for each additional person (up to eight people in total).

   a Draw up a table of values showing the number of people (p) and the cost (c).

   b Which is the independent variable?

   c Write the function for the relationship between the number of people and the cost of the meal and graph the relationship.

Reasoning

13 The length of cardboard (c) needed to make a carton is four times the required side length (s) plus an overlap of 25 cm that is needed to glue it together. Use side length values of 20, 40, 60, 80, 100 to draw up a table of the side length and the length of cardboard required. Write down the function for the relationship between the length of cardboard and the side length.

14 Draw a graph of the function $y = x^2 - 1$ for $x = 0$ to 4.

15 Draw a graph of the function $y = x(x - 2)$ for $x = 0$ to 4.

See Example 13

Chapter 12 summary

- The **Cartesian plane** has two axes at right angles. It is also called a **number lattice**.
- The **first axis** is horizontal and is called the **x-axis**. The **second axis** is vertical and is the **y-axis**. The axes cross at the **origin**.
- Every point on the Cartesian plane is opposite a number on each axis. The points are named using numbers that are called **coordinates**.
- Coordinates are shown in parentheses (round brackets) with a comma between them. The origin is the point (0, 0). The **x-coordinate** is always shown *first* and the **y-coordinate** *last*.
- Each quarter of the Cartesian plane is called a **quadrant**.
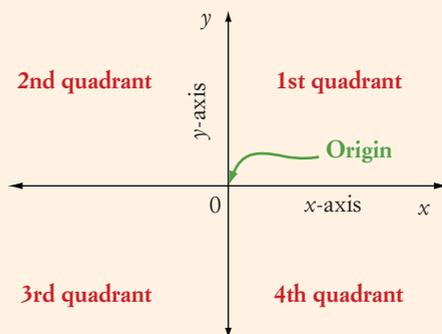
- The **rule** for a number pattern is written with the term number as the input and the term as the **output** number. A number pattern can be written as a table with the input numbers in the first row and the output numbers in the second row.
- A number pattern can be thought of as a **number machine** that produces an output number from an input number.
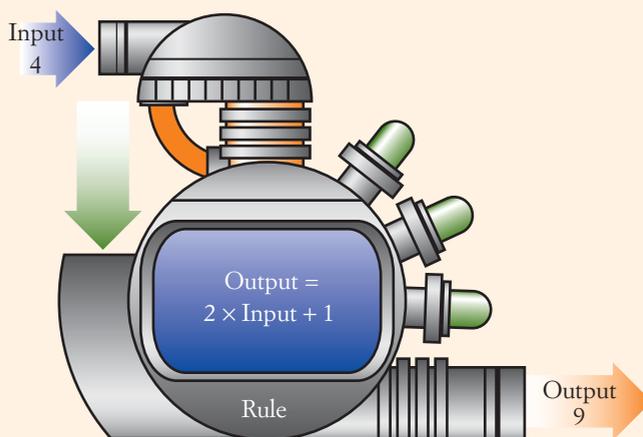
- A **linear** number pattern goes up or down by the same amount from one term to the next.
- A number pattern can be graphed on the Cartesian plane. The graph of a linear number pattern is a straight line.
- **Linear rules** are of the form: output = □ × input + △

  or   output = □ × input − △.

- A mathematical rule that changes numbers into new ones is called a **function**. The starting number is called the **input** or **independent variable**. The number produced is called the **output** or **dependent variable**.
- The rule for a function is usually written with the dependent variable on the left-hand side of the equals sign and the rule on the right in terms of the input variable.
- If the x-coordinate of a point produces the y-coordinate when it is put into a function, then we say that the point **satisfies** the function.
- A **travel graph** shows the distance travelled vertically and time horizontally.

Understanding

See Example 1

1 For each of the following points, state its position and quadrant or axis.
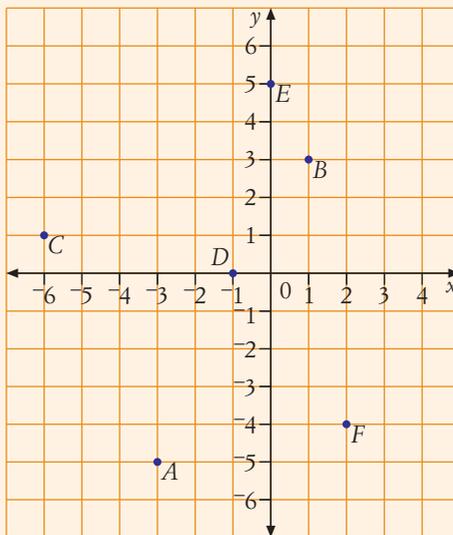
See Example 2

2 Plot this group of points on a grid and join them in order.
$A(2, 2), B(2, 4), C(5, 4), D(5, 5), E(7, 3), F(5, 1), G(5, 2), A$ – That-a-way!

3 Plot this group of points on a grid and join them in order.
$A(2, 6), B(^-4, 0), C(2, 0), A, D(6, 0), C, E(2, ^-1), F(^-5, ^-1), G(^-2, ^-3), H(6, ^-3), I(7, ^-1),$
E – Sailing!

See Example 4,5

4 Look at the pattern below.
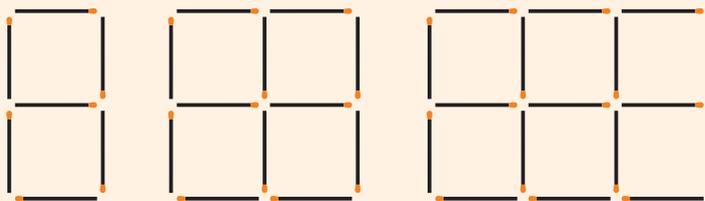
a Extend the pattern and count the matchsticks to complete the following table.

Term number (t)	1	2	3	4	5	6
Number of matches (m)	7	12				

b Draw a graph of the pattern using the horizontal axis to show the term number.
c Use your graph to find the number of matches in the 10th term.
d Is the graph linear?

5 For the pattern in question **4**, find a rule for the number of matches (m) for any given term number (t).

6 Refer to the pattern below.
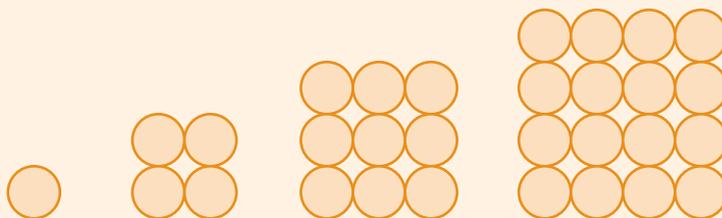
a Extend the pattern and count the discs to complete this table of values.

Term number (T)	1	2	3	4	5	6
Number of discs (D)	1	4				

b Draw a graph of this pattern using the horizontal axis for the term number.
c Is the graph linear?

7 Here are the first four terms of a pattern.
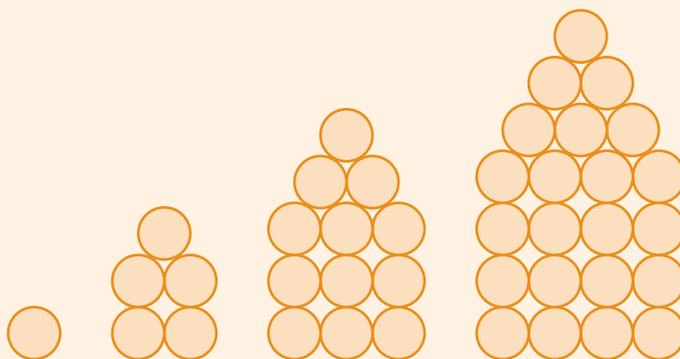
Use the pattern to complete this table.

Term number	1	2	3	4	5	6	7
Number of discs	1	5					

8 Look at this hexagonal pattern.
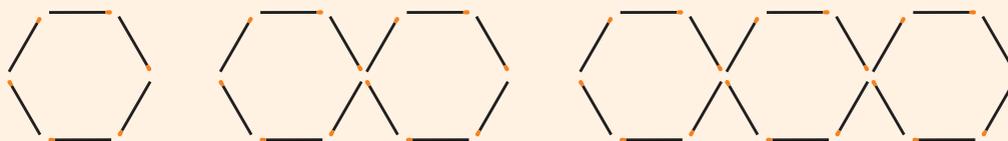
a Use the pattern to complete this table.

Term number (t)	1	2	3	4	5
Number of matches (m)	6	12			

b Use the information in the table to form a rule for the number of matches (m) using the term number (t).

Fluency

9 Look at this pattern.
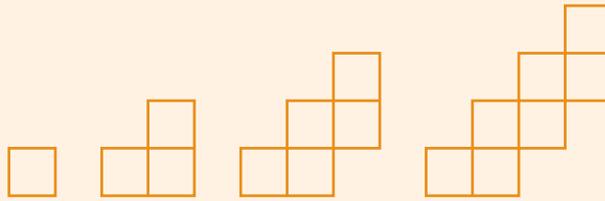
a Use the pattern to complete this table.

Term number (t)	1	2	3	4	5	6
Number of squares (s)	1	3	5			

b Use the information in the table to form a rule for the number of squares (s) using the term number (t).

c Use your rule to calculate the number of squares that would be in the 12th and 28th terms of the pattern.

d Which term would have 127 squares?

See Example 6

10 For each of the following tables, use the rule to complete the table.

a Output $= 4 \times$ input $+ 1$

Input	Output
1	
2	
3	
4	
5	

b Output $= 3 \times$ input $- 2$

Input	Output
2	
3	
4	
5	

11 Calculate the output for each of the following input values.

a 1   **b** 10   **c** 4

Input

$5n - 3$
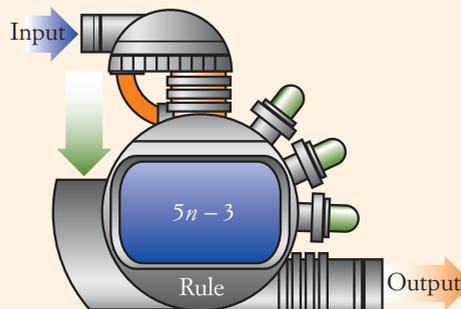
Rule

Output

12 Find the rule that relates output and input values in the following tables. In each case, use the rule to complete the table.

a

Input	Output
1	4
2	8
3	12
5	
9	
30	

b

Input	Output
2	10
6	14
14	22
7	
9	
12	

13 Which of the following rules was used to produce this table?

Input	Output
0	5
3	26

A output $= 3 \times$ input $+ 5$
B output $= 8 \times$ input $+ 2$
C output $= 7 \times$ input $+ 5$
D output $= 9 \times$ input $- 2$

14 Complete the output tables for the following functions.

a $h = 3r + 5$

r	0	2	3
h			

b $m = 4n - 7$

n	2	3	5
m			

c $y = {}^-3x + 8$

x	0	1	3
y			

d $k = 2(3j - 1)$

j	$^-2$	0	2
k			

15 Name the dependent and independent variables for each function in question **14**.

16 Use the function $h = 2 + 5g$ to find the value of h when:

a $g = 4$ **b** $g = 11$

17 Name the independent variable in question **16**.

18 Complete the table and use the values to draw a graph of the function $b = 2a + 3$.

a	0	3	5
b			

19 Draw a graph of the function $y = 13 - 2x$ by first completing the following table.

x	0	2	5
y			

20 Draw up a table of values for $y = 3x + 2$ using x from 1 to 4 and plot the graph.

21 In each of the following, state if the ordered pair satisfies the given function.

a $y = 2x + 3$ $(2, 8)$ **b** $y = 3x - 4$ $(7, 17)$
c $y = 3 - 5x$ $(1, {}^-2)$ **d** $y = 4x - 9$ $(^-2, ^-5)$

22 For each function, complete the ordered pair that satisfies the given function.

   a $y = 3x - 2$ $(2, \ldots)$ **b** $y = 4x + 3$ $(^-3, \ldots)$

Problem solving

23 Cans are stacked in a supermarket as shown here.

   a Copy and complete the following table.

Number of rows (r)	1	2	3	4	5	6
Number of cans (c)	1	3	6			

   b Draw a graph showing the number of cans for different numbers of rows.
   c Extend your table to find the number of cans that would be in a stack with eight rows.

24 Find the rule that relates output and input values in the following tables. In each case, use the rule to complete the table.

a

Input	Output
0	5
1	7
2	9
5	15
7	
10	

b

Input	Output
2	3
3	6
5	12
8	21
6	
11	

See Example 8

25 Here are the inputs and corresponding outputs for a function. Find the rule for each function.

a

Input, a	1	2	3	4
Output, p	5	8	11	14

b

Input, x	1	2	4	6
Output, y	1	3	7	11

See Example 11

26 A resort charges $120 for accommodation for two people and an extra $40 for each additional person (up to 10 people in total).

   a Draw up a table of values showing the number of people (p) and the cost (c) of accommodation.
   b Which is the independent variable?
   c Find the function for the relationship between the number of people and the cost of accommodation and graph the rule.

27 Explain what is meant by a linear rule.

28 The graph shows Ahmed and Patrick cycling towards each other from different towns. Explain the graph and any events shown.

See Example 12
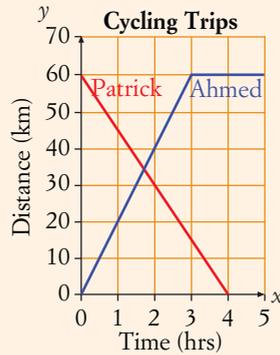
29 Explain how the rule is found for a straight line from 3 points on a graph with first coordinates of 1, 2 and 3.

30 Explain how the rule is found for a linear function from 2 points of the function.

Chapter 1

Exercise 1.1

1 a Continuous **b** Categorical **c** Discrete
d Continuous **e** Discrete **f** Categorical

2 a Sample **b** Sample **c** Population

3 a The method is biased because only Year 3 students were asked.
b Probably fair, provided the supermarket is open
c Probably fair, but would be clearer if it asked whether they want daytime or night-time.

4 a Sample **b** Population
c Sample **d** Sample

5 a Staff and students who use the tuckshop
b Workers and families who are coming to the picnic
c People who buy pocket diaries
d Tradesmen and do-it-yourself plumbers in Gympie

6 a Only people in the affected suburbs were asked, not all those in Townsville. The question is also biased. You could ask a more complete sample and include questions about alternatives as well.
b The only real problem is that not all people will return the survey and more of those who do would probably be unhappy customers. This could not be improved without costing a great deal more.
c The sample is very restricted. A better sample could be chosen using the electoral roll.

7 No, because the results of surveys are unlikely to be exactly the same for different groups.

8 As the groups get larger, the results will get more like each other.

Exercise 1.2

1 a 7:24 a.m. **b** 11:06 p.m. **c** 9:20 a.m.
d 16 minutes **e** 14 minutes

2 a 3 **b** 8 **c** None
d Tigers (262) **e** Wests (67) **f** Easts (310)

3 a 2 L **b** 3 L
c 3 L **d** **i** 1 L
ii 1.5 L

4

Score	Tally	Frequency
0	I	1
1		0
2	II	2
3	I	1
4	III	3
5	IIII	5
6	IIII	4
7	IIII II	7
8	IIII I	6
9	I	1
	Total	30

5

Score	Tally	Frequency
1	IIII	4
2	IIII IIII I	11
3	IIII I	6
4	II	2
5		0
6	III	3
7	I	1
	Total	27

6

Score	Tally	Frequency
SA	IIII III	8
A	IIII II	7
I	IIII	5
D	IIII IIII II	12
SD	IIII III	8
	Total	40

7 a 41 minutes **b** 45 minutes **c** 23 minutes

8 a Souths **b** Wests
c Yes, because there have been nine matches. **d** No

9

Score	Tally	Frequency
0–4	I	1
5–9	IIII	4
10–14	IIII IIII I	11
15–19	IIII I	6
20–24	III	3
	Total	25

10

Score	Tally	Frequency
80–99	JHT	5
100–119	JHT II	7
120–139	JHT JHT I	11
140–159	JHT	5
160–179	II	2
	Total	30

11 Any three, with some paint left.

12 See table below.

Exercise 1.3

1 a
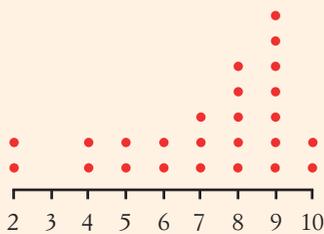
b
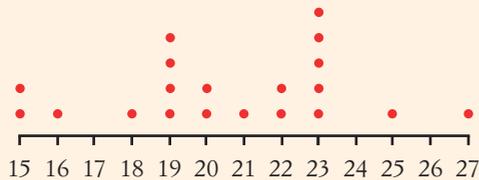
c
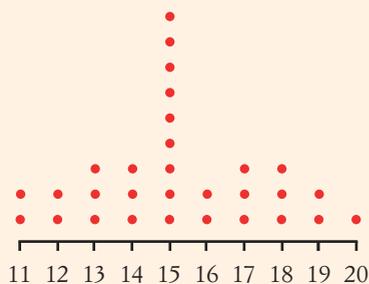
2 a 500 frogs/picture **b** 2000 **c** 1000

3 a 2 people/cm **b** 6 **c** 5 **d** 3

4 a 1000 km^2/picture or 2000 km^2/picture

b

Queensland coastal land	
Conservation	= 1000 km^2
Military	
Grazing	
Urban/mining	
Aboriginal reserves	
Agriculture	
Forestry	
Unused	

c Using a scale of 2000 km^2/cm.
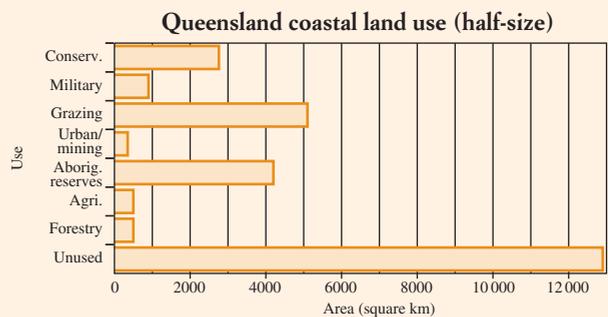
Exercise 1.2

12

							Games		Sets	
	T.S	S.A.	W.C	J.L.	N.S.	P.G.	W	L	W	L
Tony Scarpelli	***	6–3	6–2	3–6	6–4	6–1	27	16	4	1
Sonja Alexopolis	3–6	***	4–6	3–6	1–6	6–4	17	28	1	4
Wendy Cohen	2–6	6–4	***	4–6	1–6	6–2	19	24	2	3
Jacqui Lee	6–3	6–3	6–4	***	3–6	6–1	27	17	4	1
Nicole Sharma	4–6	6–1	6–1	6–3	***	7–5	29	16	4	1
Patty Green	1–6	4–6	2–6	1–6	5–7	***	13	31	0	5

5 a 40 or 50 doughnuts/picture

b

Sales of doughnuts	
Plain	
Cinnamon	
Iced	
Cream-filled	
Chocolate	
Fancy	⬭ = 40 doughnuts

c

Sales of doughnuts
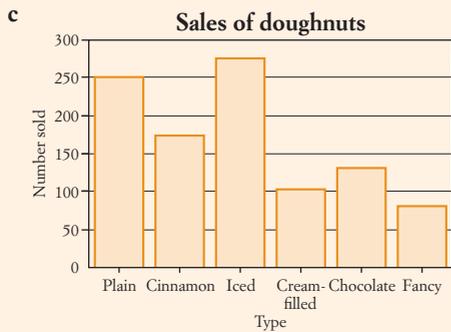
6 a 5°C/cm and 20 g/100 mL/cm
  b i 32 g   **ii** 60 g   **iii** 14 g
    iv 100 g at 60°C **v** 38 g at 25°C
  c 280 g
  d 3900 mL

7

Family pets (Quarter-scale)
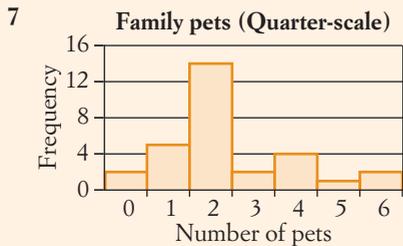
8 a 10 000 marriages/cm
  b i 23 000 marriages
    ii 12 000 marriages
    iii 14 000 marriages

9 Words per paragraph

0	8
1	3 7 9
2	0 0 2 4 5 6
3	0 1 4 7
4	2 3 8

10 DVDs owned

0	0 1 3 9
1	0 2 8 8
2	2 9
3	4 4 4 5 8 8 9
4	0 1 2 3 3
5	1 4 5
6	0 5 5 6
7	2 6 8
8	
9	8

11 a Vertical 2/cm, horizontal $5/cm
  b

Pocket money
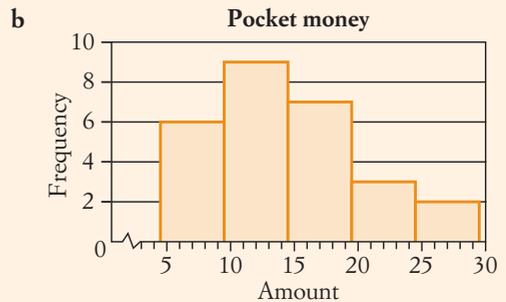
12 Preferred holiday
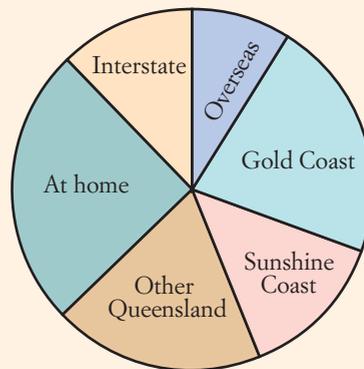
13

Density of battery acid
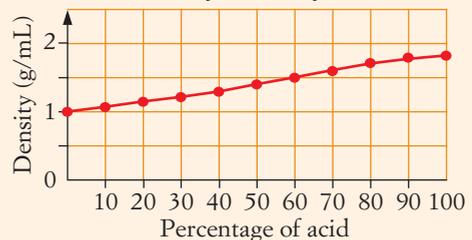
14 House of representatives, 1901: 75 Members

NSW   Vic.   Qld SA WA Tas.
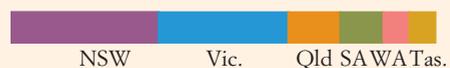
15 a It has declined.
  b Introduction of cane toads, building of more houses and roads etc.

16 a 28 **b** 15 **c** 72

17 a

Distance	Tally	Frequency
0	I	1
1	JHT III	8
2	JHT	5
3	JHT	5
4	I	1
5	III	3
6	II	2
7		0
8	I	1
	Total	26

b

Distances from school
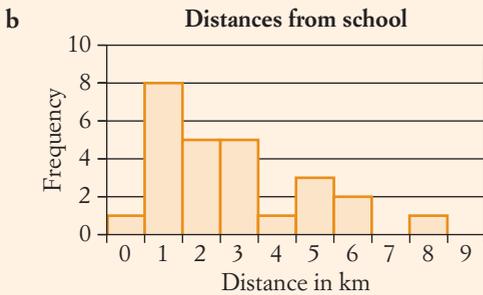
c 1 km

18 a 9°/min **b** 5 minutes
  c 10 minutes **d** 15 minutes

19 a Continuous

b

Blood alcohol (g/100 mL)	0–0.04	0.05–0.09	0.10–0.14
Frequency	22	18	12

Blood alcohol (g/100 mL)	0.15–0.19	0.20–0.24
Frequency	7	1

c

Accident blood alcohol

20 a

Heating sugar solution
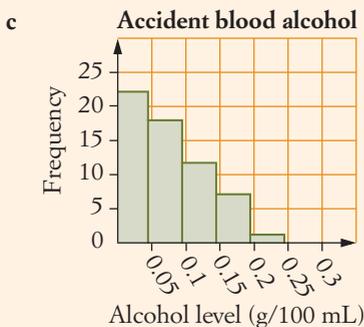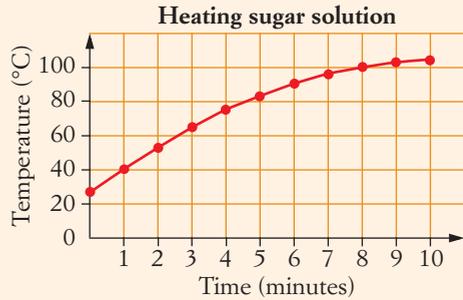
b No, it rises more quickly at the start.

21 a

Average maximum monthly temperature at Davis Base
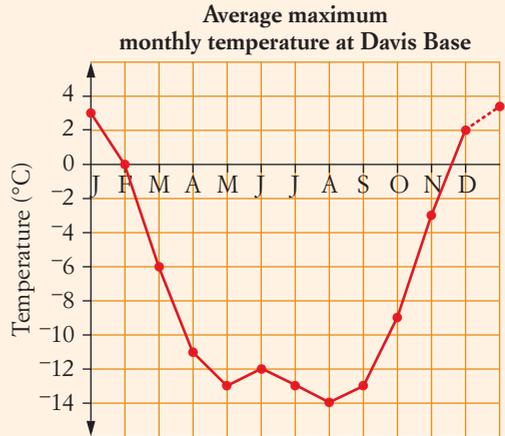
b 76 days (2½ months)

22 a The vertical scale does not start from zero.
  b To emphasise the rise in production.

Exercise 1.4

1 a Mode = 4, median = 5
  b Mode = 7, median = 6
  c Mode = 26, median = 26
  d Modes = 22 and 24, median = 24
  e Mode = 55, median = 54

2 a Mean ≈ 5.3, range = 5
  b Mean ≈ 5.8, range = 8
  c Mean ≈ 26.1, range = 7
  d Mean ≈ 23.3, range = 10
  e Mean ≈ 52.2, range = 18

3 a Mode = 6, median = 6
  b Modes = 120, 137, 144, median = 140.5
  c Mode = 77, median = 77.5
  d Mode = 44, median = 49

Answers

4

Time for Questionnaire
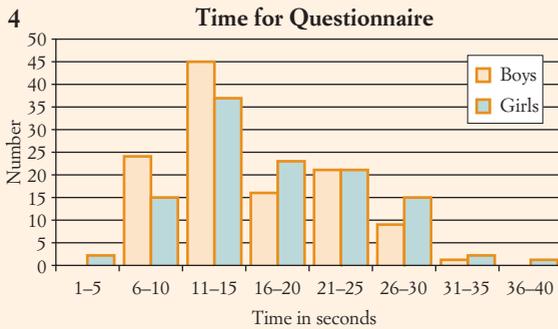
5 a

Score x	Frequency f	xf	Cumulative frequency
3	2	6	2
4	4	16	6
5	5	25	11
6	6	36	17
7	7	49	24
8	5	40	29
9	4	36	33
10	2	20	35
Total	**35**	**228**	

Mean ≈ 6.5, median = 7, mode = 7, range = 7

b

Score x	Frequency f	xf	Cumulative frequency
12	6	72	6
13	7	91	13
14	8	112	21
15	5	75	26
16	3	48	29
17	4	68	33
18	2	36	35
19	5	95	40
20	3	60	43
21	2	42	45
22	1	22	46
Total	**46**	**721**	

Mean ≈ 15.7, median = 15, mode = 14, range = 10

c

Score x	Frequency f	xf	Cumulative frequency
7	2	14	2
8	3	24	5
9	5	45	10
10	7	70	17
11	9	99	26
12	11	132	37
13	8	104	45
14	6	84	51
15	5	75	56
Total	**56**	**647**	

Mean ≈ 11.6, median = 12, mode = 12, range = 8

d

Score x	Frequency f	xf	Cumulative frequency
42	2	84	2
43	4	172	6
44	5	220	11
45	7	315	18
46	8	368	26
47	9	423	35
48	7	336	42
49	4	196	46
50	3	150	49
51	1	51	50
Total	**50**	**2315**	

Mean = 46.3, median = 45.5, mode = 47, range = 9

e

Score x	Frequency f	xf	Cumulative frequency
22	2	44	2
23	5	115	7
24	9	216	16
25	11	275	27
26	6	156	33
27	4	108	37
28	6	168	43
Total	**43**	**1082**	

Mean ≈ 25.2, median = 25, mode = 25, range = 6

6 a Mean = 65, median = 62, mode = 62
b Median, 62

7 a Median = $460 000, mean = $632 000, range = $1 020 000
  b Median, $460 000
  c Because a few very high prices can dramatically alter the mean.
  d about $12 000 000

8 a Median = 7, mode = 6
  b Median changes to 7.5, mode unchanged.
  c The mean increases from about 7.5 to about 7.8.

9 a Median = 101 cm, mode = 100 cm
  b The median drops by 1 cm and the mode stays the same.
  c The mean drops by about 3.6 cm, from 102.6 to 99 cm.

10 a About 31 years 9 months.
  b Because it is just one score, and it may be one of the new people.
  c 37
  d The median is higher than the mean, so putting in two extra low scores reduces both.

11 27

12 a Mean ≈ 18.6, median = 18, mode = 17
  b Median, 18

13 a Mean = $428, median = $350, mode = $300, range = $800
  b Median, $350
  c About $1750
  d The incomes are skewed to the left (the high incomes stretch out further).

14 The girls' times are spread more than the boys', with some girls faster than all the boys and at least one girl slower than all the boys. However, the boys appear to be generally faster than the girls.

15 There are a larger number of people in age groups about 20–50 than in other age groups. After 50, the number in each age group is much smaller than the previous age group. The number of males is greater than the number of females in each age group up to about 40, but after that the number of women is more than the number of men. There are about twice as many women over 85 as men.

16 10

Chapter review

1 a Continuous   **b** Categorical
  c Discrete   **d** Continuous

2 a Sample   **b** Population

3 a

Score	Tally	Frequency
9	I	1
10	II	2
11	II	2
12	III	3
13	IIII	5
14	IIII I	6
15	IIII	4
16	I	1
	Total	24

b

Back door steps — bar graph of Frequency vs Number of steps.
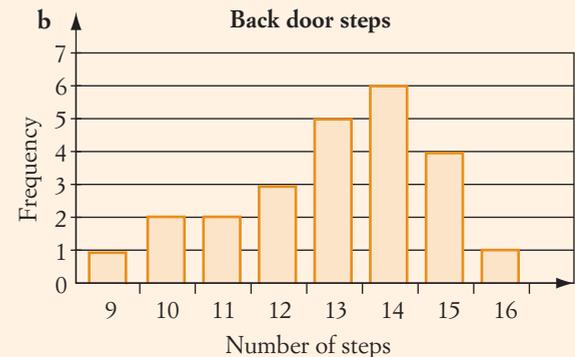
4 a 200 **b** Hereford **c** 40/picture or 50/picture
  d

Cattle on a farm pictograph. Angus, Hereford, Brahman, Simmental, Charolais. = 40 cattle

5

Stem	Leaf
1	7 9
2	2 4 8 9
3	2 2 2 4 5 6 6 7 7 8
4	2 5 7 7
5	1 3 5
6	1 4

6 a Median = 49, mode = 51
  b Mean ≈ 48.7, range = 15

7 a Biased, because only customers of that bank were asked.
  b Biased, because Mt Isa is an inland town.

8
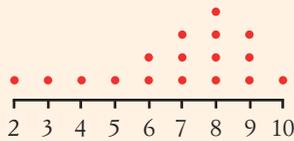
9 Drink preference (total = 60)
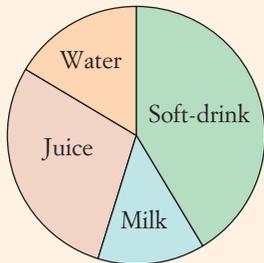
10 a Sample
  b Population (some could want kosher, halal or vegetarian meals)

11 a Parents of Students in Queensland schools
  b The teachers at the school

12 a

Tyre pressure	Frequency
15–19	2
20–24	7
25–29	5
30–34	8
35–39	6
40–44	2

  b
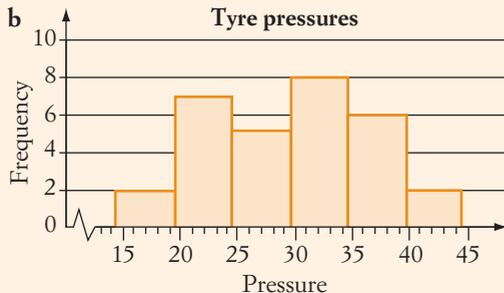
  c They would be about the same, since the distribution is symmetrical

13

Score x	Frequency f	xf	Cumulative frequency
6	1	6	1
7	3	21	4
8	6	48	10
9	5	45	15
10	6	60	21
11	4	44	25
12	3	36	28
13	2	26	30
Total	30	286	

Mean ≈ 9.5, median = 9.5, mode = 8, 10, range = 7

14 a 3 **b** 16 **c** 12 **d** 35

15 a 4% **b** 30 and 40

16 a 5°/cm and 0.01 g/L/cm **b** 0.054 g/L
  c 13 °C **d** 24–31°C

17 a The median, 17, as the mean would be affected by a few high ages
  b There is probably a height restriction.
  c Under 20, since the median is 17

18 a
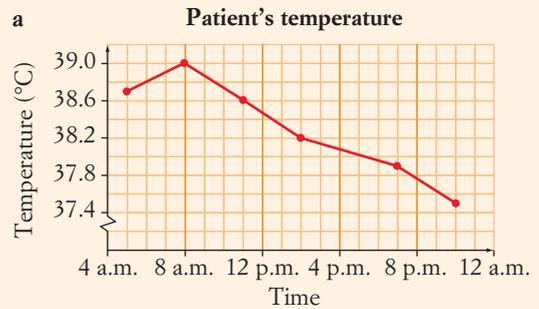
  b About 38.5°C

19 a Mode = 15, median = 14
  b Yes, 3
  c It would increase the mean
  d The mean would be lower than the median because the distribution is skewed left.

20 The percentage of females in the lower weekly earnings groups is much higher than for males. The males' earnings are also more evenly spread higher and lower in the middle than the females', which are spread more below the middle than above the middle.

21 The sample could be biased by surnames (people whose names begin with Z could be mostly of central European or Chinese origin), and it is biased towards people listed in the *White Pages*. The question is not phrased fairly.

22 In summer, Roma is a little warmer than Brisbane during the day and slightly colder at night. In winter, Roma is a little cooler than Brisbane during the day and much colder at night. The temperatures in Roma are more extreme than the temperatures in Brisbane.

Chapter 2
Exercise 2.1

1 a 9 has a value of 9 (or 9 units)
    0 means there are no tens
    6 has a value of 600 (or 6 hundreds)
    six hundred and nine

b 9 has a value of 9 (or 9 units)
3 has a value of 30 (or 3 tens)
0 means there are no hundreds
1 has a value of 1000 (or 1 thousand)
one thousand and thirty-nine
c 4 has a value of 4 (or 4 units)
0 means there are no tens
1 has a value of 100 (or 1 hundred)
0 means there are no thousands
7 has a value of 70 000 (or 7 ten thousands)
seventy thousand, one hundred and four
d 0 means there are no units
6 has a value of 60 (or 6 tens)
8 has a value of 800 (or 8 hundreds)
4 has a value of 4000 (or 4 thousands)
0 means there are no ten thousands
5 has a value of 500 000 (or 5 hundred thousands)
five hundred and four thousand, eight hundred and sixty
e 1 has a value of 1 (or 1 unit)
7 has a value of 70 (or 7 tens)
6 has a value of 600 (or 6 hundreds)
4 has a value of 4000 (or 4 thousands)
3 has a value of 30 000 (or 3 ten thousands)
1 has a value of 100 000 (or 1 hundred thousands)
9 has a value of 9 000 000 (or 9 millions)
nine million, one hundred and thirty-four thousand, six hundred and seventy-one
f 0 means there are no units
0 means there are no tens
0 means there are no hundreds
7 has a value of 7000 (or 7 thousands)
3 has a value of 30 000 (or 3 ten thousands)
8 has a value of 800 000 (or 8 hundred thousands)
5 has a value of 5 000 000 (or 5 millions)
five million, eight hundred and thirty-seven thousand
g 1 has a value of 1 (or 1 unit)
0 means there are no tens
0 means there are no hundreds
4 has a value of 4000 (or 4 thousands)
four thousand and one
h 9 has a value of 9 (or 9 units)
8 has a value of 80 (or 8 tens)
6 has a value of 600 (or 6 hundreds)
5 has a value of 5000 (or 5 thousands)
0 means there are no ten thousands
2 has a value of 200 000 (or 2 hundred thousands)

two hundred and five thousand, six hundred and eighty-nine
i 6 has a value of 6 (or 6 units)
3 has a value of 30 (or 3 tens)
0 means there are no hundreds
0 means there are no thousands
0 means there are no ten thousands
0 means there are no hundred thousands
4 has a value of 4 000 000 (or 4 millions)
3 has a value of 30 000 000 (or 3 ten millions)
thirty-four million and thirty-six

2 a 8796 **b** 3 000 088 **c** 2385
d 6907 **e** 420 830 **f** 309 211
g 1 280 460 **h** 12 000 901

3 a $(5 \times 10) + (6 \times 1)$
b $3 \times 10^3 + 5 \times 10^2 + 6 \times 10 + 2 \times 1$
c $4 \times 10^2 + 1 \times 10 + 6 \times 1$
d $5 \times 10^2 + 0 \times 10 + 2 \times 1$
e $1 \times 10^3 + 0 \times 10^2 + 0 \times 10 + 1 \times 1$
f $1 \times 10^4 + 0 \times 10^3 + 2 \times 10^2 + 5 \times 10 + 3 \times 1$
g $3 \times 10^4 + 8 \times 10^3 + 0 \times 10^2 + 0 \times 10 + 2 \times 1$
h $5 \times 10^4 + 9 \times 10^3 + 6 \times 10^2 + 4 \times 10 + 4 \times 1$
i $3 \times 10^3 + 8 \times 10^2 + 0 \times 10 + 9 \times 1$
j $1 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 4 \times 10^2 + 3 \times 10 + 5 \times 1$

4 a 524 **b** 6537 **c** 429
d 6473 **e** 82 343 **f** 3057
g 76 001 **h** 10 999 **i** 344 220
j 900 999

5 a 4093 **b** 188 **c** 718
d 1564 **e** 2637

6 a 17, 321, 8000
b 17, 27, 63, 707
c 16, 246, 432, 3596, 5369, 6125
d 123, 132, 213, 231, 321
e 82, 145, 450, 1045
f 43, 243, 721, 722, 4372
g 300 806, 308 022, 380 211, 392 084
h 1 429 950, 3 006 853, 4 766 372, 4 856 231

7 a 10 times **b** 100 times
c 1000 times **d** 1000 times

8 a booleroo booleroo **b** guliba guliba
c bularr guliba **d** booleroo booleroo
e guliba **f** guliba

9 1905

10 6

Answers

11 a 1893 km   **b** 1987 km   **c** 3135 km
   d 2718 km   **e** 5191 km

12 a $54   **b** $13.50

13 1008 words

14 119 089

15 a 35 754   **b** 3486   **c** $107 262

16 118 024 points

17 $3312

18 a $44   **b** $2288

19 a $13 312   **b** $53 248   **c** $13 248

20 At least 36 trips

21 a 5225 km   **b** 15 720 items

22 a $149   **b** $6768   **c** $14 301
   d $10 368   **e** $288

23 a 173 308 568 carats   **b** 15 019 604
   c $524 367 669   **d** $412 413 684

24 $4.50 per week

25 480 apples

26 4256 buses

27 There are 20 different combinations. The smallest number of items possible is 12 and the greatest number possible is 28.

Exercise 2.2

1 a 96   **b** 26   **c** 12   **d** 5
   e 42   **f** 2   **g** 91   **h** 9
   i 19   **j** 12   **k** 245   **l** 0

2 a 13   **b** 10   **c** 4   **d** 2
   e 30   **f** 19   **g** 39   **h** 1
   i 25   **j** 23   **k** 118   **l** 48

3 C

4 a 11   **b** 30   **c** 100   **d** 17
   e 171   **f** 121   **g** 12   **h** 43
   i 21   **j** 2   **k** 83   **l** 4
   m 3   **n** 3   **o** 122   **p** 37

5 a $(5 - 2) \times 4 = 12$ **b** $3 + 8 - 7 = 4$
   c $(15 - 3) \times 5 = 60$ **d** $15 - 3 \times 5 = 0$
   e $8 + (4 - 3) \times 2 = 10$ **f** $8 + 4 - 3 \times 2 = 6$

g $(8 + 4 - 3) \times 2 = 18$ **h** $6 + 4 \times 0 = 6$
   i $(6 + 4) \times 0 = 0$ **j** $100 \div (10 + 10) = 5$
   k $100 \div 10 + 10 = 20$

6 a $84 \div 3 + 9 \times 15 - 11 = 152$
   b $84 \div 3 + 9 \times (15 - 11) = 64$
   c $84 \div (3 + 9) \times 15 - 11 = 94$

7 a $8 + 9 - 7 + 2 = 12$
   b $3 \times 5 + 2 + 1 = 18$
   c $6 \times 8 - (3 + 4) = 41$
   d $6 + 11 + 8 \div 2 = 21$
   e $8 \div (4 - 2) + 6 = 10$
   f $10 \times 5 - 8 + 2 = 44$
   g $(7 + 9) \div (3 + 5) = 2$
   h $9 \div (4 + 5) + 7 = 8$
   i $10 \div 2 \times (5 + 7) = 60$ (other solutions may exist)

Exercise 2.3

1

+	−	×	÷
plus	minus	times	divide
and	subtract	multiply	share
increase	decrease	product	quotient
total	difference	groups of	
more than	less		
	take away		

2 a $15 - 6$ **b** $48 + 12$ **c** $12 > 5$
   d $5 \neq 3 + 6$ **e** 7×8 **f** $\sqrt{16}$
   g $36 \div 4$ **h** 5^2 **i** $12 + 8$
   j $13 - 6$ **k** $3 + 13$ **l** $39 \div 3$
   m $25 - 8$ **n** $\sqrt[3]{125}$ **o** $13 \neq 3$
   p $999 \approx 1000$

3 a Six squared   **b** Seven cubed
   c Six to the power of seven   **d** Two to the power of ten
   e Eight to the power of four   **f** Twenty-four to the power of four

4 a 25   **b** 216   **c** 49   **d** 81   **e** 128
   f 1024   **g** 81   **h** 729   **i** 125   **j** 8

5 a 4096   **b** 2401   **c** 7776   **d** 38 416
   e 4096   **f** 3375   **g** 59 049   **h** 59 049
   i 1296   **j** 1296

6 a 5.10   **b** 4.90   **c** 10.95   **d** 7.35
   e 3.61   **f** 12.25   **g** 24.49   **h** 8.66
   i 16.61   **j** 12.81

Answers

7 a 5   **b** 14   **c** 9   **d** 8
   e 24   **f** 11   **g** 4   **h** 7
   i 15   **j** 30

8 a 12   **b** 40   **c** 36   **d** 72
   e 49   **f** 9   **g** 22   **h** 22
   i 110   **j** 91   **k** 4   **l** 106

9 D

10 a T   **b** F   **c** F   **d** F
   e T   **f** T   **g** T   **h** F
   i F   **j** T   **k** F   **l** T
   m F   **n** F   **o** T   **p** T
   q F   **r** F

11 a $7130 > 860$ **b** $2001 < 2010$
   c $352\,140 < 4\,082\,716$ **d** $2651 > 2561$
   e $3602 > 3206$ **f** $13\,253 > 1353$
   g $8079 < 8097$ **h** $1432 < 1483$

12 a $5 \times 10^3 + 8 \times 10^2 + 9 \times 10^1 + 6$
   b $3 \times 10^5 + 1 \times 10^4 + 4 \times 10^3 + 5 \times 10^2 + 2 \times 10^1 + 8$
   c $5 \times 10^2 + 0 \times 10^1 + 3$
   d $6 \times 10^4 + 0 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 7$
   e $9 \times 10^5 + 3 \times 10^4 + 0 \times 10^3 + 2 \times 10^2 + 0 \times 10^1 + 0$

13 a 2^3 **b** 2^7 **c** 2^7
   d 2^2 **e** 2^3

14 a 3^3 **b** 3^1 **c** 3^0 **d** 3^3

15 a 36, 41   **b** 2, 3
   c 2, 3, 7, 11, 36, 41   **d** 41
   e 2   **f** 11, 36, 41
   g 8   **h** 3, 7, 8, 11, 36, 41

16 a The powers of 4 are the same as every 2nd power of 2, and the powers of 8 are the same as every 3rd power of 2. Every 3rd power of 4 is the same as every 2nd power of 8.
   b The powers of 9 are the same as every 2nd power of 3, and the powers of 27 are the same as every 3rd power of 3. Every 3rd power of 9 is the same as every 2nd power of 27.
   c The powers of 25 are likely to be the same as every 2nd power of 5, and the powers of 125 are likely to be the same as every 3rd power of 5. Every 3rd power of 25 is likely to be the same as every 2nd power of 125. In general, this will apply to the first three powers of any number.

17 a 9 dots

b The number of dots in the square is the square of the number of dots along the side.

18 $5^4 \times 5^6 = 5^{(4+6)} = 5^{10}$

19 $7^8 \div 7^2 = 7^{(8-2)} = 7^6$

Exercise 2.4

1 a Missing values are ¯5, ¯4, ¯2, 4, 5, 7, 8
   b Missing values are ¯50, ¯40, ¯30, 20, 30, 50, 60, 70
   c Missing numbers are ¯35, ¯30, ¯25, ¯20, ¯15, ¯10
   d Missing numbers are ¯400, ¯300, ¯200, 200
   e Missing numbers are ¯33, ¯30, ¯27, ¯24, ¯21, ¯18, ¯15, ¯12, ¯9, ¯6

2 a >   **b** <   **c** >   **d** <   **e** >
   f >   **g** <   **h** <   **i** <   **j** <
   k >   **l** >   **m** <   **n** <   **o** <
   p >   **q** <   **r** >

3 a ¯6   **b** ¯1   **c** 10
   d ¯11   **e** 0   **f** 2

4 a down   **b** less   **c** up
   d right   **e** descending   **f** north
   g deposit   **h** increase   **i** east

5 a ¯3, ¯1, 2, 3   **b** ¯8, ¯5, ¯3, 2, 5
   c ¯10, ¯6, ¯4, ¯3, 0   **d** ¯5, ¯3, ¯2, 4, 6
   e ¯48, ¯24, ¯11, 0, 8, 36   **f** ¯26, ¯10, ¯2, 3, 12, 15

6 a 5, 4, 3, ¯1, ¯2   **b** 8, 0, ¯2, ¯4, ¯7
   c 4, 1, ¯1, ¯3, ¯5, ¯11   **d** 8, 3, ¯2, ¯4, ¯18
   e 1, ¯3, ¯6, ¯15, ¯48   **f** 33, 1, ¯36, ¯58, ¯100

7 a 2   **b** 2   **c** 4   **d** 3
   e 6   **f** 20   **g** 7   **h** 7

8 a 15   **b** ¯1   **c** 3   **d** ¯7
   e 0   **f** 3   **g** ¯11   **h** ¯4
   i ¯26   **j** 20   **k** ¯8   **l** 0
   m ¯2   **n** ¯9   **o** ¯1   **p** 6

9 a 2   **b** 1   **c** ¯11   **d** 6
   e 12   **f** ¯1   **g** 0   **h** ¯10
   i ¯10   **j** 5   **k** ¯6   **l** 6
   m ¯13   **n** ¯6   **o** 12   **p** ¯9

10 a 0   **b** 0   **c** 7   **d** ¯20
   e ¯5   **f** ¯14   **g** 70   **h** ¯21
   i ¯19   **j** 8   **k** ¯32   **l** 8

Answers

11 a 11   **b** ⁻11   **c** 29   **d** ⁻29
   e ⁻11   **f** 3   **g** 16   **h** ⁻10
   i ⁻8   **j** 8   **k** 10   **l** ⁻10

12 a ⁻7   **b** 6   **c** ⁻4   **d** 1
   e ⁻5   **f** 31   **g** 26   **h** ⁻178
   i 120

13 a ⁻10   **b** ⁻4   **c** ⁻2   **d** 0
   e ⁻14   **f** ⁻10   **g** ⁻8   **h** ⁻4

14 a ⁻10   **b** ⁻14   **c** ⁻20   **d** ⁻21
   e ⁻10   **f** ⁻2   **g** 7   **h** 6

15 a ⁻7   **b** 3   **c** ⁻3   **d** ⁻7
   e 7   **f** ⁻7   **g** ⁻6   **h** 6
   i 20   **j** ⁻16   **k** ⁻9   **l** 5

16 $⁻7 ($7 overdrawn)

17 a 6610 m   **b** 2316 m
   c 311 m   **d** 9237 m

18 13°C

19 4 km west

20 7 km west

21 4602 m

22 155 m

23 a 2002, 2003, 2006, 2007   **b** 2004, 2005
   c −$100 million   **d** Profit, $200 million

24 11 m

Exercise 2.5

1 a ⁻18   **b** ⁻24   **c** 21   **d** ⁻32
   e 63   **f** ⁻45   **g** 42   **h** 42
   i ⁻24   **j** ⁻9   **k** 1   **l** ⁻54
   m ⁻10   **n** ⁻42   **o** 44   **p** ⁻81
   q 60   **r** ⁻60   **s** 100   **t** 56

2 a ⁻3   **b** ⁻2   **c** 4   **d** ⁻3
   e ⁻9   **f** 3   **g** ⁻4   **h** ⁻50
   i ⁻4   **j** 17   **k** ⁻9   **l** ⁻12
   m 16   **n** ⁻5   **o** ⁻14   **p** ⁻6
   q ⁻3   **r** 20

3 a ⁻6   **b** ⁻2   **c** 6   **d** 33
   e ⁻21   **f** ⁻9   **g** 20   **h** 9

4 a 24   **b** 30   **c** 81
   d ⁻33   **e** ⁻16   **f** ⁻243

---

   g 18 900   **h** 4160   **i** ⁻99 000
   j 5600   **k** ⁻1   **l** ⁻52
   m ⁻30 000   **n** ⁻16   **o** 36
   p 90   **q** 112 000   **r** ⁻1300
   s 0   **t** 360

5 a 16   **b** 9   **c** 49
   d ⁻128   **e** 125   **f** ⁻8
   g 4   **h** ⁻200   **i** 216

6 a ⁻48   **b** 44   **c** 240
   d ⁻65   **e** ⁻418   **f** 300
   g 25   **h** ⁻512   **i** ⁻735

7 a ⁻20   **b** ⁻2   **c** ⁻21   **d** 3
   e ⁻200   **f** 28   **g** ⁻23   **h** 27
   i 3   **j** 42   **k** 4   **l** 13
   m ⁻200   **n** 5

8 a ⁻10   **b** 112   **c** 1   **d** 1440
   e ⁻6   **f** ⁻20   **g** ⁻48 960   **h** ⁻192

9 a ⁻2   **b** 2   **c** ⁻12   **d** 2
   e ⁻14   **f** ⁻25   **g** ⁻21   **h** ⁻3
   i 11   **j** 8   **k** ⁻6   **l** ⁻9
   m ⁻3   **n** ⁻22   **o** ⁻1   **p** ⁻5

10 a ⁻18   **b** 12   **c** 9   **d** ⁻2
   e 50   **f** 5   **g** ⁻48   **h** ⁻8
   i 10   **j** 1   **k** 30   **l** 10
   m ⁻16   **n** 8   **o** ⁻4

11 a ⁻7   **b** ⁻25   **c** ⁻36
   d 10   **e** 14   **f** 3
   g 5   **h** 8   **i** 2
   j 8   **k** ⁻11   **l** 2

12 a ⁻38   **b** 10   **c** 84
   d 133   **e** 117   **f** ⁻204
   g ⁻16   **h** ⁻3   **i** 5
   j ⁻6   **k** 3   **l** 1
   m ⁻26   **n** ⁻9   **o** 15
   p 22   **q** 11   **r** 42

13 a ⁻19   **b** ⁻84   **c** 34
   d 1   **e** ⁻60   **f** ⁻17
   g 74   **h** 61   **i** 16

14 Positive

15 a Correct
   b Incorrect, 6 − 8 = ⁻2
   c Incorrect, ⁻6 − 8 = ⁻14
   d Correct
   e Correct

f Incorrect, $(^-2 + 6) \times 2 = 8$
g Incorrect, $^-7 - 6 + 10 = ^-3$
h Incorrect, $(^-12 - 3) \div (^-3) = 5$
i Incorrect, $12 - 3 \times 5 = ^-3$
j Incorrect, $2 - 9 \times (^-3) - (^-18) = 47$

Exercise 2.6

1 a Yes   **b** Yes   **c** No
   d Yes   **e** No   **f** Yes
   g Yes   **h** Yes   **i** No
   j Yes

2 a 1, 2, 4, 8, 16
   b 1, 3, 7, 21
   c 1, 2, 3, 4, 6, 8, 12, 24
   d 1, 2, 3, 4, 6, 9, 12, 18, 36
   e 1, 5, 7, 35
   f 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
   g 1, 2, 4, 13, 26, 52
   h 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
   i 1, 2, 4, 7, 8, 14, 16, 28, 56, 112
   j 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72 144
   k 1, 2, 4, 7, 14, 28
   l 1, 2, 4, 5, 10, 20, 25, 50, 100
   m 1, 3, 5, 9, 15, 45
   n 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200
   o 1, 3, 11, 33, 121, 363

3 a 1, 2   **b** 1, 3
   c 1, 2   **d** 1, 2, 4
   e 1, 2, 5, 10, 25, 50   **f** 1, 23
   g 1, 5   **h** 1, 2, 4
   i 1, 2, 5, 10   **j** 1, 2, 3, 6
   k 1, 2, 3, 5, 6, 10, 15, 30   **l** 1, 13
   m 1, 3, 5, 15   **n** 1, 3
   o 1   **p** 1, 2, 5, 7, 10, 14, 35, 70

4 a 1, 2   **b** 1, 2, 5, 10
   c 1, 11   **d** 1, 2, 3, 4, 6, 12
   e 1, 3   **f** 1, 2, 4, 8
   g 1, 2, 7, 14   **h** 1, 3, 5, 15
   i 1, 2, 5, 10

5 a 12   **b** 11   **c** 12   **d** 3
   e 3   **f** 16   **g** 4   **h** 2
   i 50   **j** 6   **k** 24   **l** 5
   m 7   **n** 9   **o** 25

6 a 37, 41, 43, 47
   b 66, 68, 69, 70, 72, 74, 75, 76, 77, 78
   c 2, 3, 5, 7, 11, 13, 17, 19
   d 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46

7 (Different factor trees are possible.)

a
$$8 = 2 \times 2 \times 2$$

b
$$63 = 7 \times 3 \times 3$$

c
$$45 = 5 \times 3 \times 3$$

d
$$36 = 2 \times 2 \times 3 \times 3$$

e
$$51 = 3 \times 17$$

f
$$49 = 7 \times 7$$

g
$$90 = 5 \times 2 \times 3 \times 3$$

h
$$27 = 3 \times 3 \times 3$$

i
$$130 = 5 \times 2 \times 13$$

j
$$200 = 2 \times 2 \times 2 \times 5 \times 5$$

k
$$275 = 5 \times 5 \times 11$$

l
$$342 = 2 \times 3 \times 3 \times 19$$

m
$$1250 = 2 \times 5 \times 5 \times 5 \times 5$$

Answers

n

$$1020$$

(factor tree)

$$5 \times 204$$
$$5 \times 2 \times 102$$
$$5 \times 2 \times 3 \times 34$$
$$5 \times 2 \times 3 \times 2 \times 17$$

$$1020 = 2 \times 2 \times 3 \times 5 \times 17$$

o

$$837$$

$$9 \times 93$$
$$3 \times 3 \times 3 \times 31$$

$$837 = 3 \times 3 \times 3 \times 31$$
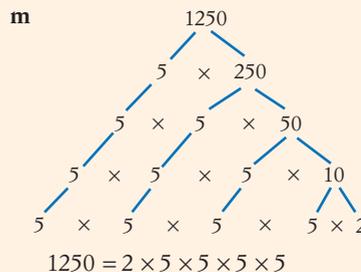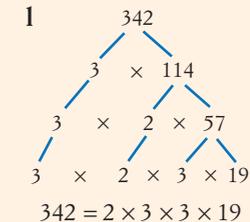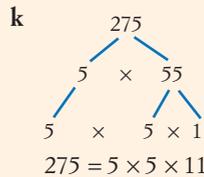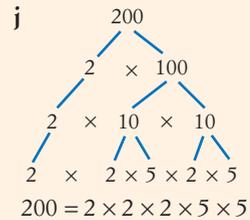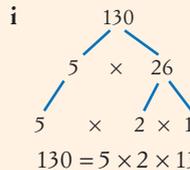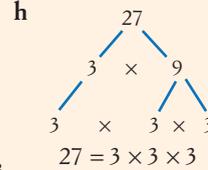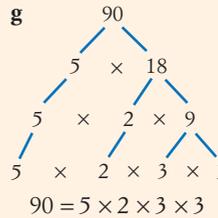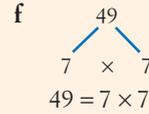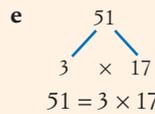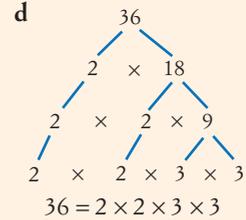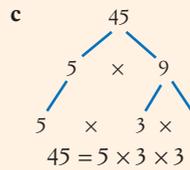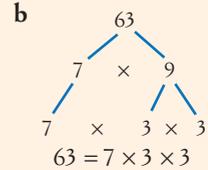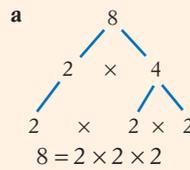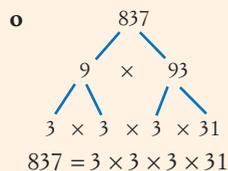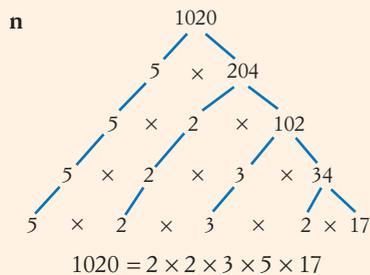
8 a 162   **b** 60   **c** 225
d 980   **e** 90   **f** 36

9 a

$$18$$
$$2 \times 9$$
$$2 \times 3 \times 3$$

$$18 = 2 \times 3^2$$

b

$$20$$
$$5 \times 4$$
$$5 \times 2 \times 2$$

$$20 = 2^2 \times 5$$

c

$$45$$
$$5 \times 9$$
$$5 \times 3 \times 3$$

$$45 = 3^2 \times 5$$

d

$$72$$
$$2 \times 36$$
$$2 \times 6 \times 6$$
$$2 \times 2 \times 3 \times 2 \times 3$$

$$72 = 2^3 \times 3^2$$

e

$$98$$
$$2 \times 49$$
$$2 \times 7 \times 7$$

$$98 = 2 \times 7^2$$

f

$$196$$
$$4 \times 49$$
$$2 \times 2 \times 7 \times 7$$

$$196 = 2^2 \times 7^2$$
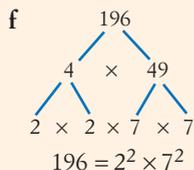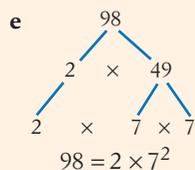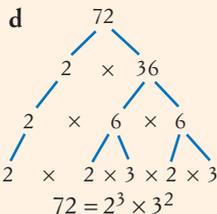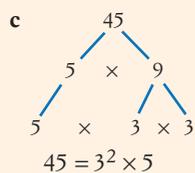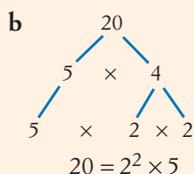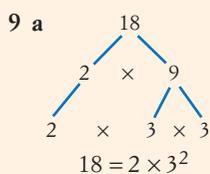
10 a They form a triangular pattern.
b 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91
c $1 + 2 + 3 + 4 + 5 = 15$
$1 + 2 + 3 + 4 + 5 + 6 = 21$
$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$
d By adding the next whole number to the sum of the previous whole numbers each time.
e 5050

11 a The numbers each form a square array.
b 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
c $1 + 3 + 5 + 7 + 9 = 25$
$1 + 3 + 5 + 7 + 9 + 11 = 36$

$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64$
d By adding consecutive odd numbers together, starting with 1 and adding the next odd number to the total for each new term.
e $50 \times 50 = 2500$
f **i** $1 + 2 + 3 + 4 + 3 + 2 + 1 = 4^2$
$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 5^2$
$1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 6^2$
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 7^2$

ii $5^2 = 4^2 + (4 + 5)$ $6^2 = 5^2 + (5 + 6)$
$7^2 = 6^2 + (6 + 7)$ $8^2 = 7^2 + (7 + 8)$

g $4 = 1 + 3$ $9 = 3 + 6$
$16 = 6 + 10$ $25 = 10 + 15$
$36 = 15 + 21$ $49 = 21 + 28$
$64 = 28 + 36$ $81 = 36 + 45$
$100 = 45 + 55$

h 1, 36

12 a

$$196$$
$$4 \times 49$$
$$2 \times 2 \times 7 \times 7$$

$$\sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7}$$
$$= 2 \times 7$$
$$= 14$$

b

$$784$$
$$4 \times 196$$
$$2 \times 2 \times 4 \times 49$$
$$2 \times 2 \times 2 \times 2 \times 7 \times 7$$

$$\sqrt{784} = \sqrt{2 \times 2 \times 2 \times 2 \times 7 \times 7}$$
$$= 2 \times 2 \times 7$$
$$= 28$$

c

$$3136$$
$$2 \times 1568$$
$$2 \times 2 \times 784$$
$$2 \times 2 \times 4 \times 196$$
$$2 \times 2 \times 2 \times 2 \times 4 \times 49$$
$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$$

$$\sqrt{3136} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7}$$
$$= 2 \times 2 \times 2 \times 7$$
$$= 56$$
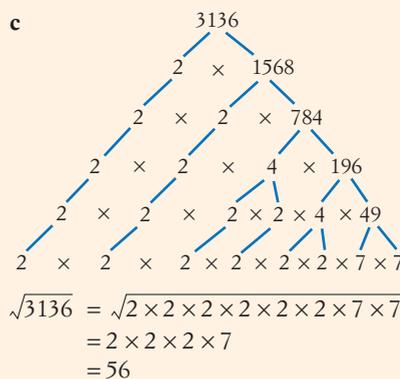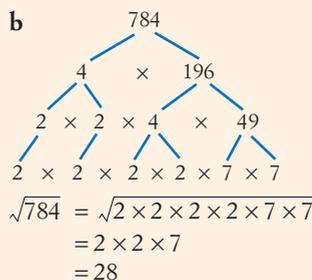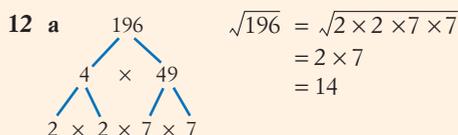
13 a 22   **b** 17   **c** 21

14 False. The number 1 has only one factor, itself.

15 Teacher to check.

16 a By starting with 1, 1, then adding the previous two numbers each time.
   b $5 + 8 = 13$
      $8 + 13 = 21$
      $13 + 21 = 34$
      $21 + 34 = 55$
      $34 + 55 = 89$
   c 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765
      i 2, 8, 34, 144, 610, 2584 (Each number divides evenly by 2)
      ii 3, 21, 144, 987, 6765 (Each number divides evenly by 3)
      iii 5, 55, 610, 6765 (Each number divides evenly by 5)
   d i 1, 3, 21, 55   **ii** 1

   e In the double spirals of a daisy wheel, the bumps on pineapples, and the leaves of many trees.

17

      1   7 21 35 21 7 1
    1 8 28 56 56 28 8 1
  1 9 36 84 112 84 36 9 1
1 10 45 120 196 196 120 45 10 1
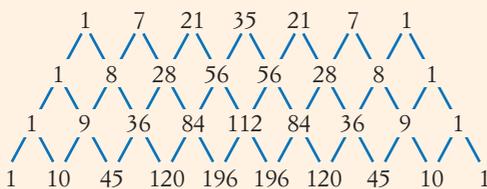
   b Add the numbers to the immediate left and immediate right in the row above.
   c 1, 2, 4, 8, 16, . . . They are the powers of 2.
   d Use the diagonal 1, 3, 6, 10, 15, 21, . . .
   e 1, 1, 2, 3, 5, 8, . . .

18 a 373, 656, 22, 191, 797
   b 1001, 1111, 1221, 1331, 1441, 1551, 1661, 1771, 1881, 1991
   c **i** one step   **ii** two steps
      iii one step   **iv** two steps
      v two steps   **vi** four steps
      vii five steps   **viii** two steps
      ix one step

   d Teacher to check.

Chapter review

1 a 612   **b** 943   **c** 5499
  d 6002   **e** 9 750 076

2 a 40 (or 4 tens)
  b 4 (or 4 units)
  c 400 (or 4 hundreds)
  d 40 000 (or 4 ten thousands)

3 a $1 \times 10 + 9 \times 1$
  b $2 \times 10^2 + 8 \times 10 + 3 \times 1$
  c $6 \times 10^2 + 6 \times 10 + 5 \times 1$
  d $4 \times 10^4 + 2 \times 10^3 + 8 \times 10^2 + 9 \times 10 + 1 \times 1$

4 a 1   **b** 21   **c** 25   **d** 15
  e 2400 **f** 1400 **g** 11   **h** 26
  i 8   **j** 10   **k** 35   **l** 1 969 999

5 a 32   **b** 81   **c** 625   **d** 7
  e 20   **f** 16

6 a 1, 2, 3   **b** $^-2, \, ^-3, \, ^-4$
  c $^-10, \, ^-12, \, ^-14$ **d** $^-4, \, ^-6, \, ^-8$
  e $^-4, 5, \, ^-6$ **f** $^-7, \, ^-9, \, ^-11$

7 a F   **b** F   **c** F   **d** T
  e T   **f** T   **g** T   **h** F
  i F   **j** F   **k** T   **l** F

8 a 2   **b** 23   **c** $^-56$ **d** 10
  e $^-8$ **f** 0

9 a $^-1$ **b** 7   **c** $^-10$ **d** $^-3$
  e 4   **f** 18   **g** $^-12$ **h** 0
  i $^-15$ **j** 3   **k** $^-11$ **l** $^-7$
  m 6   **n** $^-2$ **o** $^-22$ **p** $^-8$
  q 17   **r** $^-11$

10 a $^-18$ **b** 48   **c** $^-10$
  d $^-400$ **e** $^-63$ **f** 48
  g $^-24$ **h** $^-42$ **i** 40

11 a 3   **b** $^-3$ **c** 1
  d $^-4$ **e** $^-2$ **f** $^-100$
  g 2   **h** $^-3$ **i** $^-8$

12 a 9, 11, 13   **b** 10, 12, 14   **c** 15, 21, 28
  d 25, 36, 49   **e** 13, 21, 34   **f** 40, 35, 30

13 a 1, 4, 9, 16, . . . (d)   **b** 1, 3, 6, 10, . . . (c)
  c 1, 1, 2, 3, 5, 8, . . . (e)

14 a 1, 2, 3, 6   **b** 4   **c** 12

15 2, 13, 17, 19, 29, 31, 37, 67, 73, 83, 89

16 a 28 561   **b** 2 000 376
  c 11.40   **d** 12.03

17 a 38, 21, 19, 16, 14

Answers

b 101, 98, 89, 88, 36
c 2635, 2534, 2533, 2356, 2300
d 12 913, 12 391, 12 300, 11 990, 11 391

18 a T **b** T **c** T
d T **e** T **f** T

19 $7 \times 10^4 + 6 \times 10^3 + 7 \times 10^2 + 2 \times 10^1 + 5$

20 a $^-12$ **b** $^-14$ **c** $^-13$ **d** $^-18$
e 0 **f** $^-19$ **g** 7 **h** $^-2$

21 a 15 **b** 34 **c** 2112
d 31 **e** 4, 6, 8, 9, 10 **f** 49

22 a

```
          24
        ╱    ╲
       4  ×  6
      ╱╲    ╱╲
     2 × 2 × 2 × 3
   24 = 2 × 2 × 2 × 3
```

b

```
              60
            ╱    ╲
           5  ×  12
                ╱╲
           5  ×  4 × 3
              ╱╲
     5  ×  2 × 2  ×  3
     60 = 2 × 2 × 3 × 5
```
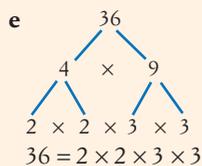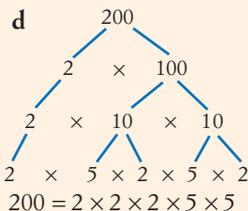
c

```
          27
        ╱    ╲
       3  ×  9
            ╱╲
     3  ×  3 × 3
    27 = 3 × 3 × 3
```

d

```
              200
            ╱    ╲
           2  ×  100
                ╱╲
       2  ×  10 × 10
            ╱╲    ╱╲
     2  ×  5 × 2 × 5 × 2
     200 = 2 × 2 × 2 × 5 × 5
```

e

```
          36
        ╱    ╲
       4  ×  9
      ╱╲    ╱╲
     2 × 2 × 3 × 3
   36 = 2 × 2 × 3 × 3
```

f

```
          45
        ╱    ╲
       5  ×  9
            ╱╲
     5  ×  3 × 3
    45 = 3 × 3 × 5
```

g

```
              72
            ╱    ╲
           2  ×  36
                ╱╲
       2  ×  6 × 6
      ╱      ╱╲  ╱╲
     2  ×  2 × 3 × 2 × 3
     72 = 2 × 2 × 2 × 3 × 3
```

h

```
              144
            ╱    ╲
           4  ×  36
          ╱╲    ╱╲
       2 × 2  × 6 × 6
            ╱╲  ╱╲
     2 × 2 × 2 × 3 × 2 × 3
   144 = 2 × 2 × 2 × 2 × 3 × 3
```
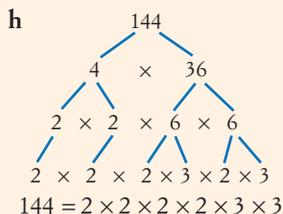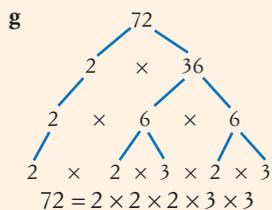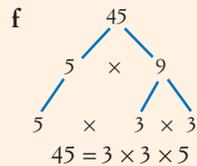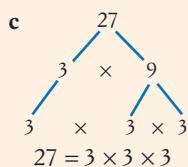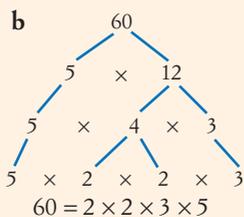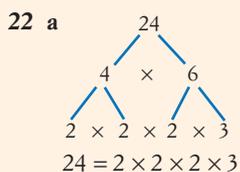
23 a $24 = 2^3 \times 3$ **b** $60 = 2^2 \times 3 \times 5$
c $27 = 3^3$ **d** $200 = 2^3 \times 5^2$
e $36 = 2^2 \times 3^2$ **f** $45 = 3^2 \times 5$
g $72 = 2^3 \times 3^2$ **h** $144 = 2^4 \times 3^2$

24

```
    1      5     10    10     5      1
   ╱ ╲    ╱ ╲   ╱ ╲   ╱ ╲    ╱ ╲    ╱ ╲
  1     6     15    20    15     6      1
 ╱ ╲   ╱ ╲   ╱ ╲   ╱ ╲   ╱ ╲   ╱ ╲    ╱ ╲
1    7     21    35    35    21     7      1
```

25 a

```
◄──┼──●──┼──●──┼──●──┼──┼──┼──●──●──┼──►
  ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1  0  1  2  3  4
```

b

```
◄──┼──┼──●⌢●⌢●⌢●⌢●⌢●──┼──┼──┼──►
  ⁻5 ⁻4 ⁻3 ⁻2 ⁻1  0  1  2  3  4  5  6
```
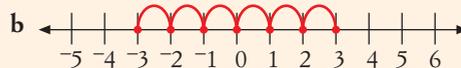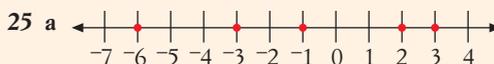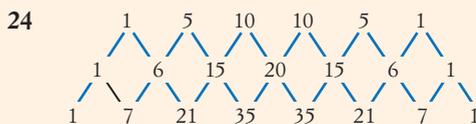
26 a $7 + 3 - 1 = 9$ **b** $10 + (5 - 5) = 10$
c $8 + 3 - 6 \div 2 = 8$ **d** $28 \div 4 \times 7 = 49$
e $6 \times 4 \div 3 \times 5 = 40$ **f** $(19 + 1) \div 5 - (3 + 1) = 0$

27 $^-120$ m, 39 m, 85 m

28 7 and 8

29 a $160 **b** $128

30 16°C

31 $112

32 5899 minutes or about 100 hours

33 86

34 a

```
                  225
                ╱    ╲
               5  ×  45
                    ╱╲
           5  ×  5 × 9
                    ╱╲
       5  ×  5  ×  3 × 3
     √225 = √5 × 5 × 3 × 3
          = 5 × 3
          = 15
```

b

```
                      256
                    ╱    ╲
                   4  ×  64
                  ╱╲    ╱╲
           2 × 2 × 4 × 16
                      ╱╲  ╱╲
         2 × 2 × 2 × 2 × 4 × 4
              ╱╲  ╱╲  ╱╲ ╱╲
     2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
     √256 = √2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
          = 2 × 2 × 2 × 2
          = 16
```

c

```
                      1764
                    ╱    ╲
                   3  ×  588
                        ╱╲
             3  ×  4  ×  147
                  ╱╲    ╱╲
         3  ×  2 × 2 × 7 × 21
                            ╱╲
     3  ×  2 × 2 × 7 × 7 × 3
     √1764 = √2 × 2 × 3 × 3 × 7 × 7
          = 2 × 3 × 7
          = 42
```

d

$$\sqrt[3]{5832} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$$
$$= 2 \times 3 \times 3$$
$$= 18$$

35 6.5 m

36 About 400

37 About 36

38 Correct answers are:

a	⁻9	**b**	⁻1	**c**	⁻6	**d**	6
e	⁻20	**f**	0	**g**	⁻1	**h**	2
i	⁻73	**j**	24				

Thus the student was correct in parts **a**, **d**, **e**, **h**, **j**.
Score: 5 out of 10

Chapter 3

Exercise 3.1

1 a ∠QPK, ∠P or ∠KPQ
b ∠OCR, ∠C or ∠RCO
c ∠GVE, ∠V or ∠EVG
d ∠TAG, ∠A or ∠GAT
e ∠PQD, ∠Q or ∠DQP
f ∠RDC, ∠D or ∠CDR

2 C

3 a ∠CAD or ∠DAC   **b** ∠PMQ or ∠QMP
c ∠TQR or ∠RQT   **d** ∠WXY or ∠YXW
e ∠AED or ∠DEA   **f** ∠FGE or ∠EGF

4 The following are examples, the sizes of the angles will differ.

a

b

c

d

5 a 64°   **b** 133°   **c** 85°   **d** 87°
e 129°   **f** 175°   **g** 157°   **h** 112°

6 a ∠AOB = 38°   **b** ∠PDQ = 162°
c ∠XPY = 84°   **d** ∠MAN = 121°
e ∠TQS = 175°   **f** ∠XYZ = 38°
g ∠LMN = 82°   **h** ∠GFD = 121°
i ∠ADM = 142°   **j** ∠BPZ = 47°
k ∠ABC = 122°   **l** ∠EFG = 38°

7 a ∠ABC = 213°   **b** ∠XYZ = 300°
c ∠LMN = 248°   **d** ∠GHK = 330°

8 a

b

c

d

e

f

g

h

9 a

b

c

d

e

f

g

h

i

Answers

10 a ∠*NAY*, ∠*AYN* or ∠*AYC*, ∠*ANY* or ∠*ANC*,
∠*AND*, ∠*AYD*, ∠*NCD*, ∠*NDY*, ∠*NCY*,
∠*DNC* or ∠*DNY*, ∠*DCY*, ∠*CDY*, ∠*NYD* or
∠*CYD*, ∠*NDC*
(Remember that ∠*NAY* and ∠*YAN* are two
ways of naming the same angle.)
b straight angle

11 a • ∠*DBC* or ∠*CBD*   **x** ∠*ADB* or ∠*BDA*
b • ∠*PSR* or ∠*RSP*   **x** ∠*PRQ* or ∠*QRP*
c • ∠*MRQ* or ∠*QRM*   **x** ∠*RNP* or ∠*PNR*
d • ∠*EIF* or ∠*FIE*   **x** ∠*FGI* or ∠*IGF*
e • ∠*DCF* or ∠*FCD*   **x** ∠*GBC* or ∠*CBG*
f • ∠*WXY* or ∠*YXW*   **x** ∠*YWZ* or ∠*ZWY*

12 a ∠*ADB* and ∠*CDB*, or ∠*CBD* and ∠*DBA*
b ∠*SRP* and ∠*QRP*, or ∠*SPR* and ∠*QPR*
c ∠*QMR* and ∠*NMR*, or ∠*RNP* and ∠*MNR*, or
∠*NPR* and ∠*QPR*, or ∠*RQP* and ∠*RQM*, or
∠*MRQ* and ∠*QRP*, or ∠*QRP* and ∠*PRN*, or
∠*PRN* and ∠*NRM*, or ∠*NRM* and ∠*MRQ*
d ∠*EIF* and ∠*FIG*, or ∠*FIG* and ∠*GIH*, or
∠*HGI* and ∠*IGF*, or ∠*GFI* and ∠*IFE*
e ∠*ECD* and ∠*DCF*, or ∠*DCF* and ∠*FCB*, or
∠*FCB* and ∠*BCE*, or ∠*BCE* and ∠*ECD*, or
∠*GBC* and ∠*CBH*, or ∠*CBH* and ∠*HBA*, or
∠*HBA* and ∠*ABG*, or ∠*ABG* and ∠*GBC*
f ∠*XWY* and ∠*YWZ*, or ∠*XYW* and ∠*WYZ*

13 B

14 a • = 153°   **x** = 27°
b • = 45°   **x** = 88°
c • = 67°   **x** = 119°
d • = 45°   **x** = 45°
e • = 114°   **x** = 78°
f • = 84°   **x** = 118°

15 Teacher to check.

16 a acute   **b** obtuse   **c** reflex
d reflex   **e** acute   **f** straight
g obtuse   **h** obtuse   **i** revolution
j acute   **k** reflex   **l** obtuse
m acute   **n** reflex   **o** right
P reflex   **q** obtuse   **r** acute

17 a obtuse **b** looks acute but is actually a right angle
c reflex **d** acute

18 D

19 D, A, H, F, C, B, G, E

20 C, H, A, E, G, B, J, I, D, F

Exercise 3.2

1 a *c*   **b** *u*   **c** *a*
d *f*   **e** *i*   **f** *p*

2 a 48°   **b** 53°

3 a ∠*MDQ*
b ∠*NDP*
c ∠*QDN*, ∠*MDP*
d ∠*MDQ* and ∠*QDP*, ∠*QDP* and ∠*PDN*, ∠*PDN*
and ∠*NDM*, ∠*NDM* and ∠*MDQ*

4 A

5 a ∠*PQR* and ∠*RQS*   **b** ∠*PQR* and ∠*RQS*
c They add to 90°.

6 a *FC* and *DA*, *GB* and *AD*
b *BG* and *CF*
c *GB* and *HE*, *FC* and *HE*, *GB* and *DA*, *FC* and
DA

7 a *FC* ⊥ *DA* or *GB* ⊥ *DA*   **b** *BG* ∥ *CF*

8 Answers should resemble these:
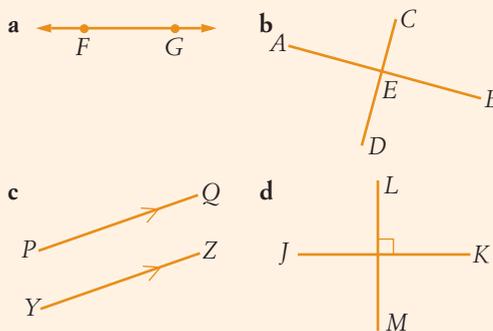
9 a *g*   **b** *d*   **c** *d*

10 Pairs of alternate angles are marked in the same
way.
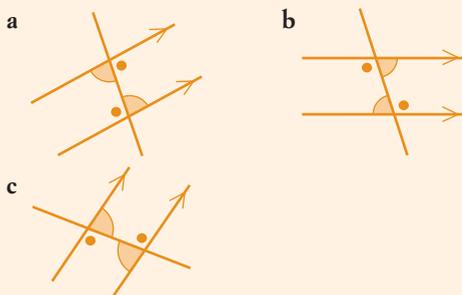
11 a *f*   **b** *c*   **c** *c*

12 a *d*   **b** *d*   **c** *c*

Answers

13 a ∠WXQ   **b** ∠XWP
   c ∠WXS   **d** ∠AWV or ∠XWP
   e ∠XWA   **f** ∠XWP

14 a $a = 70°$ **b** $m = 110°$
   c $k = 85°$ **d** $m = 90°$
   e $x = 135°$ **f** $f = 25°$
   g $w = 133°$ **h** $n = 29°$
   i $q = 62°$ **j** $t = 163°$
   k $h = 20°, g = 160°$
   l $q = 90°, r = 90°, s = 90°$

15 a $a = 60°$, supplementary angles
   b $a = 20°$, complementary angles
   c $y = 100°$, vertically opposite angles
   d $m = 135°$, supplementary angles
   e $p = 120°$, angles at a point
   f $a = 120°$, angles at a point
   g $m = 71°$, complementary angles
   h $x = 41°$, vertically opposite angles
   i $f = 165°$, supplementary angles
   j $h = 170°$, vertically opposite angles
   k $a = 45°$, complementary angles
   l $t = 90°$, supplementary angles
   m $a = 32°$, vertically opposite angles;
     $b = 148°$, supplementary angles
   n $y = 53°$, angles at a point
   o $d = 135°$, angles at a point
   p $x = 70°$, complementary angles;
     $y = 90°$, supplementary angles
   q $e = 68°$, supplementary angles;
     $f = 112°$, supplementary angles
   r $j = 132°$, supplementary angles;
     $k = 48°$, supplementary angles;
     $l = 132°$, supplementary angles
   s $x = 62°$, supplementary angles;
     $y = 105°$, supplementary angles
   t $p = 60°$, angles at a point
   u $e = 60°$, supplementary angles

16 a $m = 110°$ **b** $a = 50°$
   c $n = 80°$ **d** $b = 122°$
   e $h = 20°$ **f** $m = 50°, n = 130°, p = 50°$

17 a $a = 120°$ **b** $y = 28°$
   c $m = 63°$ **d** $c = 108°$
   e $t = 74°$ **f** $a = 50°, b = 60°$

18 $a = 75°, b = 105°, c = 75°, d = 105°, e = 75°,$
   $f = 105°, g = 75°$

19 a $a = 130°$ **b** $m = 90°$
   c $b = 105°$ **d** $d = 68°$
   e $m = 112°$ **f** $a = 82°, b = 98°$

20 a 65°   **b** 71°   **c** 105°
   d 120°   **e** 110°   **f** 132°
   g 28°   **h** 72°   **i** 95°

21 a $a = 113°, b = 113°$
   b $j = 133°, k = 133°, l = 133°$
   c $m = 52°, n = 52°, p = 128°$
   d $y = 42°, z = 42°$ **e** $l = 95°, m = 85°$
   f $b = 45°, c = 30°$ **g** $p = 105°, q = 105°$
   h $k = 85°, m = 85°$ **i** $p = 63°, w = 117°$

22 a $p = 71°$ **b** $m = 35°$ **c** $k = 76°$ **d** $x = 51°$
   e $a = 52°$ **f** $y = 33°$ **g** $d = 82°$ **h** $t = 105°$
   i $e = 72°$

23 Teacher to check.

24 C

25 a ∠BEF and ∠CFE
     AB ∥ CD, alternate angles are equal
   b ∠AGH and ∠GHD
     AB ∥ CD, alternate angles are equal
   c ∠BEF and ∠EFC
     AB is not parallel to CD, alternate angles are not equal

26 a ∠GEB and ∠DFE
     AB is not parallel to CD, corresponding angles are not equal
   b ∠AEF and ∠CFG
     AB ∥ CD, corresponding angles are equal
   c ∠CEF and ∠AFG
     AB is not parallel to CD, corresponding angles are not equal

27 a ∠AEF and ∠EFC
     AB ∥ CD, co-interior angles add up to 180°
   b ∠BEF and ∠EFD
     AB is not parallel to CD, co-interior angles do not add up to 180°.
   c ∠AEF and ∠EFC
     AB ∥ CD, co-interior angles add up to 180°

28 a MN ∥ PQ, co-interior angles add up to 180°
   b PQ ∥ MN, corresponding angles are equal
   c MN ∥ PQ, alternate angles are equal, corresponding angles are equal
   d PQ is not parallel to MN, corresponding angles are not equal, ∠PCB = 115° and ∠MAD = 120°

29 B

Exercise 3.3

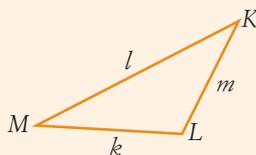**1 a** EH and EI   **b** EH ⊥ EI   **c** DF ∥ GJ

2 a and **b**
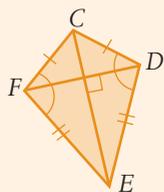
Answers
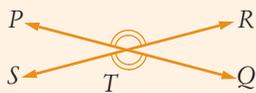
c They are vertically opposite angles.

3

4 *EF*, the line, is infinite. The interval *EF* is the section of the line between point *E* and point *F*.

5 a, c, d, and e   **b** *FE*

6 a, b and **d** Teacher to check.   **c** side *g*

7 a and **b**
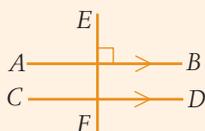
c True

8 a Teacher to check.
  b *Y*
  c *WX* ∥ *ZY* or *WZ* ∥ *XY*
  d *WX* ⊥ *XY*, *XY* ⊥ *YZ*, *YZ* ⊥ *ZW*, or *ZW* ⊥ *WX*
  e **i** Side *WX* is perpendicular to side *XY*.
      ii The length of side *WX* is equal to the length of side *XY*.

9 a 6   **b** 4   **c** 9   **d** 10   **e** 7
  f 5   **g** 12   **h** 8   **i** 11

10 D

11 Teacher to check.

12 a △*PQT*, △*PRS*
  b **i** *PQ*
      ii *TS*
  c *TQ* ∥ *SR*

d and **e**
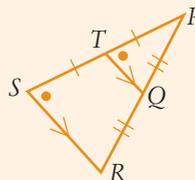
f Corresponding angles on parallel lines are equal.

13 a False   **b** False
   c True   **d** *ST* and *VT*

14 a square   **b** triangle
   c pentagon   **d** kite
   e rectangle   **f** parallelogram
   g trapezium   **h** hexagon, non-convex
   i quadrilateral   **j** equilateral triangle
   k pentagon   **l** hexagon

15 C and D

16 B, E, F

17 a 2   **b** 5   **c** 9

18

Acute	Obtuse	Right	Equilateral	Isosceles	Scalene
c	a	b	g	d	a
f	d	i	j	e	b
g	e			f	c
h	l			h	l
j				i	
k				k	

19 Teacher to check.

20 a Isosceles **b** Scalene **c** Equilateral
   d Isosceles **e** Scalene **f** Scalene

21 C

22 Teacher to check.

23 a convex   **b** convex
   c non-convex   **d** convex
   e non-convex   **f** non-convex

24 a rhombus   **b** parallelogram
   c square   **d** trapezium
   e rectangle   **f** kite

25 d one pair of adjacent sides equal; one pair of opposite angles equal; diagonals intersect at right angles

26

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Diagonals are equal				✓	✓	
Diagonals bisect each other		✓	✓	✓	✓	
Diagonals intersect at right angles			✓		✓	✓
Diagonals bisect the angles of the quadrilateral			✓		✓	

27

| Shape	Properties
Trapezium	• One pair of parallel sides
Kite	• Two pairs of adjacent sides are equal • One pair of opposite angles are equal • Diagonals intersect at right angles
Parallelogram	• Opposite sides are equal and parallel • Opposite angles are equal • Diagonals bisect each other
Rhombus	• All four sides are equal • Opposite sides are parallel • Opposite angles are equal • Diagonals bisect at right angles • Diagonals bisect the angles of the rhombus
Rectangle	• All four angles measure 90° • Opposite sides are equal and parallel • Diagonals are equal • Diagonals bisect each other

Shape	Properties
Square	• All four sides are equal • All four angles measure 90° • Diagonals are equal and bisect each other at right angles • Diagonals bisect the angles of the square

28 a square, triangle
b 2 rectangles
c rectangle, trapezium, and pentagon
d rectangle and two trapeziums
e rectangle, triangle
f two triangles and two trapeziums (Other answers possible.)

29 a   **b**   **c**

d   **e**   **f**

g   (Other answers possible.)
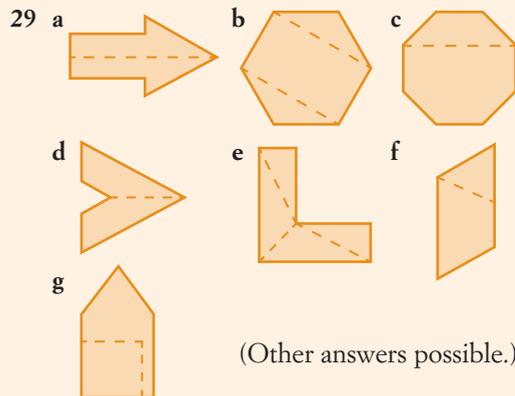
30 a rectangle, square
b square, rhombus
c parallelogram, rectangle, square, rhombus
d parallelogram, rhombus, rectangle, square
e rectangle, square
f parallelogram, rectangle, square, rhombus
g rhombus, square

31 No. In an equilateral triangle, all angles are equal.

32 No, two obtuse angles would add up to more than 180°

33 a Yes, a square is a rectangle
b Yes, a rhombus is a parallelogram.

Exercise 3.4

1 a 60°–30°   **b** 45°
   c 45°   **d** 60°–30°

2 Teacher to check.

3 Teacher to check.

4 Teacher to check.

5 Teacher to check.

6 Teacher to check

7 **a–c** Teacher to check   **d** isosceles

8 Teacher to check

9 a b   **b** Two angles are equal

10

| Shape	Properties
Equilateral triangle	• All three sides are equal • All three angles measure 60°
Isosceles triangle	• Two sides are equal • Two angles equal (opposite the equal sides)
Scalene triangle	• No sides or angles are equal

11 Teacher to check.

12 Teacher to check.

13 Teacher to check.

14 Teacher to check.

15 d Yes   **e** parallelogram

16 b rhombus

17 a *PS* and *QR*
   c $\angle P = \angle Q = 121°$, $\angle R = \angle S = 59°$
   d $\angle P = \angle Q$, $\angle S = \angle R$

18 Teacher to check.

19 b trapezium

20 Teacher to check.

21 Teacher to check.

22 Teacher to check.

23 c They are the same.

24 kite

25 Perpendicular lines intersect to form a right angle. A vertical line is perpendicular to the horizontal.

26 Teacher to check.

27 a i $\angle Q, \angle R, SR, QS$ **ii** $\angle S, QR$
   b i $\angle N, OM$ **ii** $\angle O, MN$
   c i $\angle D, EC$ **ii** $\angle E, CD$
   d i All angles and all sides are equal.
   e i $\angle M, KL$ **ii** $\angle K, \angle L, ML, KM$
   f i $\angle X, YZ$ **ii** $\angle Y, XZ$

28 opposite, opposite, smallest

29 a d
   b They are the same, all equal 60°.

30 d equal, equal, 60

31 b $7 + 5 < 15$ so the triangle inequality rule does not hold.

Exercise 3.5

1 a $N \leftrightarrow Q, L \leftrightarrow P, M \leftrightarrow R$
   b $\triangle NLM \equiv \triangle QPR$

2 a $A \leftrightarrow H, C \leftrightarrow G, B \leftrightarrow I, D \leftrightarrow F$
   b $ACBD \,|||\, HGIF$

3 $a \equiv m, c \equiv l$

4 $a \,|||\, d, m \,|||\, d, b \,|||\, f, c \,|||\, h, e \,|||\, i, l \,|||\, h$

5 a The figures have the   **b** 3/2 (or 2/3)
   same shape.

6 a $X \to A$: 4 down; $X \to B$: 2 right, 2 up; $X \to C$: 5 right, 2 down; $X \to D$: 4 left, 2 up; $X \to E$: 5 left, 3 down
   b i 9 right, 4 down
      ii 5 left, 2 down
      iii 7 right, 5 up
      iv 4 left, 6 up
      v 4 right, 2 down

7 a 90° clockwise about *S*
   b 90° clockwise about *R*
   c 90° clockwise about *Q*
   d 270° clockwise about *R*
   e 180° clockwise about *P*
   f 90° clockwise about *T*

8

Answers

9

10
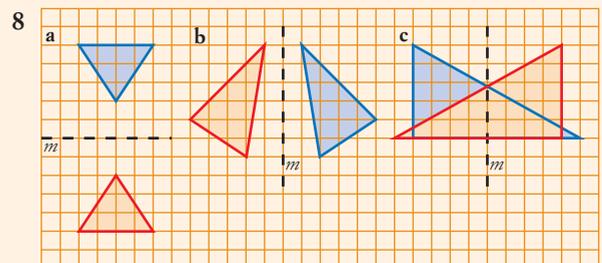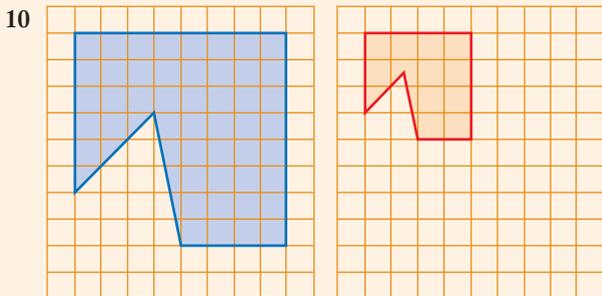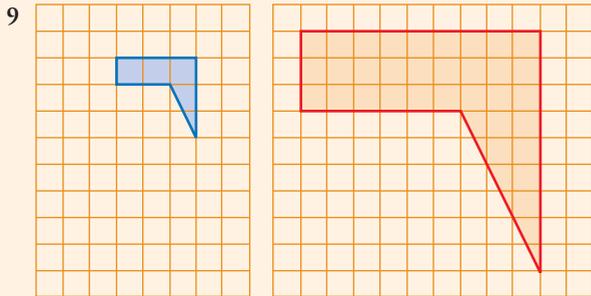
11 a $A(1, 1) \to A'(1, 0), B(1, 2) \to B'(1, 1)$
 b $A(1, 1) \to A'(0, 1), B(1, 2) \to B'(0, 2)$
 c $A(1, 1) \to A'(0, 1), B(1, 2) \to B'(1, 1)$
 d $A(1, 1) \to A'(1, 1), B(1, 2) \to B'(1, 2)$

12 Teacher to check.

13 Teacher to check.

14
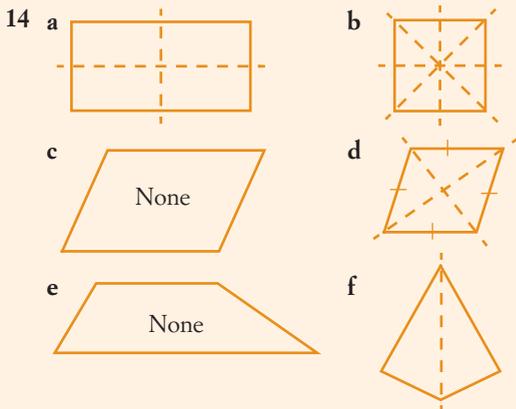
15 Corresponding angles are equal, so shapes are similar.

16 B

17 parallelogram

18 a

b

c
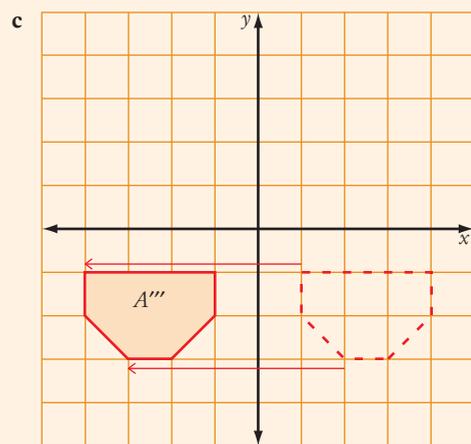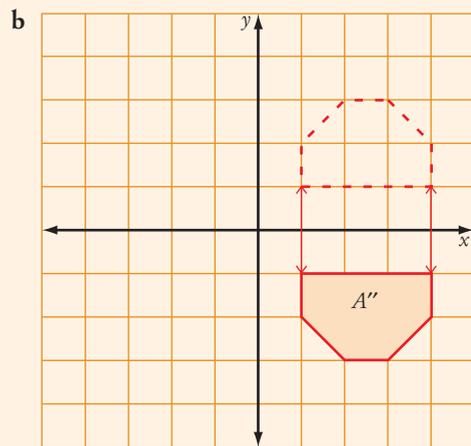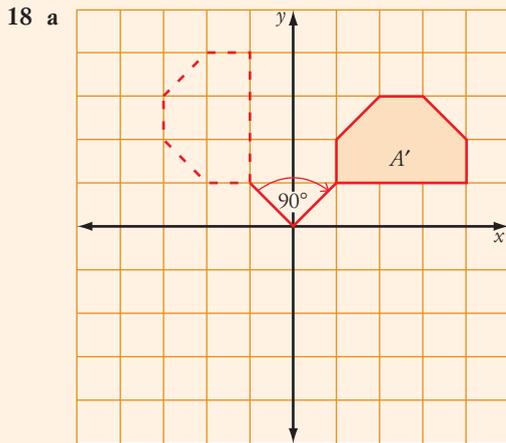
 d Rotation of 270° clockwise around the origin or rotation of 90° anticlockwise around the origin.

19 Yes, any equilateral triangle.

Answers

Chapter review

1 Teacher to check

2 Teacher to check

3 **a** $\angle WHI$ or $\angle IHW$, acute angle
  b $\angle DAR$ or $\angle RAD$, obtuse angle
  c $\angle GLU$ or $\angle ULG$, acute angle
  d $\angle PVR$ or $\angle RVP$, acute angle
  e $\angle NPE$ or $\angle EPN$, right angle
  f $\angle SAM$ or $\angle MAS$, obtuse angle
  g $\angle MTV$ or $\angle VTM$, straight angle
  h $\angle XYZ$ or $\angle ZYX$, reflex angle
  i $\angle MPQ$ or $\angle QPM$, reflex angle

4 **a i** $55°$ **ii** $12°$ **iii** $86°$
  b i $135°$ **ii** $80°$ **iii** $2°$
  c i $a = 220°$ **ii** $m = 65°$ **iii** $y = 325°$

5 **a** AB and GH
  b EF and GH, EF and AB
  c AB and CD, AB and EF, GH and CD, GH and EF

6 **a i**

   ii
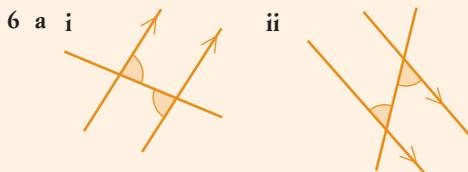
  b i $a = 38°$
   ii $b = 120°$
   iii $a = 126°$, $b = 126°$

7 **a i**

   ii

  b i $a = 112°$ **iii** $p = 150°$, $n = 30°$
   ii $p = 117°$

8 **a i**

   ii
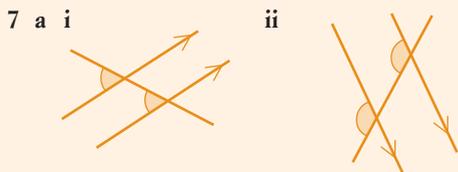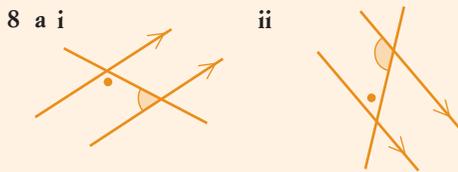
  b i $k = 108°$ **iii** $x = 98°$, $y = 98°$
   ii $x = 68°$

9 **a** alternate   **b** co-interior
  c corresponding   **d** corresponding
  e alternate   **f** corresponding

10 **a** AD and BC
  b AD and DC, BC and DC
  c $\angle ADC$ or $\angle CDA$ and $\angle BCD$ or $\angle DCB$

11 decagon

12 circle, ellipse, etc.

13 Teacher to check.

14 **a** equilateral, acute-angled
  b scalene, right-angled
  c isosceles, acute-angled
  d scalene, obtuse-angled
  e scalene, acute-angled
  f isosceles, right-angled

15 **a** square   **b** rhombus
  c trapezium   **d** parallelogram
  e rectangle   **f** kite

16 A quadrilateral with all sides equal.

17 Teacher to check.

18 $a \mathbin{\|\!\|} j$, $c \mathbin{\|\!\|} k$, $d \mathbin{\|\!\|} i$, $b \equiv h$, $e \equiv l$, $f \equiv g$

19 **a**

   b

   c

   d

   e

   f

   g

   h

   i
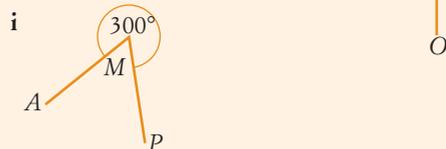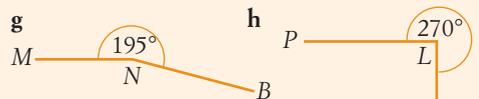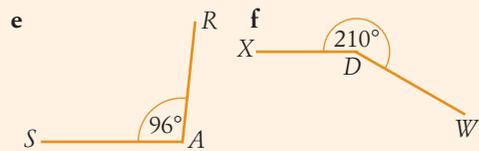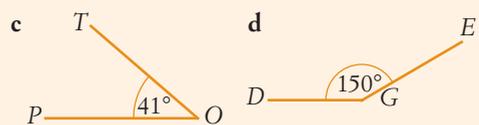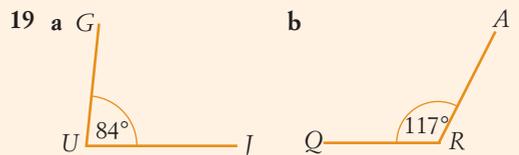
20 **a** $a = 100°$ **b** $m = 44°$ **c** $a = 95°$, $b = 85°$

21 **a** $m = 62°$
  b $k = 133°$
  c $x = 58°$, $y = 122°$
  d $p = 145°$

e $x = 42°$
f $f = 70°$
g $t = 83°$
h $p = 155°$, $q = 155°$, $r = 25°$
i $x = 30°$

22 a $a = 115°$
b $m = 35°$
c $k = 115°$
d $d = 125°$
e $x = 50°$
f $q = 62°$
g $x = 37°$, $y = 37°$
h $m = 62°$, $p = 118°$, $z = 118°$
i $a = 112°$, $d = 68°$, $t = 112°$

23 a $x = 50°$, $y = 50°$ **b** $x = 26°$, $m = 64°$
c $a = 72°$, $c = 70°$, $z = 38°$ **d** $a = 38°$, $c = 142°$
e $x = 35°$, $y = 88°$, $z = 92°$

24 a $x = 63°$ **b** $y = 103°$

25 Teacher to check.

26 a isosceles, acute-angled
b $\angle F$ and $\angle H$

27 Teacher to check.

28 b The longest side is opposite the largest angle.

29 Teacher to check.

30

31

32
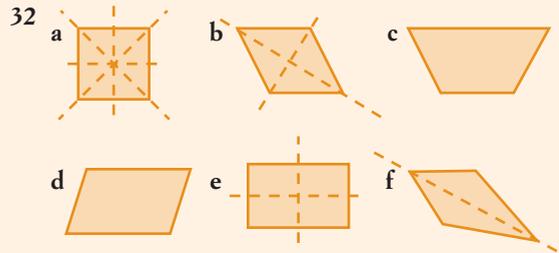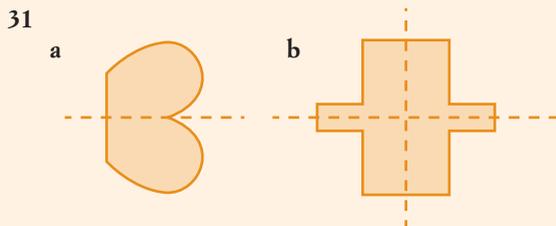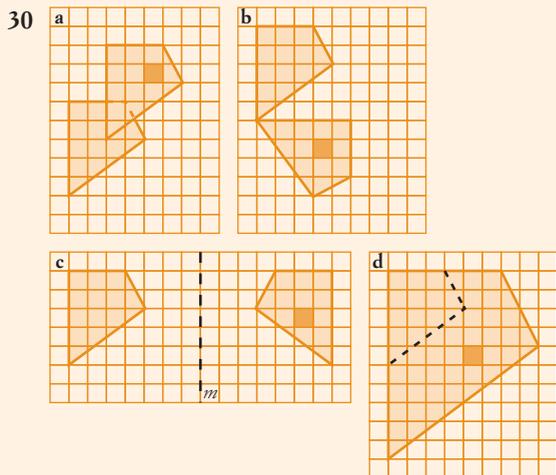
b **a** 4, **b** 2, **d** 2, **e** 2

33 a $AB \parallel CD$, co-interior angles add up to 180°
b AB is not parallel to CD, alternate angles are not equal
c $AB \parallel CD$, corresponding angles are equal

34 Teacher to check.

35 Teacher to check.

36 a equilateral triangle
b parallelogram, rectangle, rhombus or square
c pentagon
d rectangle, parallelogram, rhombus or square
e trapezium
f scalene acute-angled triangle

37 Yes, because all their angles are the same (60°).

Chapter 4
Exercise 4.1

1

	Hundreds	Tens	Units	tenths	hundredths	thousandths
a		1	4	8	2	
b			6	0	1	4
c	9	3	1	0	2	
d		7	0	8		
e	2	9	7	8	6	
f		1	1	1	4	
g	5	0	3	9	2	
h			8	3		
i			0	3	7	5
j	2	0	0	0	4	7
k			4	0	2	5
l			0	8	1	

2 a 5.4   **b** 6.15   **c** 8.3
d 11.38   **e** 14.06   **f** 402.003

Answers

g 6.23   **h** 3.7   **i** 19.9
j 14.39   **k** 24.09   **l** 2.06
m 8.75   **n** 0.73   **o** 0.009
p 0.859

3 a $1 \times 1 + 2 \times \frac{1}{10} + 3 \times \frac{1}{100} + 4 \times \frac{1}{1000}$
   b $1 \times 100 + 0 \times 10 + 2 \times 1 + 3 \times \frac{1}{10} + 4 \times \frac{1}{100}$
   c $3 \times 10 + 0 \times 1 + 1 \times \frac{1}{10} + 2 \times \frac{1}{100}$
   d $7 \times \frac{1}{10} + 5 \times \frac{1}{100} + 1 \times \frac{1}{1000}.$
   e $2 \times 1 + 0 \times \frac{1}{10} + 9 \times \frac{1}{100}$
   f $1 \times 10 + 2 \times 1 + 7 \times \frac{1}{10} + 1 \times \frac{1}{100}$
   g $8 \times 1 + 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 3 \times \frac{1}{1000}$
   h $4 \times 1 + 5 \times \frac{1}{10} + 0 \times \frac{1}{100} + 9 \times \frac{1}{1000}$
   i $0 \times \frac{1}{10} + 4 \times \frac{1}{100}$

4 a 4 hundreds   **b** 4 hundredths
   c 4 hundredths   **d** 4 units
   e 4 thousandths   **f** 4 tenths
   g 4 thousandths   **h** 4 ten thousands
   i 4 thousandths   **j** 4 hundredths
   k 4 hundredths   **l** 4 units

5 a 7 tenths   **b** 7 thousandths
   c 7 hundreds   **d** 7 hundredths
   e 7 tenths   **f** 7 units
   g 7 ten-thousandths   **h** 7 tens
   i 7 tens   **j** 7 tenths
   k 7 ten-thousandths   **l** 7 hundredths

6 a 4.1   **b** 3.8   **c** 4.35
   d 12.93   **e** 5.279   **f** 0.0461
   g 19.009   **h** 2.703   **i** 11.6002
   j 3.27   **k** 270.601   **l** 76.05
   m 0.341   **n** 0.907   **o** 27.23
   p 40.09   **q** 9.041   **r** 500.403

7 a 2   **b** 3   **c** 4   **d** 3   **e** 5
   f 1   **g** 6   **h** 5   **i** 7

8 a 9.998, 23.89, 43.89, 56.314, 56.324, 80.879, 400
   b 0.0586, 0.099, 0.568, 0.586, 0.684, 1.002, 5.608
   c 0.222, 0.89, 0.891, 1.23, 1.814, 7.007
   d 0.005, 0.05, 0.5
   e 3.004, 3.044, 3.4, 3.404, 3.44, 3.441
   f 0.022, 0.2, 0.202, 0.22
   g 1.002, 1.01, 1.012, 1.21
   h 0.007, 0.07, 0.67, 0.71, 7

9 a 4000.99, 1256.3, 570.25, 400.099, 125.63, 0.9899
   b 6.539, 5.639, 5.37, 5.369, 5.367, 3.66, 3.659
   c 1.61, 1.601, 1.6, 1.599, 1.509

d 6.6, 6, 0.6, 0.06
e 7.07, 0.77, 0.707, 0.7, 0.07, 0.007
f 0.44, 0.404, 0.4004, 0.044
g 0.65, 0.1, 0.08, 0.029
h 0.921, 0.92, 0.099, 0.09

10 a $0.2 < 0.25$ **b** $0.731 > 0.73$
   c $0.035 < 0.305$ **d** $0.007 < 0.070$
   e $1.59 > 1.059$ **f** $0.099 < 0.99$
   g $44.44 > 4.444$ **h** $0.7932 > 0.7239$

11 a–g (number lines — teacher to check)
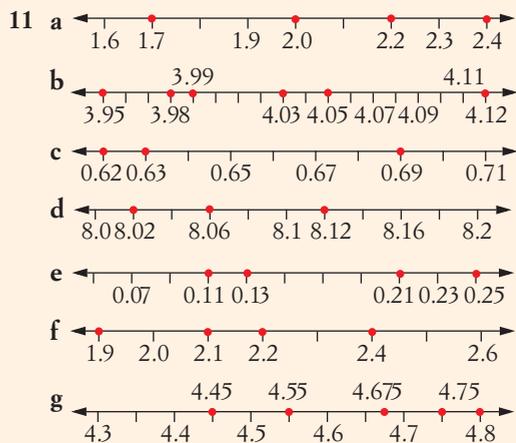
12 a 43.4 L   **b** 139.9c
   c $50.00   **d** $20.00
   e $9.30   **f** 1.88 m
   g 14.4 kg   **h** 15.0 cm
   i 20.0 kg   **j** 0.25 kg
   k 85.0 km/h   **l** 2300.0 m
   m 10.5 m   **n** 8.0 m
   o 1.1 times   **p** 13.4 km
   q 250.0 km   **r** 3.00 h

13 Teacher to check.

Exercise 4.2

1 a 0.7   **b** 0.5   **c** 0.5   **d** 0.4
2 a 0.9   **b** 0.15   **c** 0.79   **d** 0.6 or 0.60
   e 0.4   **f** 0.23   **g** 0.6   **h** 0.411
   i 0.704   **j** 0.87   **k** 0.07   **l** 0.14
   m 0.8   **n** 0.235   **o** 0.247   **p** 0.17
   q 0.368   **r** 0.9345   **s** 0.493

3 a 0.67   **b** 0.3   **c** 0.04
   d 0.011   **e** 0.08   **f** 0.006
   g 0.01   **h** 0.32   **i** 0.005
   j 0.0017   **k** 0.02   **l** 0.057
   m 0.0033   **n** 0.00046   **o** 0.000 007

4 a 1.67 **b** 4.47 **c** 23.9
d 6.8 **e** 45.023 **f** 2.3
g 6.011 **h** 89.143 **i** 42.0 or 42
j 5.00 or 5 **k** 4.006

5 a $\frac{7}{10}$ **b** $\frac{4}{10}$ **c** $\frac{39}{100}$ **d** $\frac{572}{1000}$
e $\frac{3}{1000}$ **f** $\frac{5}{100}$ **g** $\frac{11}{100}$ **h** $\frac{309}{1000}$
i $\frac{9}{10}$ **j** $\frac{999}{1000}$ **k** $\frac{13}{1000}$ **l** $\frac{4}{10\,000}$
m $\frac{471}{10\,000}$ **n** $\frac{3333}{10\,000}$ **o** $\frac{5001}{10\,000}$ **p** $\frac{91}{100}$
q $\frac{87}{1000}$ **r** $1\frac{9}{10}$ **s** $27\frac{33}{100}$ **t** $2\frac{7}{1000}$
u $10\frac{349}{1000}$ **v** $7\frac{41}{100}$ **w** $101\frac{3}{10}$ **x** $6\frac{102}{10\,000}$

6 a 0.4 **b** 0.5 **c** 0.8 **d** 0.7
e 2.6 **f** 0.3 **g** 0.9 **h** 2.9

7 a 0.48 **b** 0.74 **c** 0.07
d 0.29 **e** 0.31 **f** 0.66
g 2.10 **h** 4.00

8 a 25.35 **b** 341.77 **c** 321.33
d 734.65 **e** 27.76 **f** 1314.12

9 See table below.

10 a 0.96 **b** 1.43 **c** 0.09 **d** 0.64
e 0.85 **f** 1.67 **g** 1.33 **h** 1.71
i 1.72 **j** 0.97 **k** 0.58 **l** 0.63

11 a 10.3337 **b** 431.5433 **c** 1.4450
d 3217.6541 **e** 4.6709 **f** 0.8889

12 a 6 **b** 22 **c** 6
d 13 **e** 3

13 Teacher to check.

Exercise 4.3

1 a 12.0 **b** 16.933 **c** 167.317
d 2.1376 **e** 591.13

2 a 41.493 **b** 5.9 **c** 6.69
d 9.06 **e** 38.649

3 a 117.42 **b** 54.98 **c** 57.55
d 3.46 **e** 64.083 **f** 10.5

4 a 81.03 m **b** 18.97 m

5 a Total = $9.54, Change = $0.46
b Total = $15.15, Change = $4.85
c Total = $34.72, Change = $15.28

6 18.5 km

7 8037.2 kg

8 a 12.45 s, 12.7 s, 13.5 s, 13.81 s, 14.62 s
b 2.17 s
c Yes, 12.7 – 0.3 = 12.4 s, which is faster than 12.45 s.

9 a $605.55 **b** $15.25

10 10.38 cm

11 6.23 m

12 738.9 km

Exercise 4.4

1 a 1, right **b** 2, right **c** 3, right
d 1, left **e** 2, left **f** 3, left

Exercise 4.2

9

Question	Calculator display	Rounded (1 decimal place)	Rounded (2 decimal places)
12.19 ÷ 3	4.063333333	4.1	4.06
12.32 ÷ 6	2.053333333	2.1	2.05
19.82 ÷ 9	2.202222222	2.2	2.20
56.85 ÷ 11	5.168181818	5.2	5.17
17.13 ÷ 4	4.2825	4.3	4.28
12.65 ÷ 12	1.05416666	1.1	1.05
4.875 ÷ 21	0.232142857	0.2	0.23
27.45 ÷ 8	3.43125	3.4	3.43
17 ÷ 12	1.416666667	1.4	1.42
254.678 ÷ 32	7.9586875	8.0	7.96

Answers

2 a 4.63   **b** 5.07   **c** 1.203
   d 0.364   **e** 0.705   **f** 310.2
   g 0.006 43   **h** 0.0643   **i** 0.004 28
   j 0.66   **k** 0.000 31   **l** 0.002
   m 2490   **n** 8.1   **o** 37 420
   p 41.6   **q** 81   **r** 2192
   s 6 045 100   **t** 602   **u** 0.31

3 a 4521.3   **b** 10 640
   c 63.04   **d** 0.005 98
   e 84 761.2   **f** 0.592
   g 0.3628   **h** 519 400
   i 4.0075   **j** 0.081 348
   k 50 200   **l** 0.0061
   m 0.0004   **n** 1701
   o 123 000   **p** 0.0066
   q 0.52   **r** 1.471

4 a 2   **b** 1   **c** 0   **d** 3
   e 3   **f** 3   **g** 4   **h** 3
   i 4   **j** 4   **k** 6   **l** 2

5 a 4.691 87   **b** 0.437 421
   c 0.000 02743   **d** 1.2
   e 23 500.0137   **f** 13.7429

6 a 214 200   **b** 2.142   **c** 2142
   d 2142   **e** 214 200   **f** 21420

7 a 20.4   **b** 204   **c** 0.204
   d 20 400   **e** 2.04   **f** 2040

8 a 244.8   **b** 244.8   **c** 2.448
   d 2448   **e** 2.448   **f** 244 800

9 a 75.6   **b** 756   **c** 0.756
   d 0.756   **e** 0.0756   **f** 75 600 000

10 a 0.72   **b** 0.72   **c** 7.2
   d 7.2   **e** 720   **f** 0.0072

11 a 12.2   **b** 7.14   **c** 18.009
   d 34.2   **e** 22.5   **f** 12.6
   g 34.75   **h** 8.0032   **i** 0.9
   j 4.8   **k** 0.012   **l** 12.6
   m 32.823   **n** 59.04   **o** 40.15

12 a 0.32   **b** 1.95   **c** 0.48
   d 0.072   **e** 0.0032   **f** 1.24
   g 0.756   **h** 0.84   **i** 3.51
   j 0.879   **k** 27.2   **l** 10.8

13 a 1101.7   **b** 46.296   **c** 525.42
   d 482.46   **e** 72.68   **f** 2.707 56

14 a 2.4   **b** 6.2   **c** 4.16
   d 4.1   **e** 14.65   **f** 2.1975
   g 0.008   **h** 2.6775   **i** 24.45
   j 3.675   **k** 0.51875   **l** 0.106

15 a 2.4   **b** 1.13   **c** 5.75
   d 8.08   **e** 11.9   **f** 13.75
   g 14.75   **h** 259.45   **i** 0.122

16 a 0.217   **b** 4.625   **c** 0.013
   d 56.25   **e** 12.763   **f** 0.855
   g 0.6875   **h** 0.0083   **i** 13.01
   j 0.157   **k** 64.21   **l** 0.058

17 a $5088 \div 12$ **b** $1782 \div 11$
   c $3330 \div 45$ **d** $17.25 \div 25$
   e $12920 \div 38$ **f** $495 \div 15$
   g $16800 \div 75$ **h** $1482.3 \div 61$
   i $660 \div 22$

18 a 8.7   **b** 36.6   **c** 4.9   **d** 7.4
   e 108   **f** 4.2   **g** 208   **h** 80
   i 5.7   **j** 7.85   **k** 152   **l** 12.6
   m 3.4   **n** 1230   **o** 62

19 $82.50

20 1.56 m

21 7.2 m by 5.8 m

22 $222.86

23 42 lengths

24 8.929 kg

25 The 0.9 m material

26 The 0.2 m tiles, which make less waste.

27 a more, there are two 0.5s in every whole
   c 36

28 a more
   c 68

29 The answer is more than the number.

Exercise 4.5

1

	Fraction	Meaning as division	Decimal
a	$\frac{3}{5}$	$3 \div 5$	0.6
b	$\frac{1}{2}$	$1 \div 2$	0.5
c	$\frac{1}{4}$	$1 \div 4$	0.25
d	$\frac{4}{5}$	$4 \div 5$	0.8
e	$\frac{2}{5}$	$2 \div 5$	0.4
f	$\frac{3}{4}$	$3 \div 4$	0.75
g	$\frac{1}{5}$	$1 \div 5$	0.2
h	$\frac{1}{8}$	$1 \div 8$	0.125

2 a 0.4 **b** 0.375 **c** 0.75
d 1 **e** 0.5 **f** 0.75
g 0.6 **h** 0.25 **i** 0.625
j 0.875 **k** 0.5 **l** 1

3 a $0.\overline{1}$ **b** $0.\overline{3}$
c $0.1\overline{6}$ **d** $0.\overline{142\,857}$
e $0.\overline{6}$ **f** $0.\overline{285\,714}$
g $0.\overline{2}$ **h** $0.\overline{428\,571}$
i $0.\overline{571\,428}$ **j** $0.\overline{4}$
k $0.\overline{714\,285}$ **l** $0.\overline{5}$
m $0.\overline{857\,142}$ **n** $0.\overline{7}$
o $0.\overline{8}$

4 $0.\overline{1}, 0.\overline{2}, 0.\overline{3}, 0.\overline{4}, 0.\overline{5}, 0.\overline{6}, 0.\overline{7}, 0.\overline{8}$

5 a $3.\overline{6}$ **b** $1.6\overline{142\,857}$
c $6.49\overline{2}$ **d** $0.31\overline{6}$
e $8.\overline{4}$ **f** $0.111\overline{6}$
g $0.\overline{285\,714}$ **h** $2.2\overline{3}$
i $21.\overline{285\,714}$

6 a 4.8 **b** 23.75 **c** 12.625
d 6.6 **e** 57.4 **f** 19.125
g 110.875 **h** 80.25

7 $43.30

8 112.9 m

9 $9

10 6 drink bottles

11 15.8 km

12 26.6 km

13 258.3 tonnes

14 3 whole shelves

15 60.84 seconds

16 41.04, 0.829, 328.24, 4.7292, 2.517 06

17 1.4 km

18 a $15.24 **b** $6.84
c $487.76 **d** $1234.05

19 400 sheets

20 a
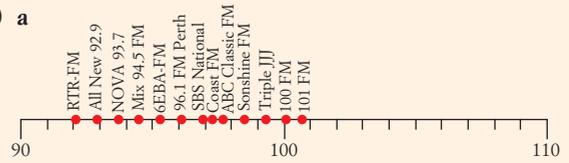
b 1.2 MHz
c 0.4 MHz (SBS National and ABC Classic FM)
d 0.8 MHz

21 The fractions are equivalent, e.g. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

22 a Queen's chamber 8.07
armoury 8.262
throne room 8.79
banquet hall 9.24
kitchen 9.96
dungeon 8.16

23 a 0.0004 **b** 0.000 008 **c** 1.331
d 0.2 **e** 0.6 **f** 0.3

24 a 3.7, 24.6 **b** 2.3, 5.7, 6
c 5.1, 0.72, 4.5, 18

25 Use a table or spreadsheet to find the result, using trial and error. See table below.

Exercise 4.5

25

Owner	Smith	Nguyen	Schulzes	Perezes	Zoghbis	Totals
Feedings	13	11	21	16	10	71
Amount (kg)	0.3	0.7	2.9	3.8	4.4	12.1
Total	3.9	7.7	60.9	60.8	44	177.3

Chapter review

1 a 40.36 **b** 520.703
 c 0.012 **d** 8.069

2 a 75.4 **b** 754 **c** 7540
 d 1.39 **e** 0.139 **f** 0.0139

3 a 65.355 **b** 104.49 **c** 11.965
 d 1037.9 **e** 396.51 **f** 228.155
 g 2.379 **h** 151.413

4

5 a 340, 56.86, 50.141, 34.98, 34.89, 3.998, 3.099
 b 8.6, 1.006, 0.86, 0.662, 0.652, 0.23, 0.136, 0.086
 c 1.293, 1.239, 1.1015, 1.015

6 a 0.4 **b** 0.13 **c** 0.07
 d 0.011 **e** 0.045

7 a $\frac{5}{10}$ **b** $\frac{89}{100}$ **c** $\frac{9}{100}$
 d $\frac{444}{1000}$ **e** $\frac{4051}{1000}$

8 a 460 **b** 126 **c** 2345.88
 d 3.9 **e** 79 000 **f** 678.4
 g 100 **h** 102.01

9 a 319.2 **b** 31.92 **c** 319.2
 d 31 920 **e** 319.2 **f** 319 200

10 a 16.5 **b** 0.6 **c** 48.92
 d 7.32 **e** 9.2 **f** 670
 g 1.3325 **h** 0.001 **i** 5.137

11 a 381.2 **b** 12.2 **c** 0.6875 **d** 16.3
12 a 25 **b** 31.8 **c** 230 **d** 25.4
13 a 0.8 **b** 0.375 **c** $0.\dot{5}$ **d** $6.\dot{6}$
14 a 1.23 km **b** $20.00
 c $4.60 **d** 0.15 h (or 1.5 h)
 e 140.0 kg **f** 45 000 people (or 4500)
 g $508 500.00 (or $50 850) **h** $11.30
 i 134.9c per L

15 $340.20

16 $8.25

17 38.8c per brick

Chapter 5

Exercise 5.1

1 a Twenty-three past three (or three twenty-three) in the afternoon
 b A quarter to two (or one forty five) in the afternoon
 c Twelve past nine (or nine twelve) in the evening
 d Twenty-four past eight (or eight twenty-four) in the morning
 e Twenty-three minutes to midday (or eleven thirty-seven) in the morning
 f A quarter past one (or one fifteen) in the morning
 g A quarter past six (or six fifteen) in the morning
 h Twenty to one (or twelve forty) in the morning
 i A quarter to one (or twelve forty-five) in the afternoon

2 a 12:40 p.m. **b** 4:16 a.m. **c** 3:45 p.m.
 d 4:40 a.m. **e** 12:26 a.m. **f** 12:15 p.m.

3 a 1240 h **b** 0416 h **c** 1545 h
 d 0440 h **e** 0026 h **f** 1215 h

4 a 1300 hours **b** 0100 hours **c** 1030 hours
 d 2310 hours **e** 1717 hours **f** 2008 hours
 g 1200 hours **h** 2350 hours **i** 0909 hours

5 a 6:00 a.m. **b** 6:00 p.m. **c** 3:30 a.m.
 d 1:40 p.m. **e** 9:45 a.m. **f** 9:50 p.m.
 g 12:20 p.m. **h** 11:40 p.m. **i** 12:05 a.m.

6 a 2:40 p.m. **b** 3:20 a.m. **c** 4:55 p.m.
 d 11:31 p.m. **e** 1:08 a.m. **f** 10:18 a.m.
 g 8:00 p.m. **h** 6:43 a.m.

7 a 0908 hours **b** 2150 hours **c** 0042 hours
 d 0739 hours **e** 0159 hours **f** 2218 hours
 g 1046 hours **h** 1743 hours

8 Start at 2135 h, finish at 0101 h.

Exercise 5.2

1 a 200 years
 b
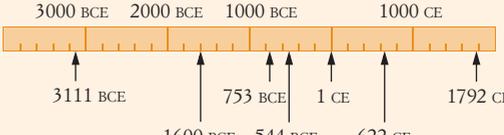
2 a 1 unit = 20 years

b A 1851, B 1982, C 1956, D 1901, E 1813, F 1974, G 1788, H 1932

3 a Teacher to check.   **b** 1 unit = 4 years
  c Sir Ninian Stephen or William Hayden (7 years)
  d Viscount Dunrossil

4 Teacher to check.

5 Teacher to check.

6 Teacher to check.

Exercise 5.3

1 a seconds   **b** hours
  c minutes or seconds   **d** days or months
  e hours   **f** minutes or hours
  g days or weeks

2 a 4 hours   **b** 12 hours   **c** 6 hours
  d 8 hours   **e** 3 hours   **f** 3 hours

3 a 18 minutes   **b** 9 minutes
  c 4 minutes   **d** 4 hours 20 minutes
  e 12 minutes   **f** 1 hour 29 minutes

4 a 360 minutes   **b** 900 seconds
  c 63 days   **d** 130 weeks
  e 72 hours   **f** 730 days
  g 336 hours   **h** 255 minutes
  i 204 hours   **j** 630 seconds
  k 720 years   **l** 42 days

5 a 8 minutes   **b** 10 weeks
  c 4 days   **d** 3 hours 20 minutes

e 9 years   **f** 9 minutes 20 seconds
g $2\frac{1}{2}$ days   **h** 18 weeks
i 5 minutes 30 seconds   **j** 12 fortnights
k 2 hours 15 minutes   **l** $4\frac{7}{10}$ (or 4.7) centuries
m 6 hours 45 minutes   **n** 2 hours 47 minutes

6 a 3600 seconds   **b** 86 400 seconds
  c 31 536 000 seconds

7 Yes

Exercise 5.4

1 a 8:00 p.m.   **b** 7:00 p.m.   **c** 7:43 p.m.
  d 1:17 p.m.   **e** 2:08 a.m.   **f** 1:32 p.m.
  g 2:14 a.m.   **h** 4:15 p.m.   **i** 6:17 a.m.
  j 3:25 p.m.

2 A

3 a 1 h 5 min   **b** 1 h 50 min
  c 5 h 44 min   **d** 3 h 53 min
  e 5 h 45 min   **f** 11 h 38 min
  g 2 h 25 min   **h** 43 min
  i 1 h 47 min   **j** 4 h 8 min
  k 7 h 39 min   **l** 13 h 25 min

4 a 6 h 47 min   **b** 11 h 52 min
  c 12 h 30 min   **d** 8 h 9 min
  e 12 h 29 min   **f** 12 h 7 min

5 a 3 h 29 min   **b** 7 h 31 min
  c 6 h 9 min   **d** 1 h 45 min
  e 4 h 51 min   **f** 1 h 43 min

Exercise 5.4

10d

Depart			
Explorer depot	11:30	1:30	2:30
City cathedral	11:38	1:38	2:38
Railway station	11:45	1:45	2:45
Parliament	11:54	1:54	2:54
Museum	12:05	2:05	3:05
City square	12:15	2:15	3:15
Zoo	12:30	2:30	3:30
Dockland shops	12:42	2:42	3:42
Arts centre	12:49	2:49	3:49
Water gardens	1:00	3:00	4:00
Hall of fame	1:08	3:08	4:08
Arrive			
Explorer depot	1:20	3:20	4:20

Answers

6

Place	Time
1st Joe	12:43 p.m.
2nd Anna	1:14 p.m.
3rd Ken	1:19 p.m.
4th Pathena	1:21 p.m.
5th Mike	1:31 p.m.
6th Gail	2:02 p.m.

7 a 2 h 35 min **b** 20 min **c** TH 038
d TH 114 may have a stop between Brisbane and Melbourne.

8 a 1 h 45 min **b** 1 h 35 min **c** 47 min
d 45 min **e** 22 h 45min

9 a 5:06 p.m. **b** 3 min **c** 33 min **d** 0 min
10 a 5 buses; Bus 1 can do the 10:00 a.m. and 12:00 noon trips.
b At Zoo at 12:45 p.m., first bus after 11:42 a.m. at station is 12:00 noon.
c Teacher to check.
d See table on previous page.

Chapter review

1 a Six thirty-four in the evening or thirty-four past six in the evening
b Nine thirty in the evening or half past nine in the evening
c Ten sixteen in the morning or sixteen past ten in the morning

2 9:20 a.m.

3 a 240 s **b** 210 min

4 a 1930 h **b** 0335 h

5 a 4:00 a.m. **b** 7:52 p.m.

6 Teacher to check

7 a 10 h **b** 3 h **c** 4 h

8 a 3 min **b** 11 min **c** 3 h 24 min

9 a 8 weeks **b** 240 min **c** 16 min
d 250 weeks **e** 168 h **f** $12\frac{1}{2}$ h

10 a 3:42 a.m. **b** 7:51 a.m.
c 6:20 p.m. **d** 5:43 p.m.
e 10:34 p.m. **f** 6:55 p.m.

11 a 4 h 19 min **b** 7 h 34 min
c 6 h 47 min **d** 6 h 35 min
e 19 min **f** 23 h 59 min

12 a 10 h 5 min **b** 1 h 50 min
c 6 h 57 min **d** 1 h 29 min

13 a Twice a day **b** 5 h 25 min **c** 6 h
d GX411 **e** 5 min

14

4 a.m.　5 a.m.　6 a.m.　7 a.m.　8 a.m.　9 a.m.　10 a.m.　11 a.m.

Arrive — Start cooking — Serve food — Clean up — Finish washing up — Pack van and leave — Arrive at depot
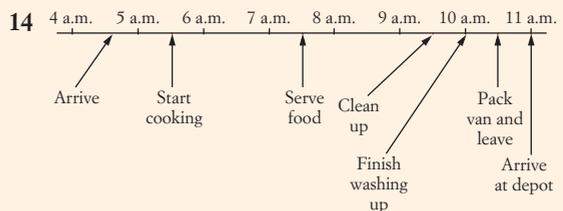
15 9:50 p.m.

16 56 h 8 min

Chapter 6
Exercise 6.1

1 a

Number of tables (t)	1	2	3	4	5	6
Number of guests (g)	5	8	11	14	17	20

b 2 more than 3 times the number of tables.
c $g = 3 \times \square + 2$
d 20 people
e 12 tables

2 a The term number plus 5; 20
b 4 times the term number; 84
c 5 more than twice the term number; 23
d 21 minus the term number 10
e 9 more than three times the term number; 45
f 14 minus three times the term number; ⁻16

3 a Five times the term number
b 2 less than the term number
c 2 more than 3 times the term number
d 1 more than double the term number
e 3 less than double the term number
f 1 more than four times the term number

4 a $3 \times \square$ **b** 24

5 a $5 \times \square - 3$ **b** 27

6 a $4 \times \square + 3$ **b** 43

Answers

7 a $3n + 14$; 44   **b** $2n - 1$; 15   **c** $4n - 3$; 45
   d $20 - 2n$; -20 **e** $4n - 7$; 49   **f** $7 - 3n$; $^-20$

8 a $h = 3 \times f - 2$ **b** $p = 5 \times m - 3$
   c $b = 3 \times m + 3$ **d** $k = 2 \times h + 2$
   e $s = 3 \times r + 1$ **f** $b = 2 \times a - 2$

9 53

10 16

11 After 29 days

12 $6.80

Exercise 6.2

1 There are many short cuts possible. Only one is shown for each part.
   a $20 \times 500 \times 154 = 1\,540\,000$
   b $80 \times 12.5 \times 136 = 136\,000$
   c $75 \times 40 \times 234 = 7\,02\,000$
   d $13 + 7 + 43 + 37 + 73 + 68 + 90 = 331$
   e $88 + 52 + 24 + 46 + 25 + 64 + 59 = 358$
   f $3 + 27 + 74 + 86 + 50 + 92 + 22 = 354$
   g $79 + 61 + 32 + 38 + 22 + 28 + 20 + 67 = 347$
   h $83 + 77 + 16 + 84 + 17 + 34 + 88 + 20 = 419$
   i $72 + 68 + 45 + 45 + 40 + 11 + 67 + 35 = 383$

2 a $14\,300 + 143 = 14\,443$
   b $15\,200 - 152 = 15\,048$
   c $32\,700 + 654 = 33\,354$
   d $68\,000 - 68 = 67\,932$
   e $10\,500 + 70 = 10\,570$
   f $24\,700 \div 2 - 247 = 12\,103$
   g $560 - 28 = 532$
   h $8440 + 422 = 8862$
   i $100\,200 - 1002 = 99\,198$

3 a $3a + 6$ **b** $2h + 4$ **c** $2m + 6$
   d $4x + 24$ **e** $4x - 8$ **f** $3m - 21$
   g $8k - 24$ **h** $5y - 25$

4 a $8m + 12$ **b** $7a + 7b$ **c** $24p + 60$
   d $5a + 10$ **e** $18x + 24$ **f** $24m + 12n$
   g $40p + 20q$ **h** $6a + 12b$

5 Use the distributive law in reverse to add the hours first, so $12 \times (3 + 5 + 4 + 3) = \$180$.

6 a $^-3x - 6$ **b** $^-7p - 7$ **c** $^-2m + 6$
   d $^-5k + 20$ **e** $^-24 + 4y$ **f** $^-9a - 36$
   g $^-k - 3$ **h** $^-m + 2$ **i** $^-6 + 2x$
   j $^-12m - 15$ **k** $^-15y + 30$ **l** $^-4 + 7x$

7 a $(100 + 3) \times 49 = 4900 + 147 = 5047$
   b $(1000 - 1) \times 104 = 104\,000 - 104 = 103\,896$
   c $205 \times (50 + 1) = 10\,250 + 205 = 10\,455$
   d $(1000 + 2) \times 458 = 458\,000 + 916 = 458\,916$
   e $(10 + 1) \times 734 = 7340 + 734 = 8074$
   f $(100 + 10 + 1) \times 75 = 7500 + 750 + 75 = 8325$

8 Show your steps to your neighbour.
   a $2m - 2p$ **b** $21f - 14g$ **c** $12m - 20$
   d $15m - 15$ **e** $18x - 24$ **f** $3 - 3k$
   g $12 - 18p$ **h** $20 - 20m$

9 Use $11x = 10x + x$. Add the number to ten times itself.
For example, $11 \times 86 = 10 \times 86 + 86 = 860 + 86 = 946$.

10 Use $99x = 100x - x$. Subtract the number from 100 times itself. For example, $99 \times 86 = 100 \times 86 - 86 = 8600 - 86 = 8514$.

Exercise 6.3

1 a 5   **b** $^-7$ **c** 6   **d** 2
   e $^-12$ **f** 1   **g** $^-1$ **h** $^-5$
   i 16   **j** 1   **k** $^-892$ **l** 412
   m 6   **n** $^-1$ **o** 7   **p** 1
   q $\frac{2}{3}$ **r** $\frac{3}{4}$

2 a 2   **b** 3   **c** 1   **d** 3
   e 2   **f** 3   **g** 1   **h** 2
   i 3   **j** 3   **k** 4   **l** 2
   m 1   **n** 2   **o** 2   **p** 1
   q 3   **r** 3

3 a $3x$ and $16x$ **b** $2mw$ and $5mw$
   c $2w$ and $23w$ **d** $2p$ and $5p$
   e $3x^2$ and x^2 **f** $4mn$, $2nm$ and mn
   g x^2y and $4x^2y$ **h** 7, 9 and 2

4 a $7m$ **b** $11k$ **c** $7ab$ **d** $7mn$
   e $2xy$ **f** $9abc$

5 a $2m$ **b** $5d$ **c** $5mk$ **d** $33abc$
   e $32fg$ **f** $19mn$

6 a $4m$ **b** $3a$ **c** $15x$
   d $9f$ **e** $9s^2$ **f** $7mn$
   g $2pq$ **h** $9abc$ **i** 0

7 a $8x + 10$ **b** $8m + 4$ **c** $7mn + 10f$
   d $4k + 8$ **e** $40xy + 6ab$ **f** $15 - r$

8 a $9m$ **b** $4k + 2$ **c** $16mx$
   d $5d + 9$ **e** $6f + 10g$ **f** $12y + 5x$
   g $10p$ **h** $4 + 5q$ **i** $3abc$

Answers

9 a $10y - 5x$ b $5w - 4u$
c $x - x^2y + 4xy^2$ d $2q - 14p$
e $3ab - 2bc - 4abc$ f $4d^3 + 5d + 2d^2$

10 a $6m^2$ b $55k^2$ c $108f^2$
d $72gh$ e $120mn$ f $84a^2$
g $84x^2$ h $30m^2$ i $4x^2$
j $3y$ k $3p$ l $\dfrac{n}{2m}$
m ^-4pq n $63e^2f^4$ o $-\dfrac{1}{3}$
p $\dfrac{-7t}{3s^2}$ q $120m^6$ r $^-4adgh^3$
s $\dfrac{1}{5tm^2}$ t $\dfrac{5g}{3m}$

11 a $2N$ or $2 \times N$ b $N \div 2$ or $\dfrac{N}{2}$ or $\frac{1}{2}N$
c $3N$ or $3 \times N$ d $N \div 4$ or $\dfrac{N}{4}$ or $\frac{1}{4}N$
e $N \div 10$ or $\dfrac{N}{10}$ or $\frac{1}{10}N$ f $N + 1$
g $5N$ or $5 \times N$ h $N + 21$
i $N - 10$ or $10 - N$ j $N + 2$
k $N + 3$ l N^2
m $\sqrt{N}$

12 a $A + B$ b $A + B + C$ c $B - C$
d AC e ABC f $A \div B$ or $\dfrac{A}{B}$
g $(A + B) \div C$ or $\dfrac{A + B}{C}$ h $C \div B$ or $\dfrac{C}{B}$

13 a 16   b 18   c 2   d 24
e 96   f 3   g 8   h $\frac{1}{2}$ or 0.5

14 a $3 + A$ or $A + 3$ b $B - 3$ c $C + 5$
d $D + 8$ or $8 + D$ e $E - 3$ f $X - F$
g $A + B + W$ h $m \div m$ or $\dfrac{m}{m}$ i R^2
j $(A + B) \div 2$ or $\dfrac{A + B}{2}$

15 a $B + G$ b $2 \times N$ or $2N$ c $T - X$
d $2N$ e $M \div 3$ or $\dfrac{M}{3}$ f $A + 2B$

16 No, it makes no difference at all.

17 C

18 a $m = 1000 M$ b $d = \dfrac{h}{24}$ or $d = h \div 24$
c $c = l + 15$

19 a $m = 1400$ mm   b $d = 3$ days   c $c = 915$ cm

20 a $c = \dfrac{5(f - 32)}{9}$ b $10°C$ c $35°C$

Exercise 6.4

1 a 6 kg   b 10 kg
2 a 11 kg   b 23 kg
3 a 14 kg   b 24 kg
4 a 23 kg   b 36 kg
5 a 14 kg   b 16 kg
6 a 12 kg   b 20 kg
7 a 4 kg   b 15 kg
8 a 27 kg   b 21 kg
9 a 14 kg   b 13 kg
10 a 6   b 11   c 13   d 4
e 5   f 3   g 3   h 5
i 2   j 14   k 7   l 5
m 6   n 5   o 27   p 4
11 a Yes   b Yes   c No   d No
e Yes   f No   g Yes   h Yes
i No   j No

12 a $\boxed{h} \xrightarrow{\times 3} \square \xrightarrow{+4} \boxed{19}$
b $\boxed{y} \xrightarrow{\times 5} \square \xrightarrow{-7} \boxed{13}$
c $\boxed{u} \xrightarrow{\times 7} \square \xrightarrow{+6} \boxed{27}$
d $\boxed{j} \xrightarrow{\times 6} \square \xrightarrow{-13} \boxed{5}$
e $\boxed{k} \xrightarrow{+6} \square \xrightarrow{\div 5} \boxed{4}$
f $\boxed{p} \xrightarrow{-8} \square \xrightarrow{\div 3} \boxed{11}$

13 a $h = 5$ b $y = 4$ c $u = 3$
d $j = 3$ e $k = 14$ f $p = 41$

14 a $\boxed{h} \overset{\times 3}{\underset{\div 3}{\leftrightarrows}} \square \overset{-8}{\underset{+8}{\leftrightarrows}} \boxed{1}$
b $\boxed{S} \overset{\times 5}{\underset{\div 5}{\leftrightarrows}} \square \overset{+12}{\underset{-12}{\leftrightarrows}} \boxed{22}$
c $\boxed{b} \overset{\div 2}{\underset{\times 2}{\leftrightarrows}} \square \overset{-9}{\underset{+9}{\leftrightarrows}} \boxed{5}$
d $\boxed{k} \overset{\div 4}{\underset{\times 4}{\leftrightarrows}} \boxed{d} \overset{+11}{\underset{-11}{\leftrightarrows}} \boxed{15}$
e $\boxed{M} \overset{\times 4}{\underset{\div 4}{\leftrightarrows}} \square \overset{-16}{\underset{+16}{\leftrightarrows}} \square \overset{\div 5}{\underset{\times 5}{\leftrightarrows}} \boxed{4}$
f $\boxed{f} \overset{\div 3}{\underset{\times 3}{\leftrightarrows}} \square \overset{+7}{\underset{-7}{\leftrightarrows}} \boxed{d} \overset{\times 2}{\underset{\div 2}{\leftrightarrows}} \boxed{22}$

15 a $h = 3$ b $S = 2$ c $b = 28$
d $k = 16$ e $M = 9$ f $f = 12$

16 a $d = 8$ **b** $k = 10$ **c** $r = 21$
   d $m = 5$ **e** $x = 17$ **f** $g = 17$
   g $n = 7$ **h** $e = 10$ **i** $f = 12$

17 a *Walk out* the door   **b** *Put down* a book
   c *Subtract* 3   **d** *Add* 2
   e Look *down*   **f** *Divide* by 7
   g *Multiply* by 3   **h** Get *up*
   i *Land* on the airstrip   **j** *Take off* a hat

18 a $7 - 7 = 0$ **b** $4 - 4 = 0$ **c** $8 \div 8 = 1$
   d $\frac{1}{4} \times 4 = 1$ **e** $12 - 12 = 0$ **f** $11 \times \frac{1}{11} = 1$
   g $2 \div 2 = 1$ **h** $a - a = 0$ **i** $d - d = 0$
   j $m \times \dfrac{1}{m} = 1$

19 a Add 6   **b** Divide by 8   **c** Subtract 5
   d Multiply by 3   **e** Divide by 10   **f** Subtract 7
   g Multiply by 4   **h** Add 12   **i** Subtract a
   j Divide by c

20 a $d + 3 - 3 = d$ **b** $d \times 4 \div 4 = d$
   c $d - 4 + 4 = d$ **d** $3d \div 3 = d$
   e $4 + d - 4 = d$ **f** $d \div 7 \times 7 = d$
   g $\dfrac{d}{13} \times 13 = d$ **h** $d \times \frac{1}{2} \times 2 = d$
   i $d + a - a = d$ **j** $\dfrac{d}{m} \times m = d$

21 a $x = 6$ **b** $y = 11$ **c** $w = 4$
   d $m = 36$ **e** $d = 8$ **f** $q = 72$

22 a $a + 5 = 14$
$$a + 5 - 5 = 14 - 5$$
$$a = 9$$
   b $6b = 18$
$$\frac{6b}{6} = \frac{18}{6}$$
$$b = 3$$
   c $d - 12 = 21$
$$d - 12 + 12 = 21 + 12$$
$$d = 33$$
   d $\dfrac{f}{7} = 13$
$$\frac{f}{7} \times 7 = 13 \times 7$$
$$f = 91$$

23 a $p = 7$ **b** $x = 18$ **c** $z = 24$
   d $s = 1$ **e** $d = 11$ **f** $f = 4$
   g $y = 20$ **h** $b = 48$ **i** $p = 7$
   j $g = 2$ **k** $a = 34$ **l** $h = 84$
   m $i = 9$ **n** $f = 8$ **o** $u = 64$

24 a $3h + 4 = 22$
$$3h + 4 - 4 = 22 - 4$$
$$\frac{3h}{3} = \frac{18}{3}$$
$$h = 6$$
   b $5m + 3 = 28$
$$5m + 3 - 3 = 28 - 3$$
$$\frac{5m}{5} = \frac{25}{5}$$
$$m = 5$$
   c $2p - 4 = 12$
$$2p - 4 + 4 = 12 + 4$$
$$\frac{2p}{2} = \frac{16}{2}$$
$$p = 8$$

25 a $\dfrac{c + 4}{5} = 3$
$$\frac{c + 4}{5} \times 5 = 3 \times 5$$
$$c + 4 = 15$$
$$c + 4 - 4 = 15 - 4$$
$$c = 11$$
   b $\dfrac{k + 2}{7} = 5$
$$\frac{k + 2}{7} \times 7 = 5 \times 7$$
$$k + 2 = 35$$
$$k + 2 - 2 = 35 - 2$$
$$k = 33$$
   c $\dfrac{r - 5}{3} \times 3 = 8 \times 3$
$$r - 5 = 24$$
$$r - 5 + 5 = 24 + 5$$
$$r = 29$$

26 a $a = 3$ **b** $s = 4$ **c** $y = 3$
   d $x = 6$ **e** $e = 6$ **f** $p = 10$
   g $m = 2$ **h** $w = 2$ **i** $y = 15$
   j $d = 4$ **k** $x = 5$ **l** $m = 6$
   m $n = 6$ **n** $b = 7$ **o** $c = 8$

27 a $a = 2$ **b** $b = 0$ **c** $c = 13$
   d $f = 16$ **e** $g = 3$ **f** $h = 3$
   g $k = 1$ **h** $j = 4.5$ **i** $m = 10$

Answers

28 a 12 **b** 4 **c** 100 **d** 7
e 26 **f** 1 **g** 52

29 a $6 + \boxed{} = 11$ or $6 + x = 11$

b $\boxed{} - 7 = 8$ or $x - 7 = 8$

c $\boxed{} \times 3 = 12$ or $x \times 3 = 12$

d $\boxed{} \times \frac{1}{2} = 9$ or $x \times \frac{1}{2} = 9$

e $\boxed{} \times 2 = 16$ or $x \times 2 = 16$

f $(3 \times \boxed{}) + 1 = 10$ or $(3 \times x) + 1 = 10$

g $(\boxed{} \times 4) - 5 = 19$ or $(x \times 4) - 5 = 19$

30 a 5 **b** 15 **c** 4 **d** 18
e 8 **f** 3 **g** 6

31 a $x + 8 = 6$; $^-2$ **b** $7x = 42$; 6

c $n - 10 = 3$; 13 **d** $\frac{n}{5} = 35$, 175

e $2x + 7 = 29$; 11

32 a $3(x + 5) = 36$; 7 **b** $7x - 5 = 51$; 8

c $\frac{m + 8}{9} = 4$; 28 **d** $4(2m + 3) = 96$; $10\frac{1}{2}$

e $\frac{y}{3} + 20 = 2$; $^-54$

33 a A, $x = 8$ **b** B, $x = 6$ **c** C, $x = ^-4$
d A, $x = 14$ **e** C, $x = 6$ **f** A, $x = ^-4$

Chapter review

1 a 2 less than 5 times the term number; 58
b 3 more than four times the term number; 83

2 a 1 more than 4 times the term number **b** 2 less than 3 times the term number

3 a $6\square - 3$ **b** 63

4 $^-3$

5 4

6 $6n - 2$; 88

7 a 12 300 **b** 90

8 3

9 $3p$ and ^-2p; $3q$ and $6q$

10 a $12e$ **b** $7k + 4m - 1$

11 a 16 **b** 5 **c** 9 **d** 14
12 a Yes **b** No

13 a $t = 13$ **b** $u = 9$

14 a $\boxed{p} \xrightarrow{\times 4} \boxed{} \xrightarrow{+ 8} \boxed{27}$

b $\boxed{k} \xrightarrow{- 11} \boxed{} \xrightarrow{\div 2} \boxed{7}$

15 a $\boxed{y} \underset{\div 4}{\overset{\times 4}{\longleftarrow}} \boxed{} \underset{+ 12}{\overset{- 12}{\longleftarrow}} \boxed{20}$

b $\boxed{f} \underset{\times 6}{\overset{\div 6}{\longleftarrow}} \boxed{} \underset{- 11}{\overset{+ 11}{\longleftarrow}} \boxed{17}$

16 a $\boxed{h = 6} \underset{\div 2}{\overset{\times 2}{\longleftarrow}} \boxed{12} \underset{- 5}{\overset{+ 5}{\longleftarrow}} \boxed{17}$

b $\boxed{m = 8} \underset{\div 4}{\overset{\times 4}{\longleftarrow}} \boxed{32} \underset{+ 14}{\overset{- 14}{\longleftarrow}} \boxed{18}$

c $\boxed{k = 20} \underset{\times 5}{\overset{\div 5}{\longleftarrow}} \boxed{4} \underset{- 3}{\overset{+ 3}{\longleftarrow}} \boxed{7}$

d $\boxed{r = 7} \underset{\div 3}{\overset{\times 3}{\longleftarrow}} \boxed{21} \underset{+ 5}{\overset{- 5}{\longleftarrow}} \boxed{16} \underset{\times 2}{\overset{\div 2}{\longleftarrow}} \boxed{8}$

17 $47 - 3n$; $^-13$

18 a $h = 7f - 5$ **b** $p = 103 - 4m$

19 a $5d + 40$ **b** $12h - 18p$

20 a $8e^2 - 7e$ **b** $6x - y - 9z + 5$

21 a ^-15xy **b** $4qr^2$ **c** $\frac{3j}{4v}$

22 a $g = 4$ **b** $p = 6$

23 a $m = 3$ **b** $w = 7$

24 32

25 46.2 L; 10 days (with 3 L left)

26 $17\,100 - 171 = 16\,929$

27 $34 \times (180 + 60 + 120) \div 10 = \$1224$ (Distributive law in reverse)

28 a $9 - g$ **b** $40 - 7f$ **c** $m + w$

29 a $n + 7 = 20$; $n = 13$ **b** $3x - 8 = 7$; $x = 5$

30 $t = 60m + s$, 201 seconds

31 \$167

32 $5 - 6x$

33 Use $201x = 200x + x$ to get 'double the number, put two zeros on the end and then add the number again'.

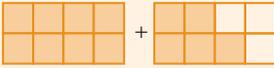**34 a** $\frac{m}{5} = ^-3$; $m = ^-15$ **b** $3x = 29 - 5$; $x = 8$
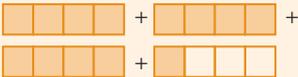
Chapter 7
Exercise 7.1

1

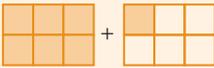| Proper fractions	Improper fractions	Mixed numerals
$\frac{3}{100}$	$\frac{100}{45}$	$1\frac{1}{2}$
$\frac{2}{3}$	$\frac{3}{2}$	$3\frac{2}{5}$
$\frac{150}{250}$	$\frac{87}{86}$	$20\frac{1}{4}$
$\frac{86}{87}$	$\frac{25}{1}$	$4\frac{1}{20}$
$\frac{3}{5}$	$\frac{1000}{25}$	
$\frac{3}{12}$	$\frac{301}{300}$	

2 a $2\frac{1}{3}$ **b** $3\frac{4}{6}$ or $3\frac{2}{3}$ **c** $1\frac{3}{4}$
d $2\frac{2}{4}$ or $2\frac{1}{2}$ **e** $3\frac{6}{12}$ or $3\frac{1}{2}$

3 a [diagram]
b [diagram]
c [diagram]
d [diagram]
e [diagram]
f [diagram]
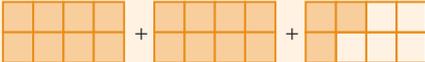
4 a $\frac{3}{2}$ **b** $\frac{13}{8}$ **c** $\frac{9}{7}$ **d** $\frac{10}{3}$
e $\frac{9}{2}$ **f** $\frac{37}{10}$ **g** $\frac{37}{6}$ **h** $\frac{23}{12}$
i $\frac{38}{5}$ **j** $\frac{142}{15}$ **k** $\frac{126}{11}$ **l** $\frac{55}{7}$

5 a $4\frac{1}{3}$ **b** $3\frac{1}{4}$ **c** $1\frac{3}{10}$ **d** $4\frac{1}{5}$
e $1\frac{3}{35}$ **f** $4\frac{3}{8}$ **g** $6\frac{5}{6}$ **h** $7\frac{1}{5}$
i $5\frac{3}{7}$ **j** $10\frac{10}{11}$ **k** $3\frac{15}{19}$ **l** $10\frac{1}{10}$

6 a 2   **b** 10   **c** 16   **d** 8
e 4   **f** 6   **g** 5   **h** 5
i 4   **j** 8   **k** 6   **l** 5

7 a 15   **b** 42   **c** 12   **d** 30
e 40   **f** 20   **g** 10   **h** 8
i 18   **j** 21   **k** 12   **l** 60

8 Teacher to check.

9 a [diagram]
b [diagram]
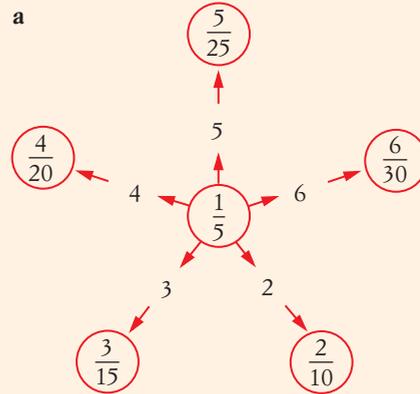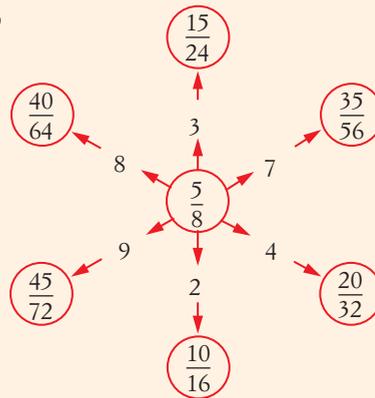
10 a $\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \frac{7}{28}$
b $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{25}{30} = \frac{30}{36} = \frac{60}{72} = \frac{100}{120}$
c $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{20}{30} = \frac{200}{300}$
d $\frac{7}{10} = \frac{14}{20} = \frac{21}{30} = \frac{35}{50} = \frac{56}{80} = \frac{70}{100} = \frac{105}{150}$
e $\frac{7}{8} = \frac{14}{16} = \frac{21}{24} = \frac{35}{40} = \frac{63}{72} = \frac{77}{88} = \frac{350}{400}$
f $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{15}{20} = \frac{24}{32} = \frac{36}{48} = \frac{75}{100}$

11 a 21   **b** 8   **c** 35
d 12   **e** 6   **f** 2
g 4   **h** 2   **i** 3
j 56   **k** 28   **l** 7
m 54   **n** 4   **o** 13

12 Teacher to check.

13 a $\frac{1}{2} = \frac{10}{20}$ **b** $\frac{1}{3} = \frac{4}{12}$ **c** $\frac{1}{4} \neq \frac{4}{12}$
d $\frac{3}{4} \neq \frac{5}{7}$ **e** $\frac{14}{21} = \frac{2}{3}$ **f** $\frac{6}{12} \neq \frac{7}{13}$
g $\frac{5}{9} \neq \frac{4}{7}$ **h** $\frac{3}{12} \neq \frac{9}{35}$

Answers

14 a $\frac{3}{8}$ **b** $\frac{1}{4}$ **c** $\frac{4}{5}$ **d** $\frac{3}{4}$
e $\frac{7}{13}$ **f** $\frac{3}{7}$ **g** $\frac{4}{5}$ **h** $\frac{1}{4}$
i $\frac{2}{3}$ **j** $\frac{4}{15}$

15 a $1\frac{1}{3}$ **b** $1\frac{1}{3}$ **c** 2 **d** $1\frac{2}{3}$
e $2\frac{1}{2}$ **f** $2\frac{1}{3}$ **g** $1\frac{1}{4}$ **h** $1\frac{1}{2}$
i $1\frac{6}{7}$ **j** $2\frac{1}{10}$

16 a $1\frac{1}{2}$ **b** $3\frac{1}{6}$ **c** $4\frac{1}{4}$
d $6\frac{3}{4}$ **e** $100\frac{2}{3}$ **f** $17\frac{7}{20}$
g $100\frac{11}{16}$ **h** $1\frac{27}{50}$ **i** $5\frac{1}{3}$
j $15\frac{4}{5}$

17 $\frac{1}{12}$

18 $\frac{1}{7}$

19 a $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{1}{20}$ **d** $\frac{1}{10}$

20 Sharon ate $\frac{2}{3}$ and Ben $\frac{1}{3}$.

21 a $\frac{1}{3}$ **b** $\frac{5}{6}$ **c** $\frac{2}{5}$ **d** $\frac{7}{10}$

Exercise 7.2

1 a $\frac{3}{10}$ **b** $\frac{2}{3}$ **c** $\frac{3}{6}$ **d** $\frac{3}{4}$
e $\frac{3}{5}$ **f** $\frac{3}{5}$

2 a $\frac{1}{2} > \frac{1}{3}$ **b** $\frac{2}{3} < \frac{3}{4}$ **c** $\frac{3}{8} < \frac{1}{2}$
d $\frac{7}{8} > \frac{3}{4}$ **e** $\frac{1}{4} < \frac{1}{3}$ **f** $\frac{3}{5} < \frac{7}{10}$
g $\frac{5}{6} > \frac{1}{2}$ **h** $\frac{1}{6} < \frac{1}{4}$ **i** $\frac{5}{12} > \frac{1}{3}$
j $\frac{17}{100} < \frac{2}{5}$ **k** $\frac{7}{10} < \frac{4}{5}$ **l** $\frac{17}{20} > \frac{6}{10}$
m $\frac{11}{15} > \frac{2}{3}$ **n** $\frac{1}{3} < \frac{2}{5}$ **o** $\frac{63}{100} < \frac{4}{5}$
p $\frac{2}{5} < \frac{3}{7}$ **q** $3\frac{3}{4} > 3\frac{5}{8}$ **r** $2\frac{3}{7} < 5\frac{2}{5}$
s $1\frac{5}{7} > 1\frac{2}{5}$ **t** $2\frac{1}{3} < 2\frac{2}{5}$

3 a $\frac{1}{10}, \frac{1}{7}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ **b** $\frac{2}{7}, \frac{3}{7}, \frac{3}{5}, \frac{5}{7}$
c $\frac{1}{8}, \frac{1}{2}, \frac{8}{9}$ **d** $\frac{2}{9}, \frac{3}{10}, \frac{3}{8}, \frac{2}{5}, \frac{4}{7}, \frac{3}{5}$
e $\frac{2}{7}, 0.5, \frac{4}{5}, 0.9$ **f** $\frac{1}{2}, \frac{3}{5}, \frac{2}{3}, 0.7$

4 a $\frac{6}{7}, \frac{6}{8}, \frac{6}{9}$ **b** $\frac{5}{4}, \frac{5}{6}, \frac{5}{9}$
c $1, \frac{4}{5}, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}$ **d** $3, \frac{12}{5}, 1\frac{1}{5}, \frac{40}{50}, \frac{3}{4}$

5 a
b
c

d
e

6
a
b
c
d
e
f
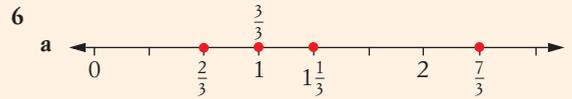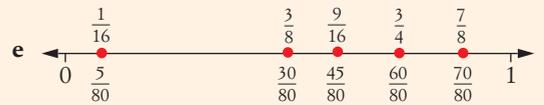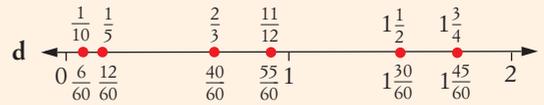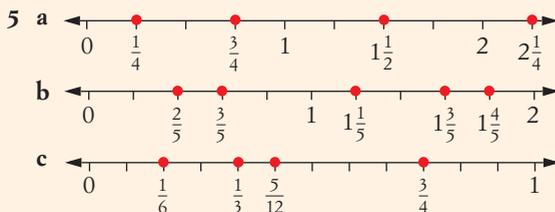
7 Ali saves the greater fraction.

8 Jenny's plane is faster.

9 a Those with denominators of 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 or 60.
b Those with denominators of 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36, 40, 45, 48, 50, 60, 72, 75, 80, 90, 100, 120, 144, 150, 180, 200, 225, 240, 300, 360, 400, 450, 600, 720, 900, 1200, 1800 and 3600

10 Cake B has more removed.

11 factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

12 6, 16 or 26—there would not be more in a kindergarten class.

Exercise 7.3

1 a 1 **b** $\frac{6}{10} = \frac{3}{5}$ **c** $\frac{3}{3} = 1$
d $\frac{4}{3} = 1\frac{1}{3}$ **e** $\frac{2}{5}$ **f** $\frac{4}{4} = 1$
g $\frac{3}{10}$ **h** $\frac{4}{12} = \frac{1}{3}$ **i** $\frac{8}{6} = 1\frac{1}{3}$
j 0 **k** $\frac{21}{6} = 3\frac{1}{2}$ **l** $\frac{9}{8} = 1\frac{1}{8}$

2 a $\frac{5}{6}$ b $\frac{13}{15}$ c $\frac{1}{4}$ d $\frac{1}{18}$
  e $1\frac{5}{24}$ f $\frac{1}{20}$ g $\frac{20}{63}$ h $\frac{23}{60}$
  i $\frac{43}{45}$ j $\frac{3}{10}$ k $\frac{26}{45}$ l $1\frac{5}{24}$
  m 0 n $1\frac{11}{20}$ o $\frac{7}{24}$ p $\frac{23}{35}$
  q $1\frac{3}{14}$ r $\frac{9}{20}$ s $1\frac{3}{20}$ t $\frac{7}{24}$
  u $\frac{13}{24}$ v $\frac{1}{24}$ w $\frac{7}{12}$ x 0
  y $\frac{19}{60}$

3 $\frac{11}{12}$

4 $\frac{1}{6}$

5 a $4\frac{1}{2}$ b $\frac{2}{3}$ c 4
  d $5\frac{2}{4} = 5\frac{1}{2}$ e $3\frac{2}{5}$ f $6\frac{3}{8}$
  g $2\frac{2}{3}$ h 6 i 10
  j $8\frac{4}{8} = 8\frac{1}{2}$ k $6\frac{5}{7}$ l $\frac{3}{5}$

6 a 3 b $4\frac{1}{4}$ c $3\frac{2}{15}$
  d $3\frac{1}{8}$ e $4\frac{1}{12}$ f $4\frac{1}{10}$
  g $2\frac{1}{2}$ h $3\frac{5}{6}$ i $2\frac{1}{8}$
  j $4\frac{3}{20}$ k $2\frac{1}{12}$ l $5\frac{10}{21}$
  m $1\frac{1}{10}$ n $1\frac{3}{8}$ o $4\frac{7}{8}$
  p $12\frac{13}{60}$

7 a $3\frac{3}{4}$ b $1\frac{1}{4}$ c $6\frac{1}{2}$
  d $5\frac{1}{4}$ e $5\frac{4}{5}$ f $5\frac{2}{5}$
  g $2\frac{3}{8}$ h $4\frac{4}{15}$ i $6\frac{1}{15}$
  j $3\frac{11}{20}$ k $4\frac{1}{4}$ l $2\frac{19}{40}$

8 a $2\frac{1}{2}$ b $1\frac{2}{5}$ c $\frac{3}{7}$
  d $3\frac{1}{3}$ e $1\frac{3}{4}$ f $2\frac{7}{12}$
  g $1\frac{7}{10}$ h $\frac{3}{4}$ i $1\frac{21}{40}$
  j $1\frac{19}{20}$ k $\frac{5}{6}$ l $1\frac{7}{10}$
  m $1\frac{1}{2}$ n $2\frac{4}{5}$ o $1\frac{11}{12}$
  p $\frac{3}{4}$

9 $\frac{8}{10} = \frac{4}{5}$

10 $\frac{5}{6}$ of an hour

11 $\frac{13}{16}$

12 $\frac{105}{200} = \frac{21}{40}$

13 a $2\frac{23}{24}$ b $1\frac{5}{18}$ c $3\frac{17}{30}$

Exercise 7.4

1 a 5 b 4 c 8 d 6
  e 5 f 18 g 3 h 18
  i 15 j 500 k 750 l 26

2 a $1 b $8 c $20
  d 20c e $0.30 f $30
  g $45 h 36c i $2.50
  j $2.25 k $200 l $780

3 a 30 minutes b 90 minutes
  c 9 hours d 18 minutes
  e 73 days f 45 seconds
  g 200 seconds h 216 seconds
  i 32 hours j 24 minutes
  k 18 hours l 336 seconds
  m 8 months n 35 minutes

4 a 250 m b 1500 m c 625 m
  d 750 m e 750 m f 3500 m
  g 3.2 cm h 3 mm i 27 mm
  j 4.5 cm k 51 mm l 15 cm
  m 45 mm n 40 km o 35 m

5   d $\frac{1}{12}$
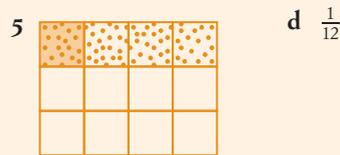
6 a $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ b $\frac{3}{5} + \frac{3}{5} = 2 \times \frac{3}{5}$
    $= 3 \times \frac{3}{4} = \frac{3}{4}$ $= \frac{6}{5} = 1\frac{1}{5}$
  c $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$ d $5 \times \frac{1}{7} = \frac{5}{7}$
    $= 5 \times \frac{3}{8} = \frac{15}{8} = 1\frac{7}{8}$
  e $7 \times \frac{3}{20} = \frac{21}{20}$ f $8 \times \frac{3}{5} = \frac{24}{5}$
    $= 1\frac{1}{20}$ $= 4\frac{4}{5}$

7 a 3 b 15 c $5\frac{3}{5}$ d $2\frac{1}{3}$
  e $13\frac{1}{3}$ f $22\frac{1}{2}$ g $4\frac{9}{10}$ h $19\frac{1}{5}$
  i $\frac{1}{4}$ j $7\frac{1}{2}$

8 a $\frac{2}{15}$ b $\frac{9}{20}$ c $\frac{35}{48}$ d $\frac{9}{80}$
  e $\frac{5}{16}$ f $\frac{9}{64}$ g $\frac{20}{63}$ h $\frac{21}{160}$
  i $\frac{4}{15}$ j $\frac{1}{5}$

9 a $\frac{1}{7}$ b $\frac{2}{7}$ c $\frac{1}{18}$ d $\frac{1}{6}$
  e $\frac{5}{16}$ f $\frac{4}{21}$ g $\frac{3}{20}$ h $\frac{1}{4}$
  i $\frac{3}{25}$ j $1\frac{3}{25}$ k $2\frac{2}{7}$ l $\frac{1}{10}$

m $\frac{11}{24}$ **n** $\frac{2}{5}$ **o** $\frac{3}{5}$ **p** $1\frac{4}{5}$

q 1 **r** $\frac{3}{10}$ **s** $6\frac{2}{5}$ **t** 15

10 a $\frac{1}{2}$ **b** $1\frac{1}{3}$ **c** 2 **d** 6

  e $2\frac{7}{9}$ **f** 2 **g** $5\frac{1}{5}$ **h** $9\frac{3}{7}$

  i 6

11 a $\frac{4}{25}$ **b** $\frac{1}{27}$ **c** $\frac{1}{4}$ **d** $\frac{4}{9}$

  e $\frac{27}{64}$ **f** $\frac{2}{3}$ **g** $\frac{5}{7}$ **h** $\frac{9}{10}$

  i $\frac{3}{4}$ **j** $\frac{3}{4}$

12 4

13 27 female, 18 male

14 2

15 270

16 7A: 18 boys, 6 girls   7B: 15 boys, 5 girls
    7C: 21 boys, 7 girls   7D: 12 boys, 4 girls

17 $\frac{3}{200\,000}$ kg $= \frac{3}{200}$ g $= 0.015$g

18 a $\frac{7}{8}$ **b** $\frac{7}{10}$ **c** $\frac{1}{3}$ **d** $\frac{3}{10}$

  e $\frac{1}{7}$ **f** $\frac{3}{7}$ **g** $\frac{4}{5}$

19 $127\frac{1}{2}$ stones

20 8 eggs, 20 cups of flour, 2 cups of sugar

21 280

Exercise 7.5

1 3

2 5

3 a 6   **b** 12   **c** 18

4 a 20   **b** 40

5 a 12 **b** 15 **c** 21 **d** 15
  e 35 **f** 40 **g** 16 **h** 4
  i 32 **j** 3 **k** 8 **l** 10

6 a $\frac{3}{8}$ **b** 4 **c** $1\frac{1}{2}$ **d** $1\frac{11}{14}$

  e $\frac{6}{7}$ **f** $\frac{24}{25}$ **g** $\frac{2}{3}$ **h** $\frac{1}{2}$

  i 2 **j** $\frac{1}{2}$ **k** $\frac{2}{5}$ **l** $2\frac{1}{2}$

  m $3\frac{3}{4}$ **n** 15 **o** 14 **p** $\frac{5}{6}$

  q $1\frac{1}{4}$ **r** $\frac{20}{21}$ **s** $\frac{5}{36}$ **t** 1

  u $1\frac{2}{3}$ **v** $1\frac{1}{6}$ **w** $\frac{27}{28}$ **x** $\frac{9}{10}$

7 a $\frac{1}{7}$ **b** $\frac{5}{49}$ **c** $\frac{1}{12}$ **d** $\frac{7}{40}$

  e $\frac{1}{4}$ **f** $\frac{2}{9}$ **g** $\frac{2}{15}$ **h** $\frac{1}{6}$

  i $\frac{1}{10}$ **j** $\frac{1}{10}$ **k** $\frac{1}{15}$ **l** $\frac{5}{12}$

8 a $3\frac{3}{4}$ **b** $2\frac{4}{5}$ **c** $\frac{7}{8}$ **d** $1\frac{1}{3}$

  e $\frac{3}{4}$ **f** $1\frac{1}{3}$ **g** 10 **h** $\frac{15}{16}$

  i $3\frac{1}{3}$ **j** $3\frac{1}{8}$ **k** $3\frac{1}{9}$ **l** $1\frac{1}{2}$

9 $17\frac{1}{3}$

10 20

11 96

12 8

13 $38\frac{2}{5}$ h or 38 h 24 min

14 Teacher to check.

15 Teacher to check.

16 7 eggs

17 a $6\frac{2}{3}$ **b** $2\frac{5}{6}$ **c** 1

  d $4\frac{5}{8}$ **e** $\frac{5}{8}$ **f** $1\frac{2}{47}$

Exercise 7.6

1 a Money : time **b** Mass : volume
  c Money : area **d** Money : mass
  e Mass : distance **f** Distance : time
  g Mass : volume **h** Volume : area
  i Distance : time

2 a 20 km/h **b** 2 kg/L **c** 20 rev/min
  d 19 words/minute **e** 4 L/s **f** 6°C/min
  g $15/person **h** $0.16/m **i** 0.075 kg/L

3 a 10 m/s **b** 50 m/s **c** 25 km/h
  d 30 km/h **e** 20 km/min **f** 100 km/h
  g 50 m/s **h** 12.5 km/h **i** 6.25 m/s

4 a 5 : 3 **b** 3 : 4 **c** 1 : 8
  d 3 : 4 **e** 2 : 3 **f** 5 : 2
  g 4 : 3 **h** 5 : 3 **i** 4 : 1

5 a 1 : 5 **b** 5 : 1 **c** 1 : 4
  d 3 : 5 **e** 3 : 20 **f** 4 : 3
  g 2 : 3 **h** 15 : 4 **i** 7 : 4

6 a 2 : 1 **b** 5 : 6 **c** 6 : 7
  d 20 : 9 **e** 72 : 35 **f** 75 : 44

7 a 20 : 3 **b** 3 : 40 **c** 3 : 25
d 20 : 3 **e** 7 : 8 **f** 1 : 15
g 10 : 3 **h** 10 : 3 **i** 20 : 1

8 a 1.5 : 1 **b** 0.8 : 1 **c** 2.4 : 1
d 1.75 : 1 **e** 2.25 : 1 **f** 3.33 : 1
g 0.27 : 1 **h** 3.65 : 1 **i** 0.75 : 1

9 a 7 **b** $4 **c** $12 : $16

10 a $10 : $5 **b** 6 L : 4 L
c 9c : 27c **d** 33 kg : 44 kg
e $25 : $10 **f** 75 mL : 125 mL
g 70 cm : 30 cm **h** 12 g : 60 g
i 36 s : 45 s

11 a $3.25/kg **b** $14/m^3
c 14 L/min **d** $300/m^2

12 a 72 L **b** 180 L

13 a $141.60 **b** $100.30 **c** $49.56

14 4 hours

15 a 50 min **b** 65 min or 1 h 5 min

16

Pair	a, e	d, f	c, d
Values	5 : 100	50 : 200	20 : 50
Simplest ratio	1 : 20	1 : 4	2 : 5

Pair	f, b	b, f	d, c
Values	200 : 10	10 : 200	50 : 20
Simplest ratio	20 : 1	1 : 20	5 : 2

Pair	f, c	e, b	b, e
Values	200 : 20	100 : 10	10 : 100
Simplest ratio	10 : 1	10 : 1	1 : 10

Pair	c, e
Values	20 : 100
Simplest ratio	1 : 5

17 a 1 : 19.17 **b** 102 : 1
c 1 : 3.7 **d** 1 : 39.42

18 15 L pineapple and 10 L lemonade

19 a $990 **b** $1650

20 a 108 g **b** 132 g

21 a 29 seconds **b** 5.1 km

22 mixture : vinegar = 5 : 1

Chapter review

1 a $\frac{7}{4}$ **b** $\frac{13}{9}$ **c** $\frac{9}{4}$ **d** $\frac{22}{7}$
e $\frac{54}{10}$ **f** $\frac{29}{8}$ **g** $\frac{38}{5}$ **h** $\frac{72}{7}$

2 a $1\frac{3}{4}$ **b** $1\frac{1}{11}$ **c** $3\frac{2}{7}$ **d** $7\frac{1}{5}$
e $5\frac{5}{8}$ **f** $8\frac{2}{3}$ **g** $1\frac{12}{24}=1\frac{1}{2}$ **h** $10\frac{1}{12}$

3 a 2 **b** 5 **c** 16

4 a 24 **b** 12 **c** 60

5 a $\frac{1}{3}=\frac{2}{6}$ **b** $\frac{3}{4}=\frac{12}{16}$ **c** $\frac{4}{5}=\frac{16}{20}$
d $\frac{5}{12}=\frac{15}{36}$ **e** $\frac{1}{2}=\frac{25}{50}$ **f** $\frac{3}{20}=\frac{15}{100}$

6 a $\frac{5}{6}$ **b** $\frac{3}{4}$ **c** $\frac{3}{4}$
d $\frac{4}{3}$ **e** $\frac{1}{2}$ **f** $\frac{4}{7}$

7 a $\frac{3}{10},\frac{2}{5},\frac{7}{10},\frac{4}{5}$ **b** $\frac{1}{3},\frac{1}{2},\frac{7}{12},\frac{11}{12}$
c $\frac{3}{8},\frac{7}{8},1\frac{1}{4},\frac{7}{4}$ **d** $\frac{3}{5},\frac{5}{8},\frac{8}{12},\frac{8}{5},2\frac{1}{4},3$

8 a 1 **b** $\frac{10}{6}=1\frac{2}{3}$ **c** $7\frac{3}{7}$
d $\frac{7}{10}$ **e** $\frac{5}{6}$ **f** $5\frac{9}{20}$
g $\frac{1}{8}$ **h** $\frac{11}{20}$ **i** $\frac{25}{30}=\frac{5}{6}$

9 a $9\frac{2}{3}$ **b** $4\frac{1}{3}$ **c** $\frac{5}{6}$
d $4\frac{3}{10}$ **e** $1\frac{1}{3}$ **f** $3\frac{2}{3}$

10 a A 9, B 8, R 12, O 7, T 20, H $3\frac{1}{2}$
b $3\frac{1}{2}$, 7, 8, 9, 12, 20; H O B A R T

11 a 18 **b** 4
c 5 **d** 12
e 6 **f** 8
g 16 hours **h** 1600 m
i $2\frac{1}{4}$ hours or 135 mins **j** 105 mm
k 50 seconds **l** 125 cm

12 a $\frac{2}{15}$ **b** 6 **c** $\frac{5}{14}$ **d** $\frac{1}{21}$
e $\frac{3}{4}$ **f** $\frac{1}{2}$ **g** $2\frac{1}{4}$ **h** 8
i 2 **j** $3\frac{3}{8}$ **k** $3\frac{3}{4}$ **l** $\frac{4}{5}$

13 15

14 a 50 **b** $\frac{1}{6}$ **c** $1\frac{1}{2}$ **d** 2
e 6 **f** $\frac{32}{35}$ **g** 3 **h** 2
i $1\frac{1}{2}$

15 a 2 : 3 **b** 10 : 7 **c** 14 : 1 **d** 5 : 2
e 10 : 29 **f** 2 : 3 **g** 15 : 4 **h** 7 : 4

16 350 mL : 280 mL

Answers

17 a $\frac{1}{21}$ **b** $5\frac{37}{45}$ **c** $1\frac{73}{84}$ **d** $8\frac{2}{5}$
  e $11\frac{2}{5}$ **f** $5\frac{1}{4}$

18 a $\frac{19}{60}$ **b** $9\frac{193}{204}$ **c** $2\frac{71}{105}$ **d** $3\frac{11}{36}$
  e $9\frac{79}{144}$ **f** $\frac{101}{114}$

19 a 300 **b** $\frac{100}{300} = \frac{1}{3}$ **c** $\frac{17}{300}$
  d classical **e** male vocal, rock groups

20 $\frac{8}{26} = \frac{4}{13}$

21 $\frac{4}{9}$

22 $6\frac{7}{12}$ litres

23 1 spotted

24 $\frac{1}{2}$

25 1.2 L of insecticide and 8.8 L of water

26 a 0.96 cm **b** 10.5 m

27 $\frac{1}{6}$

28 Because multiplying by the reciprocal of a number undoes multiplication by the number, so is the same as dividing by a number.

29 Because division is the same as multiplication by the reciprocal, and it doesn't matter in what order you do multiplications, the answer is the same.

30 a $^-$0.24 **b** 27

Chapter 8

Exercise 8.1

1 a 200 cm **b** 1500 cm
  c 50 cm **d** 350 cm
  e 25 cm **f** 125 cm
  g 1548 cm **h** 1260 cm
  i 300 000 cm **j** 2 500 000 cm
  k 1 250 000 cm **l** 2 025 000 cm
  m 42 cm **n** 6.5 cm
  o 0.7 cm **p** 12.5 cm

2 a 30 mm **b** 8000 mm
  c 400 mm **d** 25 000 mm
  e 115 mm **f** 4200 mm
  g 32.5 mm **h** 15 750 mm
  i 5 000 000 mm **j** 12 000 000 mm
  k 8 400 000 mm **l** 32 750 000 mm
  m 780 mm **n** 2000 mm
  o 1250 mm **p** 700 mm

3 a 2 m **b** 3 m **c** 8.5 m
  d 9.8 m **e** 12.5 m **f** 2.75 m
  g 0.8 m **h** 0.25 m **i** 0.75 m
  j 0.325 m **k** 6000 m **l** 18 000 m
  m 23 600 m **n** 48 250 m **o** 13.8 m
  p 800 m

4 a 2 km **b** 1.385 km **c** 1.4 km
  d 2.9387 km **e** 0.375 km **f** 0.098 km
  g 0.235 km **h** 0.0648 km **i** 0.5 km
  j 2.7 km **k** 3.2 km **l** 6.07 km

5 a 1200 cm **b** 2800 cm **c** 20 m
  d 24 km **e** 1.5 km **f** 0.65 km
  g 2.3 km **h** 6 m **i** 250 cm
  j 0.2 km

6 a 200 cm, 2500 mm, 3 m
  b 240 cm, 3200 mm, 4 m
  c 700 mm, 0.8 m, 900 cm
  d 0.045 km, 4800 cm, 450 m
  e 60 cm, 650 mm, 0.69 m
  f 1500 cm, 150 m, 1.5 km
  g 0.18 m, 1750 mm, 180 cm
  h 0.011 m, 120 cm, 1.3 m

7 D

8 a kilo
  b centi
  c milli

9 a 1 : 200 **b** 1 : 250 000 **c** 1 : 100 000
  d 1 : 10 000 **e** 1 : 20 000 **f** 1 : 5000

10 a lookout **b** gate **c** Visitor Centre **d** toilets

11 a 1550 m **b** 2000 m **c** 1650 m
  d 1075 m **e** 400 m **f** 950 m

12 a 210 m **b** 450 m **c** 520 m
  d 465 m **e** 180 m **f** 620 m

13 Teacher to check.

14 Teacher to check.

15 a Time, minutes and seconds
  b Temperature, °C
  c Distance, kilometres
  d Mass or volume, kilograms or cubic centimetres
  e Time, hours and minutes
  f Distance, kilometres
  g Time, hours and minutes
  h Speed, kilometres per hour
  i Capacity, millilitres
  j Area, square metres

16 a Number of fish, number or mass, kilograms or length, centimetres
   b Speed, kilometres per hour
   c Height, metres or centimetres
   d Temperature, °C
   e Capacity, number of people

17 Teacher to check.

18 a pencil 120 mm, screen 330 mm, door 800 mm, ruler 35 mm, room 7400 mm
   b pencil 0.12 m, screen 0.33 m, door 0.8 m, ruler 0.035 m, room 7.4 m
   c Teacher to check.

19 Teacher to check.

20 Teacher to check.

21 a *CD*   **b** *EF*   **c** *AB*   **d** *IJ*

22 Teacher to check.

23 Teacher to check.

24 Teacher to check.

25 Approximately 3.7 km

26 Teacher to check.

Exercise 8.2

1 a 46 mm   **b** 44 cm   **c** 118 m
   d 39 cm   **e** 246 cm   **f** 2000 mm
   g 31.8 km   **h** 26 cm   **i** 208 mm
   j 60 mm   **k** 48 m

2 a 87 mm   **b** 171 mm
   c 120 mm   **d** 112 mm

3 a 44 m   **b** 30 cm   **c** 48 mm
   d 40 m   **e** 36 km   **f** 54 mm

4 a 32 m   **b** 36 mm   **c** 25.4 cm
   d 42.2 cm   **e** 6.56 m   **f** 14.8 km

5 a 34 cm   **b** 14.4 m   **c** 60 cm
   d 4.8 m   **e** 38.4 cm   **f** 6.2 km

6 a 62.8 cm   **b** 94.2 mm   **c** 5.65 m   **d** 110.6 cm

7 a 94.2 cm   **b** 47.1 m   **c** 30.2 km
   d 251 cm   **e** 44 mm   **f** 3.93 m

8 a 4 cm   **b** 8 cm   **c** 14 cm
   d 18 cm   **e** 9.5 cm   **f** 7.5 cm

9 Any two numbers that add up to 15

10 54 cm

11 $13 475

12 77.4 m

13 80 cm

Exercise 8.3

1 a Some possible shapes
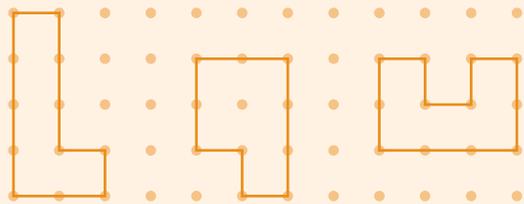
   b Some possible shapes

2 C

3 Some possible shapes

   a

   b

4 a 10 cm^2 **b** 8 cm^2 **c** 10 cm^2
   d 12.5 cm^2 **e** 4 cm^2 **f** 4 cm^2
   g 6.5 cm^2 **h** 28 cm^2 **i** 15 cm^2

5 a 1500 mm^2 **b** 1.5 m^2
   c 6.9 cm^2 **d** 65 000 cm^2
   e 500 000 mm^2 **f** 1.22 m^2
   g 12.5 cm^2 **h** 790 mm^2
   i 750 000 mm^2 **j** 86.5 m^2
   k 0.69 m^2 **l** 4700 cm^2

Answers

6 a 245 ha   **b** 3445.2 ha
  c 1 275 020 ha   **d** 0.15 ha
  e 0.0854 ha   **f** 0.2 ha

7 a 2.45 km^2 **b** 34.452 km^2
  c 12 750.2 km^2

8 a 5 cm^2 **b** 8 cm^2 **c** 9 cm^2
  d 6 cm^2 **e** 9 cm^2 **f** 5 cm^2
  g 10 cm^2 **h** 10 cm^2

9 a 102 400 mm^2 **b** 0.1024 m^2

10 250 000 cm^2, 25 000 000 mm^2

11 108 cm^2, 0.0108 m^2

12 a 6.25 cm^2, 625 mm^2
  b 11 cm^2, 1100 mm^2
  c 10.5 cm^2, 1050 mm^2
  d 22.5 cm^2, 2250 mm^2

13 3 002 800 ha

14 1348.05 km^2

15 About 6.2.

16 About 186 times.

17 About 32 'Italys'.

18 About $\frac{1}{3}$

19 $\frac{1}{80}$

20 C

Exercise 8.4

1 a 55.8 m^2 **b** 58.6 m^2 **c** 96 cm^2
  d 324 m^2 **e** 1.44 cm^2 **f** 4 cm^2
  g 96 mm^2 **h** 225 m^2 **i** 154 mm^2
  j 1890 cm^2 **k** 448 m^2

2 a 15 cm^2 **b** 16 cm^2 **c** 36 mm^2
  d 18 cm^2 **e** 10 cm^2 **f** 48.59 m^2
  g 8.82 m^2 **h** 9.01 mm^2 **i** 7.68 cm^2
  j 45.72 cm^2 **k** 21.76 m^2 **l** 1750 mm^2

3 a 30.68 m^2 **b** 105 cm^2
  c 8.96 cm^2 or 896 mm^2 **d** 120 m^2
  e 34.44 cm^2 **f** 1.032 m^2 or 10 320 cm^2

4 a 153.9 cm^2 **b** 67.9 m^2 **c** 430.1 cm^2

5 a 17 500 mm^2 **b** 15.6 m^2 **c** 300 mm^2
  d 48 cm^2 **e** 7.8 m^2 **f** 50 cm^2

---

6 D

7 C

8 a 152 cm^2 **b** 372 cm^2 **c** 60 cm^2
  d 11 cm^2

9 a 11 250 cm^2 **b** 500 cm^2 **c** 1300 cm^2

10 a i 12 km^2 **ii** 1200 ha
  b i 40 000 m^2 **ii** 4 ha
  c i 64 km^2 **ii** 6400 ha
  d i 7200 m^2 **ii** 0.72 ha
  e i 9600 m^2 **ii** 0.96 ha
  f i 16 km^2 **ii** 1600 ha

11 422.5 m^2

12 a i 20 m^2 **ii** 20 m^2 **iii** 12.8 m^2
  b 80 tiles   **c** 3.6 L

13 42.8 m^2

14 60 m^2

15 D

16 a 10 m   **b** 20 m   **c** 25 m
  d 5 m   **e** 8 m   **f** 2 m

17 a 10 000 m   **b** 6250 m   **c** 4000 m
  d 2000 m   **e** 12 500 m   **f** 500 m

Chapter review

1 a mm   **b** km (but for some it will be m)
  c cm   **d** m

2 Teacher to check.

3 a 300 cm   **b** 8000 mm   **c** 3 000 000 m
  d 6.5 km   **e** 75 cm   **f** 25 mm
  g 7.8 m   **h** 5.2 m

4 B

5 a 92 mm, 9.2 cm   **b** 91 mm, 9.1 cm
  c 81 mm, 8.1 cm

6 a 52 m   **b** 36 mm   **c** 78.5 mm
  d 100.5 mm   **e** 64 cm   **f** 88 mm

7 a 45 m   **b** 122 m   **c** 45.2 m
  d 38 m   **e** 70 m

8 a 6 cm^2 **b** 9 cm^2 **c** 7 cm^2

9 a 91 000 cm^2 **b** 250 mm^2
c 40 000 m^2 **d** 1 000 000 m^2
e 3.8 m^2 **f** 8 600 000 mm^2
g 2.4 cm^2 **h** 17.5 m^2

10 a 16 m^2 **b** 24 cm^2 **c** 360 mm^2
d 112 mm^2 **e** 5.2 cm^2 **f** 187 km^2
g 162 m^2 **h** 177 km^2 **i** 15.2 m^2

11 a 9 cm^2 **b** 84 m^2

12 28 cm

13 8.5 sq units

14 a 3910 km^2 **b** About 113 **c** 6 800 000 ha

15 6 L

16 a Teacher to check. **b** Teacher to check.
c 15cm × 15 cm

17 a Teacher to check. **b** 6 cm × 6 cm

Chapter 9

Exercise 9.1

1 Impossible, almost impossible, (very unlikely/highly improbable), (unlikely/improbable), (even/fifty-fifty), (probable/likely), (very likely/highly probable), almost certain, certain

2 a Unlikely **b** Even
c Likely **d** Impossible

3 a Unlikely **b** Very unlikely
c Likely **d** Certain

4 a Impossible **b** Even
c Likely **d** Unlikely

5 a Unlikely **b** Unlikely
c Fifty-fifty **d** Certain

6 a Unlikely **b** Likely **c** Certain
d Impossible **e** Likely **f** Unlikely

7 a {C, C, C, C, C, A, A, A, A, A, A, R, R, R, W, W}
b Comedy **c** Western
d Action **e** Equally likely

8 a Equally likely **b** Number **c** Ace
d Above 5 **e** Below 6 **f** Red
g Equally likely

9 a Almost certain **b** Impossible
c Certain **d** Very likely
e Very unlikely (there are free tourist buses in some cities)

10 a Odd **b** Equally likely
c 4 **d** Even

11 a {HH, HT, TH, TT} **b** Equally likely
c 1 head **d** Even
e Unlikely

12 a {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
b Fifty-fifty **c** Unlikely
d Certain **e** Unlikely

13 Examples: Students will write with blue biros, on a weekday a student will be in school, A card from a normal pack will be a number card, A student's name will have an 'e' in it, . . .

14 Blue is likely, green is unlikely and red is very unlikely

15 Lee (17) is least likely to win while Jo (18) and Ahmed (18) are equally likely to win.

16 Assuming boys and girls are equally likely, the sample space is {GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB}, so at least 2 girls (4) has an even chance but exactly 2 girls (3) is unlikely.

Exercise 9.2

1 a 30 **b**

Result	Frequency
H	17
T	13
Total	30

c Likely **d** Heads are more likely
e About 0.567

2 a 20 **b**

Result	Frequency
HH	3
HT	7
TH	3
TT	7
Total	20

c Fifty-fifty **d** Two tails is more likely
e 0.15 **f** 0.35

Answers

3 a 24

b

Result	Frequency
1	3
2	5
3	6
4	3
5	3
6	4
Total	24

c 3   **d** Equally likely

e About 0.167   **f** 0.25

g $6/23 \approx 0.261$

4 a 0.4   **b** About 0.267

c No, it could be a fluke   **d** About 15

5 a 0.1   **b** 0.1   **c** 0.125   **d** 1 or 2 (1.5)

6 1: 8, 2: 11, 3: 11, 4: 13, 5: 16, 6: 21

7 About 17

8 a 18

b

Result	Frequency
10	2
11	5
12	3
13	2
14	3
15	2
16	1
Total	18

c 11   **d** 14 is more likely

e 0.111   **f** 0.167

g About 8   **h** 13

9 a 25   **b** Nokia

c Apple: 0.08, LG: 0.12, Nokia: 0.4, Samsung: 0.32, Sony: 0.08

d 20, taking account of those without mobiles

10 About 0.233

11 About 15

12 24

Exercise 9.3

1 a Equal   **b** Unequal   **c** Equal

d Unequal   **e** Unequal

2 a $\frac{1}{3}$, About 0.333 or 33.3%

b 0

c $\frac{1}{2}$, 0.5 or 50%

d $\frac{1}{6}$, about 0.167 or 16.7%

3 a $\frac{1}{6}$, about 0.167 or 16.7%   **b** $\frac{1}{5}$, 0.2 or 20%

c $\frac{1}{3}$, About 0.333 or 33.3%   **d** $\frac{1}{2}$, 0.5 or 50%

e $\frac{3}{8}$, 0.375 or 37.5%

4 a $\frac{1}{6}$, about 0.167 or 16.7%   **b** $\frac{1}{5}$, 0.2 or 20%

c 0   **d** 0   **e** $\frac{1}{8}$, 0.125 or 12.5%

5 a 0   **b** $\frac{1}{5}$, 0.2 or 20%

c $\frac{1}{3}$, About 0.333 or 33.3%   **d** $\frac{1}{4}$, 0.25 or 25%

e $\frac{1}{8}$, 0.125 or 12.5%

6 a $\frac{2}{5}$ **b** $\frac{3}{10}$

c $\frac{1}{5}$ **d** $\frac{3}{5}$

e 0

7 a 0.25   **b** 0.125   **c** About 0.167

d 0.2   **e** 0

8 a 0.3   **b** 0.1   **c** 0.4

d 0.2   **e** 1   **f** 0

9 About 0.167 $(\frac{1}{6})$

10 About 0.0769 $(\frac{1}{13})$

11 a About 0.417   **b** About 0.0833

c About 0.0833   **d** About 0.167

e 0.75   **f** 0.25

12 22

13 a 25   **b** 19   **c** 8

d 10   **e** 17

14 0.706

15 48.7%

16 1 in 50, 0.02, 2% or $\frac{1}{50}$

17 35

18 0.3

19 0.1875, assuming it is random

20 0.48

21 0.5

22 win $\frac{2}{9}$; lose $\frac{7}{36}$

23 $\frac{15}{51}$ or about 0.2941

24 a Yes, if one of the other aces turns up.

b $\frac{3}{20}$ or 0.15

c $\frac{12}{25}$ or 0.48

d $\frac{1}{221}$ or about 0.004 525

25 a $\frac{1}{2}$ or 0.5   **b** 0.25   **c** $\frac{1}{64}$

26 2

27 0.3125

28 About 0.46

29 $\frac{2}{9}$ or about 22.2%

30 $\frac{4}{25}$ or about 0.16

Chapter review

1 a Fifty-fifty   **b** Impossible
   c Unlikely   **d** Likely

2 a Unlikely   **b** Likely
   c Unlikely   **d** Impossible

3 a 26   **b**

Time	Frequency
8:00	3
8:30	4
9:00	7
9:30	2
10:00	6
10:30	3
11:00	1
Total	26

   c 9 p.m. is more likely   **d** About 0.154
   e About 6

4 a I and III   **b** I: $\frac{1}{2}$, II: $\frac{1}{4}$, III: $\frac{1}{4}$, IV: $\frac{3}{8}$
   c 0, 0.25, 0.125   **d** About 6

5 a {7♠, 8♠, 9♠, 10♠, J♠, Q♠, K♠, A♠, 7♥, 8♥, 9♥, 10♥, J♥, Q♥, K♥, A♥, 7♦, 8♦, 9♦, 10♦, J♦, Q♦, K♦, A♦, 7♣, 8♣, 9♣, 10♣, J♣, Q♣, K♣, A♣}
   b Number
   c Higher

6 a {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
   b More than 7   **c** Bigger than 5

7 a 0.2
   b 0.3
   c No, they may not have come up
   d About 8

8 About 0.333, about 0.267, 0.2 and 0.2 respectively.

9 a {SS, SH, SD, SC, HS, HH, HD, HC, DS, DH, DD, DC, CS, CH, CD, CC}
   b Different suits   **c** 0.4375

10 97%

11 Example

12 Examples: A student is likely to be living with at least one parent, A coin has an even chance of landing on heads, A student is unlikely to get over 80% for an English essay.

13 a 9   **b**

Colour	Frequency
Black	3
Blue	4
Red	2
Total	9

   c Blue   **d** About 0.222
   e About 13

14 200, as 1 in 25 is the same as 8 in 200.

15 In the city, because more people get on at city stops.

16 a Impossible
   b About even (knowing nothing about them)
   c Likely
   d Even
   e Certain

17 3

18 About 0.167

19 Unlikely because just less than half will be yellow.

20 About 2

21 For 27 students in 10 weeks, with a chance of 1 in 15 per student per week, about 18.

22 $\frac{1}{5}$, 0.2, 20%

Chapter 10
Exercise 10.1

1 $1.89

2 $4.65

3 $7.50

4 $18.30

Answers

5 $9.55

6 $5.60

7 $7.35

8 $3.80

9 The bill is $36.10 ($36.11) and the change is $13.90.

10 The bill is $41.55 ($41.57) and the change is $8.45.

11 The bill is $23.10 ($23.12) and the change is $6.90.

12 $5, $1, 50c, 20c, 10c, 5c; or 3 × $2, 4 × 20c, 5c or other combinations

13 $10, $2, 50c, 20c; or 2 × $5, 2 × $1, 3 × 20c, 10c or other combinations

14 $73.22

15 $247.48

16 $124.32

17 $88.35

Exercise 10.2

1 a 75% **b** 50% **c** 37.5%
  d 34% **e** 80% **f** 40%
  g 125% **h** 25% **i** 148%

2 a 25% **b** 37% **c** 24%
  d 78% **e** 23% **f** 3%
  g 20% **h** 70% **i** 6%

3 a $\frac{9}{20}$ **b** $\frac{2}{25}$ **c** $\frac{4}{25}$
  d $\frac{1}{5}$ **e** $\frac{12}{25}$ **f** $\frac{19}{25}$
  g $\frac{3}{25}$ **h** $\frac{3}{4}$ **i** $\frac{3}{10}$

4 a 0.15 **b** 0.28 **c** 0.16
  d 0.07 **e** 0.4 **f** 0.21
  g 0.34 **h** 0.85 **i** 0.98

5 a 20 kg **b** 6 L **c** $32
  d 6 g **e** $30 **f** 16.4 kg
  g 50 mL **h** 2 km **i** 6 kg

6 a 300 kg **b** 6 g **c** $35.10
  d 81.6 mL **e** 42.9 km **f** $62.28
  g 41.4 L **h** 28 g **i** 16.32 m

7 a 62.2% **b** 93.3% **c** 98.3%
  d 37.5% **e** 62.5% **f** 58.3%
  g 183.3% **h** 232.5% **i** 51.4%

8 a 5% **b** 9.6% **c** 24%
  d 124% **e** 385% **f** 160%
  g 60% **h** 0.4% **i** 200%

9 a 75% **b** 40% **c** 52%
  d 63.0% **e** 44.6% **f** 42.9%
  g 91.7% **h** 84.4% **i** 13.5%

10 a $2\frac{9}{20}$ **b** $1\frac{63}{100}$ **c** $1\frac{9}{20}$
  d $\frac{1}{125}$ **e** $1\frac{2}{5}$ **f** $3\frac{3}{5}$

11 a 0.046 **b** 0.138 **c** 1.26
  d 0.425 **e** 0.184 **f** 1.86
  g 0.0942 **h** 1.09 **i** 0.536

12 a $135 **b** 21 kg **c** 11.2 L
  d 192 g **e** 48 km **f** $18
  g 72 mL **h** $360 **i** 517.5 m

13 a $15.96 **b** 60 g **c** $0.81
  d 4.76 km **e** 10.8 L **f** 65.5 km
  g $408 **h** 0.3724 L **i** 27.14 m
  j $5.92 **k** $7.57 **l** $702

14 a 60.42 mL **b** 595.2 kg **c** $110.42
  d 1.2474 mg **e** 208.56 m **f** $24.20
  g 428.06 km **h** 4.62 mL **i** $33

15 50% lead and 25% tin

16 a 125 **b** 36% **c** 64% **d** 100%

17 a 19.1%, 3.2%, 3.5%, 18.6%
  b No, it is not urban.

18 a 26% **b** 14% **c** 60%

19 Ciara did best with 67.1% compared to Lucy's 64.4%.

20 25.9 tonnes

21 $10 850 **22** 1.89 L

Exercise 10.3

1 a $50.00 **b** $170.00 **c** $35.00
  d $8.80 **e** $31.20 **f** $10.20
  g $2.75 **h** $31.75 **i** $11.95
  j $2.25

2 a $7,425.00 **b** $4,950.00 **c** $7,920.00
  d $8,910.00 **e** $6,930.00 **f** $9,405.00
  g $8,415.00

3 a $13.00 **b** $49.00 **c** $47.00
  d $7.00 **e** $28.00 **f** $208.00

g $18.00 **h** $132.50 **i** $17.92
j $0.33

4 a $110.00 **b** $1,320.00 **c** $176.00
d $19.80 **e** $380.60 **f** $128.70
g $131.45 **h** $569.25 **i** $20.35
j $101.75

5 a $112.50 **b** $675.00 **c** $1,845.00
d $204.00 **e** $1,116.25 **f** $627.25

6 a 450 g **b** 750 g **c** 100
d 956 g **e** Twin pack

7 a $9.00 **b** $15.20 **c** $3.65
d $42.13 **e** $45.05 **f** $48.19

8 $263.12

9 $244.04

10 20%

11 45%

12 560 g

13 1.5 L

14 $\frac{1}{8}$

15 a $96.00 **b** $76.80
c The overall discount is less than 45% because the further 20% discount is applied to the lesser amount after the 25% discount has already been applied. Overall discount is 40%.

16 a $34.20
b The overall discount is less than 40% because the further 20% discount is applied to the lesser amount after the 20% discount has already been applied. Overall discount is 36%.

17 The overall discount is less than 60% because the second 30% discount is applied to the lesser amount after the first 30% discount has already been applied. Overall discount is $0.7 \times 0.7 = 0.49$ or 49%.

18 a $30 **b** $\frac{1}{9}$

19 $36.00

20 $340.00

21 Lloyd calculated 25% of $420 instead of the original price of the van which must have been $560. His discount was $140.

22 9

23 The prices are $8.40/kg for the 250g, $7.80/kg for the 500g and $4.80/kg for the 1.5 kg. He should get the 500g, even though it is not the cheapest, because 1.5 kg would be far too much and 250g is not enough (even though it would be cheaper for 1 tray).

24 The prices are $15/L, $14.50/L, $14/L and $12/L respectively. If she is only doing 1 coat she should buy a 2 L tin because she'll need more than 1 L. If she's doing 2 coats she could buy a 2 L and a 1 L tin for $43, but it will be very tight, so she'd be safer with a 4 L tin for $52.

Chapter review

1 $9.88

2 $7.30

3 $14.30

4 a 65% **b** 80% **c** 46% **d** 80%

5 a $\frac{11}{20}$ **b** $\frac{3}{10}$ **c** $\frac{3}{50}$

6 a 0.68 **b** 0.05 **c** 0.4

7 a $15 **b** 19 kg

8 a 54 L **b** $21

9 $11

10 a $288.00 **b** $342.00

11 a 43.75% **b** 287.5% **c** 145%
d 6.7% **e** 70% **f** 46.7%

12 a $2\frac{1}{20}$ **b** $\frac{3}{500}$

13 a 0.0545 **b** 1.25

14 a 350 km **b** 63 kg **c** $150

15 a 262.4 mm **b** $35.81

16 a $592.93 **b** 28.665 g

17 $38.25

18 2 L

19 $80.75

20 $78.40

21 a $14 **b** $154

22 Total bill is $26.82 \approx $26.80 and the change is $23.20.

23 $5, $2, 50c, 2 × 20c; or 3 × $2, $1, 4 × 20c, 10c; or other combinations

24 $230.72

25 67.9%

26 She did better in Maths (80%) than English (70%).

27 13.215 kg

28 30%

29 680 g

30 28%

31 $20

Chapter 11

Exercise 11.1

1 d **i** cube   **ii** cylinder
   iii square pyramid   **iv** triangular prism
   v L-shaped prism   **vi** rectangular prism
   vii rectangular pyramid   **viii** cone
   ix T-shaped prism   **x** triangular pyramid
e **i** convex   **ii** convex
   iii convex   **iv** convex
   v non-convex   **vi** convex
   vii convex   **viii** convex
   ix non-convex   **x** convex

2 Teacher to check.

3 Teacher to check.

4 a

b

c
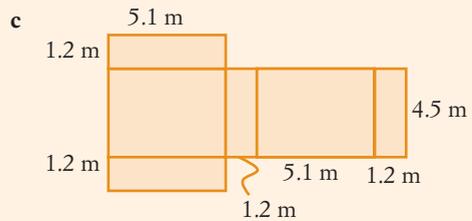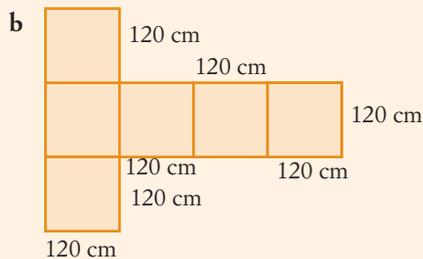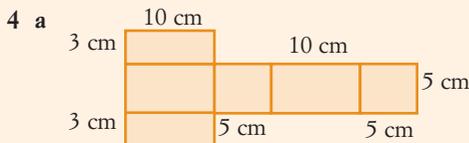
5 a convex   **b** non-convex
c non-convex   **d** convex
e convex   **f** non-convex
g convex   **h** non-convex
i convex

6 a

b

c
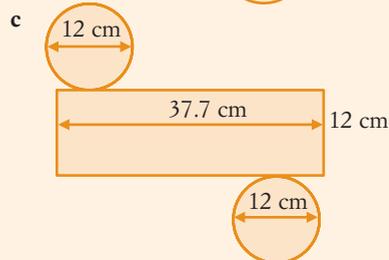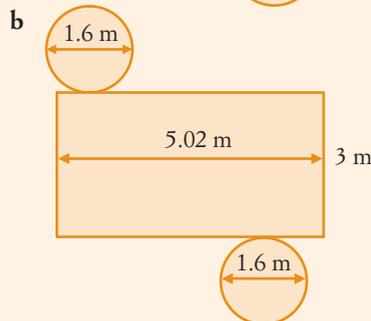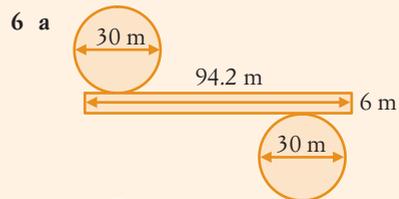
7 a rectangular prism   **b** trapezoidal prism
c cube   **d** triangular prism
e square pyramid   **f** cylinder
g triangular prism   **h** rectangular prism

8 A

9 a 214 cm^2 **b** 16.56 m^2

10 135 cm^2

Exercise 11.2

1 triangular pyramid

2 A, B, D

3 a i a cube and a square pyramid
   ii polyhedron
   iii convex
 b i a rectangular prism and a cylinder
   ii It is not a polyhedron.
   iii not convex
 c i a cylinder and a cone
   ii It is not a polyhedron.
   iii convex
 d i a cone and a hemisphere
   ii It is not a polyhedron.
   iii convex
 e i two triangular pyramids
   ii It is a polyhedron.
   iii convex
 f i two rectangular prisms
   ii It is a polyhedron.
   iii not convex

4 Teacher to check.

5 See table below.

6 a 4   **b** 8   **c** 6   **d** 5

7 a pentahedron   **b** hexahedron
 c heptahedron   **d** pentahedron
 e nonahedron   **f** octahedron

8 D

9 Other answers are possible. (Teacher to check.)
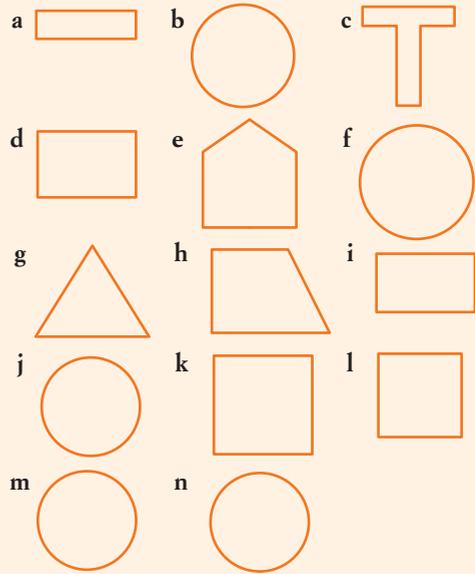
10 a a, c, d, e, g, h, i, k, l
  b a, c, e, g, h, i, l
  c d, k
  d b, f, j, m, n

11 B

12 Teacher to check.

13 Teacher to check.

14 Teacher to check.

15 a i octagon   **ii** octagonal prism
  b i T-shape or octagon
   ii T-prism or octagonal prism
  c i pentagon   **ii** pentagonal prism

Exercise 11.2

5

Solid	No. of faces	Shapes of faces	No. of identical faces	No. of parallel flat faces
Cube	6	square	6	6
Cylinder	3	circle, rectangle	2	2
Square pyramid	5	square, triangle	4	0
Triangular prism	5	triangle, rectangle	2	2
Rectangular prism	6	rectangle	3 lots of 2	6
Rectangular pyramid	5	triangle, rectangle	2 lots of 2	0
Cone	2	circle, sector	0	0
Triangular pyramid	4	triangle	0	0

16 a triangular pyramid
  b hexagonal pyramid
  c trapezoidal pyramid
  d square pyramid
  e rectangular pyramid

17 a triangular pyramid **b** cone
  c sphere **d** cylinder

18 a B, C, D, E, F, H, I, J, K, L, M, N, O, P, R, S, U, V, W, X, Y, Z
  b A, B, E, I, M, O, S, Z
  c C, F, K, P, R, U, V

19 B

20 b 4 **c** triangular pyramid
  d equilateral triangle **e** 4 **f** 6
  g i No **ii** Yes **iii** Yes

21 b 8 **c** equilateral triangle
  d 2 square pyramids **e** 12
  f i Yes **ii** Yes **iii** Yes

22 b 20 **c** equilateral triangle
  d 30 **e** 12
  f i Yes **ii** Yes

23 b 6 **c** square **d** cube

24 b 12 **c** regular pentagon
  d 30 **e** 20
  f i Yes **ii** Yes

25 D

26 See table below.

27 a dodecahedron **b** tetrahedron
  c octahedron **d** icosahedron
  e hexahedron **f** dodecahedron

28 D

29 a 20 cans **b** about 79%

30 A tetrapak is a box in the shape of a triangular pyramid and has four faces. Tetris uses shapes made up of four squares.

31 4

32 Because they all have six faces.

33 a Rectangles
  b Triangles

34 The base, a circle, is not a polygon.

Exercise 11.3

1 Teacher to check.

2 Teacher to check.

3 Teacher to check.

4 Teacher to check.

5 Teacher to check.

6 **a** **b**

  c **d** **e**
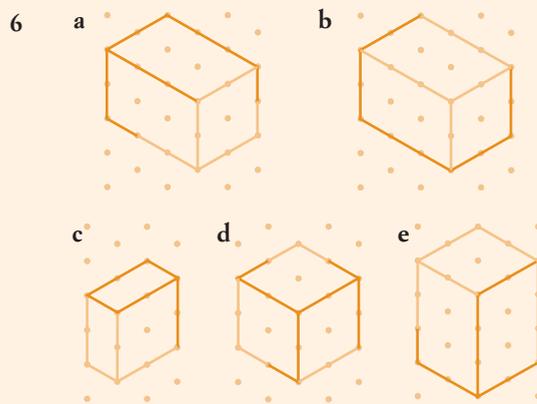
Exercise 11.2

26

Platonic solid	Description	Shape of each face	No. of faces
Octahedron	two square pyramids joined at bases	equilateral triangle	8
Dodecahedron	like a soccer ball pattern	regular pentagon	12
Icosahedron	like a crystal	equilateral	20
Tetrahedron	triangular pyramid	equilateral triangle	4
Hexahedron	cube	square	6

7 a i **ii** **iii**

b i **ii** **iii**

c i **ii** **iii**

d i **ii** **iii**

e i **ii** **iii**

f i **ii** **iii**

8

9 A **B** **C**
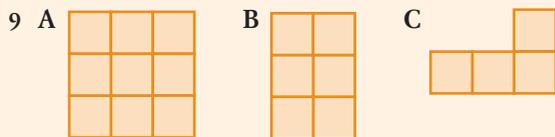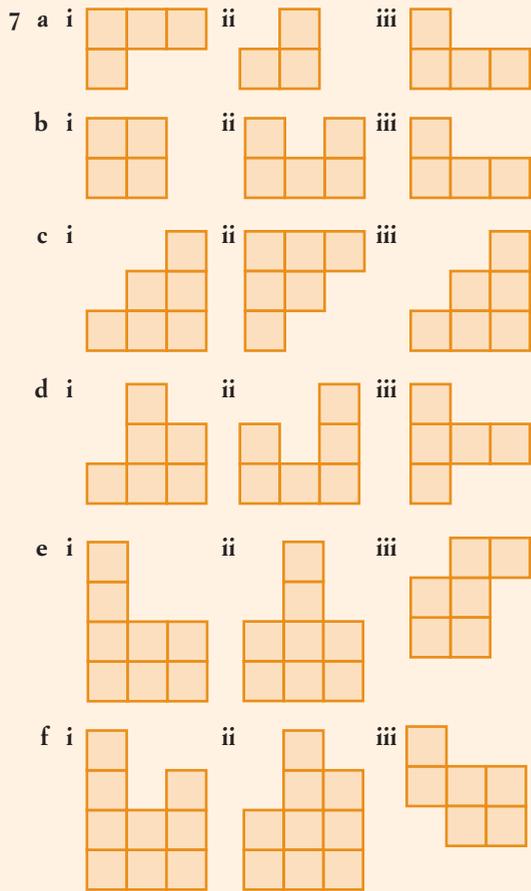
10 Teacher to check.

11 a iv, vii **b** i **c** v, x **d** ii, vi, ix

12 Compare your answers with those of other students. (There are more than five.)

13 D

14 a i **ii**

b i **ii** **iii**

c i **ii** **iii**

d i **ii**

e i **ii**

iii **iv**

f i **ii**
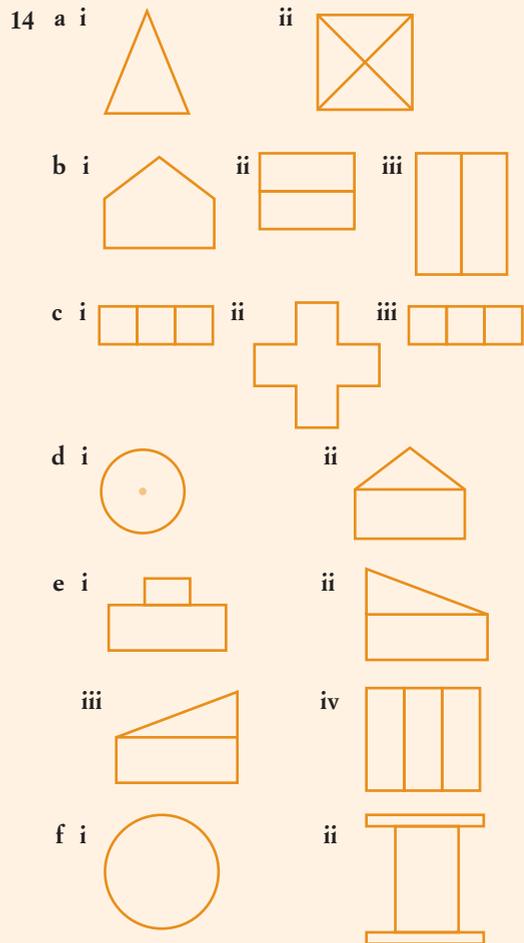
15 Teacher to check.

16 D

17 C

18 Teacher to check.

Exercise 11.4

1 a flour **b** fruit **c** rice
d salt **e** water **f** biscuits
g medicine **h** soft-drink **i** sugar
j bricks **k** books **l** detergent
(Other answers possible.)

2 a cm^3 **b** cm^3 **c** m^3
d m^3 **e** mm^3 **f** m^3
(Other answers possible.)

Answers

3 a 3000 mm³ **b** 5 000 000 cm³
 c 2600 mm³ **d** 4 cm³
 e 7 200 000 cm³ **f** 66 cm³
 g 1 000 000 000 mm³ **h** 0.0023 m³
 i 126 m³ **j** 3.45 cm³
 k 25 000 000 000 mm³ **l** 0.000 078 m³
 m 0.063 m³ **n** 0.0014 cm³

4 a 225 cm³ **b** 1071 cm³
 c 576 cm³ **d** 2295 cm³
 e 27 cm³ **f** 39.6 m³
 g 40500 cm³ **h** 144.72 m³

5

	Length	Breadth	Height	Volume
a	50 cm	50 cm	50 cm	125 000 cm³
b	5 cm	10 cm	18 cm	900 cm³
c	4 m	2.5 m	1.4 m	14 m³
d	24 mm	16 mm	11 mm	4224 mm³
e	20 cm	10 cm	10 cm	2000 cm³
f	5 mm	2 mm	10 mm	100 mm³
g	6 m	1.5 m	3 m	27 m³
h	22 cm	8 cm	5 cm	880 cm³
i	70 mm	10 mm	100 mm	70 000 mm³
j	1.8 m	0.5 m	10 m	9 m³

6 a 1 L, 600 mL, 300 mL
 b 375 mL, 750 mL, 1250 mL, 2 L
 c 375 mL
 d 250 mL
 e 50 L, 30 L
 f 2 000 000 L
 g 9000 L
 h 250 mL

7 a mL **b** kL
 c L **d** mL
 e L (other answers possible.)

8 a 7 L **b** 2000 mL **c** 3500 mL
 d 10 L **e** 2.5 L **f** 1500 mL
 g 4 L **h** 8500 mL **i** 6200 mL
 j 1.75 L **k** 5000 L **l** 9 kL
 m 25 000 000 L **n** 0.52 L
 o 0.0023 L **p** 0.000 006 kL

9 a 252 cm³ **b** 50 mm³ **c** 49.4 m³
 d 7.82 m³ **e** 7440 cm³ **f** 2744 mm³
 g 4.08 m³ **h** 4335 cm³

10 a 200 cm³
 b 5625 mm³ or 5.625 cm³
 c 5000 cm³ or 0.005 m³

 d 3 m³
 e 0.6 m³

11 A

12 a D **b** G **c** J **d** H **e** I
 f A **g** F **h** E **i** C **j** B

13 Teacher to check.

14 D

15 a E **b** G **c** F **d** C
 e A **f** B **g** D

16 a 62 cm³ **b** 126 mm³ **c** 498 m³
 d 6400 cm³ **e** 23 552 m³ **f** 23 500 cm³
 g 28 700 mm³

17 8

18 a 54 L **b** 27

19 10 days

20 a 0.2 mL **b** 12 mL
 c 2160 mL or 2.16 L
 d 17 280 mL or 17.28 L

21 $6\frac{1}{4}$ h

22 a 2700 cm³ **b** 2700 mL

23 a 48 m³ **b** 48 kL = 48 000 L

24 a 72 000 cm³ **b** 72 L

25 a 28 **b** 3 **c** about 61%

Exercise 11.5

1 a gram **b** tonne **c** kilogram
 d tonne **e** kilogram

2 Teacher to check.

3 a 3 kg **b** 2000 kg **c** 4000 g
 d 9 t **e** 7500 kg **f** 10 g
 g 1500 g **h** 3.8 t **i** 3000 mg

4 C

5 a > **b** > **c** = **d** =
 e = **f** < **g** < **h** >

6 a H **b** E **c** I **d** J **e** D
 f A **g** B **h** C **i** F **j** G

7 Teacher to check.

Chapter review

1
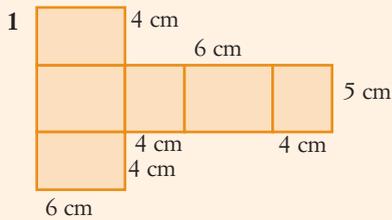
2 84 cm^3

3 a non-convex   **b** convex

4 See table below.

5 Teacher to check.

6

7 656 cm^2

8 B

9 a Prism   **b** Not a prism   **c** Not a prism
d Prism   **e** Not a prism   **f** Prism
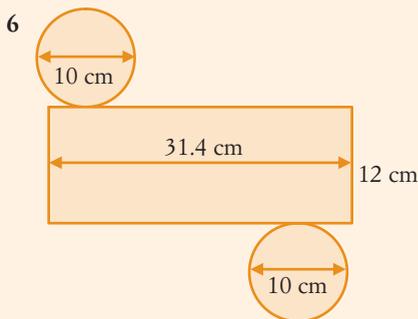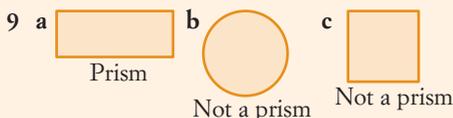
10 a Octagonal prism
b Cylinder
c Square pyramid

11 10f and 11b

12 a D   **b** C   **c** I   **d** F   **e** A
f H   **g** B   **h** E   **i** J   **j** G

13 A polyhedron with faces that are identical regular polygons. Regular tetrahedron, Regular hexahedron, Regular octahedron, Regular dodecahedron or Regular icosahedron.

14 a 16   **b** 2   **c** 6

15 a Two square pyramids.
b All faces are equilateral triangles.

16 a   **b**

17 a i   **ii**   **iii**

b i   **ii**   **iii**
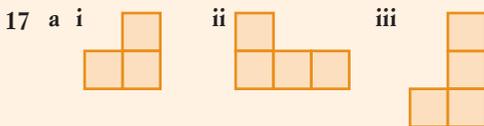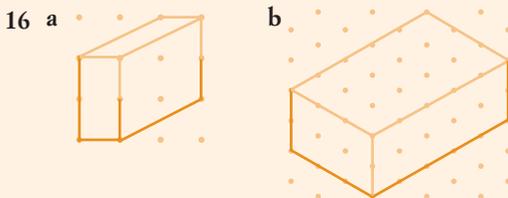
18 a 20 000 mm^3 **b** 500 000 cm^3
c 7.5 cm^3 **d** 0.000 23 m^3

19 a 80 m^3 **b** 320 cm^3
c 672 mm^3 **d** 360 m^3
e 3000 cm^3 **f** 870 m^3

Chapter 11 review

4

	Solid A	Solid B	Solid C
No. of faces	5	4	7
No. of congruent faces	2	2	3 lots of 2
Shapes of faces	triangle, rectangle	rectangle, semicircle	pentagon, rectangle
No. of parallel faces	2	2	2 lots of 2
Shape of cross-section	triangle	semicircle	pentagon
Are the cross-sections uniform?	Yes	Yes	Yes
Is the solid a polyhedron?	Yes	No	Yes

20 a 2 L   **b** 3000 L
   c 7000 mL   **d** 3.3 kL
   e 1.75 L   **f** 2500 mL

21 a 5 kg   **b** 2000 mg
   c 1500 kg   **d** 6.5 t
   e 4 g   **f** 1500 g

22 720 cm^2

23 Teacher to check.

24 a c i   **ii**   **iii**

    b i   **ii**   **iii**
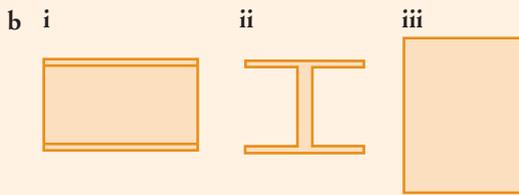
25 3150 mL or 3.15 L

26 a 45 m^2 **b** 13.5 m^3 **c** $10.80

27 a 3072 cm^3 **b** 870 m^3

28 6 cm

29 B

30 216 t

31 $2 \times 10^{15} = 2\,000\,000\,000\,000\,000$ L

32 8 cm

33 $54.16 \approx 54$ drops

34 a 500   **b** 12.5 tonnes

35 a 30 m^3
   b 240 m^3
   c 810 m^3
   d 1920 m^3
   e 8 : 1, 27 : 1, 64 : 1
   f n^3
   g If it was touching a rectangular prism on each side, the prism would increase by 5^3 times, so it must as well. The volume will increase by 125 times.

Chapter 12
Exercise 12.1

1 $A(2, 5)$ first quadrant
  $B(3, ^-4)$ fourth quadrant
  $C(^-2, ^-3)$ third quadrant
  $D(7, 2)$ first quadrant
  $E(^-4, 9)$ second quadrant
  $F(0, 4)$ y-axis
  $G(^-9, ^-3)$ third quadrant
  $H(0, ^-7)$ y-axis
  $I(^-8, 2)$ second quadrant
  $J(^-2, 0)$ x-axis
  $K(^-7, 6)$ second quadrant
  $L(8, ^-6)$ fourth quadrant
  $M(5, 0)$ x-axis
  $N(9, 8)$ first quadrant
  $P(1, ^-2)$ fourth quadrant
  $Q(^-6, ^-8)$ third quadrant
  $R(9, 0)$ x-axis
  $S(0, ^-3)$ y-axis
  $T(6, ^-2)$ fourth quadrant
  $U(^-2, 6)$ second quadrant

2   **a**   **b**   **c**
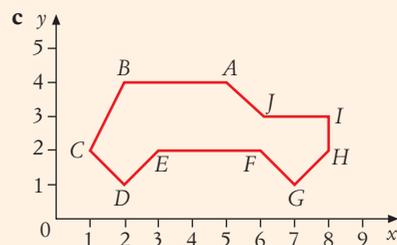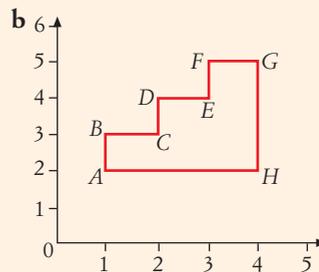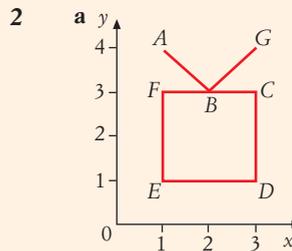
3 a
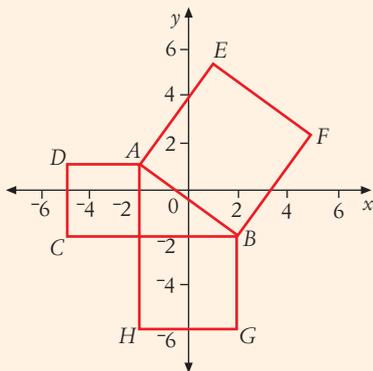
b
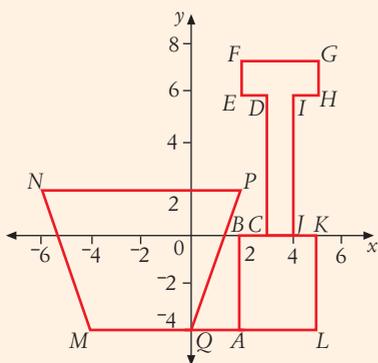
4 a

Number of squares (*s*)	1	2	3	4	5	6
Number of matches (*m*)	4	7	10	13	16	19

b
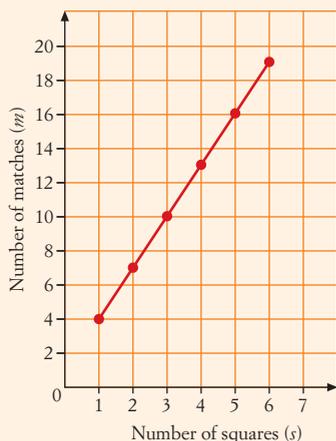
c $m = 3s + 1$

5 a

Term number (*T*)	1	2	3	4	5	6
Number of discs (*D*)	1	3	6	10	15	21

b
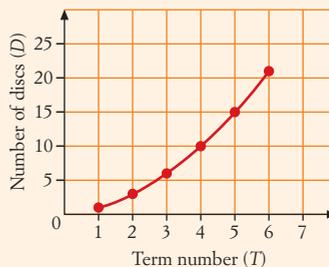
c No, the graph is non-linear.

6 a

Quadrilateral term number (*t*)	1	2	3	4	5	6
Number of matches (*m*)	4	7	10	13	16	19

Pentagonal term number (*t*)	1	2	3	4	5	6
Number of matches (*m*)	5	9	13	17	21	25

Hexagonal term number (*t*)	1	2	3	4	5	6
Number of matches (*m*)	6	11	16	21	26	31

b
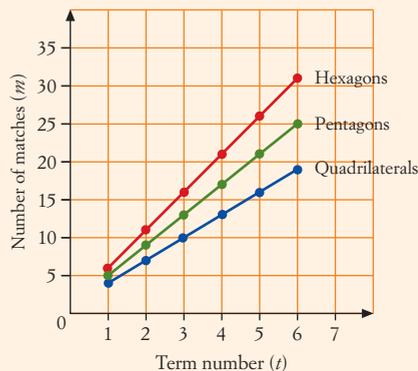
c Quadrilaterals: $m = 3t + 1$;
pentagons: $m = 4t + 1$;
hexagons: $m = 5t + 1$

7 a

Term number (*t*)	1	2	3	4	5	6
Number of matches (*m*)	5	12	19	26	33	40

Answers

b

Number of matches (m) vs Term number (t)
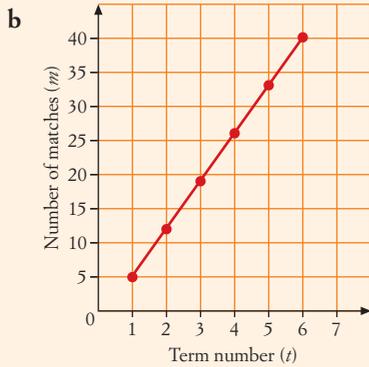
c 68 matches

d $m = 7t - 2$

8 a

Arm length (a)	1	2	3	4	5	6
Number of tiles (t)	4	7	10	13	16	19

b

Number of tiles (t) vs Arm length (a)
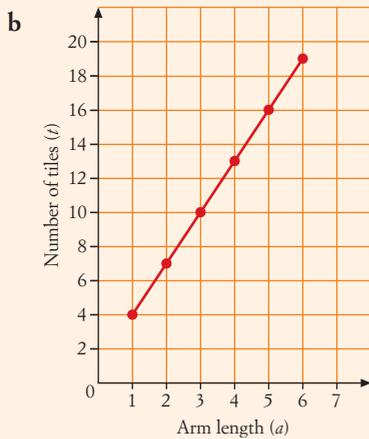
c $t = 3a + 1$

9 a

Number of tables (t)	1	2	3	4	5	6
Number of people (p)	6	10	14	18	22	26

b

Number of people (p) vs Number of tables (t)
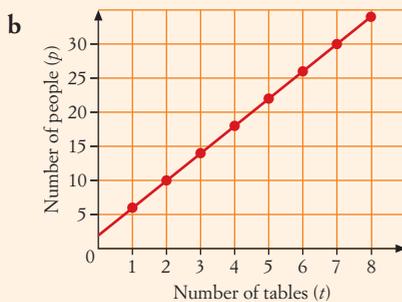
c 10 tables

d $p = 4t + 2$

10 Check your partner's answers.

11 a

Number of people (p)	1	2	3	4	5	6
Number of handshakes (h)	0	1	3	6	10	15

b

Number of handshakes (h) vs Number of people (p)
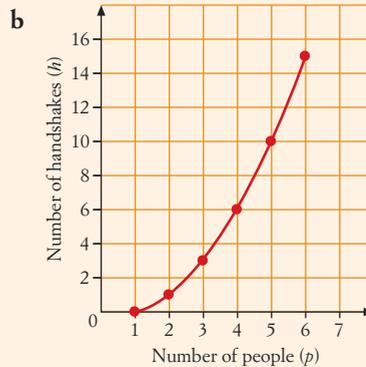
c 28 handshakes

12 The graph is not linear, but the number of rainy days has a smooth change from highest in February to lowest in August–September.

Rainy days in Cairns
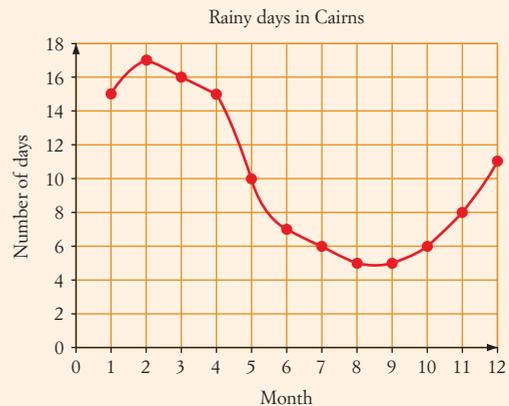
Exercise 12.2

1 a

Input	Output
1	5
2	8
3	11
4	14
5	17

b

Input	Output
2	1
3	3
4	5
5	7
6	9

c

Input	Output
2	13
3	18
4	23
6	33
11	58

d

Input	Output
1	48
2	46
3	44
8	34
13	24

e

A	B
0	7
1	17
2	27
6	67
8	87
12	127

f

y	x
3	13
4	19
5	25
8	43
11	61
20	115

2 a 13   **b** 9   **c** 5
    d 25   **e** 45   **f** 37

3 a 1   **b** 0   **c** 4
    d 10   **e** 8   **f** 2.5

4 C output $= 3 \times$ input $+ 7$

5 D $B = 4 \times A - 5$

6 a

Input	Output
1	5
2	10

Input	Output
3	15
5	25
7	35
9	45

output $= 5 \times$ input

b

Input	Output
4	15
7	18
5	16
9	20
17	28
11	22

output $=$ input $+ 11$

c

Input	Output
2	11
6	15
14	23
7	16
9	18
12	21

output $=$ input $+ 9$

d

Input	Output
2	6
11	33
6	18
9	27
18	54
16	48

output $= 3 \times$ input

e

A	B
7	43
11	39
4	46
22	28
16	34
34	16

output $= 50 -$ input
or $B = 50 - A$

f

m	n
24	3
56	7
112	14
72	9
88	11
64	8

output = input ÷ 8

or $m = n \div 8$

or $m = \dfrac{n}{8}$

7 a $t = 4b - 3$

b	1	2	3	4
t	1	5	9	13

b $g = 5n + 2$

n	0	2	3	5
g	2	12	17	27

c $y = 2(x + 1)$

x	1	3	5	7
y	4	8	12	16

d $q = 14 - 3p$

p	0	1	3	4
q	14	11	5	2

8 a b independent, t dependent
b n independent, g dependent
c x independent, y dependent
d p independent, q dependent

9 a $h = {}^{-}2g + 7$

g	0	1	3	6
h	7	5	1	${}^{-}5$

b $k = 2(j + 4)$

j	${}^{-}2$	${}^{-}1$	0	1
k	4	6	8	10

c $y = {}^{-}4(2x - 3)$

x	1	3	5	7
y	4	${}^{-}12$	${}^{-}28$	${}^{-}44$

d $d = 3e(e + 6)$

e	${}^{-}2$	0	3	5
d	${}^{-}24$	0	81	165

10 a 13   **b** 22   **c** 1

11 a 3   **b** 9   **c** 19

12 Independent variables are b (question **10**) and N (question **11**).

13 a output = 3 × input + 4

Input	Output
0	4
1	7
2	10
5	19
7	25
10	34

b output = 5 × input + 6

Input	Output
2	16
3	21
5	31
8	46
6	36
11	61

c output = 4 × input − 1

Input	Output
2	7
3	11
4	15
7	27
9	35
14	55

d output = 7 × input − 2

Input	Output
6	40
2	12
7	47
10	68
8	54
15	103

e $T = 2 \times S + 11$

S	T
1	13
8	27
6	23
11	33
9	29
2	15

f $b = 9 \times a - 1$

a	b
10	89
1	8
7	62
9	80
13	116
5	44

14 a $m = 4n - 3$ **b** $p = 2m + 3$ **c** $d = 3c - 2$
d $k = 5d + 1$ **e** $y = 20 - 3x$

15

Sides	4	5	6	. . .	15
Lines	6	10	15	. . .	105

Exercise 12.3

1 a $t = 5s$

s	0	3	5
t	0	15	25
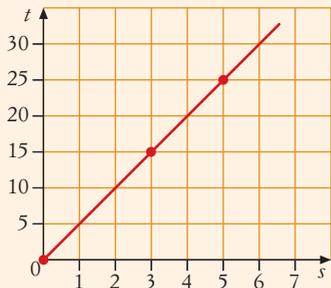
b $P = 4K - 1$

K	1	3	6
P	3	11	23
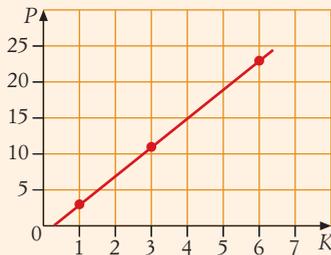
c $a = 15 - 3b$

b	0	2	4
a	15	9	3
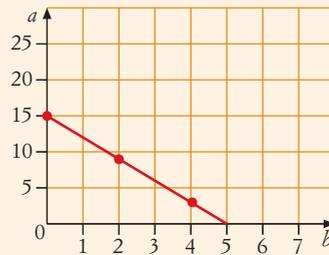
d $d = 2f + 7$

f	1	3	6
d	9	13	19
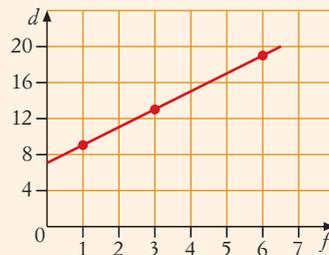
2 a $y = x + 3$

x	0	3	5
y	3	6	8
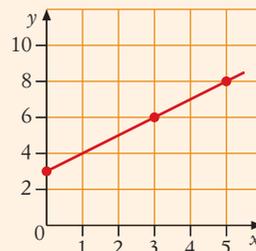
b $y = 3x + 5$

x	0	2	4
y	5	11	17
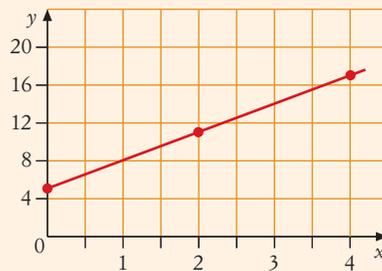
c $y = 13 - 2x$

x	0	3	6
y	13	7	1
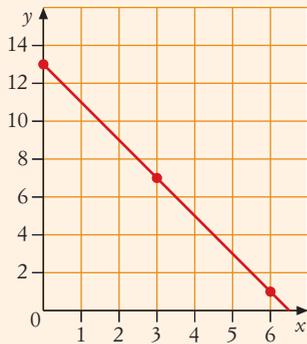
d $y = 7x - 5$

x	1	3	5
y	2	16	30
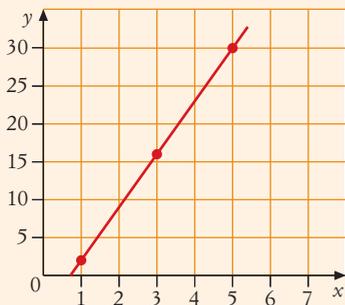
3 a $y = 2x + 3$

x	1	2	3	4
y	5	7	9	11
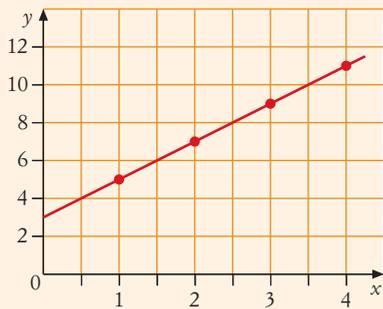
b $y = 3x + 1$

x	1	2	3	4
y	4	7	10	13
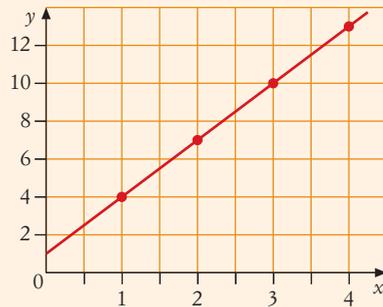
c $y = 4x - 3$

x	1	2	3	4
y	1	5	9	13
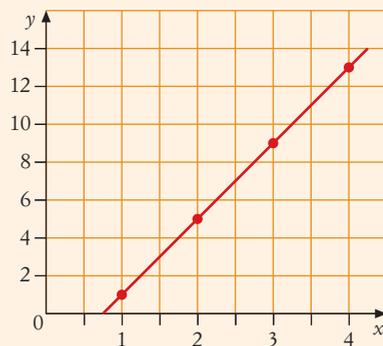
d $y = 13 - 2x$

x	1	2	3	4
y	11	9	7	5
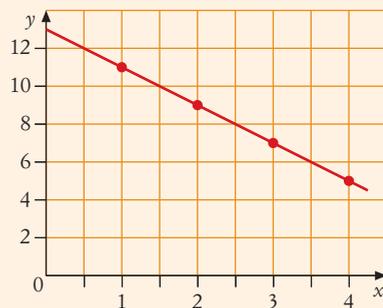
e $y = 3x - 3$

x	1	2	3	4
y	0	3	6	9
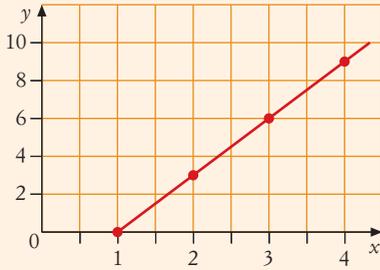
f $y = 16 - 3x$

x	1	2	3	4
y	13	10	7	4
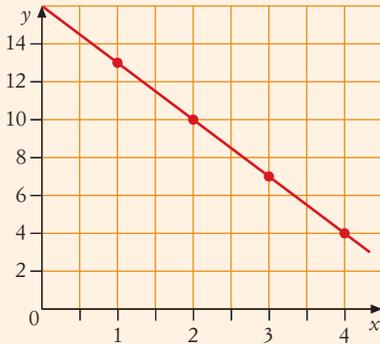
g $y = 5x - 4$

x	1	2	3	4
y	1	6	11	16
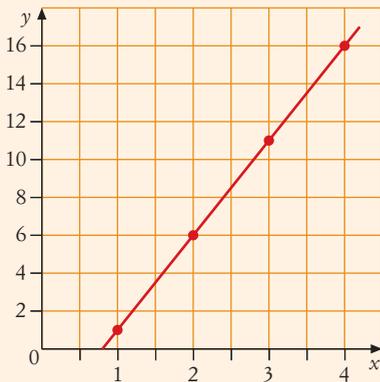
h $y = 9 - 2x$

x	1	2	3	4
y	7	5	3	1
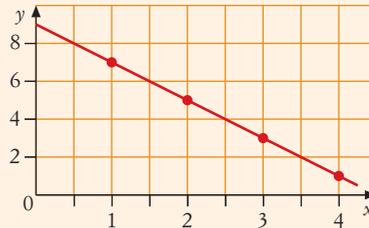
4 a Yes **b** Yes **c** Yes
d No **e** No **f** Yes

5 a $(3, 1)$ **b** $(1, 4)$ **c** $(2, 0)$
d $(4, 11)$ **e** $(6, 31)$ **f** $(3, 4)$

6 a Yes **b** Yes **c** No
d Yes **e** No **f** No

7 a $(^-3, ^-17)$ **b** $(2, 0)$ **c** $(^-2, 5)$
d $(^-4, 27)$ **e** $(^-2, 4)$ **f** $(^-2, 16)$

8 a 11 a.m. **b** ½ hour
c In the first 2 hours, near Geelong.
d About 12:50 p.m. **e** About 210 km

9 See table below.

Walking to school
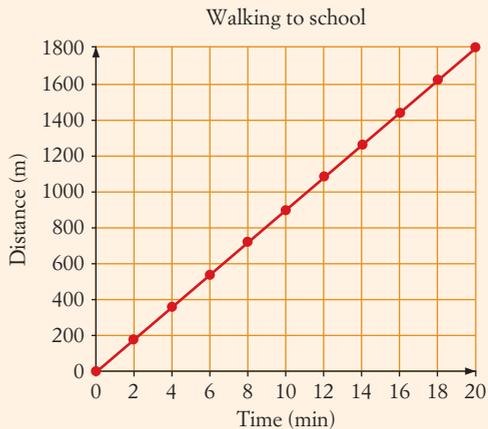
Exercise 12.3

9

Time (t) – minutes	0	2	4	6	8	10	12	14	16	18	20
Distance from home (d) – metres	0	180	360	540	720	900	1080	1260	1440	1620	1800

Answers

10

Bicycle trip
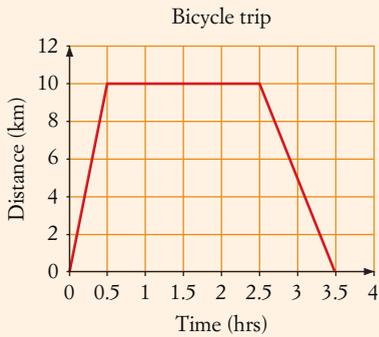
The graph is initially steeper because Julia rode faster on the way to Sam's than when she rode home.

11 a $P = 4t - 500$

b

t	200	250	300	350	400
P	300	500	700	900	1100

c
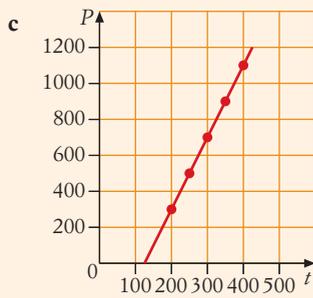
About 230 tickets (225)

d 125 tickets

12 a

People (p)	2	3	4	5	6	7	8
Cost (c)	50	60	70	80	90	100	110

b The number of people is the independent variable.

c $c = 10p + 30$, for $p = 2$ or more.
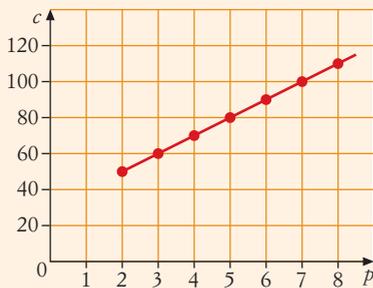
13

Side length (s)	20	40	60	80	100
Cardboard length (c)	105	185	265	345	425

$c = 4s + 25$

14

15
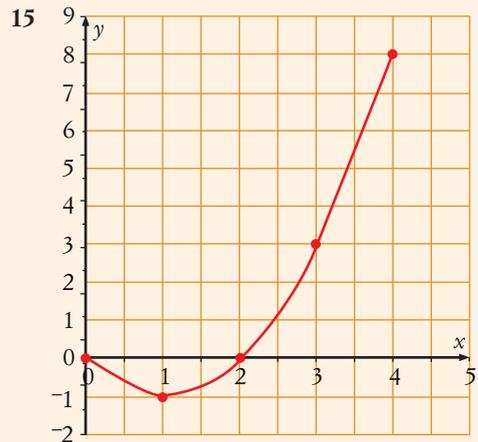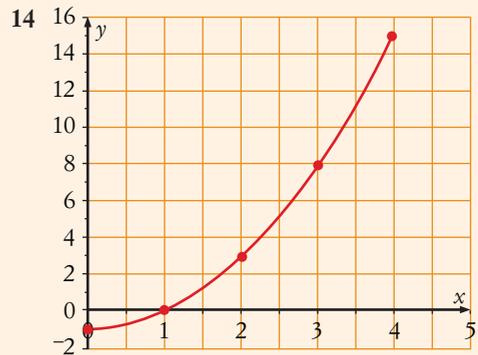
Chapter review

1 $A(^-3, ^-5)$, third quadrant
$B(1, 3)$, first quadrant
$C(^-6, 1)$, second quadrant
$D(^-1, 0)$, x-axis
$E(0, 5)$, y-axis
$F(2, ^-4)$, fourth quadrant

2
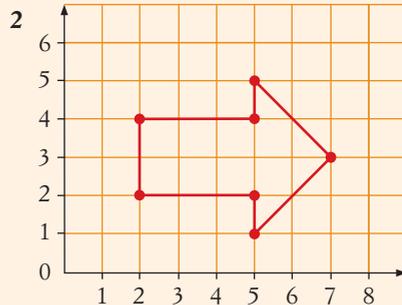
3
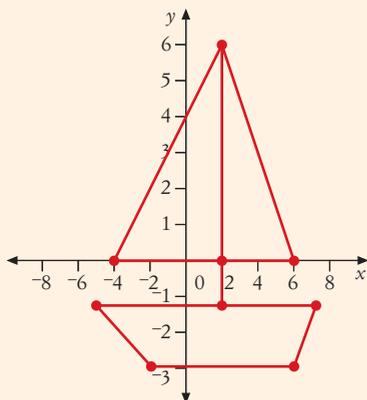
4 a

Term number (*t*)	1	2	3	4	5	6
Number of matches (*m*)	7	12	17	22	27	32

b
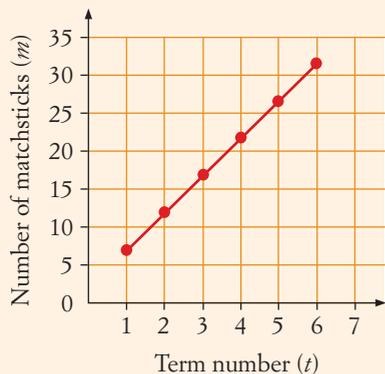
c 52 matches
d Yes

5 $m = 5t + 2$

6 a

Term number (*T*)	1	2	3	4	5	6
Number of discs (*D*)	1	4	9	16	25	36

b
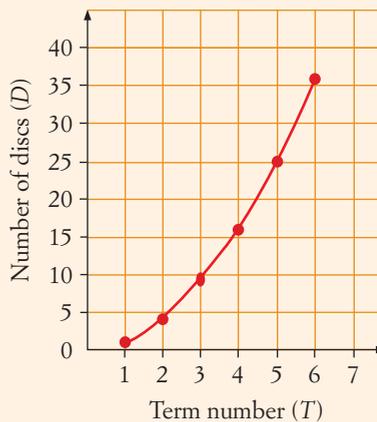
c No

7

Term number (*t*)	1	2	3	4	5	6	7
Number of discs	1	5	12	22	35	51	70

8 a

Term number (*t*)	1	2	3	4	5
Number of matches (*m*)	6	12	18	24	30

b $m = 6t$

9 a

Term number (*t*)	1	2	3	4	5	6
Number of squares (*s*)	1	3	5	7	9	11

b $s = 2t - 1$
c 23, 55
d 64th

10 a Output = 4 × input + 1.

Input	Output
1	5
2	9
3	13
4	17
5	21

b Output = 3 × input − 2.

Input	Output
2	4
3	7
4	10
5	13
6	16

Answers

11 a 2 **b** 47 **c** 17

12 a Output = input × 4

Input	Output
1	4
2	8
3	12
5	20
9	36
30	120

b Output = input + 8

Input	Output
2	10
6	14
14	22
7	15
9	17
12	20

13 C output = 7 × input + 5

14 a $h = 3r + 5$

r	0	2	3
h	5	11	14

b $m = 4n - 7$

n	2	3	5
m	1	5	13

c $y = {}^-3x + 8$

x	0	1	3
y	8	5	$^-1$

d $k = 2(3j - 1)$

j	$^-2$	0	2
k	$^-14$	$^-2$	10

15 a r (independent), h (dependent)
  b n (independent), m (dependent)
  c x (independent), y (dependent)
  d j (independent), k (dependent)

16 a $h = 22$ **b** $h = 57$

17 g

18

a	0	3	5
b	3	9	13
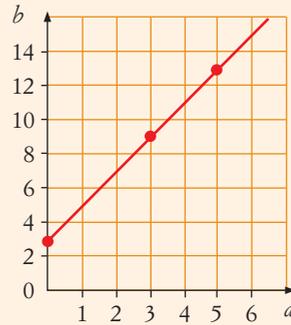
19

x	0	2	5
y	13	9	3
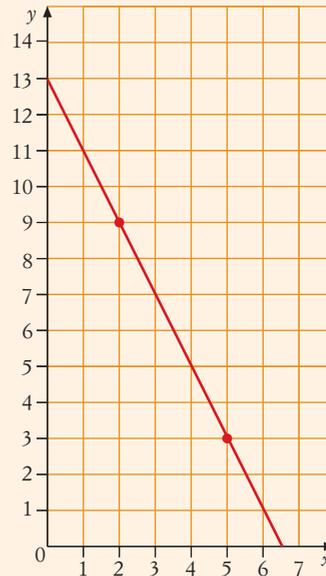
20

x	1	2	3	4
y	5	8	11	14
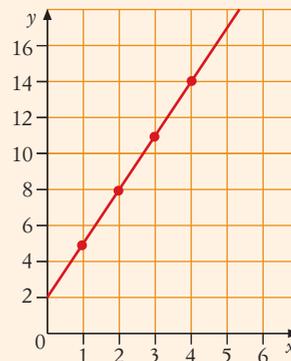
21 a No   **b** Yes   **c** Yes   **d** No

22 a (2, 4)   **b** ($^-$3, $^-$9)

23 a

Number of rows (r)	1	2	3	4	5	6
Number of cans (c)	1	3	6	10	15	21

b
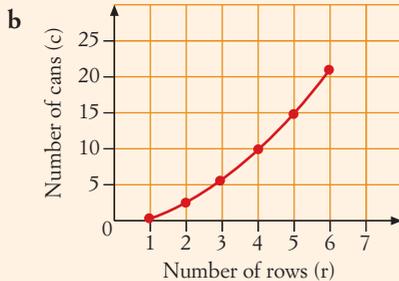
c

Number of rows (r)	1	2	3	4	5	6	7	8
Number of cans (c)	1	3	6	10	15	21	28	36

36 cans

24 a Output = 2 × input + 5

Input	Output
0	5
1	7
2	9
5	15
7	19
10	25

b Output = 3 × input – 3

Input	Output
2	3
3	6
5	12
8	21
6	15
11	30

25 a $p = 3a + 2$ **b** $y = 2x - 1$

26 a See table below.
    b p
    c $c = 40p + 40$

27 A rule which has a graph that is a straight line.

28 Ahmed travelled at 20 km/h and Patrick travelled at 15 km from towns 60 km apart. They started at the same time and passed each other after about 1 h 40 min (1 h 43 m) at about 34 km from the town Ahmed started from. Ahmed finished an hour before Patrick.

29 Find the difference between the second coordinates. The rule is the number times x plus the difference between the value at the first coordinate and the number.

The first coordinate, or x-coordinate, is the independent variable. Each x-coordinate has a corresponding y-coordinate. If we subtract the second y-coordinate from the first, then we obtain the multiplier of x. We can then subtract the multiplier from the first y-coordinate to obtain the other part of the rule. Using the third coordinate, we can check the rule for correctness.

30 Divide the difference between the y-coordinates by the difference between the x-coordinates to get the number that is multiplied by x. Then substitute either of the x values to find what must be added or subtracted to get the corresponding y value.

Chapter 12 review

26 a

Number of people p	2	3	4	5	6	7	8	9	10
Cost of accommodation c	120	160	200	240	280	320	360	400	440

Appendix

Keys

Key	Name	Use
+ − × ÷	Plus, minus, multiply, and divide	Carry out basic operations
=	Equals sign	Gives the answer
.	Decimal point	Inserts a decimal point
DEL	Delete	Deletes previous entry
ANS	Answer	Retrieves the previous answer
REPLAY	Arrow pad	Moves around the screen
SHIFT MODE 2ndF	Shift, mode, second function	Accesses other operations
(−) or +/−	Negative number	Makes a number negative
$a^{b/c}$ or ▤ or a/b	Fraction	Allows fractions to be entered
( ) or ( )	Bracket	Inserts brackets
x^2	Square	Squares a number
$\sqrt{}$	Square root	Finds the square root of a number
x^3	Cube	Cubes a number
$\sqrt[3]{}$	Cube root	Finds the cube root of a number
y^x a^b ^	y to the power of x a to the power of b Power	Working with powers
$\sqrt[n]{x}$ or $\sqrt[x]{}$ or $x^{1/y}$	nth root of x	Working with roots other than 2 and 3
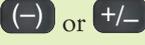
Basic scientific calculator skills

Key	Name	Use
EXP or ×10^x	Exponent key	To enter numbers in scientific notation
○'" or DMS	Degrees-minutes-seconds	Working with degrees, minutes and seconds or hours, minutes and seconds
STO	Store	Stores an answer
A	Memory A	Used with STO and RCL keys to store and retrieve a number from memory
RCL	Recall	Recalls an answer
π	Pi	Inserts the value of Pi

Basic operations

Performing a calculation

Question	Calculator steps	Answer
$34 + 6 - 16$	34 + 6 − 16 =	24
$5 + 6 \times 7$	5 + 6 × 7 =	47
7.4×9.1	7.4 × 9.1 =	67.34
$234 \div 1.5$	234 ÷ 1.5 =	156
$45 \div (0.2 + 4.3)$	45 ÷ (0.2 + 4.3) =	10
$45 \div 0.2 + 4.3$	45 ÷ 0.2 + 4.3 =	229.3

Appendix

Basic scientific calculator skills

Correcting a wrong entry

Question	Calculator steps	Answer
$34 + 6 - 16$	Wrong entry: 34 [+] 8 [−] 16 [=] To correct this entry, press the left side of the arrow pad [REPLAY] to go back to the 8, then press [DEL] 6 [=].	26 ✗ 24 ✓

Performing calculations involving integers

Question	Calculator steps	Answer
$^-4 + 9$	[(−)] 4 [+] 9 [=]	5
$17 \times\ ^-2$	17 [×] [(−)] 2 [=]	$^-34$
$18 -\ ^-3 \times 4$	18 [−] [(−)] 3 [×] 4 [=]	30
$25 - (7 \times (5 - 3) + 4)$	25 [−] [(] 7 [×] [(] 5 [−] 3 [)] [+] 4 [)] [=]	7
$^-12 - 7 +\ ^-2 -\ ^-8$	[(−)] 12 [−] 7 [+] [(−)] 2 [−] [(−)] 8 [=]	$^-13$
$^-5 \times (5 - 7)$	[(−)] 5 [×] [(] 5 [−] 7 [)] [=]	10

Performing calculations involving powers or roots

Question	Calculator steps	Answer
16^2	16 [x^2] [=]	256
9^3	9 [x^3] [=]	729
5^4	5 [a^b] 4 [=] or 5 [y^x] 4 [=]	625
$\sqrt{196}$	[$\sqrt{\ }$] 196 [=]	14

Basic scientific calculator skills

Question	Calculator steps	Answer
$\sqrt[3]{4913}$	$\sqrt[3]{}$ 4913 =	17
$\sqrt[5]{200}$	5 $\sqrt[n]{x}$ 200 =	$2.885399812 \approx 2.885$

Converting fractions and decimals

Question	Calculator steps	Answer
Simplify $\frac{16}{20}$	16 $a^{b}/_{c}$ 20 =	$\frac{4}{5}$
Change $1\frac{3}{4}$	1 $a^{b}/_{c}$ 3 $a^{b}/_{c}$ 4 = SHIFT $a^{b}/_{c}$	$\frac{7}{4}$
Change $\frac{19}{4}$ to a proper fraction	19 $a^{b}/_{c}$ 4 =	$4\frac{3}{4}$
Change 1.7 to a fraction	1.7 = $a^{b}/_{c}$	$1\frac{7}{10}$
Change $\frac{5}{6}$ to a decimal	5 ÷ 6 =	0.8333333333 $= 0.8\dot{3}$

Performing operations involving fractions

Question	Calculator steps	Answer
$\frac{1}{2}+\frac{1}{3}$	1 $a^{b}/_{c}$ 2 + 1 $a^{b}/_{c}$ 3 =	$\frac{5}{6}$
$\frac{2}{3}-\frac{1}{2}$	2 $a^{b}/_{c}$ 3 − 1 $a^{b}/_{c}$ 2 =	$\frac{1}{6}$
$\frac{2}{3}\times\frac{2}{5}$	2 $a^{b}/_{c}$ 3 × 2 $a^{b}/_{c}$ 5 =	$\frac{4}{15}$
$\frac{1}{2}\div\frac{1}{3}$	1 $a^{b}/_{c}$ 2 ÷ 1 $a^{b}/_{c}$ 3 =	$1\frac{1}{2}$
$\frac{1}{2}\div\frac{1}{3}-\frac{2}{3}\times\frac{2}{5}$	1 $a^{b}/_{c}$ 2 ÷ 1 $a^{b}/_{c}$ 3 − 2 $a^{b}/_{c}$ 3 × 2 $a^{b}/_{c}$ 5 =	$1\frac{7}{30}$

Basic scientific calculator skills

Question	Calculator steps	Answer
$\frac{1}{2} \div (\frac{1}{3} - \frac{2}{3}) \times \frac{2}{5}$	1 [aᵇ/c] 2 [÷] [(] 1 [aᵇ/c] 3 [−] 2 [aᵇ/c] 3 [)] [×] 2 [aᵇ/c] 5 [=]	$\frac{-3}{5}$
$9\frac{2}{3} - 3\frac{3}{4}$	9 [aᵇ/c] 2 [aᵇ/c] 3 [−] 3 [aᵇ/c] 3 [aᵇ/c] 4 [=]	$5\frac{11}{12}$
$3\frac{2}{5} \times 1\frac{2}{3}$	3 [aᵇ/c] 2 [aᵇ/c] 5 [×] 1 [aᵇ/c] 2 [aᵇ/c] 3 [=]	$5\frac{2}{3}$

Storing and recalling answers

Question	Calculator steps	Answer
Find the angle required to show 210 out of 1200 Year 7 students on a circle graph	Find degrees per student: 360 [÷] 1200 [=]	0.3
	Then store the answer: [ANS]	
	Multiply by the stored answer to find the angle: 210 [×] [ANS] [=]	63 (angle is 63°)

Performing calculations involving time

Question	Calculator steps	Answer
Find 7 hours 5 mins − 3 hours 24 mins	7 [° ' "] 5 [° ' "] [−] 3 [° ' "] 24 [° ' "] [=]	3° 41′ = 3 h 41 mins

Converting units

Question	Calculator steps	Answer
Convert 23 km/h to m/s	23 [×] 1000 [÷] 60 [÷] 60 [=]	6.388888889 ≈ 6.4 m/s
Convert 23° 16′ 30″ to decimal degrees	23 [° ' "] 16 [° ' "] 30 [° ' "] [=] [SHIFT] [° ' "]	23.275°

Basic scientific calculator skills

Performing calculations involving π

Question	Calculator steps	Answer
Find the area of a circle with $r = 20$ $(A = \pi r^2)$	$\boxed{\pi}$ $\boxed{\times}$ 20 $\boxed{x^2}$ $\boxed{=}$	$1256.637061 \approx 1257$

Performing calculations involving percentages

Question	Calculator steps	Answer
Change $\frac{15}{37}$ to a percentage	15 $\boxed{\div}$ 37 $\boxed{\times}$ 100 $\boxed{=}$	40.54054054 $\approx 40.54\%$
Find 36% of $380	36 $\boxed{\div}$ 100 $\boxed{\times}$ 380 $\boxed{=}$	136.8

Performing calculations involving numbers in scientific notation

Question	Calculator steps	Answer
$5.6 \times 10^6 \times 6.3 \times 10^8$	5.6 $\boxed{\text{EXP}}$ 6 $\boxed{\times}$ 6.3 $\boxed{\text{EXP}}$ 8 $\boxed{=}$ OR 5.6 $\boxed{\times 10^x}$ 6 $\boxed{\times}$ 6.3 $\boxed{\times 10^x}$ 8 $\boxed{=}$	3.528^{15} or 3.528E15 $= 3.528 \times 10^{15}$

Word	Definition
acute (angle)	Angle between 0° and 90°.
allied (angles)	Inside angles on the same side of a transversal over two lines. *See also* co-interior angles.
alternate (angles)	Inside opposite angles on a transversal over two lines.
approximation	Result close to the true value.
arc	Curved line, particularly one that is part of a circle.
area	The amount of a surface enclosed by the boundary of a shape.
arm	One of the lines forming half of an angle.
associative (law)	Addition (or multiplication) of three numbers is the same for either operation order.
axis	A vertical or horizontal line with markings showing values; plural *axes*.
axis of symmetry	A line that divides a shape into halves like reflections in a mirror.
bar chart, bar graph	A graph of values shown by vertical or horizontal columns/bars.
base (power)	The bottom number of a power, such as the 5 in 5^4.
base (prism)	'Bottom' of a prism. All parallel cross-sections are identical to this face (side).
base (pyramid)	'Bottom' of a pyramid. It is opposite the apex.
bias (data, sample)	Data or sample that is generally different from the population; not fair.
bias (survey question, method)	Survey question or method that favours particular results.
bimodal	Having two modes; two scores with equal highest frequency.
bisect	Divide into halves.
Cartesian plane	A flat surface with horizontal and vertical axes intersecting at 0.
categorical (data)	Data that can be put into categories, such as names of colours.
centre of rotation	The point around which a rotation occurs.
centre of symmetry	The centre of a shape with point symmetry; it rotates about this point.
certain	Will always happen.
circle graph	Graph where a circle is divided into sectors in proportion to values. *See also* pie graph, sector graph.
circumference	The boundary of a circle; the length of this boundary; the perimeter of a circle.
closed (2D shape)	Flat shape with a continuous boundary that must be crossed to get from the inside to the outside.
coefficient	Number in front of a variable; number that is multiplied by a variable.

Glossary

Word	Definition
co-interior (angles)	Inside angles on the same side of a transversal over two lines. *See also* allied angles.
column graph	Graph with vertical bars with heights in proportion to values. *See also* vertical bar graph
common denominator	Number that is a multiple of each of some given denominators.
common factor (of numbers)	Number that is a factor of each of some given numbers.
common fraction	Fraction shown with a denominator and numerator; as $\frac{5}{8}$. *See also* vulgar fraction.
common multiple (of numbers)	Number that is a multiple of each of some given numbers.
commutative (law)	Addition (or multiplication) of two numbers is the same for either number order.
compasses	Drawing instrument with two legs on a pivot, with a point on one and a pencil on the other.
complementary (angles)	Adjacent angles that together form a right angle (90°).
complementary (events)	Two separate events that together make the entire sample space.
composite number	A whole number with factors other than 1 and itself; having at least 3 factors.
concave (shape)	Shape with at least one internal angle more than 180°; an inward-pointing vertex.
cone	A shape equivalent to a pyramid with a circular base.
congruent (shapes), congruence	Shapes that are the same. They may be exactly superimposed, possibly by turning one over.
constant	A number, particularly a term of an expression that is by itself.
construct, construction	A shape drawn (constructed) using only a straight-edge and compasses.
continuous (data)	Data that can have any values, such as heights.
convex (shape)	Shape with internal angles all less than 180°; vertices all point outwards.
coordinates	The numbers showing position in a Cartesian plane; e.g. (3, 5).
corresponding (angles)	Angles on the same side of a transversal and the same sides of lines it crosses.
counting numbers	The numbers 1, 2, 3, 4, . . . Some people include zero. *See also* natural numbers.
cross-section	The 2D shape on a flat surface made by cutting through a solid (3D) shape.
cube (number)	The third power of a number, such as 5^3.

Word	Definition
cube root	The number whose cube is the original number, such as $\sqrt[3]{216} = 6$.
cumulative frequency	The progressive total of frequencies of ordered numeric data.
cylinder	Shape with a circular base and identical cross sections; like a prism with a circular base.
data	Information collected or used for analysis or reference; singular *datum*.
degree	Angle measure such that a complete revolution is 360 degrees, written as 360°.
denominator	The bottom number of a fraction; the number of parts in one whole.
dependent (variable)	The variable of the output of a rule or function; its value is calculated from the input(s).
diameter	The distance from one side of a circle to the other through the centre; double the radius.
directed number	Any of zero, positive or negative whole numbers. *See also* integer.
discount	An amount taken off the usual price of an article.
discrete (data)	Data that can only have particular values, such as shoe sizes.
distributive (law)	The product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each of the two numbers; $a(b \pm c) = ab \pm ac$.
divided bar chart	A single bar divided into parts in proportion to values.
dodecahedron	Regular polyhedron with 12 regular pentagon faces.
dot plot	A graph of frequencies made by placing equally spaced dots for each score above a scale of values.
edge	Straight or curved line where two faces of a solid (3D) shape meet.
EFTPOS	Electronic funds transfer at point of sale; electronic payment using a credit/debit card.
element	A basic probability outcome; '5' on a die. *See also* simple outcome, sample point.
equilateral (triangle)	A triangle with all sides (and angles) the same.
equivalent fractions	Fractions with different denominators that have the same value; $\frac{4}{6}$ and $\frac{10}{15}$.
evaluate	Calculate the answer to an expression.
even chance	Happens half the time in the long run.
event (probability)	An individual outcome or collection of outcomes.
event space	List of all the simple outcomes of a situation. *See also* sample space.

Glossary

Word	Definition
expected frequency	The number of times an event is expected from a particular number of trials.
experimental probability	The frequency of an outcome divided by the total frequency.
exponent	The raised number of a power, such as the 4 in 5^4. *See also* index.
expression	Variables and/or numbers connected by operations.
face	Flat or curved surface that forms part of the boundary of a solid (3D) shape.
factor (of a number)	Whole number that divides evenly into a given number; 5 is a factor of 20. *See also* multiple.
fair (sample)	A sample that truly represents a population; not biased.
fair (survey question, method)	A question/method that does not favour particular results.
favourable outcome	The particular outcome being considered.
frequency	The number of times a particular value/outcome occurs.
frequency distribution	The frequencies for all the scores of a data set.
function	A rule that changes a number (input) to a new one (output).
Goods and Services Tax (GST)	Tax paid on the selling price of goods or service; currently 10% in Australia.
hectare	The area of a square 100 m by 100 m; 10 000 m^2.
highest common factor (HCF)	The biggest number that is a common factor of given numbers.
histogram	A column graph of frequencies of continuous data. It has no gaps between columns.
icosahedron	Regular polyhedron with 20 equilateral triangle faces.
impossible	Will never happen.
improbable	Happens less than half the time in the long run. *See also* unlikely.
improper fraction	A fraction more than 1; the numerator is bigger than the denominator.
independent (variable)	Variable used to find the value of a rule or function.
index	The raised number of a power, such as the 4 in 5^4; plural *indices*. *See also* exponent.
integer	Any of zero, positive or negative whole numbers. *See also* directed number.
intersection	Common point of lines that cross each other.
interval	A line between two points called the ends of the interval.
inverse operations	Operations that do the reverse of each other; $\times$ and $\div$, etc.
isosceles (triangle)	Triangle with two equal sides.

Word	Definition
kite	Quadrilateral with two pairs of equal sides, each pair joined at one vertex.
left-hand side (LHS)	The side of an equation on the left of the equals sign.
length	A distance, particularly from one end of a shape to the other.
like terms	Terms of an expression containing exactly the same variable parts.
likely	Happens more than half the time in the long run. *See also* probable.
line graph	A graph where points joined by line(s) show data.
line symmetry	Property of a shape with one or more lines of symmetry.
linear (rule, function)	A rule or function whose graph is a straight line.
linear graph	A line graph that is a straight line.
lowest common denominator (LCD)	The smallest number that is a common denominator of given denominators.
lowest common multiple (LCM)	The smallest number that is a common multiple of given numbers.
mean	The arithmetic average, $\overline{x} =$ total $\div$ number; for 3, 6, 12, $\overline{x} = 21 \div 3 = 7$.
measurement	The value or act of finding a property such as length or mass.
median	The central value, by count, of ordered statistical data.
metric system	The system of measurement based on powers of 10 first devised in France about 1790.
mixed numeral	A number with whole and fraction parts; like $3\frac{1}{4}$.
mode	The value that occurs most often in statistical data; the score with the highest frequency. *See also* bimodal.
multiple (of a number)	Whole number that divides evenly by a given number; 24 is a multiple of 8. *See also* factor.
natural numbers	The numbers 1, 2, 3, 4, . . . Some people include 0. *See also* counting numbers.
negative (number)	Number less than 0; $^{-}6$ or $^{-}23.75$.
net	A flat (2D) shape that can be folded or rolled to form a 3D shape.
number line	A straight line with numbers marked evenly along it.
numerator	The top number of a fraction; the number of parts from one whole.
numeric, numerical (data)	Data that are numbers.
obtuse (angle)	Angle between 90° and 180°.
octahedron	Regular polyhedron with eight equilateral triangle faces.

Glossary

Word	Definition
order (of symmetry)	For point symmetry, the number of different rotations that make the identical shape.
outcomes	The possible results when different things could happen.
outlier	A data value well outside the range of most numerical data.
parallel	Lines or surfaces in the same direction that do not intersect.
parallelogram	A quadrilateral whose opposite sides are parallel.
percentage	A number written as a fraction out of 100, with the denominator (100) replaced by %; $\frac{2}{5} = 40\%$.
perimeter	The length of the boundary of a (flat) closed shape.
perpendicular	A line drawn through another line or surface at a angle of 90°.
picture graph	A graph showing values using different sizes or numbers of pictures.
pie graph (pie chart)	Graph where a circle is divided into sectors in proportion to values. *See also* circle graph, sector graph.
Platonic solid	Regular polyhedron; tetrahedron, cube, octahedron, dodecahedron or icosahedron.
point symmetry	The property that a shape can be rotated less than 360° to make the same shape. *See also* rotational symmetry.
polygon	Flat (2D) closed shape with straight sides.
polyhedron	Solid shape with faces that are polygons; plural *polyhedra* or *polyhedrons*.
population	The entire group that could be relevant to a survey.
positive (number)	Number more than 0, sometimes shown with a + sign; $^{+}7$, $^{+}42$, 5.63.
power	A number multiplied by itself, written as 5^4, say, for $5 \times 5 \times 5 \times 5$.
prime decomposition	Number written as a product of powers of prime numbers.
prime number	Number with exactly two factors, 1 and itself; $2, 3, 5, 7, 11, 13, 17, \ldots$
prism	Polyhedron where every cross-section parallel to one face is identical to that face.
probability	The chance of an event; experimental or theoretical probability.
probable	Happens more than half the time in the long run. *See also* likely.
product	The result of multiplying two or more numbers.
proper fraction	A fraction less than 1; the denominator is bigger than the numerator.
pyramid	Polyhedron such that cross-sections parallel to one face are similar and reduce evenly in size to a vertex called the apex opposite this face.
quadrant	One of the four sections of a Cartesian plane bounded by the axes.

Word	Definition
quadrilateral	Flat (2D) closed shape with four straight sides.
radius	The length of a line from the centre to the circumference of a circle.
range	The difference between the highest and lowest scores in a set of numeric data.
rate	The result of dividing one quantity by a different related quantity; $2.40/kg.
ratio	Comparison of quantities of the same kind; 7 : 2.
reciprocal	The result of turning a fraction upside down; its product with the fraction is 1.
recurring decimal	A decimal with a pattern of digits that repeats endlessly. *See also* repeating decimal.
reflex (angle)	Angle between 180° and 360°.
regular (polygon, polyhedron)	Polygon (or polyhedron) with identical sides, angles (faces and edges).
repeating decimal	A decimal with a pattern of digits that repeats endlessly. *See also* recurring decimal.
revolution	The angle made by rotating a line through a complete circle; 360°.
rhombus	A quadrilateral with all sides the same.
right (angle)	Angle of 90°.
right-hand side (RHS)	The side of an equation on the right of the equals sign.
rotational symmetry	The property that a shape can be rotated less than 360° to appear exactly the same. *See also* point symmetry.
round	Find an approximation with only some place values, such as $5.87 \approx 5.9$ or $7245 \approx 7000$.
sample	Part of a population (usually used for a survey).
sample point	A basic probability outcome; '5' on a die. *See also* element, simple outcome.
sample space	List of all the simple outcomes of a situation. *See also* event space.
satisfy	Make true; (3, 7) satisfies $y = 2x + 1$, $m = 4$ satisfies $3m = 12$.
scale drawing	A drawing where all the shapes are drawn at the same scale.
scale factor	The ratio of the sizes of an enlargement or reduction of a shape. Also called magnification.
scalene (triangle)	A triangle with no sides the same.
score	A possible value for a data item.
similar	Shapes that are enlargements or reductions of each other.
similarity	Property of shapes that are enlargements or reductions of each other.

Glossary

Word	Definition
simple outcome	A basic probability outcome; '5' on a die. *See also* element, sample point.
simplest form (fraction)	The equivalent form with the lowest possible denominator; $\frac{10}{15} = \frac{2}{3}$.
simplify	Multiply out brackets and/or collect like terms.
skew lines	Lines than are not intersecting or parallel.
solid	Having length, width and breadth; occupying a space. *See also* three-dimensional (3D).
solution	The answer to a problem, particularly an equation.
solve	Find the answer to a problem, particularly an equation.
sphere	3D shape such that every point on the surface is the same distance (the radius) from a single point called the centre.
spread	Measure of the differences between the values of a data set.
square (number)	The product of a number with itself.
square root	The number whose square is the original number, such as $\sqrt{25} = 5$.
statistical (data)	Data that is counted to obtain information.
stem-and-leaf plot	Data display with high place values ordered vertically and the other places horizontally.
straight angle	An angle of 180°.
substitution	Calculation of the value of an expression for a particular value(s) of the variable(s).
sum	The result of adding two or more numbers.
supplementary (angles)	Adjacent angles that form a straight line; their sum is 180°.
surface area	The area of a surface, particularly the total area of the faces of a solid shape.
term (number patterns)	One of the values of a number pattern; 6 is the second term of 3, 6, 9, 12, 15, . . .
term (equations and expressions)	Part of an expression separated from other parts by addition or subtraction.
terminating decimal	Decimal with a finite number of digits after the decimal point; it ends.
tetrahedron	Regular polyhedron with four identical faces that are equilateral triangles.
theoretical probability	The number of simple outcomes in a probability event divided by the total number of simple outcomes.
three-dimensional (3D)	Having length, width and height; occupying a space. *See also* solid.
time line	A line with times and events placed along it.

| Word	Definition
time zone	A region of the Earth where all the clocks are set to the same time.
translation	Sliding movement of a shape; all points slide the same amount in the same direction.
transversal	A line that crosses two or more lines.
trapezium	A quadrilateral with one pair of opposite sides parallel.
two-dimensional (2D)	Having length and width; occupying part of a flat surface.
unknown	A symbol, particularly a letter, which stands for a number. *See also* pronumeral, variable.
unlikely	Happens less than half the time in the long run. *See also* improbable.
variable	A symbol, particularly a letter, which stands for a number. *See also* unknown.
vertex	The intersection of two sides of a polygon or three or more faces of a solid shape; plural *vertices*.
vertically opposite (angles)	Opposite angles at the intersection of two lines; they are not adjacent.
vinculum	The line between the numerator and denominator of a fraction.
vulgar fraction	Fraction shown with a denominator and numerator; as $\frac{5}{8}$. *See also* common fraction.
x-axis	The horizontal axis of a Cartesian plane.
x-coordinate	The first coordinate of a point in the Cartesian plane; the 4 in (4, 3).
y-axis	The vertical axis of a Cartesian plane.
y-coordinate	The second coordinate of a point in the Cartesian plane; the 3 in (4, 3).

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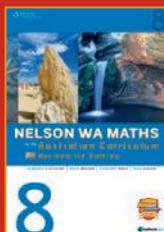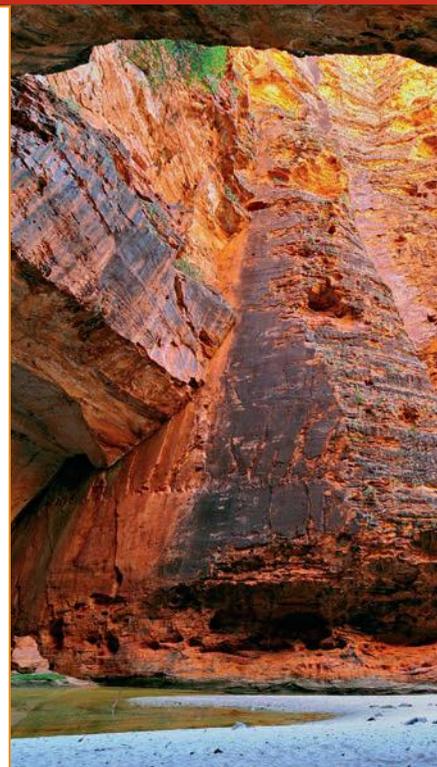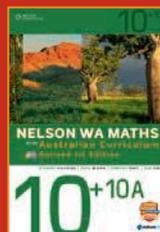
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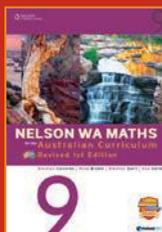
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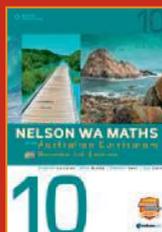
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