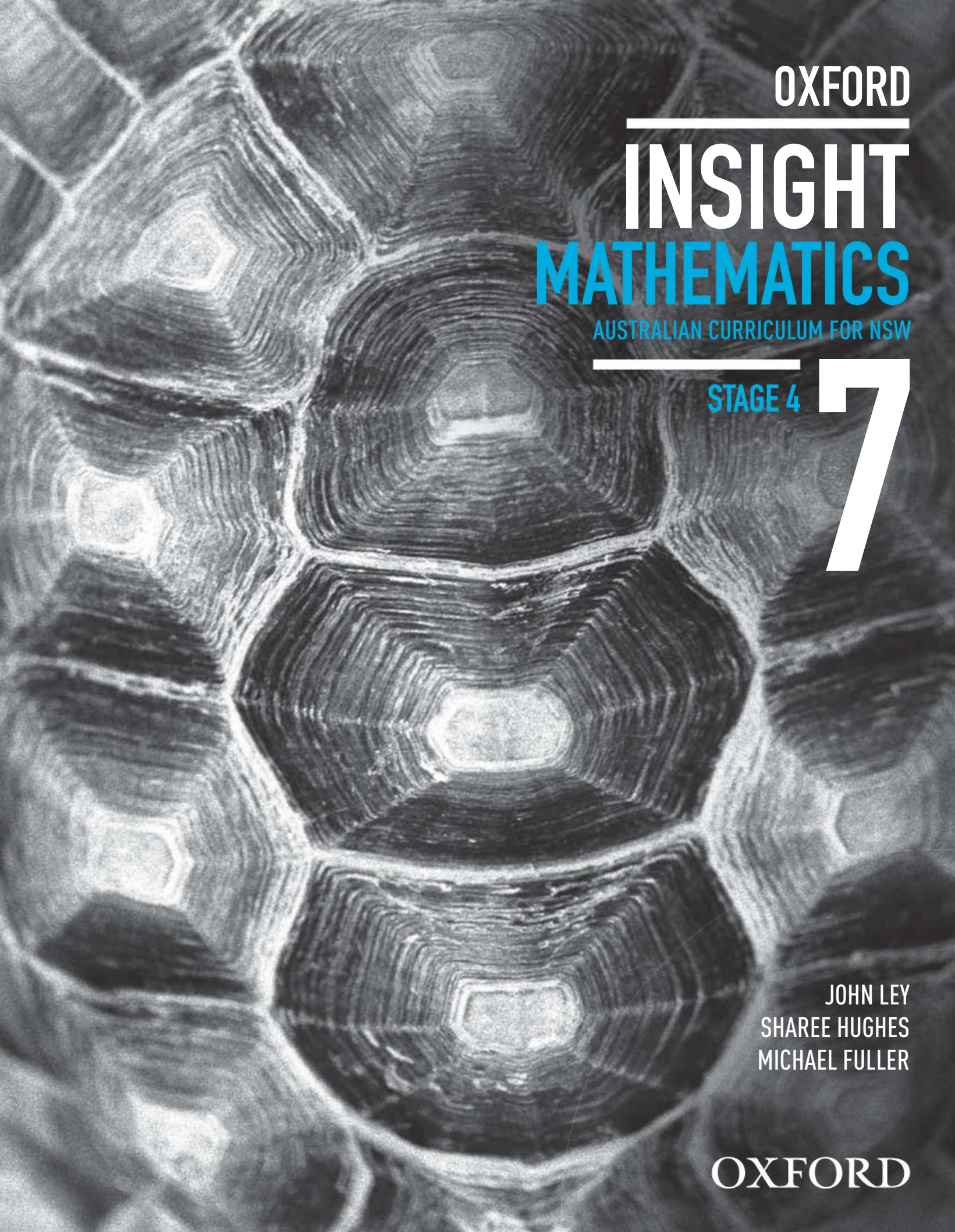

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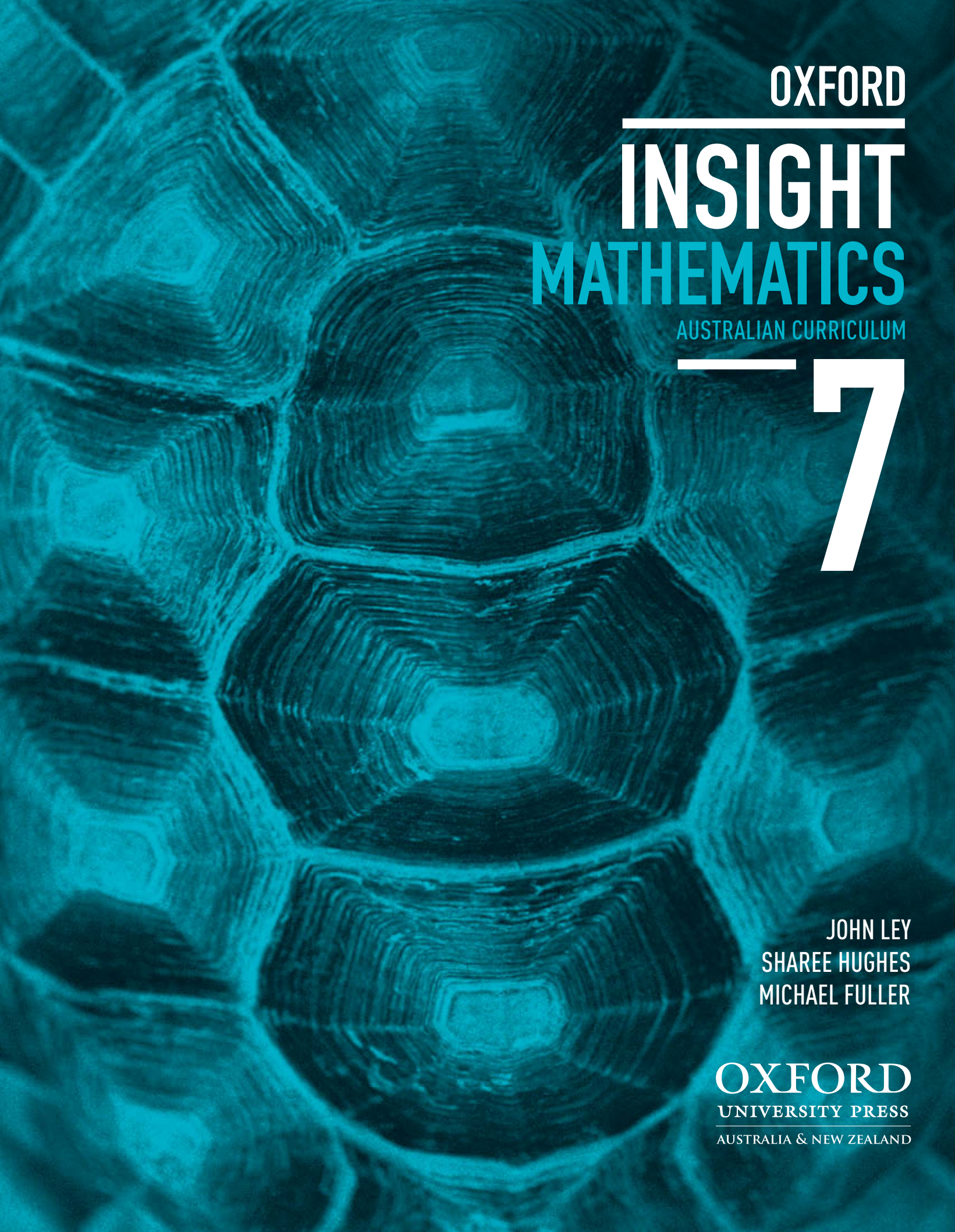

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AUSTRALIAN CURRICULUM

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JOHN LEY
SHAREE HUGHES
MICHAEL FULLER

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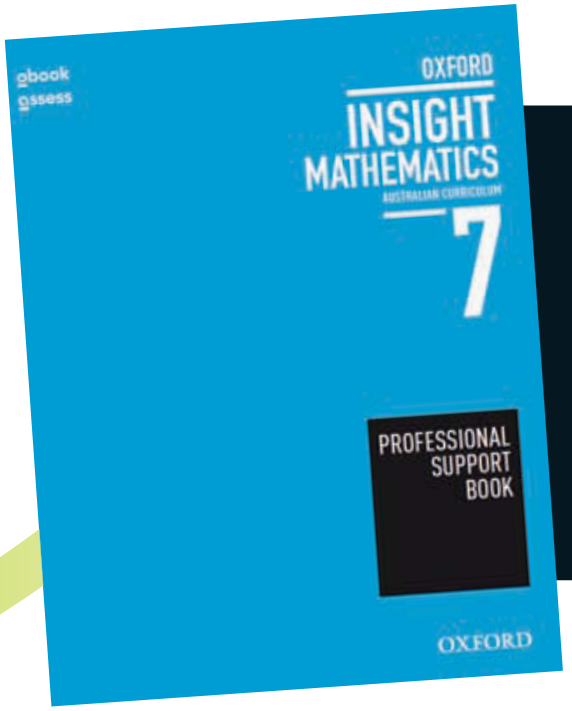

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SYLLABUS GRID

Chapter	Title	Australian Curriculum references	NSW Syllabus references
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3	Angles and parallel lines	ACMMG 163, 164	M&G Angle Relationships
4	Number and indices	ACMNA 149, 150, 151	N&A Indices, Computation with Integers
5	Fractions	ACMNA 152, 153, 154	N&A Fractions, Decimals and Percentages
6	Drawing and building solids	ACMMG 161	M&G Volume
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1

Review of Year 6

This chapter reviews Year 6.

At the end of this chapter you should be able to:

- recall number and place value
- recall fractions and decimals
- recall money and financial mathematics
- recall patterns and algebra
- recall using units of measurement
- recall shape
- recall location and transformation
- recall geometric reasoning
- recall chance
- recall data representation and interpretation.

Diagnostic test

1 The first 4 triangular numbers are shown in the diagram below.

The sixth triangular number is:

A 28 **B** 10 **C** 15 **D** 21

2 What type of number is 25?

A prime **B** square
C even **D** triangular

3 The first 4 prime numbers are:

A 1, 2, 3, 4 **B** 2, 4, 6, 8
C 1, 3, 5, 7 **D** 2, 3, 5, 7

4 Chloe has $3 and Chrissie has $5. How much do they have altogether?

A $2 **B** $8 **C** $35 **D** $15

5 Susan has $23 and gives Chantelle $7. How much does Susan have left?

A $6 **B** $16 **C** $30 **D** $15

6 There are 685 students in a school. If 156 students are in Year 7, the number of students in the other years is:

A 841 **B** 529 **C** 531 **D** 539

7 There are 387 students in the hall. Another 156 students go into the hall. The number of students in the hall is:

A 443 **B** 543 **C** 433 **D** 443

8 The best estimate for 1687 + 489 is:

A 2000 **B** 2200 **C** 2500 **D** 3000

9 Melissa has 3 red envelopes each containing $35. The total amount is:

A $105 **B** $35 **C** $5 **D** $38

10 The whole school is having an assembly. The 15 classes, each containing 24 students, are sent to the hall. The number of students sent to the hall is:

A 360 **B** 120 **C** 320 **D** 220

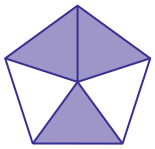
11 The temperature was 8°C. It dropped by 10°C. The temperature is now:

A 18°C **B** 2°C **C** −2°C **D** −18°C

12 The temperature overnight was −5°C. It rose by 12°C. The new temperature is:

A 17°C **B** −17°C **C** −7°C **D** 7°C

13 What fraction is shaded in the diagram below?



A $\frac{3}{5}$ **B** $\frac{2}{5}$ **C** $\frac{1}{5}$ **D** $\frac{4}{5}$

14 Name the numerator in $\frac{8}{9}$.

A 9 **B** 89 **C** 98 **D** 8

15 Express $\frac{11}{5}$ as a mixed numeral.

A $3\frac{1}{5}$ **B** $\frac{5}{11}$ **C** $2\frac{1}{5}$ **D** $2\frac{4}{5}$

16 Which fraction is equivalent to $\frac{1}{3}$?

A $\frac{4}{12}$ **B** $\frac{3}{10}$ **C** $\frac{6}{15}$ **D** $\frac{3}{6}$

17 When simplified, $\frac{8}{10}$ is equivalent to:

A $\frac{2}{5}$ **B** $\frac{4}{2}$ **C** $\frac{5}{4}$ **D** $\frac{4}{5}$

18 Simplify $4 + \frac{2}{3}$.

A $\frac{42}{3}$ **B** $4\frac{2}{3}$ **C** $\frac{34}{2}$ **D** $3\frac{2}{4}$

19 Simplify $\frac{3}{10} + \frac{7}{20}$.

A $\frac{10}{30}$ **B** $\frac{37}{20}$ **C** $\frac{13}{20}$ **D** $\frac{73}{200}$

20 Simplify $\frac{4}{5} - \frac{1}{10}$.

A $\frac{7}{10}$ **B** $\frac{3}{5}$ **C** $\frac{4}{10}$ **D** $\frac{4}{50}$

21 There are 51 people travelling to a sports carnival. If each car carries 4 people, the number of cars needed is:

A 12 **B** $12\frac{3}{4}$ **C** 13 **D** $13\frac{1}{4}$

22 As an improper fraction, $1\frac{4}{7}$ is:

A $\frac{14}{7}$ **B** $\frac{11}{7}$ **C** $1\frac{7}{4}$ **D** $\frac{7}{11}$

23 As a mixed number, $\frac{29}{12}$ is:

A $\frac{29}{12}$ **B** $2\frac{9}{12}$ **C** $2\frac{5}{12}$ **D** $\frac{12}{29}$

24 $3 - \frac{5}{12}$ is:

A $3\frac{5}{12}$ **B** $3\frac{7}{12}$ **C** $2\frac{5}{12}$ **D** $2\frac{7}{12}$

25 $\frac{6}{12} + \frac{5}{12} =$

A $\frac{11}{24}$ **B** $\frac{1}{24}$ **C** $\frac{11}{12}$ **D** $\frac{1}{12}$

26 $\frac{1}{3} + \frac{2}{5} =$

A $\frac{3}{5}$ **B** $\frac{3}{15}$ **C** $\frac{5}{6}$ **D** $\frac{11}{15}$

27 Simplify $5 \times \frac{3}{4}$.

A 4 **B** $3\frac{1}{4}$ **C** $2\frac{2}{5}$ **D** $3\frac{3}{4}$

28 Find $\frac{2}{3}$ of \$90.

A \$33 **B** \$60 **C** \$20 **D** \$30

29 Write $5 + \frac{8}{100}$ in decimal form.

A 5.08 **B** 50.8 **C** 0.508 **D** 508

30 Evaluate $1.2 + 3.06$.

A 4.8 **B** 3.2 **C** 3.26 **D** 4.26

31 Evaluate $8.7 - 5.31$.

A 3.39 **B** 2.66 **C** 2.76 **D** 3.49

32 Evaluate 7×1.4.

A 9.8 **B** 8.3 **C** 7.8 **D** 9.1

33 Evaluate $84.2 \div 2$.

A 4.21 **B** 21.0 **C** 24.1 **D** 42.1

34 Evaluate $8 \div 10$.

A 8 **B** 0.8 **C** 0.08 **D** 80

35 Evaluate 0.18×1000.

A 18 **B** 1.8 **C** 180 **D** 0.0018

36 Express 75% as a simplified fraction.

A $\frac{1}{4}$ **B** $\frac{3}{4}$ **C** $\frac{75}{100}$ **D** $\frac{15}{20}$

37 What fraction is equivalent to $33\frac{1}{3}\%$?

A 33 **B** $\frac{1}{3}$ **C** $33\frac{1}{3}$ **D** 3

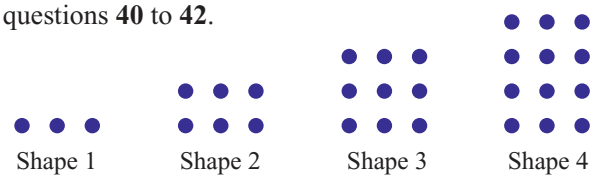
38 A \$70 shirt has 10% off. The new *price* is:

A \$70 **B** \$63 **C** \$10 **D** \$7

39 A \$250 camera is to be discounted by 20%. The *discount* is:

A \$200 **B** \$50 **C** \$230 **D** \$270

Use the pattern of counters shown below to answer questions **40** to **42**.



40 The number of counters needed to make shape 5 is:

A 15 **B** 25 **C** 5 **D** 10

41 A rule that could *not* be used to find the number of counters needed for each shape is:

A it's the 3 times table
B start with 3 and add 3
C multiply the shape number by 3
D add 3 to the shape number

42 The number of counters needed to make shape 100 is:

A 100 **B** 300 **C** 103 **D** 130

For questions **43** to **45** consider the number pattern 6, 7, 8, 9, …

43 The 5th term of the number pattern is:

A 5 **B** 8 **C** 10 **D** 30

44 A rule that could be used to find each term given its position number is:

A position number + 5
B position number + 6
C position number × 6
D position number × 6 + 1

45 The 100th term of the pattern is:

A 105 **B** 106 **C** 600 **D** 109

46 $6 + 3 \times 4 =$

A 36 **B** 18 **C** 13 **D** 7

47 $4 \times (2 + 5) - 1 =$

A 27 **B** 12 **C** 6 **D** 0

48 5 cm 3 mm is equal to:

A 53 cm **B** 5.3 cm
C 0.53 cm **D** 5300 cm

49 43.2 cm is equal to:
A 4.32 mm **B** 432 mm
C 0.432 mm **D** 4320 mm

50 7 m is equal to:
A 70 cm **B** 7000 cm
C 0.7 cm **D** 700 cm

51 4 m 65 cm is equal to:
A 46.5 cm **B** 465 cm
C 4650 cm **D** 4.65 cm

52 3.25 m is equal to:
A 32.5 cm **B** 3.25 cm
C 325 cm **D** 3250 cm

53 300 cm is equal to:
A 30 m **B** 300 m
C 3000 m **D** 3 m

54 In millilitres, 0.038 L is:
A 0.38 mL **B** 3.8 mL
C 38 mL **D** 380 mL

55 0.035 t in kg is:
A 0.35 kg **B** 3.5 kg
C 35 kg **D** 350 kg

56 The mass of 4 L 300 mL of water is:
A 4.3 g **B** 4.3 kg
C 304 g **D** 4300 kg

57 The number of square metres in 3 ha is:
A 3000 **B** 9000
C 30 000 **D** 90 000

58 160 000 m^2 is:
A 400 ha **B** 16 ha
C 160 ha **D** 1600 ha

59 5.832 km is equal to:
A 5 km 83 m 2 cm
B 5 km 832 m
C 5 km 8 m 3 cm 2 mm
D 5 km 832 cm

60 Three measurements in ascending order are:
A 90 mm, 8.5 cm, 0.88 m
B 8.5 cm, 90 mm, 0.88 m
C 8.5 cm, 0.88 m, 90 mm
D 0.88 m, 8.5 cm, 90 mm

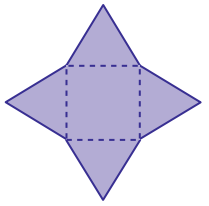
61 The area of a square with sides of 5 cm is:
A 5 cm^2 **B** 20 cm^2
C 25 cm^2 **D** 10 cm^2

62 A rectangle with area 56 cm^2 has length 8 cm. The breadth is:
A 8 cm **B** 48 cm
C 20 cm **D** 7 cm

63 Find the capacity of a bottle with a volume of 155 cm^3.
A 155 mL **B** 155 L
C 155 ML **D** 1.55 mL

64 Gavin catches the 7:45 am bus from Wentworthville station to school. If the journey takes 35 min, at what time does he arrive at school?
A 8:20 am **B** 8:18 am
C 8:13 am **D** 8:15 am

65 Ferries run every 42 min. If the first ferry departs at 8:13 am. when does the fourth depart?
A 11:01 am **B** 8:55 am
C 10:19 am **D** 9:37 am

66 Which of the following shapes could be the net of a triangular prism?
A

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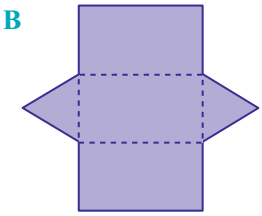
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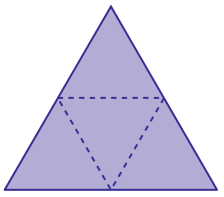
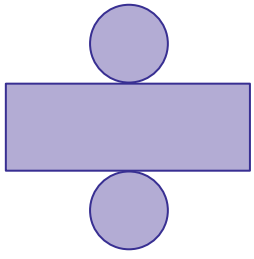
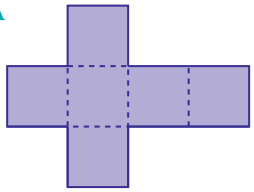

D


67 The diagram below shows the net of a:
A cylinder **B** cone
C sphere **D** cube

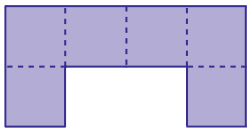


68 Which of the following shapes could *not* be the net of a cube?

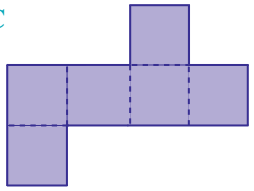
A



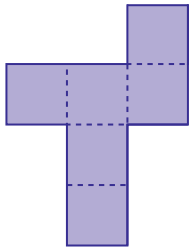
B



C

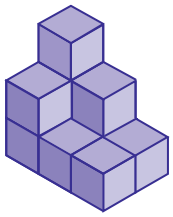


D

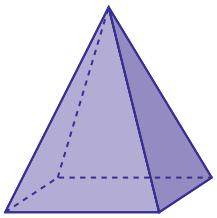


69 The number of cubes used to make the solid below is:

A 7 **B** 8 **C** 9 **D** 10



Use the diagram of a solid shape below to answer questions **70** and **71**.



70 The number of edges in the solid is:

A 5 **B** 8 **C** 4 **D** 1

71 The base of the solid is a:

A rectangle **B** triangle
C circle **D** rectangle or triangle

72 The following is a view of a solid from above.

The solid could be a:

A rectangular pyramid
B rectangular prism
C triangular pyramid
D triangular prism

73 Diagram 2 below is formed from diagram 1.

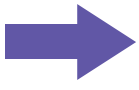


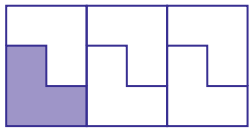
Diagram 1

Diagram 2

The transformation is:

A reflection **B** rotation
C translation **D** reversal

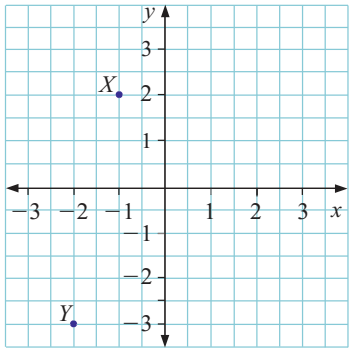
74 The pattern below is formed from  .



The pattern is made using:

A reflection only
B rotation only
C translation only
D combined transformations

Use the diagram of a number plane below to answer questions **75** and **76.**



75 The coordinates of X are:

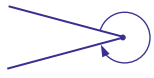
A $(2, -1)$ **B** $(-1, 2)$
C $(1, 2)$ **D** $(2, 1)$

76 The coordinates of Y are:

A $(-2, -3)$ **B** $(-3, -2)$
C $(2, 3)$ **D** $(3, 2)$

77 The type of angle drawn below is:

A acute **B** obtuse
C reflex **D** revolution



78 An obtuse angle is:

A $> 0°$ but $< 90°$ **B** equal to $90°$
C $> 90°$ but $< 180°$ **D** $> 180°$ but $< 360°$

79 A straight angle is:

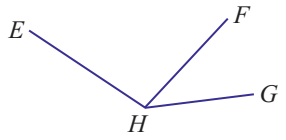
A equal to 90°

B $>$ 180° but $<$ 360°

C equal to 180°

D $>$ 90° but $<$ 180°

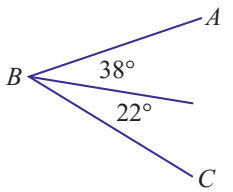
80 Name the pair of adjacent angles in the diagram below.



A $\angle EHG$ $\angle EHF$ **B** $\angle FGH$ $\angle GHE$

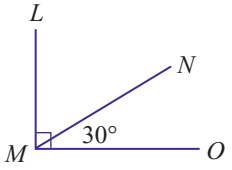
C $\angle EHG$ $\angle GFE$ **D** $\angle EHF$ $\angle FHG$

81 Calculate the size of $\angle ABC$.



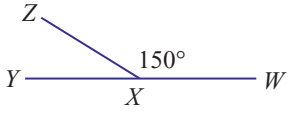
A 60° **B** 38° **C** 22° **D** 50°

82 Calculate the size of $\angle LMN$.



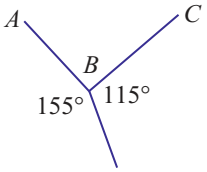
A 90° **B** 60° **C** 26° **D** 54°

83 Calculate the size of $\angle YXZ$.



A 180° **B** 150° **C** 30° **D** 210°

84 What is the size of $\angle ABC$?



A 30° **B** 90° **C** 110° **D** 50°

85 A normal die is thrown once. The number of elements in the sample space is:

A 1 **B** 3 **C** 5 **D** 6

86 A ticket is selected from a hat containing blue, red and green tickets. The sample space is:

A {blue, red, green} **B** {blue}

C {red} **D** {green}

87 A bag contains 1 pink, 1 purple and 1 orange ball. The probability of selecting a purple ball is:

A 1 **B** 3 **C** $\frac{1}{3}$ **D** $\frac{2}{3}$

88 The probability of an event is $\frac{2}{5}$. As a percentage this is:

A 20% **B** 40%

C 50% **D** none of these

89 A hat contains 3 blue and 5 black tickets. One ticket is chosen from the hat. The probability that it is blue is:

A $\frac{1}{2}$ **B** $\frac{3}{5}$ **C** 3 **D** $\frac{3}{8}$

90 A letter is chosen from the word PROBABILITY. The probability that it is a B is:

A $\frac{1}{11}$ **B** $\frac{2}{11}$ **C** 1 **D** $\frac{2}{9}$

91 A term describing a probability of about 80% is:

A certain **B** highly probable

C evens **D** low probability

92 A coin is tossed once. An impossible event would be getting:

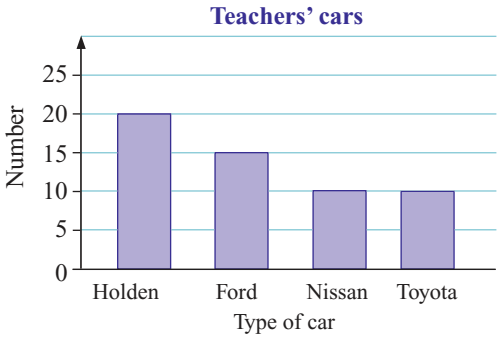
A heads **B** tails

C heads or tails **D** a six

93 Using the key '  represents 10 chairs', the number represented by the diagram below is:

A 20 **B** 30 **C** 40 **D** 45

Use the graph below to answer questions **94** to **96**.



94 The title of this graph is:

A Number **B** Type of car
C Teachers' cars **D** Holden

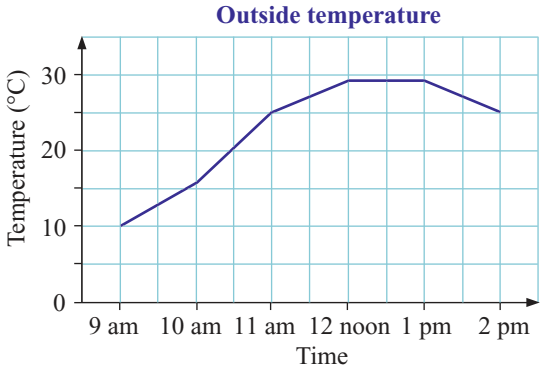
95 The label of the horizontal axis is:

A Number **B** Type of car
C Teachers' cars **D** Holden

96 The number of Nissans is:

A 20 **B** 15 **C** 10 **D** 55

Use the graph below for questions **97** to **100**.



97 The type of graph is:

A sector **B** column **C** bar **D** line

98 The temperature at 11 am is:

A 10°C **B** 20°C **C** 25°C **D** 30°C

99 The vertical axis goes up by:

A temperature **B** 1°C
C 5°C **D** 10°C

100 An estimate for the temperature at 9:30 am is:

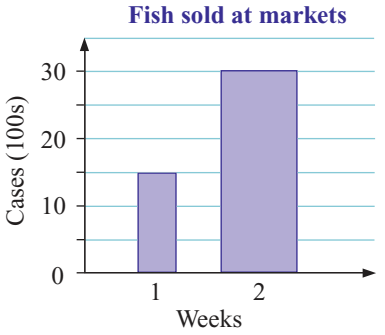
A 10°C **B** 13°C **C** 15°C **D** 20°C

101 In a survey, 85% of people stated that smoking should be allowed in restaurants. Which group was probably surveyed?

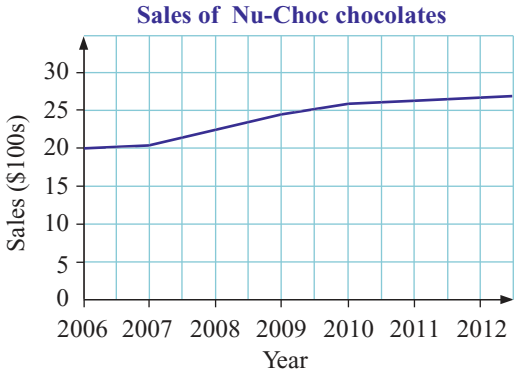
A any member of the population
B people in a restaurant
C people exiting a tobacco store
D restaurant owners

102 Which graph accurately represents the information?

A

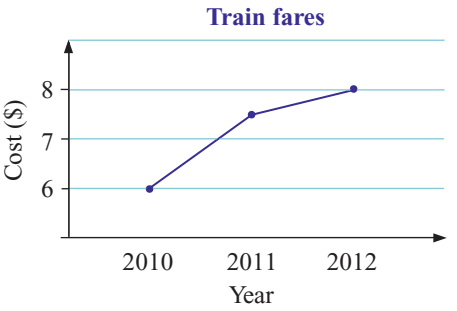


B



C

D



The Diagnostic Test questions refer to the Year 6 syllabus as indicated in the following table.

Question number	1–12	13–37	38–39	40–47	48–65
Year 6 syllabus section	Number and place value	Fractions and decimals	Money and financial mathematics	Patterns and algebra	Using units of measurement
Question number	66–72	73–76	77–84	85–92	95–102
Year 6 syllabus section	Shape	Location and transformation	Geometric reasoning	Chance	Data representation and interpretation

2 Integers

This chapter deals with operations with positive and negative integers.

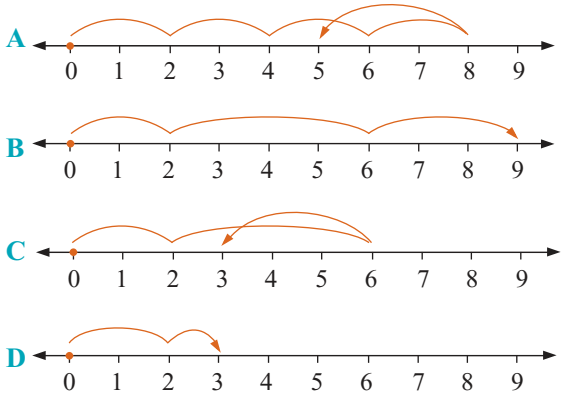
At the end of this chapter you should be able to:

- plot and order numbers using the number line
- use the number line to show operations with numbers
- understand the need for negative numbers
- describe situations involving negative numbers
- understand that the number line can be extended to include negative numbers
- perform addition and subtraction with positive and negative numbers
- use grouping symbols and order of operations to simplify expressions
- investigate, interpret and analyse graphs.

NUMBER & ALGEBRA – ACMNA 178, 180, 280

Diagnostic test

1 Which of the following number lines represents the expression: $2 + 2 + 2 + 2 - 3$?



2 The temperature at midday was 15°C. By 5 am the following morning, the temperature had dropped 20°C. The temperature at 5 am was:

A −5°C **B** 5°C
C 35°C **D** −20°C

3 A lift goes up 4 floors, then down 7 floors. Where is the lift in relation to where it started?

A 11 floors up **B** 3 floors down
C 11 floors down **D** 3 floors up

4 In ascending order 3, −5, 0, −2, −1, 1 are:

A −5, 3, −2, −1, 1, 0
B −2, −1, 0, 1, 3, −5
C 0, −1, 1, −2, 3, −5
D −5, −2, −1, 0, 1, 3

5 Which of the following numbers is not an integer?

A 0 **B** 0.5
C 3 **D** −2

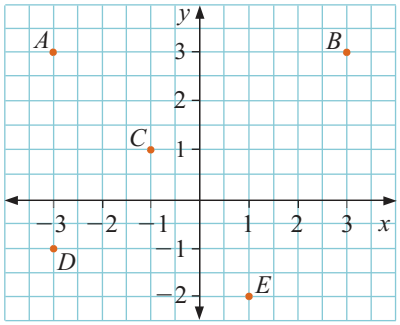
6 Start at position −7 on a number line, then move 2 places to the left. Your final position will be:

A −5 **B** 5
C 9 **D** −9

7 Start at position −2 on a number line, then move 5 places to the left and 6 places to the right. Your final position will be:

A 13 **B** −13
C −1 **D** 1

Use this number plane for questions **8** to **12**.



8 Points A and C are in the:

A 1st quadrant **B** 2nd quadrant
C 3rd quadrant **D** 4th quadrant

9 The point where the two axes cross is called the:

A intersection **B** horizontal axis
C origin **D** vertical axis

10 The coordinates of the point E are:

A (3, 3) **B** (1, –2)
C (−1, 1) **D** (−3, −1)

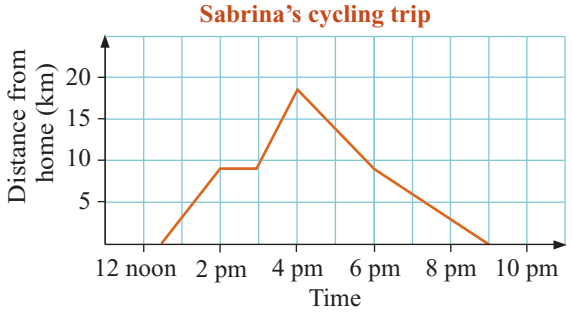
11 Which of these two points have the same x-coordinate?

A A and B **B** A and D
C C and E **D** D and E

12 Tom is looking for all the points that have a negative x-coordinate and a positive y-coordinate. Which points are possible?

A D and E **B** A and C
C A and B **D** B and E

Refer to the distance–time graph below for questions **13** to **16**.



13 The label on the x-axis is:

A Distance from home **B** Time
C Sabrina's cycling trip **D** km

14 The time Sabrina started her journey was:

A 12 noon **B** 1 pm
C 12:30 pm **D** 4 pm

15 For part of the journey, Sabrina travelled away from home, and for part of it she travelled towards home. Which statement is *not* true?

A At 1 pm she was travelling away from home.
B At 4 pm she was travelling away from home.
C At 5 pm she was travelling towards home.
D Sabrina arrived home at 9 pm.

16 The reason for the horizontal section of the graph is that Sabrina:

A has stopped cycling
B is travelling really fast
C has turned around
D has slowed down.

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–7	8–16
Section	ACMNA 124	ACMMG 143

Number line review

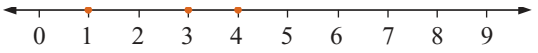
As you move right on a **number line**, the numbers increase (**ascend**). As you move left, the numbers decrease (**descend**).

EXAMPLE 1

Plot each group of numbers on a separate number line, then write the numbers in ascending order.

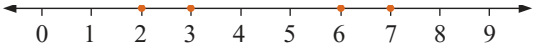
a 3, 4, 1 **b** 2, 6, 7, 3 **c** $2, 2\frac{1}{2}, 1\frac{1}{2}$

a



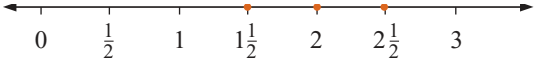
1, 3, 4

b



2, 3, 6, 7

c



$1\frac{1}{2}, 2, 2\frac{1}{2}$

Use a dot to show the position of each number.

Exercise 2A

1 Plot each group of numbers on a separate number line, then write the numbers in ascending order.

a 5, 3, 6, 2 **b** 3, 1, 5, 8 **c** 15, 12, 16, 10
d $3\frac{1}{2}, 1, 2\frac{1}{2}, 4$ **e** 1.5, 2.8, 0.3 **f** $5\frac{1}{2}, 4\frac{1}{4}, 4\frac{3}{4}$

2 Plot the following numbers on separate number lines.

a the first eight multiples of 2 **b** the factors of 6
c the first five multiples of 4 **d** the factors of 10
e the first six multiples of 3 **f** the factors of 12
g five numbers starting at 3, going up by 2s **h** five numbers starting at 1, going up by 3s
i the numbers starting at 12 and going down by 4s until you reach zero
j the numbers starting at 15 and going down by 3s until you reach zero

EXAMPLE 2

Using separate number lines to help you, insert $>$ or $<$ symbols to make the statements true.

a $5 \_\_\_ 3$ **b** $3\frac{1}{4} \_\_\_ 3\frac{1}{2}$

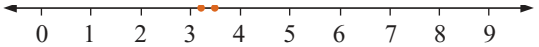
$>$ means 'is larger than'.
$<$ means 'is smaller than'.

The symbols $>$ and $<$ point to the smaller number.

a

5 is to the right of 3, so it is larger than 3: $5 > 3$.

b



$3\frac{1}{4}$ is to the left of $3\frac{1}{2}$, so it is smaller than $3\frac{1}{2}$: $3\frac{1}{4} < 3\frac{1}{2}$.

3 Using separate number lines to help you, insert $>$ or $<$ symbols to make the statements true.

a $6 \_\_\_ 4$ **b** $3 \_\_\_ 8$ **c** $7 \_\_\_ 5$
d $4 \_\_\_ 3$ **e** $3\frac{3}{4} \_\_\_ 2\frac{1}{2}$ **f** $2\frac{1}{3} \_\_\_ 2\frac{2}{3}$
g $15 \_\_\_ 14\frac{1}{2}$ **h** $4.3 \_\_\_ 5.2$

4 **a** Plot the numbers 4 and 7 on a number line.
b Write a statement using $<$ to describe the numbers.
c Write a statement using $>$ to describe the numbers.
d Write two whole numbers between 4 and 7.
e Write three other numbers between 4 and 7.

5 **a** Plot the numbers 9 and 10 on a number line.
b Write a statement using $<$ to describe the numbers.
c Write a statement using $>$ to describe the numbers.
d Write three numbers between 9 and 10.

6 Plot each pair of numbers on a separate number line, and write two other numbers between them.

a 5 and 6 **b** 10 and 11 **c** $\frac{1}{2}$ and 1 **d** 0 and $\frac{1}{4}$
e 1.3 and 1.5 **f** 2.7 and 2.8 **g** 0.1 and 0.2 **h** 0 and 0.1

7 How many numbers are there between any two numbers on a number line?

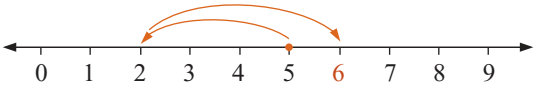
EXAMPLE 3

Use number lines to show the following operations and hence find the answers.

a $5 - 3 + 4$ **b** $3 \times 4 - 2$

a Start at 5. Move 3 left, then 4 right.

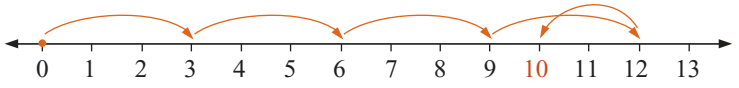
When adding, move to the right.
When subtracting, move to the left.



$5 - 3 + 4 = 6$

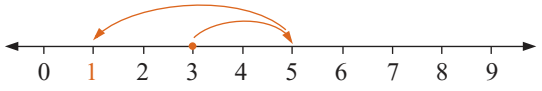
b Start at 0. Move 3 right four times, then 2 left.

When multiplying first, start at 0.



$3 \times 4 - 2 = 10$

8 Copy the number line showing $3 + 2 - 4$ and hence find the answer.



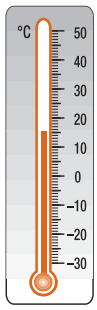
$3 + 2 - 4 =$ _____

9 Represent each set of operations on a number line and hence find the answer.

a $4 + 5 - 3$	**b** $2 + 4 - 5$	**c** $3 + 7 - 4$	**d** $6 - 4 + 1$
e $3 \times 4 + 2$	**f** $2 \times 5 - 4$	**g** $4 \times 2 - 5$	**h** $2 + 3 \times 5$
i $6 + 3 - 5$	**j** $3 \times 4 - 5$	**k** $1 + 5 - 3$	**l** $2 \times 3 - 5$

Investigation 1 Directed numbers

1 Temperature



Winter weather reports sometimes describe the temperature as 'below zero'; for example, '3 degrees below zero'. We measure temperature in degrees Celsius. The freezing point of water is the zero for the Celsius system. So if the temperature is below the freezing point of water, it is below zero. What is the new temperature if it is:

a 5°C and the thermometer drops by 4°C?
b 5°C and the thermometer drops by 5°C?
c 5°C and the thermometer drops by 6°C?
d 5°C and the thermometer drops by 7°C?
e 5°C and the thermometer drops by 8°C?
f 10°C and the thermometer drops by 13°C?

2 Golf

In golf, each hole is given a **par** score. The par score is the number of strokes or shots that it should take a golfer to complete the hole. For example, if a hole is rated as a par 4, the golfer should expect to need 4 strokes to complete the hole. The number of strokes taken is the golfer's score. If the golfer's score is greater than par, it is said to be 'over par'. If the golfer's score is less than par, it is said to be 'under par'. When two or more golfers compete, the *lowest* score wins!

a If the hole is:

i par 4 and the golfer's score is 5, the player is _____ over par.
ii par 4 and the golfer's score is 6, the player is _____ over par.
iii par 4 and the golfer's score is 4, the player is _____ with par.
iv par 4 and the golfer's score is 3, the player is _____ under par.
v par 4 and the golfer's score is 2, the player is _____ under par.
vi par 4 and the golfer's score is 1, the player is _____ under par.
vii par 5 and the golfer's score is 6, the player is _____ _____ par.
viii par 5 and the golfer's score is 3, the player is _____ _____ par.
ix par 3 and the golfer's score is 2, the player is _____ _____ par.

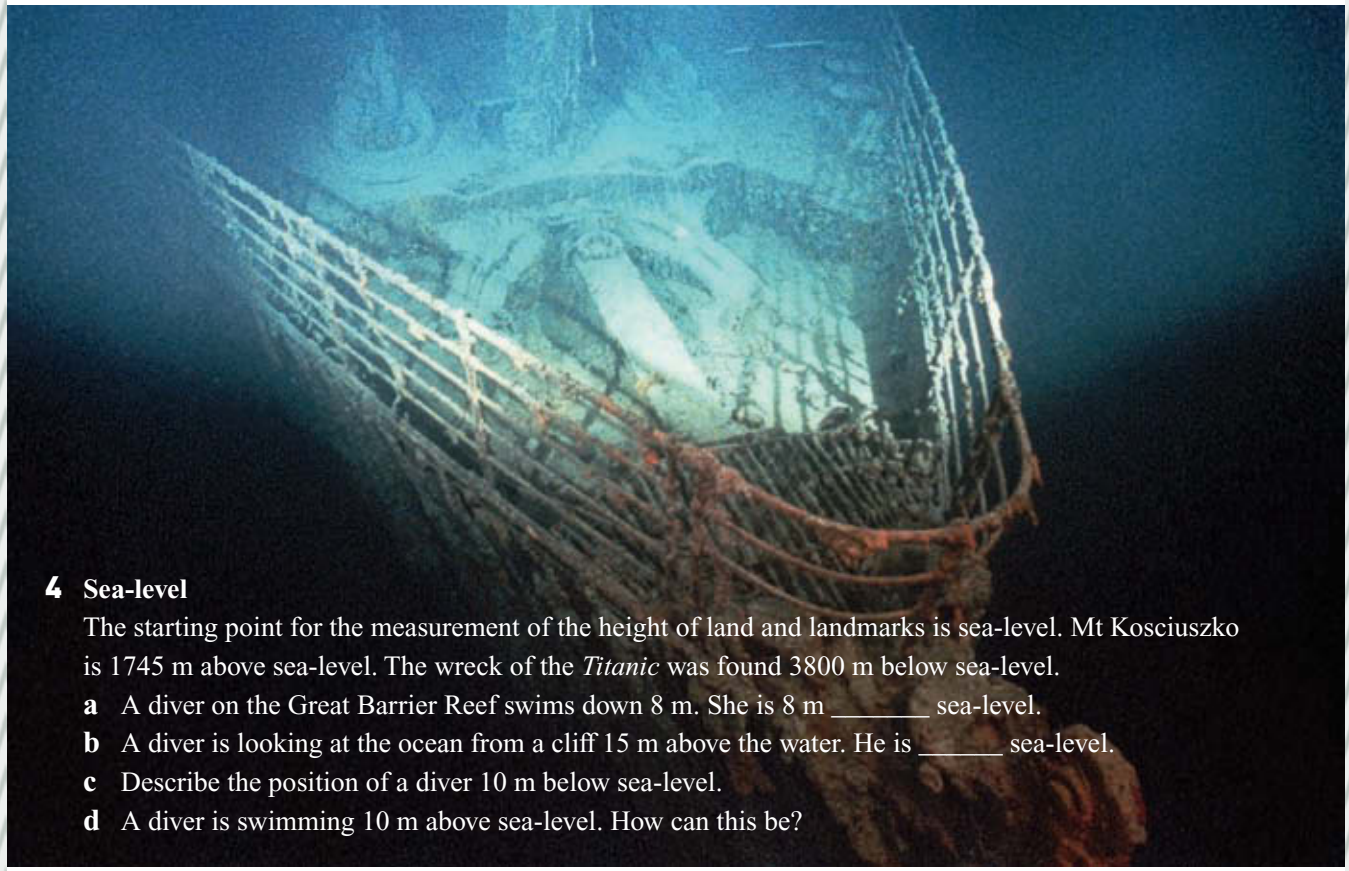
b Find the meanings of the golfing terms Eagle, Albatross and Birdie.

c Find some other golfing terms to describe scores.

3 Time

The modern western calendar commences with the birth of Christ. Times before this are said to be BC. Times after this are said to be AD. The pyramids in Egypt were built in about 2600 BC. The Sydney Olympic Games were held in 2000 AD.

a Stonehenge was built about 1650 years before Christ, or in about _____ BC.
b The Parthenon temple was built 438 years before Christ, or in _____ BC.
c Give the date of an event occurring 150 years after Christ.
d Give the date of an event occurring 550 years after Christ.
e Give the date of an event occurring 250 years before Christ.
f Give the date of an event occurring 1050 years before Christ.
g Find the meanings of AD and BC.
h In recent years, the terms BCE and CE have been used instead of BC and AD. Find the meanings of BCE and CE.



4 Sea-level

The starting point for the measurement of the height of land and landmarks is sea-level. Mt Kosciuszko is 1745 m above sea-level. The wreck of the *Titanic* was found 3800 m below sea-level.

a A diver on the Great Barrier Reef swims down 8 m. She is 8 m _____ sea-level.
b A diver is looking at the ocean from a cliff 15 m above the water. He is _____ sea-level.
c Describe the position of a diver 10 m below sea-level.
d A diver is swimming 10 m above sea-level. How can this be?

Investigation 2 Temperature scales

There are several temperature scales: Celsius (or Centigrade), Fahrenheit and Kelvin.

1 For each of these scales, find the value of the freezing point of water at sea-level.

2 For each of these scales, find the value of the boiling point of water at sea-level.

3 For each scale, find the reasoning behind the position of zero.

4 Find out about absolute zero.

5 Air is mostly ($\frac{4}{5}$ or 80%) nitrogen. Find the boiling point of liquid nitrogen.

B Negative numbers

Investigation 1 has shown that **positive numbers** do not describe all situations. **Negative numbers** are the opposite of positive numbers. The symbol (or sign) for a positive number is +, and the symbol for a negative number is −.

There are many examples of opposites.

- Batteries have a + end and a − end.
- Magnets have a north pole and a south pole.

There are many terms that describe positives and negatives. For example:

- positives: above, increase, right, fast, win
- negatives: below, decrease, left, slow, loss.

EXAMPLE 1

Write the opposites of the following statements.

a 3 kg decrease in weight
b 10 seconds after take-off
c 4 floors above ground level
d 3 hours before dinner

a 3 kg increase in weight
b 10 seconds before take-off
c 4 floors below ground level
d 3 hours after dinner

Exercise 2B

1 Write the opposite of each of the following statements.

a depositing $10
b a decrease of 10 cm in length
c a gain of 5 kg in weight
d a fall of 5°C in temperature
e 200 m above sea-level
f 5 km north
g going up 3 flights of stairs
h 3 degrees below zero
i 10 km east

EXAMPLE 2

State the combined effect of each of the following.

a a deposit of $10 then a withdrawal of $13
b a $15 withdrawal then a $10 withdrawal

a a $3 withdrawal
b a $25 withdrawal

2 State the combined effect of each of the following.

a a deposit of $10 then a withdrawal of $14
b a deposit of $10 then a withdrawal of $18
c a deposit of $10 then a withdrawal of $7
d a deposit of $10 then a withdrawal of $10
e a deposit of $8 then a withdrawal of $13
f a $10 withdrawal then an $8 withdrawal
g a $10 withdrawal then a $16 withdrawal
h a $5 withdrawal then a $12 withdrawal
i a withdrawal of $5 then a deposit of $10
j a withdrawal of $10 then a deposit of $6

EXAMPLE 3

If north is the positive direction, write **directed numbers** for these journeys:

a 3 km north **b** 5 km south **c** 4 km south then 2 km north

a $+3$ -5 $-4 + 2$

A directed number has a + or − sign.

3 If north is the positive direction, write directed numbers for these journeys:

a 4 km north **b** 6 km south **c** 3 km south then 7 km north

d 5 km south then 2 km north **e** 6 km north then 4 km south **f** 3 km south then 5 km north

4 If east is the positive direction, write directed numbers for these journeys:

a 4 km west **b** 6 km east **c** 5 km east then 7 km west

d 8 km west then 3 km east **e** 10 km east then 7 km west **f** 4 km west then 2 km west

5 If down is the negative direction, write directed numbers for someone travelling in a lift:

a 3 floors up **b** 6 floors down

c 3 floors down then 7 floors up **d** 8 floors down then 5 floors up

e 4 floors up then 7 floors down **f** 3 floors down then 4 floors down

6 If position above sea-level is the positive direction, write directed numbers for a diver's position:

a 65 m above sea-level **b** 15 m below sea-level

c 25 m above sea-level followed by 8 m below sea-level

7 Write descriptions of the following directed numbers.

a -3 **b** $+9$ **c** -95 **d** $-6 + 8$

EXAMPLE 4

Write as a directed number the opposite of:

a $+6$ **b** -5 **c** 3 **d** $3\frac{4}{5}$ **e** -4.76

The + sign may be omitted if there is no confusion.

a -6 **b** $+5$ **c** -3 **d** $-3\frac{4}{5}$ **e** $+4.76$

8 Write as a directed number the opposite of:

a -4 **b** $+9$ **c** -2.45 **d** $+4\frac{1}{3}$

EXAMPLE 5

Write a number sentence describing each situation, and state the final resulting floor.

a I am on the 3rd floor of a building and travel down 5 floors to the carpark. I then go up 10 floors to my office.

b I park my car on the 5th basement level of the underground carpark, 5 floors below the ground floor. I take the lift up 8 floors, then walk up 3 more floors.

The ground floor is the zero floor.

a $3 - 5 + 10 = 8$. I am on the 8th floor.

b $-5 + 8 + 3 = 6$. I am on the 6th floor.

9 Write a number sentence describing each situation, and state the final resulting floor.

a I am on the 4th floor of a building and travel down 6 floors to the carpark. I then go up 9 floors to my office.

b I am on the 7th floor of a building and travel down 15 floors to the carpark. I then go up 20 floors to my office.

c I park my car on the 6th basement level of the underground carpark, 6 floors below the ground floor. I take the lift up 18 floors, then walk up 2 more floors.

d I park my car on the 8th basement level of the underground carpark, 8 floors below the ground floor. I take the lift up 15 floors, then walk down 2 floors.

10 Write stories about a lift, describing the following number sentences.

a $+6 - 8 + 5$ **b** $3 - 8 + 12$

c $-3 + 8 + 5$ **d** $-2 + 10 + 4$

e $-3 + 8 - 4$ **f** $-2 - 4 + 2$

C Extending the number line

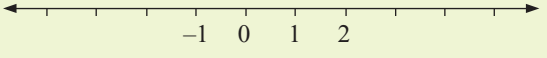
The number line may be extended to the left to include numbers less than zero.

Numbers less than zero are negative numbers.

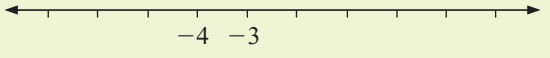
EXAMPLE 1

Complete the following number lines.

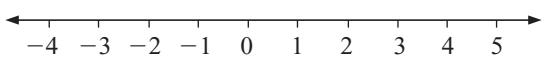
a



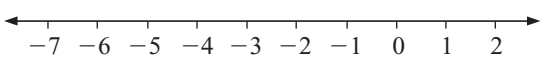
b



a

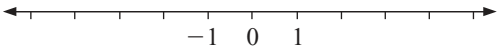


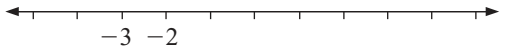
b

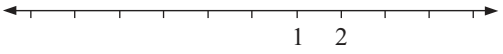


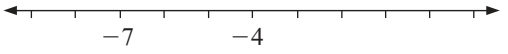
Exercise 2C

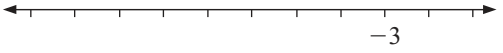
1 Complete the following number lines.

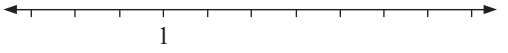
a


b


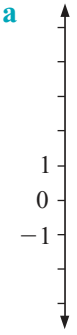
c


d


e


f


2 Number lines can be vertical. Up is considered the positive direction. Complete the following number lines.

a


b


c

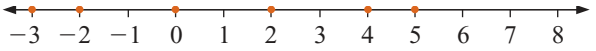

d


e

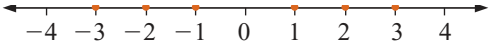

EXAMPLE 2

Plot each set of numbers on a number line, then write them in ascending order.

a 4, −2, 2, −3, 5, 0 **b** 2, −3, 1, −2, −1, 3

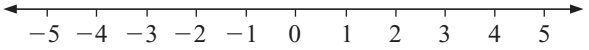
a


In ascending order the numbers are −3, −2, 0, 2, 4, 5.

b


In ascending order the number are −3, −2, −1, 1, 2, 3.

3 Plot the numbers −4, 0, 3, 1, −2 on this number line, then write them in ascending order.



In ascending order the number are ___, ___, ___, ___, ___.

4 Plot each set of numbers on a number line, then write them in ascending order.

a −11, −7, −6, −14, −8 **b** −3, −5, −8, 0, 5

c −4, 3, −3, 0, 4, 2 **d** 7, −4, 4, −1, 0, −5

e −1, −3, 0, 1, 4, −4 **f** −2, 0, −3, −4, 1, −1

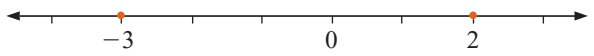
g 1, −1, −2, 2, 4, −4 **h** −58, −55, −40, −42, −51, −61

EXAMPLE 3

Using a number line to help you, insert a $>$ or $<$ symbol to make a true statement for each pair of numbers.

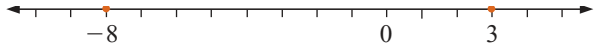
a $2 \_\_\_ -3$ **b** $-8 \_\_\_ 3$ **c** $-3 \_\_\_ -4$ **d** $2 \_\_\_ 0$

a



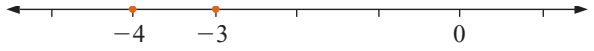
2 is to the right of -3, so it is greater than -3: $2 > -3$.

b



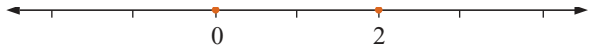
-8 is to the left of 3, so it is less than 3: $-8 < 3$.

c



-3 is to the right of -4, so it is greater than -4: $-3 > -4$.

d



2 is to the right of 0, so it is greater than 0: $2 > 0$.

$>$ means 'is greater than'.
$<$ means 'is less than'.
The symbol $>$ or $<$ points to the lesser number.

5 Using a number line to help you, insert a $>$ or $<$ symbol to make a true statement for each pair of numbers.

a $-6 \_\_\_ -5$ **b** $-2 \_\_\_ 1$ **c** $0 \_\_\_ -3$ **d** $3 \_\_\_ -1$

e $-102 \_\_\_ -110$ **f** $-1 \_\_\_ 2$ **g** $-7 \_\_\_ -5$ **h** $13 \_\_\_ -11$

6 For each set of letters, write the letters below their correct positions on a number line. What is the message?

a V -2, A 3, S 6, L -4, I -6, H 5, E -1, M 2, O -3, T 4

b I -3, H -8, O -6, T 4, G 0, S -2, S -10, O -7, E 2, R 1, L -5, C -9, A 3

c D 3, T -2, R 1, T 9, Y 5, A 4, I -6, F 0, E 8, Y 7, I 2, S -5, I -3

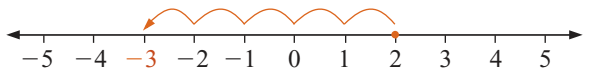
d H -7, O -15, S -10, T -1, B -4, E -6, E -17, I -11, S -2, E -3, T -8, H -20, W -16, R -14, O -19, M -18, K -13

EXAMPLE 4

Use a number line to:

a decrease 2 by 5 **b** increase -11 by 7

a



Start at 2, then move left 5 steps. $2 - 5 = -3$

b

Start at -11, then move right 7 steps. $-11 + 7 = -4$

To increase, move right.
To decrease, move left.

7 Use a number line to:

a increase 2 by 3 **b** increase -1 by 3 **c** decrease 5 by 2

d decrease -1 by 3 **e** increase -4 by 3 **f** increase -2 by 1

g decrease 3 by 6 **h** decrease -2 by 2 **i** increase -7 by 3

D Using directed numbers

A number line helps in adding and subtracting directed numbers.

- When *adding* two numbers, locate the first number on the number line, then move to the *right*.
- When *subtracting* two numbers, locate the first number on the number line, then move to the *left*.

EXAMPLE 1

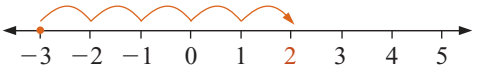
Simplify using a number line.

a $-3 + 5$ **b** $-3 - 5$

c $-5 + 2$ **d** $2 - 4$

To add, move right.
To subtract, move left.

a

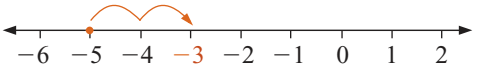


$-3 + 5 = 2$

b

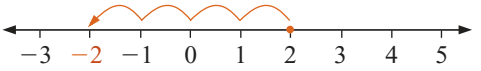
$-3 - 5 = -8$

c



$-5 + 2 = -3$

d



$2 - 4 = -2$

Exercise 2D

1 Simplify using a number line, by moving to the right by the number added.

a $4 + 3$	**b** $-4 + 2$	**c** $5 + 3$	**d** $-5 + 2$
e $-4 + 4$	**f** $0 + 3$	**g** $-3 + 1$	**h** $-5 + 6$
i $0 + 4$	**j** $-6 + 6$	**k** $-4 + 7$	**l** $-8 + 5$

2 Simplify using a number line, by moving to the left by the number subtracted.

a $6 - 3$	**b** $5 - 2$	**c** $4 - 6$	**d** $2 - 4$
e $-3 - 4$	**f** $0 - 2$	**g** $-3 - 5$	**h** $-2 - 1$
i $0 - 3$	**j** $-5 - 0$	**k** $-1 - 3$	**l** $1 - 5$

3 Simplify using a number line.

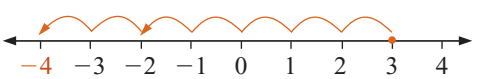
a $-2 + 5$	**b** $-5 + 8$	**c** $-6 + 2$	**d** $-8 - 2$
e $-3 + 6$	**f** $-4 - 3$	**g** $-6 - 3$	**h** $-8 + 5$
i $-4 + 6$	**j** $-4 - 2$	**k** $-7 + 5$	**l** $-6 - 9$

EXAMPLE 2

Simplify using a number line.

a $3 - 5 - 2$ **b** $-3 + 6 - 4$

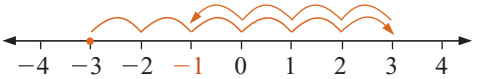
a



Move 5 left from 3, then 2 more left.

$3 - 5 - 2 = -4$

b



Move 6 right from -3, then 4 left.

$-3 + 6 - 4 = -1$

4 Simplify using a number line.

a $3 - 2 + 4$	**b** $6 - 3 - 4$	**c** $4 - 5 - 2$	**d** $-3 - 2 + 5$
e $-4 + 6 - 3$	**f** $5 - 4 - 3$	**g** $-4 - 2 + 3$	**h** $-5 - 2 + 3$
i $-2 + 5 - 4$	**j** $3 - 8 + 4$	**k** $2 - 5 + 7$	**l** $-3 - 2 - 1$

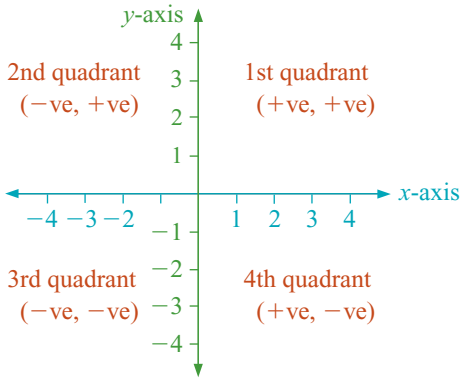
5 **a** Jana had \$20 in the bank and withdrew \$30. How much does she need to deposit to have \$10 in the bank?

b The temperature at sunset was 16°C. Overnight it fell 20°C. What was the new temperature?

c At 5:00 am it was −4°C. The temperature rose 8°C by midday, then fell 6°C by 7:00 pm. What was the temperature at 7:00 pm?

E The number plane

The **number plane**, also referred to as a **Cartesian plane**, consists of four **quadrants**. Two number lines, the ***x*-axis** and ***y*-axis**, intersect at right angles to form the number plane.



Coordinate numbers are used to locate a point in the number plane.

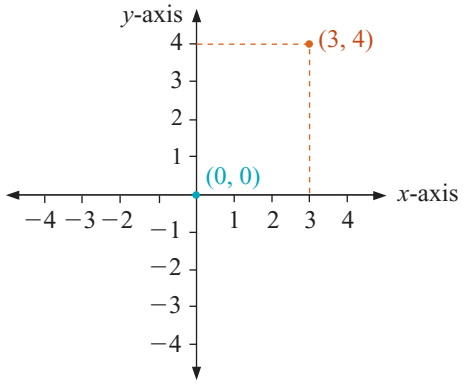
- The **horizontal** (x) position of the point is given first.
- The **vertical** (y) position of the point is given second.

Each quadrant contains coordinates that are specific to that quadrant. For example, in the 1st quadrant both the x- and y-coordinates are positive.

So the point at (3, 4), where x and y are both positive, must be in the first quadrant.

- '3' is first, so it gives the horizontal (x) position.
- '4' is second, so it gives the vertical (y) position.

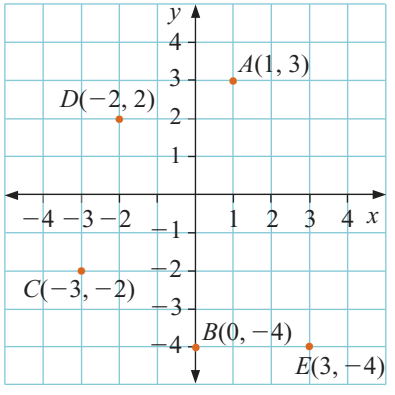
The intersection of the x and y values is where the point (3, 4) lies.



The point (0, 0) in the middle, where the axes intersect, is called the **origin**.

EXAMPLE 1

Plot points $A(1, 3)$, $B(0, -4)$, $C(-3, -2)$, $D(-2, 2)$ and $E(3, -4)$ on a number plane.

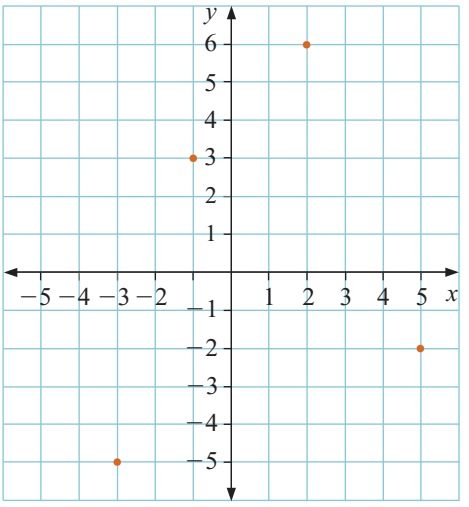


Don't forget to label the axes.

Exercise 2E

1 Copy the number plane and label the following points.

$A(-3, -5)$ $B(2, 6)$ $C(5, -2)$ $D(-1, 3)$



2 Plot the following points on a number plane.

$E(1, 1)$ $F(-2, -5)$ $G(0, 2)$ $H(-3, 0)$

3 Plot the following points on a number plane.

$I(3, -5)$ $J(4, 0)$ $K(4, 2)$ $L(-4, 5)$

4 Plot the following points on a number plane.

$M(0, -2)$ $N(4, 4)$ $O(0, 0)$
$P(-3, -2)$ $Q(-5, 2)$ $R(-5, -5)$
$S(-1, 3)$ $T(-1, 2)$ $U(2, -5)$

5 Complete each sentence by correctly inserting 'positive' or 'negative'.

a i In the 1st quadrant, the x-coordinate is _____ and the y-coordinate is _____.

ii Give four examples of points that could lie in the 1st quadrant.

b i In the 2nd quadrant, the x-coordinate is _____ and the y-coordinate is _____.

ii Give four examples of points that could lie in the 2nd quadrant.

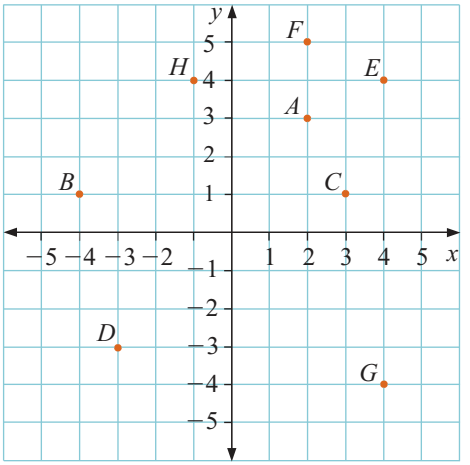
c i In the 3rd quadrant, the x-coordinate is _____ and the y-coordinate is _____.

ii Give four examples of points that could lie in the 3rd quadrant.

d i In the 4th quadrant, the x-coordinate is _____ and the y-coordinate is _____.

ii Give four examples of points that could lie in the 4th quadrant.

6 Eight points have been plotted on this number plane.



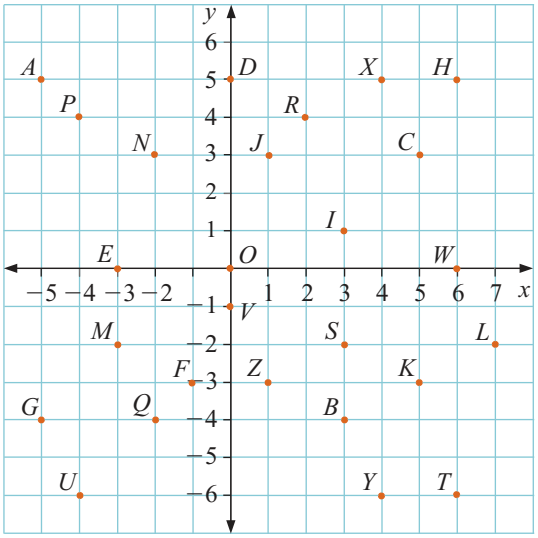
a Write the coordinates of each point.

b Name two points with the same x-coordinate. What do you notice about their positions on the number plane?

c Name two points with the same y-coordinate. What do you notice about their positions on the number plane?

d Name two points with the x-coordinate equal to the y-coordinate. What do you notice about their positions on the number plane?

7 Write the coordinates of the letters A to Z shown on the number plane; for example, $A(-5, 5)$.



8 **a** Plot points $A(-3, 3)$, $B(1, 3)$ and $C(1, -1)$ on a number plane.

b If $ABCD$ is a square, find the coordinates of point D.

9 **a** Plot points $P(-4, 0)$, $Q(-4, 5)$ and $R(3, 5)$ on a number plane.

b If $PQRS$ is a rectangle, find the coordinates of point S.

10 **a** Plot points $A(-3, -2)$, $B(-2, -1)$, $C(-1, 0)$, $D(0, 1)$ and $E(1, 2)$ on the same number plane. Join the points.

b What do you notice?

c What are the coordinates of the next three points, F, G and H, if the pattern continues?

11 **a** Plot points $A(5, 3)$, $B(4, 2)$, $C(3, 1)$, $D(2, 0)$ and $E(1, -1)$ on the same number plane. Join the points.

b What are the next three points, F, G and H, if the pattern continues?

12 Plot and join the following points in the order given and describe each shape.

a (1, 3), (1, 5), (3, 5), (3, 3), (1, 3)

b (1, 6), (5, 6), (5, 8), (1, 8), (1, 6)

c (8, 9), (7, 6), (9, 6), (10, 9), (8, 9)

d (4, 2), (9, 2), (10, 4), (5, 4), (4, 2)

13 **a** Draw a grid with numbers up to 30 on the x-axis and up to 40 on the y-axis. Plot the coordinates in parts **i** to **xii** on the number plane. Join the points in each part together.

i (12, 36), (13, 38), (16, 39), (18, 39), (22, 37), (21, 36), (24, 34), (26, 30), (26, 27), (25, 25), (23, 23), (20, 22)

ii (22, 37), (24, 39), (26, 37), (29, 30), (28, 28), (26, 27)

iii (12, 36), (11, 37), (10, 37), (8, 36), (6, 34), (5, 32), (6, 30), (6, 27), (8, 25), (9, 26), (10, 25), (11, 30), (12, 34)

iv (9, 26), (10, 22), (14, 20), (16, 20)

v (10, 22), (9, 20), (8, 18), (7, 15), (7, 11), (7, 9), (8, 5), (10, 4), (12, 3), (14, 3), (16, 3)

vi (17, 3), (18, 5), (18, 8), (17, 11), (16, 12), (14, 13)

vii (13, 3), (14, 1), (16, 1), (18, 1), (19, 2), (20, 3), (18, 5)

viii (8, 5), (6, 6), (5, 7), (4, 8), (3, 9), (2, 11), (2, 14), (2, 15), (3, 17), (3, 14), (4, 12), (5, 10), (7, 9)

ix (20, 22), (19, 20), (24, 21), (26, 19), (26, 15), (24, 13), (21, 15), (20, 17), (21, 19)

x (14, 16), (16, 12), (18, 12), (20, 15), (20, 17), (19, 18), (18, 19)

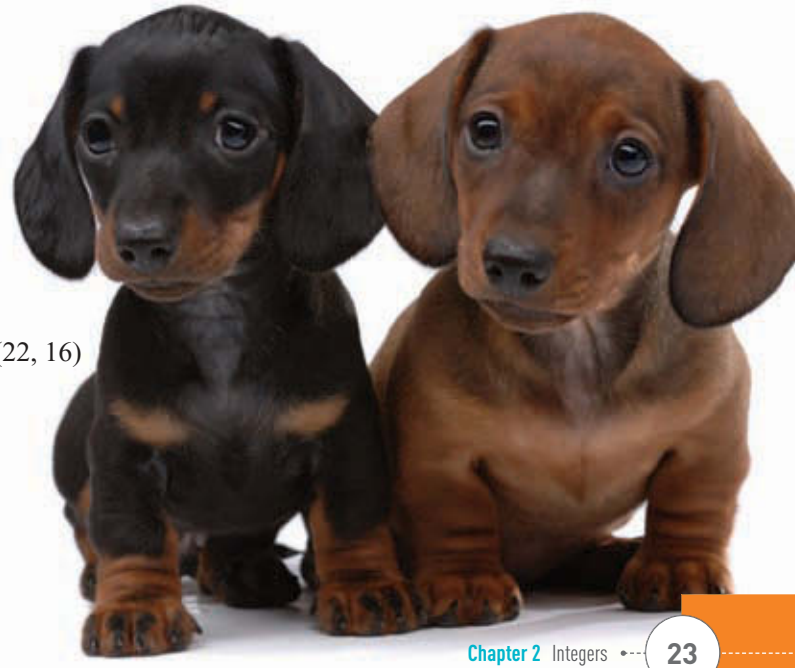
xi (24, 13), (25, 10), (24, 7), (26, 6), (25, 4), (24, 3), (19, 5)

xii (19, 32), (21, 29), (19, 27), (17, 28), (19, 32)

b Join the following pairs of points.

(24, 3) to (23, 5) (25, 4) to (24, 5) (22, 14) to (22, 16)
(24, 13) to (24, 15) (25, 14) to (25, 16)
(17, 1) to (16, 2) (18, 1) to (17, 2)
(19, 2) to (18, 3) (19, 27) to (19, 24)
(19, 24) to (23, 25) (19, 24) to (14, 24)

c Draw in two eyes.



14 Write the coordinates of points A to V on this number plane. Use fractions $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ for coordinates that are not whole numbers.

15 Plot the following points on a number plane. Use at least 2 cm between whole numbers. You may need to estimate the positions of some points.

$A(1\frac{1}{2}, 2\frac{3}{4})$ $B(\frac{1}{2}, -\frac{3}{4})$

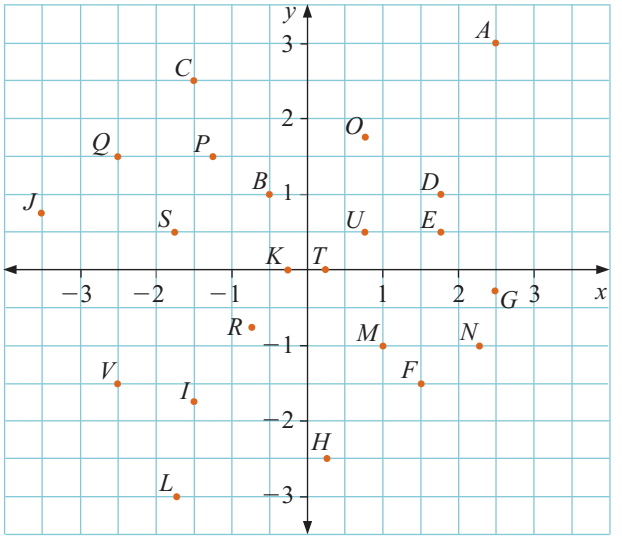
$C(-\frac{1}{4}, 2\frac{1}{2})$ $D(0.6, 1.8)$

$E(-0.4, -2.1)$ $F(-3.2, 1.8)$

$G(-1.5, 1.6)$ $H(2\frac{1}{5}, -1\frac{1}{2})$

$I(\frac{4}{5}, -\frac{3}{4})$ $J(-1\frac{1}{5}, -2)$

$K(0, -\frac{2}{3})$ $L(1.3, 0)$



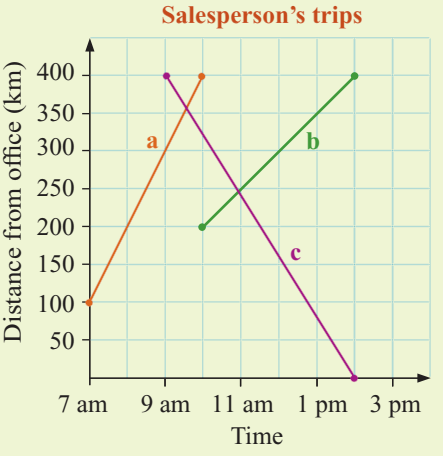
F Interpreting distance–time graphs

A distance–time graph shows the relative position, at different times, of a person or object as it moves. The position is indicated by the distance of the person (or object) from a starting point.

EXAMPLE 1

Each of the straight-line segments drawn on the graph represents a trip made by a salesperson, not necessarily on the same day. The graph shows the distance of the salesperson from his office at different times of the day. For each trip determine:

i the time taken for the trip
ii the distance travelled
iii the speed at which he travelled.



a i Started at 7 am and arrived at 10 am, so time taken is 3 hours.

ii Started 100 km from office, finished 400 km from office, so distance travelled is $400 - 100 = 300$ km.

iii Speed $= \frac{\text{distance travelled}}{\text{time taken}} = \frac{300 \text{ km}}{3 \text{ h}} = 100$ km/h

b i Started at 10 am and arrived at 2 pm, so time taken is 4 hours.

ii Started 200 km from office, finished 400 km from office, so distance travelled is $400 - 200 = 200$ km.

iii Speed $= \frac{\text{distance travelled}}{\text{time taken}} = \frac{200 \text{ km}}{4 \text{ h}} = 50$ km/h

c i Started at 9 am and arrived at 2 pm, so time taken is 5 hours.

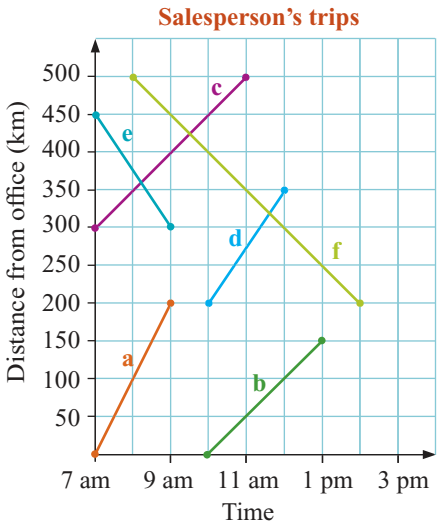
ii Started 400 km from office, finished at office, so distance travelled $= 400$ km.

iii Speed $= \frac{\text{distance travelled}}{\text{time taken}} = \frac{400 \text{ km}}{5 \text{ h}} = 80$ km/h

Exercise 2F

1 Each of the straight-line segments drawn on the graph represents a trip made by a salesperson, not necessarily on the same day. The graph shows the distance of the salesperson from her office at different times of the day. For each trip determine:

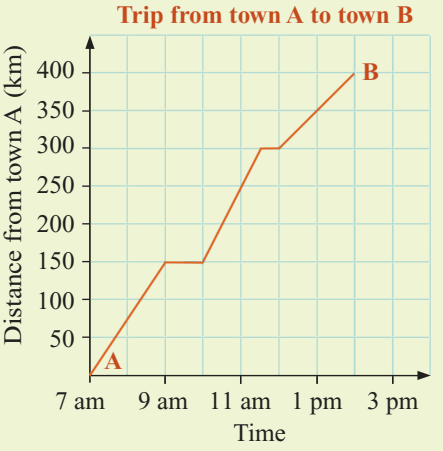
- **i** the time taken for the trip
- **ii** the distance travelled
- **iii** the speed at which she travelled.



EXAMPLE 2

A motorist drives from town A to town B. The graph shows the journey.

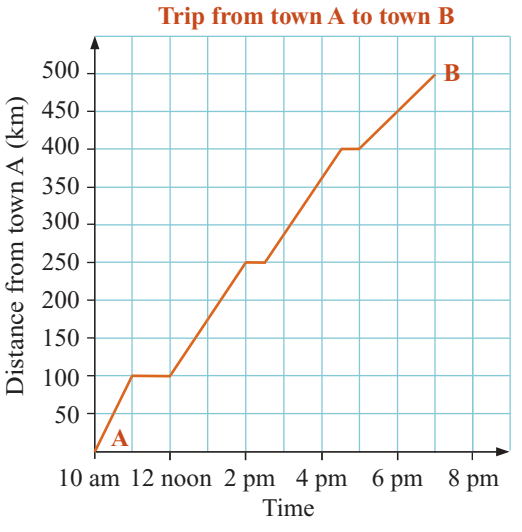
- **a** What is the meaning of the horizontal sections of the graph?
- **b** How many times did the motorist stop?
- **c** When did the motorist stop and for how long?



- **a** Each horizontal section of the graph indicates that the motorist's distance from town A did not change during this time interval; that is, they were stopped.
- **b** The motorist stops two times.
- **c** The motorist stops at 9 am for 1 hour and at 11:30 am for $\frac{1}{2}$ hour.

2 A motorist drives from town A to town B. The graph shows the journey.

- **a** What is the meaning of the horizontal sections of the graph?
- **b** How many times did the motorist stop?
- **c** When did the motorist stop and for how long?

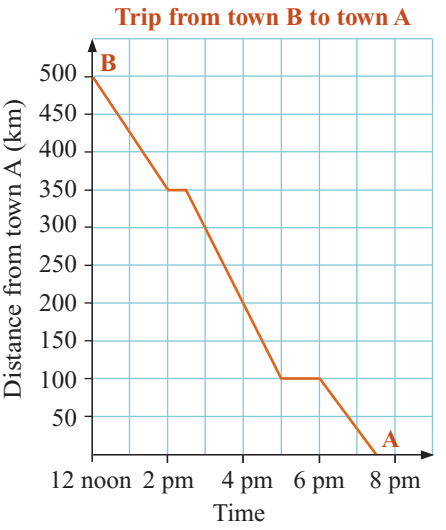


3 A motorist drives from town B to town A. The graph shows the journey.

a What is the meaning of the horizontal sections of the graph?

b How many times did the motorist stop?

c When did the motorist stop and for how long?



4 Alison walks from home to school. On the way she stops at the houses of two school friends, Michelle and Lara, in that order. The graph shows her journey to school.

a When did she stop at Michelle's house?

b How long did she stay?

c How far from home is Michelle's house?

d At what speed did she walk on this section of her journey?

e How long did it take to reach Lara's house from:

i home? **ii** Michelle's house?

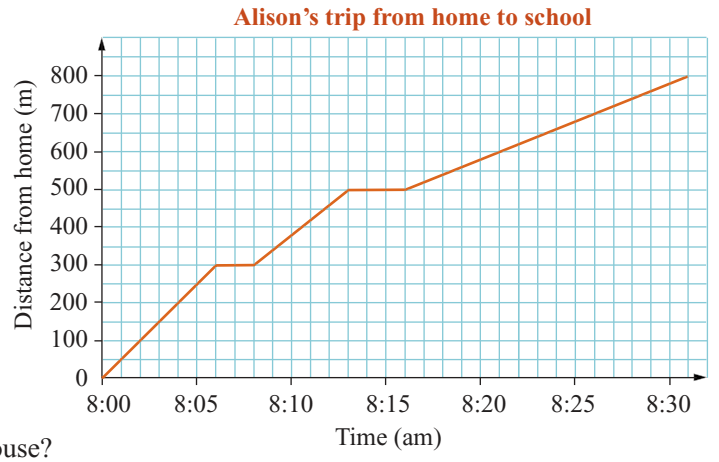
f How far is Lara's house from:

i home? **ii** Michelle's house?

g At what speed did she walk from Michelle's house to Lara's house?

h How far is it from home to school?

i What was Alison's average speed for the whole journey, excluding the times she stopped?



5 Bill rides his bike from home to the shops and back home again. The graph shows Bill's journey.

a At what time did Bill leave home?

b Where was Bill at 4:18 pm?

c At what times was Bill 1 kilometre from home?

d How far did he travel in the first 5 minutes?

e What was his speed in the first 5 minutes?

f What is the meaning of the horizontal section of the graph?

g When did Bill stop?

h How far are the shops from Bill's home?

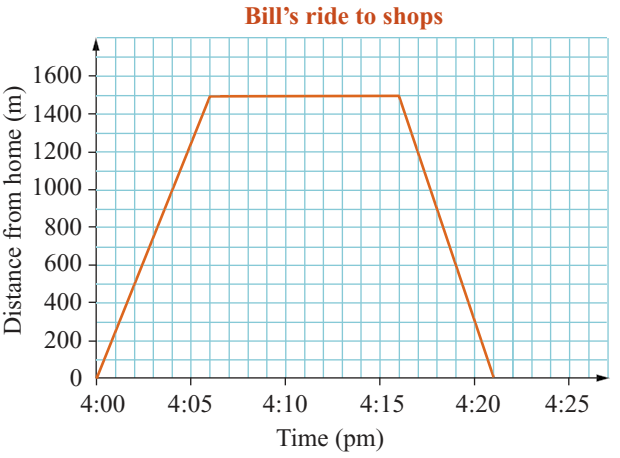
i How long did he spend at the shops?

j How long did the return journey home take?

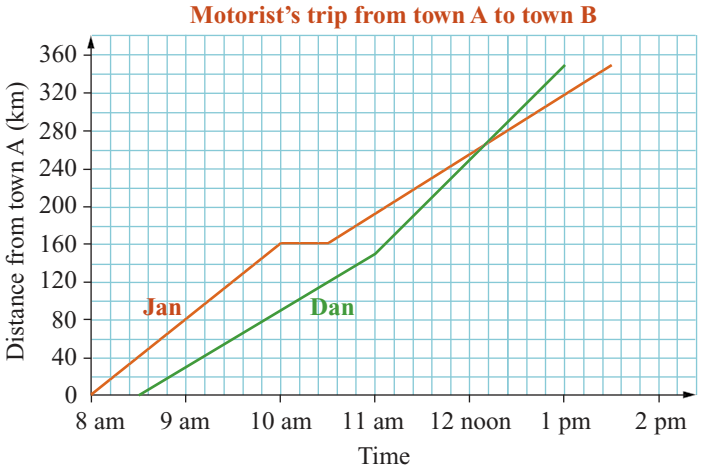
k What was his speed for the journey home?

l What was the total distance travelled by Bill for the whole journey?

m What was the average speed for the whole journey, not including the time he was at the shops?

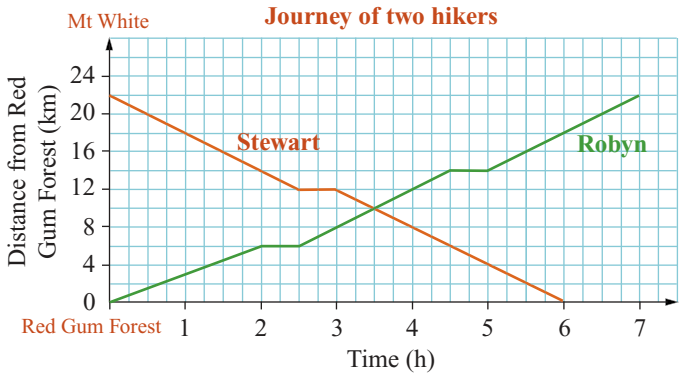


6 Two motorists, Jan and Dan, set out from town A at different times and travel along the same road to town B. The graph shows the journey of each motorist.



- **a** How far is town B from town A?
- **b** When did Jan begin her journey?
- **c** What was her speed on the first section of her journey?
- **d** For how long did she stop?
- **e** What was her speed for the last section of her journey?
- **f** How long after Jan did Dan set out?
- **g** Describe Dan's journey.
- **h** When did Dan overtake Jan?
- **i** At this time, how far did they still have to travel to get to town B?
- **j** How much sooner than Jan did Dan arrive at town B?
- **k** How far apart were they at 10 am?
- **l** How far did Jan still have to travel when Dan arrived at town B?

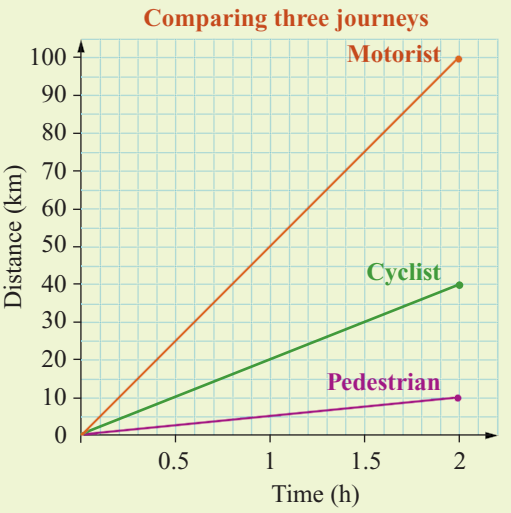
7 The graph shows the journeys of two hikers, Robyn and Stewart. Robyn starts at Red Gum Forest and walks to Mount White. Stewart starts at Mount White and walks to Red Gum Forest along the same track.



- **a** How far is Red Gum Forest from Mount White?
- **b** Who completed the hike in the least time?
- **c** How many rest breaks did Robyn have?
- **d** For how long did she stop?
- **e** Calculate Robyn's speed on each section of her journey.
- **f** How long did it take Robyn to complete the hike, including rest breaks?
- **g** What was Robyn's average speed:
  - **i** including the time for rest breaks?
  - **ii** excluding the time for rest breaks?
- **h** Describe Stewart's journey.
- **i** When did Robyn and Stewart pass each other?
- **j** At this time, how far was each hiker from their destination?
- **k** How far apart were they after 3 hours?
- **l** How far did Robyn still have to travel when Stewart arrived at Red Gum Forest?

EXAMPLE 3

A pedestrian, a cyclist and a motorist set out from the same starting point at the same time. The graph shows the distance of each from the starting point for the first 2 hours of their trip.



a **i** How far did the pedestrian walk in the first 2 hours?
ii What was her speed for these 2 hours?
b **i** How far did the cyclist ride in the first 2 hours?
ii What was her speed for these 2 hours?
c **i** How far did the motorist drive in the first 2 hours?
ii What was her speed for these 2 hours?
d **i** Which line is the steepest?
ii Who travelled the fastest?
e **i** Which line is the least steep?
ii Who travelled the slowest?
f Another car starts from the same point and travels at 100 km/h for 2 hours. Describe the line for this motorist.
g Complete this sentence: The steeper a line is on a distance–time graph, the ____ the speed.
h **i** Are the slopes (or gradients) of these lines positive or negative?
ii Are the distances from the origin (starting point) increasing or decreasing?

a **i** 10 km
ii Speed $= \frac{\text{distance}}{\text{time}} = \frac{10 \text{ km}}{2 \text{ h}} = 5 \text{ km/h}$

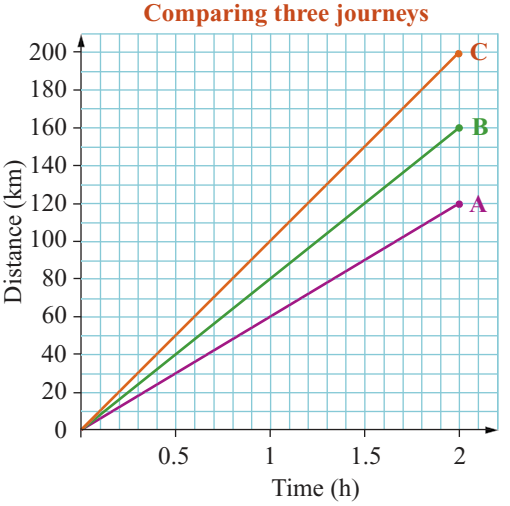
b **i** 40 km
ii Speed $= \frac{\text{distance}}{\text{time}} = \frac{40 \text{ km}}{2 \text{ h}} = 20 \text{ km/h}$

c **i** 100 km
ii Speed $= \frac{\text{distance}}{\text{time}} = \frac{100 \text{ km}}{2 \text{ h}} = 50 \text{ km/h}$

d **i** The steepest line represents the motorist.
ii The motorist travels the fastest.
e **i** The line that is the least steep represents the pedestrian.
ii The pedestrian travels the slowest.
f The line would be much steeper than the other lines.
g The steeper a line is on a distance–time graph, the *greater* the speed.
h **i** The lines slope up, from left to right, hence they all have positive slopes.
ii The distances from the origin are increasing.

The slope (or gradient) a line is $\frac{\text{rise}}{\text{run}}$ as we move from left to right.

8 Three motorists set out from the same starting point at the same time. The graph shows the distance of each motorist from the starting point for the first 2 hours of their trip.



a **i** How far did motorist A travel in the first 2 hours?
ii What was her speed for these 2 hours?
b **i** How far did motorist B travel in the first 2 hours?
ii What was her speed for these 2 hours?
c **i** How far did motorist C travel in the first 2 hours?
ii What was her speed for these 2 hours?
d **i** Which line is the steepest?
ii Which motorist travelled the fastest?
e **i** Which line is the least steep?
ii Which motorist travelled the slowest?

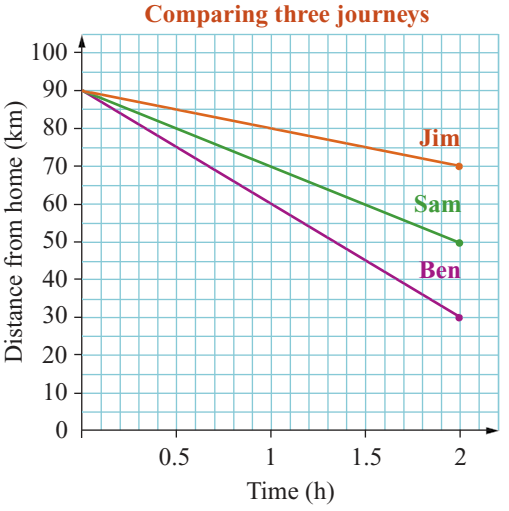
f Another motorist starts at the same point and travels at 50 km/h for 2 hours. Describe the line for this trip.

g Complete this sentence: The steeper a line is on a distance–time graph, the ____ the speed of the vehicle.

h **i** Are the slopes (or gradients) of these lines positive or negative?

ii Are the distances from the origin (starting point) increasing or decreasing?

9 Three cyclists, Jim, Sam and Ben, set out from the same starting point to travel home. The graph shows the distance from home of each cyclist for the first 2 hours of their journey.



a How far from home did the cyclists start their journey?

b How far was Jim from home after 1 hour?

c When was Sam 80 km from home?

d How long did it take Ben to travel 45 km?

e **i** How far was Jim from home after 2 hours?

ii How far had Jim travelled in the first 2 hours?

iii What was Jim's speed for the first 2 hours?

f **i** How far was Sam from home after 2 hours?

ii How far had Sam travelled in the first 2 hours?

iii What was Sam's speed for the first 2 hours?

g **i** How far was Ben from home after 2 hours?

ii How far had Ben travelled in the first 2 hours?

iii What was Ben's speed for the first 2 hours of his journey?

h **i** Which line is the steepest?

ii Which cyclist travelled the fastest?

i **i** Which line is the least steep?

ii Which cyclist travelled the slowest?

j Another cyclist starts at the same point and travels at 15 km/h for 2 hours. Describe the line for this cyclist.

k Complete this sentence: The steeper a line is on a distance-time graph, the ____ the speed of the vehicle.

l **i** Are the slopes (or gradients) of these lines positive or negative?

ii Are the distances from the origin (home) increasing or decreasing?

From these questions you will have discovered that on a distance–time graph consisting of straight-line segments:

- the slope of the line segment indicates the speed of the object
- the steeper the line segment, the greater the speed of the object
- horizontal sections indicate that the object is not moving
- a positive gradient indicates the distance from the origin is increasing
- a negative gradient indicates the distance from the origin is decreasing.

Investigation 3 Modelling motion

This is a practical activity for the class. Have a member of the class move along a 50 m tape for 30 seconds using a variety of activities that involve a constant rate. Here are some possible activities for the mover:

Walk slowly for 30 seconds.
Walk quickly for 30 seconds.
Walk for 10 seconds, stop for 10 seconds, walk (at the same rate) for 10 seconds.
Walk for 10 seconds, jog for 10 seconds, walk for 10 seconds.
Walk for 10 seconds, stop for 10 seconds, walk back to the start.
Stop for 10 seconds, walk slowly for 10 seconds, jog for 10 seconds.
Start walking slowly and then speed up until reaching the end of the tape.
Run at an increasing then decreasing speed.
Start at the other end of the tape and repeat some of the above activities.

You will need: 50 m measuring tape, stopwatch, markers (such as flags or witches' hats)

1 Lay a 50 m measuring tape out in a straight line.
   a Appoint a member of the class as the timekeeper to call out the time every 5 seconds.
   b Appoint a member of the class as the 'mover', who moves along the tape according to the instructions given.
   c Appoint six other members of the class to mark the position of the mover every 5 seconds.

2 Record the distance of the mover from the start at fixed time intervals in a table like the one below.

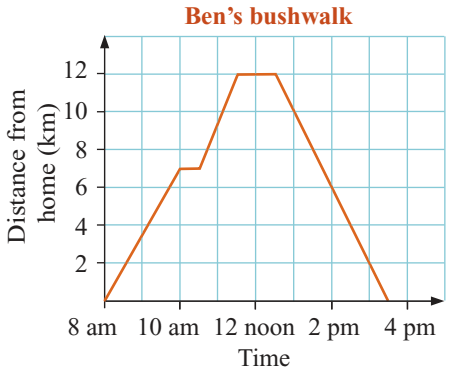
Time (s)	5	10	15	20	25	30
Distance (m)						

3 Draw a distance–time graph to represent each situation using the recorded table of values.

G Drawing distance–time graphs

EXAMPLE 1

Ben went bushwalking. He started at 8 am and walked 7 km in the first 2 hours. He rested for 30 minutes, then walked a further 5 km in the next hour. He spent an hour having lunch and then began the return trip to his starting point. On the return trip he walked without stopping for 3 hours.
Draw a distance–time graph to represent Ben's hike.



Exercise 2G

1 Simon has a parcel delivery service in the city. Starting at 9 am, he rode 4 km in 12 minutes. He stopped for 3 minutes to deliver a letter to a legal firm and then returned to his starting point in 15 min. Draw a distance–time graph to represent his trip.

2 Rachelle has a pick-up ironing service. One morning she left home at 8 am and drove 3 km in 5 minutes to her first customer. She spent 1 minute at her first customer's place. She then drove a further 2 km to her next customer's place. This took 3 minutes and she spent 2 minutes with this customer before returning home. The return trip took 7 minutes. Draw a distance–time graph for her journey.

EXAMPLE 2

Tiffany rode her bike to town and back again. She left home at 9 am and rode at a constant speed, covering 12 km in the first 2 h of her journey. She then rested for 30 minutes before continuing at an average speed of 10 km/h for $1\frac{1}{2}$ h to reach town. She spent 1 hour shopping and having lunch, and then rode back home at an average speed of 9 km/h, without stopping.

a How far is it from home to town?

b How long did it take Tiffany to ride home from town?

c Draw a distance–time graph to represent Tiffany's journey.

a Distance travelled = speed × time

At 10 km/h for $1\frac{1}{2}$ h:

Distance travelled $= 10 \times 1\frac{1}{2} = 15$ km

Total distance to town $= 12$ km $+ 15$ km

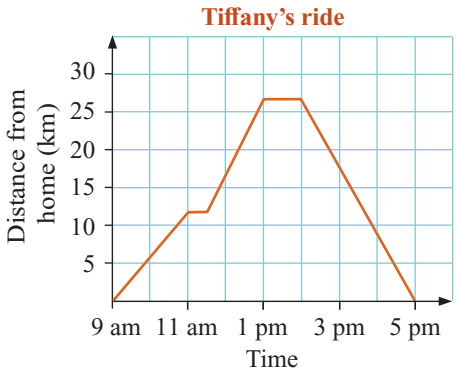
$= 27$ km

b $\text{Time} = \dfrac{\text{distance}}{\text{speed}}$

$= \dfrac{27}{9}$

$= 3$ h

c



3 Karen went on a car trip. She started at 10 am and drove 140 km in the first 2 hours. She rested for 30 minutes and then drove at an average speed of 50 km/h for the next 2 hours. She rested again for 30 minutes before beginning her trip home. She did not stop on the return trip and averaged 80 km/h.

a How far did Karen drive before she began the return trip?

b How long did it take her to drive home?

c Draw a distance–time graph to represent Karen's trip.

4 Draw a travel graph representing this journey. Lauren left home at midday and walked 5 km in $1\frac{1}{2}$ hours. She rested for half an hour, then started back home. Lauren walked 2 km in half an hour, then stopped for 45 minutes, finally arriving home at 4:30 pm.

5 A country bus service begins its journey at town A at 7 am. It travels the 150 km to town B at an average speed of 75 km/h. It stops in town B for 15 minutes before continuing to town C, which takes 2 hours at 60 km/h. The bus stops in town C for 1 hour before making the return journey to town B at 80 km/h and continuing without stopping to town A, where it arrives at 3:30 pm.

a How long does it take the bus to get from town A to town B?

b How far is it from town B to town C?

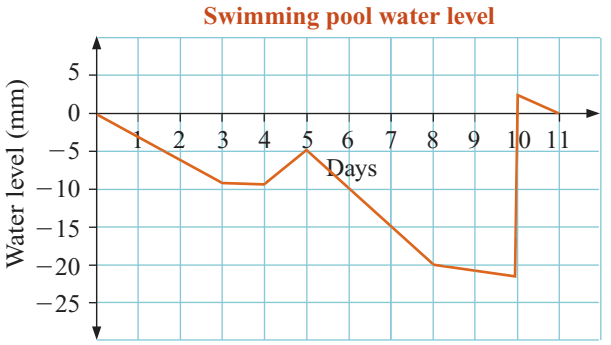
c How long does it take to travel from town C to town B at 80 km/h?

d Draw a distance–time graph to represent the bus's journey.

6 Lucy catches the bus to the station one morning. There are five bus stops between Lucy's home and the station, and the bus stops at all of them. Draw a distance–time graph that could represent Lucy's bus trip.

7 Draw a distance–time graph to represent your journey to and from school. Compare and discuss your graph with another student.

8 Jade is investigating the rate at which water evaporates from their swimming pool. One evening she fills the pool and marks a line at the maximum level. Each day at the same time she measures the distance from that mark to the current water level. Her results are shown in the graph.



a What was the water level on the day Jade marked the pool?

b What was the water level after 3 days?

c What was the rate of evaporation in mm per day for the first 3 days?

d It rained a little on one day and then more on the next. Which days were these?

e Which 3 days were the hottest? (The evaporation is greater then.)

f Jade filled the pool with the hose one evening but forgot to turn it off. When was this?

9 Construct a graph for this information One evening a pool is filled to maximum. Over 5 days the level goes down by 23 mm. On the sixth day it rains, and the level increases by 14 mm. Over the next 3 days another 12 mm of water evaporates. On the next day it drizzles, adding 2 mm to the level. That evening the pool is refilled to the maximum level.

Investigation 4 Adding and subtracting

1 Complete the following number patterns.

a	**b**	**c**	**d**
$4 + 3 =$ ____	$5 + 3 =$ ____	$4 - 3 =$ ____	$5 - 3 =$ ____
$4 + 2 =$ ____	$5 + 2 =$ ____	$4 - 2 =$ ____	$5 - 2 =$ ____
$4 + 1 =$ ____	$5 + 1 =$ ____	$4 - 1 =$ ____	$5 - 1 =$ ____
$4 + 0 =$ ____	$5 + 0 =$ ____	$4 - 0 =$ ____	$5 - 0 =$ ____
$4 + (-1) =$ ____	$5 + (-1) =$ ____	$4 - (-1) =$ ____	$5 - (-1) =$ ____
$4 + (-2) =$ ____	$5 + (-2) =$ ____	$4 - (-2) =$ ____	$5 - (-2) =$ ____
$4 + (-3) =$ ____	$5 + (-3) =$ ____	$4 - (-3) =$ ____	$5 - (-3) =$ ____
$4 + (-4) =$ ____	$5 + (-4) =$ ____	$4 - (-4) =$ ____	$5 - (-4) =$ ____

Looking at these patterns, we can see the following:

- *Subtracting* a negative number is the same as adding a positive number.
  − **negative number** = + **positive number**
  Example: $-(-2) = +2$
- *Adding* a negative number is the same as subtracting a positive number.
  + **negative number** = − **positive number**
  Example: $+(-2) = -2$

H Adding and subtracting directed numbers

From Investigation 4, we can see that $3 + (-4)$ is the same as $3 - 4$, and that $5 - (-3)$ is the same as $5 + 3$. It is also clear that $3 - (+2)$ is the same as $3 - 2$.

EXAMPLE 1

Simplify and evaluate the following.

a $5 + (-2)$ **b** $3 - (-6)$ **c** $-3 + (-2)$
d $-4 - (-3)$ **e** $-4 - (-2)$ **f** $-3 - (+5)$

a $5 + (-2)$
$= 5 - 2$
$= 3$

b $3 - (-6)$
$= 3 + 6$
$= 9$

c $-3 + (-2)$
$= -3 - 2$
$= -5$

d $-4 - (-3)$
$= -4 + 3$
$= -1$

e $-4 - (-2)$
$= -4 + 2$
$= -2$

f $-3 - (+5)$
$= -3 - 5$
$= -8$

Exercise 2H

1 Complete the following.

a $5 + (-1)$
$= 5$ ____ 1
$= 4$

b $6 - (-2)$
$=$ ____ $+$ ____
$=$ ____

c $-2 + (-3)$
$= -2 -$ ____
$= -$____

2 Simplify and evaluate the following.

a $6 + (-2)$ **b** $5 - (-2)$ **c** $-3 + (-4)$ **d** $-2 + (-6)$
e $4 - (+3)$ **f** $7 - (-2)$ **g** $4 - (-10)$ **h** $-3 - (+5)$
i $10 - (-4)$ **j** $-6 + (-1)$ **k** $7 + (-3)$ **l** $-5 + (-2)$

3 Evaluate the following.

a $5 + (-10)$ **b** $3 + (-5)$ **c** $12 - (+5)$ **d** $4 - (+8)$
e $-3 + (-7)$ **f** $-8 + (-5)$ **g** $-4 + (-12)$ **h** $-10 - (+5)$
i $6 - (-2)$ **j** $8 - (-4)$ **k** $4 + (+8)$ **l** $9 + (+2)$
m $-3 - (-2)$ **n** $-2 - (-11)$ **o** $-8 - (+3)$ **p** $-4 - (+5)$
q $12 - (+2)$ **r** $-4 + (-7)$ **s** $-7 - (-9)$ **t** $3 - (-4)$
u $7 - (+5)$ **v** $5 + (-13)$ **w** $-8 - (+3)$ **x** $4 - (+5)$

EXAMPLE 2

Simplify the following.

a $3 - (-2) + 5$ **b** $15 - (-3) + (-5)$

a $3 - (-2) + 5$
$= 3 + 2 + 5 = 10$
$= 10$

b $15 - (-3) + (-5)$
$= 15 + 3 - 5 = 13$
$= 13$

4 Complete the following.

a
$4 - (-2) + 1$
$= 4 + \_\_\_\_ + 1$
$= 6 + \_\_\_\_ = \_\_\_\_$

b
$3 - (-4) + (-2)$
$= 3 + \_\_\_\_ \_\_\_\_ 2$
$= 7 - \_\_\_\_ = \_\_\_\_$

5 Simplify the following.

a $2 - (-1) + 7$ **b** $11 - (-4) + 8$ **c** $4 - (-2) + 5$
d $4 - (-6) + 3$ **e** $12 - (-9) + 10$ **f** $9 - (-4) - 12$
g $10 - (-3) - 8$ **h** $20 - (-5) - 15$ **i** $12 - (-4) + (-2)$
j $15 - 5 + (-7)$ **k** $10 - 6 + (-4)$ **l** $19 - 4 + (-3)$
m $14 - 9 + (-2)$ **n** $15 - 8 - (-2)$ **o** $21 - 11 - (-5)$

6 Simplify the following.

a $-3 - (-1) + 2$ **b** $-4 + (-3) - 1$ **c** $-3 - 5 + (-2)$
d $-10 - (-6) - 4$ **e** $-14 - 2 - (-5)$ **f** $-6 - (-3) + (-3)$

7 A helicopter rises 30 m, falls 45 m, rises 20 m, falls 10 m, rises 15 m, and falls 12 m. How far is it now above or below its original position?

8 The morning temperature of $-2°C$ rose by $7°C$ by midday, then dropped by $10°C$ in the evening. Find:

a the midday temperature
b the evening temperature.

9 **AEST** means **Australian Eastern Standard Time**. This is the time in the eastern states of Australia (Qld, NSW, Vic, Tas) without daylight saving. The table gives the time differences between AEST and the standard time in some other countries. For example, Tonga time is 3 hours ahead of AEST, PNG time is the same as AEST, and Malaysia time is 5 hours behind. Use the information in the table to answer the following questions.

Country	Difference from AEST in hours
Lebanon	−8
Malaysia	−5
Japan	−1
PNG	0
New Zealand	+2
Tonga	+3

a Find the time difference between:

i NSW and New Zealand **ii** NSW and PN
iii Tonga and New Zealand **iv** Tonga and Lebanon
v Malaysia and PNG **vi** Japan and Malaysia.

b If it is 3 am, Wednesday AEST, find the time and day in:

i New Zealand **ii** Japan **iii** Malaysia.

Calculator activities

To make a number negative on the calculator, use the ± or the (−) key.

Find out how your calculator works.

On a calculator with 'natural display', you may be able to use the − key before the number.

EXAMPLE 1

Use your calculator to evaluate the following.

a $-28 - 57$ **b** $-38 + 5 \times (-19)$ **c** $(-36 - 15) \times (-11)$

a $-28 - 57 = -85$

28 ± − 57 =

or DAL ± 28 − 57 = or − 28 − 57 =

b $-38 + 5 \times (-19) = -133$

38 ± + 5 × 19 ± =

or DAL ± 38 + 5 × ± 19 =

or − 38 + 5 × − 19 =

c $(-36 - 15) \times (-11) = 561$

(36 ± − 15) × 11 ± =

or DAL (± 36 − 15) × ± 1 =

or (− 36 − 15) × − 11 =

1 Use your calculator to evaluate the following.

a $28 - 147$ **b** $-53 + 27$ **c** -28×37 **d** -41×38

e $-1484 \div 14$ **f** $-39 - 53$ **g** $18 \times (-47)$ **h** $48 - 211$

i $83 - 9 \times 21$ **j** $-28 + 14 \times 32$ **k** $-43 - 21 \times 72$ **l** $49 + 63 \times (-28)$

m $(27 - 35) \times 14$ **n** $(63 - 85) \times (14 - 26)$ **o** $(-38 - 57) \times (-81)$

2 Simplify the following using your calculator.

a $-428 + 192 - 845 + 1291 - 478 + 293 - 482 - 311$

b $6852 - 4915 - 3211 - 1698 - 2139 + 4821$ **c** $242 \times (-83) + (-93) \times 45 - 26 \times 89$

3 The Olympic Games originated in ancient Greece. From 776 BC to 392 AD they were held every 4 years at Olympia in Greece. In the year 2000 the Olympic Games were held at Sydney.

a How many times were the Olympic Games held at Olympia?

b If the games had been held every 4 years, how many games would there have been by the year 2000?

4 Sally has $673 in the bank and she makes the following transactions: a withdrawal of $517, a deposit of $263, a deposit of $143 and a withdrawal of $317. What is her new bank balance?

5 The table gives the average monthly temperature in °C for three places for a year.

Month	North Cape, Norway	Beijing, China	Alice Springs, Australia
J	−3.6	−4.7	28.5
F	−4.2	−1.5	27.8
M	−3.4	5.0	24.8
A	−0.3	13.7	20.1
M	2.8	19.9	15.4
J	6.8	24.5	12.4
J	9.9	26.0	11.4
A	10.0	24.7	14.7
S	6.6	19.8	18.6
O	2.1	12.5	22.9
N	−1.1	3.6	26.9
D	−2.9	−2.6	27.9

Give all answers correct to 1 decimal place.

a What is the difference between the highest and lowest temperatures in:
i North Cape? **ii** Beijing? **iii** Alice Springs?

b What is the difference between the highest and lowest temperatures in the table?

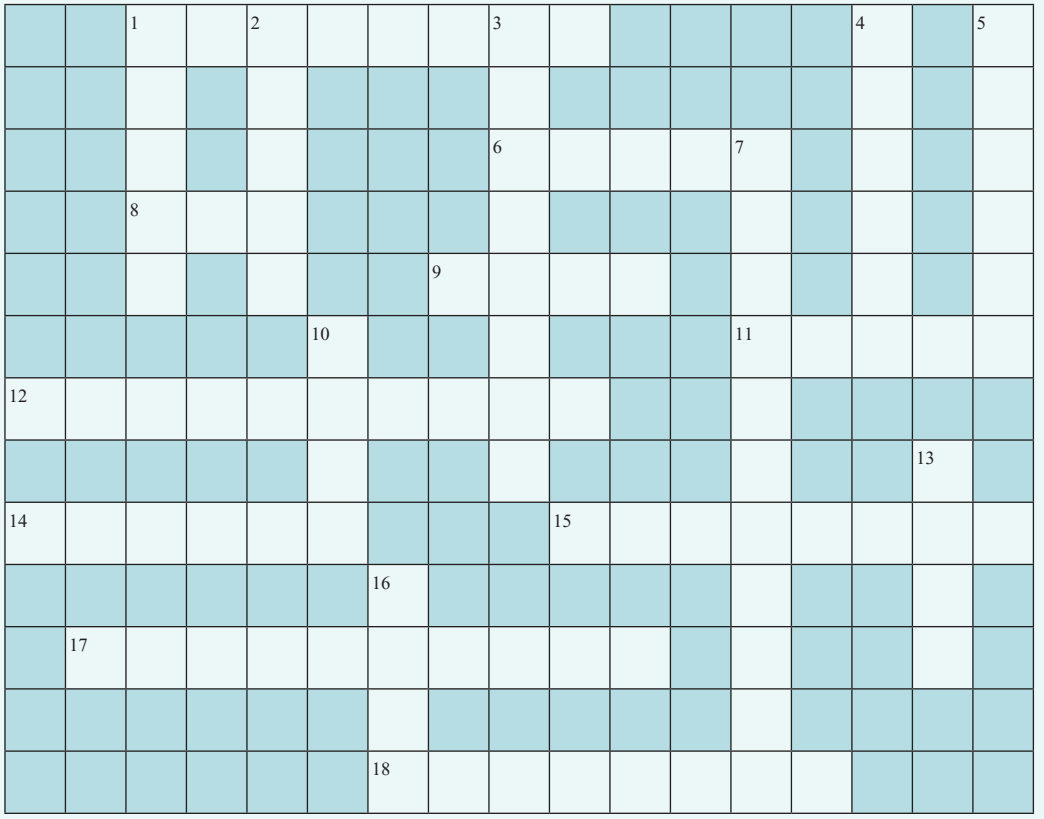
c If the lowest average monthly temperature increased by 2°C, what would this new temperature be in:
i North Cape? **ii** Beijing? **iii** Alice Springs?

d If the average monthly temperature in July was 10°C less, what would this new temperature be in:
i North Cape? **ii** Beijing? **iii** Alice Springs?

e To find the *average* of the average monthly temperatures for a year in one location, you need to add the 12 average monthly temperatures for that location, then divide the sum by 12. Find the average of the average monthly temperatures for the year in:
i North Cape **ii** Beijing **iii** Alice Springs.

Language in mathematics

This crossword uses many of the terms from this chapter. Use the clues to complete the crossword.



Across

1 Greater than zero
6 Opposite of left
8 A number that is negative is _____ positive.
9 A symbol used to show whether a number is positive or negative
11 There are two of these, a north and a south one.
12 Sideways, like the *x*-axis
14 Three is one of these.
15 Less than zero
17 The opposite of deposit
18 Having a positive or negative sign

Down

1 A flat surface
2 Opposite of north
3 Up and down, like the *y*-axis
4 Move from place to place
5 The *x*- and *y*-axes intersect at right _____ .
7 This makes you feel hot or cold.
10 The number of quadrants on a number plane
13 Opposite of odd
16 The pattern of straight lines on graph paper

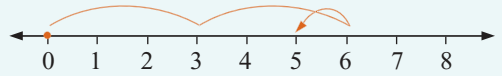
Terms

addition	AEST	ascending	Cartesian plane	coordinates
decrease	descending	directed number	division	increase
integer	multiplication	negative number	number line	number plane
origin	order of operations	par	positive number	quadrant
subtraction	temperature	travel graph	*x*-axis	*y*-axis

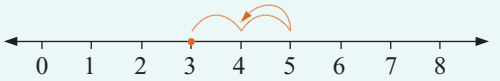
Check your skills

1 The operation $3 \times 2 - 1$ on a number line is:

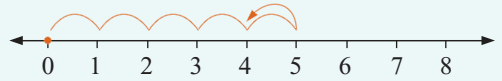
A



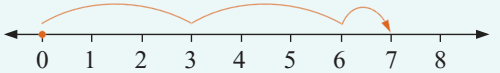
B



C



D



2 A deposit of \$10 followed by a withdrawal of \$6 changes the balance by:

A \$6 **B** \$10 **C** \$16 **D** \$4

3 If down is the negative direction, write directed numbers for someone travelling in a lift 3 floors down then 5 floors up.

A $+3 + 5$ **B** $+3 - 5$ **C** $-3 + 5$ **D** $-3 - 5$

4 The numbers $-7, 4, -4, 1, 0, 5$ in ascending order are:

A $-7, 5, 4, -4, 1, 0$ **B** $0, 1, 4, 5, -4, -7$ **C** $5, 4, 1, 0, -4, -7$ **D** $-7, -4, 0, 1, 4, 5$

5 $-4 + 3 =$

A -1 **B** -7 **C** 7 **D** 1

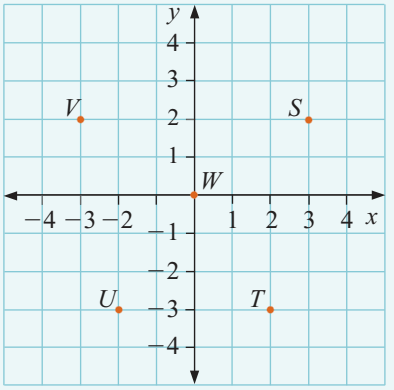
6 $-3 - 5 =$

A 8 **B** 2 **C** -2 **D** -8

7 $-3 + 7 - 5 =$

A -9 **B** -1 **C** 1 **D** 15

Use the number plane shown for questions **8** and **9**.



8 The point with x-coordinate -3 and y-coordinate 2 is:

A S **B** T **C** U **D** V

9 Which statement is *not* true?

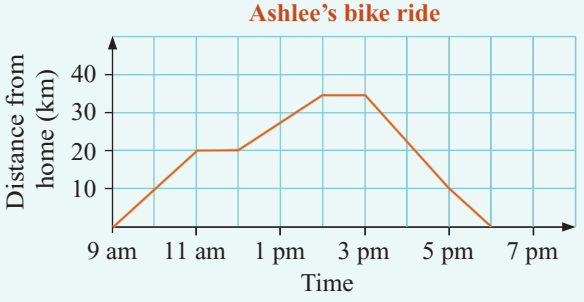
A Point S has coordinates (2, 3).

B Points U and T have the same y-coordinate.

C Point V is in the 2nd quadrant.

D Point W has coordinates (0, 0).

Use this graph for questions **10** to **13**.



10 The total distance travelled was:

A 35 km **B** 9 h

C 70 km **D** 0 km

11 The number of times Ashlee stopped was:

A 0 **B** 1 **C** 2 **D** 3

12 Ashlee headed for home at:

A 9:00 am **B** 11:00 am **C** 2:00 pm **D** 3:00 pm

13 Ashlee was travelling fastest between:

A 9:00 am and 11:00 am **B** 12 noon and 2:00 pm

C 3:00 pm and 5:00 pm **D** 5:00 pm and 6:00 pm

14 $7 + (-2) =$

A 9 **B** 5 **C** -5 **D** -9

15 $-3 - (-5) =$

A 2 **B** -8 **C** 8 **D** -2

16 $2 - (-3) + 8 =$

A 13 **B** 7 **C** -7 **D** -13

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2, 3	4	5–7	8, 9	10–13	14–16
Section	A	B	C	D	E	F, G	H

2A Review set

1 Plot each set of numbers on a number line.

a 2, 5, 8, 9 **b** 6, 11, 2, 4

2 **a** Plot the numbers 3 and 9 on a number line.

b Write a statement to describe the numbers 3 and 9:

i using $<$ **ii** using $>$

c Write two whole numbers between 3 and 9.

d Write two numbers between 3 and 9 that are not whole numbers.

3 Represent the following operations on a number line and hence find the answer.

a $3 + 5$ **b** $4 + 7 - 5$ **c** $3 \times 5 - 6$

4 Write the opposite of each statement.

a depositing \$15 **b** a decrease of 3 cm in length

c a gain of 4 kg in weight **d** a fall of 3°C in temperature

5 State the combined effect of a deposit of \$5 followed by a withdrawal of:

a \$7 **b** \$3 **c** \$14

6 If east is the positive direction, write directed numbers for these journeys:

a 8 km west **b** 5 km east **c** 2 km east, then 5 km west

7 Plot each set of numbers on a number line, then write them in ascending order.

a $-10, -3, -7, -11, -5$ **b** $-6, -3, -9, 0, 6$

8 Plot each set of numbers on a number line, then write them in ascending order.

a $1\frac{2}{3}, 2\frac{2}{3}, -\frac{2}{3}, -1\frac{1}{3}$ **b** $-1.2, -1.5, -1, -1.1, -0.6$

9 Using a number line to help you, insert a $>$ or $<$ symbol to make a true statement for each pair of numbers.

a $-6 \_\_\_ -7$ **b** $-3 \_\_\_ 3$ **c** $0 \_\_\_ -6$

10 Simplify using a number line.

a $-4 - 3$ **b** $-7 - (-2)$ **c** $2 - (-3) + 4$

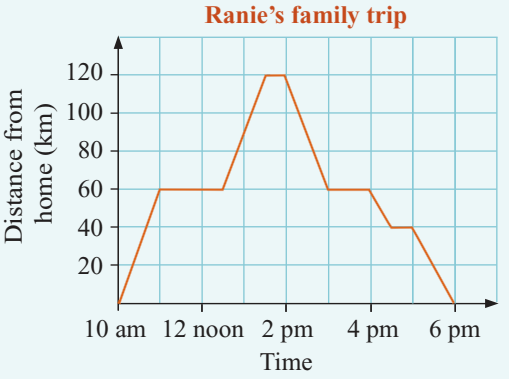
11 **a** Plot these points on a number plane: $A(3, 1)$ $B(-2, 5)$ $C(-4, -2)$ $D(3, -4)$ $E(0, -2)$

b Which points have the same x-coordinate?

c Which points have the same y-coordinate?

12 Draw a travel graph to describe this journey: Jack started from home at 8:00 am and travelled 35 km in 1 hour. He stopped for 30 minutes, then continued for another 2 hours, travelling another 90 km. He stopped for 45 minutes, then returned home, taking $2\frac{1}{2}$ hours.

13 The travel graph represents a car trip taken by Ranie's family. Use the graph to answer the following questions.



a At what time did Ranie's family start the trip?

b **i** At what time did they have their first rest stop?

ii For how long did they stop?

c What distance did they travel before their first rest stop?

d Between what times did they have their second rest stop?

e Between their second and third rest stops:

i What distance did the family travel?

ii How long did it take?

iii What was their average speed?

f For how long did the family stop during the third rest stop?

g After the third rest stop the family decided to head directly home. Unfortunately, they got a flat tyre.

i At what time did they get the flat tyre?

ii How long did it take to change the flat tyre?

h **i** At what time did the family arrive home?

ii At what average speed did they travel after changing the tyre?

i Calculate the total amount of time spent:

i travelling **ii** stopped.

j Calculate the total distance of the trip.

k For every 4 km travelled, 1 L of petrol was used. Calculate the number of litres used on the trip.

2B Review set

1 Plot each set of numbers on a number line.

a 3, 6, 7, 8 **b** 5, 3, 9, 4

2 **a** Plot the numbers 5 and 10 on a number line.

b Write a statement to describe the numbers 5 and 10:

i using $<$ **ii** using $>$

c Write two whole numbers between 5 and 10.

d Write two numbers between 5 and 10 that are not whole numbers.

3 Represent the following operations on a number line and hence find the answer.

a $2 + 4$ **b** $3 + 5 - 2$ **c** $2 \times 7 - 3$

4 Write the opposite of each statement.

a withdrawing \$5 **b** an increase of 2 cm in length

c a loss of 1 kg in weight **d** a rise of 4°C in temperature

5 State the combined effect of a deposit of \$6 followed by a withdrawal of:

a \$7 **b** \$3 **c** \$14

6 If north is the positive direction, write directed numbers for these journeys:

a 7 km south **b** 6 km north **c** 8 km north, then 4 km south

7 Plot each set of numbers on a number line, then write them in ascending order.

a $-8, -5, -6, -9, -4$ **b** $-7, -2, -8, 0, 4$

8 Plot each set of numbers on a number line, then write them in ascending order.

a $1\frac{1}{2}, 2\frac{1}{4}, 0, 1, -\frac{1}{2}$ **b** $-1.3, 0.2, -1.6, 0, -1.4$

9 Using a number line to help you, insert a $>$ or $<$ symbol to make a true statement for each pair of numbers.

a -5 ___ -6 **b** -2 ___ 2 **c** 0 ___ -3

10 Simplify using a number line.

a $-5 - 2$ **b** $-6 - (-3)$ **c** $4 - (-3)$

11 **a** Plot these points on a number plane: $A(3, 0), B(-3, -2), C(-3, 0), D(3, 2)$

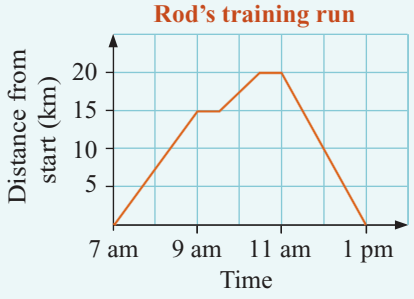
b Join $ABCD$. What shape have you made?

c What are the coordinates of the centre of shape $ABCD$?

12 Draw a travel graph to describe this journey.

Georgia left home at 9:00 am to go to the shopping centre. She walked $1\frac{1}{2}$ km to the bus stop, taking 15 minutes, then waited 15 more minutes for a bus. The bus journey took 30 minutes to travel 10 km. Georgia shopped for $1\frac{1}{4}$ hours, then met a friend. Her friend's family gave her a lift home, taking only 15 minutes.

13 Rod is training to run a marathon. Use this graph to answer these questions.



a How many rest stops did Rod have during his run?

b Between what times did Rod take his rest stops?

c Between 7 am and 9 am:

i How far did Rod run? **ii** How long did it take?

iii What was his average speed?

d Between 9:30 am and 10:30 am:

i How far did Rod run? **ii** How long did it take?

iii What was his average speed?

e At what time did Rod begin returning to the start of his run?

f Did Rod have any rest stops on his return journey?

g Calculate the number of kilometres completed by Rod.

h How long did it take Rod to complete his run?

i Between what times did Rod run fastest?

j Between what times did Rod run slowest?

2C Review set

1 Plot each set of numbers on a number line.

a 1, 3, 8, 9 **b** 1, 9, 5, 4

2 **a** Plot the numbers 2 and 7 on a number line.

b Write a statement to describe the numbers 2 and 7:

i using $<$ **ii** using $>$

c Write two whole numbers between 2 and 7.

d Write two numbers between 2 and 7 that are not whole numbers.

3 Represent the following operations on a number line and hence find the answer.

a $5 + 2$ **b** $8 + 3 - 4$ **c** $5 \times 2 - 4$

4 Write the opposite of each statement.

a depositing \$25 **b** a decrease of 9 cm in length

c a loss of 6 kg in weight **d** a rise of 8°C in temperature

5 State the combined effect of a deposit of \$9 followed by a withdrawal of:

a \$7 **b** \$3 **c** \$14

6 If east is the positive direction, write directed numbers for these journeys:

a 3 km west **b** 2 km east **c** 9 km east, then 12 km west

7 Plot each set of numbers on a number line, then write them in ascending order.

a $-9, -7, -4, -10, -3$ **b** $2, 0, -5, -2, 1$

8 Plot each set of numbers on a number line, then write them in ascending order.

a $1\frac{1}{4}, -1\frac{1}{2}, 2\frac{3}{4}, 0, \frac{1}{2}$ **b** $-1.5, 1.3, 0, 1, -2.5$

9 Using a number line to help you, insert a $>$ or $<$ symbol to make a true statement for each pair of numbers.

a -4 ___ -3 **b** -4 ___ 4 **c** 0 ___ -5

10 Simplify using a number line.

a $-7 - 4$ **b** $-9 - (-3)$ **c** $5 - (-2)$

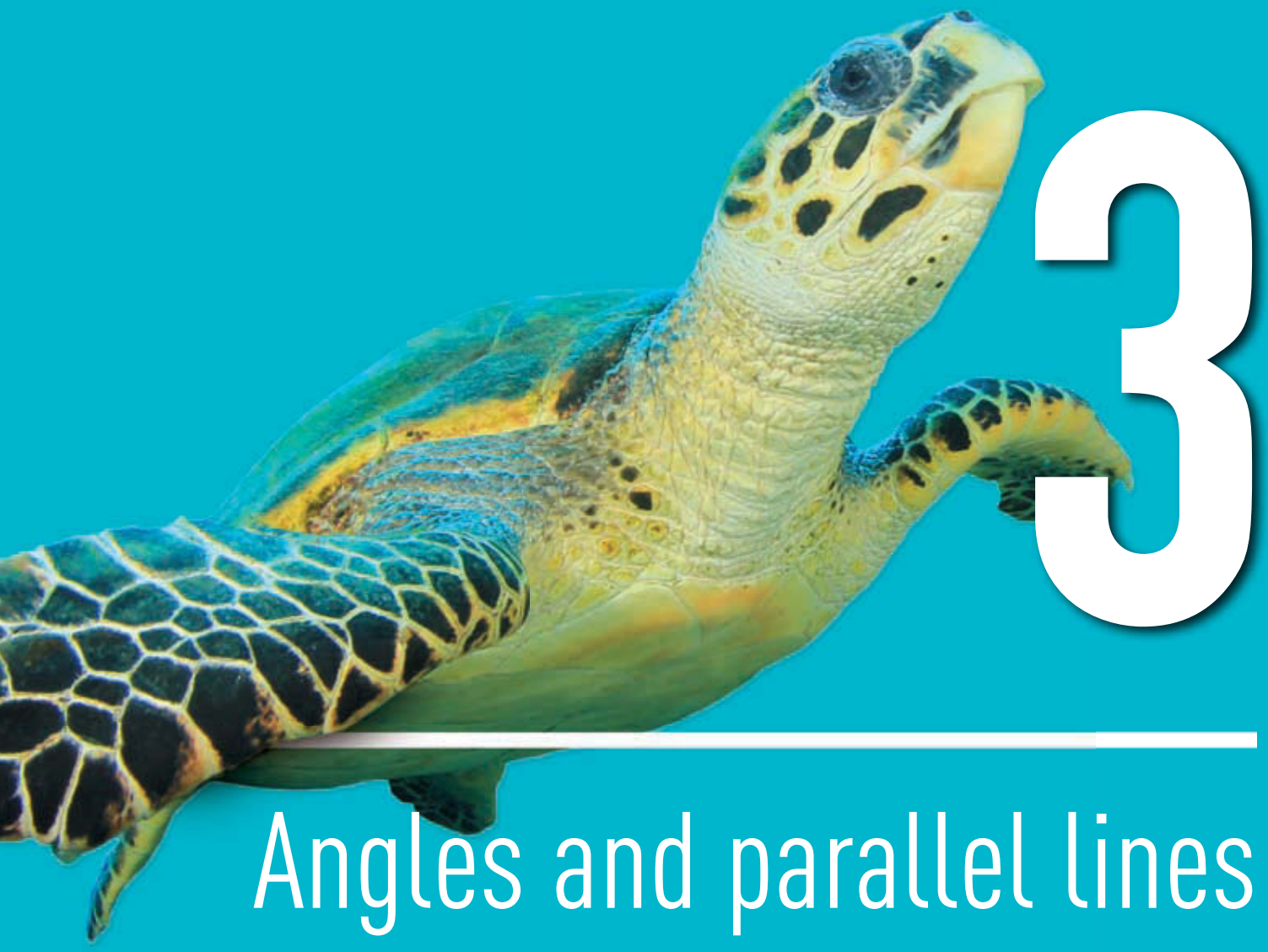
11 **a** Plot these points on a number plane: $A(0, 0)$, $B(-4, 4)$, $C(2, -1)$, $D(3, 3)$, $E(-4, -4)$

b Through which three points could you draw a straight line?

c What are the coordinates of the point exactly halfway between B and E?

12 Draw a travel graph to describe this journey.

Carly started from home half an hour before noon and drove 80 km in 1 hour. She had lunch with a friend, which took 2 hours, then started back home. After 30 minutes, halfway home, she stopped for fuel and a coffee, which took 15 minutes. Then she drove right home at the same speed as before.



3

Angles and parallel lines

This chapter deals with angles and parallel lines.

At the end of this chapter you should be able to:

- identify and name the vertex and arms of an angle
- name angles using correct notation
- classify angles
- determine conditions for two lines to be parallel
- identify and name properties of adjacent, complementary, supplementary, vertically opposite, co-interior, alternate and corresponding angles
- solve numerical and non-numerical problems using angle facts.

Diagnostic test

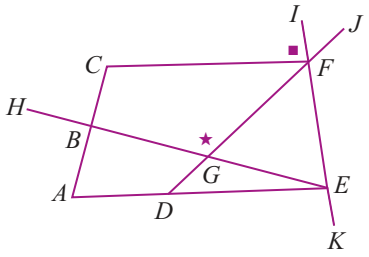
1 Name the vertex of $\angle FGH$.

A F **B** GH **C** FG **D** G

2 Name the arms of $\angle STU$.

A TS, SU **B** TS, TU
C UT, US **D** SU, TS

Questions **3** and **4** relate to the diagram below.



3 Name the angle marked with a ★.

A $\angle BGF$ **B** $\angle DGE$
C $\angle BGD$ **D** $\angle FGE$

4 Name the angle marked with a ■.

A $\angle IFJ$ **B** $\angle CFG$
C $\angle CFE$ **D** $\angle CFI$

5 The type of angle drawn below is:

A acute **B** obtuse
C reflex **D** revolution

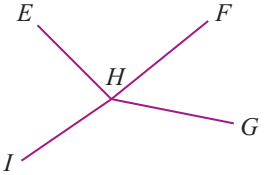
6 An obtuse angle is:

A $> 0°$ but $< 90°$ **B** equal to $90°$
C $> 90°$ but $< 180°$ **D** $> 180°$ but $< 360°$

7 A straight angle is:

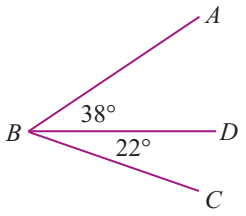
A equal to $90°$ **B** $> 180°$ but $< 360°$
C equal to $180°$ **D** $> 90°$ but $< 180°$

8 If IHF was a straight angle in this figure, which two angles would add to $180°$?



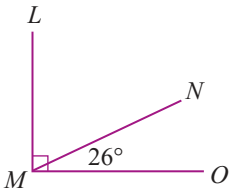
A $\angle EHF$ and $\angle FHG$ **B** $\angle IHE$ and $\angle FHG$
C $\angle EHF$ and $\angle IHG$ **D** $\angle IHE$ and $\angle EHF$

9 Calculate the size of $\angle ABC$.



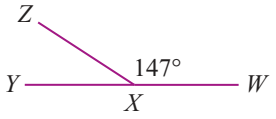
A $60°$ **B** $38°$
C $22°$ **D** $50°$

10 Calculate the size of $\angle LMN$.



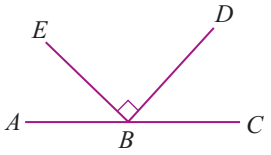
A $90°$ **B** $64°$
C $26°$ **D** $54°$

11 Calculate the size of $\angle YXZ$.



A $180°$ **B** $147°$
C $33°$ **D** $213°$

Questions **12** to **14** relate to the diagram below.



12 $\angle ABE + \angle EBD$ would be what type of angle?

A reflex **B** obtuse
C right **D** straight

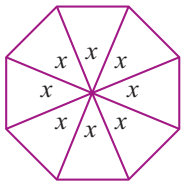
13 $\angle EBC - \angle DBC$ would be what type of angle?

A right **B** obtuse
C acute **D** straight

14 $\angle ABE + \angle DBC$ would be what type of angle?

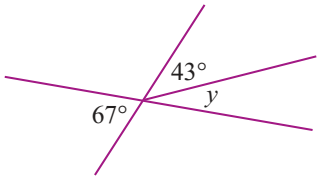
A obtuse **B** right
C acute **D** reflex

15 Calculate the value of x.



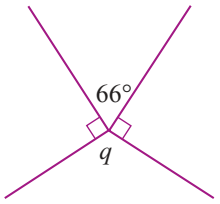
A 45° **B** 60° **C** 30° **D** 55°

16 Calculate the value of y.



A 43° **B** 24° **C** 67° **D** 134°

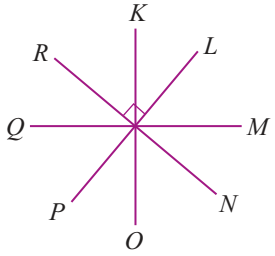
17 Calculate the value of q.



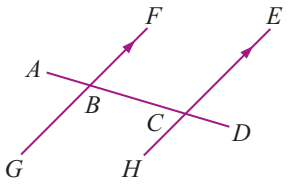
A 156° **B** 66° **C** 294° **D** 114°

18 Which one of these statements about the lines in the diagram below is true?

A $LP \perp QM$ **B** $RN \perp LP$
C $QM \perp RN$ **D** $KO \perp QM$



Questions **19** to **22** relate to the diagram below.



19 An angle vertically opposite to $\angle FBC$ is:

A $\angle BCE$ **B** $\angle ABF$
C $\angle ABG$ **D** $\angle HCD$

20 Which two angles would add to 180°?

A $\angle BCH$ and $\angle DCH$
B $\angle ECD$ and $\angle FBC$
C $\angle ABG$ and $\angle BCH$
D $\angle GBC$ and $\angle ECB$

21 The four angles at the point B add to 360°. Three of these angles are $\angle ABF$, $\angle GBC$ and $\angle ABG$. The fourth angle is:

A $\angle BCF$ **B** $\angle ABC$
C $\angle ACE$ **D** $\angle CBF$

22 If $\angle CBF$ measured as 65°, the size of $\angle GBC$ would be:

A 65° **B** 25°
C 130° **D** 115°

23 Which of the following statements is incorrect?

A A right angle is larger than an acute angle, and smaller than an obtuse angle.
B An obtuse angle is smaller than a revolution, but larger than a reflex angle.
C A reflex angle is larger than an acute angle, but smaller than a revolution.
D An acute angle is smaller than an obtuse angle, and smaller than a straight angle.

The Diagnostic test questions refer to the Year 6 content description below.

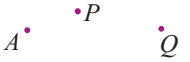
Question	1–23
Section	ACMMG 141

A Naming angles

Points, lines, intervals, rays and angles

We often use a dot to represent a **point** and name the point using a capital letter.

The points A, P and Q are shown.



A **line** is determined by any two points. Hence a line is named using any two points on it.

This line could be named AP or PA.

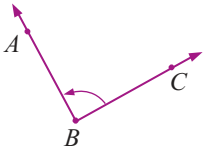
A line extends indefinitely in both directions. This is sometimes emphasised using arrowheads as shown. We cannot measure the length of the line.

An **interval** is a section of a straight line. The interval AP is shown. It is the set of points between and including the endpoints A and P.

An interval has finite length. Hence we can measure the length of an interval. Sometimes an interval is referred to as a **line segment**.

A **ray** is a part of a straight line that starts at a point and continues in one direction only. The ray shown would be named AP, as A is the endpoint of the ray. Always begin with the endpoint when naming a ray.

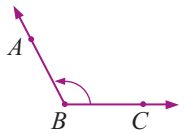
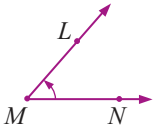
An **angle** is formed by two rays sharing a common endpoint. The diagram below shows the angle formed by the rays BA and BC. The common endpoint, B, is called the **vertex** and the rays BA and BC are called the **arms** of the angle.



The size of the angle is the amount that the ray BC must be turned through to meet the ray BA.

Angle naming

The table below outlines the ways in which angles can be named.

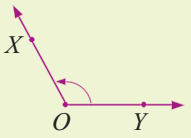
Angle	Method 1	Method 2	Method 3
	$\angle ABC$ or $\angle CBA$	$\hat{ABC}$ or $\hat{CBA}$	$\hat{B}$ or $\angle B$
	$\angle LMN$ or $\angle NML$	$\hat{LMN}$ or $\hat{NML}$	$\hat{M}$ or $\angle M$

Note: Method 3 may be used *only* if there is no possibility of confusion. If there are more than two rays at the vertex, method 3 may *not* be used.

EXAMPLE 1

For the angle drawn, name the:

a vertex **b** arms **c** angle



a The vertex is O.

b OX and OY; that is, rays $\overrightarrow{OX}$ and $\overrightarrow{OY}$.

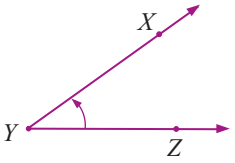
c $\angle XOY$ or $\angle YOX$ or $\angle O$ or $\hat{XOY}$ or $\hat{YOX}$ or $\hat{O}$.

Rays are sometimes called arms.

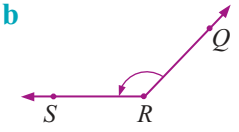
Exercise 3A

1 For each angle name the: **i** vertex **ii** arms **iii** angle.

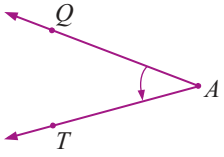
a



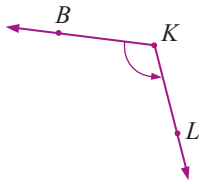
b



c



d



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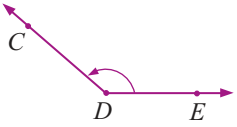

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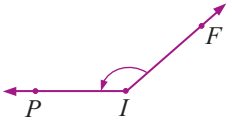
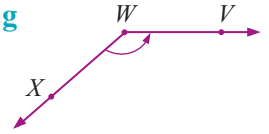
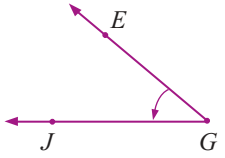


g



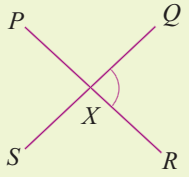
h



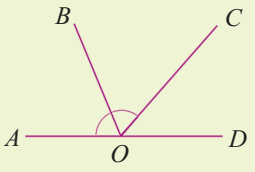
EXAMPLE 2

Name the angles marked in these diagrams.

a



b



Arms are often shown without arrows, and vertices without dots.

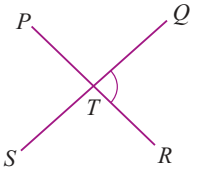
a $\angle QXR$ (or $\angle RXQ$) or $\hat{QXR}$ (or $\hat{RXQ}$)

b $\angle AOC$ (or $\angle COA$) or $\hat{AOC}$ (or $\hat{COA}$)

Using three letters makes it easier to identify the angle.

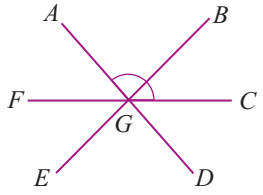
2 Complete each of the following to name the marked angle.

a



The vertex is ____.
The arms are ____ and ____.
The angle is ____.

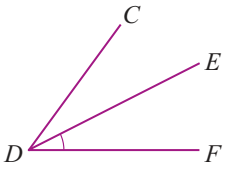
b



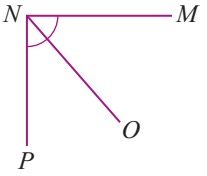
The vertex is ____.
The arms are ____ and ____.
The angle is ____.

3 Name the marked angles.

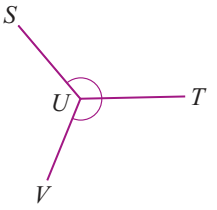
a



b



c



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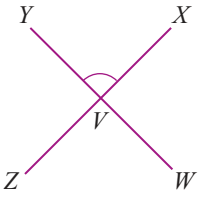

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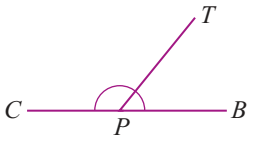

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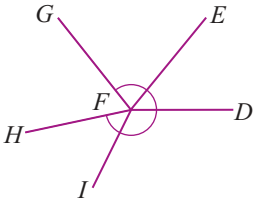
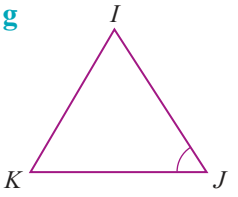
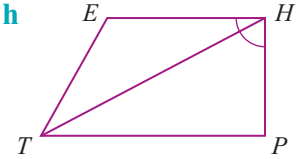


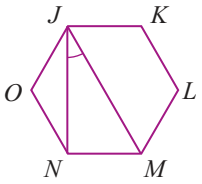
g



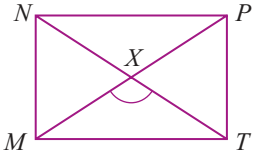
h



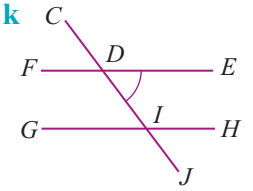
i



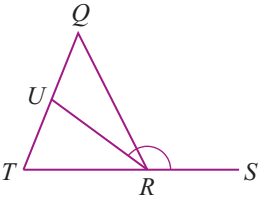
j



k

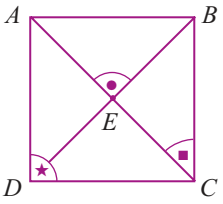


l

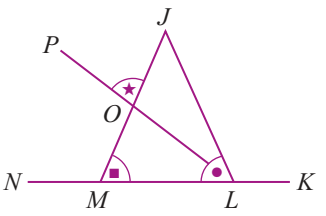


4 Name the angles marked ●, ★ or ■, using the nearest letters.

a



b



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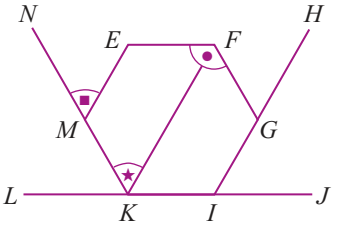
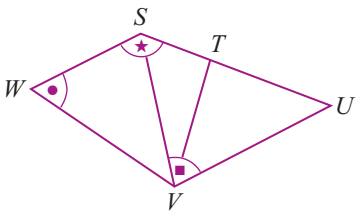


d



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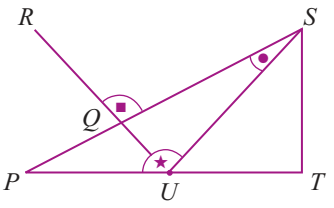

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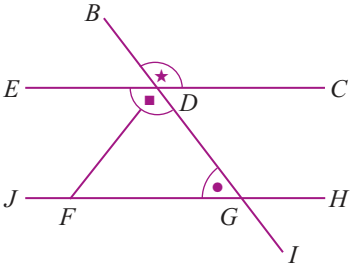
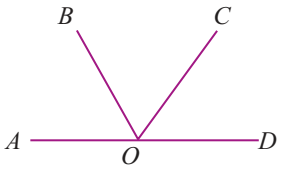
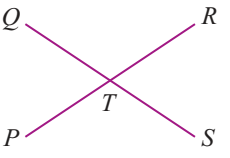


5 Copy these diagrams and mark the angles named.

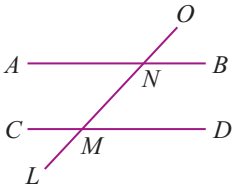
a $\angle BOD$



b $\angle QTS$

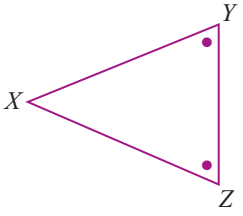


c $\angle ANM$, $\angle MNB$ and $\angle CML$

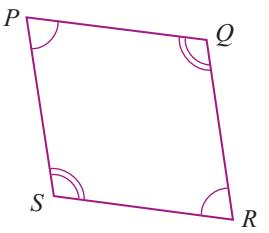


6 Name the pairs of equal angles in the following diagrams. (Equal angles have identical markings on them.)

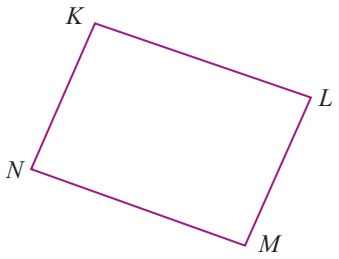
a



b

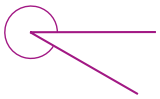


7 Copy the diagram and indicate on it that $\angle KNM = \angle KLM$.



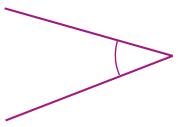
B Types of angles

Angles are classified according to their size.

Angle type	Diagram	Description
Acute angle		between 0° and 90°
Right angle		equal to 90°
Obtuse angle		between 90° and 180°
Straight angle (line)		equal to 180°
Reflex angle		between 180° and 360°
Revolution		equal to 360°

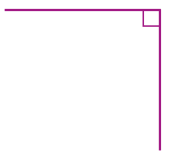
Exercise 3B

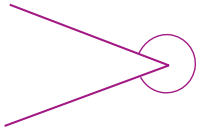
1 Classify the following angles.

a 

b 

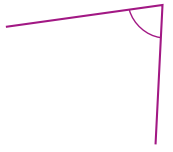
c 

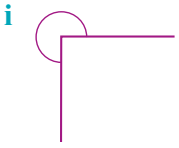
d 

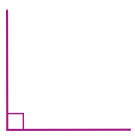
e 

f 

g 

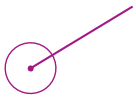
h 

i 

j 

k 

l 

m 

n 

o 

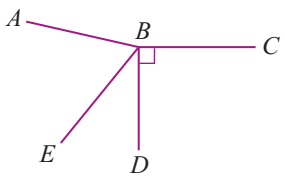
p 

2 Classify these angles given their sizes.

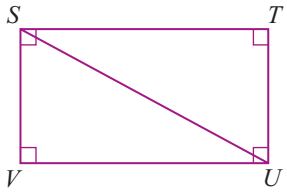
a 58°	**b** 175°	**c** 90°	**d** 108°	**e** 2°
f 89°	**g** 360°	**h** 149°	**i** 241°	**j** 93°
k 180°	**l** 224°	**m** 22°	**n** 305°	**o** 136°

3 Classify the angles shown in these diagrams.

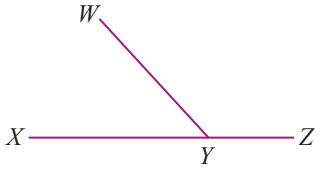
a **i** $\angle ABC$
ii $\angle CBD$
iii $\angle EBD$
iv $\angle ABE$



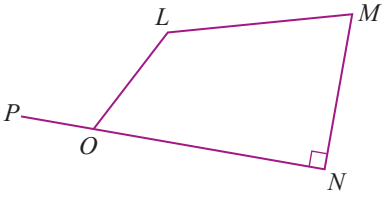
b **i** $\angle V$
ii $\angle T$
iii $\angle VSU$
iv $\angle TUS$



c **i** $\angle XYZ$
ii $\angle WYZ$
iii $\angle XYW$



d **i** $\angle L$
ii $\angle M$
iii $\angle N$
iv $\angle LOP$
v $\angle LON$
vi $\angle NOP$



4 Decide whether each statement is true or false. Correct any false statements.

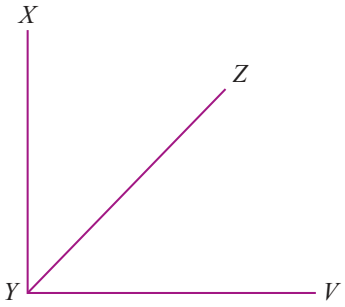
a All acute angles are less than 90°.
b Some obtuse angles are greater than 180°.
c All straight angles are 180°.
d Reflex angles are smaller in measure than obtuse angles.
e Two right angles make a straight angle.
f An acute angle plus a right angle makes a reflex angle.

Investigation 1 Types of angles

Measure each angle as stated in Sets A, B and C. Find the total of angles in each set. State what type of angle is formed in each set.

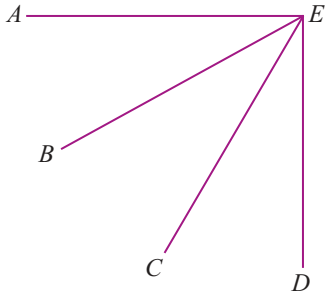
Set A

a



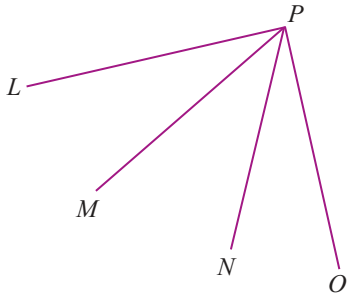
$\angle XYZ =$ ____
$\angle ZYV =$ ____
$\therefore \angle XYV =$ ____

b



$\angle AEB =$ ____
$\angle BEC =$ ____
$\angle CED =$ ____
$\therefore \angle AED =$ ____

c

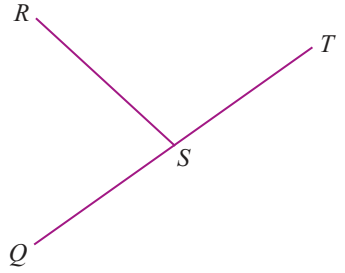


$\angle LPM =$ ____
$\angle MPN =$ ____
$\angle NPO =$ ____
$\therefore \angle LPO =$ ____

Type of angle formed in Set A: _____

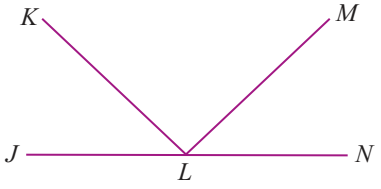
Set B

a



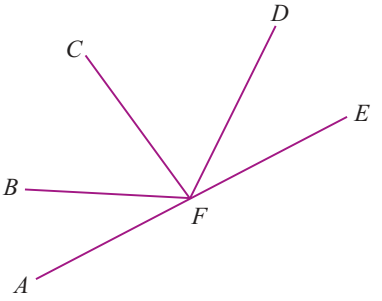
$\angle QSR =$ ____
$\angle RST =$ ____
$\therefore \angle QST =$ ____

b



$\angle JLK =$ ____
$\angle KLM =$ ____
$\angle MLN =$ ____
$\therefore \angle JLN =$ ____

c

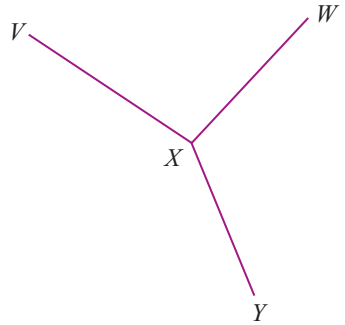


$\angle AFB =$ ____
$\angle BFC =$ ____
$\angle CFD =$ ____
$\angle DFE =$ ____
$\therefore \angle AFE =$ ____

Type of angle formed in Set B: _____

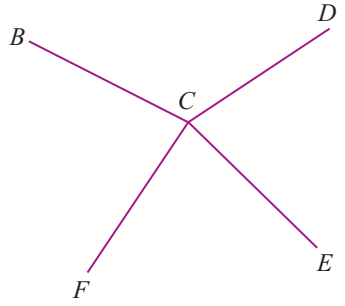
Set C

a



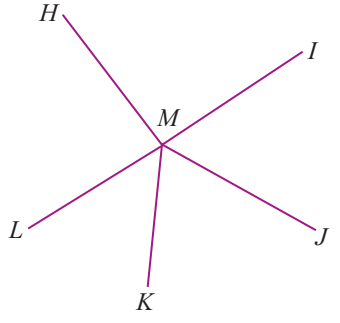
$\angle VXW =$ ____
$\angle WXY =$ ____
$\angle YXV =$ ____
$\therefore$ Total of angles = ____

b



$\angle BCD =$ ____
$\angle DCE =$ ____
$\angle ECF =$ ____
$\angle FCB =$ ____
$\therefore$ Total of angles = ____

c



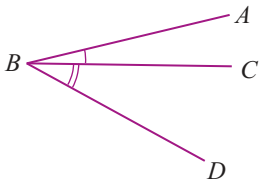
$\angle HMI =$ ____
$\angle IMJ =$ ____
$\angle JMK =$ ____
$\angle KML =$ ____
$\angle LMH =$ ____
$\therefore$ Total of angles = ____

Type of angle formed in Set C: _____

C Adjacent angles

Adjacent angles must have:
- a common vertex
- a common arm and
- lie on opposite sides of the common arm.

For example, $\angle ABC$ and $\angle CBD$ are adjacent.
- B is the common vertex.
- BC is the common arm.
- $\angle ABC$ and $\angle CBD$ lie on opposite sides of BC.

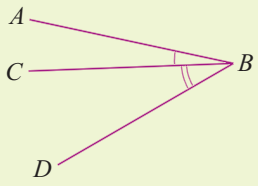


'Common' means shared or belonging to more than one owner. Arm BC and vertex B belong to both ∠ABC and ∠CBD.

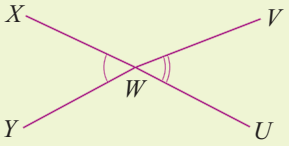
EXAMPLE 1

For each diagram, state whether the marked angles are adjacent. Give a reason.

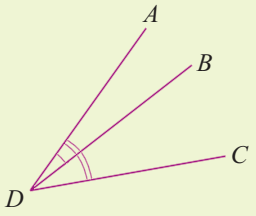
a $\angle ABC$ and $\angle CBD$



b $\angle XWY$ and $\angle VWU$



c $\angle ADB$ and $\angle ADC$



a Common vertex is B.
Common arm is BC.
The angles lie on opposite sides of BC.
$\therefore$ $\angle ABC$ and $\angle CBD$ are adjacent because all criteria are satisfied.

b Common vertex is W.
No common arm.
$\therefore$ $\angle XWY$ and $\angle VWU$ are *not* adjacent because not all criteria are satisfied.

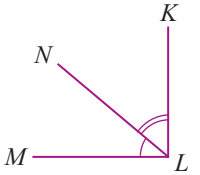
c Common vertex is B.
Common arm is AD.
The angles do not lie on opposite sides of AD.
$\therefore$ $\angle ADB$ and $\angle ADC$ are *not* adjacent because not all criteria are satisfied.

Exercise 3C

1 Complete the statements to determine whether the marked angles are adjacent.

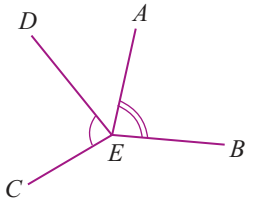
a $\angle MLN$ and $\angle NLK$

Common vertex is _____.
Common arm is _____.
The angles do/do not lie on opposite sides of _____.
$\therefore$ The angles are/are not adjacent because _____.



b $\angle DEC$ and $\angle AEB$

Common vertex is _____.
There _____ a common arm.
$\therefore$ The angles are/are not adjacent because _____.



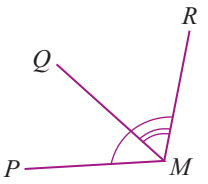
c $\angle PMR$ and $\angle QMR$

Common vertex is _____.

Common arm is _____.

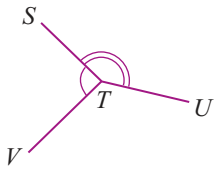
The angles do/do not lie on opposite sides of _____.

$\therefore$ The angles are/are not adjacent because _____.

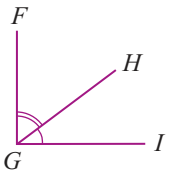


2 State whether the marked angles in each diagram are adjacent. Give a reason.

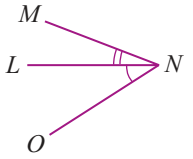
a $\angle STV$ and $\angle STU$



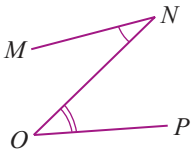
b $\angle FGH$ and $\angle HGI$



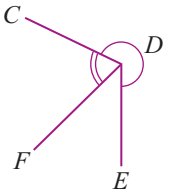
c $\angle MNL$ and $\angle LNO$



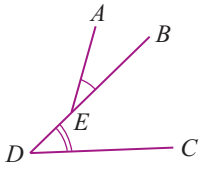
d $\angle MNO$ and $\angle NOP$



e $\angle CDE$ and $\angle FDC$

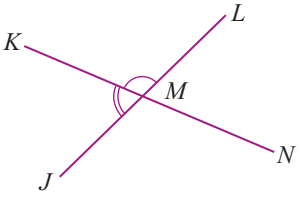


f $\angle AEB$ and $\angle BDC$

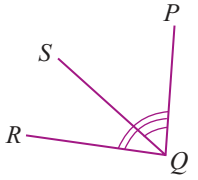


g $\angle IGF$ and $\angle HGI$

h $\angle KMJ$ and $\angle KML$

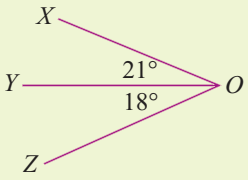


i $\angle PQS$ and $\angle PQR$

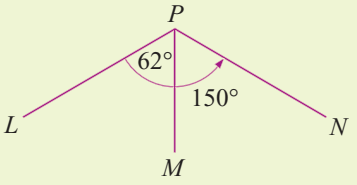


EXAMPLE 2

a In the diagram $\angle XOY = 21°$ and $\angle YOZ = 18°$. Find the size of $\angle XOZ$.



b In the diagram $\angle LPM = 62°$ and $\angle LPN = 150°$. Find the size of $\angle MPN$.



a $\angle XOY$ and $\angle YOZ$ are adjacent angles.

$\therefore \angle XOZ = 21° + 18° = 39°$

b $\angle LPM$ and $\angle MPN$ are adjacent angles that add to 150°.

$\therefore \angle MPN = 150° - 62° = 88°$

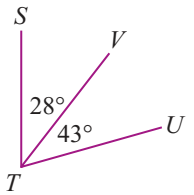
3 Complete the following to find the size of $\angle STU$.

a In the diagram, $\angle STV = 28°$ and $\angle VTU = 43°$.

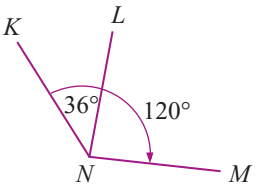
$\angle STV$ and $\angle VTU$ are _____ angles.

$\therefore \angle STU$ = _____° + _____°

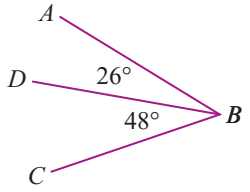
= _____°

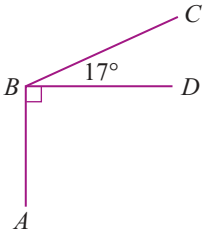


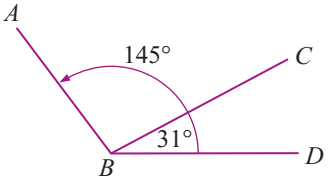
b In the diagram, $\angle KNL = 36°$ and $\angle KNM = 120°$.
$\angle KNL$ and $\angle LNM$ are _____ angles that add to _____°.
$\therefore \angle LNM =$ _____° − _____°
= _____°

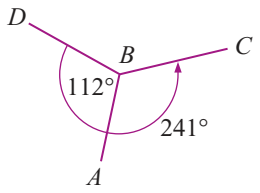


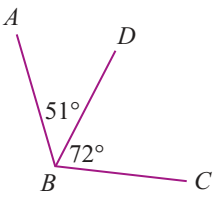
4 Calculate the size of $\angle ABC$ in each diagram.

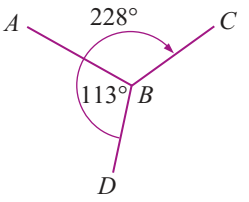
a


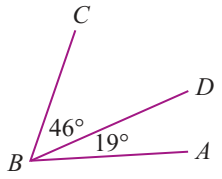
b


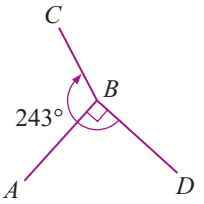
c


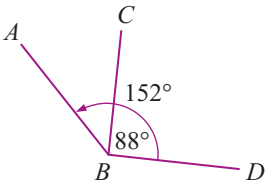
d


e


f


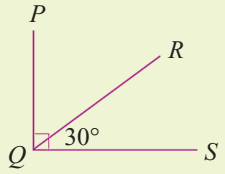
g


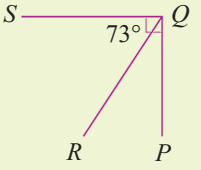
h


i


EXAMPLE 3

Find the size of $\angle PQR$ in these diagrams.

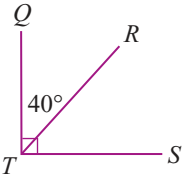
a


b


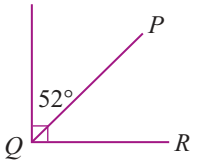
a $\angle PQR$ and $\angle RQS$ are adjacent angles that add to 90°.
$\therefore \angle PQR = 90° - 30 = 60°$

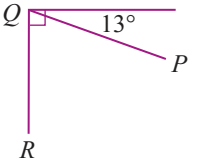
b $\angle PQR$ and $\angle RQS$ are adjacent angles that add to 90°.
$\therefore \angle PQR = 90° - 73° = 17°$

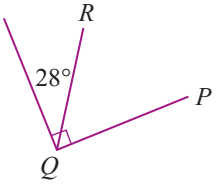
5 Complete the following to find the size of $\angle RTS$.
$\angle QTR$ and $\angle RTS$ are _____ angles that add to ____°.
$\therefore \angle RTS =$ ____° − 40°
= ____°

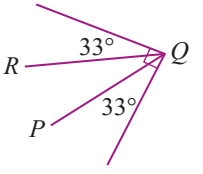


6 Find the size of $\angle PQR$ in each diagram.

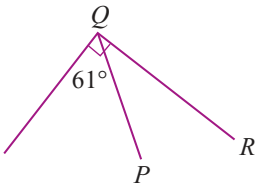
a


b


c


d


e



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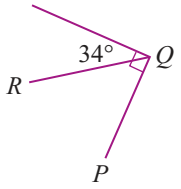

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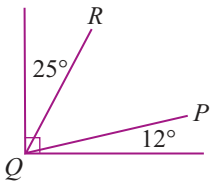

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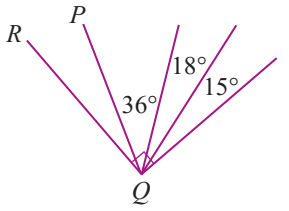

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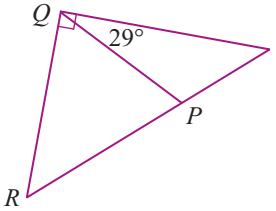

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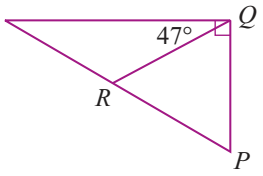

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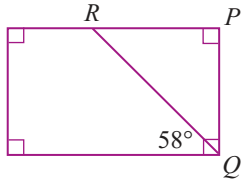

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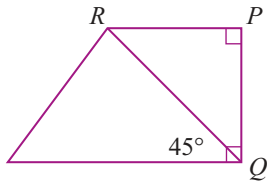
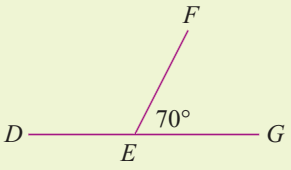


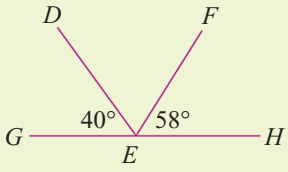
EXAMPLE 4

Find the size of $\angle DEF$ in each diagram.

a



b



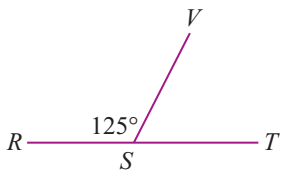
a $\angle DEF$ and $\angle FEG$ are adjacent angles on a straight line.
$\therefore \angle DEF = 180° - 70° = 110°$
because adjacent angles on a straight line add to 180°.

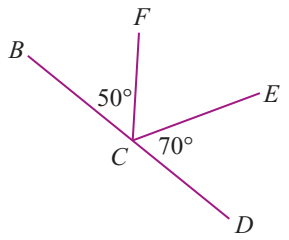
b $\angle GED$ is adjacent to $\angle DEF$, and $\angle DEF$ is adjacent to $\angle FEH$. We may say that the angles are adjacent angles on a straight line.
$\therefore \angle DEF = 180° - 40° - 58° = 82°$
because adjacent angles on a straight line add to 180°.

7 Complete the following to find the size of the unknown angle.

a $\angle RSV$ and $\angle VST$ are _____ angles on a _____ line.
$\therefore \angle VST =$ ___° − 125°
= ___°
because _____ angles on a straight line add to ___°.

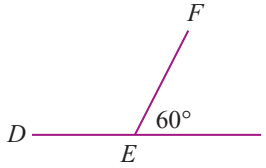
b $\angle BCF$, $\angle FCE$ and $\angle ECD$ are _____ angles on a _____ line.
$\therefore \angle FCE =$ ___° − 50° − 70°
= ___°
because _____ angles on a straight line add to ___°.



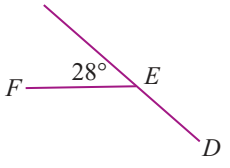


8 Find the size of $\angle DEF$ in each diagram.

a



b



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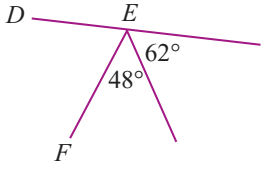

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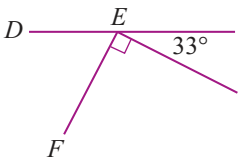

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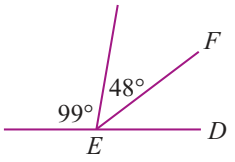

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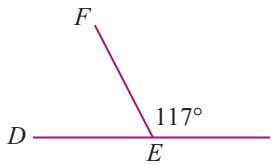

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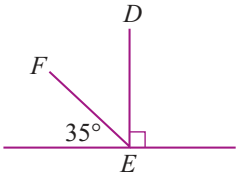

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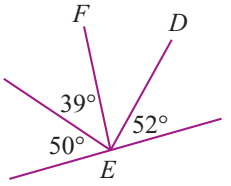

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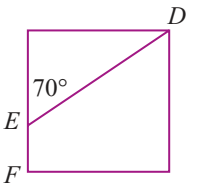

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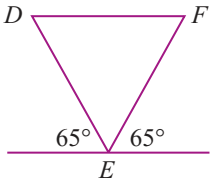

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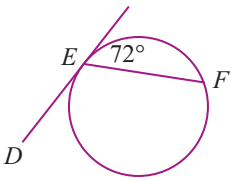

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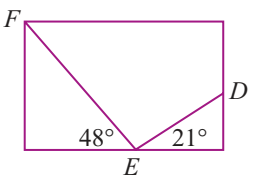
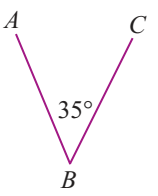
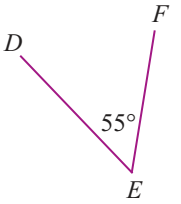


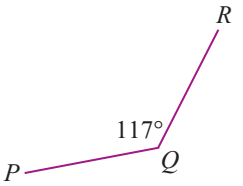
D Complementary and supplementary angles

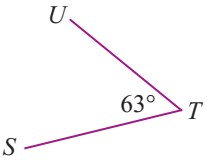
Complementary angles are any two angles that add up to 90°. They do not need to be adjacent. Since each angle is said to be the **complement** of the other. As 35° + 55° = 90°, then $\angle ABC$ and $\angle DEF$ (or 35° and 55°) are a pair of complementary angles. The complement of 35° is 55°, and the complement of 55° is 35°.





Supplementary angles are any two angles that add up to 180°. They do not need to be adjacent. Each angle is said to be the **supplement** of the other. As 117° + 63° = 180°, then $\angle PQR$ and $\angle STU$ (or 117° and 63°) are a pair of supplementary angles. The supplement of 117° is 63°, and the supplement of 63° is 117°.

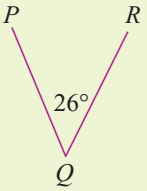


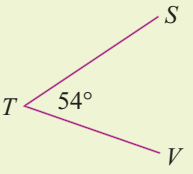


EXAMPLE 1

a State whether the following pairs of angles are complementary.

i

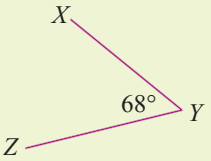


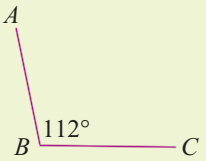


ii 71° and 19°

b State whether the following pairs of angles are supplementary.

i





ii 98° and 92°

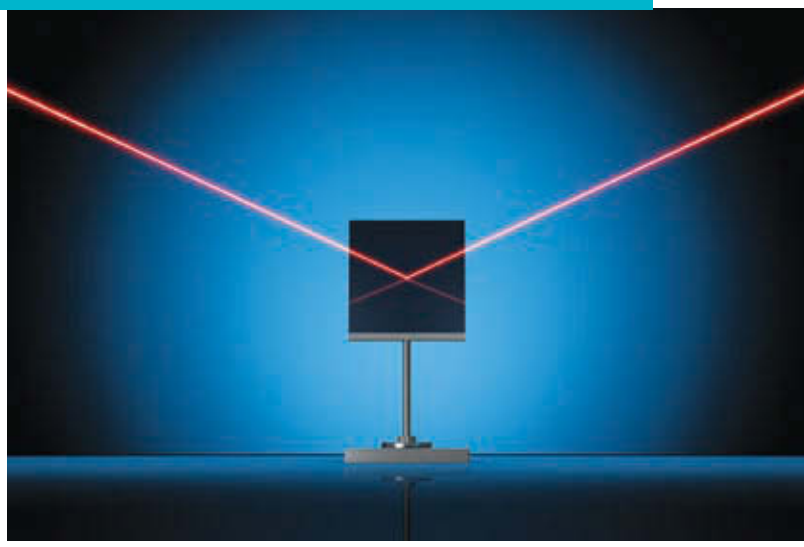
a **i** 26° + 54° = 80° (not 90°) ∴ The angles are not complementary.

ii 71° + 19° = 90° ∴ The angles are complementary.

b **i** 68° + 112° = 180° ∴ The angles are supplementary.

ii 98° + 92° = 190° (not 180°) ∴ The angles are not supplementary.

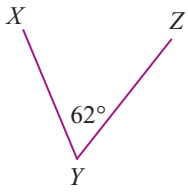
Exercise 3D

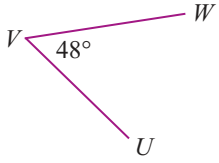


1 Complete the following to determine whether the given pairs of angles are complementary.

a $62° + 48° =$ ____°

$\therefore$ 62° and 48° are/are not complementary angles because they do/do not add to ____°.





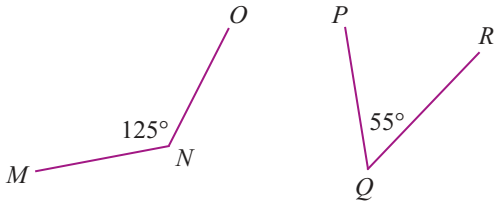
b $43° + 47° =$ ____°

$\therefore$ 43° and 47° are/are not complementary angles because they do/do not add to ____°.

2 Complete the following to determine whether the given pairs of angles are supplementary.

a $125° + 55° =$ ____°

$\therefore$ 125° and 55° are/are not supplementary angles because they do/do not add to ____°.

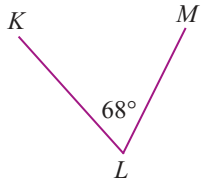


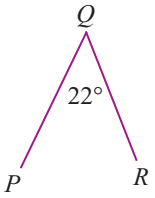
b $79° + 111° =$ ____°

$\therefore$ 79° and 111° are/are not supplementary angles because they do/do not add to ___°.

3 State whether these pairs of angles are complementary.

a

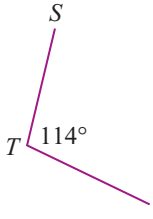


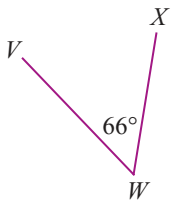


b 73° and 7° **c** 41° and 49° **d** 13° and 67°

4 State whether these pairs of angles are supplementary.

a





b 85° and 95° **c** 136° and 34° **d** 8° and 172°

5 State whether the following pairs of angles are complementary, supplementary or neither.

a 46° and 134° **b** 46° and 34° **c** 92° and 88° **d** 65° and 25° **e** 10° and 80°

EXAMPLE 2

a Determine the complement of 57°.

a Complement of $57° = 90° - 57° = 33°$ because $57° + 33° = 90°$.

b Determine the supplement of 84°.

b Supplement of $84° = 180° - 84° = 96°$ because $84° + 96° = 180°$.

6 Complete the following.

a Complement of $75° = 90° - \_\_\_\_°$
$= \_\_\_\_°$
because $75° + \_\_\_\_° = 90°$.

b Supplement of $110° = 180° - \_\_\_\_°$
$= \_\_\_\_°$
because $110° + \_\_\_\_° = 180°$.

7 Write down the:

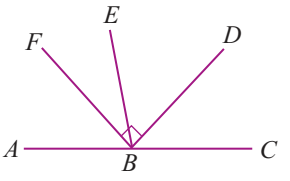
a complement of 40°
b supplement of 100°
c complement of 72°
d supplement of 60°
e supplement of 15°
f complement of 5°
g complement of 41°
h supplement of 48°
i supplement of 90°

8 **a** If a pair of complementary angles are placed next to each other (so that they form a pair of adjacent angles), what type of angle is formed?

b If a pair of supplementary angles are placed next to each other, what type of angle is formed?

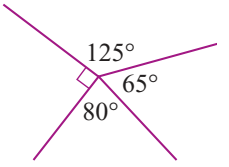
9 State whether the following pairs of angles are complementary, supplementary or neither.

a $\angle FBE$ and $\angle DBE$
b $\angle ABE$ and $\angle CBE$
c $\angle FBE$ and $\angle DBC$
d $\angle FBC$ and $\angle FBA$
e $\angle FBA$ and $\angle DBC$
f $\angle FBA$ and $\angle EBD$



E Angles at a point and vertically opposite

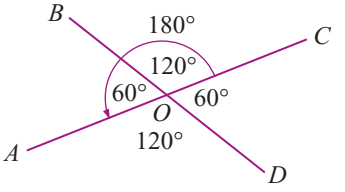
This diagram shows four angles sharing a common vertex. They are referred to as **angles at a point**.



Because the angles at a point form a revolution, we conclude that:

The sum of all angles at a point is 360°.

A special case of angles at a point occurs when two straight lines cut each other. Since the sum of angles on a straight line is 180°, two pairs of equal supplementary angles are formed.
Since $\angle AOB = 60°$, then $\angle BOC = 120°$, $\angle COD = 60°$ and $\angle DOA = 120°$.
$\angle AOB$ and $\angle COD$ are said to be vertically opposite.
Likewise, $\angle BOC$ and $\angle DOA$ are vertically opposite.

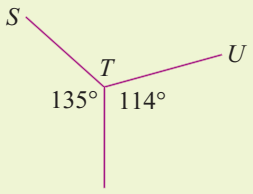


Vertically opposite angles are equal.

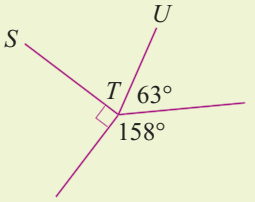
EXAMPLE 1

Find the size of $\angle STU$.

a



b



a The sum of angles at a point is 360°.

$\therefore \angle STU = 360° - 135° - 114°$

$= 111°$

b The sum of angles at a point is 360°.

$\therefore \angle STU = 360° - 158° - 90° - 63°$

$= 49°$

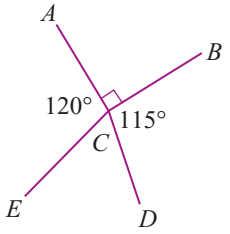
Exercise 3E

1 Complete the following.

$\angle ECD =$ ____° − 120° − 90° − 115°

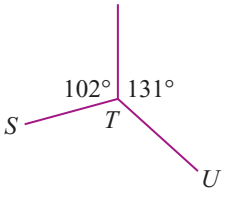
= ____°

because the sum of angles at a point is ____°.

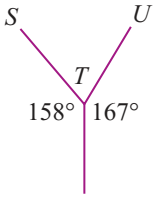


2 Find the size of $\angle STU$ in each of the following.

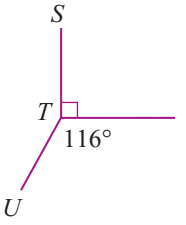
a



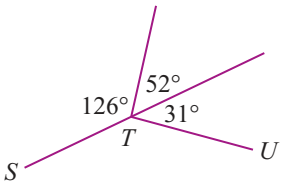
b



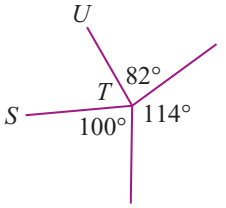
c



d



e



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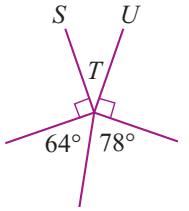
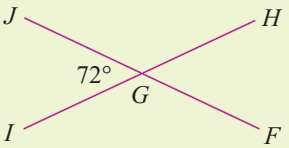


EXAMPLE 2

Find the value of $\angle FGH$ in each diagram.

a



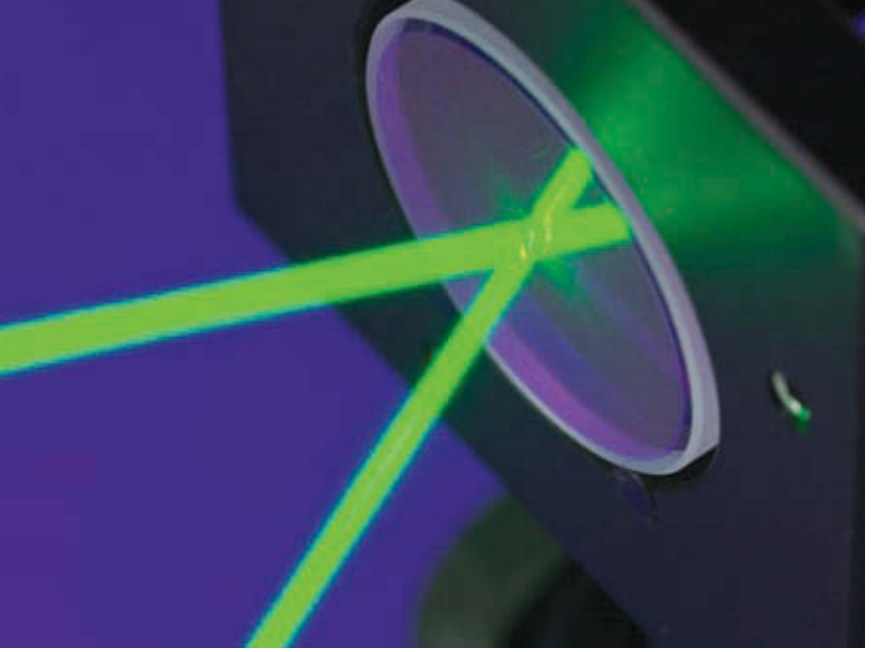
b

a $\angle FGH = 72°$

because it is vertically opposite $\angle JGI$.

b $\angle FGH = 143°$

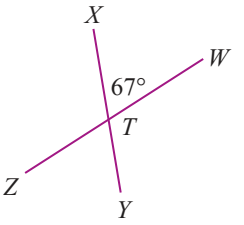
because it is vertically opposite $\angle TGR$.



3 Complete the following.

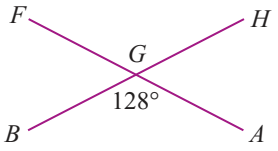
$\angle ZTY =$ ____°

because it is vertically opposite $\angle$____.



4 Find the size of $\angle FGH$ in each diagram.

a



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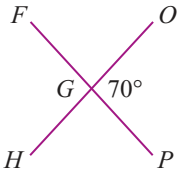

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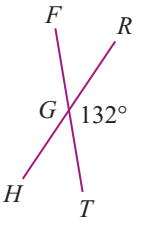

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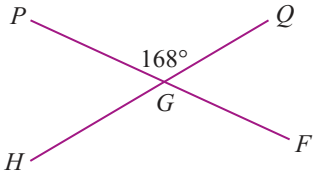

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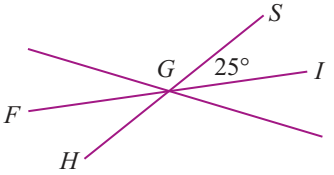

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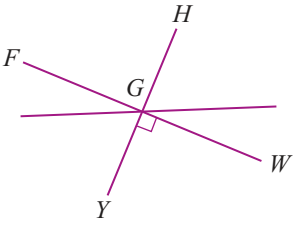


Using pronumerals in angles

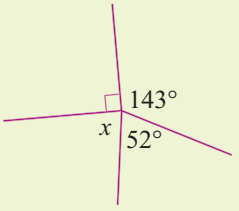
A **pronumeral** is a letter (or other symbol) that is used instead of a number. Sometimes pronumerals are used to show the sizes of the angles to be found.

Pronumerals (sometimes called **variables**) are used a lot in **algebra**, which you will learn about in Chapter 7. If necessary, come back to the remaining part of this exercise after studying Chapter 7 *Algebra*.

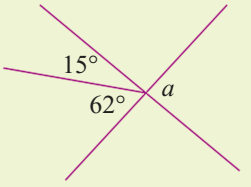
EXAMPLE 3

Find the values of the pronumerals, giving reasons.

a



b

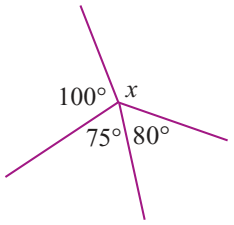


a $x = 360° - 90° - 143° - 52° = 75°$
because angles at a point add to 360°.

b $a = 62° + 15° = 77°$
because vertically opposite angles are equal.

5 Complete the following.

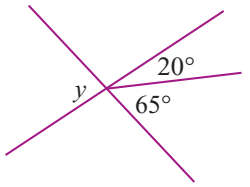
a



$x =$ ____° − 100° − 75° − 80° = ____°

because angles at a point add to ____°.

b

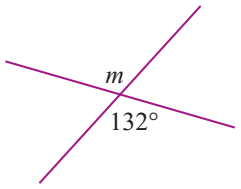


$y =$ ____° + ____° = ____°

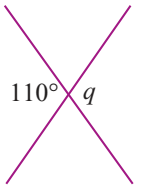
because _____ _____ angles are equal.

6 Find the values of the pronumerals in these diagrams, giving reasons.

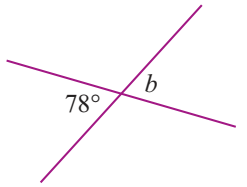
a



b



c



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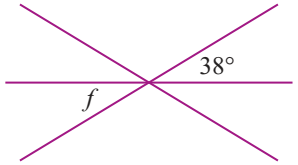

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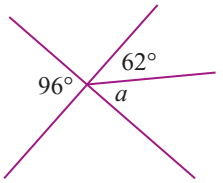

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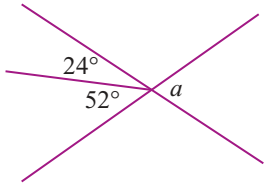
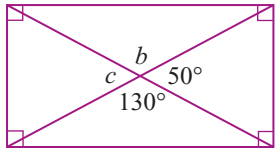


g



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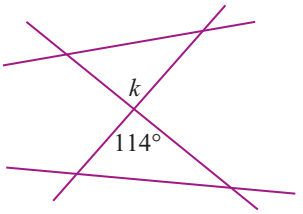

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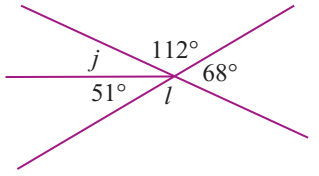
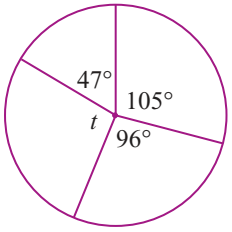
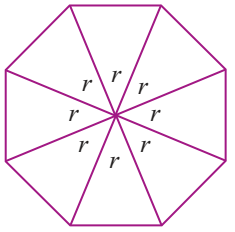


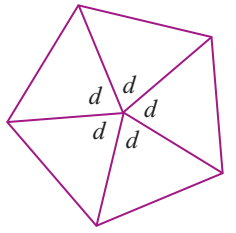
j



k



l



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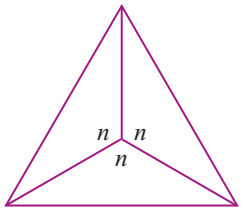

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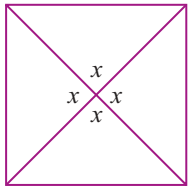
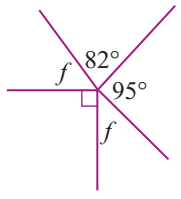
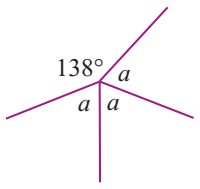


o



p



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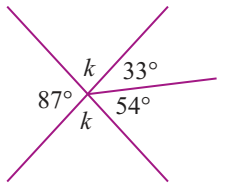
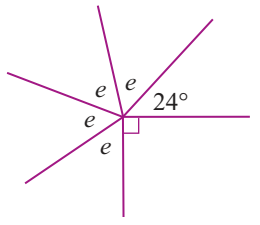


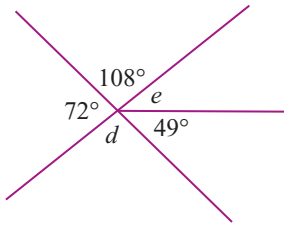
r



7 Find the values of the pronumerals in the following, using your knowledge that:

- angles on a straight line add to 180°
- angles at a point add to 360° and
- vertically opposite angles are equal.

a



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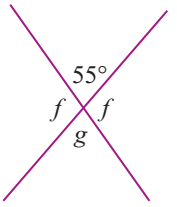
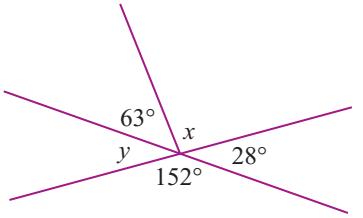
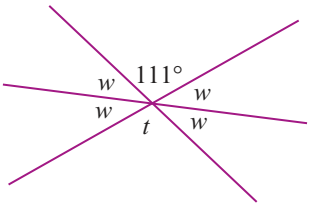


c



d



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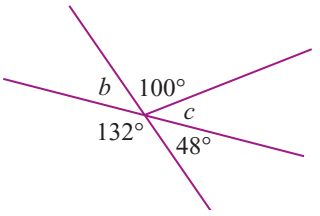

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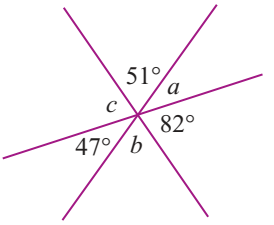

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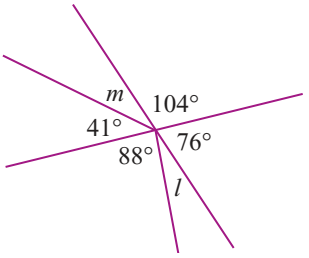

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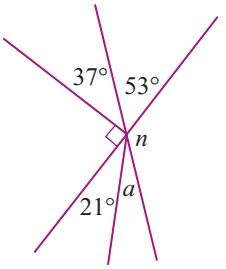

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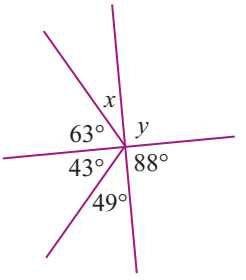
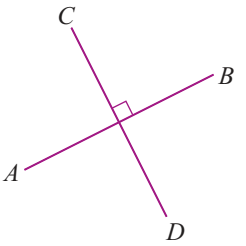


F Perpendicular lines

Perpendicular lines are lines that intersect at right angles.

In the diagram, AB and DC are perpendicular lines because they make a right angle at the point where they intersect.

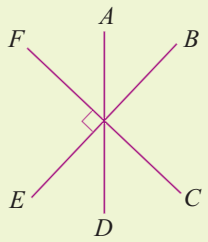
We say that line AB is perpendicular to line CD. This is written $AB \perp CD$.



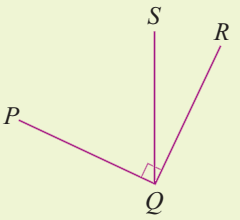
EXAMPLE 1

Name the perpendicular lines in the following diagrams.

a



b



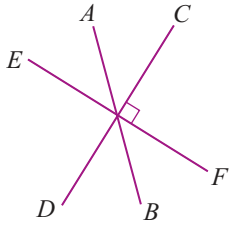
a BE and FC are perpendicular lines because they make a right angle at the point where they intersect ($BE \perp FC$).

b PQ and QR are perpendicular lines because they make a right angle at point Q ($PQ \perp QR$).

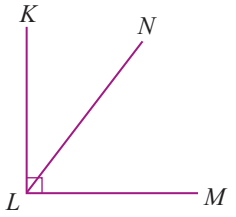
Exercise 3F

1 Name the perpendicular lines in the following diagrams.

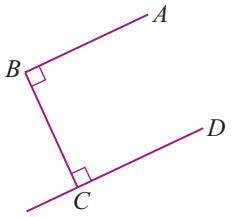
a



b



c



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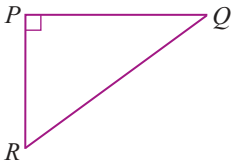

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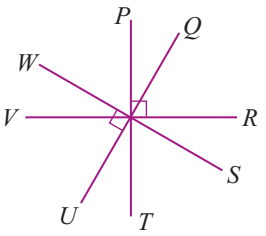

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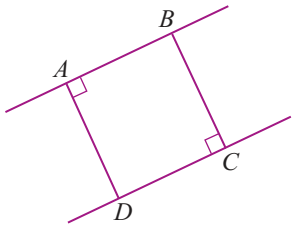
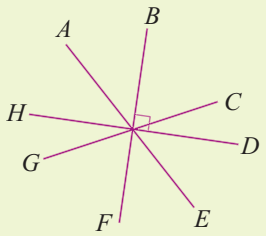


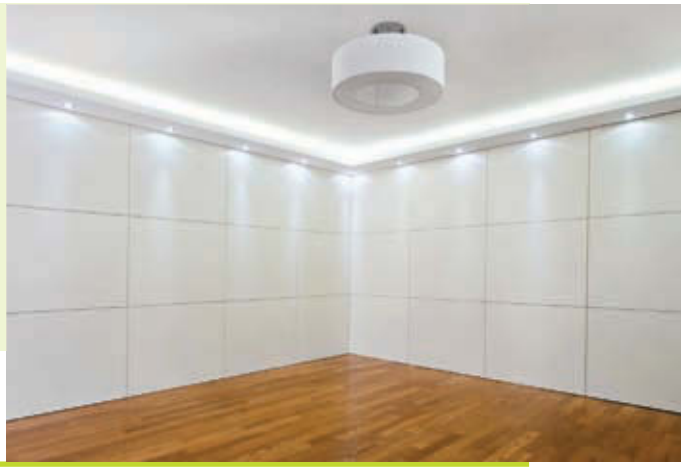
EXAMPLE 2

Determine whether the following statements are true or false.

a $AE \perp GC$

b $BF \perp HD$





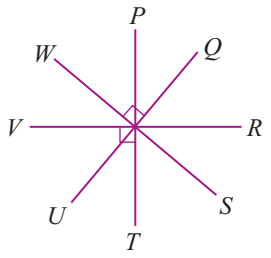
a False because AE and GC do not intersect at right angles.

b True, because BF and HD intersect at right angles.

2 Determine whether the following statements are true or false.

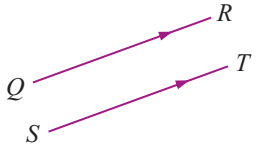
a $PT \perp VR$ **b** $WS \perp QU$

c $VR \perp WS$ **d** $QU \perp PT$



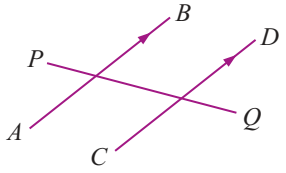
Investigation 2 Parallel lines

Straight lines that lie in the same plane and do not intersect are described as **parallel lines**. Arrowheads on the lines indicate parallel lines. In the diagram, QR is parallel to ST. This may be written $QR \parallel ST$. A line cutting through a set of parallel lines is called a **transversal**.

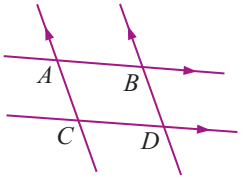


1 Name the parallel lines in the following diagrams.

a



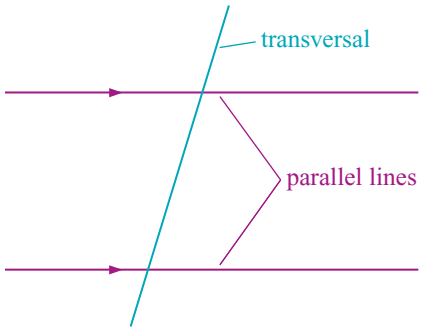
b



2 In the diagram at right a transversal cuts a pair of parallel lines.

a Use your protractor to measure the eight angles formed.

b What can you say about the angles?



3 **a** Use the opposite sides of your ruler to draw a pair of parallel lines. Then draw a transversal. Measure all the angles.

b What can you say about the angles?

c Repeat for another pair of parallel lines and transversal.

4 **a** When parallel lines are cut by a transversal, what conclusions can you draw about the pairs of angles formed:

i above (or below) the parallel lines and on the same side of the transversal?

ii between the parallel lines and on opposite sides of the transversal?

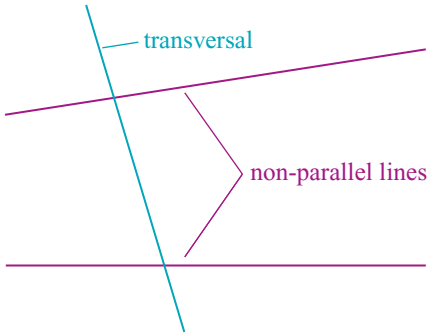
iii between the parallel lines and on the same side of the transversal?

b What is the relationship between the non-equal angles?

5 Here are two *non-parallel* lines cut by a transversal.

a Measure the eight angles formed.

b What can you say about the angles?



6 **a** Draw your own pair of *non-parallel* lines and a transversal.

b Measure all the angles. What do you notice?

c Repeat for another pair of *non-parallel* lines and a transversal.

d Is there a relationship between the non-equal angles?

7 **a** What does the term 'perpendicular' mean?

b Draw a line AB 5 cm long.

c Use the opposite sides of your ruler to draw another line parallel to AB called CD.

d Use your protractor to draw a line EF that is perpendicular to AB and cuts CD.

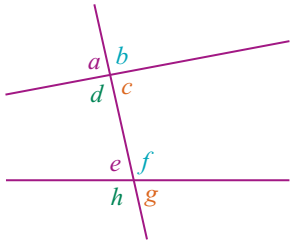
e Measure all the angles. What do you notice?

G Angle pairs on parallel lines

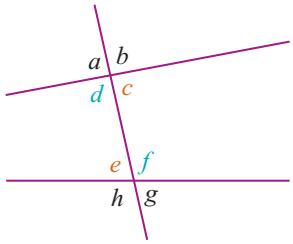
For sections G and H you need to be comfortable working with **pronumerals**. If necessary, come back to these sections after studying Chapter 7 *Algebra*.

When two straight lines are cut by a third straight line (called a **transversal**), three special pairs of angles are formed.

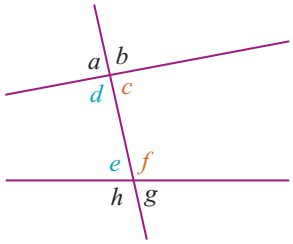
- **Corresponding angles** lie above the lines and on the same side of the transversal, or below the lines and on the same side of the transversal. In the diagram, b and f, c and g, a and e, and d and h are pairs of corresponding angles.



- **Alternate angles** lie between the lines and on opposite sides of the transversal. In the diagram, d and f, and c and e are pairs of alternate angles.



- **Co-interior angles** lie between the lines and on the same side of the transversal. In the diagram, d and e, and c and f are pairs of co-interior angles.



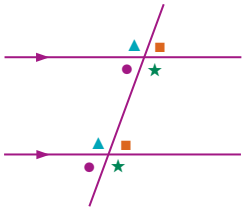
As you saw in Investigation 2, if two *parallel* lines are cut by a transversal, then:
- the pairs of corresponding angles are equal
- the pairs of alternate angles are equal
- the pairs of co-interior angles are supplementary.

Sometimes, lower-case letters (such as a, b, c) look like angle names, but they are still actually pronumerals. They have a value.

Exercise 3G

Corresponding angles

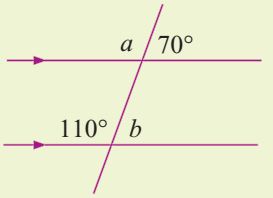
This diagram shows four pairs of corresponding angles on parallel lines. The corresponding angles are equal.



Corresponding angles have the same positioning.

EXAMPLE 1

Find the values of the pronumerals in the diagram.

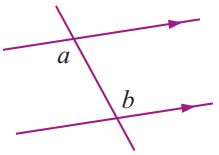


The lines are parallel, so the corrresponding angles are equal.

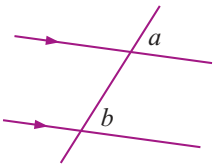
a is corresponding to 110° $\therefore a = 110°$ b is corresponding to 70° $\therefore b = 70°$

1 Are a and b corresponding angles?

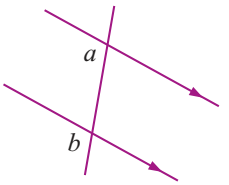
a



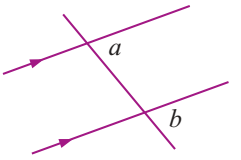
b



c

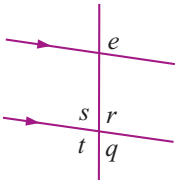


d

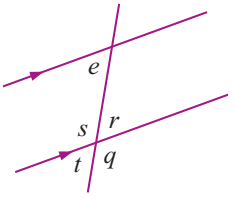


2 Which of the angles q, r, s or t is corresponding to e?

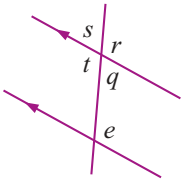
a



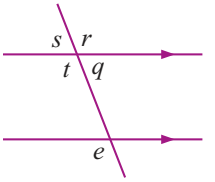
b



c

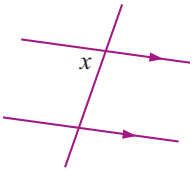


d

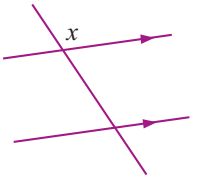


3 Sketch the following diagrams and use a dot to mark the corresponding angle to x.

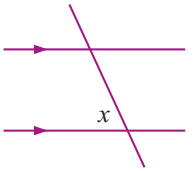
a



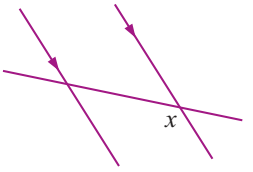
b



c

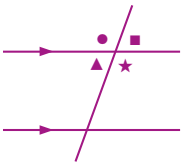


d

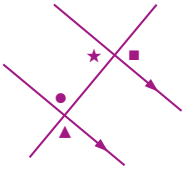


4 Sketch the following diagrams and use ■, ●, ▲ and ★ to mark the corresponding angles.

a

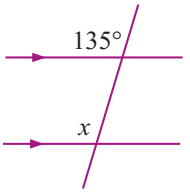


b

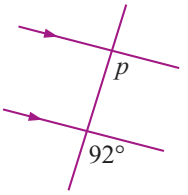


5 Find the values of the pronumerals.

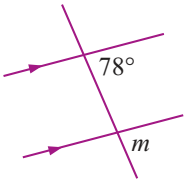
a



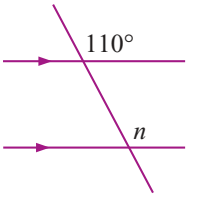
b



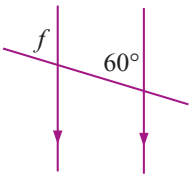
c



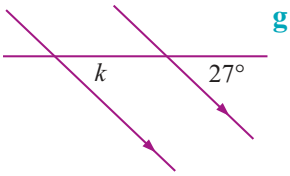
d



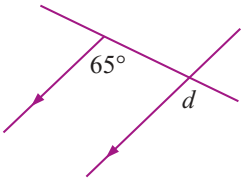
e



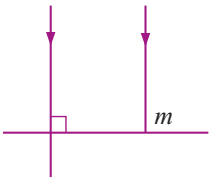
f



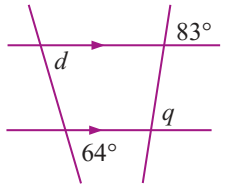
g



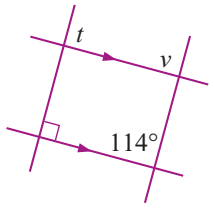
h



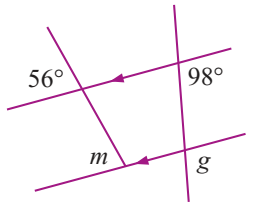
i



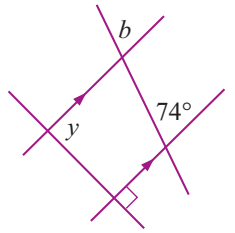
j



k

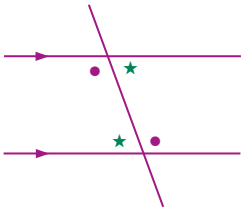


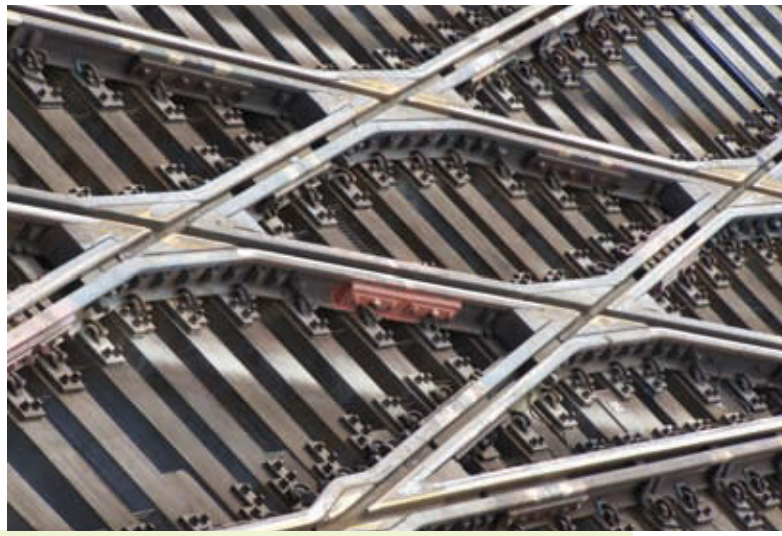
l



Alternate angles

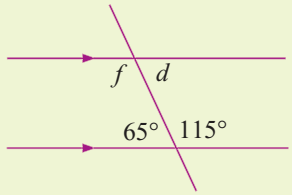
This diagram shows two pairs of alternate angles on parallel lines. The alternate angles are equal.





EXAMPLE 2

Find the values of the pronumerals in the diagram.

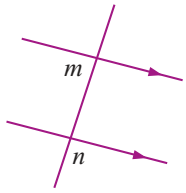


The lines are parallel, so the alternate angles are equal.
d is alternate to 65° $\therefore d = 65°$ f is alternate to 115° $\therefore f = 115°$

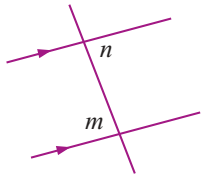
Alternate angles have different or opposite positioning.

6 Are m and n alternate angles?

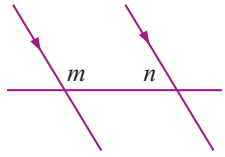
a



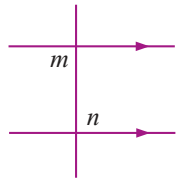
b



c

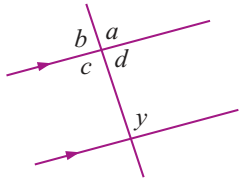


d

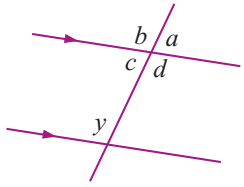


7 Which angle, a, b, c or d, is alternate to y in each diagram?

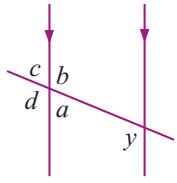
a



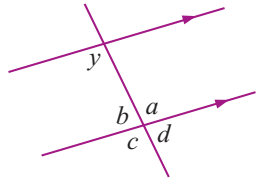
b



c

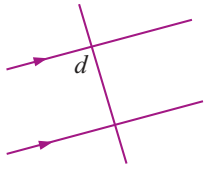


d

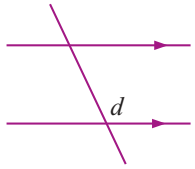


8 Sketch the following diagrams and use a dot to mark the alternate angle to d.

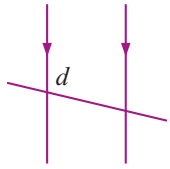
a



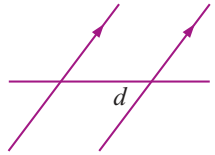
b



c

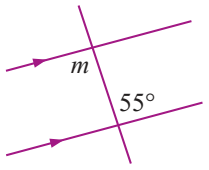


d

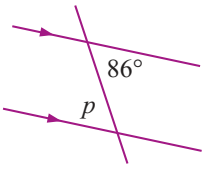


9 Find the values of the pronumerals in these alternate angles.

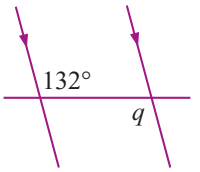
a



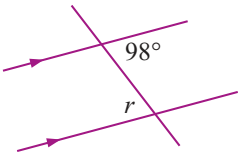
b



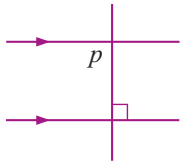
c



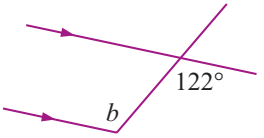
d



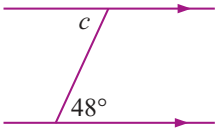
e



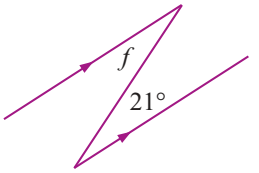
f



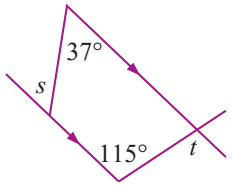
g



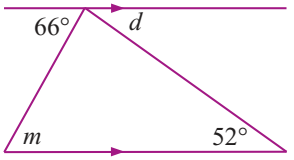
h



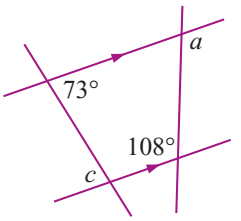
i



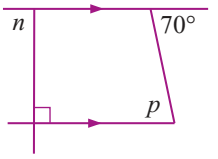
j



k

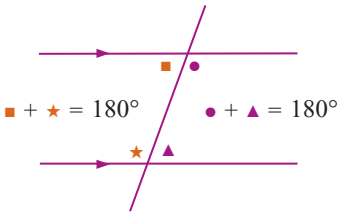


l



Co-interior angles

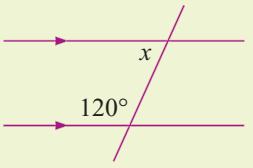
This diagram shows two pairs of co-interior angles on parallel lines.
The co-interior angles add up to 180°. They are supplementary.



Co-interior angles are on the inside of the lines.

EXAMPLE 3

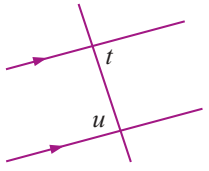
Find the value of the pronumeral in the diagram.



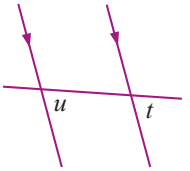
The lines are parallel, so the co-interior angles are supplementary.
x is co-interior to 120°
$\therefore x = 180° - 120° = 60°$

10 Are the angles marked t and u co-interior angles?

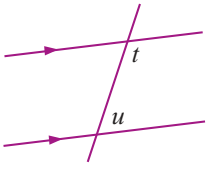
a



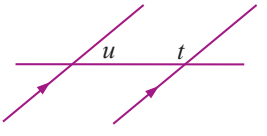
b



c

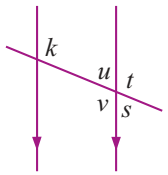


d



11 Which angle, s, t, u or v, is co-interior to k in each diagram?

a



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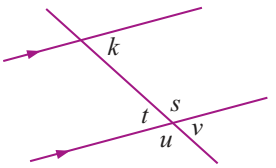

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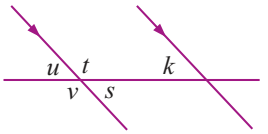

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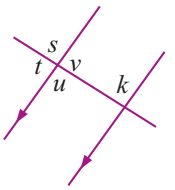
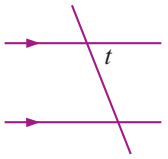


12 Sketch the following diagrams and use a dot to mark the co-interior angle to t.

a



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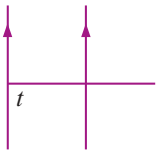

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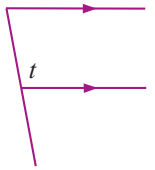

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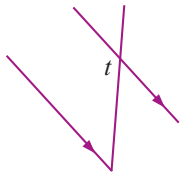
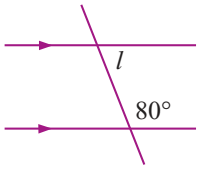
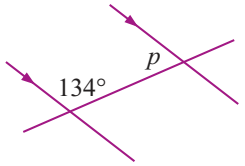


13 Find the values of the pronumerals for these co-interior angles.

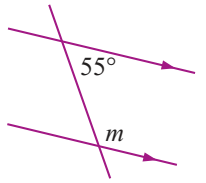
a



b



c



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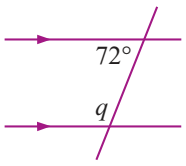

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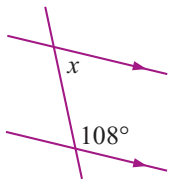

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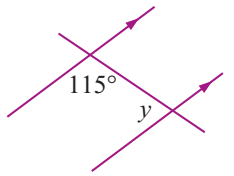
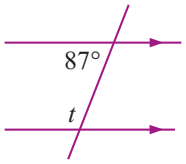


g



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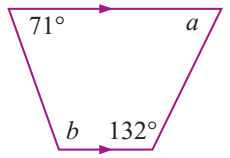
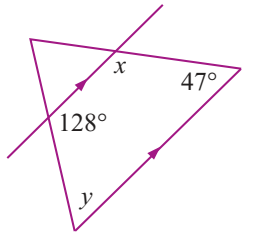


i



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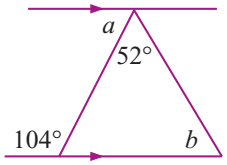
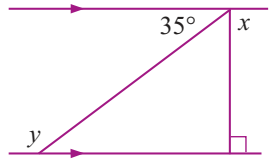
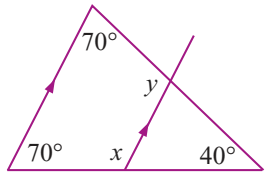


k



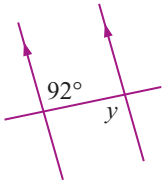
l



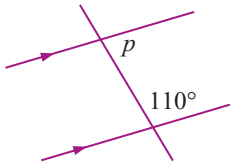
Corresponding, alternate and co-interior angles

14 Find the value(s) of the pronumeral(s) and give a reason for each answer.

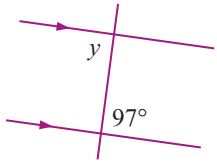
a



b



c



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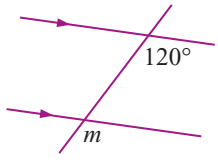

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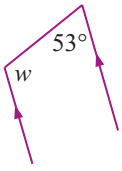

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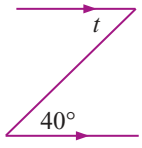

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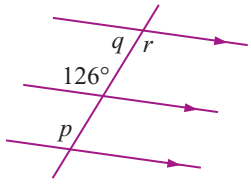

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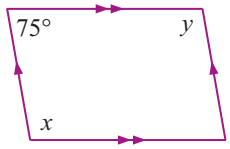
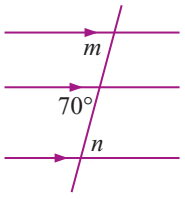


i



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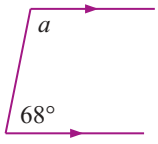

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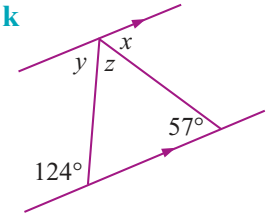

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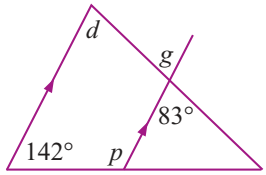

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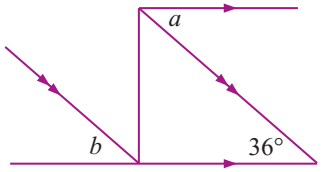

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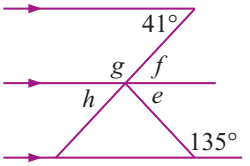
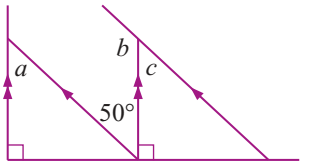


o



Determining whether lines are parallel

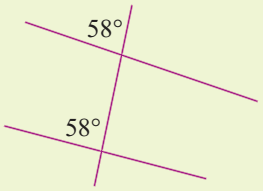
Lines are parallel if:

- corresponding angles are equal, or
- alternate angles are equal, or
- co-interior angles are supplementary.

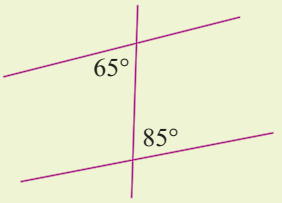
EXAMPLE 4

Determine whether each diagram contains a pair of parallel lines. State a reason.

a



b

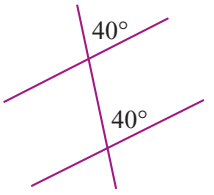


a A pair of corresponding angles are equal, therefore the lines are parallel.

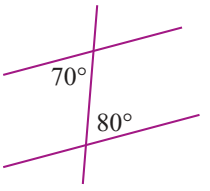
b A pair of alternate angles are *not* equal, therefore the lines are *not* parallel.

15 Complete the following.

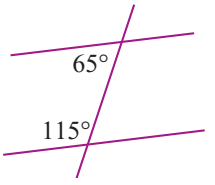
a The given angles are a pair of _____ angles.
Since these angles are/are not equal, the lines are/are not parallel.



b The given angles are a pair of _____ angles.
Since these angles are/are not equal, the lines are/are not parallel.

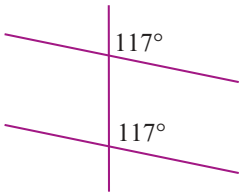


c The given angles are a pair of _____ angles.
Since these angles are/are not supplementary, the lines are/are not parallel.

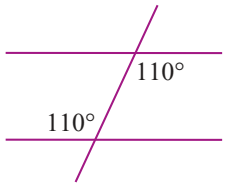


16 Which diagrams contain a pair of parallel lines? Give reasons for your answers.

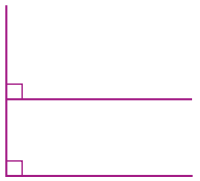
a



b



c



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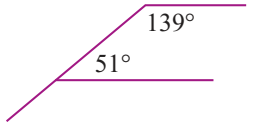

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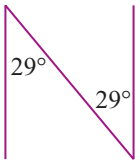

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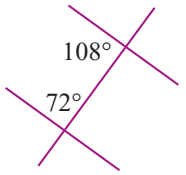

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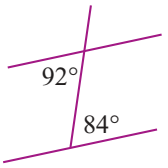
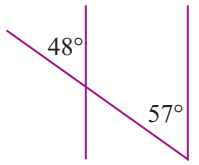
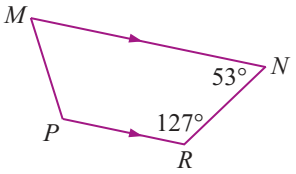


h



17 For each diagram, state why *MN* is parallel to *PR*.

a



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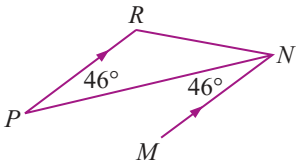
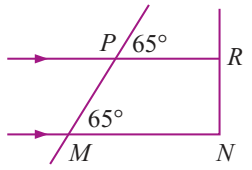
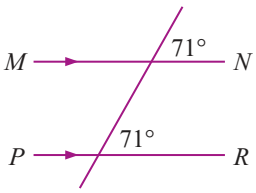


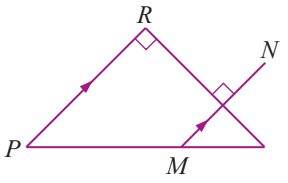
c



d



e



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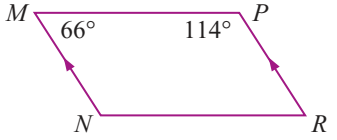
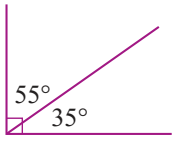


H Combinations of angles

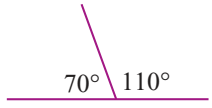
This section combines information you have learnt in previous sections about the different types of angles and their properties. This is a summary of the different types of angles.

Important facts **Example**

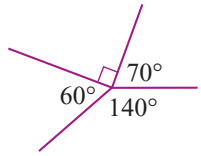
1 The sum of the angles making up a right angle is 90°.
Two angles that make up a right angle are called *complementary angles*.



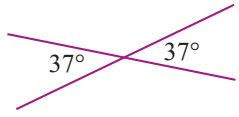
2 The sum of the angles on a straight line (making a straight angle) is 180°.
Two angles that make up a straight angle are called *supplementary angles*.



3 The sum of the *angles at a point* is 360°.

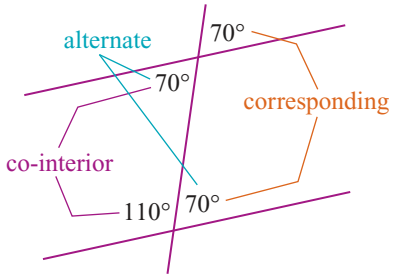


4 *Vertically opposite angles* are equal.



5 When parallel lines are crossed by a transversal:

- *corresponding angles* are equal
- *alternate angles* are equal
- *co-interior angles* are supplementary (their sum is 180°).

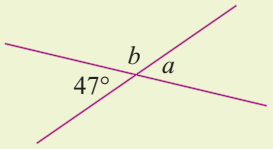


To find the value of a pronumeral you may need to use one or more of these facts.

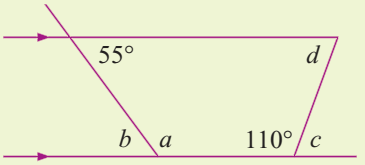
EXAMPLE 1

Find the value of the pronumerals in alphabetical order and give reasons for your answers.

a



b



The symbol ∥ means parallel.

a

$a = 47°$ (vertically opposite angles)

$b + 47° = 180°$ (straight angle)

$\therefore b = 133°$

b

$a = 125°$ (co-interior angles on ∥ lines)

$b = 55°$ (alternate angles on ∥ lines)

or $b + a = 180°$ (straight angle)

$\therefore b = 55°$

$c + 110° = 180°$ (straight angle)

$\therefore c = 70°$

$d = 70°$ (co-interior angles on ∥ lines)

or $d = c = 70°$ (alternate angles on ∥ lines)

Exercise 3H

1 Complete the following.

If you have a choice of reasons, use them to check your answers.

a **i** $a + 130° =$ _____° (straight angle)

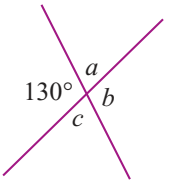
$\therefore a =$ _____°

ii $b = 130°$ (_____ opposite angles)

iii $c + 130° = 180°$ (_____ angle)

$\therefore c =$ _____°

or $c = a =$ _____° (_____ _____ angles)



b **i** $a + 50° =$ _____° (co-interior angles on ∥ lines)

$\therefore a =$ _____°

ii $b + 50° =$ _____° (straight angle)

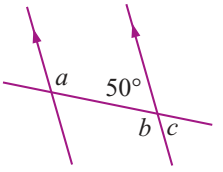
$\therefore b =$ _____°

or $b = a =$ _____° (_____ angles on ∥ lines)

iii $c =$ _____° (_____ opposite angles)

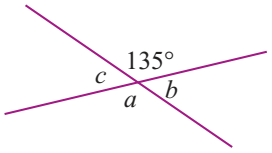
or $c + b = 180°$ (_____ angle)

$\therefore c =$ _____°

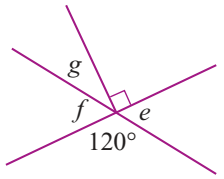


2 Find the values of the pronumerals in alphabetical order, giving reasons for your answers.

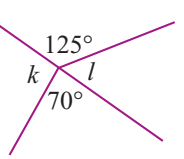
a



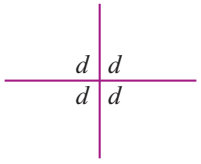
b



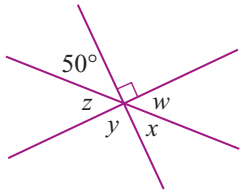
c



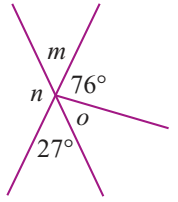
d



e

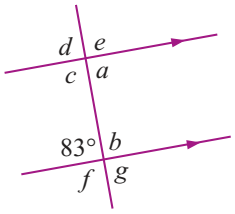


f

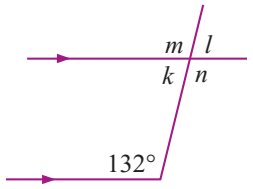


3 Find the values of the pronumerals in alphabetical order, giving reasons for your answers.

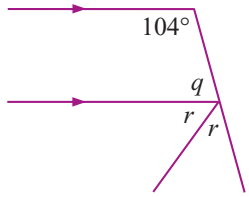
a



b



c



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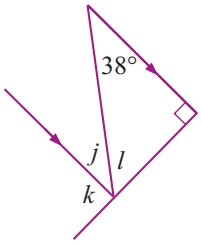

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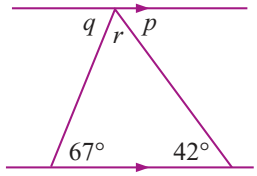

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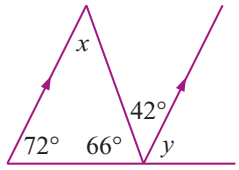
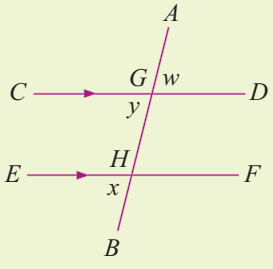


Non-numerical problems

Reasoning in geometry can also be applied to exercises that do not use numerical values. The same rules apply, and pronumerals are used to indicate angles of unknown size.

EXAMPLE 1

In the diagram, prove that $w = x$.



$y = w$ (vertically opposite angles)

and $y = x$ (corresponding angles, $CD \parallel EF$)

$\therefore w = x$ (both equal to y)

$CD \parallel EF$ reads 'CD is parallel to EF'.

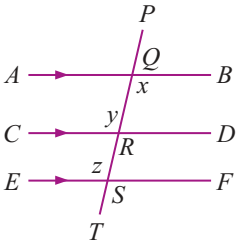
Exercise 3I

1 Complete the following to prove that $x = z$.

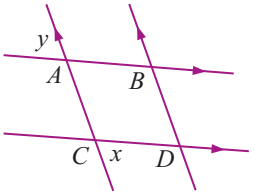
$x = y$ (_____ angles, $AB \parallel CD$)

$y =$ ____ (corresponding angles, ____ $\parallel$ ____)

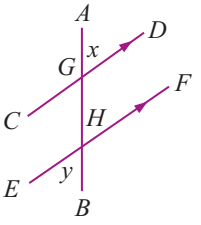
$\therefore x = z$ (both equal to ____)



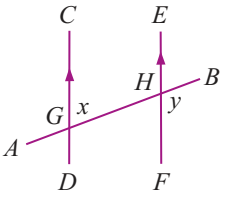
2 **a** Prove that $x = y$.



b Prove that $x = y$.

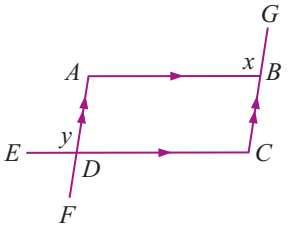


c Prove that $y = 180° - x$.

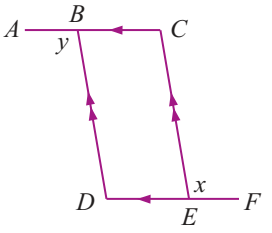


3 In each of the following prove that $x = y$.

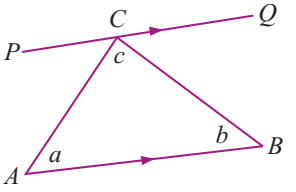
a



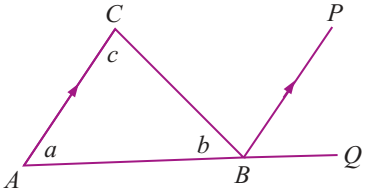
b



4 In the diagram opposite, prove that $a + b + c = 180°$.



5 In the diagram opposite, prove that $a + b + c = 180°$.



6 **a** Write out in sentence form what you have deduced in questions **4** and **5**.

b Check your deduction by drawing several triangles of different shapes and measuring the angles.

7 **a** Copy and complete the following.

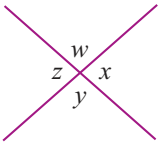
$w + z =$ ____ (_____)

$\therefore z = 180° - w$

$w + x =$ ____ (_____)

$\therefore x = 180° - w$

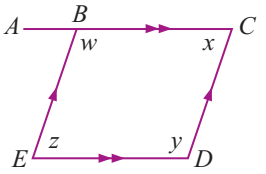
$\therefore x = z$



b Using the same diagram, prove that $w = y$.

8 In the given diagram, prove that:

a $w = y$ **b** $z = x$



Language in mathematics

To solve the following word puzzles, start at the arrow marked IN. Follow the letters vertically or horizontally, but not diagonally, to the arrow marked OUT to reveal some words you have been using in this chapter. All letters are used, and no letter is used more than once.

Write the letters discovered in the squares provided. This will help you to see the length of the next word to be found.

1

IN →	A	N	G	T	A	
	S	E	L	N	R	
	S	I	G	E	M	
	I	R	H	C	S	→ OUT
	Z	E	T	A	J	
	R	O	C	A	D	
	R	P	O	G	N	
	E	S	N	D	I	

A	N	G	L	E	S							
S	I	Z	E									
R	I	G	H	T								

2

	L	E	M	U	R	P	P
	P	N	E	S	E	O	O
	P	T	A	A	E	M	S
	U	Y	R	T	R	R	I
IN →	S	A	R	O	A	O	T
	D	D	P	E	C	T	E
	P	A	R	T	O	X	V
	L	L	A	A	B	E	E
	E	L	T	N	T	T	R
	L	A	E	R	U	S	E → OUT

S	U	P	P	L	E	M	E	N	T	A	R	Y
A												
P												
A												
P												

3

IN ↓ OUT ↑

E	R	I	L	Y	R
V	E	N	M	E	A
O	L	U	E	N	T
O	I	T	L	P	M
N	A	C	U	T	O
N	I	O	C	E	C
T	E	R	I	O	R

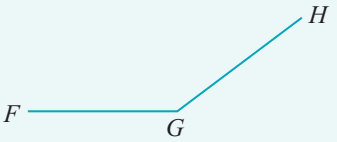
L												

Terms

acute angle	adjacent	alternate	angle	angles at a point	arm
co-interior	common	complement	complementary	corresponding	interval
line	line segment	obtuse angle	parallel	perpendicular	point
pronumeral	ray	reflex angle	revolution	right angle	straight angle
supplement	supplementary	transversal	vertex	vertically opposite	vertices

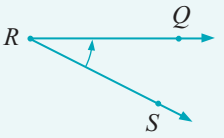
Check your skills

1 Name the vertex of the angle shown.



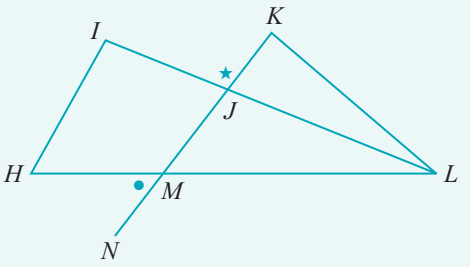
A F **B** GF
C G **D** GH

2 Name the arms of the angle shown.



A RS, SQ **B** SR, QS
C QR, SQ **D** RS, RQ

Questions **3** and **4** refer to the diagram opposite.



3 Name the angle marked with a ●.

A $\angle LMN$ **B** $\angle LMJ$
C $\angle HMN$ **D** $\angle HMJ$

4 Name the angle marked with a ★.

A $\angle IJK$ **B** $\angle IJM$
C $\angle LJK$ **D** $\angle LJM$

5 The type of angle shown is:

A acute **B** obtuse
C reflex **D** revolution

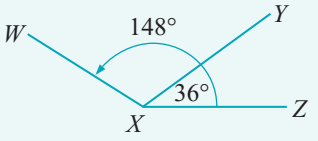
6 A revolution equals:

A 90° **B** 180° **C** 270° **D** 360°

7 An acute angle is:

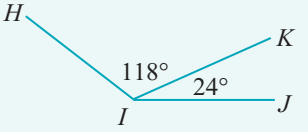
A $> 0°$ but $< 90°$ **B** equal to 90° **C** $> 90°$ but $< 180°$ **D** equal to 180°

8 Calculate the size of $\angle WXY$.



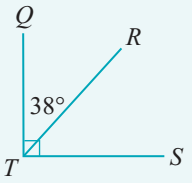
A 184° **B** 112°
C 32° **D** 176°

9 Calculate the size of $\angle HIJ$.



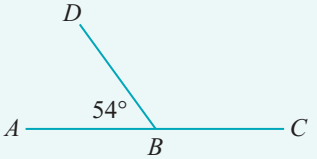
A 38° **B** 142°
C 94° **D** 218°

10 Calculate the size of $\angle RTS$.



A 42° **B** 90°
C 52° **D** 38°

11 Calculate the size of $\angle DBC$.



A 336° **B** 144°
C 208° **D** 126°

12 What is the complement of 72°?

A 72° **B** 18° **C** 108° **D** 288°

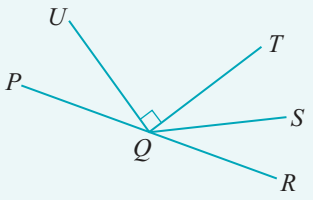
13 What is the supplement of 87°?

A 273° **B** 267° **C** 177° **D** 93°

14 In the diagram, $\angle PQT$ and $\angle RQT$ are:

A co-interior **B** complementary

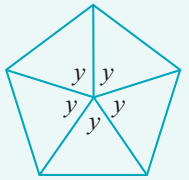
C supplementary **D** opposite



15 Calculate the value of y.

A 60° **B** 36°

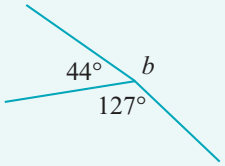
C 72° **D** 30°



16 Calculate the value of b.

A 189° **B** 109°

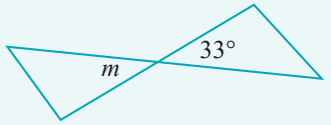
C 351° **D** 171°



17 Calculate the value of m.

A 327° **B** 33°

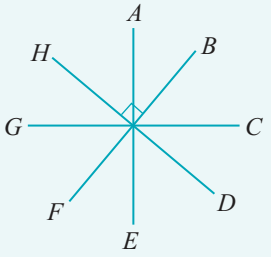
C 147° **D** 294°



18 Which one of these statements about the lines in the diagram is true?

A $AE \perp BF$ **B** $CG \perp AE$

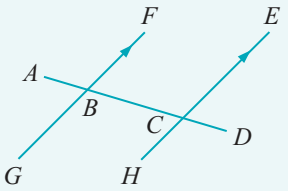
C $HD \perp BF$ **D** $CG \perp HD$



19 $\angle GBC$ and $\angle ECB$ are a pair of:

A alternate angles **B** corresponding angles

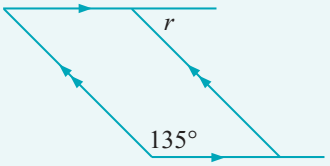
C co-interior angles **D** vertically opposite angles



20 The value of r in the diagram is:

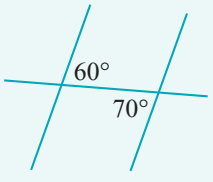
A 180° **B** 45°

C 135° **D** 120°

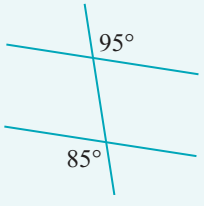


21 Which diagram shows a pair of parallel lines?

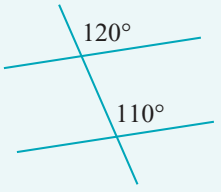
A



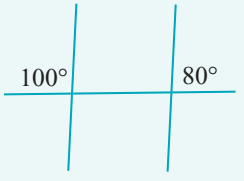
B



C



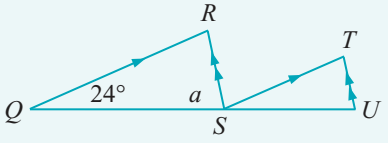
D



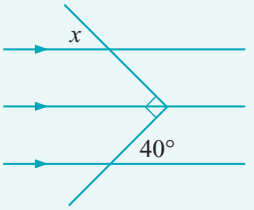
22 The line RS divides $\angle QST$ evenly. Use this to calculate the value of a.

A 24° **B** 12°

C 78° **D** 156°



23 Which three steps of reasoning could be used to find the value of x in the diagram?



A alternate angles, complementary angles, alternate angles

B alternate angles, complementary angles, corresponding angles

C alternate angles, vertically opposite angles, alternate angles

D co-interior angles, vertically opposite angles, corresponding angles

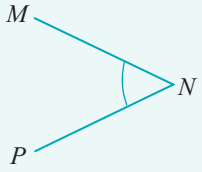
If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–4	5–7	8, 9	10–14	15–17	18	19–21	22, 23
Section	A	B	C	D	E	F	G	H

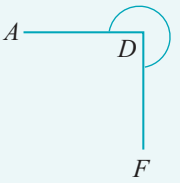
3A Review set

1 Name the vertex for each angle.

a



b



2 Draw an acute angle and label it $\angle FGK$.

3 Classify these angles.

a 170° **b** 180° **c** 80° **d** 260°

4 What is a reflex angle?

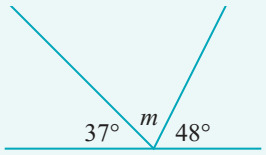
5 What are adjacent angles? (Give the three characteristics.)

6 Find the angle supplementary to:

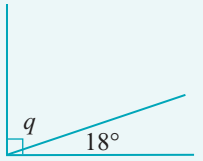
a 145° **b** 70° **c** 63° **d** 128°

7 Calculate the value of the pronumeral and give a reason.

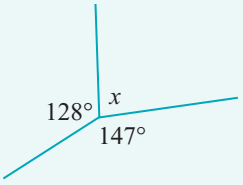
a



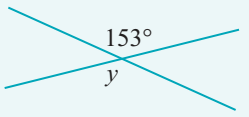
b



c

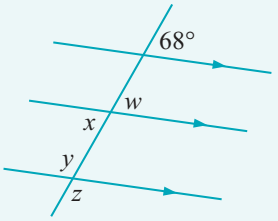


d

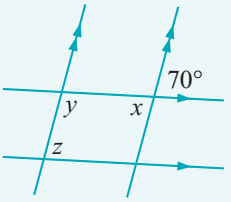


8 Calculate the values of the pronumerals.

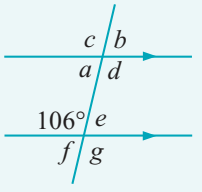
a



b

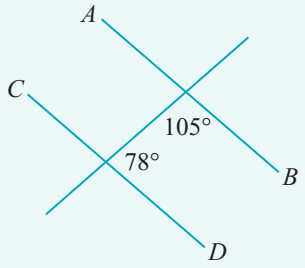


c

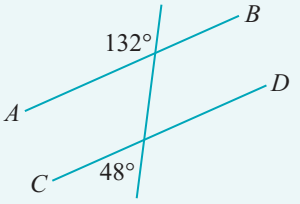


9 Are the lines AB and CD parallel? Give a reason.

a



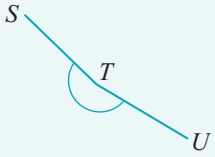
b



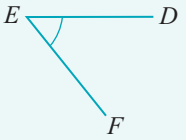
3B Review set

1 Name the angles shown using: **i** three letters **ii** the vertex

a



b

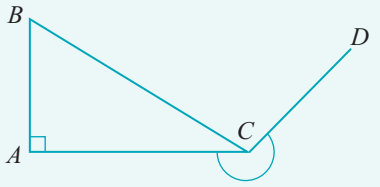


2 Draw a reflex angle and label it $\angle QRS$.

3 Use the diagram to classify each angle according to size.

a $\angle BAC$ **b** $\angle BCA$

c $\angle BCD$ **d** $\angle ACD$ (marked)



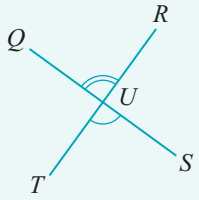
4 What is a revolution?

5 **a** What is the complement of 50°?

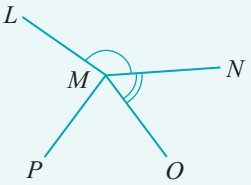
b What is the supplement of 50°?

6 Are the following marked pairs of angles adjacent?

a $\angle QUR$ and $\angle TUS$

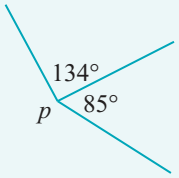


b $\angle LMN$ and $\angle NMO$

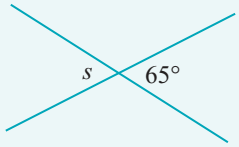


7 Calculate the value of the pronumeral and give a reason.

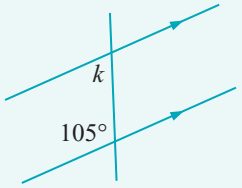
a



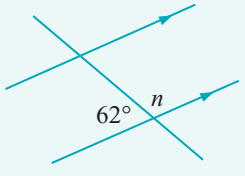
b



c

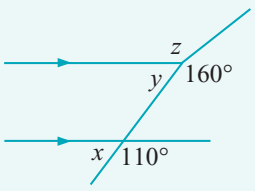


d



8 Calculate the values of the pronumerals.

a



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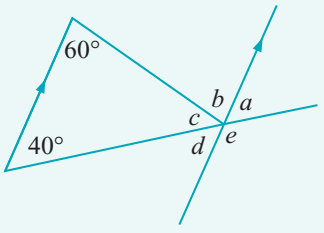
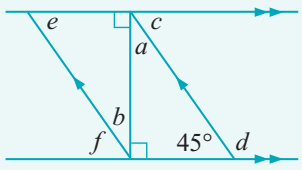


c



3C Review set

1 Name the vertex of each angle.

a $\angle R$ **b** $\angle Q$ **c** $\angle WST$ **d** $\angle JKL$

2 Correct the errors in each part below.

a $\angle BPQ$: vertex is Q and rays are BP, PQ **b** $\angle AFG$: vertex is F and rays are AF, AG

c $\angle HIJ$: vertex is J and rays are HJ, JI

3 Classify the following angles according to size.

a 90° **b** 125° **c** 337° **d** 78°

4 **a** Describe adjacent complementary angles.

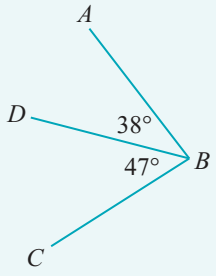
b Draw a diagram showing adjacent complementary angles.

5 Find the angle complementary to:

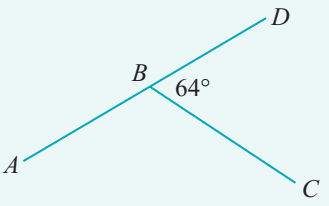
a 50° **b** 73° **c** 47° **d** 48°

6 Calculate the size of $\angle ABC$, giving a reason.

a

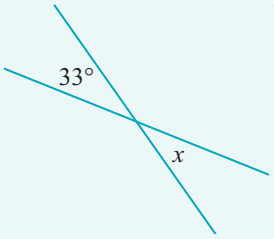


b

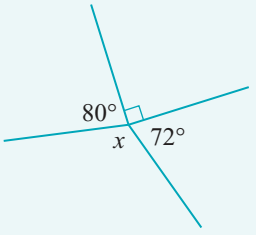


7 Find the value of the pronumeral, giving a reason.

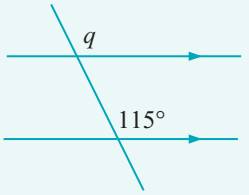
a



b



c



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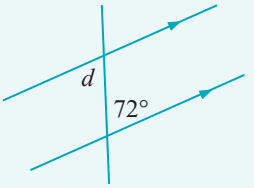
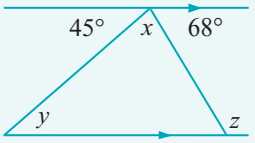
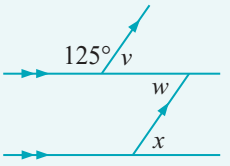


8 Calculate the values of the pronumerals.

a

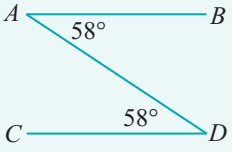


b



9 Are the lines AB and CD are parallel? Give a reason.

a



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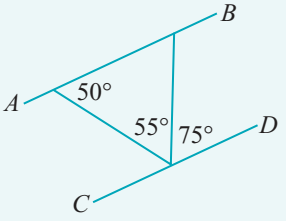


4

Number and indices

This chapter deals with indices.

At the end of this chapter you should be able to:

- understand some basic number facts
- determine tests for divisibility
- use a non-calculator method to divide by a 2-digit number
- list multiples of given numbers
- classify numbers as prime or composite
- determine and list factors of a number
- use index notation to find HCF and LCM
- work with and estimate square and cube roots using correct notation
- apply the associative, commutative and distributive laws of computation.

NUMBER & ALGEBRA – ACMNA 149, 150, 151

Diagnostic test

1 The number 81 cannot be written as which of the following?
A 1×81 **B** 9×3
C 3×27 **D** 9×9

2 $10 \times 10 \times 10 \times 10 \times 10$ is equal to:
A 100 **B** 1000
C 10 000 **D** 100 000

3 Which number is divisible by 9?
A 1314 **B** 2719
C 9103 **D** 8099

4 Which number is divisible by 6?
A 920 **B** 923 **C** 924 **D** 926

5 What is $364 \div 14$?
A 24 **B** 26 **C** 34 **D** 36

6 The number 32 is not divisible by:
A 4, 8 and 32 **B** 2 and 16
C 4 and 32 **D** 3 and 16

7 The fifth multiple of 7 is:
A 5 **B** 7 **C** 12 **D** 35

8 The factors of 10 are:
A only 1 and 10 **B** only 2 and 5
C 1, 2, 5 and 10 **D** 1, 2 and 5

9 The total number of factors 12 has is:
A 6 **B** 5 **C** 4 **D** 2

10 One reason the number 39 is not divisible by 6 is that:
A it is not divisible by 3
B it is not a multiple of 2
C it is not a multiple of 3
D it is a multiple of 13.

11 The largest factor of 24 less than 12 is:
A 12 **B** 10 **C** 6 **D** 8

12 Which of the following is not a square number?
A 1 **B** 16 **C** 64 **D** 20

13 The next larger triangular number after 3 is:
A 6 **B** 4 **C** 5 **D** 10

14 The number 36 can be written as a product of its prime factors as:
A 4×9 **B** $2 \times 2 \times 3 \times 3$
C 6×6 **D** $2 \times 3 \times 6$

15 $2 \times 2 \times 2 \times 2 \times 2 \times 2$ is equal to:
A 32 **B** 64
C 128 **D** 12

16 The number 1 can be classed as which of the following?
A a prime number
B a composite number
C an even number
D neither a prime nor a composite number

17 The number 50 has how many different prime factors?
A 2 **B** 1 **C** 4 **D** 3

18 The value of $20 - 12 \div 2$ is:
A 4 **B** 14 **C** 16 **D** 26

19 Calculate the value of $80 \div 10 \times 2$.
A 4 **B** 16
C 400 **D** 1600

20 $48 \div (6 - 4) \times 4 =$
A 96 **B** 16 **C** 6 **D** 8

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–17	18–20
Section	ACMNA 122	ACMNA 134

NUMBERS

People have been fascinated by numbers for thousands of years. Numbers have been associated with sayings, mystical numerology, luck and more.

This chapter examines some of the interesting facts about our number system. Some of them will surprise you!

Investigation 1 Groups of numbers

This investigation examines some properties of numbers.

7 is considered a lucky number. I wonder why 13 is considered unlucky?

1 Write the next four numbers in each pattern.

a 2, 4, 6, 8, 10, … **b** 1, 3, 5, 7, … **c** 2, 6, 10, … **d** 1, 5, 9, 13, …

2 **a** Write the first 20 even numbers. **b** Write the first 20 odd numbers.

3 Complete the following.

a Even numbers end in _____. **b** Odd numbers end in _____.

c Even numbers can all be divided by ___ without remainder.

4 True or false?

a There is an odd number between any two even numbers.

b An odd number is formed by adding 1 to an even number.

c Some even numbers end in 3.

d All even numbers end in 2.

e Between any two even numbers there is an even number.

5 Here is a sequence of dot patterns.

a Write the numbers represented by the numbers of dots.

b Write the next five numbers in the sequence.

c These numbers are called **triangular numbers**.
Why do you think they have that name?

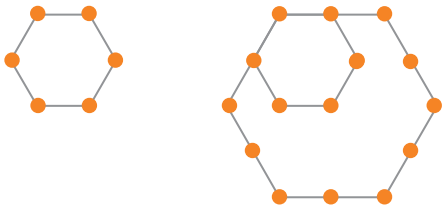
6 Here is a different sequence of dot patterns.

a Write the numbers represented by the numbers of dots.

b Write the next five numbers in the sequence.

c These numbers are called **square numbers**.
Why do you think they have that name?

7 Here is another sequence of dot patterns.



a Write the numbers represented by the numbers of dots.

b Write the next two numbers in the sequence.

c These numbers are called **hexagonal numbers**.
Why do you think they have that name?

8 There are numbers called **pentagonal numbers**. Draw the first five and find their values.

A Index notation and powers

A **square number** (or **square**) is formed when any number is multiplied by itself. Since $3 \times 3 = 9$, we can say that 3 squared is equal to 9. There is a special way of writing 3 squared. This is called **index notation**.

3^{2} ← Index or power
← Base

This is sometimes read as '3 to the power of 2'.

A **cubic number** (or **cube**) is formed when three of the same numbers are multiplied together. Since $5 \times 5 \times 5 = 125$, we can say that 5 cubed is equal to 125, or $5^3 = 125$.

The **power** of a number is how many of that number are multiplied together.

Exercise 4A

1 **a** Find 5×5.
b What is five squared?
c Write 5^2 in words.

2 **a** Find 8×8.
b What is eight squared?
c Write 8^2 in words.

3 **a** Find 6×6.
b Complete: $6^2 =$ ___
c Write six squared in index notation.

4 Belinda suggested that, if $6 = 2 \times 3$, then $6^2 = (2 \times 3)^2 = 2^2 \times 3^2$.
a Find $(2 \times 3)^2$.
b Find 2^2 and 3^2, then $2^2 \times 3^2$.
c Is $(2 \times 3)^2 = 2^2 \times 3^2$?

5 Check Belinda's suggestion using these squares.
a $12^2 = (3 \times 4)^2$ **b** $10^2 = (2 \times 5)^2$ **c** $21^2 = (3 \times 7)^2$

6 Write the first ten square numbers.

EXAMPLE 1

Write $2 \times 2 \times 2$ in index notation, and find its value.

This is 2 cubed or '2 to the power of 3'.

$2 \times 2 \times 2 = 2^3 = 8$

7 Write in index notation and find the value of:
a $4 \times 4 \times 4$ **b** $3 \times 3 \times 3$ **c** $6 \times 6 \times 6$ **d** $7 \times 7 \times 7$

8 Belinda further suggested that, since $15 = 3 \times 5$, then $15^3 = (3 \times 5)^3 = 3^3 \times 5^3$. Was she correct?

EXAMPLE 2

Write 3 to the power of 4 in index notation, and find its value.

$3 \times 3 \times 3 \times 3 = 3^4 = 81$

9 Write in index notation and find the value of:

a 2 to the power of 5

b 11 to the power of 2

c 5 to the power of 4

d 7 to the power of 1

10 The first four powers of 10 are 10, 100, 1000, 10 000.

a Write these numbers in index notation.

b What power of 10 is one million? Write one million in index notation.

Investigation 2 Divisibility of numbers

Sue-Lin wants to know how to find numbers that divide into other numbers. This investigation examines patterns in multiples of numbers. The purpose is to develop divisibility rules.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100	101	102			

1 **a** Copy this table and colour all the multiples of 5.

b What do you notice about the last digit of all the multiples of 5?

c How can you tell whether a number is divisible by 5?

2 **a** Colour all the multiples of 3 in another colour.

b Add together all the digits of each multiple of 3 greater than 10.

c Write these sums as a number pattern.

d What do you notice about the sums?

e How can you tell whether a number is divisible by 3?

3 **a** Use another colour for all the multiples of 9.

b Add together all the digits of each multiple of 9 greater than 10.

c What do you notice about the sums?

d How can you tell whether a number is divisible by 9?

4 **a** Use a fourth colour for all the multiples of 4.

b Notice that 100 divided by 4 is 25. This means that multiples of 100 have no effect on whether a number is divisible by 4.

c What do you notice about the last two digits of the multiples of 4?

d How can you tell whether a number is divisible by 4?

B Divisibility tests

Before the widespread use of calculators, mathematicians devised a set of rules for divisibility of numbers so that factors of numbers could be found without actually dividing. Numbers needed to be easily written as a product of prime factors so that a large number could be classified as prime or composite. The tests for divisibility by primes were the most important. Here is a list of some of the rules.

Number	Rule	Example
2	The last digit of the number is even: 2, 4, 6, 8 or 0.	5896 ends in an even number, so it is divisible by 2.
3	The sum of the digits of the number is divisible by 3.	Sum of digits of 4584 is $4 + 5 + 8 + 4 = 21$, which is divisible by 3; so 4584 is divisible by 3.
4	The last two digits of the number are divisible by 4.	1932 has last two digits 32, which is divisible by 4; so 1932 is divisible by 4.
5	The last digit is a 5 or 0.	4765 ends in 5, so it is divisible by 5.
6	The number is divisible by 2 and 3.	1164 is even, so it is divisible by 2; and $1 + 1 + 6 + 4 = 12$, which is divisible by 3; so 1164 is divisible by 6.
7	The sum of the number groups is divisible by 7.	Separate the digits into groups of three. From the right, alternately place a $+$ then a $-$ sign in front of each group. Find the sum.
8	The last three digits form a number divisible by 8.	35 096 has last three digits 096, which is divisible by 8; so 35 096 is divisible by 8.
9	The sum of the digits is divisible by 9.	Sum of digits of 45 792 is $4 + 5 + 7 + 9 + 2 = 27$, which is divisible by 9; so 45 792 is divisible by 9.
10	The number ends in 0.	4670 ends in 0, so it is divisible by 10.
11	The difference between sum of the digits in the 1st, 3rd, … places and the sum of the 2nd, 4th … places is a multiple of 11 or 0.	914 166: odd placed digits $= 9 + 4 + 6 = 19$; even placed digits $= 1 + 1 + 6 = 8$; difference $= 19 - 8 = 11$; so 914 166 is divisible by 11.
12	The number is divisible by 3 and 4.	170 472 has last two digits 72, which is divisible by 4; and $1 + 7 + 0 + 4 + 7 + 2 = 21$, which is divisible by 3; so 170 472 is divisible by 12.

Exercise 4B

1 Which of the following numbers are divisible by 2?

a 43 568 **b** 23 533 **c** 16 900 **d** 303 030

2 Which of the following numbers are divisible by 3?

a 25 961 **b** 29 505 **c** 26 320 **d** 169 026

3 Which of the following numbers are divisible by 4?
a 28 424 **b** 63 954 **c** 32 866 **d** 147 008

4 Which of the following numbers are divisible by 5?
a 34 675 **b** 64 980 **c** 55 842 **d** 167 312

5 Which of the following numbers are divisible by 6?
a 9690 **b** 14 858 **c** 195 738 **d** 39 735

6 Which of the following numbers are divisible by 8?
a 20 026 **b** 45 972 **c** 108 376 **d** 674 312

7 Which of the following numbers are divisible by 9?
a 34 243 **b** 26 130 **c** 84 277 **d** 431 262

8 Which of the following numbers are divisible by 10?
a 42 801 **b** 63 580 **c** 29 375 **d** 148 600

9 Which of the following numbers are divisible by 11?
a 2717 **b** 4199 **c** 110 055 **d** 323 323

10 Which of the following numbers are divisible by 12?
a 2340 **b** 10 013 **c** 39 780 **d** 110 143

11 Which of the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 12 are factors of 4824?

12 **a** The number 52__7 is divisible by 3. What digits can replace ___?
b The number 2__718 is divisible by 9. What digits can replace ___?

13 Use your divisibility rules to find the first number greater than 2000 that is divisible by:
a 3 **b** 6 **c** 9 **d** 11 **e** 12

14 Use the divisibility tests to find all the factors up to 12 of:
a 720 **b** 396 **c** 550

15 The rule for divisibility by 12 is that the number is divisible by 3 and 4. Find a rule for divisibility by 15 and test it on the following.
a 7290 **b** 85 361 **c** 17 694

EXAMPLE 1

Is 37 091 341 divisible by 7?

Group: 37 091 341
Signs: +37 −091 +341
Sum: 37 − 91 + 341 = 287
287 ÷ 7 = 41; 287 is divisible by 7, so 37 091 341 is divisible by 7.

−091 is the same as −91, as there are no hundreds.

Using a calculator makes doing the sum easier.

16 Which of the following numbers are divisible by 7?
a 10 947 118 **b** 181 908 **c** 48 548 297 **d** 523 763 217 125 141

17 Explain how each of the divisibility tests used in this section works.

Investigation 3 Number facts

1 Explain how 4×98 can be thought of as $4 \times 100 - 4 \times 2$. Use this to calculate the following.

a 6×98 **b** 8×97 **c** 7×99

2 Find an easy way to calculate the following.

a 8×102 **b** 6×103 **c** 9×604

3 When multiplying numbers the order does not matter. Change the order and calculate the following.

a $7 \times 5 \times 2$ **b** $20 \times 17 \times 5$ **c** $5 \times 19 \times 20$

d $4 \times 39 \times 25$ **e** $81 \times 50 \times 2$ **f** $76 \times 53 \times 0 \times 18$

4 What happens when a number is multiplied by:

a 0? **b** 1?

5 Write true or false for the following.

a $5 + 8 = 8 + 5$ **b** $7 + 2 = 2 + 7$ **c** $6 + 12 = 12 + 6$

d $98 + 2 = 2 + 98$ **e** $15 + 7 = 7 + 15$ **f** $43 + 7 = 7 + 43$

g $5 \times 2 = 2 \times 5$ **h** $8 \times 3 = 3 \times 8$ **i** $6 \times 5 = 5 \times 6$

j $20 - 5 = 5 - 20$ **k** $16 - 3 = 3 - 16$ **l** $4 - 1 = 1 - 4$

m $20 \div 4 = 4 \div 20$ **n** $12 \div 3 = 3 \div 12$ **o** $18 \div 6 = 6 \div 18$

p Order does not matter when adding. **q** Order does not matter when multiplying.

r Order does not matter when subtracting. **s** Order does not matter when dividing.

6 A method for multiplying by 12 is to first multiply by 6 then double the answer. Use this method to find:

a 123×12 **b** 485×12 **c** 691×12

7 A method for multiplying by 13 is to first multiply by 10 then add three times the original number. Use this method to find:

a 203×13 **b** 481×13 **c** 722×13

8 To divide by 20, first halve the number then divide by 10. Use this method to find:

a $460 \div 20$ **b** $584 \div 20$ **c** $673 \div 20$

9 Explain why multiplying by 9 can be done by first multiplying a number by 10 then subtracting the number. Use this method to find:

a 238×9 **b** 411×9 **c** 968×9

Investigation 4 Dividing

The Year 7 class was asked to find $779 \div 54$.

1 Claire used this method.

There are two 50s per 100, so 700 has fourteen 50s. I will try 15.

$$\begin{array}{r} 54 \\ \times\ 15 \\ \hline 270 \\ 540 \\ \hline 810 \end{array}$$

But this is too big.

$810 - 54 = 756$, which is smaller than 779. This means 54 goes 14 times. Why does it go 14 times?

The remainder is $779 - 756 = 23$. Why is this the remainder? The answer is $14\frac{23}{54}$.

Use Claire's method to find $645 \div 56$.

2 Keely used a different method.

$779 \div 54$ is the same as $54 \times \_\_\_ = 779$.

Keely finds $54 \times 10 = 540$, so

$$\begin{array}{r} 7\,7\,9 \\ -\ 5\,4\,0 \\ \hline 2\,3\,9 \end{array}$$

She then finds $54 \times 3 = 162$, so

$$\begin{array}{r} 2\,3\,9 \\ -\ 1\,6\,2 \\ \hline 7\,7 \end{array}$$

Now $77 - 54 = 23$.

So, $54 \times (10 + 3 + 1) + 23 = 779$; that is, $54 \times (14) + 23 = 779$ or $779 \div 54 = 14$ remainder 23.

Use Keely's method to find $836 \div 56$.

3 Joseph uses long division:

$$\begin{array}{r} 1\,4 \\ 54\overline{)\,7\,7\,9} \\ 5\,4 \\ \hline 2\,3\,9 \\ 2\,1\,6 \\ \hline 2\,3 \end{array}$$

so, $54 \times 14 + 23 = 779$; that is, $779 \div 54 = 14\frac{23}{54}$.

Use Joseph's method to find $811 \div 56$.

4 Choose any method and complete these divisions.

a $427 \div 31$ **b** $581 \div 48$ **c** $239 \div 28$ **d** $871 \div 93$

C Long division

This section examines division by two-digit numbers. Two approaches can be used. The first method is the traditional long division, while the second is the preferred multiples method.

EXAMPLE 1

Complete this division which does not have a remainder: $532 \div 14$.

Method 1

$$\begin{array}{r} 3\,8 \\ 14\overline{)\,5\,3\,2} \\ 4\,2 \\ \hline 1\,1\,2 \\ 1\,1\,2 \\ \hline 0 \end{array}$$

$3 \times 14 = 42$

$8 \times 14 = 112$

$532 \div 14 = 38$ or $14 \times 38 = 532$

Method 2

$$\begin{array}{r|l} 14\overline{)\,5\,3\,2} & \\ 1\,4\,0 & 10 \\ \hline 3\,9\,2 & \\ 1\,4\,0 & 10 \\ \hline 2\,5\,2 & \\ 1\,4\,0 & 10 \\ \hline 1\,1\,2 & \\ 1\,1\,2 & 8 \\ \hline 0 & 38 \end{array}$$

Exercise 4C

1 Complete these divisions which do not have remainders.

a $325 \div 13$ **b** $966 \div 23$ **c** $806 \div 31$ **d** $646 \div 17$

e $728 \div 26$ **f** $608 \div 32$ **g** $615 \div 41$ **h** $722 \div 19$

EXAMPLE 2

Complete this division which has a remainder: 601 ÷ 34.

Method 1

$$\begin{array}{r} 17\tfrac{23}{34} \\ 34\overline{)\,601} \\ 34 \\ \hline 261 \\ 238 \\ \hline 23 \end{array}$$

$8 \times 34 = 272$
too big, try
$7 \times 34 = 238$

$601 \div 34 = 17\frac{23}{34}$ or $17 \times 34 + 23 = 601$

Method 2

$$\begin{array}{r|l} 34\overline{)\,601} & \\ 340 & 10 \\ \hline 261 & \\ 170 & 5 \\ \hline 91 & \\ 68 & 2 \\ \hline 23 & 17 \end{array}$$

2 Complete these divisions, writing the remainders in fraction form.

a 711 ÷ 15 **b** 695 ÷ 21
c 800 ÷ 33 **d** 526 ÷ 14
e 949 ÷ 28 **f** 989 ÷ 37
g 680 ÷ 43 **h** 637 ÷ 18

3 Check the answer to question **2** using a calculator.

D Multiples, factors, primes and composites

We will revise multiples and factors and look at numbers that are not multiples.

EXAMPLE 1

Find the first six **multiples** of 3.

The multiples of a number are its products. They are the answers in the multiplication table.

The first six multiples of 3 are 3, 6, 9, 12, 15, 18.

Exercise 4D

1 Complete the first ten multiples of 5: 5, 10, 15, ___, ___, 30, ___, ___, ___, ___

2 Find the first ten multiples of:

a 4 **b** 7 **c** 8 **d** 9 **e** 11

3 Write the multiples of 6 between 23 and 55.

4 Write the multiples of 7 between 20 and 60.

5 Write the multiples of 9 that are less than 55.

EXAMPLE 2

a Write the first twelve multiples of 6.
b Write the first twelve multiples of 5.
c List the common multiples you have found.
d What is the **lowest common multiple (LCM)** of 6 and 5?

A common multiple of two numbers is a product they both have.

a 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72
b 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60
c Common multiples are 30, 60.
d The LCM is 30.

Multiplying two numbers together gives a common multiple but not necessarily the *lowest* common multiple. For example, the LCM of 4 and 6 is 12, not 24. There is more on LCM in section F.

6 **a** Complete the multiples of 3 that are less than 50.
3, 6, 9, ___, ___, ___, 21, 24, ___, ___, ___, ___, ___, 42, ___, ___ .
b Complete the multiples of 5 that are less than 50.
5, 10, 15, ___, ___, ___, 35, 40, ___
c List the common multiples of 3 and 5 that are less than 50: ___, 30, ___ .
d The LCM of 3 and 5 is ___ .

What does LCM stand for?

7 **a** Write the first ten multiples of 3.
b Write the first ten multiples of 4.
c What is the LCM of 3 and 4?

8 Write the first ten multiples of each number, then find the LCM of:
a 7 and 5 **b** 8 and 6 **c** 9 and 6

EXAMPLE 3

Write the **factors** of 24.

The factors of a product are the numbers that can multiply together to give the product.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

9 Complete the factors of 48: 1, 2, ___, ___, ___, 8, ___, 16, ___, 48

Divide to find the factors.

10 Find the factors of these numbers.
a 10 **b** 18 **c** 13 **d** 30 **e** 20

EXAMPLE 4

a Write the factors of 27.
b Write the factors of 36.
c List the common factors of 27 and 36.
d What is the **highest common factor** (**HCF**) of 27 and 36?

A common factor of two products is a factor they both have.

a Factors of 27 are 1, 3, 9, 27.
b Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.
c Common factors are 1, 3, 9.
d The highest common factor is 9.

The HCF is also referred to as the **greatest common divisor** (**GCD**). There is more on HCF in section F.

11 **a** Complete the factors of 12: ____, 2, ____, 4, ____, ____
b Complete the factors of 36: 1, ____, ____, ____, 9, ____, 18, ____
c List the common factors of 12 and 36: ____, ____, 3, ____, ____, 12
d The HCF of 12 and 36 is ____ .

What does HCF stand for?

12 **a** Write the factors of 30.
b Write the factors of 45.
c List the common factors of 30 and 45.
d What is the HCF of 30 and 45?

13 **a** Write the factors of 12.
b Write the factors of 18.
c What is the HCF of 12 and 18?

14 **a** Write the factors of 20.
b Write the factors of 30.
c What is the HFC of 20 and 30?

15 **a** List a pair of numbers that have a common multiple of 18.
b List another pair of numbers that have a common multiple of 18.
c Explain a method for determining all the pairs of numbers with 18 as a common multiple. List them.
d If 18 is the lowest common multiple (LCM), what are all the possible pairs of numbers?
e Explain why the lists for parts **c** and **d** are different.

16 Repeat question **15** using 30 as the common multiple.

17 **a** List a pair of numbers with a common factor of 8.
b List five pairs of numbers with a common factor of 8.
c Is it possible to list all the pairs of numbers with a common factor of 8? Explain.
d Is it possible to list all the pairs of numbers with 8 as the HCF? Explain.

18 Two numbers have a HCF of 6 and a LCM of 90. Find the numbers.

19 Find two numbers with:
a HCF of 4 and LCM of 60
b HCF of 8 and LCM of 48
c HCF of 3 and LCM of 180
d HCF of 90 and LCM of 5400

A **prime number** has exactly two factors, itself and 1. A number with more than two factors is **composite**.

20 By finding the factors of the following numbers, decide which of them are prime numbers.
a 3 **b** 15 **c** 21 **d** 14 **e** 29
f 11 **g** 35 **h** 23 **i** 39 **j** 19

21 **a** Write the factors of 1.
b The number 1 is neither prime nor composite. Explain.
c Write the factors of 2.
d Is 2 prime or composite?
e Look at the prime numbers from question **20**. What is special about 2?
f Copy and complete the following statement.
Except for 2, all prime numbers are _____ .
g Why is 2 a unique prime number? How can you be sure?

22 From the first 30 counting numbers, write all the composite numbers that have a pair of factors other than the number and 1.

Investigation 5 Codes

Codes use large prime numbers.

1 Investigate the largest prime number. How many digits does it have?

2 Investigate why codes need prime numbers.

Investigation 6 Sieve of Eratosthenes

Eratosthenes (pronounced Erra-toss-tha-nees) lived between 275 BC and 194 BC. He was born at Cyrene in North Africa and was a mathematician, historian, astronomer, poet and geographer. He became head of the largest library of the time, at Alexandria in Egypt. He was probably the first to make a calculation of the circumference of the Earth. He used the lengths of shadows.

Eratosthenes devised a method to find prime numbers. He did this by sieving out the composite numbers. This was done by crossing out 1, then all the even numbers except 2, all the multiples of 3 except 3, all the multiples of 5 except 5, all the multiples of 7 except 7, and so on.

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49	50	51	52
53	54	55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76	77	78
79	80	81	82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	100	101	102	103	104

1 Copy this table and sieve out the composite numbers, leaving the primes.

2 Write a list of primes less than 100.

Why don't you cross out the 4s and 6s?

E Factor trees

From Investigation 6, the primes less than 100 are:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

When finding the HCF and LCM of large numbers, a factorisation into **prime factors** can make it easier.

Factorisation means writing a number as a product of its factors.

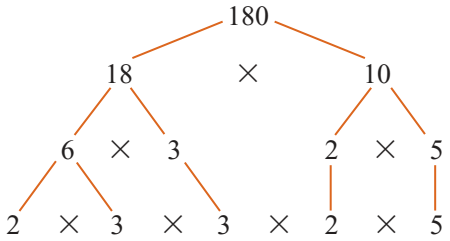
The **factor tree** method finds two factors at a time until all the factors are prime numbers.

EXAMPLE 1

Find the prime factors of:
a 180 **b** 168

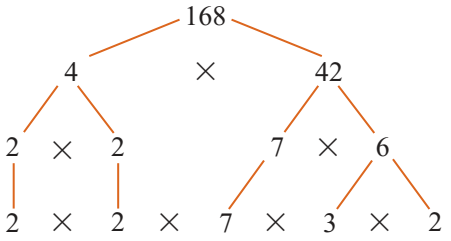
Prime factors are factors that are prime numbers.

a



$180 = 2 \times 3 \times 3 \times 2 \times 5$
$= 2 \times 2 \times 3 \times 3 \times 5$
$= 2^2 \times 3^2 \times 5$

b



$168 = 2 \times 2 \times 7 \times 3 \times 2$
$= 2 \times 2 \times 2 \times 3 \times 7$
$= 2^3 \times 3 \times 7$

The **division by primes** method reduces numbers to the product of prime factors. Just divide by 2 until you cannot any more, then divide by 3, then 5 and so on.

EXAMPLE 2

Find the prime factors of:
a 180 **b** 168

a

$2\overline{)180}$
$2\overline{)\ 90}$
$3\overline{)\ 45}$
$3\overline{)\ 15}$
$5\overline{)\ 5}$
$\quad 1$

$180 = 2 \times 2 \times 3 \times 3 \times 5$
$= 2^2 \times 3^2 \times 5$

b

$2\overline{)168}$
$2\overline{)\ 84}$
$2\overline{)\ 42}$
$3\overline{)\ 21}$
$7\overline{)\ 7}$
$\quad 1$

$168 = 2 \times 2 \times 2 \times 3 \times 7$
$= 2^3 \times 3 \times 7$

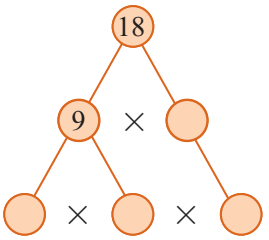
Remember: 1 is not a prime number.

Index notation makes writing the numbers shorter.

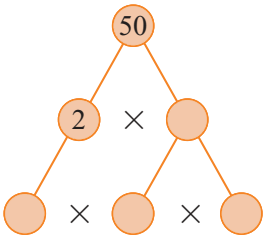
Exercise 4E

1 Complete the following factor trees.

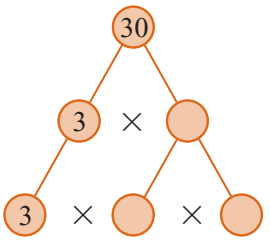
a



b



c



2 Use a factor tree to write each number as a product of prime factors.

a 60	**b** 90	**c** 140	**d** 84	**e** 126
f 252	**g** 315	**h** 784	**i** 1764	**j** 2200

3 Use the method of division by primes to write each number as a product of prime factors.

a 63	**b** 136	**c** 195	**d** 405	**e** 528
f 156	**g** 504	**h** 216	**i** 2640	**j** 2600

4 If you are ready for a challenge, write 2 167 074 000 as a product of prime factors.

F HCF and LCM by prime factors

If the numbers are large, it may be difficult and time consuming to find all the factors in order to find the HCF. It may take many multiples and difficult multiplications to find the LCM.

HCF and LCM are used in the simplification of fractions (chapter 5) and in algebra (chapter 7).

Here is a method that is easier and quicker for finding the HCF and LCM.

Step 1: Write each number as a product of prime factors.
Step 2: See which factors are the same and which are different.
Step 3: The common factors give the HCF.
Step 4: All the factors, used *once* only, give the LCM.

The HCF is smaller than or equal to the largest factor. The LCM is greater than or equal to the largest number.

Common factors are used only once.

EXAMPLE 1

a Write 300 and 630 as products of prime factors.
b Find the HCF of 300 and 630.
c Find the LCM of 300 and 630.

a $300 = 2 \times 2 \times 3 \times 5 \times 5$ $\quad$ $630 = 2 \times 3 \times 3 \times 5 \times 7$

b The HCF has all the factors common to both numbers. Here they are circled.

$300 = \textcircled{2} \times 2 \times \textcircled{3} \times \textcircled{5} \times 5$ $\quad$ $630 = \textcircled{2} \times \textcircled{3} \times 3 \times \textcircled{5} \times 7$

So the HCF is $2 \times 3 \times 5 = 30$.

c The LCM uses all the factors *once*, so each of the common factors is included only once.

Again circle the common factors.

$300 = \textcircled{2} \times 2 \times \textcircled{3} \times \textcircled{5} \times 5$ $\quad$ $630 = \textcircled{2} \times \textcircled{3} \times 3 \times \textcircled{5} \times 7$

Write *all* the factors of the bigger number, then include any *uncircled* factors of the smaller number.

$630 = 2 \times 3 \times 3 \times 5 \times 7$ $\quad$ $\text{LCM} = 2 \times 3 \times 3 \times 5 \times 7 \times 2 \times 5$

$= 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 6300$

Here are all the factors of 300 and 630. It took ages to find them all!

The factors of 300 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150, 300.
The factors of 630 are: 1, 2, 3, 5, 6, 7, 9, 10, 14, 15, 18, 21, 30, 35, 42, 45, 63, 70, 90, 105, 126, 210, 315, 630.
It is much more difficult to find all the factors of these numbers than it is to use prime factors.

For the LCM you would need the first 21 multiples of 300 and the first 10 multiples of 630.

Exercise 4F

1 **a** Write 30 and 42 as products of prime factors.
b Find the HCF of 30 and 42.
c Find the LCM of 30 and 42.

2 **a** Write 140 and 90 as products of prime factors.
b Find the HCF of 140 and 90.
c Find the LCM of 140 and 90.

3 Below are some numbers already written as products of prime factors. Copy and complete the table.

Prime factors	Index form	Number
	$2^2 \times 3 \times 5 \times 7$	
$2 \times 2 \times 3 \times 3 \times 5 \times 11$		1980
	$2^4 \times 5^2 \times 7$	2800
$3 \times 5 \times 7 \times 11 \times 11$		12 705
$3 \times 5 \times 5 \times 5 \times 7 \times 7$		18 375
	$2 \times 3^3 \times 5 \times 11^2$	32 670

4 Use your table from question **3** to find the HCF of each pair of numbers.
a 420 and 1980 **b** 1980 and 12 705 **c** 2800 and 12 705
d 32 670 and 420 **e** 18 375 and 1980 **f** 12 705 and 32 670
g 420 and 2800 **h** 12 705 and 18 375 **i** 1980 and 32 670

5 Use your table from question **3** to find the LCM of each pair of numbers. Leave the LCM in factor form.
a 420 and 1980 **b** 1980 and 12 705 **c** 2800 and 12 705
d 32 670 and 420 **e** 18 375 and 1980 **f** 12 705 and 32 670
g 420 and 2800 **h** 12 705 and 18 375 **i** 1980 and 32 670

6 Find the HCF and LCM of each pair of numbers. Write the numbers as products of prime factors first.
a 84 and 136 **b** 315 and 195 **c** 405 and 140
d 528 and 252 **e** 126 and 504 **f** 1736 and 2640

7 **a** Find a pair of numbers that has the HCF equal to one of the numbers.
b Find two other pairs of numbers that have the HCF equal to one of the numbers.
c Write a rule so that other pairs can be found that have the HCF equal to one of the numbers.

8 **a** Find a pair of numbers that has the LCM equal to one of the numbers.
b Find two other pairs of numbers that have the LCM equal to one of the numbers.
c Write a rule so that other pairs can be found that have the LCM equal to one of the numbers.

Investigation 7 Types of numbers

There are many classifications of numbers. Some are based on factors, while others are based on other number properties. This investigation examines some classifications of numbers based on the sum of their factors.

In each of these number investigations, the number itself as a factor is not included.

1 An **abundant number** is a number for which the sum of its factors, excluding itself, is *more* than the number. An example of an abundant number is 18. The factors of 18 (excluding 18) are 1, 2, 3, 6, 9, which sum to 21. Find the factors of 24 and show that it too is an abundant number.

2 A **deficient number** is a number for which the sum of its factors, excluding itself, is *less* than the number. An example of a deficient number is 16. The factors of 16 (excluding 16) are 1, 2, 4, 8, which sum to 15. Find the factors of 8 and show that it too is a deficient number.

3 A **perfect number** is a number that is *equal* to the sum of its factors, excluding itself.
An example of a perfect number is 28. The factors of 28 (excluding itself) are 1, 2, 4, 7, 14, which sum to 28. Find the factors of 14 and show that it is *not* a perfect number.

4 Find the factors of the numbers 4, 6, 8, 10, 12, 14, 16, 18 and 20 and classify them as abundant, deficient or perfect.

5 Show that 496 is a perfect number.

6 All perfect numbers known so far end in 28 or 6, preceded by an odd numeral. The fifth perfect number is 33 550 336.
- **a** Show that this number is perfect.
- **b** Find out how many perfect numbers are known.

G Square and cube roots

In section A you examined squares and cubes.

The opposite of squaring a number is finding the **square root** of the number (symbol $\sqrt{\ }$), and the opposite of cubing a number is finding the **cube root** of the number (symbol $\sqrt[3]{\ }$).

The symbols $\sqrt{\ }$ and $\sqrt[3]{\ }$ are called **radical signs**.

It is easier to find the square root if the number is written as a product of its prime factors.

Mathematicians always want to be able to undo what they have done. That's radical!

Exercise 4G

1 Find the missing values.

a $\_\_\_^2 = 9$ **b** $\_\_\_^2 = 25$ **c** $\_\_\_^2 = 16$ **d** $\_\_\_^2 = 49$ **e** $\_\_\_^2 = 100$

EXAMPLE 1

Another way of asking question **1** is to use the square root symbol $\sqrt{\ }$. Find:

a $\sqrt{9}$ **b** $\sqrt{25}$ **c** $\sqrt{16}$

$3^2 = 9$, so $\sqrt{9} = 3$. 3 is the square root of 9.

a $\sqrt{9} = \sqrt{3 \times 3}$
$= 3$

b $\sqrt{25} = \sqrt{5 \times 5}$
$= 5$

c $\sqrt{16} = \sqrt{4 \times 4}$
$= 4$

2 Find the following.

a $\sqrt{49}$ **b** $\sqrt{100}$ **c** $\sqrt{81}$ **d** $\sqrt{64}$ **e** $\sqrt{4}$

3 Courtney suggested that, since $36 = 4 \times 9$, then $\sqrt{36} = \sqrt{4 \times 9} = \sqrt{4} \times \sqrt{9}$.

a Find $\sqrt{4 \times 9}$.

b Find $\sqrt{4}$ and $\sqrt{9}$, then $\sqrt{4} \times \sqrt{9}$.

c Is $\sqrt{4 \times 9} = \sqrt{4} \times \sqrt{9}$?

4 Check Courtney's suggestion using these square roots.

a $\sqrt{100} = \sqrt{4 \times 25}$ **b** $\sqrt{225} = \sqrt{9 \times 25}$ **c** $\sqrt{144} = \sqrt{9 \times 16}$

5 Complete the following.

a $\_\_\_^3 = 27$ **b** $\_\_\_^3 = 64$ **c** $\_\_\_^3 = 125$ **d** $\_\_\_^3 = 1000$

EXAMPLE 2

Another way of asking question **5** is to use the cube root symbol $\sqrt[3]{\ }$. Find:

a $\sqrt[3]{1000}$ **b** $\sqrt[3]{64}$

$10^3 = 1000$, so $\sqrt[3]{1000} = 10$. 10 is the cube root of 1000.

a $\sqrt[3]{1000} = \sqrt[3]{10 \times 10 \times 10}$
$= 10$

b $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4}$
$= 4$

6 Find the following.

a $\sqrt[3]{27}$ **b** $\sqrt[3]{125}$ **c** $\sqrt[3]{216}$ **d** $\sqrt[3]{729}$

7 Courtney further suggested that, since $8000 = 8 \times 1000$, then $\sqrt[3]{8000} = \sqrt[3]{8 \times 1000} = \sqrt[3]{8} \times \sqrt[3]{1000}$. Was she correct?

EXAMPLE 3

Find the square root of 1764 using prime factors.

$\sqrt{2^2} = \sqrt{2 \times 2} = 2$

$1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$
$= 2^2 \times 3^2 \times 7^2$
$\sqrt{1764} = 2 \times 3 \times 7 = 42$

or

$1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$
$= (2 \times 3 \times 7) \times (2 \times 3 \times 7)$
$\sqrt{1764} = (2 \times 3 \times 7) = 42$

Find the square root of each factor and multiply.

8 Complete the following.

a $196 = \_\_\_\_ \times 2 \times \_\_\_\_ \times 7$

$= \_\_\_\_ \times 7^2$

$\sqrt{196} = \_\_\_\_ \times \_\_\_\_ = \_\_\_\_$

b $100 = \_\_\_\_ \times \_\_\_\_ \times 5 \times \_\_\_\_$

$= (\_\_\_\_) \times (\_\_\_\_)$

$\sqrt{100} = (\_\_\_\_) = \_\_\_\_$

c $225 = 3 \times \_\_\_\_ \times \_\_\_\_ \times 5$

$= 3^2 \times \_\_\_\_$

$\sqrt{225} = \_\_\_\_ \times \_\_\_\_ = \_\_\_\_$

d $900 = 2 \times \_\_\_\_ \times 3 \times \_\_\_\_ \times 5 \times \_\_\_\_$

$= (\_\_\_\_) \times (\_\_\_\_)$

$\sqrt{900} = (\_\_\_\_) = \_\_\_\_$

9 Find the square root of each of the following numbers.

a $441 = 3 \times 3 \times 7 \times 7$

b $5625 = 3 \times 3 \times 5 \times 5 \times 5 \times 5$

c $9801 = 3 \times 3 \times 3 \times 3 \times 11 \times 11$

d $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

10 **a** Complete the following sequence of the first ten cubic numbers.

1, 8, ____, ____, 125, ____, 343, ____, 729, ____

b $8 = 2 \times 2 \times 2$, so $\sqrt[3]{8} = \_\_\_\_$.

c Find: **i** $\sqrt[3]{27}$ **ii** $\sqrt[3]{1}$ **iii** $\sqrt[3]{125}$ **iv** $\sqrt[3]{64}$

11 Complete the following.

a $729 = 9 \times 9 \times 9$

$\therefore \sqrt[3]{729} = \_\_\_\_$

b $512 = 8 \times 8 \times 8$

$\therefore \sqrt[3]{512} = \_\_\_\_$

c $343 = 7 \times 7 \times 7$

$\therefore \sqrt[3]{343} = \_\_\_\_$

d $2744 = 14 \times 14 \times 14$

$\therefore \sqrt[3]{2744} = \_\_\_$

e $3375 = 15 \times 15 \times 15$

$\therefore \sqrt[3]{3375} = \_\_\_\_$

f $4913 = 17 \times 17 \times 17$

$\therefore \sqrt[3]{4913} = \_\_\_\_$

g $216 = (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$\therefore \sqrt[3]{216} = (\_\_\_\_) = \_\_\_\_$

h $1000 = (2 \times 5) \times (2 \times 5) \times (2 \times 5)$

$\therefore \sqrt[3]{1000} = (\_\_\_\_) = \_\_\_\_$

EXAMPLE 4

Find the cube root of 729 000 given that $729\,000 = 2^3 \times 3^6 \times 5^3$.

$\sqrt[3]{2^3} = \sqrt[3]{2 \times 2 \times 2} = 2$

$\sqrt[3]{729\,000} = \sqrt[3]{2^3 \times 3^6 \times 5^3}$

$= \sqrt[3]{2^3} \times \sqrt[3]{3^6} \times \sqrt[3]{5^3}$

$= 2 \times 3^2 \times 5$

$= 2 \times 9 \times 5$

$= 90$

$\sqrt[3]{3^6} = \sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= \sqrt[3]{3^2 \times 3^2 \times 3^2}$

$= 3^2$

12 Find the cube root of each of the following numbers.

a $2744 = 2^3 \times 7^3$

b $10\,648 = 2^3 \times 11^3$

c $1125 = 3^3 \times 5^3$

d $27\,000 = 2^3 \times 3^3 \times 5^3$

e $343\,000 = 2^3 \times 5^3 \times 7^3$

f $1728 = 2^6 \times 3^3$

g $46\,656 = 2^6 \times 3^6$

h $216\,000\,000 = 2^9 \times 3^3 \times 5^6$

i $272\,097\,792 = 2^9 \times 3^{12}$

13 Anadeep is trying to find $\sqrt{13}$ and knows it is not a whole number.

a What square numbers are closest to 13? Find the one smaller and the one larger.

b Find the square root of each of these numbers.

c Between which two numbers does $\sqrt{13}$ lie?

d Use a calculator to find $\sqrt{13}$ to 2 decimal places.

14 Chris is trying to find $\sqrt{24}$, but is having difficulty.

a Write 24 as a product of prime factors.

b Explain why $\sqrt{24}$ cannot be a whole number.

c Between which two whole numbers does $\sqrt{24}$ lie? Explain.

d Use a calculator to find $\sqrt{24}$.

15 **i** Find between which two whole numbers the following square roots lie.

ii Check with a calculator.

a $\sqrt{30}$ **b** $\sqrt{40}$ **c** $\sqrt{11}$ **d** $\sqrt{53}$ **e** $\sqrt{72}$

16 Angela says that $\sqrt[3]{150}$ is between 5 and 6.

a Explain why $\sqrt[3]{150}$ is not a whole number.

b Is Angela correct? Explain.

c Find $\sqrt[3]{150}$ using a calculator.

17 Find between which two whole numbers the following cube roots lie.

a $\sqrt[3]{6}$ **b** $\sqrt[3]{30}$ **c** $\sqrt[3]{45}$ **d** $\sqrt[3]{100}$ **e** $\sqrt[3]{250}$ **f** $\sqrt[3]{500}$

Investigation 8 The order of operations

The students in Year 7 Red were asked to find the value of the expression $15 + 9 \div 3$. Sue-Lin said that the answer was 8 and Ace said that the answer was 18. Work out how each student arrived at their answer.

What to do:

1 Using Sue-Lin's method and Ace's method, find two answers to the following questions.

a $25 - 15 \div 5$ **b** $39 + 26 \div 13$ **c** $15 \times 2 + 3$

d $10 \times 6 + 3$ **e** $15 - 9 \div 3$ **f** $11 + 5 \times 2$

2 The correct answer to the first question was Ace's answer of 18. The correct answers to question **1** are:

a 22 **b** 41 **c** 33 **d** 63 **e** 12 **f** 21

Work out a rule for finding the correct answer to each of the questions.

3 Test your rule on the following questions.

a $27 - 18 \div 9$ **b** $11 + 5 \times 3$ **c** $30 - 10 \div 5$

d $8 \times 5 + 2$ **e** $6 + 8 \times 3$ **f** $20 - 8 \div 2$

4 Discuss your rule with the person next to you and see if the rules are the same. Find the best rule using both your ideas. Share your rule with the class.

5 Try your rule on the following calculations.

a $100 \div 5 \times 2 = 40$ **b** $60 \div 10 \times 3 = 18$ **c** $80 \div 4 \times 2 = 40$ **d** $80 \div 2 \times 4 = 160$

6 Mary says that the order does not matter if all the operations are the same.

a Calculate the following. Are all the answers the same?

i $2 \times 3 \times 5$ **ii** $3 \times 5 \times 2$ **iii** $5 \times 2 \times 3$

b Calculate the following using the same method each time. Are the answers the same?

i $80 \div 8 \div 2$ **ii** $80 \div 2 \div 8$

c Comment on the statement: Order does not matter when multiplying or dividing numbers.

H Order of operations

Investigation 8 shows a need for a specific order of operations. Each calculation must result in the same answer. Mathematicians developed a set of rules called the **order of operations**. The order of operations are:

- Complete calculations within grouping symbols.
- Complete calculations involving powers and roots.
- Complete multiplication and division from left to right.
- Complete addition and subtraction from left to right.

Brackets [ ], braces { } and parentheses ( ) are grouping symbols.

EXAMPLE 1

Complete the following using the correct order of operations.

a $30 - 20 \div 4$ **b** $15 \times 2 + 20 \div 5$ **c** $3 + 16 \times 3 \div 4$ **d** $80 \div 5 \times 2$

a	$30 - 20 \div 4$	division first
	$= 30 - 5 = 25$	then subtraction
b	$15 \times 2 + 20 \div 5$	multiplication and division first (from left to right)
	$= 30 + 4 = 34$	then addition
c	$3 + 16 \times 3 \div 4$	multiplication first (from left to right)
	$= 3 + 48 \div 4$	then division
	$= 3 + 12 = 15$	then addition
d	$80 \div 5 \times 2$	division first (from left to right)
	$= 16 \times 2 = 32$	then multiplication

Exercise 4H

1 Complete the following using the correct order of operations.

a $40 + 10 \div 2$ **b** $18 + 4 \times 8$ **c** $30 - 5 \times 3$ **d** $16 + 16 \div 2$
e $35 - 16 \div 4$ **f** $4 \times 6 - 7 \times 3$ **g** $8 \times 3 + 12 \div 4$ **h** $10 \times 4 + 3 \times 7$
i $4 \times 8 + 5 - 8$ **j** $60 \div 5 \times 2$ **k** $80 \div 4 \times 5$ **l** $14 + 100 \div 4 - 12$
m $40 \div 5 - 50 \div 10$ **n** $7 \times 4 \div 2 + 8$ **o** $50 \div 5 \times 10 \div 2$

EXAMPLE 2

Complete the following using the correct order of operations.

a $(14 + 11) \div 5$ **b** $18 \times [4 - 2]$ **c** $\{25 - 7\} \div [2 + 4]$ **d** $(15 - (3 + 4)) \times 2$.

a	$(14 + 11) \div 5$	grouping symbols first
	$= 25 \div 5 = 5$	then division
b	$18 \times [4 - 2]$	grouping symbols first
	$= 18 \times 2 = 36$	then multiplication
c	$\{25 - 7\} \div [2 + 4]$	grouping symbols first
	$= 18 \div 6 = 3$	then division
d	$(15 - (3 + 4)) \times 2$	inside grouping symbols first
	$= (15 - 7) \times 2$	then grouping symbols
	$= 8 \times 2 = 16$	then multiplication

When there is more than one pair of grouping symbols work from the inside to the outside.

2 Complete the following using the correct order of operations.

a $(8 + 12) \div 5$
b $100 \div [7 + 3]$
c $60 \div \{12 - 7\}$
d $(27 - 12) \times 3$
e $(16 - 10) \div 2$
f $[18 - 7] \times [3 + 8]$
g $\{8 + 2\} \div (7 - 5)$
h $(9 + 7) \div (15 - 7)$
i $(6 + 5 \times 2) \div 8$
j $[11 - 4 \times 2] + 6$
k $45 \div (9 - 4) \times 3$
l $(3 + 4 \times 2) + (2 \times 7)$
m $5 \times [14 - 8] \div 3$
n $\{18 \div 3 + 4\} \div (3 \times 2 - 1)$
o $(10 - 3 \times 2) \div [15 - 13]$
p $(32 - (4 + 8)) \div 5$
q $((15 - 7) \div 4) \times 3$
r $[4 \times \{9 - 7\}] \times 5$
s $(22 \div (7 + 4)) \times 8$
t $((15 - 12) \times (4 + 6)) \div (4 + 11)$

There are other grouping symbols. These include powers, radical signs and the fraction bar.

EXAMPLE 3

Simplify the following.

a $\dfrac{15 - 7}{2 \times 4}$ **b** $\sqrt{7 + 9} \times 3$ **c** $20 \div 2^2 + 3$

a $\dfrac{15 - 7}{2 \times 4}$
$= \dfrac{8}{8}$
$= 1$

b $\sqrt{7 + 9} \times 3$
$= \sqrt{16} \times 3$
$= 4 \times 3$
$= 12$

c $20 \div 2^2 + 3$
$= 20 \div 4 + 3$ (division first)
$= 5 + 3$
$= 8$

3 Simplify the following.

a $\dfrac{20 - 8}{2 + 2}$ **b** $\dfrac{16 + 4}{15 \div 3}$ **c** $\dfrac{9 \times 8}{16 - 12}$

d $\sqrt{11 + 5} \times 2$ **e** $6 \times \sqrt{3 + 1}$ **f** $45 \div \sqrt{17 + 8}$

g $50 \div 5^2 + 8$ **h** $100 \div (4 + 1)^2$ **i** $\dfrac{5 \times (3 + 1)^2}{17 + 3}$

j $\sqrt{\dfrac{100 - 64}{9}}$ **k** $\sqrt{\dfrac{5^2 - 3^2}{2}}$ **m** $4^3 + 2^2$

n $3^3 + 2 \times 4^2$ **o** $5^3 \div \sqrt{9 + 16}$ **p** $\sqrt{2^3 + 4^2 + 1}$

4 **a** Calculate $\sqrt{36} + \sqrt{64}$.
b Calculate $\sqrt{36 + 64}$.
c Is $\sqrt{36} + \sqrt{64} = \sqrt{36 + 64}$? Explain.

5 Is $\sqrt{144 + 25} = \sqrt{144} + \sqrt{25}$? Explain.

Calculator activities

1 Complete the following using your calculator.

a $43 + 65$ **b** $94 - 38$ **c** $136 + 58$ **d** $439 - 288$

e 53×42 **f** $4806 \div 54$ **g** 875×49 **h** $32\,964 \div 67$

i $9752 - 3571$ **j** $4924 + 3981$ **k** 2391×3595 **l** $2\,677\,136 \div 4592$

m $245 + 573 + 911 + 347 + 537 + 223 + 446$ **n** $14 \times 13 \times 34 \times 23 \times 41 \times 25 \div 17 \div 20 \div 65$

2 For each root below, estimate between which two numbers the root lies, then use your calculator to find the answer, correct to 2 decimal places.

a $\sqrt{10}$ **b** $\sqrt{18}$ **c** $\sqrt{40}$ **d** $\sqrt{130}$

e $\sqrt[3]{20}$ **f** $\sqrt[3]{55}$ **g** $\sqrt[3]{100}$ **h** $\sqrt[3]{140}$

3 Find out how to use the automatic constant on your calculator, then use it to list the first 20 multiples of:

a 13 **b** 14 **c** 23 **d** 28 **e** 46

4 All scientific calculators obey the order of operations rules. Use your calculator to answer the following.

a $345 + 23 \times 16$ **b** $459 \div 27 + 234$ **c** $145 + 37 \times 16 - 211$

d $(375 - 219) \times 47$ **e** $(561 - 398) \times 53 - 2581$ **f** $((256 - 184) \times 34) \times 18$

g $16 + 28 \times 43 - 540 \div 9 + 38$ **h** $(158 - (375 - 288)) \times 51$

5 Evaluate the following.

a $(-2)^3$ **b** $(-2)^4$ **c** $(-2)^5$ **d** $(-2)^6$ **e** $(-3)^2$ **f** $(-3)^3$ **g** $(-3)^4$ **h** $(-3)^5$

6 Complete the following statements.

a When a negative number is raised to an even power the answer is always _____.

b When a negative number is raised to a _____ power the answer is always negative.

Language in mathematics

Here is a find-a-word puzzle. Find all the words in the following list:

cardinal, code, composite, counting, cube, cubic, divisibility, Eratosthenes, even, factors, Fibonacci, index, multiples, notation, numbers, odd, patterns, perfect, previous, prime, radical, sieve, sign, term, unique, zero.

H	I	U	O	D	N	X	I	C	C	A	N	O	B	I	F
P	C	N	U	M	B	E	R	S	N	N	R	U	B	F	F
L	U	I	T	U	T	D	V	T	F	P	G	E	Y	D	S
A	B	Q	U	L	C	N	L	E	R	A	D	I	C	A	L
N	I	U	A	T	E	I	C	P	N	T	C	N	S	M	F
I	C	E	E	I	F	E	O	N	O	T	A	T	I	O	N
D	D	O	E	P	R	I	M	E	E	E	M	S	O	A	Q
R	N	R	R	L	E	H	P	K	F	R	V	Z	X	R	E
A	C	U	B	E	P	C	O	D	E	N	A	E	E	A	S
C	X	I	U	S	Z	R	S	T	K	S	D	Q	I	V	K
D	I	V	I	S	I	B	I	L	I	T	Y	E	S	S	Q
L	O	A	C	O	U	N	T	I	N	G	P	N	F	H	X
E	T	S	U	O	I	V	E	R	P	T	R	E	B	K	D
N	A	S	G	S	E	N	E	H	T	S	O	T	A	R	E
A	U	E	D	K	G	I	D	B	L	H	B	K	F	J	X

Terms

base
composite number
cube root
divisible
even number
expanded form
factor
factorial number
factorisation
Fibonacci numbers
highest common factor
index notation
lowest common multiple
multiple
odd number
palindromic number
Pascal's triangle
power
prime number
radical sign
simplify
square numbers
square root
triangular numbers

Check your skills

1 In 5^3, the numeral 5 is the:
A power **B** base **C** index **D** cube

2 3 to the power of 2 is:
A 3×2 **B** 2 cubed **C** 2^3 **D** 3 squared

3 Which number is divisible by 3?
A 24 300 **B** 18 521 **C** 30 323 **D** 9191

4 Which number is divisible by 12?
A 1242 **B** 1244 **C** 1246 **D** 1248

5 What is $943 \div 23$?
A 45 **B** 43 **C** 41 **D** 39

6 What is $758 \div 28$?
A $27\frac{2}{28}$ **B** $29\frac{6}{28}$ **C** $25\frac{8}{28}$ **D** $28\frac{4}{28}$

7 The sixth multiple of 8 is:
A 6 **B** 8 **C** 14 **D** 48

8 The LCM of 10 and 6 is:
A 60 **B** 30 **C** 10 **D** 6

9 The number of factors that 24 has is:
A 2 **B** 4 **C** 6 **D** 8

10 A common factor of 30 and 28 is:
A 2 **B** 4 **C** 6 **D** 7

11 The HCF of 20 and 24 is:
A 2 **B** 4 **C** 8 **D** 20

12 Two numbers with a HCF of 3 and LCM of 30 are:
A 6 and 30 **B** 6 and 15 **C** 6 and 90 **D** 3 and 60

13 Which number is prime?
A 21 **B** 23 **C** 25 **D** 27

14 The prime factorisation of 120 is:
A $2 \times 3 \times 5$
B $2 \times 2 \times 3 \times 5$
C $2 \times 2 \times 2 \times 3 \times 5$
D $2 \times 2 \times 2 \times 3 \times 3 \times 5$

15 The HCF of $100 \times 2 \times 2 \times 5 \times 5$ and $80 \times 2 \times 2 \times 2 \times 2 \times 5$ is:

A 2×5 **B** $2 \times 2 \times 5$

C $2 \times 2 \times 5 \times 5$ **D** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

16 The LCM of 100 and 80 is:

A $2 \times 2 \times 5 \times 5$ **B** $2 \times 2 \times 5$

C $2 \times 2 \times 2 \times 2 \times 5 \times 5$ **D** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

17 Find $\sqrt{2 \times 2 \times 3 \times 3 \times 11 \times 11}$.-

A $2 \times 3 \times 11$ **B** $2 \times 2 \times 3 \times 11$

C $2 \times 2 \times 3 \times 3 \times 11$ **D** $2 \times 2 \times 3 \times 3 \times 11 \times 11$

18 $1728 \times 2^6 \times 3^3$, so $\sqrt[3]{1728}$ is:

A 2×3 **B** $2^2 \times 3$ **C** $2^3 \times 3$ **D** $2^3 \times 3^2$

19 What is $150 \div 5 \times 3$?

A 10 **B** 30 **C** 90 **D** 250

20 What is $4^2 + 3 \times 2^3$?

A 27 **B** 40 **C** 152 **D** 392

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3, 4	5, 6	7–13	14	15, 16	17, 18	19, 20
Section	A	B	C	D	E	F	G	H

4A Review set

1 Write the multiples of:

a 6 between 11 and 35 **b** 8 between 23 and 65 **c** 5 that are less than 56

2 **a** List the multiples of 4 that are less than 50.

b List the multiples of 6 that are less than 50.

c Write the common multiples of 4 and 6 that are less than 50.

d What is the LCM of 4 and 6?

3 **a** Write the factors of 24. **b** Write the factors of 36.

c What is the HCF of 24 and 36?

4 Use a factor tree to write each number as a product of prime factors.

a 40 **b** 120 **c** 352

5 Use the method of division by primes to write each number as a product of prime factors.

a 80 **b** 154 **c** 440

6 **a** Write 24 and 40 as products of prime factors.

b Find the HCF of 24 and 40. **c** Find the LCM of 24 and 40.

7 **a** Write 180 and 120 as products of prime factors.

b Find the HCF of 180 and 120. **c** Find the LCM of 180 and 120.

8 Find the HCF and LCM of each pair of numbers. Write the numbers as products of prime factors first.

a 84 and 63 **b** 75 and 40 **c** 240 and 400

9 Find the following.
a $\sqrt{16}$ **b** $\sqrt{64}$ **c** $\sqrt{121}$ **d** $\sqrt[3]{64}$ **e** $\sqrt[3]{125}$ **f** $\sqrt[3]{216}$

10 Which of the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 12 are factors of:
a 11 730? **b** 16 120? **c** 61 380?

11 Between which two whole numbers does $\sqrt{54}$ lie?

12 Find the following.
a $918 \div 27$ **b** $584 \div 32$

13 Find the following.
a $14 - 3 \times 4$ **b** $5^2 + 7 \times 3^2$ **c** $\sqrt{\dfrac{10^2 - 5^2}{3}}$

4B Review set

1 Write the multiples of:
a 4 between 23 and 49 **b** 7 between 20 and 71 **c** 9 that are less than 64

2 **a** List the multiples of 9 that are less than 100.
b List the multiples of 12 that are less than 100.
c Write the common multiples of 9 and 12 that are less than 100.
d What is the LCM of 9 and 12?

3 **a** Write the factors of 18. **b** Write the factors of 30. **c** What is the HCF of 18 and 30?

4 Use a factor tree to write each number as a product of prime factors.
a 60 **b** 100 **c** 420

5 Use the method of division by primes to write each number as a product of prime factors.
a 90 **b** 168 **c** 480

6 **a** Write 36 and 60 as products of prime factors.
b Find the HCF of 36 and 60. **c** Find the LCM of 36 and 60.

7 **a** Write 140 and 200 as products of prime factors.
b Find the HCF of 140 and 200. **c** Find the LCM of 140 and 200.

8 Find the HCF and LCM of each pair of numbers. Write the numbers as products of prime factors first.
a 40 and 96 **b** 70 and 100 **c** 210 and 360

9 Find the following.
a $\sqrt{25}$ **b** $\sqrt{81}$ **c** $\sqrt{144}$ **d** $\sqrt[3]{8}$ **e** $\sqrt[3]{27}$ **f** $\sqrt[3]{343}$

10 Which of the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 12 are factors of:
a 1830? **b** 53 940? **c** 79 112?

11 Between which two whole numbers does $\sqrt[3]{350}$ lie?

12 Find the following.
a $684 \div 18$ **b** $729 \div 24$

13 Find the following.
a $1000 \div 10 \div 2$ **b** $1000 \div (10 \div 2)$
c Why are the answers different? Explain.

5

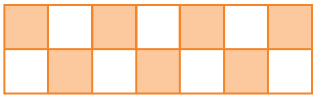
Fractions

This chapter deals with the operations of fractions. At the end of this chapter you should be able to:

- understand the concept of fractions
- recognise types of fractions
- convert mixed numerals to improper fractions
- convert improper fractions to mixed numerals
- simplify and compare fractions using equivalence
- locate and represent fractions and mixed numerals on a number plane
- solve problems involving fractions
- use a calculator to perform skills learned in fractions.

Diagnostic test

1 What fraction of the diagram shown below has been shaded?



A $\frac{1}{4}$ **B** $\frac{7}{12}$ **C** $\frac{1}{2}$ **D** $\frac{7}{18}$

2 How many eighths in 4 wholes?

A 32 **B** 30 **C** 28 **D** 20

3 Which fraction below is a proper fraction?

A $\frac{9}{8}$ **B** $7\frac{1}{4}$ **C** $\frac{4}{5}$ **D** $\frac{13}{13}$

This list of fractions relates to questions **4** and **5**.

$\frac{4}{10}, \frac{7}{10}, \frac{9}{10}, \frac{5}{10}, \frac{1}{10}$

Using a number line may help you answer the questions.

4 The largest fraction is:

A $\frac{4}{10}$ **B** $\frac{9}{10}$ **C** $\frac{5}{10}$ **D** $\frac{1}{10}$

5 The smallest fraction is:

A $\frac{5}{10}$ **B** $\frac{1}{10}$ **C** $\frac{4}{10}$ **D** $\frac{9}{10}$

6 Which statement is true?

A $\frac{3}{5} > \frac{4}{5}$ **B** $\frac{1}{10} > \frac{7}{10}$

C $\frac{4}{5} > \frac{3}{5}$ **D** $\frac{7}{10} > \frac{8}{10}$

7 Which statement is true?

A $\frac{3}{7} < \frac{5}{7}$ **B** $\frac{4}{7} < \frac{3}{7}$ **C** $\frac{5}{7} < \frac{4}{7}$ **D** $\frac{2}{7} < \frac{1}{7}$

8 Find $\frac{4}{11} + \frac{3}{11} =$

A $\frac{7}{22}$ **B** $\frac{12}{22}$ **C** $\frac{43}{11}$ **D** $\frac{7}{11}$

9 $\frac{2}{7} + \frac{3}{7} =$

A $\frac{5}{14}$ **B** $\frac{6}{14}$ **C** $\frac{23}{77}$ **D** $\frac{5}{7}$

10 Find $\frac{11}{13} - \frac{5}{13}$.

A 6 **B** $\frac{6}{13}$ **C** $\frac{16}{13}$ **D** $\frac{6}{26}$

11 Find $\frac{8}{10} - \frac{5}{10}$.

A 3 **B** $\frac{3}{10}$ **C** $\frac{13}{10}$ **D** $\frac{3}{20}$

12 Find $\frac{4}{5} + \frac{1}{10}$.

A $\frac{5}{15}$ **B** $\frac{5}{10}$ **C** $\frac{9}{10}$ **D** $\frac{5}{50}$

13 Find $\frac{1}{4}$ of 20.

A 16 **B** 5 **C** 1 **D** 15

14 Find $\frac{3}{5}$ of 20.

A 12 **B** 8 **C** 4 **D** 1

15 Find $\frac{2}{7}$ of 28.

A 1 **B** 4 **C** 8 **D** 5

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–7	8–12	13–15
Section	ACMNA 125	ACMNA 126	ACMNA 127

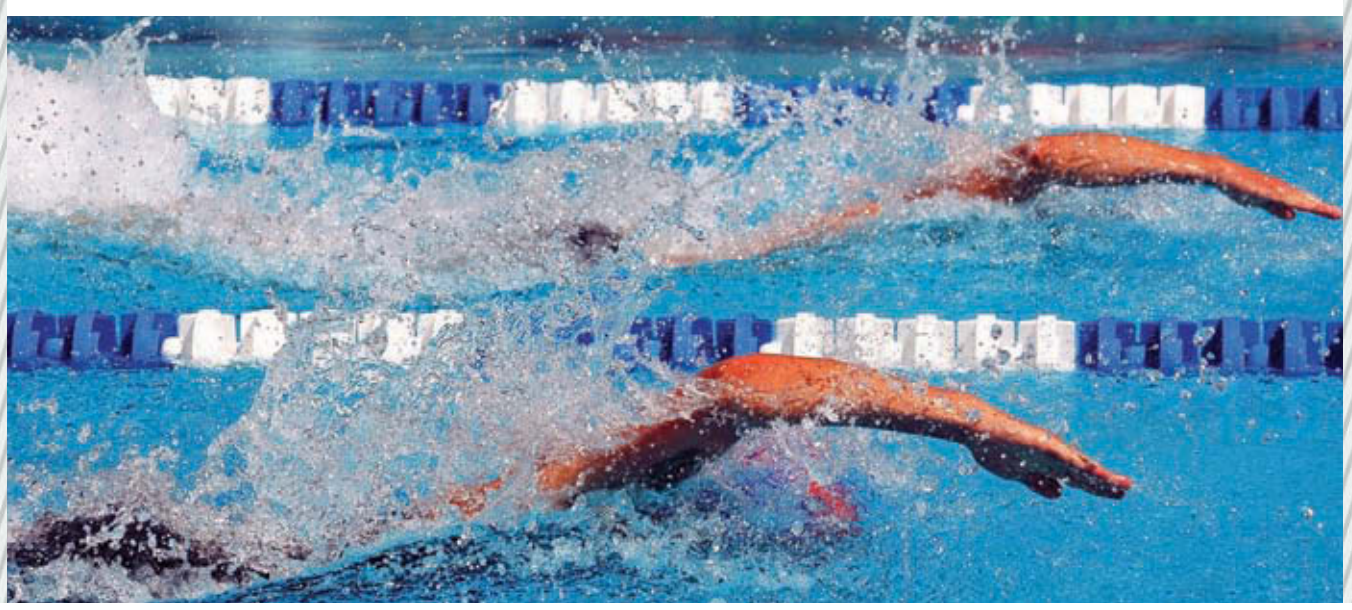
WHAT IS A FRACTION?

When whole numbers are 'broken' into equal parts, fractions are formed. Fractions are parts of whole numbers.

Investigation 1 Fractions in everyday life

1 Look at your local or daily newspaper. Are there any examples of fractions in the:
a advertisements? **b** news articles?
Select examples of fractions used for advertisements and news articles.

2 Discuss this use of fractions with your class:
'Ace's swimming time was a fraction of a second outside his personal best!'



A Types of fractions

The parts of a fraction are:

$\frac{2}{3}$

numerator (top number)

vinculum (bar or dividing line) ($\frac{2}{3}$ means the same as $2 \div 3$. The bar represents division.)

denominator (bottom number)

There are three types of fractions, listed in this table.

Proper	Improper	Mixed numeral
$\frac{3}{4}$	$\frac{3}{3}, \frac{4}{3}$	$2\frac{1}{2}$
Numerator smaller than denominator	Numerator equal to or larger than denominator	Whole number and a fraction part

Exercise 5A

1 Classify each fraction as proper, improper or mixed numeral.

a $\frac{11}{14}$ **b** $2\frac{3}{8}$ **c** $\frac{19}{5}$ **d** $\frac{18}{7}$

e $6\frac{1}{4}$ **f** $\frac{22}{9}$ **g** $\frac{11}{11}$ **h** $\frac{5}{13}$

i $\frac{8}{15}$ **j** $20\frac{1}{3}$ **k** $\frac{25}{25}$ **l** $\frac{31}{12}$

EXAMPLE 1

Express each improper fraction as a mixed numeral.

a $\frac{9}{4}$ **b** $\frac{22}{5}$

a $9 \div 4 = 2$ remainder 1

$= 2\frac{1}{4}$

b $22 \div 5 = 4$ remainder 2

$= 4\frac{2}{5}$

2 Express each improper fraction as a mixed numeral.

a $\frac{14}{3} = 14 \div \_\_\_$

$=$ 4 remainder $\_\_\_$

$= 4\frac{\square}{3}$

b $\frac{73}{10} = \_\_\_ \div 10$

$=$ $\_\_\_$ remainder 3

$= \square\frac{3}{10}$

c $\frac{21}{5} = 21 \div \square$

$=$ $\_\_\_$ remainder $\_\_\_$

$= \square\frac{\square}{\square}$

3 Express the following as mixed numerals.

a $\frac{10}{3}$ **b** $\frac{15}{2}$ **c** $\frac{11}{6}$ **d** $\frac{19}{4}$ **e** $\frac{31}{8}$

f $\frac{37}{5}$ **g** $\frac{41}{7}$ **h** $\frac{29}{4}$ **i** $\frac{46}{9}$ **j** $\frac{57}{10}$

EXAMPLE 2

Express each mixed numeral as an improper fraction.

a $3\frac{2}{5}$ **b** $7\frac{4}{9}$

$\frac{(\text{denominator} \times \text{whole number}) + \text{numerator}}{\text{denominator}}$

a $3\frac{2}{5} = \frac{(5 \times 3) + 2}{5}$

$= \frac{17}{5}$

b $7\frac{4}{9} = \frac{(9 \times 7) + 4}{9}$

$= \frac{67}{9}$

4 Express each mixed numeral as an improper fraction.

a $3\frac{1}{4} = \frac{4 \times \square + 1}{4}$

$= \frac{\square}{4}$

b $2\frac{3}{5} = \frac{\square \times 2 + \square}{5}$

$= \frac{\square}{\square}$

c $5\frac{1}{2} = \frac{2\square 5\square 1}{\square}$

$= \frac{11}{\square}$

5 Express the following as improper fractions.

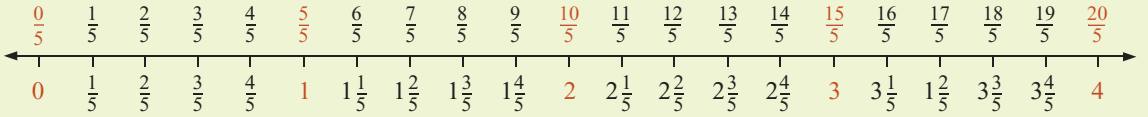
a $2\frac{1}{4}$ **b** $3\frac{1}{3}$ **c** $1\frac{5}{7}$ **d** $4\frac{2}{9}$ **e** $8\frac{4}{5}$

f $7\frac{1}{6}$ **g** $5\frac{2}{5}$ **h** $1\frac{13}{15}$ **i** $6\frac{4}{7}$ **j** $2\frac{9}{10}$

EXAMPLE 3

Use the number line below to answer the following questions.

Improper fractions



Mixed numerals

a Write a mixed numeral for each improper fraction.

i $\frac{7}{5}$ **ii** $\frac{9}{5}$ **iii** $\frac{12}{5}$ **iv** $\frac{19}{5}$

b Write an improper fraction for each mixed numeral.

i $1\frac{3}{5}$ **ii** $2\frac{1}{5}$ **iii** $2\frac{4}{5}$ **iv** $3\frac{3}{5}$

c How many fifths are there in:

i 1 whole? **ii** 2 wholes? **iii** 3 wholes? **iv** 4 wholes?

a **i** $\frac{7}{5} = 1\frac{2}{5}$ **ii** $\frac{9}{5} = 1\frac{4}{5}$ **iii** $\frac{12}{5} = 2\frac{2}{5}$ **iv** $\frac{19}{5} = 3\frac{4}{5}$

b **i** $1\frac{3}{5} = \frac{8}{5}$ **ii** $2\frac{1}{5} = \frac{11}{5}$ **iii** $2\frac{4}{5} = \frac{14}{5}$ **iv** $3\frac{3}{5} = \frac{18}{5}$

c **i** 1 whole = 5 fifths **ii** 2 wholes = 10 fifths

iii 3 wholes = 15 fifths **iv** 4 wholes = 20 fifths

6 **a** Draw a number line extending from 2 to 6 and divide it into quarters. Using 1 cm subdivisions, label all points using whole numbers or mixed numerals below the line and improper fractions above it (as shown in Example 3).

b Using mixed numerals for your answers, find the number that is:

i $\frac{3}{4}$ more than $\frac{16}{4}$ **ii** $2\frac{1}{4}$ more than $3\frac{2}{4}$ **iii** $\frac{2}{4}$ less than 5

iv $1\frac{3}{4}$ less than $5\frac{1}{4}$ **v** 3 less than $\frac{22}{4}$

Ascending is smallest to largest.
Descending is largest to smallest.

c Place the following fractions in **ascending** order.

i $3\frac{1}{4}, \frac{9}{4}, 4\frac{2}{4}, \frac{16}{4}$ **ii** $5, 2\frac{1}{4}, \frac{22}{4}, \frac{13}{4}$

d Place the following fractions in **descending** order.

i $3\frac{3}{4}, \frac{11}{4}, 6, \frac{20}{4}, \frac{18}{4}$ **ii** $2\frac{1}{4}, \frac{23}{4}, 3\frac{3}{4}, 5, \frac{19}{4}$

7 How many:

a quarters in 1 whole? **b** quarters in 2 wholes? **c** sevenths in 1 whole?

d sevenths in 4 wholes? **e** twelfths in 2 wholes? **f** ninths in 8 wholes?

Investigation 2 Zero as a denominator

Work in groups of four.

Complete the following to determine whether it is possible to have zero as a denominator.

a $4 \div 0 =$ **b** $5 \div 0 =$ **c** $\frac{8}{0} =$ **d** $\frac{12}{0} =$

Share your conclusions with the class. What have you discovered?

Hint: Suppose that there are four lollies to be shared between zero people. Does this make sense?

Investigation 3 Equivalent fractions

1 Use 5 mm grid paper to construct five identical squares with sides 4 cm long. Use vertical and horizontal lines only.

1 2 3 4 5

a Divide square 1 into two equal parts as shown. Each part is half (shaded).

b Divide square 2 into quarters as shown. $\therefore \frac{1}{2} = \frac{2}{4}$.

c Divide square 3 into eighths, square 4 into sixteenths and square 5 into thirty-two equal parts.

d Now complete: $\frac{1}{2} = \frac{2}{4} = \frac{\square}{8} = \frac{\triangle}{16} = \frac{\diamond}{32}$.

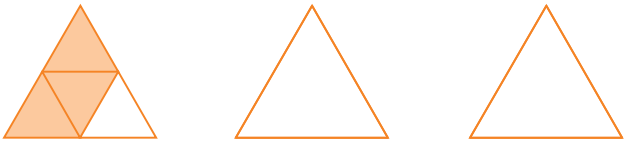
2 Try the same exercise constructing five identical 4-cm sided squares. Start with diagonal lines to create the halves and quarters. Name the equal fractions created in each square.

$\frac{1}{2}$ $\frac{1}{2} = \frac{\square}{4}$ $\frac{1}{2} = \frac{\square}{8}$ $\frac{1}{2} = \frac{\triangle}{16}$ $\frac{1}{2} = \frac{\diamond}{32}$

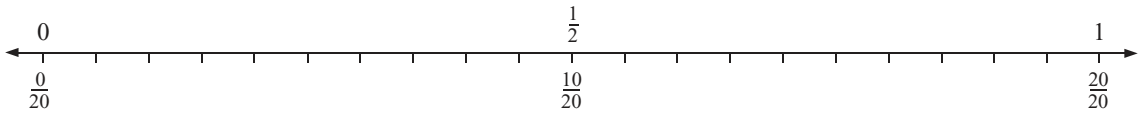
3 Construct four identical rectangles and show that $\frac{2}{5} = \frac{4}{10} = \frac{8}{20} = \frac{16}{40}$.

4 Construct three identical equilateral triangles and show that $\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$.

An equilateral triangle has three equal sides.



5 This number line is divided into 20 equal parts.



Use the line to show that:

a $\frac{1}{2} = \frac{10}{20}$ **b** $\frac{2}{20} = \frac{1}{10}$ **c** $\frac{1}{4} = \frac{5}{20}$ **d** $\frac{3}{4} = \frac{15}{20}$ **e** $\frac{1}{5} = \frac{4}{20}$ **f** $\frac{3}{5} = \frac{12}{20}$

6 Construct a number line with 24 equal intervals. Use it to show that:

a $\frac{1}{2} = \frac{12}{24}$ **b** $\frac{1}{3} = \frac{8}{24}$ **c** $\frac{1}{4} = \frac{6}{24}$ **d** $\frac{3}{4} = \frac{18}{24}$ **e** $\frac{2}{3} = \frac{16}{24}$ **f** $\frac{5}{12} = \frac{10}{24}$

7 Construct a number line with 30 equal intervals. Use it to show that:

a $\frac{1}{2} = \frac{15}{30}$ **b** $\frac{1}{3} = \frac{10}{30}$ **c** $\frac{1}{5} = \frac{6}{30}$ **d** $\frac{1}{6} = \frac{5}{30}$ **e** $\frac{3}{5} = \frac{18}{30}$ **f** $\frac{5}{6} = \frac{25}{30}$

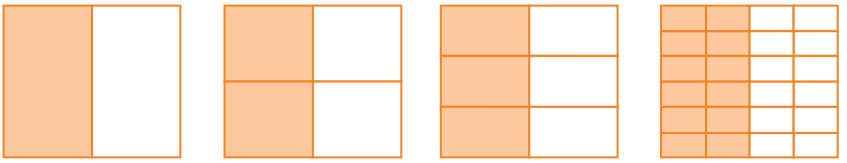
8 Take a square piece of paper and fold it in half 4 times. Unfold it, then complete the following.

$\frac{1}{2} = \frac{2}{4} = \frac{\square}{8} = \frac{\square}{16}$

B Equivalent fractions

Equivalent fractions are equal fractions written a different way.

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ and $\frac{6}{12}$ are equivalent fractions as they represent an equal amount.

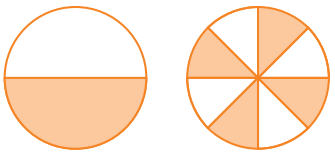


Colour one-half of a sheet of paper. Try folding it as shown above. What do you notice?

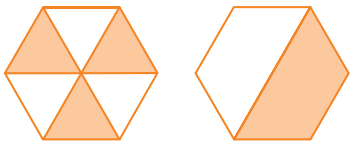
Exercise 5B

1 Write down the following pairs of equivalent fractions.

a

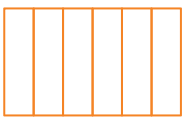


b

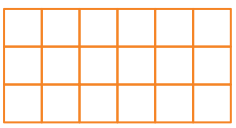


2 These diagrams represent chocolate bars of the same size that have been divided differently.
Copy the diagrams and shade $\frac{1}{3}$ of each chocolate bar.

a



b



3 Complete the following by referring to the diagrams in questions **2a** and **2b**.

a $\frac{1}{3} = \frac{\square}{6}$ **b** $\frac{1}{3} = \frac{\square}{18}$

EXAMPLE 1

Complete the following equivalent fractions.

Multiply the numerator and denominator by the same number.

a $\frac{1}{2} \overset{\times 6}{=} \frac{\square}{\square}$ (× 6 on both) **b** $\frac{4}{5} = \frac{\square}{10}$ **c** $\frac{3}{4} = \frac{15}{\square}$

a $\frac{1}{2} \overset{\times 6}{=} \frac{6}{12}$ (× 6 on both)

b $\frac{4}{5} = \frac{\square}{10}$ $5 \times 2 = 10$

$\frac{4}{5} \overset{\times 2}{=} \frac{8}{10}$ (× 2 on both)

c $\frac{3}{4} = \frac{15}{\square}$ $3 \times 5 = 15$

$\frac{3}{4} \overset{\times 5}{=} \frac{15}{20}$ (× 5 on both)

4 For each fraction, multiply the numerator and denominator by the number shown to find an equivalent fraction.

a $\frac{1}{2} \overset{\times 6}{=} \frac{\triangle}{\square}$ **b** $\frac{4}{9} \overset{\times 3}{=} \frac{\triangle}{\square}$ **c** $\frac{6}{11} \overset{\times 4}{=} \frac{\triangle}{\square}$

d $\frac{3}{7} \overset{\times 2}{=} \frac{\triangle}{\square}$ **e** $\frac{3}{8} \overset{\times 5}{=} \frac{\triangle}{\square}$ **f** $\frac{2}{3} \overset{\times 3}{=} \frac{\triangle}{\square}$

5 Complete the following equivalent fractions.

a $\frac{3}{4} = \frac{\square}{12}$ **b** $\frac{1}{2} = \frac{\square}{10}$ **c** $\frac{4}{7} = \frac{\square}{21}$ **d** $\frac{3}{5} = \frac{\square}{25}$

e $\frac{7}{12} = \frac{\square}{36}$ **f** $\frac{1}{11} = \frac{\square}{55}$ **g** $\frac{2}{17} = \frac{12}{\square}$ **h** $\frac{9}{10} = \frac{81}{\square}$

i $\frac{10}{13} = \frac{20}{\square}$ **j** $\frac{6}{7} = \frac{18}{\square}$ **k** $\frac{4}{11} = \frac{24}{\square}$ **l** $\frac{13}{15} = \frac{39}{\square}$

m $\frac{4}{15} = \frac{36}{\square}$ **n** $\frac{8}{9} = \frac{88}{\square}$ **o** $\frac{17}{20} = \frac{51}{\square}$ **p** $\frac{6}{11} = \frac{\square}{110}$

EXAMPLE 2

Complete the following equivalent fractions.

Divide the numerator and denominator by the same number.

a $\frac{6}{10} \overset{\div 2}{=} \frac{\triangle}{\square}$ (÷ 2 on both) **b** $\frac{15}{20} = \frac{\square}{4}$ **c** $\frac{21}{35} = \frac{3}{\square}$

a $\frac{6}{10} \overset{\div 2}{=} \frac{3}{5}$ (÷ 2 on both)

b $\frac{15}{20} = \frac{\square}{4}$ $20 \div 5 = 4$

$\frac{15}{20} \overset{\div 5}{=} \frac{3}{4}$ (÷ 5 on both)

c $\frac{21}{35} = \frac{3}{\square}$ $21 \div 7 = 3$

$\frac{21}{35} \overset{\div 7}{=} \frac{3}{5}$ (÷ 7 on both)

6 For each fraction, divide the numerator and denominator by the number shown to find an equivalent fraction.

a $\frac{10}{15} \overset{\div 5}{=} \frac{\triangle}{\square}$ **b** $\frac{6}{22} \overset{\div 2}{=} \frac{\triangle}{\square}$ **c** $\frac{55}{66} \overset{\div 11}{=} \frac{\triangle}{\square}$

d $\frac{14}{35} \overset{\div 7}{=} \frac{\triangle}{\square}$ **e** $\frac{16}{36} \overset{\div 4}{=} \frac{\triangle}{\square}$ **f** $\frac{20}{90} \overset{\div 10}{=} \frac{\triangle}{\square}$

7 Complete the following equivalent fractions.

a $\frac{6}{9} = \frac{\square}{3}$ **b** $\frac{12}{21} = \frac{\square}{7}$ **c** $\frac{22}{66} = \frac{\square}{3}$ **d** $\frac{30}{100} = \frac{\square}{10}$

e $\frac{6}{24} = \frac{\square}{4}$ **f** $\frac{6}{27} = \frac{\square}{9}$ **g** $\frac{22}{77} = \frac{2}{\square}$ **h** $\frac{12}{32} = \frac{3}{\square}$

i $\frac{36}{39} = \frac{12}{\square}$ **j** $\frac{100}{110} = \frac{\square}{11}$ **k** $\frac{72}{99} = \frac{8}{\square}$ **l** $\frac{50}{65} = \frac{\square}{13}$

Lowest common multiple

To compare fractions with different denominators, you first need to form equivalent fractions with the *same denominator*. To do this you need to find the **lowest common multiple** (**LCM**) of the denominators.

EXAMPLE 3

Find the LCM for each pair of numbers.

a 5 and 8 **b** 7 and 14

The lowest common multiple (LCM) of two numbers is the smallest number that both can divide into with no remainder.

a Multiples: 5, 10, 15, 20, 25, 30, 35, 40, …
8, 16, 24, 32, 40, 48, …
LCM = 40

b Multiples: 7, 14, 28, …
14, 28, …
LCM = 14

Note: In part **a** the LCM is a product of the two numbers ($5 \times 8 = 40$), but in part **b** the LCM is *not* a product of the two numbers.

8 Complete to find the LCM for each pair of numbers.

a 5 and 2
5, 10, ___, 20, …
2, ___, ___, 8, 10, 12, ___, …
LCM = ____

b 7 and 6
7, 14, 21, 28, 35, ___, …
6, ___, 18, 24, 30, ___, 42, …
LCM = ____

9 Find the LCM for each pair of numbers.

a 3 and 2 **b** 4 and 7
c 8 and 14 **d** 9 and 5
e 10 and 6 **f** 3 and 12
g 8 and 6 **h** 2 and 6
i 5 and 15 **j** 9 and 3
k 20 and 4 **l** 16 and 8

Converting to equivalent fractions with the same denominator

To convert two (or more) fractions to equivalent fractions that have the same denominator, follow these steps:
Step 1: *Change mixed numerals to improper fractions.*
Step 2: Find the **lowest common denominator** (**LCD**) by finding the LCM of the denominators.
Step 3: Make the denominators the same (change to the LCD)

EXAMPLE 4

Convert each pair of fractions to equivalent fractions that have the same denominator.

a $\frac{4}{5}$ and $\frac{1}{6}$

b $1\frac{1}{4}$ and $2\frac{2}{3}$

a $\frac{4}{5}$ and $\frac{1}{6}$

Multiples: 5, 10, 15, 20, 25, 30, 35, …

6, 12, 18, 24, 30, …

LCD = LCM = 30

$\frac{4}{5} \xrightarrow{\times 6} \frac{24}{30}$ and $\frac{1}{6} \xrightarrow{\times 5} \frac{5}{30}$

b $1\frac{1}{4} = \frac{4 \times 1 + 1}{4} = \frac{5}{4}$ $\quad 2\frac{2}{3} = \frac{3 \times 2 + 2}{3} = \frac{8}{3}$

Multiples: 4, 8, 12, 16, …

3, 6, 9, 12, …

LCD = LCM = 12

$\frac{5}{4} \xrightarrow{\times 3} \frac{15}{12}$ and $\frac{8}{3} \xrightarrow{\times 4} \frac{32}{12}$

$\therefore 1\frac{1}{4} = \frac{15}{12}$ and $2\frac{2}{3} = \frac{32}{12}$

10 Convert each pair of fractions to equivalent fractions with the same denominator.

a $\frac{3}{4}$ and $\frac{7}{9}$

Multiples: 4, 8, __, __, 20, 24, __, 32, 36, …

9, __, __, __, 45, …

LCD = LCM = ____

$\frac{3}{4} \xrightarrow{\times \square} \frac{27}{\square}$ and $\frac{7}{9} \xrightarrow{\times 4} \frac{\square}{36}$

b $3\frac{5}{7}$ and $3\frac{2}{5}$

$3\frac{5}{7} = \frac{7 \times 3 + 5}{7} = \frac{26}{7}$ $\quad 3\frac{2}{5} = \frac{5 \times 3 + 2}{5} = \frac{17}{5}$

Multiples: 7, __, 21, __, 35, …

5, 10, __, 20, __, 30, __, …

LCD = LCM = ____

$\frac{26}{7} \xrightarrow{\times \square} \frac{130}{\square}$ and $\frac{17}{5} \xrightarrow{\times 7} \frac{\square}{35}$

$\therefore 3\frac{5}{7} = \frac{130}{\square}$ and $3\frac{2}{5} = \frac{\square}{35}$

11 Convert each pair of fractions to equivalent fractions with the same denominator.

a $\frac{4}{5}$ and $\frac{7}{8}$ **b** $\frac{2}{9}$ and $\frac{3}{10}$ **c** $\frac{11}{6}$ and $\frac{2}{7}$

d $\frac{5}{12}$ and $\frac{5}{3}$ **e** $\frac{7}{10}$ and $\frac{3}{25}$ **f** $\frac{5}{6}$ and $\frac{4}{15}$

12 Convert each pair of mixed numerals to equivalent fractions with the same denominator.

a $1\frac{1}{3}$ and $1\frac{2}{5}$ **b** $2\frac{5}{6}$ and $1\frac{1}{4}$ **c** $1\frac{2}{7}$ and $2\frac{1}{3}$

d $1\frac{3}{4}$ and $1\frac{1}{2}$ **e** $1\frac{4}{5}$ and $2\frac{1}{2}$ **f** $2\frac{1}{6}$ and $4\frac{1}{3}$

Comparing fractions

EXAMPLE 5

Use a $<$ or $>$ sign to make each statement true.

a $\frac{2}{3}$ — $\frac{7}{9}$ **b** $\frac{3}{5}$ — $\frac{8}{15}$

$<$ means 'is less than' and $>$ means 'is greater than'.

a $\frac{2}{3}$ can be changed to ninths.

$\frac{2}{3} \xrightarrow{\times 3} \frac{6}{9}$

$\frac{6}{9} < \frac{7}{9}$ $\quad \therefore \frac{2}{3} < \frac{7}{9}$

b $\frac{3}{5}$ can be changed to fifteenths.

$\frac{3}{5} \xrightarrow{\times 3} \frac{9}{15}$

$\frac{9}{15} > \frac{8}{15}$ $\quad \therefore \frac{3}{5} > \frac{8}{15}$

13 Use a $<$ or $>$ sign to make each statement true.

a $\frac{2}{3} \text{ — } \frac{8}{15}$

$\frac{2}{3}$ can be changed to fifteenths.

$\frac{2}{3} \overset{\times 5}{=} \frac{\square}{15}$ (multiply by $\times \square$)

$\frac{10}{15} \text{ — } \frac{8}{15}$

$\therefore \frac{2}{3} \text{ — } \frac{8}{15}$

b $\frac{1}{4} \text{ — } \frac{5}{12}$

$\frac{1}{4}$ can be changed to twelfths.

$\frac{1}{4} \overset{\times \square}{=} \frac{\square}{12}$

$\frac{\square}{12} \text{ — } \frac{5}{12}$

$\therefore \frac{1}{4} \text{ — } \frac{5}{12}$

14 Use a $<$ or $>$ or $=$ sign to make each statement true.

a $\frac{2}{3} \text{ — } \frac{5}{12}$ **b** $\frac{1}{4} \text{ — } \frac{3}{8}$ **c** $\frac{3}{5} \text{ — } \frac{6}{20}$ **d** $\frac{2}{7} \text{ — } \frac{14}{49}$

e $\frac{6}{11} \text{ — } \frac{32}{44}$ **f** $\frac{4}{9} \text{ — } \frac{10}{27}$ **g** $\frac{3}{10} \text{ — } \frac{9}{30}$ **h** $\frac{1}{2} \text{ — } \frac{3}{8}$

i $\frac{8}{11} \text{ — } \frac{15}{22}$ **j** $\frac{4}{7} \text{ — } \frac{10}{21}$ **k** $\frac{2}{13} \text{ — } \frac{7}{52}$ **l** $\frac{5}{6} \text{ — } \frac{30}{36}$

C Simplifying fractions

A fraction is in its **simplest form** when the numerator and denominator have no **common factors** other than 1.

For example, $\frac{3}{5}, \frac{7}{9}, \frac{11}{13}$ are in simplest form.

The factors of a number are smaller whole numbers that can multiply together to make the larger number.

To reduce a fraction to its simplest form, follow these steps:

Step 1: List the **factors** of the numerator and denominator.

Step 2: Find the **highest common factor (HCF)** of the numerator and denominator.

Step 3: Simplify the fraction by dividing both numerator and denominator by the HCF.

EXAMPLE 1

Reduce $\frac{12}{18}$ to its simplest form.

List the factors. 12: 1, 2, 3, 4, 6, 12
18: 1, 2, 3, 6, 9, 18

HCF = 6

Simplify. $\frac{12}{18} \overset{\div 6}{=} \frac{2}{3}$

A common factor of two numbers is a factor they both have. The highest common factor (HCF) is the largest common factor.

Exercise 5C

1 a Complete the following to reduce $\frac{20}{25}$ to its simplest form.

i List the factors. 20: 1, 2, ___, 5, □, 20 and 25: 1, 5, ___

ii HCF = ___

iii Simplify. $\frac{20}{25} \overset{\div 5}{=} \frac{\square}{5}$

b Complete the following to reduce $\frac{12}{30}$ to its simplest form.

i List the factors. 12: 1, 2, ___, 4, 6, ___ and 30: 1, 2, 3, 5, 6, ___, ___, 30

ii HCF = ___

iii Simplify. $\frac{12}{30} \overset{\div \square}{=} \frac{2}{\square}$

2 List the factors of each number.

a 8	**b** 10	**c** 12	**d** 15	**e** 20
f 24	**g** 16	**h** 14	**i** 21	**j** 36
k 48	**l** 100	**m** 9	**n** 54	**o** 60

3 Use your answers from question **2** to list the HCF of each pair of numbers.

a 8 and 10	**b** 12 and 15	**c** 15 and 20	**d** 14 and 21	**e** 24 and 36
f 9 and 54	**g** 60 and 100	**h** 48 and 54	**i** 20 and 100	**j** 39 and 52

4 Use your answers from question **3** to simplify the following fractions.

a $\frac{8}{10}$	**b** $\frac{12}{15}$	**c** $\frac{15}{20}$	**d** $\frac{14}{21}$	**e** $\frac{24}{36}$
f $\frac{9}{54}$	**g** $\frac{60}{100}$	**h** $\frac{48}{54}$	**i** $\frac{20}{100}$	**j** $\frac{39}{52}$

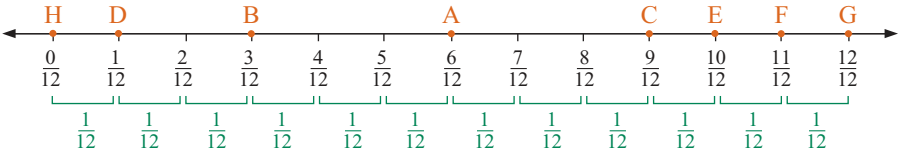
5 Simplify the following fractions.

a $\frac{5}{15}$	**b** $\frac{2}{10}$	**c** $\frac{6}{9}$	**d** $\frac{12}{14}$	**e** $\frac{15}{18}$
f $\frac{4}{10}$	**g** $\frac{6}{18}$	**h** $\frac{21}{24}$	**i** $\frac{30}{40}$	**j** $\frac{200}{800}$
k $\frac{8}{24}$	**l** $\frac{36}{48}$	**m** $\frac{7}{28}$	**n** $\frac{50}{200}$	**o** $\frac{16}{22}$

EXAMPLE 2

Determine the values plotted on the number line. Simplify all fractions where possible.

The number line has been divided into 12 equal parts; thus each section represents $\frac{1}{12}$.



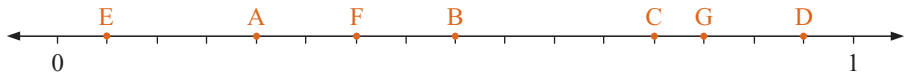
A $= \frac{6}{12} = \frac{1}{2}$ B $= \frac{3}{12} = \frac{1}{4}$ C $= \frac{9}{12} = \frac{3}{4}$ D $= \frac{1}{12}$

E $= \frac{10}{12} = \frac{5}{6}$ F $= \frac{11}{12}$ G $= \frac{12}{12} = 1$ H $= \frac{0}{12} = 0$

6 Determine the fractional value of each letter plotted on each number line. Simplify all fractions where possible.

a

b



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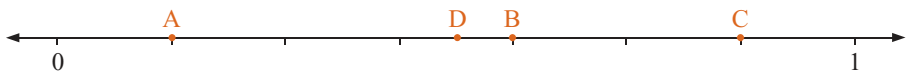

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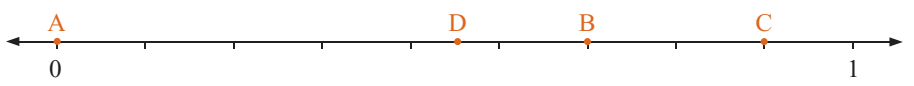


EXAMPLE 3

Think carefully for part **d**. *Can you halve each interval?*

Create a number line to plot the following fractions: $\frac{3}{8}, \frac{1}{2}, \frac{3}{4}, \frac{15}{16}$.

Step 1: Think of an appropriate scale.

Step 2: This can be done by finding the LCM of all denominators and creating equivalent fractions.

Step 3: The LCM of 8, 2, 4 and 16 is **16**.

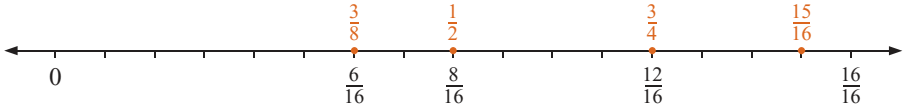
8: 8, **16**, 24, … 2: 2, 4, 6, 8, 10, 12, 14, **16**, 18, …

4: 4, 8, 12, **16**, 20, … 16: **16**, 32, 48, …

Step 4: Create a number line with 16 even intervals.

Step 5: Form equivalent fractions and plot the fractions.

$\frac{3}{8} \overset{\times 2}{=} \frac{6}{16}$ and $\frac{1}{2} \overset{\times 8}{=} \frac{8}{16}$ $\frac{3}{4} \overset{\times 4}{=} \frac{12}{16}$ $\frac{15}{16}$



7 Using an appropriate scale create a number line to plot the following fractional values. Form equivalent fractions by finding the LCM of the denominators.

a $\frac{3}{4}, \frac{1}{3}, \frac{7}{12}, \frac{5}{6}$ **b** $\frac{1}{2}, \frac{3}{4}, \frac{2}{5}, \frac{9}{10}$ **c** $\frac{2}{3}, \frac{4}{5}, \frac{5}{6}, \frac{1}{10}$ **d** $\frac{7}{9}, \frac{1}{2}, \frac{5}{6}, \frac{17}{18}, \frac{1}{4}$

D First quantity as a fraction of the second

Sometimes one quantity needs to be expressed as a fraction of a second quantity. To do this, follow these steps:

Step 1: Make the measurement units the same.

Step 2: Write the numbers as a fraction, with the first as the numerator and the second as the denominator.

Step 3: Simplify the fraction if necessary.

EXAMPLE 1

Using quick mental strategies, express the first quantity as a fraction of the second in simplest form.

a \$30, \$90 **b** 10 min, 1 h

a \$30 and \$90 have the same units.

$\frac{30}{90} = \frac{1}{3}$

$\therefore$ \$30 is $\frac{1}{3}$ of \$90.

b 10 min and 1 h have different units.

$\frac{10}{60} = \frac{1}{6}$ min

$\therefore$ 10 min is $\frac{1}{6}$ of 1 h.

Exercise 5D

1 Express the first quantity as a fraction of the second quantity using quick mental strategies.

a \$50, \$150	**b** 60 c, \$5	**c** 20 s, 3 min	**d** 40 min, 2 h
e 25 min, 1 h	**f** 750 m, 1 km	**g** 520 mL, 1.2 L	**h** 700 g, 1.4 kg

EXAMPLE 2

Express each first quantity as a fraction of the second quantity, in the simplest form.

a 5 km, 12 km **b** 24 min, 1 h

a 5 km and 12 km have the same units.

$\frac{5\text{ km}}{12\text{ km}} = \frac{5}{12}$

$\therefore$ 5 km is $\frac{5}{12}$ of 12 km.

b 24 min and 1 h have different units.

$\frac{24\text{ min}}{1\text{ h}} = \frac{24\text{ min}}{60\text{ min}} = \frac{24}{60}$

HCF = 12: $\frac{24}{60} \overset{\div 12}{=} \frac{2}{5}$

$\therefore$ 24 min is $\frac{2}{5}$ of 1 h.

2 Express the first quantity as a fraction of the second, in the simplest form.

a 4 kg, 9 kg	**b** 7 m, 11 m	**c** 2 L, 9 L	**d** 9 m, 14 m
e \$25, \$35	**f** 3GB, 6GB	**g** 4 km, 18 km	**h** 6 g, 21 g

3 Express the first quantity as a fraction of the second, in the simplest form.

a 20 min, 1 h	**b** 40c, \$1.20	**c** 80c, \$2.00	**d** 30 min, $1\frac{1}{2}$ h
e 600 g, 1 kg	**f** 900 m, 1.6 km	**g** 840 g, 1.4 kg	**h** 720 m, 2.1 km
i 64c, \$1.22	**j** 40 min, $2\frac{1}{4}$ h	**k** \$1.20, \$4.80	**l** 35 g, 0.085 kg

4 In the simplest form, what fraction is:

a \$1.10 of \$2.00?	**b** 80c of \$1.40?	**c** 500 m of 1.2 km?	**d** 400 m of 2.4 km?
e 600 g of 1.4 kg?	**f** 120 g of 3.6 kg?	**g** 55c of \$2.75?	**h** 126 m of 12.6 km?
i 480 mL of 1 L?	**j** 375 mL of 1.5 L?		

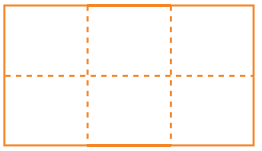
Concrete examples means examples using real objects.

Investigation 4 Using concrete materials to add and subtract fractions

1 Paper can easily be folded to solve fraction problems. For example: $\frac{1}{3} + \frac{1}{2} \neq \frac{2}{5}$, so what does it equal?

Step 1: Fold a piece of paper into thirds.

Step 2: Fold the same sheet of paper in half so that there are six equal parts.

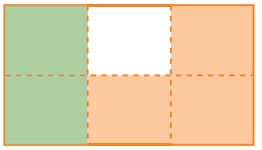


Step 3: Shade the sections.

$\frac{1}{3}$ is the same as $\frac{2}{6}$, so shade 2 squares.

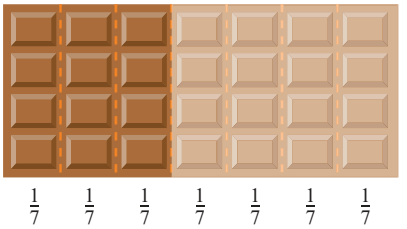
$\frac{1}{2}$ is the same as $\frac{3}{6}$, so shade 3 squares.

Thus we can see that $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$.

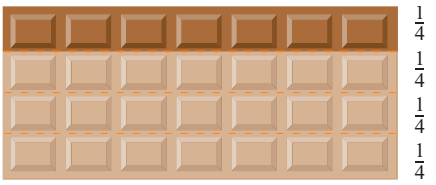


2 A block of chocolate could be used to show that $\frac{3}{7} + \frac{1}{4} \neq \frac{4}{11}$. So what does it equal?

Step 1: Consider a block of chocolate containing 28 pieces.
Divide the bar into 7 equal parts.
This shows that 4 squares of chocolate is equal to $\frac{1}{7}$ of the bar.
Shade $\frac{3}{7}$ of the bar. This is 12 pieces of chocolate.



Step 2: Divide the chocolate bar into 4 equal parts.
This shows that 7 squares of chocolate is equal to $\frac{1}{4}$ of the bar.
Shade $\frac{1}{4}$ of the bar. This is 7 pieces of chocolate.
So together $\frac{3}{7} + \frac{1}{4} = \frac{12 \text{ pieces} + 7 \text{ pieces}}{28 \text{ pieces}} = \frac{19}{28}$



3 Show how you could use concrete materials to explain the following fraction problems.

b $\frac{1}{5} + \frac{2}{3} \neq \frac{3}{8}$ **b** $\frac{2}{9} + \frac{1}{2} \neq \frac{3}{11}$ **c** $\frac{11}{12} - \frac{3}{4} \neq \frac{8}{8}$ or 1 **d** $\frac{7}{8} - \frac{2}{3} \neq \frac{5}{5}$ or 1

4 Create your own algorithm and use concrete materials to derive the answer.

E Addition and subtraction

You can add or subtract fractions *only if denominators are the same*.

EXAMPLE 1

Simplify the following.

a $\frac{2}{5} + \frac{1}{5}$ **b** $\frac{6}{8} - \frac{2}{8}$

a $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$ **b** $\frac{6}{8} - \frac{2}{8} = \frac{4}{8} = \frac{1}{2}$ (when simplified)

Exercise 5E

1 Complete to simplify the following.

a $\frac{5}{13} + \frac{6}{13} = \frac{\square}{13}$ **b** $\frac{14}{17} - \frac{9}{17} = \frac{5}{\square}$

c $\frac{8}{10} - \frac{6}{10} = \frac{2}{10} = \frac{\square}{5}$ (when simplified) **d** $\frac{8}{14} + \frac{2}{14} = \frac{\square}{\square} = \frac{5}{\square}$ (when simplified)

2 Simplify the following.

a $\frac{7}{10} + \frac{2}{10}$ **b** $\frac{5}{7} + \frac{1}{7}$ **c** $\frac{3}{12} + \frac{4}{12}$ **d** $\frac{6}{9} + \frac{2}{9}$

e $\frac{1}{6} + \frac{1}{6}$ **f** $\frac{3}{15} + \frac{9}{15}$ **g** $\frac{10}{17} + \frac{4}{17}$ **h** $\frac{13}{18} + \frac{2}{18}$

i $\frac{15}{20} - \frac{5}{20}$ **j** $\frac{8}{10} - \frac{2}{10}$ **k** $\frac{11}{18} - \frac{7}{18}$ **l** $\frac{29}{32} - \frac{13}{32}$

m $\frac{13}{21} - \frac{6}{21}$ **n** $\frac{26}{43} - \frac{9}{43}$ **o** $\frac{42}{57} - \frac{27}{57}$ **p** $\frac{19}{20} - \frac{16}{20}$

Different denominators

If denominators are different, they need to be made the same by forming equivalent fractions using the **lowest common denominator (LCD)**. This is the same as the lowest common multiple (LCM) of the denominators.

When adding or subtracting fractions with different denominators, follow these steps:

Step 1: Find the LCD.

Step 2: Form equivalent fractions with the same denominator.

Step 3: Evaluate by adding or subtracting the numerators.

Step 4: Simplify if necessary.

EXAMPLE 2

Simplify the following.

a $\frac{2}{3} + \frac{1}{4}$ **b** $\frac{7}{9} - \frac{1}{5}$

a $\frac{2}{3} + \frac{1}{4}$

LCD = 12

$\frac{2}{3} = \frac{8}{12}$ (×4 top and bottom) and $\frac{1}{4} = \frac{3}{12}$ (×3 top and bottom)

$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

b $\frac{7}{9} - \frac{1}{5}$

LCD = 45

$\frac{7}{9} = \frac{35}{45}$ (×5 top and bottom) and $\frac{1}{5} = \frac{9}{45}$ (×9 top and bottom)

$\frac{35}{45} - \frac{9}{45} = \frac{26}{45}$

3 Complete to simplify the following.

a $\frac{1}{4} + \frac{2}{5}$

LCD = ___

$\frac{1}{4} = \frac{\square}{20}$ (×□ top, ×5 bottom) and $\frac{2}{5} = \frac{8}{\square}$ (×4 top, ×□ bottom)

$\frac{\square}{20} + \frac{8}{\square} = \frac{\square}{\square}$

b $\frac{2}{3} - \frac{1}{2}$

LCD = ___

$\frac{2}{3} = \frac{\square}{\square}$ (×□ top and bottom) and $\frac{1}{2} = \frac{\square}{\square}$ (×□ top and bottom)

$\frac{\square}{\square} - \frac{3}{6} = \frac{\square}{6}$

4 Simplify the following.

a $\frac{1}{3} + \frac{1}{2}$ **b** $\frac{1}{3} + \frac{2}{5}$ **c** $\frac{1}{4} + \frac{3}{7}$ **d** $\frac{4}{9} + \frac{2}{5}$

e $\frac{2}{7} + \frac{3}{8}$ **f** $\frac{3}{5} + \frac{1}{7}$ **g** $\frac{1}{6} + \frac{2}{3}$ **h** $\frac{3}{14} + \frac{3}{7}$

i $\frac{3}{4} - \frac{1}{3}$ **j** $\frac{7}{10} - \frac{1}{5}$ **k** $\frac{6}{8} - \frac{1}{4}$ **l** $\frac{2}{3} - \frac{1}{2}$

m $\frac{4}{5} - \frac{1}{6}$ **n** $\frac{11}{12} - \frac{3}{4}$ **o** $\frac{5}{9} - \frac{1}{3}$ **p** $\frac{3}{5} - \frac{1}{3}$

5 Simplify the following, expressing your answers as mixed numerals.

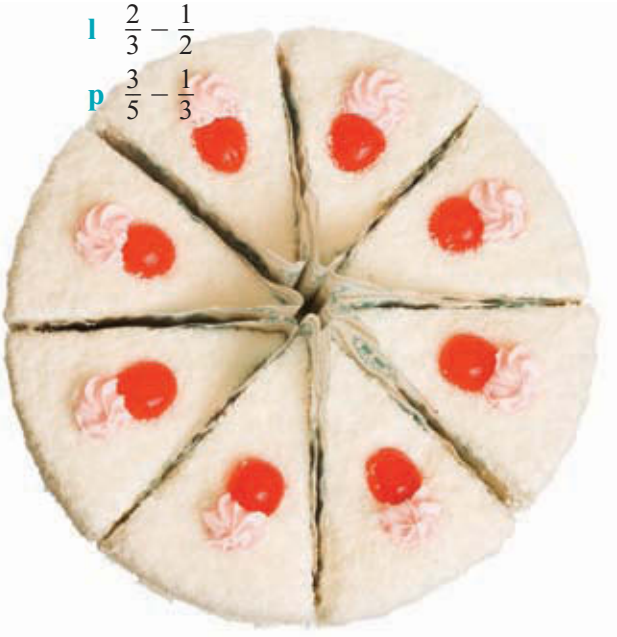
a $\frac{1}{2} + \frac{3}{4}$ **b** $\frac{2}{5} + \frac{2}{3}$

c $\frac{5}{9} + \frac{2}{3}$ **d** $\frac{5}{6} + \frac{3}{4}$

6 Rod and Ace shared a cake. Rod ate $\frac{1}{8}$ and Ace ate $\frac{1}{2}$.

a What fraction was eaten?

b What fraction remained?



7 Ria owns $\frac{1}{4}$ of a business. Delta owns $\frac{1}{5}$ and Alicia owns $\frac{1}{2}$. The bank owns the rest.

a Who owns the largest proportion of the business?

b Who owns the smallest?

c What fraction does the bank own?

EXAMPLE 3

Simplify the following.

a $3 - \frac{2}{3}$ **b** $6 - \frac{5}{9}$

a $3 - \frac{2}{3} = 2 + (1 - \frac{2}{3})$

$= 2 + \frac{1}{3} = 2\frac{1}{3}$

b $6 - \frac{5}{9} = 5 + (1 - \frac{5}{9})$

$= 5 + \frac{4}{9} = 5\frac{4}{9}$

8 Complete the following.

a $4 - \frac{1}{3} = 3 + (\square - \frac{1}{3})$

$= \square + \frac{\square}{3} = \square\frac{\square}{3}$

b $2 - \frac{4}{5} = \square + (1 - \frac{\square}{5})$

$= \square + \frac{\square}{5} = \square\frac{\square}{5}$

9 Simplify the following.

a $10 - \frac{4}{9}$ **b** $7 - \frac{6}{7}$ **c** $8 - \frac{3}{5}$ **d** $9 - \frac{2}{3}$ **e** $3 - \frac{8}{9}$ **f** $12 - \frac{2}{11}$

g $6 - \frac{4}{5}$ **h** $5 - \frac{3}{4}$ **i** $4 - \frac{7}{8}$ **j** $2 - \frac{3}{10}$ **k** $17 - \frac{11}{15}$ **l** $18 - \frac{14}{17}$

Mixed numerals

When adding or subtracting mixed numerals, follow these steps:

Step 1: Change to improper fractions.

Step 2: Find the LCD.

Step 3: Form equivalent fractions with the same denominator.

Step 4: Evaluate by adding or subtracting the numerators.

Step 5: Simplify to a mixed numeral if necessary.

A mixed numeral is simpler than an improper fraction.

EXAMPLE 4

Simplify the following.

a $4\frac{1}{2} + 3\frac{1}{5}$ **b** $6\frac{1}{3} - 5\frac{4}{5}$

a $4\frac{1}{2} + 3\frac{1}{5} = \frac{9}{2} + \frac{16}{5}$

LCD = 10

$\frac{9}{2} \xrightarrow{\times 5} \frac{45}{10}$ and $\frac{16}{5} \xrightarrow{\times 2} \frac{32}{10}$

$\frac{45}{10} + \frac{32}{10} = \frac{77}{10} = 7\frac{7}{10}$

b $6\frac{1}{3} - 5\frac{4}{5} = \frac{19}{3} - \frac{29}{5}$

LCD = 15

$\frac{19}{3} \xrightarrow{\times 5} \frac{95}{15}$ and $\frac{29}{5} \xrightarrow{\times 3} \frac{87}{15}$

$\frac{95}{15} - \frac{87}{15} = \frac{8}{15}$

10 Complete the following.

a $3\frac{1}{3} + 2\frac{2}{5} = \frac{10}{\square} + \frac{12}{5}$

LCD = ____

$\frac{10}{\square} \xrightarrow{\times 5} \frac{50}{\square}$ and $\frac{12}{5} \xrightarrow{\times 3} \frac{\square}{\square}$

$\frac{50}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = 5\frac{11}{\square}$

b $4\frac{1}{4} - 1\frac{1}{2} = \frac{\square}{4} - \frac{3}{2}$

LCD = ____

$\frac{3}{2} \xrightarrow{\times 2} \frac{\square}{\square}$

$\frac{\square}{4} - \frac{\square}{\square} = \frac{11}{\square} = \square\frac{3}{\square}$

11 Simplify the following, showing all working.

a $3\frac{1}{4} + 2\frac{1}{2}$ **b** $2\frac{1}{5} + 6\frac{1}{4}$ **c** $1\frac{1}{2} + 3\frac{2}{3}$ **d** $4\frac{2}{3} + 5\frac{3}{4}$

e $6\frac{2}{5} + 1\frac{1}{3}$ **f** $1\frac{5}{6} + 4\frac{1}{2}$ **g** $5\frac{7}{8} + 2\frac{1}{3}$ **h** $6\frac{3}{7} + 3\frac{2}{5}$

i $4\frac{1}{4} - 2\frac{1}{2}$ **j** $6\frac{1}{3} - 2\frac{2}{5}$ **k** $7\frac{1}{2} - 3\frac{1}{3}$ **l** $8\frac{2}{5} - 6\frac{1}{6}$

m $3\frac{1}{2} - 1\frac{2}{3}$ **n** $4\frac{4}{5} - 1\frac{3}{7}$ **o** $6\frac{1}{2} - 2\frac{4}{9}$ **p** $11\frac{1}{3} - 8\frac{1}{4}$

12 Venus and Arian went berry picking. Venus picked $3\frac{1}{3}$ buckets of berries and Arian picked $5\frac{4}{5}$ buckets.

a How many buckets of berries do they have altogether?

b If they need 11 full buckets, how many more buckets of berries do they need?

13 In a race, a cyclist travels $\frac{3}{8}$ of the distance in the first hour and $\frac{1}{3}$ of the distance in the second hour.

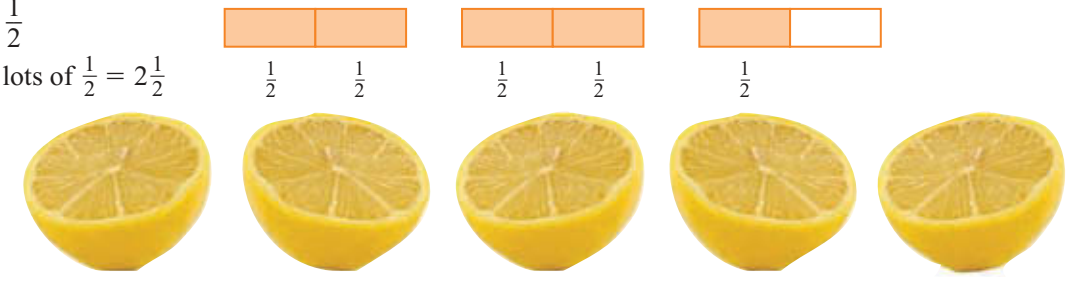
a What fraction of the total distance has he travelled?

b What fraction of the total distance does he need to travel in the third hour to complete the race?

Investigation 5 Multiplication of fractions

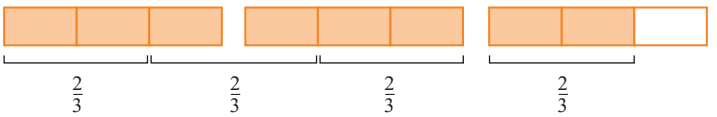
1 a $5 \times \frac{1}{2}$

$\therefore$ 5 lots of $\frac{1}{2} = 2\frac{1}{2}$



b $4 \times \frac{2}{3}$

$\therefore$ 4 lots of $\frac{2}{3} = 2\frac{2}{3}$



2 Construct diagrams to show the following.

a $6 \times \frac{4}{5}$ **b** $9 \times \frac{1}{3}$ **c** $2 \times \frac{8}{9}$ **d** $3 \times \frac{3}{7}$

F Multiplication of fractions

For multiplication, the denominators do not have to be the same. Multiply the numerators and multiply the denominators.

EXAMPLE 1

Simplify the following.

a $5 \times \frac{1}{3}$ **b** $\frac{7}{8} \times \frac{4}{5}$

a $5 \times \frac{1}{3} = \frac{5}{1} \times \frac{1}{3}$

$= \frac{5}{3}$ or $1\frac{2}{3}$

b $\frac{7}{{}_{2}\cancel{8}} \times \frac{\cancel{4}^{1}}{5} = \frac{7}{2} \times \frac{1}{5}$

$= \frac{7}{10}$

5 is the same as $\frac{5}{1}$.

Cancelling at the start makes it a lot easier.

Exercise 5F

1 Complete to find the solution.

a $7 \times \frac{1}{2} = \frac{7}{1} \times \frac{1}{2}$

$= \frac{\square}{2}$ or $3\frac{\square}{2}$

b $\frac{3}{5} \times \frac{1}{4} = \frac{\square}{\square}$

c $\frac{7}{12} \times \frac{4}{5} = \frac{7}{{}_{3}\cancel{12}} \times \frac{\cancel{4}^{1}}{5}$

$= \frac{7}{\square}$

2 Simplify the following.

a $4 \times \frac{4}{9}$ **b** $7 \times \frac{5}{8}$ **c** $9 \times \frac{4}{5}$ **d** $\frac{2}{7} \times 3$ **e** $2 \times \frac{7}{8}$

f $6 \times \frac{1}{3}$ **g** $5 \times \frac{4}{7}$ **h** $8 \times \frac{6}{11}$ **i** $\frac{7}{9} \times \frac{4}{7}$ **j** $\frac{8}{9} \times \frac{9}{10}$

k $\frac{1}{6} \times \frac{6}{7}$ **l** $\frac{4}{5} \times \frac{2}{3}$ **m** $\frac{3}{5} \times \frac{4}{7}$ **n** $\frac{1}{12} \times \frac{2}{9}$ **o** $\frac{7}{8} \times \frac{2}{3}$

EXAMPLE 2

Simplify $4\frac{3}{4} \times 1\frac{1}{5}$.

$4\frac{3}{4} = \frac{4 \times 4 + 3}{4} = \frac{19}{4}$ and $1\frac{1}{5} = \frac{5 \times 1 + 1}{5} = \frac{6}{5}$ *Step 1:* Change to improper fractions.

$4\frac{3}{4} \times 1\frac{1}{5} = \frac{19}{{}_{2}\cancel{4}} \times \frac{\cancel{6}^{3}}{5}$ *Step 2:* Cancel where possible.

$= \frac{57}{10}$ *Step 3:* Multiply numerators, multiply denominators.

$= 5\frac{7}{10}$ *Step 4:* Simplify to a mixed numeral.

3 Complete the following.

a $5\frac{1}{2} \times 1\frac{1}{3} = \frac{11}{\square} \times \frac{\square}{3}$

$= \frac{\square}{6}$

$= 7\frac{\square}{6} = 7\frac{\square}{3}$

b $4\frac{2}{5} \times 3 = \frac{22}{\square} \times \frac{3}{1}$

$= \frac{\square}{\square}$

$= 13\frac{1}{\square}$

4 Simplify the following, expressing your answers as mixed numerals.

a $1\frac{1}{2} \times 3\frac{2}{5}$ **b** $2\frac{1}{4} \times 6\frac{1}{3}$ **c** $4\frac{2}{5} \times 1\frac{1}{2}$ **d** $3\frac{1}{3} \times 5\frac{4}{5}$

e $\frac{5}{9} \times 7\frac{2}{3}$ **f** $8\frac{1}{2} \times 1\frac{3}{7}$ **g** $\frac{2}{3} \times 6\frac{1}{2}$ **h** $10\frac{1}{7} \times 1\frac{1}{2}$

i $8 \times 6\frac{1}{4}$ **j** $2 \times 2\frac{5}{8}$ **k** $3\frac{4}{5} \times 1\frac{2}{3}$ **l** $8\frac{4}{5} \times 2$

5 Seven bottles each contain $2\frac{1}{2}$ L of water. How many litres of water are there altogether?

6 Belinda, Elisha, Suzannah and Kool each purchased $5\frac{1}{4}$ m of fabric. How much fabric did they purchase in total?

7 Simplify the following.

a $\frac{3}{4}$ of 16 m **b** $\frac{2}{5}$ of 40 kg

c $\frac{4}{5}$ of 100 L **d** $\frac{6}{7}$ of 35

e $\frac{2}{9}$ of \$45 **f** $\frac{11}{12}$ of 36 km

g $\frac{3}{5}$ of \$60 **h** $\frac{4}{7}$ of 105 kg

i $\frac{9}{10}$ of 140 L **j** $\frac{1}{3}$ of 120

k $\frac{2}{7}$ of 56 m **l** $\frac{8}{9}$ of 720

8 Rod purchased a piece of rope that was $2\frac{1}{4}$ m in length. He realised that he required only $\frac{2}{3}$ of this length.

a How much rope did Rod require? **b** How much excess rope did he purchase?

9 A bottle of drinking water contained 600 mL. Carissa drank $\frac{5}{8}$ of the water.

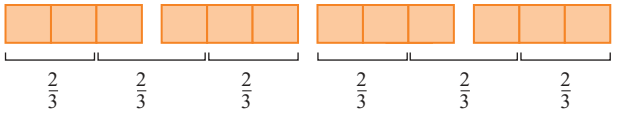
a How much water did Carissa drink? **b** How much water remains?

Investigation 6 Fraction division

1 $4 \div \frac{2}{3}$

There are 4 wholes.

Make each whole into thirds.



There are 12 thirds.

There are 6 lots of $\frac{2}{3}$. $\therefore 4 \div \frac{2}{3} = 6$

2 Construct diagrams to show the following.

a $3 \div \frac{1}{4}$ **b** $4 \div \frac{4}{7}$ **c** $6 \div 3.5$ **d** $5 \div \frac{5}{6}$

G Fraction division

Reciprocals are numbers that when multiplied together equal 1.

When a number is written in fraction form, the reciprocal is formed when the fraction is tipped 'upside down'.

Thus the reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$ or 4. It therefore follows that the reciprocal of 4 is $\frac{1}{4}$.

EXAMPLE 1

Find the reciprocal of each number.

a $\frac{3}{8}$ **b** 5 **c** $2\frac{3}{4}$

Change a mixed numeral to an improper fraction first.

a $\frac{3}{8} \times \frac{8}{3}$
$= \frac{24}{24}$
$= 1$
$\therefore$ Reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$.

b $\frac{5}{1} \times \frac{1}{5}$
$= \frac{5}{5}$
$= 1$
$\therefore$ Reciprocal of 5 is $\frac{1}{5}$.

c $2\frac{3}{4} = \frac{11}{4}$
$\frac{11}{4} \times \frac{4}{11} = \frac{44}{44}$
$= 1$
$\therefore$ Reciprocal of $2\frac{3}{4}$ (or $\frac{11}{4}$) is $\frac{4}{11}$.

Exercise 5G

1 Complete to give the reciprocal of each number.

a $\frac{5}{8} = \frac{8}{\square}$ **b** $\frac{4}{9} = \frac{\square}{4}$ **c** $\frac{12}{13} = \frac{13}{\square}$

d $\frac{9}{7} = \frac{\square}{9}$ **e** $5 = \frac{1}{\square}$ **f** $\frac{1}{2} = \frac{\square}{1}$

2 Give the reciprocal of each number.

a $\frac{4}{7}$ **b** $\frac{2}{3}$ **c** $\frac{1}{5}$ **d** $\frac{17}{10}$ **e** $\frac{14}{9}$

f 8 **g** 3 **h** 57 **i** $1\frac{1}{2}$ **j** $4\frac{2}{3}$

EXAMPLE 2

Simplify the following.

a $7 \div \frac{1}{3}$ **b** $\frac{3}{4} \div \frac{1}{5}$ **c** $2\frac{1}{3} \div 2\frac{4}{5}$

a $7 \div \frac{1}{3}$
$= \frac{7}{1} \times \frac{3}{1}$
$= \frac{21}{1}$
$= 21$

b $\frac{3}{4} \div \frac{1}{5}$
$= \frac{3}{4} \times \frac{5}{1}$
$= \frac{15}{4}$
$= 3\frac{3}{4}$

c $2\frac{1}{3} \div 2\frac{4}{5}$
$= \frac{7}{3} \div \frac{14}{5}$
$= \frac{1\cancel{7}}{3} \times \frac{5}{\cancel{14}_{2}}$
$= \frac{5}{6}$

Remember to cancel if possible.

Steps for division

Consider part **a** in Example 2: $7 \div \frac{1}{3}$.

So $7 \div \frac{1}{3}$ is the same as 7×3.

Seven wholes are each divided into thirds. The result is 21 equal parts.

So to divide two fractions, follow these steps:

Step 1: Rewrite mixed numerals as improper fractions.

Step 2: Make a reciprocal of the second fraction.

Step 3: Change the division sign to a multiplication sign.

Step 4: Solve as a multiplication: that is, multiply numerators, multiply denominators.

Step 5: Simplify.

3 Complete the following.

a $4 \div \frac{2}{5} = \frac{4}{1} \times \frac{5}{\square}$

$= \frac{\square}{2} = \square$

b $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \square \frac{2}{1}$

$= \frac{\square}{\square}$

$= 1\frac{2}{4} = 1\frac{\square}{2}$

c $3\frac{1}{4} \div 1\frac{2}{3} = 3\frac{1}{4} \div 1\frac{2}{3}$

$= \frac{\square}{4} \div \frac{5}{\square}$

$= \frac{\square}{4} \times \frac{\square}{5}$

$= \frac{\square}{20} = 1\frac{\square}{20}$

4 Simplify the following.

a $6 \div \frac{3}{5}$ **b** $5 \div \frac{3}{8}$ **c** $2 \div \frac{1}{3}$ **d** $7 \div \frac{6}{7}$ **e** $8 \div \frac{6}{11}$

f $4 \div \frac{2}{5}$ **g** $\frac{1}{2} \div \frac{3}{8}$ **h** $\frac{4}{9} \div \frac{1}{3}$ **i** $\frac{2}{5} \div \frac{1}{4}$ **j** $\frac{5}{12} \div \frac{1}{6}$

k $\frac{4}{7} \div \frac{1}{3}$ **l** $\frac{9}{14} \div \frac{1}{3}$ **m** $\frac{8}{9} \div \frac{2}{3}$ **n** $8 \div 2\frac{1}{4}$ **o** $5 \div 1\frac{1}{2}$

p $6 \div 2\frac{1}{3}$ **q** $1\frac{1}{2} \div 1\frac{2}{7}$ **r** $2\frac{2}{3} \div 1\frac{1}{5}$ **s** $3\frac{3}{7} \div 1\frac{1}{2}$ **t** $7\frac{1}{2} \div 2\frac{1}{3}$

5 The stride length of a galloping racehorse is about $7\frac{1}{2}$ m. How many strides would a horse take in race that is $2\frac{2}{5}$ km long?

6 A piece of fabric is $2\frac{2}{3}$ m in length. How many scarves can Rachel make if she requires $\frac{2}{9}$ m for each scarf?

Investigation 7 Using concrete examples to solve fractions

Blocks, buttons, matches or other real objects can be used to solve fraction problems.

1 $4 \div \frac{4}{9} = 4 \times \frac{9}{4}$. Is this statement correct? Use concrete examples to explain your findings.

a

1 whole $= \frac{9}{9}$. There are 4 wholes represented in the diagram above.

Dividing 4 wholes by $\frac{4}{9}$ gives 9 lots of $\frac{4}{9}$.

$\therefore 4 \div \frac{4}{9} = 9$

b

The diagram shows 4 lots of 9 quarters. In total there are 36 quarters: $\frac{36}{4} = 9$.

$\therefore 4 \times \frac{9}{4} = 9$

Using concrete examples it is shown that $4 \div \frac{4}{9} = 4 \times \frac{9}{4}$.

2 $5 \div \frac{1}{2} = 5 \times \frac{2}{1}$. Is this statement correct? Use concrete examples to explain your findings.

3 $3 \div \frac{3}{5} = 3 \times \frac{5}{3}$. Is this statement correct? Use concrete examples to explain your findings.

4 What can be said about multiplication and division of fractions from the above examples?

H Calculator skills

Your calculator can be a useful tool when working with fractions and decimals. Calculators vary, so make sure you are familiar with the functions on your calculator.

SHIFT allows you to access other functions on your calculator

$\square\frac{\square}{\square}$ numerator / denominator

SHIFT $\square\frac{\square}{\square}$ mixed numeral

SHIFT <=> allows conversion of an improper fraction to a mixed numeral

<=> allows fractions to be expressed in decimal form

Inserting a fraction followed by = simplifies the fraction.

EXAMPLE 1

Simplify the following fractions using your calculator.

a $\frac{15}{27}$ **b** $\frac{33}{84}$ **c** $\frac{342}{698}$

a $\frac{15}{27} = \frac{5}{9}$

$\square\frac{\square}{\square}$ 1 5 ▼ 2 7 =

b $\frac{33}{84} = \frac{11}{28}$

$\square\frac{\square}{\square}$ 3 3 ▼ 8 4 =

c $\frac{342}{698} = \frac{171}{349}$

$\square\frac{\square}{\square}$ 3 4 2 ▼ 6 9 8 =

Exercise 5H

1 Simplify these fractions.

a $\frac{18}{34}$ **b** $\frac{21}{96}$ **c** $\frac{15}{114}$ **d** $\frac{49}{161}$

e $\frac{28}{244}$ **f** $\frac{66}{234}$ **g** $\frac{98}{126}$ **h** $\frac{175}{215}$

EXAMPLE 2

Express each mixed numeral as an improper fraction.

a $4\frac{9}{11}$ **b** $12\frac{3}{7}$

a $4\frac{9}{11} = \frac{53}{11}$

SHIFT $\square\!\!\square$ 4 ▼ 9 ▼ 11 =

b $12\frac{3}{7} = \frac{87}{7}$

SHIFT $\square\!\!\square$ 1 2 ▼ 3 ▼ 7 =

2 Express each mixed numeral as an improper fraction.

a $7\frac{4}{9}$ **b** $15\frac{7}{8}$ **c** $23\frac{4}{5}$ **d** $82\frac{3}{14}$

e $93\frac{7}{17}$ **f** $65\frac{45}{52}$ **g** $161\frac{18}{31}$ **h** $137\frac{4}{27}$

EXAMPLE 3

Express each improper fraction as a mixed numeral.

a $\frac{9}{4}$ **b** $\frac{154}{9}$

a $\frac{9}{4} = 2\frac{1}{4}$

$\square\!\!\square$ 9 ▼ 4 = SHIFT ⇔

b $\frac{154}{9} = 17\frac{1}{9}$

$\square\!\!\square$ 1 5 4 ▼ 9 = SHIFT ⇔

3 Express each improper fraction as a mixed numeral.

a $\frac{18}{5}$ **b** $\frac{23}{3}$ **c** $\frac{71}{12}$ **d** $\frac{184}{15}$

e $\frac{237}{34}$ **f** $\frac{104}{18}$ **g** $\frac{366}{50}$ **h** $\frac{729}{46}$

EXAMPLE 4

Express the following as decimals.

a $\frac{4}{5}$ **b** $\frac{23}{7}$ **c** $8\frac{4}{9}$

a $\frac{4}{5} = 0.8$

$\square\!\!\square$ 4 ▼ 5 = ⇔

b $\frac{23}{7} = 3.29$ (rounded to 2 decimal places)

$\square\!\!\square$ 2 3 ▼ 7 = ⇔

c $8\frac{4}{9} = 8.\dot{4}$ (8.444… is expressed as $8.\dot{4}$)

SHIFT $\square\!\!\square$ 8 ▼ 4 ▼ 9 = ⇔

4 Express each fraction as a decimal.

a $\frac{6}{15}$ **b** $\frac{18}{24}$ **c** $\frac{24}{40}$ **d** $\frac{46}{80}$

5 Express these fractions as decimals correct to 2 decimal places.

a $\frac{56}{62}$ **b** $\frac{94}{13}$ **c** $12\frac{13}{15}$ **d** $\frac{18}{52}$

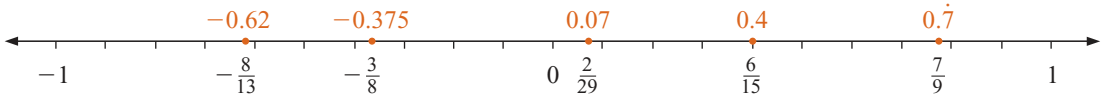
e $8\frac{17}{36}$ **f** $\frac{137}{14}$ **g** $\frac{31}{43}$ **h** $\frac{108}{19}$

EXAMPLE 5

Convert each fraction to a decimal and plot on a number line: $\frac{7}{9}$, $-\frac{3}{8}$, $\frac{6}{15}$, $-\frac{8}{13}$, $\frac{2}{29}$

$\frac{7}{9} = 0.\dot{7}$, $-\frac{3}{8} = -0.375$, $\frac{6}{15} = 0.4$, $-\frac{8}{13} \approx -0.62$, $\frac{2}{29} \approx 0.07$

The plotted values are in ascending order from left to right



6 Convert these sets of fractions to decimals correct to 2 decimal places, then plot each set on a number line.

a $\frac{6}{13}, \frac{8}{11}, -\frac{12}{19}, \frac{3}{10}, -\frac{17}{20}$ **b** $\frac{3}{14}, -\frac{6}{7}, \frac{17}{21}, -\frac{32}{35}, \frac{48}{72}$

c $\frac{41}{87}, -\frac{61}{65}, -\frac{1}{8}, \frac{32}{55}, \frac{29}{41}$ **d** $-\frac{25}{29}, \frac{2}{11}, \frac{14}{15}, -\frac{17}{33}, \frac{18}{47}$

7 Refer to the number lines in question **6** to complete the following statement by inserting a $>$ or $<$ sign.

a **i** $\frac{6}{13} \_\_ \frac{3}{10}$ **ii** $-\frac{17}{20} \_\_ -\frac{12}{19}$

b **i** $\frac{17}{21} \_\_ \frac{48}{72}$ **ii** $\frac{3}{14} \_\_ -\frac{32}{35}$

c **i** $-\frac{1}{8} \_\_ -\frac{61}{65}$ **ii** $\frac{29}{41} \_\_ \frac{41}{87}$

d **i** $-\frac{17}{33} \_\_ -\frac{25}{29}$ **ii** $\frac{2}{11} \_\_ \frac{18}{47}$

8 Use your calculator to solve these problems. Express the answers in fraction form (mixed numerals).

a $3\frac{1}{5} + 7\frac{2}{3}$ **b** $14\frac{7}{12} + 6\frac{4}{9}$ **c** $-11\frac{3}{4} + 10\frac{4}{11}$ **d** $5\frac{1}{4} - 20\frac{6}{7}$

e $6 - 2\frac{13}{15}$ **f** $-10\frac{3}{7} - 2\frac{5}{9}$ **g** $15\frac{11}{12} - 7\frac{3}{4}$ **h** $-13\frac{21}{25} - 9\frac{5}{13}$

i $7 \times \frac{2}{3}$ **j** $-\frac{4}{9} \times 16$ **k** $-\frac{10}{11} \times -\frac{3}{5}$ **l** $10\frac{3}{8} \times -2\frac{1}{2}$

m $-\frac{7}{10} \div \frac{1}{3}$ **n** $-15\frac{1}{2} \div -3\frac{2}{3}$ **o** $7\frac{4}{9} \div \frac{8}{11}$ **p** $-18\frac{3}{4} \div 2\frac{4}{5}$

9 **a** Carl purchased three pieces of timber measuring $\frac{3}{5}$ m, $\frac{11}{12}$ m and $\frac{1}{4}$ m respectively.

i Calculate the total length of these pieces of timber.

ii If the total length of timber Carl needs is 2 m, what length of timber is still required? (Answer to the nearest cm.)

b On Sunday morning Laura began her walk from home. She walked $3\frac{1}{5}$ km south then had a break. She walked a further $2\frac{9}{10}$ km south, before realising she was going in the wrong direction. She then walked $4\frac{7}{8}$ km north. How far away is she from home? (Answer in m.)

c John has three bags of seed weighing $12\frac{1}{7}$ kg, two bags weighing $18\frac{1}{4}$ kg and another weighing $26\frac{11}{12}$ kg.

i What is the total weight of the seed?

ii If he shares the seed between five farmers, how much does each receive? (Answer to the nearest kg.)

Language in mathematics

Too much money

Many people would not complain about being in John Willey's shoes – except maybe John Willey! Over his 73 years John, or 'Jack' to his mates, has acquired a massive fortune from his highly profitable mining company, which he started in 1964.

His enormous wealth has caused Jack immense concern. Already enjoying a lavish lifestyle, Jack is intent on giving away 8.5 million dollars of his wealth for the benefit of others. His extended family, as well as a number of notable charities, are to be the recipients.

Below is a list of lucky people and charities along with the fraction of Jack's 8.5 million dollars that they are set to acquire.

The 'Jack' Willey benefit fund

1 Allan Smith $\frac{1}{20}$	**2** Beatrice Xuereb $\frac{1}{50}$	**3** Cardiac Research $\frac{3}{40}$
4 Maria Smith $\frac{1}{20}$	**5** Gordon Galea $\frac{3}{50}$	**6** Children's Hospital $\frac{3}{40}$
7 Betty Willey $\frac{1}{20}$	**8** Michelle Admen $\frac{3}{25}$	**9** Sports Clinic Universal $\frac{1}{20}$
10 Sam Willey $\frac{1}{10}$	**11** Bruno Gossling $\frac{1}{5}$	**12** Carmela Willey $\frac{1}{20}$
13 Archmid Bzolvick $\frac{1}{10}$		

1 Complete the following questions using the information contained above.

- **a** What is John Willey's name among his mates?
- **b** How old is John Willey?
- **c** By what means has John Willey made his fortune?
- **d** Find three words in the passage that mean 'extremely big'.
- **e** What word from the passage means all the individuals and charities that will receive a benefit?
- **f** Write '8.5 million dollars' in numbers.
- **g** Use your knowledge of fractions to calculate how much each individual and charity listed will receive.
- **h** **i** What is the total amount to be given to charities?
  ii What fraction is this?
- **i** Who will receive the greatest proportion of John Willey's benefit fund?
- **j** 8.5 million dollars is just $\frac{1}{200}$ of John Willey's total wealth. Calculate his total wealth.

2 **a** Imagine that you win 1.5 million dollars. Write this amount as a numeral.

b If you invest $\frac{3}{8}$ of the money, how much do you invest?

c If you spend $\frac{1}{4}$ of the money purchasing a home, how much does the home cost?

d If you give $\frac{1}{5}$ of the winnings to a favourite aunt, how much does the aunt receive?

e If you donate $\frac{1}{20}$ of the money to charity, how much do you donate?

f Suppose that you use the money as in parts **b** to **e**, and place the remainder in a savings account.

i What is the remainder as a fraction?

ii What amount do you place in the savings account?

3 Write a creative newspaper report that includes the use of fractions. For example:
'Sporting Event – Athlete reduces personal best time by $\frac{4}{5}$ of a second!'
'Boy genius can evaluate fractions in seconds!'

Terms

ascending	convert	denominator	descending	equivalent	factor
fraction	highest common factor (HCF)		improper fraction	lowest common denominator (LCD)	
lowest common multiple (LCM)			mixed numeral	number line	numerator
proper fraction	reciprocal	simplify	vinculum		

Check your skills

1 Name the denominator in $16\frac{4}{9}$.

A 1 **B** 16 **C** 4 **D** 9

2 How many fifths in 25?

A 5 **B** 25 **C** 100 **D** 125

3 Express $\frac{47}{3}$ as a mixed numeral.

A $15\frac{2}{3}$ **B** $15\frac{1}{3}$ **C** $16\frac{1}{3}$ **D** $16\frac{2}{3}$

4 Express $12\frac{1}{4}$ as an improper fraction.

A $\frac{17}{4}$ **B** $\frac{49}{4}$ **C** $\frac{37}{4}$ **D** $\frac{46}{4}$

5 What sign could be inserted to make the following true: $\frac{31}{3}$ ____ 11?

A = **B** < **C** > **D** ×

6 Which of the following is equivalent to $\frac{4}{9}$?

A $\frac{12}{28}$ **B** $\frac{28}{64}$ **C** $\frac{20}{45}$ **D** $\frac{12}{36}$

7 Which of the following is equivalent to $\frac{3}{7}$?

A $\frac{21}{28}$ **B** $\frac{18}{42}$ **C** $\frac{9}{20}$ **D** $\frac{12}{29}$

8 Which of the following *cannot* be simplified to $\frac{4}{5}$?

A $\frac{32}{40}$ **B** $\frac{60}{75}$ **C** $\frac{88}{110}$ **D** $\frac{200}{245}$

9 Which fraction when simplified is $\frac{9}{11}$?

A $\frac{54}{66}$ **B** $\frac{35}{44}$ **C** $\frac{90}{101}$ **D** $\frac{18}{20}$

10 Express \$20 as a fraction of \$64, in simplest form.

A $\frac{5}{16}$ **B** $\frac{10}{32}$ **C** $\frac{4}{16}$ **D** $\frac{2}{9}$

11 Express 350 g as a fraction of 1.8 kg, in simplest form.

A $194\frac{2}{5}$ **B** $1\frac{17}{18}$ **C** $\frac{36}{7}$ **D** $\frac{7}{36}$

12 Simplify $\frac{11}{13} + \frac{9}{13}$.

A $\frac{20}{13}$ **B** $\frac{2}{13}$ **C** $1\frac{7}{13}$ **D** $\frac{99}{13}$

13 Simplify $10\frac{4}{5} - 8\frac{2}{3}$.

A $2\frac{6}{15}$ **B** $2\frac{2}{15}$ **C** $2\frac{8}{15}$ **D** $2\frac{4}{5}$

14 Simplify $\frac{4}{9} \times 6\frac{1}{2}$.

A $24\frac{1}{3}$ **B** $2\frac{8}{9}$ **C** $\frac{52}{54}$ **D** $36\frac{9}{12}$

15 Simplify $8\frac{1}{4} \div 3\frac{1}{3}$.

A $3\frac{4}{12}$ **B** $2\frac{13}{40}$ **C** $2\frac{7}{12}$ **D** $2\frac{19}{40}$

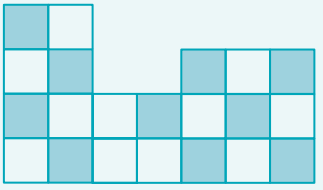
If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–6	7	8, 9	10, 11	12, 13	14	15
Section	A	B	C	D	E	F	G

5A Review set

1 What fraction of this diagram is:

a shaded? **b** unshaded?



2 If $\frac{5}{13}$ of a diagram is shaded, what fraction is unshaded?

3 Convert $\frac{47}{9}$ to a mixed numeral.

4 Convert $8\frac{4}{7}$ to an improper fraction.

5 Complete $\frac{7}{12} = \frac{49}{\square}$.

6 Arrange in ascending order: $\frac{7}{10}, \frac{4}{5}, \frac{9}{20}$.

7 State the reciprocal of $\frac{5}{8}$.

8 Simplify the following.

a $\frac{4}{5} + \frac{1}{3}$ **b** $\frac{11}{12} - \frac{1}{3}$ **c** $\frac{3}{7} \times \frac{1}{4}$ **d** $\frac{5}{6} \div \frac{1}{4}$

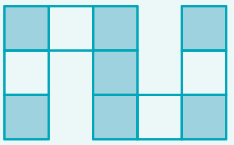
9 Simplify $\frac{1}{2} \times \frac{2}{3} + \frac{1}{4}$.

10 Sue-Lin donated $\frac{2}{11}$ of her weekly income to charity. If her income is $495 per week, how much did she donate?

5B Review set

1 What fraction of this diagram is:

a shaded? **b** unshaded?



2 If Maria ate $\frac{4}{9}$ of a chocolate cake, what fraction would remain?

3 Convert $\frac{108}{7}$ to a mixed numeral.

4 Convert $5\frac{2}{9}$ to an improper fraction.

5 Complete $\frac{80}{100} = \frac{\square}{5}$.

6 Arrange in descending order: $\frac{3}{8}, \frac{2}{3}, \frac{7}{12}$.

7 State the reciprocal of $5\frac{1}{2}$.

8 Simplify the following.

a $2\frac{1}{2} + 6\frac{3}{5}$ **b** $12\frac{1}{3} - 4\frac{1}{4}$

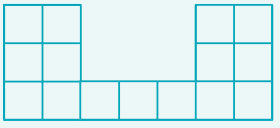
9 Simplify the following.

a $3\frac{4}{5} \times 2\frac{1}{2}$ **b** $8\frac{1}{3} \div 2\frac{1}{6}$

10 Simplify $15\frac{3}{5} \div 2\frac{2}{3} - 3\frac{1}{4}$.

5C Review set

1 Copy the diagram and shade $\frac{1}{3}$.



2 Jennifer inherited $\frac{5}{12}$ of her grandfather's estate and Pauline inherited the remainder. What fraction of the estate did Pauline inherit?

3 Convert $\frac{58}{19}$ to a mixed numeral.

4 Convert $12\frac{3}{4}$ to an improper fraction.

5 Complete $\frac{75}{120} = \frac{\square}{8}$.

6 Simplify $\frac{48}{72}$.

7 Arrange in ascending order: $\frac{1}{2}, \frac{2}{3}, \frac{1}{6}$.

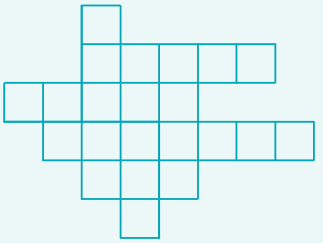
8 State the reciprocal of $8\frac{5}{7}$.

9 Simplify $20\frac{1}{2} \div 5\frac{1}{4} + 6\frac{1}{3}$.

10 A bowl contains $7\frac{3}{4}$ L of punch. After 3 hours, $\frac{3}{4}$ of the punch has been drunk. How many litres are left?

5D Review set

1 Copy the diagram and shade $\frac{6}{11}$.



2 In a class of 30 students, $\frac{1}{3}$ play soccer, $\frac{1}{5}$ play netball and the remainder play football. What fraction of the class plays football?

3 Convert $\frac{146}{13}$ to a mixed numeral.

4 Convert $20\frac{4}{5}$ to an improper fraction.

5 Complete $\frac{155}{\square} = \frac{31}{40}$.

6 Simplify $\frac{185}{240}$.

7 Arrange in descending order: $\frac{3}{5}, \frac{8}{15}, \frac{2}{3}$.

8 State the reciprocal of $5\frac{1}{6}$.

9 Calculate $\frac{5}{8}$ of 592 kg.

10 Liam earns \$870 per week. He banks $\frac{1}{5}$, spends $\frac{2}{3}$ on rent and food, and puts the remaining money to personal use.

a How much does Liam bank each week?

b How much does he spend weekly on rent and food?

c What fraction of Liam's weekly wage is for personal use?

d How much does he spend weekly on personal use?

6

Drawing and building solids

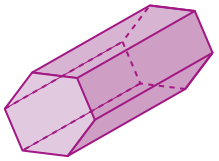
This chapter deals with three-dimensional solids.

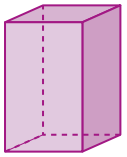
At the end of this chapter you should be able to:

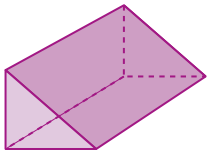
- represent three-dimensional objects in two dimensions from different views
- identify and draw the cross-section of a prism
- determine whether a solid has a uniform cross-section
- visualise, construct and draw various prisms from a given cross-sectional diagram.

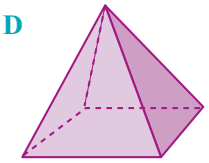
Diagnostic test

1 Which of the following solids is *not* a prism?

A 

B 

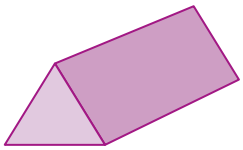
C 

D 

2 The geometrical name that best describes the shape of a matchbox is:

A rectangular pyramid **B** square prism
C sphere **D** rectangular prism

Use this diagram to answer questions **3** to **5**.



3 What is the name of the prism?

A rectangular prism **B** square prism
C cubic prism **D** triangular prism

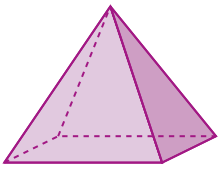
4 How many faces does the prism have?

A 2 **B** 3 **C** 5 **D** 9

5 How many sides does the prism have?

A 3 **B** 5 **C** 9 **D** 6

Use the diagram to answer questions **6** and **7**.



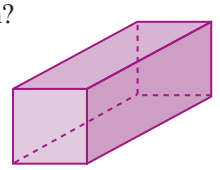
6 Name the solid.

A rectangular prism **B** rectangular pyramid
C triangular prism **D** triangular pyramid

7 How many vertices does the solid have?

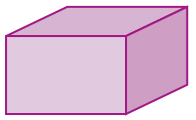
A 2 **B** 5 **C** 7 **D** 8

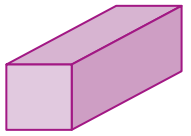
8 Which face is the same shape as the top face of this prism?

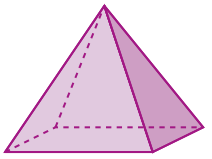


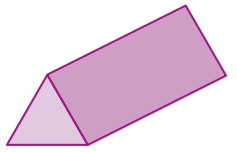
A front face **B** left face
C right face **D** bottom face

9 Which of these diagrams does *not* show a prism?

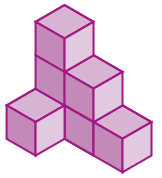
A 

B 

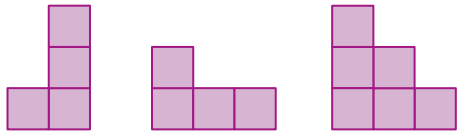
C 

D 

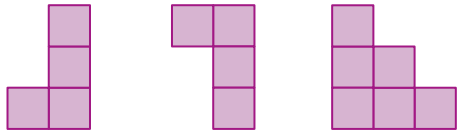
10 The views of this solid from the front, top and left side are:



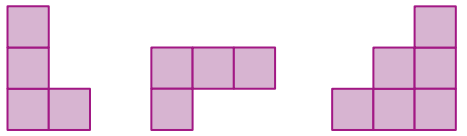
A



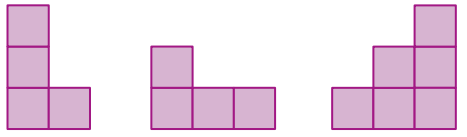
B



C



D



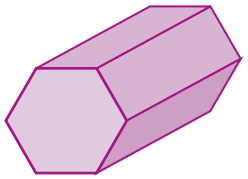
The Diagnostic test questions refer to the Year 6 content description below.

Question	1–10
Section	ACMMG 140

A Identifying solids

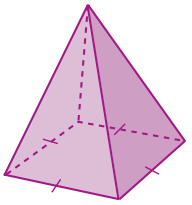
A **plane** (flat) shape with all sides straight is called a **polygon**. A solid that has only polygonal surfaces is called a **polyhedron** (plural is **polyhedra**).

- Solids that have identical polygonal ends joined by rectangular faces are known as **prisms**. The flat end is sometimes called the **base**.
  Prisms are named according to the shape of their flat ends.
  Shape 1 is called a hexagonal prism because its ends are identical hexagons, which are joined by rectangular faces.



Shape 1

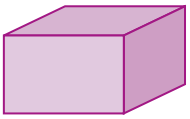
- Solids that have a polygon for the base and triangular faces meeting at a point are called **pyramids**.
  Pyramids are named according to the shape of their base.
  Shape 2 is called a square pyramid because its base is a square.



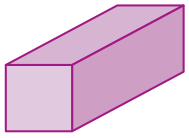
Shape 2

Summary of common solids and their names

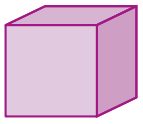
Prisms



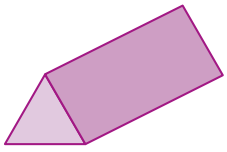
Rectangular prism



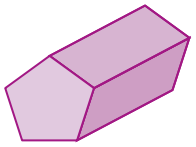
Square prism



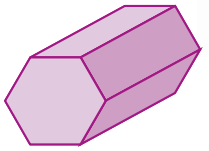
Cube



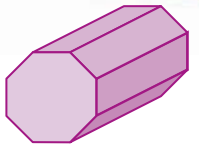
Triangular prism



Pentagonal prism

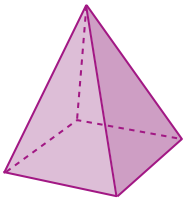


Hexagonal prism

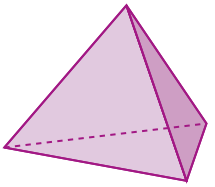


Octagonal prism

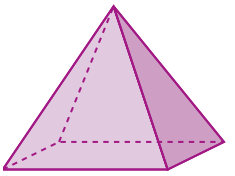
Pyramids



Square pyramid

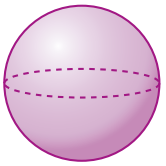


Triangular pyramid

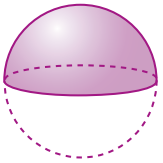


Rectangular pyramid

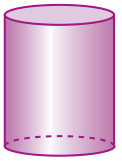
Solids with curved surfaces



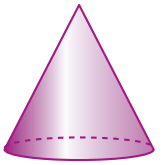
Sphere



Hemisphere



Cylinder



Cone

Exercise 6A

1 What is the geometrical name of the shape that best describes a:

a netball? **b** soup can? **c** matchbox?

d small tent? **e** funnel? **f** tetra juice pack?

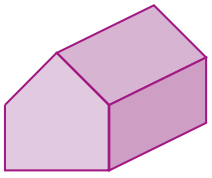
2 Give an example of a solid with a shape resembling a:

a rectangular prism **b** cone **c** cylinder

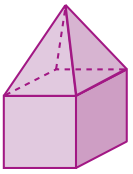
d cube **e** pyramid **f** hemisphere

3 List the solids that have been combined to make each of the following objects. They are sometimes referred to as **composite shapes**.

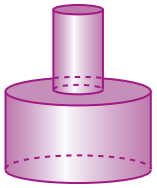
a



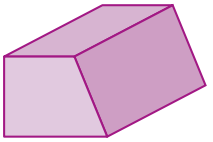
b



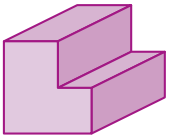
c



d



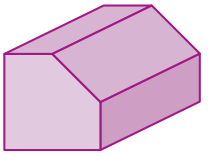
e



f

g

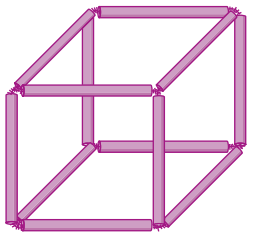
h



4 **a** **Skeletal models** of solids, like that of the cube shown, can be made using straws and pipe cleaners. Use this method to make a:

i rectangular prism **ii** triangular prism **iii** square pyramid

b Can you make a cylinder or sphere using this method? Explain your answer.



B Sketching solids

Here are some simple methods that can be used to draw three-dimensional shapes.

EXAMPLE 1

Sketch a triangular prism on ordinary grid paper.

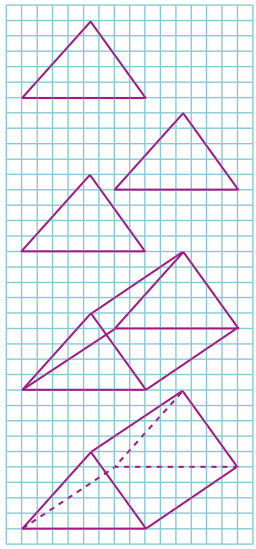
On ordinary grid paper the lines are at right angles.

Step 1: Draw a triangle.

Step 2: Draw an identical triangle a short distance to the right of and up from the first.

Step 3: Join the corresponding corners of these triangles with straight lines.

Step 4: Use dashed lines for all the edges of the solid that are hidden from view.

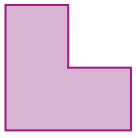
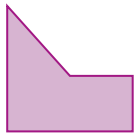
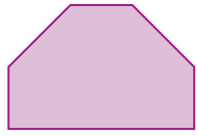


Exercise 6B

1 Using the method in Example 1, sketch a:

a triangular prism **b** cube **c** square prism
d rectangular prism **e** pentagonal prism **f** octagonal prism

2 Use the same method to copy and complete the following prisms whose front face is given.

a  **b**  **c**  **d** 

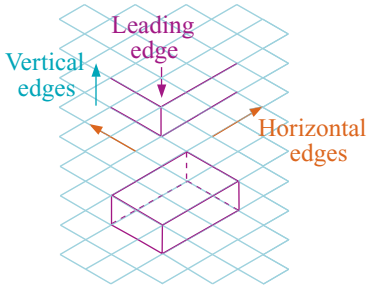
EXAMPLE 2

Sketch a rectangular prism on **isometric** grid paper.

On isometric grid paper the lines are at 60° to each other.

Step 1: Draw one vertical edge to be the leading edge. Then draw horizontal edges along the grid lines, away from the leading edge.

Step 2: Draw the further-away vertical edges and horizontal edges.

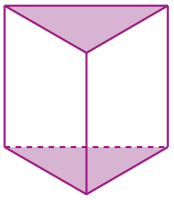


3 Using the method in Example 2, sketch a:

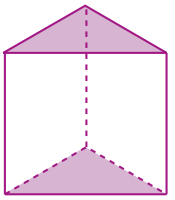
a rectangular prism **b** cube **c** square prism

4 Copy these sketches of a triangular prism onto isometric grid paper.

a



b

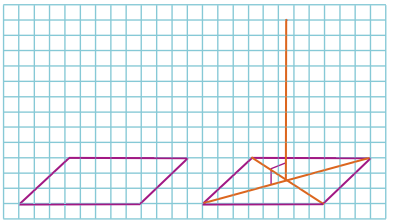


EXAMPLE 3

Sketch a rectangular pyramid on ordinary grid paper.

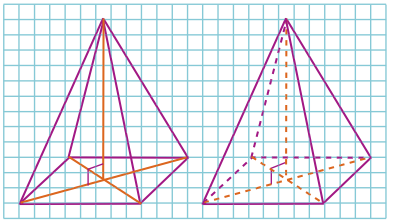
Step 1: Draw a parallelogram to represent the rectangular base.

Step 2: Lightly draw the diagonals of the parallelogram and find their point of intersection. Sketch a perpendicular line upward from the base at that point.



Step 3: Choose a point on the perpendicular line and join the corners of the parallelogram to that point.

Step 4: Use dashed lines for all the edges of the solid that are hidden from view.



5 Using the method in Example 3, sketch a square pyramid.

You may like to use shading to emphasise the 3-dimensional nature of the solid.

6 Follow the steps below to sketch a triangular pyramid on ordinary grid paper.

Step 1: Draw a triangle.

Step 2: Find the centre of the triangle and mark a point above it.

Step 3: Join this point to the three corners of the triangle.

Step 4: Use dashed lines for all the edges of the solid that are hidden from view.

EXAMPLE 4

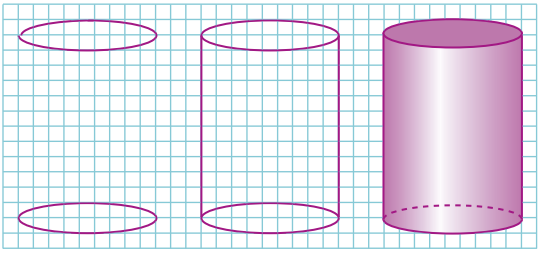
Sketch a cylinder on ordinary grid paper.

When drawing solids, circular faces are usually drawn as ellipses (ovals).

Step 1: Draw two identical ellipses, one directly above the other.

Step 2: Join the corresponding ends of the ellipses.

Step 3: Use dashed lines for all the edges of the solid that are hidden from view.



7 Using the method in Example 4, sketch a cylinder that is wider than it is high.

8 Follow the steps below to sketch a cone on ordinary grid paper.

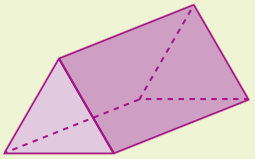
Step 1: Draw an ellipse.

Step 2: Find the centre of the ellipse and mark a point above it.

Step 3: Join this point to the ends of the ellipse.

Step 4: Use dashed lines for all the edges of the solid that are hidden from view.

EXAMPLE 5



For this solid draw the:

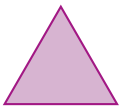
a front view

b side view

c top view

Be careful as you orientate the top view. The top view should be as wide as the front view.

a

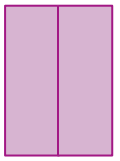


Front

b

Side

c



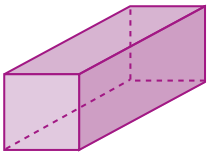
Top

9 For each solid given, draw the:

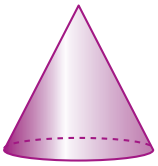
i front view ii side view(s) iii top view

If the right-side view and the left-side view are different, draw both.

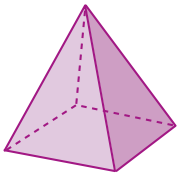
a



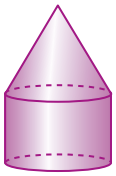
b



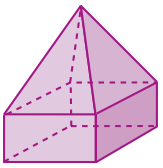
c



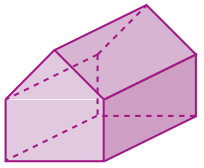
d



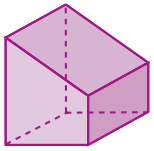
e



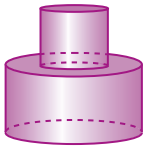
f



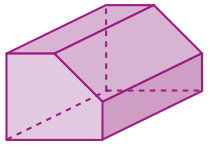
g



h

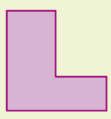


i



EXAMPLE 6

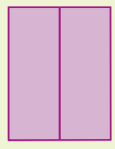
Given the views of a solid, make a neat sketch of the solid.



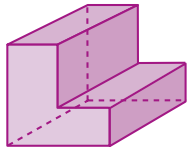
Front

Right side

Left side

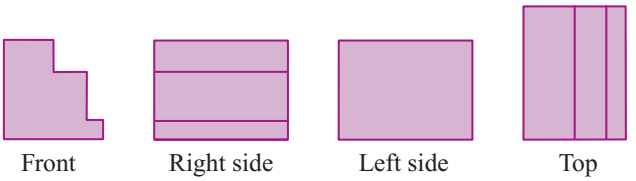


Top

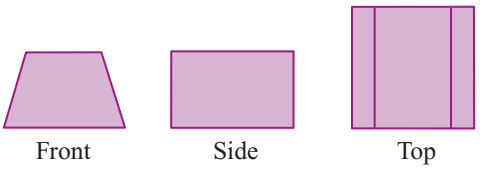


10 Given the following views of solids, make a neat sketch of each solid.

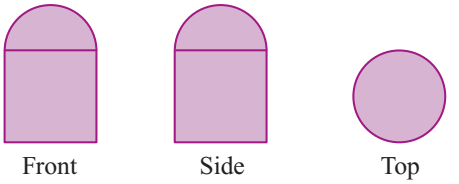
a



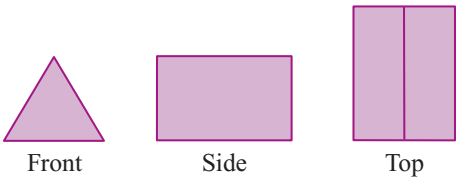
b



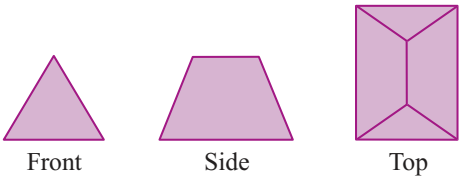
c



d

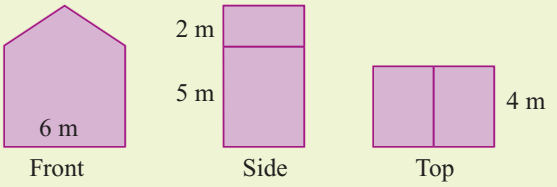


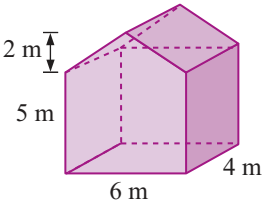
e



EXAMPLE 7

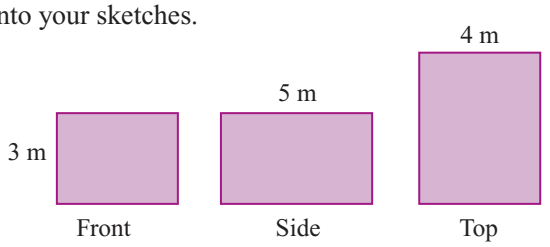
Given the following views of a solid, make a neat sketch of the solid, transferring the dimensions given onto your sketch.



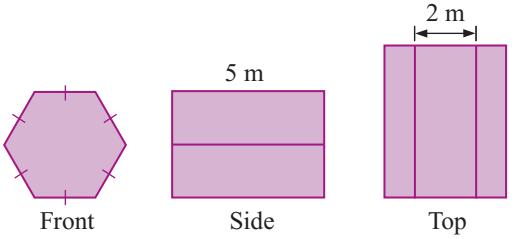


11 Given the following views of solids, make a neat sketch of each solid, transferring the dimensions given onto your sketches.

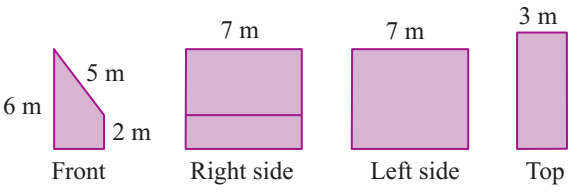
a



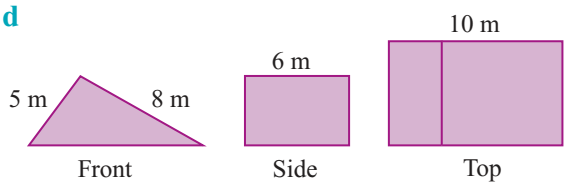
b



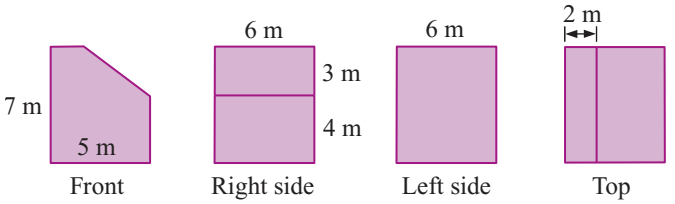
c



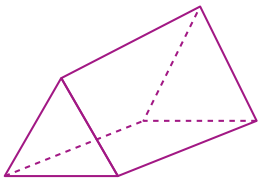
d



e



C Solids and cross-sections

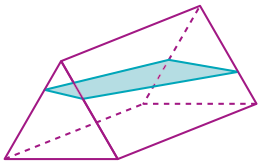


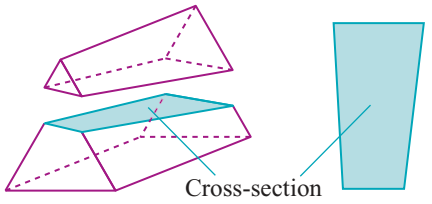
The **cross-section** of a solid is the shape of the surface formed when the solid is sliced through.

For example, consider this triangular prism.

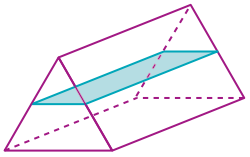
Parallel flat faces never meet.

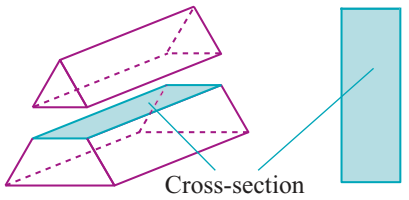
- When you take an oblique slice as shown, the resulting cross-section is a trapezium.



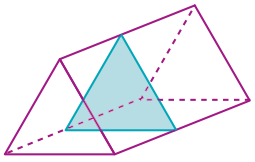


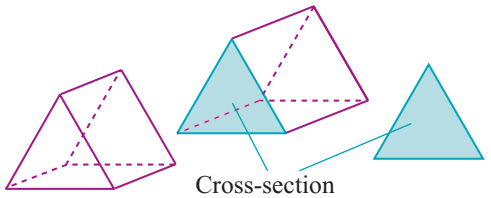
- When you take slices parallel to a side face, the cross-sections are rectangles of different sizes.





- When you take slices parallel to the front face (base), the cross-sections are identical triangles. This is called a **uniform (constant) cross-section**.





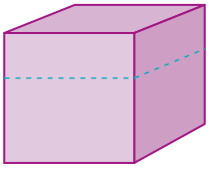
In all prisms, a face can be found such that all slices taken parallel to this face (the base) result in a uniform (the same) cross-section.

The prism is named according to the shape of the uniform cross-section.

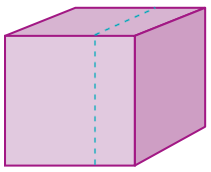
Exercise 6C

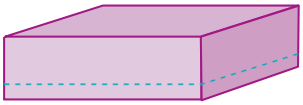
1 Sketch the cross-sections when the following solids are sliced as shown.

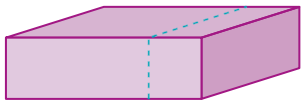
a

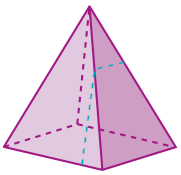


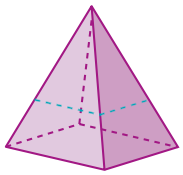
b

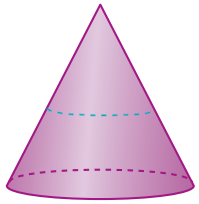


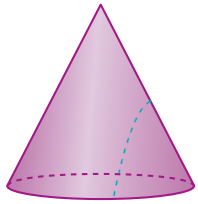
c 

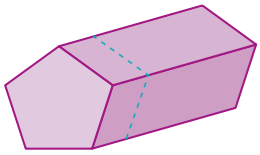
d 

e 

f 

g 

h 

i 

2 **a** Which of the solids in question **1** have a uniform cross-section? That is, which are prisms?

b Name these prisms.

3 **a** Does a cylinder have a uniform cross-section? If so, sketch it.

b Is a cylinder a prism? Explain your answer.

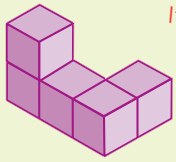
D Building solids from cubes

Using cubes enables us to build solids without too much difficulty.

EXAMPLE 1

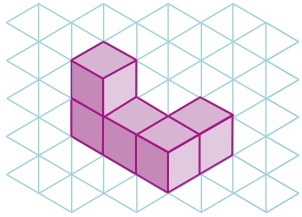
a How many cubes are needed to build this solid?

b Sketch the solid on isometric grid paper.



It may be helpful to build a model of the solid.

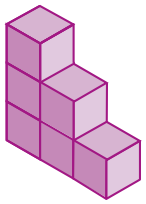
a 5 cubes

b 

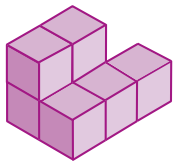
Exercise 6D

1 **i** How many cubes are needed to build each of the following solids?
ii Build the solid using cubes, then sketch the solid on isometric grid paper.

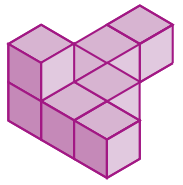
a



b

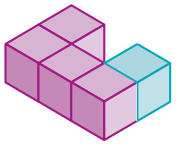


c

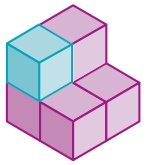


2 Build each solid, then *remove* the cube(s) shaded blue. Sketch the resulting solid on isometric grid paper.

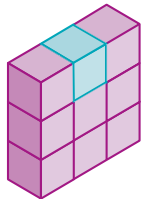
a



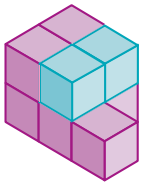
b



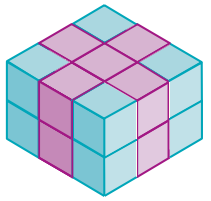
c



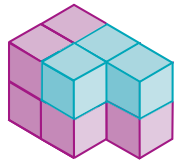
d



e

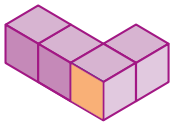


f

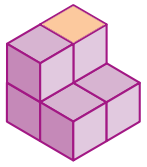


3 Build each solid, then *add* a cube to each face shaded orange. Sketch the resulting solid on isometric grid paper.

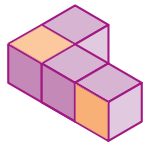
a



b



c



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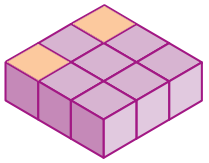
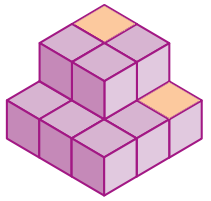


e



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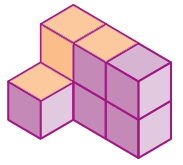
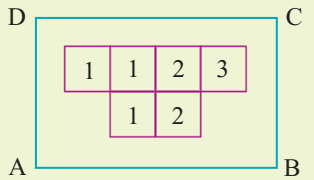
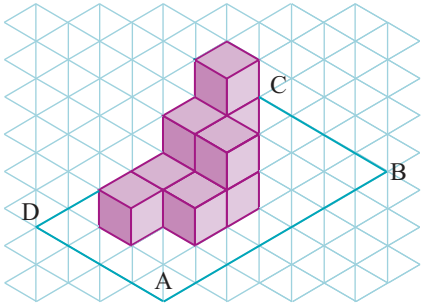


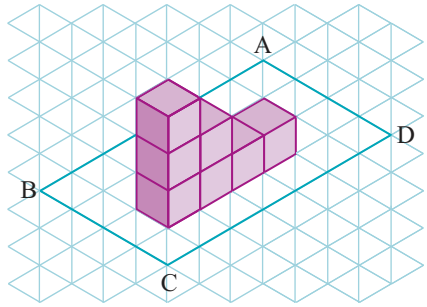
EXAMPLE 2

a Build a solid, using cubes, given the plan shown. The numbers indicate how many cubes there should be in each stack.
b Sketch, on isometric grid paper, the views of the solid from corners A and C.



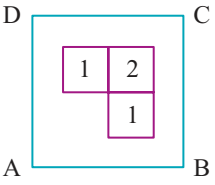
b



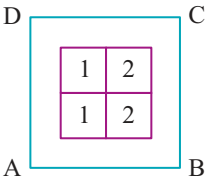


4 Build a solid, using cubes, for each of the following plans. Sketch, on isometric grid paper, the views of the solid from corners A and C.

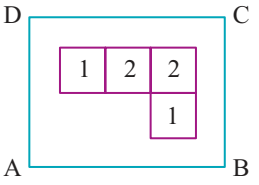
a



b



c



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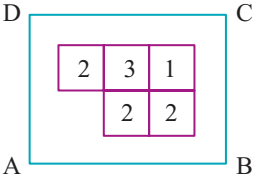
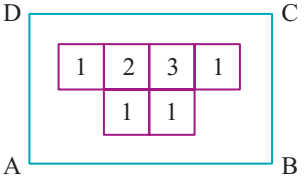


e



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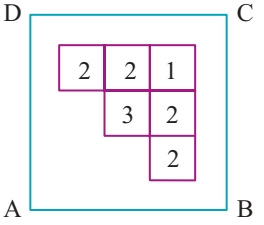
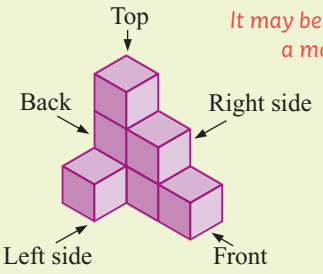


EXAMPLE 3

Sketch the view of the solid shown from the:

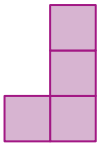
a front **b** back **c** top

d left side **e** right side



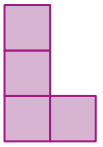
It may be helpful to build a model of the solid.

a



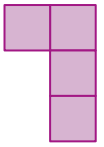
Front

b



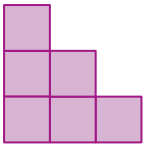
Back

c



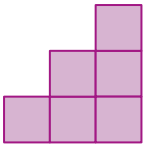
Top

d



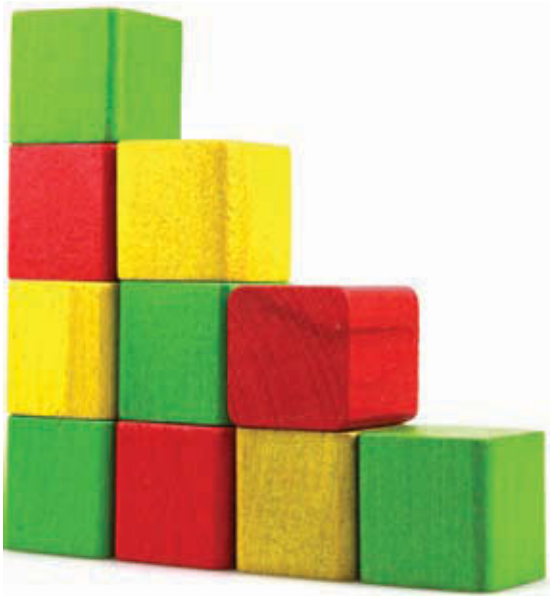
Left side

e

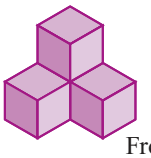


Right side

5 For each of the solids drawn below, build the solid and sketch the view from the front, back, top, left side and right side.

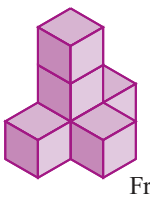


a



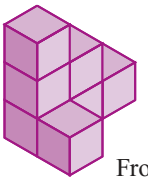
Front

b



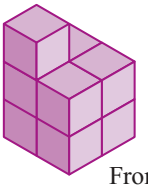
Front

c



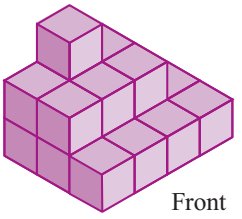
Front

d



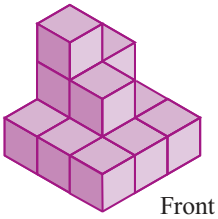
Front

e



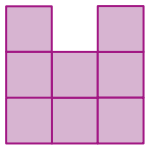
Front

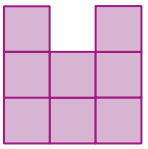
f

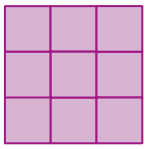


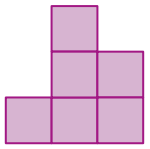
Front

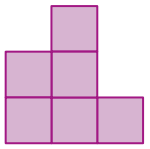
6 A solid was built from unit cubes. From the views given below, build the solid, then sketch the solid and mark the front.


Front


Back

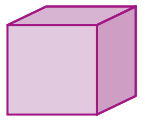

Top


Left side

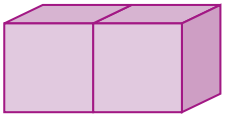

Right side

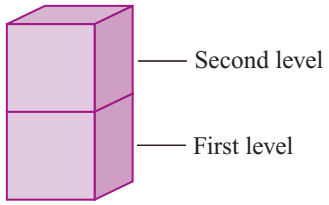
Investigation 1 Cubes and cones

1 The Humble House factory manufactures cubic living quarters for use in central Australia where the conditions most of the year are dry and hot. Heat enters every roof and exposed wall by exactly the same amount. For a house made of one cube, heat enters in equal amounts from five sides, but not from the floor.

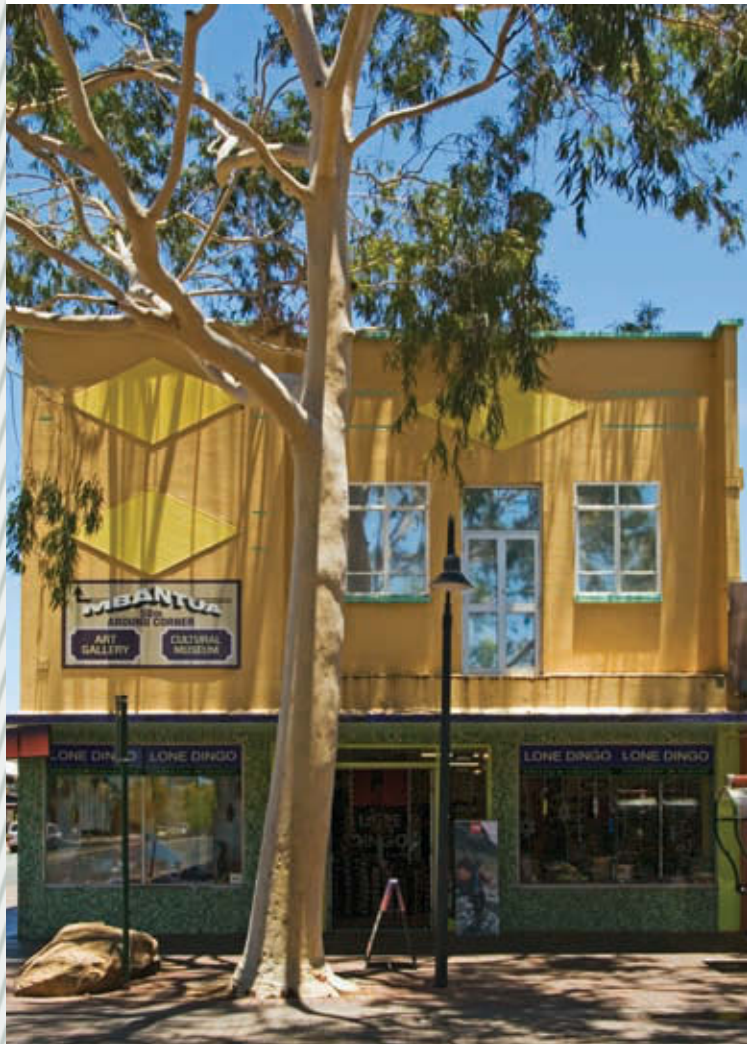


There are two possible house designs made from two cubes placed together:


Design 1



Design 2



a How many exposed faces can be seen in Design 1 and Design 2? Which design would be more suitable for hot conditions?

b Draw the four possible house designs using three cubes placed together. This investigation is best done with actual cubes. Remember, the blocks must touch face to face, and the building must be free-standing. (Columns for support are not acceptable; and in any case, you do not want heat to come in through the floor.)

c Determine, from your models, the 'best' three-cube design to minimise heat intake.

d Investigate possible four-cube buildings and determine the design that would take in the least amount of heat.

e Are you game to try five-cube buildings? How many different five-cube designs are possible, and which one is 'best'?

2 Investigate the various shapes of the cross-sections of a cone, known as the **conic sections**.

Language in mathematics

1 In the find-a-word puzzle below, find all the words in the following list:
circular, combination, composite, cone, constant, cube, cylinder, edge, ellipse, fold, hexagon, identical, isometric, link, pattern, pentagonal, plane, polygon, polyhedra, polyhedron, prism, pyramid, rectangular, right, sketch, solid, sphere, square, stack, straight, triangular, uniform, vertex, vertices, view.

S	P	P	G	H	A	N	O	I	T	A	N	I	B	M	O	C
P	O	L	Y	H	E	D	R	A	V	R	S	Q	U	A	R	E
H	L	A	L	K	C	E	X	W	E	I	V	F	A	P	E	V
E	Y	N	V	I	Y	N	J	I	R	G	G	Z	R	Y	C	Y
R	G	E	E	D	L	O	F	E	T	H	G	I	A	R	T	S
E	O	Z	R	S	I	C	U	B	E	T	S	Z	E	A	A	K
T	N	A	T	S	N	O	C	V	X	M	T	R	U	M	N	E
X	J	A	I	H	D	L	A	C	I	T	N	E	D	I	G	T
K	C	F	C	P	E	N	T	A	G	O	N	A	L	D	U	C
K	L	X	E	X	R	A	L	U	C	R	I	C	E	B	L	H
X	Y	B	S	E	N	O	R	D	E	H	Y	L	O	P	A	A
D	E	T	I	S	O	P	M	O	C	U	N	I	F	O	R	M
Z	I	S	O	M	E	T	R	I	C	N	O	G	A	X	E	H
J	M	L	O	X	E	T	N	R	E	T	T	A	P	T	D	E
R	S	T	O	R	Q	P	R	A	L	U	G	N	A	I	R	T
S	T	V	E	S	P	I	L	L	E	L	O	N	O	G	O	N

2 Complete the given sentences using words from the following list:
polygon, point, identical, cross-section, prism, triangular, uniform.

a A solid that has _____ polygonal ends joined by rectangular faces is called a _____.

b Prisms are named according to the shape of their _____ _____ .

c Solids that have a _____ for the base and _____ faces meeting at a _____ are called pyramids.

3 **a** Look up 'isometric' in a dictionary. What does 'iso' mean?

b Look up 'hemisphere' in a dictionary. What does 'hemi' mean?

4 Use a dictionary to find two different meanings for each of the following words:

a solid **b** uniform **c** square **d** base

5 Write down the plural of polyhedron.

6 Three of the words in the following list are spelt incorrectly. Rewrite them with the correct spelling:
circlar cylinder right skech straght view

7 Describe in your own words a:

a cube **b** sphere **c** cone **d** square pyramid

8 Describe in your own words how to draw a triangular prism.

9 Explain the difference between a cube and a square prism.

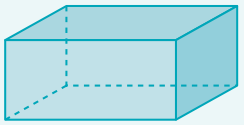
10 How many words of three or more letters can you make from RECTANGULAR? (No proper nouns allowed.)

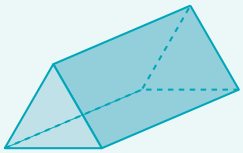
Terms

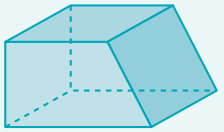
cone	conic	composite	cube	cross-section	cylinder	ellipse
hemisphere	hexagon	horizontal	isometric	octagon	parallel	parallelogram
pentagon	plane	polygon	polyhedra	polyhedron	prism	pyramid
rectangle	solid	sphere	trapezium	uniform	vertical	view

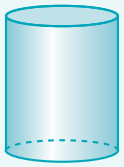
Check your skills

1 Which of the following solids is *not* a prism?

A 

B 

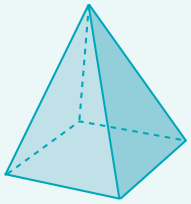
C 

D 

2 The geometrical name of this solid is a:

A triangular prism **B** triangular pyramid

C square pyramid **D** square prism



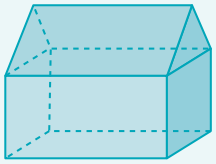
3 The solids that have been combined to make this object are:

A a triangular prism and a rectangular prism

B a pyramid and a rectangular prism

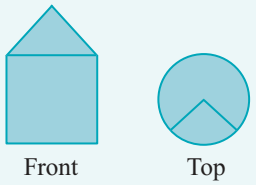
C a pyramid and a cube

D a triangular prism and a rectangular pyramid

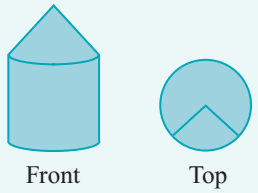


4 The views of the solid below from the front and top are:

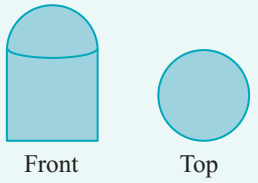
A



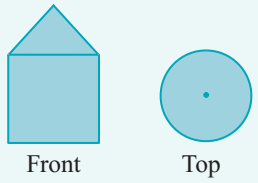
B



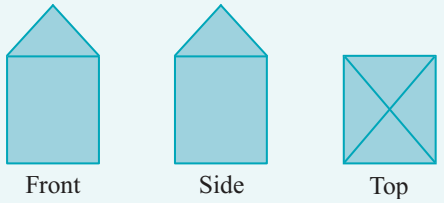
C

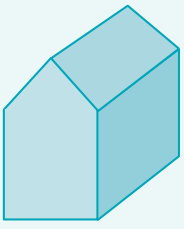


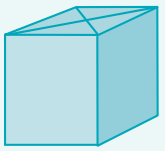
D

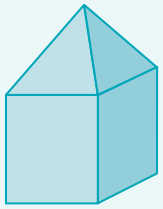


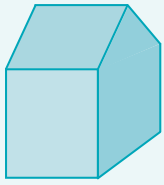
5 Given the views of a solid shown, the solid could be which of the following?



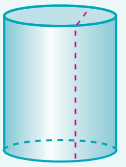
A 

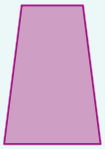
B 

C 

D 

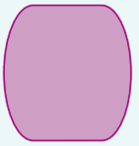
6 The cross-section when this solid is sliced along the dashed line is:



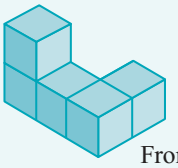
A 

B 

C 

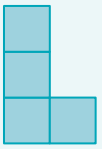
D 

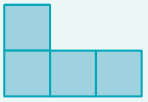
7 The views of this solid from the front, top and left side are:



A

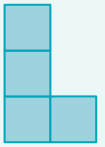

Front

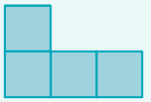

Top


Left side

B

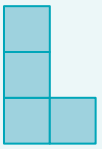

Front

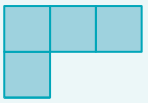

Top


Left side

C

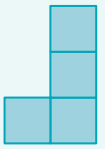

Front

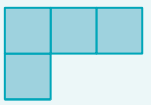

Top


Left side

D


Front


Top

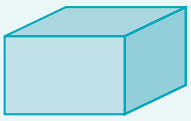

Left side

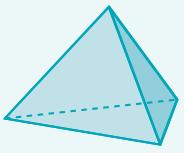
If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

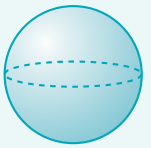
Question	1–3	4, 5	6	7
Section	A	B	C	D

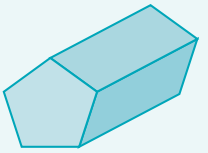
6A Review set

1 Name the solids below.

a 

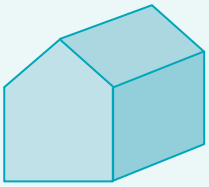
b 

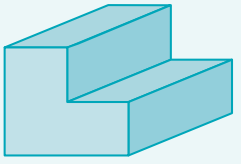
c 

d 

2 Sketch a hexagonal prism.

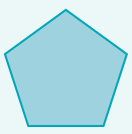
3 Sketch the views of the following solids from the front, side(s) and top.

a 

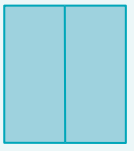
b 

4 Given the following views of solids, make a neat sketch of each solid.

a

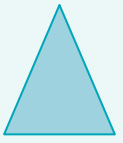

Front


Side


Top

b

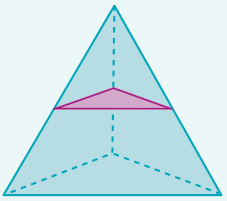

Front


Side

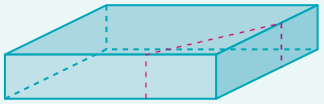

Top

5 Sketch the cross-section when each solid is sliced as shown.

a

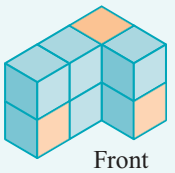


b



6 **a** How many cubes are needed to build the given solid?

b Build the solid and sketch it on isometric paper.



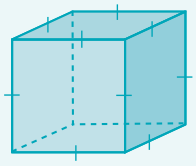
7 **a** For the solid in question **6**, add a cube to each of the faces shaded orange and sketch the resulting solid on isometric paper.

b Sketch the views of the resulting solid from the front, back, top, left side and right side.

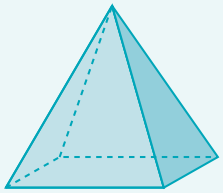
6B Review set

1 Name the solids below.

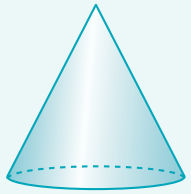
a



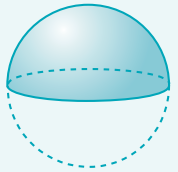
b



c



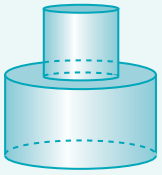
d



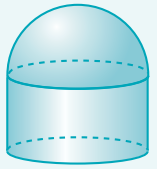
2 Sketch a cylinder that is lying with its end faces vertical.

3 Sketch the views of the following solids from the front, side and top.

a



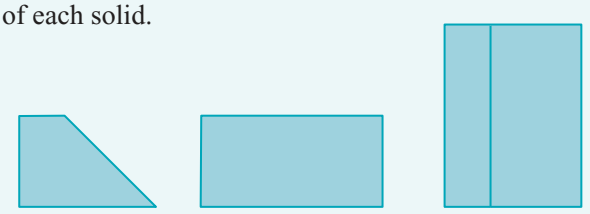
b



4 Given the following views of solids, make a neat sketch of each solid.

a

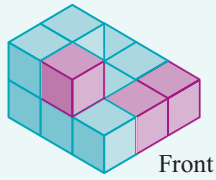
b



5 **a** How many cubes are needed to build the given solid?

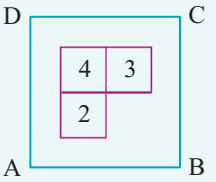
b Build the solid and sketch it on isometric grid paper.

c Sketch the views of the solid from the front, back, top, left side and right side.



6 For the solid in question **5**, remove the cubes shaded purple and sketch the resulting solid on isometric grid paper.

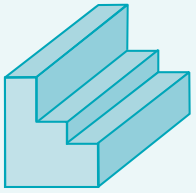
7 The numbers on the plan give the number of cubes in each stack of a solid. Build the solid and sketch, on isometric grid paper, the view of the solid from corner B.



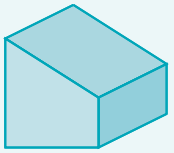
6C Review set

1 Name the common solids that have been combined to make each of the following solids.

a

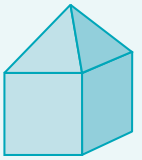


b



c

d



2 Explain the differences between:

a a cylinder and a prism

b a cone and a pyramid

3 Sketch the views of each solid in question **1** from the front, side(s) and top.

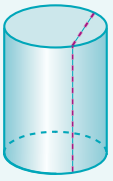
4 Given the following views of solids, make a neat sketch of each solid.

a

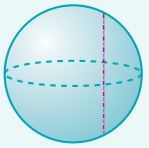
b

5 Sketch the cross-section when each solid is sliced as shown.

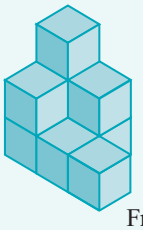
a



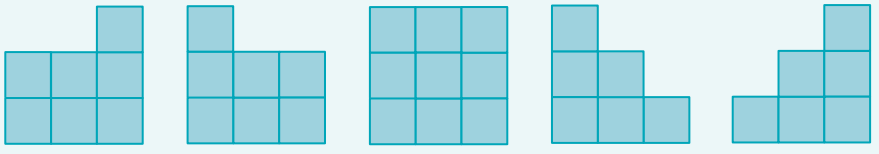
b



6 Build the given solid and then sketch the views from the front, back, top, left side and right side.

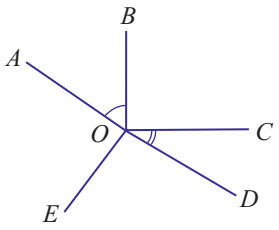


7 A solid was built from cubes. From the views given below, build the solid and sketch it on isometric grid paper.



Chapters 2–6 Cumulative review

1 **a** Name the marked angles in this diagram.



b Classify these angles.

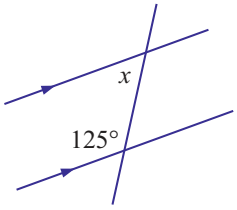
i 48° **ii** 153° **iii** 248°

c What are adjacent angles?

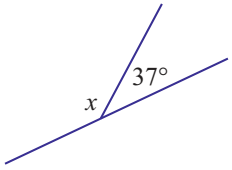
d What are complementary angles?

e Calculate the value of x, giving a reason.

i

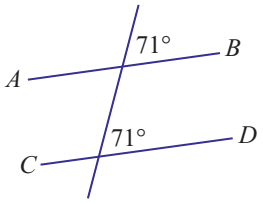


ii



f Are the lines AB and CD parallel? Give a reason.

i



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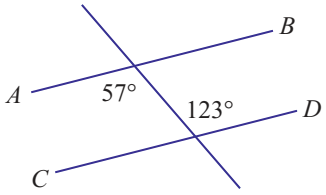


2 **a** Plot 7, 4, 3, 9, on a number line.

b Use a number line to find five numbers between 3 and 6.

c Represent $4 + 3 \times 5$ on a number line.

d What is the opposite of a rise of 5°C in temperature?

e If north is positive, write a directed number for 3 km north then 10 km south.

f Use a number line to help you write each set of numbers in ascending order.

i $-11, -4, -8, -10, -6$ **ii** $-7, -4, -10, 0, 7, 9$ **iii** $1\frac{1}{4}, -1\frac{1}{4}, -1\frac{1}{2}, 2, 0, \frac{1}{2}$

g Insert $>$ or $<$ to make each of the following a true statement.

i $-7 \_\_ -8$ **ii** $-4 \_\_ 4$ **iii** $0 \_\_ -3$

3 Simplify the following.

a $-5 - 2$ **b** $-6 - (-3)$ **c** $2 - (-4)$ **d** $4 - (-4) + 3$

e $-6 + (-5) + 2$ **f** $-3 + (-4) + 2$ **g** $-35 + 27$ **h** $-40 + (-5)$

i $16 + (-8)$ **j** $7 - 3 - 2\,5$ **k** $48 + (4 - 10)$ **l** $-2 + 20 - 6$

4 Plot the following points on a number plane.

$A(2, -1)$, $B(3, -5)$, $C(5, 0)$, $D(2, -4)$, $E(3, 4)$, $F(0, -2)$, $G(-2, 3)$, $H(-4, -3)$

1 **a** Write the multiples of 6 between 17 and 61.

b List the multiples of 7 less than 50.

c Find the following.

i factors of 24 **ii** factors of 42 **iii** HCF of 24 and 42

d Write 180 as a product of prime factors.

e Find the LCM of 24 and 42.

f Find: **i** $\sqrt{36}$ **ii** $\sqrt[3]{64}$

g Which of the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 12 are factors of 44 880?

6 **a** Rod ate $\frac{7}{12}$ of his Easter eggs. What fraction remains?

b Convert $\frac{42}{17}$ to a mixed numeral.

c Convert $3\frac{8}{11}$ to an improper fraction.

d Complete: $\frac{25}{120} = \frac{\square}{24}$

e Arrange in ascending order: $\frac{1}{2}, \frac{1}{4}, \frac{3}{8}, \frac{3}{4}$

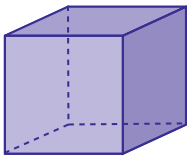
f State the reciprocal of:

i $\frac{3}{5}$ **ii** $2\frac{3}{4}$

g Simplify $4\frac{2}{3} - 2\frac{1}{2} + 1\frac{1}{4}$.

7 **a** Name the following solids.

i



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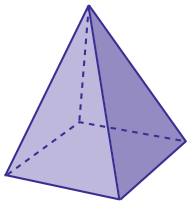

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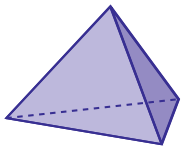
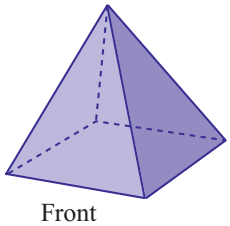


b Sketch a rectangular prism.

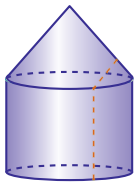
c Sketch the view of this solid from the front, side and top.



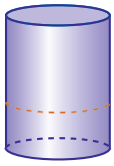
Front

d Sketch the cross-section when each solid is cut as shown.

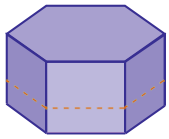
i



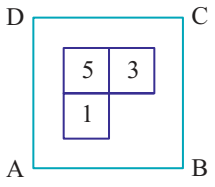
ii



iii



e The numbers on the plan give the number of cubes in each stack of a solid. Build the solid and sketch, on isometric grid paper, the view of the solid from corner B.



7

Algebra

This chapter deals with using letters to represent numbers, the simplification of algebraic expressions and the translation of word statements into algebraic symbols and vice versa.

After completing this chapter you should be able to:

- use letters to represent numbers
- model algebraic expressions using concrete materials
- recognise and use equivalent algebraic expressions
- adding and subtracting like terms
- simplify algebraic expressions involving multiplication and division
- translating word statements into algebraic expressions.

Diagnostic test

1 What is the result when 5 is added to 3?
A 8 **B** 7 **C** 2 **D** 53

2 What is the result when 4 is subtracted from 9?
A 5 **B** 13 **C** −5 **D** 3

3 What is the result when 7 is multiplied by 4?
A 11 **B** 28 **C** 3 **D** 47

4 What is the result when 24 is divided by 8?
A $\frac{1}{3}$ **B** 3 **C** 8 **D** 16

5 What is the sum of 5 and 8?
A 3 **B** 13 **C** $\frac{5}{8}$ **D** 40

6 What is the difference between 13 and 6?
A 19 **B** $\frac{13}{6}$ **C** 7 **D** 78

7 What is the produce of 8 and 4?
A 2 **B** 12 **C** 84 **D** 32

8 What is the quotient of 24 and 4?
A 6 **B** 20 **C** 28 **D** 96

9 The missing number in $5 + \square = 12$ is:
A 17 **B** 7 **C** 5 **D** 1

10 The missing number in $4 \times \square = 28$ is:
A 112 **B** 32 **C** 24 **D** 7

11 The missing number in $\square - 8 = 3$ is:
A 5 **B** 11 **C** 24 **D** 8/3

12 The missing number in $\frac{\square}{4} = 8$ is:
A 32 **B** 12 **C** 4 **D** 2

13 The next number in 3, 5, 7, … is:
A 3 **B** 8 **C** 9 **D** 10

14 The next number in 20, 17, 14, … is:
A 23 **B** 11 **C** 12 **D** 10

15 The next number in $\frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \ldots$ is:
A $\frac{5}{6}$ **B** $\frac{6}{5}$ **C** $\frac{7}{5}$ **D** $\frac{8}{5}$

16 The next number in 0.4, 0.9, 1.4, … is:
A 0.5 **B** 1.8 **C** 1.9 **D** 2.3

17 The rule is add 6. The next number after 15 is:
A 6 **B** 21 **C** 9 **D** 90

18 The rule is divide by 3. The next number after 21 is:
A 3 **B** 7 **C** 21 **D** 63

19 The rule is multiply by $\frac{1}{2}$. The next number after 16 is:
A 64 **B** 32 **C** 16 **D** 8

20 Start with 6 and add 3 and then multiply by 2. The answer is:
A 18 **B** 9 **C** 12 **D** 6

21 Start with 8 and divide by 4 then add 7. The answer is:
A 9 **B** 2 **C** 39 **D** 15

22 Start with 10 and multiply by 3 then subtract 8. The answer is:
A 22 **B** 38 **C** 30 **D** 2

23 Which statement is incorrect?
A $12 \div 4 = 6$ **B** $12 + 4 = 16$
C $12 \times 4 = 48$ **D** $12 - 4 = 8$

24 Which statement is incorrect?
A $4 \times 3 + 1 = 13$ **B** $4 \times 3 + 1 = 16$
C $4 + 3 \times 1 = 7$ **D** $4 \times 3 - 1 = 11$

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–23	24
Section	ACMNA 133	ACMNA 134

The ancient Egyptians used algebra to solve problems. The unknown value, which today we refer to by using a pronumeral (letter), often x, was referred to as the 'heap' or the 'aha'. We often call these letters 'variables' because the numbers they represent can vary—that is, we can substitute one or more numbers for the letters in the expression. The Egyptians relied on algebra to communicate mathematical ideas, much as we do today!

A The concept of variables

A concept is a thought or idea.

Mathematicians simplify statements by using a language in which letters or other symbols represent numbers of objects. This language is called **algebra**.

Letters and other symbols, when used to take the place of numbers, are called **pronumerals**. Sometimes they are referred to as **variables** because the number that they replace can vary.

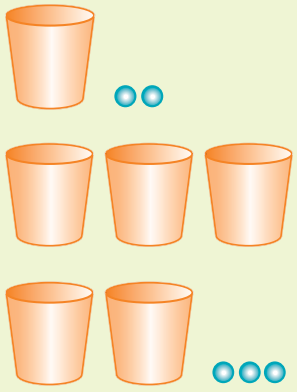
Statements in which letters are used to represent numbers are called **algebraic expressions**.

EXAMPLE 1

a The diagram shows 1 cup and 2 marbles. How many marbles are there altogether if the cup contains:
i 5 marbles? **ii** 8 marbles? **iii** w marbles?

b The diagram shows 3 cups. What is the total number of marbles if each cup contains:
i 5 marbles? **ii** 8 marbles? **iii** w marbles?

c The diagram opposite shows 2 cups and 3 marbles. How many marbles are there altogether if each cup contains:
i 5 marbles? **ii** 8 marbles? **iii** w marbles?



a **i** Number of marbles $= 5 + 2 = 7$
ii Number of marbles $= 8 + 2 = 10$
iii Number of marbles $= w + 2$

b **i** Number of marbles $= 5 + 5 + 5 = 3 \times 5 = 15$
ii Number of marbles $= 8 + 8 + 8 = 3 \times 8 = 24$
iii Number of marbles $= w + w + w = 3 \times w$

c **i** Number of marbles $= 5 + 5 + 3 = 2 \times 5 + 3 = 13$
ii Number of marbles $= 8 + 8 + 3 = 2 \times 8 + 3 = 19$
iii Number of marbles $= w + w + 3 = 2 \times w + 3$

Exercise 7A

1 The diagram shows 1 cup and 4 marbles. Complete the following statements.

If the cup contains:

a 6 marbles, the total number of marbles = ___ + 4 = ____

b 20 marbles, the total number of marbles = ___ + 4 = ____

c w marbles, the total number of marbles = ___ + 4

d z marbles, the total number of marbles = ___ + ____

2 The diagram shows 4 cups. Complete the following statements.

If each cup contains:

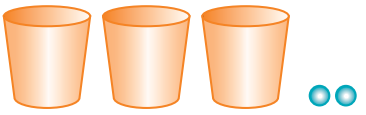
a 5 marbles, the total number of marbles = ___ + ___ + ___ + ___ = 4 × ___ = ____

b 8 marbles, the total number of marbles = ___ + ___ + ___ + ___ = 4 × ___ = ____

c w marbles, the total number of marbles = ___ + ___ + ___ + ___ = 4 × ___

d z marbles, the total number of marbles = ___ + ___ + ___ + ___ = 4 × ___

3 The diagram shows 3 cups and 2 marbles. Complete the following statements.



If each cup contains:

a 5 marbles, the total number of marbles = ___ + ___ + ___ + 2 = 3 × ___ + 2 = ____

b 10 marbles, the total number of marbles = ___ + ___ + ___ + 2 = 3 × ___ + 2 = ____

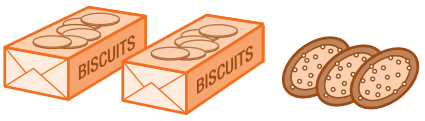
c k marbles, the total number of marbles = ___ + ___ + ___ + 2 = 3 × ___ + 2

d z marbles, the total number of marbles = ___ + ___ + ___ + 2 = 3 × ___ + 2

4 The diagram shows 2 packets and 3 biscuits. How many biscuits are there altogether if each packet contains:

a 20 biscuits? **b** 25 biscuits?

c 30 biscuits? **d** k biscuits?



5 The diagram shows 4 bottles and 6 pills.

a Find the total number of pills if each bottle contains:

i 20 pills **ii** 40 pills

iii 50 pills

b If t = the number of pills in each bottle, write an algebraic expression for the total number of pills.

6 **a** Draw a diagram to represent 3 packets and 10 nails.

b How many nails are there altogether if each packet contains:

i 25 nails? **ii** 60 nails?

iii 100 nails?

c If m = the number of nails in each packet, write an algebraic expression for the total number of nails.

7 **a** Draw a diagram to represent 5 balls of string plus a length of 0.3 m of string.

b Find the total length of string if each ball has a length of:

i 2 m **ii** 6 m

iii 10 m

c If q = the length of string, in metres, in each ball, write an expression for the total length of string.

8 **a** Draw a diagram to represent 6 cartons plus 4 cans of soup.

b Find the total number of cans of soup if each carton contains:

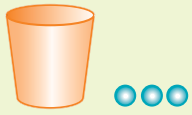
i 20 cans **ii** 30 cans

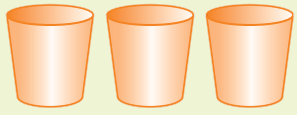
iii 50 cans

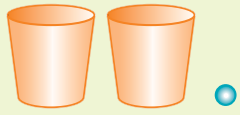
c If d = the number of cans of soup in each carton, write an expression for the total number of cans.

EXAMPLE 2

Let y = the number of marbles in each cup. Write an algebraic expression for the total number of marbles shown by each diagram.

a 

b 

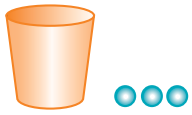
c 

a Number of marbles $= y + 3$

b Number of marbles $= y + y + y$
$= 3 \times y$

c Number of marbles $= y + y + 1$
$= 2 \times y + 1$

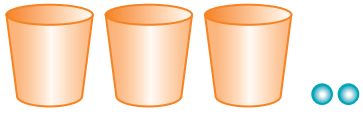
9 Let z = the number of marbles in each cup. Complete the following to write an algebraic expressions for the total number of marbles shown by each diagram.

a 

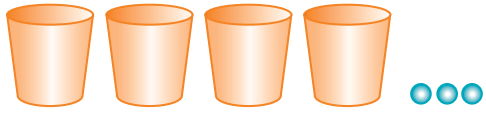
Number of marbles = __ + 3

b 

Number of marbles = __ + __ + __ + __
= 4 × __

c 

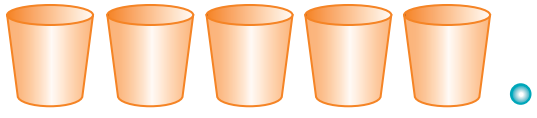
Number of marbles = __ + __ + __ + 2
= 3 × __ + 2

d 

Number of marbles = __ + __ + __ + __ + 3
= 4 × __ + 3

e 

Number of marbles = __ + __ + 4
= 2 × __ + 4

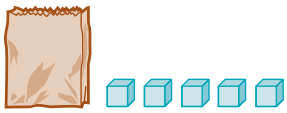
f 

Number of marbles = __ + __ + __ + __ + __ + 1
= 5 × __ + 1

10 Let k = the number of blocks in each paper bag. Write an algebraic expression for the total number of blocks shown by each diagram.

a

b

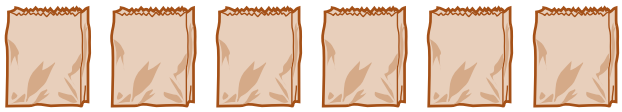


c

d

e

f

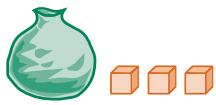


11 Let m = the number of blocks in each bag. Match each algebraic expression below with the total number of blocks shown by each diagram.

A $m + 2$ **B** $2 \times m$ **C** $m + 3$ **D** $3 \times m$ **E** $3 \times m + 1$ **F** $5 \times m + 4$

a

b



c

d

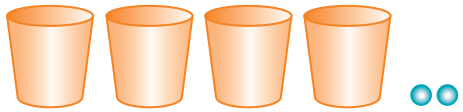
e

f

EXAMPLE 3

Draw a diagram to represent the algebraic expression $4 \times z + 2$ if z represents the number of marbles in a cup.

If z represents the number of marbles in a cup, there must be 4 cups and an extra 2 marbles.



12 Let z = the number of marbles in a cup. Complete this statement:
To draw a diagram to represent the algebraic expression $7 \times z + 4$, there must be ____ cups and an extra ____ marbles.

13 Let d = the number of marbles in a cup. Draw a diagram to represent each of these algebraic expressions.

a $d + 5$ **b** $2 \times d + 1$ **c** $4 \times d$
d $3 \times d + 2$ **e** $5 \times d + 3$

14 Draw a diagram to represent the expression $2 \times a + 7$, if a = the number of chocolates in a box.

15 If p = the number of pieces of chewing gum in a packet, draw a diagram to represent the expression $5 \times p + 2$.

16 Draw a diagram to represent the algebraic expression $3 \times n + 5$, if n = the number of bottles in a crate.

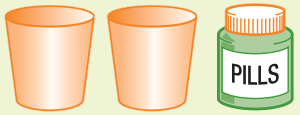
17 Rod wrote down the expression $5 \times n + 4$ for the total number of marbles he had. What could:

a n represent? **b** 5 represent? **c** 4 represent?

EXAMPLE 4

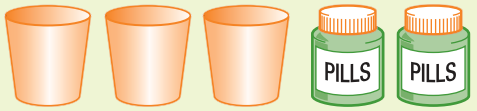
Let s = the number of pills in each cup and t = the number of pills in each bottle, write an algebraic expression for the total number of pills shown by each diagram.

a



b

c



a Number of pills $= s + s + t = 2 \times s + t$

b Number of pills $= s + s + t + t = 2 \times s + 2 \times t$

c Number of pills $= s + s + s + t + t = 3 \times s + 2 \times t$

18 Let s = the number of pills in each cup and t = the number of pills in each bottle. Complete the following to write an algebraic expression for the total number of pills shown by each diagram.

a

Number of pills $=$ ___ $+$ ___ $+$ ___

$=$ ___ $+ 2 \times$ ___

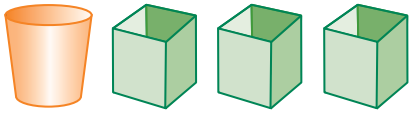
b

Number of pills $=$ ___ $+$ ___ $+$ ___ $+$ ___ $+$ ___

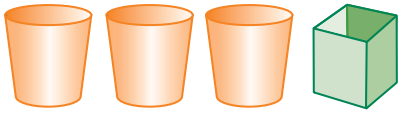
$= 2 \times$ ___ $+ 3 \times$ ___

19 Let h = the number of dice in each cup and k = the number of dice in each box. Write an algebraic expression for the total number of dice shown by each diagram.

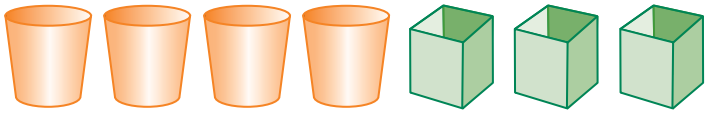
a



b



c



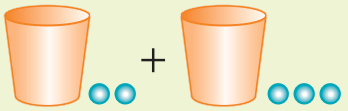
20 Let p = the number of dice in a cup and q = the number of dice in a box. Draw a diagram to represent each of these algebraic expressions.

a $p + q$ **b** $2 \times p + q$ **c** $p + 2 \times q$ **d** $2 \times p + 3 \times q$ **e** $3 \times p + 4 \times q$

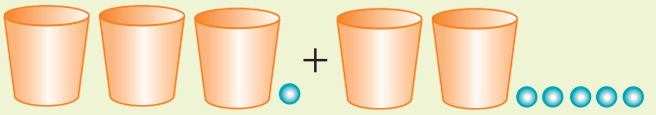
EXAMPLE 5

Let m = the number of marbles in each cup. Write an algebraic expression for the total number of marbles shown by each diagram.

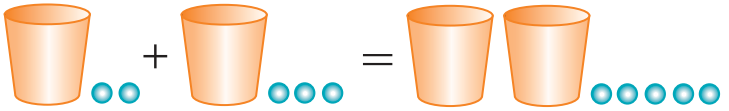
a



b

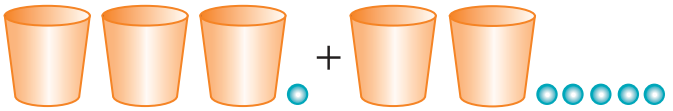


a



$(m + 2)$ + $(m + 3)$ = $2 \times m + 5$

b

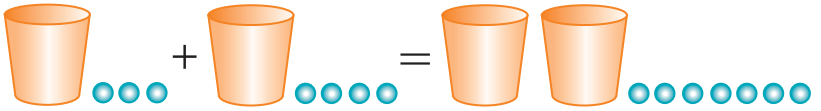


$(3 \times m + 1)$ + $(2 \times m + 5)$

$= 5 \times m + 6$

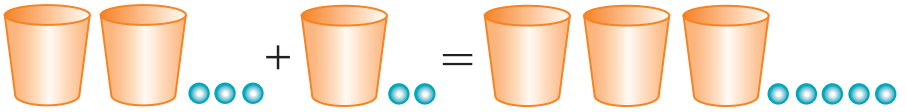
21 Let w = the number of marbles in each cup. Complete the following to write an algebraic expression for the total number of marbles shown by each diagram.

a



$(w + 3)$ + $(w + 4)$ = $2 \times \_\_\_ + \_\_\_$

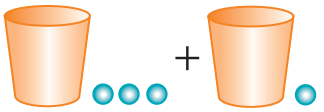
b



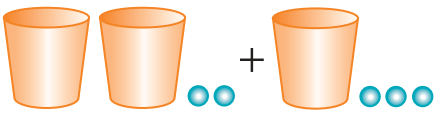
$(2 \times w + 3)$ + $(w + 2)$ = $3 \times \_\_\_ + \_\_\_$

22 Let z = the number of marbles in each cup. Write an algebraic expression for the total number of marbles shown by each diagram.

a



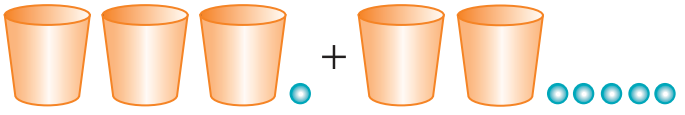
b



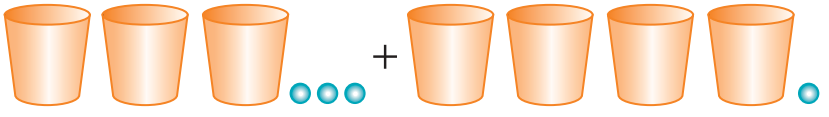
c

d

e



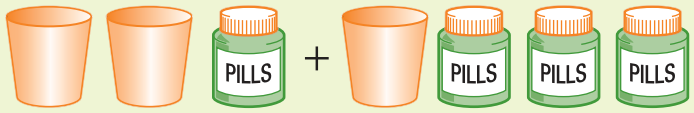
f



EXAMPLE 6

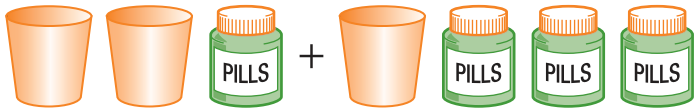
Let k = the number of pills in each cup and m = the number of pills in each bottle. Write an algebraic expression for the total number of pills shown by each diagram.

a



b

a



$(2 \times k + m) \quad + \quad (k + 3 \times m)$

$= 3 \times k + 4 \times m$

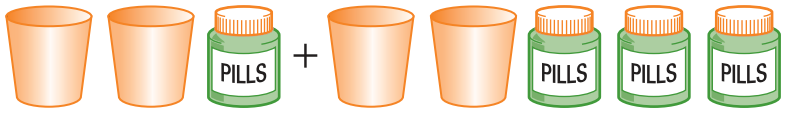
b

$(k + 2 \times m) \quad + \quad (3 \times k + 2 \times m)$

$= 4 \times k + 4 \times m$

23 Let k = the number of pills in each cup and m = the number of pills in each bottle. Complete the following to write an algebraic expression for the total number of pills shown by each diagram.

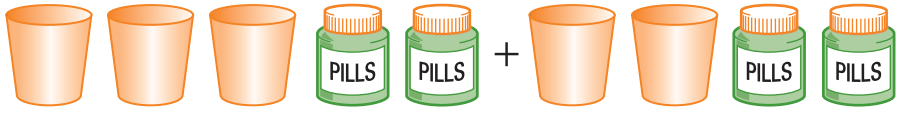
a



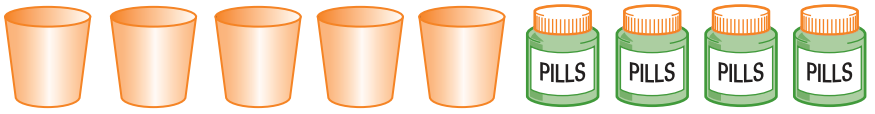
$(2 \times k + m) \quad + \quad (2 \times k + 3 \times m)$

$= \_\_\_ \times k + \_\_\_ \times m$

b



$(\_\_\_ \times k + \_\_\_ \times m) \quad + \quad (\_\_\_ \times k + \_\_\_ m)$

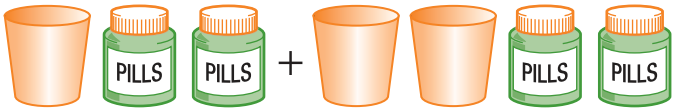


$= \_\_\_ \times k + \_\_\_ \times m$

24 Let v = the number of pills in each cup and y = the number of pills in each bottle. Write an algebraic expression for the total number of pills shown by each diagram.

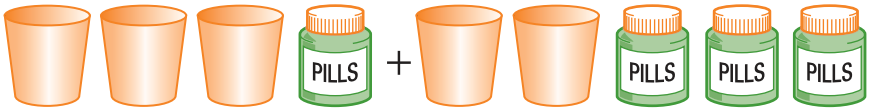
a

b

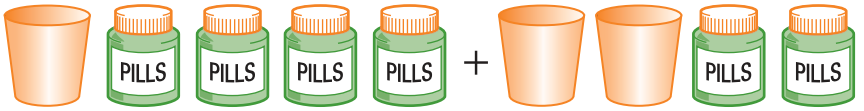


c

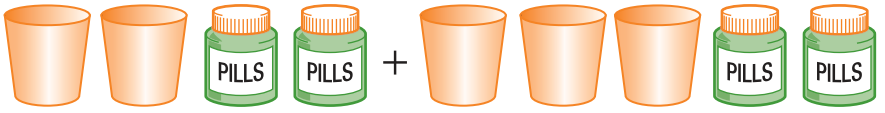
d



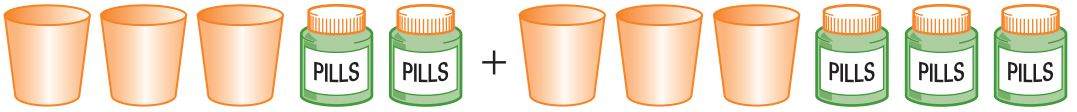
e



f



g



B Substitution into algebraic expressions

Substitution into an algebraic expression means replacing a pronumeral by a number, in order to **evaluate** the expression.

Evaluate means find the value of.

EXAMPLE 1

Evaluate the algebraic expression $3 \times z + 4$ if:

a $z = 2$ **b** $z = 5$

a Replace z by the number 2.
If $z = 2$, $3 \times z + 4$
$= 3 \times 2 + 4 = 10$

b Replace z by the number 5.
If $z = 5$, $3 \times z + 4$
$= 3 \times 5 + 4 = 19$

Exercise 7B

1 Complete the following to find the value of the algebraic expression $z + 6$.

a If $z = 2$, $z + 6$
$= \_\_\_ + 6 = \_\_\_$

b If $z = 5$, $z + 6$
$= \_\_\_ + 6 = \_\_\_$

2 Complete the following to find the value of the algebraic expression $2 \times w - 5$.

a If $w = 6$, $2 \times w - 5$
$= 2 \times \_\_\_ - 5 = \_\_\_$

b If $w = 10$, $2 \times w - 5$
$= 2 \times \_\_\_ - 5 = \_\_\_$

3 Evaluate the algebraic expression $20 - 3 \times k$ if:

a $k = 2$ **b** $k = 4$ **c** $k = 5$

4 Evaluate the algebraic expression $3 \times (t - 1) + 5$ if:

a $t = 2$ **b** $t = 5$ **c** $t = 10$

EXAMPLE 2

Substitute $y = 4$ into each expression and evaluate.

a $5 \times y + 2$ **b** $2 \times (y - 1) + 5$

This is another way of asking us to replace y by the number 4 in each expression, then simplify.

a $5 \times y + 2$
$= 5 \times 4 + 2 = 22$

b $2 \times (y - 1) + 5$
$= 2 \times (4 - 1) + 5$
$= 2 \times 3 + 5 = 11$

5 Complete the following by substituting $p = 5$ into each expression, then simplifying.

a $p + 9$
$= \_\_\_ + 9 = \_\_\_$

b $p - 5$
$= \_\_\_ - 5 = \_\_\_$

c $6 \times p$
$= 6 \times \_\_\_ = \_\_\_$

d $10 \times p + 2$
$= 10 \times \_\_\_ + 2 = \_\_\_$

e $7 \times p - 4$
$= 7 \times \_\_\_ - 4 = \_\_\_$

f $12 - 2 \times p$
$= 12 - 2 \times \_\_\_ = \_\_\_$

6 Substitute $m = 12$ into each expression and evaluate.

a $15 - m$ **b** $m + 4$ **c** $2 \times m + 3$ **d** $3 \times (m + 2)$ **e** $m \times m$

7 Substitute $k = 2$ into each expression and evaluate.

a $5 \times k - 3$ **b** $4 \times (k + 3)$ **c** $5 + 6 \times k$ **d** $(k + 1) \times 6$ **e** $4 \times k - 3 \times k$

EXAMPLE 3

If $a = 9$, $b = 4$ and $c = 10$, find the value of each expression.

a $a + b - c$ **b** $2 \times a + 3 \times b$ **c** $a \times b \times c$ **d** $2 \times a \div 6$

Again we are being asked to replace each pronumeral by a numerical value.

a $a + b - c$
$= 9 + 4 - 10 = 3$

b $2 \times a + 3 \times b$
$= 2 \times 9 + 3 \times 4 = 30$

c $a \times b \times c$
$= 9 \times 4 \times 10 = 360$

d $2 \times a \div 6$
$= 2 \times 9 \div 6 = 3$

8 If $a = 5$, $b = 6$ and $c = 8$, find the value of:

a $a + b + c$ **b** $a + b - c$ **c** $12 - a$ **d** $c - 8$
e $10 \times b$ **f** $2 \times c$ **g** $c \times 3$ **h** $a \times b$
i $b \times c$ **j** $a \times b \times c$ **k** $a \times a$ **l** $b \times b$
m $10 \div a$ **n** $2 \times a + 5$ **o** $7 + 3 \times b$ **p** $4 \times a - 2 \times c$
q $c \div 2$ **r** $a \times b \div 10$ **s** $(a + 1) \times 3$ **t** $4 \times (b - 2)$
u $(a + 1) \times (c - 1)$ **v** $(b + 2) \div 4$ **w** $(c - 2) \div b$ **x** $(b + c) \div (a + 2)$

9 Complete the following tables to generate a number pattern from the algebraic expression.

a

x	1	2	3	4	5	…	10	…	100
$x + 3$	4	5							

b

x	1	2	3	4	5	…	10	…	100
$x - 2$	-1	0							

c

x	1	2	3	4	5	…	10	…	100
$4 \times x$	4	8							

d

x	1	2	3	4	5	…	10	…	100
$3 \times x + 1$	4	7							

e

x	1	2	3	4	5	…	10	…	100
$2 \times x - 3$	-1	1							

C Expressions involving multiplication

When writing algebraic expressions we introduce certain **conventions** for particular operations. For example:

- $3 \times p$ may be written $3p$.
  The multiplication sign may be left out (omitted).
- $p \times 3$ is the same as $3 \times p$ and is written $3p$.
  The number is *always* written in front of the pronumeral.
- $p \times q$ is the same as $q \times p$ and may be written pq.
  Pronumerals are usually written in alphabetical order.
- $p \times p$ may be written pp or p^2.

Conventions are rules that everybody agrees to follow.

EXAMPLE 1

Simplify these algebraic expressions.

a $5 \times k$ **b** $k \times 5$ **c** $k \times m$ **d** $k \times k$
e $5 \times k \times m$ **f** $3 \times q + 1$ **g** $5 \times m - 7 \times n$ **h** $n \times m$

By leaving out the multiplication signs, the answers are:

a $5k$ **b** $5k$ **c** km **d** kk or k^2
e $5km$ **f** $3q + 1$ **g** $5m - 7n$ **h** mn

The multiplication sign may be left out *only* when there is no chance of any confusion about the meaning of the expression. That is, it may be left out between a numeral and a pronumeral, as in $5k$, or between two pronumerals, as in km. It may *never* be left out between two numerals. For example:

35 means 3 lots of 10 plus 5. It never means 3×5.

Exercise 7C

1 Simplify the following algebraic expressions by writing them without multiplication signs.

a $4 \times m$	**b** $6 \times t$	**c** $3 \times k$	**d** $10 \times z$	**e** $p \times 7$
f $m \times 5$	**g** $k \times 8$	**h** $n \times 2$	**i** $w \times z$	**j** $c \times v$
k $p \times q$	**l** $a \times b$	**m** $t \times t$	**n** $q \times q$	**o** $s \times s$
p $a \times b \times c$	**q** $2 \times s \times t$	**r** $6 \times q \times p$	**s** $8 \times p \times q \times r$	**t** $3 \times a + 1$
u $7 \times d - 8$	**v** $23 + 6 \times p$	**w** $15 - 5 \times t$	**x** $4 \times m + 2 \times n$	**y** $3 \times z - 8 \times k$

2 Simplify by omitting the multiplication signs.

a $2 \times v \times v$	**b** $5 \times w \times w$	**c** $2 \times p \times p$	**d** $4 \times t \times t$	**e** $3 \times m \times m$
f $6 \times d \times d$	**g** $9 \times p \times p$	**h** $8 \times k \times k$	**i** $10 \times s \times s$	**j** $7 \times n \times n$

EXAMPLE 2

Show the meaning of each algebraic expression by inserting the multiplication sign(s).

a $7w$	**b** $36z$	**c** pq	**d** a^2
e $5pq$	**f** $5pqr$	**g** $5w + 4$	**h** $3k - 7m$

a $7 \times w$	**b** $36 \times z$	**c** $p \times q$	**d** $a \times a$
e $5 \times p \times q$	**f** $5 \times p \times q \times r$	**g** $5 \times w + 4$	**h** $3 \times k - 7 \times m$

3 Show the meaning of each algebraic expression by inserting multiplication sign(s).

a $9w$	**b** $3k$	**c** $5g$	**d** $2d$	**e** $24b$
f $52m$	**g** $48z$	**h** tz	**i** bc	**j** km
k v^2	**l** b^2	**m** $3pq$	**n** $8ab$	**o** $5pq$
p $12mn$	**q** $35hs$	**r** krz	**s** $3abc$	**t** $6t + 5$
u $12 + 4d$	**v** $2w - 9$	**w** $7 - 8k$	**x** $2a + 3b$	**y** $4k - 9m$

4 Show the meanings of the following expressions by inserting multiplication signs.

a $2k^2$	**b** $6t^2$	**c** $4q^2$	**d** $7a^2$	**e** $4w^2$
f $5m^2$	**g** $8z^2$	**h** $9q^2$	**i** $12s^2$	**j** $5y^2$

EXAMPLE 3

If $w = 0$, $y = 5$ and $z = 4$, find the value of:

a $3z$	**b** yz	**c** $10yz$	**d** $7w$	**e** y^2

a $3z$ $= 3 \times z$ $= 3 \times 4$ $= 12$	**b** yz $= y \times z$ $= 5 \times 4$ $= 20$	**c** $10yz$ $= 10 \times y \times z$ $= 10 \times 5 \times 4$ $= 200$	**d** $7w$ $= 7 \times w$ $= 7 \times 0$ $= 0$	**e** y^2 $= y \times y$ $= 5 \times 5$ $= 25$

5 If $x = 3$ and $y = 4$, complete the following to evaluate each expression.

a $5x = 5 \times x$	**b** $xy = x \times y$	**c** $25x = 25 \times x$	**d** $y^2 = y \times y$
$= 5 \times \_\_\_ = \_\_\_$	$= \_\_\_ \times \_\_\_ = \_\_\_$	$= 25 \times \_\_\_ = \_\_\_$	$= \_\_\_ \times \_\_\_ = \_\_\_$

6 If $m = 6$, $n = 5$, $p = 2$ and $q = 0$, find the value of:

a $4m$	**b** $2n$	**c** $3p$	**d** $5q$	**e** $10m$	**f** $8p$	**g** $3m$	**h** $4n$
i $24p$	**j** $317q$	**k** mn	**l** mp	**m** mq	**n** np	**o** nq	**p** pq
q mnp	**r** $3mn$	**s** $5mp$	**t** $4np$	**u** $8nq$	**v** $45np$	**w** m^2	**x** n^2

EXAMPLE 4

If $y = 5$ and $z = 4$, find the value of:

a $2z + 3$ **b** $6y - 7z$ **c** $6 + yz$

a $2z + 3$
$= 2 \times z + 3$
$= 2 \times 4 + 3 = 11$

b $6y - 7z$
$= 6 \times y - 7 \times z$
$= 6 \times 5 - 7 \times 4 = 2$

c $6 + yz$
$= 6 + y \times z$
$= 6 + 5 \times 4 = 26$

7 If $x = 2$ and $y = 3$, complete the following to evaluate each expression.

a $5x + 4$
$= 5 \times x + 4$
$= 5 \times \_\_\_ + 4 = \_\_\_$

b $20 - 4y$
$= 20 - 4 \times y$
$= 20 - 4 \times \_\_\_ = \_\_\_$

c $4x + 5y$
$= 4 \times x + 5 \times y$
$= 4 \times \_\_\_ + 5 \times \_\_\_ = \_\_\_$

8 If $a = 2$, $b = 4$ and $c = 3$, find the value of:

a $3a + 5$ **b** $5b + 1$ **c** $2c + 5$ **d** $2a - 3$ **e** $4b - 2$
f $5c - 8$ **g** $6 + 3a$ **h** $13 + 2b$ **i** $1 + 4c$ **j** $20 - 4a$
k $100 - 5c$ **l** $29 - 6b$ **m** $ab + 3$ **n** $7 + bc$ **o** $ac - 5$
p $3a + 2b$ **q** $2c + 3b$ **r** $5c + 4a$ **s** $10b - 10a$ **t** $bc - ab$
u $a + 2b + c$ **v** $5a + b - 3c$ **w** $10c - b + 2a$ **x** $5b - 8a - c$ **y** $6a + bc - ac$

9 The teacher asked the class to find the value of $3y^2$ if $y = 5$. Who was right? Discuss your answer.

Rod did it this way:
$3y^2 = 35^2$
$= 35 \times 35$
$= 1225$

Sylvanna's answer was:
$3y^2 = 3 \times 5^2$
$= 3 \times 5 \times 5$
$= 75$

Sue-Lin's working was:
$3y^2 = 3 \times 5^2$
$= 15^2 = 15 \times 15$
$= 225$

EXAMPLE 5

If $m = 3$, find the value of:

a m^2 **b** $2m^2$ **c** $(2m)^2$

a $m^2 = m \times m$
$= 3 \times 3$
$= 9$

b $2m^2 = 2 \times m \times m$
$= 2 \times 3 \times 3$
$= 18$

c $(2m)^2 = (2 \times m)^2$
$= (2 \times 3)^2$
$= 6^2 = 36$

10 If $k = 3$, complete the following to evaluate each expression.

a $k^2 = k \times k$
$= \_\_\_ \times \_\_\_$
$= \_\_\_$

b $4k^2 = 4 \times k \times k$
$= 4 \times \_\_\_ \times \_\_\_$
$= \_\_\_$

c $(4k)^2 = (4 \times k)^2$
$= (4 \times \_\_\_)^2$
$= \_\_\_^2 = \_\_\_$

11 If $a = 5$, $b = 4$ and $c = 2$, find the value of:

a a^2 **b** $2a^2$ **c** $(2a)^2$ **d** $3a^2$ **e** $(3a)^2$ **f** b^2 **g** $2b^2$ **h** $(2b)^2$
i $3b^2$ **j** $(3b)^2$ **k** c^2 **l** $3c^2$ **m** $(3c)^2$ **n** $10c^2$ **o** $(10c)^2$

12 **a** Complete the following table.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
x^2	25	16									

b Compare the values of x^2 for values of x with the same magnitude but opposite sign; that is, compare the values of $(-5)^2$ and 5^2, $(-4)^2$ and 4^2, etc. What do you notice? Explain why this happens.

13 By substituting values for k, find whether each of the following is true or false.

a $1 \times k = k \times 1 = k$ **b** $0 \times k = k \times 0 = 0$ **c** $k + 0 = 0 + k = k$

14 Simplify the following.

a $1 \times t$	**b** $1 \times k$	**c** $d \times 1$	**d** $z \times 1$	**e** $1 \times 5y$
f $1 \times pq$	**g** $3v \times 1$	**h** $ab \times 1$	**i** $1 \times 2wyz$	**j** $5klm \times 1$
k $0 \times a$	**l** $w \times 0$	**m** $0 \times 2b$	**n** $5z \times 0$	**o** $0 \times 7pq$
p $m + 0$	**q** $0 + 4a$	**r** $6xy + 0$	**s** $m \times 0 + n$	**t** $5n - 0$

D Expressions involving division

The division of two numbers is often written in **fraction form**.

Just as $1 \div 8$ may be written as $\frac{1}{8}$, then $a \div 8$ may be written as $\frac{a}{8}$.

Also, $a \div b$ may be written as $\frac{a}{b}$ and $3 \div a$ may be written as $\frac{3}{a}$.

Fraction form is: $\frac{\text{numerator}}{\text{denominator}}$

EXAMPLE 1

Write the following expressions in fraction form.

a $4 \div 5$ **b** $m \div 9$ **c** $12 \div z$

d $5t \div 3$ **e** $vw \div 2$

a $\frac{4}{5}$ **b** $\frac{m}{9}$ **c** $\frac{12}{z}$ **d** $\frac{5t}{3}$ **e** $\frac{vw}{2}$

Exercise 7D

1 Complete to write the following expressions in fraction form.

a $2 \div 5 = \frac{2}{\square}$ **b** $t \div 5 = \frac{\square}{5}$ **c** $5 \div q = \frac{\square}{\square}$ **d** $2k \div 5 = \frac{\square}{\square}$

2 Write the following expressions in fraction form.

a $7 \div 20$	**b** $a \div 3$	**c** $m \div 4$	**d** $k \div 2$
e $z \div 4$	**f** $y \div 6$	**g** $8 \div k$	**h** $6 \div w$
i $2 \div s$	**j** $9 \div t$	**k** $ab \div 3$	**l** $pq \div 7$
m $3t \div 4$	**n** $7y \div 3$	**o** $4k \div 9$	**p** $7tv \div 2$

EXAMPLE 2

Show the meanings of the following expressions by writing each with a division sign.

a $\frac{8}{9}$ **b** $\frac{h}{2}$ **c** $\frac{15}{a}$ **d** $\frac{7d}{8}$ **e** $\frac{ab}{c}$

a $8 \div 9$ **b** $h \div 2$ **c** $15 \div a$ **d** $7d \div 8 = 7 \times d \div 8$ **e** $ab \div c = a \times b \div c$

3 Complete to show the meanings of the following expressions by writing each with a division sign.

a $\frac{3}{10} = \_\_ \div 10$ **b** $\frac{m}{6} = m \div \_\_$ **c** $\frac{8}{n} = \_\_ \div \_\_$ **d** $\frac{3w}{7} = \_\_ \div \_\_$

4 Show the meanings of the following expressions by writing each with a division sign.

a $\frac{1}{9}$ **b** $\frac{h}{3}$ **c** $\frac{m}{7}$ **d** $\frac{a}{5}$ **e** $\frac{t}{4}$

f $\frac{z}{2}$ **g** $\frac{9}{d}$ **h** $\frac{4}{z}$ **i** $\frac{2}{p}$ **j** $\frac{6}{t}$

k $\frac{pq}{2}$ **l** $\frac{km}{5}$ **m** $\frac{5t}{6}$ **n** $\frac{8b}{9}$ **o** $\frac{4t}{5}$

EXAMPLE 3

If $a = 12$ and $b = 4$, find the value of each expression.

a $\frac{a}{6}$ **b** $\frac{a}{b}$ **c** $\frac{20}{b}$ **d** $\frac{2a}{b}$ **e** $\frac{a}{3} + 1$

a $\frac{a}{6}$
$= a \div 6$
$= 12 \div 6$
$= 2$

b $\frac{a}{b}$
$= a \div b$
$= 12 \div 4$
$= 3$

c $\frac{20}{b}$
$= 20 \div b$
$= 20 \div 4$
$= 5$

d $\frac{2a}{b}$
$= 2 \times a \div b$
$= 2 \times 12 \div 4$
$= 24 \div 4$
$= 6$

e $\frac{a}{3} + 1$
$= a \div 3 + 1$
$= 12 \div 3 + 1$
$= 4 + 1$
$= 5$

5 If $x = 3$ and $y = 12$, complete to find the value of each expression.

a $\frac{y}{2} = y \div \_\_\_$
$= \_\_\_ \div \_\_\_$
$= \_\_\_$

b $\frac{y}{x} = \_\_\_ \div x$
$= \_\_\_ \div \_\_\_$
$= \_\_\_$

c $\frac{6x}{y} = 6x \div \_\_\_$
$= (6 \times x) \div \_\_\_$
$= (6 \times \_\_\_) \div \_\_\_$
$= \_\_\_ \div \_\_\_ = \_\_\_$

d $\frac{2y}{8x} = \_\_\_ \div 8x$
$= (2 \times y) \div (8 \times x)$
$= (2 \times \_\_\_) \div (8 \times \_\_\_)$
$= \_\_\_ \div \_\_\_ = \_\_\_$

e $5 - \frac{y}{6} = 5 - y \div \_\_\_$
$= 5 - \_\_\_ \div \_\_\_$
$= 5 - \_\_\_$
$= \_\_\_$

6 If $m = 20$, $n = 5$ and $p = 3$, find the value of:

a $\frac{m}{10}$ **b** $\frac{n}{5}$ **c** $\frac{p}{3}$ **d** $\frac{15}{n}$ **e** $\frac{9}{p}$

f $\frac{40}{m}$ **g** $\frac{2p}{6}$ **h** $\frac{2m}{8}$ **i** $\frac{5p}{n}$ **j** $\frac{3m}{10}$

k $\frac{m}{4n}$ **l** $\frac{p}{3} + 2$ **m** $\frac{m}{10} - 3$ **n** $3 + \frac{m}{20}$ **o** $7 - \frac{3n}{5}$

The convention for the order of operations in a numerical expression involving multiplication and division only is to work from left to right. For example, $20 \div 5 \times 2 = (20 \div 5) \times 2 = 4 \times 2 = 8$.

If we follow this convention, $a \div bc$ would mean $(a \div b) \times c$ or $\frac{a}{b} \times c$.

In algebra, we make an exception to this convention. We take $a \div bc$ to mean $a \div (bc)$ or $a \div (b \times c)$ or $\frac{a}{bc}$.

EXAMPLE 4

Write the following expressions in fraction form.

a $5 \div 3m$ **b** $2x \div 5y$ **c** $x \div yz$

a $5 \div 3m = \frac{5}{3m}$ **b** $2x \div 5y = \frac{2x}{5y}$ **c** $x \div yz = \frac{x}{yz}$

7 Write the following expressions in fraction form.

a $4 \div 7k$ **b** $3p \div 4q$ **c** $t \div uv$ **d** $xy \div 3z$ **e** $4m \div pq$

8 Show the meaning of the following expressions by writing them with a division sign.

a $\frac{6}{5m}$ **b** $\frac{2w}{7z}$ **c** $\frac{k}{mn}$ **d** $\frac{uv}{3z}$ **e** $\frac{9p}{qr}$

9 If $x = 2$ and $y = 3$, find the value of:

a $\frac{12}{3x}$ **b** $\frac{6x}{4y}$ **c** $\frac{18}{xy}$ **d** $\frac{10y}{3x}$ **e** $\frac{8y}{xy}$

E Expressions involving grouping symbols

We can leave out the multiplication sign between a numeral and a grouping symbol, or between a pronumeral and a grouping symbol.

EXAMPLE 1

Simplify each expression by omitting the multiplication sign.

a $3 \times (m + 5)$ **b** $(7 - b) \times 9$ **c** $2p \times (3q - 5m)$ **d** $(z + 4) \times w$

3(m + 5) means that the value of (m + 5) is multiplied by 3.

a $3 \times (m + 5)$
$= 3(m + 5)$

b $(7 - b) \times 9$
$= (7 - b)9$ usually written as $9(7 - b)$

c $2p \times (3q - 5m)$
$= 2p(3q - 5m)$

d $(z + 4) \times w$
$= (z + 4)w$ usually written as $w(z + 4)$

Exercise 7E

1 Simplify each expression by omitting the multiplication sign(s).

a $3 \times (f + 9)$ **b** $(n - 4) \times 7$ **c** $(s - 6) \times 4$
d $d \times (z + 1)$ **e** $(k - 7) \times m$ **f** $y \times (4 + p)$
g $2 \times p \times (4 \times q - 3)$ **h** $7 \times t \times (8 - 5 \times w)$ **i** $4 \times z \times (3 \times w + 2 \times y)$

EXAMPLE 2

Show the meaning of each expression by inserting multiplication sign(s).

a $2(v + 5)$ **b** $a(b - 1)$ **c** $3k(2k - 7m)$

a $2(v + 5)$
$= 2 \times (v + 5)$

b $a(b - 1)$
$= a \times (b - 1)$

c $3k(2k - 7m)$
$= 3k \times (2k - 7m)$
$= 3 \times k \times (2 \times k - 7 \times m)$

2 Show the meaning of each expression by inserting multiplication sign(s).

a $4(b + 1)$ **b** $2(m - 5)$ **c** $3(s + 4)$ **d** $w(z + 6)$ **e** $k(k - 1)$
f $5t(t + 8)$ **g** $z(2z - 4)$ **h** $2a(3b + 2)$ **i** $7p(3q - 5)$ **j** $6k(7m + 3n)$

EXAMPLE 3

Given $k = 5$ and $m = 6$, evaluate each expression.

Evaluate inside brackets first.

a $3(k + 2)$ **b** $4(3 - m)$ **c** $5m(k - 3)$

a $3(k + 2)$
$= 3 \times (k + 2)$
$= 3 \times (5 + 2)$
$= 3 \times 7$
$= 21$

b $4(3 - m)$
$= 4 \times (3 - m)$
$= 4 \times (3 - 6)$
$= 4 \times (-3)$
$= -12$

c $5m(k - 3)$
$= 5m \times (k - 3)$
$= 5 \times m \times (k - 3)$
$= 5 \times 6 \times (5 - 3)$
$= 5 \times 6 \times 2 = 60$

3 Given $a = 3$, $b = 4$ and $c = 5$, complete to evaluate each expression.

a $3(b + 7)$
$= 3 \times (b + 7)$
$= 3 \times (\_\_\_ + 7)$
$= 3 \times \_\_\_ = \_\_\_$

b $a(b + c)$
$= a \times (b + c)$
$= \_\_\_ \times (\_\_\_ + \_\_\_)$
$= \_\_ \times \_\_\_ = \_\_\_$

c $2c(4a - 7)$
$= 2 \times c \times (4 \times a - 7)$
$= 2 \times \_\_\_ \times (4 \times \_\_\_ - 7)$
$= \_\_\_ \times \_\_\_ \times (\_\_\_ - 7)$
$= \_\_\_ \times \_\_\_ \times \_\_\_ = \_\_\_$

d $ab(2 - c)$
$= a \times b \times (2 - c)$
$= \_\_\_ \times \_\_\_ \times (2 - \_\_\_)$
$= \_\_\_ \times \_\_\_ \times (\_\_\_)$
$= \_\_\_$

4 Given $r = 2$, $s = 4$ and $t = 5$, evaluate the following.

a $4(s - 1)$ **b** $7(t + 1)$ **c** $6(r + 2)$ **d** $8(t - 4)$
e $10(s - 2)$ **f** $5(r - 10)$ **g** $9(s - 3)$ **h** $t(s - 5)$
i $s(r + t)$ **j** $3r(t - 1)$ **k** $2s(8 - t)$ **l** $rs(8s - 4t)$

5 **a** **i** Given $m = 5$, evaluate $2(m + 3)$ and $2m + 3$. **ii** Does $2(m + 3)$ mean the same as $2m + 3$?
b **i** Given $p = 4$, evaluate $5(p - 1)$ and $5p - 1$. **ii** Does $5(p - 1)$ mean the same as $5p - 1$?
c **i** Predict whether $10(k + 2) = 10k + 2$. **ii** Test your prediction by evaluation, given $k = 3$.

EXAMPLE 4

Write each expression in fraction form.

a $(z + 3) \div 5$ **b** $w \div (k + 3)$ **c** $(a - 4) \div (b + 9)$

a $(z + 3) \div 5$
$= \dfrac{z + 3}{5}$

b $w \div (k + 3)$
$= \dfrac{w}{k + 3}$

c $(a - 4) \div (b + 9)$
$= \dfrac{a - 4}{b + 9}$

6 Complete to write each expression in fraction form.

a $(p + 5) \div 3 = \dfrac{p + 5}{\square}$ **b** $(4m - 3) \div 10 = \dfrac{\square}{10}$

c $8 \div (a + 3) = \dfrac{\square}{\square}$ **d** $(m + 3) \div (p + 2) = \dfrac{\square}{\square}$

7 Write the following expressions in fraction form.

a $(b + 3) \div 7$ **b** $(m + n) \div 2$ **c** $(3k + 1) \div 5$
d $(a + 6) \div b$ **e** $15 \div (4a)$ **f** $6 \div (5y)$
g $10 \div (b + 1)$ **h** $12 \div (w + 5)$ **i** $p \div (q - 3)$
j $(t + 12) \div (p + q)$ **k** $(3a - 2b) \div (d + 1)$ **l** $(4d - 1) \div 7e$

EXAMPLE 5

If $v = 7$ and $w = 4$, find the value of:

a $\frac{v+1}{2}$ **b** $\frac{16}{v+1}$ **c** $\frac{v-13}{w+2}$ **d** $\frac{w+6}{12-v}$.

Evaluate each numerator and denominator, before dividing.

a $\frac{v+1}{2}$
$= \frac{7+1}{2}$
$= \frac{8}{2} = 4$

b $\frac{16}{v+1}$
$= \frac{16}{7+1}$
$= \frac{16}{8} = 2$

c $\frac{v-13}{w+2}$
$= \frac{7-13}{4+2}$
$= \frac{-6}{6} = -1$

d $\frac{w+6}{12-v}$
$= \frac{4+6}{12-7}$
$= \frac{10}{5} = 2$

8 If $a = 3$ and $b = 5$, complete to find the value of each expression.

a $\frac{a+7}{2} = \frac{\square + 7}{2}$
$= \frac{\square}{2}$
$=$ ___

b $\frac{16}{a+b} = \frac{16}{\square + \square}$
$= \frac{16}{\square}$
$=$ ___

c $\frac{b+1}{a-1} = \frac{\square + 1}{\square - 1}$
$= \frac{\square}{\square}$
$=$ ___

d $\frac{a+11}{12-b} = \frac{\square + 11}{12 - \square}$
$= \frac{\square}{\square}$
$=$ ___

9 If $d = 20$, $e = 5$ and $f = 12$, find the value of each expression.

a $\frac{f-2}{5}$ **b** $\frac{d+4}{6}$ **c** $\frac{10-d}{2}$ **d** $\frac{f+2}{7}$ **e** $\frac{10}{6-e}$ **f** $\frac{15}{f+3}$

g $\frac{f}{22-d}$ **h** $\frac{50}{2e}$ **i** $\frac{12-e}{e+2}$ **j** $\frac{f-6}{e+1}$ **k** $\frac{5-d}{f-7}$ **l** $\frac{d+f}{e+3}$

10 Given the values in this table, evaluate the following expressions.

a	b	c	d	e
6	2	8	0	10

a $a + b + c$ **b** $a - b + c$ **c** $3a$ **d** $9d$ **e** $5c$

f b^2 **g** $4ab$ **h** $\frac{a}{3}$ **i** $\frac{e}{b}$ **j** $5a + 1$

k $3e - 7$ **l** $9 + 3b$ **m** $100 - 5c$ **n** $bc - 5a$ **o** $abcde$

p $5(b + 4)$ **q** $2e(c - 7)$ **r** $3b(a + b - c)$ **s** $(c - a)(b + e)$ **t** $\frac{5a}{b}$

u $\frac{a+b}{4}$ **v** $\frac{c-e}{c-a}$ **w** $\frac{3b+14}{c+2}$ **x** $(a + b)^2$ **y** $5e^2$

11 **a** **i** Given $a = 2$, evaluate $10 \div (a + 3)$ and $\frac{10}{a} + 3$ and $\frac{10}{a+3}$.

ii Which expression means the same as $10 \div (a + 3)$?

b **i** Given $b = 6$, evaluate $(12 - b) \div 3$ and $12 - \frac{b}{3}$ and $\frac{12-b}{3}$.

ii Which expression means the same as $(12 - b) \div 3$?

c Which answer is correct for $(a + 3) \div (4 - b) =$ ___?

A $\frac{a+3}{4} - b$ **B** $a + \frac{3}{4-b}$ **C** $a + \frac{3}{4} - b$ **D** $\frac{a+3}{4-b}$

12 **i** By substituting values for the pronumerals, check which of the following statements are true.
ii Where possible, give an example showing when the statement is true and when it is false.
iii Discuss your answers with a friend.

a $k + m = m + k$ **b** $k - m = m - k$ **c** $\frac{k}{m} = \frac{m}{k}$

d $(j + k) + m = j + (k + m)$ **e** $(jk)m = j(km)$ **f** $(j + k) - m = j + (k - m)$

g $(j - k) + m = j - (k + m)$ **h** $m(j + k) = (j + k)m$ **i** $m \div (j + k) = \frac{m}{j + k}$

j $(j + k) \div m = \frac{j + k}{m}$ **k** $a \div cb = \frac{a}{bc}$ **l** $a \div bc = \frac{a}{b} \times c$

Adding and subtracting algebraic terms

EXAMPLE 1

Simplify the following by adding or subtracting.

a $4k + 6k$ **b** $7w - 3w$ **c** $4k + 7w$

a If k represents the number of marbles in a cup, then $4k$ or $4 \times k$ = number of marbles in 4 cups and $6k$ or $6 \times k$ = number of marbles in 6 cups.
Thus $4k + 6k$ = number of marbles in $(4 + 6)$ cups $= 10 \times k$ or $10k$.

b If w represents the number of marbles in a box, then $7w$ or $7 \times w$ = number of marbles in 7 boxes and $3w$ or $3 \times w$ = number of marbles in 3 boxes.
Thus $7w - 3w$ = number of marbles in $(7 - 3)$ boxes $= 4 \times w$ or $4w$.

c If k represents the number of marbles in a cup and if w represents the number of marbles in a box, then $4k + 7w$ = number of marbles in 4 cups plus the number of marbles in 7 boxes.
Because the units (cups and boxes) are different, we cannot simplify this algebraic expression.

Algebraic terms with the same pronumerals (same units) are called *like terms*. Like algebraic terms can be added or subtracted.
Algebraic terms with different pronumerals (different units) are called *unlike terms*. Unlike terms cannot be added or subtracted. For example:

- $5a$, $14a$ and $23a$ are like terms (same pronumeral a)
- $5a$ and $14a^2$ are unlike terms (different pronumeral forms, a and a^2)
- $5p$, $14a^2q$ and $73d$ are unlike terms (different pronumerals p, a^2q, d)

EXAMPLE 2

Simplify the following where possible.

a $7x + 3x$ **b** $9w - 3w$ **c** $7x + 9w$ **d** $8y - y$ **e** $6k + 7$

a $7x + 3x = 7 \times x + 3 \times x$
$= (7 + 3) \times x$
$= 10 \times x = 10x$

b $9w - 3w = 9 \times w - 3 \times w$
$= (9 - 3) \times w$
$= 6 \times w = 6w$

c $7x + 9w$ cannot be simplified as $7x$ and $9w$ are unlike terms.

d $8y - y = 8 \times y - 1 \times y$
$= (8 - 1) \times y = 7 \times y = 7y$

e $6k + 7$ cannot be simplified as $6k$ and 7 are unlike terms.

Exercise 7F

1 Complete the following to simplify.

a $6a + 9a = \_\_\_ \times a + \_\_\_ \times a$
$= (\_\_\_ + \_\_\_) \times a$
$= \_\_\_ \times a = \_\_\_$

b $14y - 6y = \_\_\_ \times y - \_\_\_ \times y$
$= (\_\_\_ - \_\_\_) \times y$
$= \_\_\_ \times y = \_\_\_$

2 Using the following values, evaluate these expressions.

i $y = 1$ **ii** $y = 5$ **iii** $y = -2$

a $3y + 6y$ **b** $9y$

Are the values of $3y + 6y$ and $9y$ always the same?

3 Using the following values, evaluate these expressions.

i $k = 5$ **ii** $k = 8$ **iii** $k = -3$

a $7k - 3k$ **b** $4k$

Are the values of $7k - 3k$ and $4k$ always the same?

The order of pronumerals in the like terms may be different, for example xy and yx.

EXAMPLE 3

Simplify the following.

a $8n - 11n$ **b** $-3b + 7b$ **c** $-4x^2 - 5x^2$

a $8n - 11n$
$= (8 - 11) \times n$
$= -3n$

b $-3b + 7b$
$= (-3 + 7) \times b$
$= 4b$

c $-4x^2 - 5x^2$
$= (-4 - 5) \times x^2$
$= -9x^2$

4 Simplify where possible.

a $4a + 7a$ **b** $6t + 5t$ **c** $8m + m$
d $10p - 3p$ **e** $8x - 5x$ **f** $4b - b$
g $6a + 3b$ **h** $7y - 4z$ **i** $11t^2 + 6t^2$
j $8m^2 - 4m^2$ **k** $9k + 5m$ **l** $5ab + 6ab$
m $5ab + 2bc$ **n** $6ab^2 - 4ab^2$ **o** $3a^2b + 5ab^2$
p $9pq - 9qp$ **q** $6g^3 + g^3$ **r** $p + p$
s $9x + 24$ **t** $24st^2 - 15t^2s$ **u** $7mn^2 - 3n^2m$
v $32x^2y - 21yx^2$ **w** $8f - 7f$ **x** $11c - c$

5 Collect like terms to simplify the following.

a $4k - 7k$
$= (\_\_\_ - \_\_\_) \times k$
$= \_\_\_$

b $-2w + 5w$
$= (\_\_\_ + \_\_\_) \times w$
$= \_\_\_$

c $-3t - 2t$
$= (\_\_\_ - \_\_\_) \times t$
$= \_\_\_$

6 Simplify by collecting like terms.

a $5n - 6n$ **b** $8p - 10p$ **c** $-7c + 5c$ **d** $-4q - 2q$
e $-2d + 5d$ **f** $3ac - 12ac$ **g** $6q - 9q$ **h** $-c + 8c$
i $yz - 5yz$ **j** $-9L + 14L$ **k** $-4p^2 + 3p^2$ **l** $15x^2 - 17x^2$
m $3xy^2 - 5xy^2$ **n** $-3pq^2 - 8pq^2$ **o** $-6t^2 + 3t^2$ **p** $b - 3b$

EXAMPLE 4

Simplify the following.

a $-4f - 5f + 6f$ **b** $qr - 8rq + 3rq$

a $-4f - 5f + 6f$
$= -9f + 6f$
$= -3f$

b $qr - 8rq + 3rq$
$= -7qr + 3qr$
$= -4qr$

7 Complete the following to simplify.

a $-3x - 5x + 4x$
$= \_\_\_ + 4x$
$= \_\_\_$

b $2pq + 4pq - 3pq$
$= \_\_\_ - 3pq$
$= \_\_\_$

c $2z - 6z + 3z$
$= \_\_\_ + 3z$
$= \_\_\_$

8 Simplify the following by collecting like terms.

a $7d + 3d - 4d$
b $8pq - 5pq - 6pq$
c $c + 3c - 5c$
d $9q^2 + 5q^2 - q^2$
e $-4m + m - 2m$
f $18r - 2r - 7r$
g $-a + 2a - 5a$
h $-3r + 2r - r$
i $4a^2b - 3a^2b - ba^2$
j $-17x^2y - 5x^2y + 3x^2y$
k $-d^2 + 7d^2 - 13d^2$
l $k^2m - 5k^2m + mk^2$

EXAMPLE 5

Simplify the following by collecting like terms.

a $6m - 7 + 2m$ **b** $4x^2y + 5p - 3yx^2$

a $6m - 7 + 2m$
$= (6m + 2m) - 7$
$= 8m - 7$

b $4x^2y + 5p - 3yx^2$
$= (4x^2y - 3x^2y) + 5p$
$= x^2y + 5p$

When collecting like terms remember to include the + or − sign in front of the like terms.

9 Complete the following.

a $5y - 3 + 4y$
$= (\_\_\_ + \_\_\_) - 3$
$= \_\_\_ - 3$

b $7p + 3q + 5p$
$= (\_\_\_ + \_\_\_) + 3q$
$= \_\_\_ + 3q$

10 Simplify the following by collecting like terms.

a $7 + 10 - 5m$
b $3a + 6 + 5a$
c $4 + 2n - n$
d $6 - 7q + 12$
e $9 - 5k + 4$
f $3f + 12 + 8f$
g $x^2y - 5 + 2yx^2$
h $13w - 6 - 4w$
i $a^2b + 9 + 11a^2b$
j $5q^2 - 6 + q^2$
k $5x - 4x - 2$
l $p + 3p - 9$
m $6t + 4 - 5t$
n $7k - k - 6L$
o $d^2 + 5d^2 - 3d$
p $4n - n + 2n^2$
q $3c + 2c^2 + 5c$
r $6 + 8n^2 - 5n^2$
s $11mn - 2m + 4nm$
t $5a^2b + 2ab^2 - 4a^2b$

EXAMPLE 6

Simplify by collecting like terms.

a $8 + 4z - 10$ **b** $-2w - 9t - 5w$ **c** $-6x - 3y + 2x$

a $8 + 4z - 10 = 8 - 10 + 4z$
$= -2 + 4z$

or

$8 + 4z - 10 = 4z + 8 - 10$
$= 4z - 2$

b $-2w - 9t - 5w = -2w - 5w - 9t$
$= -7w - 9t$

or

$-2w - 9t - 5w = -9t - 2w - 5w$
$= -9t - 7w$

c $-6x - 3y + 2x = -6x + 2x - 3y$
$= -4x - 3y$ (or $-3y - 4x$)

11 Simplify the following by collecting like terms.

a $6 + 5k - 9$
b $3 - 4s - 7$
c $5m + 2n - 7m$
d $2x - 8y - 5x$
e $5t - 9t + 7u$
f $2a - 4b - 7a$
g $-5d + 4e + 3d$
h $-8z - 4w + 2z$
i $-a - 3b - 2a$
j $6m - 8n - 2n$
k $-3v - 2w - v$
l $-6k + m + 2k$

EXAMPLE 7

Simplify the following by collecting like terms.

a $9x + 7 - 3x + 10$ **b** $5a + 7b - 3a + 2b$

a $9x + 7 - 3x + 10$
$= 9x - 3x + 7 + 10$
$= 6x + 17$

b $5a + 7b - 3a + 2b$
$= 5a - 3a + 7b + 2b$
$= 2a + 9b$

12 Complete the following by collecting like terms.

a $5x + 9 - 2x - 7$
$= 5x - 2x + 9 - 7$
$= \_\_\_ + \_\_\_$

b $7w - 4z - 3w + 2z$
$= 7w - 3w - 4z + 2z$
$= \_\_\_ - \_\_\_$

c $-3 + 4d + 7 - d$
$= -3 + 7 + 4d - d$
$= \_\_\_ + \_\_\_$

13 Simplify the following expressions by collecting like terms.

a $4a + 6 + 2a + 5$
b $11c^2 + c - 2c^2 + c$
c $x + y + x + y$
d $q^2 + 5 + q^2 - 3$
e $6t + 3v - 2v + 3t$
f $q + 4d + 3q - d$
g $12p + p^2 + 5p^2 - 8\text{p}$
h $-8l + 4 - 5l + 6$
i $-n - 2 - 5n - 1$
j $m^2 - m - m^2 - 3m$
k $-d^2 - 7d + 3d^2 - 4d$
l $25 + 8m - 9 + 2m$
m $-13 + 6n - n + 5$
n $6p - 3m + 6p - 3m$

14 Simplify the following by collecting like terms.

a $3c - 7 + 2d - 9 + 6c$
b $n - dc + 3cd + 5n - 8 + 2$
c $-4q - 2 + r - q + 8 + 2q$
d $7e + e^2 + 3 - e^2 - 5e + 6$
e $8 - 3c + 9 - 6 - 4c + 2d - d$
f $15l + 2 - 3l + 7c - 12c + 6$
g $-7 - c + a + 10 + c - 11 - a - c - 8$
h $d - qr + r - q - qr + 6r - 3q + 4d$
i $-12r + s + 6 - t + 5 - 9s + 12r + 7t - 3$
j $ac + 2pa - 6a + 7 - 5ap + 9ac + 12$

G Multiplying algebraic terms

EXAMPLE 1

Simplify the following.

a $p \times s$ **b** $p \times p$ **c** $3p \times s$
d $5 \times 2k$ **e** $3p \times 2s$ **f** $4pq \times 3qs$

a $p \times s = ps$ (leave out the multiplication sign between the pronumerals)

b $p \times p = pp$ or p^2

c $3p \times s = 3 \times p \times s$ (leave out the multiplication sign between the numbers and pronumerals)
$= 3ps$

d $5 \times 2k = 5 \times 2 \times k$ (split into numerical and pronumeral parts)
$= 10 \times k$
$= 10k$

e $3p \times 2s = 3 \times p \times 2 \times s$ (split into numerical and pronumeral parts)
$= 3 \times 2 \times p \times s$ (group the numbers and pronumerals together)
$= 6 \times ps$
$= 6ps$

f $4pq \times 3ps = 4 \times p \times q \times 3 \times p \times s$ (split into numerical and pronumeral parts)
$= 4 \times 3 \times p \times q \times p \times s$ (group the numbers and pronumerals together)
$= 4 \times 3 \times p \times p \times q \times s$ (order the pronumerals alphabetically)
$= 12 \times p^2qs$
$= 12p^2qs$

To multiply algebraic terms, multiply the numbers and multiply the pronumerals. We usually write the pronumerals in alphabetical order.

Exercise 7G

1 Complete the following to simplify.

a $5t \times w = \_\_\_ \times \_\_\_ \times w$
$= \_\_\_$

b $4 \times 3m = 4 \times \_\_\_ \times \_\_\_$
$= \_\_\_$

c $7y \times 2 = \_\_\_ \times y \times \_\_\_$
$= \_\_\_ \times \_\_\_ \times y$
$= \_\_\_$

d $3p^2 \times 7q = \_\_\_ \times p^2 \times \_\_\_ \times q$
$= \_\_\_ \times \_\_\_ \times p^2 \times q$
$= \_\_\_$

e $2ab \times 3a = \_\_\_ \times a \times b \times \_\_\_ \times a$
$= \_\_\_ \times \_\_\_ \times a \times a \times b = \_\_\_$

2 Simplify the following.

a $4x \times y$ **b** $3k \times m$ **c** $x \times 5y$
d $4 \times 7w$ **e** $5 \times 4k$ **f** $6 \times 10p$
g $2x \times 8$ **h** $6z \times 3$ **i** $3m \times 4n$
j $6v \times 2w$ **k** $4p^2 \times 7q$ **l** $5a \times 6b^2$
m $4ab \times 5c$ **n** $3xz \times 6xy$ **o** $10pq \times 2qr$
p $5bc \times 7bc$ **q** $2 \times 3a \times 4b$ **r** $3 \times 5k \times 2m$
s $2a \times 3b \times 4c$ **t** $4p \times 5q \times 2r$ **u** $3a \times 4a \times 3c$

H Dividing algebraic terms

When dividing algebraic terms containing pronumerals and numbers, follow these steps.

Step 1: Write the division as a fraction.

Step 2: Cancel the numbers and pronumerals if possible.

Step 3: Write your answer as a fraction.

Notice that $\frac{3}{1} = 3$, but $\frac{1}{3} \neq 3$.

EXAMPLE 1

Simplify the following.

a $5x \div 10$ **b** $21x \div 7x$ **c** $\frac{5f}{15f}$ **d** $\frac{8xy}{6y}$

a $5x \div 10 = \frac{{}^{1}\cancel{5}x}{\cancel{10}_{2}} = \frac{x}{2}$

b $21x \div 7x = \frac{{}^{3}\cancel{21}x}{\cancel{7}x_{1}} = 3$

c $\frac{{}^{1}\cancel{5}f}{{}^{3}\cancel{15}f} = \frac{1}{3}$

d $\frac{{}^{4}\cancel{8}xy}{{}^{3}\cancel{6}y} = \frac{4x}{3}$

Exercise 7H

1 Complete the following.

a $10y \div 15 = \frac{10y}{\square} = \frac{2y}{\square}$

b $8m \div 12m = \frac{\square}{12m} = \frac{\square}{3}$

c $6x \div 8xy = \frac{\square}{8xy} = \frac{\square}{4y}$

2 Simplify the following.

a $9x \div 18$ **b** $3m \div 12$ **c** $5p \div 25$ **d** $16d \div 4$ **e** $\frac{10c}{2}$

f $\frac{8a}{4}$ **g** $\frac{6a}{12a}$ **h** $\frac{44m}{22m}$ **i** $\frac{12a}{15a}$ **j** $\frac{20d}{10d}$

k $\frac{3f}{9f}$ **l** $\frac{4t}{20t}$ **m** $\frac{18p}{20d}$ **n** $\frac{6xy}{15x}$ **o** $\frac{24ab}{36bc}$

p $\frac{16r}{20qr}$ **q** $\frac{8yz}{40xyz}$ **r** $\frac{70dkl}{10klm}$ **s** $\frac{15pqr}{12q}$ **t** $\frac{14mn}{35mp}$

I Words into algebraic expressions

EXAMPLE 1

Translate the following word statements into algebraic expressions.

a the sum of A and B
b L divided by 7
c a number that exceeds x by k
d the product of 8 and m
e the sum of H and 9 multiplied by 2
f a number 4 times greater than D
g the difference between Q and R
h half of D

a $A + B$ **b** $L \div 7$ or $\frac{L}{7}$ **c** $x + k$ **d** $8 \times m$ or $8m$

e $2(H + 9)$ **f** $4D$ **g** $Q - R$ **h** $\frac{1}{2} \times D$ or $\frac{1}{2}D$ or $\frac{D}{2}$

Exercise 7I

1 Complete the following statements.

a The sum of 3 and 5 is ___ + ___, hence the sum of p and q is ___ + ___.

b The product of 2 and 6 is ___ × ___, hence the product of k and m is ___ × ___.

c The difference between 10 and 7 is ___ − ___, hence the difference between x and y is ___ − ___.

d 5 more than 4 is ___ + ___, hence 5 more than w is ___ + ___.

e 2 less than 8 is ___ − ___, hence 2 less than c is ___ − ___.

f 9 squared is ___, hence k squared is ___.

g 2 cubed is ___, hence z cubed is ___.

h Double 7 is ___, hence double a is ___.

i A half of 10 is ___, hence a half of b is ___.

j A quarter of 12 is ___, hence a quarter of w is ___.

2 Write algebraic expressions for the following statements.

a the sum of m and n

b the product of 6 and q

c 4 more than x

d decrease y by 2

e increase z by 1

f 5 less than p

g double k

h the difference between w and 2

i n squared

j t cubed

k half of p

l a third of z

3 Write the following in algebraic form.

a double x, add 7

b twice the sum of b plus 5

c three times y minus 4

d half of p plus 11

e product of T squared and 3

f J more than 8

g four times the difference of p and s

h y minus three is multiplied by 10. This is then divided by x.

i the average of x and y

4 Write the following in algebraic form.

a m squared times j

b fifteen minus the product of b and three

c a quarter of k plus the product of seven and q

d the square root of d divided by two

e four less than the product of eight and f

f the whole of twice t minus four, cubed

5 Write an expression for:

a	the number of cents in	**i** \$3	**ii** \$$x$
b	the number of centimetres in	**i** 5 m	**ii** y m
c	the number of grams in	**i** 4 kg	**ii** z kg
d	the number of seconds in	**i** 6 min	**ii** k min

6 Write an expression for each of the following.

a There are 12 people on a bus and x get off. How many people are left on the bus?

b A train has 88 passengers and at the next station y people get on. How many passengers are now on the train?

c A can of lemonade costs c cents. How much do 5 cans of lemonade cost in:

i cents **ii** dollars?

d If my ferry pass cost z dollars, how much change would I receive from \$10.00?

e There are E apples in a bag. If I take out seven, how many apples remain?

f A length of rope is x metres long. If I cut off four lengths of y metres each, how much rope remains?

7 Allison buys p pencils and b books. Find the total cost:

a in cents, if each pencil costs 45c and each book costs 85c

b in dollars, if each pencil costs 52c and each book costs 96c

8 Fred travelled a km in 4 hours and b km in the next 3 hours.

a How far did Fred travel altogether?

b What is Fred's average speed for the whole trip?

9 Complete the following.

a If a car travels 200 km in 4 hours, its average speed is ____.

b If a car travels 200km in z hours, its average speed is ____.

c If a car travels p km in t hours, its average speed is ____.

10 Complete the following.

a The perimeter of a rectangle of length 5 cm and breadth 3 cm is $2 \times$ ___ $+ 2 \times$ ___ $=$ ___

b The perimeter of a rectangle of length x cm and breadth 2 cm is $2 \times$ ___ $+ 2 \times$ ___ $=$ ___

11 Write the meaning of the following algebraic expressions.

a $x + y$	**b** $x - y$	**c** xy	**d** $2x$
e $2x + 7$	**f** $2(x + 7)$	**g** $\frac{x}{3}$	**h** $\frac{x}{3} + 1$
i $\frac{x+1}{3}$	**j** $\frac{x-y}{4}$	**k** x^2	**l** $\sqrt{x}$

Investigation 1 Spreadsheet formulas

What operation(s) are performed when the following statements are typed into the formula bar of an Excel spreadsheet.

a =A4+B4	**b** =3*C2	**c** =A3*B3	**d** =B5/3
e A6/B6	**f** =SUM(B1:B10)	**g** =AVERAGE(C3:C8)	

Calculator activities

1 If $a = 35$, $b = 61$ and $c = 7$, find the value of:

a $8a$ **b** $9b + 17$ **c** $9a - 14$ **d** $\frac{6a}{5}$ **e** $\frac{a - b}{13}$

2 If $w = \frac{3}{4}$, $v = 2\frac{1}{2}$ and $z = \frac{5}{6}$, evaluate:

a $2w + v$ **b** $\frac{w}{z}$ **c** vwz **d** $v - z$ **e** v^2

3 If $k = 0.75$, $m = 1.2$ and $n = 0.46$, evaluate:

a $m - n$ **b** $13k + 4m$ **c** $-\frac{m}{k}$ **d** $2.8(3m + 4n)$ **e** $(k - n)^2$

Language in mathematics

1 Find-a-word puzzle. Find the following words in the puzzle below:

abbreviation, algebra, algebraic, alternative, area, brought, combine, expression, function, grouping, identical, index, indices, notation, process, pronumeral, raise, result, reverse, review, roll, rule, simplify, statement, substitution, symbol, value.

R	A	B	B	R	E	V	I	A	T	I	O	N	C	B	A
O	L	A	D	O	E	A	G	R	E	V	I	E	W	F	G
T	G	S	H	L	I	L	R	U	J	D	N	A	P	X	E
C	E	E	K	L	M	U	O	L	N	O	I	N	D	E	X
A	B	T	R	U	P	E	U	E	S	S	E	C	O	R	P
F	R	S	V	Q	X	W	P	Y	E	S	R	E	V	E	R
Z	A	S	U	B	S	T	I	T	U	T	I	O	N	A	E
L	I	I	B	H	D	F	N	J	K	L	O	B	M	Y	S
A	C	M	I	C	G	E	G	N	M	O	L	R	P	A	S
C	R	P	R	O	N	U	M	E	R	A	L	O	Q	R	I
I	T	L	S	N	O	I	T	A	T	O	N	U	T	B	O
T	L	I	N	D	I	C	E	S	U	V	E	G	W	E	N
N	U	F	U	N	C	T	I	O	N	X	S	H	Y	G	A
E	S	Y	A	C	O	M	B	I	N	E	I	T	Z	L	E
D	E	B	Q	D	T	N	E	M	E	T	A	T	S	A	R
I	R	R	C	P	E	V	I	T	A	N	R	E	T	L	A

2 Explain the meanings of:

a alternative and alternate **b** bought and brought **c** roll and role

3 Complete the following sentences using words from the list (use one word twice):

numeral, two, pronumerals, multiplication, expressions, variables, algebraic.

a When letters are used to take the place of numbers, they are called _____. Sometimes they are referred to as _____ because the numbers that they replace can vary. Mathematical statements connecting letters used in this way are called _____ _____.

b When simplifying algebraic expressions, the _____ sign may be left out only when there is no chance of confusion about the meaning of the expression. That is, it may be left out between a _____ and a pronumeral or between two _____. It may never be left out between _____ numerals.

4 Three of the words in the following list are spelt incorrectly. Find these words and write the correct spelling.
identicle notation pronumeral simplefy symbal value

5 Write in your own words the meaning of:
a insert **b** omit **c** substitute

6 Complete the following words used in this chapter by adding the vowels.
a __ l g __ b r __ **b** p r __ c __ s s **c** __ b b r __ v __ __ t __
d m __ l t __ p l __ c __ t __ __ n **e** __ v __ l __ __ t __ **f** c __ n v __ n t __ __ n

7 How many words of three or more letters can you make from the word SUBSTITUTION?
(No proper names or plurals allowed.)

8 Give two examples of grouping symbols.

9 Explain in words, with examples, the meaning of like and unlike terms.

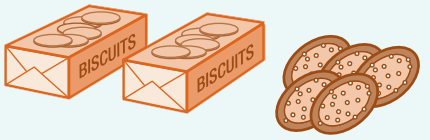
10 Three of the words in the following list are spelt incorrectly. Find these words and write the correct spelling.
algebrake difference distributive numericle remainder variabel

Terms

addition	algebra	algebraic	convention	difference	division	expression
fraction form	grouping	half	like terms	multiplication	numerical	pronumeral
product	replace	represent	simplify	substitution	subtraction	sum
symbol	translate	unlike terms	variable			

Check your skills

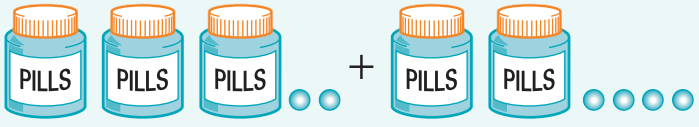
1 The diagram shows 2 packets and 5 biscuits. If there are n biscuits in each packet, an expression for the total number of biscuits is:



A $7 \times n$ **B** 7
C $2 \times n + 5$ **D** $5 \times n + 2$

2 The algebraic expression $y + y + y + y + y$ is equivalent to:
A y^5 **B** $5 + y$ **C** $5 \times y$ **D** $yyyyy$

3 Using the diagram shown below, if there are k pills in each bottle, then $(3 \times k + 2) + (2 \times k + 4) =$
A $11 \times k$ **B** $5 \times k + 6$ **C** $6 \times k + 6$ **D** $6 \times k + 8$



4 The value of the expression $5 \times z + 2$ if $z = 3$ is:
A 25 **B** 10 **C** 17 **D** 55

5 When simplified, $1 \times 5y =$

A $15y$ **B** $6y$ **C** 1 **D** $5y$

6 When simplified, $6z \times 0 =$

A $6z$ **B** $60z$ **C** 0 **D** 1

Use the values $x = 5$ and $y = 2$ for questions **7** to **9**.

7 The value of $3xy$ is:

A 352 **B** 30 **C** 90 **D** 13

8 The value of $3x - 2y$ is:

A 13 **B** 31 **C** 3 **D** 11

9 The value of $2x^2$ is:

A 625 **B** 20 **C** 50 **D** 100

10 Which of the following expressions is *not* equivalent to $\frac{4m}{5n}$?

A $4m \div 5n$ **B** $4 \times m \div 5n$ **C** $4 \times m \div (5 \times n)$ **D** $5n \div 4m$

11 If $k = 4$, then $\frac{6k}{2} =$

A 32 **B** 12 **C** 34 **D** 62

12 If $k = 8$, the value of $5(k - 2)$ is:

A 30 **B** 38 **C** 56 **D** 6

13 If $k = 8$, the value of $(10 - k) \times k$ is:

A -54 **B** 16 **C** 72 **D** 54

14 If $k = 6$, the value of $\frac{k + 18}{6}$ is:

A 19 **B** 9 **C** 4 **D** 18

15 If $k = 6$, the value of $\frac{k}{10 - k}$ is:

A 24 **B** $-\frac{9}{10}$ **C** 26 **D** $1\frac{1}{2}$

16 $4a + 3a =$

A $7aa$ **B** $43a$ **C** $7a$ **D** $7aa$

17 $5k - k =$

A 4 **B** 5 **C** $4kk$ **D** $4k$

18 $4w + 3v - 6w =$

A $10w + 3v$ **B** $10w - 3v$ **C** $2w + 3v$ **D** $-2w + 3v$

19 $3w - 5 + 4w - 1 =$

A $7w - 6$ **B** $7w - 4$ **C** $-w - 6$ **D** $-w - 4$

20 $-4x \times -5y =$

A $-20xy$ **B** $45xy$ **C** $20xy$ **D** $9xy$

21 $8 \div 2k =$

A $4k$ **B** $\frac{4}{k}$ **C** $\frac{1}{4k}$ **D** $\frac{k}{4}$

22 $\frac{-8x^2}{12xy} =$

A $\frac{-2x}{3y}$ **B** $\frac{2x}{3y}$ **C** $\frac{2}{3xy}$ **D** $\frac{2y}{3x}$

23 Three lengths of b metres are cut off a piece of rope which is a metres long. The length of rope remaining is:

A $\frac{3a}{b}$ **B** $\frac{a}{3b}$ **C** $a - 3b$ **D** $3b - a$

24 If bread costs \$$m$ per loaf and milk costs \$$n$ per litre, the total cost of 2 loaves of bread and 3 litres of milk, in dollars, is:

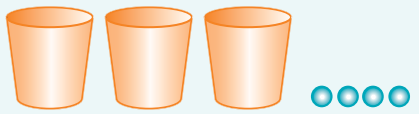
A $3m + 3n$ **B** $2m + 3n$ **C** $5m + 5n$ **D** $5mn$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4	5–9	10, 11	12–15	16–19	20	21, 22	23, 24
Section	A	B	C	D	D	D	D	D	E

7A Review set

1 The diagram shows 3 cups and 4 marbles. How many marbles are there altogether if each cup contains:



a 2 marbles? **b** 5 marbles?

c x marbles? **d** z marbles?

2 Simplify the following.

a $5 \times y$ **b** $x \times y$ **c** $x \times x$ **d** $3 \times a \times b$ **e** $6 \times m - 2$

3 Show the meanings of the following expressions by inserting multiplication signs.

a $3k$ **b** mn **c** y^2 **d** $5pq$ **e** $3z^2$

4 Simplify the following.

a $p + p + p + p$ **b** $t + t + t + t + t + t$ **c** $1 \times w$

d $3x \times 1$ **e** $6z \times 0$ **f** $m + n + n$

5 If $x = 3$ and $y = 4$, evaluate the following.

a $5x$ **b** $2x + 3y$ **c** $-2xy$ **d** x^2 **e** $2x^2$

6 Write the following in fraction form.

a $z \div 4$ **b** $m \div n$ **c** $3 \div k$ **d** $5x \div 7$ **e** $y^2 \div z$

7 Show the meanings of the following expressions by writing each with a division sign.

a $\frac{k}{2}$ **b** $\frac{2}{k}$ **c** $\frac{x}{y}$ **d** $\frac{3t}{4}$ **e** $\frac{2m}{3n}$

8 If $m = 12$ and $n = 4$, evaluate the following.

a $\frac{m}{3}$ **b** $\frac{16}{n}$ **c** $\frac{m}{n}$ **d** $\frac{2m}{n}$ **e** $\frac{m}{3n}$

9 If $k = 2$ and $m = 3$, find the value of:

a $4(k + 5)$ **b** $5(m - 1)$ **c** $m(5 - m)$ **d** $3k(2 - m)$ **e** $km(k + m)$

10 If $m = 24$ and $n = 8$, evaluate:

a $\frac{n + 7}{3}$ **b** $\frac{m - 4}{5}$ **c** $\frac{m}{10 - n}$ **d** $\frac{m + 4}{n - 1}$ **e** $\frac{6 - m}{n + 1}$

11 Simplify the following.

a $4s + 7s$ **b** $10k - 4k$ **c** $5w + 3 + 2w + 8$

d $5p - 2q + 3p$ **e** $7x + 1 - 4x + 7$ **f** $3a - 2b - 4a + b$

12 Simplify the following.

a $4x \times 5y$ **b** $-2p \times 7$ **c** $-3a \times 6b$

13 Simplify the following.

a $\frac{8h}{2}$ **b** $\frac{g}{3g}$ **c** $\frac{12a}{4a}$ **d** $\frac{14w}{21vw}$ **e** $\frac{6a}{9ab}$

14 Write algebraic expressions for:

a the sum of a and b **b** the product of 9 and k **c** 3 more than m

d 7 less than p **e** the average of a and b **f** the difference between n and 5

15 Write in words the meaning of:

a $p + q$ **b** $p - q$ **c** pq **d** $\frac{p}{2}$ **e** p^2

7B Review set

1 The diagram shows 2 cartons and 3 cans. How many cans are there if each carton contains:

a 6 cans? **b** 12 cans?

c m cans? **d** y cans?

2 Simplify the following.

a $3 \times z$ **b** $m \times n$ **c** $y \times y$ **d** $3 \times p \times q$ **e** $3 \times m + 2 \times n$

3 Insert multiplication sign(s) to show the meaning of:

a $5t$ **b** xy **c** h^2 **d** $3h^2$ **e** $4fg$

4 Simplify the following.

a $k + k + k$ **b** $m + m + m + m + m$ **c** $1 \times p$ **d** $4q \times 1$ **e** $ab \times 0$

5 If $p = 4$ and $q = 5$, evaluate the following.

a $3p$ **b** $2pq$ **c** $2q - 6p$ **d** p^2 **e** $3p^2$

6 Write the following in fraction form.

a $k \div 2$ **b** $k \div m$ **c** $m \div k$ **d** $3t \div 5$ **e** $xy \div z$

7 Show the meanings of the following expressions by writing each with a division sign.

a $\frac{m}{3}$ **b** $\frac{4}{p}$ **c** $\frac{y}{x}$ **d** $\frac{3x}{5}$ **e** $\frac{3p}{4q}$

8 If $x = 10$ and $y = 6$, evaluate the following.

a $\frac{x}{2}$ **b** $\frac{12}{y}$ **c** $\frac{3x}{y}$ **d** $\frac{3x}{5y}$ **e** $\frac{6x}{5y}$

9 If $p = 3$ and $q = 4$, evaluate the following.

a $3(p + 2)$ **b** $4(1 - q)$ **c** $p(q + 1)$ **d** $2q(7 - p)$ **e** $pq(p - 2)$

10 If $k = 36$ and $m = 4$, find the value of:

a $\frac{m + 6}{5}$ **b** $\frac{6 - k}{10}$ **c** $\frac{36}{m + 2}$ **d** $\frac{k + 4}{5m}$ **e** $\frac{k - 1}{9 - m}$

11 Simplify the following.

a $5q + 4q$ **b** $9m - 4m$ **c** $4a + 7 + 2a + 5$

d $7x - 4y + x$ **e** $6m - 5 - 3m + 9$ **f** $5w - 3z - 6w + 4z$

12 Simplify the following.

a $5k \times 4m$ **b** $-3d \times 5c$ **c** $-2m \times -3n$

13 Simplify the following.

a $\frac{9y}{3}$ **b** $\frac{a}{5a}$ **c** $\frac{10x}{5x}$ **d** $\frac{9ab}{12a}$ **e** $\frac{10xy}{10y}$

14 **a** Write algebraic expressions for:

i the number of millimetres in x centimetres

ii the number of minutes in w hours

b If sausages cost \$$a$ per kg and steak costs \$$b$ per kg, write an algebraic expression for the total cost of 4 kg of sausages and 2 kg of steak.

15 Write in words the meaning of:

a $w + 3$ **b** $5w$ **c** $\frac{w}{3}$ **d** $\sqrt{w}$ **e** $\frac{w + x}{2}$

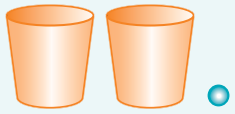
7C Review set

1 If there are z marbles in each cup, write an algebraic expression for the total number of marbles shown by each diagram.

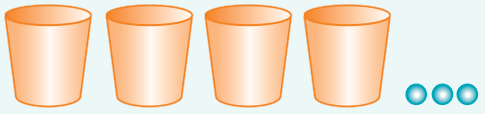
a

b

c



d



2 Simplify the following.

a $6 \times y$ **b** $g \times h$ **c** $8 \times y \times z$ **d** $4 \times m \times m$ **e** $4 \times a + 3 \times b$

3 Insert multiplication sign(s) to show the meaning of:

a $4m$ **b** pq **c** v^2 **d** $4v^2$ **e** $5mn$

4 Simplify the following.

a $p + p$ **b** $y + y + y + y + y + y + y$ **c** $z \times 1$ **d** $2pq \times 1$ **e** $0 \times 4t$

5 If $m = 5$ and $n = 2$, evaluate the following.

a mn **b** $-5mn$ **c** $7m - 3n$ **d** n^2 **e** $4n^2$

6 Write the following in fraction form.

a $t \div 3$ **b** $g \div h$ **c** $h \div g$ **d** $4w \div 3$ **e** $5 \div 2x$

7 Show the meanings of the following expressions by writing each with a division sign.

a $\frac{k}{4}$ **b** $\frac{3}{m}$ **c** $\frac{a}{b}$ **d** $\frac{4e}{5}$ **e** $\frac{mn}{p}$

8 If $p = 8$ and $q = 10$, evaluate the following.

a $\frac{q}{5}$ **b** $\frac{24}{p}$ **c** $\frac{5p}{q}$ **d** $\frac{4q}{p}$ **e** $\frac{5p}{2q}$

9 If $p = 7$ and $q = 5$, evaluate the following.

a $3(p + 1)$ **b** $4(q - 3)$ **c** $q(q + 1)$ **d** $5p(4 - q)$ **e** $pq(10 - p)$

10 If $p = 7$ and $q = 5$, evaluate the following.

a $\frac{p + 8}{3}$ **b** $\frac{q - 3}{2}$ **c** $\frac{24}{p - 15}$ **d** $\frac{p + 5}{q + 1}$ **e** $\frac{3p + 3}{q + 3}$

11 Simplify the following.

a $10h + h$ **b** $9d - d$ **c** $8z + 9 + 2z + 3$

d $7p - 3q - 2q$ **e** $4a - 3 + 3a - 4$ **f** $x - 3y - 4x + 2y$

12 Simplify the following.

a $5r \times 3s$ **b** $4g \times -3$ **c** $-4p \times -4q$

13 Simplify the following.

a $\frac{6y}{2}$ **b** $\frac{w}{3w}$ **c** $\frac{10a}{2a}$ **d** $\frac{12p}{15pq}$ **e** $\frac{8ab}{12b}$

14 Write an algebraic expression for:

a If I spend \$$p$ at the supermarket, how much change will I get from \$20?

b A bus has 15 people on board. If x people get on at the next stop, how many people are now on the bus?

c Hayley has 2 kg of flour in a jar. She uses 3 cups of flour for a recipe. If each cup holds t grams of flour, how much flour does she have left?

15 If the cost of potatoes is \$$x$ per kg and the cost of onions is \$$y$ per kg, write in words a possible interpretation of the meaning of the algebraic expression $5x + 2y$.

8

Fractions, decimals and percentages

This chapter deals with the operations of fractions, decimals and percentages.

At the end of this chapter you should be able to:

- recognise ratios and solve ratio problems using fractions
- round decimals to a specific number of decimal places
- connect fractions, decimals and percentages and carry out conversions
- find percentages of quantities
- express one quantity as a percentage of another
- investigate and calculate 'best buys'.

NUMBER & ALGEBRA – AMCNA 154, 155, 156, 157, 158, 173, 174, 184

Diagnostic test

1 $0.8 + 0.3 =$
A 0.83 **B** 0.5 **C** 1.1 **D** 2.4

2 $3.8 - 1.6 =$
A 5.4 **B** 2.4 **C** 1.2 **D** 2.2

3 $1.7 + 3.8 =$
A 5.5 **B** 4.5 **C** 2.1 **D** 6.5

4 $6 - 3.27 =$
A 2.83 **B** 3.83 **C** 3.73 **D** 2.73

5 $5.37 + 1.06 =$
A 6.97 **B** 4.31 **C** 6.43 **D** 6.31

6 $2.3 \times 4 =$
A 8.2 **B** 9.2 **C** 6.3 **D** 8.12

7 $8.15 \times 6 =$
A 48.9 **B** 48.09 **C** 48.090 **D** 138

8 $8.4 \div 2 =$
A 4.2 **B** 4.4 **C** 8.6 **D** 16.4

9 $4.7 \div 5 =$
A 0.3 **B** 1.32 **C** 0.9 **D** 0.94

10 $7.43 \div 4 =$
A 1.8575 **B** 1.85 **C** 1.857 **D** 29.72

11 $6.34 \times 100 =$
A 63 400 **B** 634 **C** 63.4 **D** 6340

12 $22.58 \div 10 =$
A 22 580 **B** 225.8 **C** 2.258 **D** 22.580

13 $0.071 \div 100 =$
A 0.000 71 **B** 0.0071
C 0.71 **D** 71

14 $0.013 \times 1000 =$
A 13 000 **B** 1300 **C** 130 **D** 13

15 $43.76 \div 1000 =$
A 0.4376 **B** 0.043 76
C 0.004 376 **D** 0.000 437 6

16 $0.5 =$
A $\frac{1}{2}$ **B** 5% **C** $\frac{1}{5}$ **D** 20%

17 $\frac{1}{4} =$
A 25% **B** 40% **C** 0.4 **D** 2.5

18 $10\% =$
A $\frac{10}{1000}$ **B** 0.1 **C** $\frac{1}{100}$ **D** 0.01

19 $0.7 =$
A 7% **B** $\frac{70}{1000}$ **C** $\frac{7}{100}$ **D** $\frac{7}{10}$

20 $0.23 =$
A $\frac{23}{100}$ **B** $\frac{23}{1000}$
C $\frac{23}{100}\%$ **D** $\frac{23}{1000}\%$

21 The discount on a \$70 dress is 10%. What is the amount of discount?
A \$7 **B** \$63 **C** \$10 **D** \$60

22 An item on sale has a discount of 25%. If the original price was \$160, what is the discount?
A \$40 **B** \$120 **C** \$25 **D** \$135

23 A \$250 camera is discounted by 50%. What is the amount of discount?
A \$500 **B** \$250 **C** \$125 **D** \$200

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–5	6–10	11–15	16–20	21–23
Section	ACMNA 128	ACMNA 129	ACMNA 130	ACMNA 131	ACMNA 132

A Fractions expressed in ratio form

A **ratio** is a comparison of like quantities. Fractions can be expressed as ratios.

Order is important.

You learned in Chapter 5 that fractions can be multiplied or simplified by cancelling (division) to form equivalent fractions. Similarly, ratios can be multiplied or simplified by division to form **equivalent ratios**.

EXAMPLE 1

John has a birthday. He eats 3 slices of cake, while Alice eats the remaining 4 slices.

a What fraction of the cake did John eat?
b Express John's share of the cake as a ratio.
c What fraction of the cake did Alice eat?
d Express Alice's share of the cake as a ratio.
e Express as a ratio John's share compared to Alice's share.
f Express as a ratio Alice's share compared to John's share.

Total number of slices $= 3 + 4 = 7$

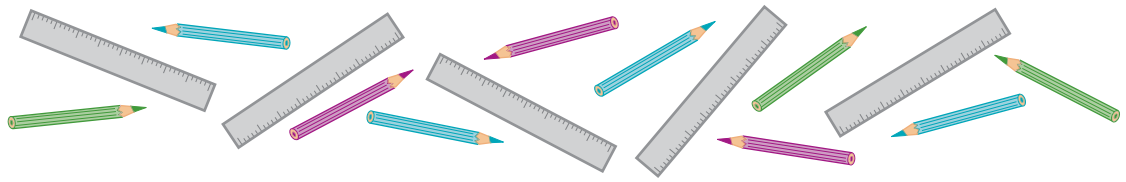
a John's share of cake $= \frac{3}{7}$
b John's share : whole cake $= 3 : 7$
c Alice's share of cake $= \frac{4}{7}$
d Alice's share : whole cake $= 4 : 7$
e John's share : Alice's share $= 3 : 4$
f Alice's share : John's share $= 4 : 3$

Exercise 8A

1

a What fraction of these shapes are stars?
b Express your answer from part **a** as a ratio.
c What fraction are squares?
d Express your answer from part **c** as a ratio.
e Express as a ratio the number of stars to the number of squares.
f Express as a ratio the number of squares to the number of stars.

2



a What fraction of these items are rulers?
b Simplify your answer from part **a**.
c **i** Express your answers from parts **a** and **b** as ratios.
ii Are these ratios equivalent?

You should be able to link equivalent fractions to equivalent ratios.

d What fraction are pencils?
e Simplify your answer from part **d**.
f **i** Express your answers from parts **d** and **e** as ratios.
ii Are these ratios equivalent?

EXAMPLE 2

Simplify the following ratios.

a 3 : 6 **b** 24 : 18 **c** $\frac{1}{2}$: 5 **d** $\frac{2}{3}$: 4

a 3 : 6 = 1 : 2 Divide both by 3.

b 24 : 18 = 4 : 3 Divide both by 6.

c $\frac{1}{2}$: 5 = 1 : 10 Multiply both by 2.

d $\frac{2}{3}$: 4 = 2 : 12 Multiply both by 3.
= 1 : 6 Divide by 2.

3 Simplify the following ratios.

a 4 : 8 **b** 6 : 12 **c** 15 : 10 **d** 6 : 9 **e** 12 : 20

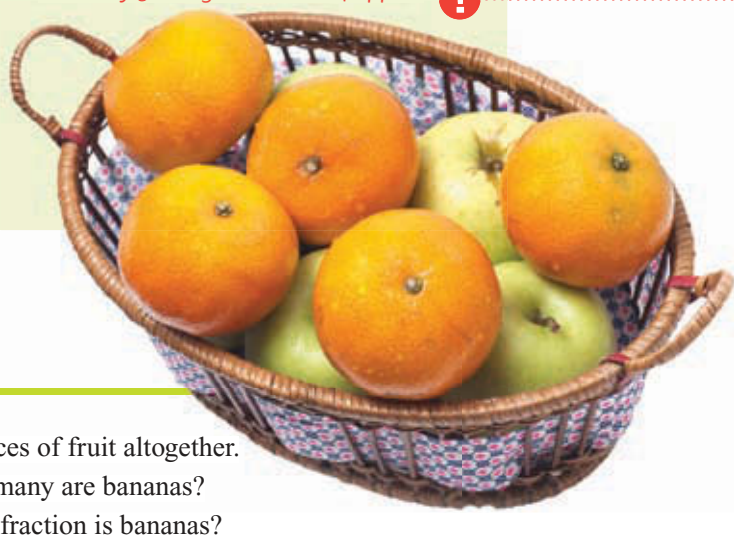
f 9 : 3 **g** $\frac{1}{2}$: 3 **h** $\frac{1}{3}$: 2 **i** $\frac{2}{5}$: 3 **j** $\frac{1}{2}$: $\frac{3}{4}$

EXAMPLE 3

The ratio of oranges to apples is 5 : 4.
There are 9 pieces of fruit in the basket.

a How many pieces of fruit are oranges?
b How many are apples?
c What fraction of the fruit is oranges?
d What fraction is apples?

For every 5 oranges there are 4 apples.



5 + 4 = 9, so all the fruit is either oranges or apples.

a There are 5 oranges. **b** There are 4 apples.

c $\frac{5}{9}$ of the fruit is oranges. **d** $\frac{4}{9}$ of the fruit is apples.

4 The ratio of pears to bananas in a bowl is 7 : 4. There are 11 pieces of fruit altogether.

a How many pieces of fruit are pears? **b** How many are bananas?

c What fraction of the fruit is pears? **d** What fraction is bananas?

5 The ratio of students to teachers in a classroom is 18 : 1 when there are 19 people present.

a How many people are teachers? **b** How many are students?

c What fraction of the people are teachers? **d** What fraction are students?

6 In a housing estate there are 22 cats and dogs.
The ratio of cats to dogs is 13 : 9.

a How many of these animals are dogs?
b What fraction are dogs?
c How many of these animals are cats?
d What fraction are cats?

EXAMPLE 4

The ratio of adults to children at a fete was 1 : 3. If 200 people attended the fete, how many of them were:

a adults? **b** children?

The ratio of adults : children was 1 : 3.

a $1 + 3 = 4$, so there was 1 adult for every 4 people (or $\frac{1}{4}$ of the people were adults).

$\frac{1}{4}$ of $200 = \frac{1}{1\ \cancel{4}} \times \frac{\cancel{200}^{\ 50}}{1} = 1 \times 50 = 50$ adults

b $1 + 3 = 4$, so there were 3 children for every 4 people (or $\frac{3}{4}$ of the people were children).

$\frac{3}{4}$ of $200 = \frac{3}{1\ \cancel{4}} \times \frac{\cancel{200}^{\ 50}}{1} = 3 \times 50 = 150$ children

Check: Adults + children = number of people at fete

$50 + 150 = 200$ ✓

7 The ratio of boy to girl competitors at a sports carnival was 5 : 3. If 256 boys and girls competed at the carnival, how many competitors were:

a boys? **b** girls?

8 The ratio of Minties to Fantails in a bag containing 80 lollies is 7 : 9. How many lollies are:

a Minties? **b** Fantails?

9 In a Year 7 class, $\frac{3}{5}$ of the class are boys.

a What fraction of the class are girls? **b** What is the ratio of boys to girls?

c What is the ratio of girls to boys?

d If there are 30 students in the class, how many are:

i boys? **ii** girls?

EXAMPLE 5

In a Year 7 class, $\frac{2}{7}$ of the class play soccer and the remainder play football. Eight students play soccer.

a What fraction of the class play football? **b** How many students play football?

c How many students are in the class?

a Soccer players $= \frac{2}{7}$ of the class

$\therefore$ Football players $= 1 - \frac{2}{7} = \frac{5}{7}$ of the class.

$\therefore \frac{5}{7}$ of the class play football.

b Soccer players $= \frac{2}{7}$ of the class

$= 8$ students

$\therefore \frac{1}{7}$ of the class $= 8 \div 2 = 4$ students

Football players $= \frac{5}{7}$ of the class

$= 5 \times 4 = 20$ students

$\therefore$ 20 students play football.

c Soccer players + football players $= 8 + 20$

$= 28$ students

$\therefore$ There are 28 students in the class.

10 Dom grows only potatoes and pumpkins. His crop of potatoes covers 15 hectares, which is $\frac{3}{5}$ of his farmland.

- **a** What fraction of the Dom's farm is used for growing pumpkins?
- **b** How many hectares are used for growing pumpkins?
- **c** What is the total size of Dom's farm?

EXAMPLE 6

In a Year 7 class the ratio of boys to girls is 5 : 3.
If there are 12 girls, calculate the number of boys.

The number of girls is 3 parts and there are 12 girls. Thus 3 parts = 12. ∴ 1 part = 4.
The number of boys is 5 parts.
∴ Number of boys = 5 × 4 = 20 boys.

11

- **a** In a Year 7 class, the ratio of boys to girls is 3 : 2. If there are 10 girls, calculate the number of boys.
- **b** The ratio of students to teachers in a school is 2 : 15. If there are 8 teachers, find the number of students.
- **c** In a box of chocolates the ratio of hard-centred chocolates to soft-centred chocolates is 4 : 7. If there are 12 hard-centred chocolates, how many chocolates have soft centres?

B Decimals

Not all numbers are whole numbers. Many problems have answers that are mixed numerals. For example, 5 lollipops are to be shared between 2 people. How many lollipops should each person receive?
$5 \div 2 = ?$
The answer is 2 and half.
$5 \div 2 = 2\frac{1}{2}$
The answer is a mixed numeral, which can be written as a **decimal**.

The **decimal point** is used to separate the whole number part from the fractional part.

whole number ← • → fraction part

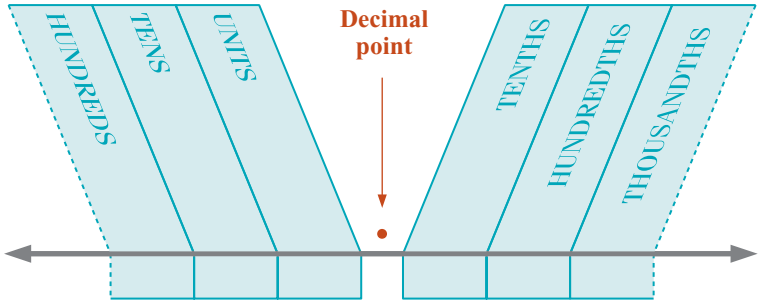
The **base 10** system is used for decimals:

$\frac{1}{10} = 0.1$, $\frac{1}{100} = 0.01$, $\frac{1}{1000} = 0.001$, $\frac{1}{10\,000} = 0.001$

A pattern is forming. Can you see it?

'deci' comes from the Ancient Greek word for tenth.

A **place-value table**, such as the one following, can help you to understand the value of decimals.



In the sharing-lollipops example, each person should receive $2\frac{1}{2}$ lollipops.
Now, $\frac{1}{2} = \frac{5}{10}$ (equivalent fractions) and $\frac{5}{10} = 0.5$, written as a decimal.
So $2\frac{1}{2}$ can be written as 2.5. Each person should receive 2.5 lollipops.
A decimal can be written in **decimal form** (using a decimal point) or in **expanded rational form** (as an addition of fractions).

EXAMPLE 1

Write each decimal in the table below in:

i decimal form ii expanded rational form

	Whole number part				Fractional part		
	hundreds 100s	tens 10s	units 1s	decimal point	tenths $\frac{1}{10}$	hundredths $\frac{1}{100}$	thousandths $\frac{1}{1000}$
a		3	5	.	2	6	
b	4	7	6	.	8	9	7

a i Decimal form is 35.26.

ii Expanded rational form is $30 + 5 + \frac{2}{10} + \frac{6}{100}$.

b i Decimal form is 476.897.

ii Expanded rational form is $400 + 70 + 6 + \frac{8}{10} + \frac{9}{100} + \frac{7}{1000}$.

Exercise 8B

1 Complete the following to write each decimal in the table in expanded rational form.

	100s	10s	1s	decimal point	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	
a			4	.	3			$= 4.3$
b		1	8	.	6	2		$= 18.62$
c	2	9	6	.	0	0	7	$= 296.007$

a $4.3 = \square + \frac{3}{\square}$ b $18.62 = 10 + \square + \frac{6}{\square} + \frac{\square}{\square}$

c $296.007 = 200 + \square + 6 + \frac{\square}{\square}$

2 Write each decimal in expanded rational form.

	Decimal form	Expanded rational form
a	8.5	$8 + \frac{5}{10}$
b	19.04	
c	57.093	
d	704.316	
e	999.104	
f	465.007	

EXAMPLE 2

Use a place-value table to write the following numbers in decimal form.

a $\frac{3}{1000}$ **b** $\frac{2}{10} + \frac{6}{1000}$ **c** $300 + 8 + \frac{4}{100}$

Fill the gaps with zeros.

	100s	10s	1s	decimal point	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	
a			0	.	0	0	3	$= 0.003$
b			0	.	2	0	6	$= 0.206$
c	3	0	8	.	0	4		$= 308.04$

3 Complete the following to write each number in decimal form.

a $1 + \frac{5}{10}$ **b** $20 + 8 + \frac{6}{100}$ **c** $100 + 30 + 7 + \frac{4}{10} + \frac{9}{1000}$

	100s	10s	1s	decimal point	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	
a			1	.	__			$= 1.\_\_$
b		__	__	.	0	6		$= 2\_\_.\_\_$
c	1	__	7	.	4	__	__	$= 13\_\_.4\_\_\_\_$

4 Write the following numbers in decimal form.

a $\frac{5}{10}$ **b** $\frac{3}{10}$ **c** $\frac{7}{100}$

d $\frac{9}{100}$ **e** $\frac{6}{1000}$ **f** $\frac{8}{1000}$

g $\frac{4}{10} + \frac{2}{100}$ **h** $\frac{3}{100} + \frac{1}{1000}$ **i** $80 + 7 + \frac{2}{10} + \frac{3}{100}$

j $15 + \frac{3}{10} + \frac{8}{1000}$ **k** $6 + \frac{1}{10} + \frac{2}{100} + \frac{3}{1000}$

5 Express these mixed numerals as decimals.

a $5\frac{3}{10}$ **b** $14\frac{2}{10}$ **c** $9\frac{4}{10}$ **d** $11\frac{8}{100}$ **e** $12\frac{1}{100}$

f $7\frac{9}{100}$ **g** $6\frac{2}{1000}$ **h** $23\frac{5}{1000}$ **i** $18\frac{6}{1000}$ **j** $2\frac{3}{10\,000}$

EXAMPLE 3

What is the value of the digit 8 in each decimal?

a 0.18 **b** 986.6 **c** 308.1 **d** 7.008

a $\frac{8}{100}$ (hundredths) **b** 80 (tens) **c** 8 (units) **d** $\frac{8}{1000}$ (thousandths)

6 Complete the following to state the value of the digit 3 in the each decimal.

a 37.06
30 (_____)

b 41.36
$\frac{3}{\square}$ (tenths)

c 8.413
$\frac{3}{\square}$ (_____)

d 5.23
$\frac{3}{\square}$ (_____)

7 **a** State the value of the digit 2 in each of the following.

i 0.02 **ii** 0.2 **iii** 2.01 **iv** 200.01

b State the value of the digit 6 in each of the following.

i 0.6 **ii** 0.006 **iii** 6.04 **iv** 6084.3

c State the value of the digit 5 in each of the following.

i 5 **ii** 853 **iii** 25 832.7 **iv** 0.005

d State the value of the digit 9 in each of the following.

i 9164 **ii** 21.9 **iii** 44.329 **iv** 3391.35

8 True or false?

> means greater than and < means less than.

a $0.5 = \frac{5}{10}$ **b** $\frac{4}{100} < 0.4$ **c** 300 = three hundredths

d two point six $> 2 + \frac{8}{100}$ **e** $3.00 = 3$ **f** twenty-two thousandths $= 0.222$

g $9 + \frac{6}{100} = 9.6$ **h** $1.9 > 1.88$ **i** $0.6 < 0.06$

EXAMPLE 4

Change these decimals to fractions.

a 4.1 **b** 0.41 **c** 0.041 **d** 0.0041

The number of zeros in the denominator corresponds to the number of digits following the decimal point.

a $\frac{41}{10}$ **b** $\frac{41}{100}$ **c** $\frac{41}{1000}$ **d** $\frac{41}{10\,000}$

9 Complete the following to change these decimals to fractions.

a $5.2 = \frac{52}{\square}$ **b** $0.52 = \frac{52}{\square}$ **c** $0.052 = \frac{52}{\square}$ **d** $0.0052 = \frac{52}{\square}$

10 Change the following decimals to fractions. (Do not simplify.)

a 0.6 **b** 0.1 **c** 0.5 **d** 0.7

e 0.02 **f** 0.03 **g** 0.04 **h** 0.08

i 0.001 **j** 0.009 **k** 0.007 **l** 0.012

m 0.000 05 **n** 0.000 06 **o** 0.000 02 **p** 0.000 01

EXAMPLE 5

Express each decimal as a fraction in simplest form.

a 0.4 **b** 0.035 **c** 0.0025

a $0.4 = \frac{\cancel{4}^{2}}{\cancel{10}_{5}} = \frac{2}{5}$ **b** $0.035 = \frac{\cancel{35}^{7}}{\cancel{1000}_{200}} = \frac{7}{200}$ **c** $0.0025 = \frac{\cancel{25}^{1}}{\cancel{10\,000}_{400}} = \frac{1}{400}$

11 Complete the following to express each decimal as a fraction in simplest form.

a $0.8 = \frac{8}{\square} = \frac{4}{\square}$ **b** $0.06 = \frac{6}{\square} = \frac{\square}{50}$ **c** $0.056 = \frac{56}{\square} = \frac{\square}{125}$

12 Express each decimal as a fraction in simplest form.

Cancel to simplify.

a 0.2 **b** 0.4 **c** 0.5 **d** 0.08

e 0.002 **f** 0.75 **g** 0.44 **h** 0.016

i 0.042 **j** 0.0014 **k** 0.0028 **l** 0.0008

EXAMPLE 6

Express each decimal as a simplified mixed numeral.

a 5.08 **b** 15.142

a $5.08 = 5 + \frac{\cancel{8}^{\,2}}{\cancel{100}_{25}}$

$= 5 + \frac{2}{25}$

$= 5\frac{2}{25}$

b $15.142 = 15 + \frac{\cancel{142}^{\,71}}{\cancel{1000}_{500}}$

$= 15 + \frac{71}{500}$

$= 15\frac{71}{500}$

13 Complete the following to express each decimal as a simplified mixed numeral.

a $6.04 = 6 + \frac{4}{\square}$

$= 6 + \frac{\square}{\square} = 6\frac{\square}{25}$

b $3.096 = 3 + \frac{96}{\square}$

$= 3 + \frac{\square}{\square} = 3\frac{\square}{125}$

14 Express each decimal as a simplified mixed numeral.

a 9.2 **b** 8.5 **c** 2.6 **d** 1.8 **e** 12.08 **f** 3.04

g 2.005 **h** 6.006 **i** 6.25 **j** 8.46 **k** 9.725 **l** 11.055

EXAMPLE 7

Change these fractions to decimals.

a $\frac{3}{4}$ **b** $\frac{13}{8}$ **c** $\frac{12}{80}$

a $\frac{3}{4}$ means $3 \div 4$

$4\overline{)3.00}$ or $4\overline{)3.020}$

$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ 28 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$ or $\begin{array}{r} 0.75 \\ 4\overline{)3.020} \end{array}$

$\therefore \frac{3}{4} = 0.75$

b $\frac{13}{8}$ means $13 \div 8$

$8\overline{)13.000}$ or $8\overline{)13.^{5}0^{2}2^{4}0}$

$\begin{array}{r} 1.625 \\ 8\overline{)13.000} \\ 8 \\ \hline 50 \\ 48 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$ or $\begin{array}{r} 1.625 \\ 8\overline{)13.^{5}0^{2}2^{4}0} \end{array}$

$\therefore \frac{13}{8} = 1.625$

c $\frac{\cancel{12}^{\,3}}{\cancel{80}_{20}} = \frac{3}{20}$ means $3 \div 20$

$20\overline{)3.00}$ or $20\overline{)3.0^{10}0}$

$\begin{array}{r} 0.15 \\ 20\overline{)3.00} \\ 20 \\ \hline 100 \\ 100 \\ \hline 0 \end{array}$ or $\begin{array}{r} 0.15 \\ 20\overline{)3.0^{10}0} \end{array}$

$\therefore \frac{12}{80} = 0.15$

Zeros can be added to the end of the fraction part of a decimal, without changing its value.

15 Complete the following to change these fractions to decimals

a $\frac{3}{8} = \_\_\_$

$\begin{array}{r} 0.37\square \\ 8\overline{)3.000} \\ 24 \\ \hline \square 0 \\ 56 \\ \hline \square 0 \end{array}$

b $\frac{7}{25} = \_\_\_$

$\begin{array}{r} 0.\square\square \\ 25\overline{)7.00} \\ 50 \\ \hline 20\square \\ 20\square \\ \hline 0 \end{array}$

c $\frac{42}{30} = \frac{\square}{\square} = \_\_\_$

$\begin{array}{r} 1.\square \\ 5\overline{)\square.\square} \\ 5 \\ \hline 20 \\ \square\square \\ \hline 0 \end{array}$

Remember to express fractions in simplest form before dividing.

16 Change the following fractions to decimals by dividing.

a $\frac{1}{5}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{8}$ **e** $\frac{11}{20}$ **f** $\frac{4}{5}$ **g** $\frac{7}{8}$ **h** $\frac{18}{15}$

i $\frac{96}{40}$ **j** $\frac{7}{8}$ **k** $\frac{9}{20}$ **l** $\frac{4}{25}$ **m** $\frac{27}{20}$ **n** $\frac{6}{25}$ **o** $\frac{28}{50}$ **p** $\frac{128}{200}$

Repeating or recurring decimals

Repeating or recurring decimals are **infinite**. The last digit (or group of digits) is repeated over and over again. To abbreviate these decimals, a dot is placed over the repeating or recurring digit (or first and last of the group of digits). For example:

0.4444… is written as $0.\dot{4}$

17.8787… is written as $17.\dot{8}\dot{7}$

6.123 123… is written as $6.\dot{1}2\dot{3}$

Infinite means having no end. Therefore the decimal goes on and on, after the decimal point.

17 Abbreviate the following repeating or recurring decimals.

a 0.6666 …	**b** 0.2222 …	**c** 1.3333 …	**d** 25.7777 …
e 0.5858 …	**f** 0.6363 …	**g** 7.8989 …	**h** 21.4747 …
i 0.176 176 …	**j** 0.843 843 …	**k** 33.319 319 …	**l** 61.873 873 …

EXAMPLE 8

Change these fractions to decimals.

a $\frac{1}{3}$ **b** $\frac{11}{12}$

a $\frac{1}{3}$ means $1 \div 3$

$\therefore \frac{1}{3} = 0.\dot{3}$

$$3\overline{)1.0^{1}0^{1}0} \quad 0.3\ 3\ 3\ \ldots$$

b $\frac{11}{12}$ means $11 \div 12$

$\therefore \frac{11}{12} = 0.91\dot{6}$

$$12\overline{)11.0^{2}0^{8}0^{8}0^{8}0} \quad 0.9\ 1\ 6\ 6\ 6\ \ldots$$

18 Complete the following to change the fractions to decimals.

a $\frac{2}{9} = \_\_\_$

$$9\overline{)\square.0\ ^{2}0^{2}0} \quad \square.2\square\square\ \ldots$$

b $\frac{5}{11} = \_\_\_$

$$\square\overline{)5.0\ ^{6}0\ ^{5}0\ ^{6}0} \quad 0.\square\ 5\ 4\square\ \ldots$$

19 Express each fraction as a repeating or recurring decimal.

a $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{5}{6}$ **d** $\frac{25}{11}$ **e** $\frac{11}{15}$ **f** $\frac{15}{36}$ **g** $\frac{11}{6}$ **h** $\frac{12}{7}$

C Comparing and ordering decimals

To compare decimals:

Step 1: Change the decimals to fractions with the same denominator.

Step 2: Compare the numerators.

EXAMPLE 1

Which decimal is smaller: 0.44 or 0.404?

Change to thousandths. $0.44 = 0.440 = \frac{440}{1000}$ $0.404 = \frac{404}{1000}$

$404 < 440$, $\therefore$ 0.404 is the smaller decimal.

Exercise 8C

1 Complete the following to show which decimal is smaller: 0.33 or 0.303?

$0.33 = 0.33\square = \frac{330}{\square}$ $\quad 0.303 = \frac{\square}{1000}$

$3\square\square < 3\square\square$ $\quad \therefore$ _____ is the smaller decimal.

2 Which decimal is smaller?

a 0.005 or 0.05 **b** 0.44 or 0.044 **c** 0.203 or 0.23

d 0.71 or 0.771 **e** 0.909 or 0.99 **f** 0.35 or 0.4

3 Which decimal is larger?

a 0.0066 or 0.606 **b** 0.717 or 0.177

c 0.157 or 0.23 **d** 0.605 or 0.61

e 0.77 or 0.707 **f** 0.31 or 0.311

EXAMPLE 2

Arrange the following decimals in ascending order:
0.03 0.303 0.33 0.003

Ascending means smallest to largest.

Change to thousandths.

$0.03 = 0.030 = \frac{30}{1000}$ $\quad 0.33 = 0.330 = \frac{330}{1000}$

$0.303 = \frac{303}{1000}$ $\quad 0.003 = \frac{3}{1000}$

Compare numerators: $3 < 30 < 303 < 330$. $\quad \therefore$ In ascending order: 0.003, 0.03, 0.303, 0.33

4 Complete the following to arrange these decimals in ascending order: 0.505, 0.05, 0.55.

$0.505 = \frac{505}{\square}$ $\quad 0.05 = 0.05\_\_ = \frac{\square}{1000}$ $\quad 0.55 = 0.55\_\_ = \frac{\square}{1000}$

_____ $< 505 <$ _____. $\quad \therefore$ In ascending order: 0.05, _____, 0.55

5 Arrange the following decimals in *ascending* order.

a 0.77, 0.7, 0.707, 0.07 **b** 0.909, 0.9, 0.009, 0.99 **c** 0.04, 0.404, 0.44, 0.004

6 Arrange the following decimals in *descending* order.

a 0.606, 0.66, 0.6, 0.006 **b** 0.507, 0.557, 0.075, 0.75

c 0.103, 0.301, 0.013, 0.31

Descending means largest to smallest.

7 Insert one of $>$ or $<$ or $=$ to make each statement true.

a 0.5 __ 0.6 **b** 0.171 __ 0.171 00

c 0.404 __ 0.440 **d** 0.02 __ 0.20

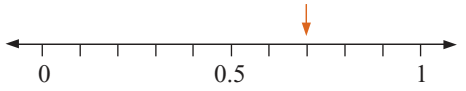
e 1.03 __ 0.93 **f** 5.71 __ 5.7100

g \$7.07 __ \$7.70 **h** \$5.60 __ \$5.06

i \$25.50 __ \$25.59

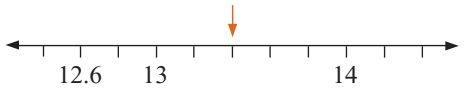
8 Determine the numbers shown by the arrows on the number lines below.

a

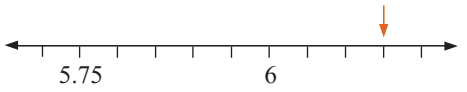


b

c



d



D Rounding decimals

Rounding decimals means reducing the number of decimal places to obtain an approximation of the original decimal.

Step 1: Determine the 'place' to which you want to round: for example, to 2 decimal places, or to the nearest hundredth.

Step 2: Look at the digit to the right of the determined decimal place.

Step 3: If the digit is less than 5, round down ($< 5 \downarrow$).

Step 4: If the digit is equal to or greater than 5, round up ($\geqslant 5 \uparrow$).

EXAMPLE 1

Round each decimal to the nearest whole number.

a 12.074 **b** 21.934 **c** 8.572

The digit in the 'tenths' place determines whether to round down or up.

a 12.074 is closer to 12 than 13. ($0 < 5$, so round down.)
$\therefore$ 12.074 $\approx$ 12

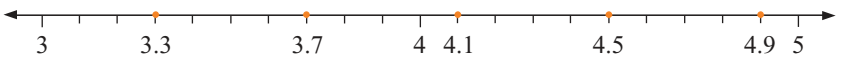
b 21.934 is closer to 22 than 21. ($9 > 5$, so round up.)
$\therefore$ 21.934 $\approx$ 22

c 8.572 is closer to 9 than 8. ($5 = 5$, so round up.)
$\therefore$ 8.572 $\approx$ 9

$\approx$ *means approximately equal to.*

Exercise 8D

1



Use the number line above to answer the following.

a Is 3.3 closer to 3 or 4?
b Is 3.7 closer to 3 or 4?
c Is 4.1 closer to 4 or 5?
d Is 4.9 closer to 4 or 5?
e Would 4.5 be rounded to 4 or 5? (Think about the rules.)

2 Complete the following to round each decimal to the nearest whole number.

a 4.8
8 > 5, so round up.
$\therefore$ 4.8 $\approx$ _____

b 12.316
3 < 5, so round ____.
$\therefore$ 12.316 $\approx$ _____

c 9.59
5 = 5, so round ____.
$\therefore$ 9.59 $\approx$ _____

d 7.21
2 __ 5, so round ____.
$\therefore$ 7.21 $\approx$ _____

3 Round the following decimals to the nearest whole number.

a 2.874	**b** 5.016	**c** 19.72	**d** 11.53
e 24.099	**f** 53.39	**g** 102.2	**h** 127.73

EXAMPLE 2

Round each decimal to the nearest tenth (to 1 decimal place).

a 8.8071 **b** 15.974 **c** 21.551

The digit in the 'hundredths' place determines whether to round down or up.

a $8.8071 \approx 8.8$ $(0 < 5$, so round down.)
b $15.974 \approx 16.0$ $(7 > 5$, so round up.)
c $21.551 \approx 21.6$ $(5 = 5$, so round up.)

Be careful! In part b, the 9 is rounded up to 10.
$\therefore$ the 5 must be rounded up to 6.
$\therefore$ 15.974 must be rounded up to 16.0.

4 Complete the following to round each decimal to the nearest tenth (to 1 decimal place).

a 0.94
$4 <$ 5, so round down.
$\therefore 0.94 \approx 0.$___

b 8.651
$5 =$ 5, so round ____.
$\therefore 8.651 \approx 8.$___

c 13.22
2 __ 5, so round ____.
$\therefore 13.22 \approx$ _____

d 24.569
6 __ 5, so round ____.
$\therefore 24.569 \approx$ _____

5 Round the following decimals to the nearest tenth (to 1 decimal place).

a 5.108	**b** 9.038	**c** 11.5701	**d** 22.345
e 62.925	**f** 43.224	**g** 109.954	**h** 35.298
i 88.143	**j** 155.8309	**k** 127.737	**l** 264.417

EXAMPLE 3

Round each decimal to the nearest hundredth (to 2 decimal places).

a 25.1824 **b** 47.9873 **c** 56.555

The digit in the 'thousandths' place determines whether to round down or up.

a $25.1824 \approx 25.18$ $(2 < 5$, so round down.)
b $47.9873 \approx 47.99$ $(7 > 5$, so round up.)
c $56.555 \approx 56.56$ $(5 = 5$, so round up.)

6 Complete the following to round each decimal to the nearest hundredth (to 2 decimal places).

a 7.8764
$6 >$ ___, so round up.
$\therefore 7.8764 \approx 7.8$___

b 0.3552
5 ___ 5, so round _____.
$\therefore 0.3552 \approx 0.3$___

c 12.6212
1 __ 5, so round _____.
$\therefore 12.6212 \approx$ _____

d 45.067
$7 >$ ___, so round _____.
$\therefore 45.067 \approx$ _____

7 Round the following decimals to the nearest hundredth (to 2 decimal places).

a 2.9101	**b** 7.1855	**c** 13.558	**d** 38.491
e 21.2156	**f** 34.146 09	**g** 21.0423	**h** 65.0299
i 82.5301	**j** 73.345	**k** 139.1332	**l** 273.345

EXAMPLE 4

Round each decimal to the nearest thousandth (to 3 decimal places).

a 0.076 43 **b** 7.149 87 **c** 55.695 54

The digit in the 'ten-thousandths' place determines whether to round down or up.

a 0.076 43 ≈ 0.076 (4 < 5, so round down.)

b 7.149 87 ≈ 7.150 (8 > 5, so round up.)

c 55.695 54 ≈ 55.696 (5 = 5, so round up.)

8 Complete the following to round each decimal to the nearest thousandth (to 3 decimal places).

a 5.113 72
7 > 5, so round _____.
∴ 5.113 72 ≈ 5.11___

b 0.0875
5 = 5, so round _____.
∴ 0.0875 ≈ 0.08___

c 19.6142
2 __ 5, so round _____.
∴ 19.6142 ≈ _____

d 24.070 09
0 __ 5, so round _____.
∴ 24.070 09 ≈ _____

9 Round the following decimals to the nearest thousandth (to 3 decimal places).

a 0.007 49 **b** 0.098 94 **c** 1.764 03 **d** 6.041 01
e 9.3334 **f** 12.0047 **g** 15.061 17 **h** 21.6688
i 34.4141 **j** 68.1842 **k** 77.7138 **l** 5.0554

10 Round the following repeating/recurring decimals to the nearest whole number.

a $3.\dot{6}$ **b** $0.\dot{3}$ **c** $3.\dot{5}\dot{6}$ **d** $5.2\dot{8}$
e $10.\dot{4}\dot{7}$ **f** $17.\dot{0}2\dot{3}$ **g** $23.\dot{5}6\dot{1}$ **h** $11.2\dot{8}\dot{7}$

11 Round the following repeating/recurring decimals to the nearest tenth (to 1 decimal place).

a 0.777 … **b** 0.444 … **c** 0.1313 … **d** 0.6464 …
e 9.5656 … **f** 4.176 176 … **g** 8.592 592 … **h** 17.581 581 …

12 Round the following repeating/recurring decimals to the nearest hundredth (to 2 decimal places).

a 0.999 … **b** 5.7777 … **c** 27.2626 … **d** 54.1414 …
e 78.8484 … **f** 0.231 231 … **g** 5.136 136 … **h** 17.192 192

Investigation 1 Value of the Australian dollar

From the television, newspaper or Internet, for a period of 1 week, record the value of the Australian dollar (AUD) in terms of:

a US dollars (USD) **b** UK pounds (GBP) **c** NZ dollars (NZD)

E Adding and subtracting decimals

Decimals are added and subtracted in a similar way to whole numbers. When decimals are added or subtracted vertically, the decimal points must be beneath each other.

An **estimate** can be obtained by rounding. It is a good way to check answers.

EXAMPLE 1

Calculate the following and check using estimates.

a $4.83 + 27.659 + 0.007$ **b** $32.876 - 9.0019$

a
$$\begin{array}{r} 4.830 \\ 27.659 \\ +\ \ 0.007 \\ \hline 32.496 \end{array}$$

Estimate:
$4.83 + 27.659 + 0.007$
$\approx 5 + 28 + 0 = 33$

b
$$\begin{array}{r} 32.8760 \\ -\ \ 9.0019 \\ \hline 23.8741 \end{array}$$

Estimate:
$32.876 - 9.0019$
$\approx 33 - 9 = 24$

Use zero place holders at the ends of decimals to avoid confusion with columns.

Exercise 8E

1 Complete the following calculations and check using estimates.

a $136.07 + 7.63$

$$\begin{array}{r} 1\,3\,6\,.\,0\,7 \\ +\ \ \square\,.\,6\,\square \\ \hline 1\,4\,\square\,.\,7\,\square \end{array}$$

Estimate:
$136 + \_\_\_\_ = 14\square$

b $5 + 0.008 + 251.6$

$$\begin{array}{r} 5\,.\,0\,0\,0 \\ 0\,.\,\square\,\square\,8 \\ +\ 2\,\square\,1\,.\,6\,0\,0 \\ \hline 2\,\square\,6\,.\,\square\,\square\,8 \end{array}$$

Estimate:
$5 + \_\_\_ + 252 = 25\square$

c $74.2 - 8.09$

$$\begin{array}{r} 7\,\square\,.\,2\,0 \\ -\ \ 8\,.\,\square\,9 \\ \hline 6\,\square\,.\,\square\,1 \end{array}$$

Estimate:
$74 - \_\_\_ = \_\_\_$

2 Estimate your answer first by rounding each decimal to the nearest tenth, then find the sums and differences.

a $6.7 + 5.0$	**b** $12.9 + 8.2$	**c** $0.5 + 6.9$
d $9.3 + 6.14$	**e** $0.86 + 1.24$	**f** $10.31 + 14.23$
g $85.821 + 12.43$	**h** $9.612 + 8.399$	**i** $4.2 - 2.8$
j $7.9 - 6.02$	**k** $11.07 - 3.49$	**l** $18.62 - 12.34$
m $39.41 - 17.85$	**n** $67.427 - 28.896$	**o** $42.223 - 16.755$

3 Find the sum of:

a 28.9, 0.0074 and 6.812 **b** 3.74, 11.063 and 0.094 **c** 15.6817, 0.98 and 3.001

4 Find the difference between:

a 16.0194 and 5.82 **b** 85 and 23.099 **c** $10.\dot{6}$ and $8.4\dot{5}$

Investigation 2 What happens?

1 Multiplying

a Complete the table below.

	Whole number	×	Whole number
i	7×4	=	_____
ii	12×3	=	_____
iii	10×5	=	_____
iv	136×11	=	_____

b What statement can be made about the outcome when whole numbers are multiplied?

c Complete the table below.

i	3.2×1.4	=	_____
ii	9.6×2.5	=	_____
iii	$2\frac{1}{2} \times 1\frac{1}{4}$	=	_____
iv	$7\frac{1}{3} \times 8\frac{2}{5}$	=	_____

d The numbers in the tables in parts **a** and **c** are greater than 1. Write a statement explaining the outcome when numbers greater than 1 are multiplied.

e Complete the table below.

i	5×0.8	=	_____
ii	32×0.3	=	_____
iii	$18 \times \frac{1}{4}$	=	_____
iv	$74 \times \frac{3}{5}$	=	_____

f The table in part **e** is multiplying whole numbers by numbers less than 1. Write a statement explaining the outcome when multiplying by a number less than 1.

2 Dividing

a Complete the table below.

i	$20 \div 4$	=	_____
ii	$18 \div 3$	=	_____
iii	$30 \div 10$	=	_____
iv	$15 \div 5$	=	_____

b What statement can be made about the outcome when dividing a whole number by a whole number?

c Complete the table below.

i	$40.6 \div 8$	=	_____
ii	$18 \div 3$	=	_____
iii	$30 \div 12.6$	=	_____
iv	$15 \div 7\frac{3}{5}$	=	_____

d The numbers in the tables in parts **a** and **c** are greater than 1. Write a statement explaining the outcome when dividing by a number greater than 1.

Round each off to 3 decimal places if required.

e Complete the table below.

i	$5 \div 0.8$	=	_____
ii	$32 \div 0.3$	=	_____
iii	$18 \div \frac{1}{4}$	≈	_____
iv	$74 \div \frac{3}{5}$	≈	_____

f The table in part **e** is dividing whole numbers by numbers less than 1. Write a statement explaining the outcome when dividing by a number less than 1.

Investigation 3 Where does the decimal point go?

1 In groups of four complete the following.

a $6 \times 3 = 18$

b $0.6 \times 3 = \frac{6}{10} \times 3 = \frac{18}{10} = 1.8$

c $0.6 \times 0.3 = \frac{6}{10} \times \frac{3}{10} = \frac{\square}{100} = 0.18$

d $0.06 \times 0.3 = \frac{\square}{100} \times \frac{\triangle}{10} = \frac{\diamond}{1000} =$ ____

e $0.06 \times 0.03 = \frac{\square}{100} \times \frac{\triangle}{100} = \frac{\diamond}{10\,000} =$ ____

f $0.006 \times 0.03 = \frac{\square}{1000} \times \frac{\triangle}{100} = \frac{\diamond}{100\,000} =$ ____

g $0.006 \times 0.003 = \frac{\square}{1000} \times \frac{\triangle}{1000} = \frac{\diamond}{1\,000\,000} = 0.000\,018$

2 If all the parts of question **1** have the decimal points removed, they all become $6 \times 3 = 18$.
If all the answers to the parts of question **1** have the decimal points removed, what are all the answers?

3 For each part of question **1**, write down the number of digits after the decimal point(s):

a in the question **b** in the answer.

For example, in question **1c** there are 2 digits after the decimal point(s) in the question and the answer.

4 Can you see a short way of multiplying decimals?

5 Complete the following, then discuss your method.

a 4×0.8 **b** 0.4×0.8 **c** 0.4×0.08

d 0.04×0.08 **e** 0.04×0.008 **f** 0.004×0.008

6 Complete the following and discuss your method.

a 0.3×0.8 **b** 0.5×0.2 **c** 0.04×0.4

d 1.2×7 **e** 1.4×3 **f** 2.5×4

F Multiplying decimals

From Investigation 3 you have found how to multiply decimals.

Estimating allows you to check the accuracy of your answer.

Step 1: Estimate the answer by rounding.

Step 2: Ignore decimal points and multiply as you would whole numbers.

Step 3: Count the number of digits after the decimal point(s) in the question.

Step 4: Insert the decimal point in the answer, so that the number of digits after the decimal point in the answer is the same as the total number of digits after the decimal point(s) in the question.

EXAMPLE 1

Estimate then calculate the following.

a 4.93×0.21 **b** 4×0.003

a *Step 1:* *Estimate:* $5 \times 0 = 0$

Step 2:

$$\begin{array}{r} 4\ 9\ 3 \\ \times \quad 2\ 1 \\ \hline 4\ 9\ 3 \\ 9\ 8\ 6\ 0 \\ \hline 1\ 0\ 3\ 5\ 3 \end{array}$$

Step 3: 4.93×0.21: 4 digits after the decimal points.

Step 4: $\therefore 4.93 \times 0.21 = 1.0353$

Note the closeness of the estimate to the answer in each example.

b *Estimate:* $4 \times 0 = 0$

$4 \times 3 = 12$

4×0.003: 3 digits after the decimal point.

$\therefore 4 \times 0.003 = 0.012$

To get the correct number of decimal places in the answer, you may need to fill the spaces after the decimal point with zeros.

Exercise 8F

1 Complete the following estimates and calculations.

If the question has 3 digits after the decimal point(s), the answer must have 3 digits after the decimal point.

a 3.41×8

Estimate: ____ $\times 8 = 24$

$$\begin{array}{r} 3\ 4\ 1 \\ \times \quad \square \\ \hline 2\ \square\square\ 8 \end{array}$$

____ digits after the decimal point.

$\therefore 3.41 \times 8 = 27.\square\square$

b 1.1×0.005

Estimate: $1 \times 0 = 0$

____ $\times 5 =$ ____

____ digits after the decimal points.

$\therefore 1.1 \times 0.005 = 0.00\square\square$

2 State the total number of digits after the decimal point(s) in each question.

a 7×2.1 **b** 6×5.9 **c** 3.1×4.3 **d** 8.2×0.4

3 **a** If there are 4 digits after the decimal point(s) in the question, how many digits must there be after the decimal point in the answer?

b Theo has written $4.2 \times 3.7 = 1554$. He has forgotten to insert the decimal point in the answer. Rewrite the answer with the decimal point.

c Insert the decimal point in the answer to make the statement correct: $5 \times 8.147 = 40\,735$.

d Maria estimated that $12.24 \times 6.9 \approx 84$. What did Maria round each decimal to for her estimate?

4 Estimate, then calculate the following.

a 1.9×3 **b** 5.7×7 **c** 3.6×6 **d** 8.5×8

e 9.2×5 **f** 10.3×9 **g** 11.1×4 **h** 15.9×2

5 Find the following products.

a 0.2×8 **b** 0.6×2 **c** 2.7×6 **d** 5.9×4

e 1.3×27 **f** 9.6×13 **g** 18×3.4 **h** 57×2.8

6 Insert the decimal point to make each answer correct.

a $0.3 \times 5 = 15$ **b** $0.3 \times 0.5 = 15$ **c** $0.3 \times 0.05 = 15$

d $0.3 \times 0.005 = 15$ **e** $0.3 \times 0.0005 = 15$ **f** $0.03 \times 0.05 = 15$

g $0.03 \times 0.005 = 15$ **h** $0.003 \times 0.005 = 15$ **i** $0.003 \times 0.000\,05 = 15$

EXAMPLE 2

Place a decimal point in each coloured decimal to make the product correct.

a $0.6 \times 213 = 12.78$ **b** $4.5 \times 829 = 37.305$

a Answer has 2 digits after the decimal point.
$\therefore 0.6 \times 21.3 = 12.78$

b Answer has 3 digits after the decimal point.
$\therefore 4.5 \times 8.29 = 37.305$

The total number of digits after the decimal point(s) in question and answer must be the same.

7 Complete the following to place a decimal point in each coloured decimal to make the product correct.

a $0.6 \times 12 = 0.72$ Answer has ___ digits after the decimal point.
$\therefore 0.6 \times \_\_\_ = 0.72$

b $0.002 \times 111 = 0.0222$ Answer has ___ digits after the decimal point.
$\therefore 0.002 \times \_\_\_ = 0.0222$

8 Place a decimal point in each coloured decimal to make the product correct.

a $0.7 \times 864 = 60.48$ **b** $9 \times 1007 = 9.063$

c $637 \times 8.5 = 541.45$ **d** $1151 \times 0.38 = 4.3738$

9 Complete the following using the words and numerals from this list:
numeral, 0, 2, multiplying, same, 6, 5.62, zero

a When _____ numerals, it does not matter in what order the numerals are multiplied as the result is always the _____. So 6×2 is the same as ___ $\times$ ___.

b When multiplying a numeral by 1, the result is always the _____. So $5.62 \times 1 =$ ___.

c When multiplying a numeral by zero, the result is always _____. So $1.36 \times 0 =$ ___.

Multiplying by multiples of 10

To multiply a decimal by a multiple of 10:

Step 1: Count the number of zeros in the multiple of 10.

Step 2: Move the decimal point in the decimal the same number of places to the *right*.

EXAMPLE 3

Calculate the following.

a 4.3×10 **b** 2.14×1000 **c** $0.8 \times 10\,000$

a $4.3 \times 10 = 43$ 1 zero, $\therefore$ move decimal point 1 place to the right.

b $2.14 \times 1000 = 2140$ 3 zeros, $\therefore$ move decimal point 3 places to the right.

c $0.8 \times 10\,000 = 8000$ 4 zeros, $\therefore$ move decimal point 4 places to the right.

To get the correct number of decimal places in the answer, you may need to fill the spaces before the decimal point with zeros.

10 Complete to calculate the following.

a $0.05 \times 10 =$ _____ ___ zero, ∴ move decimal point ___ place to the right.

b $0.05 \times 100 =$ _____ ___ zeros, ∴ move decimal point ___ places to the _____.

c $0.05 \times$ _____ $=$ _____ 3 zeros, ∴ move decimal point ___ places to the _____.

d $0.05 \times 10\,000 =$ _____ 4 ____, ∴ move decimal point ___ places to the right.

11 Calculate the following.

a 3.8×10 **b** 41.6×10 **c** 2.09×10

d 0.7×100 **e** 2.51×100 **f** 6.4×100

g 43.6×1000 **h** 0.009×1000 **i** 0.78×1000

i $0.63 \times 10\,000$ **k** $0.4 \times 10\,000$ **l** $21.647 \times 10\,000$

G Dividing decimals

- To divide a decimal by a whole number, ignore the decimal point and divide as you would a whole number. Keep the decimal points aligned.
- To divide a decimal by a decimal, first move the decimal point to the *right* in both decimals, by the same number of places, to make the divisor a whole number.

EXAMPLE 1

Calculate the following.

a $0.06 \div 3$ **b** $1.47 \div 7$ **c** $57.06 \div 4$

a $0.06 \div 3$

$$3\overline{)0.06} \quad 0.02$$

$\therefore 0.06 \div 3 = 0.02$

b $1.47 \div 7$

$$7\overline{)1.47} \quad 0.21$$

$\therefore 1.47 \div 7 = 0.21$

c $57.06 \div 4$

$$4\overline{)57.060} \quad 14.265$$

$\therefore 57.06 \div 4 = 14.265$

Exercise 8G

1 Calculate the following by filling in the missing values.

a $0.08 \div 2$

$$2\overline{)0.08} \quad \square.0\square$$

$\therefore 0.08 \div 2 =$ _____

b $25.5 \div 5$

$$5\overline{)25.5} \quad 5.\square$$

$\therefore 25.5 \div 5 =$ _____

c $48.12 \div 4$

$$4\overline{)48.12} \quad 1\square.0\square$$

$\therefore 48.12 \div 4 =$ _____

2 Calculate the following.

a $0.4 \div 2$ **b** $0.6 \div 3$ **c** $1.5 \div 5$

d $2.7 \div 9$ **e** $4.8 \div 8$ **f** $0.56 \div 7$

g $10.8 \div 12$ **h** $1.24 \div 2$ **i** $5.24 \div 4$

j $21.56 \div 7$ **k** $36.9 \div 3$ **l** $50.25 \div 5$

m $484.8 \div 8$ **n** $7.224 \div 6$ **o** $8.03 \div 5$

EXAMPLE 2

Calculate the following.

a 36.04 ÷ 0.4 **b** 8.163 ÷ 0.02 **c** 0.0512 ÷ 0.005

a $36.04 \div 0.4$
$= 360.4 \div 4$
$= 90.1$

Multiply both numbers by 10 to make 0.4 a whole number.

$$4\overline{)360.4} = 90.1$$

b $8.163 \div 0.02$
$= 816.3 \div 2$
$= 408.15$

Multiply both numbers by 100 to make 0.02 a whole number.

$$2\overline{)816.30} = 408.15$$

c $0.0512 \div 0.005$
$= 51.2 \div 5$
$= 10.24$

Multiply both numbers by 1000 to make 0.005 a whole number.

$$5\overline{)51.20} = 10.24$$

3 Calculate the following by filling in the missing values.

a $4.8 \div 0.2$
$= 4.8 \div \_\_ = \_\_\_\_\_$

Multiply both numbers by ____ to make 0.2 a whole number.

b $12.5 \div 0.05$
$= 1.25\square \div \_\_ = \_\_\_\_\_$

Multiply both numbers by ____ to make ____ a whole number.

c $3.672 \div 0.006$
$= \_\_\_\_\_ \div 6 = \_\_\_\_\_$

Multiply both numbers by ____ to make ____ a whole number.

4 Calculate the following by first adjusting the divisor to a whole number.

a 0.36 ÷ 0.3 **b** 0.55 ÷ 0.5 **c** 0.049 ÷ 0.07
d 0.054 ÷ 0.06 **e** 0.002 08 ÷ 0.004 **f** 4.92 ÷ 0.06
g 0.362 ÷ 0.05 **h** 0.476 ÷ 0.04 **i** 9.296 ÷ 0.08

5 Calculate the cost of the following.

a If 58 L of petrol costs $73.08, calculate the cost of 1 L of petrol.
b If 3 kg of oranges cost $5.25, calculate the cost of 1 kg of oranges.
c If 10 hamburgers cost $53, calculate the cost of 1 hamburger.
d If 9 tennis courts are hired at a cost of $86.40, calculate the cost of hiring 1 tennis court.

Dividing by multiples of 10

To divide a decimal by a multiple of 10:

Step 1: Count the number of zeros in the multiple of 10.

Step 2: Move the decimal point in the decimal the same number of places to the *left*.

EXAMPLE 3

Calculate the following.

a 150.2 ÷ 10 **b** 286.4 ÷ 100 **c** 0.6 ÷ 1000

a 150.2 ÷ 10 = 15.02 — 1 zero, ∴ move decimal point 1 place to the left.

b 286.4 ÷ 100 = 2.864 — 2 zeros, ∴ move decimal point 2 places to the left.

c 0.6 ÷ 1000 = 0.0006 — 3 zeros, ∴ move decimal point 3 places to the left.

To get the correct number of decimal places in the answer, you may need to fill the spaces after the decimal point with zeros.

6 Complete to calculate the following.

a 53.2 ÷ 10 = _____ 1 zero, ∴ move decimal point __ place to the left.

b 53.2 ÷ 100 = _____ __ zeros, ∴ move decimal point __ places to the _____.

c 53.2 ÷ 1000 = _____ 3 zeros, ∴ move decimal point __ places to the _____.

d 53.2 ÷ 10 000 = _____ __ zeros, ∴ move decimal point __ places to the _____.

7 Calculate the following.

a 37.4 ÷ 10 **b** 214.3 ÷ 10 **c** 0.7 ÷ 10 **d** 0.03 ÷ 10

e 8.9 ÷ 100 **f** 42.9 ÷ 100 **g** 0.8 ÷ 100 **h** 0.55 ÷ 100

i 54.1 ÷ 1000 **j** 387.2 ÷ 1000 **k** 0.86 ÷ 1000 **l** 4.3 ÷ 1000

Investigation 4 Use of percentages

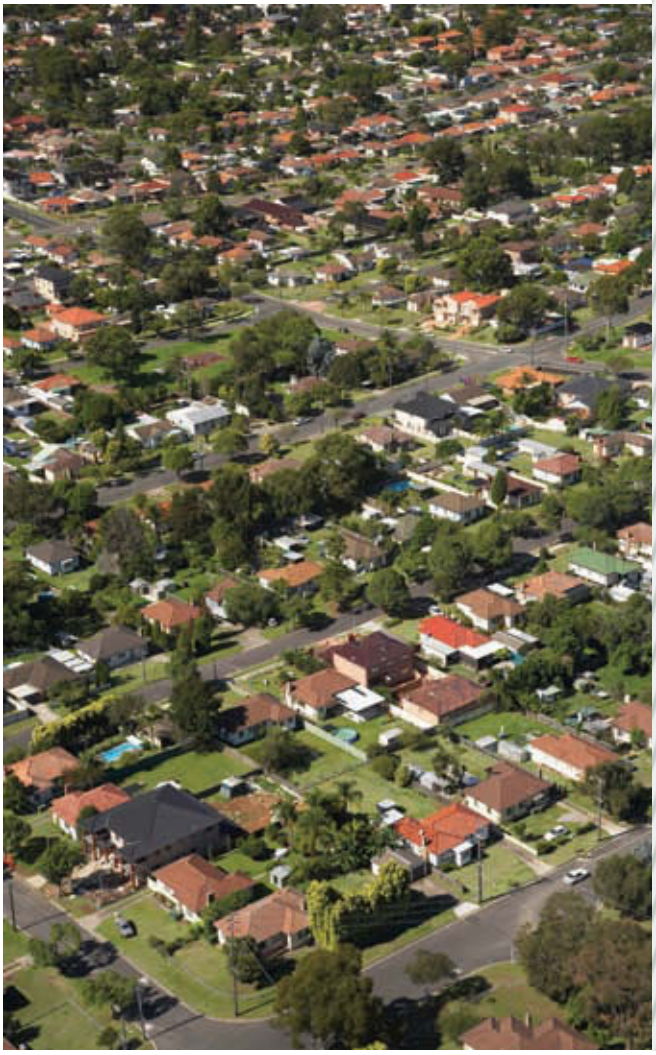
1 Percentages are widely used in everyday life Over a week, create a scrapbook containing:

- newspaper ads/articles
- sporting facts
- retail brochures or
- radio ads/news
- bank/financial institution statistics
- any other evidence of percentages in daily life.

Present your findings to the class. Are percentages really a part of our everyday lives? Do we need to understand the concept of percentages?

2 In groups, read the following examples of everyday percentage use and suggest how and why these percentages have been worked out.

a In our class, 54% of the students are male.
b 22% of primary school-aged children do not eat fruit.
c Australia's inflation rate is 3.5%.
d House prices in the inner city have risen by 36.5%.
e Brent improved 22% in his weekly spelling test.
f Australia's unemployment rate dropped to 5.3%.
g 73% of all houses in our suburb are single-storey.
h Over 61% of 5 to14-year-old children living in Australia play sport outside school hours.
i Term deposit rates are set to increase by 0.25%.
j After the recent heavy rains, the Cataract Dam was at 82% capacity.



H Changing percentages to fractions

A **percentage** is a way of writing a fraction with a denominator of 100.

'Per cent' means 'out of 100'.

$$\frac{\text{Percentage}}{100} = \text{Fraction}$$

EXAMPLE 1

Express each percentage as a fraction.

a 13% **b** 47% **c** 89%

Write the percentage numeral over 100, and remove the % symbol.

a $13\% = \frac{13}{100}$ **b** $47\% = \frac{47}{100}$ **c** $89\% = \frac{89}{100}$

Exercise 8H

1 Complete the following to express each percentage as a fraction.

a $7\% = \frac{7}{\square}$ **b** $19\% = \frac{19}{\square}$ **c** $41\% = \frac{\square}{100}$

2 Express the following as fractions.

a 17% **b** 49% **c** 51% **d** 77% **e** 91%

f 11% **g** 83% **h** 69% **i** 33% **j** 27%

EXAMPLE 2

Express each percentage as a fraction in simplest form.

a 25% **b** 96% **c** 45%

a $25\% = \frac{\cancel{25}^{\,1}}{\cancel{100}_{\,4}}$
$= \frac{1}{4}$

b $96\% = \frac{\cancel{96}^{\,24}}{\cancel{100}_{\,25}}$
$= \frac{24}{25}$

c $45\% = \frac{\cancel{45}^{\,9}}{\cancel{100}_{\,20}}$
$= \frac{9}{20}$

3 Express the following as fractions in simplest form.

a 10% **b** 20% **c** 15% **d** 90% **e** 60%

f 55% **g** 50% **h** 8% **i** 46% **j** 58%

EXAMPLE 3

Express each percentage as a whole number or mixed numeral in simplest form.

a 300% **b** 264% **c** 180%

a $300\% = \frac{\cancel{300}^{\,3}}{\cancel{100}_{\,1}}$
$= 3$

b $264\% = 200\% + 64\%$
$= \frac{\cancel{200}^{\,2}}{\cancel{100}_{\,1}} + \frac{\cancel{64}^{\,16}}{\cancel{100}_{\,25}}$
$= 2 + \frac{16}{25}$
$= 2\frac{16}{25}$

c $180\% = 100\% + 80\%$
$= \frac{100}{100} + \frac{\cancel{80}^{\,4}}{\cancel{100}_{\,5}}$
$= 1 + \frac{4}{5}$
$= 1\frac{4}{5}$

4 Complete to express each percentage as a whole number or mixed numeral in simplest form.

a $700\% = \frac{\square}{100}$
$= \_\_\_$

b $950\% = 900\% + \_\_\%$
$= \frac{\square}{100} + \frac{50}{\square}$
$= \_\_\_ + \frac{1}{\square}$
$= \_\_\_$

c $615\% = \_\_\% + 15\%$
$= \frac{600}{\square} + \frac{\square}{100}$
$= 6 + \frac{\square}{20}$
$= \_\_\_$

5 Express the following as whole numbers or mixed numerals in simplest form.

a 1100% **b** 1500% **c** 2900% **d** 3450% **e** 130%

f 125% **g** 175% **h** 148% **i** 254% **j** 222%

EXAMPLE 4

Express each percentage as a fraction in simplest form.

a $\frac{1}{4}\%$ **b** $7\frac{1}{2}\%$ **c** $5\frac{1}{3}\%$

a $\frac{1}{4}\% = \frac{\frac{1}{4}}{100}$
$= \frac{1}{400}$

Multiply the numerator and denominator by 4.

Multiply the numerator (top) and denominator (bottom) by the same number to remove the fraction within the fraction.

b $7\frac{1}{2}\% = \frac{\frac{15}{2}}{100}$
$= \frac{\cancel{15}^{3}}{\cancel{200}_{40}}$
$= \frac{3}{40}$

Multiply the numerator and denominator by 2.

c $5\frac{1}{3}\% = \frac{\frac{16}{3}}{100}$
$= \frac{\cancel{16}^{4}}{\cancel{300}_{75}}$
$= \frac{4}{75}$

Multiply the numerator and denominator by 3.

6 Complete to express each percentage as a fraction in simplest form.

a $5\frac{1}{2}\% = \frac{\square}{200}$ **b** $9\frac{1}{3}\% = \frac{28}{\square} = \frac{7}{75}$ **c** $12\frac{1}{2}\% = \frac{\square}{200} =$ ____

7 Express the following as fractions in simplest form.

a $6\frac{2}{3}\%$ **b** $15\frac{1}{3}\%$ **c** $18\frac{2}{3}\%$ **d** $4\frac{2}{5}\%$ **e** $8\frac{4}{5}\%$

f $20\frac{1}{2}\%$ **g** $2\frac{1}{3}\%$ **h** $\frac{1}{10}\%$ **i** $13\frac{3}{4}\%$ **j** $10\frac{5}{8}\%$

I Changing percentages to decimals

A percentage can be written as a decimal as well as a fraction.

Percentage ÷ **100** = **Decimal**

EXAMPLE 1

Express each percentage as a decimal.

a 6% **b** 84% **c** 10%

Divide the percentage numeral by 100, and remove the % symbol.

Move the decimal point 2 places to the left.

a $6\% = \frac{6}{100}$
$= 6 \div 100$
$= 0.06$

b $84\% = \frac{84}{100}$
$= 84 \div 100$
$= 0.84$

c $10\% = \frac{10}{100}$
$= 10 \div 100$
$= 0.1$

Exercise 8I

1 Complete the following to express each percentage as a decimal.

a $5\% = \frac{5}{\square}$
$= 5 \div \_\_\_$
$= 0.\_\_5$

b $13\% = \frac{\square}{100}$
$= 13 \div \_\_\_$
$= 0.\_\_\_$

c $60\% = \frac{\square}{100}$
$= 60 \div \_\_\_$
$= \_\_\_$

2 Express each percentage as a decimal.

a 38% **b** 2% **c** 44% **d** 9% **e** 57%

f 21% **g** 66% **h** 52% **i** 8% **j** 80%

EXAMPLE 2

Express each percentage as a decimal.

a 204% **b** 8.4% **c** 11.04%

a $204\% = \frac{204}{100}$
$= 204 \div 100$
$= 2.04$

b $8.4\% = \frac{8.4}{100}$
$= 8.4 \div 100$
$= 0.084$

c $11.04\% = \frac{11.04}{100}$
$= 11.04 \div 100$
$= 0.1104$

3 Complete to express each percentage as a decimal.

a $250\% = \frac{250}{\square}$
$= 250 \div \_\_\_$
$= \_\_\_\_$

b $3.6\% = \frac{\square}{\square}$
$= 3.6 \div 100$
$= 0.\square 36$

c $87.25\% = \frac{87.25}{100}$
$= \_\_\_ \div \_\_\_$
$= \_\_\_\_$

4 Express each percentage as a decimal.

a 187% **b** 150% **c** 132% **d** 173% **e** 245%

f 0.9% **g** 91.6% **h** 47.4% **i** 41.09% **j** 73.94%

EXAMPLE 3

Express each percentage as a decimal.

a $10\frac{1}{2}\%$ **b** $9\frac{3}{4}\%$

Remove each fraction part by writing it as a decimal: $\frac{1}{2} = 0.5$ and $\frac{3}{4} = 0.75$

a $10\frac{1}{2}\% = 10.5\%$
$= \frac{10.5}{100}$
$= 10.5 \div 100$
$= 0.105$

b $9\frac{3}{4}\% = 9.75\%$
$= \frac{9.75}{100}$
$= 9.75 \div 100$
$= 0.0975$

5 Using the information in this table, complete the following to express each percentage as a decimal.

Fraction	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
Decimal	0.2	0.4	0.6	0.8	0.25	0.75	0.5

a $7\frac{1}{4}\% = 7.\_\_\%$
$= \frac{\square}{100}$
$= 7.25 \div 100$
$= 0.\square 725$

b $105\frac{2}{5}\% = 105.\_\%$
$= \frac{\square}{100}$
$= 105.4 \div 100$
$= \_\_\_\_$

6 Using information from the table in question **5**, express the following as decimals.

a $15\frac{1}{2}\%$ **b** $23\frac{3}{5}\%$ **c** $9\frac{1}{4}\%$ **d** $42\frac{2}{5}\%$ **e** $8\frac{3}{4}\%$

f $7\frac{2}{5}\%$ **g** $86\frac{4}{5}\%$ **h** $2\frac{1}{2}\%$ **i** $140\frac{1}{5}\%$ **j** $366\frac{1}{2}\%$

J Changing to percentages

Any fraction or decimal can be written as a percentage.

Fraction or Decimal x **100** = **Percentage**

EXAMPLE 1

Express each fraction as a percentage.

a $\frac{4}{5}$ **b** $\frac{5}{8}$ **c** $\frac{5}{6}$

Multiply the fraction by 100 and add a % symbol.

a $\frac{4}{5} = \frac{4}{5} \times \frac{100}{1}\%$
$= \frac{^{80}\cancel{400}}{_{1}\cancel{5}}\%$
$= 80\%$

b $\frac{5}{8} = \frac{5}{8} \times \frac{100}{1}\%$
$= \frac{^{125}\cancel{500}}{_{2}\cancel{8}}\%$
$= \frac{125}{2}\%$
$= 62\frac{1}{2}\%$ or 62.5%

c $\frac{5}{6} = \frac{5}{6} \times \frac{100}{1}\%$
$= \frac{^{250}\cancel{500}}{_{3}\cancel{6}}\%$
$= \frac{250}{3}\%$
$= 83\frac{1}{3}\%$ or $83.\dot{3}\%$

The 3 is repeating or recurring.

Exercise 8J

1 Complete the following.

Remember to cancel.

a $\frac{5}{\cancel{100}} \times \frac{\cancel{100}}{1} = \_\_\_$ **b** $\frac{17}{\cancel{100}} \times \frac{\cancel{100}}{1} = \_\_\_$ **c** $\frac{24}{\cancel{100}} \times \frac{\cancel{100}}{1} = \_\_\_$

d $\frac{76}{\cancel{100}} \times \frac{\cancel{100}}{1} = \_\_\_$ **e** $\frac{7}{1\cancel{0}} \times \frac{10\cancel{0}}{1} = \_\_\_$ **f** $\frac{9}{1\cancel{0}} \times \frac{10\cancel{0}}{1} = \_\_\_$

2 Express each fraction as a percentage.

a $\frac{1}{4}$ **b** $\frac{3}{10}$ **c** $\frac{47}{50}$ **d** $\frac{19}{25}$ **e** $\frac{11}{10}$ **f** $\frac{63}{50}$

3 Express the following as percentages correct to 2 decimal places.

a $\frac{4}{9}$ **b** $\frac{2}{3}$ **c** $\frac{3}{7}$ **d** $\frac{4}{11}$ **e** $\frac{28}{9}$ **f** $\frac{11}{6}$

EXAMPLE 2

Express each decimal as a percentage.

a 0.14 **b** 0.385 **c** 7.1 **d** 5.0

Multiply the decimal by 100 and add a % symbol.

Move the decimal point 2 places to the right.

a $0.14 = 0.14 \times 100\%$
$= 14\%$

b $0.385 = 0.385 \times 100\%$
$= 38.5\%$

c $7.1 = 7.1 \times 100\%$
$= 710\%$

d $5.0 = 5.0 \times 100\%$
$= 500\%$

4 Complete the following to express each decimal as a percentage.

a $0.09 \times 100\% =$ ____%

b $0.87 \times$ ____% $= 87\%$

c $0.003 \times$ ____% $= 0.$ __%

d $0.592 \times 100\% =$ ____%

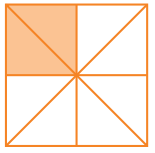
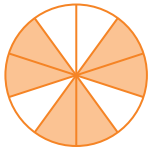
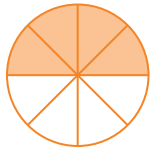
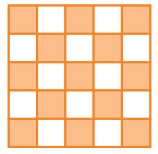
5 Express the following decimals as percentages.

a 0.27	**b** 0.56	**c** 0.71	**d** 0.99	**e** 2.03
f 0.06	**g** 0.615	**h** 0.211	**i** 0.909	**j** 0.444
k 9.6	**l** 6.3	**m** 10.4	**n** 0.6	**o** 7
p 3	**q** 1	**r** 0.004	**s** 0.662	**t** 0.91

6 In every dollar there are 100 cents. So 52 cents in the dollar is the same as 52%. Express each amount of money as a percentage of $1.

a 31 cents	**b** 18 cents	**c** $0.27	**d** $0.63	**e** $1
f $5	**g** $7.81	**h** $9.52	**i** $12.05	**j** $10.10

7 Copy and complete the following table.

	a	**b**	**c**	**d**	**e**
Figure					
Fraction shaded	$\frac{1}{5}$				
Percentage shaded		25%			
Percentage unshaded			40%		

8 Change the following to percentages.

a $\frac{4}{10}$ **b** 0.05 **c** $\frac{6}{50}$ **d** $\frac{13}{20}$ **e** $\frac{73}{100}$

f $\frac{31}{50}$ **g** 0.2 **h** 0.047 **i** $\frac{3}{20}$ **j** 5.08

9 Complete the table below, writing equivalent fractions, decimals and percentages on each line.

	Fraction	Decimal	Percentage
a	$\frac{3}{20}$		
b		0.8	
c			0.4%
d	$\frac{5}{8}$		
e		4.3	
f			$9\frac{1}{2}\%$
g		0.65	
h	$1\frac{2}{5}$		
i			21%
j	$6\frac{3}{4}$		
k		5.025	
l			0.5%

10 By converting each set of numbers to either percentages or decimals, arrange them in ascending order.

a 72%, $\frac{1}{2}$, 0.86 **b** $\frac{3}{4}$, 86%, 0.79 **c** 0.32, $\frac{5}{8}$, 55%

d 21%, 0.27%, $\frac{1}{4}$ **e** $\frac{3}{5}$, 0.582, $57\frac{1}{2}\%$ **f** $19\frac{1}{3}\%$, 0.16, $\frac{1}{5}$

11 Convert each set of numbers to percentages and plot them on a number line.

a $\frac{3}{5}$, 70%, 0.43

b 55%, $\frac{9}{20}$, 0.82

c 0.85, $\frac{3}{4}$, 23%

d 47%, 0.74, $\frac{18}{20}$

e $\frac{3}{4}$, 0.65, 41%

f $\frac{5}{8}$, 73%, 0.48

Investigation 5 Converting percentages to fractions

Complete this table before starting section K. Refer to these simplified fractions to help you in section K.

Percentage	Fraction	Simplified fraction
1%	$\frac{1}{100}$	
5%	$\frac{5}{100}$	$\frac{1}{20}$
10%		
$12\frac{1}{2}$%	$\frac{12\frac{1}{2}}{100} = \frac{25}{200}$	$\frac{1}{8}$
20%		
25%		
$33\frac{1}{3}$ %		$\frac{1}{3}$
$37\frac{1}{2}$%		
50%		
$62\frac{1}{2}$%		
$66\frac{2}{3}$%	$\frac{66\frac{2}{3}}{100} = \frac{200}{300}$	$\frac{2}{3}$
75%		
$87\frac{1}{2}$%		

Finding a percentage of a quantity

To find a percentage of a quantity:

- express each percentage as a fraction in simplest form
- replace 'of' by '×'
- calculate the answer.

EXAMPLE 1

Calculate the following.

a 12% of \$75 **b** $66\frac{2}{3}$% of 180 kg **c** 9% of 189 m

a 12% of 75 $= \frac{\cancel{12}^{3}}{\cancel{100}_{25}} \times 75$

$= \frac{3}{\cancel{25}_{1}} \times \cancel{75}^{3} = 9$

∴ 12% of \$75 is \$9.

b $66\frac{2}{3}$% simplifies to $\frac{2}{3}$.

$\frac{2}{\cancel{3}_{1}} \times \cancel{180}^{60} = 120$

∴ $66\frac{2}{3}$% of 180 kg is 120 kg.

c 9% of 189 $= \frac{9}{100} \times 189$

$= \frac{1701}{100} = 17.01$

∴ 9% of 189 m is 17.01 m.

Exercise 8K

1 Complete to calculate each percentage.

Use % conversions from Investigation 5.

a 20% of 40 kg $= \frac{\cancel{1}^{1}}{\cancel{5}_{4}} \times \frac{\cancel{40}^{8}}{1} =$ ___ $\times 8 =$ ___ kg

b 36% of 150 m $= \frac{\cancel{36}^{9}}{\cancel{100}_{25}} \times \frac{\square}{1} = \frac{\square}{\cancel{25}_{1}} \times \frac{\cancel{150}^{6}}{1} =$ ___ $\times$ ___ $= 54$ m

c $12\frac{1}{2}$% of \$60 $= \frac{\cancel{1}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{60}^{15}}{1} = \frac{1}{\square} \times 15 =$ \$7.□□

EXAMPLE 2

Refer to Investigation 5 and use quick mental strategies to determine these percentages.

a 25% of 200 kg

a 25% simplifies to $\frac{1}{4}$

$\frac{1}{4} \times 200 = \frac{200}{4}$

$= 50$

$\therefore$ 25% of 200 kg is 50 kg.

b $37\frac{1}{2}$% of 64 L

b $37\frac{1}{2}$% simplifies to $\frac{3}{8}$

$\frac{3}{8} \times 64 = \frac{64}{8} \times 3$

$= 8 \times 3 = 24$

$= 24$

$\therefore$ $37\frac{1}{2}$% of 64 L is 24 L.

2 Refer to Investigation 5 and use quick mental strategies to determine these percentages.

a 5% of 80 L
b 10% of 995 km
c $12\frac{1}{2}$% of 56 m
d 20% of 50 s
e 25% of \$16 000
f $33\frac{1}{3}$% of \$9 000 000
g $37\frac{1}{2}$% of 72 t
h 50% of 38 cm
i $62\frac{1}{2}$% of 240 mL
j $66\frac{2}{3}$% of 600 paces
k 75% of 120 days
l $87\frac{1}{3}$% of 4000 kg

3 Calculate the following.

a 50% of 100 g
b 20% of 40 m
c 10% of 70 m
d 25% of 80 kg
e 5% of 20 L
f 1% of 300 km
g $12\frac{1}{2}$% of \$24
h $33\frac{1}{3}$% of \$9
i $37\frac{1}{2}$% of \$120
j $62\frac{1}{2}$% of \$160
k $66\frac{2}{3}$% of 90 m
l 75% of \$120

4 Find the following.

a 17% of 25 km
b 12% of 32 kg
c 18% of 40 m
d 45% of \$130
e 64% of 250 L
f 32% of 154 kg
g 72% of 105 L
h 21% of \$272
i 13% of \$63

When calculating percentages, you can express the fraction part either as a fraction or a decimal.

5 Convert each percentage to a decimal, then solve.

a 4% of \$60
b 9% of 110 L
c 6% of 20 m
d 15% of 300 kg
e 13% of \$80
f 52% of 800 km

6 Tristy scored 95% in her maths exam. If the exam was out of 120, how many marks did she score?

7 Bianca earns $600 per week. She pays 25% in tax, and 40% for rent and household expenditure, and the remainder is placed in a savings account.

a How much tax does Bianca pay?

b What is the weekly expenditure for rent and household purposes?

c How much does Bianca save each week?

Investigation 6 What is a rational number?

A rational number is a number that can be written as a simple fraction. Consider these examples.

Number	*Simplified fraction*
4	$\frac{4}{1}$
1.7	$\frac{17}{10}$
0.009	$\frac{9}{1000}$
$0.\dot{2}\dot{7}$	$\frac{3}{11}$

A **rational number** is a number that can be expressed as the ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$. An **irrational number** cannot be written as a simple fraction.

1 Complete the following table.

	a	b	$\frac{a}{b}$	=	**Rational?**
a	7	20	$\frac{7}{20}$	0.35	Yes
b	5	1000			Yes
c	2	5			
d	1	9			
e	13	5		2.6	
f	12	0			Not rational

2 Explain why $\frac{12}{0}$ is not a rational number.

3 In groups, determine if the following are rational or irrational numbers. Explain your reasoning for each.

a 2.4 **b** $3\frac{1}{2}$ **c** $\sqrt{5}$ **d** π **e** 0.052

f 0.625 **g** $\frac{9}{0}$ **h** $\sqrt{3}$ **i** 1.7

4 Below is an interesting fact about rational and irrational numbers. Use the internet to find two more interesting facts about rational and irrational number. Share your results with your class.

Interesting fact

Pythagoras, an ancient Greek mathematician, believed that all numbers could be written in fraction form and were consequently rational. Hippasus, one of his students, used geometry to prove that $\sqrt{2}$ could not be represented as a fraction and was therefore irrational.

Pythagoras, however, firmly believed that all numbers had perfect values, so were indeed rational. He could not disprove Hippasus' theory, so Hippasus was thrown overboard and drowned.

L Comparing prices – the best buy

When purchasing items, people generally want *value for money*. However, it is often not obvious which item represents the best value. The range of sizes and prices of articles can be confusing and makes direct comparisons misleading.

The best way to compare prices is to convert the cost of an item into a **unit cost** (for example, cost per gram, cost per kilogram, cost per litre) and then compare the unit costs. Some supermarkets list the *cost per 100 g* on shelf labels to assist customers when choosing between similar products.

Brand X

500 g

$7.50

Cost per kg = $15.00

Brand Y

250 g

$6.08

Cost per kg = $24.32

Brand Z

750 g

$12.47

Cost per kg = $16.63

It is easy to see that Brand X is the 'best buy' at only $15.00 per kg.

EXAMPLE 1

Calculate the cost of 100 g of coffee for each pack size shown below to determine the 'best buy'.

Pack A

500 g

$19.33

Pack B

200 g

$9.75

Pack C

1 kg

$32.54

Convert the cost of each pack into a cost per 100 g.

500 g $= \$19.33$

$\therefore 100\text{ g} = \dfrac{\$19.33}{5}$

$\approx \$3.87$

200 g $= \$9.75$

$\therefore 100\text{ g} = \dfrac{\$9.75}{2}$

$\approx \$4.88$

1000 g $= \$32.54$

$\therefore 100\text{ g} = \dfrac{\$32.54}{10}$

$\approx \$3.25$

Pack C is the 'best buy', followed by Pack A, then Pack B: the most expensive.

Exercise 8L

1 Which is the better buy: Box A or Box B? Calculate the cost of 100 g for each to determine the answer.

Box A

500 g

$5.70

Box B

750 g

$7.60

2 Calculate the cost of 100 g for each product. Which is the better buy in each case?

a	Ricey Bubbles:	200 g costing $2.30	or	500 g costing $4.70
b	peanuts :	750 g costing $16	or	1 kg costing $20.50
c	flour :	750 g costing $2.50	or	400 g costing $1.24
d	Doggie Chews:	550 g costing $9.35	or	820 g costing $15.99
e	potatoes:	800 g costing $2.56	or	1.3 kg costing $2.90
f	carrots:	500 g costing $1.10	or	1.2 kg costing $2.28

Convert to the same units (grams) before calculating.

EXAMPLE 2

Which can of baked beans (A, B or C) is the best value for money?
A: 225 g can costing $0.80 B: 440 g can costing $1.06 C: 900 g can costing $1.83

Convert the cost of each can into a cost per gram, to compare the costs of one unit.

Can A: $\frac{\$0.80}{225\text{ g}} \approx \$0.0036$ per g or $\frac{80\text{ cents}}{225} \approx 0.36$ cents per g

Can B: $\frac{\$1.06}{440\text{ g}} \approx \$0.0024$ per g or $\frac{106\text{ cents}}{440} \approx 0.24$ cents per g

Can C: $\frac{\$1.83}{900\text{ g}} \approx \$0.0020$ per g or $\frac{183\text{ cents}}{900} \approx 0.20$ cents per g

Can C is the best value for money as the cost per gram is least.

$\frac{\text{Cost}}{\text{Size}}$ = *cost per unit*

3 **a** For each of the following grocery items at Supamarket, which pack is the best value for money?

i *Mocha coffee*
250 g jar costs $10.60
500 g jar costs $16.10

ii *Coffee*
250 g jar costs $10.50
400 g tin costs $16.40

iii *Chocolate*
55 g block costs $1.56
250 g block costs $4.76
375 g block costs $7.48

iv *Chocolate drink powder*
220 g tin costs $3.26
375 g tin costs $4.98
750 g tin costs $10.18
1.25 kg tin costs $16.16

v *Cheese spread*
115 g jar costs $2.86
175 g jar costs $3.92
235 g jar costs $4.98
455 g jar costs $7.66

Do not forget to convert to the same units where necessary.

vi *Baked beans*
445 g tin costs $4.10
680 g tin costs $6.60

vii *Tuna*
95 g tin costs $2.26
185 g tin costs $3.94
425 g tin costs $6.60

viii *Cordial*
750 mL bottle costs $2.24
2 L bottle costs $6.20
4 L bottle costs $12.12

b Most of these items are cheaper per unit as you buy the bigger size. Can you suggest why this is the case?

4 Using the prices in question **3**, how much do you save by buying:

a a 500 g jar of coffee rather than two 250 g jars?

b a 4 L bottle of cordial rather than two 2 L bottles?

c two 375 g tins of chocolate drink powder rather than one 750 g tin?

5 Which is the better value for money?

a 4 L of paint for $79.90 or 10 L of paint for $163.80?

b 5 DVDs for $40 or 3 DVDs for $25?

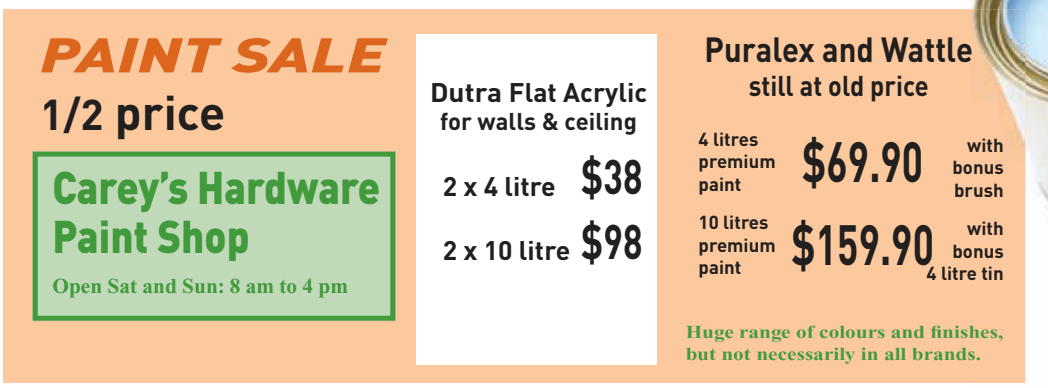
c 30 tickets for $750 or 50 tickets for $1200?

6 Cinema ticket prices are: adult $18, child $6. A family ticket is $40 for 2 adults and 2 children.

a A family of 1 adult and 2 children wishes to see a movie at the cinema. Which is the better buy: a family ticket, or paying for each member of the family?

b How much would a family of 2 adults and 3 children save by purchasing a single child's ticket plus a family ticket, rather than 5 individual tickets?

7



a In the advertisement above, find the cost per litre and hence the 'best buy' for:

i Dutra flat acrylic paint **ii** Wattle paint

b If you needed 20 L of flat acrylic paint, and the 4 L tins *are not* available singly, which is cheaper: six 4 L tins or two 10 L tins?

Do not be fooled into thinking that the 'biggest' product is always the 'best buy'.

c If, in part **b**, the 4 L tins *are* available singly, at $19 each, which is cheaper: five 4 L tins or two 10 L tins?

8 Examine the following Gold Coast holiday accommodation packages.

a Find the cost for a family of 2 adults and 2 children to stay for 5 nights at:

i Ocean Palms **ii** Tradewinds

b Which resort charges the cheaper rate per night for a family (2 adults and 2 children) staying for:
  i 5 nights? **ii** 8 nights?

c Which resort is the best value for 2 adults staying in a twin room for:
  i 5 nights? **ii** 8 nights?

d Find the rates per night for both cases in part **c**.

Investigation 7 Nutritional information

Work in groups for this investigation. All packing contains nutritional information expressed as a percentage of an individual's daily intake per serving.

1 Collect as many everyday products that you can that give nutritional information.

2 Discuss how the *percentage* for each nutritional aspect has been achieved. Is it accurate?

3 Do similar products with different brands have similar or contrasting nutritional information? Discuss and explain.

4 From reading the nutritional information, what aspects do you think are vital in determining the product's nutritional value? What should the consumer be avoiding or checking?

5 Select a scale or ranking system to order or group your products according to their degree of good nutrition.

6 Present your product groups to the class. Explain your scale/ranking system and justify how each product was judged according to its nutritional value. Also comment on the accuracy of percentages.

7 Are the nutritional information labels misleading, confusing or informative to consumers? Present a statement to the class regarding your findings after investigating/analysing the nutritional information on the many products your group collected.

Woolworths Home brand Cheese dip and crispbread

Nutrition information

Servings per package 12	Per serving 25 g	% DI* per serving	Per 100 g
	Serving size 25 g		
Energy	400 kJ	5%	1600 kJ
Protein	3.1 g	6%	12.4 g
Fat – Total	5.8 g	8%	23.1 g
– Saturated	2.9 g	12%	11.4 g
– Trans	0.2 g		0.7 g
– Polyunsaturated	0.7 g	-	2.7 g
– Monounsaturated	2.1 g	-	8.3 g
– Carbohydrate	7.5 g	2%	29.8 g
– Sugars	1.4 g	2%	5.5 g
Dietary fibre	0.9 g	3%	3.4 g
Sodium	275 mg	12%	1100 mg

Quantities stated above are averages only.
* Percentage Daily Intakes are based on an average adult diet of 8 700 kJ
Your daily intakes may be higher or lower depending on your energy need

Kellogg's LCMs

Nutrition information (average)

Servings per package – 6
Average serving size – 23 g (1 BAR*)

	Quantity per serving	% daily intake per serving †	Quantity per 100 g
Energy	420 kJ	5%	1830 kJ
Protein	0.8 g	2%	3.3 g
Fat total	3.2 g	5%	13.7 g
Saturated	2.5 g	10%	10.9 g
Carbohydrate	17.1 g	6%	74.2 g
Sugars	7.9 g	9%	34.2 g
Dietary fibre	0.1 g	0.3%	0.4 g
Sodium #	73 mg	3%	320 mg
Potassium	19 mg	–	85 mg

* Bar weight is approximate and is only to be used as a guide. If you have any specific dietary requirements please weigh your serving.
† % Daily intakes are based on an average adult diet of 8 700 kJ. Your daily intakes may be higher or lower depending on your energy needs.
# 73 mg of sodium per serve is equivalent to 0.2 g salt.

Calculator activities

1 Simplify the following.

a $\frac{25}{35}$ **b** $\frac{18}{24}$ **c** $\frac{164}{256}$ **d** $\frac{102}{8}$ **e** $\frac{215}{55}$

2 Convert each to a percentage.

a $\frac{59}{100}$ **b** 0.6 **c** 5.1 **d** $\frac{3}{8}$ **e** $5\frac{1}{2}$

3 Convert each to a simplified fraction.

a 8% **b** 55% **c** 42.6% **d** $12\frac{1}{2}\%$ **e** $20\frac{3}{5}\%$

4 Convert each to a decimal.

a 92% **b** 405% **c** 0.07% **d** 6.4% **e** 9.375%

5 Change each to an improper fraction.

a $1\frac{1}{4}$ **b** $2\frac{2}{3}$ **c** $5\frac{4}{5}$ **d** $6\frac{1}{3}$ **e** $10\frac{1}{11}$

6 Change each to a mixed numeral.

a $\frac{21}{5}$ **b** $\frac{33}{4}$ **c** $\frac{63}{6}$ **d** $\frac{92}{7}$ **e** $\frac{67}{9}$

7 Evaluate the following.

a $\frac{2}{9} + \frac{4}{11}$ **b** $18\frac{1}{2} - 10\frac{5}{7}$ **c** $\frac{7}{8} + \frac{1}{5}$

d $\frac{8}{9} \div 1.4 + 5\frac{1}{9}$ **e** $8\frac{1}{2} \times 6$ **f** $18 \div \frac{2}{3} \times 9\frac{1}{5}$

8 Calculate each quantity.

a $\frac{2}{3}$ of \$18.60 **b** $\frac{4}{5}$ of \$245 **c** $\frac{3}{7}$ of \$59.50

d $\frac{7}{9}$ of 396 L **e** $\frac{6}{11}$ of 154 m **f** $\frac{4}{15}$ of 390 kg

9 Convert each fraction to a decimal by dividing.

a $\frac{13}{20}$ **b** $\frac{24}{30}$ **c** $\frac{9}{12}$ **d** $\frac{21}{25}$ **e** $\frac{344}{22}$

10 Simplify the following.

a 0.7×6.1 **b** $4.9 - 3.6$ **c** $18.7 + 5.007$

d 2.34×0.9 **e** $9.12 - 5.6$ **f** $25.5 \div 5.1$

g $46.7 - 31.9 + 8.86$ **h** $14.3 + 2.6 - 5.7$ **i** $6.1 \times 8.2 \div 10.25$

j $(19.1 + 6.895) - (2.5 \times 3)$ **k** $(25.8 - 3.4) \div 0.04$ **l** $(6.2 + 3.9 - 5.7) \times (4.2 \div 0.7)$

11 Solve the following problems.

a Bianca weighs 58.63 kg and Sylvanna weighs 47.8 kg. What is the difference in their weights?

b Calculate the cost of 16 pens at \$1.56 each and 4 folders at \$5.65 each.

c Steak costs \$19.80 per kg. Calculate the cost of purchasing $6\frac{1}{2}$ kg of steak.

d A length of ribbon is 6.03 m. How many lengths of 9 cm can be cut to make bows for a charity dinner?

12 Calculate the following quantities.

a 15% of 764 m **b** 33% of 8050 t **c** 18% of 60 km

d 75% of 205 L **e** 85% of \$280 **f** $16\frac{1}{2}\%$ of 200 m

13 Use your calculator to round off each to 2 decimal places

a $\frac{4}{9}$ **b** $\frac{2}{3}$ **c** $\frac{5}{7}$ **d** $\frac{11}{19}$

e $38.6 \div 9.4$ **f** 7.5×9.87 **g** $0.5 \times 45.1 \div 3$ **h** $(3.3 + 8.7) \div 5.2$

14 Use your calculator to check that $0.\dot{9} \approx 1$. Explain the reasoning for this answer.

Language in mathematics

1 Circle the words below in the 'find-a-word', then write out the remaining letters in order. What message do they reveal?

add, addition, align, approximate, convert, cost, decimals, decimal point, divide, hundredth, invented, maths, money, multiply, place-value, repeating, subtract, tenth

I	A	P	P	R	O	X	I	M	A	T	E	S
N	I	T	M	O	N	S	T	A	E	H	V	E
V	G	R	N	C	S	B	L	O	O	T	K	S
E	N	E	T	N	O	I	T	I	D	D	A	U
N	I	V	H	E	G	S	T	E	N	E	T	B
T	T	N	H	N	S	H	T	A	M	R	E	T
E	A	O	S	L	A	M	I	C	E	D	E	R
D	E	C	I	M	A	L	P	O	I	N	T	A
E	P	X	M	P	A	L	A	V	T	U	I	C
N	E	E	D	O	T	D	I	H	H	H	E	T
W	R	O	R	K	N	D	D	I	N	G	O	F
D	E	U	L	A	V	E	C	A	L	P	E	C
M	U	L	T	I	P	L	Y	I	M	A	L	S

2 Use a dictionary to find the meaning of:

a align **b** approximate **c** convert

3 Write two synonyms for the word 'repeating'.

4 The following passage contains seven spelling errors:

Rod and Sue-Lin loved working with decimals. In fact, it was there favorite topic in Mathamatics. They desided to have a little compatition to see who was the best. Their techer gave them this problem:

Calculate: $\frac{4.8 \times 100 + 0.56 \times 1000 + 0.3 \times 10}{100}$

Rod's anser was 5.39. Sue-Lin's was 10.43.

a Write the incorrect words correctly.

b Who had the correct answer and was therefore the winner?

5 Use the information to discover the word.

a D E C I __ __ __ __ __ one-tenth of a litre (100 mL)

b D E C I __ __ a one-tenth block of a set of scores

c D E C I __ __ __ a unit for measuring the intensity of sound

d D E C I __ __ __ __ to kill one in ten

6 Write a creative newspaper report that includes the use of fractions. For example:
- Sporting Event: Athlete reduces personal best time by $\frac{4}{5}$ of a second!
- Boy genius can evaluate fractions in seconds!

7 Bartholomeus Pitiscus invented the decimal point in 1612. What else can you find out about him?

8 In some European countries, a decimal point is represented by a comma.
For example, 80.14 is written as 80,14. Find out which countries use a comma.

Terms

ascending order	base 10	best buy	compare	decimal
decimal place	decimal point	denominator	descending order	equivalent fraction
equivalent ratio	estimate	express	fraction	hundredth
improper fraction	infinite	mixed numeral	numerator	percentage
place-value	proper fraction	quantity	ratio	rational form
recurring decimal	repeating decimal	rounding	thousandth	unit cost

Check your skills

1

The ratio of circles to rectangles is:

A 3 : 8 **B** 3 : 5 **C** 5 : 3 **D** 5 : 8

2 A fruit bowl contains only oranges and apples in the ratio 7 : 2. What fraction of the fruit are oranges?

A $\frac{2}{9}$ **B** $\frac{7}{2}$ **C** $\frac{2}{7}$ **D** $\frac{7}{9}$

3 The ratio of pens to pencils in a pencil case is 1 : 4. If there are 16 pencils, how many pens are there?

A 20 **B** 4 **C** 24 **D** 30

4 Express as a decimal: $\frac{8}{10} + \frac{6}{100} + \frac{3}{10\,000}$.

A 0.8063 **B** 0.863 **C** 0.8603 **D** 8.63

5 Convert $\frac{188}{10\,000}$ to a decimal.

A 0.188 **B** 0.0188 **C** 0.001 88 **D** 0.000 188

6 Convert 2.63 to a fraction.

A $2\frac{63}{10}$ **B** $\frac{263}{1000}$ **C** $\frac{263}{100}$ **D** $\frac{263}{10}$

7 The value of the digit 2 in 3.0029 is:

A $\frac{2}{100}$ **B** $\frac{2}{1000}$ **C** 2 **D** 2000

8 What type of decimal is 8.323 232 323 …?

A infinite **B** recurring **C** repeating **D** continuous

9 Which statement below is incorrect?

A $0.47 = \frac{47}{1000}$ **B** $0.47 > 0.047$ **C** $0.47 < 0.477$ **D** $0.47 = \frac{47}{100}$

10 Round 7.355 correct to 1 decimal place.

A 7.3 **B** 7.5 **C** 7.4 **D** 7.6

11 Simplify $0.8 + 28.39$.

A 36.39 **B** 108.39 **C** 28.19 **D** 29.19

12 Simplify $4.3 - 2.932$.

A 1.368 **B** 2.632 **C** 1.638 **D** 7.232

13 Simplify 0.7×0.0021.

A 0.000 147 **B** 0.00147 **C** 0.01 47 **D** 0.147

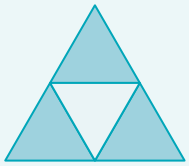
14 Simplify $0.02 \div 0.5$.

A 0.0004 **B** 0.4 **C** 0.004 **D** 0.04

15 What percentage of the shape is shaded?

A $\frac{3}{4}$ **B** 0.75%

C 75% **D** 0.34%



16 Express 53% as a fraction.

A $\frac{53}{100}$ **B** 0.53 **C** $\frac{5}{10}$ **D** 0.053

17 Express 84% as a simplified fraction.

A 0.84 **B** $\frac{42}{50}$ **C** $\frac{84}{100}$ **D** $\frac{21}{25}$

18 Express $10\frac{1}{5}\%$ as a decimal.

A 10.5 **B** 0.102 **C** 0.1015 **D** 0.105

19 Express $\frac{4}{9}$ as a percentage.

A $0.\dot{4}\%$ **B** $4.\dot{4}\%$ **C** $44.\dot{4}\%$ **D** $444.\dot{4}\%$

20 Express 0.0327 as a percentage.

A 0.327% **B** 3.27% **C** 32.7% **D** 327%

21 Arrange 0.386, $\frac{3}{8}$ and $41\frac{1}{2}\%$ in ascending order.

A $\frac{3}{8}$, 0.386, $41\frac{1}{2}\%$ **B** $41\frac{1}{2}\%$, 0.386, $\frac{3}{8}$ **C** $\frac{3}{8}$, $41\frac{1}{2}\%$, 0.386 **D** 0.386, $41\frac{1}{2}\%$, $\frac{3}{8}$

22 Find 26% of \$5800.

A \$15.08 **B** \$150.80 **C** \$1508 **D** \$15 080

23 Which is the best buy for Corn Flake Apples cereal?

A 250 g at \$3.80 **B** 500 g at \$7.10 **C** 750 g at \$10.20 **D** 1.2 kg at \$15.50

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4–8	9	10	11, 12	13	14	15–17	18	19–21	22	23
Section	A	B	C	D	E	F	G	H	I	J	K	L

8A Review set

1 **a** If $\frac{5}{13}$ of a diagram is shaded, what fraction is unshaded?

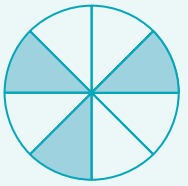
b Express as a ratio the shaded area to the unshaded area.

2 Shade 25% of this diagram.

3 Write 56 out of 100 as a percentage.

4 What percentage of the diagram is:

a shaded? **b** unshaded?



5 Convert 41% to a fraction.

6 Convert 63% to a decimal.

7 Express $22 + \frac{4}{10} + \frac{9}{1000}$ as a decimal.

8 **a** Express 5.6 as a percentage. **b** Express $\frac{1}{4}$ as a percentage.

9 Convert to percentages and arrange in ascending order: $\frac{2}{5}$, 63%, 0.13

10 State the value of the digit 3 in 4013.

11 Express 4.325 55 correct to the nearest hundredth.

12 Insert one of $>$ or $<$ or $=$ to make each statement correct.

a $\frac{3}{5}$ __ 0.6 **b** 9.3 __ 9.33

13 Simplify the following.

a 3.6×5 **b** 14.9×100 **c** 12×16.47

14 Simplify the following.

a $1.4 \div 0.2$ **b** $84.7 \div 10$ **c** $37.6 \div 0.4$

15 Simplify the following.

a $12.6 - 15.7 + 3.84$ **b** $15.5 \div 0.05 + 42.8$ **c** $16.2 \div 0.8 + 5.7 - 1.9$

16 Find 25% of 32 m.

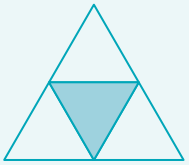
17 Which is the better buy?

500 g of Aaroma Coffee at \$5.70 or 750 g of Aaroma Coffee at \$7.25

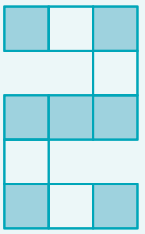
8B Review set

1 What percentage of this diagram is:

a shaded? **b** unshaded?



2 Express as a ratio the shaded area to the unshaded area in this diagram.



3 Express the following as simplified fractions.

a 28% **b** 42%

4 State the value of the digit 5 in 1.058.

5 Express $16 + \frac{7}{100} + \frac{5}{1000}$ as a decimal.

6 Write 2.1 as a fraction.

7 Write 12.05 as a fraction.

8 Express $\frac{3}{8}$ as a decimal.

9 **a** Express $\frac{11}{25}$ as a percentage. **b** Express $6\frac{1}{8}\%$ as a decimal.

10 Express 9.8742 correct to the nearest whole number.

11 Arrange the following decimals in descending order: 0.21, 0.122, 0.02, 0.12

12 Simplify the following.

a $15.7 + 6.3 + 8.2$ **b** $19.2 + 0.84 + 2$

13 Simplify the following.

a $58.6 - 17.43$ **b** $8.99 - 5.632$

14 Simplify the following.

a $9.6 \div 3 + 4.8 \times 5$ **b** $4.3 \times 4.7 - 16.48 + 8.2$

15 In the morning the temperature was 18.38C, but by noon it had risen by 5.678C. What was the temperature at noon?

16 Express this statement as a percentage: '18 students from a class of 30 students play football'.

17 Calculate the cost of 100 g if 850 g costs $14.50.

8C Review set

1 Write a simplified ratio to compare the number of moons to the number of stars.

2 In Class 7A, the ratio of netball players to soccer players is 4 : 9. If 12 girls play netball, how many play soccer?

3 **a** Express 156% as a decimal. **b** Express $7\frac{1}{2}$% as a decimal.

4 **a** Find 25% of 600 kg. **b** Find $12\frac{1}{4}$% of \$5000

5 State the value of the digit 8 in 148 507.

6 Express $\frac{7}{100} + \frac{9}{1000}$ as a decimal.

7 Express 0.08 as a fraction.

8 Express $\frac{7}{9}$ as a decimal.

9 Convert each number to a percentage and plot the numbers on a number line.
0.37, $\frac{30}{32}$, 74%, $\frac{12}{15}$, 0.586

10 Write 15.0775 correct to the nearest hundredth.

11 Express $\frac{5}{6}$ as a decimal correct to 2 decimal places.

12 Insert one of $>$ or $<$ or $=$ to make each statement correct.
a 21.12 ___ 21.012 **b** 19.6 ___ 19.60

13 Which decimal number is closest to 0.4?
A 0.45 **B** 0.41 **C** 0.5 **D** 0.05

14 Simplify the following.
a $4.8 \div 0.4 \times 6$ **b** $1.2 \times 0.86 \times 3$ **c** $16.6 + 2.38 + 4.7$

15 Simplify the following.
a $(2.1 + 5) \times (16.4 - 5.9)$ **b** $(10.42 - 8.7) + (0.6 \times 9)$

16 Find $10\frac{5}{8}$% of \$5000.

17 Sara purchased 15.4 m of fabric to make three shirts. If each shirt requires 4.25 m of fabric:
a how much fabric is used to make the shirts? **b** how much fabric remains?

9

Transformations and symmetry

This chapter deals with transformations on the Cartesian number plane and the symmetry of shapes.

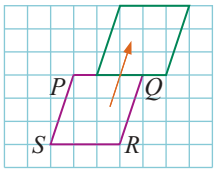
At the end of the chapter you should be able to:

- translate, rotate and reflect plane shapes
- identify the properties of these transformations
- recognise and describe transformations and combinations of transformations
- perform transformations and combinations of transformations on the number plane using coordinates
- understand and identify line and rotational symmetry.

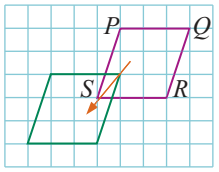
Diagnostic test

Use these diagrams for questions **1** to **3**.

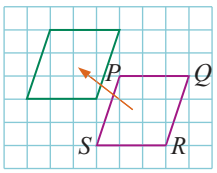
A



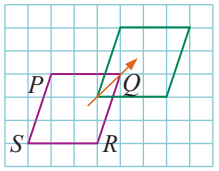
B



C



D



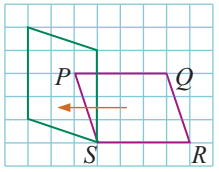
1 Which diagram shows the parallelogram *PQRS* translated 3 units left and 2 units up?

2 Which diagram shows the parallelogram *PQRS* translated 2 units to the right and 3 units up?

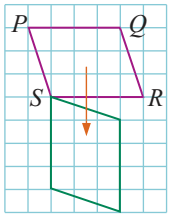
3 Which diagram shows the parallelogram *PQRS* translated 2 units up and 3 units left?

Use these diagrams for questions **4** and **5**.

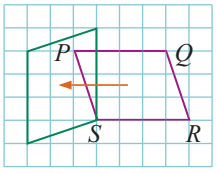
A



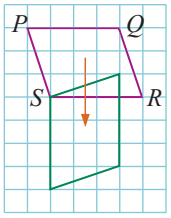
B



C



D

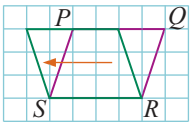


4 Which diagram shows parallelogram *PQRS* rotated anticlockwise through 90° about *S*?

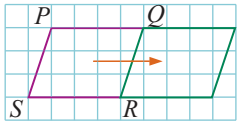
5 Which diagram shows parallelogram *PQRS* rotated clockwise through 90° about *S*?

Use these diagrams for questions **6** and **7**.

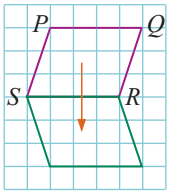
A



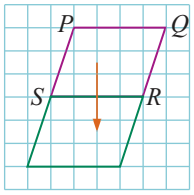
B



C



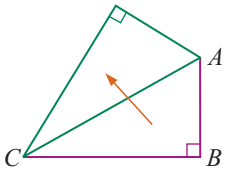
D



6 Which diagram shows *PQRS* reflected in *SR*?

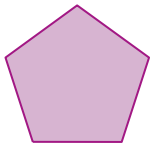
7 Which diagram shows *PQRS* translated?

8 The triangle *ABC* has been:



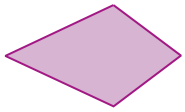
A translated
B reflected in *AC*
C rotated clockwise about *A*
D rotated anticlockwise about *A*

9 What type of symmetry does this shape have?



A horizontal
B vertical
C vertical and horizontal
D none

10 What type of symmetry does this shape have?



A horizontal
B vertical
C vertical and horizontal
D none

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–10
Section	ACMMG 142

MEASUREMENT & GEOMETRY

TRANSFORMATIONS

Interesting patterns can often be formed by sliding, flipping and turning plane shapes.
The mathematical terms for these processes are translation, reflection and rotation.

- Sliding a shape is **translation**.
- Flipping a shape is **reflection**.
- Turning a shape is **rotation**.

These processes are called **geometrical transformations**. The second shape is called an **image** of the **original**.

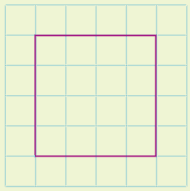
A Translation

A translation is a sliding movement.

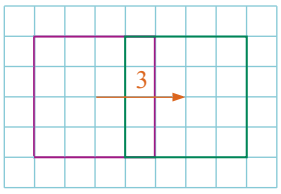
EXAMPLE 1

Draw this square on grid paper, make a copy and cut it out. Place the copy on top of the square and translate it as follows. In each case, draw the image on your grid paper.

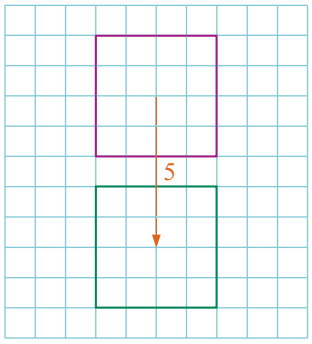
a 3 units right
b 5 units down
c 5 units right and 3 units up
d 2 units left and 4 units up



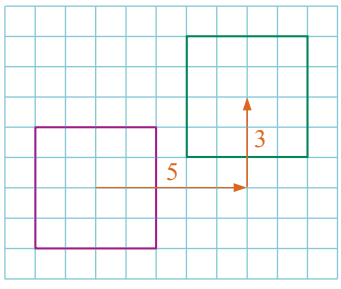
a



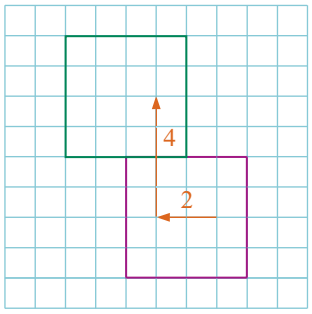
b



c

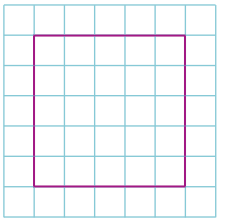


d



Exercise 9A

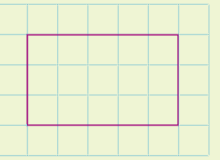
1 Draw this square on grid paper, make a copy and cut it out. Place the copy on top of the square and translate it as follows. In each case, draw the image on your grid paper.



a 4 units right
b 3 units down
c 4 units right and 3 units down
d 3 units left and 2 units up

EXAMPLE 2

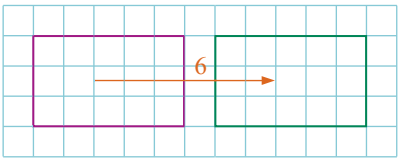
a Translate this rectangle as follows.



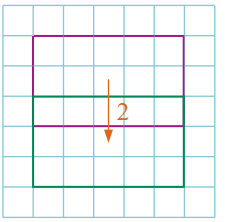
i 6 units right **ii** 2 units down **iii** 3 units left and 2 units down

b i Compare each image with its original. Are the matching sides the same length?
ii Are the matching angles the same size? **iii** Are the areas the same?

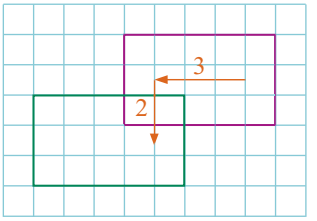
a i Move each vertex 6 units right. Join the new positions of the vertices.



ii Move each vertex 2 units down. Join the new positions of the vertices.

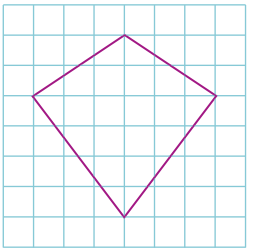


iii Move each vertex 3 units left and 2 units down. Join the new positions of the vertices.



b i Yes. The length and breadth remain 5 units and 3 units.
ii Yes. All angles remain 90°.
iii Yes. Area remains $5 \times 3 = 15$ square units.

2 a Copy this kite onto grid paper and show the new position when it is translated as follows.



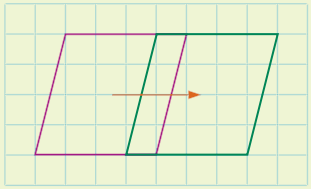
i 3 units right **ii** 2 units left
iii 3 units down **iv** 2 units up
v 1 unit right and 1 unit down
vi 2 units left and 3 units up

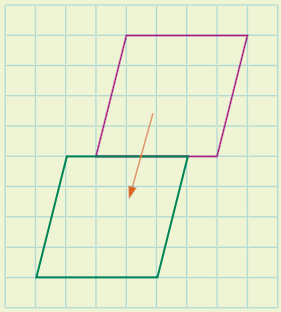
b Compare each image with its original.
i Are the matching sides the same length?
ii Are the matching angles the same size?
iii Are the areas the same?

3 a Consider the kite and its image drawn in question **2** part **v**. Mark a point P on the kite and translate it 1 unit right and 1 unit down. Is the new position of P on the image? Is this true for any point P on the kite?
b Consider the kite and its image drawn in question **2** part **vi**. Mark a point P on the kite and translate it 2 units left and 3 units up. Is the new position of P on the image? Is this true for any point P on the kite?
c Complete the statement:
'When a shape is translated, every point on it moves the _____ distance in the _____ direction'.

EXAMPLE 3

Describe the following transformations.

a 

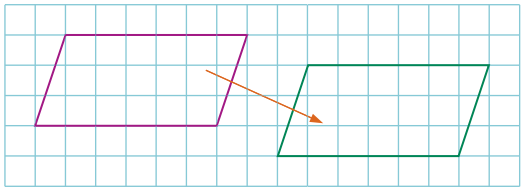
b 

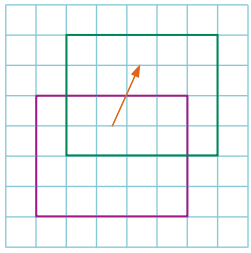
Look at each vertex of the shape and relate it to its new position.

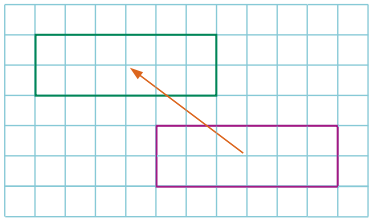
a Each vertex has been translated 3 units right. The parallelogram has been translated 3 units right.

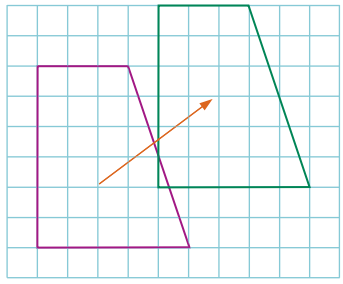
b Each vertex has been translated 2 units left and 4 units down. The parallelogram has been translated 2 units left and 4 units down.

4 Describe the following transformations.

a 

b 

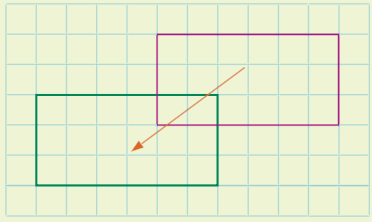
c 

d 

EXAMPLE 4

a Describe this transformation.

b What transformation is needed to move the image back to its original position?



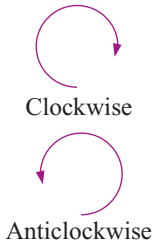
a The rectangle has been translated 4 units left and 2 units down.

b The image needs to be translated 4 units right and 2 units up to return to its original position.

5 For each of the transformations shown in question **4**, what transformation is needed to move the image back to its original position?

B Rotation

A rotation is a turn around a fixed point, called the **centre of rotation**. The direction of rotation can be clockwise or anticlockwise.



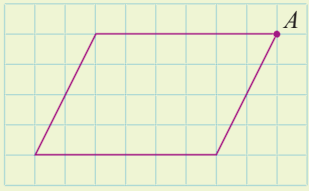
EXAMPLE 1

Draw this parallelogram on grid paper, make a copy and cut it out. Place the copy on top of the parallelogram and rotate it clockwise about A through the following turns. In each case, draw the image on your grid paper.

a $\frac{1}{4}$ turn (1 right angle or 90°)

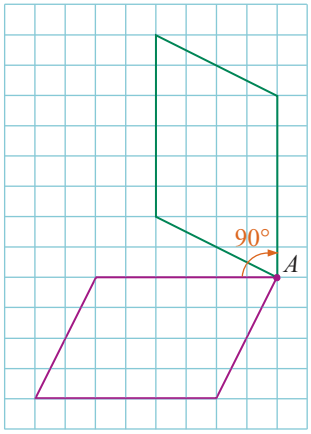
b $\frac{1}{2}$ turn (2 right angles or 180°)

c $\frac{3}{4}$ turn (3 right angles or 270°)

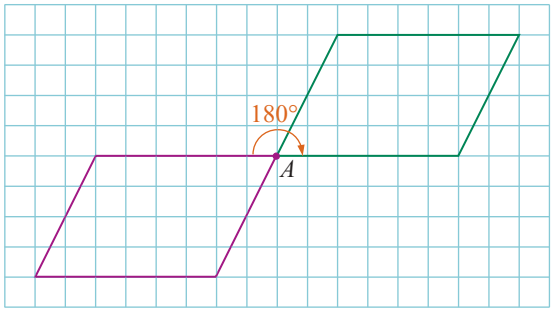


The point A is the centre of rotation.

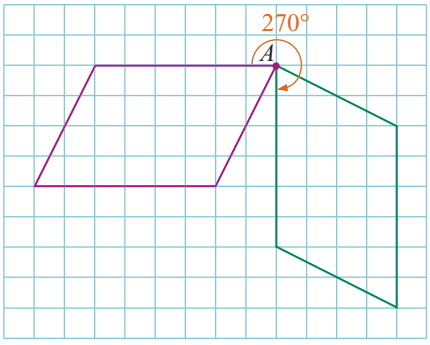
a



b



c

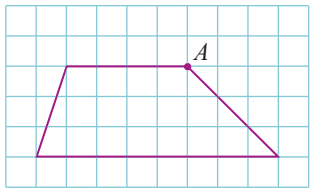


Exercise 9B

1 Draw this trapezium on grid paper, make a copy and cut it out. Place the copy on top of the trapezium and rotate it clockwise about A through the following turns. In each case, draw the image on your grid paper.

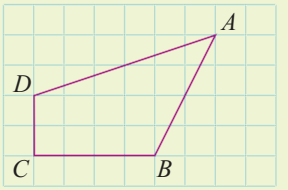
a $\frac{1}{4}$ turn (1 right angle or 90°) **b** $\frac{1}{2}$ turn (2 right angles or 180°)

c $\frac{3}{4}$ turn (3 right angles or 270°)

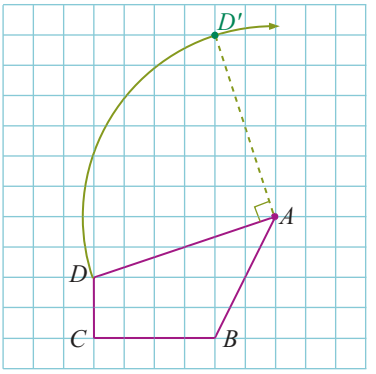


EXAMPLE 2

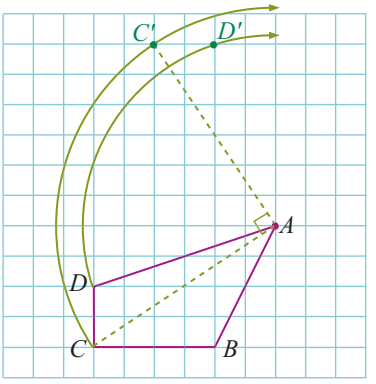
Rotate this shape clockwise about A through 90° (1 right angle).



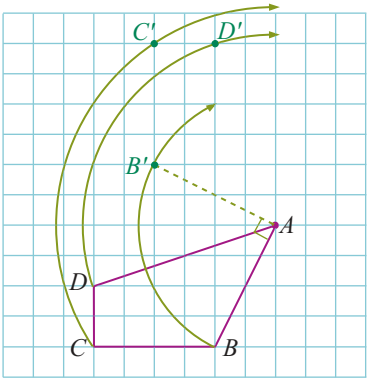
Step 1: Place the point of your compasses at A, open to width AD and draw an arc in a clockwise direction. Using your protractor, measure an angle of 90° from AD, as shown. Label the point D' where the arm of the right angle cuts this arc.



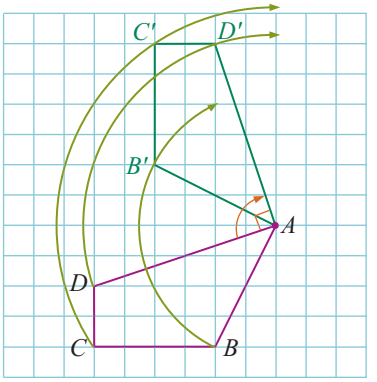
Step 2: Place the point of your compasses at A, open to width AC and draw an arc in a clockwise direction. Using your protractor, measure an angle of 90° from AC, as shown. Label the point C' where the arm of the right angle cuts this arc.



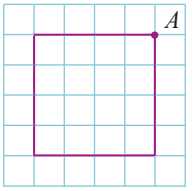
Step 3: Place the point of your compasses at A, open to width AB and draw an arc in a clockwise direction. Using your protractor, measure an angle of 90° from AB, as shown. Label the point B' where the arm of the right angle cuts this arc.



Step 4: Join the new points $AB'C'D'$.



2 **a** Copy this square onto grid paper and rotate it clockwise about A through:



i 90° (1 right angle) **ii** 180° (2 right angles)
iii 270° (3 right angles) **iv** 360° (4 right angles)

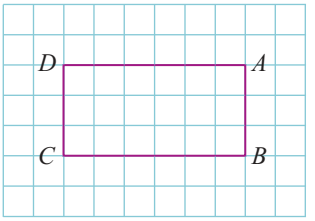
b Compare each image with its original.
i Are the matching sides the same length?
ii Are the matching angles the same size?
iii Are the areas the same?

3 **a** Consider the square and its image drawn in question **2** part **ii**. Mark a point P on the square and rotate it through 180°. Is the new position of P on the image? Is this true for any point P on the square? Are P and its new position the same distance from the centre of rotation?

b Consider the square and its image in question **2** part **iii**. Mark a point P on the square and rotate it through 270°. Is the new position of P on the image? Is this true for any point P on the square? Are P and its new position the same distance from the centre of rotation?

c Complete these statements:
i 'When a shape is rotated, every point on it moves through the _____ angle in the _____ direction'.
ii 'The distance of any point and its image from the centre of rotation _____ _____ _____.'

4 Copy this rectangle onto grid paper and rotate it:



a clockwise about A through
i 1 right angle **ii** 2 right angles
iii 3 right angles **iv** 4 right angles

b clockwise about C through
i 1 right angle **ii** 2 right angles
iii 3 right angles **iv** 4 right angles

c anticlockwise about B through
i 1 right angle **ii** 2 right angles **iii** 3 right angles

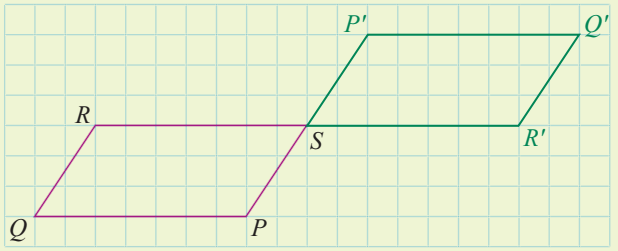
d anticlockwise about D through
i 1 right angle **ii** 2 right angles **iii** 3 right angles

Anticlockwise is in the opposite direction to clockwise. !

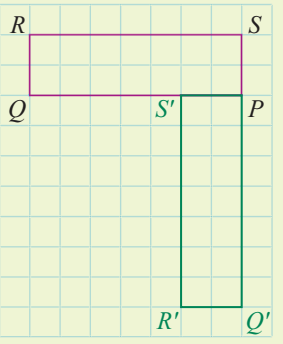
EXAMPLE 3

For each transformation shown:
i Describe the smallest rotation needed for the transformation.
ii State what transformation is needed to move the image back to its original position.

a



b



a **i** Clockwise (or anticlockwise) rotation about S through 180°.
ii Anticlockwise (or clockwise) rotation about S through 180°.

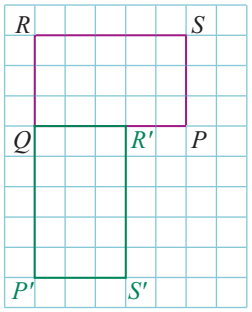
b **i** Anticlockwise rotation about P through 90°.
ii Clockwise rotation about P through 90°.

5 For each transformation shown:

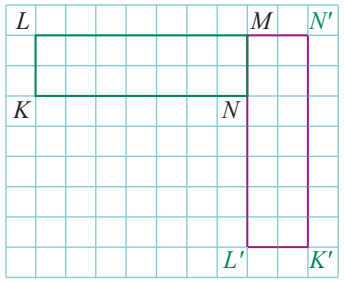
i Describe the smallest rotation needed for the transformation.

ii State what transformation is needed to move the image back to its original position.

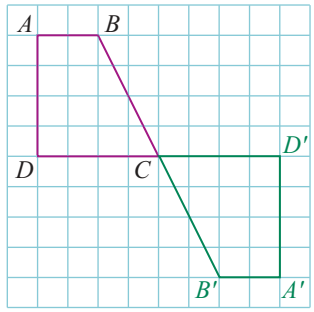
a



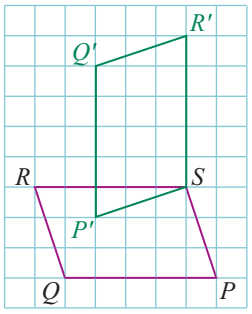
b



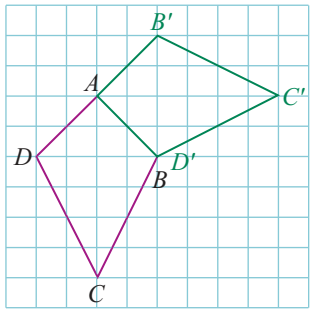
c



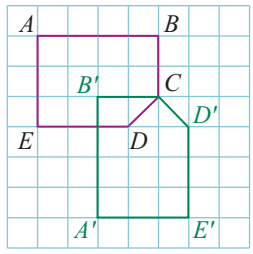
d



e



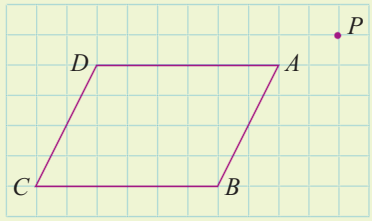
f



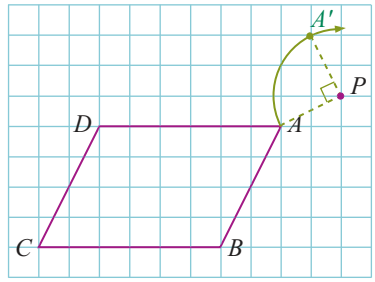
The centre of rotation does not have to be a point on the shape.

EXAMPLE 4

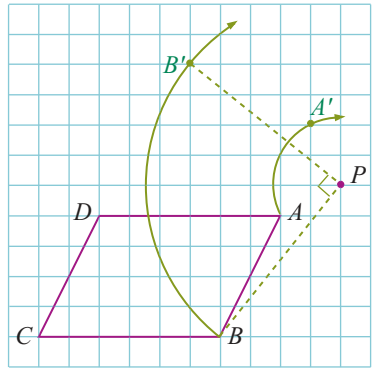
Rotate this parallelogram clockwise through 90° (1 right angle) about the point P.



Step 1: Place the point of your compasses at P. Draw an arc in a clockwise direction from A. Use your protractor to measure an angle of 90° from PA, as shown. Mark the new point A'.

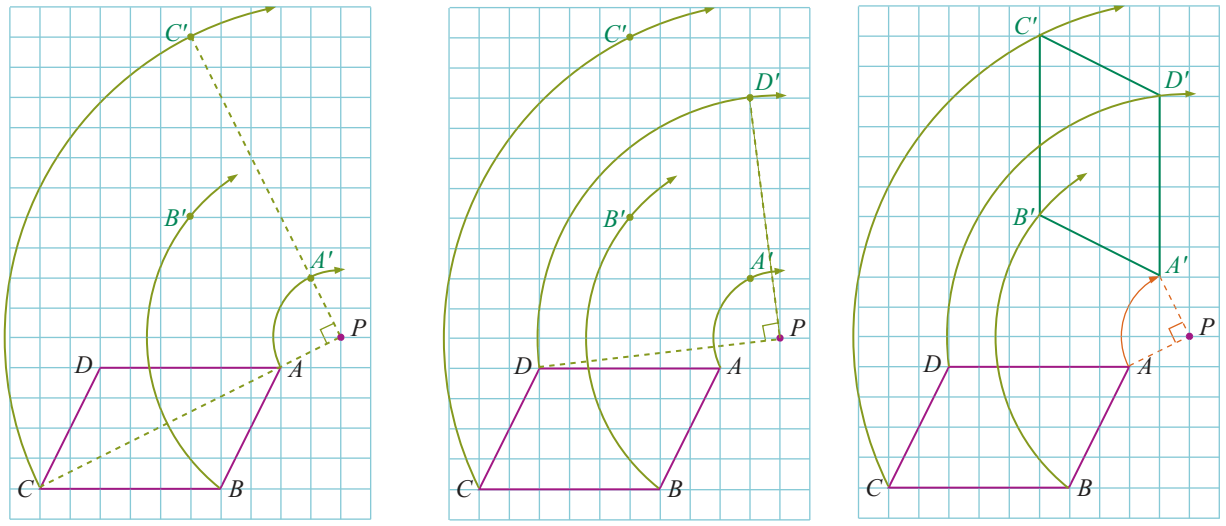


Step 2: Do similar for point B. Mark the new point B'.

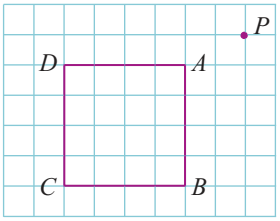


Step 3: Repeat for points C and D. Mark the new points C' and D' respectively.

Step 4: Join the new points $A'B'C'D'$.



6 Repeat questions **2a** and **2b**, except rotate this square about P, a point outside the square.



C Reflection

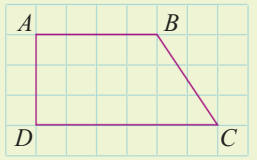
A reflection produces a mirror image of the shape. It is made by 'flipping' the shape over a fixed line called the **line of reflection**.

EXAMPLE 1

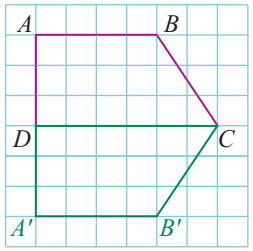
Draw the trapezium $ABCD$ on grid paper, make a copy and cut it out.
Place the copy on top of the trapezium and flip it over the edge:

a DC **b** AD

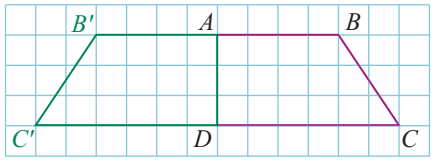
In each case draw the image on your grid paper.



a



b



*In part **a**, DC is the line of reflection.*
*In part **b**, AD is the line of reflection.*

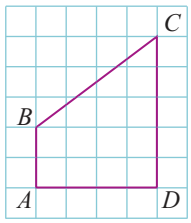
Check by placing a mirror along the edges DC and AD.
Does the image in the mirror each time coincide with the flipped-over shape?

Exercise 9C

1 Draw the trapezium $ABCD$ on grid paper, make a copy and cut it out. Place the copy on top of the trapezium and, by flipping it over, reflect it in the line:

a AD **b** CD **c** BA

In each case, draw the image on your grid paper.



'Flip over the edge AD' is usually expressed mathematically as 'reflect in the line AD'.

EXAMPLE 2

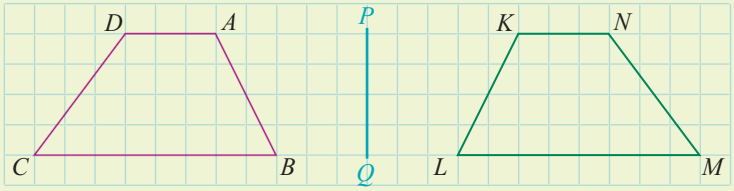
The line of reflection does not have to be an edge of the shape.

The diagram shows trapezium $ABCD$ and its reflection in the line PQ.

a What point is the image of vertex:

i A? **ii** B?

iii C? **iv** D?



b **i** Join each vertex to its image. What angle does each of these joining lines make with PQ?

ii Complete this statement: 'The line joining each point and its image makes a _____ angle with the line of reflection.'

c **i** Measure the distance of each vertex and its image from PQ.

ii Complete this statement: 'Every point and its image are the _____ distance from the line of reflection.'

d What transformation is needed to move the image back to its original position?

a **i** K **ii** L **iii** M **iv** N

b **i** 90°

ii The line joining each point and its image makes a right angle with the line of reflection.

c **i** $PA = PK = 5$ units, $QB = QL = 3$ units, $QC = QM = 11$ units, $PD = PN = 8$ units.

ii Every point and its image are the same distance from the line of reflection.

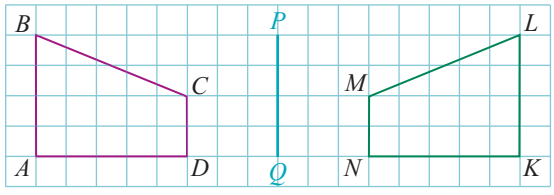
d Another reflection in the line PQ.

2 The diagram shows the trapezium $ABCD$ and its reflection in the line PQ.

a What point is the image of vertex:

i A? **ii** B?

iii C? **iv** D?



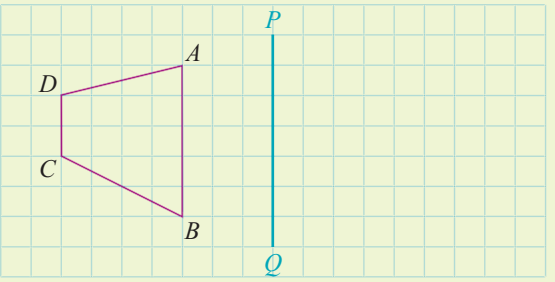
b Join each vertex to its image. What angle does each joining line make with the line of reflection?

c Measure the distance of each vertex and its image from the line of reflection. What do you notice?

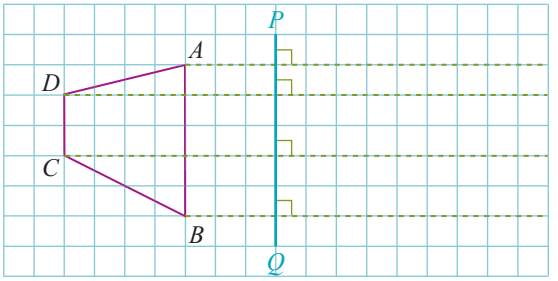
d What transformation is needed to move the image back to its original position?

EXAMPLE 3

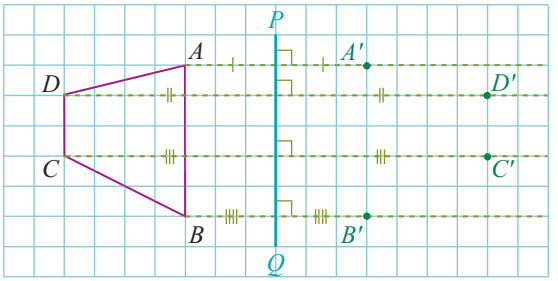
Reflect $ABCD$ in the line PQ.



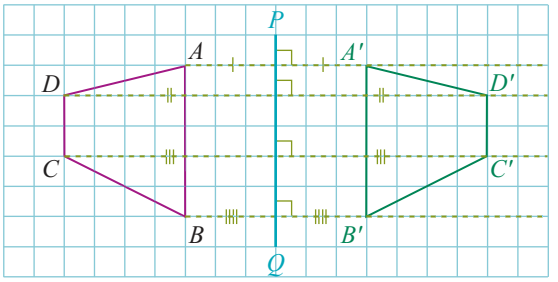
Step 1: Draw a perpendicular line from each vertex through PQ, as shown.



Step 2: Measure the distance of each vertex from PQ. Mark the position of its image on the perpendicular line the same distance away on the opposite side of PQ. Label the images of vertices A, B, C and D as A', B', C' and D' respectively.

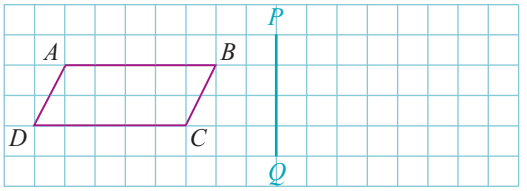


Step 3: Join the new points $A'B'C'D'$.

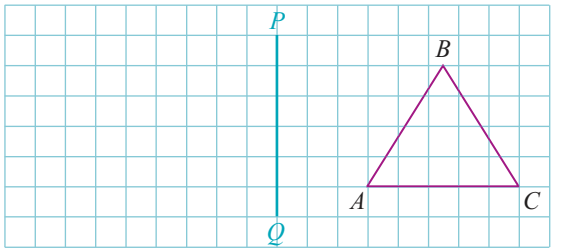


3 Copy each diagram onto grid paper and reflect each shape in the line PQ given.

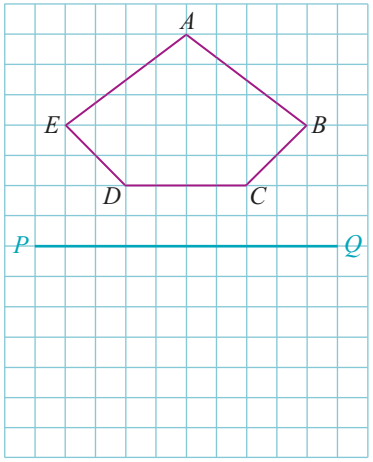
a



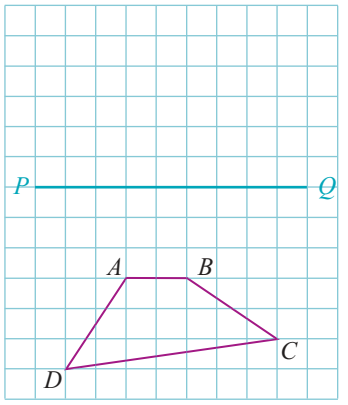
b



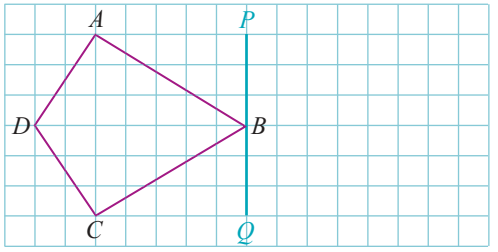
c



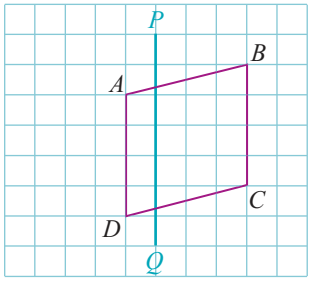
d



e



f



4 Compare each shape in question **3** with its image.

a Are matching sides the same length?

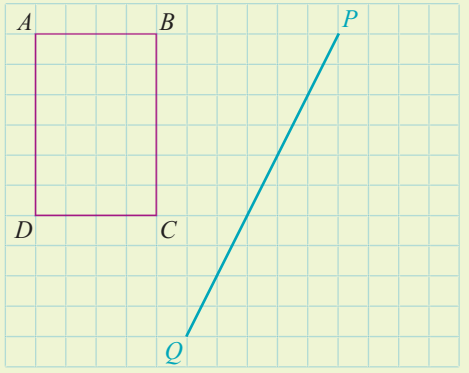
b Are matching angles the same size?

c Are the areas the same?

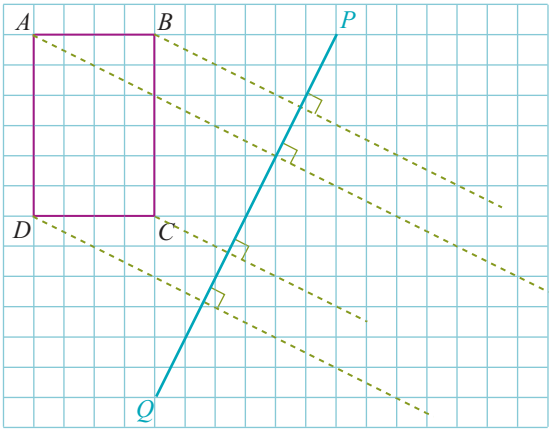
5 For each of the transformations shown in question **3**, what transformation is needed to move the image back to its original position?

EXAMPLE 4

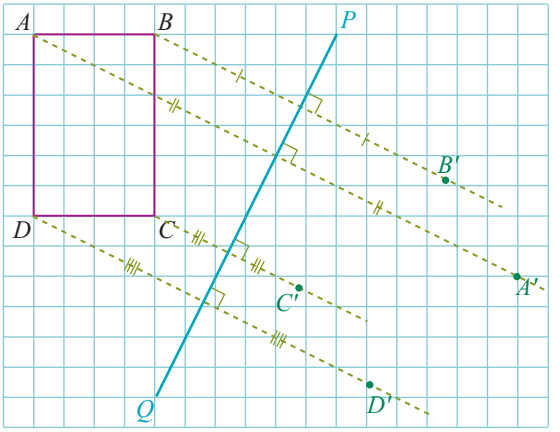
Reflect the rectangle $ABCD$ in the line PQ.



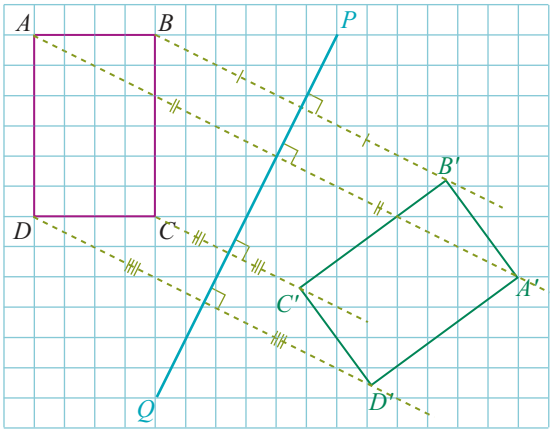
Step 1: Draw a perpendicular line from each vertex through PQ, as shown.



Step 2: Measure the distance of each vertex from PQ. Mark the position of its image on the perpendicular line the same distance away on the opposite side of PQ. Label the images of vertices A, B, C and D as A', B', C' and D' respectively.

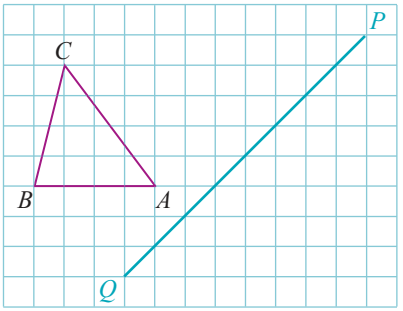


Step 3: Join the new points $A'B'C'D'$.

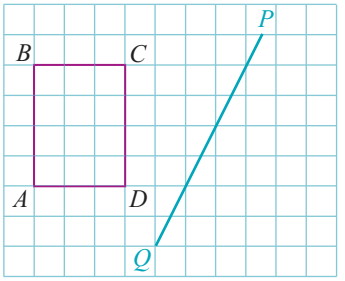


6 Copy each diagram onto grid paper and reflect the shape in the line PQ given.

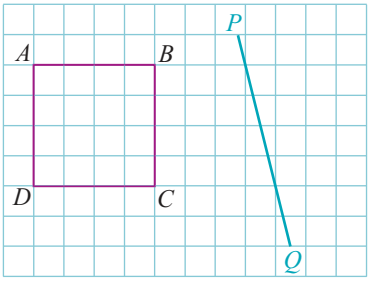
a



b

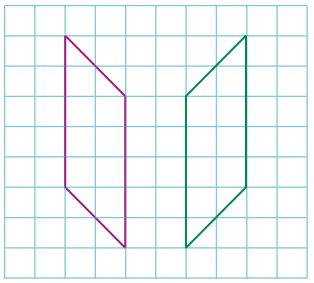


c

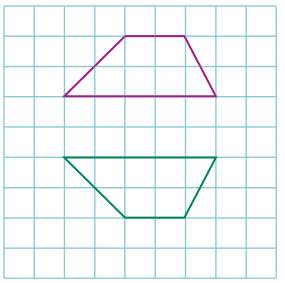


7 Each diagram shows a shape and its image. Copy onto grid paper and draw the line of reflection.

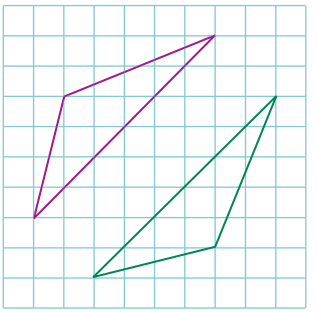
a



b



c

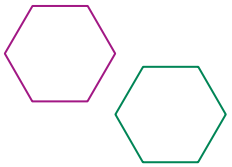


D Transformations and patterns

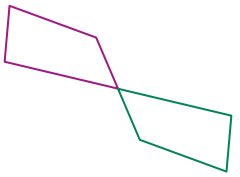
Exercise 9D

1 For each of the following, state whether the original (purple) shape has been translated, rotated or reflected.

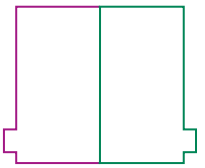
a



b

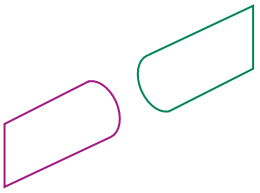


c

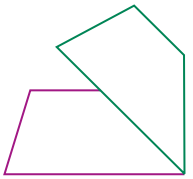


d

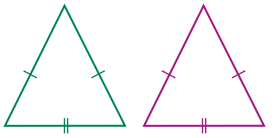
e



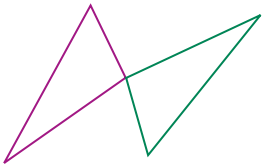
f



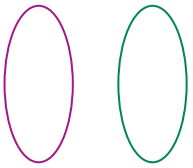
g



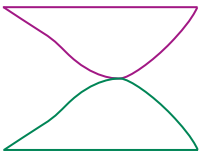
h



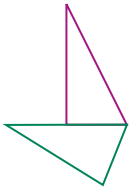
i



j



k



l

2 Use the results from question **1** to answer these questions.

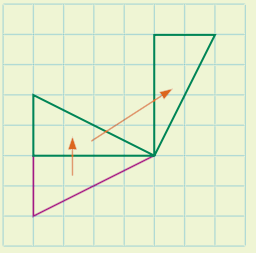
a Can a translation produce the same result as a rotation and vice versa?

b Can a translation produce the same result as a reflection and vice versa?

c Can a reflection produce the same result as a rotation and vice versa?

EXAMPLE 1

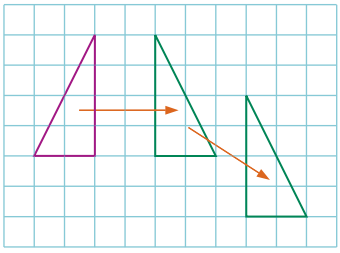
What combination of transformations is shown in this diagram?



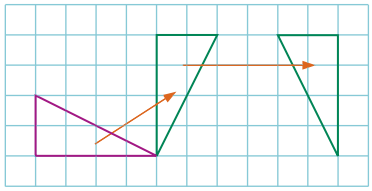
A reflection followed by a clockwise rotation of 90°.

3 What combination of transformations is shown in each diagram?

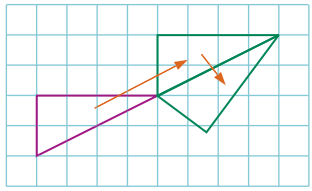
a



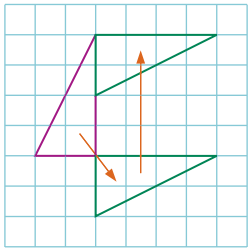
b



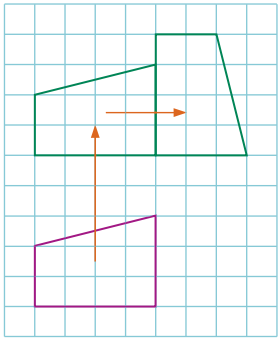
c



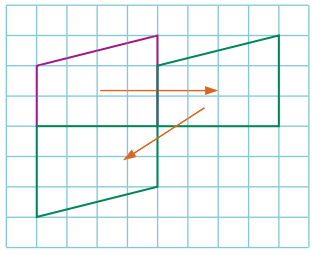
d



e

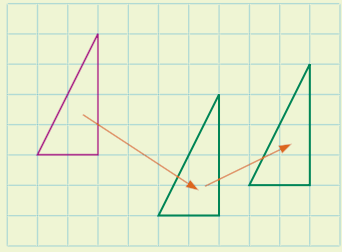


f



EXAMPLE 2

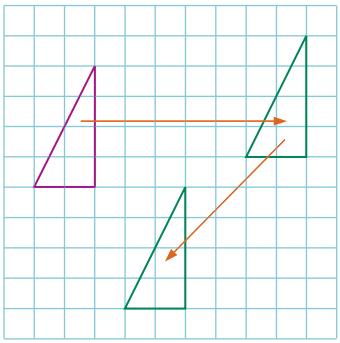
a What combination of transformations is shown in this diagram?
b What single transformation could be used to produce the same result?



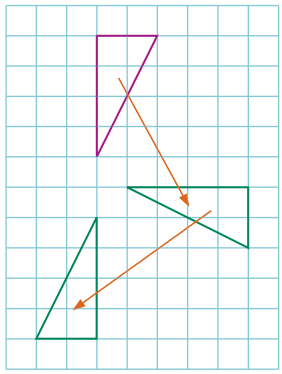
a Translation 4 units right and 2 units down followed by translation 3 units right and 1 unit up
b Translation 7 units right and 1 unit down

4 For each diagram, state:
i what combination of transformations is shown
ii what single transformation could be used to produce the same result.

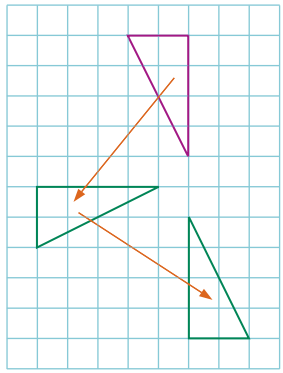
a



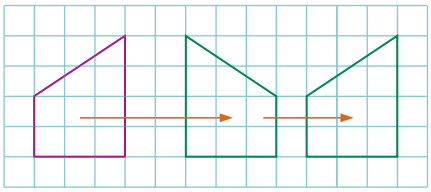
b



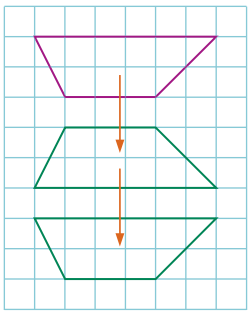
c



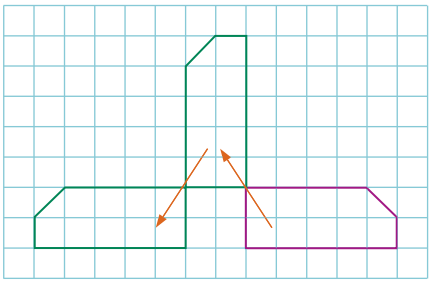
d



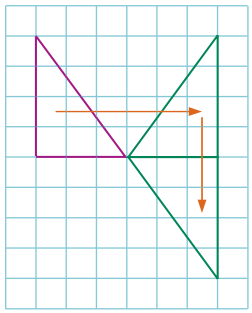
e



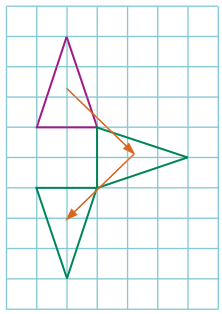
f



g



h

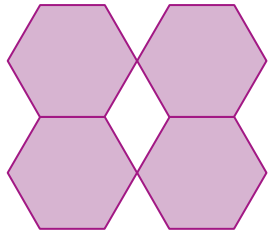


5 **a** Look at the answers to question **4**. Can two successive reflections produce a rotation?

b Can two successive rotations produce a reflection?

c Check the answers to parts **d** and **e** of question **4**. Are two successive reflections always equivalent to a single translation? Draw some shapes on grid paper to investigate.

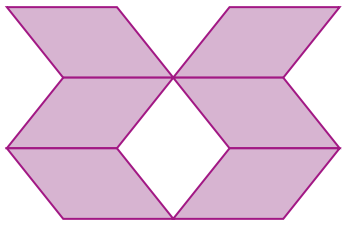
6 **a** The following pattern has been formed by translating a hexagon horizontally and vertically. Copy the pattern onto isometric grid paper and extend it so that it contains at least 16 hexagons.



b What other shape(s) is present?

c Colour the shapes to create a design.

7 The following pattern was formed by reflecting a parallelogram horizontally and vertically.

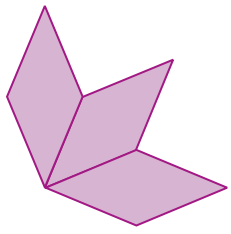


a Copy the pattern onto ordinary grid paper and extend it so that it contains at least 16 parallelograms.

b What other shape(s) is present?

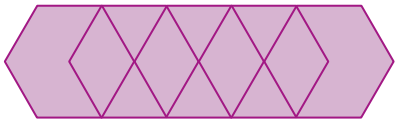
c Colour the shapes to create a design.

8 The following pattern has been started by rotating a rhombus about one of its corners. Complete the pattern and shade it to form a design.

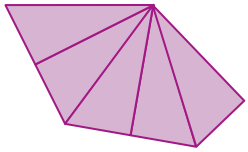


9 Identify the shape, and the transformation used on it, to create each of the following patterns.

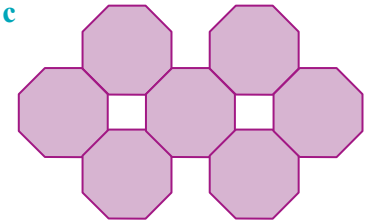
a



b



c



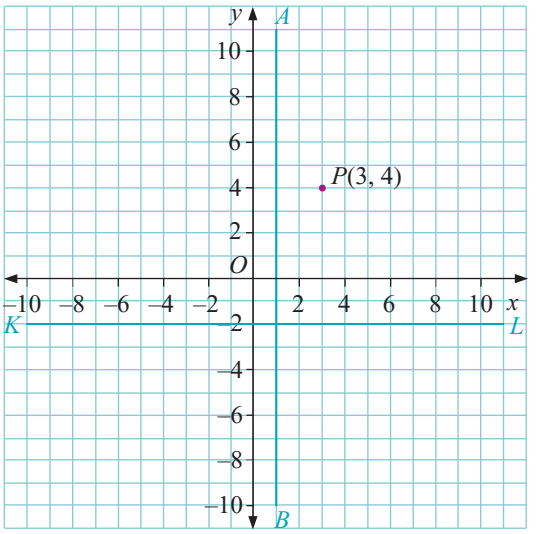
10 Make a pattern by translating, rotating or reflecting:

a a triangle **b** an octagon **c** a trapezium

E Transformations and the number plane

Exercise 9E

1 P is the point (3, 4) in the number plane and O is the origin. Write down the coordinates of the image of P if it is:



a translated:

i 2 units right **ii** 4 units left

iii 2 units up **iv** 6 units down

v 5 units left and 1 unit down

vi 3 units right and 4 units down

vii 3 units left and 5 units down

viii 1 unit right and 3 units up

b reflected:

i in the x-axis **ii** in the y-axis

iii in the line AB **iv** in the line KL

v in the x-axis then the y-axis

vi in the y-axis then the x-axis

vii in the line KL then the line AB **viii** in the line AB then the line KL

ix in the x-axis then the line AB **x** in the line KL then the y-axis

c rotated clockwise about O through:

Use your compass and protractor.

i 90° **ii** 180° **iii** 270° **iv** 360°

d **i** translated 3 units right then reflected in the x-axis

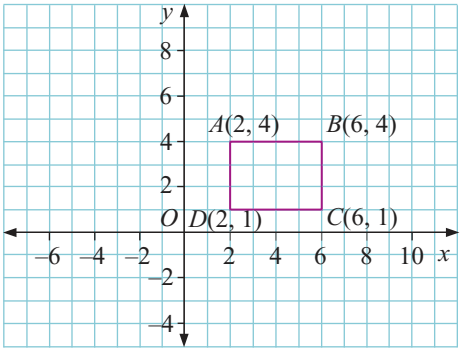
ii rotated clockwise about O through 90° then reflected in the line KL

iii reflected in the y-axis then translated 2 units right and 3 units down

iv translated 3 units left and 2 units down then rotated anticlockwise about O through 90°

v reflected in the line AB then rotated clockwise about O through 90°

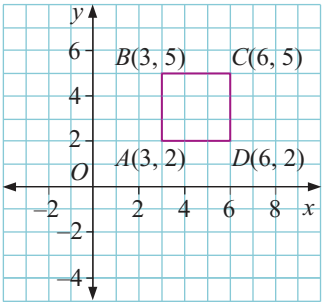
2 $A(2, 4)$, $B(6, 4)$, $C(6, 1)$ and $D(2, 1)$ are the vertices of a rectangle and O is the origin. What are the coordinates of the images of A, B, C and D if $ABCD$ is reflected in the:



a line AB? **b** line BC? **c** line AD?

d line DC? **e** x-axis? **f** y-axis?

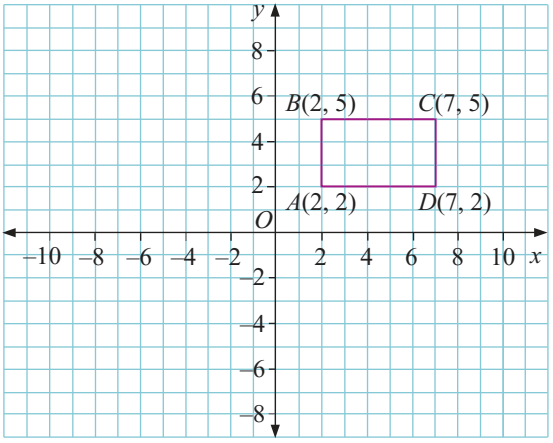
3 $A(3, 2)$, $B(3, 5)$, $C(6, 5)$ and $D(6, 2)$ are the vertices of a square and O is the origin. What are the coordinates of the images of A, B, C and D if $ABCD$ is rotated clockwise about A through:



a 90°? **b** 180°? **c** 270°?

4 $A(2, 2)$, $B(2, 5)$, $C(7, 5)$ and $D(7, 2)$ are the vertices of a rectangle and O is the origin. What are the coordinates of the images of A, B, C and D if $ABCD$ is rotated clockwise about O through:

a 90°? **b** 180°? **c** 270°?

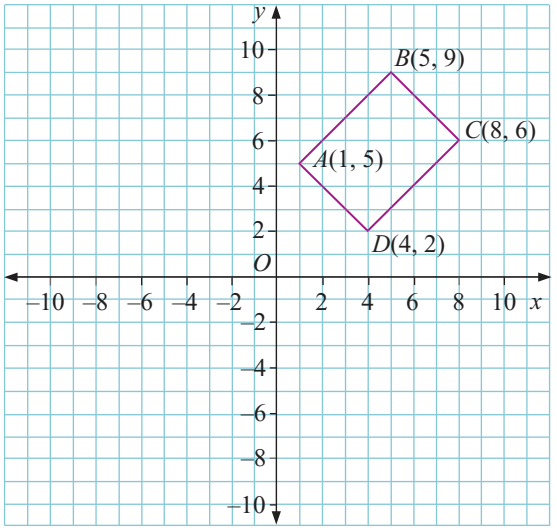


5 $ABCD$ is a rectangle and O is the origin. Determine the coordinates of the images of A, B, C and D if the rectangle is:

a translated 2 units right and 1 unit up then reflected in the x-axis

b rotated clockwise about O through 90° then translated 3 units left

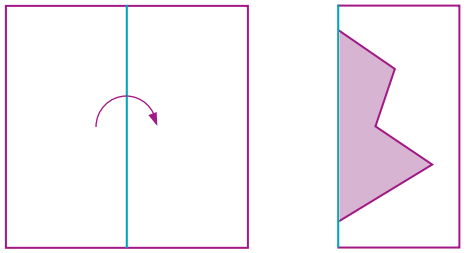
c reflected in the y-axis then reflected in the x-axis.



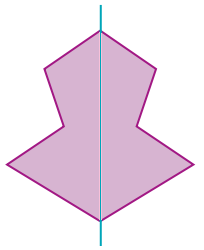
Investigation 1 Line symmetry

Step 1: Take a piece of paper and fold it.

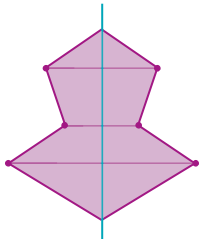
Step 2: Cut out a shape along the fold line. Open and close the new shape to see that one half fits exactly on top of the other.



Step 3: Open out the paper to display the new shape formed. Place a mirror along the fold line and look at the image in the mirror. Does the image coincide with the other side of the shape?



Step 4: Mark at least three points along one side of the shape. Use the mirror to mark the images of these points on the other side of the shape. Join each point and its image with a straight line.
What angle does this line make with the fold line? Are these pairs of points (each point and its image) the same distance away from the fold line?



Repeat steps 1 to 4, cutting out a different shape.

F Line symmetry

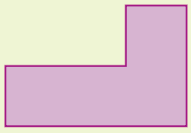
Shapes with the property demonstrated in Investigation 1 are said to have **line symmetry**. The shape can be divided into two halves that are the exact reflection of each other. The line of reflection is called the **axis (or line) of symmetry**. Shapes can have more than one axis of symmetry.

EXAMPLE 1

Find the axis (or axes) of symmetry of each of these shapes.

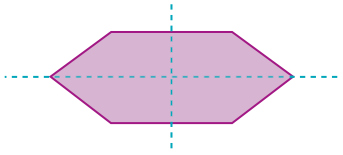
a 

b 

c 

a 

1 axis of symmetry

b 

2 axes of symmetry

c 

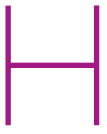
no axes of symmetry

Exercise 9F

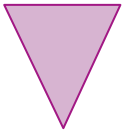
1 Copy the following shapes and draw all the axes of symmetry.

a 

b 

c 

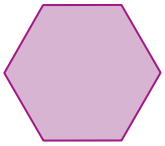
d 

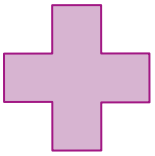
e 

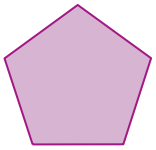
f 

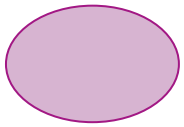
g 

h 

i 

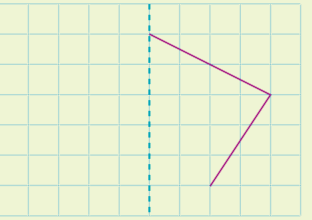
j 

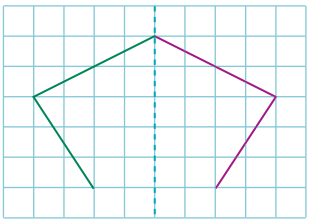
k 

l 

EXAMPLE 2

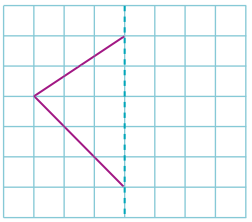
Complete this shape, given one half of the shape and its axis of symmetry.



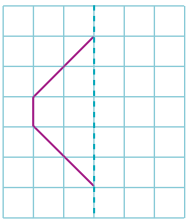


2 Copy and complete the following shapes, given one half of the shape and its axis of symmetry.

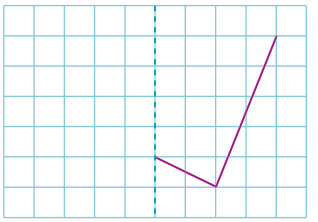
a



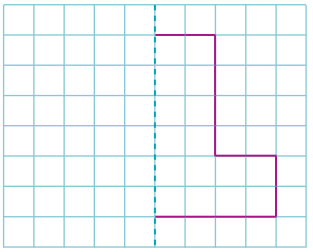
b



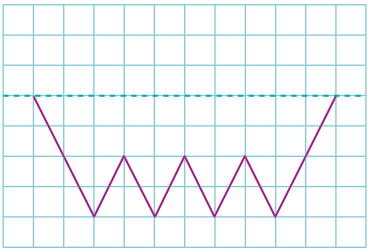
c



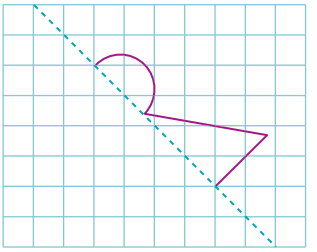
d



e

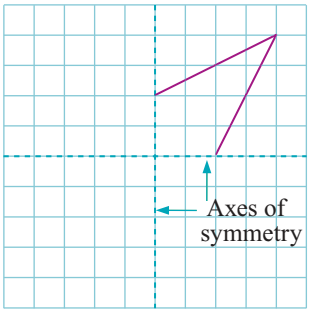


f



3 Copy and complete these shapes from the information given.

a



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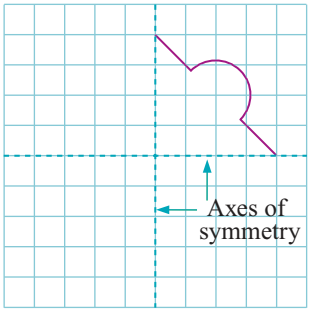
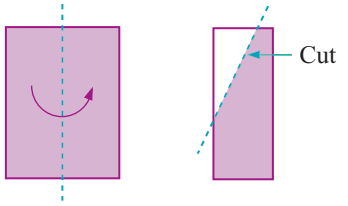


4 Copy and complete these drawings using a mirror.

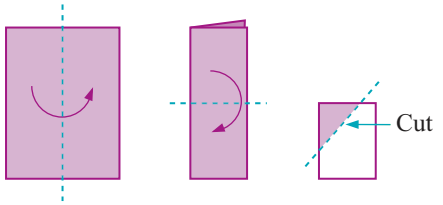
a

b

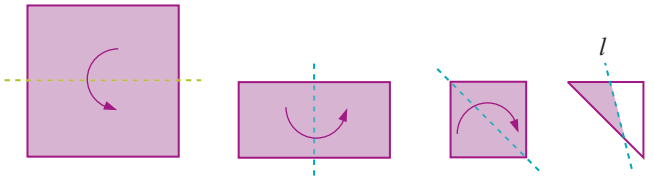
5 **a** Fold a rectangular piece of paper, as shown. Cut off a folded corner and open out the remaining paper.
b Draw the shape formed and mark its axis of symmetry.



6 **a** Fold a rectangular piece of paper twice, as shown. Cut off the double-folded corner and open out this corner.
b What is the name of the shape formed?
c How many axes of symmetry does this shape have?



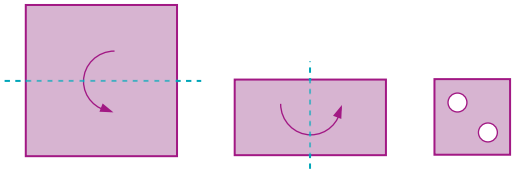
7 **a** Fold a square piece of paper three times, as shown. Cut off the triple-folded corner along the line l and open out this corner.
b What is the name the shape formed?
c Are the fold lines axes of symmetry?



8 Repeat question **7** but cut along the line m, as shown.

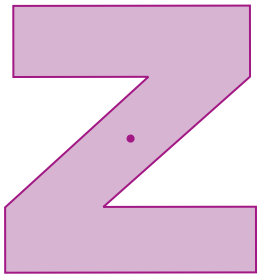
9 **a** How many axes of symmetry does a circle have?
b Cut out a circle and find the centre by folding.

10 **a** Fold a square into four and punch two holes, as shown.
b Unfold the square and draw the resulting pattern. Mark in any axes of symmetry.
c Experiment with different foldings and hole placements. Investigate the resulting patterns for symmetry. Draw the results.

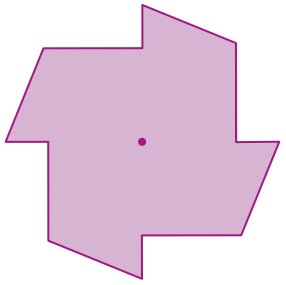


Investigation 2 Rotational symmetry

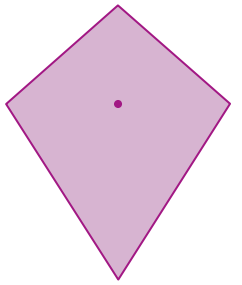
There are some shapes that do not have line symmetry but still appear to have a certain balance.



Shape 1



Shape 2



Shape 3

1 Make two copies of Shape 1 and cut them out. Paste one into your book, place the other exactly on top of it, and put a pin through the centres of both.
a Rotate the top shape in a clockwise direction until it again fits exactly on top of the other. Did you rotate the shape through half a revolution (180°)?
b Rotate the top shape further until it again exactly covers the other. Through what angle did you rotate the shape this time? Is the shape now back in its original position?
c How many times did you rotate the shape onto itself in completing one revolution?

2 **a** Make two copies of Shape 2 and cut them out. Paste one into your book, place the other exactly on top of it, and put a pin through the centres of both.

b Rotate the top shape until it exactly covers the other. Through what angle did you rotate the shape?

c Rotate the top shape further until it again exactly covers the other. Did you rotate the shape through the same angle as in part **b**?

d Repeat until the shape is back in its original position.

e How many times did you rotate the shape onto itself in completing one revolution?

3 Repeat question **2** for Shape 3. How many times did you rotate the shape onto itself in completing one revolution?

G Rotational symmetry

Shape 1 and Shape 2 in Investigation 2 do not have line symmetry but have a certain balance and regularity when rotated. This is called **rotational symmetry**. The point about which the shape is turned is called the **centre of symmetry**.

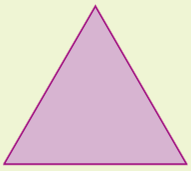
The **order** of rotational symmetry is the number of times (> 1) a shape will rotate onto itself in completing one full revolution.

- Shape 1 has rotational symmetry of order 2, since the shape rotated onto itself twice in one revolution.
- Shape 2 has rotational symmetry of order 4, since the shape rotated onto itself 4 times in one revolution.
- Shape 3 rotated onto itself only once in one revolution. It is said to have *no* rotational symmetry.

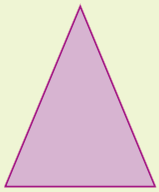
EXAMPLE 1

Which of the following shapes have rotational symmetry? For those that do, state the order of symmetry and mark the centre of symmetry.

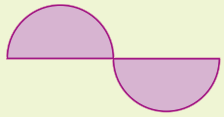
a



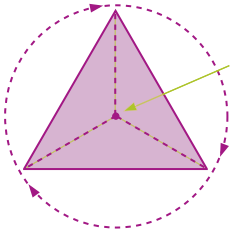
b



c

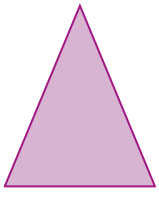


a



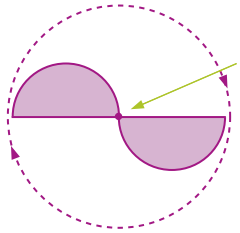
Rotational symmetry of order 3

b



No rotational symmetry

c



Rotational symmetry of order 2

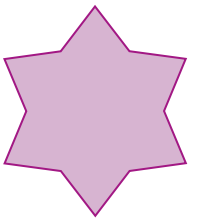
Exercise 9G

1 Consider the star drawn.

a How many times can this shape rotate onto itself in one complete turn?

b What is the order of rotational symmetry?

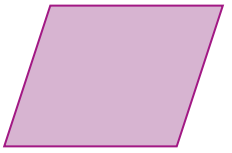
c Sketch the star and mark the centre of symmetry.



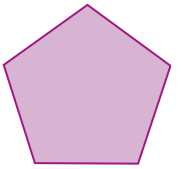
2 Which of the following shapes have rotational symmetry? For those that do, state the order of symmetry, draw a sketch and mark the centre of symmetry. Copy, cut out and rotate any shapes you are not sure of.

a

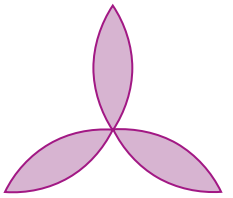
b



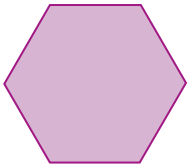
c



d



e



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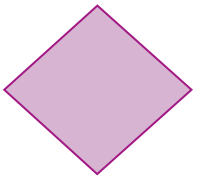
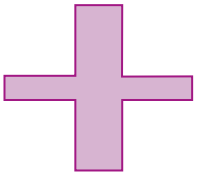


g



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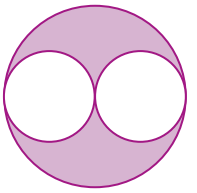
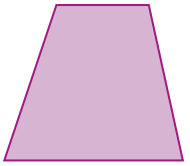


i



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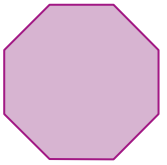

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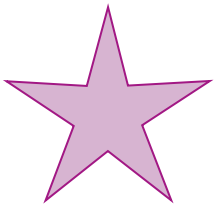


l

3 Draw a shape that has rotational symmetry of order:

a 2 **b** 3 **c** 4 **d** 5

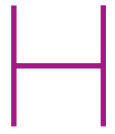
4 A shape is said to have **point symmetry** if it rotates onto itself when rotated through half a complete revolution (180°). Which of the shapes in question **2** have point symmetry?

5 Draw a shape that has rotational symmetry but not point symmetry.

6 Investigate the following shapes for both line and rotational symmetry, and complete the following table. (If no rotational symmetry, write 'None'.)

Shape	Number of axes of symmetry	Order of rotational symmetry
a		
b		
:		

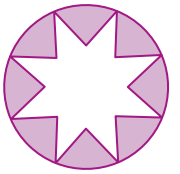
a



b

c

d



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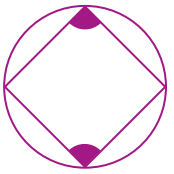
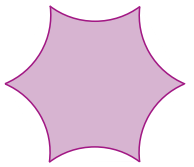
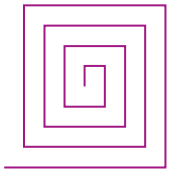
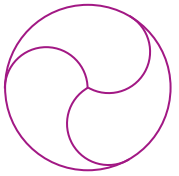
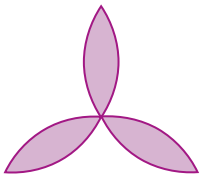
f 

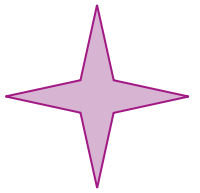
g 

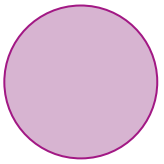
h 

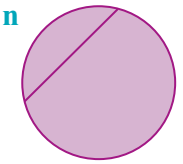
i 

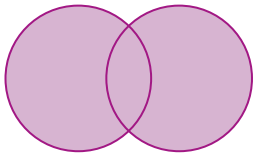
j 

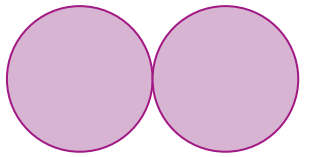
k 

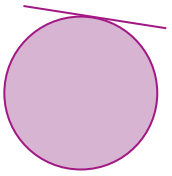
l 

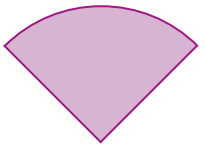
m 

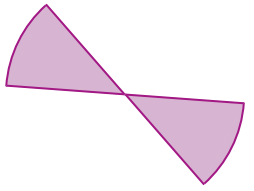
n 

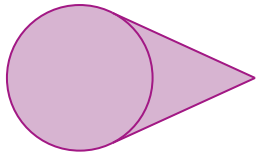
o 

p 

q 

r 

s 

t 

8 Investigate the following quadrilaterals for line and rotational symmetry and complete the table.

Quadrilateral	Number of axes of symmetry	Order of rotational symmetry
Trapezium		
Parallelogram		
Rectangle		
Rhombus		
Square		
Kite		

Investigation 3 Symmetry in life

1 Collect logos from newspapers and magazines that have line and/or rotational symmetry.
Make a display to show your class.

2 Create your own logo using a symmetrical design.
Explain the design to your class.

3 Investigate how symmetry is used in architecture.
Collect pictures to discuss with your class.

4 Investigate where symmetry occurs in nature. Collect pictures to show your class.

5 Investigate the symmetry of container lids. Report to your class.

6 Collect pictures of flags that have symmetry and make a display to show your class.

7 Investigate the drawings of the Dutch artist Maurits Escher (1898–1972). Make a presentation to your class.

Language in mathematics

1 Complete the following statements using words from the list:

image, transformations, centre, right, same, reflection, translation, turning

a Sliding a shape is called _____.

b _____ a shape is called rotation.

c Flipping a shape is called _____.

d Translation, rotation and reflection are called geometrical _____.

e When a shape is translated, every point on it moves the _____ distance in the _____ direction.

f The transformed shape is called the _____ of the original.

g The point about which a shape is rotated is called the _____ of rotation.

h When a shape is rotated, every point on it moves through the _____ angle in the _____ direction.

i When a shape is reflected, the line joining each point and its image makes a _____ angle with the line of reflection.

j When a shape is reflected, every point and its image is the _____ distance from the line of reflection.

2 Describe in your own words, using diagrams, the meaning of:

a line symmetry

b rotational symmetry

3 What is the plural of vertex?

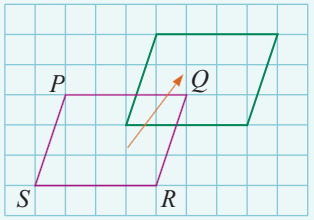
Terms

anticlockwise	axis	clockwise	flipping	image	original
reflection	revolution	rotation	sliding	symmetry	transformation
translation	turning	vertex	vertices		

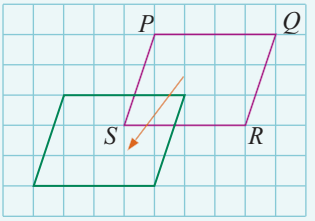
Check your skills

1 When the parallelogram *PQRS* is translated 3 units right and 2 units down, the result is:

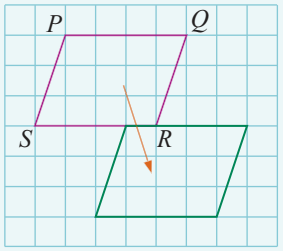
A



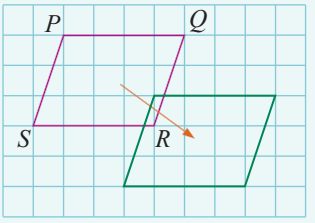
B



C

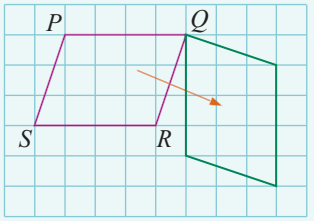


D

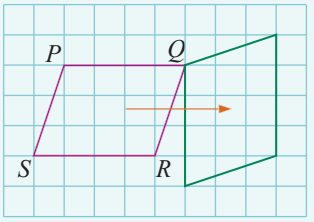


2 When the parallelogram $PQRS$ is rotated clockwise about Q through 90°, the result is:

A



B



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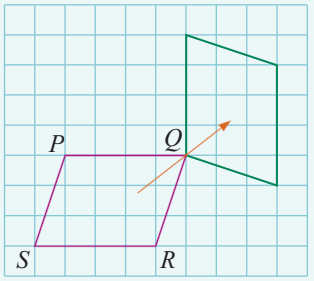
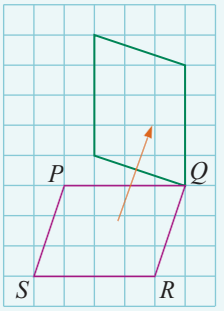
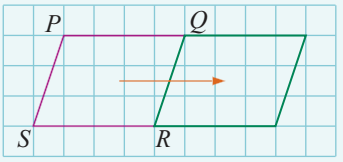


D

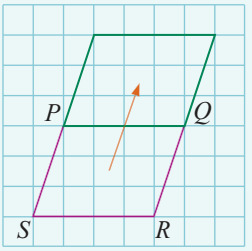


3 When the parallelogram $PQRS$ is reflected in the line PQ, the result is:

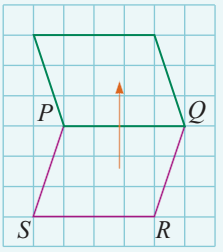
A



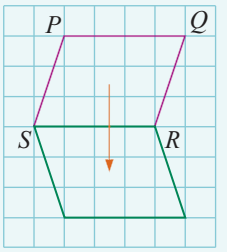
B



C



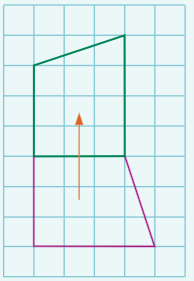
D



4 This trapezium has been:

A rotated **B** reflected

C translated **D** none of these



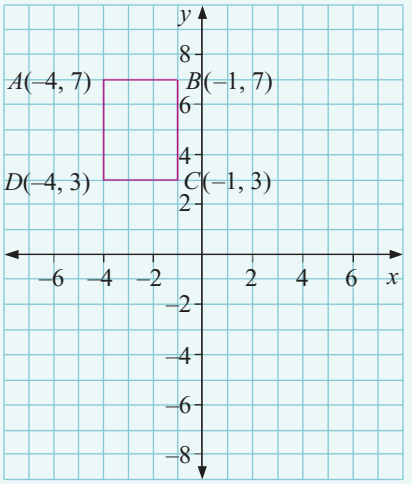
5 When the point $P(-5, 2)$ is translated 4 units right and 3 units down, the coordinates of its image are:

A $(-9, 5)$ **B** $(-1, 5)$ **C** $(-8, -2)$ **D** $(-1, -1)$

6 When rectangle $ABCD$ is reflected in the x-axis, the coordinates of the image of B are:

A $(1, 7)$ **B** $(-1, -7)$

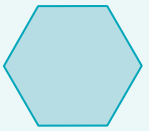
C $(1, -7)$ **D** $(-1, -1)$



7 Which one of the following shapes has line symmetry?

A  **B**  **C**  **D** 

8 How many axes of symmetry does this shape have?

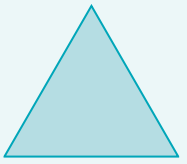


A 0 **B** 3
C 6 **D** 12

9 Which one of these shapes has rotational symmetry?

A  **B**  **C**  **D** 

10 The order of rotational symmetry of this shape is:



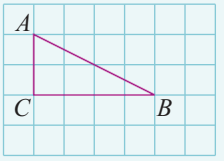
A 2 **B** 3
C 6 **D** 12

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2	3	4	5, 6	7, 8	9, 10
Section	A	B	C	D	E	F	G

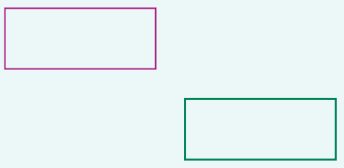
9A Review set

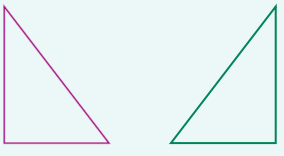
1 Copy the triangle onto grid paper and show its new position when it is:



a translated 2 units right and 1 unit down
b rotated clockwise about C through 90°
c reflected in the line CB

2 State whether the original (purple) shape has been translated, rotated or reflected to its new position.

a 

b 

c 

3 Point P has coordinates $(-6, 8)$. Write down the coordinates of the image of P if it is:

a translated 5 units right and 3 units down
b reflected in the y-axis
c reflected in the x-axis
d rotated clockwise about the origin O through 90°

4 Copy the following shapes and draw all the axes of symmetry for each.

a 

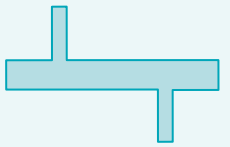
b 

c 

5 Which of the following shapes have rotational symmetry? For those that do, state the order of symmetry.

a 

b 

c 

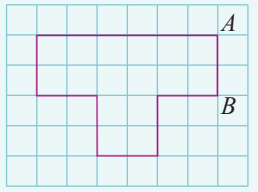
9B Review set

1 Copy this shape onto grid paper and show its new position when it is:

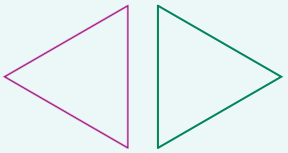
a translated 3 units right and 2 units down

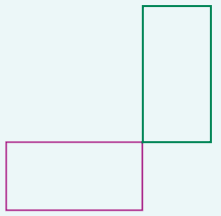
b rotated about A in a clockwise direction through 90°

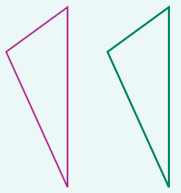
c reflected in the line AB



2 State whether the original (purple) figure has been translated, rotated or reflected to its new position.

a 

b 

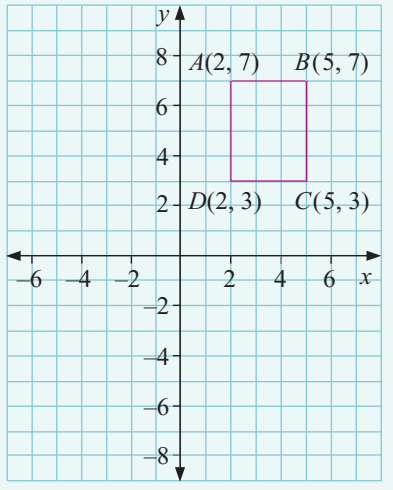
c 

3 What are the coordinates of the image $A'B'C'D'$ of rectangle $ABCD$ if $ABCD$ is:

a translated 5 units left and 3 units down?

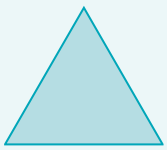
b reflected in the y-axis?

c reflected in the line DC?



4 Copy the following shapes and draw all the axes of symmetry for each.

a 

b 

c 

5 Which of the shapes in question **4** have rotational symmetry? For those that have rotational symmetry, state the order of symmetry.

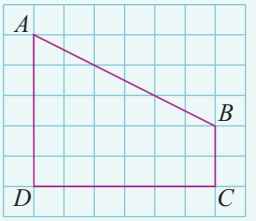
9C Review set

1 Copy this trapezium onto grid paper and show its new position when it is:

a translated 3 units right and 4 units down

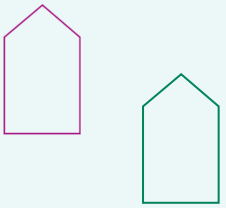
b rotated clockwise about C through 90°

c reflected in the line BC

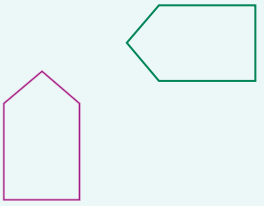


2 State whether the original (purple) shape has been translated, rotated or reflected to its new position.

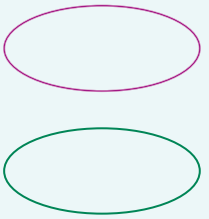
a



b



c



3 P is the point with coordinates (5, 12). Write down the coordinates of the image of P if it is:

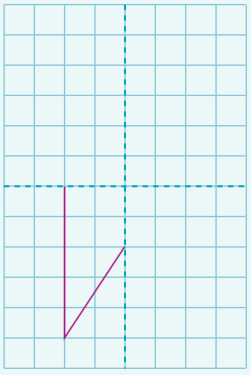
a translated 3 units right and 10 units down

b reflected in the x-axis

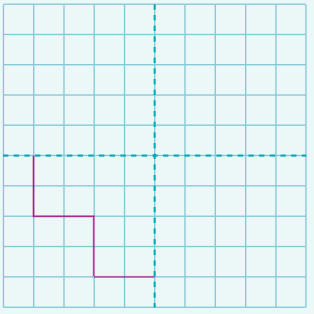
c rotated clockwise about the origin O through 90°

4 Copy and complete the following shapes, given the axes of symmetry.

a



b



5 Draw a shape with two axes of symmetry and rotational symmetry of order 2.

10

Probability

This chapter deals with chance.

At the end of this chapter you should be able to:

- list all possible outcomes of a simple event
- find the sample space
- find the probability of simple events
- understand that probabilities take values from 0 and 1
- order words used to informally describe chance

Diagnostic test

1 Which is not describing an even chance of an event occurring?

A $\frac{1}{2}$ **B** 50% **C** 0.5 **D** $\frac{5}{6}$

2 A probability of 25% is the same as

A 25 **B** 2.5 **C** $\frac{1}{4}$ **D** $\frac{1}{25}$

3 A bag contains 1 pink, 1 purple and 1 orange ball. The probability of randomly selecting a purple ball is:

A 1 **B** 3 **C** $\frac{1}{3}$ **D** $\frac{2}{3}$

4 The probability of an event is $\frac{2}{5}$. As a percentage this is:

A 20% **B** 40%
C 50% **D** none of these

5 A normal six-sided die is thrown. What is the probability of getting a 5?

A $\frac{5}{6}$ **B** $\frac{3}{6}$ **C** $\frac{2}{6}$ **D** $\frac{1}{6}$

6 A fair coin is tossed. What is the probability of tails?

A 1 **B** $\frac{1}{2}$ **C** 0.25 **D** 0.1

7 A term describing a probability of about 80% is:

A certain **B** highly probable
C evens **D** low probability

8 A coin is tossed once. An impossible event would be getting:

A heads **B** tails
C heads or tails **D** a six

9 When a normal six-sided die is rolled, the probability of getting a 7 is:

A $\frac{7}{6}$ **B** $\frac{6}{7}$ **C** $\frac{1}{6}$ **D** 0

10 If an event is certain to happen, its probability is:

A 0 **B** $\frac{1}{2}$ **C** 1 **D** 2

The Diagnostic test questions refer to the Year 6 content description below.

Question	2–6	1, 7–10
Section	ACMSP 144	ACMSP 145

PROBABILITY

We use probability every day in our lives. People often say things like:

- 'It is likely to rain today.'
- 'I will probably come to your party.'
- 'I am unlikely to pass this exam.'
- 'The Suns will certainly beat the Eagles this game.'

These statements talk about the **chance** of something occurring. Many terms are used to discuss the likelihood of something happening, such as: certain, very probable, highly likely, not likely, no chance, small chance, even chance.

A The sample space

One of the most common examples of chance is the tossing of a coin at the start of a game of sport. In probability, the action of tossing a coin is referred to as a **chance experiment**. (Other examples of a chance experiment are rolling a die and selecting a ticket from a hat.) The possible results of a chance experiment are called **outcomes** and the **sample space**, S, is a list of all the possible outcomes.

When a coin is tossed, heads or tails are possible outcomes, and hence S = {head, tail}. If the coin is unbiased, the outcomes are **equally likely**. This means there is an equal chance of each outcome occurring. When the outcomes of tossing a coin are equally likely, the coin is referred to as being a **fair** or **normal** coin.

EXAMPLE 1

A normal six-sided die with the numbers 1 to 6 on its faces is thrown.

a What is the chance experiment in this example?
b Write down two outcomes of this experiment.
c List the sample space.
d Are the outcomes equally likely?

a Throwing a die
b 1, 2 (or 3, 4, 5, 6)
c S = {1, 2, 3, 4, 5, 6}
d Yes because it is a normal die.

Exercise 10A

1 The letters of the word MATHS are written on cards and put in a hat. One card is selected at random.

a What is the chance experiment in this example?
b Write down two outcomes of this experiment.
c List the sample space.
d Are the outcomes equally likely?

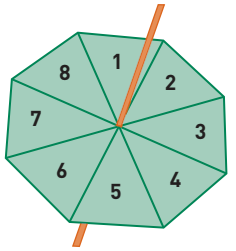
At random or randomly means without any bias.

2 A ball is selected at random from a bag containing 1 black, 1 white and 1 green ball.

- **a** What is the chance experiment in this example?
- **b** Write down two outcomes of this experiment.
- **c** List the sample space.
- **d** Are the outcomes equally likely?

3 A spinner with eight edges has the numbers 1 to 8 listed on it.

- **a** What is the sample space when it is spun?
- **b** Are these outcomes equally likely?

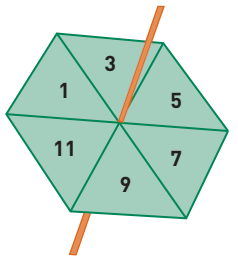


4 A hat contains 1 red, 1 white and 1 blue ticket.

- **a** List the sample space when a ticket is drawn at random.
- **b** Are these outcomes equally likely?

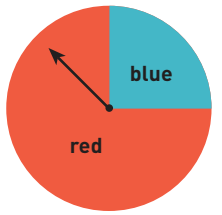
5

- **a** List the sample space when this spinner is spun.
- **b** Are these outcomes equally likely?



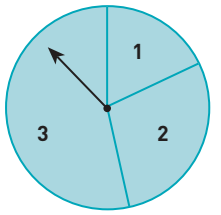
6

- **a** List the sample space when this spinner is spun.
- **b** Are these outcomes equally likely?



7

- **a** List the sample space when this spinner is spun.
- **b** Are these outcomes equally likely?



8 Design a spinner with two colours, red and blue, with the following outcomes.

- **a** Red and blue are equally likely to occur.
- **b** Red is twice as likely to occur as blue.
- **c** Red is three times as likely to occur as blue.

9 Design a spinner with three colours, red, blue and green, with the following outcomes.

- **a** Red and blue are equally likely to occur and green is twice as likely as red.
- **b** Red is twice as likely to occur as blue, and green is three times as likely as blue.
- **c** Red is twice as likely to occur as green, and green is three times as likely as blue.

10 **a** List the sample space when a child is born.
b Are these outcomes equally likely?

11 **a** List the sample space for the colour of the traffic light as a car approaches it.
b Are these outcomes equally likely?

12 **a** List the sample space for the result (not the score) of a soccer match.
b Are these outcomes equally likely?

13 A ticket is selected from a bag containing red, green and blue tickets.
a Is a red ticket in the sample space?
b Is a pink ticket an outcome?
c List the sample space.

14 A ten-sided die with the numbers 1 to 10 is tossed.
a Is 6 in the sample space?
b Is 12 an outcome in the sample space?

B Probability

EXAMPLE 1

What is the probability of getting 5 when a normal six-sided die is thrown?

The sample space is {1, 2, 3, 4, 5, 6}. There are 6 equally likely outcomes.
There is only one 5, so the probability of getting 5 is 1 out of 6. This is written as $P(5) = \frac{1}{6}$.

Exercise 10B

1 A normal six-sided die is thrown. Complete the following.
a Sample space S = {_____}
b Number of equally likely outcomes = ____
c **i** P(1) = ____ **ii** P(3) = ____ **iii** P(4) = ____ **iv** P(6) = ____

2 A bag contains 1 blue, 1 red and 1 green ball. A ball is selected at random.

a List the sample space.

b How many equally likely outcomes are there?

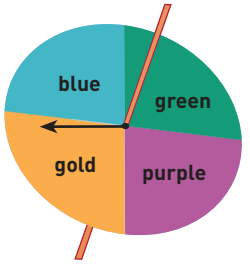
c What is the probability of selecting a red ball?

3 This spinner is spun once.

a List the sample space.

b How many equally likely outcomes are there?

c What is the probability that the colour spun is green?



4 Ten cards with the numbers 0 to 9 written on them are shuffled, then one card is selected at random.

a List the sample space.

b How many outcomes are in the sample space?

c What is the probability that the number selected is:

i 5? **ii** 8?

5 The letters of the alphabet are written on cards and the cards shuffled. A card is chosen at random.

a How many equally likely outcomes are there?

b What is the probability that the letter chosen is:

i T? **ii** M? **iii** Z?

Investigation 1 Fractions, decimals and percentages

Fractions, decimals and percentages are different ways of showing the same probability. For example, a probability of $\frac{1}{4}$ may be written as 0.25 or as 25%.

Copy and complete this table. Which method of showing probability is easiest? Explain.

Fraction	Decimal	Percentage
$\frac{1}{10}$		
	0.2	
		30%
$\frac{2}{5}$		
		35%
	0.43	
$\frac{1}{2}$		
$\frac{5}{8}$		
		60%
$\frac{3}{4}$		
	0.85	
		100%

C Theoretical probability

In probability, an **event** is either one outcome or a collection of outcomes in the sample space. For example, in the chance experiment of rolling a normal six-sided die once, obtaining the outcome '6' is an event and obtaining the outcome 'an even number' is also an event. The outcomes that make up an event are said to be **favourable** for that event. We formalise the chance of an event happening by defining its **theoretical probability** as:

$$\text{Probability (event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

The probability of an event E occurring can be written P(E).

EXAMPLE 1

A normal six-sided die is rolled. Calculate the probability of these events:

a 4

b 5 or 6

c an even number

S = {1, 2, 3, 4, 5, 6}, so the number of possible outcomes = 6.

a Favourable outcome = {4}, so the number of favourable outcomes = 1.

$P(4) = \frac{1}{6}$

Remember to simplify fractions.

b Favourable outcomes = {5, 6}, so the number of favourable outcomes = 2.

$P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$

c Favourable outcomes = {2, 4, 6}, so the number of favourable outcomes = 3.

$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$

Exercise 10C

1 A normal six-sided die is rolled. Complete the following to calculate the probability of the event:

a 6 **b** 2 or 3 **c** an odd number

d a number < 3 **e** a number > 2

S = {_____}, so number of possible outcomes = ____

a Favourable outcome = {_____}, so number of favourable outcomes = ____.

$P(6) = \frac{\square}{\square}$

b Favourable outcomes = {2, 3}, so number of favourable outcomes = ____.

$P(2 \text{ or } 3) = \frac{\square}{\square} = \frac{\square}{\square}$

c Favourable outcomes = {_____}, so number of favourable outcomes = ____.

$P(\text{odd number}) = \frac{\square}{\square} = \frac{\square}{\square}$

d Favourable outcomes = {_____}, so number of favourable outcomes = ____.

$P(\text{number} < 3) = \frac{\square}{\square} = \frac{\square}{\square}$

e Favourable outcomes = {_____}, so number of favourable outcomes = ____

$P(\text{number} > 2) = \frac{\square}{\square} = \frac{\square}{\square}$

2 The numbers 0 to 9 are written on cards. The cards are shuffled and one is chosen at random. Complete the following to find the probability (as a decimal) of the event:

a 5 **b** 0 or 9 **c** an odd number

d an even number **e** a number < 5 **f** a number > 7

S = {_____}, so number of possible outcomes = ____

a Favourable outcome = {_____}, so number of favourable outcomes = ____.

$P(5) = \frac{\square}{\square} =$ ____

b Favourable outcomes = {_____}, so number of favourable outcomes = ____.

$P(0 \text{ or } 9) = \frac{\square}{\square} =$ ____

c Favourable outcomes = {_____}, so number of favourable outcomes = ____.

$P(\text{odd number}) = \frac{\square}{\square} =$ ___

d Favourable outcomes = {_____}, so number of favourable outcomes = ____.

$P(\text{even number}) = \frac{\square}{\square} =$ ____

e Favourable outcomes = {_____}, so number of favourable outcomes = ____.

$P(\text{number} < 5) = \frac{\square}{\square} =$ ____

f Favourable outcomes = {_____}, so number of favourable outcomes = ____.

$P(\text{number} > 7) = \frac{\square}{\square} =$ ____

3 The letters of the word RANDOM are written on cards, then one is selected at random. Find the probability that it is:

a N **b** N or M **c** a vowel **d** a consonant

4 A bag of lollies contains 5 red and 2 green lollies. One lolly is chosen at random from the bag. Complete the following to find the probability that it is:

a red **b** green

S = {r, r, r, r, r, g, g}, so number of possible outcomes (= number of lollies in the bag) = ____.

a Favourable outcomes = {_____},
so number of favourable outcomes = ____.

$P(\text{red}) = \frac{\square}{\square}$

b Favourable outcomes = {_____},
so number of favourable outcomes = ____.

$P(\text{green}) = \frac{\square}{\square}$

5 A hat contains 4 black and 5 green tickets. One ticket is chosen at random from the hat. Complete the following to find the probability that it is:

a black **b** green

S = {b, b, b, b, g, g, g, g, g}, so number of possible outcomes (= number of tickets in hat) = ____

a Favourable outcomes = {_____}, so number of favourable outcomes = ____.

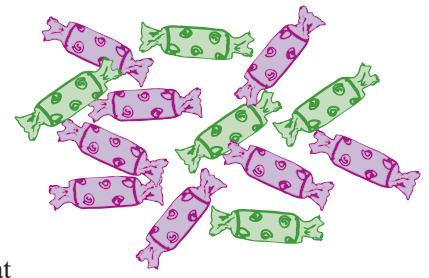
$P(\text{black}) = \frac{\square}{\square}$

b Favourable outcomes = {_____}, so number of favourable outcomes = ____.

$P(\text{green}) = \frac{\square}{\square}$

6 A bag contains 5 green and 8 purple lollies. One lolly is chosen at random from the bag. Find the probability that it is:

a green **b** purple



7 A hat contains 4 green, 6 red and 5 blue tickets. If one ticket is chosen at random from the hat, find the probability of getting:

a a red ticket **b** a blue ticket **c** a green or red ticket

8 A box of ice-blocks contains 6 green, 5 red and 3 blue ice-blocks. One ice-block is chosen at random from the box. Find the probability that it is:

a blue **b** red or green **c** blue or green

9 A purse contains seven 10-cent coins, eight 20-cent coins and nine dollar coins. A coin is chosen at random from the purse. Find:

a P(20-cent coin)

b P(10-cent coin)

c P(value of coin $>$ 10 cents)

10 A hat contains 3 red, 4 white and 5 blue tickets. A ticket is selected at random from the hat. Find the probability that the ticket is:

a red **b** white

c blue **d** green

e not red **f** not white

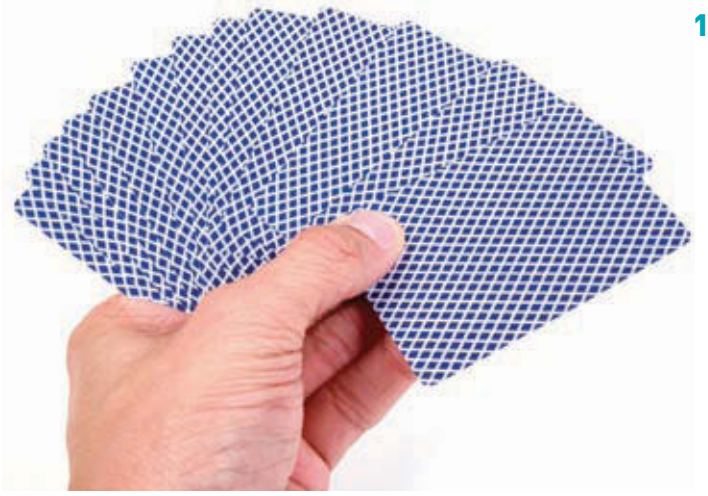
g red or blue **h** not red or white

i red, white or blue

11 A letter is chosen at random from the word MATHEMATICS. Find the probability that the letter is:

a H **b** I **c** M

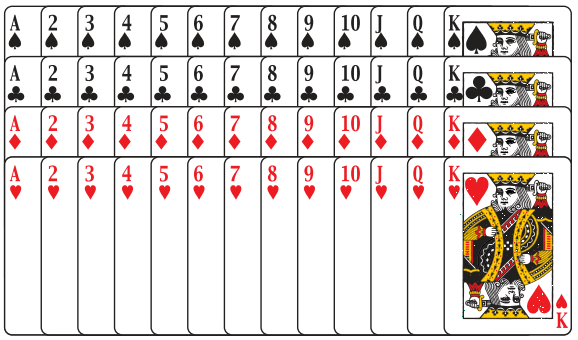
d A **e** a vowel **f** not repeated



12 Mark is holding 13 cards, from a normal playing pack. They are all the same suit. One card is selected at random. Find the probability that it is:

a an ace **b** 3, 4 or 5

c a king or queen **d** a number $>$ 5



13 Adam has spread out a pack of playing cards. He picks a card at random. Hearts and diamonds are red. Clubs and spades are black. Picture cards are king, queen and jack. Find:

a P(spade)
b P(4 of hearts)
c P(club)
d P(black 10)
e P(ace)
f P(picture card)
g P(5 or 6)
h P(green card)
i P(not a picture card)
j P(number < 10)

14 In Year 7, 40% of students walk to school, 15% ride a bicycle, 25% travel by bus and 20% by car. For a student, chosen at random, find as a percentage:

a P(travels by bus)
b P(rides a bicycle)
c P(walks)
d P(does not travel by bus)

15 Rewrite the percentage probabilities in question **14** as decimals.

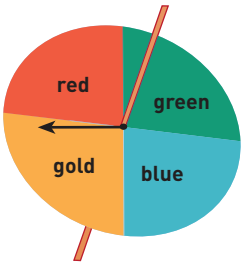
16 Year 7 were surveyed on pocket money received. 35 students received less than \$10, 58 received from \$10 to \$20 and 7 received more than \$20. A student is chosen at random. Find, as a decimal, the probability that they received as pocket money:

a less than \$10
b more than \$20
c \$20 or less

17 Rewrite your decimal probabilities in question **16** as percentages.

18
a A normal die is rolled once. List the sample space.
b How many times does a 7 appear in the sample space?
c Write down the probability of getting a 7.
d How many times does a number less than 7 appear in the sample space?
e Write down the probability of getting a number less than 7.

19
a This spinner is spun once. List the sample space.
b How many times does the outcome purple appear in the sample space?
c What is the probability of getting purple?
d How many outcomes of the sample space are red or green or blue or gold?
e Write down the probability of getting red or green or blue or gold.



20 A set of 9 cards have the numbers 1 to 9 written on them. One card is selected at random. What is the probability that the number on the card is:

a 15?
b odd or even?

21 Discuss the results of questions **18**, **19** and **20** and complete the following.

a If an event is **impossible**, its probability is _____.
b If an event is **certain**, its probability is _____.

D Informal concept of chance

Everyday conversations include many references to probability in statements such as 'I am unlikely to win a prize in a raffle'. They give an indication of how likely something is to happen. We see from question **21** of Exercise **10C** that:

- If an event is impossible then its probability is 0.
- If an event is certain then its probability is 1.

Hence $0 \leqslant P(E) \leqslant 1$.

That is, the probability of an event always lies between 0 and 1, inclusive. The closer the probability is to 0, the less likely the event is to occur. The closer the probability is to 1, the more likely the event is to occur.

Exercise 10D

1 A bag contains 99 black and 1 white marbles. A marble is selected at random from the bag.

a Is it likely that the marble is black?

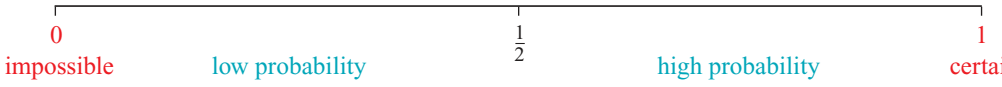
b Is the chance of selecting a black marble the same as the chance of selecting a white one?

c Is it likely that the marble will be white?

d Is it impossible to select a white marble?

e Is it impossible to select a green marble? Explain.

2 Copy this number line, then locate each of the terms below on the line.



likely	small chance	even chance	possible	sure	often
rarely	fifty-fifty	good chance	little chance	no hope	sure thing
high hopes	slim chance	unlikely	no chance		

3 Use one of these terms to describe each event described below:

impossible — low probability
even chance — high probability
certain

a 25 December being a holiday

b snow falling in Darwin

c your family going on an overseas trip this year

d the sun rising tomorrow

e you being struck by lightning

f living to 200 years of age

g getting Mathematics homework tonight

h winning Lotto

i winning the toss next time you try

j getting 6 if you throw a normal six-sided die once

4 Use one of the percentages 0%, 10%, 50%, 80%, 100% to describe each event in question **3**.

5 Write your own statement to describe an event that is:

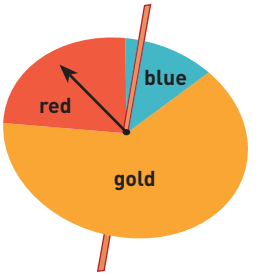
a impossible **b** low probability **c** even chance **d** high probability **e** certain

6 For this spinner, describe each event below using one of the terms:

certain high probability even chance
low probability impossible

The spinner is spun and the colour is:

a blue **b** red **c** gold
d green **e** red or gold **f** red, gold or blue



7 Describe these percentage probabilities using the same terms as in question **6**.

a 10% **b** 50% **c** 100% **d** 80%
e 30% **f** 49% **g** 0% **h** 5%

EXAMPLE 1

A normal six-sided die is thrown once. Describe an event that would be:

a certain
b impossible
c even chance

a Getting 1 or 2 or 3 or 4 or 5 or 6 would be certain. P(1, 2, 3, 4, 5 or 6) = 1
b Getting tails would be impossible. P(tails) = 0.
c The chance of getting 1 or 2 or 3 would be even. P(1, 2 or 3) = $\frac{1}{2}$.

8 A coin is tossed once. Describe an event that would be:

a certain **b** impossible **c** even chance

9 A hat contains 10 tickets numbered 1 to 10. Describe an event that would be:

a certain **b** impossible **c** even chance **d** probability $\frac{1}{10}$

10 A tennis racquet with sides labelled rough and smooth is spun once. Describe an event with a probability of:

a 1 **b** 0 **c** $\frac{1}{2}$

11 A spinner has 4 equal sections coloured red, blue, green and yellow. The spinner is spun once. Describe an event with a probability of:

a 1 **b** 0 **c** $\frac{1}{2}$
d $\frac{1}{4}$ **e** $\frac{3}{4}$

Calculator activities

1 Lotto is a game where six numbers are selected from 44.

a The number of ways of choosing six numbers from 44 is $\frac{44 \times 43 \times 42 \times 41 \times 40 \times 39}{1 \times 2 \times 3 \times 4 \times 5 \times 6} =$ _____.

b Write the probability of winning Lotto as a fraction.

2 Year 7 students could buy one item from the canteen each day for a week. Each student could choose from a pie, hot-dog or pizza. The canteen sold 320 pies, 170 hot-dogs and 410 pizzas in the week.

a Find the total number of food items.

b Find the probability that a given student would have purchased a:

i hot-dog **ii** pie **iii** pizza

3 A bag contains 480 red marbles and 270 black marbles. What is the probability of selecting at random a:

a a red marble? **b** a black marble?

4 In a raffle there are 280 white, 150 blue and 320 green tickets. What is the probability of drawing a:

a white ticket? **b** blue ticket? **c** green ticket?

5 A game requires you to throw a 6 to start. A normal six-sided die is thrown.

a What is the probability of starting on the first attempt?

b What is the probability of *not* starting on the first attempt?

c Calculate $\frac{5}{6}$ 3 $\frac{5}{6}$, which is the probability of not starting after 2 throws of the die.

d The probability of not starting after 4 throws of the die is $\left(\frac{5}{6}\right)^4$. Calculate this probability.

Language in mathematics

Here is a list of some of the terms used in this chapter:

certainty, chance, equally likely, even chance, event, good chance, high probability, impossible, likely, low probability, no chance, no hope, often, outcome, probability, rarely, sample space, unlikely

1 Find a term from the word list that is the opposite to:

a unlikely **b** high probability **c** certain **d** good chance **e** rarely

2 Complete these terms by filling in the missing vowels.

pr__b__b__l__ty __v__n ch__nc__ __v__nt __ __ tc __ m __

__ft__n __mp__ss__bl__ r__r__ly s__mpl__ sp__c__

3 Find a term from the list that describes each probability.

a $\frac{1}{100}$ **b** $\frac{25}{50}$ **c** $\frac{2}{10}$ **d** $\frac{99}{100}$ **e** 0 **f** 1

4 Explain the difference between experiments, events, outcomes and the sample space in chance situations.

Terms

certain	chance	event	experiment	favourable	impossible
occur	outcome	probability	random	sample space	theoretical

Check your skills

1 A coin is tossed once. The number of outcomes in the sample space is:
A 0 **B** 1 **C** 2 **D** 3

2 A ticket is selected from a hat containing 4 tickets numbered from 1 to 4. The sample space is:
A {tickets} **B** {1, 4} **C** {1, 2, 3, 4} **D** {4}

3 The probability of randomly selecting ticket number 3 from the hat in question **2** is:
A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** 1

4 The probability of an event is 0.3. As a fraction this is:
A 3 **B** $\frac{3}{10}$ **C** $\frac{1}{3}$ **D** 30

5 A hat contains 8 red and 5 blue tickets. One ticket is chosen at random. The probability that it is red is:
A $\frac{1}{2}$ **B** $\frac{5}{8}$ **C** $\frac{5}{13}$ **D** $\frac{8}{13}$

6 A letter is chosen at random from the word ALGEBRA. The probability that it is A is:
A $\frac{1}{7}$ **B** $\frac{1}{6}$ **C** $\frac{2}{7}$ **D** $\frac{2}{6}$

7 Which percentage probability approximates an event that is unlikely?
A 0% **B** 10% **C** 50% **D** 80%

8 A normal six-sided die is thrown. A certain event is:
A {1} **B** {6} **C** {7} **D** {1, 2, 3, 4, 5 or 6}

9 When a coin is tossed, the probability of getting 6 is:
A 0 **B** $\frac{1}{6}$ **C** $\frac{2}{6}$ **D** 6

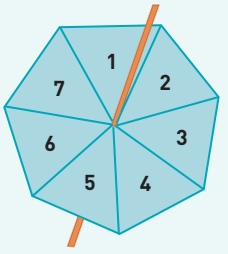
10 When a normal six-sided die is rolled, the probability of getting a number less than 7 is:
A $\frac{6}{7}$ **B** $\frac{7}{6}$ **C** 1 **D** 6

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3, 4	5, 6	7–10
Section	A	B	C	D

10A Review set

1 A seven-sided spinner with the numbers 1 to 7 is spun once.
a List the sample space.
b What is the probability of getting 5?



2 A letter is chosen at random from the word NUMBER.
a List the sample space.
b What is the probability of choosing B?

3 Complete the table.

	Fraction	Decimal	Percentage
a	$\frac{1}{5}$		
b		0.4	
c			50%

4 A bag contains 8 orange, 12 black and 5 gold marbles. One marble is selected at random.
- **a** How many marbles are in the bag?
- **b** What is the probability of selecting a black marble?

5 A letter is chosen from the word STUDENT. What is:
- **a** P(E)? **b** P(T)? **c** P(consonant)?

6
- **a** Write a statement describing an event with a probability of 1.
- **b** Estimate a percentage probability for the phrase 'very likely'.
- **c** Write a term for an event with a probability of about 15%.

7 A spinner has four equal sections coloured pink, purple, blue and brown. The spinner is spun once. Describe an event that would be:
- **a** certain **b** impossible **c** even chance **d** probability of $\frac{1}{4}$

10B Review set

1 A hat contains 1 red, 1 blue and 1 green ticket. One ticket is chosen at random.
- **a** List the sample space.
- **b** What is the probability of selecting the red ticket?

2 Ten cards with the numbers 1 to 10 written on them are shuffled, and one card is chosen at random.
- **a** List the sample space.
- **b** What is the probability that the chosen card has 5 written on it?

3 Complete the table.

	Fraction	Decimal	Percentage
a		0.6	
b			70%
c	$\frac{3}{8}$		

4 A bag contains 5 green, 7 red and 8 blue marbles. One marble is chosen at random.
- **a** How many marbles are in the bag?
- **b** What is the probability of choosing a red marble?

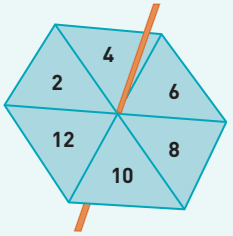
5 One card is selected at random from a normal deck of 52 cards. What is:
- **a** P(heart)? **b** P(red card)?
- **c** P(ace)?

6 **a** Write a statement describing an event with a probability of 0.
b Estimate a percentage probability for the phrase 'even chance'.
c Write a phrase for an event with a probability of about 90%.

7 A coin with sides coloured red and blue is tossed once. Describe an event that would be:
a certain **b** impossible **c** even chance

10C Review set

1 **a** List the sample space of this spinner.
b What is the probability of spinning 10?



2 This spinner is spun once.
a List the sample space.
b What is the probability that the colour spun is gold?

3 Complete the table.

	Fraction	Decimal	Percentage
a			80%
b		0.35	
c	$\frac{1}{25}$		

4 A hat contains 7 orange and 8 black tickets. One ticket is selected at random.
a How many tickets are in the hat?
b What is the probability of selecting an orange ticket?

5 In Year 7, 55% of students walk to school, 25% catch the bus, 15% come by car and 5% bicycle. A student is chosen at random. Find the probability that the student:
a walks **b** comes by car
c does not walk **d** walks or bicycles

6 **a** Write a statement describing an event with a probability of $\frac{1}{2}$.
b State the percentage probability for the phrase 'no chance'.
c Write a phrase for an event with a probability of 100%.

7 A bag contains 8 tickets labelled 1 to 8. Describe an event with a probability of:
a 1 **b** 0 **c** $\frac{1}{2}$ **d** $\frac{1}{4}$

Chapters 7–10 Cumulative review

1 **a** Simplify the following.

i $3 \times p$ **ii** $q \times 4$ **iii** $3 \times r \times s$ **iv** $m \times m$

v $6 \times a - 3$ **vi** $8 + 4 \times t$ **vii** $3 \times x - 2 \times y$ **viii** $2 \times a \times 1 \times b$

b Insert multiplication symbols to show the meaning of:

i $8x$ **ii** $7mn$ **iii** $4b - 3$ **iv** $5x - 7z$

c If $m = 5$ and $n = 3$, find the value of:

i $3m$ **ii** $8n$ **iii** $2m - 3n$ **iv** m^2

v $\frac{6m}{n}$ **vi** $\frac{m + n}{4}$ **vii** $\frac{2m + 3n}{4m - 1}$ **viii** $\frac{m^2 + n^2}{2}$

d Write in fraction form.

i $m \div 6$ **ii** $a \div b$ **iii** $(x + y) \div (p + q)$

e Simplify the following.

i $f + f + f + f$ **ii** $t + t + t + t + t + t$ **iii** $a + a + a + b + b + b + b$

2 **a** Simplify the following.

i $n \times n$ **ii** $5 \times p \times p \times p \times p$

b Show the meaning of:

i p^2 **ii** $3a^2$

c If $w = 3$, evaluate:

i w^2 **ii** $3w^2$

d Simplify the following.

i $4 \times 5t$ **ii** $6m \times 5$ **iii** $6a \times b \times 5c$

e Simplify the following.

i $6p + 5p$ **ii** $8m - m$ **iii** $6p - 6p$ **iv** $6x^2 - 3x^2 - x^2$

f If $a = 5$ and $b = 12$, find the value of:

i $5ab$ **ii** $\frac{3ab}{5}$ **iii** $2a - b$ **iv** $\frac{b}{a + 1}$

g Simplify the following.

i $2a + 5a - 3b$ **ii** $2pq + 5pq - 3pq$ **iii** $8k + 5 - 3k$ **iv** $-5y + 2y$

v $3y^2 - 8y^2$ **ii** $-3x^2 + 5x^2$ **vii** $-a - a$ **viii** $9a - 9a + 9a$

3 Simplify the following.

a $9h + h$ **b** $3d - d$ **c** $7z + 8 + 3z$ **d** $4p + 7p - 6q$

e $4a - 5 + 7a - 3$ **f** $5p \times 3q$ **g** $7g \times 5$ **h** $5m \times 5n$

i $\frac{12y}{4}$ **j** $\frac{12a}{3a}$ **k** $\frac{12mn}{15m}$ **l** $\frac{20pq}{5p}$

4 **a** Write an algebraic expression for each of the following.

i If I spend \$$c$ at a shop, how much change will I get from \$10?

ii If cheese pizza costs \$$x$ each and supreme pizza costs \$$t$ each, find an expression for the total cost of 4 cheese and 5 supreme pizzas.

b Write in words the meaning of:

i $a + b$ **ii** $a - b$ **iii** b^2

5 **a** Express $\frac{47}{100}$ as a decimal.

b Write 3.8 as a fraction.

c Express $\frac{77}{10\,000}$ as a decimal.

d Arrange in ascending order: 0.37, 0.365, 0.38, 0.04

6 Simplify the following.

a 4.5×5 **b** 63.8×100 **c** 11×14.83 **d** $1.8 \div 0.2$

e $93.4 \div 10$ **f** $1.74 \div 0.06$ **g** $(2.8 \times 5.3) - 4.1 + (3.8 \div 0.2)$

7 **a** What percentage of the diagram is:

i shaded? **ii** unshaded?

b Express 48% as a simplified fraction.

c Kylie scored 45 out of 90 in her exam. Express her result as a percentage.

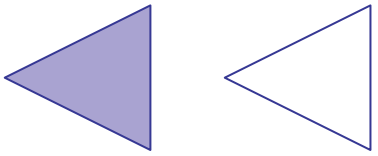
d Write $\frac{24}{32}$ as a percentage. **e** Write 85 g as a percentage of 250 g.

f Find 47% of \$600. **g** Find $4\frac{1}{4}$% of 400 kg.

h Convert $6\frac{1}{4}$% to a simplified fraction. **i** Convert 0.8% to a decimal.

8 **a** Has the shaded figure been translated, rotated or reflected?

i



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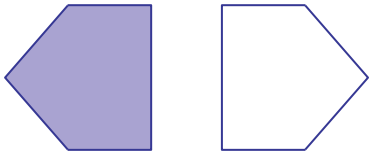
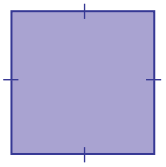


b Copy the following and draw all axes of symmetry.

i



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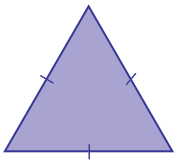


c Which shape(s) in part **b** have rotational symmetry? State the order.

d Point R has coordinates $(-5, 3)$. Write the coordinates of the image of R if it is:

i translated 4 units to the left **ii** reflected in the y-axis

iii reflected in the x-axis **iv** rotated anticlockwise about O through 90°

9 **a** An eight-sided die with the numbers 1 to 8 is thrown once.

i List the sample space. **ii** What is the probability of throwing 6?

b A box of ice-creams has 5 chocolate, 3 strawberry, 2 caramel and 2 vanilla.

i How many ice-creams are in the box? **ii** How many are strawberry?

iii What is the probability of selecting a strawberry ice-cream at random?

c A letter is chosen at random from the word FLUFFY. What is the probability that it is:

i U? **ii** F?

d Write a statement describing a probability of $\frac{1}{2}$.

e Estimate the percentage probability for the statement 'very likely'.

f A normal six-sided die is rolled once. Describe an event with a probability of:

i $\frac{1}{6}$ **ii** 0 **iii** $\frac{1}{2}$ **iv** 1

g A money box contains 10-cent, 20-cent and \$1 coins. A coin is selected at random. The probability of selecting a 10-cent coin is $\frac{7}{12}$ and the probability of selecting a 20-cent coin is $\frac{1}{6}$.

i What is the probability of selecting a \$1 coin?

ii What is the complement of selecting a 10-cent coin?

h Explain the statement $P(E) + P(\tilde{E}) = 1$.

11

Data investigation

This chapter deals with the display of statistical information.

At the end of this chapter you should be able to:

- identify the difference between primary and secondary data
- understand how data is collected using a census or sample
- understand how a sample can be biased
- recognise data as quantitative or categorical
- draw frequency distribution tables, histograms and polygons
- draw and use dot plots
- draw and use stem-and-leaf plots
- draw and use scatter plots
- use the terms 'cluster' and 'outlier' when describing data
- interpret data from graphs.

Diagnostic test

Use this two-way table to answer questions **1** to **8**.

Lunch choices

Food	Boys	Girls
Sandwich	10	12
Wrap	5	4
Salad	4	1

1 How many boys were surveyed?
A 19 **B** 10 **C** 5 **D** 17

2 How many girls were surveyed?
A 19 **B** 17 **C** 12 **D** 36

3 What is the most popular lunch choice for girls?
A sandwich **B** wrap
C salad **D** wrap and salad

4 What is the most popular lunch choice for boys?
A wrap **B** sandwich
C salad **D** wrap and salad

5 What fraction of boys selected a wrap?
A 5 **B** $\frac{5}{14}$ **C** $\frac{5}{19}$ **D** $\frac{14}{19}$

6 How many more girls chose a sandwich over a wrap?
A 12 **B** 11 **C** 8 **D** 16

7 What fraction of all students chose a salad?
A $\frac{10}{36}$ **B** $\frac{12}{36}$ **C** $\frac{22}{19}$ **D** $\frac{5}{36}$

8 This information is drawn in a-side by-side column graph. Which column will be the tallest?
A girl's sandwich **B** boy's sandwich
C sandwich **D** girl's salad

Use this bar graph to answer questions **9** to **11**.

Favourite pizza

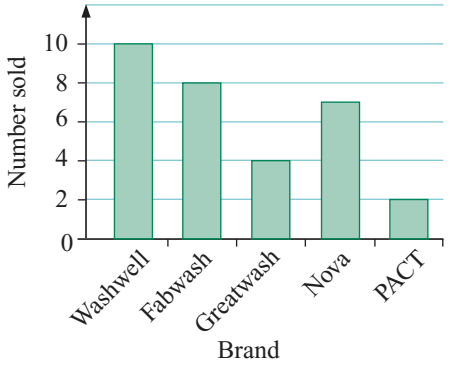
9 What is the most popular pizza?
A Supreme **B** Vegetarian
C Chicken **D** Meat

10 What is the least popular pizza?
A Supreme **B** Vegetarian
C Chicken **D** Meat

11 The fraction of people choosing the Supreme pizza is:
A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{2}{5}$

This graph refers to questions **12** to **15**.

Dishwasher sales



12 What type of graph is shown?
A pictograph **B** line graph
C sector graph **D** column graph

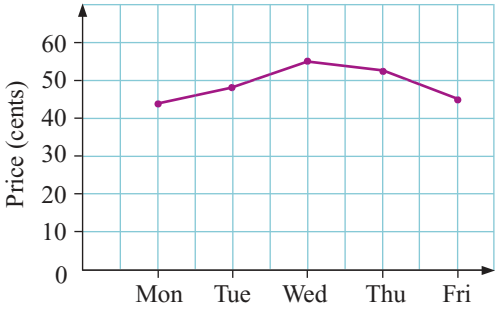
13 What is the title on the vertical axis?
A Number sold **B** Brand
C Dishwasher sales **D** Washwell

14 Which dishwasher had the fewest sales?
A PACT **B** Nova
C Greatwash **D** Washwell

15 What was the total number of dishwasher sales?
A 2 **B** 10 **C** 30 **D** 31

This graph refers to questions **16** to **19**.

Price of share (at 4 pm)



16 What type of graph is shown?

A pictograph **B** line graph
C sector graph **D** column graph

17 On which day was the closing price of the share highest?

A Thursday
B Wednesday
C Tuesday
D Monday

18 What scale is used on the vertical axis?

A 1 unit represents 1 cent
B 1 unit represents 10 cents
C 1 unit represents 100 cents
D 1 unit represents price

19 What would be an appropriate name for the horizontal axis?

A Share price **B** Cost of shares
C Time **D** Days of the week

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–8	9–19
Section	ACMSP 147	ACMSP 120, 148

STATISTICS

Statistics is an important branch of mathematics. We use statistics to collect and display information. Examples include the top 10 pop songs, the average monthly temperature and the number of people using a product.

The building blocks of statistics are the items of **data** collected. When this data is organised in some way that is useful, it has become **information**. When numerical data is organised in a meaningful way, the data is statistics. Statistics may be used to help people make informed decisions. This chapter examines some of these aspects.

A Primary and secondary data

Primary data is data that you collect. There are a number of ways you can collect primary data, such as by:

- *Direct observation:* This allows you to specifically focus on the details or aspects that are important to your research.
- *Surveys:* Considerable amounts of data can be collected by surveys. However, there is an assumption that the person completing the survey is trustworthy with their responses.
- *Interviews:* Although time consuming and expensive, interviews allow for in-depth questions and responses. They also allow interpretation of facial expressions, body language and sarcastic remarks. Again honesty can be an issue, as the person being interviewed may feel inclined to say what they think the interviewer wants to hear.
- *Logs:* These can produce a lot of valuable data regarding particular performances over time under various conditions. Examples are complaint logs, transaction logs and fault logs.

Secondary data is data collected from external sources such as the internet, television, radio, newspapers, magazines, journals, research papers, reviews or by hearsay. It is usually in the form of a ready-made product, so averages, conclusions, etc. are already calculated and stated.

Using secondary data can save time and costs, but the data can lack originality as it has been collected by other agencies. It may also not be suitable or relate directly to what is being investigated.

Primary data is more costly, but it is also likely to be more suitable and trustworthy. Secondary data, although cheaper and relatively easy to obtain, needs to be used cautiously.

Exercise 11A

1 **a** Define primary data.
b List two positive features of primary data.
c List two negative features of primary data.
d List two forms of primary data.

2 **a** Define secondary data.
b List two positive features of secondary data.
c List two negative features of secondary data.
d List two forms of secondary data.

3 Consider the illustrations shown below. State whether each is an example of primary or secondary data.

a

b

Beckham's career			
Years	**Team**	**Games**	**Goals**
1993–2003	Manchester United	265	62
1995	Preston North End (loan)	5	2
2003–2007	Real Madrid	116	13
2007	Los Angeles Galaxy	102	18
2009–2010	Milan (loan)	29	2
National team			
1996–2009	England	115	17

c

d

4 Create a collage in your workbook by collecting as many images as you can that show examples of primary data and secondary data. Use a separate page for each.

5 Respond to this statement: 'Primary data is preferable to secondary data.' Outline your response by justifying why you agree or disagree with the statement.

6 Joanne wanted to find out which was the most popular sport within her year group. Would she use primary or secondary data? Explain your answer.

7 Gary wanted to investigate the number of accidents over the Easter holiday weekend in NSW over the last five years. Would he use primary or secondary data? Explain your answer.

B Data collection

One of the first decisions to be made when collecting data is to decide from whom, or from what, the information is to be collected. There are two types of data collection: a **census** and a **sample survey**.

Census

A census involves collecting information from every individual in the whole **population**. The population is all the people or objects you want data about. For example, if all new cars are tested before sale, this is a census. The Australian Bureau of Statistics (ABS) conducts a census of the entire population of Australia every 5 years. A census is accurate and detailed, but also expensive, time consuming and often impractical.

Sample survey

A sample survey involves collecting data from only a part of the population. It is cheaper and quicker than a census, but not as detailed or accurate. Conclusions drawn from sample surveys always involve some error. Often this error is due to a bias in the sample or method of data collection.

- A **biased** sample does not truly represent the whole population.
- A **random** sample is fair. It truly represents the whole population.

EXAMPLE 1

State whether a census or sample survey be used to investigate each situations.

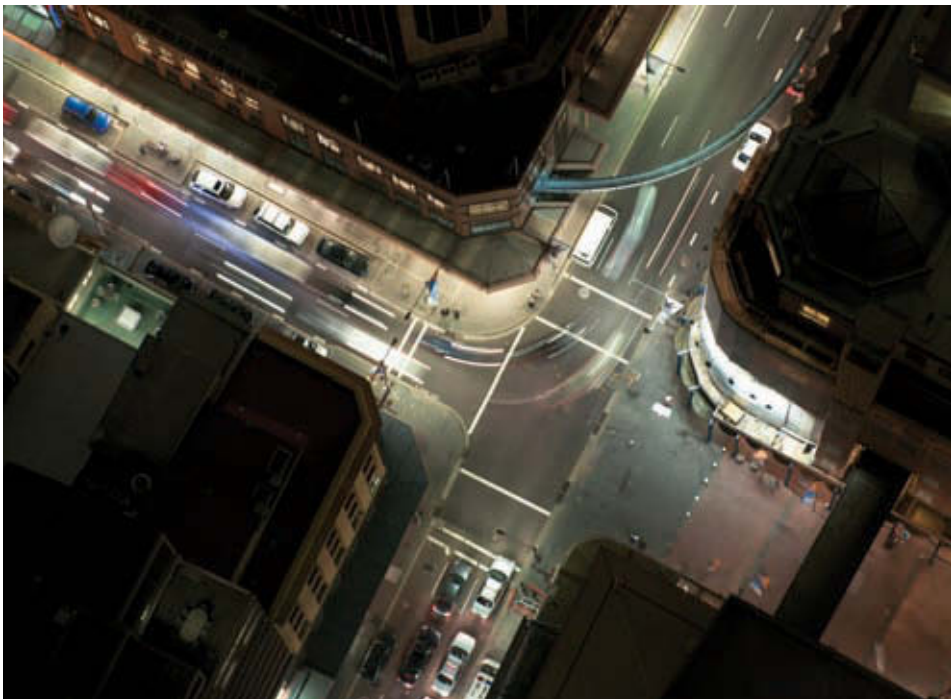
a the length of time an electric light globe will last
b the causes of car accidents in NSW
c the number of people who use Bright Teeth toothpaste

a *Sample:* It would obviously be impractical to test every light globe produced until it failed—there would be none to sell!
b *Census:* An accurate analysis of all accidents would be important.
c *Sample:* It would be very time consuming and expensive to interview the whole population to find out who uses Bright Teeth toothpaste.

Exercise 11B

1 State whether a census or sample survey would be used for each of these investigations. Discuss your answers in groups.

a the number of goals scored each week by a netball team
b the heights of the members of a football team
c the most popular radio station
d the number of children in an Australian family
e the number of loaves of bread bought each week by a family
f the types of pets owned by a class of students
g the star ratings of different brands of washing machines
h the numbers of leaves on the stems of plants
i the amount of sunshine each day
j the number of people who die from cancer each year
k the amount of rainfall each month
l the time spent doing homework each night
m the countries of origin of immigrants
n the most popular colours of cars
o the numbers of pets owned by students in a class
p the genders of school principals
q the number of cars passing through an intersection
r the sports played by students in high schools
s the stopping distances of cars
t the marks scored in a class test
u the items sold at a school canteen
v the number of matches in a box
w the amounts raised by school raffles
x the reasons people use taxis
y the fuel consumption of different cars



EXAMPLE 2

Suggest the possible bias in each of the following samples.

a people surveyed by phone during the day
b people surveyed at a train station
c people selected from a football crowd
d ten people tested with a new drug

a The sample would be biased towards people who are at home during the day and have a phone. It does not include people who go to work in the daytime or do not have a phone.
b The sample would be biased towards people who catch the train at that station. It does not include people who use other forms of transport or do not travel or use a different station.
c The sample would be biased towards people who attend football matches. For example, there would probably be more males than females at football matches.
d The sample would be biased by the characteristics of the ten people. A larger sample is needed.

2 Explain and discuss any possible bias in the following samples.

a people surveyed by phone on a Saturday night
b people at a bus stop
c people in a supermarket carpark
d people at the beach
e people in your street
f businesses selected from the Yellow Pages phone directory
g people selected from the electoral roll

3 Comment on any possible bias in the following situations.

a Year 7 students are interviewed about school uniform changes.
b Motorists stopped in peak hour are interviewed about traffic problems.
c Real estate agents are interviewed about house prices.
d Politicians are interviewed about the state of the country's economy.
e People are asked to phone in to register their vote on an issue.
f An opinion poll is conducted by posting a questionnaire to people.
g A manufacturing company tests a sample of its products every Monday morning.
h A survey of 20 people indicates that 80% of people watch the Channel 9 News.
i A company claims that 4 out of 5 dentists recommend their brand of toothbrush.
j An advertisement claims that 'Dog breeders recommend Buddy dog food'.

Biased samples

Sometimes people use biased samples to enhance their claims for selling their product or to support a particular point of view.

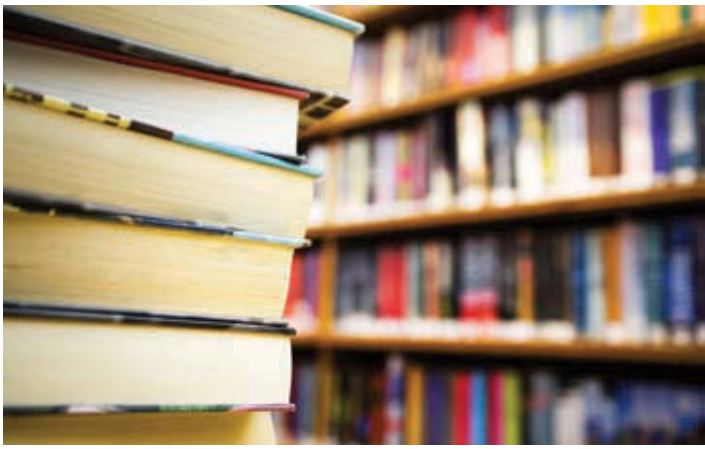
EXAMPLE 3

If you wanted the local council to upgrade its swimming pools, what sample of people would you choose to survey?

A sample of people at the swimming pool. You would expect the people who use the pool to be biased towards this proposal.

4 Describe the sample you would choose if you wanted to gather support to:

- **a** stop smoking in public areas
- **b** improve the local bus service
- **c** increase the number of books in the public library
- **d** improve the facilities in the school's senior common room
- **e** stop logging in national parks
- **f** increase unemployment benefits
- **g** reduce tariffs on farm products
- **h** reduce bank fees



5 Describe samples with the opposite bias to those in question **4**.

Sometimes people want a sample that is biased towards a certain group.

EXAMPLE 4

What would be a suitable sample to survey if you wanted to:

- **a** investigate the most popular movies?
- **b** collect information about the cleanliness of trains?

- **a** People at a cinema, people hiring DVDs, cinema managers, DVD stores.
- **b** People who travel on trains, train cleaners.

6 What would be a suitable sample to survey if you wanted to:

- **a** investigate the suitability of bus timetables on a certain route?
- **b** collect information about the effects of road tax on company profits?
- **c** gather data about the effects on a town of closing its only bank?
- **d** investigate the success of a new treatment for ticks and fleas on dogs?
- **e** find the most popular brand of dishwasher?
- **f** investigate the reasons for bank charges?

Investigation 1 Sampling techniques

Many techniques are used to ensure that samples are random. Five of these techniques are:

- simple random sampling
- interval sampling (or systematic sampling)
- stratified random sampling
- cluster sampling
- multi-stage sampling.

1 Investigate each of these sampling techniques and how they are used.

2 Tables of random numbers are used to help with samples.

a Investigate random numbers.

b How are tables of random numbers used in sampling?

C Key questions

In any census or sample survey it is very important to ask clear and relevant questions. The questions need to be structured so that the data obtained is easy to use.

EXAMPLE 1

Year 7 students are conducting a sample survey to investigate people's eating habits at their evening meal. Comment on the suitability of the following questions.

a *Is the evening meal your main meal of the day?*

b *Do you eat in front of the television?*

c *What do you watch?*

d *How often do you eat take-away food?*

e *How would you describe your evening meal?*

f *At what time do you eat your evening meal?*

a Relevant question, but it should include 'usually' to eliminate the once-in-a-while meals that are not typical. The expected responses would be Yes or No.

b Somewhat relevant question, but it should be reworded to ask the usual location of the evening meal. The expected responses might include: at the dining table, in front of the TV, etc.

c Irrelevant question. It also assumes that the answer to the previous question was Yes.

d Relevant question, but it needs to be reworded. Does it mean in a week, or over a month? Does the question refer to the evening meal only, or does it include other meals as well?

e Relevant question, but it may be difficult to use the responses. The question needs to include 'usual' and be restructured to include choices like: 'red meat and vegetables', 'chicken and vegetables', 'vegetarian', etc.

f Relevant question, but it needs to include time slots to tick.

Exercise 11C

1 These questions have been suggested for use in a survey about the different methods of transport used by people travelling to work. Comment on their appropriateness and possible responses. Reword if necessary.

a *Do you own a car?*
b *What colour are trains?*
c *How often do you drive to work?*
d *What type of public transport do you use?*
e *How often do you travel to work by public transport?*

2 These questions have been suggested for use in a survey about school uniforms. Comment on their appropriateness and possible responses. Reword if necessary.

a *Do you like the present uniform?*
b *Do you want to wear a uniform?*
c *What is your favourite colour?*
d *How old are you?*
e *Have you attended a school that doesn't have a uniform?*
f *Is your uniform comfortable?*

3 These questions have been suggested for use in a survey of what people watch on TV. Comment on their appropriateness and possible responses. Reword if necessary.

a *Do you own a TV?*
b *Do you watch TV?*
c *How many TVs are there in your house?*
d *What is your favourite program?*
e *Do you like sport?*
f *Which is your favourite channel?*
g *Do you lie down while watching TV?*

4 These questions have been suggested for use in a survey on marriage. Comment on their appropriateness and possible responses. Reword if necessary.

a *Are you married?*
b *Do you want to get married?*
c *Do you go to church?*
d *Would you wear a white wedding dress?*
e *Do you have a special person in your life at the moment?*

5 These questions have been suggested for use in a survey on people's knowledge and use of computers. Comment on their appropriateness and possible responses. Reword if necessary.

a *How does a computer work?*
b *Do you own a computer?*
c *What do you use it for?*
d *How much did it cost?*
e *Which room is it in?*
f *Do you work with computers?*
g *How old are you?*

6 These questions have been suggested for use in a survey about pets. Comment on their appropriateness and possible responses. Reword if necessary.

a *Do you own a pet?*
b *What is it?*
c *Do you prefer cats or dogs?*
d *Do you have a TV?*
e *What do you think of people with lots of pets?*

Types of questions

The main types of questions are:

- Free-response or open-ended questions, like *What is your favourite TV program?* The person answers the question in their own words.
- Yes or No questions, like *Did you watch program X last week?* The person answers Yes or No to the question.
- True or False questions, which are similar to Yes or No questions.
- Tick-box questions, like *What do you like watching on TV? Tick one or more boxes.*

  ☐ *Nothing* ☐ *News* ☐ *Sport* ☐ *Drama* ☐ *Comedy* ☐ *Soapies* ☐ *Other*

  Options must include all possible responses.
- Scaled-response questions, like *Do you think that there should be more Australian programs on TV? Circle a number.*

  1 Strongly disagree *2 Disagree* *3 Don't know* *4 Agree* *5 Strongly agree*

  The number of options must be odd and there must be a neutral option.

EXAMPLE 2

You want to find the top five TV programs for students in Year 7, so you interview 20 people from Year 7. Which type of question would you ask? Write an appropriate question.

The most obvious question is:

- *What is your favourite TV program?* However, there could be 20 different answers from the 20 people, and you wouldn't know the top five.

Another questions could be:

- *List your three favourite TV programs.* This would give more opportunity to compile the top five.

If you had a list of likely programs, you could ask:

- *Rate each of these TV programs on a scale of 1 to 5.*

  1 Strongly dislike *2 Dislike* *3 No opinion* *4 Like* *5 Strongly like*

This would be easy to analyse by adding the scores for each program. The five programs with the highest score would be the most popular.

7 Write open-ended questions on each survey topic.

- **a** favourite FM radio station
- **b** school uniform
- **c** food at the canteen
- **d** pedestrian safety around your school
- **e** career choices

8 Write Yes or No questions for each topic in question **7**.

9 Write tick-box questions for each topic in question **7**.

10 Write questions that require a response on a scale of 1 to 5 for each topic in question **7**.

11 **a** Select from your answers to questions **7** to **10** the most appropriate questions to use for each survey topic. Give reasons for your choice.

b Give reasons why you rejected the other questions.

12 Form groups of four or five, and ask each member of your group to respond to your questions on each survey topic. Comment on the responses and the suitability of your questions.

D Variables

The word 'variable' is used to describe several things in many areas of mathematics, including algebra, graphing and statistics. In statistics, a **variable** is something that has different values for different people or objects.

When collecting data we are interested in a particular property or characteristic of a group of people or objects. The particular characteristic that we are interested in is the variable.

Examples of variables include: age, weight, height, method of travelling to school, eye colour, marks in a test.

- A **nominal** or **categorical variable** describes categories that are the names of things. Examples include method of transport to school, eye colour and gender.
- A **numerical** or **quantitative variable** has a numerical value.

Further, a numerical variable can be discrete or continuous.

- The values of **discrete variables** are *exact* and increase in steps. They are usually a result of *counting*. Examples include the number of children in a family, marks in a test and shoe size.
- The values of **continuous variables** can vary by any amount. They are usually the result of *measuring* and lie within a certain *range*. For example, if we say someone weighs 42 kg to the nearest kilogram, their exact weight could be anywhere between 41.5 kg and 42.5 kg. Other examples include height and distance from home.

EXAMPLE 1

Classify the following variables as nominal, discrete numerical or continuous numerical.

a the number obtained when a die is rolled

b the brand of runners worn by students in a class

c the weight of students in a class

Nominal → name
Numerical → number

a Values are exact and are obtained by counting the number of dots on the upper face of the die: 1, 2, 3, 4, 5 or 6. The variable is discrete numerical.

b The variable describes the brand of runners. It is a nominal variable.

c Values can vary by any amount and are obtained by measuring. The results are determined by the accuracy of the measurement. The variable is continuous numerical.

Exercise 11D

1 Indicate whether the following are nominal or numerical variables.

a age

b weight

c the method of travel to school

d eye colour

e the time spent talking on the telephone

f the number of people in your family

2 Indicate whether the following are nominal or numerical variables.

a the speed of a car
b hair colour
c favourite TV show
d type of pet
e the temperature during the day
f the number of aces served by a tennis player
g the number of goals scored by a soccer team
h the mass of grapes in a bunch
i the number of grapes in a bunch
j the length of a line

3 Give three examples of the following (not mentioned in questions **1** or **2**).

a nominal variables
b numerical variables

4 Are the following numerical variables continuous or discrete?

Continuous is a value within a possible range of values. Discrete is an exact value.

a age
b height
c the number of people in your family
d the speed of a car
e the number of words in a book
f the amount of money in your pocket
g the time taken to run 100 m
h the length of your hair
i the time spent talking on the telephone
j the number of male students in your school

5 Give three examples of the following. (List examples not mentioned in questions **1** to **4**).

a continuous variables
b discrete variables

6 For each of the following investigations, classify the variable as categorical (nominal), discrete (numerical) or continuous (numerical).

a the number of goals scored each week by a netball team
b the heights of the members of a football team
c the most popular radio station
d the number of children in an Australian family
e the number of loaves of bread bought each week by a family
f the types of pets owned by a class of students
g the star ratings of different brands of washing machines
h the numbers of leaves on the stems of plants
i the amount of sunshine each day
j the number of people who die from cancer each year
k the number of cars passing through an intersection
l the amount of rainfall each month
m the countries of origin of immigrants
n the most popular colours of cars
o the numbers of pets owned by students in a class
p the genders of school principals
q the time spent doing homework each night
r the sports played by students in high schools
s the stopping distances of cars
t the marks scored in a class test
u the items sold at a school canteen
v the number of matches in a box
w the amounts raised by school raffles
x the reasons people use taxis
y the fuel consumption of different cars

E Frequency distribution tables

Once data has been collected, the next step is to organise the data before it is analysed and displayed.

The value of one piece of data is called a **score**. A score can be nominal, such as blue, or numerical, such as the number of heads tossed. The **frequency** of a score is the number of times it occurs; for example, 16 people have blue eyes or a head is tossed 12 times.

A **frequency distribution table** is a table that displays the frequency for each score.

Tally marks are often used to help record the data in the table.

- Every fifth tally mark is crossed through the four preceding tally marks. (This makes counting easier.)
- The frequency of a score is the number of tally marks for that score.

They are called 'gate post' tally marks because of how they look.

The sum of all the frequencies is the same as the number of observations in the data.

EXAMPLE 1

The method by which the 30 students in class 7A travelled to school on a particular day is recorded below, using the code: walk (W), cycle (C), bus (B), car (M) and train (T).

WCBWC BBBWB BBCBT CMCBT MMTMM MWCCB

Rearrange this information into a frequency distribution table using a tally column.

Method of travel (score)	Tally	Frequency
Walk	𝍸	4
Cycle	𝍸 ||	7
Bus	𝍸 𝍸	10
Car	𝍸 |	6
Train	|||	3
	Total	**30**

Exercise 11E

1 a Three coins were tossed simultaneously and the number of heads recorded each time, using tally marks. Complete the table of results below.

Always check that the total of the frequency column is the same as the total number of rally marks.

Number of heads (score)	Tally	Frequency
0	𝍸 𝍸 |||	
1	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	
2	𝍸 𝍸	
3	𝍸 𝍸 ||	
	Total	

b The numbers of days absent in a week for the students in Year 7 are shown in the table, using tally marks. Complete the table.

Days absent (score)	Tally	Frequency
0	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	
1	𝍸 |||	
2	||	
3	|||	
4	𝍸 𝍸 ||	
5	|||	
	Total	

2 The maximum daily temperatures in February (in °C) were:

42 41 40 42 43 42 41 43 39 41 41 43 41 40
40 42 40 43 39 40 39 40 38 39 41 40 42 37

Complete this table using tally marks, then answer the questions.

Temperature (°C) (score)	Tally	Frequency
37		
38		
39		
40		
41		
42		
43		
	Total	

a On how many days in February was the maximum temperature recorded?

b On how many days did the highest maximum temperature occur?

c How many days had a maximum temperature of 40°C or more?

3 The types of vehicles passing a certain point during a 10-minute period were recorded using the code: car (C), bus (B), truck (T) and motorcycle (M). Complete the table below using tally marks.

CCMCC TBCCC TTMMC CTBCC CCTMC CCTCM
CCMTT CCCMC CCCCT CMBCC CCCTM TCCCC

Vehicle (score)	Tally	Frequency
Car		
Motorcycle		
Bus		
Truck		
	Total	

a How many cars passed this point?

b What was the total number of trucks?

c How many more trucks than buses were there?

4 Complete the table below, using tally marks, for the month of birth of the students in a Year 7 class.

Jan	Mar	Dec	May	Jun	Jul	Jan	Dec	Oct	Apr	Jan	Jul	Sep	Aug	Jul
Oct	Jul	May	Nov	Mar	Jul	Oct	May	Jul	May	Aug	Sep	Dec	Dec	Sep

Month (score)	Tally	Frequency
January		
February		
⋮		
December		
	Total	

a How many students are in the class?
b What is the most common month for birthdays?
c Which month has no birthdays?
d How many students were born in winter?

5 The colours of cars passing the front of a school in a 30-minute period are recorded below, using the code: white (W), blue (B), grey (G), red (R), other (O).

BWROW	BRGWO	BGRWW	GBRWO	GBRWG
BRRGW	BRGOW	BWGRB	WWBRG	RRWWW

a Rearrange this information into a frequency distribution table, using a tally column.
b How many cars passed the front of the school in this time period?
c What was the most frequent colour observed?
d Calculate the percentage of each colour.

6 The eye colours of a group of students were recorded, using the code: blue (B), brown (b), green (G), grey (g).

BBgBb	bGbBg	bBgbG
bbBgB	BBbBG	BbgbB

a Organise this data into a frequency distribution table, using a tally column.
b How many students were observed?
c What was the most common eye colour in this group of students?

7 The marks out of 10 for a mental arithmetic test were:

9	10	4	9	7	9	3	2	9	5
8	7	1	7	3	6	3	10	5	9
0	8	7	4	6	1	6	5	7	7
5	3	3	5	8	2	6	7	9	8

a Rearrange the data into a frequency distribution table, using a tally column.
b How many students sat for the test?
c How many scored full marks?
d How many scored zero?
e What percentage of students scored 5 or more?

Grouped data

Sometimes numerical data covers a large range of values (scores). When this is the case, it is often desirable to group the data into **classes** and determine the frequency of each class. Otherwise the frequency distribution table would have too many rows. Classes must all be the same size.

EXAMPLE 1

Measurements of the maximum temperature on Mondays were taken over 1 year, with the results shown below (in °C).

14	29	32	21	19	23	14	20	25	31	24	16	26	24	21	20	15	26
23	30	16	19	27	22	23	26	16	24	35	25	27	14	21	20	27	21
14	32	23	19	26	29	20	15	22	25	31	19	23	14	25	19		

a Draw a frequency distribution table with classes of 11–15, 16–20, 21–26°C and so on.
b What is the size of each class?
c How many Mondays had a maximum temperature of 26°C or more?
d How many Mondays had a maximum temperature of 20°C or less?
e How many Mondays had a maximum temperature of 27°C?

a

Temperature (°C)	Tally	Frequency
11–15	𝍸 ||	7
16–20	𝍸 𝍸 ||	12
21–25	𝍸 𝍸 𝍸 |||	18
26–30	𝍸 𝍸	10
31–35	𝍸	5
	Total	**52**

b The class size is 5°C as classes have a range of 5°C.
c 10 + 5 = 15 days
d 7 + 12 = 19 days
e Cannot answer from the table. To answer this question the raw data must be used. By looking through the raw data, the answer is 3 days.

This frequency distribution table would have 25 rows if classes were not used.

Exercise 11F

1 The marks out of 100 for students in a mathematics exam were:

69	90	61	58	94	68	77	80	64	87	59	81	73	56	72	66	85	60	62	75
78	75	87	73	81	70	73	58	85	73	76	92	82	86	70	75	82	71	79	80

a Using classes 50–59, 60–69, 70–79, 80–89, 90–99, tabulate the data using columns for the score, tally and frequency.

Marks	Tally	Frequency
50–59	𝍸	4
60–69	𝍸 ||	7
70–79		
80–89		
90–99		
	Total	

b What is the size of each class?
c How many students scored in the 70–79 class?
d How many students scored less than 80?
e How many students scored 70 or better?

2 A group of young footballers held a long-kicking competition. The following results in metres were obtained.

55	30	51	49	58	31	32	47	41	36	40	57
45	48	49	69	47	43	35	49	41	45	45	46
49	53	41	40	30	51	35	37	23	33	68	49
49	31	61	48	57	47	37	48	56	38	40	29
45	65	45	45	41	40	39	36	36	39	30	39
35	42	58	37	47	43	43	41	53	42	42	55
48	59	45	46	57	49	48	66	42	43	37	46
68	42	41	37	31	39	49	36	23	47	53	40

a Complete the table below using the classes given.
b What is the size of each class?
c How many footballers kicked less than 44 m?
d How many footballers kicked at least 51 m?
e Which class has the most footballers in it?

Distance (m)	Tally	Frequency
23–29		
30–36		
37–43		
44–50		
51–57		
58–64		
65–71		
	Total	

3 A plant inspector took a random sample of 2-week-old seedlings from a nursery and measured their height to the nearest millimetre. The results were:

333	423	349	425	370	404	370	323	351	351	438	398	390	398	326
311	368	360	411	389	381	378	377	408	359	357	374	301	350	361
329	370	375	373	321	340	358	400	448	366	380	348	385	371	308
440	351	390	379	340	402	366	380	342	369	368	300	370	330	424

a Complete the table shown.
b What is the size of each class?
c How many seedlings were measured?
d How many seedlings were:
  i from 375 to 399 mm high?
  ii between 349 and 400 mm high?
e How many seedlings were:
  i 400 mm or more high?
  ii less than 400 mm high?
f What fraction of seedlings were less than 400 mm high?

Height (mm)	Tally	Frequency
300–324		
325–349		
350–374		
375–399		
400–424		
425–449		
	Total	

g If the total number of seedlings was 1462, estimate the number of seedlings less than 400 mm high.

G Presenting data

Graphs are used to display data in a way that is both visually attractive and informative. Although there is some loss of detail compared with a table, it is often easier to see trends and relationships. Often both a table and a graph are used to convey the maximum amount of information.

This section examines picture, sector, divided bar, column, bar and line graphs.

Exercise 11G

Picture graphs

Picture graphs or **pictographs** are probably the simplest of all graphs. A symbol or picture is used to represent a specific number of items. It is important to always read the scale/key carefully before interpreting a picture graph.

EXAMPLE 1

Month	Number of sunny days
January	☀ ☀ ☀ ☀ ☀ ☀
February	☀ ☀ ☀ ☀
March	☀ ☀ ☀ ½☀
April	☀ ☀

Scale: ☀ represents 4 sunny days.

a How many sunny days occurred during:
- **i** January?
- **ii** February?
- **iii** March?
- **iv** April?

b How many sunny days occurred during the first 4 months of the year?

a

Month	Number of symbols	Number of sunny days
January	6	$6 \times 4 = 24$
February	4	$4 \times 4 = 16$
March	$3\frac{1}{2}$	$3\frac{1}{2} \times 4 = 14$
April	2	$2 \times 4 = 8$

b Sunny days occurred during the first 4 months:
$24 + 16 + 14 + 8 = 62$ sunny days

1 The picture graph shows the preferred sports of Year 7 students.

Key: 🧍 represents 10 students.

Sport	Number of students
Soccer	🧍🧍🧍
Football	🧍🧍🧍🧍🧍
Netball	🧍🧍🧍
Basketball	🧍🧍🧍
Cricket	🧍🧍
Swimming	🧍🧍
Golf	
Tennis	🧍🧍
Other	🧍🧍🧍

a How many students preferred:

i soccer? **ii** football?

iii basketball? **iv** cricket?

b Which sport did only 5 students prefer?

c How many more students selected football than cricket as their preferred sport?

d How many Year 7 students were surveyed, if each student nominated only one sport?

2 This frequency distribution table contains data on the number of mobile phones in each household surveyed.

Number	Tally	Frequency
0	||	
1	𝍸	
2	𝍸 |	
3	𝍸 𝍸 𝍸 |	
4	𝍸 𝍸 ||	
5	𝍸	
more than 5	||	
	Total	

a Complete the table.

b Draw a picture graph using the symbol 📱 to represent 4 households.

c How many households had 3 mobile phones?

d How many households had no mobile phones?

e How many households were surveyed?

Sector graphs

A **sector graph** or **pie chart** is useful for illustrating how a whole (the total number) is divided into parts. Each category (part) is represented by a sector of a circle. The area of each sector is proportional to the size of each category, so each sector angle is proportional to the size of each category.

EXAMPLE 2

In a survey, 50 salespeople were asked the country of manufacture of their car. The results are given in the table.

a What fraction of cars were manufactured in each country?

b For a sector graph, calculate the size of the sector angle for each country.

c Draw a sector graph to illustrate this information.

Country	Frequency
Australia	17
Japan	19
Korea	6
Germany	3
Other	5
Total	**50**

a The fraction of cars manufactured in:

Australia $= \frac{17}{50}$ Japan $= \frac{19}{50}$ Korea $= \frac{6}{50}$

Germany $= \frac{3}{50}$ Other $= \frac{5}{50}$

b The size of the sector angle for each country (rounded the nearest degree) is:

Australia $= \frac{17}{50} \times 360° \approx 122°$ Japan $= \frac{19}{50} \times 360° \approx 137°$ Korea $= \frac{6}{50} \times 360° \approx 43°$

Germany $= \frac{3}{50} \times 360° \approx 22°$ Other $= \frac{5}{50} \times 360° = 36°$

c Use a protractor and compasses to draw the sector graph.

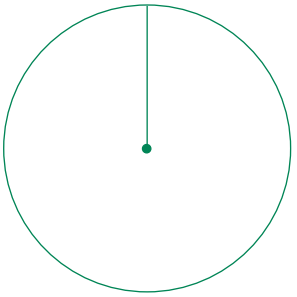
Step 1: Draw a circle using compasses.

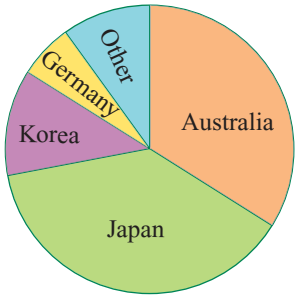
Step 2: Mark the radius at the top of the circle, and use this as the starting point for measuring each sector.

Step 3: Using a protractor, measure the angle for each sector.

Step 4: Label each sector and add a graph title.

Step 5: Colour each sector differently.





3 The table shows the quantities of meats sold by a butcher.

Meat	Sales (kg)
Beef	160
Lamb	120
Chicken	90
Pork	80
Total	**450**

a Complete the following to find the fraction of sales for each type of meat.

Beef: $\frac{160}{450} = \frac{\square}{45}$ Lamb: $\frac{\square}{450} = \frac{\square}{15}$

Chicken: $\frac{\square}{450} = \frac{\square}{5}$ Pork: $\frac{\square}{450} = \frac{\square}{\square}$

b For a sector graph, calculate the size of the sector angle for each type of meat.

Beef: $\frac{\square}{45} \times 360° = 128°$ Lamb: $\frac{\square}{15} \times 360° =$ ____

Chicken: $\frac{\square}{5} \times 360° =$ ____ Pork: $\frac{\square}{\square} \times 360° =$ ____

c Draw a sector graph to illustrate this information, using the angle sizes from part **b**.

4 The industries in which a group of Year 7 parents were employed are given in the table.

a For a sector graph, calculate the size of the sector angle for each industry.

The sector angle for each industry is the percentage of 360°. Sometimes, because of rounding, the sector angles do not add to exactly 360° and you need to use the best approximation to half a degree.

b Draw a sector graph to illustrate the information.

Industry	% of parents
Agriculture	5
Mining	26
Construction	12
Hospitality	35
Finance	16
Public service	6
Total	**100**

Divided bar graphs

Divided bar graphs also are useful for illustrating how a whole is divided into parts. They have all the advantages of sector graphs, apart from perhaps being less attractive visually. The calculations and measurements required are easier.

EXAMPLE 3

In a survey, 50 salespeople were asked the country of manufacture of their car. The results are given in the table. (This is the same data as in Example 2.)

Country	Australia	Japan	Korea	Germany	Other
Frequency	17	19	6	3	5

a Calculate the percentage of cars manufactured in each country.

b Draw a divided bar graph of length 100 mm to illustrate this data.

a

Country	Frequency	Percentage of total
Australia	17	$\frac{17}{50} \times 100\% = 34\%$
Japan	19	$\frac{19}{50} \times 100\% = 38\%$
Korea	6	$\frac{6}{50} \times 100\% = 12\%$
Germany	3	$\frac{3}{50} \times 100\% = 6\%$
Other	5	$\frac{5}{50} \times 100\% = 10\%$
Total	**50**	**100%**

Sometimes, because of rounding, the percentages do not add to exactly 100% and you need to use the best decimal approximation.

b *Step 1:* Use a ruler to draw a rectangle bar 100 mm in length.

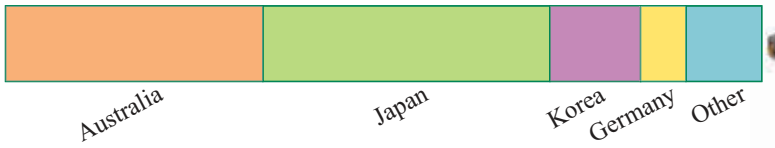
Step 2: Divide the bar into the percentages given in the table above.

Australia = 34% of 100 mm = 34 mm
Japan = 38% of 100 mm = 38 mm
Korea = 12% of 100 mm = 12 mm
Germany = 6% of 100 mm = 6 mm
Other = 10% of 100 mm = 10 mm

Step 3: Label each section of the bar and add a graph title.

Step 4: Colour the sections differently.

Country of manufacture of cars



5 The table shows the quantities of meats sold by a butcher. (This is the same data as in question **3**.) Complete the table and draw a divided bar graph of length 100 mm to illustrate this information.

Meat	Sales (kg)	Percentage of total	Length on 100 mm bar
Beef	160	$\frac{160}{450} \times 100\% \approx 35.6\%$	$35.6\% \times 100 = 35.6$ mm ≈ 35.5 mm
Lamb	120	$\frac{120}{450} \times 100\% \approx 26.7\%$	$26.7\% \times 100 = 26.7$ mm ≈ 26.5 mm
Chicken	90	$\frac{90}{450} \times 100\% =$ ___%	___% × 100 = ___ mm
Pork	80	$\frac{80}{450} \times 100\% \approx$ ___%	___% × 100 = ___ mm ≈ ___ mm
Total	**450**	**100%**	**100 mm**

6 The industries in which a group of Year 7 parents were employed are given in the table (the same data as in question **4**). Complete the table and draw a divided bar graph of length 100 mm to illustrate this information.

Industry	% of parents	Length on 100 mm bar
Agriculture	5	$5\% \times 100 = 5$ mm
Mining	26	$26\% \times 100 =$ ___ mm
Construction	12	$12\% \times 100 =$ ___ mm
Hospitality	35	$35\% \times 100 =$ ___ mm
Finance	16	$16\% \times 100 =$ ___ mm
Public service	6	$6\% \times 100 =$ ___ mm
Total	**100**	**100 mm**

Column graphs

A **column graph** is useful for displaying nominal (categorical) data or discrete numerical data.

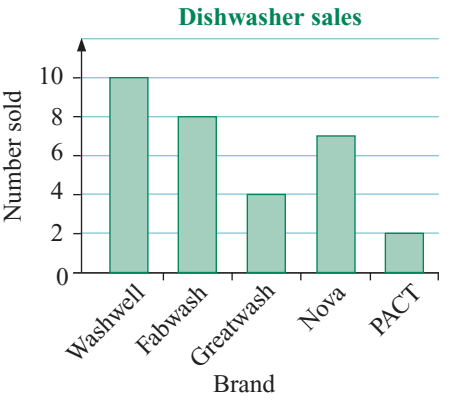
- Categories or discrete scores are shown on the horizontal axis.
- The vertical axis shows the scale, so the frequency for each category or discrete score is shown by the height of the column.
- The rectangular columns are the same width and evenly spaced. The gaps indicate that there is no connection between categories or discrete scores.
- When the columns represent consecutive discrete numbers or moments in time, the columns may be joined.

EXAMPLE 4

The number of each brand of dishwasher sold by an electrical store in 1 week is shown in the table. Draw a column graph to illustrate this information.

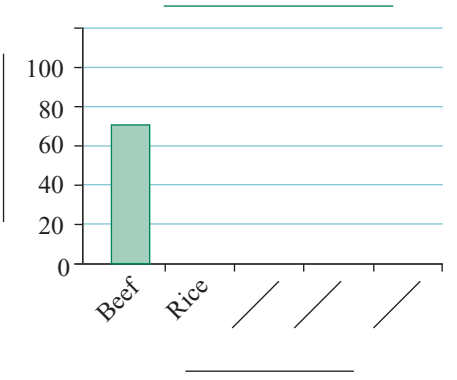
Brand	Number sold
Washwell	10
Fabwash	8
Greatwash	4
Nova	7
PACT	2
Total	**31**

Put brand on the horizontal axis and number sold on the vertical axis.
Show a grid so that the graph can be read easily.
Add a title.



7 The table shows the average percentage of protein in certain foods. Complete the column graph to illustrate this information. Use percentage protein on the vertical scale.

Food	% protein
Beef	70
Rice	80
Eggs	95
Fish	70
Milk	80



8 The sales of a real estate company for 1 year are shown in the table. Draw a column graph to illustrate this data. Use sales in millions on the vertical axis, with increments of 1 unit.

Month	Sales ($ millions)
Jan–Feb	6.4
Mar–Apr	7.0
May–Jun	2.7
Jul–Aug	3.6
Sep–Oct	2.9
Nov–Dec	5.1

9 The table shows the monthly sales figures for a computer firm. Draw a column graph to illustrate this data.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales ($000s)	54	36	30	30	28	34	25	26	31	38	44	56

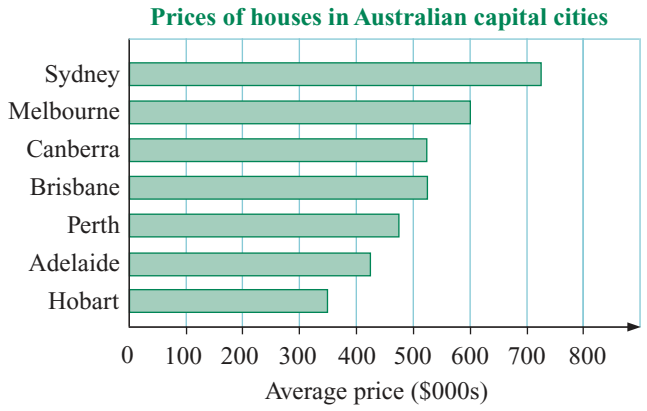
Bar graphs

When the rectangles of a column graph are drawn horizontally, the graph is usually called a **bar graph**.

EXAMPLE 5

City	Average price ($000s)
Sydney	725
Melbourne	600
Canberra	525
Brisbane	525
Perth	475
Adelaide	425
Hobart	350

The table shows the average prices for houses in the capital cities of Australia in a certain year. Illustrate this information on a bar graph.



Put city on the vertical axis and average price on the horizontal axis. Show a grid so that the graph can be read easily. Add a title

10 This table shows the number of injuries in each of the nine main sports played in Australia, in a certain year. Illustrate this information on a bar graph. Put number of injuries on the horizontal axis.

Sport	Number of injuries
AFL	270
Soccer	160
Cricket	99
Rugby league	88
Rugby union	58
Netball	40
Hockey	32
Indoor cricket	30
Basketball	23

Line graphs

Line graphs are useful for showing upward and downward trends in a variable, especially over time. The data collected is plotted as dots on the graph. The lines between the dots show the expected values between the data collected. Line graphs may only be drawn for quantitative (numerical) continuous data. Otherwise the in-between values do not exist.

EXAMPLE 6

This table shows the variation in price of a share for 1 week in 2012. The prices were taken at the close of trading at 4 pm.

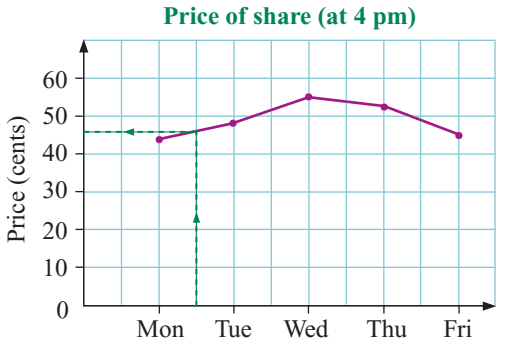
Day of the week	Mon	Tue	Wed	Thu	Fri
Price (cents)	44	48	55	52	46

a Draw a line graph to show this information.
b Using the graph, approximate the share value halfway through trading on Tuesday.

a Put time (day of the week) on the horizontal axis and price on the vertical axis.

Step 1: Mark an appropriate scale for the horizontal axis.
Step 2: Mark an appropriate scale for the vertical axis.
Step 3: Write a heading and label each axis.
Step 4: Plot the points and join them with straight lines.

b *Step 1:* Locate the position 'halfway through trading on Tuesday' on the horizontal axis.
Step 2: Rule a line up to the graph, then across to the vertical axis.
Step 3: Read the value off the vertical axis: about 46 cents.



11 The height of a seedling was measured at the same time each day over a week.

Days	0	1	2	3	4	5	6	7
Height (cm)	4	6	8	8	10	11	13	14

a Draw a line graph for this information.
i What heading should go on the graph?
ii What label should go on the horizontal axis?
iii What label should go on the vertical axis?
iv The first two coordinates to plot are (0, 4) and (1, 6). Write the remaining coordinates.
b What was the initial height of the seedling? On what day was this?
c Using the graph, estimate the height of the seedling after $5\frac{1}{2}$ days.

12 The weight of a baby at various ages is shown below.

Age (months)	0	3	6	9	12	15	18	21	24
Weight (kg)	3.2	5.1	7.0	8.8	10.0	10.7	11.2	11.8	12.5

a Illustrate this information on a line graph.
b Estimate the baby's weight at 10 months.

H Histograms and polygons

Continuous numerical data may be displayed using a frequency histogram or frequency polygon.

- A **frequency histogram** is a column graph in which the numerical values of the variable (the scores) are placed on the horizontal axis and the frequencies of the values on the vertical axis.
- A **frequency polygon** is a line graph with scores on the horizontal axis and frequency on the vertical axis, where the first and last points are joined to the horizontal axis, forming a polygon.

In a histogram, the columns are joined. The area of each column represents the frequency of each score. Thus the total area of the histogram represents the total number of scores.

The area under a frequency polygon is equal to the area under the corresponding histogram.

EXAMPLE 1

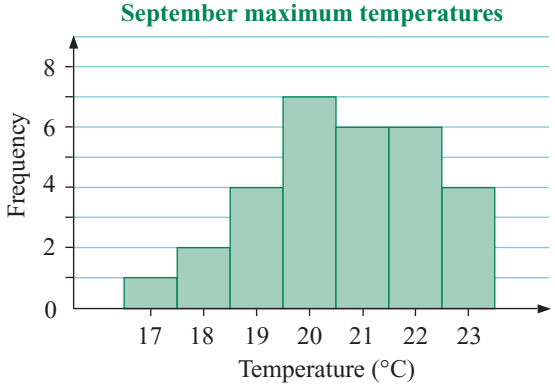
The maximum temperature on each day in September was recorded, and the results were summarised in a frequency table, as shown.

Temperature (°C)	17	18	19	20	21	22	23
Frequency	1	2	4	7	6	6	4

a Draw a frequency histogram.

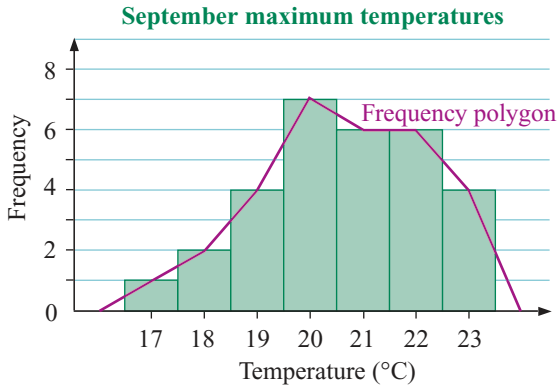
b Using the graph, approximate the share value halfway through trading on Tuesday.

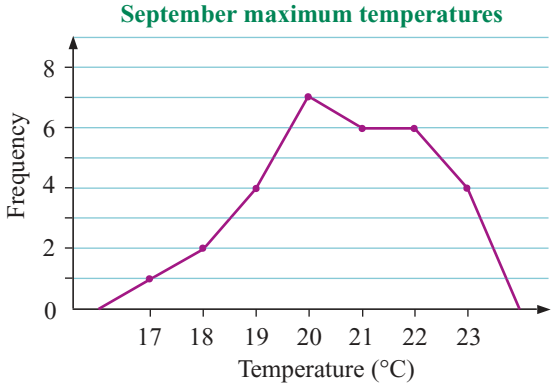
a The columns are centred on the scores they represent. They are the same width and are joined. Both axes are labelled. The frequency histogram has a heading.



b The frequency polygon can be drawn either:

- by joining the midpoints of the frequency histogram columns, or
- by plotting the points as a line graph.





In both cases, the first and last points are joined to the positions on the horizontal axis where the next score would be placed.

When drawing the histogram, leave at least half a column space at each end so that the polygon can be drawn accurately.

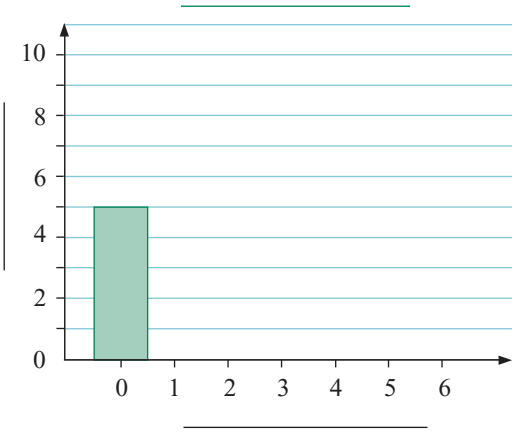
Exercise 11H

1 Complete the graph below to draw a frequency histogram for this frequency distribution.

Number of snacks	0	1	2	3	4	5	6
Frequency	5	7	9	7	6	4	2

a Label each axis.
b Give the graph a heading.
c Complete the frequency histogram shown using the data provided.

A frequency histogram is a column graph.



2 Draw a frequency histogram for the following distribution.

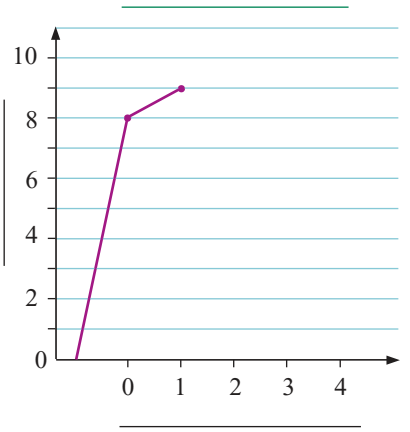
Score	10	11	12	13	14
Frequency	7	11	16	0	5

3 Complete the graph to draw a frequency polygon for this frequency distribution.

A frequency polygon is a line graph.

Number of goals	0	1	2	3	4
Frequency	8	9	6	3	1

a Label each axis.
b Give the graph a heading.
c Complete the frequency polygon shown using the data provided.



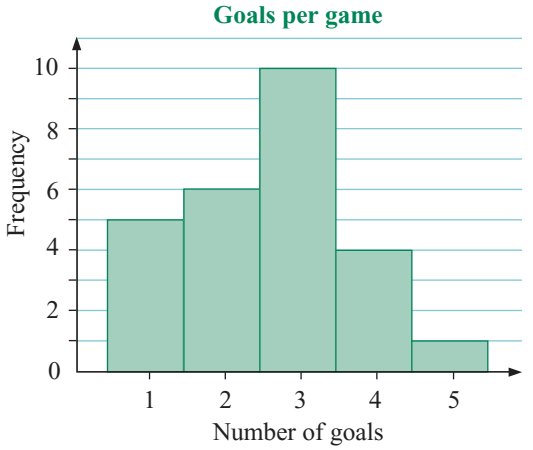
4 Draw a frequency polygon for the following distribution.

Salary ($)	Frequency
30 000	8
40 000	6
50 000	7
60 000	3
70 000	0
80 000	1

5 For the data given in the table, draw a frequency histogram and a frequency polygon on the same diagram.

Marks	14	15	16	17	18	19	20
Frequency	4	6	9	0	2	3	1

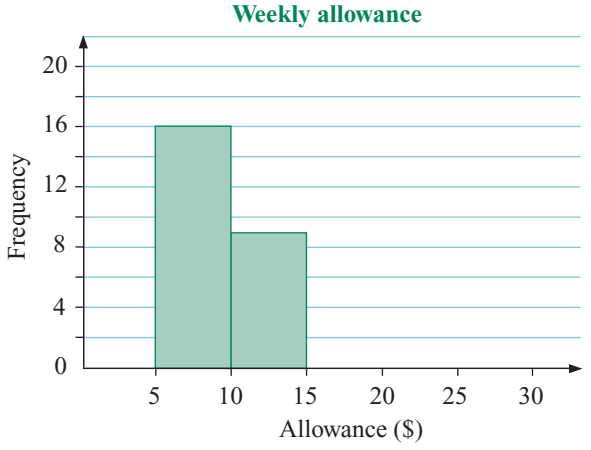
6 Construct a frequency distribution table of values from the data shown on this frequency histogram.



7 **a** A sample survey of the weekly allowances of 800 students from a certain school was taken by randomly choosing 38 students and asking each of them: 'How much pocket money do you receive per week?' The results are given in the table. Complete the table.

Weekly allowance ($)	Tally	Frequency
5 to 9.99	𝍸 𝍸 𝍸 |	
10 to 14.99	𝍸 ||||	
15 to 19.99	|||	
20 to 24.99	𝍸	
25 to 29.99	𝍸	
	Total	

b Use the table to complete the frequency histogram shown.



c Draw a frequency polygon for this information.

d What fraction of the sample received an allowance from $5 to $9.99?

e Why can't you say exactly how many students received an allowance of $7.50?

8 Consider the following set of data.

55	30	51	49	58	31	32	47	41	36	40	57	45	48
49	69	48	47	43	35	49	41	45	45	46	49	53	41
40	30	51	35	37	43	23	33	68	49	49	31	61	48
57	47	37	48	56	38	40	29	45	65	45	45	41	40
39	36	36	39	30	39	35	42	58	37	47	43	43	41
53	42	42	55	48	59	45	46	57	49	48	66	40	42
43	37	46	68	42	41	37	31	39	49	36	23	47	53

a Sort these scores into a frequency distribution table using:

i classes: 23–29, 30–36, 37–43, 44–50, 51–57, etc.

ii classes: 21–27, 28–34, 35–41, 42–48, etc.

b Draw a frequency histogram and polygon to represent each frequency distribution table.

c Compare the two histograms.

I Dot plots

A **dot plot** is a special type of column graph used for small sets of data. It displays frequencies using a dot for each piece of data. It is especially useful when the scores are a large number of categories or discrete numerical values.

- The horizontal axis must contain *all* categories or discrete numbers in the range, evenly spaced, even if there is no data for some scores.
- There must be a dot for each piece of data, and dots must be evenly spaced upwards.

A dot plot makes it easy to see gaps in the data and where scores are **clustered** (bunched) together. This type of graph also highlights any **outliers**: extreme scores that are much different from the rest.

EXAMPLE 1

The maximum temperature (in °C) in a town for each day in November is as follows.

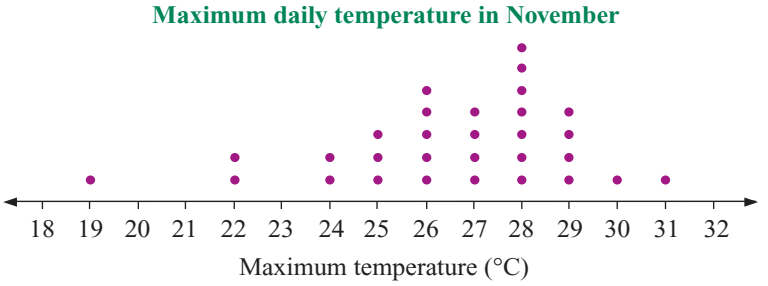
27	28	26	25	26	24	27	29	28	27	27	22	30	28	29
28	26	24	22	28	19	25	26	29	29	26	28	28	31	25

a Draw a dot plot to show this information.

b Decide where the temperatures are clustered.

c Identify any outliers.

a There must be one dot for each recorded temperature.



b The temperatures are clustered between 24°C and 29°C.

c 19°C is an outlier as it is separated from the main group of dots.

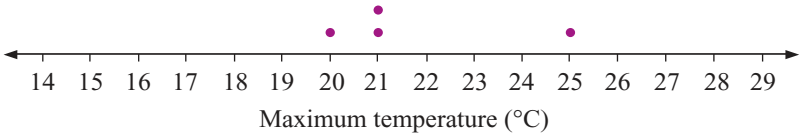
Exercise 11I

1 The maximum temperature (in °C) at a holiday resort for each day in May is as follows.

20	21	25	21	18	19	18	17	23	27	16	18	21	25	28
17	16	17	25	21	15	16	18	19	18	17	21	23	26	18

Complete the dot plot to show this information.

Maximum daily temperature in May



2 Melanie played 22 competition basketball games last year. She threw these numbers of goals:

4 5 1 2 3 0 3 9 4 6 5 4 1 1 4 4 2 5 3 1 1 0

a Draw a dot plot representing this data.

b Find the total number of goals she threw for the year.

c Is there an outlier in this data?

3 This table represents the sales of six different drink types at the school canteen in 1 day.

a Draw a dot plot representing this information.

b Use the dot plot to find the two most popular drink types.

Flavour	Number sold
Orange	7
Lemon	5
Mineral water	2
Cola	10
Apple juice	3
Lemonade	4
Total	**31**

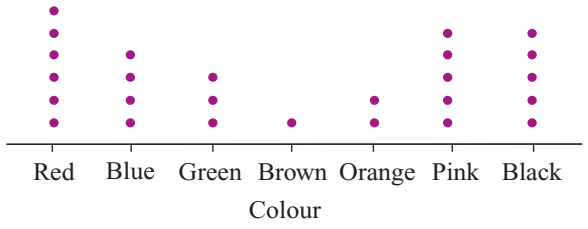
4 Year 7 were asked which colour ice-block was their favourite. The results are listed using the abbreviations R for red, B for blue, G for green, W for white, O for orange and Y for yellow.

RBGOO RROYG WRRBB GBRWO
RWYRR GWROY RORYG GGBBW
OORBO GBBBY

Draw a dot plot showing this information.

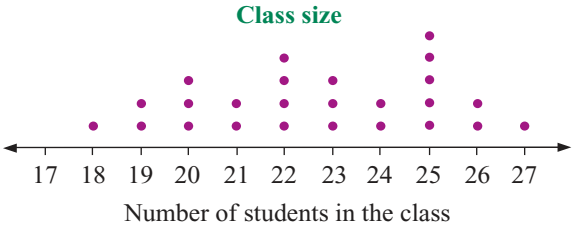
5 Use the dot plot shown to complete the table.

Favourite colours of class 7C



Colour	Frequency
Red	
Blue	
Green	
Brown	
Orange	
Pink	
Black	
Total	

6 Look at the data shown on this dot plot for the sizes of 25 classes at a school.

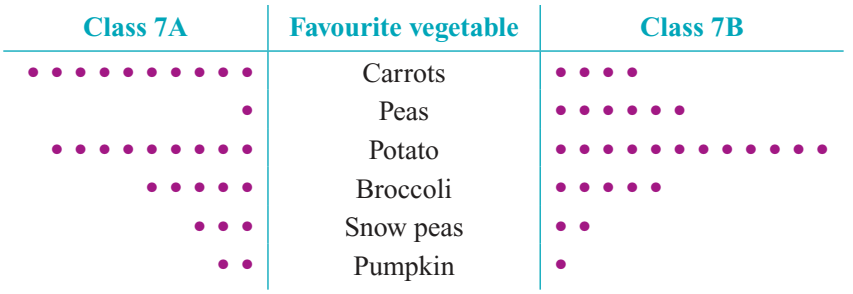


a How many students are in the smallest class?

b What class size is most common in the school?

c How many classes have more than 20 students?

7 Students in class 7A and class 7B were surveyed about their favourite vegetable. The results are shown in the **back-to-back dot plot** below.



a Which is the most popular vegetable among class 7A students?

b Which vegetable is the most popular among classes 7A and 7B combined? Explain.

c Redraw the dot plot combining the information from both classes.

J Stem-and-leaf plots

A **stem-and-leaf plot** is usually used when there is a large number of numerical scores that are spread out. If there are more than 15 different values for scores, a stem-and-leaf plot is useful. Each score is broken into two parts. The **leaf** is the last digit of the score (the units digit) and the **stem** contains all other digits.

The stem-and-leaf plot looks like a histogram or column graph on its side, with the length of the leaf row corresponding to the height of the histogram or column graph's column.

With a stem-and-leaf plot:

- all of the data is used and displayed
- the largest and smallest measurements can be found
- the clustering of data can be easily seen
- the length of the leaf row indicates the number of scores belonging to that stem.

EXAMPLE 1

Complete this table, showing each score broken into a stem and a leaf.

A leaf has only one digit, but a stem may have more than one digit.

Score	Stem	Leaf
28		
153		
91		
8		
	1	9
	2	8
	18	6
	204	9
	0	6

Score	Stem	Leaf
28	2	8
153	15	3
91	9	1
8	0	8
19	1	9
28	2	8
186	18	6
2049	204	9
6	0	6

Exercise 11J

1 Complete this table, showing each score broken into a stem and a leaf.

Score	Stem	Leaf
39		
47		
125		
	8	3
	11	4
	9	3
	0	4
350		
5		
1384		

EXAMPLE 2

The results in a mathematics class test out of 70 are given below. Draw a stem-and-leaf plot of the results.

43	45	46	22	65	65	23	53	45	26	46	61	51	57	55
55	66	57	42	41	63	70	57	65	48	23	67	62	70	46

For this stem-and-leaf plot, the tens digit forms the stem and the units digit forms the leaf.
For the value 43, the stem is the 4 and the leaf is the 3.

Step 1: List the stems in numerical order.

Step 2: Write the leaves as they occur in the data.

Stem	Leaf	
2	2 3 6 3	← This row represents the numbers 22, 23, 26 and 23.
3		← There were no scores in the 30s range of numbers.
4	3 5 6 5 6 2 1 8 6	
5	3 1 7 5 5 7 7	
6	5 5 1 6 3 5 7 2	
7	0 0	

Step 3: Put each row of leaves into ascending numerical order.

In each row, put the smallest leaf closest to the stem.

Stem	Leaf
2	2 3 3 6
3	
4	1 2 3 5 5 6 6 6 8
5	1 3 5 5 7 7 7
6	1 2 3 5 5 5 6 7
7	0 0

2 The first three scores have been placed in the stem-and-leaf plot. Complete the stem-and-leaf plot by adding the remaining 17 scores, then sorting into ascending numerical order.

(34)	(49)	(41)	57	38	59	33	31	61	68
55	39	51	53	63	61	58	33	49	60

Stem	Leaf
3	4
4	9 1
5	
6	

3 Complete the stem-and-leaf plot using these 20 scores.

135	148	157	169	160	111	123	129	138	125
128	155	133	129	158	114	119	133	144	122

Stem	Leaf
11	
12	
13	
14	
15	
16	

4 Draw a stem-and-leaf plot using stems of 3, 4, 5 and 6 for these 20 scores.

39	45	52	68	37	44	59	62	66	40
58	35	49	62	41	59	32	52	47	48

5 Draw a stem-and-leaf plot with stems of 4, 5, 6, 7, 8 and 9 for these 32 scores.

49	68	74	89	49	96	83	98	98	92	48	95	47	93	48	79
78	93	67	96	91	42	66	74	45	71	77	44	65	78	70	41

6 Draw a stem-and-leaf plot with stems of 11, 12, 13 and 14 for these 40 scores.

138	146	126	140	146	134	119	136	142	146	144	132	140	113
124	144	139	117	130	130	142	125	140	133	112	118	146	118
143	136	146	143	146	122	113	131	112	112	124	140		

7 Draw a stem-and-leaf plot for each set of scores.

a

27	49	40	40	13	21	30	30	4	49
38	29	27	38	38	32	47	44	36	45

b

109	145	111	111	130	137	127	112	135	116	127	121	124	106	122
120	136	129	110	141	140	141	133	134	131	141	111	135	138	138

c

216	225	225	203	206	223	221	190	190	193
208	203	193	225	224	208	205	225	194	223

8 **a** Draw a stem-and-leaf plot for the 50 scores below.

44	59	59	107	65	43	85	60	64	86	64	46	62	57	71	43	62
84	84	63	79	70	46	81	104	44	73	105	106	65	97	108	107	45
103	59	40	41	52	101	107	102	105	75	46	105	105	102	70	43	

b Construct a grouped frequency distribution table with classes 40–49, 50–59, etc.

c Draw a frequency distribution histogram and polygon.

d What can be seen on the stem-and-leaf plot that cannot be seen on the histogram or in the frequency distribution table?

9 A **back-to-back stem-and-leaf plot** has been drawn comparing the marks of class 7L and class 7R in a mathematics test.

Class 7L Leaf	Stem	Class 7R Leaf
	0	2
9 9 9 8 8 5 5 5 4 4 2	1	9
9 9 8 7 6 5 5 5 1 0 0 0 0	2	0 0 2 2 3 3 5 6 6 7 9
9	3	0 0 1 2 2 4 4 4 5 8 8 9

In a back-to-back stem-and-leaf plot, the smallest leaves are still closest to the centre.

a In class 7L, what was the:

i highest score? **ii** second-highest score?

b The highest score in class 7L is an outlier.
What is the outlier score in class 7R? Give a reason.

An outlier is a score that is much smaller or larger than the rest of the scores.

c In class 7R the scores are clustered around the stems 2 and 3.
Describe the cluster of the scores in class 7L.

Cluster is a statistical term for grouped.

d Do you think the classes are of equal ability? Explain.

10 A back-to-back stem-and-leaf plot has been drawn comparing the marks of class 7L and class 7R in an English test.

Class 7L Leaf	Stem	Class 7R Leaf
3	2	0 7 9 9
9 8 5 4 4 4 4 3 2 1 1 1 1 0 0	3	0 1 1 3 5 7
9 8 5 4 4 4 4 3 2 1 1 1 1 0 0	4	2 4 4 4 8 8 9
4 3 3 3 2 2 1 1 1	5	2 4 4 4 8 8 9
	6	2 3 4

a Describe the test scores in class 7L, using the terms 'cluster' and 'outlier'.

b Are the scores in class 7R clustered? Explain.

c In class 7R, the lowest score is 20. Is it an outlier? Explain.

d Compare the results of the two classes.

Investigation 2 What's the origin of my classmates?

1. Survey the birth country of students in your class.
2. Present this information as a visual display (table or graph).
3. Use the Internet to find similar data about students living in China or the Philippines.
4. Compare the information. What differences would be noted? Why? Discuss as a class.
5. Write a paragraph to explain reasons for these differences.

K Interpreting graphs

Graphs enable data to be interpreted easily. Some graphs are better suited to particular data. For example, sector graphs and divided bar graphs are better suited to categorical data rather than numerical data. The following exercise allows you to interpret a variety of graphs.

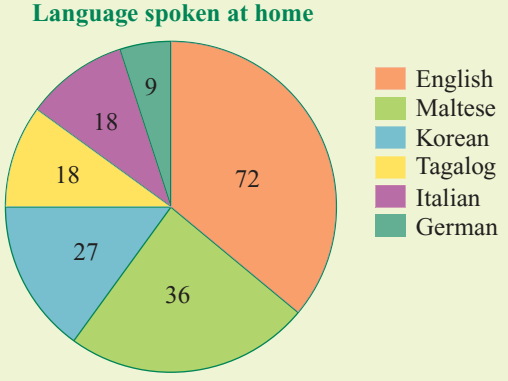
Sector graph

Sector graphs show comparisons, with the largest sector representing the largest amount and the smallest sector showing the smallest amount. Often the percentage amount of each category is shown in the graph.

EXAMPLE 1

This sector graph represents the native languages spoken at home by the parents of 180 Year 7 students. Answer these questions using the information in the graph.

a. How many different native languages are spoken?
b. What is the most common language spoken?
c. Which two languages were equal in use?
d. What fraction of parents spoke English as their native language?
e. What is the angle at the centre for the Maltese sector?



a By counting the sectors there are six different languages spoken.

b English, as it has the biggest sector.

c Tagalog and Italian have equal-sized sectors.

d 72 families spoke English as their native language. $\therefore$ Fraction $= \frac{72}{180} = \frac{2}{5}$

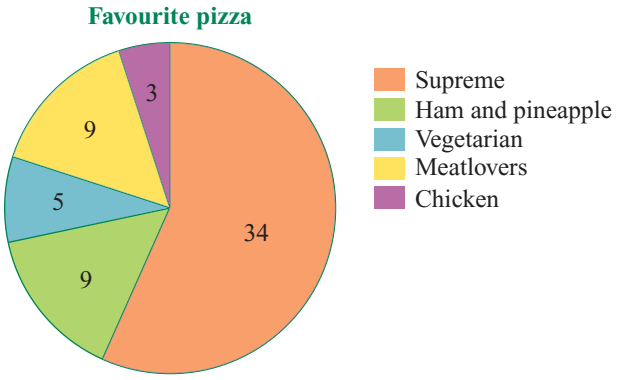
e 36 families spoke Maltese. $\therefore$ Angle $= \frac{36}{180} \times 360° = 36 \times 2 = 72°$

The angle at the centre will be 72°.

A circle contains 360°.

Exercise 11K

1 The sector graph shows the types of pizza ordered by Year 7 students while on camp.



a How many different types of pizza were ordered?

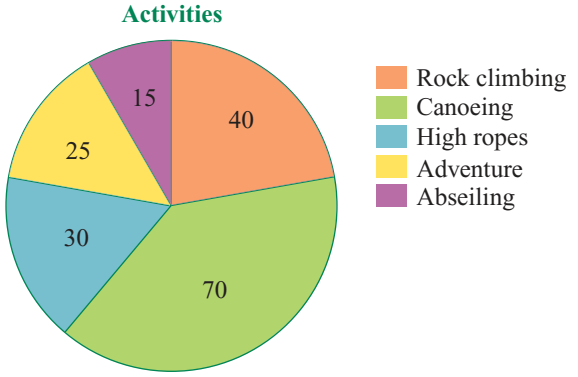
b What was the most popular type of pizza ordered?

c Which two varieties were equally popular? Explain.

d What fraction of the total number of pizzas was vegetarian?

e Calculate the angle at the centre for chicken pizza.

2 The sector graph shows the different activities chosen by Year 7 while at camp.



a Which activity was the least popular?

b Were any two activities equally popular? Explain.

c What fraction of students chose the high ropes course?

d Calculate the angle at the centre of the sector for rock climbing.

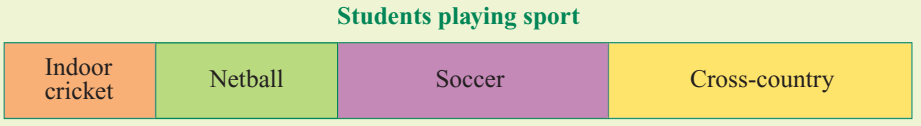
e What fraction of students on the camp did not do the adventure course?

Divided bar graph

A divided bar graph can be used to show information in another way. The data can easily be obtained by measurement and calculation.

EXAMPLE 2

This divided bar graph represents the number of students playing a sport for their school.



a Which sport had the least number of participants?

b What fraction of these students participated in cross-country?

c What percentage of these students participated in netball? Answer to the nearest per cent.

d If there were 180 participants in total, how many were cross-country runners?

a Indoor cricket, as it is the smallest part of the graph.

b Cross-country measures 40 mm. The total length is 120 mm.

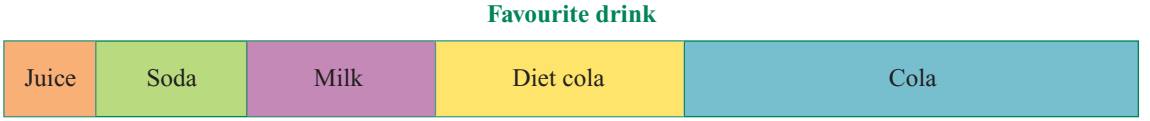
$$\text{Cross-country} = \frac{40}{120} = \frac{1}{3}$$

c Netball measures 24 mm. The total length is 120 mm.

$$\text{Netball} = \frac{24}{120} \times \frac{100}{1} = 20\%$$

d $\text{Cross-country} = \frac{1}{3} \times 180 = 60$ participants.

4 The divided bar graph shows information about the favourite drink of 150 Year 7 students.



a Which is the most popular drink? Explain your answer.

b Which is the least popular drink? Explain your answer.

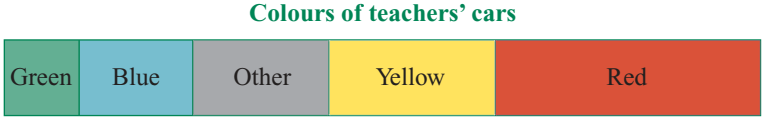
c By measuring the length of the milk section, find the fraction of students who preferred milk.

d How many students preferred milk?

e What percentage of Year 7 students drink cola?

f How many students drink cola?

5 The divided bar graph shows the colours of teachers' cars in the school car park.



a Is red the most popular colour for a teacher's car? Explain your answer.

b What fraction of teachers' cars are red?

c What fraction of teachers' cars are green?

d If there are four green cars in the car park, how many cars are there in the car park?

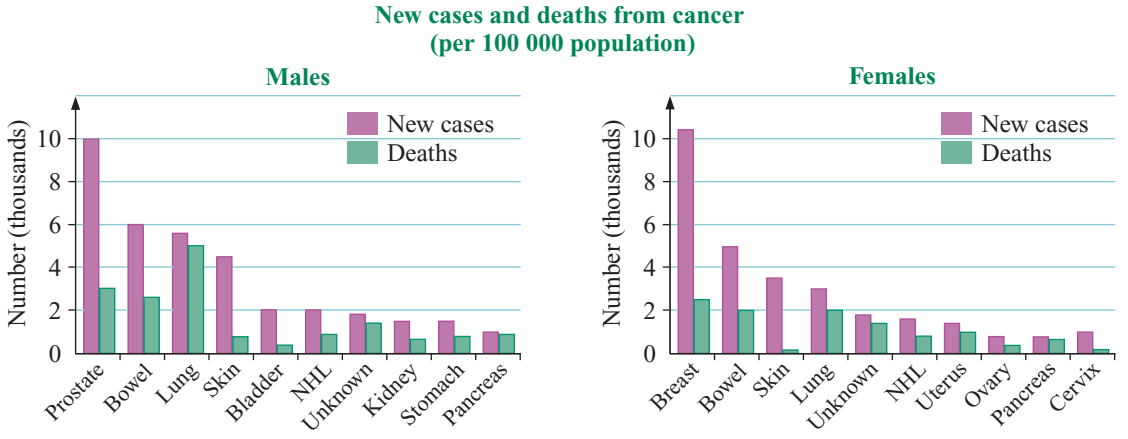
e How many cars are red?

f What percentage of teachers' cars are blue?

g How many cars are blue?

Bar or column graphs

Bar or column graphs can be very useful, especially when making comparisons. Be sure to read the values on each axis carefully.



EXAMPLE 3

Look at the graphs of deaths by cancers per 100 000 people.

The vertical axis is in thousands so 2 really means 2000 and 10 really means 10 000.

a What are the graphs describing?
b Which cancer has the highest number of new cases for males
c Which cancer has the highest number of new cases for females?
d Which cancer has the highest number of deaths among males?
e Which cancer has the highest number of deaths among females?

a The graphs show the types of cancers causing death.
b Prostrate cancer has the highest number of new cases for males.
c Breast cancer has the highest number of new cases for females.
d Lung cancer has the highest number of deaths among males.
e Breast cancer has the highest number of deaths among females.

6 Look at the graphs of deaths by cancers per 100 000 people.

a Lung cancer is described as a deadly cancer. How do the graphs show this?
b Which other cancers are particularly deadly?
c Which cancer has the highest survival rate for:
  i males? **ii** females?
d How many more male patients contracted prostrate cancer than bowel cancer?
e Find out what NHL cancer stands for. What percentage survival rate could be assumed for:
  i males? **ii** females?
f Why do you think there are more cases of skin cancer for males than females? Approximately how many cases are there for:
  i males? **ii** females?

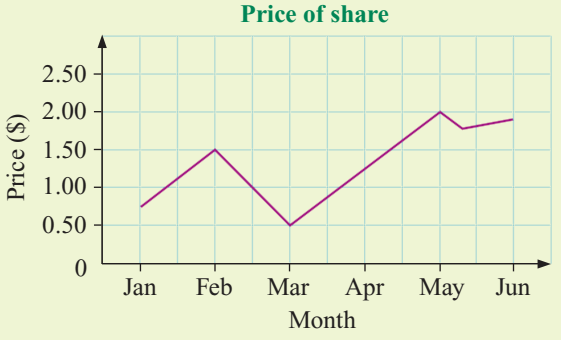
Line graph

A line graph is usually used to show how data has changed over a period of time. It is a useful way for showing upwards and downwards trends.

EXAMPLE 4

This line graph shows the price of a share over a number of months.

a What was the maximum share price?
b Which month shows the lowest share price?
c How many months are represented by this graph?
d What was the share price in February?



a Maximum share price is $2.00. It is the highest point on the graph.
b The lowest share price is in March.
c There are six months shown.
d The share price in February was $1.50.

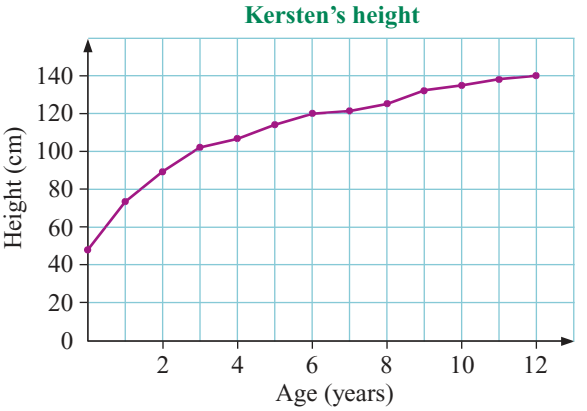
7 Kersten's height was measured each year on her birthday. The line graph shows this information.

a What was Kersten's height on her 10th birthday?

b When was Kersten about 120 cm tall?

c Between which two birthdays did she grow the most?

d At what age was Kersten double her birth height?



8 Racine was admitted to hospital at 6 am and her temperature was measured every 2 hours. The line graph shows this information.

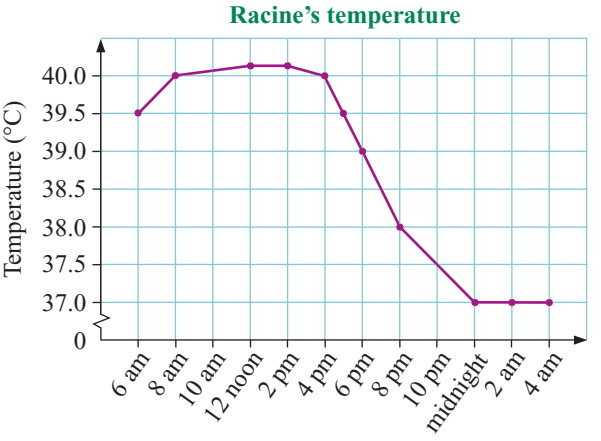
a When was Racine's temperature 39°C?

b At what times was Racine's temperature 40°C?

c What was Racine's temperature at 5 pm?

d What was Racine's maximum temperature, and when was this?

e Estimate Racine's temperature at 4 am.



L Selecting methods of displaying data

When we choose a method to display data, there are a number of things to consider. Some types of displays are better than others in certain situations. This section considers different types of displays.

Exercise 11L

1 **a** The results of a survey of the heights of 14-year-old boys are shown. Complete the table.

Height (cm)	Tally	Frequency
Less than 140	𝍸	
140–149	𝍸 𝍸 |	
150–159	𝍸 𝍸 𝍸 𝍸 |||	
160–169	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	
170–179	𝍸 𝍸 𝍸 𝍸 𝍸 ||	
180–189	𝍸 𝍸 𝍸 ||	
190 and over	||	
	Total	

b Draw a frequency histogram representing this information.

c Draw a frequency polygon representing this information.

d Draw a picture graph representing this information.

e Which type(s) of graph(s) best show the information? Why?

2 Twenty households in a street were surveyed to find the number of people in each household. The results were:

4 2 3 5 4 4 2 3 1 5
4 4 3 7 2 3 4 4 2 4

a Draw a stem-and-leaf plot displaying this data.
b Draw a dot plot displaying this information.
c What was the greatest number of people in one household?
d What was the most common number of people in a household?
e Which display was easiest to use? Explain.
f Write a question that uses one of the displays to get the answer 1.

3 The local under-14 soccer team has listed the number of goals scored in each of their 15 games.

0 1 2 2 0 0 1 1 0 8 0 3 1 1 1

a Display this data in a dot plot.
b Display this data in stem-and-leaf plot.
c Put the information into a frequency distribution table and draw a histogram.
d Are there any outlier scores? Which display(s) show(s) this best? Explain.
e List the advantages of the histogram over the stem-and-leaf plot for this data.
f Write a question for this information that has the answer 3.

4 The marks in the Year 7 final mathematics exam out of 100 were:

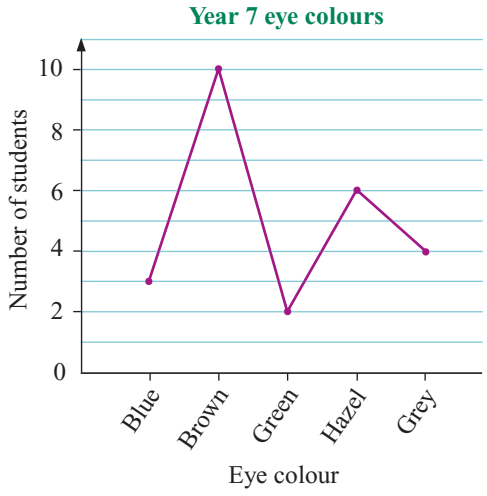
79 70 71 68 74 78 87 90 74 97 69 91
83 66 95 70 72 85 88 85 97 83 91 63

a Display this data in a stem-and-leaf plot.
b Would it be a good idea to draw a dot plot?
c Display this data in a frequency distribution table using the classes 60–69, 70–79, 80–89, 90–99.
d Is it possible to use the frequency distribution table to find whether any student scored 75? Explain.
e Give a reason why this data should be displayed using a stem-and-leaf plot.
f Write a question for this data that has the answer 85.

5 This table shows the eye colour of 25 Year 7 students.

Eye colour	Number of students
Blue	3
Brown	10
Green	2
Hazel	6
Grey	4

Carmine drew a line graph showing this information.



a Does it make sense to join each point on the graph? Explain.
b Display this information in a dot plot.
c What type of data should *not* be displayed using a line graph?

6 List the advantages and disadvantages of:

a stem-and-leaf plots
b dot plots
c frequency distribution tables
d frequency distribution tables using classes
e line graphs

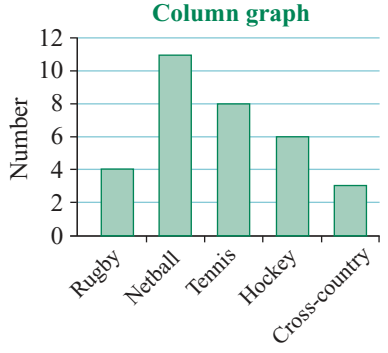
Investigation 3 Spreadsheet exercise

Here is some data to use with a spreadsheet package.

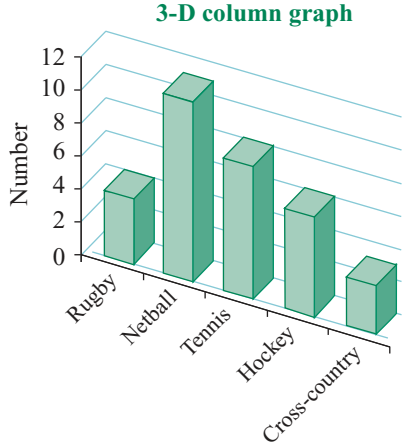
Favourite sport	Number
Rugby	4
Netball	11
Tennis	8
Hockey	6
Cross-country	3

Enter the information into a spreadsheet, then use the graphing function to show different types of graphs. Some examples are shown. Other graphs and different versions of these graphs are available through the spreadsheet. Experiment with different graphs.

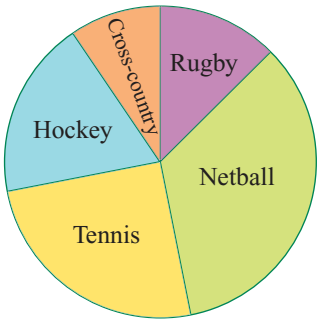
Column graph



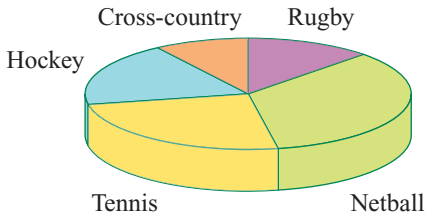
3-D column graph



Pie chart



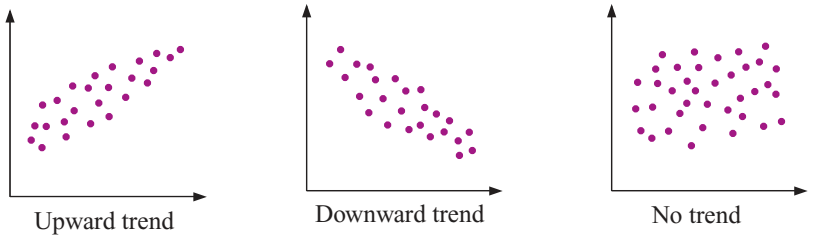
3-D pie chart



1. Collect data from secondary sources that is presented as tables or graphs about a topic of interest. This could be sport, politics, the economy, music, education, health of different countries, food production of countries or another topic of interest to you.
2. Create a series of questions for each secondary source.
3. Swap your table or graph and questions with another classmate.
4. As a group discuss the variety of questions asked and the manner in which the data was displayed. Was it misleading? Was it easy to interpret? What scale was used? Could the data have been presented in a better graphical form?
5. Finally, discuss the ethics of collecting data from secondary sources. Are there any ethical issues that arise from using secondary data? Discuss.

M Scatter graphs

Scatter graphs or **scatter plots** compare two variables. They can be drawn by entering information into a spreadsheet. They are used to find **trends**, if trends exist. These diagrams show the three types of trends.

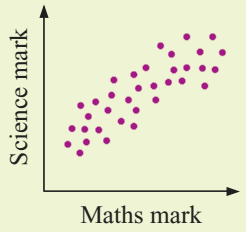


EXAMPLE 1

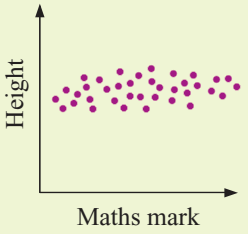
Describe any relationship between the two variables plotted on each graph.

Each point represents both variables.

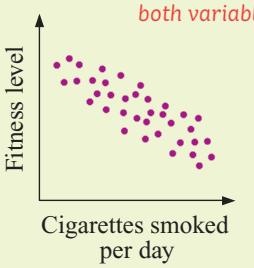
a



b



c



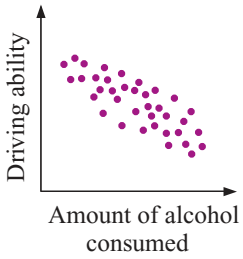
a As Maths marks increase, so do Science marks.

b There seems to be no correlation between Maths mark and height.

c As the number of cigarettes smoked per day increases, the level of fitness decreases.

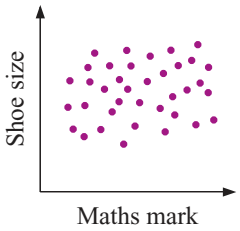
Exercise 11M

1 a



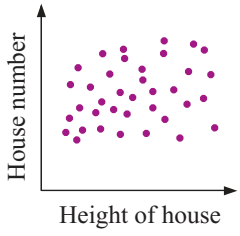
As the amount of alcohol consumed increases, what happens to driving ability?

b



Is there any relationship shown between Maths mark and shoe size?

c



Is there any relationship shown between the height of a house and the house number? Explain.

d

As the hours of training increase, what is the impact on fitness level?

EXAMPLE 2

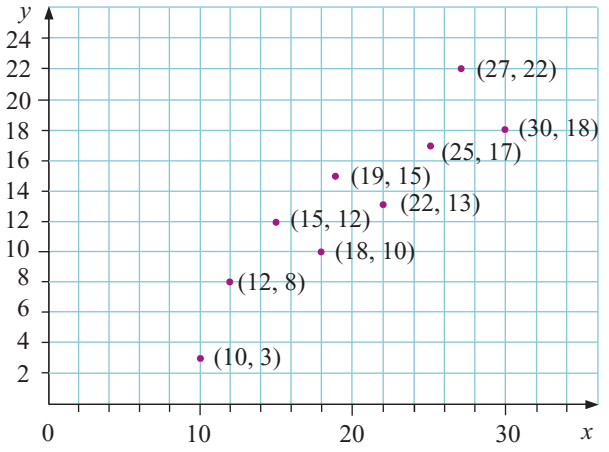
a Draw a scatter graph of this data.

x	10	12	15	18	19	22	25	27	30
y	3	8	12	10	15	13	17	22	18

b Is there an upwards trend, a downwards trend or no trend?

a Plot each pair of values on a number plane.

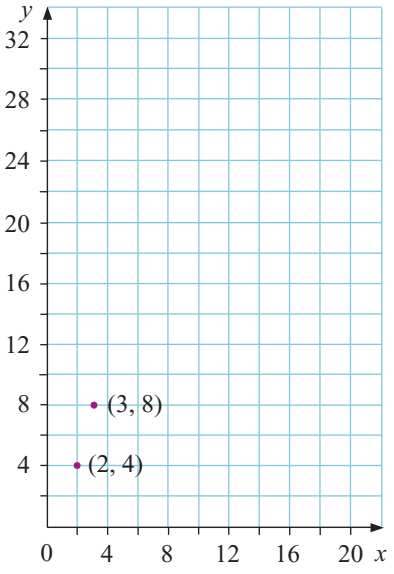
b The points are moving upwards to the right, so the scatter plot shows an upwards trend.



2 **a** Complete the scatter graph shown, using the data given.

x	2	3	5	8	10	15
y	4	8	13	17	19	31

b Complete: The graph shows a(n) _____ trend.



3 Draw a scatter graph for each set of data.

a

x	5	7	10	12	15	18	20
y	23	20	18	15	11	11	7

b

x	5	8	11	13	20	23
y	16	4	15	8	5	18

4 Describe the type of trend, upwards, downwards or no trend, for the scatter graphs in question **3**.

5 The ages and values of a sample of a particular model of car are listed in the table.

Age (years)	1	1	2	2	3	3	4	4	5	5	6
Value ($000s)	24	23	24	22	19	18	20	17	15	14	14

Age (years)	6	6	7	7	8	8	9	10	10	10
Value ($000s)	13	15	12	14	11	10	10	8	7	6

a Plot this data in a scatter graph, with age on the horizontal axis. Give your graph a title.

b Is there a trend? Explain.

6 The marks in Maths and Science for 10 students are shown below.

Maths mark	50	60	65	70	72	74	78	78	80	81
Science mark	42	53	58	64	66	64	71	75	71	75

a Draw a scatter graph displaying this information, with Maths mark on the horizontal axis. Give your graph a title.

b Is there a relationship between results in Maths and Science for these 10 students? Explain.

Misleading graphs

Misleading graphs give a false impression of the data, either by mistake or deliberately. The main causes of graphs being misleading are:

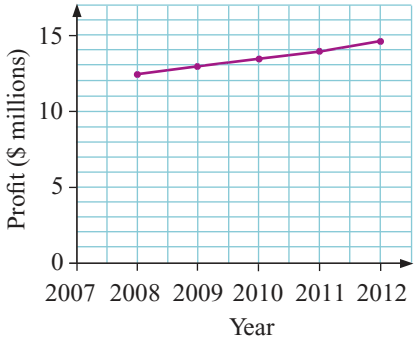
- The scale on the vertical axis does not start at zero.
- The scale on the vertical axis is irregular.
- The scale on the vertical axis is missing.
- The use of area or volume creates a false impression.

EXAMPLE 1

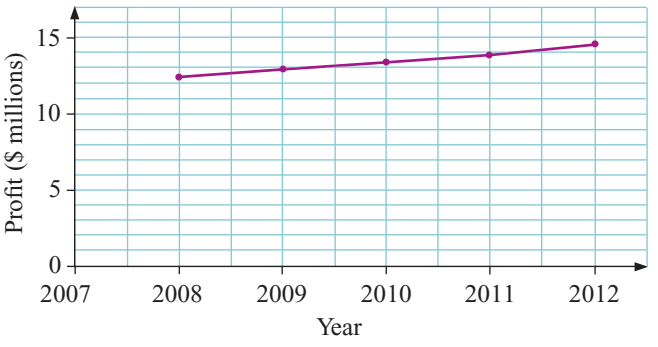
The table shows the profits of a company over the 5-year period from 2008 to 2012. Graphs A to E, shown below and at the top of the next page, are five ways of presenting this information graphically. What features, if any, are misleading?

Year	2008	2009	2010	2011	2012
Profit ($ millions)	12.3	12.9	13.2	13.8	14.6

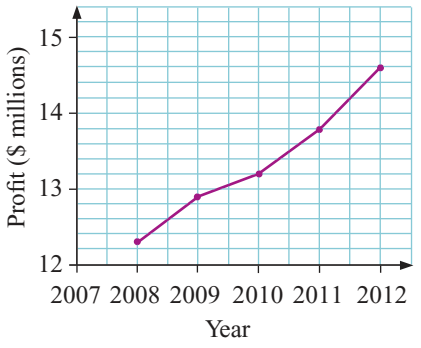
Graph A



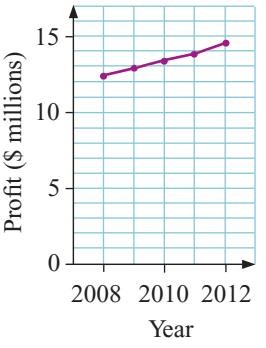
Graph B



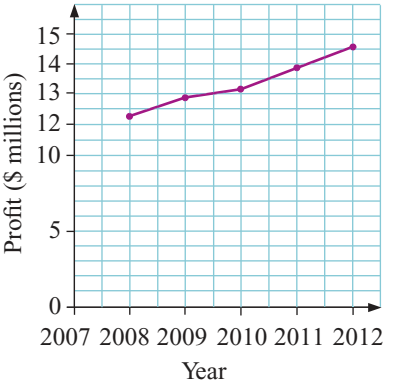
Graph C



Graph D



Graph E



Graphs A, B and D are fair, although each gives a different impression:

- Graph B makes the increase in profit look smaller as the horizontal scale is enlarged.
- Graph D makes the increase in profit look larger as the horizontal scale is compressed.

Graphs C and E are misleading:

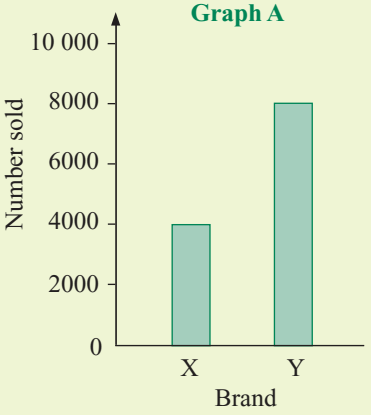
- Graph C has exaggerated the increase in profit by not starting the vertical scale at 0, thus enlarging this scale.
- Graph E has an irregular vertical scale, so the graph is incorrect.

EXAMPLE 2

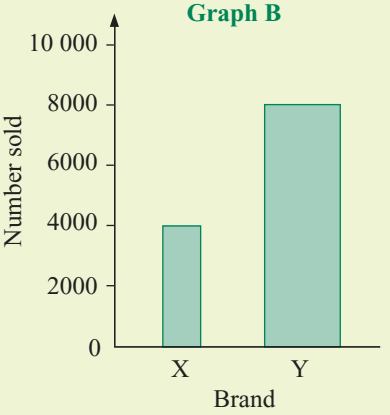
Here are three ways of presenting the information in the table graphically. Comment on any misleading features of the graphs.

Brand	Number of TVs sold
X	4000
Y	8000

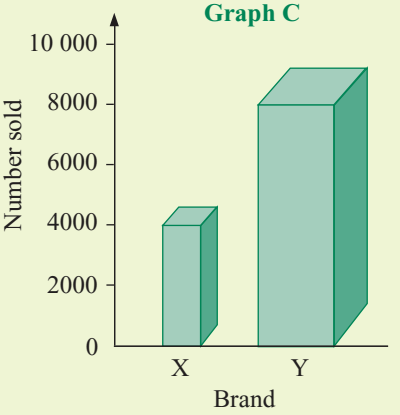
Graph A



Graph B



Graph C



Graph A: The columns correctly show that sales of brand X are double those of brand Y (as in the table). The widths of the columns are the same, and the height of the Y column is twice that of the X column.

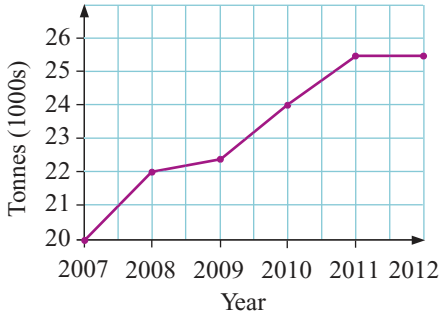
Graph B: Both dimensions of the brand X column have been doubled for brand Y, creating the impression that sales of brand Y are $(2 \times 2 =)$ 4 times those of brand X.

Graph C: All three dimensions of the brand X column have been doubled for brand Y, creating the impression that sales of brand Y are $(2 \times 2 \times 2 =)$ 8 times those of brand X.

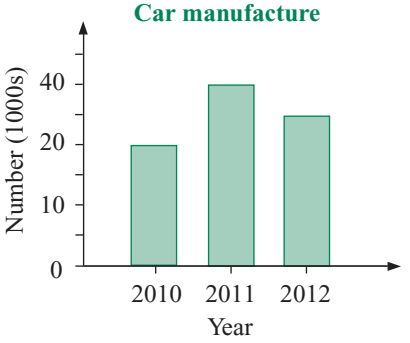
Exercise 11N

1 Describe the misleading or poor features of the following graphs.

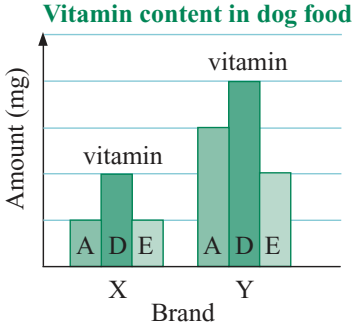
a



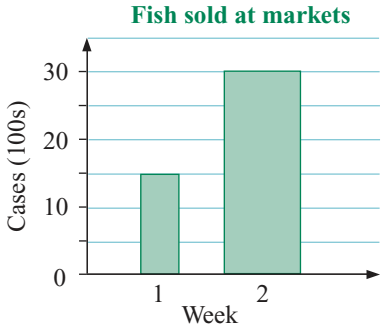
b



c

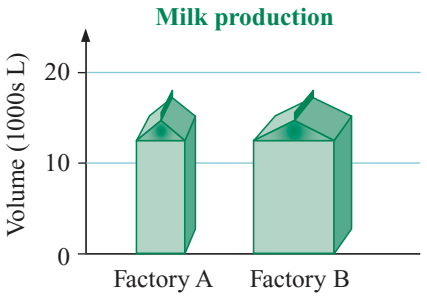


d



e

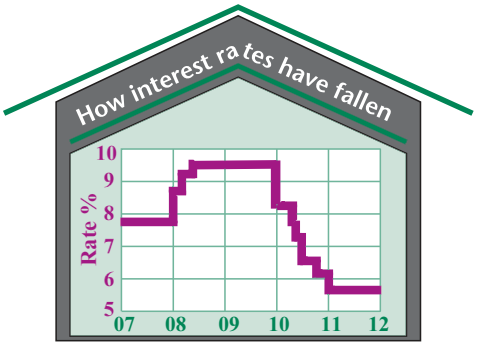
f



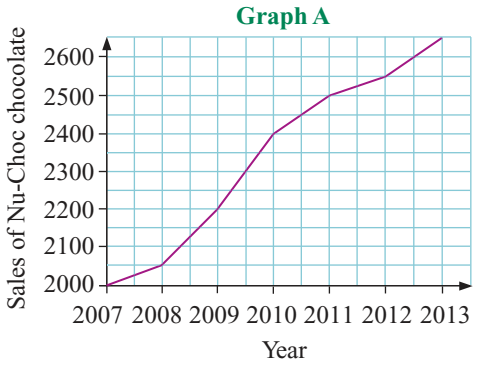
g

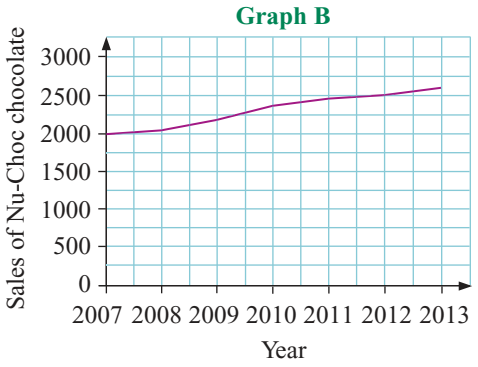
2 **a** Explain how this graph is misleading.

b Redraw the step graph without the bias.



3 **a** Which graph gives the impression of rapidly increasing sales?





b Have sales in fact rapidly increased over this 6-year period?

c In graph A, the sales for 2010 appear to be double those of 2009. Is this true?

Investigation 4 Relationship between the wealth, education and health of a population

Global awareness

1 Visit the following website and download the pdf.

http://www.nwabr.org/education/pdfs/HIV_CURRICULUM_08/Lesson_Five_0108.pdf

- The above information highlights the impact that wealth and education have on the health of a nation, in particular HIV.
- The spread of HIV is epidemic in some parts of the world.
- Create a PowerPoint highlighting how limited access to wealth and education impact upon an individual's quality of life and health.

2 Visit the following websites:

http://www.globalissues.org/article/4/poverty-around-the-world
http://huebler.blogspot.com/2008/08/hh-wealth.html
http://en.wikipedia.org/wiki/IQ_and_the_Wealth_of_Nations

- What conclusions can be drawn between the relationship between wealth, education and health?
- Present your findings in the form of a written report.

The Australian population

Use the Internet to visit the sites listed below, as well as any others sites you may find, to create a statistical presentation regarding the relationship between wealth, education and health within Australian communities.

- Does a relationship exist?
- If so, what are the implications for each community?
- What responsibility do other members of the community have to assist in addressing this issue?
- What changes can be made? How?
- Present your findings as a written report or PowerPoint.
- Ensure that you include statistical data and graphs to display your findings.

Internet sites

- Income and Wealth Distribution in Australia:
  http://www.abs.gov.au/AUSSTATS/abs@.nsf/lookup/4704.0Chapter365Oct+2010
- Poverty and wealth in extremes: identifying global patterns, A ...
  www.skwirk.com.au/p-c_s-16_u-185_t-495_c-1821/NSW/7/Poverty-and-wealth-in-extremes...
- Statistics: Definition from Answers.com
  www.answers.com/topic/statistics
- Health, income and inequality:
  www.nber.org/reporter/spring03/health.html
- Magnitude of economic inequality, causes of inequality, mitigating factors:
  http://en.wikipedia.org/wiki/Economic_inequality
- Longitudinal study of ageing reveals health and wealth relationship:
  www.ucl.ac.uk/media/library/elsareporttwo

Aboriginal/Indigenous communities

As a group, use the Internet sites listed below, and others that you find, to research the link between wealth, education and health in Aboriginal and Indigenous communities of Australia.

- Present your findings as a PowerPoint.
- Ensure that statistics are used to substantiate your findings.
- Highlight incentives and programs that have been implemented to combat the disparities.
- Comment on their effect.
- List possible solutions that could be implemented in the future to address this community concern?

Remember, you will need to complete additional research.

Internet sites

- Aboriginal health information:
  www.ahmrc.org.au/AboriginalHealthInformation.htm
- Summary of selected social indicators:
  www.healthinfonet.ecu.edu.au/determinants/social

Language in mathematics

1 The last two letters of some words from the Terms list below are given. What are the terms?

a _____am	**b** _____on	**c** _____ot	**d** _____le
e _____er	**f** _____ph	**g** _____ta	**h** _____cy
i _____ve	**j** _____or	**k** _____te	**l** _____us

2 Complete the following passage by filling in the blanks with words or numbers from this list:

categorical continuous numerical outlier scores small together

a A distribution has clustered _____ when they are grouped _____.

b An _____ is a score that is very large or _____ when compared to the other scores.

c Quantitative data is also called _____ data.

d The heights of students are _____ numerical data.

e Line graphs should not be drawn for _____ data.

Terms

back-to-back	bar graph	biased	categorical	census
class	cluster	column graph	continuous	data
discrete	divided bar graph	dot plot	fair	frequency
frequency distribution table		frequency polygon	grouped data	histogram
line graph	misleading	nominal	numerical	outlier
pictograph	picture graph	population	primary data	quantitative
random	sample survey	secondary data	scale	scatter graph
score	sector graph	statistics	stem-and-leaf plot	tally marks
trend	variable			

Check your skills

1 An example of secondary data is:

A a survey **B** a questionnaire **C** a journal article **D** an interview

2 Interviewing 20 students from a school of 300 is:

A a sample survey **B** a census **C** biased **D** random

3 Which of the following is an example of data that could *not* be collected by a census?

A the pets owned by a class of students **B** the causes of car accidents in Australia

C the time a candle takes to burn completely **D** the sports played by all students in a school

4 What type of survey question is the following?

Do you like pizza?

☐ *yes* ☐ *no* ☐ *it depends on the pizza*

A yes or no **B** free-response **C** tick-box **D** scaled-response

5 Country of origin is a:

A categorical variable **B** discrete numerical variable

C continuous numerical variable **D** numerical variable

Organise this data for the number of children in a family into a frequency distribution table, then use it to answer questions **6** and **7**.

2	1	3	2	3	0	4	6	2	1	3	2	4	0	2	3	2	5	0	2
0	3	3	1	0	3	1	4	2	1	2	4	3	3	0	4	5	2	2	4

6 The number of families with two children is:

A 9 **B** 10 **C** 11 **D** 12

7 The two scores with the same frequency are:

A 2 and 3 **B** 0 and 3 **C** 0 and 4 **D** 2 and 4

8 A possible score in the 30–35 class is:

A 4 **B** 25 **C** 30 **D** 39

9 Which two graphs can be used to represent parts of a whole?

A column graph and bar graph
B sector graph and divided bar graph
C sector graph and bar graph
D column graph and divided bar graph

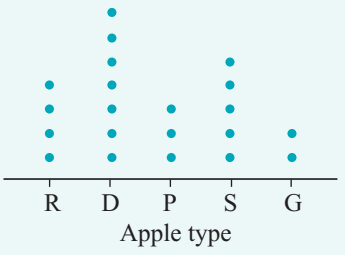
10 Which looks the most like a frequency histogram?

A divided bar graph
B column graph
C sector graph
D line graph

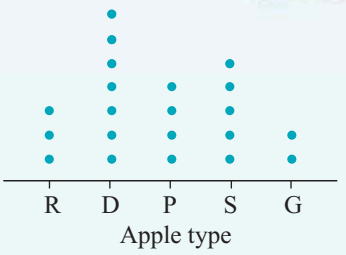
11 Which is the correct dot plot for the data for favourite apple type?

Apple type	Number
Royal Gala (R)	3
Delicious (D)	7
Pink Lady (P)	4
Granny Smith (S)	5
Golden Delicious (G)	2

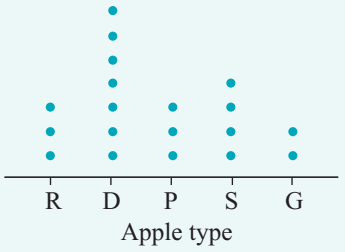
A



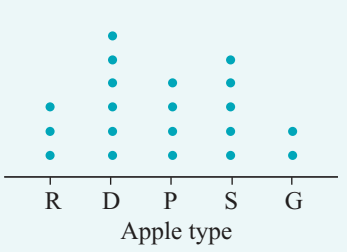
B



C



D



Use this stem-and-leaf plot to answer questions **12** and **13**.

Stem	Leaf
2	3 3 4 5 6 6 9
3	0 1 2 2 2 2 3 4 5 8 8
4	0 0 1 1 1 2
5	9

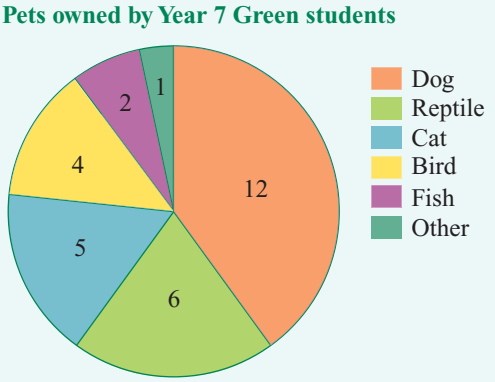
12 The highest and lowest scores are:

A 0 and 9 **B** 2 and 5 **C** 23 and 59 **D** 9 and 3

13 The outlier score is:

A 32 **B** 59 **C** 9 **D** there is none

Use the graph of number of pets owned by students in Year 7 Green to answer questions **14** and **15**.



14 What was the second most popular pet?

A Dog **B** Cat
C Reptile **D** Bird

15 What fraction of 7 Green's pets were birds?

A $\frac{4}{30}$ **B** $\frac{6}{30}$
C $\frac{5}{30}$ **D** $\frac{12}{30}$

16 The display that does not allow easy comparison of actual scores is:

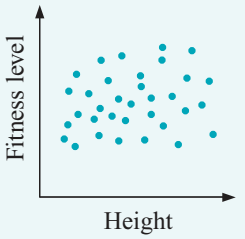
A dot plot **B** stem-and-leaf plot
C frequency distribution table **D** frequency distribution table with classes

17 Which display cannot be used with categorical data?

A dot plot **B** stem-and-leaf plot
C frequency distribution table **D** column graph

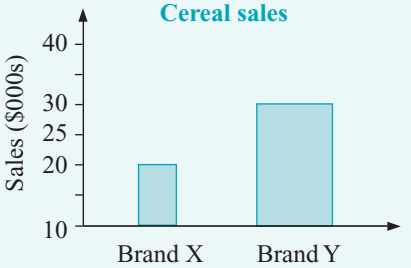
18 This scatter graph shows:

A no trend **B** an upwards trend
C a downwards trend **D** cannot tell



19 This graph is misleading because:

A the columns are not the same width
B the vertical scale is uneven
C the vertical scale doesn't start at zero
D both A and C



If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2, 3	4	5	6, 7	8	9	10	11	12, 13	14, 15	16, 17	18	19
Section	A	B	C	D	E	F	G	H	I	J	K	L	M	N

11A Review set

1 Define the statistical term 'census'.

2 Would a census or a sample survey be used to investigate the number of people who use a particular brand of deodorant? Why?

3 Describe the sample you would use if you wanted to gather support for improved netball facilities at your local park.

4 For a survey on the quality of children's television programs, write:

a a 'yes or no' question

b a question requiring a response on a scale of 1 to 5

5 Describe each of these variables as nominal, discrete numerical or continuous numerical.

a eye colour **b** monthly sales **c** temperature

6 **a** Construct a frequency distribution table for this data showing the winning margins (in goals) in a series of soccer matches.

2	1	3	0	1	2	0	4	2	1	3	5	0	4	3	3	4	1	2	2
2	0	3	1	0	0	2	1	3	4	5	3	2	1	0	2	5	0	4	3
2	2	3	0	1	3	4	0	1	0	2	0	1	2	3	2	1	4	0	2

b Draw a frequency histogram and polygon for this data.

c **i** How many soccer matches were played?

ii How many winning margins of 2 were there?

iii What does a winning margin of 0 mean?

7 **a** Construct a frequency distribution table to show the following scores.

8	9	5	10	7	6	6	7	5	1	8	6	2	8	6
2	6	4	9	4	7	4	9	2	6	5	9	10	10	1

b How many scores were 8 or more?

8 Draw a stem-and-leaf plot for the following data. Use stems of 2, 3, 4, 5, 6.

46	45	28	38	61	50	49	42	32	42
50	45	51	31	48	45	32	45	44	53

9 **a** Draw a column graph for the data in this table.

b Draw a dot plot showing this data.

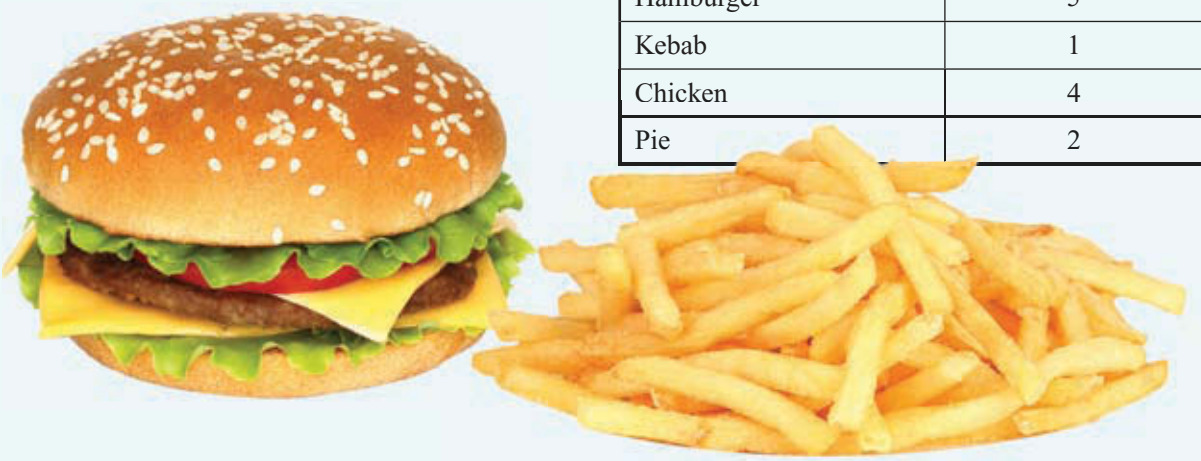
Drink choice	Frequency
Soft drink	14
Still water	10
Juice	6
Tea/coffee	8
Other	3

10 The table shows the favourite winter activity of 36 Year 7 students. Draw a sector graph to represent the results of the survey.

Favourite winter activity	Number of students
Skiing	8
Ice skating	14
Soccer	10
Netball	4

11 The table shows the favourite snacks of 30 Year 7 students. Draw a divided bar graph 15 cm long to represent the results of the survey.

Favourite snack	Number of students
Hot chips	18
Hamburger	5
Kebab	1
Chicken	4
Pie	2



12 The price of a share over a fortnight varied as shown. Draw a line graph to show this variation in price.

Day	M	T	W	T	F	M	T	W	T	F
Price ($)	8.25	8.27	8.30	8.00	8.06	8.06	8.14	7.99	8.08	8.04

11B Review set

1 Define categorical data.

2 State the advantages and disadvantages of using a census to collect information.

3 Explain the term 'statistical bias'.

4 For a survey about mobile phone use among Year 7 students, write:
a a free-response question **b** a tick-box question

5 Describe each of these variables as nominal, discrete numerical or continuous numerical.
a length of time **b** country of origin **c** number of words on a page

6 The Mathematics results for five students are given in the table. Draw a column graph representing this data.

Student	Mark
Renee	75
Jake	65
Emily	89
Harry	94
Yousef	62

7 **a** Complete the grouped frequency distribution table below for this data.

15	19	22	22	20	24	15	21	26	21	25	17	27	25	22
21	16	27	24	20	17	20	28	23	24	27	19	21	16	11

Score	Tally	Frequency
10–14		
15–19		
20–24		
25–29		
	Total	

b How many scores were 24 or less?

8 **a** Draw a stem-and-leaf plot for the information in question **7**.

b Does the frequency distribution table or the stem-and-leaf plot show the information more clearly?

9 From the frequency distribution table in question **3**, draw:

a a sector graph **b** a frequency histogram **c** a frequency polygon

10 **a** The table shows the temperatures in a playground. use the information to draw a line graph.

b Use the graph to estimate the temperature at 9:30 am.

c When was the temperature 14°C?

Time	Temperature (°C)
8:00 am	68
10:00 am	10
12:00 noon	12
2:00 pm	15
4:00 pm	13

11 A survey was conducted on the number of matches contained in a box. The results obtained from the first 60 boxes were as follows.

47	52	49	51	50	47	50	49	48	51	49	50	48	49	50
52	20	52	48	50	51	49	50	51	25	52	50	50	48	50
51	50	49	51	49	48	49	50	48	50	51	47	50	48	50
51	48	49	50	48	50	50	51	52	50	50	51	50	49	50

a Draw a stem-and-leaf plot using the stems 2, 3, 4, 5.

b Are there any clusters? If so, where are they located?

c Are there any outlier scores? Explain how these scores may have occurred.

11C Review set

1 Describe each of these variables as nominal, discrete numerical or continuous numerical.

a hair colour **b** weight **c** number of peas in a pod

2 **a** Construct a frequency distribution table for these marks out of 10 for a mental arithmetic test.

9	10	6	0	8	7	7	8	6	2	9	7	0	7	5	10	5	8	5	10
3	7	6	10	9	3	5	3	9	9	8	8	5	4	6	5	6	10	9	7

b Draw a frequency histogram and polygon for the data.

c **i** How many students sat for the test? **ii** How many scored full marks?

iii How many scored zero? **iv** How many scored 1 mark?

v If a score of 5 or better was considered a pass, how many students passed?

3 The number of milligrams of cholesterol per 100 g of various foods is shown in the table. Draw a horizontal bar graph to illustrate this data.

Food	Cholesterol (mg/100 g)
Lobster	70
Beef	80
Chicken	100
Duck	105
Prawns	145

4 The months of birth of the students in a Year 7 class were recorded as follows.

Jan	Mar	Dec	May	Jun	Jul	Jan	Mar	May	Aug	Oct	Apr	Jan	Jul	Sep
Aug	Jul	Oct	Sep	Dec	Jul	May	Nov	Mar	Jul	Oct	May	Jul	Jul	Sep

a Use a tally column to put this data into a frequency distribution table.
b How many students are in the class?
c What was the most common month for birthdays?
d Which month had no birthdays?
e How many students were born in winter?
f Organise this data into a dot plot.

5 The numbers of jellybeans in 40 packets were counted, and the data appears below.

28	32	29	32	33	29	31	32	27	28	27	30	26	31
27	28	29	31	32	28	31	30	29	30	27	32	29	18
31	29	28	29	27	31	28	32	33	32	31	27		

a Draw a stem-and-leaf plot for this information, using the stems 1, 2, 3.
b What comment can you make about the packet containing 18 jellybeans?

6 The table shows the percentage of the audience watching each of the major TV channels, for Sydney and nationally. Draw a sector graph to illustrate the results:
a for Sydney **b** nationally

Channel	TV audience (%)	
	Sydney	Nationally
9	36.0	33.6
7	26.4	29.1
10	21.3	20.4
ABC	13.2	13.8
SBS	3.1	3.2

12

Area, surface area and volume

This chapter deals with area, surface area and volume.

At the end of this chapter you should be able to:

- calculate the areas of squares, rectangles, triangles and parallelograms
- understand units of area
- calculate the surface areas of rectangular and triangular prisms
- calculate the volumes of cubes and rectangular and triangular prisms
- solve questions relating to area, surface area and volume
- understand units of volume and capacity.

MEASUREMENT & GEOMETRY – ACMMG 159, 160

Diagnostic test

1 What is 1.35 m in centimetres?

A 135 cm **B** 13.5 cm
C 0.135 cm **D** 0.0135 cm

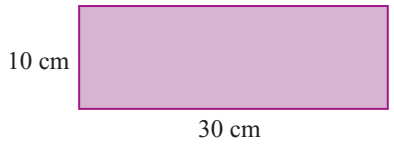
2 What is 3720 m in km?

A 0.3720 km **B** 372 km
C 37.20 km **D** 3.720 km

3 What is 50 mm + 40 cm + 1.3 m?

A 46.3 m **B** 1.75 m
C 46.3 mm **D** 1.75 mm

4 What is the perimeter of this rectangle?

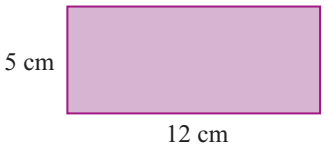


A 40 cm **B** 60 cm
C 80 cm **D** 300 cm

5 How many m^2 in 1 hectare?

A 100 000 m^2 **B** 10 000 m^2
C 1000 m^2 **D** 100 m^2

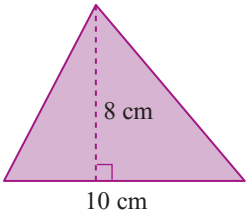
6 What is the area of this rectangle?



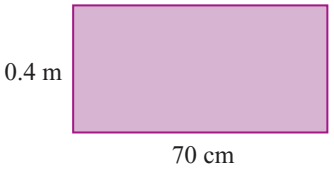
A 34 cm^2 **B** 17 cm^2
C 120 cm^2 **D** 60 cm^2

7 What is the area of this triangle?

A 80 cm^2
B 60 cm^2
C 40 cm^2
D 18 cm^2

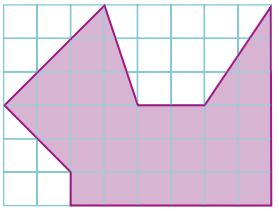


8 What is the area of this rectangle?



A 28 m^2 **B** 2.8 m^2
C 0.28 m^2 **D** 0.028 m^2

9 Estimate the area of the shaded shape, to the nearest square unit.



A 29 units^2 **B** 23 units^2
C 35 units^2 **D** 25 units^2

10 The area of a rectangle is 84 cm^2. Calculate its length if the breadth is 7 cm.

A 8 cm **B** 14 cm
C 7 cm **D** 12 cm

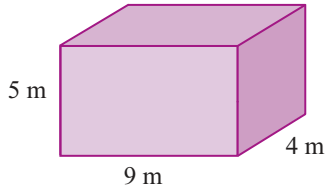
11 What is 425 mL in litres?

A 42.5 L **B** 4.25 L
C 0.425 L **D** 0.0425 L

12 What is 2 L 45 mL in mL?

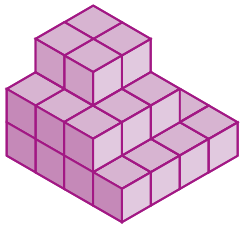
A 245 mL **B** 2450 mL
C 2045 mL **D** 2405 mL

13 Calculate the volume of this rectangular prism.



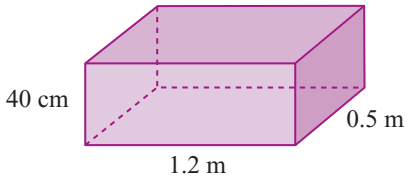
A 18 m^3 **B** 180 m^3
C 72 m^3 **D** 36 m^3

14 If this solid is made from 1 cm^3 cubes, calculate the volume of the solid.



A 18 cm^3 **B** 29 cm^3
C 16 cm^3 **D** 30 cm^3

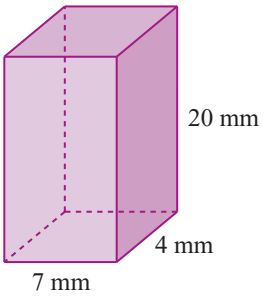
15 Calculate the volume of this rectangular prism.



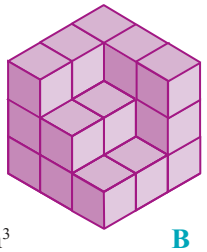
A 240 m^3 **B** 24 m^3
C 2.4 m^3 **D** 0.24 m^3

16 Calculate the volume of this prism.

A 464 mm^3
B 276 mm^3
C 560 mm^3
D 385 mm^3

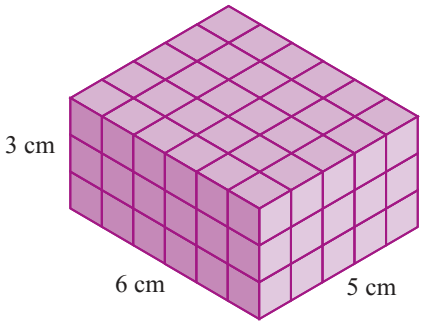


17 The diagram shows a solid made up of 1 cm^3 cubes. What is the volume of this solid?



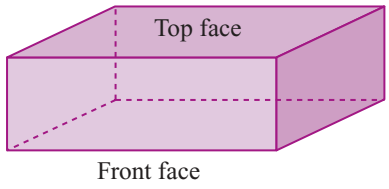
A 14 cm^3 **B** 21 cm^3
C 22 cm^3 **D** 27 cm^3

18 What is the volume of this rectangular prism?



A 14 cm^3 **B** 28 cm^3
C 33 cm^3 **D** 90 cm^3

Use this diagram to answer questions **19** and **20**.



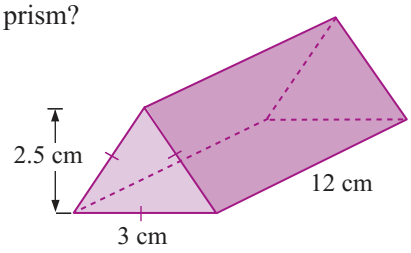
19 Which face is the same shape and size as the front face?

A back face **B** left face
C bottom face **D** right face

20 Which face is the same shape and size as the top face?

A back face **B** left face
C bottom face **D** right face

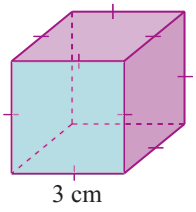
21 What plane shapes make up this triangular prism?



A 2 equal sized triangles and 3 equal sized rectangles
B 2 equal sized triangles and 3 unequal sized rectangles
C 2 unequal sized triangles and 3 equal sized rectangles
D 2 unequal sized triangles and 3 unequal sized rectangles

22 What is the volume of this cube?

A 3 cm^3
B 9 cm^3
C 27 cm^3
D 54 cm^3



The Diagnostic test questions refer to the Year 6 content description below.

Question	1–4	5–10	11–22
Section	ACMMG 135, 136	ACMMG 137	ACMMG 138

A Area

The **area** of a shape is the number of **square units** that it covers. The most common metric units for measuring area are square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) and square kilometres (km^2).

A square millimetre (1 mm^2) is the area of a square with side length 1 mm.

1 mm^2

Actual size

A square centimetre (1 cm^2) is the area of a square with side length 1 cm.

1 cm

1 cm^2 1 cm

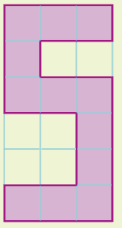
Actual size

A square metre (1 m^2) is the area a square with side length 1 m.

A square kilometre (1 km^2) is the area of a square with side length 1 km.

EXAMPLE 1

Find the area of the number 5 by counting squares.

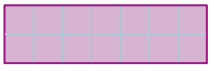


Area = 12 square units ($units^2$)

Exercise 12A

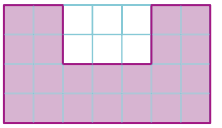
1 By counting squares, find the area of each shape in square units.

a

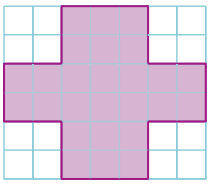


b

c



d



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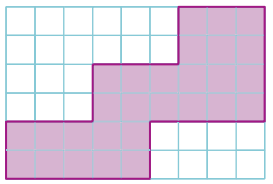

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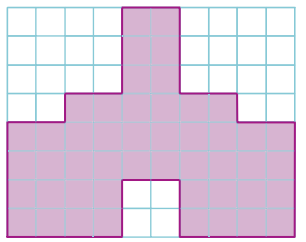
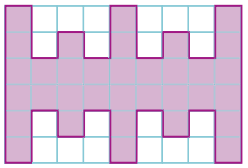
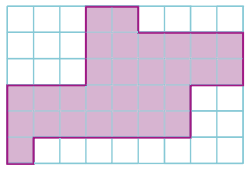


2 By counting squares, find the area of each shape in square units.

a



b



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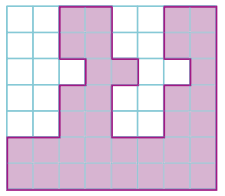

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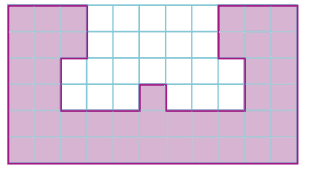
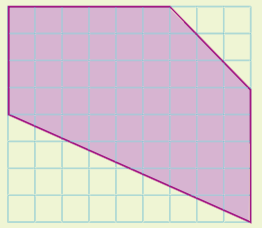


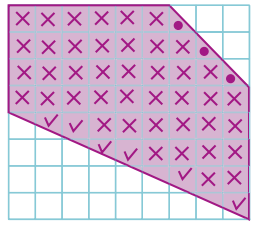
EXAMPLE 2

Estimate the area of the given shape by counting squares.



To estimate the area by counting squares:

- count whole squares as 1 square unit
- count squares with more than half shaded as 1 square unit
- count squares with exactly half shaded as $\frac{1}{2}$ square unit
- do not count squares with less than half shaded.



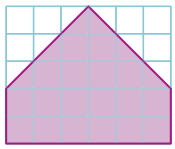
There are 42 complete squares.	42	(×)
There are 6 with more than half shaded.	+ 6	(✓)
There are 3 with exactly half shaded.	$+ 1\frac{1}{2}$	(•)
	$49\frac{1}{2}$	

$\therefore$ Area $\approx 42 + 6 + 1\frac{1}{2} = 49\frac{1}{2}$ square units.

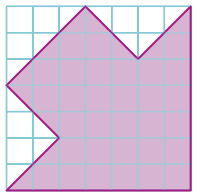
≈ means approximately equal to.

3 By counting squares, estimate the area of each shape.

a



b



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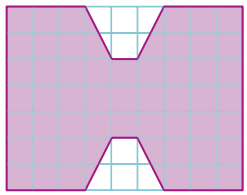

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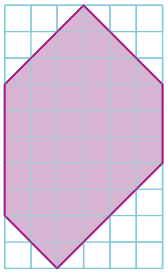

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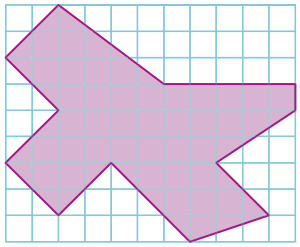

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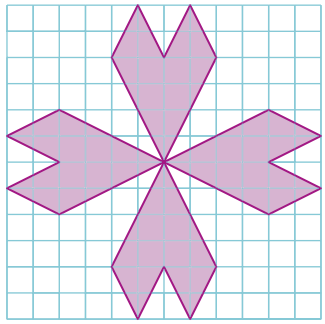
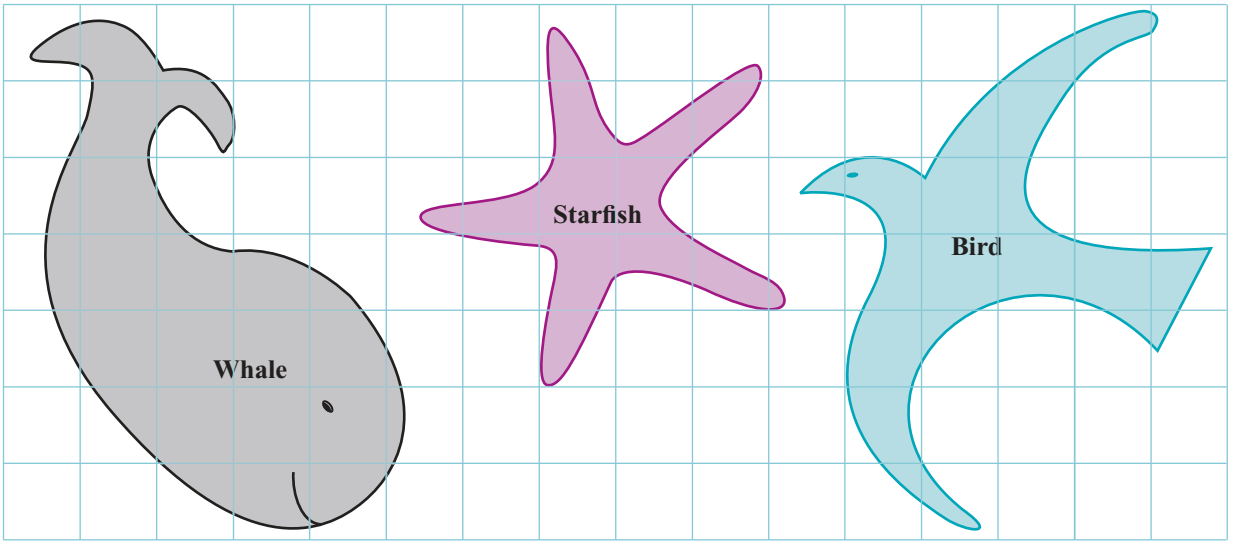
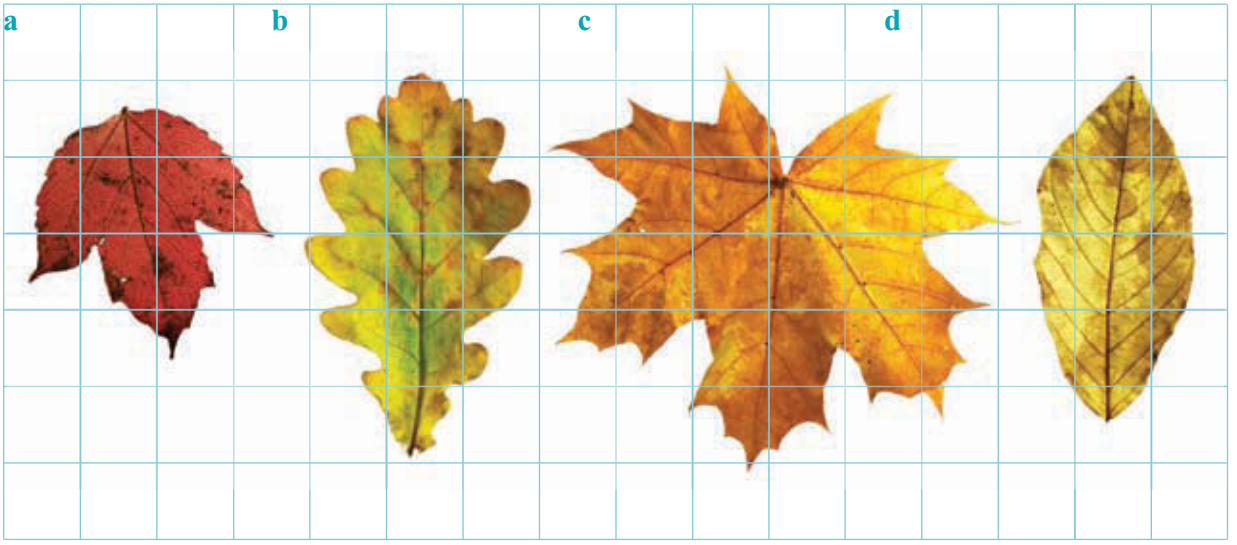


4 By counting squares on the grid paper, find an approximation for the area of each shape.



5 Approximate the areas of the leaves drawn on the 1 cm grid paper below by counting the squares.



6 **a** Explain the meaning of a square metre. Illustrate with a diagram.

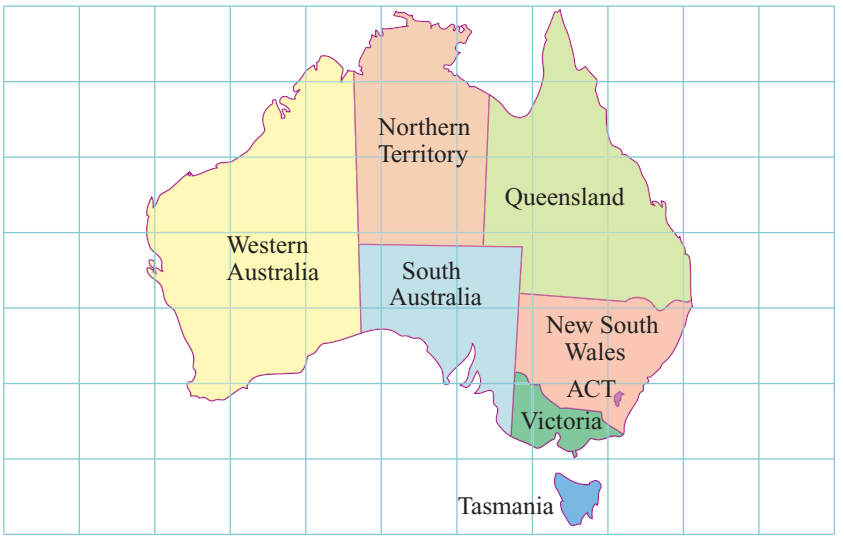
b Explain the meaning of a square kilometre.

Investigation 1 Areas in life

1 Use newspaper or cardboard to make a square metre.

- **a** Use this square metre to estimate the areas of the walls in your classroom.
- **b** If 1 L of paint covers about 10 m^2, how many litres of paint are needed to paint your classroom's walls?
- **c** How many litres of paint are needed to paint the ceiling with two coats of paint?

2 Below is a map of Australia, overlaid by a square grid.



- **a** Use the grid paper to estimate the area of each state and territory in square units.
- **b** List the states in order of size.
- **c** Estimate the number of Tasmanias that will 'fit' into each state.
- **d** If the area of Tasmania is 70 000 km^2, write an estimate for the area of each state and territory.

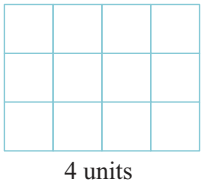
3 Collect leaves or other objects and find their areas.

4 Discuss which unit (mm^2, cm^2, m^2 or km^2) would be the most suitable to measure the area of a:

- **a** book page
- **b** calculator key
- **c** bathroom floor
- **d** postage stamp
- **e** state of Australia
- **f** football field

B Area of a rectangle

Consider a rectangle that is 4 units by 3 units as shown.
Counting the squares gives an area of 12 square units, or $4 \times 3 = 12$.

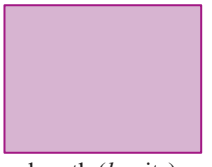


To find the area of any rectangle, multiply the lengths of the two sides.

Area of a rectangle $=$ length $\times$ breadth

$A = l \times b$

$A = lb$

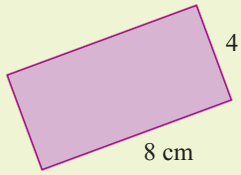


EXAMPLE 1

Find the area of each rectangle.

a

b



a $A = lb$
$= 6 \times 3 = 18 \text{ cm}^2$

b $A = lb$
$= 8 \times 4 = 32 \text{ cm}^2$

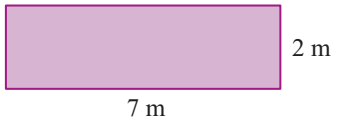
Exercise 12B

1 Complete the following to find the areas of the rectangles.

a

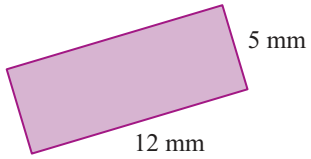
$A = l \times b$
$= 5 \times \_\_\_$
$= \_\_\_ \text{ km}^2$

b



$A = l \times b$
$= \_\_\_ \times 2$
$= \_\_\_ \text{ m}^2$

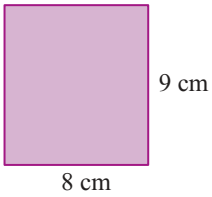
c



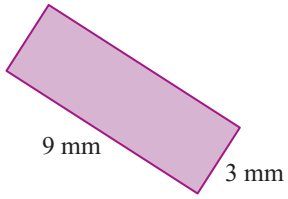
$A = l \times b$
$= \_\_\_ \times \_\_\_$
$= 60 \text{ mm}^2$

2 Find the areas of the following rectangles.

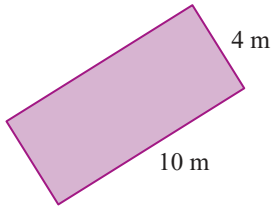
a



b

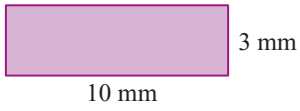


c

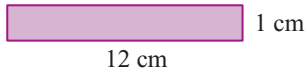


d

e

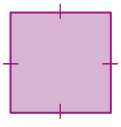


f



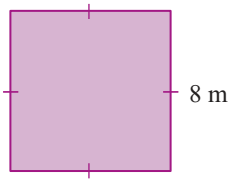
3 A square is a rectangle where length and breadth are equal.
Hence the formula is $A = s^2$. Find the areas of the following squares.

a



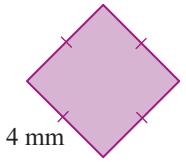
$A = s^2$
$= 5 \times 5$
$= \_\_\_ \text{ cm}^2$

b

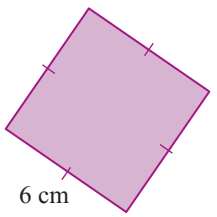


$A = s^2$
$= \_\_\_ \times 8$
$= \_\_\_ \text{ m}^2$

c

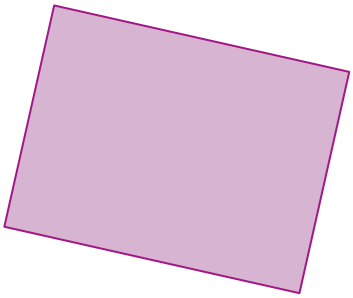


d



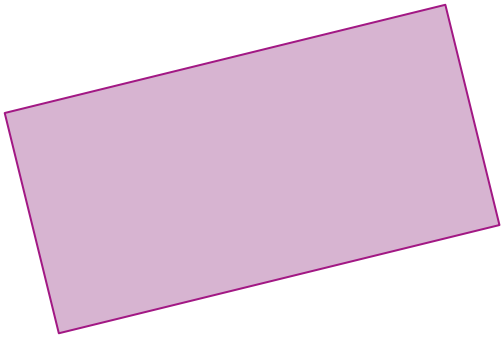
4 By measurement and calculation, find the areas of the following rectangles.

a



b

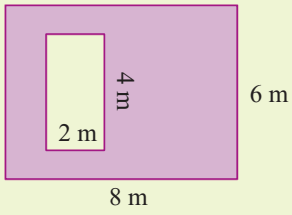
c



d

EXAMPLE 2

Find the shaded area.



The shaded area is found by subtraction.

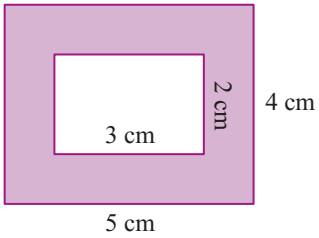
Area of large rectangle $= 8 \times 6 = 48$ m^2

Area of unshaded rectangle $= 4 \times 2 = 8$ m^2

$\therefore$ Shaded area $= 48 - 8 = 40$ m^2

5 Complete the following to find the shaded areas.

a



Area of large rectangle $= 5 \times$ ___

$=$ ___ cm^2

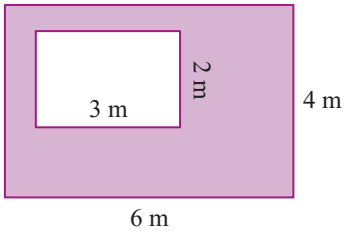
Area of unshaded rectangle $=$ ___ $\times 2$

$=$ ___ cm^2

$\therefore$ Shaded area $=$ ___ $-$ ___

$=$ ___ cm^2

b



Area of large rectangle $= 6 \times$ ___

$=$ ___ m^2

Area of unshaded rectangle $=$ ___ $\times 2$

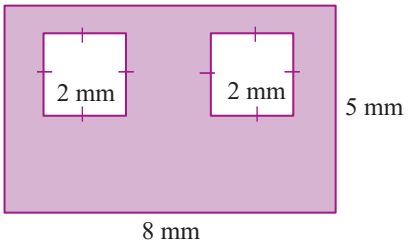
$=$ ___ m^2

$\therefore$ Shaded area $=$ ___ $-$ ___

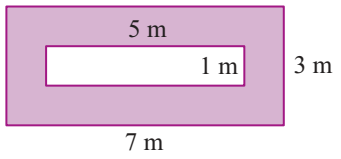
$=$ ___ m^2

6 Find the shaded areas.

a



b

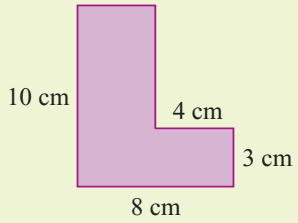


EXAMPLE 3

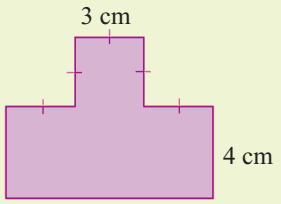
A composite shape is a combination of two or more standard geometric shapes.

Find the areas of the following **composite shapes**.

a

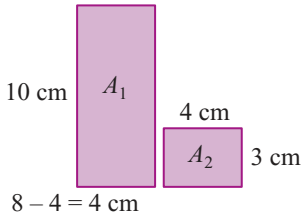


b



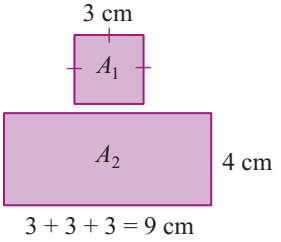
Divide the shapes into separate rectangles. Subtract or add to find any missing measurements. The area of each shape is the area of two rectangles.

a



$A = A_1 + A_2$
$= (10 \times 4) + (4 \times 3)$
$= 40 + 12 = 52 \text{ cm}^2$

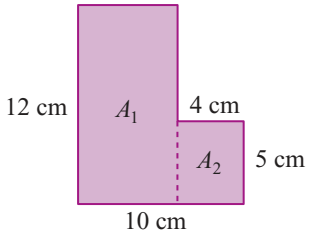
b



$A = A_1 + A_2$
$= (3 \times 3) + (9 \times 4)$
$= 9 + 36 = 45 \text{ cm}^2$

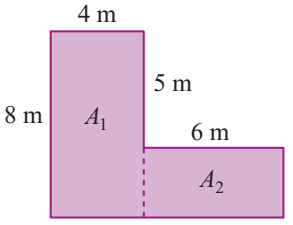
7 Complete to find the area of each composite shape by first dividing the shape into rectangles as shown.

a



$A = A_1 + A_2$
$= (\_\_\_ \times 6) + (\_\_\_ \times 4)$
$= \_\_\_ + \_\_\_ = \_\_\_ \text{ cm}^2$

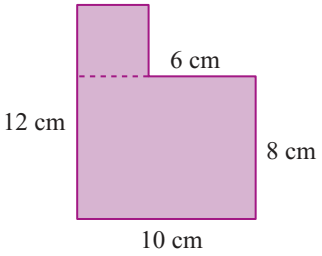
b



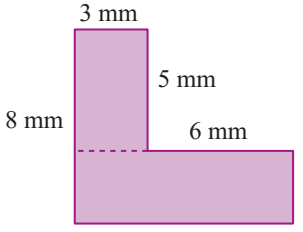
$A = A_1 + A_2$
$= (8 \times \_\_\_) + (6 \times \_\_\_)$
$= \_\_\_ + \_\_\_ = \_\_\_ \text{ m}^2$

8 Find the areas of the following shapes by first dividing each into rectangles as shown.

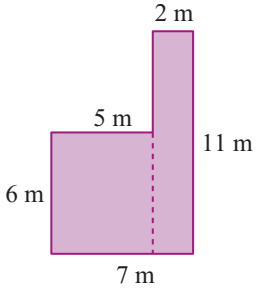
a



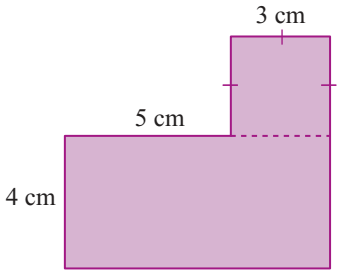
b



c

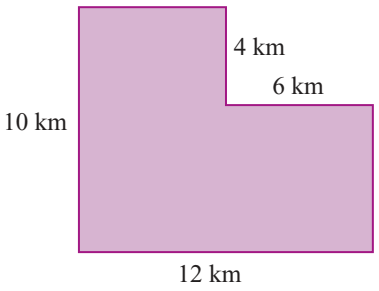


d

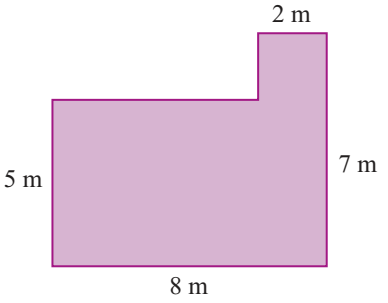


9 Find the areas of the following shapes.

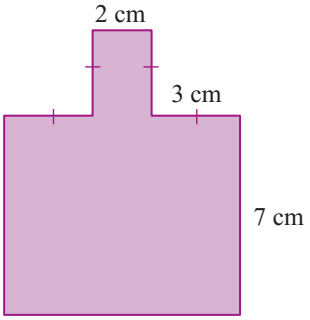
a



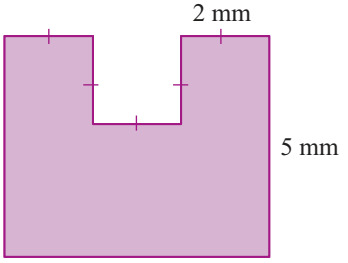
b



c



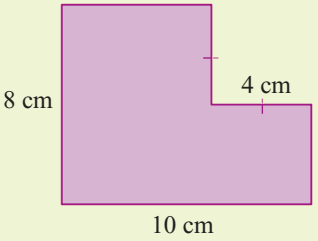
d



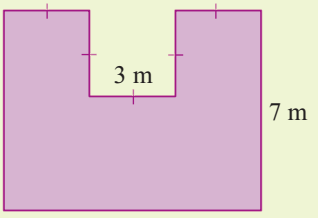
EXAMPLE 4

Find the areas of the following shapes by subtraction.

a

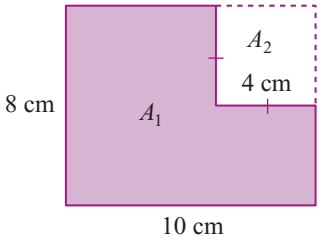


b



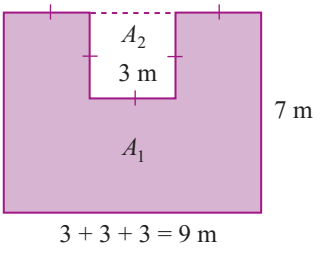
The area of each shape is the area of the largest rectangle minus the area of the missing square. Subtract or add to find any missing measurements.

a



$A = A_1 - A_2$
$= (10 \times 8) - (4 \times 4)$
$= 80 - 16 = 64 \text{ cm}^2$

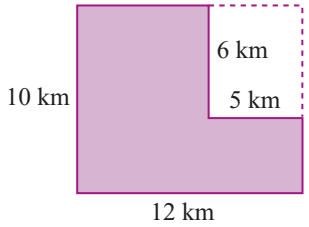
b



$A = A_1 - A_2$
$= (9 \times 7) - (3 \times 3)$
$= 63 - 9 = 54 \text{ m}^2$

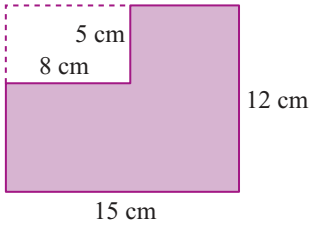
10 Complete to find the area of each shape by subtraction.

a



A = area largest − area smallest
= (12 × ___) − (6 × ___)
= ___ − ___ = ___ km²

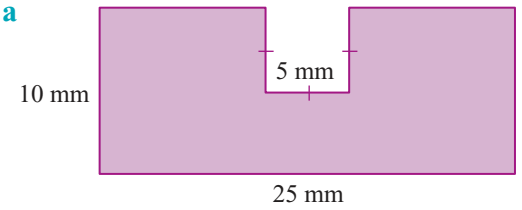
b



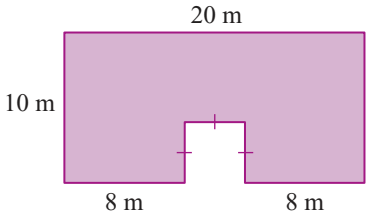
A = area largest − area smallest
= (15 × ___) − (8 × ___)
= ___ − ___ = ___ cm²

11 Find the areas of the following shapes by subtraction.

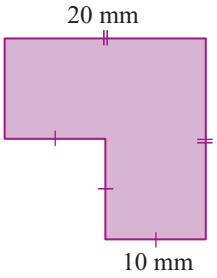
a



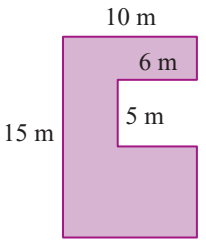
b



c



d

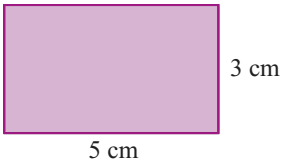


Investigation 2 Area of a triangle

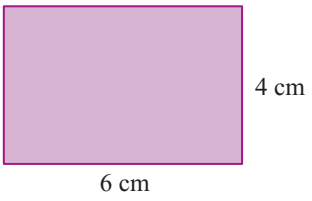
Follow the steps to develop a rule for finding the area of a triangle.

1 *Step 1*: Find the area of each rectangle.

a

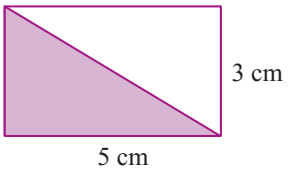


b

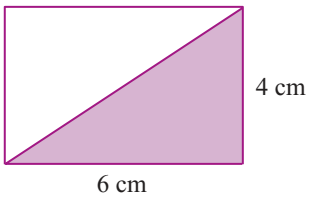


Step 2: Find the shaded area within each rectangle.

a

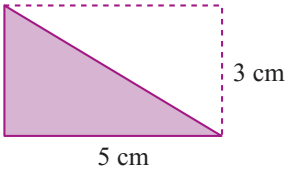


b

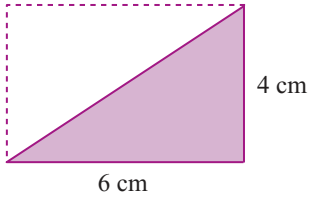


Step 3: Find the area of each shaded triangle.

a

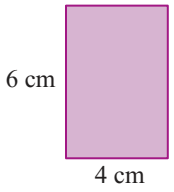


b

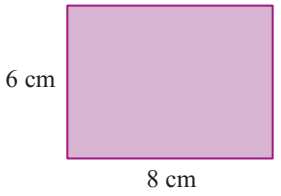


2 *Step 1*: Find the area of each rectangle.

a

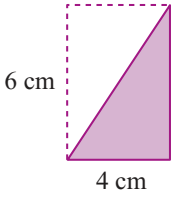


b

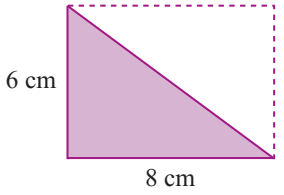


Step 2: Find the area of each shaded triangle.

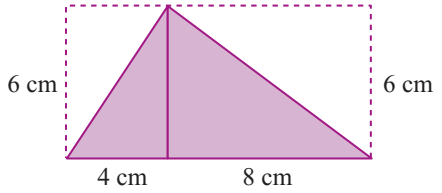
a



b

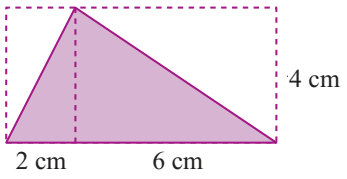


3 The rectangles in question **2** can be put together as shown. What is the area of the shaded triangle formed?

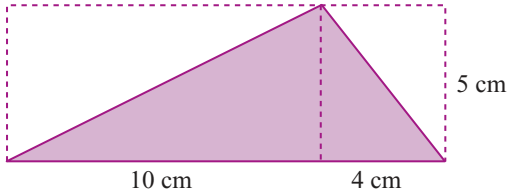


4 Find the area of each shaded triangle.

a

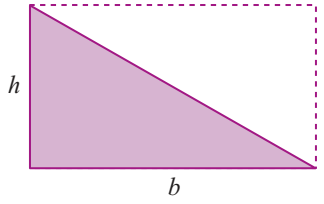


b

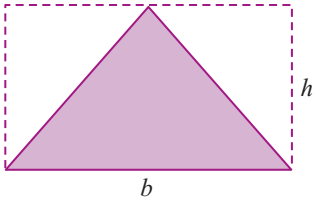


5 Find an expression for the area of each shaded triangle.

a



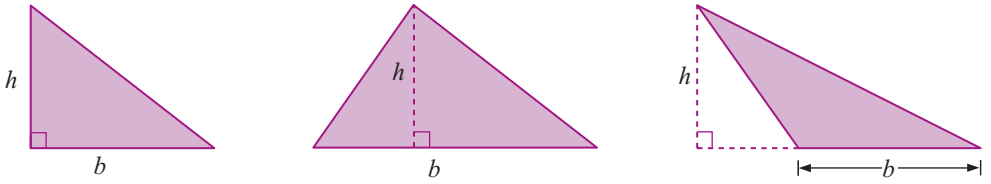
b



6 Copy and complete the following:
The area of a triangle with base b and perpendicular height h is $A = \frac{\square}{2}$.

C Area of a triangle

In Investigation 2 you developed a rule for finding the area of a triangle.
The area of a triangle is half the area of the rectangle enclosing it.



Area of triangle $= \frac{1}{2}$(base $\times$ height)

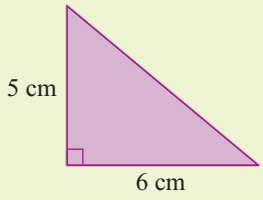
$$A = \frac{1}{2}bh \text{ or } A = \frac{bh}{2}$$

With obtuse-angled triangles, the perpendicular height is shown outside the triangle.

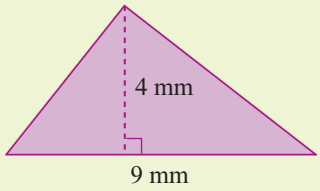
EXAMPLE 1

Find the areas of the following triangles.

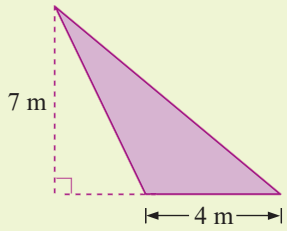
a



b



c



a $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 5$

$= 15 \text{ cm}^2$

b $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 9 \times 4$

$= 18 \text{ mm}^2$

c $= \frac{1}{2}bh$

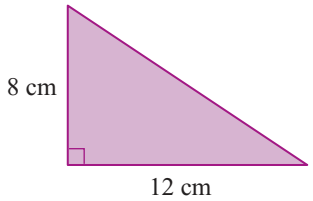
$= \frac{1}{2} \times 4 \times 7$

$= 14 \text{ m}^2$

Exercise 12C

1 Complete to find the areas of the following triangles.

a

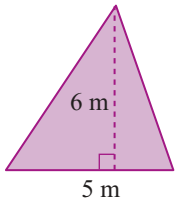


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \_\_\_ \times 8$

$= \_\_\_ \text{ cm}^2$

b

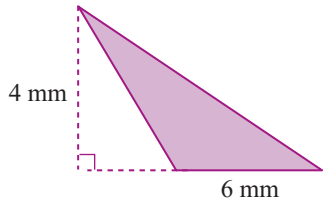


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times \_\_\_$

$= \_\_\_ \text{ m}^2$

c



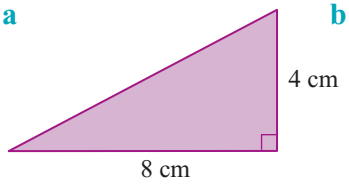
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \_\_\_ \times \_\_\_$

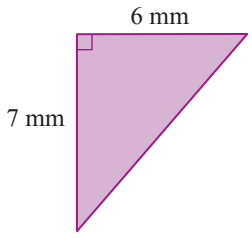
$= \_\_\_ \text{ mm}^2$

2 Find the areas of the following triangles.

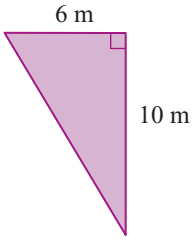
a



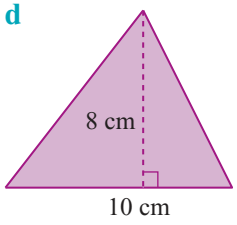
b



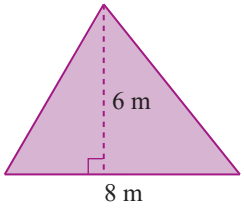
c



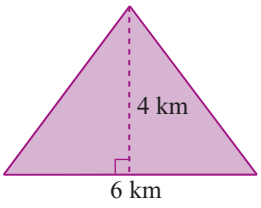
d



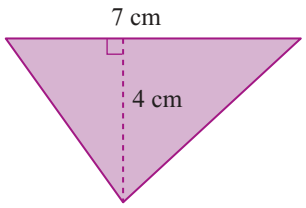
e



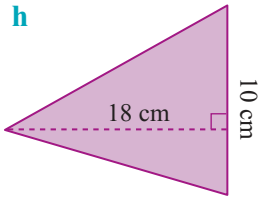
f



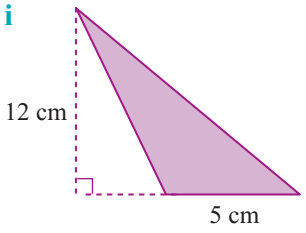
g



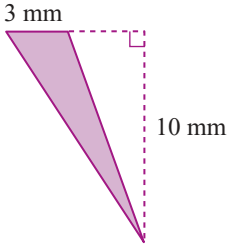
h



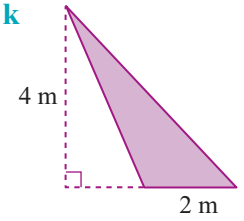
i



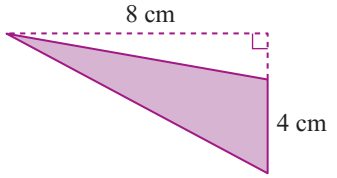
j



k



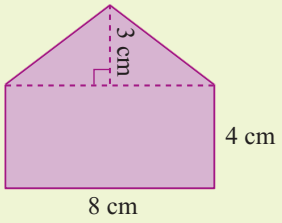
l



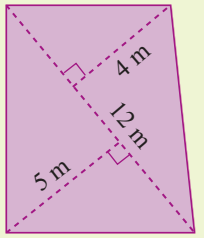
EXAMPLE 2

Find the areas of the following shapes.

a



b



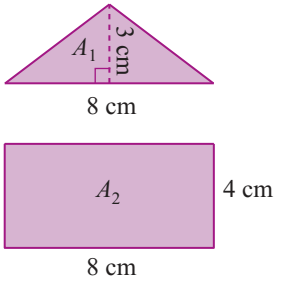
a This shape is made up of a rectangle and a triangle.

$A = A_1 + A_2$

$= \frac{1}{2}(8 \times 3) + (8 \times 4)$

$= 12 + 32$

$= 44 \text{ cm}^2$



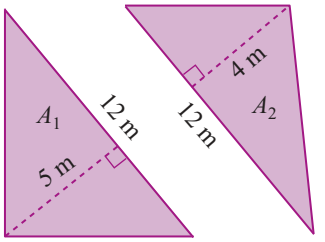
b This shape is made up of two triangles.

$A = A_1 + A_2$

$= \frac{1}{2}(12 \times 5) + \frac{1}{2}(12 \times 4)$

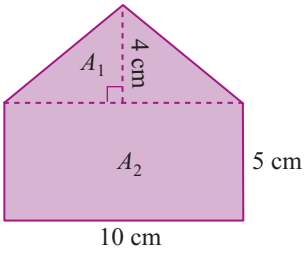
$= 30 + 24$

$= 54 \text{ m}^2$



3 Complete the following to find the area of these shapes.

a



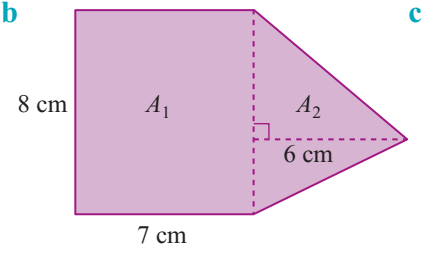
$A = A_1 + A_2$

$= \frac{1}{2}(\_\_\_ \times 4) + (10 \times \_\_\_)$

$= \_\_\_ + \_\_\_$

$= \_\_\_ \text{ cm}^2$

b



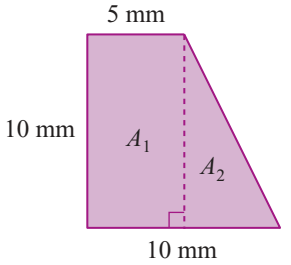
$A = A_1 + A_2$

$= (8 \times \_\_\_) + \frac{1}{2}(8 \times \_\_\_)$

$= \_\_\_ + \_\_\_$

$= \_\_\_ \text{ cm}^2$

c



$A = A_1 + A_2$

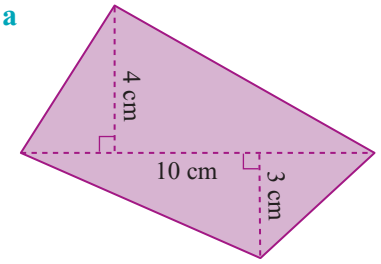
$= (10 \times \_\_\_) + \frac{1}{2}(\_\_\_ \times \_\_\_)$

$= \_\_\_ + \_\_\_$

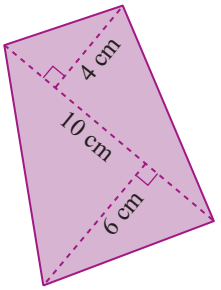
$= \_\_\_ \text{ mm}^2$

4 Find the areas of the following shapes.

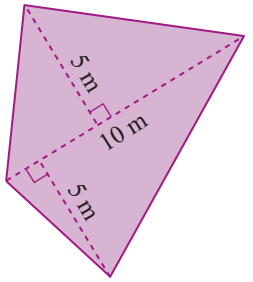
a



b



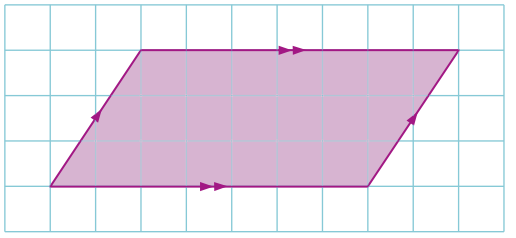
c



Investigation 3 Area of a parallelogram

One way to find the area of a shape is to divide the shape into a number of unit squares and count the squares.

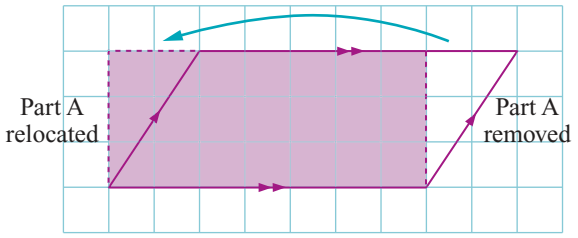
1 Consider the parallelogram shown. By counting the squares determine the area of the parallelogram.



2 Now consider this diagram.

a What shape is formed when part A is removed and relocated?

b What do you now notice about the area of the new shape and the original shape?

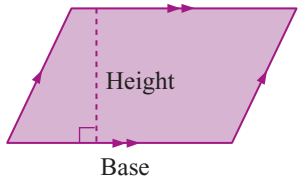


3 Using the terms 'base' and 'height', develop a formula to determine the area of a parallelogram.

D Area of a parallelogram

In Investigation 3 you developed a rule to find the area of a parallelogram.
The area of a parallelogram is:

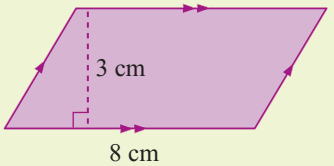
$$\text{Area} = \text{base} \times \text{height}$$
$$A = bh$$



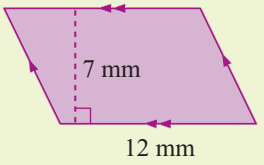
EXAMPLE 1

Find the areas of the following parallelograms.

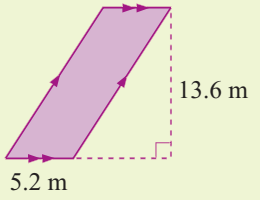
a



b



c



a $A = bh$
$= 8 \times 3 = 24 \text{ cm}^2$

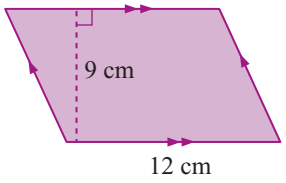
b $A = bh$
$= 12 \times 7 = 84 \text{ mm}^2$

c $A = bh$
$= 5.2 \times 13.6 = 70.72 \text{ m}^2$

Exercise 12D

1 Complete the following to find the areas of these parallelograms.

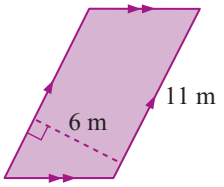
a



$A = bh$

$= 12 \times \_\_\_ = \_\_\_ \text{ cm}^2$

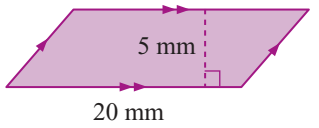
b



$A = bh$

$= \_\_\_ \times 6 = \_\_\_ \text{ m}^2$

c

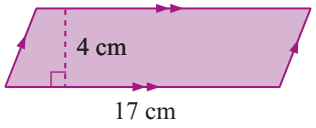


$A = bh$

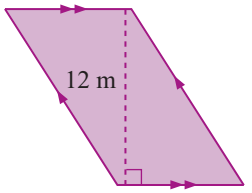
$= \_\_\_ \times \_\_\_ = 100 \text{ mm}^2$

2 Find the areas of the following parallelograms.

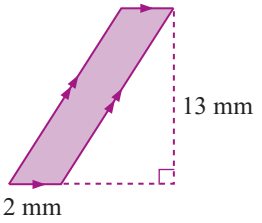
a



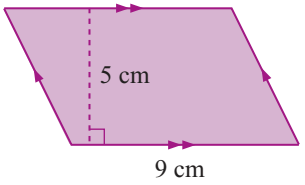
b



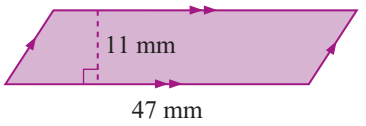
c



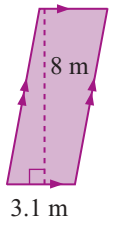
d



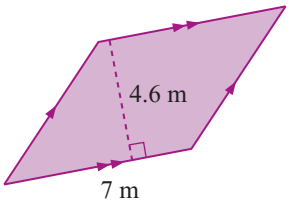
e



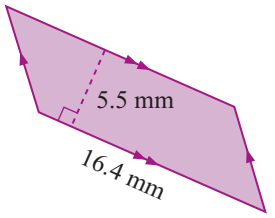
f



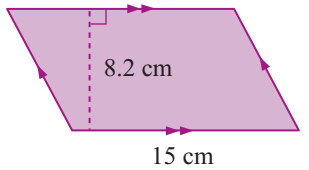
g



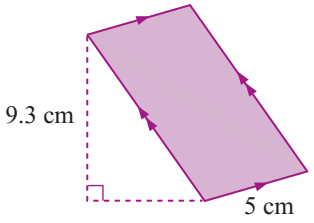
h



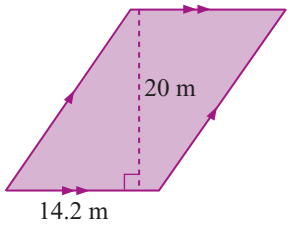
i



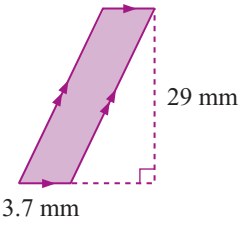
j



k



l



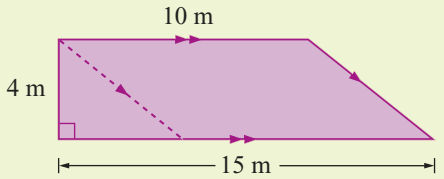
3 Find the area of each parallelogram with the dimensions given.

a $b = 14$ cm, $h = 10$ cm
b $b = 15$ m, $h = 8$ m
c $b = 7$ cm, $h = 13$ cm
d $b = 21$ mm, $h = 12$ mm
e $b = 9$ m, $h = 30$ m
f $b = 4.1$ cm, $h = 5$ cm
g $b = 9$ cm, $h = 2.1$ cm
h $b = 8.5$ m, $h = 7.2$ m
i $b = 2.7$ m, $h = 9.3$ m
j $b = 12.4$ m, $h = 8.6$ m

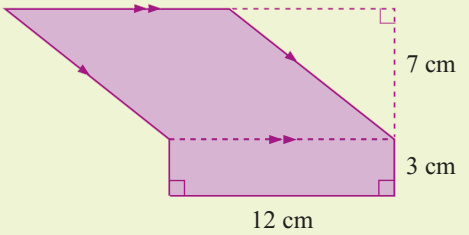
EXAMPLE 2

Find the areas of the following shapes.

a



b



a This shape is made up of a triangle and a parallelogram.

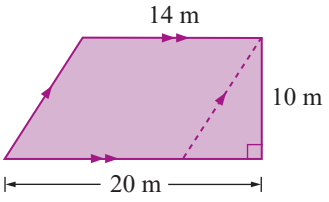
$A = A_1 + A_2$
$= \frac{1}{2}bh + bh$
$= \frac{1}{2}(5 \times 4) + (10 \times 4)$
$= 10 + 40$
$= 50$ m^2

b The shape is made up of a rectangle and a parallelogram.

$A = A_1 + A_2$
$= lb + bh$
$= (12 \times 3) + (12 \times 7)$
$= 36 + 84$
$= 120$ cm^2

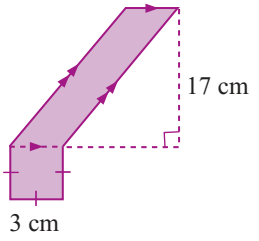
4 Complete the following to find the area of these composite shapes.

a



$A = A_1 + A_2$
$=$ parallelogram $+$ _____
$= bh +$ ____
$= (14 \times$ ___$) + \frac{1}{2}($___ $\times 10)$
$=$ ___ $+$ ___
$=$ ___ m^2

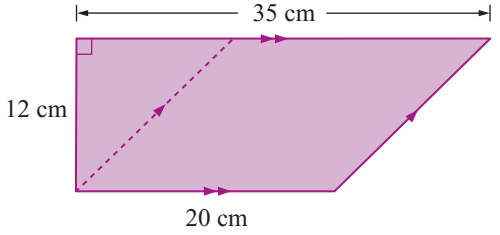
b



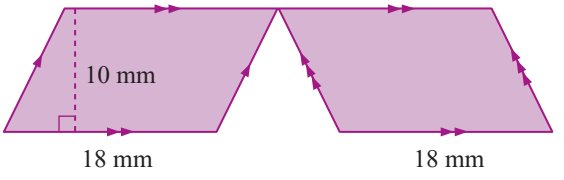
$A = A_1 + A_2$
$=$ square $+$ _____
$= s^2 +$ _____
$=$ ___$^2 +$ (___ $\times$ ___)
$=$ ___ $+$ ___
$=$ ___ cm^2

5 Find the area of the following composite shapes.

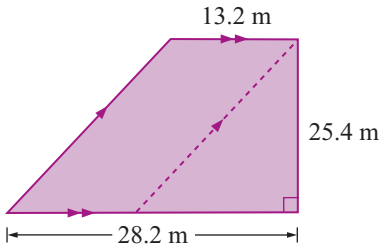
a



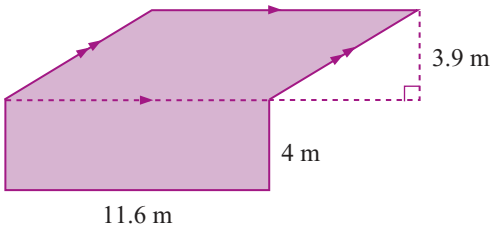
b



c



d

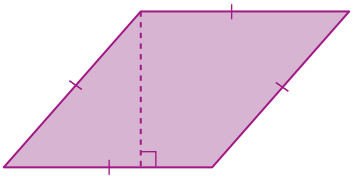


Investigation 4 Making rectangles

Many plane shapes can be made into rectangles. This gives a method of finding their areas.

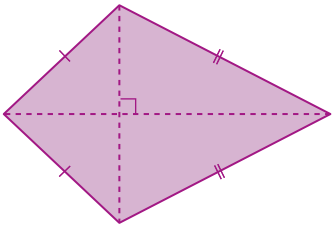
An isosceles trapezium has both non-parallel sides equal in length.

1 *Step 1:* Copy and cut out each of the following shapes.

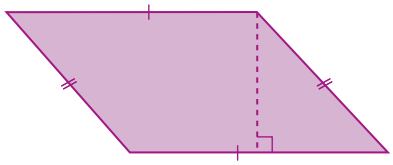


Rhombus

Isosceles trapezium



Kite



Parallelogram

Step 2: Cut along the dotted line(s) and arrange the pieces to make each shape into a rectangle.
Step 3: Find the area of each rectangle and hence the area of each original shape.

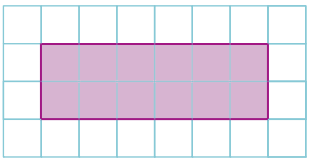
2 Draw your own rhombus, isosceles trapezium, kite and parallelogram and find their areas.

Investigation 5 How many possibilities are there?

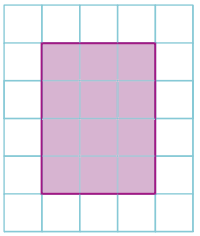
Consider the examples shown below.

1 Determine the area of each shape. What do you notice?

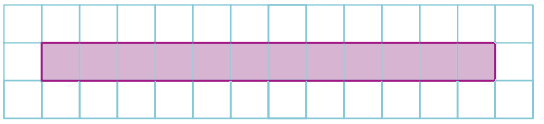
a



b



c



2 On grid paper, show the number of ways you could make shapes of the following area.

a 18 units^2 **b** 20 units^2 **c** 36 units^2

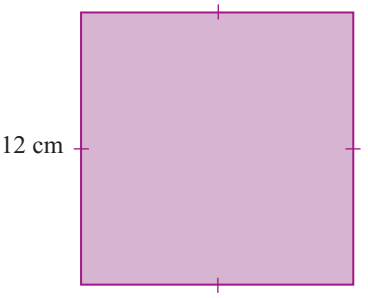
4 Consider the square shown.

$A = s^2$

$= 12 \times 12$

$= 144$ cm^2

If we change the side length, how does this affect the area?

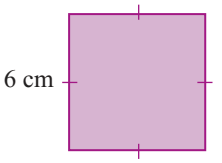


If we divide the side length of 12 cm by 2, the side length becomes 6 cm.

$\therefore A = 6 \times 6$

$= 36$ cm^2

Original area $\div 2^2 = 144 \div 4 = 36$ cm^2

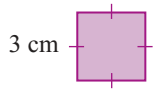


If we divide the side length of 12 cm by 4, the side length becomes 3 cm.

$\therefore A = 3 \times 3$

$= 9$ cm^2

Original area $\div 4^2 = 144 \div 16 = 9$ cm^2

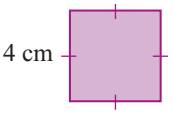


If we divide the side length of 12 cm by 3, the side length becomes 4 cm.

$\therefore A = 4 \times 4$

$= 16$ cm^2

Original area $\div 3^2 = 144 \div 9 = 16$ cm^2



a Can you explain why this pattern occurs?

b Repeat the process starting with a square with the following side lengths.

i 15 cm **ii** 30 cm **iii** 48 cm

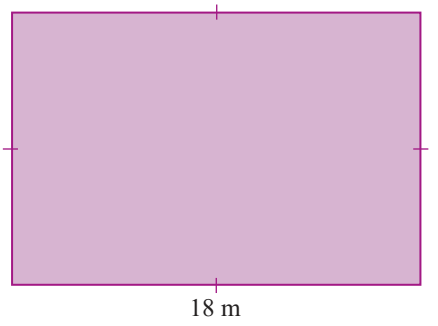
c What conclusions can you make from your investigations in part **b**?

5 Consider the rectangle shown. Repeat the process in question **4** to determine the relationship between dividing side lengths and the area.

$A = lb$

$= 18 \times 12$

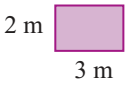
$= 216$ m^2



If we divide each side by 6:

$\therefore\ A = 3 \times 2$

$= 6\ \text{cm}^2$



Original area $\div 6^2 = 216 \div 36 = 6\ \text{cm}^2$

a Continue this process by dividing each side length by:

i 4 **ii** 2 **iii** 3

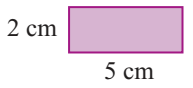
b From your investigations what conclusions can you draw?

6 Now try increasing the side length of the original shape.

$A = lb$

$A = 5 \times 2$

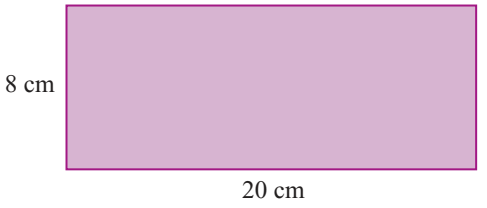
$= 10\ \text{cm}^2$



If we multiply the side lengths by 4:

$\therefore\ A = 20 \times 8$

$= 160\ \text{cm}^2$



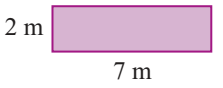
Original area $\times 4^2 = 10 \times 16 = 160\ \text{cm}^2$

a Continue this process by multiplying each side length by:

i 5 **ii** 7 **iii** 10

b From your investigations what conclusions can you draw?

7 Consider the rectangle shown.



$A = lb$

$= 7 \times 2$

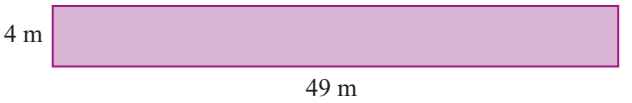
$= 14\ \text{m}^2$

Repeat the process in question **6** but this time square the side lengths.

$\therefore\ A = 49 \times 4$

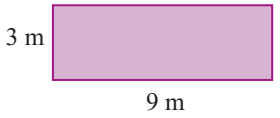
$= 196\ \text{m}^2$

$\sqrt{\text{Original area}} = \sqrt{196} = 14\ \text{m}^2$

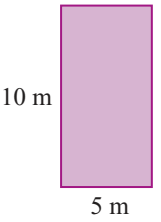


a Continue investigating this pattern using each rectangle below as the original shape.

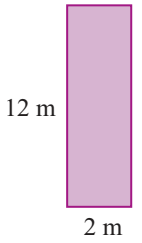
i



ii



iii



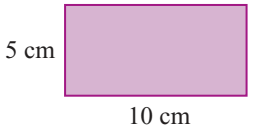
b Apply the same strategy to squares with the following side lengths.

i 20 cm **ii** 4 mm **iii** 15 m

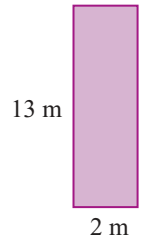
c What conclusions can you drawn about squaring side lengths and the square root of the area of the new shape?

8 The rectangles below all have a perimeter of 30 cm.

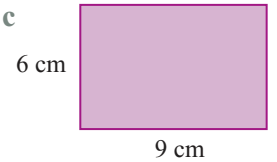
a



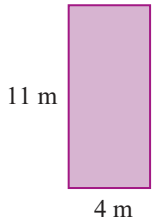
b



c



d



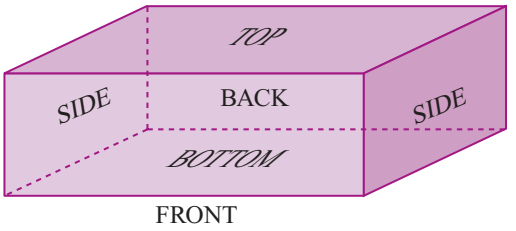
i Calculate the area of each rectangle.

ii If shapes have the same perimeter, do they have the same area? Explain.

E Identifying faces to calculate surface area

When calculating the **surface area** of a solid, you need to be able to identify all faces along with their dimensions. This rectangular prism has three pairs of **corresponding faces**:

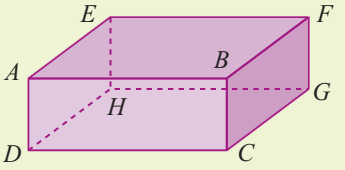
- two sides
- front and back
- top and bottom.



Corresponding faces are equal and parallel.

EXAMPLE 1

Find the pairs of corresponding faces, given that:
Front = *ABCD*
Side = *BFGC*
Bottom = *DHGC*

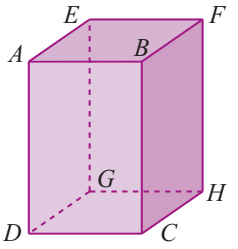


Given	*Corresponding face*
Front = *ABCD*	Back = *EFGH*
Side = *BFGC*	Side = *AEHD*
Bottom = *DHGC*	Top = *AEFB*

Exercise 12E

1 Find each corresponding face, given that:

a Front = *ABCD* **b** Side= *BFHC* **c** Top = *AEFB*



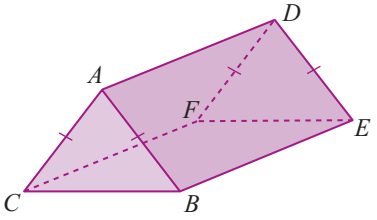
2 For each triangular prism shown:

i Identify the remaining faces, given that the front = *ABC*.

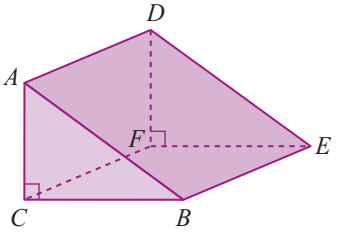
ii Which faces are corresponding?

iii Which faces are equal?

a



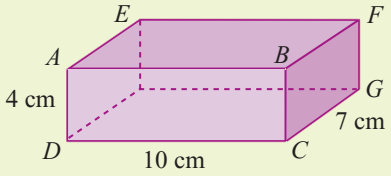
b



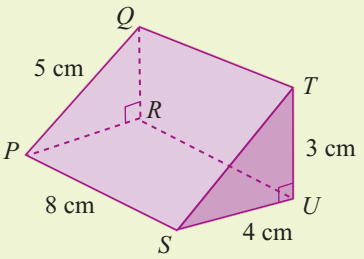
EXAMPLE 2

Using the diagram, state the dimensions of each face in a table.

a



b



a

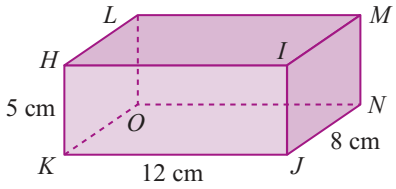
Face	Length	Breadth
ABCD	10 cm	4 cm
EFGH	10 cm	4 cm
AEHD	7 cm	4 cm
BFGC	7 cm	4 cm
AEFB	10 cm	7 cm
DHGC	10 cm	7 cm

b

Face	Base	Height
PQR	4 m	3 m
STU	4 m	3 m
Face	**Length**	**Breadth**
PQTS	8 m	5 m
QTUR	8 m	3 m
PRUS	8 m	4 m

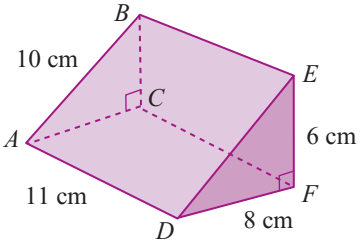
3 Complete the tables for the diagrams shown.

a



Face	Length	Breadth
LMNO		
HIJK		
IMNJ		
HLOK		
HLMI		
KONJ		

b

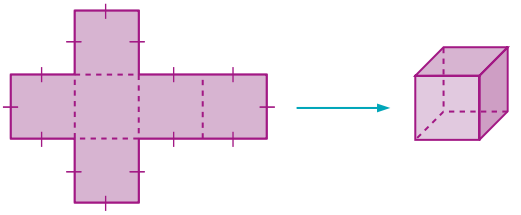


Face	Base	Height
ABC		
DEF		
Face	**Length**	**Breadth**
ABED		
BEFC		
ACFD		

F Surface area using nets

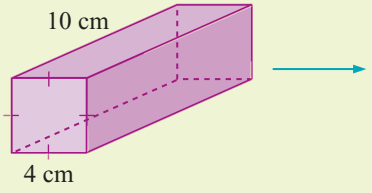
The **net** of a solid is a plane shape that can be folded to form the solid. For example, here is the net of a cube.

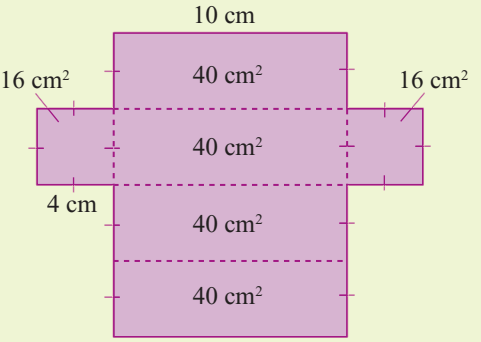
A net can be used to calculate the surface area of a solid, since surface area is the sum of the areas of all faces. Example 1 outlines how useful nets can be when calculating surface area.



EXAMPLE 1

Calculate the surface area of this square prism by referring to its net.



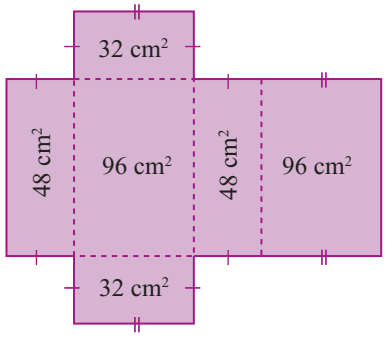


Surface area = the sum of the areas of the faces
$= (16 \times 2) + (40 \times 4)$
$= 32 + 160 = 192 \text{ cm}^2$

Exercise 12F

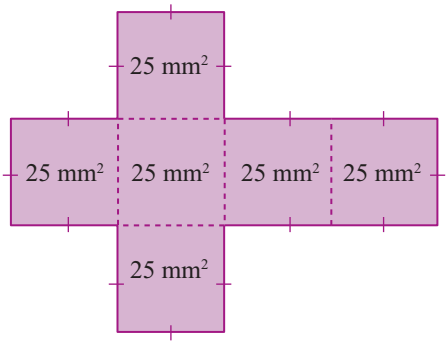
1 Calculate the surface areas of the prisms whose nets are shown.

a



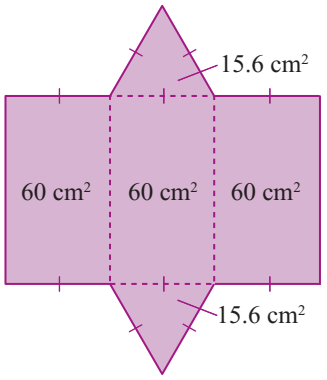
Rectangular prism

b



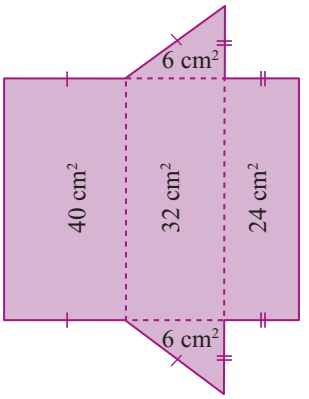
Cube

c



Triangular prism

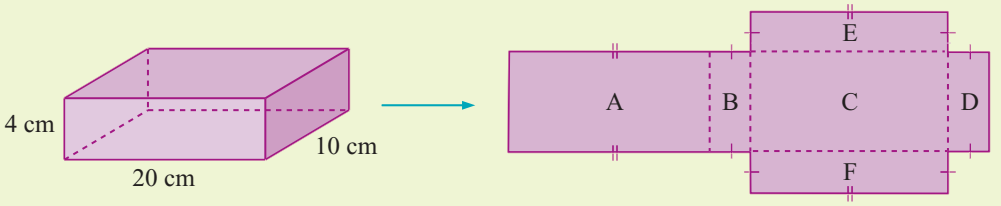
d



Triangular prism

EXAMPLE 2

Calculate the surface area of this rectangular prism by referring to its net.



Step 1: Mark the dimensions on the net.

Step 2: Calculate the area of each section (three pairs of rectangles).

A and C: $2 \times 20 \times 10 = 400$ cm²

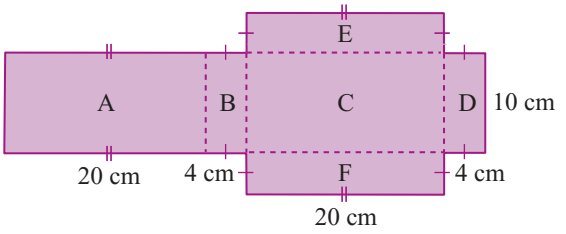
B and D: $2 \times 10 \times 4 = 80$ cm²

E and F: $2 \times 20 \times 4 = 160$ cm²

Step 3: Calculate the total surface area.

Surface area $= 400 + 80 + 160$In

$= 640$ cm²



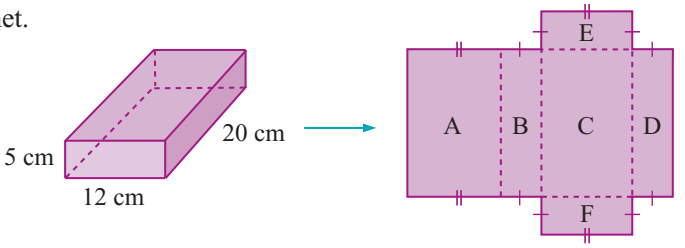
Remember:
Area of a rectangle is A = lb.

2 The diagram shows a rectangular prism and its net.

a Copy the net and show the dimensions on each section.

b Calculate the area of each section labelled A to F.

c Calculate the surface area of the prism.



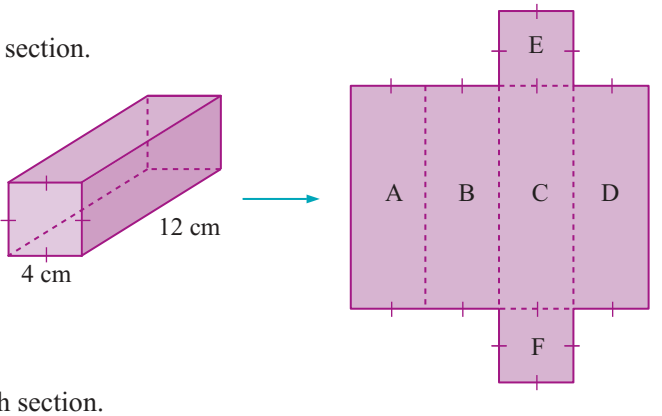
3 The diagram shows a square prism and its net.

a Copy the net and show the dimensions on each section.

b What do you notice about the dimensions of areas A to D?

c Calculate the area of each section labelled A to F.

d Calculate the surface area of the prism.



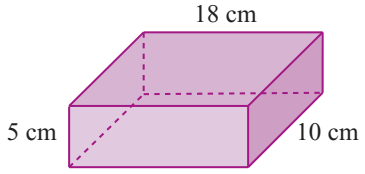
4 For each prism shown:

i Draw a net and label the dimensions of each section.

ii Calculate the area of each section.

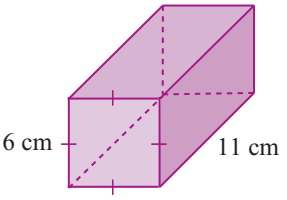
iii Calculate the surface area of the prism.

a



Rectangular prism

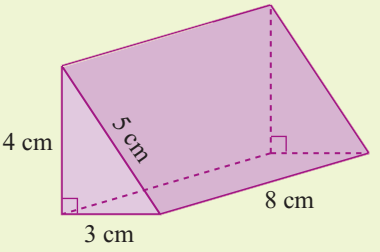
b



Square prism

EXAMPLE 3

Calculate the surface area of this triangular prism.



Step 1: Draw the net and mark the dimensions on the net.

Step 2: Calculate the area of each section (two same-size triangles plus three different-size rectangles).

A and B: $2 \times \frac{1}{2}(3 \times 4) = 12 \text{ cm}^2$

C: $8 \times 3 = 24 \text{ cm}^2$

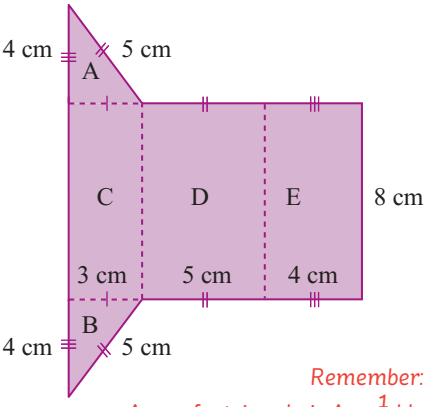
D: $8 \times 5 = 40 \text{ cm}^2$

E: $8 \times 4 = 32 \text{ cm}^2$

Step 3: Calculate the total surface area.

Surface area $= 12 + 24 + 40 + 32$

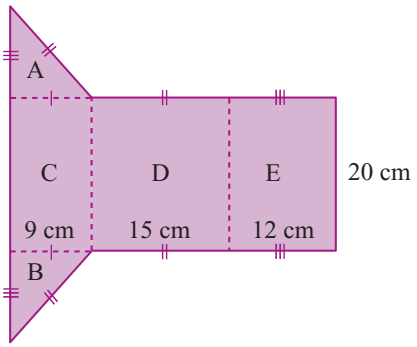
$= 108 \text{ cm}^2$



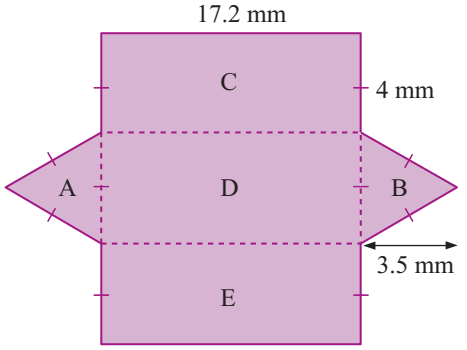
Remember:
Area of a triangle is $A = \frac{1}{2}bh$.

5 Calculate the surface area of the triangular prisms whose nets are shown.

a



b

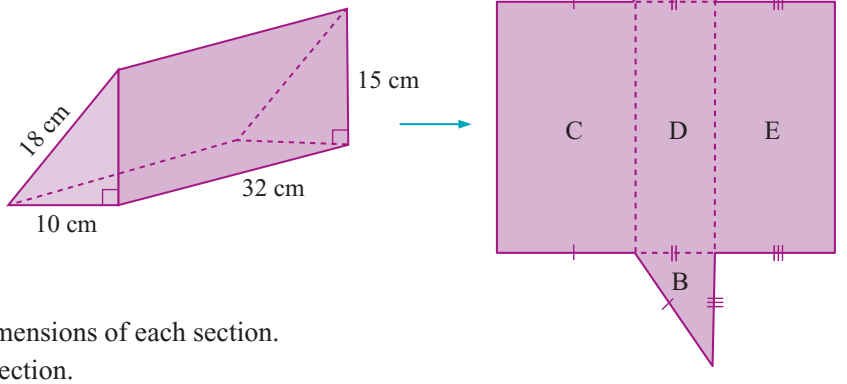


6 The diagram shows a triangular prism and its net.

a Copy the net and show the dimensions on each section.

b Calculate the area of each section labelled A to E.

c Calculate the surface area of the prism.



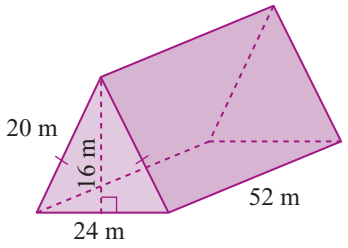
7 For each triangular prism shown:

i Draw a net and label the dimensions of each section.

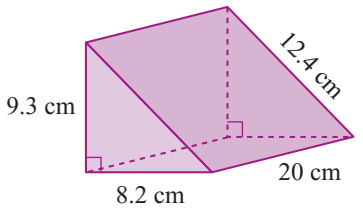
ii Calculate the area of each section.

iii Calculate the surface area of the prism.

a



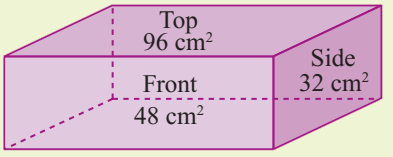
b



G Calculating surface area

EXAMPLE 1

Calculate the surface area of this rectangular prism.



The rectangular prism has 6 faces.

Area of two sides $= 2 \times 32 = 64$ cm^2

Area of front + area of back $= 2 \times 48 = 96$ cm^2

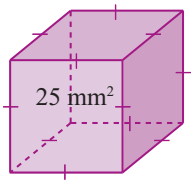
Area of top + area of bottom $= 2 \times 96 = 192$ cm^2

Surface area $= 64 + 96 + 192 = 352$ cm^2

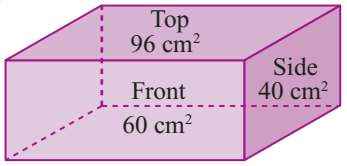
Exercise 12G

1 Calculate the surface area of each prism using the areas given.

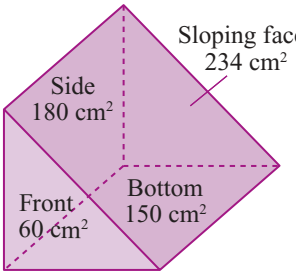
a



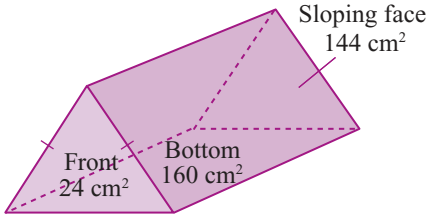
b



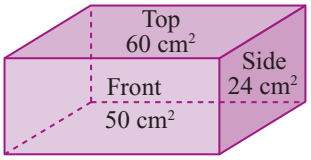
c



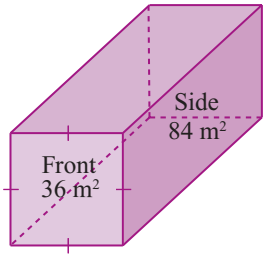
d



e



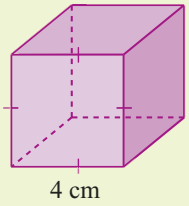
f



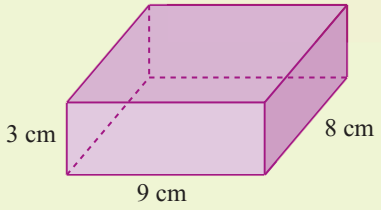
EXAMPLE 2

Calculate the surface area of each prism.

a



b



a Surface area $= 6 \times$ area of 1 face
$= 6 \times 4 \times 4$
$= 96 \text{ cm}^2$

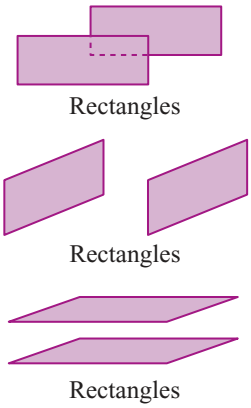
b Area of front and back $= 2 \times$ area of front
$= 2 \times 9 \times 3$
$= 54 \text{ cm}^2$

Area of sides $= 2 \times$ area of 1 side
$= 2 \times 8 \times 3$
$= 48 \text{ cm}^2$

Area of top and bottom $= 2 \times$ area of top
$= 2 \times 9 \times 8$
$= 144 \text{ cm}^2$

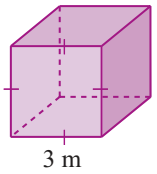
Surface area $= 54 + 48 + 144$
$= 246 \text{ cm}^2$

A rectangular prism has six faces and its opposite faces are equal in area.

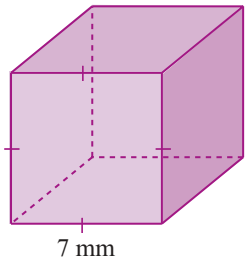


2 Calculate the surface areas of these cubes.

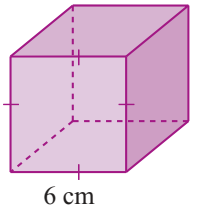
a



b



c

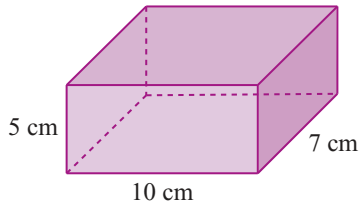


3 Calculate the surface area of a cube with sides of length:

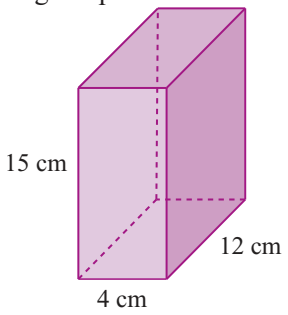
a 10 cm **b** 4.5 m **c** 9.8 mm

4 Calculate the surface areas of these rectangular prisms.

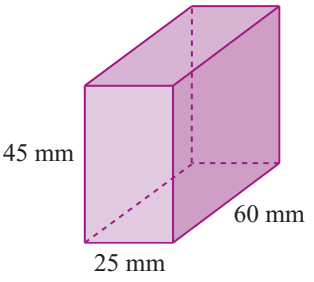
a



b



c



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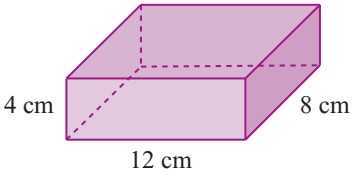
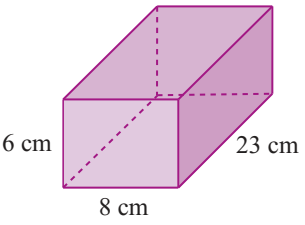
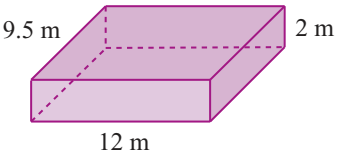


e

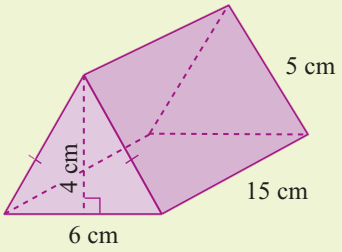


f



EXAMPLE 3

Calculate the surface area of this triangular prism.



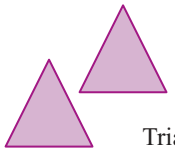
Drawing a diagram can help.

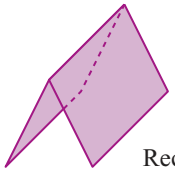
Area of 2 triangles (front and back) $= 2 \times$ area of front
$= 2 \times \frac{1}{2}(6 \times 4)$
$= 24 \text{ cm}^2$

Area of 2 rectangles (identical sides) $= 2 \times$ area of 1 side
$= 2 \times 15 \times 5$
$= 150 \text{ cm}^2$

Area of 1 rectangle (bottom) $= 15 \times 6$
$= 90 \text{ cm}^2$

Surface area $= 24 + 150 + 90$
$= 264 \text{ cm}^2$

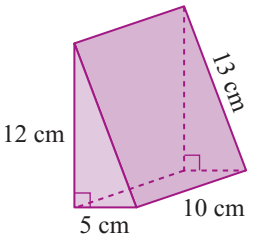

Triangles


Rectangles

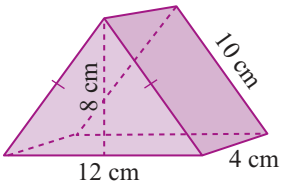

Rectangle

5 Calculate the surface areas of these triangular prisms.

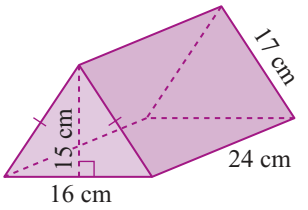
a



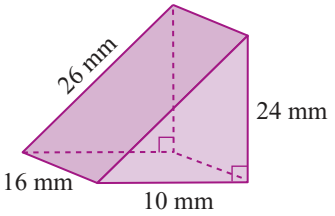
b



c



d



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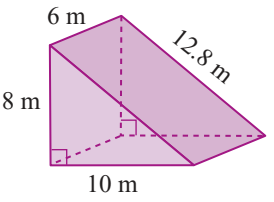
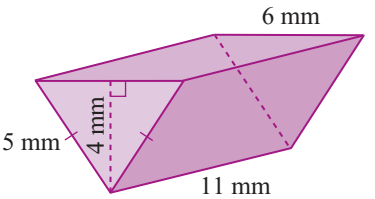


f



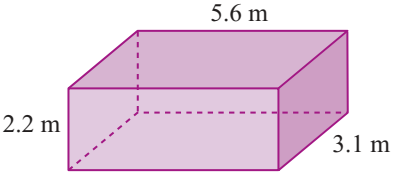
6 This chocolate box is in the shape of a triangular prism. The cross-section of the box is an equilateral triangle. Calculate the surface area of the chocolate box.

7 A shipping container has the shape of a rectangular prism. If it is 6 m long, 3 m wide and 4 m high, calculate the number of square metres of metal required to build the container.

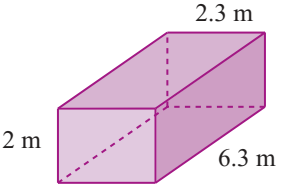
8 Which cereal box has the greater surface area?

9 **a** Calculate the wall area of Belinda's bedroom.

b Calculate the wall area of Tania's bedroom.



Belinda's room



Tania's room

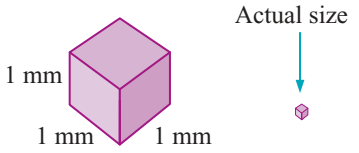
c Which bedroom has the greater wall area?

H Volume

Volume refers to the amount of space that an object occupies. The volume of a solid is the number of **cubic units** that it occupies. The most common metric units for measuring volume are:

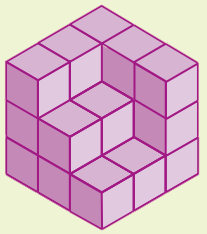
- cubic millimetres (mm^3)
- cubic centimetres (cm^3)
- cubic metres (m^3).

A cubic millimetre (1 mm^3) is the amount of space occupied by a cube of side length 1 mm.



EXAMPLE 1

Find the volume of this solid by counting cubes.



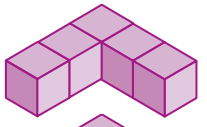
Method 1: Count the cubes in each layer separately.

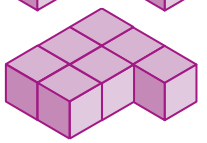
3rd layer = 5 cubes

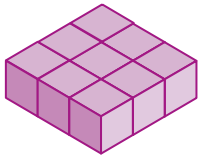
2nd layer = 7 cubes

1st layer = 9 cubes

Total = 21 cubes


3rd layer


2nd layer


1st layer

Method 2: Alternatively, make a plan of the solid and count the number of cubes in each stack.

Total = $(3 \times 5) + (2 \times 2) + (1 \times 2) = 21$ cubes

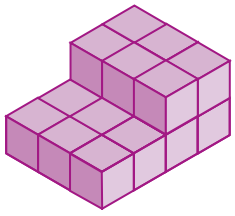
3	3	3
2	2	3
1	1	3

By both methods, the volume of the solid is 21 cubic units.

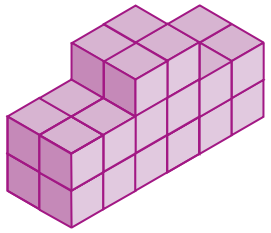
Exercise 12H

1 If the solids below are made from 1 cm^3 cubes, calculate the volume of each solid.

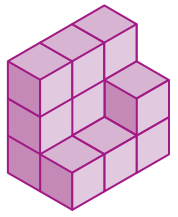
a



b



c



2 **a** Explain the meaning of a cubic centimetre. Illustrate with a diagram.

b Explain the meaning of a cubic metre.

Investigation 6 Volumes in life

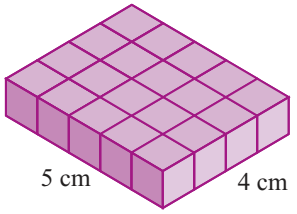
1 **a** Make a cube of side length 1 m. Use the cube to estimate the volume of your classroom, in cubic metres.
b Write a short report on how you made the cube and the method your group used to find the volume of your classroom.

2 Discuss which unit (mm^3, cm^3 or m^3) would be the most suitable to measure the volume of a:

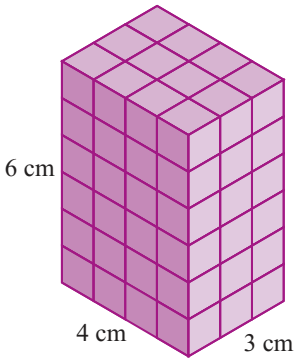
a fruit juice carton **b** key **c** caravan
d refrigerator **e** garage **f** house brick

Investigation 7 Volume of a rectangular prism

1 **a** Calculate the volume of this layer of cubes.
b What would be the volume of the rectangular prism formed by stacking up:
i 3 layers? **ii** 5 layers? **iii** 10 layers?

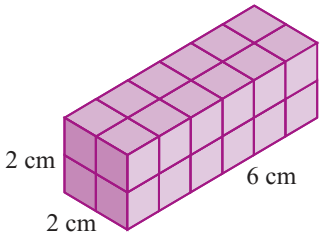


2 This rectangular prism was formed by stacking layers of cubes on top of each other.
a How many cubic centimetres are there in the bottom layer?
b How many layers are stacked on top of each other?
c What is the volume of this prism?

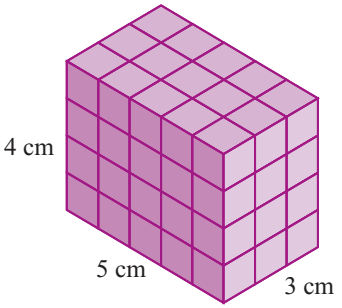


3 Repeat question **2** for each of these rectangular prisms.

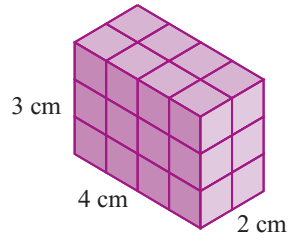
a



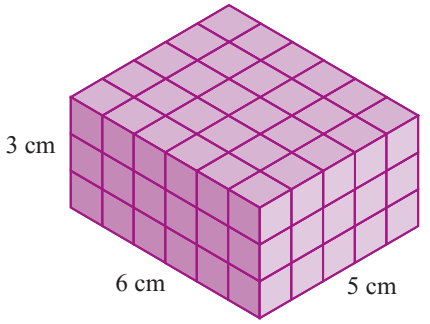
b



c



d



4 **a** Using the results of questions **2** and **3**, write in words and symbols a rule for calculating the volume of a rectangular prism.

b Discuss your rule with other groups and modify your rule if you wish.

12I Calculating volume

Volume of a rectangular prism

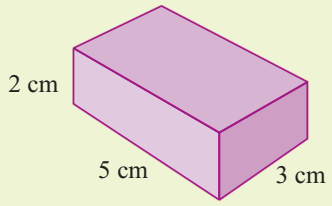
In Investigation 7 you developed a rule for finding the volume of a rectangular prism. One way of writing this rule is:

$$V = l \times b \times h \qquad \text{or} \qquad V = lbh$$

where V = the volume, b = the breadth, l = the length and h = the height.

EXAMPLE 1

Find the volume of this rectangular prism.



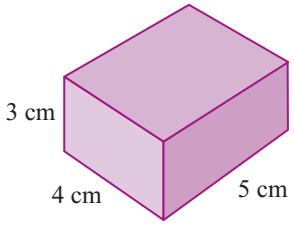
$V = lbh$

$= 5 \times 3 \times 2 = 30 \text{ cm}^3$

Exercise 12I

1 Complete the following to calculate the volume of each rectangular prism.

a

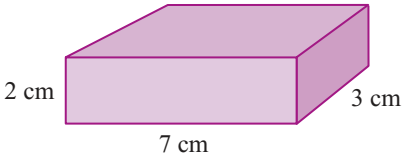


$V = lbh$

$= 5 \times \_\_\_\_ \times \_\_\_\_$

$= \_\_\_\_ \text{ cm}^3$

b



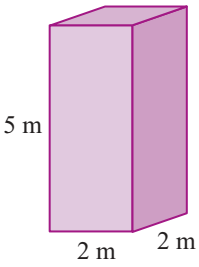
$V = lbh$

$= \_\_\_\_ \times 3 \times \_\_\_\_$

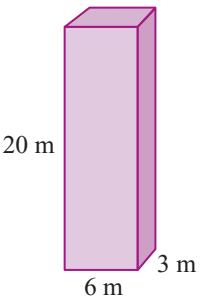
$= \_\_\_\_ \text{ cm}^3$

2 Calculate the volumes of these rectangular prisms.

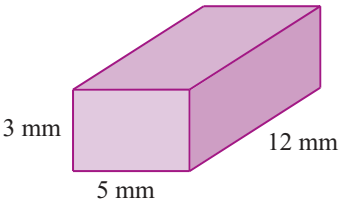
a



b



c



Volume using a base

When a solid has a uniform cross-section (base), the volume can be determined using the rule:

Volume = area of the uniform cross-section multiplied by the height

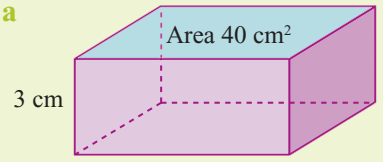
or Volume = area of base × height

The height is always measured *perpendicular to the base*. The base may be positioned on the top, bottom or side of a solid, so the height is not always vertical. The base may be a polygon (as in a prism) or round (as in a cylinder) or an irregular shape.

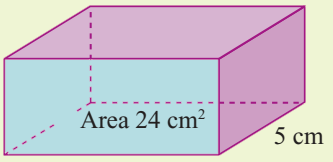
EXAMPLE 2

Calculate the volume of this rectangular prism. A different face has been selected as the base each time.

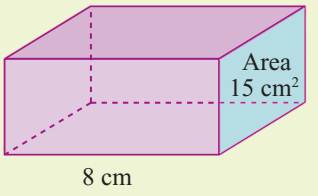
a



b



c



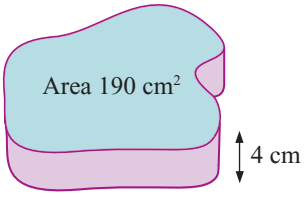
a V = area of base × height
= 40 × 3
= 120 cm³

b V = area of base × height
= 24 × 5
= 120 cm³

c V = area of base × height
= 15 × 8
= 120 cm³

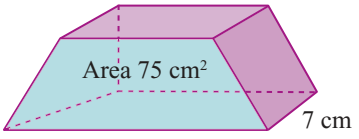
3 Calculate the volume of each solid, given the area of its base.

a



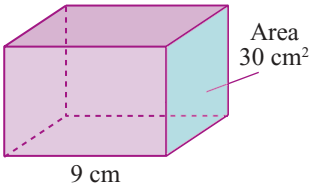
V = area of base × height
= 190 × ____
= ____ cm³

b



V = area of base × height
= ____ × 7
= ____ cm³

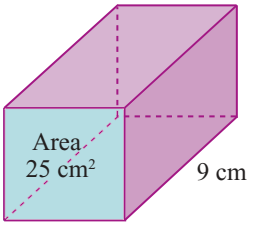
c



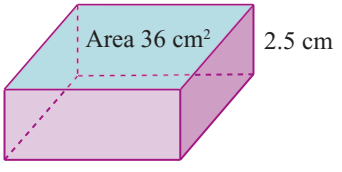
V = area of base × height
= ____ × ____
= ____ cm³

4 Calculate the volume of each solid, given the area of its base.

a



b



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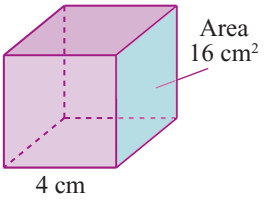

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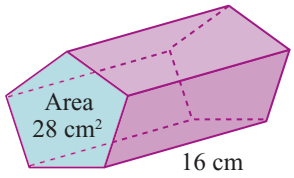
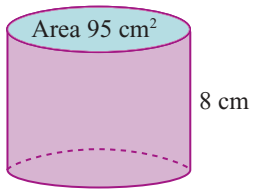
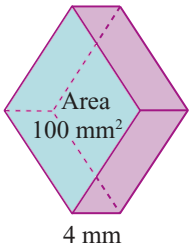


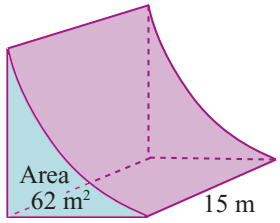
e



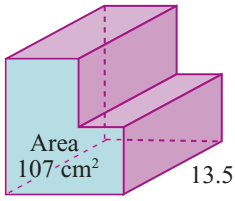
f



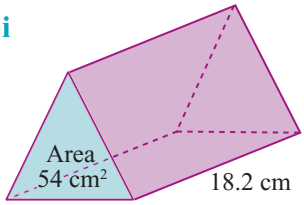
g



h



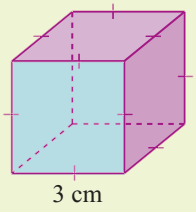
i



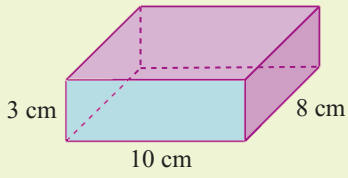
EXAMPLE 3

Calculate the volume of each rectangular prism.

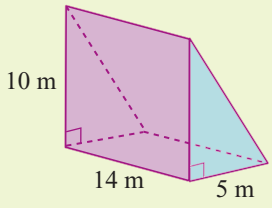
a



b



c



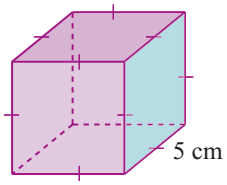
a V = area of base × height
$= (3 \times 3) \times 3$
$= 9 \times 3 = 27$ cm³

b V = area of base × height
$= (10 \times 8) \times 3$
$= 80 \times 3 = 240$ cm³

c V = area of base × height
$= \frac{1}{2}(5 \times 10) \times 14$
$= 25 \times 14 = 350$ cm³

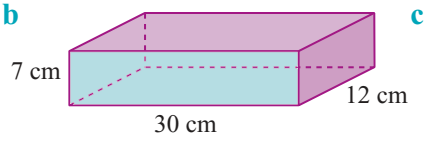
5 Complete the following to calculate the volume of each prism.

a



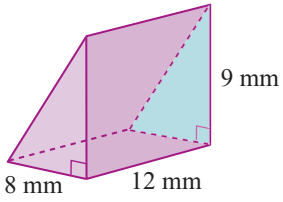
V = area of base × height
$= (5 \times 5) \times$ ____
$=$ ____ cm³

b



V = area of base × height
$= (30 \times$ ____$) \times$ ____
$=$ ____ cm³

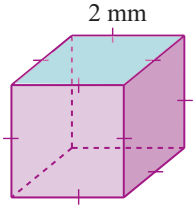
c



V = area of base × height
$= \frac{1}{2}(8 \times$ ____$) \times$ ____
$=$ ____ mm³

6 Calculate the volumes of the following prisms.

a



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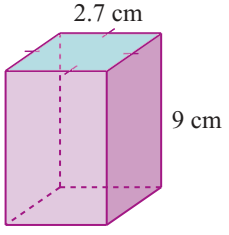

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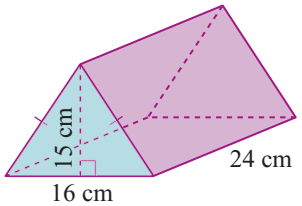
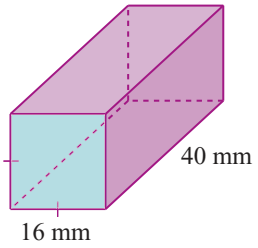


d



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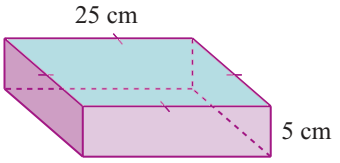

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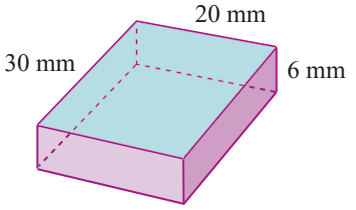

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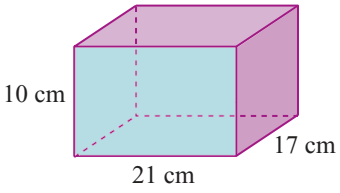

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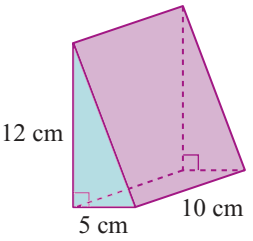

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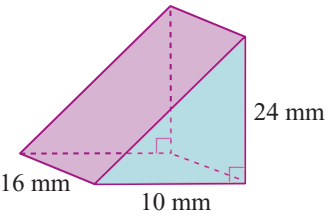

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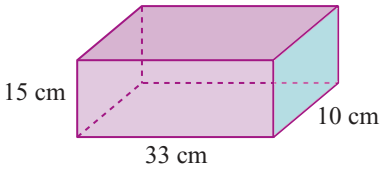
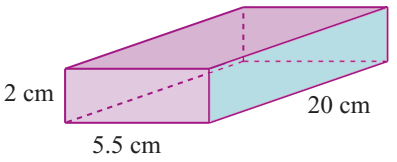
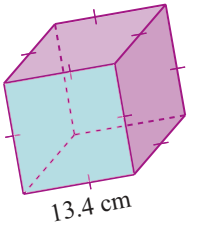


k



l

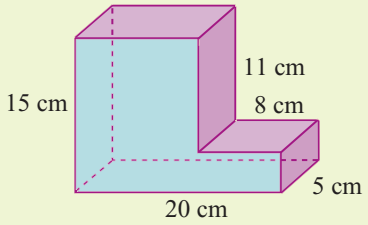


EXAMPLE 4

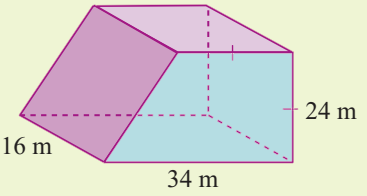
A composite solid is a combination of two or more standard geometric solids.

Calculate the volume of each **composite solid**.

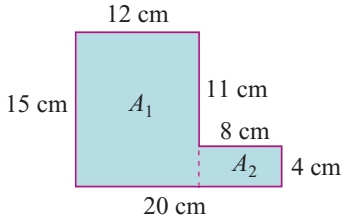
a

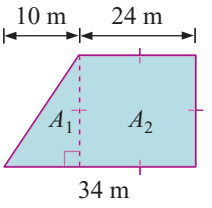


b



First draw the base of the solid. Use addition or subtraction to find any missing dimensions.





a Area of base $= A_1 + A_2$

$= (15 \times 12) + (8 \times 4)$

$= 180 + 32$

$= 212 \text{ cm}^2$

Height $= 5$ cm

Volume $=$ area of base $\times$ height

$= 212 \times 5$

$= 1060 \text{ cm}^3$

b Area of base $= A_1 + A_2$

$= \frac{1}{2}(10 \times 24) + (24 \times 24)$

$= 120 + 576$

$= 696 \text{ m}^2$

Height $= 16$ m

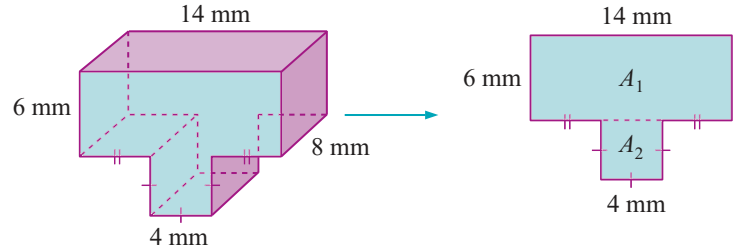
Volume $=$ area of base $\times$ height

$= 696 \times 16$

$= 11\,136 \text{ m}^3$

7 Complete the calculate the volumes of these composite shapes.

a

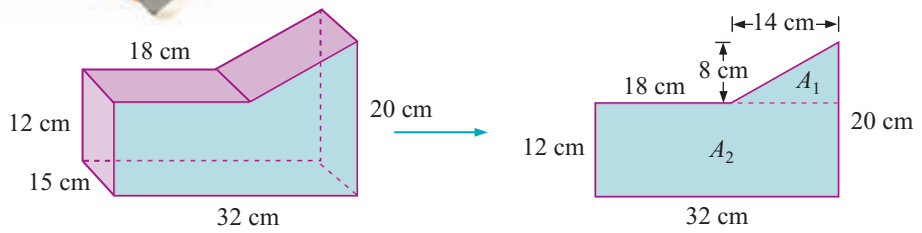


Area of base $= A_1 + A_2$
$= (14 \times \_\_\_) + (4 \times \_\_\_)$
$= \_\_\_ + \_\_\_ = \_\_\_ \text{ mm}^2$

Height = ___ mm

Volume = area of base × height
$= \_\_\_ \times \_\_\_ = \_\_\_ \text{ mm}^3$

b



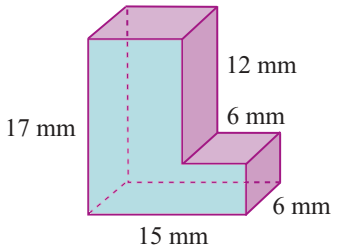
Area of base $= A_1 + A_2$
$= \frac{1}{2}(14 \times \_\_\_) + (32 \times \_\_\_)$
$= \_\_\_ + \_\_\_ = \_\_\_ \text{ cm}^2$

Height = ___ cm

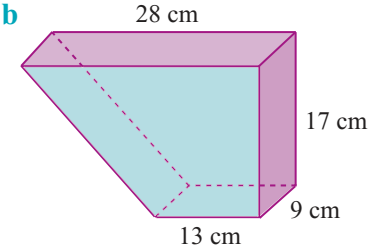
Volume = area of base × height
$= \_\_\_ \times \_\_\_ = \_\_\_ \text{ cm}^3$

8 Calculate the volumes of the following composite shapes.

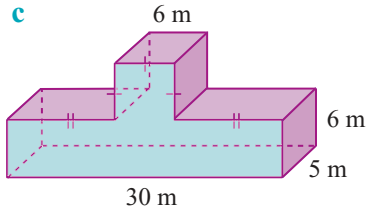
a



b



c



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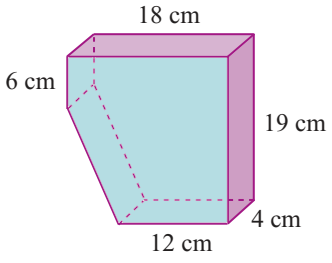

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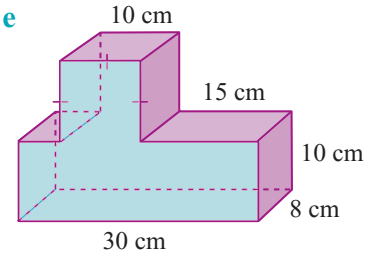

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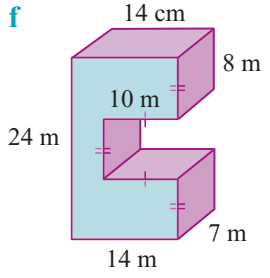

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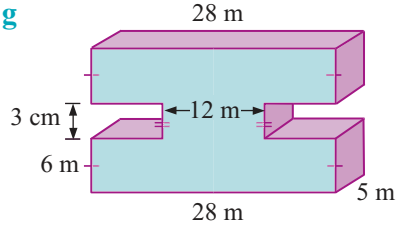
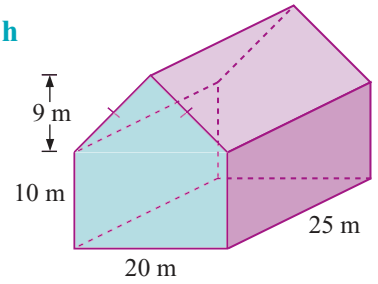
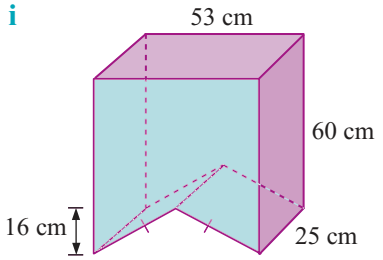


h



i



Investigation 8 Volume and capacity

1 *Step 1*: Take an empty milk carton and open the top so that it is in the shape of a rectangular prism.
Step 2: Measure the length and breadth of the base (bottom).
Step 3: Pour one litre (1 L) of water into the carton and measure the height of the water in the carton.
Step 4: Calculate the volume of water in cubic centimetres. How much space does 1 L of water occupy?

2 *Step 1:* Take a small box of fruit-drink that is in the shape of a rectangular prism.
Step 2: Measure the dimensions of the box, and calculate its volume in cubic centimetres.
Step 3: Compare this volume with the number of millilitres (mL) marked on the box (the **capacity** of the box). What do you notice?

3 Repeat question **2** for a 1 L box of fruit-drink or milk.

4 *Step 1*: Pour some water into a measuring cylinder and accurately note the level of the water in the cylinder in millilitres (mL).
Step 2: Drop 10 plastic centicubes into the cylinder. Record the new level of the water.
Step 3: Calculate how much water has been displaced by the 10 centicubes. This is the amount of water that occupies 10 cm^3. How much water would occupy 1 cm^3?

5 Repeat question **4** for 20 and 30 centicubes. How many centicubes would it take to displace:
a 100 mL of water? **b** 500 mL of water? **c** 1 L of water?

6 From the investigation, what conclusions can be drawn about the quantities 1 mL and 1 L and the volume that each occupies?

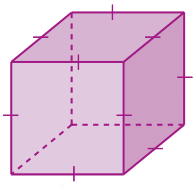
There are 1000 mL in 1 L.

7 Complete these definitions of volume and capacity:
a _____ is the amount of space occupied by a solid or quantity of liquid.
b _____ is the volume of liquid that a container can hold.

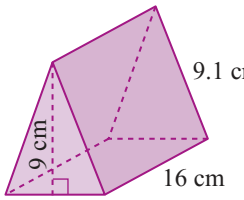
Investigation 9 Relationship between surface area and volume

Complete the following questions to determine whether there is a relationship between surface area and volume.

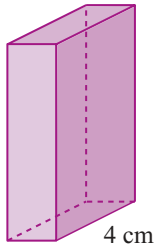
1 **a** Calculate the surface area and volume of each prism.



Prism 1



Prism 2



Prism 3

b What do you notice about the surface area of each prism?
c What do you notice about the volume of each prism?

2 **a** Construct two prisms that are different yet have the same volume.
b Draw a sketch of each, showing its dimensions.
c Calculate the surface areas of your prisms. Are they the same or different?

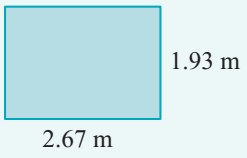
3 What do you notice about the relationship between surface area and volume?

Calculator activities

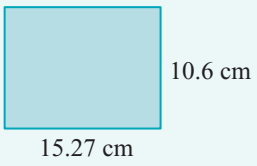
Calculators can be very useful, if you know how to use them! But remember, you still have to know the process.

1 Find the areas of the following squares, rectangles, triangles and parallelograms.

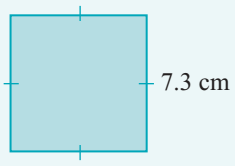
a



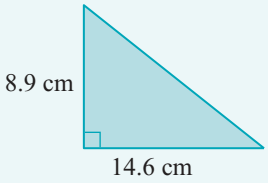
b



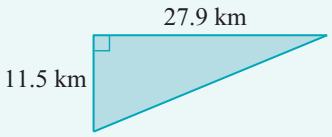
c



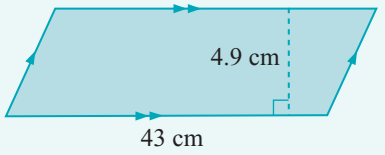
d



e

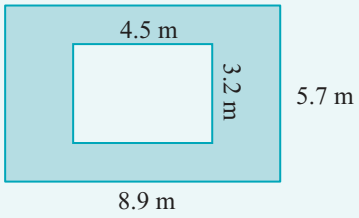


f

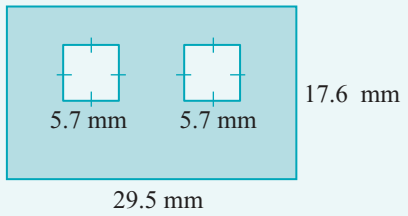


2 Find the shaded area in each of these shapes.

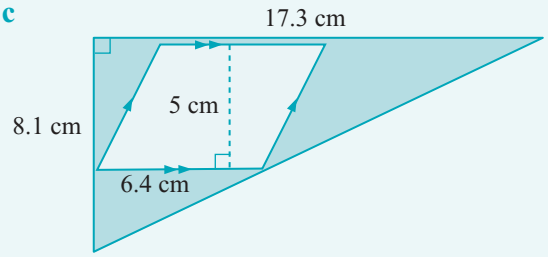
a



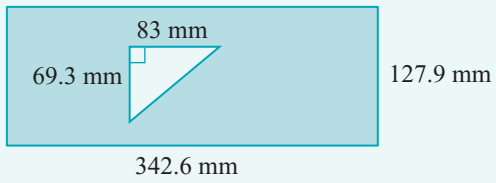
b



c

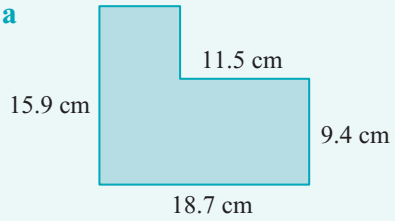


d

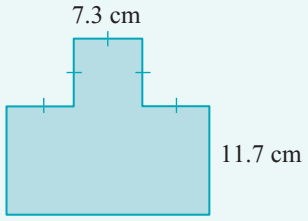


3 Find the areas of the following shapes.

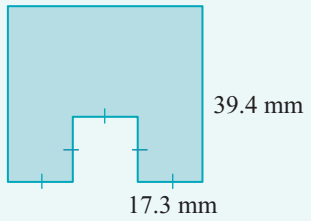
a



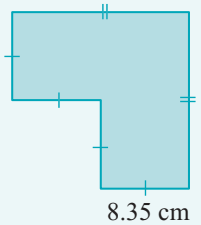
b



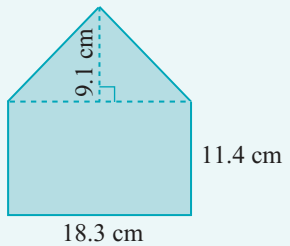
c



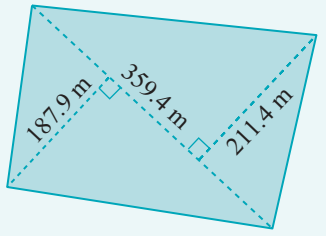
d



e



f



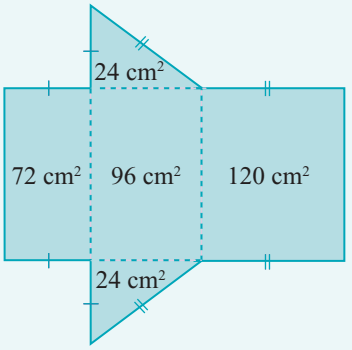
4 Kelly is redecorating her room. The dimensions of her bedroom are 3.4 m by 4.8 m by 3 m high.

a Calculate the total area of the four walls.

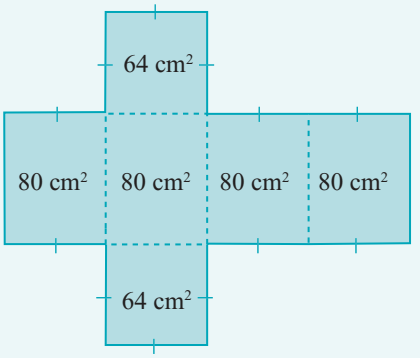
b The total area of the windows is 15 m^2 and of the doors is 9 m^2. Find the wall area to be painted.

5 Calculate the surface area of each solid whose net is shown.

a

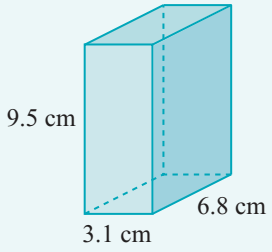


b

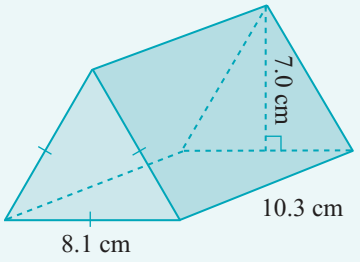


6 Calculate the surface area of each prism shown.

a

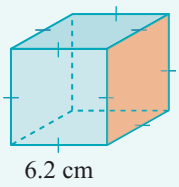


b



7 Calculate the volume of each prism shown.

a



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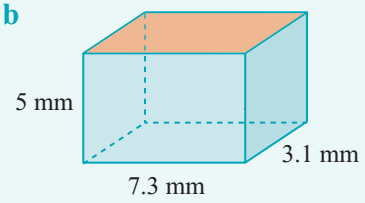
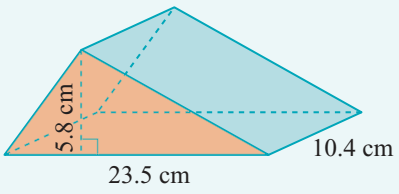
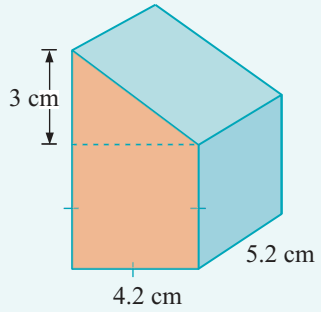


c

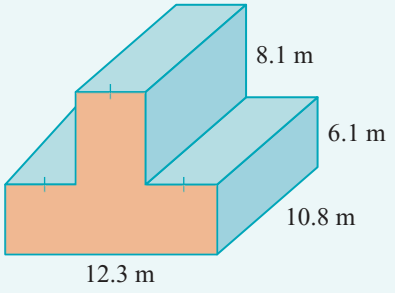


8 Calculate the volume of each composite solid shown.

a



b



Language in mathematics

1 Here is a find-a-word puzzle. Find all the words in the following list:
breadth, capacity, container, conversion, convert, cylinder, dimensions, estimate, gram, gross, height, kilolitre, length, liquid, mass, measure, milligram, millilitre, modify, net, object, prism, recommend, rectangular, tonne, volume, weight

K	I	L	O	L	I	T	R	E	A	X	D	S	S	O	R	G
T	H	D	A	E	R	B	A	S	E	F	N	T	B	J	D	F
C	D	B	H	E	I	G	H	T	C	T	O	C	K	G	I	E
C	X	E	Z	A	F	E	D	I	G	H	I	E	M	H	M	I
R	G	R	A	M	J	N	G	M	I	G	S	J	I	U	E	L
A	M	E	A	S	U	R	E	A	Y	I	R	B	L	Y	N	E
L	B	N	W	K	V	L	L	T	H	E	E	O	L	O	S	R
U	L	I	Q	U	I	D	I	E	T	W	V	N	I	J	I	T
G	E	A	A	T	Y	C	T	N	M	Q	N	Z	G	P	O	I
N	N	T	U	X	A	P	R	I	S	M	O	M	R	I	N	L
A	G	N	W	P	T	R	E	V	N	O	C	Q	A	H	S	I
T	T	O	A	S	T	O	N	N	E	D	O	U	M	S	L	L
C	H	C	Y	L	I	N	D	E	R	I	B	T	A	P	C	L
E	W	D	V	Z	R	A	S	Q	N	F	O	M	Q	K	U	I
R	E	C	O	M	M	E	N	D	E	Y	N	R	R	M	S	M

2 Write two common metric units of:

a area **b** volume **c** capacity

3 What does each of these prefixes mean?

a milli- **b** centi- **c** kilo-

4 Complete the following words, used in this chapter, by inserting the vowels.

sp__c__ pr__sm c__nt__ __n__r s__rf__c__

cr__ss-s__ct__ __n v__l__m__ l__tr__ c__p__c__ty

5 Three of the words in the following list are spelt incorrectly. Find these words and write the correct spelling.
heigt lenth liquid mililitre occupy uniform

6 Use a dictionary to find two meanings of each word.

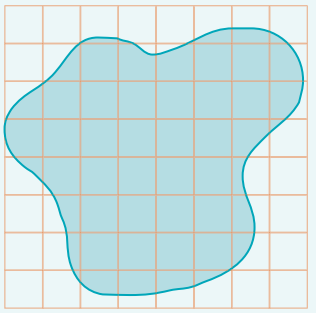
a net **b** solid **c** volume

Terms

area base breadth capacity composite corresponding cross-section
cubic units dimensions face height irregular net perpendicular
prism quantity solid square units surface area uniform volume

Check your skills

1 Estimate the area of the shaded shape, to the nearest square unit.

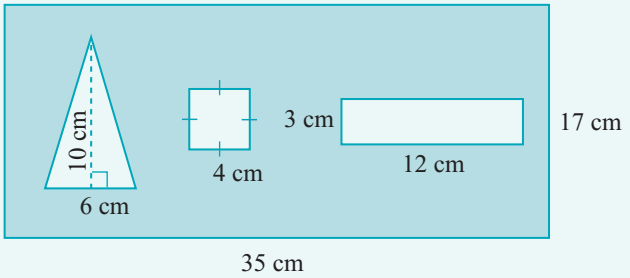


A 33 **B** 37
C 40 **D** 48

2 The area of a square is 25 cm^2. What is its side length?

A 10 cm **B** 15 cm
C 5 cm **D** 25 cm

3 Calculate the shaded area.

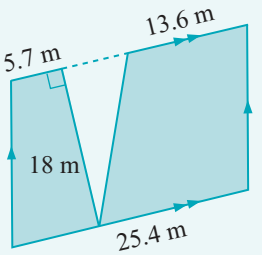


A 595 cm^2 **B** 513 cm^2
C 483 cm^2 **D** 479 cm^2

4 Calculate the area of a parallelogram with a base length of 9 m and a height of 5.5 m.

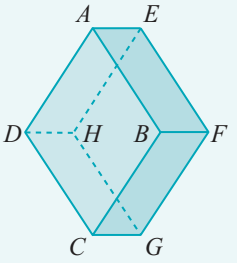
A 49 m^2 **B** 54 m^2
C 45.9 m^2 **D** 49.5 m^2

5 Calculate the area of the composite shape shown.



A 347.4 m^2 **B** 405.9 m^2
C 250.5 m^2 **D** 402.3 m^2

Questions **6** to **8** refer to the prism on the right.



6 The face *AEFB* corresponds to the face:

A *BFGC* **B** *FBCG*
C *DHCG* **D** *DHGC*

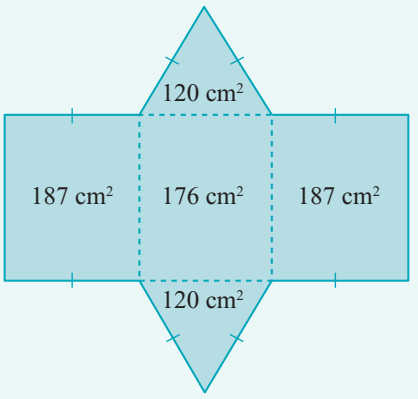
7 The face *EFGH* corresponds to the face:

A *ADBC* **B** *ABFE*
C *ABCD* **D** *DHGC*

8 The face *BFGC* corresponds to the face:

A *BADC* **B** *AEHD* **C** *BCGF* **D** *ADEH*

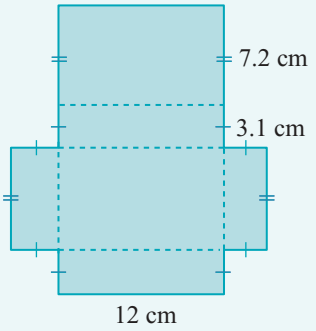
9 The surface area of the solid whose net is shown is:



A 670 cm^2 **B** 483 cm^2
C 772 cm^2 **D** 790 cm^2

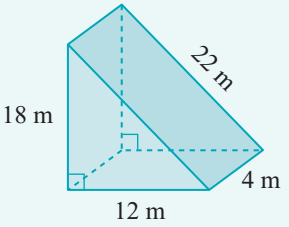
10 The surface area of the solid made from this net is:

A 237.65 cm^2 **B** 193.44 cm^2
C 291.84 cm^2 **D** 274.14 cm^2



11 Calculate the surface area of this triangular prism.

A 424 m^2 **B** 640 m^2
C 304 m^2 **D** 1008 m^2



12 Calculate the surface area of a cube with sides 5.5 cm.

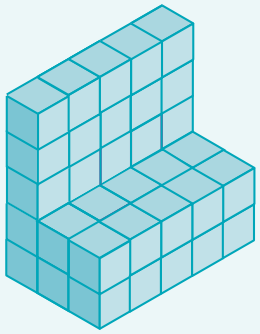
A 166.375 cm^2 **B** 181.5 cm^2 **C** 166.375 cm^3 **D** 181.5 cm^3

13 A shipping container is in the shape of a rectangular prism. It is 8 m long, 4.2 m wide and 4 m high. Calculate the number of square metres of metal required to build the container. (Calculate its surface area.)

A 132.8 m^2 **B** 164.8 m^2 **C** 131.2 m^2 **D** 153.8 m^2

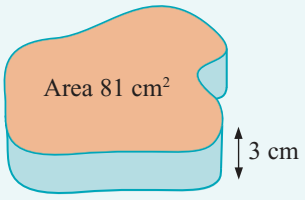
14 If the following solid is made from 1 cm^3 cubes, calculate its volume.

A 35 cm^3 **B** 30 cm^3
C 25 cm^3 **D** 45 cm^3



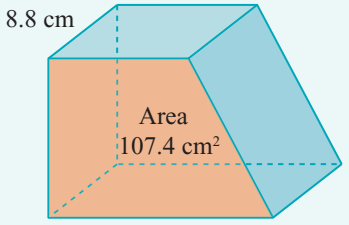
15 Calculate the volume of the following solid.

A 84 cm^3 **B** 243 cm^3
C 27 cm^3 **D** 6564 cm^3



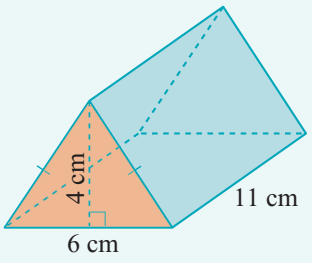
16 Calculate the volume of the solid shown on the right.

A 907.8 cm^3 **B** 921.36 cm^3
C 945.12 cm^3 **D** 973.51 cm^3



17 Calculate the volume of this prism.

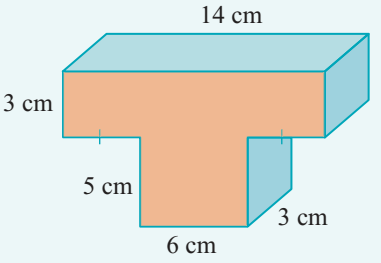
A 110 cm^3 **B** 70 cm^3
C 264 cm^3 **D** 132 cm^3



18 Calculate the volume of this composite solid.

A 216 cm^3 **B** 196 cm^3

C 236 cm^3 **D** 266 cm^3

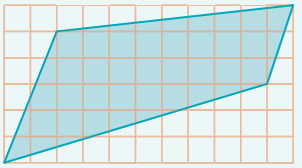


If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2	3	4, 5	6–8	9, 10	11–13	14	15–18
Section	A	B	C	D	E	F	G	H	I

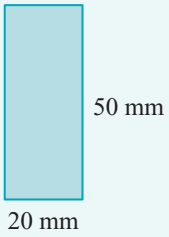
12A Review set

1 By counting squares, estimate the area of the shaded shape.

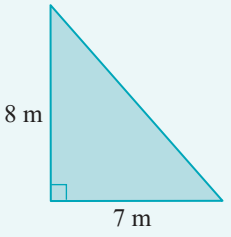


2 Find the areas of the following shapes.

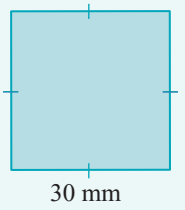
a



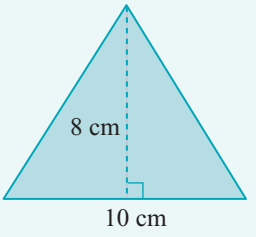
b



c

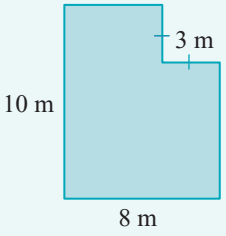


d

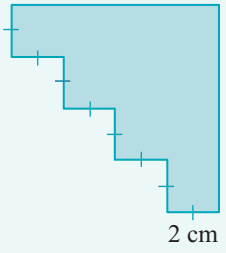


3 Find the areas of these composite shapes.

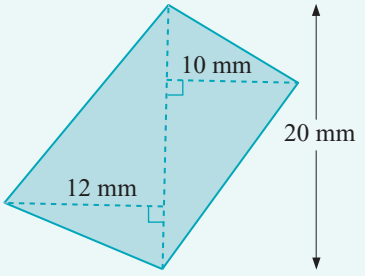
a



b

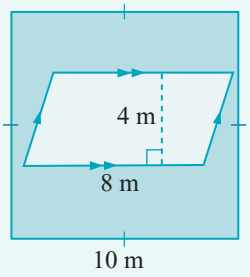


c

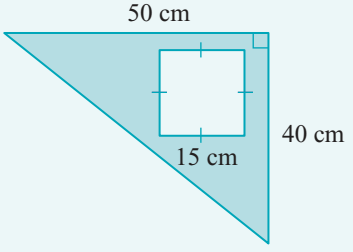


4 Find the shaded area in each shape.

a



b

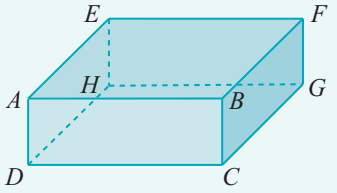


5 *ABCD* is the front face of this rectangular prism.

a Name the back face.

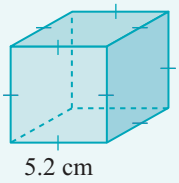
b Name the top and bottom faces.

c Name the two sides.

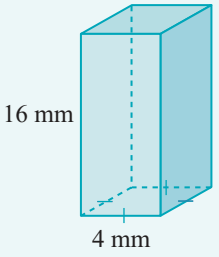


6 **a** Construct a net of the cube shown.

b Use the net to calculate the surface area of the cube.



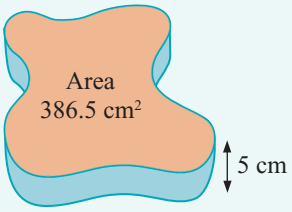
7 Calculate the surface area of this prism.



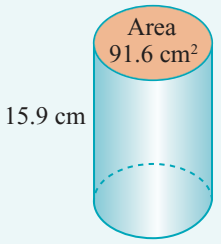
8 Define the term 'surface area'.

9 Calculate the volumes of the following solids.

a

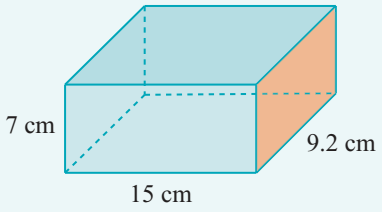


b

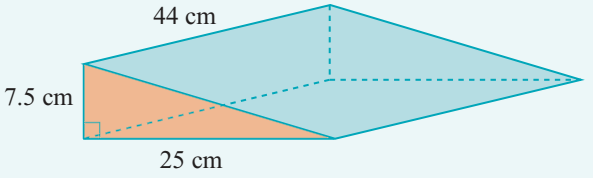


10 Calculate the volumes of the following prisms.

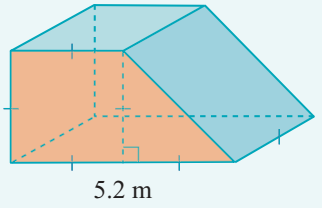
a



b

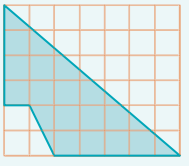


11 Calculate the volume of this composite solid.



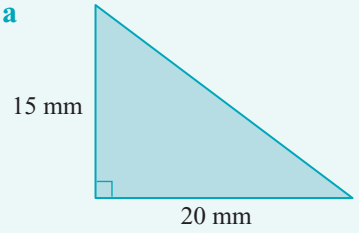
12B Review set

1 By counting squares, estimate the area of the shaded shape.

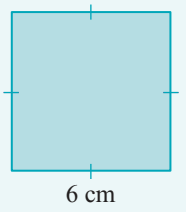


2 Find the areas of the following shapes.

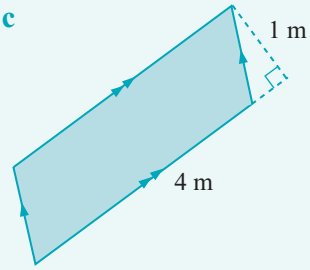
a



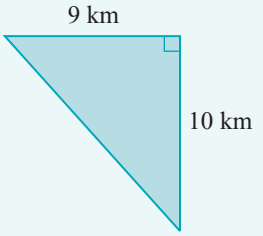
b



c

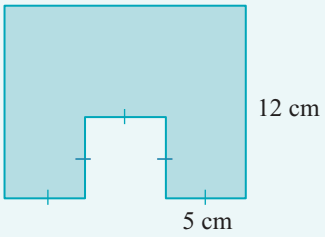


d

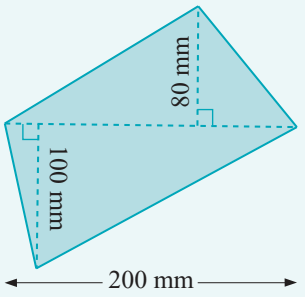


3 Find the areas of these composite shapes.

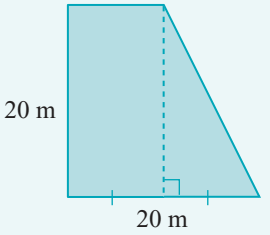
a



b

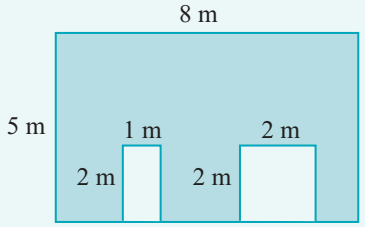


c

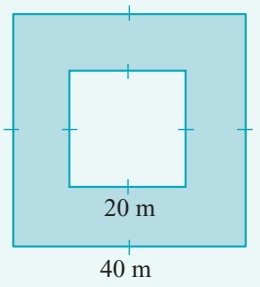


4 Find the shaded area in each shape.

a



b



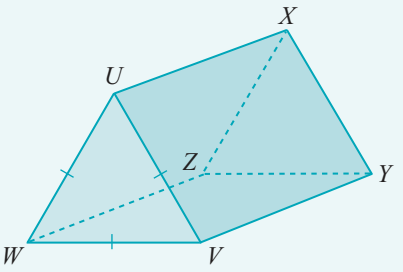
5 The diagram shows a triangular prism.

a Name the front face of the prism.

b What type of triangle is used to form the front face?

c Name the back face of the prism.

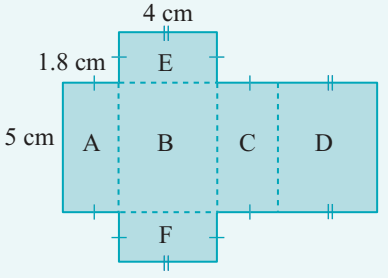
d What shape is the face *WZYV*?



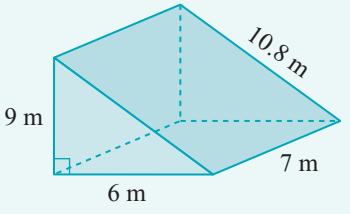
6 **a** Name the prism formed from the net shown.

b Calculate the area of each section labelled A to F.

c Calculate the total surface area of the prism.

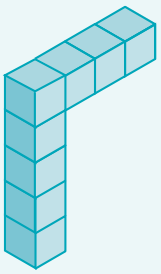


7 Calculate the surface area of this triangular prism.

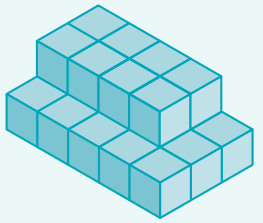


8 What is the volume of each construction if 1 cm^3 cubes were used?

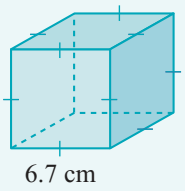
a



b

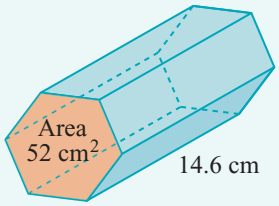


9 Calculate the volume of the following solid.

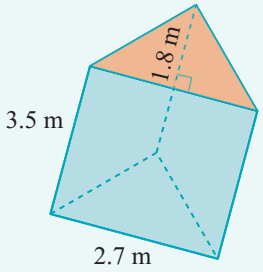


10 Calculate the volumes of the following prisms.

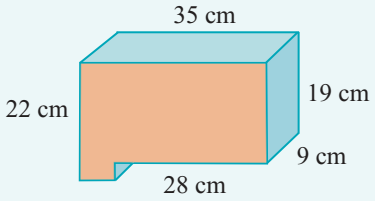
a



b

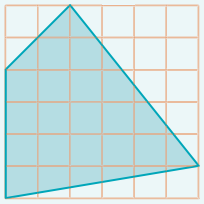


11 Calculate the volume of this composite solid.



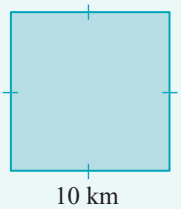
12C Review set

1 By counting squares, estimate the area of the shaded shape.

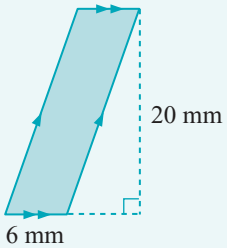


2 Find the areas of the following shapes.

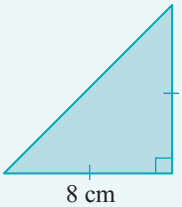
a



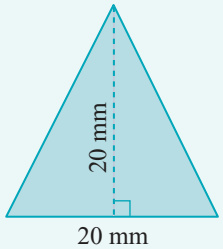
b



c

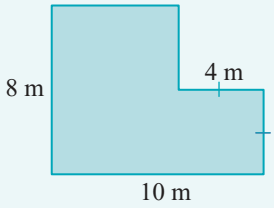


d

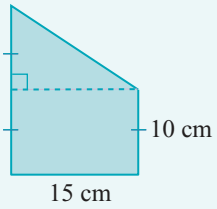


3 Find the areas of these composite shapes.

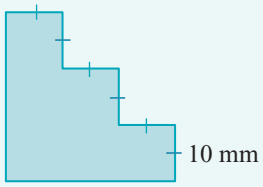
a



b

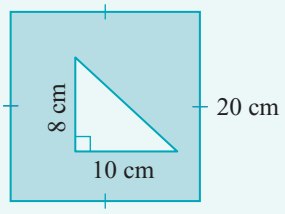


c

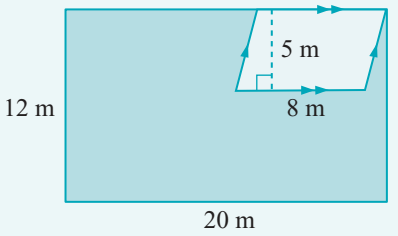


4 Find the shaded area in each shape.

a

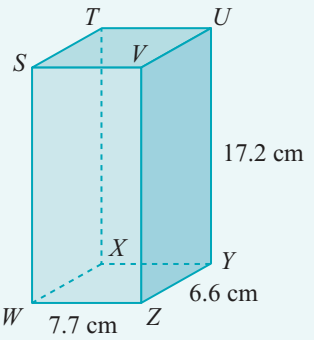


b



5 Complete the following table showing the dimensions of each face of this prism.

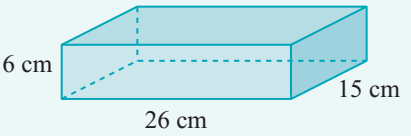
Face	Length	Breadth
STUV		
WXYZ		
SVZW		
TUYX		
VUYZ		
STXW		



6 Calculate the surface area of the rectangular prism shown in question **5**.

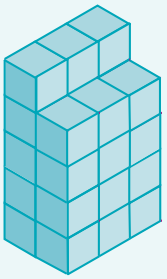
7 **a** Construct a net of this rectangular prism and show the dimensions on each section.

b From the net, calculate the surface area of the prism.

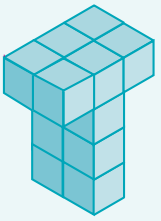


8 Find the volume in cubic centimetres of each solid, built using cubes of 1 cm^3.

a

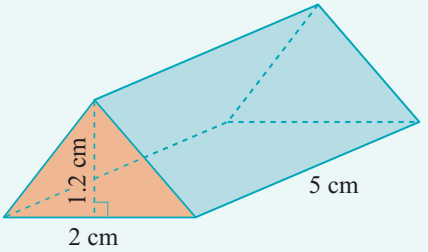


b



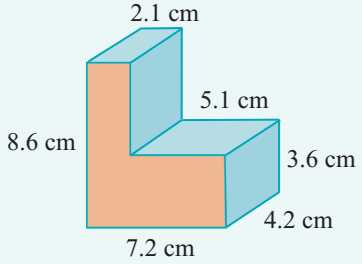
9 Calculate the volume of a rectangular prism with a length of 0.9 m, breadth of 5.3 m and height of 1.6 m.

10 Calculate the volume of this triangular prism.

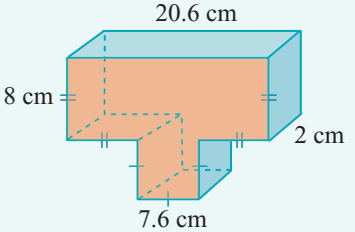


11 Calculate the volumes of these composite solids.

a



b



13

Data measures

This chapter deals with the interpretation of statistics.

At the end of this chapter you should be able to:

- find the mean, mode, median and range for scores in a variety of displays
- know which of mean, mode and median is appropriate in a particular situation
- compare data using mean, mode, median and range.

Diagnostic test

Use the following graph to answer questions **1** to **3**.

Month	Number of sunny days
January	☀☀☀☀☀☀
February	☀☀☀☀
March	☀☀☀☀
April	☀☀

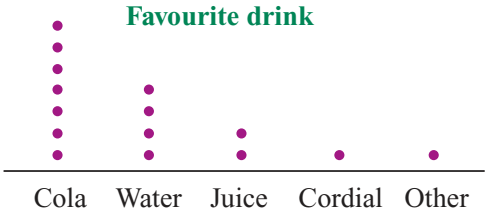
Scale: ☀ represents 4 sunny days.

1 How many sunny days occurred in March?
A 12 **B** 14 **C** 16 **D** 18

2 Which month had 8 sunny days?
A January **B** February
C March **D** April

3 The scale is changed so that the sun symbol represents 3 sunny days. How many sun symbols would January have with the new scale?
A 3 **B** 6 **C** 8 **D** 10

Use the following graph to answer questions **4** to **6**.



4 What type of graph is shown?
A sector graph **B** dot plot
C line graph **D** bar chart

5 How many people chose water as their favourite drink?
A 7 **B** 5 **C** 4 **D** 1

6 How many people were surveyed?
A 7 **B** 14 **C** 15 **D** 16

Use this two-way table to answer questions **7** to **10**.

Favourite colour

Colour	Boys	Girls
Blue	8	2
Pink	1	7
Black	10	6
Other	1	5

7 How many boys were surveyed?
A 19 **B** 20 **C** 21 **D** 22

8 What was the most popular colour choice for boys?
A black **B** blue
C pink **D** other

9 What fraction of girls chose pink?
A $\frac{6}{20}$ **B** $\frac{7}{20}$ **C** $\frac{6}{15}$ **D** $\frac{7}{15}$

10 This information is drawn in a side-by-side column graph. Which column will be the tallest?
A boy's blue **B** girl's pink
C girl's black **D** boy's black

Questions **11** to **14** refer to the following data set:
4, 6, 3, 5, 7, 4, 7, 4, 5.

11 What is the middle number when the data is put in order?
A 4 **B** 5 **C** 6 **D** 7

12 Which number appears the most times?
A 7 **B** 6 **C** 5 **D** 4

13 What is the range of the numbers?
A 2 **B** 3 **C** 4 **D** 5

14 If this data is displayed in a dot plot, which numbers have the same number of dots?
A 3 and 4 **B** 3 and 5
C 5 and 7 **D** 3 and 6

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–6	7–10	11–14
Section	ACMSP 120	ACMSP 147	ACMSP 148

A Mean

To make collected data easier to understand and read, it is often condensed into a single number that is thought to be representative of all the data. The common name for this number is the **average**. Statisticians refer to the average as a measure of **central tendency**. There are three commonly used methods of calculating measures of central tendency: the mean, the mode and the median. They may also be described as measures of centre or measures of location.

This section examines the mean. The **mean** of a number of scores is the sum of all the scores divided by the number of scores. The symbol for the mean is $\bar{x}$. The mean can only be calculated from numerical data.

EXAMPLE 1

Find the mean of each set of scores.

a 3, 9, 5, 7, 10, 8 **b** 12, 15, 15, 11, 13, 10, 8, 6, 9, 7

a $\text{Mean} = \dfrac{3+9+5+7+10+8}{6} = \dfrac{42}{6} = 7$

b $\text{Mean} = \dfrac{12+15+15+11+13+10+8+6+9+7}{10} = \dfrac{106}{10} = 10.6$

The mean does not have to be one of the scores.

Exercise 13A

1 Complete the following to find the mean of 8, 9, 10, 11, 11, 12.

$\bar{x} = \dfrac{8+9+\ldots}{6} = \dfrac{\square}{6} = \_\_\_$

2 Nikki completed question **1** using her calculator and her answer was 51. What mistake did she make?

3 Find the mean (to 1 decimal place if necessary) of each set of data.

a 2, 4, 5, 6, 9, 9, 10 **b** 2, 3, 3, 4, 5, 6, 7, 8, 9

c 11, 13, 13, 16, 17 **d** 27, 28, 29, 27, 30, 31, 27, 31, 30

e 0, 2, 4, 5, 7, 6, 4, 5, 4, 0, 1 **f** 20, 20, 20, 23, 25, 27

g 51, 52, 54, 55, 57, 57, 58, 59 **h** 1, 1, 2, 4, 4, 4, 4, 7, 7, 8, 9, 10

i 240, 243, 245, 246, 244, 243 **j** 104, 101, 104, 102, 104, 105, 106, 101

EXAMPLE 2

Use the statistics function of your calculator to find the mean of 7, 5, 9, 6, 5, 7, 4.

Here is one way of finding the mean, using a CASIO calculator. Check the instruction booklet for *your* calculator to determine the appropriate steps.

Step 1: Set the calculator to statistics mode (SD) by pressing MODE and the key for statistics functions.

Step 2: Make sure the statistics memory is clear by pressing SHIFT AC.

Step 3: Press 7 M+ 5 M+ 9 M+ 6 M+ 5 M+ 7 M+ 4 M+ to enter the scores.

Step 4: Press the keys for the mean. $\bar{x} = 7.2$ (to 1 decimal place)

4 Use your calculator to check the mean of each part of question **3**.

EXAMPLE 3

Find the mean of the scores given in this frequency distribution table.

Score	4	5	6	7	8
Frequency	3	2	4	8	6

For simplicity, use x for the values of the scores and f for the frequencies. Add an $f \times x$ column to the table.

Score (x)	Frequency (f)	$f \times x$	
4	3	$3 \times 4 = 12$	This is the sum of all the 4s.
5	2	$2 \times 5 = 10$	This is the sum of all the 5s.
6	4	$4 \times 6 = 24$	This is the sum of all the 6s.
7	8	$8 \times 7 = 56$	This is the sum of all the 7s.
8	6	$6 \times 8 = 48$	This is the sum of all the 8s.
	$\Sigma f = \mathbf{23}$	$\Sigma fx = \mathbf{150}$	This is the sum of all the 4s, 5s, 6s, 7s and 8s.

Σf = the sum of the frequencies = the total number of scores = 23

Σfx = the sum of the subtotals 12, 10, 24, 56 and 48
= the sum of all the scores = 150

$\therefore$ Mean $(\bar{x}) = \dfrac{\text{sum of all scores}}{\text{number of scores}} = \dfrac{150}{23} = 6.5$ (to 1 decimal place)

The Greek letter Σ is used to mean the 'sum of'.

EXAMPLE 4

Find the mean for the frequency distribution shown.

x	23	24	25	26	27
f	5	8	9	12	3

Add an $f \times x$ column to the table.

x	f	fx
23	5	115
24	8	192
25	9	225
26	12	312
27	3	81
	$\Sigma f = \mathbf{37}$	$\Sigma fx = \mathbf{925}$

$\therefore \bar{x} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{925}{37} = 25$

5 **a** Complete this frequency distribution table.
b Calculate the mean, correct to 1 decimal place.

Score (x)	Frequency (f)	$f \times x$
8	6	48
9	8	
10	15	
11	11	121
12	3	
	$\Sigma f =$	$\Sigma fx =$

6 **a** Complete this frequency distribution table.
b Calculate the mean, correct to 1 decimal place.

Score (x)	Frequency (f)	$f \times x$
18	3	
19	5	95
20	10	
21	15	
22	8	
23	1	
	$\Sigma f =$	$\Sigma fx =$

7 For each of the following frequency distribution tables:
i Copy the table and add an fx column.
ii Calculate the mean.

a

x	13	14	15	16	17
f	2	3	6	4	1

b

x	2	3	4	5	6
f	4	6	5	3	2

c

x	50	51	52	53	54	55
f	3	5	8	6	2	4

d

x	18	19	20	21	22
f	12	28	25	26	9

EXAMPLE 5

Use your calculator to find the mean of this distribution.

x	43	44	45	46	47
f	6	8	11	9	7

Here are possible steps, using a CASIO calculator (check the instruction booklet for your own calculator).

Step 1: Set the calculator to statistics mode (SD) by pressing MODE and the key for statistics functions.

Step 2: Press SHIFT AC to clear the contents of the memory.

Step 3: Press 43 × 6 M+ 44 × 8 M+ 45 × 11 M+ 46 × 9 M+ 47 × 7 M+ .

Step 4: Press the keys for the mean. $\bar{x} = 45.1$ (to 1 decimal place)

8 Use your calculator to find the mean of the data given in each part of question **7**.

B Mode and range

The **mode** is the score that occurs most often, or the score with the highest frequency. It may be used for numerical *or* categorical data. It is the only measure that can be used for categorical data.

The **range** is the simplest measure of the spread of scores. It is calculated using:

range = highest score − lowest score.

EXAMPLE 1

Find the mode for each set of scores.

a 3, 7, 8, 9, 9 **b** 10, 6, 14, 9, 14, 7 **c** 4, 5, 8, 7, 5, 4, 3, 4, 6, 5

a The score 9 occurs twice, while all other scores occur once. The mode is 9.

The mode is always one of the scores.

b First arrange the scores in order: 6, 7, 9, 10, 14, 14. The mode is 14.

c First arrange the scores in order: 3, 4, 4, 4, 5, 5, 5, 6, 7, 8. Two scores occur three times.
This means that there are two modes: 4 and 5.
This data set is **bimodal**.

Bimodal means there are two modes.

Exercise 13B

1 Complete the following to find the mode of 6, 3, 8, 4, 5, 9, 5.
First arrange the scores in order: 3, 4, ___ , ___ , ___ , 8, 9.
The mode is ___ .

2 Find the mode of each set of data.

a 2, 4, 5, 6, 9, 9, 10
b 2, 3, 3, 4, 5, 6, 7, 8, 9
c 11, 13, 13, 16, 17
d 27, 28, 29, 27, 30, 31, 27, 31, 30
e 0, 2, 4, 5, 7, 6, 4, 5, 4, 0, 1
f 20, 20, 20, 23, 25, 27
g 51, 52, 54, 55, 57, 57, 58, 59
h 1, 1, 2, 4, 4, 4, 4, 7, 7, 8, 9, 10
i 240, 243, 245, 246, 244, 243
j 104, 101, 104, 102, 104, 105, 106, 101

3 Find the mode of each set of data.

a ABAC CDBA BDCC ABDA
b Mon, Tue, Mon, Sun, Thu, Tue, Fri, Mon, Sat
c blue, green, red, green, green, red, blue

EXAMPLE 2

Find the range of each set of scores.

a 3, 7, 8, 9, 9
b 10, 6, 14, 9, 14, 7

a Range = highest score − lowest score
= 9 − 3 = 6

b Range = highest score − lowest score
= 14 − 6 = 8

4 Complete the following to find the range of 6, 8, 5, 3, 9.

Range = ____ score − lowest ____

= 9 − ____ = ____

5 Find the range of each data set in question **2**.

6 Can you find the range for each data set in question **3**? Explain.

EXAMPLE 3

Find the mode and range of each set of scores.

a

Score	Frequency
4	2
5	3
6	5
7	8
8	3

b

Score	Frequency
18	1
19	3
20	6
21	4
22	2

a The score with the highest frequency is 7 (7 occurs 8 times), so the mode is 7.

Range = highest score − lowest score

= 8 − 4 = 4

To find the range, use scores, not frequencies.

b The score with the highest frequency is 20 (20 occurs 6 times), so the mode is 20.

Range = highest score − lowest score

= 22 − 18 = 4

7 Use this frequency distribution to answer the questions.

Complete the following.

a The highest frequency is ___ .

This means the mode is ___ .

b The highest score is ___ .

The _____ score is 9.

Range = ___ − 9 = ___

Score	Frequency
9	8
10	12
11	6
12	3
13	4

8 Find the mode for each frequency distribution.

a

x	3	4	5	6	7
f	2	3	6	4	1

b

x	12	13	14	15	16
f	4	6	5	3	2

c

x	f
24	9
25	11
26	18
27	0
28	12

d

Mass (kg)	Frequency
23	2
24	5
25	7
26	4
27	2

e

Score	Frequency
100	3
101	0
102	10
103	8
104	7
105	2

f

Number of patients/day	Frequency
33	13
34	15
35	16
36	10
37	4
38	2

9 Find the range of each set of scores in question **8**.

10 Find the mode for each frequency distribution.

a

Car type	Frequency
Nissan	18
Toyota	22
Holden	24
Peugeot	4

b

Order	Number sold
Pizza	81
Burger	87
Pie	72
Sandwich	63

C Median

The **median** is the middle score after the scores have been arranged in ascending order (that is, from smallest to largest). The median divides the data into two parts such that there are equal numbers of scores below it and above it. The median cannot be used for categorical data.

The median strip is down the middle of the road.

EXAMPLE 1

Find the median for each set of scores.

a 2, 3, 4, 4, 6, 7, 7 **b** 7, 9, 3, 9, 8

a The scores are already in order.
Cross off scores from both ends until one remains: ~~2~~, ~~3~~, ~~4~~, (4), ~~6~~, ~~7~~, ~~7~~
This is the median. The median is 4.
Note: There are seven scores, so the 4th score is in the middle.

b First arrange the scores in order: 3, 7, 8, 9, 9
Cross off scores from both ends: ~~3~~, ~~7~~, (8), ~~9~~, ~~9~~
The median is 8.
Note: There are five scores, so the 3rd score is in the middle.

Exercise 13C

1 Complete the following to find the median of 4, 5, 8, 8, 9, 11, 11.
First cross off the 4 and 11, then the 5 and other 11. Continue until the median remains.
Median = ____

2 Find the median of each set of scores. They are already in ascending order.

a 2, 4, 5, 6, 9, 9, 10
b 2, 3, 3, 4, 5, 6, 7, 8, 9
c 11, 13, 13, 16, 17
d 27, 27, 27, 28, 29, 30, 30, 31, 31
e 0, 0, 1, 2, 4, 4, 4, 5, 5, 6, 7
f 51, 52, 54, 55, 57, 57, 58, 58, 59

3 Arrange each set of scores in order, then find the median.

a 240, 243, 245, 241, 246, 244, 243
b 25, 20, 21, 20, 27, 23, 20
c 104, 101, 102, 104, 102, 104,05, 106, 101
d 1, 4, 7, 10, 2, 4, 7, 1, 7, 8, 4, 9, 4

EXAMPLE 2

Find the median for each set of scores.

a 3, 4, 8, 9, 8, 9, 3, 5
b 3, 4, 8, 2, 9, 8, 9, 10, 3, 8

a Arrange the scores in order: 3, 3, 4, 5, 8, 8, 9, 9
Cross off from both ends: ~~3~~, ~~3~~, ~~4~~, (5), (8), ~~8~~, ~~9~~, ~~9~~
There are two scores left. The median is the average of these two scores (in the middle of them).

$$\text{Median} = \frac{5 + 8}{2} = 6.5$$

Note: There are eight scores, so the 4th and 5th scores are in the middle.

b Arrange the scores in order: 2, 3, 3, 4, 8, 8, 8, 9, 9, 10
Cross off from both ends: ~~2~~, ~~3~~, ~~3~~, ~~4~~, (8), (8), ~~8~~, ~~9~~, ~~9~~, ~~10~~
There are two scores remaining. They are both 8, so the median is 8.
Note: There are ten scores, so the 5th and 6th scores are in the middle.

4 Complete the following to find the median of 6, 12, 8, 11, 10, 10, 8, 5.
Arrange the scores in order: 5, 6, ___ , ___ , ___ , ___ , 11, 12
Cross out from both ends, leaving ___ and 10.

$$\text{Median} = \frac{\square + 10}{2} = \text{___}$$

5 For each set of data, rearrange the scores into numerical order where necessary, and hence find the median.

a 6, 6, 7, 7, 8, 9, 10, 10, 11, 12
b 20, 21, 21, 22, 22, 23, 23, 25
c 15, 15, 16, 16, 17, 17, 18, 19
d 48, 49, 49, 49, 50, 50, 50, 51, 52, 52
e 4, 5, 7, 8, 2, 4, 6, 7, 7, 5
f 12, 18, 14, 16, 12, 16, 13, 12
g 48, 49, 50, 51, 53, 49, 50, 50, 51, 55
h 18, 17, 17, 20, 23, 17, 18, 20, 23, 21
i 2, 1, 0, 3, 4, 5, 4, 4, 3, 1, 0, 2
j 35, 40, 37, 36, 33, 38, 36, 37, 38, 37

EXAMPLE 3

Find the median of the scores in this frequency distribution table.

Score	Frequency
4	3
5	2
6	4
7	8
8	6

To find the median it is useful to add a **cumulative frequency** column to the table.

Score	Frequency	Cumulative frequency	Scores
4	3	3	1st, 2nd, 3rd
5	2	$3 + 2 = 5$	4th, 5th
6	4	$5 + 4 = 9$	6th, 7th, 8th, 9th
7	8	$9 + 8 = 17$	10th, 11th, 12th, …, 16th, 17th
8	6	$17 + 6 = 23$	18th, 19th, 20th, …, 22nd, 23rd
	$\Sigma f = 23$		

The scores are already arranged in numerical order.

From the table, $n = \Sigma f = 23$. Hence, $\frac{n + 1}{2} = \frac{23 + 1}{2} = 12$

The median is the 12th score (there are 11 scores below it and 11 scores above it).

From the cumulative frequency column, you can see that the 12th score is 7.

So the median is 7.

Since there are only 23 scores, it is possible to check the answer by writing the scores in order:

~~4~~, ~~4~~, ~~4~~, ~~5~~, ~~5~~, ~~6~~, ~~6~~, ~~6~~, ~~6~~, ~~7~~, ~~7~~, (7), ~~7~~, ~~7~~, ~~7~~, ~~7~~, ~~7~~, ~~8~~, ~~8~~, ~~8~~, ~~8~~, ~~8~~, ~~8~~

Crossing off from both sides leaves 7 as the median.

EXAMPLE 4

Find the median of this set of scores.

x	20	21	22	23	24
f	5	7	11	9	4

Add a cumulative frequency column to the table.

x	f	Cumulative frequency	Scores
20	5	5	1st, 2nd, 3rd, 4th, 5th
21	7	12	6th, 7th, 8th, ..., 11th, 12th
22	11	23	13th, 14th, 15th, ..., 22nd, 23rd
23	9	32	24th, 25th, 26th, ..., 31st, 32nd
24	4	36	33rd, 34th, 35th, 36th
	$\Sigma f = 36$		

From the table, $n = \Sigma f = 36$. Hence, $\frac{n+1}{2} = \frac{36+1}{2} = 18.5$

The median is the number midway between the 18th and 19th scores, or the average of the 18th and 19th scores. From the cumulative frequency column, you can see that the 18th and 19th scores are both 22. So the median is 22.

Check:

~~20~~, ~~20~~, ~~20~~, ~~20~~, ~~20~~, ~~21~~, ~~21~~, ~~21~~, ~~21~~, ~~21~~, ~~21~~, ~~21~~, ~~22~~, ~~22~~, ~~22~~, ~~22~~, ~~22~~, (22),
(22), ~~22~~, ~~22~~, ~~22~~, ~~22~~, ~~23~~, ~~23~~, ~~23~~, ~~23~~, ~~23~~, ~~23~~, ~~23~~, ~~23~~, ~~24~~, ~~24~~, ~~24~~, ~~24~~

Writing the scores in order, then crossing out, leaves two scores, both 22, so the median is 22.

6 Find the median of each set of scores.

a

x	3	4	5	6	7
f	2	3	7	4	1

b

x	12	13	14	15	16
f	4	7	5	3	2

c

x	50	51	52	53	54	55
f	3	5	8	6	2	4

d

x	33	34	35	36	37
f	2	4	9	10	5

e

x	8	9	10	11	12
f	12	28	25	26	9

f

x	24	25	26	27	28
f	9	11	18	0	12

D Stem-and-leaf plots

Information displayed in stem-and-leaf plots can be used to calculate the measures of central tendency and range.

EXAMPLE 1

For this stem-and-leaf plot, find the:

a mode **b** median **c** range

Stem	Leaf
2	2 3 3 6
3	
4	1 2 3 5 5 6 6 6 8
5	1 5 5 7 7 7 7
6	1 2 3 5 5 5 6 7
7	0 0

a To find the mode, find the leaf that occurs most often: 57 occurs four times, so the mode is 57.

b To find the median, cross off the numbers from top and bottom until you reach the middle score or scores. There are 30 scores, so the median is the average of the 15th and 16th scores. They are both 55, so the median is 55.

Stem	Leaf
2	~~2~~ ~~3~~ ~~3~~ ~~6~~
3	
4	~~1~~ ~~2~~ ~~3~~ ~~5~~ ~~5~~ ~~6~~ ~~6~~ ~~6~~ ~~8~~
5	~~1~~ (5) (5) ~~7~~ ~~7~~ ~~7~~ ~~7~~
6	~~1~~ ~~2~~ ~~3~~ ~~5~~ ~~5~~ ~~5~~ ~~6~~ ~~7~~
7	~~0~~ ~~0~~

c Range = highest score − lowest score
= 70 − 22 = 48

Exercise 13D

1 Use this stem-and-leaf plot to answer the questions.

Stem	Leaf
4	0 1 2
5	1 3 4 6 8
6	2 2 2 5
7	0 4

Complete the following.

a The leaf of 2 in the 6 stem occurs ___ times. This means the mode is ___ .

b Cross off 74 with 40, then 70 with 41, continuing until 56 and ___ are left.

$$\text{Median} = \frac{56 + \square}{2} = \_\_\_$$

c Highest score is ___. _____ score is 40. Range = ___ − 40 = ___

2 Find the mode of the information in each stem-and-leaf plot.

a

Stem	Leaf
5	1 2 3 5
6	0 2 2 4 6 8
7	1 1 1 1 2 2 3 5
8	2 3 5 8
9	0 2

b

Stem	Leaf
11	8 9
12	0 0 1 3 5 8
13	4 5 6 9
14	2 2 2 5 6 8
15	4 6 6 9
16	0 3

3 Find the median of the information in each stem-and-leaf plot in question **2**.

4 For each stem-and-leaf plot in question **2**, find the range.

EXAMPLE 2

Find the mean of the data displayed in this stem-and-leaf plot.

Stem	Leaf
2	2 3 3 6
3	
4	1 2 3 5 5 6 6 6 8
5	1 5 5 7 7 7 7
6	1 2 3 5 5 5 6 7
7	0 0

Write the stem-and-leaf plot as a table. Add three extra columns labelled 'f', '$f \times$ stem' and 'Sum of leaves', as shown.

Stem	Leaf	f	$f \times$ stem	Sum of leaves
2	2 3 3 6	4	$20 \times 4 = 80$	$2 + 3 + 3 + 6 = 14$
3		0	$30 \times 0 = 0$	0
4	1 2 3 5 5 6 6 6 8	9	$40 \times 9 = 360$	$1 + 2 + 3 + 5 + 5 + 6 + 6 + 6 + 8 = 42$
5	1 5 5 7 7 7 7	7	$50 \times 7 = 350$	$1 + 5 + 5 + 7 + 7 + 7 + 7 = 39$
6	1 2 3 5 5 5 6 7	8	$60 \times 8 = 480$	$1 + 2 + 3 + 5 + 5 + 5 + 6 + 7 = 34$
7	0 0	2	$70 \times 2 = 140$	$0 + 0 = 0$
	Total	**30**	**1410**	**129**

Note: f is the number of leaves, and $f \times$ stem is similar to fx in frequency distribution tables. To find the mean, add the totals of the '$f \times$ stem' and 'Sum of leaves' columns, then divide by the total of the f column.

$$\text{Mean} = \frac{1410 + 129}{30} = 51.3$$

5 Complete the following to find the mean, mode, median and range for each stem-and-leaf plot.

Leaves must be in order.

a

Stem	Leaf	f	$f \times$ stem	Sum of leaves
5	2 2 3 3 5 8 8	7	350	31
6	1 2 4 6 7 8 9 9	8	480	46
7	1 1 1 1 3 4 6 8	8		
8	1 1 2 4			
9	4 7			

b

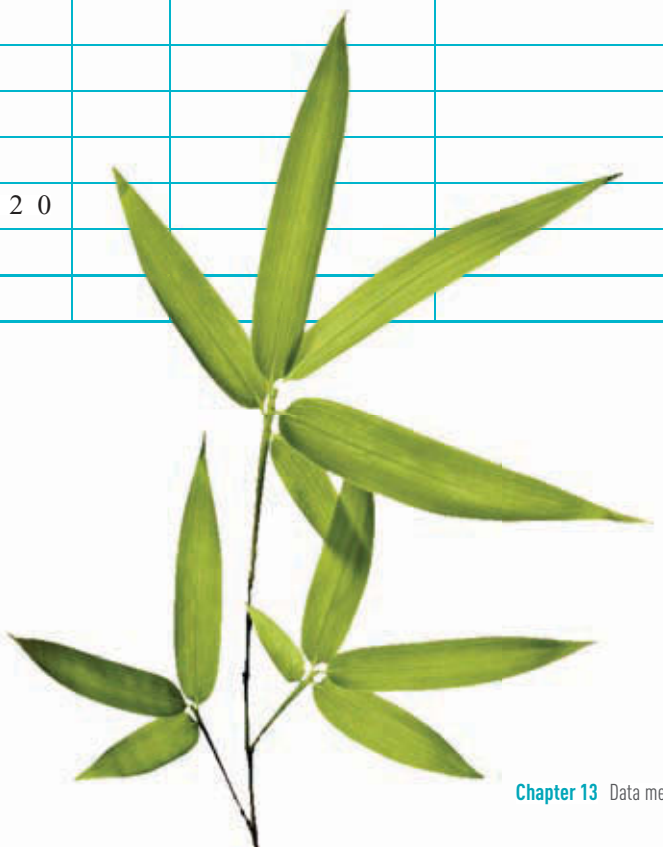
Stem	Leaf	f	$f \times$ stem	Sum of leaves
10	4 5 7	3	$100 \times 3 = 300$	$4 + 5 + 76$
11	4 7 8 9			
12	0 0 1 2 5 8 8 8 8			
13	4			
14	1 1 2 2 4 6 7 7 7			
15	2 2 3 3 4 5 6			

c

Stem	Leaf	f	$f \times$ stem	Sum of leaves
1	7 1 9 2 9 4 3 1			
2	7 3 1 4 6 3 8 8 2 0 2 3			
3	2 8 4 2			
4	9 8 3 8 2 4 5 9 9 9			
5	9 1 0 3 7 9 9			
6	8 1 8 4 3 7 4 2			

d

Stem	Leaf	f	$f \times$ stem	Sum of leaves
110	4 9 7			
111	0 3 0 5 5 5 8 9 5 4 7			
112	8 8 1 4 5 6 1 3 3 8 3			
113	1 8 1 2 9 1 2 9 7 1			
114	0 5 1 2 1 8 9 8 9 6 9 2 0			
115	1 1 9 3 2 1 8 0 4 2			
116	5 6 3 8 4 6 9 8 5 9 0			



E Mean, mode and median

The mean, mode and median were introduced in this chapter as measures of central tendency; that is, as single numbers that are central or typical of all the numbers under consideration. Often we need to consider which measure is the most appropriate in a given situation.

There is no general rule that can be used. Rather it depends on the nature of the data and the relative merits of these measures. For categorical data, the mode is the only measure that can be used.

Exercise 13E

1 For the scores 15, 15, 16, 17, 18, 20, 22 find the:

a mean **b** mode **c** median **d** range

2 For the scores 28, 18, 12, 25, 29, 17, 15, 22, 23, 25 find the:

a mean **b** mode **c** median **d** range

EXAMPLE 1

Find the mode of each set of scores, and discuss the merits of the mode in each case.

a 14, 15, 15, 16, 16, 16, 17, 18 **b** 14, 15, 16, 17, 19, 21 **c** 14, 15, 16, 17, 26, 26, 26

a Mode = 16. The number 16 occurs the most often. It is central and typical of this set of scores.

b No mode. This set of scores has no score that occurs more often than any other. The mode is obviously not a satisfactory measure of central tendency for this set of scores.

c Mode = 26. This set of scores does have a mode but it occurs at one end of the set. It is not a central value.

There are cases when the mode is important, even if it is not a central value. For example, if the numbers are the sizes of shoes sold by a shop or the sizes of dresses sold in a fashion house, the size sold most often would be important to the manufacturer of the items and to anyone ordering stock for the shop owner and fashion house.

EXAMPLE 2

Find the mean of each set scores, and discuss the merits of the mean in each case.

a 5, 5, 6, 7, 8 **b** 5, 5, 6, 7, 80

a Mean = 6.2. The mean is central and typical of the data. It is a good measure of central tendency.

b Mean = 20.6. Four out of five scores are less than the mean, so it is not a central value, nor is it typical of the size of any of the scores.

The mean has the disadvantage of being affected by extremely large or small values, such as the score 80 in Example **2b**. Very small or very large atypical scores are called **outliers**. In some cases it may be useful to ignore outliers when calculating the mean. In Example **2b**, if the outlier 80 is ignored, mean = 5.75.

EXAMPLE 3

Find the median of each set of scores, and discuss the merits of the median in each case.

a 8, 8, 9, 10, 11, 11, 13, 15, 15 **b** 1, 2, 2, 3, 6, 6, 6, 6, 6, 6

a Median = 5th score = 11. The median is central and typical of the data. It is a good measure of central tendency.

b Median = average of 5th and 6th scores = 6. For this set of scores, the median is the end score (which is also the mode). It is not a good measure of central tendency.

3 For each of the following sets of scores:

a 5, 6, 6, 7, 7, 7, 8, 9 **b** 2, 3, 4, 5, 7, 9 **c** 23, 24, 25, 26, 35, 35, 35

i Find the mode.

ii State whether the mode is a satisfactory measure of central tendency for the set of scores.

iii Find the mean and median, then discuss the relative merits of each measure for the data.

4 For each of the following sets of scores:

a 8, 8, 9, 10, 11 **b** 8, 8, 9, 10, 110

i Find the mean.

ii State whether the mean is a satisfactory measure of central tendency for the set of scores.

iii Find the median and mode, then discuss the relative merits of each measure for the data.

5 For each of the following sets of scores:

a 12, 13, 14, 14, 15, 15, 16, 18, 18 **b** 2, 3, 3, 3, 5, 5, 5, 5, 5

i Find the median.

ii State whether the median is a satisfactory measure of central tendency for the set of scores.

iii Find the mode and mean, then discuss the relative merits of each measure for the data.

6 A shoe store had a special on women's running shoes and sold the following shoe sizes.

6 10 4 7 8 7 6 5 7 8 7 5 6 4 3 7

a For the shoe sizes sold, find the mean, mode and median.

b Which of these measures would be of most use to the shop owner?

7 The Beacon Lamp Company employs 46 people, whose annual salaries are listed below.

1 general manager	$260 000
2 accountants	$120 000 each
1 engineer	$100 000
1 marketing manager	$100 000
1 warehouse manager	$100 000
15 tradespeople	$60 000 each
25 production workers	$50 000 each

a What is the total annual wages bill for this company?

b Calculate the mean wage for the employees.

c How many employees earn:

i less than the average wage? **ii** more than the average wage?

d What is the median wage?

e What is the modal wage?

f In a wage determination case for the employees of this company, which 'average' would you use to support your argument if you were:

i the general manager? **ii** the production workers' representative?

g Which measure of central tendency is the most appropriate to represent the wages of the employees of this company? Give reasons.

8 Gerry scored the following marks in a series of topic tests: 6, 3, 6, 2, 6, 2, 6, 1.

a Find the mean, mode and median for this set of marks.

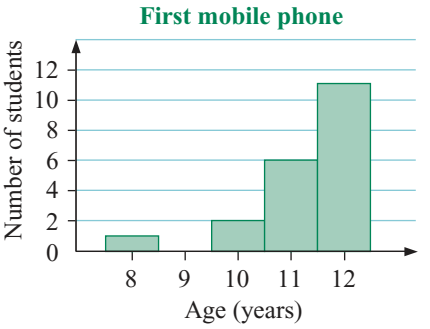
b Which of these 'averages' would Gerry prefer to tell his parents?

9 The prices of houses sold by a real estate agent in a particular suburb for 1 month were $510 000, $630 000, $960 000, $540 000 and $560 000.

a The real estate agent claimed that, for this month, the 'average' price of houses in the suburb was $640 000. Which average did the estate agent use?

b Is this an appropriate average to use? Give reasons, with reference to the other measures of central tendency.

10 Students in a Year 7 class who have a mobile phone were surveyed and asked at what age they received their first mobile phone. The data is shown in the frequency histogram.



a How many students in the class have a mobile phone?

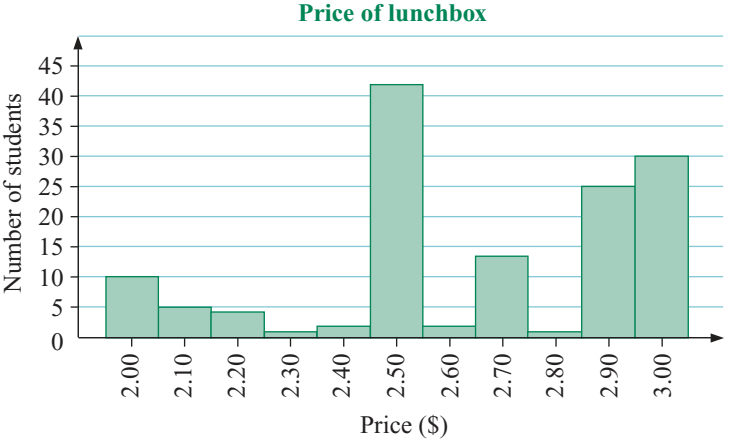
b Calculate the mean, mode, median and range for the data. It may be easier to put the information into a frequency distribution table first.

c From this data, at what average age did these students receive their first mobile phone? Which measure(s) would you use to justify your answer?

d The average age of students starting Year 7 is 12.3 years. Sheldon says that most students are given a mobile phone when they start high school. Does the data agree with his statement? Explain.

e Conduct this survey in your class, and compare your results with the data in this question.

11 As a fundraising activity, Year 7 students are selling lunchboxes at the canteen. Year 7 students were surveyed about the price of the new lunchbox and asked to suggest the price in a range from $2 to $3. The results are shown in the column graph.



Your task is to decide on a price and present your findings to the principal.

a From the graph, calculate the mean, mode and median.

b What is the range?

c Give a reason why the mode price should be used.

d Discuss the use of the mean as the price.
e Give a reason why the median would be a good choice for the price.
f Amalea says that the price should be $3, to make the most profit. How would you use the survey data to support her?
g What price would you charge? Give reasons.

Summary: Choosing the appropriate measure

When selecting which of the three measures of central tendency to use as a representative value for a set of data, consider the following characteristics of each measure.

Mean

- It is the most commonly used, is easy to understand and is easy to calculate.
- Its value depends on the value of every score in the data set.
- Its main disadvantage is that its value is easily distorted by very large or very small atypical scores (outliers).

Mode

- It is the most frequent value in a set of data.
- Its value is easy to determine.
- It is not affected by outliers.
- There may be no mode, or more than one mode, in a data set.
- It may not be a central value.

Median

- It is easy to understand: there are equal numbers of scores above and below it.
- It is not affected by outliers, since its value depends on the number of scores above and below it, not the values of these scores.
- For some unusual patterns of scores it may not be a central value; for example, 0, 1, 1, 5, 5, 5, 5.

Investigation 1 Using statistical functions

This spreadsheet shows the formulas to calculate mean, mode, median and range. There are ten scores. If more scores are used, change the formula A3:A12 to range to the required values, depending on the number of scores.

	A	B	C
1	**Using statistical functions**		
2	**Scores**		
3	2	Mean	=AVERAGE(A3:A12)
4	5	Mode	=MODE.MULT(A3:A12)
5	6	Median	=MEDIAN(A3:A12)
6	3	Range	=MAX(A3:A12)–MIN(A3:A12)
7	1		
8	4		
9	5		
10	6		
11	6		
12	8		

F Comparing data

Often, two sets of data are to be compared. The measures of central tendency—mean, mode and median—may be useful. The range as a measure of spread may show some information.

Exercise 13F

1 The marks scored by Tiffany and Con in eight topic tests in mathematics are shown below.

Test	1	2	3	4	5	6	7	8
Tiffany	82	81	91	84	82	75	88	54
Con	81	80	86	83	88	72	86	79

a Find the total marks scored by each student.
b Find the mean, mode and median for each student.
c In how many tests did Con score more marks than Tiffany?
d Find the range of each student's scores and comment.
e Which student scored the highest mark?
f Which student scored the lowest mark?
g The mathematics prize is to be given to the better student. Who should get it? Give three reasons.

2 The marks for a class in English, Mathematics and Science are given.

Mark	English frequency	Mathematics frequency	Science frequency
5	2	6	8
6	8	6	6
7	10	6	2
8	8	6	6
9	2	6	8

a For each subject, find the mean, mode, median and range.
b Which of the measures in part **a** show differences in the results?
c In which subject did the class perform best? Explain.
d Display this information in frequency histograms.

3 The ages (in years) of concert-goers in the front row are compared with those of concert-goers in the back row in this back-to-back stem-and-leaf plot.

Back row Leaf	Stem	Front row Leaf
	0	8
8 6 5 5 5 2	1	3 6 9 9
7 3 2 2 1 0 0 0	2	0 0 1 2 2 3 7 7
9 4 2	3	1 2 2 2 5 8
4 5	4	6
1	5	

a For each row, find the mean, mode, median and range.
b Comment on the differences between the front and back rows, if any.

Language in mathematics

1 Put the words in order to make a sentence. The first word has a capital letter.

a the occurs most mode score that The is often

b by mean scores To add of all and find scores the divide then the together number the

c order median scores the middle the The when in score is are

d score The does to mean not have score be a

e be must mode score The a

2 Write questions that will have these words as answers.

a median **b** range **c** mode **d** mean

Terms

average	bias	bimodal	central tendency	cumulative frequency	data
frequency distribution		mean	median	mode	outlier
random	range	score	spread	stem-and-leaf plot	

Check your skills

Use this data to answer questions **1** to **4**.

13 8 6 5 4 14 5 9

1 The mean of the data is:

A 5 **B** 6 **C** 7 **D** 8

2 The mode of the data is:

A 5 **B** 6 **C** 7 **D** 8

3 The range of the data is:

A 6 **B** 7 **C** 8 **D** 10

4 The median of the data is:

A 3.5 **B** 6 **C** 7 **D** 8

Use this frequency distribution table to answer questions **5** to **8**.

Score	6	7	8	9	10
Frequency	9	11	8	15	7

5 The mean from the table is:

A 4 **B** 7 **C** 8 **D** 9

6 The mode from the table is:

A 4 **B** 7 **C** 8 **D** 9

7 The range from the table is:

A 4 **B** 7 **C** 8 **D** 9

8 The median from the table is:

A 4 **B** 7 **C** 8 **D** 9

Use this stem-and-leaf plot to answer questions **9** to **12**.

Stem	Leaf
0	7 7 8 9
1	0 2 3 3 3 4 5 7 8
2	1 2 4 6 7 9
3	0 0 1 1 4 5 7
4	3 5 6 7

9 The mean from the stem-and-leaf plot is:

A 2 **B** 24
C 40 **D** 47

10 The mode from the stem-and-leaf plot is:

A 3 **B** 13 **C** 23 **D** 47

11 The median from the stem-and-leaf plot is:

A 5 **B** 23 **C** 24 **D** 25

12 The range from the stem-and-leaf plot is:

A 2 **B** 24 **C** 40 **D** 47

13 The best measure of central tendency to use when there is an outlier score is the:

A mean **B** mode **C** median **D** range

14 The mode is the best measure when:

A the data is categorical
B there are more than 50 scores
C the scores are clustered
D the mean is less than 10

15 Carla and Sam's results on 7 tests are shown in this back-to-back stem-and-leaf plot. Which of the following is *not* true?

Carla Leaf	Stem	Sam Leaf
9	2	6 7
3 2 1	3	3 3 5 6
4 2 0	4	1

A Sam's measures of central tendency are all 33.
B Carla and Sam have the same median and range.
C Carla's marks are more clustered than Sam's.
D Carla is probably the better student as her mean is higher than Sam's.

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 5	2, 3, 6, 7	4, 8	9–12	13–14	15
Section	A	B	C	D	E	F

13A Review set

1 List an advantage and a disadvantage of using the mean or median as the measure of central tendency of a group of scores.

2 For the scores 12, 14, 16, 19, 19, 21, find the:

a mean **b** mode **c** median **d** range

3 For the scores in the frequency distribution table below, find the:

a mean **b** mode **c** median **d** range

Score	19	20	21	22	23	24
Frequency	6	5	12	8	3	6

4 For the scores in this stem-and-leaf plot, find the:

a mean **b** mode

c median **d** range

Stem	Leaf
2	8 8 9
3	0 0 1 2 3 4 5 9
4	1 2 4 4 4 6 8
5	3 5 9
6	2

5 The back-to-back stem-and-leaf plot compares the marks gained by two classes, A and B, in their half-yearly mathematics exam.

a Find the mean, mode, median and range for each class.

b Which class performed better? Explain.

Class A Leaf	Stem	Class B Leaf
6 1	2	7 9
6 4 2 1	3	0 3 5 6
8 5 3 1 0	4	0 2 6 6 8
1 1 0	5	2 5 8
7	6	7

13B Review set

1 For the scores 35, 31, 34, 31, 31, 32, find the:

a mean **b** mode **c** median **d** range

2 For the scores 11, 12, 12, 13, 14, 18, 53:

a Find the mean.

b Find the median.

c Discuss the relative merits of the mean and median for these scores.

3 For the scores in this frequency distribution table, find the:

a mean **b** mode **c** median **d** range

Score	55	56	57	58	59	60
Frequency	6	10	10	13	6	2

4 For the scores in this stem-and-leaf plot, find the:

a mean **b** mode

c median **d** range

Stem	Leaf
23	3 4 5 8
24	0 0 0 6 9 9
25	1 3 4 8 8
26	0 0 3 4 8 9
27	1 2 3 4 8 8

5 The marks scored by Anthony and Mark in six mathematics topic tests are shown in the table.

Anthony	71	64	62	55	68	34
Mark	66	63	68	52	66	59

a Find the total marks scored by each student.

b Find the mean, mode, median and range for each student.

c In how many tests did Anthony score more marks than Mark?

d Which student scored the highest mark?

e Which student scored the lowest mark?

f Who should win the mathematics prize? Explain.

b Which class performed better? Explain.

13C Review set

1 Explain the term 'outlier' when used in statistics. Which measure of central tendency does it affect the most?

2 For the scores 18, 22, 21, 18, 19, 20, 18, find the:

a mean **b** mode **c** median **d** range

3 For the scores in the frequency distribution table below, find the:

a mean **b** mode **c** median **d** range

Score	2	3	4	5	6	7	8
Frequency	3	12	6	10	5	4	10

4 For the scores in this stem-and-leaf plot, find the:

a mean **b** mode

c median **d** range

Stem	**Leaf**
7	4 8 6 1 2
8	3 5 9 8 8 6 6 5 1 6
9	7 2 9 4 3
10	0 5 6 1 4
11	8 8 5 9 8 1 2
12	3 4 7 7 9 6 5 0 1
13	1 5 0 0

5 The back-to-back stem-and-leaf plot compares the temperatures in two cities X and Y taken at the same time and day each week for the 40 school weeks.

a Find the mean, mode, median and range of the temperatures for each city.

b City X is described as having a higher average temperature than city Y. Do you agree? Explain.

City X												**City Y**	
							Leaf					**Stem**	**Leaf**
							9	9	8	7	7	1	7 7 7 8 9
9	9	8	7	5	5	5	5	2	1	1	1	2	0 0 1 1 1 2 2 3 3 4 4 4 5 5 5 6 7 7 8 8 9 9 9
							9	7	5	4	3	3	0 0 0 0 0 1 1 2 3 8 9
							3	3	3	2	0	4	0

6 For the scores 1, 1, 1, 1, 1, 1, 2, 3, 3, 4:

a Find the mean, mode and median.

b Which is the most appropriate measure of central tendency? Explain.

7 Matthew's and Melissa's marks for 5 tests are given in the table.

a Is the mode a good measure of central tendency for both students? Explain.

b Is Melissa's mean and range greater than Matthew's?

c Are Melissa's marks more spread out than Matthew's? Explain.

d Is Matthew the better student? Explain.

Mathew	**Melissa**
27	25
28	29
30	29
30	32
16	33

14

Linear equations

This chapter deals with the solution of simple linear equations.

After completing this chapter you should be able to:

- use the guess, check and improve method, concrete materials, backtracking strategies and the balance model to solve equations
- solve equations using algebraic methods involving one- or two-step processes
- check solutions by substitution
- generate equations with a given solution
- solve real-life problems by using pronumerals to represent unknowns.

Diagnostic test

1 Calculate the missing number in $5 \times \square = 4 \times 10$.

A 1 **B** 2 **C** 4 **D** 8

2 Calculate the missing number in $5 \times \square = 30 - 10$.

A 1 **B** 2 **C** 4 **D** 6

3 Which statement represents the sum of 5 and 8?

A 5×8 **B** $5 - 8$
C $5 \div 8$ **D** $5 + 8$

4 Which statement represents the product of 7 and 9?

A 7×9 **B** $7 - 9$
C $7 \div 9$ **D** $7 + 9$

5 A number is multiplied by 4 then has 1 added to it. The answer is 13. What is the original number?

A 3 **B** 4 **C** 1 **D** $3\frac{1}{2}$

6 What is $7 + 3 \times 5$?

A 50 **B** 22 **C** 15 **D** 38

7 What is $3 \times 6 - 5 + 4 + 2 \times 6$?

A 29 **B** 39 **C** 6 **D** 61

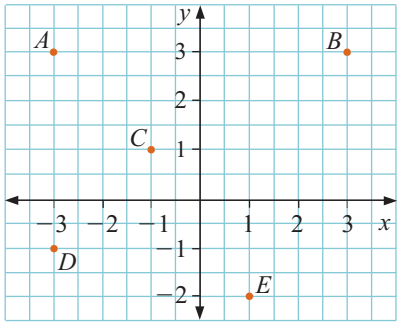
8 Start with 8, add 4, subtract 7 then divide the answer by 5. What is the final answer?

A 4 **B** 3 **C** 2 **D** 1

9 What is $5 \times (7 - 4) + 8$?

A 39 **B** 63 **C** 23 **D**

This diagram refers to questions **10** and **11**.



10 Which quadrant contains point E?

A 1st quadrant **B** 2nd quadrant
C 3rd quadrant **D** 4th quadrant

11 Which quadrant contains the point $(-3, -1)$?

A 1st quadrant **B** 2nd quadrant
C 3rd quadrant **D** 4th quadrant

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–7	8–11
Section	ACMNA 121, 134	ACMMG 143

LINEAR EQUATIONS

An algebraic expression is one in which letters are used to represent numbers. We refer to these letters as variables (see Chapter 7).

An **equation** is a mathematical statement that two expressions are equal. The **solution** of the equation is the value of the unknown (variable) that makes these expressions equal in value.

For example, to solve the equation $4x + 7 = 19 + x$ means to find the value of the unknown x such that the value of the expression $4x + 7$ is the same as the value of the expression $19 + x$. In this case, $x = 4$ is the solution.
If $x = 4$, then $4x + 7 = 4 \times 4 + 7 = 23$ and $19 + x = 19 + 4 = 23$.

In this chapter we compare and contrast several methods of solving simple equations.

Guess, check and improve

To solve an equation by the guess, check and improve method:
Step 1: Make a reasonable guess of the value of the pronumeral and substitute it into the equation.
Step 2: Use the result to improve the next guess.
Step 3: Repeat until the required result is found.

Solving equations by the guess, check and improve method can be tedious and time consuming. The method works most simply when the solution is an integer (whole number).

EXAMPLE 1

Solve the equation $2x + 11 = 29$ by the guess, check and improve method.

Try $x = 4$, $2x + 11 = 2 \times 4 + 11 = 19$. The first guess is too small.
$x = 10$, $2x + 11 = 2 \times 10 + 11 = 31$. The second guess is too big.
$x = 8$, $2x + 11 = 2 \times 8 + 11 = 27$. The third guess is too small.
$x = 9$, $2x + 11 = 2 \times 9 + 11 = 29$. This is the required result.

The solution of the equation is $x = 9$.
This could be set out in a table like the one below.

x	4	10	8	9
$2x + 11$	19	31	27	29

EXAMPLE 2

Use the table below to solve $3x - 5 = 19$ by the guess, check and improve method.

x	5	10		
$3x - 5$	10			

x	5	10	7	8
$3x - 5$	10	25	16	19

Solution is $x = 8$.

Exercise 14A

1 Complete the tables to solve the given equations by the guess, check and improve method.

a $2x + 17 = 25$

x	2	5		
$2x + 17$	21			

Solution is $x =$ ___.

b $3x - 15 = 12$

x	10	5		
$3x - 15$	15			

Solution is $x =$ ___.

2 Use tables to solve these equations by the guess, check and improve method.

a $5x - 9 = 51$ **b** $4x + 9 = 37$ **c** $3x - 11 = 25$ **d** $5x + 8 = 53$

B Using concrete materials

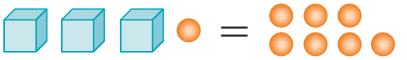
Another method of solving equations is to use **concrete materials**, such as centicubes and counters, to model the equation and 'do the same thing to both sides'.

The concrete materials method can be used for equations with pronumerals on both sides, but does not work if any of the numbers are negative.

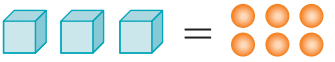
EXAMPLE 1

Using centicubes to represent the unknown x and counters to represent units, solve the equation $3x + 1 = 7$ by doing the same thing to both sides. Check the solution by substitution.

Step 1: Form the equation using centicubes and counters.

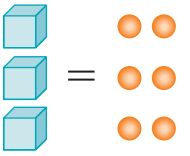


Step 2: Take 1 (counter) from both sides.



Step 3: Since there are 3 cubes left on the left-hand side, divide the right-hand side into 3 equal groups.

Step 4: Hence each cube represents 2 counters; that, is $x = 2$.



Check: $3x + 1 = 3 \times 2 + 1 = 7$ √

Exercise 14B

1 Complete the following to solve the equations using concrete materials and the method of Example 1. Use centicubes to represent the unknown x and counters to represent units. Draw diagrams for steps 2 and 3.

a $2x + 3 = 11$

Step 1: Form the equation using centicubes and counters.

Step 2: Take ____ from both sides.

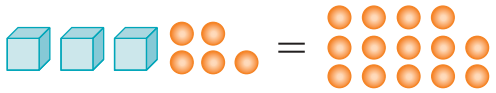
Step 3: Since there are ____ cubes left on the left-hand side, divide the right-hand side into ____ equal groups.

Step 4: Hence each cube represents ____ counters; that is, $x =$ ____.

Check: $2x + 3 = 2 \times$ ____ $+ 3 =$ ____

b $3x + 5 = 14$

Step 1: Form the equation using centicubes and counters.



Step 2: Take ____ from both sides.

Step 3: Since there are ____ cubes left on the left-hand side, divide the right-hand side into ____ equal groups.

Step 4: Hence, each cube represents ____ counters; that is, $x =$ ____.

Check: $3x + 5 = 3 \times$ ____ $+ 5 =$ ____

2 Solve the following equations using the method of Example 1, and check your solutions by substitution.

a $4x + 2 = 10$ **b** $2x + 5 = 11$ **c** $3x + 4 = 10$

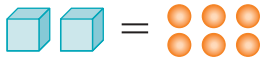
EXAMPLE 2

Use centicubes and counters to solve $3x + 1 = x + 7$. Check the solution by substitution.

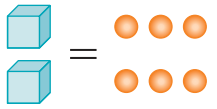
Step 1: Form the equation using centicubes and counters.

Step 2: Take 1 (counter) from both sides.

Step 3: Take x (1 cube) from both sides.



Step 4: Since there are 2 cubes left on the left-hand side, divide the right-hand side into 2 equal groups.



Step 5: Hence, each cube represents 3 counters; that is, $x = 3$.

Check: $3x + 1 = 3 \times 3 + 1 = 10$ and $x + 7 = 3 + 7 = 10$ √

3 Solve the following equations using the method shown in Example 2, and check your solutions by substitution.

a $3x + 3 = x + 5$ **b** $4x + 3 = 2x + 7$ **c** $3x + 2 = 2x + 6$

C Solving equations by backtracking

Backtracking is a useful method of solving equations by representing operations on a **flowchart**, and then computing backwards. However, it does not work when the equation has pronumerals on both sides.

EXAMPLE 1

Starting with x, write the expression produced if you:

a add 7 **b** subtract 6 **c** multiply by 4

d divide by 4 **e** multiply by 3 then add 2 **f** divide by 5 then subtract 1

g multiply by 2 then divide by 3 **h** add 4 then multiply by 6 **i** subtract 8 then divide by 3

a $x + 7$ **b** $x - 6$ **c** $4x$ **d** $\frac{x}{4}$ **e** $3x + 2$

f $\frac{x}{5} - 1$ **g** $\frac{2x}{3}$ **h** $6(x + 4)$ **i** $\frac{x - 8}{3}$

Exercise 14C

1 Starting with x, write the expression produced if you:

a add 9 **b** subtract 7 **c** multiply by 3

d divide by 8 **e** multiply by 2 then add 5 **f** divide by 3 then subtract 1

g multiply by 5 then divide by 7 **h** add 7 then multiply by 5 **i** subtract 10 then divide by 4

EXAMPLE 2

Illustrate the processes in Example 1 above using flowcharts.

a $\boxed{x} \xrightarrow{+7} \boxed{x+7}$ **b** $\boxed{x} \xrightarrow{-6} \boxed{x-6}$ **c** $\boxed{x} \xrightarrow{\times 4} \boxed{4x}$

d $\boxed{x} \xrightarrow{\div 4} \boxed{\frac{x}{4}}$ **e** $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+2} \boxed{3x+2}$ **f** $\boxed{x} \xrightarrow{\div 5} \boxed{\frac{x}{5}} \xrightarrow{-1} \boxed{\frac{x}{5}-1}$

g $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{\div 3} \boxed{\frac{2x}{3}}$ **h** $\boxed{x} \xrightarrow{+4} \boxed{x+4} \xrightarrow{\times 6} \boxed{6(x+4)}$ **i** $\boxed{x} \xrightarrow{-8} \boxed{x-8} \xrightarrow{\div 3} \boxed{\frac{x-8}{3}}$

2 Complete the following flowcharts.

a $\boxed{x} \xrightarrow{\times 9} \boxed{}$ **b** $\boxed{w} \xrightarrow{-6} \boxed{}$ **c** $\boxed{z} \xrightarrow{\div 8} \boxed{}$

d $\boxed{m} \xrightarrow{+13} \boxed{}$ **e** $\boxed{y} \xrightarrow{\times 2} \boxed{} \xrightarrow{-11} \boxed{}$ **f** $\boxed{a} \xrightarrow{+3} \boxed{} \xrightarrow{\times 2} \boxed{}$

g $\boxed{x} \xrightarrow{\div 5} \boxed{} \xrightarrow{+27} \boxed{}$ **h** $\boxed{w} \xrightarrow{+7} \boxed{} \xrightarrow{\div 6} \boxed{}$ **i** $\boxed{x} \xrightarrow{\times 3} \boxed{} \xrightarrow{\div 2} \boxed{}$

j $\boxed{k} \xrightarrow{\times 3} \boxed{} \xrightarrow{+2} \boxed{}$

k $\boxed{z} \xrightarrow{+2} \boxed{} \xrightarrow{\times 3} \boxed{}$

l $\boxed{y} \xrightarrow{\div 5} \boxed{} \xrightarrow{-4} \boxed{}$

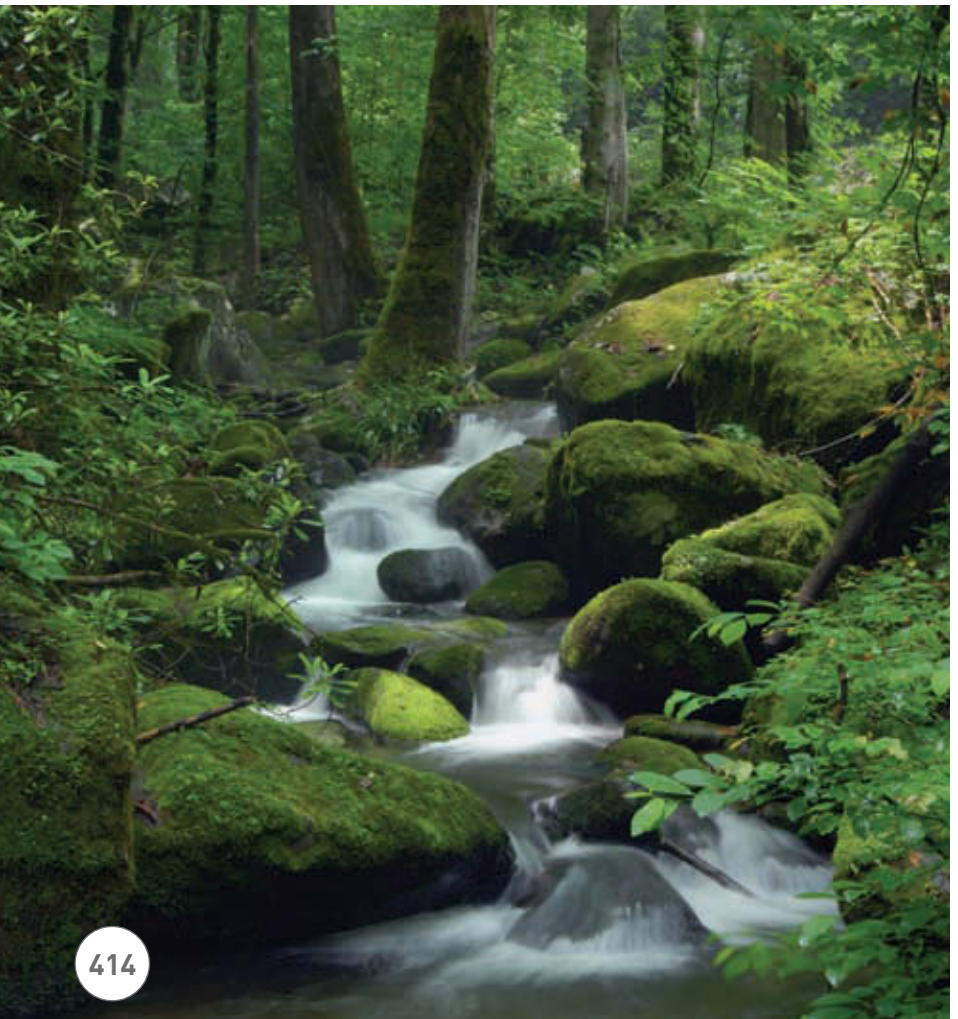
m $\boxed{z} \xrightarrow{-4} \boxed{} \xrightarrow{\div 5} \boxed{}$

n $\boxed{x} \xrightarrow{\times 5} \boxed{} \xrightarrow{+6} \boxed{}$

o $\boxed{w} \xrightarrow{\times 4} \boxed{} \xrightarrow{\div 7} \boxed{}$

p $\boxed{p} \xrightarrow{\div 7} \boxed{} \xrightarrow{\times 4} \boxed{}$

q $\boxed{n} \xrightarrow{-4} \boxed{} \xrightarrow{\times 3} \boxed{}$



EXAMPLE 3

Draw flowcharts for the following expressions.

a $x + 5$ **b** $y - 8$ **c** $3m$ **d** $\frac{k}{2}$

e $2x + 7$ **f** $5z - 9$ **g** $\frac{x}{6} + 2$ **h** $\frac{p}{7} - 1$

i $3(u + 2)$ **j** $\frac{y - 3}{7}$ **k** $\frac{3w}{4}$ **l** $\frac{7t}{3}$

a $x \xrightarrow{+5} x + 5$

b $y \xrightarrow{-8} y - 8$

c $m \xrightarrow{\times 3} 3m$

d $k \xrightarrow{\div 2} \frac{k}{2}$

e $x \xrightarrow{\times 2} 2x \xrightarrow{+7} 2x + 7$

f $z \xrightarrow{\times 5} 5z \xrightarrow{-9} 5z - 9$

g $x \xrightarrow{\div 6} \frac{x}{6} \xrightarrow{+2} \frac{x}{6} + 2$

h $p \xrightarrow{\div 7} \frac{p}{7} \xrightarrow{-1} \frac{p}{7} - 1$

i $u \xrightarrow{+2} u + 2 \xrightarrow{\times 3} 3(u + 2)$

j $y \xrightarrow{-3} y - 3 \xrightarrow{\div 7} \frac{y - 3}{7}$

k $w \xrightarrow{\times 3} 3w \xrightarrow{\div 4} \frac{3w}{4}$

l $t \xrightarrow{\div 3} \frac{t}{3} \xrightarrow{\times 7} \frac{7t}{3}$

In parts k and l you can do × before ÷ or ÷ before ×.

3 Draw flowcharts for the following expressions.

a $x + 9$ **b** $y - 7$ **c** $8z$ **d** $\frac{q}{6}$ **e** $5x - 3$ **f** $2x + 1$

g $2(x + 1)$ **h** $\frac{w}{7} + 2$ **i** $\frac{m}{5} - 3$ **j** $\frac{q + 2}{3}$ **k** $\frac{5z}{7}$ **l** $\frac{9q}{4}$

EXAMPLE 4

Solve the following equations by backtracking.

a $m + 19 = 32$ **b** $w - 18 = 17$ **c** $2x = 27$ **d** $\frac{p}{8} = 13$

a $m + 19 = 32$

Step 1: Draw a flowchart for $m + 19$.

$m \xrightarrow{+19} m + 19$

Step 2: Replace $m + 19$ by 32.

$m \xrightarrow{+19} 32$

Step 3: Compute backwards to find m by 'undoing' each arithmetic operation. Undo 'add 19' by 'subtract 19' ($32 - 19 = 13$).

$13 \xleftarrow{-19} 32$

Solution is $m = 13$.

b $w - 18 = 17$

Step 1: Draw a flowchart for $w - 18$.

$w \xrightarrow{-8} w - 18$

Step 2: Replace $w - 18$ by 17.

$w \xrightarrow{-18} 17$

Step 3: Compute backwards to find w by 'undoing' each arithmetic operation. Undo 'subtract 18' by 'add 18' ($17 + 18 = 35$).

$35 \xleftarrow{+18} 17$

Solution is $w = 35$.

c $2x = 27$

Step 1: Draw a flowchart for $2x$.

$x \xrightarrow{\times 2} 2x$

Step 2: Replace $2x$ by 27.

$x \xrightarrow{\times 2} 27$

Step 3: Compute backwards to find x by 'undoing' each arithmetic operation. Undo 'multiply by 2' by 'divide by 2' ($27 \div 2 = 13.5$).

$13.5 \xleftarrow{\div 2} 27$

Solution is $x = 13.5$.

d $\frac{p}{8} = 13$

Step 1: Draw a flowchart for $\frac{p}{8}$.

$p \xrightarrow{\div 8} \frac{p}{8}$

Step 2: Replace $\frac{p}{8}$ by 13.

$p \xrightarrow{\div 8} 13$

Step 3: Compute backwards to find p by 'undoing' each arithmetic operation. Undo 'divide by 8' by 'multiply by 8' ($13 \times 8 = 104$).

$104 \xleftarrow{\times 8} 13$

Solution is $p = 104$.

4 Complete to solve the following equations by backtracking.

a $x + 23 = 51$

Step 1: Draw a flowchart for $x + 23$.

$x \longrightarrow x + 23$

Step 2: Replace $x + 23$ by ____.

$x \longrightarrow \square$

Step 3: Compute backwards to find x by 'undoing' each arithmetic operation.

Solution is $x =$ ____.

$\square \longleftarrow \square$

b $q - 38 = 25$

Step 1: Draw a flowchart for $q - 38$.

$q \longrightarrow q - 38$

Step 2: Replace $q - 38$ by ____.

$q \longrightarrow \square$

Step 3: Compute backwards to find q by 'undoing' each arithmetic operation.

Solution is $q =$ ____.

$\square \longleftarrow \square$

c $7w = 98$

Step 1: Draw a flowchart for $7w$.

$w \longrightarrow 7w$

Step 2: Replace $7w$ by ____.

$w \longrightarrow \square$

Step 3: Compute backwards to find w by 'undoing' each arithmetic operation.

Solution is $w =$ ____.

$\square \longleftarrow \square$

d $\frac{n}{13} = 6$

Step 1: Draw a flowchart for $\frac{n}{13}$.

$n \longrightarrow \frac{n}{13}$

Step 2: Replace $\frac{n}{13}$ by ____.

$n \longrightarrow \square$

Step 3: Compute backwards to find n by 'undoing' each arithmetic operation.

Solution is $n =$ ____.

$\square \longleftarrow \square$

5 Solve the following equations by backtracking.

a $n + 13 = 41$ **b** $w - 17 = 28$

c $9x = 108$ **d** $\frac{z}{7} = 18$

e $p - 11 = 8$ **f** $\frac{w}{4} = 7$

g $z + 5 = 12$ **h** $6x = 54$

EXAMPLE 5

Solve the following equations by backtracking, and check each solution by substituting into the equation.

a $4x + 11 = 31$ **b** $\frac{z}{4} - 5 = 11$ **c** $\frac{y - 3}{4} = 2$

a $4x + 11 = 31$

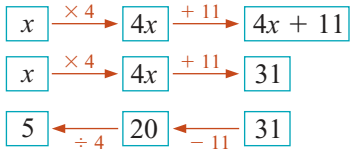
Step 1: Draw a flowchart for $4x + 11$.

Step 2: Replace $4x + 11$ by 31.

Step 3: Compute backwards to find x by 'undoing' each operation.
Undo 'add 11' by 'subtract 11' ($31 - 11 = 20$), then
undo 'multiply by 4' by 'divide by 4' ($20 \div 4 = 5$)

Solution is $x = 5$.

Check: $4x + 11 = 4 \times 5 + 11 = 20 + 11 = 31$ √



b $\frac{z}{4} - 5 = 11$

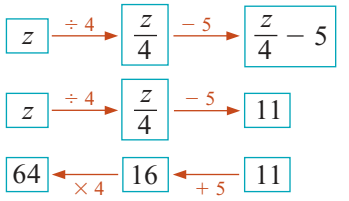
Step 1: Draw a flowchart for $\frac{z}{4} - 5$.

Step 2: Replace $\frac{z}{4} - 5$ by 11.

Step 3: Compute backwards to find z by 'undoing' each operation.
Undo 'subtract 5' by 'add 5' ($11 + 5 = 16$), then
undo 'divide by 4' by 'multiply by 4' ($16 \times 4 = 64$).

Solution is $z = 64$.

Check: $\frac{z}{4} - 5 = \frac{64}{4} - 5 = 16 - 5 = 11$ √



c $\frac{y - 3}{4} = 2$

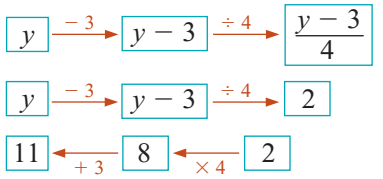
Step 1: Draw a flowchart for $\frac{y - 3}{4}$.

Step 2: Replace $\frac{y - 3}{4}$ by 2.

Step 3: Compute backwards to find y by 'undoing' each operation.
Undo 'divide by 4' by 'multiply by 4' ($2 \times 4 = 8$), then
undo 'subtract 3' by 'add 3' ($8 + 3 = 11$).

Solution is $y = 11$.

Check: $\frac{y - 3}{4} = \frac{11 - 3}{4} = \frac{8}{4} = 2$ √



6 Complete to solve the following equations by backtracking and check the solutions.

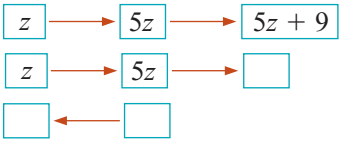
a $5z + 9 = 54$

Step 1: Draw a flowchart for $5z + 9$.

Step 2: Replace $5z + 9$ by ____.

Step 3: Compute backwards to find z by 'undoing' each operation.

Solution is $z =$ ____. *Check:* $5z + 9 = 5 \times$ ____ $+ 9 =$ ____



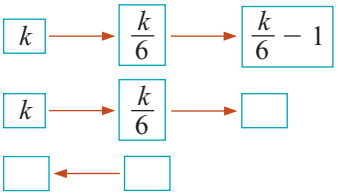
b $\frac{k}{6} - 1 = 10$

Step 1: Draw a flowchart for $\frac{k}{6} - 1$.

Step 2: Replace $\frac{k}{6} - 1$ by ____.

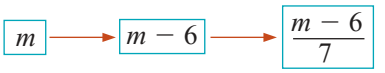
Step 3: Compute backwards to find k by 'undoing' each operation.

Solution is $k =$ ____. *Check:* $\frac{k}{6} - 1 = \frac{\square}{6} - 1 =$ ____

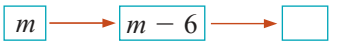


c $\frac{m-6}{7} = 9$

Step 1: Draw a flowchart for $\frac{m-6}{7}$.



Step 2: Replace $\frac{m-6}{7}$ by ____.



Step 3: Compute backwards to find m by 'undoing' each operation.

Solution is $m =$ ____.

Check: $\frac{m-6}{7} = \frac{\square - 6}{7} =$ ____

7 Solve the following equations by backtracking, and check your solutions by substitution.

a $2w + 5 = 19$ **b** $6t - 11 = 19$ **c** $8a + 14 = 30$ **d** $\frac{x}{8} - 6 = 1$ **e** $\frac{m}{5} + 11 = 31$

f $\frac{k}{9} + 17 = 21$ **g** $\frac{y-4}{3} = 7$ **h** $\frac{z+5}{2} = 14$ **i** $3(x + 7) = 24$ **j** $5(y - 4) = 40$

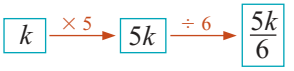
EXAMPLE 6

Solve the following equations by backtracking, and check each solution by substituting into the equation.

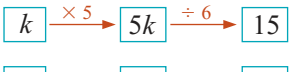
a $\frac{5k}{6} = 15$ **b** $\frac{7x}{2} = 14$

a $\frac{5k}{6} = 15$

Step 1: Draw a flowchart for $\frac{5k}{6}$.



Step 2: Replace $\frac{5k}{6}$ by 15.



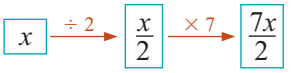
Step 3: Compute backwards to find x by 'undoing' each operation.

Solution is $k = 18$.

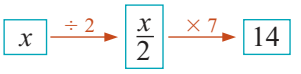
Check: $\frac{5k}{6} = \frac{5 \times 18}{6} = 15$ √

b $\frac{7x}{2} = 14$

Step 1: Draw a flowchart for $\frac{7x}{2}$.



Step 2: Replace $\frac{7x}{2}$ by 14.



Step 3: Compute backwards to find x by 'undoing' each operation.

Solution is $x = 4$.

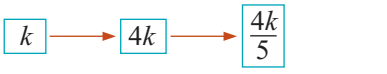
Check: $\frac{7x}{2} = \frac{7 \times 4}{2} = 14$ √

*For both parts **a** and **b** you can do × before ÷ or ÷ before ×.*

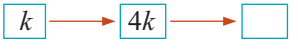
8 Complete to solve the following equations by backtracking and check the solutions.

a $\frac{4k}{5} = 20$

Step 1: Draw a flowchart for $\frac{4k}{5}$.



Step 2: Replace $\frac{4k}{5}$ by ____.



Step 3: Compute backwards to find k by 'undoing' each operation.

Solution is $k =$ ____.

Check: $\frac{4k}{5} = \frac{4 \times \square}{5} =$ ____

b $\frac{5y}{2} = 25$

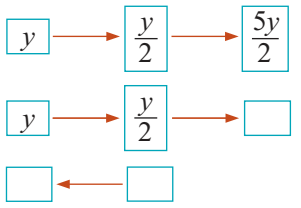
Step 1: Draw a flowchart for $\frac{5y}{2}$.

Step 2: Replace $\frac{5y}{2}$ by ____.

Step 3: Compute backwards to find x by 'undoing' each operation.

Solution is $y =$ ____.

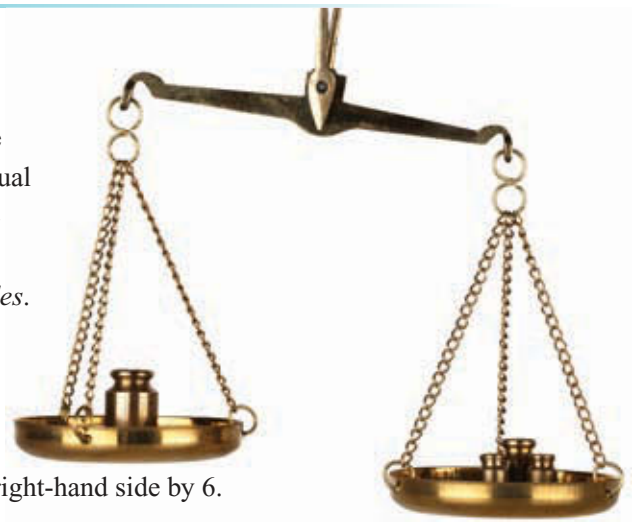
Check: $\frac{5y}{2} = \frac{5 \times \square}{2} =$ ____



9 Solve the following equations by backtracking and check your solutions by substitution.

a $\frac{2x}{3} = 8$ **b** $\frac{3y}{2} = 18$ **c** $\frac{3w}{4} = 6$ **d** $\frac{6z}{5} = 12$ **e** $\frac{11x}{3} = 33$

D The balance method



The **balance** method of solving equations involves representing the equation on a balance with the point of balance representing the equal sign. The pronumeral is isolated by performing the same arithmetic operation on both sides.

To keep an equation balanced, *always do the same thing to both sides*. For example, if you add 10 to the left-hand side, you must also add 10 to the right-hand side.

$3x - 10 = 26$

$3x - 10 + 10 = 26 + 10$

If you multiply the left-hand side by 6, you must also multiply the right-hand side by 6.

$\frac{5x}{6} = 10$

$\frac{5x}{6} \times 6 = 10 \times 6$

Representing equations on a real-life balance can help you solve equations.

This process is called balancing an equation.

EXAMPLE 1

Solve the following by representing the equation on a balance.

a $x + 17 = 33$ **b** $m - 26 = 29$ **c** $2k = 19$ **d** $\frac{q}{7} = 12$

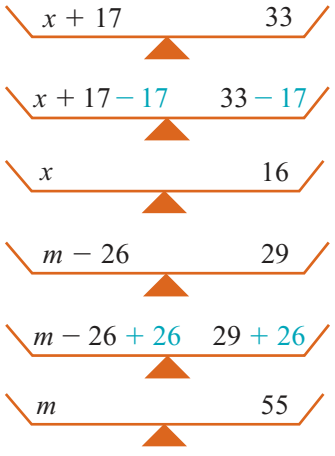
a $x + 17 = 33$

Represent the equation on a balance as shown.

To isolate the x on the left-hand side, you need to undo 'add 17'; that is, 'subtract 17'

Subtract 17 from both sides to keep the equation balanced.

$\therefore x = 33 - 17 = 16$



b $m - 26 = 29$

Represent the equation on a balance as shown.

To isolate the m on the left-hand side, you need to undo 'subtract 26'; that is, 'add 26'.

Add 26 to both sides to keep the equation balanced.

$\therefore m = 29 + 26 = 55$

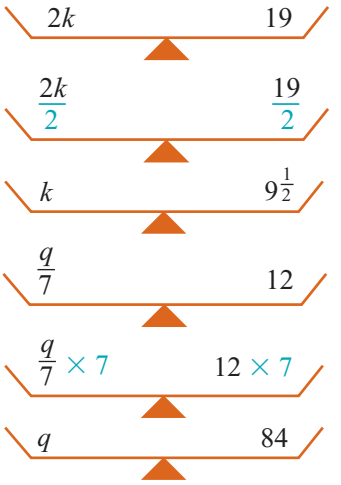
c $2k = 19$

Represent the equation on a balance as shown.

To undo 'multiply by 2'; you need to 'divide by 2'.

Divide by 2 on both sides to keep the equation balanced.

$\therefore k = \frac{19}{2} = 9\frac{1}{2}$



d $\frac{q}{7} = 12$

Represent the equation on a balance as shown.

To undo 'divide by 7' you need to 'multiply by 7'.

Multiply by 7 on both sides to keep the equation balanced.

$\therefore q = 12 \times 7 = 84$

Exercise 14D

1 Complete the following to solve the equations using the balance method.

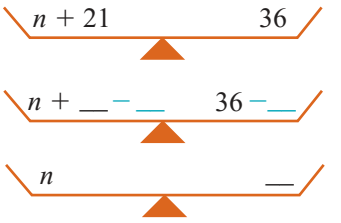
a $n + 21 = 36$

Represent the equation on a balance.

To undo 'add 21' you need to '_____'.

_____ from both sides.

$\therefore n = 36 - \_\_\_\_ = \_\_\_\_$



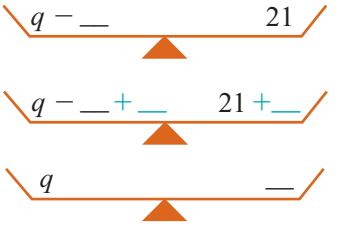
b $q - 35 = 21$

Represent the equation on a balance.

To undo 'subtract 35' you need to '_____'.

_____ to both sides.

$\therefore q = 21 + \_\_\_\_ = \_\_\_\_$



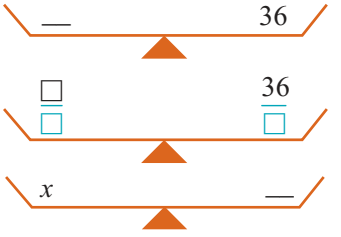
c $9x = 36$

Represent the equation on a balance.

To undo 'multiply by 9' you need to '_____'.

_____ on both sides.

$\therefore x = \frac{36}{\square} = \_\_\_\_$



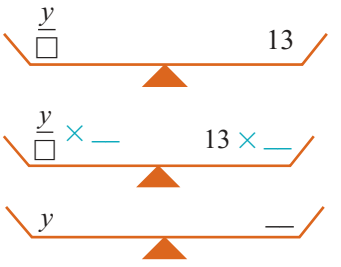
d $\frac{y}{3} = 13$

Represent the equation on a balance.

To undo 'divide by 3' you need to '_____'.

_____ on both sides.

$\therefore y = 13 \times \_\_\_\_ = \_\_\_\_$



2 Solve the following equations using the balance method.

a $z + 36 = 53$ **b** $q - 18 = 44$ **c** $5x = 100$ **d** $\frac{w}{6} = 7$

e $k - 15 = 27$ **f** $2w = 23$ **g** $p + 37 = 81$ **h** $\frac{k}{7} = 13$

E Algebraic solution of equations

Equations can be solved algebraically by:
- adding the same number to both sides
- subtracting the same number from both sides
- multiplying both sides by the same number
- dividing both sides by the same number

or a combination of two or more of the above.

EXAMPLE 1

Solve the following equations. Write out the steps used.

a $y + 55 = 81$ **b** $84 = w - 37$ **c** $13k = 91$ **d** $15 = \frac{m}{9}$ **e** $14 + z = 23$

a
$$y + 55 = 81$$
$$y + 55 - 55 = 81 - 55 \quad \text{Undo '+ 55'}$$
$$y = 26$$

b $84 = w - 37$ is the same as $w - 37 = 84$.
$$w - 37 = 84$$
$$w - 37 + 37 = 84 + 37 \quad \text{Undo '− 37'.}$$
$$w = 121$$

c
$$13k = 91$$
$$\frac{13k}{13} = \frac{91}{13} \quad \text{Undo '× 13'.}$$
$$k = 7$$

d $15 = \frac{m}{9}$ is the same as $\frac{m}{9} = 15$.
$$\frac{m}{9} = 15$$
$$\frac{m}{9} \times 9 = 15 \times 9 \quad \text{Undo '÷ 9'.}$$
$$m = 135$$

e $14 + z = 23$ is the same as $z + 14 = 23$.
$$z + 14 = 23$$
$$z + 14 - 14 = 23 - 14 \quad \text{Undo '+ 14'.}$$
$$z = 9$$

*In parts **b** and **d**, rearrange the equation to put the pronumeral term on the left-hand side.*

Exercise 14E

1 Complete the following to solve the equations.

a
$$m + 33 = 51$$
$$m + 33 - \_\_ = 51 - \_\_$$
$$m = \_\_$$

b
$$w - 12 = 15$$
$$w - 12 + \_\_ = 15 \_\_ \_\_$$
$$w = \_\_$$

c
$$7z = 56$$
$$\frac{7z}{\square} = \frac{56}{\square}$$
$$z = \_\_$$

d
$$\frac{n}{4} = 11$$
$$\frac{n}{4} \times \_\_ = 11 \_\_ \_\_$$
$$n = \_\_$$

2 Solve the following equations.

a $q + 38 = 61$ **b** $p - 42 = 35$ **c** $9z = 81$ **d** $\frac{m}{2} = 54$

e $77 = 7w$ **f** $16 + v = 24$ **g** $7 = \frac{n}{5}$ **h** $k - 12 = 19$

3 Solve the following equations.

a $h + 6.7 = 9.5$ **b** $k - 3.8 = 4.9$ **c** $5w = 17$ **d** $\frac{q}{3} = 2.8$

e $2\frac{1}{2} + x = 7$ **f** $y - \frac{2}{3} = \frac{5}{8}$ **g** $3z = 1$ **h** $\frac{p}{5} = \frac{2}{3}$

4 Solve the following equations.

a $w + 7 = 2$ **b** $m - 6 = -2$ **c** $4z = -12$ **d** $\frac{n}{4} = -7$

e $k - 8 = -1$ **f** $3t = -6$ **g** $\frac{x}{3} = -8$ **h** $y + 6 = 1$

EXAMPLE 2

Solve the following equations and check each solution by substituting into the equation.

a $5x + 2 = 17$ **b** $5(x + 2) = 45$

a

$5x + 2 = 17$

$5x + 2 - 2 = 17 - 2$ Undo '+ 2' first.

$5x = 15$

$\frac{5x}{5} = \frac{15^{3}}{5_{1}}$ Now undo '× 5'.

$x = 3$

Check: $5x + 2 = 5 \times 3 + 2 = 17$ √

b

$5(x + 2) = 45$

$\frac{5(x + 2)}{5} = \frac{45^{9}}{5_{1}}$ Undo '× 5' first.

$x + 2 = 9$

$x + 2 - 2 = 9 - 2$ Now undo '+ 2'.

$x = 7$

Check: $5(x + 2) = 5 \times (7 + 2) = 45$ √

5 Complete the following to solve the equations and check the solutions.

a

$3m + 2 = 20$

$3m + 2 - \_\_ = 20 \_\_ \_\_$

$3m = \_\_$

$\frac{3m}{\square} = \frac{\square}{\square}$

$m = \_\_$

Check: $3m + 2 = 3 \times \_\_ + 2 = \_\_$

b

$3(m + 2) = 21$

$\frac{3(m + 2)}{\square} = \frac{21}{\square}$

$m + 2 = \_\_$

$m + 2 - \_\_ = \_\_ - \_\_$

$m = \_\_$

Check: $3(m + 2) = 3 \times (\_\_ + 2) = \_\_$

6 Solve the following equations and check your solutions by substitution.

a $2x + 5 = 19$ **b** $2(x + 5) = 14$ **c** $3z - 2 = 10$ **d** $3(z - 2) = 18$

e $5y - 4 = 16$ **f** $5(y - 4) = 35$ **g** $20 = 2w + 14$ **h** $4k - 11 = 5$

i $56 = 7(x + 1)$ **j** $4(n - 5) = 24$ **k** $27 = 3 + 8t$ **l** $10(q - 3) = 40$

7 Solve the following equations and check your solutions by substitution.

a $3x + 4 = 1$ **b** $2n - 3 = -7$ **c** $4 = 5k + 9$ **d** $2t - 6 = -2$

e $2(v + 7) = 6$ **f** $3(z - 5) = -9$ **g** $5(k + 3) = 10$ **h** $4(m - 3) = -16$

EXAMPLE 3

Solve the following equations and check each solution by substituting into the equation.

a $\frac{y}{5} + 12 = 16$ **b** $\frac{y + 12}{5} = 8$

a

$\frac{y}{5} + 12 = 16$

$\frac{y}{5} + 12 - 12 = 16 - 12$ Undo '+ 12' first.

$\frac{y}{5} = 4$

$\frac{y}{5} \times 5 = 4 \times 5$ Now undo '÷ 5'.

$y = 20$

Check: $\frac{y}{5} + 12 = \frac{20}{5} + 12 = 16$ √

b

$\frac{y + 12}{5} = 8$

$\frac{y + 12}{5} \times 5 = 8 \times 5$ Undo '÷ 5' first.

$y + 12 = 40$

$y + 12 - 12 = 40 - 12$ Now undo '+ 12'.

$y = 28$

Check: $\frac{y + 12}{5} = \frac{28 + 12}{5} = 8$ √

8 Complete the following to solve the equations and check the solutions.

a $\frac{w}{3} + 5 = 9$

$\frac{w}{3} + 5 - \_\_ = 9 \_\_ \_\_$

$\frac{w}{3} = \_\_$

$\frac{w}{3} \_\_ \_\_ = \_\_ \_\_ \_\_$

$w = \_\_$

Check: $\frac{w}{3} + 5 = \frac{\square}{3} + 5 = \_\_$

b $\frac{w + 5}{3} = 9$

$\frac{w + 5}{3} \times \_\_ = 9 \_\_ \_\_$

$w + 5 = \_\_$

$w + 5 \_\_ \_\_ = \_\_ \_\_ \_\_$

$w = \_\_$

Check: $\frac{w + 5}{3} = \frac{\square + 5}{3} = \_\_$

9 Solve the following equations and check your solutions by substitution.

a $\frac{m}{2} + 5 = 8$ **b** $\frac{m + 5}{2} = 12$ **c** $\frac{n}{3} - 7 = 8$

d $\frac{n - 7}{3} = 8$ **e** $\frac{x}{4} + 7 = 27$ **f** $\frac{x + 7}{4} = 3$

g $\frac{w}{2} - 13 = 2$ **h** $10 = \frac{z}{3} + 9$ **i** $8 = \frac{k - 6}{4}$

j $\frac{n + 3}{2} = 9$ **k** $\frac{8 + t}{7} = 10$ **l** $7 = \frac{t - 5}{4}$

10 Solve the following equations and check your solutions by substitution.

a $\frac{x}{3} + 5 = 1$ **b** $\frac{n}{2} - 4 = -3$ **c** $\frac{m}{5} + 7 = 3$

d $\frac{w + 6}{3} = 1$ **e** $\frac{z - 3}{2} = -5$ **f** $\frac{k + 9}{4} = 2$

g $-2 = \frac{x + 8}{2}$ **h** $5 = \frac{n}{10} + 7$ **i** $\frac{x - 2}{3} = 6$

EXAMPLE 4

Solve the following equations and check each solution by substituting into the equation.

a $\frac{2t}{5} = 6$ **b** $\frac{7x}{3} = 28$

a

$\frac{2t}{5} = 6$

$\frac{2t}{\cancel{5}} \times \cancel{5} = 6 \times 5$ Undo '$\div$ 5' first.

$2t = 30$

$\frac{\cancel{2}t}{\cancel{2}} = \frac{\overset{15}{\cancel{30}}}{\cancel{2}_{1}}$ Now undo '$\times$ 2'.

$t = 15$

Alternative method

$\frac{2t}{5}$ is equivalent to $\frac{2}{5} \times t$.

So $\frac{2}{5} \times t = 6$

$\frac{2}{5} \times t \div \frac{2}{5} = 6 \div \frac{2}{5}$ Undo '$\times \frac{2}{5}$'.

$\frac{\cancel{2}}{\cancel{5}} \times t \times \frac{\cancel{5}}{\cancel{2}} = \overset{3}{\cancel{6}} \times \frac{5}{\cancel{2}_{1}}$

$t = 15$

Check: $\frac{2t}{5} = \frac{2 \times 15}{6} = 5$ √

b

$\frac{7x}{3} = 28$

$\frac{7x}{\cancel{3}} \times \cancel{3} = 28 \times 3$ Undo '$\div$ 3' first.

$7x = 84$

$\frac{\cancel{7}x}{\cancel{7}} = \frac{\overset{12}{\cancel{84}}}{\cancel{7}_{1}}$ Now undo '$\times$ 7'.

$x = 12$

Alternative method

$\frac{7x}{3}$ is equivalent to $\frac{7}{3} \times x$.

So $\frac{7}{3} \times x = 28$

$\frac{7}{3} \times x \div \frac{7}{3} = 28 \div \frac{7}{3}$ Undo '$\times \frac{7}{3}$'.

$\frac{\cancel{7}}{\cancel{3}} \times x \times \frac{\cancel{3}}{\cancel{7}} = \overset{4}{\cancel{28}} \times \frac{3}{\cancel{7}_{1}}$

To divide by a fraction, multiply by its reciprocal.

$x = 12$

Check: $\frac{7x}{3} = \frac{7 \times 12}{3} = 28$ √

11 Complete the following to solve the equations and check the solutions.

a $\frac{4m}{9} = 8$

$\frac{4m}{9} \times \_\_ = 8 \_\_ \_\_$

$4m = \_\_$

$\frac{4m}{\square} = \frac{\square}{\square}$

$m = \_\_$

Check: $\frac{4m}{9} = \frac{4 \times \square}{9} = \_\_$

b $\frac{11z}{24} = 22$

$\frac{11z}{24}$ is equivalent to $\frac{11}{24} \times z$.

$\frac{11}{24} \times z = 22$

$\frac{11}{24} \times z \div \frac{\square}{\square} = 22 \div \frac{\square}{\square}$

$\frac{11}{24} \times z \times \frac{\square}{\square} = 22 \times \frac{\square}{\square}$

$z = \_\_$

Check: $\frac{11z}{24} = \frac{11 \times \square}{24} = \_\_$

12 Solve the following equations and check your solutions by substitution.

a $\frac{3w}{5} = 6$ **b** $\frac{5k}{2} = 15$ **c** $10 = \frac{5z}{6}$ **d** $\frac{6k}{5} = 12$ **e** $4 = \frac{4t}{7}$

13 Solve the following equations and check your solutions by substitution.

a $\frac{2t}{5} = -4$ **b** $\frac{7w}{3} = -14$ **c** $\frac{4x}{7} = -8$ **d** $-10 = \frac{5x}{11}$ **e** $-6 = \frac{4n}{9}$

EXAMPLE 5

Solve the following equations and check each solution by substituting into the equation.

a $\frac{2k}{3} = \frac{5}{7}$ **b** $\frac{6x}{5} = \frac{2}{3}$

a

$\frac{2k}{3} = \frac{5}{7}$

$\frac{2k}{\cancel{3}} \times \cancel{3} = \frac{5}{7} \times 3$ Undo '÷ 3' first.

$2k = \frac{15}{7}$

$2k \div 2 = \frac{15}{7} \div 2$ Now undo '× 2'.

$\cancel{2}k \times \frac{1}{\cancel{2}} = \frac{15}{7} \times \frac{1}{2}$

$k = \frac{15}{14} = 1\frac{1}{14}$

Alternative method

$\frac{2k}{3}$ is equivalent to $\frac{2}{3} \times k$.

$\frac{2}{3} \times k = \frac{5}{7}$

$\frac{2}{3} \times k \div \frac{2}{3} = \frac{5}{7} \div \frac{2}{3}$ Undo '× $\frac{2}{3}$'.

$\frac{\cancel{2}}{\cancel{3}} \times k \times \frac{\cancel{3}}{\cancel{2}} = \frac{5}{7} \times \frac{3}{2}$

$k = \frac{15}{14} = 1\frac{1}{14}$

Check: $\frac{2k}{3} = \frac{2}{3} \times \frac{15}{14} = \frac{5}{7}$ √

b

$\frac{6x}{5} = \frac{2}{3}$

$\frac{6x}{\cancel{5}} \times \cancel{5} = \frac{2}{3} \times 5$ Undo '÷ 5' first.

$6x = \frac{10}{3}$

$6x \div 6 = \frac{10}{3} \div 6$ Now undo '× 6'.

$\cancel{6}x \times \frac{1}{\cancel{6}} = \frac{{}^{5}\cancel{10}}{3} \times \frac{1}{\cancel{6}_{3}}$

$= \frac{5}{9}$

Alternative method

$\frac{6x}{5}$ is equivalent to $\frac{6}{5} \times x$.

$\frac{6}{5} \times x = \frac{2}{3}$

$\frac{6}{5} \times x \div \frac{6}{5} = \frac{2}{3} \div \frac{6}{5}$ Undo '× $\frac{6}{5}$'.

$\frac{\cancel{6}}{\cancel{5}} \times x \times \frac{\cancel{5}}{\cancel{6}} = {}^{1}\frac{2}{3} \times \frac{5}{\cancel{6}_{3}}$

$x = \frac{5}{9}$

Check: $\frac{6x}{5} = \frac{6}{5} \times \frac{5}{9} = \frac{2}{3}$ √

14 Complete to solve the following equations and check the solutions.

a $\frac{4m}{3} = \frac{5}{8}$

$\frac{4m}{3} \times \_\_ = \frac{5}{8} \times \_\_$

$4m = \frac{15}{8}$

$4m \div \_\_ = \frac{15}{8} \div \_\_$

$4m \times \frac{1}{4} = \frac{15}{8} \times \frac{1}{4}$

$m = \frac{\square}{\square}$

Check: $\frac{4m}{3} = \frac{4}{3} \times \frac{\square}{\square} = \frac{\square}{\square}$

b $\frac{5y}{2} = \frac{4}{7}$

$\frac{5y}{2}$ is equivalent to $\frac{5}{2} \times y$.

$\frac{5}{2} \times y = \frac{4}{7}$

$\frac{5}{2} \times y \div \frac{\square}{\square} = \frac{4}{7} \div \frac{\square}{\square}$

$\frac{5}{2} \times y \times \frac{\square}{\square} = \frac{4}{7} \times \frac{\square}{\square}$

$y = \frac{\square}{\square}$

Check: $\frac{5y}{2} = \frac{5}{2} \times \frac{\square}{\square} = \frac{\square}{\square}$

15 Solve the following equations and check your solutions by substitution.

a $\frac{2x}{3} = \frac{5}{8}$ **b** $\frac{4k}{3} = \frac{2}{5}$ **c** $\frac{2}{3} = \frac{3t}{5}$ **d** $\frac{1}{2} = \frac{9p}{5}$ **e** $\frac{7x}{10} = \frac{2}{5}$

EXAMPLE 6

Solve $7 - 2x = 13$ and check the solution by substituting into the equation.

Method 1

$7 - 2x = 13$

Subtract 7 from both sides. $7 - 2x - 7 = 13 - 7$

$-2x = 6$

This is $-2 \times x = 6$

Divide both sides by -2. $\frac{-2x}{-2} = \frac{6^{3}}{-2}$

$x = -3$

Check: $7 - 2x = 7 - 2 \times (-3) = 13$ √

Method 2

$7 - 2x = 13$

Add $2x$ to both sides. $7 - 2x + 2x = 13 + 2x$

$7 = 13 + 2x$

Reverse sides. $2x + 13 = 7$

Subtract 13 from both sides. $2x + 13 - 13 = 7 - 13$

$2x = -6$

Divide both sides by 2. $\frac{2x}{2} = -\frac{6^{3}}{2}$

$x = -3$

16 Complete the following to solve $5 - 3x = 11$ and check the solution.

Method 1

$5 - 3x = 11$

Subtract 5 from both sides. $5 - 3x - 5 = 11 - 5$

$\_\_ = 6$

Divide both sides by -3. $\frac{\square}{-3} = \frac{6}{-3}$

$x = \_\_$

Check: $5 - 3x = 5 - 3 \times (\_\_) = \_\_$

Method 2

$5 - 3x = 11$

Add $3x$ to both sides. $5 - 3x + 3x = 11 + 3x$

$\_\_ = \_\_ + \_\_$

Reverse sides. $\_\_ + \_\_ = \_\_$

Subtract 11 from both sides. $\_\_ + \_\_ - 11 = \_\_ - 11$

$3x = -6$

Divide both sides by 3. $\frac{\square}{\square} = \frac{\square}{\square}$

$x = \_\_\_$

17 Solve the following equations and check your solutions by substitution.

a $8 - 2x = 10$ **b** $5 - 4x = 15$ **c** $7 - 3x = -8$ **d** $9 - x = -2$

F Generating equations with a given solution

In section E you discovered that equations can be solved algebraically by:

- adding the same number to both sides
- subtracting the same number from both sides
- multiplying both sides by the same number
- dividing both sides by the same number

or by a combination of two or more of the above.

These steps can also be used to **generate** (create) equations, given the solution.

EXAMPLE 1

Generate an equation that can be solved in one step and has the solution $x = 3$.

Start with $x = 3$. Carry out any one of the steps listed at the start of this section.

- Add the same number to both sides of the equation. For example:
  Add 4 to both sides. $x + 4 = 3 + 4$
  $x + 4 = 7$
- Subtract the same number from both sides of the equation. For example:
  Subtract 7 from both sides. $x - 7 = 3 - 7$
  $x - 7 = -4$
- Multiply both sides of the equation by the same number. For example:
  Multiply both sides by 5. $5 \times x = 5 \times 5$
  $5x = 15$
- Divide both sides of the equation by the same number. For example:
  Divide both sides by 2. $x \div 2 = 3 \div 2$
  $\frac{x}{2} = \frac{3}{2}$ or $\frac{x}{2} = 1.5$

You can add, subtract, multiply and divide by any number, so there is an infinite number of correct answers to this question.

Exercise 14F

1 Do the following to generate equations that can be solved in one step and have the solution $x = 5$.

a Add 6 to both sides.
b Subtract 4 from both sides.
c Multiply both sides by 3.
d Divide both sides by 5.

2 Generate an equation that can be solved in one step and has the solution:

a $x = 10$ **b** $x = 6$ **c** $x = -2$ **d** $x = \frac{7}{2}$

Check your equations with another person.

EXAMPLE 2

Generate an equation that can be solved in two steps and has the solution $x = 4$.

Start with $x = 4$. Carry out any two of the steps listed at the start of this section. For example:

- Multiply both sides of the equation by 2, then add 3 to both sides.
  $2 \times x = 2 \times 4$
  $2x = 8$
  then $2x + 3 = 8 + 3$
  So $2x + 3 = 11$

- Divide both sides of the equation by 2, then subtract 3 from both sides.

$$x \div 2 = 4 \div 2$$
$$\frac{x}{2} = 2$$
then $$\frac{x}{2} - 3 = 2 - 3$$
So $$\frac{x}{2} - 3 = -1$$

- Subtract 1 from both sides of the equation, then multiply both sides by 2.

$$x - 1 = 4 - 1$$
$$x - 1 = 3$$
then $$2 \times (x - 1) = 2 \times 3$$
So $$2(x - 1) = 6$$

- Multiply both sides of the equation by 5, then divide both sides by 2.

$$5 \times x = 5 \times 4$$
$$5x = 20$$
then $$5x \div 2 = 20 \div 2$$
So $$\frac{5x}{2} = 10$$

3 Do the following to generate equations that can be solved in two steps and have the solution $x = 6$.

a Multiply both sides by 3 then add 5 to both sides.
b Multiply both sides by 2 then subtract 7 from both sides.
c Add 3 to both sides then multiply both sides by 5.
d Subtract 1 from both sides then multiply both sides by 4.
e Divide both sides by 3 then add 5 to both sides.
f Divide both sides by 2 then subtract 4 from both sides.
g Add 8 to both sides then divide both sides by 4.
h Subtract 6 from both sides then divide both sides by 5.

4 Generate an equation that can be solved in two steps and has the solution:

a $x = 3$ **b** $x = 9$
c $x = -4$ **d** $x = \frac{2}{3}$

Check your equations with another person.

G Solving problems using equations

To solve problems using equations:
Step 1: Write the equation from the question.
Step 2: Substitute known values into the equation.
Step 3: Solve the equation by a suitable method (guess, check and improve; backtracking; balancing; or algebraic methods).
Step 4: Write the answer in the words of the question.

Sometimes you may be given an equation or rule. At other times you may need to work out an equation from the information in the question.

EXAMPLE 1

The formula $C = 1.5P$ can be used to find the cost ($\$C$) of buying P litres of petrol. How many litres of petrol can be bought for \$60?

$C = 1.5P$

Substitute $C = 60$.

$60 = 1.5P$ or $1.5P = 60$

$$\frac{1.5P}{1.5} = \frac{60}{1.5}$$

$$P = 40$$

$\therefore$ 40 L of petrol can be bought for \$60.

Exercise 14G

1 The formula for converting between kilometres (K) and miles (M) is $K = 1.6M$. Complete the following to convert 640 kilometres into miles.

$K = 1.6M$

$640 = 1.6M$ or $1.6M = 640$

$$\frac{1.6\,M}{\square} = \frac{640}{\square}$$

$M =$ ___

$\therefore$ ___ kilometres = ___ miles

2 Bricklayers use the formula $N = 50A$ to estimate the number of bricks needed (N) to build a wall of area A m^2. What area of wall could be built with 1000 bricks?

3 Ben earns \$18/h. He uses the formula $w = 18t$ to calculate his wages ($\$w$) each week, where t is the number of hours he works. How many hours did he work if he earned \$540?

4 The diameter (d) of a circle can be approximated by the formula $d = \frac{C}{3.1}$, where C is the circumference. Find the approximate circumference of a circle with a diameter of 1.7 m.

5 The number of litres of paint (P) needed to cover an area of A m^2 is given by the formula $P = \frac{A}{16}$. Calculate the area that can be painted with a 20 L can of paint.

6 The body mass index of a person of height 1.6 m is approximated by the formula $B = \frac{2m}{5}$, where B is the body mass index and m is the person's mass in kilograms. What would be the mass of a person of height 1.6 m with a body mass index of 24.4?

7 The cost ($\$C$) of hiring a taxi is given by the formula $C = 2d + 3$, where d is the length of the journey in kilometres.

a The distance from Kelly's home to the airport is 22 km. How much would it cost Kelly to hire a taxi for this journey?

b When Kelly caught a taxi home from the local railway station, it cost her $12. How far does she live from the station?

8 The time needed to bake a piece of meat is 20 min plus 30 min for every $\frac{1}{2}$ kg of meat to be cooked. This information can be expressed by the formula $T = 20 + 30w$, where T is the number of minutes needed and w is the number of $\frac{1}{2}$ kg of meat.

a Calculate the time needed to bake a piece of meat that weighs 1.5 kg.

b A piece of meet was baked for 80 min, using this formula. What was the weight of meat cooked?

9 Cherry is paid a weekly commission ($\$C$), which is calculated by the formula $C = 200 + \frac{S}{20}$, where S is the value, in dollars, of her sales (the goods she sells).

a How much did she earn in the week when the value of her sales was $12 000?

b One week her commission was $960. What was the value of her sales?

10 George is paid a weekly commission ($\$C$), which is calculated from the formula $C = \frac{S - 2000}{10}$, where S is the value, in dollars, of his sales.

a Calculate his commission if his sales are $7890.

b One week his commission was $960. What were his sales?

11 To build a pattern of rectangles, the number of matches needed (N) is given by $N = 5r + 1$, where r is the number of rectangles.

a How many matches are needed to build 9 rectangles?

b How many rectangles could be built using 86 matches?

12 To build a pattern of triangles, the number of matches needed (N) is given by $N = 3 + 2(t - 1)$, where t is the number of triangles.

a How many matches are needed to build 10 triangles?

b How many triangles could be built using 25 matches?

EXAMPLE 2

The average of two numbers is 45. If one of the numbers is 53, what is the other number?

Let the other number be n. Write an equation from which the value of n can be found.

$$\frac{n + 53}{2} = 45$$

Solve the equation.

$$\frac{n + 53}{2} \times 2 = 45 \times 2$$
$$n + 53 = 90$$
$$n + 53 - 53 = 90 - 53$$
$$n = 37$$

$\therefore$ The other number is 37.

Remember: Average $= \frac{\text{sum of scores}}{\text{number of scores}}$

13 The sum of two numbers is 83. If one of the numbers is 45, let the other number be n.

a Complete the following to write an equation from which n be found.

$n +$ __ $=$ __

b Solve the equation to find n.

14 The difference between two numbers is 17. If the smaller number is 53, let the other number be n.

a Write an equation from which n can be found.

b Solve the equation.

15 The average of two numbers is 34. If one of the numbers is 39, let the other number be n.

a Write an equation from which n can be found.

b Solve the equation.

16 When a number is multiplied by 5 and 3 is added to the result, the answer is 38. Let the number be n.

a Write an equation from which n can be found.

b Solve the equation.

17 When a number is divided by 3 and 2 is subtracted from the result, the answer is 2. Let the number be n.

a Write an equation from which n can be found.

b Solve the equation.

18 When 2 is subtracted from a number and the result is divided by 3, the answer is 5. Let the number be n.

a Write an equation from which n can be found.

b Solve the equation.

19 Georgia is 3 years older than her brother. The total of their ages is 33. Let n be her brother's age.

a Write an equation from which n can be found.

b Solve the equation.

20 Daniel is 4 years younger than his brother. The total of their ages is 30. Let n be the age of Daniel's brother.

a Write an equation from which n can be found.

b Solve the equation.

Language in mathematics

1 Explain in words how to produce the following algebraic expressions, starting with x.

a $x + 3$ **b** $x - 5$ **c** $9x$ **d** $\frac{x}{4}$ **e** $\frac{6x}{7}$

f $4x + 5$ **g** $4(x + 5)$ **h** $\frac{x}{5} - 7$ **i** $\frac{x - 7}{5}$

2 Explain in words the steps needed to solve each of these equations (you do not need to solve them).

a $3x - 1 = 8$ **b** $3(x - 1) = 8$ **c** $\frac{x + 2}{3} = 5$ **d** $\frac{x}{3} + 2 = 7$ **e** $\frac{8x}{9} = 4$

3 Explain the guess, check and improve method of solving equations.

Terms

arithmetic	backtrack	balance	check	compute	concrete materials
equation	expression	flowchart	generate	guess	improve
operation	reciprocal	solution	substitution	undo	

Check your skills

1 $x = 6$ is a solution of which equation?

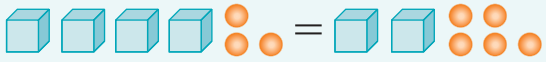
A $x - 8 = 2$ **B** $4x = 46$ **C** $4x - 3 = 12$ **D** $4(x - 3) = 12$

2 The guess, check and improve method is being used to solve the equation $143x + 324 = 1761$. The first two guesses are shown in the table. Using this information, an improved guess for the solution would be:

x	5	9
$143x + 324$	1039	1611

A 1040 **B** 1657 **C** 8 **D** 10

3 In this equation, cubes represent the unknown x and counters represent units. Which statement is not correct?

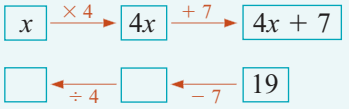


A $2x = 2$ **B** $6x = 8$ **C** $4x + 3 = 2x + 5$ **D** $x = 1$

4 The expression produced by this flowchart is:

A $\frac{x}{2} - 3$ **B** $x - \frac{3}{2}$ **C** $\frac{x - 2}{3}$ **D** $\frac{x - 3}{2}$

5 A flowchart to solve the equation $4x + 7 = 19$ is shown. The missing numbers, backtracking from right to left, are:



A 26, $6\frac{1}{2}$ **B** 12, 3

C 26, 104 **D** 12, 48

6 To solve the equation $\frac{y}{2} - 3 = 1$ in two steps, you could perform the following operations on both sides.

A Add 3 then divide by 2. **B** Add 3 then multiply by 2.

C Subtract 3 then multiply by 2. **D** Subtract 3 then divide by 2.

7 The solution of the equation $\frac{z - 5}{3} = 8$ is $z =$ ___.

A 19 **B** 29 **C** 39 **D** $4\frac{1}{3}$

8 The solution of the equation $\frac{2n}{5} = \frac{3}{4}$ is $n =$ ___.

A $1\frac{7}{8}$ **B** $\frac{3}{10}$ **C** $7\frac{1}{2}$ **D** $1\frac{1}{2}$

9 The solution of the equation $13 - 2x = -1$ is:

A $x = 7$ **B** $x = -7$ **C** $x = 6$ **D** $x = -6$

10 The equation with solution $x = 10$, generated by dividing both sides by 5 then subtracting 3 from both sides, is:

A $x - 3 = -1$ **B** $\frac{x}{5} - 3 = 2$ **C** $\frac{x-3}{5} = \frac{7}{5}$ **D** $\frac{x}{5} - 3 = -1$

11 Peta is 5 years older than her brother. The total of their ages is 33. Let n be her brother's age. An equation that could be used to find n is:

A $n + 5 = 33$ **B** $2n = 33$ **C** $2n - 5 = 33$ **D** $2n + 5 = 33$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3	4, 5	6	7–9	10	11
Section	A	B	C	D	E	F	G

14A Review set

1 Complete the tables below to solve the given equations by the guess, check and improve method.

a $3x - 5 = 31$

Solution is $x =$ ____.

x	6	10		
$3x - 5$	13			

b $\frac{m}{3} + 6 = 11$

Solution is $m =$ ____.

x	6	9		
$\frac{m}{3} + 6$	8			

2 Write the equation represented by these centicubes (unknown x) and counters (units). What is the first step towards solving the equation?

3 Starting with x, write the expression produced if you:

a multiply by 5 then add 3 **b** add 3 then multiply by 5

c divide by 4 then subtract 1 **d** subtract 1 then divide by 4

4 Draw flowcharts for the following expressions.

a $3x + 7$ **b** $2z - 5$ **c** $\frac{x}{3} + 9$ **d** $\frac{p}{2} - 3$

e $5(u + 4)$ **f** $\frac{y-6}{5}$ **g** $\frac{2w}{3}$

5 Solve the following equations by drawing flowcharts and backtracking. Check solutions by substitution.

a $4x - 3 = 17$ **b** $4(x - 3) = 12$ **c** $\frac{x}{2} - 4 = 1$ **d** $\frac{x-4}{2} = 1$ **e** $\frac{3k}{4} = 9$

6 Solve the following equations and check your solutions by substitution.

a $y + 5 = 2$ **b** $z - 5 = -2$ **c** $9 = 4n$ **d** $\frac{m}{6} = -5$ **e** $\frac{2k}{5} = \frac{1}{3}$

f $11 = 4w + 7$ **g** $\frac{y}{3} - 6 = 7$ **h** $\frac{k-5}{2} = 7$ **i** $6 - 5x = 21$ **j** $5 - x = -4$

7 To generate an equation that can be solved in two steps and has the solution $x = 12$, you divide both sides by 4 then subtract 1 from both sides. Write the equation.

8 Sylvie is a salesperson. She is paid a weekly commission ($\$C$). Her commission is calculated by the formula $C = 200 + \frac{S}{20}$, where S is the value, in dollars, of her sales (the goods she sells).

a How much did she earn in the week when the value of her sales was \$14 000?

b One week her commission was \$840. What was the value of her sales?

9 The average of the numbers 17, 22 and n is 20.

a Write an equation from which n can be found. **b** Solve the equation.

14B Review set

1 Complete the tables below to solve the given equations by the guess, check and improve method.

a $3(x - 17) = 42$

Solution is $x =$ ____.

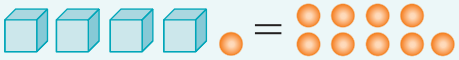
x	20	25		
$3(x - 17)$	9			

b $\frac{n + 15}{4} = 8$

Solution is $n =$ ____.

x	10	15		
$\frac{n + 15}{4}$	$6\frac{1}{4}$			

2 An equation is represented by centicubes (unknown x) and counters (units) as shown. What are the next two steps towards solving the equation?



3 Starting with x, write the expression produced if you:

a multiply by 3 then add 5 **b** add 1 then multiply by 5

c divide by 6 then subtract 1 **d** subtract 2 then divide by 7

4 Draw flowcharts for the following expressions.

a $2x + 9$ **b** $3z - 7$ **c** $\frac{n}{2} + 8$ **d** $\frac{m}{5} - 1$

e $6(u - 3)$ **f** $\frac{m + 3}{4}$ **g** $\frac{4x}{9}$

5 Solve the following equations by drawing flowcharts and backtracking. Check solutions by substitution.

a $3x + 1 = 16$ **b** $3(x + 1) = 12$ **c** $\frac{x}{3} - 4 = 1$ **d** $\frac{x - 4}{3} = 1$ **e** $\frac{2m}{7} = \frac{3}{5}$

6 Solve the following equations and check your solutions by substitution.

a $y + 7 = 2$ **b** $m - 6 = -2$ **c** $3n = 17$ **d** $\frac{k}{7} = -5$

e $\frac{1}{3} = \frac{3n}{5}$ **f** $5t + 1 = 11$ **g** $7 = \frac{v}{3} - 8$ **h** $\frac{y - 2}{2} = 9$

i $9 - 4x = 17$ **j** $2 - x = 10$

7 To generate an equation that can be solved in two steps and has the solution $x = 5$, you subtract 8 from both sides then multiply both sides by 3. Write the equation.

8 To build a pattern of hexagons, the number of matches needed (N) is given by $N = 6 + 5(h - 1)$, where h is the number of hexagons.

a How many matches are needed to build 5 hexagons?

b How many hexagons could be built using 51 matches?

9 The average of two numbers is 37. One of the numbers is 45. Let the other number be n.

a Write an equation from which n can be found.

b Solve the equation to find n.

14C Review set

1 Complete the tables below to solve the given equations by the guess, check and improve method.

a $4x + 13 = 57$

Solution is $x =$ ____.

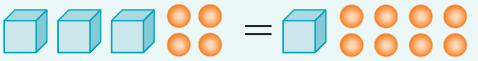
x	5	10		
$4x + 13$	33			

b $5x - 11 = 89$

Solution is $x =$ ____.

x	5	10		
$5x - 11$	14			

2 Write the equation represented by these centicubes (unknown x) and counters (units), and find the solution.



3 Starting with x, write the expression produced if you:

a multiply by 7 then add 1

b add 4 then multiply by 9

c divide by 3 then subtract 4

d subtract 1 then divide by 5

4 Draw flowcharts for the following expressions.

a $3m + 8$ **b** $2x - 5$ **c** $\frac{m}{4} + 3$ **d** $\frac{w}{9} - 1$

e $3(w + 7)$ **f** $\frac{b - 6}{5}$ **g** $\frac{9v}{4}$

5 Solve the following equations by drawing flowcharts and backtracking. Check solutions by substitution.

a $4x + 3 = 27$ **b** $4(x - 5) = 28$ **c** $\frac{y}{4} - 3 = 1$ **d** $\frac{y - 3}{4} = 1$ **e** $\frac{n}{5} = \frac{3}{4}$

6 Solve the following equations and check your solutions by substitution.

a $x + 3 = 1$ **b** $-3 = m - 8$ **c** $5w = 13$ **d** $\frac{n}{3} = -4$

e $\frac{2x}{3} = \frac{6}{7}$ **f** $13 = 4t - 7$ **g** $\frac{q}{5} - 3 = 2$ **h** $\frac{z - 6}{2} = 7$

i $6 - 2x = -24$ **j** $11 - x = 7$

7 To generate an equation that can be solved in two steps and has the solution $x = -4$, you add 9 to both sides then divide both sides by 5. Write the equation.

8 A salesperson is paid a weekly commission ($\$C$), which is calculated from the formula $C = \frac{S - 1500}{10}$, where S is the value, in dollars, of the goods he sells.

a Calculate his commission if his sales are \$9000.

b One week his commission was \$1100. What were his sales?

9 Katie is 4 years older than her brother. The total of their ages is 34. Let n be her brother's age.

a Write an equation from which n can be found.

b Solve the equation.

15

Triangles and quadrilaterals

This chapter deals with the properties of geometrical figures.

At the end of this chapter you should be able to:

- use correct terminology and notation
- construct triangles
- classify triangles based on their sides and angles
- use the angle sum of a triangle property
- use the properties of types of triangles
- use the exterior angle of a triangle theorem
- classify quadrilaterals
- use the properties of quadrilaterals including angle sum.

Diagnostic test

1 How many degrees in a right angle?

A 360° **B** 270° **C** 90° **D** 180°

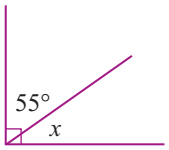
2 How many degrees in a straight angle?

A 360° **B** 270° **C** 90° **D** 180°

3 How many degrees in a revolution?

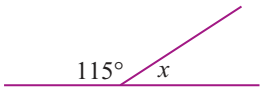
A 360° **B** 270° **C** 90° **D** 180°

4 What is the value of x?



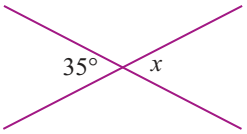
A 35° **B** 55° **C** 135° **D** 305°

5 What is the value of x?



A 35° **B** 65° **C** 145° **D** 245°

6 What is the value of x?



A 35° **B** 55° **C** 135° **D** 325°

7 What is this type of angle?

A revolution **B** acute
C obtuse **D** reflex

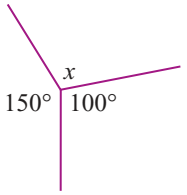
8 What is this type of angle?

A revolution **B** acute
C obtuse **D** reflex

9 What is this type of angle?

A revolution **B** acute
C obtuse **D** reflex

10 What is the value of x?

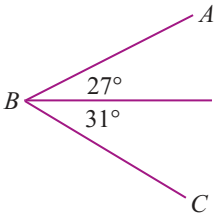


A 110° **B** 80° **C** 30° **D** 10°

11 An acute angle is:

A equal to 180° **B** $>$ 90° but $<$ 180°
C $>$ 0° but $<$ 90° **D** equal to 90°

12 Calculate the size of $\angle ABC$.



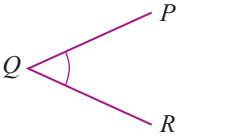
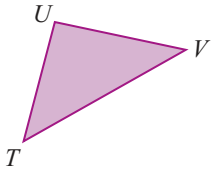
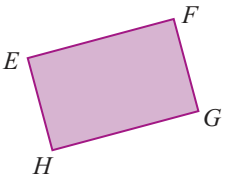
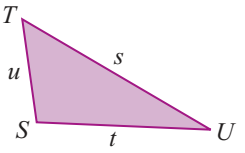
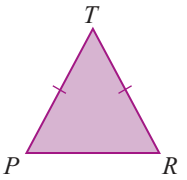
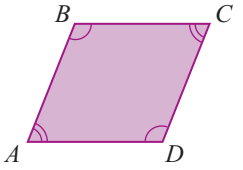
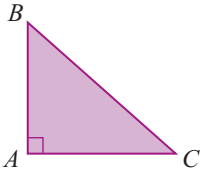
A 4° **B** 32° **C** 58° **D** 122°

The Diagnostic test questions refer to the Year 6 content description below.

Question	1–12
Section	ACMMG 141

A Notation: a review

Conventions are used to label diagrams and describe geometric figures and their properties. This section reviews these conventions and consolidates some others.

Convention	Example
A point is labelled with a capital letter. The example shows point B.	
An interval is named using its endpoints. The example shows the interval DC.	
A line is named using any two points on it. The example shows the line PR.	
An angle is named using its arms and vertex. The example could be named angle PQR, angle RQP or angle Q (written $\angle PQR$, $\angle RQP$ or $\angle Q$, or $\hat{PQR}$, $\hat{RQP}$ or $\hat{Q}$ respectively).	
Triangles are named using the three angles or vertices. The example shows $\triangle TUV$.	
Other figures are labelled using the vertices in a cyclic order. The example shows a quadrilateral $EFGH$.	
Sides of triangles are named in two ways. In the example the longest side can be named using two points, TU or UT, or the lower case letter of the opposite vertex, s.	
Equal sides are labelled with the same markings. The example shows $TR = TP$.	
Equal angles are labelled with the same markings. The example shows $\angle B = \angle D$ and $\angle A = \angle C$.	
Right angles are shown with a square symbol. The example shows $\angle BAC = 90°$.	

Exercise 15A

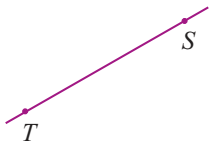
1 Draw a diagram showing each feature.

a Point Q **b** Interval XY **c** Line EF **d** Angle BAC

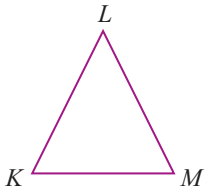
e Triangle MNO **f** Rectangle $PQRS$ **g** Hexagon $ABCDEF$ **h** Triangle with sides g, h and i

2 Describe the features shown in the following diagrams.

a

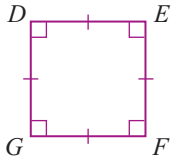


b



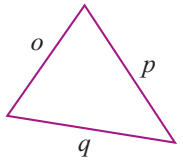
c

d

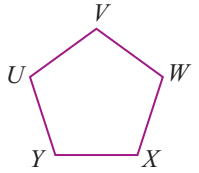


e

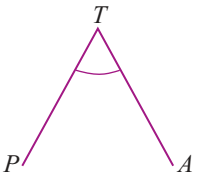
f



g

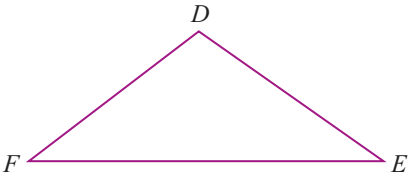


h

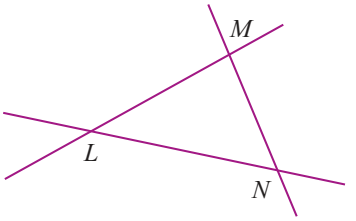


3 **a** Name this line in three different ways.

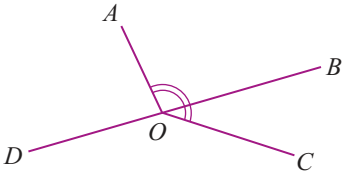
b Name the side opposite angle D in two different ways.



c Name the three lines in this diagram.

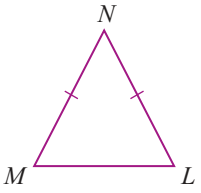


d Name the angle marked and explain why it is not $\angle O$.

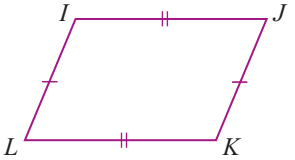


4 List the equal sides in each diagram.

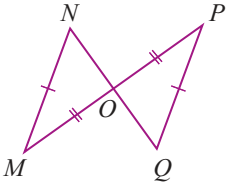
a



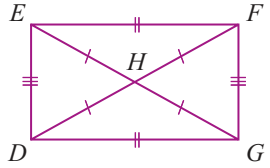
b



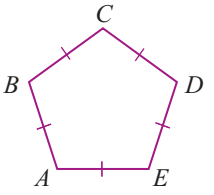
c



d



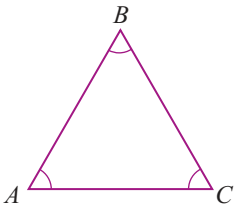
e



More than one marking may be used if more than one pair of sides are equal.

5 List the equal angles in each diagram.

a



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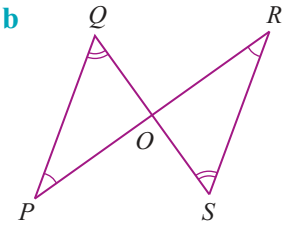
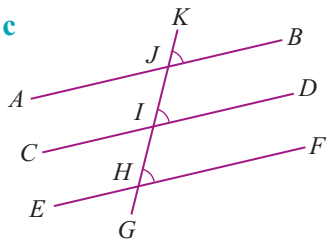
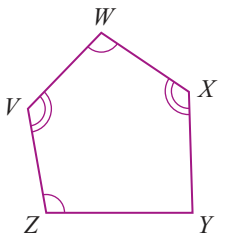


c



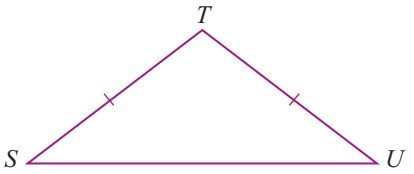
d



EXAMPLE 1

Draw a sketch of triangle STU with $\angle STU$ obtuse and $ST = TU$.

$\angle STU$ is greater than 90°.
ST and TU are labelled as equal.



6 Draw sketches showing the following information.

a Triangle DEF with $DE = DF$

b Triangle XYZ with $\angle Y = \angle Z$ and side YZ the longest side

c Triangle LMN with $\angle NLM = 90°$ and $NL = LM$

d Quadrilateral $PQRS$ with $PQ = RS$ and $\angle PSR = 90°$

e Quadrilateral $DEFG$ with $DE = GF$, $EF = DG$ and $\angle DEF = 90°$

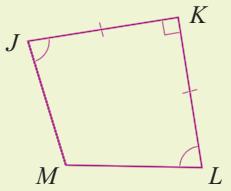
f Quadrilateral $ABCD$ with $AB = AD = 5$ cm, $\angle BAD = 80°$, $BC = 6$ cm and $CD = 7$ cm

g Quadrilateral $PQRS$ with $PQ = 5$ cm, $PS = 6$ cm, $QR = RS = 7$ cm and $\angle QPS = 110°$

h Quadrilateral $KLMN$ with $KL = 6$ cm, $LM = 5$ cm, $KN = 7$ cm, $\angle LKN = 70°$ and $\angle KLM = 130°$

EXAMPLE 2

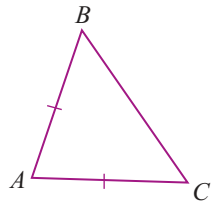
Describe the information shown in the diagram.



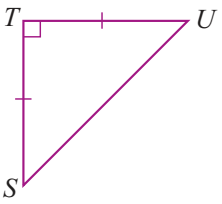
Quadrilateral $JKLM$ with $JK = KL$, $\angle MJK = \angle KLM$ and $\angle JKL = 90°$.

7 Describe the information shown in the following diagrams.

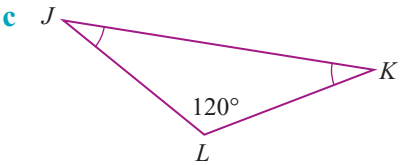
a



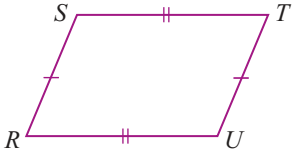
b



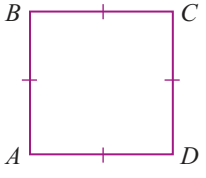
c



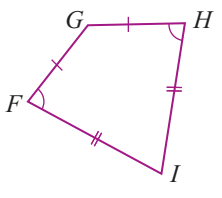
d



e



f



B Constructing triangles

A **triangle** is a three-sided plane shape with straight sides.

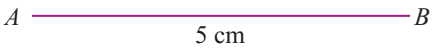
To construct a triangle, you need information about the lengths of the sides or the sizes of the angles. You also need a ruler, compass and protractor.

EXAMPLE 1

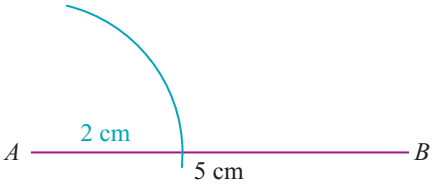
Constructing a triangle given three sides

Use a ruler and compasses to construct $\triangle ABC$ with side lengths $AB = 5$ cm, $BC = 2$ cm and $AC = 4$ cm.

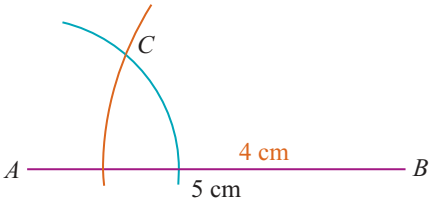
Step 1: Draw an interval AB 5 cm in length.



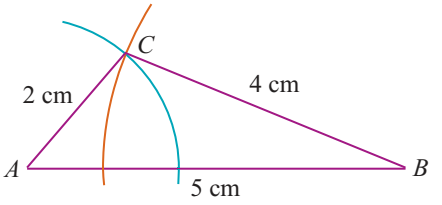
Step 2: Set the compasses to 2 cm. Draw an arc from point A.



Step 3: Set the compasses to 4 cm. Draw an arc from point B. Mark the point C where the arcs intersect.



Step 4: Draw the remaining sides (join points A and B to point C with straight lines).



Exercise 15B

1 Complete the following steps to construct $\triangle PQR$ with side lengths $PQ = 2$ cm, $QR = 3$ cm and $PR = 4$ cm.

Step 1: Draw an interval PQ 2 cm in length.

Step 2: Set the compasses to 3 cm. Draw an arc from point Q.

Step 3: Set the compasses to 4 cm. Draw an arc from point P. Mark point R where the arcs intersect.

Step 4: Join points Q and P to point R.

2 Construct the following triangles.

a $\triangle LMN$ with $LM = 6$ cm, $MN = 4$ cm, $LN = 5$ cm

b $\triangle DEF$ with $DE = 5$ cm, $EF = 3.5$ cm, $DF = 4.5$ cm

c $\triangle DKM$ with $DK = 4$ cm, $KM = 2.5$ cm, $DM = 4.2$ cm

d $\triangle OPR$ with $OP = 4$ cm, $PR = 6.5$ cm, $OR = 4$ cm

3 **a** For each triangle constructed in question **2**, measure the three angles and write them on the diagram.

b What do you notice about the angle opposite the:

i longest side? **ii** shortest side?

c Copy and complete:

The _____ angle is opposite the longest side.

4 Use the triangles constructed in question **2** to complete this table.

	Longest side	Sum of other 2 sides	Is longest side greater than sum of the other 2 sides?
a			
b			
c			
d			

5 Would it be possible to construct a triangle with side lengths:

a 1 cm, 7 cm and 9 cm? **b** 5.2 cm, 8 cm and 6 cm?

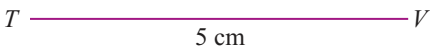
Explain your answer in each case.

EXAMPLE 2

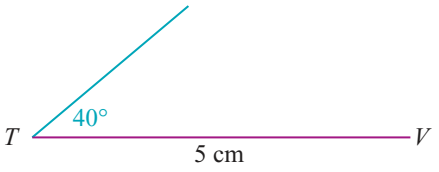
Constructing a triangle given two sides and the included angle

Use a ruler and protractor to construct $\triangle TVU$ with side lengths $TV = 5$ cm and $TU = 2$ cm and $\angle VTU = 40°$.

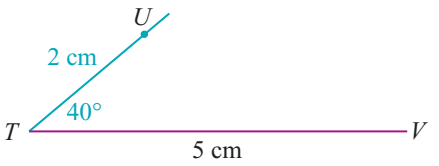
Step 1: Draw a line interval TV of length 5 cm.



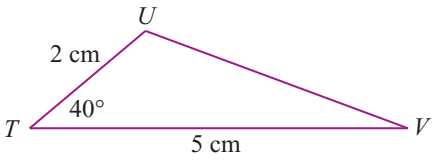
Step 2: At point T use a protractor to construct an angle of 40° with line TV.



Step 2: Measure 2 cm along the arm of the angle. Mark this point U.



Step 4: Join points U and V to form the triangle.



6 Complete the following steps to construct $\triangle ABC$ with $AB = 6$ cm, $BC = 4$ cm and $\angle B = 70°$.

Step 1: Draw a line interval AB of length 6 cm.

Step 2: At point B, use a protractor to construct an angle of 70° with line AB.

Step 3: Measure 4 cm along the arm of the angle. Mark this point C.

Step 4: Join points C and A to form the triangle.

7 Construct the following triangles by drawing the angle first.

a $\triangle ABC$ with $AB = 3$ cm, $BC = 4$ cm, $\angle B = 110°$

b $\triangle KLM$ with $KL = 4$ cm, $LM = 2$ cm, $\angle L = 75°$

c $\triangle XTR$ with $XT = 3$ cm, $TR = 4$ cm, $\angle T = 135°$

d $\triangle LST$ with $LT = 5$ cm, $TS = 1.5$ cm, $\angle T = 65°$

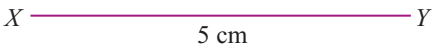
EXAMPLE 3

Constructing a triangle given two angles and a side

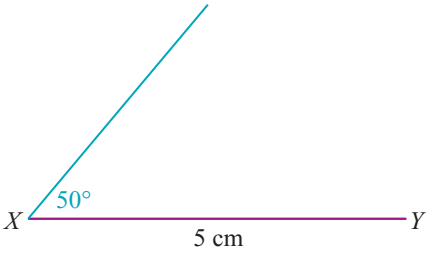
Use a ruler and protractor to construct $\triangle XYZ$ where XY is 5 cm in length, $\angle X = 50°$ and $\angle Y = 40°$.

The given side is called the **baseline**.

Step 1: Draw a line interval XY of length 5 cm.

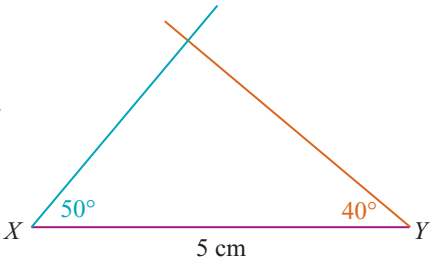


Step 2: At point X, use a protractor to construct an angle of 50° with line XY.

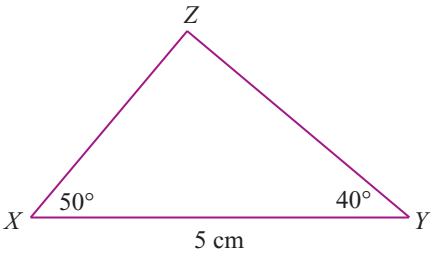


Step 3: At point Y, use a protractor to construct an angle of 40° with line XY.

Extend the arms of the angles until they cross at point Z.



Step 4: Join points X, Y and Z to form a triangle.



8 Complete the following steps to construct $\triangle PQR$ with $PQ = 4$ cm, $\angle P = 60°$ and $\angle Q = 40°$.

Step 1: Draw a line interval PQ of length 4 cm.

Step 2: At point P, use a protractor to construct an angle of 60° with line PQ.

Step 3: At point Q, use a protractor to construct an angle of 40° with line PQ.
Extend the arms of each angle until they cross at point R.

Step 4: Join points P, Q and R to form a triangle.

9 Construct the following triangles by drawing the baseline first.

a $\triangle XYZ$ with $XZ = 5$ cm, $\angle X = 50°$, $\angle Z = 30°$

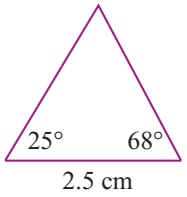
b $\triangle ABC$ with $AB = 4$ cm, $\angle B = 50°$, $\angle A = 70°$

c $\triangle MRL$ with $ML = 6$ cm, $\angle M = 90°$, $\angle L = 25°$

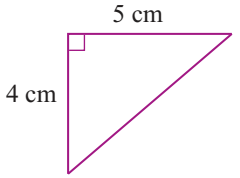
d $\triangle PQR$ with $PR = 5$ cm, $\angle P = 20°$, $\angle R = 110°$

10 Make accurate full-sized drawings of these triangles using the dimensions given.

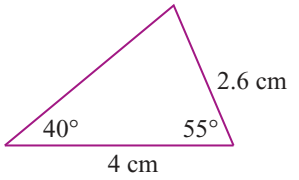
a



b



c



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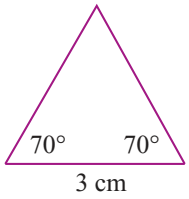
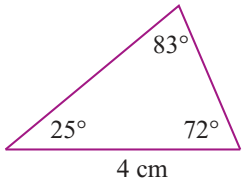


e



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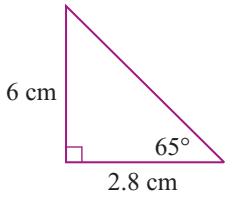
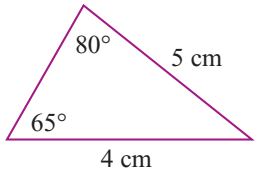


11 Is it possible to construct the triangle shown? Explain.



12 **a** Construct two different triangles with sides 5 cm, 4 cm and an angle of 50°.

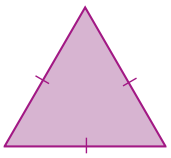
b Write a set of instructions to construct each triangle.

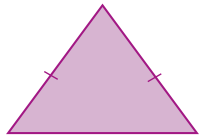
C Classifying triangles

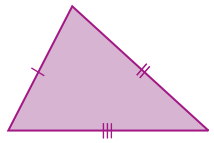
Triangles are classified according to their sides, or their angles, or their sides and angles.

Sides

An **equilateral** triangle has all sides equal in length.
An **isosceles** triangle has two sides equal in length.
A **scalene** triangle has no sides equal in length.

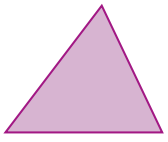

Equilateral

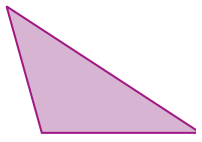

Isosceles

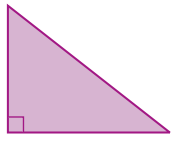

Scalene

Angles

An **acute-angled** triangle has all three angles acute.
An **obtuse-angled** triangle has one obtuse angle.
A **right-angled** triangle has one right angle.


Acute-angled


Obtuse-angled

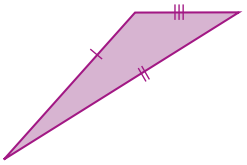

Right-angled

When describing triangles, give the angle name first, followed by the side name. For example, we write obtuse-angled isosceles triangle, *not* isosceles obtuse-angled triangle.

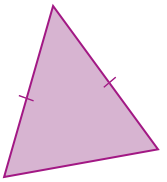
Exercise 15C

1 Write the side name for each of these triangles.

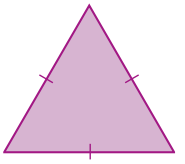
a



b

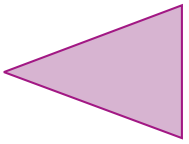


c

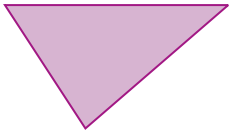


2 By measuring lengths of sides, classify the following triangles.

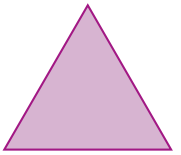
a



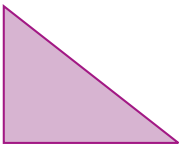
b



c

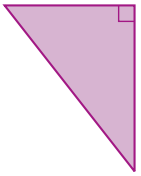


d

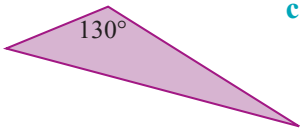


3 Write the angle name for each of these triangles.

a



b



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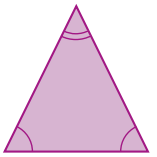

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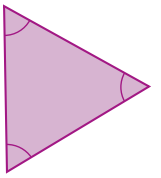
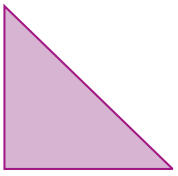


4 By measuring angles, classify the following triangles.

a



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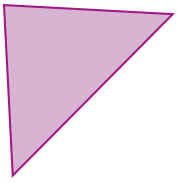

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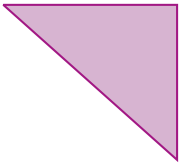

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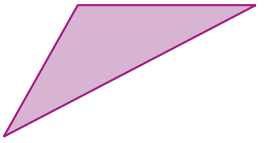
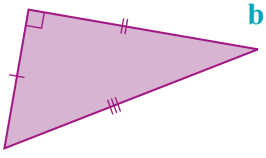


5 Classify each triangle by both its angles and its sides.

a



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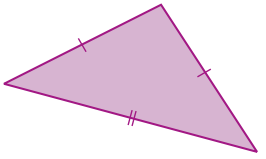

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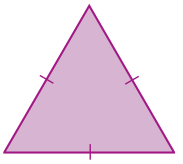

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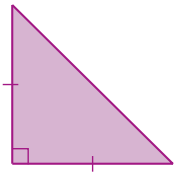
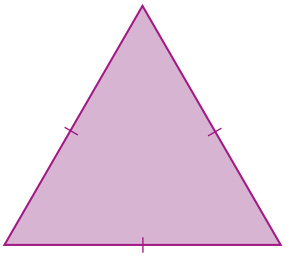
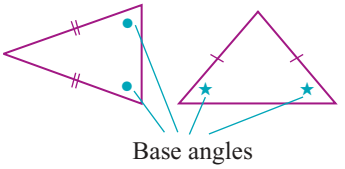
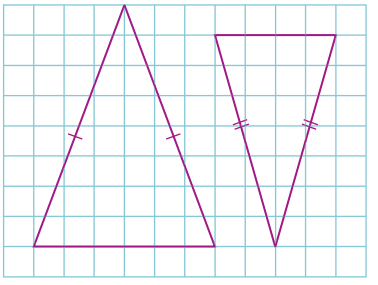


6 For the given equilateral triangle, measure the interior angles.
Copy and complete:
The interior angles of an equilateral triangle are _____ _____ .



7 In an isosceles triangle, the angles opposite the two equal sides are called **base angles**.
By measuring the base angles in the large triangles given on the right, copy and complete the following:
The base angles of an isosceles triangle are _____ .



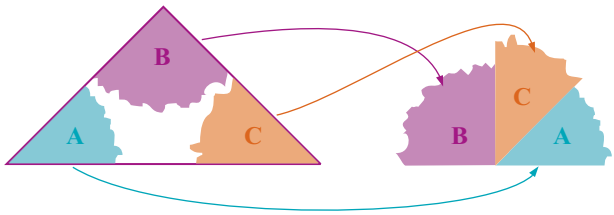


Investigation 1 Sum of the angles of a triangle

Equipment needed: ruler, paper, scissors.

1. On a piece of paper draw any triangle and label the angles A, B and C on the inside of the triangle. Cut out the triangle.

2. Tear off each of the three angles and place them adjacent to each other in any order. What do you notice?



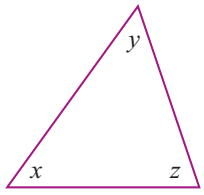
3. Repeat with triangles of different dimensions. Do not forget obtuse-angled triangles.

4. What proposal can you make about the sum of the angles of a triangle?

D Angle sum of a triangle

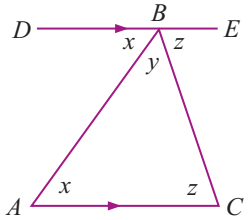
The **angle sum** of a triangle is 180°; that is, in any triangle, no matter what shape, the sum of the angles is 180°.

In the triangle shown, $x + y + z = 180°$.



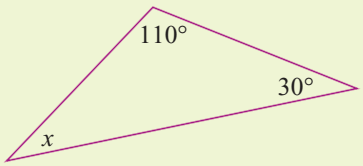
You would have discovered this in Investigation 1.

Proof: Consider any $\triangle ABC$ with interior angles x, y and z.
Draw DE parallel to AC through B.
Now $\angle ABD = x$ (equal alternate angles, $DE \parallel AC$)
Also $\angle CBE = z$ (equal alternate angles, $DE \parallel AC$)
$x + y + z = 180°$ (adjacent angles on a straight line)
$\therefore$ The angle sum of a triangle is 180°.



EXAMPLE 1

Calculate the value of the pronumeral. Show working and provide a reason for your answer.



$x + 110° + 30° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore x + 140° = 180°$
$x = 180° - 140° = 40°$
Alternatively: $x = 180° - 110° - 30°$ (Angle sum of a triangle is 180°.)
$\therefore x = 180° - 140° = 40°$

Exercise 15D

1 Complete the following to calculate the value of the pronumeral.

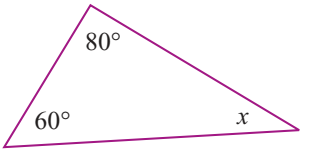
x + ___ + ___ = 180° (Angle sum of a triangle is ___.)

$\therefore x$ + ___ = 180°

x = 180° − ___ = ___

Alternatively: x = 180° − ___ − ___ (___ ___ of a triangle is ___.)

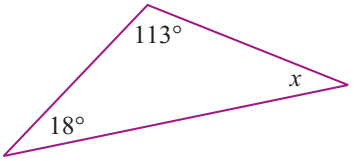
$\therefore x$ = 180° − ___ = ___



Use the fact that the angles of a triangle always add up to 180°.

2 Calculate the value of the pronumeral in each of the following. Show all working.

a



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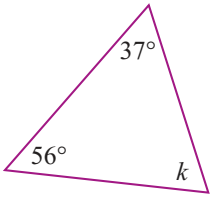

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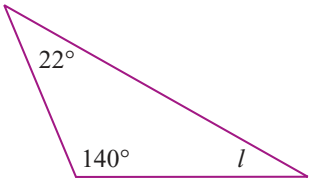

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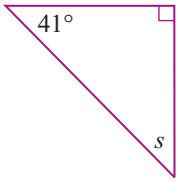

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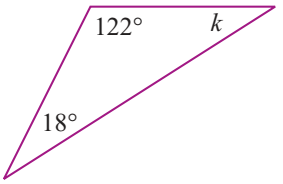

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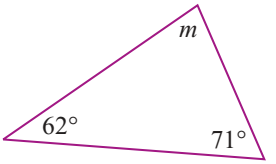

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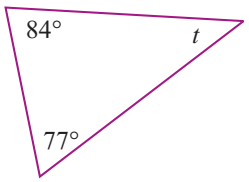

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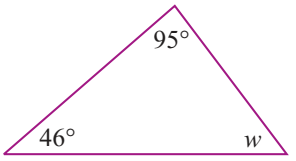
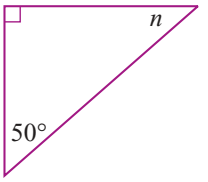


i



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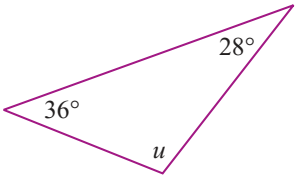

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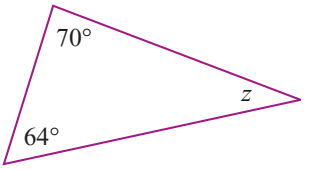

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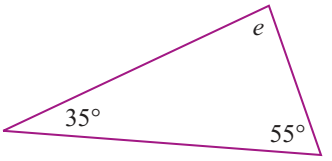

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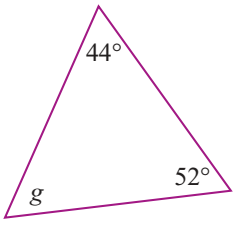

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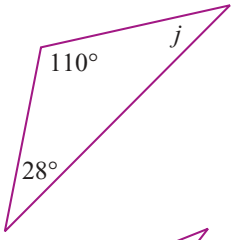
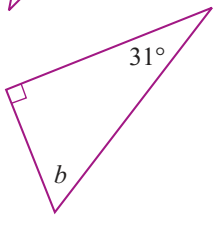
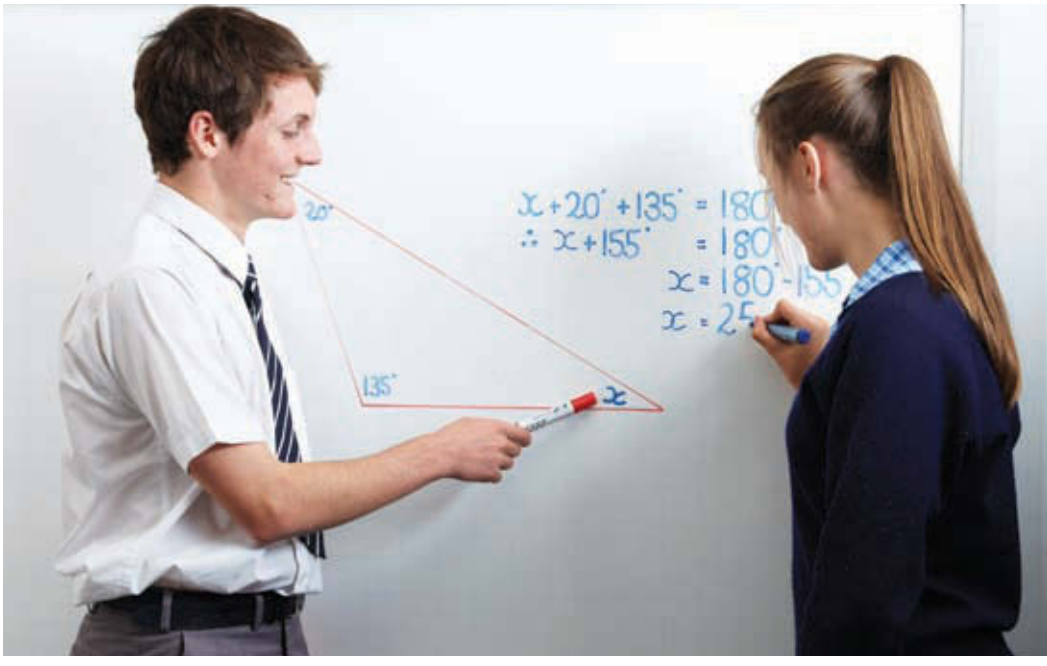


o

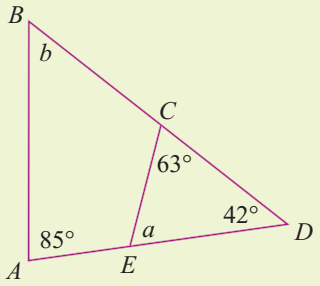




EXAMPLE 2

Find the values of the pronumerals in each of the following, giving reasons for your answers.

a



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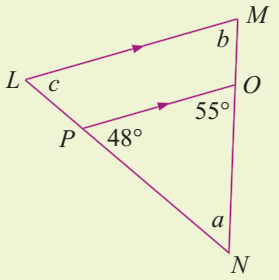


a In $\triangle CDE$, $a + 63° + 42° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore a + 105° = 180°$

$a = 180° - 105° = 75°$

In $\triangle BDA$, $b + 85° + 42° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore b + 127° = 180°$

$b = 180° - 127° = 53°$

b In $\triangle PON$, $a + 48° + 55° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore a + 103° = 180°$

$a = 180° - 103° = 77°$

Also $b = 55°$ (corresponding angles, $PO \parallel LM$)

and $c = 48°$ (corresponding angles, $PO \parallel LM$)

3 Complete the following to find the values of the pronumerals.

a In $\triangle BCD$,

$a + \_\_\_ + \_\_\_ = 180°$ (Angle sum of a triangle is ___.)

$\therefore a + \_\_\_ = 180°$

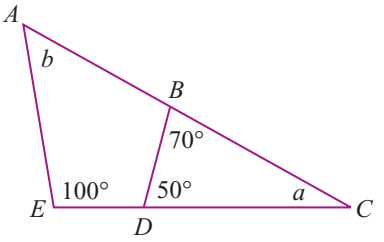
$a = 180° - \_\_\_ = \_\_\_$

In $\triangle ACE$,

$b + 100° + \_\_\_ = 180°$ (Angle sum of a triangle is ___.)

$\therefore b + \_\_\_ = 180°$

$b = 180° - \_\_\_ = \_\_\_$



b In $\triangle QRS$,

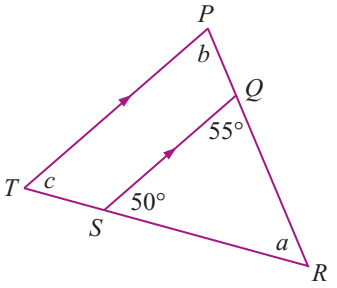
$a + \_\_\_ + \_\_\_ = 180°$ (Angle sum of a triangle is ___.)

$\therefore a + \_\_\_ = 180°$

$a = 180° - \_\_\_ = \_\_\_$

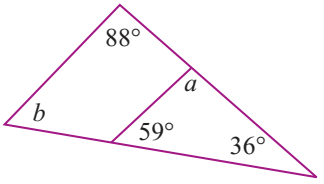
Also, $b = \_\_\_$ (corresponding angles, $TP \parallel SQ$)

and $c = \_\_\_$ (_____ angles, ___ $\parallel$ ___)



4 Find the values of the pronumerals in each of the following, giving reasons for your answers.

a



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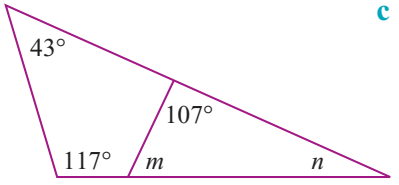

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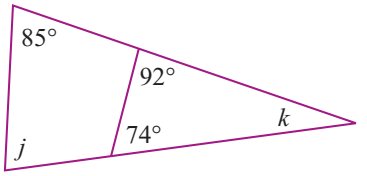

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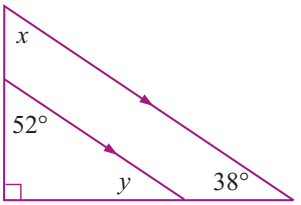

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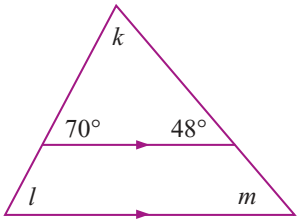

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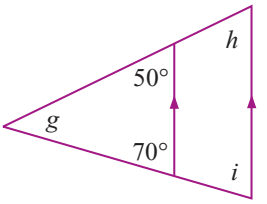
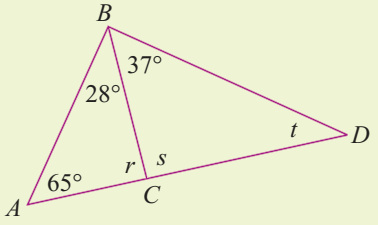


EXAMPLE 3

Find the values of the pronumerals in each of the following, giving reasons for your answers.

a



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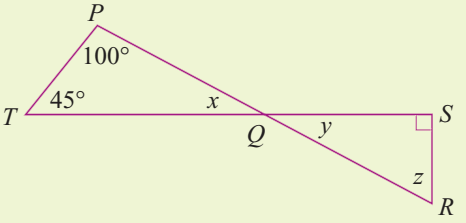


a In $\triangle ABC$, $28° + 65° + r = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 93° + r = 180°$

$r = 180° - 93° = 87°$

At C, $s = 180° - 87° = 93°$ (adjacent angles on a straight line)

In $\triangle BDC$, $37° + 93° + t = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 130° + t = 180°$

$t = 180° - 130° = 50°$

b In $\triangle PQT$, $x + 45° + 100° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore x + 145° = 180°$

$x = 180° - 145° = 35°$

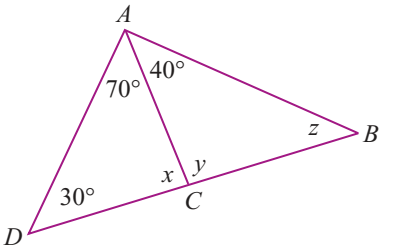
At Q, $y = 35°$ (Vertically opposite angles are equal.)

In $\triangle QSR$, $z + 90° + 35° = 180°$ (Angle sum of a triangle is ___.)

$\therefore z + 125° = 180°$

$z = 180° - 125° = 55°$

5 Complete the following to find the values of the pronumerals.



a In $\triangle DAC$,

$x + 70° + 30° =$ ___ (Angle sum of a triangle is ___.)

$\therefore x + 100° =$ ___

$x =$ ___ $- 100° =$ ___

At C, $y = 180° -$ ___ (adjacent angles on a straight line)

$=$ ___

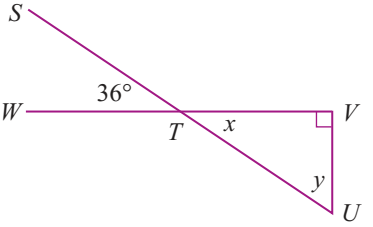
In $\triangle ABC$,

$z + 40° +$ ___ $= 180°$ (Angle sum of a triangle is ___.)

$\therefore z +$ ___ $= 180°$

$z = 180° -$ ___ $=$ ___

b At T, $x =$ ___ (____ ____ angles are equal.)



In $\triangle TVU$,

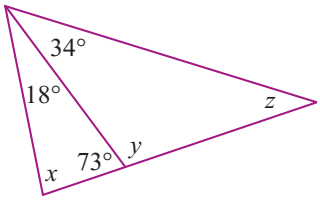
$y + 90° +$ ___ $= 180°$ (Angle sum of a triangle is ___.)

$\therefore y +$ ___ $= 180°$

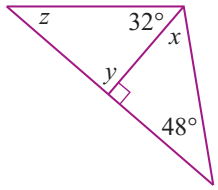
$y = 180° -$ ___ $=$ ___

6 Find the values of the pronumerals in each of the following, giving reasons for your answers.

a



b



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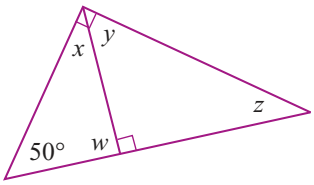

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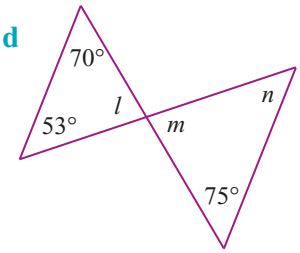

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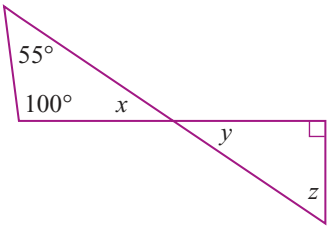

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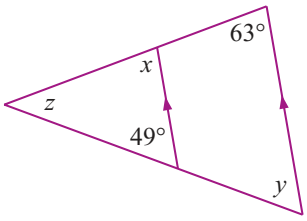

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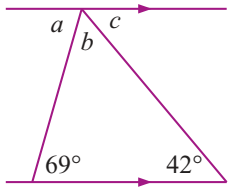

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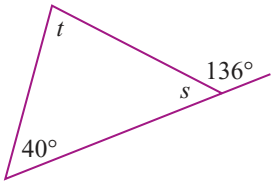

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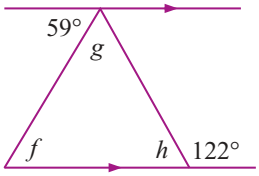
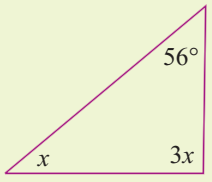


You should always give reasons to justify your working.

EXAMPLE 4

Write an equation for each and solve it to find the value of the pronumeral. Hence find the unknown angles.

a



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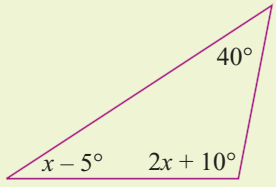


a $x + 3x + 56° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 4x + 56° = 180°$

$4x = 180° - 56° = 124°$

$x = \frac{124°}{4} = 31°$

$\therefore$ Unknown angles are: $x = 31°$

and $3x = 3 \times 31° = 93°$

b $(x - 5°) + (2x + 10°) + 40° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 3x + 45° = 180°$

$3x = 180° - 45° = 135°$

$x = \frac{135°}{3} = 45°$

$\therefore$ Unknown angles are: $x - 5° = 45° - 5° = 40°$

and $2x + 10° = 2 \times 45° + 10° = 90° + 10° = 100°$

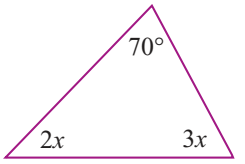
7 Complete the following to find the value of each pronumeral. Hence find the unknown angles.

a $2x + 3x + 70° =$ ___ (Angle sum of a triangle is ___.)

$\therefore$ ___ $x + 70° =$ ___

___ $x =$ ___ $- 70° =$ ___

$x = \frac{\square}{5} =$ ___



$\therefore$ Unknown angles are:

$2x = 2 \times$ ___ $=$ ___

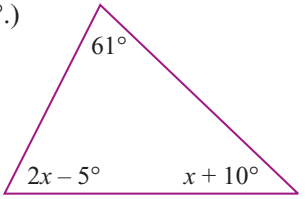
and $3x = 3 \times$ ___ $=$ ___

b $(2x - 5°) + (x + 10°) +$ ___ $= 180°$ (Angle sum of a triangle is 180°.)

$\therefore 3x +$ ___ $= 180°$

$3x =$ ___ $-$ ___ $=$ ___

$x = \frac{\square}{3} =$ ___



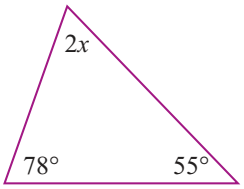
$\therefore$ Unknown angles are:

$2x - 5° = 2 \times$ ___ $- 5° =$ ___

and $x + 10° =$ ___ $+$ ___ $=$ ___

8 Write an equation for each and solve it to find the value of the pronumeral. Hence find the unknown angles.

a



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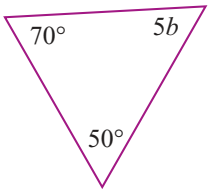

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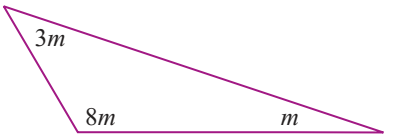

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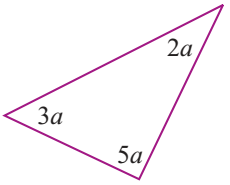

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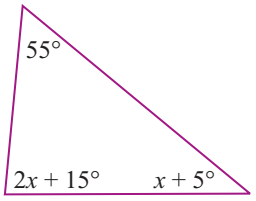

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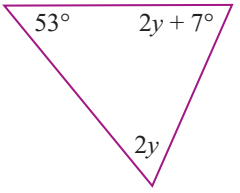
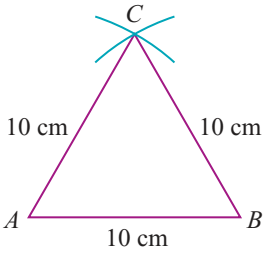


Investigation 2 Equilateral triangle

Equipment needed: compasses, ruler, paper, scissors.

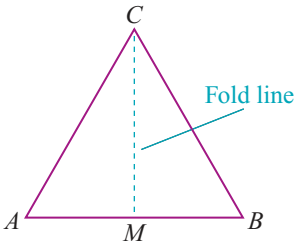
1 Construct an equilateral triangle with sides of length 10 cm, using your compasses and ruler (see section B of this chapter).

2 Accurately cut out the triangle and name the vertices A, B and C.



3 Fold the triangle in half so that vertex A matches exactly with vertex B. Unfold the triangle.

- **a** Comment on the lengths of AM and BM.
- **b** Explain why $\angle CMA = \angle CMB$. What is their degree measure? Why?
- **c** What can be said about $\angle CAM$ and $\angle CBM$?



4 Repeat the procedure in question **3**, folding so that vertices B and C match up.

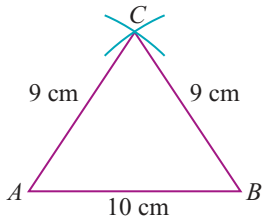
5 Repeat the procedure in question **3**, folding so that vertices A and C match up.

6 What conclusions can you make about:
   a the three angles at A, B and C?
   b each fold line?

Investigation 3 Isosceles triangle

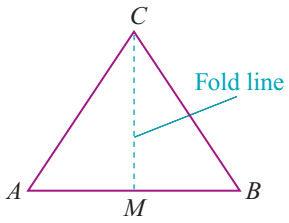
Equipment needed: compasses, ruler, paper, scissors.

1 Construct, using compasses and ruler, an isosceles triangle ABC with AB of length 10 cm and $AC = BC = 9$ cm.



2 Cut out the triangle.

3 Fold the triangle in half so that vertices A and B coincide. Unfold the triangle. What can be said about:
   a the lengths of AM and BM?
   b the measures of angles CAM and CBM?
   c the measures of angles ACM and BCM?
   d the measures of angles AMC and BMC?



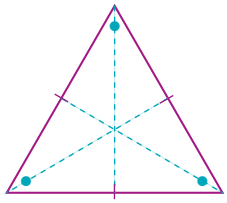
4 Repeat question **3**, folding so that vertices A and C coincide. What do you notice?

E Properties of equilateral and isosceles triangles

From Investigations 2 and 3, you have discovered the following properties of equilateral and isosceles triangles.

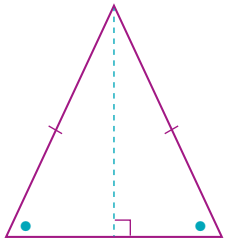
Properties of an equilateral triangle

- All sides are equal.
- All angles are 60°.
- It has three axes of symmetry.



Properties of an isosceles triangle

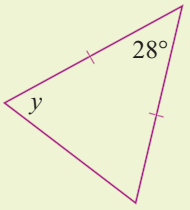
- Two sides are equal.
- The angles opposite the equal sides (the base angles) are equal.
- It has one axis of symmetry.



EXAMPLE 1

Find the value of the pronumeral in each triangle, giving a reason.

a



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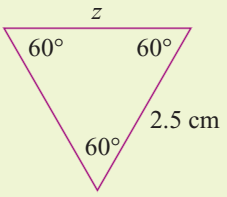

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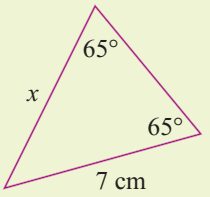

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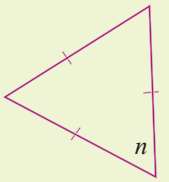


a The third angle of the triangle is also y. (Base angles of an isosceles triangle are equal.)

$\therefore 28° + y + y = 180°$ (Angle sum of a triangle is 180°.)

$2y = 180° - 28°$

$= 152°$

$y = \frac{152°}{2} = 76°$

b $z = 2.5$ cm (Sides of an equilateral triangle are equal.)

c $x = 7$ cm (Sides opposite base angles of an isosceles triangle are equal.)

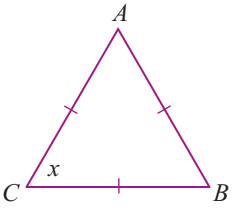
d $n = 60°$ (Angles of an equilateral triangle are 60°.)

Exercise 15E

1 Complete the following to find the value of the pronumeral in each triangle.

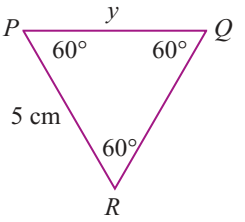
a $\triangle ABC$ is an ____ triangle. (____ sides are equal.)

$\therefore x =$ ____ (Angles of an ____ triangle are ____.)



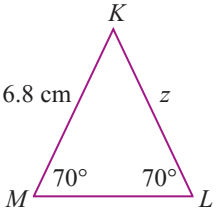
b $\triangle PQR$ is an ____ triangle. (____ angles are equal.)

$\therefore y =$ ____ cm (sides of an ____ triangle are ____.)



c $\triangle KLM$ is an ____ triangle. (____ angles are equal.)

$\therefore z =$ ____ (Sides opposite ____ angles of an ____ triangle are ____.)



d $\triangle XYZ$ is an ____ triangle. (____ sides are equal.)

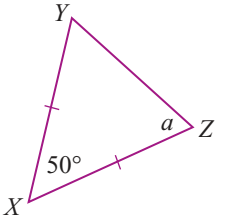
$\therefore \angle Y = a$ (____ angles of an ____ triangle are ____.)

$a + a + 50° =$ ____ (Angle sum of a triangle is ____.)

$2a + 50° =$ ____

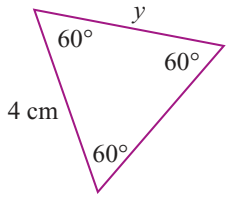
$2a =$ ____ $- 50° =$ ____

$a = \frac{\square}{2} =$ ____



2 Find the values of the pronumerals, giving reasons for your answers.

a



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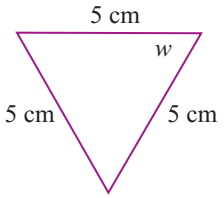

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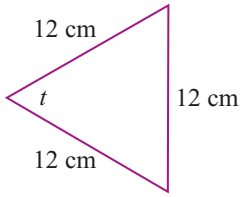

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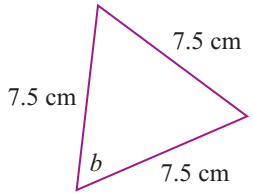

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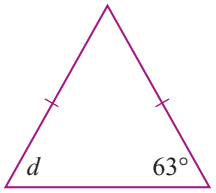

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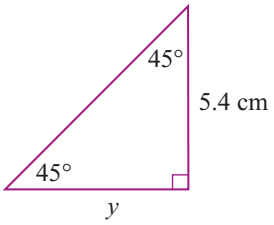

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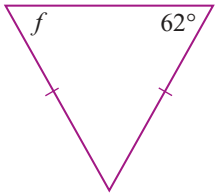

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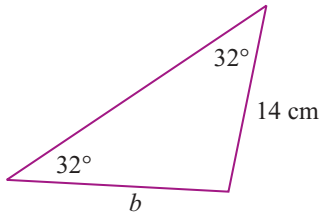

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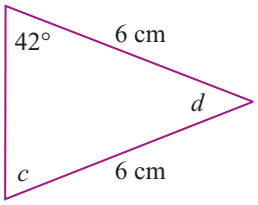

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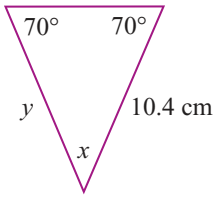

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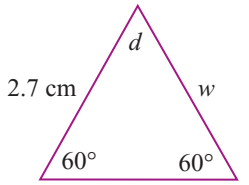

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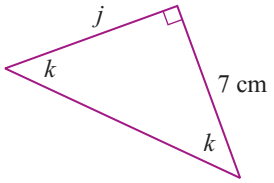

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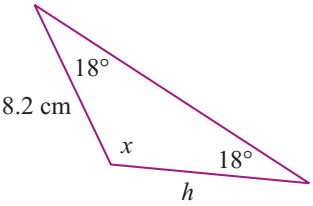

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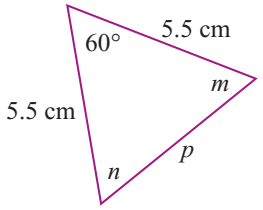
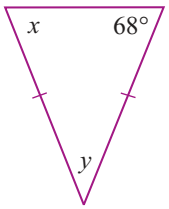


3 Find the values of x and y in each of the following. Provide reasons for your answers.

a



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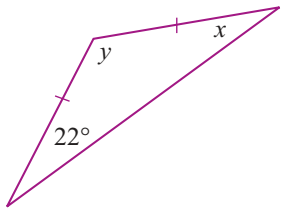

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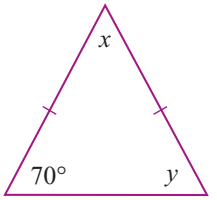

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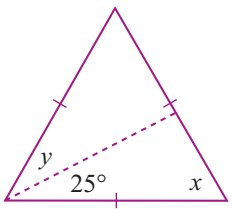

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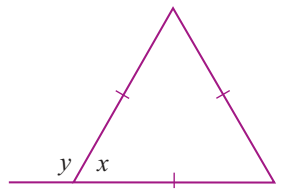

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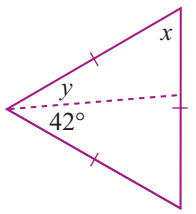

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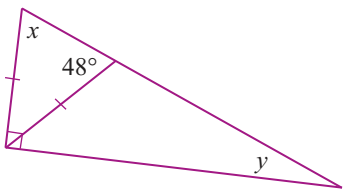

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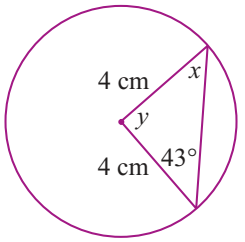

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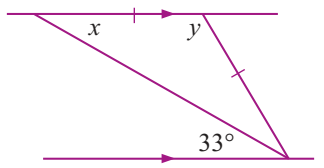
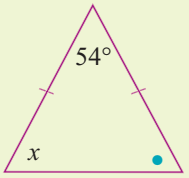


EXAMPLE 2

Write an equation for each and solve it to find the value of the pronumeral. Hence find the unknown angles.

a



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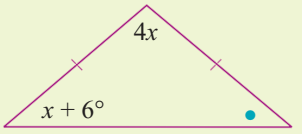


a Isosceles triangles have base angles equal.

$\therefore$ Angle marked • $= x$

$x + x + 54° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 2x + 54° = 180°$

$2x = 180° - 54° = 126°$

$x = \frac{126°}{2} = 63°$

$\therefore$ Unknown angles are both 63° (base angles).

b Isosceles triangles have base angles equal.

$\therefore$ Angle marked • $= x + 6°$

$4x + (x + 6°) + (x + 6°) = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 6x + 12° = 180°$

$6x = 180° - 12° = 168°$

$x = \frac{168°}{6} = 28°$

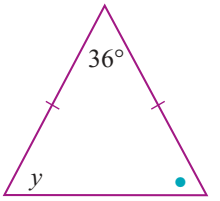
$\therefore$ Unknown angles are:

$4x = 4 \times 28° = 112°$

and $x + 6° = 28° + 6° = 34°$ (base angles)

4 Complete the following to find the values of the pronumerals. Hence find the unknown angles.

a Isosceles triangles have base angles equal.



$\therefore$ Angle marked • $=$ ____

$y + y + 36° =$ ____ (____ ____ of a triangle is ____.)

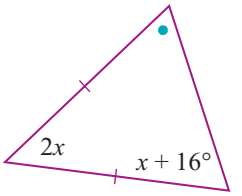
$\therefore$ ___$y + 36° =$ ____

___$y =$ ____ $- 36° =$ ____

$y = \frac{\square}{\square} =$ ____

$\therefore$ Unknown angles are both ____ (base angles).

b Isosceles triangles have ____ angles ____.



$\therefore$ Angle marked • $=$ ____

$2x + (x + 16°) +$ ____ $= 180°$ (____ ____ of a triangle is ____.)

$\therefore$ ___$x + 32° = 180°$

___$x = 180° - 32° = 148°$

$x = \frac{148°}{\square} =$ ____

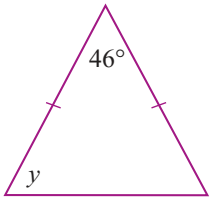
$\therefore$ Unknown angles are:

$2x = 2 \times$ ____ $=$ ____

and $x + 16° =$ ____ $+ 16° =$ ____ (base angles)

5 Write an equation for each and solve it to find the value of the pronumeral. Hence find the unknown angles.

a



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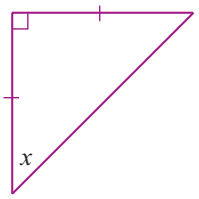

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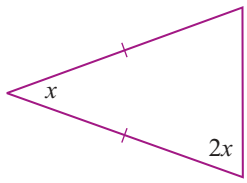

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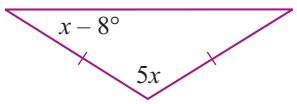

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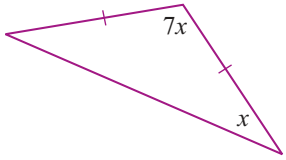

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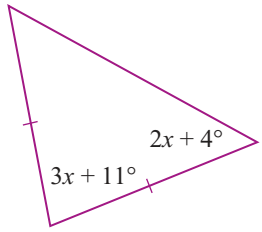
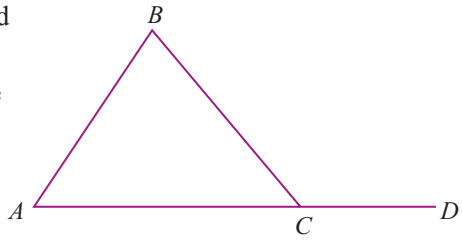


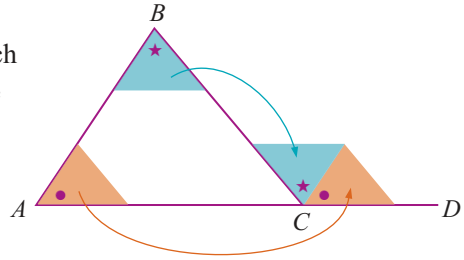
Investigation 4 Exterior angle of a triangle

An **exterior angle** of a triangle is the angle formed outside the triangle when a side is extended. Equipment needed: ruler, paper, scissors.

1 Draw any triangle ABC and extend the side AC to D. $\angle BCD$ is an exterior angle of the triangle.



2 Cut off the angles A and B and place them next to each other on the exterior angle BCD, as shown.
Is $\angle BCD = \angle A + \angle B$?



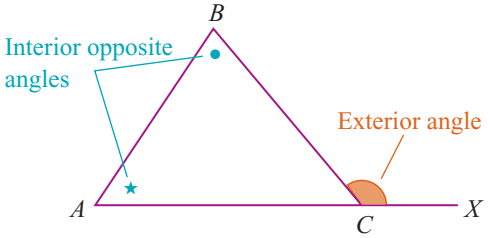
3 Repeat with triangles of different dimensions. Do not forget obtuse-angled triangles.

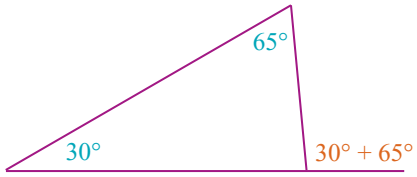
5 What proposal can you make about the exterior angle of a triangle?

F Exterior angle of a triangle

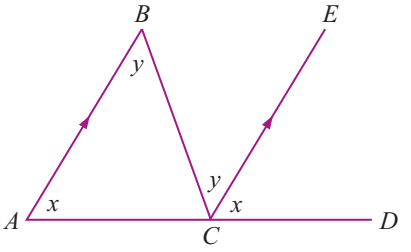
The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

You would have discovered this fact in Investigation 4.





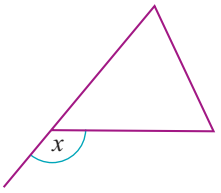
Proof: Consider any $\triangle ABC$ with $\angle BAC = x$ and $\angle ABC = y$.
Extend AC to D and draw CE parallel to AB.
Now $\angle BAC = \angle ECD = x$ (equal corresponding angles, $AB \parallel CE$)
Also $\angle ABC = \angle BCE = y$ (equal alternate angles, $AB \parallel CE$)
$\therefore \angle BCD = x + y = \angle BAC + \angle ABC$
$\therefore$ The exterior angle of a triangle is equal to the sum of the two interior opposite angles.



Exercise 15F

1 Copy these diagrams and shade the two interior angles opposite to the exterior angle labelled x.

a



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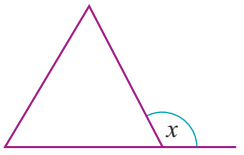

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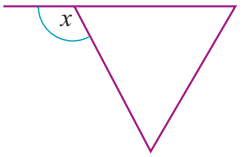

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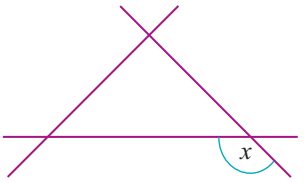

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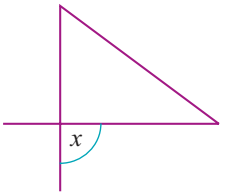

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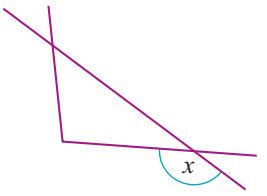
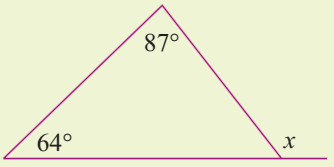


EXAMPLE 1

Find the value of the pronumeral in each triangle, giving a reason.

a



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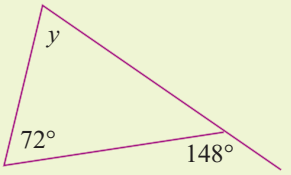
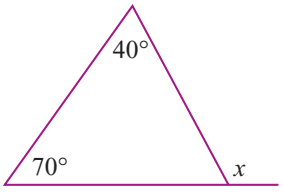


a $x = 87° + 64°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
$\therefore x = 151°$

b $148° = y + 72°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
or $y + 72° = 148°$
$\therefore y = 148° - 72° = 76°$

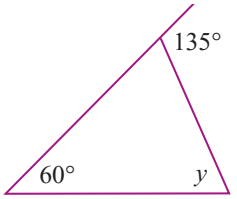
2 Complete the following to find the value of the pronumeral in each triangle.

a



$x =$ ___ + ___ (Exterior angle of a triangle equals
$\therefore x =$ ___ sum of two ____ ____ angles.)

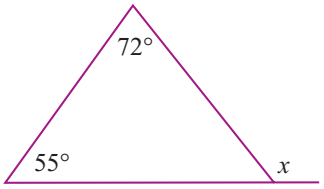
b



$135° = y +$ ___ (____ angle of a triangle equals
or $y +$ ___ $= 135°$ sum of two ____ ____ angles.)
$\therefore y = 135° -$ ___ = ___

3 Find the value of the pronumeral in each triangle.

a



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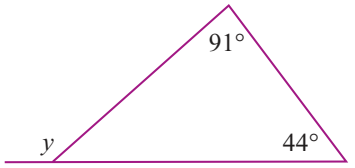

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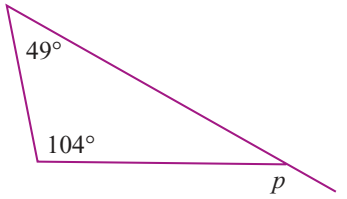

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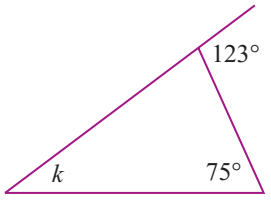

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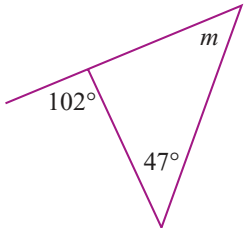

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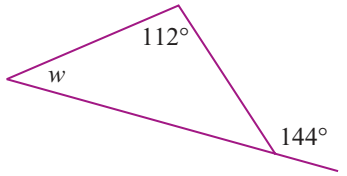
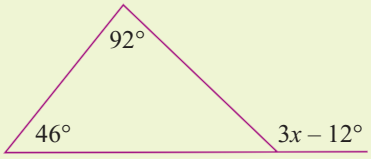


EXAMPLE 2

Write an equation for each and solve it to find the value of the pronumeral. Hence find the unknown angles with pronumerals.

a



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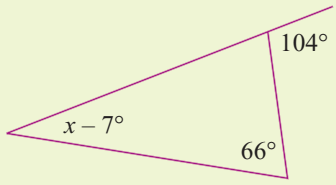


a $3x - 12° = 92° + 46° = 138°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)

$\therefore 3x = 138° + 12° = 150°$

$x = \frac{150°}{3} = 50°$

$\therefore$ Unknown angle is:

$3x - 12° = 3 \times 50° - 12° = 138°$

b $104° = (x - 7°) + 66°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)

$104° = x + 59°$

or $x + 59° = 104°$

$x = 104° - 59° = 45°$

$\therefore$ Unknown angle is:

$x - 7° = 45° - 7° = 38°$

4 Complete the following to find the value of the pronumeral. Hence find the unknown angles with pronumerals.

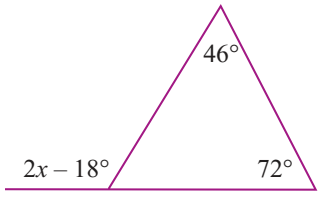
a $2x - 18° = \_\_\_ + \_\_\_ = \_\_\_$ (Exterior angle of a triangle equals sum of ____ ____ ____ angles.)

$\therefore 2x = \_\_\_ + 18° = \_\_\_$

$x = \frac{\square}{2} = \_\_\_$

$\therefore$ Unknown angle is:

$2x - 18° = 2 \times \_\_\_ - 18° = \_\_\_$



b $140° = (x + 10°) + \_\_\_$ (____ angle of a triangle equals ___ of ____ ____ ____ angles.)

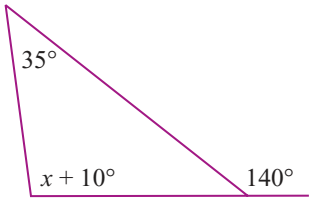
$140° = x + \_\_\_$

or $x + \_\_\_ = 140°$

$x = 140° - \_\_\_ = \_\_\_$

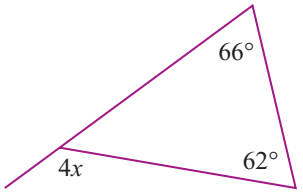
$\therefore$ Unknown angle is:

$x + 10° = \_\_\_ + \_\_\_ = \_\_\_$



5 Find the value of the pronumeral in each by first forming an equation. Hence find the unknown angles.

a



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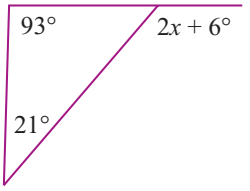

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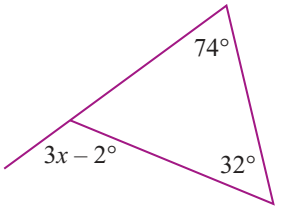

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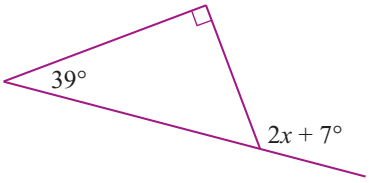

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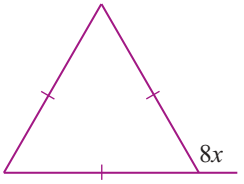

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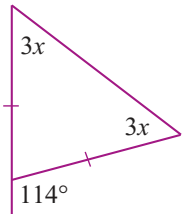
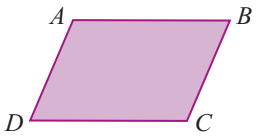


Investigation 5 Convex and concave quadrilaterals

1 A **quadrilateral** is a four-sided plane shape with straight sides. Sketch each of these quadrilaterals.

a



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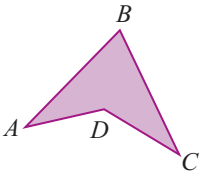

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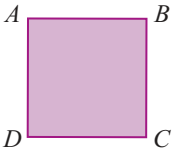

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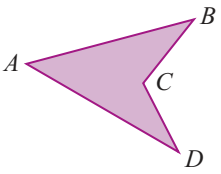


2 In each quadrilateral, draw in both diagonals AC and BD.

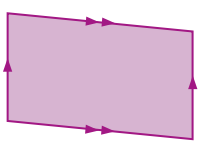
3 If all diagonals are inside the quadrilateral, it is **convex**. Which quadrilaterals are convex?

4 If one of the diagonals lies outside the quadrilateral, it is **concave**. Which quadrilaterals are concave?

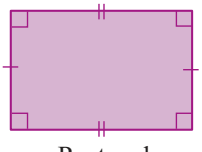
5 Examine the angles in each quadrilateral. Make a comment about the angles in a:

- **a** convex quadrilateral
- **b** concave quadrilateral

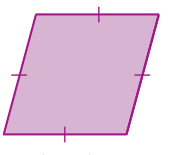
G Properties of special quadrilaterals



Parallelogram

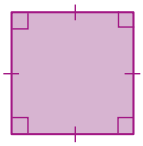


Rectangle



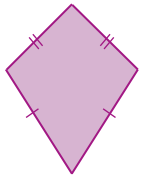
Rhombus

- A **parallelogram** is a quadrilateral that has opposite sides parallel.
- A **rectangle** is a quadrilateral with four equal angles of 90°.
- A **rhombus** is a quadrilateral in which all sides are equal in length (an **equilateral quadrilateral**).



Square

Trapezium



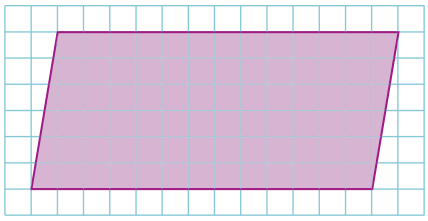
Kite

- A **square** is a rhombus with four equal angles of 90°, or a rectangle with all sides equal in length.
- A **trapezium** is a convex quadrilateral that has a pair of parallel opposite sides.
- A **kite** is a convex quadrilateral with two pairs of adjacent sides equal.

Investigation 6 Properties of quadrilaterals

Equipment needed: compasses, ruler, square grid paper, lined paper.

1 Draw a parallelogram using a ruler and square grid paper as shown. Check the following **properties of a parallelogram**:

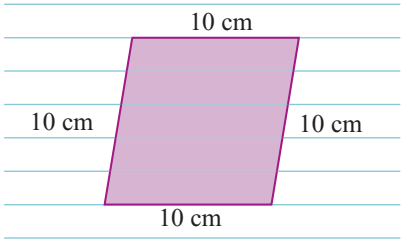


- Opposite sides are parallel.
- Opposite sides are equal in length.
- Opposite angles are equal.
- Diagonals **bisect** each other (i.e. cut each other in half).

2 Draw a rectangle on square grid paper and use it to check the following **properties of a rectangle**:

- Opposite sides are parallel.
- All angles are right angles.
- Diagonals bisect each other.
- Opposite sides are equal in length.
- Diagonals are equal in length.

3 Construct a rhombus using a ruler, compasses and lined paper as shown. Why is the figure a rhombus? Check the following **properties of a rhombus**:



- Opposite sides are parallel.
- All sides are equal in length.
- Opposite angles are equal.
- Diagonals bisect each other at right angles.
- Diagonals bisect the angles at each vertex.

4 A square is a special case of a rhombus and a rectangle and therefore has all of their properties. On squared grid paper, draw a square and check the following **properties of a square**:

- Opposite sides are parallel.
- All angles are right angles.
- Diagonals bisect each other at right angles.
- All sides are equal in length.
- Diagonals are equal in length.
- Diagonals bisect the angles at each vertex.

5 Draw a trapezium on square grid paper and check the following **properties of a trapezium**:

- Exactly one pair of opposite sides are parallel.

6 Draw a kite on square grid paper and check the following **properties of a kite**:

- Two pairs of adjacent sides are equal.
- One pair of opposite angles are equal.
- Diagonals meet at right angles, one diagonal being bisected.
- One diagonal bisects the angles at its endpoints.

Properties of geometrical figures are important facts that need to be remembered.

Exercise 15G

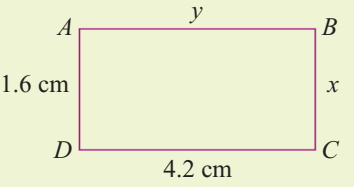
1 Copy and complete the following yes or no table.

Property	Parallelogram	Rectangle	Rhombus	Square	Trapezium	Kite
Opposite sides parallel						
Opposite sides equal						
Opposite angles equal						
All sides equal						
Adjacent sides perpendicular						
Diagonals equal						
Adjacent sides equal						
Diagonals bisect each other						
Diagonals meet at right angles						
Diagonals bisect vertices						
All angles 90°						

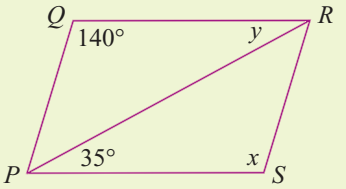
EXAMPLE 1

Use the properties of special quadrilaterals to find the values of the pronumerals, giving your reasons.

a Given $ABCD$ is a rectangle, find x and y.



b Given $PQRS$ is a parallelogram, find x and y.



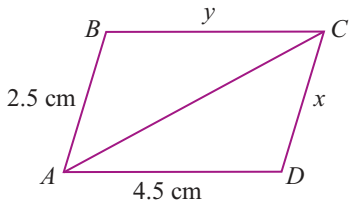
a $x = 1.6$ cm and $y = 4.2$ cm (Opposite sides of a rectangle are equal.)

b $x = 140°$ (Opposite angles of a parallelogram are equal.)

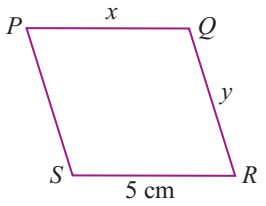
$y = 35°$ (equal alternate angles, $QR \parallel PS$)

2 Use the properties of special quadrilaterals to find the values of the pronumerals, giving your reasons.

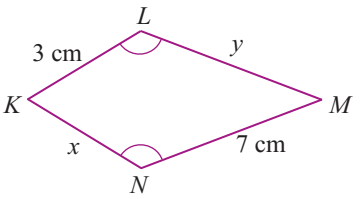
a *ABCD* is a parallelogram.



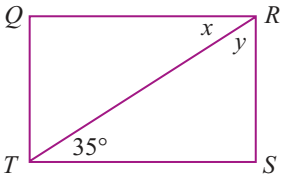
b *PQRS* is a rhombus.



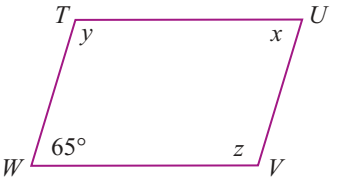
c *KLMN* is a kite.



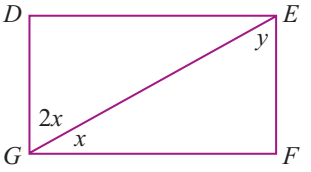
d *QRST* is a rectangle.



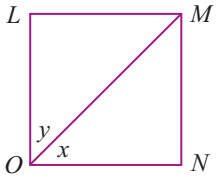
e *TUVW* is a parallelogram.



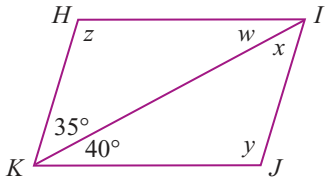
f *DEFG* is a rectangle.



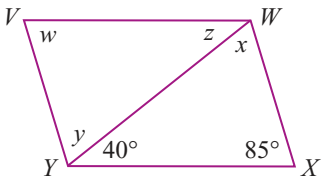
g *LMNO* is a square.



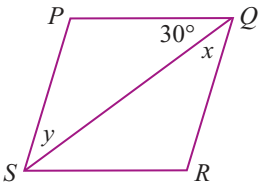
h *HIJK* is a parallelogram.



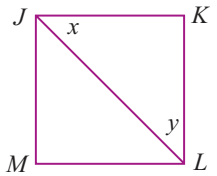
i *VWXY* is a parallelogram.



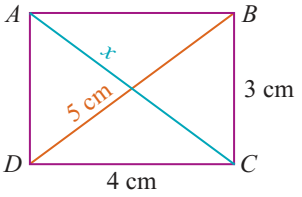
j *PQRS* is a rhombus.



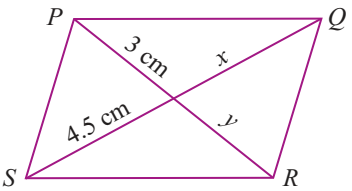
k *JKLM* is a square.



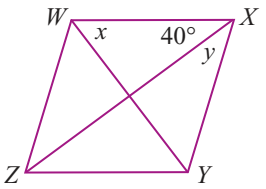
l *ABCD* is a rectangle.



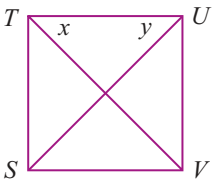
m *PQRS* is a parallelogram.



n *WXYZ* is a rhombus.



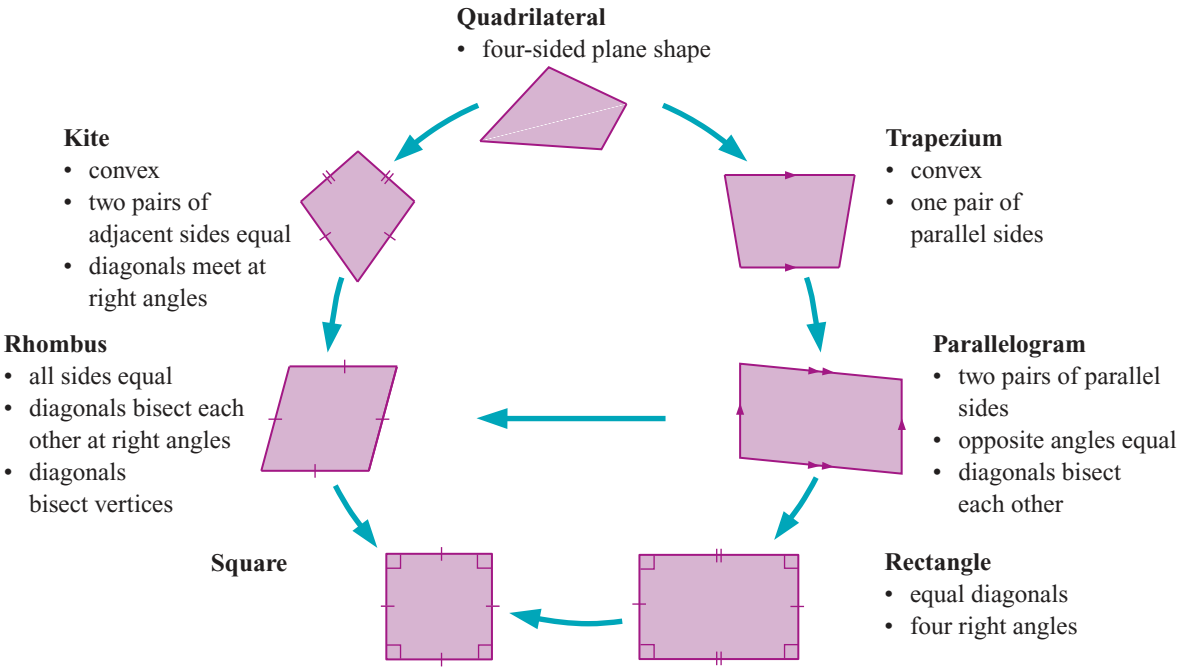
o *STUV* is a square.



3 Identify and sketch quadrilaterals with the following properties.

a One pair of sides are parallel.
b Two pairs sides are of parallel.
c One pair of sides are parallel and one pair of opposite sides are equal.
d One pair of sides are parallel and one pair of adjacent sides are equal.
e One pair of sides are parallel and one angle is a right angle.
f One pair of sides are parallel and opposite angles are equal.
g All angles are right angles.
h Diagonals meet at right angles.
i Diagonals meet at right angles and one pair of opposite angles are equal.
j Diagonals bisect each other.
k Diagonals bisect each other at right angles.
l Diagonals are equal.
m Diagonals are equal and bisect each other.
n Diagonals are equal and bisect each other at right angles.

Investigation 7 Flowchart for quadrilaterals



The flowchart shows the properties of each quadrilateral. Each quadrilateral has its own properties along with the properties of the shape before it. For example:

- A rhombus has its own properties as well as those of a kite and a quadrilateral.
- A square has all the properties of a rhombus and a rectangle.

Are the following statements true or false?

1 All rectangles are parallelograms.

2 All parallelograms are rectangles.

3 All squares are rectangles.

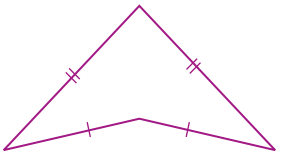
4 All rectangles are squares.

5 Every rhombus is a parallelogram.

6 Every parallelogram is a rhombus.

7 A rhombus is a special case of a kite.

8 This quadrilateral is a kite.

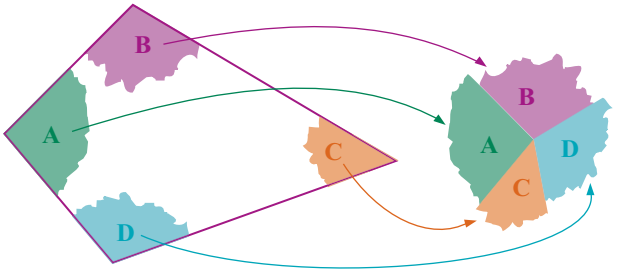


Investigation 8 Angle sum of a quadrilateral

Equipment needed: paper, scissors

1 Draw any quadrilateral on a piece of paper and label the vertices A, B, C and D on the inside of the quadrilateral. Cut out the quadrilateral.

2 Tear off the four angles and place them adjacent to each other, in any order, with all vertices meeting and no overlapping. What do you notice?



3 Repeat this experiment with a few other quadrilaterals, including a concave quadrilateral. What do you notice?

4 Write a statement about the sum of the angles of a quadrilateral.

H Angle sum of a quadrilateral

The angle sum of a quadrilateral is 360°; that is, in any quadrilateral, no matter what shape, the sum of the angles is 360°.

You would have discovered this in Investigation 8.

As you have already learned:

- the angle sum of a triangle equals 180°
- all quadrilaterals can be divided into two triangles.

It would appear reasonable that the angle sum of a quadrilateral is equal to twice the angle sum of a triangle; that is, $2 \times 180° = 360°$.

Proof: Consider any quadrilateral $ABCD$.

Draw the diagonal BD.

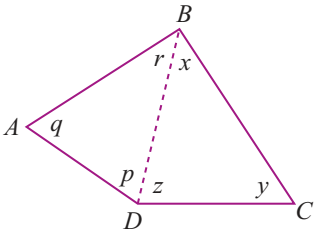
$p + q + r = 180°$ (Angle sum of $\triangle ABD = 180°$.)

$x + y + z = 180°$ (Angle sum of $\triangle BCD = 180°$.)

$\therefore q + (r + x) + y + (z + p) = 360°$

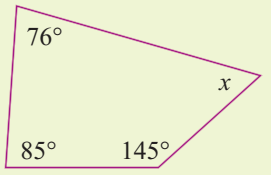
$\therefore \angle A + \angle B + \angle C + \angle D = 360°$

$\therefore$ The angle sum of a quadrilateral is 360°.



EXAMPLE 1

Find the value of the pronumeral, giving a reason for your answer.



$x + 145° + 85° + 76° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore x + 306° = 360°$

$x = 360° - 306° = 54°$

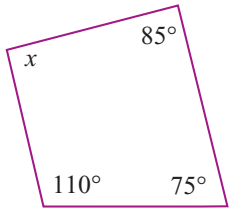
Exercise 15H

1 Complete the following to find the value of the pronumeral.

$x + 85° + 75° + 110° =$ ___ (Angle sum of a quadrilateral is ___.)

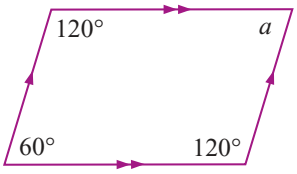
$\therefore x +$ ___ $=$ ___

$x =$ ___ $-$ ___ $=$ ___

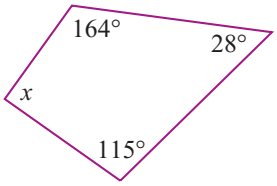


2 Find the values of the pronumerals, giving reasons for your answers.

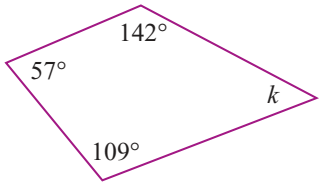
a



b



c



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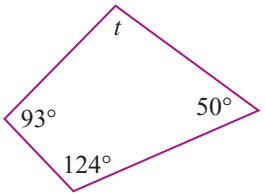
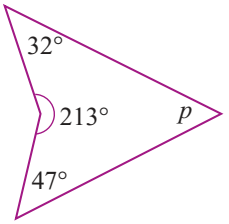


e



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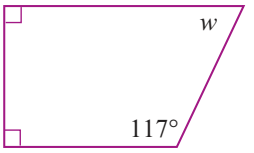
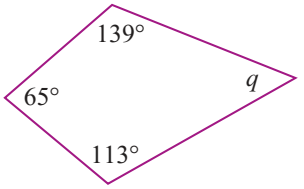
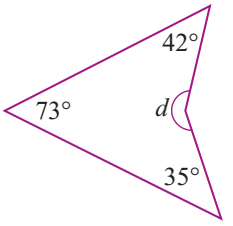


g

h

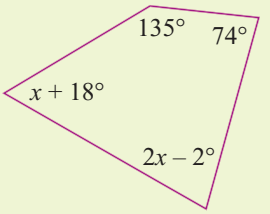


i



EXAMPLE 2

Write an equation and solve it to find the value of the pronumeral.
Hence find the unknown angles.



$(x + 18°) + (2x - 2°) + 74° + 135° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore 3x + 225° = 360°$

Collect like terms and solve the equation.

$3x = 360° - 225° = 135°$

$x = \frac{135°}{3} = 45°$

$\therefore$ Unknown angles are:

$x + 18° = 45° + 18° = 63°$

and $2x - 2° = 2 \times 45° - 2° = 88°$

3 Complete the following to find the value of the pronumeral. Hence find the unknown angles.

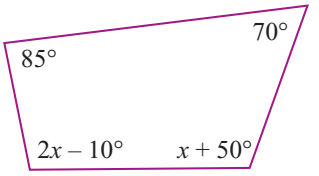
$(2x - 10°) + (x + 50°) + \_\_\_ + \_\_\_ = \_\_\_$ (Angle sum of a quadrilateral is ___.)

$\therefore 3x + \_\_\_ = \_\_\_$

$3x = \_\_\_ - \_\_\_$

$= \_\_\_$

$x = \frac{\square}{\square} = \_\_\_$



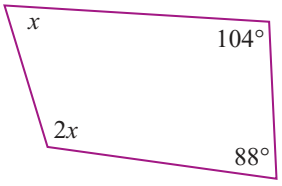
$\therefore$ Unknown angles are:

$2x - 10° = 2 \times \_\_\_ - 10° = \_\_\_$

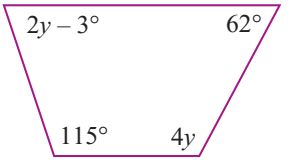
and $x + 50° = \_\_\_ + 50° = \_\_\_$

4 Write an equation for each of the following and solve it to find the value of the pronumeral. Hence find the unknown angles.

a



b



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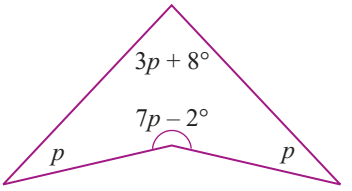
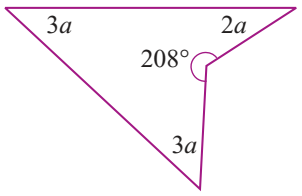


d



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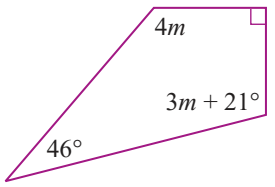
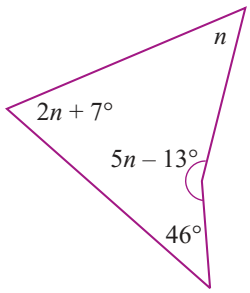


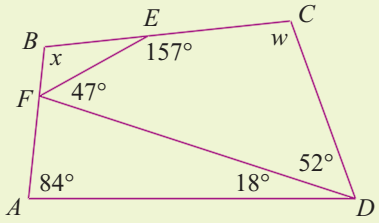
f



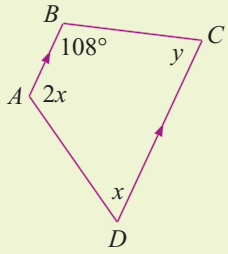
EXAMPLE 3

Find the values of the pronumerals in each quadrilateral, giving reasons for your answers.

a



b



a In quadrilateral $FECD$,

$w + 157° + 47° + 52° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore w + 256° = 360°$

$w = 360° - 256° = 104°$

In quadrilateral $ABCD$,

$84° + x + 104° + 52° + 18° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore x + 258° = 360°$

$x = 360° - 258° = 102°$

b $\angle B + \angle C = 180°$ (Sum of co-interior angles is 180°, $AB \parallel DC$.)

$\therefore 108° + y = 180°$

$y = 180° - 108 = 72°$

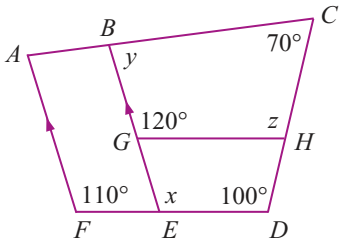
$\angle A + \angle D = 180°$ (Sum of co-interior angles is 180°, $AB \parallel DC$.)

$\therefore 2x + x = 180°$

$3x = 180°$

$x = \frac{180°}{3} = 60°$

5 Complete the following to find the values of the pronumerals.



$x =$ ___ (corresponding angles, $AF \parallel$ ___)

In quadrilateral $BCDE$,

$y + 70° + 100° +$ ___ $=$ ___ (Angle sum of a quadrilateral is ___.)

$\therefore y +$ ___ $=$ ___

$y =$ ___ $-$ ___

$=$ ___

In quadrilateral $BCHG$,

$z + 120° +$ ___ $+ 70° =$ ___ (Angle sum of a quadrilateral is ___.)

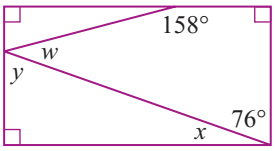
$\therefore z +$ ___ $=$ ___

$z =$ ___ $-$ ___

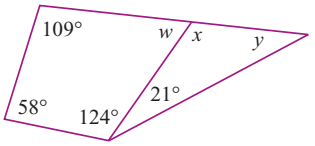
$=$ ___

6 Find the values of the pronumerals, giving reasons for your answers.
Hint: In this question, find the values of the pronumerals in alphabetical order, for example w, x, y.

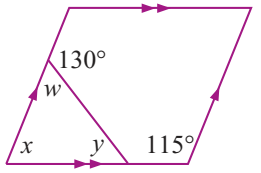
a



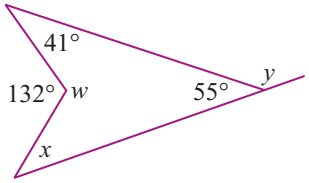
b



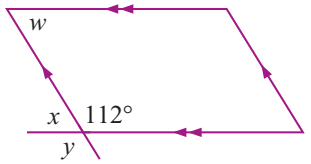
c



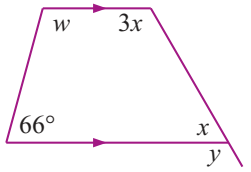
d



e



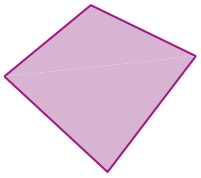
f



Investigation 9 Composite figures

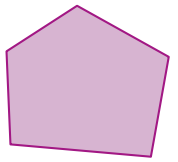
Copy each of the following figures and divide them into the plane shapes specified.

1



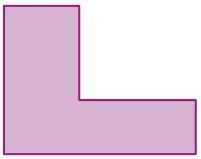
a 2 triangles
b 4 triangles

2



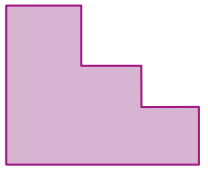
a 3 triangles
b 5 triangles
c a quadrilateral and a triangle

3



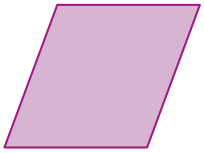
2 rectangles (2 ways)

4



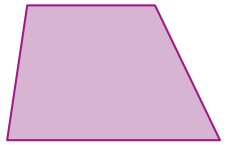
3 rectangles. How many ways can you do this?

5



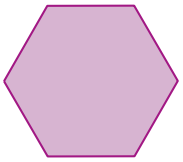
a 2 triangles
b a rectangle and 2 triangles
c a trapezium and a triangle
d a rhombus and a parallelogram

6



a 2 triangles
b a rectangle and 2 triangles
c a parallelogram and a triangle

7



a 2 quadrilaterals
b 4 triangles
c 6 triangles
d a quadrilateral and 2 triangles

8

a a square and a rectangle
b 3 triangles
c a trapezium and 2 triangles
d a parallelogram and 2 triangles
e a kite and 4 triangles

Language in mathematics

Here is a find-a-word puzzle.

P	T	I	D	F	I	S	E	L	E	C	S	O	S	I	N	S
R	R	R	N	O	I	T	A	U	Q	E	M	D	U	T	Q	E
O	A	E	E	R	I	N	A	T	U	U	N	G	M	U	X	L
N	H	A	T	E	G	S	C	S	I	D	E	S	A	T	L	E
U	C	S	X	L	A	U	N	Z	L	D	A	R	E	Q	A	L
M	W	O	E	U	R	A	E	D	A	K	E	R	D	R	U	G
E	O	N	I	T	L	P	R	A	T	T	I	E	C	R	Q	N
R	L	I	S	M	A	A	A	L	E	O	S	T	F	A	E	A
A	F	N	S	R	W	C	D	P	R	O	V	E	E	I	N	I
L	O	G	T	A	A	D	T	I	A	S	U	B	M	O	H	R
C	N	G	Q	U	A	D	R	I	L	A	T	E	R	A	L	T

1 Find all the words in the following list:

add, angle, arc, arm, construct, draw, equal, equation, equilateral, extend, exterior, flowchart, isosceles, kite, pronumeral, prove, quadrilateral, reasoning, rhombus, sides, square, sum, trapezium, triangle

2 The remaining letters reveal a message. Working from the top, left to right, place the remaining letters in the order in which they appear to reveal the message.

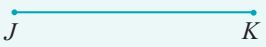
Terms

acute angle	adjacent	angle	angle sum	base angles	baseline
bisect	compasses	concave	construct	convex	diagonal
equilateral	exterior	interior	interval	isosceles	kite
line	obtuse angle	parallelogram	protractor	quadrilateral	rectangle
rhombus	right angle	scalene	square	trapezium	triangle

Check your skills

Note: Due to the practical nature of this topic, not all outcomes can be assessed here.

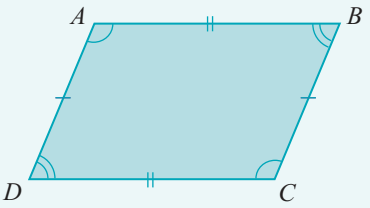
1 The diagram shows:



A line J **B** interval J

C line JK **D** interval JK

2 This diagram shows:



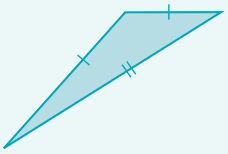
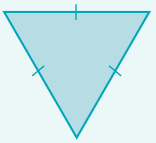
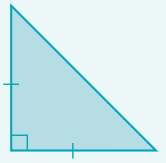
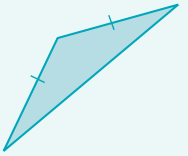
A quadrilateral $ABCD$

B a quadrilateral with four equal sides

C a quadrilateral with no equal angles

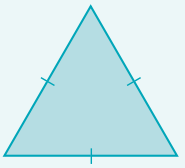
D all of the above

3 Which diagram shows a right-angled isosceles triangle?

A  **B**  **C**  **D** 

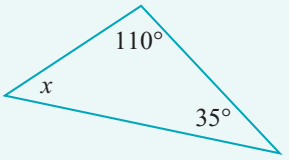
4 The side name for this triangle is:

A acute-angled **B** obtuse-angled
C right-angled **D** equilateral



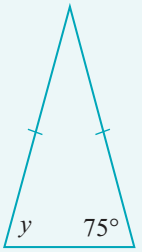
5 The value of x in the diagram is:

A 145° **B** 35°
C 215° **D** 180°



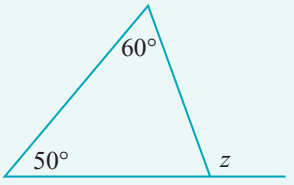
6 The value of y in the diagram is:

A 75° **B** 150°
C 105° **D** cannot find



7 The value of z in the diagram is:

A 50° **B** 60°
C 70° **D** 110°



8 A concave quadrilateral has:

A all sides equal **B** all angles equal
C all diagonals inside the figure **D** none of the above properties

9 Which quadrilateral has all its angles equal?

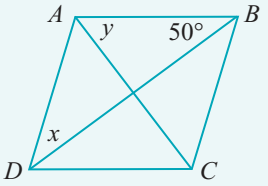
A rhombus **B** rectangle **C** parallelogram **D** kite

10 Which quadrilateral has exactly two sides parallel?

A rhombus **B** rectangle **C** parallelogram **D** trapezium

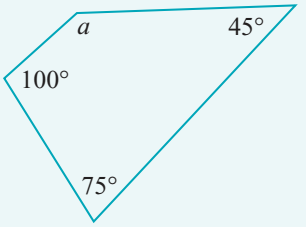
11 The values of x and y in rhombus $ABCD$ are:

A $x = 50°, y = 50°$ **B** $x = 50°, y = 40°$
C $x = 40°, y = 50°$ **D** $x = 40°, y = 40°$



12 The value of a in the diagram is:

A 140° **B** 75°
C 220° **D** 360°



If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3, 4	5	6	7	8–11	12
Section	A	C	D	E	F	G	H

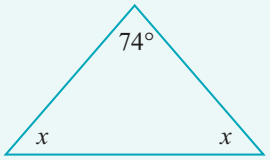
15A Review set

1 Construct a triangle with side lengths 10 cm, 8 cm and 6 cm.

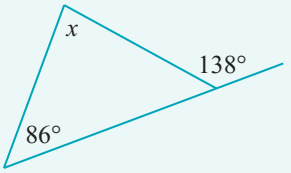
2 Draw an obtuse-angled isosceles triangle.

3 Find the value of x, giving reasons.

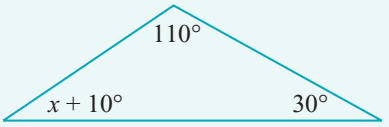
a



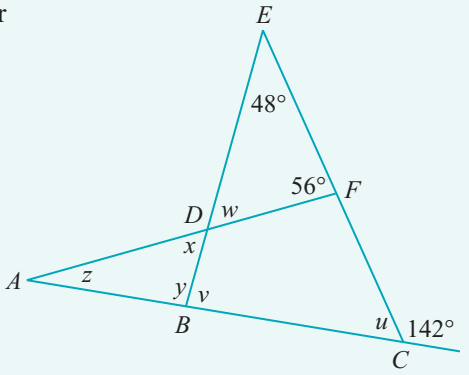
b



4 Write an equation and solve it to determine the value of the unknown angle.



5 Find the values of the pronumerals, in alphabetical order (u to z), giving reasons for your answers.

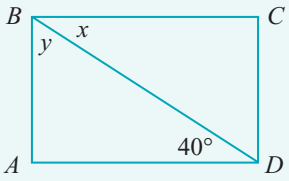


6 **a** Draw a parallelogram.
b List the properties of a parallelogram.

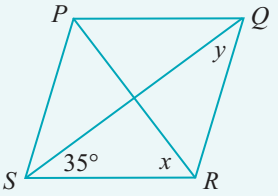
7 **a** Draw a convex quadrilateral.
b Comment on the diagonals of your diagram.

8 Find the values of the pronumerals, giving reasons.

a $ABCD$ is a rectangle.



b $PQRS$ is a rhombus.



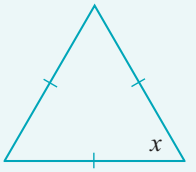
15B Review set

1 Construct a triangle with side lengths 4 cm and 7 cm that form an angle of 65°.

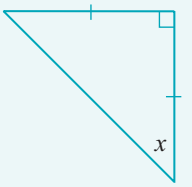
2 Draw an acute-angled scalene triangle.

3 Find the value of x, giving reasons.

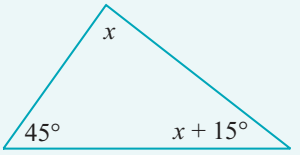
a



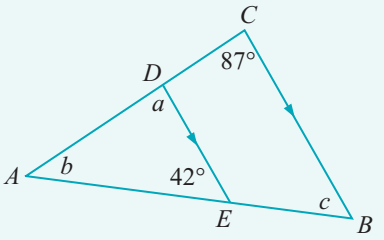
b



4 Write an equation and solve it to determine the values of the unknown angles.



5 Find the value of the pronumerals, giving reasons for your answers.



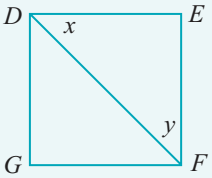
6 **a** Draw a rectangle.

b List the properties of a rectangle.

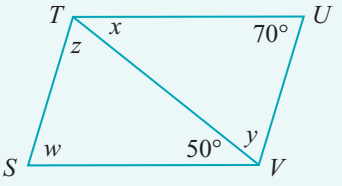
7 Give a definition for a concave quadrilateral.

8 Find the values of the pronumerals, giving reasons.

a *DEFG* is a square.



b *STUV* is a parallelogram.



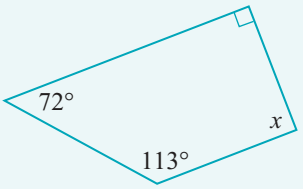
15C Review set

1 Construct a triangle with angles of 30° and 120°, joined by a side of length 6 cm.

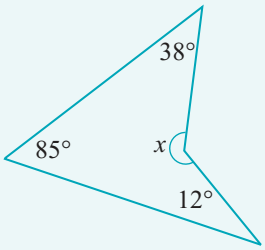
2 Draw a right-angled isosceles triangle.

3 Find the value of x, giving reasons.

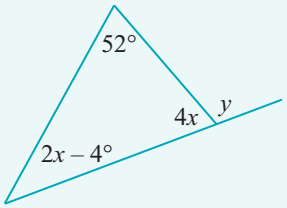
a



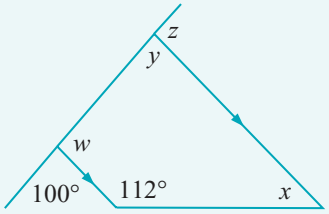
b



4 Write an equation and solve it to determine the values of the unknown angles.



5 Find the values of the pronumerals, giving reasons for your answers.



6 **a** Draw a trapezium.

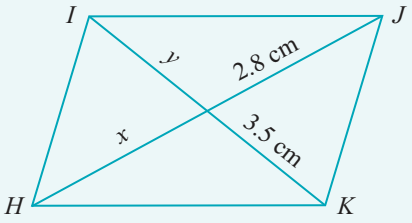
b List the properties of a trapezium.

7 **a** Sketch a quadrilateral in which the diagonals bisect each other at right angles.

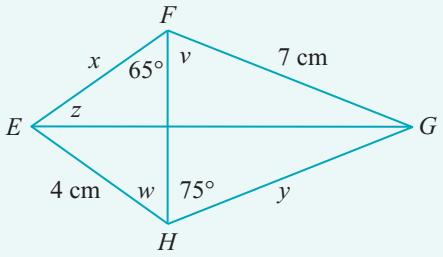
b What is the name given to this quadrilateral?

8 Find the values of the pronumerals, giving reasons.

a $HIJK$ is a parallelogram.



b $EFGH$ is a kite.



Chapters 11–15 Cumulative review

1 Classify each of these variables as nominal, continuous numerical or discrete numerical.

a hair colour **b** height **c** temperature **d** number of apples on a tree

2 **a** Draw a scatter plot for this information.

x	6	10	13	19	21	31
y	6	7	9	12	14	19

b Is there a trend? Explain.

3 **a** Draw a frequency distribution table to show the following information.

8 9 5 10 7 6 6 7 5 1 8 6
2 8 6 2 6 4 9 4 7 4 9 2
6 5 9 10 10 1 6 4 8 9 9 10

b How many scores were 8 or more?

4 Draw a stem-and-leaf plot for the following information. Use stems of 2, 3, 4, 5, 6.

46 45 28 38 61 50 49 42 32 42 28 22
50 45 51 31 48 45 32 45 44 53 40 35

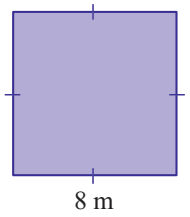
5 **a** Draw a column graph for the data in this table.

b Draw a dot plot showing this information.

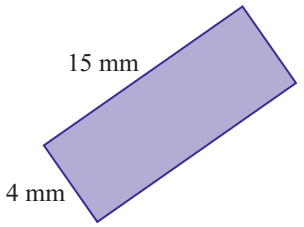
Choice of drink	Frequency
Soft drink	18
Still water	12
Juice	10
Tea/coffee	3
Other	7

6 **a** Find the area of each of the following.

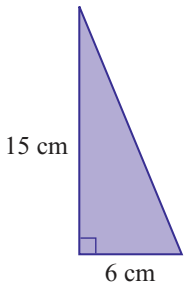
i



ii



iii



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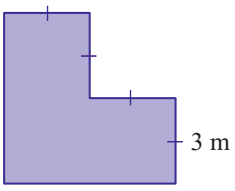
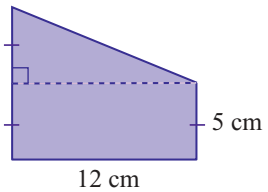
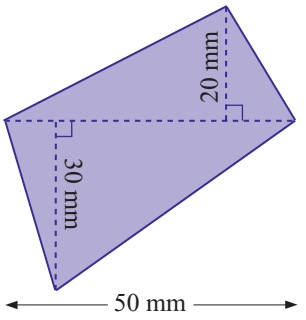


v

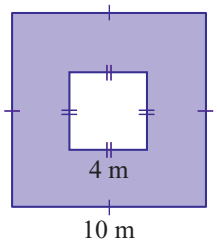


vi

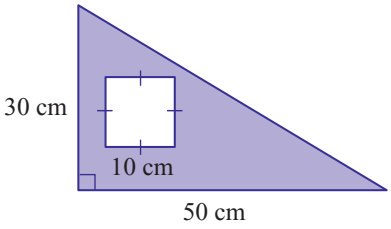


b Find the shaded area.

i

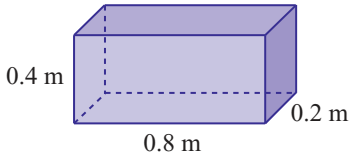


ii

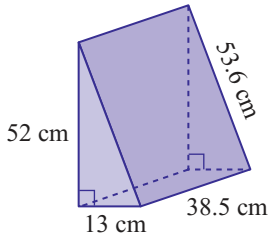


7 **a** Calculate the surface area of each prism.

i



ii



b Calculate the volume of each prism in part **a**.

8 **a** For the scores 23, 25, 24, 23, 26, 29, 23, 21, find the:

i mean **ii** mode **iii** median **iv** range

b For the scores in the frequency distribution table below, find the:

i mean **ii** mode **iii** median **iv** range

Score	Frequency
19	11
20	23
21	15
22	6
23	5
24	1

9 For the scores in this stem-and-leaf plot, find the:

a mean **b** mode

c median **d** range

Stem	Leaf
8	2 3 4 9
9	1 1 1 6 8 8
10	1 2 5 7 7 9 9
11	1 1 2 3 7 8
12	1 4 4 4 4 5 7

10 **a** Complete the table to solve the equation $3x + 8 = 35$ by guess and check.

x	5	10	
$3x + 8$	23	38	

b Starting with x, write the expression produced if you multiply by 5 and subtract 1.

c Draw a flowchart for $\frac{x}{3} - 7$.

d Solve $\frac{x - 4}{3} = 1$ by drawing a flowchart and backtracking.

e Solve the following.

i $4t - 5 = 19$ **ii** $\frac{r}{3} - 5 = 1$

f Lily is 8 years older than her brother. The total of their ages is 32. Let n be her brother's age.

i Write an equation from which n can be found.

ii Solve the equation.

11 **a** Construct a triangle with side lengths 5 cm, 12 cm and 13 cm.

b Draw these triangles.

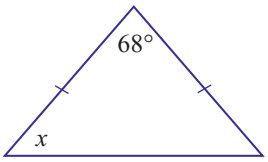
i Obtuse-angled scalene triangle

ii Right-angled isosceles triangle

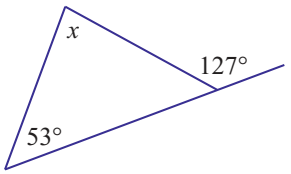
iii Acute-angled triangle

c Find the value of x, giving reason(s).

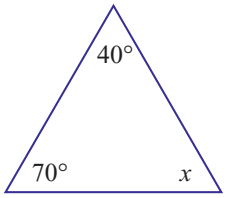
i



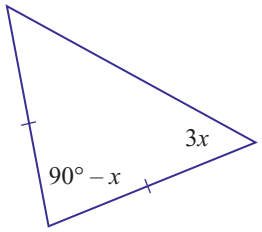
ii



iii



d Write an equation and solve it to find the value of x.
Hence find the unknown angles.

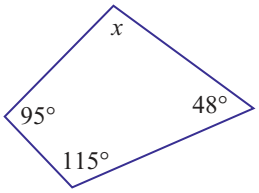


12 **a** Draw each of these plane shapes and list their properties.

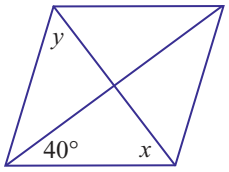
i Trapezium **ii** Rhombus **iii** Kite **iv** Rectangle

b Find the values of the pronumerals, giving reason(s).

i

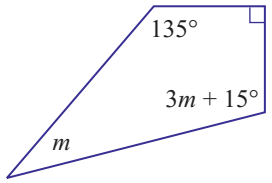


ii



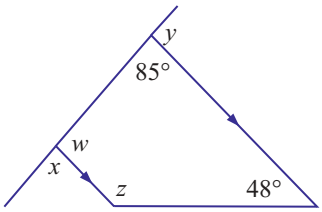
Rhombus

c Write an equation and solve it to find the value of m.
Hence find the unknown angles.

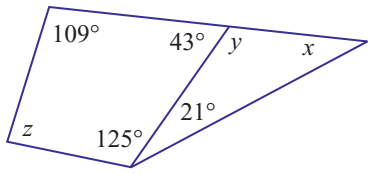


13 Find the values of the pronumerals, giving reasons.

a



b



CHAPTER 1 REVIEW OF YEAR 6

Diagnostic test

1 D	**2** B	**3** D	**4** B	**5** B
6 B	**7** B	**8** B	**9** A	**10** A
11 C	**12** D	**13** A	**14** D	**15** C
16 A	**17** D	**18** B	**19** C	**20** A
21 C	**22** B	**23** C	**24** D	**25** C
26 D	**27** D	**28** B	**29** A	**30** D
31 A	**32** A	**33** D	**34** B	**35** C
36 B	**37** B	**38** B	**39** B	**40** A
41 D	**42** B	**43** C	**44** A	**45** A
46 B	**47** A	**48** B	**49** B	**50** D
51 B	**52** C	**53** D	**54** C	**55** C
56 B	**57** C	**58** B	**59** B	**60** B
61 C	**62** D	**63** A	**64** A	**65** C
66 B	**67** A	**68** B	**69** D	**70** B
71 A	**72** D	**73** B	**74** D	**75** B
76 A	**77** C	**78** C	**79** C	**80** D
81 A	**82** B	**83** C	**84** B	**85** D
86 AC	**87** C	**88** B	**89** D	**90** B
91 B	**92** D	**93** D	**94** C	**95** B
96 C	**97** D	**98** C	**99** C	**100** B
101 C	**102** B			

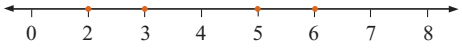
CHAPTER 2 INTEGERS

Diagnostic test

1 A	**2** A	**3** B	**4** D	**5** B
6 D	**7** C	**8** B	**9** C	**10** B
11 B	**12** B	**13** B	**14** C	**15** B
16 A				

Exercise 2A

1 a

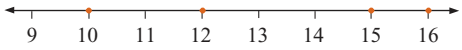


2, 3, 5, 6

b

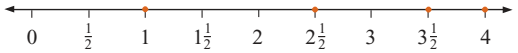
1, 3, 5, 8

c



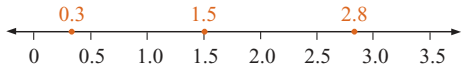
10, 12, 15, 16

d



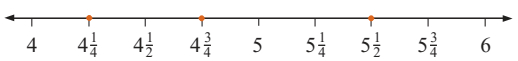
$1, 2\frac{1}{2}, 3\frac{1}{2}, 4$

e



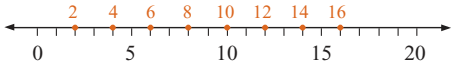
0.3, 1.5, 2.8

f

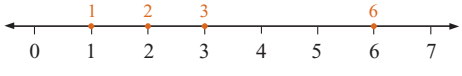


$4\frac{1}{4}, 4\frac{3}{4}, 5\frac{1}{2}$

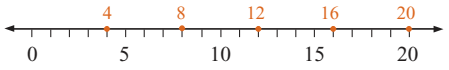
2 a



b

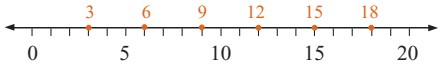


c



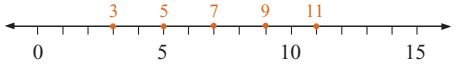
d

e



f

g



h

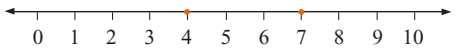
i

j

3 a $6 > 4$ **b** $3 < 8$ **c** $7 > 5$ **d** $4 > 3$

e $3\frac{3}{4} > 2\frac{1}{2}$ **f** $2\frac{1}{3} < 2\frac{2}{3}$ **g** $15 > 14\frac{1}{2}$ **h** $4.3 < 5.2$

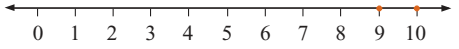
4 a



b $4 < 7$ **c** $7 > 4$ **d** 5, 6

e Examples: 5.5, 5.9, 6.1

5 a



b $9 < 10$ **c** $10 > 9$

d Examples: 9.1, 9.6, 9.7

6 a Examples: 5.3, 5.7 **b** Examples: 10.2, 10.8

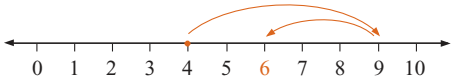
c Examples: $\frac{2}{3}, \frac{3}{4}$ **d** Examples: $\frac{1}{8}, \frac{1}{5}$

e Examples: 1.35, 1.4 **f** Examples:2.75, 2.79

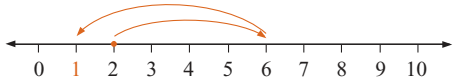
g Examples:0.13, 0.15 **h** Examples: 0.05, 0.07

7 An infinite number **8** 1

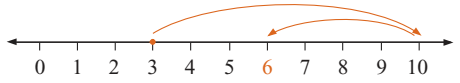
9 a 6



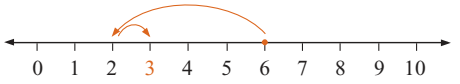
b 1



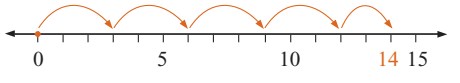
c 6



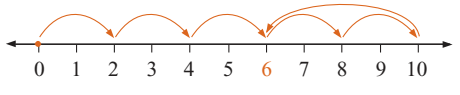
d 3



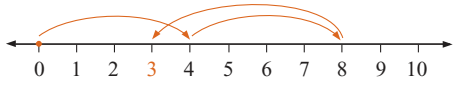
e 14



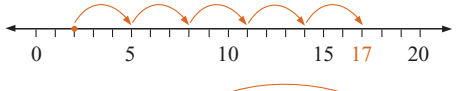
f 6



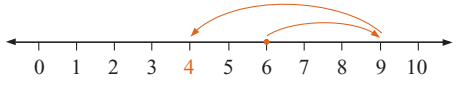
g 3



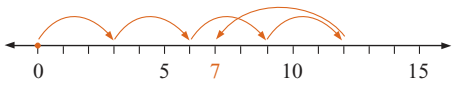
h 17



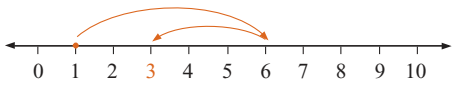
i 4



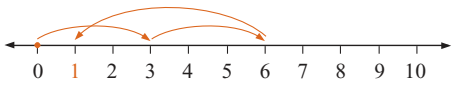
j 7



k 3



l 1



Exercise 2B

1 **a** Withdrawing $1 **b** Increase of 10 cm in length
c Loss of 5 kg in weight
d Rise of 5°C in temperature
e 200 m below sea-level **f** 5 km south
g Going down 3 flights of stairs
h 3 degrees above zero **i** 10 km west

2 **a** $4 withdrawal **b** $8 withdrawal
c Deposit $3 **d** Zero (no change)
e $5 withdrawal **f** $18 withdrawal
g $26 withdrawal **h** $17 withdrawal
i $5 deposit **j** $4 withdrawal

3 **a** $+4$ **b** -6 **c** $-3 + 7$
d $-5 + 2$ **e** $+6 - 4$ **f** $-3 + 5$

4 **a** -4 **b** $+6$ **c** $+5 - 7$
d $-8 + 3$ **e** $+10 - 7$ **f** $-4 + 2$

5 **a** $+3$ **b** -6 **c** $-3 + 7$
d $-8 + 5$ **e** $+4 - 7$ **f** $-3 - 4$

6 **a** $+65$ m **b** -15 m **c** $+25$ m $-$ 8 m

7 **a** Examples: 3 floors down **b** Examples: 9 km north
c Examples: 95 m below sea-level
d Examples:. 6 paces west then 8 paces east

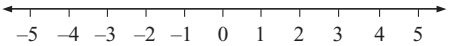
8 **a** $+4$ **b** -9 **c** $+2.45$ **d** $-4\frac{1}{3}$

9 **a** $+4 - 6 + 9 = 7$th floor
b $+7 - 15 + 20 = 12$th floor
c $-6 + 18 + 2 = 14$th floor
d $-8 + 15 - 2 = 5$th floor

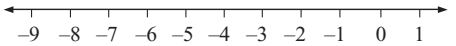
10 **a** Start on 6th floor, down 8, up 5 floors
b Start on 3rd floor, down 8, up 12 floors
c Start on 3rd basement level, up 8, up 5 floors
d Start on 2nd basement level, up 10, up 4 floors
e Start on 3rd basement level, up 8, then down 4 floors
f Start on 2nd basement level, down 4, then up 12 floors

Exercise 2C

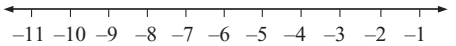
1 **a–c**



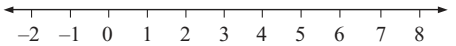
d



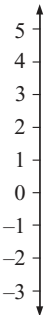
e



f



2 **a**



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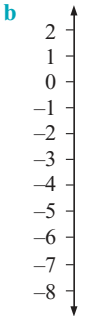

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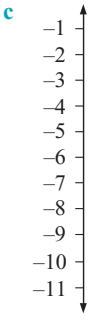

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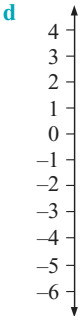

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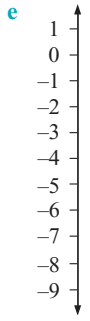

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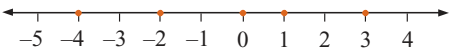
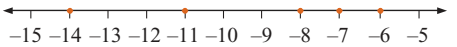


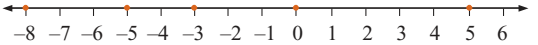
−4, −2, 0, 1, 3

4 **a**



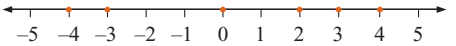
−14, −11, −8, −7, −6

b



−8, −5, −3, 0, 5

c

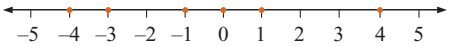


−4, −3, 0, 2, 3, 4

d

−5, −4, −1, 0, 4, 7

e

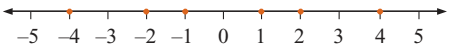


−4, −3, −1, 0, 1, 4

f

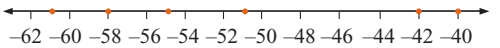
−4, −3, −2, −1, 0, 1

g



−4, −2, −1, 1, 2, 4

h



−61, −58, −55, −51, −42, −40

5 **a** $-6 < -5$ **b** $-2 < 1$ **c** $0 > -3$
d $3 > -1$ **e** $-102 > -110$
f $-1 < 2$ **g** $-7 < -5$ **h** $13 > -11$

6 **a** I love maths. **b** School is great.
c Is it Friday yet? **d** Homework is the best.

7 **a** 5 **b** 2 **c** 3 **d** -4 **e** -1
f -1 **g** -3 **h** -4 **i** -4

Exercise 2D

1 **a** 7 **b** -2 **c** 8 **d** -3 **e** 0 **f** 3
g -2 **h** 1 **i** 4 **j** 0 **k** 3 **l** -3

2 **a** 3 **b** 3 **c** -2 **d** -2 **e** -7 **f** -2
g -8 **h** -3 **i** -3 **j** -5 **k** -4 **l** -4

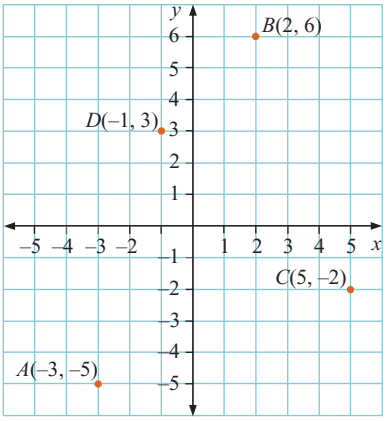
3 **a** 3 **b** 3 **c** -4 **d** -10 **e** 3 **f** -7
g -9 **h** -3 **i** 2 **j** -6 **k** -2 **l** -15

4 **a** 5 **b** -1 **c** -3 **d** 0 **e** -1 **f** -2
g -3 **h** -4 **i** -1 **j** -1 **k** 4 **l** -6

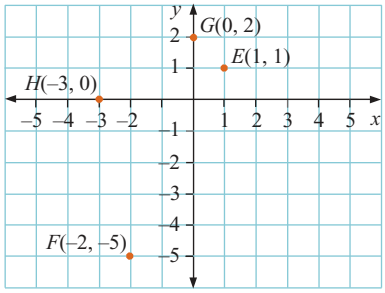
5 **a** $20 **b** -4°C **c** -2°C

Exercise 2E

1



2



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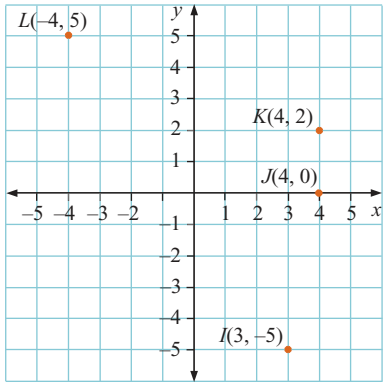

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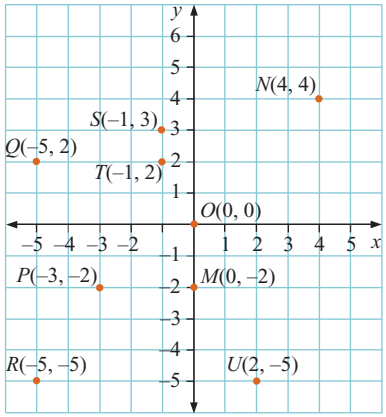
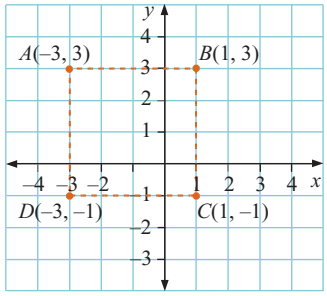


5 **a** **i** positive, positive
ii Both coordinates must be positive, for example (7, 4).
b **i** negative, positive
ii First coordinate must be negative, second must be positive, for example (−8, 4).
c **i** negative, negative
ii Both coordinates must be negative, for example (−7, −4).
d **i** positive, negative
ii First coordinate must be positive, second must be negative, for example (5, −10).

6 **a** $A(2, 3)$ $B(-4, 1)$ $C(3, 1)$ $D(-3, -3)$ $E(4, 4)$ $F(2, 5)$ $G(4, -4)$ $H(-1, 4)$
b *A, F* or *E, G*. They are on the same vertical line.
c *B, C* or *E, H*. They are on the same horizontal line.
d *D* and *E*. They are diagonally opposite each other.

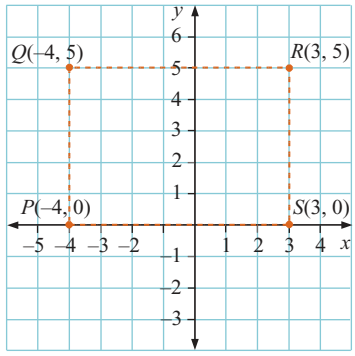
7 $A(-5, 5)$ $B(3, -4)$ $C(5, 3)$ $D(0, 5)$ $E(-3, 0)$ $F(-1, -3)$ $G(-5, -4)$ $H(6, 5)$ $I(3, 1)$ $J(1, 3)$ $K(5, -3)$ $L(7, -2)$ $M(-3, -2)$ $N(-2, 3)$ $O(0, 0)$ $P(-4, 4)$ $Q(-2, -4)$ $R(2, 4)$ $S(3, -2)$ $T(6, -6)$ $U(-4, -6)$ $V(0, -1)$ $W(6, 0)$ $X(4, 5)$ $Y(4, -6)$ $Z(1, -3)$

8 **a**



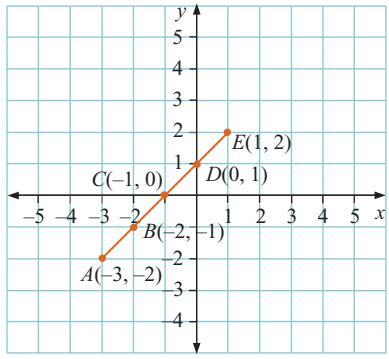
b $D(-3, -1)$

9 **a**



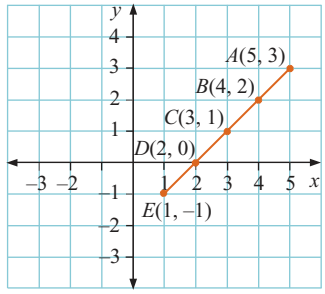
b $S(3, 0)$

10 **a**



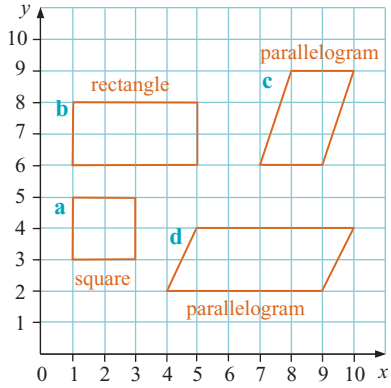
b They form a straight line.
c $F(2, 3)$, $G(3, 4)$, $H(4, 5)$

11 **a**

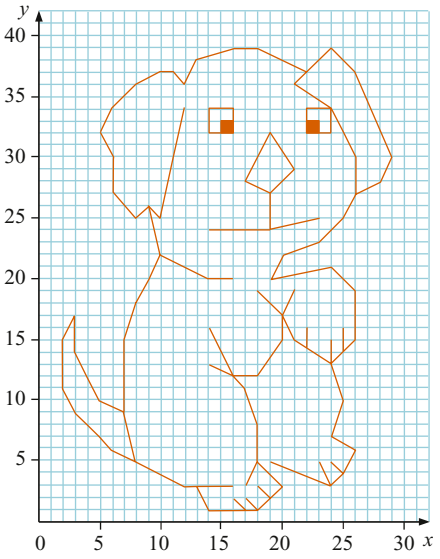


b $F(0, -2)$, $G(-1, -3)$, $H(-2, -4)$

12

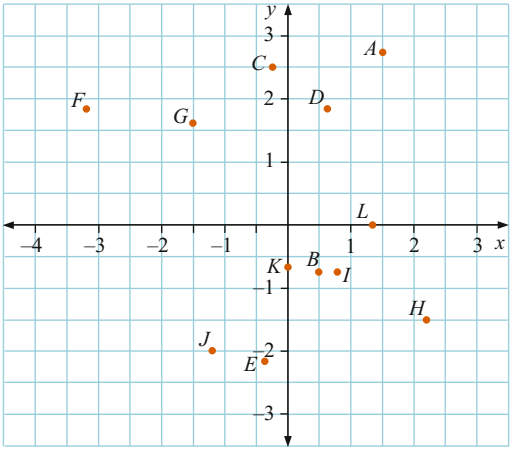


13



14 $A(2\frac{1}{2}, 3)$ $B(-\frac{1}{2}, 1)$ $C(-1\frac{1}{2}, 2\frac{1}{2})$ $D(1\frac{3}{4}, 1)$
$E(1\frac{3}{4}, \frac{1}{2})$ $F(1\frac{1}{2}, -1\frac{1}{2})$ $G(2\frac{1}{2}, -\frac{1}{4})$ $H(\frac{1}{4}, -2\frac{1}{2})$
$I(-1\frac{1}{2}, -1\frac{3}{4})$ $J(-3\frac{1}{2}, \frac{3}{4})$ $K(-\frac{1}{4}, 0)$ $L(-1\frac{3}{4}, -3)$
$M(1, -1)$ $N(2\frac{1}{4}, -1)$ $O(\frac{3}{4}, 1\frac{3}{4})$ $P(-1\frac{1}{4}, 1\frac{1}{2})$
$Q(-2\frac{1}{2}, 1\frac{1}{2})$ $R(-\frac{3}{4}, -\frac{3}{4})$ $S(-1\frac{3}{4}, \frac{1}{2})$ $T(\frac{1}{4}, 0)$
$U(\frac{3}{4}, \frac{1}{2})$ $V(-2\frac{1}{2}, -1\frac{1}{2})$

15



Exercise 2F

1 **a** **i** 2 h **ii** 200 km **iii** 100 km/h
b **i** 3 h **ii** 150 km **iii** 50 km/h
c **i** 4 h **ii** 200 km **iii** 50 km/h
d **i** 2 h **ii** 150 km **iii** 75 km/h
e **i** 2 h **ii** 150 km **iii** 75 km/h
f **i** 6 h **ii** 300 km **iii** 50 km/h
g **i** 1 h **ii** 0 km **iii** 0 km/h

2 **a** The motorist was stationary. **b** 3 times
c 11 am for 1 h, 2 pm for $\frac{1}{2}$ h, 4:30 pm for $\frac{1}{2}$ h

3 **a** The motorist was stationary. **b** 2 times
c After 2 h for $\frac{1}{2}$ h, after 5 h for 1 h.

4 **a** 8:06 am **b** 2 min **c** 300 m **d** 50 m/min
e **i** 13 min **ii** 5 min
f **i** 500 m **ii** 200 m
g 40 m/min **h** 800 m **i** 31 m/min

5 **a** 4 pm **b** 900 m from home
c 4:04 and 4:17 pm **d** 1250 m
e 250 m/min **f** Bill was stationary.
g At 4:06 pm **h** 1500 m
i 10 min **j** 5 min
k 300 m/min **l** 3000 m
m 273 m/min

6 **a** 350 km **b** 8 am **c** 80 km/h
d 30 min **e** 60 km/h **f** 30 min after
g Dan left home at 8:30 am, drove for $2\frac{1}{2}$ h until he was 150 km from Town A (at 60 km/h) then increased his speed and drove to Town B in 2 h (at 100 km/h).
h At 12:10 pm **i** 90 km **j** $\frac{1}{2}$ h earlier
k 70 km apart **l** 30 km

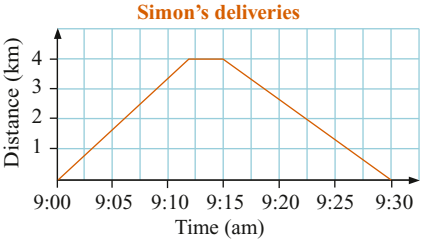
7 **a** 22 km **b** Stewart **c** 2 breaks
d $\frac{1}{2}$ h each time **e** 3 km/h, 4 km/h, 4 km/h
f 7 h
g **i** 3.1 km/h **ii** 3.7 km/h
h He left Mt White and walked 10 km in $2\frac{1}{2}$ h, rested for $\frac{1}{2}$ h, walked remaining distance of 12 km to Red Gum Forest in 3 h.
i After $3\frac{1}{2}$ h **j** Robyn 12 km, Stewart 10 km
k 4 km **l** 4 km

8 **a** **i** 120 km **ii** 60 km/h
b **i** 160 km **ii** 80 km/h
c **i** 200 km **ii** 100 km/h
d **i** line C **ii** motorist C
e **i** line A **ii** motorist A
f under line A **g** greater
h **i** positive **ii** increasing

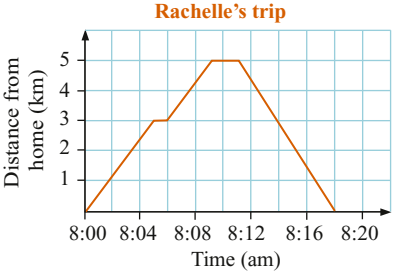
9 **a** 90 km **b** 80 km from home
c After $\frac{1}{2}$ h **d** $1\frac{1}{2}$ h
e **i** 70 km **ii** 20 km **iii** 10 km/h
f **i** 50 km **ii** 40 km **iii** 20 km/h
g **i** 30 km **ii** 60 km **iii** 30 km/h
h **i** Ben's line **ii** Ben
i **i** Jim's line **ii** Jim
j Between Jim and Sam line **k** greater
l **i** negative **ii** decreasing

Exercise 2G

1

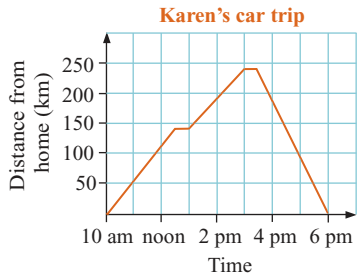


2

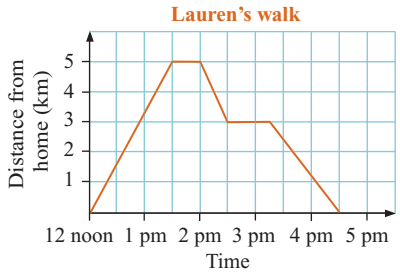


3 **a** 240 km **b** 3 h

c

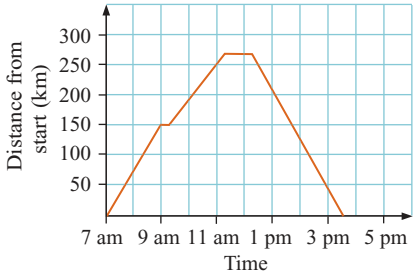


4



5 **a** 2 h **b** 120 km **c** $1\frac{1}{2}$ h

d

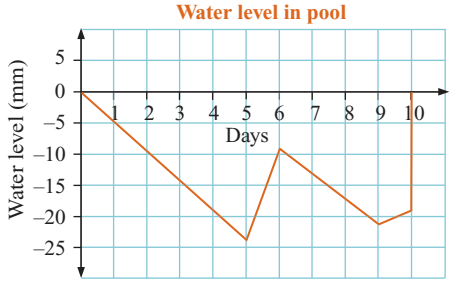


7 Class discussion.

8 Class discussion.

9 **a** 0 mm **b** −9 mm
c 3 mm/day **d** Days 4 and 5
e Days 6, 7,8 **f** Day 10

10



Exercise 2H

1 **a** 5 − 1 = 4 **b** 6 + 2 = 8 **c** −2 − 3 = −5

2 **a** 4 **b** 7 **c** −7 **d** −8
e 1 **f** 9 **g** 14 **h** −8
i 14 **j** −7 **k** 4 **l** −7

3 **a** −5 **b** −2 **c** 7 **d** −4
e −10 **f** −13 **g** −16 **h** −15
i 8 **j** 12 **k** 12 **l** 11
m −1 **n** 9 **o** −11 **p** −9
q 10 **r** −11 **s** 2 **t** 7
u 2 **v** −8 **w** −11 **x** −1

4 **a** 4 + 2 + 1= 6 + 1 = 7
b 3 + 4 − 2 = 7 − 2 = 5

5 **a** 10 **b** 23 **c** 11 **d** 13 **e** 31
f 1 **g** 5 **h** 10 **i** 14 **j** 3
k 0 **l** 12 **m** 3 **n** 9 **o** 15

6 **a** 0 **b** −8 **c** −10
d −8 **e** −11 **f** −6

7 2 m below

8 **a** 5°C **b** −5°C

9 **a** **i** 2 h **ii** 0 h **iii** 1 h
iv 11 h **v** 5 h **vi** 4 h
b **i** 5 am Wednesday **ii** 2 am Wednesday
iii 10 pm Tuesday

Calculator activities

1 **a** −119 **b** −26 **c** −1036
d 1558 **e** −106 **f** −92
g −846 **h** −163 **i** −106
j 420 **k** −1555 **l** −1715
m −112 **n** 264 **o** 7695

2 **a** −768 **b** −290 **c** −26 585

3 **a** 293 times **b** 695 times

4 $245

5 **a** **i** 14.2°C **ii** 30.7°C **iii** 17.1°C
b 33.2°C
c **i** −2.2°C **ii** −2.7°C **iii** 13.4°C
d **i** −0.1°C **ii** 16.0°C **iii** 1.4°C
e **i** 1.9°C **ii** 11.7°C **iii** 21°C

Language in mathematics

		P	O	S	I	T	I	V	E					T		A
		L		O				E						R		N
		A		U				R	I	G	H	T		A		G
		N	O	T				T				E		V		L
		E		H			S	I	G	N		M		E		E
					F			C				P	O	L	E	S
H	O	R	I	Z	O	N	T	A	L			E				
					U			L				R			E	
N	U	M	B	E	R				N	E	G	A	T	I	V	E
						G						T			E	
	W	I	T	H	D	R	A	W	A	L		U			N	
						I						R				
						D	I	R	E	C	T	E	D			

Check your skills

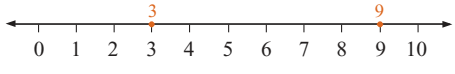
1 A **2** D **3** C **4** D **5** A
6 D **7** B **8** D **9** A **10** C
11 C **12** D **13** C **14** B **15** A
16 A

Review set 2A

1 **a**

b

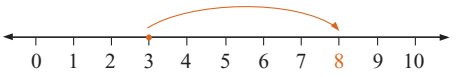
2 **a**



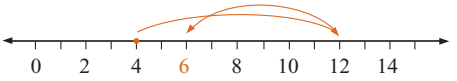
b **i** 3 < 9 **ii** 9 > 3

c Examples: 4, 6 **d** Examples: $5\frac{1}{2}$, 8.1

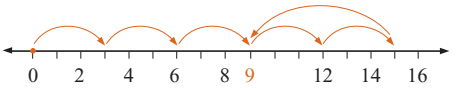
3 a 8



b 6



c 9

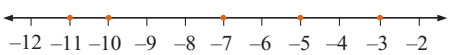


4 a Withdrawing \$15 **b** Increase of 3 cm
c Loss of 4 kg **d** Rise of 3°C

5 a \$2 withdrawal **b** \$2 deposit
c \$9 withdrawal

6 a -8 **b** $+5$ **c** $+2 - 5 = -3$

7 a

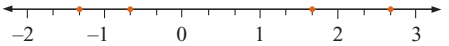


$-11, -10, -7, -5, -3$

b

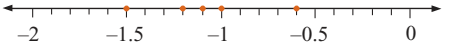
$-9, -6, -3, 0, 6$

8 a



$-1\frac{1}{3}, -\frac{2}{3}, 1\frac{2}{3}, 2\frac{2}{3}$

b

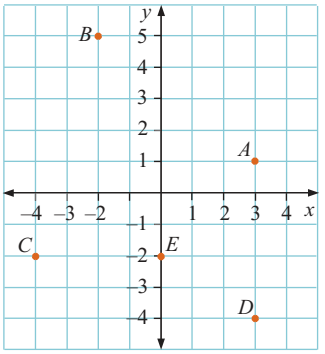


$-1.5, -1.2, -1.1, -1, -0.6$

9 a $-6 > -7$ **b** $-3 < 3$ **c** $0 > -6$

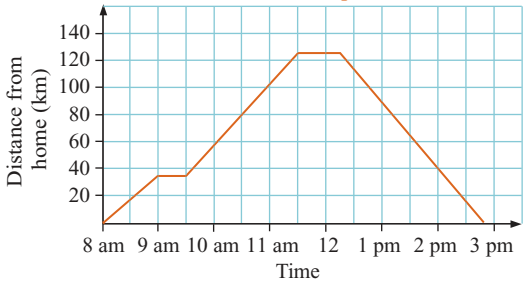
10 a -7 **b** -5 **c** 9

11 a



b A and D **c** C and E

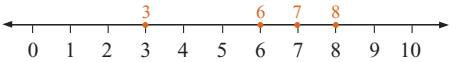
12



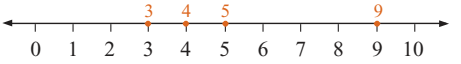
13 a 10:00 am **b i** 11:00 am **ii** $1\frac{1}{2}$ h
c 60 km **d** 1:30 pm–2:00 pm
e i 60 km **ii** 1 h **iii** 60 km/h
f 1 h **g i** 4:30 pm **ii** 30 min
h i 6:00 pm **ii** 40 km/h
i i $4\frac{1}{2}$ h **ii** $3\frac{1}{2}$ h
j 240 km **k** 60 L

Review set 2B

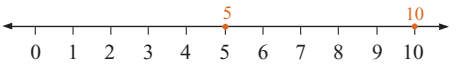
1 a



b

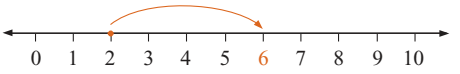


2 a

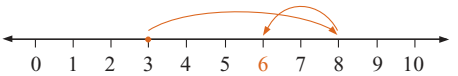


b i $5 < 10$ **ii** $10 > 5$
c Examples: 6, 7 **d** Examples: 8.4, $7\frac{1}{2}$

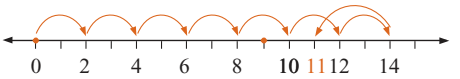
3 a 6



b 6



c 11

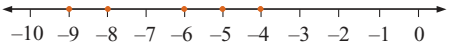


4 a Depositing \$5 **b** Decrease of 2 cm
c Gain of 1 kg **d** Fall of 4°C

5 a \$1 withdrawal **b** \$3 deposit **c** \$8 withdrawal

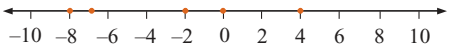
6 a -7 **b** $+6$ **c** $+8 - 4 = +4$

7 a



$-9, -8, -6, -5, -4$

b

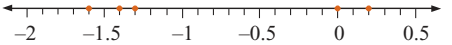


$-8, -7, -2, 0, 4$

8 a

$-\frac{1}{2}, 0, 1, 1\frac{1}{2}, 2\frac{1}{4}$

b

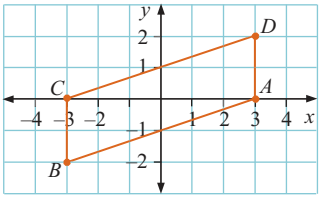


$-1.6, -1.4, -1.3, 0, 1.2$

9 a $-5 > -6$ **b** $-2 < 2$ **c** $0 > -3$

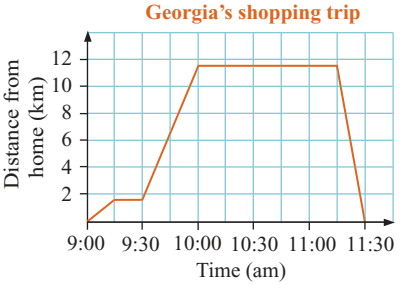
10 a -7 **b** -3 **c** 7

11 a



b Parallelogram
c (0, 0)

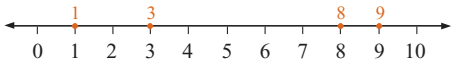
12



13 a 2 **b** 9:00–9:30 am, 10:30–11:00 am
c i 15 km **ii** 2 h **iii** $7\frac{1}{2}$ km/h
d i 5 km **ii** 1 h **iii** 5 km/h
e 11:00 am **f** No **g** 40 km **h** 6 h
i 11:00 am–1:00 pm **j** 9:30 am–10:30 am

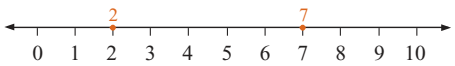
Review set 2C

1 a



b

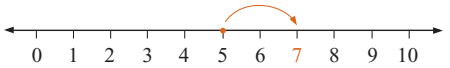
2 a



b i $2 < 7$ ii $7 > 2$

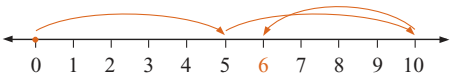
c Examples: 3, 5 **d** Examples: $3\frac{1}{2}$, 5.2

3 a 7



b 7

c 6



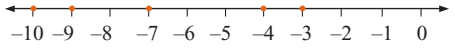
4 a Withdrawing \$25 **b** Increase of 9 cm

c Gain of 6 kg **d** Fall of 8°C

5 a \$2 deposit **b** \$6 deposit **c** \$5 withdrawal

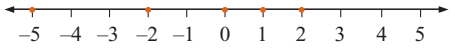
6 a -3 **b** $+2$ **c** $+9 - 12 = -3$

7 a



$-10, -9, -7, -4, -3$

b

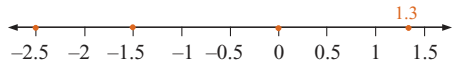


$-5, -2, 0, 1, 2$

8 a

$-1\frac{1}{2}, 0, \frac{1}{2}, 1\frac{1}{4}, 2\frac{3}{4}$

b

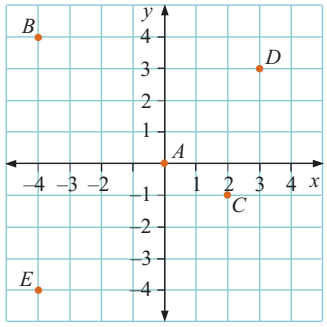


$-2.5, -1.5, 0, 1, 1.3$

9 a $-4 < -3$ **b** $-4 < 4$ **c** $0 > -5$

10 a -11 **b** -6 **c** 7

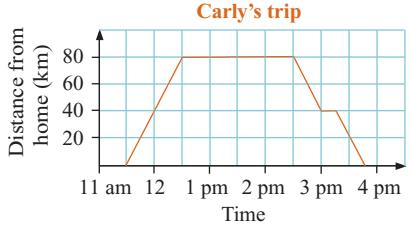
11 a



b *E*, *A*, *D*

c $(-4, 0)$

12



13 a -30 **b** 54 **c** 40

14 a -2 **b** 5 **c** 9

15 a 7 **b** -5 **c** 16 **d** 12 **e** 14 **f** 8

CHAPTER 3 ANGLES AND PARALLEL LINES

Diagnostic test

1 D	**2** B	**3** A	**4** D	**5** C
6 C	**7** C	**8** D	**9** A	**10** B
11 C	**12** B	**13** A	**14** B	**15** A
16 B	**17** D	**18** B	**19** C	**20** A
21 D	**22** D	**23** B		

Exercise 3A

1 a i *Y* ii *YX*, *YZ* iii $\angle XYZ$

b i *R* ii *RS*, *RQ* iii $\angle QRS$

c i *A* ii *AT*, *AQ* iii $\angle QAT$

d i *K* ii *KB*, *KL* iii $\angle BKL$

e i *D* ii *DC*, *DE* iii $\angle CDE$

f i *I* ii *IP*, *IF* iii $\angle PIF$

g i *W* ii *WX*, *WV* iii $\angle XWV$

h i *G* ii *GJ*, *GE* iii $\angle EGJ$

i i *P* ii *PR*, *PQ* iii $\angle RPQ$

j i *I* ii *IL*, *IF* iii $\angle LIF$

k i *T* ii *TU*, *TS* iii $\angle UTS$

l i *O* ii *OM*, *OT* iii $\angle MOT$

2 a The vertex is *T*. The arms are *TQ* and *TR*. The angle is $\angle QTR$.

b The vertex is *G*. The arms are *GA* and *GC*. The angle is $\angle AGC$.

3 a $\angle EDF$ **b** $\angle MNP$ **c** $\angle SUV$ **d** $\angle YVX$

e $\angle CPB$ **f** $\angle GFH$ **g** $\angle IJK$ **h** $\angle EHP$

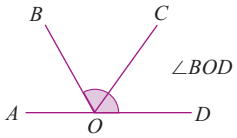
i $\angle NJM$ **j** $\angle MXT$ **k** $\angle EDI$ **l** $\angle URS$

4 a $\angle AEB$, $\angle ADC$, $\angle BCE$ **b** $\angle JLM$, $\angle JOP$, $\angle OML$

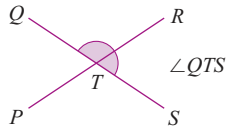
c $\angle EFG$, $\angle MKF$, $\angle NME$ **d** $\angle SWV$, $\angle WST$, $\angle UVS$

e $\angle USQ$, $\angle SUP$, $\angle RQS$ **f** $\angle FGD$, $\angle BDC$, $\angle EDG$

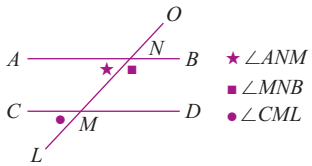
5 a



b



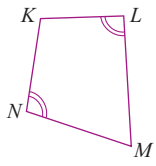
c



6 a $\angle XYZ = \angle XZY$

b $\angle SPQ = \angle SRQ$, $\angle PSR = \angle PQR$

7



Exercise 3B

1 a Acute **b** Obtuse **c** Straight

d Right **e** Reflex **f** Reflex

g Obtuse **h** Acute **i** Reflex

j Right **k** Acute **l** Straight

m Revolution **n** Obtuse **o** Reflex

p Reflex

2 a Acute **b** Obtuse **c** Right

d Obtuse **e** Acute **f** Acute

g Revolution **h** Obtuse **i** Reflex

j Obtuse **k** Straight **l** Reflex
m Acute **n** Reflex **o** Obtuse

3 a **i** Obtuse **ii** Right **iii** Acute
iv Acute
b **i** Right **ii** Right **iii** Acute
iv Acute
c **i** Straight **ii** Obtuse **iii** Acute
d **i** Obtuse **ii** Acute **iii** Right
iv Obtuse **v** Acute **vi** Straight

4 a True **b** False. $90° < \text{obtuse} < 180°$
c True **d** False. $180° < \text{reflex} < 360°$
e True
f False; acute angle + right angle = obtuse angle

Exercise 3C

1 a Common vertex is L. Common arm is LN.
The angles do lie on opposite sides of LN.
$\therefore$ The angles are adjacent because all criteria are satisfied.
b Common vertex is E.
There is not a common arm.
$\therefore$ The angles are not adjacent because all criteria are not satisfied.
c Common vertex is M. Common arm is MR.
The angles do not lie on opposite sides of MR.
$\therefore$ The angles are not adjacent because all criteria are not satisfied.

2 a, b, c, e, g, h Yes, because all criteria are satisfied
d, f No, because the angles do not have a common vertex
i No, because the angles do not lie on opposite sides of the common arm PQ

3 a $\angle STV$ and $\angle VTU$ are adjacent angles.
$\therefore \angle STU = 28° + 43° = 71°$
b $\angle KNL$ and $\angle LNM$ are adjacent angles that add to 120°.
$\therefore \angle LNM = 120° - 36° = 84°$

4 a 74° **b** 107° **c** 114° **d** 129° **e** 123°
f 115° **g** 65° **h** 153° **i** 64°

5 $\angle QTR$ and $\angle RTS$ are adjacent angles that add to 90°.
$\therefore \angle RTS = 90° - 40° = 50°$

6 a 38° **b** 77° **c** 62° **d** 24° **e** 29° **f** 56°
g 53° **h** 21° **i** 61° **j** 43° **k** 32° **l** 45°

7 a $\angle RSV$ and $\angle VST$ are adjacent angles on a straight line.
$\therefore \angle VST = 180° - 125° = 55°$ because adjacent angles on a straight line add to 180°.
b $\angle BCF$, $\angle FCE$ and $\angle ECD$ are adjacent angles on a straight line.
$\therefore \angle FCE = 180° - 50° - 70° = 60°$ because adjacent angles on a straight line add to 180°.

8 a 120° **b** 152° **c** 70° **d** 57°
e 33° **f** 63° **g** 55° **h** 39°
i 110° **j** 50° **k** 108° **l** 111°

Exercise 3D

1 a $62° + 48° = 110°$ $\therefore$ 62° and 48° are not complementary angles because they do not add to 90°.
b $43° + 47° = 90°$ $\therefore$ 43° and 47° are complementary angles because they do add to 90°.

2 a $125° + 55° = 180°$ $\therefore$ 125° and 55° are supplementary angles because they do add to 180°.
b $79° + 111° = 190°$ $\therefore$ 79° and 111° are not supplementary angles because they do not add to 180°.

3 a Yes **b** No **c** Yes **d** No
4 a Yes **b** Yes **c** No **d** Yes
5 a Supplementary **b** Neither
c Supplementary **d** Complementary
e Complementary
6 a Complement of $75° = 90° - 75° = 15°$ because $75° + 15° = 90°$.
b The supplement of $110° = 180° - 110° = 70°$ because $110° + 70° = 180°$.
7 a 50° **b** 80° **c** 18° **d** 120° **e** 165°
f 85° **g** 49° **h** 132° **i** 90°
8 a Right angle **b** Straight angle
9 a Complementary angles **b** Supplementary angles
c Neither **d** Supplementary angles
e Complementary angles **f** Neither

Exercise 3E

1 $\angle ECD = 360° - 120° - 90° - 115° = 35°$ because the sum of angles at a point is 360°.

2 a 127° **b** 35° **c** 154°
d 151° **e** 64° **f** 38°

3 $\angle ZTY = 67°$ because it is vertically opposite $\angle XTW$.

4 a 128° **b** 70° **c** 132°
d 168° **e** 25° **f** 90°

5 a $x = 360° - 100° - 75° - 80° = 105°$ because angles at a point add to 360°.
b $y = 20° + 65° = 85°$ because vertically opposite angles are equal.

6 Reasons include vertically opposite angles and/or the sum of angles at a point.
a $m = 132°$ **b** $q = 110°$
c $b = 78°$ **d** $f = 38°$
e $a = 34°$ **f** $a = 76°$
g $b = 130°, c = 50°$ **h** $k = 114°$
i $l = 112°, j = 17°$ **j** $t = 112°$
k $r = 45°$ **l** $d = 72°$
m $n = 120°$ **n** $x = 90°$
o $f = 46.5°$ **p** $a = 74°$
q $k = 93°$ **r** $e = 61.5°$

7 a $d = 108°, e = 23°$ **b** $f = 125°, g = 55°$
c $x = 89°, y = 28°$ **d** $t = 111°, w = 34.5°$
e $b = 48°, c = 32°$ **f** $a = 47°, b = 51°, c = 82°$
g $l = 16°, m = 35°$ **h** $a = 32°, n = 127°$
i $x = 25°, y = 92°$

Exercise 3F

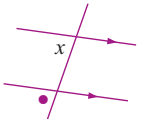
1 a CD, EF **b** KL, LM
c AB, BC and BC, CD **d** PQ, PR
e PT, VR and WS, UQ **f** BA, AD and BC, CD

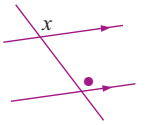
2 a True **b** True **c** False **d** False

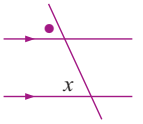
Exercise 3G

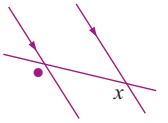
1 a No **b** Yes **c** Yes **d** Yes

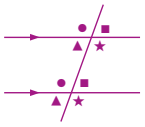
2 a r **b** t **c** r **d** t

3 a


b


c


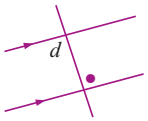
d


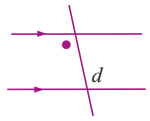
4 **a**  **b** 

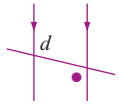
5 **a** $x = 135°$ **b** $p = 92°$ **c** $m = 78°$ **d** $n = 110°$
e $f = 60°$ **f** $k = 27°$ **g** $d = 65°$ **h** $m = 90°$
i $d = 64°, q = 83°$ **j** $t = 90°, v = 114°$
k $g = 98°, m = 56°$ **l** $b = 74°, y = 90°$

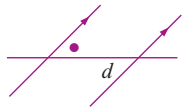
6 **a** No **b** Yes **c** No **d** Yes

7 **a** c **b** d **c** b **d** a

8 **a** 

b 

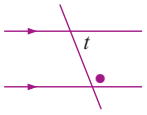
c 

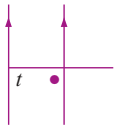
d 

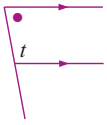
9 **a** $m = 55°$ **b** $p = 86°$ **c** $q = 132°$
d $r = 98°$ **e** $p = 90°$ **f** $b = 122°$
g $g = 48°$ **h** $f = 21°$
i $s = 37°, t = 115°$ **j** $d = 52°, m = 66°$
k $a = 108°, c = 73°$ **l** $n = 90°, p = 70°$

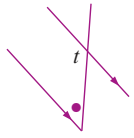
10 **a** No **b** No **c** Yes **d** Yes

11 **a** u **b** s **c** t **d** v

12 **a** 

b 

c 

d 

13 **a** $l = 100°$ **b** $p = 46°$ **c** $m = 125°$
d $q = 108°$ **e** $x = 72°$ **f** $y = 65°$
g $t = 93°$ **h** $a = 48°, b = 109°$
i $x = 133°, y = 52°$ **j** $a = 76°, b = 52°$
k $x = 90°, y = 145°$ **l** $x = 110°, y = 110°$

14 **a** $y = 92°$ alternate angles
b $p = 70°$ co-interior angles
c $y = 97°$ alternate angles
d $m = 120°$ corresponding angles
e $w = 127°$ co-interior angles
f $t = 40°$ alternate angles
g $p = 126°$ corresponding angles, $q = 54°$ co-interior angles, $r = 126°$ alternate angles
h $x = 105°$ co-interior angles, $y = 105°$ co-interior angles
i $m = 70°$ corresponding angles, $n = 70°$ alternate angles
j $a = 112°$ co-interior angles
k $x = 57°$ alternate angles, $y = 56°$ co-interior angles, $z = 67°$ straight angle
l $d = 83°$ corresponding angles, $g = 83°$ alternate angles or vertically opposite angles, $p = 38°$ co-interior angles
m $a = 36°$ alternate angles, $b = 36°$ corresponding angles
n $e = 45°$ co-interior angles, $f = 41°$ alternate angles, $g = 139°$ co-interior angles, $h = 41°$ corresponding angles or straight angle
o $a = 50°$ alternate angles, $b = 130°$ co-interior angles, $c = 50°$ alternate angles or straight angle

15 **a** The given angles are a pair of corresponding angles. Since these angles are equal, the lines are parallel.
b The given angles are a pair of alternate angles. Since these angles are not equal, the lines are not parallel.
c The given angles are a pair of co-interior angles. Since these angles are supplementary, the lines are parallel.

16 **a** Yes, corresponding angles equal
b Yes, alternate angles equal
c Yes, corresponding angles equal
d No, co-interior angles not supplementary
e Yes, alternate angles equal
f Yes, co-interior angles supplementary
g No, alternate angles not equal
h No, corresponding angles not equal

17 **a** Co-interior angles supplementary
b Alternate angles equal
c Corresponding angles equal
d Corresponding angles
e Alternate angles equal
f Co-interior angles supplementary

Exercise 3H

1 **a** **i** $a + 130° = 180°$ (straight angle), $\therefore a = 50°$
ii $b = 130°$ (vertically opposite angles)
iii $c + 130° = 180°$ (straight angle), $\therefore c = 50°$ or $c = a = 50°$ (vertically opposite angles)
b **i** $a + 50° = 180°$ (co-interior angles on $\parallel$ lines), $\therefore a = 130°$
ii $b + 50° = 180°$ (straight angle), $\therefore b = 130°$ or $b = a = 130°$ (alternate angles on $\parallel$ lines)
iii $c = 50°$ (vertically opposite angles) or $c + b = 180°$ (straight angle), $\therefore c = 50°$

2 Other reasons may be possible.
a $a = 135°$ vertically opposite angles, $b = 45°$ straight angle, $c = 45°$ straight angle or vertically opposite angles
b $e = 60°$ straight angle, $f = 60°$ straight angles or vertically opposite angles, $g = 30°$ complementary angles
c $k = 110°$ straight angle, $l = 55°$ straight angle
d $d = 90°$ angles at a point
e $w = 40°$ straight angle, $x = 50°$ vertically opposite angles, $y = 90°$ vertically opposite angles, $z = 40°$ straight angle or vertically opposite angles
f $m = 27°$ vertically opposite angles, $n = 153°$ straight angle, $o = 77°$ angles at a point or straight angle

3 Other reasons may be possible.
a $a = 83°$ alternate angles, $b = 97°$ straight angle, $c = 97°$ co-interior angles, $d = 83°$ corresponding angles, $e = 97°$ vertically opposite angles, $f = 97°$ straight angle, $g = 83°$ vertically opposite angles
b $k = 48°$ co-interior angles, $l = 48°$ vertically opposite angles, $m = 132°$ corresponding angles, $n = 132°$ alternate angles or vertically opposite angles
c $q = 76°$ co-interior angles, $r = 52°$ corresponding angles ($2r = 104°$)
d $j = 38°$ alternate angles, $k = 90°$ corresponding angles, $l = 52°$ straight angle
e $p = 42°$ alternate angles, $q = 67°$ alternate angles, $r = 71°$ straight angle
f $x = 42°$ alternate angles, $y = 72°$ corresponding angles

Exercise 3I

1 $x = y$ (alternate angles, $AB \parallel CD$)
$y = z$ (corresponding angles, $CD \parallel EF$)
$\therefore x = z$ (both equal to y)

2 Other proofs may be possible.
a $\angle EHG = x$ (vertically opposite angles)
$\angle EHG = y$ (corresponding angles)
$\therefore x = y$ (both equal to $\angle EHG$)
b $\angle GHF = y$ (vertically opposite angles)
$\angle GHF = x$ (corresponding angles)
$\therefore x = y$ (both equal to $\angle GHF$)
c $\angle EHG = y$ (vertically opposite angles)
$\angle EHG + x = 180°$ (co-interior angles)
$\therefore \angle EHG = 180° - x$
$\therefore y = 180° - x$ (both equal to $\angle EHG$)

3 Other proofs may be possible.
a $\angle DCB = y$ (corresponding angles, $AD \parallel BC$)
$\angle DCB = x$ (corresponding angles, $AB \parallel DC$)
$\therefore x = y$
b $\angle BDE = y$ (alternate angles, $BC \parallel DE$)
$\angle BDE = x$ (corresponding angles, $CE \parallel BD$)
$\therefore x = y$

4 $\angle PCA = a$ (alternate angles, $PQ \parallel AB$)
$\angle QCB = b$ (alternate angles, $PQ \parallel AB$)
$\angle PCA + c + \angle QCB = 180°$ (straight angle)
$\therefore a + b + c = 180°$

5 $\angle PBC = c$ (alternate angles, $AC \parallel BP$)
$\angle PBQ = a$ (corresponding angles, $AC \parallel BP$)
$\angle PBQ + b + \angle PBC = 180°$ (straight angle)
$\therefore a + b + c = 180°$

6 The sum of the three angles in a triangle is 180°.

7 **a** $w + z = 180°$ (straight angle)
$\therefore z = 180° - w$
$w + x = 180°$ (straight angle)
$\therefore x = 180° - w$
$\therefore x = z$
b $w + x = 180°$ (straight angle)
$\therefore w = 180° - x$
$x + y = 180°$ (straight angle)
$\therefore y = 180° - x$
$\therefore w = y$

8 **a** $w + x = 180°$ (co-interior angles, $BE \parallel CD$)
$\therefore w = 180° - x$
$x + y = 180°$ (co-interior angles, $BC \parallel ED$)
$\therefore y = 180° - x$
$\therefore w = y$
b $w + z = 180°$ (co-interior angles, $BC \parallel ED$)
$\therefore z = 180° - w$
$x + w = 180°$ (co-interior angles, $BE \parallel CD$)
$\therefore x = 180° - w$
$\therefore z = x$

Language in mathematics

1

A	N	G	L	E	S
S	I	Z	E		
R	I	G	H	T	
C	O	R	R	E	S	P	O	N	D	I	N	G
A	D	J	A	C	E	N	T					
A	R	M	S									

2

S	U	P	P	L	E	M	E	N	T	A	R	Y
A	D	D										
P	A	R	A	L	L	E	L					
A	L	T	E	R	N	A	T	E				
P	R	O	T	R	A	C	T	O	R			
M	E	A	S	U	R	E						
O	P	P	O	S	I	T	E					
V	E	R	T	E	X							
O	B	T	U	S	E							

3

L	I	N	E							
R	E	V	O	L	U	T	I	O	N	
A	C	U	T	E						
C	O	I	N	T	E	R	I	O	R	
C	O	M	P	L	E	M	E	N	T	A	R	Y

Check your skills

1 C	**2** D	**7** C	**4** A	**5** B
6 D	**7** A	**8** B	**9** B	**10** C
11 D	**12** B	**13** D	**14** C	**15** C
16 A	**17** B	**18** C	**19** A	**20** B
21 D	**22** C	**23** B		

Review set 3A

1 **a** N **b** D

2 $\angle FGK > 0°$ and $< 90°$

3 **a** Obtuse **b** Straight
c Acute **d** Reflex

4 An angle $> 180°$ but $< 360°$

5 Common arm; common vertex; lie on opposite sides of common arm.

6 **a** 35° **b** 110° **c** 117° **d** 52°

7 **a** $m = 95°$ (adjacent angles on a straight line)
b $q = 72°$ (adjacent angles that add to 90°)
c $x = 85°$ (angles at a point)
d $y = 153°$ (vertically opposite angles)

8 **a** $w = 68°, x = 68°, y = 112°, z = 112°$
b $x = 70°, y = 110°, z = 70°$
c $a = 74°, b = 74°, c = 106°, d = 106°, e = 74°, f = 74°, g = 106°$

9 **a** No, co-interior angles are not supplementary.
b Yes, $48° + 132° = 180°$, so corresponding angles are equal and co-interior angles are supplementary.

Review set 3B

1 **a** **i** $\angle STU$ **ii** $\angle T$
b **i** $\angle DEF$ **ii** $\angle E$

2 $\angle QRS > 180°$ and $< 360°$

3 **a** Right **b** Acute
c Obtuse **d** Reflex

4 One full turn, 360°

5 **a** 40° **b** 130°

6 **a** No **b** Yes

7 **a** $p = 141°$ (angles at a point)
b $s = 65°$ (vertically opposite angles)
c $k = 75°$ (co-interior angles)
d $n = 118°$ (straight angle)

8 **a** $x = 70°, y = 70°, z = 130°$
b $a = 40°, b = 60°, c = 80°, d = 40°, e = 140°$
c $a = 45°, b = 45°, c = 45°, d = 135°, e = 45°, f = 45°$

Review set 3C

1 **a** R **b** Q **c** S **d** K
2 **a** Vertex is P; rays PB, PQ
b Vertex is F; rays FA, FG
c Vertex is I; rays IH, IJ
3 **a** Right **b** Obtuse **c** Reflex **d** Acute
4 **a** Adjacent angles that add to 90°
5 **a** 40° **b** 17° **c** 43° **d** 42°
6 **a** 85° (sum of adjacent angles)
b 116° (sum of adjacent angles on a straight line)
7 **a** $x = 33°$ (vertically opposite angles)
b $x = 118°$ (angles at a point)
c $q = 115°$ (corresponding angles)
d $d = 72°$ (alternate angles)
8 **a** $x = 67°$, $y = 45°$, $z = 112°$
b $v = 55°$, $w = 55°$, $x = 55°$
9 **a** Yes, alternate angles are equal.
b Yes, co-interior angles are supplementary.

CHAPTER 4 NUMBER AND INDICES

Diagnostic test

1 B	**2** D	**3** A	**4** C	**5** B
6 D	**7** D	**8** C	**9** A	**10** B
11 D	**12** D	**13** A	**14** B	**15** B
16 D	**17** A	**18** B	**19** B	**20** A

Exercise 4A

1 **a** 25 **b** 25 **c** five squared
2 **a** 64 **b** 64 **c** eight squared
3 **a** 36 **b** $6^2 = 36$ **c** 6^2
4 **a** 36 **b** 4, 9, 36 **c** Yes
5 **a** Both 144 **b** Both 100 **c** Both 441
6 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
7 **a** $4^3 = 64$ **b** $3^3 = 27$ **c** $6^3 = 216$ **d** $7^3 = 343$
8 Yes
9 **a** $2^5 = 32$ **b** $11^2 = 121$ **c** $5^4 = 625$ **d** $7^1 = 7$
10 **a** 10^1, 10^2, 10^3, 10^4 **b** 6, 10^6

Exercise 4B

1 **a** Yes **b** No **c** Yes **d** Yes
2 **a** No **b** Yes **c** No **d** Yes
3 **a** Yes **b** No **c** No **d** Yes
4 **a** Yes **b** Yes **c** No **d** No
5 **a** Yes **b** No **c** Yes **d** No
6 **a** No **b** No **c** Yes **d** Yes
7 **a** No **b** No **c** No **d** Yes
8 **a** No **b** Yes **c** No **d** Yes
9 **a** Yes **b** No **c** Yes **d** Yes
10 **a** Yes **b** No **c** Yes **d** No
11 2, 3, 4, 6, 8, 9, 12
12 **a** 1, 4, 7 **b** 0, 9
13 **a** 2001 **b** 2004 **c** 2007 **d** 2002 **e** 2004
14 **a** 1, 2, 3, 4, 5, 6, 8, 9, 10, 12
b 1, 2, 3, 4, 6, 9, 11, 12 **c** 1, 2, 5, 10, 11
15 Divisible by 3 and 5 **a** Yes **b** No **c** No
16 **a** Yes **b** No **c** Yes **d** Yes

Exercise 4C

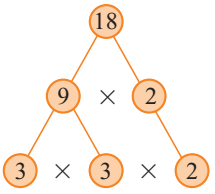
1 **a** 25 **b** 42 **c** 26 **d** 38
e 28 **f** 19 **g** 15 **h** 38
2 **a** $47\frac{6}{15}$ **b** $33\frac{2}{21}$ **c** $24\frac{8}{33}$ **d** $37\frac{4}{7}$
e $33\frac{25}{28}$ **f** $26\frac{27}{37}$ **g** $15\frac{35}{43}$ **h** $35\frac{7}{18}$

Exercise 4D

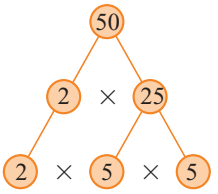
1 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
2 **a** 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
b 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
c 8, 16, 24, 32, 40, 48, 56, 64, 72, 80
d 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
e 11, 22, 33, 44, 55, 66, 77, 88, 99, 110
3 24, 30, 36, 42, 48, 54
4 21, 28, 35, 42, 49, 56
5 9, 18, 27, 36, 45, 54
6 **a** 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48
b 5, 10, 15, 20, 25, 30, 35, 40, 45
c 15, 30, 45 **d** 15
7 **a** 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
b 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
c 12
8 **a** 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
7, 14, 21, 28, 35, 42, 49, 56, 63, 70, LCM = 35
b 8, 16, 24, 32, 40, 48, 56, 64, 72, 80
6, 12, 18, 24, 30, 36, 42, 48, 54, 60, LCM = 24
c 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
6, 12, 18, 24, 30, 36, 42, 48, 54, 60, LCM = 18
9 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
10 **a** 1, 2, 5, 10 **b** 1, 2, 3, 6, 9, 18 **c** 1, 13
d 1, 2, 3, 5, 6, 10, 15, 30 **e** 1, 2, 4, 5, 10, 20
11 **a** 1, 2, 3, 4, 6, 12 **b** 1, 2, 3, 4, 6, 9, 12, 18, 36
c 1, 2, 3, 4, 6, 12 **d** 12
12 **a** 1, 2, 3, 5, 6, 10, 15, 30 **b** 1, 3, 5, 9, 15, 45
c 1, 3, 5, 15 **d** 15
13 **a** 1, 2, 3, 4, 6, 12 **b** 1, 2, 3, 6, 9, 18 **c** 6
14 **a** 1, 2, 4, 5, 10, 20 **b** 1, 2, 3, 5, 6, 10, 15, 30 **c** 10
15 **a–c** 1 and 18, 2 and 9, 2 and 6. They are any two of the factors of 18.
d 1 and 18, 2 and 9 **e** Some pairs have a smaller LCM.
16 **a–c** Any two factors of 30 will have a common multiple of 30.
d 1 and 30, 2 and 15, 3 and 10, 5 and 6
e Some pairs have a smaller LCM.
17 **a–b** Any two multiples of 8. Examples: 8 and16, 48 and 200
c–d Not possible, because the number of multiples of 8 is unlimited
18 6 and 90, 18 and 30
19 **a** 12 and 20, 4 and 60 **b** 16 and 24, 8 and 48
c 3 and 180, 9 and 60, 12 and 45, 15 and 36
d 90 and 5400, 270 and 1800, 360 and 1350, 450 and 1080
20 **a** Prime **b** Composite **c** Composite
d Composite **e** Prime **f** Prime
g Composite **h** Prime **i** Composite
j Prime
21 **a** 1 **b** It has only one factor: itself.
c 1, 2 **d** Prime
e It is the only even prime.
f Odd
g It is even. Its factors are only itself and 1, unlike all other even numbers.
22 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30

Exercise 4E

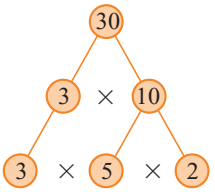
1 a



b



c



2 a $2 \times 2 \times 3 \times 5$ **b** $2 \times 3 \times 3 \times 5$
c $2 \times 2 \times 5 \times 7$ **d** $2 \times 2 \times 3 \times 7$
e $2 \times 3 \times 3 \times 7$ **f** $2 \times 2 \times 3 \times 3 \times 7$
g $3 \times 3 \times 5 \times 7$ **h** $2 \times 2 \times 2 \times 2 \times 7 \times 7$
i $2 \times 2 \times 3 \times 3 \times 7 \times 7$
j $2 \times 2 \times 2 \times 5 \times 5 \times 11$

3 a $3 \times 3 \times 7$ **b** $2 \times 2 \times 2 \times 17$
c $3 \times 5 \times 13$ **d** $3 \times 3 \times 3 \times 3 \times 5$
e $2 \times 2 \times 2 \times 2 \times 3 \times 11$
f $2 \times 2 \times 3 \times 13$
g $2 \times 2 \times 2 \times 3 \times 3 \times 7$
h $2 \times 2 \times 2 \times 3 \times 3 \times 3$
i $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 11$
j $2 \times 2 \times 2 \times 5 \times 5 \times 13$

4 $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7 \times 13$

Exercise 4F

1 a $30 = 2 \times 3 \times 5$; $42 = 2 \times 3 \times 7$
b 6 **c** 210

2 a $140 = 2 \times 2 \times 5 \times 7$; $90 = 2 \times 3 \times 3 \times 5$
b 10 **c** 1260

3

Prime factors	Index form	Number
$2 \times 2 \times 3 \times 5 \times 7$	$2^2 \times 3 \times 5 \times 7$	420
$2 \times 2 \times 3 \times 3 \times 5 \times 11$	$2^2 \times 3^2 \times 5 \times 11$	1980
$2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7$	$2^4 \times 5^2 \times 7$	2800
$3 \times 5 \times 7 \times 11 \times 11$	$3 \times 5 \times 7 \times 11^2$	12 705
$3 \times 5 \times 5 \times 5 \times 7 \times 7$	$3 \times 5^3 \times 7^2$	18 375
$2 \times 3 \times 3 \times 3 \times 5 \times 11 \times 11$	$2 \times 3^3 \times 5 \times 11^2$	32 670

4 a 60 **b** 165 **c** 35 **d** 30 **e** 15
f 1815 **g** 140 **h** 105 **i** 990

5 a $2^2 \times 3^2 \times 5 \times 7 \times 11$ **b** $2^2 \times 3^2 \times 5 \times 7 \times 11^2$
c $2^4 \times 3 \times 5^2 \times 7 \times 11^2$ **d** $2^2 \times 3^3 \times 5 \times 7 \times 11^2$
e $2^2 \times 3^2 \times 5^3 \times 7^2 \times 11$ **f** $2 \times 3^3 \times 5 \times 7 \times 11^2$
g $2^4 \times 3 \times 5^2 \times 7$ **h** $3 \times 5^3 \times 7^2 \times 11^2$
i $2^2 \times 3^3 \times 5 \times 11^2$

6 a 4, $2^2 \times 3 \times 7 \times 17$ **b** 15, $3^2 \times 5 \times 7 \times 13$
c 5, $2^2 \times 3^4 \times 5 \times 7$ **d** 12, $2^4 \times 3^2 \times 7 \times 11$
e 126, $2^3 \times 3^2 \times 7$
f 8, $2^4 \times 3 \times 5 \times 7 \times 11 \times 31$

7 a 4 and 8 and others **b** 3 and 6 and others
c One is the multiple of the other.

8 a 2 and 8 and others
b 3 and 6 and others
c One is a multiple of the other.

Exercise 4G

1 a 3 **b** 5 **c** 4 **d** 7 **e** 10
2 a 7 **b** 10 **c** 9 **d** 8 **e** 2
3 a 6 **b** 2, 3, 6 **c** Yes
4 a Yes **b** Yes **c** Yes
5 a 3 **b** 4 **c** 5 **d** 10
6 a 3 **b** 5 **c** 6 **d** 9
7 Yes

8 a $196 = 2 \times 2 \times 7 \times 7 = 2^2 \times 7^2$
$\sqrt{196} = 2 \times 7 = 14$
b $100 = 2 \times 2 \times 5 \times 5 = (2 \times 5) \times (2 \times 5)$
$\sqrt{100} = (2 \times 5) = 10$
c $225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$
$\sqrt{225} = 3 \times 5 = 15$
d $900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 = (2 \times 3 \times 5) \times (2 \times 3 \times 5)$
$\sqrt{900} = (2 \times 3 \times 5) = 30$

9 a 21 **b** 75 **c** 99 **d** 36

10 a 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000
b 2
c i 3 **ii** 1 **iii** 5 **iv** 4

11 a 9 **b** 8 **c** 7 **d** 14 **e** 15
f 17 **g** $(2 \times 3) = 6$ **h** $(2 \times 5) = 10$

12 a 14 **b** 22 **c** 15 **d** 30 **e** 70
f 12 **g** 36 **h** 600 **i** 648

13 a 9, 16 **b** 3, 4 **c** 3 and 4 **d** 3.61

14 a $24 = 2^3 \times 3$
b Its prime factors are not squares.
c 4 and 5, as $4^2 = 16$ and $5^2 = 25$
d 4.9

15 a 5 and 6 **b** 6 and 7 **c** 3 and 4
d 7 and 8 **e** 8 and 9

16 a $150 = 2 \times 3 \times 5^2$. Its prime factors are not cubes.
b Yes, as $5^3 = 125$ and $6^3 = 216$
c 12.2

17 a 1 and 2 **b** 3 and 4 **c** 3 and 4
d 4 and 5 **e** 6 and 7 **f** 7 and 8

Exercise 4H

1 a 45 **b** 50 **c** 15 **d** 24 **e** 31
f 3 **g** 27 **h** 61 **i** 29 **j** 24
k 100 **l** 27 **m** 3 **n** 22 **o** 50

2 a 4 **b** 10 **c** 12 **d** 45 **e** 3
f 121 **g** 5 **h** 2 **i** 2 **j** 9
k 27 **l** 25 **m** 10 **n** 2 **o** 3
p 4 **q** 6 **r** 40 **s** 16 **t** 2

3 a 21 **b** 16 **c** 18 **d** 3 **e** 15
f 17 **g** 8 **h** 11 **i** 14 **j** 4
k 2 **l** 2 **m** 68 **n** 59 **o** 25
p 5

4 a 14 **b** 10 **c** No, $\sqrt{\ }$ is a grouping symbol.

5 No, $13 \neq 12 + 5$

Calculator activities

1 a 108 **b** 56 **c** 194
d 151 **e** 2226 **f** 89
g 42 875 **h** 492 **i** 6181
j 8905 **k** 8 595 645 **l** 583
m 3282 **n** 6601

2 a 3 and 4, 3.16 **b** 4 and 5, 4.24
c 6 and 7, 6.32 **d** 11 and 12, 11.40
e 2 and 3, 2.71 **f** 3 and 4, 3.80
g 4 and 5, 4.64 **h** 5 and 6, 5.19

3 **a** 13, 26, 39, 52, 65, 78, 91, 104, 117, 130, 143, 156, 169, 182, 195, 208, 221, 234, 247, 260
b 14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196, 210, 224, 238, 252, 266, 280
c 23, 46, 69, 92, 115, 138, 161, 184, 207, 230, 253, 276, 299, 322, 345, 368, 391, 414, 437, 460
d 28, 56, 84, 112, 140, 168, 196, 224, 252, 280, 308, 336, 364, 392, 420, 448, 476, 504, 532, 560
e 46, 92, 138, 184, 230, 276, 322, 368, 414, 460, 506, 552, 598, 644, 690, 736, 782, 828, 874, 920

4 **a** 713 **b** 251 **c** 526 **d** 7332
e 6058 **f** 44 064 **g** 1198 **h** 3621

5 **a** -8 **b** 16 **c** -32 **d** 64
e 9 **f** -27 **g** 81 **h** -243

6 **a** Positive **b** Odd

Language in mathematics

H	I	U	O	D	N	X	I	C	C	A	N	O	B	I	F
P	C	N	U	M	B	E	R	S	N	N	R	U	B	F	F
L	U	I	T	U	T	D	V	T	F	P	G	E	Y	D	S
A	B	Q	U	L	C	N	L	E	R	A	D	I	C	A	L
N	I	U	A	T	E	I	C	P	N	T	C	N	S	M	F
I	C	E	E	I	F	E	O	N	O	T	A	T	I	O	N
D	D	O	E	P	R	I	M	E	E	E	M	S	O	A	Q
R	N	R	R	L	E	H	P	K	F	R	V	Z	X	R	E
A	C	U	B	E	P	C	O	D	E	N	A	E	E	A	S
C	X	I	U	S	Z	R	S	T	K	S	D	Q	I	V	K
D	I	V	I	S	I	B	I	L	I	T	Y	E	S	S	Q
L	O	A	C	O	U	N	T	I	N	G	P	N	F	H	X
E	T	S	U	O	I	V	E	R	P	T	R	E	B	K	D
N	A	S	G	S	E	N	E	H	T	S	O	T	A	R	E
A	U	E	D	K	G	I	D	B	L	H	B	K	F	J	X

Check your skills

1 B **2** D **3** A **4** D **5** C
6 A **7** D **8** B **9** D **10** A
11 B **12** B **13** B **14** C **15** B
16 C **17** A **18** B **19** C **20** B

Review set 4A

1 **a** 12, 18, 24, 30 **b** 24, 32, 40, 48, 56, 64
c 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55

2 **a** 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48
b 6, 12, 18, 24, 30, 36, 42, 48
c 12, 24, 36, 48 **d** 12

3 **a** 1, 2, 3, 4, 6, 8, 12, 24
b 1, 2, 3, 4, 6, 9, 12, 18, 36 **c** 12

4 **a** $40 = 2^3 \times 5$ **b** $120 = 2^3 \times 3 \times 5$
c $352 = 2^5 \times 11$

5 **a** $80 = 2^4 \times 5$ **b** $154 = 2 \times 7 \times 11$
c $440 = 2^3 \times 5 \times 11$

6 **a** $24 = 2^3 \times 3$; $40 = 2^3 \times 5$
b $2^3 = 8$ **c** $2^3 \times 3 \times 5 = 120$

7 **a** $180 = 2^2 \times 3^2 \times 5$; $120 = 2^3 \times 3 \times 5$
b $2^2 \times 3 \times 5 = 60$
c $2^3 \times 3^2 \times 5 = 360$

8 **a** $84 = 2^2 \times 3 \times 7$; $63 = 3^2 \times 7$; HCF $= 3 \times 7 = 21$; LCM $= 2^2 \times 3^2 \times 7 = 252$
b $75 = 3 \times 5^2$; $40 = 2^3 \times 5$; HCF $= 5$; LCM $= 2^3 \times 3 \times 5^2 = 600$
c $240 = 2^4 \times 3 \times 5$; $400 = 2^4 \times 5^2$; HCF $= 2^4 \times 5 = 80$; LCM $= 2^4 \times 3 \times 5^2 = 1200$

9 **a** 4 **b** 8 **c** 11 **d** 4 **e** 5 **f** 6

10 **a** 2, 3, 5, 6, 10 **b** 2, 4, 5, 8, 10
c 2, 3, 4, 5, 6, 9, 10, 11, 12

11 7 and 8

12 **a** 34 **b** $18\frac{8}{32}$ $(18\frac{1}{4})$

13 **a** 2 **b** 88 **c** 5

Review set 4B

1 **a** 24, 28, 32, 36, 40, 44, 48
b 21, 28, 35, 42, 49, 56, 63, 70
c 9, 18, 27, 36, 45, 54, 63

2 **a** 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
b 12, 24, 36, 48, 60, 72, 84, 96
c 36, 72 **d** 36

3 **a** 1, 2, 3, 6, 9, 18 **b** 1, 2, 3, 5, 6, 10, 15, 30 **c** 6

4 **a** $60 = 2^2 \times 3 \times 5$ **b** $100 = 2^2 \times 5^2$
c $420 = 2^2 \times 3 \times 5 \times 7$

5 **a** $90 = 2 \times 3^2 \times 5$ **b** $168 = 2^3 \times 3 \times 7$
c $480 = 2^5 \times 3 \times 5$

6 **a** $36 = 2^2 \times 3^2$; $60 = 2^2 \times 3 \times 5$
b $2^2 \times 3 = 12$ **c** $2^2 \times 3^2 \times 5 = 180$

7 **a** $140 = 2^2 \times 5 \times 7$; $200 = 2^3 \times 5^2$
b $2^2 \times 5 = 20$ **c** $2^3 \times 5^2 \times 7 = 1400$

8 **a** $40 = 2^3 \times 5$; $96 = 2^5 \times 3$; HCF $= 2^3 = 8$; LCM $= 2^5 \times 3 \times 5 = 480$
b $70 = 2 \times 5 \times 7$; $100 = 2^2 \times 5^2$; HCF $= 2 \times 5 = 10$; LCM $= 2^2 \times 5^2 \times 7 = 700$
c $210 = 2 \times 3 \times 5 \times 7$; $360 = 2^3 \times 3^2 \times 5$; HCF $= 2 \times 3 \times 5 = 30$; LCM $= 2^3 \times 3^2 \times 5 \times 7 = 2520$

9 **a** 5 **b** 9 **c** 12 **d** 2 **e** 3 **f** 7

10 **a** 2, 3, 5, 6, 10 **b** 2, 3, 4, 5, 6, 10, 12
c 2, 4, 8, 11

11 7 and 8

12 **a** 38 **b** $30\frac{9}{24}$ $(30\frac{3}{8})$

13 **a** 50 **b** 200 **c** Order of operations

CHAPTER 5 FRACTIONS

Diagnostic test

1 C **2** A **3** C **4** B **5** B
6 C **7** A **8** D **9** D **10** B
11 B **12** C **13** B **14** A **15** C

Exercise 5A

1 **a** Proper **b** Mixed numeral **c** Improper
d Improper **e** Mixed numeral **f** Improper
g Improper **h** Proper **i** Proper
j Mixed numeral **k** Improper **l** Improper

2 **a** $4\frac{2}{3}$ **b** $7\frac{3}{10}$ **c** $4\frac{1}{5}$

3 **a** $3\frac{1}{3}$ **b** $7\frac{1}{2}$ **c** $1\frac{5}{6}$ **d** $4\frac{3}{4}$ **e** $3\frac{7}{8}$
f $7\frac{2}{5}$ **g** $5\frac{6}{7}$ **h** $7\frac{1}{4}$ **i** $5\frac{1}{9}$ **j** $5\frac{7}{10}$

4 **a** $\frac{13}{4}$ **b** $\frac{13}{5}$ **c** $\frac{11}{2}$

5 **a** $\frac{9}{4}$ **b** $\frac{10}{3}$ **c** $\frac{12}{7}$ **d** $\frac{38}{9}$ **e** $\frac{44}{5}$
f $\frac{43}{6}$ **g** $\frac{27}{5}$ **h** $\frac{28}{15}$ **i** $\frac{46}{7}$ **j** $\frac{29}{10}$

6 a $\frac{8}{4}\ \frac{9}{4}\ \frac{10}{4}\ \frac{11}{4}\ \frac{12}{4}\ \frac{13}{4}\ \frac{14}{4}\ \frac{15}{4}\ \frac{16}{4}\ \frac{17}{4}\ \frac{18}{4}\ \frac{19}{4}\ \frac{20}{4}\ \frac{21}{4}\ \frac{22}{4}\ \frac{23}{4}\ \frac{24}{4}$

$2\ 2\frac{1}{4}\ 2\frac{2}{4}\ 2\frac{3}{4}\ 3\ 3\frac{1}{4}\ 3\frac{2}{4}\ 3\frac{3}{4}\ 4\ 4\frac{1}{4}\ 4\frac{2}{4}\ 4\frac{3}{4}\ 5\ 5\frac{1}{4}\ 5\frac{2}{4}\ 5\frac{3}{4}\ 6$

b i $4\frac{3}{4}$ **ii** $5\frac{3}{4}$ **iii** $4\frac{2}{4}$ **iv** $3\frac{2}{4}$ **v** $2\frac{2}{4}$

c i $\frac{9}{4}, 3\frac{1}{4}, \frac{16}{4}, 4\frac{2}{4}$ **ii** $2\frac{1}{4}, \frac{13}{4}, 5, \frac{22}{4}$

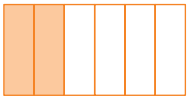
d i $6, \frac{20}{4}, \frac{18}{4}, 3\frac{3}{4}, \frac{11}{4}$ **ii** $\frac{23}{4}, 5, \frac{19}{4}, 3\frac{3}{4}, 2\frac{1}{4}$

7 a 4 **b** 8 **c** 7 **d** 28 **e** 24 **f** 72

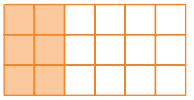
Exercise 5B

1 a $\frac{1}{2}, \frac{4}{8}$ **b** $\frac{3}{6}, \frac{1}{2}$

2 a



b



3 a $\frac{2}{6}$ **b** $\frac{6}{18}$

4 a $\frac{6}{12}$ **b** $\frac{12}{27}$ **c** $\frac{24}{44}$ **d** $\frac{6}{14}$ **e** $\frac{15}{40}$ **f** $\frac{6}{9}$

5 a $\frac{9}{12}$ **b** $\frac{5}{10}$ **c** $\frac{12}{21}$ **d** $\frac{15}{25}$ **e** $\frac{21}{36}$ **f** $\frac{5}{55}$

g $\frac{12}{102}$ **h** $\frac{81}{90}$ **i** $\frac{20}{26}$ **j** $\frac{18}{21}$ **k** $\frac{24}{66}$ **l** $\frac{39}{45}$

m $\frac{36}{135}$ **n** $\frac{88}{99}$ **o** $\frac{51}{60}$ **p** $\frac{60}{110}$

6 a $\frac{2}{3}$ **b** $\frac{3}{11}$ **c** $\frac{5}{6}$ **d** $\frac{2}{5}$ **e** $\frac{4}{9}$ **f** $\frac{2}{9}$

7 a $\frac{2}{3}$ **b** $\frac{4}{7}$ **c** $\frac{1}{3}$ **d** $\frac{3}{10}$ **e** $\frac{1}{4}$ **f** $\frac{2}{9}$

g $\frac{2}{7}$ **h** $\frac{3}{8}$ **i** $\frac{12}{13}$ **j** $\frac{10}{11}$ **k** $\frac{8}{11}$ **l** $\frac{10}{13}$

8 a 5, 10, 15, 20, …
2, 4, 6, 8, 10, 12, 14, … LCM = 10

b 7, 14, 21, 28, 35, 42, …
6, 12, 18, 24, 30, 36, 42, … LCM = 42

9 a 6 **b** 28 **c** 56 **d** 45
e 30 **f** 12 **g** 24 **h** 6
i 15 **j** 9 **k** 20 **l** 16

10 a 4, 8, 12, 16, 20, 24, 28, 32, 36, …
9, 18, 27, 36, 45, … LCD = 36
$\times 9, \frac{27}{36}$ and $\times 4, \frac{28}{36}$

b 7, 14, 21, 28, 35, …
5, 10, 15, 20, 25, 30, 35, … LCD = 35
$\times 5, \frac{130}{35}$ and $\times 7, \frac{119}{35}$

11 a $\frac{32}{40}$ and $\frac{35}{40}$ **b** $\frac{20}{90}$ and $\frac{27}{90}$ **c** $\frac{77}{42}$ and $\frac{12}{42}$

d $\frac{5}{12}$ and $\frac{20}{12}$ **e** $\frac{35}{50}$ and $\frac{6}{50}$ **f** $\frac{25}{30}$ and $\frac{8}{30}$

12 a $1\frac{1}{3} = \frac{20}{15}, 1\frac{2}{5} = \frac{21}{15}$ **b** $2\frac{5}{6} = \frac{34}{12}, 1\frac{1}{4} = \frac{15}{12}$

c $1\frac{2}{7} = \frac{27}{21}, 2\frac{1}{3} = \frac{49}{21}$ **d** $1\frac{3}{4} = \frac{7}{4}, 1\frac{1}{2} = \frac{6}{4}$

e $1\frac{4}{5} = \frac{18}{10}, 2\frac{1}{2} = \frac{25}{10}$ **f** $2\frac{1}{6} = \frac{13}{6}, 4\frac{1}{3} = \frac{26}{6}$

13 a $\times 5, \frac{10}{15}$
$\frac{10}{15} > \frac{8}{15}$
$\therefore \frac{2}{3} > \frac{8}{15}$

b $\times 3, \frac{3}{12}$
$\frac{3}{12} < \frac{5}{12}$
$\therefore \frac{1}{4} < \frac{5}{12}$

14 a > **b** < **c** > **d** =
e < **f** > **g** = **h** >
i > **j** > **k** > **l** =

Exercise 5C

1 a i 20: 1, 2, 4, 5, 10, 20 and 25: 1, 5, 25
ii HCF = 5 **iii** $\frac{4}{5}$

b i 12: 1, 2, 3, 4, 6, 12 and 30: 1, 2, 3, 5, 6, 10, 15, 30
ii HCF = 6 **iii** $\div 6, \frac{2}{5}$

2 a 8: 1, 2, 4, 8 **b** 10: 1, 2, 5, 10
c 12: 1, 2, 3, 4, 6, 12 **d** 15: 1, 3, 5, 15
e 20: 1, 2, 4, 5, 10, 20 **f** 24: 1, 2, 3, 4, 6, 8, 12, 24
g 16: 1, 2, 4, 8, 16 **h** 14: 1, 2, 7, 14
i 21: 1, 3, 7, 21
j 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
k 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
l 100: 1, 2, 4, 5, 10, 20, 25, 50, 100
m 9: 1, 3, 9 **n** 54: 1, 2, 3, 6, 9, 18, 27, 54
o 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

3 a 2 **b** 3 **c** 5 **d** 7 **e** 12
f 9 **g** 20 **h** 6 **i** 20 **j** 13

4 a $\frac{4}{5}$ **b** $\frac{4}{5}$ **c** $\frac{3}{4}$ **d** $\frac{2}{3}$ **e** $\frac{2}{3}$

f $\frac{1}{6}$ **g** $\frac{3}{5}$ **h** $\frac{8}{9}$ **i** $\frac{1}{5}$ **j** $\frac{3}{4}$

5 a $\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $\frac{2}{3}$ **d** $\frac{6}{7}$ **e** $\frac{5}{6}$

f $\frac{2}{5}$ **g** $\frac{1}{3}$ **h** $\frac{7}{8}$ **i** $\frac{3}{4}$ **j** $\frac{1}{4}$

k $\frac{1}{3}$ **l** $\frac{3}{4}$ **m** $\frac{1}{4}$ **n** $\frac{1}{4}$ **o** $\frac{8}{11}$

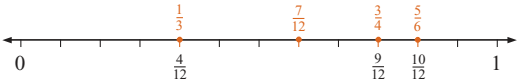
6 a $A = \frac{1}{5}, B = \frac{3}{5}, C = \frac{5}{5} = 1$

b $A = \frac{4}{16} = \frac{1}{4}, B = \frac{8}{16} = \frac{1}{2}, C = \frac{12}{16} = \frac{3}{4}, D = \frac{15}{16},$
$E = \frac{1}{15}, F = \frac{6}{15} = \frac{2}{5}, G = \frac{13}{15}$

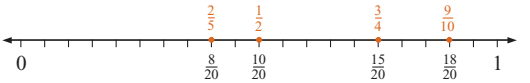
c $A = \frac{1}{7}, B = \frac{4}{7}, C = \frac{6}{7}, D = \frac{7}{14} = \frac{1}{2}$

d $A = \frac{0}{9} = 0, B = \frac{6}{9} = \frac{2}{3}, C = \frac{8}{9}, D = \frac{9}{18} = \frac{1}{2}$

7 a LCM = 12 $\therefore \frac{3}{4} = \frac{9}{12}, \frac{1}{3} = \frac{4}{12}, \frac{7}{12}, \frac{5}{6} = \frac{10}{12}$

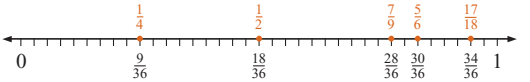


b LCM = 20 $\therefore \frac{1}{2} = \frac{10}{20}, \frac{3}{4} = \frac{15}{20}, \frac{2}{5} = \frac{8}{20}, \frac{9}{10} = \frac{18}{20}$



c LCM = 30 $\therefore \frac{2}{3} = \frac{20}{30}, \frac{4}{5} = \frac{24}{30}, \frac{5}{6} = \frac{25}{30}, \frac{1}{10} = \frac{3}{30}$

d LCM = 36 $\therefore \frac{7}{9} = \frac{28}{36}, \frac{1}{2} = \frac{18}{36}, \frac{5}{6} = \frac{30}{36}, \frac{17}{18} = \frac{34}{36}, \frac{1}{4} = \frac{9}{36}$



Exercise 5D

1 a $\frac{1}{3}$ **b** $\frac{3}{25}$ **c** $\frac{1}{9}$ **d** $\frac{1}{3}$

e $\frac{5}{12}$ **f** $\frac{3}{4}$ **g** $\frac{13}{30}$ **h** $\frac{1}{2}$

2 a $\frac{4}{9}$ **b** $\frac{7}{11}$ **c** $\frac{2}{9}$ **d** $\frac{9}{14}$

e $\frac{25}{35} = \frac{5}{7}$ **f** $\frac{3}{6} = \frac{1}{2}$ **g** $\frac{4}{18} = \frac{2}{9}$ **h** $\frac{6}{21} = \frac{2}{7}$

3 **a** $\frac{20}{60} = \frac{1}{3}$ **b** $\frac{40}{120} = \frac{1}{3}$ **c** $\frac{80}{200} = \frac{2}{5}$
d $\frac{30}{90} = \frac{1}{3}$ **e** $\frac{600}{1000} = \frac{3}{5}$ **f** $\frac{900}{1600} = \frac{9}{16}$
g $\frac{840}{1400} = \frac{3}{5}$ **h** $\frac{720}{2100} = \frac{12}{35}$ **i** $\frac{64}{122} = \frac{32}{61}$
j $\frac{40}{135} = \frac{8}{27}$ **k** $\frac{120}{480} = \frac{1}{4}$ **l** $\frac{35}{85} = \frac{7}{17}$

4 **a** $\frac{11}{20}$ **b** $\frac{4}{7}$ **c** $\frac{5}{12}$ **d** $\frac{1}{6}$ **e** $\frac{3}{7}$
f $\frac{1}{30}$ **g** $\frac{1}{5}$ **h** $\frac{1}{100}$ **i** $\frac{12}{25}$ **j** $\frac{1}{4}$

Exercise 5E

1 **a** $\frac{11}{13}$ **b** $\frac{5}{17}$ **c** $\frac{1}{5}$ **d** $\frac{10}{14} = \frac{5}{7}$

2 **a** $\frac{9}{10}$ **b** $\frac{6}{7}$ **c** $\frac{7}{12}$ **d** $\frac{8}{9}$
e $\frac{2}{6} = \frac{1}{3}$ **f** $\frac{12}{15} = \frac{4}{5}$ **g** $\frac{14}{17}$ **h** $\frac{15}{18} = \frac{5}{6}$
i $\frac{10}{20} = \frac{1}{2}$ **j** $\frac{6}{10} = \frac{3}{5}$ **k** $\frac{4}{18} = \frac{2}{9}$ **l** $\frac{16}{32} = \frac{1}{2}$
m $\frac{7}{21} = \frac{1}{3}$ **n** $\frac{17}{43}$ **o** $\frac{15}{57} = \frac{5}{19}$ **p** $\frac{3}{20}$

3 **a** LCD = 20
$\times 5$, $\frac{5}{20}$ and $\times 4$, $\frac{8}{20}$
$\frac{5}{20} + \frac{8}{20} = \frac{13}{20}$

b LCD = 6
$\times 2$, $\frac{4}{6}$ and $\times 3$, $\frac{3}{6}$
$\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$

4 **a** $\frac{5}{6}$ **b** $\frac{11}{15}$ **c** $\frac{19}{28}$ **d** $\frac{38}{45}$
e $\frac{37}{56}$ **f** $\frac{26}{35}$ **g** $\frac{5}{6}$ **h** $\frac{9}{14}$
i $\frac{5}{12}$ **j** $\frac{1}{2}$ **k** $\frac{1}{2}$ **l** $\frac{1}{6}$
m $\frac{19}{30}$ **n** $\frac{1}{6}$ **o** $\frac{2}{9}$ **p** $\frac{4}{15}$

5 **a** $1\frac{1}{4}$ **b** $1\frac{1}{15}$ **c** $1\frac{2}{9}$ **d** $1\frac{7}{12}$

6 **a** $\frac{5}{8}$ **b** $\frac{3}{8}$

7 **a** Alicia **b** bank **c** $\frac{1}{20}$

8 **a** $3 + (1 - \frac{1}{3})$
$= 3 + \frac{2}{3} = 3\frac{2}{3}$

b $1 + (1 - \frac{4}{5})$
$= 1 + \frac{1}{5} = 1\frac{1}{5}$

9 **a** $9\frac{5}{9}$ **b** $6\frac{1}{7}$ **c** $7\frac{2}{5}$ **d** $8\frac{1}{3}$
e $2\frac{1}{9}$ **f** $11\frac{9}{11}$ **g** $5\frac{1}{5}$ **h** $4\frac{1}{4}$
i $3\frac{1}{8}$ **j** $1\frac{7}{10}$ **k** $16\frac{4}{15}$ **l** $17\frac{3}{17}$

10 **a** $3\frac{1}{3} + 2\frac{2}{5} = \frac{10}{3} + \frac{12}{5}$ LCD = 15
$\frac{10}{3} = \frac{50}{15}$ and $\frac{12}{5} = \frac{36}{15}$
$\frac{50}{15} + \frac{36}{15} = \frac{86}{15} = 5\frac{11}{15}$

b $4\frac{1}{4} - 1\frac{1}{2} = \frac{17}{4} - \frac{3}{2}$ LCD = 4
$\frac{3}{2} = \frac{6}{4}$
$\frac{17}{4} - \frac{6}{4} = \frac{11}{4} = 2\frac{3}{4}$

11 **a** $5\frac{3}{4}$ **b** $8\frac{9}{20}$ **c** $5\frac{1}{6}$ **d** $10\frac{5}{12}$
e $7\frac{11}{15}$ **f** $6\frac{1}{3}$ **g** $8\frac{5}{24}$ **h** $9\frac{29}{35}$
i $1\frac{3}{4}$ **j** $3\frac{14}{15}$ **k** $4\frac{1}{6}$ **l** $2\frac{7}{30}$
m $1\frac{5}{6}$ **n** $3\frac{13}{35}$ **o** $4\frac{1}{18}$ **p** $3\frac{1}{12}$

12 **a** $9\frac{2}{15}$ **b** $1\frac{13}{15}$

13 **a** $\frac{17}{24}$ **b** $\frac{7}{24}$

Exercise 5F

1 **a** $\frac{7}{2}$ or $3\frac{1}{2}$ **b** $\frac{3}{20}$ **c** $\frac{7}{15}$

2 **a** $\frac{16}{9}$ or $1\frac{7}{9}$ **b** $\frac{35}{8}$ or $4\frac{3}{8}$ **c** $\frac{36}{5}$ or $7\frac{1}{5}$
d $\frac{6}{7}$ **e** $\frac{7}{4}$ or $1\frac{3}{4}$ **f** 2
g $\frac{20}{7}$ or $2\frac{6}{7}$ **h** $\frac{48}{11}$ or $4\frac{4}{11}$ **i** $\frac{4}{9}$
j $\frac{4}{5}$ **k** $\frac{1}{7}$ **l** $\frac{8}{15}$
m $\frac{12}{35}$ **n** $\frac{1}{54}$ **o** $\frac{7}{12}$

3 **a** $\frac{11}{2} \times \frac{4}{3} = \frac{44}{6}$
$= 7\frac{2}{6} = 7\frac{1}{3}$

b $\frac{22}{5} \times \frac{3}{1} = \frac{66}{5}$
$= 13\frac{1}{5}$

4 **a** $5\frac{1}{10}$ **b** $14\frac{1}{4}$ **c** $6\frac{3}{5}$ **d** $19\frac{1}{3}$ **e** $4\frac{7}{27}$ **f** $12\frac{1}{7}$
g $4\frac{1}{3}$ **h** $15\frac{3}{14}$ **i** 50 **j** $5\frac{1}{4}$ **k** $6\frac{1}{3}$ **l** $17\frac{3}{5}$

5 $17\frac{1}{2}$ L **6** 21 m

7 **a** 12 m **b** 16 kg **c** 80 L **d** 30
e \$10 **f** 33 km **g** \$36 **h** 60 kg
i 126 L **j** 40 **k** 16 m **l** 640

8 **a** $1\frac{1}{2}$ m **b** $\frac{3}{4}$ m

9 **a** 375 mL **b** 225 mL

Exercise 5G

1 **a** $\frac{8}{5}$ **b** $\frac{9}{4}$ **c** $\frac{13}{12}$ **d** $\frac{7}{9}$ **e** $\frac{1}{5}$ **f** $\frac{2}{1}$

2 **a** $\frac{7}{4}$ **b** $\frac{3}{2}$ **c** $\frac{5}{1} = 5$ **d** $\frac{10}{17}$ **e** $\frac{9}{14}$
f $\frac{1}{8}$ **g** $\frac{1}{3}$ **h** $\frac{1}{57}$ **i** $\frac{2}{3}$ **j** $\frac{3}{14}$

3 **a** $\frac{4}{1} \times \frac{5}{2} = \frac{20}{2} = 10$ **b** $\frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = 1\frac{1}{2}$
c $\frac{13}{4} \div \frac{5}{3} = \frac{13}{4} \times \frac{3}{5} = \frac{39}{20} = 1\frac{19}{20}$

4 **a** 10 **b** $13\frac{1}{3}$ **c** 6 **d** $8\frac{1}{6}$ **e** $14\frac{2}{3}$
f 10 **g** $1\frac{1}{3}$ **h** $1\frac{1}{3}$ **i** $1\frac{3}{5}$ **j** $2\frac{1}{2}$
k $1\frac{5}{7}$ **l** $1\frac{13}{14}$ **m** $1\frac{1}{3}$ **n** $3\frac{5}{9}$ **o** $3\frac{1}{3}$
p $2\frac{4}{7}$ **q** $1\frac{1}{6}$ **r** $2\frac{2}{9}$ **s** $2\frac{2}{7}$ **t** $3\frac{3}{14}$

5 40 **6** 12

Exercise 5H

1 **a** $\frac{9}{17}$ **b** $\frac{7}{32}$ **c** $\frac{5}{38}$ **d** $\frac{7}{23}$
e $\frac{7}{61}$ **f** $\frac{11}{39}$ **g** $\frac{7}{9}$ **h** $\frac{35}{43}$

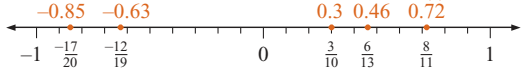
2 **a** $\frac{67}{9}$ **b** $\frac{127}{8}$ **c** $\frac{119}{5}$ **d** $\frac{1151}{14}$
e $\frac{1588}{17}$ **f** $\frac{3425}{52}$ **g** $\frac{5009}{31}$ **h** $\frac{3703}{27}$

3 **a** $3\frac{3}{5}$ **b** $7\frac{2}{3}$ **c** $5\frac{11}{12}$ **d** $12\frac{4}{15}$
d $6\frac{33}{34}$ **f** $5\frac{7}{9}$ **g** $7\frac{8}{25}$ **h** $15\frac{39}{46}$

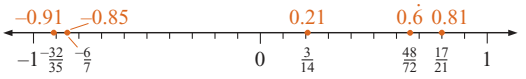
4 **a** 0.4 **b** 0.75 **c** 0.6 **d** 0.575

5 **a** 0.90 **b** 7.23 **c** 12.87 **d** 0.35
e 8.47 **f** 9.79 **g** 0.72 **h** 5.68

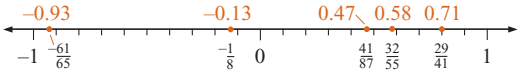
6 **a** $\frac{6}{13} = 0.46$, $\frac{8}{11} = 0.\dot{7}\dot{2}$, $-\frac{12}{19} = -0.63$, $\frac{3}{10} = 0.3$,
$-\frac{17}{20} = -0.85$



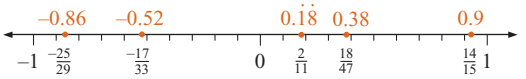
b $\frac{3}{14} = 0.21$, $-\frac{6}{7} = -0.85$, $\frac{17}{21} = 0.81$, $-\frac{32}{35} = -0.91$, $\frac{48}{72} = 0.6\dot{}$



c $\frac{41}{87} = 0.47$ $-\frac{61}{65} = -0.93$, $-\frac{1}{8} = -0.13$, $\frac{32}{55} = 0.58$, $\frac{29}{41} = 0.71$



d $-\frac{25}{29} = -0.86$, $\frac{2}{11} = 0.1\dot{8}$, $\frac{14}{15} = 0.9\dot{3}$, $-\frac{17}{33} = -0.52$, $\frac{18}{47} = 0.38$



7 **a** **i** > **ii** <
b **i** > **ii** >
c **i** > **ii** >
d **i** > **ii** <

8 **a** $10\frac{13}{15}$ **b** $21\frac{1}{36}$ **c** $-1\frac{17}{44}$ **d** $-15\frac{17}{28}$
e $3\frac{2}{15}$ **f** $-12\frac{62}{63}$ **g** $8\frac{1}{6}$ **h** $-23\frac{73}{325}$
i $4\frac{2}{3}$ **j** $-7\frac{1}{9}$ **k** $\frac{6}{11}$ **l** $-25\frac{15}{16}$
n $-2\frac{1}{10}$ **n** $4\frac{5}{22}$ **o** $10\frac{17}{72}$ **p** $-6\frac{39}{56}$

9 **a** **i** $1\frac{23}{30}$ m **ii** ≈ 23 cm
b −1.225 km ∴ 1225 m
c **i** $99\frac{71}{84}$ kg or ≈ 99.85 kg
ii 19.9609 kg ∴ 20 kg each

Language in mathematics

1 **a** Jack **b** 73
c Mining company
d Massive, enormous, immense
e Recipients **f** $8 500 000
g **1** $425 000 **2** $170 000 **3** $637 500
4 $425 000 **5** $510 000 **6** $637 500
7 $425 000 **8** $1 020 000 **9** $425 000
10 $850 000 **11** $1 700 000 **12** $425 000
13 $850 000
h **i** $1 700 000 **ii** $\frac{1}{5}$
i Bruno Gossling
j $1 700 000 000 or 1.7 billion dollars

2 **a** $1 500 000 **b** $562 500 **c** $375 000
d $300 000 **e** $75 000
f **i** $\frac{1}{8}$ **ii** $187 500

Check your skills

1 D **2** D **3** A **4** B **5** B
6 C **7** B **8** D **9** A **10** A
11 D **12** C **13** B **14** B **15** D

Review set 5A

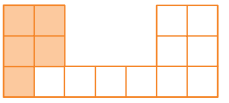
1 **a** $\frac{10}{21}$ **b** $\frac{11}{21}$
2 $\frac{8}{13}$ **3** $5\frac{2}{9}$
4 $\frac{60}{7}$ **5** $\frac{49}{84}$
6 $\frac{9}{20}, \frac{7}{10}, \frac{4}{5}$ **7** $\frac{8}{5}$
8 **a** $1\frac{2}{15}$ **b** $\frac{7}{12}$ **c** $\frac{3}{28}$ **d** $3\frac{1}{3}$
9 $\frac{7}{12}$ **10** $90

Review set 5B

1 **a** $\frac{7}{11}$ **b** $\frac{4}{11}$
2 $\frac{5}{9}$ **3** $15\frac{3}{7}$
4 $\frac{47}{9}$ **5** $\frac{4}{5}$
6 $\frac{2}{3}, \frac{7}{12}, \frac{3}{8}$ **7** $\frac{2}{11}$
8 **a** $9\frac{1}{10}$ **b** $8\frac{1}{12}$
9 **a** $9\frac{1}{2}$ **b** $3\frac{11}{13}$
10 $2\frac{3}{5}$

Review set 5C

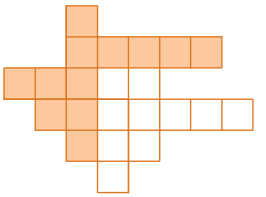
1 Shade any 5 squares.



2 $\frac{7}{12}$
3 $3\frac{1}{19}$ **4** $\frac{51}{4}$
5 $\frac{5}{8}$ **6** $\frac{2}{3}$ **7** $\frac{1}{6}, \frac{1}{2}, \frac{2}{3}$
8 $\frac{7}{61}$ **9** $10\frac{5}{21}$ **10** $1\frac{15}{16}$ L

Review set 5D

1 Shade any 12 squares.



2 $\frac{7}{15}$ **3** $11\frac{3}{13}$
4 $\frac{104}{5}$ **5** $\frac{155}{200}$
6 $\frac{37}{48}$ **7** $\frac{2}{3}, \frac{3}{5}, \frac{8}{15}$
8 $\frac{6}{31}$ **9** 370 kg
10 **a** $174 **b** $580 **c** $\frac{2}{15}$ **d** $116

CHAPTER 6 DRAWING AND BUILDING SOLIDS

Diagnostic test

1 D **2** D **3** D **4** C **5** C
6 B **7** B **8** D **9** C **10** B

Exercise 6A

1 **a** Sphere **b** Cylinder
c Rectangular prism **d** Triangular prism
e Cone **f** Triangular pyramid

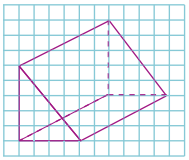
3 **a** Rectangular prism and triangular prism
b Cube and square pyramid
c Cylinder and cylinder
d Rectangular prism and triangular prism
e 2 rectangular prisms
f Cylinder and cone
g Cylinder and 2 hemispheres
h 2 rectangular prisms and triangular prism

4 **b** No, because cylinders and spheres do not have straight edges

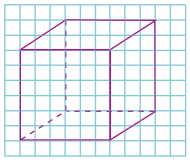
Exercise 6B

1 Examples:

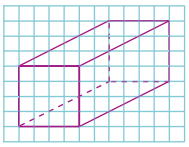
a



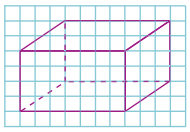
b



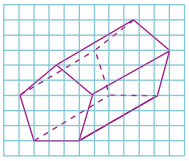
c



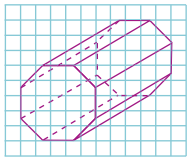
d



e

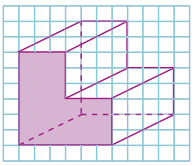


f

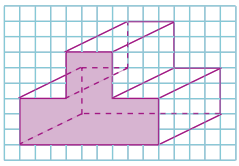


2 Examples:

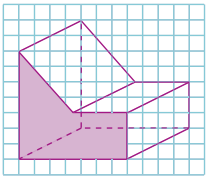
a



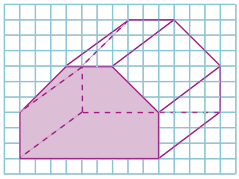
b



c

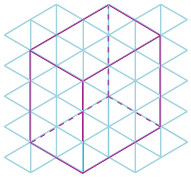


d

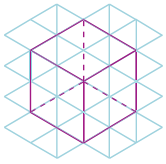


3 Examples:

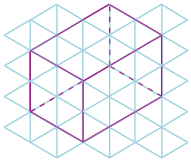
a



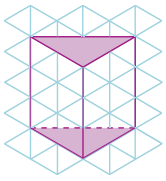
b



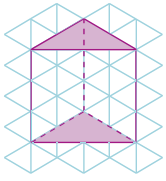
c



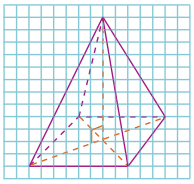
4 **a**



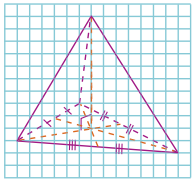
b



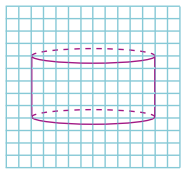
5 Example:



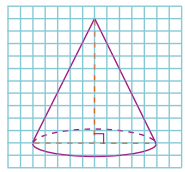
6 Example:



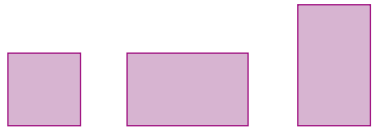
7 Example:



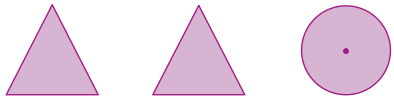
8 Example:



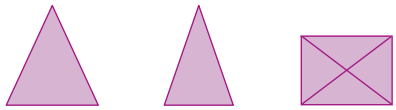
9 **a**



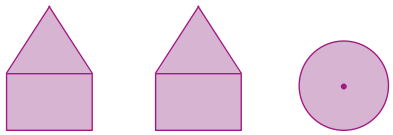
b



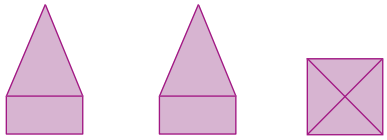
c



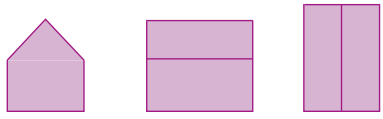
d



e



f



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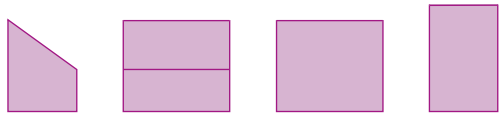

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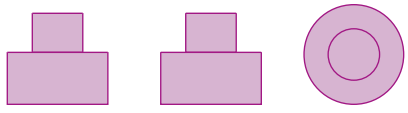
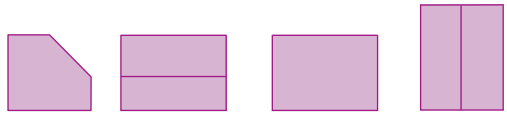
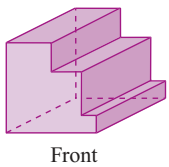


i



10 **a**



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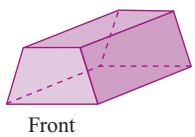

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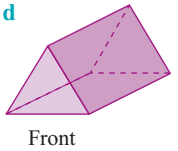

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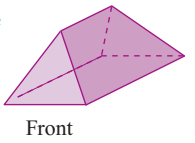
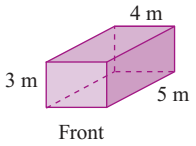


11 a



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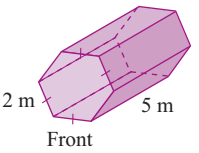

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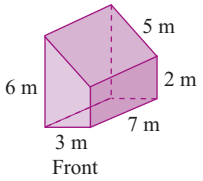

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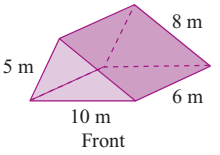

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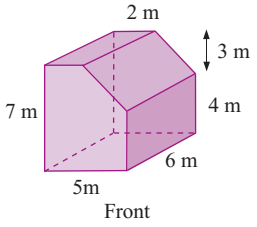


Exercise 6C

1 a

b

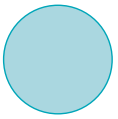
c

d

e

f

g



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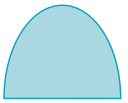
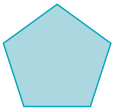


i

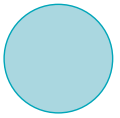


2 a a, b, c, d, i

b a and b are cubes, c and d are rectangular prisms, i is a pentagonal prism.

3 a Yes, a circle

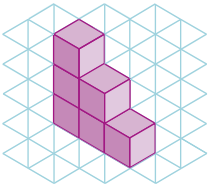
b No, as it does not have rectangles joining identical polygonal ends



Exercise 6D

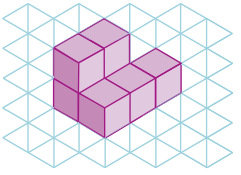
1 a i 6

ii



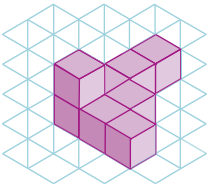
b i 7

ii

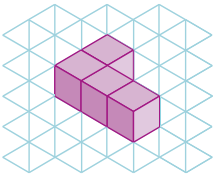


c i 8

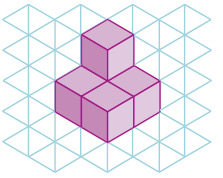
ii



2 a



b



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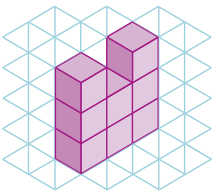

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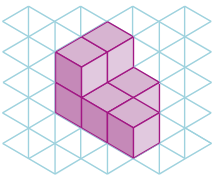

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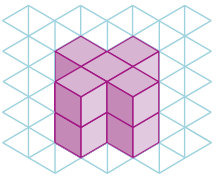

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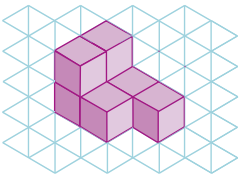
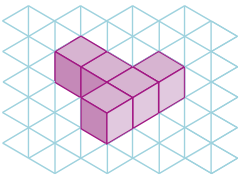


3 a



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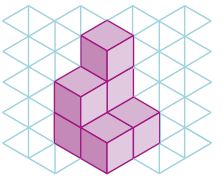

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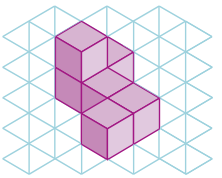

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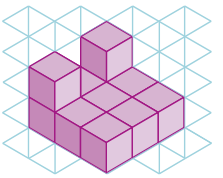

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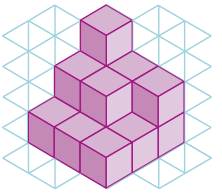

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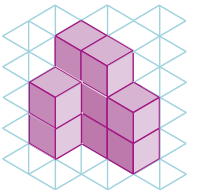
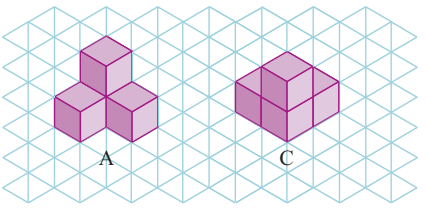


4 a



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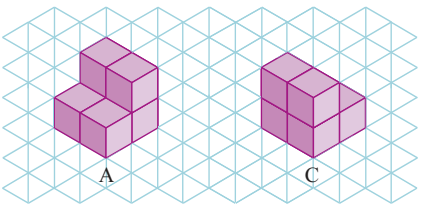

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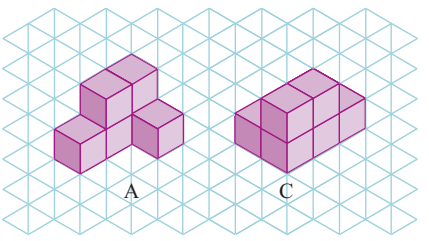

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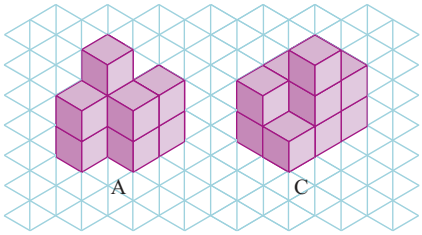

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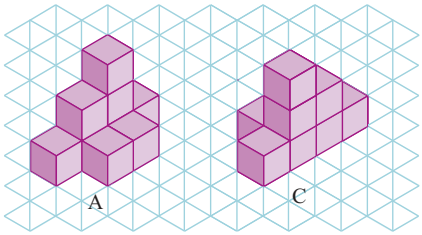

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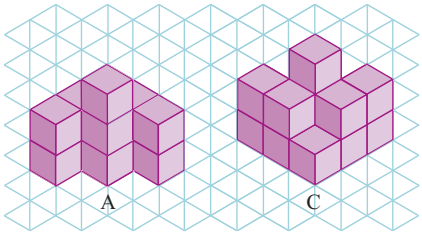
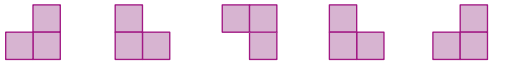


5 a



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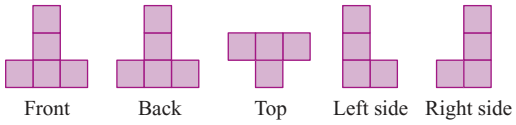

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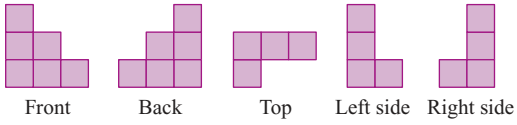

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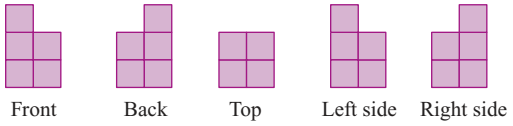

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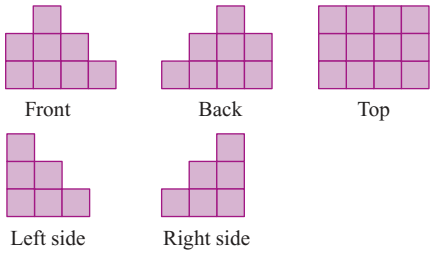

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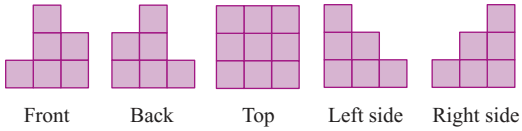
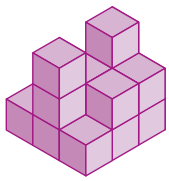


6 One possible solid is:



Language in mathematics

S	P	P	G	H	A	N	O	I	T	A	N	I	B	M	O	C
P	O	L	Y	H	E	D	R	A	V	R	S	Q	U	A	R	E
H	L	A	L	K	C	E	X	W	E	I	V	F	A	P	E	V
E	Y	N	V	I	Y	N	J	I	R	G	G	Z	R	Y	C	Y
R	G	E	E	D	L	O	F	E	T	H	G	I	A	R	T	S
E	O	Z	R	S	I	C	U	B	E	T	S	Z	E	A	A	K
T	N	A	T	S	N	O	C	V	X	M	T	R	U	M	N	E
X	J	A	I	H	D	L	A	C	I	T	N	E	D	I	G	T
K	C	F	C	P	E	N	T	A	G	O	N	A	L	D	U	C
K	L	X	E	X	R	A	L	U	C	R	I	C	E	B	L	H
X	Y	B	S	E	N	O	R	D	E	H	Y	L	O	P	A	A
D	E	T	I	S	O	P	M	O	C	U	N	I	F	O	R	M
Z	I	S	O	M	E	T	R	I	C	N	O	G	A	X	E	H
J	M	L	O	X	E	T	N	R	E	T	T	A	P	T	D	E
R	S	T	O	R	Q	P	R	A	L	U	G	N	A	I	R	T
S	T	V	E	S	P	I	L	L	E	L	O	N	O	G	O	N

2 a identical, prism **b** uniform, cross-section
c polygon, triangular, point
3 a Equal or identical **b** Half
5 Polyhedra
6 Circular, sketch, straight

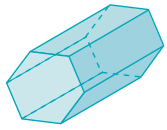
Check your skills

1 D **2** C **3** A **4** D **5** C
6 C **7** A

Review set 6A

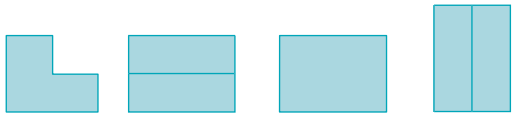
1 a Rectangular prism **b** Triangular pyramid
c Sphere **d** Pentagonal prism

2



3 a

b



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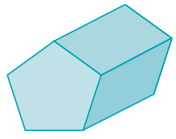

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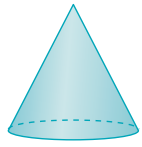
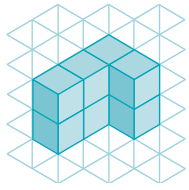


5 a

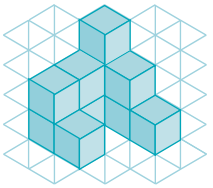
b

6 a 8

b



7 a



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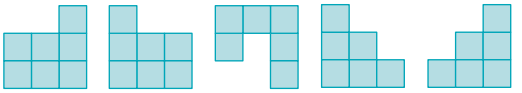
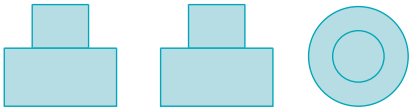


Review set 6B

1 a Cube
b Rectangular pyramid
c Cone
d Hemisphere

2

3 a



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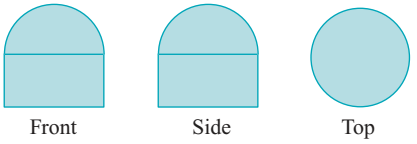

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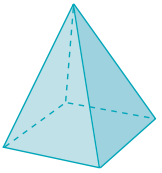

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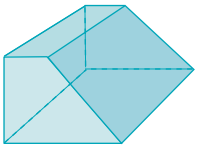
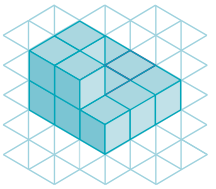
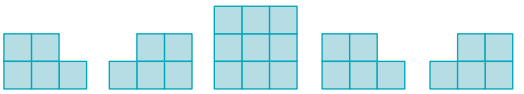


5 a 12

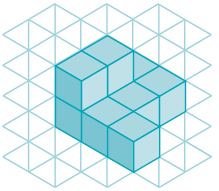
b



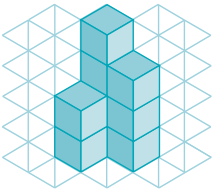
c



6



7

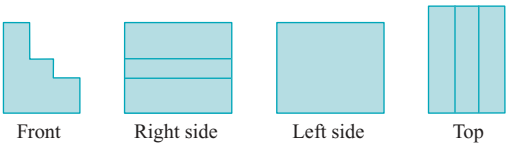


Review set 6C

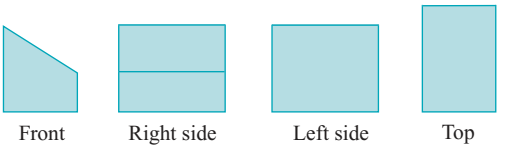
1 a 3 rectangular prisms
b Rectangular prism and triangular prism
c Cone and cylinder
d Cube and square pyramid

2 a A cylinder has circular ends and one curved face. A prism has polygonal ends and rectangular faces.
b A cone has a circular base and one curved face that comes to a point. A pyramid has a polygonal base and triangular faces that meet at a point.

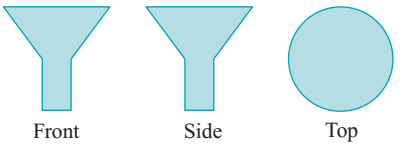
3 a



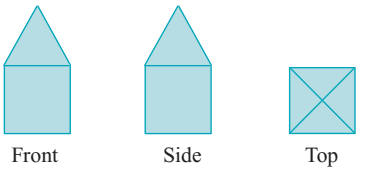
b



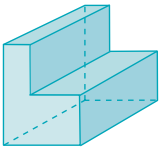
c



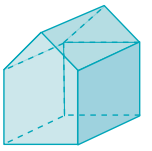
d



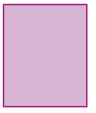
4 a



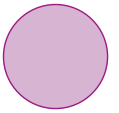
b



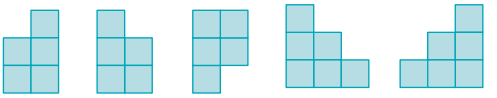
5 a



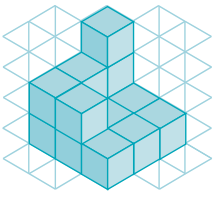
b



6



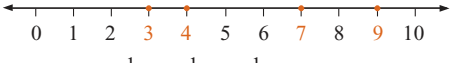
7



CUMULATIVE REVIEW CHAPTERS 2 TO 6

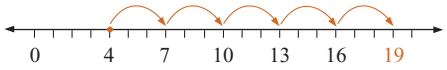
1 a $\angle AOB$ or $\angle BOA$, $\angle COD$ or $\angle DOC$
b **i** Acute **ii** Obtuse **iii** Reflex
c Adjacent angles have a common vertex and common arm and lie on opposite sides of the common arm.
d Complementary angles add to 90°.
e **i** $x = 55°$ (co-interior angles on $\parallel$ lines)
ii $x = 143°$ (adjacent angles on a straight line)
f **i** Yes, corresponding angles are equal.
ii No, alternate angles are not equal.

2 a



b Examples: $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$

c



d Fall of 5°C

e $3 - 10 = -7$

f **i** $-11, -10, -8, -6, -4$

ii $-10, -7, -4, 0, 7, 9$

iii $-1\frac{1}{2}, -1\frac{1}{4}, 0, \frac{1}{2}, 1\frac{1}{4}, 2$

g **i** $-7 > -8$ **ii** $-4 < 4$

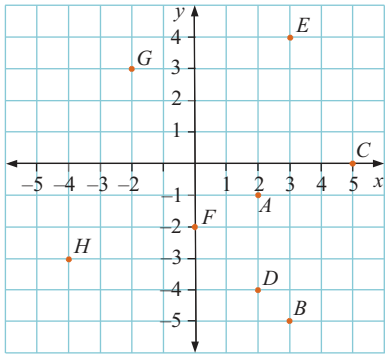
iii $0 > -3$

3 **a** -7 **b** -3 **c** 6 **d** 11

e -9 **f** -5 **g** -8 **h** -45

i 8 **j** -1 **k** -42 **l** 12

4



5 **a** 18, 24, 30, 36, 42, 48, 54, 60

b 7, 14, 21, 28, 35, 42, 49

c **i** 1, 2, 3, 4, 6, 8, 12, 24

ii 1, 2, 3, 6, 7, 14, 21, 42

iii 6

d $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$

e 168

f **i** 6 **ii** 4

g 2, 3, 4, 5, 6, 8, 10, 11, 12

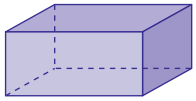
6 **a** $\frac{5}{12}$ **b** $2\frac{8}{17}$ **c** $\frac{41}{11}$ **d** $\frac{5}{24}$ **e** $\frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4}$

f **i** $\frac{5}{3}$ **ii** $\frac{4}{11}$ **g** $3\frac{5}{12}$

7 **a** **i** Cube **ii** Square pyramid

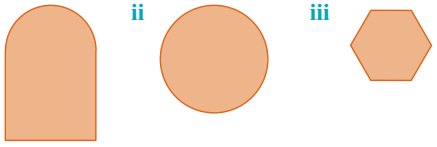
iii Triangular pyramid

b Example:

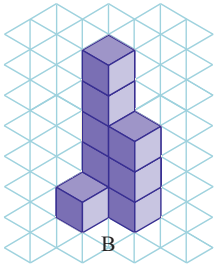


c

d **i** **ii** **iii**



e



CHAPTER 7 ALGEBRA

Diagnostic test

1 A	**2** A	**3** B	**4** B	**5** B
6 C	**7** D	**8** A	**9** B	**10** D
11 B	**12** A	**13** C	**14** B	**15** B
16 C	**17** B	**18** B	**19** D	**20** A
21 A	**22** A	**23** A	**24** B	

Exercise 7A

1 **a** $6 + 4 = 10$ **b** $20 + 4 = 24$

c $w + 4$ **d** $z + 4$

2 **a** $5 + 5 + 5 + 5 = 4 \times 5 = 20$

b $8 + 8 + 8 + 8 = 4 \times 8 = 32$

c $w + w + w + w = 4 \times w$

d $z + z + z + z = 4 \times z$

3 **a** $5 + 5 + 5 + 2 = 3 \times 5 + 2 = 17$

b $10 + 10 + 10 + 2 = 3 \times 10 + 2 = 32$

c $k + k + k + 2 = 3 \times k + 2$

d $z + z + z + 2 = 3 \times z + 2$

4 **a** 43 **b** 53

c 63 **d** $2 \times k + 3$

5 **a** **i** 86 **ii** 166 **iii** 206

b $4 \times t + 6$

6 **a**

b **i** 85 **ii** 190 **iii** 310

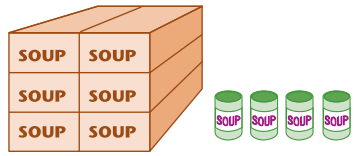
c $3 \times m + 10$

7 **a**

b **i** 10.3 m **ii** 30.3 m **iii** 50.3 m

c $5 \times q + 0.3$

8 **a**



b **i** 124 **ii** 184 **iii** 304

c $6 \times d + 4$

9 **a** $z + 3$

b $z + z + z + z = 4 \times z$

c $z + z + z + 2 = 3 \times z + 2$

d $z + z + z + z + 3 = 4 \times z + 3$

e $z + z + 4 = 2 \times z + 4$

f $z + z + z + z + z + 1 = 5 \times z + 1$

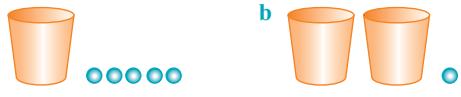
10 **a** $2 \times k + 3$ **b** $k + 5$ **c** $5 \times k$

d $3 \times k + 2$ **e** $4 \times k + 1$ **f** $6 \times k + 4$

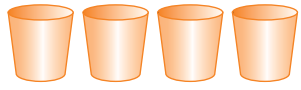
11 **a** B **b** C **c** E **d** A **e** F **f** D

12 7 cups and an extra 4 marbles

13 **a** **b**



c

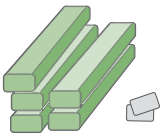


d

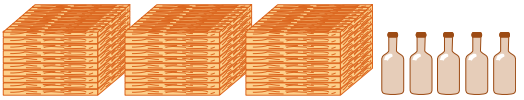
e

14

15



16

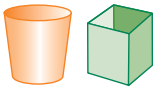


17 **a** Number of marbles in a cup or bag
b Number of cups or bags
c Number of separate marbles

18 **a** $s + t + t = s + 2 \times t$
b $s + s + t + t + t = 2 \times s + 3 \times t$

19 **a** $h + k + k + k = h + 3 \times k$
b $h + h + h + k = 3 \times h + k$
c $h + h + h + h + k + k + k = 4 \times h + 3 \times k$

20 **a**

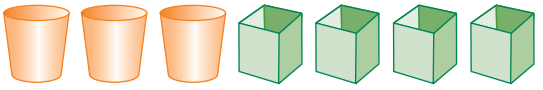


b

c

d

e



21 **a** $(w + 3) + (w + 4) = 2 \times w + 7$
b $(2 \times w + 3) + (w + 2) = 3 \times w + 5$

22 **a** $2 \times z + 4$ **b** $3 \times z + 5$
c $3 \times z + 7$ **d** $5 \times z + 5$
e $5 \times z + 6$ **f** $7 \times z + 4$

23 **a** $(2 \times k + m) + (2 \times k + 3 \times m) = 4 \times k + 4 \times m$
b $(3 \times k + 2 \times m) + (2 \times k + 2 \times m)$
$= 5 \times k + 4 \times m$

24 **a** $3 \times v + 3 \times y$ **b** $3 \times v + 4 \times y$
c $3 \times v + 3 \times y$ **d** $5 \times v + 4 \times y$
e $3 \times v + 6 \times y$ **f** $5 \times v + 4 \times y$
g $6 \times v + 5 \times y$

Exercise 7B

1 **a** $2 + 6 = 8$ **b** $5 + 6 = 11$

2 **a** $2 \times 6 - 5 = 7$ **b** $2 \times 10 - 5 = 15$

3 **a** 14 **b** 8 **c** 5

4 **a** 8 **b** 17 **c** 32

5 **a** $5 + 9 = 14$ **b** $5 - 5 = 0$
c $6 \times 5 = 30$ **d** $10 \times 5 + 2 = 52$
e $7 \times 5 - 4 = 31$ **f** $12 - 2 \times 5 = 2$

6 **a** 3 **b** 16 **c** 27 **d** 42 **e** 144

7 **a** 7 **b** 20 **c** 17 **d** 18 **e** 2

8 **a** 19 **b** 3 **c** 7 **d** 0 **e** 60
f 16 **g** 24 **h** 30 **i** 48 **j** 240
k 25 **l** 36 **m** 2 **n** 15 **o** 25
p 4 **q** 4 **r** 3 **s** 18 **t** 16
u 42 **v** 2 **w** 1 **x** 2

9 **a**

x	1	2	3	4	5	…	10	…	100
$x + 3$	4	5	6	7	8	…	13	…	103

b

x	1	2	3	4	5	…	10	…	100
$x - 2$	-1	0	1	2	3	…	8	…	98

c

x	1	2	3	4	5	…	10	…	100
$4 \times x$	4	8	12	16	20	…	40	…	400

d

x	1	2	3	4	5	…	10	…	100
$3 \times x + 1$	4	7	10	13	16	…	31	…	301

e

x	1	2	3	4	5	…	10	…	100
$2 \times x - 3$	-1	1	3	5	7	…	17	…	197

Exercise 7C

1 **a** $4m$ **b** $6t$ **c** $3k$
d $10z$ **e** $7p$ **f** $5m$
g $8k$ **h** $2n$ **i** wz
j cv **k** pq **l** ab
m t^2 **n** q^2 **o** s^2
p abc **q** $2st$ **r** $6pq$
s $8pqr$ **t** $3a + 1$ **u** $7d - 8$
v $23 + 6p$ **w** $15 - 5t$ **x** $4m + 2n$
y $3z - 8k$

2 **a** $2v^2$ **b** $5w^2$ **c** $2p^2$ **d** $4t^2$ **e** $3m^2$
f $6d^2$ **g** $9p^2$ **h** $8k^2$ **i** $10s^2$ **j** $7n^2$

3 **a** $9 \times w$ **b** $3 \times k$ **c** $5 \times g$
d $2 \times d$ **e** $24 \times b$ **f** $52 \times m$
g $48 \times z$ **h** $t \times z$ **i** $b \times c$
j $k \times m$ **k** $v \times v$ **l** $b \times b$
m $3 \times p \times q$ **n** $8 \times a \times b$
o $5 \times p \times q$ **p** $12 \times m \times n$
q $35 \times h \times s$ **r** $k \times r \times z$
s $3 \times a \times b \times c$ **t** $6 \times t + 5$
u $12 + 4 \times d$ **v** $2 \times w - 9$
w $7 - 8 \times k$ **x** $2 \times a + 3 \times b$
y $4 \times k - 9 \times m$

4 **a** $2 \times k \times k$ **b** $6 \times t \times t$
c $4 \times q \times q$ **d** $7 \times a \times a$
e $4 \times w \times w$ **f** $5 \times m \times m$
g $8 \times z \times z$ **h** $9 \times q \times q$
i $12 \times s \times s$ **j** $5 \times y \times y$

5 **a** $5 \times 3 = 15$ **b** $3 \times 4 = 12$
c $25 \times 3 = 75$ **d** $4 \times 4 = 16$

6 **a** 24 **b** 10 **c** 6 **d** 0 **e** 60 **f** 16
g 18 **h** 20 **i** 48 **j** 0 **k** 30 **l** 12
m 0 **n** 10 **o** 0 **p** 0 **q** 60 **r** 90
s 60 **t** 40 **u** 0 **v** 450 **w** 36 **x** 25

7 **a** $5 \times 2 + 4 = 14$ **b** $20 - 4 \times 3 = 8$
c $4 \times 2 + 5 \times 3 = 23$

8 **a** 11 **b** 21 **c** 11 **d** 1 **e** 14
f 7 **g** 12 **h** 21 **i** 13 **j** 12
k 85 **l** 5 **m** 11 **n** 19 **o** 1
p 14 **q** 18 **r** 23 **s** 20 **t** 4
u 13 **v** 5 **w** 30 **x** 1 **y** 18

9 Sylvanna

10 **a** $3 \times 3 = 9$ **b** $4 \times 3 \times 3 = 36$
c $(4 \times 3)^2 = 12^2 = 144$

11 **a** 25 **b** 50 **c** 100 **d** 75 **e** 225
f 16 **g** 32 **h** 64 **i** 48 **j** 144
k 4 **l** 12 **m** 36 **n** 40 **o** 400

12 **a**

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
x^2	25	16	9	4	1	0	1	4	9	16	25

b They are the same. When multiplying integers, if the signs are the same the answers are positive.

13 **a** True **b** True **c** True

14 **a** t **b** k **c** d **d** z **e** $5y$
f pq **g** $3v$ **h** ab **i** $2wyz$ **j** $5klm$
k 0 **l** 0 **m** 0 **n** 0 **o** 0
p m **q** $4a$ **r** $6xy$ **s** n **t** $5n$

Exercise 7D

1 **a** $\frac{2}{5}$ **b** $\frac{t}{5}$ **c** $\frac{5}{q}$ **d** $\frac{2k}{5}$

2 **a** $\frac{7}{20}$ **b** $\frac{a}{3}$ **c** $\frac{m}{4}$ **d** $\frac{k}{2}$ **e** $\frac{z}{4}$ **f** $\frac{y}{6}$
g $\frac{8}{k}$ **h** $\frac{6}{w}$ **i** $\frac{2}{s}$ **j** $\frac{9}{t}$ **k** $\frac{ab}{3}$ **l** $\frac{pq}{7}$
m $\frac{3t}{4}$ **n** $\frac{7y}{3}$ **o** $\frac{4k}{9}$ **p** $\frac{7tv}{2}$

3 **a** $3 \div 10$ **b** $m \div 6$
c $8 \div n$ **d** $3w \div 7$

4 **a** $1 \div 9$ **b** $h \div 3$ **c** $m \div 7$
d $a \div 5$ **e** $t \div 4$ **f** $z \div 2$
g $9 \div d$ **h** $4 \div z$ **i** $2 \div p$
j $6 \div t$ **k** $pq \div 2$ **l** $km \div 5$
m $5t \div 6$ **n** $8b \div 9$ **o** $4t \div 5$

5 **a** $y \div 2 = 12 \div 2 = 6$
b $y \div x = 12 \div 3 = 4$
c $6x \div y = (6 \times x) \div y = (6 \times 3) \div 12 = 1\frac{1}{2}$
d $2y \div 8x = (2 \times 12) \div (8 \times 3) = 24 \div 24 = 1$
e $5 - y \div 6 = 5 - 12 \div 6 = 5 - 2 = 3$

6 **a** 2 **b** 1 **c** 1 **d** 3 **e** 3
f 2 **g** 1 **h** 5 **i** 3 **j** 6
k 1 **l** 3 **m** −1 **n** 4 **o** 4

7 **a** $\frac{4}{7k}$ **b** $\frac{3p}{4q}$ **c** $\frac{t}{uv}$ **d** $\frac{xy}{3z}$ **e** $\frac{4m}{pq}$

8 **a** $6 \div 5m$ **b** $2w \div 7z$ **c** $k \div mn$
d $uv \div 3z$ **e** $9p \div qr$

9 **a** 2 **b** 1 **c** 3 **d** 5 **e** 4

Exercise 7E

1 **a** $3(f + 9)$ **b** $7(n - 4)$ **c** $4(s - 6)$
d $d(z + 1)$ **e** $m(k - 7)$ **f** $y(4 + p)$
g $2p(4q - 3)$ **h** $7t(8 - 5w)$ **i** $4z(3w + 2y)$

2 **a** $4 \times (b + 1)$ **b** $2 \times (m - 5)$
c $3 \times (s + 4)$ **d** $w \times (z + 6)$
e $k \times (k - 1)$ **f** $5 \times t \times (t + 8)$
g $z \times (2 \times z - 4)$ **h** $2 \times a \times (3 \times b + 2)$
i $7 \times p \times (3 \times q - 5)$ **j** $6 \times k \times (7 \times m + 3 \times n)$

3 **a** $3 \times (4 + 7) = 3 \times 11 = 33$
b $3 \times (4 + 5) = 3 \times 9 = 27$
c $2 \times 5 \times (4 \times 3 - 7) = 2 \times 5 \times (12 - 7)$
$= 2 \times 5 \times 5 = 50$
d $3 \times 4 \times (2 - 5) = 3 \times 4 \times (-3) = -36$

4 **a** 12 **b** 42 **c** 24 **d** 8
e 20 **f** −40 **g** 9 **h** −5
i 28 **j** 24 **k** 24 **l** 96

5 **a** **i** 16, 13 **ii** No
b **i** 15, 19 **ii** No
c **i** No **ii** 50, 32

6 **a** $\frac{p + 5}{3}$ **b** $\frac{4m - 3}{10}$ **c** $\frac{8}{a + 3}$ **d** $\frac{m + 3}{p + 2}$

7 **a** $\frac{b + 3}{7}$ **b** $\frac{m + n}{2}$ **c** $\frac{3k + 1}{5}$ **d** $\frac{a + 6}{b}$
e $\frac{15}{4a}$ **f** $\frac{6}{5y}$ **g** $\frac{10}{b + 1}$ **h** $\frac{12}{w + 5}$
i $\frac{p}{q - 3}$ **j** $\frac{t + 12}{p + q}$ **k** $\frac{3a - 2b}{d + 1}$ **l** $\frac{4d - 1}{7e}$

8 **a** $\frac{3 + 7}{2} = \frac{10}{2} = 5$ **b** $\frac{16}{3 + 5} = \frac{16}{8} = 2$
c $\frac{5 + 1}{3 - 1} = \frac{6}{2} = 3$ **d** $\frac{3 + 11}{12 - 5} = \frac{14}{7} = 2$

9 **a** 2 **b** 4 **c** −5 **d** 2
e 10 **f** 1 **g** 6 **h** 5
i 1 **j** 1 **k** −3 **l** 4

10 **a** 16 **b** 12 **c** 18 **d** 0 **e** 40
f 4 **g** 48 **h** 2 **i** 5 **j** 31
k 23 **l** 15 **m** 60 **n** −14 **o** 0
p 30 **q** 20 **r** 0 **s** 24 **t** 15
u 2 **v** −1 **w** 2 **x** 64 **y** 500

11 **a** **i** 2, 8, 2 **ii** $\frac{10}{a + 3}$
b **i** 2, 10, 2 **ii** $\frac{12 - b}{3}$
c D

12 **a** Always true **b** False, except if $m = k$
c False, except if $m = k$ **d** Always true
e Always true **f** Always true
g False, except if $m = 0$ **h** Always true
i True, except if $j + k = 0$
j True, except if $m = 0$ **k** Always true
l False, except if $c = 1$ or -1

Exercise 7F

1 **a** $6 \times a + 9 \times a = (6 + 9) \times a$
$= 15 \times a = 15a$
b $14 \times y - 6 \times y = (14 - 6) \times y$
$= 8 \times y = 8y$

2 **a** **i** 9 **ii** 45 **iii** −18
b **i** 9 **ii** 45 **iii** −18
Yes

3 **a** **i** 20 **ii** 32 **iii** −12
b **i** 20 **ii** 32 **iii** −12
Yes

4 **a** $11a$ **b** $11t$ **c** $9m$
d $7p$ **e** $3x$ **f** $3b$
g $6a + 3b$ **h** $7y - 4z$ **i** $17t^2$
j $4m^2$ **k** $9k + 5m$ **l** $11ab$
m $5ab + 2bc$ **n** $2ab^2$ **o** $3a^2b + 5ab^2$
p 0 **q** $7g^3$ **r** $2p$
s $9x + 24$ **t** $9st^2$ **u** $4mn^2$
v $11x^2y$ **w** f **x** $10c$

5 **a** $(4 - 7) \times k = -3k$ **b** $(-2 + 5) \times w = 3w$
c $(-3 - 2) \times t = -5t$

6 **a** $-n$ **b** $-2p$ **c** $-2c$ **d** $-6q$
e $3d$ **f** $-9ac$ **g** $-3q$ **h** $7c$
i $-4yz$ **j** $5L$ **k** $-p^2$ **l** $-2x^2$
m $-2xy^2$ **n** $-11pq^2$ **o** $-3t^2$ **p** $-2b$

7 **a** $-8x + 4x = -4x$ **b** $6pq - 3pq = 3pq$
c $-4z + 3z = -z$

8 **a** $6d$ **b** $-3pq$ **c** $-c$ **d** $13q^2$
e $-5m$ **f** $9r$ **g** $-4a$ **h** $-2r$
i 0 **j** $-19x^2y$ **k** $-7d^2$ **l** $-3k^2m$

9 **a** $(5y + 4y) - 3 = 9y - 3$
b $(7p + 5p) + 3q = 12p + 3q$

10 **a** $17 - 5m$ **b** $8a + 6$ **c** $4 + n$
d $18 - 7q$ **e** $13 - 5k$ **f** $11f + 12$
g $3x^2y - 5$ **h** $9w - 6$ **i** $12a^2b + 9$
j $6q^2 - 6$ **k** $x - 2$ **l** $4p - 9$
m $t + 4$ **n** $6k - 6L$ **o** $6d^2 - 3d$
p $3n + 2n^2$ **q** $8c + 2c^2$ **r** $6 + 3n^2$
s $15mn - 2m$ **t** $a^2b + 2ab^2$

11 **a** $-3 + 5k$ or $5k - 3$ **b** $-4 - 4s$ or $-4s - 4$
c $-2m + 2n$ or $2n - 2m$ **d** $-3x - 8y$ or $-8y - 3x$
e $-4t + 7u$ or $7u - 4t$ **f** $-5a - 4b$ or $-4b - 5a$
g $-2d + 4e$ or $4e - 2d$ **h** $-6z - 4w$ or $-4w - 6z$
i $-3a - 3b$ or $-3b - 3a$ **j** $6m - 10n$
k $-4v - 2w$ or $-2w - 4v$ **l** $-4k + m$ or $m - 4k$

12 **a** $3x + 2$ **b** $4w - 2z$ **c** $4 + 3d$

13 **a** $6a + 11$ **b** $9c^2 + 2c$ **c** $2x + 2y$
d $2q^2 + 2$ **e** $9t + v$ **f** $4q + 3d$
g $4p + 6p^2$ **h** $-13l + 10$ **i** $-6n - 3$
j $-4m$ **k** $2d^2 - 11d$ **l** $16 + 10m$
m $5n - 8$ **n** $12p - 6m$

14 **a** $9c + 2d - 16$ **b** $6n + 2cd - 6$
c $-3q + r + 6$ **d** $2e + 9$
e $11 - 7c + d$ **f** $12l - 5c + 8$
g $-16 - c$ **h** $5d - 2qr + 7r - 4q$
i $-8s + 6t + 8$ **j** $10ac - 3ap - 6a + 19$

Exercise 7G

1 **a** $5 \times t \times w = 5tw$
b $4 \times 3 \times m = 12m$
c $7 \times y \times 2 = 7 \times 2 \times y = 14y$
d $3 \times p^2 \times 7 \times q = 3 \times 7 \times p^2 \times q = 21p^2q$
e $2 \times a \times b \times 3 \times a = 2 \times 3 \times a \times a \times b = 6a^2b$

2 **a** $4xy$ **b** $3km$ **c** $5xy$ **d** $28w$
e $20k$ **f** $60p$ **g** $16x$ **h** $18z$
i $12mn$ **j** $12vw$ **k** $28p^2q$ **l** $30ab^2$
m $20abc$ **n** $18x^2yz$ **o** $20pq^2r$ **p** $35b^2c^2$
q $24ab$ **r** $30km$ **s** $24abc$ **t** $40pqr$
u $36a^2c$

Exercise 7H

1 **a** $\frac{10y}{15} = \frac{2y}{3}$ **b** $\frac{8m}{12m} = \frac{2}{3}$ **c** $\frac{6x}{8xy} = \frac{3}{4y}$

2 **a** $\frac{x}{2}$ **b** $\frac{m}{4}$ **c** $\frac{p}{5}$ **d** $4d$ **e** $5c$
f $2a$ **g** $\frac{1}{2}$ **h** 2 **i** $\frac{4}{5}$ **j** 2
k $\frac{1}{3}$ **l** $\frac{1}{5}$ **m** $\frac{9p}{10d}$ **n** $\frac{2y}{5}$ **o** $\frac{2a}{3c}$
p $\frac{4}{5q}$ **q** $\frac{1}{5x}$ **r** $\frac{7d}{m}$ **s** $\frac{5pr}{4}$ **t** $\frac{2n}{5p}$

Exercise 7I

1 **a** $3 + 5, p + q$ **b** $2 \times 6, k \times m = km$
c $10 - 7, x - y$ **d** $4 + 5, w + 5$
e $8 - 2, c - 2$ **f** $9^2, k^2$
g $2^3, z^3$ **h** $2 \times 7, 2 \times a = 2a$
i $\frac{1}{2} \times 10, \frac{1}{2} \times b$ or $\frac{1}{2}b$ or $\frac{b}{2}$
j $\frac{1}{4} \times 12, \frac{1}{4} \times w$ or $\frac{1}{4}w$ or $\frac{w}{4}$

2 **a** $m + n$ **b** $6q$ **c** $x + 4$ **d** $y - 2$
e $z + 1$ **f** $p - 5$ **g** $2k$ **h** $w - 2$
i n^2 **j** t^3 **k** $\frac{1}{2}p$ or $\frac{p}{2}$ **l** $\frac{1}{3}z$ or $\frac{z}{3}$

3 **a** $2x + 7$ **b** $2(b + 5)$ **c** $3y - 4$
d $\frac{p}{2} + 11$ **e** $3T^2$ **f** $8 + J$
g $4(p - s)$ **h** $\frac{10(y - 3)}{x}$ **i** $\frac{x + y}{2}$

4 **a** m^2j **b** $15 - 3b$ **c** $\frac{k}{4} + 7q$
d $\frac{\sqrt{d}}{2}$ **e** $8f - 4$ **f** $(2t - 4)^3$

5 **a** **i** 300 **ii** $100x$
b **i** 500 **ii** $100y$
c **i** 4000 **ii** $1000z$
d **i** 360 **ii** $60k$

6 **a** $12 - x$ **b** $88 + y$
c **ii** $5c$ **ii** $\frac{5c}{100} = \frac{c}{20}$
d $\$(10 - z)$ **e** $E - 7$ **f** $x - 4y\ m$

7 **a** $45p + 85b$ **b** $\frac{52p + 96b}{100}$

8 **a** $a + b$ km **b** $\frac{a + b}{7}$ km/h

9 **a** 50 km/h **b** $\frac{200}{z}$ km/h **c** $\frac{p}{t}$ km/h

10 **a** $2 \times 5 + 2 \times 3 = 16$ cm
b $2 \times x + 2 \times 2 = 2x + 4$ cm

11 **a** the sum of x and y
b the difference between x and y
c the product of x and y **d** twice x
e double x plus 7 **f** twice the sum of x plus 7
g one third of x **h** one third of x plus 1
i one third of the sum of x plus 1
j one quarter of the difference between x and y
k x squared **l** square root of x

Calculator activities

1 **a** 280 **b** 566 **c** 301 **d** 42 **e** -2

2 **a** 4 **b** $\frac{9}{10}$ **c** $1\frac{9}{16}$ **d** $1\frac{2}{3}$ **e** $6\frac{1}{4}$

3 **a** 0.74 **b** 14.55 **c** -1.6 **d** 15.232 **e** 0.0841

Language in mathematics

1 Find-a-word

R	A	B	B	R	E	V	I	A	T	I	O	N	C	B	A
O	L	A	D	O	E	A	G	R	E	V	I	E	W	F	G
T	G	S	H	L	I	L	R	U	J	D	N	A	P	X	E
C	E	E	K	L	M	U	O	L	N	O	I	N	D	E	X
A	B	T	R	U	P	E	U	E	S	S	E	C	O	R	P
F	R	S	V	Q	X	W	P	Y	E	S	R	E	V	E	R
Z	A	S	U	B	S	T	I	T	U	T	I	O	N	A	E
L	I	I	B	H	D	F	N	J	K	L	O	B	M	Y	S
A	C	M	I	C	G	E	G	N	M	O	L	R	P	A	S
C	R	P	R	O	N	U	M	E	R	A	L	O	Q	R	I
I	T	L	S	N	O	I	T	A	T	O	N	U	T	B	O
T	L	I	N	D	I	C	E	S	U	V	E	G	W	E	N
N	U	F	U	N	C	T	I	O	N	X	S	H	Y	G	A
E	S	Y	A	C	O	M	B	I	N	E	I	T	Z	L	E
D	E	B	Q	D	T	N	E	M	E	T	A	T	S	A	R
I	R	R	C	P	E	V	I	T	A	N	R	E	T	L	A

3 **a** pronumerals, variables, algebraic expressions
b multiplication, numeral, pronumerals, two
4 identical, symbol, simplify
6 **a** algebra **b** process
c abbreviate **d** multiplication
e evaluate **f** convention
8 Brackets, vinculum (division line)
10 Algebraic, numerical, symbol

Check your skills

1 C	**2** C	**3** B	**4** C	**5** D
6 C	**7** B	**8** D	**9** C	**10** D
11 B	**12** A	**13** B	**14** C	**15** D
16 C	**17** D	**18** D	**19** A	**20** C
21 B	**22** A	**23** C	**24** B	

Review set 7A

1 **a** 10 **b** 19 **c** $3x + 4$
d $3z + 4$
2 **a** $5y$ **b** xy **c** x^2
d $3ab$ **e** $6m - 2$
3 **a** $3 \times k$ **b** $m \times n$ **c** $y \times y$
d $5 \times p \times q$ **e** $3 \times z \times z$
4 **a** $4p$ **b** $6t$ **c** w
d $3x$ **e** 0 **f** $m + 2n$
5 **a** 15 **b** 18 **c** -24 **d** 9 **e** 18
6 **a** $\frac{z}{4}$ **b** $\frac{m}{n}$ **c** $\frac{3}{k}$ **d** $\frac{5x}{7}$ **e** $\frac{y^2}{z}$
7 **a** $k \div 2$ **b** $2 \div k$ **c** $x \div y$
d $3t \div 4$ **e** $2m \div 3n$
8 **a** 4 **b** 4 **c** 3 **d** 6 **e** 1
9 **a** 28 **b** 10 **c** 6 **d** -6 **e** 30
10 **a** 5 **b** 4 **c** 12 **d** 4 **e** -2
11 **a** $11s$ **b** $6k$ **c** $7w + 11$
d $8p - 2q$ **e** $3x + 8$ **f** $-a - b$
12 **a** $20xy$ **b** $-14p$ **c** $-18ab$
13 **a** $4h$ **b** $\frac{1}{3}$ **c** 3 **d** $\frac{2}{3v}$ **e** $\frac{2}{3b}$
14 **a** $a + b$ **b** $9k$ **c** $m + 3$
d $p - 7$ **e** $\frac{a+b}{2}$ **f** $n - 5$
15 **a** the sum of p and q
b the difference between p and q
c the product of p and q
d half of p **e** p squared

Review set 7B

1 **a** 15 **b** 27 **c** $2m + 3$ **d** $2y + 3$
2 **a** $3z$ **b** mn **c** y^2
d $3pq$ **e** $3m + 2n$
3 **a** $5 \times t$ **b** $x \times y$ **c** $h \times h$
d $3 \times h \times h$ **e** $4 \times f \times g$
4 **a** $3k$ **b** $5m$ **c** p **d** $4q$ **e** 0
5 **a** 12 **b** 40 **c** -14 **d** 16 **e** 48
6 **a** $\frac{k}{2}$ **b** $\frac{k}{m}$ **c** $\frac{m}{k}$ **d** $\frac{3t}{5}$ **e** $\frac{xy}{z}$
7 **a** $m \div 3$ **b** $4 \div p$ **c** $y \div x$
d $3x \div 5$ **e** $3p \div 4q$
8 **a** 5 **b** 2 **c** 5 **d** 1 **e** 2
9 **a** 15 **b** -12 **c** 15 **d** 32 **e** 12
10 **a** 2 **b** -3 **c** 6 **d** 2 **e** 7
11 **a** $9q$ **b** $5m$ **c** $6a + 12$
d $8x - 4y$ **e** $3m + 4$ **f** $-w + z$
12 **a** 20 km **b** $-15cd$ **c** $6mn$
13 **a** $3y$ **b** $\frac{1}{5}$ **c** 2 **d** $\frac{3b}{4}$ **e** x
14 **a** **i** $10x$ **ii** $60w$
b $\$(4a + 2b)$
15 **a** the sum of w and 3 **b** five times w
c one third of w **d** the square root of w
e the average of w and x

Review set 7C

1 **a** $z + 2$ **b** $3z$ **c** $2z + 1$ **d** $4z + 3$
2 **a** $6y$ **b** gh **c** $8yz$
d $4m^2$ **e** $4a + 3b$
3 **a** $4 \times m$ **b** $p \times q$ **c** $v \times v$
d $4 \times v \times v$ **e** $5 \times m \times n$
4 **a** $2p$ **b** $7y$ **c** z **d** $2pq$ **e** 0
5 **a** 10 **b** -50 **c** 29 **d** 4 **e** 16
6 **a** $\frac{t}{3}$ **b** $\frac{g}{h}$ **c** $\frac{h}{g}$ **d** $\frac{4w}{3}$ **e** $\frac{5}{2x}$
7 **a** $k \div 4$ **b** $3 \div m$ **c** $a \div b$
d $4e \div 5$ **e** $mn \div p$
8 **a** 2 **b** 3 **c** 4 **d** 5 **e** 2
9 **a** 24 **b** 8 **c** 30 **d** -35 **e** 105
10 **a** 5 **b** 1 **c** -3 **d** 2 **e** 3
11 **a** $11h$ **b** $8d$ **c** $10z + 12$
d $7p - 5q$ **e** $7a - 7$ **f** $-3x - y$
12 a $15rs$ **b** $-12g$ **c** $16pq$
13 **a** $3y$ **b** $\frac{1}{3}$ **c** 5 **d** $\frac{4}{5q}$ **e** $\frac{2a}{3}$
14 **a** $\$(20 - p)$ **b** $15 + x$ **c** $2000 - 3t$ g
15 The total cost of 5 kg of potatoes and 2 kg of onions

CHAPTER 8 FRACTIONS, DECIMALS AND PERCENTAGES

Diagnostic test

1 C	**2** D	**3** A	**4** D	**5** C
6 B	**7** A	**8** A	**9** D	**10** A
11 B	**12** C	**13** A	**14** D	**15** B
16 A	**17** A	**18** B	**19** D	**20** A
21 A	**22** A	**23** C		

Exercise 8A

1 **a** $\frac{6}{10} = \frac{3}{5}$ **b** 6 : 10 or 3 : 5 **c** $\frac{4}{10} = \frac{2}{5}$
d 4 : 10 or 2 : 5 **e** 6 : 4 or 3 : 2 **f** 4 : 6 or 2 : 3
2 **a** $\frac{5}{15}$ **b** $\frac{1}{3}$
c **i** 5 : 15 and 1 : 3 **ii** Yes
d $\frac{10}{15}$ **e** $\frac{2}{3}$
f **i** 10 : 15 and 2 : 3 **ii** Yes
3 **a** 1 : 2 **b** 1 : 2 **c** 3 : 2 **d** 2 : 3 **e** 3 : 5
f 3 : 1 **g** 1 : 6 **h** 1 : 6 **i** 2 : 15 **j** 2 : 3
4 **a** 7 **b** 4 **c** $\frac{7}{11}$ **d** $\frac{4}{11}$
5 **a** 1 **b** 18 **c** $\frac{1}{19}$ **d** $\frac{18}{19}$
6 **a** 9 **b** $\frac{9}{22}$ **c** 13 **d** $\frac{13}{22}$
7 **a** 160 **b** 96
8 **a** 35 **b** 45
9 **a** $\frac{2}{5}$ **b** 3 : 2 **c** 2 : 3
d **i** 18 **ii** 12
10 **a** $\frac{2}{5}$ **b** 10 ha **c** 25 ha
11 **a** 15 **b** 60 **c** 21

Exercise 8B

1 **a** $4.3 = 4 + \frac{3}{10}$

b $18.62 = 10 + 8 + \frac{6}{10} + \frac{2}{100}$

c $296.007 = 200 + 90 + 6 + \frac{7}{1000}$

2

	Decimal form	Expanded rational form
a	8.5	$8 + \frac{5}{10}$
b	19.04	$10 + 9 + \frac{4}{100}$
c	57.093	$50 + 7 + \frac{9}{100} + \frac{3}{1000}$
d	704.316	$700 + 4 + \frac{3}{10} + \frac{1}{100} + \frac{6}{1000}$
e	999.104	$900 + 90 + 9 + \frac{1}{10} + \frac{4}{1000}$
f	465.007	$400 + 60 + 5 + \frac{7}{1000}$

3

	100s	10s	1s	decimal point	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	
a			1	.	5			= 1.5
b		2	8	.	0	6		= 28.06
c	1	3	7	.	4	0	9	= 137.409

4 **a** 0.5 **b** 0.3 **c** 0.07 **d** 0.09
e 0.006 **f** 0.008 **g** 0.42 **h** 0.031
i 87.23 **j** 15.308 **k** 6.123

5 **a** 5.3 **b** 14.2 **c** 9.4 **d** 11.08 **e** 12.01
f 7.09 **g** 6.002 **h** 23.005 **i** 18.006 **j** 2.0003

6 **a** 30 (tens) **b** $\frac{3}{10}$ (tenths)
c $\frac{3}{1000}$ (thousandths) **d** $\frac{3}{100}$ (hundredths)

7 **a** **i** $\frac{2}{100}$ **ii** $\frac{2}{10}$ **iii** 2 **iv** 200
b **i** $\frac{6}{10}$ **ii** $\frac{6}{1000}$ **iii** 6 **iv** 6000
c **i** 5 **ii** 50 **iii** 5000 **iv** $\frac{5}{1000}$
d **i** 9000 **ii** $\frac{9}{10}$ **iii** $\frac{9}{1000}$ **iv** 90

8 **a** True **b** True **c** False **d** True **e** True
f False **g** False **h** True **i** False

9 **a** $5.2 = \frac{52}{10}$ **b** $0.52 = \frac{52}{100}$
c $0.052 = \frac{52}{1000}$ **d** $0.0052 = \frac{52}{10\,000}$

10 **a** $\frac{6}{10}$ **b** $\frac{1}{10}$ **c** $\frac{5}{10}$ **d** $\frac{7}{10}$
e $\frac{2}{100}$ **f** $\frac{3}{100}$ **g** $\frac{4}{100}$ **h** $\frac{8}{100}$
i $\frac{1}{1000}$ **j** $\frac{9}{1000}$ **k** $\frac{7}{1000}$ **l** $\frac{12}{1000}$
m $\frac{5}{100\,000}$ **n** $\frac{6}{100\,000}$ **o** $\frac{2}{100\,000}$ **p** $\frac{1}{10\,000}$

11 **a** $0.8 = \frac{8}{10} = \frac{4}{5}$ **b** $0.06 = \frac{6}{100} = \frac{3}{50}$
c $0.056 = \frac{56}{000} = \frac{7}{125}$

12 **a** $\frac{1}{5}$ **b** $\frac{2}{5}$ **c** $\frac{1}{2}$ **d** $\frac{2}{25}$
e $\frac{1}{500}$ **f** $\frac{3}{4}$ **g** $\frac{11}{25}$ **h** $\frac{2}{125}$
i $\frac{21}{500}$ **j** $\frac{7}{5000}$ **k** $\frac{7}{2500}$ **l** $\frac{1}{1250}$

13 **a** $6 + \frac{4}{100} = 6 + \frac{1}{25} = 6\frac{1}{25}$
b $3 + \frac{96}{1000} = 3 + \frac{12}{125} = 3\frac{12}{125}$

14 **a** $9\frac{1}{5}$ **b** $8\frac{1}{2}$ **c** $2\frac{3}{5}$ **d** $1\frac{4}{5}$ **e** $12\frac{2}{25}$ **f** $3\frac{1}{25}$
g $2\frac{1}{200}$ **h** $6\frac{3}{500}$ **i** $6\frac{1}{4}$ **j** $8\frac{23}{50}$ **k** $9\frac{29}{40}$ **l** $11\frac{11}{200}$

15 **a** $\frac{3}{8} = 0.375$

$$8\overline{)3.000}\quad \begin{array}{r}0.375\\ 24\\ \hline 60\\ 56\\ \hline 40\end{array}$$

b $\frac{7}{25} = 0.28$

$$25\overline{)7.00}\quad \begin{array}{r}0.28\\ 50\\ \hline 200\\ 200\\ \hline 0\end{array}$$

c $\frac{42}{30} = \frac{7}{5} = 1.4$

$$5\overline{)7.0}\quad \begin{array}{r}1.4\\ 5\\ \hline 20\\ 20\\ \hline 0\end{array}$$

16 **a** 0.2 **b** 0.25 **c** 0.5 **d** 0.125
e 0.55 **f** 0.8 **g** 0.875 **h** 1.2
i 2.4 **j** 0.875 **k** 0.45 **l** 0.16
m 1.35 **n** 0.24 **o** 0.56 **p** 0.64

17 **a** $0.\dot{6}$ **b** $0.\dot{2}$ **c** $1.\dot{3}$ **d** $25.\dot{7}$
e $0.\dot{5}\dot{8}$ **f** $0.\dot{6}\dot{3}$ **g** $7.\dot{8}\dot{9}$ **h** $21.\dot{4}\dot{7}$
i $0.\dot{1}7\dot{6}$ **j** $0.\dot{8}4\dot{3}$ **k** $33.3\dot{1}\dot{9}$ **l** $61.\dot{8}7\dot{3}$

18 **a** $\frac{2}{9} = 0.\dot{2}$

$$9\overline{)2.0^{2}0^{2}0}\quad \begin{array}{r}0.222\ldots\\ \\ \\ \\ \\ \end{array}$$

b $\frac{5}{11} = 0.\dot{4}\dot{5}$

$$11\overline{)5.0^{6}0^{5}0^{6}0}\quad \begin{array}{r}0.4545\ldots\\ \\ \\ \\ \\ \end{array}$$

19 **a** $0.\dot{6}$ **b** $0.\dot{5}$ **c** $0.8\dot{3}$ **d** $2.2\dot{7}$
e $0.7\dot{3}$ **f** $0.41\dot{6}$ **g** $1.8\dot{3}$ **h** $1.\dot{7}14\,285\dot{7}$

Exercise 8C

1 $0.33 = 0.330 = \frac{330}{1000}$ $0.303 = \frac{303}{1000}$
$303 < 330$, $\therefore$ 0.303 is the smaller decimal.

2 **a** 0.005 **b** 0.044 **c** 0.203
d 0.71 **e** 0.909 **f** 0.35

3 **a** 0.606 **b** 0.717 **c** 0.23
d 0.61 **e** 0.77 **f** 0.311

4 $0.505 = \frac{505}{1000}$ $0.05 = 0.050 = \frac{50}{1000}$
$0.55 = 0.550 = \frac{550}{1000}$
$50 < 505 < 550$
$\therefore$ In ascending order: 0.05, 0.505, 0.55

5 **a** 0.07, 0.7, 0.707, 0.77 **b** 0.009, 0.9, 0.909, 0.99
c 0.004, 0.04, 0.404, 0.44

6 **a** 0.66, 0.606, 0.6, 0.006 **b** 0.75, 0.557, 0.507, 0.075
c 0.31, 0.301, 0.103, 0.013

7 **a** $0.5 < 0.6$ **b** $0.171 = 0.171\,00$
c $0.404 < 0.440$ **d** $0.02 < 0.20$
e $1.03 > 0.93$ **f** $5.71 = 5.7100$
g \$7.07 < \$7.70 **h** \$5.60 > \$5.06

8 **a** 0.7 **b** 7.6 **c** 13.4 **d** 6.15

Exercise 8D

1 **a** 3 **b** 4 **c** 4 **d** 5 **e** 5

2 **a** $8 > 5$, so round up. $\therefore 4.8 \approx 5$
b $3 < 5$, so round down. $\therefore 12.316 \approx 12$
c $5 = 5$, so round up. $\therefore 9.59 \approx 10$
d $2 > 5$, so round down. $\therefore 7.21 \approx 7$

3 **a** 3 **b** 5 **c** 20 **d** 12
e 24 **f** 53 **g** 102 **h** 128

4 **a** $4 < 5$, so round down. $\therefore 0.94 \approx 0.9$
b $5 = 5$, so round up. $\therefore 8.651 \approx 8.7$
c $2 < 5$, so round down. $\therefore 13.22 \approx 13.2$
d $6 > 5$, so round up. $\therefore 24.569 \approx 25.0$

5 **a** 5.1 **b** 9.0 **c** 11.6 **d** 22.3
e 62.9 **f** 43.2 **g** 110.0 **h** 35.3
i 88.1 **j** 155.8 **k** 127.7 **l** 264.4

6 **a** $6 > 5$, so round up. $\therefore 7.8764 \approx 7.88$
b $5 = 5$, so round up. $\therefore 0.3552 \approx 0.36$
c $1 < 5$, so round down. $\therefore 12.6212 \approx 12.62$
d $7 > 5$, so round up. $\therefore 45.067 \approx 45.07$

7 **a** 2.91 **b** 7.19 **c** 13.56 **d** 38.49
e 21.22 **f** 34.15 **g** 21.04 **h** 65.03
i 82.53 **j** 73.35 **k** 139.13 **l** 273.35

8 **a** $7 > 5$, so round up. $\therefore 5.113\,72 \approx 5.114$
b $5 = 5$, so round up. $\therefore 0.0875 \approx 0.088$
c $2 < 5$, so round down. $\therefore 19.6142 \approx 19.614$
d $0 < 5$, so round down. $\therefore 24.070\,09 \approx 24.070$

9 **a** 0.007 **b** 0.099 **c** 1.764 **d** 6.041
e 9.333 **f** 12.005 **g** 15.061 **h** 21.669
i 34.414 **j** 68.184 **k** 77.714 **l** 5.055

10 **a** 4 **b** 0 **c** 4 **d** 5
e 10 **f** 17 **g** 24 **h** 11

11 **a** 0.8 **b** 0.4 **c** 0.1 **d** 0.6
e 9.6 **f** 4.2 **g** 8.6 **h** 17.6

12 **a** 1.00 **b** 5.78 **c** 27.26 **d** 54.14
e 78.85 **f** 0.23 **g** 5.14 **h** 17.19

Exercise 8E

1 **a**
$$\begin{array}{r} 136.07 \\ +\ 7.63 \\ \hline 143.70 \end{array}$$
Estimate: $136 + 8 = 144$

b
$$\begin{array}{r} 5.000 \\ +\ 0.008 \\ \hline 251.600 \\ \hline 256.608 \end{array}$$
Estimate: $5 + 0 + 252 = 257$

c
$$\begin{array}{r} 74.20 \\ -\ 8.09 \\ \hline 66.11 \end{array}$$
Estimate: $74 - 8 = 66$

2 **a** $7 + 5 = 12$, 11.7 **b** $13 + 8 = 21$, 21.1
c $1 + 7 = 8$, 7.4 **d** $9 + 6 = 15$, 15.44
e $1 + 1 = 2$, 2.1 **f** $10 + 14 = 24$, 24.54
g $86 + 12 = 98$, 98.251 **h** $10 + 8 = 18$, 18.011
i $4 - 3 = 1$, 1.4 **j** $8 - 6 = 2$, 1.88
k $11 - 3 = 8$, 7.58 **l** $19 - 12 = 7$, 6.28
m $39 - 18 = 21$, 21.56 **n** $67 - 29 = 38$, 38.531
o $42 - 17 = 25$, 25.468

3 **a** 35.7194 **b** 14.897 **c** 19.6627

4 **a** 10.1994 **b** 61.901 **c** $2.2\dot{1}\dot{1}$

Exercise 8F

1 **a** *Estimate:* $3 \times 8 = 24$
$$\begin{array}{r} 341 \\ \times\ \ 8 \\ \hline 2728 \end{array}$$
2 digits after the decimal point.
$\therefore 3.41 \times 8 = 27.28$

b *Estimate:* $1 \times 0 = 0$
$11 \times 5 = 55$
4 digits after the decimal points.
$\therefore 1.1 \times 0.005 = 0.0055$

2 **a** 1 **b** 1 **c** 2 **d** 2

3 **a** 4 **b** 15.54 **c** 40.735 **d** 12×7

4 **a** 6, 5.7 **b** 42, 39.9 **c** 24, 21.6
d 72, 68 **e** 45, 46 **f** 90, 92.7
g 44, 44.4 **h** 32, 31.8

5 **a** 1.6 **b** 1.2 **c** 16.2 **d** 23.6
e 35.1 **f** 124.8 **g** 61.2 **h** 159.6

6 **a** 1.5 **b** 0.15 **c** 0.015
d 0.0015 **e** 0.000 15 **f** 0.0015
g 0.000 15 **h** 0.000 015 **i** 0.000 000 15

7 **a** Answer has 2 digits after the decimal point.
$\therefore 0.6 \times 1.2 = 0.72$
b Answer has 4 digits after the decimal point.
$\therefore 0.002 \times 11.1 = 0.0222$

8 **a** 86.4 **b** 1.007 **c** 63.7 **d** 11.51

9 **a** When multiplying numerals, it does not matter in what order the numerals are multiplied as the result is always the same. So 6×2 is the same as 2×6.
b When multiplying a numeral by 1, the result is always the numeral. So $5.62 \times 1 = 5.62$.
c When multiplying a numeral by zero, the result is always zero. So $1.36 \times 0 = 0$.

10 **a** $0.05 \times 10 = 0.5$
1 zero, $\therefore$ move decimal point 1 place to the right.
b $0.05 \times 100 = 5$
2 zeros, $\therefore$ move decimal point 2 places to the right.
c $0.05 \times 1000 = 50$
3 zeros, $\therefore$ move decimal point 3 places to the right.
d $0.05 \times 10\,000 = 500$
4 zeros, $\therefore$ move decimal point 4 places to the right.

11 **a** 38 **b** 416 **c** 20.9
d 70 **e** 251 **f** 640
g 43 600 **h** 9 **i** 780
j 6300 **k** 4000 **l** 216 470

Exercise 8G

1 **a**
$$2\overline{)0.08}\quad 0.04$$
$\therefore 0.08 \div 2 = 0.04$

b
$$5\overline{)25.5}\quad 5.1$$
$\therefore 25.5 \div 5 = 5.1$

c
$$4\overline{)48.12}\quad 12.03$$
$\therefore 48.12 \div 4 = 12.03$

2 **a** 0.2 **b** 0.2 **c** 0.3
d 0.3 **e** 0.6 **f** 0.08
g 0.9 **h** 0.62 **i** 1.31
j 3.08 **k** 12.3 **l** 10.05
m 60.6 **n** 1.204 **o** 1.606

3 **a** $4.8 \div 0.2 = 48 \div 2 = 24$
Multiply both numbers by 10 to make 0.2 a whole number.
b $1.25 \div 0.05 = 1250 \div 5 = 250$
Multiply both numbers by 100 to make 0.05 a whole number.
c $3.672 \div 0.006 = 3672 \div 6 = 612$.
Multiply both numbers by 1000 to make 0.006 a whole number.

4 **a** 1.2 **b** 1.1 **c** 0.7
d 0.9 **e** 0.52 **f** 82
g 7.24 **h** 11.9 **i** 116.2

5 **a** \$1.26 **b** \$1.75 **c** \$5.30 **d** \$9.60

6 **a** $53.2 \div 10 = 5.32$
1 zero, $\therefore$ move decimal point 1 place to the left.
b $53.2 \div 100 = 0.532$
2 zeros, $\therefore$ move decimal point 2 places to the left.
c $53.2 \div 1000 = 0.0532$
3 zeros, $\therefore$ move decimal point 3 places to the left.
d $53.2 \div 10\,000 = 0.005\,32$
4 zeros, $\therefore$ move decimal point 4 places to the left.

7 **a** 3.74 **b** 21.43 **c** 0.07
d 0.003 **e** 0.089 **f** 0.429
g 0.008 **h** 0.0055 **i** 0.0541
j 0.3872 **k** 0.000 86 **l** 0.0043

Exercise 8H

1 **a** $7\% = \frac{7}{100}$ **b** $19\% = \frac{19}{100}$ **c** $41\% = \frac{41}{100}$

2 **a** $\frac{17}{100}$ **b** $\frac{49}{100}$ **c** $\frac{51}{100}$ **d** $\frac{77}{100}$ **e** $\frac{91}{100}$
f $\frac{11}{100}$ **g** $\frac{83}{100}$ **h** $\frac{69}{100}$ **i** $\frac{33}{100}$ **j** $\frac{27}{100}$

3 **a** $\frac{1}{10}$ **b** $\frac{1}{5}$ **c** $\frac{3}{20}$ **d** $\frac{9}{10}$ **e** $\frac{3}{5}$
f $\frac{11}{20}$ **g** $\frac{1}{2}$ **h** $\frac{2}{25}$ **i** $\frac{23}{50}$ **j** $\frac{29}{50}$

4 **a** $700\% = \frac{700}{100} = 7$

b $950\% = 900\% + 50\%$
$= \frac{900}{100} + \frac{50}{100}$
$= 9 + \frac{1}{2}$
$= 9\frac{1}{2}$

c $615\% = 600\% + 15\%$
$= \frac{600}{100} + \frac{15}{100}$
$= 6 + \frac{3}{20}$
$= 6\frac{3}{20}$

5 **a** 11 **b** 15 **c** 29 **d** $34\frac{1}{2}$
e $1\frac{3}{10}$ **f** $1\frac{1}{4}$ **g** $1\frac{3}{4}$ **h** $1\frac{12}{25}$
i $2\frac{27}{50}$ **j** $2\frac{11}{50}$

6 **a** $5\frac{1}{2}\% = \frac{11}{200}$ **b** $9\frac{1}{3}\% = \frac{28}{300} = \frac{7}{75}$
c $12\frac{1}{2}\% = \frac{25}{200} = \frac{1}{8}$

7 **a** $\frac{1}{15}$ **b** $\frac{23}{150}$ **c** $\frac{14}{75}$ **d** $\frac{11}{250}$
e $\frac{11}{125}$ **f** $\frac{41}{200}$ **g** $\frac{7}{300}$ **h** $\frac{1}{1000}$
i $\frac{11}{80}$ **j** $\frac{17}{160}$

Exercise 8I

1 **a** $5\% = \frac{5}{100} = 5 \div 100 = 0.05$

b $13\% = \frac{13}{100} = 13 \div 100 = 0.13$

c $60\% = \frac{60}{100} = 60 \div 100 = 0.6$

2 **a** 0.38 **b** 0.02 **c** 0.44 **d** 0.09
e 0.57 **f** 0.21 **g** 0.66 **h** 0.52
i 0.08 **j** 0.8

3 **a** $250\% = \frac{250}{100} = 250 \div 100 = 2.5$

b $3.6\% = \frac{3.6}{100} = 3.6 \div 100 = 0.036$

c $87.25\% = \frac{87.25}{100} = 87.25 \div 100 = 0.8725$

4 **a** 1.87 **b** 1.5 **c** 1.32 **d** 1.73
e 2.45 **f** 0.009 **g** 0.916 **h** 0.474
i 0.4109 **j** 0.7394

5 **a** $7\frac{1}{4}\% = 7.25\% = \frac{7.25}{100} = 7.25 \div 100 = 0.0725$

b $105\frac{2}{5}\% = 105.4\% = \frac{105.4}{100} = 105.4 \div 100 = 1.054$

6 **a** 0.155 **b** 0.236 **c** 0.0925 **d** 0.424
e 0.0875 **f** 0.074 **g** 0.868 **h** 0.025
i 1.402 **j** 3.665

Exercise 8J

1 **a** $\frac{5}{100} \times \frac{100}{1} = 5\%$ **b** $\frac{17}{100} \times \frac{100}{1} = 17\%$
c $\frac{24}{100} \times \frac{100}{1} = 24\%$ **d** $\frac{76}{100} \times \frac{100}{1} = 76\%$
e $\frac{7}{10} \times \frac{100}{1} = 70\%$ **f** $\frac{9}{10} \times \frac{100}{1} = 90\%$

2 **a** 25% **b** 30% **c** 94%
d 76% **e** 110% **f** 126%

3 **a** 44.44% **b** 66.67%
c 42.86% **d** 36.36%
e 311.11% **f** 183.33%

4 **a** $0.09 \times 100\% = 9\%$ **b** $0.87 \times 100\% = 87\%$
c $0.003 \times 100\% = 0.3\%$ **d** $0.592 \times 100\% = 59.2\%$

5 **a** 27% **b** 56% **c** 71%
d 99% **e** 203% **f** 6%
g 61.5% **h** 21.1% **i** 90.9%
j 44.4% **k** 960% **l** 630%
m 1040% **n** 60% **o** 700%
p 300% **q** 100% **r** 0.4%
s 66.2% **t** 91%

6 **a** 31% **b** 18% **c** 27%
d 63% **e** 100% **f** 500%
g 781% **h** 952% **i** 1205%
j 1010%

7

	a	b	c	d	e
Figure					
Fraction shaded	$\frac{1}{5}$	$\frac{2}{8} = \frac{1}{4}$	$\frac{6}{10} = \frac{3}{5}$	$\frac{4}{8} = \frac{1}{2}$	$\frac{13}{25}$
Percentage shaded	20%	25%	60%	50%	52%
Percentage unshaded	80%	75%	40%	50%	48%

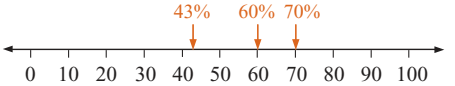
8 **a** 40% **b** 5% **c** 12% **d** 65% **e** 73%
f 62% **g** 20% **h** 4.7% **i** 15% **j** 508%

9

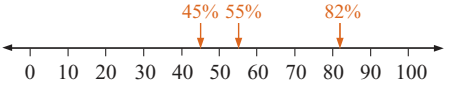
	Fraction	Decimal	Percentage
a	$\frac{3}{20}$	0.15	15%
b	$\frac{8}{10} = \frac{4}{5}$	0.8	80%
c	$\frac{4}{1000} = \frac{1}{250}$	0.004	0.4%
d	$\frac{5}{8}$	0.625	$62\frac{1}{2}\%$
e	$4\frac{3}{10}$	4.3	430%
f	$\frac{95}{1000} = \frac{19}{200}$	0.095	$9\frac{1}{2}\%$
g	$\frac{65}{100} = \frac{13}{20}$	0.65	65%
h	$1\frac{2}{5}$	1.4	140%
i	$\frac{21}{100}$	0.21	21%
j	$6\frac{3}{4}$	6.75	675%
k	$5\frac{1}{40}$	5.025	502.5%
l	$\frac{1}{200}$	0.005	0.5%

10 **a** $\frac{1}{2}$, 72%, 0.86 **b** $\frac{3}{4}$, 0.79, 86%
c 0.32, 55%, $\frac{5}{8}$ **d** 0.27%, 21%, $\frac{1}{4}$
e $57\frac{1}{2}\%$, 0.582, $\frac{3}{5}$ **f** 0.16, $19\frac{1}{3}\%$, $\frac{1}{5}$

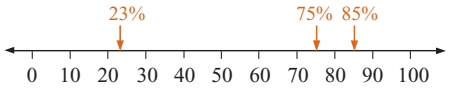
11 **a** $\frac{3}{5} = 60\%$, 70%, $0.43 = 43\%$



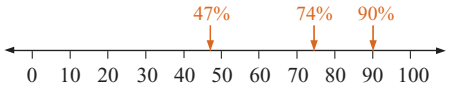
b 55%, $\frac{9}{20} = 45\%$, $0.82 = 82\%$



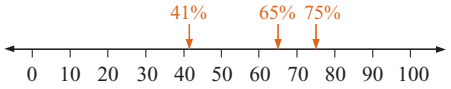
c 0.85 = 85%, $\frac{3}{4}$ = 75%, 23%



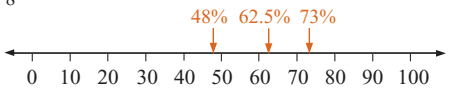
d 47%, 0.74 = 74%, $\frac{18}{20}$ = 90%



e $\frac{3}{4}$ = 75%, 0.65 = 65%, 41%



f $\frac{5}{8}$ = 62.5%, 73%, 0.48 = 48%



Exercise 8K

1 a 20% of 40 kg = $\frac{1}{5} \times \frac{40}{1} = 1 \times 8 = 8$ kg

b 36% of 150 m = $\frac{36}{100} \times \frac{150}{1} = \frac{9}{25} \times \frac{150}{1} = 9 \times 6 = 54$ m

c $12\frac{1}{2}$% of \$60 = $\frac{1}{8} \times \frac{60}{1} = \frac{1}{2} \times 15 = \$7.50$

2 a 4 L **b** 99.5 km **c** 7 m **d** 10 s
e \$4000 **f** \$3 000 000 **g** 27 t **h** 19 cm
i 150 mL **j** 400 paces **k** 90 days **l** 3500 kg

3 a 50 g **b** 8 m **c** 7 m **d** 20 kg
e 1 L **f** 3 km **g** \$3 **h** \$3
i \$45 **j** \$100 **k** 60 m **l** \$90

4 a 4.25 km **b** 3.84 kg **c** 7.2 m
d \$58.50 **e** 160 L **f** 49.28 kg
g 75.6 L **h** \$57.12 **i** \$8.19

5 a \$2.40 **b** 9.9 L **c** 1.2 m
d 45 kg **e** \$10.40 **f** 416 km

6 114

7 a \$150 **b** \$240 **c** \$210

Exercise 8L

1 Box A = \$1.14 per 100 g and Box B = \$1.01 per 100 g, so Box B is the better buy.

2 a 500 g at \$4.70 **b** 1 kg at \$20.50
c 400 g at \$1.24 **d** 550 g at \$9.35
e 1.3 kg at \$2.90 **f** 1.2 kg at \$2.28

3 a i 500 g jar at \$16.10 **ii** 400 g tin at \$16.40
iii 250 g block at \$4.76 **iv** 1.25 kg tin at \$16.16
v 455 g jar at \$7.66 **vi** 445 g tin at \$4.10
vii 425 g tin at \$6.60 **viii** 750 mL bottle at \$2.24
b Less packaging per volume, leading to cheaper storage and transport

4 a \$4.60 **b** \$0.28 **c** \$0.22

5 a 10 L of paint for \$163.80 **b** 5 DVDs for \$40
c 50 tickets for \$1200

6 a Paying for each member of the family (\$30 total)
b \$8

7 a 2 × 4 L at \$38 **b** 2 × 10 L (total \$98)
c 5 × 4 L (total \$95)

8 a i \$1228 **ii** \$1300
b i Ocean Palms **ii** Ocean Palms
c i Trade Winds **ii** Trade Winds
d i Ocean Palms \$205.60, Trade Winds \$190
ii Ocean Palms \$180, Trade Winds \$178.75

Calculator activities

1 a $\frac{5}{7}$ **b** $\frac{3}{4}$ **c** $\frac{41}{64}$
d $\frac{51}{4} = 12\frac{3}{4}$ **e** $\frac{43}{11} = 3\frac{10}{11}$

2 a 59% **b** 60% **c** 510% **d** 37.5% **e** 550%

3 a $\frac{2}{25}$ **b** $\frac{11}{20}$ **c** $\frac{213}{5000}$ **d** $\frac{1}{8}$ **e** $\frac{103}{500}$

4 a 0.92 **b** 4.05 **c** 0.0007 **d** 0.064 **e** 0.093 75

5 a $\frac{5}{4}$ **b** $\frac{8}{3}$ **c** $\frac{29}{5}$ **d** $\frac{19}{3}$ **e** $\frac{111}{11}$

6 a $4\frac{1}{5}$ **b** $8\frac{1}{4}$ **c** $10\frac{1}{2}$ **d** $13\frac{1}{7}$ **e** $7\frac{4}{9}$

7 a $\frac{58}{99}$ **b** $7\frac{11}{14}$ **c** $1\frac{3}{40}$
d $5\frac{47}{63}$ **e** 51 **f** $248\frac{2}{5}$

8 a \$12.40 **b** \$196 **c** \$25.50
d 308 L **e** 84 m **f** 104 kg

9 a 0.65 **b** 0.8 **c** 0.75 **d** 0.84 **e** $15.\dot{6}\dot{3}$

10 a 4.27 **b** 1.3 **c** 23.707 **d** 2.106
e 3.52 **f** 5 **g** 23.66 **h** 11.2
i 4.88 **j** 18.495 **k** 560 **l** 26.4

11 a 10.83 kg **b** \$47.56 **c** \$128.70 **d** 67

12 a 114.6 m **b** 2656.5 t **c** 10.8 km **d** 153.75 L
e \$238 **f** 33 m

13 a 0.44 **b** 0.67 **c** 0.71 **d** 0.58
e 4.11 **f** 74.03 **g** 7.52 **h** 2.31

14 Place into calculator a 'fix' to zero decimal places and the answer is 1. Applying the rule that $9 > 5$, $0.\dot{9}$ rounds up to 1.

Language in mathematics

I	A	P	P	R	O	X	I	M	A	T	E	S
N	I	T	M	O	N	S	T	A	E	H	V	E
V	G	R	N	C	S	B	L	O	O	T	K	S
E	N	E	T	N	O	I	T	I	D	D	A	U
N	I	V	H	E	G	S	T	E	N	E	T	B
T	T	N	H	N	S	H	T	A	M	R	E	T
E	A	O	S	L	A	M	I	C	E	D	E	R
D	E	C	I	M	A	L	P	O	I	N	T	A
E	P	X	M	P	A	L	A	V	T	U	I	C
N	E	E	D	O	T	D	I	H	H	H	E	T
W	R	O	R	K	N	D	D	I	N	G	O	F
D	E	U	L	A	V	E	C	A	L	P	E	C
M	U	L	T	I	P	L	Y	I	M	A	L	S

1 Find-a-word message:
Simon Steven's book the tenth explained the working of decimals.

4 a their, favourite, Mathematics, decided, competition, teacher, answer
b Sue-Lin

5 a decilitre **b** decile
c decibel **d** decimate

Check your skills

1 B	**2** D	**3** B	**4** C	**5** B
6 C	**7** B	**8** B	**9** A	**10** C
11 D	**12** A	**13** B	**14** D	**15** C
16 A	**17** D	**18** B	**19** C	**20** B
21 A	**22** C	**23** D		

Review set 8A

1 **a** $\frac{8}{13}$ **b** 5 : 8

2

3 56%

4 **a** 37.5% **b** 62.5%

5 $\frac{41}{100}$ **6** 0.63

7 22.409

8 **a** 560% **b** 25%

9 13%, 40%, 63% **10** 3

11 4.33

12 **a** = **b** <

13 **a** 18 **b** 1490 **c** 197.64

14 **a** 7 **b** 8.47 **c** 94

15 **a** 0.74 **b** 352.8 **c** 24.05

16 8 m **17** 750 g at $7.25

Review set 8B

1 **a** 25% **b** 75%

2 7 : 4

3 $\frac{28}{100} = \frac{7}{25}$ $\frac{42}{100} = \frac{21}{50}$

4 $\frac{5}{100}$ **5** 16.075

6 $\frac{21}{10}$ **7** $\frac{1205}{100} = \frac{241}{20}$

8 0.375

9 **a** 44% **b** 0.061 25

10 10 **11** 0.21, 0122, 0.12, 0.02

12 **a** 30.2 **b** 22.04

13 **a** 41.17 **b** 3.358

14 **a** 27.2 **b** 11.93

15 23.97°C **16** $\frac{18}{30} \times \frac{100}{1} = 60\%$

17 $1.71

Review set 8C

1 4 : 6 = 2 : 3 **2** 27

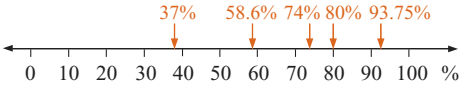
3 **a** 1.56 **b** 0.075

4 **a** 150 kg **b** $612.50

5 8000 **6** 0.079

7 $\frac{8}{100} = \frac{2}{25}$ **8** $0.\dot{7}$

9 37%, 93.75%, 74%, 80%, 58.6%



10 15.08 **11** 0.83

12 **a** > **b** =

13 B

14 **a** 72 **b** 3.096 **c** 23.68

15 **a** 74.55 **b** 7.12

16 $531.25

17 **a** 12.75 m **b** 2.65 m

CHAPTER 9 TRANSFORMATIONS AND SYMMETRY

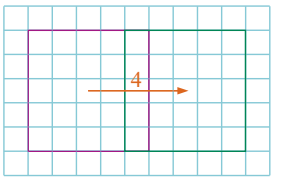
Diagnostic test

1 C **2** A **3** C **4** C **5** D

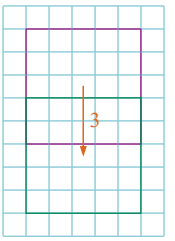
6 C **7** B **8** B **9** B **10** A

Exercise 9A

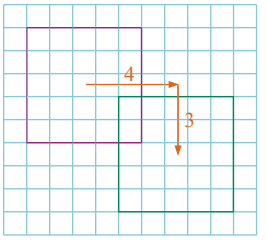
1 **a**



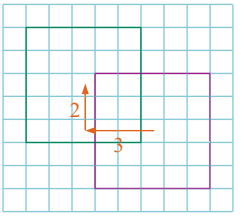
b



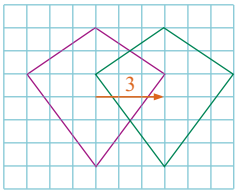
c



d



2 **a** **i**



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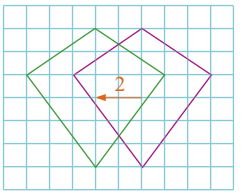

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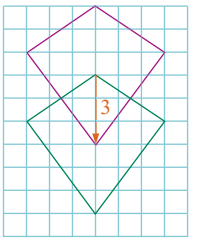

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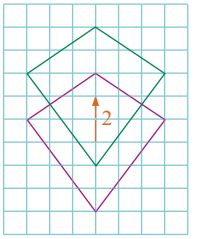

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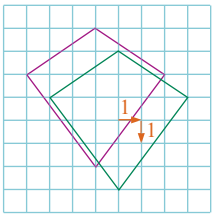

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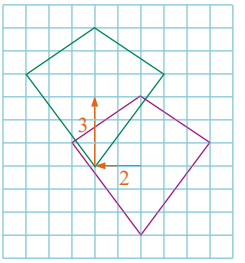


b **i** Yes **ii** Yes **iii** Yes

3 **a** Yes, yes **b** Yes, yes

c When a shape is translated, every point on it moves the same distance in the same direction.

4 **a** Translation 8 right then 1 down

b Translation 1 right then 2 up

c Translation 4 left then 3 up

d Translation 4 right then 2 up

5 **a** Translation 8 left then 1 up

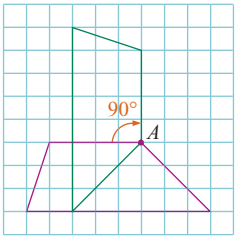
b Translation 1 left then 2 down

c Translation 4 right then 3 down

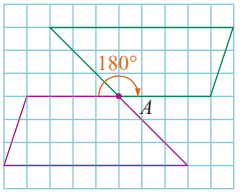
d Translation 4 left then 2 down

Exercise 9B

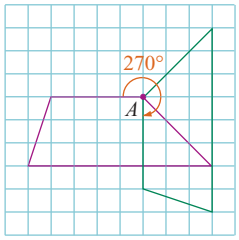
1 **a**



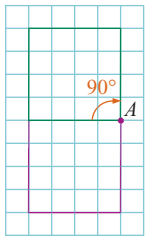
b



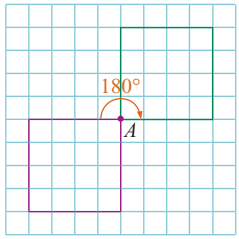
c



2 a i



ii



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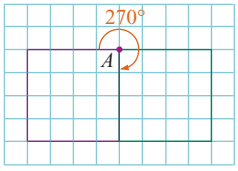
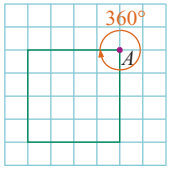


iv



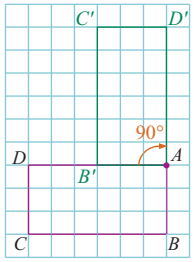
b i Yes **ii** Yes **iii** Yes

3 a Yes, yes, yes **b** Yes, yes, yes

c i When a shape is rotated, every point on it moves through the same angle in the same direction.

ii The distance from the centre of rotation to any point on the shape stays the same.

4 a i



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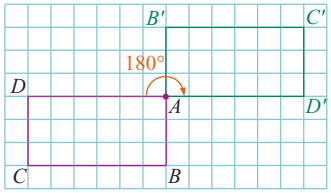

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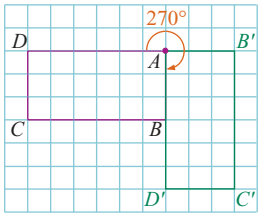

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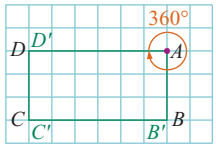
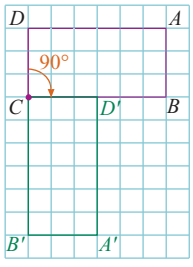


b i



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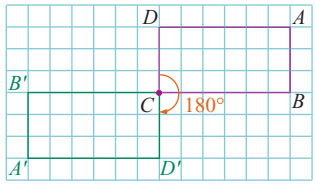

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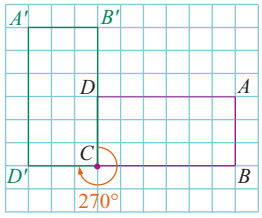

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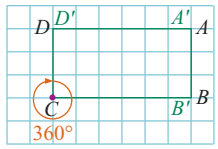
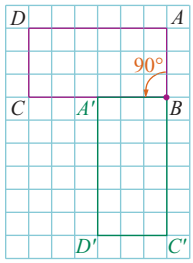


c i



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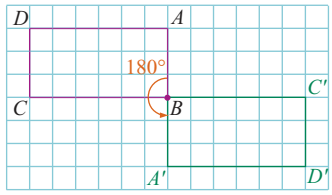

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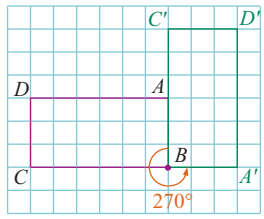
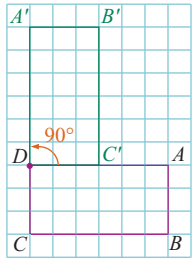


d i



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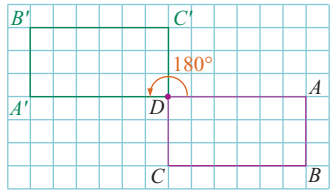

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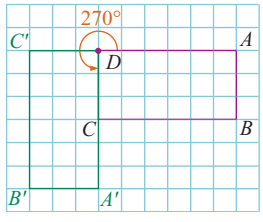
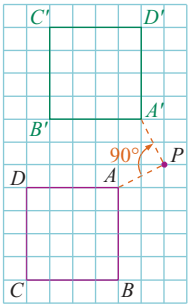
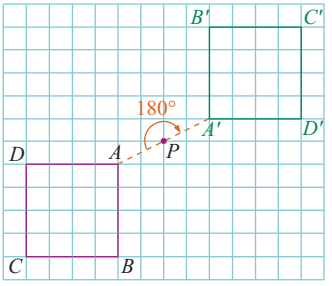


5 a i Rotation clockwise about Q through 90°
ii Rotation anticlockwise about Q through 90°
b i Rotation anticlockwise about M through 90°
ii Rotation clockwise about M through 90°
c i Rotation clockwise (or anticlockwise) about C through 180°
ii Rotation anticlockwise (or clockwise) about C through 180°
d i Rotation clockwise about S through 90°
ii Rotation anticlockwise about S through 90°
e i Rotation anticlockwise about A through 90°
ii Rotation clockwise about A through 90°
f i Rotation anticlockwise about C through 90°
ii Rotation clockwise about C through 90°

6 a i



ii



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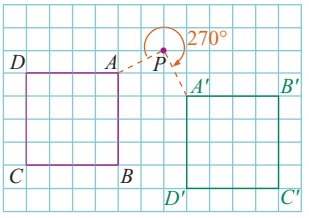
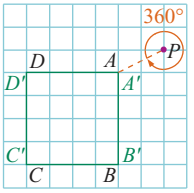


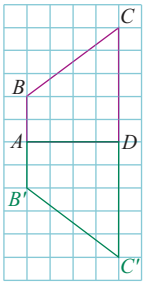
iv.,



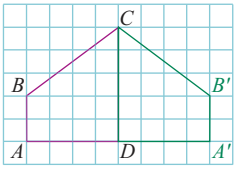
b i Yes **ii** Yes **iii** Yes

Exercise 9C

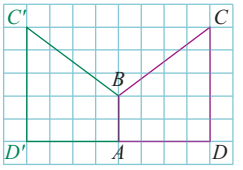
1 a



b

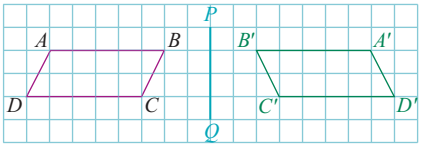


c

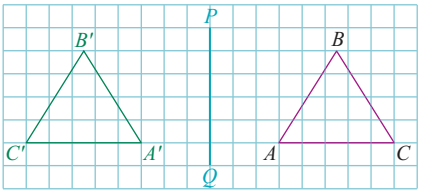


2 a i K **ii** L **iii** M **iv** N
b 90° **c** Distances are equal.
d Another reflection in line PQ

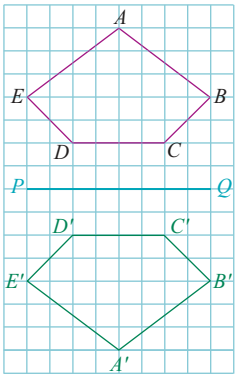
3 a



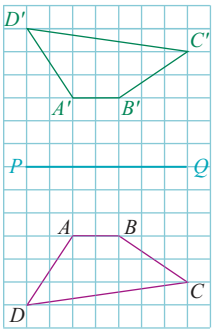
b



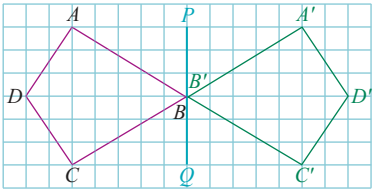
c



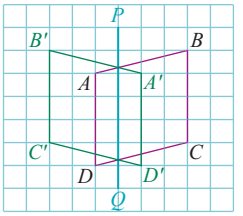
d



e

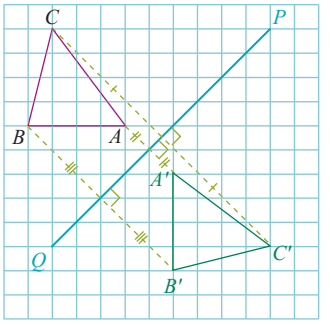


f

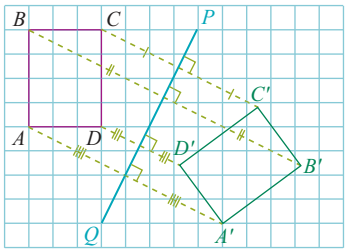


4 a Yes **b** Yes **c** Yes
5 Another reflection in line PQ

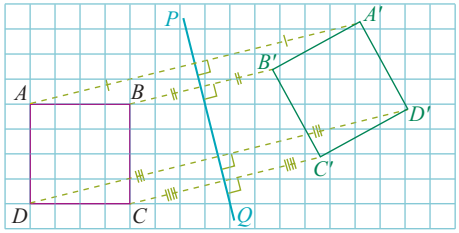
6 a



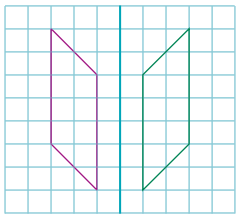
b



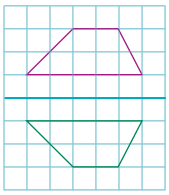
c



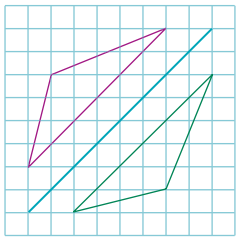
7 a



b



c



Exercise 9D

1 a Translated or rotated **b** Rotated
c Reflected **d** Rotated **e** Rotated
f Rotated **g** Translated or reflected
h Rotated **i** Translated or reflected or rotated
j Reflected **k** Rotated **l** Reflected

2 a Yes (part **1a**) **b** Yes (part **1g**) **c** Yes (part **1i**)

3 a Reflection then translation 3 right 2 down
b Rotation 90° clockwise then reflection
c Translation 4 right 2 up then reflection
d Reflection then translation 4 up
e Translation 5 up then rotation 90° clockwise
f Translation 4 right then rotation 180° clockwise or anticlockwise

4 a i Translation 7 right and 1 up then translation 4 left and 5 down
ii Translation 3 right and 4 down
b i Clockwise rotation through 90° then another clockwise rotation through 90°
ii Clockwise (or anticlockwise) rotation through 180°
c i Anticlockwise rotation through 90° then anticlockwise rotation through 90°
ii Clockwise (or anticlockwise) rotation through 180°
e i Reflection then another reflection
d i Reflection then another reflection
ii Translation 9 right
ii Translation 6 down
f i Anticlockwise rotation through 90° then reflection
ii Reflection
g i Reflection followed by reflection
ii Rotation clockwise through 180°
h i Rotation clockwise through 90° followed by rotation clockwise through 90°
ii Reflection

5 a Yes (part **4g**) **b** Yes (part **4h**)
c Yes, if the lines of reflection are parallel.

6 b Rhombus

7 b Rhombus

9 a Hexagon, translation or reflection or rotation
b Right-angled triangle, reflection
c Octagon, translation or reflection or rotation

Exercise 9E

1 a i (5, 4) **ii** (−1, 4) **iii** (3, 6) **iv** (3, −2)
v (−2, 3) **vi** (6, 0) **vii** (0, −1) **viii** (4, 7)
b i (3, −4) **ii** (−3, 4) **iii** (−1, 4)
iv (3, −8) **v** (−3, −4) **vi** (−3, −4)
vii (−1, −8) **viii** (−1, −8) **ix** (−1, −4)
x (−3, −8)
c i (4, −3) **ii** (−3, −4) **iii** (−4, 3)
iv (3, 4)
d i (6, −4) **ii** (4, −1) **iii** (−1, 1)
iv (−2, 0) **v** (4, 1)

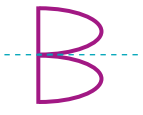
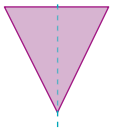
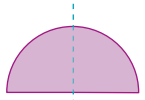
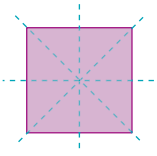
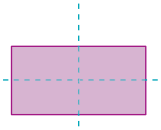
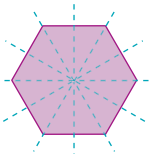
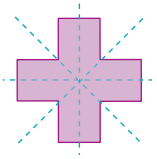
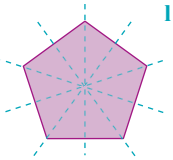
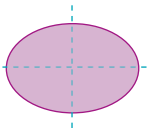
2 a $A'(2, 4)$, $B'(6, 4)$, $C'(6, 7)$, $D'(2, 7)$
b $A'(10, 4)$, $B'(6, 4)$, $C'(6, 1)$, $D'(10, 1)$
c $A'(2, 4)$, $B'(-2, 4)$, $C'(-2, 1)$, $D'(2, 1)$
d $A'(2, -2)$, $B'(6, -2)$, $C'(6, 1)$, $D'(2, 1)$
e $A'(2, -4)$, $B'(6, -4)$, $C'(6, -1)$, $D'(2, -1)$
f $A'(-2, 4)$, $B'(-6, 4)$, $C'(-6, 1)$, $D'(-2, 1)$

3 a $A'(3, 2)$, $B'(6, 2)$, $C'(6, -1)$, $D'(3, -1)$
b $A'(3, 2)$, $B'(3, -1)$, $C'(0, -1)$, $D'(0, 2)$
c $A'(3, 2)$, $B'(0, 2)$, $C'(0, 5)$, $D'(3, 5)$

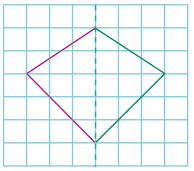
4 a $A'(2, -2)$, $B'(5, -2)$, $C'(5, -7)$, $D'(2, -7)$
b $A'(-2, -2)$, $B'(-2, -5)$, $C'(-7, -5)$, $D'(-7, -2)$
c $A'(-2, 2)$, $B'(-5, 2)$, $C'(-5, 7)$, $D'(-2, 7)$

5 a $A'(3, -6)$, $B'(7, -10)$, $C'(10, -7)$, $D'(6, -3)$
b $A'(2, -1)$, $B'(6, -5)$, $C'(3, -8)$, $D'(-1, -4)$
c $A'(-1, -5)$, $B'(-5, -9)$, $C'(-8, -6)$, $D'(-4, -2)$

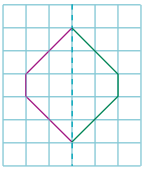
Exercise 9F

1 a 
b 
c 
d none
e 
f 
g 
h 
i 
j 
k 
l 

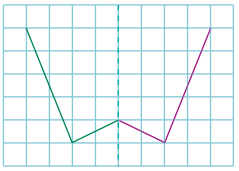
2 a



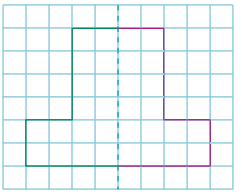
b



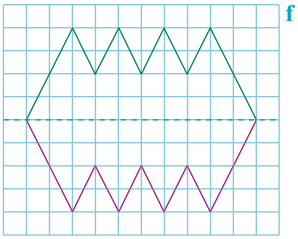
c



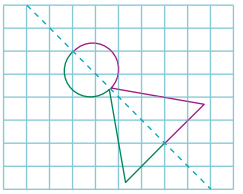
d



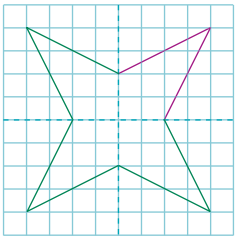
e



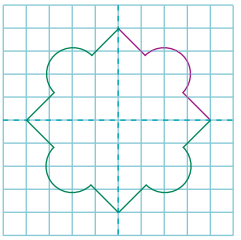
f



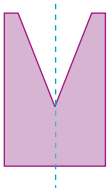
3 a



b



5 b

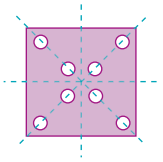


6 b Rhombus **c** 2

7, 8 b Octagon **c** Yes

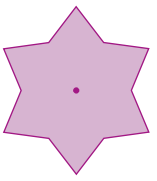
9 a Infinite **b** Fold the circle twice.

10 b

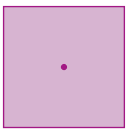


Exercise 9G

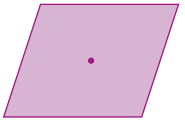
1 a 6 times **b** 6 **c**



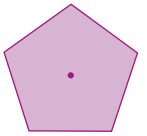
2 a Order 4



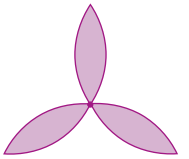
b Order 2



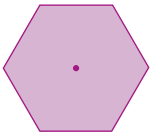
c Order 5



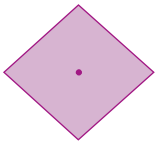
d Order 3



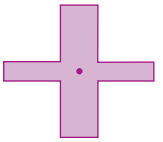
e Order 6



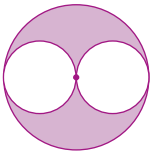
f Order 2



g Order 2

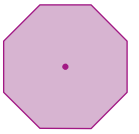


h Order 2

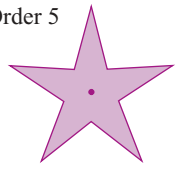


i No rotational symmetry

j Order 8



k Order 5



l Order 2

4 a, b, e, f, g, h, j and l

5 Any shape with order of symmetry an odd number

6

Shape	Number of axes of symmetry	Order of rotational symmetry
a	2	2
b	1	None
c	4	4
d	8	8
e	1	None
f	2	2
g	6	6
h	0	None
i	0	5
j	0	3
k	3	3
l	4	4
m	Infinite	Infinite
n	1	None
o	2	2
p	2	2
q	1	None
r	1	None
s	2	2
t	1	None

7

Quadrilateral	Number of axes of symmetry	Order of rotational symmetry
Trapezium	0	None
Parallelogram	0	2
Rectangle	2	2
Rhombus	2	2
Square	4	4
Kite	1	None

Language in mathematics

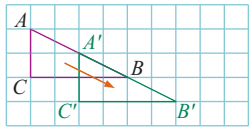
1 **a** translation **b** turning **c** reflection
d transformations **e** same, same **f** image
g centre **h** same, same **i** right
j same
3 vertices

Check your skills

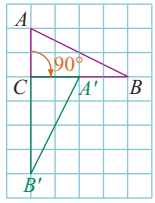
1 D **2** D **3** C **4** A **5** D
6 B **7** C **8** C **9** C **10** B

Review set 9A

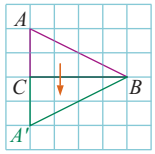
1 **a**



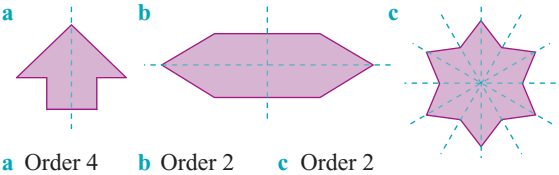
b



c



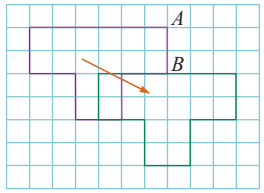
2 **a** Translated or rotated **b** Reflected
c Rotated
3 **a** (−1, 5) **b** (6, 8) **c** (−6, −8) **d** (8, 6)
4 **a** **b** **c**



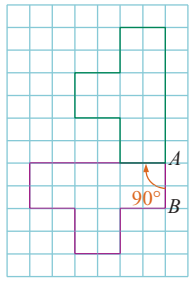
5 **a** Order 4 **b** Order 2 **c** Order 2

Review set 9B

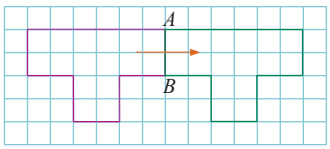
1 **a**



b

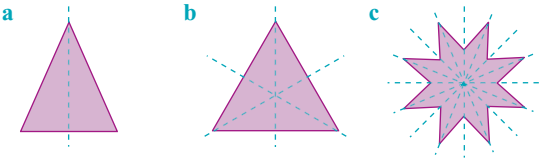


c



2 **a** Reflected or rotated **b** Reflected or rotated
c Translated
3 **a** $A'(-3, 4)$, $B'(0, 4)$, $C'(0, 0)$, $D'(-3, 0)$
b $A'(-2, 7)$, $B'(-5, 7)$, $C'(-5, 3)$, $D'(-2, 3)$
c $A'(2, -1)$, $B'(5, -1)$, $C'(5, 3)$, $D'(2, 3)$

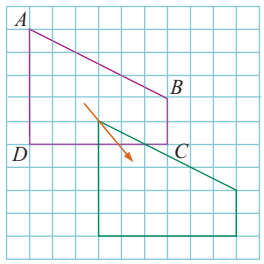
4 **a** **b** **c**



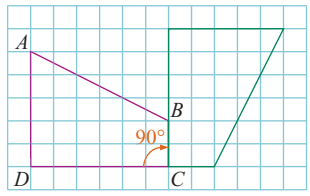
5 **a** No rotational symmetry
b Order 3 **c** Order 8

Review set 9C

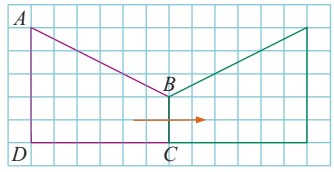
1 **a**



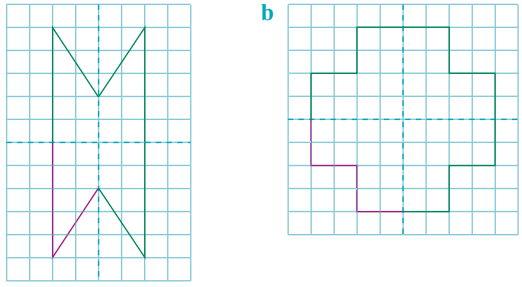
b



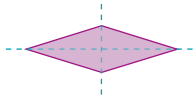
c



2 **a** Translated **b** Rotated
c Translated or reflected or rotated
3 **a** (8, 2) **b** (5, −12) **c** (12, −5)
4 **a** **b**



5 Example:



CHAPTER 10 PROBABILITY

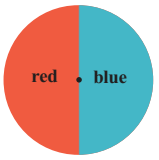
Diagnostic test

1 D **2** C **3** C **4** B **5** D
6 B 7 B **8** D **9** D **10** C

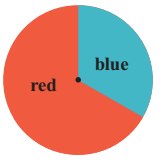
Exercise 10A

1 **a** Selecting a card from a hat
b M, A (or T, H, S)
c S = {M, A, T, H, S}
d Yes, the card is selected at random.

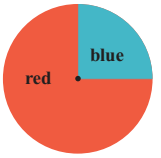
2 **a** Selecting a ball from a bag
b black, white (or green)
c S = {black, white, green}
d Yes, the ball is selected at random.
3 **a** {1, 2, 3, 4, 5, 6, 7, 8} **b** Yes
4 **a** {red, white, blue} **b** Yes
5 **a** {1, 3, 5, 7, 9, 11} **b** Yes
6 **a** {red, blue} **b** No
7 **a** {1, 2, 3} **b** No
8 **a**



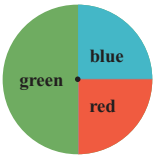
b



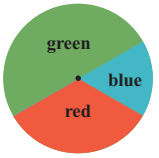
c



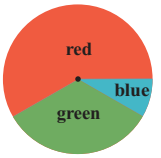
9 **a**



b



c



10 **a** {boy, girl} **b** Yes (very close to equal)
11 **a** {green, amber, red} **b** No
12 **a** {win, lose, draw} **b** No
13 **a** Yes **b** No **c** {red, green, blue}
14 **a** Yes **b** No

Exercise 10B

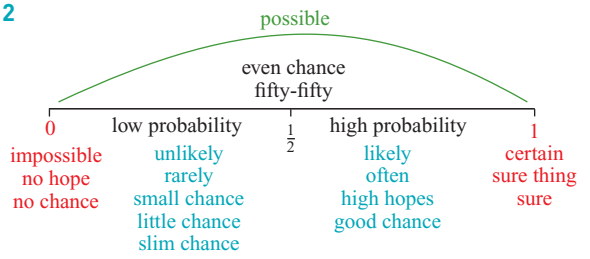
1 **a** {1, 2, 3, 4, 5, 6} **b** 6
c **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{6}$
2 **a** {blue, red, green} **b** 3 **c** $\frac{1}{3}$
3 **a** {blue, green, purple, gold} **b** 4 **c** $\frac{1}{4}$
4 **a** {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} **b** 10
c **i** $\frac{1}{10}$ **ii** $\frac{1}{10}$
5 **a** 26
b **i** $\frac{1}{26}$ **ii** $\frac{1}{26}$ **iii** $\frac{1}{26}$

Exercise 10C

1 {1, 2, 3, 4, 5, 6}, 6
a {6}, 1, $\frac{1}{6}$ **b** {2, 3}, 2, $\frac{2}{6} = \frac{1}{3}$
c {1, 3, 5}, 3, $\frac{3}{6} = \frac{1}{2}$ **d** {1, 2}, 2, $\frac{2}{6} = \frac{1}{3}$
e {3, 4, 5, 6}, 4, $\frac{4}{6} = \frac{2}{3}$
2 {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, 10
a {5}, 1, $\frac{1}{10} = 0.1$ **b** {0, 9}, 2, $\frac{2}{10} = 0.2$
c {1, 3, 5, 7, 9}, 5, $\frac{5}{10} = 0.5$
d {2, 4, 6, 8}, 4, $\frac{4}{10} = 0.4$ **e** {0, 1, 2, 3, 4}, 5, $\frac{5}{10} = 0.5$
f {8, 9}, 2, $\frac{2}{10} = 0.2$
3 **a** $\frac{1}{6}$ **b** $\frac{1}{3}$ **c** $\frac{1}{3}$ **d** $\frac{2}{3}$
4 7
a {r, r, r, r, r}, 5, $\frac{5}{7}$ **b** {g, g}, 2, $\frac{2}{7}$
5 9
a {b, b, b, b}, 4, $\frac{4}{9}$ **b** {g, g, g, g, g}, 5, $\frac{5}{9}$
6 **a** $\frac{5}{13}$ **b** $\frac{8}{13}$
7 **a** $\frac{2}{5}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$
8 **a** $\frac{3}{14}$ **b** $\frac{11}{14}$ **c** $\frac{9}{14}$
9 **a** $\frac{1}{3}$ **b** $\frac{7}{24}$ **c** $\frac{17}{24}$
10 **a** $\frac{3}{12} = \frac{1}{4}$ **b** $\frac{4}{12} = \frac{1}{3}$ **c** $\frac{5}{12}$
d 0 **e** $\frac{9}{12} = \frac{3}{4}$ **f** $\frac{8}{12} = \frac{2}{3}$
g $\frac{2}{3}$ **h** $\frac{5}{12}$ **i** 1
11 **a** $\frac{1}{11}$ **b** $\frac{1}{11}$ **c** $\frac{2}{11}$ **d** $\frac{2}{11}$
e $\frac{4}{11}$ **f** $\frac{5}{11}$
12 **a** $\frac{1}{13}$ **b** $\frac{3}{13}$ **c** $\frac{2}{13}$ **d** $\frac{5}{13}$
13 **a** $\frac{13}{52} = \frac{1}{4}$ **b** $\frac{1}{52}$ **c** $\frac{13}{52} = \frac{1}{4}$ **d** $\frac{2}{52} = \frac{1}{26}$
e $\frac{4}{52} = \frac{1}{13}$ **f** $\frac{12}{52} = \frac{3}{13}$ **g** $\frac{8}{52} = \frac{2}{13}$ **h** 0
i $\frac{40}{52} = \frac{10}{13}$ **j** $\frac{36}{52} = \frac{9}{13}$
14 **a** 25% **b** 15% **c** 40% **d** 75%
15 **a** 0.25 **b** 0.15 **c** 0.4 **d** 0.75
16 **a** 0.35 **b** 0.07 **c** 0.93
17 **a** 35% **b** 7% **c** 93%
18 **a** {1, 2, 3, 4, 5, 6} **b** 0 **c** $\frac{0}{6} = 0$
d 6 **e** $\frac{6}{6} = 1$
19 **a** {red, green, blue, gold}
b 0 **c** $\frac{0}{4} = 0$ **d** 4 **e** $\frac{4}{4} = 1$
20 **a** 0 **b** 1
21 **a** 0 **b** 1

Exercise 10D

1 **a** Yes **b** No **c** No **d** No
e Yes, as there are no green marbles in the bag.
2



3 **a** Certain **b** Impossible
c No answer **d** Certain
e Low probability **f** Impossible
g High probability **h** Low probability
i Even chance **j** Low probability
4 **a** 100% **b** 0% **c** No answer
d 100% **e** 10% **f** 0% **g** 80%
h 10% **i** 50% **j** 10%
6 **a** Low probability **b** Low probability
c High probability **d** Impossible
e High probability **f** Certain
7 **a** Low probability **b** Even chance
c Certain **d** High probability
e Low probability **f** Even chance
g Impossible **h** Low probability
8 **a** Getting a head or tail
b Example: getting 6
c Example: getting a head or getting a tail
9 **a** Selecting any ticket numbered from 1 to 10
b Example: selecting a ticket numbered 11
c Example: selecting an even-numbered ticket
d Selecting any single number

10 **a** It lands on the smooth or rough side.
b Example: it lands on an edge.
c Example: it lands on the rough side.

11 **a** Spinning red, blue, green or yellow
b Example: spinning purple
c Example: spinning red or blue
d Spinning any single colour
e Example: spinning red or blue or green

Calculator activities

1 **a** 7 059 052 **b** $\frac{1}{7\,059\,052}$

2 **a** 900
b **i** $\frac{17}{90}$ **ii** $\frac{16}{45}$ **iii** $\frac{41}{90}$

3 **a** $\frac{480}{750} = \frac{16}{25}$ **b** $\frac{270}{750} = \frac{9}{25}$

4 **a** $\frac{280}{750} = \frac{28}{75}$ **b** $\frac{150}{750} = \frac{1}{5}$ **c** $\frac{320}{750} = \frac{32}{75}$

5 **a** $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{25}{36}$ **d** $\frac{625}{1296}$

Language in mathematics

1 **a** Likely, good chance, high probability, often
b Unlikely, rarely, low probability
c Impossible, no hope, no chance
d Unlikely, rarely, low probability
e Likely, good chance, high probability, often

2 Probability, even chance, event, outcome, often, impossible, rarely, sample space

3 **a** Unlikely, rarely, low probability
b Even chance, often, equally likely
c Low probability, unlikely
d Good chance, high probability, likely
e Impossible, no chance, no hope
f Certainty

Check your skills

1 C **2** C **3** A **4** B **5** D
6 C **7** B **8** D **9** A **10** C

Review set 10A

1 **a** {1, 2, 3, 4, 5, 6, 7} **b** $\frac{1}{7}$

2 {N, U, M, B, E, R} **b** $\frac{1}{6}$

3

	Fraction	Decimal	Percentage
a	$\frac{1}{5}$	0.2	20%
b	$\frac{2}{5}$	0.4	40%
c	$\frac{1}{2}$	0.5	50%

4 **a** 25 **b** $\frac{12}{25}$

5 **a** $\frac{1}{7}$ **b** $\frac{2}{7}$ **c** $\frac{5}{7}$

6 **a** The event is certain to happen.
b Example: 80%
c Example: unlikely

7 **a** Spinning a pink, purple, blue or brown
b Example: spinning red
c Example: spinning pink or purple
d Example: spinning pink

Review set 10B

1 **a** {red, blue, green} **b** $\frac{1}{3}$

2 **a** {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} **b** $\frac{1}{10}$

3

	Fraction	Decimal	Percentage
a	$\frac{3}{5}$	0.6	60%
b	$\frac{7}{10}$	0.7	70%
c	$\frac{3}{8}$	0.375	$37\frac{1}{2}$%

4 **a** 20 **b** $\frac{7}{20}$

5 **a** $\frac{13}{52} = \frac{1}{4}$ **b** $\frac{26}{52} = \frac{1}{2}$ **c** $\frac{4}{52} = \frac{1}{13}$

6 **a** The event is impossible.
b 50%
c Example: highly likely

7 **a** Tossing red or blue
b Example: tossing orange
c Example: tossing red

Review set 10C

1 **a** {2, 4, 6, 8, 10, 12} **b** $\frac{1}{6}$

2 **a** {red, green, blue, gold} **b** $\frac{1}{4}$

3

	Fraction	Decimal	Percentage
a	$\frac{4}{5}$	0.80	80%
b	$\frac{7}{20}$	0.35	35%
c	$\frac{1}{25}$	0.04	4%

4 **a** 15 **b** $\frac{7}{15}$

5 **a** $\frac{55}{100} = \frac{11}{20}$ **b** $\frac{15}{100} = \frac{3}{20}$
c $\frac{45}{100} = \frac{9}{20}$ **d** $\frac{60}{100} = \frac{3}{5}$

6 **a** The event has an even chance of occurring.
b 0% **c** Certain

7 **a** Selecting number 1, 2, 3, 4, 5, 6, 7 or 8
b Example: selecting number 9
c Example: selecting an even number
d Example: selecting 1 or 2

CUMULATIVE REVIEW CHAPTERS 7 TO 10

1 **a** **i** $3p$ **ii** $4q$ **iii** $3rs$ **iv** m^2
v $6a - 3$ **vi** $8 + 4t$ **vii** $3x - 2y$ **viii** $2ab$
b **i** $8 \times x$ **ii** $7 \times m \times n$
iii $4 \times b - 3$ **iv** $5 \times x - 7 \times z$
c **i** 15 **ii** 24 **iii** 1 **iv** 25
v 10 **vi** 2 **vii** 1 **viii** 17
d **i** $\frac{m}{6}$ **ii** $\frac{a}{b}$ **iii** $\frac{x+y}{p+q}$
e **i** $4f$ **ii** $6t$ **iii** $3a + 4b$

2 **a** **i** n^2 **ii** $5p^4$
b **i** $p \times p$ **ii** $3 \times a \times a$
c **i** 9 **ii** 27
d **i** $20t$ **ii** $30m$ **iii** $30abc$
e **i** $11p$ **ii** $7m$ **iii** 0 **iv** $2x^2$

f i 300 ii 36 iii -2 iv 2
g i $7a - 3b$ ii $4pq$ iii $5k + 5$ iv $-3y$
v $-5y^2$ vi $2x^2$ vii $-2a$ viii $9a$

3 **a** $10h$ **b** $2d$ **c** $10z + 8$ **d** $11p - 6q$
e $11a - 8$ **f** $15pq$ **g** $35g$ **h** $25mn$
i $3y$ **j** 4 **k** $\frac{4n}{5}$ **l** $4q$

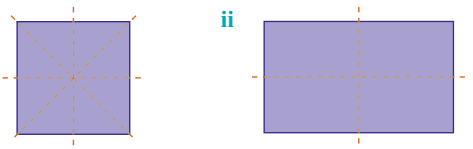
4 **a** i $\$(10 - c)$ ii $4x + 5t$
b i Sum of a and b
ii Difference between a and b
iii The square of b

5 **a** 0.47 **b** $\frac{38}{10} = \frac{19}{5}$
c 0.0077 **d** 0.04, 0.365, 0.37, 0.38

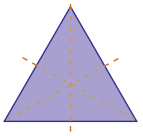
6 **a** 22.5 **b** 6380 **c** 163.13 **d** 9
e 9.34 **f** 29 **g** 29.74

7 **a** i 40% ii 60%
b $\frac{12}{25}$ **c** 50% **d** 75% **e** 34%
f \$282 **g** 17 kg **h** $\frac{1}{16}$ **i** 0.008

8 **a** i Translated ii Reflected or rotated
b i ii



iii



c All i Order 4 ii Order 2 iii Order 3
d i $(-9, 3)$ ii $(5, 3)$ iii $(-5, -3)$ iv $(-3, -5)$

9 **a** i $\{1, 2, 3, 4, 5, 6, 7, 8\}$ ii $\frac{1}{8}$
b i 12 ii 3 iii $\frac{3}{12} = \frac{1}{4}$
c i $\frac{1}{6}$ ii $\frac{3}{6} = \frac{1}{2}$
d Examples: even chance, fifty-fifty
e Example: 90%
f i Rolling a single number
ii Example: rolling 7
iii Example: rolling an even number
iv Rolling a number from 1 to 6
g i $\frac{3}{12} = \frac{1}{4}$
ii 'Selecting a 20-cent or \$1 coin' or 'not selecting a 10-cent coin'
h The sum of the probabilities of an event and its complement is 1.

CHAPTER 11 DATA INVESTIGATION

Diagnostic test

1 A	**2** B	**3** A	**4** B	**5** C
6 C	**7** D	**8** A	**9** A	**10** D
11 D	**12** D	**13** A	**14** A	**15** D
16 B	**17** B	**18** B	**19** D	

Exercise 11A

1 **a** Primary data is data that you collect.
b Positive features: original, targeted specifically to investigation, able to read body language, clarify responses, etc.
c Negative features: costly, time consuming
d Direct observations, surveys, interviews, logs

2 **a** Secondary data is data collected from external sources.
b Positive features: saves costs and time
c Negative features: lacks originality, may not be suitable for the enquiry, must be cautious, is in the form of a ready-made product
d Internet, television, radio, newspapers, magazines, journals, research papers, reviews, people's stories

3 **a** Primary **b** Secondary **c** Primary **d** Secondary

6 Primary: greater ability to be specific with data. No means to source this with secondary data.

7 Secondary: most material already available, would in this case be reliable for the research enquiry.

Exercise 11B

1 **a** Census **b** Census **c** Sample **d** Census
e Sample **f** Census **g** Census **h** Sample
i Census **j** Census **k** Census **l** Sample
m Census **n** Sample **o** Census **p** Census
q Sample **r** Sample **s** Sample **t** Census
u Census **v** Sample **w** Census **x** Sample
y Sample

2 **a** Includes only people at home on Saturday night with a phone
b Includes only people who travel by bus on this route
c Includes only people who use the supermarket and travel by car
d Does not include non-beachgoers
e These people are not representative of people in all streets.
f Includes only businesses that advertise in Yellow Pages
g Includes only over-18s.

3 **a** Year 7 students are not representative of whole school.
b Motorists do not represent all peak-hour travellers.
c Only one employee group that works in that business is interviewed.
d Politicians are not representative of whole population and it is their job to affect the economy.
e Only interested people with phones will respond.
f Only people who want to respond will do so.
g Sample is not representative of full week's production.
h Sample is too small.
i Only one company's claim is given, we do not know how the dentists were selected and the sample is too small.
j How many dog breeders were surveyed, how were they selected and what does 'recommend' mean?

4 **a** Non-smokers in public areas
b People who catch the local bus
c People in the library
d Senior students who use the common room
e Greenies **f** Unemployed people
g Farmers **h** People with a bank account.

5 **a** Smokers **b** People who drive
c People without library cards
d Junior students **e** Logging companies
f Employed people **g** Importers
h Banks

6 **a** People on the bus **b** Major companies
c People using the bank **d** Vets
e Stores that sell dishwashers
f Bank managers

Exercise 11C

1 **a** Relevant: yes or no response.
b Probably not relevant.
c Relevant, but assumes ownership of a car; time period not clear. Reword: *Do you ever drive to work?*, then *How many days per week do you usually drive to work?*
d Relevant, but assumes use of public transport. Combine with the question from **e**.
e Relevant, but assumes use of public transport. Reword similar to **c**.

2 **a** Relevant; yes or no response.
b Relevant; yes or no response.
c Possibly relevant with respect to a new colour for the uniform. Reword to *What colour would you prefer for a school uniform?*
d Relevant; answer in years.
e Relevant, but most students attend the same school for their whole time at school.
f Relevant; responses would affect the new uniform.

3 **a** Relevant; yes or no answer.
b Reword to *How many hours per day or week do you watch TV?*
c Relevant, but combine with **a**.
d Relevant if the survey is to determine popular programs. Responses would vary immensely. Reword to ask about categories of programs.
e Reword to *Do you like watching sport on TV?*, then *How many hours per week do you spend watching sport on TV?* Other categories could be included.
f Relevant, but limited responses as channels are limited.
g Not relevant.

4 **a** Relevant; yes or no response.
b Relevant, but needs an age-specific question as well.
c Not necessarily relevant as many marriages are performed outside church. Reword to *Would you like to get married in a church?* Include other options.
d Not relevant.
e Relevant, as someone's view on marriage may depend on that fact.

5 **a** Not relevant, as most people do not know.
b Relevant; yes or no responses.
c Relevant; answers too varied, so give categories.
d Not necessarily relevant but could provide some information.
e Not necessarily relevant.
f Relevant, as perhaps ownership is related to work.
g Relevant.

6 **a** Relevant; yes or no response.
b Reword to *What type of pet do you own?*, providing of course they own one!
c Too narrow. Asking for which animals would not be too varied.
d Not relevant.
e Relevant; varied responses.

7 Examples:
a *What is your favourite FM radio station?*
b *What do you think of your school uniform?*
c *What do you think of the food at the canteen?*
d *How safe do you feel when walking to and from school?*
e *What job would you like in the future?*

8 Examples:
a *Is Triple M your favourite FM radio station?*
b *Do you like your school uniform?*
c *Do you buy food from the canteen each day?*
d *Do you feel safe when walking to and from school?*
e *Have you decided on a career?*

9 Examples:
a *What is your favourite FM station?*
☐ *104* ☐ *104.9*
☐ *105.7* ☐ *107.9* ☐ *other*
b *What do you think of your school uniform?*
☐ *like it* ☐ *it's ok*
☐ *replace it* ☐ *have no uniform*
c *What is your favourite food at the canteen?*
☐ *pies* ☐ *sandwiches/rolls*
☐ *chips* ☐ *ice-creams*
☐ *lollies* ☐ *other*
d *How do you feel when walking to and from school?*
☐ *safe* ☐ *at risk from traffic*
☐ *at risk from people*
e *What career are you interested in?*
☐ *professional* ☐ *service industry*
☐ *unskilled labour* ☐ *other*

10 Examples:
a *What do you think of the music on 104.9 Triple M?*
(1) strongly like *(2) like* *(3) no opinion*
(4) dislike *(5) strongly dislike*
b *What do you think of the current school uniform?*
(1) like it a lot *(2) like it* *(3) neutral*
(4) don't like it *(5) dislike it a lot*
c *The food at the canteen is appropriate and excellent.*
(1) strongly agree *(2) agree* *(3) no opinion*
(4) disagree *(5) strongly disagree*
e *Do you want a professional career?*
(1) yes *(2) maybe yes* *(3) don't know*
(4) maybe not *(5) no*

Exercise 11D

1 **a** Numerical **b** Numerical **c** Nominal
d Nominal **e** Numerical **f** Numerical

2 **a** Numerical **b** Nominal **c** Nominal
d Nominal **e** Numerical **f** Numerical
g Numerical **h** Numerical **i** Numerical
j Numerical

3 **a** Examples: favourite food, favourite colour, colour of car
b Examples: height, shoe size, number of books read

4 **a** Continuous **b** Continuous **c** Discrete
d Continuous **e** Discrete **f** Discrete
g Continuous **h** Continuous **i** Continuous
j Discrete

5 **a** Examples: weight, temperature, distance from school to home
b Examples: number of cars in your household, shoe size, marks in a test

6 **a** Discrete **b** Continuous **c** Categorical
d Discrete **e** Discrete **f** Categorical
g Discrete **h** Discrete **i** Continuous
j Discrete **k** Discrete **l** Continuous
m Categorical **n** Categorical **o** Discrete
p Categorical **q** Continuous **r** Categorical
s Continuous **t** Discrete **u** Categorical
v Discrete **w** Discrete **x** Categorical
y Continuous

Exercise 11E

1 a

Number of heads (score)	Frequency
0	13
1	38
2	10
3	12
Total	**73**

b

Days absent (score)	Frequency
0	57
1	8
2	2
3	3
4	12
5	3
Total	**85**

2

Temperature (score) (°C)	Tally	Frequency
37	|	1
38	|	1
39	||||	4
40	𝍸 ||	7
41	𝍸 |	6
42	𝍸	5
43	||||	4
	Total	**28**

a 28 **b** 4 **c** 22

3

Vehicle (score)	Tally	Frequency
Car	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	37
Motorcycle	𝍸 ||||	9
Bus	|||	3
Truck	𝍸 𝍸 |	11
	Total	**60**

a 37 **b** 11 **c** 8

4

Month (score)	Tally	Frequency
Jan	|||	3
Feb		0
Mar	||	2
Apr	|	1
May	||||	4
Jun	|	1
Jul	𝍸 |	6
Aug	||	2
Sep	|||	3
Oct	|||	3
Nov	|	1
Dec	||||	4
	Total	**30**

a 30 **b** July
c February **d** 9 (Jun–Aug)

5 a

Colour (score)	Tally	Frequency
White	𝍸 𝍸 𝍸	15
Blue	𝍸 𝍸	10
Grey	|||	9
Red	𝍸 𝍸 ||	12
Other	||||	4
	Total	**50**

b 50 **c** White
d White 30%, blue 20%, grey 18%, red 24%, other 8%

6 a

Eye colour (score)	Tally	Frequency
Blue	𝍸 𝍸 ||	12
Brown	𝍸 𝍸	10
Green	|||	3
Grey	𝍸	5
	Total	**30**

b 30 **c** Blue

7

Mark (score)	Tally	Frequency
0	|	1
1	||	2
2	||	2
3	𝍸	5
4	||	2
5	𝍸	5
6	||||	4
7	𝍸 ||	7
8	||||	4
9	𝍸 |	6
10	||	2
	Total	**40**

b 40 **c** 2 **d** 1 **e** 70%

Exercise 11F

1 a

Marks	Tally	Frequency
50–59	||||	4
60–69	𝍸 ||	7
70–79	𝍸 𝍸 𝍸	15
80–89	𝍸 𝍸 |	11
90–99	|||	3
	Total	**40**

b 10 marks **c** 15 **d** 26 **e** 29

2 a

Distance (m)	Tally	Frequency
23–29	|||	3
30–36	𝍸 𝍸 𝍸	15
37–43	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	30
44–50	𝍸 𝍸 𝍸 𝍸 𝍸 |||	28
51–57	𝍸 𝍸 |	11
58–64	||||	4
65–71	𝍸	5
	Total	**96**

b 7 m **c** 48 **d** 20 **e** 37–43 m

3

Height (mm)	Tally	Frequency
300–324	𝍸 |	6
325–349	𝍸 ||||	9
350–374	𝍸 𝍸 𝍸 𝍸 |	21
375–399	𝍸 𝍸 |||	13
400–424	𝍸 ||	7
425–499	||||	4
	Total	**60**

b 25 mm **c** 60
d **i** 13 **ii** 34
e **i** 11 **ii** 49
f $\frac{49}{60}$ **g** 1194

Exercise 11G

1 **a** **i** 35 **ii** 50 **iii** 30 **iv** 25
b Golf **c** 25 **d** 250

2 **a**

Number	Tally	Frequency
0	||	2
1	𝍸	5
2	𝍸 |	6
3	𝍸 𝍸 𝍸 |	16
4	𝍸 𝍸 ||	12
5	||||	4
more than 5	||	2
	Total	**47**

b

Number of mobile phones	Number of households
0	📱
1	📱 📱
2	📱 📱
3	📱 📱 📱 📱
4	📱 📱 📱
5	📱
more than 5	📱

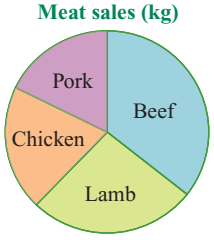
📱 = 4 households

c 16 **d** 2 **e** 47

3 **a** Beef $\frac{160}{450} = \frac{16}{45}$, lamb $\frac{120}{450} = \frac{4}{15}$,
chicken $\frac{90}{450} = \frac{1}{5}$, pork $\frac{80}{450} = \frac{8}{45}$

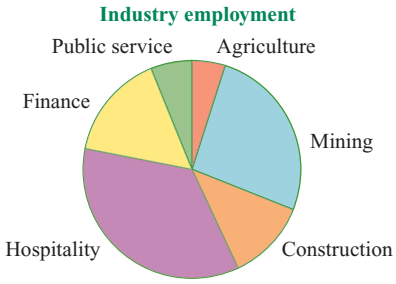
b Beef $\frac{16}{45} \times 360° = 128°$, lamb $\frac{4}{15} \times 360° = 96°$,
chicken $\frac{1}{5} \times 360° = 72°$, pork $\frac{8}{45} \times 360° = 64°$

c

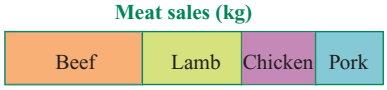


4 **a** Agriculture 18°, mining 94°, construction 43°, hospitality 126°, finance 58°, public service 22°

c

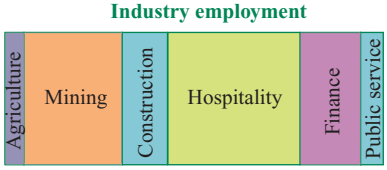


5 Beef 35.6% (35.5 mm), lamb 26.7% (26.5 mm), chicken 20% (20 mm), pork 17.8% (18 mm)



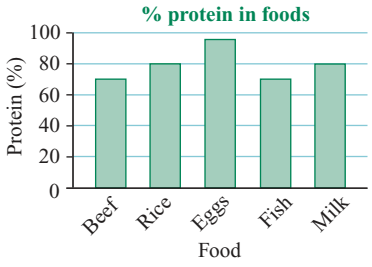
Shown half size

6 Agriculture 5 mm, mining 26 mm, construction 12 mm, hospitality 35 mm, finance 16 mm, public service 6 mm



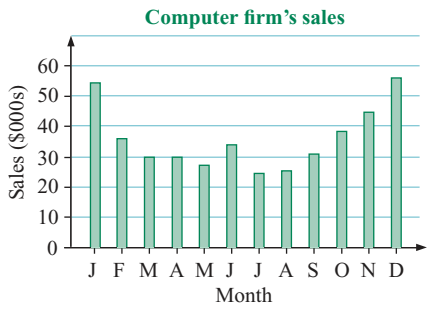
Shown half size

7

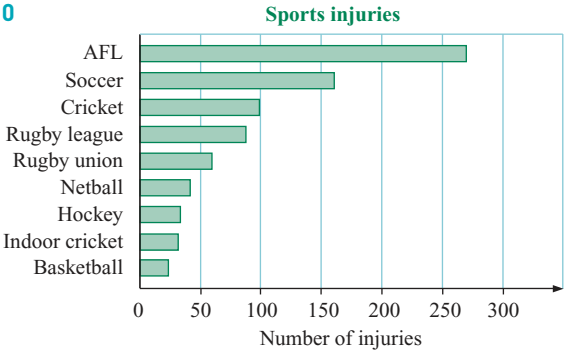


8

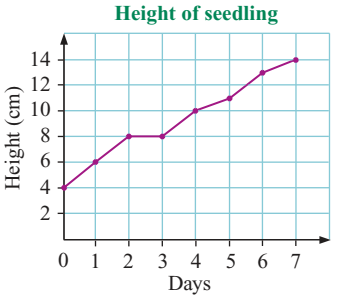
9



10

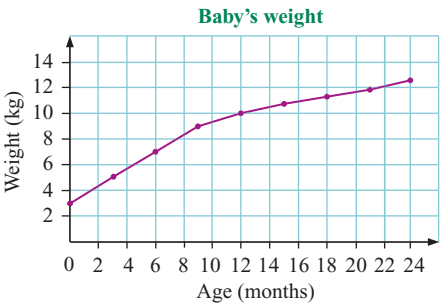


11 a iv (2, 8), (3, 8), (4, 10), (5, 11), (6, 13)



b 4 cm, day 0 **c** 12 cm

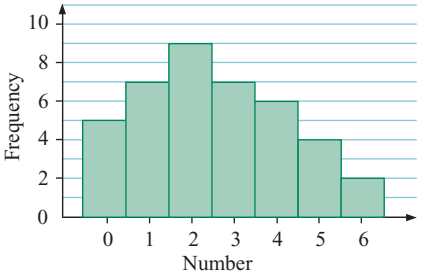
12



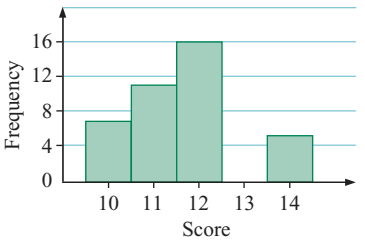
b 9.2 kg

Exercise 11H

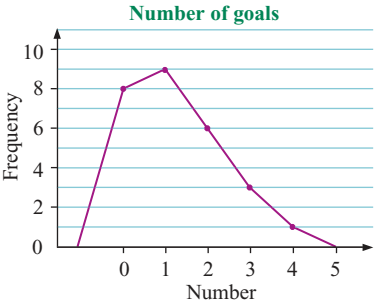
1



2

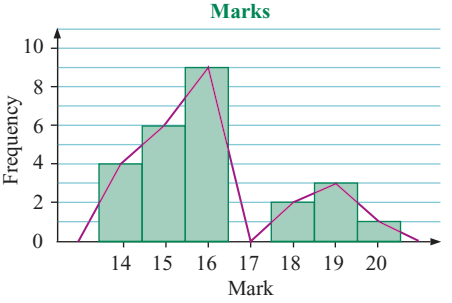


3



4

5



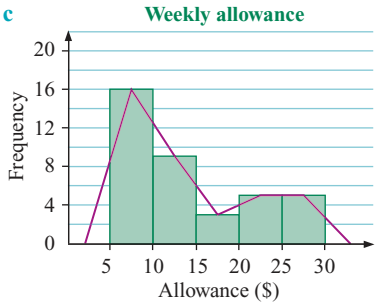
6

Number of goals	Frequency
1	5
2	6
3	10
4	4
5	1
Total	**26**

7 a

Weekly allowance ($)	Tally	Frequency
5 to 9.99	𝍸 𝍸 𝍸 |	16
10 to 14.99	𝍸 ||||	9
15 to 19.99	|||	3
20 to 24.99	𝍸	5
25 to 29.99	𝍸	5
	Total	**38**

b, c



d $\frac{16}{38} = \frac{8}{19}$ **e** Because the data is grouped

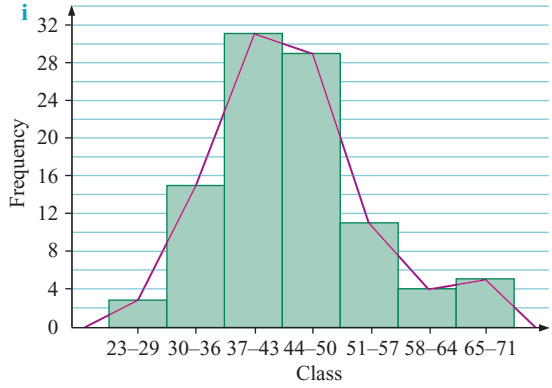
8 a i

Class	Tally	Frequency
23–29	𝍸	3
30–36	𝍸 𝍸 𝍸	15
37–43	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	31
44–50	𝍸 𝍸 𝍸 𝍸 ||||	29
51–57	𝍸 𝍸 |	11
58–64	||||	4
65–71	𝍸	5
	Total	**98**

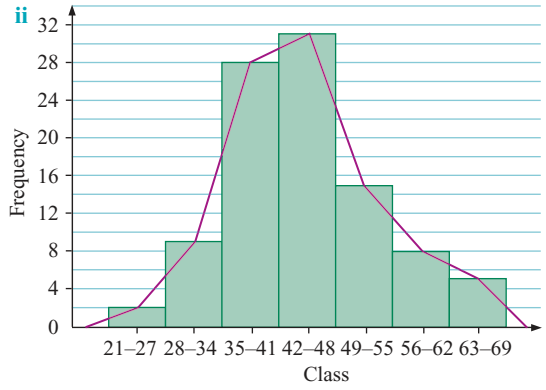
ii

Class	Tally	Frequency
21–27	||	2
28–34	𝍸 ||||	9
35–41	𝍸 𝍸 𝍸 𝍸 𝍸 |||	28
42–48	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	31
49–55	𝍸 𝍸 𝍸	15
56–62	𝍸 |||	8
63–69	𝍸	5
	Total	**98**

b i



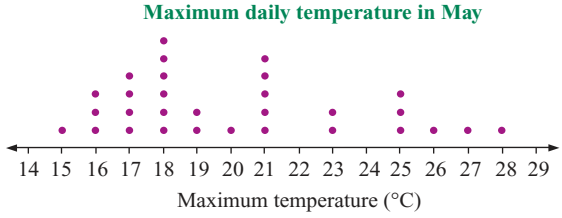
ii



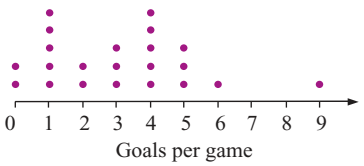
c The shapes are different although the data is the same. Therefore the grouping of the data affects the results.

Exercise 11I

1

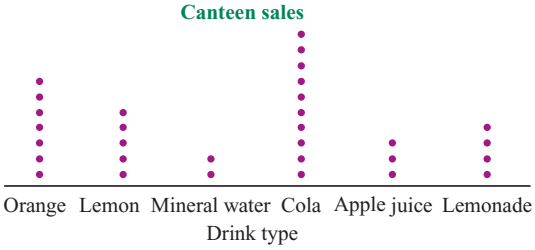


2 a



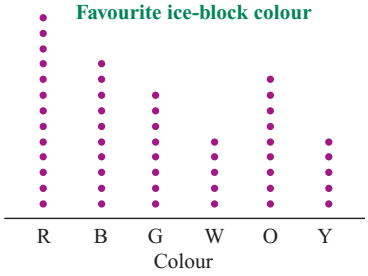
b 68 **c** Yes, 9

3 a



b Cola and orange

4



5

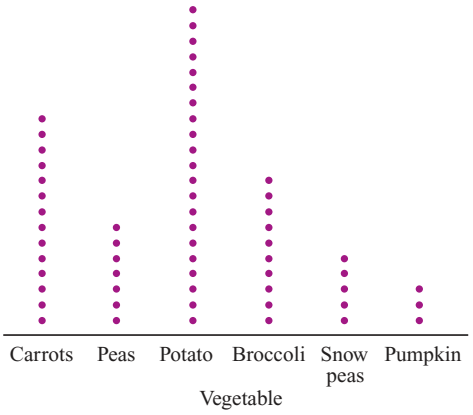
Colour	Frequency
Red	6
Blue	4
Green	3
Brown	1
Orange	2
Pink	5
Black	5
Total	**26**

6 a 18 **b** 25 **c** 19

7 a Carrots

b Potato, as it has the largest number of dots

c



Exercise 11J

1

Score	Stem	Leaf
39	3	9
47	4	7
125	12	5
83	8	3
114	11	4
93	9	3
4	0	4
350	35	0
5	0	5
1384	**138**	**4**

2

Stem	Leaf
3	1 3 3 4 8 9
4	1 9 9
5	1 3 5 7 8 9
6	0 1 1 3 8

3

Stem	Leaf
11	1 4 9
12	2 3 5 8 9 9
13	3 3 5 8
14	4 8
15	5 7 8
16	0 9

4

Stem	Leaf
3	2 5 7 9
4	0 1 4 5 7 8 9
5	2 2 8 9 9
6	2 2 6 8

5

Stem	Leaf
4	1 2 4 5 7 8 8 9 9
5	
6	5 6 7 8
7	0 1 4 4 7 8 8 9
8	3 9
9	1 2 3 3 5 6 6 8 8

6

Stem	Leaf
11	2 2 2 3 3 7 8 8 9
12	2 4 4 5 6
13	0 0 1 2 3 4 6 6 8 9
14	0 0 0 0 2 2 3 3 4 4 6 6 6 6 6 6

7 a

Stem	Leaf
0	4
1	3
2	1 7 7 9
3	0 0 2 6 8 8 8
4	0 0 4 5 5 7 9 9

b

Stem	Leaf
10	6 9
11	0 1 1 1 2 6
12	0 1 2 4 7 7 9
13	0 1 3 4 5 5 6 7 8 8
14	0 1 1 1 5

c

Stem	Leaf
19	0 0 3 3 4
20	3 3 5 6 8 8
21	6
22	1 3 3 4 5 5 5 5

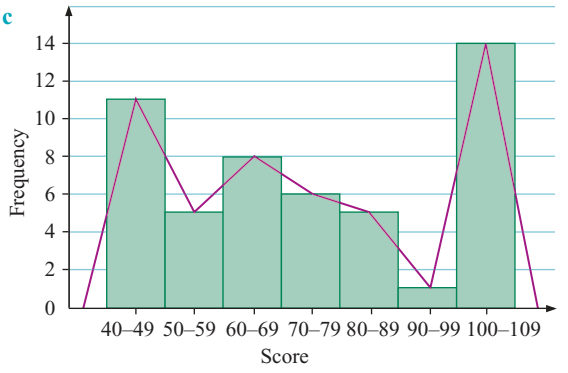
8 a

Stem	Leaf
4	0 1 3 3 3 4 4 5 6 6 6
5	2 7 9 9 9
6	0 2 2 3 4 4 5 5
7	0 0 1 3 5 9
8	1 4 4 5 6
9	7
10	1 2 2 3 4 5 5 5 5 6 7 7 7 8

b

Class	Tally	Frequency
40–49	𝍸 𝍸 |	11
50–59	𝍸	5
60–69	𝍸 |||	8
70–79	𝍸 |	6
80–89	𝍸	5
90–99	|	1
100–109	𝍸 𝍸 ||||	14
	Total	**50**

c



d In a stem-and-leaf plot, individual scores that recur within a class can be seen.

9 a **i** 39 **ii** 29

b 2, as it is much smaller than the rest of the scores

c Around stems 1 and 2

d No, as class 7R scored well in the 20s (stem 2) and 30s (stem 3), whereas class 7L scored well only in the 10s and 20s

10 a In the class 7L test scores there is a cluster in stems 3 and 4 and an outlier of 23.

b No, as they are more evenly distributed

c No, as there are a number of other scores relatively close to 20

d Scores in class 7R are distributed well in all stems, but in class 7L scores are clustered in stems 3 and 4.

Exercise 11K

1 a 5 **b** Supreme

c Meatlovers and ham and pineapple

d $\frac{5}{60} = \frac{1}{12}$ **e** 18°

2 a Abseiling **b** No **c** $\frac{30}{80} = \frac{3}{8}$

d 80° **e** $\frac{155}{180} = \frac{31}{36}$

3 a Cola: longest section

b Orange juice: shortest section

c $\frac{25}{150} = \frac{1}{6}$ **d** 25 students

e 40% **f** 60 students

5 a Yes, as it is the largest segment.

b $\frac{35}{100} = \frac{7}{20}$ **c** $\frac{1}{10}$ **d** 40 cars

e 14 cars **f** $\frac{15}{100} = 15\%$ **g** 6 cars

6 **a** The number of new cases and the number of deaths are very close.
b Pancreas
c **i** Skin **ii** Skin
d About 4000 more cases
e NHL is non-Hodgkin lymphoma
i about 40% **ii** about 50%
f Males are more likely to work at outside jobs.
i 4500 cases **ii** 3500 cases

7 **a** 135 cm **b** Between 6 and 7 years old
c Between 0 and 1 year old
d Between 2 and 3 years old

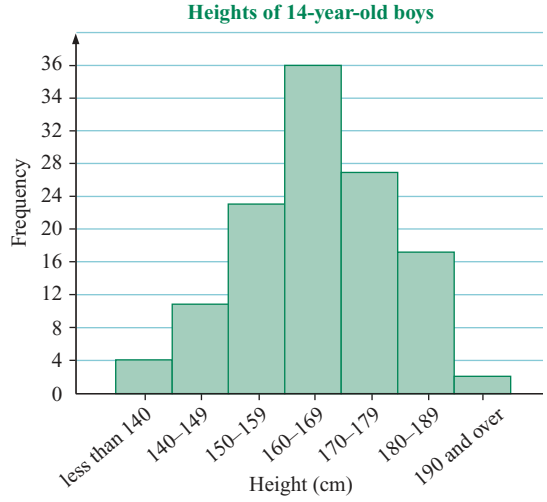
8 **a** 6 pm **b** 8 am, 4 pm **c** 39.5°C
d 40.1°C, 12 noon–2 pm **e** 39°C

Exercise 11L

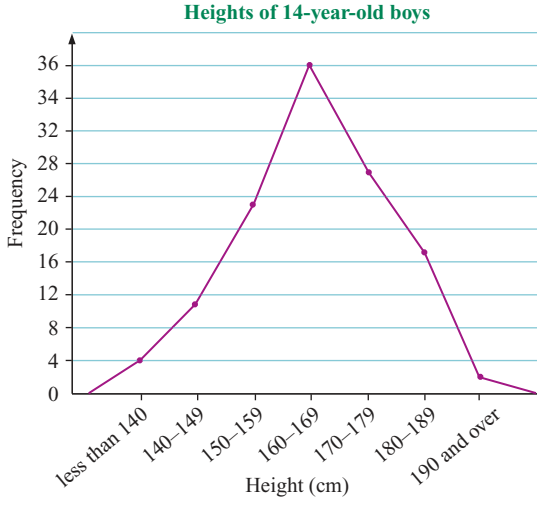
1 **a**

Height (cm)	Tally	Frequency
less than 140	𝍸	4
140–149	𝍸 𝍸 |	11
150–159	𝍸 𝍸 𝍸 𝍸 |||	23
160–169	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	36
170–179	𝍸 𝍸 𝍸 𝍸 ||	27
180–189	𝍸 𝍸 𝍸 ||	17
190 and over	||	2
	Total	**120**

b



c



d

Height (cm)	Frequency
less than 140	🧍
140–149	🧍🧍🧍
150–159	🧍🧍🧍🧍🧍🧍
160–169	🧍🧍🧍🧍🧍🧍🧍🧍🧍
170–179	🧍🧍🧍🧍🧍🧍🧍
180–189	🧍🧍🧍🧍🧍
190 and over	🧍

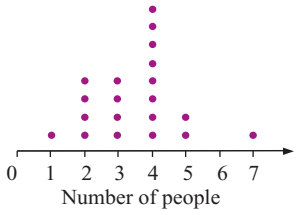
🧍 = 4 boys

e They are all accurate and all represent the data well.

2 **a**

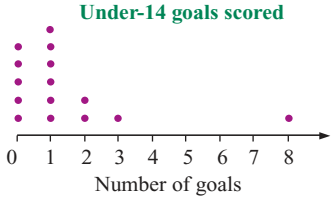
Stem	Leaf
0	1 2 2 2 2 3 3 3 3 3 4 4 4 4 4 4 4 4 5 5 7

b People in each household **c** 7 **d** 4



e Dot plot, as the stem-and-leaf plot did not separate the scores much
f Example: What is the number of households with 7 occupants?

3 **a**



b

Stem	Leaf
0	0 0 0 0 0 1 1 1 1 1 1 2 2 3 8

c

Number of goals	Frequency
0	5
1	6
2	2
3	1
8	1
Total	**15**

d Yes, 8 goals. Frequency histogram and dot plot, as the stem-and-leaf plot had only one stem
e The histogram shows the data better, and it's easier to see the outlier.
f Example: Which score occurred the same number of times as the 8-goal score?

4 a

Stem	Leaf
6	3 6 8 9
7	0 0 1 2 4 4 8 9
8	3 3 5 5 7 8
9	0 1 1 5 7 7

b No, as the ranges of scores is from 63 to 97: too many.

c

Marks	Frequency
60–69	4
70–79	8
80–89	6
90–99	6
Total	**24**

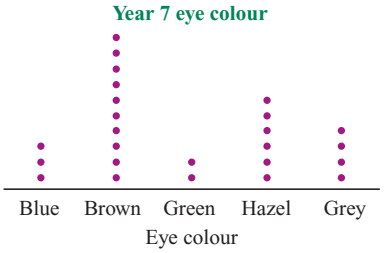
d No, because individual scores are not recorded

e A stem-and-leaf plot shows all the scores.

f Example: Which score besides 70, 74, 83, 91 and 97 occurs twice?

5 a No, as there is nothing between different eye colours

c



d Nominal (categorical) variables

6 a Advantages: shows all data, easy to see trends.
Disadvantages: may be unsuitable for very clustered scores.

b Advantages: shows comparisons and outliers.
Disadvantages: not useful if there are more than 10 scores.

c Advantages: good if many of the same score, shows trends.
Disadvantages: if there's a large range of scores the table is too large.

d Advantages: can be used for broad range of scores.
Disadvantages: individual scores not shown.

e Advantages: shows trends.
Disadvantages: unsuitable for categorical data.

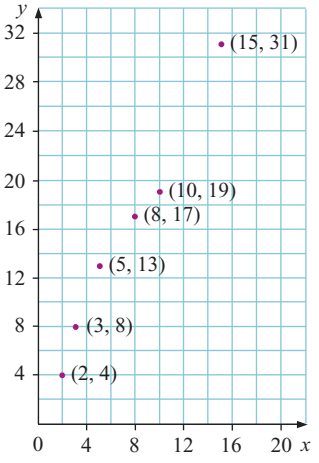
Exercise 11M

1 a It decreases. **b** No

c No, as the pattern of dots does not show a trend.

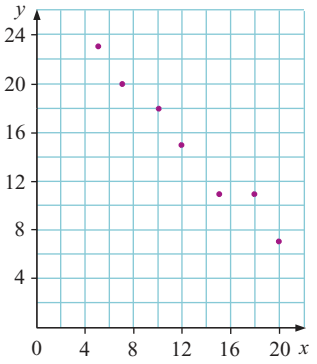
d It increases.

2 a

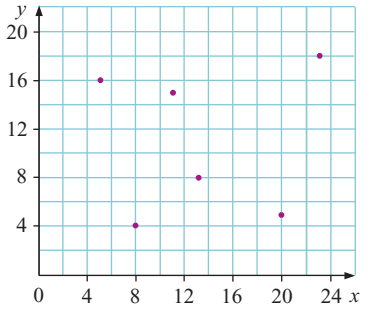


b upward

3 a

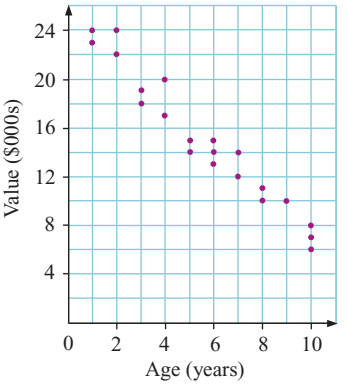


b



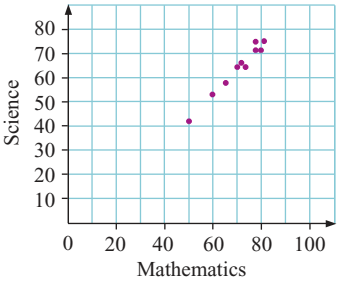
4 a Downward **b** No trend

5 a



b Downward trend. Values decrease as age increases.

6 a



b Yes. Higher Maths results correspond with higher Science results (upward trend).

Exercise 11N

1 a Scale on vertical axis does not start at zero.

b Scale on vertical axis has irregular intervals.

c No scale on vertical axis

d Width of columns is inconsistent.

e Width and depth of objects are inconsistent.

f Width and depth of objects are inconsistent.

g Scale on horizontal axis is irregular.

2 a The vertical axis does not start at zero.

b

3 a Graph A **b** No
c No

Language in mathematics

1 a histogram **b** polygon
c plot **d** variable
e outlier, scatter **f** graph
g data **h** frequency
i quantitative **j** sector
k discrete **l** continuous

2 a scores, together **b** outlier, small
c numerical **d** continuous
e categorical

Check your skills

1 C	**2** A	**3** C	**4** C	**5** A
6 C	**7** C	**8** C	**9** B	**10** B
11 B	**12** C	**13** B	**14** C	**15** A
16 D	**17** B	**18** A	**19** D	

Review set 11A

1 A census involves collecting information about every individual in the whole population.

2 Sample survey, as it would be very time consuming and expensive to interview the whole population.

3 People who play or support netball

4 Examples:

a *Does anyone in your household watch children's television?*

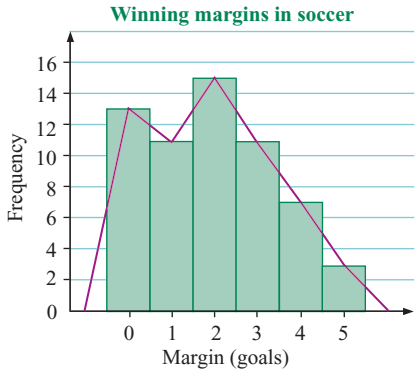
b *The quality of children's television programs is:*
(1) very good *(2) good* *(3) no opinion*
(4) poor *(5) very poor*

5 a Nominal
b Discrete numerical
c Continuous numerical

6 a

Margin (goals)	Tally	Frequency
0	𝍸 𝍸 |||	13
1	𝍸 𝍸 |	11
2	𝍸 𝍸 𝍸	15
3	𝍸 𝍸 |	11
4	𝍸 ||	7
5	|||	3
	Total	**60**

b



c **i** 60 **ii** 15 **iii** Draw

7 a

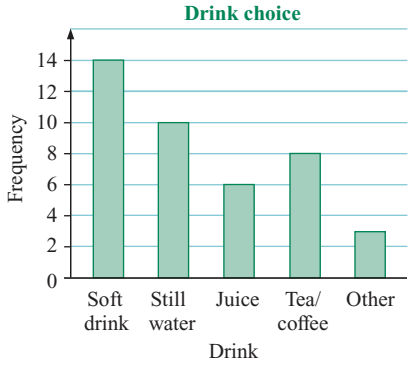
Score	Tally	Frequency
1	||	2
2	|||	3
3		0
4	|||	3
5	|||	3
6	𝍸 |	6
7	|||	3
8	|||	3
9	||||	4
10	|||	3
	Total	**30**

b 10

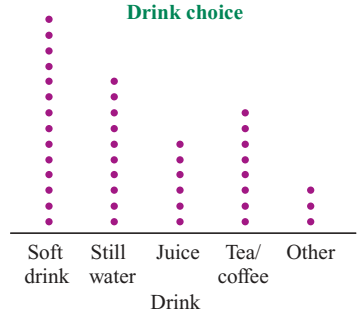
8 a

Stem	Leaf
2	8
3	1 2 2 8
4	2 2 4 5 5 5 5 6 8 9
5	0 0 1 3
6	1

9 a



b

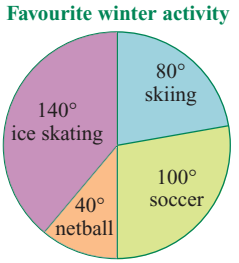


10 Skiing: $\frac{8}{36} \times 360 = 80°$

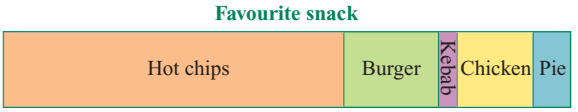
Ice skating: $\frac{14}{36} \times 360 = 140°$

Soccer: $\frac{10}{36} \times 360 = 100°$

Netball: $\frac{4}{36} \times 360 = 40°$



11



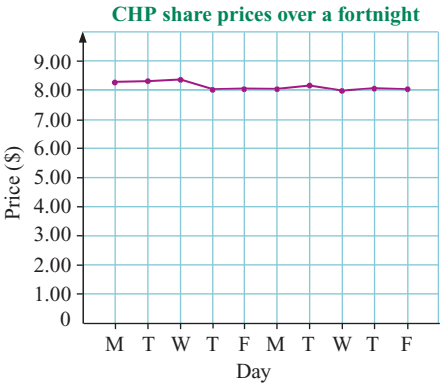
Shown half size

Hot chips: $\frac{18}{30} \times 15 = 9$ cm Burger: $\frac{5}{30} \times 15 = 2.5$ cm

Kebab: $\frac{1}{30} \times 15 = 0.5$ cm Chicken: $\frac{4}{30} \times 15 = 2$ cm

Pie: $\frac{2}{30} \times 15 = 1$ cm

12



Review set 11B

1 Categorical data describes categories that are names of things.

2 A census may be more accurate but information is difficult and expensive to collect.

3 Bias in statistics means that the information collected does not reflect the true situation.

4 **a** *What do you use a mobile phone for?*

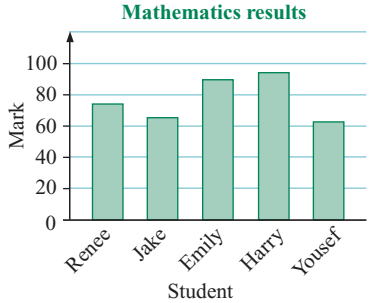
b *Who pays for your mobile phone use?*

☐ *you* ☐ *parents* ☐ *other*

5 **a** Continuous numerical **b** Nominal

c Discrete numerical

6



7 **a**

Score	Tally	Frequency
10–14	|	1
15–19	𝍸 |||	8
20–24	𝍸 𝍸 ||||	14
25–29	𝍸 ||	7
	Total	**30**

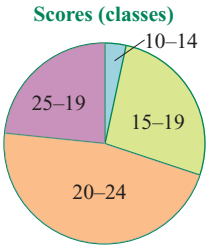
b 23

8 **a**

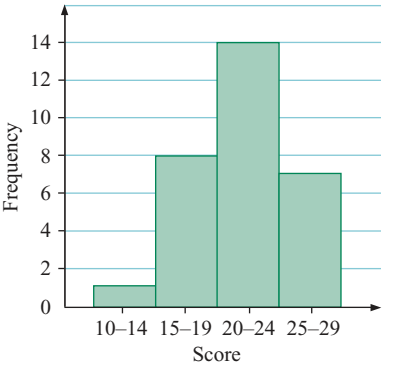
Stem	Leaf
1	1 5 5 6 6 7 7 9 9
2	0 0 0 1 1 1 1 2 2 2 3 4 4 4 5 5 6 7 7 7 8

b Frequency distribution table

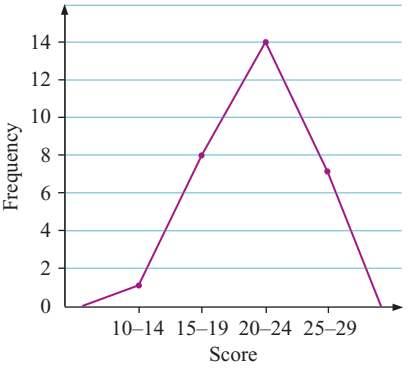
9 **a**



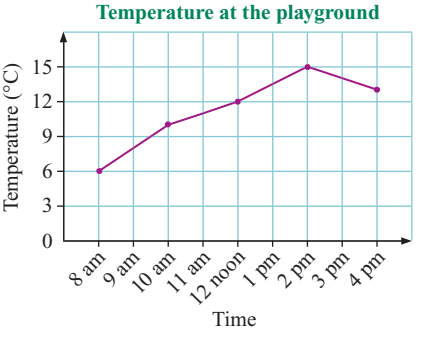
b



c



10 **a**



b 9°C **c** 1:30 pm, 3 pm

11 **a**

Stem	Leaf
2	0 5
3	
4	7 7 7 8 8 8 8 8 8 8 8 9 9 9 9 9 9 9 9 9
5	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
	1 1 1 1 1 1 1 1 1 2 2 2 2 2

b Yes, stems 4 and 5

c Yes, stem 2. The boxes were not filled properly.

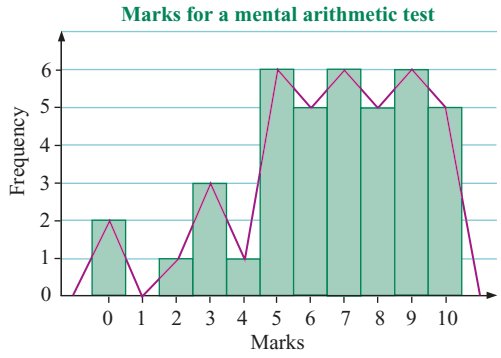
Review set 11C

1 **a** Nominal **b** Continuous numerical
c Discrete numerical

2 **a**

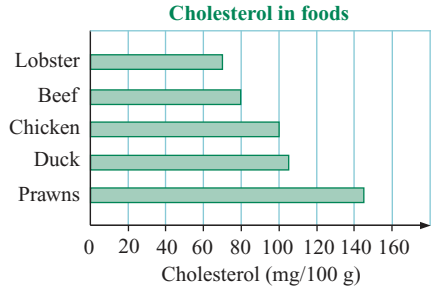
Mark	Tally	Frequency
0	||	2
1		0
2	|	1
3	|||	3
4	|	1
5	𝍸 |	6
6	𝍸	5
7	𝍸 |	6
8	𝍸	5
9	𝍸 |	6
10	𝍸	5
	Total	**40**

b



c **i** 40 **ii** 5 **iii** 2 **iv** 0 **v** 33

3

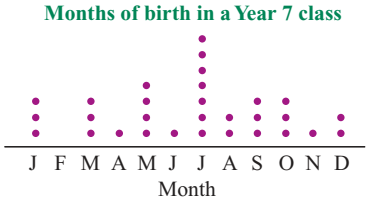


4 **a**

Month	Tally	Frequency
Jan	|||	3
Feb		0
Mar	|||	3
Apr	|	1
May	|||	4
Jun	|	1
Jul	𝍸 ||	7
Aug	||	2
Sep	|||	3
Oct	|||	3
Nov	|	1
Dec	||	2
	Total	**30**

b 30 **c** July
d February **e** 10 (Jun–Aug)

f

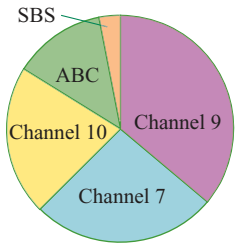


5 **a**

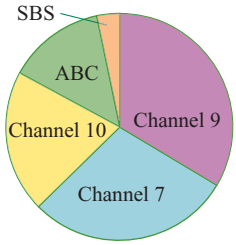
Stem	Leaf
1	8
2	6 7 7 7 7 7 7 8 8 8 8 8 8 9 9 9 9 9 9 9
3	0 0 0 1 1 1 1 1 1 1 2 2 2 2 2 2 2 3 3

b An outlier—the packet must not have been full.

6 **a**



b



CHAPTER 12 AREA, SURFACE AREA AND VOLUME

Diagnostic test

1 A	**2** D	**3** B	**4** C	**5** B
6 D	**7** C	**8** C	**9** A	**10** D
11 C	**12** C	**13** B	**14** B	**15** D
16 C	**17** B	**18** D	**19** A	**20** C
21 A	**22** C			

Exercise 12A

1 **a** 14 units^2 **b** 18 units^2 **c** 22 units^2
d 26 units^2 **e** 28 units^2 **f** 48 units^2

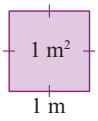
2 **a** 34 units^2 **b** 29 units^2 **c** 35 units^2
d 43 units^2

3 **a** 21 units^2 **b** $36\frac{1}{2}$ units^2 **c** 55 units^2
d 41 units^2 **e** $49\frac{1}{2}$ units^2 **f** 48 units^2

4 Whale 16–17 units^2, starfish 5–6 units^2, bird 10–11 units^2

5 **a** 6 cm^2 **b** 9 cm^2 **c** 15 cm^2 **d** 6 cm^2

6 **a** A square metre (1 m^2) is the area of a square with side length 1 m.
b A square kilometre (1 km^2) is the area of a square with side length 1 km.



Exercise 12B

1 **a** $A = 5 \times 3 = 15$ km^2 **b** $A = 7 \times 2 = 14$ m^2
c $A = 12 \times 5 = 60$ mm^2

2 **a** 72 cm^2 **b** 27 mm^2 **c** 40 m^2
d 77 cm^2 **e** 30 mm^2 **f** 12 cm^2

3 **a** $A = 5 \times 5 = 25\ cm^2$ **b** $A = 8 \times 8 = 64\ m^2$
c $16\ mm^2$ **d** $36\ cm^2$
4 **a** $12\ cm^2$ **b** $20\ cm^2$ **c** $18\ cm^2$ **d** $10\ cm^2$
5 **a** Area of large rectangle $= 5 \times 4 = 20\ cm^2$
Area of unshaded rectangle $= 3 \times 2 = 6\ cm^2$
$\therefore$ Shaded area $= 20 - 6 = 14\ cm^2$
b Area of large rectangle $= 6 \times 4 = 24\ m^2$
Area of unshaded rectangle $= 3 \times 2 = 6\ m^2$
$\therefore$ Shaded area $= 24 - 6 = 18\ m^2$
6 **a** $32\ mm^2$ **b** $16\ m^2$
7 **a** $A = (12 \times 6) + (5 \times 4) = 72 + 20 = 92\ cm^2$
b $A = (8 \times 4) + (6 \times 3) = 32 + 18 = 50\ m^2$
8 **a** $96\ cm^2$ **b** $42\ mm^2$
c $52\ m^2$ **d** $41\ cm^2$
9 **a** $96\ km^2$ **b** $44\ m^2$
c $62\ cm^2$ **d** $38\ mm^2$
10 **a** $A = (12 \times 10) - (6 \times 5) = 120 - 30 = 90\ km^2$
b $A = (15 \times 12) - (8 \times 5) = 180 - 40 = 140\ cm^2$
11 **a** $225\ mm^2$ **b** $184\ m^2$
c $300\ mm^2$ **d** $120\ m^2$

Exercise 12C

1 **a** $A = \frac{1}{2} \times 12 \times 8 = 48\ cm^2$
b $A = \frac{1}{2} \times 5 \times 6 = 15\ m^2$
c $A = \frac{1}{2} \times 6 \times 4 = 12\ mm^2$
2 **a** $16\ cm^2$ **b** $21\ mm^2$ **c** $30\ m^2$ **d** $40\ cm^2$
e $24\ m^2$ **f** $12\ km^2$ **g** $14\ cm^2$ **h** $90\ cm^2$
i $30\ cm^2$ **j** $15\ m^2$ **k** $4\ mm^2$ **l** $16\ cm^2$
3 **a** $A = \frac{1}{2}(10 \times 4) + (10 \times 5) = 20 + 50 = 70\ cm^2$
b $A = (8 \times 7) + \frac{1}{2}(8 \times 6) = 56 + 24 = 80\ cm^2$
c $A = (10 \times 5) + \frac{1}{2}(10 \times 5) = 50 + 25 = 75\ mm^2$
4 **a** $35\ cm^2$ **b** $50\ cm^2$ **c** $50\ m^2$

Exercise 12D

1 **a** $12 \times 9 = 108\ cm^2$ **b** $6 \times 11 = 66\ m^2$
c $20 \times 5 = 100\ mm^2$
2 **a** $68\ cm^2$ **b** $96\ m^2$ **c** $26\ mm^2$
d $45\ cm^2$ **e** $517\ mm^2$ **f** $24.8\ m^2$
g $32.2\ m^2$ **h** $90.2\ mm^2$ **i** $123\ cm^2$
j $46.5\ cm^2$ **k** $284\ m^2$ **l** $107.3\ mm^2$
3 **a** $140\ cm^2$ **b** $120\ m^2$ **c** $91\ cm^2$
d $252\ mm^2$ **e** $270\ m^2$ **f** $20.5\ cm^2$
g $18.9\ cm^2$ **h** $61.2\ m^2$ **i** $25.11\ m^2$
j $106.64\ m^2$
4 **a** triangle, $bh + \frac{1}{2}bh$
$= 14 \times 10 + \frac{1}{2} \times 6 \times 10$
$= 140 + 30 = 170\ m^2$
b parallelogram, $s^2 + bh$
$= 3^2 + 3 \times 17$
$= 9 + 51 = 60\ cm^2$
5 **a** $330\ cm^2$ **b** $360\ mm^2$
c $525.78\ m^2$ **d** $91.64\ m^2$

Exercise 12E

1 **a** Back $= EFHG$ **b** Side $= AEGD$
c Bottom $= DGHC$
2 **a** **i** Back $= FDE$, side 1 $= ADFC$, side 2 $= ADEB$, bottom $= CFEB$
ii Front and back
iii Front and back, side 1 and side 2
b **i** Back $= DEF$, side 1 $= ADFC$, side 2 $= ADEB$, bottom $= CFEB$
ii Front and back **iii** Front and back

3 **a**

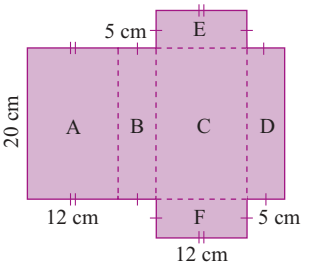
Face	Length	Breadth
LMNO	12 cm	5 cm
HIJK	12 cm	5 cm
IMNJ	8 cm	5 cm
HLOK	8 cm	5 cm
HLMI	12 cm	8 cm
KONJ	12 cm	8 cm

b

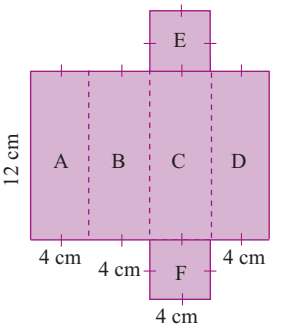
Face	Base	Height
ABC	8 cm	6 cm
DEF	8 cm	6 cm
Face	**Length**	**Breadth**
ABED	11 cm	10 cm
BEFC	11 cm	6 cm
ACFD	11 cm	8 cm

Exercise 12F

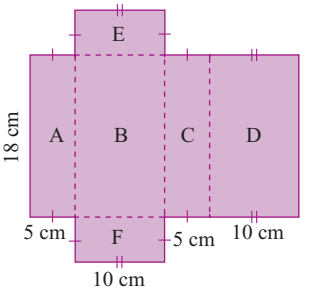
1 **a** $352\ cm^2$ **b** $150\ mm^2$
c $211.2\ cm^2$ **d** $108\ cm^2$
2 **a**



b A $= 240\ cm^2$, B $= 100\ cm^2$, C $= 240\ cm^2$, D $= 100\ cm^2$, E $= 60\ cm^2$, F $= 60\ cm^2$
c SA $= 800\ cm^2$
3 **a**

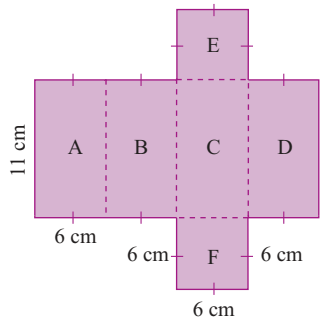


b They are the same.
c A, B, C and D are all $48\ cm^2$; E and F are $16\ cm^2$.
d SA $= 224\ cm^2$
4 **a** **i**



ii A $= 90\ cm^2$, B $= 180\ cm^2$, C $= 90\ cm^2$, D $= 180\ cm^2$, E $= 50\ cm^2$, F $= 50\ cm^2$
iii SA $= 640\ cm^2$

b i

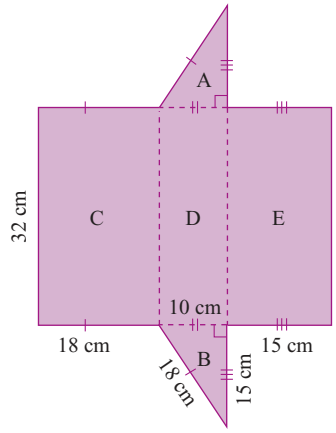


ii A = 66 cm², B = 66 cm², C = 66 cm², D = 66 cm², E = 36 cm², F = 36 cm²

iii SA = 336 cm²

5 a 828 cm² **b** 220.4 mm²

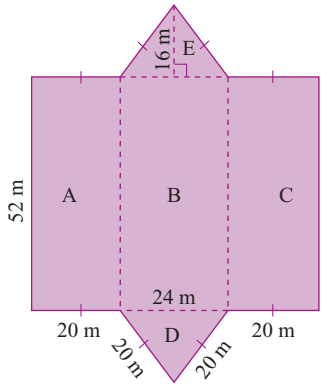
6 a



b A = 75 cm², B = 75 cm², C = 576 cm², D = 320 cm², E = 480 cm²

c SA = 1526 cm²

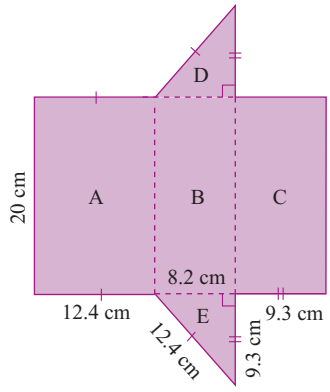
7 a i



ii A = 1040 cm², B = 1248 cm², C = 1040 cm², D = 192 cm², E = 192 cm²

iii SA = 3712 cm²

b i



ii A = 248 m², B = 164 m², C = 186 m², D = 38.13 m², E = 38.13 m²

iii SA = 674.26 m²

Exercise 12G

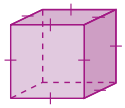
1 a 150 mm² **b** 392 cm² **c** 684 cm²
d 496 cm² **e** 268 cm² **f** 408 m²
2 a 54 m² **b** 294 mm² **c** 216 cm²
3 a 600 cm² **b** 121.5 m² **c** 576.24 mm²
4 a 310 cm² **b** 576 cm² **c** 10 650 mm²
d 352 cm² **e** 740 cm² **f** 314 m²
5 a 360 cm² **b** 224 cm² **c** 1440 cm²
d 1200 mm² **e** 264.8 m² **f** 200 mm²
6 167.04 cm² **7** 108 m² **8** Sultana Bran
9 a 38.28 m² **b** 34.4 m² **c** Belinda's

Exercise 12H

1 a 18 cm³ **b** 28 cm³ **c** 13 cm³

2 a A cubic centimetre (1 cm³) is the amount of space occupied by a cube of side length 1 cm.

b A cubic metre (1 m³) is the amount of space occupied by a cube of side length 1 m.



Exercise 12I

1 a $V = 5 \times 4 \times 3 = 60$ cm³
b $V = 7 \times 3 \times 2 = 42$ cm³
2 a 20 m³ **b** 360 m³ **c** 180 mm³
3 a $V = 190 \times 4 = 760$ cm³
b $V = 75 \times 7 = 525$ cm³
c $V = 30 \times 9 = 270$ cm³
4 a 225 cm³ **b** 90 cm³ **c** 64 cm³
d 448 cm³ **e** 760 cm³ **f** 400 mm³
g 930 m³ **h** 1444.5 cm³ **i** 982.8 cm³
5 a $V = (5 \times 5) \times 5 = 125$ cm³
b $V = (30 \times 12) \times 7 = 2520$ cm³
c $V = \frac{1}{2}(8 \times 9) \times 12 = 432$ mm³
6 a 8 mm³ **b** 65.61 cm³ **c** 2880 cm³
d 10 240 mm³ **e** 3125 cm³ **f** 3600 mm³
g 3570 cm³ **h** 300 cm³ **i** 1920 mm³
j 4950 cm³ **k** 220 cm³
l 2406 cm³ (rounded)
7 a Area of base = (14 × 6) + (4 × 4)
= 84 + 16 = 100 mm²
Height = 8 mm
Volume = 100 × 8 = 800 mm³
b Area of base = $\frac{1}{2}$(14 × 8) + (32 × 12)
= 56 + 384 = 440 cm²
Height = 15 cm
Volume = 440 × 15 = 6600 cm³
8 a 1098 mm³ **b** 3136.5 cm³ **c** 1080 m³
d 1212 cm³ **e** 3200 cm³ **f** 1792 m³
g 1860 m³ **h** 7250 m³ **i** 68 900 cm³

Calculator activities

1 a 5.1531 m² **b** 161.862 cm²
c 53.29 cm² **d** 64.97 cm²
e 160.425 km² **f** 210.7 cm²
2 a 36.33 cm² **b** 454.22 mm²
c 38.065 cm² **d** 40 942.59 mm²

3 **a** 222.58 cm^2 **b** 309.52 cm^2
c 1745.57 mm^2 **d** 209.1675 cm^2
e 291.885 cm^2 **f** 71 754.21 m^2
4 **a** 49.2 m^2 **b** 25.2 m^2
5 **a** 336 cm^2 **b** 448 cm^2
6 **a** 230.26 cm^2 **b** 306.99 cm^2
7 **a** 238.328 cm^3 **b** 113.15 mm^3 **c** 708.76 cm^3
8 **a** 124.488 cm^3 **b** 1168.992 m^2

Language in mathematics

1

K	I	L	O	L	I	T	R	E	A	X	D	S	S	O	R	G
T	H	D	A	E	R	B	A	S	E	F	N	T	B	J	D	F
C	D	B	H	E	I	G	H	T	C	T	O	C	K	G	I	E
C	X	E	Z	A	F	E	D	I	G	H	I	E	M	H	M	I
R	G	R	A	M	J	N	G	M	I	G	S	J	I	U	E	L
A	M	E	A	S	U	R	E	A	Y	I	R	B	L	Y	N	E
L	B	N	W	K	V	L	L	T	H	E	E	O	L	O	S	R
U	L	I	Q	U	I	D	I	E	T	W	V	N	I	J	I	T
G	E	A	A	T	Y	C	T	N	M	Q	N	Z	G	P	O	I
N	N	T	U	X	A	P	R	I	S	M	O	M	R	I	N	L
A	G	N	W	P	T	R	E	V	N	O	C	Q	A	H	S	I
T	T	O	A	S	T	O	N	N	E	D	O	U	M	S	L	L
C	H	C	Y	L	I	N	D	E	R	I	B	T	A	P	C	L
E	W	D	V	Z	R	A	S	Q	N	F	O	M	Q	K	U	I
R	E	C	O	M	M	E	N	D	E	Y	N	R	R	M	S	M

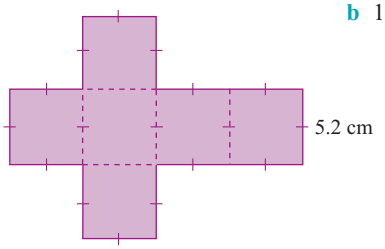
2 Two each of:
a mm^2, cm^2, m^2, km^2 **b** mm^3, cm^3, m^3, km^3
c mL, L
3 **a** one-thousandth ($\frac{1}{1000}$) **b** one-hundredth ($\frac{1}{100}$)
c one-thousand (1000)
4 space, prism, container, surface, cross-section, volume, litre, capacity
5 height, length, millilitre

Check your skills

1 B **2** C **3** B **4** D **5** D
6 C **7** C **8** B **9** D **10** C
11 A **12** B **13** B **14** D **15** B
16 C **17** D **18** A

Review set 12A

1 35 units^2
2 **a** 1000 mm^2 **b** 28 m^2
c 900 mm^2 **d** 40 cm^2
3 **a** 71 m^2 **b** 40 cm^2 **c** 220 mm^2
4 **a** 68 m^2 **b** 775 cm^2
5 **a** *EFGH* **b** *AEFB* and *DHGC* **c** *AEHD* and *BFGC*
6 **a** **b** 162.24 cm^2



7 288 mm^2
8 Surface area is the sum of the areas of all faces of a solid.
9 **a** 1932.5 cm^3 **b** 1456.44 cm^3
10 **a** 966 cm^3 **b** 4125 cm^3
11 210.912 m^3

Review set 12B

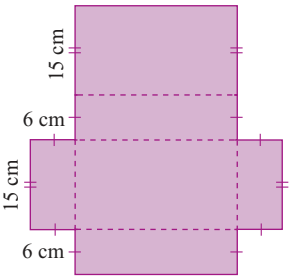
1 18 units^2
2 **a** 150 mm^2 **b** 36 cm^2 **c** 4 m^2 **d** 45 km^2
3 **a** 155 cm^2 **b** 18 000 mm^2 **c** 300 m^2
4 **a** 34 m^2 **b** 1200 m^2
5 **a** *UVW* **b** Equilateral triangle
c *XYZ* **b** Rectangle
6 **a** Rectangular prism
b A = 9 cm^2, B = 20 cm^2, C = 9 cm^2, D = 20 cm^2, E = 7.2 cm^2, F = 7.2 cm^2
c 72.4 cm^2
7 234.6 m^2
8 **a** 8 cm^3 **b** 23 cm^3
9 300.763 cm^3
10 **a** 759.2 cm^3 **b** 8.505 m^3
11 6174 cm^3

Review set 12C

1 21 units^2
2 **a** 100 km^2 **b** 120 mm^2 **c** 32 cm^2
d 200 mm^2
3 **a** 64 m^2 **b** 225 cm^2 **c** 600 mm^2
4 **a** 360 cm^2 **b** 200 m^2
5

Face	Length	Breadth
STUV	7.7 cm	6.6 cm
WXYZ	7.7 cm	6.6 cm
SVZW	17.2 cm	7.7 cm
TUYX	17.2 cm	7.7 cm
VUYZ	17.2 cm	6.6 cm
STXW	17.2 cm	6.6 cm

6 593.56 cm^2
7 **a** **b** 1272 cm^2



8 **a** 27 cm^3 **b** 12 cm^3
9 7.632 m^3
10 6 cm^3
11 **a** 152.964 cm^3 **b** 445.12 cm^3

CHAPTER 13 DATA MEASURES

Diagnostic test

1 B **2** D **3** C **4** B **5** C
6 C **7** B **8** A **9** B **10** D
11 B **12** D **13** C **14** C

Exercise 13A

1 $\bar{x} = \frac{8 + 9 + 10 + 11 + 11 + 12}{6} = \frac{61}{6} = 10.2$
2 She did not use brackets.
3,4 **a** 6.4 **b** 5.2 **c** 14 **d** 28.9
e 3.5 **f** 22.5 **g** 55.4 **h** 5.1
i 243.5 **j** 103.4

5 a

Score (x)	Frequency (f)	$f \times x$
8	6	48
9	8	72
10	15	150
11	11	121
12	3	36
	$\Sigma f = 43$	$\Sigma fx = 427$

b 9.9

6 a

Score (x)	Frequency (f)	$f \times x$
18	3	54
19	5	95
20	10	200
21	15	315
22	8	176
23	1	23
	$\Sigma f = 42$	$\Sigma fx = 863$

b 20.5

7, 8 a i

x	f	$f \times x$
13	2	26
14	3	42
15	6	90
16	4	64
17	1	17
	$\Sigma f = 16$	$\Sigma fx = 239$

ii 14.9

b i

x	f	$f \times x$
2	4	8
3	6	18
4	5	20
5	3	15
6	2	12
	$\Sigma f = 20$	$\Sigma fx = 73$

ii 3.7

c i

x	f	$f \times x$
50	3	150
51	5	255
52	8	416
53	6	318
54	2	108
55	4	220
	$\Sigma f = 28$	$\Sigma fx = 1467$

ii 52.4

d i

x	f	$f \times x$
18	12	216
19	28	532
20	25	500
21	26	546
22	9	198
	$\Sigma f = 100$	$\Sigma fx = 1992$

ii 19.9

Exercise 13B

1 3, 4, 5, 5, 6, 8, 9. The mode is 5.

2 a 9 **b** 3 **c** 13 **d** 27 **e** 4
f 20 **g** 57 **h** 4 **i** 243 **j** 104

3 a A **b** Mon **c** green

4 Range = highest score − lowest score = 9 − 3 = 6

5 a 8 **b** 7 **c** 6 **d** 4 **e** 7
f 7 **g** 8 **h** 9 **i** 6 **j** 5

6 No, because the data is categorical

7 a The highest frequency is 12. This means the mode is 10.
b The highest score is 13. The lowest score is 9.
Range = 13 − 9 = 4.

8 a 5 **b** 13 **c** 26 **d** 25 kg **e** 102
f 35 patients/day

9 a 4 **b** 4 **c** 4 **d** 4 **e** 5 **f** 5

10 a Holden **b** Burger

Exercise 13C

1 Median = 8

2 a 6 **b** 5 **c** 13 **d** 29 **e** 4 **f** 57

3 a 243 **b** 21 **c** 104 **d** 4

4 5, 6, 8, 8, 10, 10, 11, 12 Leaving 8 and 10.
Median $= \frac{8 + 10}{2} = 9$

5 a $8\frac{1}{2}$ **b** 22 **c** 16.5 **d** 50 **e** $5\frac{1}{2}$
f 13.5 **g** 50 **h** 19 **i** 2.5 **j** 37

6 a 5 **b** 13 **c** 52 **d** 35.5 **e** 10 **f** 26

Exercise 13D

1 a The leaf of 2 in the 6 stem occurs 3 times.
This means the mode is 62.
b Cross off until 56 and 58 are left.
Median $= \frac{56 + 58}{2} = 57$
c Highest score is 74. Lowest score is 40.
Range = 74 − 40 = 34

2 a 71 **b** 142

3 a 71 **b** $140\frac{1}{2}$

4 a 41 **b** 45

5 a

Stem	Leaf	f	$f \times$ stem	Sum of leaves
5	2233588	7	350	31
6	12467899	8	480	46
7	11113468	8	560	25
8	1124	4	320	8
9	47	2	180	11
	Total	**29**	**1890**	**121**

Mean = 69.3 (to 1 decimal place), mode = 71, median = 69, range = 45

b

Stem	Leaf	f	$f \times$ stem	Sum of leaves
10	457	3	300	16
11	4789	4	440	28
12	001258888	9	1080	40
13	4	1	130	4
14	112246777	9	1260	37
15	2233456	7	1050	25
	Total	**33**	**4260**	**150**

Mean = 133.6 (to 1 decimal place), mode = 128, median = 134, range = 52

c

Stem	Leaf	f	$f \times$ stem	Sum of leaves
1	11234799	8	80	36
2	012233346788	12	240	47
3	2248	4	120	16
4	2345889999	10	400	66
5	0137999	7	350	38
6	12344788	8	480	37
	Total	**49**	**1670**	**240**

Mean = 39.0 (to 1 decimal place), mode = 49, median = 42, range = 57

d

Stem	Leaf	f	$f \times$ stem	Sum of leaves
110	479	3	3300	20
111	00345555789	11	12210	51
112	11333456888	11	12320	50
113	1111227899	10	11300	41
114	0011225688999	13	14820	60
115	0111223489	10	11500	31
116	03455668899	11	12760	63
	Total	**69**	**78210**	**316**

Mean = 1138.1 (to 1 decimal place), modes = 1115 and 1131, median = 1139, range = 65

Exercise 13E

1 **a** 17.6 **b** 15 **c** 17 **d** 7

2 **a** 21.4 **b** 25 **c** 22.5 **d** 17

3 **a** **i** 7 **ii** Yes
iii Mean = 6.9, median = 7; both good measures
b **i** No mode **ii** No
iii Mean = 5, median = 4.5; both good measures
c **i** 35 **ii** No, not a central value
iii Mean = 29, median = 26; both good measures

4 **a** **i** 9.2 **ii** Yes
iii Median = 9 is a good measure.
Mode = 8 is not a central value.
b **i** 29 **ii** No, affected by outlier 110
iii Median = 9 is satisfactory.
Mode = 8 is not a central value.

5 **a** **i** 15 **ii** Yes
iii Mode = 14, 15 and 18 is not a good measure.
Mean = 15 is satisfactory.
b **i** 5 **ii** No, not a central value
iii Mode = 5 is not central.
Mean = 4 is satisfactory.

6 **a** Mean = 6.25, mode = 7, median = 6.5
b Mode

7 **a** \$2 950 000 **b** \$64 130
c **i** 40 **ii** 6
d \$50 000 **e** \$50 000
f **i** Mean **ii** Median or mode
g Median: it is not affected by the outlier or the few very high scores.

8 **a** Mean = 4, mode = 6, median = 4.5 **b** Mode

9 **a** Mean = \$640 000
b The mean has been affected by the high price \$960 000. No mode. Median = \$560 000 is the most appropriate.

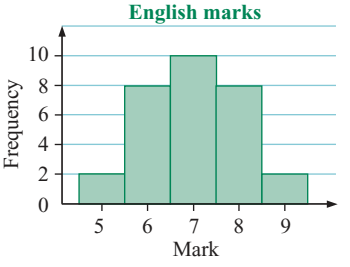
10 **a** 20
b Mean = 11.3, mode = 12, median = 12, range = 4
c 12 years, median and mode
d Yes, as most start high school and receive a mobile phone at age 12

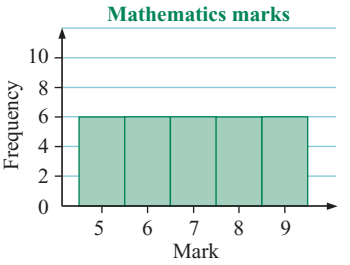
11 **a** Mean \$2.60 (or \$2.65 to nearest 5 cents), mode \$2.50, median \$2.70
b \$1
c Most popular choice and in the middle of the prices
d Mean could be used as \$2.60 (or \$2.65) is just above the mode but still less than the maximum price. Giving change at the canteen for \$2.65 might be an issue.
e Median could be used as \$2.70 is just above the mode but still less than the maximum price.
f The second and third most popular choices were \$2.90 and \$3, so a high price is more popular than even \$2.50.

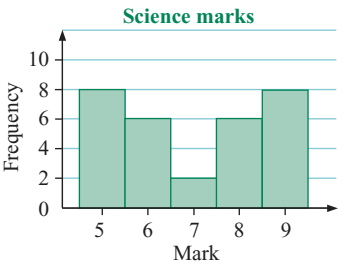
Exercise 13F

1 **a** Tiffany 637, Con 655
b Tiffany: mean = 79.6, mode = 82, median = 82
Con: mean = 81.9, mode = 86, median = 82
c 2 **d** Tiffany 37, Con 16
e Tiffany **f** Tiffany
g Con, because he has a higher mean and is more consistent with a smaller range. Tiffany has a weakness in the topic tested by test 8.
Or Tiffany did better on more tests. Ignoring the outlier (she may have been sick) would give her higher averages. She had the highest score.

2 **a** English: mean = 7, mode = 7, median = 7, range = 4
Mathematics: mean = 7, no mode, median = 7, range = 4
Science: mean = 7, mode = 5 and 9, median = 7, range = 4
b Mode **c** English, as there were fewer low scores
d







3 **a** Back row: mean = 25.55, mode = 15 and 20, median = 21.5, range = 39
Front row: mean = 25.15, mode = 32, median = 22.5, range = 38
b Younger people sat in the front row.

Language in mathematics

1 **a** The score that occurs most often is the mode; or the mode is the score that occurs most often.
b To find the mean, add all the scores together then divide by the number of scores.
c The median is the middle score when the scores are in order.
d The mean does not have to be a score.
e The mode must be a score.

Check your skills

1 D	**2** A	**3** D	**4** C	**5** C
6 D	**7** A	**8** C	**9** B	**10** B
11 B	**12** C	**13** C	**14** A	**15** C

Review set 13A

1 A mean uses all the scores but is affected by outliers. The median is not affected by outlier scores, but does not include all the data.

2 **a** 16.8	**b** 19	**c** 17.5	**d** 9	
3 **a** 21.4	**b** 21	**c** 21	**d** 5	
4 **a** 40.3	**b** 44	**c** 40	**d** 34	

5 **a** Class A: mean = 41.1, mode = 51, median = 41, range = 46
Class B: mean = 42.9, mode = 46, median = 42, range = 40
b Class B has a higher mean.

Review set 13B

1 **a** 32.3	**b** 31	**c** 31.5	**d** 4	
2 **a** 19		**b** 13		

c The mean is affected by an outlier score but the median is not.

3 **a** 57.2	**b** 58	**c** 57	**d** 5	
4 **a** 255.9	**b** 240	**c** 258	**d** 45	

5 **a** Anthony: 354, Mark: 374
b Anthony: mean = 59, no mode, median = 63, range = 37
Mark: mean = 62.3, mode = 66, median = 64.5, range = 16
c 4 **d** Anthony **e** Anthony
f Mark has a higher mean and his marks are more clustered. Or, if you ignore the outlier, Anthony has the higher mean. He also scored better on more topics.

Review set 13C

1 An outlier is significantly larger or smaller than all the other scores. It affects the mean most.

2 **a** 19.4	**b** 18	**c** 19	**d** 4	
3 **a** 5.08	**b** 3	**c** 5	**d** 6	
4 **a** 104	**b** 118	**c** 104	**d** 64	

5 **a**

City	Mean	Mode	Median	Range
City X	27.4	25, 40	26	29
City Y	18.94	30	25.5	23

b Using the mean as the average, City X has a higher mean temperature. The medians are close to equal. The range of City X is higher, indicating a greater variety of temperatures, so City Y has a more consistent temperature.

6 **a** Mean = 1.8 **b** Mode = 1 **c** Median = 1
b The mean is the most appropriate measure as the mode and the median are the first score.

7 **a** No, as the mode is Matthew's highest score.
b Melissa's mean is higher, but her range is lower.
c No, Melissa's marks are more consistent.
d No, Melissa has a higher mean.

CHAPTER 14 LINEAR EQUATIONS

Diagnostic test

1 D	**2** C	**3** D	**4** A	**5** A
6 B	**7** A	**8** D	**9** C	**10** D
11 C				

Exercise 14A

1 Examples:

a

x	2	5	3	4
$2x + 17$	21	27	23	25

$x = 4$

b

x	10	5	8	9
$3x - 15$	15	0	9	12

$x = 9$

2 **a** $x = 12$ **b** $x = 7$ **c** $x = 12$ **d** $x = 9$

Exercise 14B

1 **a** 3 counters, 2 cubes, 2 equal groups, 4 counters, $x = 4$.
$2x + 3 = 2 \times 4 + 3 = 11$
b 5 counters, 3 cubes, 3 equal groups, 3 counters, $x = 3$.
$3x + 5 = 3 \times 3 + 5 = 14$

2 **a** $x = 2$ **b** $x = 3$ **c** $x = 2$

3 **a** $x = 1$ **b** $x = 2$ **c** $x = 4$

Exercise 14C

1 **a** $x + 9$ **b** $x - 7$ **c** $3x$ **d** $\frac{x}{8}$ **e** $2x + 5$
f $\frac{x}{3} - 1$ **g** $\frac{5x}{7}$ **h** $5(x + 7)$ **i** $\frac{x - 10}{4}$

2 **a** $x \xrightarrow{\times 9} 9x$

b $w \xrightarrow{-6} w - 6$

c $z \xrightarrow{\div 8} \frac{z}{8}$

d $m \xrightarrow{+13} m + 13$

e $y \xrightarrow{\times 2} 2y \xrightarrow{-11} 2y - 11$

f $a \xrightarrow{+3} a + 3 \xrightarrow{\times 2} 2(a + 3)$

g $x \xrightarrow{\div 5} \frac{x}{5} \xrightarrow{+27} \frac{x}{5} + 27$

h $w \xrightarrow{+7} w + 7 \xrightarrow{\div 6} \frac{w + 7}{6}$

i $x \xrightarrow{\times 3} 3x \xrightarrow{\div 2} \frac{3x}{2}$

j $k \xrightarrow{\times 3} 3k \xrightarrow{+ 2} 3k + 2$

k $z \xrightarrow{+ 2} z + 2 \xrightarrow{\times 3} 3(z + 2)$

l $y \xrightarrow{\div 5} \frac{y}{5} \xrightarrow{- 4} \frac{y}{5} - 4$

m $z \xrightarrow{- 4} z - 4 \xrightarrow{\div 5} \frac{z - 4}{5}$

n $x \xrightarrow{\times 5} 5x \xrightarrow{+ 6} 5x + 6$

o $w \xrightarrow{\times 4} 4w \xrightarrow{\div 7} \frac{4w}{7}$

p $p \xrightarrow{\div 7} \frac{p}{7} \xrightarrow{\times 4} \frac{4p}{7}$

q $n \xrightarrow{- 4} n - 4 \xrightarrow{\times 3} 3(n - 4)$

3 a $x \xrightarrow{+ 9} x + 9$

b $y \xrightarrow{- 7} y - 7$

c $z \xrightarrow{\times 8} 8z$

d $q \xrightarrow{\div 6} \frac{q}{6}$

e $x \xrightarrow{\times 5} 5x \xrightarrow{- 3} 5x - 3$

f $x \xrightarrow{\times 2} 2x \xrightarrow{+ 1} 2x + 1$

g $x \xrightarrow{+ 1} x + 1 \xrightarrow{\times 2} 2(x + 1)$

h $w \xrightarrow{\div 7} \frac{w}{7} \xrightarrow{+ 2} \frac{w}{7} + 2$

i $m \xrightarrow{\div 5} \frac{m}{5} \xrightarrow{- 3} \frac{m}{5} - 3$

j $q \xrightarrow{+ 2} q + 2 \xrightarrow{\div 3} \frac{q + 2}{3}$

k $z \xrightarrow{\times 5} 5z \xrightarrow{\div 7} \frac{5z}{7}$

or $z \xrightarrow{\div 7} \frac{z}{7} \xrightarrow{\times 5} \frac{5z}{7}$

l $q \xrightarrow{\div 4} \frac{q}{4} \xrightarrow{\times 9} \frac{9q}{4}$

or $q \xrightarrow{\times 9} 9q \xrightarrow{\div 4} \frac{9q}{4}$

4 a *Step 1:* $x \xrightarrow{+ 23} x + 23$

Step 2: Replace $x + 23$ by 51.

$x \xrightarrow{+ 23} 51$

Step 3: $28 \xleftarrow[- 23]{} 51$ $\quad x = 28$

b *Step 1:* $q \xrightarrow{- 38} q - 38$

Step 2: Replace $q - 38$ by 25.

$q \xrightarrow{- 38} 25$

Step 3: $63 \xleftarrow[+ 38]{} 25$ $\quad q = 63$

c *Step 1:* $w \xrightarrow{\times 7} 7w$

Step 2: Replace $7w$ by 98.

$w \xrightarrow{\times 7} 98$

Step 3: $14 \xleftarrow[\div 7]{} 98$ $\quad w = 14$

d *Step 1:* $n \xrightarrow{\div 13} \frac{n}{13}$

Step 2: Replace $\frac{n}{13}$ by 6.

$n \xrightarrow{\div 13} 6$

Step 3: $78 \xleftarrow[\times 13]{} 6$ $\quad n = 78$

5 a $n = 28$ **b** $w = 45$ **c** $x = 12$
d $z = 126$ **e** $p = 19$ **f** $w = 28$
g $z = 7$ **h** $x = 9$

6 a *Step 1:* $z \xrightarrow{\times 5} 5z \xrightarrow{+ 9} 5z + 9$

Step 2: Replace $5z + 9$ by 54.

$z \xrightarrow{\times 5} 5z \xrightarrow{+ 9} 54$

Step 3: $9 \xleftarrow[\div 5]{} 45 \xleftarrow[- 9]{} 54$ $\quad z = 9$

Check: $5z + 9 = 5 \times 9 + 9 = 54$

b *Step 1:* $k \xrightarrow{\div 6} \frac{k}{6} \xrightarrow{- 1} \frac{k}{6} - 1$

Step 2: Replace $\frac{k}{6} - 1$ by 10.

$k \xrightarrow{\div 6} \frac{k}{6} \xrightarrow{- 1} 10$

Step 3: $66 \xleftarrow[\times 6]{} 11 \xleftarrow[+ 1]{} 10$ $\quad k = 66$

Check: $\frac{k}{6} - 1 = \frac{66}{6} - 1 = 10$

c *Step 1:* $m \xrightarrow{- 6} m - 6 \xrightarrow{\div 7} \frac{m - 6}{7}$

Step 2: Replace $\frac{m - 6}{7}$ by 9.

$m \xrightarrow{- 6} m - 6 \xrightarrow{\div 7} 9$

Step 3: $69 \xleftarrow[+ 6]{} 63 \xleftarrow[\times 7]{} 9$ $\quad m = 69$

Check: $\frac{m - 6}{7} = \frac{69 - 6}{7} = 9$

7 a $w = 7$ **b** $t = 5$ **c** $a = 2$ **d** $x = 56$
e $m = 100$ **f** $k = 36$ **g** $y = 25$ **h** $z = 23$
i $x = 1$ **j** $y = 12$

8 a *Step 1:* $k \xrightarrow{\times 4} 4k \xrightarrow{\div 5} \frac{4k}{5}$

Step 2: Replace $\frac{4k}{5}$ by 20.

$k \xrightarrow{\times 4} 4k \xrightarrow{\div 5} 20$

Step 3: $25 \xleftarrow[\div 4]{} 100 \xleftarrow[\times 5]{} 20$ $\quad k = 25$

Check: $\frac{4k}{5} = \frac{4 \times 25}{5} = 20$

b *Step 1:* $y \xrightarrow{\div 2} \frac{y}{2} \xrightarrow{\times 5} \frac{5y}{2}$

Step 2: Replace $\frac{5y}{2}$ by 25.

$y \xrightarrow{\div 2} \frac{y}{2} \xrightarrow{\times 5} 25$

Step 3: 10 $\xleftarrow{\times 2}$ 5 $\xleftarrow{\div 5}$ 25 $y = 10$

Check: $\frac{5y}{2} = \frac{5 \times 10}{2} = 25$

9 **a** $x = 12$ **b** $y = 12$ **c** $w = 8$
d $z = 10$ **e** $x = 9$

Exercise 14D

1 **a** $n + 21$ | 36

To undo 'add 21' you need to 'subtract 21'.
Subtract 21 from both sides.

$n + 21 - 21$ | $36 - 21$ $\quad$ n | 15

$\therefore n = 36 - 21 = 15$

b $q - 35$ | 21

To undo 'subtract 35' you need to 'add 35'.
Add 35 to both sides.

$q - 35 + 35$ | $21 + 35$ $\quad$ q | 56

$\therefore q = 21 + 35 = 56$

c $9x$ | 36

To undo 'multiply by 9' you need to 'divide by 9'.
Divide by 9 on both sides.

$\frac{9x}{9}$ | $\frac{36}{9}$ $\quad$ x | 4

$\therefore x = \frac{36}{9} = 4$

d $\frac{y}{3}$ | 13

To undo 'divide by 3' you need to 'multiply by 3'.
Multiply by 3 on both sides.

$\frac{y}{3} \times 3$ | 13×3 $\quad$ y | 39

$\therefore y = 13 \times 3 = 39$

2 **a** $z = 17$ **b** $q = 62$ **c** $x = 20$
d $w = 42$ **e** $k = 42$ **f** $w = 11\frac{1}{2}$
g $p = 44$ **h** $k = 91$

Exercise 14E

1 **a**
$m + 33 = 51$
$m + 33 - 33 = 51 - 33$
$m = 18$

b
$w - 12 = 15$
$w - 12 + 12 = 15 + 12$
$w = 27$

c
$7z = 56$
$\frac{7z}{7} = \frac{56}{7}$
$z = 8$

d
$\frac{n}{4} = 11$
$\frac{n}{4} \times 4 = 11 \times 4$
$n = 44$

2 **a** $q = 23$ **b** $p = 77$ **c** $z = 9$
d $m = 108$ **e** $w = 11$ **f** $v = 8$
g $n = 35$ **h** $k = 31$

3 **a** $h = 2.8$ **b** $k = 8.7$ **c** $w = 3.4$
d $q = 8.4$ **e** $x = 4\frac{1}{2}$ **f** $y = 1\frac{7}{24}$
g $z = \frac{1}{3}$ **h** $p = 3\frac{1}{3}$

4 **a** $w = -5$ **b** $m = 4$ **c** $z = -3$
d $n = -28$ **e** $k = 7$ **f** $t = -2$
g $x = -24$ **h** $y = -5$

5 **a**
$3m + 2 = 20$
$3m + 2 - 2 = 20 - 2$
$3m = 18$
$\frac{3m}{3} = \frac{18}{3}$
$m = 6$
Check: $3m + 2 = 3 \times 6 + 2 = 20$

b
$3(m + 2) = 21$
$\frac{3(m + 2)}{3} = \frac{21}{3}$
$m + 2 = 7$
$m + 2 - 2 = 7 - 2$
$m = 5$
Check: $3(m + 2) = 3 \times (5 + 2) = 21$

6 **a** $x = 7$ **b** $x = 2$ **c** $z = 4$
d $z = 8$ **e** $y = 4$ **f** $y = 11$
g $w = 3$ **h** $k = 4$ **i** $x = 7$
j $n = 11$ **k** $t = 3$ **l** $q = 7$

7 **a** $x = -1$ **b** $n = -2$ **c** $k = -1$ **d** $t = 2$
e $v = -4$ **f** $z = 2$ **g** $k = -1$ **h** $m = -1$

8 **a**
$\frac{w}{3} + 5 = 9$
$\frac{w}{3} + 5 - 5 = 9 - 5$
$\frac{w}{3} = 4$
$\frac{w}{3} \times 3 = 4 \times 3$
$w = 12$
Check: $\frac{w}{3} + 5 = \frac{12}{3} + 5 = 9$

b
$\frac{w + 5}{3} = 9$
$\frac{w + 5}{3} \times 3 = 9 \times 3$
$w + 5 = 27$
$w + 5 - 5 = 27 - 5$
$w = 22$
Check: $\frac{w + 5}{3} = \frac{22 + 5}{3} = 9$

9 **a** $m = 6$ **b** $m = 19$ **c** $n = 45$
d $n = 31$ **e** $x = 80$ **f** $x = 5$
g $w = 30$ **h** $z = 3$ **i** $k = 38$
j $n = 15$ **k** $t = 62$ **l** $t = 33$

10 **a** $x = -12$ **b** $n = 2$ **c** $m = -20$
d $w = -3$ **e** $z = -7$ **f** $k = -1$
g $x = -12$ **h** $n = -20$ **i** $x = 20$

11 **a**
$\frac{4m}{9} = 8$
$\frac{4m}{9} \times 9 = 8 \times 9$
$4m = 72$
$\frac{4m}{4} = \frac{72}{4}$
$m = 18$
Check: $\frac{4m}{9} = \frac{4 \times 18}{9} = 8$

b
$\frac{11z}{24} = 22$
$\frac{11}{24} \times z = 22$
$\frac{11}{24} \times z \div \frac{11}{24} = 22 \div \frac{11}{4}$
$\frac{11}{24} \times z \times \frac{24}{11} = 22 \times \frac{24}{11}$
$z = 48$
Check: $\frac{11z}{24} = \frac{11 \times 48}{24} = 22$

12 **a** $w = 10$ **b** $k = 6$ **c** $z = 12$
d $k = 10$ **e** $t = 7$

13 **a** $t = -10$ **b** $w = -6$ **c** $x = -14$
d $x = -22$ **e** $n = -13\frac{1}{2}$

14 **a**
$$\frac{4m}{3} = \frac{5}{8}$$
$$\frac{4m}{3} \times 3 = \frac{5}{8} \times 3$$
$$4m = \frac{15}{8}$$
$$4m \div 4 = \frac{15}{8} \div 4$$
$$4m \times \frac{1}{4} = \frac{15}{8} \times \frac{1}{4}$$
$$m = \frac{15}{32}$$
Check: $\frac{4m}{3} = \frac{4}{3} \times \frac{15}{32} = \frac{5}{8}$

b
$$\frac{5y}{2} = \frac{4}{7}$$
$$\frac{5}{2} \times y = \frac{4}{7}$$
$$\frac{5}{2} \times y \div \frac{5}{2} = \frac{4}{7} \div \frac{5}{2}$$
$$\frac{5}{2} \times y \times \frac{2}{5} = \frac{4}{7} \times \frac{2}{5}$$
$$y = \frac{8}{35}$$
Check: $\frac{5y}{2} = \frac{5}{2} \times \frac{8}{35} = \frac{4}{7}$

15 **a** $x = \frac{15}{16}$ **b** $k = \frac{3}{10}$ **c** $t = 1\frac{1}{9}$
d $p = \frac{5}{18}$ **e** $x = \frac{4}{7}$

16 *Method 1*
$$-3x = 6$$
$$\frac{-3x}{-3} = \frac{6}{-3}$$
$$x = -2$$
Method 2
$$5 = 11 + 3x$$
$$3x + 11 = 5$$
$$3x + 11 - 11 = 5 - 11$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$x = -2$$

17 **a** $x = -1$ **b** $x = -2\frac{1}{2}$
c $x = 5$ **d** $x = 11$

Exercise 14F

1 **a** $x + 6 = 11$ **b** $x - 4 = 1$
c $3x = 15$ **d** $\frac{x}{5} = 1$

3 **a** $3x + 5 = 23$ **b** $2x - 7 = 5$
c $5(x + 3) = 45$ **d** $4(x - 1) = 20$
e $\frac{x}{3} + 5 = 7$ **f** $\frac{x}{2} - 4 = -1$
g $\frac{x + 8}{4} = 3.5$ **h** $\frac{x - 6}{5} = 0$

Exercise 14G

1 $K = 1.6M$
$640 = 1.6M$ or $1.6M = 640$
$$\frac{1.6M}{1.6} = \frac{640}{1.6}$$
$$M = 400$$
$\therefore$ 640 kilometres = 400 miles

2 20 m² **3** 30 h
4 5.3 m **5** 320 m²
6 61 kg
7 **a** \$47 **b** 4.5 km
8 **a** 110 min **b** 1 kg
9 **a** \$800 **b** \$15 200
10 **a** \$589 **b** \$11 600
11 **a** 46 **b** 17
12 **a** 21 **b** 12
13 **a** $n + 45 = 83$ **b** $n = 38$
14 **a** $n - 53 = 17$ **b** $n = 70$
15 **a** $\frac{n + 39}{2} = 34$ **b** $n = 29$
16 **a** $5n + 3 = 38$ **b** $n = 7$
17 **a** $\frac{n}{3} - 2 = 2$ **b** $n = 12$
18 **a** $\frac{n - 2}{3} = 5$ **b** $n = 17$
19 **a** $2n + 3 = 33$ **b** $n = 15$
20 **a** $2n - 4 = 30$ **b** $n = 17$

Language in mathematics

1 **a** Add 3. **b** Subtract 5.
c Multiply by 9. **d** Divide by 4.
e Multiply by 6 then divide by 7, or multiply by six-sevenths.
f Multiply by 4 then add 5.
g Add 5 then multiply by 4.
h Divide by 5 then subtract 7.
i Subtract 7 then divide by 5.

2 **a** Add 1 to both sides then divide both sides by 3.
b Divide both sides by 3 then add 1 to both sides.
c Multiply both sides by 3 then subtract 2 from both sides.
d Subtract 2 from both sides then multiply both sides by 3.
e Multiply both sides by 9 then divide both sides by 8, or divide both sides by $\frac{8}{9}$.

Check your skills

1 D **2** D **3** C **4** D **5** B
6 B **7** B **8** A **9** A **10** D
11 D

Review set 14A

1 Examples:

a

x	6	10	11	12
$3x - 5$	13	25	28	31

$x = 12$

b

m	6	9	12	15
$\frac{m}{3} + 6$	8	9	10	11

$m = 15$

2 $3x + 3 = x + 5$. Take 3 counters from both sides.

3 **a** $5x + 3$ **b** $5(x + 3)$
c $\frac{x}{4} - 1$ **d** $\frac{x - 1}{4}$

4 **a** $x \xrightarrow{\times 3} 3x \xrightarrow{+ 7} 3x + 7$
b $z \xrightarrow{\times 2} 2z \xrightarrow{- 5} 2z - 5$
c $x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{+ 9} \frac{x}{3} + 9$

d $p \xrightarrow{\div 2} \frac{p}{2} \xrightarrow{-3} \frac{p}{2} - 3$

e $u \xrightarrow{+4} u + 4 \xrightarrow{\times 5} 5(u + 4)$

f $y \xrightarrow{-6} y - 6 \xrightarrow{\div 5} \frac{y - 6}{5}$

g $w \xrightarrow{\times 2} 2w \xrightarrow{\div 3} \frac{2w}{3}$ or $w \xrightarrow{\times \frac{2}{3}} \frac{2w}{3}$

or $w \xrightarrow{\div 3} \frac{w}{3} \xrightarrow{\times 2} \frac{2w}{3}$

5 **a** $x = 5$ **b** $x = 6$ **c** $x = 10$
d $x = 6$ **e** $k = 12$

6 **a** $y = -3$ **b** $z = 3$ **c** $n = 2\frac{1}{4}$ **d** $m = -30$
e $k = \frac{5}{6}$ **f** $w = 1$ **g** $y = 39$ **h** $k = 19$
i $x = -3$ **j** $x = 9$

7 $\frac{x}{4} - 1 = 2$

8 **a** \$900 **b** \$12 800

9 **a** $\frac{17 + 22 + n}{3} = 20$, i.e. $\frac{39 + n}{3} = 20$
b $n = 21$

Review set 14B

1 Examples:

a

x	20	25	30	31
$3(x - 17)$	9	24	39	42

$x = 31$

b

n	10	15	20	17
$\frac{n + 15}{4}$	$6\frac{1}{4}$	$7\frac{1}{2}$	$8\frac{3}{4}$	8

$n = 17$

2 Take 1 counter from both sides, then divide the right-hand side into 4 equal groups.

3 **a** $3x + 5$ **b** $5(x + 1)$ **c** $\frac{x}{6} - 1$ **d** $\frac{x - 2}{7}$

4 **a** $x \xrightarrow{\times 2} 2x \xrightarrow{+9} 2x + 9$

b $z \xrightarrow{\times 3} 3z \xrightarrow{-7} 3z - 7$

c $n \xrightarrow{\div 2} \frac{n}{2} \xrightarrow{+8} \frac{n}{2} + 8$

d $m \xrightarrow{\div 5} \frac{m}{5} \xrightarrow{-1} \frac{m}{5} - 1$

e $u \xrightarrow{-3} u - 3 \xrightarrow{\times 6} 6(u - 3)$

f $m \xrightarrow{+3} m + 3 \xrightarrow{\div 4} \frac{m + 3}{4}$

g $x \xrightarrow{\times 4} 4x \xrightarrow{\div 9} \frac{4x}{9}$ or $x \xrightarrow{\times \frac{4}{9}} \frac{4x}{9}$

or $x \xrightarrow{\div 9} \frac{x}{9} \xrightarrow{\times 4} \frac{4x}{9}$

5 **a** $x = 5$ **b** $x = 3$ **c** $x = 15$
d $x = 7$ **e** $m = 2\frac{1}{10}$

6 **a** $y = -5$ **b** $m = 4$ **c** $n = 5\frac{2}{3}$ **d** $k = -35$
e $n = \frac{5}{9}$ **f** $t = 2$ **g** $v = 45$ **h** $y = 20$
i $x = -2$ **j** $x = -8$

7 $3(x - 8) = -9$

8 **a** 26 **b** 10

9 **a** $\frac{n + 45}{2} = 37$ **b** $n = 29$

Review set 14C

1 Examples:

a

x	5	10	12	11
$4x + 13$	33	53	61	57

$x = 11$

b

x	5	10	15	20
$5x - 11$	14	39	64	89

$x = 20$

2 $3x + 4 = x + 8$, $x = 2$

3 **a** $7x + 1$ **b** $9(x + 4)$ **c** $\frac{x}{3} - 4$ **d** $\frac{x - 1}{5}$

4 **a** $m \xrightarrow{\times 3} 3m \xrightarrow{+8} 3m + 8$

b $x \xrightarrow{\times 2} 2x \xrightarrow{-5} 2x - 5$

c $m \xrightarrow{\div 4} \frac{m}{4} \xrightarrow{+3} \frac{m}{4} + 3$

d $w \xrightarrow{\div 9} \frac{w}{9} \xrightarrow{-1} \frac{w}{9} - 1$

e $w \xrightarrow{+7} w + 7 \xrightarrow{\times 3} 3(w + 7)$

f $b \xrightarrow{-6} b - 6 \xrightarrow{\div 5} \frac{b - 6}{5}$

g $v \xrightarrow{\times 9} 9v \xrightarrow{\div 4} \frac{9v}{4}$ or $v \xrightarrow{\times \frac{9}{4}} \frac{9v}{4}$

or $v \xrightarrow{\div 4} \frac{v}{4} \xrightarrow{\times 9} \frac{9v}{4}$

5 **a** $x = 6$ **b** $x = 12$ **c** $y = 16$
d $y = 7$ **e** $n = 3\frac{3}{4}$

6 **a** $x = -2$ **b** $m = 5$ **c** $w = 2\frac{3}{5}$ **d** $n = -12$
e $x = 1\frac{2}{7}$ **f** $t = 5$ **g** $q = 25$ **h** $z = 20$
i $x = 15$ **j** $x = 4$

7 $\frac{x + 9}{5} = 1$

8 **a** \$750 **b** \$12 500

9 **a** $2n + 4 = 34$ **b** $n = 15$

CHAPTER 15 TRIANGLES AND QUADRILATERALS

Diagnostic test

1 C **2** D **3** A **4** A **5** B
6 A **7** D **8** C **9** B **10** A
11 C **12** C

Exercise 15A

1 **a** Q **b** X Y

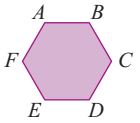
c E F

d A B C

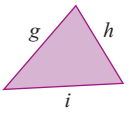
e M N O

f P Q R S

g



h



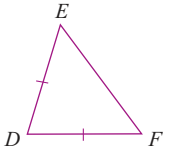
2 a Line TS **b** Triangle KLM
c Point R **d** Square $DEFG$
e Interval OP
f Triangle with sides o, p, q or triangle OPQ
g Pentagon $UVWXY$ **h** Angle PTA

3 a TP, PT, TR, RP, RT, PR **b** FE, d, EF
c LM, MN, LN
d $\angle AOC$ as more than one angle has its vertex at O.

4 a $MN = NL$
b $IJ = LK$, $IL = JK$
c $MN = QP$, $MO = OP$
d $EF = DG$, $FG = ED$, $HF = HG = HE = HD$
e $BC = CD = DE = EA = AB$

5 a $\angle BAC = \angle CBA = \angle ACB$, or $\angle A = \angle B = \angle C$
b $\angle PQO = \angle OSR$, $\angle ORS = \angle OPQ$, or $\angle Q = \angle S$, $\angle R = \angle P$
c $\angle KJB = \angle JID = \angle IHF$
d $\angle WVZ = \angle WXY$, $\angle VZY = \angle VWX$, or $\angle V = \angle X$, $\angle Z = \angle W$

6 a



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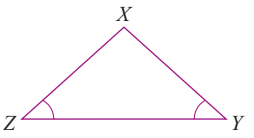

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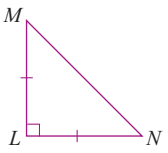

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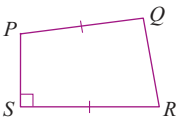

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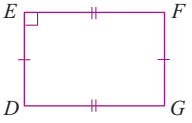

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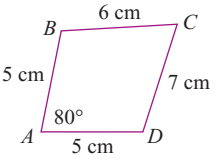

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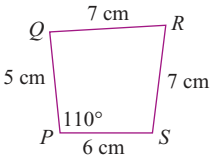

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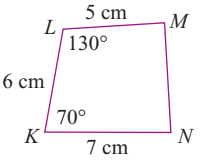
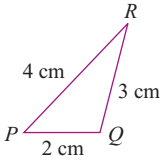


7 a Triangle ABC, $AB = AC$
b Triangle STU, $ST = UT$, $\angle T = 90°$
c Triangle JKL, $\angle L = 120°$, $\angle J = \angle K$
d Quadrilateral $RSTU$, $ST = UR$, $SR = TU$
e Quadrilateral $ABCD$, $AD = DC = CB = AB$
f Quadrilateral $FGHI$, $FG = GH$, $\angle F = \angle H$, $FI = IH$

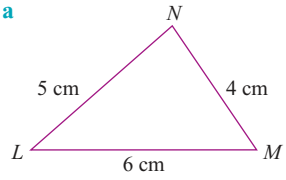
Exercise 15B

Constructions are shown at half size.

1



2 a



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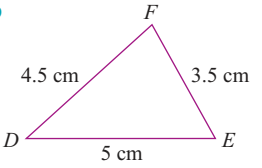
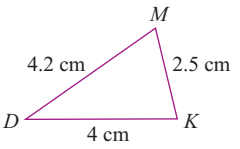
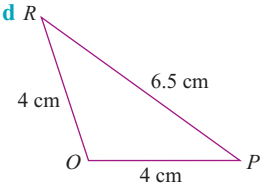


c



d



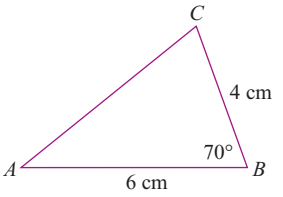
3 b i Largest angle **ii** Smallest angle
c The largest angle is opposite the longest side.

4

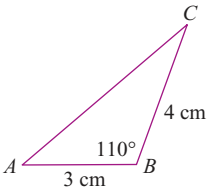
	Longest side	Sum of other 2 sides	Is longest side greater than sum of other 2 sides?
a	$LM = 6$ cm	9 cm	No
b	$DE = 5$ cm	8 cm	No
c	$DM = 4.2$ cm	6.5 cm	No
d	$PR = 6.5$ cm	8 cm	No

5 a No, $1 + 7 < 9$ **b** Yes, $5.2 + 6 > 8$

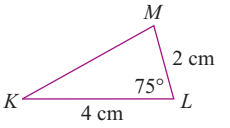
6



7 a



b



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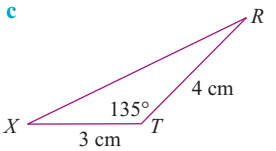

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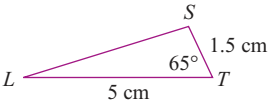

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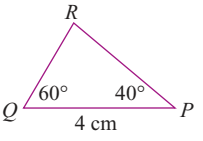
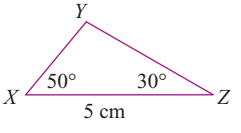
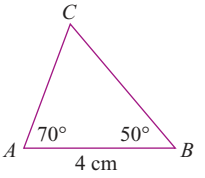


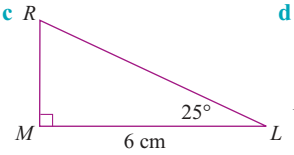
9 a



b



c



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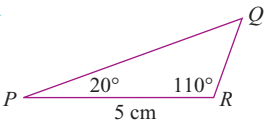


11 No, the largest angle (80°) should be opposite the longest side.

Exercise 15C

1 **a** Scalene **b** Isosceles
c Equilateral
2 **a** Isosceles **b** Scalene
c Equilateral **d** Scalene
3 **a** Right-angled **b** Obtuse-angled
c Acute-angled **d** Acute-angled
4 **a** Right-angled **b** Acute-angled
c Right-angled **d** Obtuse-angled
5 **a** Right-angled scalene **b** Obtuse-angled isosceles
c Acute-angled equilateral **d** Right-angled isosceles
6 The interior angles of an equilateral triangle are all 60°.
7 The base angles of an isosceles triangle are equal.

Exercise 15D

1 $x + 60° + 80° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore x + 140° = 180°$
$x = 180° - 140° = 40°$
or $x = 180° - 60° - 80°$ (Angle sum of a triangle is 180°.)
$\therefore x = 180° - 140° = 40°$

2 **a** $x = 49°$ **b** $k = 87°$ **c** $l = 18°$
d $s = 49°$ **e** $k = 40°$ **f** $m = 47°$
g $t = 19°$ **h** $w = 39°$ **i** $n = 40°$
j $u = 116°$ **k** $z = 46°$ **l** $e = 90°$
m $g = 84°$ **n** $j = 42°$ **o** $b = 59°$

3 **a** $a + 50° + 70° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore a + 120° = 180°$
$a = 180° - 120° = 60°$
$b + 100° + 60° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore b + 160° = 180°$
$b = 180° - 160° = 20°$
b $a + 50° + 55° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore a + 105° = 180°$
$a = 180° - 105° = 75°$
$b = 55°$ (corresponding angles, $TP \parallel SQ$)
$c = 50°$ (corresponding angles, $TP \parallel SQ$)

4 **a** $a = 85°$ (Angle sum of a triangle is 180°.)
$b = 56°$ (Angle sum of a triangle is 180°.)
b $n = 20°$ (Angle sum of a triangle is 180°.)
$m = 53°$ (Angle sum of a triangle is 180°.)
c $k = 14°$ (Angle sum of a triangle is 180°.)
$j = 81°$ (Angle sum of a triangle is 180°.)
d $x = 52°$ (Angle sum of a triangle is 180°, or corresponding angles in parallel lines are equal.)
$y = 38°$(Angle sum of a triangle is 180°, or corresponding angles in parallel lines are equal.)
e $k = 62°$ (Angle sum of a triangle is 180°.)
$l = 70°$ (Corresponding angles in parallel lines are equal.)
$m = 48°$ (Corresponding angles in parallel lines are equal.)
f $g = 60°$ (Angle sum of a triangle is 180°.)
$h = 50°$ (Corresponding angles in parallel lines are equal.)
$i = 70°$ (Corresponding angles in parallel lines are equal.)

5 **a** $x + 70° + 30° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore x + 100° = 180°$
$x = 180° - 100° = 80°$
$y = 180° - 80° = 100°$
(adjacent angles on a straight line.)
$z + 40° + 100° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore z + 140° = 180°$
$z = 180° - 140° = 40°$
b $x = 36°$ (Vertically opposite angles equal.)
$y + 90° + 36° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore y + 126° = 180°$
$y = 180° - 126° = 54°$

6 **a** $x = 89°$ (Angle sum of a triangle is 180°.)
$y = 107°$ (adjacent angles on a straight line)
$z = 39°$ (Angle sum of a triangle is 180°.)
b $x = 42°$ (Angle sum of a triangle is 180°.)
$y = 90°$ (adjacent angles on a straight line)
$z = 58°$ (Angle sum of a triangle is 180°.)
c $w = 90°$ (adjacent angles on a straight line)
$x = 40°$ (Angle sum of a triangle is 180°.)
$y = 50°$ (Sum of complementary adjacent angles is 90°.)
$z = 40°$ (Angle sum of a triangle is 180°.)
d $l = 57°$ (Angle sum of a triangle is 180°.)
$m = 57°$ (Vertically opposite angles are equal.)
$n = 48°$ (Angle sum of a triangle is 180°.)
e $x = 25°$ (Angle sum of a triangle is 180°.)
$y = 25°$ (Vertically opposite angles are equal.)
$z = 65°$ (Angle sum of a triangle is 180°.)
f $x = 63°$ (Corresponding angles in parallel lines are equal.)
$y = 49°$ (Corresponding angles in parallel lines are equal.)
$z = 68°$ (Angle sum of a triangle is 180°.)
g $a = 69°$ (Alternate angles are equal.)
$b = 69°$ (Angle sum of a triangle is 180°.)
$c = 42°$ (Alternate angles are equal, or adjacent angles on a straight line.)
h $s = 44°$ (adjacent angles on a straight line)
$t = 96°$ (Angle sum of a triangle is 180°.)
i $f = 59°$ (Alternate angles in parallel lines are equal.)
$h = 58°$ (adjacent angles on a straight line)
$g = 63°$ (Angle sum of a triangle is 180°.)

7 **a** $2x + 3x + 70° = 180°$
(Angle sum of a triangle is 180°.)
$\therefore 5x + 70° = 180°$
$5x = 180° - 70° = 110°$
$x = \frac{110°}{5} = 22°$
$\therefore$ Unknown angles are:
$2x = 2 \times 22° = 44°$
and $3x = 3 \times 22° = 66°$
b $(2x - 5) + (x + 10°) + 61° = 180°$
(Angle sum of a triangle is 180°.)
$\therefore 3x + 66° = 180°$
$3x = 180° - 66° = 114°$
$x = \frac{114°}{3} = 38°$
$\therefore$ Unknown angles are:
$2x - 5° = 2 \times 38° - 5° = 71°$
and $x + 10° = 38° + 10° = 48°$

8 **a** $2x + 55° + 78° = 180°$, $x = 23\frac{1}{2}°$; 47°
b $5b + 70° + 50° = 180°$, $b = 12°$; 60°
c $3m + 8m + m = 180°$, $m = 15°$; 15°, 45°, 120°
d $3a + 5a + 2a = 180°$, $a = 18°$; 36°, 54°, 90°
e $(x + 5°) + (2x + 15°) + 55° = 180°$, $x = 35°$; 40°, 85°
f $2y + (2y + 7°) + 53° = 180°$, $y = 30°$; 60°, 67°

Exercise 15E

1 a $\triangle ABC$ is an equilateral triangle. (All sides are equal.)
$\therefore x = 60°$ (Angles of an equilateral triangle are 60°.)
b $\triangle PQR$ is an equilateral triangle. (All angles are equal.)
$\therefore y = 5$ cm (Sides of an equilateral triangle are equal.)
c $\triangle KLM$ is an isosceles triangle. (Base angles are equal.)
$\therefore z = 6.8$ cm (Sides opposite base angles of an isosceles triangle are equal.)
d $\triangle XYZ$ is an isosceles triangle. (Two sides are equal.)
$\therefore \angle Y = a$ (Base angles of an isosceles triangle are equal.)
$a + a + 50° = 180°$ (Angle sum of a triangle is 180°.)
$2a + 50° = 180°$
$2a = 180° - 50° = 130°$
$a = \frac{130°}{2} = 65°$

2 a $y = 4$ cm (Sides of equilateral triangle are equal.)
b $w = 60°$ (Angles of an equilateral triangle are 60°.)
c $t = 60°$ (Angles of an equilateral triangle are 60°.)
d $b = 60°$ (Angles of an equilateral triangle are 60°.)
e $d = 63°$ (Base angles of an isosceles triangle are equal.)
f $p = 8$ cm (Sides opposite base angles of an isosceles triangle are equal.)
g $y = 5.4$ cm (Sides opposite base angles of an isosceles triangle are equal.)
h $f = 62°$ (Base angles of an isosceles triangle are equal.)
i $b = 14$ cm (Sides opposite base angles of an isosceles triangle are equal.)
j $c = 42°$ (Base angles of an isosceles triangle are equal.)
$d = 96°$ (Angle sum of a triangle is 180°.)
k $x = 40°$ (Angle sum of a triangle is 180°.)
$y = 10.4$ cm (Sides opposite base angles of an isosceles triangle are equal.)
l $d = 60°$ (Angle sum of a triangle is 180°.)
$w = 2.7$ cm (Sides of an equilateral triangle are equal.)
m $j = 7$ cm (Sides opposite base angles of an isosceles triangle are equal.)
$k = 45°$ (Base angles of an isosceles triangle are equal, and angle sum of a triangle is 180°.)
n $h = 8.2$ cm (Sides opposite base angles of an isosceles triangle are equal.)
$x = 144°$ (Angle sum of a triangle is 180°.)
o $m = n$ (Base angles of an isosceles triangle are equal.)
$m = n = 60°$ (Angle sum of a triangle is 180°.)
$p = 5.5$ cm (Triangle is equilateral, and sides of an equilateral triangle are equal.)

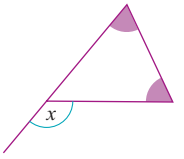
3 a $x = 68°$ (Base angles of an isosceles triangle are equal.)
$y = 44°$ (Angle sum of a triangle is 180°.)
b $x = 22°$ (Base angles of an isosceles triangle are equal.)
$y = 136°$ (Angle sum of a triangle is 180°.)
c $y = 70°$ (Base angles of an isosceles triangle are equal.)
$x = 40°$ (Angle sum of a triangle is 180°.)
d $x = 60°$ (Angles of an equilateral triangle are 60°.)
$y = 35°$ (Adjacent angles adding to 60°, and angles of an equilateral triangle are 60°.)
e $x = 60°$ (Angles of an equilateral triangle are 60°.)
$y = 120°$ (adjacent angles on a straight line)
f $x = 60°$ (Angles of an equilateral triangle are 60°.)
$y = 18°$ (Adjacent angles adding to 60°, and angles of an equilateral triangle are 60°.)
g $x = 48°$ (Base angles of an isosceles triangle are equal.)
$y = 42°$ (Angle sum of a triangle is 180°.)
h $x = 43°$ (Base angles of an isosceles triangle are equal.)
$y = 94°$ (Angle sum of a triangle is 180°.)
i $x = 33°$ (Alternate angles in parallel lines are equal.)
$y = 114°$ (Base angles of an isosceles triangle are equal, and angle sum of a triangle is 180°.)

4 a Isosceles triangles have base angles equal.
$\therefore$ Angle marked • $= y$
$y + y + 36° = 180°$
(Angle sum of a triangle is 180°.)
$\therefore 2y + 36° = 180°$
$2y = 180° - 36° = 144°$
$y = \frac{144°}{2} = 72°$
$\therefore$ Unknown angles are both 72° (base angles).
b Isosceles triangles have base angles equal.
$\therefore$ Angle marked • $= x + 16°$
$2x + (x + 16°) + (x + 16°) = 180°$
(Angle sum of a triangle is 180°.)
$\therefore 4x + 32° = 180°$
$4x = 180° - 32° = 148°$
$x = \frac{148°}{4} = 37°$
$\therefore$ Unknown angles are:
$2x = 2 \times 37° = 74°$
and $x + 16° = 37° + 16° = 53°$ (base angles)

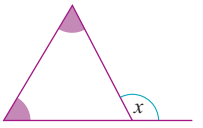
5 a $2y + 46° = 180°$, $y = 67°$; 67°, 67°
b $2x + 90° = 180°$, $x = 45°$; 45°, 45°
c $x + 2x + 2x = 180°$, $x = 36°$; 36°, 72°, 72°
d $(x - 8°) + (x - 8°) + 5x = 180°$,
$x = 28°$; 140°, 20°, 20°
e $7x + x + x = 180°$, $x = 20°$; 140°, 20°, 20°
f $(3x + 11°) + (2x + 4°) + (2x + 4°) = 180°$,
$x = 23°$; 80°, 50°, 50°

Exercise 15F

1 a



b



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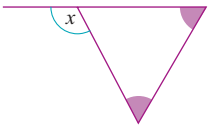

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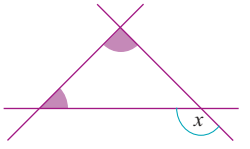
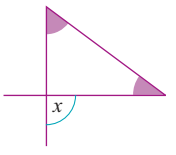
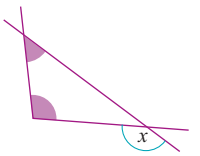


e



f



2 a $x = 70° + 40° = 110°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
b $135° = y + 60°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
or $y + 60° = 135°$
$\therefore y = 135° - 60° = 75°$

3 a $x = 127°$ **b** $y = 135°$ **c** $p = 153°$
d $k = 48°$ **e** $m = 55°$ **f** $w = 32°$

4 a $2x - 18° = 46° + 72° = 118°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)

$\therefore 2x = 118° + 18° = 136°$

$x = \frac{136°}{2} = 68°$

$\therefore$ Unknown angle is:

$2x - 18° = 2 \times 68° - 18° = 118°$

b $140° = (x + 10°) + 35°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)

$140° = x + 45°$

or $x + 45° = 140°$

$\therefore x = 140° - 45° = 95°$

$\therefore$ Unknown angle is:

$x + 10° = 95° + 10° = 105°$

5 a $4x = 128°$, $x = 32°$; 128°

b $2x + 6° = 114°$, $x = 54°$; 114°

c $3x - 2° = 106°$, $x = 36°$; 106°

d $2x + 7° = 129°$, $x = 61°$; 129°

e $8x = 120°$, $x = 15°$; 120°

f $3x + 3x = 114°$, $x = 19°$; 57°, 57°

Exercise 15G

1

Property	Parallelogram	Rectangle	Rhombus
Opposite sides parallel	Y	Y	Y
Opposite sides equal	Y	Y	Y
Opposite angles equal	Y	Y	Y
All sides equal	N	N	Y
Adjacent sides perpendicular	N	Y	N
Diagonals equal	N	Y	N
Adjacent sides equal	N	N	Y
Diagonals bisect each other	Y	Y	Y
Diagonals meet at right angles	N	N	Y
Diagonals bisect vertices	N	N	Y
All angles 90°	N	Y	N

Property	Square	Trapezium	Kite
Opposite sides parallel	Y	One pair	N
Opposite sides equal	Y	N	N
Opposite angles equal	Y	N	One pair
All sides equal	Y	N	N
Adjacent sides perpendicular	Y	N	N
Diagonals equal	Y	N	N
Adjacent sides equal	Y	N	Two pairs
Diagonals bisect each other	Y	N	One diagonal
Diagonals meet at right angles	Y	N	Y
Diagonals bisect vertices	Y	N	One diagonal
All angles 90°	Y	N	N

2 a $x = 2.5$ cm, $y = 4.5$ cm
(Opposite sides of a parallelogram are equal.)

b $x = y = 5$ cm (All sides of a rhombus are equal.)

c $x = 3$ cm, $y = 7$ cm (Adjacent sides of a kite are equal.)

d $x = 35°$ (Alternate angles, $QR \parallel TS$.)
$y = 55°$ (All angles of a rectangle are 90°, and adjacent complementary angles.)

e $x = 65°$
(Opposite angles of a parallelogram are equal.)
$y = 115°$ (Co-interior angles add to 180°, $TU \parallel WV$.)
$z = 115°$ (Co-interior angles add to 180°, $TW \parallel UV$, or opposite angles of a parallelogram are equal.)

f $x = 30°$ ($2x + x = 90°$; all angles of a rectangle are 90°.)
$y = 60°$ ($y = 2x$; alternate angles, $DQ \parallel EF$, or angle sum of a triangle, $\angle F = 90°$.)

g $x = y = 45°$ (All angles of a square are 90°, and the diagonals of a square bisect its vertices.)

h $w = 40°$ (Alternate angles, $HI \parallel KJ$.)
$x = 35°$ (Alternate angles, $HK \parallel IJ$.)
$y = 105°$ (Angle sum of a triangle is 180°.)
$z = 105°$ (Opposite angles of a parallelogram are equal; or co-interior angles add to 180°, $HI \parallel KJ$.)

i $w = 85°$
(Opposite angles of a parallelogram are equal.)
$x = 55°$ (Angle sum of a triangle is 180°.)
$y = 55°$ (Alternate angles, $VY \parallel WX$.)
$z = 40°$ (Alternate angles, $VW \parallel YX$.)

j $x = 30°$ (Diagonals of a rhombus bisect its vertices.)
$y = 30°$ (Alternate angles, $PS \parallel QR$, or base angles of an isosceles triangle are equal.)

k $x = y = 45°$ (All angles of a square are 90°, and diagonals of a square bisect its vertices.)

l $x = 5$ cm (Diagonals of a rectangle are equal.)

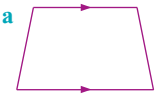
m $x = 4.5$ cm, $y = 3$ cm (Diagonals of a parallelogram bisect each other.)

n $x = 50°$ (Diagonals of a rhombus bisect each other at right angles, and angle sum of a triangle is 180°.)
$y = 40°$ (Diagonals of a rhombus bisect its vertices.)

o $x = y = 45°$ (All angles of a square are 90°, and diagonals of a square bisect its vertices.)

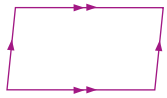
3 Examples:

a



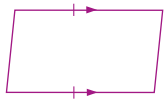
Trapezium

b



Parallelogram

c

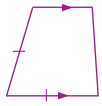


Parallelogram

or

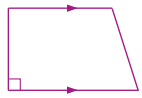
Trapezium (isosceles)

d



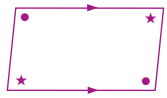
Trapezium

e



Trapezium

f

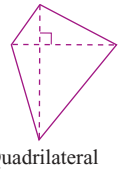


Parallelogram

g

Rectangle

h

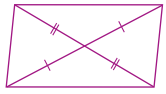


Quadrilateral

i

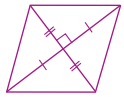
Kite

j



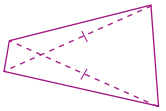
Parallelogram

k



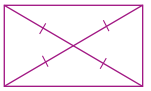
Rhombus

l



Quadrilateral

n



Rectangle

m

Square

Exercise 15H

1 $x + 85° + 75° + 110° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore x + 270° = 360°$

$x = 360° - 270° = 90°$

2 a $x = 60°$ (Opposite angles in a parallelogram are equal.)

b $x = 53°$ (Angle sum of a quadrilateral is 360°.)

c $k = 52°$ (Angle sum of a quadrilateral is 360°.)

d $t = 93°$ (Angle sum of a quadrilateral is 360°.)

e $p = 68°$ (Angle sum of a quadrilateral is 360°.)

f $w = 63°$ (Angle sum of a quadrilateral is 360°.)

g $m = 120°$ (Angle sum of a quadrilateral is 360°.)

h $g = 43°$ (Angle sum of a quadrilateral is 360°.)

i $d = 210°$ (Angle sum of a quadrilateral is 360°.)

3 $(2x - 10°) + (x + 50°) + 70° + 85° = 360°$ (Angle sum of a quadrilateral is 360°.)

$3x + 195° = 360°$

$\therefore 3x = 360° - 195° = 165°$

$x = \frac{165°}{3} = 55°$

$\therefore$ Unknown angles are:

$2x - 10° = 2 \times 55° - 10° = 100°$

and $x + 50° = 55° + 50° = 105°$

4 a $x + 2x + 88° + 104° = 360°$, $x = 56°$; 56°, 112°

b $(2y - 3°) + 62° + 4y + 115° = 360°$, $y = 31°$; 59°, 124°

c $(3p + 8°) + p + (7p - 2°) + p = 360°$, $p = 29.5°$; 29.5°, 29.5°, 96.5°, 204.5°

d $3a + 3a + 2a + 208° = 360°$, $a = 19°$; 38°, 57°, 57°

e $4m + 90° + (3m + 21°) + 46° = 360°$, $m = 29°$; 116°, 108°

f $(2n + 7°) + n + (5n - 13°) + 46° = 360°$, $n = 40°$; 40°, 87°, 187°

5 $x = 110°$ (corresponding angles, $AF \parallel BE$)

$y + 70° + 100° + 110° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore y + 280° = 360°$

$y = 360° - 280° = 80°$

$z + 120° + 80° + 70° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore z + 270° = 360°$

$z = 360° - 270° = 90°$

6 a $w = 36°$ (Angle sum of a quadrilateral is 360°.)

$x = 14°$ (adjacent complementary angles)

$y = 76°$ (Angle sum of a triangle is 180°.)

b $w = 69°$ (Angle sum of a quadrilateral is 360°.)

$x = 111°$ (adjacent angles on a straight line)

$y = 48°$ (Angle sum of a triangle is 180°.)

c $w = 50°$ (adjacent angles on a straight line)

$x = 65°$ (Sum of co-interior angles is 180°.)

$y = 65°$ (Angle sum of a triangle is 180°.)

d $w = 228°$ (Angle sum at a point is 360°.)

$x = 36°$ (Angle sum of a quadrilateral is 360°.)

$y = 125°$ (adjacent angles on a straight line)

e $w = 68°$ (Sum of co-interior angles is 180°.)

$x = 68°$ (Alternate angles are equal, or adjacent angles on a straight line.)

$y = 112°$ (Vertically opposite angles are equal, or adjacent angles on a straight line.)

f $w = 114°$ (Sum of co-interior angles is 180°.)

$x = 45°$ ($3x + x = 180°$, sum of co-interior angles is 180°; or $3x + x + 114° + 66° = 360°$, angle sum of a quadrilateral is 360°.)

$y = 135°$ (Corresponding angles are equal, or adjacent angles on a straight line.)

Language in mathematics

1

P	T	I	D	F	I	S	E	L	E	C	S	O	S	I	N	S
R	R	R	N	O	I	T	A	U	Q	E	M	D	U	T	Q	E
O	A	E	E	R	I	N	A	T	U	U	N	G	M	U	X	L
N	H	A	T	E	G	S	C	S	I	D	E	S	A	T	L	E
U	C	S	X	L	A	U	N	Z	L	D	A	R	E	Q	A	L
M	W	O	E	U	R	A	E	D	A	K	E	R	D	R	U	G
E	O	N	I	T	L	P	R	A	T	T	I	E	C	R	Q	N
R	L	I	S	M	A	A	A	L	E	O	S	T	F	A	E	A
A	F	N	S	R	W	C	D	P	R	O	V	E	E	I	N	I
L	O	G	T	A	A	D	T	I	A	S	U	B	M	O	H	R
C	N	G	Q	U	A	D	R	I	L	A	T	E	R	A	L	T

2 I find triangles and quadrilaterals fascinating.

Check your skills

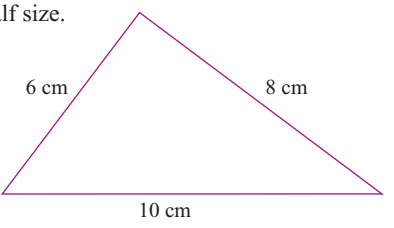
1 D **2** A **3** C **4** D **5** B

6 A **7** D **8** D **9** B **10** D

11 B **12** A

Review set 15A

1 Shown at half size.



2 Example:

3 a $x = 53°$ (adjacent angles on a straight line)

b $x = 52°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)

4 $(x + 10°) + 110° + 30° = 180°$

$x = 30°$

$\therefore$ Unknown angle is 40°.

5 $u = 38°$ (adjacent angles on a straight line)

$v = 94°$ (Angle sum of a triangle is 180°.)

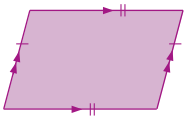
$w = 76°$ (Angle sum of a triangle is 180°.)

$x = 76°$ (Vertically opposite angles are equal.)

$y = 86°$ (Adjacent angles on a straight line; or exterior angle of $\triangle EBC$ equals sum of two interior angles.)

$z = 18°$ (Angle sum of a triangle is 180°.)

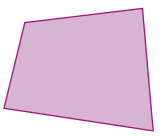
6 a Example:



b Two pairs of equal and parallel sides; opposite angles are equal; diagonals bisect each other.

7 a Example:

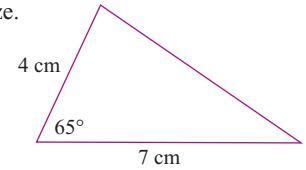
b They are all inside.



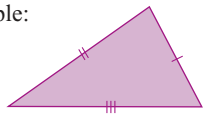
8 **a** $x = 40°$ (Alternate angles, $BC \parallel AD$.)
$y = 50°$ (All angles of a rectangle equal 90°, and adjacent complementary angles.)
b $x = 55°$ (Diagonals of a rhombus bisect each other at right angles, and angle sum of a triangle is 180°.)
$y = 35°$ (Sides of a rhombus are equal, $SR \parallel RQ$, and base angles in an isosceles triangle are equal.)

Review set 15B

1 Shown at half size.



2 Example:

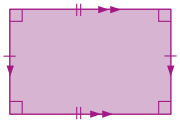


3 **a** $x = 60°$ (Angles of an equilateral triangle are 60°.)
b $x = 45°$ (Base angles of an isosceles triangle are equal, and angle sum of a triangle is 180°.)

4 $x + 45° + (x + 15°) = 180°$
$x = 60°$
$\therefore$ Unknown angles are 60° and 75°.

5 $a = 87°$ (Corresponding angles are equal, $DE \parallel CB$.)
$b = 51°$ (Angle sum of a triangle is 180°.)
$c = 42°$ (Corresponding angles are equal, $DE \parallel CB$.)

6 **a** Example:



b Opposite sides are equal and parallel; diagonals are equal and bisect each other; all angles are 90°.

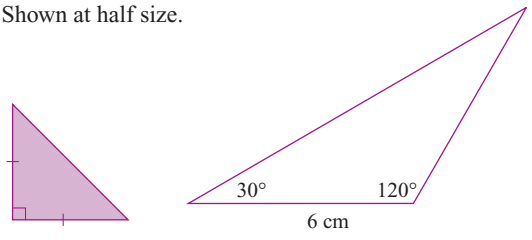
7 A concave quadrilateral has one diagonal outside the quadrilateral, or one interior angle greater than 180°.

8 **a** $x = y = 45°$ (All angles in a square equal 90°, and diagonals bisect the angles at each vertex.)
b $w = 70°$ (Opposite angles of a parallelogram are equal.)
$x = 50°$ (alternate angles, $TU \parallel SV$)
$y = 60°$ (Angle sum of a triangle is 180°; or sum of co-interior angles is 180°, $TU \parallel SV$.)
$z = 60°$ (alternate angles, $TS \parallel UV$; or angle sum of a triangle is 180°.)

Review set 15C

1 Shown at half size.

2



3 **a** $x = 85°$ (Angle sum of a quadrilateral is 360°.)
b $x = 225°$ (Angle sum of a quadrilateral is 360°.)

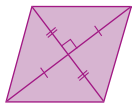
4 $(2x - 4°) + 52° + 4x = 180°$
(Angle sum of a triangle is 180°.) $\therefore x = 22°$
$\therefore$ Unknown angles are 40° and 88°.
$y = 92°$ (Exterior angle of a triangle equals sum of two interior opposite angles, or adjacent angles on a straight line.)

5 $w = 80°$ (adjacent angles on a straight line)
$x = 68°$ (Sum of co-interior angles in parallel lines is 180°.)
$y = 100°$ (Corresponding angles in parallel lines are equal, or angle sum of a quadrilateral is 360°.)
$z = 80°$ (Corresponding angles in parallel lines are equal, or adjacent angles on a straight line.)

6 **a** Example:

b One pair of opposite sides parallel

7 **a** Example:



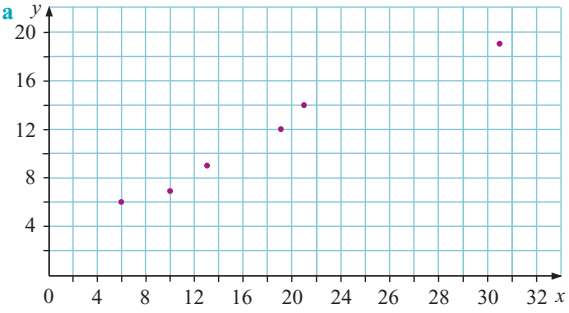
b Rhombus

8 **a** $x = 2.8$ cm, $y = 3.5$ cm
(Diagonals of a parallelogram bisect each other.)
b $x = 4$ cm, $y = 7$ cm
(Two pairs of adjacent sides of a kite are equal.)
$v = 75°$ (Base angles of an isosceles triangle are equal, $GF = GH$.)
$w = 65°$ (Base angles of an isosceles triangle are equal, $EF = EH$.)
$z = 25°$ (Diagonals of a kite meet at right angles, and angle sum of a triangle is 180°.)

CUMULATIVE REVIEW CHAPTERS 11 TO 15

1 **a** Nominal **b** Continuous numerical
c Continuous numerical **d** Discrete numerical

2 **a**



b Yes, upwards. y increases as x increases.

3 **a**

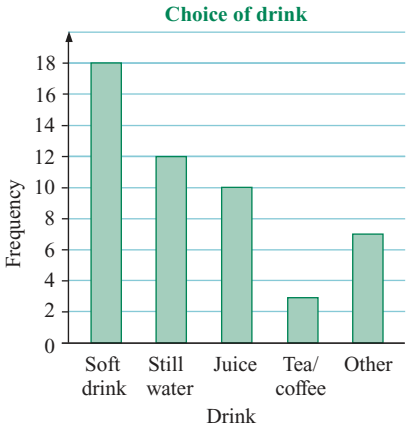
Score	Tally	Frequency
1	| |	2
2	| | |	3
3		0
4	| | | |	4
5	| | |	3
6	| | | | | |	7
7	| | |	3
8	| | | |	4
9	| | | | |	6
10	| | | |	4
	Total	**36**

b 14

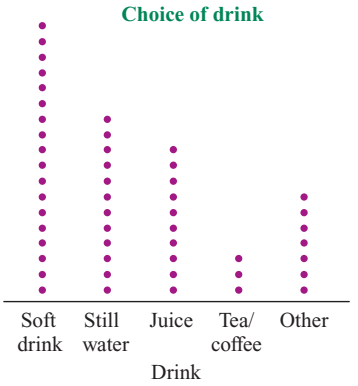
4

Stem	Leaf
2	2 8 8
3	1 2 2 5 8
4	0 2 2 4 5 5 5 5 6 8 9
5	0 0 1 3
6	1

5 a



b



6 a i 64 m² **ii** 60 mm² **iii** 45 cm²
iv 27 m² **v** 90 cm² **vi** 1250 mm²
b i 84 m² **ii** 650 cm²

7 a i 1.12 m² **ii** 5242.1 cm²
b i 0.0064 m³ **ii** 13 013 cm³

8 a i 24.25 **ii** 23 **iii** 23.5 **iv** 8
b i 20.57 **ii** 20 **iii** 20 **iv** 5

9 a 106.47 **b** 124 **c** 108 **d** 45

10 a

x	5	10	9
$3x + 8$	23	38	35

$x = 9$

b $5x - 1$

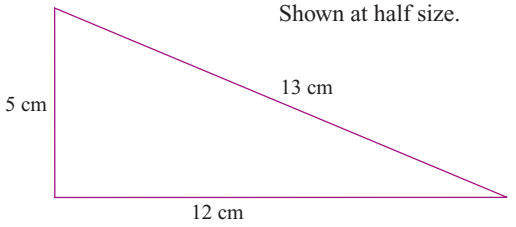
c $x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{-7} \frac{x}{3} - 7$

d $x \xrightarrow{-4} x - 4 \xrightarrow{\div 3} \frac{x-4}{3}$

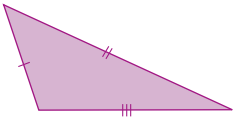
$7 \xleftarrow{+4} 3 \xleftarrow{\times 3} 1$ $\quad x = 7$

e i $t = 6$ **ii** $r = 18$
f i $2n + 8 = 32$ **ii** $n = 12$

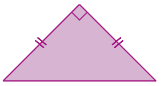
11 a Shown at half size.



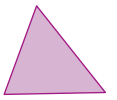
b i Example:



ii Example:



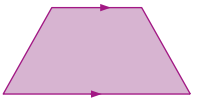
iii Example:



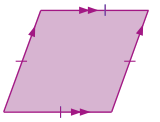
c i $2x = 180° - 68° = 112°$, $\therefore x = 56°$
(Base angles of an isosceles triangle are equal, and angle sum of a triangle is 180°.)
ii $127° = x + 53°$, $\therefore x = 74°$
(Exterior angle of a triangle equals sum of the two interior opposite angles.)
iii $x = 70°$ (Angle sum of a triangle is 180°.)

d $(90° - x) + 3x + 3x = 180°$
$\therefore x = 18°$
$\therefore$ Unknown angles are 72° and 54°.

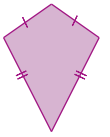
12 a i One pair of parallel sides.



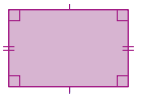
ii Two pairs of parallel sides.
All sides equal.
Opposite angles equal.
Diagonals bisect at right angles.
Diagonals bisect vertices.



iii Two pairs of adjacent sides equal.
One pair of opposite angles equal.
Diagonals intersect at right angles.
One diagonal bisects vertices and bisects other diagonal.



iv Two pairs of equal parallel sides.
All angles 90°.
Equal diagonals that bisect each other.



b i $x + 48° + 115° + 95° = 360°$, $\therefore x = 102°$
(Angle sum of a quadrilateral is 360°)
ii $x + 40° = 90°$, $\therefore x = 50°$
(Diagonals of a rhombus intersect at 90°, and angle sum of a triangle is 180°.)
$y = x = 50°$ (Sides of a rhombus are equal, and base angles of an isosceles triangle are equal.)

c $m + 135° + 90° + (3m + 15°) = 360°$
$\therefore m = 30°$
$\therefore$ Unknown angles are 30° and 105°.

13 a $w = 95°$ (Sum of co-interior angles is 180°.)
$x = 85°$ (Corresponding angles are equal, or adjacent angles on a straight line.)
$y = 95°$ (Adjacent angles on a straight line, or corresponding angles are equal.)
$z = 132°$ (Sum of co-interior angles is 180°, or angle sum of a quadrilateral is 360°.)

b $x = 22°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
$y = 137°$ (Adjacent angles on a straight line, or angle sum of a triangle is 180°.)
$z = 83°$ (Angle sum of a quadrilateral is 360°.)

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