CAMBRIDGE

GENERAL MATHEMATICS

UNITS 1 & 2

SECOND EDITION

CAMBRIDGE SENIOR MATHEMATICS FOR QUEENSLAND

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INCLUDES INTERACTIVE TEXTBOOK POWERED BY CAMBRIDGE HOTMATHS

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Online appendices accessed through the Interactive or PDF Textbook

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Appendix B Online guides to using technology
Appendix C Guide to PSMTs

About the lead author and consultants

About the lead author

Kay Lipson has a PhD in Statistics Education, and Diploma of Education. Her experience includes over 30 years teaching mathematics and statistics at both secondary and tertiary level, as well as extensive periods as a VCE examiner. Kay is an experienced author of textbooks for the Years 11 and 12 Mathematics courses, specialising in the areas of Probability and Statistics.

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Introduction and overview

Cambridge Senior Mathematics for Queensland General Mathematics Units 1 & 2 provides complete and aligned coverage of the QCAA syllabus to be implemented in Year 11 from 2025. Its four components – the print book, downloadable PDF textbook, online Interactive Textbook (ITB) and Online Teaching Resource (OTS) – contain a huge range of resources, available to schools in a single package.

New features in the second edition:

- **Learning intentions** complemented by a Skills Checklist at the end of each chapter that allows you to check your understanding and tick off your achievements,
- **Section summaries** at the end of each section further learning intentions by highlighting the key points and skills needed to progress.
- **The problem-solving and modelling task (Appendix A):** This is a new appendix at the end of the book, written by consultant Joel Speranza, providing advice on how to complete problem-solving and modelling tasks (PSMTs). This is supported by video resources accessed through QR codes and in the Interactive Textbook.

The Second Edition also features significantly revised and updated material from the first edition, including:

Detailed worked examples which go step by step through a problem together with precise explanations. For every worked example also has a video to encourage independent learning and is linked to exercises.

Degree of difficulty classification of questions: in the exercises, questions are classified as simple familiar SF, complex familiar CF, or complex unfamiliar CU questions and are indicated by a strip along the margin. The revision chapters described below also contain model questions for each of these categories, and tests are also provided in the teacher resources, made up of such categorised model questions.

Two revision chapters each covering one unit. Revision chapters are divided into simple familiar multiple-choice and short-response questions that have been designed with each degree of difficulty in mind to help prepare students for assessment.

Assessment: Examination practice questions and assessment tasks are provided in the revision chapters and the Online Teaching Suite.

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Interactive Textbook (ITB)

The Interactive Textbook (ITB) is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available as a separate purchase.

Updated and revised for the new syllabus, the Interactive Textbook includes:

- **Video demonstrations** of all worked examples
- **Quick quizzes** containing auto-marked multiple-choice questions that have been thoroughly updated and revised, enabling students to check their understanding.
- A **success criteria** checklist at the end of each chapter with linked questions and examples available for download
- Comprehensive **worked solutions** for all questions are provided in the Interactive Textbook as an option that teacher can choose to enable for their students.
- **Downloadable skillsheets** can be used for homework or in class to focus on a single skill or small set of related skills.
- **Definitions** pop up for key terms in the text, and are also provided in a dictionary.

The Online Teaching Suite (OTS)

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the teacher resources are in one place for easy access. The features include:

- A teacher's view of a student's working and self-assessment which enables them to modify the student's self-assessed marks, and respond where students flag that they had difficulty.
- The **task manager** allowing to direct students on a custom activity sequence based on their scores in measurable activities
- Quickly create customised tests from a bank of multiple-choice questions using the **test generator**. Tests are auto-marked in the Interactive Textbook or can be printed and used for homework or assessment practice.
- An expanded and revised suite of **chapter tests** and **assignments**
- Editable **curriculum grids** and **teaching programs**.
- A brand-new **Exam Generator**, allowing the creation of customised printable and online trial exams (see below for more).

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More about the Exam Generator

The Online Teaching Suite, available from 2025, will include a comprehensive bank of QCAA exam questions, augmented by exam-style questions written by experts, to allow teachers to create custom trial exams.

Custom exams can model end-of-year exams, or target specific topics or types of questions that students may be having difficulty with.

Features include:

- Filtering by question-type, topic and degree of difficulty
- Searchable by key words
- Answers provided to teachers
- Worked solutions for all questions
- QCAA marking scheme
- Multiple-choice exams can be auto-marked if completed online, with filterable reports
- All custom exams can be printed and completed under exam-like conditions or used as revision.

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Chapter 1

Consumer arithmetic

Chapter questions

UNIT 1 MONEY, MEASUREMENT, ALGEBRA AND LINEAR EQUATIONS

Topic 1 Consumer arithmetic

- How do we understand the meaning of rates and percentages?
- How do we calculate income payments from a salary?
- How do we calculate wages using hourly rate, overtime rates and allowances and earnings based on commission and piecework?
- How do we determine payments based on government allowances and pensions?
- How do we prepare a personal budget?
- How do we compare prices using the unit cost method?
- How do we determine the new price when percentage discounts or increases are applied?
- How do we calculate percentage profit and loss?
- How do we calculate prices with and without GST?
- How do we apply percentage increase to calculate simple interest and compound interest?
- What is inflation and how does it affect costs and wages?
- How do we apply exchange rates to determine the cost of items given in a foreign currency?
- What are shares and how do we understand their value?

In this chapter we explore a range of concepts associated with budgeting, income and investments which have real world application.

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1A Rates and percentages

Learning intentions

- To be able to understand the concept of a rate.
- To be able to convert rates and fractions to percentages and vice-versa.
- To be able to calculate percentages of quantities.

Rates

In general terms a **rate** refers to one quantity divided by another quantity, where the two quantities do not have the same unit of measurement. Because a rate compares different types of quantities the units must be shown. Some examples of rates with which are we quite familiar are:

- speed measured in kilometres per hour
- cost of oranges measured in dollars per kilogram
- growth rate of an investment measured as dollars per year.

Calculations involving using rates usually require you to determine one of the quantities, given the value of the other quantity, as illustrated in the following example.

Example 1 Calculations using rates

a Tyler paid $2.40 per kg for bananas. Calculate the cost of a 5 kg bag of bananas.

b Jordan paid $5.80 for a 2 kg bag of oranges. Calculate the cost of the oranges per kg.

Solution	Explanation
a Cost	Write the quantity to be found.
$= 2.40 \times 5$	Multiply the rate by weight bought in kg.
$= \$12$	Evaluate and write using correct units.
b Cost per kg	Write the quantity to be found.
$= \frac{5.80}{2}$	Divide the cost by the weight bought.
$= \$2.90$ per kg	Evaluate and write using correct units.

Percentages

A **percentage** is a fraction or a proportion expressed as part of one hundred. The symbol used to indicate percentage is %. Percentages can be expressed as common fractions or as decimals.

For example: 17% (17 per cent) means 17 parts out of every 100.

$$17\% = \frac{17}{100} = 0.17$$

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Conversions

1. To convert a fraction or a decimal to a percentage, multiply by 100.
2. To convert a percentage to a decimal or a fraction, divide by 100.

Example 2 Converting fractions to percentages

Express $\frac{36}{90}$ as a percentage.

Solution

$\frac{36}{90} \times 100$

$= 40\%$

Explanation

Multiply the fraction $\frac{36}{90}$ by 100.

Evaluate and write your answer.

Example 3 Converting decimals to a percentage

Express 0.75 as a percentage.

Solution

0.75×100

$= 75\%$

Explanation

Multiply 0.75 by 100.

Evaluate and write your answer.

Example 4 Converting a percentage to a fraction

Express 62% as a fraction.

Solution

$62\% = \frac{62}{100}$

$= \frac{62 \div 2}{100 \div 2}$

$= \frac{31}{50}$

Explanation

As 62% means 62 out of 100, this can be written as a fraction $\frac{62}{100}$.

Simplify the fraction by dividing both the numerator and the denominator by 2.

Example 5 Converting a percentage to a decimal

Express 72% as a decimal.

Solution

$\frac{72}{100} = 0.72$

Explanation

Write 72% as a fraction over 100 and express this as a decimal.

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Finding a percentage of a quantity

To find a percentage *of* a number or a quantity, remember that in mathematics *of* means *multiply*.

Example 6 Finding a percentage of a quantity

Find 15% of $140.

Solution	Explanation
$15\%\text{ of }140$	Write out the problem and rewrite 15% as a fraction out of 100.
$= \frac{15}{100}\text{ of }140$	Change *of* to *multiply*.
$= \frac{15}{100} \times 140$	
$= 21$	Perform the calculation and write your answer.

Comparing two quantities

Unlike rates, to express one quantity or number as a percentage of another quantity or number both quantities must always be in the same units. Divide the quantity by what you are comparing it with and then multiply by 100 to convert it to a percentage.

Example 7 Expressing a quantity as a percentage of another quantity

There are 18 girls in a class of 25 students. What percentage of the class are girls?

Solution	Explanation
$\text{Girls} = \frac{18}{25}$	Work out the fraction of girls in the class.
$\frac{18}{25} \times 100 = 72$	Convert the fraction to a percentage by multiplying by 100.
72% of the class are girls.	Evaluate and write your answer.

Section Summary

- A **rate** refers to one quantity divided by another quantity, where the two quantities do not have the same unit of measurement.
- A **percentage** is a fraction or a proportion expressed as part of one hundred.
- Percentages can be expressed as fractions or as decimals.

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Exercise 1A

SF

Example 1

1 Express each of the following as a simplified rate.

a \$8.40 for 6 kg
b 280 km in 4 hours
c \$210 in 7 hours
d 6 cm in 12 months

Example 2
Example 3

2 Express the following as percentages.

a $\frac{1}{4}$ **b** $\frac{2}{5}$ **c** $\frac{3}{20}$ **d** $\frac{7}{10}$
e 0.19 **f** 0.79 **g** 2.15 **h** 39.57
i 0.073 **j** 1 **k** 1.25 **l** 2

Example 4
Example 5

3 Express the following as:

i fractions, in their lowest terms
ii decimals.

a 25% **b** 50% **c** 75% **d** 5.75%
e 27.2% **f** 0.45% **g** 0.03% **h** 0.0065%

Example 6

4 Calculate the following, correct to three significant figures.

a 15% of \$760
b 22% of \$500
c 17% of 150 m
d $13\frac{1}{2}$% of \$10 000
e 2% of 79.34 cm
f 19.6% of 13.46
g 22.4% of \$346 900
h 1.98% of \$1 000 000

Example 7

5 From a class of 35 students, 28 wanted to take part in a project. Calculate the percentage of the class who wanted to take part.

6 A farmer lost 450 sheep out of a flock of 1200 during a drought. Calculate the percentage of the flock that was lost.

7 After three rounds of a competition, a basketball team had scored 300 points and 360 points had been scored against them. Calculate (to two decimal places) the points scored by the team as a percentage of the points scored against them.

8 Express 75 cm as a percentage of 2 m.

9 In a population of 3.25 million people, 2 115 000 are under the age of 16. Calculate the percentage, to two decimal places, of the population who are under the age of 16.

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1B Salary and wages

Learning intentions

- To be able to calculate weekly, fortnightly or monthly pay from an annual salary,
- To be able to calculate wages based on an hourly rate, which may include overtime, penalty rates and other allowances.
- To be able to calculate earnings based on commission or piecework.

The money paid to an individual for the work they carry out can be expressed as a **salary** or a **wage**.

Salary

A **salary** is payment for a year's work, generally paid as equal weekly, fortnightly or monthly payments. People who earn a salary work a pre-agreed number of hours per week, which is generally from 36 hours to 40 hours for a full-time worker. They are also entitled to benefits such as sick leave and holiday pay, which are factored into the salary.

Example 8 Calculating from a salary

Mitchell earns a salary of $65 208 per year. He is paid fortnightly. How much does he receive each fortnight? Assume there are 52 weeks in the year.

Solution	Explanation
Fortnightly pay $= 65\,208 \div 26$ $= \$2508.00$	Write the quantity to be found. Divide the salary by the number of fortnights in a year (26). Evaluate and write using correct units.

Wages

A **wage** describes payment for work calculated on an hourly basis, with the amount earned dependent on the number of hours actually worked. There are no additional payments such sick leave or holiday pay.

Example 9 Calculating a wage

Jasmine is paid at a rate of $27.45 per hour. Determine the wage Jasmine will receive for a week in which she works 38 hours.

Solution	Explanation
Wage for 38 hours $= 27.45 \times 38$ $= \$1043.10$	Write the quantity to be found. Multiply the rate by the number of hours worked. Evaluate and write using correct units.

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Salary and Wages

Salary

A payment for a year's work, which is then divided into equal monthly, fortnightly or weekly payments.

Wage

A payment for a week's work that is calculated on an hourly basis.

Of course, the amount you earn is not necessarily the amount that you are actually paid. Amounts will be deducted for taxation and superannuation, and there may be other deductions depending on the type of work.

The amount that you earn is called your **gross** income, and the amount that you are paid once the deductions have been made is called your **net** income.

Overtime, penalty rates and allowances

Overtime

Overtime rates apply when employees work beyond the normal working day. Payment for overtime is more than the normal pay rate, often paid at 150% (referred to as time-and-a-half) or 200% (double time). So, a person whose normal pay rate is $10 per hour receives:

Time-and-a-half: $\$10 \times 150\% = \$10 \times 1.5 = \$15$

Double time: $\$10 \times 200\% = \$10 \times 2.0 = \$20$

Example 10 **Calculating wages including overtime**

John works for a building construction company. Determine John's wage during one week in which he works 35 hours at the normal rate of $38 per hour, 3 hours at time-and-a-half rates and 1 hour at double time rates.

Solution	Explanation
Wage	Write the quantity to be found.
$= (35 \times 38)$ normal pay	Normal wage is 35 multiplied by $38.
$+ (3 \times 38 \times 1.5)$ time-and-a-half pay	Payment for time-and-a-half is 3 multiplied by $38 multiplied by 1.5.
$+ (1 \times 38 \times 2)$ double time pay	Payment for double time is 1 multiplied by $38 multiplied by 2.
$= \$1577.00$	Evaluate and write using correct units.

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Penalty rates

As well as earning a higher pay rate when working overtime, employees often get a higher pay rate, called a **penalty rate**, for working weekends, public holidays or night shifts.

In Australia, penalty rates are determined by the Fair Work Commission and can be found on their website. The following table shows an example of the penalty rates that may be payable for those working in the hospitality industry.

Hospitality penalty rates

Note: These rates are indicative only, they may change at any time.

Column 1 Time of ordinary hours worked	Column 2 Full-time and part-time employees	Column 3 Casual employees
	% of ordinary hourly rate	% of ordinary hourly rate (inclusive of casual loading)
Monday to Friday – 7.00 am to 7.00 pm	100%	125%
Monday to Friday – 7.00 pm to midnight	100% plus $2.62 per hour or part of an hour	125% plus $2.62 per hour or part of an hour
Monday to Friday— midnight to 7.00 am	100% plus $3.93 per hour or part of an hour	125% plus $3.93 per hour or part of an hour
Saturday	125%	150%
Sunday	150%	175%
Public holiday	225%	250%

Source: *https://library.fairwork.gov.au/award/?krn=MA000009#_Toc154048184*

Note that the penalty rate is higher for **casual** workers, people who are paid an hourly rate, and who may be asked to work differing numbers of hours each week, as required by the business. There is generally no guarantee of ongoing work for a casual worker.

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Example 11 Calculating wages including penalty rates

Milan is employed on a casual basis in hospitality. His pay rate is $21 per hour with penalty rates as shown in the table above. Last week Milan worked from 3 p.m. until 7 p.m. on Thursday, from 5 p.m. until 9 p.m. on Friday and from 12 noon until 4 p.m. on Sunday. Determine how much Milan earned last week.

Solution	Explanation
Wages last week =	Write down the quantity to be found.
$6 \times \$21$	The number of normal hours worked is 4 hours on Thursday and 2 hours on Friday (from 5 p.m. until 7 p.m.)
$+ 2 \times \$21 \times 1.25 + 2 \times \$2.62$	The number of evening hours worked is 2 hours on Friday (from 7 p.m. until 9 p.m.).
$+ 4 \times \$21 \times 1.75$ $= \$330.74$	The number of Sunday hours worked is 4. Evaluate and write your answer with correct units.

Allowances

Allowances are extra payments that may be made to employees who have a particular skill, use their own tools or equipment at work, or work in unpleasant or dangerous conditions.

Common allowances include:

- uniforms and special clothing
- tools and equipment
- travel and fares
- car and phone.

Example 12 Calculating pay including an allowance

Barbara works as a builder and is paid $35 per hour, plus an extra $8.50 per hour when she supplies her own tools. Last week she worked 3 days on which the tools were supplied, and two days on which she supplied her own tools. If she worked 8 hours each day, calculate how much she earned.

Solution	Explanation
Wages last week $= 3 \times 8 \times \$35$	Write down the quantity to be found. Determine the number of normal hours = 8 hours on each of the three days.
$+ 2 \times 8 \times (\$35 + \$8.50)$	Determine the number of allowance hours = 8 hours on each of the two days.
$= \$1536$	Evaluate and write your answer with correct units.

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Commission and piecework

Commission

Commission is a percentage of the value of the goods or services sold. People such as real estate agents and salespersons are often paid a commission. The advantage of this is that a very good salesperson will be able to earn a higher income, but the disadvantage is that it is very hard to plan, as the income may vary from week to week.

Commission

Commission = percentage of the value of the goods sold

Example 13 Finding the commission

Zoë sold a house for $650 000. Determine the commission from the sale if her rate of commission was 1.25%.

Solution	Explanation
Commission	Write the quantity to be found.
= 1.25% of $650 000	Multiply 1.25% by $650 000.
= 0.0125 × 650 000	Evaluate and write using correct units.
= $8125	

Example 14 Finding the commission

An electrical goods salesman is paid $670.50 per week plus 6% commission on all sales over $2000 a week. Determine his earnings in a week in which his sales amounted to $6800.

Solution	Explanation
Commission paid on = 6800 – 2000	Commission is paid on sales of over $2000; that is, on $4800.
= 4800	
Earnings	Write the quantity to be found.
= 670.50 + (6% of $4800)	Add the weekly payment and commission of 6% on $4800.
= 670.50 + (0.06 × 4800)	
= $958.50	Evaluate and write using correct units.

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Piecework

Piecework is when a worker is paid a fixed payment, called a **piece rate**, for each unit produced or action completed. For example, a dressmaker may be paid a fixed price for each dress produced, regardless of the time spent sewing each one. The advantage of piecework is that harder work is rewarded with higher pay, while the disadvantage is the lack of permanent employment and holiday and sick pay.

Piecework

Piecework = number of units of work × amount paid per unit

Example 15 Calculating a piecework payment

Noah is a tiler and charges $47 per square metre to lay tiles. Calculate how much he will earn for laying tiles in a room with an area of 14 square metres.

Solution	Explanation
Earnings	Write the quantity to be found.
$= 14 \times \$47$	Multiply the number of square metres (14) by the charge ($47) per square metre.
$= \$658$	Evaluate and write using correct units.

Section Summary

- A **salary** is payment for a year's work, generally paid as equal weekly, fortnightly or monthly payments.
- A **wage** describes payment for work calculated on an hourly basis, with the amount earned dependent on the number of hours actually worked.
- **Overtime rates** apply when employees work beyond the normal working day. Payment for overtime is more than the normal pay rate, often paid at 150% (referred to as time-and-a-half) or 200% (double time).
- **Penalty rates** of pay are higher rates paid for working weekends, public holidays or night shifts.
- **Allowances** are extra payments that may be made to employees who have a particular skill, use their own tools or equipment at work, or work in unpleasant or dangerous conditions.
- **Commission** is a percentage of the value of the goods or services sold.
- **Piecework** is when a worker is paid a fixed payment, called a **piece rate**, for each unit produced or action completed.

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Exercise 1B

Assume that there are 52 weeks, 26 fortnights and 12 months in a year unless otherwise specified.

Calculating from a salary

Example 8

1 Emily earns a salary of $92 648. Calculate, to the nearest dollar, her salary per:

a week **b** fortnight **c** month.

2 The annual salaries for 4 people are shown in the table below. Calculate their weekly and fortnightly payments. (Answer correct to the nearest dollar.)

	Name	Salary	Week	Fortnight
a	Abbey	$57 640		
b	Blake	$78 484		
c	Chloe	$107 800		
d	David	$44 240		

3 Calculate the annual salary for the following people.

a Amber earns $580 per week. **b** Tyler earns $1520 per fortnight.

c Samuel earns $3268 per month. **d** Ava earns $2418 per week.

4 Laura is paid $1235 per fortnight and Ebony $2459 per month. Determine which person receives the higher annual salary and by how much.

Calculating wages

Example 9

5 Joshua works in a coffee shop and is paid $25.50 per hour. Calculate how much he earns for working the following hours (assuming no penalty rates apply).

a 5 hours **b** 15 hours **c** 37 hours

6 Determine the wage for a 37-hour week for each of the following hourly rates.

a $12.00 **b** $9.50 **c** $23.20 **d** $13.83

7 Alyssa is paid $36.90 per hour and Connor $320 per day. Alyssa works a 9-hour day. Determine who earns more per day and by how much.

8 Suchitra works at the local supermarket. She gets paid $22.50 per hour. Her time card is shown opposite.

a Determine how many hours Suchitra worked this week.

b Calculate her weekly wage.

Day	Start	Finish
Monday	9:00 a.m.	5:00 p.m.
Tuesday	9:00 a.m.	6:00 p.m.
Wednesday	8:30 a.m.	5:30 p.m.
Thursday	9:00 a.m.	4:30 p.m.
Friday	9:00 a.m.	4:00 p.m.

SF

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Overtime

Example 10

9 Calculate the payment for working 4 hours overtime at time-and-a half given the following normal pay rates.

a $18.00 **b** $39.50 **c** $63.20 **d** $43.83

10 Calculate the payment for working 3 hours overtime at double time given the following normal pay rates.

a $37.99 **b** $19.05 **c** $48.78 **d** $61.79

11 Jack works for a concreting company. Calculate his wage during one week in which he works 37 hours at the normal rate of $35 per hour, 5.5 hours at 125% of the normal rate and 6 hours at 225% of the normal rate.

12 Carla is paid time-and-a-half for the first 10 hours of overtime outside of her normal hours, and and double time for any extra hours after that. Determine her wage if during one week she works 30 hours at the normal rate of $28.40 per hour and 15 hours of overtime.

Penalty rates

Example 11

13 Abbey's normal pay is $18.80 per hour during the week, with penalty rates of 150% for Saturdays and 200% for Sundays. Abbey is not paid for meal breaks. Determine her weekly wage based on the timesheet shown below.

Day	Start	Finish	Meal break
Monday	8:30 a.m.	5:30 p.m.	1 hour
Tuesday	8:30 a.m.	3:00 p.m.	1 hour
Wednesday	8:30 a.m.	5:30 p.m.	1 hour
Thursday	8:30 a.m.	9:00 p.m.	2 hour
Friday	4:00 p.m.	7:00 p.m.	No break
Saturday	8:00 a.m.	4:00 p.m.	No break
Sunday	10:00 a.m.	3:00 p.m.	30 minutes

Allowances

Example 12

14 A window washer is paid $22.50 per hour and a height allowance of $55 per day. If he works 9 hours each week day on a high-rise building, calculate the:

a amount earned each week day

b total weekly earnings for five days of work.

15 Anna works in a factory and is paid $18.54 per hour. If she operates the oven she is paid a temperature allowance of $4.22 per hour in addition to her normal rate. Calculate her weekly pay if she works a total of 42 hours, including 10 hours working the oven.

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16 Cal is a painter who is paid a normal rate of $36.80 per hour plus a height allowance of $21 per day. If Cal works 9 hours per day for 5 days on a tall building, calculate his total earnings.

17 Kathy is a scientist who is working in a remote part of Australia. She earns a salary of $86 840 plus a weekly allowance of $124.80 for working under extreme and isolated conditions. Calculate Kathy's fortnightly pay.

Commission

Example 13

18 Jake earns a commission of 4% of the sales price. Determine his commission on the following sales.

a $8820 **b** $16 740 **c** $34 220

19 Olivia sold a car valued at $54 000. Determine Olivia's commission from the sale if her rate of commission is 3%.

Example 14

20 Sophie earns a weekly retainer of $355 plus a commission of 10% on sales. Determine Sophie's total earnings for each week if she made the following sales.

a $760 **b** $2870 **c** $12 850

21 Chris earns $240 per week plus 25% commission on sales. Calculate Chris's weekly earnings if he made sales of $2880.

22 Ella is a salesperson for a cosmetics company. She is paid $500 per week and a commission of 3% on sales in excess of $800.

a Calculate what Ella earns in a week in which she makes sales of $1200.

b Calculate what Ella earns in a week in which she makes sales of $600.

Piecework

Example 15

23 A dry cleaner charges $15 to clean a dress and $12 to clean a shirt. Calculate how much they earn by dry cleaning:

a 10 dresses **b** 12 shirts **c** 7 dresses and 14 shirts.

24 Angus works part-time by addressing envelopes at home and is paid $23 per 100 envelopes completed, plus $40 to deliver them to the office. Calculate his pay for delivering 2000 addressed envelopes.

25 Abbey is an artist who is paid $180 for each large portrait and $100 for each small portrait she paints. Calculate how much she will earn if she paints 13 large and 28 small portraits.

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Applications

26 Feng is retiring and will receive 7.6 times the average of his salary over the past three years. In the past three years he was paid $84 640, $83 248 and $82 960. Determine the amount of his payout.

27 Liam's salary in 2024 is $76 000. He receives salary increases as follows: a 5% increase on 1 January, 2025, followed by another 5% increase on 1 July 2025.

a Determine his new salary after the second increase.

b Calculate how much more he earned in 2025 compared to 2024.

Spread-sheet

28 Create the spreadsheet below.

	A	B	C	D	E
1	Worksheet to calculate the wages of employees				
2					
3	Family name	First name	Hours	Hourly pay rate	Weekly wage
4	Cini	Olivia	35	$21.80	=C4*D4
5	Croft	Liam		$22.50	
6	Griffen	Lily		$24.42	
7	Hong	Tin		$35.75	
8	Lang	Molly		$35.75	
9	Sitou	Noth		$24.42	
10	Taylor	Nathan		$21.80	
11	Woods	Joshua		$26.90	
12					

- Cell E4 has a formula that multiplies cells C4 and D4. Enter this formula.
- Enter the hours worked for the following employees:

  Liam – 20 Lily – 26 Tin – 38 Molly – 40

  Noth – 37.5 Nathan – 42 Joshua – 38.5
- Fill down the contents of E4 to E11.

Use the spreadsheet to determine the total weekly wages for the company.

29 Connor works a 35-hour week and is paid $18.25 per hour, with overtime paid at time-and-a-half. Connor wants to work enough overtime to earn at least $800 each week. Determine, to the nearest hour, the minimum number of hours overtime that Connor needs to work.

30 Max works in a shop and earns $21.60 per hour at the normal rate. Each week he works 15 hours at the normal rate and 4 hours at time-and-a-half.

a Calculate Max's weekly wage.

b Max aims to increase his weekly wage to $540 by working extra hours at the normal rate. Determine how many extra hours Max must work.

c Max's rate of pay increased by 5%. Determine his new hourly rate for normal hours.

d Determine Max's new weekly wage, assuming he maintains the extra working hours.

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CF

31 Use the table of Hospitality penalty rates on page 8 to determine the weekly pay for the following casual workers last week. The ordinary rates of pay for their roles are given in brackets.

a Teale ($22.50/h), last week she worked from 2 p.m. until 6 p.m. on Tuesday and Thursday, and 8 p.m. until 11 p.m. on Friday.

b Kiana ($23.40/h), last week she worked from 9 a.m. until 1 p.m. on Monday, Tuesday, Saturday and Sunday.

c Scott ($32.90/h), last week he worked from 4 p.m. until 9 p.m. on Monday and Friday, and from 2 p.m. until 6 p.m. on Saturday and Sunday.

32 Dave was paid $412.50 for 5 hours work at his normal rate, and 5 hours work at double his normal rate. Determine his normal hourly rate.

33 Ravi's penalty rate for working on Saturday is 175% of his normal rate. If he was paid for $151.20 for 3 hours of Saturday work, determine his normal hourly rate.

34 The information in the spreadsheet below gives the hours worked in one week by a group of employees. Create the spreadsheet as shown.

Spread-sheet

a Cell E5 has the formula that multiplies C5 by 1.5. Enter this formula, and fill down to E10.

	A	B	C	D	E	F
1						
2	Worksheet to calculate wages including overtime					
3						
4	Employee	Normal hours	Normal pay rate	Hours worked at time and a half	Time and a half pay rate	Weekly wage
5	Sacha	30	$47.00	8	=C5*1.5	
6	Janelle	34	$32.50	6		
7	Gavin	38	$41.70	0		
8	Ivar	38	$38.90	12		
9	Nicola	34	$35.60	8		
10	Samar	16	$29.50	16		
11						

Write down the hourly pay rate for Nicola when she is paid time-and-a-half.

b Cell F5 has the formula that multiplies B5 by C5, D5 by E5, and then adds these two amounts together.

	A	B	C	D	E	F
1						
2	Worksheet to calculate wages including overtime					
3						
4	Employee	Normal hours	Normal pay rate	Hours worked at time and a half	Time and a half pay rate	Weekly wage
5	Sacha	30	$47.00	8	$70.50	=B5*C5+D5*E5
6	Janelle	34	$32.50	6		
7	Gavin	38	$41.70	0		
8	Ivar	38	$38.90	12		
9	Nicola	34	$35.60	8		
10	Samar	16	$29.50	16		

Enter this formula as shown and fill the contents down to F10. Determine Ivar's weekly wage.

c Determine the total wage for these employees this week.

35 Jade is a real estate agent and is paid an annual salary of $18 000 plus a commission of 2.5% on all sales. She is also paid a car allowance of $50 per week. Determine Jade's total yearly income if she sold $1 200 000 worth of property.

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36 The commission to a salesperson is based on the selling price and is shown in the table. Determine the commission paid on properties with the following sales:

Selling price	Commission
First $20 000	5%
Next $120 000	3%
Thereafter	1%

a $100 000 **b** $150 000 **c** $200 000

37 Harry is a salesperson. He earns a basic wage of $300 per week and receives commission on all sales. Last week he sold $20 000 worth of goods and earned $700. Determine Harry's rate of commission.

38 Caitlin and her assistant, Holly, sell perfume. Caitlin earns 20% commission on her own sales, as well as 5% commission on Holly's sales. Calculate Caitlin's commission last month when she made sales of $1800 and Holly made sales of $2000.

39 The commission charged by a real estate agency for selling a property is based on the selling price as shown in the table. Bailey is paid $500 per week by the real estate agency plus 10% of the commission received by the agency.

Commission rates	
Up to $300 000	4%
$300 000 and over	5%

Spread-sheet

The information in the spreadsheet gives the sales made by Bailey in one month, together with the agency's commission rates. Create the spreadsheet as shown.

	A	B	C	D	E
1					
2	Worksheet to calculate commission				
3					
4	Week	Sale price	Agency commission	Bailey's commission	
5	1	$440,000.00	=300000*0.04+(B5-300000)*0.05		
6	1	$560,000.00			
7	2	$840,000.00			
8	2	$580,000.00			
9	3	$1,200,000.00			
10	4	$790,000.00			

a Cell C5 has the formula that determines the agency's commission. Enter this formula, and fill down to C10. Determine how much commission the agency receives from the properties sold by Bailey in week 3.

b In cell D5, enter a formula that multiplies C5 by 0.10 to give the amount of commission paid to Bailey, and fill the contents down to D10. Determine the total commission paid to Bailey in week 1.

c Determine the total amount paid to Bailey over that four week period.

40 Valerie is a dressmaker. She is paid $135 for shortening three pairs of jeans and three dresses. If she charges twice as much to shorten a dress as she does to shorten a pair of jeans, determine how much she charges to shorten a dress.

41 Kristy earns $25 for making a skirt, $35 for a shirt, and $55 for making a dress. It takes Kristy on average 60 minutes to make a skirt, 90 minutes to make a shirt, and 140 minutes to make a dress. If she wants to earn $1000 as quickly as possible, use mathematical reasoning to determine what she should make.

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1C Incomes from the government

Learning intentions

- To be able to determine income support payments based on government allowances and pensions.

Some people who are unable to work full-time receive a pension, allowance or benefit from the government. The eligibility requirements, and the amount paid, vary from time to time according to the priorities of the current government. Some of the allowances available to young people or those in full-time study are discussed in this section. Details about the eligibility requirements and current payments are available from Services Australia.

Youth Allowance

Subject to meeting requirements you may be eligible for Youth Allowance. To get this you must be one of the following:

- 18 to 24 and studying full-time
- 16 to 17, studying full-time and either independent or needing to live away from home to study
- 16 to 17, studying full-time and have completed year 12 or equivalent
- 16 to 24 and doing a full-time Australian Apprenticeship.

The Youth Allowance payment scale is updated on January 1 each year. At the time of publication the scale was as follows:

Your situation	Your maximum fortnightly payment from 1 January 2024
Single, no children, younger than 18, and live at your parent's home	$395.30
Single, no children, younger than 18, living away from your parent's home to study, train or look for work	$639.00
Single, no children, 18 or older and live at your parent's home	$455.20
Single, no children, 18 or older and need to live away from your parent's home	$639.00
Single, with children	$806.00
A couple, with no children	$639.00
A couple, with children	$691.80

Source: *https://www.servicesaustralia.gov.au/how-much-youth-allowance-for-students-and-apprentices-you-can-get?context=43916*

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Example 16 Youth Allowance

Ryan is eligible for Youth Allowance. How much does he receive in a year if he is over 18 and living at home while studying?

Solution	Explanation
Yearly allowance $= \$455.20 \times 26$ $= \$11\,835.20$	Write the quantity to be found. Multiply the allowance per fortnight ($288.10) by 26. Evaluate and write using correct units.

Disability Support Pension

You may get a Disability Support Pension if you have a permanent and diagnosed disability or medical condition that stops you from working.

At the time of publication the Disability Support Pension scale for those younger than 21 without dependent children was as follows (allowances change yearly):

Your situation	Maximum rate per fortnight
Single, younger than 18, and dependent	$548.80
Single, younger than 18, and independent	$792.50
Single, 18 to 20, and dependent	$608.70
Single, 18 to 20, and independent	$792.50
A couple, and younger than 21	$792.50

Source: *https://www.servicesaustralia.gov.au/payment-rates-for-disability-support-pension?context=22276*

Example 17 Disability

Mike is 16 years old and living with his parents. How much more will he receive each fortnight from his Disability Support Pension after he turns 18 if he moves out of home at that time?

Solution	Explanation
Additional Disability Support Pension $= \$792.50 - \$548.80$ $= \$243.70$	Write the quantity to be found. Subtract Mike's allowance when he is 18 and independent ($792.50) from his allowance when he is 16 and living at home ($548.80). Evaluate and write using correct units.

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Austudy

Austudy is available to those who wish to study or retrain as adults. To be eligible for Austudy you must be:

- 25 or older
- a full-time student in an approved course or Australian Apprenticeship
- under the income test limits.

At the time of publication the Austudy payment scale was as follows:

Your situation	Your maximum fortnightly payment from 1 January 2024
Single, no children	$639.00
Single, with children	$806.00
A couple, no children	$639.00
A couple, with children	$691.80

Source: *https://www.servicesaustralia.gov.au/how-much-austudy-you-can-get?context=22441*

Example 18 Austudy

Jen and Brad are both full-time students, living as a couple, with no children. What is their combined annual income from Austudy?

Solution	Explanation
Annual Austudy allowance	Write the quantity to be found.
$= (\$639.00 + \$639.00) \times 26$	Determine what they receive in total each fortnight ($639.00) and multiply by 26.
$= \$33\,228$	Evaluate and write using correct units.

Section Summary

- You may be eligible for Youth Allowance if you're 24 or younger and a student or Australian Apprentice, or 21 or younger and looking for work.
- You may be eligible for a Disability Pension if you are unable to work due to a disability or medical condition.
- You may be eligible for Austudy if you are 25 or older, and wish to study or retrain full-time.

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Exercise 1C

Use the scales given in the section to answer the following questions.

Youth Allowance

Example 16

1 Nikki is eligible for Youth Allowance. She is 17 years old, single with no children and living away from home.

- **a** Determine how much she receives each fortnight.
- **b** Determine how much she receives over the year in total.

2 Alan and Ann are both eligible for Youth Allowance. They are a couple with no children.

- **a** Determine how much they receive in total each fortnight.
- **b** Determine how much more they would receive in total each fortnight if they had a child.

3 Ben begins receiving Youth Allowance payments on 1 January. He is 17 years old, and his birthday is on 1 July, when he will turn 18. He is living at home.

- **a** Determine how much he receives between 1 January and 31 December in total.
- **b** Determine how much he receives between 1 January and 31 December in total if he decided to move out of home on his 18th birthday.

Disability Support Pension

Example 17

4 Connie receives a Disability Support Pension. She is 19 years old and living at home.

- **a** Determine how much she receives each fortnight.
- **b** Determine how much she receives over the year in total.

5 Benny is eligible for a Disability Support Pension. He is 18 years old, single and living independently.

- **a** Determine how much he receives each fortnight.
- **b** Determine how much more he receives each fortnight than someone of his age and circumstances who receives a Youth Allowance.

Austudy

Example 18

6 Madison, Oscar and Sam are all eligible for Austudy.

- **a** Determine how much Madison receives in a year if she is single with no children.
- **b** Oscar and Sam are a couple with children. Determine how much they receive in total in a year.

7 Martina is 24 years old, living independently and studying full-time.

- **a** Identify which allowance she is eligible for.
- **b** Determine how much she would receive each fortnight.
- **c** Identify which allowance she would be eligible for after she turns 25.
- **d** Determine how much more she would receive each fortnight after she turned 25.

SF

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1D Budgeting

Learning intentions

- To be able to prepare a personal budget for a given income, taking into account fixed and discretionary spending.

A **budget** is a spending plan based on income and expenses. The best way to manage your finances is to prepare a budget, balancing income and expenses to ensure that you have enough to pay the essential bills and start putting money towards your future goals.

Expenses can be further classified into:

- **Fixed expenses**: Expenses associated with essential living costs, such as rent, electricity and insurance. These expenses are paid on a regular basis (weekly, fortnightly, monthly) and are less able to be varied.
- **Discretionary expenses**: Expenses which can be varied according to circumstances. Examples include the cost of food, clothing, entertainment and hobbies.

Whatever remains after we subtract expenses from our income can be considered as **savings**.

Here are some simple steps to preparing and using a budget:

1. Choose a time period for your budget that suits your lifestyle; for example, a week, a fortnight or a month.
2. List all the income for the time period. This should include income from work, investments and any other allowances.
3. List all of your expenses for the time period. It helps to determine your annual expense in some categories, and put aside money for this each time period. For example, if your annual car insurance payment is $900, and you are preparing a monthly budget, then you need to allow $900/12 = $75 in each calendar month for car insurance.
4. Calculate the total of the income and expenses.
5. Balance the budget, ensuring that you are not spending more than you have. This might mean that you need to modify spending in the categories over which you have control, your discretionary spending.

There are many ways in which you can approach setting up a budget. One recommendation is is to start by determining your income (after tax and other deductions), then apply the 50/30/20 budget principles: 50% towards fixed payments, 30% towards discretionary spending and 20% towards savings. It is highly unlikely that you are going to be able to anticipate all your expenses in advance, so having some savings is going to be important when you are financially independent.

The process of comparing your income to your expenses is called **balancing** the budget. When income and expenses are the same, we say the budget is **balanced**.

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Example 19 Balancing a budget

Balance the following weekly budget.

Income		Expenses	
Salary	$1726.15	Clothing	$ 73.08
Bonus	$ 20.00	Gifts and Christmas	$ 114.80
Investment	$ 156.78	Groceries	$ 467.31
Part-time work	$ 393.72	Insurance	$ 171.34
		Loan repayments	$ 847.55
		Motor vehicle costs	$ 105.96
		Phone	$ 38.26
		Power and heating	$ 51.82
		Rates	$ 54.82
		Recreation	$ 216.79
		Work-related costs	$ 68.76
		Balance	
Total		**Total**	

Solution

Income = 1726.15 + . . . + 393.72
= $2296.65

Expenses = 73.08 + . . . + 68.92
= $2210.49

Balance = Income – Expenses
= 2296.6 – 2210.49
= $86.16

Explanation

Add all the income.

Add all the expenses excluding 'balance'.

Subtract the total expenses from the total income.

Example 20 Creating a budget

Maya and Logan have a combined weekly net wage of $954. Their monthly expenses are home loan repayment $1032, car loan repayment $600, electricity $102, phone $66 and car maintenance $120. Their other expenses include insurance $2160 annually, rates $1800 annually, food $180 weekly, petrol $48 fortnightly and train fares $36 weekly. Maya and Logan allow $72 for miscellaneous items weekly and need to save $84 per week for a holiday next year.

a Prepare a monthly budget for Maya and Logan. Assume there are four weeks in a month.

b What is the balance?

c How can Maya and Logan ensure they have their holiday next year?

Continued on next page

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Solution

a Solution is shown below.

Income		Expenses	
Wage	$3816	Home loan repayment	$1032
		Car loan repayment	$600
		Electricity	$102
		Phone	$66
		Car maintenance	$120
		Insurance	$180
		Rates	$150
		Food	$720
		Petrol	$96
		Train fares	$144
		Miscellaneous	$288
		Holiday	$336
		Balance	−$18
	$3816		$3816

b Total income = $3816

Total expenses = $3834

c Balance = $3816 − $3834

= −$18

d Maya and Logan need to either increase income or reduce expenses by $18 per month.

Explanation

Draw up a table with columns to list income and expenses.
List all the monthly income categories.
List all the monthly expenses categories.

Calculate the total income and expenses categories.

Subtract the total expenses from the total income to calculate the balance.

The balance is −$18, indicating that they don't have enough income for their expenses.

Section Summary

- A **budget** is a spending plan based on income and expenses.
- Expenses include both **fixed** and **discretionary** spending.
- The process of determining the difference between income and expenditure is called **balancing** the budget.
- A budget is **balanced** when income is equal to expenses.

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Exercise 1D

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1 Arlo pays rent of $240 per week, and must budget for his share of the household expenses, which are $180 per month. Determine his annual expenditure on these two items.

2 Ollie and Jill are living in a unit. Part of their budget is shown below.

Item	When	Cost
Electricity	Quarterly	$ 384
Food	Weekly	$ 360
Insurance	Biannually	$1275
Rent	Monthly	$1950

a Calculate the total amount paid over one year for:

- **i** electricity
- **ii** insurance
- **iii** food
- **iv** rent.

b Determine the weekly amount they should set aside for the expenses listed.

3 Some of Pete's bills are shown in the following table.

Item	When	Cost
Mortgage	Monthly	$2185.33
Car payment	Monthly	$1045.67
Electricity	Quarterly	$ 484.50
Rates	Annually	$ 955.46

a Calculate the total amount paid over one year for:

- **i** mortgage
- **ii** car payments
- **iii** electricity.

b Determine the fortnightly amount that he should set aside for the expenses listed.

Example 19

4 Adam has constructed a yearly budget as shown below.

Net Income		Expenses	
Wage	$60 786.22	Clothing	$ 4634.42
Interest	$ 674.15	Council rates	$ 1543.56
		Electricity	$ 1956.87
		Entertainment	$ 4987.80
		Food	$17 543.90
		Gifts and Christmas	$ 5861.20
		Insurance	$ 2348.12
		Loan repayments	$16 789.34
		Motor vehicle costs	$ 2458.91
		Telephone	$ 832.98
		Work-related costs	$ 812.67
		Balance	
Total		**Total**	

a Calculate the total income.

b Calculate the total expenses.

c Balance the budget.

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SF

5 Martha's net salary after deductions each fortnight is $2034.45, and Eden's net salary after deductions each fortnight is $1943.47. Their annual budget is:

Income after deductions		Expenses	
Martha		Clothing	$ 6608.00
Eden		Council rates	$ 2543.56
		Electricity	$ 1856.87
		Gym membership	$ 2736.00
		Insurance	$ 2348.12
		Mortgage repayments	$25 090.52
		Car loan repayments	$ 5558.88
		Telephone & internet	$ 2488.00
		Petrol & car servicing	$ 5497.40
		Health insurance	$ 4560.00
		Balance	
Total		**Total**	

a Calculate their total net income.

b Calculate the total of the expenses listed.

c Balance the budget.

d Martha and Eden would like to use 50% of the balance after paying these expenses for food and entertainment, and put the other 50% towards saving. Determine how much they will have to spend on food and entertainment each week if they follow this plan.

e Calculate the percentage of their combined income that Martha and Eden spend on their mortgage. Give your answer correct to one decimal place.

f Martha and Eden have just found out that their mortgage repayments will increase by 5%.

i Determine their new annual mortgage repayment.

ii Calculate the percentage of their combined income that Martha and Eden spend on the increased mortgage repayment. Give your answer correct to one decimal place.

Example 20

6 Dimitri has a total weekly income of $155 made up of a part-time job earning $125 and an allowance of $30. He decided to budget his expenses in the following way: games – $40, entertainment – $40, food – $55 and savings – $20.

a Prepare a weekly budget showing income and expenses.

b Calculate the percentage of his weekly income that Dimitri spends on food.

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CF

7 Ava has a gross fortnightly pay of $1896.

- **a** Ava has a mortgage with an annual repayment of $13 676. Calculate the amount that Ava must budget each fortnight for her mortgage.
- **b** Ava has budgeted $180 per week for groceries, $60 per week for entertainment, $468 per year for medical expenses and $80 per week to run a car. Express these as fortnightly amounts and calculate their total.
- **c** Ava has an electricity bill of $130 per quarter, a telephone bill of $91 per quarter and council rates of $1118 per annum. Express these amounts annually and convert to fortnightly amounts. Determine the total of these fortnightly amounts.
- **d** Prepare a fortnightly budget showing income and expenses.

Spread-sheet

8 Ryan has a full-time job, and he also earns money umpiring on the weekends. He is living at home and paying off his car loan. His budget is shown in the following spreadsheet.

	A	B	C	D	E
1					
2	Worksheet to calculate the monthly budget				
3			Annual	Fortnightly	Percentage
4	Income	Salary (net)	$58,458.00		=C4/C6*100
5		Part-time job (net)	$4,000.00		
6		Total	$62,458.00		
7					
8	Expenses	Board	$7,800.00		
9		Car running costs	$6,150.00		
10		Car repayment	$12,500.00		
11		Eating out	$7,800.00		
12		Entertainment	$5,200.00		
13		Clothing	$10,400.00		
14		Sport	$1,280.00		
15		Savings	$11,328.00		
16		Total expenses	$62,458.00		
17					
18		Balance	$0.00		

- Create the spreadsheet, and complete column D to calculate Ryan's fortnightly income and expenses.
- Enter the formula shown in cell E4 to calculate the percentage of his income that comes from his salary. Fill down to E5.
- Enter a formula in cell E to calculate the percentages of his expenses for each item. Fill down to E16.

- **a** Calculate the amount Ryan puts in the bank for savings each fortnight.
- **b** Suppose that the amount spent per fortnight on eating out increases from $300 to $350, and that the amount allocated to savings decreases so that the budget remains balanced. Determine the percentage of his income he is now saving.
- **c** Fortunately, to help with the increase in his eating out expenses, Ryan receives a salary increase of 4.6%. Determine the amount he can save each fortnight now if all other expenses remain unchanged, and the percentage of his income that represents.

CU

9 Bri's fixed expenses currently account for 40% of her fortnightly income. If her fixed expenses increase by 5% of their current value, use mathematics reasoning to determine the new percentage of her income taken up by fixed expenses.

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1E Comparing prices using the unit cost method

Learning intentions

- To be able to compare prices and values using the unit cost method.

Unit cost method

Many products may be bought individually, or in packets containing multiple items. For example, chocolate truffles might be sold individually as well as in packets of 12 truffles.

Suppose a single chocolate truffle in a particular supermarket is sold for 85 cents, and a bag of 12 chocolate truffles is sold in the same supermarket for $5.28.

Would you buy 12 individual chocolate truffles or a bag of 12 chocolate truffles?

We can answer this question by calculating the **unit cost**, or the cost of one single chocolate truffle from the bag.

$$\text{One truffle from bag} = \frac{\text{bag cost}}{12} = \frac{\$5.28}{12} = \$0.44$$

It is obviously better value to buy a bag of truffles because the unit cost of a truffle from the bag is much less than the individual price.

To be able to calculate the cost of multiple items, we start by determining the **unit cost**.

Example 21 Using the unit cost method

If 24 golf balls cost $86.40, how much do 7 golf balls cost?

Solution	Explanation
$\$86.40 \div 24 = \$3.60$	Find the cost of one golf ball (the unit cost) by dividing $86.40 (the total cost) by 24 (the number of golf balls).
$\$3.60 \times 7 = \$25.20$	Multiply the unit cost ($3.60) by 7.

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Using the unit cost method to compare items

The unit cost method can be used to compare the cost of items. This enables us to calculate which item is the best buy.

Example 22 Using the unit cost method to compare items

Two different brands of kitchen plastic wrap are sold in a shop.

- Brand A contains 50 metres of plastic wrap and costs $4.48.
- Brand B contains 90 metres of plastic wrap and costs $5.94.

Which brand is the better value?

Solution

$$\text{Unit cost brand A} = \frac{\$4.48}{50}$$
$$= \$0.0896 \text{ per metre}$$

$$\text{Unit cost brand B} = \frac{\$5.94}{90}$$
$$= \$0.066 \text{ per metre}$$

Brand B has the lower unit cost per metre of plastic wrap, so it is the better value brand.

Explanation

The 'unit' in each pack is a metre of plastic wrap. The prices of both brands can be compared based on this unit.
Calculate the unit cost, per metre, for each brand by dividing the package cost by the number of units inside.

Choose the brand that has the lower unit cost.

Section Summary

- To compare the cost of items using the **unit cost** method, determine the cost per unit.
- The choice of unit may be a unit of weight (such as kilogram), or length (such a metre) or another measure depending on the particular item.

Exercise 1E

Example 21

1 Use the unit cost method to answer the following questions.

- **a** If 12 cupcakes cost $14.40, how much do 13 cupcakes cost?
- **b** If a clock gains 20 seconds in 5 days, how much does the clock gain in three weeks?
- **c** If 17 textbooks cost $501.50, how much would 30 textbooks cost?
- **d** If an athlete can run 4.5 kilometres in 18 minutes, how far could she run in 40 minutes at the same pace?

2 If one tin of red paint is mixed with four tins of yellow paint, it produces five tins of orange paint. How many tins of the red and yellow paint would be needed to make 35 tins of paint of the same shade of orange?

SF

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SF

3 If a train travels 165 kilometres in 1 hour 50 minutes at a constant speed, calculate how far it could travel in the following times.

a 3 hours **b** $2\frac{1}{2}$ hours **c** 20 minutes
d 70 minutes **e** 3 hours and 40 minutes **f** $\frac{3}{4}$ hour

4 You need six large eggs to bake two chocolate cakes. Determine the number of eggs needed to bake 17 chocolate cakes.

Example 22

5 Ice creams are sold in two different sizes. A 35 g cone costs $1.25 and a 73 g cone costs $2.00. Determine which is the better buy.

6 A shop sells 2 L containers of brand A milk for $2.99, 1 L of brand B milk for $1.95 and 600 mL of brand C milk for $1.42. Calculate the best buy.

7 Monica is planning a holiday for herself and her friends, six people in total. Since each person wants their own bedroom her choices are:

- three two-bedroom apartments, which cost $180 per night each, or
- two three-bedroom apartments which cost $260 per night each, or
- one four-bedroom apartments which costs $370 per night, plus one two-bedroom apartment which costs $180 per night.

Determine which would be the cheapest option, and how much per night each person will pay for that option.

CF

8 Soft drink can be bought in all of the following options:

- A box of 30×375 ml cans for $26.00, from Shop A
- A box of 12×1.25 litre bottles for $32.40, from Shop B
- 1×2 litre bottle for $4.90, from Shop C.

a Use mathematical reasoning to determine which shop is offering the best buy.

b Jaimi wants 90 litres of the soft drink delivered to her home. If Shop A charges a $30 delivery fee, Shop B charges a $15 delivery fee, and Shop C offers free delivery, determine the minimum amount Jaimi will pay for her order.

9 Sam can choose between different brands of knitting wool.

- Brand A contains 200 grams of knitting wool and costs $12.
- Brand B contains 200 metres of knitting wool and costs $6.

If a 100 gram ball of knitting wool contains 270 metres of yarn, determine which brand is the better buy, and by how much, measured in cents per metre.

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1F Percentages and applications

Learning intentions

- To be able to apply percentage mark-ups and discounts to the cost of an item.
- To be able to determine the percentage mark-up or discount which has been applied to the cost of an item.
- To be able to determine the amount of GST applied to the cost of an item or service.
- To be able to determine percentage profit and loss.

Discounts and mark-ups

Suppose an item is discounted, or **marked down**, by 10%. The amount of the **discount** and the new price are:

discount = 10% of original price = $0.10 \times$ original price

and

new price = 100% of old price − 10% of old price = 90% of old price = $0.90 \times$ old price

Applying discounts

In general, if r% discount is applied:

$$\text{discount} = \frac{r}{100} \times \text{original price}$$

$$\text{new price} = \text{original price} - \text{discount} = \frac{(100 - r)}{100} \times \text{original price}$$

Example 23 Calculating the discount and the new price

a How much is saved if a 10% discount is offered on an item marked $50.00?

b What is the new discounted price of this item?

Solution

a $\text{Discount} = \frac{10}{100} \times \$50 = \$5.00$

b New price = original price − discount

$= \$50.00 - \$5.00 = \$45.00$

or

$\text{New price} = \frac{90}{100} \times \$50 = \$45.00$

Explanation

Evaluate the discount.

Evaluate the new price by either:

- subtracting the discount from the original price

  or
- calculating 90% of the original price.

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Sometimes, prices are increased or marked *up*.

If a price is increased by 10%:

increase = 10% of original price = 0.10 × original price *and* new price = 100% of old price + 10% of old price = 110% of old price = 1.10 × old price

Applying mark-ups

In general, if r% increase is applied:

$$\text{increase} = \frac{r}{100} \times \text{original price}$$

$$\text{new price} = \text{original price} + \text{increase} = \frac{(100 + r)}{100} \times \text{original price}$$

Example 24 Calculating the increase and the new price

a How much is added if a 10% increase is applied to an item marked $50?

b What is the new increased price of this item?

Solution

a $\text{Increase} = \frac{10}{100} \times 50 = \$5.00$

b New price = original price + increase
$= \$50.00 + 5.00 = \$55.00$

or

$\text{New price} = \frac{110}{100} \times 50 = \$55.00$

Explanation

Evaluate the increase.

Evaluate the new price by either:

- adding the increase to the original price, or
- calculating 110% of the original price.

Calculating the percentage change

Given the original and new price of an item, we can work out the **percentage discount**, when the new price is less than the old price, or the **percentage increase**, when the new price is more than the old price.

Calculating percentage discount or increase

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original price}} \times \frac{100}{1}\%$$

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original price}} \times \frac{100}{1}\%$$

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Example 25 Calculating the percentage discount or increase

a The price was reduced from \$50 to \$45. What percentage discount was applied?

b The price was increased from \$50 to \$55. What percentage increase was applied?

Solution	Explanation
a Discount = original price − new price = 50.00 − 45.00 = \$5.00	Determine the amount of the discount.
Percentage discount $= \frac{5.00}{50.00} \times \frac{100}{1}$ $= 10\%$	Express this amount as a percentage of the original price.
b Increase = new price − original price = 55.00 − 50.00 = \$5.00	Determine the amount of the increase.
Percentage increase $= \frac{5.00}{50.00} \times \frac{100}{1}$ $= 10\%$	Express this amount as a percentage of the original price.

Calculating the original price

Sometimes we are given the new price and the percentage increase or decrease ($r\%$), and asked to determine the original price. Since we know that:

- for a discount, new price $= \frac{(100 - r)}{100} \times$ original price
- for an increase, new price $= \frac{(100 + r)}{100} \times$ original price

we can rearrange these formulas to give rules for determining the original price as follows.

Calculating the original price

When $r\%$ discount has been applied: original price = new price $\times \frac{100}{(100 - r)}$

When $r\%$ increase has been applied: original price = new price $\times \frac{100}{(100 + r)}$

Example 26 Calculating the original price

Suppose that Cate has a \$50 gift voucher from her favourite shop.

a If the store has a '10% off' sale, what is the original value of the goods she can now purchase?

b If the store raises its prices by 10%, what is the original value of the goods she can now purchase?

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Solution	Explanation
a Original price $= 50 \times \frac{100}{90} = \$55.555\ldots$ $= \$55.56$ to nearest cent	Substitute new price = 50 and $r = 10$ into the formula for an $r\%$ discount.
b Original price $= 50 \times \frac{100}{110} = \$45.454\ldots$ $= \$45.45$ to nearest cent	Substitute new price = 50 and $r = 10$ into the formula for an $r\%$ increase.

Goods and services tax (GST)

The **goods and services tax (GST)** is a tax of 10% that is added to the price of most goods (such as cars) and services (such as insurance). We can consider this a special case of the previous rules, where $r = 10$.

Consider the cost of an item after GST is added – this is the same as finding the new price when there has been a 10% increase in the cost of the item. Thus:

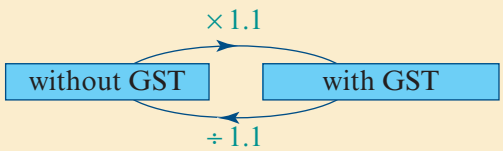
$$\text{cost with GST} = \text{cost without GST} \times \frac{110}{100} = \text{cost without GST} \times 1.1$$

Similarly, finding the cost of an item before GST was added is the same as finding the original cost when a 10% increase has been applied. Thus:

$$\text{cost without GST} = \text{cost with GST} \times \frac{100}{110} = \frac{\text{cost with GST}}{1.1}$$

Finding the cost with and without GST

- Cost with GST = cost without GST × 1.1

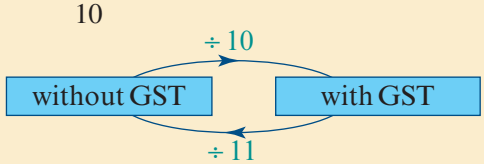


- Cost without GST $= \dfrac{\text{cost with GST}}{1.1}$

We can also directly calculate the actual amount of GST from either the cost without GST or the cost with GST.

Finding the amount of GST

- Amount of GST $= \dfrac{\text{cost without GST}}{10}$



- Amount of GST $= \dfrac{\text{cost with GST}}{11}$

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Example 27 Calculating GST

a If the cost of electricity supplied in one quarter is $288.50, how much GST will be added to the bill?

b If the selling price of a washing machine is $990:

i what is the price without GST? **ii** how much of this is GST?

Solution	Explanation
a GST= 288.50 ÷ 10 = $28.85	Substitute $288.50 into the rule for GST from cost without GST.
b Cost without GST = 990 ÷ 1.1 = $900 GST = 990 – 900 = 90 or GST = 990 ÷ 11 = $90	**i** Substitute $990 into the rule for cost with GST. **ii** We can determine the amount of the GST either by subtraction or by direct substitution into the formula.

Profit and loss

Profit is income (e.g. from selling an item) minus costs (e.g. from buying or producing the item). Loss is the same kind of calculation, but when costs are greater than the income, so the calculation is reversed, loss is cost minus income.

Profit and loss can be expressed in absolute terms (the amount of money) or percentage terms (the amount of money as a percentage of the cost).

Percentage profit and loss

When an item is sold for more than it cost: $\text{Percentage profit} = \dfrac{\text{income} - \text{cost}}{\text{cost}}$

When an item is sold for less than it cost: $\text{Percentage loss} = \dfrac{\text{cost} - \text{income}}{\text{cost}}$

Example 28 Calculating profit and loss

The cost for you to make a cake is $1.

a You sell each cake for $1.20. What is your absolute profit and percentage profit?

b If you can only sell each cake for 80c, what is your absolute loss and percentage loss?

Solution	Explanation
a $1.20 – $1.00 = $0.20c The absolute profit is 20 cents. = 0.20/1.00 × 100/1% = 20% The percentage profit is 20%.	Subtract cost from income. Write the answer for absolute profit. Percentage profit is absolute profit divided by cost, as a percentage.

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b $1.00 – $0.80 = $0.20 The absolute loss is 20 cents. = $0.20/$1.00 × 100/1% = 20% The percentage loss is 20%.	Subtract income from cost. Write the answer for absolute loss. Percentage loss is absolute loss divided by cost, as a percentage.

Section Summary

- When increasing or decreasing a quantity by a certain percentage, the actual change is expressed as a percentage of the original price.
- The formula for percentage change is: **Percentage change** = $\frac{\text{change}}{\text{original value}} \times 100$.
- **GST** is a tax of 10% added to the cost of goods and services.
- To find the cost with GST, multiply the cost without GST by 1.1.
- To find the cost without GST, divide the cost with GST by 1.1.
- Profit and loss can be expressed as a percentage of the cost of an item.
- The formula for percentage profit is: **Percentage profit** = $\frac{\text{income} - \text{cost}}{\text{cost}}$.
- The formula for percentage loss is: **Percentage loss** = $\frac{\text{cost} - \text{income}}{\text{cost}}$.

Exercise 1F

Review of percentages

1 Calculate the following as percentages. Give answers correct to one decimal place.

a $200 of $410 **b** $6 of $24.60 **c** $1.50 of $13.50
d $24 of $260 **e** 30c of 90c **f** 50c of $2

2 Calculate the amount of the following percentage increases and decreases. Give answers to the nearest cent.

a 10% increase on $26 000 **b** 5% increase on $4000
c 12.5% increase on $1600 **d** 15% increase on $12
e 10% decrease on $18 650 **f** 2% decrease on $1 000 000

Discounts, mark-ups and mark-downs

Example 23

3 Calculate the amount of the discount for the following, to the nearest cent.

a 24% discount on $360 **b** 72% discount on $250
c 6% discount on $9.60 **d** 9% discount on $812

Example 24

4 Calculate the new increased price for each of the following.

a $260 marked up by 12% **b** $580 marked up by 8%
c $42.50 marked up by 60% **d** $5400 marked up by 17%

SF

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SF

5 Calculate the new discounted price for each of the following.

a $2050 discounted by 9%
b $11.60 discounted by 4%
c $154 discounted by 82%
d $10 600 discounted by 3%
e $980 discounted by 13.5%
f $2860 discounted by 8%

Example 25

6 **a** The price of an item was reduced from $25 to $19. Determine the percentage discount applied.

b The price of an item was increased from $25 to $30. Determine the percentage increase applied.

Example 26

7 Determine the original prices of the items that have been marked down as follows.

a Marked down by 10%, now priced $54.00
b Marked down by 25%, now priced $37.50
c Marked down by 30%, now priced $50.00
d Marked down by 12.5%, now priced $77.00

8 Determine the original prices of the items that have been marked up as follows.

a Marked up by 20%, now priced $15.96
b Marked up by 12.5%, now priced $70.00
c Marked up by 5%, now priced $109.73
d Marked up by 2.5%, now priced $5118.75

9 Mikki has a card that entitles her to a 7.5% discount at the store where she works. Calculate how much she will pay for boots marked at $230.

10 The price per litre of petrol was $1.80 on Friday. When Rafik goes to fill up his car on Saturday, he finds that the price has increased by 2.3%. If the tank holds 50 L of petrol, calculate how much he will pay to fill the tank.

GST calculations

Example 27

11 Calculate the GST payable on each of the following (give your answer correct to the nearest cent).

a A gas bill of $121.30
b A telephone bill of $67.55
c A television set costing $985.50
d Gardening services of $395

12 The following prices are without GST. Calculate the price of each after GST has been added.

a A dress priced at $139
b A bedroom suite priced at $2678
c A home video system priced at $9850
d Painting services of $1395

13 If a computer is advertised for $2399 including GST, calculate how much the computer would cost without GST.

14 Determine the amount of the GST that has been added if the price of a car is advertised as $39 990 including GST.

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15 The telephone bill is $318.97 after GST is added. SF

a Determine the price before GST was added.

b Determine how much GST must be paid.

Profit and loss

Example 28

16 Andrew bought a rare model train for $450. He later sold the train for $600.

a Calculate the profit Andrew in dollars made on the sale of the train.

b Calculate the profit Andrew made as a percentage of the purchase price of the train, correct to one decimal place.

17 A bookseller bought eight copies of a book for $12.50 each. They were eventually sold for $10.00 each.

a Calculate the loss in dollars and cents that the bookseller made on the sale of the books.

b Determine the loss that the bookseller made as a percentage of the purchase price of the books.

18 The cost of producing a chocolate bar that sells for $1.50 is 60c. Calculate the profit made on a bar of chocolate as a percentage of the production cost of the bar.

19 The cost to a retailer of a pair of shoes is $64.50. CF

a Determine the price that the shoes should be sold for if the retailer wants to make 30% profit (include GST in the price).

The retailer has a sale advertising 20% off the price of the shoes.

b Determine the sale price of the shoes.

c Determine the amount of profit the retailer now makes.

20 The cost of making a cafe latte is $1.50 for the ingredients and $2.00 labour.

a Determine the percentage profit to the coffee shop if the cafe latte is sold for $5.70 (which includes GST). Give your answer as a percentage rounded to one decimal place.

b Suppose the cost of the ingredients rises by 12%. Determine the new cost of a cafe latte:

i for the actual profit to stay the same

ii for the percentage profit is to stay the same.

21 Nathan wants to produce t-shirts which he will sell to a shop with a 30% mark-up on his production costs. The shop will add their own 25% mark-up to the price they pay Nathan, and then add a further 10% for GST to determine the price the customer will pay. Determine the maximum amount that the t-shirts can cost for Nathan to produce in order for the final price of the t-shirts to be less than $30. CU

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1G Simple interest

Learning intentions

- To understand simple interest as an application of percentage increase.

When you borrow money, you have to pay for the use of that money. When you invest money, someone else will pay you for the use of your money. The amount you pay when you borrow, or the amount you are paid when you invest, is called **interest**. There are many different ways of calculating interest. The simplest of all is called, rather obviously, **simple interest**. Simple interest is a fixed percentage of the amount invested or borrowed and is *calculated on the original amount*.

Suppose we invest \$1000 in a bank account that pays simple interest at the rate of 5% per annum. This means that for each year we leave the money in the account, interest of 5% of the original amount will be paid to us. Remember 5% is equal to $\frac{5}{100}$ which is equal to 0.05.

In this instance, the amount of interest paid to us is 5% of \$1000 or $\$1000 \times 0.05 = \$50$.

If the money is left in the account for several years, the interest will be paid yearly.

To calculate simple interest we need to know:

- the initial investment, called the **principal**
- the **interest rate**, as a decimal interest rate per annum (such as 0.1) or more usually as % per annum (p.a.) (such as 10%)
- the length of time the money is invested.

Example 29 **Calculating simple interest from first principles**

How much interest will be earned if investing \$1000 at 5% p.a. (0.05 p.a. as a decimal) simple interest for 3 years?

Solution	Explanation
Interest $= 1000 \times 0.05 = \$50$	Calculate the interest for the first year.
Interest $= 1000 \times 0.05 = \$50$	Calculate the interest for the second year.
Interest $= 1000 \times 0.05 = \$50$	Calculate the interest for the third year.
Interest for 3 years $= 50 + 50 + 50$ $= \$150$	Calculate the total interest.

The same rules apply when simple interest is applied to a loan rather than an investment.

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The simple interest formula

Since the amount of interest in a simple interest investment is the same each year, we can apply a general rule.

Interest = amount invested or borrowed × interest rate (decimal rate per annum) × length of time (in years)

This rule gives rise to the following formula.

Simple interest formula

To calculate the simple interest earned or owed:

$$I = Pin$$

where I = the total interest earned or paid, in dollars

P = the principal (the initial amount borrowed or invested), in dollars

i = the decimal interest rate per annum

n = the time in years of the loan or investment.

However, interest rates are more often quoted as percentage interest rate per annum, $r\%$, where

$$i = \frac{r}{100}$$

So $I = \dfrac{P \times r \times n}{100}$.

Example 30 Calculating simple interest for periods other than one year

Calculate the amount of simple interest that will be paid on an investment of $5000 at 10% simple interest per annum for 3 years and 6 months.

Solution

$I = Pin$

$= 5000 \times 0.1 \times 3.5$

$= \$1750$

The simple interest on the investment is $1750.

Explanation

Apply the formula with $P = \$5000$, $i = \dfrac{r}{100} = \dfrac{10}{100} = 0.1\%$ and $n = 3.5$ (since 3 years and 6 months is equal to 3.5 years).

Write the answer.

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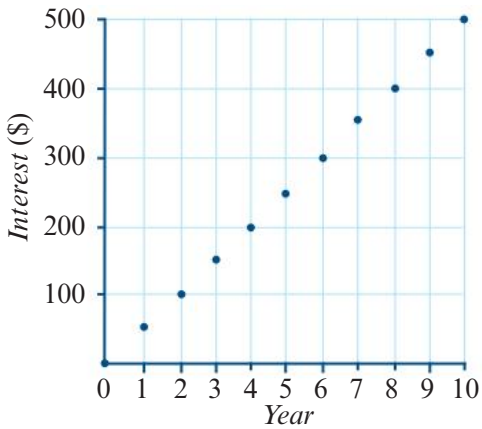
The graph below shows the total amount of interest earned after 1, 2, 3, 4, ... years, when $1000 is invested at 5% per annum simple interest for a period of years.

As we would expect from the simple interest rule, the graph is linear.

The slope of a line that could be drawn through these points is equal to the amount of interest added each year, in this case $50.



Technology enables us to investigate the growth in simple interest with time, using tables and graphs.

Desmos widget 1G: Simple interest calculator

Spreadsheet activity 1G: Calculating simple interest with a spreadsheet

Calculating the amount of a simple interest loan or investment

To determine the total value or amount of a simple interest loan or investment, the total interest amount is added to the initial amount borrowed or invested (the principal).

Total value of a simple interest loan

Total amount after t years (A) = principal (P) + total interest (I)

or $A = P + I$

Example 31 Calculating the total amount owed on a simple interest loan

Find the total amount owed on a simple interest loan of $16 000 at 8% per annum after 2 years.

Solution

$I = Pin$

$= 16\,000 \times 0.08 \times 2 = \$2560$

$A = P + I$

$= 16\,000 + 2560 = \$18\,560$

Explanation

Apply the formula with $P = \$16\,000$, $i = \frac{r}{100} = \frac{8}{100} = 0.08$ and $t = 2$ to find the total interest accrued.

Find the total amount owed by adding the interest to the principal.

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The simple interest formula can be rearranged to find any one of the four variables as long as the other three are known.

Calculating the interest rate

Decimal interest rate

To find the decimal interest rate i given the values of P, I and n:

$$i = \frac{I}{Pn}$$

where P is the principal, I is the amount of interest accrued in n years.

Percentage interest rate

To find the percentage interest rate, $r\%$, given the values of P, I and n:

$$r = 100i = 100\left(\frac{I}{Pn}\right) = \frac{100I}{Pn}$$

Example 32 Calculating the interest rate

Find the rate of simple interest per annum if:

a a principal of \$8000 increases to \$11 040 in 4 years

b a principal of \$5000 increases to \$5500 in 9 months.

Solution	Explanation
a Interest:	Find the amount of interest earned on the investment.
$I = 11\,040 - 8000$	
$= \$3040$	
$i = \frac{I}{Pn} = \frac{3040}{8000 \times 4}$	Apply the formula $i = \frac{I}{Pn}$ with $P = \$8000$, $I = \$3040$ and $n = 4$, then convert decimal rate to percentage ($r = 100i$).
$= 0.095$	
Interest rate is 9.5% per annum.	
b Interest:	Find the amount of interest earned on the investment.
$I = 5500 - 5000$	
$= \$500$	
$i = \frac{I}{Pn} = \frac{500}{5000 \times 0.75}$	Apply the same formula with $P = \$5000$, $I = \$500$ and $n = \frac{9}{12} = 0.75$ years.
$= 0.133$ to three decimal places	
$= 13.3\%$ to one decimal place	Convert the decimal rate to a percentage.
Interest rate is 13.3% per annum.	

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Calculating the time period

Time period

To find the number of periods or term of an investment given P, I and i:

$$n = \frac{I}{Pi}$$

where P is the principal, I is the amount of interest and i is the decimal interest rate.

To use the percentage interest rate, $r\%$:

$$n = \frac{I}{Pi} = \frac{I}{P\left(\frac{r}{100}\right)} = \frac{100I}{Pr}$$

Example 33 Calculating the time period of a loan or investment

Find the length of time it would take for \$5000 invested at an interest rate of 12% per annum to:

a earn \$1800 interest **b** earn \$404 interest.

For part **b** give the answer in days to the nearest day.

Solution

a $i = \frac{12}{100}$

$n = \frac{I}{Pi} = \frac{1800}{5000 \times 0.12}$

$= 3$ years

b $n = \frac{I}{Pi} = \frac{404}{5000 \times 0.12}$

$= 0.673\ldots$ years

$= 365 \times 0.673$

$= 245.766\ldots$ days

$= 246$ days (to the nearest day)

Explanation

Convert the percentage interest rate r to decimal interest rate i. Apply the formula $n = \frac{I}{Pi}$ with $P = \$5000$, $I = \$1800$ and $i = 0.12$.

Apply the same formula with $P = \$5000$, $I = \$404$ and $i = 12$. We can convert years into days by assuming that there are 365 days in a year.

Calculating the principal, P

Calculating the principal

- To find the value of the principal, P, given the values of I, i and n use the formula:

$$P = \frac{I}{in}$$

where I is the amount of interest accrued, i is the decimal interest rate and n is the number of time periods.

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- To find the value of the principal, P, given the values of A, r and n:

$$P = \frac{A}{1 + in} \quad \text{where } i = \frac{r}{100}$$

where A is the amount of the investment or loan, $r\%$ is the annual percentage interest rate and n is the time in years.

- To find the value of the principal using the percentage interest rate:

$$P = \frac{I}{in} \quad \text{where } i = \frac{r}{100}$$

Example 34 Calculating the principal of a loan or investment

a Find the amount that should be invested in order to earn \$1500 interest over 3 years at an annual interest rate of 5%.

b Find the amount that should be invested at an annual interest rate of 5% if you require the value of the investment to be \$15 600 in 4 years' time.

Solution

a $P = \frac{I}{in} = \frac{1500}{0.05 \times 3}$

$= \$10\,000$

b $P = \frac{A}{1 + in}$

$= \frac{15\,600}{1 + (0.05 \times 4)}$

$= \frac{15\,600}{1.2}$

$= \$13\,000$

Explanation

We are given the value of the interest, I, so use the formula $P = \frac{I}{in}$ with $I = \$1500$, $i = 0.05$ and $n = 3$ years.

Here we are *not* given the value of the interest, I, but the value of the total investment, A.

Use the formula $P = \frac{A}{1 + in}$

with $A = \$15\,600$, $i = 0.05$ and $n = 4$.

Section Summary

- **Simple interest** is calculated using the formula $I = Pin$
  where I = the total interest earned or paid, in dollars
  P = the principal (the initial amount borrowed or invested), in dollars
  i = the decimal interest rate per annum
  n = the time in years of the loan or investment
- The formula can be rearranged to find any one of the four variables as long as the other three are known.

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Skill-sheet

Exercise 1G

Calculating simple interest

Example 29
Example 30

1 Calculate the amount of interest earned from each of the following simple interest investments. Give answers correct to the nearest cent.

	Principal	Interest rate	Time
a	$400	5%	4 years
b	$750	8%	5 years
c	$1000	7.5%	8 years
d	$1250	10.25%	3 years
e	$2400	12.75%	15 years
f	$865	15%	2.5 years
g	$599	10%	6 months
h	$85.50	22.5%	9 months
i	$15 000	33.3%	1.25 years

Simple interest loans and investments

Example 31

2 Calculate the total amount to be repaid for each of the following simple interest loans. Give answers correct to the nearest cent.

	Principal	Interest rate	Time
a	$500	5%	4 years
b	$780	6.5%	3 years
c	$1200	7.25%	6 months
d	$2250	10.75%	8 months
e	$2400	12%	18 months

3 A simple interest loan of $20 000 is taken out for 5 years.

a Calculate the simple interest owed after 5 years if the rate of interest is 12% per annum.

b Calculate the total amount to be repaid after 5 years.

4 A sum of $10 000 was invested in a fixed term account for 3 years paying a simple interest rate of 6.5% per annum.

a Calculate the total amount of interest earned after 3 years.

b Calculate the total amount of the investment at the end of 3 years.

5 A loan of $1200 is taken out at a simple interest rate of 14.5% per annum. Determine how much is owed, in total, after 3 months.

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6 A company invests $1 000 000 in the short-term money market at 11% per annum simple interest. Determine how much interest is earned by this investment in 30 days. Give your answer to the nearest cent.

Simple interest: calculating interest rate

Example 32

7 Find the annual interest rate if a simple interest investment of $5000 amounts to $6500 in 2.5 years.

8 Find the annual interest rate if a simple interest investment of $500 amounts to $550 in 8 months.

Simple interest: calculating time

Example 33

9 Calculate the time taken for $2000 to earn $975 at 7.5% simple interest per annum.

10 Calculate the time in days for $760 to earn $35 at 4.75% simple interest per annum.

11 Geoff invests $30 000 at 10% per annum simple interest until he has $42 000. Calculate how many years he will need to invest the money.

Simple interest: calculating principal

Example 34

12 Calculate the principal that earns $514.25 in 10 years at 4.25% simple interest.

13 Calculate the principal that earns $780 in 100 days at 6.25% per annum simple interest.

Simple interest: mixed problems

14 Calculate the answers to complete the following table.

Principal	Rate	Time	Simple interest	Total investment
$600	6%	5 years	**a**	**b**
$880	6.5%	**c**	$171.60	**d**
$1290	**e**	6 months	$45.15	**f**
g	10%	4 months	$150.00	**h**
$3600	**i**	200 days	$98.63	**j**
$980	7.5%	**k**	**l**	$1200.50
m	7.25%	6 months	$52.50	**n**

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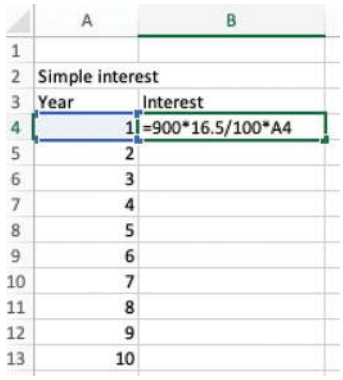
Exploring the growth of interest in a simple interest loan or investment

Spread-sheet

15 A loan of $900 is taken out at a simple interest rate of 16.5% per annum. Create the following spreadsheet to illustrate this investment. Enter the formula shown in B4 to multiply the amount borrowed by the interest rate per year, and fill down to B13.



a Determine how much interest is owed after 5 years.

b Determine how many years it will take for the interest owed to be $1336.50.

SF

Applications

16 A building society offers the following interest rates for its cash management accounts.

CF

	Interest rate (per annum) on term (months)				
Balance	1– <3	3– <6	6– <12	12– <24	24– <36
$20 000–$49 999	2.85%	3.35%	3.85%	4.35%	4.85%
$50 000–$99 999	3.00%	3.50%	4.00%	4.50%	5.00%
$100 000–$199 999	3.40%	3.90%	4.40%	4.90%	5.40%
$200 000 and over	4.00%	4.50%	5.00%	5.50%	6.00%

Using this table, find the simple interest earned by each of the following investments. Give your answers to the nearest cent.

a $25 000 for 2 months

b $125 000 for 6 months

c $37 750 for 18 months

d $200 000 for 2 years

e $74 386 for 8 months

f $145 000 for 23 months

17 Josh decides to put $5000 into an investment account that pays 5.0% per annum simple interest. Calculate how long it will take for his money to double.

18 A personal loan of $15 000 over a 3-year period costs $500 per month to repay.

a Calculate how much money will be repaid in total.

b Calculate how much of the money repaid is interest.

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1H Compound interest

Learning intentions

- To understand compound interest as an application of percentage increase.

We have seen that simple interest is calculated only on the original amount borrowed or invested. A more common form of interest, known as **compound interest**, calculates the interest by applying the interest rate to the original amount plus any interest already earned.

Calculating compound interest

Consider, for example, \$250 invested at 10% per annum, with the percentage increase added to the account each year.

After 1 year:

$$\begin{aligned} I &= Pin \\ &= \$250 \times 10\% \times 1 \\ &= 250 \times 0.01 \times 1 \\ &= \$25 \end{aligned}$$

So, after one year, the amount of money in the account is:

$$\begin{aligned} \text{amount} &= \text{amount at the start of year} + \text{interest earned} \\ &= \$250 + \$25 \\ &= \$275 \end{aligned}$$

After 2 years:

$$I = Pin = \$275 \times 0.01 \times 1 = \$27.50$$

So, after two years, the amount of money in the account is:

$$\begin{aligned} \text{amount} &= \text{amount at the start of year} + \text{interest earned} \\ &= \$275 + \$27.50 \\ &= \$302.50 \end{aligned}$$

After 3 years:

$$I = Pin = \$302.50 \times 0.01 \times 1 = \$30.25$$

So after three years, the amount of money in the account is:

$$\begin{aligned} \text{amount} &= \text{amount at the start of year} + \text{interest earned} \\ &= \$302.50 + \$30.25 \\ &= \$332.75 \end{aligned}$$

When the interest is paid each year, we say that the interest is **compounding annually**. If we tabulate this information, we will see that by applying the percentage increase, which is called **compounding** the interest, the amount of interest owed or paid increases each year.

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After	Amount invested	Interest earned	Total amount of investment
1 year	\$250	\$25	\$(250 + 25) = \$275
2 years	\$275	\$27.50	\$(275 + 27.50) = \$302.50
3 years	\$302.50	\$30.25	\$(302.50 + 30.25) = \$332.75

Developing the compound interest formula

We can use a formula to determine the amount of the investment which results from applying the percentage increase to the total amount at then end of the previous time period.

Start by recalling that the multiplying factor to increase a quantity by 10% is $\left(1 + \frac{10}{100}\right) = 1.1$

Using this factor we have the value of the investment, A, is:

$A = \$250 \times 1.1 = \$275$ (after 1 year)

$A = \$250 \times 1.1 \times 1.1$

$= \$250 \times (1.1)^2 = \$302.50$ (after 2 years)

$A = \$250 \times 1.1 \times 1.1 \times 1.1$

$= \$250 \times (1.1)^3 = \$332.75$ (after 3 years)

and so on, giving the general formula:

$A = \$250 \times (1.1)^n$ (after n years)

Thus the value of the investment after 10 years would be: $A = \$250 \times 1.1^{10} = \$648.44$

Following this pattern, we can write down a general formula for calculating the amount of an investment after a given amount of time when the percentage increase is applied to the preceding total amount.

The compound interest formula interest compounding annually

In general, the amount, A, of a compound interest investment is given by:

$$A = P(1 + i)^n$$

- P is the initial amount invested (the principal)
- i is the decimal annual interest rate
- n is the number of years.

To find the total amount of interest earned, subtract the initial investment from the final amount.

Determining the interest earned

Interest earned (I) = value of the investment (A) − initial amount invested (P)

$$I = A - P$$

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To find the overall percentage change in the investment, express this amount as a percentage of the initial investment.

Determining the percentage increase in the investment

$$\text{percentage change} = \frac{I}{P} \times 100$$

where I is the interest earned and P is this initial amount invested

Example 35 Calculating the amount and percentage increase of an investment

a Determine, to the nearest dollar, the amount of money accumulated after 3 years when $2000 is invested at an interest rate of 8% per annum, compounded annually.

b Determine, to the nearest dollar, the total amount of interest earned, and hence the percentage increase over three years.

Solution

a $i = \frac{r}{100} = \frac{8}{100} = 0.08$

$A = P \times (1 + i)^n = 2000 \times (1 + 0.08)^3$

$= \$2519$ to the nearest dollar

```
2000 * (1 + 0.08) ^ 3
                2519.424
```

b $I = A - P = 2519 - 2000$

$= \$519$

$$\text{percentage increase} = \frac{519}{2000} \times 100$$

$= 25.95\%$

Explanation

The annual percentage interest rate ($r\%$) is given so convert to the decimal interest (i) rate first.

Substitute $P = \$2000, i = 0.08, n = 3$, into the formula giving the amount of the investment. Enter the information into a scientific calculator and evaluate.

Subtract the principal from this amount to determine the interest earned.

Calculate the percentage increase in the amount of the investment.

The formula for compound interest can also be applied when money is borrowed, as shown in the following example.

Example 36 Calculating the amount of the debt and interest owed

a Determine, to the nearest dollar, the amount of money owed after 2 years when $10 000 is borrowed at an interest rate of 10% per annum, compounded annually.

b Determine the amount of interest owed, and hence the percentage increase in the amount owed over two years.

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Solution	Explanation
a $i = \frac{r}{100} = \frac{10}{100} = 0.10$	The annual percentage interest rate ($r\%$) is given so convert to the decimal interest (i) rate first.
$A = P \times (1 + i)^n = 10\,000 \times (1 + 0.1)^2$ $= \$12\,100$	Substitute $P = \$10\,000, i = 0.10, n = 2$, into the formula giving the amount of the debt.
b $I = A - P = 12\,100 - 10\,000$ $= \$2100$ percentage increase$= \frac{2100}{10\,000} \times 100$ $= 21\%$	Subtract the principal from this amount to determine the interest owed. Calculate the percentage increase in the amount owed.

Other ways to determine compound interest, apart from using a scientific calculator, are to use the Desmos widget, or a spreadsheet, provided in the Interactive Textbook.

Spread-sheet **Spreadsheet 1H:** Calculating compound interest

Compounding periods other than one year

So far in this section we have considered percentage increases that are calculated and added every year. It is very common for percentage increase to be calculated and added to an account, either loan or investment, more often than this. The compound interest formula is the same for any **compounding time period**, as long as the decimal interest rate is expressed for the same compounding period. The interest rate is generally given as an annual one, so this is adjusted to take into account the more frequent interest calculations.

The compound interest formula used with any compounding period

In general, the amount, A, of a compound interest investment is given by

$$A = P(1 + i)^n$$

where:

- P is the initial amount invested or borrowed (the principal)
- i is the interest rate per compounding period written as a decimal
- n is the total number of compounding periods (sometimes called **'rests'**) for the loan or investment.

If the compounding period is less than a year the annual percentage interest rate, can be converted to a percentage interest rate for the compounding period concerned.

Suppose for example a particular bank loan charges interest at an annual rate of 4.8% for five years, but the bank calculates the interest owing and adds this to the loan after every month. Here the rate of interest we use in our calculations should be a monthly one. Since there are 12 months in a year, the yearly interest rate is converted to a monthly rate by dividing by 12.

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Annual interest rate = 4.8% = 0.048 per year

Monthly interest rate $i = \frac{0.048}{12} = 0.004$ per month

We also need to calculate n, the number of compounding time periods. Since in this example the interest rate is compounding monthly for 5 years then we multiply the number of years by 12 to determine the number of months.

n = number of years of the loan × number of compounding time periods per year = $5 \times 12 = 60$

For simplicity we will assume that, in one year, there are:

- 365 days (ignore the possibility of leap years)
- 52 weeks (even though there are slightly more than this)
- 26 fortnights (even though there are slightly more than this)
- 12 months
- 4 quarters

Example 37 Compound interest with compounding periods other than one year

Courtney invests $50 000 in an account that earns interest at the rate of 4.5% per annum, compounding monthly (monthly rests).

a Determine the amount in her account after three and a half years. Round your answer to the nearest cent.

b Determine the percentage increase in the investment over this time.

Solution	Explanation
a $P = \$50\,000$ $r = 4.5\% = 0.045$ per year The investment lasts 3.5 years	Write the given information.
$i = \frac{0.045}{12} = 0.00375$	Calculate the monthly interest rate i by dividing the annual interest rate by 12
$n = 3.5 \times 12 = 42$	Calculate the number of time periods n by multiplying the number of years by 12
$A = P(1 + i)^n$ $A = 50\,000(1 + 0.00375)^{42}$ $A = 58\,511.799\,17$	Calculate A using $A = P(1 + i)^n$.
$50\,000(1 + 0.00375)^{42}$ $58\,511.79917$	Use a calculator to evaluate.
After three and a half years, the amount in the account is $58 511.80.	Write your answer rounded to the nearest cent.

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b $I = A - P = \$58\,511.80 - 50\,000$
$= \$8511.80$
percentage increase $= \frac{8511.80}{50\,000} \times 100$
$= 17.02\%$

Subtract the principal from this amount to determine the interest earned.

Calculate the percentage increase in the investment.

Section Summary

- **Compound interest** is an application of percentage increase, where the percentage increase is applied to the principal and the interest accumulated.
- In general, the amount, A, of a compound interest investment is given by

$$A = P(1 + i)^n$$

where:

  - P is the initial amount invested or borrowed (the principal)
  - i is the interest rate per compounding period written as a decimal
  - n is the total number of compounding periods (sometimes called **'rests'**) for the loan or investment.
- The amount of interest on the investment or loan $I = A - P$
- The percentage increase in the investment or loan $= \frac{I}{P} \times 100$

Skill-sheet

Exercise 1H

In the following exercises, give all answers correct to the nearest cent and to the nearest 0.1 per cent.

Compound interest investments and loans with interest compounded annually

Example 35

SF

1 An amount of \$3500 is invested at 5% compound interest per annum for 5 years.

a Determine the final value of this investment.

b Calculate the total amount of interest earned.

c Calculate the percentage increase in the investment over this time.

2 An amount of \$7000 is invested at 8% compound interest per annum for 4 years.

a Determine the final value of this investment.

b Calculate the total amount of interest earned.

c Calculate the percentage increase in the investment over this time.

3 An amount of \$100 000 is invested at 3.45% compound interest per annum for 2 years.

a Determine the final value of this investment.

b Calculate the percentage increase in the investment over this time.

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Example 36

4 A person borrows $1250 at 7.5% compound interest per annum for 3 years.

a Determine the total amount of money owed after 3 years.

b Calculate the amount of interest owed.

c Calculate the percentage increase in the amount owed over this time.

5 Gina borrows $45 500 at 5.5% compound interest per annum for 2 years.

a Determine the total amount of money owed after 2 years.

b Calculate the amount of interest owed.

c Calculate the percentage increase in the amount owed over this time.

6 A person borrows $1000 at 6.0% compound interest per annum for 5 years.

a Determine the total amount of money owed after 5 years.

b Calculate the percentage increase in the amount owed over this time.

Exploring compound interest loans and investments with a Spreadsheet

Spread-sheet

7 Ben invests his savings of $1850 from his holiday job for ten years at 5.25% per annum compound interest.

Create the following spreadsheet to illustrate this investment. Enter the formula shown in B5 which multiplies the amount in B4 ($1850) by (1 + 5.25/100) to give the new amount with compound interest added at the end of year 1, and fill down to B14.

	A	B
2	$1850 at 5.25% compound interest	
3	Year	Amount
4	0	$1,850.00
5	1	=B4*(1+5.25/100)
6	2	
7	3	
8	4	
9	5	
10	6	
11	7	
12	8	
13	9	
14	10	

a Determine the amount of the investment after 10 years.

b Determine the amount of interest earned after 5 years.

Spread-sheet

8 $850 is invested at 13.25% per annum compound interest for 8 years.

Create a spreadsheet to illustrate this investment. Enter the formula shown, which multiplies the amount in B4 ($850) by (1 + 13.25/100) to give the new amount with compound interest added at the end of year 1, and fill down.

	A	B
2	$850 at 13.25% compound interest	
3	Year	Amount
4	0	$850.00
5	1	=B4*(1+13.25/100)
6	2	
7	3	
8	4	
9	5	
10	6	
11	7	

a Determine the amount of the investment after 7 years.

b Determine the amount of interest earned after 5 years.

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Spread-sheet

9 Peter invests $3000 at 5.65% per annum compound interest. Create the following spreadsheet to illustrate this investment. Enter the formula shown which multiplies the amount in B4 ($3000) by (1 + 5.65/100) to give the new amount with compound interest added at the end of year 1, and fill down.

	A	B
1		
2	$3000 at 5.65% compound interest	
3	Year	Amount
4	0	$3,000.00
5	1	=B4*(1+5.65/100)
6	2	
7	3	
8	4	
9	5	
10	6	
11	7	
12		

a Determine the amount of the investment after 5 years.

b Peter decides to leave his money invested until the amount of interest he has earned is more than the amount he invested. Determine the minimum number of years this would take.

SF

Compound interest with compounding periods (rests) other than one year

Example 37

10 A bank offers a loan with an interest rate of 3.6% per annum, compounding monthly.

a Calculate the monthly interest rate for this loan.

b If $10 000 is borrowed, calculate how much is owed at the end of six months.

c Suppose $25 000 is borrowed for a period of 3 years.

i Calculate how much must be paid back at the end of three years.

ii Calculate the percentage increase in the amount owed over this time.

11 A bank will pay interest at the rate of 5.2% per annum, compounding fortnightly.

a Calculate the fortnightly interest rate for this investment.

b Suppose $5000 is invested for a period of 5 years.

i Determine the final value of the investment.

ii Calculate the percentage increase in the investment over the five years.

12 Millicent invests $18 000 in an account that pays compound interest at the rate of 3.8% per annum.

a Calculate the amount in the account after 1 year if the interest compounds:

i monthly **ii** fortnightly **iii** daily

b Calculate how much extra Millicent will earn in interest over the first year if she chooses fortnightly rests instead of monthly rests.

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Applications

13 Suppose $3000 is invested for 5 years. CF

a Calculate the difference between the interest at 7.9% per annum simple interest, and at 7.9% per annum compounding annually.

b Determine the simple interest rate per annum required to earn the same amount of interest over 5 years as would be earned at 7.9% per annum compounding annually. Give your answer as a percentage rounded to two decimal places.

14 An amount of $100 000 is invested at 3.5% compound interest per annum for 4 years.

a Determine the final value of this investment.

b Determine how much interest was earned in the fourth year.

c Determine the simple interest rate per annum required to earn the same amount of interest over 4 years as would be earned at 3.5% per annum compounding annually. Give your answer as a percentage rounded to two decimal places.

15 Coco invested $40 000 at r% interest per year, compounding annually for 5 years. At the end of this time there was $60 000 in her account. Determine the value of r, correct to one decimal place. CU

16 Maisie invested $20 000 at r% interest per year, compounding monthly for 6 years. At the end of this time there was $30 000 in her account. Determine the value of r, correct to one decimal place.

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1I Inflation

Learning intentions

- To understand inflation as an application of percentage increase.
- To be able to determine the effect of inflation on costs and wages.

Effect of inflation on prices

Inflation is a term that describes the continuous upwards movement in the general level of costs and prices.

Inflation

Inflation is the rate of increase in prices over a given period of time. It is generally expressed as a percentage increase per year.

Because cost and prices increase, the effect of inflation is to steadily reduce the **purchasing power** of your money; that is, what you can actually buy with your money.

In the early 1970s, inflation rates were very high, up to around 16% and 17%. Inflation in Australia has been relatively low in recent years, although there was an increase in 2022.

- Since 1970, inflation has averaged 5.1% per year.
- Since 2013, it has averaged 2.4% per year.
- In 2022, inflation rose to 6.6%.

Example 38 Determining the effect of inflation on prices over a short period of time

Suppose that inflation is recorded as 2.9% in 2021 and 6.6% in 2022 and that a loaf of bread cost $2.70 at the end of 2020. If the price of bread increases with inflation, what was the price of the loaf at the end of 2022?

Solution	Explanation
Increase in price (2021) $= 2.70 \times \frac{2.9}{100}$ $= 0.08$	Determine the increase in price of the loaf of bread during 2021 after a 2.7% increase.
Price (2021) $= 2.70 + 0.08 = \$2.78$	Calculate the price at the end of 2021.
Increase in price (2022) $= 2.78 \times \frac{6.6}{100}$ $= 0.18$	Determine the price of the loaf of bread at the end of 2022 after a further 6.6% increase.
Price (2022) $= 2.78 + 0.18 = \$2.96$	Calculate the price at the end of 2022.

Although the difference in price seen in the previous example does not seem a lot, you will be aware from earlier compound interest examples that even if inflation holds steady at a low

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2.1% per year for 20 years, prices will still increase a lot. The compound interest formula introduced in the last section can be used to determine the size of this increase.

Future price prediction

In general, the future value or cost of an item due to inflation, A, is:

$A = P(1 + i)^n$

where:

- P is the current cost of the item
- i is average rate of inflation per year
- n is the number of years into the future we are predicting.

Example 39 Determining the effect of inflation on prices over a long period

Suppose that a one-litre carton of milk costs $1.70 today.

a Determine the price of the one-litre carton of milk in 20 years' time if the average annual inflation rate is 2.1%.

b Determine the price of the one-litre carton of milk in 20 years' time if the average annual inflation rate is 6.8%.

Solution	Explanation
a $A = P \times (1 + i)^n$	This is the equivalent of investing \$1.70 at 2.1% interest compounding annually, so we can use the compound interest formula, with n as the number of years, since we are dealing with annual inflation.
$i = \frac{r}{100} = \frac{2.1}{100} = 0.21$	Convert the annual percentage interest rate to an annual decimal interest rate.
Price $= 1.70 \times (1 + 0.021)^{20}$ $=$ \$2.58 to the nearest cent	Substitute $P =$ \$1.70, $n = 20$ and $r = 2.1$ in the formula to find the price in 20 years.
b Price $= 1.70 \times (1 + 0.068)^{20}$ $=$ \$6.34 to the nearest cent	Substitute $P = 1.70$, $n = 20$ and $r = 6.8$ in the formula and evaluate.

Effect of inflation on the purchasing power of money

Another way of looking at the effect of inflation on our money is to consider what a sum of money today would buy in the future. Suppose, for example, that you put \$100 in a box under the bed and leave it there for 10 years. When you go back to the box, there is still \$100, but what could you buy with this amount in 10 years' time? To find out we need to 'deflate' this to reflect the impact of inflation on the purchasing power of this amount.

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We can do this by using the compound interest formula. In this scenario we wish to find the value of P, given the value of A, so the formula needs to be rearranged.

Future purchasing power

In general, the future purchasing power, P, of an amount of money is:

$$P = \frac{A}{(1+i)^n}$$

where:

- A is actual amount of money
- i is the average inflation rate per year
- n is the number of years into the future we are predicting

Suppose there has been an average inflation rate of 4% over the 10-year period.

Substituting $A = 100$, $r = 4$ and $t = 10$ gives:

$$P = \frac{100}{(1+0.04)^{10}} = \frac{100}{1.04^{10}}$$
$$= \$67.57 \text{ to the nearest cent}$$

100/(1.04^10)	67.556 . . .

That is, the money that was worth \$100 when it was put away has a purchasing power of only \$67.56 after 10 years, if the inflation rate has averaged 4% per annum. This means that it will buy about as much as we could buy for \$67.56 today.

Example 40 Investigating purchasing power

If savings of \$100 000 are hidden under a mattress in 2022, determine the purchasing power of this amount 8 years later if the average inflation rate over this period is 3.7%. Give your answer to the nearest dollar.

Solution

$$i = \frac{3.7}{100}$$

$$A = P \times (1+i)^n$$
$$100\,000 = P \times (1+0.037)^8$$

$$P = \frac{100\,000}{(1+0.037)^8} = 74\,777.329$$

The purchasing power of \$100 000 in 8 years is \$74 777, to the nearest dollar.

Explanation

Convert the annual percentage interest rate to an annual decimal interest rate.

Write the compound interest formula with P (the purchasing power, which is unknown), $A = 100\,000$ (current value), $i = 0.037$ and $t = 8$.

Rearrange the formula, and use your calculator to solve this equation for P. Round to the nearest dollar and write your answer.

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Section Summary

- **Inflation** is the increase over time in cost and prices, usually expressed as a percentage increase per year.
- The predicted effect of inflation is an increase in price equal to the annual inflation percentage.
- The **purchasing power** of money is how much you could buy with your money at a particular time.
- The **compound interest formula** can be used to calculate predicted price or value of money when given an average inflation rate over the projected time period.

Exercise 1I

Effect of inflation on prices

Example 38

1 Suppose that inflation was recorded as 2.7% in Year 1 and 3.5% in Year 2, and that a magazine cost $3.50 at the beginning of Year 1. Assume that the price increases with inflation.

a Calculate the price of the magazine at the end of Year 1.

b Calculate the price of the magazine at the end of Year 2.

2 Suppose that Henry receives a salary increase at the end of each year equal to the rate of inflation for that year. Inflation was recorded as 3.2% in Year 1 and 5.3% in Year 2, and Henry's weekly salary was $825 at the beginning of Year 1.

a Calculate Henry's salary at the end of Year 1.

b Calculate Henry's salary at the end of Year 2.

3 Suppose that inflation is recorded as 4.5% in Year 1, 2.1% in Year 2 and 3.3% in Year 3, and that a kilo of oranges costs $4.95 at the beginning of Year 1. If the price of oranges increases with inflation, determine the price per kilo at the end of Year 3.

Example 39

4 Suppose that the cost of petrol per litre is $1.80 today.

a Determine the predicted price of petrol per litre in 20 years' time if the average annual inflation rate is 1.9%.

b Determine the predicted price of petrol per litre in 20 years' time if the average annual inflation rate is 7.1%.

5 A unit is sold at auction for $500 000. If the price of the unit increases with the inflation rate, determine the predicted price of the unit in 12 years' time, if the average inflation rate over the 12-year period is:

a 2.6% **b** 6.9%.

SF

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SF

6 Suppose that the cost of a mobile phone is $1200. If the price of the phone increases with the inflation rate, determine the predicted price of the mobile phone in 5 years' time, if the average inflation rate over the 5-year period is:

a 1.6% **b** 5.6%.

Effect of inflation on purchasing power

Example 40

7 If savings of $200 000 are hidden in a mattress today, determine the predicted price purchasing power of that money in 10 years' time, if the average inflation rate over the 10-year period is:

a 3% **b** 13%

8 Hank puts $2000 into an envelope which he hides to keep safe, but unfortunately he forgets where he put it. Luckily, five years later, he accidentally comes across the envelope and the $2000 is still there. If the average inflation rate of the five years for which the money was lost was 3.2% per year, determine the purchasing power of that money now.

9 Jo puts $1000 saving in her safe. Determine the predicted purchasing power of this money in 20 years' time, if the average inflation rate over the 20-year period is:

a 2.6% **b** 6.9% **c** 14.3%

Applications

CF

10 The price today of a particular car is $48 500. Suppose the price of the car increases at the rate of inflation, and the average inflation rate is 3% per year.

a Calculate the price of the car in five years.

b Determine the minimum number of years it takes for the price of the car to have more than doubled.

11 Suppose the price of milk is $3.90 per litre. If the price of milk increases at the rate of inflation, and the average inflation rate is 2.1% per year:

a Calculate the price of the milk in 10 years.

b Determine the minimum number of years it takes for the price of the milk to have increased by at least 25%.

CU

12 Suppose that the current price of petrol is $2 per litre. Due to inflation, the price of petrol per litre is expected to double over the next 10 years. Determine the average annual inflation rate over that time period that would give this result. Give your answer as a percentage, correct to one decimal place.

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1J Currency and exchange rates

Learning intentions

- To be able to use currency exchange rates to convert between Australian dollars and foreign currencies.

The money that you use to pay for goods and services in one country cannot usually be used in any other country. If you take Australian dollars to New Zealand, for example, they must be exchanged with, or converted to, New Zealand dollars. Even though the Australian dollar (AUD) and the New Zealand dollar (NZD) have the same name, they have different values.

Exchange rate

An **exchange rate** is the price of one currency expressed in terms of another currency.

The table below shows the exchange rate between Australian dollars and other currencies on a particular day, rounded to five decimal places.

Currency exchange: Australian dollar (AUD)				
Country	Currency name	Symbol	Code	Units per AUD
United States of America	Dollar	$	USD	0.67187
European Union	Euro	€	EUR	0.61349
United Kingdom	Pound	£	GBP	0.52700
Japan	Yen	¥	JPY	96.91477
South Africa	Rand	R	ZAR	12.48702
China	Yuan Renminbi	¥	CNY	4.79510

Source: *http://www.xe.com*

Note: Exchange rates may change on a daily basis

The numbers in the column 'Units per AUD' are the **exchange rates** for each currency and are used to convert between Australian dollars and other currencies, in a similar way to converting between units of measurement.

The units per AUD for the Japanese yen is 96.91477, which means that one AUD will be exchanged for 96.91477 yen in Japan.

$1 AUD = 96.91477 JPY

Ten Australian dollars would be exchanged for ten times this amount.

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Converting between currencies

An **exchange rate** between Australian dollars and other currencies is given as units per AUD.

- Convert Australian dollars to other currencies by **multiplying** the amount by the exchange rate.
- Convert other currencies to Australian dollars by **dividing** the amount by the exchange rate.

In Australia, the dollar consists of 100 cents. Most countries divide their main currency unit into 100 smaller units, and so it is usual to round currency amounts to two decimal places, even though the conversion rates are usually expressed with many more decimal places than this.

Example 41 Converting between Australian dollars and other currencies

Use the table of currency exchange for the Australian dollar (on page 61) to convert these currencies.

a 300 AUD into British pounds

b 2500 ZAR into Australian dollars

Solution	Explanation
a 1 AUD = 0.52700 GBP	Write the exchange rate for AUD to GBP.
300 AUD = 300 × 00.52700 GBP = 158.100 GBP	Multiply 300 AUD by the exchange rate to convert to GBP.
$300 AUD is converted to £156.10	Round your answer to two decimal places.
b 1 AUD = 12.48702 ZAR	Write the exchange rate for AUD to ZAR.
$2500 \text{ ZAR} = \frac{2500}{12.48702} \text{AUD}$ = 200.208	Divide 2500 ZAR by the exchange rate to convert to AUD.
2500 ZAR is converted to $200.21 AUD.	Round your answer to two decimal places.

Section Summary

- **Exchange rate** is the price of one currency in terms of another currency.
- Given the value of another currency in Australian dollars:
  1. To convert from AUD to that currency **multiply** by the exchange rate.
  2. To convert from that currency to AUD **divide** by the exchange rate.

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Exercise 1J

Example 41

1 Use the table of currency exchange values below to convert the following amounts into the currency in brackets. Round your answer to two decimal places.

Currency exchange: Australian dollar (AUD)				
Country	Currency name	Symbol	Code	Units per AUD
United States of America	Dollar	$	USD	0.67187
European Union	Euro	€	EUR	0.61349
United Kingdom	Pound	£	GBP	0.52700
Japan	Yen	¥	JPY	96.91477
South Africa	Rand	R	ZAR	12.48702
China	Yuan Renminbi	¥	CNY	4.79510

Source: *http://www.xe.com*

a $750 AUD (EUR) **b** $4800 AUD (USD) **c** $184 AUD (CNY)
d ¥5000 JPY (AUD) **e** R8500 ZAR (AUD) **f** £8500 GBP (AUD)

2 Use the table in Question 1 to determine in AUD the cost of a cafe latte which costs
a $6 in the USA. **b** €4.50 in Austria. **c** £3.25 in London.

3 On a particular day, one Australian dollar was worth 8.6226 Botswana pula (BWP). Calculate how many pula Tapiwa would need to exchange if she wanted to receive $2000 AUD.

4 On a particular day, $850 AUD could be exchanged to €581.40. Calculate how many euros would be exchanged for $480 AUD.

5 Katia is shopping in Paris. She sees a pair of shoes she really likes, costing €129.50. She can purchase the shoes with her credit card, which will charge her in AUD at the current exchange rate plus a 2.6% foreign transaction fee. If that day the exchange rate is 1 Euro for $1.63000 AUD, determine how much she pays for the shoes in $AUD.

6 Parris has found three places online which stock the book he wants to buy.
- Supplier 1 is in the USA, where he can buy the book for $39 USD, plus an international transaction fee of 2%, and delivery of $15 USD.
- Supplier 2 is in China, where he can buy the book for ¥510 CNY, plus an international transaction fee of 1.8%, and delivery of ¥20 CNY with no transaction fee on the delivery.
- Supplier 3 is in Australia, where he can buy the book for $99 AUD, and delivery of $10 AUD.

Use the table of exchange rates in Question **1** to determine the total cost of the book in Australian dollars from each supplier, and hence from which supplier he should order the book.

SF

CF

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1K Shares and dividends

Learning intentions

- To be able to determine the dividend paid on shares given the percentage dividend or dividend per share.
- To be able to compare shares using a price-to-earnings ratio.

Investors often choose to invest their money in shares. A **share** is a unit of ownership in a company. All shares are equal in value, and each share entitles the person who owns it to an equal claim on the company's **profits**. The company may decide to use some of the profits to re-invest in the company, and the rest may be paid to the shareholders. For example:

- if there are 100 shares in a company and you own 20, then you own 20% of the shares in the company
- if the company pays out a profit of $100 000 in one year then you are entitled to 20% of that profit (which would be $20 000).

Example 42 Calculating profit from shares

There are 500 shares in the Kanz Electrical Company. Phil owns 25 shares.

a Calculate the percentage of the company Phil owns.

b The company declares an annual profit of $780 000. Calculate how much profit Phil is entitled to.

Solution

a Percentage ownership

$$= \frac{25}{500} \times \frac{100}{1}$$

$$= 5\%$$

b $\text{Profit} = 780\,000 \times \frac{5}{100}$

$$= \$39\,000$$

Explanation

We need to convert 25 out of 500 into a percentage.

Phil is entitled to 5% of the profit.

In practice, companies do not share all of their profits with shareholders, but they do pay **dividends**.

Dividends

A **dividend** is the amount of profit that is paid to shareholders annually. Dividends can be specified in one of two ways:

- as the number of dollars each share receives
- as a percentage of the current price of the shares, called the **dividend yield** where:

$$\text{Dividend yield} = \frac{\text{dividend}}{\text{share price}} \times \frac{100}{1}\%$$

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Example 43 Dividends

Miller has 3000 shares in Alphabet Childcare Centres. The current market price of the shares is $3.50 each and the company has recently paid a dividend of 40 cents per share.

a Calculate how much Miller receives in dividends in total.

b Calculate the percentage dividend yield for this share. Answer to one decimal place.

Solution

a Total dividend $= 3000 \times 0.40$

$= \$1200$

b Dividend yield $= \frac{0.40}{3.50} \times \frac{100}{1}\%$

$= 11.4\%$

Explanation

Total dividend
= number of shares × dividend per share

Use the percentage dividend rule and substitute $0.40 for the share dividend and $3.50 for the share price.

Investors are not only interested in the amount of profit they are entitled to, they also want to interpret this profit in light of the amount that they have invested in shares. One measure of this is the **price-to-earnings ratio** of the shares.

Price-to-earnings ratio

$$\text{Price-to-earnings ratio} = \frac{\text{market price per share}}{\text{annual earnings per share}}$$

Earnings per share is a measure of companies profitability, but it is not the same as profit or dividend. The lower the price-to-earnings ratio the less you are investing for each dollar of profit, which is better for the investor.

Example 44 Price-to-earnings ratio

Share in Company A have a market price of $20, and an annual earnings per share of $1.85, while shares in Company B have a market price of $50, and an annual earnings per share of $3.30.

a Determine the price-to-earnings ratio for each company over that time period.

b Determine which shares have been a better investment, and why.

Solution

a A: price-to-earnings ratio $= \frac{20}{1.85} = 10.8$

B: price-to-earnings ratio

$= \frac{50}{3.30} = 15.2$ to one decimal place

b Company A, because the price-to-earnings ratio is lower.

Explanation

Substitute in the formula above.

Compare the ratios.

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Section Summary

- A **dividend** is the amount of profit that is paid to shareholders annually.
- Dividends can be specified in one of two ways:
  - as the number of dollars each share receives
  - as a percentage of the current price of the shares, called the **dividend yield** where:

$$\text{Dividend yield} = \frac{\text{dividend}}{\text{share price}} \times \frac{100}{1}\%$$

- **Price-to-earnings ratio** $= \dfrac{\text{market price per share}}{\text{annual earnings per share}}$

Exercise 1K

Shares and dividends

Example 42

1 Nick owns 500 of the 100 000 shares available in the Lucky Insurance Company.

a Calculate the percentage of the company that Nick owns.

b If the company declares annual profit of $2 500 000, determine how much of this profit Nick is entitled to.

2 There are 50 000 shares available in the Get Rich Quick investment company, and Joe owns 4% of them.

a Calculate the number of shares Joe owns.

b If the company declares annual profit of $1.25 million dollars, determine how much Joe would be entitled to,

c Determine the number of shares would Joe have to buy in order for his annual earnings share to be $80 000.

Example 43

3 Taj has 500 shares in Bunyip Plumbing Supplies. The current market price of the shares is $4.60 each, and the company declares a dividend of 50 cents per share.

a Calculate how much Taj receives in dividends in total.

b Determine the percentage dividend yield for this share, correct to one decimal place.

4 Lacey has 2000 shares in the Yummy Chocolate Factory. The current market price of the shares is $12.50, and the company recently paid a percentage dividend yield of 10%.

a Determine how much dividend was paid per share.

b Calculate the amount Lacey receives in dividends in total.

5 Dale has 3000 shares in the Bonza Bank. The current market price of the shares is $34.60 each, and the company has recently declared a dividend of 80 cents per share.

a Calculate how much Dale receives in dividends in total.

b Determine the percentage dividend yield for this share, correct to one decimal place.

SF

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SF

6 Scarlet owns 400 shares in the Fantastic Gold Mine. The current market price of the shares is $4.80, and the company recently paid a percentage dividend yield of 5%.

- **a** Determine how much dividend was paid per share.
- **b** Calculate the amount Scarlet receives in dividends in total.

Price to earnings ratio

Example 44

7 Suppose shares in Company A have a market value of $42.50, and the company has annual earnings of $4.85 per share, while shares in Company B have a market value of $8, and have annual earnings per share of $0.80 for the same 12-month period.

- **a** Calculate the price-to-earnings ratio for each company for that time period, correct to one decimal place.
- **b** Determine which shares have been a better investment.

CF

8 After their last financial report the Bank of Brizbane had a price-to-earnings ratio of 20. If the annual earnings declared were $10 000 000, and there are 500 000 shares, calculate the current price of the shares.

9 Nitika owns 5000 shares in Company A and 3000 shares in Company B.

- **a** The price-to-earnings ratio for Company A is 5.5 and the annual earnings per share is $2.20.
  - **i** Calculate how much Nitika received in her dividend from Company A.
  - **ii** Calculate the share price for company A.
- **b** The price-to-earnings ratio for Company B is 8 and the annual earnings per share is also $2.20.
  - **i** Calculate how much Nitika received in her dividend from Company B.
  - **ii** Calculate the share price for company B.

10 Michael has $5000 to invest in shares. He has decided to invest in either the Alpha Oil Company or Omega Mining.

- **a** The price-to-earnings ratio for the Alpha Oil Company is 10. If the share price is $5.00, calculate the earnings per share.
- **b** The price-to-earnings ratio for Omega Mining is also 10. If the share price is $10.00, calculate the earnings per share.
- **c** Michael decides to spend his money equally between the two share investments. Calculate how many shares in each company he buys.
- **d** Suppose that in the next 12 months the share price of:
  - Alpha Oil is expected to increase by 10%, while the price-to-earnings ratio is expected to remain at 10.
  - Omega Mining is expected to increase by 8%, while the price-to-earnings ratio is expected to reduce to 8.

  Calculate the expected increase in Michael's annual earnings from these changes.

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Key ideas and chapter summary

Rates

A **rate** refers to one quantity divided by another quantity, where the two quantities do not have the same unit of measurement. Because a rate compares different types of quantities the units must be shown.

Percentages

A **percentage** is a fraction or a proportion expressed as part of one hundred.

Salary and wages

A **salary** is payment for a year's work, generally paid as equal weekly, fortnightly or monthly payments.
A **wage** describes payment for work calculated on an hourly basis, with the amount earned dependent on the number of hours actually worked.

Overtime, penalty rates and allowances

Overtime is when employees work beyond the normal working day.
A **penalty rate** is paid for working weekends, public holidays, late night shifts or early morning shifts.
Allowances are extra payments for use of own tools, or for working in unpleasant or dangerous conditions.

Commission

Commission is a percentage of the value of the goods or services sold.

Piecework

Piecework is when a worker is paid a fixed payment, called a **piece rate**, for each unit produced or action completed.

Youth Allowance

Youth Allowance may be paid by the government to 18–24 years olds studying full-time, 16–24 years olds undertaking a full-time Australian Apprenticeship, or 16–20 years old and looking for full-time work.

Disability Support Pension

A **Disability Support Pension** is paid to someone who has a permanent and diagnosed disability or medical condition that stops them from working.

Austudy

Austudy is an allowance paid to someone who is aged 25 years or older and studying full-time or undertaking a full-time Australian Apprenticeship or traineeship.

Budgeting

Budgeting involves balancing income and expenditure over a specified time frame, such as a week or month.

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Review

Unit cost method

The **unit cost method** is used to compare the cost of items using cost per unit. The choice of unit may be a unit of weight (such as kilogram), or length (such a metre) or another measure depending on the particular item.

Percentage increase or decrease

Percentage increase or decrease is the amount of the increase or decrease of value or quantity expressed as a percentage of the original value or quantity.

GST

GST (goods and services tax) is a 10% tax that is added to most purchases.

Profit and loss

Profit is the difference between income (e.g. sales revenue) and cost (e.g. cost or purchase or production). When income is less than cost, then this becomes a loss.

Simple interest

Simple interest is interest paid on an investment or loan on the basis of the original amount invested or borrowed. The amount of simple interest is constant from year to year.

Compound interest

Under **compound interest**, the interest paid on a loan or investment is credited or debited to the account at the end of each period. The interest earned for the next period is based on the principal plus previous interest earned. The amount of interest earned increases each year.

Inflation

Inflation is the increase over time in cost and prices, usually expressed as a percentage increase per year.

Currency and exchange rates

Exchange rates allow you to convert the cost of items in another currency to the cost in Australian dollars, and vice-versa.

Shares

A **share** is a unit of ownership of a company.

Dividend

A **dividend** is the amount paid to shareholders annually.

Dividend yield

Dividend yield is the dividend paid by a company expressed as a percentage of the share price.

Price-to-earnings ratio

The **price-to-earnings ratio** is a measure of the profit of a company, given by the market share price/earnings per share. A lower value of the price-to-earnings ratio may indicate a better investment.

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Skills checklist

Check-list

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills.

1A	1	I can find quantities involving rates.	☐
		See Example 1 and Exercise 1A Question 1.	
1A	2	I can convert fractions to percentages.	☐
		See Example 2 and Exercise 1A Question 2.	
1A	3	I can convert decimals to percentages.	☐
		See Example 3 and Exercise 1A Question 2.	
1A	4	I can convert a percentage to a fraction.	☐
		See Example 4 and Exercise 1A Question 3.	
1A	5	I can convert a percentage to a decimal.	☐
		See Example 5 and Exercise 1A Question 3.	
1A	6	I can find the percentage of a quantity.	☐
		See Example 6 and Exercise 1A Question 4.	
1A	7	I can express a quantity as a percentage of another quantity.	☐
		See Example 7 and Exercise 1A Question 5.	
1B	8	I can calculate payments from a salary.	☐
		See Example 8 and Exercise 1B Question 1.	
1B	9	I can calculate wages from an hourly rate.	☐
		See Example 9 and Exercise 1B Question 7.	
1B	10	I can calculate wages including overtime.	☐
		See Example 10 and Exercise 1B Question 12.	
1B	11	I can calculate wages including penalty rates.	☐
		See Example 11 and Exercise 1B Question 18.	
1B	12	I can calculate wages including allowances.	☐
		See Example 12 and Exercise 1B Question 23.	

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Review

1B	13	I can calculate pay including commission.

See Examples 13 and 14, and Exercise 1B Questions 27 and 29.

1B	14	I can calculate pay based on piecework.

See Example 15 and Exercise 1B Question 37.

1C	15	I can determine income based on Youth Allowance.

See Example 16 and Exercise 1C Question 1.

1C	16	I can determine income based on a Disability Support Pension.

See Example 17 and Exercise 1C Question 4.

1D	17	I can determine income based on Austudy.

See Example 18 and Exercise 1C Question 6.

1D	18	I can balance a budget.

See Example 19 and Exercise 1D Question 4.

1D	19	I can create a budget.

See Example 20 and Exercise 1D Question 6.

1E	20	I can compare prices using the unit cost method.

See Examples 21 and 22, and Exercise 1E Questions 1 and 5.

1F	21	I can calculate the amount of a percentage discount and the new price.

See Example 23 and Exercise 1F Question 3.

1F	22	I can calculate the amount of a percentage increase and the new price.

See Example 24 and Exercise 1F Question 4.

1F	23	I can calculate percentage increase and decrease.

See Example 25 and Exercise 1F Question 6.

1F	24	I can calculate the original price from a discounted price.

See Example 26 and Exercise 1F Question 7.

1F	25	I can calculate GST.

See Example 27 and Exercise 1F Question 11.

1F	26	I can calculate profit and loss.

See Example 28 and Exercise 1F Question 16.

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1G	27	I can calculate simple interest.	☐

See Examples 29 and 30, and Exercise 1G Questions 1 and 5.

1G	28	I can solve for any unknown in the simple interest formula.	☐

See Examples 32, 33 and 34, and Exercise 1G Questions 9, 11 and 13.

1H	29	I can apply the compound interest formula.	☐

See Examples 35 and 36, and Exercise 1H Questions 9, 11 and 13.

1H	30	I can apply the compound interest formula with compounding periods other than one year.	☐

See Example 37 and Exercise 1H Question 10.

1I	31	I can calculate the effect of inflation on prices year by year.	☐

See Example 38 and Exercise 1I Question 1.

1I	32	I can determine the effect of inflation on prices over a long period.	☐

See Example 39 and Exercise 1I Question 4.

1I	33	I can determine the effect of inflation on purchasing power.	☐

See Example 40 and Exercise 1I Question 7.

1J	34	I can apply currency exchange rates.	☐

See Example 41 and Exercise 1J Question 1.

1K	35	I can determine the dividend paid on shares.	☐

See Example 43 and Exercise 1K Question 1.

1K	36	I can apply price-to-earnings ratio.	☐

See Example 44 and Exercise 1K Question 7.

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Multiple-choice questions

1 The amount saved when a 10% discount is offered on an item marked $120 is:

A $20 **B** $12 **C** $1.20 **D** $10

2 If a 20% discount is offered on an item marked $30, the new discounted price of the item is:

A $10 **B** $24 **C** $6 **D** $28

3 If a 15% increase is applied to an item marked $60, the new price of the item is:

A $69 **B** $9 **C** $75 **D** $70

4 If GST is applied to an item marked $121.50, the new price of the item is:

A $111.50 **B** $12.15 **C** $133.65 **D** $110.45

5 The telephone bill including GST is $318.97. The price before GST was:

A $289.97 **B** $31.90 **C** $350.87 **D** $29.00

6 Shares in Company A have a market value of $22.50. If the company makes a 12-month earnings of $2.85 per share, the price-to-earnings ratio for that time period is:

A 12.67 **B** 0.13 **C** 7.89 **D** $25.35

7 Calculate much interest is earned if $2000 is invested for 1 year at a simple interest rate of 4% per annum.

A $2080 **B** $80 **C** $8 **D** $800

8 The total value of an investment of $1000 after 3 years if simple interest is paid at the rate of 5.5% per annum is:

A $1055 **B** $1165 **C** $1174.24 **D** $3165

9 Determine the interest rate, per annum, for a deposit of $1500 if it earns interest of $50 over a period of 6 months.

A 5.45% **B** 5.55% **C** 6.45% **D** 6.67%

10 $2400 is invested at a rate of 4.25% compound interest, paid annually. The value of this investment after 6 years is closest to:

A $3081 **B** $3074 **C** $3012 **D** $680

11 $3600 is invested at a rate of 5% p.a. compounding annually. The value of the investment after 4 years is given by:

A $3600(1 + (5/100)^4)$ **B** $3600(1 + 5/100)^4$

C $3600(1 + 4 \times 0.05)$ **D** $3600 + 4 \times 3600 \times 0.05$

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12 An amount of $8500 is invested at 6% p.a. compounding annually. The percentage increase in this investment after five years is closest to:

A 25.3% **B** 26.2% **C** 30.0% **D** 33.8%

13 The cost of producing and selling a phone that sells for $1499 is $1099. The profit made on the phone as a percentage of the production cost is closest to:

A 400% **B** 36% **C** 27% **D** 73%

The following information relates to Questions 14 *and* 15.

Janet buys a car costing $23 000. She pays a $5000 deposit and then makes payments of $440 per month for the next 5 years.

14 Determine how many payments Janet makes under this arrangement.

A 12 **B** 20 **C** 40 **D** 60

15 Determine much interest Janet pays under this arrangement.

A $3400 **B** $8400 **C** $3120 **D** $1600

16 Alyssia receives an annual salary of $85 640. Calculate how much she receives each fortnight.

A $3293.84 **B** $3293.85 **C** $1646.92 **D** $1646.93

17 Christopher receives a normal hourly rate of $22.60 per hour. Calculate his pay when he works 8 hours at a normal rate and 3 hours at time-and-a-half.

A $180.80 **B** $248.60 **C** $282.50 **D** $296.60

18 Bonnie is employed on a casual basis. She earns $15 per hour normally, with time-and-a-half for Sundays. Last week Bonnie worked from 2 p.m. until 6 p.m. on Monday, Tuesday and Wednesday, and from 12 noon until 5 p.m. on Sunday. Determine how much she earned last week.

A $180 **B** $270 **C** $255 **D** $292.50

19 Taylah earns a weekly retainer of $425 plus commission of 8% on sales. Calculate her weekly earnings when she makes sales of $8620.

A $1114.60 **B** $459.00 **C** $493.96 **D** $689.60

20 Ahmet is a carpet layer and charges $37.50 per square metre of carpet laid. Calculate how much he will earn for laying carpet in a room that is 9 square metres.

A $337.50 **B** $46.50 **C** $112.50 **D** $225.00

21 Three different brands of gift wrap are sold in a store. Brand A contains 10 metres of gift wrap and costs $12, brand B contains 12 metres of gift wrap and costs $15, and brand C contains 20 metres of gift wrap and costs $20. Putting the brands in order from cheapest to most expensive per metre we get:

A A, B, C **B** B, A, C **C** C, B, A **D** C, A, B

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Review

22 On a particular day, the exchange rate between the Australian dollar and the Thai baht (THB) is 26.305. $550 Australian dollars converted to Thai baht would be:

A 20.91 baht **B** 26.31 baht **C** 14 467.75 baht **D** 576.31 baht

23 Mikki is planning a holiday to Bali. She has found some accommodation that will cost her 350 000 Indonesian rupiah (IDR) per night, and she intends to stay for 7 nights. If the exchange rate between the Australian dollar and Indonesian rupiah is 8863.12, then her total accommodation cost in Australian dollars is:

A $34.49 **B** $276.43 **C** $340.49 **D** $564.14

24 Adam has the following bills: electricity $250 per quarter, phone $70 per month, petrol $1200 per year and rent $300 per week. Determine the total amount Adam should budget for the year.

A $720 **B** $1553 **C** $6640 **D** $18 640

The following information relates to Questions 25 *and* 26.

Thomas wants to save up $10 000 for a new car. He earns $850 per week, and has expenses as shown in the table:

Electricity	$280 per quarter
Food	$185 per week
Rent	$1000 per month
Travel	$50 per week

25 Thomas's annual expenses are:

A $18 180 **B** $18 860 **C** $25 060 **D** $25 340

26 If Thomas puts all the rest of his money into savings, determine how many weeks it will take him to save for the car.

A 12 weeks **B** 21 weeks **C** 26 weeks **D** 28 weeks

27 Jason has 1000 shares in a golf retailer. The current market price of the shares is $15.29, and the company recently paid a percentage dividend yield of 3%. Determine how much dividend he received.

A $458.70 **B** $509.67 **C** $1.96 **D** $0.46

28 Suppose shares in a company have a market value of $33.50, and the company has annual earnings of $1.47 per share. Determine the price-to-earnings ratio for the company.

A 4.39% **B** 32.1% **C** 25.7% **D** 22.8%

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SF

Short-response questions

1 Jake earns $96 470.40 per annum and works an average of 48 hours per week.
   a Determine his weekly salary.
   b Determine the hourly rate of pay that this would equate to.

2 Alex works for a fast-food company and is paid $13.50 per hour for a 35-hour week. He gets time-and-a-half pay for overtime worked on the weekdays and double time for working on weekends. Last week he worked a normal 35-hour week plus three hours of overtime during the week and four hours of overtime on the weekend. Calculate his wage last week.

3 Carlo's employer has decided to reward all employees with a bonus. The bonus awarded is 6.25% of their annual salary. Calculate Carlo's bonus if his annual salary is $85 940.

4 Patrick is a comedian who makes $120 for a short performance and $260 for a long performance. Calculate how much he earns if he completes 11 short and 12 long performances.

5 After Easter, Rabbit Easter eggs were reduced from $2.99 to $2.37 and Bilby Easter eggs were reduced from $4.79 to $3.83. Determine which type of Easter egg had the larger percentage reduction, and by how much.

6 After Christmas, all stock in JDs was discounted by 20%. The sale price of a pair of cross trainers is now $110.
   a Calculate the original marked price.
   b If the cost of production of the cross trainers is $120, calculate the percentage profit or loss before and after they were discounted, correct to one decimal place.

7 Farad takes out a loan of $25 000, for which he has to make repayments of $600 per month for 5 years.
   a Calculate how much he will repay in total.
   b Calculate how much of the money repaid is interest.

8 William works as a builder. His annual union fees are $278.20. William has his union fees deducted from his weekly pay. Calculate William's weekly union deduction.

9 Quan received a gross fortnightly salary of $2968. His pay deductions were $765.60 for income tax, $345.15 for superannuation and $23.40 for union fees.
   a Calculate his fortnightly net pay.
   b Determine the percentage of his gross income that was deducted for income tax. (Answer correct to one decimal place.)

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Review

10 Joel is a carpet layer and is paid $16 per square metre to lay carpet. Calculate how much he will earn for laying carpet in a house with an area of 32 square metres. SF

11 Daniel has a gross monthly wage of $3640. He has the following deductions taken from his pay: $764 for income tax, $71.65 for superannuation and $23.23 for union membership. Calculate Daniel's net pay.

12 Hannah has budgeted $210 per week for groceries, $70 per week for leisure, $23 per fortnight for medical expenses and $90 per week to run a car. Calculate her monthly expenses, assuming 4 weeks in a month.

13 Amelie earns $90 345 annually. She has budgeted 30% of her salary for a loan repayment. Calculate how much she should put aside for her loan repayment each fortnight.

14 On a particular day, one Australian dollar (AUD) can be exchanged for 0.7562 United States dollars (USD).

a Calculate the equivalent amount of USD for $350.00 AUD.

b A tourist from the US is visiting Australia. A tour to Phillip Island will cost $140.00 USD per person. Calculate the cost in AUD.

15 An online shop sells computer equipment and lists the prices of items in Australian dollars, US dollars and British pounds (GBP).
One Australian dollar exchanges for $0.842 USD and £0.53 GBP on a particular day.
If a hard drive is listed with a price of $125.60 AUD, calculate the price for a customer in:

a the US

b the UK.

16 Angelo has 1000 shares in Capital Investments. The current price of the shares is $22.80, and the company declares a dividend of $1.10 per share.

a Calculate how much Angelo receives in dividends in total.

b Determine the percentage dividend yield for this share, correct to one decimal place.

17 Lucy works a 37-hour week for a company. The company provides all employees with a 17% holiday loading on four weeks normal wages. She is paid a normal hourly rate of $32.40. Calculate how much Lucy will receive in holiday loading. CF

18 Suppose that the maximum Youth Allowance is reduced by 20 cents for every dollar your parents combined income exceeds $55 624. Determine by how much Hannah's youth allowance will be reduced if her parents earn a combined income of $62 634.

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CF

19 **a** The wholesale price of a digital camera is $350. The maximum profit that a retailer is allowed to make when selling this particular camera is 75% of the wholesale price. Calculate the maximum retail price of the camera.

b Suppose that the wholesale price of the camera increases at 1.3% per annum each year.

i Calculate by how much the wholesale price will have increased at the end of 5 years.

ii Determine the new retail price of the camera (with 75% profit).

iii Determine the percentage increase this is on the retail price determined in part **a**.

20 Chelsea is a real estate agent and charges the following commission for selling the property: 3% on the first $45 000, then 2% for the next $90 000 and 1.5% thereafter. Calculate Chelsea's commission if she sells a property for $240 000.

21 A bank offers a loan with a compound interest rate of 3.5% per annum, compounding monthly. Determine the percentage increase in a loan of $50 000 after 3 years.

22 Gavin finds $10 000 that his grandfather had hidden in the attic 20 years ago. If the average annual inflation rate over that time period was 2%, determine the loss in purchasing power of that money over that 20-year period.

23 Lou has $50 000 to invest for 2 years.

a She can put the money in an investment account account paying 3.6% per annum, compounding monthly. Determine the interest earned under this plan after 2 years.

b Lou's sister would like her to invest the money in her new business. She offers Lou 3.65% simple interest for 2 years. Determine the interest earned under this plan after 2 years.

c What rate of simple interest should Lou's sister pay her to ensure she earns as much interest from lending the money to her as she would receive from the bank after 2 years? Give your answer as a percentage rounded to two decimal places.

CU

24 Phil purchases a new car, costing $53 190, with no deposit. He is required to make repayments of $1235 per calendar month for five years to pay off the loan. If he travels 15 000 km per year in the car, he must allow for petrol ($80 per week), tyres (one set during the five-year period, which will cost $908), servicing (one service every 6 months costing $320 each time), and insurance and registration ($2400 per year).

a Calculate how much Phil spends on the car over the five-year period.

b If Phil sells the car at the end of the five-year period for $28 500, calculate how much it will have cost him in total.

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c In order to help pay his weekly car costs (from part **a**), Phil takes on some additional overtime at work. His normal pay rate is $32.50 per hour and he earns time-and-a-half for overtime. Assume he can work overtime for 48 weeks each year.

i Calculate how many hours overtime per week he should work to pay for the car. Give your answer rounded to one decimal place.

ii Calculate how many hours overtime per week he should work to earn the amount he pays for the car, given that he pays 30% income tax on the overtime payment. Give your answer rounded to one decimal place.

25 A truck is for sale at a price of $36 000. The dealer is offering finance terms of a 20% cash payment and repayments of $276 per week for 3 years, which includes simple interest and payment for the truck. Assume there are 52 weeks in the year.

a Calculate the total cost of the truck if bought on the dealer's finance terms.

b Calculate the total interest paid.

c Calculate the simple interest rate for the loan, correct to one decimal place.

d Assume you have enough cash to pay 20% of the price of the truck. Suppose you borrow the difference from a bank that offers a loan with interest compounded monthly, and allows you to pay off the entire balance and interest in one lump sum after 3 years. Determine the maximum annual compound interest rate for the lump sum repayment to be less than the total repayments to the dealer as calculated in part **b**.

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Chapter 2

Shape and measurement

Chapter questions

UNIT 1 MONEY, MEASUREMENT AND RELATIONS

Topic 2: Shape and measurement

- What is Pythagoras' theorem?
- How do we use Pythagoras' theorem?
- How do we convert units of measurement?
- How do we calculate the perimeter of a shape?
- How do we calculate the area of a shape?
- What is a composite shape?
- How do we calculate the volume of a shape?
- How do we calculate the surface area of a shape?
- How do we apply Pythagoras' theorem, perimeter, area, volume and surface area in practical situations?
- How do we solve practical problems involving shape and measurement?

This geometry chapter covers perimeter and area of two-dimensional shapes, and surface area and volume of three-dimensional solids.

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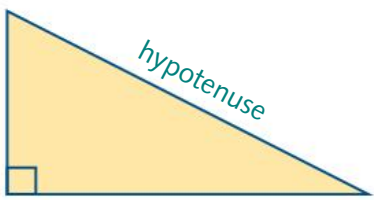
2A Pythagoras' theorem

Learning intentions

- To review Pythagoras' theorem.
- To show understanding of how Pythagoras' theorem relates the side lengths of a right-angled triangle.
- To be able to use Pythagoras' theorem to solve for the length of any unknown side of a right-angled triangle.
- To be able to use Pythagoras' theorem to solve practical problems in two dimensions.

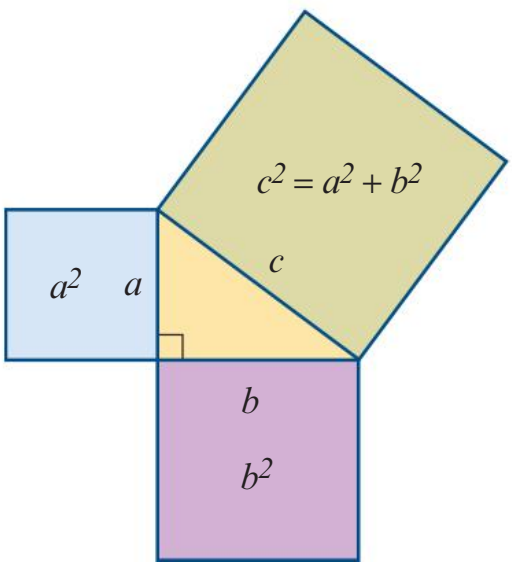
Pythagoras' theorem is a relationship connecting the side lengths of a right-angled triangle. In a right-angled triangle, the side opposite the **right angle** is called the **hypotenuse**. The hypotenuse is always the longest side of a right-angled triangle.



Pythagoras' theorem

Pythagoras' theorem states that the square of the hypotenuse (c) of any right-angled triangle equals the sum of the squares of the lengths of the other two sides (a and b).

$$c^2 = a^2 + b^2$$



Pythagoras' theorem can be used to calculate the length of the hypotenuse in a right-angled triangle.

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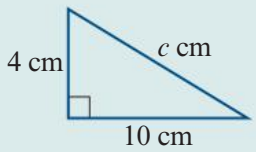
Example 1 Using Pythagoras' theorem to calculate the length of the hypotenuse

Calculate the length of the hypotenuse in the triangle shown, correct to two decimal places.



Solution

$c^2 = a^2 + b^2$

$c^2 = 4^2 + 10^2$

$c = \sqrt{4^2 + 10^2}$

$= 10.770\ldots$

The length of the hypotenuse is 10.77 cm, correct to two decimal places.

Explanation

Write Pythagoras' theorem.

Substitute known values.

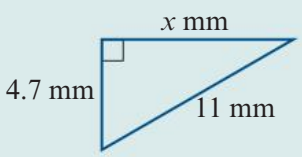
Take the square root of both sides, then evaluate.

Write your answer correct to two decimal places, with correct units.

Pythagoras' theorem can also be rearranged to calculate the lengths of the sides other than the hypotenuse.

Example 2 Using Pythagoras' theorem to calculate the length of an unknown side

Determine the length of the unknown side, x, in the triangle shown. Give the answer correct to one decimal place.



Solution

$a^2 + b^2 = c^2$

$x^2 + 4.7^2 = 11^2$

$x = \sqrt{11^2 - 4.7^2}$

$= 9.945\ldots$

The length of x is 9.9 mm, correct to one decimal place.

Explanation

Write Pythagoras' theorem.

Substitute known values.

Rearrange the formula to make x the subject, then evaluate.

Write your answer correct to one decimal place, with correct units.

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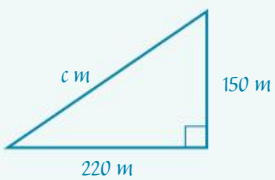
Pythagoras' theorem can be used to solve many practical problems.

Example 3 Using Pythagoras' theorem to solve a practical problem

A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 220 m from a landing pad. Calculate the direct distance of the helicopter from the landing pad, correct to two decimal places.

Solution



$c^2 = a^2 + b^2$

$c^2 = 150^2 + 220^2$

$c = \sqrt{150^2 + 220^2}$

$= 266.270\ldots$

The helicopter is 266.27 m from the landing pad, correct to two decimal places.

Explanation

Draw and label a diagram to show the unknown distance being calculated.

Write Pythagoras' theorem.

Substitute known values.

Take the square root of both sides, then evaluate.

Write your answer correct to two decimal places, with correct units.

Section Summary

- Pythagoras' theorem relates the sides of any right-angled triangle using the rule $c^2 = a^2 + b^2$ where c = the length of the hypotenuse and a and b are the lengths of the two perpendicular sides.
- Pythagoras' theorem can be used to solve for an unknown side of any right-angled triangle.
- Pythagoras' theorem can be used to solve practical problems involving right-angled triangles.

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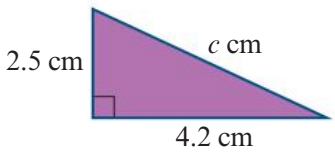
Exercise 2A

SF

1 Determine the length of the unknown side in each of the triangles shown, giving the answer correct to one decimal place.

Example 1
Example 2

a



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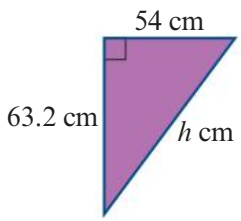

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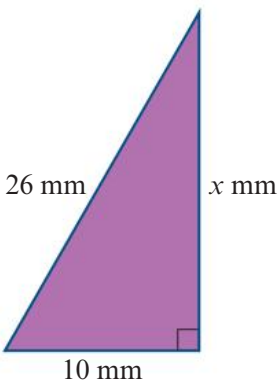

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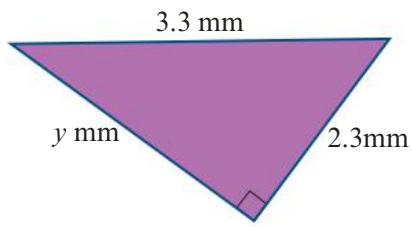

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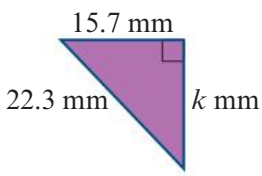

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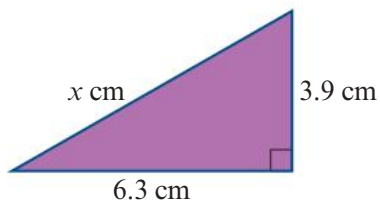

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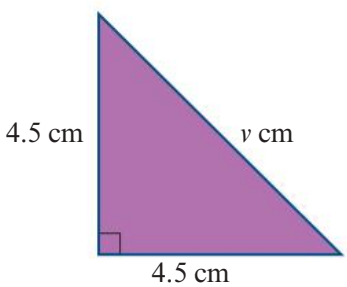

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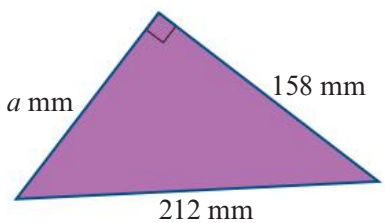

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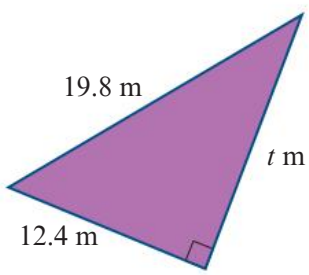

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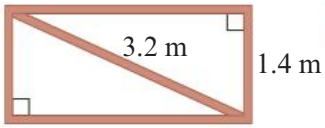
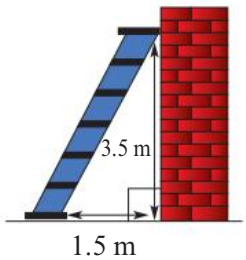
Applications of Pythagoras' theorem

Example 3

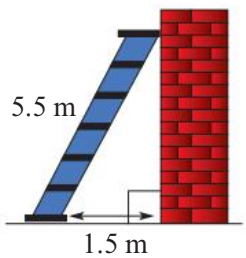
2 A farm gate that is 1.4 m high is supported by a diagonal bar of length 3.2 m. Calculate the width of the gate, correct to one decimal place.



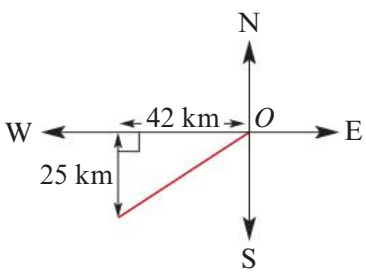
3 A ladder rests against a brick wall as shown in the diagram on the right. The base of the ladder is 1.5 m from the wall, and the top reaches 3.5 m up the wall. Calculate the length of the ladder, correct to one decimal place.



4 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. If the ladder is 5.5 m long, determine how high the top of the ladder is from the ground, correct to one decimal place.



5 A ship sails 42 km due west and then 25 km due south. Calculate how far the ship is from its starting point. Give the answer correct to two decimal places.



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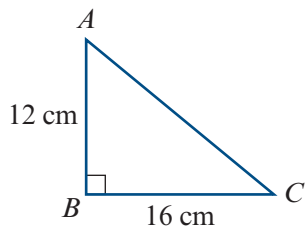
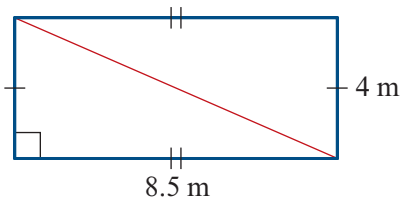
6 In triangle ABC, there is a right angle at B. AB is 12 cm and BC is 16 cm. Determine the length of AC.



7 Calculate the length of the diagonal of a rectangle with dimensions 8.5 m by 4 m. Give the answer correct to one decimal place.



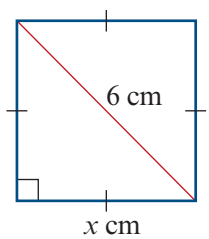
8 A rectangular block of land measures 28 m by 55 m. John wants to put a fence along the diagonal. Determine the length of this fence. Give the answer correct to three decimal places.

9 A yacht sails 12 km due east and then 9 km due north. Determine the distance between the ship and its starting point.

10 A hiker walks 10 km due west and then 8 km due north. Calculate how far the hiker is from their starting point. Give the answer correct to two decimal places.

11 A zip line starts from 25 m off the ground in a tower at a school camp. If the students are told that the zip line is 103 m long, calculate the distance between where the zip line connects to the ground and the base of the tower. Give the answer to the nearest metre.

12 A square has diagonals of length 6 cm. Determine the length of its sides, correct to two decimal places.



13 A farmer would like to divide his paddock in half by getting a fence installed along the diagonal. His paddock measures 25 m by 30 m. He has two offers. Great Fencing will install the fence at $44 per metre. Ultimate Farmers will install the fence for a flat fee of $1500. Identify which company provides the best value for money. Justify your answer using mathematical reasoning.

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2B Pythagoras' theorem in three dimensions

Learning intentions

- To be able to recognise right-angled triangles in solid figures.
- To be able to use Pythagoras' theorem to calculate unknown lengths of right-angled triangles in three dimensions.
- To be able to solve simple applications of Pythagoras' theorem in three dimensions.

When solving three-dimensional problems, it is essential to draw careful diagrams.

In general, to determine lengths in solid figures, we must first identify the correct right-angled triangle in the plane containing the unknown side. Remember, a plane is a flat surface, such as the cover of a book or a tabletop.

Once it has been identified, the right-angled triangle should be drawn separately from the solid figure, displaying as much information as possible.

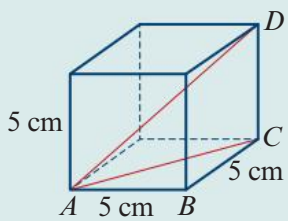
Example 4 Using Pythagoras' theorem in three dimensions

The cube in the diagram has sides of length 5 cm.

Calculate these lengths, correct to two decimal places:

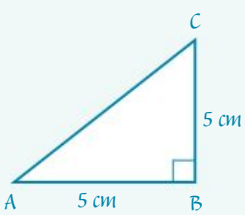
a AC

b AD.



Solution

a



$AC^2 = AB^2 + BC^2$

$\therefore AC = \sqrt{5^2 + 5^2}$

$= 7.071\ldots$

The length AC is 7.07 cm, correct to two decimal places.

Explanation

Locate the relevant right-angled triangle in the diagram.

Draw the right-angled triangle ABC that contains AC, and then label the known side lengths.

Using Pythagoras' theorem, calculate the length AC.

Write your answer with correct units and correct to two decimal places.

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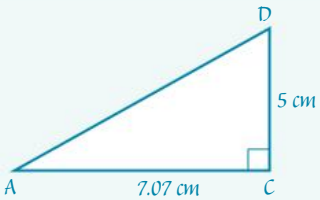
b



$AD^2 = AC^2 + CD^2$

$\therefore\ AD = \sqrt{50 + 5^2}$

$= 8.660\ldots$

The length AD is 8.66 cm, correct to two decimal places.

Locate the relevant right-angled triangle in the diagram.

Draw the right-angled triangle ACD that contains AD and label the known side lengths. (From part **a**, $AC^2 = 50\,\text{cm}^2$).

Using Pythagoras' theorem, calculate the length AD.

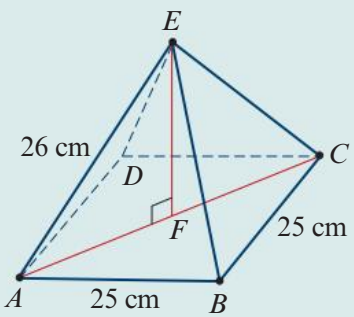
Write your answer with correct units and correct to two decimal places.

Example 5 Using Pythagoras' theorem in three-dimensional problems

For the square pyramid shown in the diagram, calculate:

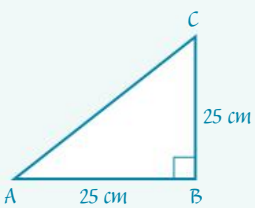
a the length AC, correct to two decimal places

b the height EF, correct to one decimal place.



Solution

a



$AC^2 = AB^2 + BC^2$

$\therefore\ AC = \sqrt{25^2 + 25^2}$

$= 35.355\ldots$

The length AC is 35.36 cm, correct to two decimal places.

Explanation

Locate the relevant right-angled triangle in the diagram.

Draw the right-angled triangle ABC that contains AC, and label the known side lengths.

Using Pythagoras' theorem, calculate the length AC.

Write your answer with correct units and correct to two decimal places.

Continued on next page

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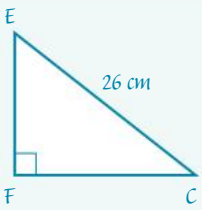
b



Locate the relevant right-angled triangle in the diagram.

Draw the right-angled triangle EFC that contains EF, and label the known side lengths.

$$FC = \frac{AC}{2}$$
$$= \frac{35.355\ldots}{2}$$
$$= 17.677\ldots$$

Find FC, which is half of AC. Use the unrounded value of AC calculated in part **a.**

$$EF^2 = EC^2 - FC^2$$
$$\therefore EF = \sqrt{26^2 - 17.677^2\ldots}$$
$$= 19.066\ldots$$

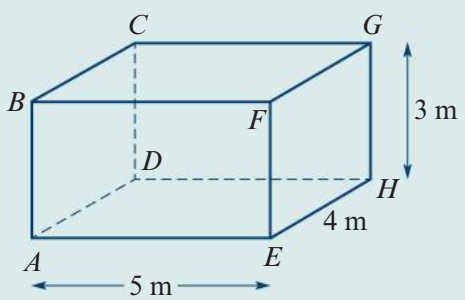
Using Pythagoras' theorem, calculate EF.

The height EF is 19.1 cm, correct to one decimal place.

Write your answer with correct units and correct to one decimal place.

Example 6 Using Pythagoras' theorem in practical three-dimensional problems

A new home entertainment system needs to be set up in a room with dimensions 4 m × 5 m × 3 m as shown in the diagram. Expensive cabling is used to wire the room from corner A to corner G.



a Determine the length of cabling required to go from A to E to H to G.

b Calculate the length of cabling required to go from A to F to G, correct to two decimal places.

c If cabling costs $9.10 per metre, calculate the cost of cabling for each option and determine which is the cheaper option – A to F to G or A to E to G.

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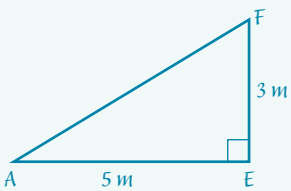
Solution	Explanation
a $5 + 4 + 3 = 12$	Add the distances from A to E (5 m), E to H (4 m) and H to G (3 m).
The distance is 12 metres.	Write your answer with correct units.
b The triangle is AFE.	First find out the distance from A to F, by locating the relevant right-angled triangle in the diagram.
	Draw the right-angled triangle AFE that contains AF, and then label the known side lengths.
$AF^2 = AE^2 + EF^2$ $AF^2 = \sqrt{5^2 + 3^2}$ $AF = 5.8309\ldots$	Using Pythagoras' theorem, calculate the length AF.
The total length $(A\text{-}F\text{-}G)$ $= 5.8309\ldots + 4$ $= 9.8309\ldots$	Add the lengths AF and FG.
The distance is 9.83 metres, correct to two decimal places.	Write your final answer with correct units and correct to two decimal places.

Continued on next page

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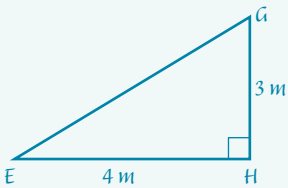
Solution	Explanation
c $9.8309\ldots \times 9.10 = 89.461$ Cost of cabling from A to F to G is \$89.46.	Calculate the cost of cabling from A to F to G by multiplying the length of cabling needed from A to F to G (9.8309 . . . m) by the cost of cabling per metre (\$9.10).
$AE = 5$ m	Calculate the distance of cable needed to go from A to E to G. The distance from A to E is 5 m. Calculate the distance from E to G by first locating the relevant right-angled triangle in the diagram.
The triangle is EGH.  	Draw the right-angled triangle EGH that contains EG and mark in known side lengths.
$EG^2 = EH^2 + HG^2$ $EG = \sqrt{4^2 + 3^2}$ $EG = 5$	Using Pythagoras' theorem, calculate the length EG.
$A\text{-}E\text{-}G = 5 + 5$ $= 10$ m	Add the lengths AE and EG to give total distance from A to E to G.
$10 \times 9.10 = 91$ Cost of cabling from A to E to G is \$91.00.	Calculate the cost of cabling from A to E to G by multiplying the length of cabling needed (10 m) by the cost of cabling per metre (\$9.10).
Cost for $A\text{-}F\text{-}G$ = \$89.46 Cost for $A\text{-}E\text{-}G$ = \$91.00 Thus the cheaper option is to wire cabling from A to F to G.	Compare the cost of cabling from A to F to G to the cost of cabling from A to E to G and decide which is the cheaper option.

Section Summary

- Draw clearly labelled diagrams to help identify the right-angled triangles in any solid figures.
- Apply Pythagoras' theorem to solve for any unknown side lengths.
- Only round at the end. Use unrounded values during intermediate steps to increase the accuracy of the final answer.

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Exercise 2B

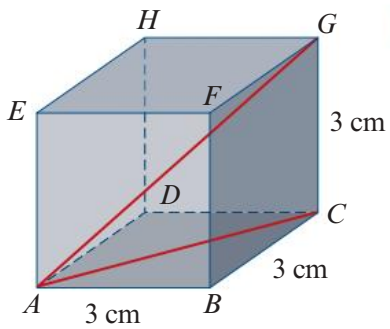
Pythagoras' theorem in three dimensions

Example 4

1 The cube shown in the diagram has sides of 3 cm. Determine these lengths, correct to three decimal places:

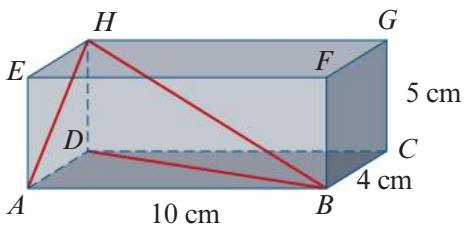
a AC

b AG



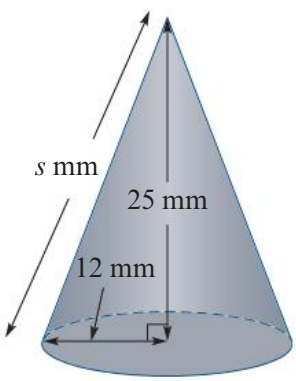
2 For this cuboid, calculate these lengths, correct to two decimal places:

a DB **b** BH **c** AH

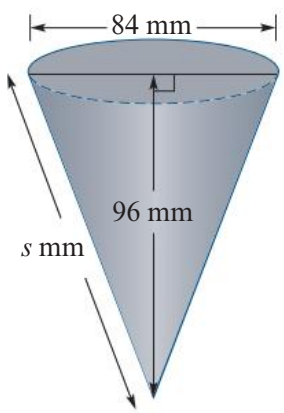


3 Calculate the slant height, s, of each of the following cones, correct to two decimal places.

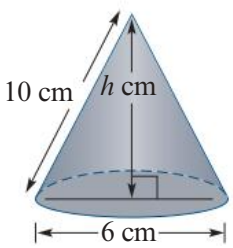
a



b



4 The slant height of this circular cone is 10 cm and the diameter of its base is 6 cm. Calculate the height of the cone, correct to two decimal places.



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SF

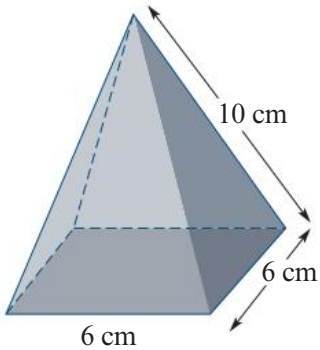
Example 5

5 For each of the following square-based pyramids, calculate, correct to one decimal place:

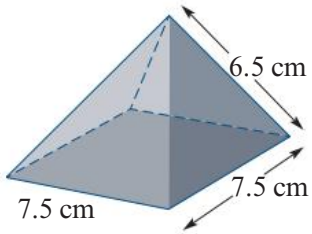
i the length of the diagonal in the base

ii the height of the pyramid.

a



b



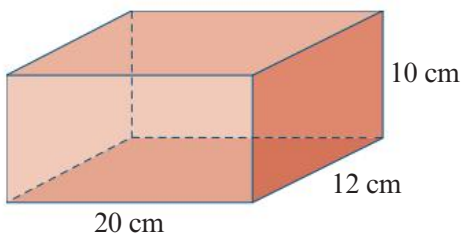
Applications of Pythagoras' theorem in three dimensions

Example 6

6 Determine the length of the longest pencil that will fit inside a cylinder of height 15 cm and with circular base 8 cm in diameter.

7 Sarah wants to put her pencils in a cylindrical pencil case. Determine the length of the longest pencil that would fit inside a cylinder of height 12 cm with a base diameter of 5 cm.

8 Chris wants to use a rectangular pencil box. Determine the length of the longest pencil that would fit inside the box shown. Give the answer rounded to the nearest centimetre.



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9 A broom is 145 cm long. State whether it would be able to fit in a cupboard measuring 45 cm by 50 cm and height 140 cm. Justify your answer using mathematical reasoning. CF

10 In order to check the accuracy of the framework and that a room is 'square', a builder often measures the length of the opposing diagonals. Calculate the distance, correct to two decimal places, from the bottom corner to the top corner diagonally opposite of a room that measures 6 m by 4 m by 3.5 m.

Example 6

11 Sophia wants to decorate her cubby house in the backyard with fairy lights along the walls. Her cubby house is 8 m tall by 5 m wide by 5 m deep. Determine the shortest length of fairy lights she would need to buy so that a string of lights can go from a corner point on the floor to the corner on the ceiling diagonally opposite where she started. The lights must run along the walls and can only be attached at any of the eight corners of the cubby house. Justify your answer with mathematical reasoning.

12 A new home entertainment system is being installed in a room that is 4 m by 6 m with a height of 2.8 m. The cable required costs $8.50 per metre. The cabling must run along the walls and/or the ceiling and can only be attached at any of the eight corners of the room. Determine the cheapest cost to run cable from a corner point on the floor, along the wall and/or ceiling, to a corner on the ceiling diagonally opposite. Round your answer to the nearest 10 cents. Comment on the reasonableness of your answer. CU

13 In the primate enclosure at the zoo, a rope is attached from the bottom corner of the enclosure to the opposite top corner for the monkeys to swing and climb on. The zoo spent $250 on the length of rope at $14.25 per metre. If the cuboid shaped enclosure measures 8 m tall by 10 m wide, determine the depth of the enclosure. Round your answer to the nearest metre.

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2C Mensuration: perimeter and area

Learning intentions

- To review the concepts of perimeter and area.
- To review metric units of length and area and how to convert between these.
- To be able to calculate the perimeter and area of standard and composite shapes.
- To be able to apply the perimeter and area of standard two-dimensional objects to practical situations.

Mensuration is a part of mathematics that looks at the measurement of length, area and volume. It comes from the Latin word *mensura*, which means 'measure'.

Conversion of units

The modern **metric system** in Australia is defined by the International System of Units (SI), which is a system of measuring that has three main units.

The three main SI units of measurement

m	the *metre* for length
kg	the *kilogram* for mass
s	the *second* for time

Larger and smaller units are based on these by the addition of a prefix. When solving problems, we need to ensure that the units we use are the same. We may also need to **convert** our answer into specified units.

Conversion of units

To convert units, remember to:

- use multiplication (×) when you convert from a larger unit to a smaller unit
- use division (÷) when you convert from a smaller unit to a larger unit.

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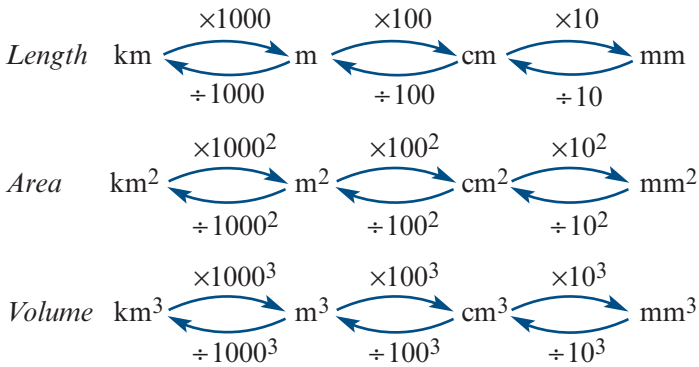
The common units used for measuring *length* are kilometres (km), metres (m), centimetres (cm) and millimetres (mm). The following chart is useful when converting units of length, and can be adapted to other metric units.



The common units for measuring *liquids* are kilolitres (kL), litres (L) and millilitres (mL).

1 kilolitre = 1000 litres

1 litre = 1000 millilitres

The common units for measuring *mass* are tonnes (t), kilograms (kg), grams (g) and milligrams (mg).

1 tonne = 1000 kilograms

1 kilogram = 1000 grams

1 gram = 1000 milligrams

Note: Strictly speaking the litre and tonne are not included in the SI, but are commonly used with SI units.

The following prefixes are useful to remember.

Prefix	Symbol	Definition	Decimal
micro	μ	millionth	0.000 001
milli	m	thousandth	0.001
centi	c	hundredth	0.01
deci	d	tenth	0.1
kilo	k	thousand	1000
mega	M	million	1 000 000
giga	G	billion	1 000 000 000

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Example 7 Converting between units

Convert these measurements into the units given in the brackets.

a 5.2 km (m) **b** 339 cm^2 (m^2) **c** 9.75 cm^3 (mm^3)

Solution	Explanation
a 5.2×1000 $= 5200$ m	As there are 1000 metres in a kilometre and we are converting from kilometres (km) to a smaller unit (m), we need to multiply 5.2 by 1000.
b $339 \div 100^2$ $= 0.0339$ m^2	As there are 100^2 square centimetres in a square metre and we are converting from square centimetres (cm^2) to a larger unit (m^2), we need to divide 339 by 100^2.
c 9.75×10^3 $= 9750$ mm^3	As there are 10^3 cubic millimetres in a cubic centimetre and we are converting from cubic centimetres (cm^3) to a smaller unit (mm^3), we need to multiply 9.75 by 10^3.

Sometimes a measurement conversion requires more than one step.

Example 8 Converting between units requiring more than one step

Convert these measurements into the units given in the brackets.

a 40 000 cm (km) **b** 0.000 22 km^2 (cm^2) **c** 0.08 m^3 (mm^3)

Solution	Explanation
a $40\,000 \div 100\,000$ $= 0.4$ km	As there are 100 centimetres in a metre and 1000 metres in a kilometre and we are converting from centimetres (cm) to a larger unit (km), we need to divide 40 000 by (100×1000) 100 000.
b $0.000\,22 \times 100^2 \times 1000^2$ $= 2\,200\,000$ cm^2	As there are 100^2 square centimetres in a square metre and 1000^2 square metres in a square kilometre and we are converting from square kilometres (km^2) to a smaller unit (cm^2), we need to multiply 0.000 22 by $(100^2 \times 1000^2)$.

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c $0.08 \times 10^3 \times 100^3$ $= 80\,000\,000 \text{ mm}^3$	As there are 10^3 cubic millimetres in a cubic centimetre and 100^3 cubic centimetres in a cubic metre and we are converting from cubic metres (m^3) to a smaller unit (mm^3), we need to multiply 0.08 by ($10^3 \times 100^3$).

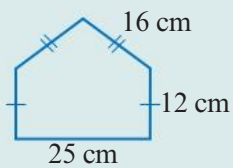
Perimeters of regular shapes

Perimeter

The **perimeter** of a two-dimensional shape is the total distance around its edge.

Example 9 Calculate the perimeter of a shape

Calculate the perimeter of the shape shown.



Solution	Explanation
Perimeter = 25 + 12 + 12 + 16 + 16 = 81 cm	To calculate the perimeter, add up all the side lengths of the shape.

Areas of regular shapes

Area

The **area** of a shape is a measure of the region enclosed by its boundaries.

When calculating area, the answer will be in *square units*, i.e. mm^2, cm^2, m^2, km^2.

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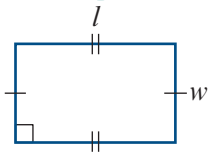
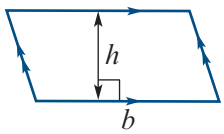
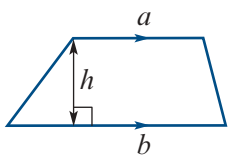
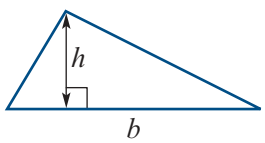
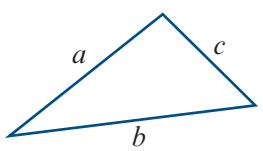
The **formulas for the areas** of some common shapes are given in the table below, along with the formula for finding the perimeter of a rectangle. Be mindful of which formulas are given in the QCAA Formula Book and which ones you will have to memorise.

Shape	Area	Perimeter
Rectangle 	$A = lw$	$P = 2l + 2w$ or $P = 2(l + w)$
Parallelogram 	$A = bh$	Sum of four sides
Trapezium 	$A = \frac{1}{2}(a + b)h$	Sum of four sides
Triangle 	$A = \frac{1}{2}bh$	Sum of three sides
Heron's rule for finding the area of a triangle with three side lengths known. 	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ (s is the half perimeter)	$P = a + b + c$

Note: More information on Heron's rule is given in *Chapter 8 Applications of trigonometry* section 8G.

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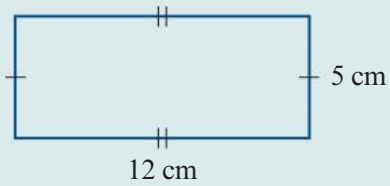
Example 10 Calculating the perimeter of a rectangle

Calculate the perimeter of the rectangle shown.



Solution

$P = 2l + 2w$
$= 2 \times 12 + 2 \times 5$
$= 34\text{ cm}$

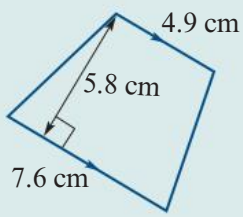
The perimeter of the rectangle is 34 cm.

Explanation

As the shape is a rectangle, use the formula $P = 2l + 2w$.
Substitute length and width values into the formula.
Evaluate.
Give your answer with correct units.

Example 11 Calculating the area of a shape

Calculate the area of the given shape.



Solution

$A = \frac{1}{2}(a + b)h$
$= \frac{1}{2}(4.9 + 7.6)(5.8)$
$= 36.25\text{ cm}^2$

The area of the shape is 36.25 cm^2.

Explanation

As the shape is a trapezium, use the formula $A = \frac{1}{2}(a + b)h$.
Substitute the values for a, b and h.
Evaluate.
Give your answer with correct units.

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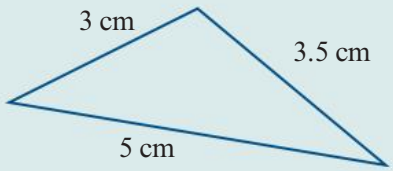
Example 12 Calculating the area of a triangle using Heron's rule

Calculate the area of the following triangle.
Give your answer correct to two decimal places.



Solution	Explanation
	As the height of the triangle is not given and the three side lengths are known, use Heron's rule.
$A = \sqrt{s(s-a)(s-b)(s-c)}$	Write down Heron's rule.
$P = 3 + 3.5 + 5$ $= 11.5$	Calculate the perimeter of the triangle by adding the three side lengths.
$s = \frac{11.5}{2}$ $= 5.75$	Divide the perimeter by 2 to find s.
$A = \sqrt{5.75(5.75-3)(5.75-3.5)(5.75-5)}$ $= 5.16562\ldots$	Substitute the value for s into Heron's rule to calculate the area of the triangle.
The area of the triangle is 5.17 cm^2, correct to two decimal places.	Give your answer correct to two decimal places and with correct units.

The formulas for area and perimeter can be applied to many practical situations.

Example 13 Calculating the area and perimeter in a practical situation

A display board for a classroom measures 150 cm by 90 cm.

a If ribbon costs \$0.55 per metre, determine the cost to add a ribbon border around the display board.

b The display board is to be covered with yellow paper. Calculate the area to be covered. Give your answer in m^2, correct to two decimal places.

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Solution	Explanation
a $P = 2l + 2w$	To determine the length of ribbon required, we need to calculate the perimeter of the display board. The display board is a rectangle so use the formula $P = 2l + 2w$.
$P = 2(150) + 2(90)$ $= 480$	Substitute $l = 150$ and $w = 90$ into the formula $P = 2l + 2w$ and evaluate to find the length of ribbon required.
The length of ribbon required is 480 cm.	
$= 480 \div 100$ $= 4.8$ m	Convert from centimetres to metres by dividing the length of the ribbon by 100.
$4.8 \times 0.55 = 2.64$	To calculate the cost of the ribbon, multiply the length of the ribbon by \$0.55.
Cost of ribbon is \$2.64.	Evaluate and write your answer.
b $A = lw$	To calculate the area of the board (which measures 150 cm by 90 cm), use the formula $A = lw$.
$A = 150 \times 90$ $= 13\,500$ cm^2	Substitute $l = 150$ and $w = 90$ and evaluate.
$A = 13\,500 \div 10\,000$ $= 1.35$	Convert your answer to m^2 by dividing by 100^2 ($100 \times 100 = 10\,000$).
Area to be covered with paper is 1.35 m^2.	Write your answer with correct units.

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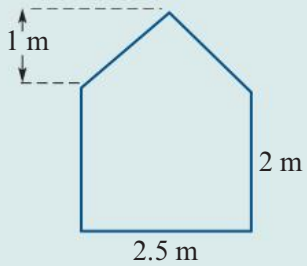
Composite shapes

Composite shape

A **composite shape** is a shape that is made up of two or more basic shapes.

Example 14 Calculating the perimeter and area of a composite shape in a practical situation

A gable window at a reception venue is to have LED lights around its perimeter (but not along the bottom of the window). The window is 2.5 m wide and the height of the room is 2 m. The height of the gable is 1 m, as shown in the diagram.

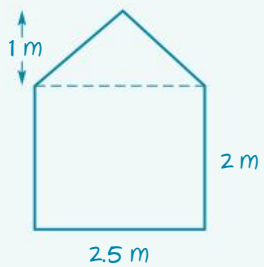


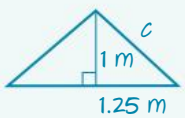
a Calculate the length of LED lights needed, correct to two decimal places.

b The glass in the window needs to be replaced. Determine the total area of the window, correct to two decimal places.

Solution

a





$c^2 = 1^2 + 1.25^2$

$\therefore c = \sqrt{1^2 + 1.25^2}$

$c = 1.6007\ldots$

$c = 1.60$ m, correct to two decimal places

$2 + 2 + 1.60 + 1.60 = 7.20$

The length of the LED lights is 7.20 m.

Explanation

The window is made of two shapes: a rectangle and a triangle. We need to calculate its perimeter and area.

First find the length of the slant edge of the triangle. Label it c on a diagram.

Use Pythagoras' theorem to calculate c.

Note: The length of the base of each triangle is 1.25 m ($\frac{1}{2}$ of 2.5 m).

Add all the outside edges of the window but do not include the bottom length.

Write your answer with correct units.

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b $A = bh$	To determine the total area of the window, first calculate the area of the rectangle by using the formula $A = bh$. Substitute the values for b and h. Evaluate and write your answer with correct units.
$= 2.5 \times 2$	
$= 5 \text{ m}^2$	
$A = \frac{1}{2}bh$	Calculate the area of the triangle by using the formula $A = \frac{1}{2}bh$.
$A = \frac{1}{2} \times 2.5 \times 1$	Substitute the values for b and h.
$A = 1.25 \text{ m}^2$	Evaluate and write your answer with correct units.
Total area = area of rectangle + area of triangle	To calculate the total area of the window, add the area of the rectangle and the area of the triangle.
$= 5 + 1.25$	
$= 6.25 \text{ m}^2$	
Total area of window is 6.25 m^2, correct to two decimal places.	Give your answer with correct units to two decimal places.

Section Summary

- **Perimeter** is the the total distance around the edges of the shape.
- **Area** is the measure of the region surrounded by the edges of the shape.
- Become familiar with the QCAA Formula Book. Know which formulas are included and which ones you will need to memorise. Practise using the formula book whenever you can.
- Use the formulas to calculate the perimeters and areas of standard shapes in practical situations, including triangles, rectangles, trapeziums, parallelograms, and composite shapes.

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Exercise 2C

SF

Conversion of units

Example 7
Example 8

1 Convert the following measurements into the units given in brackets.

a 5.7 m (cm) **b** 1.587 km (m) **c** 8 cm (mm) **d** 670 cm (m)
e 0.0046 km (cm) **f** 289 mm^2 (cm^2) **g** 5.2 m^2 (cm^2) **h** 0.08 km^2 (m^2)
i 3700 mm^2 (cm^2) **j** 6 m^2 (mm^2) **k** 500 mL (L) **l** 0.7 kg (g)
m 2.3 kg (mg) **n** 567 000 mL (kL) **o** 793 400 mg (g) **p** 0.5 L (mL)

2 Convert the following measurements into the units indicated in brackets and give your answer in standard form.

a 5 tonne (kg) **b** 6000 mg (kg) **c** 27 100 km^2 (m^2) **d** 33 m^3 (cm^3)
e 487 m^2 (km^2) **f** 28 mL (L) **g** 6 km (cm) **h** 1125 mL (kL)
i 50 000 m^3 (km^3) **j** 340 000 mm^3 (m^3)

3 Determine the total sum of these measurements. Express your answer in the units given in brackets.

a 14 cm, 18 mm (mm) **b** 589 km, 169 m (km)
c 3.4 m, 17 cm, 76 mm (cm) **d** 300 mm^2, 10.5 cm^2 (cm^2)

4 A wall in a house is 7860 mm long. Convert this measurement to metres.

5 A truck weighs 3 tonnes. State this weight in kilograms.

6 An Olympic swimming pool holds approximately 2.25 megalitres of water. Calculate the equivalent volume in litres.

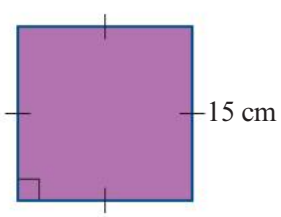
Perimeters and areas

Example 9
Example 10

7 For each of the following shapes, calculate, correct to one decimal place:

i the perimeter **ii** the area

a



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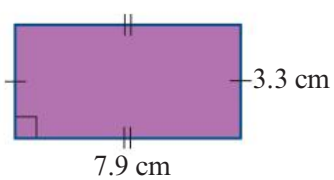

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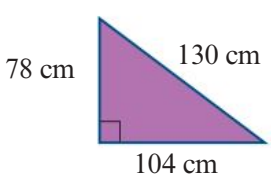

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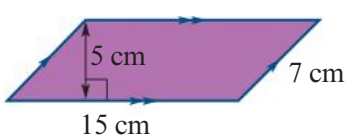

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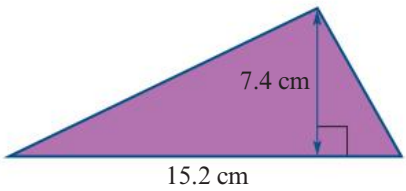
SF

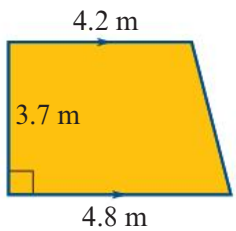
Example 11

8 Determine the areas of the given shapes, correct to one decimal place, where appropriate.

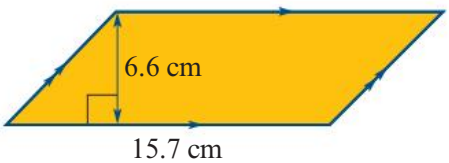
a



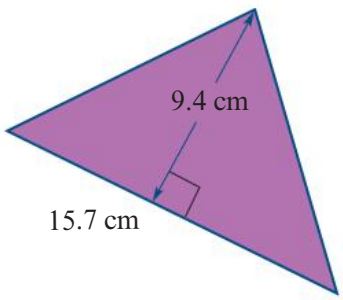
b



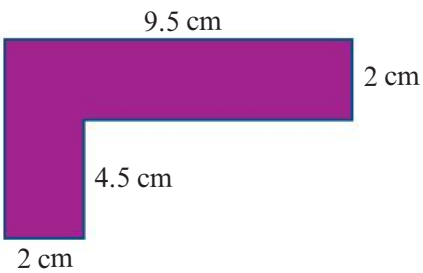
c



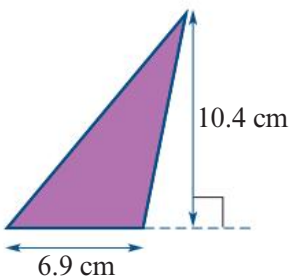
d



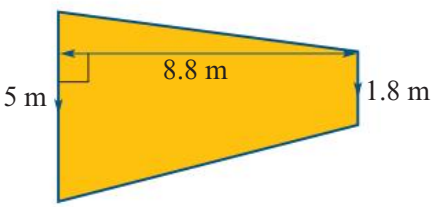
e



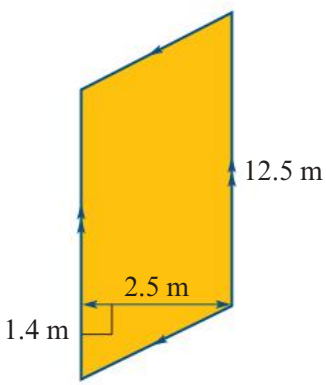
f



g



h



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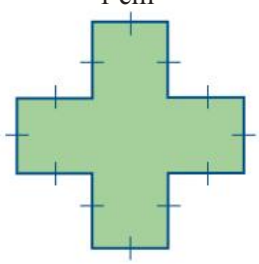
Composite shapes

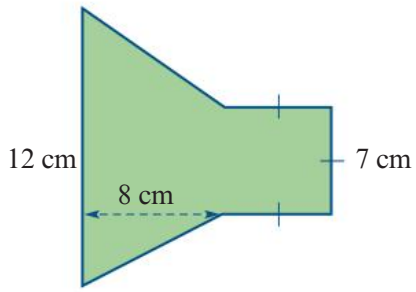
9 Calculate the area of the following composite shapes.

CF

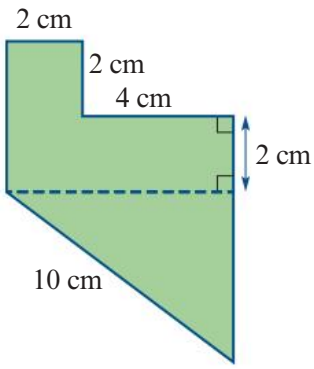
a



b



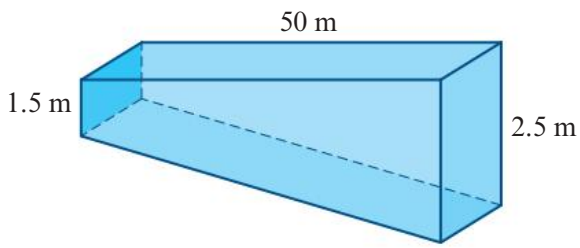
c



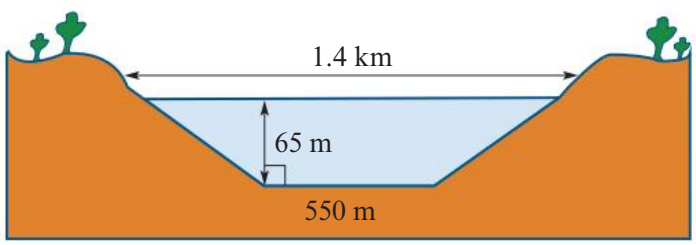
Applications of perimeters and areas

Example 13
Example 14

10 A 50 m swimming pool increases in depth from 1.5 m at the shallow end to 2.5 m at the deep end, as shown in the diagram (*not* to scale). Calculate the area of a side wall of the pool.



11 A dam wall is built across a valley that is 550 m wide at its base and 1.4 km wide at its top, as shown in the diagram (*not* to scale). The wall is 65 m deep. Calculate the area of the dam wall.



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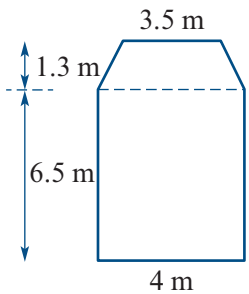
CF

12 Ray wants to tile a rectangular area measuring 1.6 m by 4 m. The tiles that he wishes to use are 40 cm by 40 cm. Determine the number of tiles that Ray will use.

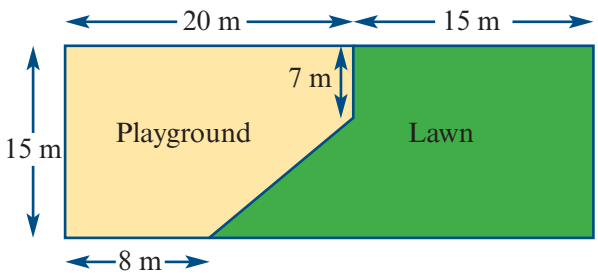
13 One litre of paint covers 9 m^2. Calculate the amount of paint that will be needed to paint a wall measuring 3 m by 12 m.

Applications of composite shapes

14 A driveway, as shown in the diagram, is to be paved. Determine the area of the driveway, correct to two decimal places.



15 The council plans to fence a rectangular piece of land to make a children's playground and a lawn as shown. (Not drawn to scale.)



a Calculate the area of the children's playground.

b Calculate the area of the lawn.

CU

16 A chef has 50 baking trays that are 30 cm wide and 32 cm long. The baking paper he buys has 3 square metres of baking paper per roll and is 30 cm wide. Determine whether the one full roll of baking paper he already has will cover all the baking trays or how many more rolls he would need to buy to make this possible. Justify your answer with mathematical reasoning.

17 The ratio of a right-angled triangle's sides are 3 : 4 : 5. If the perimeter of this triangle is 72 cm, calculate the area of this triangle.

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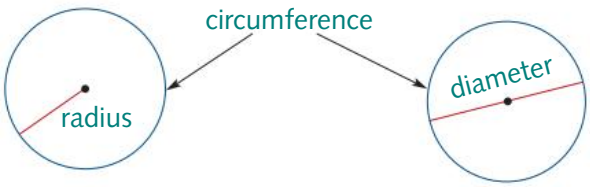
2D Circles

Learning intentions

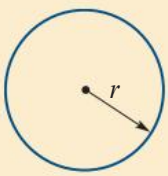
- To review the concept of circumference.
- To be able to calculate the circumference and area of a circle.
- To be able to apply the circumference and area of a circle to practical situations.

The circumference and area of a circle

The perimeter of a circle is also known as the **circumference** (C) of the circle.



Formulas for area and circumference of a circle

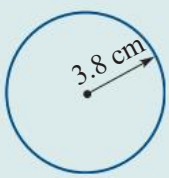
Circle	Area	Circumference
	$A = \pi r^2$ where r is the **radius**	$C = 2\pi r$ or $C = \pi d$ where d is the **diameter**

Example 15 Calculating the circumference and area of a circle

For the circle shown, calculate, correct to one decimal place:

a the circumference

b the area.



Solution

a $C = 2\pi r$

$C = 2\pi \times 3.8$

$C = 23.876\ldots$

The circumference of the circle is 23.9 cm, correct to one decimal place.

Explanation

For the circumference, use the formula $C = 2\pi r$.

Substitute $r = 3.8$ and evaluate.

Give your answer correct to one decimal place and with correct units.

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b $A = \pi r^2$

To calculate the area of the circle, use the formula $A = \pi r^2$.

$A = \pi \times 3.8^2$

Substitute $r = 3.8$ and evaluate.

$A = 45.364\ldots$

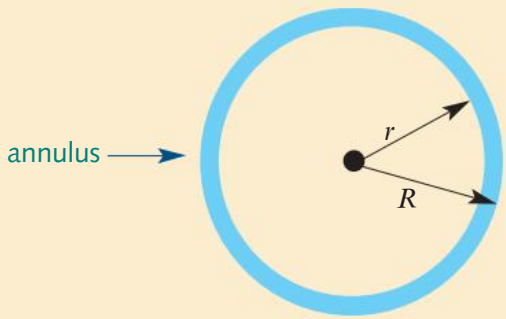
The area of the circle is 45.4 cm^2, correct to one decimal place.

Give your answer correct to one decimal place and with correct units.

The annulus

Annulus

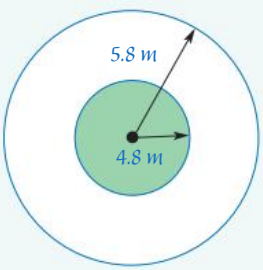
An **annulus** is a flat ring shape bounded by two circles that have the same centre.



Example 16 Calculating the area of an annulus in a practical situation

A path 1 metre wide is to be built around a circular lawn of radius 4.8 m. Calculate the area of the path, correct to two decimal places.

Solution



Explanation

Draw a diagram to represent the situation.
The two circles form an annulus.
The smaller circle has radius of 4.8 m.
With the path of width of 1 m, the larger circle has a radius of 5.8 m.

$A = \pi r^2$

Calculate the area of the larger circle using the formula $A = \pi r^2$.

$A = \pi \times 5.8^2$

Substitute $r = 5.8$ and evaluate.

$A = 105.683\ldots$ m

Continued on next page

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$A = \pi r^2$	Calculate the area of the smaller circle using the formula $A = \pi r^2$.
$A = \pi \times 4.8^2$ $A = 72.382\ldots$ m	Substitute $r = 4.8$ and evaluate.
Required area = area of large circle − area of small circle $= 105.683\ldots - 72.382\ldots$ $= 33.300\ldots$	Subtract the area of the smaller circle from the area of the larger circle to give the required area (the area of the annulus).
Area of circular path is 33.30 m^2, correct to two decimal places.	Give your answer correct to two decimal places and with correct units.

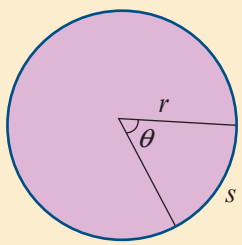
Arc length and the sector of a circle

Arc of a circle

An **arc** is the length of a circle between two points on the circle. The length of the arc, s, is given by:

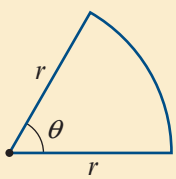
$$s = \frac{\theta}{180}\pi r$$

where r is the radius of the circle and $\theta°$ is the central angle of the circle.



Perimeter and area of a sector

A **sector** is a portion of a circle enclosed by two radii and an arc.



The **perimeter of a sector** is given by:

$$P = 2r + \frac{\theta}{180}\pi r$$

The **area of a sector** is given by:

$$A = \frac{\theta}{360}\pi r^2$$

where r is the radius of the circle and $\theta°$ is the central angle of the circle.

Note: Although arc length itself is not specifically listed in the syllabus, it is part of calculating the perimeter of a sector and a useful idea for the study of Earth geometry in Unit 3.

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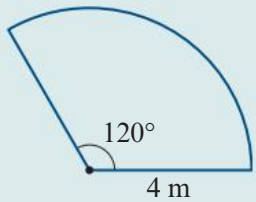
Example 17 Calculating the arc length, the perimeter and area of a sector

For the sector shown, calculate, correct to two decimal places:

a the arc length

b the perimeter

c the area.



Solution	Explanation
a $s = \frac{\theta}{180}\pi r$	For the arc length, use the formula $s = \frac{\theta}{180}\pi r$.
$s = \frac{120}{180} \times \pi \times 4$	Substitute $r = 4$, $\theta = 120°$ and evaluate.
$s = 8.3775\ldots$	
The arc length of the sector is 8.38 m, correct to two decimal places.	Give your answer correct to two decimal places and with correct units.
b $P = 2r + \frac{\theta}{180}\pi r$	To calculate the perimeter of the sector, use the formula $P = 2r + \frac{\theta}{180}\pi r$.
$P = 2 \times 4 + \frac{120}{180} \times \pi \times 4$	Substitute $r = 4$, $\theta = 120°$ and evaluate.
$P = 16.377\ldots$	
The perimeter of the sector is 16.38 m, correct to two decimal places.	Give your answer correct to two decimal places and with correct units.
c $A = \frac{\theta}{360}\pi r^2$	To calculate the area of the sector, use the formula $A = \frac{\theta}{360}\pi r^2$.
$A = \frac{120}{360} \times \pi \times 4^2$	Substitute $r = 4$, $\theta = 120°$ and evaluate.
$A = 16.755\ldots$	
The area of the sector is 16.76 m^2, correct to two decimal places.	Give your answer correct to two decimal places and with correct units.

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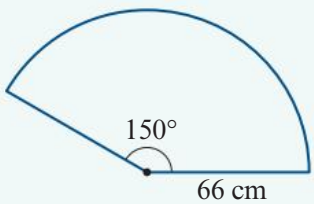
Example 18 Calculating the area of a sector in a practical situation

One of the windscreen wipers on a car is 66 cm long and moves through an angle of 150°. Calculate the amount of windscreen that can be cleared with one pass of the wiper. Round your answer to the nearest whole square centimetre.

Solution



$A = \frac{\theta}{360}\pi r^2$

$A = \frac{150}{360} \times \pi \times 66^2$

$A = 5701.9906\ldots$

The area of windscreen is 5702 cm^2, correct to the nearest whole number.

Explanation

Draw a diagram to represent the situation. We will need to calculate the area of a sector where the radius is 66 cm and θ is 150°.

Use the formula $A = \frac{\theta}{360}\pi r^2$.

Substitute $r = 66$, $\theta = 150°$ and evaluate.

Give your answer correct to the nearest whole number and with correct units.

Section Summary

- The perimeter of a circle is called the **circumference**.
- A **sector** is a portion of a circle.
- The **arc length** is part of the perimeter of a sector and will be useful when studying Earth geometry in Unit 3.
- Become familiar with the QCAA Formula Book. Know which formulas are included and which ones you will need to memorise. Practise using the formula book whenever you can.
- Use the formulas to calculate the circumference and area of circles in practical situations.
- Use the formulas to calculate the perimeter and area of sectors in practical situations.

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Skill-sheet

Exercise 2D

SF

Calculating the circumference and area of a circle

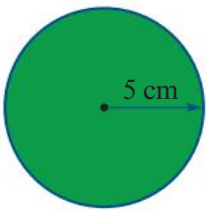
Example 15

1 For each of the following circles, calculate, correct to one decimal place:

i the circumference

ii the area.

a



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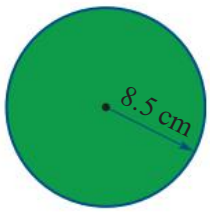

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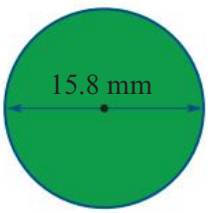

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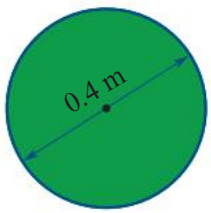


Calculating the perimeter and area of a sector

Example 17

2 A circle has a radius of 10 cm and central angle of 50°. Calculate the arc length, correct to two decimal places.

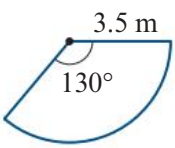
Example 17

3 For each of the following sectors, calculate, correct to two decimal places:

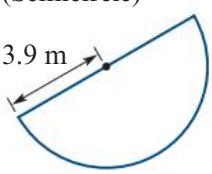
i the perimeter

ii the area.

a



b (Semicircle)



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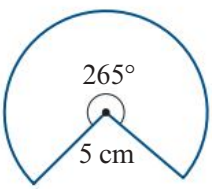

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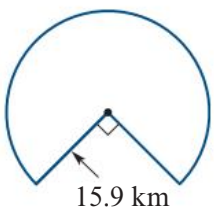

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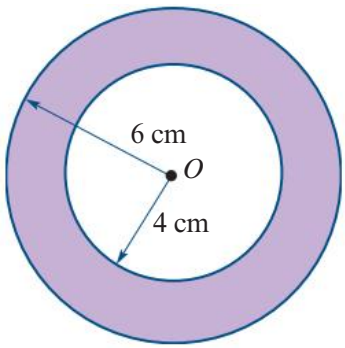
Example 16

4 The diagram shows two circles with centre O. The radius of the inner circle is 4 cm and the radius of the outer circle is 6 cm. Determine the area of the annulus (shaded area), correct to two decimal places.



SF

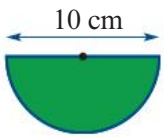
Calculating the perimeter and area of shapes involving circles

5 For each of the following shapes, calculate, correct to two decimal places:

i the perimeter

ii the area.

a



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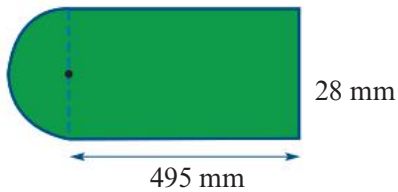

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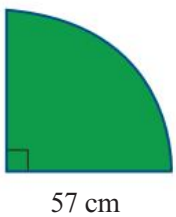

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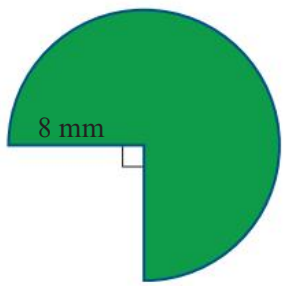
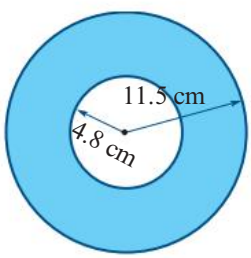


6 Calculate the area of the shaded regions in the following diagrams, correct to one decimal place.

CF

a



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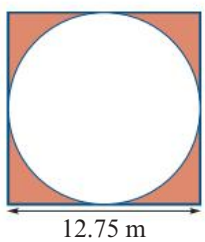

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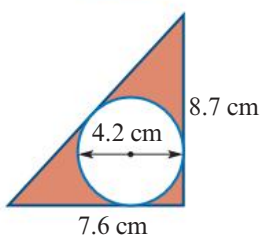

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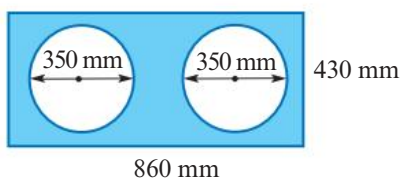

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Applications of perimeters and areas involving circles

7 A fence needs to be built around an athletics track that has straights 400 m long and semicircular ends of diameter 80 m. Give answers correct to two decimal places.

a Determine the length of fencing that will be required.

b Calculate the amount of track area that will be enclosed by the fencing.

8 A couple wish to decorate an arch for their wedding. The width of the arch is 1.4 metres and the semicircle at the top begins at a height of 1.9 metres.

a Material is to be attached around the perimeter of the arch but not including the base. Determine the length of material required, rounded to the nearest metre.

b If material is to cover the whole arch so that you cannot see through the arch, determine the minimum number of square metres of material required, correct to one decimal place.

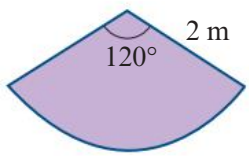
9 Three juggling rings cut from a thin sheet are to be painted. The diameter of the outer circle of the ring is 25 cm and the diameter of the inside circle is 20 cm. If both sides of the three rings are to be painted, determine the total area to be painted. (Ignore the inside and outside edges.) Round your answer to the nearest cm^2.

10 A path 1.2 m wide surrounds a circular garden bed whose diameter is 7 m. Determine the area of the path. Give your answer correct to two decimal places.

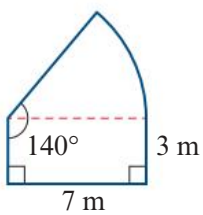
Sector applications

Example 18

11 Maria wishes to place edging around the perimeter of her garden bed. The garden bed is in the shape of a sector as shown. Determine the perimeter of her garden bed, correct to two decimal places.



12 The diagram shows the shape of the top of the VIP seating area at a stadium. It is cut out of a piece of steel that is 7 m by 8 m. If the steel costs $46 per square metre, calculate the value of the steel that is not used for the top.



13 A circle is drawn inside a square so that it touches the centre of each side. If the square has side lengths 14 cm each, calculate the percentage of the square that is outside the circle.

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2E Volume of a prism

Learning intentions

- To review the concept of volume.
- To be able to calculate the volume of a prism.
- To be able to calculate the capacity of a prism.
- To be able to apply the volume and capacity of a prism to practical situations.

Volume

Volume is the amount of space occupied by a three-dimensional object.

For **prisms** and **cylinders**:

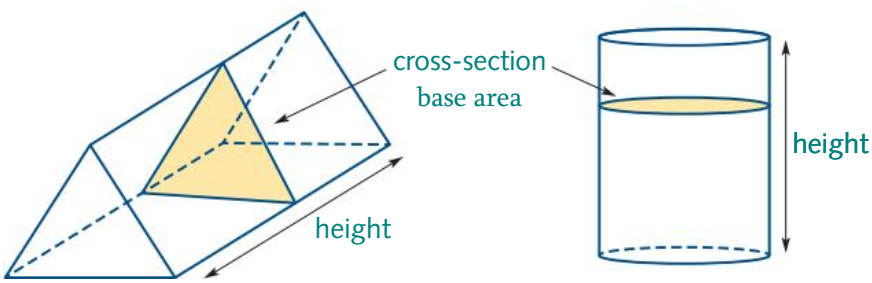
$$V = Ah$$

where A is the base area and h is the perpendicular height of the prism.

Prisms and cylinders are three-dimensional objects that have a uniform cross-section along their entire height.

The volume of a prism or cylinder is found by using the area of its *cross-sectional area* and multiplying by its perpendicular height. The **perpendicular height** is the measurement of the height at 90° to the cross-sectional base area. This cross-sectional area can be simply known as the **base area** of the prism.

$$\text{volume} = \text{base area} \times \text{ perpendicular height}$$



When calculating volume, the answer will be in *cubic units*, e.g. mm^3, cm^3, m^3.

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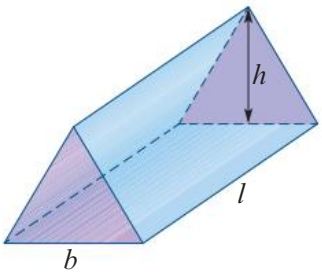
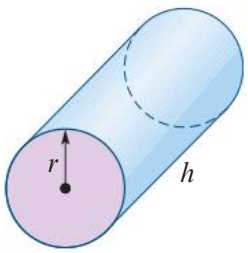
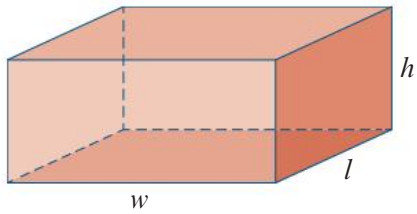
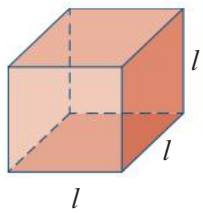
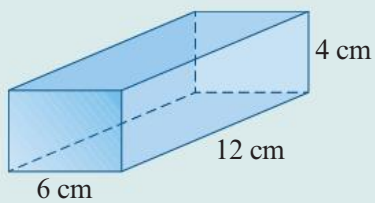
The **formulas for the volumes** of regular prisms and a cylinder are given in the table below. Be mindful of which formulas are given in the QCAA Formula Book and which ones you will need to memorise.

Shape	Volume
Triangular prism 	$V = \frac{1}{2}bhl$
Cylinder 	$V = \pi r^2 h$

Shape	Volume
Rectangular prism (cuboid) 	$V = lwh$
Square prism (cube) 	$V = l^3$

Example 19 Calculating the volume of a rectangular prism

Calculate the volume of this rectangular prism.



Solution	Explanation
$V = lwh$	Use the formula $V = lwh$.
$V = 12 \times 6 \times 4$	Substitute in $l = 12, w = 6$ and $h = 4$.
$V = 288$ cm^3	Evaluate.
The volume of the rectangular prism is 288 cm^3.	Give your answer with correct units.

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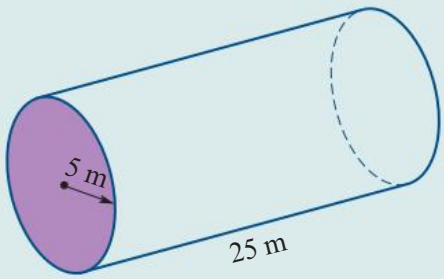
Example 20 Calculating the volume of a cylinder

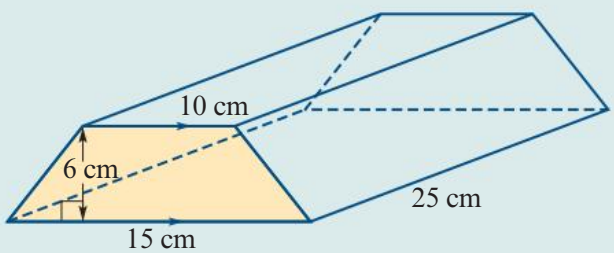
Calculate the volume of this cylinder in cubic metres. Give your answer correct to two decimal places.



Solution	Explanation
$V = \pi r^2 h$	Use the formula $V = \pi r^2 h$.
$V = \pi \times 5^2 \times 25$	Substitute $r = 5$, $h = 25$ and evaluate.
$V = 1963.495\ldots$	
The volume of the cylinder is 1963.50 m^3, to two decimal places.	Write your answer correct to two decimal places and with correct units.

Example 21 Calculating the volume of a three-dimensional shape

Calculate the volume of the three-dimensional shape shown.



Solution	Explanation
Identify that the cross-section is in the shape of a trapezium.	This must be known so that the base area can be calculated.
Area of trapezium $= \frac{1}{2}(a + b)h$	Use the formula $A = \frac{1}{2}(a + b)h$.
$= \frac{1}{2}(10 + 15)(6)$	Substitute $a = 10, b = 15, h = 6$ and evaluate.
$= 75\ cm^2$	
V = base area × pendicular height	To calculate the volume, multiply the base area by the perpendicular height of the shape (25 cm).
$= 75 \times 25$	
$= 1875\ cm^3$	
The volume of the shape is 1875 cm^3.	Give your answer with correct units.

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Capacity

Capacity

Capacity is the amount of substance that an object can hold.

For example, a bucket might have a capacity of 7 litres.

The difference between volume and capacity is that volume is the space available and capacity is the amount of substance that fills the volume.

Examples:

- A cube that measures 1 metre on each side has a volume of one cubic metre (m^3) and is able to hold 1000 litres (L) (capacity).
- A bucket of volume 7000 cm^3 can hold 7000 mL (or 7 L) of water.

Capacity conversions

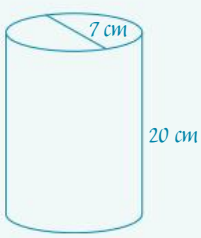
The following conversions are useful to remember.

$$1\ m^3 = 1000 \text{ litres (L)}$$
$$1\ cm^3 = 1 \text{ millilitre (mL)}$$
$$1000\ cm^3 = 1 \text{ litre (L)}$$

Example 22 Calculating the capacity of a cylinder

A drink container is in the shape of a cylinder. Calculate the number of litres of water it can hold if the height of the cylinder is 20 cm and the diameter is 7 cm. Give your answer correct to two decimal places.

Solution



$V = \pi r^2 h$

$V = \pi \times 3.5^2 \times 20$

Explanation

Draw a diagram to represent the situation.

Use the formula for finding the volume of a cylinder $V = \pi r^2 h$.

The diameter is 7 cm so the radius is 3.5 cm.
Substitute $h = 20$ and $r = 3.5$.

Continued on next page

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$V = 769.6902\ldots$ The volume of the cylinder is $769.69\ \text{cm}^3$.	Evaluate to find the volume of the cylinder.
$\frac{769.69}{1000} = 0.76969\ldots$	As there are $1000\ \text{cm}^3$ in a litre, divide the volume by 1000 to convert to litres.
Cylinder has capacity of 0.77 litres, correct to two decimal places.	Give your answer correct to two decimal places and with correct units.

Section Summary

- **Volume** is the space available inside a three-dimensional object.
- To calculate the volume of a prism, determine the base area and multiply it by the perpendicular height of the shape.
- **Capacity** is the amount of substance that a three-dimensional object can hold.
- Become familiar with the QCAA Formula Book. Know which formulas are included and which ones you will need to memorise. Practice using the formula book whenever you can.
- Use the formulas to calculate the volumes and capacities of standard solids in practical situations, including rectangular prisms and cylinders.

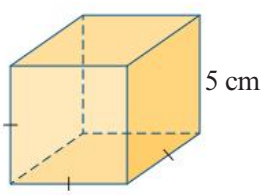
Exercise 2E

Volumes of prisms

Example 19
Example 21

1 Calculate the volumes of the following solids. Give your answers correct to one decimal place, where appropriate.

a



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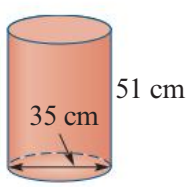

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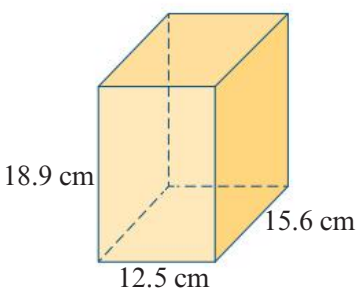

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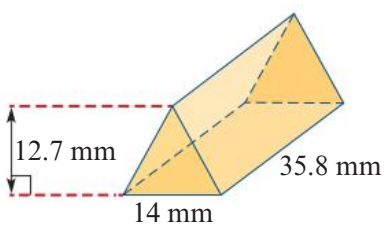

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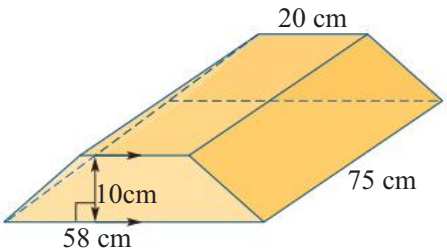
SF

e



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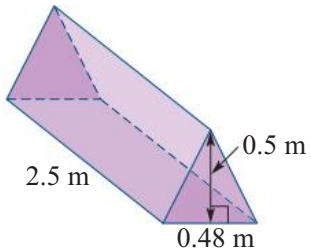

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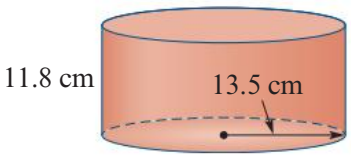

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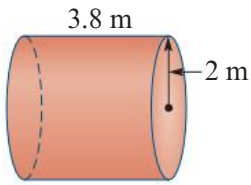


Example 20

2 A cylindrical plastic container is 15 cm high and its circular end surfaces each have a radius of 3 cm. Determine its volume, to the nearest cm^3.

3 Determine the volume, to the nearest cm^3, of a rectangular box with dimensions 5.5 cm by 7.5 cm by 12.5 cm.

Applications of volume and capacity

Example 22

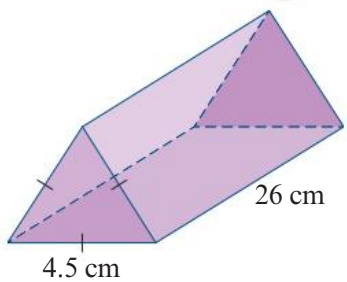
CF

4 Calculate the number of litres of water that a fish tank with dimensions 50 cm by 20 cm by 24 cm would hold when full.

5 **a** Calculate the volume, correct to two decimal places, of a cylindrical paint tin with height 33 cm and diameter 28 cm.

b Determine the number of litres of paint that would fill this paint tin. Give your answer to the nearest litre.

6 The box for a chocolate bar is made in the shape of an equilateral triangular prism. Calculate the volume of the box if the length is 26 cm and the side length of the triangle is 4.5 cm. Give your answer to the nearest cm^3.



CU

7 A cylindrical container is available at the store to collect honey. The container is 8 cm tall with a diameter of 4.75 cm. If the price of honey is $2.07 per 100 mL, determine the cost to completely fill the container.

8 A rectangular prism has dimensions 16 cm by 27 cm by 32 cm. Determine the side length of a cube that has the same volume.

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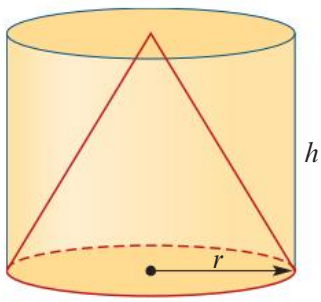
2F Volume of other solids

Learning intentions

- To be able to calculate the volume of a cone, pyramid and sphere.
- To be able to calculate the capacity of a cone, pyramid and sphere.
- To be able to apply the volume and capacity of a cone, pyramid and sphere to practical situations.

Volume of a cone

A cone can fit inside a cylinder, as shown in the diagram. The cone occupies one-third of the volume of the cylinder containing it.



Therefore, the formula for calculating the **volume of a cone** is:

$$\text{volume of cone} = \tfrac{1}{3} \times \text{volume of its cylinder}$$

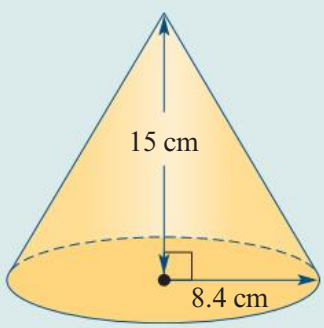
$$\text{volume of cone} = \tfrac{1}{3} \times \text{ base area} \times \text{perpendicular height}$$

$$V = \tfrac{1}{3}\pi r^2 h$$

The cone in the above diagram is called a right circular cone because a line drawn from the centre of the circular base to the vertex at the top of the cone is perpendicular to the base.

Example 23 Calculating the volume and capacity of a cone

Calculate the volume and capacity of this right circular cone. Give your answers correct to two decimal places.



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Solution	Explanation
$V = \frac{1}{3}\pi r^2 h$	Use the formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$.
$V = \frac{1}{3} \times \pi \times (8.4)^2 \times 15$ $V = 1108.353\ldots$	Substitute $r = 8.4$, $h = 15$ and evaluate.
The volume of the cone is 1108.35 cm^3, correct to two decimal places.	Give your answer correct to two decimal places and with correct units.
$\frac{1108.35}{1000} = 1.10835\ldots$ The cone has a capacity of 1.11 litres, correct to two decimal places.	As there are 1000 cm^3 in a litre, divide the volume by 1000 to convert to litres. Give your answer correct to two decimal places and with appropriate units.

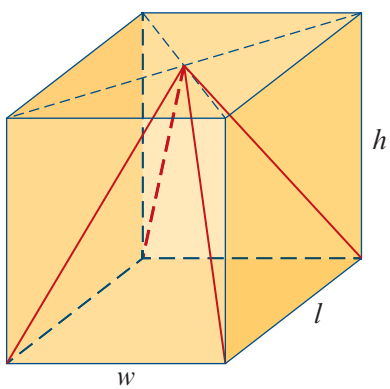
Volume of a pyramid

In general, the **volume of a pyramid** with any shaped base can be found using the following formula:

$$V = \frac{1}{3}Ah$$

where A is the base area and h is the perpendicular height.

To illustrate, a square pyramid can fit inside a prism, as shown in the diagram. The pyramid occupies one-third of the volume of the prism containing it.



The formula for calculating the **volume of a square pyramid** is therefore:

$$\text{volume of pyramid} = \tfrac{1}{3} \times \text{volume of its prism}$$

$$\text{volume of pyramid} = \tfrac{1}{3} \times \text{base area} \times \text{perpendicular height}$$

$$V = \tfrac{1}{3}lwh$$

The pyramid in the above diagram is called a square right pyramid because a line drawn from the centre of the square base to the vertex at the top of the pyramid is perpendicular to the base.

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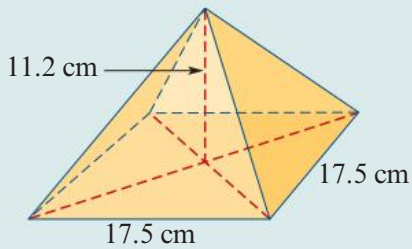
Example 24 Calculating the volume and capacity of a square right pyramid

Calculate the volume and capacity of a square right pyramid of height 11.2 cm and base 17.5 cm. Give your answers correct to two decimal places.



Solution

$$V = \frac{1}{3}lwh$$
$$= \frac{1}{3} \times 17.5^2 \times 11.2$$
$$= 1143.333\ldots$$

The volume of the pyramid is 1143.33 cm^3, correct to two decimal places.

$$\frac{1143.33}{1000} = 1.14333\ldots$$

The pyramid has a capacity of 1.14 litres, correct to two decimal places.

Explanation

Use the formula: $V = \frac{1}{3}Ah$.
Substitute the values for the base area (in this example, the base is a square) and perpendicular height of the pyramid.
Evaluate.

Give your answer correct to two decimal places and with correct units.

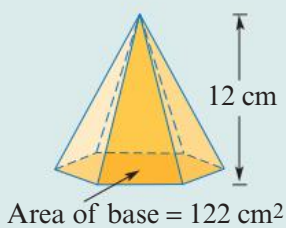
As there are 1000 cm^3 in a litre, divide the volume by 1000 to convert to litres.

Give your answer correct to two decimal places and with appropriate units.

The general formula can be used for a pyramid with a base of any shape.

Example 25 Calculating the volume of a hexagonal pyramid

Calculate the volume of this hexagonal pyramid that has a base of area 122 cm^2 and a height of 12 cm.



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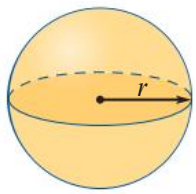
Solution	Explanation
$V = \frac{1}{3}Ah$	Use the formula: $V = \frac{1}{3} \times$ base area $\times$ perpendicular height
$V = \frac{1}{3} \times 122 \times 12$ $= 488 \text{ cm}^3$	Substitute the values for base area (122 cm^2) and perpendicular height (12 cm). Then evaluate.
The volume of the pyramid is 488 cm^3.	Give your answer with correct units.

Volume of a sphere

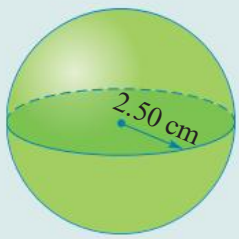
The volume of a sphere of radius r can be found by using the formula:

$$V = \frac{4}{3}\pi r^3$$



Example 26 Calculating the volume and capacity of a sphere

Calculate the volume and capacity of this sphere, giving your answer correct to two decimal places.



Solution	Explanation
$V = \frac{4}{3}\pi r^3$ $= \frac{4}{3}\pi \times 2.5^3$ $= 65.449 \ldots$	Use the formula $V = \frac{4}{3}\pi r^3$. Substitute $r = 2.5$ and evaluate.
The volume of the sphere is 65.45 cm^3, correct to two decimal places.	Give your answer correct to two decimal places and with correct units.
$65.449 \ldots \times 1 = 65.449 \ldots$ The sphere has a capacity of 65.45 mL, correct to two decimal places.	As $1 \text{ cm}^3 = 1$ millilitre (mL), multiply by 1 to convert to millilitres. Give your answer correct to two decimal places and with appropriate units.

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Section Summary

- Use the appropriate formula to calculate the volume of a cone, pyramid and sphere.
- Become familiar with the QCAA Formula Book. Know which formulas are included and which ones you will need to memorise. Practice using the formula book whenever you can.
- Use the formulas to calculate the volumes and capacities of standard solids in practical situations, including cones, pyramids and spheres.

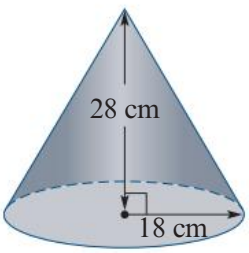
Exercise 2F

Volumes of cones

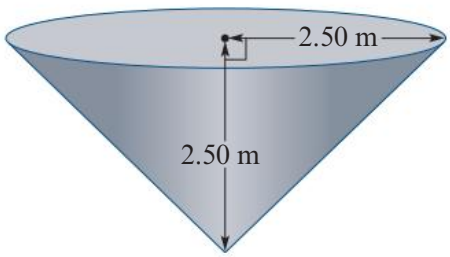
Example 23

1 Calculate the volume of these cones, correct to two decimal places.

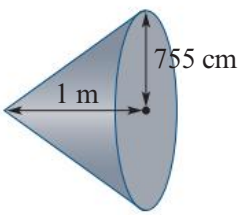
a



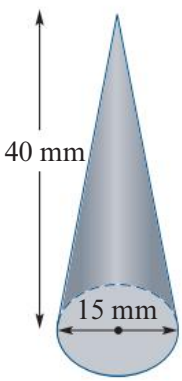
b



c



d



2 Calculate the volume of the cones with the following dimensions, correct to two decimal places.

a Base radius 3.50 cm, perpendicular height 12 cm

b Base radius 7.90 m, perpendicular height 10.80 m

c Base diameter 6.60 cm, perpendicular height 9.03 cm

d Base diameter 13.52 cm, perpendicular height 30.98 cm

SF

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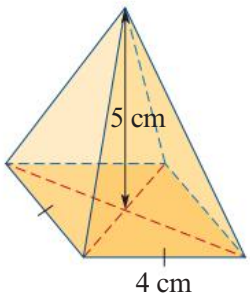
Volumes of pyramids

Example 24
Example 25

3 Calculate the volumes of the following right pyramids, correct to two decimal places where appropriate.

a



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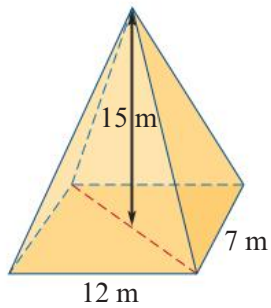

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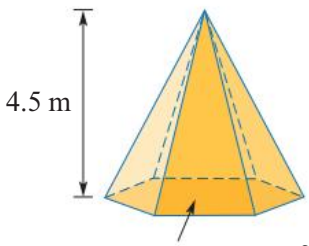

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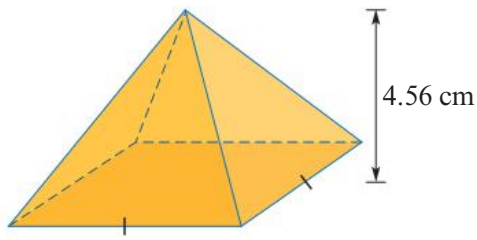


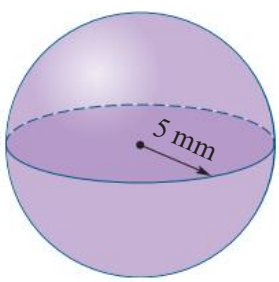
4 A square-based pyramid has a base side length of 8 cm and a height of 10 cm. Determine its volume. Answer correct to three decimal places.

Volumes of spheres and hemispheres

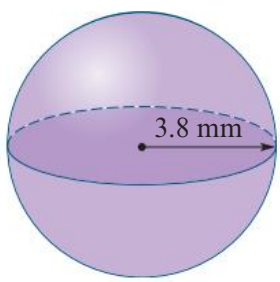
Example 26

5 Calculate the volumes of these spheres, giving your answers correct to two decimal places.

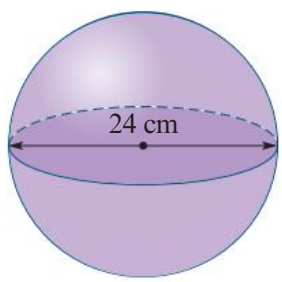
a



b



c



6 Calculate the volumes, correct to two decimal places, of the following balls.

a Tennis ball, radius 3.5 cm

b Basketball, radius 14 cm

c Golf ball, radius 2 cm

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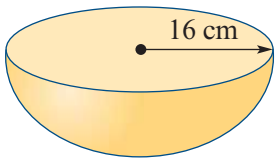
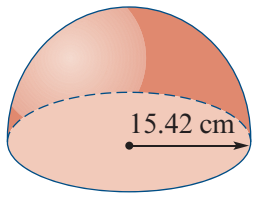
7 Calculate the volumes, correct to two decimal places, of the following hemispheres. SF

a



b



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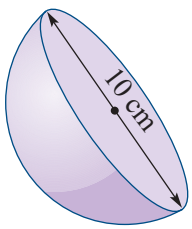
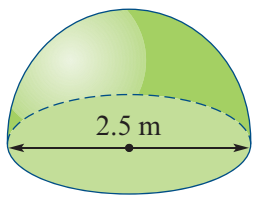


d



Applications

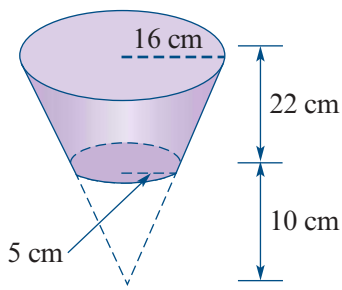
8 Determine the volume of crushed ice that will fill a snow cone level to the top, if the snow cone has a radius of 5 cm and a height of 15 cm. Give your answer to the nearest cm^3. SF

9 A tepee is a conical-shaped tent. Calculate the volume, correct to two decimal places, of a tepee with height 2.6 m and diameter of 3.4 m.

10 Determine the number of litres of water, correct to 2 decimal places, that can be poured into a conical flask with a diameter 2.8 cm and a height of 10 cm.

11 The first true pyramid in Egypt is known as the Red Pyramid. It has a square base approximately 220 m long and is about 105 m high. Determine its volume.

12 A solid figure is *truncated* when a portion of the bottom is cut and removed. Find the volume, correct to two decimal places, of the truncated cone shown in the diagram. CF



13 A flat-bottomed silo for grain storage is made of a cylinder with a cone on top. The cylinder has a circumference of 53.4 m and a height of 10.8 m. The total height of the silo is 15.3 m. Calculate the capacity of the silo. Give your answer to the nearest litre.

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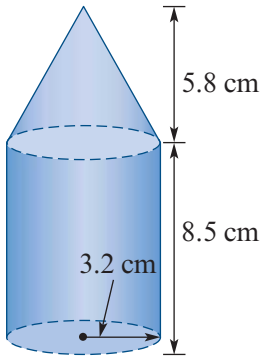
CF

14 Calculate the volumes of these composite objects, correct to one decimal place.

a



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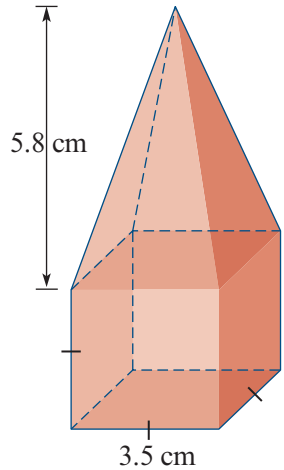
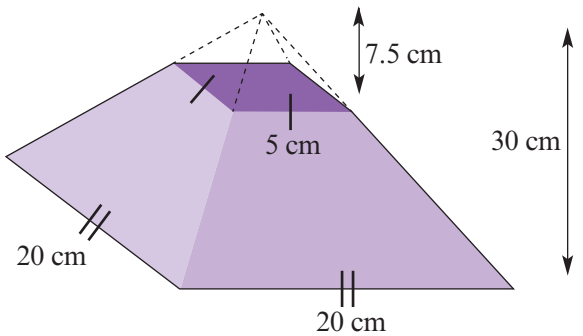


15 Calculate the volume of the following truncated pyramid, correct to one decimal place.



16 An orange is cut into quarters. If the radius is 35 mm, calculate the volume of one-quarter to the nearest mm^3.

17 Lois wants to serve punch at Christmas lunch in her new hemispherical bowl with diameter of 38 cm. Calculate the number of litres of punch that could be served, given that 1 millilitre (mL) is the amount of fluid that fills 1 cm^3. Answer to the nearest litre.

CU

18 The radius of a sphere is the same as the radius of the base of a cone. They both have the same volume. State the ratio of the height and the radius in the cone.

19 A time capsule is in the shape of a cylinder in the centre with a hemisphere on each end. The cylinder is 320 mm long and the hemispheres have radii of 4.5 cm. The archaeologist has 2.6 litres of soil to preserve. Justify whether or not this will fit in this capsule. Comment on the reasonableness of your answer.

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2G Surface area

Learning intentions

- To review the concept of surface area.
- To be able to calculate the surface area of solids with plane faces.
- To be able to calculate the surface area of solids with curved surfaces.
- To be able to apply the surface area of standard three-dimensional objects to practical situations.

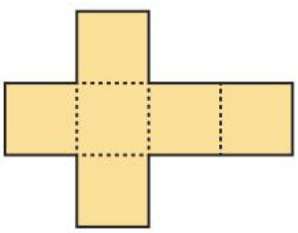
To determine the **surface area (*SA*)** of a solid, you need to calculate the area of each of the surfaces of the solid and then add these all together.

Solids with plane faces (prisms and pyramids)

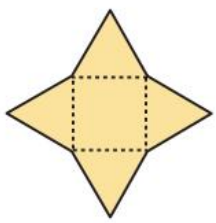
It is often useful to draw the *net* of a solid to ensure that all sides have been considered.

A *net* is a flat diagram consisting of the plane faces of a polyhedron, arranged so that the diagram may be folded to form the solid.

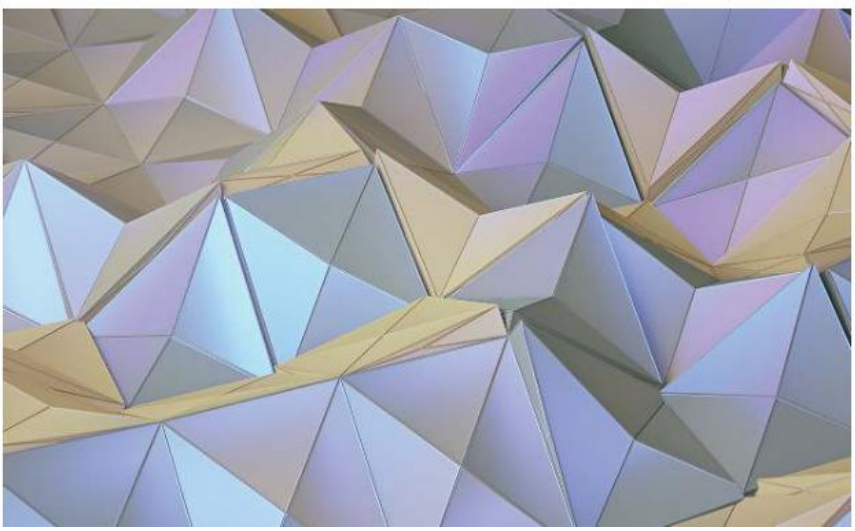
The net of a cube and of a square pyramid are shown below.



Net of a cube



Net of a square pyramid



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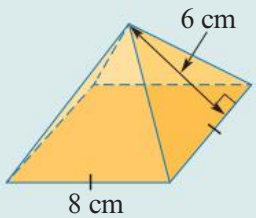
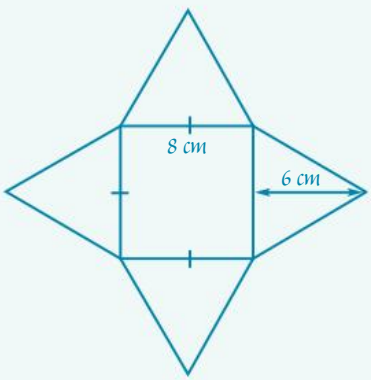
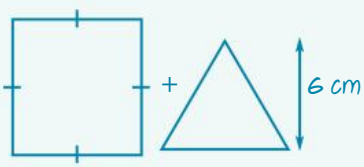
Example 27 Calculating the surface area of a pyramid

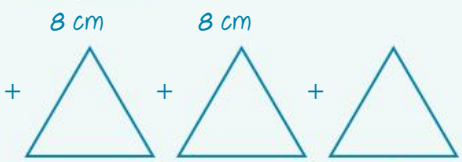
Calculate the surface area of this square-based pyramid.



Solution







Surface area = area of + 4

$= 8 \times 8 + 4 \times (\frac{1}{2} \times 8 \times 6)$

$= 160$

The surface area of the square pyramid is 160 cm^2.

Explanation

Draw a net of the square pyramid. Note that the net is made up of one square and four identical triangles, as shown.

To calculate the area of the square, multiply the length by the width (8×8).

To calculate the area of the triangles, use $A = \frac{1}{2}bh$, where b is 8 and h is 6.

Write down the formula for the total surface area, using the net as a guide, and evaluate.

Give your answer with correct units.

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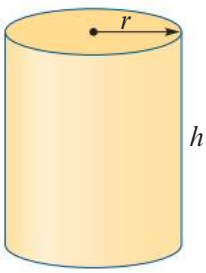
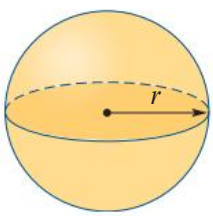
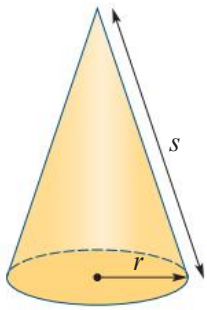
Solids with curved surfaces (cylinder, cone, sphere)

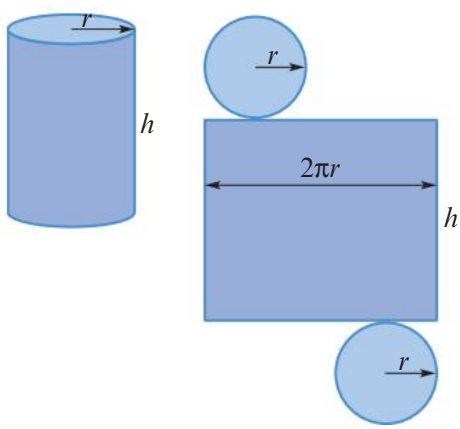
For some special objects, such as a cylinder, cone and sphere, formulas to calculate the surface area can be developed. The formulas for the surface area of a cylinder, cone and sphere are given below. Be mindful of which formulas are given in the QCAA Formula book and which ones you will need to memorise.

Shape	Surface area
Cylinder 	$SA = 2\pi rh + 2\pi r^2$ $= 2\pi r(h + r)$
Sphere 	$SA = 4\pi r^2$

Shape	Surface area
Cone 	$SA = \pi rs + \pi r^2$

To develop the formula for the surface area of a cylinder, we first draw a net, as shown. The **surface area (SA) of a cylinder** can therefore be found using:

$$\begin{aligned} SA &= \text{area of curved surface} + \text{area of ends} \\ &= \text{area of rectangle} + \text{area of 2 circles} \\ &= 2\pi rh + 2\pi r^2 \\ &= 2\pi r(h + r) \end{aligned}$$



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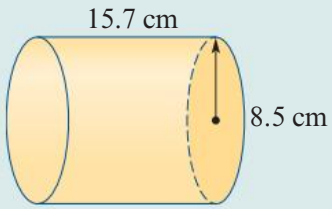
Example 28 Calculating the surface area of a cylinder

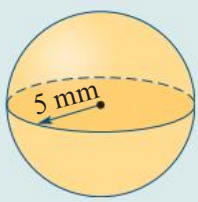
Calculate the surface area of this cylinder, correct to one decimal place.



Solution	Explanation
$SA = 2\pi rh + 2\pi r^2$	Use the formula for the surface area of a cylinder, $SA = 2\pi rh + 2\pi r^2$.
$SA = 2 \times \pi \times 8.5 \times 15.7 + 2 \times \pi \times (8.5)^2$ $= 1292.451 \ldots$	Substitute $r = 8.5$, $h = 15.7$ and evaluate.
The surface area of the cylinder is 1292.5 cm^2, correct to one decimal place.	Give your answer correct to one decimal place and with correct units.

Example 29 Calculating the surface area of a sphere

Calculate the surface area of a sphere with radius 5 mm, correct to two decimal places.



Solution	Explanation
$SA = 4\pi r^2$	Use the formula $SA = 4\pi r^2$.
$SA = 4\pi \times 5^2$ $= 314.159 \ldots$	Substitute $r = 5$ and evaluate.
The surface area of the sphere is 314.16 mm^2, correct to two decimal places.	Give your answer correct to two decimal places and with correct units.

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Section Summary

- The **total surface area** of a solid is the sum of all the areas of all the surfaces.
- A **net** is a two-dimensional representation of all the surfaces of the solid.
- To calculate the surface area of a solid, use the net to help ensure all sides have been considered.
- Become familiar with the QCAA Formula Book. Know which formulas are included and which ones you will need to memorise. Practise using the formula book whenever you can.
- Use the formulas to calculate the surface area of standard solids in practical situations, including a cylinder, cone and sphere.

Skill-sheet

Exercise 2G

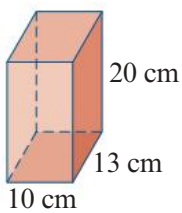
Surface areas of prisms and pyramids

Example 27

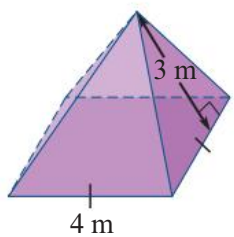
1 Calculate the surface areas of these prisms and pyramids. Where appropriate, give your answer correct to one decimal place.

SF

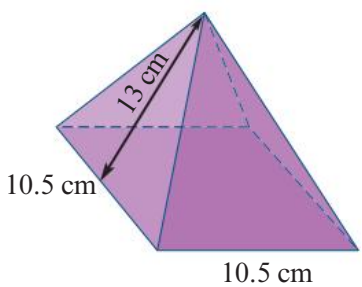
a



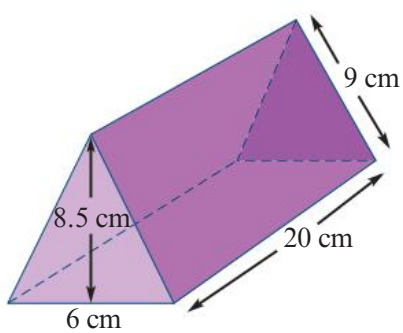
b



c



d



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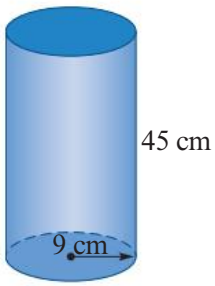
Surface area of curved surfaces

Example 28
Example 29

SF

2 Calculate the surface area of each of these solids with curved surfaces, correct to two decimal places.

a



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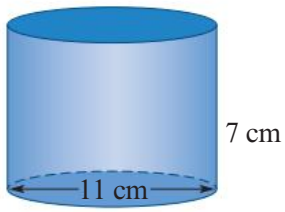

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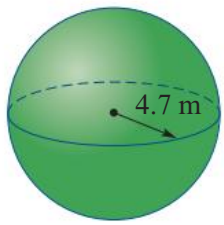

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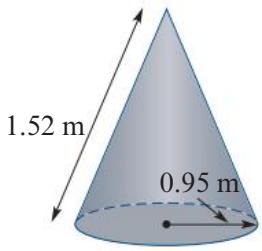
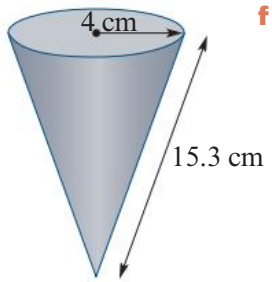
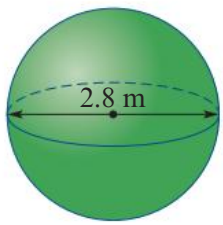


e



f

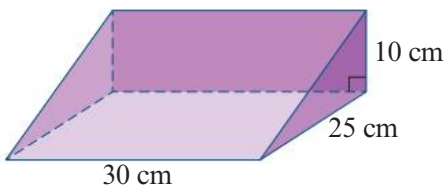


Surface area of composite surfaces

CF

3 Calculate the surface area of each of these solids, correct to two decimal places.

a



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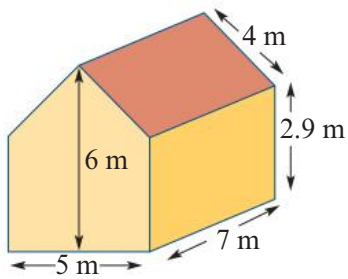

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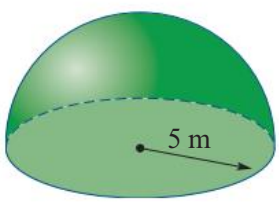
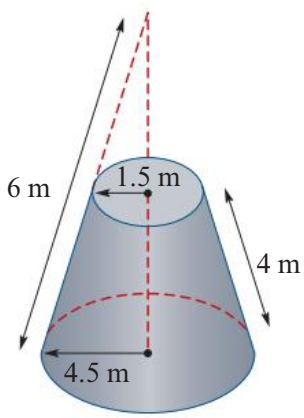


d



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Applications of surface areas

CF

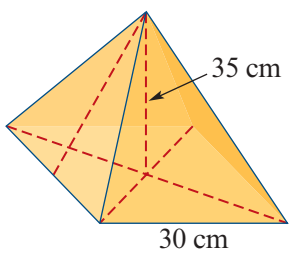
4 A tennis ball has a radius of 3.5 cm. A manufacturer wants to provide sufficient material to cover 100 tennis balls. Determine the amount of material that is required. Give your answer correct to the nearest cm^2.

5 A set of 10 conical paper hats are to be covered with material. The height of a hat is 35 cm and the diameter is 19 cm.

a Determine the amount of material, in m^2, that will be needed. Give your answer correct to two decimal places.

b Tinsel is to be placed around the base of the hats. Determine the amount of tinsel, to the nearest metre, that is required.

6 For a project, Mark has to cover all sides of a square-based pyramid with material, excluding the base. The pyramid has the dimensions as shown in the diagram. Calculate the amount of material that Mark will need to cover the sides of the pyramid. Give your answer in square metres, correct to two decimal places.



CU

7 Ethan made a Christmas ornament in the shape of a tetrahedron. A tetrahedron is a triangular prism that has four equilateral triangular faces. He sealed the outside using 12.5 mL of paint. If the paint he used has coverage of 5 cm^2 per mL, determine the side length of each equilateral triangle used in the tetrahedron.

8 A cube and a sphere have the same surface area. If the cube has sides of length 8 cm, determine the radius of the sphere.

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2H Problem-solving and modelling

Learning intentions

- To gain some experience with solving practical problems involving shape and measurement.
- To gain some experience with questions that require skills up to and including those from Unit 1 Topic 2.
- To gain some experience with questions that combine multiple skills into one question.
- To gain some experience with complex and unfamiliar questions.

Exercise 2H

1 Brandon is building a new rectangular deck that will measure 5 m by 3 m. He has to order concrete for the 20 rectangular stump holes that measure 350 mm by 350 mm by 500 mm. CF

a Determine the amount of concrete (in m^3) he will need to order. Give your answer correct to two decimal places.

b If the decking boards are 90 mm wide, determine the number of 3 m length boards that he should order.

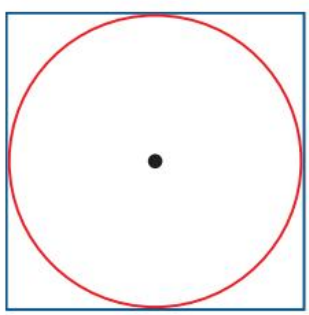
c The decking is to be stained. Calculate the total area of the deck.

2 Gloria has four wooden planter boxes. The boxes have a square base with side lengths of 40 cm and a height of 60 cm. CU

a Determine the volume of dirt, in cubic metres, required to fill these boxes if she fills them up to 10 cm from the top.

b She wants to paint the outside surface of the planter boxes, but not the base or the insides. Determine the total surface area in m^2 of the planter boxes that she needs to paint. Give the answer correct to one decimal place.

3 A 1 m piece of wire is to be cut into two pieces, one of which is bent into a circle (red). The other piece is bent into a square around the circle (blue).



a Determine the length of the side of the square, rounded to the nearest centimetre.

b Calculate the lengths of the two pieces of wire.

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4 Farmer Green owns an irregularly shaped paddock, *APQBRSTA*, as shown in the diagram.

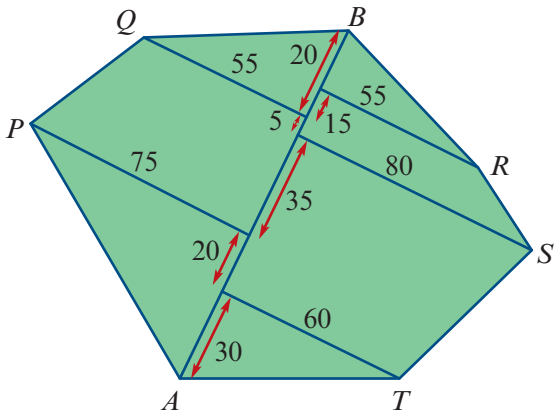
Starting at *A*, he has measured distances in metres along *AB* as well as the distances at right angles from *AB* to each other corner of his paddock.

Use this information to calculate:

a the total area of his paddock in hectares, correct to two decimal places

(Note: 1 hectare = 10 000 m^2)

b the length of fencing needed to enclose the paddock, correct to two decimal places.



5 A steel pipe that is 90 cm long has an outside diameter of 100 mm and an inside diameter of 80 mm. The pipe needs to be coated on the outside to protect against rust. If the protective material has a coverage of 16 m^2 per litre, calculate the amount of this protective material that is required to coat the outside of the pipe (not including the ends). Give your answer in millilitres.

6 Jess owns a company that creates the eyes for toy dolls where each eye is in the shape of a hemisphere and each doll has two eyes. The iris of the eye is a blue annulus painted in the centre of the flat circular end of the hemisphere. The centre of the annulus is painted black for the pupil of the eye. The radius of the pupil is 3 mm and the width of the iris is 7 mm. The plastic hemispherical eyeballs cost her $1.80 each. The blue paint costs her $1.50 per mL and the black paint costs her $0.75 per mL. Both paints can cover 8 cm^2 per mL. Jess pays herself piecework rates at $4.27 per eye. She can make eyes for a maximum of 250 dolls per day and expects to make $2000 each day. Comment on the reasonableness of her situation.

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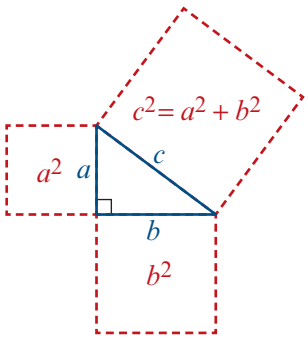
Key ideas & chapter summary

Pythagoras' theorem

Pythagoras' theorem states that for any right-angled triangle, the sum of the areas of the squares of the two shorter sides (a and b) equals the area of the square of the hypotenuse (c): $c^2 = a^2 + b^2$.



Conversion of measurements and capacities

Length: km $\overset{\times 1000}{\underset{\div 1000}{\rightleftarrows}}$ m $\overset{\times 100}{\underset{\div 100}{\rightleftarrows}}$ cm $\overset{\times 10}{\underset{\div 10}{\rightleftarrows}}$ mm

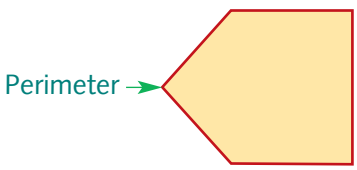
Area: $\text{km}^2 \overset{\times 1000^2}{\underset{\div 1000^2}{\rightleftarrows}} \text{m}^2 \overset{\times 100^2}{\underset{\div 100^2}{\rightleftarrows}} \text{cm}^2 \overset{\times 10^2}{\underset{\div 10^2}{\rightleftarrows}} \text{mm}^2$

Volume: $\text{km}^3 \overset{\times 1000^3}{\underset{\div 1000^3}{\rightleftarrows}} \text{m}^3 \overset{\times 100^3}{\underset{\div 100^3}{\rightleftarrows}} \text{cm}^3 \overset{\times 10^3}{\underset{\div 10^3}{\rightleftarrows}} \text{mm}^3$

1 kilolitre = 1000 litres
1 litre = 1000 millilitres
1 tonne = 1000 kilograms
1 kilogram = 1000 grams
1 gram = 1000 milligrams
1 m^3 = 1000 litres
1 cm^3 = 1 millilitre
1000 cm^3 = 1 litre

Perimeter (P)

Perimeter is the distance around the edge of a two-dimensional shape.



Perimeter formulas

Perimeter of polygon = sum of all sides

Perimeter of rectangle = $2l + 2w$

Perimeter of a sector = $2r + \frac{\theta}{180}\pi r$

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Review

Circumference (C)

Circumference is the perimeter of a circle: $C = 2\pi r$.

Area (A)

Area is the measure of the region enclosed by the boundaries of a two-dimensional shape.

Area formulas

Area of rectangle = lw

Area of parallelogram = bh

Area of triangle = $\frac{1}{2}bh$

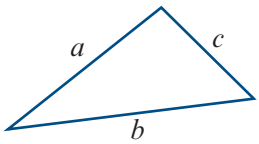
Area of trapezium = $\frac{1}{2}(a + b)h$

Area of a circle = πr^2

Area of a sector = $\frac{\theta}{360}\pi r^2$

Heron's rule

For the area of a triangle with three side lengths known:



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where

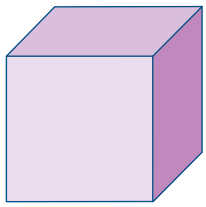
$$s = \frac{a+b+c}{2}$$

(s is the half perimeter)

Volume (V)

Volume is the amount of space occupied by a three-dimensional object.

- For prisms and cylinders:
  Volume = base area × perpendicular height
- For pyramids and cones:
  Volume = $\frac{1}{3}$ × base area × perpendicular height



Volume formulas

Volume of a prism = Ah

Volume of cube = s^3

Volume of rectangular prism = lwh

Volume of cylinder = $\pi r^2 h$

Volume of triangular prism = $\frac{1}{2}bhl$

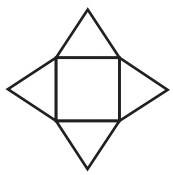
Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of pyramid = $\frac{1}{3}lwh$

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area (SA)

Surface area is the total of the areas of all the surfaces of a solid. When calculating surface area, it is often useful to draw the net of the shape.



Surface area formulas

Surface area of cylinder = $2\pi rh + 2\pi r^2$

Surface area of cone = $\pi rs + \pi r^2$

Surface area of sphere = $4\pi r^2$

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Skills checklist

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills.

2A	**1** I can use Pythagoras' theorem to solve for the length of the hypotenuse or an unknown side.	☐
	See Examples 1 and 2, and Exercise 2A Question 1.	
2A	**2** I can use Pythagoras' theorem to solve practical problems.	☐
	See Example 3 and Exercise 2A Questions 2 to 13.	
2B	**3** I can recognise right-angled triangles in solid figures and calculate unknown lengths in three dimensions.	☐
	See Examples 4 and 5, and Exercise 2B Questions 1 to 5.	
2B	**4** I can solve simple applications of Pythagoras' theorem in three dimensions.	☐
	See Example 6 and Exercise 2B Questions 6 to 13.	
2C	**5** I can convert between units.	☐
	See Examples 7 and 8, and Exercise 2C Questions 1 to 6.	
2C	**6** I can calculate the perimeter of regular shapes.	☐
	See Examples 9, 10 and 11, and Exercise 2C Question 7.	
2C	**7** I can calculate the area of regular shapes.	☐
	See Example 11 and Exercise 2C Questions 7 and 8.	
2C	**8** I can calculate the area of a triangle using Heron's rule.	☐
	See Example 12 and Exercise 2C Question 7.	
2C	**9** I can apply perimeter and area in practical situations.	☐
	See Example 13 and Exercise 2C Questions 10 to 17.	
2C	**10** I can calculate the perimeter and area of a composite shape in practical situations.	☐
	See Example 14 and Exercise 2C Questions 9, 14 and 15.	
2D	**11** I can calculate the circumference and area of a circle.	☐
	See Example 15 and Exercise 2D Question 1.	
2D	**12** I can apply calculating the perimeter and area of a circle in practical situations.	☐
	See Example 16 and Exercise 2D Questions 4, 6 to 10 and 13.	

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Review

2D	13	I can calculate the perimeter and area of a sector.	☐

See Example 17 and Exercise 2D Questions 2, 3 and 5.

2D	14	I can apply calculating the perimeter and area of a sector in practical situations.	☐

See Example 18 and Exercise 2D Questions 11 and 12.

2E	15	I can calculate the volume and capacity of a prism and cylinder.	☐

See Examples 19, 20 and 21, and Exercise 2E Questions 1 to 3.

2E	16	I can apply volume and capacity to a three-dimensional shape in practical situations.	☐

See Example 22 and Exercise 2E Questions 4 to 8.

2F	17	I can calculate the volume and capacity of a cone.	☐

See Example 23 and Exercise 2F Questions 1 and 2.

2F	18	I can calculate the volume and capacity of a pyramid.	☐

See Examples 24 and 25, and Exercise 2F Questions 3 and 4.

2F	19	I can calculate the volume and capacity of a sphere.	☐

See Example 26 and Exercise 2F Questions 5 to 7.

2F	20	I can apply volume and capacity to practical situations.	☐

See Exercise 2F and Questions 8 to 19.

2G	21	I can calculate the surface area of solids with plane faces (prisms and pyramids).	☐

See Example 27 and Exercise 2G Question 1.

2G	22	I can calculate the surface area of solids with curved surfaces (cylinder, cone, sphere).	☐

See Examples 28 and 29, and Exercise 2G Question 2.

2H	23	I can apply surface area of solids to practical problems.	☐

See Exercise 2G Questions 3 to 8.

2H	24	I can apply shape and measurement to practical problems.	☐

See Exercise 2H Questions 1 to 5.

2H	25	I can apply consumer arithmetic to practical problems involving shape and measurement.	☐

See Exercise 2H Question 6.

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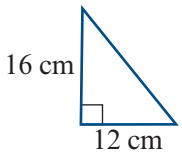
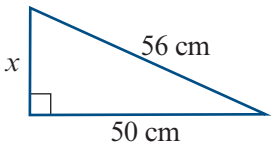
Multiple-choice questions

1 The length of the hypotenuse for the triangle shown is:



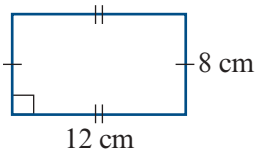
A 7.46 cm **B** 10.58 cm
C 20 cm **D** 28 cm

2 The value of x in the triangle shown is:



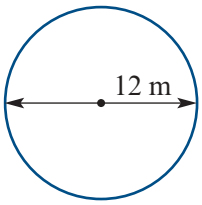
A 6 cm **B** 25.22 cm
C 75.07 cm **D** 116 cm

3 The perimeter of the rectangle shown is:



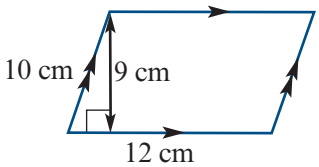
A 28 cm **B** 32 cm
C 40 cm **D** 96 cm

4 The circumference of a circle with diameter 12 m is closest to:



A 37.70 m **B** 113.10 m
C 118.44 m **D** 453.29 m

5 The area of the shape shown is:



A 44 cm^2 **B** 90 cm^2
C 108 cm^2 **D** 120 cm^2

6 The area of a circle with radius 3 cm is closest to:

A 18.85 cm^2 **B** 28.27 cm^2 **C** 31.42 cm^2 **D** 113.10 cm^2

7 The volume of a box with length 11 cm, width 5 cm and height 6 cm is:

A 44 cm^3 **B** 302 cm^3 **C** 330 cm^3 **D** 1650 cm^3

8 The volume of a cylinder with radius 3 m and height 4 m is closest to:

A 12.57 m^3 **B** 37.70 m^3 **C** 113.10 m^3 **D** 452.39 m^3

9 The volume of a sphere with radius 16 mm is closest to:

A 268.08 mm^3 **B** 1072.33 mm^3 **C** 3217 mm^3 **D** 17 157.28 mm^3

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Review

10 The volume of a cone with base diameter 12 cm and height 8 cm is closest to:

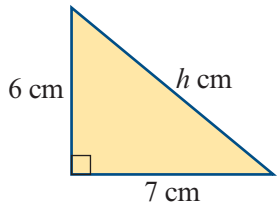
A 301.59 cm^3 **B** 904.78 cm^3 **C** 1206.37 cm^3 **D** 1809.56 cm^3

Short-response questions

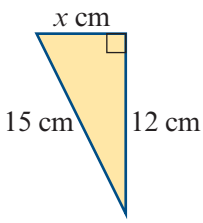
ST

1 Determine the lengths of the unknown sides, correct to two decimal places, in the following triangles.

a

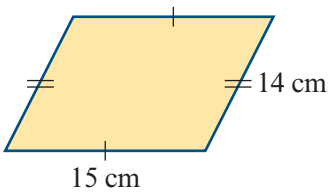


b

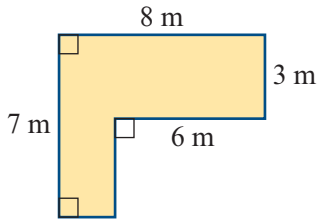


2 Calculate the perimeters of these shapes.

a



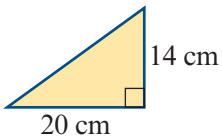
b



3 Determine the perimeter of a square with side length 9 m.

4 Calculate the areas of the following shapes.

a



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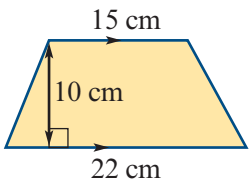
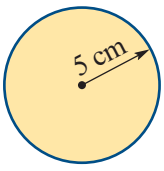


5 Determine the volume of a rectangular prism with length 3.5 m, width 3.4 m and height 2.8 m.

6 Determine the surface area of a cube with side length 2.5 m.

7 Calculate the circumference and area of the following circles, correct to two decimal places.

a



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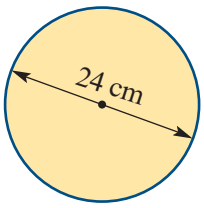

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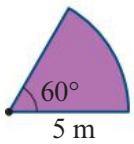
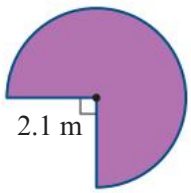
SF

8 Calculate the perimeter and area of the following sectors, correct to two decimal places.

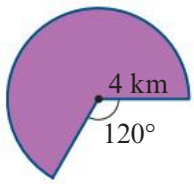
a



b



c



9 Convert the following measurement to the units given in the brackets.

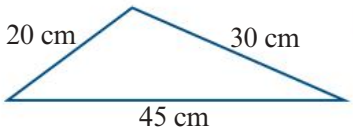
a 3.9 m (cm)
b 275 mm^2 (cm^2)
c 8 m^2 (cm^2)
d 0.05 m^3 (cm^3)
e 2800 mL (L)
f 189 cm^3 (mL)

10 The diameter of the base of an oil can in the shape of a cone is 12 cm and its height is 10 cm.

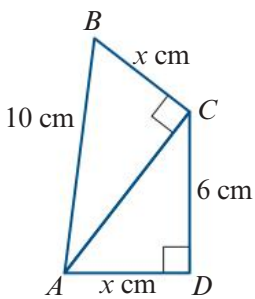
a Determine its volume in cubic centimetres, correct to two decimal places.
b Calculate its capacity to the nearest millilitre.

11 Calculate the area of the triangle shown.

CF



12 In the diagram, angles ACB and ADC are right angles. If BC and AD each have a length of x cm, calculate the value of x.



13 A circular swimming pool has a diameter of 4.5 m and a depth of 2 m. Determine the amount of water the pool will hold, to the nearest litre.

14 A soup can has a diameter of 7 cm and a height of 13.5 cm.

a Determine the area of metal sheeting, correct to two decimal places, that is needed to make the can.
b A paper label is made for the outside cylindrical shape of the can. Determine the amount of paper, in m^2, that is needed for 100 cans. Give your answer correct to two decimal places.
c Calculate the capacity of one can. Give your answer in litres, correct to two decimal places.

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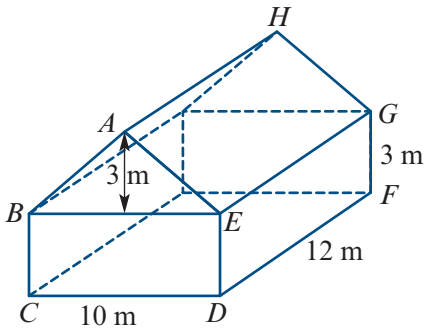
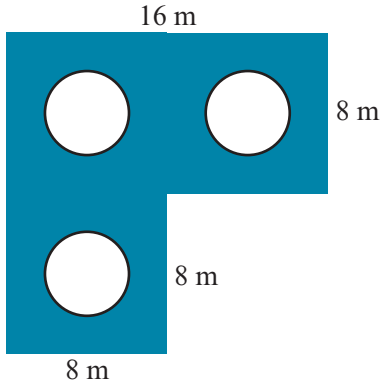
15 Calculate the total surface area of the solid shown. Give the answer correct to two decimal places.



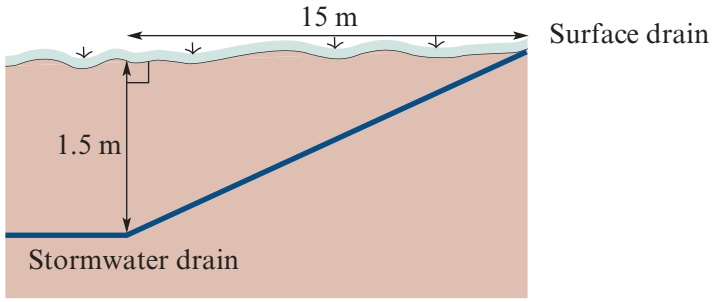
16 A lawn has three circular flowerbeds in it, as shown in the diagram. Each flowerbed has a radius of 2 m. A gardener has to mow the lawn and trim all the edges. Calculate:

a the area to be mown, correct to two decimal places.

b the length of the edges to be trimmed. Give your answer correct to two decimal places.



17 A builder is digging a trench for a cylindrical water pipe. From a drain at ground level, the water pipe goes 1.5 m deep, where it joins a stormwater drain. The horizontal distance from the surface drain to the stormwater drain is 15 m, as indicated in the diagram below *(not* to scale).



a Calculate the length of water pipe required to connect the surface drain to the stormwater drain, correct to two decimal places.

b If the radius of the water pipe is 20 cm, determine the capacity of the water pipe in litres. Give your answer correct to two decimal places.

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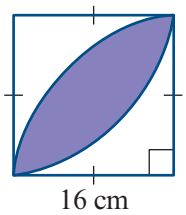
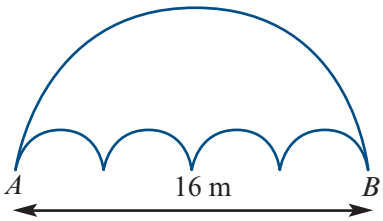
18 A right pyramid with a square base of side length 8 m has a height of 3 m. Determine the slant height of a sloping face, correct to one decimal place.

19 Determine the perimeter and area of the shaded region, correct to two decimal places.

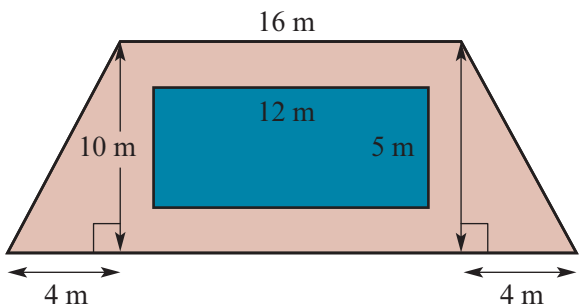


20 For a path from A to B, there is one along the four semicircles or another one along the larger semicircle. Determine which is the shortest path. Justify your answer with mathematical reasoning.



21 Chris and Gayle decide to build a swimming pool on their new house block. The design layout of the pool and surrounding area is shown in the diagram. They have budgeted $30 000 for the pool. Using the specifications listed below, comment on the reasonableness of their budget. Justify your answer with mathematical reasoning.

- The pool will measure 12 m by 5 m and have a constant depth of 2 m. The interior of the pool is to be sealed with special white paint. The paint has coverage of 8 m^2 per litre and costs $21 per litre.
- The pool will be surrounded by timber decking. The timber costs $180 per square metre.
- A safety fence will surround the decking. Fencing costs $82 per metre.
- The safety fence will need to include a gate that is 1 metre wide. The gate costs $50.



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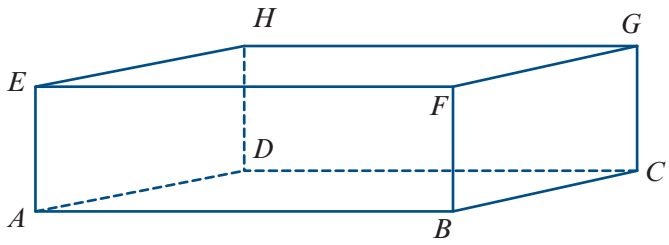
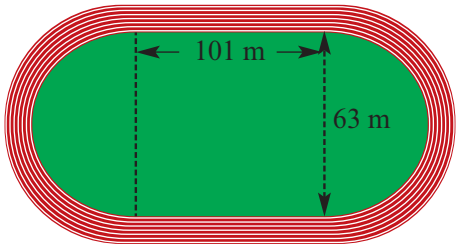
22 The length of a rectangular prism is eight times its height. The width is four times the height. The length of the diagonal between two opposite vertices (AG) is 36 cm. Calculate the volume of the prism.



23 An athletics track is made up of a straight stretch of 101 m and two semicircles on the ends as shown in the diagram. There are six lanes, each 1 metre wide.



If six athletes run around the track keeping to their own lane, indicate on the diagram at which point each runner should start so that they all run the same distance. Justify your answer with mathematical reasoning.

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Chapter 3

Similarity and scale

Chapter questions

UNIT 1 MONEY, MEASUREMENT AND RELATIONS

Topic 3: Similarity and scale

- What does it mean when we say that two figures are similar?
- What are the tests for similarity for triangles?
- What is a scale factor?
- How do we use a scale factor?
- What are scale drawings?
- How do we determine measurements from scale drawings?
- How do we know whether two solids are similar?
- How do we solve scaling problems?

This chapter covers similarity within two-dimensional shapes and three-dimensional solids.

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3A Similarity

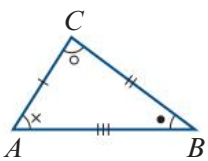
Learning intentions

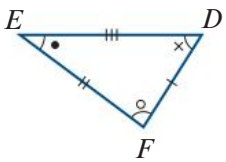
- To understand the conditions for similarity of two-dimensional figures.
- To understand the naming conventions of corresponding sides and angles of similar figures.
- To be able to calculate the scale factor between two similar figures.

Shapes that are similar have the same shape but are different sizes. These three frogs are **similar figures**.

For two figures, the corresponding sides and angles of the different figures are in the same position. For the two triangles shown, the corresponding angles and corresponding sides are listed.

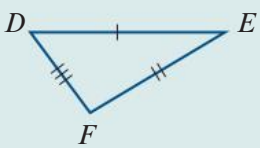
Corresponding sides	Corresponding angles
AB corresponds to DE	$\angle A = \angle D$
BC corresponds to EF	$\angle B = \angle E$
AC corresponds to DF	$\angle C = \angle F$

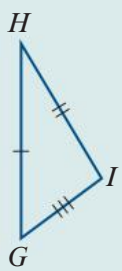




Example 1 Recognising corresponding sides and angles

Identify the corresponding sides and angles in these two figures.





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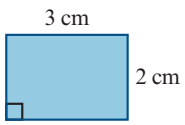
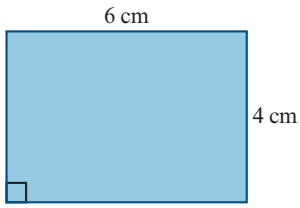
Solution	Explanation
DE corresponds to GH EF corresponds to HI FD corresponds to IG	Ensure the sides match when the two triangles are in the same orientation. Write the letters in the same order.
$\angle D = \angle G$ $\angle E = \angle H$ $\angle F = \angle I$	Ensure the angles match when the two triangles are in the same orientation.

Polygons (closed plane figures with straight sides), such as these rectangles, are similar if:

- corresponding angles are equal, and
- corresponding sides are proportional (each pair of corresponding side lengths are in the same ratio).



Rectangle A



Rectangle B

For example, the two rectangles above have corresponding angles that are equal and their side lengths are in the same ratio or scale factor.

$$\text{Ratio of side length} = 6 : 3 \text{ or } \frac{6}{3} = \frac{2}{1} = 2$$

$$\text{Ratio of side length} = 4 : 2 \text{ or } \frac{4}{2} = \frac{2}{1} = 2$$

When we enlarge or reduce a shape by a **scale factor**, the first shape (the *original*) and the second shape (the *image*) are **similar**.

Shapes can be scaled up or scaled down. When a shape is made larger, it is scaled up and when it is made smaller, it is scaled down. In the diagram above, the original rectangle A has been enlarged by a scale factor, $k = 2$, to give the image rectangle B. We could say that rectangle A has been scaled up to give rectangle B.

Scale factor

The **scale factor** $= \dfrac{\text{image length}}{\text{original length}}$.

- If the scale factor is between 0 and 1, the image will be smaller than the original.
- If the scale factor is greater than 1, the image will be larger than the original.
- If the scale factor is equal to 1, the image will be the same size as the original.

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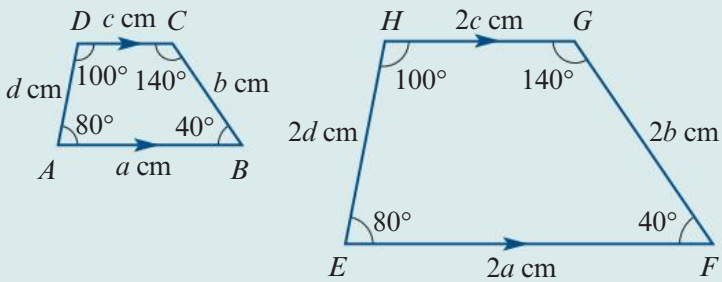
When determining scale factors, the numerator is the length of a side of the second shape (or image) and the denominator is the length of the corresponding side of the first shape (or original).

Example 2 Recognising similarity

State reasons why these two figures are similar.



Solution	Explanation
$\angle D = \angle H = 100°$	Compare corresponding angles.
$\angle C = \angle G = 140°$	
$\angle A = \angle E = 80°$	
$\angle B = \angle F = 40°$	
scale factor $= \frac{EF}{AB} = \frac{10}{5} = 2$	Determine the scale factor.
Since the corresponding angles are equal and the shapes are the same but different sizes, the figures are similar.	Explain.

Example 3 Calculating the scale factor

The two shapes shown are similar.

a Identify whether the first shape has been scaled up or down to give the second shape.

b Determine the scale factor.

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Solution	Explanation
a Shape has been scaled down.	Since the shape is made smaller it has been scaled down.
b $\frac{4}{6} = \frac{2}{3}$	The shapes are similar so their side lengths are in the same ratio. Compare the corresponding side lengths.
The scale factor is $\frac{2}{3}$.	Write your answer.

Section Summary

- Figures are similar when they are the same shape but different size.
- Corresponding angles and sides are in the same position of two similar figures.
- The scale factor is a ratio where the numerator is the side length of the image shape and the denominator is the side length of the original shape.

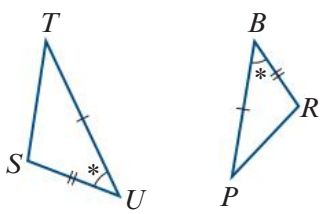
Exercise 3A

Similarity and ratios

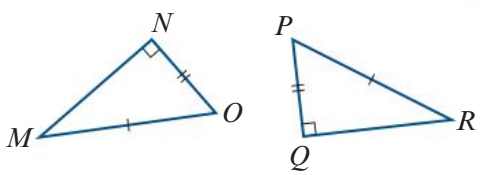
Example 1

1 List the corresponding sides and angles in these similar triangles.

a



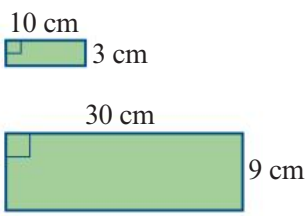
b



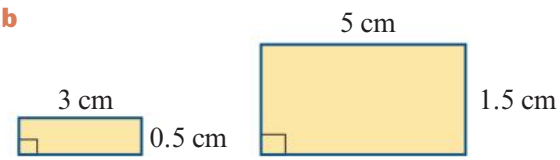
Example 2

2 Identify which of the following pairs of figures are similar. For those that are similar, determine the ratios of the corresponding sides. Assume that angles which look equal in these diagrams are equal.

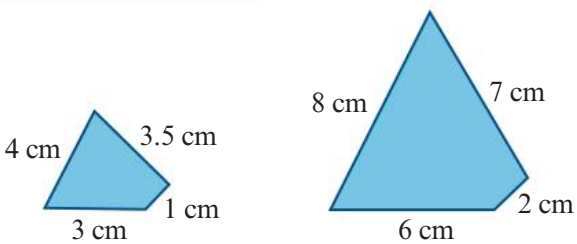
a



b



c



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SF

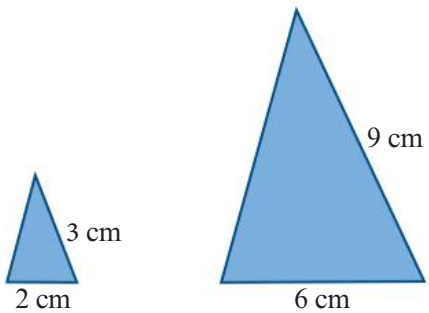
3 Identify which of the following pairs of figures are similar. State the scale factor of the corresponding sides where relevant.

- **a** Two rectangles 8 cm by 3 cm and 16 cm by 4 cm
- **b** Two rectangles 4 cm by 5 cm and 16 cm by 20 cm
- **c** Two rectangles 30 cm by 24 cm and 10 cm by 8 cm
- **d** Two triangles, one with sides measuring 3 cm, 4 cm and 5 cm and the other 4.5 cm, 6 cm and 7.5 cm.

Example 3

4 The following triangles are similar.

- **a** Identify whether the first shape has been scaled up or down to give the second shape.
- **b** Determine the scale factor.



Applications of similarity and ratios

5 A photo is 12 cm by 8 cm. It is to be enlarged by a scale factor of 2. State the dimensions of the new photo.

6 Determine the scale factor if a company's logo has been enlarged from 15 cm by 9 cm to 25 cm by 15 cm. Give your answer correct to two decimal places.

CF

7 A magazine advertisement is scaled from an A4 sheet of paper that is 297 mm by 210 mm to a roadside billboard that is 14.85 m by 10.5 m. Determine the scale factor required for this to happen.

8 A graphic designer claims he enlarged a diagram by a scale factor of 4.28 to make it fit on the larger screen. If the diagram on the screen is 34.5 cm by 19.4 cm, determine the dimensions of the original diagram. Round your answers to one decimal place.

CU

9 A triangle has sides in the ratio 3:4:5. If the sides of a larger similar triangle are four times as long and it has a perimeter of 96 cm, determine the side lengths of the smaller triangle.

10 A street name sign is 850 mm wide by 200 mm high. The letters printed on the sign are 120 mm tall. Determine the largest scale factor that the letters can be enlarge by before it they are too tall to fully appear on the sign.

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3B Similar triangles

Learning intentions

- To understand the conditions for two triangles to be similar.
- To know the four tests for similar triangles.
- To be able to use any of the tests to show two triangles are similar.

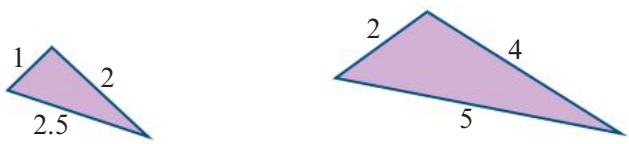
Two **triangles** are said to be **similar** if they have the same shape but different sizes. Two triangles can be tested for similarity by considering the following necessary conditions.

- Corresponding angles are equal (AAA).

Remember: If two pairs of corresponding angles are equal, then the third pair of corresponding angles is also equal.

- Corresponding sides are in the same ratio (SSS).

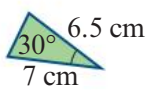
$\frac{5}{2.5} = \frac{4}{2} = \frac{2}{1} = 2$

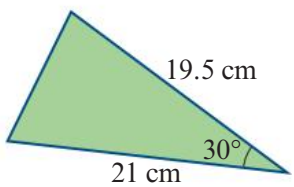


- Two pairs of corresponding sides are in the same ratio and the included corresponding angles are equal (SAS).

$\frac{19.5}{6.5} = \frac{21}{7} = 3$

Both triangles have an included corresponding angle of 30°.

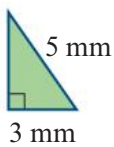


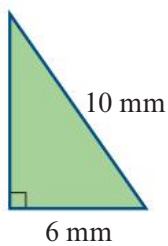


- The hypotenuse of two right-angled triangles and another pair of corresponding sides are in the same ratio (RHS).

$\frac{10}{5} = \frac{6}{3} = 2$

Both triangles have corresponding right angles.





Note: If the test AAA is not used, then at least two pairs of corresponding sides in the same ratio are required for all other three tests.

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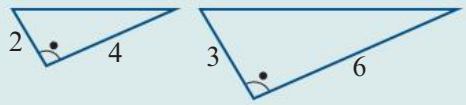
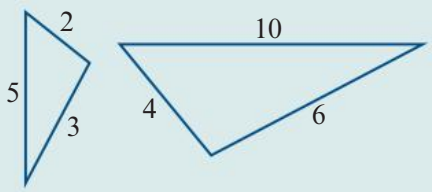
Example 4 Checking if triangles are similar

Explain why the triangles in each set are similar.

a



b



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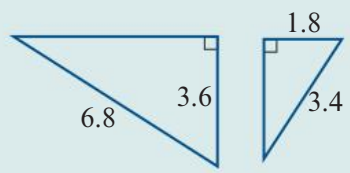

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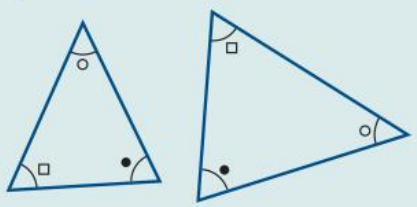

Solution

a $\frac{6}{4} = \frac{3}{2}$ and two corresponding angles are equal.
So the two triangles are similar by SAS.

b $\frac{6}{3} = \frac{4}{2} = \frac{10}{5} = 2$
So the two triangles are similar by SSS.

c $\frac{3.4}{6.8} = \frac{1.8}{3.6} = \frac{1}{2}$ and two corresponding right angles are equal.
So the two triangles are similar by RHS.

d Three corresponding angles are equal.
So the two triangles are similar by AAA.

Explanation

Show that two corresponding sides are in the same ratio and the included angles are equal.

Show that three corresponding sides are in the same ratio.

Show that two corresponding sides are in the same ratio and the right angles are equal.

Show that three corresponding angles are equal.

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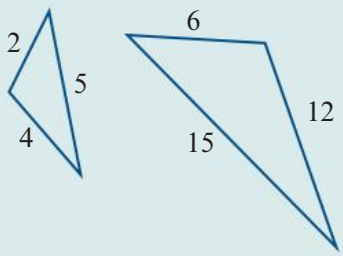
Example 5 Calculating the scale factor

The two triangles shown are similar.



a Identify whether the first shape has been scaled up or down to give the second shape.

b Determine the scale factor.

Solution	Explanation
a Shape has been scaled up.	Since the shape is made larger it has been scaled up.
b $\frac{15}{5} = \frac{12}{4} = \frac{6}{2} = 3$	The shapes are similar as their side lengths are in the same ratio. Compare the corresponding side lengths.
The scale factor is 3.	Write your answer.

Section Summary

- Two triangles are similar if they have the same shape but different sizes.
- There are four tests that can be used to show that two triangles are similar: AAA, SSS, SAS, and RHS.
- If three corresponding angles cannot be used to show similarity, then at least two pairs of corresponding sides in the same ratio are required.

Skill-sheet

Exercise 3B

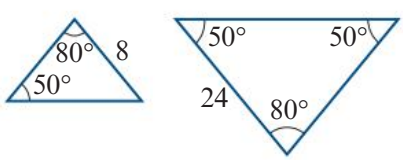
Similar triangles

Example 4
Example 5

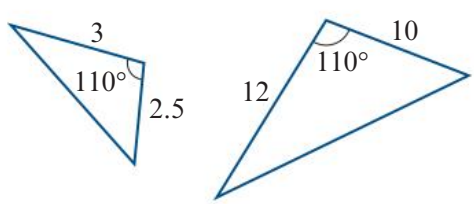
1 For each pair of triangles:

i explain why the triangles are similar **ii** determine the scale factor.

a



b



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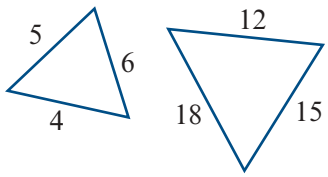
c



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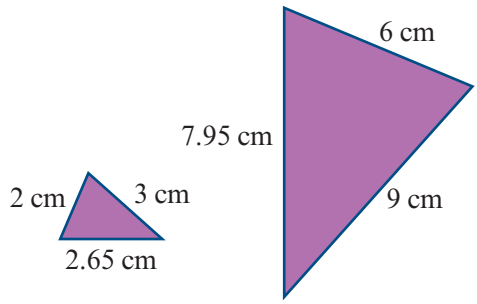
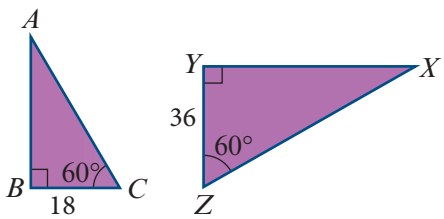


e



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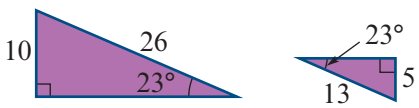
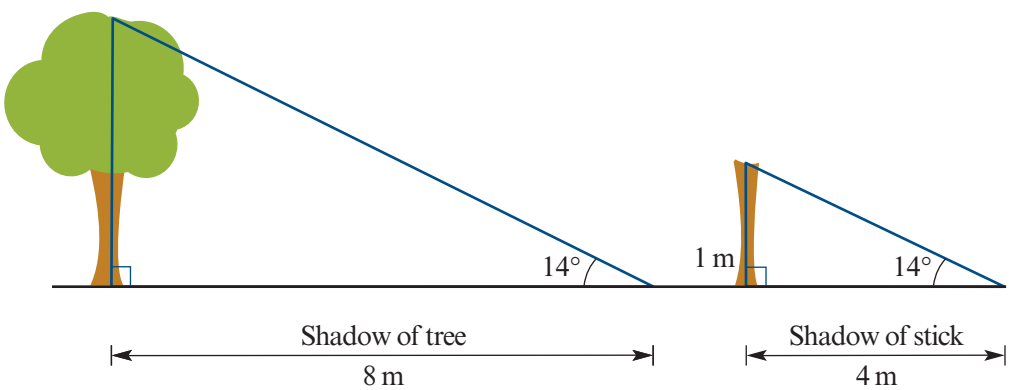


Applications of similar triangles

2 A tree and a 1 m vertical stick cast their shadows at a particular time in the day. The shadow lengths are shown in the diagram below (*not* drawn to scale).

a Give reasons why the two triangles shown are similar.

b Determine the scale factor for the side lengths of the triangles.



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3C Scaling similar figures

Learning intentions

- To be able to use the scale factor to determine side lengths of similar figures.
- To be able to use the scale factor to determine the areas of similar figures.

Once we know that two figures are similar and the scale factor that relates them, we can use the scale factor to calculate unknown side lengths.

Scaling lengths

When a shape is scaled, we multiply the lengths in the original figure by the scale factor, k, to determine the lengths in the image figure.

- If the scale factor is between 0 and 1, the image length will be smaller than the original.
- If the scale factor is greater than 1, the image length will be bigger than the original.
- If the scale factor is equal to 1, the image length will be the same as the original.

Example 6 Using a scale factor to determine unknown values

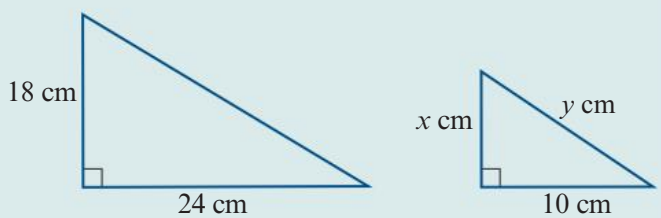
The following two triangles are similar.

a Determine the scale factor.

Use the scale factor to calculate:

b the value of x.

c the value of y.



Solution

a $\frac{10}{24} = \frac{5}{12}$

Triangle has been scaled down by a scale factor of $\frac{5}{12}$.

Explanation

As the triangles are similar, we can compare corresponding side lengths and simplify the fraction.

Remember: The numerator of the fraction is the length of a side of the second (or image) shape and the denominator is the length of the corresponding side of the first (or original) shape.

Write your answer.

Note: In this case, the triangle has been scaled down since the image is smaller than the original.

Continued on next page

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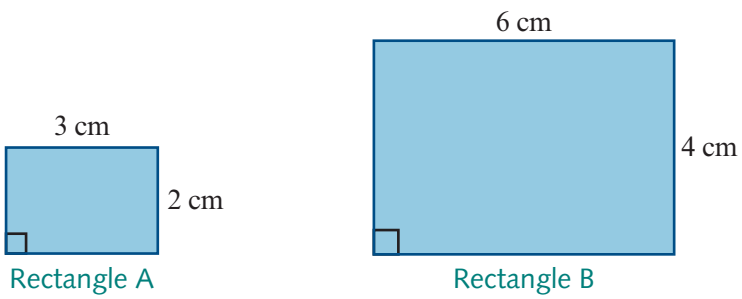
b $x = 18 \times \frac{5}{12}$	Calculate the value of x by multiplying the corresponding side by the scale factor.
$x = \frac{90}{12}$	
$x = 7.5$ cm	Evaluate and give your answer with correct units.
c $c^2 = 18^2 + 24^2$	Use Pythagoras' theorem to solve for the hypotenuse of the original triangle.
$c = \sqrt{18^2 + 24^2}$	
$c = 30$ cm	
$y = c \times \frac{5}{12}$	Calculate the value of y by multiplying the corresponding side by the scale factor.
$y = 30 \times \frac{5}{12}$	
$y = 12.5$ cm	Evaluate and give your answer with correct units.

We can also compare the ratio of the areas of similar figures. The two rectangles shown are similar since their corresponding side lengths are proportional. We notice that, as the length dimensions are enlarged by a scale factor of 2, the area is enlarged by a scale factor of $2^2 = 4$.



Area of rectangle A $= 6$ cm^2

Area of rectangle B $= 24$ cm^2

$$\text{Ratio of areas} = 24 : 6 = \frac{24}{6} = \frac{4}{1} = 4$$

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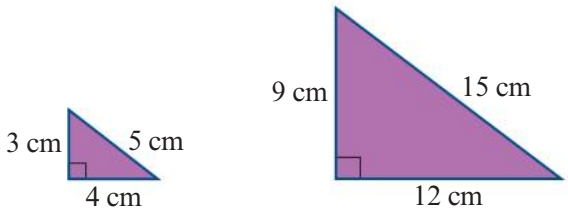
Scaling areas

When all the dimensions are multiplied by a scale factor of k, the area is multiplied by a scale factor of k^2.



For example, these two triangles are similar since their corresponding side lengths are in the same ratio.

$$\text{Scale factor, } k = \text{ratio of lengths} = \frac{15}{5} = \frac{9}{3} = \frac{12}{4} = \frac{3}{1} = 3$$

We would expect the area scale factor, k^2, or the ratio of the areas of the triangles to be 9 ($= 3^2$).

$$\text{Area of small triangle} = 6 \text{ cm}^2$$

$$\text{Area of large triangle} = 54 \text{ cm}^2$$

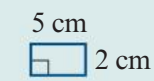
$$\text{Area scale factor, } k^2 = \text{Ratio of areas} = \frac{54}{6} = \frac{9}{1} = 9$$

Example 7 Calculating the ratio (scale factor) of dimension and area

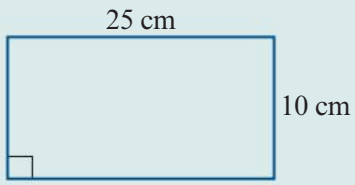
The rectangles shown are similar.

a Determine the ratio of their side lengths.

b Determine the ratio of their areas.



Rectangle A



Rectangle B

Solution

a $\frac{25}{5} = \frac{10}{2} = \frac{5}{1}$

The ratio of the side lengths is $\frac{5}{1}$.

Explanation

Since the rectangles are similar, their side lengths are in the same ratio. Compare the corresponding side lengths.

Write your answer.

Note: We can also say that the second rectangle has been scaled up by a factor of 5.

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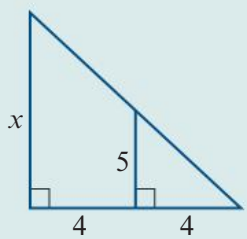
b $5^2 = 25$	Since the dimensions are multiplied by a scale factor of 5, the area will be multiplied by a scale factor of 5^2. Square the ratio of the side lengths.
The ratio of the areas is $\frac{25}{1}$.	Write your answer. Note: We can also say that the area of the second rectangle has been scaled up by a factor of 25.

Example 8 Using a scale factor to determine unknown areas

The following two triangles are similar, with measurements given in centimetres.

a Use the scale factor to calculate the area of the larger triangle.

b Comment on the reasonableness of your answer.



Solution	Explanation
a $\frac{8}{4} = 2$	As the triangles are similar, we can compare corresponding side lengths and simplify the fraction. Note: In this case, the side of the larger triangle is 4 + 4 = 8.
The larger triangle has been scaled up by a scale factor of 2.	
So, the area of the larger triangle will be scaled up by a factor of $2^2 = 4$.	Determine the scale factor of the areas.
A = area of small triangle × (scale factor)^2 $A = \left(\frac{1}{2} \times 4 \times 5\right) \times (2)^2$ $A = 40$	Calculate the area using the square of the scale factor.
Therefore the area of the larger triangle is 40 cm^2.	Evaluate and give your answer with correct units.

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b $x = 5 \times 2$
$x = 10$

As the triangles are similar and the second triangle has been scaled up, multiply by the scale factor to calculate the value of x.

$A = \frac{1}{2}bh$
$A = \frac{1}{2} \times 8 \times 10$
$A = 40 \text{ cm}^2$

Calculate the area of the larger triangle using the area formula.

The value of the area of the larger triangle found is reasonable since the area calculated using the scale factor and the area using the area formula are equal.

Compare the two methods and state your conclusion.

Section Summary

- The image shape is similar to the original shape with dimensions multiplied by the value of a scale factor.
- The numerator of the scale factor is from the image shape and the denominator of the scale factor is from the original shape.
- When a shape is scaled, multiply the length in the original figure by the scale factor, k, to determine the length in the image figure.
  - If the scale factor is between 0 and 1, the image length will be smaller than the original.
  - If the scale factor is greater than 1, the image length will be bigger than the original.
  - If the scale factor is equal to 1, the image length will be the same as the original.
- When there is a scale factor of k, the areas have a scale factor of k^2.

Exercise 3C

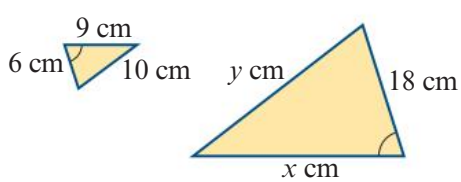
Similarity and ratios

Example 6

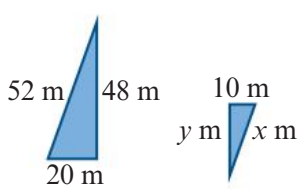
1 Calculate the missing dimensions, marked x and y, in these pairs of similar figures.

SF

a

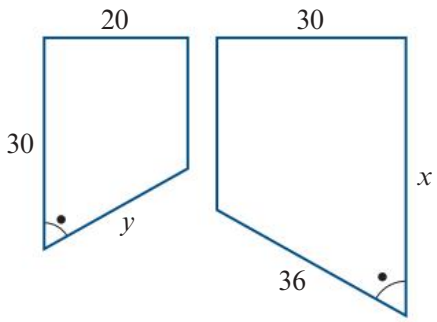


b

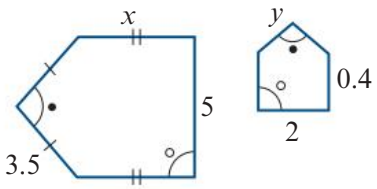


SF

c



d



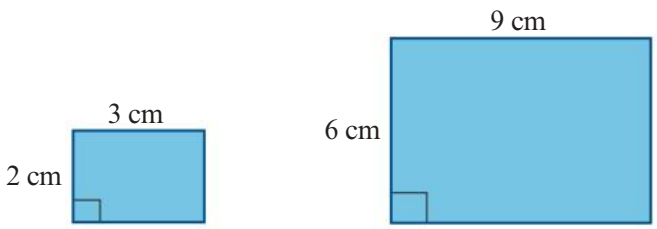
Example 7

2 The following pairs of figures are similar.

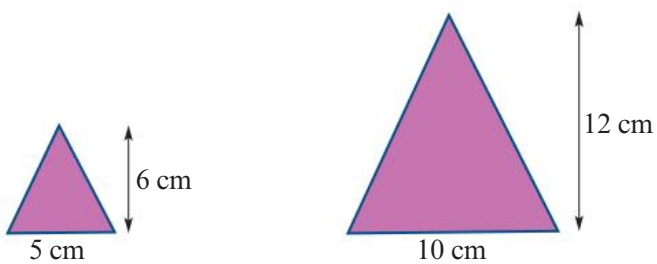
i Determine the ratio of their side lengths.

ii Determine the ratio of their areas.

a



b



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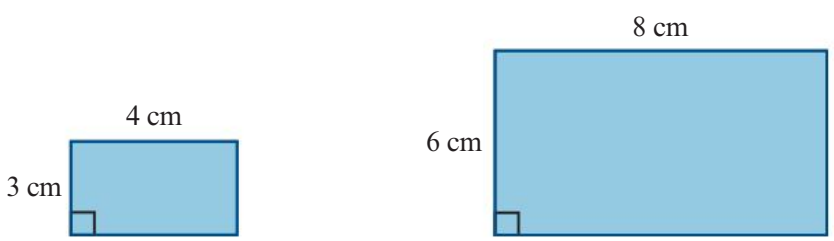

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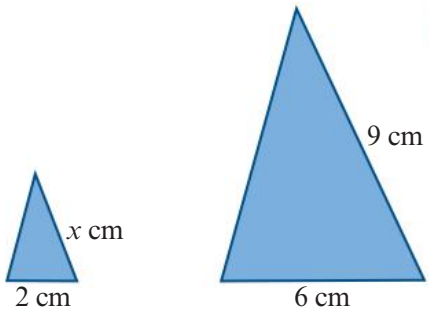
Example 6

Example 7

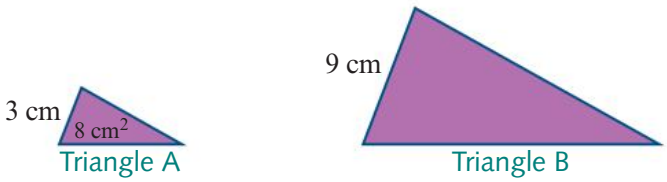
3 The triangles shown to the right are similar.

a Determine the value of x.

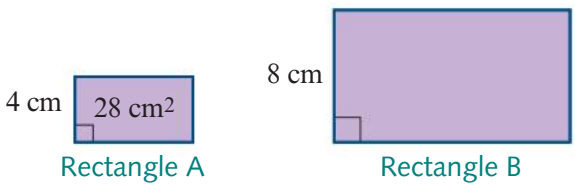
b Calculate the ratio of their areas.



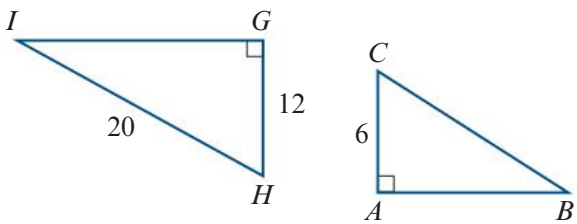
4 The area of triangle A is 8 cm^2. Triangle B is similar to triangle A. Determine the area of triangle B.



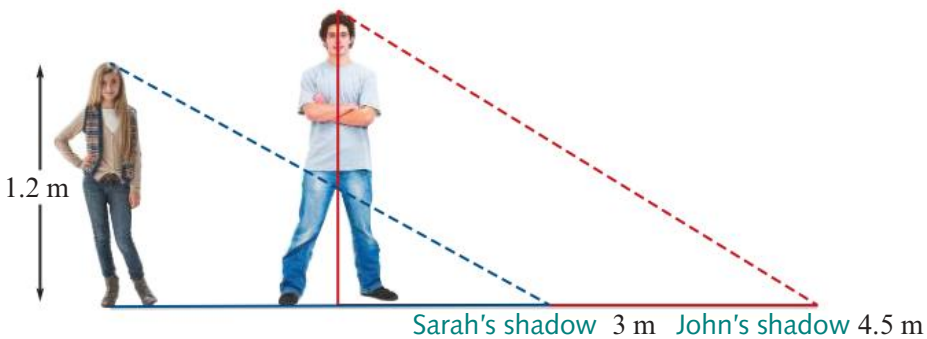
5 The two rectangles shown to the right are similar. The area of rectangle A is 28 cm^2. Determine the area of rectangle B.



6 The two triangles shown are similar. The area of the first triangle is 96 cm^2. Determine the area of the second triangle.



7 John and his younger sister, Sarah, are standing side by side. Sarah is 1.2 m tall and casts a shadow 3 m long. Determine John's height if his shadow is 4.5 m long.



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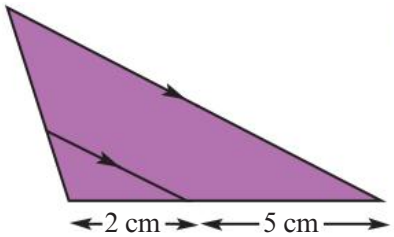
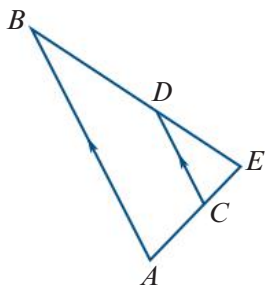
Example 8

8 Given that the area of the small triangle in the following diagram is 2.4 cm^2, and the two triangles are similar, determine the area of the larger triangle, correct to two decimal places.



CF

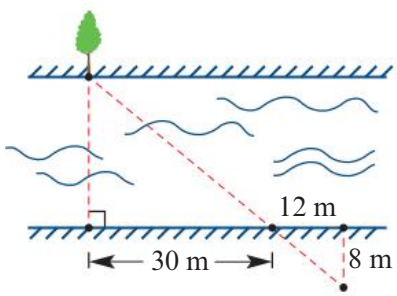
9 In triangle ABE, point C lies on side AE and point D lies on side BE. The lines CD and AB are parallel. The length of ED is 5 cm, the length of DB is 7 cm and the length of CD is 6 cm. Calculate the length of AB.



Applications of similarity and ratios

10 A photo is 12 cm by 8 cm. It is to be enlarged and then framed. If the dimensions are tripled, determine the area of the new photo.

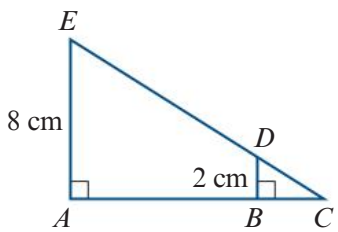
11 Audrey would like to know the width of this river without crossing it. She measures out four pegs as shown in the diagram. Calculate the width of the river.



CU

12 A triangle with sides 5 cm, 4 cm and 8 cm is similar to a larger triangle with a longest side of 56 cm. Calculate the perimeter of the larger triangle.

13 If the two triangles are similar and the area of ΔEAC is 100 cm^2, determine the length of AB.



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3D Scale drawings

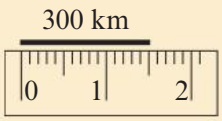
Learning intentions

- To be able to interpret the scale given in a map or scale drawing.
- To be able to use the map scale to determine measurements in real life.
- To be able to determine measurements from scale drawings to solve problems.

Using scale drawings to calculate actual measurements is a useful skill. This can be helpful when using a map. The scale ratio on the map indicates the relationship between the distance on the scale drawing and the distance in real life.

Map scale

The **scale** on the map shows the ratio of the map distance to the actual distance. This can be presented on the map with numbers or a picture.



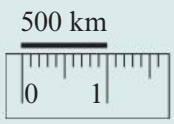
is a **line segment scale** or

map scale ratio = map distance : actual distance

The **actual distance** is the distance measured on the ground in real life. The **map distance** is the distance measured with a ruler on the map.

Example 9 Converting using a line segment scale

A map has the following line segment scale, marked in mm and cm:



Calculate the actual distance for each of the following scaled distances.

a 45 mm **b** 4 cm

Solution

a The scale indicates that 500 km in real life is 10 mm on the map. So, 1 mm on the map is equal to 50 km in real life.

Actual distance = 45 mm × 50
= 22 500 km

Explanation

Interpret the line segment scale given.

Multiply 45 mm by the scale factor of 50.

Continued on next page

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b The scale indicates that 500 km in real life is 10 mm on the map. So 1 cm on the map is equal to 500 km in real life since 10 mm = 1 cm.	Interpret the line segment scale given.
Actual distance = 4 cm × 500 = 2000 km	Multiply 4 cm by the scale factor 500.

Example 10 Converting using a scale ratio

A map has a scale of 1 : 30 000. Calculate the actual distance for each of the following scaled distances.

a 220 mm **b** 15.9 cm

Solution	Explanation
The scale indicates that 30 000 units in real life is 1 unit on the map.	Interpret the map scale given.
a Actual distance = 220 mm × 30 000 = 6 600 000 mm = 6600 m = 6.6 km	Multiply 220 mm by the scale factor of 30 000 giving an answer in mm. Then convert mm to km.
b Actual distance = 15.9 cm × 30 000 = 477 000 cm = 4770 m = 4.77 km	Multiply 15.9 cm by the scale factor 30 000 to give an answer in cm. Then convert cm to km.

Example 11 Using scaling in maps

A landscape gardener is planning the layout of a garden. She sketches a plan using a scale of 1:150.

1 A shrub on her plan is drawn with a diameter of 3 cm. Determine the diameter of the shrub in metres.

2 The largest feature of the garden is the rear fence, which is 30 metres long. Determine the measurement for this fence on her plan in centimetres.

Solution	Explanation
a Actual distance = 3 × 150 = 450 cm = 4.5 m	Multiply this distance by 150 to get the actual diameter in cm. Then convert to m.
The shrub has a diameter of 4.5 m.	Write your answer.

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b Plan distance = 30 ÷ 150 = 0.2 m = 20 cm The fence will be 20 cm long on the plan.	Divide this distance by 150. Then convert to cm. Write your answer.

The map scale can be used to calculate actual distances in real life. Adventurers must be able to calculate distances using national park maps, orienteering maps and topological maps.

Example 12 Using a line segment scale in maps

Answer the following questions using the map of New Zealand shown.

a Measure the line segment scale given in millimetres and interpret the scale.

b Calculate the actual distance between Auckland and Invercargill, rounding your answer to the nearest 10 km.

Solution	Explanation
a The scale indicates that 200 km in real life is 12 mm on the map. So, 1 mm on the map is equal to 16.7 km in real life.	Interpret the map scale given.

Continued on next page

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b Map distance = 72 mm	Measure directly on the map to determine the distance between Auckland and Invercargill in millimetres.
Actual distance = 72 mm × 16.7 = 1202.4 km	Multiply 72 mm by the scale factor 16.7 to give the distance in km.
The distance between Auckland and Invercargill is 1200 km, rounded to the nearest 10 km.	Write your answer, rounded as required.

Measuring distance from a map can be useful for estimating the time it will take to travel between places.

Example 13 Applying scaling

A map has a scale of 1 : 20 000. The measurement on the map between two towns is 5.4 cm.

a Calculate the actual distance between these two towns. Give your answer in kilometres, correct to two decimal places.

b Determine the number of minutes it would take to cycle from one town to the other if travelling at 12 kilometres per hour.

Solution	Explanation
a Method 1	
A scale of 1 : 20 000 means that 1 cm on the map represents 20 000 cm on the ground.	Interpret the map scale given.
Actual distance = 5.4 × 20 000 = 108 000 cm = 1080 m = 1.08 km	Multiply 5.4 cm by 20 000 to calculate the actual distance between the two towns in centimetres. Then convert cm to km.
Method 2	
A scale of 1 : 20 000 can also be written as a scale factor of $\frac{1}{20\,000}$.	Interpret the map scale given.
Let x be the actual distance. $\frac{5.4}{x} = \frac{1}{20\,000}$	The ratio of the distance on the map to the actual distance will be the same as the scale factor of $\frac{1}{20\,000}$. Write out the corresponding ratios.

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$5.4 \times 20\,000 = x$ $x = 108\,000$ cm $= 1.08$ km	Solve for x by cross-multiplying. Then convert to kilometres.
The distance between the two towns is 1.08 km.	Write your answer.
b $d = r \times t$	Use the rule distance = rate × time.
$1.08 \text{ km} = 12 \text{ km/h} \times t$ $t = \frac{1.08}{12}$ $= 0.09$ hours	Solve for time t.
$0.09 \times 60 = 5.4$ minutes	Covert to appropriate units and write your answer.

Section Summary

- A map scale shows the relationship between the distance on the map and the distance in real life.
- Typically, use multiplication with the map scale to determine the distance in real life and use division to determine the distance on the map.

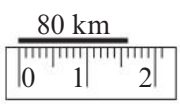
Exercise 3D

Converting using map scales

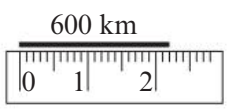
Example 9

1 The following line segment scales have a ruler showing cm and mm. Interpret the scale given and calculate the actual distance for the scaled distance given.

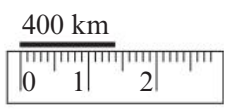
a 32 mm



b 14 mm



c 29 cm



Example 10

2 Calculate the actual distance for each of the scaled distances. Give your answer in appropriate units.

a scale 1 : 10 000

i 8 cm **ii** 230 mm

b scale 1 : 2000

i 6 cm **ii** 800 mm

c scale 1 : 8500

i 190 mm **ii** 2.86 cm

SF

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SF

Example 11

3 A scale on a map is 1 : 500 000.

a Determine the actual distance between two towns if the distance on the map is 7.2 cm. Give your answer in kilometres.

b If the actual distance between two landmarks is 15 km, calculate the distance between them on the map.

4 A plan of a rectangular shed has been drawn to a scale of 1 : 50.

a If the plan length of the shed is 8.5 cm, determine the actual length of the shed in metres.

b The actual perimeter of the shed is 25 m. Calculate the plan perimeter in centimetres.

Example 12

5 Answer the following questions using the map of Queensland shown below.

a Measure the line segment scale given in millimetres and interpret the scale, rounded to one decimal place.

b Calculate the actual distance between the two locations listed, rounding your answer to the nearest 10 km.

i Mt Isa and Brisbane

ii Rockhampton and Longreach

iii Toowoomba and Weipa

iv Mt Isa and Cairns

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Example 13

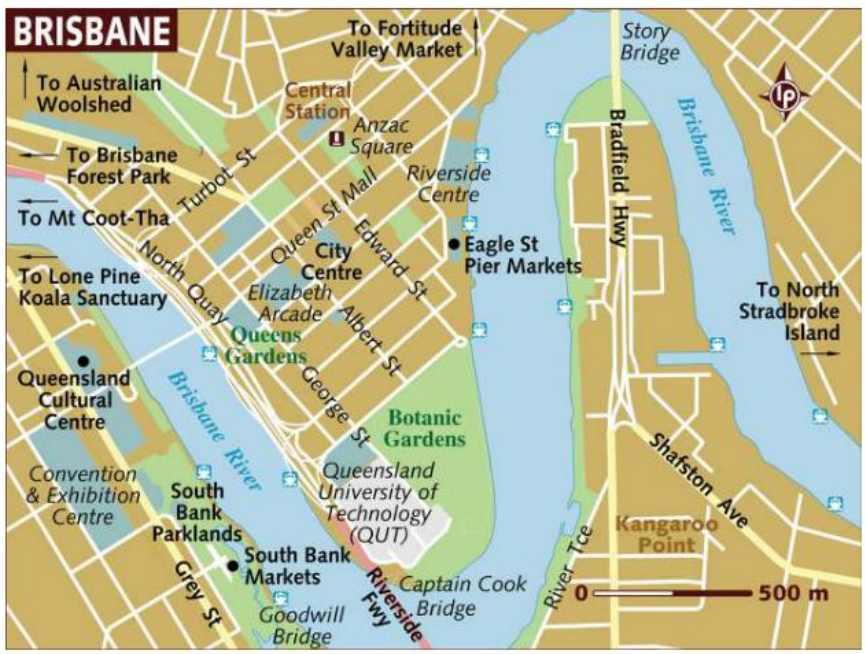
6 Answer the following questions using the map of Brisbane shown below.

a Calculate the actual distance between the Queensland Cultural Centre and South Bank Markets.

b Determine the number of minutes it would take to walk between the Queensland Cultural Centre and South Bank Markets if travelling at 5 kilometres per hour.

c Calculate the actual distance between the ferry stops of South Bank Parklands and the QUT campus across the river, marked on the map by little boat symbols.

d Determine the approximate time it would take for a ferry to get from South Bank Parklands to QUT if the ferry travels at 10 km/h.



7 A map has a scale of 1 : 2500. The measurement on the map between two buildings in a city is 86 mm.

a Calculate the actual distance between these two buildings. Give your answer in metres, correct to one decimal place.

b Determine the number of seconds it would take for a helicopter to fly from one building to the other if travelling at 50 metres per second.

8 A rectangular swimming pool has a width of 3.5 m and an area of 52.5 m^2. It is represented on a 1 : 25 scale plan. Determine the plan area of the pool in cm^2.

9 A concreter reads a 1 : 120 scale plan showing a square helipad. The plan length of the side of the helipad is 12.5 cm. If the helipad is constructed to a depth of 15 cm, calculate the volume of concrete required in m^3.

CF

CU

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3E Similar solids

Learning intentions

- To be able to use the scale factor to determine the surface area of similar solids.
- To be able to use the scale factor to determine the volume and capacity of similar solids.

Two solids are similar if they have the same shape and the ratios of their corresponding linear dimensions are equal. We already know that when scaling areas, multiplying by the scale factor of k^2 gives the enlarged area. This also applies to the surface areas of similar solids.

Scaling surface areas

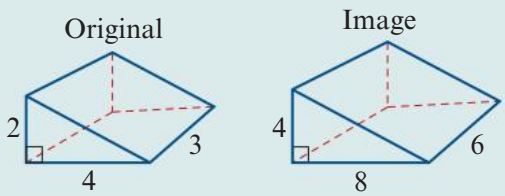
When all the dimensions are multiplied by a scale factor of k, the surface area is multiplied by a scale factor of k^2.

Example 14 Using a scale factor to determine unknown surface areas

The following two triangular prisms are similar with measurements given in centimetres.

a Use the scale factor to calculate the surface area of the image triangular prism.

b Comment on the reasonableness of your answer.



Solution

a $\frac{4}{2} = \frac{8}{4} = \frac{6}{3} = 2$

The image triangular prism has side lengths that have been enlarged by a scale factor of 2.

So the surface area of the image triangular prism will be scaled up by a factor of $2^2 = 4$.

Explanation

As the triangles are similar, we can compare corresponding side lengths and simplify the fractions.

Determine the scale factor of the surface areas.

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$$SA_{\text{original}} = \left(\frac{1}{2} \times 4 \times 2\right) \times 2 + 4 \times 3 + 2 \times 3 + 3 \times \sqrt{20}$$
$$= 8 + 12 + 6 + 3\sqrt{20}$$
$$= 39.4164\ldots$$

Calculate the surface area of the original triangular prism.

Note: The hypotenuse of the front triangular face is $\sqrt{20}$ by Pythagoras' theorem.

$$SA_{\text{image}} = 39.4164\ldots \times 2^2$$
$$= 157.6656\ldots$$

Since the prisms are similar, use the square of the scale factor to calculate the surface area of the image triangular prism.

Therefore the surface area of the image triangular prism is 157.67 cm^2.

Evaluate and give your answer with correct units.

b $$SA_{\text{image}} = \left(\frac{1}{2} \times 4 \times 8\right) \times 2 + 8 \times 6 + 4 \times 6 + 6 \times \sqrt{80}$$
$$= 32 + 48 + 24 + 6\sqrt{80}$$
$$= 157.6656\ldots$$

Calculate the surface area of the image triangular prism.

Note: The hypotenuse of the front triangular face is $\sqrt{80}$ by Pythagoras' theorem.

The value of the surface area of the image triangle found is reasonable since the surface area calculated using the scale factor and the surface area using its faces give equal values.

Compare the two methods and state your conclusion.

Scaling volumes

When all the dimensions are multiplied by a scale factor of k, the volume is multiplied by a scale factor of k^3.

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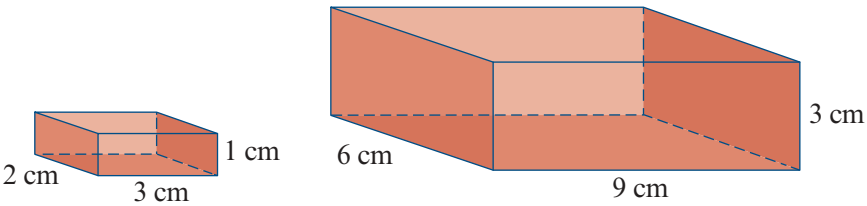
Rectangular prisms



Rectangular Prism B Rectangular Prism A

The two rectangular prisms are similar because:

- they are the same shape (both are rectangular prisms)
- the ratios of the corresponding dimensions are the same.

$$\frac{\text{length of prism A}}{\text{length of prism B}} = \frac{\text{width of prism A}}{\text{width of prism B}} = \frac{\text{height of prism A}}{\text{height of prism B}}$$

$$\frac{6}{2} = \frac{9}{3} = \frac{3}{1} = 3$$

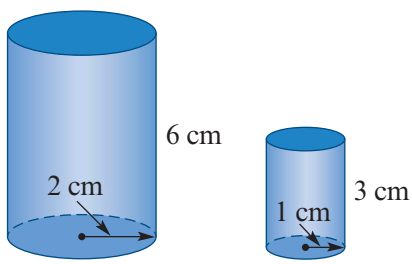
$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{6 \times 9 \times 3}{2 \times 3 \times 1} = \frac{162}{6} = 27 = 3^3$$

As the length dimensions are enlarged by a scale factor of 3, the volume is enlarged by a scale factor of $3^3 = 27$.

Cylinders

These two cylinders are similar because:

- they are the same shape (both are cylinders)
- the ratios of the corresponding dimensions are the same.



Cylinder A Cylinder B

$$\frac{\text{height of cylinder B}}{\text{height of cylinder A}} = \frac{\text{radius of cylinder B}}{\text{radius of cylinder A}}$$

$$\frac{3}{6} = \frac{1}{2}$$

$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{\pi \times 1^2 \times 3}{\pi \times 2^2 \times 6} = \frac{3}{24} = \frac{1}{8} = \left(\frac{1}{2}\right)^3$$

As the length dimensions are reduced by a scale factor of $\frac{1}{2}$, the volume is reduced by a scale factor of $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

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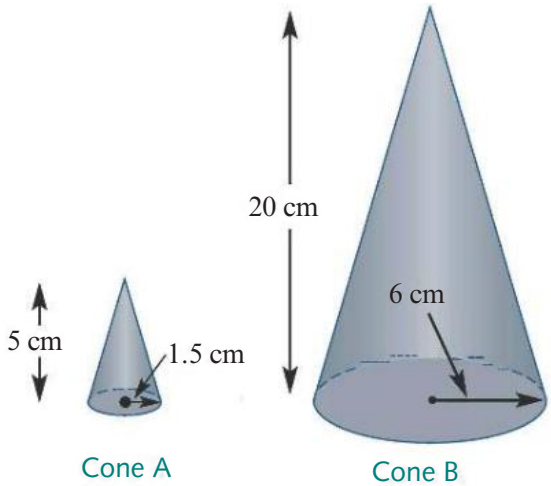
Cones

These two cones are similar because:

- they are the same shape (both are cones)
- the ratios of the corresponding dimensions are the same.

$$\frac{\text{height of cone B}}{\text{height of cone A}} = \frac{\text{radius of cone}}{\text{radius of cone}}$$

$$\frac{20}{5} = \frac{6}{1.5}$$



$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{\frac{1}{3} \times \pi \times 6^2 \times 20}{\frac{1}{3} \times \pi \times 1.5^2 \times 5} = \frac{720}{11.25} = 64 = 4^3$$

As the length dimensions are enlarged by a scale factor of 4, the volume is enlarged by a scale factor of $4^3 = 64$.

Example 15 Comparing volumes of similar solids

Two solids are similar such that the larger solid has all of its dimensions three times that of the smaller solid. Determine the number of times bigger the larger solid's volume is compared to the smaller solid.

Solution	Explanation
$3^3 = 27$	Since all of the larger solid's dimensions are 3 times those of the smaller solid, the volume will be 3^3 times as large. Evaluate 3^3.
The volume of the larger solid is 27 times the volume of the smaller solid.	Write your answer.

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Example 16 Calculating capacities of similar solids

A property has a water tank in the shape of cylinder which has a diameter of 7 m and height of 3 m. The grain silo on the property is a similar cylinder that has dimensions four times the size of the water tank. Use a scale factor to determine the capacity of the grain silo. Give your answer in litres, rounded to the nearest thousand.

Solution	Explanation
$4^3 = 64$	Since all of the larger solid's dimensions are 4 times those of the smaller solid, the volume will be 4^3 times larger.
The volume of the grain silo is 64 times the volume of the water tank.	Determine the scale factor of the volumes.
$V_{\text{water tank}} = \pi \times 3.5^2 \times 3$ $= 115.45353\ldots$	Calculate the volume of the water tank. Note: The radius of the water tank is 3.5 m.
$V_{\text{grain silo}} = V_{\text{water tank}} \times (\text{scale factor})^3$ $= 115.45353\ldots \times 64$ $= 7389.025921\ldots$	Calculate the volume of the grain silo using the cube of the scale factor.
So, the volume of the grain silo is 7389.03 m^3.	Evaluate.
Capacity = 7389.03 $m^3 \times 1000$ litres/m^3 = 7 389 000 litres	Convert to litres and give your answer in the correct units. Remember: 1 m^3 = 1000 litres

Section Summary

- When there is a scale factor of k, the surface areas have a scale factor of k^2.
- When there is a scale factor of k, the volumes have a scale factor of k^3.

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Skill-sheet

Exercise 3E

Scaling volumes and surface areas

Example 14

1 Determine the scale factor of the surface areas of two similar pyramids whose sides have a scale factor of 8.

Example 15

2 Two cylindrical water tanks are similar such that the height of the larger tank is 3 times the height of the smaller tank. Determine the number of times bigger the volume of the larger tank is compared to the volume of the smaller tank.

3 Determine the scale factor of the volumes of two similar rectangular prisms whose sides have a scale factor of 3.

4 Two similar spheres have diameters of 6 cm and 12 cm, respectively.

a Calculate the ratio of their surface areas.

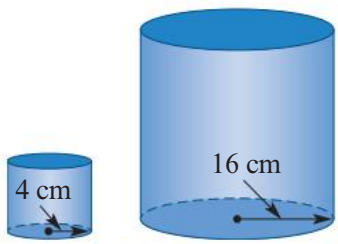
b Determine the ratio of their volumes.

5 Two cylinders are similar and have radii of 4 cm and 16 cm, respectively.

a Determine the ratio of their heights.

b Determine the ratio of their surface areas.

c Determine the ratio of their volumes.



6 Two rectangular prisms have side lengths with a scale factor of 10. If the surface area of the smaller rectangular prism is 18 m^2, determine the surface area of the larger rectangular prism.

7 A cone with radius 10 cm and slant height 60 cm is similar to one that has measurements one-fifth as long. Use the scale factor to calculate the surface area of the smaller cone.

8 Two similar cubes have side lengths with a scale factor of 8. If the smaller cube has a volume of 64 cm^3, determine the volume of the larger cube.

9 A trapezoidal prism has a volume of 45 m^3. If a similar one has corresponding side lengths half as long, determine the volume of this smaller trapezoidal prism.

SF

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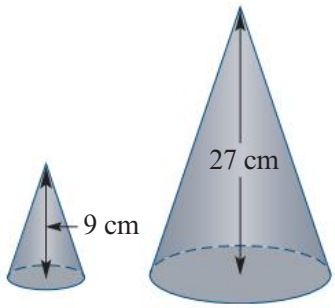
Example 14
Example 16

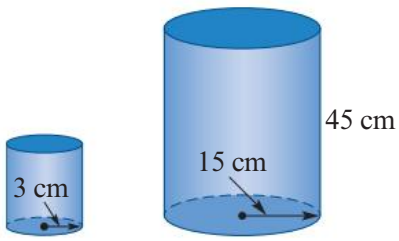
10 Two similar cones are shown at right. The scale factor of their heights is 3.



a The surface area of the larger cone is 1336.8 cm^2. Calculate the surface area of the smaller cone.

b The volume of the smaller cone is 120 cm^3. Calculate the capacity of the larger cone.

11 The radii of the bases of two similar cylinders have a scale factor of 5. The height of the larger cylinder is 45 cm. Use the scale factor to:



a determine the surface area of the smaller cylinder.

b determine the volume of the smaller cylinder.

12 A pyramid has a square base of side 4 cm and a volume of 16 cm^3. Determine the height and the length of the side of the base of a similar pyramid with a volume of 1024 cm^3.

13 Chef Gordon has a 3000 litre chest freezer. If the dimensions of the freezer are doubled, determine the capacity, in litres, of the larger chest freezer.

14 The radii of the bases of two similar cylinders are in the ratio 3 : 4. The height of the larger cylinder is 8 cm.

a Calculate the height of the smaller cylinder.

b Determine the ratio of the volumes of the two cylinders.

15 James has a cylindrical container that holds 27.4 litres of juice in his food truck. He sells a cup of juice at $4.50 for 500 mL. He wants to upgrade to a container that has measurements which are twice the size of his original container. If he only fills this new container to 80% capacity but sells it at the same price, calculate the amount of money he would earn if he sold all of the juice in this new container.

CF

CU

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3F Problem-solving and modelling

Learning intentions

- To gain some experience with solving practical problems using similarity and scaling.
- To gain some experience with questions that require skills up to and including those from Unit 1 Topic 3.
- To gain some experience with questions that combine multiple skills into one question.
- To gain some experience with complex and unfamiliar questions.

Exercise 3F

CD

1 Harriet stands 8 m from a vertical lamp post and casts a 2 m shadow. The shadow from the lamp post and from Harriet end at the same place. Determine the height of the lamp post if Harriet is 1.5 metres tall.

2 A person who is 1.9 metres tall stands 230 cm in front of a light that sits on the floor and casts a shadow on the wall behind them. Calculate the distance between the wall and the person if the height of the shadow on the wall is 2.5 times as tall as the person.

3 A map is drawn to a scale of 1 : 20 000. A park drawn on the map has an area of 6 cm^2. Calculate the actual area of the park. Convert your answer into hectares.
(1 hectare = 10 000 m^2)

4 An architect uses a scale of 1 cm : 3 m for the plans of a house she is designing. On the plans, a room has an area of 3.3 cm^2.

a Calculate the actual area of the room in m^2.

b Another room in the same house is to have an actual area of 3.5 m^2. Determine the area, in cm^2, this would be on the plans.

5 A cubic box has sides of length 10 cm. Identify the number of times you would have to enlarge the box by a scale factor of 2 before it is too big for a room that is 3 m high with length 4 m and width 3 m.

6 Using a cube of side length 10 cm, identify the number of times that you can reduce it by a scale factor of $\frac{1}{2}$ before it becomes smaller than 1 mm by 1 mm by 1 mm.

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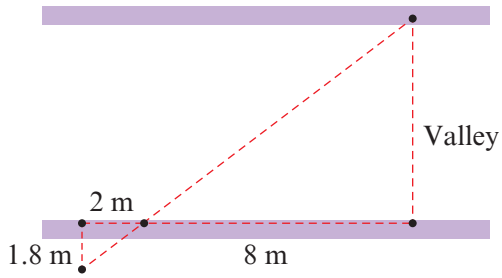
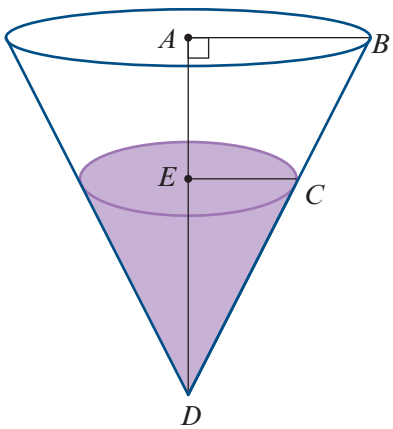
7 A national park would like to build a bridge across a narrow valley, as shown. They are requesting a grant to fund this project. The materials for the bridge will cost $420 per metre. The connection at each end costs $1500 each. Labour is estimated at 900 hours at $65 per hour. Determine the total cost of this project.



8 A cone with radius 6 cm and height 10 cm is filled with water so it is half full. Determine the radius and height of the water in the cone.



9 A cube has a surface area in cm^2 that is twice the volume in cm^3. Use algebraic procedures to determine the maximum capacity of the cube, in mL, if it can only be filled to 85% of the total capacity.

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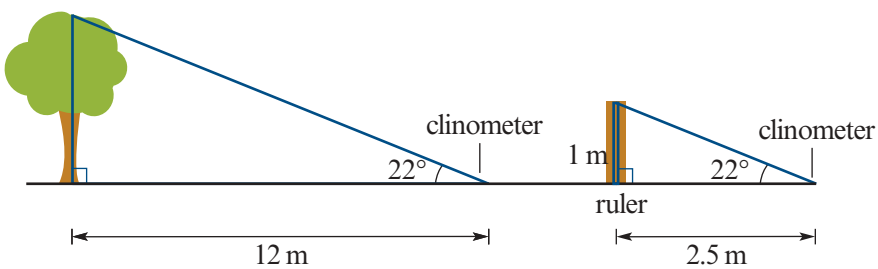
Using a clinometer

10 In this activity, you will find the vertical height of a tree or other object using a clinometer, which measures angles. You can make one using directions given on the internet, such as those on the NRICH website (https://nrich.maths.org/5382). Alternatively, you can download a clinometer app for a smartphone – just search for 'clinometer' on an app store website. This converts the phone into a clinometer. You will also need a metre ruler or a long straight stick or pole that you can measure and a long tape measure – at least 6 metres is best.

Using the clinometer from a convenient position, measure the angle between a horizontal line to the base of the tree and a sloping line to the top of the tree. Measure the distance from the clinometer to the tree. In the diagram below, the clinometer is 12 m from the tree and the angle to its top is 22°.

Set the metre rule vertically and place the clinometer level with the base of the ruler at a distance from it so the angle from the horizontal to the top of the rule is the same as the angle found to the top of the tree (22° in this case). In the diagram below right, this has been done and the distance from the ruler was found to be 2.5 m.



Use similar triangles to find the height of your tree.

If you can't do this activity, find the height of the tree in this diagram.

Note: The baselines of the two triangles do not have to be on the same level, but they have to be horizontal. If the ground to the tree is sloping, the clinometer has to be level with the base of the tree. The ruler could be set up on a level table.

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Review

Key ideas and chapter summary

Similar figures

Similar figures are shapes that are the same shape but different sizes.

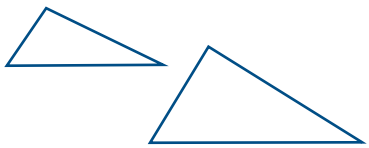
Scale factor

The **scale factor** $= \dfrac{\text{image length}}{\text{original length}}$.

- If a scale factor is between 0 and 1, the image will be smaller than the original.
- If a scale factor is greater than 1, the image will be larger than the original.
- If a scale factor is equal to 1, the image will be the same size as the original.

Similar triangles

Triangles are shown to be **similar** if:



- corresponding angles are equal (AAA)
- corresponding sides are in the same ratio (SSS)
- two pairs of corresponding sides are in the same ratio and the included corresponding angles are equal (SAS)
- the hypotenuse and one side of a right angled triangle is in the same ratio as the corresponding sides and the corresponding right angles are equal (RHS).

Scaling lengths

When a shape is scaled, multiply the lengths of the original figure by the scale factor, k, to determine the lengths of the image figure.

Scaling areas

When all the dimensions of similar shapes are multiplied by a scale factor of k, the areas are multiplied by a scale factor of k^2.

Map scale

The **scale** on the map shows the ratio of the map distance to the actual distance in real life. This is shown in a **line segment scale** or with numbers in a **scale ratio**.

map scale ratio = map distance : actual distance

Scaling surface areas

When all the dimensions of similar shapes are multiplied by a scale factor of k, the areas are multiplied by a scale factor of k^2.

Scaling volumes

When all the dimensions of similar shapes are multiplied by a scale factor of k, the volumes are multiplied by a scale factor of k^3.

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Skills checklist

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills.

3A	1	I can recognise corresponding sides and angles in two figures.	☐
		See Example 1 and Exercise 3A Question 1.	
3A	2	I can recognise similarity between two figures.	☐
		See Example 2 and Exercise 3A Question 2.	
3A	3	I can calculate the scale factor between two shapes.	☐
		See Example 3 and Exercise 3A Question 4.	
3B	4	I can explain why two triangles are similar.	☐
		See Example 4 and Exercise 3B Question 1.	
3B	5	I can calculate the scale factor between two similar triangles.	☐
		See Example 5 and Exercise 3B Question 1.	
3C	6	I can use a scale factor to determine unknown values in similar figures.	☐
		See Example 6 and Exercise 3C Questions 1 and 3.	
3C	7	I can calculate the scale factor of lengths and areas.	☐
		See Example 7 and Exercise 3C Questions 2 and 3.	
3C	8	I can use a scale factor to determine unknown areas in similar figures.	☐
		See Example 8 and Exercise 3C Question 9.	
3D	9	I can convert distances using a line segment scale.	☐
		See Example 9 and Exercise 3D Question 1.	
3D	10	I can convert distances using a scale ratio.	☐
		See Example 10 and Exercise 3D Question 2.	
3D	11	I can use scaling in maps.	☐
		See Example 11 and Exercise 3D Question 3.	
3D	12	I can apply scaling in maps.	☐
		See Example 12 and Exercise 3D Question 5.	

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Review

3D	13	I can use a line segment scale in maps.	☐

See Example 13 and Exercise 3D Question 6.

3E	14	I can use a scale factor to determine unknown surface areas of similar prisms.	☐

See Example 14 and Exercise 3E Questions 1 and 10.

3E	15	I can use a scale factor to determine unknown volumes of similar prisms.	☐

See Example 15 and Exercise 3E Question 2.

3E	16	I can use a scale factor to determine unknown capacities of similar prisms.	☐

See Example 16 and Exercise 3E Question 10.

3F	17	I can solve practical problems using similarity and scaling.	☐

See Exercise 3F.

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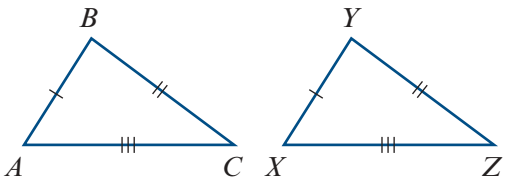
Multiple-choice questions

1 Determine which are corresponding sides in these similar triangles.



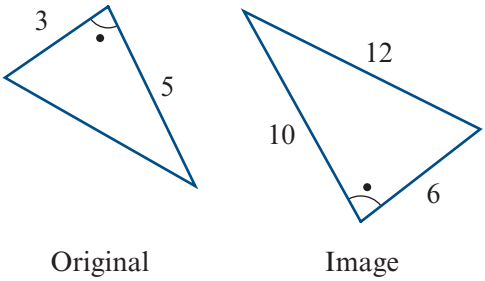
A AB corresponds to XY

B AC corresponds to ZZ

C BA corresponds to YZ

D CB corresponds to XY

2 Determine the scale factor between these two similar triangles.



A $\frac{1}{2}$

B $\frac{3}{5}$

C $\frac{5}{3}$

D 2

3 Choose the reason that explains how these two triangles are similar.

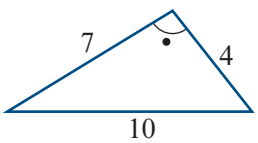
A AAA

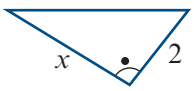
B SSS

C SAS

D RHS

4 For these two similar triangles, calculate the value of x.





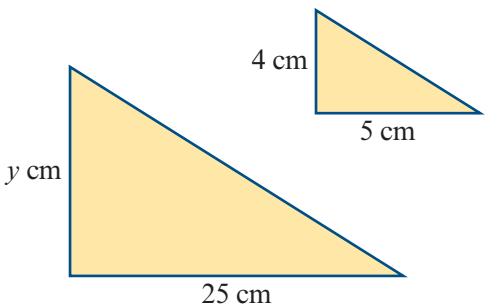
A 2 units

B 2.5 units

C 3.5 units

D 5 units

5 The two triangles shown are similar. The value of y is:



A 9

B 16

C 20

D 21

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Review

6 The diameter of a large sphere is 4 times the diameter of a smaller sphere. It follows that the ratio of the volume of the large sphere to the volume of the smaller sphere is:

A 8 : 1 **B** 16 : 1 **C** 32 : 1 **D** 64 : 1

7 A map has a scale of 1 : 35 000. Calculate the actual distance for a scaled distance of 34 mm.

A 1.19 km **B** 1190 cm **C** 119 000 mm **D** 1 190 000 km

8 A plan has a scale of 1 : 250. If the garden bed is 20 metres long in real life, determine the measurement for this garden bed on the plan.

A 0.08 cm **B** 0.8 cm **C** 8 cm **D** 80 cm

9 Determine the scale factor of the surface areas of two similar prisms whose sides have a scale factor of 6.

A 12 **B** 18 **C** 36 **D** 216

10 Determine the scale factor of the volumes of two similar solids whose sides have a scale factor of 4.

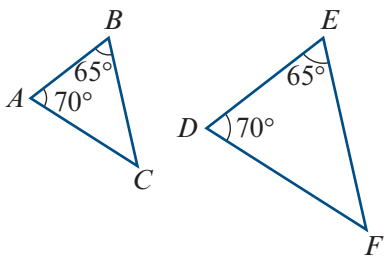
A 8 **B** 16 **C** 64 **D** 256

Short-response questions

SF

1 Two rectangles are 6 cm by 2 cm (original) and 18 cm by 6 cm (image). State the scale factor of the corresponding sides.

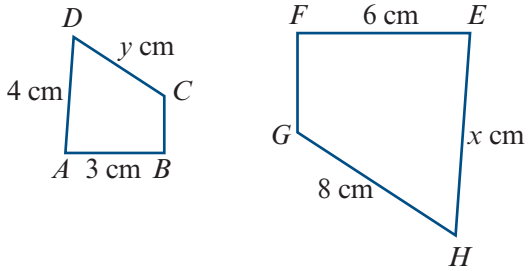
2 Explain why these triangles are similar.



3 Use the scale factor for these similar figures to determine the value of:

a x

b y.



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4 A photo is 5 cm by 12 cm. It is to be enlarged by a scale factor of 3. State the dimensions of the new photo.

5 You are given a circle of radius 4 cm. The radius increases by a scale of factor of 2. Determine the area of the larger circle.

6 Triangle A has an area of 36 cm^2. In Triangle B the side lengths decrease by a scale factor of $\frac{1}{2}$. Determine the area of Triangle B.

7 A scale on a map is 1 : 35 000.

a Determine the actual distance if the distance on the map is 8 cm. Give your answer in kilometres.

b If the actual distance between two landmarks is 9 km, calculate the distance between them on the map. Give your answer rounded to the nearest cm.

8 Answer the following questions using the map of Australia below.

a Measure the line segment scale given in millimetres and interpret the scale, rounded to one decimal place.

b Calculate the actual distance between Hobart and Darwin, rounded to the nearest 10 km.

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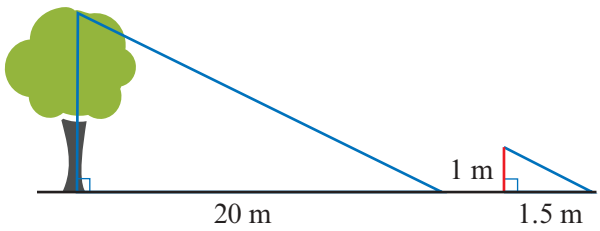
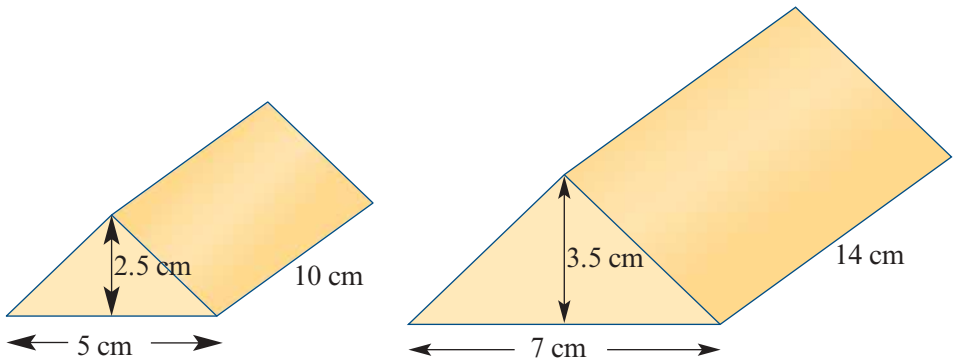
Review

SF

9 A biologist studying trees wanted to calculate the height of a particular tree. She placed a wooden stake vertically in the ground, sticking exactly 1 m above the ground, with the base level with the base of the tree. The stake cast a shadow on the ground measuring 1.5 m. The tree cast a shadow of 20 m, as shown in the diagram below *(not* to scale). Calculate the height of the tree. Give your answer correct to two decimal places.



10 Two similar triangular prisms are shown below.



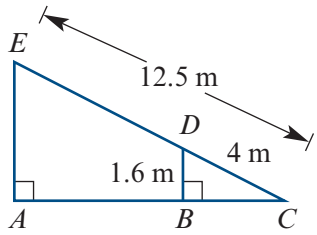
a Determine the ratio of their surface areas.

b Determine the ratio of their volumes.

c Calculate the volume of the smaller prism to the nearest cm^3.

CF

11 A conveyor belt used to load luggage onto a plane is 12.5 m long. A vertical support 1.6 m high is placed under the conveyor belt such that it is 4 m along the conveyor belt as shown. Calculate the height of the luggage door above the ground, AE.



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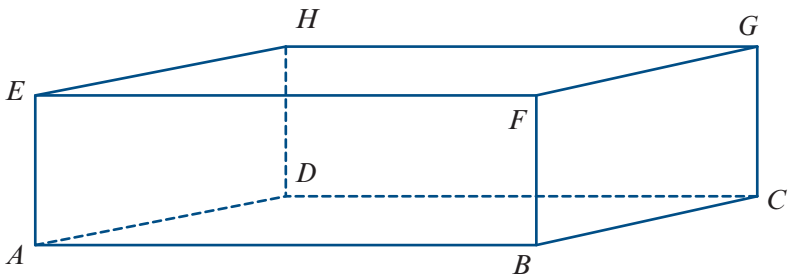
12 Tom has an insulated lunch box that has a capacity of 500 mL. If the dimensions of the lunch box are doubled, determine the capacity, in litres, of the larger lunch box.

13 The length of a rectangular prism is eight times its height. The width is four times the height. The length of the diagonal between two opposite vertices (AG) is 36 cm. Calculate the volume of the prism.



14 The volume of a cone of height 28.4 cm is 420 cm^3. Determine the height of a similar cone whose volume is 120 cm^3, correct to two decimal places.

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Chapter 4

Algebra: Linear and non-linear relationships

Chapter questions

UNIT 2: APPLIED TRIGONOMETRY, ALGEBRA, MATRICES AND UNIVARIATE DATA

Topic 4: Linear and non-linear relationships

- How do we substitute numerical values into linear and non-linear algebraic expressions?
- How do we find the value of a pronumeral in linear and simple non-linear equations, given other values?
- How do we transpose an equation?
- How do we create a table of values?
- How do we use a spreadsheet to construct a table of values?
- How do we use linear and non-linear formulas?

Linear relationships and equations connect two or more variables such that they yield a straight line when graphed. The unknown values are always to the power of 1. Linear relationships have many applications in technology, science and business.

A non-linear relationship or equation is one where unknown values are not all to the power of 1.

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4A Substituting values into an algebraic expression

Learning intentions

- To be able to substitute a numerical value into a linear algebraic expression.
- To be able to substitute a numerical value into a non-linear algebraic expression.

An **algebraic expression** (or formula) is a mathematical relationship connecting two of more variables.

For example, $C = 45t + 150$ is an algebraic expression for relating the cost, C dollars, to t, the number of hours they were hired for. C and t are the variables. It is a linear relationship.

By **substituting** all known values into an algebraic expression, we are able to determine the value of an unknown variable.

Example 1 Using a linear algebraic expression

The cost of hiring a windsurfer is given by the rule:

$$C = 40t + 10$$

where C is the cost in dollars and t is the time in hours. Determine how much it will cost to hire a windsurfer for 2 hours.

Solution	Explanation
$C = 40t + 10$ $C = 40(2) + 10$	Write the algebraic expression. To determine the cost of hiring a windsurfer for 2 hours, substitute $t = 2$ into the algebraic expression. Note: $40(2)$ means 40×2.
$C = 90$ It will cost \$90 to hire a windsurfer for 2 hours.	Evaluate. Write your answer.

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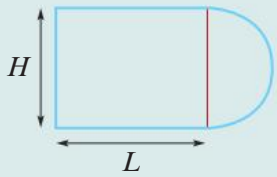
Example 2 Using a linear algebraic expression

The perimeter of this shape can be given by the expression:

$$P = 2L + H\left(1 + \frac{\pi}{2}\right)$$



In this expression, L is the length of the rectangle and H is the height. Calculate the perimeter, correct to one decimal place, if $L = 16.1$ cm and $H = 3.2$ cm.

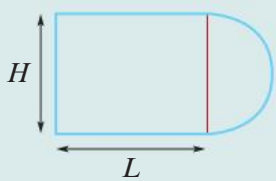
Solution	Explanation
$P = 2L + H\left(1 + \frac{\pi}{2}\right)$	Write the expression.
$P = 2 \times 16.1 + 3.2\left(1 + \frac{\pi}{2}\right)$	Substitute values for L and H into the expression.
$P = 40.4$ (correct to one decimal place)	Evaluate.
The perimeter of the shape is 40.4 cm.	Give your answer with correct units.

The same process can be applied for non-linear expressions. For example, $A = \pi r^2$ is an algebraic expression for the area of a circle, where A is the area, r is the radius and π is a constant approximately equal to 3.142. It is a non-linear relationship as r has a power of 2.

Example 3 Using a non-linear algebraic expression (polynomial)

The area, A, of the shape shown can be given by the expression:

$$A = HL + \frac{1}{2}\pi\left(\frac{H}{2}\right)^2$$



In this expression, L is the length of the rectangle and H is the height. Calculate the area, correct to one decimal place, if $H = 5.2$ cm and $L = 18.4$ cm.

Solution	Explanation
$A = HL + \frac{1}{2}\pi\left(\frac{H}{2}\right)^2$	Write the expression.
$A = 5.2 \times 18.4 + \frac{1}{2}\pi\left(\frac{5.2}{2}\right)^2$	Substitute values for H and L into the expression.
$A = 106.3$ (correct to one decimal place)	Evaluate.
The area of the shape is 106.3 cm^2.	Give your answer with correct units.

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Example 4 Using a non-linear algebraic expression (inverse proportion)

The current, I amperes, that flows in an electrical appliance depends on the resistance of the appliance, R ohms, where $I = \frac{240}{R}$. Calculate the current that flows through an appliance with a resistance of 100 ohms.

Solution	Explanation
$I = \frac{240}{R}$	Write the expression.
$I = \frac{240}{100}$	Substitute $R = 100$.
$I = 2.4$	Evaluate.
The current that flows in the appliance is 2.4 amperes.	Give your answer with the correct units.

Example 5 Using a non-linear algebraic expression

Use the expression $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ to determine f when $u = 5$ and $v = 3$.

Solution	Explanation
$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$	Write the expression.
$\frac{1}{f} = \frac{1}{5} + \frac{1}{3}$	Substitute $u = 5$ and $v = 3$ into the expression then simplify.
$= \frac{3}{15} + \frac{5}{15}$	
$= \frac{8}{15}$	
$\frac{f}{1} = \frac{15}{8}$	Invert both sides.
$f = \frac{15}{8}$	Write your answer.

Section Summary

- An **algebraic expression** is a mathematical relationship connecting two or more variables.
- A **linear relationship** has unknown values that are always to the power of one.
- A **non-linear relationship** or equation is one whose unknown values are not all to the power of one.
- By substituting all known variables into algebraic expressions, we are able to determine the value of an unknown variable.

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Exercise 4A

SF

Example 1

1 The cost of hiring a dance hall is given by the rule:

$$C = 50t + 1200$$

where C is the total cost in dollars and t is the number of hours for which the hall is hired.
Calculate the cost of hiring the hall for:

a 4 hours **b** 6 hours **c** 4.5 hours

2 Taxi fares are calculated using the expression:

$$F = 1.3K + 4$$

where K is the distance travelled in kilometres and F is the cost of the fare in dollars.
Calculate the costs of the following trips.

a 5 km **b** 8 km **c** 20 km

3 The formula used to convert temperature from degrees Fahrenheit to degrees Celsius is:

$$C = \frac{5}{9}(F - 32)$$

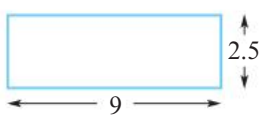
Use this formula to convert the following temperatures to degrees Celsius. Give your answers correct to one decimal place where appropriate.

a 50°F **b** 0°F **c** 212°F **d** 92°F

Example 2

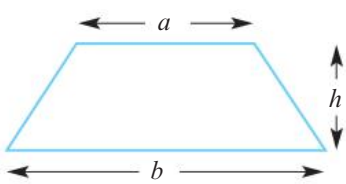
4 The perimeter of a rectangle is given by $P = 2(L + W)$. Calculate the value of P for the following rectangles.

a $L = 3$ and $W = 4$ **b** $L = 15$ and $W = 8$ **c**



5 The area of a trapezium as shown is $A = \frac{1}{2}h(a + b)$.
Calculate A when:

a $h = 1, a = 3, b = 5$
b $h = 5, a = 2.5, b = 3.2$
c $h = 2.7, a = 4.1, b = 8.3$



6 The distance, d km, travelled by a car in t hours at an average speed of v km/h is given by the expression:

$$d = v \times t$$

Determine the distance travelled by a car travelling at a speed of 95 km/h for 4 hours.

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Example 3

7 The circumference, C, and area, A, of a circle with radius r can be calculated using the expressions:

$$C = 2\pi r \text{ and } A = \pi r^2$$

For the following circles, calculate, correct to two decimal places:

i the circumference **ii** the area

a A circular stained glass window with $r = 25$ cm

b An earring with $r = 3$ mm

c A DVD with $r = 5.4$ cm

d A circular garden bed with $r = 7.2$ m

8 The speed v in metres per second of a bullet fired from a gun after t seconds is given by $v = 6600 + 61t + 3t^2$. Calculate the speed of the bullet after 2 seconds.

9 The surface area, A, and volume of a sphere, V, can be calculated using the rules:

$$V = \frac{4}{3}\pi r^3 \text{ and } A = 4\pi r^2$$

where r is the radius of the sphere.

For the following spheres, calculate, correct to two decimal places:

i the volume **ii** the surface area

a A basketball with radius $r = 12.1$ cm

b A soap bubble with radius $r = 12.5$ mm

c A plastic sphere with radius $r = 1.35$ m

d An orange with radius $r = 6.3$ cm

Example 4

10 The current, I amperes, that flows through a toaster of resistance R ohms is given by $I = \frac{240}{R}$. Calculate the current that flows through a toaster of resistance 150 ohms.

11 The average speed of a car, s km/h, that travels a distance of d kilometres in t hours is given by $s = \frac{d}{t}$. A car travels 250 km in 3.5 hours. Calculate the average speed of the car, correct to one decimal place.

Example 5

12 Use the expression $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ to calculate f when:

a $u = 4$ and $v = 7$ **b** $u = 6$ and $v = 10$

13 Use the expression $F = G\frac{m_1 m_2}{r^2}$ to calculate F, correct to two decimal places, when:

a $G = 6.673$, $m_1 = 3$, $m_2 = 4.5$ and $r = 7$

b $G = 6.673$, $m_1 = 2$, $m_2 = 5.5$ and $r = 12$

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CF

14 The formula for calculating simple interest is:

$$I = \frac{Prt}{100}$$

where P is the principal (amount invested or borrowed), r is the interest rate per annum and t is the time (in years). Calculate the simple interest for the following.

a Frank borrows \$5000 at 12% for 4 years. Calculate how much interest he will pay.

b Chris borrows \$1500 at 6% for 2 years. Calculate how much interest he will pay.

c Henry invests \$8500 for 3 years with an interest rate of 7.9%. Calculate how much interest he will earn.

15 In Australian Rules football, a goal is worth 6 points and a behind is worth 1 point. The total number of points is given by:

$$\text{total number of points} = 6 \times \text{number of goals} + \text{number of behinds}$$

a Determine the number of points for the following scores:

i 2 goals and 3 behinds

ii 5 goals and 7 behinds

iii 8 goals and 20 behinds

b In a match, Redteam scores 4 goals and 2 behinds and Greenteam scores 3 goals and 10 behinds. Determine the winner of the match.

16 In Rugby League, a try is worth 4 points, a conversion or penalty kick is worth 2 points, and a field goal (drop kick) is worth one point. In a match, the Gold team scores 4 tries, 2 conversions, a penalty kick and a field goal. The Silver team scores 3 tries, 3 conversions and two penalty kicks. Determine the final score in the match.

17 In Rugby Union, a try is worth 5 points, a conversion kick is worth 2 points, and a penalty goal and field goal are both worth 3 points. Calculate the score in the previous question if the Gold and Silver teams were playing Rugby Union.

18 The number of matchsticks used for each shape below follows the pattern $3, 5, 7, \ldots$

Shape 1 Shape 2 Shape 3

The rule for finding the number of matches used in this sequence is:

$$\text{number of matches} = a + (n - 1)d$$

where a is the number of matches in the first shape ($a = 3$), d is the number of extra matches used for each shape ($d = 2$) and n is the shape number.
Determine the number of matches in the:

a 6th shape **b** 11th shape **c** 50th shape

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CF

19 Suggested cooking times for roasting x kilograms of meat are given in the following table.

Meat type	Cooking time
Chicken (well done)	45 min/kg + 20 min
Lamb (medium)	55 min/kg + 25 min
Lamb (well done)	65 min/kg + 30 min
Beef (medium)	55 min/kg + 20 min
Beef (well done)	65 min/kg + 30 min

a Determine the cooking time, to the nearest minute, of the following:

i a 2 kg chicken.

ii a 2.25 kg beef roast well done.

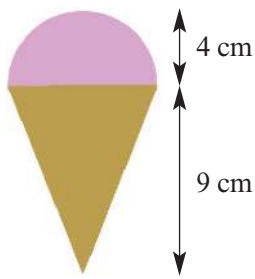
iii a piece of lamb weighing 2.4 kg cooked well done.

iv a 2.5 kg beef roast cooked medium.

b Calculate the time at which you should put a 2 kg leg of lamb into the oven to be able to serve it medium at 7:30 p.m.

CU

20 An ice cream consists of a wafer in the shape of a cone and a sphere of ice cream on the top, as shown in the diagram on the right. The circular end of the cone has a radius of 4 cm. The formulas for the surface area and volume of cones and spheres are shown below:



surface area of cone $= \pi r\left(r + \sqrt{h^2 + r^2}\right)$

volume of cone $= \frac{1}{3}\pi r^2 h$

surface area of sphere $= 4\pi r^2$

volume of sphere $= \frac{4}{3}\pi r^3$

where r is the radius of the cone or sphere and h is the height of the cone.
Use the formulas to determine the following, correct to two decimal places.

a The surface area of ice cream that is above the cone

b The surface area of the wafer

c The volume of ice cream

d The volume of air in the cone (assume all of the bottom half of the sphere of ice cream is inside the cone)

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4B Transposition of equations

Learning intentions

- To be able to transpose a linear equation.
- To be able to transpose a non-linear equation.
- To be able to transpose an equation with multiple variables.

The formula $C = 2\pi r$ can be used to calculate the circumference, C, of a circle of radius r. In this formula, C is the **subject**.

Dividing both sides of this equation by 2π gives $r = \frac{C}{2\pi}$ and now r is the subject of the formula.

The process of rearranging a formula in order to make a different pronumeral the subject of the equation is called **transposition**.

Example 6 Transposing a linear equation

The cost of hiring a windsurfer is given by the rule $C = 40t + 10$, where C is the cost in dollars and t is the time in hours.

a Rearrange the equation to make t the subject.

b Determine how long a windsurfer can be hired for a cost of $130.

Solution	Explanation
a $C = 40t + 10$	Write the formula.
$C - 10 = 40t + 10 - 10$	Subtract 10 from both sides of the equation.
$C - 10 = 40t$	
$\frac{C - 10}{40} = \frac{40t}{40}$	Divide both sides of the equation by 40.
$t = \frac{C - 10}{40}$	
b $t = \frac{C - 10}{40}$	Write the formula.
$t = \frac{130 - 10}{40}$	Substitute $C = 130$ into the formula.
$t = \frac{120}{40}$	Evaluate.
$t = 3$	
The windsurfer can be hired for 3 hours for a cost of $130.	Write your answer.

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Example 7 Transposing a non-linear equation

The formula $S = 4\pi r^2$ gives the surface area, S, of a sphere of radius r. Calculate the radius of a sphere, correct to one decimal place, which has a surface area of 36.2 cm^2.

Solution	Explanation
$S = 4\pi r^2$	Write the formula.
$\frac{S}{4\pi} = \frac{4\pi r^2}{4\pi}$ $r^2 = \frac{S}{4\pi}$	Divide both sides of the equation by 4π and reverse the sides of the equation.
$\sqrt{r^2} = \sqrt{\frac{S}{4\pi}}$ $r = \sqrt{\frac{S}{4\pi}}$	Take the square root of both sides of the equation.
$r = \sqrt{\frac{36.2}{4\pi}}$	Substitute $S = 36.2$ into the formula.
$r = 1.6972\ldots$	Evaluate and round to one decimal place.
$r = 1.7$ correct to one decimal place	
The sphere has a radius of 1.7 cm.	Write your answer.

Example 8 Transposing a non-linear equation

The current, I amperes, that flows in an electrical appliance depends on the resistance of the appliance, R ohms: $I = \frac{240}{R}$.

a Transpose the formula to make R the subject.

b If a kettle has a current of 1.2 amperes running through it, calculate the resistance of the kettle.

Solution	Explanation
a $I = \frac{240}{R}$	Write the formula.
$I \times R = \frac{240}{R} \times R$ $IR = 240$	Multiply both sides of the equation by R.
$\frac{IR}{I} = \frac{240}{I}$ $R = \frac{240}{I}$	Divide both sides of the equation by I.

Continued on next page

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b $R = \frac{240}{I}$	Write the formula.
$R = \frac{240}{1.2}$	Substitute $I = 1.2$ into the formula.
$R = 200$	Evaluate.
A kettle that has a current of 1.2 amperes running through it has a resistance of 200 ohms.	Write your answer.

Example 9 Transposing an equation with more than two variables

The formula for calculating simple interest is

$$I = \frac{Prt}{100}$$

where P is the principal (amount invested or borrowed), r is the interest rate per annum and t is the time (in years).

Dan invests $4000 at 6% per annum. He earns $720 in interest. Calculate the length of his investment in years.

Solution	Explanation
$I = \frac{Prt}{100}$	Write the formula.
$100 \times I = 100 \times \frac{Prt}{100}$	Multiply both sides of the equation by 100.
$100I = Prt$	
$\frac{100I}{Pr} = \frac{Prt}{Pr}$	Divide both sides of the equation by Pr then reverse the sides of the equation.
$t = \frac{100I}{Pr}$	
$t = \frac{100 \times 720}{4000 \times 6}$	Substitute $P = 4000$, $r = 6$ and $I = 720$ into the formula.
$t = 3$	Evaluate.
The length of the investment was 3 years.	Write your answer.

Section Summary

- Linear and non-linear equations can be **transposed** to make a different pronumeral the **subject** of the equation.
- Transposing an equation can make it easier to determine the value of a missing pronumeral when it isn't originally the subject.

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Skill-sheet

Exercise 4B

SF

Example 6

1 The cost of hiring a dance hall is given by the rule:

$$C = 50t + 1200$$

where C is the total cost in dollars and t is the number of hours the hall is hired.

a Rearrange the equation to make t the subject.

b Calculate the length of time the hall is hired if the total cost is $1450.

2 Taxi fares are calculated using the formula:

$$F = 1.3K + 4$$

where K is the distance travelled in kilometres and F is the cost of the fare in dollars.

a Transpose the formula to make K the subject.

b Determine the distance travelled if the fare for a taxi ride was $32.60.

Example 7

3 The formula, $S = 4\pi r^2$, can be used to calculate the surface area, S, of a sphere of radius r. Calculate, correct to the nearest whole number, the radius of a baseball that has a surface area of 452.4 cm^2.

4 The circumference, C, of a circle of radius r can be calculated using the formula $C = 2\pi r$. Calculate the radius of a circular plate with a circumference 81.7 cm. Give your answer correct to the nearest whole number.

5 The area, A, of a circle of radius r can be calculated using the formula $A = \pi r^2$. A mysterious crop circle has an area of 725.83 m^2. Calculate the radius of this crop circle, correct to one decimal place.

Example 8

6 The current, I amperes, that flows in an electrical appliance depends on the resistance of the appliance, R ohms, where:

$$I = \frac{240}{R}.$$

a Transpose the formula to make R the subject.

b Calculate the resistance of an oven if the current is 1.5 amperes.

Example 9

7 The distance, d km, travelled by a car in t hours at an average speed of v km/h is given by the formula:

$$d = v \times t$$

a Transpose the formula to make t the subject.

b Calculate how long the journey takes for a car that travels 275 km at 100 km/h. Give your answer in hours and minutes.

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SF

8 The perimeter of a rectangle is given by $P = 2L + 2W$.

a Transpose this formula to make W the subject.

b Determine the width of a rectangle of length 16 m which has a perimeter of 84 m.

CF

9 The formula used to convert temperature from degrees Fahrenheit to degrees Celsius is:

$$C = \frac{5}{9}(F - 32)$$

A recipe requires a cake to be cooked at 180°C for 40 minutes. Andy's oven has a temperature scale marked in degrees Fahrenheit. Determine the temperature that Andy should set his oven to cook this cake. Give your answer to the nearest 10 degrees.

10 A car travels a fixed distance of 260 km. A formula for finding the average speed of the car, s km/h, when it takes t hours to travel this distance is:

$$s = \frac{260}{t}.$$

Calculate the time taken for a car to travel this distance at a speed of 100 km/h. Give your answer in hours and minutes.

11 The formula for calculating simple interest is:

$$I = \frac{Prt}{100}$$

where P is the principal (amount invested or borrowed), r is the interest rate per annum and t is the time (in years).

a Madeleine borrows \$8000 at a rate of 5.5% per annum and pays \$2200 in interest. Determine the length of her loan.

b Kyley invests an amount of money for 3 years at 2.3% per annum and earns \$372.60 in interest. Determine how much she invested.

c Samuel invests \$2000 for 4 years and earns \$240 in interest. Determine the per annum interest rate for the investment.

12 In Rugby League football, a try, T, is worth 4 points and a conversion, C, is worth 2 points. If no penalties or field goals are scored in a game, the total number of points, P, is given by $P = 4T + 2C$. In one game, the Far North Salties scored a total of 44 points, which included 4 conversions and no penalties or field goals. Calculate the number of tries that they scored in this match.

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4C Constructing a table of values

Learning intentions

- To be able to construct a table of values by hand.
- To be able to construct a table of values using technology.
- To be able to construct a table of values with two variables.

We can use an algebraic expression or formula to construct a *table of values*. This can be done by substituting a series of values into the formula, either by hand or with technology.

Example 10 Constructing a table of values by hand

The formula for converting degrees Celsius to degrees Fahrenheit is given by:

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

Solution

If $C = 0, F = \frac{9}{5}(0) + 32 = 32$

If $C = 10, F = \frac{9}{5}(10) + 32 = 50$

and so on.

Construct the table.

Explanation

Substitute values of $C = 0, 10, 20, 30, \ldots$ into the formula $F = \frac{9}{5}C + 32$, to calculate values of F.

C	0	10	20	30	40	50	60	70	80	90	100
F	32	50	68	86	104	122	140	158	176	194	212

How to construct a table of values using a spreadsheet

Spread-sheet

The formula for converting degrees Celsius to degrees Fahrenheit is given by:

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

1

	A	B
1	c	f

Open a new spreadsheet.

In cell A1, type 'c' and then in cell B1 type 'f'.

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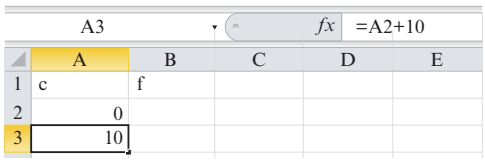
2



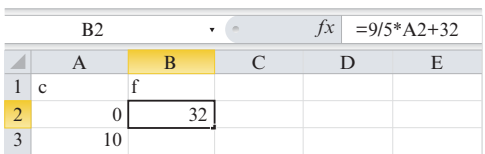
In cell A2, type '0' and then in cell A3 type '= a2 + 10'.

3

	A	B
1	c	f
2	0	
3	10	
4	20	
5	30	
6	40	
7	50	
8	60	
9	70	
10	80	
11	90	
12	100	

Highlight cells A3 to A12 then press CTRL D to fill down.

4



In cell B2, type '= 9/5 ∗ a2 + 32'.

5

	A	B
1	c	f
2	0	0
3	10	50
4	20	68
5	30	86
6	40	104
7	50	122
8	60	140
9	70	158
10	80	176
11	90	194
12	100	212

Highlight cells B2 to B12 and press CTRL D to fill down.

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Example 11 Constructing a table of values by hand

A car travels a fixed distance of 480 km. A formula for finding the average speed of the car, s km/h, when it takes t hours to travel this distance is $s = \frac{480}{t}$.

Use this formula to construct a table of values for s using values of t in intervals of 1 between $t = 4$ and $t = 12$. Give answers correct to one decimal place.

Solution

If $t = 4$, $s = \frac{480}{4}$

$= 120$

If $t = 5$, $s = \frac{480}{5}$

$= 96$

Construct the table.

Explanation

Substitute values of $t = 4, 5, 6, \ldots$ into the formula $s = \frac{480}{t}$, to determine values of s.

t	4	5	6	7	8	9	10	11	12
s	120	96	80	68.6	60	53.3	48	43.6	40

Alternatively use an Excel spreadsheet to construct this table of values

Spread-sheet

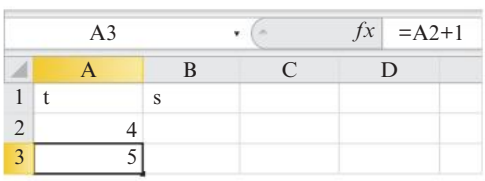
1

	A	B
	t	s

Open a new Excel spreadsheet.

In cell A1, type 't' and in cell B1, type 's'.

2



In cell A2, type 4 then in cell A3, type '=a2 + 1.'

3

	A	B
1	t	s
2	4	
3	5	
4	6	
5	7	
6	8	
7	9	
8	10	
9	11	
10	12	

Highlight cells A3 to A10 then press CTRL D to fill down.

Continued on next page

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4

B2	fx	=480/A2			
	A	B	C	D	E
1	t	s			
2	4	120			

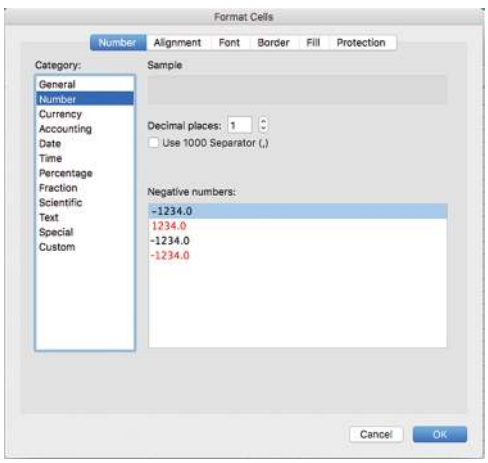
In cell B2, type '= 480/a2'.

5

	A	B
1	t	s
2	4	120
3	5	96
4	6	80
5	7	68.57143
6	8	60
7	9	53.33333
8	10	48
9	11	43.63636
10	12	40

Highlight cells B2 to B10 and press CTRL D to fill down.

6



Cells can be formatted to give answers correct to one decimal place as shown.

	A	B
1	t	s
2	4	120.0
3	5	96.0
4	6	80.0
5	7	68.6
6	8	60.0
7	9	53.3
8	10	48.0
9	11	43.6
10	12	40.0

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Example 12 Constructing a table of values with two variables

The formula for the body mass index, or BMI, of a person is:

$$\text{BMI} = \frac{m}{h^2}$$

where m is the mass of the person in kilograms and h is the height of the person in metres.

a Construct a table of values for BMI using values of m in intervals of 10 between $m = 50$ and $m = 100$, and values of h in intervals of 0.2 between $h = 1$ and $h = 2$.

b Use the table to calculate the BMI for a man with mass 80 kg and height 1.8 m.

Solution

a The values of m will start at 50 and increase by 10, up to 100:
$m = 50, 60, 70, 80, 90, 100$
The values of h will start at 1 and increase by 0.2, up to 2:
$h = 1, 1.2, 1.4, 1.6, 1.8, 2.0$

BMI		Height (m)					
		1	1.2	1.4	1.6	1.8	2.0
Mass (kg)	50						
	60						
	70						
	80						
	90						
	100						

BMI		Height (m)					
		1	1.2	1.4	1.6	1.8	2.0
Mass (kg)	50	50	34.72	25.51	19.53	15.43	12.50
	60	60	41.67	30.61	23.44	18.52	15.00
	70	70	48.61	35.71	27.34	21.60	17.50
	80	80	55.56	40.82	31.25	24.69	20.00
	90	90	62.50	45.92	35.16	27.78	22.50
	100	100	69.44	51.02	39.06	30.86	25.00

b The BMI for a man of mass 80 kg and height 1.8 m is 24.69.

Explanation

Determine the values of m to go in the table.

Determine the values of h to go in the table.

Draw up a table with one variable in the rows and one variable in the columns.
It won't matter which way it is arranged. Here, the mass is in the rows and height is in the columns.

Calculate the BMI for each pair of mass and height values and enter them into the table.
For example, the value in the shaded box has been calculated as

$$\text{BMI} = \frac{70}{1.6^2} = 27.34$$

Round answers to two decimal places.

Read the value in the row for 80 kg and the column for 1.8 m, as indicated by the red lines on the table.

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Spread-sheet

Using a spreadsheet to construct a table of values with two variables

1

	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50						
4	Mass (kg)	60						
5		70						
6		80						
7		90						
8		100						

Set up a table with masses in the rows and heights in the columns as previously outlined.

2

C3 · fx =B3/C2^2

	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50	50					
4	Mass (kg)	60						
5		70						
6		80						
7		90						
8		100						

In cell C3, type the formula '= b3/(c2^2)'.

3

C3 · fx =B3/(C2^2)

	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50	50					
4	Mass (kg)	60	60					
5		70	70					
6		80	80					
7		90	90					
8		100	100					

Highlight cells C3 to C8 and press CTRL D to fill down.

4

C3 · fx =B3/(D2^2)

	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50	50	34.72				
4	Mass (kg)	60	60	41.67				
5		70	70	48.61				
6		80	80	55.56				
7		90	90	62.50				
8		100	100	69.44				

In cell D3, type the formula '= b3/(d2^2)' and then highlight cells D3 to D8 and fill down. Answers can be given correct to two decimal places by highlighting cells and changing them to 'Number' format.

5

	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50	50	34.72	25.51	19.53	15.43	12.50
4	Mass (kg)	60	60	41.67	30.61	23.44	18.52	15.00
5		70	70	48.61	35.71	27.34	21.60	17.50
6		80	80	55.56	40.82	31.25	24.69	20.00
7		90	90	62.50	45.92	35.16	27.78	22.50
8		100	100	69.44	51.02	39.06	30.86	25.00

In cell E3, type the formula '= b3/(e2^2)' and then highlight cells E3 to E8 and fill down. Continue this process until columns F, G and H are all filled.

Section Summary

- Tables of values can be constructed by hand or using technology by substituting values into a formula.

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Skill-sheet

Exercise 4C

Example 10

SF

1 A football club wishes to purchase pies at a cost of $2.15 each. If C is the cost ($) and x is the number of pies, complete the table showing the amount of money needed to purchase from 40 to 50 pies.

x	40	41	42	43	44	45	46	47	48	49	50
C ($)	86	88.15	90.3								

2 The circumference of a circle is given by:

$$C = 2\pi r$$

where r is the radius. Complete the table of values to show the circumferences of circles with radii from 0 to 1 cm in intervals of 0.1 cm. Give your answers correct to three decimal places.

r (cm)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C (cm)	0	0.628	1.257	1.885							

3 A phone bill is calculated using the formula:

$$C = 40 + 0.18n$$

where C is the total cost and n represents the number of calls made. Complete the table of values to show the cost for 50, 60, 70, ... 130 calls.

n	50	60	70	80	90	100	110	120	130
C ($)	49	50.80	52.60						

4 The amount of energy (E) in kilojoules expended by an adult male of mass M at rest can be estimated using the formula:

$$E = 110 + 9M$$

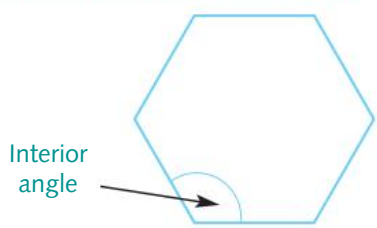
Complete the table of values in intervals of 5 kg for males of mass 60–120 kg to show the corresponding values of E.

M (kg)	60	65	70	75	80	85	90	95	100	105	110	115	120
E (kJ)	650	695											

5 The sum, S, of the interior angles of a polygon with n sides is given by the formula:

$$S = 90(2n - 4)$$

Construct a table of values showing the sum of the interior angles of polygons with 3 to 10 sides.



n	3	4	5					
S	180°	360°						

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6 The height, h, of a triangle of base length b and area A is given by the formula $h = \frac{2A}{b}$.

a Rewrite this formula for triangles of area 24 cm^2.

b Use this formula to complete the table of values that gives the height of triangles of area 24 cm^2.

b	2	4	8	12	16	24	48	96
h								

Spreadsheet

7 Taxi fares are calculated using the formula:

$$F = 1.3K + 4$$

where K is the distance travelled in kilometres and F is the cost of the fare in dollars.
The spreadsheet shows a table of values created using this formula.

a Write the formula that was used in cell B2 to calculate this value.

b Calculate the fare for a taxi journey of 30 kilometres.

	A	B
1	K	F
2	0	4.00
3	5	10.50
4	10	17.00
5	15	23.50
6	20	30.00
7	25	36.50
8	30	43.00
9	35	49.50
10	40	56.00

8 A car salesman's weekly wage, E dollars, is given by the formula:

$$E = 60n + 680$$

where n is the number of cars sold.

a Use a spreadsheet to construct a table of values to show how much his weekly wage will be if he sells from 0 to 10 cars.

b Using your table of values, calculate the number of cars a salesman sells if he earns $1040 in a week.

Example 11

9 Kaela uses a geometry package on a computer to construct rectangles of area 3200 cm^2. Let L cm be the length of a rectangle and W cm be the width of this rectangle.
The formula $W = \frac{3200}{L}$ can be used to calculate the width of one of these rectangles if its length is known.
Complete the following table of values that gives possible lengths and widths of Kaela's rectangles.

L	10	20	40	80	100	320	640
W							

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CF

Example 12

10 Use the rule $P = M \times T$ to complete the table on the right.

P		T			
		1	2	3	4
M	1				
	2				
	3				
	4				

11 Use the rule $H = \dfrac{R+2}{Z}$ to complete the table on the right. Write the table values as fractions.

H		Z			
		1	2	3	4
R	1				
	2				
	3				
	4				

Spread-sheet

12 Anita has \$10 000 that she wishes to invest at a rate of 4.5% per annum. She wants to know how much interest she will earn after $1, 2, 3, \ldots 10$ years. The formula:

$$I = \frac{Prt}{100}$$

where P is the principal, and r is the interest rate (%) and t is the number of time periods gives the total amount of interest earned. Construct a table of values with a spreadsheet to show how much interest, I, she will have after $t = 1, 2, 3, \ldots 10$ years.

13 The formula for finding the amount, A, accumulated at compound interest is given by:

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

where P is the principal, r is the interest rate per period (%) and t is the number of time periods. Construct a table of values using a spreadsheet showing the amount accumulated when \$5000 is invested at a rate of 5.5% over 5, 10, 15, 20 and 25 years. Give your answers to the nearest dollar.

Example 8

14 The formula for the cost of sending messages on a particular mobile phone plan, \$$C$, is:

$$C = 0.05t + 0.2p$$

where t is the number of text messages sent and p is the number of photo messages sent.

a Use the formula and a spreadsheet to construct a table of values for the cost of sending messages using values of t in intervals of 2 between $t = 0$ and $t = 10$ and values of p in intervals of 2 between $p = 0$ and $p = 10$.

b Use the table to determine the cost of sending 8 text messages and 4 photo messages.

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CF

15 The formula for the cost of limousine hire, C, using a particular company is:

$$C = 250 + 2t + 0.5d$$

where t is the time, in minutes, that the limousine is hired and d is the distance, in kilometres, that the limousine travels during the booking.

a Use the formula and a spreadsheet to construct a table of values for the cost of hiring a limousine using values of t between $t = 0$ and $t = 120$ in intervals of 20, and values of d from $d = 20$ to $d = 60$ in intervals of 5.

b Use the table to determine the cost of hiring a limousine for one hour and twenty minutes and travelling 30 kilometres.

c Use the table to determine the maximum number of kilometres you can travel if you hire a limousine for 2 hours and spend $515.

CU

16 David would like to borrow $5000 from a bank. The amount of simple interest charged on this loan is given by the formula:

$$I = 50r \times t$$

where I is the interest charged, r, is the annual interest rate and t is the number of years before the loan is repaid.

a Use the formula and a spreadsheet to construct a table of values for the interest charged on the loan using values of r in intervals of 0.2 between $r = 3$ and $r = 4$ and values of t in intervals of 1 between $t = 1$ and $t = 5$.

b Use the table to determine the interest charged if the loan has an annual interest rate of 3.8% and is repaid after 4 years.

17 Saskia has $6000 that she wishes to invest at a rate of 5% per annum.
The formula for interest earned is:

$$I = \frac{Prt}{100}$$

where P is the principal, r is the interest rate per period (%) and t is the number of time periods. By constructing a table of values, determine the minimum length of time, in years, that Saskia will need to invest her money to earn total interest of at least $2400.

18 Xanthe wants to take out a loan to buy a new car. The formula for interest is given as:

$$I = \frac{Prt}{100}$$

where I is the total interest payable, P is the amount that is borrowed and r is the annual interest rate. The bank offers an interest rate of 7.2% on car loans.
By first transposing the loan to make P the subject, construct a table of values with different values of t and I. Use the table of values that you have constructed to determine how much Xanthe can borrow for a car loan if the loan is for 3 years and she pays back $4536 in interest.

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Key ideas and chapter summary

Algebraic expression

An **algebraic expression**, also called a formula, is a mathematical relationship connecting two or more variables.

Substitution

Substitution involves replacing variables in a formula with values.

Transposition

Transposition involves making a different pronumeral the subject of a formula.

Linear equation

A **linear equation** is an equation whose unknown values are always to the power of 1.

Non-linear equation

A **non-linear equation** is one whose unknown values are *not* all to the power of 1.

Skills checklist

Check-list

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills.

4A **1** I can substitute values into a linear algebraic expression. ☐

See Examples 1 and 2, and Exercise 4A Questions 1 and 4.

4A **2** I can substitute values into a non-linear equation. ☐

See Examples 3, 4 and 5, and Exercise 4A Questions 7, 10 and 12.

4B **3** I can transpose a linear equation. ☐

See Example 6 and Exercise 4B Question 1.

4B **4** I can transpose a non-linear equation. ☐

See Examples 7 and 8, and Exercise 4B Questions 3 and 6.

4B **5** I can transpose an equation with multiple variables. ☐

See Example 9 and Exercise 4B Question 7.

4C **6** I can construct a table of values by hand and using technology. ☐

See Examples 10 and 11, and Exercise 4C Questions 1, 7 and 9.

4C **7** I can construct a table of values with two variables. ☐

See Example 12 and Exercise 4C Question 10.

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Multiple-choice questions

1 If $a = 4$, then $3a + 5 =$

A 39 **B** 12 **C** 17 **D** 27

2 If $b = 1$, then $2b - 9 =$

A -11 **B** -7 **C** 12 **D** 21

3 If $C = 50t + 14$ and $t = 8$, then $C =$

A 522 **B** 1100 **C** 72 **D** 414

4 If $P = 2L + 2W$, then the value of P when $L = 6$ and $W = 2$ is:

A 48 **B** 16 **C** 12 **D** 30

5 If $x = -2$, $y = 3$ and $z = 7$, then $\frac{z - x}{y} =$

A 3 **B** $\frac{5}{3}$ **C** $-\frac{5}{3}$ **D** -3

6 If $a = 2$, $b = 5$, $c = 6$ and $d = 10$, then $bd - ac =$

A 38 **B** 24 **C** 7 **D** 484

7 The area of a circle is given by $A = \pi r^2$. If $r = 6$ cm, then the area of the circle is closest to:

A 18.84 cm^2 **B** 37.70 cm^2 **C** 35 531 cm^2 **D** 113.10 cm^2

8 Using the expression $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, when $u = 6$ and $v = 3$ gives $f =$

A 6 **B** 2 **C** 3 **D** 9

9 The area of a trapezium is given by $A = \frac{(a + b)h}{2}$.
An expression for h is:

A $\frac{2A}{a + b}$ **B** $\frac{A - 2}{a + b}$ **C** $\frac{A - (a + b)}{2}$ **D** $2A - (a + b)$

10 If the formula $R = 5S - P$ is transposed to make S the subject then:

A $S = \frac{R - P}{5}$ **B** $S = \frac{R + P}{5}$ **C** $S = \frac{R}{5} + P$

D $S = \frac{P - R}{5}$

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11 The cost of hiring a community centre is given by the rule

$$C = 120t + 200$$

where C is the total cost in dollars and t is the number of hours for which the community centre is hired. The cost of hiring the centre for 6 hours is

A \$120 **B** \$200 **C** \$800 **D** \$920

12 The speed v, in metres per second, of a cannonball fired from a cannon after t seconds is given by $v = 800 + 50t + 2t^2$. The speed of the cannonball after 5 seconds is

A 800 m/s **B** 850 m/s **C** 852 m/s **D** 1100 m/s

Short-response questions

SF

1 If $P = 2l + 2b$, state P when:

a $l = 12$ and $b = 8$

b $l = 40$ and $b = 25$

2 If $A = \frac{1}{2}bh$, calculate A when:

a $b = 6$ and $h = 10$

b $b = 12$ and $h = 9$

3 The formula for calculating the circumference of a circle is given by $C = 2\pi r$, where r is the radius.
Calculate the circumference of a circle with radius 15 cm, correct to two decimal places.

4 The formula for the volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the circle cross-section of the cylinder and h is the height of the cylinder.
Calculate the volume of a cylinder with a cross-section radius of 3 cm and a height of 15 cm.
Round your answer to two decimal places.

5 The curved surface area, S, of a cylinder of radius r and height h is given by the formula $S = 2\pi rh$.
Transpose the formula to make r the subject and hence determine the radius of a cylinder with surface area 225 cm^2 and height 20 cm.
Give your answer correct to one decimal place.

6 The cost, C, of hiring a boat is given by $C = 8h + 25$ where h represents hours.

a Determine the cost if the boat is hired for 4 hours.

b Calculate the number of hours that the boat is hired for if the cost was \$81.

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7 A phone bill is calculated using the formula $C = 25 + 0.50n$, where n is the number of calls made. SF

a Complete the table of values below for values of n from 60 to 160.

n	60	70	80	90	100	110	120	130	140	150	160
C											

b Calculate the cost of making 160 phone calls.

8 A ticket to see a play costs \$89 per adult and \$42 per child. The total price, \$$P$, for a adult tickets and c child tickets is calculated using the formula $P = 89 \times a + 42 \times c$.

a Construct a table of values that shows the price of play tickets for values of a and c in intervals of 1 between 0 and 5.

b Determine the total price of tickets to the play for 3 adults and 1 child.

9 Consider the equation $y = 33x - 56$.

a Construct a table of values for values of x in intervals of 5 from -20 to 25.

b Determine the value of x when $y = 274$.

c Determine the value of x when $y = -221$.

10 For the equation $k = 2h + g$, construct a table of values for values of h and g in intervals of 1 from -2 to 2.

11 I think of a number, double it and add 4. The result is 6. By constructing an equation and then transposing the equation, determine the original number. CF

12 Four less than three times a number is 11. By constructing an equation and then transposing the equation, determine the original number.

13 An electrician charges \$80 up front and \$45 for each hour, h, that he works.

a Write a linear equation for the total charge, C, of any job.

b Calculate the cost of a 3-hour job.

14 A model rocket is fired from a height of 1 metre above the ground. The height of the rocket, h metres, at time t seconds after it is fired is given by the equation $h = 1 + 20t - 5t^2$.

a Use a spreadsheet to complete the following table of values.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
h									

b Determine the time that the rocket reaches its greatest height.

c Determine the greatest height reached by the rocket.

d Calculate the times the rocket is 16 metres above the ground.

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15 Tyson wants to take out a loan to buy a new car. The amount to be repaid is given by:

$$A = P\left(1 + \frac{r}{100}\right)^t$$

where P is the principal (the loan amount) and r is the interest rate (%) and t is the number of years.

a Transpose the equation to make P the subject.

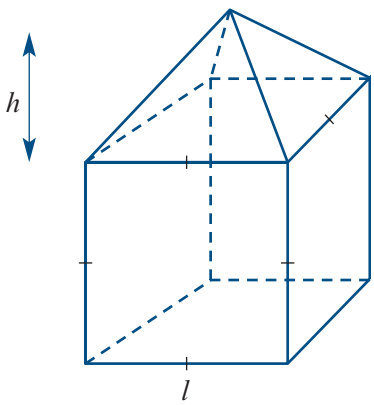
Assume that the annual interest rate is 9.6%.

b Construct a table of values in a spreadsheet for the amount that Tyson can borrow. Tyson is able to repay between \$15 000 and \$20 000 (use an interval of \$1000) over 1–3 years (use an interval of 1 year). Give all answers correct to two decimal places.

c Determine the amount that Tyson can borrow if he repays \$15 000 over 3 years.

d Calculate the additional amount that Tyson will have to pay on an \$18 000 loan over 4 years if the interest rate goes up by 0.9%.

16 A house consists of a cube with a pyramid on top, as shown in the diagram on the right.



The formulas for the surface area and volume of a cube and pyramid are shown below:

surface area of a cube $= 6l^2$

volume of a cube $= l^3$

surface area of a pyramid $= l^2 + 2l\sqrt{\left(\frac{l}{2}\right)^2 + h^2}$

volume of a pyramid $= \frac{1}{3}l^2h$

where l is the length of the square and h is the height of the pyramid.

Note: When a cube and pyramid are added together, the joined faces do not count in the total surface area.

a Construct a table of values in a spreadsheet for the surface area of the house using values of l between $l = 8$ and $l = 12$ in intervals of 1 and values of h between $h = 2$ and $h = 6$ in intervals of 1. Give your answers correct to 2 decimal places.

b Construct a table of values in a spreadsheet for the volume of the house using values of l between $l = 8$ and $l = 12$ in intervals of 1 and values of h between $h = 2$ and $h = 6$ in intervals of 1. Give your answers correct to 2 decimal places.

c From the tables, determine the surface area and volume of a house with a side length of 11 m and a roof height of 3 m.

d Determine the roof height and side length that will give a house volume of at least 2400 m^3 and a surface area of no more than 1050 m^2.

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Chapter 5

Linear equations and their graphs

Chapter questions

UNIT 1 MONEY, MEASUREMENT, ALGEBRA AND LINEAR EQUATIONS

Topic 5: Linear equations and their graphs

- How do we solve linear equations?
- How do we develop linear equations from word descriptions?
- How do we solve practical problems involving linear equations?
- What is a linear graph?
- What is the slope–intercept form of a linear equation?
- How do we construct a straight-line graph from its equation?
- How do we determine the slope and intercepts of a straight-line graph?
- How do find the equation of a straight line from its graph?
- How do we use straight-line graphs to model practical situations?
- How do we analyse a straight-line graph that is used to model linear relationships?

Many everyday situations can be described and investigated using a linear graph and its equation. Examples include the depreciating value of a newly purchased car, and the short-term growth of a newly planted tree. In this chapter, you will revise the properties of linear graphs and their equations and apply these ideas to modelling linear growth and decay in the real world.

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5A Solving linear equations with one unknown

Learning intentions

- To be able to solve a linear equation involving one step.
- To be able to solve a linear equation involving two or more steps.
- To be able to solve a linear equation involving brackets.
- To be able to solve a linear equation with variables on both sides.

Practical applications of mathematics often involve the need to be able to solve **linear equations**. An **equation** is a mathematical statement that says that two expressions have the same value. It consists of two expressions that are separated by an equals sign (=). For example, $x - 3 = 5$.

Linear equations come in many different forms in mathematics but are easy to recognise because the powers on the unknown values are always 1. For example: $4y - 3 = 17$ is a linear equation with unknown value y while $3x = 18$ is a linear equation with unknown value x. $x^2 + 3 = 9$ is **not** a linear equation as x has a power of 2.

The process of finding the unknown value is called **solving the equation**. To solve an equation, we can perform an operation to **both** sides of an equation, thus keeping the equation balanced. To balance an equation we add or subtract the same number on both sides of the equation or multiply or divide both sides of the equation by the same non-zero number. We perform the opposite or inverse operation to find the unknown value.

Example 1 Solving a linear equation

Solve the following equations.

a $x + 6 = 10$ **b** $3y = 18$ **c** $\frac{t}{4} = 2.1$

Solution

a $x + 6 = 10$

$x + 6 - 6 = 10 - 6$

$\therefore x = 4$

LHS $= x + 6 = 4 + 6 = 10$

RHS $= 10$

Explanation

Write the equation.

Subtract 6 from both sides of the equation. This is the opposite process to adding 6.

Check that the solution is correct by substituting $x = 4$ into the original equation.

Continued on next page

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b	$3y = 18$	Write the equation.
	$\frac{3y}{3} = \frac{18}{3}$ $\therefore y = 6$	The opposite process of multiplying by 3 is dividing by 3. Divide both sides of the equation by 3.
	LHS $= 3y = 3 \times 6 = 18$ RHS $= 18$	Check that the solution is correct by substituting $y = 6$ into the original equation.
c	$\frac{t}{4} = 2.1$	Write the equation.
	$\frac{4t}{4} = 2.1 \times 4$ $\therefore t = 8.4$	The opposite process of dividing by 4 is multiplying by 4. Multiply both sides of the equation by 4.
	LHS $= \frac{t}{4} = \frac{8.4}{4} = 2.1$ RHS $= 2.1$	Check that the solution is correct by substituting $t = 8.4$ into the original equation.

Sometimes more than one step is required to find the unknown value. In this case, the order of the operations is important.

Example 2 Solving a linear equation involving two or more steps

Solve the following equations.

a $3a + 1 = 10$ **b** $14 - 2b = -14$

Solution	Explanation
a $3a + 1 = 10$	Write the equation.
$3a + 1 - 1 = 10 - 1$	Subtract 1 from both sides of the equation. This is the opposite process to adding 1.
$3a = 9$ $\frac{3a}{3} = \frac{9}{3}$ $\therefore a = 3$	Divide both sides by 3. This is the opposite to multiplying by 3.

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b	$14 - 2b = -14$	Write the equation.
	$14 - 14 - 2b = -14 - 14$	Subtract 14 from both sides. This is the opposite process to adding 14.
	$-2b = -28$	
	$\frac{-2b}{-2} = \frac{-28}{-2}$	Divide both sides by -2. This is the opposite to multiplying both sides by -2.
	$b = 14$	

To solve an equation with brackets, there are two options. First, the brackets can be expanded or alternatively, both sides can be divided by the number that is outside of the brackets. Both methods are shown in Example 3 below.

Example 3 Solving a linear equation with brackets

Solve the equation $4(x - 3) = 24$.

Solution	Explanation
Method 1	
$4(x - 3) = 24$	Write the equation.
$4x - 12 = 24$	Expand the brackets.
$4x - 12 + 12 = 24 + 12$	Add 12 to both sides of the equation.
$4x = 36$	Divide by 4.
$\frac{4x}{4} = \frac{36}{4}$	
$\therefore x = 9$	
Method 2	
$4(x - 3) = 24$	Write the equation.
$\frac{4(x - 3)}{4} = \frac{24}{4}$	Divide both sides of the equation by 4.
$x - 3 = 6$	
$x - 3 + 3 = 6 + 3$	Add 3.
$\therefore x = 9$	

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To solve an equation with variables on both sides, we need to add or subtract the terms with variables so that they are all on the same side.

Example 4 Solving a linear equation with variables on both sides

Solve the equations.

a $4x + 1 = 3x + 6$ **b** $3.4q + 2.5 = 2.2q + 3.7$

Solution	Explanation
a $4x + 1 = 3x + 6$	Write the equation.
$4x + 1 - 3x = 3x + 6 - 3x$ $x + 1 = 6$	Subtract $3x$ from both sides of the equation to collect like terms.
$x + 1 - 1 = 6 - 1$ $x = 5$	Subtract 1 from each side.
b $3.4q + 2.5 = 2.2q + 3.7$	Write the equation.
$3.4q + 2.5 - 2.2q = 2.2q + 3.7 - 2.2q$ $1.2q + 2.5 = 3.7$	Subtract $2.2q$ from both sides of the equation to collect like terms.
$1.2q + 2.5 - 2.5 = 3.7 - 2.5$ $1.2q = 1.2$	Subtract 2.5 from both sides of the equation.
$\frac{1.2q}{1.2} = \frac{1.2}{1.2}$ $q = 1$	Divide both sides by 1.2.

Section Summary

- An **equation** is a mathematical statement that tells us that two expressions have the same value.
- Equations can be solved by applying the same actions to both sides of the equation.

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Skill-sheet

Exercise 5A

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Example 1

1 Solve the following linear equations.

a $x + 6 = 15$ **b** $y + 11 = 26$ **c** $t + 5 = 10$ **d** $m - 5 = 1$

e $g - 3 = 3$ **f** $f - 7 = 12$ **g** $f + 5 = 2$ **h** $v + 7 = 2$

i $x + 11 = 10$ **j** $g - 3 = -2$ **k** $b - 10 = -5$ **l** $m - 5 = -7$

m $2 + y = 8$ **n** $6 + e = 9$ **o** $7 + h = 2$ **p** $3 + a = -1$

q $9 - k = 2$ **r** $5 - n = 1$ **s** $3 - a = -5$ **t** $10 - b = -11$

2 Solve the following linear equations.

a $5x = 15$ **b** $3g = 27$ **c** $9n = 36$ **d** $2x = -16$

e $6j = -24$ **f** $4m = 28$ **g** $2f = 11$ **h** $2x = 7$

i $3y = 15$ **j** $3s = -9$ **k** $-5b = 25$ **l** $4d = -18$

m $\frac{r}{3} = 4$ **n** $\frac{q}{5} = 6$ **o** $\frac{x}{8} = 6$ **p** $\frac{t}{-2} = 6$

q $\frac{h}{-8} = -5$ **r** $\frac{m}{-3} = -7$ **s** $\frac{14}{a} = 7$ **t** $\frac{24}{f} = -12$

Example 2

3 Solve the following linear equations.

a $2a + 15 = 27$ **b** $\frac{y}{4} - 10 = 0$ **c** $13 = 3r - 11$

d $\frac{x + 1}{3} = 2$ **e** $\frac{3m}{4} = 6$ **f** $\frac{2x - 1}{3} = 4$

g $3a + 5 = 17$ **h** $7c - 2 = 19$ **i** $2y - 5 = 18$

j $4f - 1 = 9$ **k** $3 + 2h = 16$ **l** $2 - 3k = 6$

Example 3

4 Solve the following linear equations.

a $2(y - 1) = 6$ **b** $8(x - 4) = 56$ **c** $3(g + 2) = 12$

d $3(4x - 5) = 21$ **e** $8(2x + 1) = 16$ **f** $3(5m - 2) = 12$

g $\frac{2(a - 3)}{5} = 6$ **h** $\frac{4(r + 2)}{6} = 10$ **i** $\frac{3(4 - b)}{2} = 15$

Example 4

5 Solve these equations by first collecting all like terms.

a $2x = x + 5$ **b** $2a + 1 = a + 4$ **c** $4b - 10 = 2b + 8$

d $7 - 5y = 3y - 17$ **e** $3(x + 5) - 4 = x + 11$ **f** $6(c + 2) = 2(c - 2)$

6 Solve the following linear equations.

a $2.5x - 1.4 = 3.2$ **b** $2 - 0.2y = 1.5$ **c** $2x - 3 = 4x - 4$

d $4 - 3x = 5x + 2$ **e** $1.8d + 2.7 = 1.4d - 1.5$ **f** $2(y + 3.5) = 15$

g $3(y + 1.2) = 5.4$ **h** $2(x + 1.1) = x - 1.5$ **i** $5(2.4 - x) = 3x + 4$

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5B Developing a linear equation from a word description

Learning intentions

- To be able to set up a linear equation from a word description.
- To be able to solve a linear equation from a word description.

In many practical problems, we often need to set up a linear equation before finding the solution to a problem. There are four steps for solving a linear equation.

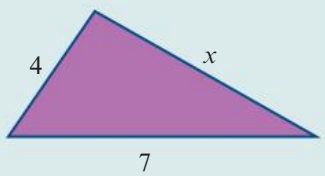
1. Choose a variable to represent the unknown number.
2. Use the information given to form an equation.
3. Solve the equation.
4. Answer the question.

To begin, we focus on the first two steps – choosing a variable and setting up an equation.

Example 5 Setting up a linear equation

State an equation for the perimeter of the triangle shown.

Note: Perimeter is the distance around the outside of a shape.



Solution	Explanation
Let P be the perimeter.	Choose a variable to represent the perimeter.
$P = 4 + 7 + x$	Add up all of the sides of the triangle and let them equal the perimeter, P.
$P = 11 + x$	Write the equation.

Once we have set up an equation, we can solve the equation and answer the question.

Example 6 Setting up and solving a linear equation

If 11 is added to a certain number, the result is 25. State the number.

Solution	Explanation
Let n be the number.	Choose a variable to represent the number.
$n + 11 = 25$	Using the information, write an equation.
$\therefore n = 14$	Solve the equation (subtract 11 from both sides).
The required number is 14.	Write your answer.

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Example 7 Setting up and solving a linear equation

At a recent show, Chris spent $100 on 8 showbags, each costing the same price.

a Using x as the cost of one showbag, write an equation showing the cost of 8 showbags.

b Use the equation to calculate the cost of one showbag.

Solution	Explanation
a Let x be the cost of one showbag. 8 showbags cost $8x$ so $8x = 100$	Choose a variable to represent the unknown number (cost of one showbag). Use the information to write an equation. Remember: $8 \times x = 8x$
b $8x = 100$ $\therefore x = 12.5$ The cost of one showbag is $12.50.	Write the equation. Solve the equation (divide both sides by 8). Write your answer.

In answering the question, it is important that we consider the context of the question. For example, noting whether the answer must be an integer.

Example 8 Setting up and solving a linear equation

A car rental company has a fixed charge of $110 plus $84 per day for the hire of a car. The Brown family has budgeted $650 to hire a car for their family holiday. Determine the number of days the car can be hired.

Solution	Explanation
Let d be the number of days the car is hired. $110 + 84d = 650$ $84d = 540$ $\therefore d = 6.428$ The Brown family could hire a car for 6 days.	Choose a variable (d) for the number of days the car is hired. Use the information to write an equation. Solve the equation (subtract 110 from each side then divide by 84). Since car hire works on a daily rate, 6.428 days is not an option. Thus we round down to 6 days to ensure that the Brown family stays within their budget of $650.

Section Summary

- Linear equations can be formed from word descriptions by first choosing a variable to represent the unknown number. An equation can then be set up and solved to answer the question.

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Exercise 5B

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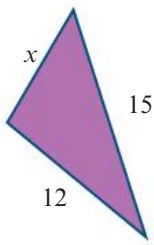
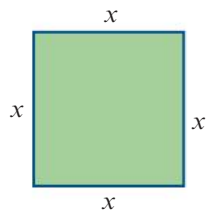
Example 5

1 Write an expression for the perimeter, P, of each of the following shapes.

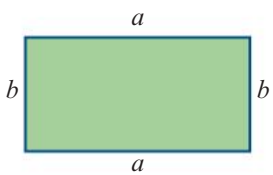
a



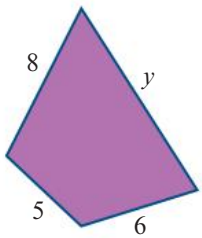
b



c



d



Example 6

2 Seven is added to a number and the result is 15.

a Write an equation using n to represent the number.

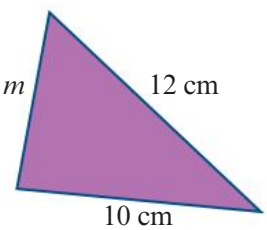
b Solve the equation for n.

3 Five is added to twtwice a number and the result is 17. State the number.

4 When a number is doubled and 15 is subtracted, the result is 103. State the number.

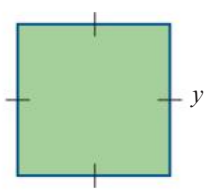
5 **a** Write an expression for the perimeter of the triangle shown.

b If the perimeter, P, of the triangle is 30 cm, solve the equation to determine the value of m.



6 **a** Write an equation for the perimeter of the square shown.

b The perimeter is 52 cm. Determine the length of one side.

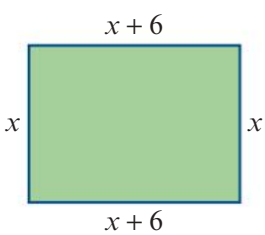


7 The perimeter of a rectangle is 84 cm. The length of the rectangle is 6 cm longer than the width, as shown in the diagram.

a Write an expression for the perimeter, P, of the rectangle.

b Calculate the value of x.

c Calculate the lengths of the sides of the rectangle.



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Example 7

8 Michael spent $126 on 15 packets of playing cards.

a Using c as the cost of one pack of playing cards, write an equation showing the cost of 15 packets of playing cards.

b Use the equation to calculate the cost of one packet of playing cards.

9 A raffle prize of $1000 is divided between Anne and Barry so that Anne receives 3 times as much as Barry. Determine how much each person receives.

Example 8

10 The cost for printing cards at Sam's Printing Company is $60 plus $2.50 per card. Kate paid $122.50 to print invitations for her party. Calculate how many invitations were printed.

11 Year 11 students want to run a school dance. The cost of hiring a band is $820 and they are selling tickets for $15 per person. The profit, P, is found by subtracting the band hire cost from the money raised from selling tickets. The students would like to make a profit of $350. Use the information to write an equation, and then solve the equation to determine how many tickets they need to sell.

12 A number is tripled then 2 is added. This gives the same result as if the number were quadrupled. State the original number.

13 Cooper is currently 20 years older than his son, Lachlan. In 12 years time, Cooper will be twice Lachlan's age. Determine Cooper's age.

14 Isabella has twice as many $10 notes as $20 notes. If she has 6 more $10 notes than $20 notes, determine the number of $20 notes that she has.

15 Bruce cycles x kilometres, then walks half as far as he cycles. If the total distance covered is 45 km, calculate the value of x.

16 Amy and Ben live 17.2 km apart. They cycle to meet each other. Ben travels at 12 km/h and Amy travels at 10 km/h.

a Determine how long, to the nearest minute, until they meet each other.

b Calculate the distance, correct to one decimal place, they each travelled.

17 The sum of three consecutive even numbers is 192. Calculate the three numbers.

18 The average of 5 numbers is 8. When one more number is added, the average becomes 12. State the number that was added.

19 Ruby and Mia compete in a sprint race. As Ruby runs 10% faster than Mia, Mia gets a 5 m headstart. The two girls are level in the race after 8 seconds. Determine the average speed of Ruby and Mia.

20 Twenty litres of a salt-water solution with unknown salt content is mixed with ten litres of salt-water solution with 3% salt. The new solution has a salt content of 3.75%. Calculate the percentage of salt in the first solution.

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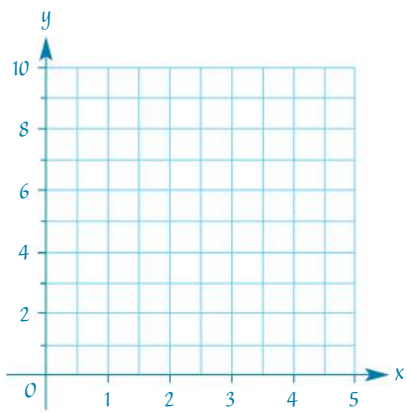
5C Constructing straight-line graphs

Learning intentions

- To be able to form a table of values from a linear function.
- To be able to draw a straight-line graph from a table of values.

Introduction to the Cartesian plane

The Cartesian plane, or number plane, includes a horizontal axis (x) and a vertical axis (y) that intersect at right angles. The point where these axes cross over is called the origin, which has the coordinates $(0, 0)$. We can graph a pair of coordinates on the Cartesian plane when given an x and a y value.



In this subject, we are generally only interested in values of x and y that are positive so we focus most of our attention on this quadrant of the Cartesian plane.

We can plot points on the Cartesian plane and if they form a straight line, we can connect up the points.

Plotting straight-line graphs

Relations of the form $y = mx + c$ are called **linear relations** because they generate **straight-line graphs**.

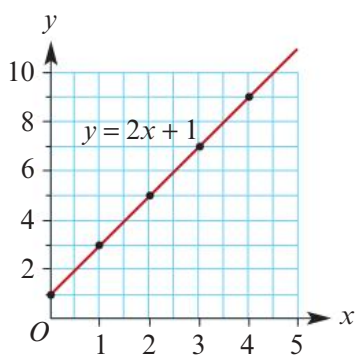
For example, consider the relation $y = 2x + 1$. To plot this graph, we can form a table.

x	0	1	2	3	4
y	1	3	5	7	9

We can then plot the values from the table on a set of axes, as shown opposite.

The points appear to lie on a straight line.

A ruler can then be used to draw the straight line to give the graph of $y = 2x + 1$.



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Example 9 Constructing a table of values

Construct a table of values of y for $x = 0, 1, 2, 3, 4$.

a $y = 2x$ **b** $y = -x + 5$

Solution

a

x	0	1	2	3	4
y	0	2	4	6	8

b

x	0	1	2	3	4
y	5	4	3	2	1

Explanation

Set up a table of values.
When $x = 0, y = 2 \times 0 = 0$.
When $x = 1, y = 2 \times 1 = 2$, and so on.

Set up a table of values.
When $x = 0, y = -0 + 5 = 5$.
When $x = 1, y = -1 + 5 = 4$, and so on.

Once we have a table of values, we can plot the points on a Cartesian plane and use a ruler to join the points.

Example 10 Constructing a graph from a table of values

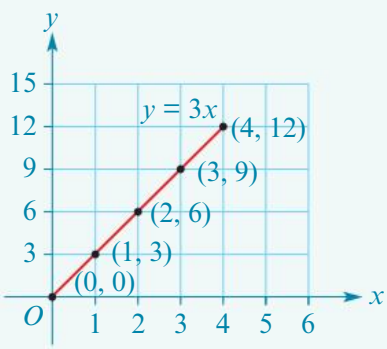
Plot the graphs of the following equations by first forming a table of values of y for $x = 0, 1, 2, 3, 4$.

a $y = 3x$ **b** $y = -2x + 8$

Solution

a

x	0	1	2	3	4
y	0	3	6	9	12



Explanation

Set up a table of values.
When $x = 0, y = 3 \times 0 = 0$.
When $x = 1, y = 3 \times 1 = 3$, and so on.

Draw, label and scale a set of axes to cover all values.
Plot the values in the table on the graph by marking each point with a dot (•). The first point is (0, 0). The second point is (1, 3), and so on.
The points will lie on a straight line.
Use a ruler to draw in the straight line.
Label the line $y = 3x$.

Continued on next page

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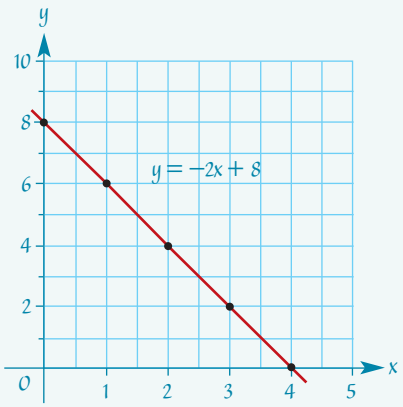
b

x	0	1	2	3	4
y	8	6	4	2	0



Set up a table of values.
When $x = 0, y = -2 \times 0 + 8 = 8$.
When $x = 1, y = -2 \times 1 + 8 = 6$, and so on.

Draw, label and scale a set of axes to cover all values.
Plot the values in the table on the graph by marking each point with a dot (•). The first point is (0, 8). The second point is (1, 6), and so on.
The points will lie on a straight line.
Use a ruler to draw in the straight line.
Label the line $y = -2x + 8$.

In the next sections, we will think more about the slope of the graph and consider other ways of graphing equations by hand that mean that we do not need to construct a table of values.

Using technology to draw straight-line graphs

Several types of technology can be used to draw a straight-line graph from a linear equation. The Interactive Textbook covers two types:

Spread-sheet **Spreadsheet activity 4C:** Drawing a straight-line graph with Microsoft Excel

Section Summary

- The **Cartesian plane**, or number plane, consists of a horizontal axis and a vertical axis that intersect at right angles. We can plot points on the Cartesian plane when given an x and a y value.
- To plot a straight line from a linear equation, we can form a table of values then plot each point. A straight line is obtained by joining the points.

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Exercise 5C

SF

Example 9

1 Construct a table of values of y for $x = 0, 1, 2, 3, 4$.

a $y = x$ **b** $y = x + 3$ **c** $y = x + 5$

d $y = 2x + 1$ **e** $y = 2x + 4$ **f** $y = -x$

g $y = -x - 8$ **h** $y = -2x + 10$ **i** $y = -3x + 12$

Example 10

2 Plot the graph of the linear equations below by first forming a table of values of y for $x = 0, 1, 2, 3, 4$.

a $y = 3x$ **b** $y = 2x + 1$ **c** $y = 2x + 5$

d $y = x + 2$ **e** $y = 3x + 1$ **f** $y = 3x + 4$

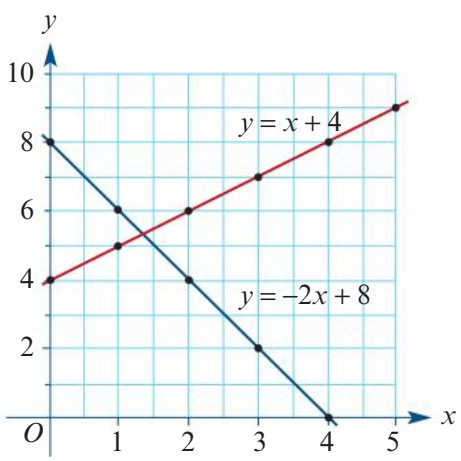
g $y = -x + 10$ **h** $y = -2x + 10$ **i** $y = -2x + 15$

CF

3 Two straight-line graphs, $y = x + 4$ and $y = -2x + 8$, are plotted as shown below.

a Reading from the graph of $y = x + 4$, determine the missing coordinates: (0, ?), (2, ?), (?, 7), (?, 9).

b Reading from the graph of $y = -2x + 8$, determine the missing coordinates: (0, ?), (1, ?), (?, 4), (?, 2).



4 Consider the equation $y = -3x + 16$. Determine:

a the value of y when $x = 3$

b the value of y when $x = 5$

c the value of x when $y = 0$

d the value of x when $y = 4$.

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5D Determining the slope of a straight line

Learning intentions

- To be able to determine the slope of a line from a graph with a positive slope.
- To be able to determine the slope of a line from a graph with a negative slope.
- To be able to calculate the slope of a line using the formula.

Introduction to slopes

Positive slopes

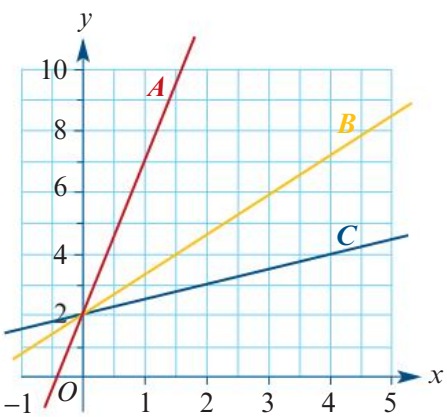
One thing that makes one straight-line graph look different from another is its steepness or slope. Another name for slope is **gradient***.

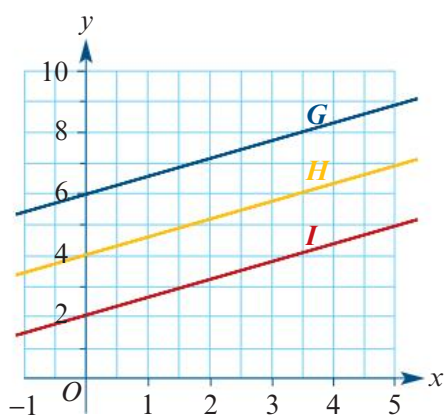
For example, the three straight lines on the graph opposite all cut the y-axis at $y = 2$, but they have quite different slopes.

Line A has the steepest slope and Line C has the gentlest slope. Line B has a slope somewhere in between.

In all cases, the lines have **positive slopes**; that is, they rise from left to right.

By contrast, the three straight lines, G, H and I, on the graph opposite cut the y-axis at different points, but they all have the *same* slope.





*Note: For linear graphs, the terms ‘slope’ and ‘gradient’ mean the same thing. However, when dealing with practical applications of linear graphs and, most particularly, in statistics applications, the word ‘slope’ is preferred.

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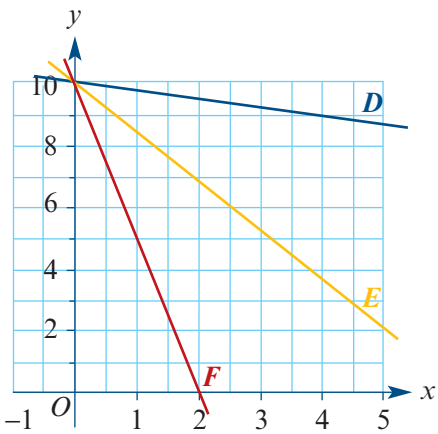
Negative slopes

Similarly, the three straight lines on the graph opposite all cut the y-axis at $y = 10$, but they have quite different slopes.

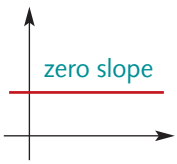
In this case, Line D has the gentlest slope and Line F has the steepest slope. Line E has a slope somewhere in between.

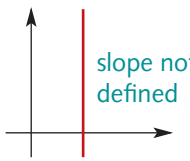
In all cases, the lines have **negative slopes**; that is, they fall from left to right.



Zero or undefined slopes

It is also posssible for a graph to have a zero slope (if it is horizontal) or for its slope to be undefined (if it is vertical).





Determining the slope

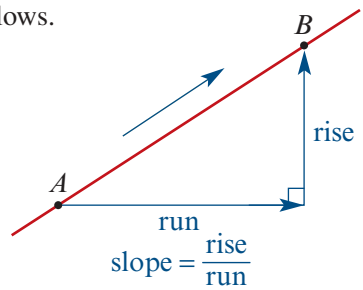
When talking about the **slope of a straight line**, we want to be able to do more than say that it has a gentle positive slope. We would like to be able to give the slope a value that reflects this fact. We do this by defining the slope of a line as follows.

First, two points A and B on the line are chosen.

As we go from A to B along the line, we move:

- **up** by a distance called the **rise**
- and **across** by a distance called the **run**.

The slope is found by dividing the rise by the run.



Note that the **rise** tells us the change in the y-coordinates, while the **run** is the change in the x-coordinates. In general, we think of the run as being positive; that is, we are asking, 'What happens to y as x increases?' When we have a negative rise, we call this a **fall**.

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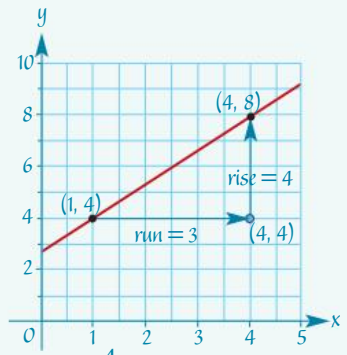
Example 11 Determining the slope of a line from a graph: positive slope

Determine the slope of the line through the points (1, 4) and (4, 8).

Solution



Slope is $\frac{4}{3}$

Explanation

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{rise} = 8 - 4 = 4$$

$$\text{run} = 4 - 1 = 3$$

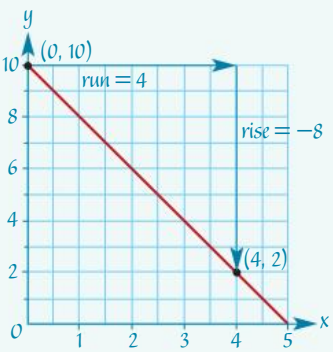
$$\therefore \text{ slope} = \frac{4}{3}$$

Note: To find the 'rise', look at the y-coordinates. To find the 'run', look at the x-coordinates.

Example 12 Determining the slope of a line from a graph: negative slope

Determine the slope of the line through the points (0, 10) and (4, 2).

Solution



Slope is −2

Explanation

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{rise} = 2 - 10 = -8$$

$$\text{run} = 4 - 0 = 4$$

$$\therefore \text{ slope} = \frac{-8}{4} = -2$$

Note: In this example, we have a negative 'rise', which represents a 'fall'.

A formula for finding the slope of a line

We can also use a formula to calculate the slope. The formula is derived as follows.

Label the coordinates of point A (x_1, y_1).

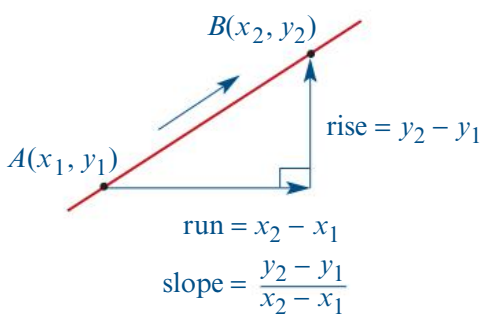
Label the coordinates of point B (x_2, y_2).

By definition: $\text{slope} = \frac{\text{rise}}{\text{run}}$

From the diagram: $\text{rise} = y_2 - y_1$

$\text{run} = x_2 - x_1$

By substitution: $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$



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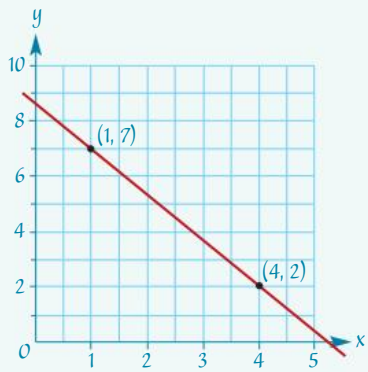
Example 13 Calculating the slope of a line using the formula for the slope

Calculate the slope of the line through the points (1, 7) and (4, 2) using the formula for the slope of a line. Give your answer correct to two decimal places.

Solution



Slope is −1.67 to two decimal places

Explanation

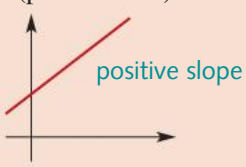
Let $(x_1, y_1) = (1, 7)$ and $(x_2, y_2) = (4, 2)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - 7}{4 - 1}$$
$$= -1.67 \text{ (to two decimal places)}$$

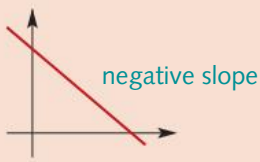
Section Summary

- The slope of a straight-line graph tells us how steep the line is by telling us how y changes as x increases.

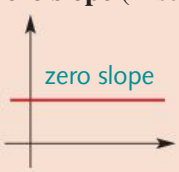
A straight-line graph that rises from left to right is said to have a **positive slope** (positive rise).



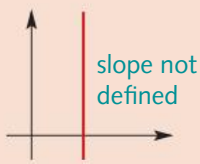
A straight-line graph that falls from left to right is said to have a **negative slope** (negative rise).



A straight-line graph that is horizontal has **zero slope** ('rise' = 0).



A straight-line graph that is vertical has **undefined** slope.



- The slope can be calculated by finding the **rise** and **run** from a graph or it can be found between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ using the formula:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

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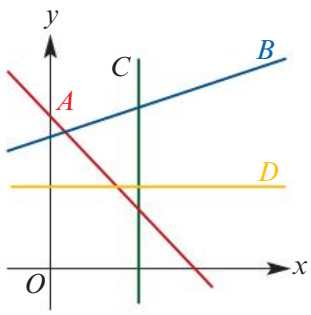
Exercise 5D

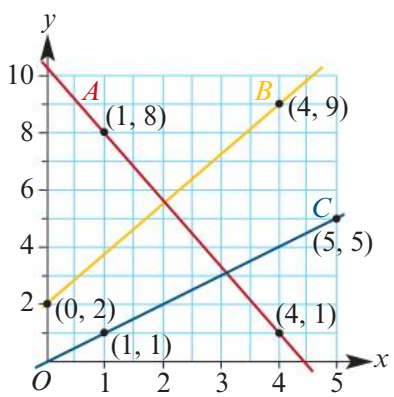
1 Identify the slope of each of the straight-line graphs A, B, C and D as positive, negative, zero, or not defined.



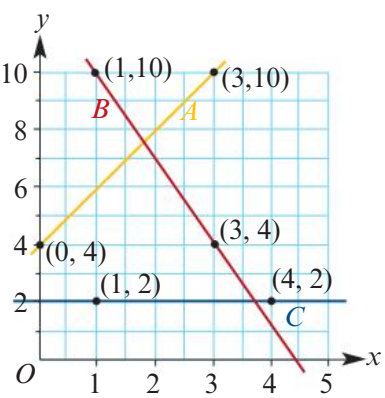
Example 11
Example 12

2 Determine the slope of each of the lines (A, B, C) shown on the graphs below.

a



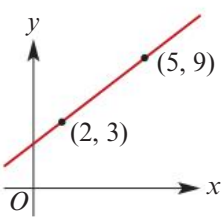
b



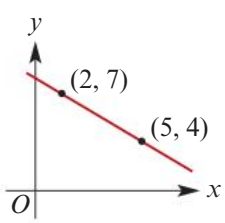
Example 13

3 Determine the slope of each of the lines shown.

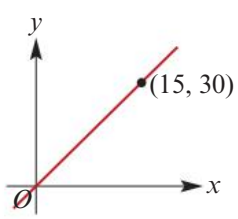
a



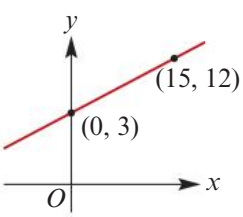
b



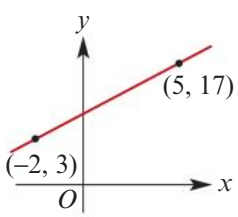
c



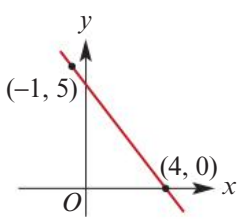
d



e



f



4 Calculate the slope of the line passing through the following two points using the formula of the line. In each case, give your answer correct to two decimal places.

a $A(2, 8)$ and $B(6, 11)$ **b** $C(1, 1)$ and $D(3, 2)$ **c** $E(16, 8)$ and $F(3, 20)$

d $G(2.5, 4)$ and $H(2.8, 3.8)$ **e** $I(-1, 10)$ and $J(3, 7.5)$ **f** $K(-2.5, 5)$ and $L(2.3, 3.2)$

5 A line has a slope of 5 and passes through the point $A(3, 8)$ and $B(x, 12)$. Calculate the value of x.

SF
CF

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5E The slope–intercept form of the equation of a straight line

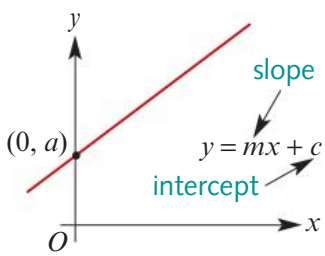
Learning intentions

- To be able to determine the slope and y-intercept of a line from an equation.
- To be able to write down an equation given its y-intercept and slope.
- To be able to calculate the x-intercept from an equation.
- To be able to sketch a straight-line graph from its equation.

Determining the slope and intercept of a straight-line graph from its equation

Recall that the most common form of a linear relation is $y = mx + c$. This form is commonly called the **slope–intercept form** of the equation of a straight line because:

- m = the **slope** of the graph
- c = the ***y*-intercept** of the graph.



The slope–intercept form of the equation can be written as $y = mx + c$.

The slope–intercept form of the equation of a straight line is useful in modelling relationships in many practical situations.

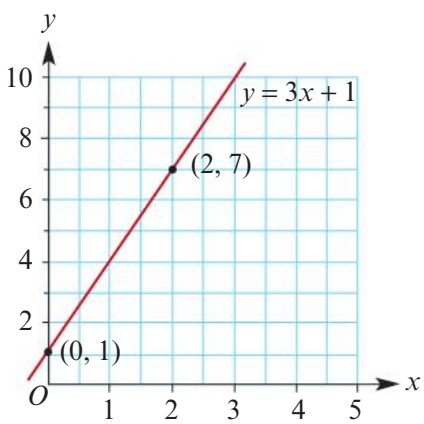
An example of the equation of a straight line written in slope–intercept form is $y = 3x + 1$.

Its graph is shown opposite.

From the graph, we see that the:

$$y\text{-intercept} = 1$$

$$\text{slope} = \frac{7 - 1}{2 - 0} = \frac{6}{2} = 3$$



That is:

- the y-intercept corresponds to the (**constant**) term in the equation (intercept = 1).
- the slope is given by the **coefficient** of x in the equation (slope = 3).

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Slope–intercept form of the equation of a straight line

If the equation of a straight line is in the slope–intercept form:

$y = mx + c$

then: m = the slope of the graph

c = the y-intercept of the graph (where the graph cuts the y-axis).

Example 14 Determining the slope and intercept of a line from its equation

Write down the slope and the y-intercept of each of the straight-line graphs defined by the following equations.

a $y = 9x - 6$ **b** $y = -5x + 10$ **c** $y = -2x$ **d** $y - 4x = 5$

Solution

a $y = 9x - 6$
slope = 9, y-intercept = -6

b $y = -5x + 10$
slope = -5, y-intercept = 10

c $y = -2x$
slope = -2, y-intercept = 0

d $y - 4x = 5$ or $y = 4x + 5$
slope = 4, y-intercept = 5

Explanation

For each equation:
Write the equation.
If it is not in slope–intercept form, rearrange the equation.
When the equation is in slope–intercept form, write the value of:
m = the slope (the coefficient of x)
c = the y-intercept (the constant term).

Example 15 Writing down the equation of a straight line given its slope and y-intercept

Write down the equation of the straight lines with the following slopes and y-intercepts in the form $y = mx + c$.

a slope = 6, y-intercept = 9 **b** slope = -5, y-intercept = 2
c slope = 2, y-intercept = -3

Solution

a slope = 6, y-intercept = 9
equation: $y = 6x + 9$

b slope = -5, y-intercept = 2
equation: $y = (-5)x + 2$
or: $y = -5x + 2$

c slope = 2, y-intercept = -3
equation: $y = 2x - 3$

Explanation

The equation of a straight line is $y = mx + c$.
Form an equation by inserting the given values of the slope and the y-intercept for m and c.

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Calculating the x-intercept of a straight line from its equation

The x-intercept is the point at which the straight line crosses the x-axis. This is the point where $y = 0$.

To find the x-intercept, we let $y = 0$ and solve the equation for x.

Example 16 Calculating the x-intercept of a straight line graph from its equation

Calculate the x-intercept of the straight-line graphs of the following equations.

a $y = 9x - 18$ **b** $y = -5x + 10$ **c** $y = -2x$

Solution	Explanation
a $y = 9x - 18$ $0 = 9x - 18$ $18 = 9x$ $2 = x$ $x = 2$	Write down the equation. Let $y = 0$. Solve the equation for x.
b $y = -5x - 10$ $0 = -5x - 10$ $10 = -5x$ $-2 = x$ $x = -2$	Write down the equation. Let $y = 0$. Solve the equation for x.
c $y = -2x$ $0 = -2x$ $0 = x$ $x = 0$	Write down the equation. Let $y = 0$. Solve the equation for x.

Note that sometimes the x-intercept is negative. While mathematically this is possible, it often doesn't fit with the context that we are dealing with. For example, if x represents **distance travelled** then it doesn't make sense to have a negative value.

Sketching straight-line graphs

Because only two points are needed to draw a straight line, all we need to do is find two points on the graph and then draw a line passing through these two points. When the equation of a straight line is written in the form $y = mx + c$, one point on the graph is immediately available: the y-intercept. A second point can then be quickly calculated by substituting a suitable value of x into the equation or by using the x-intercept.

When we draw a graph in this manner, we call it a **sketch graph**.

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Example 17 Sketching a straight-line graph from its equation

Sketch the graph of $y = 2x + 8$.

Solution

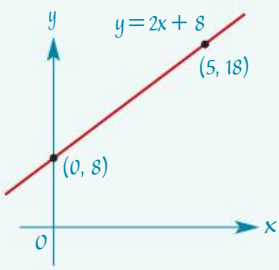
Method 1: Two points

y-intercept $= 8$

$\therefore$ (0, 8) is a point on the line.

When $x = 5, y = 2(5) + 8 = 18$

$\therefore$ (5, 18) is a point on the line.



Method 2: Both intercepts

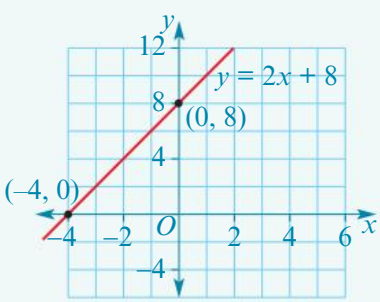
$\therefore$ (0, 8) is a point on the line.

$0 = 2x + 8$

$-8 = 2x$

$-4 = x$

$\therefore$ (−4, 0) is a point on the line.



Explanation

As the equation is written in the slope-intercept form, the y-intercept is given by the constant term, which is 8.
Calculate a second point on the graph by substituting any x value (not 0) into the equation. Here we choose $x = 5$.
Draw a set of labelled axes.
Mark in the two points with coordinates.
Draw a straight line through the points.
Label the line with its equation.

Write the y-intercept as a coordinate as above.

Calculate the x-intercept by solving the equation when $y = 0$.
Write the x-intercept as a coordinate.

Draw a set of labelled axes.
Mark in the two points with coordinates.
Draw a straight line through the points.
Label the line with its equation.

Section Summary

- The slope–intercept form of a linear equation is $y = mx + c$ where m is the slope and c is the y-intercept.
- The x-intercept can be found by solving the equation for x when $y = 0$.

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Skill-sheet

Exercise 5E

Determining the slope and *y*-intercept from an equation

Example 14

1 Write down the slopes and *y*-intercepts of the straight lines with the following equations.

a $y = 5 + 2x$ **b** $y = 6 - 3x$ **c** $y = 15 - 5x$
d $y + 3x = 10$ **e** $y = 3x$ **f** $4y + 8x = -20$
g $x = y - 4$ **h** $x = 2y - 6$ **i** $2x - y = 5$
j $y - 5x = 10$ **k** $2.5x + 2.5y = 25$ **l** $y + 3x - 6 = 0$
m $10x - 5y = 20$ **n** $4x - 5y - 8 = 7$

Determining the equation of a straight-line graph given its slope and *y*-intercept

Example 15

2 Write down the equation of a line that has:

a slope = 5, *y*-intercept = 2 **b** slope = 10, *y*-intercept = 5
c slope = 4, *y*-intercept = −2 **d** slope = −3, *y*-intercept = 12
e slope = −5, *y*-intercept = −2 **f** slope = −0.4, *y*-intercept = 1.8
g slope = −2, *y*-intercept = 2.9 **h** slope = −0.5, *y*-intercept = −1.5

Calculating the *x*-intercept of an equation

Example 16

3 Calculate the *x*-intercept for each of the following straight lines.

a $y = 2x + 5$ **b** $y = -3x + 6$ **c** $y = -5x + 15$
d $y + 3x = 10$ **e** $y = 3x$ **f** $4y + 8x = -20$
g $x = y - 4$ **h** $x = 2y - 6$ **i** $2x - y = 5$
j $y - 5x = 10$

Sketching a straight-line graph from its equation

Example 17

4 Sketch the graphs of the straight lines with the following equations, clearly showing the *y*-intercepts and the coordinates of one other point.

a $y = 2x + 5$ **b** $y = 5x + 5$ **c** $y = -2x + 20$
d $y = 10x - 10$ **e** $y = 4x$ **f** $y = -2x + 16$

5 Sketch the graphs of the straight lines with the following equations, using the *y*-intercept and the *x*-intercept.

a $y = 4 - 2x$ **b** $y = 10 + 2x$ **c** $y = -5 + 3x$

The slope and *x*-intercept

6 An equation has a gradient of 1 and an *x*-intercept of 8. Determine the equation of the line in the form $y = mx + c$.

7 An equation has a gradient of 2 and an *x*-intercept of −4. Determine the equation of the line in the form $y = mx + c$.

SF

CF

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5F Determining the features and equation of a straight line from its graph

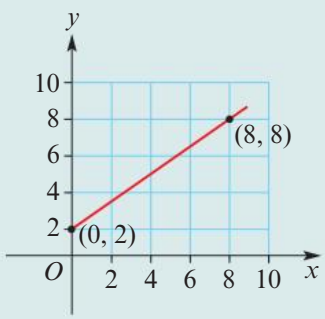
Learning intentions

- To be able to determine the slope and y-intercept from a graph.
- To be able to write an equation from its graph.

We have already determined how to find the slope of a straight line from its graph in Section 5D. In this section, we review this and also determine the y-intercept.

Example 18 Determining the slope and y-intercept from a graph

Determine the slope and y-intercept from the graph below.



Solution

Rise = 8 − 2 = 6

Run = 8 − 0 = 8, slope = $\frac{\text{rise}}{\text{run}} = \frac{6}{8} = \frac{3}{4}$

y-intercept is 2 or (0, 2)

Explanation

Calculate the rise.

Calculate the run.

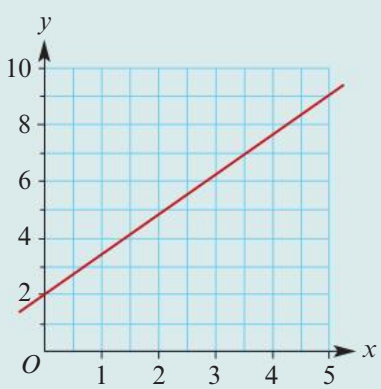
Calculate the slope.

Read the y-intercept off the graph.

We have learned how to construct a straight-line graph from its equation. We can also determine the equation from a graph. In particular, if the graph shows the y-intercept, it is a relatively straightforward procedure.

Example 19 Determining the equation of a line: slope–intercept method

Determine the equation of the straight-line graph shown opposite.



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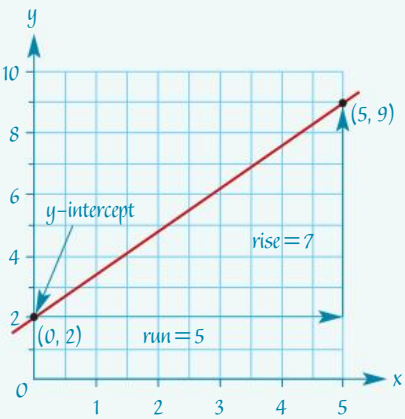
Solution	Explanation
$y = mx + c$ y-intercept $= 2$ so $c = 2$	Write the general equation of a line in slope–intercept form. Read the y-intercept from the graph.
	Calculate the slope using two well-defined points on the line, for example, (0, 2) and (5, 9).
slope $= \frac{\text{rise}}{\text{run}} = \frac{7}{5} = 1.4$ so $m = 1.4$	
$y = 1.4x + 2$	Substitute the values of c and m into the equation.
$y = 1.4x + 2$ is the equation of the line.	Write your answer.

Calculating the equation of a straight-line graph from its slope and intercept

To find the equation of a straight line in slope–intercept form ($y = mx + c$) from its graph:

1. identify the y-intercept (c)
2. use two points on the graph to find the slope (m)
3. substitute these two values into the standard equation $y = mx + c$.

Unfortunately, not all straight-line graphs show the y-intercept. When this happens, we have to use the two-point method for finding the equation of the line.

Calculating the equation of a straight-line graph using two points

The general equation of a straight-line graph is $y = mx + c$.

1. Use the coordinates of the two points to determine the slope (m).
2. Substitute this value for the slope into the equation. There is now only one unknown, c.
3. Substitute the coordinates of one of the two points on the line into this new equation and solve for the unknown (c).
4. Substitute the values of m and c into the general equation $y = mx + c$ to obtain the equation of the straight line.

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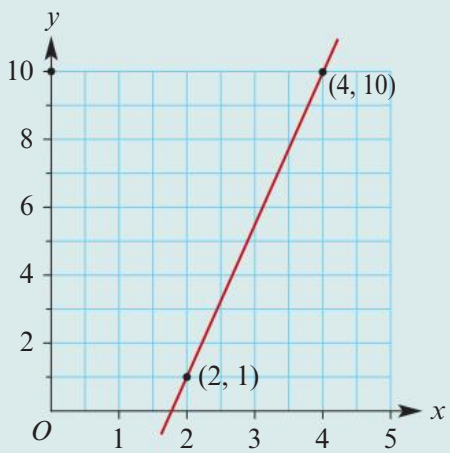
Example 20 Determining the equation of a straight-line using two points on the graph

Determine the equation of the line that passes through the points (2, 1) and (4, 10).



Solution

$y = mx + c$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{10 - 1}{4 - 2} = \frac{9}{2} = 4.5$$

so $m = 4.5$

$y = 4.5x + c$

Using the point (2, 1):

$1 = 4.5 \times 2 + c$

$= 9 + c$

$c = -8$

Thus, the equation of the line is:
$y = 4.5x - 8$.

Explanation

Write down the general equation of a straight-line graph.

Use the coordinates of the two points on the line to find the slope (m).

Substitute the value of m into the general equation.

To find the value of c, substitute the coordinates of one of the points on the line (either will do) and solve for c.

Substitute the value of c into the general equation $y = mx + c$ to find the equation of the line.

Section Summary

- The slope can be calculated using the coordinates of any two points on the graph to give the value of m in the slope–intercept form of a linear equation.
- If the y-intercept cannot be read directly from a graph, the value of c can be found by substituting in a point into the equation $y = mx + c$ once m has been calculated.

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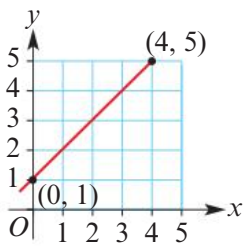
Exercise 5F

Determining the slope and *y*-intercept from a graph

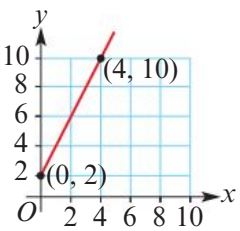
Example 18

1 Determine the slope and *y*-intercept in each of the graphs below.

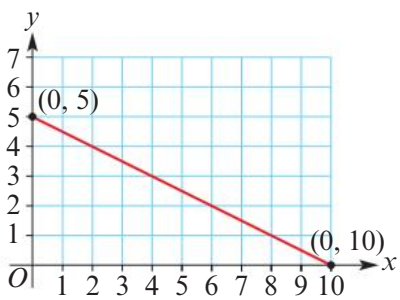
a



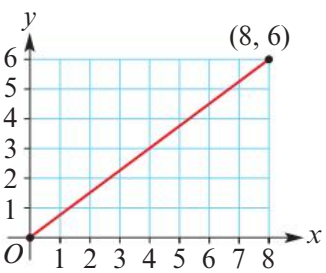
b



c



d

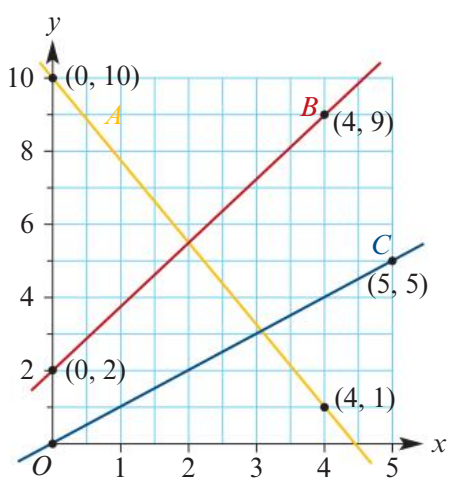


Determining the equation of a line from its graph using the slope–intercept method

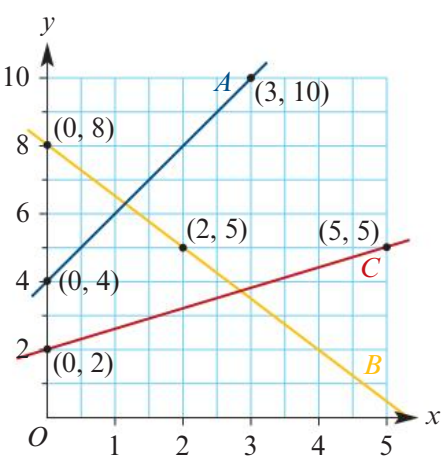
Example 19

2 Determine the equation of each of the lines (A, B, C) shown on the graphs below. Write your answers in the form $y = mx + c$.

a



b



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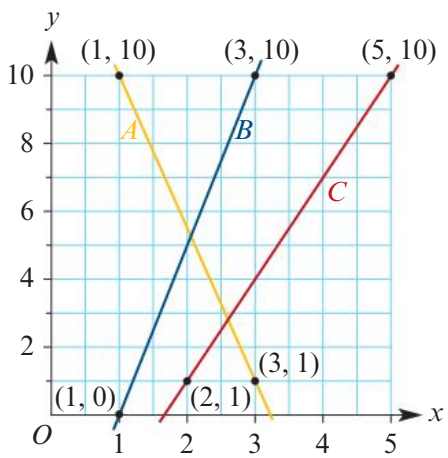
Determining the equation of a line given any two points on its graph

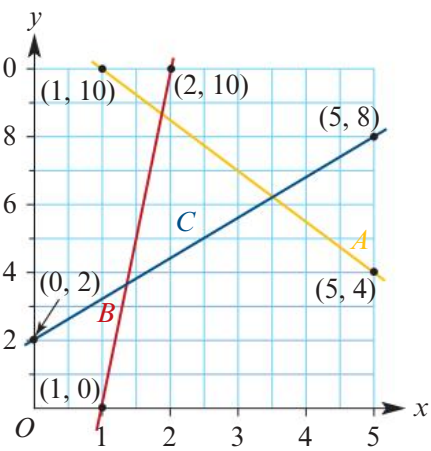
Example 20

3 Determine the equation of each of the lines (A, B, C) on the graphs below. Write your answers in the form $y = mx + c$.

a



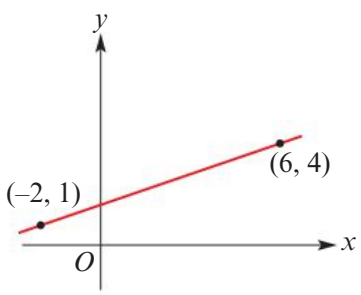
b



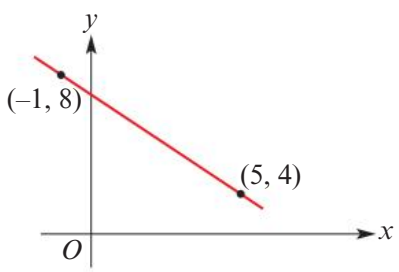
SF

4 Determine the equation of each of the lines in the graphs below. Write your answers in the form $y = mx + c$.

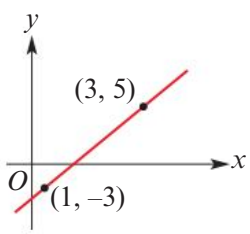
a



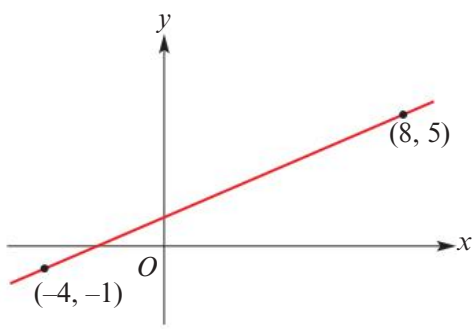
b



c



d



CF

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5G Constructing linear models

Learning intentions

- To be able to construct a linear model from a real-life example.
- To be able to use a linear model to make predictions.

Many real-life relationships between two variables can be described mathematically by linear (straight-line) equations. This is called *linear modelling*.

These linear models can be used to solve problems such as finding the time taken to completely fill a partially filled swimming pool with water, estimating the depreciating value of a car over time or describing the growth of a plant over time. Equations of linear models take the form $y = mx + c$, although other variables are often used.

Example 21 Constructing a linear model of plant growth

A plant is initially 5 cm when first measured. For the next 10 weeks, it grows at a constant rate of 6 cm per week.

Let h be the height of the plant (in cm).

Let t be the time in weeks after the plant was first measured.

a Write down a linear model in terms of h and t to represent this situation.

b Sketch the graph showing the coordinates of the intercept and the end point.

Solution

a $c = 5$

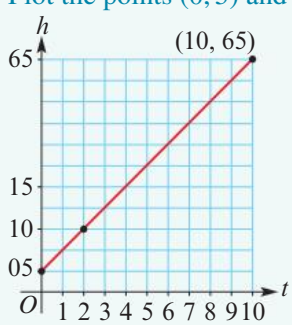
$m = 6$

$h = 6t + 5, \quad \text{for } 0 \leq t \leq 10$

b When $t = 0$, $h = 6 \times 0 + 5 = 5$

When $t = 1$, $h = 6 \times 1 + 5 = 11$

Plot the points $(0, 5)$ and $(1, 11)$



Explanation

The initial height of the plant was 5 cm. In graphical terms, this is the y-intercept.

The plant grows at a constant rate of 6 cm per week. In graphical terms, this is the slope of the graph.

Hence, the linear model can be written.

To sketch a graph, we need two points.

Find the value of h when $t = 0$ and $t = 1$.

Plot the two points and then connect them to form a straight line.

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Three important features of the linear model $h = 6t + 5$ for $0 \leq t \leq 10$ should be noted.

- The *h-intercept* gives the height of the plant at the start; that is, its height when $t = 0$. The plant was 5 cm tall when it was first planted.
- The *slope* of the graph gives the growth rate of the plant. The plant grows at a rate of 6 cm per week; that is, each week the height of the plant increases by 6 cm.
- The graph is only plotted for $0 \leq t \leq 10$. This is because the model is only valid for the time when the plant is growing at the constant rate of 6 cm a week.

Note: The expression $0 \leq t \leq 10$ is included to indicate the range of number of weeks for which the model is valid. In more formal language, this would be called the domain of the model.

Using a linear model to make predictions

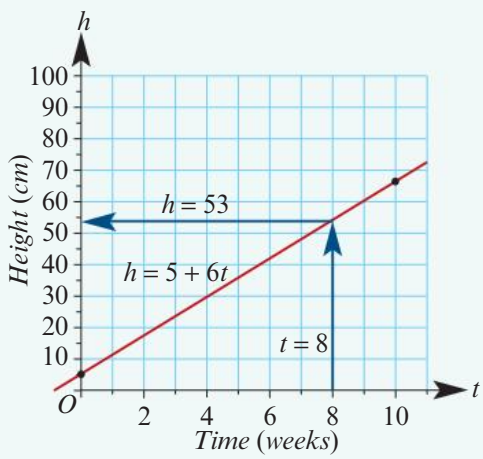
To use the mathematical model to make predictions, we simply substitute a value of t into the model and evaluate.

Example 22 Using a linear model to make predictions

Given the model of plant growth used in Example 21, predict the height of the plant after 8 weeks.

Solution

$h = 6(8) + 5 = 53$ cm



Height = 53 cm

Explanation

Eight weeks corresponds to when $t = 8$ so substitute $t = 8$ into the linear model. This value could also be read directly from the graph, as shown.

Section Summary

- Linear equations can often be used to model real-life situations.
- Variables should be defined carefully to indicate what they are representing. We often use letters other than x or y and instead choose something related to the information (e.g. h for height).

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Exercise 5G

Constructing linear models

Example 21
Example 22

SF

1 A tree is 910 cm tall when first measured. For the next five years its height increases at a constant rate of 16 cm per year.
Let h be the height of the tree (in cm).
Let t the time in years after the tree was first measured.

a Write down a linear model in terms of h and t to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the height of the tree 4.5 years after it was first measured.

2 An empty 20 L cylindrical water jug is to be filled with water at a constant rate of 5 litres per minute.
Let V be the volume of water in the jug after t minutes.

a The water jug is filled in 4 minutes; write down a linear model in terms of V and t to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the volume of water in the jug after 3.2 minutes.

3 A home waste removal service charges $80 to attend a property. It then charges $120 for each cubic metre of waste that is removed. The maximum amount of waste that can be removed in one visit is 8 cubic metres.
Let c be the total charge for removing w cubic metres of waste.

a Write down a linear model in terms of c and w to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the cost of removing 5 cubic metres of waste.

4 A motorist fills the tank of her car with unleaded petrol, which costs $1.57 per litre. Her tank can hold a maximum of 60 litres of petrol. When she started filling the tank, there was already 7 litres in it.
Let c be the cost of adding v litres of petrol to the tank.

a Write down a linear model in terms of c and v to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the cost of filling the tank of her car with petrol.

5 A business buys a new photocopier for $25 000. It plans to depreciate its value by $4000 per year for 5 years, at which time it will be sold.
Let V be the value of the photocopier after t years.

a Write down a linear model in terms of V and t to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the depreciated value of the photocopier after 2.6 years.

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SF

6 A swimming pool, when full, contains 10 000 litres of water. Due to a leak, it loses on average 200 litres of water per day.
Let V be the volume of water remaining in the pool after t days.

a The pool continues to leak. Determine how long will it take to empty the pool.

b Write down a linear model in terms of V and t to represent this situation.

c Sketch the graph showing the coordinates of the intercept and its end point.

d Use the model to predict the volume of water left in the pool after 30 days.

7 A graphic designer starts their own business and earns $42 000 in the first year. The business grows by $6000 each year.
Let B be the amount the business earns annually after t years.

a Write down a linear model in terms of B and t to represent this situation.

b Use the model to predict how much the graphic designer will earn from their business annually after ten years.

c Sketch the graph showing the coordinates of the intercept(s) and your prediction after ten years.

CF

8 Sally invests $2000 in a savings account and then adds $150 each month. Determine how much money Sally will have in her savings account after 3 years.

9 A scaffold hire company charges a fixed fee of $350 for delivery then $180 per day. Calculate how much it costs to hire scaffolding for 9 days, including the fixed cost.

CU

10 An empty 6000 L swimming pool takes 2.5 hours to fill. Calculate how much water is in the pool after 40 minutes.

11 Tyson is choosing whether to hire a bobcat on a daily or fortnightly basis. The daily rate is $40 fixed cost then $60 per day. The fortnightly rate is $50 fixed cost then $406 for the fortnight. Determine the cheapest way for Tyson to hire the bobcat if he requires it for 2, 4, 8 and 10 days.

12 Jackson plans to invest $10 000 into a simple interest account that pays 5% per annum. Determine the number of years that it would take for Jackson's account to double in value.

13 Riley bought a ute for $58 000. The value of the ute depreciates by an amount each year for five years, after which time the ute is worth $41 615. Calculate the annual percentage of depreciation and how many years it will take for the ute to first be worth less than $10 000.

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5H Analysing and interpreting graphs of linear models

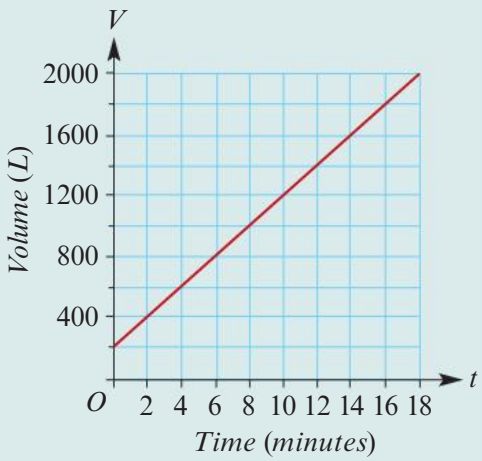
Learning intentions

- To be able to interpret and analyse a linear model from a graph.

Sometimes we are given graphs of linear models that we need to be able to interpret so we can understand a real-life situation.

Example 23 Graphs of linear models with a positive slope

Water is pumped into a partially full tank. The graph gives the volume of water V (in litres) after t minutes.



a Determine how much water is in the tank at the start ($t = 0$).

b Calculate how much water is in the tank after 10 minutes ($t = 10$).

c The tank holds 2000 L. Calculate how long it takes to fill the tank.

d Write down the equation of the line in terms of V and t.

e Use the equation to calculate the volume of water in the tank after 15 minutes.

f Calculate the rate at which the water is pumped into the tank; that is, how many litres are pumped into the tank each minute.

Solution

a 200 L

b 1200 L

c 18 minutes

d $V = mt + c$

$c = 200$

$m = \text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{2000 - 200}{18 - 0}$

$= 100$

$\therefore\ V = 100t + 200\ \ (t \geq 0)$

e $V = 100(15) + 200 = 1700$ L

f 100 L/min

Explanation

Read from the graph (when $t = 0$, $V = 200$).

Read from the graph (when $t = 10$, $V = 1200$).

Read from the graph (when $V = 2000$, $t = 18$).

The equation of the line is $V = a + bt$.

- c is the V-intercept. Read from the graph.
- m is the slope. Calculate using two points on the graph, say, (0, 200) and (18, 2000).

Substitute $t = 15$ into the equation. Evaluate.

The rate at which water is pumped into the tank is given by the slope of the graph, 100 (from **d**).

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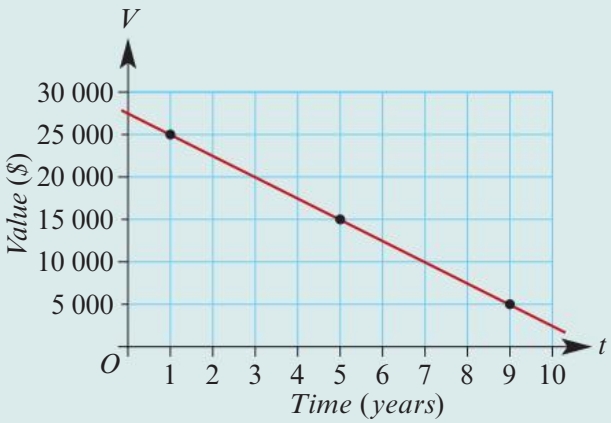
Example 24 Graphs of linear models with a negative slope

The value of new cars depreciates with time. The graph shows how the value V (in dollars) of a new car depreciates with time t (in years).



a Determine the value of the car when it was new.

b Determine the value of the car when it was 5 years old.

c Write down the equation of the line in terms of V and t.

d Calculate the rate that the value of the car depreciates over time; that is, by how much its value decreases each year.

e Using the equation, predict when the car will have no (zero) value.

Solution

a \$27 500

b \$15 000

c $V = mt + c$

$c = 27\,500$

$$m = \text{slope} = \frac{25\,000 - 5000}{1 - 9}$$

$$= -2500$$

$\therefore V = 27\,500 - 2500t \ (t \geq 0)$

d \$2500 per year

e $0 = -2500t + 27\,500$

$2500t = 27\,500$

$$\therefore t = \frac{27\,500}{2500} = 11 \text{ years}$$

Explanation

Read from the graph (when $t = 0$, $V = 27\,500$).

Read from the graph (when $t = 5$, $V = 15\,000$).

The equation of the line is $V = mt + c$.

- c is the V-intercept. Read from the graph.
- m is the slope. Calculate using two points on the graph, say (1, 25 000) and (9, 5000).

The slope of the line is –2500, so the car depreciates in value by \$2500 per year.

Substitute $V = 0$ into the equation and solve for t.

Section Summary

- Straight-line graphs can be interpreted by reading off the information and taking into account the information given on each axis.

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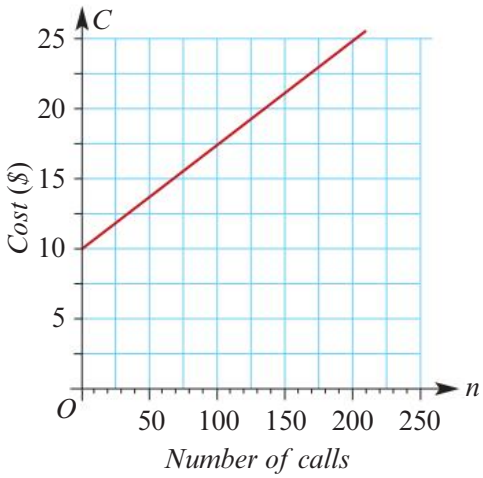
Exercise 5H

Interpreting the graphs of linear models in their context

SF

Example 23

1 A phone company charges a monthly service fee plus the cost of calls. The graph opposite gives the total monthly charge, C dollars, for making n calls. This includes the service fee.



a State the monthly service fee ($n = 0$).

b Determine how much the company charges if you make 100 calls a month.

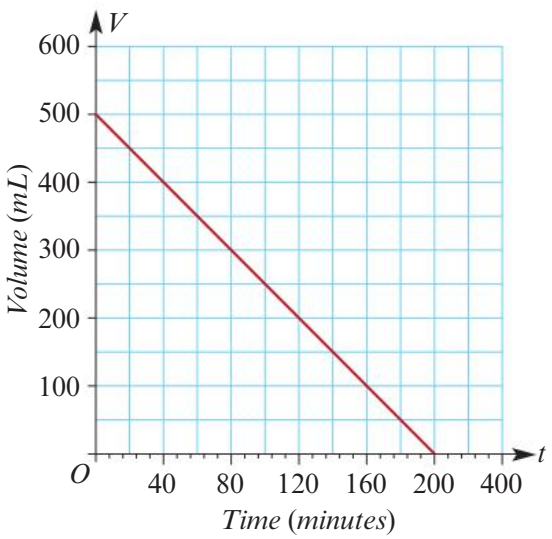
c Write down the equation of the line in terms of C and n.

d Use the equation to calculate the cost of making 300 calls in a month.

e Calculate how much the company charges per call.

Example 24

2 The graph opposite shows the volume of saline solution, V mL, remaining in the reservoir of a saline drip after t minutes.



a State how much saline solution was in the reservoir at the start.

b Determine how much saline solution remains in the reservoir after 40 minutes. Read the result from the graph.

c Calculate how long it takes for the reservoir to empty.

d Write down the equation of the line in terms of V and t.

e Use the equation to calculate the amount of saline solution in the reservoir after 115 minutes.

f Calculate the rate (in mL/min) that the saline solution is flowing out of the drip.

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SF

3 A party planner is organising an 18th birthday party for a client and needs to determine the budget. She needs to rent a space, and then also organise food and drinks for each guest. The graph below shows the total cost, C, and the number of guests, g.

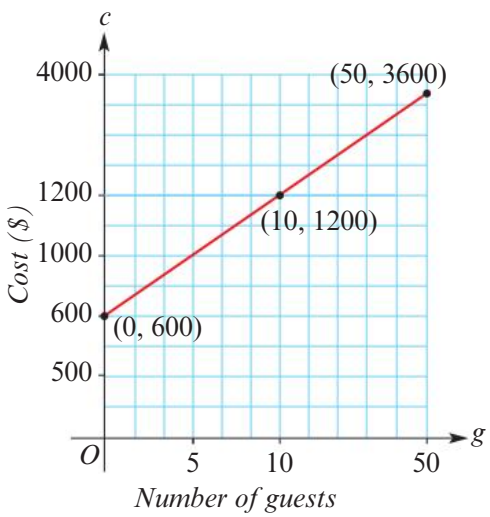
a State how much it costs to rent the space for the party.

b Determine how much food and drink costs for each guest.

c Write down the equation of the line in terms of C and g.

d Calculate how much it costs to host a party with 40 guests.

e Calculate the maximum number of guests that the client can invite if she wishes to spend no more than $4000.



CF

4 The graph opposite can be used to convert temperatures in degrees Celsius (C) to temperatures in degrees Fahrenheit (F).

a Write down the equation of the line in terms of F and C.

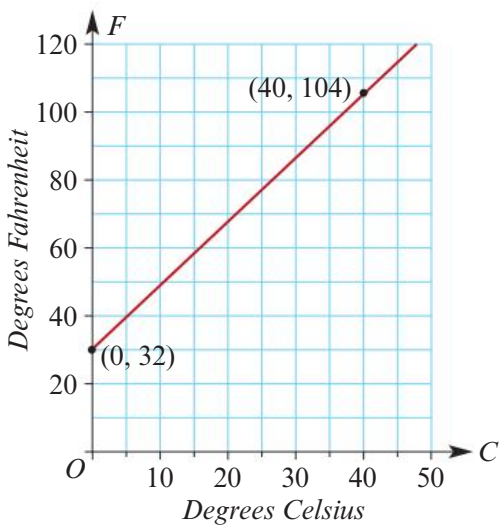
b Use the equation to predict the temperature in degrees Fahrenheit when the temperature in degrees Celsius is:

i 50°C

ii 150°C

iii −40°C

c Complete the following sentence by filling in the box.
When the temperature in Celsius increases by 1 degree, the temperature in Fahrenheit increases by ☐ degrees.



CU

5 The amount that a real estate agent is paid each month consists of a base salary plus a commission which is a percentage of the value of the property that they sold in the month. In the first month they are paid $7500 for selling $8 000 000 worth of property and in the second month they are paid $17 750 for selling $28 500 000 worth of property.
Show on a linear graph the amount that the real estate agent would earn if they sell $10 000 000 worth of property in a month.

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Key ideas and chapter summary

Slope of a straight-line graph

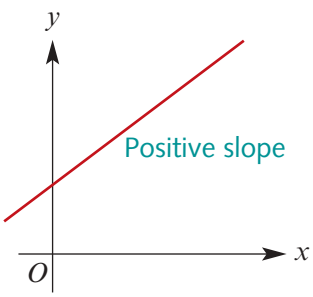
Slope of a straight-line graph is defined to be:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

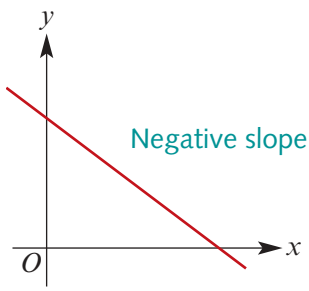
where (x_1, y_1) and (x_2, y_2) are two points on the line.

Positive and negative slope

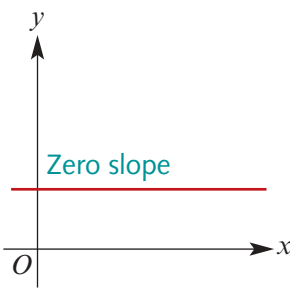
If the line rises to the right, the slope is **positive**.



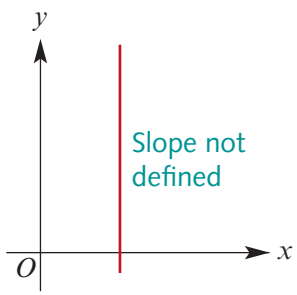
If the line falls to the right, the slope is **negative**.



If the line is horizontal, the slope is **zero**.



If the line is vertical, the slope is **undefined**.



Equation of straight-line graph: the slope–intercept form

The equation of a straight line can take several forms.
The **slope–intercept form** is:

$$y = mx + c$$

where m is the **slope** of the line and c is the ***y*-intercept**.

Linear model

A **linear model** has a linear equation or relation of the form:

$$y = mx + c \quad \text{where} \quad a \leq x \leq b$$

where m, c, a and b are constants.

Linear equation

A linear equation is one whose unknown values are always to the power of 1.

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Review

Skills checklist

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills. ☑

5A	1	I can solve a linear equation involving one step.	☐
		See Example 1 and Exercise 5A Questions 1 and 2.	
5A	2	I can solve a linear equation involving two or more steps.	☐
		See Example 2 and Exercise 5A Question 3.	
5A	3	I can solve a linear equation involving brackets.	☐
		See Example 3 and Exercise 5A Question 4.	
5A	4	I can solve a linear equation with variables on both sides.	☐
		See Example 4 and Exercise 5A Question 5.	
5B	5	I can set up a linear equation from a worded description.	☐
		See Example 5 and Exercise 5B Question 1.	
5B	6	I can set up and solve a linear equation from a worded description.	☐
		See Examples 6, 7 and 8, and Exercise 5B Questions 2 to 11.	
5C	7	I can form a table of values from a linear function.	☐
		See Example 9 and Exercise 5C Question 1.	
5C	8	I can draw a straight-line graph from a table of values.	☐
		See Example 10 and Exercise 5C Question 2.	
5D	9	I can determine the slope of a line from a graph with a positive slope.	☐
		See Example 11 and Exercise 5D Question 2.	
5D	10	I can determine the slope of a line from a graph with a negative slope.	☐
		See Example 12 and Exercise 5D Question 2.	
5D	11	I can calculate the slope of a line using the formula	☐
		See Example 13 and Exercise 5D Questions 3 and 4.	
5E	12	I can determine the slope and y-intercept of a line from an equation.	☐
		See Example 14 and Exercise 5E Question 1.	

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5E	13	I can write down the equation of a straight line given its y-intercept and slope.	☐

See Example 15 and Exercise 5E Question 2.

5E	14	I can calculate the x-intercept from an equation.	☐

See Example 16 and Exercise 5E Question 3.

5E	15	I can sketch a straight-line graph from its equation	☐

See Example 17 and Exercise 5E Questions 4 and 5.

5F	16	I can calculate the slope and y-intercept of a line from a graph.	☐

See Example 18 and Exercise 5F Question 1.

5F	17	I can write down the equation of a straight line from its graph.	☐

See Examples 19 and 20, and Exercise 5F Questions 2 and 3.

5G	18	I can construct a linear model from a real-life example.	☐

See Example 21 and Exercise 5G Question 1.

5G	19	I can use a linear model to make predictions.	☐

See Example 22 and Exercise 5G Question 1.

5H	20	I can interpret and analyse a linear model from a graph.	☐

See Examples 23 and 24, and Exercise 5H Question 1.

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Multiple-choice questions

1 The solution to $4x = 24$ is:

A $x = 2$ **B** $x = 6$ **C** $x = 20$ **D** $x = 96$

2 The solution to $\frac{x}{3} = -8$ is:

A $x = \frac{8}{3}$ **B** $x = 24$ **C** $x = -\frac{8}{3}$ **D** $x = -24$

3 The solution to $2v + 5 = 11$ is:

A $v = 8$ **B** $v = 3$ **C** $v = 6$ **D** $v = 16$

4 The solution to $3k - 5 = -14$ is:

A $k = 115.67$ **B** $k = 19$ **C** $k = 3$ **D** $k = -3$

5 The cost of hiring a car for a day is \$60 plus 25c per kilometre. Michelle travels 750 kilometres. Her total cost is:

A \$810 **B** \$187.50 **C** \$247.50 **D** \$188.10

6 The equation of a straight line is $y = 3x + 4$. When $x = 2$, y is:

A 2 **B** 4 **C** 6 **D** 10

7 The equation of a straight line is $y = 4x + 5$. The y-intercept is:

A 2 **B** 3 **C** 4 **D** 5

8 The coordinates of the x-intercept of the line given by the equation $y = 2x + 8$ are:

A $(-4, 0)$ **B** $(2, 0)$ **C** $(4, 0)$ **D** $(8, 0)$

9 The coordinates of the x-intercept of the line given by the equation $4x + 6y = 12$ are:

A $(2, 0)$ **B** $(3, 0)$ **C** $(4, 0)$ **D** $(6, 0)$

10 The equation of a straight line is $y = -3x + 10$. The slope is:

A -3 **B** 0 **C** 3 **D** 7

11 The equation of a straight line is $y - 2x = 3$. The slope is:

A -3 **B** -2 **C** 2 **D** 3

12 The slope of the line passing through the points (5, 8) and (9, 5) is:

A -1.3 **B** -0.75 **C** 0.75 **D** 1.3

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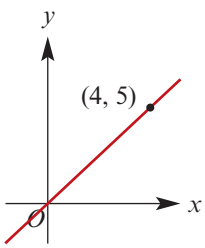
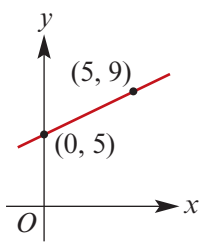
13 The graph of $y = 5x + 4$ is:

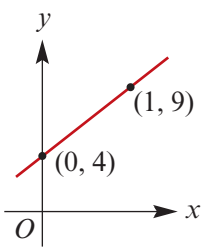
A



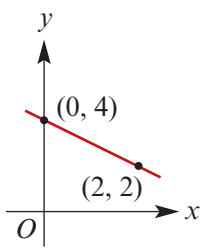
B



C

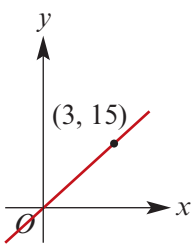


D

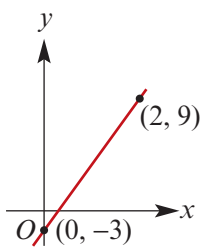


14 The graph of $y = -3x + 15$ is:

A



B



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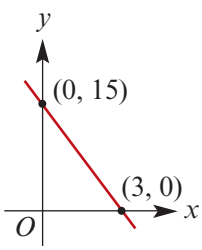
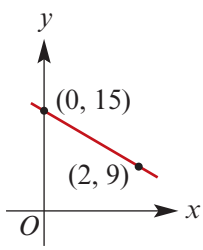
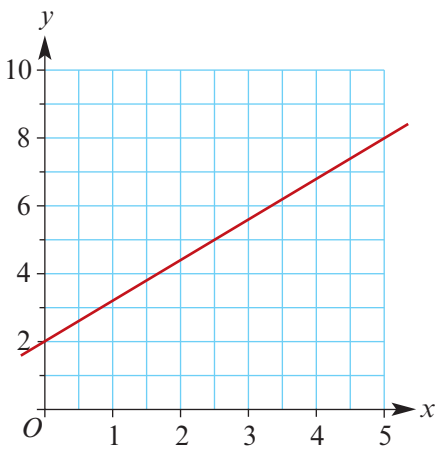


D



Questions 15 *and* 16 *relate to the following graph.*



15 The y-intercept is:

A −2 **B** 0 **C** 2 **D** 5

16 The slope is:

A 1.6 **B** 1.2 **C** 2 **D** 8

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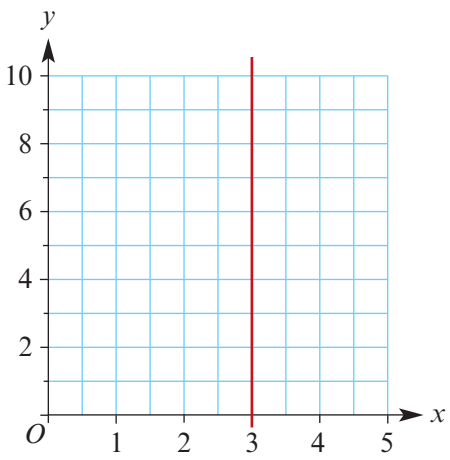
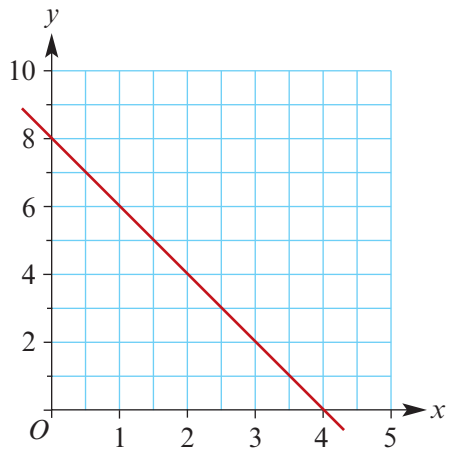
17 The slope of the line in the graph shown opposite is:

A negative
B zero
C positive
D not defined



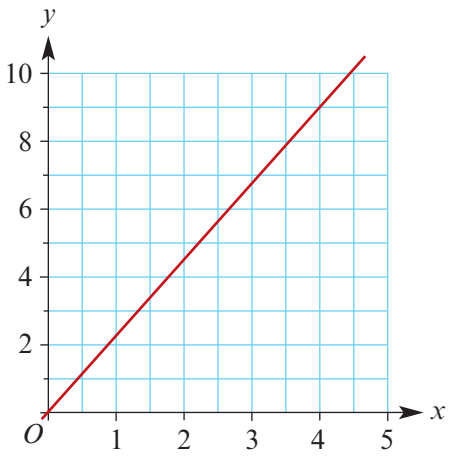
18 The equation of the graph shown opposite is:

A $y = 8x + 2$
B $y = -2x + 4$
C $y = -2x + 8$
D $y = 2x + 8$



19 The equation of the graph shown opposite is:

A $y = -2.25x$
B $y = 2.25x$
C $y = -9x$
D $y = 9x$



20 Determine which of the following points lies on the line $y = -5 + 10x$.

A $(1, -5)$ **B** $(1, 5)$ **C** $(1, 15)$ **D** $(2, 20)$

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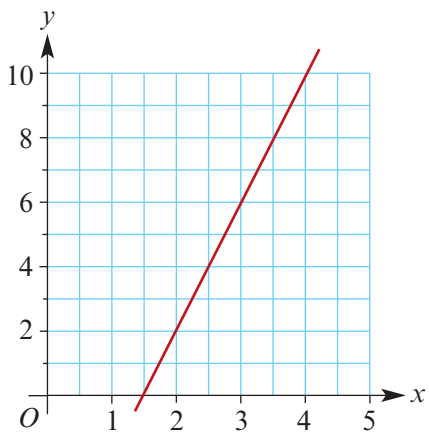
21 The equation of the graph shown opposite is:

A $y = 4x - 6$

B $y = 4x - 8$

C $y = -4x - 4$

D $y = -4x + 2.5$



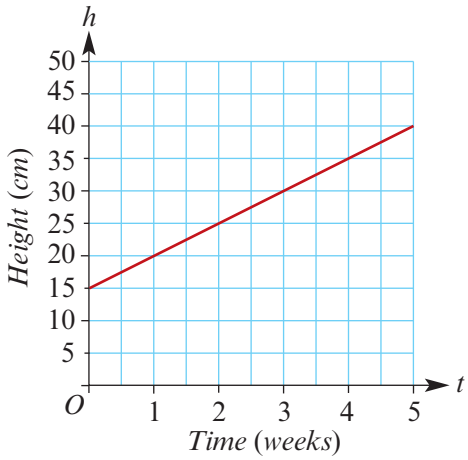
22 The graph opposite shows the height of a small sapling, h, as it increases with time, t. Its growth rate is:

A 1 cm/week

B 3 cm/week

C 5 cm/week

D 8 cm/week



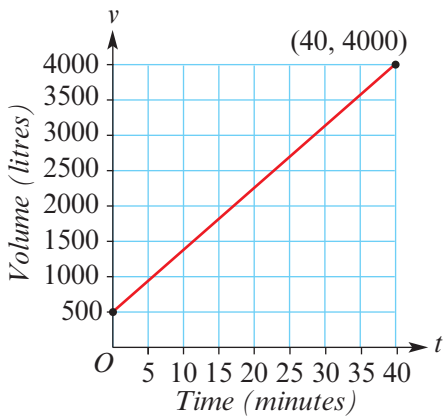
23 The graph opposite shows the volume of water (L) in a tank after t minutes. The number of litres in the tank after 20 minutes is:

A 500 litres

B 2250 litres

C 2000 litres

D 4000 litres



24 The value of a new car depreciates over time. This is modelled by the equation $V = 32\,500 - 2800t$ where V is the value of the car after t years.
Based on the equation, the rate of depreciation is:

A −$3250 per year **B** −$2800 per year **C** $2800 per year **D** $3250 per year

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Review

Short-response questions

SF

1 Solve the following equations for x.

a $x + 5 = 15$ **b** $x - 7 = 4$ **c** $16 + x = 24$

d $9 - x = 3$ **e** $2x + 8 = 10$ **f** $3x - 4 = 17$

g $x + 4 = -2$ **h** $3 - x = -8$ **i** $6x + 8 = 26$

j $3x - 4 = 5$ **k** $\frac{x}{5} = 3$ **l** $\frac{x}{-2} = 12$

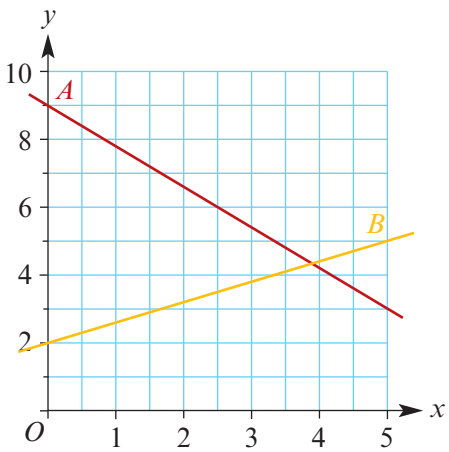
2 I think of a number, double it and add 4 and the result is 6. Determine the original number.

3 Four less than three times a number is 11. State the number.

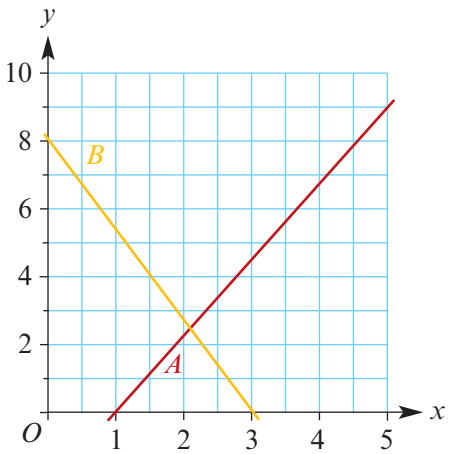
4 Plot the graphs of these linear relations by hand.

a $y = 5x + 2$ **b** $y = -x + 12$ **c** $y = 4x - 2$

5 Calculate the slope of each of the lines A and B shown on the graph below. Give your answers correct to one decimal place.



6 Calculate the slope of each of the lines A and B shown on the graph below. Give your answers correct to two decimal places.



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7 A line passes through the points (1, 6) and (2, 14).

- **a** Determine the slope of the line.
- **b** State the coordinates of the y-intercept of the line.
- **c** State the equation of the line in the form $y = mx + c$.
- **d** State the coordinates of the x-intercept of the line.

8 A linear model for the amount, C, in dollars, charged to deliver w cubic metres of builder's sand is given by $C = 110w + 95$ for $0 \leq w \leq 7$.

- **a** Use the model to determine the total cost of delivering 6 cubic metres of sand.
- **b** The delivery cost is \$95. Determine the cost for each additional cubic metre of builder's sand.

9 The perimeter of a rectangle is 10 times the width. The length is 9 metres more than the width. Determine the width of the rectangle.

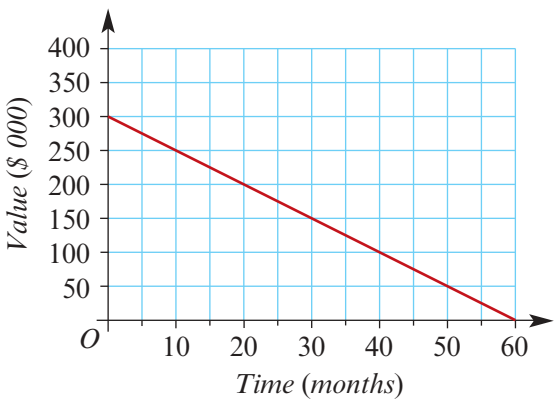
10 Kristen purchased four more than twice as many bread rolls as Simon. In total, they purchased 37 bread rolls. Determine how many they each purchased.

11 A bag of 70 jelly beans is shared between two friends, Tyson and Ethan. Tyson receives four times as many jelly beans as Ethan. Calculate how many jelly beans each boy receives.

12 Amelia and Jacob both collect basketball cards. Amelia has 3 more than twice the number than Jacob has. Together, they have a total of 219 basketball cards. Calculate how many basketball cards Jacob has.

13 A new piece of machinery is purchased by a business for \$300 000. Its value is then depreciated each month using the graph below.

- **a** State the value of the machine after 20 months.
- **b** Determine when the line predicts that the machine will have no value.
- **c** Determine the equation of the line in terms of value, V, and time in months, t.
- **d** Use the equation to predict the value of the machine after 3 years.
- **e** Calculate by how much the machine depreciates in value each month.



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Review

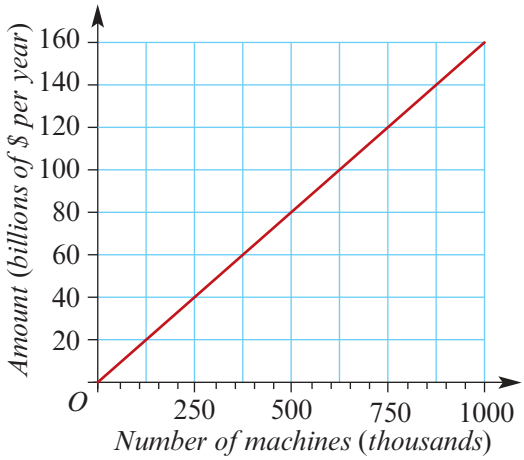
SF

14 The amount of money transacted through ATMs has increased with the number of ATMs available. The graph charts this increase.

a Determine the amount of money transacted through ATMs when there were 500 000 machines.

b Determine the equation of the line in terms of amount of money transacted, A, and number of ATMs, N. (Leave A in billions per year and N in thousands).

c Use the equation to predict the amount transacted per year when there were 600 000 machines.



d If the same rule applies, calculate how much money is predicted to be transacted through ATM machines annually when there are 1 500 000 machines.

e Determine by how much the amount of money transacted through ATMs annually increases with each 1000 extra ATMs.

CF

15 A secondary school offers three languages: French, Indonesian and Japanese. In Year 9, there are 105 students studying one of these languages. The Indonesian class has two-thirds the number of students in the French class and the Japanese class has five-sixths the number of students in the French class. Calculate how many students study each language. (No student is studying more than one language.)

16 To conserve water, one charging system increases the amount people pay as the amount of water used increases. The charging system is modelled by:

$$C = 0.4x + 5 \ (0 \leq x < 30) \quad \text{and} \quad C = 1.6x - 31 \ (x \geq 30)$$

C is the charge in dollars and x is the amount of water used in kilolitres (kL).

a Use the appropriate equation to determine the charge for using:

i 20 kL **ii** 30 kL **iii** 50 kL

b Calculate how much a kilolitre of water costs when you use:

i less than 30 kL **ii** more than 30 kL.

c Use the equations to construct a segmented graph for $0 \leq x \leq 50$.

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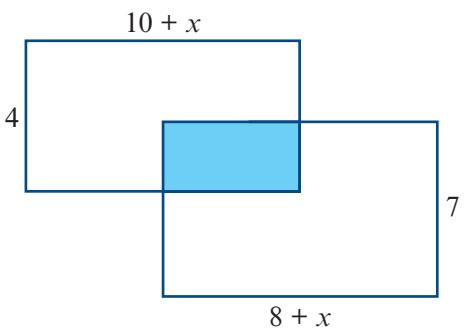
17 Sienna is choosing between two investment options.
Option A is to make an initial investment of $5000 with simple interest paid annually of 5%.
Option B is to make an initial investment of $4200 with simple interest paid annually of 6%.
Determine the value of each investment after eight years and show both options on a linear graph, highlighting which investment has the highest value after eight years.

18 Joshua wants to open a new wellness retreat and is considering different packages to offer to potential clients. All packages include an induction and cleaning fee (fixed cost) as well as accommodation and meals for each day.
One option is a 4-day 'Technology detox' package which costs $1690, while the second option is a 7-day 'Nature and nurture' package which costs $2800.
If Joshua wants to charge the same fixed cost and per day cost, calculate how much he should charge for the 'Ultimate Relaxation' package that lasts for 10 days.

19 Lucy has been offered two jobs as a travel agent. Both jobs pay a base salary as well as commission based on sales.
'Travel with us' offer Lucy a monthly salary of $5800 and 5% commission.
'Fly with us' offer Lucy a monthly salary of $6400 and 2% commission.
If Lucy sells $40 000 worth of travel products per month, calculate how much she will expect to earn from each job in a year.
Calculate how much Lucy will need to sell per month from each job if she wishes to earn $108 000 in one year.

20 Harrison and Toby decide to buy a new excavator for their landscaping business.
The excavator costs $80 000 brand new.
Harrison expects that it will depreciate in value by 15% each year but Toby thinks that it will depreciate by 12% each year.
Calculate the difference in their predicted values of the excavator after 5 years and show each of their predictions on the Cartesian plane by showing a straight line of the equation for each of Harrison and Toby.

21 The following diagram has a total area of 103 units squared. The shaded area is 15 units squared. Determine the value of x.



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Chapter 6

Revision of Unit 1 Chapters 1–5

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6A Revision of Chapter 1: Consumer arithmetic

Multiple-choice questions

1 Catherine earns an annual salary of $75 932. Determine how much she earns per fortnight.

A $1460.23 **B** $2564.13 **C** $2920.46 **D** $6327.67

2 Every week Samuel works 7 hours each day for 5 days. For 5 hours each day Samuel is getting paid a normal rate of $24, and for 2 hours each day he is paid at time and a half. Determine how much he earns in a week.

A $840 **B** $960 **C** $1080 **D** $1260

3 Three different brands of gift wrap are sold in a store. Brand A contains 10 metres of gift wrap and costs $12, brand B contains 12 metres of gift wrap and costs $14 and brand C contains 20 metres of gift wrap and costs $23. Put the brands in order from the cheapest to the most expensive per metre.

A A, B, C **B** B, A, C **C** C, B, A **D** C, A, B

4 Currently the exchange rate between the Australian dollar and the Hong Kong Dollar (HKD) is 8.541. Therefore, $330 Australian dollars, converted to HKD would be:

A $38.64 **B** $338.54 **C** $1543.35 **D** $2818.53

5 Madeline has the following bills: Electricity per quarter $443, internet $65 per month, food $83 per week and $270 rent per fortnight. Determine the total amount that Madeline should have in her budget for bills per year.

A $13 888 **B** $15 668 **C** $20 465 **D** $20 908

6 After GST is added, an item costs $155.95. Calculate the original price of the item.

A $133.94 **B** $141.77 **C** $159.42 **D** $171.55

7 Ben invested $1500 into his saving account which earns an annual simple interest rate of 6.7%. Determine how much Ben has in his account after 7 years.

A $703.50 **B** $1600.50 **C** $1825.50 **D** $2203.50

8 Determine the simple interest rate per annum if a deposit of $3750 earns interest of $100 over a period of 6 months.

A 5.3% **B** 5.4% **C** 2.7% **D** 6.3%

9 The percentage increase in $25 000 compounding annually for 5 years at 4.5% p.a. is closest to:

A 24.6% **B** 80.2% **C** 19.8% **D** 22.5%

10 Betty has 3000 shares in a company. The current market price of the shares is $1.30, and the company recently paid a percentage dividend yield of 14%. Determine how much dividend she received.

A $278.57 **B** $64.29 **C** $182 **D** $546

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Short-response questions

Simple familiar

1 Matt works in a department store. He gets paid $25 per hour for the first 30 hours he works each week, then time and a half for additional hours. He is required to take a 30 minute break during each day that he works, for which he is not paid. His time card for this week is as follows:

Day	Start	Finish
Monday	8.30	5.00
Tuesday	8.30	6.00
Wednesday	8.30	5.00
Friday	8.30	6.00

a Calculate the number of hours Matt worked this week.

b Determine how much he earned.

2 Fatima is a salesperson. She is paid $26 000 per year, plus a commission of 1% of all sales she makes. Last week she sold $64 900 worth of goods. Calculate how much she earned last week.

3 An investment of $4000 attracts an interest rate of 5% per annum simple interest for a period of 4 years. Calculate how much interest is earned in total.

4 Determine the amount that should be invested in order to earn $2400 over 5 years at an annual simple interest rate of 3.5%. Give your answer to the nearest dollar.

5 An amount of $5000 is invested at 2% compound interest per annum for 2 years. Determine:

a the final value of this investment

b the percentage increase in the investment over the two year period.

6 Kira has 1000 shares in an art gallery. The current market price of the shares is $11.50 each, and the gallery has recently paid a dividend of 83 cents per share.

a Calculate how much Kira receives in dividends in total.

b Calculate the percentage yield for this share. Give your answer correct to one decimal place.

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Complex familiar

7 Several online stores sell the same running shoe, with the following prices listed for a range of countries:

Australia	$149.95 + $15 delivery (AUD)
US	$129.99 + $20 delivery (USD)
New Zealand	$154.95 + $20 delivery (NZD)
Great Britain	£85.00 + £20 delivery (GBP)

Connor is in Australia, but he can order from any of the online stores.

a Where do you recommend he purchase the shoes if on the day he wants to purchase them $1 USD is equal to $1.26 AUD, $1.38 NZD, and £0.73 GBP? Use mathematical reasoning to explain your choice.

b Calculate how much the cheapest shoes cost in AUD (include cost of delivery).

8 Juliet has recorded all her expenses for a year as shown:

a Calculate the total of her expenses for the year, and then the average she spends per month her expenses.

b Juliet's income is $3950 per month.

i Calculate how much she is able to save per month on average.

ii Calculate how much she could save during the year. Give your answer to the nearest dollar.

Expenses	
Rent	$12 600.00
Electricity	$923.40
Clothes	$5240.00
Entertainment	$12 459.00
Health Insurance	$1479.40
Petrol &servicing	$2120.60
Car insurance	$1288.60
Telephone	$972.00
Gifts	$2500.00
Food & Groceries	$5560.00

c Juliet really wants to save to buy a new car. She wants to increase the amount she earns by $300 per week by working overtime in her current role. She can work extra hours at time and a half. If her annual salary is $58 000, and she normally works 38 hours per week, determine how many hours overtime she will need to work each week to achieve her goal. Give your answer to the nearest half hour.

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9 When Alan buys his shares in an IT company the purchase price is $35.50 per share, and he pays 1.8% commission on the purchase. He sells them 1 year later for $48.75, and again pays 1.8% commission on the sale.

a Calculate his profit per share.

b Calculate his percentage gain on the shares. Give your answer correct to two decimal places.

10 Determine how much additional interest is earned if $10 000 is invested for 10 years at 3.5% when interest is compounded monthly, as compared with simple interest paid at the same rate over the same time period. Give your answer to the nearest dollar.

11 Tom invests $8000 for 3 years at 2.4% per annum interest compounding monthly.

a How much does he have in the account after 3 years?

b At the time that he made the investment Tom could have used the money to buy a painting for the same amount ($8000). The price of the painting has increased according to inflation, which has averaged 2.5% per annum since then.

i How much will the painting cost Tom if he decides to buy it at the end of the investment period, i.e. after 3 years?

ii Is he better off to have bought the painting for $8000 in the first place, or to have invested his money and bought it after 3 years?

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Complex unfamiliar

12 Tim wishes to change 2000 Singapore Dollars (SGD) to EUR. He decides to look around before changing his money. He gets information from three banks as shown in the table – commission is the amount charged by the bank for exchanging the currency.

Bank	Exchange rate	Commission
Eastern	0.57	1.7%
Centre	0.54	1.5%
Western	0.60	2%

a Determine where Tim should go to exchange his money. Explain your answer.

b On another trip Tim decides to exchange the money at the airport. He notices that they offer two different exchange rates:

- The **sell rate**, which according to the company 'is the rate at which we sell foreign currency in exchange for local currency. For example, if you were heading to Europe, you would exchange Australian dollars for euros at the sell rate'.
- The **buy rate**, which according to the company 'is the rate at which we buy foreign currency back from you into your local currency. For example, if you were returning from America, we would exchange your US dollars (USD) back into Australian dollars at the buy rate'.

This company also charges 1.75% commission, on both the buy and sell rates. Tim is travelling from Australia to the US, and then back to Australia. The exchange rates offered by the company are as follows:

Currency	Sell rate	Buy rate
USD United States Dollars	0.7636	0.8322

i If Tim changes $3000 AUD to US dollars on his way to the US, determine how much he receives. Give your answer to the nearest cent.

ii If Tim exchanges $535 USD back to AUD on his return to Australia, determine how much money he now has. Give your answer to the nearest cent.

iii Calculate how much he spent in total on commission. Give your answer in AUD to the nearest cent.

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13 Ali planned to invest $8500 in a term deposit. He had two investment plans from which to choose.

Plan 1: Simple interest at 5% per annum

Plan 2: Compound interest at 5% per annum with interest added to the account every month.

a Calculate the total interest earned by Ali if he invested his money using Plan 1 for one year.

b Calculate, to the nearest cent, the total interest earned by Ali if he invested his money using Plan 2 for one year.

c Calculate the simple interest rate that would provide the same total interest as earned under Plan 2 for an investment of $8500 for one year. Give your answer correct to two decimal places.

d Determine the number of years it would take for the investment to double if it is invested under Plan 1.

e Determine the number of months it would take for the investment to double if it is invested under Plan 2.

14 Brad buys a coffee machine with an initial value of $12 000.

a He thinks that the resale value of the coffee machine will decrease by 17% of the purchase price in year 1, 14% of the purchase price in Year 2 and 8% of the purchase price in Year 3. Determine the value of the coffee machine after three years. Write your answer rounded to the nearest dollar.

b His business partner James thinks instead that the value of the machine will decrease of current value by 15% in each year. Determine the value of the machine after three years. Write your answer rounded to the nearest dollar.

c If Brad sells 35 000 cups of coffee per year, calculate how much it costs him per coffee to cover the loss in value of the coffee machine after three years (give your answer in cents, rounded to one decimal place):

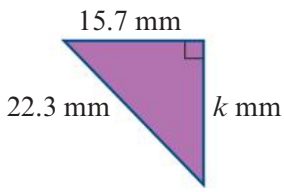
i using Brad's figures for the value

ii using James's figures for the value.

6B Revision of Chapter 2: Shape and measurement

Multiple-choice questions

1 Find the length of the unknown side.



A 6.6 **B** 15.84

C 27.27 **D** 38

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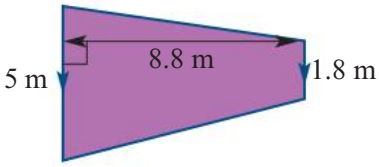
2 Find the length of the longest pencil that will fit into the cylinder, given that its radius is 3 cm and its height is 14 cm.

A 12.65 cm
B 14.32 cm
C 15.23 cm
D 17 cm

3 Calculate the perimeter of a square with side length 18 mm.

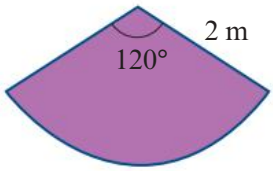
A 22 mm **B** 36 mm **C** 72 mm **D** 324 mm

4 Calculate the area of the following shape. Round to the nearest m^2.



A 16 m^2 **B** 30 m^2
C 44 m^2 **D** 60 m^2

5 Determine the area of the following shape.

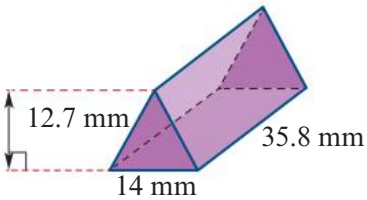


A 4.19 m^2 **B** 8.38 m^2 **C** 12.57 m^2 **D** 25.13 m^2

6 Calculate the volume of a cylinder with a diameter of 18 cm and a height of 22 cm.

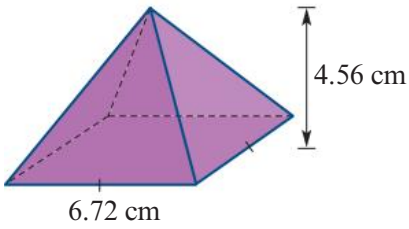
A 622.04 cm^3 **B** 1244.07 cm^3 **C** 5598.32 cm^3 **D** 11 196.64 cm^3

7 Find the volume of the following solid.



A 454.66 mm^3
B 501.2 mm^3
C 3182.62 mm^3
D 635.24 mm^3

8 Find the volume of the following solid.



A 34.32 cm^3
B 41.184 cm^3
C 68.64 cm^3
D 102.96 cm^3

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9 Calculate the surface area of a rectangular prism that has dimensions 15 mm by 9 mm by 10 mm.

A 52 mm^2 **B** 375 mm^2 **C** 750 mm^2 **D** 1350 mm^2

10 Find the surface area of a hemisphere (sphere cut in half) which has a diameter of 16 m.

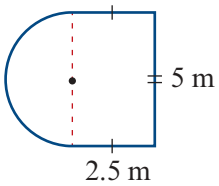
A 402.12 m^2 **B** 603.19 m^2 **C** 1608.50 m^2 **D** 2412.74 m^2

Short-response questions

Simple familiar questions

1 A right-angled triangle has a hypotenuse of length 20 cm and another side of length 16 cm. Calculate the length of the remaining side.

2 Calculate the perimeter of this composite shape.



3 Calculate the area of the following shapes.

a A circle with diameter 7 cm.

b A triangle with a base of 14.2 cm and height 7 cm.

4 Calculate the volume of the following shapes.

a A triangular pyramid with a base area of 32 cm^2 and height 9 cm.

b A right square-based pyramid with a base side length of 5 cm and height of 8 cm.

c A hemisphere with radius 3 cm.

5 Calculate the number of millilitres of water, correct to two decimal places, that can be poured into a conical flask with diameter of 3.6 cm and height of 9.6 cm.

6 Calculate the surface area of a sphere with a diameter of 22 cm.

Complex familiar questions

7 The radius of the base of a cylinder is 4 cm and its height is 12 cm. Determine the amount of metal required to make the cylinder in cm^2.

8 A path 1.3 metres wide surrounds a circular swimming pool whose radius is 6 m. Determine the area of the path. Give your answer correct to two decimal places.

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Complex unfamiliar questions

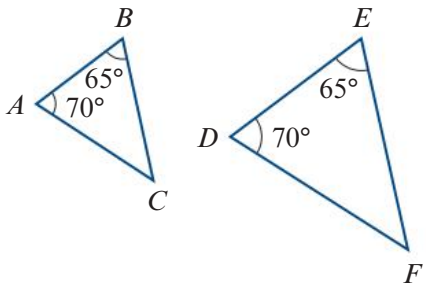
9 A scalene triangle has sides of lengths 5 cm, 7 cm and 8 cm. Determine the height of the triangle perpendicular to the side of length 8 cm.

10 Determine the radius of the sphere whose surface area is equal to its volume.

6C Revision of Chapter 3: Similarity and scale

Multiple-choice questions

1 Determine the corresponding sides in these similar triangles.

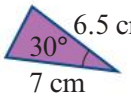


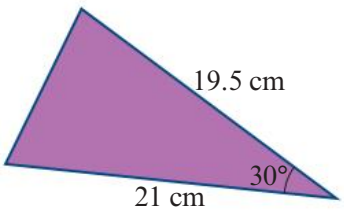
A AC corresponds to FD.
B BA corresponds to DE.
C EF corresponds to BC.
D CB corresponds to FD.

2 Identify the scale factor between two rectangles 36 cm by 52 cm and 9 cm by 13 cm.

A 1.4 **B** 4 **C** 5.8 **D** 2.8

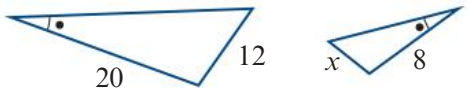
3 Explain why these two triangles are similar.





A Two pairs of the corresponding sides are in the same ratio.
B All corresponding sides are in the same ratio.
C Two pairs of the corresponding sides are in the same ratio and the included corresponding angles are equal.
D They are not similar.

4 For these two similar triangles, calculate the value of x.



A 4.8 units **B** 13.3 units **C** 30 units **D** 40 units

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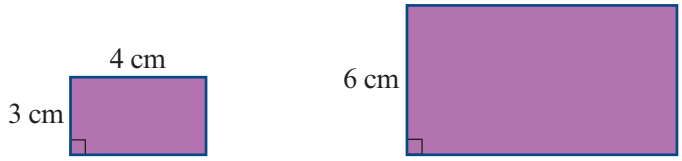
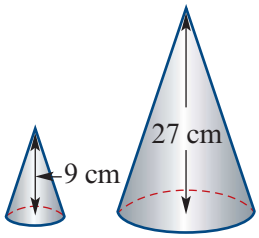
5 Given that the two rectangles are similar, find the area of the second rectangle.



A 12 m^2 **B** 18 m^2 **C** 24 m^2 **D** 48 m^2

6 Given that the cones shown are similar and the larger cone has a radius of 12 cm, determine the volume of the smaller cone.



A 75.40 cm^3
B 150.80 cm^3
C 1357.17 cm^3
D 4071.50 cm^3

7 A map has a scale of 1 : 600. Calculate the actual distance for a scaled distance of 18 cm.

A 10 800 mm **B** 108 m **C** 108 000 cm **D** 1.08 km

8 A floor plan has a scale of 1 : 325. If the hallway between the kitchen and the garage is 5 metres long in real life, determine the measurement for this hallway on the floor plan.

A 1.5 cm **B** 6.5 cm **C** 15 cm **D** 65 cm

9 Two similar prisms have sides with a scale factor of 0.75. Determine the scale factor of their surface areas.

A 0.422 **B** 0.5625 **C** 0.8660 **D** 1.5

10 Determine the scale factor of volumes of two similar solids whose sides have a scale factor of 2.

A 4 **B** 6 **C** 8 **D** 10

Short-response questions

Simple familiar questions

1 Identify the scale factor between two triangles, the original with sides measuring 7.5 cm, 10 cm and 12.5 cm, and the image with sides measuring 3 cm, 4 cm and 5 cm.

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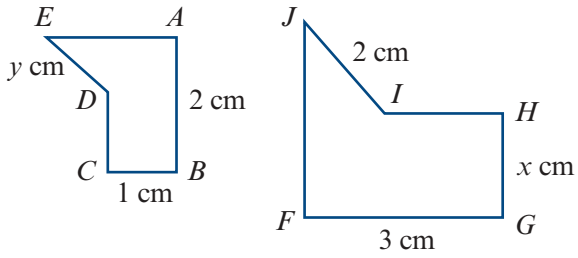
2 Use the scale factor for these similar figures to determine the value of:

a x

b y.



3 Answer the following questions using the map of Western Australia below.

a Measure the line segment scale given in millimetres and interpret the scale, rounded to one decimal place.

b Calculate the actual distance between Perth and Broome, rounded to the nearest 10 km.

4 Prism A has a volume of 875 m^3. In a similar Prism B, the side lengths decrease by a scale factor of 0.2. Determine the volume of Prism B.

5 A short cylinder has a volume of 216 cm^3 and a taller cylinder with the same radius has a volume of 512 cm^3. Given the taller cylinder has a height of 16 cm, calculate the height of the shorter cylinder.

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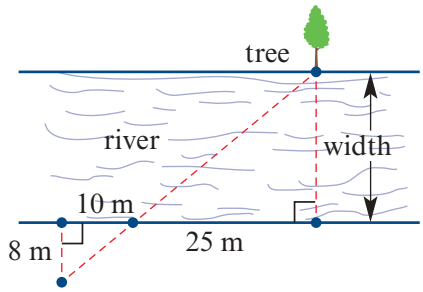
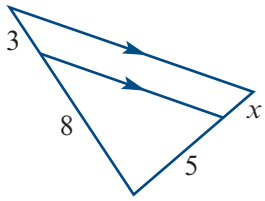
6 Rocks are placed strategically near a river, as indicated by dots on the diagram opposite. Use this to calculate the river's width.



Complex familiar questions

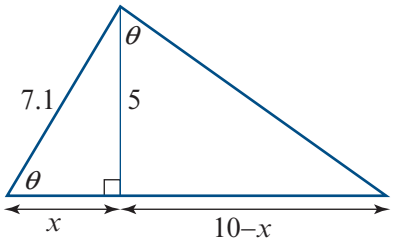
7 Calculate the value of the pronumeral in the similar triangles opposite.



8 A photocopy machine enlarged a diagram by a scale factor of 4 to make it fit into a wall poster. If the wall poster is 210 mm by 297 mm, determine the dimensions of the original diagram. Round your answers to one decimal place.

Complex unfamiliar questions

9 Consider this diagram. Use similar triangles to calculate x.



10 A rectangular prism has length 10 cm, width 5 cm and height 2 cm. A similar object has a surface area of 640 cm^2. Determine the volume of the similar object.

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6D Revision of Chapter 4: Algebra: Linear and non-linear relationships

Multiple-choice questions

1 If $a = 2$, then $9a + 3 =$

A 3 **B** 9 **C** 21 **D** 95

2 If $b = -3$, then $3b - 13 =$

A -13 **B** -22 **C** -46 **D** -13

3 If $V = 40t - 18$ and $t = 6$, then $V =$

A 240 **B** 406 **C** 388 **D** 222

4 The speed, v, in metres per second, of a bullet fired from a gun after t seconds is given by $v = 7400 + 58t + 2t^2$. The speed of the bullet after 3 seconds is:

A 7418 m per second **B** 7574 m per second
C 7580 m per second **D** 7592 m per second

5 If $x = -4$, $y = 2$ and $z = 5$, then $\dfrac{z - x}{y} =$

A $\dfrac{1}{2}$ **B** $\dfrac{9}{2}$ **C** $-\dfrac{1}{2}$ **D** $-\dfrac{9}{2}$

6 The volume of a cylinder is given by $V = \pi r^2 h$. If $r = 3$ cm and $h = 8$, then the volume of the cylinder is closest to:

A 28.27 cm^3 **B** 150.796 cm^3 **C** 603.19 cm^3 **D** 226.19 cm^3

7 The formula for converting temperature from degrees Fahrenheit to degrees Celsius is:

$$C = \frac{5}{9}(F - 32)$$

An expression for F is:

A $F = \dfrac{9C}{5}$ **B** $F = \dfrac{5C}{9}$ **C** $F = \dfrac{9C}{5} + 32$ **D** $F = \dfrac{9(C + 32)}{5}$

8 The perimeter of a rectangle is given by $P = 2L + 2W$. A rectangle of length 14 m which has a perimeter of 52 m has a width of:

A 12 m **B** 14 m **C** 38 m **D** 52 m

9 The cost of hiring a jumping castle is given by the rule:

$$C = 130t + 220$$

where C is the total cost in dollars and t is the number of hours for which the jumping castle is hired. The cost of hiring the jumping castle for 4 hours is:

A \$130 **B** \$400 **C** \$520 **D** \$740

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10 The cost of hiring an e-scooter is given by the rule:

$$S = 0.5t + 3$$

where S is the total cost in dollars and t is the number of minutes for which the scooter is hired. If the customer paid \$9, then the customer hired the scooter for:

A 12 minutes **B** 7.5 minutes **C** 24 minutes **D** 28 minutes

Short-response questions

Simple familiar questions

1 If $A = \frac{(a+b)}{2}h$, state A when:

a $a = 5$, $b = 3$ and $h = 9$

b $a = 11$, $b = 7$ and $h = 2$.

2 The formula for the area of a circle is given by $A = \pi r^2$, where r is the radius of the circle.

Calculate the area of a circle with radius 12 mm, correct to two decimal places.

3 Transpose the following equations for the letter indicated in brackets.

a $A = P + PRT$ (R)

b $C = 2\pi r$ (r)

4 The cost of hiring a taxi, in dollars, is given by $C = 3.20k + 5.5$ when k kilometres are travelled.

a Determine the cost of hiring a taxi to travel 6 kilometres.

b Calculate how far the taxi travelled if the cost was \$50.30.

5 The cost of hiring a bicycle is calculated using the formula $C = 12n + 5$, where n is the number of hours the bicycle is hired for.

a Complete the table of values below for values of n from 1 to 8.

n	1	2	3	4	5	6	7	8
C								

b Determine the cost of hiring two bikes for 3 hours.

6 Consider the equation $y = 32x - 14$.

a Construct a table of values for values of x in intervals of 5 from -10 to 10.

b Determine the value of x when y is 498.

c Calculate the value of x when y is -110.

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Complex familiar questions

7 The local convenience store sells pies for $4.95 each and tomato sauce packets for $0.50 each.

a Write a formula for the total cost, C, if x pies and y tomato sauce packets are purchased.

b Katie bought 3 pies and 2 packets of tomato sauce. Calculate the cost of her purchase.

c Will spent $26.25 for 5 pies and an unknown number of packets of tomato sauce. Calculate the number of packets of tomato sauce that he purchased.

8 A plumber charges an upfront cost of $90 plus $40 for each hour, t.

a Write an equation for the total cost C in terms of t.

b The total cost was $230. Calculate how long the plumber worked for.

Complex unfamiliar questions

9 Jacob currently has an average mark of 88% across his first four maths tests. Calculate the mark he requires on his fifth test if he wishes to achieve an average of 90%.

10 Ally's piggy bank contains $9.50 in 50 cent and $1 coins. She has 13 coins in total. Determine the number of 50 cent coins and $1 coins she has.

6E Revision of Chapter 5: Linear equations and their graphs

Multiple-choice questions

1 The solution to $8x = 24$ is:

A $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 8$

2 The solution to $7k - 3 = 9$ is:

A $k = -3$ **B** $k = 12$ **C** $k = \frac{12}{7}$ **D** $k = \frac{7}{12}$

3 The solution to $5v + 11 = 1$ is:

A $v = -1$ **B** $v = -2$ **C** $v = \frac{12}{5}$ **D** $v = 2$

4 The cost of hiring a car for a day is $80 plus $0.30 per kilometre. James travels 480 kilometres. His total cost is:

A $224 **B** $244 **C** $380 **D** $830

5 The equation of a straight line is $y = 2x + 6$. The y-intercept is:

A -3 **B** 2 **C** 3 **D** 6

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6 The coordinates of the x-intercept of the line given by the equation $y = 2x + 8$ are:

A $(-4, 0)$ **B** $(2, 0)$ **C** $(4, 0)$ **D** $(8, 0)$

7 Determine which of the following points lies on the line $y = 4x - 5$.

A $(1, -5)$ **B** $(4, 5)$ **C** $(4, -5)$ **D** $(-2, -13)$

8 The slope of the line passing through the points $(-1, 4)$ and $(3, -4)$ is:

A $-\frac{1}{2}$ **B** -2 **C** $\frac{1}{2}$ **D** 2

9 The slope of the line given by the equation $4x + 3y = -7$ is:

A -7 **B** $\frac{3}{4}$ **C** 4 **D** $-\frac{4}{3}$

10 Danny invests a certain amount of money. Each year, the investment earns a certain amount of interest. The value of the investment, V, after t years is given by $V = 5000 + 200t$.
Based on the equation, the value of the investment after 6 years is:

A \$200 **B** \$5000 **C** \$5200 **D** \$6200

Short-response questions

Simple familiar questions

1 Write down the equation of the line that:

a has y-intercept -3, slope 7

b passes through the points $(0, 3)$ and $(2, 5)$

c has slope 2 and passes through $(1, 3)$.

2 Sketch the following graphs.

a $2y - x = 4$ **b** $x = y + 3$

c $\frac{y}{2x} + 3 = 1$ **d** $y = \frac{1}{3}x - \frac{2}{3}$

3 State the y-intercept and slope of the graphs of the following equations.

a $y = 4x + 1$ **b** $7x = 4y + 2$

c $x = -\frac{1}{3}y + 4$ **d** $8x + 4y = 12$

4 Four more than three times a number is 55.

a Write down an equation to represent the situation.

b Calculate the number.

5 The length of a rectangle is 5 metres more than the width. The perimeter is 38 metres.

a Write down an equation for the perimeter of the rectangle in terms of the width, w.

b Calculate the area of the rectangle.

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6 Lisa and James both collect marbles. Lisa has eight more than twice the number of marbles that James has. They have 104 marbles in total.

a Write down an equation for the total number of marbles that Lisa and James have.

b Calculate the number of marbles that Lisa and James each have.

Complex familiar questions

7 The formula for converting Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$.
The temperature in degrees Celsius increases by one degree. Calculate how much the temperature in Fahrenheit increases by.

8 A graduate student has a starting salary of $52 000. Their salary increases by $4000 every year.
Let S be the salary after t years.

a Write down a linear model in S and t to represent this situation.

b Sketch the graph, showing the coordinates of the intercept and end point. Assume the model stops at 10 years.

Complex unfamiliar questions

9 Daniel buys a new car for $40 000. Its value depreciates by $8000 per year for 4 years, at which time it will be sold.
Let V be the value of the car after t years.
At the same time, Jane invests $10 000 in shares. The value of the shares appreciates by $4000 per year for 4 years, at which time she intends to sell them.
Let J be the value of the shares after t years.
Calculate when Daniel's car has the same value as Jane's shares.

10 A food delivery driver earns a base amount of $3, and $7 for each delivery they make. The driver gives his flatmate 20% of all his earnings in exchange for borrowing his car for deliveries.
Let T be the driver's net earnings after n deliveries.
By first writing down a linear model in terms of T and n, determine the minimum number of deliveries that the driver must make to have net earnings of at least $100.

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Chapter 7

Applications of linear equations and their graphs

Chapter questions

UNIT 2 APPLICATIONS OF LINEAR EQUATIONS AND TRIGONOMETRY, MATRICES AND UNIVARIATE DATA ANALYSIS

Topic 1: Applications of linear equations and their graphs

- How do we solve simultaneous equations algebraically?
- How do we find the point of intersection of two linear graphs?
- How do we use simultaneous equations to solve practical problems?
- What is a piece-wise linear graph?
- What is a step graph?
- How do we sketch piece-wise linear graphs and step graphs?
- How do we interpret piece-wise linear graphs and step graphs to model practical situations?

Simultaneous equations are important as we often have multiple pieces of information (represented in equations) and two (or more) unknown quantities that we need to find. By defining variables, setting up equations and using either algebraic or graphical techniques, we can find the values of key variables.

Piece-wise linear graphs show us the relationship between two variables when the relationship between the variables depends on the value of the x variable. For example, hiring a car for more than 2 weeks might have a cheaper daily rate than hiring a car for less than 2 weeks. A step graph is a special type of piece-wise linear graph where the function takes on constant values over intervals on the x-axis.

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7A Solving simultaneous linear equations graphically

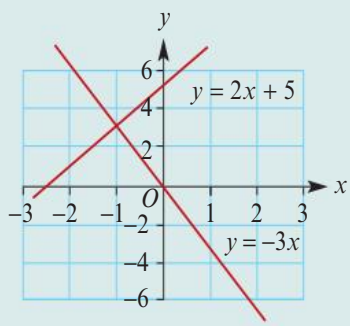
Learning intentions

- To be able to find the point of intersection of two linear graphs.

The aim when solving simultaneous linear equations is to find the unique pair of numbers that makes both equations true. One way of doing this to sketch the graphs of the two equations on the one set of axes and then read off the coordinates at the point of intersection (two straight lines will always intersect unless they are parallel). When we find the **point of intersection**, we are said to be **solving the equations simultaneously**.

Example 1 Finding the point of intersection of two linear graphs

Use the diagram on the right to determine the solution of the simultaneous equations $y = 2x + 5$ and $y = -3x$.



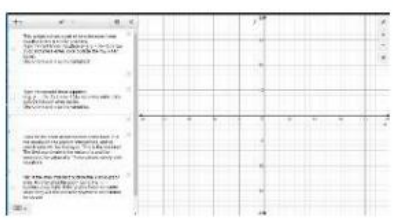
Solution	Explanation
The solution is $(-1, 3)$.	The solution is the intersection point on the graph.

Solving simultaneous equations in Desmos by finding the point of intersection

The online Desmos graphing calculator will solve simultaneous linear equations by finding the coordinates of the point of intersection. The equations must be in terms of x and y. If they are not, rewrite the equations so that this is the case. Desmos 4K Widget is a good guide. For this example, we will use the equations $y = 2x - 2$ and $y = -x + 10$.

Steps

1. Go to the Interactive Textbook, locate this section and page in it, and click on the Desmos widget icon above this box. Its window will open.



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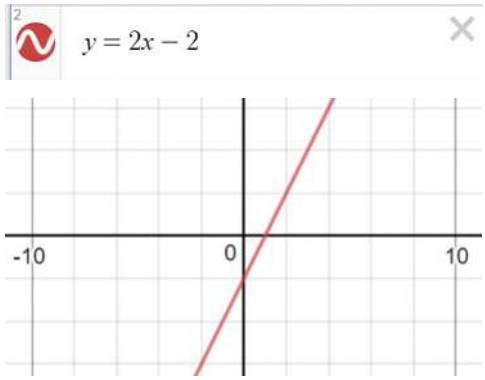
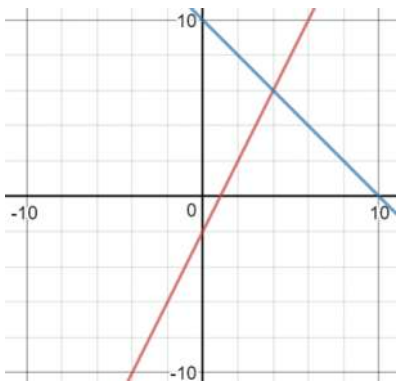
2 Type the first equation into row 2. Don't press Enter after typing, which will insert another row, just click anywhere outside the row to complete the equation. Its graph will appear.



3 Type the second equation into row 4. Its graph will also appear.

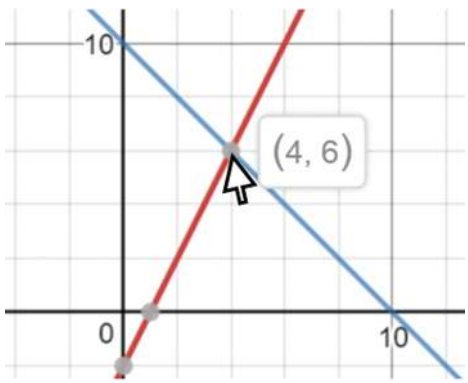


4 Click the mouse on the point of intersection. Its coordinates will display. This is the solution.

The first coordinate is the value of x and the second is the value of y. These values satisfy both equations.

In this example, the solution of the simultaneous equations is:

$x = 4, y = 6.$



If the lines intersect outside the visible graph area, try zooming out using the − button at top right. Click the default zoom button to return to the original zoom.

Note: Once you have learned the method, you can use a blank Desmos graphing calculator window to enter the two equations, rather than returning to the widget. Click on the Desmos icon at the top right of any page in a section of the Interactive Textbook.

Section Summary

- The solution to two simultaneous linear equations is found at the **point of intersection** of the two lines on a graph.
- The lines should be graphed by hand or using technology on the same set of axes.

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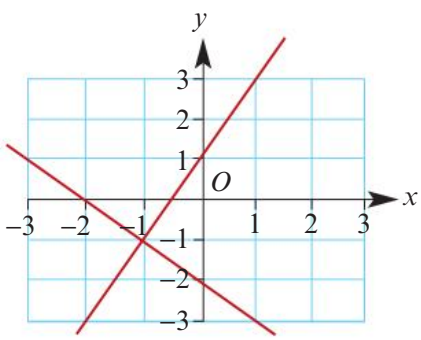
Exercise 7A

SF

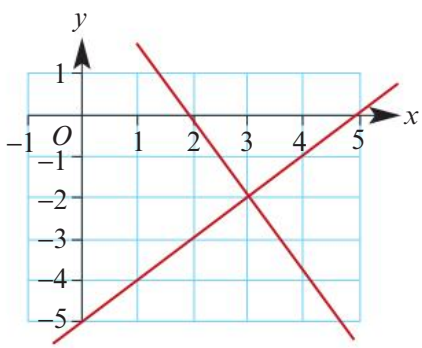
Example 1

1 Determine the solution to each of the following pairs of equations in the graphs by identifying the point of intersection.

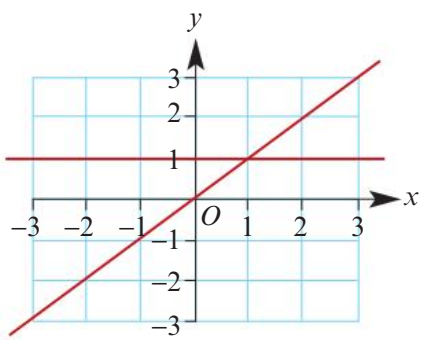
a



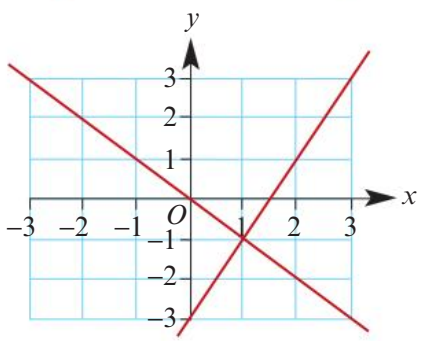
b



c



d



2 Using technology, determine the point of intersection of each of these pairs of lines.

a $y = x - 6$ and $y = -2x$

b $y = x + 5$ and $y = -x - 1$

c $y = 3x - 2$ and $y = -x + 4$

d $y = x - 1$ and $y = 2x - 3$

e $y = 2x + 6$ and $y = x + 6$

f $x - y = 5$ and $y = 2$

g $x + 2y = 6$ and $y = -x + 3$

h $2x + y = 7$ and $y - 3x = 2$

i $3x + 2y = -4$ and $y = x - 3$

j $y = 4x - 3$ and $y = 3x + 4$

k $y = x - 12$ and $y = 2x - 4$

l $y + x = 7$ and $2y + 5x = 5$

3 Using grid paper and your skills from Chapter 5, sketch the graphs of the following pairs of lines. Show the x-intercept, y-intercept of each line as well as the point of intersection, if it exists.

a $y = x - 1$ and $y = 2$

b $y = x - 4$ and $y = -3$

c $y = 2x - 4$ and $y = x - 10$

d $y = 2x + 1$ and $y = 8 - x$

e $x + 2y = 8$ and $y = x + 6$

f $2x + y = 1$ and $y = 5 - 2x$

g $2x + 3y = 12$ and $3x + 2y = 15$

h $x + y = 7$ and $2x + 3y = 23$

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7B Solving simultaneous linear equations using the substitution method

Learning intentions

- To be able to solve a pair of simultaneous equations algebraically using the substitution method.

If we have two linear simultaneous equations, we can also use algebra to solve them. There are two algebraic methods that we can use – the substitution method and the elimination method.

In this section, we will focus on the simplest algebraic method for solving equations simultaneously, the substitution method.

When solving simultaneous equations by substitution, the process is to substitute one variable from one equation into the other equation.

Example 2 Solving simultaneous equations using substitution

Solve the pair of simultaneous equations $y = 5 + x$ and $3x - 2y = 4$.

Solution	Explanation
$y = 5 + x \quad (1)$	Number the two equations as (1) and (2).
$3x - 2y = 4 \quad (2)$	
$3x - 2(5 + x) = 4$	Substitute the y-value from equation (1) into equation (2).
$3x - 10 - 2x = 4$	Expand the brackets and collect the like terms then solve for x by adding 10 to both sides.
$x - 10 = 4$	
$x = 14$	
$y = 5 + (14)$	To find y, substitute $x = 14$ back into equation (1).
$y = 19$	
$(14, 19)$	Write your solution.
$\text{LHS} = 3(14) - 2(19)$	Check your solution by substituting (14, 19) into equation (2).
$= 42 - 38$	
$= 4$	
$= \text{RHS}$	

Checking your solution by substituting the values into the equation to ensure they satisfy the equation is a helpful and necessary step.

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If both equations are of the form $y = mx + c$, then the right-hand sides of each expression can be made equal to each other.

Example 3 Solving simultaneous equations both in the form $y = mx + c$

Solve the pair of simultaneous equations $y = x + 5$ and $y = -3x + 9$.

Solution	Explanation
$y = x + 5 \quad (1)$	Number the two equations as (1) and (2).
$y = -3x + 9 \quad (2)$	
$x + 5 = -3x + 9$	Both equations contain expressions for y, so they can be made equal to each other.
$x + 5 + 3x = -3x + 9 + 3x$	Solve for x by adding $3x$ to both sides of the equation, subtracting 5 from both sides of the equation and then dividing both sides of the equation by 4.
$4x + 5 = 9$	
$4x + 5 - 5 = 9 - 5$	
$4x = 4$	
$\frac{4x}{4} = \frac{4}{4}$	
$\therefore x = 1$	
$y = (1) + 5$	Substitute $x = 1$ into equation 1 to solve for y.
$y = 6$	
$(1, 6)$	Write your solution.
$\text{RHS} = -3(1) + 9$	Check your solution by substituting $(1, 6)$ into equation (2).
$= -3 + 9$	
$= 6$	
$= \text{RHS}$	

If neither equation is in the form $y = mx + c$, then we can still use the substitution method but we first need to rearrange one of the equations into the form $y = mx + c$. Note that this is not always the best method to use as we will see in the next section.

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Example 4 Solving simultaneous equations

Solve the pair of simultaneous equations $x + y = 3$ and $2x - y = 9$.

Solution	Explanation
$x + y = 3 \quad (1)$ $2x - y = 9 \quad (2)$	Number the two equations.
$x + y = 3$ $x + y - x = 3 - x$ $y = 3 - x \quad (3)$	Make y the subject in equation (1) by subtracting x from both sides.
$2x - (3 - x) = 9$ $2x - 3 + x = 9$ $3x - 3 = 9$	Substitute the expression for y in equation (3) into equation (2) then expand the brackets and collect like terms.
$3x - 3 + 3 = 9 + 3$ $3x = 12$ $\frac{3x}{3} = \frac{12}{3}$ $x = 4$	Solve for x by adding 3 to both sides then dividing both sides by 3.
$y = 3 - 4$ $y = -1$	To find y, substitute $x = 4$ into equation (3).
$x = 4, \quad y = -1$	Write your solution.
LHS $= 2(4) - (-1)$ $= 8 + 1$ $= 9$ $=$ RHS	Check your solution by substituting $(4, -1)$ into equation (2).

Section Summary

- If we have two linear equations with two unknown variables, we can use algebra to solve them.
- In the **substitution method**, we need to have one equation in the form $y = mx + c$ to perform the substitution.
- If both equations are in the form $y = mx + c$, then we can equate the right-hand sides and solve.
- If neither equation is in the form $y = mx + c$, then one equation must be rearranged for the substitution method to be used.

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Exercise 7B

SF

Example 2

1 Solve the following pairs of simultaneous equations using the substitution method.

a $y = x - 1$
$3x + 2y = 8$

b $y = x + 3$
$6x + y = 17$

c $x + 3y = 15$
$x = 1 + y$

d $x + y = 10$
$x = y + 8$

e $2x - y = 11$
$y = -3x + 9$

f $3x - 5y = 8$
$y = 2x - 1$

Example 3

2 Solve the following pairs of simultaneous equations using the substitution method.

a $y = 6 - x$
$y = 2 + 3x$

b $y = 12 + x$
$y = 8 - 3x$

c $y = 9 - x$
$y = 2 - 4x$

d $y = 12 + 5x$
$y = 3$

e $y = 8 + 6x$
$y = 7 - 2x$

f $x = y - 12$
$x = 3y - 20$

Example 4

3 Solve the following pairs of simultaneous equations using the substitution method.

a $2y = 6 - 4x$
$3x + y = 8$

b $3x + 2y = 0$
$-3x - y = 3$

c $3x + y = 4$
$4y = 2 - 4x$

d $3x + 5y = 9$
$5y = 3$

e $2x + 3y = 5$
$3x + 2y - 7 = 0$

f $4x + 3y = -28$
$5x - 6y + 35 = 0$

CF

4 Solve the following pairs of simultaneous equations using the substitution method.

a $2y = \dfrac{6 - 4x}{3}$
$2x + y = 8$

b $y = \dfrac{2x - 3}{3}$
$y = \dfrac{4x + 3}{5}$

c $y = \dfrac{2x + 10}{3}$
$4y = 2 - 4x$

d $3x + 5y = 9$
$y = \dfrac{3 + x}{2}$

e $x = \dfrac{10 - 2y}{4}$
$3x = 7 - 2y$

f $y = \dfrac{7x - 1}{2}$
$x = \dfrac{4 - 2y}{3}$

CU

5 Determine if the following pairs of simultaneous solutions have a single solution. If not, explain why not.

a $y = 2x + 1$
$y - 10 = x$

b $x + y = 8$
$y = 14 - x$

c $2y = 10 - x$
$4x + 8y = 10$

6 Solve the following system of simultaneous equations for all three variables.

a $y = x + 1$
$z = x + 3$
$y = 2x + z + 5$

b $x + y + z = 8$
$y = 14 - x$
$y = 3 + z$

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7C Solving simultaneous linear equations using the elimination method

Learning intentions

- To be able to solve a pair of simultaneous equations algebraically using the elimination method.

In this section, we will focus on another method for solving simultaneous equations algebraically, the elimination method. The elimination method is often used when both equations are in the form $ax + by = c$ and involves adding the two equations together or subtracting one equation from the other so that we can eliminate a variable.

Example 5 Solving simultaneous equations using elimination – addition

Solve the pair of simultaneous equations $2x + 3y = 19$ and $3x - 3y = 6$.

Solution	Explanation
$2x + 3y = 19 \quad (1)$ $3x - 3y = 6 \quad (2)$	Label the two equations (1) and (2).
$2x + 3y + 3x - 3y = 19 + 6$ $5x = 25$ $x = 5$	Add equations (1) and (2) and simplify. Solve the equation for x by dividing both sides by 5.
$2(5) + 3y = 19$ $10 + 3y = 19$ $3y = 9$ $y = 3$	Substitute $x = 5$ into equation (1). Solve for y by then subtracting 10 from both sides and then dividing by 3.
$(5, 3)$	Write down the solution.
$\text{LHS} = 3(5) - 3(3)$ $= 15 - 9$ $= 6$ $= \text{RHS}$	Check your solution by substituting $(5, 3)$ into equation (2).

For the elimination method to work, we need the **coefficient** of the variable to be eliminated to be the same in each equation. In Example 4, we had $3y$ in both equations. Note that in equation (1) the term was added ($+3y$), while in equation (2), the term was subtracted ($-3y$). Since the signs were **different**, we **added** the two equations together.
If the signs were the **same**, we would need to **subtract** one equation from the other.

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Example 6 Solving simultaneous equations using elimination – subtraction

Solve the pair of simultaneous equations $2x + 3y = 13$ and $3x + 3y = 14.5$.

Solution	Explanation
$2x + 3y = 13 \quad (1)$ $3x + 3y = 14.5 \quad (2)$	Label the two equations (1) and (2).
$3x + 3y - (2x + 3y) = 14.5 - 13$ $x = 1.5$	Subtract equation (1) from (2) and simplify.
$2(1.5) + 3y = 13$ $y = \frac{10}{3}$ $\left(\frac{3}{2}, \frac{10}{3}\right)$	Solve the equation for y by substituting $x = 1.5$ into equation (1). Write down the solution.
$\text{LHS} = 3(1.5) + 3\left(\frac{10}{3}\right) = \text{RHS}$	Check your solution by substituting $\left(\frac{3}{2}, \frac{10}{3}\right)$ into equation (2).

Sometimes the coefficients are different. In this case, we need to multiply one (or both) equations by a constant so that the coefficients are the same.

Example 7 Solving simultaneous equations using elimination – different coefficients

Solve the pair of simultaneous equations $x + 4y = 15$ and $3x - 2y = -11$.

Solution	Explanation
$x + 4y = 15 \quad (1)$ $3x - 2y = -11 \quad (2)$	Label the two equations (1) and (2).
$3(x + 4y) = 3 \times 15$ $3x + 12y = 45 \quad (3)$	Multiply the first equation by 3 to eliminate x.
$3x + 12y - (3x - 2y) = 45 - (-11)$ $3x + 12y - 3x + 2y = 56$ $y = 4$	Solve the equation for y by subtracting (2) from (3).
$x + 4(4) = 15$ $x + 16 = 15$ $x = -1$ $(3, 4)$	Substitute $y = 4$ into equation (1) to solve for x. Write down the solution.
$\text{LHS} = 3(-1) - 2(4) = \text{RHS}$	Check your solution by substituting $(-1, 4)$ into equation (2).

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Example 8 Solving simultaneous equations

Solve the pair of simultaneous equations $3x + 2y = 2.3$ and $8x - 3y = 2.8$.

Solution	Explanation
$3x + 2y = 2.3 \quad (1)$ $8x - 3y = 2.8 \quad (2)$	Label the two equations (1) and (2).
$9x + 6y = 6.9 \quad (3)$ $16x - 6y = 5.6 \quad (4)$	Multiply equation (1) by 3 and equation (2) by 2 to give $6y$ in both equations.
$9x + 6y + 16x - 6y = 6.9 + 5.6$ $25x = 12.5$	Add equation (3) and equation (4) together to eliminate $6y$.
$\frac{25x}{25} = \frac{12.5}{25}$ $x = 0.5$	Solve for x by dividing both sides by 25.
$3(0.5) + 2y = 2.3$ $1.5 + 2y = 2.3$ $1.5 + 2y - 1.5 = 2.3 - 1.5$ $\frac{2y}{2} = \frac{0.8}{2}$ $y = 0.4$	To find y, substitute $x = 0.5$ into equation (1) and solve for y.
$(0.5, 0.4)$	Write your solution.
$\text{LHS} = 8(0.5) - 3(0.4)$ $= 4 - 1.2$ $= 2.8 = \text{RHS}$	Check by substituting $x = 0.5$ and $y = 0.4$ into equation (2).

Note that for this example, we could also have multiplied equation (1) by 8 and equation (2) by 3 to get $24x$ in both equations. We then could have subtracted equation (2) from equation (1) to solve for y.

In general, to help decide whether to add or subtract we can use the acronym **DASS**.

- Signs **D**ifferent: **A**dd
- Signs **S**ame: **S**ubtract

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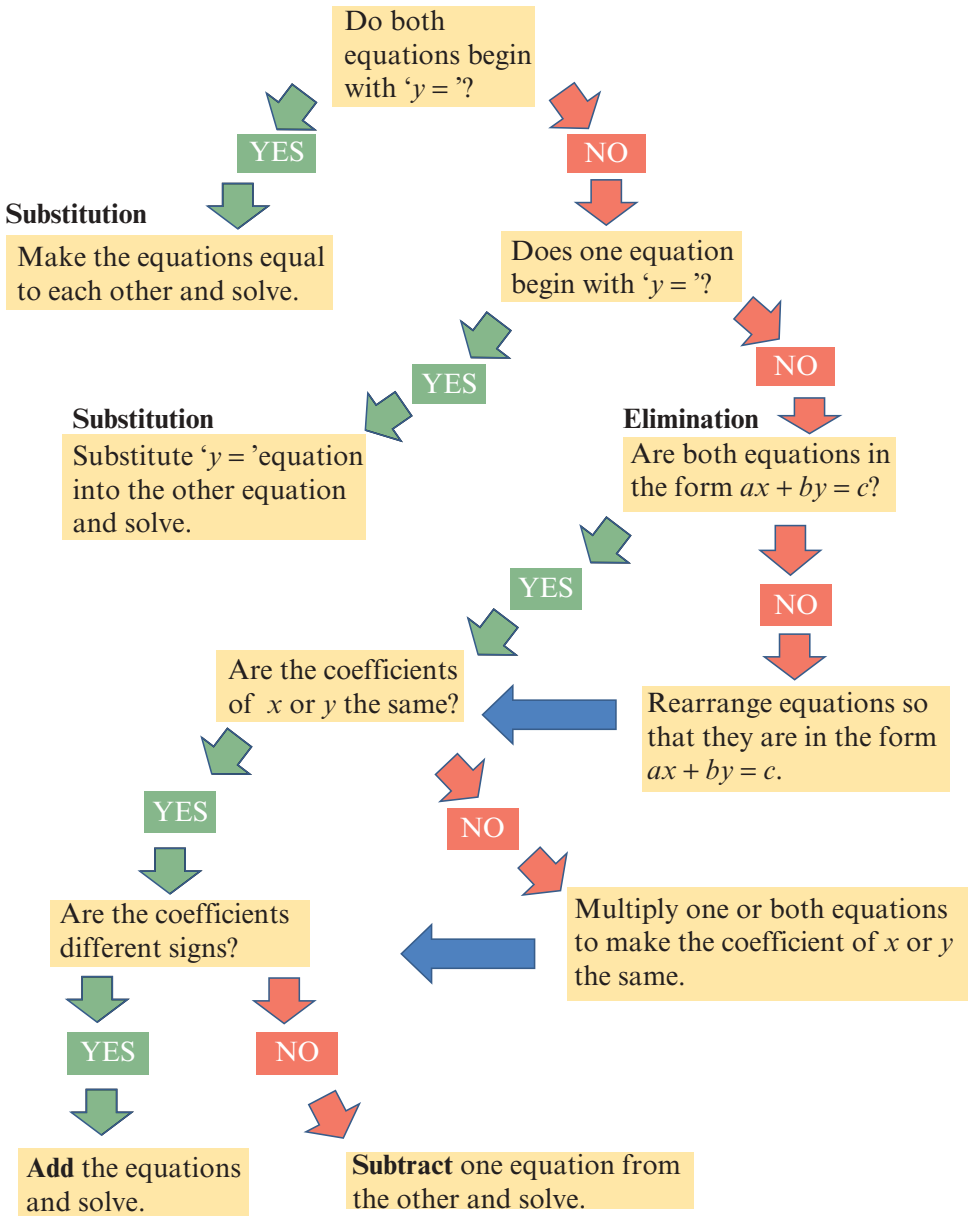
Deciding whether to use the substitution or elimination method



Section Summary

- In the **elimination** method, we **add** or **subtract** the two equations.
- We may need to multiple one or both equations by a constant so that we can eliminate a variable.

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Exercise 7C

SF

Example 5

1 Solve the following pairs of simultaneous equations using the elimination method.

a $5x - 2y = 4$
$3x + 2y = 0$

b $6x - 3y = 37$
$5x + 3y = 29$

c $x + 3y = 24$
$2x - 3y = 12$

d $x + y = 10$
$x - y = 8$

e $2x + y = 11$
$3x - y = 9$

f $3x + 5y = 40$
$x - 5y = 0$

Example 6

2 Solve the following pairs of simultaneous equations using the elimination method.

a $5x + y = 42$
$2x + y = 18$

b $3x + 2y = 27$
$10x + 2y = 62$

c $3x + y = 14$
$3x + 6y = 24$

d $3x + 5y = 38$
$8y + 3x = 41$

e $2x + 3y = 20$
$3y + 10x = 76$

f $4x + 3y = 24$
$3y + 5x = 27$

Example 7

3 Solve the following pairs of simultaneous equations using the elimination method.

a $2x + 3y = 40$
$x + 5y = 62$

b $3x + y = 29$
$2y + 5x = 49$

c $3x + y = 17$
$7x - 2y = -21$

d $y + x = 7$
$2x - 3y = -26$

e $2x + 3y = -9$
$5x - 6y = 45$

f $3y + 4x = 9$
$5x - 6y = 1.5$

Example 8

4 Solve the following pairs of simultaneous equations using the elimination method.

a $2x + 3y = 16$
$4y + 3x = 23$

b $3x + 2y = -1$
$8x + 5y = 2$

c $3x + 5y = -1$
$2x - 7y = 20$

d $3x - 9y = -6$
$5y + 2x = 29$

e $3y + 2x = -7$
$-3x - 7y = 18$

f $4x + 3y = 11$
$5x - 2y = 8$

5 Solve the following pairs of simultaneous equations using your choice of method.

a $y = 10 - x$
$2x + y = 14$

b $3x + 2y = 0$
$-3x - y = 3$

c $3x + y = 10$
$y - 8 = -4x$

d $y = 3x - 1$
$y = 3 + 2x$

e $2x + 3y + 6 = 0$
$-2x - y - 6 = 0$

f $4x + 3y - 13 = 0$
$5x - 7y + 59 = 0$

CU

6 Solve the following system of equations for all three variables using the elimination method.

a $2x + 3y + z = 19$
$2x + y + 4z = 24$
$2y + 3z = 13$

b $3x + 2y - z = 9$
$-3x - y + 2z = -12$
$4y - 3z = 2$

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7D Problem-solving with simultaneous equations

Learning intentions

- To be able to use simultaneous equations to solve practical problems.
- To be able to use simultaneous equations to solve break-even problems.
- To be able to use simultaneous equations to compare pricing options.

Simultaneous equations can be used to solve problems in real situations using four key steps that are adapted from Chapter 5. First, variables must be defined to represent unknown numbers. Second, equations are formed using the information given in the question. Third, simultaneous equations are solved using one of the techniques covered in the preceeding sections. Fourth, the answer to the question is clearly written with units.

Example 9 Using simultaneous equations to solve a practical problem

Tickets for a movie cost $19.50 for adults and $14.50 for children. Two hundred tickets were sold, giving a total of $3265. How many children's tickets were sold?

Solution	Explanation
Let a be the number of adult's and c be the number of children's tickets sold.	Choose appropriate variables to represent the number of adult and child tickets sold.
$a + c = 200$ (1) $19.5a + 14.5c = 3265$ (2)	Write two equations using the information given in the question. Label the equations as (1) and (2).
$a = 200 - c$ (3) $19.5(200 - c) + 14.5c = 3265$ $3900 - 19.5c + 14.5c = 3265$ $3900 - 5c = 3265$	To solve, first rearrange equation (1) to make a the subject then substitute a from (3) into equation (2). Expand the brackets and then collect like terms.
$3900 - 5c - 3900 = 3265 - 3900$ $-5c = -635$ $\frac{-5c}{-5} = \frac{-635}{-5}$ $\therefore c = 127$	Solve for c by subtracting 3900 from both sides of the equation then divide both sides of the equation by -5.
In this question we are not asked for a, but calculating it allows us to check our answer. $a + (127) = 200$ (1) $a + 127 - 127 = 200 - 127$ $a = 73$	To solve for a, substitute $c = 127$ into equation (1) then subtract 127.
127 children's tickets were sold.	Write your solution.

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Example 10 Using simultaneous equations to solve a practical problem

The perimeter of a rectangle is 48 cm. If the length of the rectangle is three times the width, determine its dimensions.

Solution

Let l be the length and w be the width of the rectangle.

$2w + 2l = 48 \quad (1)$

$l = 3w \quad (2)$

$2w + 2(3w) = 48$

$2w + 6w = 48$

$8w = 48$

$\therefore w = 6$

$l = 3(6)$

$\therefore l = 18$

The length is 18 cm and the width is 6 cm.

Explanation

Choose appropriate variables to represent the dimensions of width and length.

Write two equations from the information given in the question. Label the equations (1) and (2).

Solve the simultaneous equations by substituting (2) into (1).
Expand the brackets and collect like terms.

Solve for w by dividing both sides by 8.

Solve for l by substituting $w = 6$ into equation (2).

Answer the question.

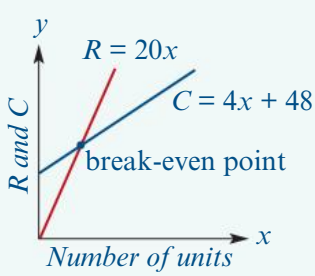
Example 11 Break-even analysis

A firm sells its product at \$20 per unit. The cost of production is given by the equation $C = 4x + 48$, where x is the number of units produced. Determine the value of x where the cost of production of x units is equal to the revenue received by the firm for selling x units.

Solution

Let the revenue from selling x units be \$$R$ and the cost be \$$C$.

$R = 20x \qquad C = 4x + 48$



$20x = 4x + 48$

$16x = 48$

$\therefore \quad x = 3$

Explanation

Define the variables.

Set up the equations.
Draw the graphs of these equations.

Find the value of x at the point of intersection, when $C = R$, by solving algebraically.

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From the previous example, if fewer than three items are sold, the cost is greater than revenue; if more than three items are sold, cost is less than revenue; the point where cost equals revenue is called the break-even point.

Note that sometimes rounding to the nearest whole number is necessary as you cannot sell part of a product.

Example 12

Anne-Marie requires a plumber to come to her house to unblock a drain. She contacts two different plumbers. The first plumber's cost, $\$C$, is given by $C = 40n + 80$ where n is the number of hours the job takes. The second plumber charges a callout fee of \$65 to visit the house and then \$45 per hour for the work.

a Write an equation that gives the second plumber's cost, $\$C$, in the form $C = mn + c$ where n is the number of hours the job takes.

b Sketch the graph of C versus n for each plumber for $0 \leq n \leq 8$. This means the horizontal axis is n from 0 to 8. C is on the vertical axis; use the range 0 to \$500.

c Determine after how many hours the first plumber becomes the cheaper option.

Solution	Explanation
a $C = mn + c$ $= 45n + 65$	Plumber 2 charges a callout fee of \$65 and an hourly rate of \$45 so $a = 65$ and $b = 45$.
b $C = 40n + 80$ $= 40(0) + 80$ $= 80$ $\therefore$ (0, 80) is a point on the line.	For the first rule, find C when $n = 0$.
$C = 40n + 80$ $= 40(8) + 80$ $= 400$ $\therefore$ (8, 400) is a point on the line.	Find C when $n = 8$.
$C = 65 + 45(0)$ $= 65$ $\therefore$ (0, 65) is a point on the line.	For the second rule, find C when $n = 0$.
$C = 65 + 45(8)$ $= 425$ $\therefore$ (8, 425) is a point on the line.	Find C when $n = 8$.

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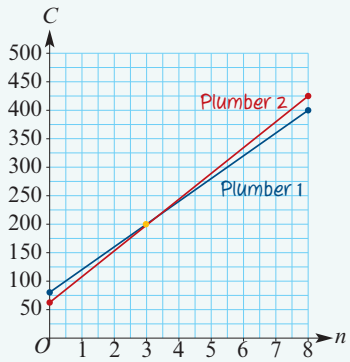

Plot the points $(0, 80)$ and $(8, 400)$ to construct the first graph.
Plot the points $(0, 65)$ and $(8, 425)$ to construct the second graph.
Find the point of intersection of the two lines by reading off the graph.

The point of intersection is at $(3, 200)$.

$40n + 80 = 45n + 65$

$80 = 5n + 65$

$15 = 5n$

$3 = n$

Algebraically, this can be solved by equating the two equations and solving for n.

The first plumber becomes a cheaper option for more than 3 hours work.

Determine when the first line is below the second line.

Section Summary

- Real world problems can be solved using simultaneous equations using the four steps for solving word problems.
  1. Define variables to represent unknown numbers.
  2. Form equations to represent the given information.
  3. Solve simultaneous equations algebraically, using either substitution or elimination techniques.
  4. Answer the question, paying careful attention to the units.
- **Break-even analysis** requires you to find when revenue and cost are equal. If the intersection point gives you a fractional answer, it may be necessary to round your answer to the nearest whole number as parts of units cannot be sold.
- Different pricing options can be compared by graphing each linear equation on the same axes.

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Skill-sheet

Exercise 7D

SF

Example 9

1 Jessica bought five marker pens and six pencils for $12.75, and Tom bought seven marker pens and one pencil for $12.30. Let t be the price of a marker pen and p be the price of a pencil.
   a Write down a pair of simultaneous equations to solve.
   b Calculate the cost of one pencil and of one marker pen.

2 Peter bought 50 litres of petrol and 5 litres of motor oil for $109. His brother Anthony bought 75 litres of petrol and 5 litres of motor oil for $146.
   a Using p for petrol and m for motor oil, write down a pair of simultaneous equations to solve.
   b Calculate the cost of one litre of petrol and of one litre of motor oil.

3 Six oranges and ten bananas cost $7.10. Six oranges and eight bananas cost $6.40.
   a Write down a pair of simultaneous equations to solve.
   b Calculate the cost of one orange and of one banana.

CF

4 The weight of a box of nails and a box of screws is 2.5 kg. Four boxes of nails and a box of screws weigh 7 kg. Determine the weight of each box.

5 An enclosure at a wildlife sanctuary contains wombats and emus. If the number of heads totals 28 and the number of legs totals 88, determine the number of each species present.

6 The owner of a service station sells unleaded petrol for $1.42 and diesel fuel for $1.54. In five days he sold a total of 10 000 litres and made $14 495.08. Determine the number of litres of each type of fuel that he sold.

Example 10

7 The perimeter of a rectangle is 36 cm. If the length of the rectangle is twice its width, determine its dimensions.

8 A chocolate thickshake costs $2 more than a fruit smoothie. Jack pays $27 for three chocolate thickshakes and four fruit smoothies. Calculate the cost of each of a chocolate thickshake and a fruit smoothie.

9 The sum of two numbers x and y is 52. The difference between the two numbers is 8. Calculate the values of x and y.

10 The sum of two numbers is 35 and their difference is 19. Determine the numbers.

11 Bruce is 4 years older than Michelle. If their combined age is 70, determine their individual ages.

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CF

12 A boy is 6 years older than his sister. In 3 years' time he will be twice her age. Determine their present ages.

13 In 4 years' time a mother will be three times as old as her son. Four years ago she was five times as old as her son. Determine their present ages.

14 The registration fees for a mathematics competition are \$1.20 for students aged 8–12 years and \$2 for students 13 years and over. One hundred and twenty-five students have already registered and an amount of \$188.40 has been collected in fees. Calculate the number of students between the ages of 8 and 12 who have registered for the competition.

15 A computer company produces two laptop models: standard and deluxe. The standard laptop requires 3 hours to manufacture and 2 hours to assemble. The deluxe model requires $5\frac{1}{2}$ hours to manufacture and $1\frac{1}{2}$ hours to assemble. The company allows 250 hours for manufacturing and 80 hours for assembly over a limited period. Calculate the number of each model that can be made in the time available.

16 The perimeter of a rectangle is 120 metres. The length is one and a half times the width. Calculate the width and the length.

17 Three classes, A, B and C, in a school are such that class A has two thirds the number of students of class B and class C has five sixths the number of students in class B. The total number of pupils in the three classes is 105. Calculate the number of students in each class.

Example 11

18 A manufacturer sells his product at \$72 per unit, selling all that he produces. His fixed cost is \$45 000 and the cost per unit is \$22. Calculate:

a the break-even point

b the profit when 1800 units are produced

c the loss when 450 units are produced

d the sales volume required in order to obtain a profit of \$90 000.

19 A manufacturer of a product sells all that she produces. If her total revenue is given by $R = 7x$ (dollars) and her total cost is given by $C = 6x + 800$ (dollars), where x represents the number of units produced and sold, then:

a determine the level of production at the break-even point

b sketch the graph of C and R against x

c determine the level of production at the break-even point if the total cost increases by 5%.

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SF

Example 12

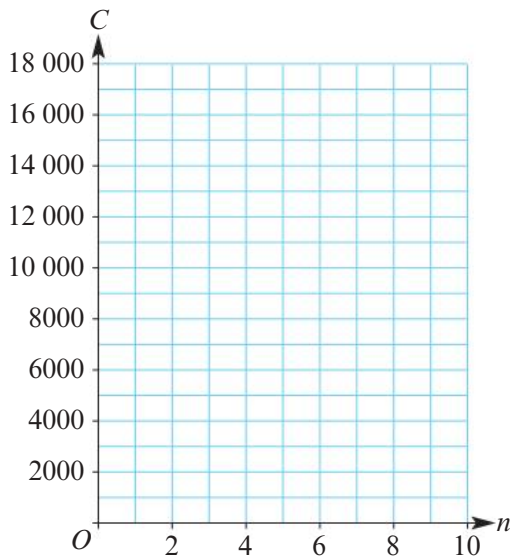
20 A machine hire company charges a $2000 delivery fee and a rental fee of $1500 per day for an excavator.

a Let C be the charge for hiring an excavator for n days. Write down a rule for the charge of hiring the generator for n days.

b On the set of axes, graph the relationship between C and n.

c A rival company charges a $4000 delivery fee and a rental fee of $500 per day. Let C be the charge for hiring an excavator for n days. Write down a rule for the charge of hiring the excavator from this company for n days.

d Sketch the graph of this rule on the same set of axes as those used in part **b**.



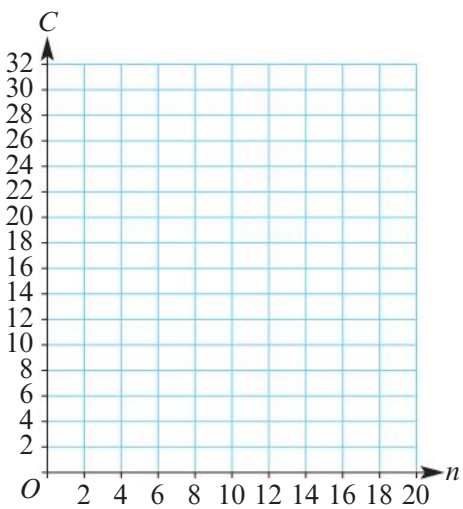
d Calculate the number of days when the two companies charge the same amount.

CF

21 The Rabbitphone telephone company offers calls to a remote island for a connection fee of $14 and thereafter $1 per minute. The Foxphone telephone company offers calls to the same island for $2 per minute and no connection fee.

a Compare the cost of a 10-minute call to the island using each company.

b By using graphical techniques, determine the call length when the two companies charge the same amount.



22 A water tank initially contains 4800 litres of water. Water is drained from the tank at a rate of 200 litres per minute.

a Write a rule for the volume of water, V litres, in the tank at time, t minutes.

b The water from this tank is drained into an irrigation channel that initially contains 600 litres of water. Write a rule for the volume of water, V litres, in the irrigation channel at time, t minutes.

c Calculate after how many minutes the tank and the channel contain the same volume of water.

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7E Sketching piece-wise linear and step graphs

Learning intentions

- To be able to sketch a piece-wise linear graph.
- To be able to sketch a step graph.

Piece-wise linear graphs

Sometimes a situation requires two linear graphs to obtain a suitable model, where each linear graph is valid for particular values of x. The graphs we use to model such situations are called **piece-wise linear graphs**.

Example 13 Sketching a piece-wise linear graph model

The amount, C dollars, charged to supply x m^3 of crushed rock is given by the equations:

$C = 40x + 50 \quad (0 \leq x < 3)$

$C = 30x + 80 \quad (3 \leq x \leq 8)$

a Use the appropriate equation to find the cost of supplying these amounts:

i 2.5 m^3 **ii** 3 m^3 **iii** 6 m^3

b Use the equations to sketch a piece-wise linear graph for $0 \leq x \leq 8$.

Solution

a i When $x = 2.5$

$C = 50 + 40x$

$C = 50 + 40(2.5) = 150$

Cost for 2.5 m^3 of crushed rock is \$150.

ii When $x = 3$

$C = 80 + 30x$

$C = 80 + 30(3) = 170$

Cost for 3 m^3 of crushed rock is \$170.

iii When $x = 6$

$C = 80 + 30x$

$C = 80 + 30(6) = 260$

Cost for 6 m^3 of crushed rock is \$260.

Explanation

For each case, choose the appropriate equation, substitute the value of x and evaluate.

Write your answer.

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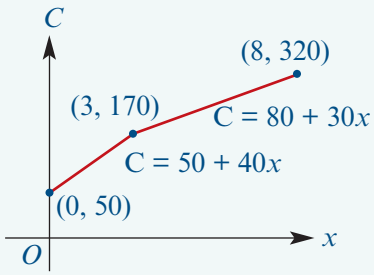
b

$x = 0 : C = 40(0) + 50 = 50$

$x = 3 : C = 40(3) + 50 = 170$

$x = 3 : C = 30(3) + 80 = 170$

$x = 8 : C = 30(8) + 80 = 320$



The graph has two line segments. Determine the coordinates of the end points of each line.

Draw a set of labelled axes and mark in the points with their coordinates. Join up the end points of each line segment with a straight line. Label each line segment with its equation.

Step graphs

A **step graph** can be used to represent information that is constant for particular intervals. One example of this is the cost of sending an airmail letter.

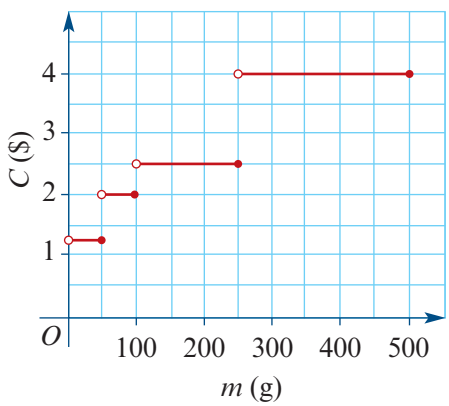
The table on the right shows that the cost of sending a letter is constant for intervals of mass. For example, a letter will cost \$2.00 to send if it has a mass of more than 50 g but less than or equal to 100 g.

Mass	Cost
Up to 50 g	\$1.20
Over 50 g up to 100 g	\$2.00
Over 100 g up to 250 g	\$2.50
Over 250 g up to 500 g	\$4.00

We must be careful when considering the cost of letters that have a mass at the end points of the mass interval.

A letter that has a mass of exactly 100 g belongs in the second interval because this interval includes masses up to and including 100 g. It does not belong in the third interval because this is for masses of more than 100 g, not equal to 100 g.

Each interval of mass from the table is shown as a horizontal line segment on the graph. The end points are open circles if the mass is *not* included in the interval and closed circles if the mass *is* included.



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Example 14 Sketching a step graph

The entry fee for a music competition depends on the age of the competitor, as shown in the table on the right.

Age of competitor	Entry fee
under 5 years	$5.00
5 years to under 10 years	$8.00
10 years to under 15 years	$15.00
15 years to under 20 years	$25.00
20 years and over	$30.00

a Determine the entry fee for a competitor who is 14 years and 9 months.

b Determine whether the circle for the horizontal line segment that ends at 20 years is open or closed.

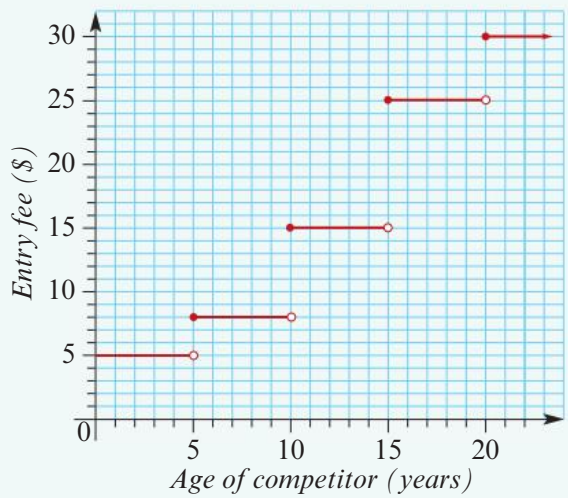
c Sketch the step graph that shows the *entry fee* against the *age of competitor*.

Solution

a The entry fee for a competitor who is 14 years and 9 months old is $15.00.

b Since 'under 20 years' does not include exactly 20 years, the circle will be open.

c



Explanation

14 years and 9 months is over 10 years and under 15 years.

The interval that ends at 20 years is 15 years to under 20 years.

For each interval of age, draw an appropriate line segment. The upper end of each interval will have an open circle.

Note: The interval '20 years and over' does not have an exact end point. An arrow can be used to indicate that this interval extends beyond the graph.

Section Summary

- A **piece-wise linear graph** can be used to model a situation with two or more linear graphs, each applicable over a different interval of x values.
- A **step graph** can be used to represent information that is constant for particular intervals.
- To form a piece-wise or step graph, find and plot the end points of each line segment and then join up the end points for each line segment.

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Exercise 7E

SF

Example 13

1 An empty tank is being filled from a mountain spring. For the first 30 minutes, the equation giving the volume, V, of water in the tank (in litres) at time t minutes is:

$$V = 15t \quad (0 \leq t \leq 30)$$

After 30 minutes, the flow from the spring slows down. For the next 70 minutes, the equation giving the volume of water in the tank at time, t, as given by the equation:

$$V = 150 + 10t \quad (30 < t \leq 100)$$

a Use the appropriate equation to determine the volume of water in the tank after:

i 20 minutes **ii** 30 minutes **iii** 60 minutes **iv** 100 minutes

b Use the equations to construct a piece-wise linear graph for $0 \leq t \leq 100$.

2 For the first 25 seconds of the journey of a train between stations, the speed, S, of the train (in metres per second) after t seconds is given by:

$$S = 0.8t \quad (0 \leq t \leq 25)$$

For the next 180 seconds, the train travels at a constant speed of 20 metres per second as given by the equation:

$$S = 20 \quad (25 < t \leq 205)$$

Finally, after travelling for 205 seconds, the driver applies the brakes and the train comes to rest after a further 25 seconds as given by the equation:

$$S = 184 - 0.8t \quad (205 < t \leq 230)$$

a Use the appropriate equation to determine the speed of the train after:

i 10 seconds **ii** 60 seconds **iii** 180 seconds **iv** 210 seconds

b Use the equations to construct a piece-wise linear graph for $0 \leq t \leq 230$.

Example 14

3 The postal rates for letters for a particular country are shown in this table. Sketch a step graph to represent this information.

Weight not over	Rate	Weight not over	Rate
60 g	34c	350 g	$1.22
100 g	48c	450 g	$1.38
150 g	62c	600 g	$1.56
200 g	76c	750 g	$2.56
250 g	90c	1000 g	$3.40
300 g	$1.06		

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SF

4 A multistorey car park has tariffs as shown. Sketch a step graph showing this information.

First 2 hours	$5.00
2–3 hours	$7.50 (more than 2, less than or equal to 3)
3–4 hours	$11.00 (more than 3, less than or equal to 4)
4–8 hours	$22.00 (more than 4, less than or equal to 8)

5 Suppose that Australia Post charged the following rates for airmail letters to Africa: $1.20 up to 25 g; $2.00 over 25 g and up to 50 g; $3.00 over 50 g and up to 150 g. Sketch a graph to represent this information.

6 Marika prints designer t-shirts in large numbers. The revenue, $\$R$, generated from the sale of t t-shirts is given by the rule $R = 12t$ for sales up to and including 500 t-shirts and $R = 6000 + 10(t - 500)$ for sales of more than 500 t-shirts.

a Calculate the revenue generated by the sale of the following number of t-shirts:

i 400 t-shirts **ii** 800 t-shirts.

b Sketch a graph of revenue for $0 \leq t \leq 1000$.

7 A sightseeing company has two modes of transport to experience the city. First, tourists could hire a bike according to the following cost schedule.

First 2 hours	$30.00
2–3 hours	$45.00 (more than 2, less than or equal to 3)
3–4 hours	$55.00 (more than 3, less than or equal to 4)
4–8 hours	$60.00 (more than 4, less than or equal to 8)

Second, tourists could hire a boat for t hours according to the following equations:

$$C = 15 + 10t \quad (0 \leq t \leq 4)$$

$$C = 30 + 5t \quad (4 < t \leq 8)$$

a Use the appropriate equation and cost schedule to determine the cheapest way to see the city if you want to hire a bike or boat for:

i 1 hour **ii** 3 hours **iii** 5 hours **iv** 7 hours

b Use the cost schedule to construct a step graph for the cost of hiring a bike and the equations to construct a piece-wise linear graph for the cost of hiring a boat for $0 \leq t \leq 8$.

CF

8 A machine hire company has the following pricing structure. If an excavator is hired for less than one week (7 days), they charge a $1800 delivery fee and a daily rental fee of $1400. If an excavator is hired for one week or more, they charge a fee of $11 600 for the first 7 days including delivery and then a daily rental fee of $1000. Sketch a graph, showing excavator hire charges, for between 0 and 10 days.

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7F Interpreting piece-wise linear and step graphs used to model practical situations

Learning intentions

- To be able to interpret a piece-wise linear graph.
- To be able to interpret a step graph.

Given a piece-wise linear graph or a step graph, we can interpret the graph and determine the equations that describe the graph.

Example 15 Interpreting a piece-wise linear graph

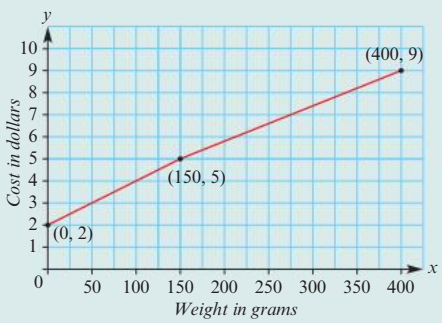
The following piece-wise graph shows the amount, C dollars, charged for x grams of frozen yoghurt.

a From the graph, determine:

i how much it costs to buy 100 grams of frozen yoghurt

ii how much frozen yoghurt you bought for $7.00.

b Determine the equation for the piece-wise graph.



Solution

a **i** $4.00

ii 275 grams

b For the first segment,

$$\frac{\text{Rise}}{\text{Run}} = \frac{5-2}{150-0} = \frac{3}{150} = \frac{1}{50}$$

The y-intercept is 2, so,

$$C = \frac{1}{50}x + 2, 0 \leq x < 150$$

For the second segment,

$$\frac{\text{Rise}}{\text{Run}} = \frac{9-5}{400-150} = \frac{4}{250} = \frac{2}{125}$$

The y-intercept is 2.6, so,

$$C = \frac{2}{125}x + 2.6, 150 \leq x < 400$$

Explanation

i Read from the graph where the weight is 100 grams.

ii Read from the graph where the cost is $7.00.

Determine the equation of the first segment ($0 \leq x < 150$) by first finding the gradient. Then read the y-intercept from the graph.

Determine the equation of the second segment ($150 \leq x < 400$) by first finding the gradient. Then find the y-intercept using methods from Chapter 5.

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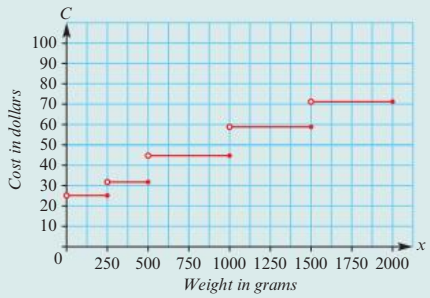
Example 16 Interpreting a step graph

The following step graph shows the cost, C, in dollars of sending a parcel that weighs x g from Australia to the UK.

a Determine how much it costs to send a parcel weighing 200 grams.
b Determine the maximum weight of a parcel that could be sent for $45.00.
c Construct a table showing the weight of a parcel and the cost of sending it to the UK.



Solution

a $25.00

b 1000 grams

c

Weight of parcel	Cost
Up to 250 grams	$25.00
250 grams up to 500 grams	$32.00
500 grams up to 1000 grams	$45.00
1000 grams up to 1500 grams	$58.00
1500 grams up to 2000 grams	$72.00

Explanation

Read from the graph when the weight is 200 grams.

Read from the graph when the cost is $45.00.

Set up a table with a column for the weight of a parcel and a column for the cost. Read the values from the graph.

Section Summary

- Piece-wise graphs and step graphs can be interpreted by reading values off the graphs.

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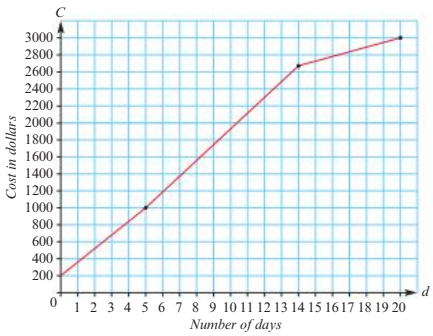
Skill-sheet

Exercise 7F

SF

Example 15

1 The graph shows the cost of hiring a car for a certain number of days.

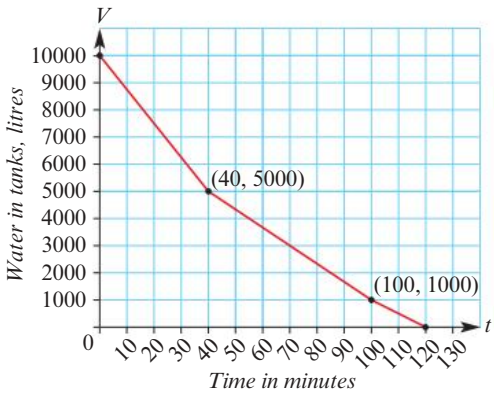


a State the cost of hiring a car for 4 days.

b State the cost of hiring a car for 17 days.

c A family paid $920 to hire a car. Determine the number of days of car hire they had.

2 The graph shows the number of litres of water in a tank t minutes after the valve has been removed.

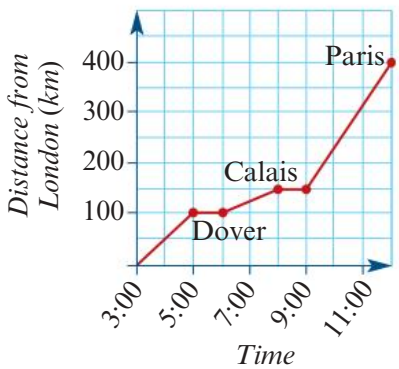


a State the volume of water in the tank after 20 minutes.

b State the volume of water in the tank after 70 minutes.

c The tank has 8000 litres remaining in it. Determine the number of minutes since the valve was removed.

3 The graph shows a man's journey by train, boat and train from London to Paris.



a Determine the time he:

i arrives at Dover.

ii arrives in Paris.

iii leaves Calais.

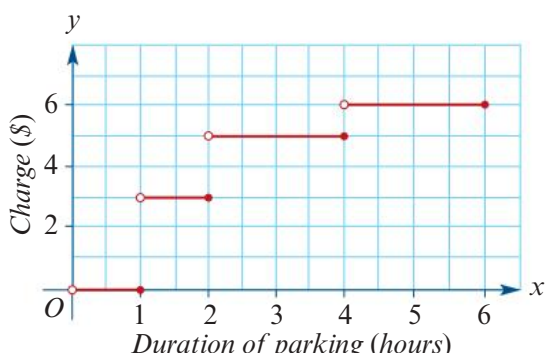
iv leaves Dover.

b Determine how long the man stops at Dover.

c Determine the time the man is exactly halfway between Calais and Paris.

Example 16

4 This step graph shows the charges for a market car park.



a Determine how much it costs to park for 40 minutes.

b Determine how much it costs to park for 2 hours.

c Determine how much it costs to park for 3 hours.

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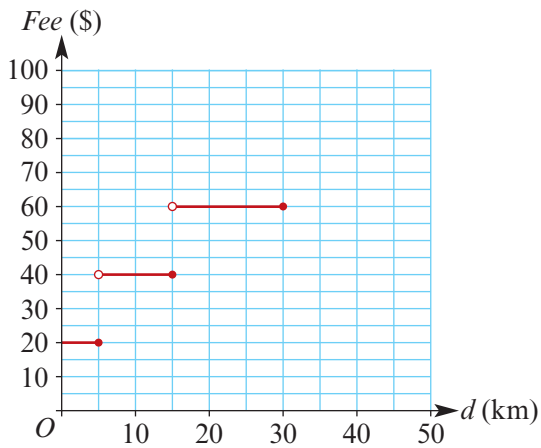
SF

5 Brad operates a parcel delivery service. The fee charged depends on the distance he has to travel in order to deliver a parcel. The fees for distances up to 30 km are shown on the step graph below.

a Determine the fee for a delivery requiring Brad to travel a distance of 20 km.

b Calculate the maximum distance Brad will travel for a delivery fee of $20.

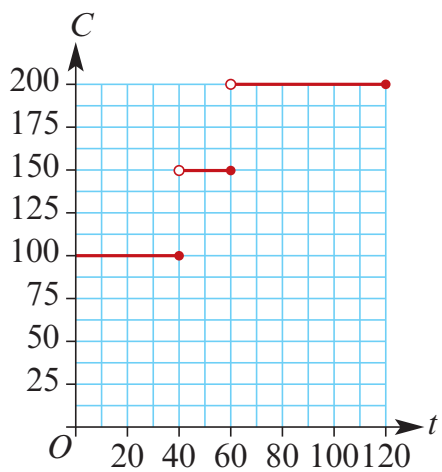
c A fee of $90 is charged to deliver a parcel requiring Brad to travel more than 30 km and no more than 40 km. Draw this information on the step graph.



6 The graph below shows the cost in dollars of home garden maintenance taking up to two hours by Kelvin the gardener.

a Determine the cost of garden maintenance that takes 60 minutes.

b Kelvin does one job that takes 30 minutes and another that takes 100 minutes. Determine the total cost of both jobs.



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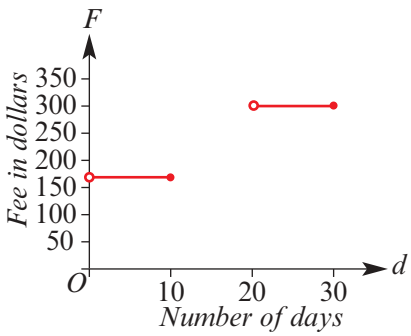
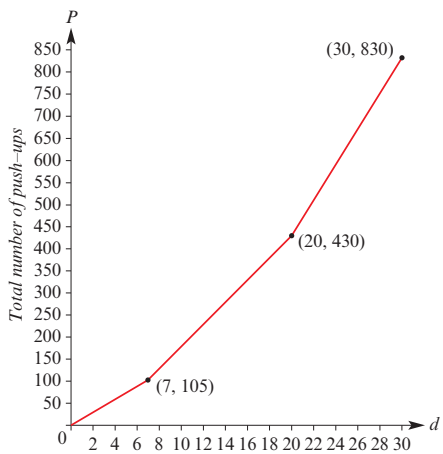
CF

7 Tilly offers a dog-walking service where a monthly fee is charged based on the number of days required each month. The step graph shows some information about her fees.



- **a** State the cost Tilly charges for 6 days of dog-walking per month.
- **b** Determine the maximum number of days Tilly is willing to walk a dog if she is paid $300.
- **c** Tilly needs to determine how much to charge customers who want between 10 and 20 days of walking of dog-walking per month. She wants to receive a minimum of $12 per day of walking. Calculate the amount that Tilly should charge.
- **d** Construct a table showing the number of days of dog-walking and the fee that Tilly charges.

8 James starts each day by completing a certain number of push-ups. The graph shows the total number of push-ups he has completed for the month, updated at the end of each day.



- **a** Determine when James increases the number of push-ups he does each day.
- **b** Determine how many push-ups James did on the twentieth day of the month.
- **c** State the total number of push-ups James has done by the end of day 6.
- **d** Determine how many days it took James to complete a total of 330 push-ups for the month.

CU

9 Cassie runs a corporate cupcake business. She charges $6 per cupcake for orders of up to 200 cupcakes. For orders of more than 200 and up to 400, she charges $5.

- **a** Determine the total cost that Cassie charges on an order of 250 cupcakes.
- **b** Determine the number of cupcakes in an order that costs $1600.

Cassie has a different price structure for orders of more than 400 cupcakes.

- **c** Determine the price charged per cupcake if Cassie charges a total of $2625 for 625 cupcakes.
- **d** Write down a set of equations for Cassie's price structure, including the conditions when each equation applies, where R is revenue in dollars and c is the number of cupcakes.
- **e** Construct a piece-wise linear graph for the cost of Cassie's cupcakes for between 0 and 500 cupcakes.

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Key ideas and chapter summary

Piece-wise linear graphs

Piece-wise linear graphs are used in practical situations where more than one linear equation is needed to model the relationship between two variables.

Step graphs

A **step graph** consists of one or more horizontal line segments. Step graphs can be used to graphically represent situations where the value of one variable is constant for intervals of another variable.

Simultaneous equations

Two straight lines will always intersect, unless they are parallel. At the point of intersection the two lines will have the same coordinates. When we find the point of intersection, we are solving the equations simultaneously. **Simultaneous equations** can be solved graphically, algebraically or by using technology.
Example:

$$y = 3x + 6$$
$$y = 4x - 9$$

are a pair of simultaneous equations.

Break-even point

The number of units of an item that need to be sold for sales revenue to equal (or just exceed) cost of producing those units.

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Skills checklist

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills.

7A	**1** I can solve a pair of simultaneous equations graphically by finding the point of intersection of two linear graphs.	☐
	See Example 1 and Exercise 7A Question 1.	
7B	**2** I can solve a pair of simultaneous equations algebraically using the substitution method.	☐
	See Examples 2, 3 and 4, and Exercise 7B Questions 1, 2 and 3.	
7C	**3** I can solve a pair of simultaneous equations algebraically using the elimination method.	☐
	See Examples 5, 6, 7 and 8, and Exercise 7C Questions 1, 2, 3 and 4.	
7D	**4** I can use simultaneous equations to solve practical problems.	☐
	See Examples 9 and 10, and Exercise 7D Questions 1 and 7.	
7D	**5** I can use simultaneous equations to solve break-even problems.	☐
	See Example 11 and Exercise 7D Question 18.	
7D	**6** I can use simultaneous equations to compare pricing options.	☐
	See Example 12 and Exercise 7D Question 20.	
7E	**7** I can sketch a piece-wise linear graph.	☐
	See Example 13 and Exercise 7E Question 1.	
7E	**8** I can sketch a step graph.	☐
	See Example 14 and Exercise 7E Question 3.	
7F	**9** I can interpret a piece-wise linear graph.	☐
	See Example 15 and Exercise 7F Question 1.	
7F	**10** I can interpret a step graph.	☐
	See Example 16 and Exercise 7F Question 4.	

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Multiple-choice questions

1 The solution to the pair of simultaneous equations

$y = 5x$

$y = 2x + 6$

is:

A $(-2, 0)$ **B** $(-1, -5)$ **C** $(3, 0)$ **D** $(2, 10)$

2 The solution to the pair of simultaneous equations

$y = 2x - 1$

$5x + 3y = 8$

is:

A $(2, -1)$ **B** $(1, -2)$ **C** $(1, 1)$ **D** $(3, 5)$

3 The solution to the pair of simultaneous equations

$2x + 3y = -6$

$x + 3y = 0$

is:

A $(-6, -2)$ **B** $(6, 2)$ **C** $(-2, 6)$ **D** $(-6, 2)$

4 The solution to the pair of simultaneous equations

$x + y = 10$

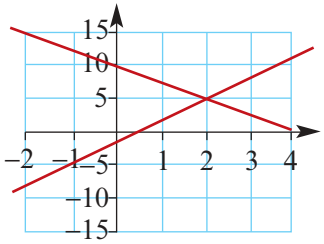
$2x + 3y = 28$

is:

A $(2, 3)$ **B** $(2, 8)$ **C** $(3, 2)$ **D** $(3, 7)$

5 The point of intersection of the lines shown in the diagram is:

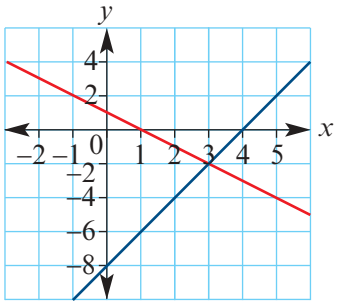
A $(5, 2)$ **B** $(0, 0)$ **C** $(0, 9)$ **D** $(2, 5)$



6 The point of intersection of the lines shown in the diagram is:

A $(3, 2)$ **B** $(3, -2)$

C $(-3, -2)$ **D** $(-2, -3)$



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7 Consider the following pairs of simultaneous equations. Determine which pair of equations has no solution.

A $y = 6 - x$
$y = 12 - 2x$

B $y = 8 + 6x$
$y = 8 - 2x$

C $2x + 3y = 0$
$3x = 7 - 2y$

D $x + y = 8$
$y = 15 - x$

8 Dennis bought five mangoes and three pineapples for $17.30, and Pam bought eight mangoes and three pineapples for $22.10. The price of two mangoes and one pineapple is:

A $3.20 **B** $4.70
C $6.30 **D** $7.80

9 The sum of two numbers is 27. The difference between the two numbers is 13. The product of the two numbers is:

A 7 **B** 13
C 20 **D** 140

10 Michelle is 17 years older than Natalie. If their combined ages are 111, then Natalie is:

A 17 years old **B** 37 years old
C 47 years old **D** 54 years old

11 The perimeter of a rectangle is 46 cm. If the length is 2 cm more than twice the width, the width is:

A 7 cm **B** 12 cm **C** 14 cm **D** 16 cm

12 A manufacturer sells his product for $18 per unit. His fixed costs are $17 500 in total and the cost per unit is $11. The number of units he must sell to break-even is:

A 972 **B** 973 **C** 1591 **D** 2500

13 An empty tank is being filled. For the first 20 minutes, the equation giving the volume, V, of water in the tank (in litres) at time t is:

$$V = 10t \quad (0 \leq t \leq 20)$$

After 20 minutes, the tank fills more slowly. For the next 40 minutes, the equation giving the volume of the tank at time t is given by:

$$V = 8t + 40 \quad (20 < t \leq 60)$$

The volume of the tank after 40 minutes is:

A 40 litres **B** 400 litres
C 360 litres **D** 560 litres

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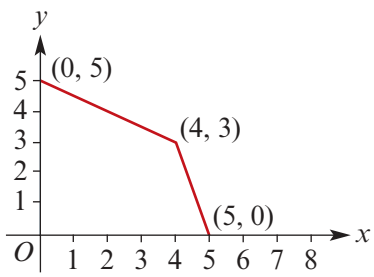
Review

14 The piece-wise linear graph shown represents the function with the rules:



A $y = \frac{1}{2}x + 5 \quad 0 \le x \le 4$

$y = x - 5 \quad 4 < x \le 5$

B $y = \frac{1}{2}x + 5 \quad 0 \le x \le 4$

$y = -2x + 11 \quad 4 < x \le 5$

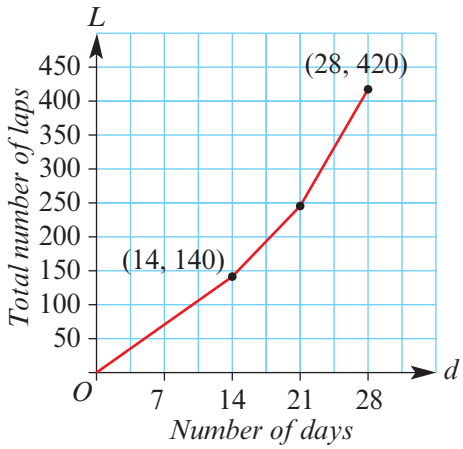
C $y = -\frac{1}{2}x + 5 \quad 0 \le x \le 4$

$y = -2x - 11 \quad 4 < x \le 5$

D $y = -\frac{1}{2}x + 5 \quad 0 \le x \le 4$

$y = -3x + 15 \quad 4 < x \le 5$

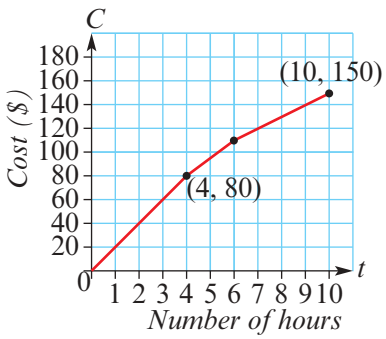
15 The graph shows Eme's swim training with the total number of laps she has completed since she began the training regime indicated. The number of laps she has completed after 8 days is:



A 50 **B** 80

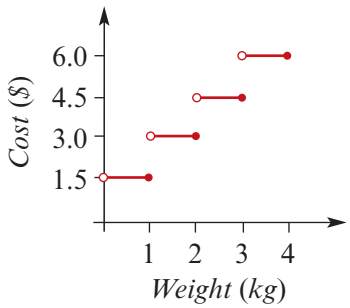
C 100 **D** 140

16 The graph shows the cost of hiring a bicycle for a certain number of hours each day. Evan hires a bicycle for 3 hours on Monday, 2 hours on Tuesday and then 5 hours on Wednesday. The amount that Evan pays **in total** for the three days is:



A \$40 **B** \$80 **C** \$175 **D** \$195

17 The graph shows the cost of posting parcels of various weights. A parcel weighing 3 kg will cost:



A \$4.50 **B** \$6.00

C \$5.00 **D** \$4.00

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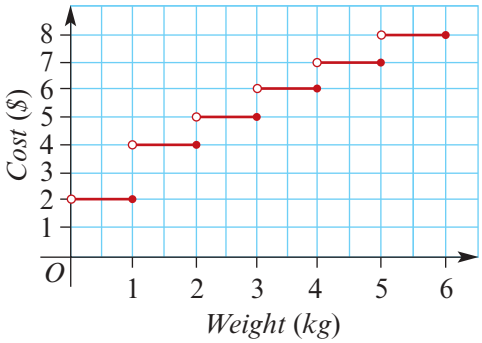
18 The graph shows the cost of posting parcels of various weights. A person posts two parcels, one weighing 3 kg and the other 1.5 kg. If each parcel is charged for separately, the cost of sending the two parcels is:



A $5.00
B $7.00
C $9.00
D $10.00

19 A shopping centre carpark has the following rates for customers.

First 2 hours	$3.00
2–3 hours	$5.00 (more than 2, less than or equal to 3)
3–4 hours	$8.00 (more than 3, less than or equal to 4)
4–8 hours	$14.00 (more than 4, less than or equal to 8)

Gillian parked at the shopping centre carpark each day during the week.

- On Monday, Tuesday and Wednesday, she was there for 2 hours each day.
- On Thursday, she spent 4 hours in the carpark.
- On Friday, she was there for 6 hours.

The total that she spent on parking for the week was:

A $3.00
B $9.00
C $31.00
D $37.00

20 Jesse hired a bobcat on four occasions during July.
The pricing to hire a bobcat is as follows:

Up to four hours	$380.00
four hours–one day	$760.00 (more than 4 hours, less than or equal to 1 day)
1-2 days	$1100.00 (more than 1 day, less than or equal to 2 days)
3–5 days	$2000.00 (more than 3 days, less than or equal to 5 days)
5–7 days	$3100.00 (more than 5 days, less than or equal to 7 days)

The first two times he hires the bobcat for 1 day only and the third time for 5 days. In total he pays $4620 for the month.
The length of time that Jesse hired the bobcat for on the fourth occasion was:

A 3 hours
B 1 day
C 2 days
D 4 days

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Review

Short-response questions

SF

1 Solve the following pairs of simultaneous equations.

a $y = 5x - 2$ and $2x + y = 12$

b $x + 2y = 8$ and $3x - 2y = 4$

c $2p + q = 12$ and $p + q = 3$

d $3p + 5q = 25$ and $2p - q = 8$

e $3p + 2q = 8$ and $p + 2q = 0$

2 Find the point of intersection of the following pairs of lines.

a $y = x + 2$ and $y = -3x + 6$

b $y = x - 3$ and $2x - y = 7$

c $x + y = 6$ and $2x - y = 9$

3 Consider the pair of simultaneous equations $x + y = 5$ and $2x - y = 16$.

a Solve the pair of simultaneous equations using the substitution method.

b Sketch each of the equations, clearly showing the x- and y-intercepts of each line.

c Show the point of intersection of the two lines.

4 Consider the pair of simultaneous equations $2x + 3y = 4$ and $5x - 2y = 29$.

a Solve the pair of simultaneous equations using the elimination method.

b Sketch each of the equations, clearly showing the x- and y-intercepts of each line.

c Show the point of intersection of the two lines.

5 Tilly and Will visit an amusement park. Tilly buys 7 stuffed animals and 2 squishy balls for $89. Will buys 3 stuffed animals and 5 squishy balls for $48.50.

a Let a be the number of stuffed toys and b be the number of balls. Write down an equation that represents Tilly's purchases and then an equation that represents Will's purchases in terms of a and b.

b Solve the two equations simultaneously to find the price of a stuffed animal and the price of a squishy ball.

c Find how much Amanda paid to buy 4 stuffed animals and 3 squishy balls.

6 Two families went to the theatre. The first family bought tickets for 3 adults and 5 children and paid $73.50. The second family bought tickets for 2 adults and 3 children and paid $46.50.

a Write down two simultaneous equations that could be used to solve the problem.

b Calculate the cost of a child's ticket.

c Calculate the cost of an adult's ticket.

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7 A hockey club charges an annual registration fee of $452 for juniors and $688 for seniors. So far, 244 people have registered to play and the club has collected $142 856. Let j be the number of junior players and s be the number of senior players.

- **a** Write down an equation that represents the number of junior and senior players who have registered so far in terms of j and s.
- **b** Write down an equation that represents the amount of registration money that has been collected from juniors and seniors.
- **c** Solve the two equations using either the substitution or the elimination method to find the number of junior and senior players who have already registered.
- **d** Sketch the two equations and show the point of intersection on your graph.

8 Sales revenue ($\$S$) for a company is given by the rule $S = 0.75x$, where x is the number of items sold, and the cost ($\$C$) is given by the rule $C = 0.25x + 100$.

- **a** Determine the fixed cost.
- **b** Determine how much revenue the company gets for each unit that it sells.
- **c** Determine the per-unit cost of making each unit.
- **d** Calculate the company's loss if it only sells 50 units.
- **e** Calculate the total profit that the company earns if it sells 300 units.
- **f** Calculate the break-even point.

9 A manufacturer produces gaming consoles which they are able to sell for $625 per unit giving them a revenue of $R = 625x$ when x units are sold. The fixed costs are $40 040 and the cost per unit is $240 per unit, meaning their costs are $C = 40\,040 + 240x$ for x units.

- **a** Calculate the break-even point.
- **b** Calculate the profit when 500 units are sold.
- **c** Calculate the loss incurred when 100 units are sold.
- **d** Calculate the number of sales that occur when a profit of $83 160 is earned.
- **e** Sketch a graph of the revenue, R, and cost lines, C, on the same axis. Show the break-even point.

10 Consider the following rules that define a piece-wise linear graph:

$$y = x + 2 \quad 0 < x \leq 2$$
$$y = 5 \quad 2 < x \leq 4$$
$$y = -\frac{1}{2}x \quad 4 < x \leq 6$$

- **a** Sketch the piece-wise linear graph defined by the rules.
- **b** Determine the value of y when $x = 5$.
- **c** Determine the value(s) of x when $y = 3$.

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11 An olive farm sells bottles of olive oil. The cost per bottle depends on the number of bottles purchased, as shown in the table below.

Number of bottles	Price per bottle
Up to 4	\$6.50
5 up to 9	\$6.00
10 up to 14	\$5.50
15 up to 20	\$5.00

a Determine the price per bottle if 15 bottles of olive oil are purchased.

b Calculate the cost to purchase 6 bottles of olive oil.

c Show the information from the table on a step graph.

Hint: You will need to show a dot for each point, rather than a solid line because only whole numbers of bottles can be purchased.

12 A swimming pool with 30 000 litres is being emptied. The volume in the pool, V, after h hours is given by:

$$V = 30\,000 - 650h \quad (0 \leq h \leq 24)$$

for the first 24 hours. After that, the volume in the pool is given by:

$$V = 24\,000 - 400h \quad (24 < h \leq a)$$

where a is an integer.

a Calculate the volume of water in the pool after 12 hours.

b Calculate the volume of water in the pool after 30 hours.

c Calculate how long it takes for the pool to be half-full. Give your answer correct to the nearest hour.

d Determine when the pool will be empty.

e Sketch a graph of the volume of water in the pool showing when it is empty.

13 A local pilates studio offers different memberships as outlined in the table below.

Name	Number of times per week	Cost per week
Basic	1	\$30
Moderate	2–3 times	\$60
Advanced	4–5 times	\$80
Urban Warrior	Unlimited	\$100

Calculate the minimum number of visits per week that an Urban Warrior needs to attend for this membership type to be the best value in terms of dollars per visit.

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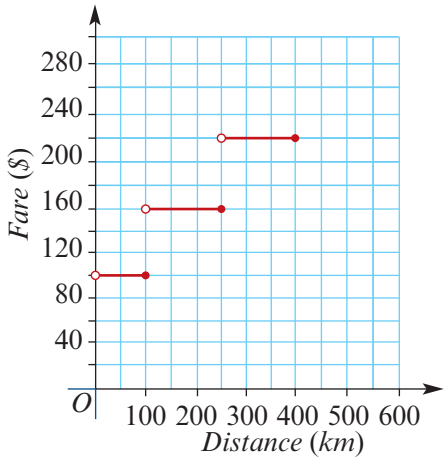
14 Fair Go Airlines offers air travel between destinations in regional Queensland.
The table shows the fares for some distances travelled.

Distance/km	Fare
$0 < \text{distance} \leq 100$	\$100
$100 < \text{distance} \leq 250$	\$160
$250 < \text{distance} \leq 400$	\$220



a The fare for a distance longer than 400 km, but not longer than 550 km, is \$280.
Draw this information on a copy of the graph opposite.

Fair Go Airlines is planning to change its fares.
A new fare will include a service fee of \$40, plus 50 cents per kilometre travelled.

b At a certain distance between 250 km and 400 km, the fare, when calculated using either the new information or the table, is the same. Calculate this distance.

c An equation connecting the maximum distance that may be travelled for each row in the table to the fare for that row can be written as:

$$fare = m \times maximum\,distance + c$$

Determine m and c.

15 The speed of a train can be modelled using a piece-wise function. Upon leaving the station, the train's speed, S, in metres per second after t seconds, is given by:

$$S = \frac{5}{6}t \quad (0 \leq t \leq 30)$$

After 30 seconds, the train travels at a constant speed of 25 metres per second for 180 seconds, given by the equation:

$$S = 25 \quad (30 < t \leq 240)$$

The train then slows down as it approaches the next station, coming to rest after a further 60 seconds, given by the equation:

$$S = mt + c \quad (240 < t \leq 300)$$

Determine the value of m and c before graphing all three equations. Hence, find the speed of the train after 280 seconds, correct to two decimal places.

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Chapter 8

Applications of trigonometry

Chapter questions

UNIT 2: APPLICATIONS OF LINEAR EQUATIONS AND TRIGONOMETRY, MATRICES AND UNIVARIATE DATA ANALYSIS

Topic 2: Applications of trigonometry

- How are $\sin\theta$, $\cos\theta$ and $\tan\theta$ defined using a right-angled triangle?
- How can the trigonometric ratios be used to calculate the side lengths or angles in right-angled triangles?
- What is meant by an angle of elevation or an angle of depression?
- How are true bearings measured?
- How can the sine and cosine rules be used to solve non-right-angled triangles?
- What are the three rules that can be used to calculate the area of a triangle?

Trigonometry can be used to solve many practical problems. How high is that tree? What is the height of the mountain we can see in the distance? What is the exact location of the fire that has just been seen by fire spotters? How wide is the lake? What is the area of this irregular-shaped paddock? In this chapter, we will review the trigonometric ratios and delve into how they are applied to solve related practical problems.

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8A Review of basic trigonometry

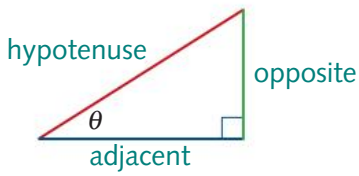
Learning intentions

- To be able to apply the conventions for naming the sides in a right-angled triangle.
- To be able to use the trigonometric ratios sin θ, cos θ and tan θ defined by a right-angled triangle.
- To be able to use a scientific calculator to evaluate sin θ, cos θ and tan θ of a right-angled triangle.

Although you are likely to have studied some trigonometry, it may be helpful to review a few basic ideas.

Naming the sides of a right-angled triangle

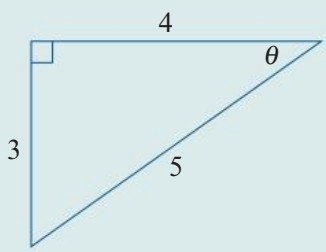
- The **hypotenuse** is the longest side in a right-angled triangle and is always opposite the right angle (90°).
- The **opposite** side is directly opposite the angle θ.
- The **adjacent** side is beside the angle θ, but it is not the hypotenuse. It runs from θ to the right angle.



The opposite and adjacent sides are located in relation to the position of angle θ. If θ was in the other corner, the sides would have to swap their labels. The letter θ is the Greek letter *theta*. It is commonly used to label an angle.

Example 1 Identifying the sides in a right-angled triangle

State the lengths of the hypotenuse, the opposite side and the adjacent side in the right-angled triangle shown.



Solution	Explanation
The hypotenuse, $h = 5$	The hypotenuse is opposite the right angle.
The opposite side, $o = 3$	The opposite side is opposite the angle θ.
The adjacent side, $a = 4$	The adjacent side is beside θ, but is not the hypotenuse.

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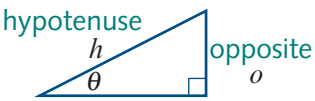
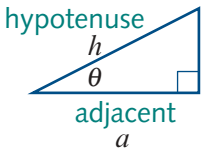
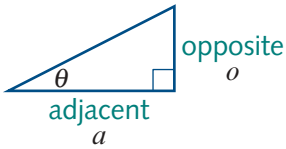
The trigonometric ratios

The **trigonometric ratios sin θ**, **cos θ** and **tan θ** can be defined in terms of the sides of a right-angled triangle. Note that we are using abbreviations here – 'sin' for sine, 'cos' for cosine, and 'tan' for tangent.

		
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{o}{h}$	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{a}{h}$	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \dfrac{o}{a}$

"SOH — CAH — TOA"

The mnemonic **SOH-CAH-TOA** is often used by students to help them remember the rule for each trigonometric ratio.

In this mnemonic:

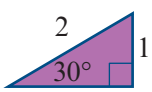
- **SOH** reminds us that sine equals opposite over hypotenuse
- **CAH** reminds us that cosine equals adjacent over hypotenuse
- **TOA** reminds us that tan equals opposite over adjacent.

Or you may prefer:

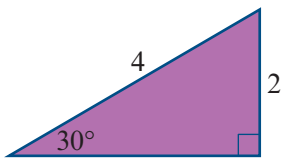
'**S**ir **O**liver's **H**orse **C**ame **A**mbling **H**ome **T**o **O**liver's **A**rms'

The meaning of the trigonometric ratios

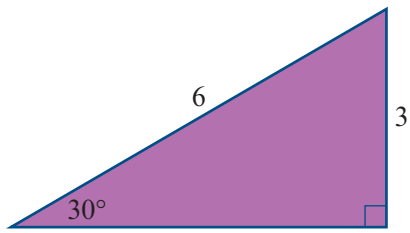
Using a calculator, we find, for example, that $\sin 30° = 0.5$. This means that in all right-angled triangles with an angle of 30°, the ratio of the length of the side opposite the 30° to the length of the hypotenuse is always 0.5.



$\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{1}{2} = 0.5$



$\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{2}{4} = 0.5$



$\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{3}{6} = 0.5$

Try drawing any right-angled triangle with an angle of 30° and check that the ratio is consistently $\dfrac{\text{opposite}}{\text{hypotenuse}} = 0.5$.

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Similarly, for any right-angled triangle with an angle of 30° the ratios cos 30° and tan 30° always have the same values:

$$\cos 30^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ is always } \frac{\sqrt{3}}{2} = 0.8660 \text{ (to four decimal places)}$$

$$\tan 30^\circ = \frac{\text{opposite}}{\text{adjacent}} \text{ is always } \frac{1}{\sqrt{3}} = 0.5774 \text{ (to four decimal places).}$$

If the triangle is truly a right-angled triangle, Pythagoras' theorem will give a true equation when tested with the corresponding size lengths given in the triangle. A scientific calculator can calculate the value of each trigonometric ratio for any angle entered.

Using a scientific calculator to evaluate trigonometric ratios

Warning!

Make sure that your calculator is set in DEGREE mode before attempting the following example. On a scientific calculator the DEGREE mode can usually be set using the **mode** key.

Scientific calculator guide This is available in the Interactive Textbook.

Example 2 Calculating the values of trigonometric ratios

Use your calculator to calculate, correct to four decimal places, the value of:

a sin 49° **b** cos 16° **c** tan 27.3°

Solution

a sin 49° = 0.7547
b cos 16° = 0.9613
c tan 27.3° = 0.5161

Explanation

Ensure that the mode is set in **Degrees**. To change, press mode and select DEG.

Press sin, type **49**, then enter.

Press cos, type **16**, then enter.

Press tan, type **27.3**, then enter.

Write your answer correct to four decimal places.

Spread-sheet **Spreadsheet activity 8A:** Using a spreadsheet to find the values of trigonometric ratios.

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Section Summary

- The **hypotenuse** is the longest side in a right-angled triangle and is always opposite the right angle (90°).

  The **opposite** side is directly opposite the angle θ.

  The **adjacent** side is beside the angle θ, but it is not the hypotenuse. It runs from θ to the right angle.
- In a right-angled triangle, the trigonometric ratios are defined as:

  $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ and $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.
- Ensure that the mode is set to **degrees** before calculating any trigonometric ratios using your scientific calculator.

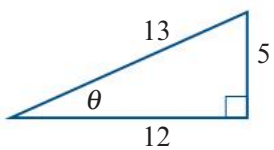
Exercise 8A

SF

Example 1

1 State the values of the hypotenuse, the opposite side and the adjacent side for each triangle.

a



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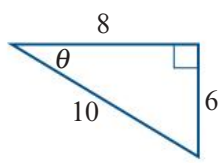

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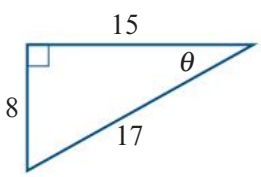
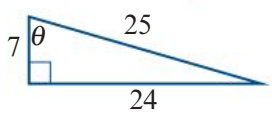


d



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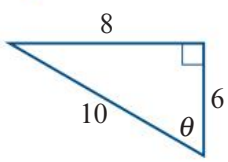

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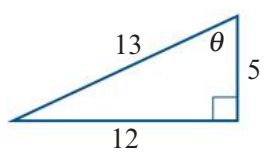

2 Write the ratios for sin θ, cos θ and tan θ for each triangle in Question **1**.

Example 2

3 Use your calculator to calculate the values of the following trigonometric ratios, correct to four decimal places.

a sin 27°	**b** cos 43°	**c** tan 62°
d cos 79°	**e** tan 14°	**f** sin 81°
g cos 17°	**h** tan 48°	**i** sin 80°
j sin 49.8°	**k** tan 80.2°	**l** cos 85.7°

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8B Calculating unknown sides in a right-angled triangle

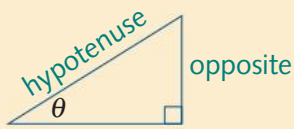
Learning intentions

- To be able to identify the most appropriate trigonometric ratio to use when solving for the length of an unknown side in a right-angled triangle.
- To be able to calculate the length of an unknown side in a right-angled triangle using trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$.

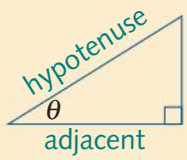
The trigonometric ratios can be used to calculate the lengths of unknown sides in a right-angled triangle, given an angle and one side.

Calculating the length of an unknown side in a right-angled triangle

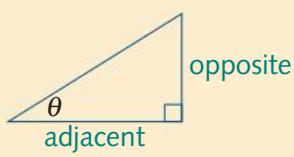
1. Draw the triangle and write in the given angle and side. Label the unknown side as x.
2. Use the trigonometric ratio that includes the given side and the unknown side.
   - **a** For the opposite side and the hypotenuse, use: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$



   - **b** For the adjacent side and the hypotenuse, use: $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$



   - **c** For the opposite and the adjacent sides, use: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

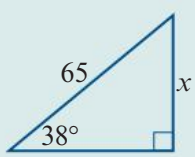


3. Rearrange the equation to make x the subject.
4. Use your calculator to calculate the value of x to the required number of decimal places.

When the unknown side is in the *numerator* (the top) of the trigonometric ratio, proceed as follows.

Example 3 Calculating an unknown side

Calculate the length of the unknown side, x, in the triangle shown, correct to two decimal places.



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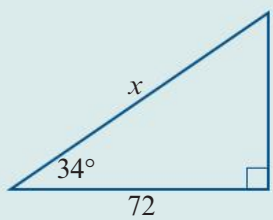
Solution	Explanation
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	The sides involved are the opposite and the hypotenuse, so use $\sin\theta$.
$\sin 38^\circ = \frac{x}{65}$	Substitute in the known values.
$65 \times \sin 38^\circ = x$ $x = 65 \times \sin 38^\circ$ $= 40.017\ldots$	Multiply both sides of the equation by 65 to obtain an expression for x. Use a calculator to evaluate.
$x = 40.02$	Write your answer correct to two decimal places.

An extra step is needed when the unknown side is in the denominator (the bottom) of the trigonometric ratio.

Example 4 Calculating the length of an unknown side in the denominator of the trigonometric ratio

Determine the value of x in the triangle shown, correct to two decimal places.



Solution	Explanation
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	The sides involved are the adjacent and the hypotenuse, so use $\cos\theta$.
$\cos 34^\circ = \frac{72}{x}$	Substitute in the known values.
$x \times \cos 34^\circ = 72$	Multiply both sides by x.
$x = \frac{72}{\cos 34^\circ}$ $= 86.847\ldots$	Divide both sides by $\cos 34^\circ$ to obtain an expression for x. Use a calculator to evaluate.
$x = 86.85$	Write your answer correct to two decimal places.

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Section Summary

- Choose the most appropriate trigonometric ratio according to the information provided in the right-angled triangle.
- When you are given the opposite side and the hypotenuse, use $\sin\theta$.
  For the adjacent side and the hypotenuse, use $\cos\theta$.
  For the opposite and the adjacent sides, use $\tan\theta$.
- The process to solve when the unknown side is in the denominator is different from when the unknown side is in the numerator of the trigonometric ratio.

Exercise 8B

Example 3

1 In each right-angled triangle below:

- decide whether the $\sin\theta$, $\cos\theta$ or $\tan\theta$ ratio should be used
- then determine the unknown side, x, correct to two decimal places.

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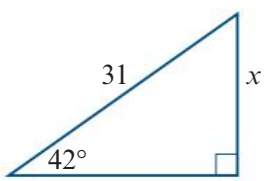

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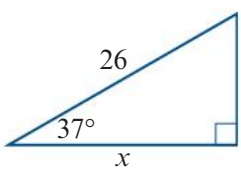

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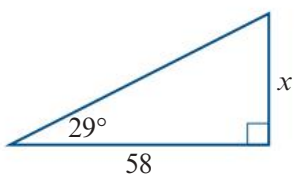

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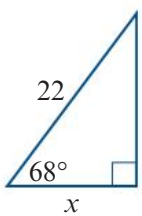

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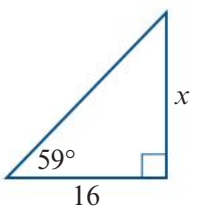

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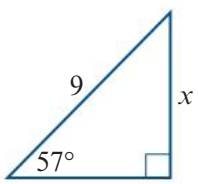

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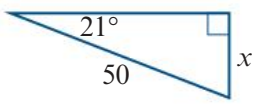

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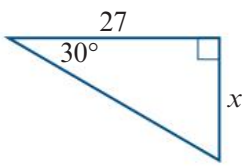

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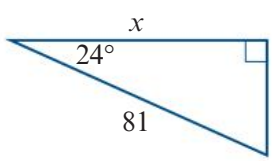

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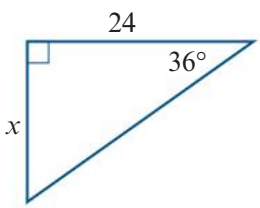

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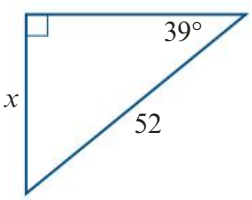

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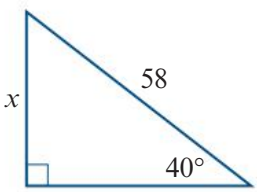

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Example 4

2 Calculate the length of the unknown side, x, in each right-angled triangle below. Give the answer correct to two decimal places.

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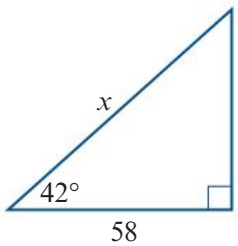

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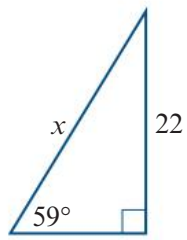

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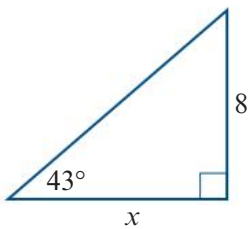

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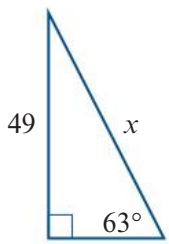

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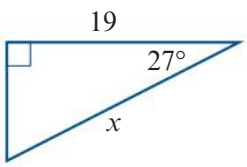

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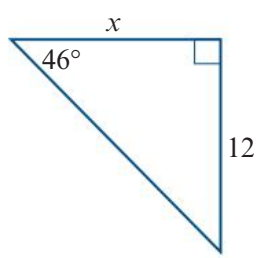

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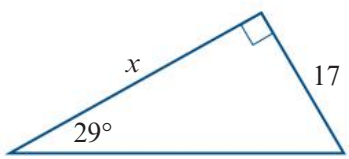

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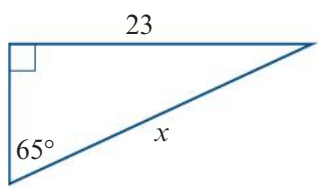

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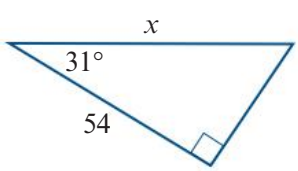

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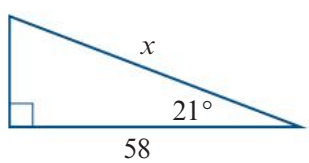

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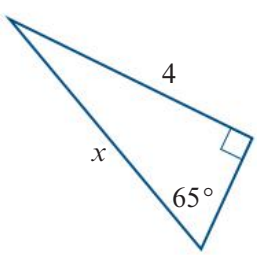

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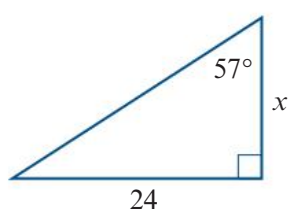

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Example 4

3 Calculate the length of the unknown side, x, in each right-angled triangle below. Give the answer correct to one decimal place.

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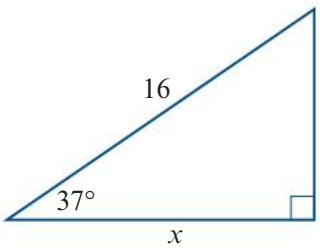

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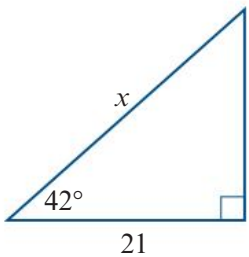

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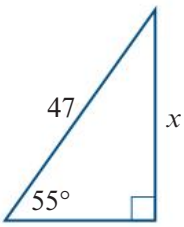

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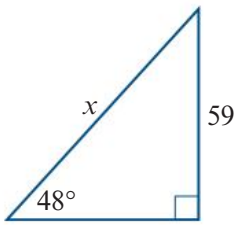

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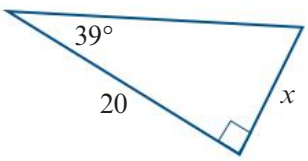

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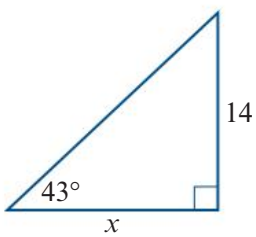

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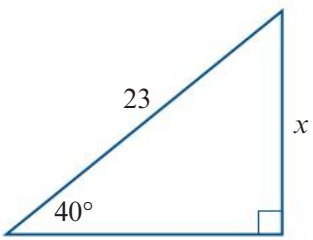

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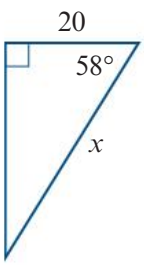

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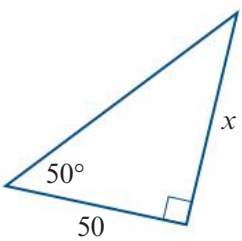

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8C Calculating unknown angles in a right-angled triangle

Learning intentions

- To be able to identify the most appropriate trigonometric ratio to use when solving for the size of an unknown angle in a right-angled triangle.
- To be able to calculate the size of an unknown angle in a right-angled triangle using trigonometric ratios sin θ, cos θ and tan θ.

Calculating an angle from a trigonometric ratio value

Before we look at how to calculate an unknown angle in a right-angled triangle, it will be useful to see how to calculate the angle when we know the value of the trigonometric ratio.

Suppose a friend told you that they found the sine value of a particular angle to be 0.8480 and challenged you to identify the mystery angle that had been used.

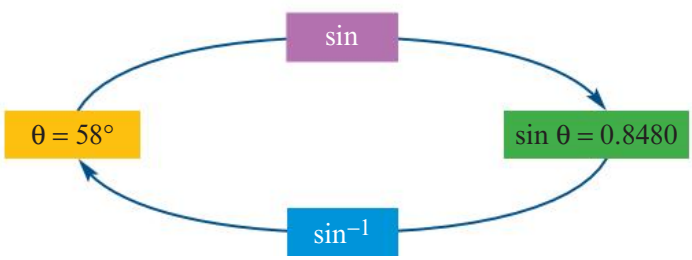
This is equivalent to saying if $\sin\theta = 0.8480$, determine the value of angle θ.

To do this, you need to work backwards from 0.8480 by undoing the sine operation to get back to the angle used. It is as if we have to apply the reverse gear to undo the effect of the sine function.

The reverse gear for sine is called the inverse of sine, written $\sin^{-1}$. The superscript −1 is not a power. It is just saying let us undo, or take one step backwards from using, the sine function.

The request to find θ when $\sin\theta = 0.8480$ can be written as $\sin^{-1}(0.8480) = \theta$.

This process is summarised in the following diagram.



- The top arrow in the diagram corresponds to 'given θ, calculate sin θ'. We use the sine function on our calculator to do this by entering sin 58° into a calculator to obtain the answer 0.8480.
- The bottom arrow in the diagram corresponds to 'given $\sin\theta = 0.8480$, calculate θ'. We use the $\sin^{-1}$ function on our calculator to do this by entering $\sin^{-1}(0.8480)$ to obtain the answer 58°.

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Similarly:

- The inverse of cosine, written as $\cos^{-1}$, is used to calculate θ when we know $\cos\theta = 0.5$ (see Example 5).
- The inverse of tangent, written $\tan^{-1}$, is used to calculate θ when we know $\tan\theta = 1.67$ (see Example 5).

You will learn how to how to use the $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ functions of your calculator in the following example.

Example 5 Calculating an angle from a trigonometric ratio

Calculate the angle θ, correct to one decimal place, given:

a $\sin\theta = 0.8480$ **b** $\cos\theta = 0.5$ **c** $\tan\theta = 1.67$

Solution	Explanation
a We need to calculate $\sin^{-1}(0.8480)$.	
$\sin^{-1}(0.848) = 57.9948$	Press **2nd** (or **SHIFT**), **sin**, type **0.848**, then press **enter** (or =).
$\theta = 58.0°$	Write your answer correct to one decimal place.
b We need to determine $\cos^{-1}(0.5)$.	
$\cos^{-1}(0.5) = 60$	Press **2nd** (or **SHIFT**), **cos**, type **0.5**, then press **enter** (or =).
$\theta = 60.0°$	Write the answer correct to one decimal place.
c We need to calculate $\tan^{-1}(1.67)$.	
$\tan^{-1}(1.67) = 59.0867$	Press **2nd** (or **SHIFT**), **tan**, type **1.67**, then press **enter** (or =).
$\theta = 59.1°$	Write the answer correct to one decimal place.

Spread-sheet **Spreadsheet activity 8C:** Calculating an angle from a trigonometric ratio.

Trigonometric terminology

The language we use when calculating an angle from a trigonometric ratio is difficult when you first meet it. Think about the examples below.

- When you see $\sin 58° = 0.8480$, think 'the sine of the angle 58° equals 0.8480'.
- When you see $\sin^{-1}(0.8480) = 58°$, think 'the angle θ where sin θ equals 0.8480 is 58°'.

Continued on next page

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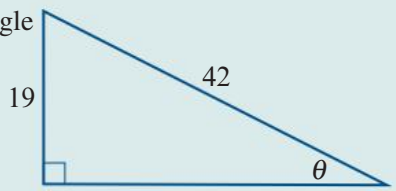
- When you see $\cos 60° = 0.5$, think 'the cosine of the angle 60° equals 0.5'.
- When you see $\cos^{-1}(0.5) = 60°$, think 'the angle θ where cos θ equals 0.5 is 60°'.
- When you see $\tan 59.1° = 1.67$, think 'the tangent of the angle 59.1° equals 1.67'.
- When you see $\tan^{-1}(1.67) = 59.1°$, think 'the angle θ where tan θ equals 1.67 is 59.1°'.

Calculating an angle given two sides

Example 6 Calculating an angle given two sides in a right-angled triangle

Calculate the angle θ, in the right-angled triangle shown, correct to one decimal place.



Solution

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{19}{42}$$

$$\theta = \sin^{-1}\left(\frac{19}{42}\right)$$

$$= 26.896...$$

$$\theta = 26.9°$$

Explanation

The sides involved are the opposite and the hypotenuse, so use sin θ.

Substitute in the known values.

Write the equation to find an expression for θ. Use a calculator to evaluate.

Write your answer correct to one decimal place.

The three angles in a triangle add to 180°. As the right angle is 90°, the other two angles must add to make up the remaining 90°. When one angle has been found, subtract it from 90° to determine the other angle. In Example 6, the other angle must be 90° – 26.9° = 63.1°.

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Calculating an angle in a right-angled triangle

1 Draw the triangle with the given sides shown. Label the unknown angle as θ.
2 Use the trigonometric ratio that includes the two known sides.
- If given the opposite side and the hypotenuse, use $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.
- If given the adjacent side and the hypotenuse, use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.
- If given the opposite side and the adjacent side, use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.

3 Divide the side lengths to calculate the value of the trigonometric ratio.
4 Use the appropriate inverse function key to determine the angle θ.

Section Summary

- Choose the most appropriate trigonometric ratio according to the information provided in the right-angled triangle.
- When you are given the opposite side and the hypotenuse, use $\sin\theta$.
  For the adjacent side and the hypotenuse, use $\cos\theta$.
  For the opposite and the adjacent sides, use $\tan\theta$.
- Apply the appropriate inverse function to determine the unknown angle.

Exercise 8C

Example 5

SF

1 Calculate the angle θ, correct to one decimal place.

a $\sin\theta = 0.4817$
b $\cos\theta = 0.6275$
c $\tan\theta = 0.8666$
d $\sin\theta = 0.5000$
e $\tan\theta = 1.0000$
f $\cos\theta = 0.7071$
g $\sin\theta = 0.8660$
h $\tan\theta = 2.500$
i $\cos\theta = 0.8383$
j $\sin\theta = 0.9564$
k $\cos\theta = 0.9564$
l $\tan\theta = 0.5774$
m $\sin\theta = 0.7071$
n $\tan\theta = 0.5000$
o $\cos\theta = 0.8660$
p $\cos\theta = 0.3414$

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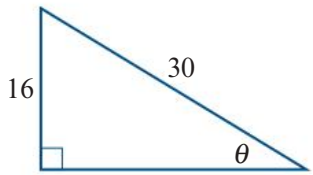
Example 6

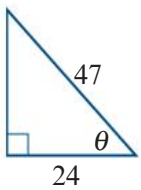
2 Calculate the unknown angle, θ, in each triangle, correct to one decimal place.

SF

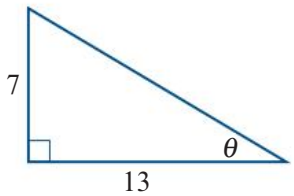
a Use $\sin^{-1}$ for this triangle.



b Use $\cos^{-1}$ for this triangle.



c Use $\tan^{-1}$ for this triangle.



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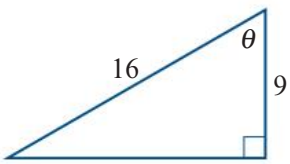

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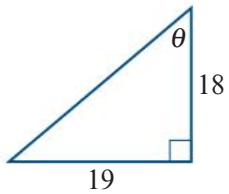

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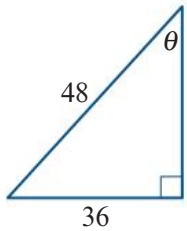

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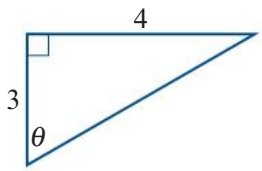

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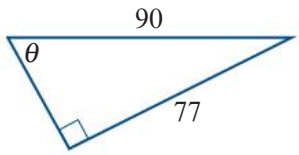

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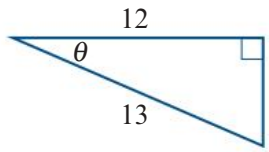

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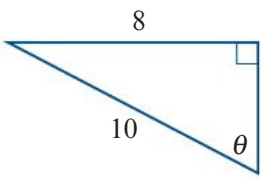

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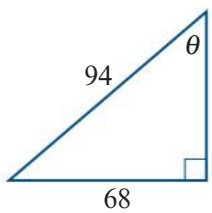

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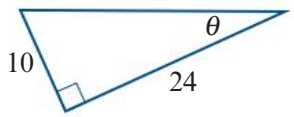

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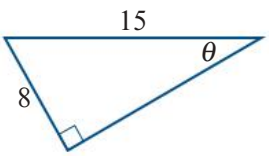

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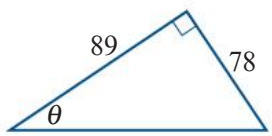

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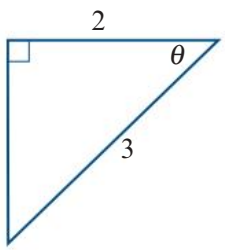

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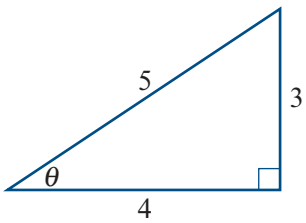
SF

3 Determine the value of θ in each triangle, correct to one decimal place.

a



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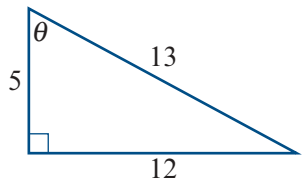

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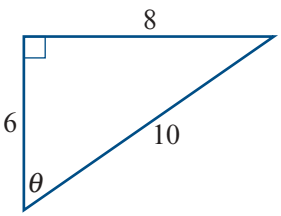

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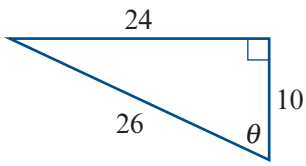

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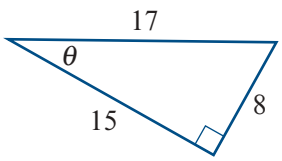

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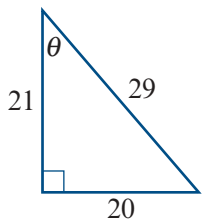
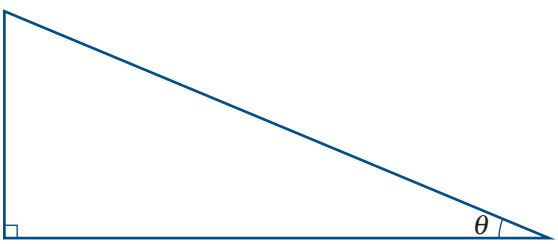


4 Use your ruler to measure appropriate side lengths of these triangles, and determine the value of θ in each triangle, correct to the nearest degree.

a



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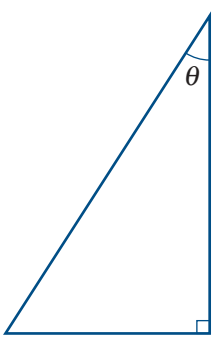

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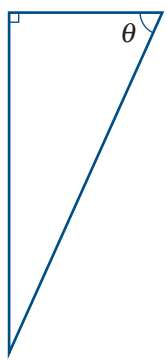

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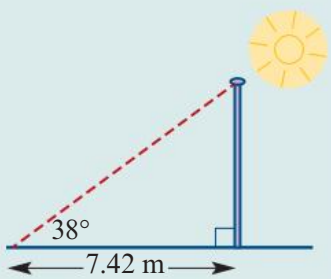
8D Applications of right-angled triangles to problem-solving

Learning intentions

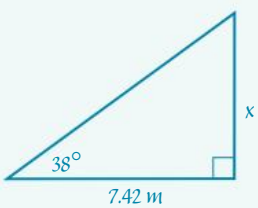
- To be able to solve practical problems using the trigonometric ratios sin θ, cos θ and tan θ to calculate an unknown side.
- To be able to solve practical problems using the trigonometric inverse functions $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ to calculate an unknown angle.

Example 7 Solving for an unknown length in an application

A flagpole casts a shadow 7.42 m long. The sun's rays make an angle of 38° with the level ground. Determine the height of the flagpole, correct to two decimal places.



Solution



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 38° = \frac{x}{7.42}$$

$$7.42 \times \tan 38° = x$$

$$x = 5.797\ldots$$

The height of the flagpole is 5.80 m.

Explanation

Draw a diagram showing the right-angled triangle. Include all the known details and label the unknown side.

The opposite and adjacent sides are involved, so use tan θ.

Substitute in the known values.

Multiply both sides by 7.42.

Use your calculator to calculate the value of x.

Write your answer correct to two decimal places, including appropriate units.

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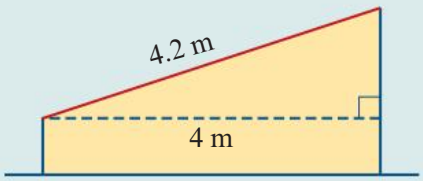
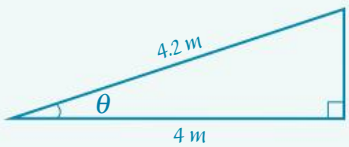
Example 8 Solving for an unknown angle in an application

A sloping roof uses sheets of corrugated iron 4.2 m long on a shed 4 m wide. There is no overlap of the roof past the sides of the walls. Calculate the angle the roof makes with the horizontal, correct to one decimal place.



Solution



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{4}{4.2}$$

$$\theta = \cos^{-1}\left(\frac{4}{4.2}\right)$$

$$\theta = 17.752\ldots$$

The roof makes an angle of 17.8° with the horizontal.

Explanation

Draw a diagram showing the right-angled triangle. Include all known details and label the required angle.

The adjacent and hypotenuse are involved so use cos θ.

Substitute in the known values.

Write the equation to calculate θ.

Use your calculator to calculate the value of θ.

Write your answer correct to one decimal place, including appropriate units.

Warning!

Always evaluate a mathematical expression as a whole, rather than breaking it into several smaller calculations. Rounding errors accumulate as more approximate answers are used in calculations.

Section Summary

- Choose the most appropriate trigonometric function according to the information given in the right-angled triangle.
- When you are solving for an unknown side, use sin θ, cos θ and tan θ. When solving for an unknown angle, use the $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$.
- Apply the appropriate function as a whole, only rounding at the end to minimise any rounding errors.
- Include a clearly labelled diagram as part of your response whenever possible.

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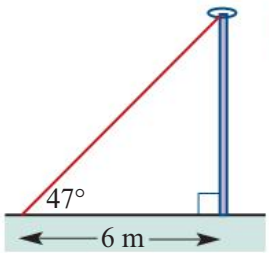
Skill-sheet

Exercise 8D

SF

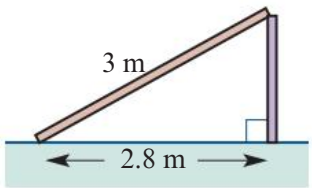
Example 7

1 A pole is supported by a wire that runs from the top of the pole to a point on the level ground 6 m from the base of the pole. The wire makes an angle of 47° with the ground. Calculate the height of the pole, correct to two decimal places.

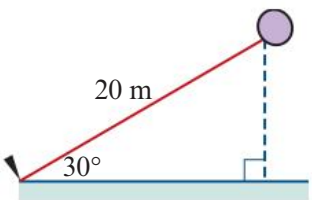


Example 8

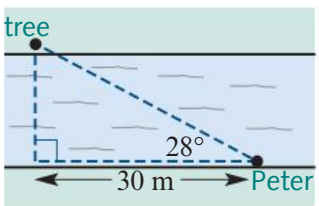
2 A 3 m log rests with one end on the top of a post and the other end on the level ground 2.8 m from the base of the post. Determine the angle the log makes with the ground, correct to one decimal place.



3 A balloon is tied to a string 20 m long. The other end of the string is secured by a peg to the surface of a level sports field. The wind blows so that the string forms a straight line making an angle of 30° with the ground. Determine the height of the balloon above the ground.



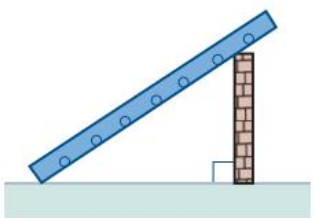
4 Peter noticed that a tree was directly opposite him on the far bank of the river. He then walked 30 m along his side of the river and found that his line of sight to the tree made an angle of 28° with the riverbank. Calculate the width of the river, to the nearest metre.



5 A ladder rests on a wall 2 m high. The foot of the ladder is 3 m from the base of the wall on level ground.

a Copy the diagram and include the given information. Label as θ the angle the ladder makes with the ground.

b Determine the angle the ladder makes with the ground, correct to one decimal place.



6 The distance measured up the sloping face of a mountain was 3.8 km. The sloping face was at an angle of 52° with the horizontal.

a Make a copy of the diagram and show the known details. Show the height of the mountain as x.

b Calculate the height of the mountain, correct to one decimal place.

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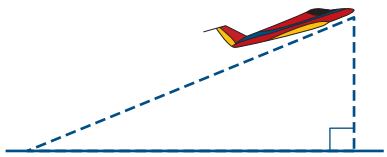
SF

7 An aeroplane maintains a flight path at an angle of 17° with the horizontal after it takes off. It travels for 2 km along that flight path.



a Show the given and required information on a copy of the diagram.

b Determine, correct to two decimal places:

i the horizontal distance of the aeroplane from its take-off point

ii the height of the aeroplane above ground level.

CF

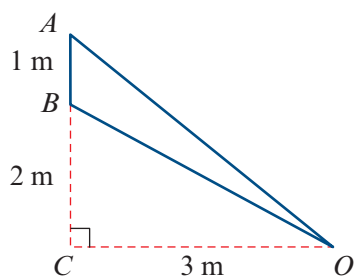
8 A 3 m ladder rests against an internal wall. The foot of the ladder is 1 m from the base of the wall. Determine the angle the ladder makes with the floor, correct to one decimal place.

9 The entrance to a horizontal mining tunnel has collapsed, trapping the miners inside. The rescue team decide to drill a vertical escape shaft from a position 200 m further up the hill. If the hill slopes at 23° from the horizontal, determine the depth that the rescue shaft needs to be to meet the horizontal tunnel. Answer correct to one decimal place.

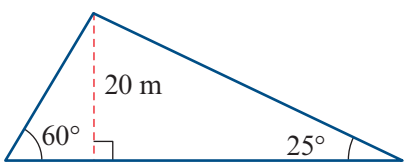
10 A strong rope needs to be fixed with one end attached to the top of a 5 m pole and the other end pegged at an angle of 60° with the level ground. Determine the required length of the rope, correct to two decimal places.

CU

11 Consider ΔOAC and ΔOBC as shown. Determine the size of angle $\angle AOB$.



12 Calculate the perimeter of the given triangle.



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8E Angles of elevation and depression

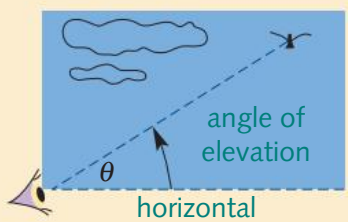
Learning intentions

- To be able to define angles of elevation and depression.
- To be able to draw and label appropriate diagrams involving angle of elevation and depression.
- To be able to solve two-dimensional practical problems involving angles of elevation and depression.

There are many situations where a two-dimensional right-angled triangle can be drawn so that trigonometry can be applied to solve a problem. Angles of elevation or depression are some examples that could be used in such practical applications.

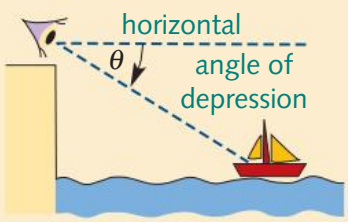
Angle of elevation

The **angle of elevation** is the angle through which you *raise* your line of sight from the horizontal when you are looking *up* at something.



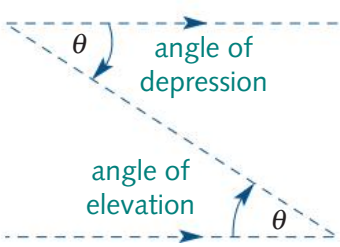
Angle of depression

The **angle of depression** is the angle through which you *lower* your line of sight from the horizontal when you are looking *down* at something.



The diagram shows that the angle of elevation and the angle of depression are alternate angles ('Z' angles), so they are equal.

angle of elevation = angle of depression



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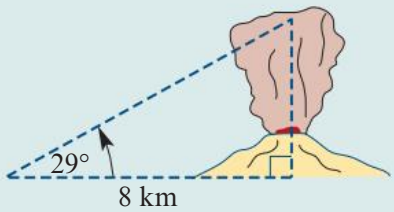
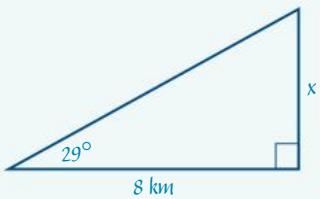
Applications of angles of elevation and depression

Example 9 Applying angle of elevation

A park ranger measured the top of a plume of volcanic ash to be at an angle of elevation of 29°. From her map she noted that the volcano was 8 km away. She calculated the height to the top of the plume to be 4.4 km. Show how she might have done this. Give your answer correct to one decimal place.



Solution



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 29° = \frac{x}{8}$$

$$8 \times \tan 29° = x$$

$$x = 4.434...$$

The height to the top of the ash plume was 4.4 km.

Explanation

Draw a right-angled triangle showing the given information. Label the required height x.

The opposite and adjacent sides are involved, so use tan θ.

Substitute in the known values.

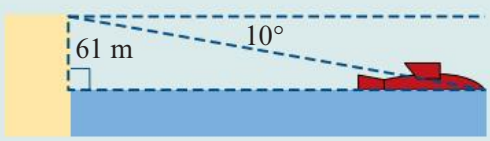
Multiply both sides by 8.

Use your calculator to calculate the value of x.

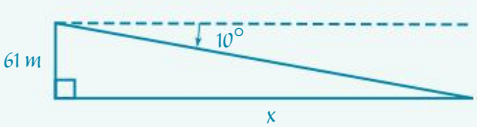
Write your answer correct to one decimal place, including the appropriate units.

Example 10 Applying angle of depression

From the top of a cliff 61 m above sea level, Chen saw a capsized yacht. He estimated the angle of depression to be about 10°. Determine the distance of the yacht from the base of the cliff, rounded to the nearest metre.



Solution



Explanation

Draw a diagram showing the given information. Label the required distance x.

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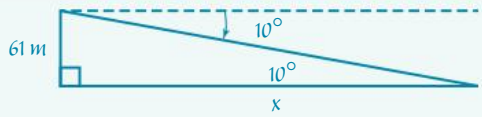


Mark in the angle at the yacht corner of the triangle. This is also 10°, because it and the angle of depression are alternate (or 'Z') angles.

Warning: The angle between the cliff face and the line of sight is *not* 10°.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The opposite and adjacent sides are involved, so use tan θ.

$$\tan 10^\circ = \frac{61}{x}$$

Substitute in the known values.

$$x \times \tan 10^\circ = 61$$

Multiply both sides by x.

$$x = \frac{61}{\tan 10^\circ}$$

Divide both sides by tan 10°.

$$x = 345.948\ldots$$

Evaluate using your calculator.

The yacht was 346 m from the base of the cliff.

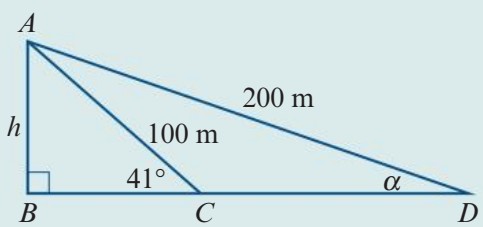
Write your answer rounded to the nearest metre.

Example 11 Application with two right-angled triangles

A cable 100 m long makes an angle of elevation of 41° with the top of a tower.

a Calculate the height, h, of the tower, to the nearest metre.

b Determine the angle of elevation, α, to the nearest degree, that a cable 200 m long would make with the top of the tower.



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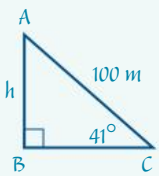
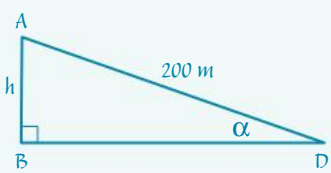
Solution	Explanation
Strategy: Calculate h in triangle ABC, then use this value to determine α in triangle ABD.	
a 	Draw triangle ABC showing the given information and label the unknown.
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	The opposite and hypotenuse are involved, so use $\sin\theta$.
$\sin 41° = \frac{h}{100}$	Substitute in the known values.
$h = 100 \times \sin 41°$	Multiply both sides by 100.
$h = 65.605\ldots$	Evaluate $100 \sin 41°$ using your calculator.
$100 \cdot \sin(41°) \rightarrow h \quad 65.6059$	Store the answer as the value of the variable h for later use.
The height of the tower is 66 m.	Write your answer rounded to the nearest metre.
b 	Draw triangle ABD showing the given and required information.
$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$	The opposite and hypotenuse are involved, so use $\sin\alpha$.
$\sin\alpha = \frac{h}{200}$	Substitute in the known values, remembering that the height of the tower was stored as h in part **a**.
$\alpha = \sin^{-1}\left(\frac{h}{200}\right)$	Write the equation to calculate α.
$100 \cdot \sin(41°) \rightarrow h \quad 65.6059$ $\sin^{-1}\left(\frac{h}{200}\right) \quad 19.1492$	Use your calculator and recall the stored value of h to calculate to evaluate α.
$\alpha = 19.149\ldots$	
The 200 m cable would have an angle of elevation of 19°.	Write your answer rounded to the nearest degree.

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Section Summary

- **Angle of elevation** is the angle when you look up at something.
- **Angle of depression** is the angle when you look down at something.
- Include a clearly labelled diagram as part of your response whenever possible.
- When you are solving for an unknown side, use $\sin\theta$, $\cos\theta$ and $\tan\theta$. When solving for an unknown angle, use the $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$.
- Apply the most appropriate function as a whole by using a stored the value from your calculator. Only round at the end to minimise any rounding errors.

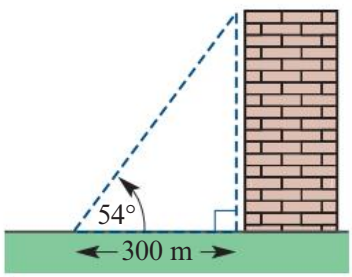
Skill-sheet

Exercise 8E

SF

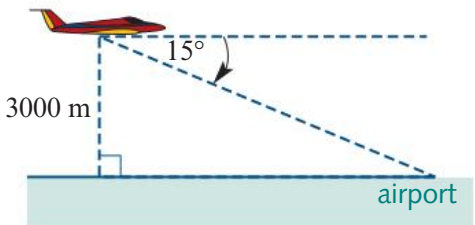
Example 9

1 From 300 m away from the base of a tall building, on level ground, Elise measured the angle of elevation from the ground to the top of the building to be 54°. Determine the height of the building, to the nearest metre.



Example 10

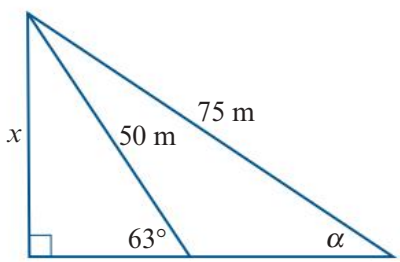
2 The pilot of an aeroplane saw an airport at sea level at an angle of depression of 15°. His altimeter showed that the aeroplane was at a height of 3000 m. Calculate the horizontal distance of the aeroplane from the airport, to the nearest metre.



Example 11

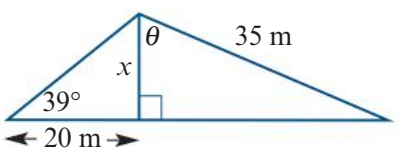
3 **a** Determine the length x, correct to one decimal place.

b Calculate the angle α, to the nearest degree.



4 **a** Calculate the length x, correct to one decimal place.

b Determine the angle θ, to the nearest degree.



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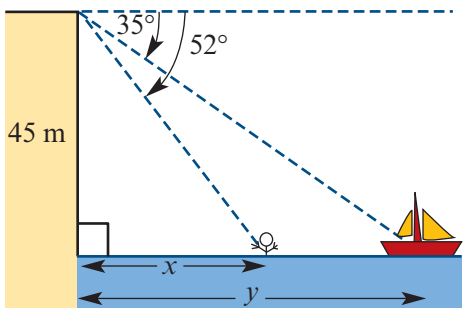
5 The angle of elevation measured from ground level to the top of a tall tree was 41°. The distance of the measurer from the base of the tree was 38 m. Determine the height of the tree. Give your answer to the nearest metre. CF

6 When Darcy looked from the top of a cliff, 60 m high, he noticed Elizabeth at an angle of depression of 20° on the ground below. Determine the distance Elizabeth was from the cliff. Answer correct to one decimal place.

7 From the top of a mountain, I could see a town at an angle of depression of 1.4° across the level plain. From my map I calculated that the town was 10 km away. Calculate the height of the mountain above the plain, to the nearest metre.

8 Determine the angle of elevation to the top of a radio transmitting tower 100 m tall and 400 m from the observer. Answer to the nearest degree.

9 From the top of a cliff 45 m high, an observer looking along an angle of depression of 52° could see a man swimming in the sea. The observer could also see a boat at an angle of depression of 35°. Calculate, to the nearest metre: CF



- **a** the distance, x, of the swimmer from the base of the cliff
- **b** the distance, y, of the boat from the base of the cliff
- **c** the distance from the man to the boat.

10 A police helicopter hovering in a fixed position at an altitude of 500 m moved its spotlight through an angle of depression of 57° onto a lost child. The pilot sighted the rescue team at an angle of depression of 31°. If the terrain was level, determine the distance between the rescue team and the child, rounded to the nearest metre. CU

11 A radio broadcasting station is in a building that is 20 m tall and on the same horizontal road as the local airport. If an aeroplane flies at a height 4 km or lower directly above the station, the broadcast is disrupted. They broadcast their morning show between 5 am and 7 am each morning.

- **a** If the aeroplane leaves the airport at 4:50 am at an angle of elevation of 16° and travels at an average speed of 190 km/h, use mathematical reasoning to justify whether or not it will disrupt the broadcast.
- **b** If the aeroplane leaves the airport at 4:55 am, calculate the minimum speed it must travel so that it does not disrupt the start of the morning show broadcast. Comment on the reasonableness of your answer.

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8F Bearings and navigation

Learning intentions

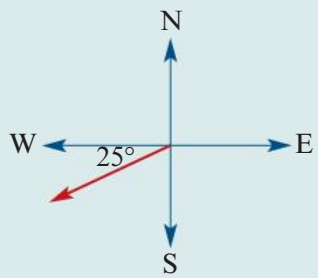
- To be able to define a true bearing.
- To be able to draw and label appropriate diagrams involving true bearings.
- To be able to solve two-dimensional practical problems involving the trigonometry of right-angled triangle and the use of true bearings.

True or three-figure bearings

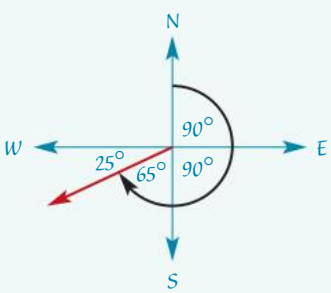
A **true bearing** is the angle measured clockwise from north around to the required direction. True bearings are also called **three-figure bearings** because they are written using three numbers or figures. For example, 090° is the direction measured 90° clockwise from north, better known as east! In this section the old style compass bearings such as S 20°E are not used.

Example 12 Determining three-figure bearings

State the three-figure bearing for the direction shown.



Solution



Method 1

$90° - 25° = 65°$

So $90° + 90° + 65 = 245°$

Explanation

Draw a diagram showing the given information and labelling the unknown.

Calculate the total angles swept out clockwise from north. There is an angle of 90° between each of the four points of the compass.

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Or Method 2 $270° - 25° = 245°$ The three-figure bearing is 245°.	Calculate the total angle swept out clockwise from north that is remaining from due west. Write your answer.

Navigation problems

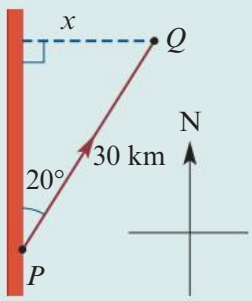
Navigation problems usually involve a consideration of the direction of travel, given as a bearing, and the distance travelled, which can be calculated using right-angled triangles.

Example 13 Navigating using a three-figure bearing

A group of bushwalkers leave point P, which is on a road that runs north–south, and walk for 30 km on a bearing 020° to reach point Q.

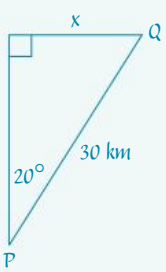
a Determine the shortest distance x from Q back to the road, correct to one decimal place.

b Looking from point Q, determine the three-figure bearing of their starting point.



Solution

a



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 20° = \frac{x}{30}$$

$$30 \times \sin 20° = x$$

$$x = 10.260...$$

The shortest distance to the road is 10.3 km.

Explanation

Show the given and required information in a right-angled triangle. The question asks for the value of x.

The opposite and hypotenuse are involved, so use $\sin\theta$.

Substitute in the known values.

Multiply both sides by 30.

Calculate the value of x using your calculator.

Write your answer correct to one decimal place.

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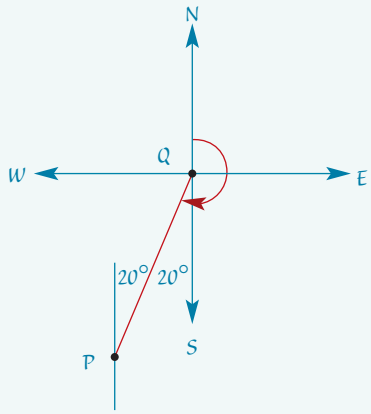
b



$180° + 20° = 200°$.

The three-figure bearing of their starting point is 200°.

The question asks for the bearing of P from Q. Draw a diagram showing the given information and labelling the unknown. Note the alternate angle 20° on the diagram.

Standing at Q, add all the angles when facing north and then turning clockwise to look at P. This gives the three-figure bearing of P when looking from Q. Write your answer.

Section Summary

- A **true bearing** is the angle measured clockwise from north around to the required direction. These are sometimes called **three-figure bearings** and always written using three numbers or figures.
- In navigation, the direction of travel is given as a bearing.
- Include a clearly labelled diagram as part of your response whenever possible and only round at the end to minimise any rounding errors.

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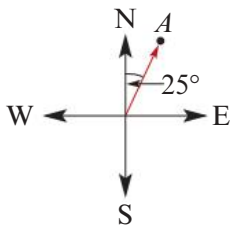
Exercise 8F

SF

Example 12

1 State the three-figure bearing of each of the points A, B, C and D.

a



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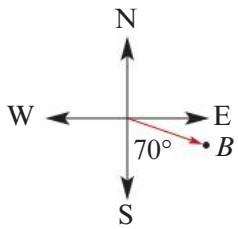

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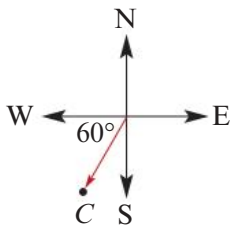
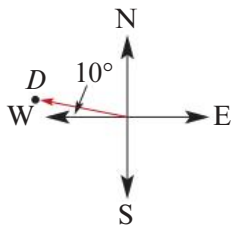
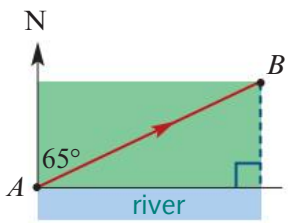


d



Example 13

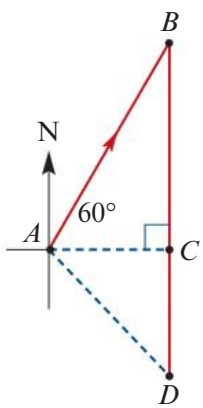
2 Eddie camped overnight at point A beside a river that ran east–west. He walked on a bearing of 065° for 18 km to point B.



a Determine the angle his direction made with the river.

b Calculate the shortest distance from B to the river, correct to two decimal places.

3 A woman walked from point A for 10 km on a bearing of 060° to reach point B. Then she walked for 15 km heading south until she was at point D. Give the following distances correct to one decimal place and directions to the nearest degree.



a State the distances walked from A to B and from B to D.

b Use trigonometry to calculate the distance from A to C.

c Use Pythagoras' theorem to calculate the distance south that she walked from B to C.

d Calculate the distance from C to D.

e Determine the three-figure bearing and distance she would need to walk to return to her starting point.

4 A ship sailed 3 km west, then 2 km south.

a State its three-figure bearings from an observer who stayed at its starting point, correct to the nearest degree.

b For a person on the ship, determine the three-figure bearings looking back to the starting point.

CF

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CF

5 An aeroplane flew 500 km south, then 600 km east. Give its three-figure bearing from its starting point, to the nearest degree.

6 A ship left port and sailed east for 5 km, then sailed north. After some time, an observer at the port could see the ship on a bearing of 050°.

a Calculate the distance north the ship had travelled. Answer correct to one decimal place.

b Looking from the ship, determine the three-figure bearing of the port.

7 A ship left port P and sailed 20 km on a bearing of 230°. It then sailed north for 30 km to reach point C. Give the following distances correct to one decimal place and directions to the nearest degree.

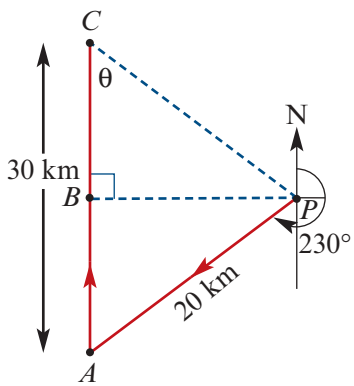
a Calculate the distance AB.

b Calculate the distance BP.

c Calculate the distance BC.

d Calculate the angle θ at point C.

e State the three-figure bearing and distance of the port P from the ship at C.



CU

8 Two cyclists leave from the same starting point. The first cyclist travels due south while the other travels on a true bearing of 223°. After travelling for 28 km, the second cyclist is due west of the first cyclist. Determine the distance that the first cyclist has travelled.

9 Boat A is 23 km from port on a true bearing of 028° and Boat B is 26 km from port on a true bearing of 074°. Boat B is in distress. Determine the bearing that Boat A should travel on to reach Boat B.

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8G The area of a triangle

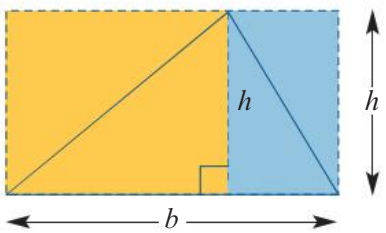
Learning intentions

- To be able to use the formula for calculating the area of a non-right-angled triangle where the lengths of two sides and the angle between them are known.
- To be able to use Heron's rule to calculate the area of any triangle where three side lengths are known.
- To be able to solve practical problems related to the area of a non-right-angled triangle.

Is this section, we will investigate three rules which can be used for calculating the area of a triangle.

Rule 1: Area of a triangle = $\frac{1}{2}bh$

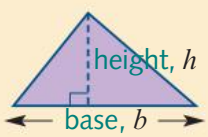
Before working with non-right-angled triangles, it is helpful to remember the previous formula for calculating the area of a triangle from Unit 1. In the diagram, we see that the area of a triangle with a base length b and height h is equal to half the area of the rectangle $b \times h$ that it fits within.

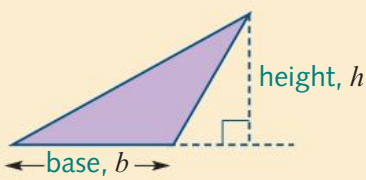


Area of a triangle

$$\text{Area of a triangle} = \frac{1}{2}bh$$

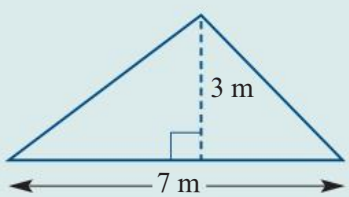
$$A = \frac{1}{2}bh$$





Example 14 Calculating the area of a triangle using $A = \frac{1}{2}bh$

Calculate the area of the triangle shown, correct to one decimal place.



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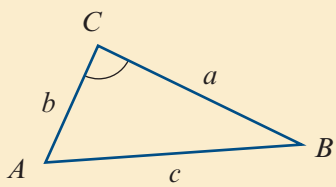
Solution	Explanation
Base length, $b = 7$	As we are given values for the base length and height of the triangle, use
Height, $h = 3$	Area $= \frac{1}{2} \times$ base length $\times$ height.
$A = \frac{1}{2}bh$	
$= \frac{1}{2} \times 7 \times 3$	Substitute the given values.
$= 10.5 \text{ m}^2$	Evaluate.
The area of the triangle is 10.5 m^2.	Write your answer rounded correctly.

Rule 2: Area of a triangle $= \frac{1}{2}bc \sin A$

If the perpendicular height of the triangle is not known or the triangle is not a right-angled triangle, there is another way to solve for the area of the triangle. For this method, the lengths of two sides and the size of the angle between them must be known.

Area of a triangle

$$\text{Area of a triangle} = \tfrac{1}{2}bc \sin A$$
$$\text{Area of a triangle} = \tfrac{1}{2}ac \sin B$$
$$\text{Area of a triangle} = \tfrac{1}{2}ab \sin C$$



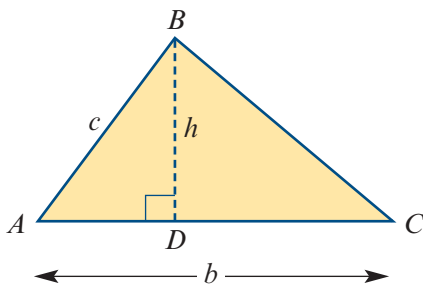
The following explanation shows how this is possible and what is used instead of the perpendicular height to allow this formula to work.

In triangle ABD,

$$\sin A = \frac{h}{c}$$
$$h = c \times \sin A$$

So we can replace h with $c \times \sin A$ in the rule:

$$\text{Area of a triangle} = \frac{1}{2} \times b \times h$$
$$\text{Area of a triangle} = \frac{1}{2} \times b \times c \times \sin A$$



Similarly, using side c or a for the base, we can make a complete set of three rules. Although only one version of the formula appears in the QCAA Formula Book, notice that each version of the rule follows the following pattern.

$$\text{Area of a triangle} = \tfrac{1}{2} \times (\text{product of two sides}) \times \sin(\text{angle between those two sides})$$

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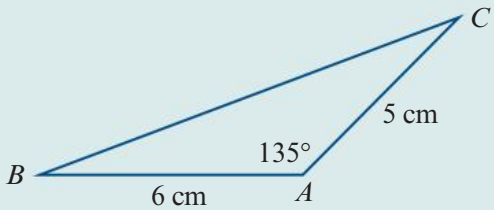
Example 15 Calculating the area of a triangle using $A = \frac{1}{2}bc\sin A$

Calculate the area of the triangle shown, correct to one decimal place.



Solution

$b = 5, c = 6, A = 135°$

Area of triangle $= \frac{1}{2}bc\sin A$

$= \frac{1}{2} \times 5 \times 6 \times \sin 135°$

$= 10.606\ldots$

The area of the triangle is $10.6\ \text{cm}^2$.

Explanation

We are given two sides b, c and the angle A between them, so use

Area of a triangle $= \frac{1}{2}bc\sin A$

Substitute values for b, c and A into the rule.

Evaluate using your calculator.

Write your answer correct to one decimal place.

Rule 3: Heron's rule for the area of a triangle

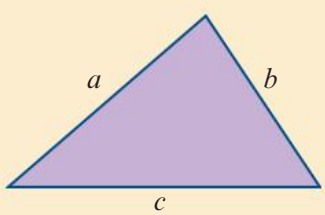
Heron's rule can be used to find the area of any triangle when we know the lengths of the three sides.

Heron's rule for the area of a triangle

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

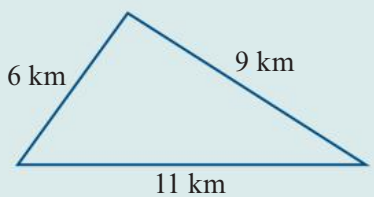
where $s = \dfrac{a+b+c}{2}$

The variable s is called the semi-perimeter because it is equal to half the sum of the sides.



Example 16 Calculating the area of a triangle using Heron's rule

The boundary fences of a farm are shown in the diagram. Calculate the area of the farm, rounded to the nearest square kilometre.



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Solution

Let $a = 6, b = 9, c = 11$.

$$s = \frac{a+b+c}{2}$$

$$s = \frac{6+9+11}{2} = 13$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{13(13-6)(13-9)(13-11)}$$

$$= \sqrt{13 \times 7 \times 4 \times 2}$$

$$= 26.981\ldots$$

The area of the farm, to the nearest square kilometre, is 27 km^2.

Explanation

As we are given the three sides of the triangle, use Heron's rule. Start by calculating s, the semi-perimeter.

Write Heron's rule.

Substitute the values of s, a, b and c into Heron's rule.

Evaluate using your calculator. Write your answer rounded to the nearest square kilometre.

Section Summary

- Area of a triangle $= \frac{1}{2}bh$ is a formula that can be used to calculate the area of a triangle where the length of the base and the perpendicular height are known.
- Area of a triangle $= \frac{1}{2}bc\sin A$ is a formula that can be used to calculate the area of a non-right-angled triangle where the lengths of two sides and the angle between them is known. There are three different versions of the rule.
- Heron's rule can be used to calculate the area of any triangle where all three sides are known.
- Include a clearly labelled diagram as part of your response whenever possible and only round at the end to minimise any rounding errors.

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Exercise 8G

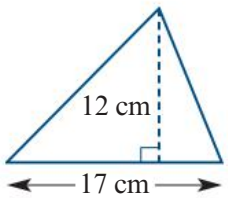
In this exercise, calculate areas correct to one decimal place where necessary.

Calculating areas using $\frac{1}{2}bh$

Example 14

1 Calculate the area of each triangle.

a



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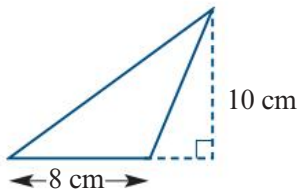

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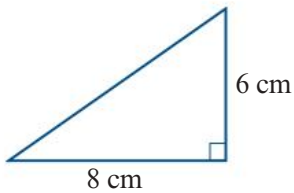

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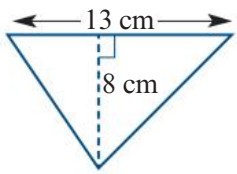

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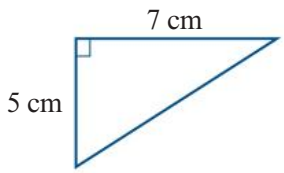

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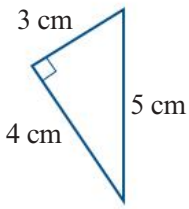
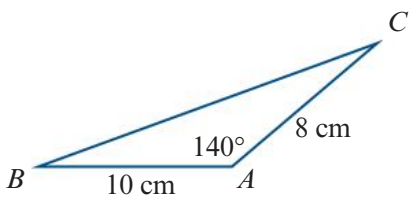


Calculating areas using $A = \frac{1}{2}bc\ \sin A$

Example 15

2 Calculate the areas of the triangles shown.

a



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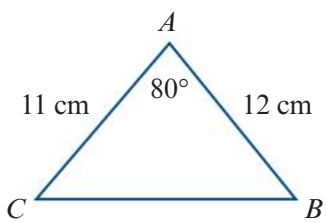

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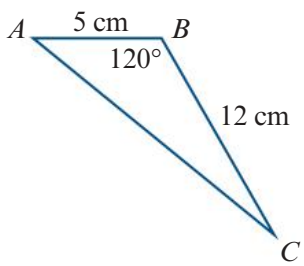

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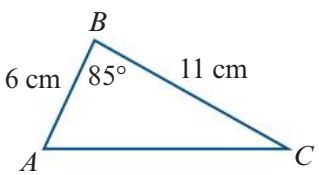

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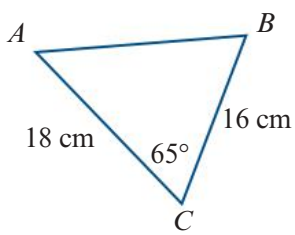

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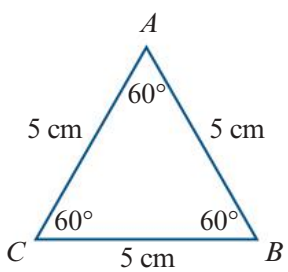

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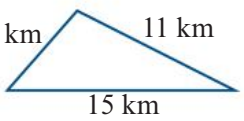
Calculating areas using Heron's rule

Example 16

3 Calculate the area of each triangle.

a



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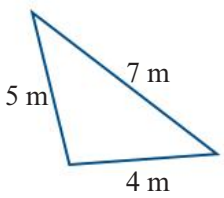

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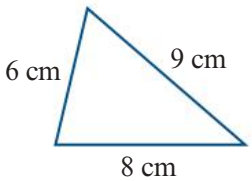

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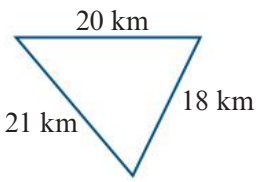

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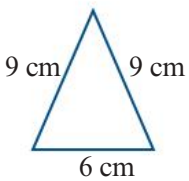

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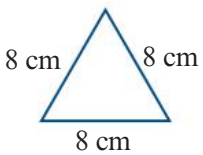


Mixed problems

4 For each triangle below, choose the rule for finding its area from rules **i** to **iv**:

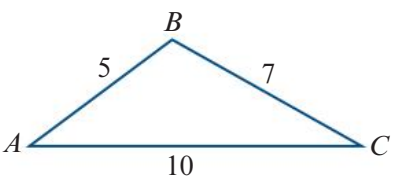
i $\frac{1}{2}$ *base* × *height*

ii $\frac{1}{2}bc \sin A$

iii $\frac{1}{2}ac \sin B$

iv $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$

a



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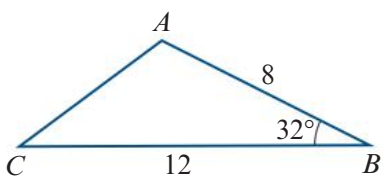

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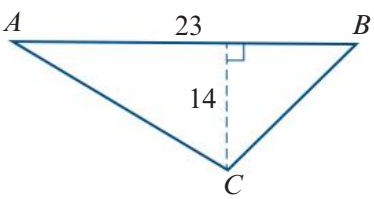

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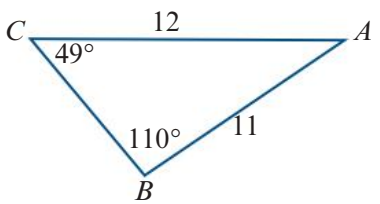

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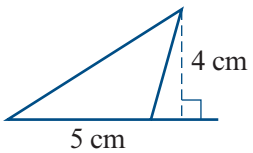
SF

5 Calculate the area of each triangle shown.

a



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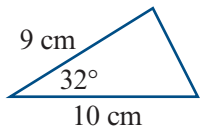

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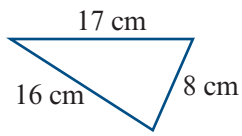

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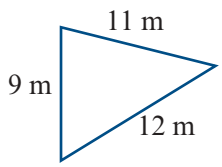

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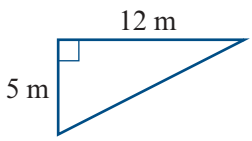

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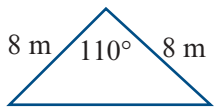

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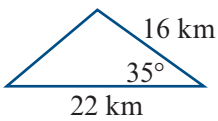

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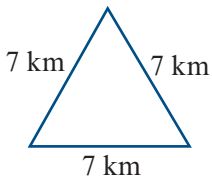

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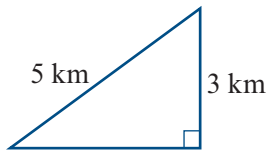


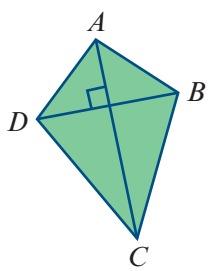
6 Calculate the area of a triangle with a base length of 28 cm and a height of 16 cm.

7 Calculate the area of triangle ABC with side a 42 cm, side b 57 cm and angle C 70°.

8 Calculate the area of a triangle with sides of 16 km, 19 km and 23 km.

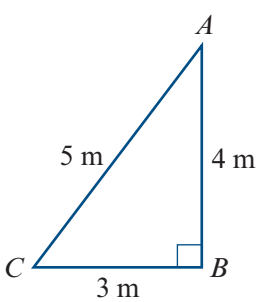
Applications

9 The kite shown is made using two sticks, AC and DB.
The length of AC is 100 cm and the length of DB is 70 cm.
Calculate the area of the kite.



10 Three students stretched a rope loop 12 m long into different triangles. Determine the area of each triangle.

a



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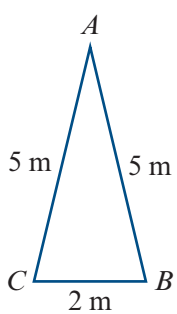

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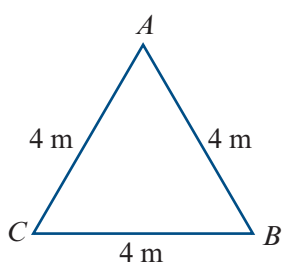

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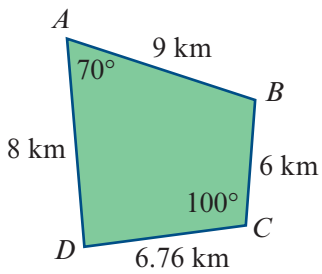
11 A farmer needs to know the area of his property with the boundary fences as shown. Write all answers correct to two decimal places.

Hint: Draw a line from B to D to divide the property into two triangles.

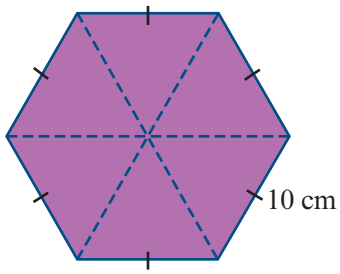


a Calculate the area of triangle ABD.

b Determine the area of triangle BCD.

c State the total area of the property.

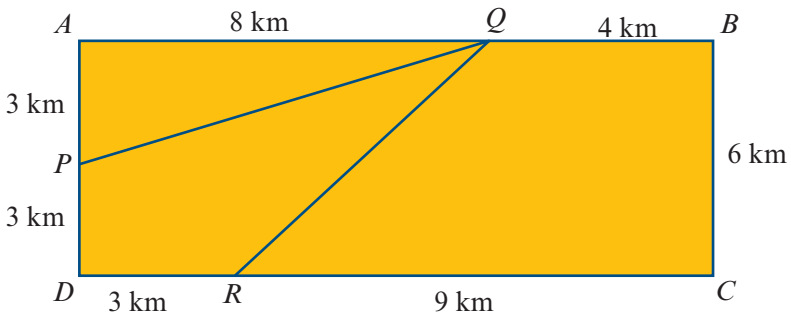
12 A regular hexagon with sides 10 cm long can be divided into six equilateral triangles. (Remember, an equilateral triangle has all sides of equal length.)



a Calculate the area of each triangle.

b State the total area of the hexagon.

13 A large rectangular area of land, $ABCD$ in the diagram, has been subdivided into three regions as shown.



a Calculate the area of:

i region PAQ **ii** region $QBCR$ **iii** region $PQRD$.

b Determine the size of angle PQR, correct to one decimal place.

14 An equilateral triangle has an area of 27.8 m^2. Determine its side length.

15 The side lengths of a triangle, measured in millimetres, are in the ratio 3:4:5. For this triangle, s from Heron's rule is equal to 87. Determine the perimeter and area of this triangle.

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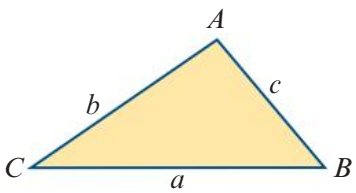
8H The sine rule

Learning intentions

- To be able to understand and use the sine rule with non-right-angled triangles.
- To be able to apply the sine rule when solving two-dimensional problems involving non-right-angled triangles.

Standard triangle notation

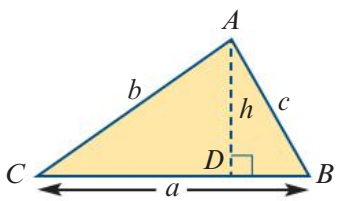
The convention for labelling a non-right-angled triangle is to use the upper case letters A, B and C for the angles at each corner. The sides are named using lower case letters, so that side a is opposite angle A, and so on.



This notation is used for the sine rule. It is also used for cosine rule in the next section. Both rules can be used to determine angles and sides in triangles that do not have a right angle.

How to develop the sine rule

In triangle ABC, show the height h of the triangle by drawing a perpendicular line from A to D on the base of the triangle.



In triangle ADC,	$\sin C = \frac{h}{b}$
So (1) first rule for h:	$h = b \times \sin C$
In triangle ABD,	$\sin B = \frac{h}{c}$
So (2) second rule for h:	$h = c \times \sin B$
Make (1) = (2):	$b \times \sin C = c \times \sin B$
Divide both sides by $\sin C$.	$b = \frac{c \times \sin B}{\sin C}$
Divide both sides by $\sin B$.	$\frac{b}{\sin B} = \frac{c}{\sin C}$

If the triangle was redrawn with side c as the base, then using similar steps we get:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

We can combine the two rules as shown in the following definition.

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The sine rule

In any triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The **sine rule** is used to determine the lengths of the sides and size of the angles in a non-right-angled triangle, given:

- two sides and an angle opposite one of the given sides
- two angles and one side.

Note: If neither of the given angles is opposite the given side, determine the size of the third angle using $A + B + C = 180°$.

The sine rule can take the form of any of these three possible equations:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \frac{a}{\sin A} = \frac{c}{\sin C}$$

Each equation has two sides and the two angles opposite those sides. If we know three of the parts, we can find the fourth.

So if we know two angles and a side opposite one of the angles, we can calculate the side opposite the other angle. Similarly, if we know two sides and an angle opposite one of those sides, we can calculate the angle opposite the other side.

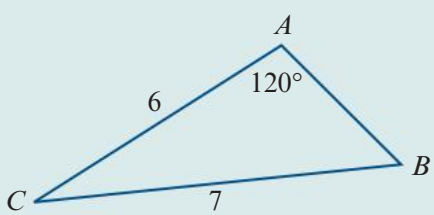
Although we have expressed the sine rule using triangle ABC, for any triangle, such as PQR, the pattern of fractions consisting of 'side / sine of angle' pairs would appear as:

$$\frac{p}{\sin P} = \frac{q}{\sin Q} \qquad \frac{q}{\sin Q} = \frac{r}{\sin R} \qquad \frac{p}{\sin P} = \frac{r}{\sin R}$$

Using the sine rule

Example 17 Using the sine rule given two sides and an opposite angle

Calculate the size of angle B in the triangle, correct to one decimal place.



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Solution

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{7}{\sin 120^\circ} = \frac{6}{\sin B}$$
$$7 \times \sin B = 6 \times \sin 120^\circ$$
$$\sin B = \frac{6 \times \sin 120^\circ}{7}$$
$$B = \sin^{-1}\left(\frac{6 \times \sin 120^\circ}{7}\right)$$
$$B = 47.928\ldots^\circ$$

Angle B is 47.9°.

Explanation

We have the pairs $a = 7$ and $A = 120^\circ$
$b = 6$ and $B = ?$ with only B unknown.

So use $\frac{a}{\sin A} = \frac{b}{\sin B}$.

Substitute in the known values.

Cross-multiply.

Divide both sides by 7.

Write the equation to calculate angle B.

Use your calculator to evaluate the expression for B.

Write your answer correct to one decimal place.

From Example 17 we know that $A = 120^\circ$ and $B = 47.9^\circ$, so we can use the fact that the angles in a triangle add to 180° to determine the size of angle C.

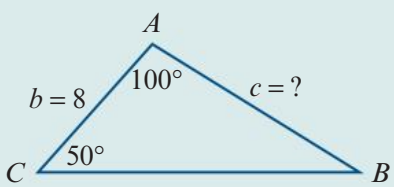
$$A + B + C = 180^\circ$$
$$120^\circ + 47.9^\circ + C = 180^\circ$$
$$167.9^\circ + C = 180^\circ$$
$$C = 180^\circ - 167.9^\circ = 12.1^\circ$$

Since $A = 120^\circ$, $a = 7$ and $C = 12.1^\circ$, we can calculate the length of side c using $\frac{a}{\sin A} = \frac{c}{\sin C}$. The steps are similar to those in the example.

If you are asked to *solve the triangle*, it is expected that your final response includes the values for the sizes of all unknown angles and the lengths of all unknown sides in the given triangle.

Example 18 Using the sine rule given two angles and one side

Calculate the length of side c in the triangle shown, correct to one decimal place.



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Solution

$$A + B + C = 180°$$
$$100° + B + 50° = 180°$$
$$B + 150° = 180°$$
$$B = 30°$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{8}{\sin 30°} = \frac{c}{\sin 50°}$$
$$c = \frac{8 \times \sin 50°}{\sin 30°}$$
$$c = 12.256...$$

Side c is 12.3 units long.

Explanation

Calculate the angle opposite the given side by using $A + B + C = 180°$.

We have the pairs $b = 8$ and $B = 30°$, $c = ?$ and $C = 50°$ with only c unknown.

So use $\frac{b}{\sin B} = \frac{c}{\sin C}$.

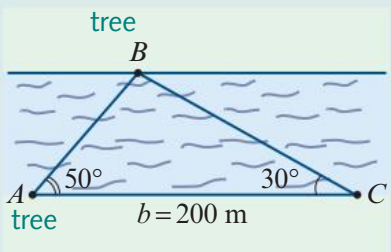
Substitute in the known values.

Multiply both sides by $\sin 50°$.

Use your calculator to evaluate c.
Write your answer correct to one decimal place.

Example 19 Application of the sine rule

Leo wants to tie a rope from a tree at point A to a tree at point B on the other side of the river. He needs to know the length of rope required. When he stood at A, he saw the tree at B at an angle of 50° with the riverbank. He then walked 200 metres east to point C and the tree was seen at an angle of 30° with the bank.



Calculate the length of rope required to reach from A to B, correct to two decimal places.

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Solution	Explanation
	To use the sine rule, we need two angle-side pairs with only one item unknown.
One part of the sine rule equation will be $\frac{c}{\sin C}$.	Angle $C = 30°$ is given. The unknown is the length of the rope, side c.
$\frac{c}{\sin C} = \frac{b}{\sin B}$	We know side $b = 200$ and need to determine angle B to use the sine rule equation.
$A + B + C = 180°$ $50° + B + 30° = 180°$ $B = 100°$	Use $A + B + C = 180°$ to determine angle B.
$\frac{c}{\sin C} = \frac{b}{\sin B}$	We have the pairs $b = 200$ and $B = 100°$, $c = ?$ and $C = 30°$ with only c unknown.
	So use $\frac{c}{\sin C} = \frac{b}{\sin B}$.
$\frac{c}{\sin 30°} = \frac{200}{\sin 100°}$	Substitute in the known values.
$c = \frac{200 \times \sin 30°}{\sin 100°}$	Multiply both sides by $\sin 30°$.
$c = 101.542...$	Evaluate c using your calculator.
The rope must be 101.54 m long.	Write your answer correct to two decimal places.

Tips for solving trigonometry problems

- Always make a rough sketch in pencil as you read the details of a problem. You may need to make changes as you read more, but it is very helpful to have a sketch to guide your understanding.
- In any triangle, the longest side is opposite the largest angle. The shortest side is opposite the smallest angle.
- When you have found a solution, re-read the question and check that your answer fits well with the given information and your diagram.
- Round answers for each part to the required decimal places. Keep more decimal places when the results are used in further calculations; otherwise, rounding off errors accumulate.

Note: Sometimes when the lengths of two sides and the size of the angle opposite one of them are given, there can be multiple solutions to the triangle. This is called *the ambiguous case*. It is not included in the syllabus and hence not considered here.

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Section Summary

- The **sine rule** is used to determine the lengths of sides and sizes of angles in a non-right-angled triangle given:
  - two sides and an angle opposite one of the given sides
  - two angles and one side.
- Include a clearly labelled diagram as part of your response whenever possible and only round at the end to minimise any rounding errors.

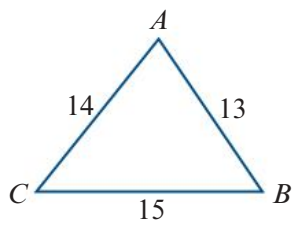
Exercise 8H

In this exercise, calculate lengths correct to two decimal places and angles correct to one decimal place where necessary.

Basic principles

1 In each triangle, state the lengths of sides a, b and c.

a



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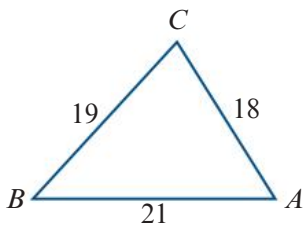

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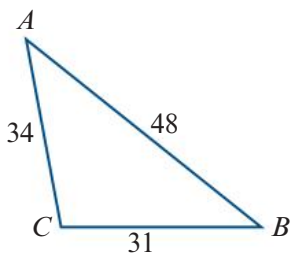
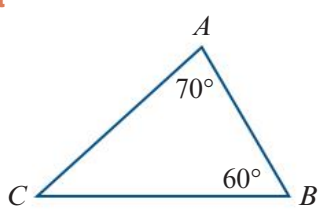


2 Calculate the size of the unknown angle in each triangle. Use $A + B + C = 180°$.

a



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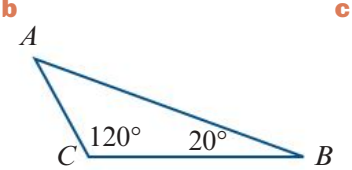

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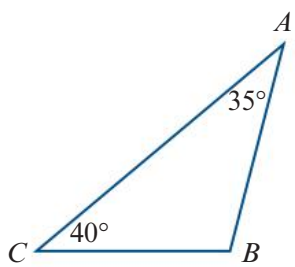

3 In each of the following, a student was using the sine rule to determine an unknown part of a triangle, but was unable to complete the final steps of the solution. Calculate the unknown value.

a $\dfrac{a}{\sin 40°} = \dfrac{8}{\sin 60°}$ **b** $\dfrac{b}{\sin 50°} = \dfrac{15}{\sin 72°}$ **c** $\dfrac{c}{\sin 110°} = \dfrac{24}{\sin 30°}$

d $\dfrac{17}{\sin A} = \dfrac{16}{\sin 70°}$ **e** $\dfrac{26}{\sin B} = \dfrac{37}{\sin 95°}$ **f** $\dfrac{21}{\sin C} = \dfrac{47}{\sin 115°}$

SF

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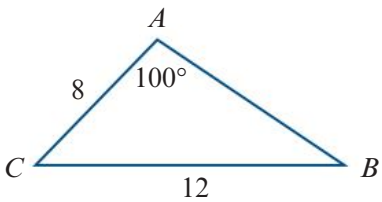
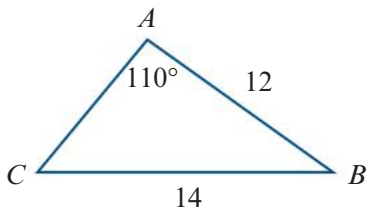
Using the sine rule to calculate angles

Example 17

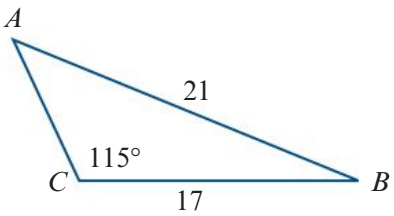
4 **a** Calculate angle B.



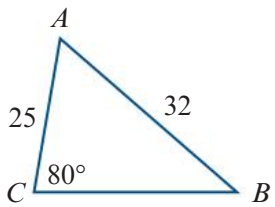
b Determine angle C.



c Determine angle A.



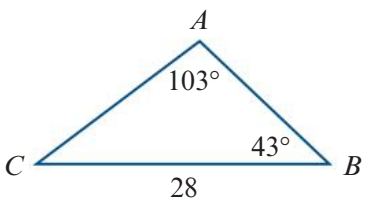
d Calculate angle B.



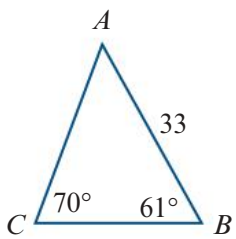
Using the sine rule to calculate sides

Example 18

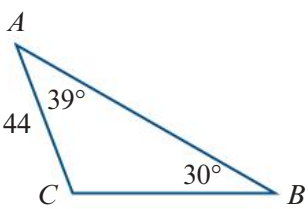
5 **a** Calculate side b.



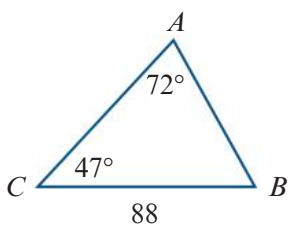
b Determine side b.



c Calculate side a.



d Determine side c.



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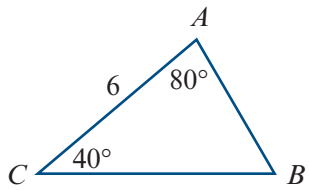
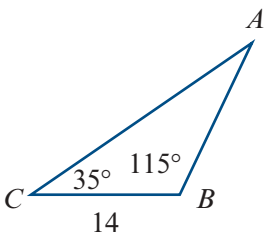
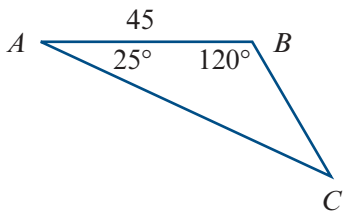
6 **a** Calculate side c.



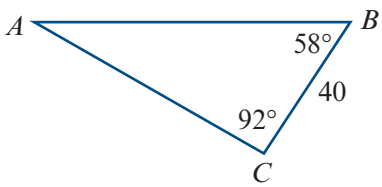
b Determine side c.



c Calculate side b.



d Determine side b.



7 In the triangle ABC, $A = 105°$, $B = 39°$ and $a = 60$. Determine the length of side b.

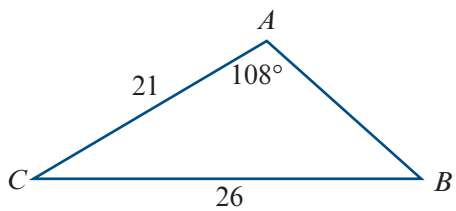
8 In the triangle ABC, $A = 112°$, $a = 65$ and $c = 48$. Calculate the size of angle C.

9 In the triangle ABC, $B = 50°$, $C = 45°$ and $a = 70$. Determine the length of side c.

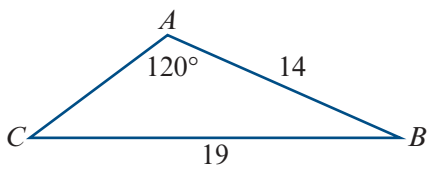
Solving triangles using the sine rule

10 Solve the following triangles. That is, determine the measurements for all the unknown sides and angles in the given triangle.

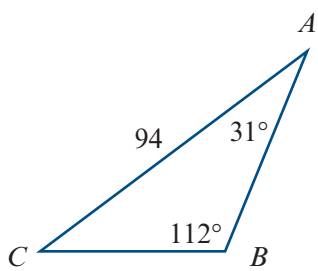
a



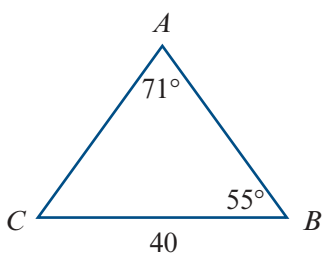
b



c



d



11 In the triangle ABC, $B = 59°$, $C = 74°$ and $c = 41$. Calculate sides a and b and angle A.

12 In the triangle ABC, $a = 60$, $b = 100$ and $B = 130°$. Determine angles A and C and side c.

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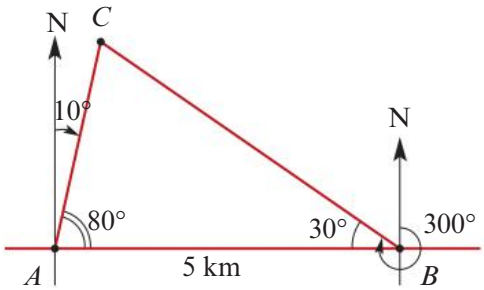
13 In the triangle ABC, $A = 130°$, $B = 30°$ and $c = 69$. Calculate the lengths of sides a and b and the size of angle C.

CF

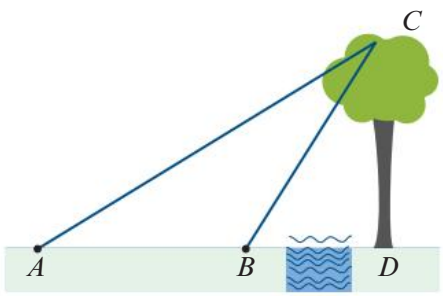
Applications

Example 19

14 A fire-spotter located in a tower at A saw a fire in the direction 010°. Five kilometres to the east of A, another fire-spotter at B saw the fire in the direction 300°. Calculate the distance of the fire from each tower.



15 Holly was recording the heights of tall trees in a State forest to have them registered for protection. A river prevented her from measuring the distance from the base of a particular tree. She recorded the angle of elevation of the top of the tree from point A as 25°. Holly walked 80 m towards the tree and recorded the angle of elevation from point B as 50°.



a Copy the diagram shown and add the given information.

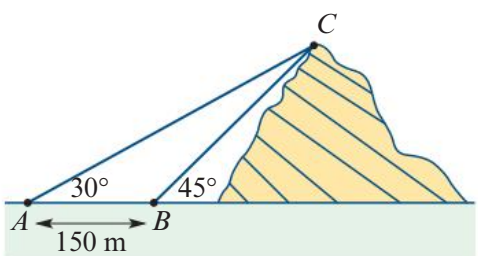
b Determine angle B in triangle ABC.

c Determine angle C in triangle ABC.

d Calculate side b (from A to C).

e Use side b as the hypotenuse in right-angled triangle ADC and the angle at A to calculate the distance DC, the height of the tree.

16 A surveyor standing at point A measured the angle of elevation to the top of the mountain as 30°. She moved 150 m closer to the mountain and at point B measured the angle of elevation to the top of the mountain as 45°. There is a proposal to have a strong cable from point A to the top of the mountain to carry tourists in a cable car. Determine the length of the required cable to one decimal place.



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17 A naval officer sighted the smoke of a volcanic island on a bearing of 044°. A navigator on another ship 25 km due east of the first ship saw the smoke on a bearing of 342°.

a Determine the distance of each ship from the volcano.

b If the ship closest to the volcano can travel at 15 km/h, calculate the amount of time it will take to reach the volcano.

18 An air-traffic controller at airport *A* received a distress call from an aeroplane low on fuel. The bearing of the aeroplane from *A* was 070°. From airport *B*, 80 km north of airport *A*, the bearing of the aeroplane was 120°.

a Identify which airport is closest to the aeroplane.

b Calculate the distance from the aeroplane to the closest airport.

c The co-pilot estimates fuel consumption to be 1525 litres per 100 km. The fuel gauge reads 1400 litres. Use mathematical reasoning to justify whether or not there is enough fuel to reach the destination.

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8I The cosine rule

Learning intentions

- To be able to understand and use the cosine rule with non-right-angled triangles.
- To be able to apply the cosine rule when solving two-dimensional problems involving non-right-angled triangles.

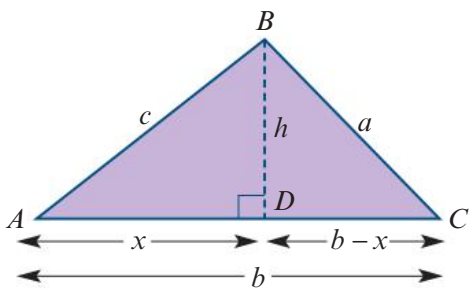
The **cosine rule** can be used to determine the length of a side in any non-right-angled triangle when two sides and the angle between them are known. When you know the three sides of a triangle, the cosine rule can be used to calculate any angle.

How to develop the cosine rule

In the triangle ABC, show the height, h, of the triangle by drawing a perpendicular line from D on the base of the triangle to B.

Let $AD = x$.

As $AC = b$, then $DC = b - x$.



$\cos A = \frac{x}{c}$		In triangle ABD,
$x = c\cos A$	(1)	Multiply both sides by c.
$x^2 + h^2 = c^2$	(2)	Use Pythagoras' theorem in triangle ABD.
$(b - x)^2 + h^2 = a^2$		Use Pythagoras' theorem in triangle CBD.
$b^2 - 2bx + x^2 + h^2 = a^2$		Expand (multiply out) the squared bracket.
$b^2 - 2bc\cos A + x^2 + h^2 = a^2$		Use (1) to replace x with $c\cos A$.
$b^2 - 2bc\cos A + c^2 = a^2$		Use (2) to replace $x^2 + h^2$ with c^2.
$a^2 = b^2 + c^2 - 2bc\cos A$		Reverse and rearrange the equation.
$b^2 = a^2 + c^2 - 2ac\cos B$		This version of the formula is developed by repeating these steps with side c as the base.
$c^2 = a^2 + b^2 - 2ab\cos C$		This version is developed by repeating these steps with side a as the base.

The three versions of the cosine rule can be rearranged to give rules for $\cos A$, $\cos B$ and $\cos C$.

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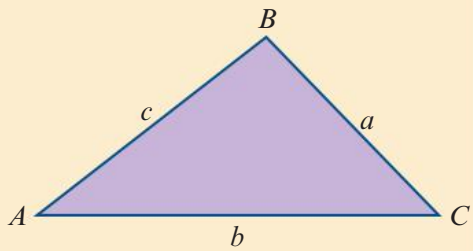
The cosine rule

In any triangle ABC, the cosine rule can be used to determine the lengths of the sides and size of the angles in a non-right-angled triangle.

- When given two sides and the angle between them, the third side can be found using one of the equations:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



- When given three sides, any angle can be found using one of the following rearrangements of the cosine rule:

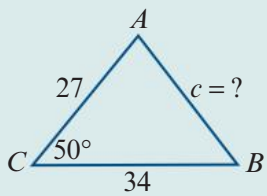
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Note: Not all of these formulas are included in the QCAA formula book. Check carefully to know which formulas are listed and which ones you will need to know.

Using the cosine rule

Example 20 Using the cosine rule given two sides and the angle between them

Calculate the length of side c in the triangle shown, correct to two decimal places.



Solution	Explanation
$a = 34, b = 27, c = ?, C = 50°$	Write down the given values and the required unknown value.
$c^2 = a^2 + b^2 - 2ab\cos C.$	We are given two sides and the angle between them. To calculate side c, use $c^2 = a^2 + b^2 - 2ab\cos C$.
$c^2 = 34^2 + 27^2 - 2 \times 34 \times 27 \times \cos 50°$	Substitute the given values into the rule.
$c = \sqrt{(34^2 + 27^2 - 2 \times 34 \times 27 \times \cos 50°)}$	Take the square root of both sides.
$c = 26.548...$	Use your calculator to evaluate c.
The length of side c is 26.55 units.	Write your answer correct to two decimal places.

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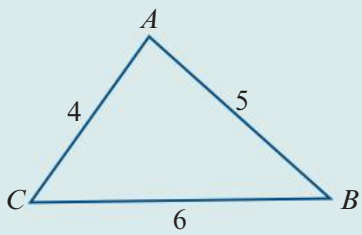
Example 21 Using the cosine rule given three sides

Determine the size of the largest angle, correct to one decimal place, in the triangle shown.



Solution

$a = 6, b = 4, c = 5$

$A = ?$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5}$$

$$A = \cos^{-1}\left(\frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5}\right)$$

$A = 82.819...°$

The size of the largest angle is 82.8°.

Explanation

Write down the given values.

The largest angle is always opposite the longest side, so calculate the size of angle A.

We are given three sides. To calculate the size of angle A, use $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

Substitute the given values into the rule.

Write the equation to determine size of angle A.

Use your calculator to evaluate the expression for A. Make sure that your calculator is in DEGREE mode.

Tip: In your calculator, put brackets around all the terms in the numerator. Also put brackets around all of the terms in the denominator.

Write your answer correct to one decimal place.

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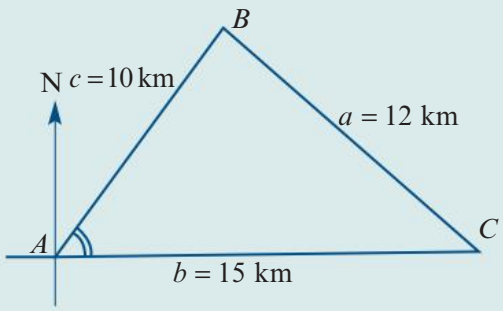
Example 22 Application of the cosine rule

A yacht left point A and sailed 15 km east to point C. Another yacht also started at point A and sailed 10 km to point B, as shown in the diagram. The distance between points B and C is 12 km.



a Determine the angle between their directions as they left point A. Give the angle correct to two decimal places.

b Determine the bearing of point B from the starting point A to the nearest degree.

Solution

a $a = 12, b = 15, c = 10$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

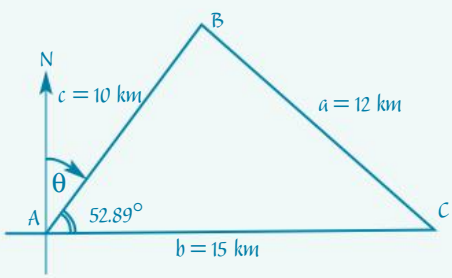
$$\cos A = \frac{15^2 + 10^2 - 12^2}{2 \times 15 \times 10}$$

$$A = \cos^{-1}\left(\frac{15^2 + 10^2 - 12^2}{2 \times 15 \times 10}\right)$$

$$A = 52.891° \ldots$$

The angle was 52.89°.

b



$$\theta + 52.89° = 90°$$

$$\theta = 90° - 52.89°$$

$$= 37.11°$$

The bearing of point B from point A is 037°.

Explanation

Write the given values.

Write the form of the cosine rule for the required angle, A.

Substitute the given values into the rule.

Write the equation to calculate angle A.
Use your calculator to evaluate the expression for A.
Give the answer to two decimal places.

The bearing, θ, of point B from the starting point A, is measured clockwise from north.

Consider the angles in the right-angle at point A.
Determine the value of θ.

Write your answer correct to the nearest degree.

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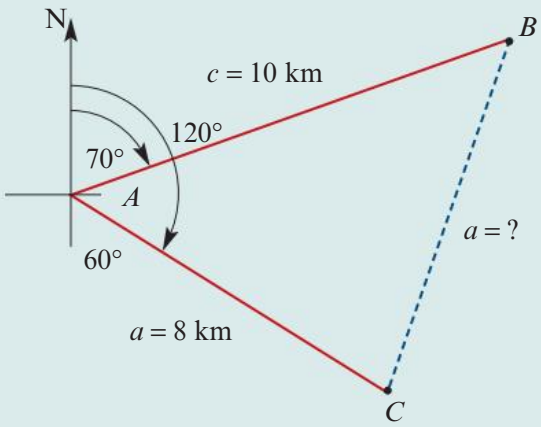
Example 23 Application of the cosine rule involving bearings

A bushwalker left his base camp and walked 10 km in the direction 070°. His friend also left the base camp but walked 8 km in the direction 120°.

a Determine the angle between their paths.

b Calculate the distance between them when they stopped walking. Give your answer correct to two decimal places.



Solution

a $60° + A + 70° = 180°$

$A + 130° = 180°$

$A = 50°$

The angle between their paths was 50°.

b $a = ?, b = 8, c = 10, A = 50$

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50°$

$a = \sqrt{(8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50°)}$

$a = 7.820...$

The distance between them was 7.82 km.

Explanation

Angles lying on a straight line add to 180°.

Write your answer.

Write down the known values and the required unknown value.

We have two sides and the angle between them. To calculate side a use $a^2 = b^2 + c^2 - 2bc\cos A$.

Substitute in the known values.

Take the square root of both sides.

Use your calculator to evaluate the expression for a.

Write your answer correct to two decimal places.

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Section Summary

- The **cosine rule** is used to determine the lengths of sides and the size of angles in a non-right-angled triangle given:
  - two sides and the angle between them
  - three sides.
- Include a clearly labelled diagram as part of your response whenever possible and only round at the end to minimise any rounding error.
- Not all formulas are included in the QCAA formula book. Check carefully to know which formulas are listed and which ones you will need to know.

Skill-sheet

Exercise 8I

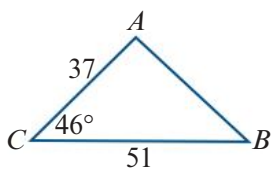
Calculate lengths correct to two decimal places and angles correct to one decimal place.

Using the cosine rule to determine sides

Example 20

1 Calculate the length of the unknown side in each triangle.

a



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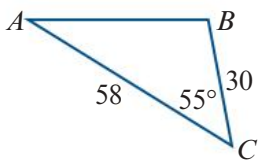

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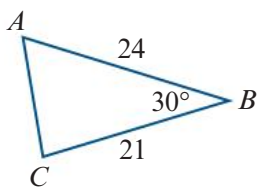

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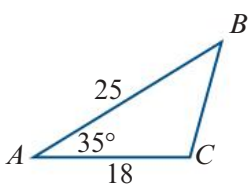

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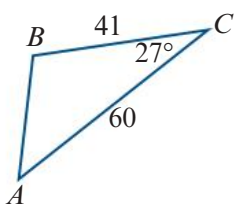

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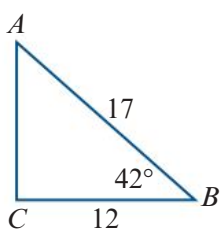

2 In the triangle ABC, $a = 27, b = 22$ and $C = 40°$. Calculate side c.

3 In the triangle ABC, $a = 18, c = 15$ and $B = 110°$. Calculate side b.

4 In the triangle ABC, $b = 42, c = 38$ and $A = 80°$. Determine side a.

SF

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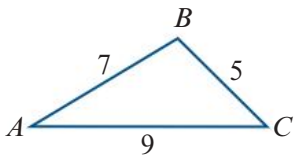
Using the cosine rule to determine angles

5 Determine the size of angle A in each triangle.

a



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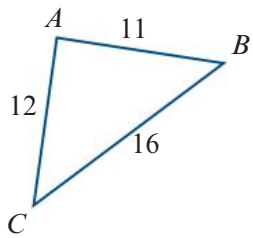

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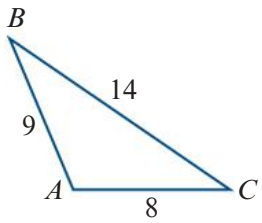

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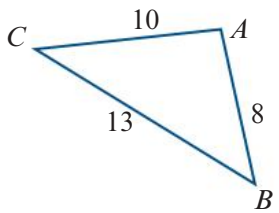

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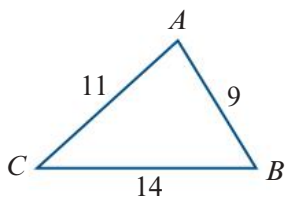

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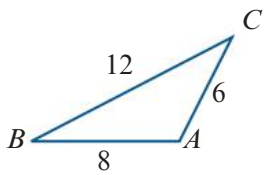


6 In the triangle ABC, $a = 9$, $b = 10$ and $c = 11$. Determine angle A.

7 In the triangle ABC, $a = 31$, $b = 47$ and $c = 52$. Determine angle B.

8 In the triangle ABC, $a = 66$, $b = 29$ and $c = 48$. Determine angle C.

Example 21

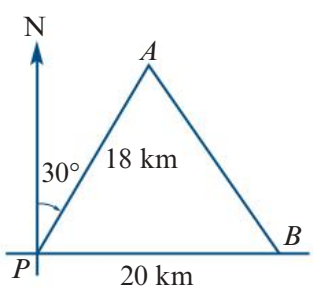
9 Solve for the size of the smallest angle in the triangle ABC, with $a = 120$, $b = 90$ and $c = 105$.

10 In the triangle ABC, $a = 16$, $b = 21$ and $c = 19$. Determine the size of the largest angle.

Applications

11 A farm has a triangular shape with fences of 5 km, 7 km and 9 km in length. Determine the size of the smallest angle between the fences. The smallest angle is always opposite the smallest side.

12 A ship left the port P and sailed 18 km on a bearing of 030° to point A. Another ship left port P and sailed 20 km east to point B. Calculate the distance from A to B, correct to one decimal place.



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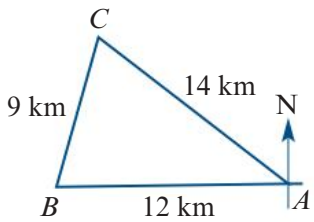
CF

Example 22

13 A bushwalker walked 12 km west from point A to point B. Her friend walked 14 km from point A to point C, as shown in the diagram. The distance from B to C is 9 km.

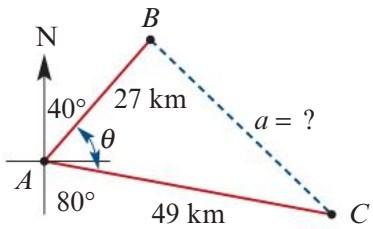


a Determine the angle at A between the paths taken by the bushwalkers, correct to one decimal place.

b Determine the bearing of point C from A. Give the bearing correct to the nearest degree.

Example 23

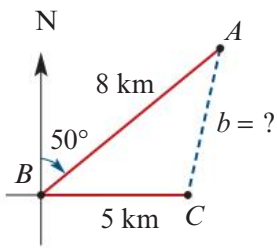
14 A ship left port A and travelled 27 km on a bearing of 40° to reach point B. Another ship left the same port and travelled 49 km on a bearing of 100° to arrive at point C.



a Determine the angle θ between the directions of the two ships.

b Calculate the distance between the two ships when they stopped.

15 A battleship B detected a submarine A on a bearing of 050° and at a distance of 8 km. A cargo ship C was 5 km due east of the battleship. Calculate the distance between the submarine and the cargo ship.



16 From a lookout tower, A, a fire-spotter saw a bushfire, B, at a distance of 15 km on a bearing of 315°. A township, C, was located 12 km on a bearing of 265° from the tower. Determine the distance between the bushfire and the township.

CU

17 Passengers, who are travelling in a car west along a road that runs east-west, see a mountain 9 km away on a bearing of 290°. After they have travelled a further 5 km west along the road, determine their distance from the mountain.

18 At a point, A, on the ground, the angle of elevation to the top of a radio transmission tower is 60°. From that point, a 40 m cable was attached to the top of the tower. At a point, B, a further 10 m away from the base of the tower, another cable is to be pegged to the ground and attached to the top of the tower. Determine the length that is required for the second cable.

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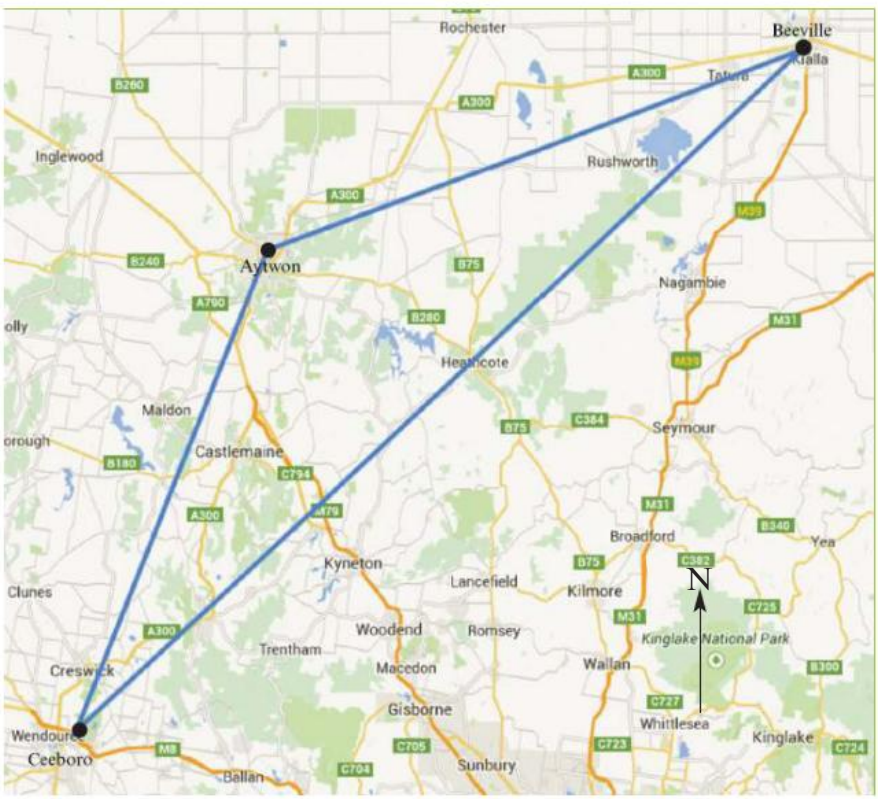
8J Further problem-solving and modelling

Learning intentions

- To be able to model a practical situation using non-right-angled triangles.
- To be able to calculate the area of non-right-angled triangles in practical situations.
- To be able to apply the sine rule to solve practical applications.
- To be able to apply the cosine rule to solve practical applications.
- To be able to gain experience with complex and unfamiliar questions.

Exercise 8J

Alice operates a charter flight service. She is planning a round trip from her base at Aytown to Beeville, Ceeboro and then returning to Aytown.



a The direct distance from Aytown to Ceeboro is known to be 92 km. By measuring the lengths of the blue lines between the three cities, calculate the direct distance from:

i Aytown to Beeville

ii Beeville to Ceeboro.

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b Her Piper Archer TX has a range of 522 nautical miles. One nautical mile equals 1.85 km. The fuel gauge indicates the tank is one-third full. Use mathematical reasoning to justify whether there is enough fuel to complete the round trip.

c The average cruising speed is 147 miles per hour. One mile equals 1.61 km. Calculate the total flight time for the round trip.

d Fuel consumption cost is 95 cents per km. Calculate her total fuel cost for the trip.

e **i** If a vertical line points north, use the map and a protractor to determine the bearing that Alice must fly from Aytown to Beeville.

ii Measure the bearing that Alice must fly from Beeville to Ceeboro.

iii Measure the bearing that Alice must fly from Ceeboro to Aytown.

f Give the direction and distance flight information that Alice will need to use on the Beeville to Ceeboro stage of the circuit.

g Use your answers from part **e** to determine the angle between the flight paths to and from Beeville.

h Show how a trigonometry distance rule can be applied to the Aytown–Beeville and Beeville–Ceeboro distances with the angle at Beeville to find the Ceeboro–Aytown distance. Use mathematical reasoning to confirm that the distance is approximately 92 km.

i Calculate the area enclosed by the flight paths.

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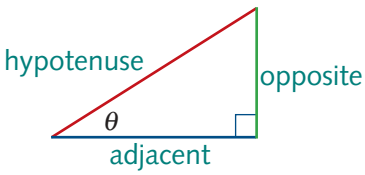
Key ideas and chapter summary

Right-angled triangles

Naming the sides of a right-angled triangle

The **hypotenuse** is the longest side and is always opposite the right angle (90°).
The *opposite* side is directly opposite the angle θ (the angle being considered).
The *adjacent* side is beside angle θ and runs from θ to the right angle.



Trigonometric ratios

The **trigonometric ratios** are sin θ, cos θ and tan θ:

$$\textbf{sin } \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \textbf{cos } \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \textbf{tan } \theta = \frac{\text{opposite}}{\text{adjacent}}$$

SOH-CAH-TOA

This helps you to remember the trigonometric ratio rules.

Degree mode

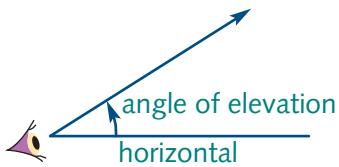
Make sure your calculator is in DEGREE mode when doing calculations with trigonometric ratios.

Applications of right-angled triangles

Always draw well-labelled diagrams showing all known sides and angles. Also label any unknown sides or angles that need to be calculated.

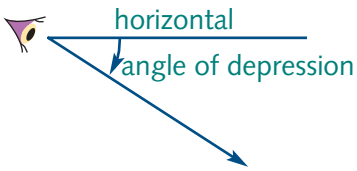
Angle of elevation

The **angle of elevation** is the angle through which you *raise* your line of sight from the horizontal, looking *up* at something.



Angle of depression

The **angle of depression** is the angle through which you *lower* your line of sight from the horizontal, looking *down* at something.



Angle of elevation = angle of depression

The angles of elevation and depression are alternate ('Z') angles so they are equal.

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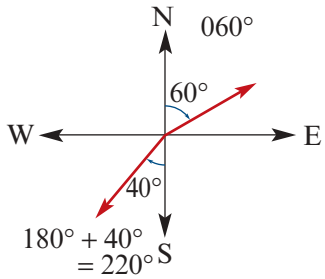
True bearings

True bearings are measured clockwise from north and always given with three digits, e.g. 060°, 220°.



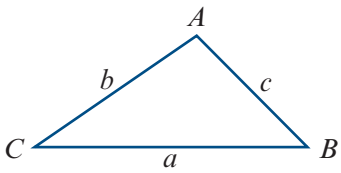
Navigation

Navigation problems usually involve consideration of the *direction* of travel, given as a bearing, and the *distance* travelled, which can be calculated using right-angled triangles.

Non-right-angled triangles

Labelling a non-right-angled triangle

Side a is always opposite angle A, side b is opposite angle B, and side c is opposite angle C.



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use the **sine rule** when given:

- two sides and an angle opposite one of those sides
- two angles and one side.

If neither angle is opposite the given side, find the third angle using $A + B + C = 180°$.

Cosine rule

The **cosine rule** has three versions.
When given two sides and the angle between them, use the rule that starts with the unknown side:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

When given three sides, any angle can be found using one of the following rearrangements of the cosine rule:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

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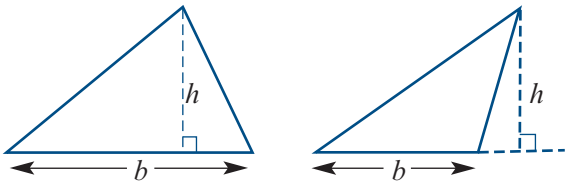
Area of a triangle

- Use the formula area of triangle $= \frac{1}{2}bh$ if the base length and height of the triangle are known.



- Use the formula area of triangle $= \frac{1}{2}bc\sin A$ if two sides and the angle between them are known.
- Use Heron's rule if the lengths of the three sides of the triangle are known.

Heron's rule

Area of triangle $= A = \sqrt{s(s-a)(s-b)(s-c)}$
where $s = \frac{a+b+c}{2}$ and a, b and c are the sides of the triangle.

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Review

Skills checklist

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills.

8A	1	I can name the sides of a right-angled triangle.	☐
		See Example 1 and Exercise 8A Question 1.	
8A	2	I can use my calculator to calculate the values of trigonometric ratios.	☐
		See Example 2 and Exercise 8A Question 3.	
8B	3	I can use trigonometric ratios to calculate an unknown side.	☐
		See Example 3 and Exercise 8B Question 1.	
8B	4	I can calculate an unknown side in the denominator of the trigonometric ratio.	☐
		See Example 4 and Exercise 8B Question 2.	
8C	5	I can use trigonometric ratios to calculate an unknown angle.	☐
		See Example 5 and Exercise 8C Question 1.	
8C	6	I can use trigonometric ratios to calculate an unknown angle given two sides.	☐
		See Example 6 and Exercise 8C Question 2.	
8D	7	I can apply trigonometric ratios to solve for an unknown length.	☐
		See Example 7 and Exercise 8D Question 1.	
8D	8	I can apply trigonometric ratios to solve for an unknown angle.	☐
		See Example 8 and Exercise 8D Question 2.	
8E	9	I understand and can apply angle of elevation.	☐
		See Example 9 and Exercise 8E Question 1.	
8E	10	I understand and can apply angle of depression.	☐
		See Example 10 and Exercise 8E Question 2.	
8E	11	I can apply angle of elevation or depression to two right-angled triangles.	☐
		See Example 11 and Exercise 8E Question 3.	
8F	12	I understand and can apply true bearings.	☐
		See Example 12 and Exercise 8F Question 1.	

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8F	13	I understand and can apply true bearings to navigation.	☐

See Example 13 and Exercise 8F Question 2.

8G	14	I can calculate the area of a triangle using the formula $A = \frac{1}{2}bh$.	☐

See Example 14 and Exercise 8G Question 1.

8G	15	I can calculate the area of a triangle using the formula Area $= \frac{1}{2}bc\sin A$.	☐

See Example 15 and Exercise 8G Question 2.

8G	16	I can calculate the area of a triangle using Heron's rule.	☐

See Example 16 and Exercise 8G Question 3.

8H	17	I understand and can apply the sine rule when given two sides and an opposite angle.	☐

See Example 17 and Exercise 8H Question 4.

8H	18	I understand and can apply the sine rule when given two angles and one side.	☐

See Example 18 and Exercise 8H Question 5.

8H	19	I can apply the sine rule to practical situations.	☐

See Example 19 and Exercise 8H Question 14.

8I	20	I understand and can apply the cosine rule when given two sides and the angle between them.	☐

See Example 20 and Exercise 8I Question 1.

8I	21	I understand and can apply the cosine rule when given three sides.	☐

See Example 21 and Exercise 8I Question 9.

8I	22	I can apply the cosine rule to practical situations.	☐

See Example 22 and Exercise 8I Question 13.

8I	23	I can apply the cosine rule to practical situations involving bearings.	☐

See Example 23 and Exercise 8I Question 14.

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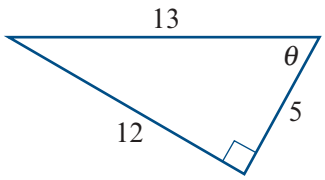
Review

Multiple-choice questions

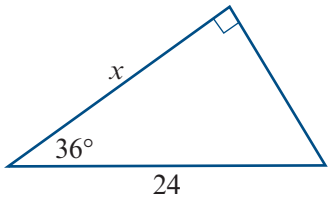
1 In the triangle shown, sin θ is:



A $\frac{5}{12}$ **B** $\frac{5}{13}$

C $\frac{12}{5}$ **D** $\frac{12}{13}$

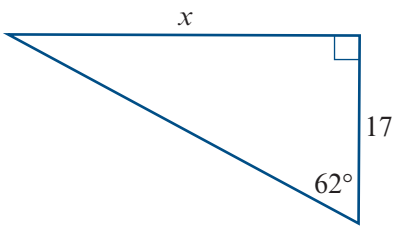
2 The length x is given by:



A $24 \sin 36°$ **B** $24 \cos 36°$

C $\frac{\sin 36°}{24}$ **D** $\frac{\cos 36°}{24}$

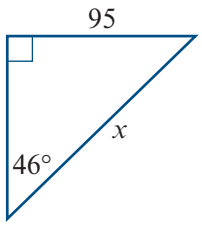
3 To calculate the length x we should use:



A $17 \sin 62°$ **B** $17 \tan 62°$

C $\frac{\sin 62°}{17}$ **D** $\frac{\tan 62°}{17}$

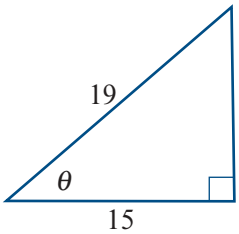
4 The side x is given by:



A $95 \sin 46°$ **B** $95 \tan 46°$

C $\frac{95}{\sin 46°}$ **D** $\frac{95}{\cos 46°}$

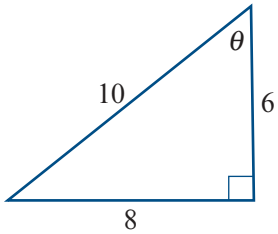
5 To determine the angle θ we need to use:



A $\cos^{-1}\left(\frac{15}{19}\right)$ **B** $\cos\left(\frac{15}{19}\right)$

C $15 \sin(19)$ **D** $19 \cos(15)$

6 The angle θ, correct to one decimal place, is:



A 36.9° **B** 38.7°

C 53.1° **D** 53.3°

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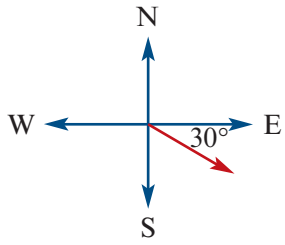
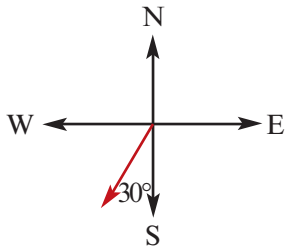
7 The direction shown has the true bearing:

A 030° **B** 060°
C 120° **D** 210°



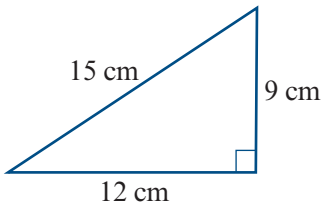
8 The direction shown has the true bearing:

A 030° **B** 060°
C 120° **D** 210°



9 The area of the triangle shown is:

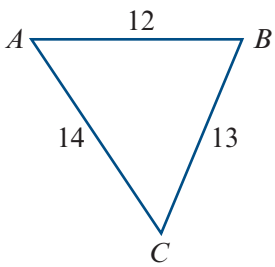
A 36 cm^2 **B** 54 cm^2
C 67.5 cm^2 **D** 90 cm^2



10 To determine angle C, we should use the rule:

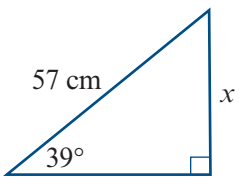
A $\sin C = \dfrac{\text{opposite}}{\text{hypotenuse}}$ **B** $\cos C = \dfrac{a^2 + c^2 - b^2}{2ac}$

C $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$ **D** $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

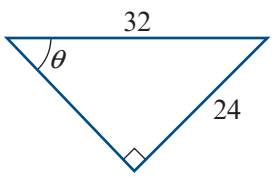


Short-response questions

1 **a** Calculate side x.



b Determine θ.



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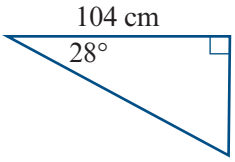
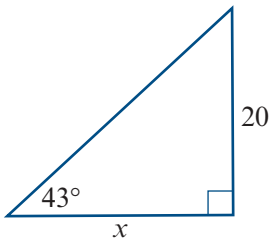
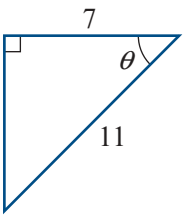
c Determine the length of the hypotenuse.



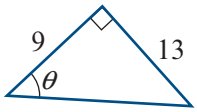
d Determine the length of side x.



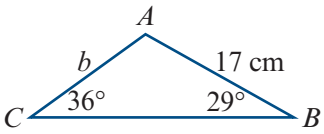
e Calculate the size of θ.



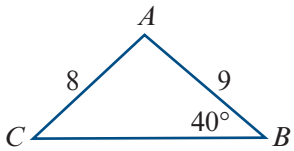
f Determine θ.



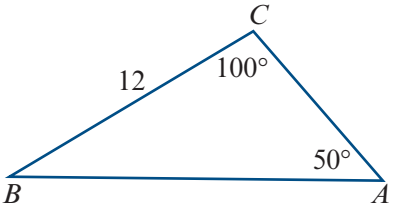
2 **a** Calculate the length of side b.



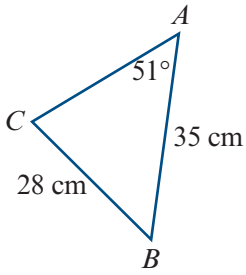
b Determine angle C.



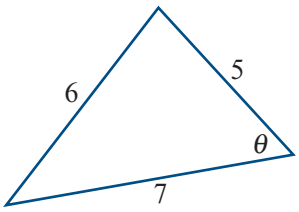
c Calculate the length of side c.



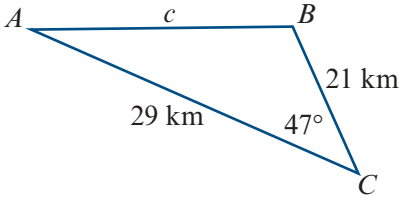
d Determine angle C.



e Calculate angle θ.



f Determine the length of the third unknown side.



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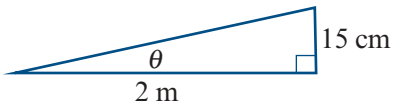
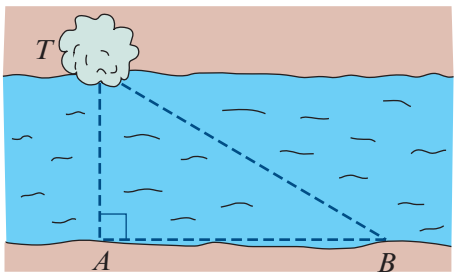
3 In a right-angled triangle for which $\cos\theta = \frac{72}{97}$ and $\tan\theta = \frac{65}{72}$, determine $\sin\theta$.

4 A road rises 15 cm for every 2 m travelled horizontally. Determine the angle of slope, θ, to the nearest degree.



5 A car travelled 30 km east, then travelled 25 km on a bearing of 070°. Determine the distance the car was from its starting point. Answer correct to two decimal places.

6 Tim was standing at point A and looking at a tree, T, directly opposite him on the far bank of the river. He walked 100 m along the riverbank to point B and noticed that his line of sight to the tree now made an angle of 27° with the riverbank. Answer the following correct to two decimal places.



- **a** Calculate the width of the river.
- **b** Calculate the distance from point B to the tree.
- **c** Standing at B, Tim measured the angle of elevation to the top of the tree to be 18°. Determine the height of the tree.

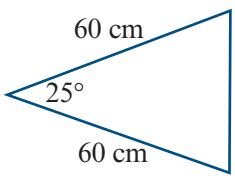
7 A yacht, P, left port and sailed 45 km on a bearing of 290°. Another yacht, Q, left the same port but sailed for 54 km on a bearing of 040°.

- **a** Determine the angle between their directions.
- **b** Calculate the distance between them at that stage, correct to two decimal places.

8 One group of bushwalkers left a road running north–south to walk along a bearing of 060°. A second group of walkers started at a point 3 km further north. They walked on a bearing of 110°. The two groups met at point C, where their paths intersected.

- **a** Determine the angle at which their paths met.
- **b** Calculate the distance walked by each group, correct to two decimal places.
- **c** The bushwalkers decided to return to the road by walking back along the path that the second group of walkers had taken. Determine the bearing they should follow.

9 A pennant flag has the dimensions shown. Calculate the area of the flag. Answer correct to one decimal place.



10 Calculate the area of an equilateral triangle with sides of 8 m, correct to one decimal place.

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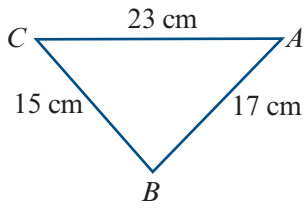
11 Calculate the size of the smallest angle in the triangle shown, correct to one decimal place.



CF

12 A triangular shade cloth must have sides of 5 m, 6 m and 7 m to cover the required area of a children's playground. The manufacturer charges according to the area of the shade cloth at $29 per square metre. Calculate the total cost of this shade cloth.

CU

13 A plane leaves Airport A on a bearing of 312°. Several minutes later the plane is spotted from Airport B at a bearing of 027°. Airport B is due west of Airport A and the two airports are 46 km apart. Determine the distance the plane had flown when it was spotted by Airport B.

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Chapter 9

Matrices and matrix arithmetic

Chapter questions

UNIT 2: APPLICATIONS OF LINEAR EQUATIONS AND TRIGONOMETRY, MATRICES AND UNIVARIATE DATA

Topic 3: Matrices

- What is a matrix?
- How is the order of a matrix defined?
- How are the positions of the elements of a matrix specified?
- What are the rules for adding and subtracting matrices?
- How do we multiply a matrix by a scalar?
- What is the method for multiplying a matrix by another matrix?
- How is the power of a matrix evaluated?
- What are the properties of an identity matrix?
- How can communications be represented by matrices?

Matrices can be used to organise, store and display information such as databases, solve sets of simultaneous equations, find optimal solutions in business, analyse networks, transform shapes in geometry, encode information and devise the best strategies in game theory. We will explore some of these applications while learning the basic theory of matrices.

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9A The basics of a matrix

Learning intentions

- To be able to determine the order, or size, of a matrix.
- To be able to identify the elements in a matrix.
- Recognise row, column and square matrices.
- To be able to interpret the information in a matrix as a model of an application.

A **matrix** (plural matrices) is a rectangular array, or group, of elements displayed in rows and columns. These elements are usually numbers. Matrices are usually named using capital letters such as A or B.

Size of a matrix

The **size** (or order) **of a matrix** with m rows and n columns is an $m \times n$ matrix.

It is written as:

number of rows × number of columns.

$$\begin{matrix} \textit{row } 1 \\ \textit{row } 2 \end{matrix} \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix}$$

Think: 'rows in a cinema'

$$\begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix}$$
$$\textit{column } 1 \quad \textit{column } 2 \quad \textit{column } 3$$

Think: 'columns of the Parthenon'

In writing down the size of a matrix, the number of rows is always given first, then the number of columns. Rows first, then columns. **Remember:** When you walk into a cinema, you go to your row first.

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For example, in matrix A shown on the right, the size of this matrix is 2×3. It has 2 rows and 3 columns. It is called a 'two by three' matrix.

$$A = \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix}$$

Two matrices are the same size if they have the same number of rows and columns.

Elements of a matrix

The numbers within a matrix are called its **elements**.

In matrix A, a_{ij} is the element in *row i*, *column j*.

For example, in matrix A shown on the right,

- element a_{13} is in row 1, column 3 and its value is 4
- element a_{22} is in row 2, column 2 and its value is 7.

$$A = \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix}$$

As an example of how to store information in a matrix, suppose that a market stall operates on Friday and Saturday with the numbers of particular items sold from the stall recorded using matrix B below.

Matrix B

$$B = \begin{array}{c} \\ \textit{Friday} \\ \textit{Saturday} \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Shirts} & \textit{Jeans} & \textit{Belts} \end{array} \\ \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix} \\ \begin{array}{ccc} \textit{column 1} & \textit{column 2} & \textit{column 3} \end{array} \end{array} \begin{array}{l} \\ \textit{row 1} \\ \textit{row 2} \\ \\ \end{array}$$

Rows	Columns
Friday sales are listed in row 1.	The number of shirts sold is listed in column 1.
Saturday sales are listed in row 2.	The number of pairs of jeans sold is listed in column 2.
	The number of belts sold is listed in column 3.

The size of matrix B in the market stall example is 2×3 or a 'two by three' matrix.

We can interpret the following information from the matrix:

- Element b_{12} tells us that on Friday, 8 pairs of jeans were sold.
- On Saturday, 1 belt was sold, given by element b_{23}.
- In row 1, the total number of items sold on Friday was $6 + 8 + 4 = 18$.
- The total number of belts sold was $4 + 1 = 5$ in column 3.

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Row matrices

A **row matrix** has a single row of elements.

In matrix B, all items from only the Friday sales from the market stall can be represented by a 1×3 row matrix.

$$B = \begin{matrix} \\ Friday \\ Saturday \end{matrix} \begin{matrix} \begin{matrix} Shirts & Jeans & Belts \end{matrix} \\ \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix} \end{matrix} \qquad \begin{matrix} \\ Friday \end{matrix} \begin{matrix} \begin{matrix} Shirts & Jeans & Belts \end{matrix} \\ \begin{bmatrix} 6 & 8 & 4 \end{bmatrix} \end{matrix}$$

Column matrices

A **column matrix** has a single column of elements.

In matrix B, the sales of only jeans over both days from the market stall can be represented by a 2×1 column matrix.

$$\begin{matrix} \\ Friday \\ Saturday \end{matrix} \begin{matrix} \begin{matrix} Jeans \end{matrix} \\ \begin{bmatrix} 8 \\ 7 \end{bmatrix} \end{matrix}$$

Although they appear to be very simple, row and column matrices have useful properties that will be explored in this chapter.

Example 1 Interpreting the elements of a matrix

Matrix C shows the number of boys and girls in Years 10 to 12 enrolled at a particular school.

$$C = \begin{matrix} \\ Year\ 10 \\ Year\ 11 \\ Year\ 12 \end{matrix} \begin{matrix} \begin{matrix} Boys & Girls \end{matrix} \\ \begin{bmatrix} 57 & 63 \\ 48 & 54 \\ 39 & 45 \end{bmatrix} \end{matrix}$$

a Give the size of matrix C.

b Interpret the information given by the element c_{12}.

c Identify the element that gives the number of girls in Year 12.

d State the number of boys in total.

e State the number of students in Year 11.

Solution

a The size of matrix C is 3×2.

Explanation

Count the rows, count the columns.
Remember: Order is rows × columns.

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b There are 63 girls in Year 10.	The element c_{12} is in row 1 and column 2. This is where the Year 10 row meets the Girls column. Write this as a sentence.
c The number of Year 12 girls is given by c_{32}.	Year 12 is row 3. Girls are column 2.
d The total number of boys is 144.	The sum of the Boys column gives the total number of boys.
e There are 102 students in Year 11.	The sum of the Year 11 row gives the total number of students in Year 11.

Square matrices

In **square matrices** the number of rows equals the number of columns.

Here are three examples.

$$\underset{1 \times 1}{[9]} \qquad \underset{2 \times 2}{\begin{bmatrix} 5 & 4 \\ 4 & 2 \end{bmatrix}} \qquad \underset{3 \times 3}{\begin{bmatrix} 0 & 4 & 3 \\ 8 & 1 & 6 \\ 2 & 0 & 7 \end{bmatrix}}$$

Section Summary

- The size, or order, of a matrix is the number of rows × the number of columns.
- Two matrices are the same size if they have the same number of rows and columns.
- The elements in matrix A are where a_{ij} is the element in *row i*, *column j*.
- Matrices are named using capital letters.
- A row matrix has a single row of elements.
- A column matrix has a single column of elements.
- A square matrix has equal number of rows and columns.

Exercise 9A

Example 1

1 Matrix C is shown on the right.

$$C = \begin{bmatrix} 2 & 4 & 16 & 7 \\ 6 & 8 & 9 & 3 \\ 5 & 6 & 10 & 1 \end{bmatrix}$$

a Write down the size of the matrix C.

b State the value of:

i c_{13} **ii** c_{24} **iii** c_{31}

c Calculate the sum of the elements in row 3.

d Calculate the sum of the elements in column 2.

SF

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SF

2 For each of the following matrices:

i state the size **ii** identify the values of the listed elements.

a $A = \begin{bmatrix} 5 & 6 & 8 \\ 4 & 7 & 9 \end{bmatrix}$ a_{12} and a_{22}

b $B = \begin{bmatrix} 6 & 8 & 2 \end{bmatrix}$ b_{13} and b_{11}

c $C = \begin{bmatrix} 4 & 5 \\ 3 & 1 \\ 8 & -4 \end{bmatrix}$ c_{32} and c_{12}

d $D = \begin{bmatrix} 8 \\ 6 \\ 9 \end{bmatrix}$ d_{31} and d_{11}

e $E = \begin{bmatrix} 10 & 12 \\ 15 & 13 \end{bmatrix}$ e_{21} and e_{12}

f $F = \begin{bmatrix} 8 & 11 & 2 & 6 \\ 4 & 1 & 5 & 7 \\ 6 & 14 & 17 & 20 \end{bmatrix}$ f_{34} and f_{23}

3 Identify the matrices that are:

a the same size **b** row matrices **c** column matrices **d** square matrices.

$J = \begin{bmatrix} 16 & 81 & 42 \end{bmatrix}$ $K = \begin{bmatrix} 6 & 9 & 0 & 4 \\ 2 & -2 & 3 & 5 \\ 4 & 12 & 15 & 18 \end{bmatrix}$ $L = \begin{bmatrix} 19 \\ -56 \\ 0 \end{bmatrix}$

$M = \begin{bmatrix} 4 & 5 \\ 3 & 6 \end{bmatrix}$ $N = \begin{bmatrix} 8 & -6 & 9 \end{bmatrix}$ $O = \begin{bmatrix} 5 & 6 \\ 4 & 2 \\ 9 & -3 \end{bmatrix}$

$P = \begin{bmatrix} -9 \\ 5 \\ 4 \end{bmatrix}$ $Q = \begin{bmatrix} 24 & -22 \\ -45 & 12 \\ 65 & 17 \end{bmatrix}$ $R = \begin{bmatrix} -12 & 10 \\ 13 & -15 \end{bmatrix}$

4 For matrix D on the right, give the values of the following elements.

a d_{23} **b** d_{45} **c** d_{11}

d d_{24} **e** d_{42}

$D = \begin{bmatrix} 3 & 4 & 6 & 11 & 2 \\ 5 & 1 & 9 & 10 & 4 \\ 8 & 7 & 2 & 0 & 1 \\ 6 & 8 & 5 & 8 & 2 \end{bmatrix}$

5 $A = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 5 & 3 \\ -3 & 4 & 8 \\ 7 & 6 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$ $C = \begin{bmatrix} 8 & -2 \end{bmatrix}$ $D = \begin{bmatrix} 4 & -3 & 0 & 1 & 9 \\ 6 & 11 & 2 & 7 & 5 \end{bmatrix}$

a Write down the size of each matrix A, B, C and D.

b Identify the elements a_{32}, b_{21}, c_{11} and d_{24} of matrices A, B, C and D respectively.

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Example 1

6 Students were asked which of four sports they preferred to play and the results were entered in the following matrix.

$$S = \begin{array}{r} \\ Year\ 10 \\ Year\ 11 \\ Year\ 12 \end{array} \begin{bmatrix} Tennis & Basketball & Football & Hockey \\ 19 & 18 & 31 & 14 \\ 16 & 32 & 22 & 12 \\ 21 & 25 & 5 & 7 \end{bmatrix}$$

a State the number of Year 11 students who preferred to play basketball.

b Write down the size of matrix S.

c Interpret the information given by s_{23}.

7 Matrix F shows the number of hectares of land used for different purposes on farms X and Y. Row 1 represents Farm X and row 2 represents Farm Y. Columns 1, 2 and 3 show the amount of land used for wheat, cattle and sheep (W, C, S) respectively, in hectares.

$$F = \begin{array}{c} \\ X \\ Y \end{array} \begin{bmatrix} W & C & S \\ 150 & 300 & 75 \\ 200 & 0 & 350 \end{bmatrix}$$

a Identify the number of hectares that are used on:

i Farm X for sheep

ii Farm X for cattle

iii Farm Y for wheat.

b Calculate the total number of hectares used on the two farms for wheat.

c Interpret the information that is given by these elements.

i f_{22} **ii** f_{13} **iii** f_{11}

d State the element of matrix F that gives the number of hectares used:

i on Farm Y for sheep

ii on Farm X for cattle

iii on Farm Y for wheat.

e State the order of matrix F.

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9B Using matrices to model practical situations

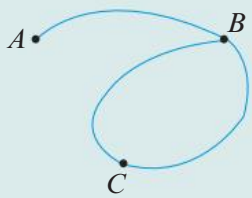
Learning intentions

- To be able to use matrices for storing and displaying information.
- To be able to interpret matrices that model practical situations.

A **network diagram** is made of points (vertices) joined by lines (edges). It can be used to show **connections** or relationships. The information in network diagrams can be recorded in a matrix and used to solve related problems.

Example 2 Using a matrix to represent connections

The network diagram drawn shows the ways to travel between three towns, A, B and C.



a Use a matrix to represent the connections. Each element should describe the number of ways to travel directly from one town to another.

b Interpret the information that is given by the sum of the second column of the matrix.

Solution

a

$$\begin{array}{cc} & \text{To} \\ & \begin{array}{ccc} A & B & C \end{array} \\ \text{From} \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & & \\ & 0 & \\ & & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \text{To} \\ & \begin{array}{ccc} A & B & C \end{array} \\ \text{From} \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \text{To} \\ & \begin{array}{ccc} A & B & C \end{array} \\ \text{From} \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & \\ 0 & & 0 \end{bmatrix} \end{array}$$

Explanation

As there are three towns, A, B and C, use a 3×3 matrix to show the direct connections.

There are no roads directly connecting any town to itself. So enter 0 where column A crosses row A, and so on.

If there was a road directly connecting town A to itself, it would be a loop from A back to A.

There is one road directly connecting B to A (or A to B). So enter 1 where column B crosses row A and where column A crosses row B.

There are no direct roads between C and A. So enter 0 where column C crosses row A and where column A crosses row C.

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		To		
		A	B	C
From	A	0	1	0
	B	1	0	2
	C	0	2	0

There are two roads between C and B. Enter 2 where column C crosses row B and where column B crosses row C.

b The sum of the second column represents the total number of roads directly connected to town B.
$1 + 0 + 2 = 3$

The second column shows the number of roads directly connected to town B.

Section Summary

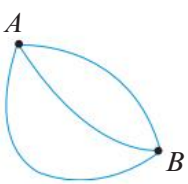
- **Matrices** can be used to organise, store and display information in practical situations.
- When matrices are used to represent connections, this relationship should be defined clearly.

Exercise 9B

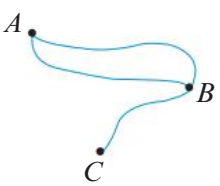
Example 2

1 The networks show roads connecting towns. In each case, use a matrix to record the number of ways of travelling *directly* from one town to another.

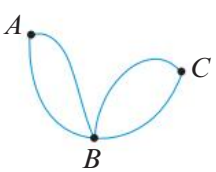
a



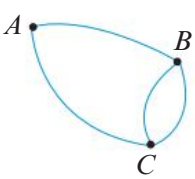
b



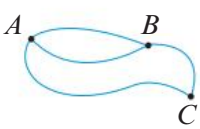
c



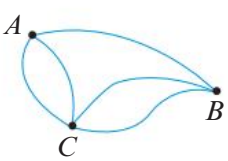
d



e



f



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CF

2 The matrices show the number of ways of going directly from one town to another. In each case, draw graphs to show the direct connections between towns A, B and C.

a
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

b
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

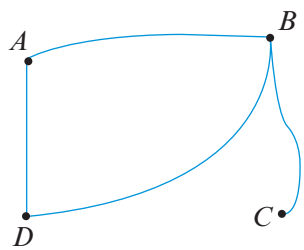
c
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \end{array}$$

d
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \end{array}$$

3 **a** For the networks given in Question **1**, interpret the information given by the sum of the second column of each matrix.

b For the networks given in Question **2**, explain what the sum of the first column of each matrix represents.

4 This network diagram has lines showing who of the four people, A, B, C and D, have met.



a Represent the graph using a matrix. Use 0 when two people have *not* met and 1 when they have met.

b Describe how to use the matrix to identify who has met the most people.

c Identify the person who has met the most people.

d Identify the person who has met the least number of people.

CU

5 The matrix gives the numbers of buses that travel between each pair of cities in each direction. Use matrix arithmetic to calculate the total number of buses that operate across all three cities.

$$\begin{array}{c} \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 0 & 8 & 9 \\ 1 & 0 & 4 \\ 6 & 13 & 0 \end{bmatrix} \end{array}$$

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9C Adding and subtracting matrices

Learning intentions

- To be able to know when matrix addition and subtraction can be performed.
- To be able to perform matrix addition.
- To be able to perform matrix subtraction.

Rules for adding and subtracting matrices

1. First, check the size of the matrices. **Matrix addition and subtraction** can only be performed if the matrices are the same size.
2. Calculate the sum of matrices by adding the corresponding elements that are in the same positions.
3. Calculate the difference of matrices by subtracting the corresponding elements that are in the same positions.

Example 3 Adding and subtracting of matrices

Complete the following addition and subtraction of matrices.

a $\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix}$ **b** $\begin{bmatrix} 7 & 3 \\ 2 & 8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 9 \\ 3 & 7 \end{bmatrix}$ **c** $\begin{bmatrix} 4 & 1 \\ 0 & 6 \end{bmatrix} + \begin{bmatrix} -14 & 22 & 19 \\ -81 & 43 & 0 \end{bmatrix}$

Solution

a $\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2+9 & 4+8 \\ 5+9 & 1+(-1) \end{bmatrix}$

$= \begin{bmatrix} 11 & 12 \\ 14 & 0 \end{bmatrix}$

b $\begin{bmatrix} 7 & 3 \\ 2 & 8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 9 \\ 3 & 7 \end{bmatrix}$

Explanation

Check that both matrices are the same size. They are both 2×2 matrices. Then proceed with the addition.

Add the corresponding elements that are in the same positions.

Evaluate each element.

Check that both matrices are the same size. They are both 3×2 matrices. Then proceed with the subtraction.

Continued on next page

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$$= \begin{bmatrix} 7-4 & 3-2 \\ 2-(-1) & 8-9 \\ 1-3 & 0-7 \end{bmatrix}$$

Subtract the corresponding elements that are in the same positions.

$$= \begin{bmatrix} 3 & 1 \\ 3 & -1 \\ -2 & -7 \end{bmatrix}$$

Evaluate each element.

c $\begin{bmatrix} 4 & 1 \\ 0 & 6 \end{bmatrix} + \begin{bmatrix} -14 & 22 & 19 \\ -81 & 43 & 0 \end{bmatrix}$

Note that the first matrix is a 2 by 2 matrix and the second matrix is a 2 by 3 matrix. So they are not the same size.

There is no solution or the resulting matrix is undefined. Matrix addition is not possible since the matrices are not the same size.

Explain.

The zero matrix, 0

In a **zero matrix** every element is zero.

The following are examples of zero matrices.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Just as in arithmetic with ordinary numbers, adding or subtracting a zero matrix does not make any change to the original matrix. For example:

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Also, subtracting any matrix from itself gives a zero matrix. For example:

$$\begin{bmatrix} 9 & 4 & 8 \end{bmatrix} - \begin{bmatrix} 9 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Section Summary

- Two matrices must be the **same size** when performing matrix addition or subtraction.
- Matrices are added by adding the elements that are in the same position.
- Matrices are subtracted by subtracting the elements that are in the same position.
- The **zero matrix** of any size has only elements of zero. When added to or subtracted from a matrix, the original matrix does not change.

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Exercise 9C

SF

Example 3

1 Calculate the sums or differences of the following matrices.

a $\begin{bmatrix} 4 & 3 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 6 & 1 \end{bmatrix}$

b $\begin{bmatrix} 8 & 6 \\ 9 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 0 \end{bmatrix}$

c $\begin{bmatrix} 3 & 5 \\ 7 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

d $\begin{bmatrix} 9 \\ 8 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

e $\begin{bmatrix} 8 & 6 \\ 2 & 9 \end{bmatrix} - \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$

f $\begin{bmatrix} 7 & 4 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 2 & -8 \end{bmatrix}$

g $\begin{bmatrix} 4 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 5 \end{bmatrix}$

h $\begin{bmatrix} 7 & -5 \end{bmatrix} - \begin{bmatrix} 7 & -5 \end{bmatrix}$

i $\begin{bmatrix} 4 & -3 \end{bmatrix} + \begin{bmatrix} -4 & 3 \end{bmatrix}$

j $\begin{bmatrix} 4 & -3 & 2 & -1 \end{bmatrix} - \begin{bmatrix} 6 & -5 & -1 & 8 \end{bmatrix}$

2 Consider the matrices below:

$$A = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 7 \\ 1 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 2 \\ 1 & 0 \\ 3 & -8 \end{bmatrix} \quad D = \begin{bmatrix} -3 & 5 \\ 4 & -2 \\ 1 & 7 \end{bmatrix} \quad E = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

Calculate, where possible. If not possible, explain.

a $A + B$ **b** $B + A$ **c** $A - B$ **d** $B - A$

e $B + E$ **f** $C + D$ **g** $B + C$ **h** $D - C$

CF

3 Two people shared the work of a telephone poll surveying voting intentions. The results for each person's survey are given in matrix form.

Sample 1:

	Liberal	*Labor*	*Democrat*	*Green*
Men	19	21	7	3
Women	18	17	11	4

Sample 2:

	Liberal	*Labor*	*Democrat*	*Green*
Men	24	21	3	2
Women	19	20	6	5

Write a matrix showing the overall result of the survey.

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CF

4 The weights and heights of four people were recorded and then checked again one year later.

2004 results:

	Aida	*Bianca*	*Chloe*	*Donna*
Weight (kg)	32	44	59	56
Height (cm)	145	155	160	164

2005 results:

	Aida	*Bianca*	*Chloe*	*Donna*
Weight (kg)	38	52	57	63
Height (cm)	150	163	167	170

a Write the matrix that gives the changes in each person's weight and height after one year.

b Identify who gained the most weight.

c Identify which person had the greatest increase in height.

CU

5 Calculate the values of x, y and z in the matrices below.

$$\begin{bmatrix} 21 & 35 & x+1 \\ 1 & 42 & 18 \end{bmatrix} - \begin{bmatrix} 2 & y-2 & 14 \\ 9 & -2 & 54 \end{bmatrix} = \begin{bmatrix} 19 & 31 & 26 \\ -8 & 44 & z-x+y \end{bmatrix}$$

6 Consider the matrices below:

$$F = \begin{bmatrix} 31 & -22 \\ 23 & 44 \end{bmatrix} \quad G = \begin{bmatrix} 53 & 46 \\ 12 & 73 \end{bmatrix} \quad H = \begin{bmatrix} 6 & 2 \\ 0 & 1 \\ 3 & -9 \end{bmatrix} \quad I = \begin{bmatrix} -4 & 2 \\ 1 & -3 \\ 6 & 0 \end{bmatrix} \quad J = \begin{bmatrix} 79 \\ 14 \end{bmatrix}$$

a If $F + Q = G$, identify matrix Q.

b If $T - I = H$, identify matrix T.

c If $J + R = J$, identify matrix R.

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9D Scalar multiplication

Learning intentions

- To be able to perform matrix multiplication by a scalar.
- To be able to apply matrix multiplication by a scalar to practical situations.

Matrix multiplication by a scalar

A **scalar** is a number that scales or multiplies some quantity. In matrix **multiplication by a scalar**, each element of the matrix is multiplied by that scalar to give a new resulting matrix. This is often called scalar multiplication of matrices.

The following is an example of scalar multiplication of a matrix.

$$5\begin{bmatrix}1 & 2\\2 & 0\end{bmatrix} = \begin{bmatrix}5\times 1 & 5\times 2\\5\times 2 & 5\times 0\end{bmatrix} = \begin{bmatrix}5 & 10\\10 & 0\end{bmatrix}$$

Example 4 Multiplication by a scalar

If $A = \begin{bmatrix}5 & 1\\-3 & 0\end{bmatrix}$, calculate $3A$.

Solution

$$3A = 3\begin{bmatrix}5 & 1\\-3 & 0\end{bmatrix}$$
$$= \begin{bmatrix}3\times 5 & 3\times 1\\3\times -3 & 3\times 0\end{bmatrix}$$
$$= \begin{bmatrix}15 & 3\\-9 & 0\end{bmatrix}$$

Explanation

If $A = \begin{bmatrix}5 & 1\\-3 & 0\end{bmatrix}$, then $3A = 3\begin{bmatrix}5 & 1\\-3 & 0\end{bmatrix}$.

Multiply each number in the matrix by 3.

Evaluate each element.

When performing scalar multiplication, the resulting matrix is the same size as the original matrix. Multiplication by a scalar does not change the size of the matrix.

Scalar multiplication can also be used in conjunction with addition and subtraction of matrices.

Example 5 Scalar multiplication and subtraction of matrices

If $A = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ and $B = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, calculate $2A - 3B$.

Continued on next page

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Solution

$$2A - 3B = 2 \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - 3 \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 2-3 \\ 0-3 & 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

Explanation

Check that both matrices are the same size. They are both 2×2 matrices. Write $2A - 3B$ in expanded matrix form.

Multiply the elements in A by 2 and the elements in B by 3.

Subtract the elements in corresponding positions.

Scalar multiplication has many practical applications. It is particularly useful in scaling up the elements of a matrix if the scalar value is greater than 1. For example, if we multiply a matrix that contains the prices of all items in a shop by 1.1, it will calculate the prices of all items including GST in the resulting matrix. Similarly, scalar multiplication can also be used to scale down the elements of a matrix if the scalar value is between 0 and 1.

Example 6 Application of scalar multiplication

A gymnasium has enrolments in the courses shown in this matrix.

	Body building	*Aerobics*	*Fitness*
Men	70	20	80
Women	10	50	60

The gym has a membership drive and the enrolments in each course double.

Show this in a matrix.

Solution

$$2 \times \begin{bmatrix} 70 & 20 & 80 \\ 10 & 50 & 60 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 70 & 2 \times 20 & 2 \times 80 \\ 2 \times 10 & 2 \times 50 & 2 \times 60 \end{bmatrix}$$

Explanation

Each element in the matrix is multiplied by 2.

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$$= \begin{matrix} \\ \textit{Men} \\ \textit{Women} \end{matrix} \begin{matrix} \begin{matrix} \textit{Body building} & \textit{Aerobics} & \textit{Fitness} \end{matrix} \\ \begin{bmatrix} 140 & 40 & 160 \\ 20 & 100 & 120 \end{bmatrix} \end{matrix}$$

Evaluate each element.

Section Summary

- A **scalar** is a number.
- To perform multiplication by a scalar on a matrix, multiply each element in the matrix by the scalar value.
- Scalar multiplication does not change the size of the matrix.

Skill-sheet

Exercise 9D

ST

Example 4

1 Calculate the resulting matrix using multiplication by the scalar.

a $2\begin{bmatrix} 7 & -1 \\ 4 & 9 \end{bmatrix}$ **b** $5\begin{bmatrix} 0 & -2 \\ 5 & 7 \end{bmatrix}$ **c** $-4\begin{bmatrix} 16 & -3 \\ 1.5 & 3.5 \end{bmatrix}$ **d** $1.5\begin{bmatrix} 1.5 & 0 \\ -2 & 5 \end{bmatrix}$

e $3\begin{bmatrix} 6 & 7 \end{bmatrix}$ **f** $6\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ **g** $\frac{1}{2}\begin{bmatrix} 4 & 6 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ **h** $-1\begin{bmatrix} 3 & 6 & -8 \end{bmatrix}$

2 Consider these matrices $A = \begin{bmatrix} 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$.

Calculate:

a $5A$ **b** $3A + 4A$ **c** $6B$ **d** $8B + B$ **e** $7B$

Example 5

3 Consider the following matrices:

$$A = \begin{bmatrix} 3 & -4 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 6 \\ 1 & -4 \end{bmatrix} \quad C = \begin{bmatrix} -3 & 4 \\ -2 & -5 \end{bmatrix} \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Calculate.

a $3A$ **b** $2B + 4C$ **c** $5A - 2B$ **d** $2O$ **e** $3B + O$

4 Consider the following matrices:

$$A = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}$$

Evaluate.

a $3A + 4B$ **b** $5C - 2D$ **c** $2(3A + 4B)$ **d** $3(5C - 2D)$

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CF

5 Consider the following matrices:

$$A = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -2 & 1 \\ 0 & 5 \end{bmatrix}$$

Evaluate.

a $17A + 14B$ **b** $29B - 21A$ **c** $5(3B + 4A)$ **d** $3(5A - 4B)$

Example 6

6 The prices of items, in dollars, sold at different shops are given in the matrix.

	shirt	*pants*	*shorts*
Shop A	16.54	49.99	14.20
Shop B	23.80	53.25	21.98

a Use scalar multiplication to calculate the price of each item from Shop A, including GST. Give your answer as a row matrix.

b If the shops sold 5 of each item, calculate the total income received from each item in each shop, excluding GST. Give your answer in matrix form.

CU

7 The expenses arising from costs and wages for each section of three stores, *A*, *B* and *C*, are shown in the Costs matrix. The Sales matrix shows the money from the sale of goods in each section of the three stores. Figures represent the nearest million dollars.

Costs:

	Clothing	*Furniture*	*Electronics*
A	12	10	15
B	11	8	17
C	15	14	7

Sales:

	Clothing	*Furniture*	*Electronics*
A	18	12	24
B	16	9	26
C	19	13	12

a Write a matrix showing the profits in each section of each store.

b If 30% tax must be paid on profits, show the amount of tax that must be paid by each section of each store. No tax needs to be paid for a section that has made a loss.

8 Zoe competed in the gymnastics rings and parallel bars events in a three-day gymnastics tournament. A win was recorded as 1 and a loss as 0. The three column matrices show the results for Saturday, Sunday and Monday.

	Sat	*Sun*	*Mon*
Gymnastics rings	1	1	1
Parallel bars	0	1	1

a Give a 2×1 column matrix which records her total wins for each of the two types of events.

b Zoe received \$50 for each win. Give a 2×1 matrix that records her total prize money for each of the two types of events.

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9E Matrix multiplication and power of a matrix

Learning intentions

- To be able to know when matrix multiplication can and cannot be performed.
- To be able to perform matrix multiplication.
- To be able to determine the power of a matrix.
- To be able to use technology to perform matrix arithmetic.
- To be able to recognise and use an identity matrix.

Matrix multiplication is the multiplication of a matrix by another matrix. It is not to be confused with scalar multiplication which is the multiplication of a matrix by a number.

The matrix multiplication of matrices A and B can be written as $A \times B$ or just AB. Although it is called multiplication and the symbol $\times$ may be used, matrix multiplication is not the simple multiplication of numbers but a routine involving the sum of pairs of numbers that have been multiplied.

For example, the method of matrix multiplication can be demonstrated by using a practical example. The numbers of DVDs and computer games sold by Fatima and Gaia are recorded in matrix N. The selling prices of the DVDs and computer games are shown in matrix P.

$$N = \begin{matrix} \\ Fatima \\ Gaia \end{matrix} \begin{matrix} \begin{matrix} DVDs & Games \end{matrix} \\ \begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \end{matrix} \qquad P = \begin{matrix} \\ DVDs \\ Games \end{matrix} \begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{bmatrix} 20 \\ 30 \end{bmatrix} \end{matrix}$$

We want a matrix, S, that shows the total value of the sales made by each person.

Fatima sold: 7 DVDs at \$20 + 4 Games at \$30.
Gaia sold: 5 DVDs at \$20 + 6 Games at \$30.

$$S = \begin{matrix} \\ Fatima \\ Gaia \end{matrix} \begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{bmatrix} 7 \times 20 + 4 \times 30 \\ 5 \times 20 + 6 \times 30 \end{bmatrix} \end{matrix}$$

The steps used in this example follow the routine for the matrix multiplication of $N \times P$.

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As we move across the *first row* of matrix N we move down the *column* of matrix P, adding the products of the pairs of numbers as we go.

Then we move across the *second row* of matrix N and down the *column* of matrix P, adding the products of the pairs of numbers as we go.

$$N \times P$$

$$\begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 7 \times 20 + 4 \times 30 \\ \square \end{bmatrix}$$

$$\begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 7 \times 20 + 4 \times 30 \\ 5 \times 20 + 6 \times 30 \end{bmatrix}$$

$$= \begin{bmatrix} 140 + 120 \\ 100 + 180 \end{bmatrix}$$

$$= \begin{matrix} \$ \\ \begin{bmatrix} 260 \\ 280 \end{bmatrix} \end{matrix} \begin{matrix} \\ \textit{Fatima} \\ \textit{Gaia} \end{matrix}$$

Rules for matrix multiplication

Because of the way the products are formed, the number of columns in the first matrix must equal the number of rows in the second matrix. Otherwise, we say that matrix multiplication is not defined, meaning it is not possible.

Matrix multiplication

For matrix multiplication to be defined:

Think of the sizes as two railway carriages that must be the same where they meet.

size of 1st matrix $m \times n$ — must be the same — size of 2nd matrix $n \times p$

In our example of the sales of DVDs and computer games:

size of 1st matrix 2×2 — the same — size of 2nd matrix 2×1

Notice that the outside numbers give the size of the product matrix, the matrix made by multiplying the two matrices. In our case, the answer is a 2×1 matrix.

Size of the product matrix

The size of the product matrix is given by:

Think: when the 'railway carriages' meet, the result has a size given by the end numbers.

size of 1st matrix $m \times n$ — order of answer — size of 2nd matrix $n \times p$

$m \times p$

We will check that these two important rules hold in the examples that follow.

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Methods of matrix multiplication

Some people like to think of the matrix multiplication of $A \times B$ using a *run and dive* description. This procedure can be very tedious and error prone, so we will only do simple cases by hand so that you understand the process.

Matrix multiplication of $A \times B$

The *run and dive* description of matrix multiplication is to add the products of the pairs made as you:

- *run* along the first row of A and *dive* down the first column of B
- repeat running along the first row of A and diving down the next column of B until all columns of B have been used
- now start running along the next row of A and repeat diving down each column of B, entering your results in a new row
- repeat this routine until all rows of A have been used.

Example 7 Matrix multiplication

Consider the following matrices:

$$A = \begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 9 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 4 & 7 \end{bmatrix} \quad D = \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$$

- Decide whether the matrix multiplication in each question below is defined.
- If matrix multiplication is defined, give the size of the product matrix and then do the matrix multiplication.

a AB **b** BA **c** CD

Solution

a $\underset{3 \times 2}{A} \quad \underset{2 \times 1}{B}$

Matrix multiplication is defined for $A \times B$.

The size of the product AB is 3×1.

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ \\ \end{bmatrix}$$

Explanation

Identify the size of each matrix.

Verify the inside numbers are the same.

The outside numbers give the size of $A \times B$.

Move across the first row of A and down the column of B, adding the products of the pairs.

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$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ 4 \times 8 + 6 \times 9 \\ \end{bmatrix}$$

Move across the second row of A and down the column of B, adding the products of the pairs.

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ 4 \times 8 + 6 \times 9 \\ 1 \times 8 + 3 \times 9 \end{bmatrix}$$

Move across the third row of A and down the column of B, adding the products of the pairs.

$$= \begin{bmatrix} 40 + 18 \\ 32 + 54 \\ 8 + 27 \end{bmatrix}$$

Simplify.

$$\text{So A} \times B = \begin{bmatrix} 58 \\ 86 \\ 35 \end{bmatrix}.$$

Write your answer.

b $\underset{2 \times 1}{B} \quad \underset{3 \times 2}{A}$

Identify the size of each matrix. Check whether the inside numbers are the same. They are not.

Multiplication is not defined for $B \times A$ since the number of columns in matrix B is not the same as the number of rows in matrix A.

Explain.

c $\underset{1 \times 3}{C} \quad \underset{3 \times 1}{D}$

Identify the size of each matrix.

Multiplication is defined for $C \times D$. The size of the product CD is 1×1.

Verify the inside numbers are the same. The outside numbers give the size of $C \times D$.

$$\begin{bmatrix} 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \times 8 + 4 \times 6 + 7 \times 5 \end{bmatrix}$$

Move across the row of C and down the column of D, adding the products of the pairs.

$$= [16 + 24 + 35]$$

Simplify.

$$\text{So } C \times \text{D} = [75].$$

Write your answer.

In the previous example, $AB \neq BA$. Usually, when we reverse (*commute*) the order of the matrices in matrix multiplication, we get a different answer. This differs from ordinary arithmetic, where multiplication is **commutative** and gives the same answer when the terms are reversed. For example, $3 \times 4 = 4 \times 3$.

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Matrix multiplication

In general, matrix multiplication is not commutative: $AB \neq BA$

Spread-sheet

Spreadsheet activity 9E: Matrix operations
Using a spreadsheet is one way to have technology assist with matrix arithmetic. Activities for demonstration of matrix operations using graphics calculators are available in the teacher resources to use for demonstrations.

Identity matrix

The **identity matrix**, I, is a matrix that behaves like the number 1 in arithmetic. When any square matrix is multiplied by the identity matrix the answer is identical to the original matrix. That is, I is such that:

$$AI = IA + A$$

The square identity matrix

The **identity matrix** for any square matrix is a square matrix of the same size with 1's in the leading diagonal (from the top left to the bottom right) and 0's in all the other positions.

[1] $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

This is illustrated in the following example for a 2×2 matrix.

Example 8 The identity matrix

Consider the following matrices.

$$A = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

a Evaluate AI.

b Evaluate IA.

Continued on next page

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Solution	Explanation
a $\underset{2\times 2}{A}\quad\underset{2\times 2}{I}$	Identify the size of each matrix. Verify that the inside numbers are the same, so matrix multiplication is defined. The outside numbers tell us that the product is a 2×2 matrix.
$A\times I=\begin{bmatrix}5 & 2\\ 8 & 3\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	Perform the matrix multiplication.
$=\begin{bmatrix}5\times 1+2\times 0 & 5\times 0+2\times 1\\ 8\times 1+3\times 0 & 8\times 0+3\times 1\end{bmatrix}$	
$=\begin{bmatrix}5 & 2\\ 8 & 3\end{bmatrix}$	
b $\underset{2\times 2}{I}\quad\underset{2\times 2}{A}$	Identify the size of each matrix. Verify that the inside numbers are the same, so matrix multiplication is defined. The outside numbers tell us that the size of the product is a 2×2 matrix.
$I\times A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}5 & 2\\ 8 & 3\end{bmatrix}$	Perform the matrix multiplication.
$=\begin{bmatrix}1\times 5+0\times 8 & 1\times 2+0\times 3\\ 0\times 5+1\times 8 & 0\times 2+1\times 3\end{bmatrix}$	
$=\begin{bmatrix}5 & 2\\ 8 & 3\end{bmatrix}$	

The **identity matrix**, I, also has the special property that it is commutative in matrix multiplication. When I is one of the matrices in the multiplication and the other is a square matrix of the same size, the answer is the same when the order of the matrices is commuted (reversed).

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Power of a matrix

The **power of a matrix** has the same meaning as a power in arithmetic or algebra.

For example:

Arithmetic	$7^3 = 7 \times 7 \times 7$
Algebra	$x^3 = x \times x \times x$
Matrices	$A^3 = A \times A \times A$

$$\begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}^3 = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

The power of a matrix

The n'th power of a matrix is calculated as A^n.

$$A^n = A \times A \times A \times \ldots \times A$$

A occurs n times in the expanded multiplication.

Only square matrices can be raised to a power of 2 or more.

Repeated multiplication involving higher powers or large matrices is time consuming and error prone. Technology such as matrix multiplication apps, spreadsheets or graphics calculators could be used when appropriate.

Example 9 Calculating powers of a matrix

Consider matrix A. $A = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$

Without using technology calculate

a A^2 **b** A^3.

Solution

a A^2

$= A \times A$

$= \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 0 \\ 8 & 1 \end{bmatrix}$

Explanation

Write the expanded form of A^2.

Perform matrix multiplication.

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b A^3
$= A \times A^2$
$= \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 9 & 0 \\ 8 & 1 \end{bmatrix}$
$= \begin{bmatrix} 27 & 0 \\ 26 & 1 \end{bmatrix}$

Expand as $A^3 = A \times A^2$.

Use A^2 from part **a**.

Multiply the matrices.

Section Summary

- **Matrix multiplication** can only be performed when the number of columns in the first matrix is equal to the number of rows in the second matrix.
- If matrix A is of order $m \times n$, and matrix B is of order $n \times p$, then the product $A \times B$ is of order $m \times p$.
- The **identity matrix**, I, is a matrix such that, for any square matrix A of the same size, $AI = IA = A$.
- In general, matrices are not commutative but the identity matrix always has commutative properties.
- The power of a matrix is calculated by multiplying the matrix by itself the number of times of the index.
- Only square matrices can be raised to a power.

Skill-sheet

Exercise 9E

Matrix multiplication

1 Consider the following matrices: $A = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 \\ 0 & 8 \\ 2 & -5 \end{bmatrix}$

- Decide whether the matrix multiplication in each question below is defined.
- If matrix multiplication is defined, give the size of the product matrix.

a AB **b** BA
c CB **d** BC
e A^2 **f** B^2
g AC **h** CA

SF

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Example 7

2 State the sizes of each pair of matrices and decide if matrix multiplication is defined. If matrix multiplication is defined, calculate the product.

a $\begin{bmatrix} 3 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ **c** $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

d $\begin{bmatrix} 8 & -2 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ **e** $\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ **f** $\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

3 Perform these matrix multiplications without using technology.

a $\begin{bmatrix} 4 & 2 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 4 \\ 5 & 9 \end{bmatrix}\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ **c** $\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 6 & 7 \end{bmatrix}$

d $\begin{bmatrix} 3 & 5 \\ 1 & 8 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -2 & -3 \end{bmatrix}$ **e** $\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **f** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 9 & 6 \\ 7 & 8 \end{bmatrix}$

g $\begin{bmatrix} 2 & 5 \\ 4 & 1 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \end{bmatrix}$ **h** $\begin{bmatrix} 7 & 4 \\ 1 & 0 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ **i** $\begin{bmatrix} 5 & 8 \end{bmatrix}\begin{bmatrix} 7 \\ 6 \end{bmatrix}$

j $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 9 \end{bmatrix}$ **k** $\begin{bmatrix} 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}$ **l** $\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$

m $\begin{bmatrix} 6 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ **n** $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix}$ **o** $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

4 Use technology to repeat the matrix multiplications in Question **3**.

5 Consider the matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$.

a Evaluate AB. **b** Evaluate BA.

c Comment on whether $AB = BA$.

6 Use these matrices to calculate the required products.

$$C = \begin{bmatrix} 9 & 8 \\ 7 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 8 & 6 \\ 4 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

a CD **b** CE **c** CF **d** DE **e** DF

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Identity matrices

7 Identify whether these matrices are identity matrices.

a $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **e** $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **f** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Example 8

8 Calculate the required products.

a $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 7 & 3 \\ 5 & 6 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 7 & 3 \\ 5 & 6 \end{bmatrix}$ **c** $\begin{bmatrix} 4 & 8 \\ 11 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 8 \\ 11 & -3 \end{bmatrix}$ **e** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 & 4 \\ 6 & 9 & 2 \\ 8 & 0 & 7 \end{bmatrix}$

Power of a matrix

Example 9

9 For each of the following matrices, calculate:

i A^2 **ii** A^3

a $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **b** $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **c** $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **e** $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

CF

10 Perform the required matrix multiplications.

a $\begin{bmatrix} 6 & 8 & 12 \\ 14 & 17 & 11 \end{bmatrix}\begin{bmatrix} 26 & 9 & 21 & 6 \\ 8 & -7 & -4 & 9 \\ 13 & 10 & 5 & 26 \end{bmatrix}$ **b** $\begin{bmatrix} 15 & 9 & 23 & 72 \end{bmatrix}\begin{bmatrix} -6 \\ 22 \\ -8 \\ 19 \end{bmatrix}$

c $\begin{bmatrix} 16 \\ 10 \\ 24 \\ -18 \end{bmatrix}\begin{bmatrix} -31 & 47 & 61 & -14 \end{bmatrix}$ **d** $\begin{bmatrix} 8 & -7 & 9 \\ 6 & 11 & 14 \\ 3 & 21 & -5 \end{bmatrix}\begin{bmatrix} 8 & -19 & 24 \\ 33 & 16 & 19 \\ 4 & 0 & 13 \end{bmatrix}$

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11 Consider $A = \begin{bmatrix} 4 & 7 & 1 \\ 0 & 6 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

a Without using technology, calculate:

i A^2 **ii** A^3

b Using technology, find A^4.

12 Consider $B = \begin{bmatrix} 2 & -1 & -1 \\ 5 & 0 & 1 \\ -2 & 2 & 8 \end{bmatrix}$

Without using technology, calculate:

a B^2 **b** B^3 **c** $B^2 \times B^2$

13 Solve for the pronumerals.

$$\begin{bmatrix} x & y \\ 0 & x \end{bmatrix}^2 = \begin{bmatrix} 4 & 12 \\ 0 & 4 \end{bmatrix}$$

14 Investigating *The Curious Case of* A^0.

$$A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$

Technology can be used to find powers of any matrix A, such as A^2, A^3 and A^4.

a Use technology to find A^0.

b Calculate $A^0 \times A$.

c Explain what is special about A^0.

d Make up another 2×2 matrix, B, and find B^0.

e Calculate $B^0 \times B$.

f State another name for A^0.

CF CU

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9F Communications and connections

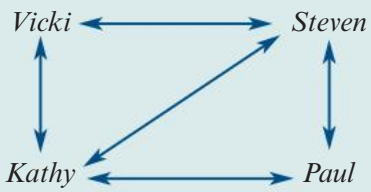
Learning intentions

- To be able to model communications and connections using matrices.
- To be able to apply matrix arithmetic to solve practical applications.

Social networks, communication pathways and connections can be represented and analysed using matrix techniques.

Example 10 Applying matrices to social networks

The diagram shows the communications within a group of friends.



- A double-headed arrow connecting two names indicates that those two people communicate with each other.
- If there is no arrow directly connecting two people, they do not communicate.

a These links are called one-step connections because there is just one direct step in making contact with the other person. Record the social links in a matrix, N, using the first letter of each name to label the columns and rows. Explain how the matrix should be read.

b Explain why there is symmetry about the leading diagonal of the matrix.

c Describe the information that is given by the sum of a column or row.

d N^2 gives the number of two-step communications between people; that is, the number of ways one person can communicate with someone via another person. Construct the matrix N^2, the square of matrix N.

e Use the matrix N^2 to calculate the number of two-step ways Kathy can communicate with Steven and write the connections.

f In the N^2 matrix there is a 3 where the S column meets the S row. This indicates that there are three two-step communications Steven can have with himself. Interpret this value.

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Solution	Explanation
a $N = \begin{array}{c} \\ \text{V} \\ \text{S} \\ \text{K} \\ P \end{array} \begin{array}{c} \begin{array}{cccc} \text{V} & \text{S} & \text{K} & P \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$	Construct the matrix described.
In the matrix, a 1 indicates the two people communicate with each other. The number 0 is used where there is no communication. For example, when reading across row S to column K, a 1 indicates that Steven communicates with Kathy.	Explain how the connections are represented.
b The symmetry occurs because the communication is two way. For example, Vicki communicates with Steven and Steven communicates with Vicki.	Explain the symmetry that is present in the matrix.
c The sum of a column or row gives the total number of people that a given person can communicate with. For example, Kathy can communicate with: $1 + 1 + 0 + 1 = 3$ people.	Explain the row and column sum values.
d $N^2 = \begin{array}{c} \\ \text{V} \\ \text{S} \\ \text{K} \\ P \end{array} \begin{array}{c} \begin{array}{cccc} \text{V} & \text{S} & \text{K} & P \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \end{array}$	Perform the matrix multiplication.
e There are two ways that there are two-step communication between Kathy and Steven.	Reading across the K row to the S column the 2 indicates there are 2 two-step communications between Kathy and Steven.
These are: Kathy → Vicki → Steven Kathy → Paul → Steven	Interpret using the arrows in the diagram.

Continued on next page

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f There are three ways Steven can communicate with himself via another person. Steven → Vicki → Steven Steven → Kathy → Steven Steven → Paul → Steven For example, using the first case above, Steven might ring Vicki and ask her to ring him back later to remind him of an appointment.	Interpret the 3 represented in the communication matrix.

The matrix N^3 would give the three-step communications between people. The number of ways of communicating with someone via two people.

The matrix methods of investigating communications can be applied to friendships, travel between towns and other types of two-way connections.

Section Summary

- A matrix N could represent the one-step connections between two people.
- The matrix N^2 will represent the two-step connections between two people.
- The matrix N^3 will represent the three-step connections between two people.
- If the connections are two-way, then the matrix will be symmetric.

Exercise 9F

Note: The multiplication of large matrices may be done using technology.

1 Road connections between towns are recorded in the matrices below. The letters represent towns. Where row C meets column B the number 1 indicates that there is a road directly connecting town C to town A. The number 0 is used to show when there is no road directly connecting two towns.

Draw a diagram corresponding to each matrix showing the roads connecting the towns.

a

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

b

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array}\begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

c

$$\begin{array}{c} \\ T \\ U \\ V \\ W \end{array}\begin{array}{c} \begin{array}{cccc} T & U & V & W \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

CF

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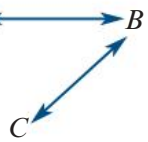
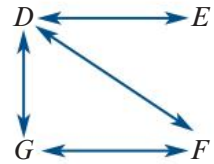
CF

2 Construct the matrix for each communications diagram. Use the number 1 when direct communication between two people exists and 0 for no direct communication.

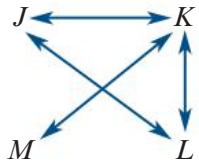
a



b

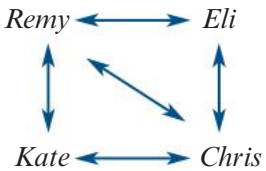


c



Example 10

3 Communication connections between Chris, Eli, Kate and Remy are shown in the diagram.



a Construct a matrix Q to represent the connections. Label the columns and rows in alphabetical order using the first letter of each name. Enter 1 to indicate that two people communicate directly or 0 if they do not.

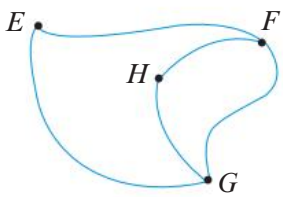
b Interpret the information given by the sum of column R.

c Calculate Q^2.

d Using matrix Q^2, evaluate the total number of ways that Eli can communicate with a person via another person.

e State the chain of connections for each way that Eli can communicate with a person via another person.

4 Roads connecting the towns Easton, Fields, Hillsville and Gorges are shown in the diagram. The first letter of each town is used.



a Construct a matrix R to represent the road connections. Label the columns and rows in alphabetical order using the first letter of the name of each town. Write 1 when two towns are directly connected by a road and write 0 if they are not connected.

b Explain what the sum of column F reveals about Fields.

c Calculate R^2.

d Determine the number of ways to travel from Fields to a town via another town. List the possible ways of starting and ending at Fields.

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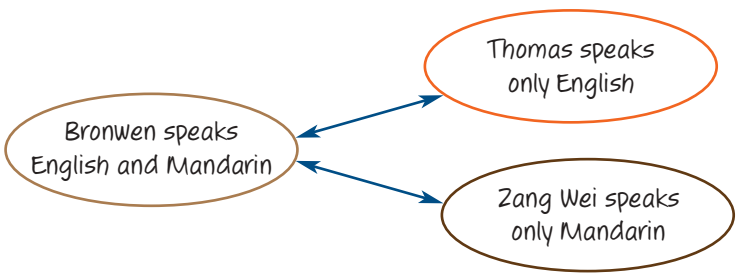
5 Assume that communications are a two-way process. So if A communicates with B then B communicates with A. The letters represent the names of people. Where row B meets column A, the number 1 indicates that B communicates with A. The number 0 is used to show when there is no direct communication between two people. Identify the error in this communications matrix.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

CU

6 The diagram shows the communications between three people. Bronwen speaks English and Mandarin. She communicates with Thomas, who only speaks English, and with Zang Wei, who only speaks Mandarin. The double-headed arrows are called one-step connections because they represent the one direct communication between two people. There is no one-step connection between Thomas and Zang Wei.



a Record the one-step connections in matrix C. Use the first letter of each person's name, B, T and Z, to label the columns and rows.

b Explain why there is symmetry about the leading diagonal of the matrix C.

c Interpret the information given by the sum of a row or column.

d The matrix C^2 gives the number of two-step communications between people. A two-step communication is from one person to another via a third person. For example: $Z \rightarrow B \rightarrow T$ and $T \rightarrow B \rightarrow T$. In the second case, Thomas rang Bronwen and left a message for her to ring him back, which she did. List all the possible two-step communications.

e Describe how the matrix C^2 could be used to check the total number of possible two-step connections.

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9G Problem-solving and modelling with matrices

Learning intentions

- To be able to model a practical situation using matrices.
- To be able to apply matrix arithmetic to solve practical applications.
- To be able to apply row and column matrices to solve practical applications.
- To be able to gain experience with complex and unfamiliar questions.

Data represented in matrix form can be multiplied to produce new useful information.

Example 11 Business application of matrices

Fatima and Gaia's store has a special sales promotion. One free cinema ticket is given with each DVD purchased. Two cinema tickets are given with the purchase of each computer game. The number of DVDs and games sold by Fatima and Gaia are given in matrix S. The selling price of a DVD and a game, together with the number of free tickets is given by matrix P.

$$S = \begin{matrix} \\ Fatima \\ Gaia \end{matrix} \begin{matrix} \begin{matrix} DVDs & Games \end{matrix} \\ \begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \end{matrix} \qquad P = \begin{matrix} \\ DVDs \\ Games \end{matrix} \begin{matrix} \begin{matrix} \$ & Tickets \end{matrix} \\ \begin{bmatrix} 20 & 1 \\ 30 & 2 \end{bmatrix} \end{matrix}$$

Determine the matrix product $S \times P$ and interpret the result.

Solution

$$\begin{matrix} \\ Fatima \\ Gaia \end{matrix} \begin{matrix} \begin{matrix} DVDs & Games \end{matrix} \\ \begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \end{matrix} \times \begin{matrix} \\ DVDs \\ Games \end{matrix} \begin{matrix} \begin{matrix} \$ & Tickets \end{matrix} \\ \begin{bmatrix} 20 & 1 \\ 30 & 2 \end{bmatrix} \end{matrix}$$

$$= \begin{matrix} \\ Fatima \\ Gaia \end{matrix} \begin{matrix} \begin{matrix} \$ & Tickets \end{matrix} \\ \begin{bmatrix} 7 \times 20 + 4 \times 30 & 7 \times 1 + 4 \times 2 \\ 5 \times 20 + 6 \times 30 & 5 \times 1 + 6 \times 2 \end{bmatrix} \end{matrix}$$

$$= \begin{matrix} \\ Fatima \\ Gaia \end{matrix} \begin{matrix} \begin{matrix} \$ & Tickets \end{matrix} \\ \begin{bmatrix} 260 & 15 \\ 280 & 17 \end{bmatrix} \end{matrix}$$

Fatima had sales of $260 and gave out 15 tickets.
Gaia had sales of $280 and gave out 17 tickets.

Explanation

Perform the matrix multiplication $S \times P$.

Interpret the product matrix.

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Properties of row and column matrices

Row and column matrices provide efficient ways of extracting information from data stored in large matrices.

Example 12 Using row and column matrices to extract information

Three rangers completed their monthly park surveys of feral animal sightings in matrix S.

$$S = \begin{array}{r} \\ Aaron \\ Barra \\ Chloe \end{array}\begin{array}{c} \begin{array}{cccc} Cats & Dogs & Foxes & Rabbits \end{array} \\ \begin{bmatrix} 27 & 9 & 34 & 59 \\ 18 & 15 & 10 & 89 \\ 35 & 6 & 46 & 29 \end{bmatrix} \end{array} \quad A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

a Evaluate $S \times B$.

b Interpret the information given in the product $S \times B$.

c Evaluate $A \times S$.

d Interpret the information given in the product $A \times S$.

Solution

a $S \times B = \begin{bmatrix} 27 & 9 & 34 & 59 \\ 18 & 15 & 10 & 89 \\ 35 & 6 & 46 & 29 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 27 + 9 + 34 + 59 \\ 18 + 15 + 10 + 89 \\ 35 + 6 + 46 + 29 \end{bmatrix} = \begin{bmatrix} 129 \\ 132 \\ 116 \end{bmatrix}$

b Each row of SB gives the sum of the rows in S. This gives the total sightings of all feral animals made by each ranger.

c $A \times S = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 27 & 9 & 34 & 59 \\ 18 & 15 & 10 & 89 \\ 35 & 6 & 46 & 29 \end{bmatrix}$

$= \begin{bmatrix} 27 + 18 + 35 & 9 + 15 + 6 \\ 34 + 10 + 46 & 59 + 89 + 29 \end{bmatrix}$

$= \begin{bmatrix} 80 & 30 & 90 & 177 \end{bmatrix}$

Explanation

Checking the sizes of the matrices, matrix multiplication of a 3×4 and a 4×1 matrix produces a 3×1 matrix.

Perform the matrix multiplication.

Interpret the product matrix.

Checking the sizes of the matrices, matrix multiplication of a 1×3 and a 3×4 matrix produces a 1×4 matrix.

Perform the matrix multiplication.

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d Each column of AS gives the sum of the columns in S. This gives the total sightings of each type of animal.	Interpret the product matrix.

Section Summary

- Multiplying matrices together can produce new useful information.
- Use row and column matrices strategically to give the sum of the rows or columns.
- Interpret the resulting product matrix in the context of the situation carefully.

Exercise 9G

General applications

1 The first matrix below shows the number of milkshakes and sandwiches that Helen had for lunch. The number of kilojoules (kJ) present in each food is given in the second matrix.

$$\begin{matrix} \\ Helen \end{matrix}\begin{bmatrix} Milkshakes & Sandwiches \\ 2 & 3 \end{bmatrix} \qquad \begin{matrix} kJ \\ Milkshakes \\ Sandwiches \end{matrix}\begin{bmatrix} 1400 \\ 1000 \end{bmatrix}$$

Use a matrix product to calculate how many kilojoules Helen had for lunch.

Example 11

2 The first matrix shows the number of cars and bicycles owned by two families. The second matrix records the wheels and seats for cars and bicycles.

$$\begin{matrix} \\ Smith \\ Jones \end{matrix}\begin{bmatrix} Cars & Bicycles \\ 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \begin{matrix} \\ Car \\ Bicycle \end{matrix}\begin{bmatrix} Wheels & Seats \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$$

Use a matrix product to produce a matrix that gives the numbers of wheels and seats owned by each family.

3 Eve played a game of darts. The parts of the dartboard that she hit during one game are recorded in matrix H. The bullseye is a small area in the centre of the dartboard. The points scored for hitting different regions of the dartboard are shown in matrix P.

$$H = \begin{matrix} \\ Hits \end{matrix}\begin{bmatrix} Bullseye & Inner\ region & Outer\ region \\ 2 & 13 & 5 \end{bmatrix} \qquad P = \begin{matrix} Bullseye \\ Inner\ region \\ Outer\ region \end{matrix}\begin{bmatrix} Points \\ 20 \\ 5 \\ 1 \end{bmatrix}$$

Use matrix multiplication to develop a matrix giving her score for the game.

CF

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Business applications

4 On a Saturday morning, Sand Café sold 18 quiches, 12 soups and 64 coffees. A quiche costs $5, soup costs $8 and a coffee costs $3.

- **a** Use a row matrix to record the number of each type of item sold.
- **b** Write the cost of each item in a column matrix.
- **c** Use matrix multiplication with parts **a** and **b** to calculate the total value of the mornings sales.

5 Han's stall at the football made the sales shown in the table.

Tubs of chips	Pasties	Pies	Sausage rolls
90	84	112	73

The selling prices were: chips $4, pastie $5, pie $5 and sausage roll $3.

- **a** Record the numbers of each product sold in a row matrix.
- **b** Write the selling prices in a column matrix.
- **c** Calculate the total value of the sales by using matrix multiplication of the row and column matrices found in parts **a** and **b**.

Using row and column matrices to extract information from a matrix

Example 12

6 The number of study hours by three students over four days is shown in matrix H.

$$H = \begin{array}{r} \\ Issie \\ Jack \\ Kaiya \end{array} \begin{array}{c} \begin{array}{cccc} Mon & Tues & Wed & Thur \end{array} \\ \begin{bmatrix} 2 & 3 & 2 & 3 \\ 1 & 4 & 0 & 2 \\ 3 & 4 & 3 & 2 \end{bmatrix} \end{array}$$

Use matrix multiplication with a suitable row or column matrix to do the following.

- **a** Produce a matrix showing the total study hours for each student.
- **b** From the total study hours, construct a matrix with the average hours of study for each student.
- **c** Obtain a matrix with the total number of hours studied on each night of the week.
- **d** From the total number of hours, develop a matrix with the average number of hours studied each night, correct to 1 decimal place.

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CF

7 Matrix R records four students' results in five tests.

$$R = \begin{array}{c} \\ \textit{Ellie} \\ \textit{Felix} \\ \textit{George} \\ \textit{Hannah} \end{array} \begin{array}{c} \begin{array}{ccccc} \textit{T1} & \textit{T2} & \textit{T3} & \textit{T4} & \textit{T5} \end{array} \\ \begin{bmatrix} 87 & 91 & 94 & 86 & 88 \\ 93 & 76 & 89 & 62 & 95 \\ 73 & 61 & 58 & 54 & 83 \\ 66 & 79 & 83 & 90 & 91 \end{bmatrix} \end{array}$$

Choose an appropriate row or column matrix and use matrix multiplication to do the following.

a Obtain a matrix with the sum of each student's results.

b From these sums, give a matrix with each student's average test score.

c Construct a matrix with the sum of the scores for each test.

d From these sums, give a matrix with the average score on each test.

8 The mobile phone bills of Anna, Boyd and Charlie for the four quarters of 2018 are recorded in matrix P. We will investigate the effect of multiplying by matrix E.

$$P = \begin{array}{c} \\ \textit{Anna} \\ \textit{Boyd} \\ \textit{Charlie} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Q1} & \textit{Q2} & \textit{Q3} & \textit{Q4} \end{array} \\ \begin{bmatrix} 47 & 43 & 52 & 61 \\ 56 & 50 & 64 & 49 \\ 39 & 41 & 44 & 51 \end{bmatrix} \end{array} \qquad E = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

a Calculate $P \times E$ and interpret the result of that matrix multiplication.

b State the matrix F needed to extract the costs for the second quarter, $Q2$.

c State the order of the matrix that displays Charlie's quarterly costs.

A matrix is needed so it can multiply matrix P to extract the four quarterly costs on Charlie's phone bill.

d State the order of matrix G that, when it multiplies a 3×4 matrix, gives a 1×4 matrix as the result.

e Identify whether matrix G should pre-multiply or post-multiply matrix P. Premultiply means it multiplies at the front (left) of matrix P; post-multiply means the matrix multiplies when written after (to the right of) matrix P.

f Suggest a suitable matrix G that when multiplied by matrix P will produce a matrix of Charlie's quarterly phone bills. Comment on the reasonableness of your result.

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9 Supermarkets sell eggs in boxes of 12, apples in bags of 8 and yoghurt tubs in sets of 4. This is represented by matrix A. The cost for each type of packet is given by matrix B.

$$A = \begin{matrix} \\ \textit{Items per packet} \end{matrix} \begin{matrix} \begin{matrix} \textit{Eggs} & \textit{Apples} & \textit{Yoghurt} \end{matrix} \\ \begin{bmatrix} 12 & 8 & 4 \end{bmatrix} \end{matrix} \qquad B = \begin{matrix} \\ \textit{Eggs} \\ \textit{Apples} \\ \textit{Yoghurt} \end{matrix} \begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{bmatrix} 7 \\ 4 \\ 3 \end{bmatrix} \end{matrix}$$

The sales of each type of packet are given by matrix C as a column matrix and by matrix D as a row matrix.

$$C = \begin{matrix} \\ \textit{Eggs} \\ \textit{Apples} \\ \textit{Yoghurt} \end{matrix} \begin{matrix} \begin{matrix} \textit{Packets} \end{matrix} \\ \begin{bmatrix} 100 \\ 50 \\ 30 \end{bmatrix} \end{matrix} \qquad D = \begin{matrix} \\ \textit{Packets} \end{matrix} \begin{matrix} \begin{matrix} \textit{Eggs} & \textit{Apples} & \textit{Yoghurt} \end{matrix} \\ \begin{bmatrix} 100 & 50 & 30 \end{bmatrix} \end{matrix}$$

Choose the appropriate matrices and use matrix multiplication to calculate:

a the total number of items sold (counting each egg, apple or yoghurt tub as an item)

b the total value of all sales.

10 Scalar multiplication occurs when a number multiplies a matrix. For a 2×2 matrix it has the general form:

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

Suggest why the matrix $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ is called a *scalar matrix*. Give examples to support your view.

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Key ideas and chapter summary

Matrix

A **matrix** is a rectangular array, or group, of elements displayed as rows and columns within square brackets. The rows are horizontal; the columns are vertical. The elements are usually numbers. Matrices are named using capital letters.

Size of a matrix

The **size (order) of a matrix** with m rows and n columns is an $m \times n$ matrix. It is written as: number of rows $\times$ number of columns. The number of rows is always given first. It is called an 'm by n' matrix.

Elements of a matrix

The **elements of a matrix** are the numbers within it. The position of an element is given by its row and column in the matrix. Element a_{ij} is in row i and column j of matrix A. The row is always given first.

Row matrices

A **row matrix** has only a single row of elements.

Column matrices

A **column matrix** has only a single column of elements.

Square matrices

In **square matrices** the number of rows is equal to the number of columns in the matrix.

Network diagram

A **network diagram** is made of points (vertices) joined by lines (edges). Information in a network diagram can be represented by a matrix.

Connections

A **connection** represents the relationship between two points in a network diagram. A matrix can be used to record various types of connections, such as social communications and roads directly connecting towns.

Adding matrices

Calculate the sum of matrices by adding the corresponding elements that are in the same positions. Only matrices that are the same size can be added. Otherwise, the sum is not possible, or undefined.

Subtracting matrices

Calculate the difference of matrices by subtracting the corresponding elements that are in the same positions. Only matrices of the same size can be subtracted. Otherwise, the difference is not possible, or undefined.

Zero matrix, *O*

A **zero matrix** is any matrix with zeroes as elements in every position.

Scalar

A **scalar** is a number that scales or multiplies some quantity.

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Review

Scalar multiplication

Scalar multiplication is the multiplication of a matrix by a scalar. Multiplying by a scalar does not change the size of the matrix.

Matrix multiplication

The process of multiplying a matrix by a matrix. Matrix multiplication can only be performed when the number of columns in the first matrix is equal to the number of rows in the second matrix. When the sizes of the two matrices are known and matrix multiplication is defined, the size of the product matrix will be the outside numbers. Otherwise, matrix multiplication is not possible, or undefined.

Identity matrix, I

An **identity matrix, I**, behaves like the number 1 in arithmetic. Any matrix multiplied by I remains unchanged. For 2×2 matrices,

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where $AI = A = IA$

In general, matrix multiplication is not **commutative**, but multiplication by the identity matrix is always commutative.

Power of a matrix, A^n

The matrix A is used n times in repeated multiplication. Only square matrices can be raised to a power.

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Skills checklist

Check-list

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills. ☑

9A	1	I can identify the size of a matrix.	☐
		See Example 1 and Exercise 9A Question 1.	
9A	2	I can identify the elements in a matrix.	☐
		See Example 1 and Exercise 9A Question 1.	
9A	3	I can identify a row or column matrix.	☐
		See Exercise 9A Question 3.	
9A	4	I can identify a square matrix.	☐
		See Exercise 9A Question 3.	
9B	5	I can use matrices to model practical situations.	☐
		See Example 2 and Exercise 9B Question 1.	
9C	6	I can add and subtract matrices.	☐
		See Example 3 and Exercise 9C Questions 1 and 2.	
9C	7	I can recognise and use the zero matrix.	☐
		See Exercise 9C Question 1.	
9D	8	I can perform scalar multiplication.	☐
		See Example 4 and Exercise 9D Question 1.	
9D	9	I can use scalar multiplication with other matrix arithmetic.	☐
		See Example 5 and Exercise 9D Questions 2, 3 and 4.	
9D	10	I can apply scalar multiplication to practical situations.	☐
		See Example 6 and Exercise 9D Question 5.	
9E	11	I can multiply matrices.	☐
		See Example 7 and Exercise 9E Questions 1, 2 and 4.	
9E	12	I can multiply matrices using technology.	☐
		See Spreadsheet activity 9E and Exercise 9E Questions 5, 12 and 15.	

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Review

9E	13	I can identify the identity matrix.	☐

See Example 8 and Exercise 9E Question 8.

9E	14	I can multiply by the identity matrix.	☐

See Example 8 and Exercise 9E Question 9.

9E	15	I can calculate the power of a matrix.	☐

See Example 9 and Exercise 9E Questions 1 and 10.

9F	16	I can apply matrices to communications and connections.	☐

See Example 10 and Exercise 9F Question 3.

9G	17	I can apply matrices to practical situations.	☐

See Example 11 and Exercise 9G Question 2.

9G	18	I can apply row and column matrices to practical situations.	☐

See Example 12 and Exercise 9G Question 6.

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Multiple-choice questions

Use matrix F with Questions 1 *and* 2.

$$F = \begin{bmatrix} 4 & 8 & 6 \\ 5 & 1 & 7 \end{bmatrix}$$

1 The size of matrix F is:

A $2 + 3$ **B** 2×3 **C** $3 + 2$ **D** 3×2

2 The element f_{21} is:

A 1 **B** 4 **C** 5 **D** 8

3 The results of a survey of the number of electronic devices owned by the families of three students are shown in the matrix.

$$\begin{array}{c} \\ \textit{Caroline} \\ \textit{Delia} \\ \textit{Emir} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{TVs} & \textit{VCRs} & \textit{PCs} \end{array} \\ \begin{bmatrix} 4 & 3 & 2 \\ 1 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix} \end{array}$$

The number of PCs owned by Emir's family is:

A 1 **B** 2 **C** 3 **D** 4

Use matrices M and N with Questions 4 *to* 7.

$$M = \begin{bmatrix} 7 & 6 \\ 4 & 3 \end{bmatrix} \quad N = \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix}$$

4 The matrix $M + N$ is:

A $\begin{bmatrix} 12 & 8 \\ 5 & 3 \end{bmatrix}$ **B** $\begin{bmatrix} 12 & 4 \\ 4 & 3 \end{bmatrix}$ **C** $\begin{bmatrix} 12 & 4 \\ 4 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 12 & 4 \\ 5 & 3 \end{bmatrix}$

5 The matrix $M - N$ is:

A $\begin{bmatrix} 2 & 8 \\ 3 & 3 \end{bmatrix}$ **B** $\begin{bmatrix} 2 & 8 \\ 3 & 0 \end{bmatrix}$ **C** $\begin{bmatrix} 2 & 4 \\ 3 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 2 & 4 \\ 3 & 3 \end{bmatrix}$

6 The matrix $N - N$ is:

A 0 **B** $\begin{bmatrix} 0 \end{bmatrix}$ **C** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix}$

7 The matrix $2N$ is:

A $\begin{bmatrix} 10 & -4 \\ 2 & 0 \end{bmatrix}$ **B** $\begin{bmatrix} 7 & 0 \\ 3 & 2 \end{bmatrix}$ **C** $\begin{bmatrix} 10 & -4 \\ 1 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 10 & -2 \\ 2 & 0 \end{bmatrix}$

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Review

Use the matrices P, Q, R, S in Questions 8 to 10.

$$P\begin{bmatrix}5 & 4 & 1\\ 7 & 6 & 8\end{bmatrix} \quad Q=\begin{bmatrix}2\\ 6\\ 0\end{bmatrix} \quad R=\begin{bmatrix}4 & 7\end{bmatrix} \quad S=\begin{bmatrix}1 & 3\\ 0 & 4\end{bmatrix}$$

8 Matrix multiplication is not defined for:

A PS **B** RS **C** SP **D** SS

9 The order of matrix QR is:

A 1×1 **B** 2×3 **C** 3×2 **D** 5

10 Identify the matrix multiplication that would give a 1×3 matrix.

A RP **B** RQ **C** PR **D** QR

Short-response questions

1 For each of the following matrices:

i state the size **ii** identify the values of the listed elements.

a $A=\begin{bmatrix}9 & 4 & 7\\ 10 & 3 & 4\end{bmatrix}$ a_{21} and a_{13}

b $B=\begin{bmatrix}12 & 16 & 18\end{bmatrix}$ b_{12} and b_{11}

c $C=\begin{bmatrix}11 & 13\\ 16 & 14\end{bmatrix}$ c_{22} and c_{12}

d $D=\begin{bmatrix}1 & 3 & 4 & 8\\ 6 & 3 & 7 & 9\\ 8 & 6 & 9 & 2\end{bmatrix}$ d_{24} and d_{32}

2 Identify the matrices that are:

a the same size **b** row matrices **c** column matrices **d** square matrices.

$$M=\begin{bmatrix}-12 & 10\\ 13 & -15\end{bmatrix} \quad N=\begin{bmatrix}19\\ -56\\ 0\end{bmatrix} \quad O=\begin{bmatrix}5 & 6\\ 4 & 2\\ 9 & -3\end{bmatrix}$$

$$P=\begin{bmatrix}16 & 81 & 42\end{bmatrix} \quad Q=\begin{bmatrix}4 & 5\\ 3 & 6\end{bmatrix} \quad R=\begin{bmatrix}8 & -6 & 9\end{bmatrix}$$

$$S=\begin{bmatrix}6 & 9 & 0 & 4\\ 2 & -2 & 3 & 5\\ 4 & 12 & 15 & 18\end{bmatrix} \quad T=\begin{bmatrix}24 & -22\\ -45 & 12\\ 65 & 17\end{bmatrix} \quad U=\begin{bmatrix}-9\\ 5\\ 4\end{bmatrix}$$

3 **a** Write the 2×2 identity matrix.

b Write the 3×3 identity matrix.

c Write a 4×4 zero matrix.

SF

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4 Consider the matrices below.

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 8 \\ 10 & 6 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

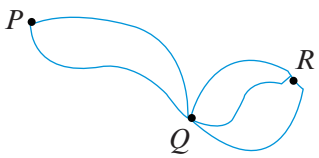
Evaluate:

a $3A$ **b** $\frac{1}{2}B$ **c** $A + B$ **d** $B - A$

e $A - A$ **f** AB **g** IB **h** AI

i A^2 **j** B^2 **k** $A^2 + B^2$ **l** $(A + B)^2$

5 Roads are shown joining towns P, Q and R. Use a matrix to record the number of roads directly connecting one town to another town.



6 Consider the matrices below.

$$A = \begin{bmatrix} 6 & -2 \\ -3 & 7 \\ 4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 9 \\ -1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

Evaluate:

a $B + C$ **b** $A + B$ **c** $C - A$ **d** $3B - 4C$

e AB **f** BA **g** A^2 **h** $3C^2$

7 Perform the required matrix multiplications.

a $\begin{bmatrix} 7 & 9 & 13 \\ 15 & 18 & 12 \end{bmatrix} \begin{bmatrix} 11 & 8 & 3 & 24 \\ 24 & 7 & 19 & 4 \\ 10 & -5 & -2 & 11 \end{bmatrix}$ **b** $\begin{bmatrix} 6 & -9 & 7 \\ 31 & 14 & 17 \\ 5 & 23 & -3 \end{bmatrix} \begin{bmatrix} 6 & -21 & 22 \\ 2 & 3 & 10 \\ 8 & 13 & 16 \end{bmatrix}$

8 For the matrix $A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$, calculate A^3.

9 Consider $A = \begin{bmatrix} 2 & 0 & 5 \\ 1 & 8 & 3 \\ 0 & 4 & 6 \end{bmatrix}$

a Without using technology, calculate:

i A^2 **ii** A^3

b Using technology, find A^4.

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10 Patsy and Geoff decided to participate in a charity fun run.

a Patsy plans to walk for 4 hours and jog for 1 hour. Geoff plans to walk for 3 hours and jog for 2 hours. Write out matrix A, filling in the missing information.

$$A = \begin{matrix} \\ \text{Patsy} \\ \text{Geoff} \end{matrix} \begin{matrix} \begin{matrix} \text{Hours walking} & \text{Hours jogging} \end{matrix} \\ \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \end{matrix}$$

b Walking raises \$2 per hour and burns 1500 kJ/h (kilojoules per hour). Jogging raises \$3 per hour and burns 2500 kJ/h. Write out matrix B, filling in the missing information.

$$B = \begin{matrix} \\ \text{Walking} \\ \text{Jogging} \end{matrix} \begin{matrix} \begin{matrix} \$ & kJ \end{matrix} \\ \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \end{matrix}$$

c Use matrix multiplication to construct a matrix that shows the money raised and the kilojoules burned by each person.

11 The matrix gives the numbers of roads directly connecting one town to another.

$$\begin{matrix} \\ D \\ E \\ F \end{matrix} \begin{matrix} \begin{matrix} D & E & F \end{matrix} \\ \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix} \end{matrix}$$

a Use a row matrix to calculate the total number of roads directly connecting town E to other towns.

b Use a column matrix to calculate the total number of roads directly connecting town D to other towns.

c Sketch a possible network diagram for the information represented in this matrix.

12 A bakery recorded the sales for Shop A and Shop B of cakes, pies and rolls in a Sales matrix, S. The prices were recorded in the Prices matrix, P.

$$S = \begin{matrix} \\ A \\ B \end{matrix} \begin{matrix} \begin{matrix} \text{Cakes} & \text{Pies} & \text{Rolls} \end{matrix} \\ \begin{bmatrix} 12 & 25 & 18 \\ 15 & 21 & 16 \end{bmatrix} \end{matrix} \qquad P = \begin{matrix} \\ \text{Cakes} \\ \text{Pies} \\ \text{Rolls} \end{matrix} \begin{matrix} \$ \\ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \end{matrix}$$

a State the number of pies that were sold by Shop B.

b State the selling price of pies.

c Calculate the matrix product SP.

d Interpret the information contained in matrix SP.

e Justify which shop had the greater income from its sales by commenting on its earnings.

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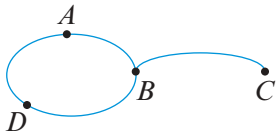
13 Roads connecting the towns Adz, Bez, Coz and Dzo are shown in the diagram. The first letter of each town is used.



a Use matrix arithmetic to identify the number of ways there are to travel from one town to another via one other town.

b List the possible ways that start and end at Bez.

14 Farms *A* and *B* have their livestock numbers recorded in the matrix shown.

$$\begin{array}{c} \\ \textit{Farm A} \\ \textit{Farm B} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Cattle} & \textit{Pigs} & \textit{Sheep} \end{array} \\ \begin{bmatrix} 420 & 50 & 100 \\ 300 & 40 & 220 \end{bmatrix} \end{array}$$

Use matrix arithmetic to show the amount of each livestock that the farms would need to buy so that Farm B has half as many cattle as Farm A, Farm B has three times the number of pigs as Farm A, and both farms have equal number of sheep. Comment on the reasonableness of your answer.

15 For these two matrices to be equal, solve for the pronumerals.

$$\begin{bmatrix} 4 & 2x + 3y \\ y & 9 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ x + 1 & 9 \end{bmatrix}$$

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Chapter 10

Univariate data analysis

Chapter questions

UNIT 2: APPLICATIONS OF LINEAR EQUATIONS AND TRIGONOMETRY, MATRICES AND UNIVARIATE DATA ANALYSIS

Topic 4: Univariate data analysis 1

- What is univariate data?
- What are categorical and numerical data?
- What is a frequency table?
- How do I construct and interpret pie, column and bar charts?
- How do I construct and interpret a histogram?
- How do I construct and interpret dot plots and a stem plots?
- What are the mean, median, mode, range, interquartile range and standard deviation?
- How do I know which summary statistics to use?

In this information age, we are increasingly required to interpret data. This data may be presented in charts, diagrams or graphs, or it may simply be lists of words or numbers. There may be a lot of useful information in the data, but the story it has to tell may not be immediately clear. There are numerous statistical procedures which can be used to help us extract that information from the data. In this chapter, we will look at some of the techniques which can be used when the data are collected from a **single variable**. This type of data is called **univariate** data.

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10A Types of data

Learning intentions

- To be able to classify data as categorical or numerical.
- To be able to classify categorical data as nominal or ordinal.
- To be able to classify numerical data as discrete or continuous.

Consider the following situation. In completing a survey, students in a senior college are asked to:

- indicate whether they were born in Australia or elsewhere by circling an 'A' for Australia or an 'E' for elsewhere
- indicate their preferred coffee cup size when buying takeaway coffee as 'small', 'medium' or 'large'
- write down the number of brothers they have
- measure and write their hand span in centimetres.

The information collected from four students is displayed in the table below.

Place of birth	*Coffee size*	*Number of brothers*	*Hand span (in cm)*
A	Small	0	23.6
E	Large	2	19.6
A	Small	1	20.2
E	Large	1	24.0

As the answers to each question in the survey will vary from student to student, each question defines a different **variable**, namely: *country of birth*, *coffee size*, *number of brothers* and *hand span*. Note that is is usual to write the variable name in italics.

The values we collect about each of these variables are called **data**.

The data in the table fall into two broad types: **categorical** or **numerical**.

Categorical data

The data arising from the students' responses to the first and second questions in the survey are called **categorical data** because the data values can be used to place the person into one of several groups or categories. However, the properties of the data generated by these two questions differ slightly.

- The question asking students to indicate where they were born will prompt a response of either A or E. This identifies the respondent as born either in Australia or elsewhere, but tells us no more. We call this **nominal data** because the values of the variable are simply names.

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- However, the question with responses ‘small’, ‘medium’ and ‘large’ that indicates the students’ preferred coffee size tells us two things. First, it names the coffee size, but it also enables us to order the students according to their preferred coffee sizes. We call this **ordinal data** because it enables us to both name and order their responses.

Numerical data

The data arising from the responses to the third and fourth questions in the survey are called **numerical data** because they have values for which arithmetic operations such as adding and averaging make sense. However, the properties of the data generated by these questions differ slightly.

- The question asking students to write down the number of brothers they have will prompt whole number responses such as $0, 1, 2, \ldots$ Because the data can only take particular numerical values it is called **discrete data**. Discrete data arises in situations where counting is involved.
- In response to the hand span question, the student who wrote 24.1 cm could have an actual hand span of anywhere between 23.05 and 24.04 cm, depending on the accuracy of the measurement and how the student rounded their answer. This is called **continuous data**, because the variable we are measuring, in this case, *hand span*, can take any numerical value within a specified range. Continuous data are generally generated when measurement is involved.

Types of variables

Categorical variables

Variables that generate categorical data are called **categorical variables** or, if we need a finer distinction, **nominal** or **ordinal** variables. For example, *country of birth* is a nominal variable, and *coffee size* is an ordinal variable.

Numerical variables

Variables that generate numerical data are called **numerical variables** or, if we need a finer distinction, **discrete** or **continuous** variables. For example, *number of brothers* is a discrete variable, while *hand span* is a continuous variable.

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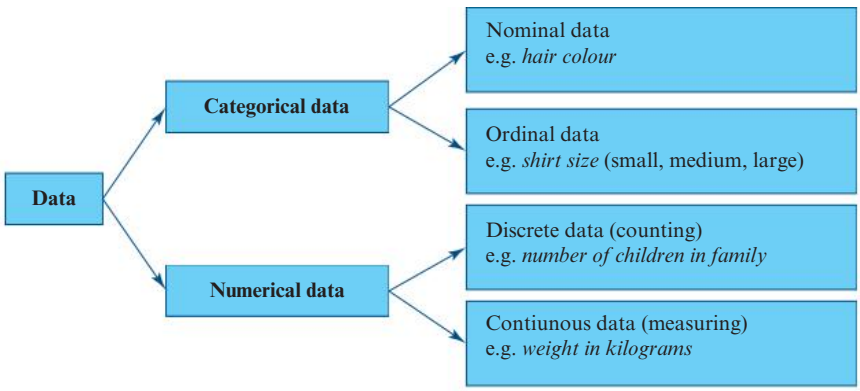
Numerical or categorical?

The relationship between categorical data (nominal or ordinal) and numerical data (discrete or continuous) is displayed in the following diagram:



Example 1 Categorical and numerical data

Classify the following data as categorical or numerical.

a Students choose their favourite pet from 'dog', 'cat', 'bird' or 'other'.

b The time taken to solve a puzzle in seconds is recorded.

Solution

a **Categorical**, as the values of the data are types of pet.

b **Numerical**, as the data takes values which represent the amount of time taken.

Example 2 Nominal and ordinal data

Classify the following data as nominal or ordinal.

a A group of people record their level of happiness as 'very happy', 'happy', 'not too happy' or 'very unhappy'.

b Students select their favourite country to visit.

Solution

a **Ordinal**, as the data takes values which represent the person's level of happiness, and there is an order to the categories.

b **Nominal**, as the data takes values which are names of countries.

Example 3 Discrete and continuous data

Classify the following numerical data as discrete or continuous.

a The number of children in the family is recorded for all the students in a school.

b The birth weight of babies measured in grams is recorded at hospital.

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Solution

a **Discrete**, as the number of children will only take whole number values.

b **Continuous**, as the data can take any value, limited only by the accuracy to which the weight can be measured.

Example 4 Classifying variables

Classify the following variables as nominal, ordinal, discrete or continuous.

a The number of students in each of 10 classes is counted.

b The time taken for 20 mice to each complete a maze is recorded in seconds.

c Diners at a restaurant were asked to rate how they felt about their meal as 1= Very satisfied, 2 = Satisfied, 3 = Indifferent, 4 = Dissatisfied, 5 = Very dissatisfied.

d Students select a colour from the list 1= Blue, 2 = Green, 3 = Red, 4 = Yellow.

e Students heights were classified as '<160 cm', '160 cm–180 cm' or '>180 cm'.

Solution

a **Discrete**, as the number of students will only take whole number values.

b **Continuous**, as the data can take any value, limited only by the accuracy to which the time can be measured.

c **Ordinal**, as the numbers in this data do not represent quantities, they represent each diner's level of approval of the meal.

d **Nominal**, as the numbers in this data represent colours, not quantities.

e **Ordinal**, as each student's height has been recorded in one of 3 categories which can be ordered.

Note the variable classification of *height* in the previous example as ordinal was determined by how we chose to measure the variable (in categories), not what the variable was measuring. If we had chosen to measure *height* in cm, then this would have been a continuous variable.

Section Summary

- Data can be classified as **categorical** (nominal or ordinal), or **numerical** (discrete or continuous).
- **Nominal data** takes values which are simply the **names** of categories.
- **Ordinal data** takes values which both **name** and **order** categories.
- **Discrete** data can only take particular numerical values, often whole numbers.
- **Continuous** data can take any numerical value within a specified range.

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Exercise 10A

Classifying data

Example 1

1 Classify the data generated in each of the following as categorical or numerical.

- **a** Kindergarten pupils bring along their favourite toys, and they are grouped together under the headings ‘dolls’, ‘soft toys’, ‘games’, ‘cars’ and ‘other’.
- **b** The number of students on each of 20 school buses are counted.
- **c** A group of people each write down their favourite colour.
- **d** Each student in a class is weighed in kilograms.
- **e** Students are weighed and then classified as ‘light’, ‘average’ or ‘heavy’.
- **f** People rate their enthusiasm for a certain rock group as ‘low’, ‘medium’ or ‘high’.

Example 2

2 Classify the categorical data arising from people answering the following questions as either nominal or ordinal.

- **a** What is your favourite football team?
- **b** How often do you exercise? Choose one of ‘never’, ‘once a month’, ‘once a week’, ‘every day’.
- **c** Indicate how strongly you agree with the statement ‘alcohol is the major cause of accidents’ by selecting one of ‘strongly agree’, ‘agree’, ‘disagree’, ‘strongly disagree’.
- **d** What language will you study next year, ‘French’, ‘Chinese’, ‘Spanish’ or ‘none’?

Example 3

3 Classify the numerical data listed below as discrete or continuous.

- **a** The number of pages in a book
- **b** The price paid to fill the tank of a car with petrol
- **c** The volume of petrol (in litres) needed to fill the tank of a car
- **d** The time between the arrival of successive customers at an ATM
- **e** The number of people at a football match.

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Example 4

4 Classify the data generated in each of the following situations as nominal, ordinal, discrete or continuous.

- **a** The different brand names of instant soup sold by a supermarket are recorded.
- **b** A group of people are asked to indicate their attitude to capital punishment by selecting a number from 1 to 5, where 1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree and 5 = strongly agree.
- **c** The number of computers per household was recorded during a census.
- **d** The annual salaries of the employees of a certain company were recorded as 'less than $60 000', '$60 000 - $90 000', 'more than $90 000'.

5 Classify the following variables as nominal, ordinal, discrete or continuous.

- **a** Amount of money in a bank account, rounded to the nearest $1000
- **b** Reading ability assessed as 'below average', 'average' or 'above average'
- **c** Letter grades on an assignment
- **d** Religion.

6 Classify data collected in each of the following situations as nominal, ordinal, discrete or continuous.

- **a** Time it takes students to travel to school
- **b** Volume of liquid in a cup
- **c** The number of coins each students has in their purse or wallet
- **d** Difficulty of a test rated by students as 1 = Easy, 2 = OK, 3 = Difficult.

7 Classify the data collected in each of the following situations as nominal, ordinal, discrete or continuous.

- **a** Eye colour recorded as 'blue', 'green', 'hazel' or 'brown'
- **b** The number of words in each of the essays submitted by a class of students
- **c** The percentage of answers each student got correct on a test
- **d** Temperatures in degrees Celsius for the month of March.

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10B Displaying and describing categorical data

Learning intentions

- To be able to construct frequency and percentage frequency tables.
- To be able to construct bar and column charts from frequency tables.
- To be able to construct pie charts from frequency tables.
- To be able to use the mode as a summary statistic for categorical data.

To make sense of a dataset, we first need to organise it into a more manageable form. For categorical data, our first step is to construct a frequency table.

The frequency table

Frequency

A **frequency table** is a listing of the values a variable takes in a dataset, along with how often (frequently) each value occurs.

Frequency can be recorded as a:

- **frequency**: a count of the number of times a value occurs
- **percentage frequency**: the percentage of times a value occurs, where:

$$\text{percentage frequency} = \frac{\text{count}}{\text{total}} \times 100\%$$

- frequency **distribution**: a listing of the values a variable takes, along with how frequently each of these values occurs.

Example 5 Constructing a frequency table for categorical data

Thirty children chose a sandwich, a salad or a pie for lunch, as follows:

sandwich, salad, salad, pie, sandwich, sandwich, salad, salad, pie, pie, pie,
salad, pie, sandwich, salad, pie, salad, pie, sandwich, sandwich, pie, salad,
salad, pie, pie, pie, salad, pie, sandwich, pie

Construct a table for the data showing both frequency and percentage frequency.

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Solution

Lunch choice	Frequency	
	Number	%
Sandwich	7	23.3
Salad	10	33.3
Pie	13	43.3
Total	30	100.0

Explanation

Set up a table as shown. The variable *lunch choice* has three categories: 'sandwich', 'salad' and 'pie'.

Count the numbers of children choosing a sandwich, a salad or a pie. Record them in the Number column.

Add the frequencies to find the total number.

Convert the frequencies into percentages and record them in the '%' column. For example, percentage frequency for pies equals $\frac{13}{30} \times 100\% = 43.3\%$.

The total frequency equals $\frac{30}{30} \times 100\% = 100\%$. Note that if we were to add the rounded percentages for sandwiches, pies, and salads, we would obtain a value of 99.9% due to rounding error.

Column charts

When there are a lot of data, a frequency table can be used to summarise the information, but we generally find that a graphical display is also useful. When the data are categorical, an appropriate display is a **column chart**.

Column charts

In a column chart:

- frequency or percentage frequency is shown on the vertical axis
- the variable being displayed is shown on the horizontal axis
- the height of the column gives the frequency (or percentage frequency)
- the columns are drawn with gaps to indicate that each value is a separate category
- there is one column for each category.

Example 6 Constructing a column chart and percentage column chart from a frequency table

Use the frequency table for lunch choice from Example 5 to construct:

a a column chart

b a percentage column chart.

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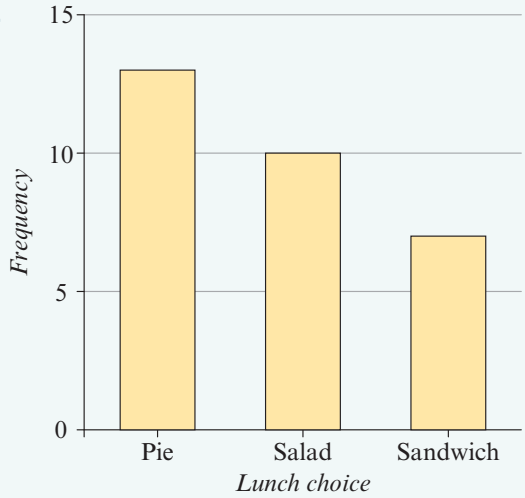
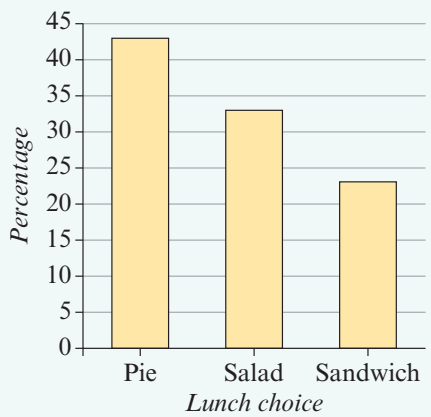
Solution

a



b



Explanation

Label the horizontal axis with the variable name *Lunch choice*. Mark the scale off into three equal intervals and label them 'Pie', 'Salad' and 'Sandwich'.
Label the vertical axis 'Frequency'. Insert a scale allowing for the maximum frequency of 13. Up to 15 would be appropriate. Mark the scale in multiples of 5.
For each interval, draw in a column as shown. Make the width of each column less than the width of the category intervals to show that the categories are quite separate.
The height of each column is equal to the frequency.

The lunch choice percentage frequencies are: sandwich 23.3%, salad 33.3%, pie 43.3%. To construct a percentage column chart of the lunch choice data, follow the procedure as for part **a** but label the vertical axis 'Percentage'. Insert a scale allowing for a maximum percentage frequency up to 45%. Mark the vertical scale in intervals of 5%.
The height of each column is equal to the percentage frequency.

Note: For nominal variables it is common, but not necessary, to list categories in decreasing order by frequency. This makes later interpretation easier.

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Bar charts

Another appropriate display for categorical data is a **bar chart**. Both the bar chart and the column charts display data using bars where the length of the bar is given by the frequency of the data values. The difference is that the bar chart is oriented horizontally, whereas a column chart is oriented vertically. This leads us to the following definition.

Bar charts

In a bar chart:

- frequency or percentage frequency is shown on the horizontal axis
- the variable being displayed is shown on the vertical axis
- the length of the bar gives the frequency (or percentage frequency)
- the bars are drawn with gaps to indicate that each value is a separate category
- there is one bar for each category.

Example 7 Constructing a bar chart and percentage bar chart from a frequency table

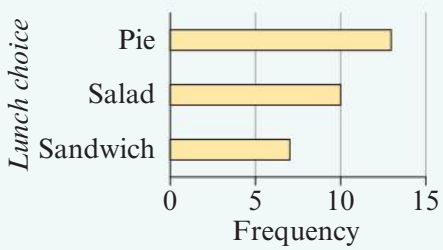
Use the frequency table for lunch choice from Example 5 to construct:

a a bar chart

b a percentage bar chart.

Solution

a



Explanation

Label the vertical axis with the variable name *Lunch choice*. Mark the scale off into three equal intervals and label them 'Pie', 'Salad' and 'Sandwich'.

Label the horizontal axis 'Frequency'. Insert a scale allowing for the maximum frequency of 13. Up to 15 would be appropriate. Mark the scale in multiples of 5.

For each interval, draw in a bar as shown. Make the width of each bar less than the width of the category intervals to show that the categories are quite separate.

The horizontal length of each bar is equal to the frequency.

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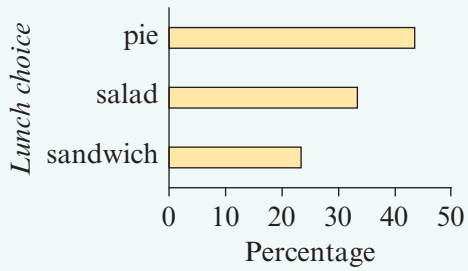
b



The lunch choice percentage frequencies are: sandwich 23.3%, salad 33.3%, pie 43.3%. To construct a percentage bar chart, follow the procedure as for part **a** but label the horizontal axis 'Percentage'. Insert a scale allowing for a maximum percentage frequency up to 45%. Mark the horizontal scale in intervals of 5%.
The horizontal length of each bar is equal to the percentage frequency.

Pie charts

Also commonly used to display categorical data is a **pie chart**. In a pie chart the data is represented in a circular graph divided into segments. Each segment of the circle represents the frequency or percentage frequency of that particular value of the variable. Generally, the segments are shown in different colours, and a key which links each segment to a value of the variable is required.

Pie charts

In a pie chart:

- the data is represented as a circle
- the circle is divided into segments, one segment for each value of the variable
- the size of the segment is proportional to the relative frequency or percentage frequency of that value
- a key is generally required.

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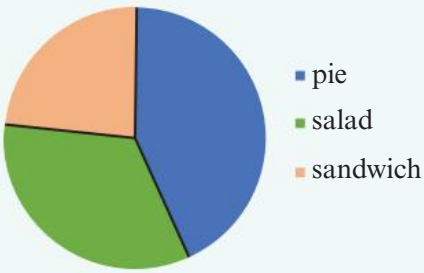
Example 8 **Constructing a pie chart from a frequency table**

Use the frequency table for lunch choice from Example 5 to construct a pie chart.

Solution



Explanation

Since the total angle at the centre of a circle is 360° we determine the angle that each segment makes at the centre of the circle as follows:

$$\text{sandwich} = \frac{7}{30} \times 360 = 84°$$

$$\text{salad} = \frac{10}{30} \times 360 = 120°$$

$$\text{pie} = \frac{13}{30} \times 360 = 156°$$

A protractor would be required to measure angles so that a pie chart can be constructed accurately. In practice, we would use a spreadsheet to construct the pie chart. The pie chart can be difficult to interpret when there are more than three segments, or if the segments are of similar sizes. For this reason, the column chart and bar chart are generally the preferred displays for a categorical variable.

The mode or modal category

One of the features of a dataset that is quickly revealed in each of the charts is the **mode** or **modal category**. This is the most frequently occurring category. In a column chart, this is given by the category with the tallest column. For the column chart in Example 6, the modal category is clearly 'pie'. That is, the most frequent or popular lunch choice was a pie.

When is the mode useful?

The mode is most useful when a single value or category in the frequency table occurs more often (frequently) than the others. Modes are of particular importance in popularity polls, when answering questions such as 'Which is the most frequently watched TV station between the hours of 6 p.m. and 8 p.m.?' or 'When is a supermarket in peak demand?'

Section Summary

- Categorical data can be summarised in a **frequency table** or a **percentage frequency table**.
- Frequency tables and percentage frequency tables can be displayed in **column charts**, **bar charts** and **pie charts**.
- The value of a categorical variable with the highest frequency is called the **mode**.

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Skill-sheet

Exercise 10B

Constructing frequency tables

Example 5

1 The *gender* of 15 people in a bus is as shown:

F M M M F M F F M M M F M M M

a State whether the variable *gender* is nominal or ordinal.

b Construct a frequency table for the data, including frequency and percentage frequency.

2 The *t-shirt sizes* for 20 males are as shown:

M L M S XL M L M L XL

S M M M L L XL M XL L

a State whether the variable *t-shirt size* is nominal or ordinal.

b Construct a frequency table for the data, including frequency and percentage frequency.

3 A group of people were asked to respond to the question 'On average, how often do you think about leaving your job?' by choosing one of the following options:
1 = never 2 = occasionally 3 = every month 4 = every week 5 = every day
The following data were collected:

1 3 5 5 1 4 2 1 1 2 5 3 3 5 1 2

a State whether the data collected are nominal or ordinal.

b Construct a frequency table for the data, including frequency and percentage frequency, giving percentage frequencies to two decimal places.

Analysing frequency tables and constructing column charts

Example 6

4 The table opposite shows the frequency distribution of the favourite type of fast food (*food type*) of a group of students.

a Complete the table.

b State whether the variable *food type* is nominal or ordinal.

c State the number of students who preferred Chinese food.

d State the percentage of students who chose chicken as their favourite fast food.

e Identify the mode.

f Construct a column chart of the frequencies (number).

Food type	Frequency	
	Number	%
Hamburger	23	33.3
Chicken	7	10.1
Fish and chips	6	
Chinese	7	10.1
Pizza	18	
Other	8	11.6
Total		100.0

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5 The following responses were received to a question regarding the return of capital punishment.

a Complete the table.

b Classify the data in the table as nominal or ordinal.

c State the number of people who said 'Strongly agree'.

d State the percentage of people who said 'Strongly disagree'.

e Identify the mode.

f Construct a column chart of the frequencies.

Capital punishment	Frequency	
	Number	%
Strongly agree	21	8.2
Agree	11	4.3
Don't know	42	
Disagree		
Strongly disagree	129	50.4
Total	256	100.0

Analysing frequency tables and constructing bar charts

Example 7

6 A bookseller noted the types of books purchased on a particular day and recorded them in the table.

a Complete the table.

b Classify the variable *type of book* as nominal or ordinal.

c State the number of 'Fiction' books.

d State the percentage of books classified as 'Children'.

e Determine the total number of books purchased.

f Construct a bar chart of the frequencies.

Type of book	Frequency	
	Number	%
Children	53	22.8
Fiction	89	
Cooking	42	18.1
Travel	15	
Other	33	14.2
Total	232	

7 A survey of the ways in which secondary school students preferred to spend their leisure time gave the results shown in the table.

a Determine the number of students surveyed.

b Classify the variable *leisure activity* as nominal or ordinal.

c State the percentage of students that preferred to spend their leisure time chatting with friends.

d Identify the mode.

e Construct a bar chart of the percentage frequencies (%).

Leisure activity	Frequency	
	Number	%
Play video games	84	42
Read	26	13
Use social media	46	23
Chat with friends	24	12
Watch a movie	8	4
Other	12	6
Total	200	100

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Constructing pie charts

Example 8

8 A group of students were asked whether they would prefer their school camp activity to be surfing or snorkelling. Their responses are summarised in the following percentage frequency table.

Activity	*%*
Surfing	75
Snorkelling	25
Total	100

a Construct a pie chart of the percentage frequencies.

b Identify the modal response to this question.

9 A group of students were asked whether they thought they should spend less, the same, or more time on social media. Their responses are summarised in the following frequency table.

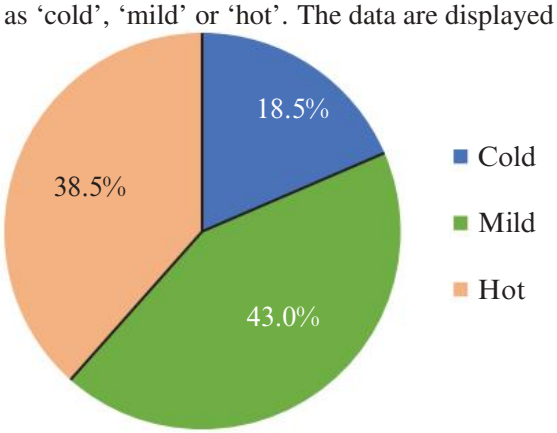
Social media time	Frequency
Less	250
Same	100
More	50
Total	400

a Construct a pie chart to display the data.

b Identify the modal response to this question.

Interpreting column and pie charts

10 In an investigation into climate type, the climate types of 200 countries were classified as 'cold', 'mild' or 'hot'. The data are displayed in the following pie chart.



a Give the variable a name, and classify it as nominal, ordinal, discrete or continuous.

b Determine the number of countries classified as 'hot'.

c Identify the modal value for the variable.

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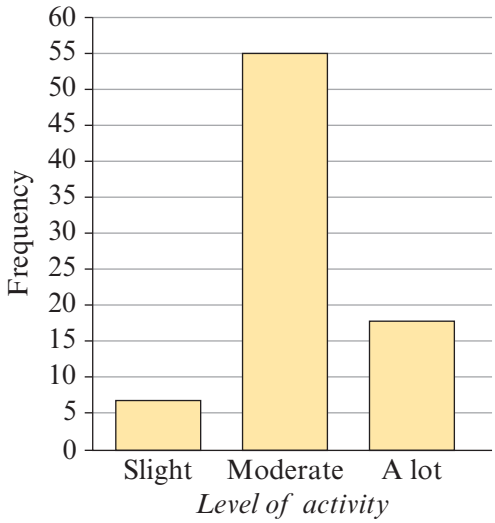
11 In a survey people were asked to select their level of activity as 'slight', 'moderate' or 'a lot'. Their responses are summarised in the following column chart.



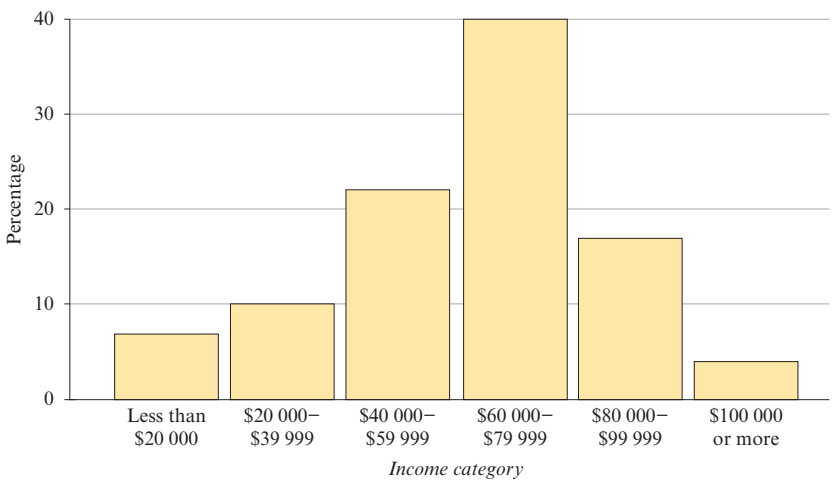
a Give the name of the variable, and classify it as nominal, ordinal, discrete or continuous.

b Determine how many people responded that they exercise 'a lot'.

c Determine the number of people who responded to this question in the survey.

d Identify the modal value for the variable, and determine the percentage of people who chose that response.

12 The following percentage bar chart shows the incomes of people in a company with 200 employees.



a Classify the variable *income category* as nominal, ordinal, discrete or continuous.

b Determine how many employees earned salaries in the modal income category.

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10C Interpreting and describing frequency tables and column charts

Learning intentions

- To be able to explain the features of a categorical variable by interpreting frequency tables and column charts.
- To be able to write a report which describes the distribution of a categorical variable.

As part of this subject you will be expected to complete a statistical investigation. Under these circumstances, constructing a frequency table or a column or bar chart is not an end in itself. The end is being able to understand something about the variables you are investigating that you didn't know before.

To complete the investigation, you will need to communicate this finding to others. To do this, you will need to know how to describe and interpret any patterns you observe in the context of your data investigation in a written report that is both systematic and concise. The purpose of this section is to help you develop such skills.

Some guidelines for describing the distribution of a categorical variable and communicating your findings

- Briefly summarise the context in which the data were collected, including the number of people (or things) involved in the study.
- If there is a clear modal category, make sure that it is mentioned.
- Include relevant counts or percentages in the report.
- If there are a lot of categories, it is not necessary to mention every category.
- Either counts or percentages can be used to describe the distribution.

These guidelines are illustrated in the following examples.

Example 9 Using a frequency table to describe the outcome of an investigation involving a categorical variable

Thirty children were offered a choice of a sandwich, a salad or a pie for lunch. Their responses were collected and summarised in the frequency table opposite.

Lunch choice	Frequency
Sandwich	7
Salad	10
Pie	13
Total	30

Use the frequency table to report on the relative popularity of the three lunch choices, quoting appropriate frequencies to support your conclusions.

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Solution

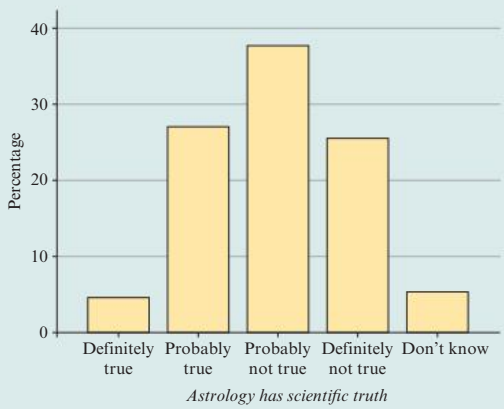
A group of 30 children were offered a choice of a sandwich, a salad or a pie for lunch. The most popular lunch choice was a pie, chosen by 13 of the children. Ten of the children chose a salad. The least popular option was a sandwich, chosen by only 7 of the children.

Example 10 Using a frequency table and a percentage column chart to describe the outcome of an investigation involving a categorical variable

A sample of 200 people were asked to comment on the statement 'Astrology has scientific truth' by selecting one of the options 'definitely true', 'probably true', 'probably not true', 'definitely not true' or 'don't know'.

The data are summarised in the following frequency table and column chart. Note that the categories in the frequency table are listed in a definite order because the data are ordinal.

Astrology has scientific truth	Frequency	
	Number	%
Definitely true	9	4.5
Probably true	54	27.0
Probably not true	75	37.5
Definitely not true	51	25.5
Don't know	11	5.5
Total	200	100.0



Write a report summarising the findings of this investigation, quoting appropriate percentages to support your conclusion.

Solution

Two hundred people were asked to respond to the statement 'Astrology has scientific truth'. The majority of respondents did not agree, with 37.5% responding that they believed that this statement was probably not true, and another 25.5% declaring that the statement was definitely not true. Over one quarter (27%) of the respondents thought that the statement was probably true, while only 4.5% of subjects thought that the statement was definitely true.

Section Summary

- Frequency tables, column, bar and pie charts all help us to understand the distribution of a categorical variable.
- Important features such as the values of the variable and their associated frequencies should be included in the report.

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Exercise 10C

Interpreting and describing frequency tables and column charts

Example 9

1 A group of 69 students were asked to nominate their preferred type of fast food. The results are summarised in the percentage frequency table opposite. Use the information in the table to complete the report below by filling in the blanks.

Fast food type	*%*
Hamburger	33.3
Chicken	10.1
Fish and chips	8.7
Chinese	10.1
Pizza	26.1
Other	11.6
Total	100.0

Report

A group of _____ students were asked their favourite type of fast food. The most popular response was _____ (33.3%), followed by pizza (_____). The rest of the group were almost evenly split between chicken, fish and chips, Chinese and other, all around 10%.

2 Two hundred and fifty-six people were asked whether they agreed that there should be a return to capital punishment in their state. Their responses are summarised in the table opposite. Use the information in the table to complete the report below by filling in the blanks.

Capital punishment	*%*
Strongly agree	8.2
Agree	4.3
Don't know	16.4
Disagree	20.7
Strongly disagree	50.4
Total	100.0

Report

A group of 256 people were asked whether they agreed that there should be a return to capital punishment in their state. The majority of these people _____ (50.4%), followed by _____ who disagreed. Levels of support for return to capital punishment were quite low, with only 4.3% agreeing and 8.2% strongly agreeing. The remaining _____ said that they didn't know.

3 A group of 200 students were asked how they prefer to spend their leisure time.
The results are summarised in the frequency table.
Use the information in the table to write a brief report of the results of this investigation, quoting appropriate percentages as mathematical reasoning to support your conclusions.

Leisure activity	*%*
Internet and digital games	42
Read	13
Listen to music	23
Watch TV or go to movies	12
Phone friends	4
Other	6
Total	100

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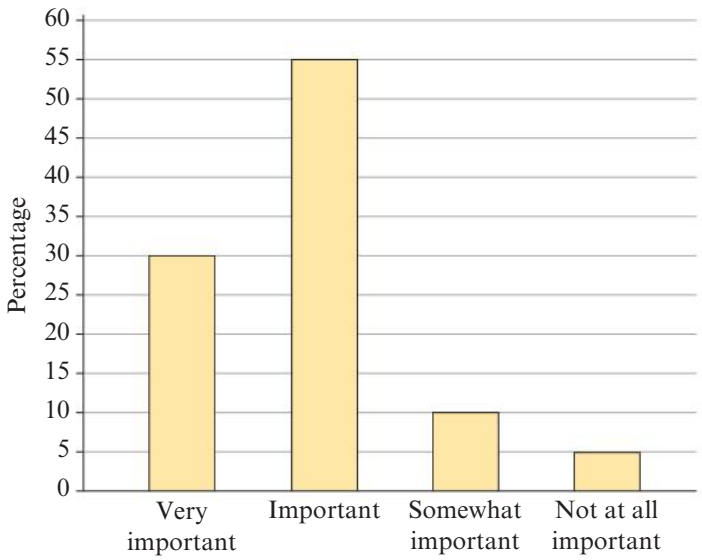
Example 10

4 A group of 600 employees from a large company were asked to rate the importance of salary in determining how they felt about their job. Their responses are shown in the following column chart.

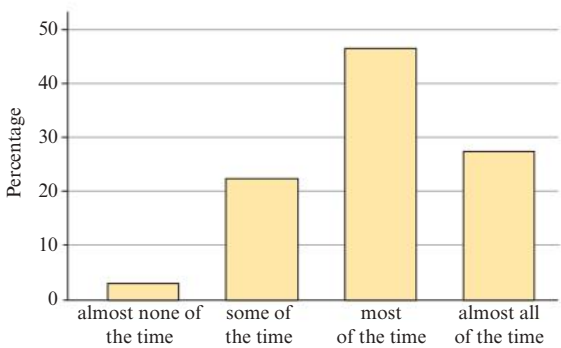


The importance of salary

Write a report describing how these employees rated the importance of salary in determining how they felt about their jobs.

5 A sample of 66 people were asked to respond to the question 'How much time have you felt happy in the past week' by selecting one of the options 'none or almost none of the time', 'some of the time', 'most of the time', or 'almost all of the time'. The data are summarised in the following frequency table and bar chart. Write a report summarising the findings of this investigation.

How much happy time?	Frequency	
	Number	*%*
almost none	2	3.0
some	15	22.7
most	31	47.0
almost all	18	27.3
Total	66	100.0



How much time have you felt happy in the past week?

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10D Displaying and describing numerical data

Learning intentions

- To be able to construct frequency tables for discrete numerical data.
- To be able to construct frequency tables for grouped numerical data (discrete and continuous).
- To be able to construct a histogram from frequency tables for numerical data.

Frequency tables can also be used to organise numerical data. For discrete numerical data which may take only a few values, the process is the same as that for categorical data, as shown in the following example.

Ungrouped data

Example 11 **Constructing a frequency table for discrete numerical data**

The number of brothers and sisters (siblings) reported by each of the 30 students in year 11 are as follows:

2 3 4 0 3 2 3 0 4 1 0 0 1 2 3
0 2 1 1 4 5 3 2 5 6 1 1 1 0 2

Construct a frequency table for these data.

Solution

Number of siblings	Frequency	
	Number	%
0	6	20.0
1	7	23.3
2	6	20.0
3	5	16.7
4	3	10.0
5	2	6.7
6	1	3.3
Total	30	100.0

Explanation

Locate the maximum and the minimum values in the dataset. Here, the minimum is 0 and the maximum is 6.
Construct a table as shown, including all the values between the minimum and the maximum.
Count the number of 0s, 1s, 2s etc. in the dataset. For example, there are seven 1s.
Record these values in the number column.
Add the frequencies to calculate the total.
Convert the frequencies to percentages, and record in the per cent (%) column.
For example, percentage of 1s equals $\frac{7}{30} \times 100 = 23.3\%$.
Record total percentage.

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Grouping data

Some variables, such as the variable *number of children in a family*, can only take on a limited range of values. For these variables, it makes sense to list each of these values individually when forming a frequency distribution.

In other cases, the variable can take on a large range of values; for example, the variable *runs scored* in a cricket match might take values from 0 to 100 or even more. Listing all possible runs scored would be tedious and would produce a large and unwieldy table. To solve this problem we **group the data** into a small number of convenient intervals (or classes).

These grouping intervals should be chosen according to the following principles.

- Every data value should be in an interval.
- The intervals should not overlap.
- There should be no gaps between the intervals.

The choice of intervals can vary but there are some guidelines.

- A division that results in about 5 to 15 groups is preferred.
- Choose an interval width that is easy for the reader to interpret, such as 10 units, 100 units or 1000 units (depending on the data).
- By convention, the beginning of the interval is given the appropriate exact value, rather than the end. As a result, intervals of 0–49, 50–99, 100–149 would be preferred over the intervals 1–50, 51–100, 101–150 etc.

Grouped discrete data

Example 12 **Constructing a grouped frequency table for a discrete numerical variable**

A group of 20 people were asked to record how many cups of coffee they drank in a particular week, with the following results:

2	0	9	10	23	25	0	0	34	32
5	0	17	14	3	6	0	33	23	0

Construct a table of these data showing both frequency (count) and percentage frequency.

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Solution

Cups of coffee	Frequency	
	Number	%
0–4	8	40
5–9	3	15
10–14	2	10
15–19	1	5
20–24	2	10
25–29	1	5
30–34	3	15
Total	20	100

Explanation

The minimum number of cups of coffee drunk is 0 and the maximum is 34. Intervals beginning at 0 and ending at 34 would ensure that all the data are included. Interval width of 5 will mean that there are 7 intervals. Note that the endpoints are within the interval, so that the interval 0 – 4 includes 5 values: 0, 1, 2, 3, 4. Set up the table as shown.

Count the number of data values in each interval to complete the number column. Convert the frequencies into percentages and record in the per cent (%) column. For example, for the interval 5–9:

$$\% \text{ frequency} = \frac{3}{20} \times 100 = 15\%$$

Record total percentage.

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Grouped continuous data

For continuous data, the variable can take any value within an interval, limited to a specific number of decimal places only by the accuracy to which we can measure. To ensure that all possible values are included in an interval, we label the upper end of the interval using the "less than"(<), symbol, as shown in the following example.

Example 13 Constructing a frequency table for a continuous numerical variable

The following are the heights of the 41 players in a basketball club, in centimetres.

178.1	185.6	173.3	193.4	183.1	184.6	202.4	170.9	183.3
180.3	185.8	189.1	178.6	194.7	185.3	191.1	189.7	191.1
180.4	180.0	193.8	196.3	189.6	183.9	177.7	178.9	193.0
188.3	189.5	182.0	183.6	184.5	188.7	192.4	203.7	180.1
170.5	179.3	184.1	183.8	174.7				

Construct a frequency table of these data.

Solution

Height (cm)	Frequency	
	Number	%
170–<175	4	9.8
175–<180	5	12.2
180–<185	13	31.7
185–<190	9	22.0
190–<195	7	17.1
195–<200	1	2.4
200–<205	2	4.9
Total	41	100.0

Explanation

Locate the minimum and maximum heights, which are 170.5 cm and 203.7 cm. A minimum value of 170.0 and a maximum of less than 205 will ensure that all the data are included.

An interval width of 5 cm will mean that there are 7 intervals from 170.0 to 205.0, which is within the guidelines of 5–15 intervals.

Set up the table as shown. All values of the variable that are from 170.0 to 174.9 have been included in the first interval. The second interval includes values from 175.0 to 179.9, and so on for the rest of the table.

The number of data values in each interval is then counted to complete the number column of the table.

Convert the frequencies to percentages and record in the per cent (%) column.

For example, for the interval 175–<180:

$$\% \text{ frequency} = \frac{5}{41} \times 100 = 12.2\%$$

Record total percentage.

The interval that has the highest frequency is again called the **modal interval**. Here the modal interval is 180.0–<185, as 13 players (31.7%) have heights that fall into this interval.

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Histograms

As with categorical data, we would like to construct a visual display of a frequency table for numerical data. The graphical display of a frequency table for a numerical variable is called a **histogram**. A histogram looks similar to a column chart but, because the data is numerical, there is a natural order to the plot and there are no gaps between the columns.

Histograms

In a histogram:

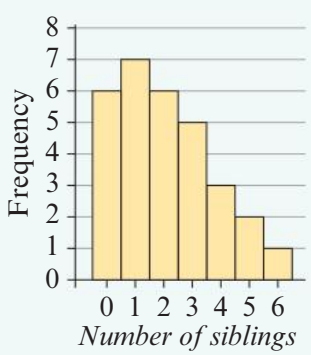
- frequency (number or percentage) is shown on the vertical axis
- the values of the variable being displayed are plotted on the horizontal axis
- each column corresponds to a data value, or a data interval if the data is grouped; alternatively, for ungrouped discrete data, the actual data value is located at the middle of the column
- the height of the column gives the frequency (number or percentage).

Example 14 Constructing a histogram for ungrouped discrete data

Construct a histogram for the data in the frequency table.

Siblings	Frequency
0	6
1	7
2	6
3	5
4	3
5	2
6	1
Total	30

Solution



Explanation

Label the horizontal axis with the variable name *Number of siblings*. Mark in the scale in units, so that it includes all possible values.

Label the vertical axis 'Frequency'. Insert a scale allowing for the maximum frequency of 7. Up to 8 would be appropriate. Mark the scale in units.

For each value for the variable, draw in a column. The data is discrete, so make the width of each column 1, starting and ending halfway between data values. For example, the column representing 2 siblings starts at 1.5 and ends at 2.5. The height of each column is equal to the frequency.

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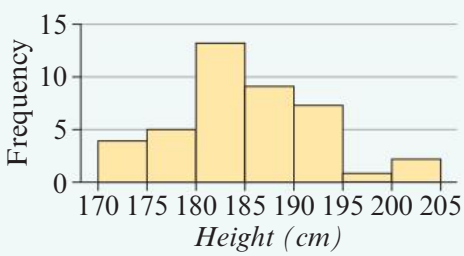
Example 15 Constructing a histogram for continuous data

Construct a histogram for the data in the frequency table.

Height (cm)	Frequency
170–<175	4
175–<180	5
180–<185	13
185–<190	9
190–<195	7
195–<200	1
200–<205	2
Total	41

Solution



Explanation

Label the horizontal axis with the variable name *Height*. Mark in the scale using the beginning of each interval as the scale points; that is, 170, 175, . . .

Label the vertical axis 'Frequency'. Insert a scale allowing for the maximum frequency of 13. Up to 15 would be appropriate.

For each interval, draw in a column. Each column starts at the beginning of the interval and finishes at the beginning of the next interval. Make the height of each column equal to the frequency.

Section Summary

- Numerical data, both discrete and continuous, can be summarised in frequency tables.
- When the data is discrete, but can take many values, the data should be grouped to form the frequency table.
- When the data is continuous, the data should be grouped to form the frequency table.
- A **histogram** is a display of a frequency table of numerical data. The values of the variable are shown on the horizontal axis, and the height of the columns give the frequency or percentage frequency.

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Exercise 10D

Constructing frequency tables for numerical data

Example 12

1 The number of magazines purchased in a month by 15 different people was as follows:

0 5 3 0 1 0 2 4 3 1 0 2 1 2 1

Construct a frequency table for the data, including both the frequency and percentage frequency.

Example 13

2 The amount of money carried by 20 students is as follows:

$4.55 $1.45 $16.70 $0.60 $5.00 $12.30 $3.45 $23.60 $6.90 $4.35
$0.35 $2.90 $1.70 $3.50 $8.30 $3.50 $2.20 $4.30 $0.00 $11.50

Construct a frequency table for the data, including both the number and percentage in each category. Use intervals of width $5, starting at $0.

Analysing frequency tables and constructing histograms

Example 14

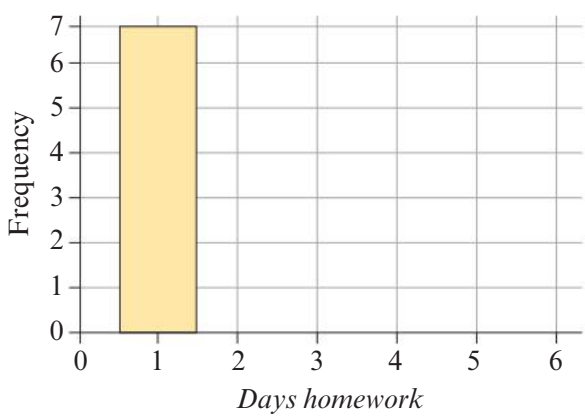
3 A group of 20 students were surveyed about the number of days they completed homework over a one-week period. The data is as shown.

1 5 3 2 1 1 2 4 3 1 4 2 4 5 3 2 2 1 1 1

a Use the data to complete the frequency table.

Days homework	Frequency
1	7
2	
3	
4	
5	
Total	20

b Use the information in the frequency table to complete the histogram shown.



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Example 15

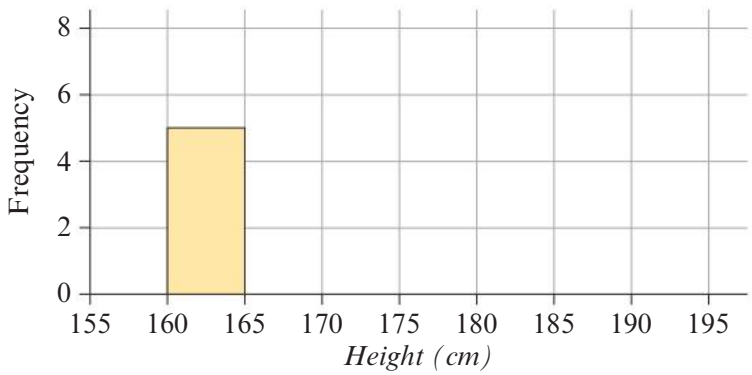
4 The following are the heights of the 25 players in a women's football team (in cms).

a Use the data to complete the grouped frequency table.

188	175	176	161	183
169	171	176	165	166
162	170	174	168	178
169	180	173	163	179
163	170	163	175	177

Height (cm)	Frequency
160–<165	5
165–<170	
170–<175	
175–<180	
180–<185	
185–<190	
Total	25

b Use the information in the frequency table to complete the histogram shown.



5 A group of 28 students were asked to draw a line that they estimated to be the same length as a 30 cm ruler. The results are shown in the frequency table below.

Length of line (cm)	Frequency	
	Number	%
28–<29	1	3.6
29–<30	2	7.1
30–<31	8	28.6
31–<32	9	32.1
32–<33	7	25.0
33–<34	1	3.6
Total	28	100.0

a State the number of students who drew a line with a length of:

- **i** 29–<30 cm
- **ii** less than 30 cm
- **iii** 32 cm or more.

b State the percentage of students who drew a line with a length of:

- **i** 29–<30 cm
- **ii** less than 31 cm
- **iii** 30 cm or more.

c Use the table to construct a histogram using the counts in the Number column.

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SF

6 The data below give the average hours worked per week in 24 countries:

35.0	48.0	45.0	43.0	38.2	50.0	39.8	40.7
40.0	50.0	35.4	38.8	40.2	45.0	45.0	40.0
43.0	48.8	43.3	53.1	35.6	44.1	34.8	38.7

a Construct a grouped frequency table of the data.

b Use the frequency table to construct a histogram.

7 The pulse rates of 28 students are given below.

59	86	82	96	71	90	78	68	71	68	88	76	74	102
70	78	69	77	64	80	83	78	68	70	86	72	79	52

Construct a histogram of the pulse rates.

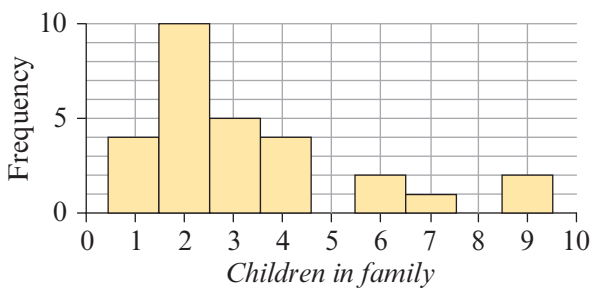
8 The following table gives the prices of units sold in a particular suburb in one month (in thousands of dollars):

356	366	375	389	432
445	450	450	495	510
549	552	579	585	590
595	625	725	760	880
940	950	1017	1180	1625

Construct a histogram of the unit prices.

Interpreting histograms

9 The number of children in the family for each student in a class is shown in the histogram.



a State the number of students who are the only child in a family.

b Determine the most common number of children in the family for this class.

c Determine how many students come from families with 6 or more children.

d Determine the number of students in the class.

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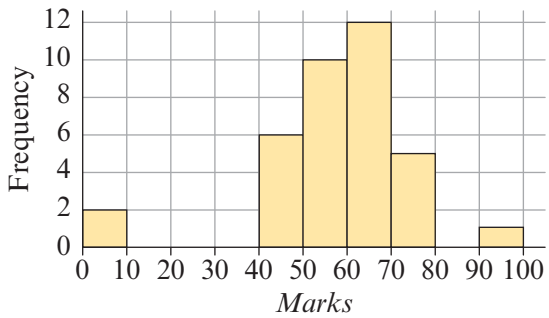
SF

10 The histogram shown gives the scores on a general knowledge quiz for a class of year 11 students.



a State the number of students who scored from 10 to less than 20 marks.

b Determine the number of students who attempted the quiz.

c State the modal interval.

d If a mark of 50 or more is designated as a pass, determine the number of students who passed the quiz.

11 A student purchased 21 new textbooks from a schoolbook supplier. The prices (in dollars) are listed below:

41.65 34.95 32.80 27.95 32.50 53.99 63.99 17.80 13.50 18.99 42.98
38.50 59.95 13.20 18.90 57.15 24.55 21.95 77.60 65.99 14.50

a Construct a histogram with column width 10 and starting point 10.

b Consider this histogram.

i Determine the range of the third interval.

ii State the frequency for the third interval.

iii Determine the modal interval.

12 The following data gives the waiting time in minutes for a group of 30 people who attended an emergency department of a hospital.

36	76	14	26	11	90	32	2	16	125
12	45	59	75	17	5	18	22	45	28
15	23	9	53	17	13	31	78	12	51

a Use the data to construct a frequency table.

b Construct a histogram of the data from the frequency table.

c Determine the modal value for the variable and the percentage of people who chose that response.

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10E Dot plots and stem plots

Learning intentions

- To be able to construct a dot plot for numerical data.
- To be able to construct a stem plot for numerical data.

Dot plots

The simplest display of numerical data is a **dot plot**.

Dot plot

A dot plot consists of a number line on which each data point is marked by a dot. When several data points have the same value, the points are stacked on top of each other.

Dot plots are a great way to display fairly small datasets in which the data takes a limited number of values.

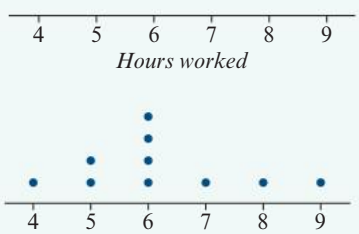
Example 16 Constructing a dot plot

The number of hours worked by each of 10 students in their part-time jobs is as follows:

6 9 5 8 6 4 6 7 6 5

Construct a dot plot of these data.

Solution



Explanation

Draw in a number line, scaled to include all data values. Label the line with the variable being displayed.
Plot each data value by marking in a dot above the corresponding value on the number as shown.

Stem plots

The **stem plot** or **stem-and-leaf plot** is another plot used for small datasets.

stem plot

A **stem plot** is a display where each data value is split into a stem (usually the leading digit or digits) and a leaf (usually the last digit). For example, 45 is split into 4 (stem) and 5 (leaf). The stem values are listed vertically, and the leaf values are listed horizontally next to their stem. The stem is usually separated from the leaves by a vertical line.

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Example 17 Constructing a stem plot

Display this set of marks on a test as an ordered stem plot.

15	2	24	30	25	19	24	33	41	60	42	35	35
28	28	19	19	28	25	20	36	38	43	45	39	

Solution

```
0 |
1 |
2 |
3 |
4 |
5 |
6 |
```

```
0 |
1 | 5
2 |
3 |
4 |
5 |
6 |
```

```
0 | 2
1 | 5
2 |
3 |
4 |
5 |
6 |
```

```
0 | 2
1 | 5 9 9 9
2 | 4 5 4 8 8 8 5 0
3 | 0 3 5 5 6 8 9
4 | 1 2 3 5
5 |
6 | 0
```

Marks 1 | 5 = 15 marks

```
0 | 2
1 | 5 9 9 9
2 | 0 4 4 5 5 8 8 8
3 | 0 3 5 5 6 8 9
4 | 1 2 3 5
5 |
6 | 0
```

Explanation

The dataset has values in the units, tens, twenties, thirties, forties, fifties and sixties. Thus, appropriate stems are 0, 1, 2, 3, 4, 5 and 6. Write these down in ascending order (smallest to largest), followed by a vertical line.

Now attach the leaves. The first data value is 15. The stem is 1 and the leaf is 5. Opposite the 1 in the stem, write the number 5, as shown.

The second data value is 2. The stem is 0 and the leaf is 2. Opposite the 0 in the stem, write the number 2, as shown.

Continue systematically working through the data, following the same procedure, until all points have been plotted. You will then have the **unordered** stem plot, as shown.

Order the leaves in increasing value as they move away from the stem to give the **ordered** stem plot, as shown. Write the name of the variable being displayed (*Marks*) at the top of the plot and add a key (1|5 = 15 marks).

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Stem plots with split stems

In some instances, using the simple process outlined above produces a stem plot that is too cramped to give an good overall picture of the variation in the data. This happens when the data values all have the same one or two first digits.

For example, consider the marks obtained by 17 students on a statistics test.

2 12 13 9 18 17 7 16 12 10 16 14 11 15 16 15 17

If we use the process described in Example 17 to form a stem plot, we end up with a 'bunched-up' plot like the one below.

0	2 7 9
1	0 1 2 2 3 4 5 5 6 6 6 7 7 8

We can solve this problem by 'splitting' the stems.

The stem should be split into halves or fifths as shown below.

Key: 1|6 = 16

0	2 7 9
1	0 1 2 2 3 4 5 5 6 6 6 7 7 8

Single stem

Key: 1|6 = 16

0	2
0	7 9
1	0 1 2 2 3 4
1	5 5 6 6 6 7 7 8

Stem split into halves

Key: 1|6 = 16

0	
0	2
0	
0	7
0	9
1	0 1
1	2 2 3
1	4 5 5
1	6 6 6 7 7
1	8

Stem split into fifths

Splitting the stems is equivalent to using an different interval when grouping data. It is useful when there are only a few different values for the stem.

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Choosing between plots

We now have three different plots that can be used to display numerical data: the histogram, the dot plot and the stem plot. They all allow us to make judgements concerning the important features of the distribution of the data, so how would we decide which one to use?

Although there are no hard and fast rules, the following guidelines are often used.

Plot	When used	How usually constructed
Dot plot	small datasets (say $n < 30$)	by hand
Stem plot	small to medium datasets (say $n < 50$)	by hand
Histogram	medium to large datasets (say $n > 30$)	with technology

Section Summary

- A **dot plot** consists of a number line on which each data point is marked by a dot. When several data points have the same value, the points are stacked on top of each other.
- A **stem plot** is a display where each data value is split into a stem (usually the leading digit or digits) and a leaf (usually the last digit). For example, 45 is split into 4 (stem) and 5 (leaf). The stem values are listed vertically, and the leaf values are listed horizontally next to their stem in ascending order. The stem is usually separated from the leaves by a vertical line.
- Dot plots are suitable for small datasets (say $n < 30$).
- Stem plots are suitable for small to medium size datasets (say $n < 50$).
- Histograms are suitable for medium to large datasets (say $n > 30$).

Exercise 10E

Constructing and analysing dot plots

Example 16

1 The number of children in each of 15 families is as follows:

0 7 2 2 2 4 1 3 3 2 2 2 0 0 1

a Construct a dot plot of the number of children.

b State the mode of this distribution.

2 A group of 20 people were asked how many times in the last week they had shopped at a particular supermarket. Their responses were as follows:

0 1 1 0 0 6 0 1 2 2 3 4 0 0 1 1 2 3 2 0

a Construct a dot plot of this data.

b Determine how many people did not shop at the supermarket in the last week.

SF

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SF

3 The number of goals scored in an AFL game by each player on one team is as follows:

0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 2 2 3 6

a Construct a dot plot of the number of goals scored.

b State the mode of this distribution.

4 In a study of the service offered at her café, Amanda counted the number of people waiting in the queue every 5 minutes from 12 noon until 1 p.m.

Time	12:00	12:05	12:10	12:15	12:20	12:25	12:30	12:35	12:40	12:45	12:50	12:55	1:00
Number	0	2	4	4	7	8	6	5	0	1	2	1	1

a Construct a dot plot of the number of people waiting in the queue.

b Determine the peak demand time at the café.

Constructing and analysing stem plots

Example 17

5 The marks obtained by a group of students on an English examination are as follows:

92 65 35 89 79 32 38 46 26 43 83 79
50 28 84 97 69 39 93 75 58 49 44 59
78 64 23 17 35 94 83 23 66 46 61 52

a Construct a stem plot of the marks.

b Determine how many students obtained 50 or more marks.

c State the lowest mark.

6 The stem plot on the right shows the ages, in years, of all the people attending a meeting.

a Determine how many people attended the meeting.

b Determine how many of these people were less than 43 years old.

Age (years) 1 | 2 represents 12 years

0	2 7
2	1 4 5 5 7 8 9
3	0 3 4 4 5 7 8 9
4	0 1 2 2 3 3 4 5 7 8 8 8
5	2 4 5 6 7 9
6	3 3 3 8
7	0

7 An investigator recorded the amount of time for which 24 similar batteries lasted in a toy. Her results (in hours) were:

26 40 30 24 27 31 21 27 20 30 33 22
4 26 17 19 46 34 37 28 25 31 41 33

a Draw a stem plot of these times.

b Determine how many of the batteries lasted for more than 30 hours.

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SF

8 The amounts of time (in minutes) that a class of students spent on homework on one particular night were:

10	27	46	63	20	33	15	21	16	14	15
39	70	19	37	56	20	28	23	0	29	10

a Draw a stem plot of these times.

b Determine how many students spent more than 60 minutes on homework.

9 The prices of a selection of shoes at a discount outlet are as follows:

$49	$75	$68	$79	$75	$39	$35	$52	$149	$84
$36	$95	$28	$25	$78	$45	$46	$76	$82	

Construct a stem plot of this data.

CF

10 The heights of a group of students (in cm) are as follows:

158	175	176	161	183
169	171	176	165	166
162	170	174	168	178
169	180	173	163	179
163	170	163	175	177

Construct a stem plot for the heights using suitable split stems.

11 The data below give the wrist circumference (in cm) of 20 men.

16.9	17.3	19.3	18.5	18.2	18.4	19.9	16.7	18.9	16.8
17.7	16.5	17.0	17.2	17.6	17.1	16.6	18.0	19.3	17.5

Construct a stem plot for wrist circumference using suitable split stems.

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10F Characteristics of distributions of numerical data

Learning intentions

- To be able to identify the shape of a distribution from a graphical display.
- To be able to understand what is meant by the centre of a distribution.
- To be able to understand what is meant by the spread of a distribution.
- To be able to understand the concept of an outlier.

The purpose of constructing a graphical display of numerical data is to identify key features of the data distribution. These features are:

- **shape**
- **centre**
- **spread**
- **outliers**

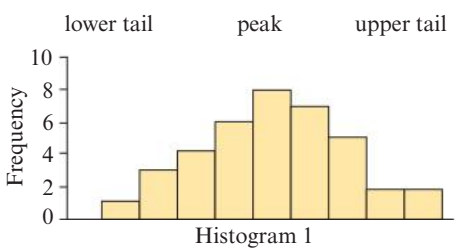
As long as there are enough data values these features can be determined from any graphical display of numerical data. In the remainder of this section we will use a histogram to illustrate these features, but dot plots and stem plots may also be used.

Shape

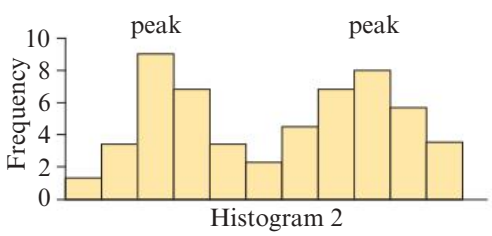
How are the data distributed? Is the histogram peaked? That is, do some data values tend to occur much more frequently than others, or is the histogram relatively flat, showing that all values in the distribution occur with approximately the same frequency?

Symmetric distributions

The shape of a distribution is said to be **symmetric** if one side is an approximate mirror image of the other. Note that exact symmetry would be very unusual when dealing with real data, so we only require the reflection to be approximate. If a histogram is single-peaked, this would mean that the histogram would tail off evenly on either side of the peak, as shown in Histogram 1.



Histogram 1



Histogram 2

A single-peaked **symmetric distribution** is characteristic of the data that derive from measuring variables such as intelligence test scores, weights of oranges, or any other data for which the values vary evenly around some central value. Note that the peak of the distribution corresponds to the value or values of the variable with the highest frequency, which is the mode or modal class.

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The double-peaked distribution (Histogram 2) is symmetric about the dip between the two peaks. A histogram that has two distinct peaks indicates that there are two modes for the distribution (they do not have to be exactly same in value), and is called a **bimodal** distribution.

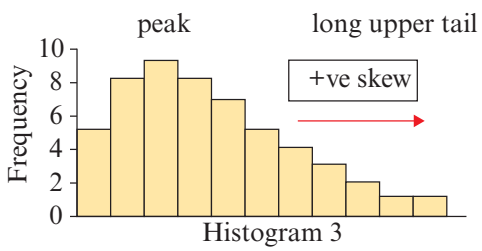
A bimodal distribution often indicates that the data have come from two different populations. For example, if we were studying the distance the discus is thrown by Olympic-level javelin throwers, we would expect a bimodal distribution if both male and female throwers were included in the study.

Symmetric distributions

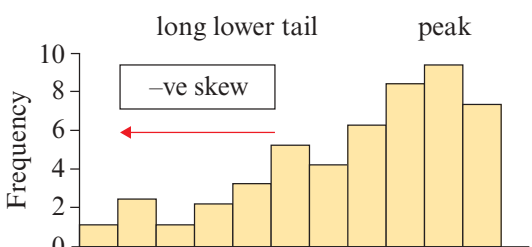
A distribution is said to be **symmetric** if one side is an approximate mirror image of the other. If the distribution is single-peaked, this means that peak is in the middle of the distribution, and the tails are approximately of the same length.

Skewed distributions

Sometimes a histogram tails off primarily in one direction. If a histogram tails off to the right, we say that it is **positively** skewed (Histogram 3). The distribution of salaries of workers in a large organisation tends to be positively skewed. Most workers earn a similar salary with some variation above or below this amount, but a few earn more and even fewer, such as the senior manager, earn even more. The distribution of house prices also tends to be positively skewed.



Histogram 3



Histogram 4

If a histogram tails off to the left, we say that it is **negatively** skewed (Histogram 4). The distribution of age at death tends to be negatively skewed. Most people die in old age, a few in middle age and fewer still in childhood.

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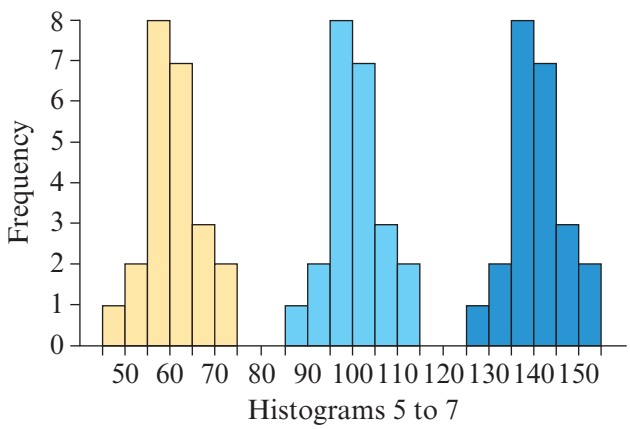
Positive and negative skew

A histogram may be positively or negatively skewed.

- It is **positively skewed** if it has a short tail to the left and a long tail pointing to the right (because of the many relatively short columns towards the positive end of the distribution).
- It is **negatively skewed** if it has a short tail to the right and a long tail pointing to the left (because of the many relatively short columns towards the negative end of the distribution).

Centre

Histograms 5 to 7 display the distribution of test scores for three different classes taking the same subject. They are identical in shape, but differ in where they are located along the axis. In statistical terms, we say that the distributions are 'centred' at different points along the axis.



But what do we mean by the **centre of a distribution**? This is an issue we will return to in more detail later in this chapter. For the present we will take the centre to be about the middle of the distribution.

The middle of a symmetric distribution is reasonably easy to locate by eye. Looking at Histograms 5 to 7, it would be reasonable to say that the centre or middle of each distribution lies roughly halfway between the extremes; half the observations would lie above this point and half below. Thus we might estimate that Histogram 5 (yellow) is centred at about 60, Histogram 6 (light blue) at about 100, and Histogram 7(dark blue) at about 140.

For skewed distributions, it is more difficult to estimate the middle of a distribution by eye. The middle is not halfway between the extremes because, in a skewed distribution, the scores tend to bunch up at one end.

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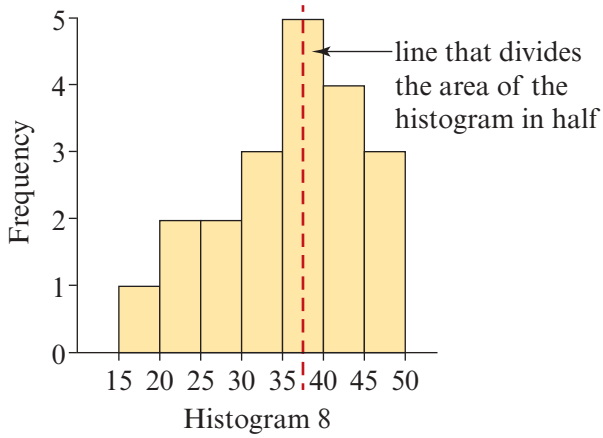
However, if we imagine a cardboard cut-out of the histogram, the midpoint lies on the line that divides the histogram into two equal areas (Histogram 8).

Using this method, we would estimate the centre of the distribution to lie somewhere between 35 and 40, but closer to 35, so we might opt for 37. However, remember that this is only an estimate.



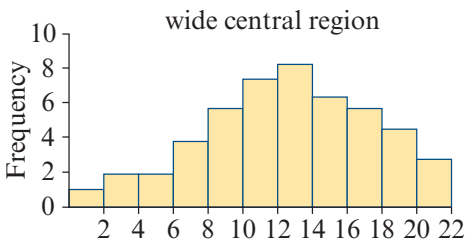
Histogram 8

Comparing centre

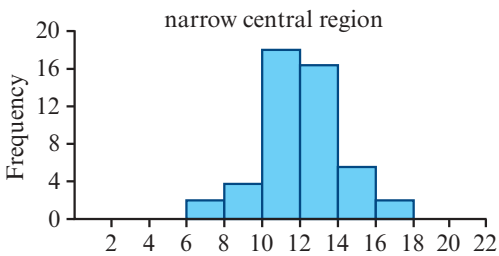
Two distributions are said to differ in **centre** if the values of the data in one distribution are generally larger than the values of the data in the other distribution.

Spread

If the histogram is single-peaked, is it narrow? This would indicate that most of the data values in the distribution are tightly clustered in a small region. Or is the peak broad? This would indicate that the data values are more widely spread out. Histograms 9 and 10 are both single-peaked. Histogram 9 has a broad peak, indicating that the data values are not very tightly clustered about the centre of the distribution. In contrast, Histogram 10 has a narrow peak, indicating that the data values are tightly clustered around the centre of the distribution.



Histogram 9



Histogram 10

Comparing spread

Two distributions are said to differ in **spread** if the values of the data in one distribution tend to be more variable (spread out) than the values of the data in the other distribution.

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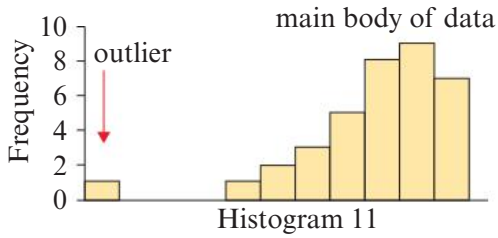
Outliers

Outliers are any data values that stand out from the main body of data. These are data values that are atypically high or low. See, for example, Histogram 11, which shows an outlier. In this case it is a data value that is atypically low compared to the rest of the data values.

In the histogram shown there appears to be an outlier, a data value which is lower than than rest of the data values. Such values should be checked. They may indicate an unusually low value, but they may also indicate an error in the data.



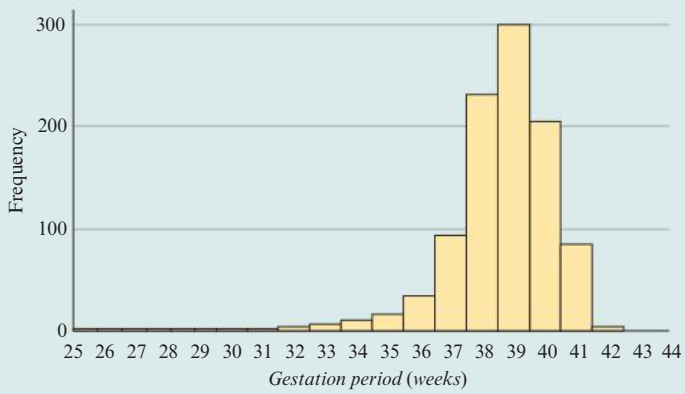
Histogram 11

Outliers

An **outlier** is a data value which is much lower or higher than the rest of the data values.

Example 18 Describing the features of a distribution from a histogram

The histogram shows the gestation period (completed weeks) for a sample for 1000 babies born in Australia one year. Describe this histogram in terms of shape, centre, spread and outliers.



Solution

The distribution is clearly negatively skewed, with a long lower tail.
The centre of the distribution is around 38–39 weeks.

The data values range from 25–42 weeks, but most of the data values are close to the centre, in the range of 36–41 weeks.

Those values that are less than 34 weeks seem to be small in comparison to the rest of the data.

Explanation

The distribution has a longer tail in the negative direction.
Locate the (approximate) centre of the distribution, the value that seems to divide the area of the histogram in half.
Consider the spread of the distribution. Are the majority of the values close to the centre, or quite spread out?
There is a long negative tail but no values that sit away from the rest of the data. We can't tell which values are outliers from this histogram.

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We can see from this example that it is very difficult to give exact values for centre and spread, and to clearly identify outliers, from a display such as a histogram. In the next section, we will define measures of centre and spread that are appropriate for this distribution, and in the next chapter introduce an exact definition of an outlier.

Section Summary

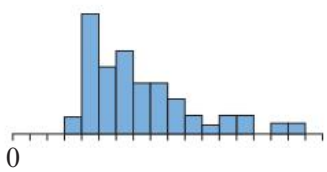
- A distribution is said to be **symmetric** if one side is an approximate mirror image of the other. If the distribution is single-peaked, this means that peak is in the middle of the distribution, and the tails are approximately of the same length.
- The shape of a distribution is **positively skewed** if it has a short tail to the left and a long tail pointing to the right, and **negatively skewed** if it has a short tail to the right and a long tail pointing to the left.
- Two distributions are said to differ in **centre** if the values of the data in one distribution are generally larger than the values of the data in the other distribution.
- Two distributions are said to differ in **spread** if the values of the data in one distribution tend to be more variable (spread out) than the values of the data in the other distribution.
- An **outlier** is a data value that is much lower or higher than the rest of the data values.

Exercise 10F

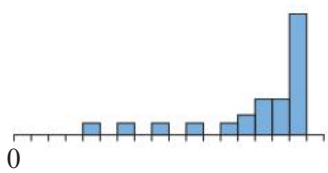
Describing shape using histograms, dot plots and stem plots

1 Classify the shape of each of the following histograms (negatively skewed, positively skewed or approximately symmetric). SF

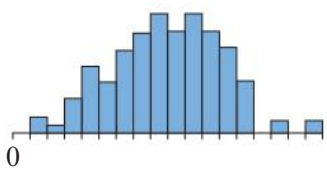
a



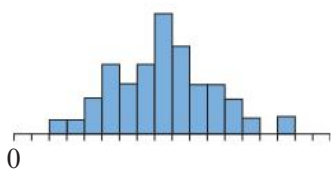
b



c



d



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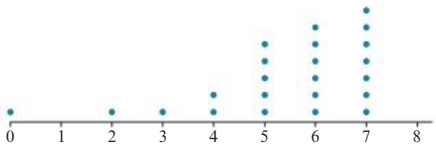
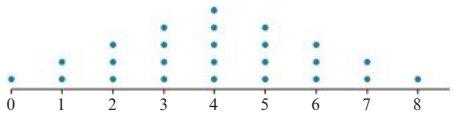
2 Classify the shape of each of the following dot plots (negatively skewed, positively skewed or approximately symmetric).

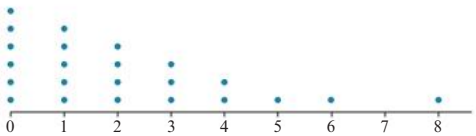
a



b



c



3 Identify the shape of each of the following stem plots (negatively skewed, positively skewed or approximately symmetric).

a Key: 3|4 represents 34

2	2
3	4 5
4	4 5 6 7
5	0 5 5 6 7 8 8 9
6	0 0 2 3 4 5 7 9
7	0 1 4 5 6
8	0 3 5 7
9	5 9

b Key: 3|1 represents 31

3	1
4	6
5	5 6
6	2 3 7 7 8
7	1 1 2 3 4 5 6 7
8	2 3 3 5 6 6 7 7 8 9
9	0 1

c Key: 1|0 represents 10

0	0 1 1 2 3 4 5 6 6 7 7
1	0 1 2 3 4 6 5 9
2	1 2 7
3	0 1
4	0

Describing distributions

Example 18

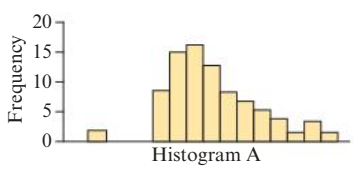
4 Classify each of the following histograms as approximately symmetric, positively skewed or negatively skewed, and mark the following.

i The mode or modes (if there is a clear mode)

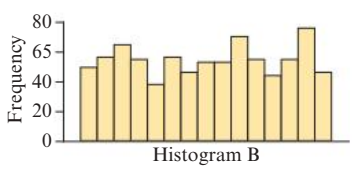
ii Any potential outliers

iii The approximate location of the centre

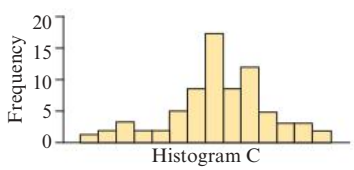
a



b



c



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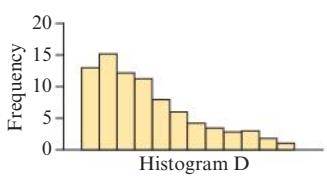
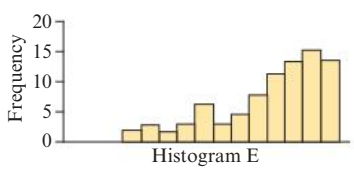
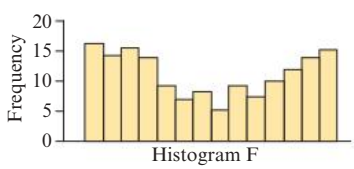


e



f



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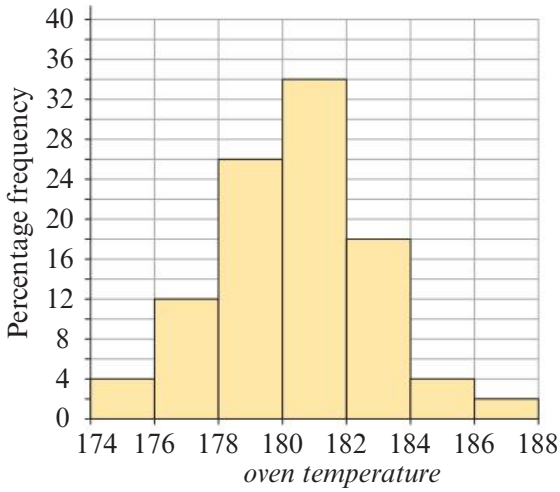
SF

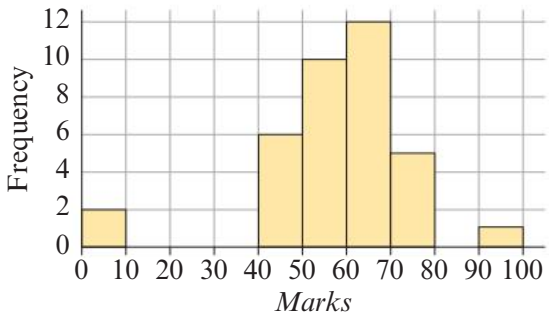
5 To test the temperature control on an oven, the control is set to 180°C and the oven is heated for 15 minutes. The temperature of the oven is then measured. The temperatures of 300 ovens tested in this way were recorded and the data displayed using the percentage frequency histogram shown.



Use the information in the histogram to complete the report below describing the distribution of the temperatures in terms of shape, centre, and spread.

Report: The distribution of temperatures is _____, centred at about _____ degrees. A wide range of temperatures was recorded, between _____ and _____ degrees.

6 The histogram shown gives the marks on a general knowledge quiz for a class of year 11 students. Use the information in the histogram to complete the report below describing the distribution of the marks in terms of shape, centre, spread and outliers.

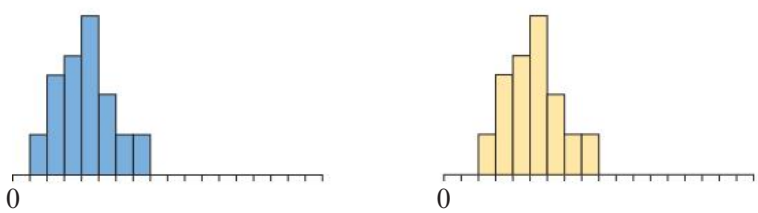


Report: The distribution of marks is _____, centred at about _____ marks. Most students scored between _____ and _____ marks. Two students scored unusually low marks, less than _____, and one student scored an unusually high mark, more than _____.

Comparing distributions

7 Do the following pairs of distributions differ in centre, spread, both or neither? Assume that each pair of histograms is drawn on the same scale.

a



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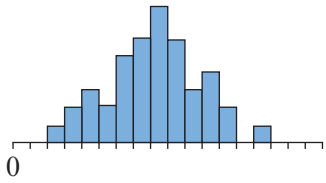
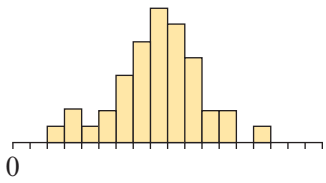
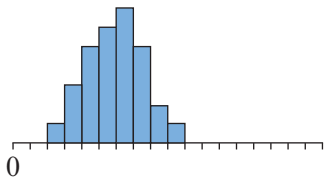
SF

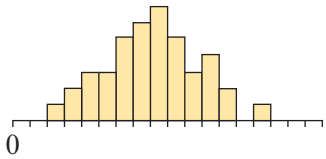
b



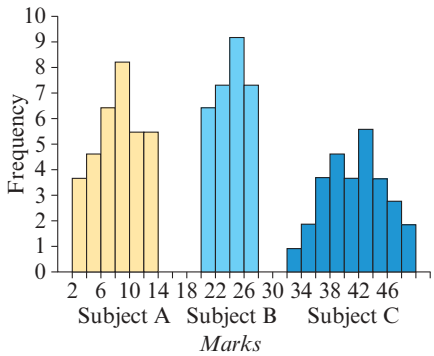


c





8 The three histograms show the marks obtained by a group of students in three subjects.



a Identify the shape of each of the distributions.

b Identify any clear outliers.

c Determine the interval containing the central mark for each of the three subjects.

d Determine in which subject the spread of marks is the least.

CF

9 **a** The minimum daily temperature (in °C) over a two–week period in a certain town was recorded as follows:

1 2 5 3 2 3 1 1 2 6 7 4 4 5

Construct a dot plot of the data.

b The maximum daily temperature (in °C) over the same two–week period in that town was also recorded as follows:

12 14 13 13 13 15 15 17 16 13 13 12 13 13

Construct a dot plot of the data, using the same scale for the axes as in part **a**.

c Compare the two distributions in terms of centre and spread.

10 A researcher determined the percentage body fat for 30 adult males as follows:

5 12 9 17 19 5 7 21 30 24

9 21 10 20 17 18 28 11 14 17

13 4 20 17 25 29 17 26 10 13

a Construct a stem plot of the data.

b After the researcher has finished collecting the data, she noted that there had been an error in the data collection, which could be fixed by subtracting 2 from each data value. Use mathematical reasoning to describe the distribution of the corrected data and compare it to that shown in part **a** in terms of centre and spread.

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10G Measures of centre

Learning intentions

- To be able to determine the mean and the median of a dataset.
- To be able to know whether to use the median or mean as a measure of centre for a dataset.

A statistic is any number computed from data. Certain special statistics are called **summary statistics** because they numerically summarise important features of the dataset. Of course, whenever any set of data is summarised into just one or two numbers, much information is lost. However, if a summary statistic is well chosen, it may reveal important information hidden in the dataset.

In this section we will consider two summary statistics that are measures of centre – the **mean** and the **median**. While the **mode** can be a statistic of interest for a numerical variable, we have seen that a distribution may have more than one mode, or that the mode may occur at the extreme values of a distribution. For these reasons, the mode is not considered a measure of centre.

The mean

The most commonly used measure of the centre of a distribution of a numerical variable is the **mean**. The mean is calculated by summing the data values and then dividing by the number of data values. The mean of a set of data is sometimes called the 'average'.

The mean

$$\text{mean} = \frac{\text{sum of data values}}{\text{total number of data values}}$$

For example, consider the set of data: 1, 5, 2, 4:

$$\text{mean} = \frac{1+5+2+4}{4} = \frac{12}{4} = 3$$

Some notation

Because the rule for the mean is relatively simple, it is easy to write in words. However, we generally use a shorthand notation that enables statistical formulas to be written out in a more compact form. In this notation we use:

- the Greek capital letter sigma, $\sum$, as a shorthand way of writing 'sum of'
- a lower case x_i to represent a data value
- a lower case x with a bar, $\overline{x}$ (pronounced 'x bar'), to represent the mean of the data values
- n to represent the total number of data values.

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Rule for calculating the mean

$$\bar{x} = \frac{\sum x_i}{n}$$

Example 19 Calculating the mean

The following dataset shows the number of premierships won by each of the current AFL teams, until the end of 2023. Calculate the mean of the number of premierships won. Round your answer to one decimal place.

Team	*Premierships*	*Team*	*Premierships*
Carlton	16	North Melbourne	4
Essendon	16	West Coast	4
Collingwood	16	Adelaide	2
Melbourne	13	Western Bulldogs	2
Hawthorn	13	Port Adelaide	1
Richmond	13	St Kilda	1
Brisbane Lions	11	Fremantle	0
Geelong	10	Gold Coast	0
Sydney	5	Greater Western Sydney	0

Solution

$\bar{x} = \frac{\sum x_i}{n}$ $\quad n = 18$

$$\bar{x} = \frac{16 + 16 + 16 + \ldots + 1 + 1 + 0 + 0 + 0}{18}$$

$$= \frac{127}{18}$$

$$= 7.1$$

Explanation

Write down the formula and the value of n. Substitute into the formula and evaluate.

We do not expect the mean to be a whole number, so give your answer to one decimal place.

The median

Another useful measure of the centre of a distribution of a numerical variable is the middle value, or **median**. To find the value of the median, all the observations are listed in ascending order and the middle one is the median.

For example, the median of the following dataset is 6, as there are five observations on either side of this value when the data are listed in order.

median = 6
↓

2 3 4 5 5 **6** 7 7 8 8 11

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When there is an even number of data values, the median is defined as the midpoint of the two middle values. For example, the median of the following dataset is 6.5, as there are six observations on either side of this value when the data are listed in order.

median = 6.5
↓
2 3 4 5 5 6 7 7 8 8 11 11

Returning to the premiership data. As the data are already given in order, it only remains to determine the middle observation.

As there are 18 entries in the table there is no actual middle observation, so the median is chosen as the value halfway between the two middle observations, in this case the ninth and tenth (5 and 4).

$$\text{Median} = \frac{1}{2}(5 + 4) = 4.5$$

The interpretation here is that, of the teams in the AFL, half (or 50%) have won the premiership 5 or more times and half (or 50%) have won the premiership 4 or less times.

The following rule is useful for locating the median in the stem plot of a larger dataset.

Determining the median

To determine the median of a distribution:

- arrange all the observations in ascending order according to size
- if n, the number of observations, is odd, then the median is the $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation from the end of the list
- if n, the number of observations, is even, then the median is found by averaging the two middle observations in the list. That is, to find the median, the $\left(\frac{n}{2}\right)^{\text{th}}$ and the $\left(\frac{n}{2}+1\right)^{\text{th}}$ observations are added together and divided by 2.

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Example 20 Determining the median when n is odd

Determine the median age of 23 people whose ages are displayed in this ordered stem plot.

Age (years) 1 | 2 represents 12 years

0	2 5
2	1 4 5 8
3	0 3 4 6
4	0 1 2 5 7
5	2 4 5 8
6	3 5 9 9

Solution

$n = 23$

Median is at the $\frac{23 + 1}{2} = 12$th position

Thus the median age is 41 years.

Explanation

Write down the number of values.

The median is located at the $\left(\frac{n + 1}{2}\right)^{\text{th}}$ position.

Evaluate and write the answer.

Note: We can check to see whether we are correct by counting the number of data values either side of the median. They should be equal.

Example 21 Determining the median when n is even

Determine the median age for a group of people whose ages are displayed in this ordered stem plot.

Age (years) 1 | 2 represents 12 years

0	5 9
2	1 3 5 8
3	0 0 4 9 9
4	0 4 5 8
5	3 7
6	3

Solution

$n = 18$

$\frac{18}{2} = 9$, the 9th value = 34

$\frac{18}{2} + 1 = 10$, the 10th value = 39

The median age is $\frac{34 + 39}{2} = 36.5$ years.

Explanation

Write down the number of values.

Since there are an even number of observations, to find the median the $\left(\frac{n}{2}\right)^{\text{th}}$ and the $\left(\frac{n}{2} + 1\right)^{\text{th}}$ observations are added together and divided by 2.

Evaluate and write the answer.

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Comparing the mean and median

In Example 19 we found that the mean number of premierships won by the 18 AFL clubs was $\bar{x} = 7.1$. We also found that the median number of premierships won was 4.5.

These two values are quite different and the interesting question is: Why are they different, and which is the better measure of centre in this situation?

To help us answer this question, consider a stem plot of these data values.

Premierships won

0	0 0 0 1 1 2 2 4 4
0	5
1	0 1 3 3 3
1	6 6 6

From the stem plot, it can be seen that the distribution of premierships won is positively skewed. This example illustrates a property of the mean. When the distribution is skewed or if there are one or two very extreme values, then the mean is pulled towards the tail of the distribution and the extreme values, giving us a value for the mean which may be far from the centre. Since the median is not so affected by unusual observations and always gives the middle value, then the median is the preferred measure of centre for a skewed distribution or one with outliers. When the distribution is symmetric, and there are no outliers, then either measure is appropriate, although we tend to prefer the mean as it is easier to calculate and more familiar to most people.

Section Summary

- **Summary statistics** are numbers calculated from a dataset which represent important features of that dataset.
- Two very useful summary statistics that give numerical values for the centre of a distribution are the **mean** and the **median**.
- The **mean** is denoted $\bar{x}$ and mean $= \dfrac{\text{sum of data values}}{\text{total number of data values}} = \dfrac{\sum x_i}{n}$
- The **median** is the middle value in the ordered data, which is the $\dfrac{n+1}{2}^{\text{th}}$ observation from the end of the list when n is odd, or the average of the $\dfrac{n}{2}$ and the $\left(\dfrac{n}{2}+1\right)^{\text{th}}$ observations when n is even.
- When the distribution is symmetric and has no outliers the **mean** is the preferred measures of centre.
- When the distribution is skewed or has outliers the **median** is the preferred measures of centre.
- The **mode** is the value (or values) with the highest frequency. It is not a useful measure of centre for a numerical variable.

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Exercise 10G

SF

Calculating the mean of a dataset

Example 19

1 Consider the following dataset:

1 2 1 0 2 3 1 1 2 6 7

a Calculate the sum of the data values.

b Hence, determine the mean of the dataset.

2 Calculate the mean for each of these datasets.

a	2	5	7	2	9	
b	4	11	3	5	6	1
c	15	25	10	20	5	
d	101	105	98	96	97	109
e	1.2	1.9	2.3	3.4	7.8	0.2

Determining the median of a dataset

Example 20

3 Consider the following dataset:

21 12 35 53 32 63 11 19 62 17 24 34 95

a Order the data from smallest to largest value.

b Hence, locate the median of the dataset.

Example 21

4 Determine the median of each of these ordered datasets.

a	2	2	5	7	9	11	12	16	23	
b	1	3	3	5	6	7	9	11	12	12
c	21	23	24	25	27	27	29	31	32	33
d	101	101	105	106	107	107	108	109		
e	0.2	0.9	1.0	1.1	1.2	1.2	1.3	1.9	2.1	2.2	2.9

Determining the mean and median from a plot

5 Determine the median of the data displayed in the following stem plots.

a 4|8 represents 48 mm

Monthly rainfall (mm)

4	8 9 9
5	0 2 7 7 8 9 9
6	0 7

b 1|7 represents 17 hours

Battery time (hours)

0	4
1	7 9
2	0 1 2 4 5 6 6 7 7 8
3	0 0 1 1 3 3 4 7
4	0 1 6

6 Determine the mean of each of the datasets displayed in the stem plots in Question **5**.

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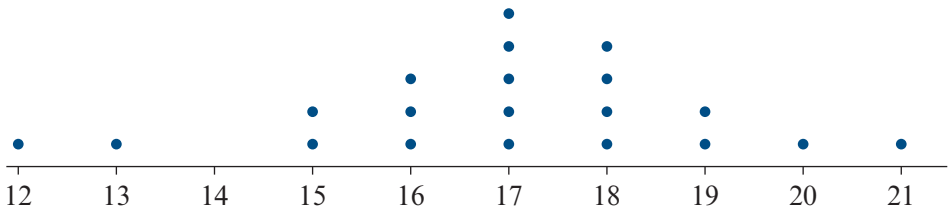
7 Find the mean and median of the dataset displayed in the following dot plot:



SF

Comparing the mean and the median

8 The following data gives the area, in hectares, of each of the suburbs of a city:

3.6 2.1 4.2 2.3 3.4 40.3 11.3 19.4 28.4 27.6 7.4 3.2 9.0

a Determine the mean and the median areas.

b Which is a better measure of centre for this dataset? Explain your answer.

9 The prices, in dollars, of apartments sold in a particular suburb during one month were:

$387 500 $329 500 $293 400 $600 000 $318 000 $368 000 $750 000
$333 500 $335 500 $340 000 $386 000 $340 000 $404 000 $322 000

a Determine the mean and the median of the prices.

b Which is a better measure of centre of this dataset? Explain your answer.

Understanding the mean and median

CF

10 Suppose that the mean of a dataset with 8 values is 23.6, and that when another data value is added the mean score becomes 24.9. Determine the data value that has been added to the dataset.

11 The mean score for Mr Miller's class in the Mathematics test was 67. The mean score for Mrs Lacey's class was 72. If 23 students took the test in Mr Miller's class, and 20 took the test in Mrs Lacey's class, calculate the mean score across both classes.

12 Jessie is completing a science experiment. She has 20 data values, and has calculated the mean of the dataset to be 15.6, and the median to be 15.0. When she is typing up her results, she mistypes the smallest data value, typing 1.6 instead of 10.6.

a Determine the mean of the new dataset.

b Determine the median of the new dataset.

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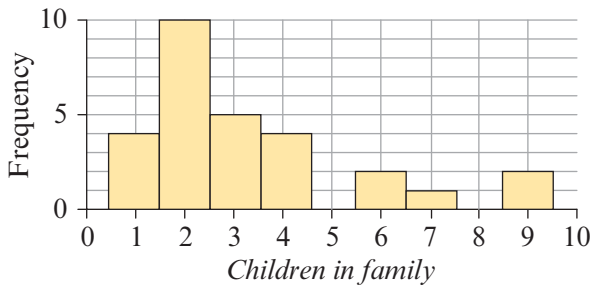
13 The number of children in the family for each student in a class is shown in the histogram.



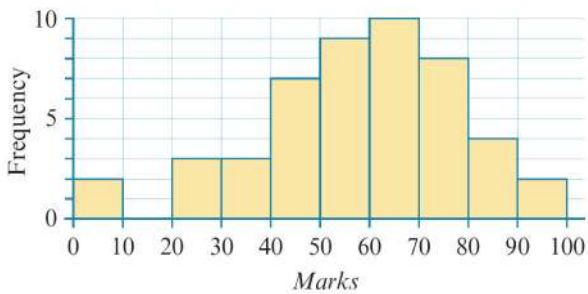
a Determine the median number of children in the family for this class.

b Determine the mean number of children in the family for this class.

14 The histogram opposite gives the scores on a general knowledge quiz for a class of year 11 students.

a In which interval is the median mark? Use mathematical reasoning to justify your answer.

b Can you identify the mean mark from this histogram? Use mathematical reasoning to justify your answer.



15 The following percentage frequency table gives the heights of members of a sporting team.

a Determine in which interval is the median height of the team members.

b Can you identify the mean height from this percentage frequency table? Justify your answer.

Height (cm)	Percentage Frequency
170–<175	5
175–<180	22
180–<185	35
185–<190	26
190–<195	10
195–<200	2
Total	100

16 Four different data values have a median of 5 and a mean of 5.5. If two of the data values are 2 and 4, determine the other values in the dataset.

17 The mean weight of five oranges is w grams. After another orange is added, the mean weight of the six oranges is increased by 10%. Determine the weight of the additional orange in terms of w.

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10H Measures of spread

Learning intentions

- To be able to determine the range, interquartile range and standard deviation of a dataset.
- To be able to know when to use a particular measure of spread for a dataset.
- To be able to use a calculator to calculate summary statistics.

A measure of spread is calculated in order to judge the *variability* of a dataset. That is, are most of the values clustered together, or are they rather spread out?

The range

The simplest measure of spread can be determined by considering the difference between the smallest and the largest observations. This is called the **range**.

The range

The **range** is the simplest measure of spread of a distribution.

The range is the difference between the largest and smallest values in the dataset.

range = largest data value – smallest data value

Example 22 Finding the range

Consider the marks awarded to a group of students for two different tasks:

Task A

2	6	9	10	11	12	13	22	23	24	26	26	27	33	34
35	38	38	39	42	46	47	47	52	52	56	56	59	91	94

Task B

11	16	19	21	23	28	31	31	33	38	41	49	52	53	54
56	59	63	65	68	71	72	73	75	78	78	78	86	88	91

Determine the range of each of these distributions.

Solution	Explanation
Range for task A = 94 – 2 = 92	For task A, the minimum mark is 2 and the maximum mark is 94.
Range for task B = 91 – 11 = 80	For task B, the minimum mark is 11 and the maximum mark is 91.

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In Example 22 the range for task A is greater than the range for task B. Is the range a useful summary statistic for comparing the spread of the two distributions? To help make this decision, consider the stem plots of the datasets:

Task A

0	2 6 9
1	0 1 2 3
2	2 3 4 6 6 7
3	3 4 5 8 8 9
4	2 6 7 7
5	2 2 6 6 9
6	
7	
8	
9	1 4

Task B

0	
1	1 6 9
2	1 3 8
3	1 1 3 8
4	1 9
5	2 3 4 6 9
6	3 5 8
7	1 2 3 5 8 8 8
8	6 8
9	1

From the stem plots of the data, it appears that the spread of marks for the two tasks is not really described by the range. It is clear that the marks for task A are more concentrated than the marks for task B, except for the two unusual values for task A.

Another measure of spread is needed, one that is not so influenced by these extreme values. The statistic we use for this task is the **interquartile range**.

The interquartile range

Determining the interquartile range

To determine the interquartile range of a distribution:

- arrange all observations in order according to size
- divide the observations into two equal-sized groups, and if n is odd, omit the median from both groups
- locate Q_1, the **first quartile**, which is the median of the lower half of the observations, and Q_3, the **third quartile**, which is the median of the upper half of the observations.

The **interquartile range** (IQR) is then: $IQR = Q_3 - Q_1$

We can interpret the interquartile range as follows:

- Since Q_1, the first quartile, is the median of the lower half of the observations, then it follows that 25% of the data values are less than or equal to Q_1, and 75% are greater than or equal to Q_1.
- Since Q_3, the third quartile, is the median of the upper half of the observations, then it follows that 75% of the data values are less than or equal to Q_3, and 25% are greater than or equal to Q_3.
- Thus, the interquartile range (IQR) gives the spread of the middle 50% of data values.

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Definitions of the quartiles of a distribution sometimes differ slightly from the one given here. Using different definitions may result in slight differences in the values obtained, but these will be minimal and should not be considered a difficulty.
Note that the median, which has 50% of the data below and 50% of the data values above, can be denoted as Q_2, but we don't commonly use this notation.

Example 23 Finding the interquartile range (*IQR*)

Find the interquartile ranges for tasks A and B in Example 22 and compare.

Solution	Explanation
Task A	There are 30 values in total. This means that there are fifteen values in the lower 'half', and fifteen in the upper 'half'. The median of the lower half (Q_1) is the 8th value.
Lower half: 2 6 9 10 11 12 13 22 23 24 26 26 27 33 34 $Q_1 = 22$	
Upper half: 35 38 38 39 42 46 47 47 52 52 56 56 59 91 94 $Q_3 = 47$	The median of the upper half (Q_3) is the 8th value.
$IQR = Q_3 - Q_1 = 47 - 22 = 25$	Determine the IQR.
Task B $Q_1 = 31$ $Q_3 = 73$ $IQR = Q_3 - Q_1 = 73 - 31 = 42$	Repeat the process for task B.
The IQR shows the variability of task A marks is smaller than the variability of task B marks.	Compare the IQR for task A to the IQR for task B.

The interquartile range describes the range of the middle 50% of observations. It measures the spread of the data distribution around the median (M). Since the upper 25% and the lower 25% of the observations are discarded, the interquartile range is generally not affected by outliers in the dataset, which makes it a reliable measure of spread for any distribution, whether skewed or symmetric.

The standard deviation

The **standard deviation** (s), measures the spread of a data distribution about the mean ($\bar{x}$).

The standard deviation

The standard deviation is defined to be:

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n - 1}}$$

where n is the number of data values (sample size) and $\bar{x}$ is the mean.

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Although it is not easy to see from the formula, the standard deviation is an average of the squared deviations of each data value from the mean. We work with the squared deviations because the sum of the deviations around the mean will always be zero. For theoretical reasons (not important here), we average by dividing by $n - 1$, not n.

Normally, you will use your calculator to determine the value of a standard deviation. However, to understand what is involved when your calculator is doing the calculation, you should know how to calculate the standard deviation from the formula.

Example 24 **Calculating the standard deviation**

Use the formula:

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

to calculate the standard deviation of the dataset: 2, 3, 4.

Solution

	x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
	2	−1	1
	3	0	0
	4	1	1
Sum	9	0	2

$$\bar{x} = \frac{\sum x_i}{n} = \frac{2 + 3 + 4}{3} = \frac{9}{3} = 3$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{2}{3 - 1}} = 1$$

Explanation

To calculate s, it is convenient to set up a table with columns for:
x_i, the data values
$(x_i - \bar{x})$, the deviations from the mean
$(x_i - \bar{x})^2$, the squared deviations.

First find the mean and then complete the table as shown.

Substitute the required values into the formula and evaluate.

Summary statistics for a dataset usually include both measures of centre and spread. Since the standard deviation measures variation around the mean, it follows that the standard deviation should be used as the measure of spread when the mean is chosen as the measure of centre. Similarly, the interquartile range should be used as the measure of spread when the median is chosen as the measure of centre.

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Using technology to calculate summary statistics

Scientific calculator activity 10H:
Using a scientific calculator to calculate summary statistics

Task

Calculate the mean and standard deviation of the following data:

2 9 10 11 12 14 15 15 19 20

Casio fx82

Change the mode to statistics with one random variable.

Press Mode > 2 > 1 and you should get a table like this:

	x
1	
2	
3	

Insert the first data by pressing 2 then =. Continue inserting the data like this.

Leave the table by pressing AC.

Press Shift > 1 [STAT] > 4 [Var] > 2 [$\bar{x}$] > = to get the mean.

$\bar{x}$
12.7

Press Shift > 1 [STAT] > 4 [Var] > 3 [σx] > = to get the standard deviation.

σx
4.9406477

TI-30XB

Press data to move to a table.

Insert the first data by pressing 2 then enter. Continue inserting the data like this.

Press 2nd > data [stat] > 1 [1-Var] > enter enter enter to bring up the 1-variable summary. Using the arrow keys to navigate, the mean can be read off from the second position as $\bar{x} = 12.7$ and the standard deviation from the fourth position as $\sigma x = 4.9406477$.

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Sharp

Enter single variable statistics mode by pressing Mode 1 0
Insert the first data by pressing 2 then M+ [DATA]. Continue inserting the data like this.
Press ALPHA > 4 [$\bar{x}$] > = to get the mean.

Press ALPHA > 6 [σx] > = to get the standard deviation.

Spread-sheet **Spreadsheet activity 10H:** Using a spreadsheet to calculate summary statistics

Section Summary

- The **range** can be used to indicate spread of a dataset. The range is the difference between the largest and smallest values in the dataset.

  range = largest data value − smallest data value

- The **quartiles** of a dataset are values which divide the dataset into quarters. The **first quartile** (Q_1)is the median of the lower half of the observations, and **third quartile** (Q_3) is the median of the upper half of the observations.
- The **interquartile range** or ***IQR*** is a measure of spread of a dataset.

  $IQR = Q_3 - Q_1$

- The **standard deviation** is another measure of spread of a dataset. It measures the variation of the data values around the mean.
- When the distribution is symmetric and has no outliers the mean and standard deviation are the preferred measures of centre and spread.
- When the distribution is skewed or has outliers the median and interquartile range are the preferred measures of centre and spread.
- The mean and standard deviation can be determined using a scientific calculator.

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Skill-sheet

Exercise 10H

SF

Determining the range and the *IQR* from data

Example 22
Example 23

1 Consider the following dataset:

1 2 1 0 2 3 1 2 6 7

a Order the data from smallest to largest value.

b Locate the minimum and maximum values, and hence find the value of the range.

c Determine the value of Q_1, the median of the lower half of the data values.

d Determine the value of Q_3, the median of the upper half of the data values.

e Hence, find the value of the interquartile range (IQR).

2 Determine the IQR and range of each of these ordered datasets.

a 2 2 5 7 9 11 12 16 23

b 1 3 3 5 6 7 9 11 12 12

c 21 23 24 25 27 27 29 31 32 33

d 101 101 105 106 107 107 108 109

e 0.2 0.9 1.0 1.1 1.2 1.2 1.3 1.9 2.1 2.2 2.9

Determining the median and *IQR* from a plot

3 Determine the IQR for the data displayed in the following stem plots.

a key: 4|8 represents 48

Monthly rainfall (mm)

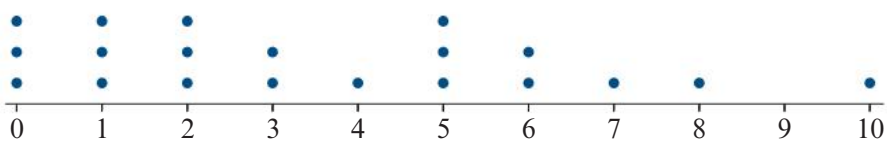
4	8 9 9
5	0 2 7 7 8 9 9
6	0 7

b key: 1|7 represents 17

Battery time (hours)

0	4
1	7 9
2	0 1 2 4 5 6 6 7 7 8
3	0 0 1 1 3 3 4
4	0 1 6

4 Determine the median and quartiles for the data displayed in the following dot plot.



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Calculating the standard deviation using a table

Example 24

5 Consider the following dataset:

1 2 3 6

a Calculate the value of the mean.

b Use a table to calculate the value of the standard deviation, s, using the formula

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

Calculating the standard deviation using a calculator

6 A manufacturer advertised that a can of soft drink contains 375 mL of liquid. A sample of 16 cans yielded the following contents:

357 375 366 360 371 363 351 369
358 382 367 372 360 375 356 371

Calculate the mean and standard deviation, median and IQR, and range for the volume of drink in the cans. Give answers rounded to one decimal place.

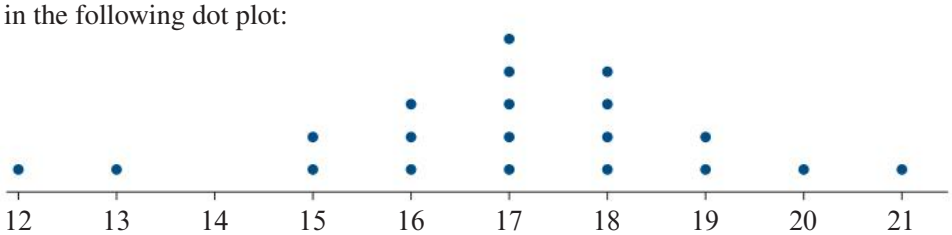
7 The serum cholesterol levels for a sample of 18 people are:

231 159 203 304 248 238 209 193 225
190 192 209 161 206 224 276 196 189

Calculate the mean and standard deviation, median and IQR, and range of the serum cholesterol levels. Give answers rounded to one decimal place.

Calculating summary statistics from a plot

8 Determine the range, interquartile range and standard deviation of the dataset displayed in the following dot plot:



9 Twenty babies were born at a local hospital on one weekend. Their birth weights are given in the stem plot.

Birth weight (kg) 3 | 6 represents 3.6 kg

2	1 5 7 9 9
3	1 3 3 4 4 5 6 7 7 9
4	1 2 2 3 5

Determine the mean and standard deviation, median and IQR, and range of the birth weights.

SF

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Understanding summary statistics

10 The results of a student's chemistry experiment were as follows:

7.3 8.3 5.9 7.4 6.2 7.4 5.8 6.1 6.0

a **i** Find the mean and the median of the results.

ii Find the *IQR* and the standard deviation of the results.

b Unfortunately, when the student was transcribing his results into his chemistry book, he made a small error and wrote:

7.3 8.3 5.9 7.4 6.2 7.4 5.8 6.1 60

i Find the mean and the median of these results.

ii Find the interquartile range and the standard deviation of these results.

c Describe the effect the error had on the summary statistics in parts **a** and **b**.

11 Consider the following ordered dataset:

$x - 3.2$ x y $y + 4.1$

If the median of the dataset is equal to 11, and the range is equal to 10.1, determine the values of x and y.

12 Consider the following ordered dataset:

0 2 p 7 8 10 q 14

If the *IQR* of the dataset is equal to 8, determine possible integer values for p and q.

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Key ideas and chapter summary

Types of data	Data can be classified as **categorical** or **numerical**.
Categorical data	**Categorical data** arises when classifying or naming some quality or attribute. Categorical data can be **nominal** or **ordinal**.
Nominal data	**Nominal** data is a type of categorical data where the values of the variable are the names of the categories, and there is no inherent order in the categories.
Ordinal data	**Ordinal** is a type of categorical data where the values of the variable are the names of groups, and there is an inherent order in the categories.
Numerical data	**Numerical data** arises from measuring or counting some quantity. Numerical data can be **discrete** or **continuous**.
Discrete data	**Discrete** data can only take particular numerical values, usually whole numbers, and often arises from counting.
Continuous data	**Continuous** numerical data describes numerical data that can take any value, sometimes in an interval, and often arises from measuring.
Frequency table	A **frequency table** is a listing of the values that a variable takes in a dataset, along with how often (frequently) each value occurs. **Frequency** can be recorded as the number of times a value occurs or as a **percentage**, the percentage of times a value occurs.
Column chart	A **column chart** is used to display the frequency distribution of a categorical variable. The frequencies are shown on the vertical axis.
Bar chart	A **bar chart** is used to display the frequency distribution of a categorical variable. The frequencies are shown on the horizontal axis.
Mode, modal category/class	The **mode** (or modal category) is the value of a variable (or the category) that occurs most frequently. The **modal interval**, for **grouped data,** is the interval that occurs most frequently.
Histogram	A **histogram** uses columns to display the frequency distribution of a numerical variable, and is suitable for medium- to large-sized datasets.
Dot plot	A **dot plot** consists of a number line with each data point marked by a dot, and is suitable for small- to medium-sized datasets.

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Review

Stem-and-leaf plot or stem plot

A **stem plot** is a visual display of a numerical dataset, formed from the actual data values, and is suitable for small- to medium-sized datasets.

Describing the distribution of a numerical variable

The **distribution of a numerical variable** can be described in terms of **shape** (**symmetric** or **skewed**: positive or negative), **centre** (the middle of the distribution), **spread** (how clustered together the values are) and **outliers**.

Summary statistics

Summary statistics are numerical values for special features of a data distribution such as centre and spread.

Mean

The **mean ($\bar{x}$)** is a summary statistic that can be used to locate the centre of a symmetric distribution. The value of the mean is determined from the formula: $\bar{x} = \frac{\sum x_i}{n}$

Range

The **range *(R)*** is the difference between the smallest and the largest data values. It is the simplest measure of spread.

Standard deviation

The **standard deviation (*s*)** is a summary statistic that measures the spread of the data values around the mean. The value of the standard deviation is determined from the formula:

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

Median

The **median *(M)*** is a summary statistic that can be used to locate the centre of a distribution. It is the midpoint of a distribution, so that 50% of the data values are less than or equal to this value and 50% are greater than or equal to it. It is sometimes denoted as Q_2.

Quartiles

Quartiles are summary statistics that divide an ordered dataset into four equal groups.

Interquartile range

The **interquartile range (*IQR*)** gives the spread of the middle 50% of data values in an ordered dataset. It is found by evaluating

$$IQR = Q_3 - Q_1$$

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Skills checklist

Check-list

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills.

10A	1	I can differentiate between categorical and numerical variables.	☐
		See Example 1 and Exercise 10A Question 1.	
10A	2	I can differentiate between nominal and ordinal variables.	☐
		See Example 2 and Exercise 10A Question 2.	
10A	3	I can differentiate between discrete and continuous variables.	☐
		See Example 3 and Exercise 10A Question 3.	
10B	4	I can construct a frequency table and a percentage frequency table.	☐
		See Example 5 and Exercise 10B Question 1.	
10B	5	I can construct a column chart from a frequency table.	☐
		See Example 6 and Exercise 10B Question 4.	
10B	6	I can construct a bar chart from a frequency table.	☐
		See Example 7 and Exercise 10B Question 6.	
10B	7	I can construct a pie chart from a frequency table.	☐
		See Example 8 and Exercise 10B Question 8.	
10B	8	I can identify the mode from a table or chart.	☐
		See Example 8 and Exercise 10B Questions 8 and 11.	
10C	9	I can write a report describing a dataset based on a a frequency table.	☐
		See Examples 9 and 10, and Exercise 10C Questions 1 and 4.	
10D	10	I can construct an (ungrouped) frequency table for discrete numerical data.	☐
		See Example 11 and Exercise 10D Question 1.	
10D	11	I can construct a grouped frequency table.	☐
		See Examples 12 and 13, and Exercise 10D Questions 1 and 2.	
10D	12	I can construct a histogram from a dataset.	☐
		See Examples 14 and 15, and Exercise 10D Questions 3 and 4.	

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Review

10E	13	I can construct a dot plot from a dataset.	☐

See Example 16 and Exercise 10E Question 1.

10E	14	I can construct a stem plot from a dataset.	☐

See Example 17 and Exercise 10E Question 5.

10F	15	I can describe the shape, centre and spread of a dataset from a plot.	☐

See Example 18 and Exercise 10F Question 4.

10G	16	I can determine the median and mean as measures of centre for a dataset, and identify when it is more appropriate to use the median.	☐

See Examples 19, 20 and 21, and Exercise 10G Questions 1, 3 and 4.

10H	17	I can determine the range and quartiles of a dataset and hence calculate the *IQR*.	☐

See Examples 22 and 23, and Exercise 10H Question 1.

10H	18	I can find the standard deviation of a dataset both using a table and using a calculator.	☐

See Example 24 and Exercise 10H Questions 5 and 6.

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Multiple-choice questions

1 In a survey, a number of people were asked to indicate how much they exercised by selecting one of these options: 'never', 'seldom', 'sometimes' or 'regularly'. The resulting variable was named *level of exercise*. The type of data generated is:

A numerical **B** nominal **C** ordinal **D** continuous

2 Respondents to a survey question 'Are you concerned about climate change?' were asked to select from the following responses:

1 = not at all 2 = a little 3 = somewhat 4 = extremely

The data which was collected in response to this question is:

A nominal **B** ordinal **C** discrete **D** numerical

The following information relates to Questions 3 *and* 4.

Data relating to the following five variables were collected from a group of university students:

- *course*
- *study mode* (1 = on campus, 2 = online)
- *study load* (1 = full-time, 2 = part-time)
- *number of contact hours scheduled per week*
- *number of subjects needed to complete degree*

3 The number of these variables that are discrete numerical variables is:

A 1 **B** 2 **C** 3 **D** 4

4 The number of these variables that are ordinal variables is:

A 0 **B** 1 **C** 2 **D** 3

5 For which of the following variables is a column chart an appropriate display?

A *Weight* in kilograms **B** *Age* in years
C *Hair colour* **D** *Reaction time* in seconds

6 For which of the following variables is a pie chart an appropriate display?

A *Distance between towns* (km) **B** *Age* in years
C *Weight* (underweight, average, overweight) **D** *Mark* on the last test

7 For which of the following variables is a histogram an appropriate display?

A *Hair colour* **B** *Gender* (male, female)
C *Distances between towns* (km) **D** *Postcode*

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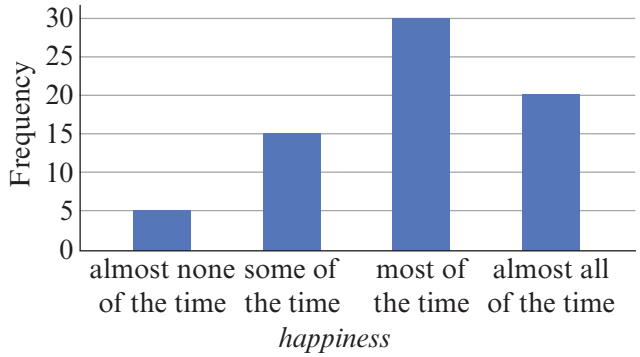
8 In a survey people were asked to select how much of the time they felt happy from the alternatives 'almost none of the time', 'some of the time', 'most of the time', or 'almost all of the time'. Their responses are summarised in the following column chart of the variable *happiness*.

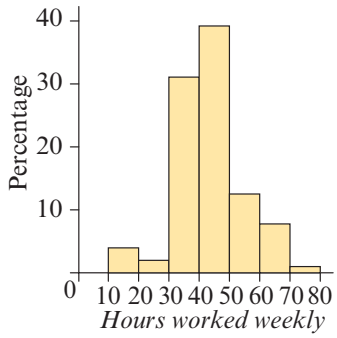


The percentage of people who chose the modal response to this question is closest to:

A 30% **B** 43% **C** 50% **D** 57%

The following information relates to Questions 9 *to* 12.

The number of hours worked per week by employees in a large company is shown in this percentage frequency histogram.



9 The percentage of employees who work from 20 to less than 30 hours per week is closest to:

A 1% **B** 2% **C** 6% **D** 10%

10 The percentage of employees who worked less than 30 hours per week is closest to:

A 2% **B** 3% **C** 4% **D** 6%

11 The modal interval for hours worked is:

A 10 to less than 20 **B** 20 to less than 30
C 30 to less than 40 **D** 40 to less than 50

12 The median number of hours worked is in the interval:

A 10 to less than 20 **B** 20 to less than 30
C 30 to less than 40 **D** 40 to less than 50

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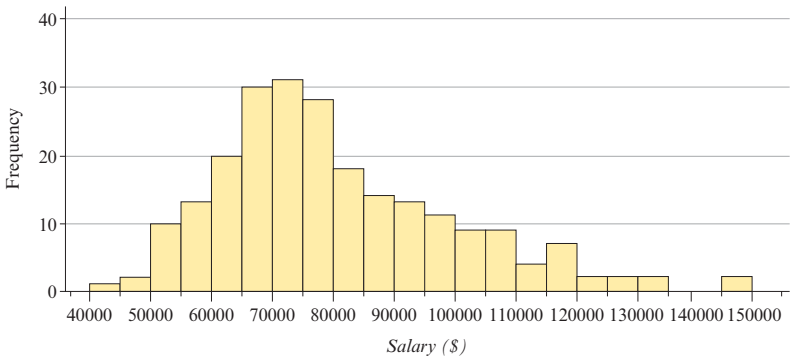
The following information relates to Questions 13 *and* 14.

The annual salaries for all the sales staff in a large company are summarised in the following histogram.



13 The number of people in the company who earn from $65 000 to less than $70 000 per year is equal to:

A 30 **B** 50 **C** 32 **D** 62

14 The shape of this histogram is best described as:

A positively skewed with possible outliers
B positively skewed with no outliers
C approximately symmetric
D negatively skewed with a possible outlier

The following information relates to Questions 15 *to* 18.

A group of 18 employees of a company were asked to record the number of meetings they had attended in the last month.

1 1 2 3 4 5 5 6 7 9 10 12 14 14 16 22 23 44

15 The range of meetings is:

A 22 **B** 24 **C** 43 **D** 44

16 The median number of meetings is:

A 7 **B** 7.5 **C** 8 **D** 9

17 The mean number of meetings is:

A 8 **B** 9 **C** 10 **D** 11

18 The interquartile range (*IQR*) of the number of meetings is:

A 4 **B** 9.5 **C** 10 **D** 14

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19 The heights of six basketball players (in cm) are:

178.1 185.6 173.3 193.4 183.1 193.0

The mean and standard deviation are closest to:

A mean = 184.4; standard deviation = 8.0
B mean = 184.4; standard deviation = 7.3
C mean = 182.5; standard deviation = 7.3
D mean = 182.5; standard deviation = 8.0

20 The following stem plot shows the distribution of the time it took (in minutes) for each of a group of 25 people to solve a complex task.

Time (minutes) key: 4|0 represents 4.0

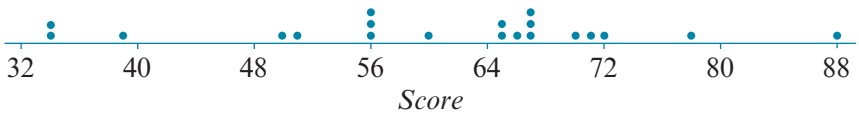
4	2 6
5	1 3 6 8
6	0 1 5 6 7
7	1 3 4 5 7 8 9
8	0 2 5 9
9	5 5
10	6

The shape of this distribution is best described as:

A positively skewed with a possible outlier
B positively skewed with no outliers
C approximately symmetric
D negatively skewed with no outliers

The following information relates to Questions 21 *and* 22.

The dot plot below gives the examination scores in mathematics for a group of 20 students.



21 The number of students who scored 56 on the examination is:

A 1 **B** 2 **C** 3 **D** 4

22 The percentage of students who scored between 40 and 80 on the exam is closest to:

A 60% **B** 70% **C** 80% **D** 90%

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Use the following information to answer Questions 23 *and* 24.

The stem plot displays the number of times each person in a sample of 16 people bought a take-away coffee in the last month.

Key: 0|1 = 1

0	0 0 8 9
1	2 3 7
2	0 1 2 5 6 6
3	0 6 7

23 The median, M, of the number of take-away coffees bought is equal to:

A 20.5 **B** 10.5 **C** 18.5 **D** 21

24 The interquartile range, IQR, of the number of take-away coffees bought is equal to:

A 15.5 **B** 10.5 **C** 15.0 **D** 36

Short-response questions

SF

1 Classify the data that arises from the following situations as nominal, ordinal, discrete or continuous.

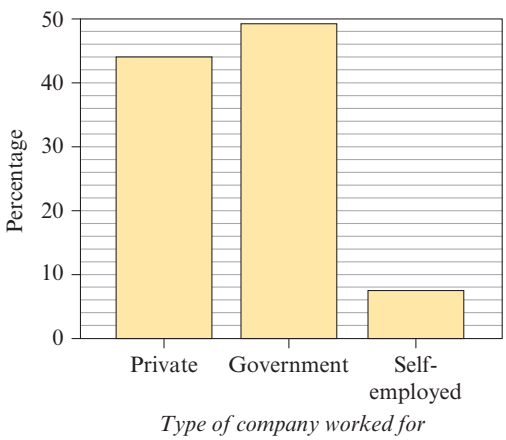
a The number of phone calls a hotel receptionist receives each day.

b Interest in politics on a scale from 1 to 5, where 1 = very interested, 2 = quite interested, 3 = somewhat interested, 4 = not very interested, and 5 = uninterested.

2 The column chart opposite shows the percentage of working people in a certain town who are employed in private companies, work for the government or are self-employed.

a Classify the data as categorical or numerical.

b Determine the approximate percentage of people who are self-employed.



3 A researcher asked a group of people to record how many cigarettes they had smoked on a particular day. Here are her results:

0	0	9	10	23	25	0	0	34	32	0	0	30	0	4
5	0	17	14	3	6	0	33	23	0	32	13	21	22	6

Using class intervals of width 5, construct a histogram of this data.

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4 A teacher recorded the time taken (in minutes) by each of a class of students to complete a test:

56	57	47	68	52	51	43	22	59	51	39
54	52	69	72	65	45	44	55	56	49	50

a Construct a dot plot of the dataset.

b Construct a stem plot of the dataset.

c Use this stem plot to find the median and quartiles for the time taken.

5 The weekly rentals, in dollars, for a group of people are given below:

285	185	210	215	320	680	280
265	300	210	270	190	245	315

Calculate the mean and standard deviation, the median and the IQR, and the range of the weekly rentals. Write your answers correct to two decimal places if they are not exact.

6 Geoff decided to record the time (in minutes) it takes him to complete his mail round each working day for four weeks. His data is recorded below:

170	189	201	183	168	182	161	166	167	173	182	167	188	211
164	176	161	187	180	201	147	188	186	176	174	193	185	183

Calculate the mean and standard deviation of the times for his mail round, correct to two decimal places.

7 The divorce rates (in percentages) of 19 countries are:

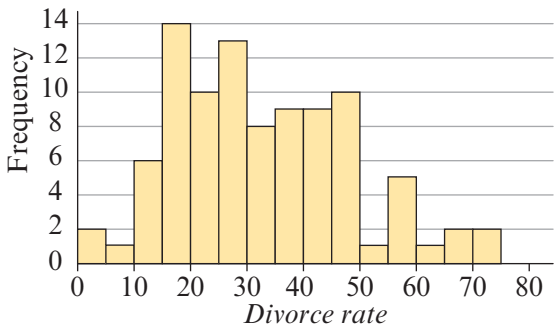
27	18	14	25	28	6	32	44	53	0	26	8	14	5	15	32	6	19	9

a Classify the data as categorical or numerical.

b Construct an ordered stem plot of divorce rate.

c Calculate the mean and median of the divorce rate.

d The histogram shows the divorce rate for a larger dataset including 93 countries:



i Describe the shape of the histogram.

ii What percentage of countries have divorce rates from 10% to less than 20%?

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8 Twenty-two students were asked the question, 'How often do you play sport?', with the possible responses: 'regularly', 'sometimes' or 'rarely'. The distribution of responses is summarised in the frequency table.

Plays sport	Frequency	
	Number	Percentage
Regularly	5	22.7
Sometimes	10	
Rarely		31.8
Total	22	

a Write down the information missing from the table.

b Use the information in the frequency table to complete the report below describing the distribution of student responses to the question, 'How often do you play sport?'

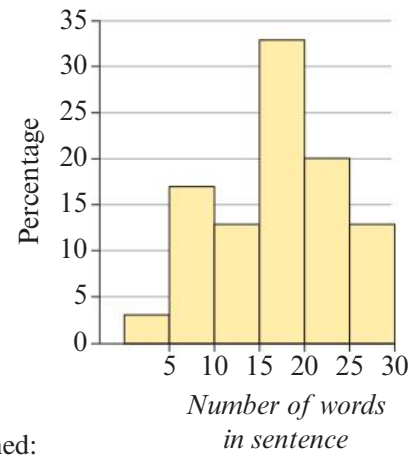
Report

When ____ students were asked the question, 'How often do you play sport', the dominant response was 'Sometimes', given by ____ % of the students. Of the remaining students, ____ % of the students responded that they played sport ____, while ____ % said that they played sport ____.

9 Construct a histogram to display the information in the frequency table opposite. Label axes and mark scales.

Population density	Frequency
0–<200	11
200–<400	4
400–<600	4
600–<800	2
800–<1000	1
Total	22

10 The histogram opposite displays the distribution of the number of words in 30 randomly selected sentences.



a Determine the percentage of these sentences that contained:

i 5–9 words

ii 25–29 words

iii fewer than 15 words.

Write answers correct to the nearest per cent.

b Calculate the number of sentences which contained:

i 20–24 words

ii more than 25 words.

c State the modal interval.

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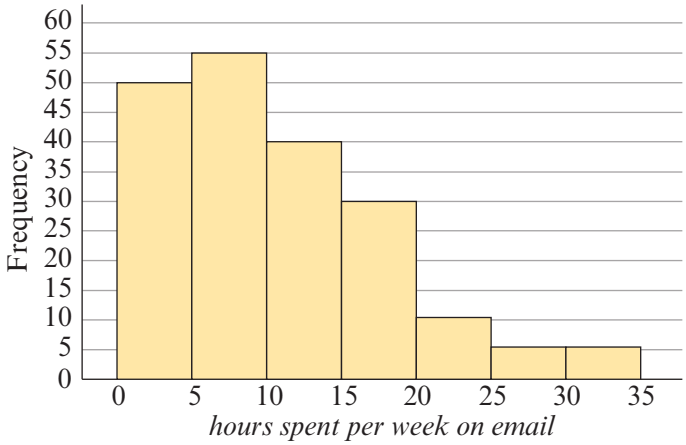
CF

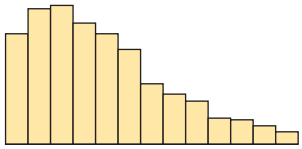
11 The histogram shows the average number of hours per week a group of 195 people spent on email. Responses were collected to the nearest 0.1 hour.

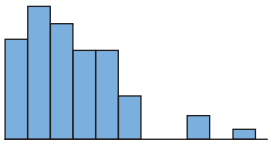


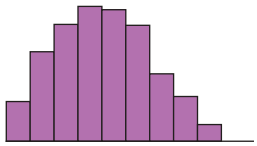
a Determine all possible values for the median.

b Determine the maximum possible value for the *IQR*.

12 State which of the following histograms you might question using the mean for as a measure of the centre of the distribution. Justify your selection.

a 

b 

c 

CU

13 Consider the following ordered dataset:

0 2 p q

If the mean of the dataset is equal to 3, and the median is equal to 2.5, determine the values of p and q.

14 Consider the following ordered dataset:

0 2 p 7 8 10 q 16

If the mean of the dataset is equal to 7.5, and the *IQR* is equal to 6.5, determine the values of p and q.

15 Kiana has completed four assessments so far in Year 11 Mathematics. Assessment 5 is not yet complete. Her scores so far are given in the following table:

Assessment	1	2	3	4	5
Mark	$\frac{8}{10}$	$\frac{17}{20}$	$\frac{7}{10}$	$\frac{25}{30}$	$\frac{?}{60}$

If each score is given equal weight, determine the minimum score she needs on the final assessment for her mean score to be 80% or more.

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Chapter 11

Comparing datasets

Chapter questions

UNIT 2: APPLICATIONS OF LINEAR EQUATIONS AND TRIGONOMETRY, MATRICES AND UNIVARIATE DATA ANALYSIS

Topic 5: Univariate data analysis 2

- How do we construct and interpret a boxplot?
- How do we identify an outlier?
- How do we use a back-to-back stem plot to compare two datasets?
- How do we use parallel boxplots to compare two or more datasets?
- How do we use summary statistics to compare two or more datasets?
- How do we report findings when comparing two or more datasets?

In Chapter 10, you learned how to display and describe the distribution of a single variable. In the process you learned how to use data to answer questions like 'What is the favourite colour of prep-grade students?' or 'How do the heights of football players vary?' In each case, we concentrated on investigating an individual statistical variable. However, questions like 'Does the new treatment for headache work more quickly than the old treatment?' or 'Do younger people have more car accidents than older people?' cannot be answered by just one dataset; we need to consider and compare two or more sets of numerical data. In this chapter, we will use both statistical displays and summary statistics to answer questions such as these.

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11A Boxplots

Learning intentions

- To introduce the boxplot as a plot for displaying the distribution of numerical data.
- To be able to define and identify an outlier.
- To be able to construct a simple boxplot and a boxplot with outliers.
- To be able to identify shape, centre, spread and outliers of a distribution from a boxplot.
- To be able to write a report describing the important features of a distribution based on a boxplot.

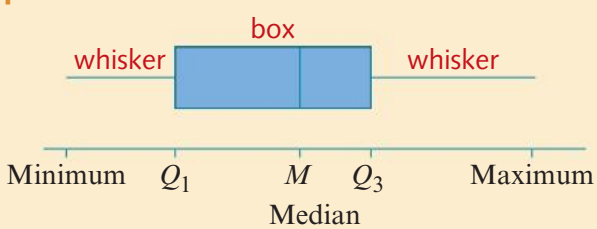
We already have three plots which we can use to display the distribution of a numerical variable, namely the histogram, the dot plot and the stem plot. Another plot that can be used to display data from a numerical variable is the **boxplot**. The boxplot is very useful as it allows us to summarise quite large datasets into concise plots, and can be used to compare two or more datasets.

The simple boxplot

Knowing the median and quartiles of a distribution means that quite a lot is known about the central region of the dataset. If something is known about the tails of the distribution as well, then a good picture of the whole dataset can be obtained. This can be achieved by knowing the **maximum** and **minimum** values of the data.

When we list the median, the quartiles and the maximum and minimum values of a datasets, we have what is known as a **five-number summary**. Its pictorial (graphical) representation is called a a box-and-whisker plot, or **boxplot**.

The simple boxplot



- A boxplot is a graphical representation of a five-number summary.
- A box is used to represent the middle 50% of scores.
- The median is shown by a vertical line drawn within the box.
- Lines (whiskers) extend out from the lower and upper ends of the box to the smallest and largest data values of the dataset respectively.

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Example 1 **Constructing a simple boxplot from a five-number summary**

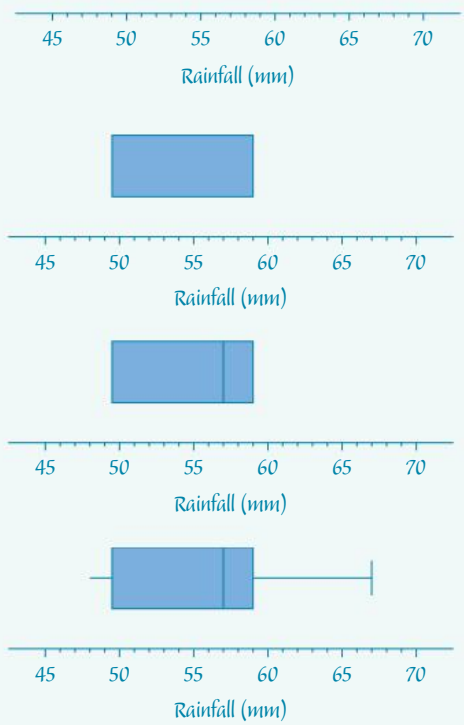
The following are the monthly rainfall figures for a year at the weather station.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall (mm)	48	57	52	57	58	49	49	50	59	67	60	59

Construct a boxplot to display this data, given the five-number summary:

Min = 48 $Q_1 = 49.5$ $M = 57$ $Q_3 = 59$ Max = 67

Solution



Explanation

Draw a labelled and scaled number line that covers the full range of values.

Draw a box starting at $Q_1 = 49.5$ and ending at $Q_3 = 59$.

Mark in the median value with a vertical line segment at $M = 57$.

Draw in the whiskers, lines joining the midpoint of the ends of the box, to the minimum and maximum values, 48 and 67 respectively.

Boxplots with outliers

An extension of the boxplot can also be used to identify possible outliers in a dataset. Sometimes it is difficult to decide whether or not a data value is an outlier. Suppose, for example, we construct a boxplot that has one extremely long whisker. How might we explain this?

- One explanation is that the data distribution is extremely skewed, with lots of data values in one tail.
- Another explanation is that the long whisker hides one or more outliers.

By modifying the boxplot so that it shows outliers we can decide which explanation is more likely, but firstly we need an exact definition of an outlier.

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Outliers

Any data point in a distribution that lies more than 1.5 interquartile ranges below the first quartile, or more than 1.5 interquartile ranges above the third quartile, is an **outlier**.

Outliers are plotted individually in the boxplot, and the whisker now ends at the largest or smallest data value that is not outside these limits.

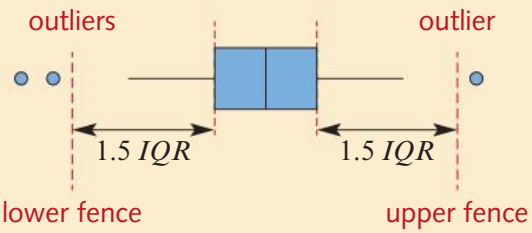
Upper and lower fences

When constructing a boxplot to display outliers, we must first determine the location of the **upper and lower fences**. These are imaginary lines drawn 1.5 interquartile ranges (or box widths) to the left and right of the ends of the box (see below). Data values outside these fences are classified as possible outliers and plotted separately.

Boxplot with outliers

In a boxplot, possible outliers are defined as those values that are:

- greater than $Q_3 + 1.5 \times IQR$ (upper fence)
- less than $Q_1 - 1.5 \times IQR$ (lower fence).



When drawing a boxplot, any data value identified as an outlier is indicated by a dot. The whiskers then end at the smallest and largest values that are not classified as outliers.

Example 2 Constructing a boxplot showing outliers

The number of hours that each of 33 students spent on a school project is shown below.

2	3	4	9	9	13	19	24	27	35	36
37	40	48	56	59	71	76	86	90	92	97
102	102	108	111	146	147	147	166	181	226	264

Construct a boxplot with outliers.

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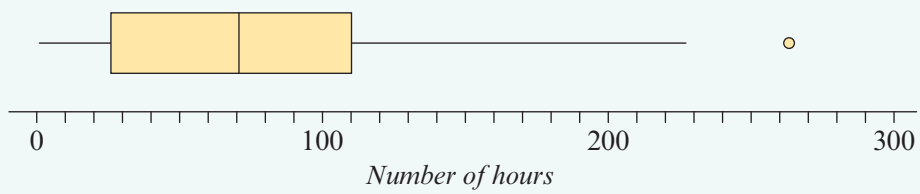
Solution	Explanation
Minimum = 2 Maximum = 264 Median = 71	From the ordered list, state the minimum and maximum values. Find the median, the $\frac{1}{2}(33 + 1)^{\text{th}} = 17^{\text{th}}$ value.
$Q_1 = \frac{24 + 27}{2} = 25.5$ $Q_3 = \frac{108 + 111}{2} = 109.5$	Determine Q_1 and Q_3. There are 33 values, so Q_1 is halfway between the 8^{th} and 9^{th} values and Q_3 is halfway between the 25^{th} and 26^{th} values.
$IQR = Q_3 - Q_1 = 109.5 - 25.5 = 84$	Determine the IQR.
Lower fence $= Q_1 - 1.5 \times IQR$ $= 25.5 - 1.5 \times 84$ $= -100.5$ Upper fence $= Q_3 + 1.5 \times IQR$ $= 109.5 + 1.5 \times 84$ $= 235.5$	Determine the upper and lower fences.
There is one outlier 264. The largest value that is not an outlier is 226.	Locate any values outside the fences, and the values that lie just inside the limits (the whiskers will extend to these values).

The boxplot can now be constructed as shown below. The upper whisker extends to the second highest value, and the circle denotes the outlier. The fences are not shown on the boxplot.



There is one possible outlier, the student who spent 264 hours on the project.

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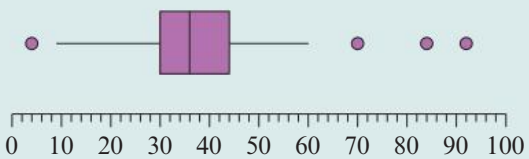
Interpreting boxplots

Constructing a boxplot is not an end in itself. The prime reason to construct a boxplot is to help us answer statistical questions. To do this, we read values from a boxplot and use them to determine statistics such as the median, the interquartile range and the range. We also use boxplots to identify possible outliers.

Example 3 Reading values from a boxplot

For the boxplot shown, write down the values of the:



a median
b quartiles Q_1 and Q_3
c interquartile range (IQR)
d minimum and maximum values
e values of any possible outliers
f smallest value in the upper end of the dataset that will be classified as an outlier
g largest value in the lower end of the dataset that will be classified as an outlier.

Solution	Explanation
a $M = 36$	The median is the vertical line in the box.
b $Q_1 = 30$, $Q_3 = 44$	The quartiles are the end points of box.
c $IQR = Q_3 - Q_1 = 44 - 30 = 14$	Interquartile range ($IQR = Q_3 - Q_1$)
d $Min = 4$, $Max = 92$	The minimum and maximum values are the extremes.
e 4, 70, 84 and 92 are possible outliers.	The possible outliers are shown as dots.
f Upper fence $= Q_3 + 1.5 \times IQR$ $= 44 + 1.5 \times 14 = 65$ Any value above 65 is an outlier.	The upper fence is given by $Q_3 + 1.5 \times IQR$
g Lower fence $= Q_1 - 1.5 \times IQR$ $= 30 - 1.5 \times 14 = 9$ Any value below 9 is an outlier.	The lower fence is given by $Q_1 - 1.5 \times IQR$

Once we know the location of the quartiles, we can use the boxplot to estimate percentages.

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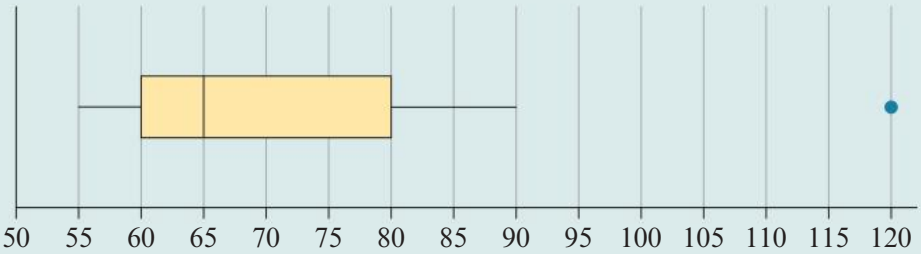
Example 4 Estimating percentages from a boxplot

Consider the boxplot shown.



Estimate the percentage of values:

a less than 60 **b** less than 65 **c** more than 80

d between 60 and 80 **e** between 60 to 120

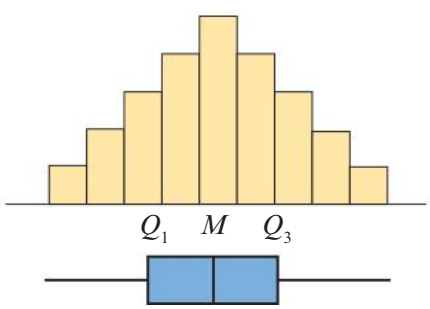
Solution	Explanation
a 25% of the data values are less than 60.	60 is the first quartile (Q_1).
b 50% of the data values are less than 65.	65 is the median (M, or sometimes Q_2).
c 75% of the data values are less than 80, so 25% of the data values are greater than 80.	80 is the third quartile (Q_3).
d 50% of the values are between 60 and 80.	75% of the data values are less than 80, and 25% of the data values are less than 60.
e 75% of the values are between 60 and 120.	100% of the data values are 120 or less, and 25% of the data values are less than 60.

Relating a boxplot to shape

When there are a reasonable number of data values, the shape of a distribution can be identified from a boxplot.

A symmetric distribution

A symmetric distribution tends to be centred on its median and have values evenly spread around the median. As a result, its boxplot will also be symmetric, its median is close to the middle of the box and its whiskers are approximately equal in length.



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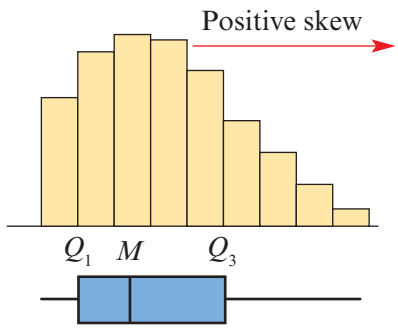
Positively skewed distributions

Positively skewed distributions are characterised by a cluster of data values around the median at the left-hand end of the distribution with a gradual tailing off to the right.

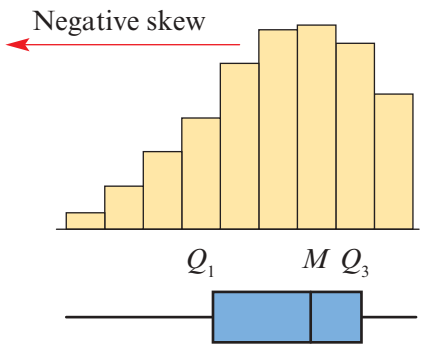
As a result, the boxplot of a positively skewed distribution will have its median off-centre and to the left-hand side of its box. The left-hand whisker will be short, while the right-hand whisker will be long, reflecting the gradual tailing off of data values to the right.



Negatively skewed distributions

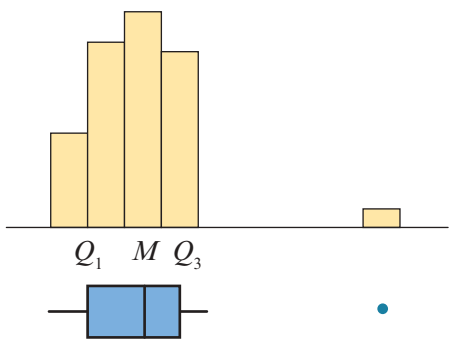
Negatively skewed distributions are characterised by a clustering of data values around the median at the right-hand end of the distribution, with a gradual tailing off of data values to the left.

As a result, the boxplot of a negatively skewed distribution has the median off-centre and in the right-hand side of its box. The right-hand whisker will be short, while the left-hand whisker will be long, reflecting the gradual tailing off of data values to the left.



Distributions with outliers

Distributions with outliers are characterised by large gaps between the main body and data values in the tails. The histogram opposite displays a distribution with an outlier. In the corresponding boxplot, the box and whiskers represent the main body of data and the dot, separated by a gap from the box and whiskers, an outlier.



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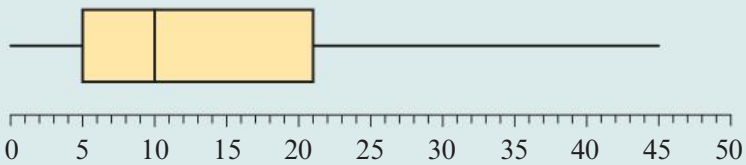
Using boxplots to describe a distribution

Because of the wealth of information contained in a boxplot, it is an extremely powerful tool for describing the features of distribution in terms of shape, centre, spread and outliers.

Example 5 Using a boxplot to describe the features of distribution without outliers

Describe the distribution represented by the boxplot in terms of shape, centre and spread. Give appropriate values.

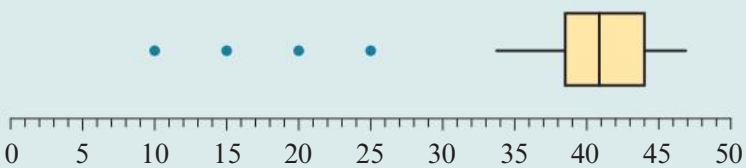


Solution

The distribution is positively skewed, centred at 10 which is the median value. The spread of the distribution as measured by the *IQR* is 16 and, as measured by the range, 45.

Example 6 Using a boxplot to describe the features of a distribution with outliers

Describe the distributions represented by the boxplot in terms of shape and outliers, centre and spread. Give appropriate values.



Solution

The distribution is symmetric but with outliers. The distribution is centred at 41, the median value. The spread of the distribution, as measured by the *IQR*, is 5.5 and, as measured by the range, 37. There are four outliers: 10, 15, 20 and 25.

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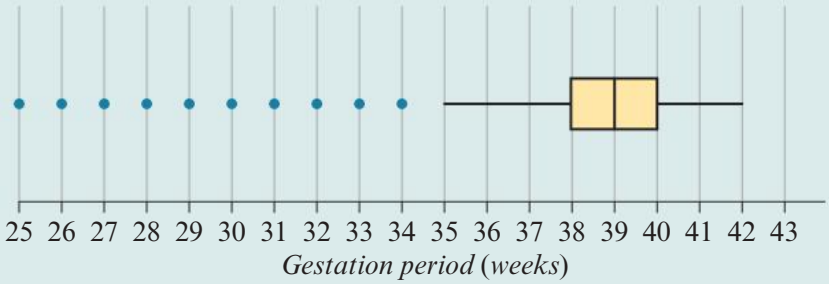
Example 7 Using a boxplot to answer statistical questions

The boxplot shows the gestation period (completed weeks) for a sample for 1000 babies born in Australia one year. Describe the distribution of gestation period in terms of shape, centre, spread and outliers.



Solution

The distribution of gestational period is negatively skewed with several outliers. The distribution is centred at 39 weeks, the median value, and 50% of the gestational periods were between 38 and 40 weeks. The range of the distribution is 17 weeks, but the interquartile range is only 2 weeks. Any gestational period less than 35 weeks or less is considered unusual, but gestational lengths of 25, 26, 27, 28, 29, 30, 31, 32, 33 and 34 weeks were recorded. No gestational period of more than 42 weeks was recorded.

Section Summary

- A **simple boxplot** is a pictorial representation of the five-number summary.
- The **five-number summary** consists of the minimum, the first quartile Q_1, the median M, the third quartile Q_3 and the maximum.
- From the boxplot, we can see:
  - 25% of the data is between the minimum and Q_1.
  - 25% of the data is between Q_1 and the median.
  - 25% of the data is between the median and Q_3.
  - 25% of the data is between Q_3 and the maximum.
- A **boxplot with outliers** shows the five-number summary as well as any outliers.
- The **lower fence** is $Q_1 - 1.5 \times IQR$.
- The **upper fence** is $Q_3 + 1.5 \times IQR$.
- Any data value less than the lower fence, or greater than the upper fence, is shown on the boxplot as an outlier.
- From a boxplot, we can write a report describing the important features of a distribution.

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Skill-sheet

Exercise 11A

Constructing a simple boxplot from a five-number summary

Example 1

1 The five-number summary for the heights (in centimetres) of a class of girls is:

Min = 123 Q_1 = 141.5 M = 154 Q_3 = 161.5 Max = 180

Use this five-number summary to construct a simple boxplot.

2 Use the following five-number summaries to construct simple boxplots.

a Min = 1, Q_1 = 4, M = 8, Q_3 = 13.5, Max = 24

b Min = 136, Q_1 = 148, M = 158, Q_3 = 169, Max = 189

3 **a** Construct a simple boxplot from the following five-number summary:

Min = 10, Q_1 = 22, M = 40, Q_3 = 70, Max = 70

b Explain why the box has no upper whisker.

Constructing a simple boxplot from data

4 The data shows how many hours each of a group of 41 players spent at training in a particular week.

1	2	2	3	3	4	4	4	4	4	4	5
5	5	6	6	6	7	7	8	8	9	10	10
11	11	12	12	12	12	13	14	14	14	15	15
16	16	17	18	24							

a Determine the five-number summary for this data.

b Use this five-number summary to construct a simple boxplot.

5 The length of time, in years, that employees have been employed by a company is:

5	1	20	8	6	9	13	15	4	2
15	14	13	4	16	18	26	6	8	2
6	7	20	2	1	1	5	8		

a Determine the five-number summary for this data.

b Use this five-number summary to construct a simple boxplot.

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Constructing a boxplot with outliers

Example 2

6 The five-number summary for a dataset is:

Min = 0 Q_1 = 13 M = 16 Q_3 = 19 Max = 35

Determine the values of the lower and upper fences.

7 The five-number summary for a dataset is:

Min = 14 Q_1 = 45 M = 55 Q_3 = 65 Max = 99

a Determine the values of the upper and lower fences.

b The smallest three values in the dataset are 14, 18, 34 and the largest are 90, 94, 99. Which of these are outliers?

8 The five-number summary for a dataset is:

Min = 14 Q_1 = 40 M = 55 Q_3 = 62 Max = 102

a Determine the values of the upper and lower fences.

b The smallest three values in the dataset are 6, 18, 34 and the largest are 90, 99, 102. Which of these are outliers?

9 The amount of pocket money paid per week to a sample of Year 8 students is:

$5.00	$10.00	$12.00	$8.00	$7.50	$12.00	$15.00
$10.00	$10.00	$0.00	$5.00	$10.00	$20.00	$15.00
$26.00	$13.50	$15.00	$5.00	$15.00	$25.00	$16.00

The five-number summary is:

Min = 0 Q_1 = 7.75 M = 12, Q_3 = 15 Max = 26

a Use the information from the five-number summary to determine the values of the lower and upper fences.

b Determine the value(s) of any outliers.

c Construct a boxplot showing any outliers.

10 University participation rates (%) in 21 countries are listed below.

3	3	7	8	9	12	13	15	17	20	21
22	25	26	26	26	27	30	36	37	55	

a Show that the five-number summary for this data is:
Min = 3, Q_1 = 10.5, M = 21, Q_3 = 26.5, Max = 55

b Show that the upper fence is equal to 50.5.

c Explain why this boxplot will show at least one outlier.

d Construct a boxplot showing the outlier.

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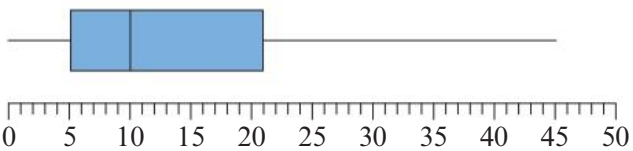
Reading values from a boxplot

Example 3

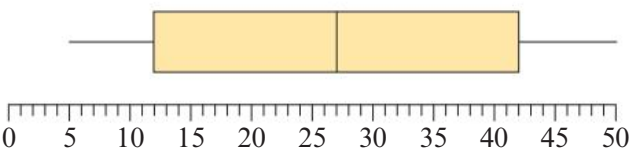
11 For each of the boxplots below, estimate the values of:

- **i** the median, M
- **ii** the quartiles Q_1 and Q_3
- **iii** the interquartile range, IQR
- **iv** the minimum and maximum values
- **v** the values of possible outliers.

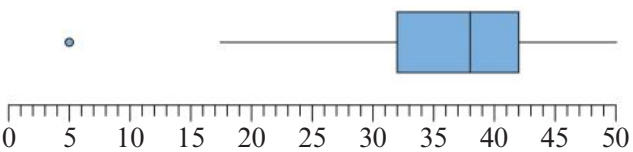
a



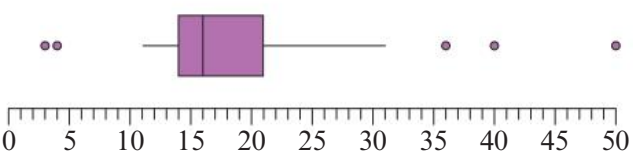
b



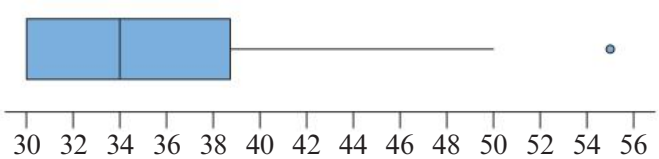
c



d



e



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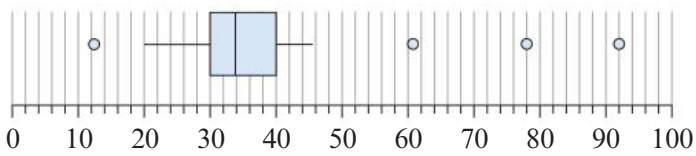
SF

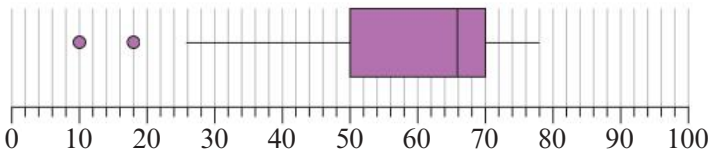
12 For the boxplots below, determine the location of:

i the upper fence **ii** the lower fence.

a



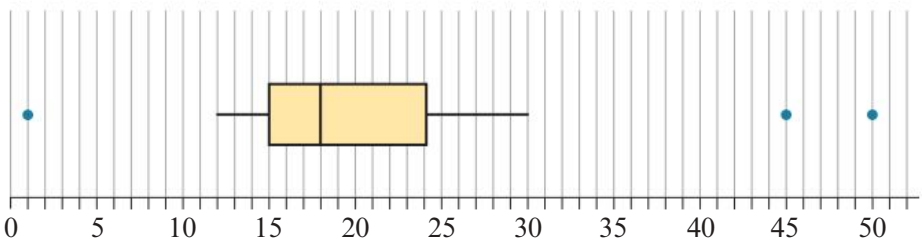
b



Estimating percentages from a boxplot

Example 4

13 Consider the boxplot below.



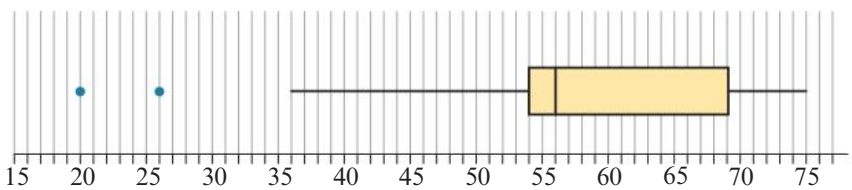
Use it to estimate the percentage of values that are:

a less than 18

b between 1 and 24

c more than 24.

14 The boxplot below displays the scores on an exam for a group of students.



Use it to estimate the percentage of students who had scores:

a less than 69

b less than 54

c between 54 and 69

d more than 75

e between 20 and 69

f between 69 and 75.

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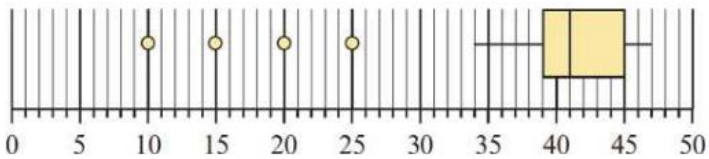
SF

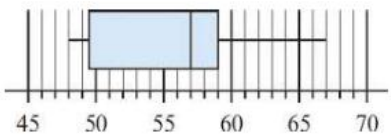
15 Use the boxplot below to estimate the percentage of values that are:

a less than 39 **b** less than 45 **c** greater than 45

d between 39 and 45 **e** between 5 and 45.



16 The boxplot displays the monthly rainfall (in mm) for 12 months.
Use the boxplot to estimate the percentage of months in which the monthly rainfall was:



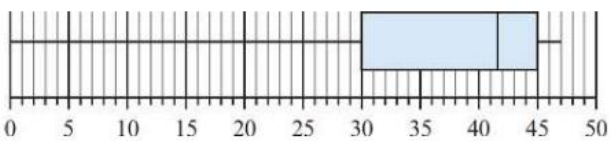
a greater than 59 mm **b** less than 49.5 mm **c** between 49.5 and 59 mm

d between 57 and 59 mm **e** less than 59 mm **f** between 57 and 70 mm.

Describing the features of a distribution from a boxplot

Example 5

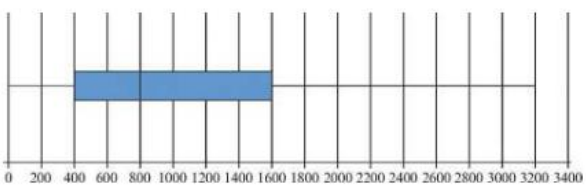
17 Use the information in the boxplot to complete the report below by filling in the blanks.



Report

The shape of the distribution is _____, centred at _____ which is the median value.
The spread of the distribution, as measured by the *IQR*, is _____ and, as measured by the range, _____.

18 Use the information in the boxplot to complete the report below by filling in the blanks.



Report

The shape of the distribution is _____, centred at _____ which is the median value.
The spread of the distribution, as measured by the *IQR*, is _____ and, as measured by the range, _____.

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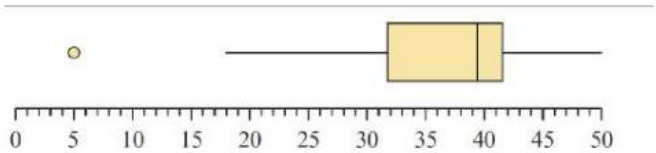
Example 6

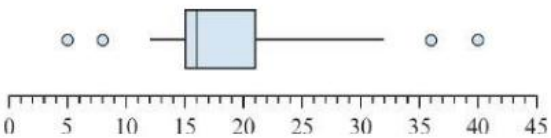
19 Use the information in the boxplot to complete the report below by filling in the blanks.



Report

The shape of the distribution is _____ with outliers. The distribution is centred at _____, the median value. The spread of the distribution, as measured by the *IQR*, is _____ and, as measured by the range, _____. There is _____ outlier at _____.

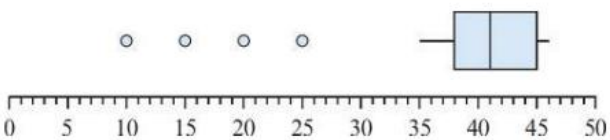
20 Use the information in the boxplot to complete the report below by filling in the blanks.



Report

The shape of the distribution is _____ with outliers. The distribution is centred at _____, the median value. The spread of the distribution, as measured by the *IQR*, is _____ and, as measured by the range, _____. There are _____ outliers at _____.

21 Use the information in the boxplot to complete the report below by filling in the blanks.



Report

The shape of the distribution is _____ with outliers. The distribution is centred at _____, the median value. The spread of the distribution, as measured by the *IQR*, is _____ and, as measured by the range, _____. There are _____ outliers at _____.

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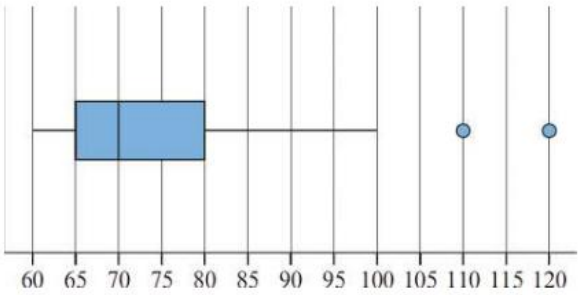
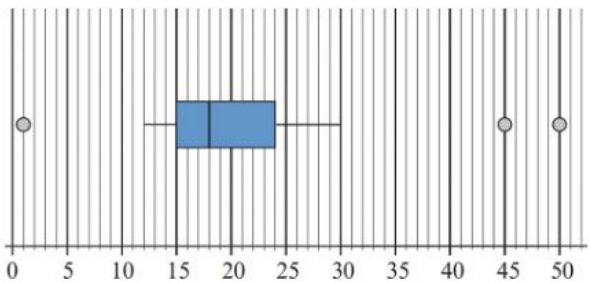
Using a boxplot to answer statistical questions

Example 7

22 Taj recorded his travel time to university (in minutes) each day for 60 days, and summarised the data in the following boxplot. Write a brief report describing the distribution of his travel time.



23 The boxplot below shows the distribution of marks scored by 200 students in a test. Write a brief report describing the distribution of test marks.



Understanding outliers

24 An ordered dataset consists of the following values:

2 4 8 17 17 21 23 25 x

Determine the largest possible value of x, if x is not an outlier.

25 An ordered dataset consists of the following values:

6.4 8.2 10.3 14.7 15.5 17.4 x 33.5

Determine all possible values of x if 33.5 is an outlier.

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11B Comparing data for a numerical variable across two or more groups

Learning intentions

- To be able to use a back-to-back stem plot to compare and interpret differences between datasets when the datasets are small.
- To be able to use parallel boxplots to compare and interpret differences between data sets when the datasets are medium to large.
- To be able to use summary to statistics compare and interpret differences between datasets.
- To be able to write a report which communicates these comparisons.

It makes sense to compare the distributions of datasets when they are concerned with the same numerical variable, for example *height* measured for different groups of people, such as a basketball team and a gymnastic team.

For example, it would be useful to compare the distributions for each of the following:

- the maximum daily temperatures in Townsville in March and the maximum daily temperatures in Brisbane in March
- the test scores for a group of students who did not have a revision class and the test scores for a group of students who had a revision class.

The outcome of such a comparison is a brief written report that compares the distribution of the numerical variable across two or more groups. The starting point for this comparison will often be a graphical display of the data, a table of summary statistics, or both. Once again, there are several points to be kept in mind when comparing the datasets and writing a report. The important point here is that the objective is to make a comparison. Thus, it is necessary not to just quote values, but to also make comparisons using terms such as 'more than', 'less than', 'similar to' and so on.

Before actually writing the report, it is a good idea to go through and make some notes under the sub-headings shape, outliers, centre and spread.

- Shape: If a graphical display is available, and the dataset is sufficiently large, then we can classify the shapes of each distribution as positively skewed, negatively skewed or symmetric. When the dataset is small then we usually do not comment on the shape.
- Outliers: If there are outliers then their values should be given.
- Centre: If any of the distributions are skewed the median is the better measure of centre for comparison, otherwise we can use either the median or the mean.
- Spread: If any of the distributions are skewed, the interquartile range is the better measure of spread for comparison, otherwise we can use either the interquartile range or the standard deviation.

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Comparing datasets using back-to-back stem plots

A back-to-back stem plot can be used to compare two numerical datasets when the datasets are small. This plot differs from the stem plots already met in that it has a single stem with two sets of leaves, one for each of the two groups being compared. The leaves for one set of data are on one side of the stem, and the leaves for the second dataset are on the opposite side. Separate keys are required for each side of the stem plot. Because it is often difficult to clearly identify the shape of a distribution with a small amount of data, we usually confine ourselves to comparing centre, spread and outliers, using the medians and *IQR*s, when describing a back-to-back stem plot.

Example 8 Comparing datasets using a back-to-back stem plot

The following back-to-back stem plot displays the distributions of life expectancies for males (in years) in several countries in the years 1970 and 2010.

Male life expectancy (in years)

1970		2010
8 | 3 = 38 years		5 | 8 = 58 years
	8 | 3	
	4	
9 4 2	5 | 8	
9 9 9 8 7 7 1 0	6 | 9	
4	7 | 1 2 4 4 6 8 9 9	
	8 | 0 0 0	

M = 67 years	M = 76 years
IQR = 12.5 years	IQR = 8 years

Use the back-to-back stem plot and the summary statistics provided to compare these datasets in terms of centre and spread, and draw an appropriate conclusion.

Solution

The median life expectancy of males in 2010 (M = 76 years) was nine years higher than in 1970 (M = 67 years).

The spread of life expectancies of males in 2010 (IQR = 8 years) was less than the spread in 1970 (IQR = 12.5).

In conclusion, the median life expectancy for these countries has increased over the last 40 years, and the variability in life expectancy between countries has decreased.

Explanation

Centre: Use the medians to compare centres.

Spread: Use the *IQR*s to compare spreads.

Conclusion: Use the above observations to write a general conclusion.

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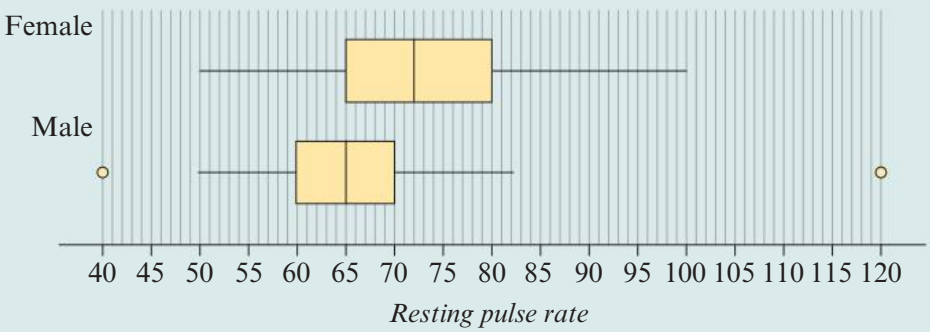
Comparing datasets using parallel boxplots

Parallel boxplots can also be used to compare datasets. Unlike a back-to-back stem plot, parallel boxplots can be used when there are more than two groups.

By drawing boxplots on the same axis, both the centre and spread for the datasets are readily identified and can be compared visually.

Example 9 Comparing two datasets using parallel boxplots

Use the following parallel boxplots to compare the pulse rates (in beats/minute) for a group of 70 male students and 90 female students.



Solution

Both distributions are approximately symmetric.

The median for females is about 72, which is higher than that for males, which is about 65.

The *IQR* for females is 15, which is more than the *IQR* for males, which is 10.

There are no outliers for the females and two for the males, one at 40 and one at 120.

Explanation

Determine the shape for each distribution.

Compare the medians by comparing the vertical line in the boxes.

Compare the spread of the two distributions using the *IQR*, which can be read directly from the boxplot (the width of the boxes).

Locate any outliers.

Report

The distributions of resting pulse rates for both male and female students were approximately symmetric. On average, the resting pulse rate for males is lower (median: male = 65, female = 72) and less variable than that for females (*IQR*: male = 10, female = 15). One male was found to have an extremely low pulse rate of 40, while another had a pulse rate of 120, higher rate than all other males and females.

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Example 10 Comparing more than two datasets using parallel boxplots

Use the parallel boxplots below to compare the salary distribution for workers in a certain industry across four different age groups: 20–29 years, 30–39 years, 40–49 years and 50–65 years.

Solution	Explanation
The shape of the distributions change with age, starting out approximately symmetric but becoming increasingly positively skewed for the older age groups.	Determine the shape for each distribution.
The median salary increased from $64 000 for 20–29 year olds, to $68 000 for 30–39 year olds, just over $70 000 for 40–49 year olds, and $72 000 for 50–65 year olds.	Compare the medians by comparing the vertical line in the boxes.
The *IQR* increased from around $12 000 for 20–29 year olds to around $20 000 for 50–65 year olds.	Compare the spread of the two distributions using the *IQR*.
There are no outliers in the 20–29 and 30–39 age groups. There is one outlier at $110 000 for the 40–49 age group, and three at $119 000, $126 000 and $140 000 for the 50–65 age group.	Locate any outliers.

Report

In this industry the median salary increased slightly across the age groups, from $64 000 for 20–29 year olds, to $68 000 for 30–39 year olds, about $70 000 for 40–49 year olds, and $72 000 for 50–65 year olds. The salaries also became more variable, with the *IQR* increasing from around $12 000 for 20–29 year olds to around $20 000 for 50–65 year olds. The shape of the distribution of salaries changes with age group, from symmetric for 20–29 year olds, and becoming progressively more positively skewed as age increases. There are no outliers in the 20–29 and 30–39 age group. There is one outlier at $110 000 for the 40–49 age group, and three at $119 000, $126 000 and $140 000 for the 50–65 age group.

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Comparing datasets using summary statistics

While it is always preferable to include a plot, summary statistics alone can also be used to compare two or more datasets.

Example 11 Using summary statistics to compare datasets

A plant manager was interested in the effect of time of day on the productivity of workers on the production line of a large assembly plant. Use the summary statistics in the following table to compare the number of units assembled per hour during the day and night shifts.

	$\bar{x}$	s	*median*	*IQR*	*R*
Day shift	44.3	4.8	46	11	13
Night shift	37.4	4.6	37	11	12

Solution

The mean number of units assembled per hour is higher during the day shift ($\bar{x} = 44.3$) than the night shift ($\bar{x} = 37.4$). The median number of units per hour during the days shift is 46, again higher than the median number produced during the night shift which is 37. The variation between the number of units produced during the two shifts is very similar, with the standard deviation during the day shift ($s = 4.8$) only slightly higher than the standard deviation during the night shift ($s = 4.6$). This similarity in variation is confirmed by the values of the *IQR* (11 for both day and night), and the ranges of values (13 for the day shift and 12 for the night shift). Since the values of the mean and median are very similar for both the datasets, we can assume that both distributions are approximately symmetric. In summary, the data provides some evidence for the manager that the day shift is more productive than the night shift.

Section Summary

- A **back-to-back stem plot** can be used to compare and interpret differences between two small datasets.
- When using a back-to-back stem plot, the centre, spread and outliers can be compared.
- **Parallel boxplots** can be used to compare and interpret differences between two or more medium to large datasets.
- When using boxplots, the shape, outliers, centre and spread of the datatsets can be compared.
- Summary statistics can be used to compare and interpret differences between datasets.

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Skill-sheet

Exercise 11B

Comparing groups using a back-to-back stem plot

Example 8

1 The stem plot displays the ages of ten females and ten males admitted to a hospital on the same day.

Age

Female		Male
1| 2 = 21 years		1 | 6 = 16 years
9	0	
5 0	1	3 6
7	2	1 4 5 6 7
7 1	3	4
3 0	4	0 7
0	5	
	6	
9	7	

a Calculate the median and the *IQR* of ages of the females and males in this sample.

b Identify any possible outliers in the data.

c Use the information in the back-to-back stem plot to complete the report below by filling in the blanks.

Report: On average, the ages of the patients admitted to this hospital are lower for males ____ than for females ____. The ages of the males are also less variable ____ than those of those of the females ____. One female was found to have an extremely ____ age of ____ years.

2 The back-to-back stem plot opposite displays the mark distribution of students from two different mathematics classes who sat a test. The test was marked out of 100.

Marks %

Class B			Class A
2 | 7 = 72			7 | 1 = 71
3 2	1	9	
	2	2	
	3	9	
	4	5 7 8	
	5	5 8	
9	6	5 8	
6 4 3 3 2 2 1 0 0	7	1 6 7 9 9	
8 8 4 4 3 2 1 1 0 0	8	0 1 2 2 5 5 9	
8 1	9	1 9	

a Determine the median and the *IQR* of the marks for each class.

b Write a report comparing the centre and spread of these datasets, quoting appropriate statistics.

3 The following back-to-back stem plot displays the distributions of the *number of hours per week spent online* by a group of students in Year 10 and another in Year 11.

Hours spent per week online

Year 10		Year 11
2 | 1 = 12 hours		1 | 5 = 15 hours
8 0 0	0	5 6 7
3 1	1	2 4
8 7 6 5	1	5 5 6 7
4 4 3 1 1 0 0	2	0 1 2 3 3 4
7 5	2	7

Compare the distributions in terms of centre and spread, quoting appropriate statistics.

SF

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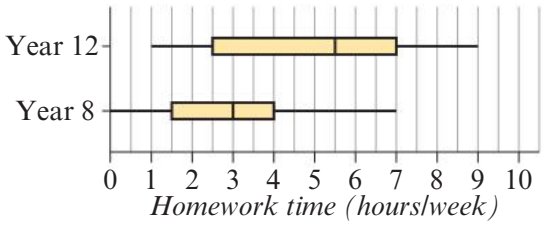
Comparing two groups using parallel boxplots

Example 9

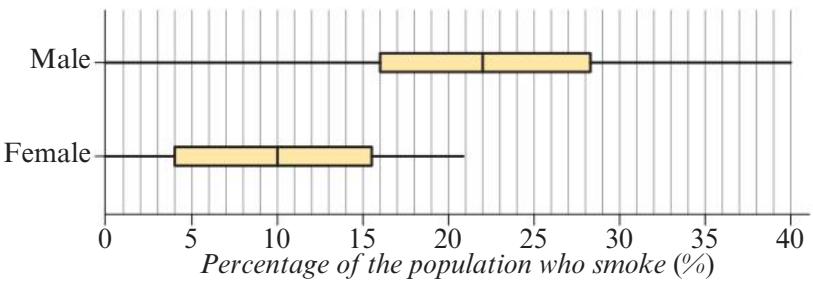
4 The boxplots below display the distributions of homework time (in hours/week) of a sample of Year 8 and a sample of Year 12 students.



a Estimate the median and *IQR*s from the boxplots.

b Write a report comparing these distributions in terms of shape, centre and spread, quoting appropriate statistics.

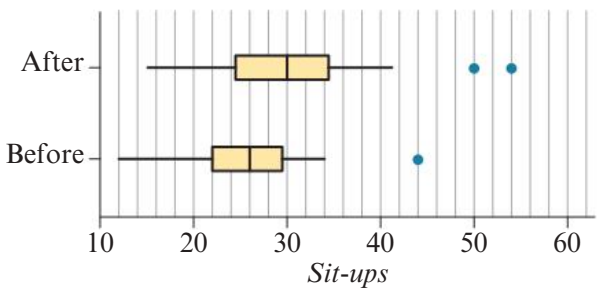
5 The boxplots display the distribution of smoking rates (%) of males and females from several countries.



a Estimate the median and *IQR*s from the boxplots.

b Write a report comparing these distributions in terms of shape, centre and spread, quoting appropriate statistics.

6 The boxplots below display the distributions of the number of sit-ups a person can do in one minute, both before and after a fitness course.



a Estimate the medians, *IQR*s and the values of any outliers from the boxplots.

b Write a report comparing these distributions in terms of shape, centre, spread and outliers, quoting appropriate statistics.

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7 To test the effect of alcohol on coordination, 20 randomly selected participants were timed to complete a task with both 0.00% blood alcohol and 0.05% blood alcohol. The times taken (in seconds) are shown in the accompanying table.

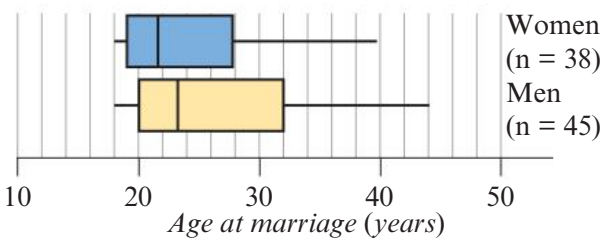
0.00% blood alcohol									
38	36	35	35	43	46	42	47	40	48
35	34	40	44	30	25	39	31	29	44

0.05% blood alcohol									
39	32	35	39	36	34	41	64	44	38
43	42	46	46	50	32	32	41	40	50

a Draw boxplots for each of the sets of times on the same scale.

b Use the information in the boxplots to write a report comparing the time taken to complete the task with 0.00% blood alcohol and 0.05% blood alcohol.

8 The parallel boxplots show the distributions of ages of 45 men and 38 women when first married.

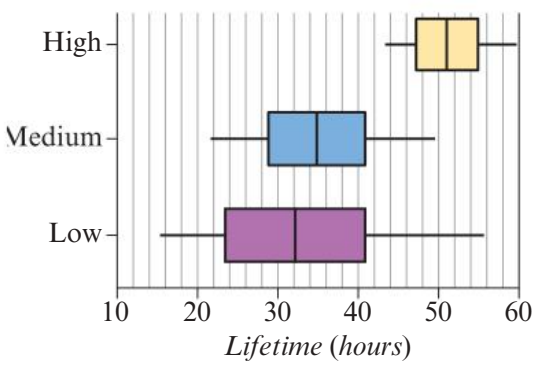


Use the information in the boxplots to write a report comparing the ages of males and females when first married.

Comparing more than two groups using parallel boxplots

Example 10

9 The parallel boxplots show the distribution of the lifetime (in hours) of three differently priced batteries (low, medium, high).



Use the information in the boxplots to write a report comparing the distributions of battery lifetime for the three different price categories.

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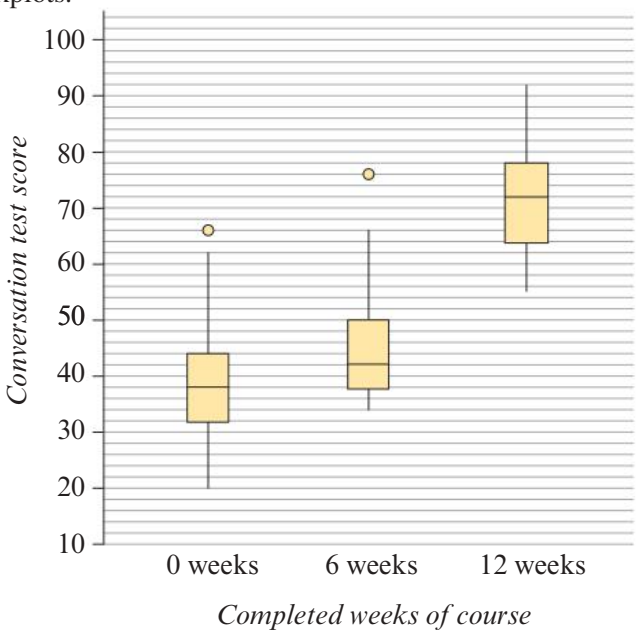
CF

10 To improve their ability in French conversation a group of 50 students who were studying French participated in a 12-week intensive conversation course. The students were given a conversation test at the start of the course, midway through the course, and at the end of the course. Their scores in each of the three tests are shown in the following boxplots.



Use the information in the boxplots to write a report comparing the distributions of the scores at each time period.

Comparing groups using summary statistics

Example 11

11 Use the summary statistic given to compare the final results of two equal size classes each of 20 students, one taught using a traditional teaching method and one taught using a new teaching method.

	$\bar{x}$	s	*median*	*IQR*	R
Traditional method	68.7	7.6	70	15	28
New method	73.4	5.6	75	12	22

12 To compare two different diets one group of 25 people follow Diet A and another group of 25 follow Diet B, and their weight loss (in kgs) after 3 months is recorded. Use the summary statistics given to compare the two diets.

	$\bar{x}$	s	*median*	*IQR*	R
Diet A	12.2	5.7	9	11	32
Diet B	15.3	6.0	12	10	33

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Key ideas and chapter summary

Five-number summary

The minimum, first quartile, median, third quartile and maximum values in a dataset are known as a **five-number summary**.

Upper and Lower fences

The **lower fence** is equal to $Q_1 - 1.5 \times IQR$.
The **upper fence** is equal to $Q_3 + 1.5 \times IQR$.

Outliers

Outliers are data values that appear to stand out from the rest of the dataset. They are values that are less than the lower fence or more than the upper fence.

Boxplot

A **boxplot** is a visual display of a five-number summary with adjustments made to display outliers separately when they are present.

Back-to-back stem plot

A **back-to-back stem plot** can be used to compare the centre and spread of two sets of numerical data when the datasets are small.

Parallel boxplots

Parallel boxplots can be used to compare the key features of two or more datasets in the context of the data when the datasets are medium to large. When using boxplot plots, the shape, outliers, centre and spread of the distributions can be compared.

Summary statistics

Summary statistics can be used to compare key features of two or more numerical variables in the context of the data.

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Review

Skills checklist

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills. ☑

11A	**1** I can construct a simple boxplot from a five-number summary.	☐
	See Example 1 and Exercise 11A Question 1.	
11A	**2** I can construct a boxplot with outliers.	☐
	See Example 2 and Exercise 11A Question 7.	
11A	**3** I can read values from a boxplot.	☐
	See Example 3 and Exercise 11A Question 12.	
11A	**4** I can estimate percentages from a boxplot.	☐
	See Example 4 and Exercise 11A Question 14.	
11A	**5** I can describe the features of a distribution from a boxplot.	☐
	See Examples 4 and 6, and Exercise 11A Questions 18 and 19.	
11A	**6** I can answer statistical questions using information from a boxplot.	☐
	See Example 7 and Exercise 11A Question 20.	
11B	**7** I can use a back-to-back stem plot to compare distributions in terms of centre and spread, and communicate this information in a written report.	☐
	See Example 8 and Exercise 11B Question 1.	
11B	**8** I can use a parallel boxplot to compare distributions in terms of shape, centre, spread and outliers, and communicate this information in a written report.	☐
	See Examples 9 and 10, and Exercise 11B Questions 5 and 10.	
11B	**9** I can use summary statistics to compare key features of datasets in the context of the data, and communicate this information in a written report.	☐
	See Example 11 and Exercise 11B Question 12.	

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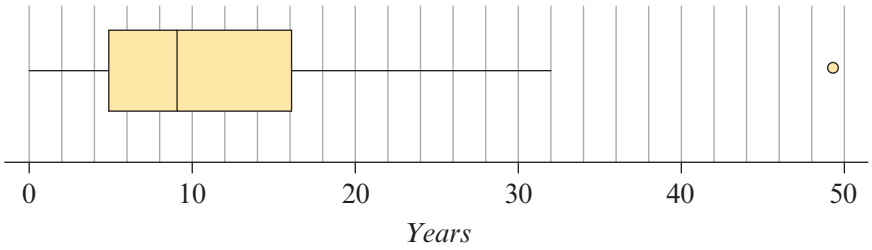
Multiple-choice questions

The following information relates to Questions 1 *to* 4.

The number of years for which a sample of people have lived at their current address is summarised in the boxplot.



1 The maximum number of years a person has lived at their address is closest to:

A 20 **B** 25 **C** 30 **D** 49

2 The median number of years lived at this address is closest to:

A 5 **B** 9 **C** 12 **D** 15

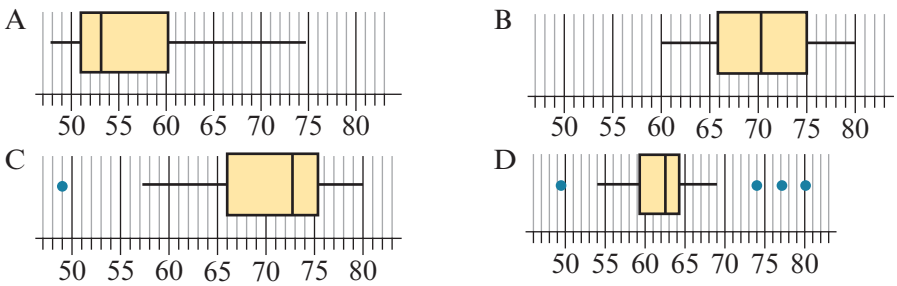
3 The interquartile range of the number of years lived at this address is closest to:

A 5 **B** 11 **C** 15 **D** 20

4 The percentage who have lived at this address for less than 16 years is closest to:

A 10% **B** 25% **C** 50% **D** 75%

The following information relates to Questions 5 *to* 12.



5 The median of boxplot A is closest to:

A 53 **B** 54.5 **C** 55 **D** 60

6 The *IQR* of boxplot B is closest to:

A 9 **B** 20 **C** 25 **D** 65

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Review

7 The range of boxplot C is closest to:

A 9 **B** 23 **C** 31 **D** 80

8 The description that best matches boxplot A is:

A symmetric **B** symmetric with outliers
C negatively skewed **D** positively skewed

9 The description that best matches boxplot B is:

A symmetric **B** negatively skewed with an outlier
C negatively skewed **D** positively skewed

10 The description that best matches boxplot D is:

A symmetric **B** negatively skewed
C symmetric with outliers **D** positively skewed

11 For the data represented by boxplot D, the percentage of data values greater than 65 is approximately:

A 25 % **B** 50 % **C** 75% **D** 100%

12 To be an outlier in boxplot D, a score must be:

A greater than 72.5 **B** either less than 55 or greater than 70
C greater than 70 **D** either less than 52.5 or greater than 72.5

13 The stem plot displays the time taken (in minutes) for two groups of 12 people to solve a complex puzzle. Before commencing the puzzle the people were divided into two groups and assigned a different activity. Group A were asked to exercise vigorously for 10 minutes, while Group B were asked to meditate for 10 minutes.

time

Group A							Group B
1|2 = 21 minutes			9	0	5 6 7		1|6 = 16 minutes
		3 1	1	2 4			
	8 6 5		1	5 5 6			
4 4 3 1			2	0 1 2 3			
		8 5	2				

Select which one of the following statements is true.

A The median time for Group A is more than the median time for Group B.
B The median times for both groups are approximately the same.
C The median time for Group B is more than the median time for Group A.
D The range of times for both groups are the same.

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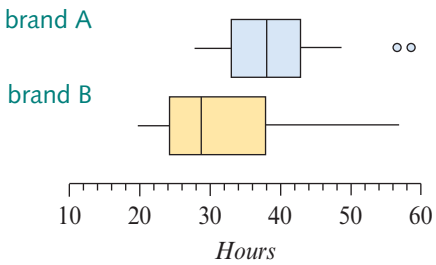
The following information relates to Questions 14 *and* 15.

The parallel boxplots shown display the distribution of battery life (in hours) for two brands of batteries (brand A and brand B).



14 From the boxplots we can say:

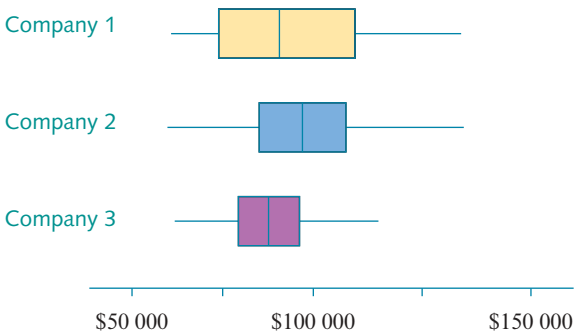
- **A** The median life of brand A is more than the median life of brand B.
- **B** The median life of brand A is less than the median life of brand B.
- **C** The median life of brand A is about the same as median life of brand B.
- **D** We cannot compare the medians because the distribution of brand B is skewed.

15 From the boxplots we can say:

- **A** The variation in life of brand A is more than the variation in life of brand B.
- **B** The variation in life of brand A is less than the variation in life of brand B.
- **C** The variation in life of brand A is about the same as the variation in life of brand B.
- **D** More brand B batteries were included in the study than brand A batteries.

The following information relates to Questions 16 *and* 17.

The amount paid per annum to the employees of each of three large companies is shown in the boxplots.



16 The company with the lowest median wage is:

- **A** Company 1
- **B** Company 2
- **C** Company 3
- **D** Company 1 and Company 2

17 The company with the largest variability (*IQR*) in wages is:

- **A** Company 1
- **B** Company 2
- **C** Company 3
- **D** Company 2 and Company 3

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Review

Short-response questions

SF

1 A group of students recorded the number of SMS messages they sent in one 24-hour period. The following five-number summary was obtained from the dataset.

$$\text{Min} = 0 \quad Q_1 = 3 \quad M = 5 \quad Q_3 = 12 \quad \text{Max} = 24$$

Use the five-number summary to construct a simple boxplot of this data.

2 The systolic blood pressure (measured in mmHg) for a group of 2000 people was measured. The results are summarised in the five-number summary below:

$$\text{Min} = 75 \quad Q_1 = 110 \quad M = 125 \quad Q_3 = 140 \quad \text{Max} = 180$$

a Use the five-number summary to construct a simple boxplot.

b Indicate on your plot where the lower and upper fences would be, and hence if there would be any outliers.

3 A researcher is interested in the number of books people borrow from a library. She selected a sample of 38 people and recorded the number of books each person had borrowed in the previous year. Here are her results:

7	28	0	2	38	18	0	0	4	0	0	5	13
2	13	1	1	14	1	8	27	0	52	4	11	0
0	12	28	15	10	1	0	2	0	1	11	0	

The five-number summary is:

$$\text{Min} = 0 \quad Q_1 = 0 \quad M = 3 \quad Q_3 = 12 \quad \text{Max} = 52$$

a Use the information from the five-number summary to determine the values of the lower and upper fences.

b Determine the value(s) of any outliers.

c Construct a boxplot showing any outliers.

d Use the boxplot to estimate the number of books borrowed by each person in the top 25% of library users.

4 The following ordered dataset gives the number of students absent from a large secondary college on each of 36 randomly chosen school days:

0	0	2	3	3	4	7	7	7	8	10	11
12	12	13	13	14	14	15	15	16	16	16	16
17	18	21	21	21	22	23	23	23	30	31	44

a Construct a boxplot of this data.

b Determine the percentage of days, correct to one decimal place, when more than 20 students were absent.

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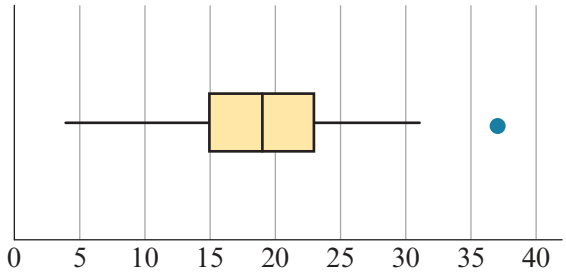
SF

5 For the boxplot below, estimate the location of:

a the upper fence **b** the lower fence.



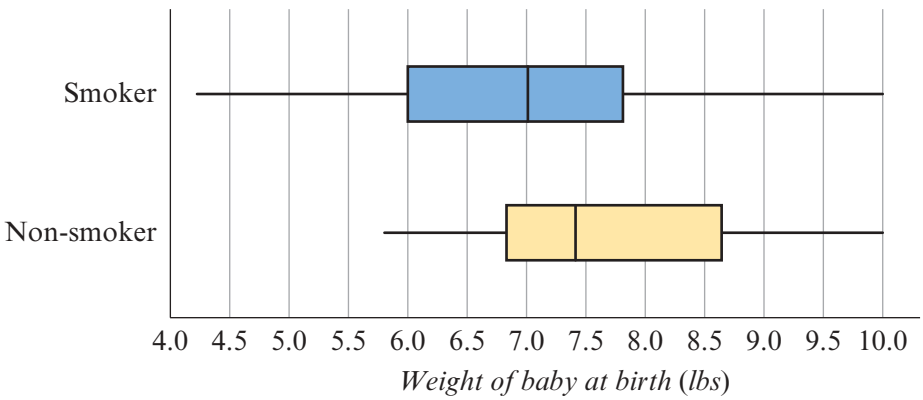
CF

6 Translink has decided to improve its service on the Gold Coast line. Trains were timed on the run from Central to Varsity Lakes Station, and their times recorded over a period of six weeks at the same time each day before and after the improvements were made. Summary statistics for the journey times are shown below (in minutes):

	$\bar{x}$	s	*median*	*IQR*	R
Before	71.8	11.8	70	17	44
After	69.6	9.3	68	14	33

Use the information in the table to write a report comparing the travel times before and after the improvements were made.

7 The data in the following boxplots was collected to investigate the association between smoking and the birth weight of babies.



Use the information from the boxplots to write a report comparing the distribution of baby birth weights for smokers and non-smokers.

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8 The following table shows the number of nights spent away from home in the past year by a group of 21 Australian tourists and by a group of 21 Japanese tourists: CF

Australian

3	14	15	3	6	17	2
7	4	8	23	5	7	21
9	11	11	33	4	5	3

Japanese

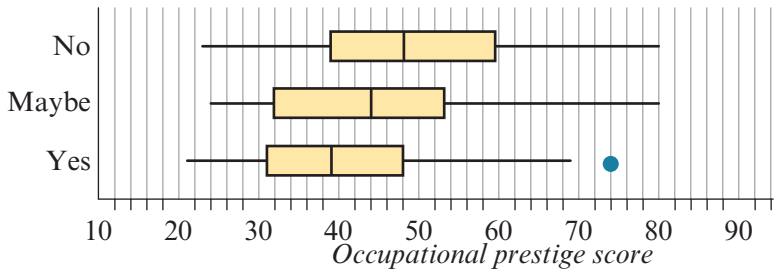
14	3	14	7	22	5	15
26	28	12	22	29	23	17
32	5	9	23	6	44	19

a Construct a back-to-back stem plot of these datasets.

b Determine the median and *IQR* for the two distributions.

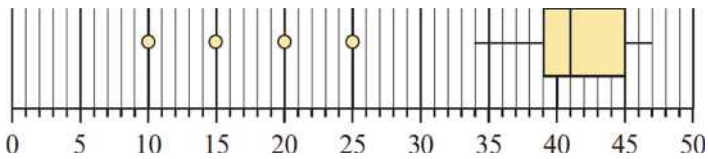
c Write a report comparing the distributions of the number of nights spent away by Australian and Japanese tourists in terms of centre and spread.

9 Occupational prestige is a numerical measure used to describe the respect that a particular occupation holds in a society. It is measured on a scale from 0 to 100, with a higher score indicating higher levels of respect for that occupation. Researchers asked a sample of 332 people if they were thinking about changing to a different kind of work, to which they could respond yes, maybe or no. They also collected the occupational prestige scores for their current occupations. The data they collected is summarised in the following parallel boxplots:



Use the information in the boxplots to write a report comparing the occupational prestige scores for each group in terms of centre and spread.

10 **a** Determine the lower fence for the boxplot opposite.



CU

b When the data were originally entered, a value of 31 was incorrectly entered as 35. Use mathematical reasoning to determine if 31 would be shown as an outlier when the error is corrected.

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Chapter 12

Revision of Unit 2 Chapters 7–11

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12A Revision of Chapter 7: Applications of linear equations and their graphs

Multiple-choice questions

1 Determine the solution to the pair of simultaneous equations:

$y = 3x$

$y = 4x + 7$

A $(3, 0)$ **B** $(4, 7)$ **C** $(-7, 21)$ **D** $(-7, -21)$

2 Determine the solution to the pair of simultaneous equations:

$y = 4x - 5$

$3x + 2y = 12$

A $(4, -5)$ **B** $(3, 2)$ **C** $(2, 3)$ **D** $(11, 39)$

3 Consider the following pairs of simultaneous equations. Determine which pair of equations has more than one solution.

A $y = 8 - x$
$y = 6 - 2x$

B $y = 6x + 8$
$y = -2x + 8$

C $4x + 3y = 0$
$2x = 7 - 3y$

D $2x + y = 5$
$y = 5 - 2x$

4 Anna bought two bottles of soft drink and three loaves of bread for $25.95, and Levi bought three bottles of soft drink and eight loaves of bread for $61.50. The price of one bottle of soft drink and two loaves of bread is:

A $3.30 **B** $6.450 **C** $16.20 **D** $9.75

5 The sum of two numbers is 25. The difference between the two numbers is 3. The product of the two numbers is:

A 2 **B** 13 **C** 25 **D** 154

6 Tiger is 21 years older than Myson. If their combined ages are 127, then Myson is:

A 21 years old **B** 53 years old **C** 63 years old **D** 74 years old

7 The perimeter of a rectangle is 36 cm. If the length is 3 cm more than twice the width, the width is:

A 2 cm **B** 5 cm **C** 10 cm **D** 13 cm

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8 A toddler swimming pool is being filled. For the first 10 minutes, the equation giving the volume, V, of water in the tank (in litres) at time t is:

$$V = 8t \quad (0 \leq t \leq 10)$$

After 10 minutes, the tank fills more slowly. For the next 20 minutes, the equation giving the volume of the tank at time t is given by:

$$V = 6t + 20 \quad (10 \leq t \leq 30)$$

The volume of the tank after 20 minutes is:

A 60 litres **B** 80 litres **C** 140 litres **D** 220 litres

9 The cost, C, in dollars, of supplying x m^3 garden mulch is given by the equations

$C = 40x + 50 \quad (0 \leq x < 4)$

$C = 37x + 62 \quad (4 \leq x \leq 10)$

The cost of purchasing 6.5 m^3 garden mulch is:

A \$40 **B** \$90 **C** \$302.50 **D** \$310

10 A shopping centre carpark has the following rates for customers.

First 2 hours	Free
2–3 hours	\$4.00 (more than 2, less than or equal to 3)
3–4 hours	\$6.00 (more than 3, less than or equal to 4)
4–8 hours	\$12.00 (more than 4, less than or equal to 8)

Simon parked at the shopping centre carpark each day during the week.

- On Monday, Tuesday and Wednesday, he was there for 3 hours each day.
- On Thursday, he spent 3.5 hours in the carpark.
- On Friday, he was there for 7 hours.

The total that he spent on parking for the week was:

A \$3.00 **B** \$12.00 **C** \$30.00 **D** \$36.00

Short-response questions

Simple familiar questions

1 Solve the following pairs of simultaneous equations.

a $x = 2y + 3$ and $2x + 3y = 7$ **b** $x - y = 7$ and $3x + 2y = 1$

2 Find the point of intersection of the following pairs of lines.

a $y = x + 12$ and $y = -5x + 6$

b $y = x - 7$ and $2x - y = 10$

c $x + 2y = 8$ and $2x - 3y = 10$

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3 Freddy and Madeleine visit the local show. Freddy buys 5 show bags and 4 rides for $94. Madeleine buys 6 show bags and 2 rides for $96.

a Let s be the number of show bags and r be the number of rides. Write down an equation that represents Freddy's purchases and then an equation that represents Madeleine's purchases in terms of s and r.

b Solve the two equations simultaneously to find the price of a show bag and the price of a ride.

c Find how much Lisa paid to buy 4 show bags and 3 rides.

4 Sales revenue ($\$S$) for a company is given by the rule $S = 0.85x$, where x is the number of items sold, and the cost ($\$C$) is given by the rule $C = 0.15x + 80$.

a Determine the fixed cost.

b Determine how much revenue the company gets for each unit that it sells.

c Determine the per-unit cost of making each unit.

d Calculate the company's loss if it only sells 60 units.

e Calculate the total profit that the company earns if it sells 150 units.

f Calculate the break-even point.

5 Consider the following rules that define a piece-wise linear graph:

$$y = x + 2 \qquad 0 < x \leq 2$$

$$y = -\frac{1}{4}x + \frac{9}{2} \qquad 2 < x \leq 8$$

$$y = -\frac{1}{2}x + \frac{13}{2} \qquad 8 < x \leq 13$$

a Sketch the piece-wise linear graph defined by the rules.

b Determine the value of y when $x = 6$.

c Determine the values of x when $y = 1.5$.

6 A delivery company in the city charges $4 for packages weighing up to 1 kg, $6 for packages weighing up to 2 kg and $10 for packages weighing up to 3 kg.

a Sketch the step graph for the cost of delivering a package, C, given the weight, w, for weights from 0 kg to 3 kg.

A legal company sends the following parcels in one day: 8 parcels weighing 700 g, 3 parcels weighing 1.4 kg and 6 parcels weighing 2.2 kg.

b Calculate the cost for sending the parcels.

The delivery company also offers a special membership deal. Customers can pay $10 per day and then pay $2 per package regardless of weight.

c Write down the equation for the cost, C in dollars, in terms of the number of packages, x.

d Calculate the total cost to the legal company if they choose the membership deal.

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Complex familiar questions

7 A graduate student, Finn, has a starting salary of $62 000. His salary increases by $4000 every year. His friend Isabella has a starting salary of $56 000, which increases by $5500 every year.
Let S be the salary after t years.

a Write down a linear model in S and t to represent this situation for Finn and for Isabella.

b Sketch the graph of S for both Finn and Isabella for the first 10 years of their working life. Show the coordinates of the y-intercept and end point for each of them.

c Determine the number of years after graduating that Isabella earns more than Finn.

8 A water tank initially contains 6600 litres of water. Water is drained from the tank at a rate of 350 litres per minute.

a Write a rule for the volume of water, V litres, in the tank at time t minutes.

b The water from this tank is drained into an irrigation channel that initially contains 800 litres of water. Write a rule for the volume of water, C litres, in the irrigation channel at time, t minutes.

c Calculate after how many minutes the tank and the channel contain the same volume of water.

Complex unfamiliar questions

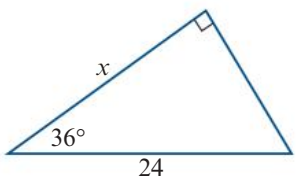
9 Christos runs a Christmas Cookie business. He charges $5 per cookie for orders of up to 150 cookies. For orders of more than 150 and up to 300, he charges $4. For orders of more than 300 cookies and up to 500 cookies, he has a different price per cookie. On the first day of December, he receives orders for 120, 170, 370, 160 and 300 cookies. Given that this generated revenue of $3860, calculate the cost per cookie that he charges on orders of more than 300 cookies but less than 500 cookies.

10 A local trainer offers two different packages to clients. The Basic package has a joining fee of $220 and allows clients to attend a personal training session for $35. The Warrior package has a joining fee of $440 and allows clients to attend a personal training session for $18. Using algebra, calculate the minimum number of sessions a client needs to attend for the Warrior session to be better value.

12B Revision of Chapter 8: Applications of trigonometry

Multiple-choice questions

1 The length x is given by:



A $24 \sin 36°$ **B** $24 \cos 36°$ **C** $36 \sin 24°$ **D** $36 \cos 24°$

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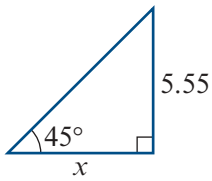
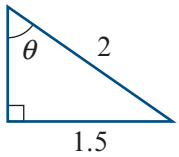
2 Calculate the length of the unknown side.



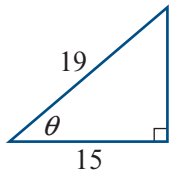
A 3.92 units
B 4.37 units
C 5.55 units
D 7.85 units

3 Calculate the size of the unknown angle.



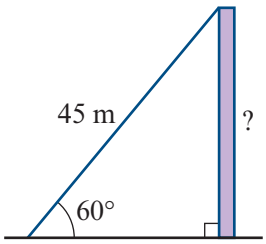
A 36.9° **B** 41.4° **C** 48.6° **D** 53.1°

4 The angle θ, to two decimal places, is:



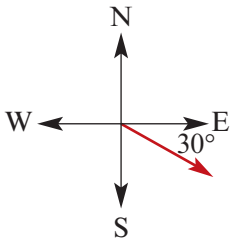
A 31°
B 35.30°
C 37.86°
D 65.22°

5 A rope 45 metres long helps to secure a vertical pole into the ground. The angle of elevation of the rope to the top of the pole is 60°. Calculate the height of the pole.



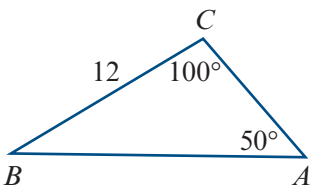
A 22.5 m **B** 39 m
C 42.5 m **D** 78 m

6 The true bearing shown in the diagram is:



A 30° **B** 120°
C 150° **D** 210°

Use the following diagram to solve Questions 7 *and* 8.



7 Use the sine rule to calculate the length of AB.

A 9.13 **B** 12.32 **C** 13.66 **D** 15.43

8 Calculate the area of the triangle.

A 28.18 units^2 **B** 32.49 units^2 **C** 41.64 units^2 **D** 46.29 units^2

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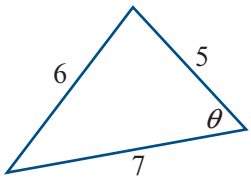
9 The size of θ is closest to:

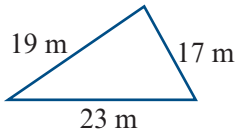
A 38.73° **B** 45.35° **C** 57.12° **D** 68.84°



10 The area of this triangle is closest to:

A 158.64 m^2 **B** 167.34 m^2

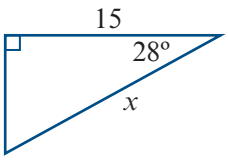
C 189.51 m^2 **D** 211.09 m^2



Short-response questions

Simple familiar questions

1 Calculate the unknown side, x, correct to two decimal places.

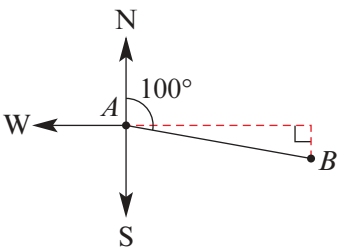


2 Determine the angle θ in the right-angled triangle shown, correct to one decimal place.

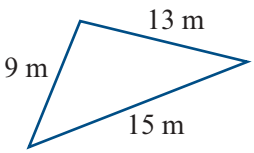
3 The true bearing from port (A) to a ship (B) is 100°. The ship is 150 m from port.

a Calculate how far east the ship is from port, rounded to two decimal places.

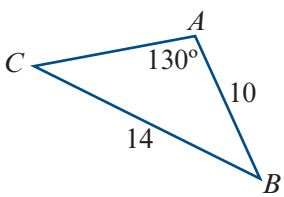
b Determine how far south the ship is from port, rounded to one decimal place.



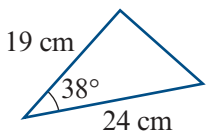
4 The boundary of a garden is shown in the diagram. Calculate the area to the nearest square metre.



5 Determine angle C in the triangle shown, correct to one decimal place.



6 Determine the length of the unknown side in the triangle shown, correct to one decimal place.



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Complex familiar questions

7 A surveyor left a road that runs north-south and walked on a true bearing of 220° for 5 km.

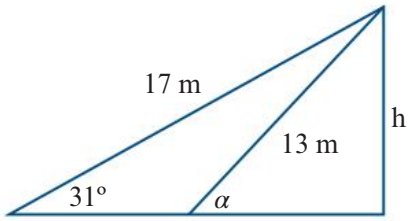
a Calculate the shortest distance back to the road, correct to one decimal place.

b State the true bearing of the shortest path back to the road.

8 A rope 17 m long makes an angle of elevation of 31° with the top of a pole.

a Calculate the height, h, of the pole, correct to one decimal place.

b Determine the angle of elevation, α, to the nearest degree, that a rope 13 m long would make with the top of the pole.



Complex unfamiliar questions

9 A bushwalker left camp A and walked on a true bearing of 020° for 2 kilometres to arrive at camp B. From camp B she walked on a bearing of 280° until she reached camp C. At camp C she took a bearing of 150° and walked back to camp A. Calculate the total distance walked, correct to two decimal places.

10 Three air control towers view an aeroplane taking off for their trip. At 5 seconds after take off, the plane is 358 m above ground level. At this time, the angles of elevation from towers A, B and C to the plane are 23°, 31° and 38°, respectively, to the horizontal. At this point in time, use mathematical reasoning to justify which tower is closest to the plane and calculate the distance between them.

12C Revision of Chapter 9: Matrices and matrix arithmetic

Multiple-choice questions

1 For $A = \begin{bmatrix} 8 & -2 \\ -7 & 1 \\ -9 & 0 \end{bmatrix}$, the order of matrix A is:

A 3 **B** 6 **C** 2×3 **D** 3×2

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2 The number of dollars in the bank accounts of three friends at the end of each year is shown in matrix form.

$$\begin{array}{c} \\ \textit{Dana} \\ \textit{Evan} \\ \textit{Faye} \end{array} \begin{array}{c} \begin{array}{ccc} 2023 & 2024 & 2025 \end{array} \\ \begin{bmatrix} 210 & 380 & 420 \\ 180 & 540 & 190 \\ 630 & 590 & 410 \end{bmatrix} \end{array}$$

The amount in Evan's account at the end of 2024 is:

A \$180 **B** \$190 **C** \$380 **D** \$540

3 For three towns, R, S and T, the below matrix gives the number of roads directly connecting one town to another.

$$\begin{array}{c} \\ R \\ S \\ T \end{array} \begin{array}{c} \begin{array}{ccc} R & S & T \end{array} \\ \begin{bmatrix} 0 & 2 & 4 \\ 2 & 0 & 3 \\ 4 & 3 & 0 \end{bmatrix} \end{array}$$

The total number of roads directly connecting town S to other towns is:

A 2 **B** 3 **C** 4 **D** 5

4 For $X = \begin{bmatrix} 5 & 4 \\ -4 & 7 \end{bmatrix}$ and $Y = \begin{bmatrix} 7 & -2 \\ -8 & 2 \end{bmatrix}$, calculate $3X - 2Y$.

A $\begin{bmatrix} 1 & 16 \\ 4 & 14 \end{bmatrix}$ **B** $\begin{bmatrix} 29 & 8 \\ -28 & 22 \end{bmatrix}$ **C** $\begin{bmatrix} 1 & 8 \\ -28 & 14 \end{bmatrix}$ **D** $\begin{bmatrix} 1 & -16 \\ -4 & 14 \end{bmatrix}$

Use matrices E, F, G and H in Questions 5, 6 *and* 7.

$$E = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad F = \begin{bmatrix} 8 & 1 \\ 0 & 4 \\ 7 & 6 \end{bmatrix}, \quad G = \begin{bmatrix} 5 & 4 \end{bmatrix}, \quad H = \begin{bmatrix} 6 & 9 \\ 4 & 3 \end{bmatrix}$$

5 Matrix multiplication is not defined for:

A EF **B** FE **C** HE **D** GH

6 The order of EG is:

A 1×1 **B** 1×2 **C** 2×1 **D** 2×2

7 The matrix multiplication GE gives the matrix:

A $\begin{bmatrix} 23 \end{bmatrix}$ **B** $\begin{bmatrix} 21 & 8 \end{bmatrix}$ **C** $\begin{bmatrix} 21 \\ 8 \end{bmatrix}$ **D** $\begin{bmatrix} 15 & 10 \\ 12 & 8 \end{bmatrix}$

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8 For $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, calculate I^2.

A $I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **B** $I = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ **C** $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **D** $I = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

9 After a night spotlight survey of a national park, rangers recorded their sightings of feral animals in the matrix shown below.

$$S = \begin{array}{c} \\ Anna \\ Ben \\ Chandru \\ Dian \end{array} \begin{array}{c} \begin{array}{ccc} Deer & Foxes & Rabbits \end{array} \\ \begin{bmatrix} 4 & 1 & 12 \\ 1 & 3 & 9 \\ 0 & 2 & 16 \\ 3 & 0 & 6 \end{bmatrix} \end{array}$$

Using one of the following matrices to form a matrix product with S, choose the matrix product that would give the total number of sightings for each type of feral animal.

$$F = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

A FS **B** GS **C** HS **D** SI

10 The matrix below shows when communication exists between people in the group Peta, Quinn, Rose and Sara. Communication between two people is shown by a 1 and no communication by a 0.

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Determine the number of people that Peta communicates with.

A 0 **B** 1 **C** 2 **D** 3

Short-response questions

Simple familiar questions

1 Identify the listed elements using the following matrix.

$$M = \begin{bmatrix} -17 & 15 & 28 & 43 \\ 81 & -32 & 54 & 26 \\ -92 & 80 & -67 & 79 \\ 64 & -24 & 39 & 99 \end{bmatrix}$$

a m_{12} **b** m_{21} **c** m_{32} **d** m_{43}

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2 Determine $2A + 3B$.

$$A = \begin{bmatrix} 7 & -4 \\ -8 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 5 & -3 \\ -5 & 2 \end{bmatrix}$$

3 The prices shown in the matrix must be increased by 10% to allow for GST. Use scalar multiplication to display the final prices in a clearly labelled matrix.

$$\begin{matrix} \\ Women \\ Men \end{matrix} \begin{matrix} \begin{matrix} Shorts & Tops & Joggers \end{matrix} \\ \begin{bmatrix} 15 & 7 & 20 \\ 10 & 8 & 25 \end{bmatrix} \end{matrix}$$

4 Determine CD.

$$C = \begin{bmatrix} 4 & -3 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 8 & -2 \\ -7 & 1 \\ -9 & 0 \end{bmatrix}$$

5 The hours Tim and Tam were jogging and swimming in the previous week are shown in matrix H. The kilojoules per hour, indicating the rate energy that was used for each activity, are listed in matrix K.

$$H = \begin{matrix} \\ Tim \\ Tam \end{matrix} \begin{matrix} \begin{matrix} Jogging & Swimming \end{matrix} \\ \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix} \end{matrix} \qquad K = \begin{matrix} \\ Jogging \\ Swimming \end{matrix} \begin{matrix} \begin{matrix} Kilojoules \end{matrix} \\ \begin{bmatrix} 2500 \\ 1600 \end{bmatrix} \end{matrix}$$

a To calculate the number of kilojoules each person used, identify whether $H \times K$ or $K \times H$ should be calculated.

b Use the appropriate matrix product to obtain a matrix showing the total kilojoules each person used.

6 Use the following terms to best classify each matrix below.

row matrix column matrix square matrix zero matrix identity matrix

a $J = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **c** $N = \begin{bmatrix} 37 \\ -25 \\ 18 \end{bmatrix}$

d $L = \begin{bmatrix} -37 & 44 \\ 82 & -25 \end{bmatrix}$ **e** $M = \begin{bmatrix} -3 & 5 & 9 \end{bmatrix}$

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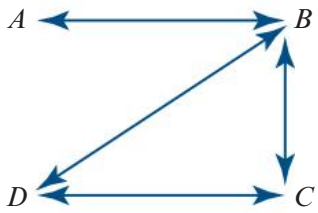
Complex familiar questions

7 **a** Construct matrix P that represents the communication diagram shown between four people. Use the number 1 when direct communication exists between two people and 0 for no direct communication.



b Interpret the information given by the sum of the column for person B.

c Using matrix P^2, evaluate the total number of ways person D can communicate with a person via another person.

d State the chain of connections for each way that person D can communicate with a person via another person.

8 The day's sales at Sam's Café were 38 sandwiches, 23 cakes and 89 coffees where a sandwich costs $7, a cake costs $3 and coffee costs $4. Use matrix multiplication to find the total value of the day's sales.

Complex unfamiliar questions

9 A family owns 5 rabbits, 2 goats and 4 sheep. The family feed their animals oats, wheat and carrots. In one week, each rabbit eats 4 serves of oats, 1 serve of wheat and 2 serves of carrots. Each goat eats 7 serves of oats, no wheat and 6 serves of carrots. Each sheep eats 2 serves of oats, 2 serves of wheat and 1 serve of carrots. Oats cost $3 per serve, wheat costs $2 per serve and carrots cost $1 per serve. Use appropriate matrix multiplication to calculate the total cost for this family to feed all their animals each week.

10 Use algebraic procedures to solve for the pronumerals.

$$\begin{bmatrix} 0 & x \\ x & y \end{bmatrix}^2 + \begin{bmatrix} x^2 & xy \\ 0 & 2x^2 \end{bmatrix} = \begin{bmatrix} 9 & 54 \\ 27 & 81 \end{bmatrix}$$

12D Revision of Chapter 10: Univariate data analysis

Multiple-choice questions

1 In a survey, a number of people were asked to indicate how often they cook at home by selecting the following options: 'never', 'seldom', 'sometimes' or 'regularly'. The resulting variable was named *level of cooking*. The type of data generated is:

A discrete **B** continuous **C** nominal **D** ordinal

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The following information applies to Questions 2, 3 *and* 4.

A group of 20 employees of a company were asked to record the number of meetings they had attended in the last month. Their answers are shown below.

1	5	5	5	6	7	7	7	8	8
9	10	12	14	14	14	16	20	21	44

2 The range of the number of meetings is:

A 6 **B** 8.5 **C** 43 **D** 44

3 The median number of meetings an employee attended is:

A 4 **B** 6 **C** 8.5 **D** 43

4 The interquartile range (*IQR*) of the number of meetings is:

A 7.5 **B** 8.5 **C** 9 **D** 11.5

5 State which variable can be displayed appropriately in a column chart.

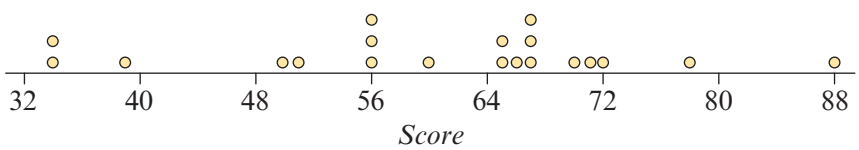
A Age of everyone in a city
B Hair colour of people in a school
C Weight (kg)
D Distance between towns (km)

6 The weight of 8 sumo wrestlers (in kg) are 115, 118, 122, 128, 134, 138, 138, 140. The mean and standard deviation are closest to:

A mean = 130.22; standard deviation = 8.74
B mean = 129.13; standard deviation = 9.82
C mean = 129.55; standard deviation = 10.23
D mean = 129.13; standard deviation = 9.18

The following information applies to Questions 7 *and* 8.

The dot plot shows the examination scores for the recent English exam.



7 The number of students who scored above 64 is equal to:

A 8 **B** 9 **C** 10 **D** 11

8 The percentage of students who scored between 40 and 80 is closest to:

A 60% **B** 70% **C** 80% **D** 90%

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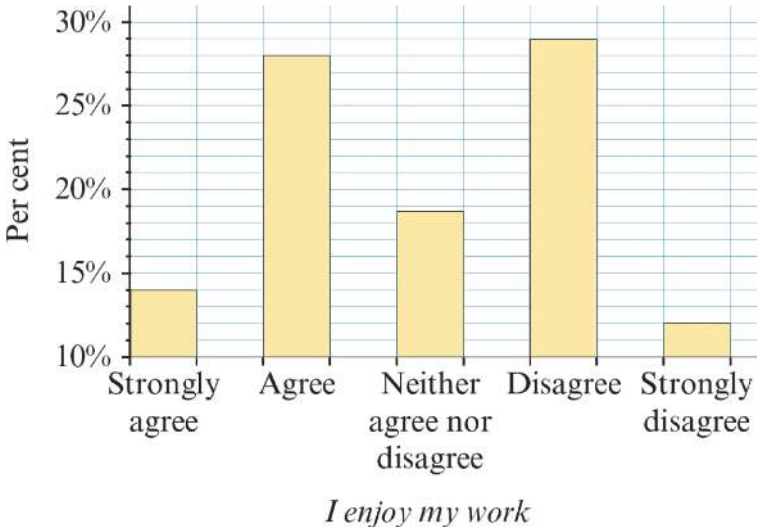
The following information applies to Questions 9 *and* 10.

In a recent survey 500 people were asked to respond to the statement 'I enjoy my work', selecting from the alternatives 'strongly disagree', 'agree', 'neither agree nor disagree', 'disagree', 'strongly disagree'. Their responses are summarised in the bar chart below.



9 Select the option which best describes this data.

A Numerical **B** Categorical **C** Infinite **D** Discrete

10 The number of people who chose the modal response is closest to:

A 145 **B** 29 **C** 28 **D** 140

Short-response questions

Simple familiar

1 Classify each of the following variables as nominal, ordinal, discrete or continuous.

a The weight of an orange in grams. **b** Eye colour.

c Coffee size (small, medium, large, extra-large).

2 The minimum daily temperatures (in degrees Celsius) in Brisbane each July day in a certain year were:

10.9	8.7	7.2	8.8	13.6	10.9	11.2	10.4	10.2	10.7	11.8
13.5	16.8	11.7	12.2	14.0	14.3	15.6	15.6	14.4	14.8	15.8
18.0	13.3	12.7	13.1	14.4	9.2	9.2	8.5	8.5		

a Construct a histogram with a column width of 1 and starting point of 7.

b Determine the percentage of days in July with a minimum temperature less than 10°C.

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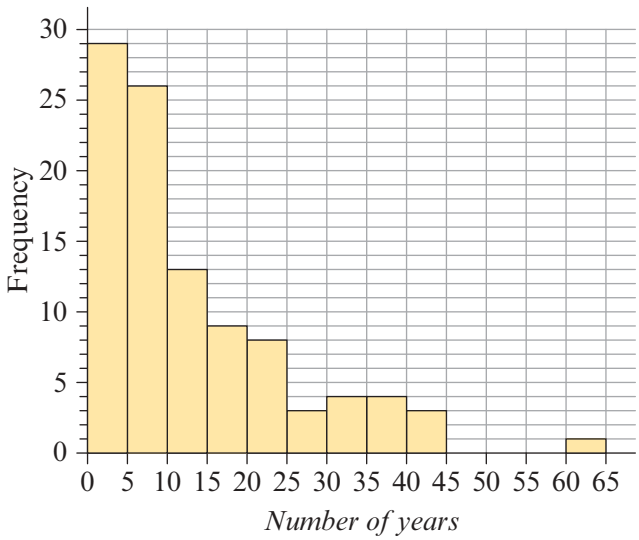
3 The height (in cm) of 20 randomly selected tomato bushes are:

8.4	7.4	8.2	13.3	7.9	9.5	9.9	9.3	8.9	9.0
11.1	10.2	10.5	8.7	9.0	9.3	8.6	7.9	9.4	9.8

Construct a stem-and-leaf plot of this data.

4 A group of 100 people were asked how many years they have been doing their current type of work. Their responses are summarised in this histogram.



a State the number of people who have been doing their current type of work for fewer than 15 years.

b State the modal interval.

c Classify the shape of the histogram (negatively skewed, positively skewed or approximately symmetric).

5 Consider the data displayed in the following stem plot.

key: 4|8 represents 48

Age (years)

2	8 9 9
3	0 2 7 7 8 9 9
4	2 7 9
5	1 3

a Determine the median and the IQR.

b Calculate the mean and the standard deviation.

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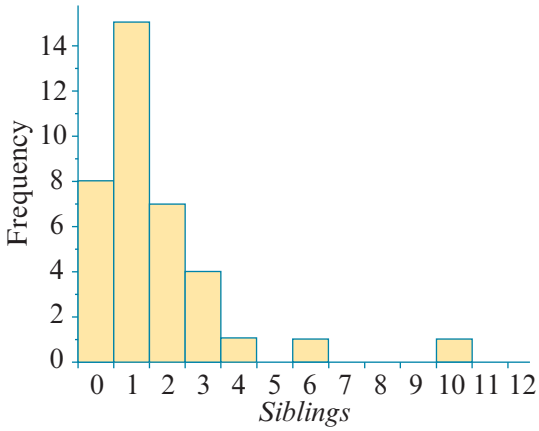
6 A group of 37 people was asked how many siblings they had. The data collected is summarised in the histogram below.



a Find the median and the interquartile range of the number of siblings.

b Describe the distribution in terms of shape, centre, spread and outliers.

Complex familiar

7 A group of 200 people was asked to describe their general happiness by choosing one of the responses 'very happy', 'pretty happy', 'not too happy'. The data is summarised in the following frequency table:

General happiness	Frequency	
	Number	%
Very happy	53	26.5
Pretty happy		59.0
Not too happy	29	
Total	200	100.0

a State the type of data collected – categorical or numerical.

b Complete the frequency table and the percentage frequency table.

c Construct a percentaged bar chart.

d Write a report summarising the responses, quoting appropriate percentages.

8 Jack is carrying out a chemistry experiment. He repeats the experiment 10 times, and records the results. He calculates the mean of his results, and records this also. Unfortunately, before he gets to write up the experiment, he spills a cup of coffee on his results sheet, and one of the data values cannot be read.
Find the missing data value if the mean $\bar{x} = 1.84$, and the nine legible data values are as given below:

1.6 1.8 2.1 2.4 2.3 1.7 1.4 1.6 1.7

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Complex unfamiliar

9 Four different data values have a median of 9 and a mean of 9.5. If two of the data values are 5 and 15, determine the possible combinations of values in the dataset. (Assume all of the data values are integers.)

10 Consider the following ordered dataset:

2 3 p 7 9 13 q 18

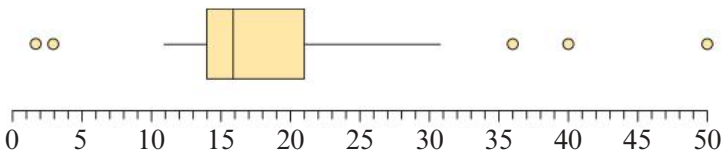
If the mean of the dataset is equal to 9, and the IQR is equal to 10, determine the values of p and q.

12E Revision of Chapter 11: Comparing datasets

Multiple-choice questions

The following information relates to Questions 1, 2 *and* 3.

For the following boxplot:



1 The median of the dataset is closest to:

A 14 **B** 15 **C** 16 **D** 21

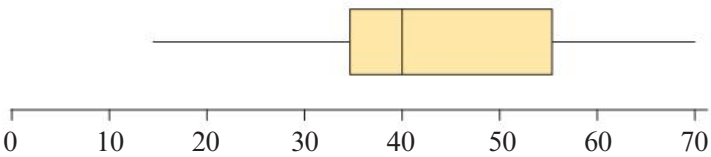
2 The interquartile range of the dataset is closest to:

A 7 **B** 14 **C** 21 **D** 49

3 The percentage of the data values less than 21 is closest to:

A 10% **B** 25% **C** 50% **D** 75%

4 Consider this box-and-whisker plot. Select which one of the following statements is true.



A The median is 45.

B Less than one-quarter of the data values are less than 30.

C Less than one-quarter of the data values are greater than 50.

D All of the data values are less than 60.

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5 A group of 500 people were asked how many hours per week they spend exercising. The five-number summary for the data collected is:

Min = 0 $Q_1 = 2$ $M = 4$ $Q_3 = 12$ Max = 42

The values of the lower and upper fences are:

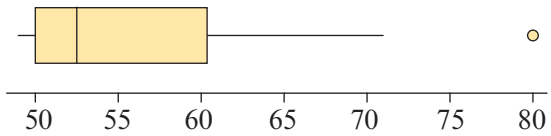
A lower fence = −8 and upper fence = 24

B lower fence = 0 and upper fence = 42

C lower fence = 2 and upper fence = 4

D lower fence = −13 and upper fence = 27

6 For the boxplot opposite, outliers are defined as data values that are:



A less than 35 or greater than 75

B less than 48 or greater than 72

C less than 48 or greater than 75

D less than 35 or greater than 72

7 Adrienne recorded the time taken for her school bus to go from her bus stop to her school in the morning, and then back again the afternoon, each school day over a 3-week period. The back-to-back stem plot displays the times (in minutes) she collected.

time

Morning								Afternoon					
					9	1							
				5	3	1	2	3	4				
8	8	7	6	6	5	2	5	5	6				
	4	4	3	1	0	3	0	1	2	3	3		
						3	5	5	6	7	7		

1|2 = 21 minutes 1|6 = 16 minutes

Select which of the following statements is true.

A The median time for the morning is more than the median time for the afternoon.

B The median times for the morning and the afternoon are approximately the same.

C The median time for afternoon is more than the median time for morning.

D The range is the same for the morning and the afternoon,

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8 The following table gives summary statistics for the time taken (in minutes) to reduce a headache for two different groups of people, one group taking Drug A and the other Drug B.

	$\bar{x}$	s	*median*	*IQR*	R
Drug A	15.5	4.3	20.3	10.4	30.2
Drug B	13.6	3.3	18.2	8.8	25.9

Use the summary statistics to select which one of the following statements is true.

A Drug B reduced the symptoms of the headache more quickly and with less variability in time than Drug A.

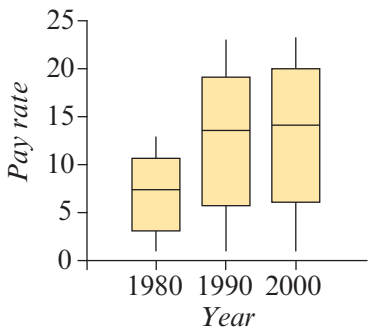
B Drug A reduced the symptoms of the headache more quickly and with less variability in time than Drug B.

C Drug B reduced the symptoms of the headache more quickly and with more variability in time than Drug A.

D Drug A reduced the symptoms of the headache more quickly and with more variability in time than Drug B.

The following information relates to Questions 9 *and* 10.

The boxplots opposite displays the distribution of the average pay rate, in dollars per hour, of workers in 35 countries for the years 1980, 1990 and 2000.



9 Which one of the following statements is **not** true?

A The *IQRs* of pay rate in 1990 and 2000 are approximately the same.

B The *IQR* of pay rate in 1980 is more than the *IQR* of pay rate in 1990 and 2000.

C All three distributions are approximately symmetric.

D The lowest pay rate each decade is $1 per hour.

10 Based on the boxplots, we could conclude (select one option):

A Median pay rate has increased steadily between 1980 and 2000.

B Pay rates are not where they should be.

C Median pay rate increased between 1980 and 1990, but remained much the same between 1990 and 2000.

D Pay rate is too high in some countries.

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Short-response questions

Simple familiar questions

1 A group of 500 people recorded the number of hours per week they watch television, and the following summary statistics were determined:

Minimum	0
Q_1	5
Median	18
Q_3	35
Maximum	92

a Determine the values of the lower and upper fences.

b The smallest three values in the dataset are 0, 0, 2. The largest three values in the dataset are 74, 85, 92. Use this information to construct a boxplot with outliers.

2 To investigate the effect of weather on depression, scores on a depression scale were collected from a group of people who live in a warm climate, and another group of people who live in a cold climate.

a Calculate the median and the *IQR* of scores for each group.

b Identify any outliers in the data.

Depression score

Cold climate						Warm climate						
4| 1 = 14						1 | 6 = 18						
				1		2	4	4				
		7	6	1		5	5	6	8	9	9	
	3	2	1	0	2	0	2	3	3			
8	8	8	6	5	2	5	6	7	7	7	8	
	4	2	2	1	3	0	3	4				
9	8	8	7	5	3							
			2	1	4							

c Use the information in the back-to-back stem plots to complete the report below by filling in the blanks.

Report: The median depression score is lower for those who live in a warm climate [ ] than for those who live in a cold climate [ ]. The depression scores for those who live in a warm climate are also less variable [ ] than those for those people who live in a cold climate [ ].

3 The following back-to-back stem plot displays the distributions of the yield of tomato plants (in kg) for two plots of plants, one of which was treated with the regular fertiliser, and another which was treated with a new fertiliser.

Yield (kg)

regular						new						
2| 1 = 1.2 kg						1 | 5 = 1.5 kg						
			9	7	7	1	8	9				
9	8	8	6	6	4	2	2	3	7	9		
		8	7	5	5	3	3	0	1	3	3	7	8
					5		4	1	2	3	6	7	

Compare the distributions in terms of centre and spread, quoting appropriate statistics.

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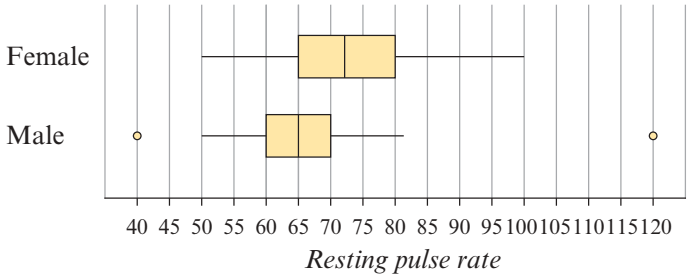
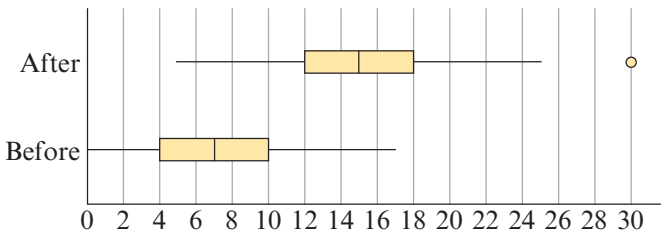
4 The parallel boxplots below display the resting pulse rates for a group of females and a group of males.



a Estimate the median, interquartile range and the values of any outliers for each boxplot.

b Write a report comparing these distributions in terms of shape, centre and spread, quoting appropriate statistics.

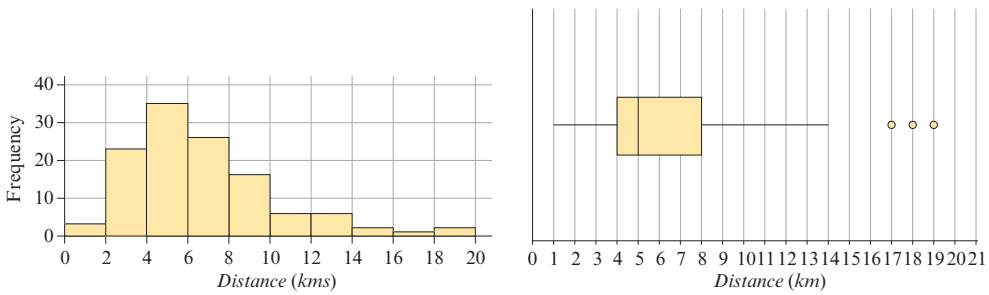
5 Wellness is the act of practicing healthy habits on a daily basis. The boxplots below display the distributions of scores on a wellness scale for a group of 100 people, before and then one month after they attended a wellness retreat.



a Estimate the medians, *IQR*s and the values of any outliers from the boxplots.

b Write a report comparing these distributions in terms of shape, centre, spread and outliers, quoting appropriate statistics.

6 The histogram and the boxplot below show the distribution of the distances 120 university students live from the campus.



a Describe the shape of the distribution of *distance*, including the values of outliers.

b Determine, approximately, how many of these students lived 4 to 5 km from the campus.

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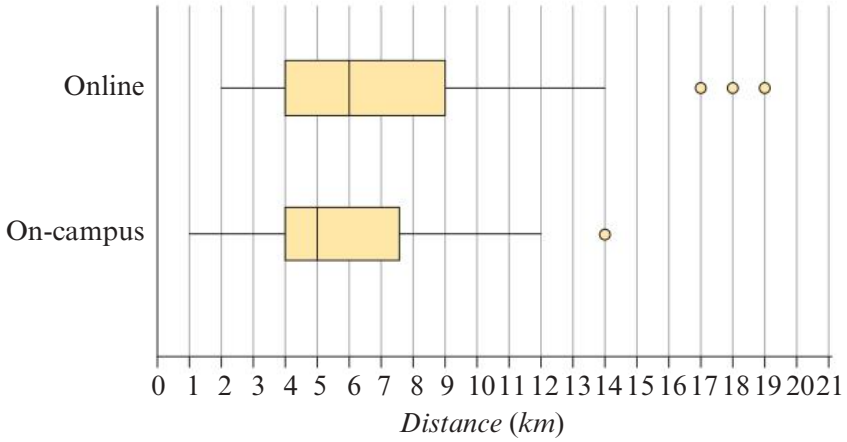
c **i** Determine the values of the upper and lower fences for the boxplot.

ii Use the fences to explain why a distance of 1 km would not be shown as an outlier.

d The boxplots below compare the *distances* students live from the campus for students who prefer to study on-campus, and those who prefer to study online.



Use the information in the boxplots to answer the following questions.

i The median *distance* for the students preferring to study online was ____ km higher than the median *distance* for students preferring to study on-campus.

ii The difference between the *IQR* for *distance* for the students preferring to study online and the *IQR* for the students preferring to study on-campus is ____.

Complex familiar questions

7 Annual salaries for a group of employees in a company were collected in 2016 and again in 2020.

a A five-number summary for the salaries in the Marketing team in 2016 was as follows:

Min = \$29 000 Q_1 = \$49 000 M = \$63 000 Q_3 = \$92 000 Max = \$133 000

i Determine the values of the upper and lower fences, and use these to confirm that there are no outliers.

ii Use the five-number summary to construct a boxplot for the salaries in the Marketing team.

iii Determine the percentage of the Marketing team who earned more than \$49 000.

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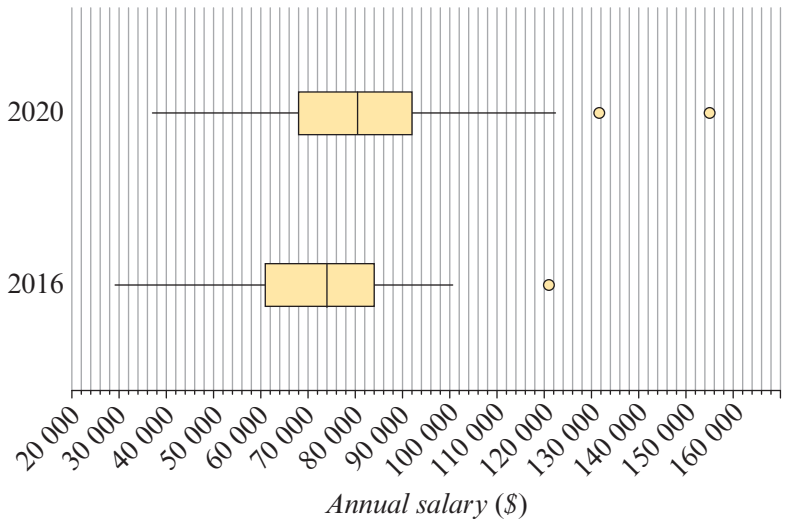
b The boxplots below display the distribution of the salaries of the employees in the Sales team in 2016 and 2020.



i Give the median salary for each year to the nearest $1000.

ii Use the information from the boxplots to write a short report comparing the salaries in the Sales team at that company in 2016 and in 2020.

8 The number of goals scored by each member of two basketball teams in the finals is as follows:

Team A

20 19 16 14 11 10 9 8 8 7 6 4 2 0 0

Team B

31 30 15 13 13 12 10 6 6 4 2 2 2 2 0

a State which of the variables *team* or *score* is a categorical variable.

b Construct back-to-back stem plots of this data, using the following stems.

Score

Team A		Team B
	0	
	0	
	1	
	1	
	2	
	2	
	3	

c Determine the five-number summary for each team.

d Identify any outliers in the scores for each team.

e How do you think the mean score for Team B would compare to the mean score for Team A? Give a reason for your answer. (You do not need to calculate the mean scores).

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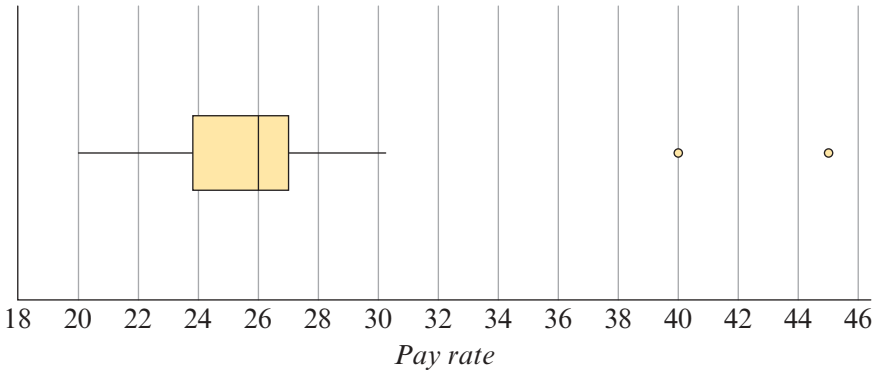
Complex unfamiliar

9 The following boxplot shows the hourly rate paid to a group of 20 workers in a restaurant.

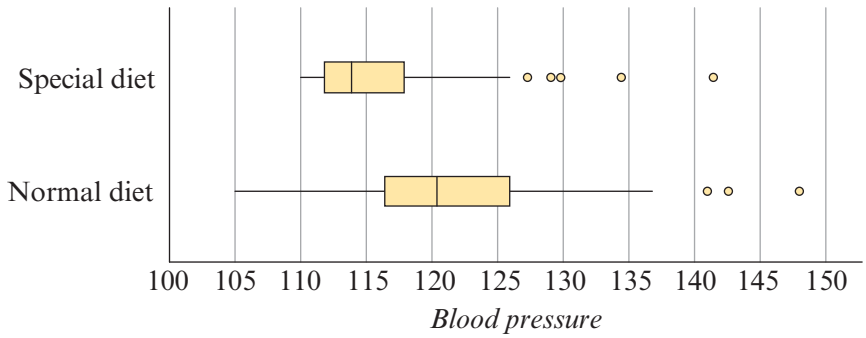


Since the restaurant was doing well. the manager offered every employee a pay rise of 10%, plus an additional loading of $2 per hour.

a On the same scale, draw a possible boxplot for the employees' new hourly wages.

b Write a report comparing the new wages to the old wages.

10 The following parallel boxplots show the blood pressure of a group of 100 people on a normal diet, and a group of 100 people on a special diet designed to reduce blood pressure.



a Of the five people with the highest blood pressure, determine how many were on the special diet.

b Determine the percentage of all 200 people with blood pressure less than or equal to 126.

c The researcher conducting the study claimed that more than half those with blood pressure less than 120 were on the special diet. Use mathematical reasoning to justify this claim.

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Appendix A

The problem-solving and modelling task

Joel Speranza

Chapter contents

- **A1** About the problem-solving and modelling task
- **A2** A content guide for a PSMT report

A1 About the problem-solving and modelling task

Mathematisation is the process of taking a real-world problem, translating it to a mathematically purposeful representation, and solving that problem. In General Mathematics you will be assessed on your ability to do this through an assessment item called a problem-solving and modelling task (PSMT).

This chapter outlines how to plan, solve and present your PSMT at a high level and provides real examples of high-level student work. Each section of this chapter is accompanied by a video lesson with additional advice, accessible via the included QR codes.

How is the PSMT marked?

Before you begin any assessment, you should consider how teachers will make judgements of your work. Your teacher will use the Instrument-specific Marking Guide (ISMG) from the syllabus to determine your mark. The ISMG is broken into four criteria (Formulate, Solve, Evaluate and Communicate) and each criterion is assessed using three to five descriptors. Throughout this chapter, we will be focusing on the top descriptor in each criterion. These top descriptors will be displayed in this appendix where needed. The full ISMG with all descriptors can accessed through the Interactive Textbook.

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A2 A content guide for a PSMT report

Video A1

A suggested set of headings for writing the PSMT report has been provided in a downloadable Word document in the Interactive Textbook. The rest of this appendix provides notes on what to include under these headings, which are reproduced here in black ('Introduction to the task', 'Formulating a solution', 'Developing a solution', and so on). Extracts from the ISMG display the criteria and descriptors covered by each heading of the report.

You also need to adhere to the required word limit and conventions of the mathematical report genre. You may have seen another report genre, the scientific report, and the two have similarities.

Word length guides of each section of the report are indicative only and will be dependent on your specific PSMT. Setting out your PSMT in this way is not compulsory but does help you to achieve this highlighted descriptor from the ISMG:

Communicate	Marks
The student response has the following characteristics:	
• correct use of appropriate mathematical language • logical organisation of the response, which can be read independently of the task sheet • justification of decisions using mathematical reasoning	3–4

1. Introduction to the task (200 words)

Video A2

These are the criteria and descriptors to be covered by the introduction:

Formulate	Marks
The student response has the following characteristics:	
• justified statements of important assumptions • justified statements of important observations • justified mathematical translation of important aspects of the task	3–4

Communicate	Marks
The student response has the following characteristics:	
• correct use of appropriate mathematical language • logical organisation of the response, which can be read independently of the task sheet • justification of decisions using mathematical reasoning	3–4

Note that, as long as you understand the task, and have written down what you need to do, you do not have to finalise the introduction at the beginning of the process. You may find it better to write the introduction alongside writing the conclusion, ensuring that the conclusion addresses the goal of the task which was stated in the introduction.

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The purpose of the introduction is twofold:

1. To introduce a report which 'can be read independently' of the task sheet. This means that a person who has never seen the task sheet will be able to understand what the task is, simply by reading your introduction. No reference to the task sheet should be made here. You should also begin with providing some context on what the problem is to be solved, and why solving the problem is important.
2. To show 'justified mathematical translation of important aspects of the task'. Mathematical translation is the process of taking a real-world problem and moving it into the mathematical world – a process known as **mathematisation**. In this part of an introduction, you should give a brief description of the mathematical techniques you will use to solve the problem. This description can include:
   - Applicable mathematical or statistical principles
   - Mathematical concepts and techniques
   - Technology that you will use throughout the task

The student sample below shows:

- how the task can be read independently of the task sheet
- justified mathematical translation of important aspects of the task.

Introduction

As the population of Australia continues to grow, it is crucial that we plan ahead. Important aspects in life such as resources, housing, and medical products and services need to account for this growing population before the demand for necessary requirements of life have such a demand that cannot be catered for (Sommerfeld, 2018). By knowing predictions for a future population count, we can plan ahead and make choices that will benefit the future now.

The purpose of this task is to determine a prediction for the rate of population growth in Australia for 2061 by exploring multiple mathematical models. A multitude of mathematical techniques will be utilised throughout this task, including; modelling of exponential, logarithmic, logistic, and sinusoidal functions, as well as deriving to find a rate of change, determining percentage error, and using technology such as GeoGebra, a Tl-84 Plus CE Graphics Calculator and Excel to model equations and manage data.

Student sample taken from the 2020 Mathematical Methods subject report

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Using mathematical language

Video A3

In the introduction and throughout the PSMT, you must demonstrate correct use of mathematical language.

These are the criteria and descriptors to be covered:

Communicate	Marks
The student response has the following characteristics:	
• correct use of appropriate mathematical language • logical organisation of the response, which can be read independently of the task sheet • justification of decisions using mathematical reasoning	3–4

The syllabus objectives elaborate on mathematical language as 'terminology, symbols, conventions and representations'. What follows is a non-exhaustive list of ways to demonstrate each aspect of mathematical language.

Terminology

- Procedural mathematical language (mathematical verbs) e.g. determine, solve, verify, calculate integrate, derive.
- Technical mathematical language (mathematical nouns) e.g. search the unit outline for terminology specific to the content being assessed.

Conventions

- Equations have a left and right hand side
- Equal signs are aligned
- Define variables before using them

Symbols

- Use mathematical symbols correctly
- If typing mathematics, use equation editor

Representations: Graphs

- An appropriate title
- Axes labelled with appropriate units
- An appropriate scale for each axis
- A legend if appropriate

Representations: Diagrams

- An appropriate title
- Labelled
- Drawn to scale or a label indicating otherwise
- Vertices of shapes labelled with capital letters
- Angles labelled with Greek letters

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2. Formulating a solution (400 words)

In the section titled 'Formulating a solution', you are aiming to demonstrate the descriptors of the ISMG shown below.

Formulate	Marks
The student response has the following characteristics:	
• justified statements of important assumptions • justified statements of important observations • justified mathematical translation of important aspects of the task	3–4

It is vital that your assumptions are both important and expressed as justified statements. This is the most important section of the PSMT, as many sections of the PSMT cannot be completed to a high-standard without them. This is demonstrated in the following flowchart.

Observations are considered and assumptions made in order to 'mathematise' the problem.

If the solution does not rely on these, then they are not 'important' and therefore do not meet the criteria. If they are not backed by justified statements, then the evidence for your solution being valid is low.

Once a solution is found, 'justified statements about the reasonableness of the solution by considering observations and assumptions' must be made. If these observations and assumptions are poorly justified or are not important, then it will be difficult to do this.

At the end of the PMST, 'justified statements of relevant strengths and limitations of the solution' are made. These are often found by examining the assumptions and observations that have been made. If assumptions and observations are not important, this section becomes more difficult to complete.

Observations vs assumptions

Students can often be confused about the difference between an observation and an assumption. Both are vital to creating a mathematical model but serve different purposes. The table below outlines the key differences between them.

Aspect	Observations	Assumptions
Definition	Data or information required to solve a mathematical problem and/or develop a mathematical model.	Conditions that are stated to be true when beginning to solve a mathematical problem and/or develop a mathematical model.

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Aspect	Observations	Assumptions
Nature and verifiability	Factual, based on actual data or information, and can be empirically verified.	While not directly verifiable, are rational and necessary for solving the problem. They are based on plausible reasoning or existing knowledge.
Role in modelling	Provides data and empirical evidence for creation of the model.	Simplifies complexity and fills data gaps. Dictates the strengths, limitations, and applicability of the model.
Flexibility	Generally rigid; they are facts that can't be altered. However, the interpretation of observations can evolve with new data.	More flexible and can be adjusted or replaced as new information becomes available or as the model evolves.
Examples	**1** Measuring the temperature and humidity levels in various regions over a year. **2** Recording the frequency and intensity of rainfall in a specific area. **3** The observable fact that warm air rises and cool air sinks, affecting weather patterns.	**1** Assuming that future weather patterns will reflect past trends due to climate consistency. **2** Assuming a certain level of accuracy in satellite data used for cloud cover analysis. **3** Assuming that ocean currents will remain relatively stable over the short-term forecasting period.
Role in validation	Used for validating the model by comparing its predictions with actual observations.	Validated indirectly through the model's performance and its ability to make accurate predictions within the defined framework of these assumptions.

2.1 Observations

Video A4

To ensure that your observations reach the level of 'justified statements' they should include:

- a discussion of how the observations affect the mathematical model/solution
- in-text referencing to a reputable source (while it is possible to justify statements without this, consider the inclusion of a reference the 'gold-standard').

The following student sample demonstrates justified statements of observations. Each observation contains:

- a statement of what the observation is
- a reference that provides justification for the observation being made
- justification for how this observation impacts the mathematical model/solution.

Structuring your observations in this way provide a high likelihood of them being considered justified statements on the ISMG.

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Observations

It was observed that the male-to-female ratio in Western Australia is 102 males for every 100 females (McCrindle, 2014). This observation directly impacts the mathematical model as the initial total population must divide by this ratio only to consider the female Western Australian population. This is relevant as males do not reproduce any offspring and cannot be factored into the Leslie matrix.

It was observed that Western Australia takes 30% of immigrants into Australia each year (Australian Bureau of Statistic, 2021). Further, 12,706 to 18,200 immigrants settled in Australia during 2018 and 2020. (Lawrence, 2018). These observations impacted the mathematical model as an increase in immigrants will impact the projected populations, causing an increase or decrease in total population growth rate.

Humans live to approximately 100 years (Vaupel, 2010); therefore, a 21×21 matrix is reasonable. From this, the age classes for the Leslie matrix were split into 5-year categories. Hence, each new generation produced by the Leslie matrix represents a 5-year gap. This observation is relevant as it impacts the scope mathematical model by investigating only eight new generations.

Student sample taken from the 2022 Specialist Mathematics subject report

2.2 Assumptions

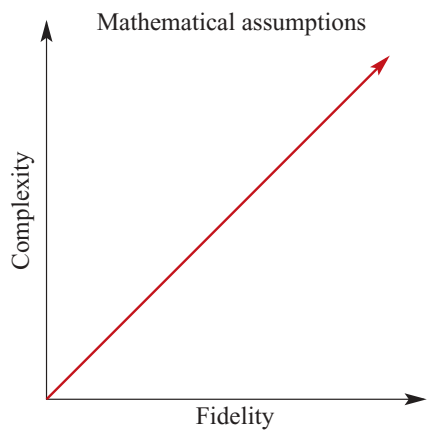
Video A5

As with observations, to justify the assumptions you make you should include discussion of how the assumptions affect the mathematical model/solution; and in-text referencing to a reputable source. While it is possible to justify statements without this, it is considered best practice to include one.

Referencing should be used to justify an assumption and it can be done in two different ways:

- use a source that brings your mathematical model closer to representing the real world
- use a source that takes your mathematical model further from representing the real world but is required to reduce the complexity of the model.

To illustrate this further, each assumption can be thought of as existing on the graph shown below, with mathematical complexity increasing as fidelity (how closely the assumption models the real world) increases. Therefore, each assumption can be thought of as a trade-off between complexity and fidelity. Adopting this approach from the beginning makes future sections of the PSMT (evaluating the reasonableness of the solution and strengths and limitations) easier to complete.



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In the student sample which follows, each assumption contains:

- a statement of what the assumption is
- a reference that provides justification for the assumption being made
- justification for how the assumption impacts the mathematical model/solution.

Assumptions

It was assumed that birth rates would only impact women aged 15–44 as in Australia. The average woman's reproductive years are between ages 15 and 44 (Watson, 2018). This assumption restricts birth rates to only six of twenty-one age classes. The assumption was made to reduce the anomalies to develop clean data.

It was assumed that the new immigrant population introduced into Western Australia was equally divided into the age classes from 18 to 34. Most migrants to Australia are young adults, with 61.2% aged between 18 and 34 years (abs.gov.au, 2018). Further, 12 706 to 18 200 immigrants settled in Australia during 2018 and 2020. (Lawrence, 2018). This assumption was made to create a realistic data spread that included the possible impact immigrants' survival and or birth rate would have on the total population.

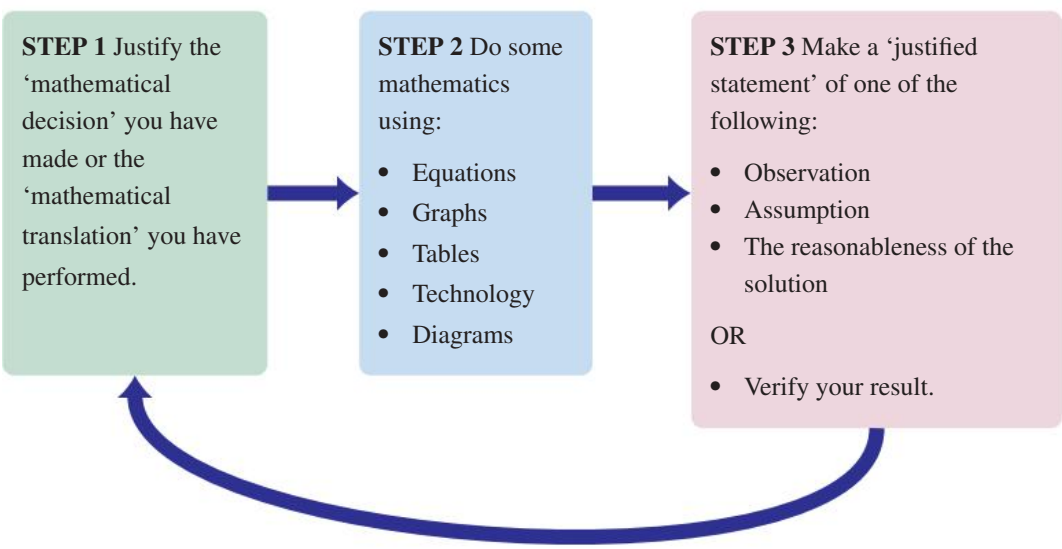
It was assumed that the investigation started during 2016 as the female population data collected was from 2016 (abs.gov.au, 2016); therefore, a 21×21 matrix is reasonable. This assumption impacts the investigation as the potential increasing or decreasing growth rate and total population can be compared to secondary data to determine the model's validity.

Student sample taken from the 2022 Specialist Mathematics subject report.

3. Developing a Solution **(700 words)**

Video A6

In this section of the PSMT, you are attempting to demonstrate almost all the descriptors of the ISMG, as shown in the following example. This is also a section where students can get a little confused about how to set things out. Below is a flowchart you can use throughout this section to ensure that you demonstrate the full range of ISMG descriptors.



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The sample following is a simple example of how this flowchart can be put into practice. It is from a PSMT in which the task is to develop a flying fox. The mathematics has been simplified and the example annotated the flow chart steps and matching ISMG descriptors.

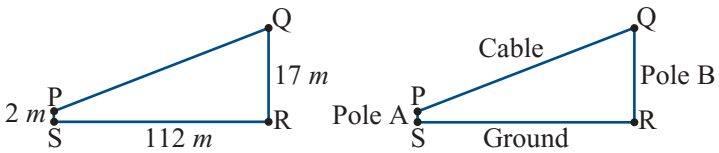
Use them to guide your solution, but do not annotate your own report with them.

3.1 Calculating the length of the cable

STEP 1

The flying fox cable, two supporting poles and the ground can be modelled as the quadrilateral, *PQRS*, as shown below.

STEP 2



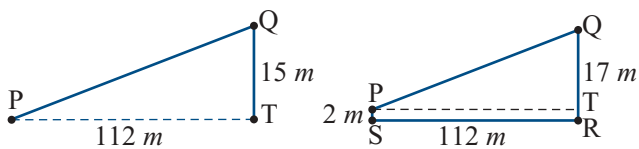
STEP 3

It is assumed that the cable is perfectly taut and has no sag. While the cable in a real flying fox will have a sag (Evans, 2017), this assumption allows the cable's length to be calculated.

STEP 1

A horizontal line drawn through point *P* creates a right triangle *PQT*. Creating a right triangle will allow the length of the cable to be calculated using Pythagoras' theorem.

STEP 2

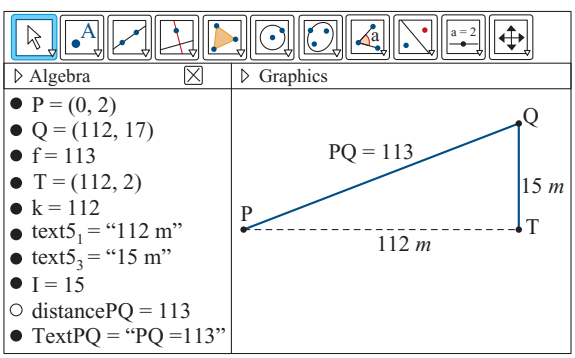


$$\text{Length of cable} = \sqrt{112^2 + 15^2}$$
$$= \sqrt{12544 + 225}$$
$$= \sqrt{12769} = 113$$

STEP 3

The length of the cable is calculated to be 113 metres.

This result can be verified using a scale diagram drawn in Geogebra, as shown in this screenshot.



Formulate
- Justified mathematical translation of important aspects of the task

Communicate
- Correct use of appropriate mathematical language

Formulate
- Justified statements of important assumptions

Communicate
- Justification of decisions using mathematical reasoning

Solve
- Accurate use of mathematical knowledge for important aspects of the task

Solve
- Efficient use of technology

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3.2 Calculating the total cost of the cable

STEP 1

The total cost of cable can now be calculated. In construction, it is common to assume that you require an additional 10% of materials to allow for wastage (Jones, 2017). 8 mm aircraft-grade galvanised cable is perfect for ziplines up to 150 metres in length and has a current cost of $8 per metre. (cable-ride.com, n.d)

STEP 2

$$\text{Total cost} = 113 \times 8 \times 1.1 = \$994.40$$

Adding an additional 10% to materials is a reasonable solution when considering the assumption made that there is no sag in the cable. In reality there will be sag, and this will increase the amount of the cable required.

STEP 3

The $994.40 total cost for the cable can be verified by comparing it to a 90 metre zipline kit available online for $1387. (cable-ride.com, n.d). While this is $392.60 more expensive than our cable, it is for a complete kit rather than just a cable. These two prices are close enough to support our result.

Formulate
- Justified statements of important assumptions
- Justified statements of important observations

Solve
- Accurate use of mathematical knowledge for important aspects of the task

Evaluate
- Justified statements about the reasonableness of the solution by considering the assumptions

Evaluate
- Verified results

Efficient use of technology

Video A7

Solve

The student response has the following characteristics:

- Efficient use of technology

Throughout your assignment, you should be looking for opportunities to demonstrate the efficient use of technology, as required by the ISMG descriptor shown. The key descriptor here is 'efficient', which can be understood as using technology in a way that minimises wasted effort and/or time. An example of efficiency would be using Excel formulas to perform repeated calculations, rather than performing each calculation by hand or on a calculator.

It is often the case that while students use quite a bit of technology throughout their PSMT, they often don't provide the evidence that they have used it. Here are some types of technology students use and the evidence that they can provide to demonstrate that they have used it.

Types of technology	Evidence provided
Spreadsheet software (e.g. Microsoft Excel)	■ Screenshots of graphs or tables ■ Samples of spreadsheet formulas used
Calculator	■ Photos or screenshots of the calculator screen ■ Descriptions of how the calculator was used
Graphing software (e.g. Desmos or Geogebra)	■ Screenshots ■ Descriptions of how the software was used
Logging software (e.g. data collectors)	■ Sample of the data collected ■ Screenshots

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The following examples which evidence efficient use of technology are taken from the 2022 Mathematics subject reports.

To verify this a larger Excel spreadsheet was created to solve the parametric equations for a variety of t values (also extending to ten years):

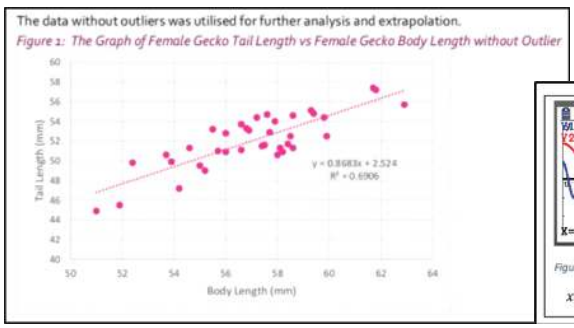
t	Xleft	Xright	Difference	Intersect?	Yleft	Yright	Difference	Intersect?	Zleft	Zright	Difference	Intersect?
0.5	0.999963	1.999997	1.0000337	FALSE	0.0086	0.001805	0.0067944	FALSE		0	-2 1.999997	FALSE
1	0.999852	1.999987	1.0001349	FALSE	0.017199	0.003611	0.0135882	FALSE		0	-1.99999 1.999987	FALSE

Figure 12: Spreadsheet

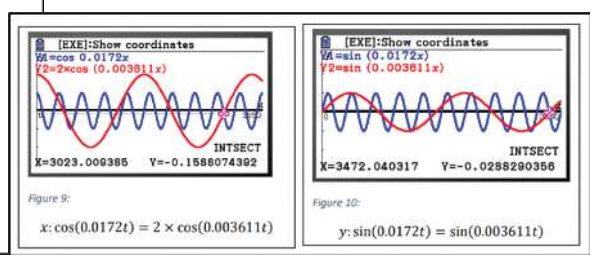
The Excel equation inserted in the 'Intersect?' column (Figure 13), calculated if the difference between the two sides of the parametric equation was less than 4.2587×10^{-5}, the conversion of Earth's radius to AU, to ensure constant units.

```
=IF(D2<(0.0000425875),"YES")
```

Student sample taken from the 2022 Specialist Mathematics subject report



Student sample taken from the 2022 General Mathematics subject report



Student sample taken from the 2022 Specialist Mathematics subject report

4. Evaluating and verifying the solution (600 words)

4.1 Verifying results

Video A8

In this section you are aiming to demonstrate the descriptor shown here, verifying the overall solution. If you have been following the advice from the previous section, you will have been verifying some of your results in the process of coming to your solution. Below, we look at four techniques students can use for verifying results.

Evaluate
The student response has the following characteristics:
• Verified results

1 **Verifying through estimation:** This method simplifies the problem before calculating an approximate solution. For instance, if you have calculated the area of a composite shape, you can verify your solution by simplifying it into a basic rectangle and recalculating the area. If the estimated area is close to your calculated area, it helps confirm the accuracy of your solution.

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2 **Verifying through research:** This method involves comparing results with reliable sources. This method is particularly useful for tasks related to historical data or correlation studies. Findings can be matched against data from textbooks, academic journals, or credible online sources. For example, in a task exploring the correlation between car weight and fuel consumption, you can verify the solution against existing research on this topic.

3 **Verifying through technology:** Using technology to redo algebraic calculations can aid in verifying results. For example, when calculating the area under a curve, employing graphing calculators or software for an approximation and comparing it to manual calculations can serve as a verification method.

4 **Verifying through an alternative method:** This method is effective in problems with multiple solutions and involves employing various techniques to solve the same issue. For example, you may initially solve an algebraic equation by factoring, then recheck the solution using the quadratic formula. If both methods produce identical results, it strongly suggests that the solution is correct.

4.2 Evaluating reasonableness of the solution

Video A9

In this section you are aiming to demonstrate the content descriptor below:

Evaluate	Marks
The student response has the following characteristics:	
• verified results • justified statements about the reasonableness of the solution by considering the assumptions • justified statements about the reasonableness of the solution by considering the observations • justified statements of relevant strengths of the solution • justified statements of relevant limitations of the solution	4–5

When evaluating the reasonableness of your solution by considering assumptions and observations you should:

- consider the solution found
- consider how it is affected by the observation or assumption
- consider how the solution might be different if the observation or assumption was altered (often with some mathematical working included).

On the next page is an example of evaluating the reasonableness of a solution by considering an assumption from a PSMT investigating the braking distance of a car dependent on the speed at which it is travelling.

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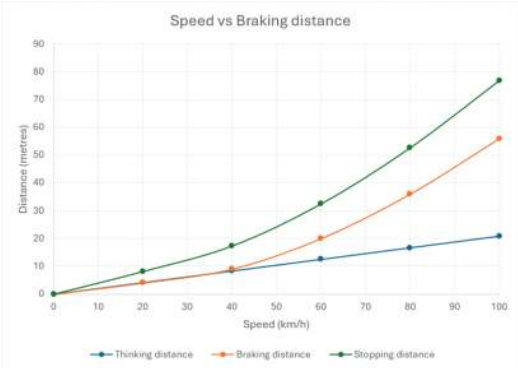
Example

The distance calculated for a car to come to a complete stop is underestimated because of the assumption that driver's response times were instantaneous. If driver response time were factored in, the braking distance would be greater than the distances calculated in this report.



Assuming average driver response time of 1.5 seconds (Muttart, 2004), we can see how the solution would change if this time was taken into account in the graph pictured. At the top speed of 100 km/h, braking distance is increased from the initial solution of 56 m to 77 m, an increase of 37.5%.

Note: Students often make the mistake of evaluating the reasonableness of their assumption, rather than evaluating the reasonableness of their solution by considering their assumption. The distinction is subtle but important. The justified statements made must refer to the solution and how it is affected by the assumption, not just the assumption itself.
Correct: The solution is reasonable because…

Incorrect: The assumption is reasonable because…

A subtle but important difference.

4.3 Strengths and limitations of the solution

In this section you are aiming to demonstrate the content descriptor below:

Video A10

Evaluate	Marks
The student response has the following characteristics:	
• verified results • justified statements about the reasonableness of the solution by considering the assumptions • justified statements about the reasonableness of the solution by considering the observations • justified statements of relevant strengths of the solution • justified statements of relevant limitations of the solution	4–5

Strengths and limitations of the solution can be thought of in the following way.

- Strengths – aspects of the model that make it useful
- Weaknesses – aspects of the model that limit its usefulness

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A series of questions to help identify these strengths and limitations is below. Do not aim to answer all these questions but use them as prompts to generate ideas.

Strengths	Limitations
What assumptions were made that closely align to the real world?	What assumptions were made that do not align closely with the real world?
What aspects of the real world does the solution consider?	What aspects of the real world does the solution not consider?
Could the method used to create this solution be easily adapted and used to solve other, related problems?	Are there other, related problems that the method used could not be easily adapted to solve?
What aspects of the solution can be verified using other observational data?	What aspects of the solution cannot be verified using observational data?
What are the potential, positive consequences of using this solution in the real world?	What are the potential, negative consequences of using this solution in the real world?
What aspects of the solution will continue to be accurate into the future?	What aspects of the solution will cease to be accurate into the future?

To make justified statements of strengths/limitations:

- state the strength/limitation
- justify why it is a strength/limitation.

The student sample of limitations below provides examples of this.

The following limitations were observed

There was a limited amount of data points that were used as a sample.This means the findings were less reliable as it may not be an accurate representation of all rugby games.

Another limitation is that when using extrapolation with regards to the regression line, it may not be accurate to predict further outcomes because the prediction is outside the sample data range.

One final limitation is that the R2 value found is not considerably strong, therefore a smaller percentage of the points scored per game can be attributed to the line breaks achieved per game, decreasing the reliability of the study.

Student sample taken from the 2022 General Mathematics subject report

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5. Conclusion (100 words)

The conclusion is another opportunity to show logical organisation of your response. In the conclusion you should:

- restate the purpose of the mathematical report
- provide a summary of your solution, stating an appropriate answer to the task.

Conclusion

The purpose of this report was to use functions and derivatives to create a reasonable prediction for the rate of change for the Australian population in 2061. It was found that the most reasonable model was solution three as it produces a reasonable population and somewhat reasonable rates of change. Therefore, using this logistic function, it is predicted that the population will reach approximately 39.6 million in 2061, with a percentage rate of change of 0.95% and an instantaneous rate of change of approximately 370 000 addition people per year.

Student sample taken from the 2022 Mathematical Methods subject report

6. Reference List (Not included in page or word count)

Use a standard referencing style. Ask your teacher for guidance on this if you need it.

7. Appendix (Not included in page or word count)

An appendix is for supporting material such as data, diagrams, calculations and screenshots or print-outs from technology, that don't form a direct part of the solution or evaluation. The appendix is not marked so don't include important items that you want a mark for. If you haven't already included your use of technology in the report, you could put a small sample into the body of your assignment to get marks for efficient use of technology.

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Glossary

Note: for definitions and discussion of cognitive verbs used in the syllabus, see the Interactive Textbook.

A

Algebraic expression [p. 195] A mathematical relationship connecting two or more variables. It is also called a formula.

Allowances [p. 9] Extra payments that may be made to employees who have a particular skill, use their own tools or equipment at work, or work in unpleasant or dangerous conditions.

Angle of depression [p. 348] The angle between the horizontal and a direction *below* the horizontal.

Angle of elevation [p. 348] The angle between the horizontal and a direction *above* the horizontal.

Annulus [p. 111] A flat ring shape bounded by two circles that have the same centre.

Arc [p. 112] The part of a circle between two given points on the circle. The length of the arc of a circle is given by $s = r\left(\frac{\theta}{180}\right)\pi$, where r is the radius of the circle and θ is the angle in degrees subtended by the arc at the centre of the circle.

Area [p. 99] The area of a shape is a measure of the region enclosed by its boundaries, measured in square units.

Austudy [p. 20] Austudy is an allowance paid to someone who is aged 25 years or older and studying full-time or undertaking a full-time Australian Apprenticeship or traineeship.

B

Back-to-back stem plot [p. 539] A type of stem plot used to compare two sets of data, with a single stem and two sets of leaves (one for each group).

Balanced [p. 24] When income is equal to expenses.

Bar chart [p. 456] A statistical graph used to display the frequency distribution of categorical data, using horizontal bars.

Bearing [p. 354] *See* Three-figure bearing.

Boxplot [p. 522] A graphical representation of a five-number summary showing outliers if present. *See* Outliers.

Budget [p. 24] A spending plan based on income and expenses.

C

Capacity [p. 121] The amount of substance that an object can hold.

Categorical data [p. 447] Data generated by a categorical variable. Even if numbers, for example, house numbers, categorical data *cannot* be used to perform meaningful numerical calculations.

Categorical variable [p. 448] Variables that are categories which are used to represent characteristics of individuals or objects, for example, place of birth, house number. Categorical variables come in types, nominal and ordinal.

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Centre of distribution [p. 486] Two distributions are said to differ in centre if the values of the data in one distribution are generally larger than the values of the data in the other distribution.

Circumference [p. 110] The circumference of a circle is the length of its boundary. The circumference, C, of a circle with radius, r, is given by $C = 2\pi r$.

Coefficient [p. 241] The slope of the linear equation $y = mx + c$ is given by the coefficient of x.

Column chart [p. 454] A statistical graph used to display the frequency distribution of categorical data, using vertical columns.

Column matrix [p. 400] A matrix with only one column.

Commission [p. 10] Percentage of the value of the goods sold.

Commutative [p. 418] Describes an operation or process that produces the same result when the order of the terms involved is changed.

Composite shape [p. 104] A shape that is made up of two or more basic shapes.

Compound interest [p. 48] Under compound interest, the interest paid on a loan or investment is credited or debited to the account at the end of each period. The interest earned for the next period is based on the principal plus previous interest earned. The amount of interest earned increases each year (it grows exponentially).

Compounding time period or **rests** [p. 51] The time period between compound interest calculations and payments; for example, a day, a month, or a year.

Connections [p. 404] A matrix can be used to record various types of connections, such as social communications and roads directly connecting towns.

Constant [p. 241] The y-intercept corresponds to the (constant) term in a linear equation.

Continuous data [p. 448] Numerical data that can take any value, usually within a range, often arising from measuring.

Continuous variable [p. 448] A numerical variable that can take any value, usually within a range, often arising from measuring.

Conversion of measurements [p. 96] The calculation required to change a value in one unit to the equivalent value in another unit.

Cosine ratio (cos θ) [p. 330] In right-angled triangles, the ratio of the side adjacent to a given angle (θ) to the hypotenuse.

Cosine rule [p. 377] In non-right-angled triangles, a rule used to find:

- the third side, given two sides and an included angle
- an angle, given three sides.

For triangle ABC and side a, the rule is:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Similar rules exist for sides b and c.

D

Data [p. 447] Information collected about a variable.

Diameter [p. 110] The distance from one side of a circle (or sphere) to the opposite side through the centre; it is equal to twice the radius.

Disability Support Pension [p. 19] A Disability Support Pension is paid to someone who has a permanent and diagnosed disability or medical condition that stops them from working.

Discount [p. 31] A reduction in the price of an item usually expressed as percentage decrease, but which may also be expressed as a fraction of the price or an amount subtracted from the price; also known as mark down.

Discrete data [p. 448] Numerical data which which only takes particular values, usually whole numbers, and often arises from counting.

Discrete variable [p. 448] A numerical variable which which only takes particular values, usually whole numbers, and often arises from counting.

Discretionary expenses [p. 22] Expenses which can be varied according to circumstances.

Distribution [p. 453] The pattern in a set of data values. It reflects how frequently different data values occur.

Dividend [p. 65] The financial return to shareholders on a share of a company. Dividends can be specified as the number of dollars each

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share receives or as a percentage of the current share price, called the *dividend yield*.

Dot plot [p. 477] A dot plot consists of a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.

E

Elements of a matrix [p. 399] The numbers of symbols displayed in a matrix.

Equation [p. 223] A mathematical statement that says that two expressions have the same value.

Exchange rate [p. 62] For a currency, the rate used to convert it into another currency.

F

Five-number summary [p. 522] A list of the five key points in a data distribution: the minimum value (Min), the first quartile (Q_1), the median (M), the third quartile (Q_3) and the maximum value (Max).

Fixed expenses [p. 22] Expenses associated with essential living costs, such as rent, electricity and insurance.

Formula [p. 195] A mathematical relationship connecting two or more variables, for example, $C = 5t + 20, P = 2L + 2W, A = \pi r^2$.

Frequency [p. 453] The number of times a value or a group of values occurs in a dataset. Sometimes known as the count.

Frequency table [p. 453] A listing of the values that a variable takes in a dataset along with how often (frequently) each value occurs. Frequency can also be recorded as a percentage.

G

Gradient of a straight line [p. 236] *See* Slope of a straight line.

Gross income [p. 7] The amount someone is paid for their employment before deductions, such as taxation and superannuation, are made.

Grouped data [p. 468] Where there are many different data values, data is grouped in intervals such as 0–9, 10–19, . . .

GST [p. 36] GST (goods and services tax) is a tax, currently at the rate of 10%, that is added to most purchases of goods and services.

H

Heron's rule (Heron's formula) [p. 100] A rule for calculating the area of a triangle from its three sides.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a + b + c)$ and is called the semi-perimeter.

Histogram [p. 471] A statistical graph used to display the frequency distribution of a numerical variable; suitable for medium- to large-sized datasets.

Hypotenuse [p. 82] The longest side in a right-angled triangle.

I

Identity matrix (I) [p. 420] A square matrix with 1s along the leading diagonal and 0s in all other positions, that behaves like the number one in arithmetic, represented by the symbol I. Any matrix multiplied by an identity matrix remains unchanged.

Inflation [p. 57] The tendency of prices to increase with time, resulting in the loss of purchasing power.

Interest [p. 41] An amount of money paid (earned) for borrowing (lending) money over a period of time. It may be calculated on a simple or compound basis.

Interest rate [p. 42] The rate at which interest is charged or paid. It is usually expressed as a percentage of the money owed or lent.

Interquartile range (IQR) [p. 501] A summary statistic that measures the spread of the middle 50% of values in a data distribution. It is defined as $IQR = Q_3 - Q_1$.

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L

Linear equation [p. 223] An equation that has a straight line as its graph. In linear equations, the unknown values are always to the power of 1, for example, $y = 2x - 3, y + 3 = 7, 3x = 8$.

Loss (financial) [p. 64] *See* profit.

M

Map distance [p. 169] The distance measured using a ruler on a map.

Map scale [p. 169] The scale of the map which shows the ratio between map distance and the actual distance.

Mark down [p. 31] *See* discount.

Mark up [p. 31] An increase in the price of an item usually expressed as percentage increase, or the calculation of the price of an item as a fixed increase from its cost.

Matrix [p. 398] *A* rectangular array of numbers or symbols set out in rows and columns within square brackets. (Plural – matrices.)

Matrix addition [p. 407] Matrices are added by adding the elements that are in the same positions.

Matrix multiplication [p. 415] The process of multiplying a matrix by a matrix.

Matrix subtraction [p. 407] Matrices are subtracted by subtracting the elements that are in the same positions.

Maximum (Max) [p. 522] The largest value in a set of numerical data.

Mean (x) [p. 492] The balance point of a data distribution. The mean is given by $\bar{x} = \frac{\Sigma x}{n}$, where Σx is the sum of the data values and n is the number of data values.

Median (M) [p. 493] The midpoint of an ordered dataset that has been divided into two equal parts, each with 50% of the data values. It is equal to the middle value (for an odd number of data values) or the average of the two middle values (for an even number of data values). It is a measure of the centre of the distribution.

Metric system [p. 96] A decimal measuring system based on the metre and other standard units.

Minimum (Min) [p. 522] The smallest value in a set of numerical data.

Modal category or interval [pp. 458] The category or data interval that occurs most frequently in a dataset.

Mode [p. 458] The most frequently occurring value in a dataset. There may be more than one.

N

Negative slope [pp. 239] A straight-line graph with a negative slope represents a decreasing y-value as the x-value increases. For the graph of a straight line with a negative slope, y decreases at a constant rate with respect to x.

Negatively skewed distribution [p. 484] A data distribution that has a long tail to the left. In negatively skewed distributions, the majority of data values fall to the right of the mean.

Net income [p. 7] The amount of pay left after deductions have been made.

Network diagram [p. 404] A set of points called vertices and connecting lines called edges, enclosing and surrounded by areas called faces.

Nominal data [p. 447] Type of categorical data, such as gender, in which categories are given names ('nominations') or labels rather than taking a numerical value.

Nominal variable [p. 448] A categorical variable that generates data values that can be used by name only, for example, eye colour: blue, green, brown.

Non-linear [p. 197] An equation or relationship with a graph that is *not* a straight line. In non-linear equations, the unknown values are not all to the power of 1, for example, $y = x^2 + 5, 3y^2 = 6, b^3 = 27$.

Numerical data [p. 448] Data obtained by measuring or counting some quantity. Numerical data can be discrete (for example, the *number* of people waiting in a queue) or continuous (for example, the *amount of time* people spent waiting in a queue).

O

Order of a matrix [p. 398] An indication of the size and shape of a matrix, written as

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$m \times n$, where m is the number of rows and n is the number of columns.

Ordinal data [p. 448] Type of categorical data, such as clothing size, in which categories are given labels that can be arranged in order, such as numbers or letters.

Ordinal variable [p. 448] A categorical variable that generates data values that can be used to both name and order; for example, house number.

Outliers [p. 487] Data values that appear to stand out from the main body of a dataset. Using boxplots, possible outliers are defined as data values greater than $Q_3 + 1.5 \times IQR$ or less than $Q_1 - 1.5 \times IQR$.

Overtime rate [p. 7] Payment for work beyond the normal working day.

P

Parallel boxplot [p. 540] A statistical graph in which two or more boxplots are drawn side by side. Used to compare distributions in terms of shape, centre and spread.

Penalty rate [p. 8] Payment for working weekends, public holidays or night shifts.

Percentage [p. 2] The number as a proportion of one hundred, indicated by the symbol %. For example, 12% means 12 per one hundred.

Percentage change [p. 36] The amount of the increase or decrease of a quantity expressed as a percentage of the original value.

Percentage discount [p. 32] When the new price is less than the old price, expressed as a percentage.

Percentage frequency [p. 453] Frequency of a value or group of values, expressed as a percentage of the total frequency.

Percentage increase [p. 32] When the new price is more than the old price, expressed as a percentage.

Percentage loss [p. 35] When an item is sold for less than it cost, expressed as a percentage of the cost.

Percentage profit [p. 35] When an item is sold for more than it cost, expressed as a percentage of the cost.

Perimeter [p. 99] The distance around the edge of a two-dimensional shape.

Perpendicular height [p. 118] The measurement of the height at 90° to the cross-sectional base area.

Piece-wise linear graph [p. 308] A graph made up of two or more parts of different straight-line graphs, each representing different overlapping intervals on the x-axis. Sometimes called a line segment graph.

Piecework [p. 11] When a worker is paid a fixed payment, called a *piece rate*, for each unit produced or action completed. Piecework = number of units of work × amount paid per unit.

Positive slope [p. 239] A positive slope represents an increasing y-value with increase in x-value. For the graph of a straight line with a positive slope, y increases at a constant rate with respect to x.

Positively skewed distribution [p. 484] A data distribution that has a long tail to the right. In positively skewed distributions, the majority of data values fall to the left of the mean.

Power of matrix A^n [p. 421] The matrix A is used n times in repeated multiplication.

Price-to-earnings ratio [p. 67] A measure of the profit of a company, given by the *current share price/profit per share*. A lower value of the price-to-earnings ratio may indicate a better investment.

Principal (P) [p. 39] The initial amount borrowed, lent or invested.

Profit [p. 66] A financial gain, such as the amount left over when costs have been subtracted from revenue leaving a positive number. If the number left is negative, that is called a loss.

Pronumeral [p. 204] A symbol (usually a letter) that stands for a numerical quantity or variable.

Purchasing power [p. 58] A measure of how much a specific good or service money can buy at different times (due to inflation, for instance), or which different currencies can buy.

Pythagoras' theorem [p. 82] A rule for calculating the third side of a right-angled triangle given the length of the other two sides. In triangle

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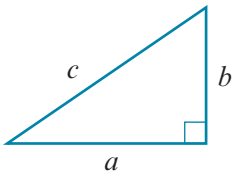
ABC, the rule is: $c^2 = a^2 + b^2$, where c is the length of the hypotenuse.



Q

Quartiles (Q_1–Q_3) [p. 501] Summary statistics that divide an ordered dataset into four equal-sized groups, each containing 25% of the scores.

R

Radius [p. 110] The distance from the centre of a circle (or sphere) to any point on its circumference (or surface); equal to half the diameter.

Range (R) [p. 500] The difference between the smallest (minimum) and the largest (maximum) values in a dataset: a measure of spread.

Ratio [p. 153] A fraction, or two numbers in the form a:b, used to numerically compare the values of two or more quantities.

Rate [p. 2] One quantity divided by another quantity, where the two quantities do not have the same unit of measurement.

Relation [p. 232] The association between two variables that can be plotted as coordinates on a number plane, often described by an equation or formula, which can also be called a relationship.

Rest [p. 53] *See* compounding time period.

Right angle [p. 82] An angle equal to 90°.

Rise [p. 240] *See* Slope of a straight line.

Row matrix [p. 400] A matrix with only one row.

Run [p. 237] *See* Slope of a straight line.

S

Salary [p. 6] Payment for a year's work, generally paid as equal weekly, fortnightly or monthly payments.

Scalar multiplication [p. 411] The multiplication of a matrix by a number.

Scale factor [p. 153] In similar figures, the ratio of the length of a side in one figure to the length of the corresponding side on the other figure.

Scale factor (areas) [p. 163] The scale factor, k^2, by which the area of a two-dimensional shape is scaled (increased or decreased) when its linear dimensions are scaled by a factor of k.

Scale factor (volumes) [p. 177] The scale factor, k^3, by which the volume of a solid shape is scaled (increased or decreased) when its linear dimensions are scaled by a factor of k.

Sector [p. 112] A portion of a circle enclosed by two radii and an arc.

Shares [p. 66] A share is a unit of ownership of a company.

Similar figures [p. 152] Figures that have exactly the same shape but differ in size.

Similar triangles [p. 157] Different sized triangles in which the corresponding angles are equal. The ratios of the corresponding pairs of sides are always the same.

Simple interest [p. 41] Interest that is calculated for an agreed period and paid only on the original amount invested or borrowed. Also called flat-rate interest.

Simultaneous linear equations [p. 289] Two or more linear equations in two or more variables, for values that are common solutions to all equations. For example, $3x - y = 7$ and $x + y = 5$ are a pair of simultaneous linear equations in x and y, with the common solution $x = 3$ and $y = 2$.

Sine ratio (sin θ) [p. 330] In right-angled triangles, the ratio of the side opposite a given angle (θ) to the hypotenuse.

Sine rule [p. 367] In non-right-angled triangles, a rule used to find:

- an unknown side, given the angle opposite and another side and its opposite angle
- an unknown angle, given the side opposite and another side and its opposite angle.

For triangle ABC the rule is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Skewness [p. 484] *See* Positively or negatively skewed distribution.

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Slope of a straight line [p. 241] The ratio of the increase in the dependent variable (y) to the increase in the independent variable (x) in a linear equation. Also known as the gradient. The slope (or gradient) rise of a straight-line graph is defined to be:
$\text{slope} = \frac{\text{rise}}{\text{run}}$.

Slope–Intercept form of the equation of a straight line [p. 241] A linear equation written in the form $y = a + bx$, where a and b are constants. In this equation, a represents the y-intercept and b represents the slope. For example, $y = 5 - 2x$ is the equation of a straight line with y-intercept of 5 and the slope of -2.

SOH–CAH–TOA [p. 330] A memory aid for remembering the trigonometric ratio rules.

Spread of a distribution [p. 486] A measure of the degree to which data values are clustered around some central point in the distribution. Measures of spread include the standard deviation (s), the interquartile range (IQR) and the range (R).

Square matrix [p. 401] A matrix with the same number of rows as columns.

Standard deviation (s) [p. 502] A summary statistic that measures the spread of the data values around the mean. It is given by
$s = \sqrt{\frac{\Sigma(x - \overline{x})^2}{n - 1}}$.

Stem plot (stem-and-leaf plot) [p. 477] A method for displaying data in which each observation is split into two parts: a 'stem' and a 'leaf'. A stem plot is an alternative display to a histogram; suitable for small-to medium-sized datasets.

Step graph [p. 309] A step-graph consists of one or more non-overlapping horizontal line segments. They can be used to graphically represent situations where the value of one variable is constant for intervals of another variable.

Subject [p. 202] The pronumeral in an expression that you are trying to solve for.

Substitution [p. 195] The process of replacing variables in a formula with values.

Substitution method [p. 292] When solving simultaneous equations by substitution, the process is to substitute one variable from one equation into the other equation.

Summary statistics [p. 492] Statistics that give numerical values to special features of a data distribution, such as centre and spread. Summary statistics include the mean, median, range, standard deviation and IQR.

Surface area [p. 132] The total of the areas of each of the surfaces of a solid.

Symmetric distribution [p. 483] A data distribution in which the data values are evenly distributed around the mean. In a symmetric distribution, the mean and the median are equal.

T

Tangent ratio (tan θ) [p. 330] In right-angled triangles, the ratio of the side opposite a given angle θ to the side adjacent to the angle, when neither side is the hypotenuse.

Three-figure bearing [p. 354] An angular direction, measured clockwise from north and written with three digits, for example, 060°, 324°. Also called a true bearing.

Total surface area (TSA) [p. 136] The total surface area (TSA) of a solid is the sum of the surface areas of all of its faces.

Transposition [p. 202] The process of rearranging a formula in order to make a different pronumeral the subject of the equation.

Trigonometric ratios [p. 330] In right-angled triangles, the ratios
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

True bearing [p. 354] *See* Three-figure bearing.

U

Undefined [p. 239] Has no meaning; has no value. The slope, or gradient, of a vertical line is undefined because $\frac{rise}{run}$ gives a zero denominator.

Unit cost [p. 28] The unit cost method is used to compare the cost of items using cost per unit.

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The choice of unit may be a unit of weight (such as kilogram), or length (such a metre) or another measure depending on the particular item.

Univariate data [p. 446] Data that are collected from a single variable.

V

Variable [p. 447] A quantity that can have many different values in a given situation. Symbols such as x, y and z are commonly used to represent variables.

Volume [p. 118] The volume of a solid is a measure of the amount of space enclosed within it, measured in cubic units.

W

Wage [p. 6] Payment for work calculated on an hourly basis, with the amount earned dependent on the number of hours actually worked.

Y

***y*-intercept** [p. 241] The point at which a graph cuts the y-axis.

Youth Allowance [p. 18] Youth Allowance may be paid by the government to 18–24 years olds studying full-time, 16–24 years olds undertaking a full-time Australian Apprenticeship, or 16–20 years old and looking for full-time work.

Z

Zero matrix (0) [p. 408] A matrix that behaves like zero in arithmetic, represented by the symbol O. Any matrix with zeros in every position is a zero matrix.

Zero slope [p. 239] A horizontal line has zero slope. The equation of this line has the form $y = c$ where c is any constant.

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Answers

Chapter 1

Exercise 1A

1 **a** \$1.40/kg **b** 70 km/h
c \$30/h
d 0.5 cm/month (or 6 cm/year or 5 mm/month)

2 **a** 25% **b** 40% **c** 15%
d 70% **e** 19% **f** 79%
g 215% **h** 3957% **i** 7.3%
j 100% **k** 125% **l** 200%

3 **a** **i** $\frac{1}{4}$ **ii** 0.25 **b** **i** $\frac{1}{2}$ **ii** 0.5
c **i** $\frac{3}{4}$ **ii** 0.75
d **i** $\frac{23}{400}$ **ii** 0.0575
e **i** $\frac{34}{125}$ **ii** 0.272
f **i** $\frac{9}{2000}$ **ii** 0.0045
g **i** $\frac{3}{10\,000}$ **ii** 0.0003
h **i** $\frac{13}{200\,000}$ **ii** 0.000 065

4 **a** \$114 **b** \$110
c 25.5 m **d** \$1350
e 1.59 cm **f** 2.64
g \$77 700 **h** \$19 800

5 80% **6** 37.5%
7 83.33% **8** 37.5%
9 65.08%

Exercise 1B

1 **a** \$1782 **b** \$3563 **c** \$7721

2 **a** \$1108, \$2217 **b** \$1509, \$3019
c \$2073, \$4146 **d** \$851, \$1702

3 **a** \$30 160 **b** \$39 520
c \$39 216 **d** \$125 736

4 Laura \$32 110.00, Ebony \$29 508.00; Laura by \$2602.00

5 **a** \$127.50 **b** \$382.50 **c** \$943.50

6 **a** \$444.00 **b** \$351.50
c \$858.40 **d** \$511.71

7 Alyssa \$332.10, Connor \$320.00; Alyssa by \$12.10

8 **a** 40.50 h **b** \$911.25

9 **a** \$108.00 **b** \$237.00
c \$379.20 **d** \$262.98

10 **a** \$227.94 **b** \$114.30
c \$292.68 **d** \$370.74

11 \$2008.13

12 \$1562.00

13 \$1052.80

14 **a** \$257.50 **b** \$1287.50

15 \$820.88 **16** \$1761 **17** \$3589.60

18 **a** \$352.80 **b** \$669.60 **c** \$1368.80

19 \$1620.00

20 **a** \$431.00 **b** \$642.00 **c** \$1640.00

21 \$960.00

22 **a** \$512.00 **b** \$500.00

23 **a** \$150.00 **b** \$144.00 **c** \$273

24 \$500

25 \$5140

26 \$635 481.60

27 **a** \$83 790.00 **b** \$5795

28 \$7503.42

29 6 hours

30 **a** \$453.60 **b** 4 hours
c \$22.68 **d** \$567.00

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31 **a** $317.24 **b** $538.20 **c** $849.43
32 $27.50
33 $28.80
34 **a** $53.40 **b** $2178.40 **c** $9952.10
35 $50 600.00
36 **a** $3400.00 **b** $4700.00 **c** $5200.00
37 2%
38 $460.00
39 **a** $57 000.00 **b** $4400.00
c $22 250.00
40 $30
41 40 skirts

Exercise 1C

1 **a** $639.00 **b** $16 614
2 **a** $1278.00 **b** $105.60
3 **a** $11 056.50 **b** $13 445.90
4 **a** $608.70 **b** $15 826.20
5 **a** $792.50 **b** $153.50
6 **a** $16 614.00 **b** $35 973.60
7 **a** Youth **b** $639.00
c Austudy **d** $0

Exercise 1D

1 $14 640
2 **a** **i** $1536 **ii** $2550
iii $18 720 **iv** $23 400
b $888.58
3 **a** **i** $26 223.96
ii $12 548.04
iii $1938
b $1602.52
4 **a** $61 460.37 **b** $59 769.77
c $1690.60
5 **a** $103 425.92 **b** $59 287.35
c $44 138.57 **d** $424.41
e 24.3%
f **i** $26 345.05 **ii** 25.5%
6 **a**

Income		Expenses	
Job	$125	Games	$40
Allow	$30	Entertain	$40
		Food	$55
		Savings	$20
	$155		$155

b 35.5%
7 **a** $526 **b** $658 **c** $77 total
d

Income		Expenses	
Job	$1896	Mortgage	$526
		Groceries	$360
		Entertain	$120
		Medical	$18
		Car	$160
		Electricity	$20
		Telephone	$14
		Rates	$43
		Balance	$635
	$1896		$1896

8 **a** $435.69 **b** 16.1% **c** $489.12, 19.5%
9 42%

Exercise 1E

1 **a** $15.60 **b** 84 seconds
c $885 **d** 10 km
2 7 red, 28 yellow
3 **a** 270 km **b** 225 km **c** 30 km
d 105 km **e** 330 km **f** 67.5 km
4 51 eggs
5 73 g cone for $2.00
6 Brand A
7 2 × 3 bedroom, $86.67 / night.
8 **a** Shop B **b** $209.40
9 Brand A, 0.78 cents per metre cheaper.

Exercise 1F

1 **a** 48.8% **b** 24.4% **c** 11.1%
d 9.2% **e** 33.3% **f** 25%
2 **a** $2600 **b** $200 **c** $200
d $1.80 **e** $1865 **f** $20 000
3 **a** $86.40 **b** $180
c $0.58 **d** $73.08
4 **a** $291.20 **b** $626.40
c $68 **d** $6318
5 **a** $1865.50 **b** $11.14 **c** $27.72
d $10 282 **e** $847.70 **f** $2631.20
6 **a** 24% **b** 20%
7 **a** $60 **b** $50 **c** $71.43 **d** $88
8 **a** $13.30 **b** $62.22
c $104.50 **d** $4993.90
9 $212.75 **10** $92.07
11 **a** $12.13 **b** $6.76 **c** $98.55 **d** $39.50
12 **a** $152.90 **b** $2945.80
c $10 835 **d** $1534.50

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13 $2180.91
14 $3635.45
15 **a** $289.97 **b** $29.00
16 **a** $150 **b** 33.3%
17 **a** $20.00 **b** 20%
18 150%
19 **a** $92.24 **b** $73.79 **c** $2.58
20 **a** 48.1%
b **i** $5.90 **ii** $6.00
21 $16.78

Exercise 1G

1 **a** $80 **b** $300 **c** $600
d $384.38 **e** $4590 **f** $324.38
g $29.95 **h** $14.43 **i** $6243.75
2 **a** $600 **b** $932.10 **c** $1243.50
d $2411.25 **e** $2832
3 **a** $12 000 **b** $32 000
4 **a** $1950 **b** $11 950
5 $1243.50 **6** $9041.10 **7** 12%
8 15% **9** 6.5 years
10 354 days
11 4 years
12 $1210 **13** $45 552
14 **a** $180 **b** $780 **c** 3 years
d $1051.60 **e** 7% **f** $1335.15
g $4500 **h** $4650 **i** 5%
j $3698.63 **k** 3 years **l** $220.50
m $1448.28 **n** $1500.78
15 **a** $742.50 **b** 9 years
16 **a** $118.75 **b** $2750 **c** $2463.19
d $24 000 **e** $1983.63 **f** $13 617.92
17 20 years
18 **a** $18 000 **b** $3000

Exercise 1H

1 **a** $4466.99 **b** $966.99 **c** 27.6%
2 **a** $9523.42 **b** $2523.42 **c** 36.0%
3 **a** $107 019.03 **b** 7.0%
4 **a** $1552.87 **b** $302.87 **c** 24.2%
5 **a** $50 642.64 **b** $5142.64 **c** 11.3%
6 **a** $1338.23 **b** 33.8%
7 **a** $3085.98 **b** $539.36
8 **a** $2030.89 **b** $733.47
9 **a** $3948.83 **b** 13 years
10 **a** 0.3% **b** $10 181.36
c **i** $27 846.69 **ii** 11.4%
11 **a** 0.2%
b **i** $6482.97 **ii** 29.7%
12 **a** **i** $18 696.04 **ii** $18 696.64
iii $18 697.13
b 60 cents
13 **a** Difference: CI − SI = $202.61
b 9.25%
14 **a** $114 752.30 **b** $3880.51
c 3.69%
15 8.45%
16 6.78%

Exercise 1I

1 **a** $3.59 **b** $3.72
2 **a** $851.40 **b** $896.52
3 $5.46
4 **a** $2.62 **b** $7.10
5 **a** $680 359.31 **b** $1 113 531.40
6 **a** $1299.12 **b** $1575.80
7 **a** $148 818.78 **b** $58 917.67
8 $1708.57
9 **a** $598.48 **b** $263.30 **c** $69.04
10 **a** $56 224.79 **b** 24 years
11 **a** $4.80 **b** 11 years
12 7.2%

Exercise 1J

1 **a** 460.12 EUR **b** $3224.98 USD
c $882.30 ¥ **d** $51.59 AUD
e $680.71 AUD **f** $16 129.03 AUD
2 **a** $8.93 AUD **b** $7.34 AUD
c $6.17 AUD
3 17 245.20 BWP
4 328.32 EUR
5 $216.57
6 Supplier 1: $81.53, Supplier 2: $112.44, Supplier 3: $109. The US supplier is cheapest.

Exercise 1K

1 **a** 0.5% **b** $12 500
2 **a** 2000 **b** $50 000 **c** 1200
3 **a** $250 **b** 10.9%
4 **a** $1.25 **b** $2500
5 **a** $2400 **b** 2.3%
6 **a** $0.24 **b** $96.00
7 **a** A: 8.8, B: 10 **b** A

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8 $400

9 a i $11 000 **ii** $12.10
b i $6600 **ii** $17.60

10 a $0.50 **b** $1.00
c 500 Alpha Oil, 250 Omega Mining
d $612.50

Chapter 1 review

Multiple-choice questions

1 B	**2** B	**3** A	**4** C
5 A	**6** C	**7** B	**8** B
9 D	**10** A	**11** B	**12** D
13 B	**14** D	**15** B	**16** B
17 C	**18** D	**19** A	**20** A
21 D	**22** C	**23** B	**24** D
25 D	**26** D	**27** A	**28** D

Short-response questions

1 a $1855.20 **b** $38.65

2 $641.25

3 $5371.25

4 $4440

5 Rabbit Easter eggs, 0.7%

6 a $137.50
b Before discount: profit of 14.6%; after discount loss of 8.3%

7 a $36 000 **b** $11 000

8 $5.35

9 a $1833.85 **b** 25.8%

10 $512

11 $2781.12

12 $1526

13 $1042.44

14 a $264.67 **b** $185.14

15 a $105.76 **b** £66.57

16 a $1100 **b** 4.8%

17 $815.18

18 $1402

19 a $612.50
b i $23.35 **ii** $653.36 **iii** $6.67%

20 $4725.00

21 11.1%

22 $3270.29

23 a $3726.98 **b** $3650 **c** 3.73%

24 a $111 008 **b** $82 508
c i 9.5 hours **ii** 13.6 hours

25 a $50 256 **b** $14 256
c 16.5% p.a. **d** 14.3% p.a.

Chapter 2

Exercise 2A

1 a 4.9 cm **b** 83.1 cm **c** 24 mm
d 2.4 mm **e** 15.8 mm **f** 7.4 cm
g 6.4 cm **h** 141.4 mm **i** 15.4 m

2 2.9 m **3** 3.8 m **4** 5.3 m
5 48.88 km **6** 20 cm **7** 9.4 m
8 61.717 m **9** 15 km **10** 12.81 km
11 100 m **12** 4.24 cm

13 Ultimate Farmers provides the best value for money since it costs $218.25 less than Great Fencing.

Exercise 2B

1 a 4.243 cm **b** 5.196 cm

2 a 10.77 cm **b** 11.87 cm **c** 6.40 cm

3 a 27.73 mm **b** 104.79 mm

4 9.54 cm

5 a i 8.5 cm **ii** 9.1 cm
b i 10.6 cm **ii** 3.8 cm

6 17 cm **7** 13 cm **8** 25 cm

9 Yes, it will fit since the length of the cupboard diagonal, 155 cm, is longer than the length of the broom, 145 cm.

10 8.02 m

11 14.43 m is the shortest length of fairy lights required.

12 The cheapest cost is $85.10 to cable the room. This is reasonable since the length using the diagonal of the 6 m by 4 m side is shorter than the diagonals of the other two sides by 0.87 m and 0.61 m, hence the cost is less by $7.40 and $5.20, respectively.

13 12 m

Exercise 2C

1 a 570 cm **b** 1587 m
c 80 mm **d** 6.7 m
e 460 cm **f** 2.89 cm^2
g 52 000 cm^2 **h** 80 000 m^2
i 37 cm^2 **j** 6 000 000 mm^2
k 0.5 L **l** 700 g
m 2 300 000 mg **n** 0.567 kL
o 793.4 g **p** 500 mL

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2 a 5000 kg **b** 0.006 kg
c 27 100 000 000 m^2 **d** 33 000 000 cm^3
e 0.000 487 km^2 **f** 0.028 L
g 600 000 cm **h** 0.001 125 kL
i 0.000 05 km^3 **j** 0.000 34 m^3

3 a 158 mm **b** 589.169 km
c 364.6 cm **d** 13.5 cm^2

4 7.86 m **5** 3000 kg

6 2 250 000 L

7 a i 60 cm **ii** 225 cm^2
b i 22.4 cm **ii** 26.07 cm^2
c i 312 cm **ii** 4056 cm^2
d i 44 cm **ii** 75 cm^2

8 a 56.2 cm^2 **b** 16.7 m^2 **c** 103.6 cm^2
d 73.8 cm^2 **e** 28 cm^2 **f** 35.9 cm^2
g 29.9 m^2 **h** 31.3 m^2

9 a 5 cm^2 **b** 125 cm^2 **c** 40 cm^2

10 100 m^2 **11** 63 375 m^2

12 40 tiles **13** 4 L

14 30.88 m^2

15 a 252 m^2 **b** 273 m^2

16 One more roll

17 216 cm^2

Exercise 2D

1 a i 31.4 cm **ii** 78.5 cm^2
b i 53.4 cm **ii** 227.0 cm^2
c i 49.6 mm **ii** 196.1 mm^2
d i 1.3 m **ii** 0.1 m^2

2 8.73 m

3 a i 14.94 m **ii** 13.90 m^2
b i 20.05 m **ii** 23.89 m^2
c i 33.13 cm **ii** 57.81 cm^2
d i 106.73 km **ii** 595.67 km^2

4 62.83 cm^2

5 a i 25.71 cm **ii** 39.27 cm^2
b i 1061.98 mm **ii** 14 167.88 mm^2
c i 203.54 cm **ii** 2551.76 cm^2
d i 53.70 mm **ii** 150.80 mm^2

6 a 343.1 cm^2 **b** 34.9 m^2
c 19.2 cm^2 **d** 177 377.4 mm^2

7 a 1051.33 m **b** 37 026.55 m^2

8 a 6 m **b** 2.3 m^2

9 1414 cm^2

10 30.91 m^2

11 8.19 m

12 $626.51

13 21.5%

Exercise 2E

1 a 125 cm^3 **b** 49 067.8 cm^3
c 3685.5 cm^3 **d** 3182.6 mm^3
e 29 250 cm^3 **f** 0.3 m^3
g 6756.2 cm^3 **h** 47.8 m^3

2 424 cm^3 **3** 516 cm^3 **4** 24 L

5 a 20 319.82 cm^3 **b** 20 L

6 228 cm^3 **7** $2.93 **8** 24 cm

Exercise 2F

1 a 9500.18 cm^3 **b** 16.36 m^3
c 59.69 m^3 **d** 2356.19 mm^3

2 a 153.94 cm^3 **b** 705.84 m^3
c 102.98 cm^3 **d** 1482.53 cm^3

3 a 26.67 cm^3 **b** 420 m^3
c 24 m^3 **d** 68.64 cm^3

4 213.333 cm^3

5 a 523.60 mm^3 **b** 229.85 mm^3
c 7238.23 cm^3

6 a 179.59 cm^3 **b** 11 494.04 cm^3
c 33.51 cm^3

7 a 8578.64 cm^3 **b** 7679.12 cm^3
c 261.80 cm^3 **d** 4.09 m^3

8 393 cm^3 **9** 7.87 m^3 **10** 0.02 L

11 1 694 000 m^3 **12** 8316.84 cm^3

13 2 791 115 L

14 a 335.6 cm^3 **b** 66.6 cm^3

15 3937.5 cm^3 **16** 44 899 mm^3

17 14 L **18** 4:1

19 The soil will not fit in the capsule. This result is reasonable since there is 0.18 L more soil than volume available in the capsule.

Exercise 2G

1 a 1180 cm^2 **b** 40 m^2
c 383.3 cm^2 **d** 531 cm^2

2 a 3053.63 cm^2 **b** 431.97 cm^2
c 277.59 m^2 **d** 7.37 m^2
e 242.53 cm^2 **f** 24.63 m^2

3 a 2107.77 cm^2 **b** 176.1 m^2
c 235.62 m^2 **d** 146.08 m^2

4 15 394 cm^2

5 a 1.08 m^2 **b** 6 m

6 0.23 m^2 **7** 6 cm **8** 4.5 cm

Exercise 2H

1 a 1.23 m^3 **b** 56 boards **c** 15 m^2

2 a 0.32 m^3 **b** 3.8 m^2

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3 a 14 cm **b** 56 cm, 44 cm

4 a 1.21 hectares **b** 416.20 m

5 35.34 mL

6 Her expectations are unreasonable since she will actually earn $788 less than she would like after taking into account the cost of the materials.

Chapter 2 review

Multiple-choice questions

1 C **2** B **3** C **4** A
5 C **6** B **7** C **8** C
9 D **10** A

Short-response questions

1 a 9.22 cm **b** 9 cm

2 a 58 cm **b** 30 m

3 36 m

4 a 140 cm^2 **b** 185 cm^2

5 33.32 m^3 **6** 37.5 cm^2

7 a C = 31.42 cm, A = 78.54 cm^2
b C = 75.40 cm, A = 452.39 cm^2

8 a P = 15.24 m, A = 13.09 m^2
b P = 14.10 m, A = 10.39 m^2
c P = 24.76 km, A = 33.51 km^2

9 a 390 cm **b** 2.75 cm^2
c 80 000 cm^2 **d** 50 000 cm^3
e 2.8 L **f** 189 mL

10 a 376.99 cm^3 **b** 377 mL

11 239.06 cm^2 **12** 5.66 cm **13** 31 809 L

14 a 373.85 cm^2 **b** 2.97 m^2 **c** 0.52 L

15 301.94 m^2

16 a 154.30 m^2 **b** 101.70 m

17 a 15.07 m **b** 1894.36 L

18 5 m

19 P = 50.27 cm, A = 146.12 cm^2

20 Both paths are the same distance, so there is no shortest path.

21 Their proposed budget is unreasonable since the actual cost is $550.33 more than they have budgeted.

22 2048 cm^3

23 After the innermost starting point, each successive starting point should be approximately 6.28 m (2π m) ahead of the previous one. So on the track, the starting distance from zero should be 0 m, 6.28 m, 12.57 m, 18.85 m, 25.13 m, 31.42 m.

Chapter 3

Exercise 3A

1 a TU corresponds to PB, TS corresponds with PR, SU corresponds to RB, $\angle STU$ corresponds to $\angle RPB$, $\angle TUS$ corresponds to $\angle PBR$, $\angle UST$ corresponds to $\angle BRP$.
b NO corresponds to QP, OM corresponds to PR, MN corresponds to RQ, $\angle MNO$ corresponds to $\angle RQP$, $\angle NOM$ corresponds to $\angle QPR$, $\angle OMN$ corresponds to $\angle PRQ$.

2 a Similar, ratio 3:1 **b** Not similar
c Similar, ratio 2:1

3 a Not similar **b** Similar, sf = 4
c Similar, sf = $\frac{1}{3}$ **d** Similar, sf = $\frac{3}{2}$

4 a Scaled up **b** sf = 3

5 24 cm by 16 cm **6** sf = 1.67

7 sf = 50 **8** 8.1 cm by 4.5 cm

9 6 cm by 8 cm by 10 cm **10** sf = $\frac{5}{3}$

Exercise 3B

1 a i AAA **ii** sf = 3
b i SAS **ii** sf = 4
c i SSS **ii** sf = 3
d i SSS **ii** sf = 3
e i AAA **ii** sf = 2
f i RHS **ii** sf = $\frac{1}{2}$

2 a AAA **b** sf = $\frac{1}{2}$

Exercise 3C

1 a x = 27, y = 30
b x = 26 m, y = 24 m
c x = 45 units, y = 24 units
d x = 1 unit, y = 1.4 units

2 a i 3 : 1 **ii** 9 : 1
b i 2 : 1 **ii** 4 : 1
c i 2 : 1 **ii** 4 : 1

3 a x = 3 **b** 9 : 1

4 72 cm^2 **5** 112 cm^2 **6** 24 cm^2

7 1.8 m **8** 29.4 cm^2 **9** 14.4 cm

10 864 cm^2 **11** 20 m **12** 119 cm

13 18.75 cm

Exercise 3D

1 a Scale = 1 : 5 000 000, 160 km
b Scale = 1 : 27 272 727, 381.82 km
c Scale = 1 : 28 571 428, 8285.7 km

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2 a i 800 m ii 2.3 km
b i 120 m ii 1.6 km
c i 1.615 km ii 243.1 m

3 a 36 km **b** 3 cm

4 a 4.25 m **b** 50 cm

5 a 1 : 20 000 000
b i 1240 km ii 520 km
iii 1580 km iv 640 km

6 a 815.8 m **b** 10 minutes
c 289.5 m **d** 2 minutes

7 a 215.0 m **b** 4.3 seconds

8 840 cm^2 **9** 33.75 m^3

Exercise 3E

1 sf = 64 **2** sf = 27 **3** sf = 27

4 a 4 : 1 **b** 8 : 1

5 a 4 : 1 **b** 16 : 1 **c** 64 : 1

6 1800 m^2 **7** 87.96 cm^2

8 32 768 cm^3 **9** 5.625 m^3

10 a 148.53 cm^2 **b** 3240 cm^3

11 a 226.19 cm^2 **b** 254.47 cm^3

12 height 12 cm, base 16 cm **13** 24 000 L

14 a 6 cm **b** 27 : 64

15 $1578.24

Exercise 3F

1 7.5 m **2** 345 cm **3** 24 hectares

4 a 29.7 m^2 **b** 0.39 cm^2

5 5 times **6** 7 times **7** $64 524

8 r = 4.76 cm, h = 7.94 cm **9** 22.95 mL

10 Diagram tree = 4.8 m

Chapter 3 review

Multiple-choice questions

1 A **2** D **3** D **4** C
5 C **6** D **7** A **8** C
9 C **10** C

Short-response questions

1 sf = 3 **2** AAA

3 a 8 **b** 4

4 15 cm by 36 cm **5** 201.06 cm^2 **6** 9 cm^2

7 a 2.8 km **b** 26 cm

8 a 1:41 666 666.7 **b** 3500 km

9 13.33 metres

10 a 1.96 **b** 2.744 **c** 62.5 cm^3

11 5 m **12** 4 L **13** 2048 cm^3

14 18.71 cm

Chapter 4

Exercise 4A

1 a $1400 **b** $1500 **c** $1425

2 a $10.50 **b** $14.40 **c** $30

3 a 10°C **b** −17.8°C
c 100°C **d** 33.3°C

4 a $P = 14$ **b** $P = 46$ **c** $P = 23$

5 a $A = 4$ **b** $A = 14.25$ **c** $A = 16.74$

6 380 km

7 a i C = 157.08 cm ii A = 1963.50 cm^2
b i C = 18.85 mm ii A = 28.27 mm^2
c i C = 33.93 cm ii A = 91.61 cm^2
d i C = 45.24 m ii A = 162.86 m^2

8 6734 m/s

9 a i V = 7420.70 cm^3
ii A = 1839.84 cm^2
b i V = 8181.23 mm^3
ii A = 1963.50 mm^2
c i V = 10.31 m^3
ii A = 22.90 m^2
d i V = 1047.39 cm^3 ii A = 498.76 cm^2

10 1.6 amperes

11 71.4 km/h

12 a $\frac{28}{11}$ **b** $\frac{15}{4}$

13 a $F = 1.84$ **b** $F = 0.51$

14 a $2400 **b** $180 **c** $2014.50

15 a i 15 points ii 37 points iii 68 points
b Greenteam

16 Gold 23, Silver 22

17 Gold 30, Silver 27

18 a 13 **b** 23 **c** 101

19 a i 1 h 50 min ii 2 h 56 min
iii 3 h 6 min iv 2 h 38 min
b 5:15 p.m.

20 a 100.53 cm^2 **b** 123.76 cm^2
c 268.08 cm^3 **d** 16.76 cm^3

Exercise 4B

1 a $t = \frac{C - 1200}{50}$ **b** 5 hours

2 a $K = \frac{F - 4}{1.3}$ **b** 22 km

3 6 cm

4 13 cm

5 15.2 m

6 a $R = \frac{240}{I}$ **b** 160 ohms

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7 a $t = \frac{d}{v}$

b 2 h 45 min

8 a $W = \frac{P - 2L}{2}$

b 26 m

9 360° F

10 2 h 36 min

11 a 5 years

b \$5400

c 3%

12 9 tries

Exercise 4C

1

x	40	41	42	43	44	45
C (\$)	86	88.15	90.3	92.45	94.6	96.75

x	46	47	48	49	50
C (\$)	98.9	101.05	103.2	105.35	107.5

2

r	0	0.1	0.2	0.3	0.4	0.5
C	0	0.628	1.257	1.885	2.513	3.142

r	0.6	0.7	0.8	0.9	1.0
C	3.770	4.398	5.027	5.655	6.283

3

n	50	60	70	80	90	100
C (\$)	49	50.8	52.6	54.4	56.2	58

n	110	120	130
C (\$)	59.8	61.6	63.4

4

M (kg)	60	65	70	75	80	85	90
E (kJ)	650	695	740	785	830	875	920

M (kg)	95	100	105	110	115	120
E (kJ)	965	1010	1055	1100	1145	1190

5

n	3	4	5	6
S	180°	360°	540°	720°

n	7	8	9	10
S	900°	1080°	1260°	1440°

6 a $h = \frac{48}{b}$

b

b	2	4	8	12	16	24	48	96
h	24	12	6	4	3	2	1	0.5

7 a = 1.3 * A2 + 4

b \$43.00

8 a

n	0	1	2	3	4	5
E (\$)	680	740	800	860	920	980

n	6	7	8	9	10
E (\$)	1040	1100	1160	1220	1280

b 6 cars

9

L	10	20	40	80	100	320	640
W	320	160	80	40	32	10	5

10

P \ T	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

11

H \ Z	1	2	3	4
1	3	$\frac{3}{2}$	1	$\frac{3}{4}$
2	4	2	$\frac{4}{3}$	1
3	5	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$
4	6	3	2	$\frac{6}{4} = \frac{3}{2}$

12

T (years)	1	2	3	4	5
I (\$)	450	900	1350	1800	2250

T (years)	6	7	8	9	10
I (\$)	2700	3150	3600	4050	4500

13

t (years)	5	10	15	20	25
A (\$)	6535	8541	11 162	14 589	19 067

14 a

C \ p	0	2	4	6	8	10
0	0	0.4	0.8	1.2	1.6	2
2	0.1	0.5	0.9	1.3	1.7	2.1
4	0.2	0.6	1	1.4	1.8	2.2
6	0.3	0.7	1.1	1.5	1.9	2.3
8	0.4	0.8	1.2	1.6	2	2.4
10	0.5	0.9	1.3	1.7	2.1	2.5

(t)

b \$1.20

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15 a *See table * at bottom of page*

b From the table, $425.

c From the table, 50 km.

16 a

	I	T				
		1	2	3	4	5
R	3	150	300	450	600	750
	3.2	160	320	480	640	800
	3.4	170	340	510	680	850
	3.6	180	360	540	720	900
	3.8	190	380	570	760	950
	4	200	400	600	800	1000

b $760.00

17 8 years

18 $21 000

Chapter 4 review

Multiple-choice questions

1 C **2** B **3** D **4** B

5 A **6** A **7** D **8** B

9 A **10** B **11** D **12** D

Short-response questions

1 a $P = 40$ **b** $P = 130$

2 a $A = 30$ **b** $A = 54$

3 94.25 cm

4 424.12 cm^2

5 $r = \frac{S}{2\pi h}$, $r = 1.8$ cm

6 a $57 **b** 7 hours

7 a

n	60	70	80	90	100	110
C	55	60	65	70	75	80

n	120	130	140	150	160
C	85	90	95	100	105

b $105

8 a

P		c					
		0	1	2	3	4	5
a	0	0	42	84	126	168	210
	1	89	131	173	215	257	299
	2	178	220	262	304	346	388
	3	267	309	351	393	435	477
	4	356	398	440	482	524	566
	5	445	487	529	571	613	655

b $309

9 a

x	−20	−15	−10	−5	0
y	−716	−551	−386	−221	−56

x	5	10	15	20	25
y	109	274	439	604	769

b $x = 10$

c $x = -5$

10

k		g				
		−2	−1	0	1	2
h	−2	−6	−5	−4	−3	−2
	−1	−4	−3	−2	−1	0
	0	−2	−1	0	1	2
	1	0	1	2	3	4
	2	2	3	4	5	6

11 1

12 5

13 a $C = 45h + 80$ **b** $215

14 a

t	0	0.5	1	1.5	2	2.5	3	3.5	4
h	1	9.75	16	19.75	21	19.75	16	9.75	1

b 2 seconds

c 21 metres

d 1 second and 3 seconds

*

	C	t						
		0	20	40	60	80	100	120
	20	260	300	340	380	420	460	500
	25	262.5	302.5	342.5	382.5	422.5	462.5	502.5
	30	265	305	345	385	425	465	505
	35	267.5	307.5	347.5	387.5	427.5	467.5	507.5
d	40	270	310	350	390	430	470	510
	45	272.5	312.5	352.5	392.5	432.5	472.5	512.5
	50	275	315	355	395	435	475	515
	55	277.5	317.5	357.5	397.5	437.5	477.5	517.5
	60	280	320	360	400	440	480	520

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15 a $P = \dfrac{A}{(1+r)^t}$

b *See table * at bottom of page*

c Reading from the table, $11\,393.56.

d $863.68

16 a

		l				
		Surface area				
		8	9	10	11	12
h	2	391.55	493.64	607.70	733.75	871.79
	3	400.00	502.35	616.62	742.83	881.00
	4	410.51	513.37	628.06	754.62	893.07
	5	422.45	526.08	641.42	768.53	907.45
	6	435.38	540.00	656.20	784.07	923.65

b *See table ** at bottom of page*

c Reading from the table, surface area is 742.83 m^2 and volume is 1452 m^3.

d Extending the spreadsheet, one set of possible values is $l = 13$ and $h = 4$.

Chapter 5

Exercise 5A

1 a $x = 9$ **b** $y = 15$ **c** $t = 5$ **d** $m = 6$
e $g = 6$ **f** $f = 19$ **g** $f = -3$ **h** $v = -5$
i $x = -1$ **j** $g = 1$ **k** $b = 5$ **l** $m = -2$
m $y = 6$ **n** $e = 3$ **o** $h = -5$ **p** $a = -4$
q $k = 7$ **r** $n = 4$ **s** $a = 8$ **t** $b = 21$

2 a $x = 3$ **b** $g = 9$ **c** $n = 4$
d $x = -8$ **e** $j = -4$ **f** $m = 7$
g $f = 5.5$ **h** $x = 3.5$ **i** $y = 5$
j $s = -3$ **k** $b = -5$ **l** $d = -4.5$
m $r = 12$ **n** $q = 30$ **o** $x = 48$
p $t = -12$ **q** $h = 40$ **r** $m = 21$
s $a = 2$ **t** $f = -2$

3 a $a = 6$ **b** $y = 40$ **c** $r = 8$
d $x = 5$ **e** $m = 8$ **f** $x = 6.5$
g $a = 4$ **h** $c = 3$ **i** $y = 11.5$
j $f = 2.5$ **k** $h = 6.5$ **l** $k = -\dfrac{4}{3}$

4 a $y = 4$ **b** $x = 11$ **c** $g = 2$
d $x = 3$ **e** $x = 0.5$ **f** $m = 1.2$
g $a = 18$ **h** $r = 13$ **i** $b = -6$

5 a $x = 5$ **b** $a = 3$ **c** $b = 9$
d $y = 3$ **e** $x = 0$ **f** $c = -4$

6 a $x = 1.84$ **b** $y = 2.5$ **c** $x = \dfrac{1}{2}$
d $x = \dfrac{1}{4}$ **e** $d = -10.5$ **f** $y = 4$
g $y = 0.6$ **h** $x = -3.7$ **i** $x = 1$

Exercise 5B

1 a $P = 27 + x$ **b** $P = 4x$
c $P = 2a + 2b$ **d** $P = 19 + y$

2 a $n + 7 = 15$ **b** 8

3 6

4 59

5 a $P = 22 + m$ **b** 8 cm

*

	Table 1					
	15000	16000	17000	18000	19000	20000
1	13686.13	14598.54	15510.95	16423.36	17335.77	18248.18
2	12487.35	13319.84	14152.33	14984.82	15817.31	16649.79
3	11393.56	12153.13	12912.71	13672.28	14431.85	15191.42

**

		l				
		House volume				
		8	9	10	11	12
h	2	554.67	783.00	1066.67	1411.67	1824.00
	3	576.00	810.00	1100.00	1452.00	1872.00
	4	597.33	837.00	1133.333	1492.23	1920.00
	5	618.67	864.00	1166.67	1532.67	1968.00
	6	640.00	891.00	1200.00	1573.00	2016.00

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6 a $P = 4y$ **b** 13 cm

7 a $P = 4x + 12$

b 18 cm

c 24 cm, 18 cm

8 a $15c = 126$

b \$8.40

9 Anne \$750, Barry \$250

10 25 invitations

11 78 tickets

12 2

13 28 years old

14 6

15 30 km

16 a 47 min

b Ben 9.4 km, Amy 7.8 km

17 62, 64, 66

18 32

19 Ruby: 6.875 m/s, Mia: 6.25 m/s

20 4.125%

Exercise 5C

1 a

x	0	1	2	3	4
y	0	1	2	3	4

b

x	0	1	2	3	4
y	3	4	5	6	7

c

x	0	1	2	3	4
y	5	6	7	8	9

d

x	0	1	2	3	4
y	1	3	5	7	9

e

x	0	1	2	3	4
y	4	6	8	10	12

f

x	0	1	2	3	4
y	0	−1	−2	−3	−4

g

x	0	1	2	3	4
y	−8	−9	−10	−11	−12

h

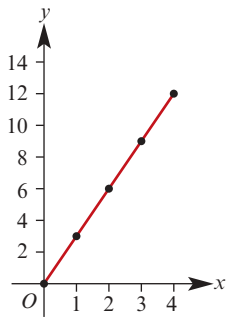
x	0	1	2	3	4
y	10	8	6	4	2

i

x	0	1	2	3	4
y	12	9	6	3	0

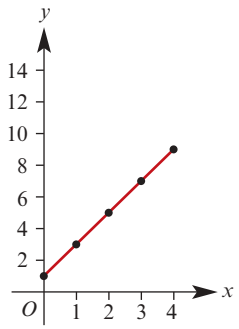
2 a

x	0	1	2	3	4
y	0	3	6	9	12



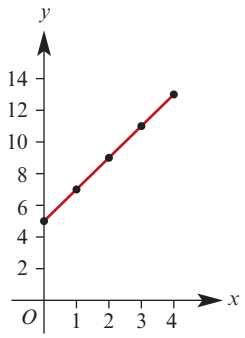
b

x	0	1	2	3	4
y	1	3	5	7	9



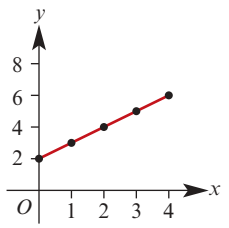
c

x	0	1	2	3	4
y	5	7	9	11	13



d

x	0	1	2	3	4
y	2	3	4	5	6



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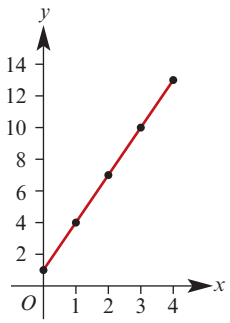
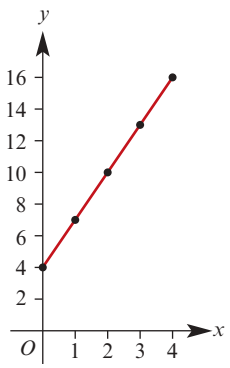
e

x	0	1	2	3	4
y	1	4	7	10	13



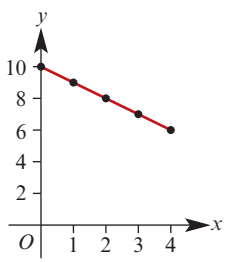
f

x	0	1	2	3	4
y	4	7	10	13	16



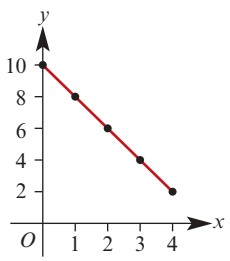
g

x	0	1	2	3	4
y	10	9	8	7	6



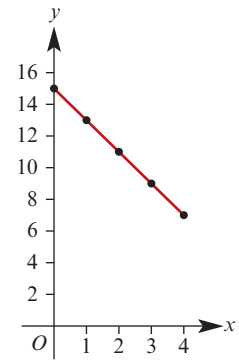
h

x	0	1	2	3	4
y	10	8	6	4	2



i

x	0	1	2	3	4
y	15	13	11	9	7



3 a (0, 4), (2, 6), (3, 7), (5, 9)
b (0, 8), (1, 6), (2, 4), (3, 2)

4 a 7 **b** 1 **c** $\frac{16}{3}$ **d** 4

Exercise 5D

1 A negative, B positive, C not defined, D zero

2 a $A\ \frac{-7}{3}$, $B\ \frac{7}{4}$, $C\ 1$
b $A\ 2$, $B\ -3$, $C\ 0$

3 a 2 **b** −1 **c** 2
d 0.6 **e** 2 **f** −1

4 a 0.75 **b** 0.5 **c** −0.92
d −0.67 **e** −0.63 **f** −0.38

5 3.8

Exercise 5E

1 a y-intercept = 5, slope = 2
b y-intercept = 6, slope = −3
c y-intercept = 15, slope = −5
d y-intercept = 10, slope = −3
e y-intercept = 0, slope = 3
f y-intercept = −5, slope = −2
g y-intercept = 4, slope = 1
h y-intercept = 3, slope = 0.5
i y-intercept = −5, slope = 2
j y-intercept = 10, slope = 5
k y-intercept = 10, slope = −1
l y-intercept = 6, slope = −3
m y-intercept = −4, slope = 2
n y-intercept = −3, slope = 0.8

2 a $y = 2 + 5x$ **b** $y = 5 + 10x$
c $y = -2 + 4x$ **d** $y = 12 - 3x$
e $y = -2 - 5x$ **f** $y = 1.8 - 0.4x$
g $y = 2.9 - 2x$ **h** $y = -1.5 - 0.5x$

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3 a x-intercept $= -2.5$ **b** x-intercept $= 2$

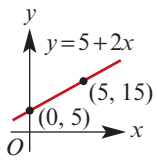
c x-intercept $= 3$ **d** x-intercept $= \frac{10}{3}$

e x-intercept $= 0$ **f** x-intercept $= -\frac{5}{2}$

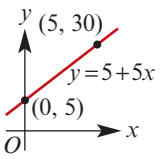
g x-intercept $= -4$ **h** x-intercept $= -6$

i x-intercept $= -\frac{5}{2}$ **j** x-intercept $= -2$

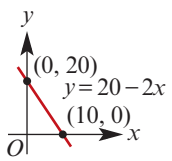
4 a



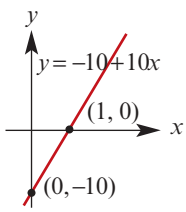
b



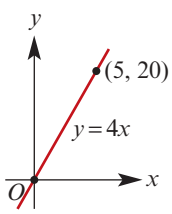
c



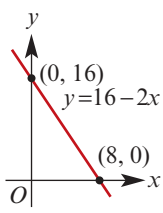
d



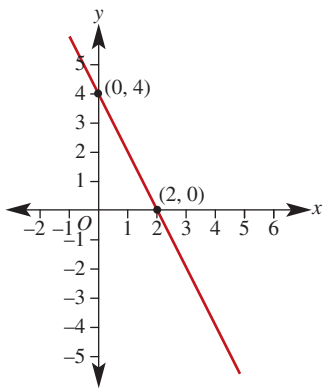
e



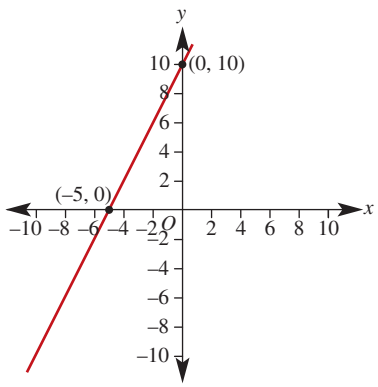
f



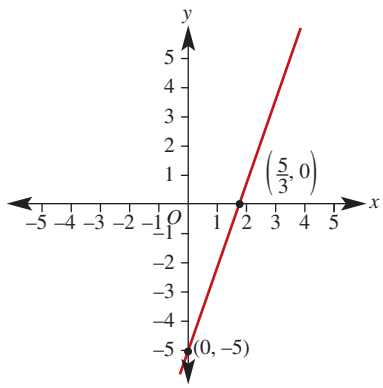
5 a



b



c



6 $y = x - 8$

7 $y = 2x + 8$

Exercise 5F

1 a slope $= 1$, y-intercept $= 1$

b slope $= 2$, y-intercept $= 2$

c slope $= -0.5$, y-intercept $= 5$

d slope $= 0.75$, y-intercept $= 0$

2 a A: $y = 10 - 2.25x$ B: $y = 2 + 1.75x$
C: $y = x$

b A: $y = 4 + 2x$ B: $y = 8 - 1.5x$
C: $y = 2 + 0.6x$

3 a A: $y = -4.5x + 14.5$ B: $y = 5x - 5$
C: $y = 3x - 5$

b A: $y = -1.5x + 11.5$ B: $y = 10x - 10$
C: $y = 1.2x + 2$

4 a $y = 0.375x + 1.75$

b $y = -\frac{2}{3}x + \frac{22}{3}$

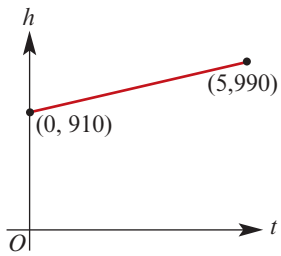
c $y = 4x - 7$

d $y = 0.5x + 1$

Exercise 5G

1 a $h = 16t + 910$ for $0 \leq t \leq 5$

b



c 982 cm

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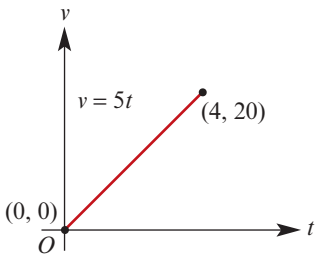
2 a $V = 5t \quad \text{for } 0 \leq t \leq 4$

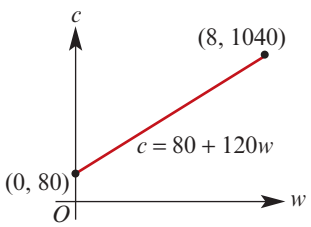
b



c 16 litres

3 a $c = 120w + 80, \quad \text{for } 0 \leq w \leq 8$

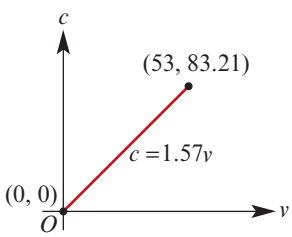
b



c $680

4 a $c = 1.57v, \quad \text{for } 0 \leq v \leq 53$

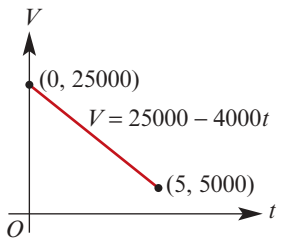
b



c $83.21

5 a $V = -4000t + 25\,000$, for $0 \leq t \leq 5$

b

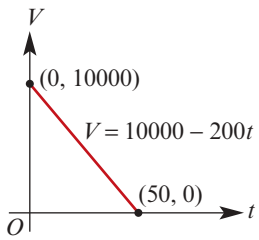


c $14 600

6 a 50 days

b $V = 200t + 10\,000, \quad \text{for } 0 \leq t \leq 50$

c

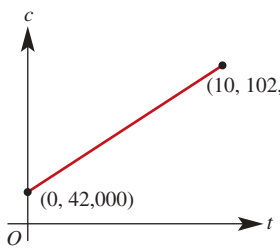


d 4000 litres

7 a $B = 6000t + 42\,000$

b $102 000

c



8 $7400

9 $1970

10 1600 litres

11 2 Days: Daily, 4 Days: Daily, 8 Days: Fortnightly, 10 Days: Fortnightly

12 20 years

13 5.65% and 15 years

Exercise 5H

1 a $10

b $17.50

c $C = 0.075n + 10$

d $32.50

e 7.5 cents

2 a 500 mL

b 400 mL

c 200 minutes

d $V = -2.5t + 500$

e 212.50 mL

f 2.5 mL/min

3 a $600

b $60 per person

c $C = 60g + 600$

d $3000

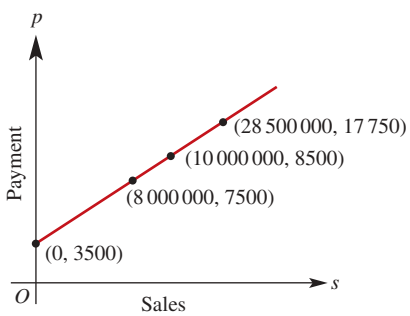
e 56 people

4 a $F = 1.8C + 32$

b **i** 122°F **ii** 302°F **iii** −40°F

c 1.8

5



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Chapter 5 review

Multiple-choice questions

1 B	**2** D	**3** B	**4** D
5 C	**6** D	**7** D	**8** A
9 B	**10** A	**11** C	**12** B
13 C	**14** D	**15** C	**16** B
17 D	**18** C	**19** B	**20** B
21 A	**22** C	**23** B	**24** C

Short-response questions

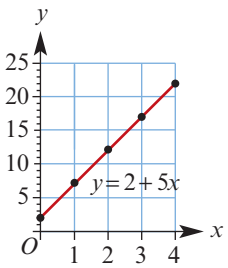
1 a $x = 10$ **b** $x = 11$ **c** $x = 8$
d $x = 6$ **e** $x = 1$ **f** $x = 7$
g $x = -6$ **h** $x = 11$ **i** $x = 3$
j $x = 3$ **k** $x = 15$ **l** $x = -24$

2 1

3 5

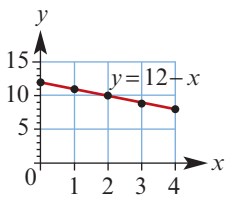
4 a

x	0	1	2	3	4
y	2	7	12	17	22



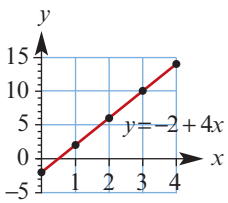
b

x	0	1	2	3	4
y	12	11	10	9	8



c

x	0	1	2	3	4
y	−2	2	6	10	14



5 A −1.2, B 0.6

6 A 2.25, B −2.67

7 a 8 **b** (0, −2)
c $y = 8x - 2$ **d** (0.25, 0)

8 a $755
b $110

9 3 m

10 Simon: 11, Kristen: 26

11 Ethan: 14, Tyson: 56

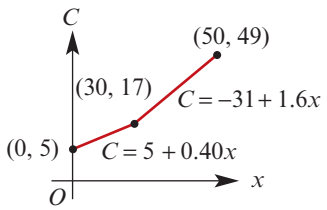
12 72

13 a $200 000
b After 60 months (5 years)
c $V = 300\,000 - 5000t$
d $120 000
e $5000

14 a $80 billion per year
b $A = 0.16N$ (with A in billions per year, N in thousands)
c $96 billion
d $240 billion
e $0.16 billion

15 Indonesian 28; French 42; Japanese 35.

16 a i $13
ii $17
iii $49
b i $0.40 (40 cents)
ii $1.60
c

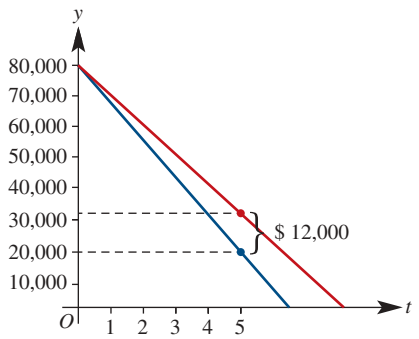


17 A: $7000, B: $6216

18 $3910

19 $93 600 or $86 400, Travel with us: $64 000, Fly with us: $130 000

20 $12 000



21 $x = 2$

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Chapter 6

Exercise 6A

6A Revision of Chapter 1: Consumer arithmetic

Multiple-choice questions

1 C **2** B **3** C **4** D
5 A **6** B **7** D **8** A
9 A **10** D

Short-response questions

1 a 34 hours **b** $900
2 $1149
3 $800
4 $13 714
5 a $5202 **b** $202
6 a $830 **b** $7.2%
7 a NZ **b** $159.74
8 a $45 143, $3761.92
b i $188.08 **ii** $2257
c 7 hours
9 a $11.73 **b** 32.47%
10 $683
11 a $8596.62
b i $8615.13
ii Better off to purchase immediately; after 3 years his investment is $18.51 short of the purchase price.
12 a Eastern: $1120.62, Centre: $1063.80, Western: $1176.00. He should go to Western.
b i $2250.71 USD **ii** $631.62 AUD
iii $63.75 AUD
13 a $425.00
b $434.88
c 5.12%
d 20 years
e 167 months
14 a $7320
b $7370
c i 4.5 cents/cup **ii** 4.4 cents/cup

6B Revision of Chapter 2: Shape and measurement

Multiple-choice questions

1 B **2** C **3** C **4** B
5 A **6** C **7** C **8** C
9 C **10** B

Short-response questions

1 12 cm
2 17.85 m
3 a 38.48 cm^2
b 49.7 cm^2
4 a 96 cm^3
b 66.67 cm^3
c 56.55 cm^3
5 32.57 cm^3
6 1520.53 cm^2
7 402.12 cm^2
8 54.32 m^2
9 4.33 cm
10 3 units

6C Revision of Chapter 3: Similarity and scale

Multiple-choice questions

1 C **2** B **3** C **4** A
5 D **6** B **7** B **8** A
9 B **10** C

Short-response questions

1 2.5
2 a 1.5 cm **b** 1.33 cm
3 a 1:2.5 million **b** 1500 km
4 7 m^3
5 6.75 cm
6 1.875 units
7 1.88 cm
8 52.5 mm, 74.3 mm
9 5.0
10 800 cm^3

6D Revision of Chapter 4 Algebra: Linear and non-linear relationships

Multiple-choice questions

1 C **2** B **3** D **4** D
5 B **6** D **7** C **8** A
9 D **10** A

Short-response questions

1 a 36 **b** 18
2 452.39 mm^2
3 a $R = \dfrac{A - P}{PT}$
b $r = \dfrac{C}{2\pi}$
4 a $24.70 **b** 14 km

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5 a

n	1	2	3	4	5	6	7	8
C	17	29	41	53	65	77	89	101

b \$82

6 a

x	−10	−5	0	5	10
y	−334	−174	−14	146	306

b $x = 16$

c $x = -3$

7 a $C = 4.95x + 0.50y$

b \$15.85

c 3 packets of tomato sauce

8 a $C = 40t + 90$ **b** 3.5 hours

9 98%

10 7 × 50 cent coins and 6 × \$1 coins

6E Revision of Chapter 5: Linear equations and their graphs

Multiple-choice questions

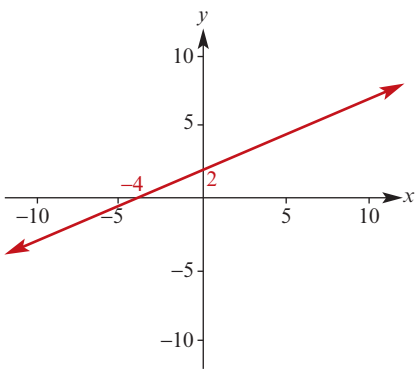
1 C **2** C **3** B **4** A
5 D **6** A **7** D **8** B
9 D **10** D

Short-response questions

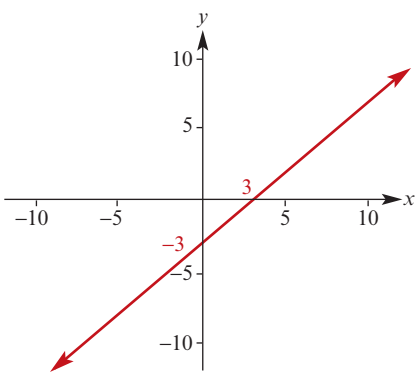
1 a $y = 7x - 3$ **b** $y = x + 3$

c $y = 2x + 1$

2 a



b



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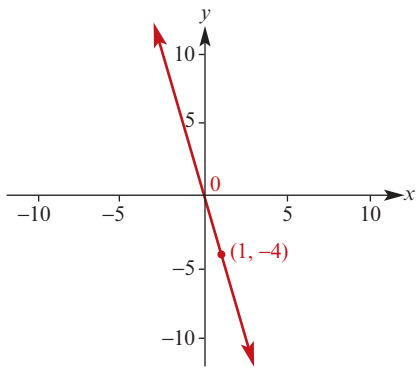
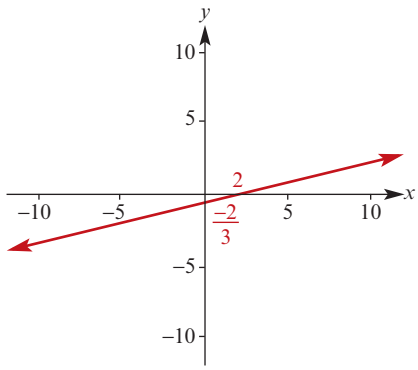


d



3 a y-intercept: 1, slope: 4

b y-intercept: $-\frac{1}{2}$, slope: $\frac{7}{4}$

c y-intercept: 12, slope: −3

d y-intercept: 3, slope: −2

4 a $3x + 4 = 55$ **b** 17

5 a $2(w + 5) + 2w = 38$ or $4w + 10 = 38$

b 84 m^2

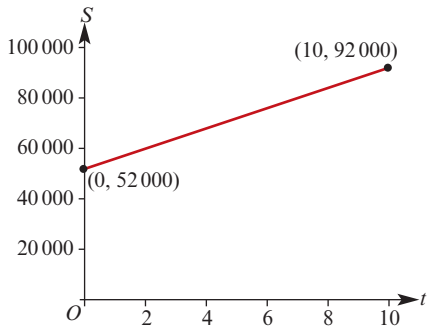
6 a $2j + 8 + j = 104$ or $3j + 8 = 104$

b Lisa has 72 marbles, James has 32 marbles

7 $\frac{9}{5}$ = 1.8 degrees Fahrenheit

8 a $S = 4000t + 52\,000$

b



9 2.5 years

10 18 deliveries

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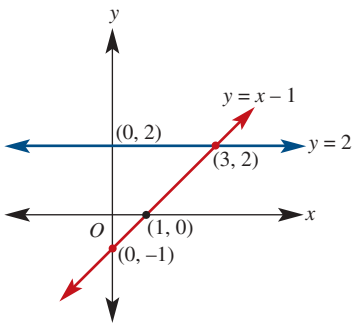
Chapter 7

Exercise 7A

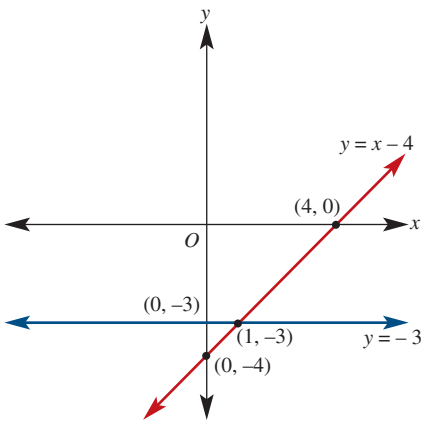
1 **a** $(-1, -1)$ **b** $(3, -2)$
c $(1, 1)$ **d** $(1, -1)$

2 **a** $(2, -4)$ **b** $(-3, 2)$ **c** $(1.5, 2.5)$
d $(2, 1)$ **e** $(0, 6)$ **f** $(7, 2)$
g $(0, 3)$ **h** $(1, 5)$ **i** $(0.4, -2.6)$
j $(7, 25)$ **k** $(-8, -20)$ **l** $(-3, 10)$

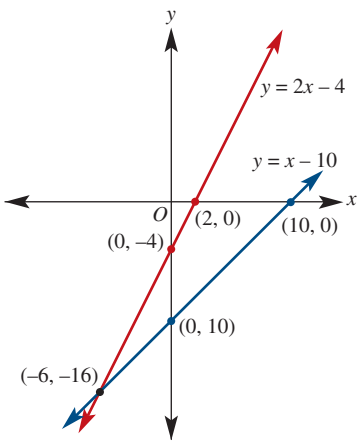
3 **a**



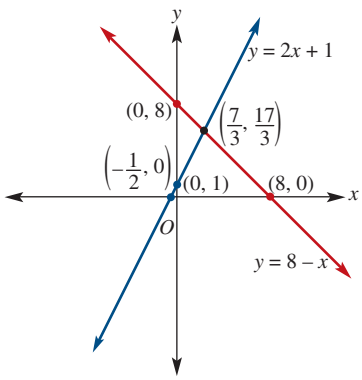
b



c



d



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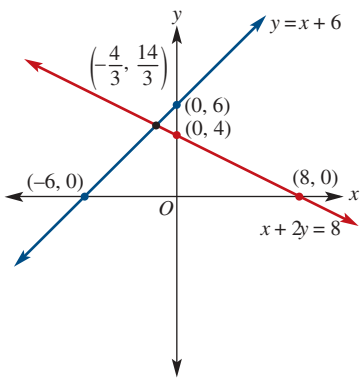

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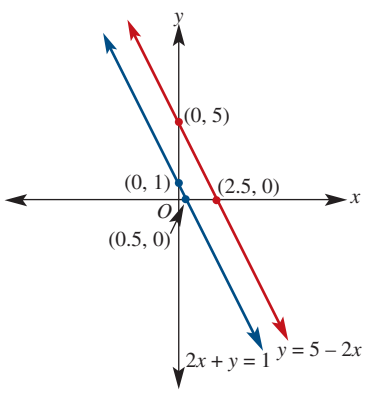

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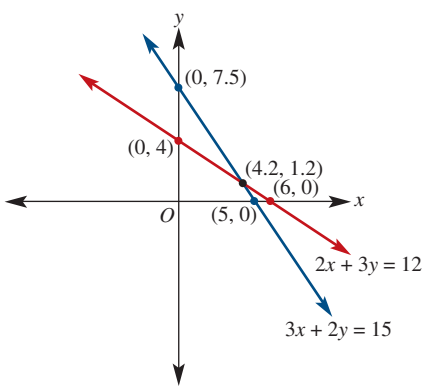

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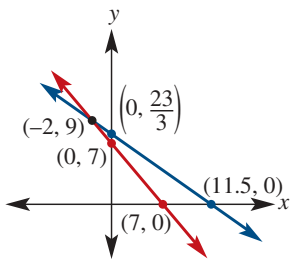
h



Exercise 7B

1 a $x = 2, y = 1$ **b** $x = 2, y = 5$
c $x = 4.5, y = 3.5$ **d** $x = 9, y = 1$
e $x = 4, y = -3$ **f** $x = -\frac{3}{7}, y = -\frac{13}{7}$

2 a $x = 1, y = 5$
b $x = -1, y = 11$
c $x = -\frac{7}{3}, y = \frac{34}{3}$
d $x = -1.8, y = 3$
e $x = -0.125, y = 7.25$
f $x = -8, y = 4$

3 a $x = 5, y = -7$
b $x = -2, y = 3$
c $x = 1.75, y = -1.25$
d $x = 2, y = 0.6$
e $x = 2.2, y = 0.2$
f $x = -7, y = 0$

4 a $x = 5.25, y = -2.5$
b $x = -12, y = -9$
c $x = -1.7, y = 2.2$
d $x = \frac{3}{11}, y = \frac{18}{11}$
e $x = 3, y = -1$
f $x = \frac{1}{2}, y = \frac{5}{4}$

5 a Single solution as different gradients.
b No solution as same gradient and different y-intercepts.
c No solution as same gradient and different y-intercepts.

6 a $x = -\frac{7}{2}, y = -\frac{5}{2}, z = -\frac{1}{2}$
b $x = 17,\ y = -3,\ z - 6$

Exercise 7C

1 a $x = \frac{1}{2}, y = -\frac{3}{4}$ **b** $x = 6, y = -\frac{1}{3}$
c $x = 12, y = 4$ **d** $x = 9, y = 1$
e $x = 4, y = 3$ **f** $x = 10, y = 2$

2 a $x = 8, y = 2$
b $x = 5, y = 6$
c $x = 4, y = 2$
d $x = 11, y = 1$
e $x = 7, y = 2$
f $x = 3, y = 4$

3 a $x = 2, y = 12$
b $x = 9, y = 2$
c $x = 1, y = 14$
d $x = -1, y = 8$
e $x = 3, y = -5$
f $x = 1.5, y = 1$

4 a $x = 5, y = 2$
b $x = 9, y = -14$
c $x = 3, y = -2$
d $x = 7, y = 3$
e $x = 1, y = -3$
f $x = 2, y = 1$

5 a $x = 4, y = 6$
b $x = -2, y = 3$
c $x = -2, y = 16$
d $x = 4, y = 11$
e $x = -3, y = 0$
f $x = -2, y = 7$

6 a $x = 5, y = 2, z = 3$
b $x = 3, y = -1, z = -2$

Exercise 7D

1 a $5t + 6p = 12.75$ and $7t + p = 12.30$
b Marker pen \$1.65, pencil \$0.75

2 a $50p + 5m = 109$ and $75p + 5m = 146$
b Petrol \$1.48/L, motor oil \$7/L

3 a $6a + 10b = 7.10$ and $6a + 8b = 6.40$
b Orange 60c, banana 35c

4 Nails 1.5 kg, screws 1 kg

5 12 emus, 16 wombats

6 7541 L unleaded, 2459 L diesel

7 6 cm, 12 cm

8 Chocolate thickshake \$5, fruit smoothie \$3

9 22, 30

10 8, 27

11 Bruce 37, Michelle 33

12 Boy is 9, sister is 3

13 Mother is 44, son is 12

14 77 students

15 10 standard, 40 deluxe

16 Width 24 m, length 36 m

17 28, 42, 35

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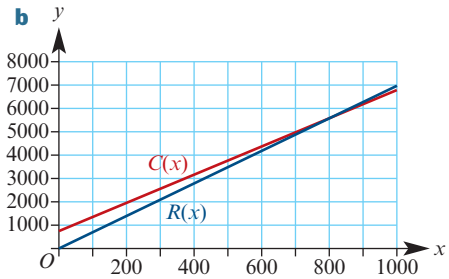
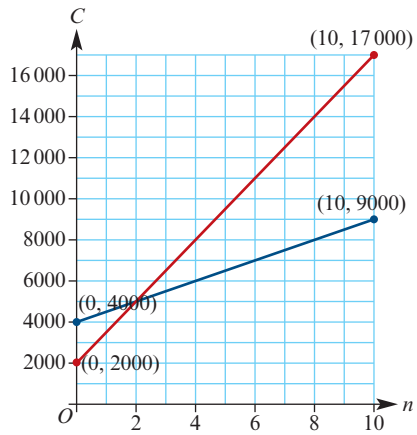
18 a 900 units ($64 800)
b $45 000
c $22 500
d 2700 units have to be sold for a profit of $90 000.

19 a 800 units
b



c 1200 units.

20 a $C = 2000 + 1500n$
b See diagram in part **d**
c $C = 4000 + 500n$
d



e 2 days

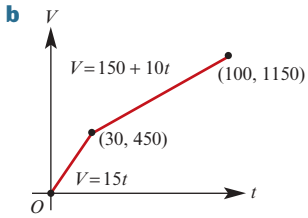
21 a Rabbitphone company $24, Foxphone company $20
b 14 minutes

22 a $V = -200t + 4800$
b $V = 200t + 600$
c 10.5 minutes

Exercise 7E

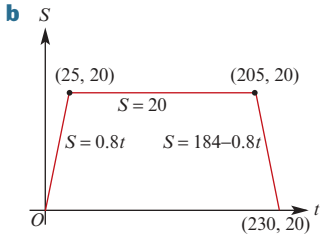
1 a **i** 300 L
ii 450 L
iii 750 L
iv 1150 L

b



2 a **i** 8 m/s **ii** 20 m/s **iii** 20 m/s
iv 16 m/s

b



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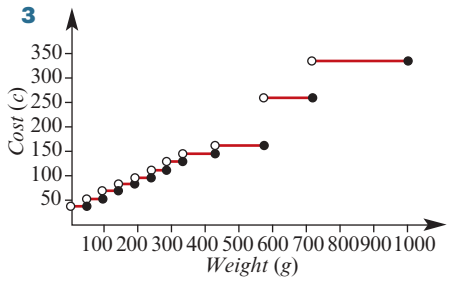

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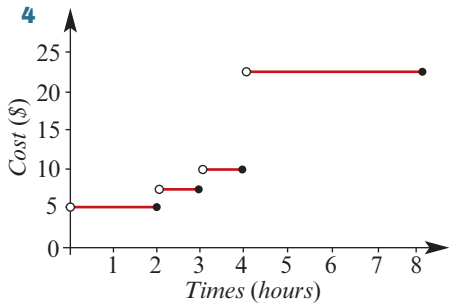

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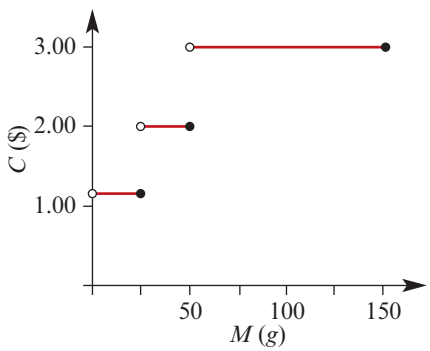

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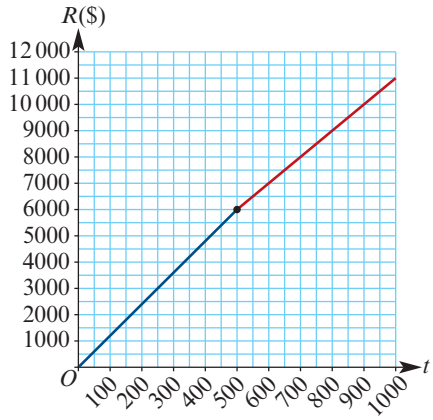
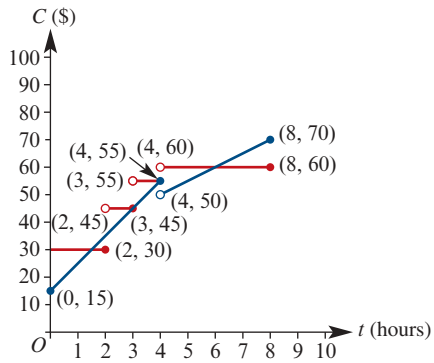
6 a **i** $4800 **ii** $9000

b



7 a **i** Boat **ii** Same price
iii Boat **iv** Bike

b



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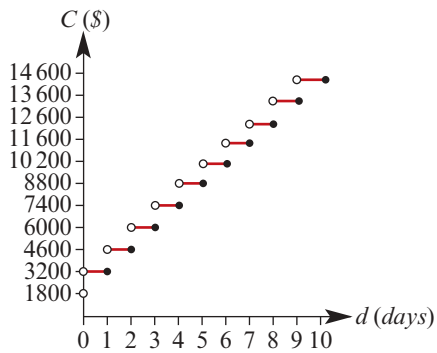


Exercise 7F

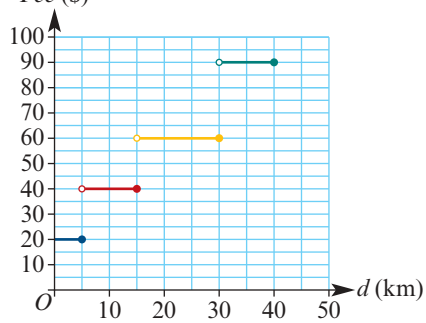
1 a $840
b $2840
c 4.5 days

2 a 7500 L
b 3000 L
c 16 minutes

3 a **i** 5:00 **ii** 12:00
iii 9:00 **iv** 6:00
b 1 hour
c 10:30

4 a Free
b $3.00
c $5.00

5 a $60
b 5 km
c *Fee* ($)



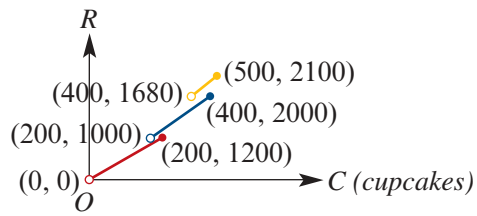
6 a $150 **b** $300

7 a $170
b 30 days
c 12 × 20 = $240
d

Days	Cost
$0 \leq d \leq 10$	$170
$10 < d \leq 20$	$240
$20 < d \leq 30$	$300

8 a After day 7 and day 20
b 25
c 90
d 16 days

9 a $1250
b 320 cupcakes
c $4.20
d $R = \begin{cases} 6n, 0 \leq n \leq 200 \\ 5n, 200 \leq n \leq 400 \\ 4.2n, 400 < n \end{cases}$
e



Chapter 7 review

Multiple-choice questions

1 D	**2** C	**3** D	**4** B
5 D	**6** B	**7** D	**8** C
9 D	**10** C	**11** A	**12** D
13 C	**14** D	**15** B	**16** D
17 A	**18** C	**19** C	**20** C

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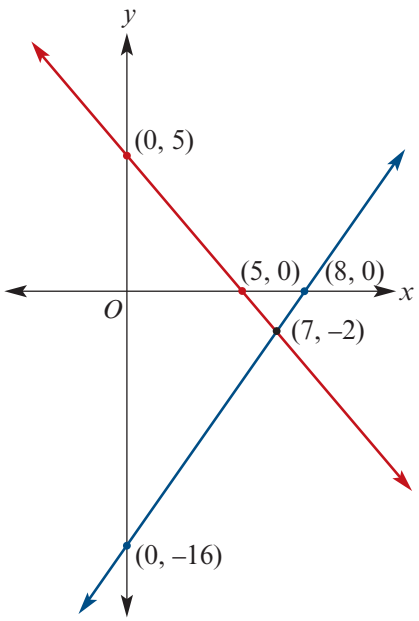
Short-response questions

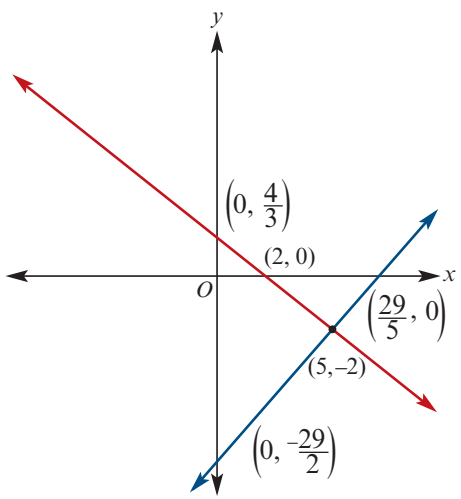
1 a $x = 2, y = 8$ **b** $x = 3, y = 2.5$
c $p = 9, q = -6$ **d** $p = 5, q = 2$
e $p = 4, q = -2$

2 a $(1, 3)$ **b** $(4, 1)$ **c** $(5, 1)$

3 a $(7, -2)$
b and **c**



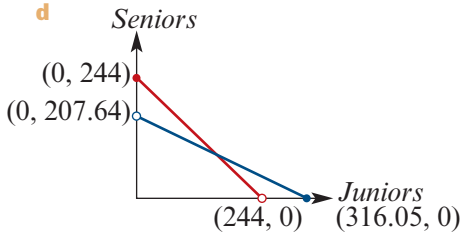
4 a $(5, -2)$
b and **c**



5 a $7a + 2b = 89$ and $3a + 5b = 48.50$
b Stuffed toy \$12, Squishy ball \$2.50
c \$55.50

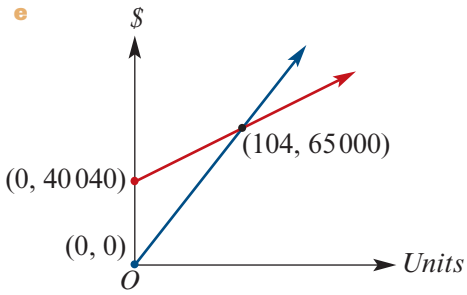
6 a $3a + 5c = 73.50$ and $2a + 3c = 46.50$
b \$7.50
c \$12

7 a $j + s = 244$
b $452j + 688s = 142\,856$
c Junior players: 106, Senior players: 138
d

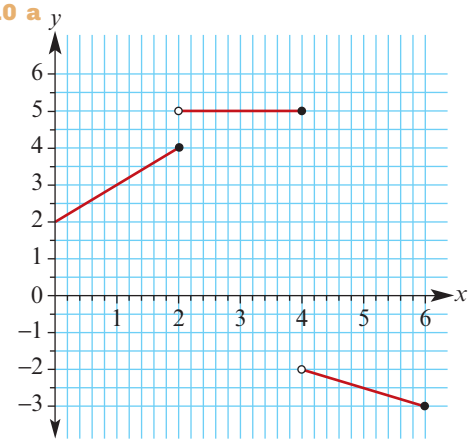


8 a \$100 **b** \$0.75 **c** \$0.25
d Loss of \$75 **e** Profit of \$50
f 200 units

9 a 104 units
b \$152 460
c Loss of \$1540
d 320 units
e



10 a



b $y = -\frac{5}{2}$
c $x = 1$

11 a \$5.00
b \$36.00

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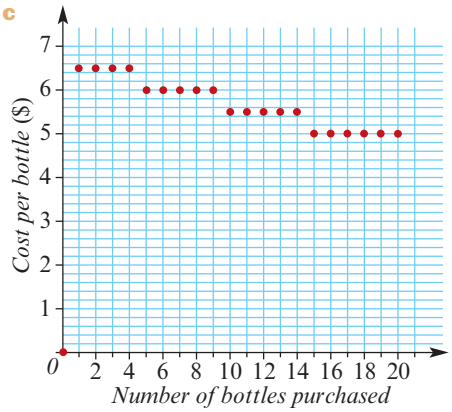
c



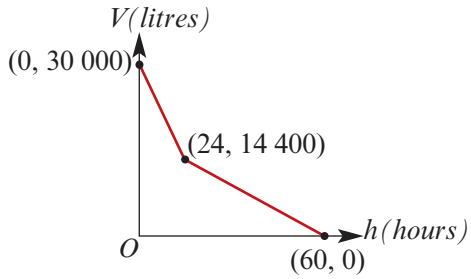
12 a 22 200 litres

b 12 000 litres

c 23 hours

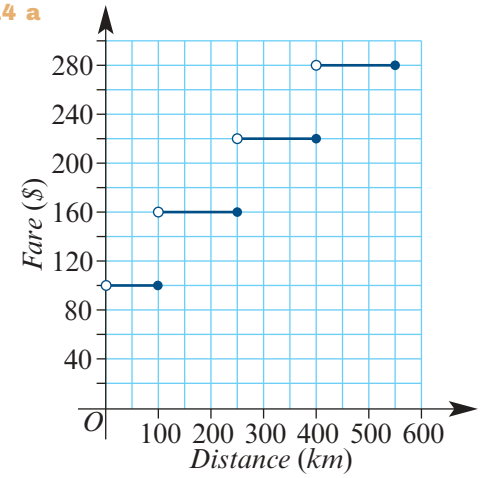
d 60 hours

e



13 Needs to visit 7 times

14 a



b 360 km

c $m = \frac{2}{5}$ and $c = 60$

15 8.33 m/s

Chapter 8

Exercise 8A

1 Answers are in order: hypotenuse, opposite, adjacent.

a 13, 5, 12 **b** 10, 6, 8

c 17, 8, 15 **d** 25, 24, 7

e 10, 8, 6 **f** 13, 12, 5

2 Answers are in order: sin θ, cos θ, tan θ.

a $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}$ **b** $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}$

c $\frac{8}{17}, \frac{15}{17}, \frac{8}{15}$ **d** $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$

e $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$ **f** $\frac{12}{13}, \frac{5}{13}, \frac{12}{5}$

3 a 0.4540 **b** 0.7314 **c** 1.8807

d 0.1908 **e** 0.2493 **f** 0.9877

g 0.9563 **h** 1.1106 **i** 0.9848

j 0.7638 **k** 5.7894 **l** 0.0750

Exercise 8B

1 a sin θ, 20.74 **b** cos θ, 20.76

c tan θ, 32.15 **d** cos θ, 8.24

e tan θ, 26.63 **f** sin θ, 7.55

g sin θ, 17.92 **h** tan θ, 15.59

i cos θ, 74.00 **j** tan θ, 17.44

k sin θ, 32.72 **l** sin θ, 37.28

2 a 78.05 **b** 25.67 **c** 8.58

d 54.99 **e** 21.32 **f** 11.59

g 30.67 **h** 25.38 **i** 63.00

j 62.13 **k** 4.41 **l** 15.59

3 a 12.8 **b** 28.3 **c** 38.5

d 79.4 **e** 16.2 **f** 15.0

g 14.8 **h** 37.7 **i** 59.6

Exercise 8C

1 a 28.8° **b** 51.1° **c** 40.9° **d** 30.0°

e 45.0° **f** 45.0° **g** 60.0° **h** 68.2°

i 33.0° **j** 73.0° **k** 17.0° **l** 30.0°

m 45.0° **n** 26.6° **o** 30.0° **p** 70.0°

2 a 32.2° **b** 59.3° **c** 28.3°

d 55.8° **e** 46.5° **f** 48.6°

g 53.1° **h** 58.8° **i** 22.6°

j 53.1° **k** 46.3° **l** 22.6°

m 32.2° **n** 41.2° **o** 48.2°

3 a 36.9° **b** 67.4° **c** 53.1°

d 67.4° **e** 28.1° **f** 43.6°

4 a 23° **b** 33° **c** 66°

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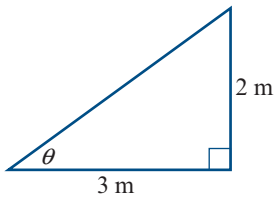
Exercise 8D

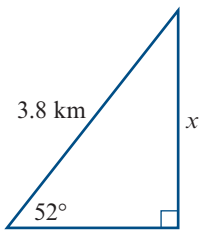
1 6.43 m **2** 21.0° **3** 10 m **4** 16 m

5 a



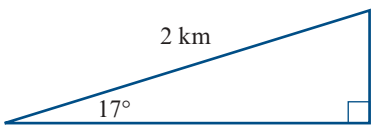
b 33.7°

6 a



b 3.0 km

7 a



b **i** Horizontal distance 1.91 km
ii Height 0.58 km

8 70.5° **9** 78.1 m **10** 5.77 m
11 11.31° **12** 124.86 m

Exercise 8E

1 413 m **2** 11 196 m
3 a 44.6 m **b** 36°
4 a 16.2 m **b** 62°
5 33 m
6 164.8 m
7 244 m
8 14°
9 a 35 m **b** 64 m **c** 29 m
10 507 m
11 a Reaches altitude 4 km at 4:54 am, so it is before 5 am, hence it will not disrupt the broadcast.
c 175 km/h

Exercise 8F

1 a 025° **b** 110° **c** 210° **d** 280°
2 a 025° T **b** 7.61 km
3 a 10 km, 15 km **b** 8.7 km
c 5 m **d** 10 km
e 319°, 13.2 km
4 a 228° T **b** 048° T
5 130°
6 a 4.2 km **b** 230°
7 a 12.9 km **b** 15.3 km **c** 17.1 km
d 42° **e** 138°, 23.0 km
8 20.5 km
9 133°

Exercise 8G

1 a 102 cm^2 **b** 40 cm^2 **c** 24 cm^2
d 52 cm^2 **e** 17.5 cm^2 **f** 6 cm^2
2 a 25.7 cm^2 **b** 65.0 cm^2
c 26.0 cm^2 **d** 32.9 cm^2
e 130.5 cm^2 **f** 10.8 cm^2
3 a 36.0 km^2 **b** 9.8 m^2
c 23.5 cm^2 **d** 165.5 km^2
e 25.5 cm^2 **f** 27.7 cm^2
4 a iv **b** iii **c** i **d** ii
5 a 10 cm^2 **b** 23.8 cm^2 **c** 63.5 cm^2
d 47.3 m^2 **e** 30 m^2 **f** 30.1 m^2
g 100.9 km^2 **h** 21.2 km^2 **i** 6 km^2
6 224 cm^2 **7** 1124.8 cm^2
8 150.4 km^2 **9** 3500 cm^2
10 a 6 m^2 **b** 4.9 m^2 **c** 6.9 m^2
11 a 33.83 km^2 **b** 19.97 km^2
c 53.80 km^2
12 a 43.30 cm^2 **b** 259.81 cm^2
13 a **i** 12 km^2 **ii** 39 km^2 **iii** 21 km^2
b 19.2°
14 8 m
15 Perimeter 174 mm, area 1261.5 mm^2

Exercise 8H

1 a $a = 15$, $b = 14$, $c = 13$
b $a = 19$, $b = 18$, $c = 21$
c $a = 31$, $b = 34$, $c = 48$
2 a $C = 50°$ **b** $A = 40°$ **c** $B = 105°$
3 a 5.9 **b** 12.1 **c** 45.1
d 86.8° **e** 44.4° **f** 23.9°
4 a 41.0° **b** 53.7° **c** 47.2° **d** 50.3°
5 a 19.6 **b** 30.7 **c** 55.4 **d** 67.7
6 a 4.5 **b** 16.1 **c** 67.9 **d** 67.8
7 39.1
8 43.2
9 49.7
10 a $c = 10.16$, $B = 50.2°$, $C = 21.8°$
b $b = 7.63$, $B = 20.4°$, $C = 39.7°$
c $a = 52.22$, $c = 61.01$, $C = 37°$
d $b = 34.65$, $c = 34.23$, $C = 54°$

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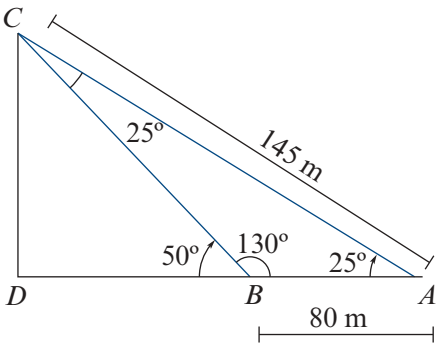
11 $A = 27.4°$, $C = 22.6°$, $c = 50.24$

12 $a = 154.54$, $b = 100.87$, $C = 20°$

13 2.66 km from A, 5.24 km from B

14 a



b 130°
c 25°
d 145 m
e 61.28 m

15 409.8 m

16 a 26.93 km from naval ship, 20.37 km from other ship
b 1.36 h (1 h 21 min)

17 a Airport A **b** 90.44 km
c Yes

Exercise 8I

1 a 36.72 **b** 47.62 **c** 12.00
d 14.55 **e** 29.95 **f** 11.39

2 17.41 **3** 27.09 **4** 51.51

5 a 33.6° **b** 88.0° **c** 110.7°
d 91.8° **e** 88.3° **f** 117.3°

6 50.5° **7** 63.2° **8** 40.9°

9 $B = 46.6°$ **10** $B = 73.2°$ **11** 33.6°

12 19.1 km **13 a** 39.6° **b** 310°

14 a 60° **b** 42.51 km

15 5.26 km **16** 11.73 km

17 4.63 km **18** 45.83 m

Exercise 8J

Answers below are approximate, based upon measured quantities. Student answers may vary.

a i 103.2 km **ii** 178.4 km

b No **c** 1 h 35 min **d** $354.92

e i 069° **ii** 226° **iii** 021°

f Fly 178.4 km on a bearing of 226°

g 23°

h 92.64 km (using 23°)

i 3526 square km

Chapter 8 review

Multiple-choice questions

1 D **2** B **3** B **4** C
5 A **6** C **7** C **8** D
9 B **10** C

Short-response questions

1 a 35.87 cm **b** 48.59° **c** 117.79 cm
d 21.45 **e** 50.48° **f** 55.30°

2 a 14.02 cm **b** 46.31° **c** 15.43
d 76.27° **e** 57.12° **f** 21.24 km

3 $\frac{65}{97}$

4 4°

5 54.17 km

6 a 50.95 m **b** 112.23 m **c** 36.47 m

7 a 110° **b** 81.26 km

8 a 50° **b** 3.68 km, 3.39 km
c 290°

9 760.7 cm^2

10 27.7 cm^2

11 40.7°

12 $426.21

13 42.43 km

Chapter 9

Exercise 9A

1 a 3×4 **b i** 16 **ii** 3 **iii** 5
c 22 **d** 18

2 a i 2×3 **ii** 6, 7
b i 1×3 **ii** 2, 6
c i 3×2 **ii** −4, 5
d i 3×1 **ii** 9, 8
e i 2×2 **ii** 15, 12
f i 3×4 **ii** 20, 5

3 a J & N, L & P, M & R, O & Q
b J & N
c L & P
d M & R

4 a 9 **b** 2 **c** 3 **d** 10 **e** 8

5 a $A : 4 \times 3$, $B : 2 \times 1$, $C : 1 \times 2$, $D : 2 \times 5$
b $a_{32} = 4$, $b_{21} = -5$, $c_{11} = 8$, $d_{24} = 7$

6 a 32 students **b** 3×4
c 22 Year 11 students preferred football.

7 a i 75 ha **ii** 300 ha **iii** 200 ha
b 350 ha

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c **i** Farm Y uses 0 ha for cattle.
ii Farm X uses 75 ha for sheep.
iii Farm X uses 150 ha for wheat.

d **i** f_{23} **ii** f_{12} **iii** f_{21}

e 2×3

Exercise 9B

1 a
$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix} \end{array}$$

b
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

c
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \end{array}$$

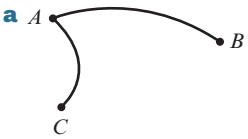
d
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \end{array}$$

e
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

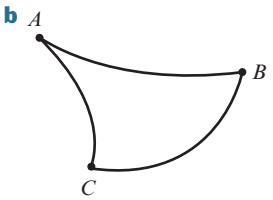
f
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix} \end{array}$$

2 Many answers are possible. Examples:

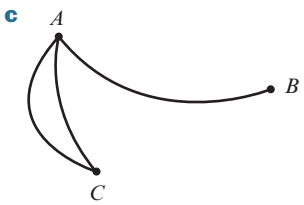
a



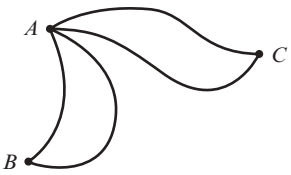
b



c



d



3 a The number of roads directly connected to town B.

b The number of roads directly connected to town A.

4 a
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

b Compare the sums of the rows (or columns). The person with the highest total has met the most people.

c Person B

d Person C

5 41 buses

Exercise 9C

1 a $\begin{bmatrix} 9 & 10 \\ 6 & 3 \end{bmatrix}$ **b** $\begin{bmatrix} 7 & 8 \\ 13 & 3 \end{bmatrix}$

c $\begin{bmatrix} 3 & 5 \\ 7 & 0 \end{bmatrix}$ **d** $\begin{bmatrix} 9 \\ 8 \end{bmatrix}$

e $\begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ **f** $\begin{bmatrix} 4 & -2 \\ 3 & 9 \end{bmatrix}$

g $\begin{bmatrix} 12 & 7 \end{bmatrix}$ **h** $\begin{bmatrix} 0 & 0 \end{bmatrix}$

i $\begin{bmatrix} 0 & 0 \end{bmatrix}$ **j** $\begin{bmatrix} -2 & 2 & 3 & -9 \end{bmatrix}$

2 a $\begin{bmatrix} 8 & 5 \\ 3 & 7 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 5 \\ 3 & 7 \end{bmatrix}$

c $\begin{bmatrix} -2 & -9 \\ 1 & 1 \end{bmatrix}$ **d** $\begin{bmatrix} 2 & 9 \\ -1 & -1 \end{bmatrix}$

e Not possible **f** $\begin{bmatrix} 3 & 7 \\ 5 & -2 \\ 4 & -1 \end{bmatrix}$

g Not possible **h** $\begin{bmatrix} -9 & 3 \\ 3 & -2 \\ -2 & 15 \end{bmatrix}$

3

	Liberal	*Labor*	*Democrat*	*Green*
Men	43	42	10	5
Women	37	37	17	9

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4 a

$$\begin{matrix} \\ \textit{Weight (kg)} \\ \textit{Height (cm)} \end{matrix}\begin{matrix} \begin{matrix} \textit{Aida} & \textit{Bianca} & \textit{Chloe} & \textit{Donna} \end{matrix} \\ \begin{bmatrix} 6 & 8 & -2 & 7 \\ 5 & 8 & 7 & 6 \end{bmatrix} \end{matrix}$$

b Bianca

c Bianca

5 $x = 39, y = 6, z = -3$

6 a $\begin{bmatrix} 22 & 68 \\ -11 & 29 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 4 \\ 1 & -2 \\ 9 & -9 \end{bmatrix}$ **c** $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Exercise 9D

1 a $\begin{bmatrix} 14 & -2 \\ 8 & 18 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & -10 \\ 25 & 35 \end{bmatrix}$

c $\begin{bmatrix} -64 & 12 \\ -6 & -14 \end{bmatrix}$ **d** $\begin{bmatrix} 2.25 & 0 \\ -3 & 7.5 \end{bmatrix}$

e $\begin{bmatrix} 18 & 21 \end{bmatrix}$ **f** $\begin{bmatrix} -12 \\ 30 \end{bmatrix}$

g $\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ **h** $\begin{bmatrix} -3 & -6 & 8 \end{bmatrix}$

2 a $\begin{bmatrix} 10 & 15 \end{bmatrix}$ **b** $\begin{bmatrix} 14 & 21 \end{bmatrix}$

c $\begin{bmatrix} 30 \\ 24 \end{bmatrix}$ **d** $\begin{bmatrix} 45 \\ 36 \end{bmatrix}$ **e** $\begin{bmatrix} 35 \\ 28 \end{bmatrix}$

3 a $\begin{bmatrix} 9 & -12 \\ 6 & 15 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 28 \\ -6 & -28 \end{bmatrix}$

c $\begin{bmatrix} 1 & -32 \\ 8 & 33 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

e $\begin{bmatrix} 21 & 18 \\ 3 & -12 \end{bmatrix}$

4 a $\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 5 & 3 & 3 \end{bmatrix}$ **c** $\begin{bmatrix} 6 \\ 14 \\ 8 \end{bmatrix}$

d $\begin{bmatrix} 0 & 15 & 9 & 9 \end{bmatrix}$

5 a $\begin{bmatrix} 23 & -3 \\ 68 & 104 \end{bmatrix}$ **b** $\begin{bmatrix} -121 & 50 \\ -84 & 103 \end{bmatrix}$

c $\begin{bmatrix} 30 & -5 \\ 80 & 115 \end{bmatrix}$ **d** $\begin{bmatrix} 69 & -27 \\ 60 & -30 \end{bmatrix}$

6 a $\begin{bmatrix} 18.19 & 54.99 & 15.62 \end{bmatrix}$

b $\begin{bmatrix} 82.70 & 249.95 & 71 \\ 119 & 266.25 & 109.90 \end{bmatrix}$

7 a

$$\begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \begin{matrix} \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \end{matrix} \\ \begin{bmatrix} 6 & 2 & 9 \\ 5 & 1 & 9 \\ 4 & -1 & 5 \end{bmatrix} \end{matrix}$$

b

$$\begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \begin{matrix} \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \end{matrix} \\ \begin{bmatrix} 1.8 & 0.6 & 2.7 \\ 1.5 & 0.3 & 2.7 \\ 1.2 & 0 & 1.5 \end{bmatrix} \end{matrix}$$

8 a

$$\begin{matrix} \\ \textit{Gymnastics rings} \\ \textit{Parallel bars} \end{matrix}\begin{matrix} \textit{Wins} \\ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \end{matrix}$$

b

$$\begin{matrix} \\ \textit{Gymnastics rings} \\ \textit{Parallel bars} \end{matrix}\begin{matrix} \$ \\ \begin{bmatrix} 150 \\ 100 \end{bmatrix} \end{matrix}$$

Exercise 9E

1 a Defined, 2×1 **b** Not defined

c Defined, 3×1 **d** Not defined

e Defined, 2×2 **f** Not defined

g Not defined **h** Defined, 3×2

2 a 1×2 and 2×1, [38]

b 1×2 and 3×1, not defined

c 1×3 and 3×1, [1]

d 1×3 and 2×1, not defined

e 1×4 and 4×1, [2]

f 1×4 and 3×1, not defined

3 a $\begin{bmatrix} 22 \\ 33 \end{bmatrix}$ **b** $\begin{bmatrix} 64 \\ 53 \end{bmatrix}$ **c** $\begin{bmatrix} 0 & -8 \\ 4 & 2 \end{bmatrix}$

d $\begin{bmatrix} -4 & -3 \\ -14 & -20 \end{bmatrix}$ **e** $\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}$

f $\begin{bmatrix} 16 & 14 \\ 16 & 14 \end{bmatrix}$ **g** $\begin{bmatrix} 31 \\ 35 \\ 21 \end{bmatrix}$ **h** $\begin{bmatrix} 11 \\ 1 \\ 7 \end{bmatrix}$

i $\begin{bmatrix} 83 \end{bmatrix}$ **j** $\begin{bmatrix} 21 \end{bmatrix}$ **k** $\begin{bmatrix} 8 \end{bmatrix}$ **l** $\begin{bmatrix} 4 \end{bmatrix}$

m $\begin{bmatrix} 30 \end{bmatrix}$ **n** $\begin{bmatrix} 36 \end{bmatrix}$ **o** $\begin{bmatrix} 3 & 3 \end{bmatrix}$

4 Use Excel to check the multiplication.

5 a $\begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$ **b** $\begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$ **c** No

6 a $\begin{bmatrix} 104 & 70 \\ 80 & 54 \end{bmatrix}$ **b** $\begin{bmatrix} 9 & 8 \\ 7 & 6 \end{bmatrix}$

c $\begin{bmatrix} 17 & 17 \\ 13 & 13 \end{bmatrix}$ **d** $\begin{bmatrix} 8 & 6 \\ 4 & 2 \end{bmatrix}$

e $\begin{bmatrix} 14 & 14 \\ 6 & 6 \end{bmatrix}$

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7 **a** No **b** No **c** Yes
d Yes **e** No **f** No

8 **a** $\begin{bmatrix} 7 & 3 \\ 5 & 6 \end{bmatrix}$ **b** $\begin{bmatrix} 12 & 9 \\ 12 & 9 \end{bmatrix}$ **c** $\begin{bmatrix} 4 & 8 \\ 11 & -3 \end{bmatrix}$

d $\begin{bmatrix} 11 & -3 \\ 4 & 8 \end{bmatrix}$ **e** $\begin{bmatrix} 3 & 1 & 4 \\ 6 & 9 & 2 \\ 8 & 0 & 7 \end{bmatrix}$

9 **a** **i** $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ **ii** $\begin{bmatrix} 37 & 54 \\ 81 & 118 \end{bmatrix}$

b **i** $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ **ii** $\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$

c **i** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **ii** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d **i** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **ii** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

e **i** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **ii** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

10 **a** $\begin{bmatrix} 376 & 118 & 154 & 420 \\ 643 & 117 & 281 & 523 \end{bmatrix}$ **b** $\begin{bmatrix} 1292 \end{bmatrix}$

c $\begin{bmatrix} -496 & 752 & 976 & -224 \\ -310 & 470 & 610 & -140 \\ -744 & 1128 & 1464 & -336 \\ 558 & -846 & -1098 & 252 \end{bmatrix}$

d $\begin{bmatrix} -131 & -264 & 176 \\ 467 & 62 & 535 \\ 697 & 279 & 406 \end{bmatrix}$

11 **a** **i** $\begin{bmatrix} 17 & 72 & 19 \\ 2 & 40 & 14 \\ 5 & 21 & 6 \end{bmatrix}$ **ii** $\begin{bmatrix} 87 & 589 & 180 \\ 22 & 282 & 96 \\ 26 & 173 & 53 \end{bmatrix}$

b $\begin{bmatrix} 528 & 4503 & 1445 \\ 184 & 2038 & 682 \\ 157 & 1326 & 425 \end{bmatrix}$

12 **a** $\begin{bmatrix} 1 & -4 & -11 \\ 8 & -3 & 3 \\ -10 & 18 & 68 \end{bmatrix}$ **b** $\begin{bmatrix} 4 & -23 & -93 \\ -5 & -2 & 13 \\ -66 & 146 & 572 \end{bmatrix}$

c $\begin{bmatrix} 79 & -190 & -771 \\ -46 & 31 & 107 \\ -546 & 1210 & 4788 \end{bmatrix}$

13 $x = 2, y = 3$ or $x = -2, y = -3$

14 **a** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$

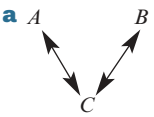
c When A is multiplied by A^0, the answer is A.

d $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **e** $B^0 \times B = B$

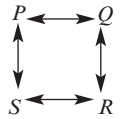
f Identity matrix

Exercise 9F

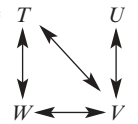
1 **a**



b



c



2 **a**

	A	B	C
A	0	1	0
B	1	0	1
C	0	1	0

b

	D	E	F	G
D	0	1	1	1
E	1	0	0	0
F	1	0	0	1
G	1	0	1	0

c

	J	K	L	M
J	0	1	1	0
K	1	0	1	1
L	1	1	0	0
M	0	1	0	0

3 **a**

$$Q = \begin{array}{c} \\ C \\ E \\ K \\ R \end{array} \begin{array}{c} \begin{array}{cccc} C & E & K & R \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

b Remy communicates with 3 people.

c

$$Q^2 = \begin{array}{c} \\ C \\ E \\ K \\ R \end{array} \begin{array}{c} \begin{array}{cccc} C & E & K & R \end{array} \\ \begin{bmatrix} 3 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 3 \end{bmatrix} \end{array}$$

d Add column E to get 6 ways.

e $E \to R \to C$
$E \to R \to E$
$E \to C \to E$
$E \to R \to K$
$E \to C \to K$
$E \to C \to R$

4 **a**

$$R = \begin{array}{c} \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{array}{cccc} E & F & G & H \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} E \\ F \\ G \\ H \end{array}$$

b Three roads directly connected to Fields.

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c
$$R^2 = \begin{matrix} \\ E \\ F \\ G \\ H \end{matrix} \begin{matrix} \begin{matrix} E & F & G & H \end{matrix} \\ \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \end{matrix} \begin{matrix} \\ E \\ F \\ G \\ H \end{matrix}$$

d 7

$F \rightarrow G \rightarrow E$

$F \rightarrow E \rightarrow F$

$F \rightarrow G \rightarrow F$

$F \rightarrow H \rightarrow F$

$F \rightarrow E \rightarrow G$

$F \rightarrow H \rightarrow G$

$F \rightarrow G \rightarrow H$

5 A communicates with D, yet D does not communicate with A.

6 a
$$C = \begin{matrix} \\ B \\ T \\ Z \end{matrix} \begin{matrix} \begin{matrix} B & T & Z \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} \\ B \\ T \\ Z \end{matrix}$$

b The communications are two-way: Bronwen communicates with Thomas (first column, second row), Thomas communicates with Bronwen (second column, first row) and so on. This gives symmetry.

c The sum of a row (or column) gives the total number of one-step connections for that person.

d $B \rightarrow T \rightarrow B$

$B \rightarrow Z \rightarrow B$

$T \rightarrow B \rightarrow T$

$T \rightarrow B \rightarrow Z$

$Z \rightarrow B \rightarrow T$

$Z \rightarrow B \rightarrow Z$

e The sum of all the numbers in matrix C^2 equals 6, which is the total number of two-step connections.

Exercise 9G

1 5800 kJ

2
$$\begin{matrix} \\ Smith \\ Jones \end{matrix} \begin{matrix} \begin{matrix} Wheels & Seats \end{matrix} \\ \begin{bmatrix} 14 & 13 \\ 12 & 9 \end{bmatrix} \end{matrix}$$

3 $\begin{bmatrix} 110 \end{bmatrix}$

4 a
$$\begin{bmatrix} Quiche & Soup & Coffee \\ 18 & 12 & 64 \end{bmatrix}$$

b
$$\begin{matrix} \\ Quiche \\ Soup \\ Coffee \end{matrix} \begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix} \end{matrix}$$

c \$378

5 a
$$\begin{bmatrix} Chips & Pasties & Pies & Sausage\ rolls \\ 90 & 84 & 112 & 73 \end{bmatrix}$$

b
$$\begin{matrix} \\ Chips \\ Pasties \\ Pies \\ Sausage\ rolls \end{matrix} \begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{bmatrix} 4 \\ 5 \\ 5 \\ 3 \end{bmatrix} \end{matrix}$$

c \$1559

6 a
$$\begin{matrix} \\ I \\ J \\ K \end{matrix} \begin{matrix} \begin{matrix} Hours \end{matrix} \\ \begin{bmatrix} 10 \\ 7 \\ 12 \end{bmatrix} \end{matrix}$$

b
$$\begin{matrix} \\ I \\ J \\ K \end{matrix} \begin{matrix} \begin{matrix} Av.\ Hours \end{matrix} \\ \begin{bmatrix} 2.5 \\ 1.75 \\ 3 \end{bmatrix} \end{matrix}$$

c
$$\begin{matrix} \\ Hours \end{matrix} \begin{matrix} \begin{matrix} M & Tu & W & Th \end{matrix} \\ \begin{bmatrix} 6 & 11 & 5 & 7 \end{bmatrix} \end{matrix}$$

d
$$\begin{matrix} \\ Av.\ Hours \end{matrix} \begin{matrix} \begin{matrix} M & Tu & W & Th \end{matrix} \\ \begin{bmatrix} 2 & 3.7 & 1.7 & 2.3 \end{bmatrix} \end{matrix}$$

7 a
$$\begin{matrix} \\ E \\ F \\ G \\ H \end{matrix} \begin{matrix} \begin{matrix} Total\ score \end{matrix} \\ \begin{bmatrix} 446 \\ 415 \\ 329 \\ 409 \end{bmatrix} \end{matrix}$$

b
$$\begin{matrix} \\ E \\ F \\ G \\ H \end{matrix} \begin{matrix} \begin{matrix} Av.\ score \end{matrix} \\ \begin{bmatrix} 89.2 \\ 83 \\ 65.8 \\ 81.8 \end{bmatrix} \end{matrix}$$

c
$$\begin{matrix} \\ Test\ total \end{matrix} \begin{matrix} \begin{matrix} T1 & T2 & T3 & T4 & T5 \end{matrix} \\ \begin{bmatrix} 319 & 307 & 324 & 292 & 357 \end{bmatrix} \end{matrix}$$

d
$$\begin{matrix} \\ Test\ av. \end{matrix} \begin{matrix} \begin{matrix} T1 & T2 & T3 & T4 & T5 \end{matrix} \\ \begin{bmatrix} 79.75 & 76.75 & 81 & 73 & 89.25 \end{bmatrix} \end{matrix}$$

8 a The mobile phone bills for each person in Quarter 3 only.

b $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

c 1×4

d 1×3

e Pre-multiply with a 1×3 matrix.

f [0 0 1]

This is reasonable since multiplying by this matrix gives the same values for Charlie as the original matrix.

9 a 1720 items sold **b** \$990

10 Multiplying by a scalar matrix has the same result as multiplying by a scalar.

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Chapter 9 review

Multiple-choice questions

1 B **2** C **3** C **4** D **5** A
6 C **7** A **8** A **9** C **10** A

Short-response questions

1 a i 2×3 **ii** 10, 7
b i 1×3 **ii** 16, 12
c i 2×2 **ii** 14, 13
d i 3×4 **ii** 9, 6

2 a M & Q, N & U, O & T, P & R
b P & R
c N & U
d M & Q

3 a $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

4 a $\begin{bmatrix} 9 & 3 \\ 12 & 6 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 4 \\ 5 & 3 \end{bmatrix}$
c $\begin{bmatrix} 3 & 9 \\ 14 & 8 \end{bmatrix}$ **d** $\begin{bmatrix} -3 & 7 \\ 6 & 4 \end{bmatrix}$
e $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **f** $\begin{bmatrix} 10 & 30 \\ 20 & 44 \end{bmatrix}$
g $\begin{bmatrix} 0 & 8 \\ 10 & 6 \end{bmatrix}$ **h** $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$
i $\begin{bmatrix} 13 & 5 \\ 20 & 8 \end{bmatrix}$ **j** $\begin{bmatrix} 80 & 48 \\ 60 & 116 \end{bmatrix}$
k $\begin{bmatrix} 93 & 53 \\ 80 & 124 \end{bmatrix}$ **l** $\begin{bmatrix} 135 & 99 \\ 154 & 190 \end{bmatrix}$

5
$$\begin{matrix} & P & Q & R \\ \end{matrix}$$
$$\begin{matrix} P \\ Q \\ R \end{matrix} \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$$

6 a $\begin{bmatrix} 8 & 10 \\ 1 & 5 \end{bmatrix}$ **b** Undefined
c Undefined **d** $\begin{bmatrix} 24 & 23 \\ -11 & 15 \end{bmatrix}$
e $\begin{bmatrix} 50 & 44 \\ -31 & 8 \\ 31 & 41 \end{bmatrix}$ **f** Undefined
g Undefined **h** $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

7 a $\begin{bmatrix} 423 & 54 & 166 & 347 \\ 717 & 186 & 363 & 564 \end{bmatrix}$

b $\begin{bmatrix} 74 & -62 & 154 \\ 350 & -388 & 1094 \\ 52 & -75 & 292 \end{bmatrix}$

8 $\begin{bmatrix} 94 & -93 \\ -31 & 32 \end{bmatrix}$

9 a i $\begin{bmatrix} 4 & 20 & 40 \\ 10 & 76 & 47 \\ 4 & 56 & 48 \end{bmatrix}$ **ii** $\begin{bmatrix} 28 & 320 & 320 \\ 96 & 796 & 560 \\ 64 & 640 & 476 \end{bmatrix}$

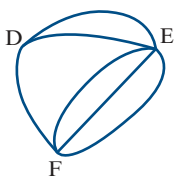
b $\begin{bmatrix} 376 & 3840 & 3020 \\ 988 & 8608 & 6228 \\ 768 & 7024 & 5096 \end{bmatrix}$

10 a
$$\begin{matrix} & \textit{Hours walking} & \textit{Hours jogging} \\ \end{matrix}$$
$$\begin{matrix} \textit{Patsy} \\ \textit{Geoff} \end{matrix} \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

b
$$\begin{matrix} & \$ & kJ \\ \end{matrix}$$
$$\begin{matrix} \textit{Walking} \\ \textit{Jogging} \end{matrix} \begin{bmatrix} 2 & 1500 \\ 3 & 2500 \end{bmatrix}$$

c
$$\begin{matrix} & \$ & kJ \\ \end{matrix}$$
$$\begin{matrix} \textit{Patsy} \\ \textit{Geoff} \end{matrix} \begin{bmatrix} 11 & 8500 \\ 12 & 9500 \end{bmatrix}$$

11 a 5 roads **b** 3 roads
c



12 a 21 pies **b** \$2 **c** $\begin{bmatrix} 104 \\ 103 \end{bmatrix}$
d Value of sales for each shop
e Shop *A* earned more since \$104 is greater than the \$103 that Shop *B* earned.

13 a
$$R^2 = \begin{matrix} & A & B & C & D \\ A \\ B \\ C \\ D \end{matrix} \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

b 3 ways $B \to A \to B$
$B \to C \to B$
$B \to D \to B$

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14

	Cattle	*Pigs*	*Sheep*
Farm A	180	0	120
Farm B	0	110	0

This is reasonable since checking the values algebraically compared to matrix arithmetic produces the same values.

15 $x = 1, y = 2$

Chapter 10

Exercise 10A

1 **a** Categorical **b** Numerical
c Categorical **d** Numerical
e Categorical **f** Categorical

2 **a** Nominal **b** Ordinal
c Ordinal **d** Nominal

3 **a** Discrete **b** Discrete
c Continuous **d** Continuous
e Discrete

4 **a** Nominal **b** Ordinal
c Discrete **d** Ordinal

5 **a** Discrete **b** Ordinal
c Ordinal **d** Nominal

6 **a** Continuous **b** Continuous
c Discrete **d** Ordinal

7 **a** Nominal **b** Discrete
c Discrete **d** Continuous

Exercise 10B

1 **a** Nominal

b

Gender	Frequency	
	Number	%
Female	5	33.3
Male	10	66.7
Total	15	100.0

2 **a** Ordinal

b

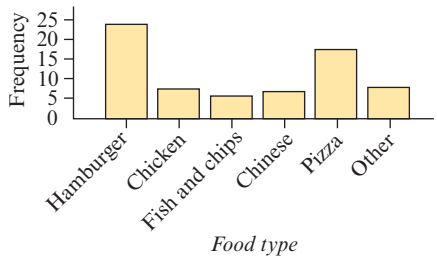
t-shirt size	Frequency	
	Number	%
S	2	10
M	8	40
L	6	30
XL	4	20
Total	20	100

3 **a** Ordinal

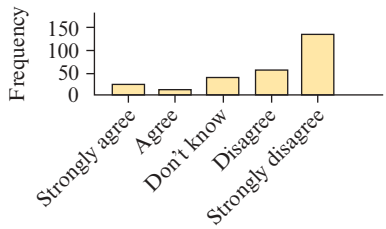
b

Thinking about quitting?	Frequency	
	Number	%
1	5	31.25
2	3	18.75
3	3	18.75
4	1	6.25
5	4	25.00
Total	16	100

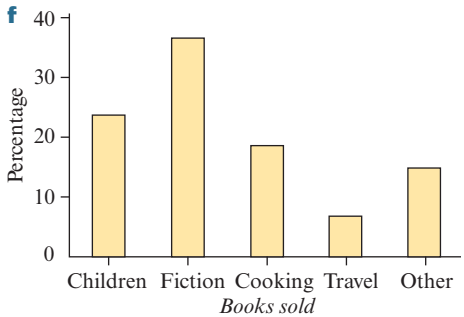
4 **a** 69; 8.7%, 26.1% **b** Nominal
c 7 students **d** 10.1%
e Hamburger
f



5 **a** 53; 16.4%, 20.7% **b** Ordinal
c 21 people **d** 50.4%
e Strongly disagree
f



6 **a** 38.4%, 6.5%, 100.0%
b Nominal
c 89 books
d 22.8%
e 232 books
f



7 **a** 200 students **b** Nominal
c 12% **d** Play video games

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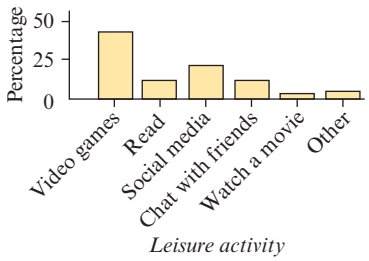
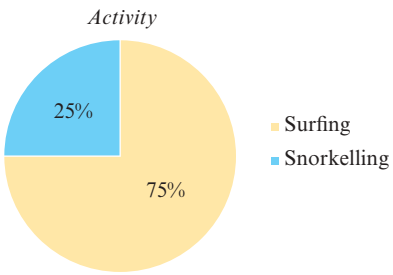
e

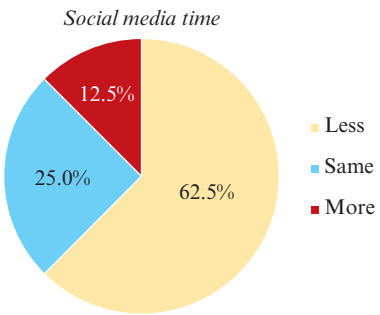


8 a



b Surfing

9 a



b Less

10 a *climate type*, ordinal **b** 77
c Mild

11 a *level of activity*, ordinal **b** 18
c 80 **d** moderate, 68.75%

12 a Ordinal **b** 80

Exercise 10C

1 69, hamburger, 26.1%

2 Strongly disagreed, 20.7%, 16.4%

3 A group of 200 students were asked how they prefer to spend their leisure time. The most popular response was using the internet and playing digital games (42%), followed by listening to music (23%), reading (13%), watching TV or going to a movie (12%) and chatting with phoning friends (4%). The remaining 6% said 'other'. For this group of students, spending time on the internet and digital games was clearly the most popular leisure time activity.

4 A group of 600 employees from a large company were asked about the importance to them of the salary that they earned in the job. The majority of employees said that it was important (55%) or very important (30%). Only a small number of employees said that it was somewhat important (10%), with even fewer saying that it was not at all important (5%). Salary was clearly important to almost all of the employees in this company.

5 A sample of 66 people were asked to respond to the question 'How much of the time have you felt happy in the last week?'. The response chosen most often was 'most of the time' chosen by 47.0% of the sample, followed by 'almost all of the time' chosen by 27.3%, and then 'some of the time' chosen by 22.7%. Very few people, only 3.0%, responded that they felt happy 'almost none of the time'.

Exercise 10D

1

Number of magazines	Frequency	
	Number	%
0	4	26.7
1	4	26.7
2	3	20.0
3	2	13.3
4	1	6.7
5	1	6.7
Total	15	100.0

2

Amount of money ($)	Frequency	
	Number	%
0.00–4.99	13	65
5.00–9.99	3	15
10.0–14.99	2	10
15.00–19.99	1	5
20.00–24.99	1	5
Total	20	100

3 a

Days homework	Frequency
1	7
2	5
3	3
4	3
5	2
Total	20

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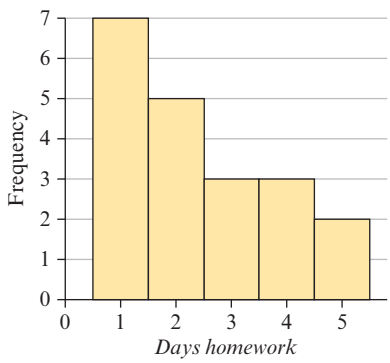
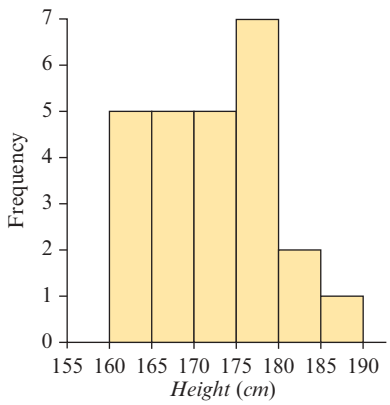
b



4 a

Height (cm)	Frequency
160–<165	5
165–<170	5
170–<175	5
175–<180	7
180–<185	2
185–<190	1
Total	25

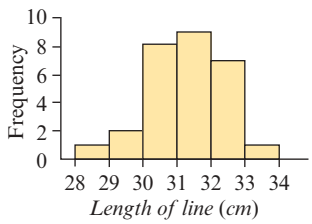
b



5 a **i** 2 students
ii 3 students
iii 8 students

b **i** 32.1%
ii 39.3%
iii 89.3%

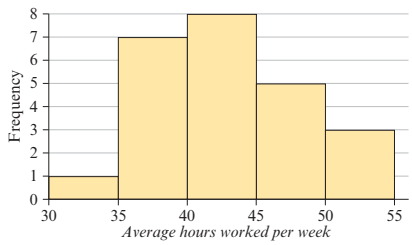
c



6 a

Av hours worked/week ($)	Number
30–<35	1
35–<40	7
40–<45	8
45–<50	5
50–<55	3
Total	24

b



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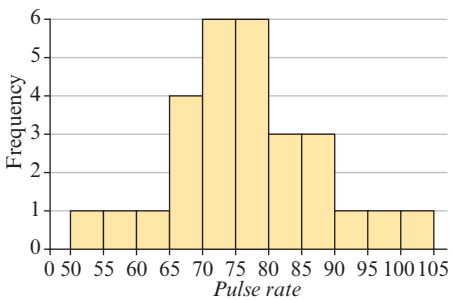

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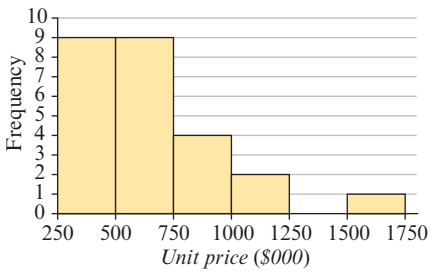
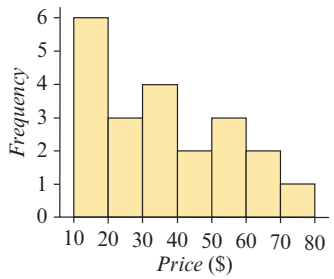


9 a 4 students **b** 2 children
c 5 students **d** 28 students

10 a 0 students **b** 36 students
c 60–69 marks **d** 28 students

11 a



b **i** $30 to <$40 **ii** 4 books
iii $10 to <$20

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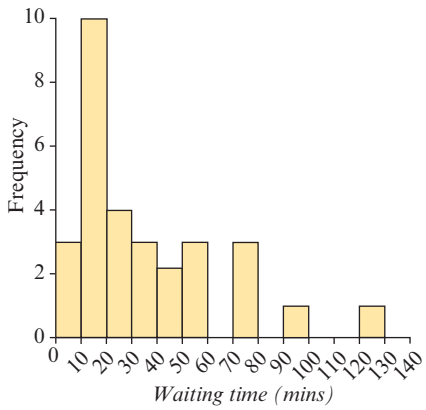
12 a

Waiting time (mins)	Frequency
0–<10	3
10–<20	10
20–<30	4
30–<40	3
40–<50	2
50–<60	3
60–<70	0
70–<80	3
80–<90	0
90–<100	1
100–<110	0
110–<120	0
120–<130	1
Total	30

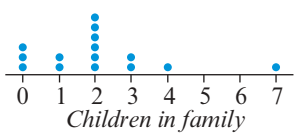
b



c 10–<20 minutes and 33.3%

Exercise 10E

1 a

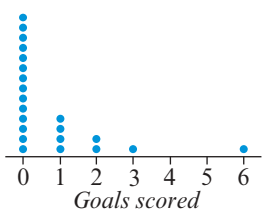


b 2 children

2 a

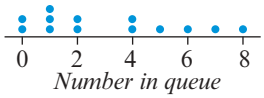
b 7 people

3 a



b 0 goals

4 a



b Around 12:25 p.m.

5 a *English marks*

1	7
2	3 3 6 8
3	2 5 5 8 9
4	3 4 6 6 9
5	0 2 8 9
6	1 4 5 6 9
7	5 8 9 9
8	3 3 4 9
9	2 3 4 7

5 | 0 represents 50 marks

b 21 students **c** 17 marks

6 a 40 people **b** 21 people

7 a *Battery time (hours)*

0	4
1	7 9
2	0 1 2 4 5 6 6 7 7 8
3	0 0 1 1 3 3 4 7
4	0 1 6

2 | 5 represents 25 hours

b 9 batteries

8 a

Homework time (minutes)

0	0
1	0 0 4 5 5 6 9
2	0 0 1 3 7 8 9
3	3 7 9
4	6
5	6
6	3
7	0

4 | 6 represents 46 minutes

b 2 students

9 *Price ($)*

2	5 8
3	5 6 9
4	5 6 9
5	2
6	8
7	5 5 6 8 9
8	2 4
9	5
10	
11	
12	
13	
14	9
15	

16 | 4 represents $164

10 *Height (cm)*

15	8
16	1 2 3 3 3
16	5 6 8 9 9
17	0 0 1 3 4
17	5 5 6 6 7 8 9
18	0 3

16 | 4 represents 164 cm

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11 *Wrist circumference (cm)*

16	5 6 7 8 9
17	0 1 2 3
17	5 6 7
18	0 2 4
18	5 9
19	3 3
19	9

16| 4 represents 16.4 cm

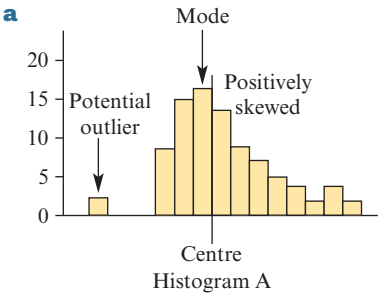
Exercise 10F

1 a Positively skewed
b Negatively skewed
c Approximately symmetric
d Approximately symmetric

2 a Negatively skewed
b Approximately symmetric
c Positively skewed

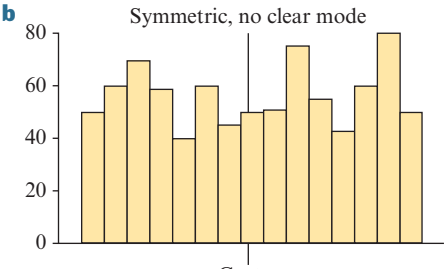
3 a Approximately symmetric
b Negatively skewed
c Positively skewed

4 a



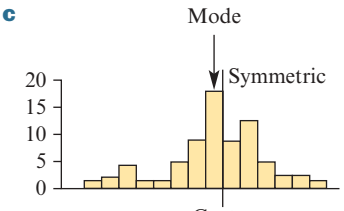
Histogram A

b



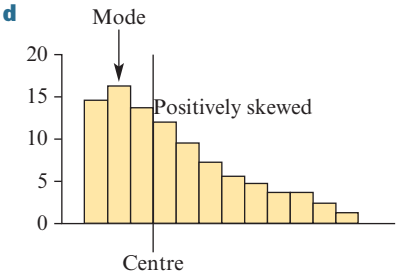
Histogram B

c



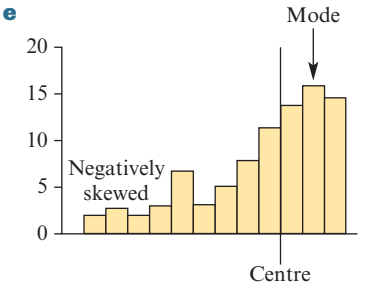
Histogram C

d



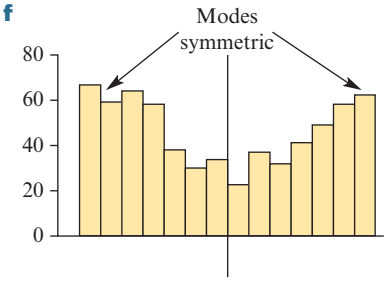
Histogram D

e



Histogram E

f



Histogram F

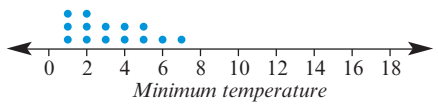
5 Approximately symmetric, 180, 174, 188

6 Approximately symmetric, 60, 40, 80, 10, 90

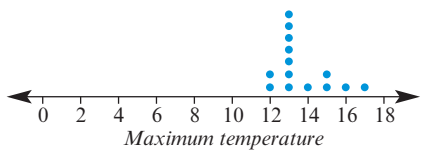
7 a Centre **b** Neither **c** Both

8 a All of the distributions are approximately symmetric.
b There are no clear outliers.
c Subject A: 8–10, Subject B: 24–26, Subject C: 40–42.
d Subject B

9 a



b



c The distribution of minimum temperature is centred much lower than the distribution of maximum temperature, as expected. It also looks to be a little more variable.

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10 a

Body fat % 1 | 2 means 12%

0	4
0	5 5 7 9 9
1	0 0 1 2 3 3 4
1	7 7 7 7 7 8 9
2	0 0 1 1 4
2	5 6 8 9
3	0

b The distribution would have exactly the same spread, but the centre would be 2% lower.

Exercise 10G

1 a 26 **b** 2.36

2 a $\bar{x} = 5$ **b** $\bar{x} = 5$ **c** $\bar{x} = 15$
d $\bar{x} = 101$ **e** $\bar{x} = 2.8$

3 a 11 12 17 19 21 24 32 34 35 53 62 63 95
b 32

4 a $M = 9$ **b** $M = 6.5$ **c** $M = 27$
d $M = 106.5$ **e** $M = 1.2$

5 a $M = 57$ mm
b $M = 27.5$ hours

6 a $\bar{x} = 55.4$ mm
b $\bar{x} = 28$ hours

7 $M = 17$, $\bar{x} = 16.95$

8 a $\bar{x} = 12.5$ ha, $M = 7.4$ ha
b The median, as it is typical of more suburbs. The median is not affected by the outlier.

9 a $\bar{x} = \$393\,386$, $M = \$340\,000$
b The median, as it is typical of more apartment prices.

10 35.3

11 69.3

12 a 15.15 **b** 15.0

13 a 2.5 **b** 3.3

14 a The median is the average of the 24th and 25th values when the data is ordered. The 24th value is in the interval 50–< 60, and the 25th value is in the interval 60–<70. Thus we could infer that the median is about 60, although we cannot give it an exact value.
b We cannot determine the exact value of the mean from a histogram where the data have been grouped. However, as the distribution is approximately symmetric we could infer that it would be similar in value to the median.

15 a The median would be in the interval where the cumulative percentage frequency is 50%. Thus, it is in the interval 180.0–< 185, although we cannot give it an exact value.
b We cannot determine the exact value of the mean from a histogram where the data have been grouped. However, as the distribution is approximately symmetric we could infer that it would be similar in value to the median.

16 6 and 10

17 $1.6w$

Exercise 10H

1 a 0 1 1 1 2 2 2 3 6 7 **b** $R = 7$
c $Q_1 = 1$ **d** $Q_3 = 3$ **e** $IQR = 2$

2 a $IQR = 10.5, R = 21$
b $IQR = 8, R = 11$
c $IQR = 7, R = 12$
d $IQR = 4.5, R = 8$
e $IQR = 1.1, R = 2.7$

3 a $IQR = 9.5$ mm
b $IQR = 11$ hours

4 $M = 3$, $Q_1 = 1$, $Q_3 = 6$

5 a $\bar{x} = 3$ **b** $s = 2.160$

6 $\bar{x} = 365.8$, $s = 8.4$, $M = 366.5$, $IQR = 12.5$, $R = 31$

7 $\bar{x} = 214.1$, $s = 36.5$, $M = 207.5$, $IQR = 39$, $R = 145$

8 $R = 9$, $IQR = 2$, $s = 2.164$

9 $\bar{x} = 3.5$ kg, $s = 0.6$ kg, $M = 3.5$ kg, $IQR = 1$ kg, $R = 2.4$ kg

10 a **i** $\bar{x} = 6.71$, $M = 6.2$
ii $IQR = 1.45$, $s = 0.90$
b **i** $\bar{x} = 12.71$, $M = 7.3$
ii $IQR = 1.85$, $s = 17.75$
c The error does not affect the median or interquartile range very much. It approximately doubles the mean, and increases the standard deviation by a factor of about 20.

11 $x = 9.6, y = 12.4$

12 $p = 2$ and $q = 10$, or $p = 3$ and $q = 11$, or $p = 4$ and $q = 12$, or $p = 5$ and $q = 13$, or $p = 6$ and $q = 14$

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Chapter 10 review

Multiple-choice questions

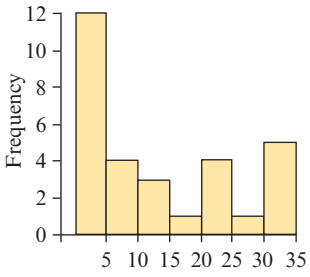
1 C	**2** B	**3** B	**4** A	**5** C
6 B	**7** C	**8** B	**9** B	**10** D
11 D	**12** D	**13** A	**14** A	**15** C
16 C	**17** D	**18** C	**19** A	**20** C
21 C	**22** C	**23** A	**24** A	

Short-response questions

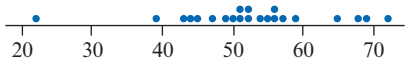
1 a Discrete **b** Ordinal

2 a Categorical **b** 7.5%

3



4 a



b

Time (minutes)

2	2
3	9
4	3 4 5 7 9
5	0 1 1 2 2 4 5 6 6 7 9
6	5 8 9
7	2

4 | 7 represents 47 minutes

c $M = 52$ minutes, $Q_1 = 47$ minutes, $Q_3 = 57$ minutes

5 $\bar{x} = \$283.57$, $s = \$122.72$, $M = \$267.50$, $IQR = \$90$, $R = \$495$

6 $\bar{x} = 178.89$ minutes, $s = 13.99$ minutes

7 a Numerical

b

Divorce rate (%)

0	0 5 6 6 8 9
1	4 4 5 8 9
2	5 6 7 8
3	2 2
4	4
5	3

3 | 2 represents 32%

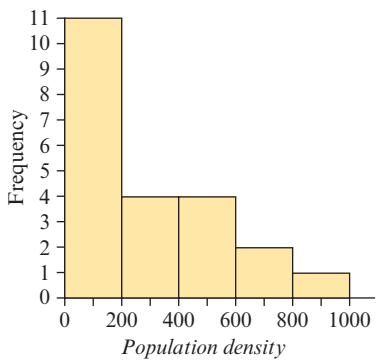
c $\bar{x} = 20.05\%$, $M = 18\%$

d i Positively skewed **ii** 21.5%

8 a 45.5%, 7, 100%

b 22, 45.5%, 31.8%, rarely, 22.7%, regularly (or 22, 45.5%, 22.7%, regularly, 31.8%, rarely)

9



10 a i 17% **ii** 13% **iii** 33%

b i 6 **ii** 4

c 15–19 words/sentence

11 a Median in the interval 5.0–9.9.

b Max $IQR = 19.9$

12 a Mean is not preferred as this is a strongly positively skewed distribution.

b Mean is not preferred as this is a positively skewed distribution with outliers.

c Mean can be used as the distribution is approximately symmetric.

13 $p = 3$, $q = 7$

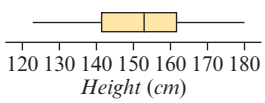
14 $p = 6$, $q = 11$

15 49

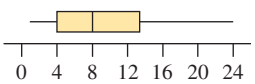
Chapter 11

Exercise 11A

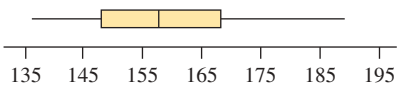
1



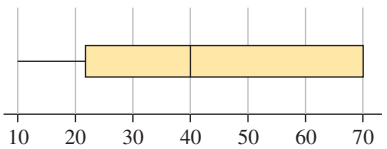
2 a



b



3 a



b Q_3 and the maximum are equal.

4 a Min = 1, $Q_1 = 4$, $M = 8$, $Q_3 = 13.5$, Max = 24

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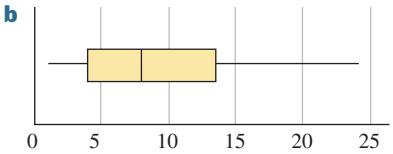
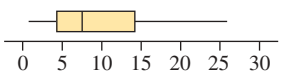
b



5



Years employed

6 Lower fence = 4, upper fence = 28

7 a Lower fence = 15, upper fence = 95

b Outliers are 14 and 99

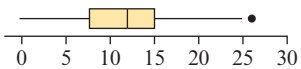
8 a Lower fence = 7, upper fence = 95.

b Outliers are 6, 99 and 102

9 a Lower fence = −$3.125, upper fence = $25.875

b Outlier = $26

c

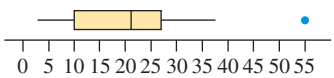


Pocket money ($)

10 b Upper fence = 26.5 + 1.5 × 16 = 50.5

c 55 is larger than the upper fence.

d



unirate

11 a i 10 **ii** 5, 21 **iii** 16
iv 0, 45 **v** None

b i 27 **ii** 12, 42 **iii** 30
iv 5, 50 **v** None

c i 38 **ii** 32, 42 **iii** 10
iv 5, 50 **v** 5

d i 16 **ii** 14, 21 **iii** 7
iv 1.5, 50 **v** 1.5, 3, 36, 40, 50

e i 34 **ii** 30, 39 **iii** 9
iv 30, 55 **v** 55

12 a i 55 **ii** 15

b i 100 **ii** 20

13 a 50% **b** 75% **c** 25%

14 a 75% **b** 25% **c** 50%
d 0% **e** 75% **f** 25%

15 a 25% **b** 75% **c** 25%
d 50% **e** 75%

16 a 25% **b** 25% **c** 50%
d 25% **e** 75% **f** 50%

17 Negatively skewed; 41.5; 15; 47

18 Positively skewed; 800; 1200; 3200

19 Negatively skewed; 39.5; 9.5; 45; one; 5

20 Positively skewed;16; 6; 35; four; 5, 8, 36 and 40

21 Symmetric; 41; 7; 36; four; 10, 15, 20, 25

22 The median time it takes Taj to travel to university is 70 minutes. The range of the distribution of travel time is 60 minutes, but the interquartile range is only 15 minutes. The distribution of travel times is positively skewed with two outliers, unusually long travel times of 110 minutes and 120 minutes respectively.

23 The median mark for these students is 18. The range of the distribution of marks is 49, but the interquartile range is only 9. The distribution of marks is slightly positively skewed with three outliers, one student who scored only one mark on the test, and two students who recorded much higher scores than the rest, with marks of 45 and 50.

24 $x \leq 51$

25 $17.4 \leq x < 20.5$

Exercise 11B

Note: The written reports should only be regarded as sample reports. There are many ways of writing the same thing.

1 a F: $M = 34$ years, $IQR = 28$ years
M: $M = 25.5$ years, $IQR = 13$ years

b 79 may be an outlier although it does not quite fall outside the upper fence.

c $M = 25.5$; $M = 34$; $IQR = 13$; $IQR = 28$; high; 79

2 a Class A: $M = 76.5$ marks, $IQR = 30.5$ marks
Class B: $M = 78$ marks, $IQR = 12$ marks

b Report: Both distributions are negatively skewed. The median mark for Class A ($M = 76.5$) was lower than the median mark for Class B ($M = 78$). The spread of marks for Class A ($IQR = 30.5$) was greater than the spread of marks for Class B ($IQR = 12$).

3 The median time spent online by the Year 10 students (20 hours) was higher than the median number of hours by the Year 11 students (16.5 hours). The variability of the hours spent online was the same for both groups ($IQR = 9.5$ hours). In conclusion, the Year 10 students spent more time online than the Year 11 students, and this time equally variable for both groups.

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4 a Year 12: $M = 5.5$ hours, $IQR = 4.5$ hours
Year 8: $M = 3$ hours, $IQR = 2.5$ hours

b Report: The distribution for Year 12 is negatively skewed and for Year 8 approximately symmetric. The median homework time for the Year 12 students ($M = 5.5$ hours/week) was higher than the median homework time for Year 8 students ($M = 3$ hours/week). The spread in the homework time for the Year 12 students ($IQR = 4.5$ hours/week) was also greater than for Year 8 students ($IQR = 2.5$ hours/week). In conclusion, the median homework time for the Year 12 students was higher than that for Year 8 students and the time they spent on homework was more variable.

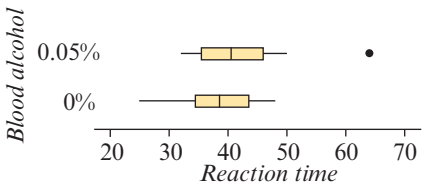
5 a Males: $M = 22\%$, $IQR = 12\%$
Females: $M = 10\%$, $IQR = 11.5\%$ (values can vary a little)

b The distribution of smoking rates is slightly negatively skewed for females and approximately symmetric for males. The median smoking rate for males ($M = 22\%$) was higher than for females ($M = 10\%$). The spread in smoking rates for males ($IQR = 12\%$) was similar to that for females ($IQR = 11.5\%$). In conclusion, median smoking rates were higher for males than females but the variability in smoking rates was similar.

6 a Before : $M = 26$, $IQR = 7$, outlier = 44;
After : $M = 30$, $IQR = 10$, outliers = 50 & 54 (values can vary a little)

b Report: The median number of sit-ups before the fitness class ($M = 26$) was lower than after the fitness class ($M = 30$). The spread in number of sit-ups before the fitness class ($IQR = 7$) was less than after the fitness class ($IQR = 10$). There was one outlier before the fitness class, the person who did 44 sit-ups. There were two outliers after the fitness class, the person who did 50 sit-ups and the person who did 54 sit-ups. In conclusion, the median number of sit-ups increased after taking the fitness class and there was more variability in the number of sit-ups that could be done.

7 a



b Report: The distribution is negatively skewed for the 0% blood alcohol group and positively skewed for the 0.05% alcohol group. The median time is slightly higher for the 0.05% blood alcohol group ($M = 40.5$) than for the 0% blood alcohol group ($M = 38.5$). The spread in time is also slightly higher for the 0.05% blood alcohol group ($IQR = 9.5$) than for 0% blood alcohol ($IQR = 9.0$). There was one outlier, the person with 0.05% blood alcohol who took a very long time of 64 seconds to complete the task.
In conclusion, there is some evidence that the time to complete the task increases with alcohol consumption, as the median time was longer for the 0.05% blood alcohol group than for the 0% blood alcohol group but the variability in times was similar.

8 In this sample, the age at marriage is higher for men ($M = 23$ years) than for women ($M = 21$ years). The variability is also greater for the men ($IQR = 12$ years) than for the women ($IQR = 9$ years). The distributions of age at marriage are positively skewed for both men and women. There are no outliers. In conclusion, the females tended to marry at a younger age than the males, and there was less variability in the age of marriage for the females than the males.

9 For this data, there is an association between the lifetime of a battery and its price.
The lifetime of the high price batteries ($M = 51$ hours) is longer than that of the medium price batteries ($M = 35$ hours), which is in turn slightly longer than that of the low price batteries ($M = 32$ hours). The variability in lifetime increased as price decreased, from $IQR = 7$ hours for the high price batteries, to $IQR = 12$ hours for the medium price batteries, and $IQR = 17$ hours for the low price batteries. All three distributions are approximately symmetric, with no outliers. In conclusion, there is some evidence that the

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higher price batteries last longer, and the time that they last is less variable.

10 The median score at the beginning of the course ($M = 38$) showed a little improvement after six weeks ($M = 42$), followed by a very large improvement by the end of the 12 week course ($M = 72$). The variability of the scores changed little over the course ($IQR = 12$ at the beginning, $IQR = 12$ at 6 weeks, $IQR = 14$ at 12 weeks). The distribution of scores at 0 weeks is approximately symmetric with an outlier at 66, positively skewed with an outlier at 76 at 6 weeks, and approximately symmetric with no outliers at 12 weeks. In conclusion, the students' scores increased a little after 6 weeks, but much more after 12 weeks of the course. There was little change in variability of the scores between the three time periods.

11 The mean score for students taught using the new method ($\bar{x} = 73.4$) is higher than the mean score for students taught using the traditional method ($\bar{x} = 68.7$). The median score is again higher for the new method ($M = 75$) than for the traditional method ($M = 70$). The variation in scores was higher for the traditional method ($s = 7.6$) than for the new method ($s = 5.6$). This higher variation in score for the traditional method compared to the new method is confirmed by the values of the IQR (15 for traditional method, 12 for the new method), and the ranges of values (28 for the traditional method and 22 for the new method). Since the values of the mean and median are very similar for both the datasets, we can assume that both distributions are approximately symmetric. In summary, the data provides some evidence that the new teaching method may be better than the traditional teaching method, in that the students taught using the new method had higher scores with less variation than those taught using the traditional method.

12 The mean weight loss for those on Diet A ($\bar{x} = 12.2$ kg) is lower than the mean weight loss for those on Diet B ($\bar{x} = 15.3$ kg). The median weight loss is again lower for Diet A ($M = 9$ kg) than for Diet B ($M = 12$ kg). The variation in weight loss was similar for both diets (Diet A: $s = 5.7$ kg, Diet B: $s = 6.0$ kg). This similarity in the variation in weight loss for both diets is confirmed by the values of the IQR (11 kg for Diet A, 10 kg for Diet B), and the ranges of weight loss (32 for Diet A and 33 for Diet B). Since the values of the means are higher than the values of the medians for both distributions, it is possible that both distributions are positively skewed. In summary, the data provides some evidence that Diet A results in a higher weight loss than Diet B, and that there is about the same variation in the weight loss.

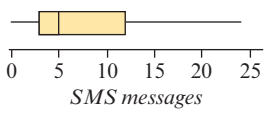
Chapter 11 review

Multiple-choice questions

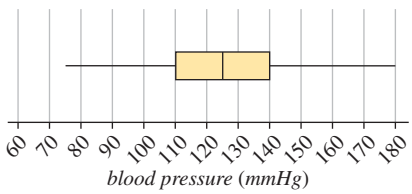
1 D	**2** B	**3** B	**4** D	**5** A
6 A	**7** C	**8** D	**9** A	**10** C
11 A	**12** D	**13** A	**14** A	**15** B
16 C	**17** A			

Short-response questions

1



2 a

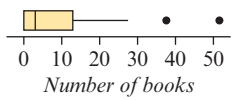


b lower fence = 65, upper fence = 185, no outliers.

3 a lower fence = −18, upper fence = 30.

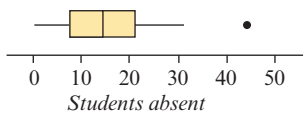
b There are two possible outliers; the people who borrowed 38 and 52 books respectively.

c



d More than 12

4 a



b 27.8%

5 Lower fence = 3, Upper fence = 35

6 The mean time after improvements ($\bar{x} = 69.6$ mins) is lower than the time before improvements ($\bar{x} = 71.8$ mins). The median time is again lower after improvements ($M = 68$ mins) than before ($M = 70$ mins). The variation in time was was higher before

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than after (Before: $s = 11.8$ mins, After: $s = 9.3$ mins). This difference in the variation in times is confirmed by the values of the IQR (17 mins before, 14 mins after), and the ranges of times (44 mins before and 33 mins after). Since the values of the means are very similar to the values of the medians for both distributions, both distributions are approximately symmetric. In summary, the data provides some evidence that the improvements have reduced the mean travel time between stations, and the variation in these travel times.

7 The distribution of birth weights is slightly positively skewed for non-smokers, and approximately symmetric for smokers. The lbs median birth weight for smokers ($M = 7.0$ lbs) was lower than for non-smokers ($M = 7.4$ lbs). The variation in birth weights for smokers ($IQR = 1.8$ kg) was similar to that for non-smokers ($IQR = 1.7$ kg). In conclusion, median birth weights were lower for smokers than for non-smokers, but the variability in birth weights was similar.

8 a

Japan						*Australia*						
			3	0	2	3	3	3	4	4	
9	7	6	5	5	0	5	5	6	7	7	8	9
		4	4	2	1	1	1	4				
		9	7	5	1	5	7					
	3	3	2	2	2	1	3					
		9	8	6	2							
			2	3	3							
				3								
			4	4								

6 | 2 represents 26 1 | 5 represents 15

b Japanese: $M = 17$ days, $IQR = 16.5$ days
Australians: $M = 7$ days, $IQR = 10.5$ days

c Report: Both distributions are positively skewed. The median time spent away from home by the Japanese ($M = 17$ days) was much higher than the median time spent away from home by the Australians ($M = 7$ days). The spread in the time spent away from home by the Japanese ($IQR = 16.5$ days) was also greater than the time spent away from home by the Australians ($IQR = 10.5$). In conclusion, the median time spent away from home by the Japanese was longer than the Australians and the length of time they spent away from home more variable.

9 People who were not thinking of changing to a different kind of work tended to work in occupations with higher median occupational prestige score ($M = 48$), than those who maybe think of changing ($M = 44$), who in turn tended to work in occupations with higher median occupational prestige scores than those who were thinking of changing ($M = 39$). The variability in occupational prestige scores was lowest for those thinking of changing work ($IQR = 17$), and similar for those not thinking of changing ($IQR = 20$) and maybe thinking of changing ($IQR = 21$). In conclusion, people who are in occupations with higher occupational prestige scores tend to be less likely to want to change to a different kind of work.

10 a Lower fence = 30

b IQR will not change, so the lower fence will stay 30, and hence 31 is not an outlier.

Chapter 12

Chapter 12 review

12A Revision of Chapter 7: Applications of linear equations and their graphs

Multiple-choice questions

1 D **2** C **3** D **4** C
5 D **6** B **7** B **8** C
9 C **10** C

Short-response questions

1 a $x = \frac{23}{7}, y = \frac{1}{7}$

b $x = 3, y = -4$

2 a $(-1, 11)$

b $(3, -4)$

c $\left(\frac{44}{7}, \frac{6}{7}\right)$

3 a $5s + 4r = 94$ and $6s + 2r = 96$

b Showbag: \$14, Ride: \$6

c \$74

4 a \$80

b 85 cents

c 15 cents

d Loss \$38

e Profit \$25

f 115 units

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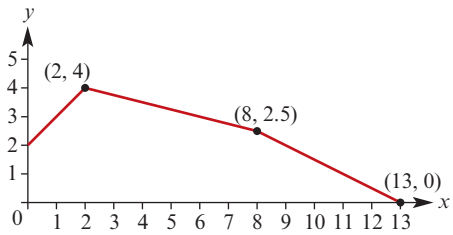
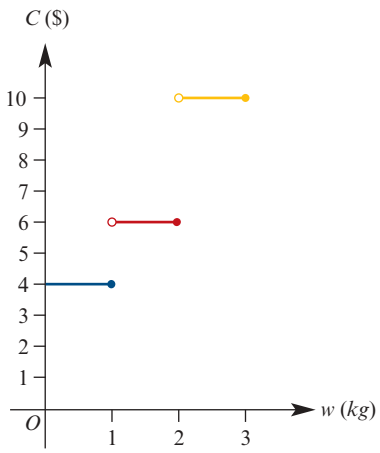
5 a



b 3

c $x = 10$

6 a



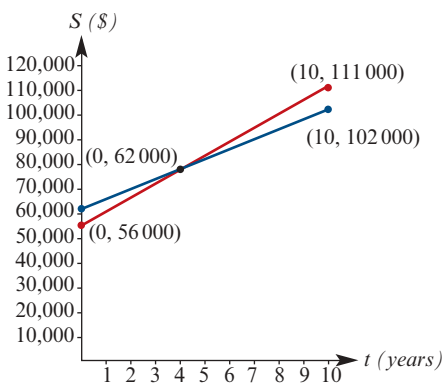
b $110

c $C = 2x + 10$

d $44

7 a Finn: $S = 4000t + 62\,000$, Isabella: $S = 5500t + 56\,000$

b



c After four years

8 a $V = -350t + 6600$

b $C = 350t + 800$

c 8 minutes

9 $2

10 Attend at least 13 sessions

12B Revision of Chapter 8: Applications of trigonometry

Multiple-choice questions

1 B **2** C **3** C **4** C
5 B **6** B **7** D **8** D
9 C **10** A

Short-response questions

1 16.99 units

2 61.4°

3 a 147.72 m **b** 26.0 m

4 58 m^2

5 33.2°

6 14.8 cm

7 a 3.2 km **b** 090°

8 a 8.8 m **b** 42°

9 6.57 km

10 Tower C is the closest to the plane at 581.49 m from the aeroplane.

12C Revision of Chapter 9: Matrices and matrix arithmetic

Multiple-choice questions

1 D **2** D **3** D **4** A
5 A **6** D **7** A **8** C
9 C **10** D

Short-response questions

1 a 15 **b** 81 **c** 80 **d** 39

2 $\begin{bmatrix} 29 & -17 \\ -31 & 10 \end{bmatrix}$

3

	Shorts	*Tops*	*Joggers*
Women	16.50	7.70	22.00
Men	11.00	8.80	27.50

4 $\begin{bmatrix} -1 & -11 \end{bmatrix}$

5 a $H \times K$ **b**

	kJ
Tim	14 800
Tam	15 500

6 a Zero matrix **b** Identity matrix
c Column matrix **d** Square matrix
e Row matrix

7 a

	A	*B*	*C*	*D*
A	0	1	0	0
B	1	0	1	1
C	0	1	0	1
D	0	1	1	0

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b There are three people that directly communicate with Person B.

c

$$P^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 3 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \end{array}$$

5 ways
$D \to B \to A$
$D \to B \to C$
$D \to C \to B$
$D \to B \to D$
$D \to C \to D$

8 $691

9 $178

10 $x = 3, y = 9$ or $x = -3, y = -9$

12D Revision of Chapter 10: Univariate data analysis

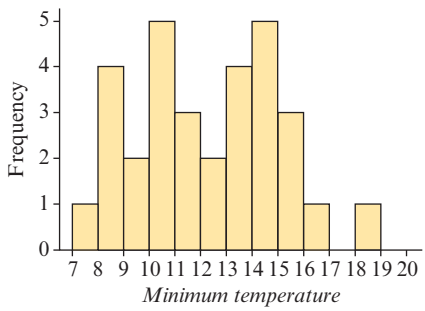
Multiple-choice questions

1 D **2** C **3** C **4** A **5** B
6 B **7** D **8** C **9** B **10** A

Short-response questions

1 a Continuous
b Nominal
c Ordinal

2 a



b 22.6%

3

Height (cm)

7	4 9 9
8	2 4 6 7 9
9	0 0 3 3 4 5 8 9
10	2 5
11	1
12	
13	3

8 | 5 represents 8.5 cm

4 a 68
b 0– <5 years
c Positively skewed

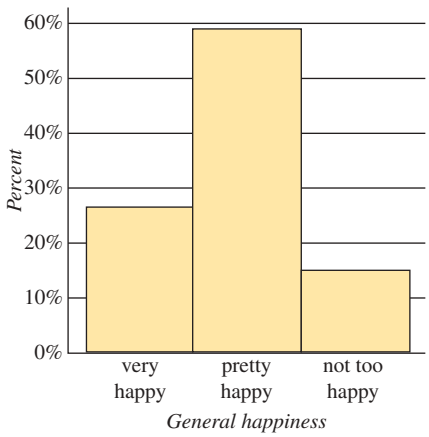
5 a $M = 38$ years, $IQR = 17$ years
b $\bar{x} = 38.67$ years, $s = 8.33$ years

6 a $M = 1$, $IQR = 1$
b The median number of siblings for this group of people is 1, and the interquartile range is 1. The distribution of number of siblings is positively skewed. There are three outliers: people with 4 siblings, 6 siblings and 9 siblings respectively.

7 a Categorical
b

General happiness	Frequency	
	Number	%
Very happy	53	26.5
Pretty happy	118	59.0
Not too happy	29	14.5
Total	200	100.0

c



d A group of 200 people was asked to describe their general happiness by choosing one of the responses very happy, pretty happy, not too happy. The highest level of response was pretty happy, chosen by 59.0% of the people. Another 26.5% of people chose very happy. Only 14.5% of people said they were not too happy.

8 1.8

9 6 and 12, 7 and 11, and 8 and 10.

10 $p = 5, q = 15$

12E Revision of Chapter 11: Comparing datasets

Multiple-choice questions

1 C **2** A **3** D **4** B **5** D
6 A **7** C **8** A **9** B **10** C

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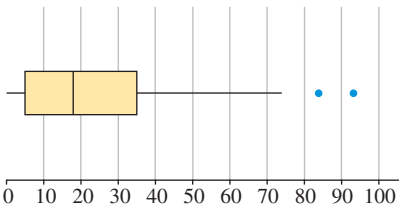
Short-response questions

1 a Lower fence = −40, Upper fence = 80

b



2 a Cold: $M = 29.5$, $IQR = 14$,
Warm: $M = 22.5$, $IQR = 11$

b Cold: LF = 2, UF = 58, no outliers
Warm: LF = −0.5, UF = 43.5, no outliers

c $M = 22.5$, $M = 29.5$, $IQR = 11$, $IQR = 14$

3 The median tomato yield is higher from the plants treated with the new fertiliser ($M = 3.3$ kg) than the plants treated with the regular fertiliser ($M = 2.8$ kg). However, the yield from the plants treated with the new fertiliser was more variable ($IQR = 1.35$ kg) than the yield from plants treated with the regular fertiliser ($IQR = 1.2$ kg).

4 a Male: Median = 65, $IQR = 10$,
two outliers = 40, 120
Female: Median = 72, $IQR = 15$,
no outliers

b The distribution of resting pulse rates is approximately symmetric for both males and females in these samples. The median pulse rate for females ($M = 72$ beats/minute) is higher than that for males ($M = 65$ beats/minute). The spread of pulse rates for females ($IQR = 15$) is higher than for males ($IQR = 10$). There was one male with an unusually low pulse rate of 40 beats/minute, and another with an unusually high pulse rate of 120 beats/minute. In summary, for this group of people, pulse rates were higher and more variable for females than for males.

5 a Before: Median = 7, $IQR = 6$, no outliers
After: Median = 15, $IQR = 6$, one outlier at 30

b The distribution of wellness score is approximately symmetric both before and after attendance at the wellness retreat. The median wellness score before the retreat ($M = 7$) is lower than that after the retreat ($M = 14$). The spread of wellness score is the same both before and after the retreat ($IQR = 6$). There was one person with an unusually high wellness score of 30 after the retreat. In summary, for this group of people, the wellness scores are higher after the retreat than before, while variability in wellness scores remains unchanged.

6 a The distribution of distance is positively skewed, with outliers at 17 km, 18 km and 19 km.

b 30

c i Lower fence = −2, Upper fence = 14.

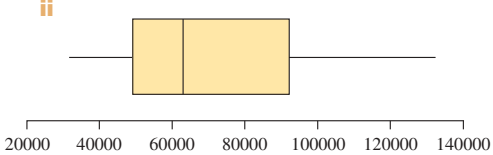
ii A distance of 1 km is within the fences.

d i 1 km

ii 1.5 km

7 a i Lower fence = −$15 500,
Upper fence = $156 500

ii



iii 75%

b i 2016 median = $74 000,
2020 median = $80 000

ii The median salary in 2020 ($M = $80 000) was higher than the median salary in 2016 ($M = $74 000). The spread of salaries in 2020 ($IQR = $24 000) was slightly higher than the spread in 2016 ($IQR = $23 000). There was one outlier in 2016, a person who had a salary of $121 000, which was high compared to the rest of the employees. There were two outliers in 2020, people with high salaries of $131 000 and $155 000 respectively. In conclusion, the median salary has certainly increased over the four years from 2016 to 2020, while at the same time salaries have become a little more variable.

8 a *team*

b

Score

Team A						Team B					
4	2	0	0	0	0	2	2	2	2	4	
9	8	8	7	6	0	6	6				
		4	1	0	1	0	2	3	3		
			9	6	1	5					
				0	2						
					2						
					3	0	1				

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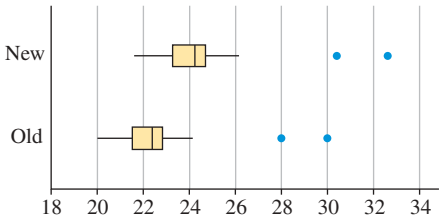
c Team A: Min = 0, Q_1 = 4, M = 8, Q_3 = 14, Max = 20.
Team B: Min = 0, Q_1 = 2, M = 6, Q_3 = 13, Max = 31

d Team A: lower fence = −11, upper fence = 29, no outliers.
Team B: lower fence = −14.5, upper fence = 29.5. Outliers 30 and 31.

e The mean would be higher for Team B because of the influence of the outliers.

9 a Old: min = 20, Q_1 = 23.8 (approx), M = 26, Q_3 = 27, upper whisker = 30.2 (approx.), outliers = 40 and 45.
New: min = 24, Q_1 = 28.2, M = 30.6, Q_3 = 31.7, upper whisker = 35.2 (approx) 46 and 51.5.



b The median hourly rate of pay increased from $26 to $30.60 after the pay increase. The spread of the hourly rates of pay increased by 10% after the pay increase, from $3.00 to $3.30. The distributions of hourly rates of pay were both approximately symmetric before and after the pay increase. Two people had hourly rates which were outliers before the pay increase ($40 and $45). These hourly rates continued to be outliers after the increase ($46 and $51.50).

10 a One

b 85%

c 50 people in the normal diet group have blood pressure less than 120, more than 75 people from the special diet group have blood pressure less than 120. Therefore the claim is correct.

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