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JACARANDA MATHS QUEST
MATHEMATICS
STANDARD

FOR NSW | FIFTH EDITION

11



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JACARANDA MATHS QUEST
MATHEMATICS
STANDARD **11**
FOR NSW | FIFTH EDITION

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Each topic starts with an engaging Discussion point

NSW
Mathematics
Syllabus
reference linked
to contents of the
topic

A brief summary in the text details the skills learnt in each topic.

2.1 Overview

Linear models can be used to help us see and understand patterns that occur in the world.



DISCUSSION

Can you think of some more real-life situations which can be modelled by linear relationships? Are there any limitations that must be put on these models in order for them to work?

LEARNING SEQUENCE

- 2.1 Overview
- 2.2 Direct variation
- 2.3 The gradient and intercepts
- 2.4 Sketching linear graphs
- 2.5 Linear models
- 2.6 Review

CURRICULUM CONTENT

Students:

- model, analyse and solve problems involving linear relationships, including constructing a straight-line graph and interpreting features of a straight-line graph, including the gradient and intercepts (ACMMM002, ACMMM003) **AAM** : $\rightarrow \sigma^2$
- construct and analyse a linear model, graphically or algebraically, to solve practical direct variation problems, including but not limited to the cost of filling a car with fuel or a currency conversion graph

TOPIC 2 Linear relationships 21

9.6 Review

9.6.1 Summary

In this topic you have learnt:

- about the difference between salary and wages
- how workers earn money
- how to calculate commissions, piecework and royalties
- about leave loading and allowances
- about taxation and deductions
- how to calculate taxable income and use the taxation table to determine the tax payable
- why workers pay the Medicare levy and how to calculate the amount required
- about superannuation and how employers calculate the amount
- how to do financial computations using a spreadsheet, such as finding the tax payable and completing wage sheets.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBook. © 2013

Summary — Topic 3: Practicalities of measurement

Units of measurement

- Standard metric volume measurements (small to large): milligrams (mL), liters (L)
- Standard metric weight measurements (small to large): milligrams (mg), grams (g), kilograms (kg), tonnes (t)
- Standard metric time measurements (small to large): seconds (s), minutes (min), hours (h), days, weeks, months, years, decades, centuries, millennia
- Ratio between units of length can be used to convert one unit to another.
- Accuracy is how closely a measured value agrees with an actual value and can be improved by measuring with smaller units.
- Precision is how closely individual measurements agree with each other.
- Limit of accuracy or range determines the highest and lowest values of a particular measurement.
- Error = estimated value - actual value
- More decimal places indicate a higher level of accuracy.
- Absolute error = |estimated value - actual value|
- Absolute relative error = $\frac{\text{estimated value} - \text{actual value}}{\text{actual value}}$

Limits of	

- Accuracy is how closely a measured value agrees with an actual value and can be improved by measuring with smaller units.
- Precision is how closely individual measurements agree with each other.
- Limit of accuracy or range determines the highest and lowest values of a particular measurement.
- Error = estimated value - actual value
- More decimal places indicate a higher level of accuracy.
- Absolute error = |estimated value - actual value|
- Absolute relative error = $\frac{\text{estimated value} - \text{actual value}}{\text{actual value}}$

WORKED EXAMPLE 5

The selling price (including GST) of a pair of runners is \$130. Calculate the price of the runners before GST was added.



THINK

- 1 Write the selling price (including GST) of the runners.
- 2 Since this price includes GST, divide it by $1.1 (100\% + 10\% \text{ (GST)})$ to obtain the price before GST was added.
- 3 Round the answer to 2 decimal places (cents).
- 4 Answer the question.

WRITE

$$\frac{130}{1.1} = \$118.1818$$
$$\approx \$118.18$$

The price of the runners before GST was added is \$118.18.

eBookplus RESOURCES

Exercise 10.3 Purchasing, running and maintaining a vehicle

Understanding, fluency and communicating

1.  A customer takes out a loan to purchase a car for \$17,995. The monthly repayments are \$313.38 and the loan is for 5 years. The customer pays a 10% deposit. Calculate:
- a. the deposit paid
 - b. the total amount paid for the car
 - c. the interest rate charged for the loan, correct to 2 decimal places.
2. A \$24,995 car is purchased through finance of 5.99% per annum. A 10% deposit is paid plus monthly repayments for 5 years. Calculate:
- a. the amount borrowed for the car (principal)
 - b. the monthly repayments for the 5 years
 - c. the total amount paid for the car.
3.  Stamp duty payable on the purchase of vehicles is:
- \$4500 or less — 3 percent (\$3 per \$100)
- \$4500 or more — \$1350, plus \$5 per \$100 over \$4500.
- Calculate the amount of stamp duty to be paid for the following vehicle purchases.
- a. \$22,995
 - b. \$65,990

TOPIC 10 Budgeting and household expenses 231

Carefully gradated exercises help students to develop skills and understanding.

Fully worked solutions to every question provided online. Answers provided at end of each print topic.

Exercise 9.6 Review

Understanding, fluency and communicating

8. A nurse is paid \$28.50 per hour for an 80-hour fortnight. Assuming 26 fortnights in a year, calculate:
 a. the nurse's annual salary
 b. the nurse's annual superannuation (assuming 9.5% superannuation)
 c. the nurse's weekly pay
 d. Which item represents getting paid for every item constructed?
 e. A novelist gets a royalty payment of 23% of gross sales on her book. Complete the following table.

Gross sales	Royalty	Net sales
\$43,000	\$12,500	\$320,000
		\$450,000
9. Convert the following annual salaries to fortnightly pay (Assume 26 fortnights per year).
 a. \$67,800 per year b. \$9,780 per year c. \$10,010 per year
 10. Calculate the amount of superannuation each fortnight, if a person (assume 9.5% superannuation) has:
 a. a salary of \$67,800 b. \$10,010 c. \$12,426
 11. An architect makes a salary of \$85,000 per year and is given 9.5% extra in the form of superannuation.
 a. How much superannuation does he get each year?
 b. How much does the architect's superannuation increase, but instead her superannuation

Fully Worked examples throughout the text demonstrate key concepts and encourages mathematical thinking with the Think/Write layout.

eBook*plus* resources contain rich interactivities and videos to support and enhance learning.

Questions classified according to the Working Mathematically strand of the NSW Mathematics Syllabus.

Each topic concludes with comprehensive Review, covering all of the material in the topic.

A range of question types at different levels of difficulty help to prepare students for examinations

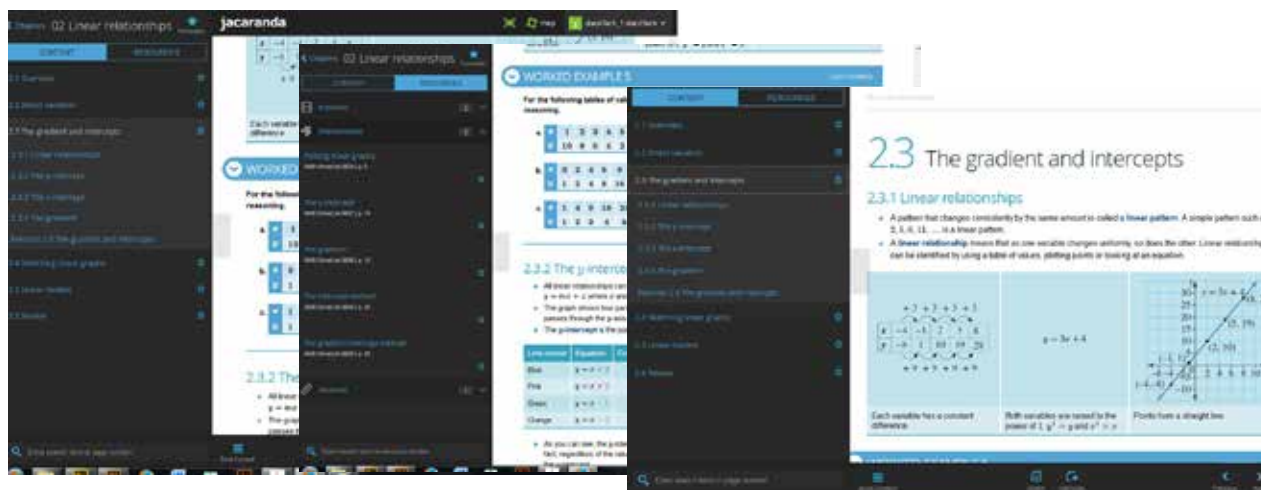
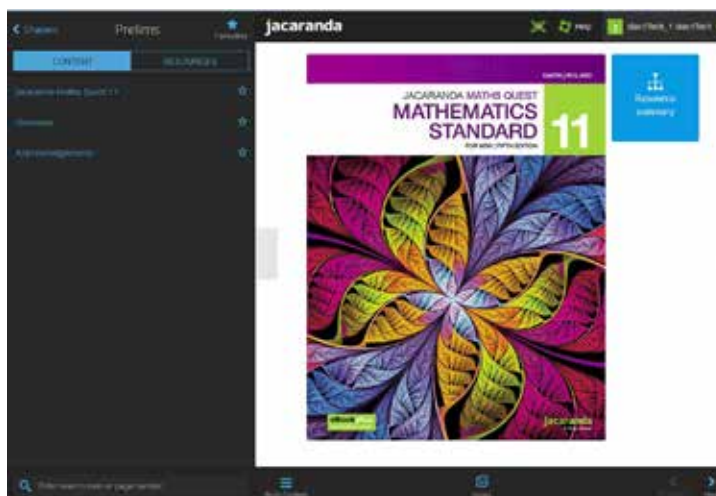
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TOPIC 1

Formulae and equations

1.1 Overview

1.1.1 Introduction

Formulae have been developed for a wide variety of theoretical and practical situations, from calculating the mortgage repayments when a new house is purchased to deciding which films should be playing at your local cinema. Once we have a formula we can use it to calculate required unknown values by substituting known values into our formula.

In this chapter we will look at some of the more practical applications of formulae, such as determining when it is safe to drive after consuming alcohol and ensuring that infants and children are given the correct medication dosages.



DISCUSSION

Can you think of other formulae that are widely used in everyday life?

LEARNING SEQUENCE

- 1.1 Overview
- 1.2 Evaluating expressions
- 1.3 Linear equations
- 1.4 Blood alcohol content
- 1.5 Medication dosages
- 1.6 Review

CURRICULUM CONTENT

- review substitution of numerical values into linear and non-linear algebraic expressions and equations (ACMGM010) ♦ 
- calculate and interpret blood alcohol content (BAC) based on drink consumption and body weight **AAM**  
- calculate required medication dosages for children and adults from packets, given age or weight, using Fried's formula, Young's formula or Clark's formula as appropriate **AAM**

1.2 Evaluating expressions

1.2.1 Substitution

- Expressions are used to describe a situation in which there are unknown numbers. Once the values of the unknown numbers are known, the expression can be evaluated and a final answer can be obtained.
- Unknown numbers are represented with letters and are called **pronumerals**.
- To **evaluate** an expression, **substitute** a number in place of the pronumeral and then use the order of operations to work out the value of the expression.
- Remember the order of operations (BIDMAS) when evaluating expressions.

WORKED EXAMPLE 1

Evaluate the following expressions by substituting $a = 2$ and $b = 5$.

a $3ab$

b $7(a + b)$

c $\sqrt{a^2 + b}$

THINK

a 1 Write the expression and substitute 2 for a and 5 for b .

2 Evaluate the expression.

b 1 Write the expression and substitute 2 for a and 5 for b .

2 Evaluate the expression. Remember to work out the part of the expression in brackets first.

c 1 Write the expression and substitute 2 for a and 5 for b .

2 Evaluate the expression. Remember the order of operations.

WRITE

$$\begin{aligned} 3ab &= 3 \times a \times b \\ &= 3 \times 2 \times 5 \\ &= 30 \end{aligned}$$

$$\begin{aligned} 7(a + b) &= 7 \times (a + b) \\ &= 7 \times (2 + 5) \\ &= 7 \times 7 \\ &= 49 \end{aligned}$$

$$\begin{aligned} \sqrt{a^2 + b} &= \sqrt{2^2 + 5} \\ &= \sqrt{4 + 5} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

1.2.2 Formulae

- Equations** are mathematical statements that show two equal **expressions**; that is, the left-hand side and the right-hand side of an equation are equal. In the equation $a + 2 = 6$, the expression on the left-hand side ($a + 2$) is equal to the right-hand side (6).
- A **formula** is a special equation or rule that describes the relationship between different quantities.
- The plural of formula is formulae.
- To be able to use a formula, you need to know:
 - what the formula is used for
 - what the pronumerals represent
 - any specific unit requirements of the pronumerals.

Formula	What it is used for	What the pronumerals represent	Unit requirements
$A = lw$	Calculating the area of a rectangle	A represents the area of the rectangle. l represents the length of the rectangle. w represents the width of the rectangle.	l and w must be in the same units.
$E = mc^2$	Calculating the energy contained in a given mass	E represents the amount of energy. m represents the mass. c represents the speed of light.	m must be in kg. c must be in m/s.

- The variable written by itself in the formula is the subject of the formula. For example, A is the subject of the formula $A = lw$.

- If you are given the values of the other pronumerals in a formula, you can evaluate the subject of the formula by substituting these values.

WORKED EXAMPLE 2

In the formula $S = ut + \frac{1}{2}at^2$, find the value of S if $a = 10$, $u = 25$ and $t = 6$.

THINK

- 1 Write the formula.
- 2 Substitute the values of a , u and t .
Calculate the value of S .
- 3 Answer the question.

WRITE

$$\begin{aligned} S &= ut + \frac{1}{2}at^2 \\ S &= 25 \times 6 + \frac{1}{2} \times 10 \times 6^2 \\ &= 150 + 180 \\ &= 330 \\ S &= 330 \end{aligned}$$

1.2.3 Changing the subject of a formula

- Sometimes you may want to change the subject of a formula. You can do this by rearranging or **transposing** the formula to get the variable you want as the stand-alone variable in the equation. This can be done by using backtracking techniques, or by performing inverse operations on both sides of the formula.

$+$ and $-$ are inverse operations

$\times$ and $\div$ are inverse operations

WORKED EXAMPLE 3

Rearrange $v = at + u$ to make the pronumeral a the subject of the formula.

THINK

- 1 Think about how the formula was built up from a .
- 2 Backtrack by performing inverse operations one at a time and simplifying.
- 3 Subtract u from both sides.
- 4 Divide both sides by t .
- 5 Write the new formula with a as the subject.

WRITE

$$\begin{aligned} &a \text{ was multiplied by } t, \text{ then } u \text{ was added to the result, giving } v. \\ v &= a \times t + u \\ v - u &= a \times t + u - u \\ v - u &= a \times t \\ \frac{v - u}{t} &= \frac{a \times t}{t} \\ \frac{v - u}{t} &= a \\ a &= \frac{v - u}{t} \end{aligned}$$

WORKED EXAMPLE 4

The area of a triangle is given by the formula $A = \frac{1}{2}bh$. Rearrange the formula to make the pronumeral b the subject of the formula.

THINK

- 1 Write out the formula and decide which operation to undo first.
- 2 Undo $\times \frac{1}{2}$ (same as $\div 2$), by multiplying both sides by its reciprocal, 2, and then simplifying.

WRITE

$$\begin{aligned} A &= \frac{1}{2}bh \\ A \times 2 &= \frac{1}{2}bh \times 2 \\ 2A &= bh \end{aligned}$$

3 Undo $\times h$ by dividing both sides by h and then simplifying.

$$\frac{2A}{h} = \frac{bh}{h}$$
$$\frac{2A}{h} = b$$

4 Rewrite the equation, placing the subject b on the LHS.

$$b = \frac{2A}{h}$$

Note: In Worked example 4, b was multiplied by both $\frac{1}{2}$ and h (same order of operation), so it would have been feasible to divide both sides by h before multiplying both sides by 2. However, it is often easier to eliminate fractions or constant multiples first.

1.2.4 Solving problems involving formulae

- We can solve problems involving formulae using two methods.

Method 1: Substitute the known values into the formula and solve for the unknown.

Method 2: Change the subject of the formula, then substitute known values into the rearranged formula and evaluate for the unknown.

- Both methods usually require solving an equation. However, rearranging (or transposing) the formula to make the unknown the subject is often more difficult.

WORKED EXAMPLE 5

The perimeter, P , of a rectangle is given by the formula $P = 2(l + w)$, where l is the length and w is the width of the rectangle.

a Find the length of a rectangle with a perimeter of 3 metres and a width of 45 centimetres.

b i Transpose the formula to make l the subject.

ii Use the transposed formula to calculate the length of the rectangle given in part a.

THINK

a 1 Write the formula and substitute the values of P and w .

2 Remember to always use the same units ($P = 300$ cm).

3 Solve using inverse operations.

4 Divide both sides by 2.

5 Subtract 45 from both sides.

6 Answer the question.

b i 1 Write the formula.

2 Change the subject to l by using inverse operations one at a time.

WRITE

$$P = 2(l + w)$$

$$300 = 2(l + 45)$$

$$\frac{300}{2} = \frac{2(l + 45)}{2}$$

$$150 = l + 45$$

$$150 - 45 = l + 45 - 45$$

$$105 = l$$

The length of the rectangle is 105 cm.

$$P = 2(l + w)$$

3 Divide both sides by 2.

$$\frac{P}{2} = \frac{2(l + w)}{2}$$

$$\frac{P}{2} = l + w$$

4 Subtract w from both sides.

$$\frac{P}{2} - w = l + w - w$$

$$\frac{P}{2} - w = l$$

5 Answer the question.

$$l = \frac{P}{2} - w$$

ii 1 Substitute for P and w (note the units need to be the same) and evaluate.

$$\begin{aligned} l &= \frac{300}{2} - 45 \\ &= 150 - 45 \\ &= 105 \end{aligned}$$

2 Answer the question.

The length of the rectangle is 105 cm.

eBookplus RESOURCES

Interactivity: Transposing linear equations (int-6449)

Exercise 1.2 Evaluating expressions

Understanding, fluency and communicating

1. **WE1** Evaluate the following expressions by substituting $x = 3$, $y = 7$ and $z = 15$.

- a. $x + 3y$ b. $3x + 2z$ c. $4z - 15$ d. $3(2z - y)$ e. $5x + 2y - z$ f. $4z - 2y + x$

2. Evaluate the following expressions by substituting $x = 3$, $y = 7$ and $z = 15$.

- a. $3x + 7z - 2x + 7$ b. $x^2 + y^2$ c. $\frac{z}{x}$
d. $\frac{x + y}{z}$ e. $x(x^2 + y)$ f. $\sqrt{2(z - y) + x^2}$

3. a. Substitute 5 for m in the expression $m + 6$.

b. If the value of k is 15, find the value of $2k - 7$.

c. Given that $l = 3$, find $10l - 27$.

d. If $j = 15$, find $11j - 62$.

e. If $s = 21$, evaluate $3s - 7$.

4. **WE2** In the formula $S = ut + \frac{1}{2}at^2$, find the value of S if $a = 8$, $u = 12$ and $t = 9$.

5. The cost of hiring a kayak (\$ C) can be calculated using the formula $C = 50 + 10h$, where h is the number of hours that the kayak is hired.

Calculate the cost of hiring the kayak for:

- a. 1 hour b. 3 hours
c. 6.5 hours d. 90 minutes.

6. The perimeter of a rectangle (P) is calculated using the formula $P = 2(l + w)$, where l is the length and w is the width of the rectangle.

a. Find the perimeter of a rectangle that has:

- i. a length of 5 cm and a width of 2 cm
ii. a length of 15 cm and a width of 3 cm.

b. Write the formula for the area of a rectangle and use it to find the areas of the two rectangles in part a.



7. **WE3** Rearrange the following formula to make the pronumeral n the subject of the formula.

$$T = a + nd$$

8. **WE4** Transpose each of the following formulae to make the variable shown in brackets the subject of the formula.

a. $C = AB + D$ (A)

b. $v = u + at$ (t)

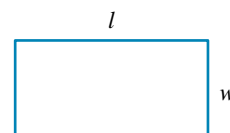
c. $V = \frac{Ah}{3}$ (h)

9. **WE5** a. Write the formula for the perimeter of the rectangle shown.

- b. Find the length of the rectangle with a perimeter of 2 metres and a width of 28 centimetres.

- c. i. Transpose the formula to make l the subject.

- ii. Use the transposed formula to calculate the length of the rectangle given in part b.



10. The total surface area of a cylinder can be found using the formula $A = 2\pi r(r + h)$ where r is the radius and h is the height of the cylinder.

- a. Rewrite the formula to make h the subject of the formula.

- b. Use your formula to calculate the height of a cylinder with a radius of 3 cm and a total surface area of 160cm^2 . Give your answer to 1 decimal place.

11. The distance, D km, travelled at a speed of s km/h for t hours is given by the formula $D = st$.

- a. Calculate the distance travelled at 80 km/h for:

- i. 4.5 hours

- ii. 150 minutes.

- b. Calculate the average speed if a car travels 120 kilometres in:

- i. 1.6 hours

- ii. 75 minutes.

- c. Find the time needed to travel a distance of 640 kilometres at an average speed of:

- i. 80 km/h

- ii. 60 km/h.

12. The formula $F = \frac{9C}{5} + 32$ converts degrees Celsius,

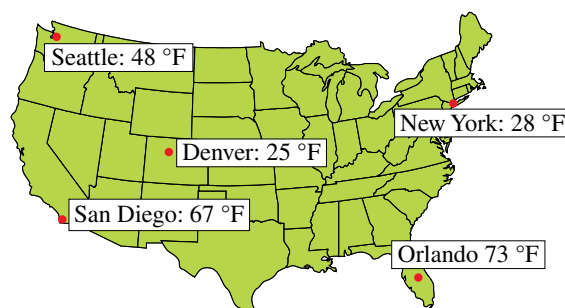
$^{\circ}\text{C}$, to degrees Fahrenheit, $^{\circ}\text{F}$. While travelling in the USA you discover that the weather reports give the weather forecast in degrees Fahrenheit.

- a. Calculate 40°C in degrees Fahrenheit.

- b. Transpose the formula to make C the subject.

- c. Use your transposed formula from part b to convert the temperatures shown to degrees Celsius.

Give your answers correct to the nearest whole number.



Problem solving, reasoning and justification

13. If $a = 2$, $b = 3$ and $c = 4$, explain why the value of $4abc$ is not 4234.

14. The cost of hiring a venue for a party includes a set amount of money, called a hiring fee, and a cost for each person attending. The cost of hiring two different venues for p people is given below.

Cost at venue A = $\$100 + \$5.00p$

Cost at venue B = $\$5.75p + \$50$

- a. What is the hiring fee for each venue?

- b. What is the cost per person for each venue?

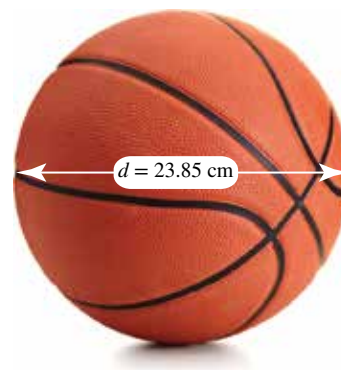
- c. If 50 people are attending a party, which venue would be cheaper to hire?



15. The volume of a sphere, V , is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere.

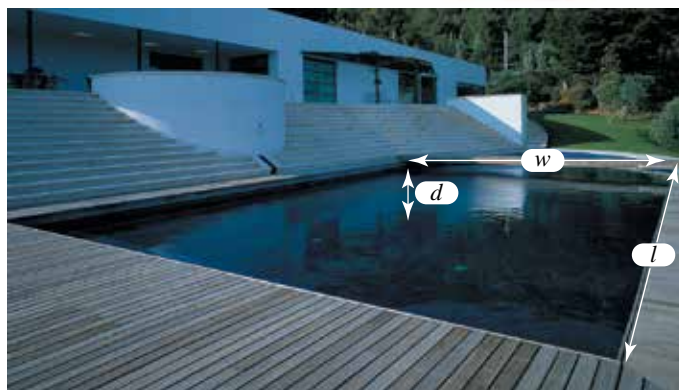
A basketball is shaped like a sphere.

- State the radius of the basketball shown in the diagram.
- Calculate the volume of the basketball shown, giving your answer correct to 2 decimal places.



16. The formula for working out how much water a rectangular swimming pool, of uniform depth, can hold is given by the formula $V = lwd$, where V is the volume of water (m^3), l is the length of the swimming pool (m), w is the width of the swimming pool (m) and d is the depth of the swimming pool (m).

- How much water would the swimming pool hold if its length is 14 m, its width is 8 m and its depth is 1.8 m when full?
- Rearrange the formula to make d the subject.
- How deep would the water be if the volume was 179.2 m^3 ?
- How many centimetres below the top of the pool is the water at the depth you found in part c?



17. When parachuting from an airplane, parachutists wait 6 seconds before they pull the ripcord to release the parachute.
- Use the formula $d = 4.9t^2$, where d is the distance in metres and t is the time in seconds, to calculate the distance that a parachutist falls during the first 6 seconds of a jump.
 - The approximate speed (s) in kilometres per hour that the parachutist travels during free fall (when the parachute is closed) can be found using the formula $s \approx 36t$, where t is the number of seconds after jumping from the plane. Find the approximate speed at which the parachutist is travelling when she pulls the ripcord.
 - The terminal velocity (maximum speed) of a parachutist is about 200 kilometres per hour. Has the parachutist reached terminal velocity at 6 seconds?
18. A student was discussing the formula for a hire car company, $C = 0.95D + 30$, where C is the total cost of the car hire and D is the distance travelled in kilometres. The student says that to hire a car from this rental company costs 95 cents a kilometre. Is the student correct? Explain.
19. Stopping distance is the distance required to bring a moving vehicle to a stop from the moment the breaks are applied. The formula to calculate the stopping distance, d metres, when travelling at a speed of v m/s is:

$$d = \frac{v^2}{2\mu g}$$

where μ is the coefficient of friction between the wheels and the road surface and g is gravitational force, where $g = 9.8 \text{ m/s}^2$.

Calculate the stopping distances under the following conditions. Give your answers to 2 decimal places.

- A car is travelling at a speed of 50 km/h on a dry road with $\mu = 0.8$.
- Road and weather conditions, such as rain, change the value of μ .
 - If $\mu = 0.45$, find the stopping distance when travelling at a speed of 100 km/h.
 - If $\mu = 0.15$, find the stopping distance when travelling at a speed of 100 km/h.
- Comment on your findings in part b.

1.3 Linear equations

1.3.1 Solving linear equations

- Equations are mathematical statements that show two equal **expressions**. This means that the left-hand side and the right-hand side of an equation are equal.
- A **linear equation** has pronumerals whose highest power is 1; for example $3a + 5 = 10$ or $6 - \frac{x}{2} = 8$.
- Linear equations can be solved using **inverse operations** and creating equivalent equations.
- When solving equations, the last operation performed on the pronumeral when building the equation is the first operation undone by applying inverse operations to both sides of the equation.

WORKED EXAMPLE 6

Solve $6(a - 3) = 72$.

THINK

- To create the equation, a had 3 subtracted from it and the result was multiplied by 6 to get 72. The last operation performed was multiplying by 6. To undo the $\times 6$ operation, divide both sides by 6, as shown in black, and simplify.
- Looking at the equivalent equation, $(a - 3 = 12)$, a has had 3 subtracted from it. To undo the $- 3$ operation, add 3 to both sides, as shown in red, and simplify.

WRITE

$$\begin{array}{l} \cancel{6}(a - 3) = \frac{72}{\cancel{6}} \\ a - 3 = 12 \end{array}$$

$$\begin{array}{l} a - 3 + 3 = 12 + 3 \\ a = 15 \end{array}$$

- Solutions to equations can be checked by substituting the solution back into the equation and comparing the left- and right-hand sides of the equation. For example, $a = 15$ is the solution for $6(a - 3) = 72$.

$$\begin{array}{lcl} \text{LHS} & = & 6(a - 3) \\ & = & 6(15 - 3) \\ & = & 6(12) \\ & = & 72 \end{array} \qquad \text{RHS} = 72$$

Therefore $a = 15$ is the solution for $6(a - 3) = 72$.

1.3.2 Equations where the pronumeral appears more than once

- When a pronumeral appears on both sides of an equation, inverse operations are used to collect the pronumerals into a single term.

WORKED EXAMPLE 7

Solve $4 + 2x = 5x - 8$.

THINK

- The pronumeral appears on both sides of the equation. Subtracting $2x$ from both sides of the equation will leave only one term containing the pronumeral: the term $3x$.
- Looking at the new equation formed, $4 = 3x - 8$, the last operation performed on x was to subtract 8. Add 8 to both sides to undo this operation, as shown in red.

WRITE

$$\begin{array}{l} 4 + 2x = 5x - 8 \\ 4 + 2x - 2x = 5x - 2x - 8 \\ 4 = 3x - 8 \end{array}$$

$$\begin{array}{l} 4 + 8 = 3x - 8 + 8 \\ 12 = 3x \end{array}$$

- 3 Looking at the new equation formed, $12 = 3x$, the last operation performed on x was to multiply by 3.

- Divide both sides by 3 to undo this operation, as shown in green.
- Simplify the fraction, as shown in pink.
- Rearrange the equation so x is on the LHS.

$$\frac{4 \cancel{12}}{\cancel{1}3} = \frac{\cancel{1}3x}{\cancel{1}3}$$

$$4 = x$$

$$x = 4$$

1.3.3 Developing equations from a word description

- To solve problems presented in words, turn the problem into an equation, solve the equation and use the solution to answer the problem.
 - **Step 1:** Read the question carefully and identify the unknown variables.
 - **Step 2:** Construct an appropriate equation using your variable.
 - **Step 3:** Solve the equation.
 - **Step 4:** Check your solution to see if it makes sense in the context of the question.
 - **Step 5:** Answer the question.

WORKED EXAMPLE 8

The ages of three sisters add to 35 years. If the eldest is 4 years older than the middle sister and the youngest is 2 years younger than the middle sister, calculate the ages of the three sisters.

THINK

- 1 Read the question carefully and identify the unknown variables.
- 2 The sum of the sisters' ages is 35 years. Construct an appropriate equation using your variable.
- 3 Solve the equation by:
 - subtracting 2 from both sides
 - dividing both sides by 3.
- 4 Check your solution to see if it makes sense in the context of the question.
It is reasonable for the middle sister to be 11 years old, which is the solution to the equation. This makes the eldest sister $11 + 4 = 15$ years old and the youngest sister $11 - 2 = 9$ years old. The ages of the three sisters add to 35 years.
- 5 Answer the question.

WRITE

Let m be the middle sister's age.
Then the eldest sister is $m + 4$ years old and the youngest sister is $m - 2$ years old.

$$\begin{aligned} &\text{Middle sister's age} \\ &+ \text{eldest sister's age} \\ &+ \text{youngest sister's age} \\ &= 35 \text{ years} \\ m + m + 4 + m - 2 &= 35 \end{aligned}$$

$$\begin{aligned} 3m + 2 &= 35 \\ 3m + 2 - 2 &= 35 - 2 \\ 3m &= 33 \\ \frac{3m}{3} &= \frac{33}{3} \\ m &= 11 \end{aligned}$$

If $m = 11$,

$$\begin{aligned} \text{LHS} &= 3m + 2 & \text{RHS} &= 35 \\ &= 3 \times 11 + 2 \\ &= 33 + 2 \\ &= 35 \end{aligned}$$

LHS = RHS
Therefore, $m = 11$ is a solution.

The ages of the three sisters are 15, 11 and 9.

WORKED EXAMPLE 9

A gardener constructs a fence with three strands of wire around a rectangular garden bed. She bought a roll of wire that was 100 m long and has 4 m left over. If the length of the garden bed is 2 m longer than its width, calculate the dimensions of the garden bed.

THINK

- 1 Read the question carefully and identify the unknown variables.
- 2 There are three strands of wire around the garden bed and 4 m left over. Construct an equation to represent this using the variables.

- 3 Solve the equation by:
 - subtracting 16 from both sides
 - dividing both sides by 12.

- 4 Check your answer to see if it makes sense in the context of the question.
If $w = 7$, the length is 9 ($w + 2$). One strand of wire would be $14 + 18 = 32$ m long. Three strands would be 96 m long. That leaves 4 m left of the 100 m of wire. So the answer is reasonable.

- 5 Answer the question.

WRITE

Let w = the width of the garden bed.

The length will be $w + 2$.

$$\begin{aligned}\text{One strand (perimeter)} &= 2(w) + 2(w + 2) \\ &= 2w + 2w + 4 \\ &= 4w + 4\end{aligned}$$

$$\begin{aligned}\text{For three strands: } &3(4w + 4) \\ &= 12w + 12\end{aligned}$$

$$\begin{aligned}\text{Add leftover wire: } &12w + 12 + 4 \\ &= 12w + 16\end{aligned}$$

$$\text{Total length of wire} = 100$$

$$\text{Equation: } 12w + 16 = 100$$

$$\begin{aligned}12w + 16 - 16 &= 100 - 16 \\ 12w &= 84 \\ \frac{12w}{12} &= \frac{84}{12} \\ w &= 7\end{aligned}$$

If $w = 7$,

$$\begin{aligned}\text{LHS} &= 12w + 16 & \text{RHS} &= 100 \\ &= 12(7) + 16 \\ &= 100\end{aligned}$$

$$\text{LHS} = \text{RHS}$$

Therefore, $w = 7$ is a solution.

The garden bed is 7 m wide and 9 m long.

eBookplus RESOURCES

Interactivity: Solving linear equations (int-6450)

Exercise 1.3 Linear equations

Understanding, fluency and communicating

1. **WE6** Solve the following equations.

a. $3(a + 6) = 24$

b. $2(b - 7) = 22$

c. $4(c + 7) = 51$

d. $7(d - 15) = 61$

2. Solve the following equations by using inverse operations and performing the same operations on both sides.

a. $2a = 18$

b. $b - 4 = 48$

c. $\frac{c}{8} = 2$

d. $4f = 16$

e. $g + 7 = 11$

f. $\frac{u}{3} = 4$

3. What is the first operation that would be undone when solving the following equations?

a. $3a + 9 = 87$

b. $(b - 7) + 3 = 5$

c. $\frac{(c - 2)}{5} + 11 = 8$

d. $\frac{3(2d + 7)}{11} = -1$

4. Solve the following equations by using inverse operations.

a. $2x + 4 = 10$

b. $3p - 2 = 7$

c. $7a + 15 = 57$

d. $11a - 13 = 119$

e. $\frac{c}{2} + 6 = 28$

f. $\frac{d + 8}{7} = 1$

g. $\frac{k - 10}{3} = 12$

5. **WE7** Solve each of the following equations and check your solutions.

a. $10a - 4 = 3a + 3$

b. $5b + 1 = 7b + 5$

c. $6 + 2c = 5c - 3$

d. $12 + 5d = 3 + 4d$

6. Solve the following equations and check your solutions.

a. $2k + 8 = -4 - k$

b. $g - 4 = 3g + 8$

c. $10r + 2 = 8r - 4$

d. $4w + 2 = 3w + 7$

e. $-2q + 7 = 4q + 19$

f. $2 - 3v = -2v - 11$

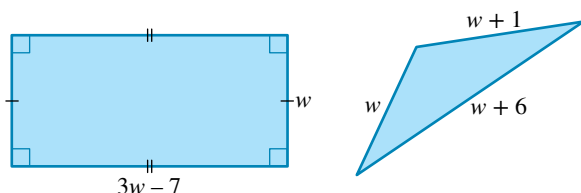
7. **WE8** The ages of three sisters add to 45 years. The eldest sister is 5 years older than the middle sister. The middle sister is 5 years older than the youngest sister. How old are the sisters?

8. A mother is five times as old as her son. Their ages add to 36 years. How old are they?

9. **WE9** A farmer constructs a fence with four strands of wire around a rectangular paddock. She bought a roll of wire that was 500 m long and has 180 m left over. If the length of the paddock is 6 m longer than its width, calculate the dimensions of the paddock.

10. A pair of shoes and matching bag cost \$175. If the shoes cost \$50 more than the bag, calculate the prices of the bag and the shoes.

11. The length of the rectangle below is 7 centimetres less than three times its width. If the perimeter of the rectangle is the same as the perimeter of the triangle, calculate the side lengths of the rectangle and triangle.



12. You have 12 more than three times the number of marker pens in your pencil case than your friend has in his pencil case. The teacher has 5 more than four times the number of marker pens in your friend's pencil case. Let your friend have x marker pens in his pencil case.

a. Write an expression for the number of marker pens in:

i. your pencil case

ii. your teacher's pencil case.

b. You have the same number of marker pens as the teacher. Write an equation to show this.

c. Calculate the number of marker pens in your friend's pencil case by solving the equation from part b.

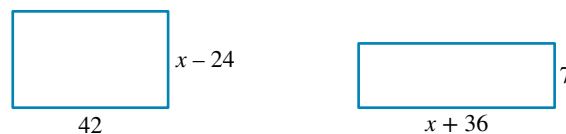
13. If a number is halved and then increased by 12, it will be six more than the original number. What is this original number?

14. During the walkathon around the school oval, you walk five laps more than your friend while your sister walks 10 laps more than you. The three of you raise \$96 at a rate of \$3 per lap. How many laps did each person walk?

Sponsor name	Number of laps	Total (\$)	Paid



15. The two rectangles pictured at right have the same area but different dimensions.



- Find the value of x .
- Find the dimensions of the rectangles.
- State the area of each rectangle.

16. For each of the following, write a linear equation and then solve it. In each case, let x equal the original number.
- When I multiply this number by 3, add 2 and divide by 4, the result is 5. Find the original number.
 - When I divide by 3, subtract 4 and multiply by 9, my answer is 9. Find the original number.
 - Three consecutive whole numbers add to 81. Find the three numbers.

Problem solving, reasoning and justification

17.



- Write a rule that could be used to calculate the cost of hiring a windsurfer for n hours.
 - Use your rule from part **a** to calculate the cost of hiring a windsurfer for 6 hours.
 - You have \$100 to spend. Write an equation to work out the number of hours you can hire a windsurfer for.
 - Solve the equation from part **c** to find the number of hours you can hire a windsurfer for. (Note: You can hire a windsurfer for a whole number of hours only.)
 - After windsurfing for as long as you can with \$100, you return the windsurfer. How much money will you have left over?
18. **a.** How many rides do you need so that option 1 and option 2 shown cost the same?
- b.** If you were planning on having only two rides, explain which option you would choose and why.
19. A classmate attempted to solve the problem $2x + 4 = 16 - 4x$. Her working is shown below. Correct her work and explain how she might solve the problem differently.

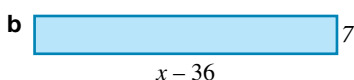
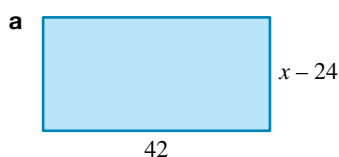
$$\begin{aligned}
 2x + 4 - 2x &= 16 - 4x - 2x \\
 4 &= 16 - 2x \\
 4 - 16 &= 16 - 2x - 16 \\
 -12 &= -2x \\
 -12 \div -2 &= -2x \div -2 \\
 x &= 6
 \end{aligned}$$



20. Your aunt from overseas sent an enormous box of chocolates for your mother's birthday. You were told not to eat chocolates. Nevertheless, sneaking into the pantry when your mother was out, you ate chocolates for five days, each day eating six more than the day before. After five days, you were caught out; it was discovered that you had eaten 100 chocolates, which made your mother extremely cross. How many chocolates had you eaten on *each* of the five days?



21. On her birthday today, a mother is three times as old as her daughter will be in eight years time. It just so happens that the mother's age today is the same as their house number, which is four times the value of thirteen minus the daughter's age.
- Write each sentence above as an expression. (*Hint: Make the daughter's age the unknown variable.*)
 - Use these two expressions to make an equation and solve it.
 - How old are the mother and the daughter?
 - What is the number of their house?
22. If the rectangles below have the same numerical value for their perimeter and area, calculate their dimensions.



1.4 Blood alcohol content

1.4.1 The blood alcohol content formulae

- Blood alcohol content (BAC)** measures the amount of alcohol that is in the bloodstream at any given time; it is measured in grams of alcohol per 100 millilitres of blood. For example, a BAC of 0.05% means there is 0.05 grams (or 50 milligrams) of alcohol in every 100 millilitres of blood.
- The formulae vary by:
 - gender
 - N , the number of standard drinks consumed
 - H , the number of hours of drinking
 - M , the person's weight, in kilograms
- To estimate BAC, the formulae are:

$$\text{BAC}_{\text{Male}} = \frac{10N - 7.5H}{6.8M} \quad \text{and} \quad \text{BAC}_{\text{Female}} = \frac{10N - 7.5H}{5.5M}$$

WORKED EXAMPLE 10

Find the BAC of a 95 kg male who consumed 4 standard drinks over a period of 3 hours.

THINK

- 1 State the BAC formula for males.
- 2 Substitute $N = 4$, $H = 3$, $M = 95$ into the formula and evaluate.
- 3 Answer the question.

WRITE

$$\begin{aligned}\text{BAC}_{\text{Male}} &= \frac{10N - 7.5H}{6.8M} \\ &= \frac{10 \times 4 - 7.5 \times 3}{6.8 \times 95} \\ &= 0.027\end{aligned}$$

1.4.2 Time to reach zero BAC

- After a person stops consuming alcohol, the time (t hours) taken for a person to reach a BAC of zero is given by the formula:

$$t = \frac{\text{BAC}}{0.015}$$

WORKED EXAMPLE 11

Marianne is at a party. She knows she has consumed 3 standard drinks.

If Marianne weighs 58 kg, calculate:

- a her blood alcohol content after 3.5 hours, giving your answer correct to 3 decimal places
- b the time she will need for her BAC to reach zero, now that she has stopped drinking.

THINK

- 1 State the BAC formula for females.
 - 2 Substitute $N = 3$, $H = 3.5$, $M = 58$ and evaluate.
 - 3 Answer the question.
- 1 State the formula.
 - 2 Substitute $\text{BAC} = 0.012$ and evaluate.
 - 3 Convert the time to minutes.
 - 4 Answer the question.

WRITE

$$\begin{aligned}\text{BAC}_{\text{Female}} &= \frac{10N - 7.5H}{5.5M} \\ &= \frac{10 \times 3 - 7.5 \times 3.5}{5.5 \times 58} \\ &= 0.011\,755\,5 \\ &\approx 0.012 \text{ (correct to 3 decimal places)}\end{aligned}$$
$$\begin{aligned}t &= \frac{\text{BAC}}{0.015} \\ &= \frac{0.012}{0.015} \\ &= 0.8 \text{ hours} \\ 0.8 \times 60 &= 48 \text{ minutes}\end{aligned}$$

The time to reach zero BAC is 48 minutes.

1.4.3 Limitations of estimating BAC

- These formulae are only estimates as other factors may be involved.
Limitations include:
 - food in the stomach, which can decrease the rate alcohol passes into the bloodstream
 - alcohol interacting with other medications, which can affect the reading of the BAC
 - underestimating the number of standard drinks consumed or the length of time drinking.

eBookplus RESOURCES

Interactivity: Blood alcohol content (int-6905)

Exercise 1.4 Blood alcohol content

Note: Unless otherwise stated, calculate blood alcohol content to 3 decimal places.

Understanding, fluency and communicating

- WE10** Find the BAC of a 105 kg male who consumed 3 standard drinks over a period of 2 hours.
- If a female, weighing 54 kg, consumed 3 standard drinks over a period of 2 hours, calculate her BAC.
- Calculate the BAC for a 93 kg male who has consumed 3 standard drinks over a period of 90 minutes.
- WE11** Jessica is at a party. She knows she has consumed 4 standard drinks.
If Jessica weighs 76 kg, calculate:
 - her blood alcohol content after 5 hours, giving your answer correct to 3 decimal places
 - the time she will need for her BAC to reach zero, now that she has stopped drinking.
- Peter is out with friends. He knows he has had 4 standard drinks in the last 2 hours and now stops drinking. Calculate:
 - his blood alcohol content, if he weighs 85 kg
 - the time he will need for his blood alcohol level to reach zero.
- If a person had a BAC reading of 0.05, how long would it take for their BAC reading to reach zero?
- A P-plate driver must have a zero blood alcohol content reading to drive legally. How long would a P-plate driver with a BAC of 0.02 need to wait without drinking alcohol to legally drive?



- At a party, friends are discussing the time they would need to wait to reach a zero blood alcohol reading. How long should they allow if the reading was 0.065?
- Matilda is a P-plate driver. She weighs 62 kg and has consumed 3 standard drinks in the past 3.5 hours.
 - Calculate her blood alcohol content.
 - How long does she need to wait, without drinking alcohol, before she can legally drive?
- Robert is a personal trainer, weighing 81 kg, and holds a P-plate licence. He has had 2 standard drinks in the last 45 minutes. Determine how long Robert needs to wait, without consuming alcohol, to legally drive. Give your answer to the nearest minute.

Problem solving, reasoning and justification

- Two friends, Andrew and Anita, are enjoying an evening relaxing. Each has had 3 standard drinks in 2 hours and each weighs 70 kg. Discuss the difference in their blood alcohol content.
- Andrew and Anita need to decide who can drive home. Both have P-plates. How long would they need to wait before driving, and who can legally drive first?



13. A male, weighing 71 kg, has been drinking for 45 minutes. He thinks he has consumed 3 standard drinks. Is he able to drive legally? Explain your answer by calculating his BAC to 4 decimal places.
14. Mary, weighing 58 kg, has consumed 2 standard drinks in 45 minutes. Is she able to legally drive? Explain your answer.
15. A person has taken 2.5 hours to reach a zero blood alcohol reading.
 - a. Calculate the blood alcohol content when they stopped consuming alcohol.
 - b. How many standard drinks would a man, weighing 93 kg, have consumed over 4 hours to reach this BAC? Give your answer to 1 decimal place.

1.5 Medication dosages

1.5.1 Fried's formula

- Fried's formula** is a method of estimating the dosage of medication for an infant under 2 years old. It is calculated by multiplying the infant's age (in months) by the adult dosage, and dividing the result by 150.
- Fried's formula is:

$$D = \frac{mA}{150}, \text{ where:}$$

D is the dosage for an infant

m is the age of the infant (in months)

A is the adult dosage.

WORKED EXAMPLE 12

Use Fried's formula to calculate the dosage of medication to be given to a 10-month-old infant if the recommended adult dose is 30 mL.

THINK

- State Fried's formula.
- Substitute $m = 10$, $A = 30$ into the formula and evaluate.
- Answer the question.

WRITE

$$\begin{aligned} D &= \frac{mA}{150} \\ &= \frac{10 \times 30}{150} \\ &= 2 \end{aligned}$$

The infant dosage is 2 mL.

1.5.2 Young's formula

- Young's formula** is a method of estimating the dose of medication for a child by multiplying the child's age (in years) by the adult dose, and dividing the result by the sum of the child's age and 12.
- Young's formula is:

$$D = \frac{yA}{(y + 12)}, \text{ where:}$$

D is the dosage for a child

y is the age of the child (in years)

A is the adult dosage.

WORKED EXAMPLE 13

Use Young's formula to calculate the dosage of medication to be given to an 11-year-old child if the recommended adult dose is 500 mg. Give your answer to the nearest 10 mg.

THINK

- 1 State Young's formula.
- 2 Substitute $y = 11$ and $A = 500$ into the formula and evaluate.
- 3 Round the answer to the nearest 10 mg and answer the question.

WRITE

$$\begin{aligned}
 D &= \frac{yA}{(y + 12)} \\
 &= \frac{11 \times 500}{(11 + 12)} \\
 &= 239.130\ 43
 \end{aligned}$$

The child's dosage is 240 mg.

1.5.3 Clark's formula

- **Clark's formula** is another method of estimating the dose of medication for a child, using the weight of the child instead of the age of the child. Originally, Clark's formula used the weight of the child in pounds, not kilograms. The formula multiplied the child's weight (in pounds) by the adult dose, and then divide the result by 150. The formula below has been converted to kilograms.
- Clark's formula is:

$$D = \frac{kA}{68}, \text{ where:}$$

D is the dosage for a child

k is the weight of the child (in kilograms)

A is the adult dosage.

WORKED EXAMPLE 14

Use Clark's formula to calculate the dosage of medication for an 11-year-old child, weighing 32 kg, if the recommended adult dose is 500 mg. Give your answer to the nearest 10 mg.

THINK

- 1 State the formula.
- 2 Substitute $k = 32$, $A = 500$ into the formula and evaluate.
- 3 Round the answer to the nearest 10 mg and answer the question.

WRITE

$$\begin{aligned}
 D &= \frac{kA}{68} \\
 &= \frac{32 \times 500}{68} \\
 &= 235.29
 \end{aligned}$$

The child's dosage is 240 mg.

eBookplus RESOURCES

Interactivity: Medication dosages (int-6906)

Exercise 1.5 Medication dosages

Understanding, fluency and communicating

1. **WE12** Use Fried's formula to calculate the dosage of medication for a 15-month-old infant if the recommended adult dose is 50 mL.
2. If an adult dosage was 35 mL, use Fried's formula to calculate the dosage for a 6-month-old infant.
3. Calculate the dosage for a 2-year-old, using Fried's formula, if the adult dosage is 60 mL.



4. **WE13** Use Young's formula to calculate the dosage of medication for a 14-year-old child if the recommended adult dose is 250 mg. Give your answer to the nearest 10 mg.
5. **WE14** If an adult dosage for a particular medication is 300 mg, use Young's formula to calculate the dosage for a 12-year-old weighing 37 kg.
6. Use Clark's formula to calculate the dosage of medication for an 8-year-old child, weighing 24 kg, if the recommended adult dose is 200 mg. Give your answer to the nearest 10 mg.
7. If an adult dosage for a particular medication was 300 mg, use Clark's formula to calculate the dosage for a 12-year-old, weighing 37 kg. Give your answer to the nearest 10 mg.
8. Sienna is 15 years old and weighs 52 kg. The adult dosage for a certain medication is 800 mg. Calculate the dosage, to the nearest 10 mg, that should be given to Sienna using:
 - a. Young's formula
 - b. Clark's formula.
9. The adult dosage for a certain medication is 1000 mg. Joshua is 17 years old and weighs 64 kg. Calculate the dosage, to the nearest 10 mg, that should be given to Joshua using:
 - a. Young's formula
 - b. Clark's formula.
10. The adult dosage for a certain medication is 150 mg. Ben is $1\frac{1}{2}$ years old and weighs 11 kg. Calculate the dosage, to the nearest milligram, that should be given to Ben using:
 - a. Fried's formula
 - b. Young's formula
 - c. Clark's formula.



Problem solving, reasoning and justification

11. The recommended adult dosage for a certain medication is 500 mg per day. Chelsea and Terry are both 15 years old. Chelsea weighs 52 kg and Terry weighs 56 kg.
 - a. Rounding to the nearest mg, compare their dosages for this medication using:
 - i. Young's formula
 - ii. Clark's formula.
 - b. Comment on your results from part a.
 - c. Could Fried's formula be used? Justify your answer.
12. A cough mixture states the recommended dosages on its label. Below are the recommended dosages.

Adults:	5 mL orally every 4 to 6 hours, not to exceed 30 mL daily
6 to 11 years:	2.5 mL orally every 4 to 6 hours, not to exceed 15 mL daily
12 years or older:	5 mL orally every 4 to 6 hours, not to exceed 15 mL daily.

 Alex is a 10-year-old weighing 34 kg.
 - a. Using the recommended dosages given, state the dosage of cough mixture that should be given to Alex.
 - b. Calculate the dosage for Alex, to 1 decimal place, using:
 - i. Clark's formula
 - ii. Young's formula.
 - c. Compare your results from parts a and b.

- 13.** The instructions on a bottle of medicine are provided below:
- If you are an adult or over 12 years old, take 5 to 10 mL every four to six hours.
 - For a child between 4 to 5 years, give 2 to 2.5 mL every four to six hours.
 - For a child between 6 to 12 years, give 2.5 to 5 mL every four to six hours.
 - Do not give this medicine to a child under the age of 4.



For a 5-year-old child:

- Calculate the dosage, to the nearest half mL, using Young's formula.
 - Compare your answer to part **a** with the instructions on the bottle.
- 14.** The typical weight of a 3-year-old child is 14 kg. The adult dosage of a certain medication is 800 mg.
- Calculate the dosage for the child, to the nearest mg, using:
 - Fried's formula
 - Young's formula
 - Clark's formula.
 - Can all of these formulae be used? Comment and justify your answer.
- 15.** Two 13-year-olds, Matthew and Melanie, are prescribed the same medication. The adult dosage for this medication is 30 mL. Matthew weighs 48 kg and Melanie weighs 43 kg.
- Calculate their dosages, to the nearest mL, using:
 - Young's formula
 - Clark's formula.
 - Comment on the results found in part **a**.

1.6 Review

1.6.1 Summary

In this topic you have learnt:

- how to substitute a number in place of a pronumeral
- how to evaluate the answer after substituting
- the relationships between different quantities
- how to take care with the specific units of the pronumerals
- how to solve problems involving formulae by:
 - substitution and solving for the unknown quantity
 - changing the subject of the formula, then substituting and evaluating
 - solving the equations found using inverse operations
- the definition of a linear equation
- how to solve linear equations using inverse operations
- how to construct an equation using your variables
- to check your solution to see if it makes sense in the context of the question
- the BAC formulae for males and females
- the time taken to reach a BAC of zero
- how to use Fried's formula
- how to use Young's formula
- how to use Clark's formula.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 1.6 Review

Understanding, fluency and communicating

1. If $m = 3$ and $n = 5$, evaluate the following expressions.
 - a. $m + n$
 - b. $4m + 2n$
 - c. $6(m + 4)$
2. If $m = 3$ and $n = 5$, evaluate the following expressions.
 - a. $4m^2 - 2n$
 - b. $\frac{m + n}{2}$
 - c. $\sqrt{m^2 + n^2 + (n - m)}$
3. Use inverse operations to solve the following equations.
 - a. $4a = 24$
 - b. $\frac{c}{3} = 21$
 - c. $2d - 6 = 14$
 - d. $\frac{e + 7}{2} = 15$
 - e. $3(f - 2) = 24$
 - f. $\frac{3(g + 4)}{5} = 6$
4. Use inverse operations to solve the following equations.
 - a. $5w - 2 = 3w + 8$
 - b. $4e + 3 = 5e - 7$
 - c. $3 - r = 2r + 9$
5. Solve the following equations.
 - a. $4x + 3 = 27$
 - b. $4(x - 1) = 12$
 - c. $5 + \frac{x}{2} = 12$
 - d. $\frac{x - 4}{3} = 5$
6. Solve the following equations.
 - a. $2x - 4 = 8x - 20$
 - b. $4(2 - x) = -3(x + 4)$
 - c. $\frac{x - 4}{3} + 4 = \frac{x + 7}{2}$
7. The area of a circle (A) in cm^2 can be found using the formula $A = \pi \times r^2$, where $\pi = 3.142$ (approximately) and r is the radius of the circle in cm. Use this formula to calculate the area of a circle with a radius of 15 cm.
8. The cost of taking a taxi (C) in dollars includes a flag fall of \$3.50 plus \$2 per km. The equation for this is $C = 2k + 3.50$, where k is the distance travelled in km. How much would it cost to travel:
 - a. 22 km
 - b. 15.5 km?



9. You have agreed to take on a babysitting job for your mother's friend. She says she will pay you according to the following formula:

$$P = 10 + 8h$$

where P is the pay for one evening's babysitting (\$) and h is the number of hours spent babysitting on that evening.

- a. On the first evening, you babysat for 4 hours. How much money did you earn?
- b. On another occasion, you earned a total of \$66. How long did you spend babysitting?

10. A local courier company uses the formula $C = 3.5h + 5$, where h is the number of kilometres and C is the cost of the delivery in dollars, to calculate the total delivery cost.
- Calculate the cost of a delivery for a journey of 18 km.
 - Transpose the formula to make h the subject of the formula.
 - If the cost of a delivery is \$43.50, calculate how many kilometres the delivery was from the courier company base.
11. The formula to calculate the body mass index is $B = \frac{m}{h^2}$, where m is the mass in kilograms and h is the height in metres.
- Calculate a person's body mass index if they weigh 58 kg and are 176 cm tall. Give your answer correct to 3 decimal places.
 - Rearrange the formula to make m the subject.
 - Calculate the weight of a person, 196 cm tall, whose body mass index is 19.5. Give your answer correct to the nearest kilogram.
12. The simple interest formula is given by $I = \frac{PRN}{100}$, where P is the principal, R the rate of interest per year, and N the number of years of the investment.
- What interest would you earn if you invested \$15 000 at 3.2% for 2 years?
 - Calculate the interest earned on an investment of \$8500 at 2.8% for 15 months?
 - Change the subject of the formula to R , the rate of interest per annum.
 - Your investment of \$5600 for 18 months earned \$210 in interest. Find the rate of interest on your investment.
13. Three children are spaced three years apart in ages. Let the youngest child be x years old.
- What are the ages of the other two children in terms of x ?
 - If the oldest is three times as old as the youngest, how old are the three children?
14. Three brothers get an average of \$25 pocket money per week. The eldest brother gets \$5 more than the middle brother, while the youngest gets \$2 less than the middle brother. How much money does each brother get?
15. The distance, D km, travelled at a speed of s km/h for t hours is given by the formula $D = st$.
- Calculate the distance travelled at 90 km/h for:
 - 2.5 hours
 - 90 minutes.
 - Calculate the average speed if a car travels 240 kilometres in:
 - 3 hours
 - 150 minutes.
 - Find the time needed to travel a distance of 890 kilometres at an average speed of:
 - 75 km/h
 - 60 km/h.
16. Calculate the blood alcohol content of a male weighing 95 kg who consumed 4 standard drinks over a period of 2 hours. Give your answer to 3 decimal places.
17. If a female, weighing 63 kg, consumes 2 standard drinks over a period of 3 hours, calculate her blood alcohol content.
18. a. Calculate the blood alcohol content, to 3 decimal places, for a 78 kg male who has consumed 2 standard drinks over a period of 90 minutes.
- b. How long would he need to wait, without drinking alcohol, for his blood alcohol content to register zero?
19. Use Fried's formula to calculate the dosage of medication to be given to a 17-month-old infant if the recommended adult dose is 35 mL. Give your answer to the nearest mL.
20. The adult dosage for a certain medication is 250 mg. Andrew is 15 years old and weighs 54 kg. Calculate the dosage, to the nearest 10 mg, that should be given to Andrew using:
- Young's formula
 - Clark's formula.

21. **MC** For the equation $2(3x - 7) = 10$, the value of x is:

A. $-\frac{11}{6}$

B. 5

C. $\frac{17}{5}$

D. 4

E. $-\frac{2}{3}$

22. **MC** An equation of motion is given by $v = u + at$. Changing the subject of the formula to t gives:

A. $t = \frac{u - v}{a}$

B. $t = \frac{v - u}{a}$

C. $t = \frac{v}{a} - u$

D. $t = u - \frac{v}{a}$

E. $t = a(v - u)$

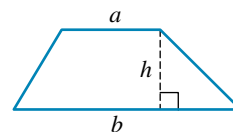
23. The area of the trapezium shown is given by the formula $A = h\left(\frac{a + b}{2}\right)$.

Rearrange the formula to make:

a. a the subject of the formula

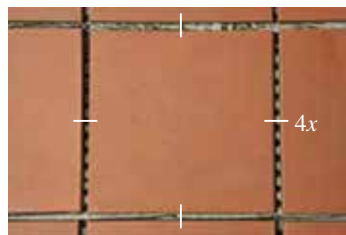
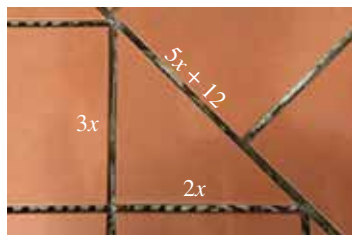
b. b the subject of the formula

c. h the subject of the formula.



Problem solving, reasoning and justification

24. A tile manufacturer produces tiles that have the following side lengths. All measurements are in centimetres.



a. Write an expression for the perimeter of each shape.

b. Calculate the value of x for which the perimeter of the triangular tile is the same as the perimeter of the square tile.

25. Let x be a number.

a. What number is five more than x ?

b. Write an expression to represent four times the result of five more than x .

c. Write an expression to represent five less than twice the number x .

d. Four times five more than a number is equal to five less than twice the number. What is the number?

26. Let x be a number.

a. What are the next two consecutive numbers after x ?

The sum of these three consecutive numbers is 81.

b. Write an equation to represent this information.

c. What are the three numbers?

27. The surface area of a sphere, A , is given by the formula

$A = 4\pi r^2$, where r is the radius of the sphere. A globe of the world is approximately spherical, with a diameter of 45 cm.

a. State the radius of the globe.

b. Calculate the surface area of the globe, giving your answer to the nearest 100.

c. The globe is to be protected with a special clear paint finish. The cost of the special finish is \$5.67 per 100 cm² or part thereof. Find the cost, to the nearest 5 cents, of protecting the globe.



28. The formula for the volume, V , of a sphere is $V = \frac{4}{3}\pi r^3$,

where r is the radius of the sphere.

The formula for the volume, V , of a cylinder is $V = \pi r^2 h$,
where r is the radius and h the height of the cylinder.



- a. A cylindrical container is 20 cm high and has a radius of 15 cm. Find the volume of the container to the nearest cm^3 .
- b. A spherical ball has a radius of 5 cm. Find the volume of the ball to the nearest cm^3 .
- c. The container from part a is full of water. Four of the balls from part b are dropped into the container, resulting in water spilling out of the container. How much water, to the nearest litre, will remain in the container?
29. Suzanna, 23 years old and weighing 56 kg, has consumed 2 standard drinks in 50 minutes.
- a. Is she able to drive legally? Explain your answer.
- b. How long would she need to wait, without drinking alcohol, for her blood alcohol content to be zero? Give your answer to the nearest minute.
30. It has taken Nick 90 minutes to reach a zero blood alcohol reading.
- a. Calculate his blood alcohol content, to 3 decimal places, when he stopped consuming alcohol.
- b. Nick weighs 86 kg and had been drinking previously for 2.5 hours. How many standard drinks had he consumed? Give your answer to 1 decimal place.
31. The typical weight of a 10-year-old girl is 33 kg with a height of 138 cm. The adult dosage of a certain medication is 425 mg.
- a. Calculate the dosage for the child, to the nearest mg, using:
- Fried's formula
 - Young's formula
 - Clark's formula.
- b. Can any of these formulae be used? Justify your answer.
32. Two six-year-olds, Angelina and Leonard, are prescribed the same medication. The adult dosage for this medication is 100 mL. Angelina weighs 19 kg and Leonard weighs 22 kg.
- a. Calculate their dosages, to the nearest mL, using:
- Young's formula
 - Clark's formula.
- b. Comment on the results found in part a.
33. Francis is 9 years of age, weighs 28 kg and is 133 cm tall. He has been given a 45 mL dose of medication.
- a. Calculate the adult dosage, to the nearest mL, using:
- Young's formula
 - Clark's formula.
- b. Comment on the results found in part a.
- c. Would it have been suitable to use Fried's formula to calculate the adult dosage? Justify your answer.

Answers

Exercise 1.2 Evaluating expressions

1. a. 24 b. 39 c. 45 d. 69 e. 14 f. 49
2. a. 115 b. 58 c. 5 d. $\frac{2}{3}$ e. 48 f. 5
3. a. 11 b. 23 c. 3 d. 103 e. 56
4. 432
5. a. \$60 b. \$80 c. \$115 d. \$65
6. a. i. 14 cm ii. 36 cm b. i. 10 cm² ii. 45 cm²
7. $n = \frac{T - a}{d}$
8. a. $A = \frac{C - D}{B}$ b. $t = \frac{v - u}{a}$ c. $h = \frac{3V}{A}$
9. a. $P = 2(l + w)$ b. 72 cm
c. i. $l = \frac{P - 2w}{2}$ ii. 72 cm
10. a. $h = \frac{A - 2\pi r^2}{2\pi r}$ b. 5.5 cm
11. a. i. 360 km ii. 200 km
b. i. 75 km/h ii. 96 km/h
c. i. 8 hours ii. 10 hours and 40 minutes
12. a. 104 °F
b. $C = \frac{5(F - 32)}{9}$
c. Seattle: 9 °C
Denver: -4 °C
San Diego: 19 °C
New York: -2 °C
Orlando: 23 °C
13. $4abc$ means $4 \times a \times b \times c = 4 \times 2 \times 3 \times 4 = 96$
14. a. Venue A: \$100 Venue B: \$50 b. Venue A: \$5 Venue B: \$5.75 c. Venue B
15. a. 11.925 cm b. 7103.36 cm³
16. a. 201.6 m³ b. $d = \frac{V}{lw}$ c. 1.6 metres d. 20 cm
17. a. 176.4 metres b. 216 km/h c. Yes
18. The student is partially correct. The company costs \$0.95 a kilometre plus a fixed fee of \$30.
19. a. 12.30 metres
b. i. 87.48 metres ii. 262.45 metres
c. When μ (the coefficient of friction between the wheels and the road) is smaller, the stopping distance at the same speed is a lot longer.

Exercise 1.3 Linear equations

1. a. $a = 2$ b. $b = 18$ c. $c = 5.75$ d. $d = \frac{166}{7}$
2. a. $a = 9$ b. $b = 52$ c. $c = 16$ d. $f = 4$ e. $g = 4$ f. $u = 12$
3. a. Subtract 9 b. Remove the brackets and simplify
c. Subtract 11 d. Multiply by 11
4. a. $x = 3$ b. $p = 3$ c. $a = 6$ d. $a = 12$
e. $c = 44$ f. $d = -1$ g. $k = 46$
5. a. $a = 1$ b. $b = -2$ c. $c = 3$ d. $d = -9$
6. a. $k = -4$ b. $g = -6$ c. $r = -3$ d. $w = 5$ e. $q = -2$ f. $v = 13$
7. 10, 15 and 20 years of age

8. The son is 6 years old and his mother is 30 years old.
9. 23 metres long and 17 metres wide
10. Bag: \$62.50, shoes: \$112.50
11. Rectangle: 4.2 cm, 5.6 cm
Triangle: 4.2 cm, 5.2 cm, 10.2 cm
12. a. i. $3x + 12$, where your friend has x marker pens
ii. $4x + 5$
b. $3x + 12 = 4x + 5$ c. 7 marker pens
13. 12
14. Your friend walked 4 laps, you walked 9 laps and your sister walked 19 laps.
15. a. $x = 36$
b. Dimensions of first rectangle: 42 cm by 12 cm
Dimensions of second rectangle: 72 cm by 7 cm
c. 504 cm^2
16. a. 6 b. 15 c. 26, 27, 28
17. a. $C = 50 + 15n$, where C = cost of hiring, n = hours hired
b. \$140 c. $100 = 50 + 15n$ d. 3 hours e. \$5
18. a. 5 rides
b. Option 2: it costs \$12 for the two rides, which is \$3 cheaper than option 1.
19. $2x + 4 = 16 - 4x$
 $2x + 4 - 2x = 16 - 4x - 2x$
 $4 = 16 - 6x$ (error with directed numbers)
 $4 - 16 = 16 - 6x - 16$
 $-12 = -6x$
 $\frac{-12}{-6} = \frac{-6x}{-6}$
 $x = 2$
20. 8, 14, 20, 26, 32
21. a. Mother's age = $3(d + 8)$
House number = $4(13 - d)$
b. $3(d + 8) = 4(13 - d)$
 $d = 4$
c. Mother: 36 years old
Daughter: 4 years old
d. 36
22. a. 42 cm by 2.1 cm b. 7 cm by 2.8 cm

Exercise 1.4 Blood alcohol content

1. 0.021
2. 0.051
3. 0.030
4. a. 0.006 b. 24 minutes
5. a. 0.043 b. 2 hours and 52 minutes
6. 3 hours and 20 minutes
7. 1 hour and 20 minutes
8. 4 hours and 20 minutes
9. a. 0.011 b. 44 minutes
10. 1 hour and 44 minutes
11. Andrew's BAC is approximately 0.032 while Anita's BAC is approximately 0.039. While both had the same number of standard drinks in the same time and weigh the same, the difference comes from the denominators of their respective BAC formulae. The BAC formula for females divides by a smaller number for the same weight, so it will give a higher BAC.

12. They would need to wait at least 2 hours and 8 minutes before Andrew drives home.
13. His BAC is 0.0505, which is greater than 0.05, so he cannot legally drive as he is over the limit.
14. Mary's BAC is 0.045, which is just less than 0.05, so she can legally drive.
15. a. 0.038 b. 5.4

Exercise 1.5 Medication dosages

1. 5 mL
2. 1.4 mL
3. 9.6 mL
4. 130 mg
5. 150 mg
6. 70 mg
7. 160 mg
8. a. 440 mg b. 610 mg
9. a. 590 mg b. 940 mg
10. a. 18 mg b. 17 mg c. 24 mg
11. a. i. Chelsea: 278 mg; Terry: 278 mg
 ii. Chelsea: 382 mg; Terry: 412 mg
 b. Using Young's formula, the medication dosage is the same for both Chelsea and Terry as it depends on their age. They are both 15 years old, so the dosage for both is 278 mg per day.
 Clark's formula depends on the weight of the child. Since Terry weighs more than Chelsea, his dosage is higher than Chelsea's dosage, by an extra 30 mg per day.
 c. No, as Fried's formula is for infants under 2 years of age.
12. a. 2.5 mL orally every 4 to 6 hours, not to exceed 15 mL daily
 b. i. 2.5 mL ii. 2.3 mL
 c. Clark's formula gave the same dosage as the recommended dosage on the label.
 Young's formula gave a slightly lower dosage than the recommended dosage on the label. It was 0.2 mL less than the recommended dosage or the dosage obtained by using Clark's formula.
13. a. 1.5 mL to 3 mL every four to six hours
 b. The instructions on the bottle of medicine have the recommended dosage for a 5-year-old child as 2 to 2.5 mL every four to six hours. Using Young's formula, the dosage would be 1.5 mL to 3 mL every four to six hours, which is a wider range of 1.5 mL, not 0.5 mL as per the instructions.
14. a. i. 192 mg ii. 160 mg iii. 165 mg
 b. Since the child is 3 years old, Fried's formula would not be appropriate, as the child is older than 2 years of age. Using this formula, the dosage is considerably higher than the other two. Using Young's formula, based on age in years, and Clark's formula, based on weight, the dosages are very close, 160 and 165 mg, respectively. They vary by only 5 mg.
15. a. i. Matthew and Melanie: 16 mL
 ii. Matthew: 21 mL; Melanie: 19 mL
 b. Using Young's formula, the medication dosage is the same for both Matthew and Melanie as it depends on their age. They are both 13 years old, so the dosage is 16 mL for each of them. Clark's formula depends on the weight of the child. Since Matthew weighs more than Melanie, his dosage is higher than her dosage by 2 mL. Both are higher than using Young's formula.

Exercise 1.6 Review

1. a. 8 b. 22 c. 42
2. a. 26 b. 4 c. 6
3. a. $a = 6$ b. $c = 63$ c. $d = 10$ d. $e = 23$ e. $f = 10$ f. $g = 6$
4. a. $w = 5$ b. $e = 10$ c. $r = -2$
5. a. $x = 6$ b. $x = 4$ c. $x = 14$ d. $x = 19$
6. a. $x = \frac{8}{3}$ b. $x = 20$ c. $x = -5$
7. 706.95 cm^2
8. a. \$47.50 b. \$34.50

9. a. \$42 b. 7 hours
10. a. \$68 b. $h = \frac{C - 5}{3.5}$ c. 11 km
11. a. 18.724 b. $m = B \times h^2$ c. 75 kg
12. a. \$960 b. \$297.50 c. $R = \frac{100I}{PN}$ d. 2.5% per annum
13. a. $(x + 3)$ and $(x + 6)$ years of age.
b. 3, 6 and 9 years of age
14. \$22, \$24 and \$29 per week (in order of age)
15. a. i. 225 km ii. 135 km
b. i. 80 km/h ii. 96 km/h
c. i. 11 hours 52 minutes ii. 14 hours 50 minutes
16. 0.039
17. 0
18. a. 0.016 b. 1 hour 4 minutes
19. 4 mL
20. a. 140 mg b. 200 mg
21. D
22. B
23. a. $a = \frac{2A - hb}{h}$ b. $b = \frac{2A - ha}{h}$ c. $h = \frac{2A}{a + b}$
24. a. Perimeter of triangle = $10x + 12$
Perimeter of square = $16x$
b. $x = 2$
25. a. $x + 5$ b. $4(x + 5)$ c. $2x - 5$ d. -12.5
26. a. $(x + 1)$ and $(x + 2)$ b. $3x + 3 = 81$ c. 26, 27 and 28
27. a. 22.5 cm b. 6400 cm^2 c. \$362.90
28. a. $14\,137 \text{ cm}^3$ b. 524 cm^3 c. 12 litres
29. a. Suzanna's BAC is less than 0.05 so she can drive legally.
b. 2 hours 59 minutes
30. a. 0.023
b. Nick consumed 3.2 standard drinks in the 2.5 hours.
31. a. i. 340 mg ii. 193 mg iii. 206 mg
b. Since the child is 10 years old, Fried's formula should not be used, as the child is over 2 years of age. Fried's formula gave a very high dosage compared with the other two, which were both around 200 mg.
32. a. i. Angelina and Leonard: 33 mL
ii. Angelina: 28 mL; Leonard: 32 mL
b. Using Young's formula, the medication dosage is the same for both Angelina and Leonard as it depends on their age. They are both 6 years old, so the dosage for both is 33 mL. Clark's formula depends on the weight of the child. Since Leonard weighs more than Angelina, his dosage is higher than her dosage, by 4 mL. Both dosages for Leonard are approximately the same.
33. a. i. 105 mL ii. 109 mL
b. Using both formulae, the adult dose is approximately 110 mL.
c. No. Fried's formula is for infants, up to the age of 2 years old. Francis is 9 years old, so using Fried's formula would not be suitable to calculate either his dosage or an adult dosage from his dosage.

TOPIC 2

Linear relationships

2.1 Overview

Through understanding linear relationships we can begin to construct and analyse linear models. These models can be used to help us see and understand patterns that occur in the world.

Linear models can be used to help us solve problems such as the distance covered by an airplane travelling at constant speed, or how much fuel can be purchased with a given amount of money.



DISCUSSION

Can you think of some more real-life situations which can be modelled by linear relationships?
Are there any limitations that must be put on these models in order for them to work?

LEARNING SEQUENCE

- 2.1 Overview
- 2.2 Direct variation
- 2.3 The gradient and intercepts
- 2.4 Sketching linear graphs
- 2.5 Linear models
- 2.6 Review

CURRICULUM CONTENT

Students:

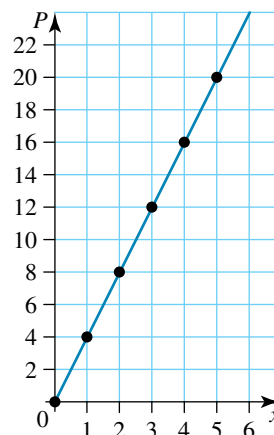
- model, analyse and solve problems involving linear relationships, including constructing a straight-line graph and interpreting features of a straight-line graph, including the gradient and intercepts (ACMMM002, ACMMM003) **AAM** ◇ 🌱 ⚙️
- construct and analyse a linear model, graphically or algebraically, to solve practical direct variation problems, including but not limited to the cost of filling a car with fuel or a currency conversion graph (ACMGM043) **AAM** ◇ 🌱 👤

2.2 Direct variation

2.2.1 Defining direct variation

- Two quantities vary directly, or are said to be directly proportional, when one quantity is a constant multiple of the other quantity.
- An example of **direct variation** can be seen in the relationship between the sides of a square and its perimeter. If the side of a square is 10 cm, then the perimeter is 40 cm. Or generally, if the side of the square is x cm, then the perimeter, P cm, is 4 times the side. This can be written simply as $P = 4x$. As x increases, P also increases and is always four times the length of the side.
- This relationship can also be illustrated in a table of values or graphically.

x	0	1	2	3	4	5
P	0	4	8	12	16	20



Since the sides of squares do not have to be integers, the dots can be joined to produce a straight-line graph.

- A direct variation relationship produces a straight-line graph which passes through the origin. It is a **linear function** in the form $y = mx$, where m is the constant multiple.

WORKED EXAMPLE 1

A special drink is being sold for \$3 per can. Anita decides to purchase up to 8 cans.

Is there a direct variation relationship between the total cost of the cans and the number of cans purchased? If there is a direct variation, determine the relationship.

THINK

- Identify the two quantities and choose a pronumeral to represent them.
- Use the information to complete a table of values. Remember the cans cost \$3 each.

WRITE

x = number of cans purchased
 y = total cost of cans

1 can (x) costs \$3 (y).
 2 cans (x) costs \$6 (y).
 and so on ...

x	0	1	2	3	4	5	6	7	8
y	0	3	6	9	12	15	18	21	24

- Are both quantities increasing at a constant rate?
- Answer the question.

The number of cans x is increasing by 1.
 The cost of the cans y is increasing by 3.

There is a direct variation between the number of cans and the total cost of the cans. It can be written as $y = 3x$.



Note: In Worked example 1 the number of cans has to be a whole number, so if we drew a graph to represent this situation we could not join the plotted points with a straight line.

WORKED EXAMPLE 2

Consider the linear function $y = 5x$.

a Complete the table of values for $y = 5x$.

x	0	1	2	3		
y					20	21

b State why this is a direct variation relationship.

THINK

a 1 Substitute the required values of either x or y into the linear function $y = 5x$.

WRITE

When $x = 0$, $y = 5 \times 0$

$$= 0$$

When $x = 1$, $y = 5 \times 1$

$$= 5$$

When $x = 2$, $y = 5 \times 2$

$$= 10$$

When $x = 3$, $y = 5 \times 3$

$$= 15$$

When $y = 20$, $20 = 5 \times x$

$$x = 20 \div 5$$

$$x = 4$$

When $y = 21$, $21 = 5 \times x$

$$x = 21 \div 5$$

$$x = 4.2$$

2 Complete the table.

x	0	1	2	3	4	4.2
y	0	5	10	15	20	21

b For a direct variation relationship, it must be a straight-line graph and pass through the origin $(0, 0)$.

Both values must increase by a constant.

For $y = 5x$, when $x = 0$:

$$y = 5 \times 0$$

$$= 0$$

So it passes through point $(0, 0)$.

The y -value is always 5 times the x -value, so as x increases, y increases.

Therefore, $y = 5x$ is a direct variation relationship.

WORKED EXAMPLE 3

A straight-line graph, starting at the origin, is shown.

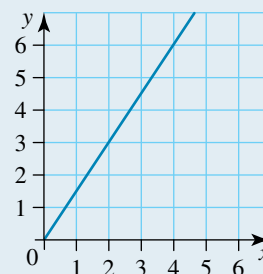
a From the graph, generate a table of values to illustrate this information.

b Do these points represent a direct variation relationship?

c State the relationship between x and y .

d Complete the following table.

x	6		35	
y		42		90



THINK

a Enter the given values of x and y from the graph.
Remember the x -axis is the horizontal axis and the y -axis is the vertical axis.

b 1 As x increases, does y increase?

2 Does the graph pass through the origin?

3 Answer the question.

c Use the table of values or the graph to find the relationship between x and y .

d 1 Enter the required values of either x or y into the linear function $y = 1.5x$.

2 Complete the table.

WRITE

x	0	1	2	3	4
y	0	1.5	3	4.5	6

The x -values increase by 1.

The y -values increase by 1.5.

The graph passes through the origin.

These points represent a direct variation relationship.

$$1 \times 1.5 = 1.5$$

$$2 \times 1.5 = 3$$

$$3 \times 1.5 = 4.5$$

$$4 \times 1.5 = 6$$

The pattern is $x \times 1.5 = y$ or $y = 1.5x$.

For $x = 6$:

$$y = 1.5 \times 6$$

$$= 9$$

For $x = 35$:

$$y = 1.5 \times 35$$

$$= 52.5$$

For $y = 42$:

$$42 = 1.5 \times x$$

$$42 \div 1.5 = x$$

$$x = 28$$

For $y = 90$:

$$90 = 1.5 \times x$$

$$90 \div 1.5 = x$$

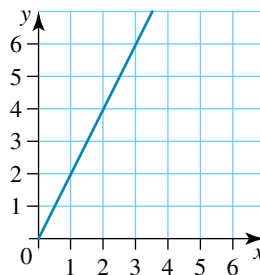
$$x = 60$$

x	6	28	35	60
y	9	42	52.5	90

2.2.2 Straight-line graphs passing through the origin

- If the line formed is a straight line passing through the origin, there is a direct variation relationship between the x -coordinates and the y -coordinates of the points on the line. Likewise, if a straight-line graph passes through the origin, then it is representing a direct variation relationship.
- The equation that describes the relationship between the x - and y -coordinates of the points can be determined by listing the coordinates in a table of values. The table of values for the points on the number plane is as shown.

x	0	1	2	3
y	0	2	4	6



- The table shows that the y -coordinate is always twice the value of the x -coordinate, so this can be described by the linear function $y = 2x$. All direct variation relationships can be represented by a linear function in the form $y = mx$.

2.2.3 Constructing straight-line graphs in the form $y = mx$

- Straight-line graphs can be plotted if you are given:
 - a table of values
 - any two points on the straight line
 - the linear function $y = mx$.

- Handy hints when constructing straight-line graphs:
 - The horizontal axis is the x -axis and the vertical axis is the y -axis.
 - A table of values can be very useful when the linear function is given.
 - Each axis needs to be marked with a suitable scale. Remember, the axes can have different scales, depending on the values to be plotted.
 - Plot each point in the table and join with a straight line.
 - Straight-line graphs can be extended past the given points and will include other points that fit the given equation.
- There are a number of graphing packages available online or through app stores, often free. Usually they ask for the relationship and then graph it on a prepared number plane. Following are two examples of using technology to construct straight-line graphs.

WORKED EXAMPLE 4

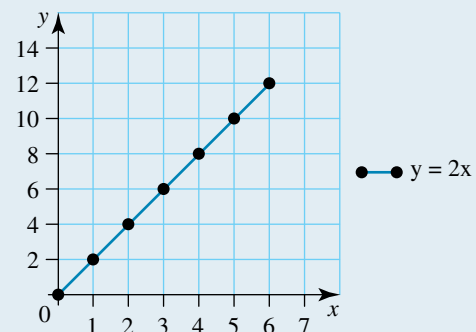
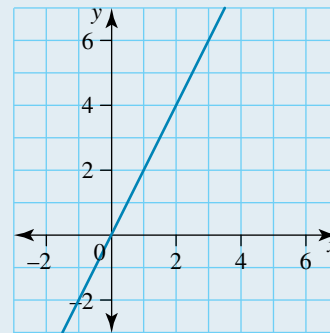
With the aid of technology, graph the straight line $y = 2x$ using:

- a the free online Desmos graphing calculator
- b Microsoft Excel.

THINK

- Follow these steps.
 - Open the Desmos graphing calculator.
 - Click 'Start Graphing'.
 - Using your keypad, enter the equation in line 1:
 $y = 2x$.
 - Use the $+$ or $-$ on the right-hand side of the graph to zoom in or out.
 - Click on the line to show the coordinates of the points.
- Follow these steps.
 - Open an Excel workbook page.
 - In column A, enter x in the first cell.
 - Enter the values of x from 0 to 6.
 - In column B, enter $y = 2x$ in the first cell.
 - In cell B2 enter the formula $= 2*A2$.
 - Press enter, then drag down the bottom right-hand corner of the cell to complete the column.
 - Highlight values in columns A and B.
 - Insert: Charts: Scatter with Straight Lines and Markers.

WRITE



eBookplus RESOURCES

Interactivity: Plotting linear graphs (int-3834)

Weblink: Desmos graphical calculator

Exercise 2.2 Direct variation

Understanding, fluency and communicating

1. **WE1** A shop has a special offer on DVDs at \$8 per DVD. Benji decides to purchase up to 10 DVDs and needs to know how many he can afford to purchase.
 - a. Is there a direct variation relationship between the total cost of the DVDs and the number of DVDs purchased?
 - b. If there is a direct variation, determine the relationship.
2. Biscuits are advertised at \$3.50 per packet. Charlotte decides to purchase a number of packets. Determine the relationship between the number of packets purchased, n , and the total cost, \$ c .
3. **WE2** Consider the linear function $y = 6x$.
 - a. State why this is a direct variation relationship.
 - b. Complete the table of values for $y = 6x$, giving values correct to 2 decimal places where necessary.



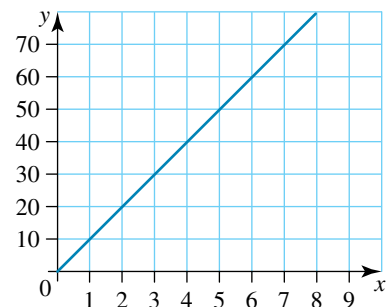
x	0	1	2	3		
y					30	40

4. Consider the linear function $y = \frac{5}{2}x$.
 - a. State why this is a direct variation relationship.
 - b. Complete the table of values for $y = \frac{5}{2}x$.

x	0	1	2	3		
y					15	20

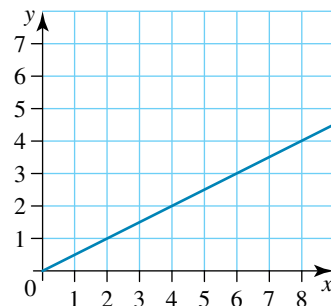
5. **WE3** A straight-line graph, starting at the origin, is as shown.
 - a. From the graph, generate a table of values to illustrate this information.
 - b. Do these points represent a direct variation relationship?
 - c. State the relationship between x and y .
 - d. Complete the following table.

x	7		15	
y		80		160



6. A straight-line graph is as shown.
 - a. From the graph, generate a table of values to illustrate this information.
 - b. Do these points represent a direct variation relationship?
 - c. State the relationship between x and y .
 - d. Complete the following table.

x	10		20	
y		15		25



7. a. Given the following tables of values, plot their straight-line graphs on separate number planes.

x	0	1	2	3	4
y	0	1	2	3	4

x	0	1	2	3	4
y	0	5	10	15	20

x	0	3	6	9	12
y	0	1	2	3	4

- b. Determine the direct variation relationship for these straight-line graphs. Write your answers in the form $y = mx$.
8. For each of the following relationships, complete a table of values from $x = 0$ to $x = 5$. Represent each of the graphs on a number plane.
- a. $y = 9x$ b. $y = \frac{7}{4}x$ c. $y = 3.5x$
9. **WE4** Using technology of your own choice, plot the straight-line graphs found in question 8.
10. Use technology of your choice to construct the straight-line graphs of:
- a. $y = \frac{7}{5}x$ b. $y = 2.25x$

Problem solving, reasoning and justification

11. Anita works casually in a cafe and is paid \$8.50 per hour. She is working out how much she will earn. Let n hours represent the number of hours worked and \$ c the total amount earned.
- a. Is there a direct variation relationship between n and c ? Justify your answer.
- b. Create a table of values for n and c , where $n = 0, 1, 2, 3, 4, 5$.
- c. Draw the graph of the number of hours worked, n , against the total earned, c .
- d. Determine the relationship between n and c .
12. The distance, d kilometres, a fast train travels in t hours is given by the formula $d = 120t$.
- a. Is there a direct variation relationship between d and t ? Justify your answer.
- b. Create a table of values for d and t , where $t = 0, 1, 2, 3, 4, 5$.
- c. Draw the graph of distance, d , against time, t .
13. Below are three tables of values relating x and y .



i.

x	1	2	3	4	5
y	3.5	7	10.5	14	17.5

ii.

x	1	2	3	4	5
y	0	3	6	9	12

iii.

x	0	1	2	3	4
y	0	1	4	9	16

- a. From these tables of values, plot the points on separate number planes to represent the relationships.
- b. For each graph, determine if it is a direct variation relationship. Justify your answer.
14. The circumference of a circle, C , is given by the formula $C = \pi d$, where d is the diameter of the circle.
- a. Find the circumference, correct to 2 decimal places, of a circle with a diameter of 5 cm.
- b. Find the circumference, correct to 2 decimal places, of a circle with a radius of 5 cm.
- c. Is there a direct variation relationship between the circumference of a circle and its diameter? Justify your answer.
- d. Construct a straight-line graph to represent this relationship.
15. Andrew is driving from Sunny Beach to Jetville, a distance of 720 km. He drives at a constant speed of 90 km/h. Let n hours represent the number of hours driving and k kilometres the total distance travelled.
- a. Is there a direct variation relationship between distance travelled and time? Justify your answer.
- b. Create a table of values for n and k , where $n = 0, 1, 2, 3, 4, 5$.
- c. Draw the graph of the number of hours driving, n , against the total distance travelled, k .
- d. Determine the relationship between n and k .
- e. How far does Andrew travel in 2.5 hours?
- f. How long does it take Andrew to drive from Sunny Beach to Jetville?

2.3 The gradient and intercepts

2.3.1 Linear relationships

- A pattern that changes consistently by the same amount is called a **linear pattern**. A simple pattern such as 2, 5, 8, 11, ... is a linear pattern.
- A **linear relationship** means that as one variable changes uniformly, so does the other. Linear relationships can be identified by using a table of values, plotting points or looking at an equation.

$+3 \quad +3 \quad +3 \quad +3$ $+9 \quad +9 \quad +9 \quad +9$ <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>-4</td> <td>-1</td> <td>2</td> <td>5</td> <td>8</td> </tr> <tr> <td>y</td> <td>-8</td> <td>1</td> <td>10</td> <td>19</td> <td>28</td> </tr> </table> Each variable has a constant difference.	x	-4	-1	2	5	8	y	-8	1	10	19	28	$y = 3x + 4$ Both variables are raised to the power of 1. $y^1 = y$ and $x^1 = x$.	 Points form a straight line.
x	-4	-1	2	5	8									
y	-8	1	10	19	28									

WORKED EXAMPLE 5

For the following tables of values, determine whether a linear relationship exists. Explain your reasoning.

a

x	1	2	3	4	5
y	10	8	6	4	2

b

x	0	2	4	6	8
y	1	2	4	8	16

c

x	1	4	9	16	25
y	1	2	3	4	5

THINK

a 1 Look for constant differences in the x -values and in the y -values.

- The x -values have a constant difference of $+1$, as shown in blue.
- The y -values have a constant difference of -2 , as shown in pink.

2 Answer the question.

b 1 Look for constant differences in the x -values and in the y -values.

- The x -values have a constant difference of 2, as shown in blue.
- The y -values do not change by a constant amount.

2 Answer the question.

c 1 Look for constant differences in the x -values and in the y -values.

- The x -values do not have a constant difference.

2 Answer the question.

WRITE

		+1	+1	+1	+1	
		↘	↘	↘	↘	
x	1	2	3	4	5	
y	10	8	6	4	2	
		↖	↖	↖	↖	
		-2	-2	-2	-2	

As the x -values increase by 1, the y -values decrease by 2. Since x is changing by a constant amount and y is changing by a constant amount, the table of values represents a linear relationship.

		+2	+2	+2	+2	
		↘	↘	↘	↘	
x	0	2	4	6	8	
y	1	2	4	8	16	
		↖	↖	↖	↖	
		+1	+2	+4	+8	

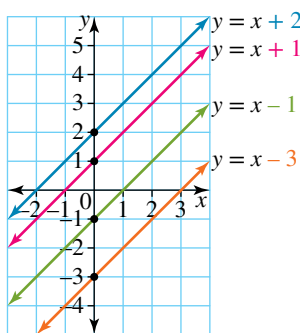
As the x -values increase by 2, the y -values increase by different amounts each time; 1, then 2, then 4 and then 8. Since y is not changing by a constant amount, the relationship is not linear.

		+3	+5	+7	+9	
		↘	↘	↘	↘	
x	1	4	9	16	25	
y	1	2	3	4	5	

Since the x -values increase by different amounts each time, this method cannot be used to determine whether the table represents a linear relationship.

2.3.2 The y-intercept

- All linear relationships can be represented by a linear function in the form $y = mx + c$, where x and y are variables and m and c are constants.
- The graph shows four parallel straight lines. Each line has the same slope but passes through the y -axis at a different point.
- The **y-intercept** is the point at which a line crosses the y -axis.



Line colour	Equation	Cuts y-axis at:	y-intercept
Blue	$y = x + 2$	2	$(0, 2)$
Pink	$y = x + 1$	1	$(0, 1)$
Green	$y = x - 1$	-1	$(0, -1)$
Orange	$y = x - 3$	-3	$(0, -3)$

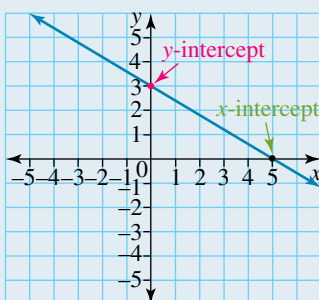
- As you can see, the y -intercept and the constant in the equation (represented by the letter c) are related. In fact, regardless of the value of m in the equation $y = mx + c$, the value of c will always be the y -value of the y -intercept.

2.3.3 The x-intercept

- From the previous table, you can see that every y -intercept has an x -value of 0.
- The y -intercept is where the straight line crosses the y -axis, and occurs when $x = 0$.
- The x -intercept is where the straight line crosses the x -axis, and occurs when $y = 0$.

WORKED EXAMPLE 6

State the coordinates of the x - and y -intercepts in the following graph.



THINK

- 1 The x -intercept is the point at which the line intersects the x -axis. At the x -intercept, the y -coordinate is equal to 0.
- 2 The y -intercept is the point at which the line intersects the y -axis. At the y -intercept, the x -coordinate is equal to 0.

WRITE

x -intercept = 5
The x -intercept is at $(5, 0)$.

y -intercept = 3
The y -intercept is at $(0, 3)$.

WORKED EXAMPLE 7

Calculate the x - and y -intercepts for the graph with the equation $y = 2x + 4$.

THINK

- To find the y -intercept, remember that the x -coordinate of the intercept is 0. Substitute $x = 0$ into the equation, as shown in black, and calculate the value of y .
- To find the x -intercept, remember that the y -coordinate of the intercept is 0. Substitute $y = 0$ into the equation, as shown in pink.
- Calculate the value of x by using inverse operations.

WRITE

$$\begin{aligned} y &= 2x + 4 \\ y &= 2 \times 0 + 4 \\ y &= 4 \end{aligned}$$

The y -intercept is the point $(0, 4)$.

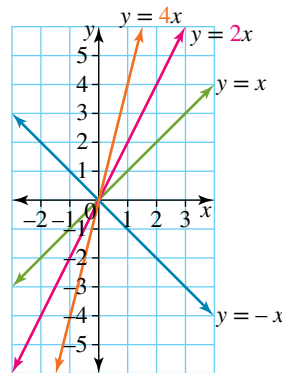
$$\begin{aligned} y &= 2x + 4 \\ 0 &= 2x + 4 \\ -4 &= 2x \\ -2 &= x \end{aligned}$$

The x -intercept is the point $(-2, 0)$.

2.3.4 The gradient

- Some lines may appear as if they are going uphill, while others will head downhill. The graph at right shows a number of straight lines with the same y -intercept but different slopes.
- The coefficient of x in the equation and the slope of the line are related.
- The slope or **gradient** of a line can be determined by measuring the change in the y -value for each increase of 1 unit in the x -value.
- The change in the y -value is referred to as the **rise**. A negative value for the rise indicates that the y -value is decreasing.
- The increase in the x -value is referred to as the **run**.

Line colour	Equation	Slope
Orange	$y = 4x$	4
Pink	$y = 2x$	2
Green	$y = x$	1
Blue	$y = -x$	-1



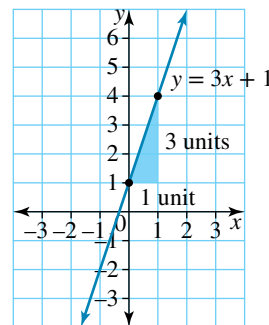
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

- If a right-angled triangle is formed (using the line itself as the hypotenuse), the 'rise' (vertical distance) and 'run' (horizontal distance) can be found.
- The gradient (m) of the line can be calculated by dividing the rise by the run.

$$m = \frac{\text{rise}}{\text{run}}$$

- In the diagram shown at right, the line has a rise of 3 units and a run of 1 unit.

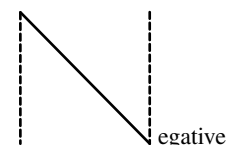
$$\begin{aligned} \text{Therefore, the slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{3}{1} \\ &= 3. \end{aligned}$$



- The gradient of a straight line with the equation $y = mx + c$ is the value of the coefficient of x , which is m . When $c = 0$, this is also known as the **constant of variation**, as the value of m is a constant in the equation.

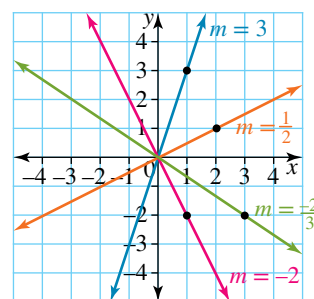
			
A positive gradient has a positive slope.	A negative gradient has a negative slope.	A horizontal line has no slope and a gradient of zero.	A vertical line has an infinite slope and an infinite or undefined gradient.

- A negative slope follows the same slope as the diagonal line in a capital letter N.



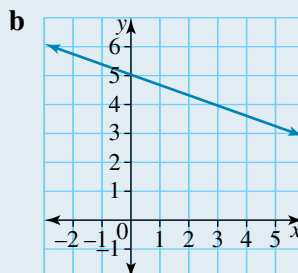
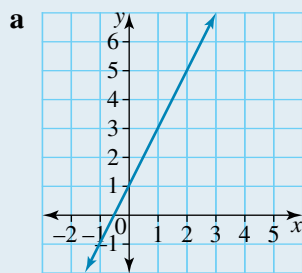
- The greater a slope's magnitude (positive or negative value), the steeper the line formed.

m	$m = \frac{\text{rise}}{\text{run}}$	Vertical distance	Horizontal distance
3	$\frac{\text{rise}}{\text{run}} = \frac{3}{1}$	Rise of 3	Run of 1
$\frac{1}{2}$	$\frac{\text{rise}}{\text{run}} = \frac{1}{2}$	Rise of 1	Run of 2
$-\frac{2}{3}$	$\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$	Fall of 2	Run of 3
-2	$\frac{\text{rise}}{\text{run}} = \frac{-2}{1}$	Fall of 2	Run of 1



WORKED EXAMPLE 8

Determine the gradient of the line for each of the graphs shown.

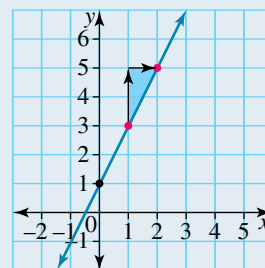


THINK

a 1 Gradient = $\frac{\text{rise}}{\text{run}}$.

- Look for two points where the x - and y -coordinates can be easily read from the grid, as shown in pink.
- Form a right-angled triangle and use this to find rise and run.
 - The y -value increases from 3 to 5, a rise of 2.
 - The x -value increases from 1 to 2, a run of 1.

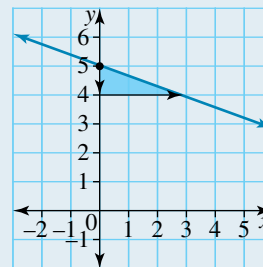
WRITE



2 Write the gradient formula and calculate the gradient.

$$\begin{aligned}\text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2}{1} \\ &= 2\end{aligned}$$

- b 1** • Look for two points where the x - and y -coordinates can be easily read from the grid.
- Form a right-angled triangle and use this to find rise and run.
 - The y -value decreases from 5 to 4, a rise of -1 .
 - The x -value increases from 0 to 3, a run of 3.



2 Write the gradient formula and calculate the gradient.

$$\begin{aligned}\text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-1}{3}\end{aligned}$$

WORKED EXAMPLE 9

State the gradient and the x - and y -intercepts of the following linear functions.

a $y = 5x$

b $y = 5x - 10$

c $y = -3x + 12$

d $2x + 3y = 6$

e $3x - 4y + 6 = 0$

THINK

- a 1** Write the equation in the form $y = mx + c$.
- 2** Identify the coefficient of x as the gradient.
- 3** Identify the value of c , the y -intercept.
- 4** For the x -intercept, substitute $y = 0$.
- 5** Answer the question.

- b 1** Write the equation in the form $y = mx + c$.
- 2** Identify the coefficient of x as the gradient.
- 3** Identify the value of c , the y -intercept.
- 4** For the x -intercept, substitute $y = 0$.
- 5** Answer the question.

WRITE

$$y = 5x$$

$$m = 5$$

$$c = 0$$

$$0 = 5x$$

$$\therefore x = 0$$

For $y = 5x$, the gradient is 5, the y -intercept is $(0, 0)$ and the x -intercept is $(0, 0)$.

$$y = 5x - 10$$

$$m = 5$$

$$c = -10$$

$$0 = 5x - 10$$

$$10 = 5x$$

$$x = 2$$

For $y = 5x - 10$, the gradient is 5, the y -intercept is $(0, -10)$ and the x -intercept is $(2, 0)$.

- c** 1 Write the equation in the form $y = mx + c$.
 2 Identify the coefficient of x as the gradient.
 3 Identify the value of c , the y -intercept.
 4 For the x -intercept, substitute $y = 0$.

$$y = -3x + 12$$

$$m = -3$$

$$c = 12$$

$$0 = -3x + 12$$

$$3x = 12$$

$$x = 4$$

For $y = -3x + 12$, the gradient is -3 , the y -intercept is $(0, 12)$ and the x -intercept is $(4, 0)$.

- d** 1 Write the equation in the form $y = mx + c$.
 2 Identify the coefficient of x as the gradient.
 3 Identify the value of c , the y -intercept.
 4 For the x -intercept, substitute $y = 0$.

$$2x + 3y = 6$$

$$3y = -2x + 6$$

$$y = \frac{-2}{3}x + \frac{6}{3}$$

$$y = \frac{-2}{3}x + 2$$

$$m = \frac{-2}{3}$$

$$c = 2$$

$$2x + 3(0) = 6$$

$$2x = 6$$

$$x = 3$$

For $2x + 3y = 6$, the gradient is $\frac{-2}{3}$, the y -intercept is $(0, 2)$ and the x -intercept is $(3, 0)$.

- e** 1 Write the equation in the form $y = mx + c$.
 2 Identify the coefficient of x as the gradient.
 3 Identify the value of c , the y -intercept.
 4 For the x -intercept, substitute $y = 0$.

$$3x - 4y + 6 = 0$$

$$3x + 6 = 4y$$

$$4y = 3x + 6$$

$$y = \frac{3}{4}x + \frac{6}{4}$$

$$y = \frac{3}{4}x + \frac{3}{2}$$

$$m = \frac{3}{4}$$

$$c = \frac{3}{2}$$

$$3x - 4(0) + 6 = 0$$

$$3x + 6 = 0$$

$$3x = -6$$

$$x = -2$$

For $3x - 4y + 6 = 0$, the gradient is $\frac{3}{4}$, the y -intercept is $\left(0, \frac{3}{2}\right)$ and the x -intercept is $(-2, 0)$.

Interactivity: The y-intercept (int-3837)

eLesson: The gradient (eles-1889)

Interactivity: The gradient (int-3836)

Exercise 2.3 The gradient and intercepts

Understanding, fluency and communicating

1. **WE5** For the following tables of values, determine whether a linear relationship exists. Explain your reasoning.

a.

x	0	1	2	3	4
y	12	9	6	3	0

b.

x	1	3	5	7	9
y	1	2	3	4	5

2. For the following tables of values, determine whether a linear relationship exists. Explain your reasoning.

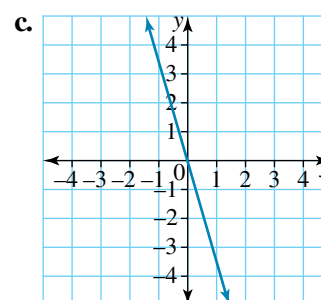
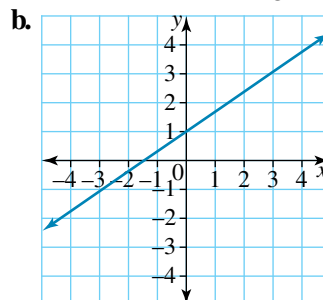
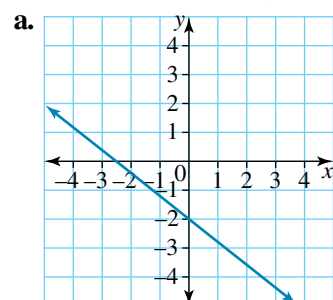
a.

x	2	4	6	8	10
y	4	2.5	1	-0.5	-2

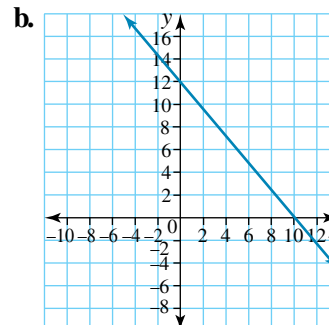
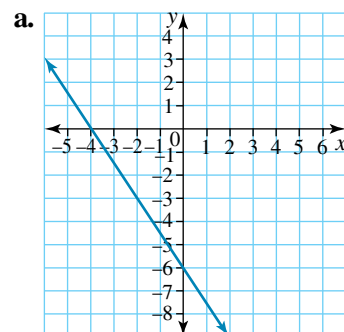
b.

x	0	1	3	6	10
y	0	1	2	3	4

3. **WE6** State the x - and y -intercepts for each of the following straight lines.



4. State the coordinates of the x - and y -intercepts in the following graphs.



5. **WE7** Calculate the x - and y -intercepts for the following straight-line graphs.

a. $y = 3x + 6$

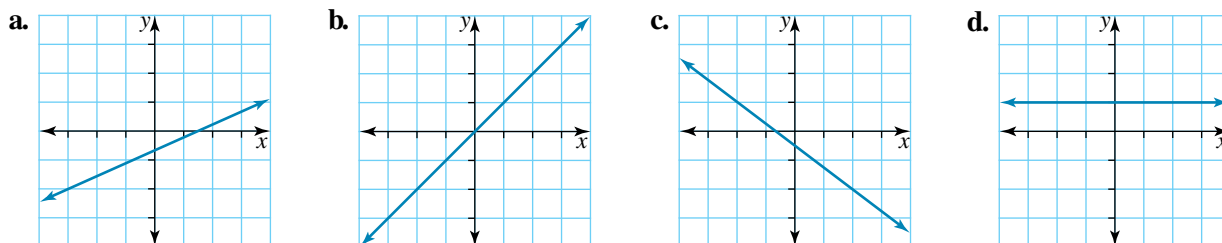
b. $y = 4x - 12$

6. Calculate the x - and y -intercepts for the following straight-line graphs.

a. $y = -2x + 5$

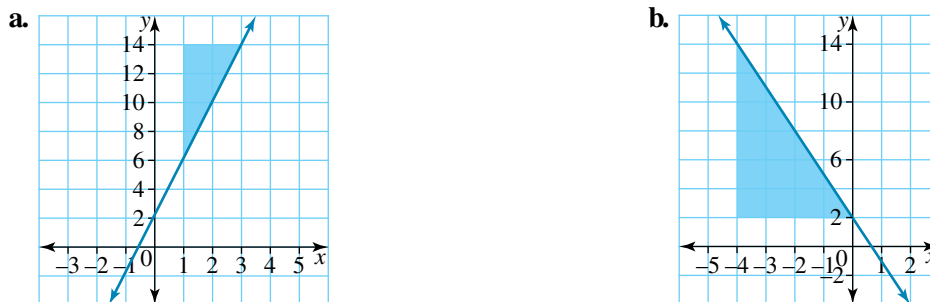
b. $y = 3x - 8$

7. For each of the linear graphs below, state whether the gradient is positive, negative or zero.

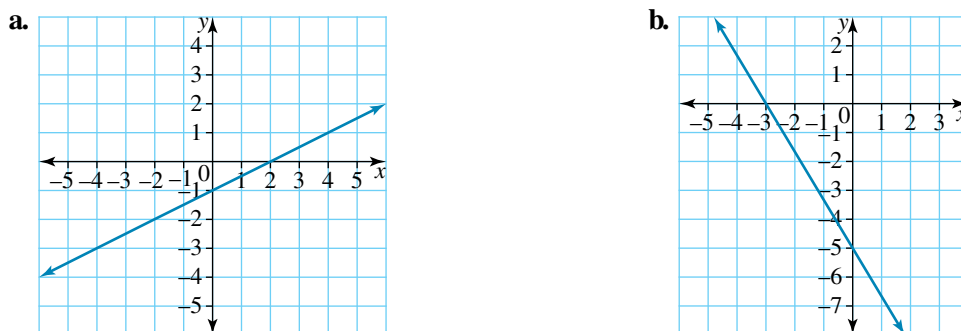


8. Use the triangles provided for each straight line graph below to determine the gradients of the lines.

Hint: Check the scale on the axes before simply counting squares.



9. **WE8** Calculate the gradient for each of the following straight line graphs.



10. **WE9** State the gradient and the x - and y -intercepts of the following linear functions.

a. $y = 2x - 1$

b. $y = -5x + 4$

c. $y = -6x$

d. $2x + 5y = 10$

e. $7x - 8y = 56$

f. $7x + 12y - 42 = 0$

Problem solving, reasoning and justification

11. Copy and complete the following table.

	Rise	Run	Gradient
a	10 metres	4 metres	
b	10 metres		5
c		10 metres	$\frac{1}{2}$

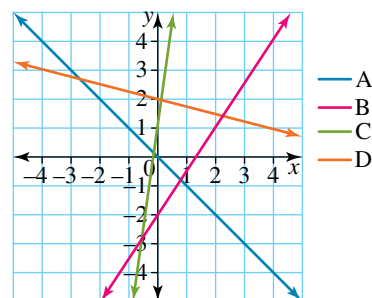
12. a. Match the descriptions given below with their corresponding lines.

i. Straight line with a y -intercept of $(0, 1)$ and a positive gradient

ii. Straight line with a gradient of $1\frac{1}{2}$

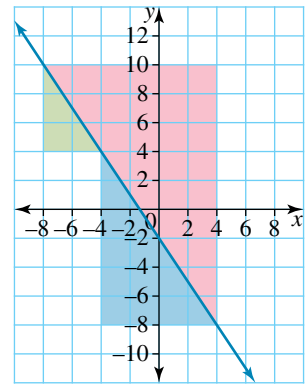
iii. Straight line with a gradient of -1

b. Write a description for the unmatched graph.

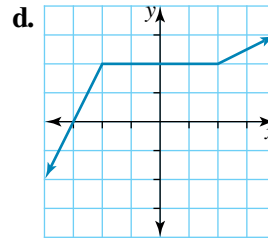
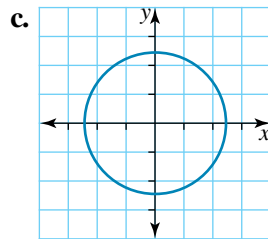
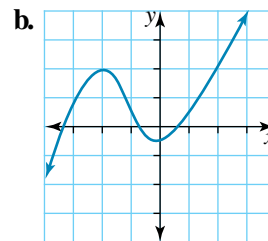
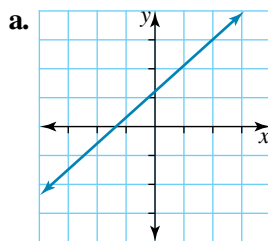


13. Three right-angled triangles have been superimposed on the graph shown.

- Use each of these to determine the gradient of the line.
- Does it matter which points are chosen to determine the gradient of a line? Explain.
- Describe the shape of the graph.

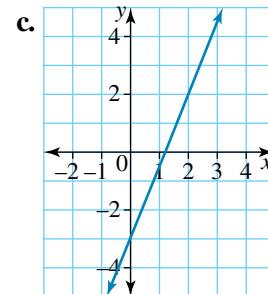
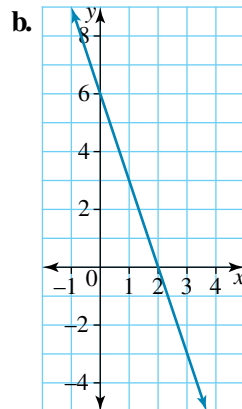
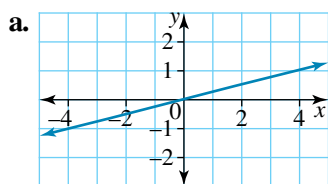


14. Which of the following graphs represents a linear function of the form $y = mx + c$?



15. For each of the straight-line graphs shown below:

- state the y-intercept
- calculate the gradient
- write a linear equation to describe the graph.
- Do any of these graphs represent direct variation? Explain your reasoning.



2.4 Sketching linear graphs

2.4.1 Sketching linear graphs from intercepts

- Handy hints when constructing linear graphs using the axis intercepts:
 - Plot the y -intercept on the y -axis.
 - Find the y -intercept by substituting $x = 0$ into the equation and solve for y .
 - Plot the x -intercept on the x -axis.
 - Find the x -intercept by substituting $y = 0$ into the equation and solve for x .
 - Join the points with a straight line, extended in both directions.
 - If both intercepts are at the origin, $(0, 0)$, choose any other x -value and calculate the corresponding y -value. Plot this second point and join to the origin.
 - Draw arrows at both ends of the line to give an indication of other points that fit the equation.
 - Name the graph with its equation.

WORKED EXAMPLE 10

By finding the x - and y -intercepts, sketch the graph of the equation $y = 2x + 4$.

THINK

- 1 The y -intercept is found by substituting $x = 0$ into the equation.
- 2 The x -intercept is found by substituting $y = 0$ into the equation.
- 3
 - Plot these two points on a suitably scaled Cartesian plane.
 - Join the two intercepts with a straight line, extending the line in both directions.
 - Write the equation of the line.

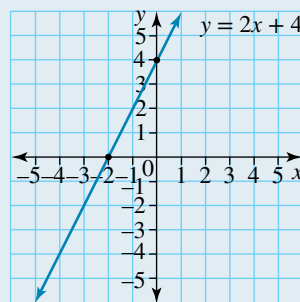
WRITE

$$\begin{aligned}y &= 2(0) + 4 \\&= 0 + 4 \\&= 4\end{aligned}$$

y -intercept: $(0, 4)$

$$\begin{aligned}0 &= 2x + 4 \\2x &= -4 \\x &= -2\end{aligned}$$

x -intercept: $(-2, 0)$



WORKED EXAMPLE 11

Sketch the graph given by the equation $y = 4x$.

THINK

- 1 The y -intercept is found by substituting $x = 0$ into the equation.

WRITE

$$\begin{aligned}y &= 4x \\y &= 4 \times 0 \\y &= 0\end{aligned}$$

The y -intercept is the point $(0, 0)$.

2 The x -intercept is found by substituting $y = 0$ into the equation.

$$y = 4x$$

$$0 = 4x$$

$$0 = x$$

The x -intercept is the point $(0, 0)$.

3 As both intercepts are at the origin, we have only one coordinate so far. Choose any other x -value, say $x = 1$, and substitute this into the original equation to find a second point.

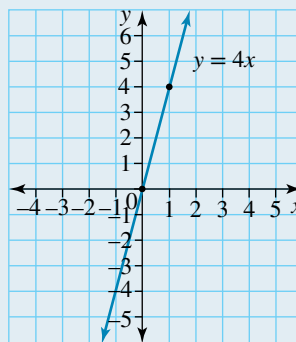
$$y = 4x$$

$$= 4 \times 1$$

$$= 4$$

Another point is $(1, 4)$.

4 Plot these two points on a suitably scaled Cartesian plane. Join the two points with a straight line, extending the line in both directions.



2.4.2 Sketching linear graphs using the gradient and y-intercept

- Handy hints when constructing linear graphs using the gradient and y-intercept:
 - Mark the y-intercept on the y-axis.
 - Another point is found by using the gradient.
 - The gradient is $\frac{\text{rise}}{\text{run}}$.
 - From the y-intercept, move horizontally by the ‘run’ value, then vertically by the ‘rise’ value. For example, for a gradient of 3, the ‘run’ is 1 and the ‘rise’ is 3.
 - If the gradient is -3 , the ‘rise’ will be down by 3.
 - Connect the two points with a straight line.
 - Remember to have arrows on the ends of the line and to name the graph.

WORKED EXAMPLE 12

For each of the linear equations below:

i state the gradient and y-intercept

ii sketch the graph of the equation.

a. $y = 4x - 11$

b. $y = -4x$

THINK

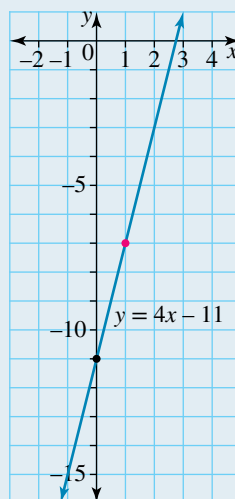
- a i Compare the equation given with the general form of a linear equation: $y = mx + c$. The coefficient of x is m (the gradient), and the constant c is the y-intercept.

WRITE

For $y = 4x - 11$

Gradient (m) = 4 and y-intercept (c) = -11 .

- ii • Construct a set of axes and mark the position of the y-intercept.
 - The y-intercept is -11 , as shown in black.
 - The gradient is 4, so $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$.
 - From the y-intercept, rise 4 and run 1, then mark in a second point, as shown in pink.
 - The two points can now be connected with a straight line.
 - Write the equation next to the line.

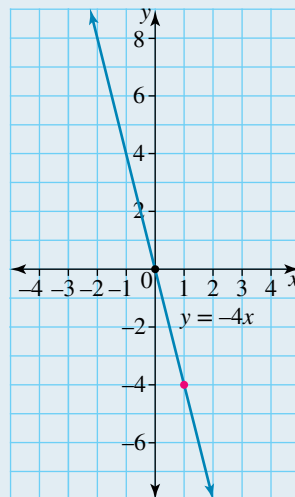


- b i Compare the equation given with the general form of a linear equation: $y = mx + c$. Identify m as the gradient and c as the y-intercept.

For $y = -4x$ or $(y = -4x + 0)$

Gradient (m) = -4 and y-intercept (c) = 0 .

- ii • Construct a set of axes.
 - Mark in the position of the y-intercept at 0 , as shown in black. The gradient is -4 , so $\frac{\text{rise}}{\text{run}} = \frac{-4}{1}$.
 - From the y-intercept, rise -4 and run 1 , then mark in a second point, as shown in pink.
 - The two points can now be connected with a straight line to form the graph.
 - Write the equation next to the line.



eBookplus RESOURCES

eLesson: Sketching linear graphs (eles-1919)

Interactivity: The intercept method (int-3840)

Interactivity: The gradient–intercept method (int-3839)

Exercise 2.4 Sketching linear graphs

Understanding, fluency and communicating

1. **WE10** For each of the linear functions below:

i. find the x - and y -intercepts

ii. sketch the graph of the function.

a. $y = 3x + 6$

b. $y = 3x - 9$

c. $y = 3x + 12$

2. For each of the straight-line graphs below:

- i. find the x - and y -intercepts
- ii. sketch the graph of the function.

a. $y = 4x + 2$

b. $y = 2x - 5$

c. $y = -2x + 1$

3. **WE11** Sketch the graphs given by the following linear equations.

a. $y = x$

b. $y = 3x$

c. $y = -3x$

4. Sketch the graphs given by the following linear equations.

a. $y = \frac{1}{2}x$

b. $y = \frac{5}{2}x$

c. $y = \frac{-5}{2}x$

5. **WE12** For each of the linear functions below:

- i. state the gradient and the y -intercept
- ii. sketch the graph of the function.

a. $y = 2x + 4$

b. $y = 2x + 6$

c. $y = 2x - 6$

d. $y = x - 6$

e. $y = x + 4$

f. $y = 4x - 8$

6. For each of the linear functions below:

- i. state the gradient and the y -intercept
- ii. sketch the straight-line graph.

a. $y = 2x + 5$

b. $y = -4x - 3$

c. $y = -3x + 5$

d. $y = \frac{1}{2}x - 3$

e. $y = \frac{-3}{2}x + 3$

f. $y = \frac{4}{5}x$

7. Using the gradient and y -intercept, sketch the following linear graphs on the same set of axes.

Gradient	y -intercept
3	2
4	-3
-2	0

8. a. Sketch the graph of the linear function with a gradient of $\frac{1}{2}$ and y -intercept of 4.

b. State the equation of this straight-line graph.

9. a. Sketch the graph of the linear function with a gradient of $\frac{-2}{3}$ and a y -intercept of -5.

b. State the equation of this straight-line graph.

10. Using technology of your choice, sketch the following linear equations on the same number plane.

a. $y = 2x + 12$

b. $y = \frac{-1}{2}x + 6$

c. $y = x - 1.5$

Problem solving, reasoning and justification

11. a. Sketch the linear equation $y = \frac{-5}{7}x - \frac{3}{4}$:

- i. using the y -intercept and the gradient
- ii. using the x - and y -intercepts.

b. Compare and contrast the methods and generate a list of advantages and disadvantages for each method. Which method do you think is best? Why?

12. a. Sketch each of the following straight lines on the same set of axes.

i. $y = \frac{5}{2}x + 5$

ii. $y = \frac{-5}{2}x + 5$

- b. Find the x -intercepts for these straight lines.
- c. What shape is formed by the three axis intercepts?
- d. Find the area enclosed by the axis intercepts.

13. a. Using an appropriate method, sketch each of the following straight lines on the same set of axes.

i. $y = 4x + 4$

ii. $y = 4x - 6$

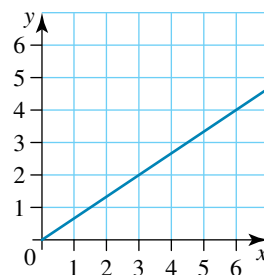
iii. $x + y = 4$

iv. $x + y + 1 = 0$

- b. What shape is formed by these four lines?
- c. Write down the coordinates of the vertices (corners) of this shape.

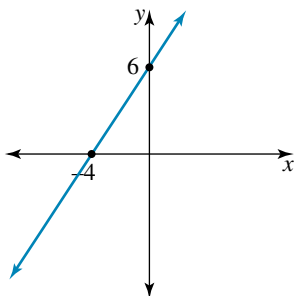
14. Refer to the diagram and answer the following questions.

- Find the gradient of the straight-line graph.
- State the equation that represents this graph.
- Is this a graph of a direct variation relationship? Justify your answer.
- Copy and complete the statement: The y -value is _____ times the x -value, so the constant of variation is _____.
- Is there a relationship between the constant of variation and the gradient? Justify your answer.

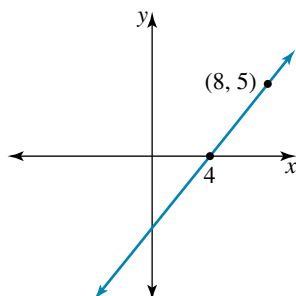


15. **MC** Which sketch below represents the equation $y = \frac{3}{2}x - 6$?

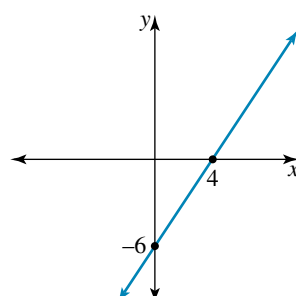
A.



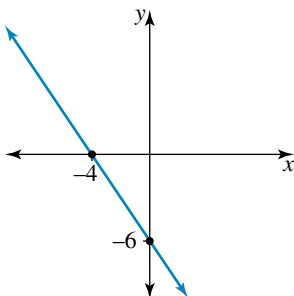
B.



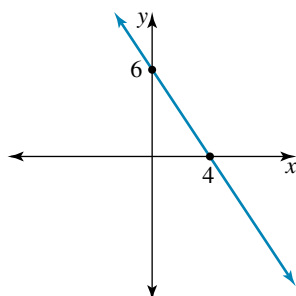
C.



D.



E.



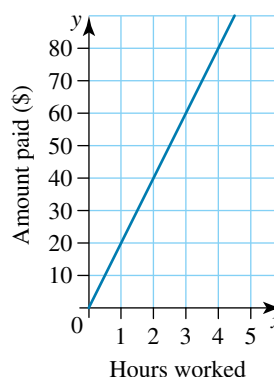
2.5 Linear models

2.5.1 Constructing a linear model

- Practical, everyday problems may often be modelled by linear equations or straight-line graphs. Examples include:
 - the cost of filling a car with petrol
 - the cost of hiring a tradesman
 - currency conversion
 - water leaking from a tank
 - distance travelled at a constant speed.
- To construct a linear model:
 - identify the two variables
 - identify the constant change between the variables, represented by the gradient, m
 - identify the initial starting point or value, represented by c , the y -intercept when $x = 0$
 - write the relationship in the form $y = mx + c$
 - sketch the graph of the linear function if necessary.
- An example of a linear model is the amount of pay a casual employee receives when employed at a rate of \$20 per hour. For every hour worked, the employee is paid \$20. The two variables are the number of hours worked, x hours, and the amount paid, \$ y . The amount paid is increasing by a constant \$20 every hour, so $m = 20$. The starting point is \$0, so $c = 0$.

This gives the linear model $y = 20x$, where y is the amount paid, in dollars, and x is the number of hours worked.

- *Note:* In a linear model we can also use pronumerals other than x and y to represent our variables. For example, we could have used n to represent the number of hours and p to represent the amount paid, giving us the linear model $p = 20n$.



2.5.2 Solving practical problems

- Once the linear relationship that models the practical problem has been found, questions can be answered by substituting into the linear equation and solving the equation for the unknown value. Alternatively, information may be read from a graph.

WORKED EXAMPLE 13

A currency converter shows that an Australian dollar buys £0.60. Eddie is planning a holiday to London.

- Find a relationship between the Australian dollar and the pound sterling.
- How much would Eddie have if he changed \$500 to pounds sterling?
- A day tour from London is advertised for £24. How much would this be in Australian dollars?
- Is this an example of direct variation? Explain your response.

THINK

- 1 Identify the variables.
- 2 Identify the constant change and the starting point.
- 3 Write the relationship between the Australian dollar and the pound sterling.

WRITE

x : amount in Australian dollars to be converted, \$
 y : value in pounds sterling, £

Constant change = £0.60 (every \$1 converts to £0.60)
 Starting point = £0

$$y = 0.6x$$

- 1 For \$500, substitute $x = 500$.
- 2 Answer the question.
- 1 To change £24 to AUD substitute $y = 24$ and solve for x .

$$\begin{aligned} y &= 0.6(500) \\ &= 300 \\ \$500 &\text{ is } \text{£}300. \end{aligned}$$

$$\begin{aligned} 24 &= 0.6x \\ \frac{24}{0.6} &= \frac{0.6}{0.6}x \\ 40 &= x \end{aligned}$$

- 2 Answer the question.
- Answer the question.

The day trip would cost Eddie the equivalent of \$40.
 The equation is $y = 0.6x$ so it is an example of direct variation of the form $y = mx$.

WORKED EXAMPLE 14

The local petrol station is selling fuel for \$1.25/litre. Tom has run out of petrol and needs to fill his car.

- Identify the relationship between the total cost and the number of litres purchased.
- If Tom needs 35 litres for work next week, how much will this cost?
- Tom checks his wallet to find only a \$20 note. How many litres of fuel can he buy?
- Is this a direct variation relationship? Explain your answer.
- Sketch the graph of the linear model.

THINK

- Identify the variables.
 - Identify the constant change and the starting point.
 - Write the relationship as an equation.
- For 35 litres, substitute $n = 35$.
 - Answer the question.
- For \$20, substitute $C = 20$ and solve.
 - Answer the question.
- Answer the question.
- State the gradient and y-intercept.
 - Plot the two points and join them with a straight line, or choose technology to graph.



WRITE

n : number of litres of fuel purchased

C : cost of fuel in dollars

Every extra litre costs \$1.25.

Constant change = 1.25

Starting point = \$0

$$C = 1.25n$$

$$\begin{aligned} C &= 1.25 \times 35 \\ &= 43.75 \end{aligned}$$

35 litres would cost \$43.75.

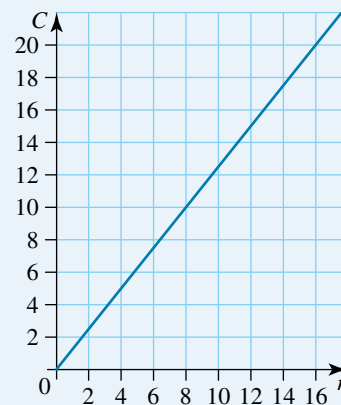
$$\begin{aligned} C &= 1.25n \\ 20 &= 1.25n \\ \frac{20}{1.25} &= \frac{1.25n}{1.25} \\ n &= 16 \end{aligned}$$

\$20 would purchase 16 litres of fuel.

The equation $C = 1.25n$ is an example of direct variation since it is of the form $y = mx$.

Gradient = 1.25

y-intercept = 0, so plot another point (16, 20).



2.5.3 The limitations of models

- Straight-line graphs have arrows at each end to indicate that the line continues.
- In modelling practical problems with straight-line graphs or linear functions, limitations may need to be placed on the variables.
- For example, in Worked example 13, since x and y represent money both x and y must be zero or positive.
- Practical problems involving variables such as time, length and volume, which can only be zero or positive, will have limitations placed on them for the linear model to be valid.

WORKED EXAMPLE 15

A water tank has developed a leak, with water flowing out at the rate of 16 litres per minute. Initially, the water tank contained 2000 litres.

- Construct a linear model that represents the amount of water, W litres, in the tank any time t minutes after the water started leaking.
- Determine how many litres of water remained in the tank after 8 minutes.
- Determine how long it would take for the water in the tank to reduce to 1000 litres.
- If the water continued flowing out, how long would it take for the water tank to be empty?
- What are the limitations required for this linear model to be valid?
- Sketch the graph of this linear model.

THINK

- 1 Use pronumerals to represent the given information.

- 2 Find the constant change and the initial information.

- 3 Write the linear equation.

- 1 Substitute $t = 8$ into the linear equation.

- 2 Answer the question.

- 1 Substitute $W = 1000$ into equation. Solve for t .

- 2 Answer the question.

- 1 To be empty, $W = 0$.
Substitute and solve for t .

- 2 Answer the question.

WRITE

Let t = time in minutes from when water starts leaking
Let W = amount of water, in litres, in the tank at any time

Constant change = -16 (negative since flowing out, decreasing)

Initially: 2000 litres in the tank

$$W = -16t + 2000$$

$$\begin{aligned} W &= -16 \times 8 + 2000 \\ &= -128 + 2000 \\ &= 1872 \end{aligned}$$

There are 1872 litres in the water tank after 8 minutes.

$$\begin{aligned} 1000 &= -16t + 2000 \\ 1000 - 2000 &= -16t + 2000 - 2000 \\ -1000 &= -16t \\ \frac{-1000}{-16} &= \frac{-16t}{-16} \\ 62.5 &= t \end{aligned}$$

It would take 62.5 minutes to reduce to 1000 litres of water.

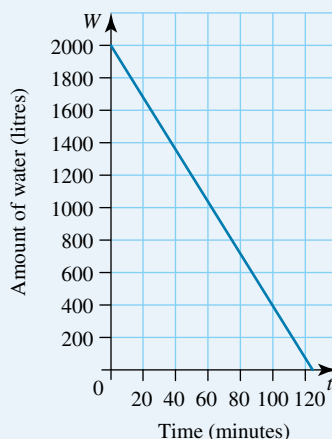
$$\begin{aligned} 0 &= -16t + 2000 \\ 16t &= 2000 \\ \frac{16}{16}t &= \frac{2000}{16} \\ t &= 125 \end{aligned}$$

It would take 125 minutes for the water tank to empty.

- e Time and capacity both have to be greater than or equal to zero.

The model is valid from 0 to 125 minutes, or $0 \leq t \leq 125$ and from 0 (empty) to 2000 litres. $0 \leq W \leq 2000$

- f Construct the straight line representing $W = -16t + 2000$.
This is a linear function of the form $y = mx + c$, with gradient $= -16$ and y-intercept $= 2000$.
Label the axes.



Exercise 2.5 Linear models

Understanding, fluency and communicating

- WE13** Maddie is planning a holiday to Europe. A currency converter shows that one Australian dollar buys €0.72.
 - Find the relationship between Australian dollars and euros.
 - How much would Maddie have if she changed \$800 to euros?
 - A day tour from Paris is advertised for €145. How much would this be in Australian dollars? Give your answer to the nearest 5 cents.
 - Is this an example of direct variation? Explain your response.
- A currency converter is showing one Australian dollar would buy 81 Japanese yen. Alan is planning a skiing trip to Japan.
 - Find the relationship between the Australian dollar and the Japanese yen.
 - How much would Alan have if he changed \$2000 to Japanese yen?
 - A weekly ski pass at the resort costs 33 500 yen. How much, in Australian dollars, would Alan have to budget for this? Give your answer to the nearest 5 cents.
 - Is this an example of direct variation? Explain.
 - Sketch the linear model relating Australian dollars and Japanese yen.
- WE14** The local petrol station is selling fuel for \$1.53/litre. Betty has run out of petrol and needs to fill her car with fuel.
 - Identify the relationship between the total cost and the number of litres purchased.
 - Betty knows her petrol tank holds 55 litres. What would it cost Betty to fill the tank from empty?
 - If Betty only has \$35 in her purse, how many litres of fuel, to 2 decimal places, can she buy?
 - Is this a direct variation relationship? Explain your answer.
 - Sketch the graph of the linear model.
- Diesel fuel is being advertised for 130.5cents/litre. Terry needs to refuel his van.
 - Identify the relationship between the total cost and the number of litres purchased.
 - If Terry needs 65 litres for the weekend, how much will this cost to the nearest 5 cents?
 - Terry only has \$50. How many litres of diesel, to 2 decimal places, can he buy?
 - Is this a direct variation relationship? Explain your answer.
 - Sketch the graph of the linear model.



5. **WE15** A water tank has developed a leak, with water flowing out at the rate of 12 litres per minute. Initially, the water tank contained 1000 litres.

- Construct a linear model that represents the amount of water, W litres, in the tank any time t minutes after the water started leaking.
- Determine how many litres of water remained in the tank after 5 minutes.
- Determine how long it would take for the water in the tank to reduce to 400 litres.
- If the water continued flowing out, how long would it take for the water tank to be empty?
- What are the limitations required for this linear model to be valid?
- Sketch the graph of this linear model.



6. Water is being pumped into a holding tank at a rate of 18 litres/minute. Initially the holding tank was empty.
- Construct a linear model that represents the amount of water, W litres, in the tank at any time t minutes after starting the pump.
 - Determine how many litres of water are in the holding tank after 5 minutes.
 - The holding tank has a capacity of 1500 litres. Determine the time taken to fill the tank.
 - Is this a direct variation relationship? Explain your answer.
 - What are the limitations required for this linear model to be valid?
 - Sketch the graph of this linear model.

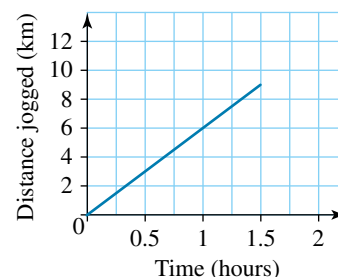
7. Australia measures speed in kilometres per hour and the United Kingdom in miles per hour. 1 km/h is equivalent to 0.6214 mph. Tony has been asked to construct a linear model so he and his friends can check their speeds. Tony decides to let x represent km/h and y mph.

- Construct the linear equation that connects km/h and mph.
- Tony knows the speed limit on the open road in Australia is 100 km/h. What would this be in the UK?
- Tony noticed a road sign in the UK that showed 50. What would this speed be in Australia to the nearest km/h?
- Is the situation an example of direct variation? Explain.

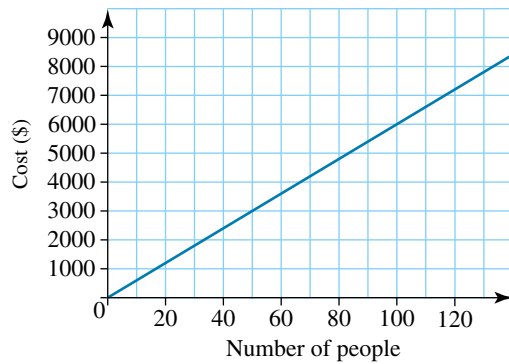


8. A jogger runs for 1.5 hours. The graph shows the distance, d km, the jogger travels in this time. Use the graph to answer the following questions.

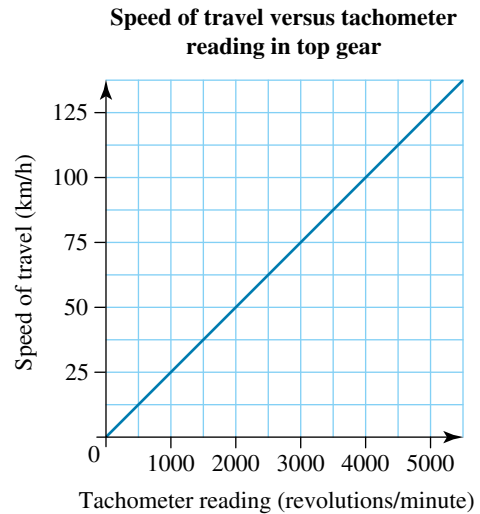
- How far did the jogger run in the first hour?
- Calculate the gradient of the straight-line graph.
- Write an equation to represent the relationship between distance jogged, d km, and time, t hours.
- How many minutes would it take for the jogger to pass the 4-kilometre mark?
- Does this represent a direct variation relationship? Explain your answer.



9. The graph shows how the cost of a school formal varies with the number of students attending.
- Is there a direct variation relationship between the cost of the formal and the number of students attending?
 - Calculate the gradient of the straight-line graph.
 - Write the equation that connects the cost, $\$C$, and the number of students, n , attending.
 - State the constant of variation.
 - Find the total cost if 165 students attended to school formal.

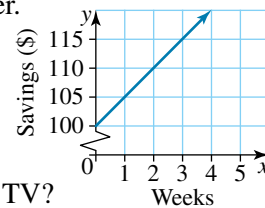


10. The graph shows the speed of travel, s km/h, versus the tachometer reading, r revolutions/minute, in top gear.
- Is there a direct variation relationship between the speed of travel and the tachometer reading?
 - Calculate the gradient of the straight-line graph.
 - Write the equation that connects the speed, s km/h, and the tachometer reading, r revolutions/minute.
 - State the constant of variation.
 - Find the speed of travel for a tachometer reading of 3450 revolutions/minute.
 - Find the tachometer reading if the speed of travel was 150 km/h.

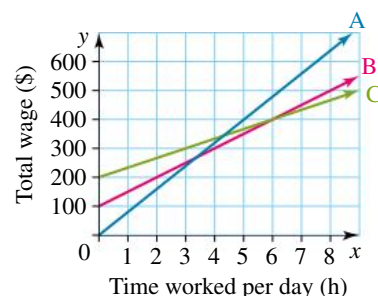


Problem solving, reasoning and justification

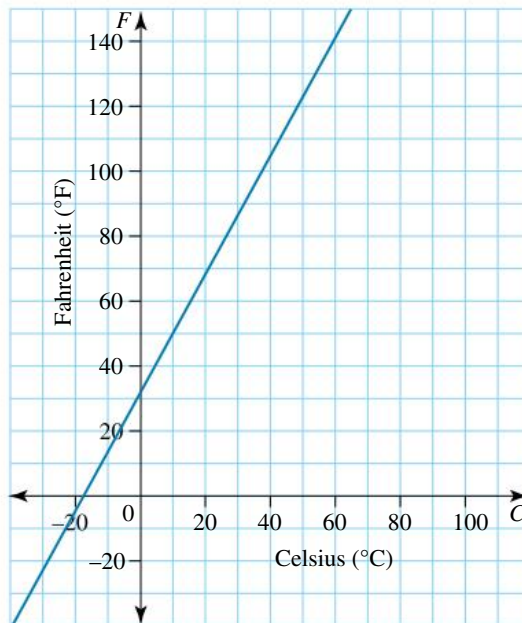
11. The manager of a manufacturing company purchased a new machine valued at $\$180\,000$. The value of the machine depreciates (decreases) constantly over a 5-year period. The value of the machine, $\$v$, at any time, t months, is given by the equation $v = 180\,000 - 3000t$.
- Sketch the graph of this relationship.
 - What is the value of the machine after 4 years?
 - How long would it take for the machine to halve in value?
 - Are there any limitations on the straight-line graph in this context? Justify your answer.
 - Does this situation represent a direct variation relationship? Justify your answer.
12. Nick is saving part of his pocket money to buy a new TV. The graph shows his total savings, $\$y$, for x weeks.
- Construct an equation of this graph, connecting $\$y$ and x weeks.
 - How much savings would Nick have after a year?
 - The TV is advertised for $\$1250$. How long will it take Nick to save to buy this TV?
 - Since the graph is increasing by a constant value each week, it represents a direct variation relationship. True or false? Justify your answer.
13. The fuel-consumption rate of a particular car is given as 10.6 km/L.
- Write an equation connecting the distance travelled, D km, with the amount of fuel consumed, n L.
 - Calculate the distance travelled on 25 litres of fuel.
 - Calculate the amount of fuel required for a trip of 850 km. Give your answer to 2 decimal places.
 - Sketch a graph of the relationship between distance and the fuel consumed.
 - Does this represent a direct variation relationship? Justify your answer.
 - Complete the statement: Since the gradient of the straight-line model is _____, then the _____ of variation is _____.



14. Abigail, Bruce and Charlotte are friends. They are discussing their wages for working up to 8 hours per day. The graph shows how their wages compare.



- Whose wage represents a direct variation relationship? Justify your answer.
 - What is Abigail's wage if she only works 5 hours?
 - Construct an equation that connects Abigail's wage, \$y\$, with the number of hours worked, x hours.
 - How many hours would Bruce and Charlotte have to work in a day to receive the same wage?
 - Construct an equation that connects Charlotte's wage, \$y\$, with the number of hours she works, x hours.
 - How much more than Charlotte would Abigail make if they both work for 7.5 hours?
15. a. The graph shown is used to convert temperature from degrees Celsius to degrees Fahrenheit. Use the graph to convert
- 25°C to degrees Fahrenheit
 - 100°F to degrees Celsius.
- b. The linear model that converts degrees Celsius to degrees Fahrenheit is given by $F = \frac{9}{5}C + 32$.
- Use the linear model to convert:
 - 25°C to degrees Fahrenheit
 - 100°F to degrees Celsius (to 2 decimal places).
 - Compare the answers from b with those in a. Comment on the results.
 - State the gradient of the straight line shown in part a.
 - Is this an example of direct variation? Justify your answer.



2.6 Review

2.6.1 Summary

- In this topic you have learnt:
 - when direct variation occurs
 - about the graphs of direct variation relationships
 - about the gradient of a straight-line graph
 - how to plot points from a table of values
 - how to draw a straight-line graph using two points
 - how to draw a straight-line graph using the gradient and y-intercept
 - how to draw a straight-line graph using the x - and y -intercepts
 - how to draw a straight-line graph with the aid of technology
 - the general equation for straight-line graphs
 - how to calculate the gradient
 - how to determine a second point using the gradient
 - how to calculate the y-intercept
 - how to calculate the x -intercept
 - how to create a linear model
 - the limitations that may need to be placed on variable for the model to be valid.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 2.6 Review

Understanding, fluency and communicating

1. Given the following tables of values, plot the straight-line graphs on separate number planes.

a.

x	0	1	2	3	4
y	2	4	6	8	10

b.

x	0	2	4	6	8
y	-4	2	8	14	20

c.

x	-2	-1	0	1	2
y	9	10	11	12	13

d.

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

2. For each of the following linear relationships, complete a table of values for $x = -2, -1, 0, 1, 2$. Sketch the graphs on separate number planes.

a. $y = 2x - 1$

b. $y = 5x + 4$

c. $y = -2x + 3$

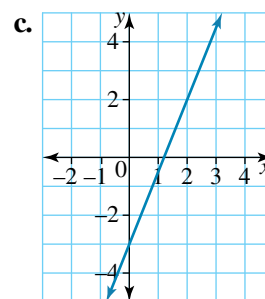
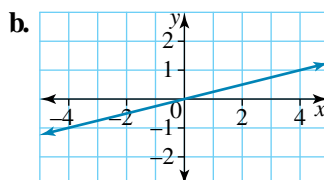
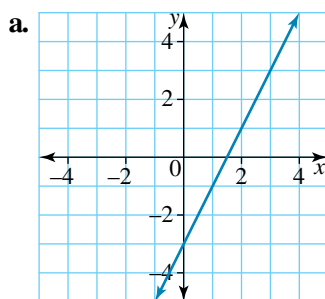
d. $y = 0.5x + 6$

e. $y = -0.5x + 1.5$

f. $y = 5 - 2x$

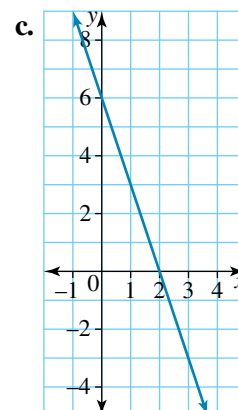
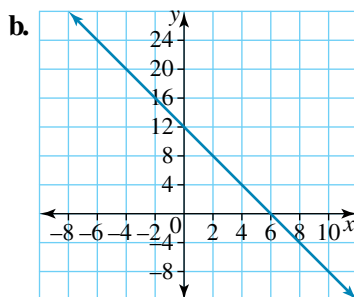
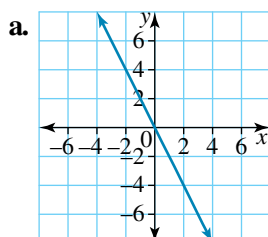
3. For each of the straight-line graphs shown below:

- state the y-intercept
- calculate the gradient
- write a linear equation to describe the graph.



4. Three straight-line graphs are shown. For each graph, find:

- the y-intercept
- the gradient
- the equation of the straight-line graph.



5. State the value of m and c in each of the following linear equations.

a. $y = -5x + 4$

b. $y = 3x + 11$

c. $y = -6x$

d. $y = \frac{5}{2}x + 5$

e. $y = \frac{2}{3}x - 6$

f. $y = 2x - 1$

6. Write the linear equations for the lines with the following properties:
- | | | |
|-------------------------------------|--------------------------------------|---|
| a. gradient = 2
y-intercept = -2 | b. gradient = -2
y-intercept = -1 | c. gradient = $-\frac{3}{8}$
y-intercept = 2 |
| d. gradient = 0
y-intercept = 4 | e. gradient = 1
y-intercept = 0 | f. gradient = -0.25
y-intercept = -4 |
7. Use the y-intercept and gradient to sketch the following straight-line graphs.
- | | |
|------------------|----------------------------|
| a. $y = 3x - 10$ | b. $y = -\frac{2}{5}x - 1$ |
|------------------|----------------------------|
8. Calculate the x- and y-intercepts of the following linear graphs.
- | | | |
|------------------|-----------------|------------------|
| a. $y = x + 2$ | b. $y = 3x + 7$ | c. $y = -2x - 3$ |
| d. $y = -4x + 5$ | e. $y = 2x - 1$ | f. $y = x - 10$ |
9. Calculate the x- and y-intercepts of the following linear graphs.
- | | | |
|------------------|-------------------|-------------------|
| a. $y = 5x - 20$ | b. $y = 20x + 20$ | c. $y = 0.5x + 1$ |
| d. $y = 10x + 5$ | e. $y = x - 12$ | f. $y = -3x + 1$ |
10. For each of the linear equations, calculate the x- and y-intercepts and sketch the graph.
- | | | |
|-------------------|-------------------|--------------------|
| a. $2x + 5y = 10$ | b. $3x + 4y = 24$ | c. $-2x - 3y = 12$ |
| d. $7y + 8x = 56$ | e. $6x + 4y = 18$ | f. $7x - 12y = 42$ |
11. For each of the following linear equations:
- rearrange in the form $y = mx + c$
 - state the gradient
 - state the y-intercept.
- | | | |
|-------------------|------------------|---------------------------|
| a. $2y = -8x + 4$ | b. $y - 5x = 11$ | c. $y = \frac{2x + 6}{8}$ |
|-------------------|------------------|---------------------------|
12. Sketch the straight-line graphs given by the following equations.
- | | | |
|-------------------|-------------------|-------------------|
| a. $y = 2x + 5$ | b. $y = 2x - 5$ | c. $y = 2x + 1.5$ |
| d. $y = 2.5x - 5$ | e. $y = -x + 0.5$ | f. $y = 4x - 5.6$ |
13. Sketch the straight-line graphs given by the following equations.
- | | | |
|--------------|--------------|---------------|
| a. $y = 3x$ | b. $y = -2x$ | c. $y = -x$ |
| d. $y = -3x$ | e. $y = 5x$ | f. $y = 2.5x$ |
14. Sketch the straight-line graphs given by the following equations.
- | | | |
|--------------------|---------------------|---------------------|
| a. $5x + 2y = 10$ | b. $-3x - 4y = -12$ | c. $2x + 3y = 12$ |
| d. $-2x - 7y = 14$ | e. $4x + 5y = 16$ | f. $11x + 44y = 22$ |
15. a. Sketch the straight-line graph with the gradient of 2.5 and the y-intercept of 4.
b. Sketch the straight-line graph with the gradient and y-intercept of 4.
16. The wages earned by Tiffany for different numbers of hours are shown in the table.

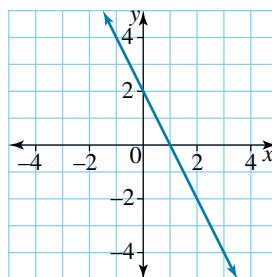
Time (hours)	1	2	3	4
Wages (\$)	25	50	75	100

- How much does Tiffany earn per hour?
- Is there a direct variation relationship between Tiffany's wage and the number of hours she works?
- Determine the relationship.



17. **MC** The equation of the straight-line graph shown is:

- A. $y = 2x - 2$
 B. $y = \frac{1}{2}x - 2$
 C. $y = -\frac{1}{2}x + 1$
 D. $y = -2x + 1$
 E. $y = -2x + 2$



18. **MC** The gradient of the linear function $10x - 2y + 5 = 0$ is:

- A. 10
 B. -2
 C. 5
 D. -5
 E. $-\frac{1}{5}$

19. **MC** The x - and y -intercepts for the straight-line graph $3x - 4y = 12$ are respectively:

- A. $(-3, 0)$ and $(0, 4)$
 B. $(3, 0)$ and $(0, -4)$
 C. $(4, 0)$ and $(0, 3)$
 D. $(4, 0)$ and $(0, -3)$
 E. $(-3, 0)$ and $(0, -4)$

20. **MC** For the straight-line graph $y = 250 + 20x$, which statement is not correct?

- A. The gradient is 20.
 B. The y -intercept is 250.
 C. When $x = 10$, $y = 450$
 D. It represents a direct variation relationship.
 E. When $y = 500$, $x = 12.5$

Problem solving, reasoning and justification

21. a. Plot the graph for each of the following equations from the table of values provided.

i. $y = 5$

x	-2	-1	0	1	2
y	5	5	5	5	5

ii. $y = -3$

x	-2	-1	0	1	2
y	-3	-3	-3	-3	-3

- b. What do you notice about the graphs in part a?
 c. State the gradient of the straight-line graph.
 d. Is there a reason for the shape of these graphs? Justify your answer.

22. a. Plot the graph for each of the following equations from the table of values provided.

i. $x = 5$

x	5	5	5	5	5
y	-2	-1	0	1	2

ii. $x = -3$

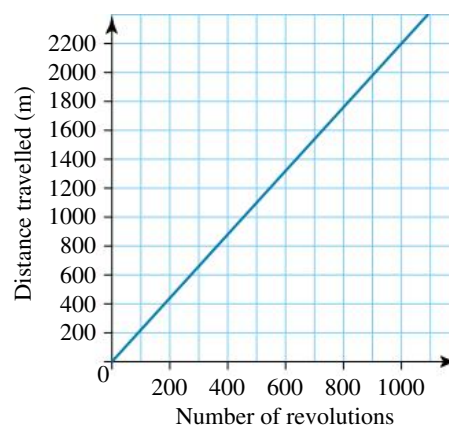
x	-3	-3	-3	-3	-3
y	-2	-1	0	1	2

- b. What do you notice about the graphs in part a?
 c. State the gradient of the straight-line graph.
 d. Is there a reason for the shape of these graphs? Justify your answer.

23. The time, T minutes, that a Year 11 student is expected to take to complete a Maths skill sheet consisting of 20 questions is given by the linear model $T = 4.5Q + 2$, where Q is the number of questions the student answers.

- a. How long is it expected to take students to complete all of the questions?
 b. State any limitations of this linear model.
 c. Sketch a graph of time taken, T , for the number of questions answered, Q .
 d. What is a possible explanation of the number 2 in the given equation.
 e. Is this a linear model of a direct variation relationship? Give reasons for your answer.

24. The graph shows the connection between the number of revolutions of a bicycle wheel and the distance travelled in metres.



- Does this graph represent a direct variation relationship between the distance travelled and the number of revolutions? Give reasons for your answer.
 - Calculate the gradient of the straight line.
 - Write an equation in the form $y = mx$, where y represents the distance travelled and x the number of revolutions.
 - Calculate the distance travelled, in kilometres, for 2000 revolutions of a bicycle wheel.
 - Calculate the number of revolutions required to travel a distance of 6.5 kilometres.
25. The linear model that converts degrees Celsius to degrees Fahrenheit is given by $F = \frac{9}{5}C + 32$.
- The average maximum and minimum temperatures for four cities are given in the following table.

City	Maximum temperature	Minimum temperature
Sydney, NSW	23°C	12°C
London, UK	19°C	5°C
New York, USA	76°F	35°F
Los Angeles, USA	72°F	56°F



- Which city has the lowest minimum temperature? Justify your answer.
 - What is the difference between the average maximum temperatures of London and Los Angeles? Give your answer in degrees Fahrenheit.
 - Convert the New York temperatures to degrees Celsius, correct to 2 decimal places.
- An old cake recipe shows an oven setting of 420°F. Your oven is in degrees Celsius. What temperature would you set your oven to? Give your answer to the nearest 5 degrees.
26. The local petrol station is showing fuel for 129.9 cents/L. Tom has run out of petrol and needs to fill his car.
- Identify the relationship between the total cost and the number of litres purchased.
 - Tom knows his petrol tank holds 75 litres. What would it cost Tom to fill the tank?
 - Is this a direct variation relationship? Explain your answer.
 - Sketch the graph of the linear model.
 - State any limitations of this linear model.
27. The fuel-consumption rate of a new car was advertised as 16.4 km/L.
- Write an equation connecting the distance travelled, D km, with the amount of fuel consumed, n litres.
 - The fuel tank holds 50 L. If the fuel tank is full, how far are you able to travel?
 - A road trip of 1250 km is being planned. How many litres of fuel, to 2 decimal places, would you expect to use?



- d. Sketch a graph of the relationship between distance and the fuel consumed.
- e. Does this represent a direct variation relationship? Justify your answer.
- f. Complete the statement: Since the gradient of the straight-line model is _____, then the _____ of variation is _____.
28. A tank has developed a leak, with water flowing out at the rate of 25 litres per minute. Initially, the water tank contained 5000 litres.
- Construct a linear model that represents the amount of water, W litres, in the tank any time t minutes after the water started leaking.
 - Sketch the graph of this linear model.
 - Determine how many litres of water remained in the tank after an hour.
 - If the water continued flowing out, how long would it take for the water tank to be empty? Give your answer in hours and minutes.
 - What are the limitations required for this linear model to be valid?
 - Is this a model of a direct variation relationship? Explain.
29. Fuel is being pumped into a petrol tank at a rate of 16 L/min. Initially the holding tank was empty.
- Construct a linear model that represents the amount of fuel, F litres, in the tank at any time t minutes after starting the pump.
 - The capacity of the petrol tank is 3400 litres. Determine the time taken to fill the tank. Give your answer in hours and minutes to the nearest minute.
 - Sketch the relationship of the linear model.
 - What are the limitations required for this linear model to be valid?
 - Is this a model of a direct variation relationship? Explain.
30. A currency converter is showing one Australian dollar would buy 50.48 Indian rupees.
- Ben is a cricket fan and plans to fly to India for a test match.
- Find a relationship between Australian dollars and Indian rupees.
 - How much would Ben have if he changed \$1200 to Indian rupees?
 - Tickets for a match range from 350 to 2500 rupees. Find this range in Australian dollars.
 - Is this an example of direct variation? Explain.
 - Sketch the linear model relating Australian dollars and Indian rupees.
31. Charlie has spent the day at the pool. When he was leaving the pool to drive home, he realised there was a linear model that connected his distance from home, d km, with the time he takes to get home, t minutes. The equation was $d = 6 - \frac{1}{2}t$.
- Sketch the graph of this equation.
 - How far is the pool from Charlie's home?
 - How long does it take for Charlie to drive home?
 - What are the limitations on this linear model?
 - Does this linear model represent a direct variation relationship? Justify your answer.



Answers

Exercise 2.2 Direct variation

1. a. Yes, there is a direct variation between the number of DVDs purchased and the total cost, as the cost is always 8 times the number of DVDs purchased.

b. $y = 8x$

2. $c = 3.5n$

3. a. It passes through $(0, 0)$ and the y -value is always 6 times the x -value, so as x increases, y increases. Therefore $y = 6x$ is a direct relationship.

b.

x	0	1	2	3	5	6.67
y	0	6	12	18	30	40

4. a. It passes through $(0, 0)$ and the y -value is always 2.5 times the x -value, so as x increases, y increases. Therefore $y = \frac{5}{2}x$ is a direct relationship.

b.

x	0	2	3	4	6	8
y	0	2.5	5	7.5	15	20

5. a.

x	0	1	2	3	4	5	6	7
y	0	10	20	30	40	50	60	70

- b. It passes through $(0, 0)$ and the y -value is always 10 times the x -value, so as x increases, y increases. Therefore it is a direct relationship.

c. $y = 10x$

d.

x	7	8	15	16
y	70	80	150	160

6. a.

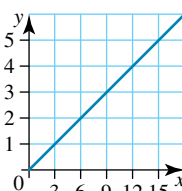
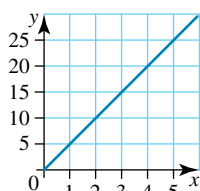
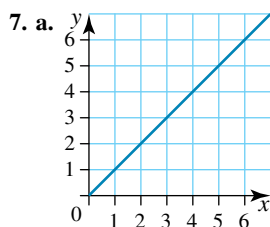
x	0	1	2	3	4	5	6	7	8
y	0	0.5	1	1.5	2	2.5	3	3.5	4

- b. It passes through $(0, 0)$ and the y -value is always 0.5 times the x -value, so as x increases, y increases. Therefore, it is a direct relationship.

c. $y = \frac{1}{2}x$

d.

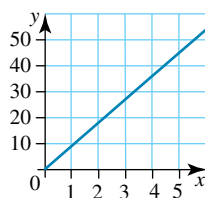
x	10	30	20	50
y	5	15	10	25



b. $y = x$
 $y = 5x$
 $y = \frac{1}{3}x$

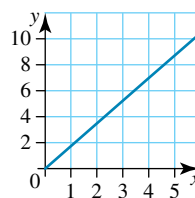
8. a.

x	0	1	2	3	4	5
y	0	9	18	27	36	45



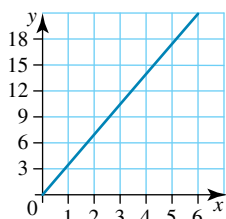
b.

x	0	1	2	3	4	5
y	0	$\frac{7}{4}$	$\frac{7}{2}$	$\frac{21}{4}$	7	$\frac{35}{4}$

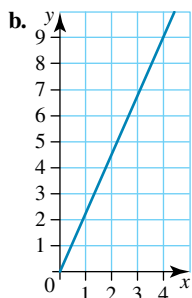
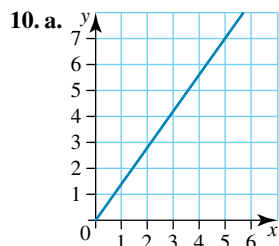


c.

x	0	1	2	3	4	5
y	0	3.5	7	10.5	14	17.5



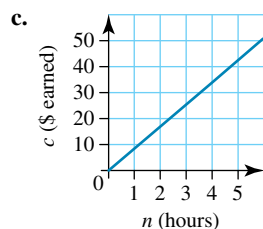
9. See the graphs from question 8.



11. a. It passes through (0,0) and the value of c is always 8.5 times the value of n , so as n increases, c increases. There is a direct relationship between n and c .

b.

n	0	1	2	3	4	5
c	0	8.5	17	25.5	34	42.5

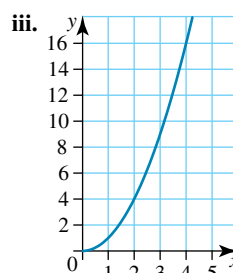
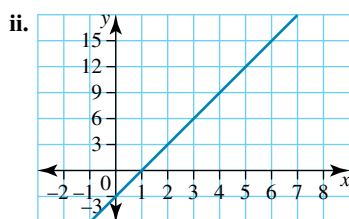
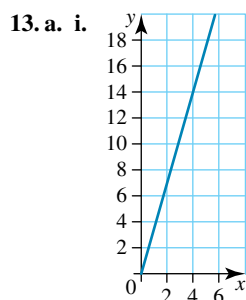
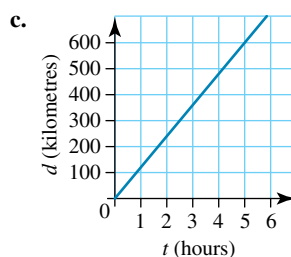


d. $c = 8.5n$

12. a. It passes through (0,0) and the value of d is always 120 times the value of t , so as t increases, d increases. There is a direct relationship between d and t .

b.

t	0	1	2	3	4	5
d	0	120	240	360	480	600

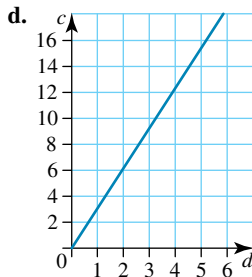


- b. i. If this graph was extended, it would pass through $(0, 0)$. The y -value is always 3.5 times the x -value, so as x increases, y increases. Therefore, it is a direct relationship.
 ii. Since this graph does not pass through $(0, 0)$, it is not a direct-variation relationship.
 iii. Even though this graph passes through $(0, 0)$, it is not linear. Therefore, it is not a direct-variation relationship.

14. a. 15.71 cm

b. 31.42 cm

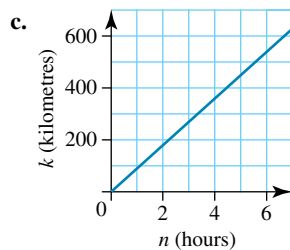
c. It passes through $(0, 0)$ and the value of C is always π (approximately 3.142) times the value of d , so as d increases, C increases. There is a direct relationship between C and d .



15. a. It passes through $(0, 0)$ and the value of k is always 90 times the value of n , so as n increases, k increases. There is a direct relationship between k and n .

b.

n	0	1	2	3	4	5
k	0	90	180	270	360	450



d. $k = 90n$

e. 225 km

f. 8 hours

Exercise 2.3 The gradient and intercepts

- a. The x -values increase by 1 and the y -values decrease by 3. Since both are changing by a constant amount, the table of values represents a linear relationship.

b. The x -values increase by 2 and the y -values increase by 1. Since both are changing by a constant amount, the table of values represents a linear relationship.
- a. The x -values increase by 2 and the y -values decrease by 1.5. Since both are changing by a constant amount, the table of values represents a linear relationship.

b. The y -values increase by 1, a constant amount; the x -values do not increase by a constant amount. The table of values does not represent a linear relationship.
- a. x -intercept = $(-2.5, 0)$
 y -intercept = $(0, -2)$

b. x -intercept = $(-1.5, 0)$
 y -intercept = $(0, 1)$

c. x -intercept = $(0, 0)$
 y -intercept = $(0, 0)$
- a. x -intercept = $(-4, 0)$
 y -intercept = $(0, -6)$

b. x -intercept = $(10, 0)$
 y -intercept = $(0, 12)$
- a. x -intercept = $(-2, 0)$
 y -intercept = $(0, 6)$

b. x -intercept = $(3, 0)$
 y -intercept = $(0, -12)$
- a. x -intercept = $(2.5, 0)$
 y -intercept = $(0, 5)$

b. x -intercept = $(\frac{8}{3}, 0)$
 y -intercept = $(0, -8)$
- a. positive

b. positive

c. negative

d. zero
- a. 4

b. -3
- a. $\frac{1}{2}$

b. $-\frac{5}{3}$

10. a. Gradient = 2, x -intercept = $(0.5, 0)$, y -intercept = $(0, -1)$
 b. Gradient = -5 , x -intercept = $(0.8, 0)$, y -intercept = $(0, 4)$
 c. Gradient = -6 , x -intercept = $(0, 0)$, y -intercept = $(0, 0)$
 d. Gradient = $-\frac{2}{5}$, x -intercept = $(5, 0)$, y -intercept = $(0, 2)$
 e. Gradient = $\frac{7}{8}$, x -intercept = $(8, 0)$, y -intercept = $(0, -7)$
 f. Gradient = $-\frac{7}{12}$, x -intercept = $(6, 0)$, y -intercept = $(0, \frac{7}{2})$

11.

	Rise	Run	Gradient
a	10 metres	4 metres	$\frac{5}{2}$
b	10 metres	2 metres	5
c	5 metres	10 metres	$\frac{1}{2}$

12. a. i. Straight line C (green line) ii. Straight line B (pink line) iii. Straight line A (blue line)
 b. The unmatched straight line, D (orange line) has a rise of 1 and a run of -4 , so it has gradient of $-\frac{1}{4}$.

13. a. $-\frac{3}{2}$

- b. No, it does not matter which points are used as the gradient is always simplified to the same value.
 c. The graph is a straight line with a negative gradient of $-\frac{3}{2}$.

14. Diagram A is a linear function of the form $y = mx + c$.

Diagram B is not linear; it has curves.

Diagram C is not linear; it is a circle.

Diagram D although it has parts of three different straight lines, it cannot be written in the required form.

15. a. i. y -intercept = $(0, 0)$

ii. $\frac{1}{4}$

iii. $y = \frac{1}{4}x$

iv. Yes, it is a straight line passing through the point $(0, 0)$.

b. i. y -intercept = $(0, 6)$

ii. -3

iii. $y = -3x + 6$

iv. No, it does not pass through the point $(0, 0)$.

c. i. y -intercept = $(0, -3)$

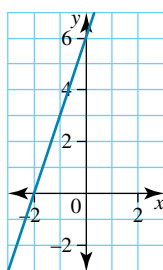
ii. 3

iii. $y = 3x - 3$

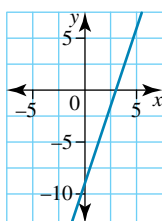
iv. No, it does not pass through the point $(0, 0)$.

Exercise 2.4 Sketching linear graphs using the gradient and y -intercept

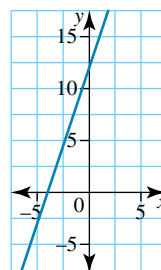
1. a. x -intercept: $(-2, 0)$
 y -intercept: $(0, 6)$



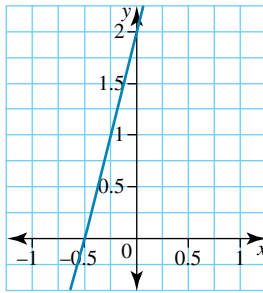
- b. x -intercept: $(3, 0)$
 y -intercept: $(0, -9)$



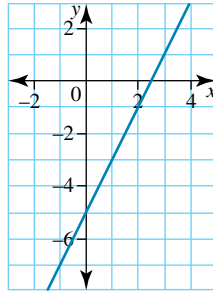
- c. x -intercept: $(-4, 0)$
 y -intercept: $(0, 12)$



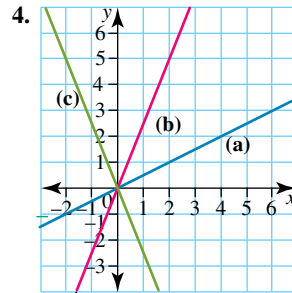
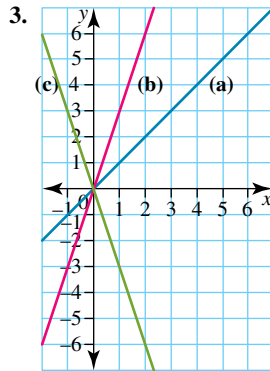
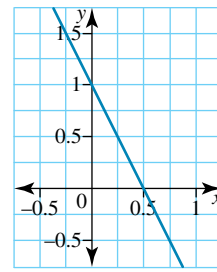
2. a. x -intercept: $(-0.5, 0)$
 y -intercept: $(0, 2)$



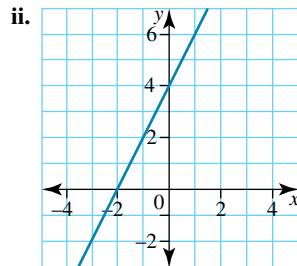
- b. x -intercept: $(2.5, 0)$
 y -intercept: $(0, -5)$



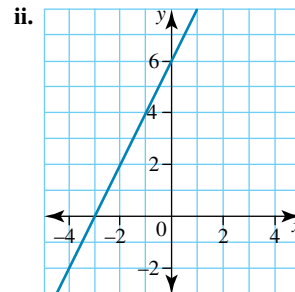
- c. x -intercept: $(0.5, 0)$
 y -intercept: $(0, 1)$



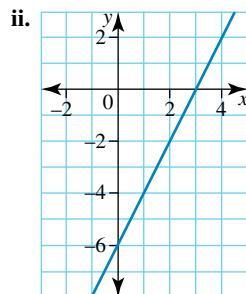
5. a. i. Gradient = 2; y -intercept = $(0, 4)$



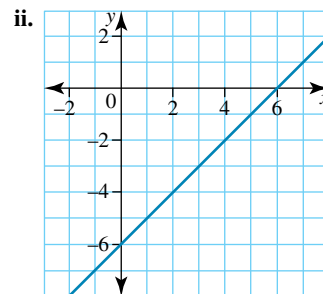
- b. i. Gradient = 2; y -intercept = $(0, 6)$



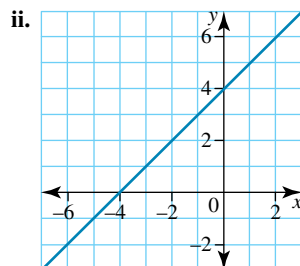
- c. i. Gradient = 2; y -intercept = $(0, -6)$



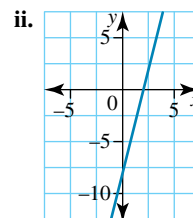
- d. i. Gradient = 1; y -intercept = $(0, -6)$



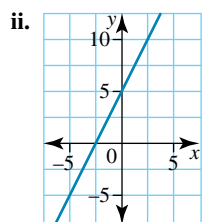
- e. i. Gradient = 1; y -intercept = $(0, 4)$



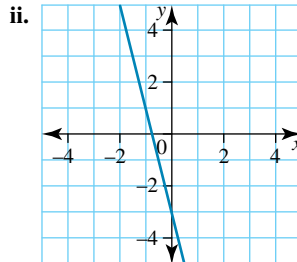
- f. i. Gradient = 4; y -intercept = $(0, -8)$



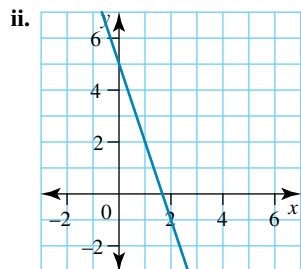
6. a. i. Gradient = 2; y-intercept = (0, 5)



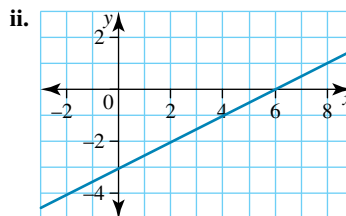
b. i. Gradient = -4; y-intercept = (0, -3)



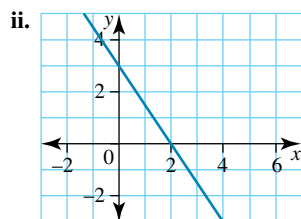
c. i. Gradient = -3; y-intercept = (0, 5)



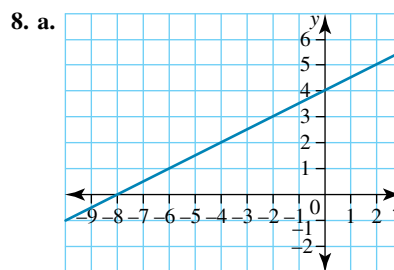
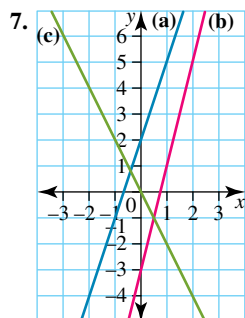
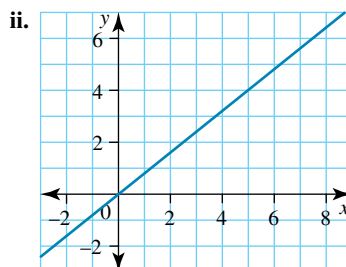
d. i. Gradient = $\frac{1}{2}$; y-intercept = (0, -3)



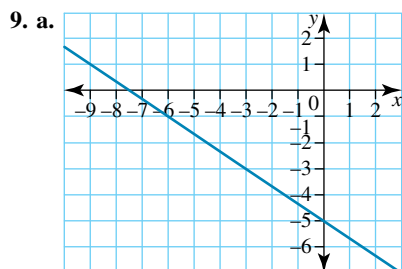
e. i. Gradient = $-\frac{3}{2}$; y-intercept = (0, 3)



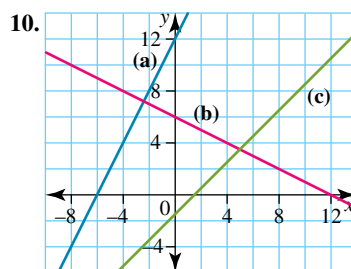
f. i. Gradient = $\frac{4}{5}$; y-intercept = (0, 0)

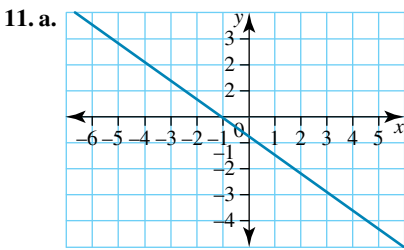


b. $y = \frac{1}{2}x + 4$

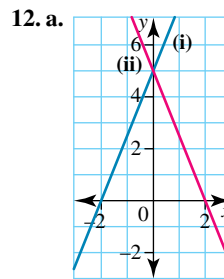


b. $y = \frac{-2}{3}x - 5$

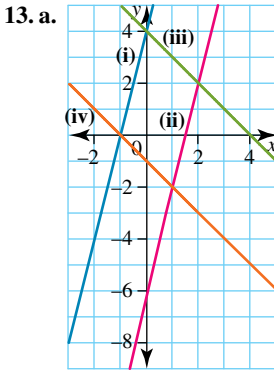




b. Answers will vary.



- b. i. $(-2, 0)$ ii. $(2, 0)$
 c. Isosceles triangle
 d. 10 units^2



b. Parallelogram

c. $(-1, 0)$, $(0, 4)$, $(2, 2)$ and $(1, -2)$

14. a. $\frac{2}{3}$ b. $y = \frac{2}{3}x$

c. The graph is of the form $y = mx$. It passes through $(0, 0)$ and the y -value is always $\frac{2}{3}$ of the x -value, so as x increases, y increases by a constant amount.

This graph is a graph of a direct-variation relationship.

d. The y -value is $\frac{2}{3}$ times the x -value, so the constant of variation is $\frac{2}{3}$.

e. Both the constant of variation and the gradient are $\frac{2}{3}$, so they are equal.

15. C

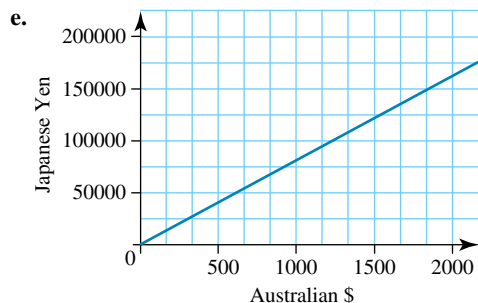
Exercise 2.5 Linear models

1. a. $y = 0.72x$ b. €576 c. \$201.40

d. Yes, this is an example of direct variation. It is of the form $y = mx$. The y -value is always 0.72 times the x -value, so it is increasing at a constant rate.

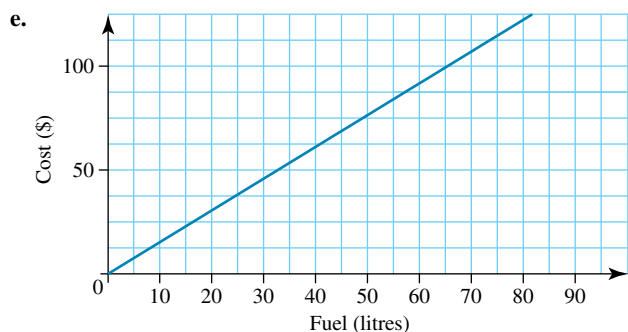
2. a. $y = 81x$ b. 162 000 yen c. \$413.60

d. Yes, this is an example of direct variation. It is of the form $y = mx$. The y -value is always 81 times the x -value, so it is increasing at a constant rate.

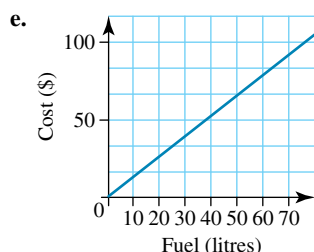


3. a. $y = 1.53x$ b. \$84.15 c. 22.88 litres

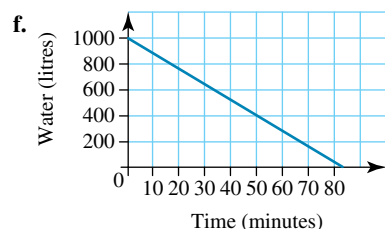
d. Yes, this is an example of direct variation. It is of the form $y = mx$. The y -value is always 1.53 times the x -value, so it is increasing at a constant rate.



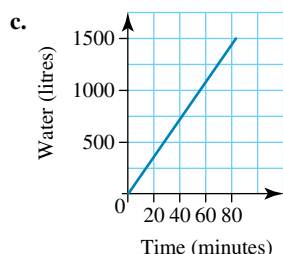
4. a. $y = 1.305x$ b. \$84.85 c. 38.31 litres
 d. Yes, this is an example of direct variation. It is of the form $y = mx$. The y -value is always 1.305 times the x -value, so it is increasing at a constant rate.



5. a. $W = -12t + 1000$ b. 940 litres c. 50 minutes d. 83 minutes 20 seconds
 e. For the model to be valid, time, t , would need to be from 0 to 83.33 minutes and the amount of water in the tank, W litres, would need to be from 0 to 1000.



6. a. $W = 18t$ b. 90 litres c. 83 minutes 20 seconds
 a. Yes, this is an example of direct variation. It is of the form $y = mx$. The W -value is always 18 times the t -value, so it is increasing at a constant rate.
 b. For the model to be valid, time, t , would need be from 0 to 83.33 minutes and the amount of water in the tank, W litres, would need to be from 0 to 1500.



7. a. $y = 0.6214x$ b. 62.14 mph c. 80 km/h
 d. Yes, this is an example of direct variation. It is of the form $y = mx$. The y -value is always 0.6214 times the x -value, so it is increasing at a constant rate.
 8. a. 6 km b. 6 c. $d = 6t$ d. 40 minutes
 e. Yes, this is an example of direct variation. It is of the form $y = mx$, the d -value is always 6 times the t -value, so d is increasing at a constant rate.

9. a. Yes, this is an example of direct variation. It is of the form $y = mx$. The cost (\$) is always a constant multiple of the number of people.

b. 60

c. $C = 60n$

d. 60

e. \$9900

10. a. Yes, this is an example of direct variation. It is of the form $y = mx$. The speed (km/h) is always a constant multiple of the tachometer reading (revolutions/minute).

b. 0.025

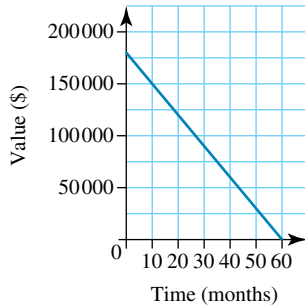
d. 0.025

f. 6000 revolutions/minute

c. $s = 0.025r$

e. 86.25 km/h

11. a.



b. \$36 000

c. 30 months (2.5 years)

d. Time can't be negative, so time is from 0 to 60 months (5 years).

The value of the machine can't be negative, so value is from \$0 to \$180 000.

e. No, this is not an example of direct variation as it is not of the form $y = mx$.

12. a. $y = 5x + 100$

b. \$360

c. 230 weeks (4 years and 22 weeks)

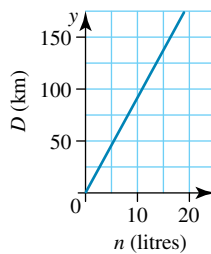
d. False, not of the form $y = mx$

13. a. $D = 10.6n$

b. 265 km

c. 80.19 litres

d.



e. Yes, this is an example of direct variation. It is of the form $y = mx$. The y-value is always 10.6 times the x-value, so it is increasing at a constant rate.

f. Since the gradient of the straight-line model is 10.6, then the constant of variation is 10.6.

14. a. Abigail. It is of the form $y = mx$.

b. \$400

c. $y = 80x$

d. 6 hours

e. $y = \frac{100}{3}x + 200$

f. \$150

15. a. i. 78° F

ii. 38° C

b. i. I. 77° F

ii. II. 37.78° C

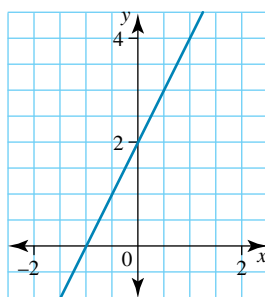
ii. The answers are very close, but it is easier to read off the graph.

iii. $\frac{9}{5}$

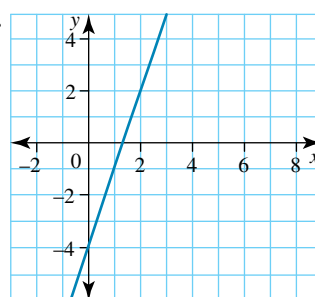
iv. No, this is not an example of direct variation as it is not of the form $y = mx$. It does not pass through the point (0, 0).

Exercise 2.6

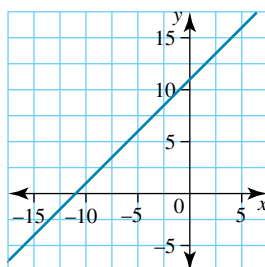
1. a.



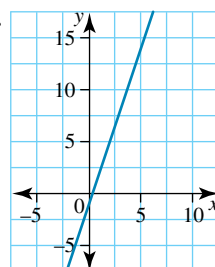
b.



c.

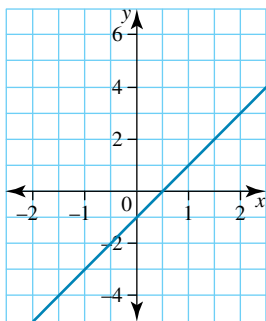


d.



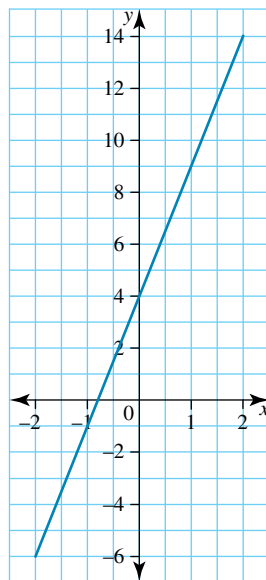
2. a.

x	-2	-1	0	1	2
y	-5	-3	-1	1	3



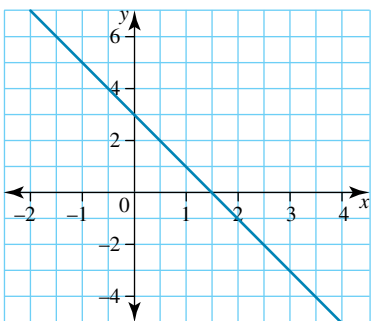
b.

x	-2	-1	0	1	2
y	-6	-1	4	9	14



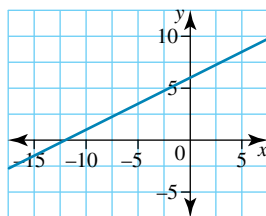
c.

x	-2	-1	0	1	2
y	7	5	3	1	-1



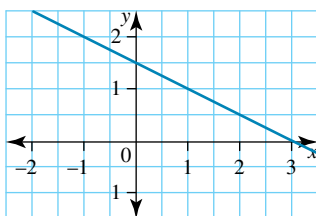
d.

x	-2	-1	0	1	2
y	5	5.5	6	6.5	7



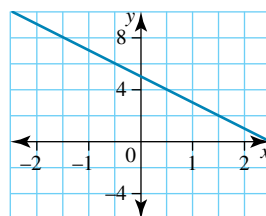
e.

x	-2	-1	0	1	2
y	2.5	2	1.5	1	0.5



f.

x	-2	-1	0	1	2
y	9	7	5	3	1



3. a. i. (0, -3)

ii. 2

iii. $y = 2x - 3$

b. i. (0, 0)

ii. 0.25

iii. $y = 0.25x$

c. i. (0, -3)

ii. $\frac{5}{2}$

iii. $y = \frac{5}{2}x - 3$

4. a. i. (0, 0)

ii. -2

iii. $y = -2x$

b. i. (0, 12)

ii. -2

iii. $y = -2x + 12$

c. i. (0, 6)

ii. -3

iii. $y = -3x + 6$

5. a. $m = -5, c = 4$

b. $m = 3, c = 11$

c. $m = -6, c = 0$

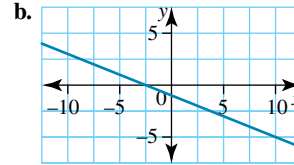
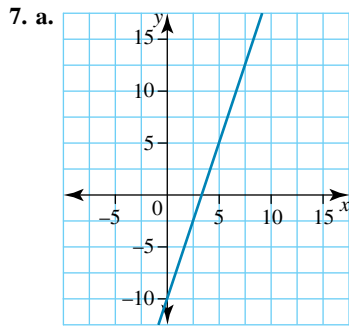
d. $m = \frac{5}{2}, c = 5$

e. $m = \frac{2}{3}, c = -6$

f. $m = 2, c = -1$

6. a. $y = 2x - 2$
 c. $y = \frac{-3}{8}x + 2$
 e. $y = x$

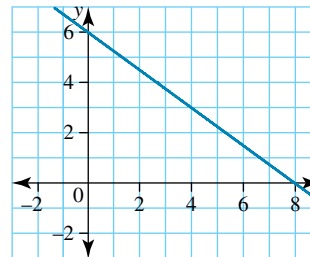
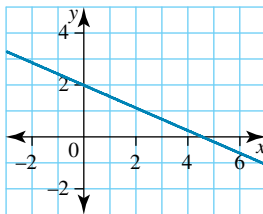
b. $y = -2x - 1$
 d. $y = 4$
 f. $y = -0.25x - 4$



8. a. x-intercept = $(-2, 0)$, y-intercept = $(0, 2)$
 c. x-intercept = $(-1.5, 0)$, y-intercept = $(0, -3)$
 e. x-intercept = $(0.5, 0)$, y-intercept = $(0, -1)$
 9. a. x-intercept = $(4, 0)$, y-intercept = $(0, -20)$
 c. x-intercept = $(-2, 0)$, y-intercept = $(0, 1)$
 e. x-intercept = $(12, 0)$, y-intercept = $(0, -12)$

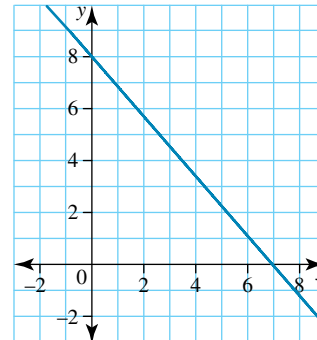
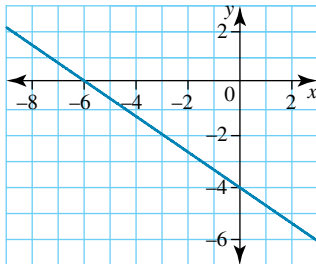
- b. x-intercept = $(\frac{-7}{3}, 0)$, y-intercept = $(0, 7)$
 d. x-intercept = $(1.25, 0)$, y-intercept = $(0, 5)$
 f. x-intercept = $(10, 0)$, y-intercept = $(0, -10)$
 b. x-intercept = $(-1, 0)$, y-intercept = $(0, 20)$
 d. x-intercept = $(-0.5, 0)$, y-intercept = $(0, 5)$
 f. x-intercept = $(\frac{1}{3}, 0)$, y-intercept = $(0, 1)$

10. a. x-intercept = $(5, 0)$, y-intercept = $(0, 2)$



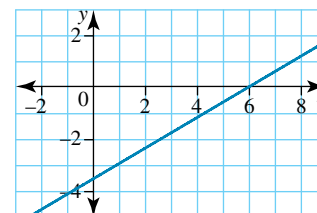
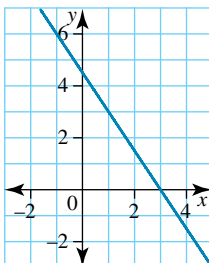
- c. x-intercept = $(-6, 0)$, y-intercept = $(0, -4)$

- d. x-intercept = $(7, 0)$, y-intercept = $(0, 8)$



- e. x-intercept = $(3, 0)$, y-intercept = $(0, 4.5)$

- f. x-intercept = $(6, 0)$, y-intercept = $(0, -3.5)$



11. a. i. $y = -4x + 2$

b. i. $y = 5x + 11$

c. i. $y = \frac{1}{4}x + \frac{3}{4}$

ii. -4

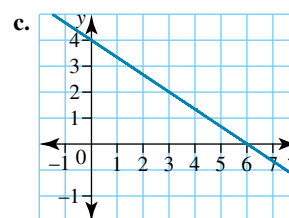
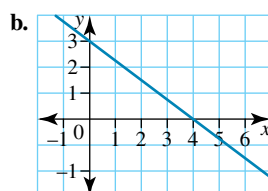
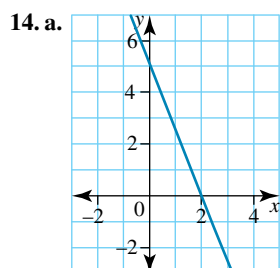
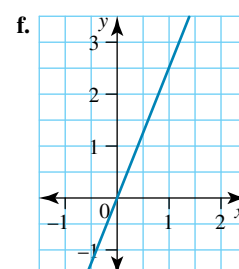
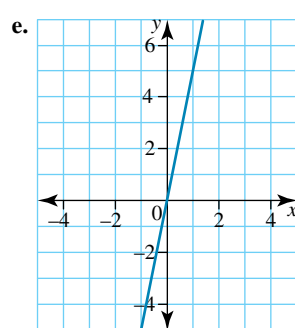
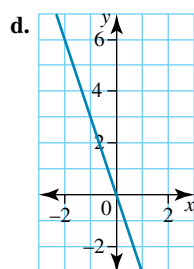
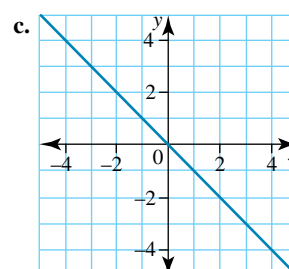
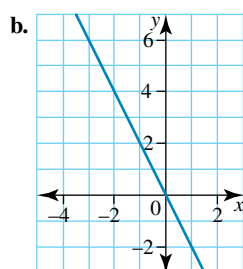
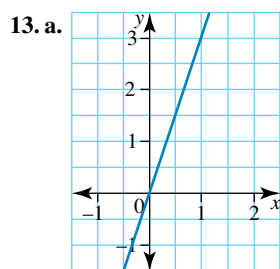
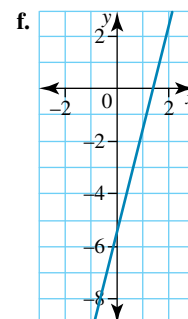
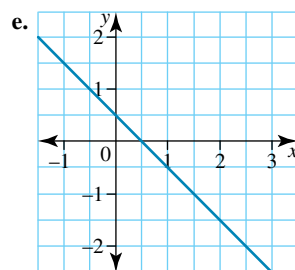
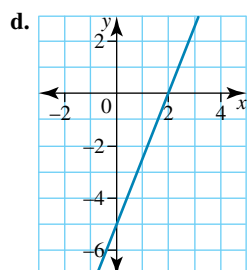
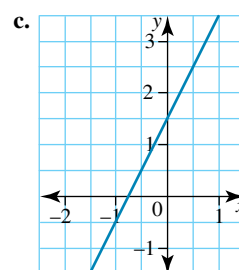
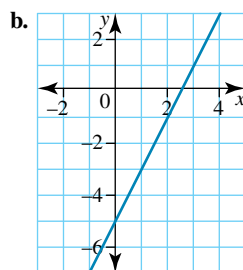
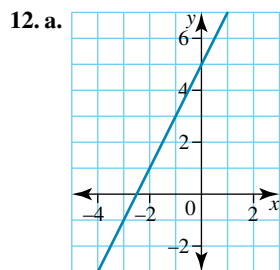
ii. 5

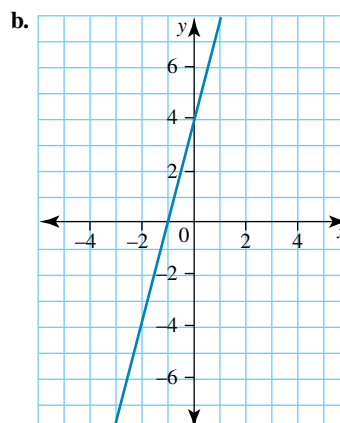
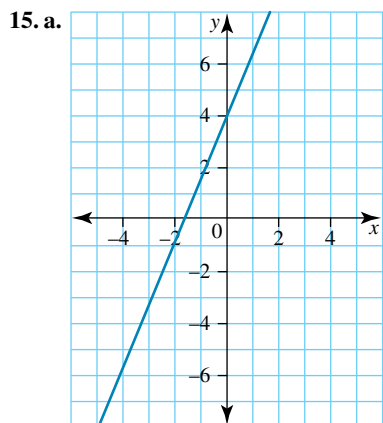
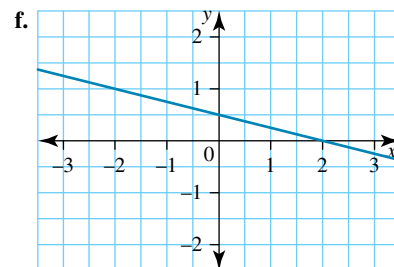
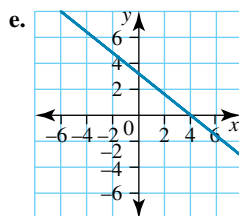
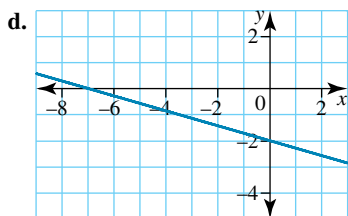
ii. $\frac{1}{4}$

iii. 2

iii. 11

iii. $\frac{3}{4}$





16. a. \$25

b. Yes

c. $w = 25n$, where w is the wage in \$, and n is number of hours worked.

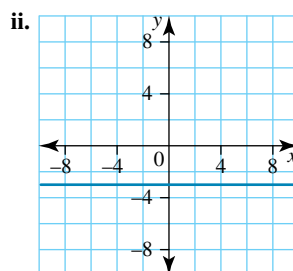
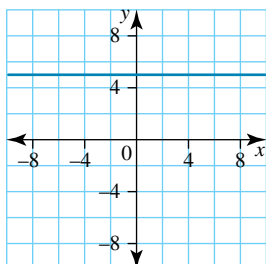
17. E

18. C

19. D

20. D

21. a. i.

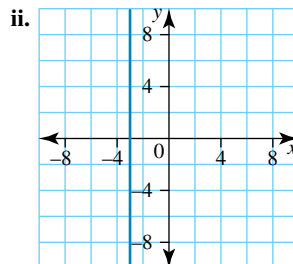
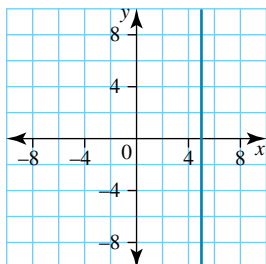


b. Both lines are horizontal. The equation only has a y ; there is no x in the equation.

c. 0

d. All of the y -values are the same; they do not change. The gradient is zero, so the line is horizontal.

22. a. i.



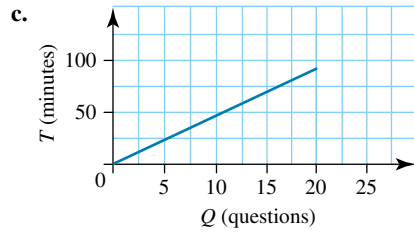
b. Both lines are vertical. The equation only has x ; there is no y .

c. The gradient is undefined.

d. All of the x -values are the same; they do not change. The gradient is undefined, so the line is vertical.

23. a. 92 minutes

- b. Q must be from 0 to 20, as there are 20 questions on the Maths skill sheet.
 T must be a positive number or zero, as it is time in minutes.



- d. Before starting the sheet, the student may have taken 2 minutes to get everything organised.
 e. No, this is not a linear model of direct variation as it is not of the form $y = mx$.

24. a. Yes, this graph does represent a direct-variation relationship as it starts at $(0, 0)$ and is of the form $y = mx$.

- b. 2.2 c. $y = 2.2x$ d. 4.4 km e. 2955 revolutions

25. a. i. New York; the temperature in Fahrenheit is the lowest (35°F).

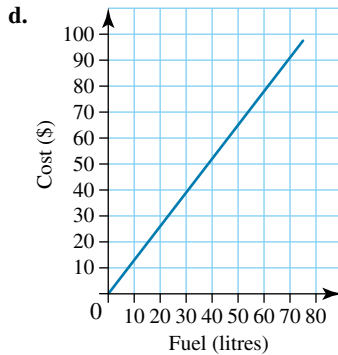
ii. 5.8°F

iii. Maximum: 24.44°C ; minimum: 1.67°C

b. 215°C

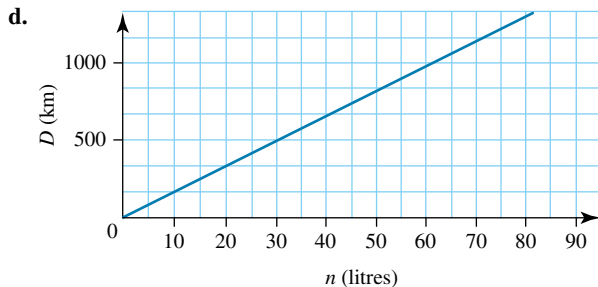
26. a. $y = 1.299x$ b. \$97.45

- c. Yes, this is an example of direct variation. It is of the form $y = mx$. The y -value is always 1.299 times the x -value, so it is increasing at a constant rate and passes through $(0, 0)$.



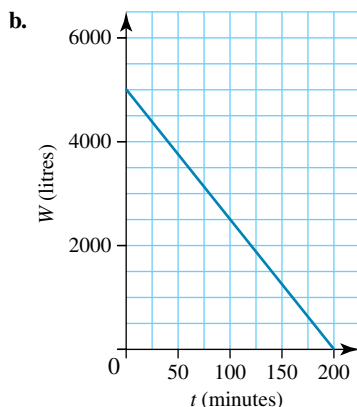
- e. The tank holds 75 litres of fuel, so the x -values lie from 0 to 75.
 The cost of the fuel must always be positive, giving the y -values from \$0 to \$97.40 for a full tank of fuel.

27. a. $D = 16.4n$ b. 820 km c. 76.22 litres



- e. Yes, this is an example of direct variation. It is of the form $y = mx$. The D -value is always 16.4 times the n -value, so it is increasing at a constant rate and passes through $(0, 0)$.
 f. 16.4, constant, 16.4

28. a. $W = -25t + 5000$



c. 3500 litres

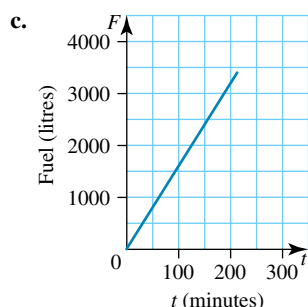
d. 200 minutes (3 hours 20 minutes)

e. For the model to be valid, t would need to be from 0 to 200 minutes and the amount of water in the tank, W litres, would need to be from 0 to 5000.

f. No, this is not an example of direct variation. It is not of the form $y = mx$, and does not start at $(0, 0)$.

29. a. $F = 16t$

b. 3 hours 30 minutes



d. For the model to be valid, t would need to be from 0 to 212.5 minutes and the amount of fuel in the tank, F litres, would need to be from 0 to 3400.

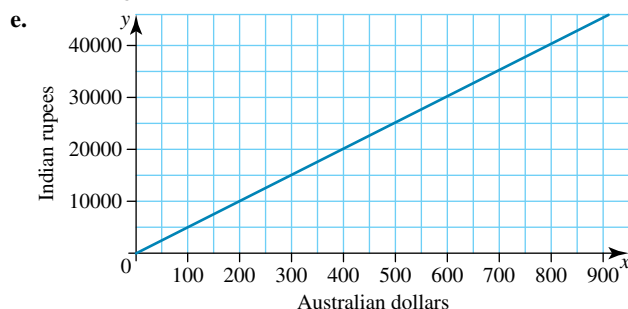
e. Yes, this is an example of direct variation. It is of the form $y = mx$. The F -value is always 16 times the t -value, so it is increasing at a constant rate and starts from $(0, 0)$.

30. a. $y = 50.48x$

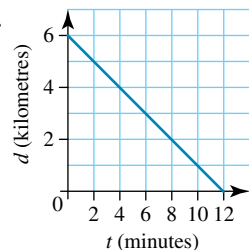
b. 60 576 Indian rupees

c. \$6.93 to \$49.52

d. Yes, this is an example of direct variation. It is of the form $y = mx$. The y -value is always 50.48 times the x -value, so it is increasing at a constant rate and starts at $(0, 0)$.



31. a.



b. 6 km

c. 12 minutes

d. The limitations on this linear model are that time, t minutes, is greater than or equal to zero, and the distance, d kilometres, is from 0 to 6 km.

e. No, this is not an example of a direct-variation relationship. It is not of the form $y = mx$, and does not start at $(0, 0)$.

TOPIC 3

Practicalities of measurement

3.1 Overview

3.1.1 Introduction

We all use measurement every day, often without even realising that we're doing so. For example, if you estimate the time that it'll take you to wash the dishes in the evening, that's a measure of time, and if you count the number of steps to walk to the corner shop, that's a measure of distance.

Understanding measurement allows us to understand the world around us, from the size of bacteria to the amount of time it would take us to travel to stars in the night sky. Think about how often you use measures every day!



DISCUSSION

Think back over the past week. What things have you had to measure or estimate the measurement of?

3.1.2 LEARNING SEQUENCE

- 3.1** Overview
- 3.2** Units of measurement
- 3.3** Limits of accuracy
- 3.4** Scientific notation
- 3.5** Review

CURRICULUM CONTENT

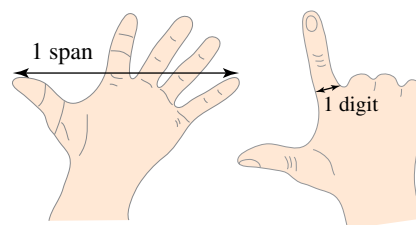
Students:

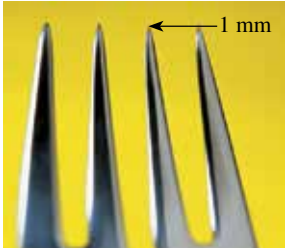
- review the use of different metric units of measurement including units of area, take measurements, and calculate conversions between common units of measurement, for example kilometres to metres or litres to millilitres (ACMEM090, ACMEM092) ◇
- calculate the absolute error of a reported measurement and state the corresponding limits of accuracy ◇
- use standard form and standard metric prefixes in the context of measurement, with and without a required number of significant figures ◇

3.2 Units of measurement

3.2.1 Units of measurement

- Units of length are used to describe:
 - the dimensions of an object, such as its length, width and height
 - the distance between two points.
- In the past, people used the sizes of their body parts as reference units to measure length, as shown.
- The **standard unit** of length in the metric system is the **metre**.
- The most common metric units of length are the millimetre (mm), centimetre (cm), metre (m) and kilometre (km).
 - 10 millimetres = 1 centimetre
 - 100 centimetres = 1 metre
 - 1000 metres = 1 kilometre



Millimetre (mm)	Centimetre (cm)	Metre (m)	Kilometre (km)
			

- While the measurement of a linear or curved distance can be measured using mm, cm, m, or km, other units of measurements are required to measure liquid, weight and time.
- In ancient Greece pottery vessels were used to trade wine or water, and balance beams were once used to measure weight. Sundials and hourglasses were used for centuries to tell the time, before advancements in technology allowed much more accurate measurements to be taken.
- In our modern society, we have standard measurements for volume (liquid, space, etc.), mass (weight) and time.
- In the metric system, the standard unit of volume is the **litre** and the standard unit of weight is the **gram**.



Metric volume measurements (small to large):

- millilitres (mL), litres (L)

Metric weight measurements (small to large):

- milligrams (mg), grams (g), kilograms (kg), tonnes (t)

Universal time measurements (small to large):

- seconds (s), minutes (min), hours (h), days, weeks, months, years, decades, centuries, millennia

3.2.2 Making measurements

- When measuring lengths, it is important to choose units that suit the situation. A suitable unit may be determined by:
 - the visual length of the measurement

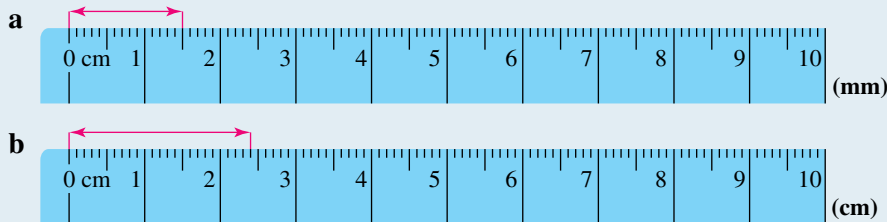


- the context in which the measurement is to be used; for example, builders, carpenters and plumbers work in millimetres and metres.
- A variety of tools can be used to help measure length.
 - A ruler can be used to measure short objects.
 - A tape measure can be used to measure longer objects or distances.
 - A car's odometer can record long measurements, such as the distance between two towns.
 - A picture with a scale, such as a map or a microscope drawing, can be used to measure very large or very small lengths.
- When using a ruler or a tape measure to measure length it is important to ensure that the zero line of the scale markings is always placed at the start of the length being measured.
- When measuring a curved line some useful tools are: a piece of string, a trundle wheel or the edge of a piece of paper, slowly rotated around the line.



WORKED EXAMPLE 1

State the measurement marked by the arrow in each of the following. Record your answer in the unit indicated in the brackets.



THINK

- a**
- 1 From left to right count how many centimetres the arrow has passed.
 - 2 Multiply that number by 10 to account for the number of mm in each cm ($1\text{ cm} = 10\text{ mm}$).
 - 3 Add the extra millimetres (mm).
- b**
- 1 Count the number of centimetres the arrow has passed. The whole numbers represent the number of cm.
 - 2 The extra 4 mm represent $\frac{4}{10}\text{ cm}$ or 0.4 cm.
 - 3 Add the whole and part cm.

WRITE

- a** 1 cm
- $$1 \times 10 = 10\text{ mm}$$
- $$10 + 5 = 15\text{ mm}$$
- b** 2 cm
- $$0.4\text{ cm}$$
- $$2 + 0.4 = 2.4\text{ cm}$$

- The measurement of volume, weight and time has become extremely accurate due to advances in technology. Volume is now measured using advanced laser systems, whereas in the past some of the volumes that occurred in nature could not be accurately measured. Digital scales are used to accurately measure very small or large weights, and atomic clocks can be accurate to the nearest second for millions of years.

3.2.3 Converting between units of measurements

- The relationships (or ratios) between the metric units of length can be used to **convert** a measurement from one unit to another.
- The following table shows how to convert a larger unit to a smaller unit.

For every 1 cm there are 10 mm.	For every 1 m there are 100 cm.	For every 1 km there are 1000 m.
5 cm = 50 mm 0.6 cm = 6 mm (Multiply by 10)	5 m = 500 cm 0.5 m = 50 cm (Multiply by 100)	25 km = 25 000 m 0.75 km = 750 m (Multiply by 1000)

- The following table shows how to convert a smaller unit to a larger unit.

For every 10 mm there is 1 cm.	For every 100 cm there is 1 m.	For every 1000 m there is 1 km.
400 mm = 40 cm 52 mm = 5.2 cm (Divide by 10)	750 cm = 7.5 m 32 cm = 0.32 m (Divide by 100)	25 000 m = 25 km 750 m = 0.750 km (Divide by 1000)

WORKED EXAMPLE 2

Convert the following lengths to the units shown.

- a 0.234 km = ____ m b 24 000 mm = ____ m

THINK

- a To convert from kilometres to metres, multiply by 1000, because there are 1000m for each 1 km.
- b To convert from millimetres to metres, first divide by 10 to convert from millimetres to centimetres, then divide by 100 to convert from centimetres to metres.

WRITE

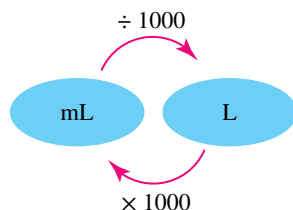
$$0.234 \text{ km} = (0.234 \times 1000) \text{ m} \\ = 234 \text{ m}$$

$$24\,000 \text{ mm} = (24\,000 \div 10 \div 100) \text{ m} \\ = (2400 \div 100) \text{ m} \\ = 24 \text{ m}$$

- All measurement conversions require the use of either multiplication (big to small units) or division (small to big units). Volume and weight measurements use the same metric prefixes as length, while time has a standard system that is the same everywhere around the world.

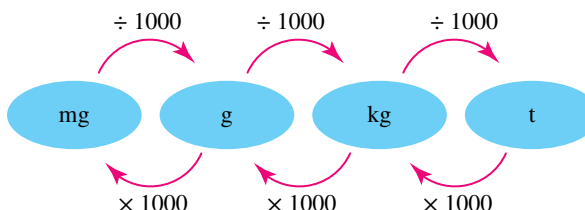
Volume

- For every 1 L there are 1000mL.



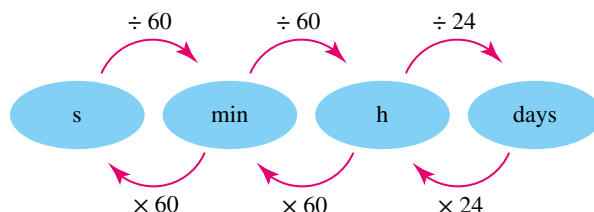
Weight

- For every 1 g there are 1000mg.
- For every 1 kg there are 1000g.
- For every 1 t there are 1000kg.



Time

- 60 seconds = 1 minute
- 7 days = 1 week
- 365 days = 1 year
- 60 minutes = 1 hour
- 12 months = 1 year
- 10 years = 1 decade
- 24 hours = 1 day
- 52 weeks = 1 year
- 100 years = 1 century



Note: There are 366 days in a leap year, which occurs every 4 years. If the last two digits form a multiple of 4 then the year is a leap year. For example, 2018 is not a leap year because 18 is not a multiple of 4.

WORKED EXAMPLE 3

Convert the following volume, weight and time measurements.

a 400 mL = ____ L

b 1500 g = ____ kg

c 5 h = ____ s

THINK

- a** To convert from millilitres (mL) to litres (L) divide by 1000.
- b** To convert from grams (g) to kilograms (kg) divide by 1000.
- c** To convert from hours to seconds, first multiply by 60 to convert from hours to minutes, then multiply by 60 again to convert from minutes to seconds.

WRITE

a 400 mL = $(400 \div 1000)$ L
= 0.4 L

b 1500 g = $(1500 \div 1000)$ kg
= 1.5 kg

c 5 h = (5×60) min
= 300 min
300 min = (300×60) s
= 18 000 s

- All measurements need to be in the same unit before they can be added or subtracted.

WORKED EXAMPLE 4

Calculate the value of $6\text{ m } 25\text{ cm} + 18\text{ m } 92\text{ cm}$ using two different methods.

THINK

- 1 Collect the like-sized measurements.
- 2 Add the like-sized measurements.
- 3 Convert 117 cm into metres and centimetres. Add 24 m and 1 m.
- 4 Convert both measurements to decimal measurements of the larger units.
- 5 Add the two decimals.

WRITE

$$\begin{aligned}
 6\text{ m } 25\text{ cm} + 18\text{ m } 92\text{ cm} &= 6\text{ m} + 25\text{ cm} + 18\text{ m} + 92\text{ cm} \\
 &= 6\text{ m} + 18\text{ m} + 25\text{ cm} + 92\text{ cm} \\
 &= 24\text{ m} + 117\text{ cm} \\
 &= 24\text{ m} + 1\text{ m} + 17\text{ cm} \\
 &= 25\text{ m} + 17\text{ cm} \\
 6\text{ m } 25\text{ cm} + 18\text{ m } 92\text{ cm} &= 6.25\text{ m} + 18.92\text{ m} \\
 &= 25.17\text{ m}
 \end{aligned}$$

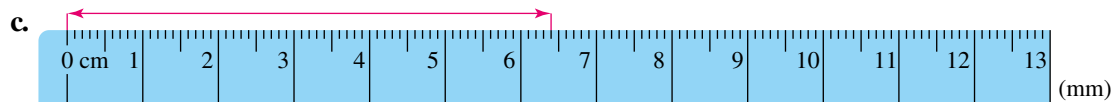
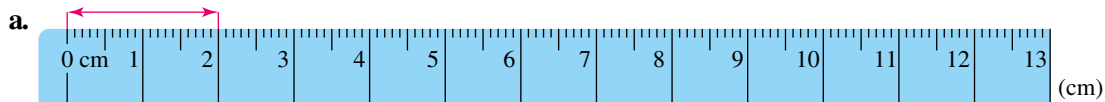
eBookplus RESOURCES

Interactivity: Converting units of length (int-4011)

Exercise 3.2 Units of measurement

Understanding, fluency and communicating

1. **WE1** State the measurement marked by the arrow in each of the following. Record your answer in the unit indicated in brackets.



2. a. Convert 5600 m to km. b. Convert 21 cm to mm. c. Convert 55 km to m.
d. Convert 2.2 m to cm. e. Convert 350 cm to m. f. Convert 25 mm to cm.

3. **WE2** Convert the following lengths to the units shown.

- a. 0.165 km = _____ m b. 780 cm = _____ m
c. 0.158 m = _____ cm d. 13 m = _____ km
e. $1\frac{1}{4}$ km = _____ cm f. 36 000 mm = _____ m
g. 1.35 km = _____ mm h. 3000 cm = _____ km

4. Convert the following measurements as indicated.

- a. Convert 5632 cm to km. b. Convert 204 310 cm to km. c. Convert 5.5 km to cm.
d. Convert 2.2 m to mm. e. Convert 35 000 cm to km. f. Convert 250 mm to m.

5. Arrange the following lengths in descending order.

- a. 89 000 cm, 7.825 m, 98 760 mm, 0.3217 km
b. 0.786 mm, 0.786 km, 0.000 786 m, 0.0786 cm

6. Arrange each of the following in ascending order.

- a. 0.15 km, 135 m, 2400 cm b. 25 cm, 120 mm, 0.5 m c. 9 m, 10 000 mm, 0.45 km
d. 32 000 cm, 1200 m, 1 km e. 1.5 m, 150 cm, 0.0015 km f. 8.25 km, 825 m, 90 000 cm

7. **WE4** Calculate the value of each of the following, in centimetres.

- a. 15 mm + 5 cm b. 1.5 m + 20 mm
c. 855 mm + 1.8 m d. 5.67 cm + 1156 mm – 0.25 m

8. Calculate the value of each of the following.

- a. 15 cm 15 mm + 27 cm 86 mm b. 6 km 58 m + 84 km 47 m
c. 75 cm 10 mm – 15 cm 5 mm d. 125 m 58 cm – 18 m 85 cm

9. Which volume measurement is the most appropriate for each of the following objects?

a.



b.



10. Which weight measurement is the most appropriate for each of the following objects?

a.



b.



c.



11. **MC** Which is the larger volume?

A. 1500mL

B. 3L

C. 0.465L

D. 858mL

E. 1.521L

12. **WE3** Convert the following volume measurements.

a. 2500mL to L

b. 3L to mL

c. 3.1 mL to L

d. 5.25L to mL

e. 7.5L to mL

f. $3\frac{3}{4}$ mL to L

13. Arrange the following weights in ascending order.

a. 2500g, 2.4kg, 400 000mg, 0.000 024t

b. 0.05kg, 54g, 52 000mg, 0.0005t

14. True or False?

a. $1555\text{g} > 2\text{g}$

b. $3.2\text{kg} = 320\text{g}$

c. $45000\text{mg} < 0.145\text{kg}$

d. $500000\text{mg} > 500\text{g}$

15. Convert the following weight measurements.

a. $505\text{g} = \underline{\hspace{1cm}}\text{mg}$

b. $4.5\text{mg} = \underline{\hspace{1cm}}\text{g}$

c. $6.4\text{kg} = \underline{\hspace{1cm}}\text{g}$

d. $5840\text{g} = \underline{\hspace{1cm}}\text{kg}$

e. $75\,821\text{mg} = \underline{\hspace{1cm}}\text{kg}$

f. $0.000\,07\text{kg} = \underline{\hspace{1cm}}\text{mg}$

g. $2\text{t} = \underline{\hspace{1cm}}\text{kg}$

h. $35\text{kg} = \underline{\hspace{1cm}}\text{t}$

i. $5\,000\,000\text{g} = \underline{\hspace{1cm}}\text{t}$

16. Fill in the blanks with the appropriate time measurement from the list below. (Use each term only once.)

centuries

second

decades

hours

weeks

minutes

months

years

day

a. The Roman Empire lasted about 5 _____.

b. The internet was invented about 3 _____ ago.

c. The speed of light is $299\,792\,458\text{m}/\underline{\hspace{1cm}}$.

d. Distance to most planets is measured in light _____.

e. A full-time job is considered to be 35–40 _____ per week.

f. There is always at least one low and one high tide every _____.

g. The _____ are based upon the lunar cycle.

h. It takes about 2 _____ to cook instant popcorn.

i. A normal pregnancy takes between 37 and 42 _____.



17. Arrange each of the following in descending order.

a. 2 decades, 1.5 centuries, 250 years, 500 months

b. 750 minutes, 12.6 hours, 4250 seconds, 0.56 days, 0.1 weeks

18. Convert the following time measurements:

a. $4.5\text{decades} = \underline{\hspace{1cm}}\text{years}$

b. $825\text{years} = \underline{\hspace{1cm}}\text{decades}$

c. $3.4\text{centuries} = \underline{\hspace{1cm}}\text{years}$

d. $485\text{years} = \underline{\hspace{1cm}}\text{centuries}$

e. $5\text{centuries} = \underline{\hspace{1cm}}\text{decades}$

f. $120\text{decades} = \underline{\hspace{1cm}}\text{centuries}$

19. Convert the following time measurements. Give answers to 2 decimal places where necessary.

- a. 240 s = _____ min b. 5.2 min = _____ s
 c. 4.6 h = _____ min d. 320 min = _____ h
 e. 30 000 s = _____ h f. 20.3 h = _____ s

20. **MC** How many days were there in 1981, 1982, 1983 and 1984 combined?

- A. 1460 B. 1461 C. 1462 D. 1459 E. 1458

21. **MC** How many hours are there in 3.6 days?

- A. 86 B. 86.6 C. 864 D. 86.4 E. 866

22. **MC** How many days are there in 396 hours?

- A. 16.5 B. 16 C. 15.5 D. 15 E. 14.5

Problem solving, reasoning and justification

23. Your friend has invented a measurement system called Measuro. In the Measuro measurement system, the smallest measurement unit is a *small*. There are 12 *smalls* in a *medium* and 8 *mediums* in a *large*.

- a. How many *smalls* are there in 4 *mediums*?
 b. How many *smalls* are there in 5 *larges*?
 c. If you are 75 *larges* tall, how tall are you in *mediums*?
 d. Explain how you convert a length measured in *larges* into *smalls*.
 e. Convert 125 *larges* into *smalls*.

24. Explain why it would not be wise to use millimetres to measure the distance between Melbourne and Perth.



25. Wayne's Concreting Crew needs to pour concrete for a driveway. They have calculated that the driveway needs 20 cubic metres (20m^3) of concrete.

- a. If 1 cubic metre equals 1000 litres, how many litres of concrete are needed?
 b. How many cubic metres can you cover if you have 2584 litres of concrete?

26. Find the metric equivalent of each of the following measurements from the imperial measurement system.

Hint: Convert to centimetres first and then to either metres or kilometres, as appropriate.

1 inch = 2.54 cm	1 mile = 1760 yards
1 foot = 12 inches	1 chain = 22 yards
1 yard = 3 feet	1 chain = 4 rods
1 furlong = 220 yards	1 rod = 25 links

27. a. Find the metric length of each of the following units and explain what they are used to measure.

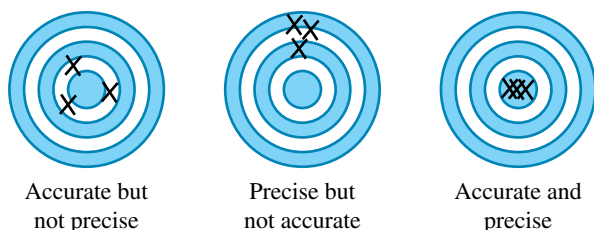
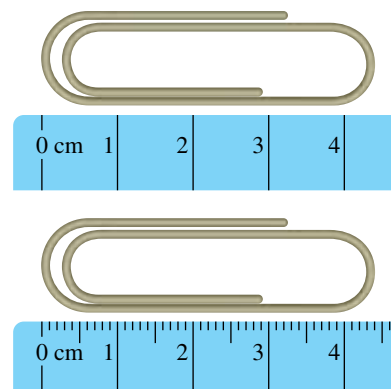
- i. A nautical mile
 ii. A light year

b. Create a poster to display in your classroom that shows the non-metric lengths from part a and their equivalent metric lengths.

3.3 Limits of accuracy

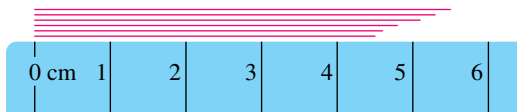
3.3.1 Accuracy

- **Accuracy** refers to how closely a measured value agrees with the actual value. The accuracy of a measurement can be improved by measuring with smaller units. For example, a ruler that has millimetre markings will give a more accurate measurement of the length of a paperclip than a ruler that has only centimetre markings, as shown.
- **Precision** refers to how closely the individual measurements agree with each other. It is possible for a measuring device to be precise but not accurate.
- The following diagram compares accuracy and precision.



3.3.2 Limits of accuracy

- The following ruler has only centimetre markings, so measurements are approximated to the nearest centimetre. Lengths between 4.5 cm and 5.5 cm are all measured as 5 cm with this ruler.



- The **limit of accuracy** of a measuring tool is *half of the smallest unit* marked on the tool. This limit determines the highest and lowest values that will result in a particular measurement being recorded. The highest and lowest values indicate the **range** within which the actual value will lie.

WORKED EXAMPLE 5

The speedometer shown at right measures the speed at which a car is travelling.

- State the limit of accuracy for this speedometer.
- Provide a range within which the car's actual speed (as indicated by the arrow) lies.



THINK

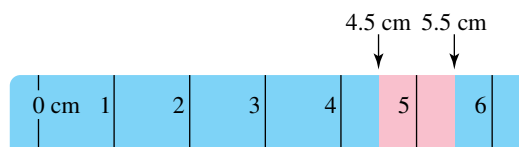
- Each marking on the scale represents 5 km/h, so the limit of accuracy is half of this measurement.
- The arrow is pointing to 80 km/h on the speedometer. The actual speed lies within the range of 80 km/h – 2.5 km/h to 80 km/h + 2.5 km/h.

WRITE

$$\begin{aligned}\text{Limit of accuracy} &= \frac{1}{2} \times 5 \\ &= 2.5\end{aligned}$$

$80 - 2.5 = 77.5$ and
 $80 + 2.5 = 82.5$
 The actual speed lies within the range of 77.5 km/h to 82.5 km/h.

- The limit of accuracy of the ruler shown is 0.5 cm, which is half of the unit shown on the scale. If an object is measured with this ruler as being 5 cm long, the actual measurement will be between 4.5 cm and 5.5 cm; that is, half a unit above and below the nearest centimetre.
- It is always possible to divide a measurement unit into smaller units. For example, metres can be divided into centimetres, centimetres can be divided into millimetres, millimetres can be divided into micrometres and so on. So the measurements that we take are never **exact**. They are **approximations** only.
- *All* measurements are approximations, not just those relating to length. For example, the measurement of the mass of a teaspoon of butter is only as accurate as the kitchen scales with which it is measured, and the measurement of a baby's temperature is only as accurate as the thermometer's scale.



3.3.3 Types of error

- Errors in measurement can be made due to a number of factors:
 - An inappropriate instrument being used. For example, it would be inappropriate to use a centimetre ruler to measure the distance from Melbourne to Sydney.
 - Human error when reading the measurement. For example, when measuring the length of a school football oval, it is possible for three different students to report three different results.
 - Inaccuracy of the instrument being used. For example, it would be more accurate to measure 20 mL with a measuring cylinder instead of a beaker.
- The difference between the measurement made (the estimated value) and the actual size of the object is called the **error**.
- The error can be positive or negative and can be calculated using the formula:



$$\text{Error} = \text{estimated value} - \text{actual value}$$

3.3.4 Accuracy of measurement

- The number of decimal places a measurement contains can give an indication of the accuracy of the measurement. For example:
 - a measurement of 5 m indicates that the length is closer to 5 m than it is to either 4 m or 6 m. It has been measured to the nearest metre.
 - a measurement of 5.00 m indicates that the length is closer to 5.00 m than it is to 4.99 m or 5.01 m. It has been measured to the nearest hundredth of a metre or the nearest centimetre.
 - a measurement of 5.000 m has been measured to the nearest thousandth of a metre or the nearest millimetre.
- The number of decimal places a measurement can have is determined by the scale on the measuring device and the units the device has.
- The more decimal places a measurement contains, the more accurately it was measured.

3.3.5 The absolute error

- The **absolute error** is the value of the error without the positive or negative sign. The calculation for the absolute error is done in a pair of vertical bars. This is called the **modulus** and means that you ignore the sign of the number. As the sign is ignored, the absolute error is never negative.

$$\text{Absolute error} = |\text{estimated value} - \text{actual value}|$$

- ## WORKED EXAMPLE 6

-

WRITE

- a** $|105 \text{ mm} - 107 \text{ mm}| = |-2 \text{ mm}|$
 $= 2 \text{ mm}$
- b** $|2.25 \text{ inches} - 2.75 \text{ inches}| = |-0.5 \text{ inches}|$
 $= 0.5 \text{ inches}$
- Yes, 0.50 inches is almost a quarter of the original measurement.

- To compare the estimated value with the actual value, the **absolute relative error** is calculated. It expresses the absolute error as a fraction of the actual value.

- The absolute relative error can be converted into the **absolute percentage error** by multiplying by 100%.

WORKED EXAMPLE 7

WRITE

- $$\begin{aligned}\text{Error} &= 19.5 - 20.5 \\ &= -1.0 \text{ cm} \\ \text{Absolute error} &= |-1.0| \\ &= 1.0 \text{ cm}\end{aligned}$$

- c The absolute relative error is the absolute error expressed as a fraction of the actual value.

$$\begin{aligned}\text{Absolute relative error} &= \frac{|\text{estimated value} - \text{actual value}|}{\text{actual value}} \\ &= \frac{|19.5 - 20.5|}{20.5} \\ &= \frac{1.0}{20.5} \\ &= 0.049\end{aligned}$$

- d The absolute percentage error is the absolute error expressed as a percentage of the actual value.

$$\begin{aligned}\text{Absolute percentage error} &= \text{absolute relative error} \times 100\% \\ &= 0.049 \times 100\% \\ &= 4.9\%\end{aligned}$$

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Weblink: Hindawi

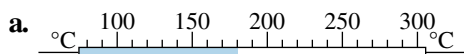
Weblink: Time.is

Exercise 3.3 Limits of accuracy

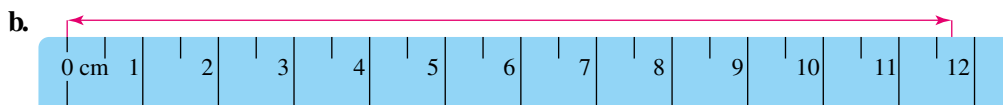
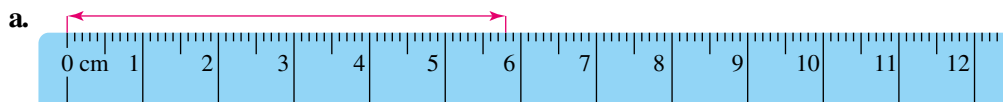
Understanding, fluency and communicating

1. **WE5** For each of the scales shown:

- state the limit of accuracy
- write a range to show the values between which the measurement shown on the scale lies.



2. Write the measurements of the following lines as accurately as possible.



- Each of the following lengths has been recorded to the nearest centimetre. Provide a range within which each measurement will lie.
 - 5 cm
 - 12 cm
 - 100 cm
 - 850 cm
 - 5 m
 - 2.5 m
- For each of the following measurements, provide the two limits between which the actual measurement will lie.
 - The length of a book is 29 cm, correct to the nearest centimetre.
 - The mass of meat purchased is 250 grams, correct to the nearest 10 grams.
 - The volume of cough medicine is 10 mL, correct to the nearest 5 mL.
 - Half an hour, to the nearest minute.
- MC** The most accurate of the following measurements is:
 - 7
 - 7.0
 - 0.7
 - 7.7
 - 7.07
- MC** The least accurate of the following measurements is:
 - 8.000
 - 8.0
 - 8
 - 8.008
 - 0.08
- A measurement of 2.500 m indicates that the measurement is accurate *to the nearest millimetre* (2.500 m = 2500 mm). Determine the unit to which each of the following measurements is accurate.
 - 35.20 m
 - 1.30 m
 - 1.000 km
 - 8.0 cm
 - 5.000 m
 - 2.520 km
- Four students recorded the following measured lengths during a Science class. The actual lengths were also recorded. Calculate the error made by each student.

Student	Estimated value	Actual value
a	10 cm	9.4 cm
b	15.5 cm	17.2 cm
c	250 km	228 km
d	37 mm	38.1 mm

- 9. WE6** a. Jordan estimated the length of his bedroom as 410 cm. However, when he measured it, he found out it was only 395 cm long. What is the absolute error of Jordan's measurement?
- b. Jamie ordered a 30 cm pizza for a party with her friends. However, when it arrived, she found that the pizza was only 29.7 cm wide. Do you think this is a significant error? Why or why not?
- 10.** Calculate the absolute error for each of the following.
- a. The distance from home to school is estimated by a student to be 2 km. The actual distance is 1.85 km.
- b. The weight stated on a packet of lollies is 250 g. The actual weight is 254 g.
- c. The scheduled arrival time of the plane from Kansas City is 7:23 pm. The actual arrival time is 7:46 pm.
- d. The estimated volume of water in a fish tank is 65 L. The actual volume is 72 L.
- 11.** You measured your friend's height to be 165 cm. If the absolute error of your measurement is 4 cm, how tall is your friend? (*Hint:* There may be more than one answer.)
- 12. WE7** Complete the table on the next page by calculating the error, the absolute error, the absolute relative error and absolute percentage error. Give your answers to the absolute relative errors correct to 2 decimal places where necessary and give your answers to the absolute percentage errors to the nearest whole number. (*Hint:* Make sure that both measurements are in the same units.)
- 



	Measurement		Error	Absolute error	Absolute relative error	Absolute percentage error
	Estimated	Actual				
a	250 m	245 m				
b	95 cm	1.02 m				
c	2 cm	18.5 mm				
d	48 seconds	1 minute				
e	1.4 kg	1 kg 300 g				
f	750 mL	0.9 L				

13. After ordering food in a local restaurant, a family estimates that their dinner will cost \$120. Compare their estimate with the actual total, as shown on the bill, and calculate their absolute percentage error to the nearest whole number.



Problem solving, reasoning and justification

14. a. Explain how you would calculate the limit of accuracy of the ruler shown.



- b. A line measured with this ruler is recorded as 23 mm. Between which two measurements will the actual length lie?
15. Explain why any measurement is an approximation and can never be exact.
16. What is the difference between a measurement of 4 km and a measurement of 4.000 km?
17. Medical researchers have found that estimating the weight of unborn babies is critical in determining the quality of their health at pregnancy. There are two methods that are used to estimate the weight of unborn babies: an ultrasound of the expecting mother, or specific measurements of her body. Both of these methods are performed over a five-month period, to ensure accuracy. Read the abstract of the scholarly report of the two methods at hindawi.com.
- What does the abstract say about the mean absolute percentage error for each method?
 - What is the mean absolute percentage error for each method?

18. The accuracy of clocks can vary significantly. Check the clock on your mobile phone and your computer at home. These clocks could be set simultaneously, yet after weeks or months might differ by seconds or even minutes. How do you know which time is the most accurate?



An atomic clock uses the resonation of a cesium atom to measure time, and it is the most accurate clock in the world. If you go to time.is on your phone or computer, you have access to an atomic clock that can compare your computer or mobile-phone time to the atomic clock. There is a problem with this method because of the inaccuracy of synchronisation of your computer or phone with the atomic clock. The accuracy of synchronisation is ± 0.425 sec.

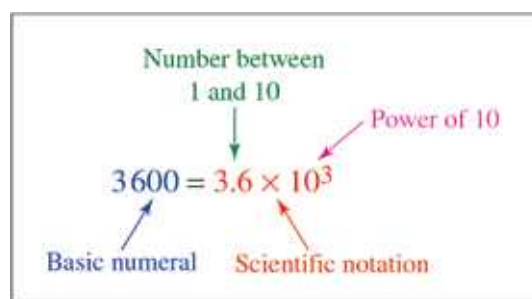
All computer or phone clocks are estimates compared to the atomic clock (actual time).

- a. Is the accuracy of synchronisation more or less significant if the estimated time is behind or ahead? Explain your answer.
- b. If your computer clock's time was one minute ahead of the atomic clock, would the accuracy of synchronisation be significant? Explain your answer.
- c. If your computer's clock time was one second behind the atomic clock, would the accuracy of synchronisation be significant? Explain your answer.

3.4 Scientific notation

3.4.1 Scientific notation

- Scientists, economists, statisticians and mathematicians often need to work with very large or very small numbers.
- Scientific notation** (sometimes called **standard form**) is a way of writing a very large or very small number using less space. In scientific notation, the number is written as a multiple of a power of 10 and looks like $a \times 10^n$, where a is a number between 1 and 10. For example,
 - $3600 = 3.6 \times 10^3$
 - $0.036 = 3.6 \times 10^{-2}$
 - $3\,600\,000\,000 = 3.6 \times 10^9$
 - $0.0000036 = 3.6 \times 10^{-6}$



WORKED EXAMPLE 8

Write the following numbers using scientific notation.

a 345

b 0.007

THINK

- a
- The non-zero digit with the largest place value is the 3 in the hundreds position, as shown in blue.
 - The 3 will become the number between 1 and 10 in scientific notation, as shown in red. The rest of the digits in 345 become decimal places in scientific notation.
 - The digit 3 must still have a value of 300, so 3.45 needs to be multiplied by 100 (10^2), as shown in green.
- b
- The non-zero digit with the largest place value is the 7 in the hundred thousandths position, as shown in blue.
 - The 7 will become the number between 1 and 10 in scientific notation, as shown in red. The rest of the digits to the right (the trailing zeros) become decimal places in scientific notation.
 - The digit 7 must still have a value of 7 thousandths, so 7.0 must be multiplied by $\frac{1}{1000}$ (10^{-3}), as shown in green.

WRITE

$$345 = 3.45 \times 10^2$$

$$0.007 = 7.0 \times 10^{-3}$$

- To write a number expressed using scientific notation as a basic numeral, simply complete the multiplication.

WORKED EXAMPLE 9

Write the following numbers as basic numerals.

a 2.897×10^8 b 6.05×10^{-6}

THINK

a Complete the multiplication by 10^8 . This can be done by moving the decimal point 8 places to the right. The result is a very large number.

b 1 Complete the multiplication by 10^{-6} . This is the same as multiplying by $\frac{1}{1\,000\,000}$.

2 Multiplying by $\frac{1}{1\,000\,000}$ can be done by moving the decimal point 6 places to the left. The result is a very small number.

WRITE

$$\begin{aligned} 2.897 \times 10^8 &= 2.897 \times 100\,000\,000 \\ &= 289\,700\,000 \\ &= 289\,700\,000 \end{aligned}$$

$$\begin{aligned} 6.05 \times 10^{-6} &= 6.05 \times \frac{1}{1\,000\,000} \\ &= 0.000\,006\,05 \\ &= 0.000\,006\,05 \end{aligned}$$

- Scientific notation is displayed on some calculators and spreadsheets using an E.
- These calculators or spreadsheets show a number between 1 and 10, then an E and then the power to which 10 is raised. For example:
 - 3.45×10^{18} will appear as 3.45E18 on a calculator or spreadsheet.
 - The display 5.9E-03 represents the number 5.9×10^{-3} .

3.4.2 Measuring very large and very small units using scientific notation

- Astronomers use very large numbers to express distances between planets, solar systems, stars and other celestial objects. If they are writing scientific papers for their research, scientific journals, or the general public, the time it takes to repeatedly refer to these distances becomes prohibitive. Imagine writing the distance to the furthest galaxies in the solar system: 145 000 000 000 000 000 000 km. It would be much easier to express this distance in scientific notation as 1.45×10^{23} km.
- Physicists have a similar problem writing numbers. They record weights that are very small and, like the astronomers, they need to describe these numbers to their readers. For example, the mass of a stationary electron is 0.000 000 000 000 000 000 000 000 000 911 kg. Try counting the 0s! Instead, the physicist uses the scientific notation of 9.11×10^{-31} kg.

WORKED EXAMPLE 10

a The approximate diameter of the Milky Way galaxy is 950 000 000 000 000 000 km. Use scientific notation to express this distance.

b A stock trade on the New York Stock Exchange can be made in 740 nanoseconds or 0.000 007 24 seconds. Use scientific notation to express this time in seconds.



WRITE

- a** 950 000 000 000 000 000 km = 9.5×10^{17} km
- b** 0.000 007 24 seconds = 7.24×10^{-6} seconds

- **Significant figures** are a method of indicating the precision of a measurement.
- The more significant figures a number has, the more precise the measurement.
- The words *significant figures* are sometimes abbreviated to sig. figs.

1. All non-zero digits are significant.
Example: 513 (3 significant figures)
2. Any zeros between two significant figures are significant.
Example: 602, 1.5003, 600.1, 100.000 02
3. Any zeros that are not between significant figures in a whole number are **not** significant.
Example: 50 000, 30, 3510
4. Zeros following the last significant figure in the decimal portion of a number are significant.
Example: 52.0030, 60.010 500, 0.340 00
5. Zeros preceding the first significant figure in the decimal portion of a number are **not** significant.
Example: 0.000 08, 0.007 80
6. Zeros in the decimal portion of a number that have no significant figures following them are significant, and therefore the zeros at the end of the whole number become significant, if they are present.
Example: 80.000, 8.0, 7080.00

- If a number is expressed using scientific notation ($a \times 10^n$), then all the digits in a are significant (see the chart on the next page).
- When working with significant figures in maths or science, do not round off to the required number of significant figures until the final answer.

- When writing a number to a specified number of significant figures, the smallest significant digit may need to be rounded, depending on the value of the next digit. For example:
 - 7.59 is written as 7.6, correct to 2 significant figures.
 - 7.54 is written as 7.5, correct to 2 significant figures.

Number	Scientific notation	Significant figures
34.56	3.456×10^1	4
3.456	3.456×10^0	4
346	3.46×10^2	3
3460	3.46×10^3	3
346000	3.46×10^5	3
0.0034	3.4×10^{-3}	2
3.4×10^4	3.4×10^4	2

WORKED EXAMPLE 11

Convert the following measurements as specified, and then write the measurements correct to the number of significant figures indicated in brackets.

a 0.000 078 1 cm to mm (2 significant figures)

b 550 900 g to kg (3 significant figures)

c 80.4586 s to min (4 significant figures)

THINK

- a**
- 10 mm = 1 cm
 - To convert from cm to mm, multiply by 10.
 - All of the non-zero digits in the number are significant; however, the three zeros to the right of the decimal point are not significant.
 - The second significant digit (8) will not need to be rounded, as 1 is less than 5.
- b**
- 1000 m = 1 km
 - To convert from m to km divide by 1000.
 - All of the digits in the number are significant; however we only need 3 significant figures, so these are the first three digits.
 - The 0 to the left of the decimal point will need to be rounded up to 1 because of the following 9.
- c**
- 1 min = 60 s
 - To convert from seconds to minutes divide by 60.
 - All the digits are significant; however we only need the first 4 significant figures, so these are the first 4 digits.
 - The fourth significant digit (4) will not need to be rounded as 0 is less than 5.

WRITE

$$\begin{aligned} \mathbf{a} \quad 0.000\,078\,1\text{ cm} &= (0.000\,078\,1 \times 10)\text{ mm} \\ &= 0.000\,781\text{ mm} \end{aligned}$$

$$0.000\,781\text{ mm} = 0.000\,78\text{ mm (2 sig. figs.)}$$

$$\begin{aligned} \mathbf{b} \quad 550\,900\text{ m} &= (550\,900 \div 1000)\text{ km} \\ &= 550.900\text{ km} \end{aligned}$$

$$550.900\text{ km} = 551\text{ km (3 sig. figs.)}$$

$$\begin{aligned} \mathbf{c} \quad 80.4586\text{ s} &= (80.4586 \div 60)\text{ min} \\ &= 1.304097\dots\text{ min} \end{aligned}$$

$$1.304\,097\dots\text{ min} = 1.304\text{ min (4 sig. figs.)}$$

Exercise 3.4 Scientific notation

Understanding, fluency and communicating

- WE8** Write the following numbers using scientific notation.
 a. 47.2 b. 3890 c. 0.56 d. -0.0067 e. 34000000
- WE9** Write the following numbers as basic numerals.
 a. 4.8×10^{-2} b. 7.6×10^3 c. 2.9×10^{-4} d. 8.1×10^0
- Which number(s) are not written in scientific notation?
 a. 10.75×10^7 b. 3.54×10^{-3} c. 0.0091×10^4 d. $7.23 \times 10^{\frac{1}{2}}$
- True or False?
 a. $0.000\,000\,51 \geq 5.1 \times 10^{-6}$ b. $3.158\,400 \times 10^7 < 3\,158\,400$
 c. $2.000\,000\,058 \times 10^9 = 2\,000\,000\,058$ d. $3.51 \times 10^4 \geq 3.502 \times 10^4$
- MC** 0.00067 written using scientific notation is:
 A. 67×10^{-5} B. 0.67×10^{-3} C. 6.7×10^{-4} D. 6.7×10^{-5} E. 0.67×10^{-4}
- WE10** a. The nearest black hole to the Earth is thought to be only 1600 light-years away, which is equal to 15 137 000 000 000 000 km. Use scientific notation to express this distance.
 b. The radius of an electron is 0.000 000 000 000 002 82 m. Use scientific notation to express this size.
- Scientists used Earth's gravitational pull on nearby celestial bodies (such as the Moon) to calculate the mass of the Earth at approximately 5.972 sextillion metric tons.
 a. Write 5.972 sextillion using scientific notation.
 b. How many significant figures does this number have?
- Write the following numbers correct to the number of significant figures specified in brackets.
 a. 0.05789 (2 significant figures)
 b. 0.05058 (3 significant figures)
 c. 55830.00001 (4 significant figures)
 d. 2.978582×10^{-17} (1 significant figures)
 e. 138000 (5 significant figures)
 f. 1.054 (2 significant figures)
- WE11** Convert the following measurements as specified, and then write the measurements correct to the number of significant figures indicated in brackets.
 a. 1200 cm to m (3 significant figures) b. 6.7481 kg to g (4 significant figures)
 c. 0.000 08 h to min (3 significant figures) d. 80.26 days to hours (4 significant figures)
 e. 51 kg to g (1 significant figure) f. 5.025 mm to m (2 significant figures)
- MC** Select the measurement that is most accurate. (*Hint*: Count the number of significant figures.)
 A. 2.5 m B. 2.51 m C. 2.059 m D. 2.5000 m E. 2.005 m
- MC** Select the measurement that is most accurate. (*Hint*: First convert measurements to grams, then count the number of significant figures.)
 A. 0.54 kg B. 500 g C. 501.0 g D. 0.541 kg E. 530 g



5 972 000 000 000 000 000 metric tons

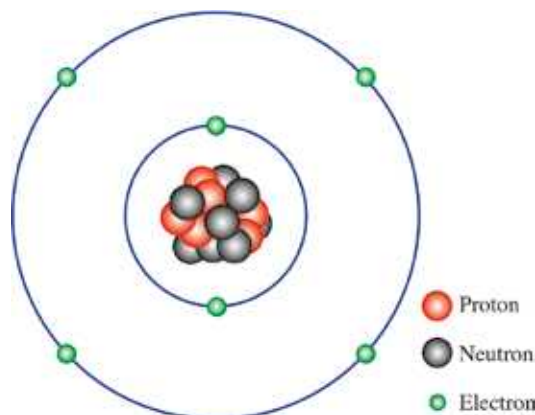
Problem solving, reasoning and justification

- 12.** What is the value of n in each of the following expressions?

a. $4793 = 4.793 \times 10^n$ **b.** $0.631 = 6.31 \times 10^n$
c. $134 = 1.34 \times 10^n$ **d.** $0.00056 = 5.6 \times 10^n$

- 13.** An atom consists of smaller particles called protons, neutrons and electrons. Electrons have a mass of $9.10938188 \times 10^{-31}$ kg correct to 9 significant figures.

- a.** Write the mass of an electron correct to 5 significant figures.
b. Protons and neutrons are the same size. They are both 1836 times the size of an electron. Use the original mass of an electron (correct to 9 sig. figs.) and your calculator to find the mass of a proton correct to 5 significant figures.
c. Use the mass of an electron correct to 3 significant figures to calculate the mass of a proton correct to 5 significant figures.
d. Why is it important to work with the original amounts and then round to the specified number of significant figures at the end of a calculation?



- 14.** How many significant figures do these numbers have?

a. 3.005×10^{-7} **b.** 3.005 **c.** 4000 **d.** 3.00×10^{-7}

- 15.** Complete the following calculations, writing your answer to the number of significant figures given in the brackets.

- a.** $2.398\text{m} + 3.5\text{m}$ (2)
b. $57.86\text{cm} + 0.157\text{m}$ (3)
c. $52.89\text{m} - 1.3 \times 10^2\text{m}$ (2)
d. $10.253\text{cm} \times 0.251\text{m}$ (3)
e. $973.48\text{m} \times 3.5 \times 10^3\text{mm}$ (2)

- 16.** Distance is equal to speed multiplied by time. If we travelled at 100km/h it would take approximately 0.44 years to reach the Moon, 89.6 years to reach Mars, 1460 years to reach Saturn and 6590 years to reach Pluto.

- a.** Assuming that there are 365 days in a year, calculate the distance (as a basic numeral) between Earth and:

- i.** the Moon **ii.** Mars **iii.** Saturn **iv.** Pluto.

- b.** Write your answers to part **a** correct to 3 significant figures.

- c.** Write your answers to part **a** using scientific notation correct to 3 significant figures.

- 17.** A light-year is the distance that light travels in one year. Light travels at approximately 300000km/s.

- a. i.** Calculate the number of seconds in a year.

- ii.** Write your answer to part **i** using scientific notation.

- b.** Calculate the distance travelled by light in one year. Express your answer:

- i.** as a basic numeral **ii.** using scientific notation.

- c.** The closest 'star' to Earth (other than our sun) is the star system Alpha Centauri, which is 4.3 light-years away.

- i.** How far is this in kilometres, correct to 4 significant figures?

- ii.** Travelling at 100km/h, how long in years would it take to reach Alpha Centauri? Give your answer to 3 significant figures.

3.5 Review

3.5.1 Summary

In this topic you have learnt to:

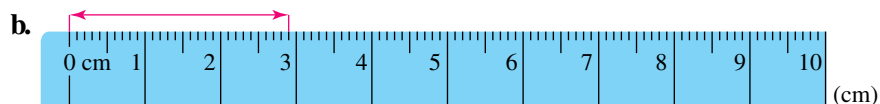
- use different units of measurement
- read a ruler using m, cm, and mm
- measure with a ruler using m, cm and mm
- calculate conversions between common units of measurement for length, weight and volume
- recognise that human error and device limitations prevent accuracy
- calculate the limit of accuracy
- calculate the range using the limit of accuracy
- calculate the error and absolute error of a reported measurement
- calculate the absolute relative error and absolute percentage error
- determine the significance of an error
- use scientific notation to express measurements with or without a required number of significant figures.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 3.5 Review

Understanding, fluency and communicating

1. State the measurement marked by the arrow in each of the following. Record your answer using the unit indicated in the brackets.



2. Match the appropriate length, weight or volume measurement with the following pictures.



(mm, cm, m, km)



(g, kg)



(mL, L)

3. Convert the following measurements into the units shown in brackets.

a. 25 cm (mm)

b. 47 mm (cm)

c. 286 cm (m)

d. 3.6 m (cm)

e. 86 m (km)

f. 1835 cm (km)

g. 1.28 km (cm)

h. 0.0845 km (mm)

4. Convert the following weight and volume measurements into the units shown in brackets.

a. 30 mL (L)

b. 4.5 L (mL)

c. 0.00089 L (mL)

d. 1289 mL (L)

e. 1400 g (kg)

f. 5.3 kg (g)

g. 0.385 g (kg)

h. 0.043 kg (g)

i. 52 t (kg)

j. 2150 kg (t)

5. Convert to the measurement indicated in brackets.
- a. 30 cm 22 mm + 42 cm 92 mm (cm) b. 3.5 cm 20 mm + 42.3 cm 53 mm (mm)
- c. 4 km 2 m + 5.2 km 4 m (km) d. 80 km 980 m + 5.6 km 225 m (m)
- e. 9.5 kg 354 g + 1520 kg 1555 g (kg) f. 850.6 kg 25 g + 376.4 kg 254 g (g)
6. Convert the following time measurements as indicated in the brackets. Give your answers to 2 decimal places where necessary.
- a. 30 min (s) b. 350 s (min) c. 45 h (s) d. 3500 s (h)
- e. 600 h (days) f. 76 days (h) g. 29 days (weeks) h. 7 weeks (days)
- i. 36 h (weeks) j. 3 weeks (h)
7. Convert the following time measurements as indicated in the brackets.
- a. 42 months (years) b. 6.5 years (months) c. 2 centuries (decades)
- d. 400 years (centuries) e. 3 decades (years) f. 45 years (decades)
8. For each of the following scales, how much is each interval worth?



9. A speedometer in a car has markings every 5 km/h.
- a. State the limit of accuracy.
- b. If the speedometer indicated 55 km/h, within what range could the speed actually lie?



10. When rounding errors, you should usually give 1 more decimal place than your measurements themselves. For example, if a typical measurement is 5.46 m, you would quote 3 decimal places in your absolute relative error measurements. Quote appropriate values for the following situations:

a. Actual value = 16.5 m	My measurement = 16.2 m
Absolute error = _____	Absolute relative error = _____
b. Actual value = 3.45 litres	My measurement = 3.5 litres
Absolute error = _____	Absolute relative error = _____
c. Actual value = 256.0 seconds	My measurement = 252.0 seconds
Absolute error = _____	Absolute relative error = _____
d. Actual value = 4.15 cm	My measurement = 4.19 cm
Absolute error = _____	Absolute relative error = _____

11. Five students measured the length of a 42.7 m netball court. Complete the following table to determine which student had the most accurate answer. Round your answers to 4 decimal places where necessary.

Student	Measurement	Error	Absolute error	Absolute relative error
A	44.8 m	2.1 m	2.1 m	0.0492
B	41.9 m			
C	43.7 m			
D	42.0 m	-0.7 m	0.7 m	
E	43.3 m			

12. Each day, your friend counts the number of steps from her front door to the bus stop. If the actual number of steps is 400, complete the following table.

Day	Number of steps	Error	Absolute error	Absolute relative error	Absolute % error
Monday	422				
Tuesday	371	-29	29	0.0725	7.25%
Wednesday	379				
Thursday	433				
Friday	411	+11			

13. The distance across a lake is 3.45 km. When a surveyor measures the distance, she comes up with 3.36 km.
- What is the error (in km)?
 - What is the error (in metres)?
 - What is the absolute error (in metres)?
 - What is the absolute relative error to 4 decimal places?
 - What is the absolute percentage error to 2 decimal places?
14. If the absolute error in measuring the width of a book is 0.2 cm, and you measure the book to be 25.5 cm, what is the actual width of the book?
15. Express the following numbers in scientific notation.
- 389 670 000
 - 0.004 860 3
 - 406 009 437
 - 0.000 000 100
 - 0.3750
 - 100 300 000 000
16. Express the following numbers as basic numerals.
- 3.56×10^6
 - 1.245×10^{-9}
 - 1.006×10^{12}
 - 2.16×10^3
 - 9.483×10^{-9}
 - 2.048×10^{-15}
17. Write the following numbers according to the indicated number of significant figures specified in brackets.
- 6500.78 (2 significant figures)
 - 73.000 080 (4 significant figures)
 - 7398.01 (2 significant figures)
 - 7000.01 (1 significant figure)
 - 8.037 (2 significant figures)
 - 0.000 600 5 (3 significant figures)
18. Write the following numbers in scientific notation and then indicate the number of significant figures.
- 35 000
 - 0.000 058 0
 - 0.071
 - 160.7154
 - 5000.1
19. Covert the following measurements as specified and then write the measurements correct to the number of significant figures indicated in brackets.
- 1400 cm to m (3 significant figures)
 - 8.5815 t to kg (4 significant figures)
 - 76 kg to g (1 significant figure)
 - 3000.100 L to mL (5 significant figures)
 - 0.00009 h to min (2 significant figures)
 - 854 h to days (4 significant figures)
20. **MC** Select the most accurate measurement. (*Hint*: Count the number of significant figures.)
- 360.005 m
 - 7000 m
 - 36.500 m
 - 3.71 m
 - 0.700 000 0 m

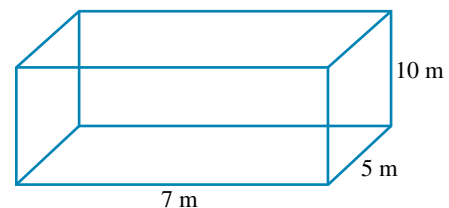
Problem solving, reasoning and justification

- 21.** Distance = rate $\times$ time is an equation used in both mathematics and science. Use this equation to solve the following questions.
- A car is traveling at an average speed of 95 km/h for 5.5 hours. How many metres did the car travel?
 - A family drove 255 km without stopping, and it took them 0.125 days to complete their journey. What was their average speed in km/h?
 - A sprint car was driving at an average speed of 225 km/h for the entire race. If the track distance was 420 m, how long in seconds did it take the driver to complete one lap?



- 22.** Marcus wants to change the dimensions on his next surfboard from imperial to metric units. Express all of the board's dimensions in cm.
- 1 inch = 2.54 cm
 - 1 foot = 12 inches
- Length of board: 6 feet 3 inches
 - Width of board: 19.75 inches
 - Thickness of board: 2.5 inches
- 23. a.** If you weigh 175 pounds (lb), do you weigh more or less in kilograms?
Hint: Conversion rate: 1 kilogram $\approx$ 2.2 pounds
- How much do you weigh in kilograms (to 2 decimal places)?
 - If you weigh 110 kg, how much do you weigh in pounds?
 - If the maximum weight for people in an elevator is 1100 lb, and the average weight of the occupants is 150 lb, how many people could fit in the elevator and what is the average weight of each person in kg (to 2 decimal places)?

- 24.** Nathan, a carpenter, tells his apprentice to build a rectangular prism with the dimensions as shown.



- What is the correct volume of the prism?
- If the length of the prism is measured incorrectly (an error of +1 m), what happens to the overall volume? What is the new incorrect volume?
- If the length and width of the prism are measured incorrectly (an error of +1 m each), what happens to the overall volume? What is the new incorrect volume?
- If the length, width and height of the prism are measured incorrectly (an error of +1 m each), what happens to the overall volume? What is the new incorrect volume?

- e. Calculate the error in volume, assuming he made +1 m errors on all three measurements.
 - f. Do you think this error is significant? Why or why not?
 - g. Why is the error above 3 m^3 , when the apprentice only made a +1 m error on each measurement?
 - h. If the apprentice made a +1 m error on the height dimension and a -1 m error on the width dimension, would the two errors affect the original volume? Give the new volume.
 - i. Calculate the absolute error, absolute relative error (to 4 decimal places) and the absolute percentage error (to 2 decimal places) for the prism assuming the apprentice made a +1 m error on each dimension.
25. When measuring area, there are two dimensions to be considered: length and width. If there are errors in the measurement of both dimensions, how does that affect the actual area?
 26. When measured on a kitchen scale, a small steak weighs 148 g. If it is known that the absolute percentage error of all measurements shown on these scales is 3%, explain how you can calculate the actual weight of the steak.
 27. The population of kangaroos is being monitored during a drought to ensure that they don't become overpopulated. If the population reaches over 300 000, culling must take place to prevent deaths from starvation and dehydration. Over the course of a year the population doubles. The initial population contains 1.83×10^5 kangaroos.
 - a. How many kangaroos are there at the beginning of the year?
 - b. What is the population after one year:
 - i. as a basic numeral
 - ii. in scientific notation?
 - c. How many kangaroos will have to be culled at the end of the year to keep the population below 300 000:
 - i. as a basic numeral
 - ii. in scientific notation?
 28. A car is travelling down a highway.
 - a. The car travels $4.3 \times 10^6 \text{ m}$; what is this in kilometres?
 - b. Calculate how long in hours it would take the car to travel $4.3 \times 10^6 \text{ m}$ at 28 metres per second. Give your answer to 2 decimal places.
 - c. The car travelled 65 309 573.05 metres over its lifetime. Express this in:
 - i. scientific notation
 - ii. kilometres to 3 decimal places
 - iii. kilometres to 3 significant figures.
 29. The X-15 supersonic jet travels at mach 6.70. Mach means the speed of sound, so the jet travels $6.70 \times (\text{speed of sound})$. The speed of sound is 340.9 m/s.
 - a. Convert the speed of sound to km/h.
 - b. How fast is the jet traveling in km/h?
 - c. Express the speed in scientific notation.
 - d. How many significant figures do you think are necessary to express this speed if you were telling a friend, without using scientific notation?
 30. The speed of light is 1 079 252 848.8 km/h.
 - a. Write the speed of light using 2 significant figures.
 - b. Write the speed in part a using scientific notation.
 - c. If the distance from Earth to Mars is $4.01 \times 10^8 \text{ km}$ (at the furthest distance in their orbits), how long would it take to travel from Earth to Mars if you could travel at the speed of light? Express your answer in minutes to 1 decimal place.
Hint: Use your answer from part b.



Answers

Exercise 3.2 Units of measurement

1. a. 2 cm b. 95 mm c. 64 mm d. 9.9 cm
2. a. 5.6 km b. 210 mm c. 55 000 m d. 220 cm e. 3.5 m f. 2.5 cm
3. a. 165 m b. 7.8 m c. 15.8 cm d. 0.013 km
e. 125 000 cm f. 36 m g. 1 350 000 mm h. 0.03 km
4. a. 0.056 32 km b. 2.0431 km c. 550 000 cm d. 2200 mm e. 0.35 km f. 0.25 m
5. a. 89 000 cm, 0.3217 km, 98 760 mm, 7.825 m b. 0.786 km, 0.786 mm = 0.000 786 m = 0.0786 cm
6. a. 2400 cm, 135 m, 0.15 km b. 120 mm, 25 cm, 0.5 m c. 9 m, 10 000 mm, 0.45 km
d. 32 000 cm, 1 km, 1200 m e. All values are equal. f. 825 m, 90 000 cm, 8.25 km
7. a. 6.5 cm b. 152 cm c. 265.5 cm d. 96.27 cm
8. a. 52.1 cm b. 90.105 km c. 60.5 cm d. 106.73 m
9. a. litres b. millimetres
10. a. grams b. kilograms c. milligrams
11. B
12. a. 2.5 L b. 3000 mL c. 0.0031 L d. 5250 mL e. 7500 mL f. 0.003 75 L
13. a. 0.000 024 t, 400 000 mg, 2.4 kg, 2500 g b. 0.05 kg, 52 000 mg, 54 g, 0.0005 t
14. a. False b. False c. True d. False
15. a. 505 000 mg b. 0.0045 g c. 6400 g d. 5.84 kg e. 0.075 821 kg
f. 70 mg g. 2000 kg h. 0.035 t i. 5 t
16. a. centuries b. decades c. second d. years e. hours
f. day g. months h. minutes i. weeks
17. a. 250 years, 1.5 centuries, 500 months, 2 decades b. 0.1 weeks, 0.56 days, 12.6 hours, 750 minutes, 4250 seconds
18. a. 45 years b. 82.5 decades c. 340 years d. 4.85 centuries e. 50 decades f. 12 centuries
19. a. 4 min b. 312 s c. 276 min d. 5.33 h e. 8.33 h f. 73 080 s
20. B
21. D
22. A
23. a. 48 smalls b. 480 smalls c. 9.375 mediums
d. This is a small-to-big conversion, which means multiply. First convert large to medium by multiplying by 8, and then to convert from medium to small multiply by 12.
e. 12 000 smalls
24. The distance from Melbourne to Perth is approximately 2721 km. Large distances are not easy to comprehend in mm, so we use the largest length measurement (km) to express long distances.
25. a. 20 000 L b. 2.584 m³
26. 1 foot = 30.48 cm
1 yard = 91.44 cm
1 furlong = 201.168 m
1 mile = 1.609 344 km
1 chain = 20.1168 m
1 rod = 5.0292 m
1 link = 20.1168 cm
27. a. i. 1 nautical mile = 1.852 km; is used for charting and navigating.
ii. 1 light-year $\approx$ 9.5 trillion km; is used to measure distance in space.
b. Answers will vary.

Exercise 3.3 Limits of accuracy

1. a. i. 5°C
ii. Between 175°C and 185°C
c. i. $\frac{1}{4}$ cup
ii. Between $6\frac{3}{4}$ cups and $7\frac{1}{4}$ cups
2. a. 58 mm b. 11.5 cm
3. a. Between 4.5 cm and 5.5 cm b. Between 11.5 cm and 12.5 cm c. Between 99.5 cm and 100.5 cm
d. Between 849.5 cm and 850.5 cm e. Between 4.995 m and 5.005 m f. Between 2.495 m and 2.505 m
4. a. Between 28.5 cm and 29.5 cm b. Between 245 g and 255 g
c. Between 7.5 mL and 12.5 mL d. Between 29.5 min and 30.5 min
5. E
6. C
7. a. 3520 cm (accurate to cm) b. 130 cm (accurate to cm) c. 1000 m (accurate to m)
d. 80 mm (accurate to mm) e. 5000 mm (accurate to mm) f. 2520 m (accurate to m)
8. a. 0.6 cm b. -1.7 cm c. 22 km d. -1.1 mm
9. a. 15 cm b. No, the error is only 1% of the advertised size.
10. a. 0.15 km b. 4 g c. 23 min d. 7 L
11. 161 or 169 cm
12. a. Error = 5 m
Absolute error = 5 m
Absolute relative error = 0.02
Absolute percentage error = 2%
c. Error = 1.5 mm
Absolute error = 1.5 mm
Absolute relative error = 0.08
Absolute percentage error = 8%
e. Error = 0.1 kg
Absolute error = 0.1 kg
Absolute relative error = 0.08
Absolute percentage error = 8%
- b. Error = -7 cm
Absolute error = 7 cm
Absolute relative error = 0.07
Absolute percentage error = 7%
d. Error = -12 s
Absolute error = 12 s
Absolute relative error = 0.2
Absolute percentage error = 20%
f. Error = -150 mL
Absolute error = 150 mL
Absolute relative error = 0.17
Absolute percentage error = 17%
13. Error = $-\$2$
Absolute percentage error = 2%
14. a. 0.5 mm (half of the smallest unit) b. 22.5 mm and 23.5 mm
15. The approximations can always be more exact by measuring to smaller and smaller increments toward infinity. There is no smallest increment.
16. The 4 km measurement is accurate to the km, while the 4.000 km is accurate to the thousandth of a km.
17. a. Statistically there is very little difference between the two methods.
b. Mother's measurements = $11.96\% \pm 9.48$
c. Ultrasound = $9.036\% \pm 7.61$
18. a. The time, whether it is behind or ahead, does not affect the significance of the accuracy of synchronisation, because the absolute error calculation forces them to be equal.
b. The absolute percentage error would be very small and therefore insignificant.
c. The absolute percentage error would be large and therefore significant.

Exercise 3.4 Scientific notation

1. a. 4.72×10^1 b. 3.890×10^3 c. 5.6×10^{-1} d. -6.7×10^{-3} e. 3.4×10^7
2. a. 0.048 b. 7600 c. 0.000 29 d. 8.1

3. a, c and d
4. a. False b. False c. True d. True
5. C
6. a. 1.5137×10^{16} km b. 2.82×10^{-15} m
7. Sextillions have 21 zeros.
a. 5.972×10^{21} b. 4 significant figures
8. a. 0.058 b. 0.0506 c. 55 830 d. 3×10^{-17} e. 138 000.00 f. 1.1
9. a. 12.0 m b. 6748 g c. 0.00480 min d. 1926 hrs e. 50 000 g f. 0.0050 m
10. D
11. C
12. a. 3 b. -1 c. 2 d. -4
13. a. 9.1094×10^{-31} kg b. 1.6725×10^{-27} kg c. 1.6726×10^{-27} kg
d. Working with the original amounts and not rounding before the final calculation ensures that the answer will be more precise.
14. a. 4 b. 4 c. 1 d. 3
15. a. 5.9 m b. 73.6 cm c. -77 m d. 257 cm^2 e. 3400 m^2
16. a. i. 385 440 km b. i. 385 000 km c. i. 3.85×10^5
ii. 78 489 600 km ii. 78 500 000 km ii. 7.85×10^7
iii. 1 278 960 000 km iii. 1 280 000 000 km iii. 1.28×10^9
iv. 138 548 160 000 km iv. 139 000 000 000 km iv. 1.39×10^{11}
17. a. i. 31 536 000 seconds b. i. 9 460 800 000 000 km c. i. 40 680 000 000 000 km
ii. 3.1536×10^7 seconds ii. 9.4608×10^{12} km ii. 46 400 000 years

Exercise 3.5 Review

1. a. 82 mm b. 2.9 cm
2. a. m b. kg c. l
3. a. 250 mm b. 4.7 cm c. 2.86 m d. 360 cm
e. 0.086 km f. 0.018 35 km g. 128 000 cm h. 84 500 mm
4. a. 0.03 L b. 4500 mL c. 0.89 mL d. 1.289 L e. 1.4 kg
f. 5300 g g. 0.000 385 kg h. 43 g i. 52 000 kg j. 2.15 t
5. a. 83.4 cm b. 531 mm c. 9.206 km d. 86 805 m e. 1531.409 kg f. 1 227 279 g
6. a. 1800 s b. 5.83 min c. 162 000 s d. 0.97 h e. 25 d
f. 1824 h g. 4.14 weeks h. 49 days i. 0.21 weeks j. 504 h
7. a. 3.5 years b. 78 months c. 20 decades d. 4 centuries e. 30 years f. 4.5 decades
8. a. 0.5 b. 0.05
9. a. 2.5 km/h b. Between 52.5 km/h and 57.5 km/h
10. a. Absolute error = 0.3 m b. Absolute error = 0.05 L
Absolute relative error = 0.019 Absolute relative error = 0.014
c. Absolute error = 4 seconds d. Absolute error = 0.04 cm
Absolute relative error = 0.02 Absolute relative error = 0.010

Student	Measurement	Error	Absolute error	Absolute relative error
A	44.8 m	2.1 m	2.1 m	0.0492
B	41.9 m	-0.8 m	0.8 m	0.0187
C	43.7 m	1 m	1 m	0.0234
D	42.0 m	-0.7 m	0.7 m	0.0164
E	43.3 m	0.6 m	0.6 m	0.0141

Student E has the most accurate answer. This is determined by finding the student who has the smallest absolute relative error.

TOPIC 4

Perimeter and area

4.1 Overview

4.1.1 Introduction

Calculating the perimeter and area of two-dimensional shapes is one of the oldest problems in mathematics, with the ancient Egyptians and Babylonians discovering various methods to solve problems involving perimeter and area. Over time rules and techniques have been developed to make these problems easier to tackle and to provide more accurate answers.

Knowing how to calculate the perimeters and areas of shapes has many practical applications. For example, it can help you to calculate the distance you run around your local park, the amount of soil you will need to evenly cover an area of garden, or the amount of wood you need to build a picture frame.



DISCUSSION

Are you able to calculate the perimeter and area of any two-dimensional shape?

LEARNING SEQUENCE

- 4.1 Overview
- 4.2 Perimeter 1
- 4.3 Perimeter 2
- 4.4 Area 1
- 4.5 Area 2
- 4.6 Applications of Pythagoras' theorem
- 4.7 Irregularly shaped blocks of land
- 4.8 Review

CURRICULUM CONTENT

- review and extend how to solve practical problems requiring the calculation of perimeters and areas of triangles, rectangles, parallelograms, trapezia, circles, sectors of circles and composite shapes (ACMGM018)   
- calculate perimeters and areas of irregularly shaped blocks of land by dissection into regular shapes including triangles and trapezia (ACMEM094) **AAM** 
- solve problems involving perimeters, area, surface area, volumes and capacity in a variety of contexts **AAM**

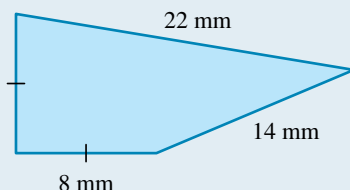
4.2 Perimeter 1

4.2.1 The perimeter

- The **perimeter** of a two-dimensional shape is the distance around the outside edge of the shape.
- The perimeter can be calculated by adding the lengths of all outside measurements.
- Ensure that all measurements are in the same units.

WORKED EXAMPLE 1

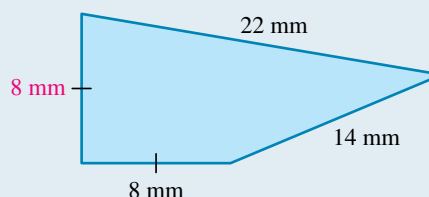
Calculate the perimeter of the following shape.



THINK

- 1 The perimeter is the distance around the outside edge of the shape. Because two sides are equal in length, the unknown side is equal to 8 mm, as shown in pink.

WRITE



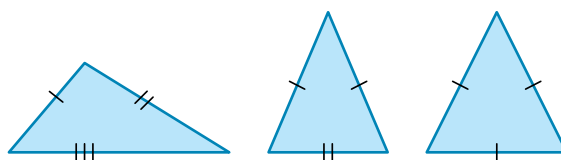
- 2 To calculate the perimeter (P) of the shape, add the lengths of all four sides. Include the units in your answer.

$$\begin{aligned} P &= 22 + 14 + 2 \times 8 \\ &= 36 + 16 \\ &= 52 \text{ mm} \end{aligned}$$

4.2.2 Common perimeters

- The perimeter of many common shapes can be found by using a formula.
- Although the formulae below use specific pronumerals, any pronumerals can be used so long as it is clear what they represent.

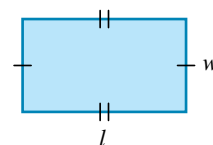
Triangles



- The formula for triangles varies according to the side lengths.
- To work out the perimeter of any triangle, especially scalene triangles (with three different side lengths), add all lengths together: $P = a + b + c$.
- For isosceles triangles (with two equal side lengths), double the congruent side length and add the third side: $P = a + a + b$. This is the same as $P = (2 \times a) + b$ or $P = 2a + b$.
- For equilateral triangles (with three equal side lengths), multiply the side length by three. $P = 3 \times a$ or $P = 3a$.

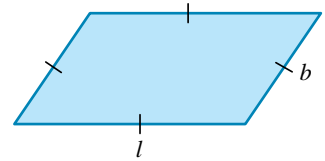
Rectangles

- The l stands for length and the w stands for width. To work out the perimeter, add all of the lengths together: $P = l + w + l + w$.
- This is the same as $P = 2 \times l + 2 \times w$ or $P = 2(l + w)$.

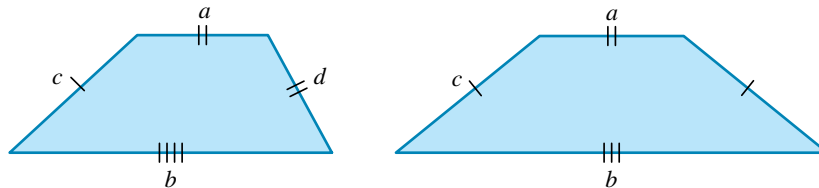


Parallelograms

- The formula for parallelograms is the same as that for rectangles, because the opposite sides are equal in length. When l stands for length (or b stands for base) and w stands for width, the formula is $P = 2(l + w)$ or $P = 2(b + w)$.
- A special formula can be used for squares and non-square rhombuses. This formula is $P = 4l$ or $P = 4s$, where l or s stands for the side length.



Trapezia

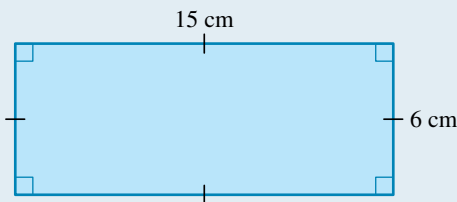


- The formulae for trapezia vary according to their side lengths.
- A general formula is to simply add all 4 sides together: $P = a + b + c + d$.
- For trapezia with exactly two equal sides, double the length of one side, then add the lengths of the other two sides: $P = a + b + (2 \times c)$ or $P = a + b + 2c$.

WORKED EXAMPLE 2

Calculate the perimeter of each shape.

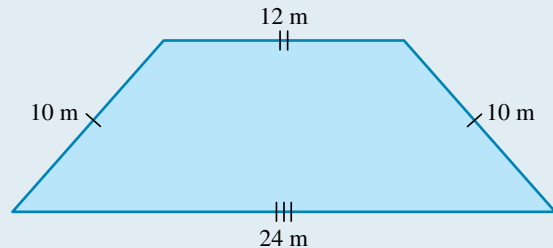
a



THINK

- a 1** Write the formula for the perimeter of a rectangle.
- 2** Substitute the values $l = 15$ and $w = 6$ into the formula.
- 3** Evaluate and include the correct units.

b



WRITE

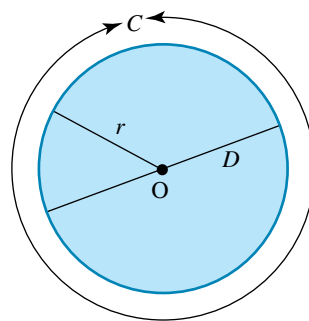
$$\begin{aligned}
 P &= 2(l + w) \\
 &= 2 \times (15 + 6) \\
 &= 2 \times 21 \\
 &= 42 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 P &= a + b + 2c \\
 &= 12 + 24 + (2 \times 10) \\
 &= 36 + 20 \\
 &= 56 \text{ m}
 \end{aligned}$$

- b 1** Write the formula for the perimeter of a trapezium with exactly two equal sides.
- 2** Substitute the values $a = 12$ and $b = 24$ and $c = 10$ into the formula.
- 3** Evaluate and include the correct units.

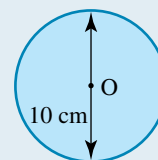
4.2.3 Circumference

- The **circumference** (C) is the perimeter of a circle.
- The **diameter** (D) of a circle is the straight-line distance across a circle through its centre (O).
- The straight-line distance from the centre of a circle to the edge of the circle is the **radius** (r).
- There is a relationship between the diameter and the circumference of a circle.
- The ratio of the circumference to the diameter has a value that is approximately 3.141 59. This ratio is given the name *pi* and is represented by the Greek letter π .
- The circumference, C , is given by the formula $C = \pi D$, where D is the diameter of the circle.
- The formula $C = 2\pi r$, where r is the radius of the circle, can also be used because $2 \times r = D$.



WORKED EXAMPLE 3

Calculate the circumference of this circle, correct to 2 decimal places.



THINK

- 1 The diameter of the circle is given so use the formula $C = \pi D$.
- 2 State the value of D .
- 3 Substitute the value of D into the formula and evaluate. Remember to include units with your answer.

WRITE

$$C = \pi D$$

$$D = 10 \text{ cm}$$

$$C = \pi \times 10 \\ \approx 31.4159 \text{ cm}$$

The circumference of the circle, correct to 2 decimal places, is 31.42 cm.

eBookplus RESOURCES

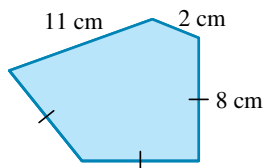
Interactivity: Area and perimeter (int-6474)

Exercise 4.2 Perimeter 1

Understanding, fluency and communicating

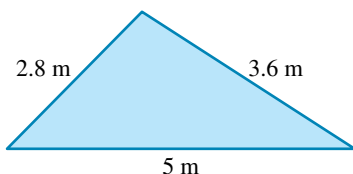
1. **MC** For which of the following situations do you need to calculate the perimeter? (*Note:* There may be more than one correct answer.)
 - A. Finding the amount of canvas needed for a painting
 - B. Finding the length of aluminium needed to make a frame for a mirror
 - C. Finding the amount of space inside a drink bottle
 - D. Finding the length of wire needed to fence a paddock
 - E. Finding the distance between two houses

2. **WE1** Calculate the perimeter of the following shape.

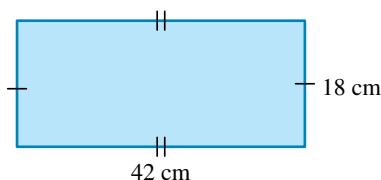


3. **WE2** Calculate the perimeter of the following shapes.

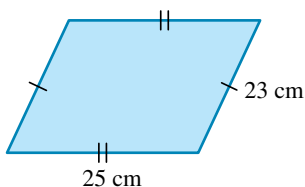
a.



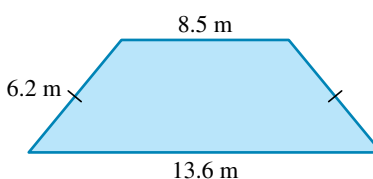
b.



c.

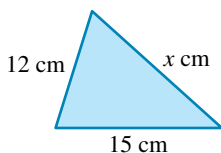


d.



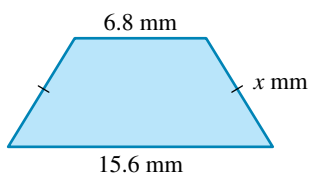
4. Calculate the unknown length in the following shapes.

a.



$$\text{Perimeter} = 43 \text{ cm}$$

b.

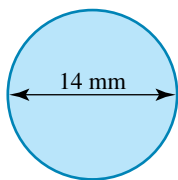


$$\text{Perimeter} = 36.8 \text{ mm}$$

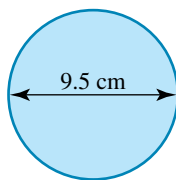
5. A quadrilateral has one side that is 32 cm long and another that is 51 cm long. The third side is twice as long as the fourth side, which is 23 cm long. Calculate the perimeter of the quadrilateral.
6. A triangle has two sides equal in length, and the sum of these sides is 18 cm. The third side is 32 mm long. What is the perimeter of the triangle?
7. A parallelogram has a perimeter of 130 mm. Its longer sides are each 38 mm. What length are the shorter sides?
8. **MC** The parallel sides of a trapezium are a and b . The other sides are c and d . Which of the following formulae can be used to find the perimeter of a trapezium with exactly 2 equal sides? (*Note: There may be more than one correct answer.*)
- A.** $P = c + d + a + b$ **B.** $P = 2(a + b)$ **C.** $P = b + 2c + a$
D. $P = 2a + 2c$ **E.** $P = 2a + b + c + d$
9. **MC** A rectangle is 7 times wider (w) than it is long. A formula for the perimeter of the rectangle could be:
- A.** $7w$ **B.** $8w$ **C.** $7(w + l)$ **D.** $14w$ **E.** $16w$
10. Answer True or False to each question about perimeter formulae.
- The formula for rectangles can also be used for squares.
 - The formula for trapezia with two exactly equal sides can be used for all parallelograms.
 - The formula for parallelograms can be used for all rectangles.
 - The formula for rectangles can be used for all trapezia.
 - The formula for scalene triangles will work for equilateral triangles.

11. **WE3** Calculate the circumference of the following circles. Give your answers correct to 2 decimal places.

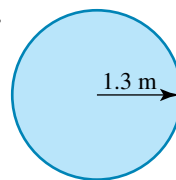
a.



b.



c.



12. **MC** How do you calculate the diameter of a circle if you only know its circumference?

- A. Divide the circumference by the radius and multiply by 2.
- B. Divide the circumference by the value of π .
- C. Multiply the radius by 2, then multiply by the value of π .
- D. Multiply the value of π by the circumference.
- E. Divide the circumference by 2, then multiply by the value of π .

13. **MC** The circumference of a circle is 50.29 cm. The radius of the circle correct to 2 decimal places is:

- A. 4.00 cm
- B. 8.00 cm
- C. 2.00 cm
- D. 16.00 cm
- E. 25.15 cm

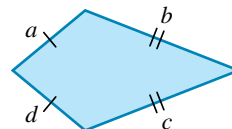
Problem solving, reasoning and justification

14. List all the possible dimensions (in whole numbers) of a rectangle with a perimeter of 24 m. A table like the one shown below may be helpful.

Width	Length
1	11
2	10

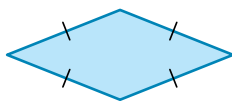
15. A garden bed needs timber edging. Two sides are each 3.4 metres long. The lengths of the other two sides are 2.6 metres and 3.8 metres. Two of the four sides also make a right angle. Which formulae for quadrilaterals could you use to calculate the length of timber that is required? Explain your thinking.
16. Calculate the radius (in cm) of a trundle wheel that measures one metre in distance in one revolution. Give your answer correct to 2 decimal places.
17. A car wheel has a radius of 60 cm. How many whole revolutions does the wheel make if the car travels a distance of 20 kilometres?
18. An equilateral triangle has side lengths given as $(x + 2)$ cm. If the perimeter is 42 cm, what is the value of x ?

19. a. Write a formula to calculate the perimeter of the shape at right. In your formula, use the fact that some sides are equal to each other.

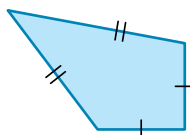


- b. **MC** For which of the shapes below could you calculate the perimeter by using your formula from part a? (Note: There may be more than one correct answer.)

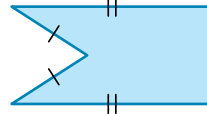
A.



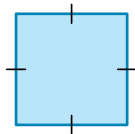
B.



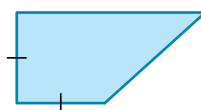
C.



D.



E.



20. A winch has 18 metres of cable wrapped around a cylinder with a 15-centimetre diameter. Calculate the maximum whole number of times the cable could be wrapped around the cylinder. Explain why the actual number of wraps may be less than the maximum that is possible.



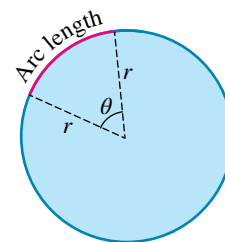
4.3 Perimeter 2

4.3.1 Arcs

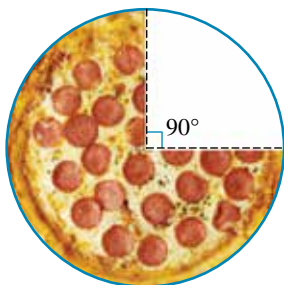
- An **arc** is part of the circumference of a circle.
- An **arc length** of a circle is calculated by finding the circumference of the circle and multiplying it by the fraction of the angle that the arc forms at the centre of the circle. The following formula can be applied:

$$\begin{aligned}\text{Arc length} &= C \times \frac{\theta}{360} \\ &= 2\pi r \times \frac{\theta}{360}\end{aligned}$$

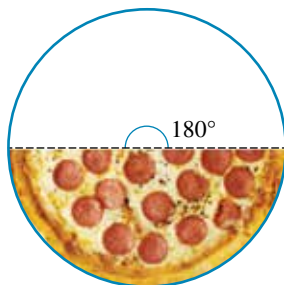
where r = radius of the circle and θ = the angle that the ends of the arc make with the centre of the circle in degrees.



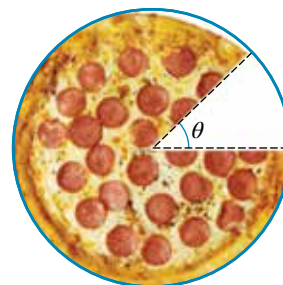
- Some common angles are:



Angle is $\frac{90^\circ}{360^\circ}$ or $\frac{1}{4}$ of a circle.



Angle is 180° or $\frac{1}{2}$ of a circle.

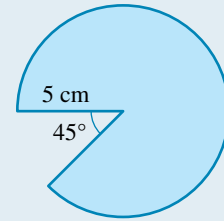


Angle is $\frac{\theta^\circ}{360^\circ}$ of a circle.

- The Greek letter θ (theta) is often used to represent the value of unknown angles.

WORKED EXAMPLE 4

Calculate the perimeter of the shape at right, giving your answer correct to 2 decimal places.



THINK

- 1 Identify which parts of the circle make the perimeter, P .
- 2 Calculate the circumference of the whole circle using the formula $C = 2\pi r$.
- 3 Calculate the arc length of the circle that has been removed.
- 4 Use the perimeter formula to calculate the perimeter.

WRITE

$P = \text{circumference} - \text{arc length} + \text{two radius lines}$

$$C = 2\pi r$$

$$= 2 \times \pi \times 5$$

$$= 2 \times 3.142 \times 5$$

$$= 31.42 \text{ cm}$$

$$\text{Arc length} = C \times \frac{\theta}{360}$$

$$= 31.42 \times \frac{45}{360}$$

$$= 3.93 \text{ cm}$$

$$P = 31.42 - 3.93 + 10$$

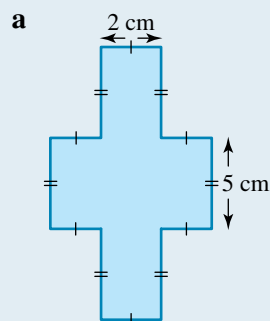
$$= 37.49 \text{ cm}$$

4.3.2 Composite shapes

- A composite shape can be thought of as being made up of a number of smaller shapes.
- If the smaller shapes are recognisable, or if the composite shape has certain features, then the perimeter of the composite shape can be easily calculated.

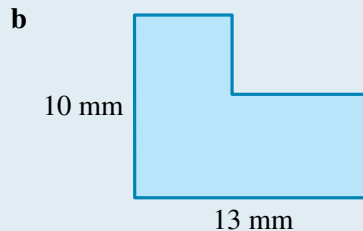
WORKED EXAMPLE 5

Calculate the missing lengths and the perimeter of each of the following shapes.

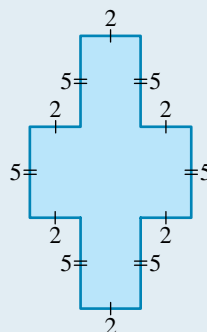


THINK

- a 1** All sides marked with the same symbol are of equal length. There are six sides of length 2 cm, so their total length is $6 \times 2 = 12 \text{ cm}$.



WRITE



2 The other six sides are 5 cm long, so their total length is $6 \times 5 = 30$ cm.

3 Add these totals to find the perimeter of the shape. Remember to include units in your answer.

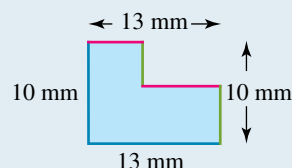
$$\begin{aligned} P &= 6 \times 2 + 6 \times 5 \\ &= 12 + 30 \\ &= 42 \text{ cm} \end{aligned}$$

The perimeter of the shape is 42 cm.

b 1 There are two horizontal sides with unknown lengths. Their total length is 13 mm, as shown in pink.

2 There are two vertical sides with unknown lengths. Their total length is 10 mm, as shown in green.

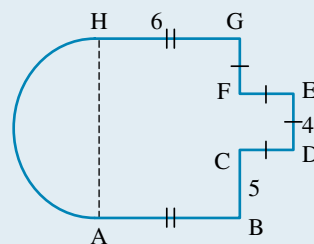
3 The perimeter is found by adding all these lengths.



$$\begin{aligned} P &= 10 \times 2 + 13 \times 2 \\ &= 20 + 26 \\ &= 46 \text{ mm} \end{aligned}$$

WORKED EXAMPLE 6

Calculate the perimeter of the shape in the figure correct to 1 decimal place. All measurements are in centimetres.



THINK

- 1 State the parts of the object that form the perimeter.
- 2 Determine the line segments that don't have a given measurement. Lines with a single dash are CD, EF, FG and ED. They are all 4 cm. Straight lines with a double dash are AB and GH. They are 6 cm.
- 3 Determine the radius of the semicircle. It is half the width of the line HA. The line HA is the same length as $GF + ED + CB$.
- 4 The perimeter of the semicircle is half the circumference of the entire circle.

WRITE

$$\text{Perimeter} = AB + BC + CD + DE + EF + FG + GH + HA \text{ (semicircle)}$$

$$CD = EF = FG = ED = 4$$

$$AB = GH = 6$$

$$\begin{aligned} \text{Radius} &= \frac{GF + ED + CB}{2} \\ &= \frac{4 + 4 + 5}{2} \\ &= 6.5 \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= \frac{\text{circumference}}{2} \\ &= \frac{2\pi r}{2} \\ &= \pi r \\ &= 6.5\pi \end{aligned}$$

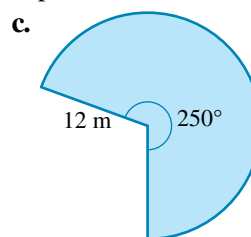
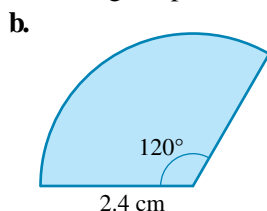
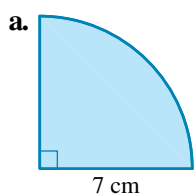
- 5 Calculate the total perimeter by adding the lengths of each side.

$$\begin{aligned}\text{Perimeter} &= 6 + 5 + 4 + 4 + 4 + 4 + 6 + 6.5\pi \\ &= 33 + 6.5\pi \\ &\approx 53.4 \text{ cm (correct to 1 decimal place)}\end{aligned}$$

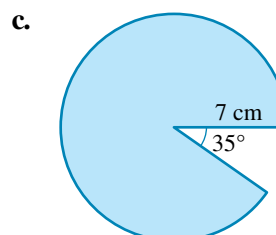
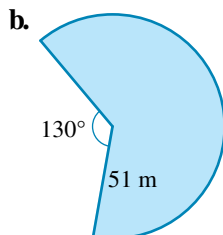
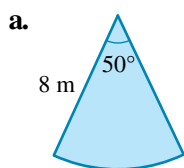
Exercise 4.3 Perimeter 2

Understanding, fluency and communicating

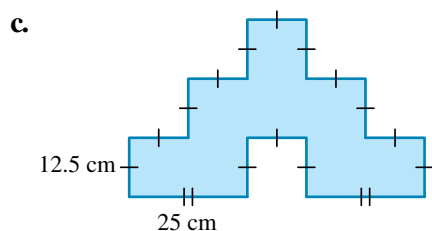
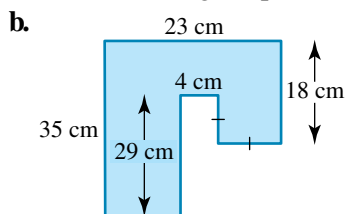
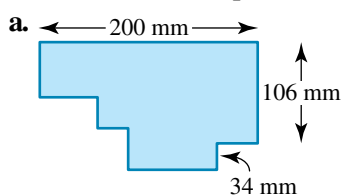
1. **WE4** Calculate the arc length of the following shapes correct to 2 decimal places.



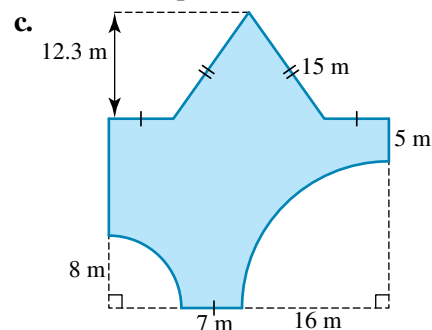
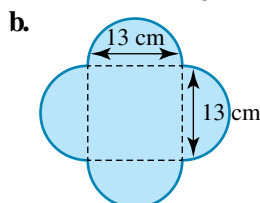
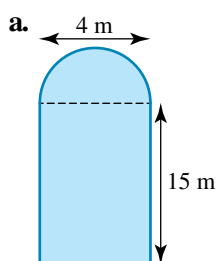
2. Calculate the perimeter of the following shapes correct to 2 decimal places.



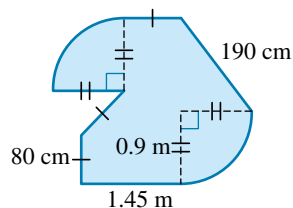
3. **WE5** Calculate the perimeter of each of the following shapes.



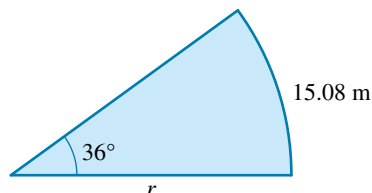
4. **WE6** Calculate the perimeter of each of the following shapes correct to 1 decimal place.



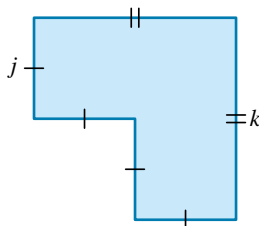
5. **MC** An arc is approximately 45 cm long and is made from a radius of 0.5 m. The angle formed at the centre is:
A. 39° **B.** 52° **C.** 162° **D.** 1° **E.** 103°
6. Determine the perimeter of the shape below correct to 2 decimal places.



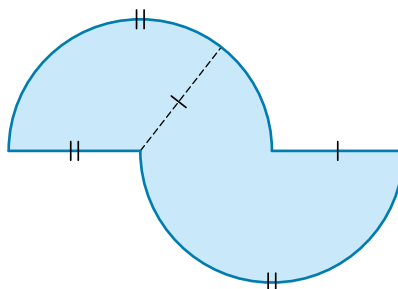
7. Calculate the radius of the shape below correct to 2 decimal places.



8. Answer True or False to each question about the shape below.



- The length of k is twice the length of j .
 - The perimeter can be calculated by multiplying j by k .
 - The perimeter can be calculated by multiplying k by itself.
 - $P = 4j + 2k$
 - The perimeter can be calculated by multiplying k by 4.
9. Which of the following formulae can be used to calculate the perimeter of the shape below, where r is radius and D is diameter of each semi-circle? (*Note:* There may be more than one correct answer.)



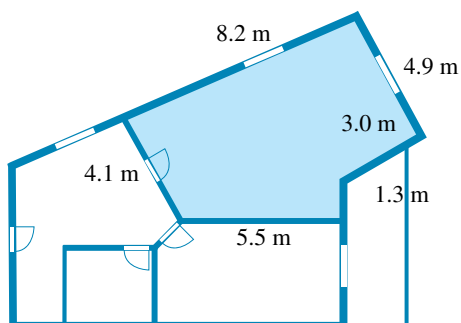
- $P = 2r + 2\pi r$
 - $P = 2\pi r$
 - $P = 2 \times 2\pi r$
 - $P = \frac{1}{2} \times (2\pi r) + \frac{1}{2} \times (2\pi r) + 2r$
 - $P = \pi D + D$
10. An equilateral triangle has a quarter-circle on two of its faces so that the radius of each quarter-circle is the same length as a side of the triangle. If the arc of each quarter-circle is 2.7 cm, what is the length of all the straight sides correct to 1 decimal place?

Problem solving, reasoning and justification

11. The minute hand on a clock is 22 centimetres in length. Calculate the distance travelled by the tip of the hand in 5 hours correct to 1 decimal place.
12. The diameter of the Earth at the equator is about 12 756 km. The cities of Quito in Ecuador and Kismayo in Somalia are both on the equator and are about 13 447 km apart. Calculate the angle formed at the centre of the Earth between these two cities correct to the nearest whole degree.



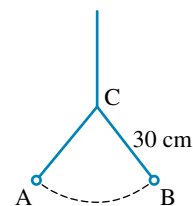
13. The room shaded below needs new skirting boards at the bottom of each wall. The doorway on the left-hand side is 80 centimetres wide. Calculate the total length of skirting board required for the room.



14. How many laps of this athletic track are required to complete a race of 2 km? Use the inside track for your calculations. Give your answer in whole laps and the whole number of metres left over.



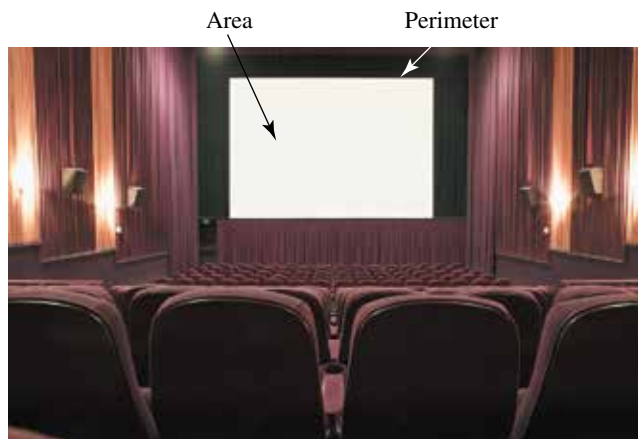
15. The pendulum of a grandfather clock swings back and forth between points A and B once per second. If $\angle ACB = 82^\circ$, then:
- calculate the arc length AB to the nearest whole centimetre
 - how far does the pendulum's weight (the small circle) move in one day?



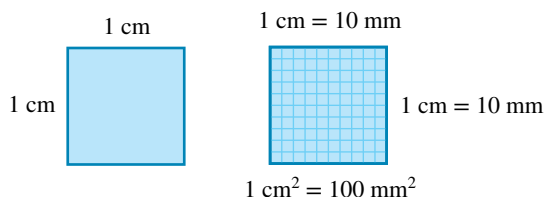
4.4 Area 1

4.4.1 Area of common shapes

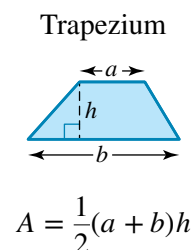
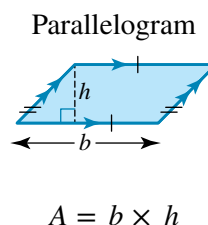
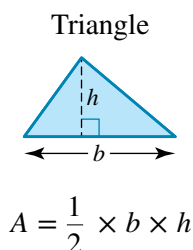
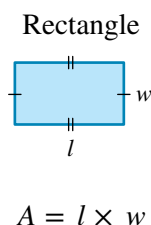
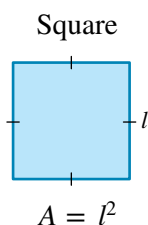
- The **area** of a two-dimensional shape is the amount of flat space enclosed by that shape. For example, the edge of a cinema screen is the perimeter and the movie is projected onto the flat area of the screen.
- Area is measured in square units. The unit used depends on the size of the object being measured.
- To convert between units of area, the conversions between units of length must be squared.



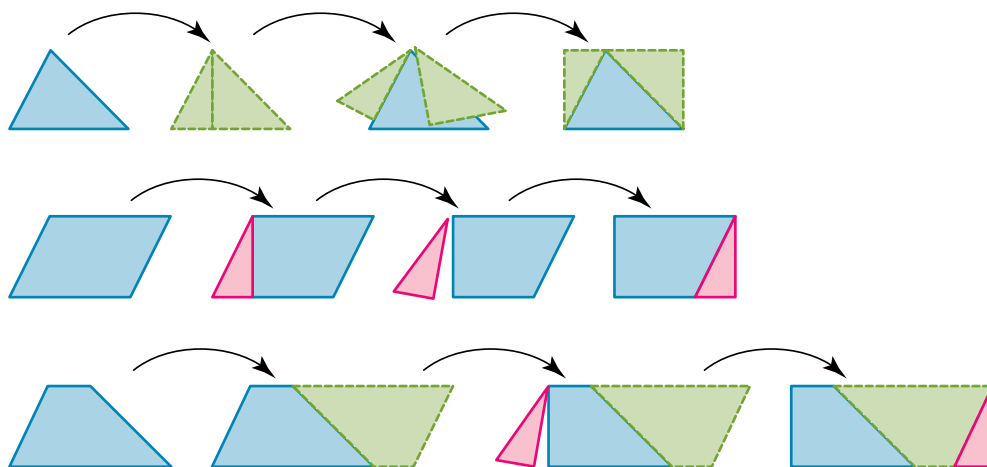
$$1 \text{ cm}^2 = 100 \text{ mm}^2$$



- The areas of common shapes can be found by using the following formulae.

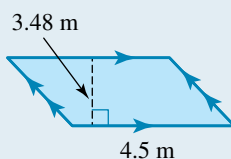


- In the formulae above, l stands for length, w stands for width, b stands for base and h stands for height. The a in the trapezium formula stands for the side opposite the base b .
- As with formulae for the perimeter, any pronumerals can be used in the formulae for area so long as it is clear what they represent.
- The formulae for triangles, parallelograms and trapezia can all be derived from the area of a rectangle (see the diagrams on the next page).



WORKED EXAMPLE 7

Calculate the area of the following shape.



THINK

- 1 The shape is a parallelogram. Write the formula for the area of a parallelogram.
- 2 Identify the values of the pronumerals.
- 3 Substitute the values of the pronumerals into the formula and evaluate. Remember to include units with your answer.

WRITE

$$A_{\text{parallelogram}} = b \times h$$

$$b = 4.5 \text{ m}$$

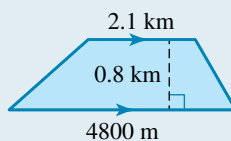
$$h = 3.48 \text{ m}$$

$$\begin{aligned} A &= 4.5 \times 3.48 \\ &= 15.66 \text{ m}^2 \end{aligned}$$

The area of the parallelogram is 15.66 m^2 .

WORKED EXAMPLE 8

Calculate the area of the following shape.



THINK

- 1 The shape is a trapezium. Write the formula for the area of a trapezium.
- 2 State the values of the pronumerals. All measurements must be in the same units, so convert 4800 m to km.
- 3 Substitute the values of the pronumerals into the formula and evaluate. Remember to include units with your answer.

WRITE

$$A_{\text{trapezium}} = \frac{1}{2}(a + b) \times h$$

$$a = 4800 \text{ m} = 4.8 \text{ km}$$

$$b = 2.1 \text{ km}$$

$$h = 0.8 \text{ km}$$

$$\begin{aligned} A &= \frac{1}{2} \times (4.8 + 2.1) \times 0.8 \\ &= \frac{1}{2} \times 6.9 \times 0.8 \\ &= 2.76 \text{ km}^2 \end{aligned}$$

The area of the trapezium is 2.76 km^2 .

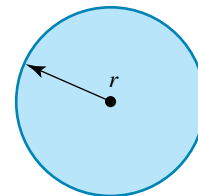
4.4.2 Area of a circle

- The area of a circle can be calculated using the formula:

$$A_{\text{circle}} = \pi \times r^2, \text{ where:}$$

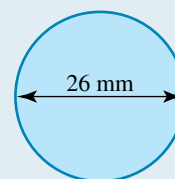
r = radius of the circle

π (pi) = the ratio of the circumference to the diameter; it is approximately equal to 3.141 59 or $\frac{22}{7}$.



WORKED EXAMPLE 9

Calculate the area of the following circle. Give your answer correct to 2 decimal places.



THINK

- The area of the circle is the space enclosed within the shape. Write the formula for calculating the area of a circle.
- Substitute the value of the radius into the formula and calculate the area.
- Give your answer correct to 2 decimal places and include the appropriate units.

WRITE

$$A = \pi \times r^2$$

$$\begin{aligned} A &= \pi \times r^2 \\ &= \pi \times \left(\frac{26}{2}\right)^2 \\ &= 169\pi \end{aligned}$$

$$A \approx 531.14 \text{ mm}^2$$

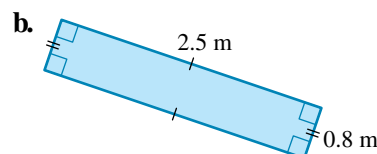
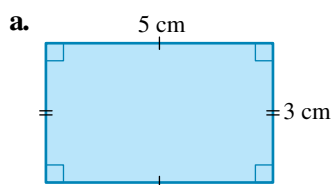
eBookplus RESOURCES

Interactivity: Conversion of units of area (int-6269)

Exercise 4.4 Area 1

Understanding, fluency and communicating

- MC** For which of the following situations do you need to calculate the area? (*Note:* There may be more than one correct answer.)
 - Finding the size of wall to be painted
 - Finding the amount of timber needed for a garden border
 - Finding the size of a field to sow with seed
 - Finding the number of tiles needed to cover a floor
 - Finding the length of ribbon to hem a tablecloth
- Calculate the area of each of the following shapes.



3. **MC** The area of a rectangle is 196 mm^2 . Which of the following could be its dimensions? (Note: There may be more than one correct answer.)

A. 1.4 cm by 14 mm

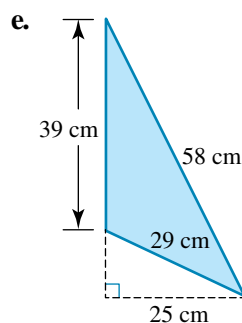
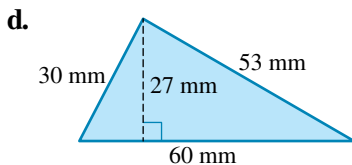
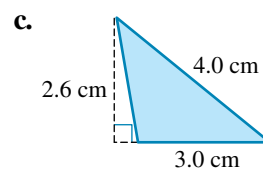
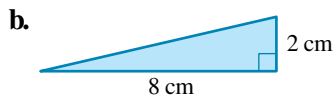
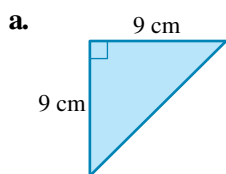
B. 4 mm by 49 mm

C. 2 mm by 9.3 cm

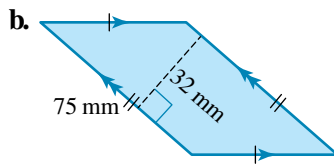
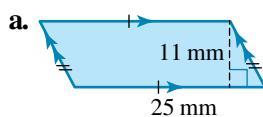
D. 19.6 mm by 1 cm

E. 0.28 cm by 7.0 mm

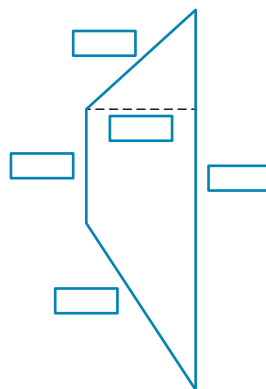
4. Calculate the area of each of the following shapes.



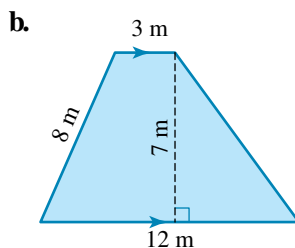
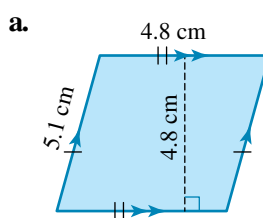
5. **WE7** Calculate the area of each of the following shapes.

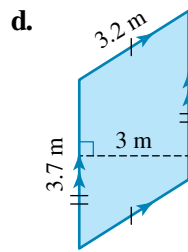
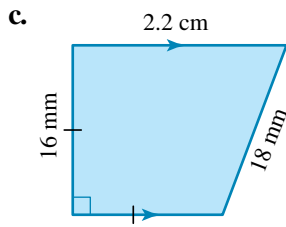


6. The formula for the area of the trapezium below can be written as $A = \frac{e(c + d)}{2}$. Label the blank spaces on the diagram c , d or e as appropriate. (Only three blank spaces on the trapezium should be used.)



7. **WE8** Calculate the area of each of the following shapes.

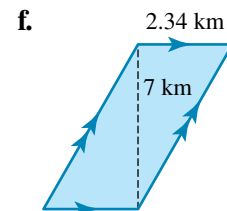
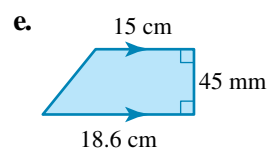
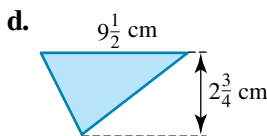
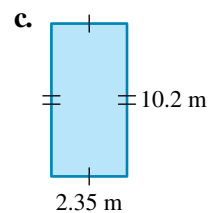
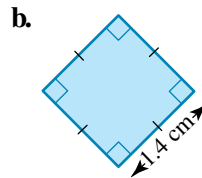
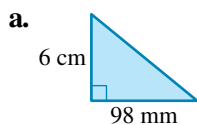




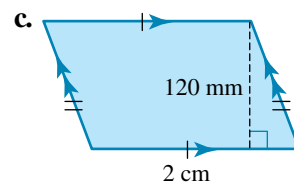
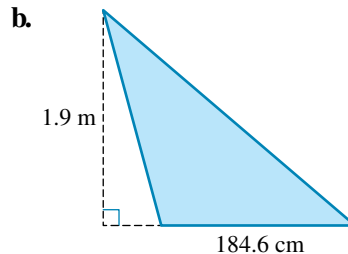
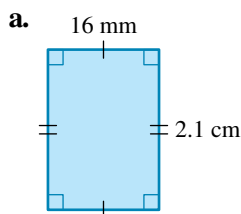
8. Answer True or False to each question about area formulae.

- The formula for squares can be used for any rectangle.
- The formula for rectangles can be used for squares.
- The formula for parallelograms can be used for all rectangles.
- For triangles, halving the base and multiplying by the height is the same as halving the height and multiplying by the base.
- The formula for trapezia can be used for squares.

9. Calculate the area of each of the following shapes.



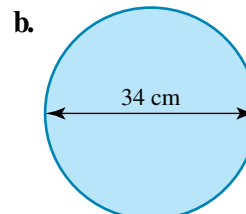
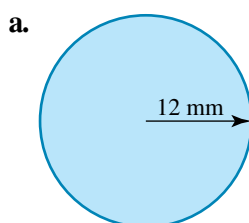
10. Calculate the area of each of the following shapes in the smaller unit of measurement.



11. A parallelogram has a base length that is five times as long as its height. If the height of the parallelogram is 2.1 m, what is the area of the parallelogram?

12. A trapezium has an area of 57 cm^2 and the lengths of its parallel sides are 9 cm and 3 cm. What is its height?

13. **WE9** Calculate the shaded area in each of the following shapes. Give your answers correct to 2 decimal places.

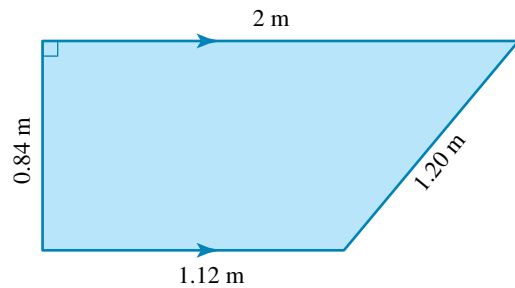


14. **MC** The area of a circle is equal to 113.14 mm^2 . The radius of the circle is closest to:
A. 36 mm **B.** 6 mm **C.** 2133 mm **D.** 7 mm **E.** 60 mm

Problem solving, reasoning and justification

15. The area of fabric used to make a triangular scarf is 600 cm^2 .
 If the base of the scarf is 50 cm, what is its height?

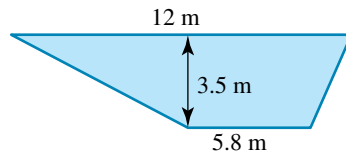
16. The benchtop shown at right needs painting. A tin of paint covers 16 square metres per litre. If you have a 500 mL tin of paint and you apply two coats, what percentage of the paint will be left? Give your answer to the nearest whole number.



17. A small rectangular photograph measuring 3 cm by 5 cm is enlarged so that its dimensions are doubled.
a. Calculate the area of the original photograph.
b. Calculate the dimensions of the new photograph.
c. Calculate the area of the new photograph.
d. Compare the area of the enlarged photo with that of the original and complete the sentence below.
 If the side lengths of the rectangle are doubled its area is _____.



18. A section of a garden is in the shape of a trapezium as shown.



- a.** Calculate the area of this section of the garden.
b. This section of the garden is to be covered with mulch. According to the information on the pack, each bag contains enough mulch to cover 5 m^2 of surface. How many bags of mulch are needed to complete the job?
c. If each bag costs \$8.95, calculate the total cost of the mulch.
19. The area of an unknown shape is 64 cm^2 . Draw three different labelled shapes that have an area of 64 cm^2 .
20. A rectangular flower bed measures 20 m by 16 m. A gravel path 2 m wide surrounds it.
a. Draw a diagram representing the flower bed and path.
b. Calculate the area of the flower bed.
c. Calculate the area of the gravel path.
d. If gravel costs \$5 per square metre, how much will it cost to cover the path?
21. A farmer has a fence surrounding a rectangular field that is 35 metres wide and 80 metres long. She then extends only one of the longer sides of the field by 24 metres and adjusts the fencing to enclose the whole field. The field now has a sloping side that is 42.4 metres long.
a. Calculate the amount of fencing that now surrounds the field.
b. Calculate the area that the field now covers.
22. When using the formula to calculate the area of a rectangle, does it matter which side is considered to be the length and which side is considered to be the width? Why?

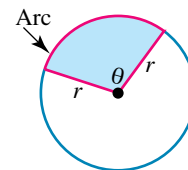
23. Circular pizza trays come in three different sizes: small, medium and large.
- Calculate the area of each tray if the diameters are 10 cm, 15 cm and 20 cm respectively. Round your answers up to the nearest whole cm^2 .
 - What is the minimum amount of metal that must be ordered to make 50 trays of each size?
24. A farm machine has connected pipes that rotate on wheels around a central point, much like the minute hand on a clock. As the pipes travel around the central point a circular area of crops is watered. If the arm of the machine is 400 metres long and is located in the centre of a square field that has a side length of 0.8 km, how much land is not watered using this machine? Give your answer in km^2 correct to 3 decimal places.



4.5 Area 2

4.5.1 Area of a sector

- A **sector** of a circle is a part of a circle shaped like a slice of pie. It is bounded by two radii and an arc.
- The area of a sector can be calculated by finding the area of the circle and multiplying it by the fraction of the angle that the sector forms at the centre of the circle.



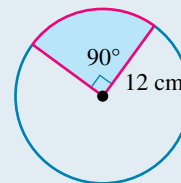
$$A = \pi r^2 \times \frac{\theta}{360}, \text{ where:}$$

r = radius of the circle

θ = angle that the sector makes with the centre of the circle in degrees.

WORKED EXAMPLE 10

Calculate the area of the shaded region.



THINK

- Write out the formula for calculating the area of a sector.
- Substitute the values into the formula. Calculate the area of the shaded region by multiplying the area of the complete circle by the fraction of the angle that forms the sector.
- Include the appropriate units in your final answer.

WRITE

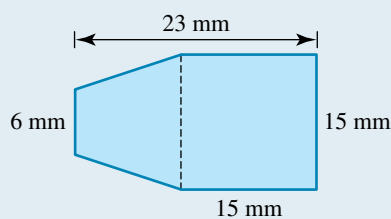
$$\begin{aligned} A_{\text{sector}} &= \pi \times r^2 \times \frac{\theta}{360} \\ A_{\text{sector}} &= \pi \times 12^2 \times \frac{90}{360} \\ &= 144\pi \times \frac{90}{360} \\ &= 133.10 \\ A_{\text{sector}} &= 113.10 \text{ cm}^2 \end{aligned}$$

4.5.2 Composite shapes

- Areas of composite shapes can be calculated by summing the areas of the components of the composite shape.

WORKED EXAMPLE 11

Calculate the area of the following shape.



THINK

- This composite shape is made up of a trapezium and a square. Therefore, the total area is equal to the area of the trapezium plus the area of the square.
- Write out the formula for calculating the area of a trapezium.
- Identify the values of a , b and h . Substitute them into the formula. Calculate your answer.
- Write out the formula for calculating the area of a square.
- Identify the value of l and substitute it into the formula. Calculate your answer.
- Add the area of the trapezium and the area of the square to find the total area. Include appropriate units in your answer.

WRITE

$$A_{\text{total}} = A_{\text{trapezium}} + A_{\text{square}}$$

$$A_{\text{trapezium}} = \frac{1}{2}(a + b) \times h$$

$$a = 6$$

$$b = 15$$

$$h = 23 - 15 = 8$$

$$\begin{aligned} A_{\text{trapezium}} &= \frac{1}{2}(6 + 15) \times 8 \\ &= 84 \text{ mm}^2 \end{aligned}$$

$$A_{\text{square}} = l \times l$$

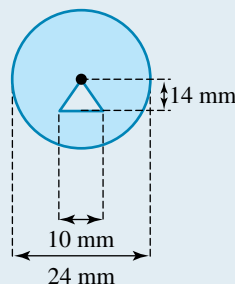
$$\begin{aligned} A_{\text{square}} &= l \times l \\ &= 15 \times 15 \\ &= 225 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{total}} &= A_{\text{trapezium}} + A_{\text{square}} \\ &= 84 + 225 \\ &= 309 \text{ mm}^2 \end{aligned}$$

- Composite shapes can also be made up of shapes that have been cut out of other shapes. To calculate the area of these composite shapes, subtract the area of the shapes that has been cut out from the area of the composite shape.

WORKED EXAMPLE 12

Calculate the shaded area in the following diagram.



THINK

- 1 The shape can be divided into a circle and a triangle.
- 2
 - Write the formula for the area of a circle.
 - Find the value of r , the radius of the circle, by halving its diameter.
 - Substitute the value of r into the formula and evaluate.
 - Write the formula for the area of a triangle.
 - Identify the values of the pronumerals. Substitute these values into the formula and evaluate.
- 3 Subtract the area of the triangle from the area of the circle to find the shaded area. Remember to include units with the answer.

WRITE

$$A_{\text{circle}} = \pi r^2$$

$$D = 24 \text{ mm}; r = \frac{D}{2} = \frac{24}{2} = 12 \text{ mm}$$

$$\begin{aligned} A_{\text{circle}} &= \pi \times 12^2 \\ &= 144\pi \\ &\approx 452.39 \text{ mm}^2 \end{aligned}$$

$$A_{\text{triangle}} = \frac{1}{2} \times b \times h$$

$$b = 10 \text{ mm}; h = 14 \text{ mm}$$

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2} \times 10 \times 14 \\ &= 70 \text{ mm}^2 \end{aligned}$$

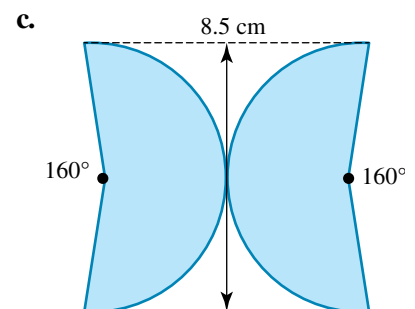
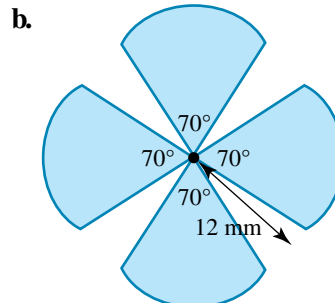
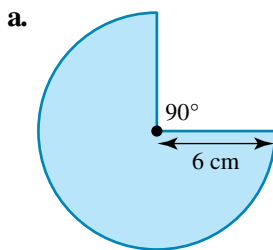
$$\begin{aligned} A &= 452.39 - 70 \\ &= 382.39 \text{ mm}^2 \end{aligned}$$

The shaded area is 382.39 mm^2 .

Exercise 4.5 Area 2

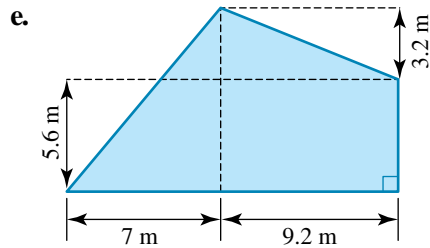
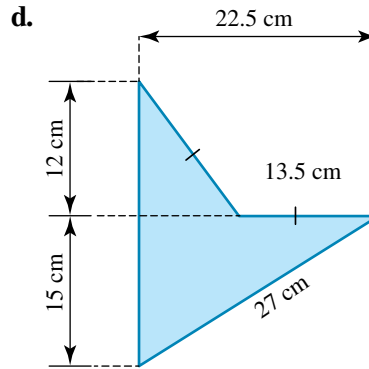
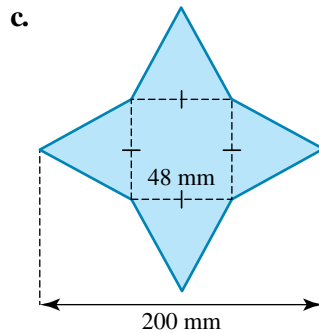
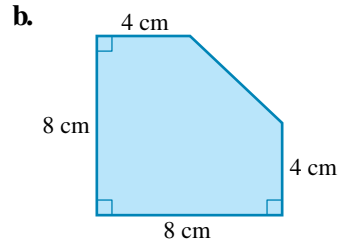
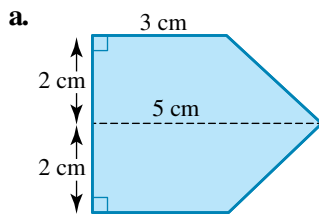
Understanding, fluency and communicating

1. **MC** A ring has an inner diameter of 5.5 cm and an outer diameter of 6.2 cm. The area of the ring, correct to 2 decimal places, is:
A. 8.19 cm^2 **B.** 2.05 cm^2 **C.** 6.43 cm^2 **D.** 3.22 cm^2 **E.** 1.54 cm^2
2. Select the words or phrases that complete the sentence.
 The area of a sector is one-third the area of the circle it sits inside. Therefore, the [radius/angle/diameter/perimeter] of the sector at the centre of the circle will be one-third of [the circle's radius/360°/the circle's diameter/the circle's circumference].
3. **WE10** Calculate the area of the shaded region in each shape, correct to 2 decimal places.



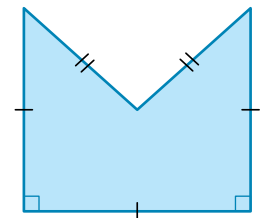
4. A sector of a circle has an area of approximately 57.73 cm^2 . The circle it sits inside has an area of approximately 153.94 cm^2 .
 - a. What proportion of the area of the circle is the area of the sector? Give your answer correct to 2 decimal places.
 - b. What is the angle of the sector at the centre of the circle?

5. **WE11** Calculate the area of each of the following shapes.

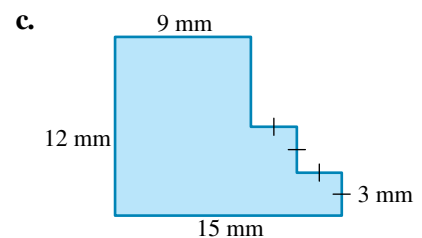
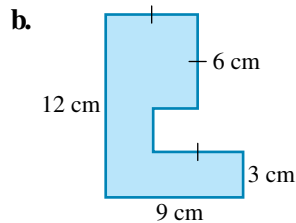
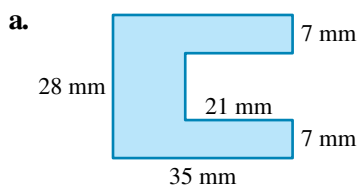


6. **MC** Which of the following can be used to calculate the area of the given shape? (*Note: There may be more than one correct answer.*)

- A. Calculate the area of two triangles, then add the area of a square.
- B. Add the area of a rectangle to the area of two triangles.
- C. Calculate the area of a parallelogram then subtract the area of a square.
- D. Calculate the area of a square then subtract the area of a triangle.
- E. Add the area of two trapezia.

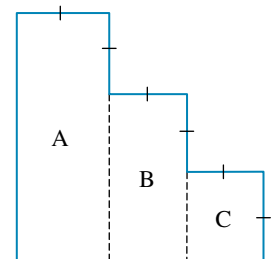


7. Calculate the area of each of the following shapes.

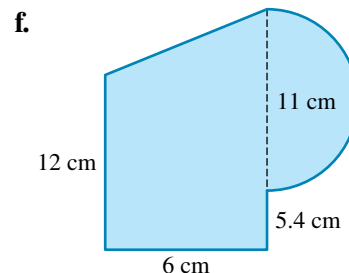
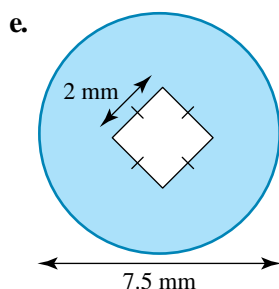
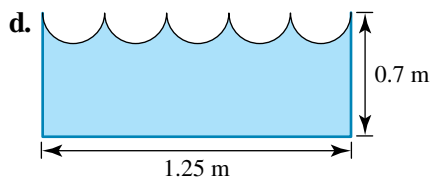
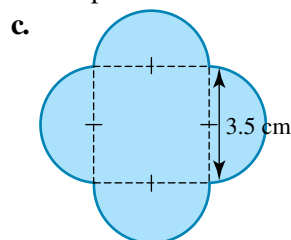
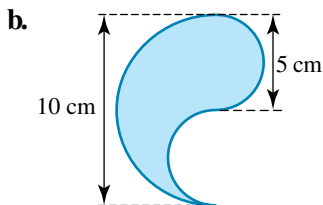
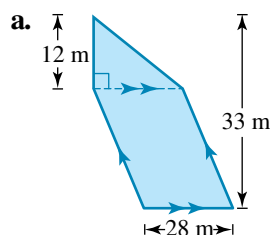


8. The shape at right is made of three rectangles A, B and C. Answer True or False to each question about the shape.

- a. The total area of A, B and C is the same as double the combined area of B and C.
- b. The total area of B and C is equal to the area of A.
- c. The total area of A, B and C can be found by squaring the length of A then subtracting the area of A.
- d. The total area of A and B is equal to double the combined area of B and C.
- e. The total area of A is equal to squaring the length of B.



9. **WE12** Calculate the area of the shaded region in each shape correct to 2 decimal places where necessary.



10. **MC** Which of the following formulae can be used to calculate the area of the sector below?

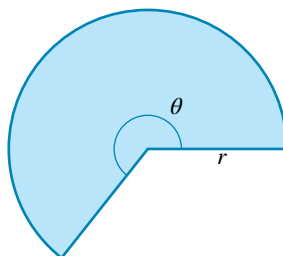
A. $A = \frac{\theta}{360} \times r \times r \times \pi$

B. $A = r^2 \times \frac{\pi}{360} \times \frac{\theta}{360}$

C. $A = \frac{\theta}{360} \times 2\pi r$

D. $A = 2r \times \pi \times \frac{\theta}{360}$

E. $A = 2\pi r^2 \times \frac{\theta}{360}$



Problem solving, reasoning and justification

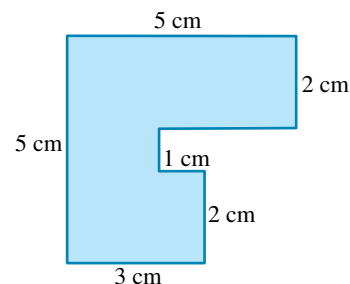
11. a. Identify three different ways to divide the composite shape at right into smaller shapes.

b. Calculate the area of the shape using each method from part a.

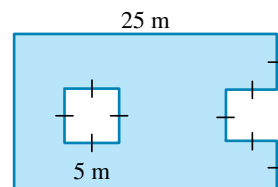
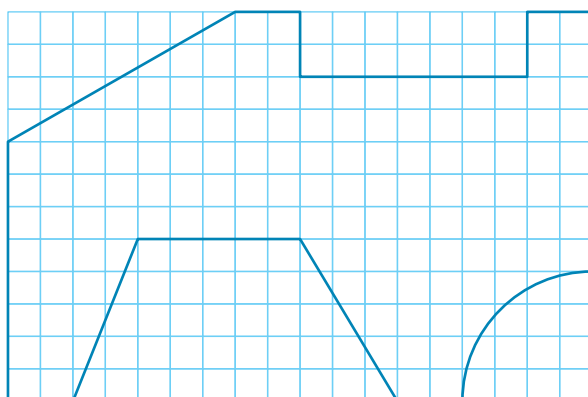
12. An outdoor entertaining area has a floor plan, as shown at right. The shaded area is to be sealed with a protective surface.

a. Use an appropriate method to calculate the floor area.

b. The sealant to be used costs \$25 per square metre. How much will it cost to seal the area?



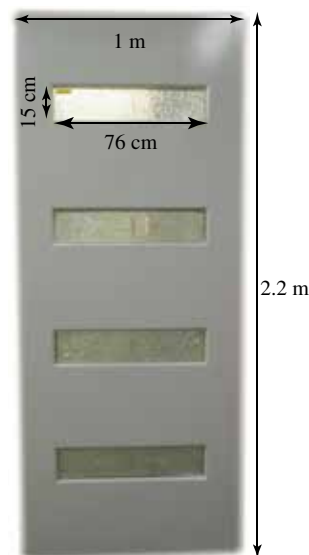
13. For a project, students cut some shapes out of a sheet of metal. The side of each grid square below is 50 mm.



- a. Calculate the amount of metal that was used and represent it as a percentage of the total area of the metal. Give your answer to the nearest whole number.
- b. Draw an alternative arrangement of the same shapes to leave the greatest rectangular amount of metal possible.

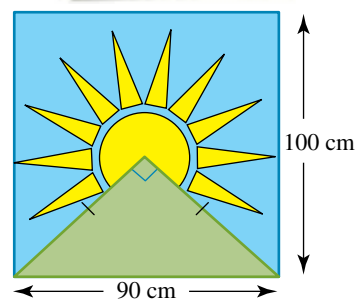
14. The front door shown at right is 1 m wide and 2.2 m high and has four identical glass panels, each 76 cm long and 15 cm wide. The door is 35 mm thick.

- a. Calculate the total area of the glass panels in m^2 , correct to 2 decimal places.
- b. The door is to be painted inside and outside, including both vertical edges. Calculate the total area to be painted.
- c. Two coats of paint are needed on each side of the door. If the paint is sold in 1-L tins at \$24.95 per liter and each liter covers 8 m^2 of the surface, calculate the total cost of painting the door.



15. The leadlight panel shown right depicts a sunrise over the mountains. The mountain is represented by a green triangle 45 cm high. The yellow sun is represented by a sector of a circle with an 18 cm radius. There are 10 yellow sunrays in the shape of isosceles triangles with a base of 3 cm and a height of 12 cm. The sky is blue. Calculate the area (correct to 2 decimal places where necessary) of the leadlight panel made of:

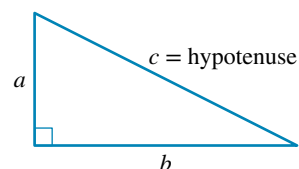
- a. green glass b. yellow glass c. blue glass.



4.6 Applications of Pythagoras' theorem

4.6.1 Pythagoras' theorem

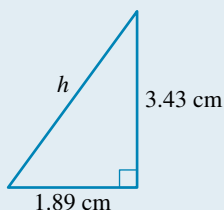
- The **hypotenuse** is the longest side of a right-angled triangle and is situated opposite the right angle.
- Pythagoras' theorem** states that for any right-angled triangle, the sum of the squares of the lengths of the two shorter sides is equal to the square of the length of the hypotenuse. $c^2 = a^2 + b^2$
- Pythagoras' theorem can be used to calculate the length of unknown sides of a right-angled triangle when the other two sides are known.



WORKED EXAMPLE 13

Calculate the value of the unknown variable correct to 2 decimal places using Pythagoras' theorem.

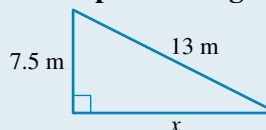
a



THINK

- a 1 • The triangle is a right-angled triangle.
- The hypotenuse is always opposite the right angle.
- In this case h is the length of the hypotenuse.

b



WRITE

$$c^2 = a^2 + b^2$$

2 Substitute in the appropriate values and solve.

$$\begin{aligned}h^2 &= 3.43^2 + 1.89^2 \\h^2 &= 11.7649 + 3.5721 \\h^2 &= 15.337 \\h &= \sqrt{15.337} \\&\approx 3.92\end{aligned}$$

3 Answer the question.

$$h = 3.92 \text{ cm (correct to 2 decimal places)}$$

- b 1**
- All of the measurements are in the same units.
 - The triangle is a right-angled triangle.
 - The hypotenuse is always opposite the right angle.
- In this case 13 m is the length of the hypotenuse.

$$c^2 = a^2 + b^2$$

2 Substitute in the appropriate values and solve.

$$\begin{aligned}13^2 &= 7.5^2 + x^2 \\13^2 - 7.5^2 &= 7.5^2 + x^2 - 7.5^2 \\169 - 56.25 &= x^2 \\112.75 &= x^2 \\\sqrt{112.75} &= x \\x &\approx 10.62\end{aligned}$$

3 Answer the question.

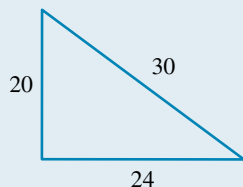
$$x = 10.62 \text{ m (correct to 2 decimal places)}$$

- The converse of Pythagoras' theorem is also true; that is, if a triangle has side lengths a , b and c , and $c^2 = a^2 + b^2$, then the triangle is right-angled. Similarly, if $c^2 \neq a^2 + b^2$ then the triangle is not right-angled.

WORKED EXAMPLE 14

Which of the following triangles are not right-angled triangles?

a



b A triangle with side lengths 3 cm, 4 cm and 5 cm.

THINK

a 1 For a right-angled triangle, Pythagoras' theorem will be true.

2 Identify the values of a , b and c . The longest side is c . Substitute the values into the left- and right-hand sides of Pythagoras' theorem to see whether the values make the equation true.

3 Pythagoras' theorem is not true for this triangle. Answer the question.

b 1 Identify the values of a , b and c . The largest value is 5 cm; it will be the hypotenuse, c . It does not matter which of the other sides are a or b .

WRITE

$$c^2 = a^2 + b^2$$

$$\begin{array}{ll}\text{LHS} = 30^2 & \text{RHS} = 20^2 + 24^2 \\&= 900 &= 976 \\&\text{LHS} \neq \text{RHS}\end{array}$$

The triangle is not a right-angled triangle.

Side lengths: $c = 5 \text{ cm}$
 $a = 3 \text{ cm}$
 $b = 4 \text{ cm}$

- 2 Substitute the values into the left- and right-hand sides of Pythagoras' theorem to see if the values make the equation true.

$$c^2 = a^2 + b^2$$

$$\begin{array}{ll} \text{LHS} = 5^2 & \text{RHS} = 3^2 + 4^2 \\ & = 25 \quad \quad = 25 \end{array}$$

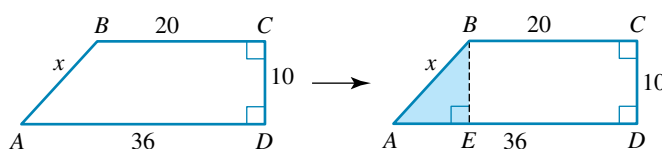
$$\text{LHS} = \text{RHS}$$

- 3 Pythagoras' theorem is true for these values. Answer the question.

The triangle is a right-angled triangle.

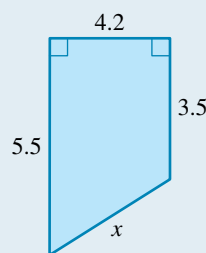
4.6.2 Using Pythagoras' theorem to solve problems

Pythagoras' theorem can be used to solve problems involving perimeter and area. For example, to calculate the value of x in the trapezium shown, a vertical line can be added to create a right-angled triangle and a rectangle. The length of x can be found using Pythagoras' theorem.



WORKED EXAMPLE 15

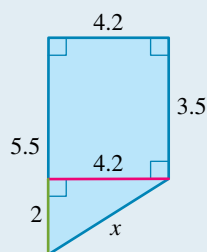
Calculate the value of x in the diagram at right, correct to 1 decimal place.



THINK

- 1 Divide the shape into smaller shapes that create a right-angled triangle, as shown in pink.
- 2 x is the hypotenuse of a right-angled triangle. To use Pythagoras' theorem, the length of the side shown in green must be known. This length can be calculated as the difference between the long and short vertical edges of the trapezium: $5.5 - 3.5 = 2$.
- 3 Substitute the values of the side lengths into Pythagoras' theorem.

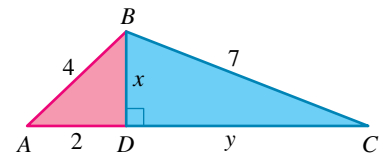
WRITE



$$\begin{aligned} c^2 &= a^2 + b^2 \\ x^2 &= 2^2 + (4.2)^2 \\ x^2 &= 4 + 17.64 \\ x^2 &= 21.64 \\ \sqrt{x^2} &= \sqrt{21.64} \\ x &\approx 4.7 \text{ (to 1 decimal place)} \end{aligned}$$

Note: In Worked Example 15 when taking the square root of 21.64 we only took the positive value, as it is not possible for a length to have a negative value.

- To find the value of y in the irregular triangle shown, the triangle can be split into two right-angled triangles: $\triangle ABD$ (pink) and $\triangle BDC$ (blue). There is enough information to calculate the missing side length from $\triangle ABD$. This newly calculated length can be used to find the value of y .



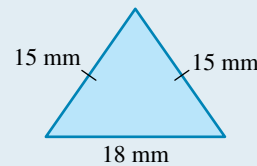
WORKED EXAMPLE 16

- Calculate the perpendicular height of the isosceles triangle whose equal sides are 15 mm each and whose third side is 18 mm long.
- Calculate the area of the triangle.

THINK

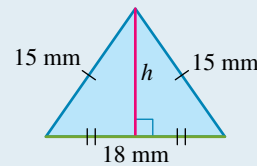
- 1 Draw the triangle and label all side lengths as described.

WRITE

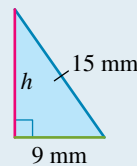


- 2 Draw an additional line to represent the height of the triangle, as shown in pink, and label it appropriately.

Since the triangle is an isosceles triangle, h bisects the base of 18 mm, as shown in green, and creates 2 right-angled triangles.



- 3 Focus on one right-angled triangle containing the height.



- 4 Substitute the values from this right-angled triangle into Pythagoras' theorem to calculate the value of h .

$$\begin{aligned}
 c^2 &= a^2 + b^2 \\
 15^2 &= h^2 + 9^2 \\
 225 &= h^2 + 81 \\
 225 - 81 &= h^2 + 81 - 81 \\
 144 &= h^2 \\
 \sqrt{144} &= \sqrt{h^2} \\
 12 &= h
 \end{aligned}$$

- 5 Answer the question, including units if appropriate.

The height of the triangle is 12 mm.

- 1 Write the formula for the area of a triangle. In this case the base is 18 mm and the height is 12 mm.

$$\begin{aligned}
 A_{\triangle} &= \frac{1}{2}bh \\
 &= \frac{1}{2} \times 18 \times 12 \\
 &= 108
 \end{aligned}$$

- 2 Answer the question, including units if appropriate.

The area of the triangle is 108 mm².

Interactivity: Finding the hypotenuse (int-3844)

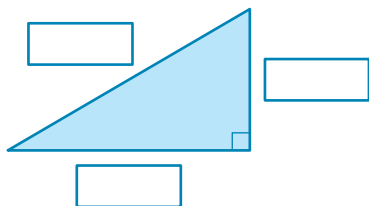
Interactivity: Finding a shorter side (int-3845)

Exercise 4.6 Applications of Pythagoras' theorem

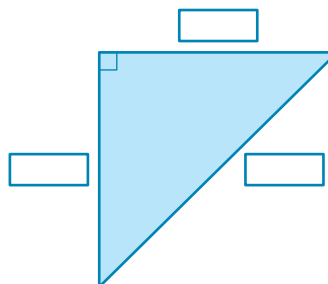
Understanding, fluency and communicating

1. Label the hypotenuse on each diagram.

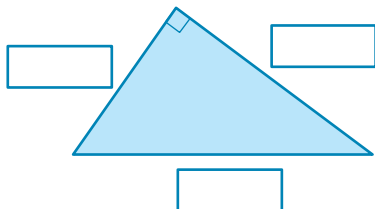
a.



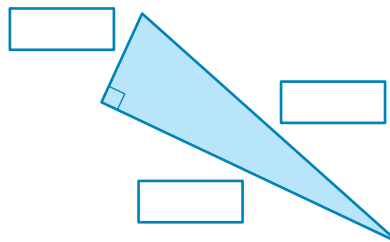
b.



c.

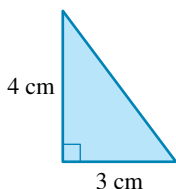


d.

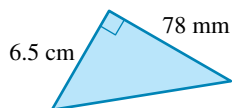


2. **WE13** For the following triangles, calculate the length of the hypotenuse correct to 2 decimal places where necessary.

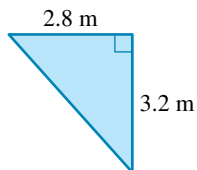
a.



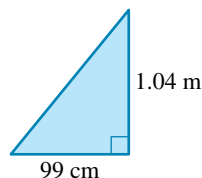
b.



c.

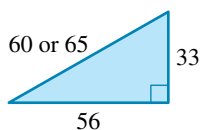


d.

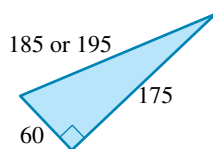


3. The triangles below are right-angled triangles. Two possible measurements have been suggested for the hypotenuse in each case. For each triangle, complete calculations to determine which of the lengths is correct for the hypotenuse.

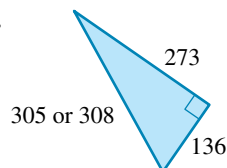
a.



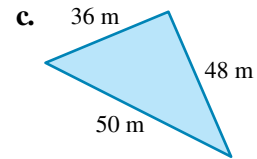
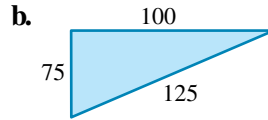
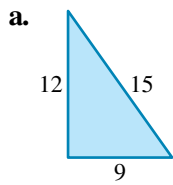
b.



c.



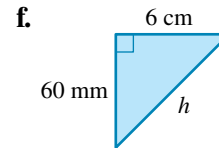
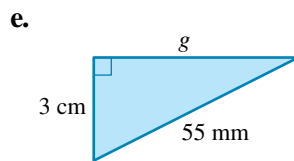
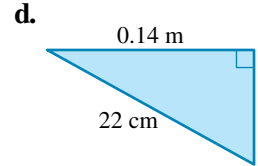
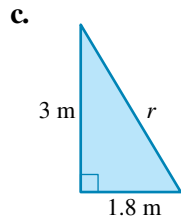
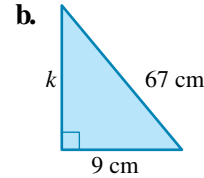
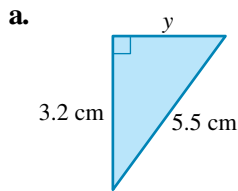
4. **WE14** Which of the following are right-angled triangles?



5. The following sets of side lengths form right-angled triangles. True or False?

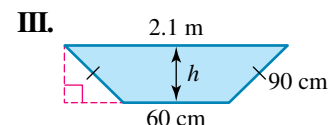
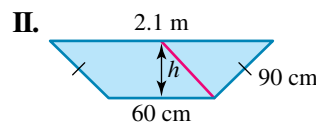
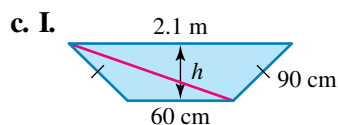
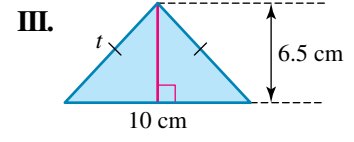
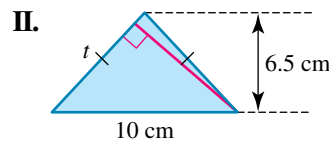
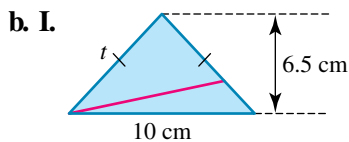
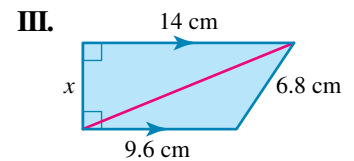
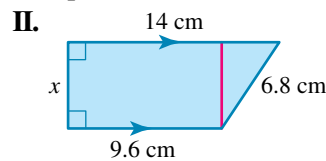
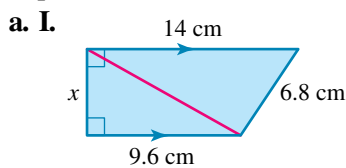
- a. 53 mm, 185 mm, 104 mm
- b. 41 mm, 4 cm, 9 mm
- c. 22.5 km, 54 km, 58.5 km

6. Identify the value or pronumeral of the hypotenuse in each of the following triangles.



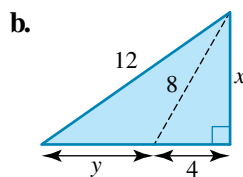
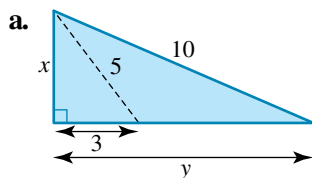
7. Calculate the value of pronumerals in triangles a, b and c in question 6, correct to 2 decimal places.

8. For each set below, state which diagram (I, II or III) represents the best use of additional lines (shown in pink) to calculate the value of the stated pronumeral.

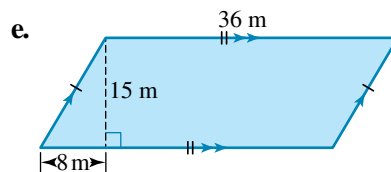
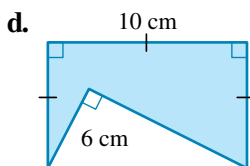
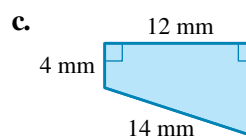
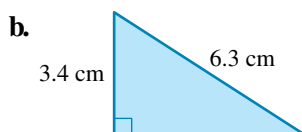
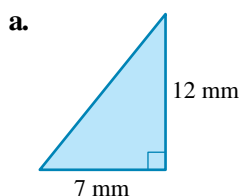


9. Calculate the values of the pronumerals in question 8 correct to 2 decimal places.

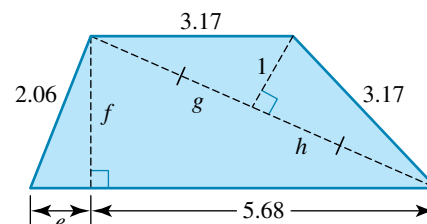
10. **WE15** For each of the following diagrams, calculate the lengths of the sides marked x and y , correct to 2 decimal places.



11. **WE16** a. Calculate the perpendicular height of the isosceles triangle whose equal sides are 25 cm long and whose base is 40 cm long.
b. Calculate the area of the triangle.
12. Use an appropriate method to calculate the perimeter of the shapes below, correct to 2 decimal places where necessary.

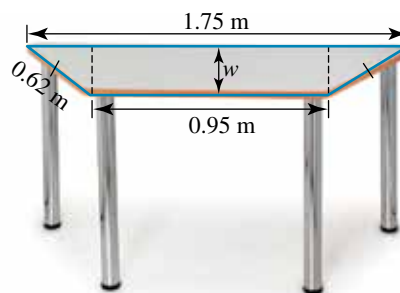


13. Use the values of some of the other pronumerals to calculate the length of e in the diagram at right. Give your answer correct to 2 decimal places.



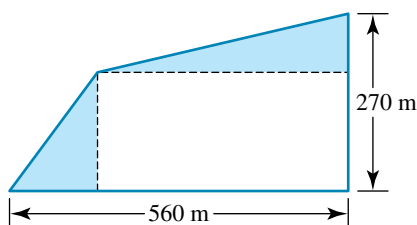
Problem solving, reasoning and justification

14. A classroom contains 20 identical desks like the one shown at right.
- Calculate the width of each desktop (labelled w), correct to 2 decimal places.
 - Calculate the area of each desktop, correct to 2 decimal places.
 - If the top surface of each desk is to be painted, what is the total area that needs to be painted?
15. A number of 1.9-metre-tall tent poles needs to be set up. It is recommended that any rope attached to the top of a tent pole should be attached to the ground at a distance from the pole of at least half the height of the pole. Allowing for an extra 50 cm of rope to tie knots, what is the minimum length of the rope needed for each pole? Give your answer correct to 2 decimal places.
16. A new housing estate is being cleared. The developer would like to work with a rectangular



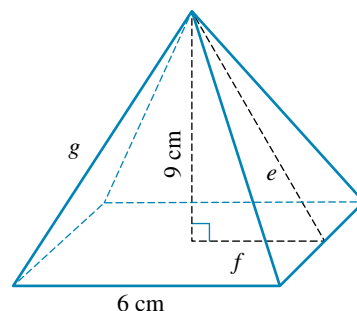
block for housing so has decided to keep the shaded areas as nature reserves. The rectangular block is 200 metres by 520 metres.

- Rounded to the nearest whole number, what percentage of the total estate is the nature reserve?
- How many metres of fencing would be needed to go around the entire estate? Give your answer correct to 2 decimal places.



17. Emily is making a net for a square-based pyramid with isosceles-triangle sides. She wants the sides of the base to be 6 cm and the height of the pyramid to be 9 cm.

- Draw a diagram representing the pyramid as a net.
- What should be the dimensions of each triangular face, correct to 1 decimal place?
- What is the area of the paper she will use to make the net?



18. Steve is on holidays at the beach. He walks 450 metres on a straight road from his hotel (marked in yellow) to the jetty which sticks out at 90 degrees from the shore. He then walks 300 metres from the shore to the end of the jetty (marked in pink). It's hot and the water looks inviting so Steve then decides to swim back to his hotel.
- To the nearest whole metre, how far will Steve have to swim?
 - If Steve can swim 50 metres in 1 minute, how long will it take him to swim back to shore, to the nearest whole minute?
 - If Steve walks at 5 km per hour, will swimming or walking back the way he came be faster, and by how much?

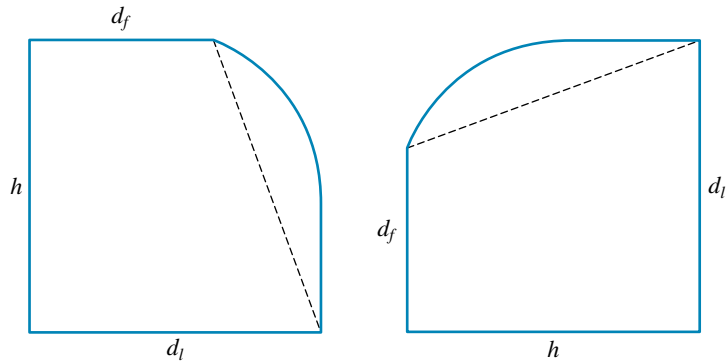


4.7 Irregularly shaped blocks of land

4.7.1 The trapezoidal rule

- Some shapes have curved boundaries that are not easily identifiable as sectors of a circle.
- The approximate perimeter and area of such shapes can be found by splitting the shape into trapezia. This involves using the **trapezoidal rule**.

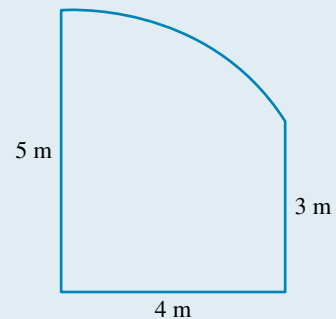
- The more trapezia that are used, the closer the approximation will be.
- The symbol $\approx$ means ‘approximately equal to’.
- The trapezoidal rule for area can be expressed as $A \approx \frac{h}{2} \times (d_f + d_l)$, where h is the height, d_f is the length of the first side and d_l is the length of the last side. Height is a label that is not affected by how the shape is oriented.



- The trapezoidal rule for area is essentially the same as the area formula for a trapezium, but in a format that allows it to be built on for more complicated calculations.

WORKED EXAMPLE 17

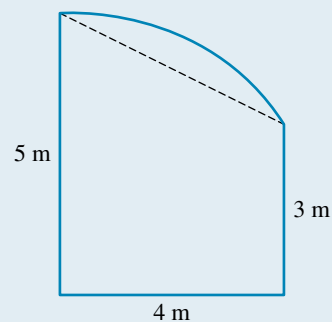
- Use the trapezoidal rule to calculate the approximate area of the shape at right using one trapezium.
- Use the trapezoidal rule to calculate the approximate area of the shape at right using two trapezia.



THINK

- Split the shape into a trapezium by drawing an extra line, as shown in black.

WRITE



- Use the formula $A \approx \frac{h}{2} \times (d_f + d_l)$.
- Substitute the values $h = 4$, $d_f = 5$ and $d_l = 3$ into the formula.

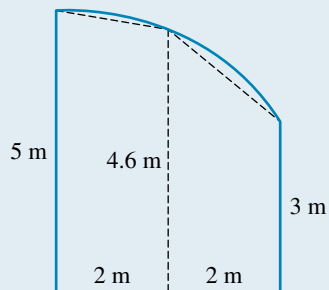
$$\begin{aligned}
 A &\approx \frac{h}{2} \times (d_f + d_l) \\
 &\approx \frac{4}{2} \times (5 + 3)
 \end{aligned}$$

4 Evaluate and include the correct units.

$$\approx 2 \times 8$$

$$\approx 16 \text{ m}^2$$

b 1 Split the shape into trapezia by drawing extra lines, as shown in black. Record the length of the vertical line.



2 Use the formula $A \approx \frac{h}{2} \times (d_f + d_l)$ for the left-hand trapezium.

$$A \approx \frac{h}{2} \times (d_f + d_l)$$

3 Substitute the values $h = 2$, $d_f = 5$ and $d_l = 4.6$ into the formula.

$$\approx \frac{2}{2} \times (5 + 4.6)$$

4 Evaluate and include the correct units.

$$\approx 1 \times 9.6$$

$$\approx 9.6 \text{ m}^2$$

5 Use the formula $A \approx \frac{h}{2} \times (d_f + d_l)$ for the right-hand trapezium.

$$A \approx \frac{h}{2} \times (d_f + d_l)$$

6 Substitute the values $h = 2$, $d_f = 4.6$ and $d_l = 3$ into the formula.

$$\approx \frac{2}{2} \times (4.6 + 3)$$

7 Evaluate and include the correct units.

$$\approx 1 \times 7.6$$

$$\approx 7.6 \text{ m}^2$$

8 Add the areas of the two trapezia.

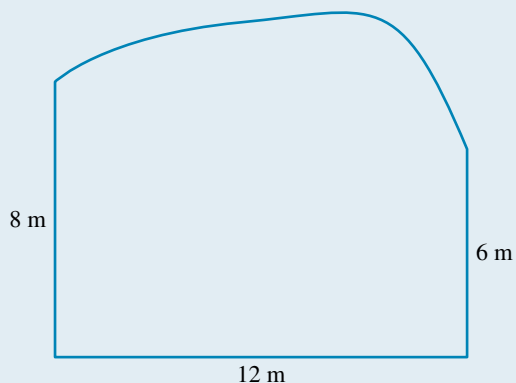
$$A \approx 9.6 + 7.6$$

$$\approx 17.2 \text{ m}^2$$

Note: In the questions in Exercise 4.7 all required heights are given in the diagrams.

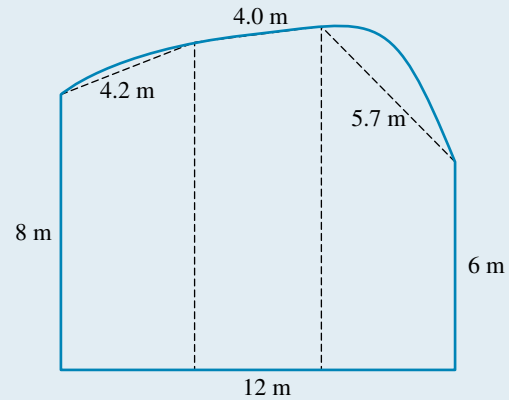
WORKED EXAMPLE 18

Calculate the approximate perimeter of the following shape using three trapezia.



THINK

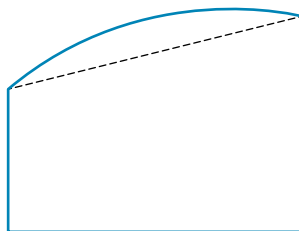
- 1 Split the shape into trapezia by drawing extra lines, as shown in black. Record the lengths of the sloping lines.

WRITE

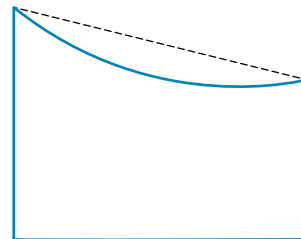
- 2 Because the shape is now a polygon with no special properties, simply add the side lengths in order and include the correct units.

$$\begin{aligned}
 P &\approx 12 + 8 + 6 + 4.0 + 4.2 + 5.7 \\
 &\approx 20 + 14.2 + 5.7 \\
 &\approx 39.9 \text{ m}^2
 \end{aligned}$$

- If the lengths of sides of the trapezia are unknown, then Pythagoras' theorem can be used to determine their length before the total perimeter is calculated.
- To know whether the approximation of the area or perimeter is too much or too little, the shape of the curve can be considered.
- If the curve is convex (i.e. above the sloping side of the trapezium) the approximation is too much. If the curve is concave (i.e. below the sloping side of the trapezium) the approximation is too little.

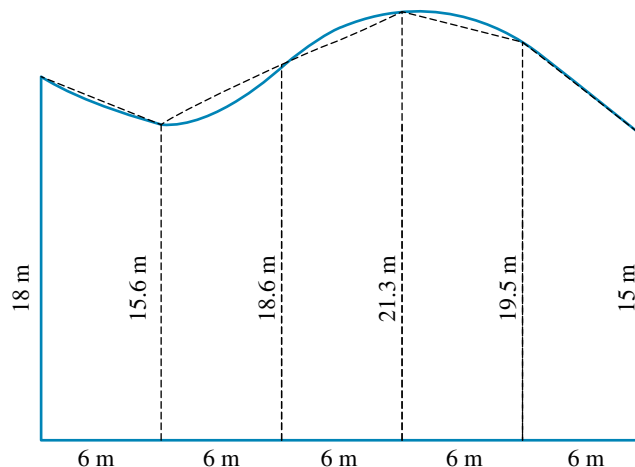


Convex



Concave

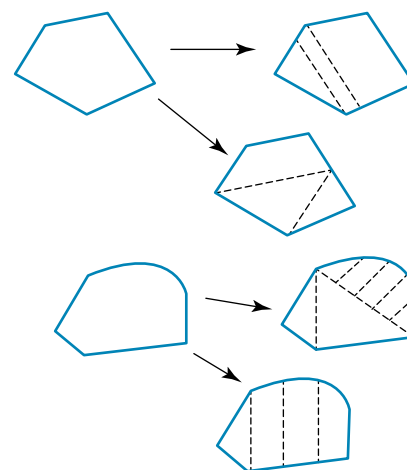
- Spreadsheet programs like Microsoft Excel can be used to help perform calculations.
- Distances can be entered in columns and Excel formulae can then be used to perform the calculations.



	A	B	C	D	E
1	Trapezium	h	d_f	d_l	Area
2	1	6	18	15.6	100.8
3	2	6	15.6	18.6	102.6
4	3	6	18.6	21.3	119.7
5	4	6	21.3	19.5	122.4
6	5	6	19.5	15	103.5
			Area sum		549.0

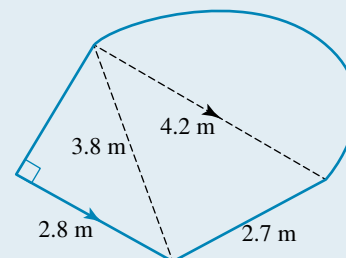
4.7.2 Solving practical problems

- Trapezia can be used in different ways to solve area and perimeter problems.
- Convex polygons can be split into trapezia and triangles in different ways according to the available information.
- Shapes with curved edges that are not sectors of a circle may require a variety of techniques, including the trapezoidal rule.
- In some cases the trapezoidal rule will create triangles. The trapezoidal rule will still work in these cases.



WORKED EXAMPLE 19

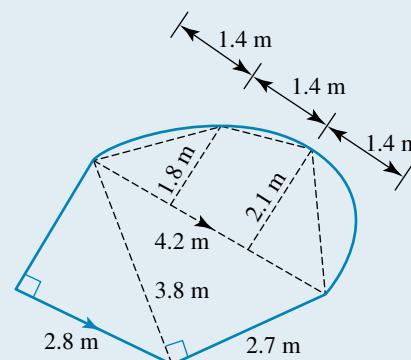
Calculate the approximate area of this garden bed, correct to 1 decimal place.



THINK

- 1 Split the shape into other shapes by drawing extra lines, as shown in black. Record the lengths of new lines. Pythagoras' theorem is needed to help determine the height of the trapezium at the bottom. The trapezoidal rule will also be used to help determine the area bounded by the curve.

WRITE



- 2 Use the formula $c^2 = a^2 + b^2$ for the bottom left-hand triangle. Substitute values, evaluate and include the correct units.

$$\begin{aligned}
 c^2 &= a^2 + b^2 \\
 3.8^2 &= 2.8^2 + b^2 \\
 14.44 &= 7.84 + b^2 \\
 6.6 &= b^2 \\
 b &= \sqrt{6.6}
 \end{aligned}$$

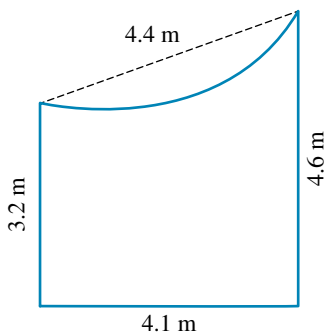
3 Use the formula $A = \frac{1}{2}(a + b)h$ for the bottom trapezium. Substitute values, evaluate and include the correct units.	$A = \frac{1}{2}(a + b)h$ $= \frac{1}{2}(4.2 + 2.8) \times \sqrt{6.6}$ $\approx 9.0 \text{ m}^2$
4 Use the formula $A \approx \frac{h}{2} \times (d_f + d_l)$ for the top left-hand trapezium (triangle).	$A \approx \frac{h}{2} \times (d_f + d_l)$
5 Substitute the values $h = 1.4$, $d_f = 0$ and $d_l = 1.8$ into the formula.	$\approx \frac{1.4}{2} \times (0 + 1.8)$
6 Evaluate and include the correct units.	$\approx 0.7 \times 1.8$ $\approx 1.3 \text{ m}^2$
7 Repeat for the middle top trapezium.	$A \approx \frac{h}{2} \times (d_f + d_l)$ $\approx \frac{1.4}{2} \times (1.8 + 2.1)$ $\approx 0.7 \times 3.9$ $\approx 2.7 \text{ m}^2$
8 Repeat for the top right-hand trapezium (triangle).	$A \approx \frac{h}{2} \times (d_f + d_l)$ $\approx \frac{1.4}{2} \times (2.1 + 0)$ $\approx 0.7 \times 2.1$ $\approx 1.5 \text{ m}^2$
9 Add the areas of all components.	$A \approx 9.0 + 1.3 + 2.7 + 1.5$ $\approx 14.5 \text{ m}^2$

Exercise 4.7 Irregularly shaped blocks of land

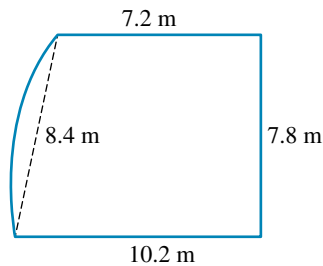
Understanding, fluency and communicating

1. **WE17** Use the trapezoidal rule to calculate the approximate area of each shape below using one trapezium, correct to 2 decimal places.

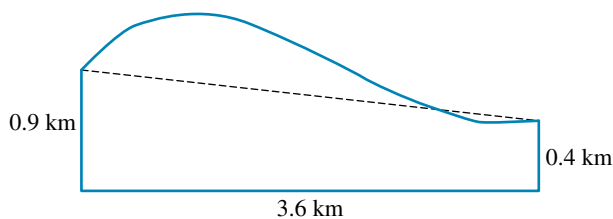
a.



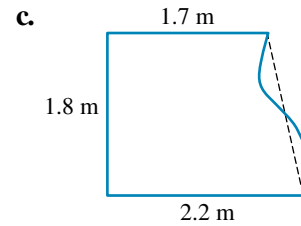
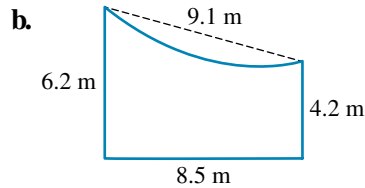
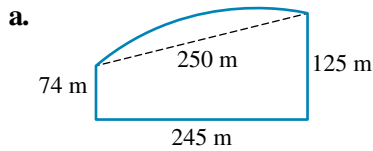
b.



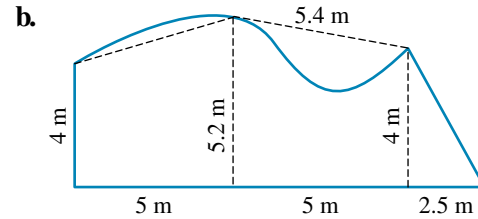
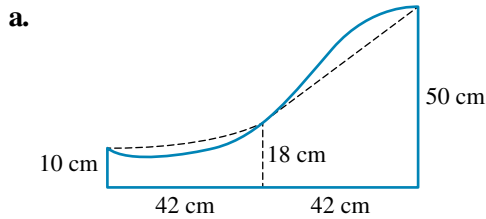
c.



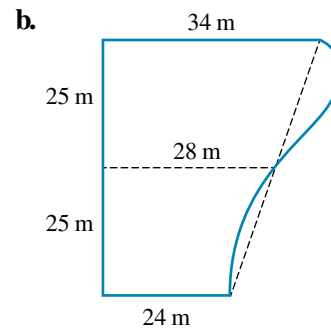
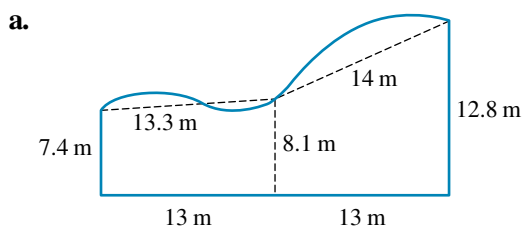
2. **WE18** Calculate the approximate perimeter of each shape below using one trapezium, correct to 1 decimal place where necessary.



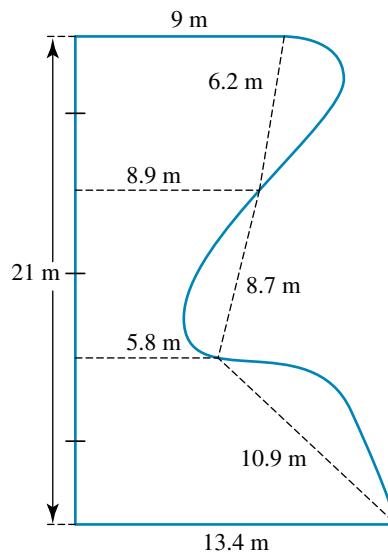
3. Use the trapezoidal rule to calculate the approximate area of each shape below.



4. Calculate the approximate perimeter of each shape below, correct to 1 decimal place where necessary.

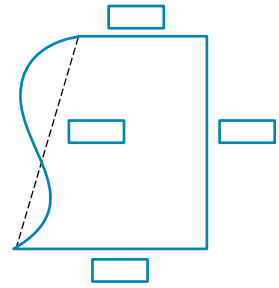


5. a. Calculate the approximate perimeter of the shape below.
b. Calculate the approximate area of the shape below, correct to 2 decimal places.



6. Label the blank spaces on the diagram *c*, *d* and *e* to match the formula

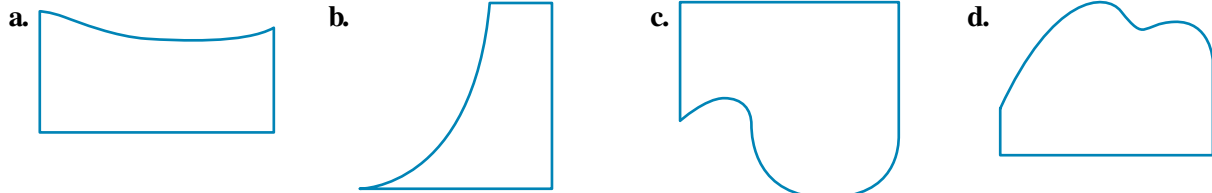
$A \approx (c + d) \times e \times \frac{1}{2}$, which calculates the approximate area of the shape. (Note: Only three blank spaces on the trapezium will be used.)



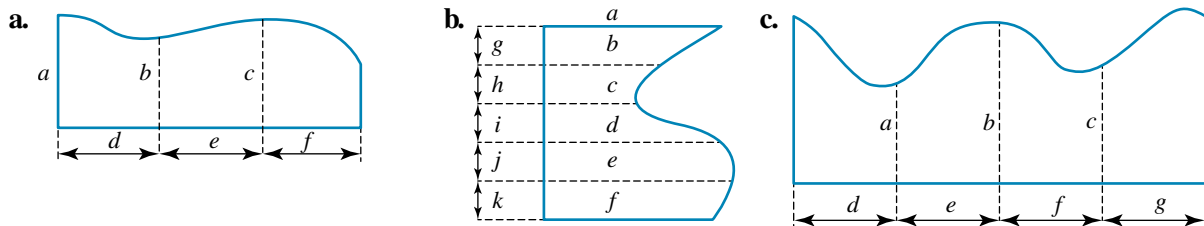
7. Answer True or False to each question about using the trapezoidal rule to calculate approximate area.

- The more trapezia that are used the closer the area is to the true value.
- The rule can't be used to calculate the area of a circle.
- Using 5 trapezia will usually give a worse estimate than using 4 trapezia.

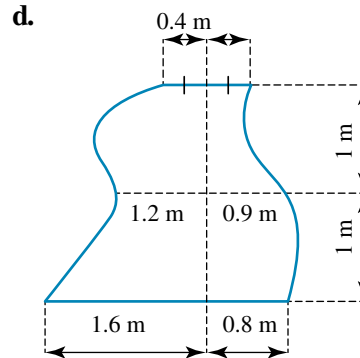
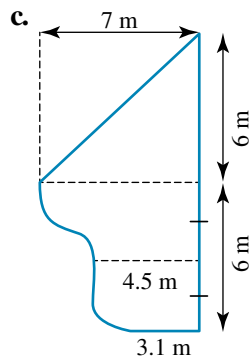
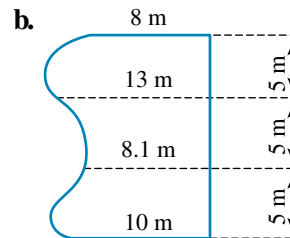
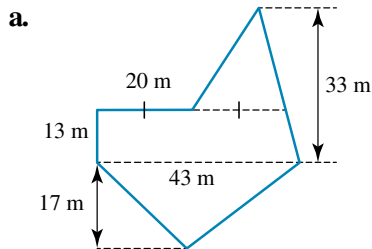
8. Look at each of the following trapezia. If only one trapezium is used, will the trapezoidal rule result in an overestimate or an underestimate?



9. Identify which of the following diagrams provide enough information to use the trapezoidal rule to calculate the approximate area if the values of the pronumerals are given. (Note: There may be more than one correct answer.)

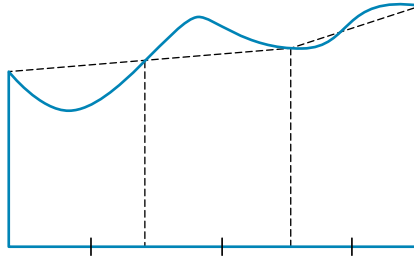


10. **WE19** Calculate the approximate area of each of the following shapes using the trapezoidal rule, correct to 1 decimal place.

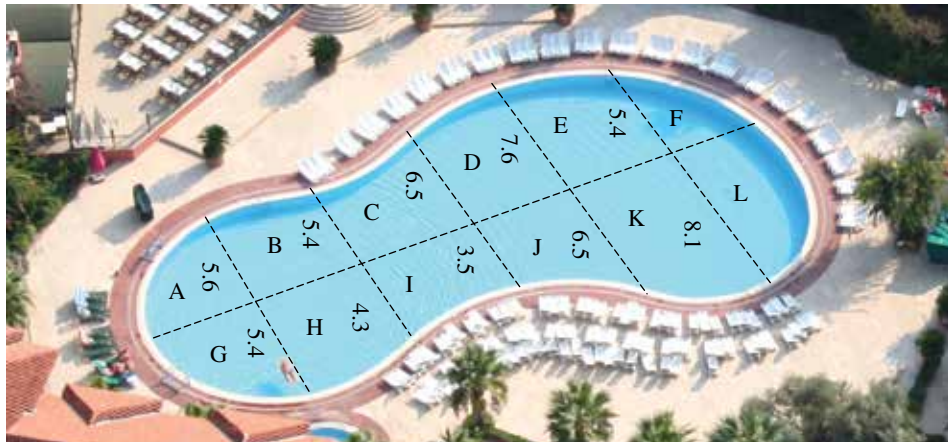


Problem solving, reasoning and justification

11. Look at the application of the trapezoidal rule below. Will it produce an underestimate, an overestimate or a very close estimate of the area of the shape? Explain your answer.



12. The measurements of a pool are shown below. The longest black line is perpendicular to all the shorter black lines and the distance of each interval is 6.5 metres.
- Use a spreadsheet to help you calculate the approximate area of the surface of the pool.
 - The volume of a pool can be calculated by multiplying the surface area by the depth of the pool. Assuming that depth of the pool is a constant 1.5 metres, calculate the approximate volume of the pool in m^3 .



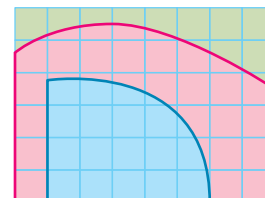
13. The Alamo is a famous landmark in San Antonio, USA.



- a. Use a spreadsheet to help you calculate the approximate area of the front of the Alamo building shown.
- b. If the front has to be gently cleaned by hand at a rate of 0.8m^2 per hour, how long will it take to clean all of it, correct to the nearest whole hour?
14. The picture below shows the top view of a garden. The side of each grid square represents 1 metre.



- a. Use a spreadsheet to help you calculate the approximate amount of ground in square metres that is covered in grass, correct to 1 decimal place.
- b. If the grass needs fertiliser at the rate of 50 grams per square metre, how much fertiliser will be used if it is applied twice a year?
15. The picture at right shows a small dam on a farm. The blue outline shows the current water level and the red outline shows the water level when the dam is full. The side of each small grid square represents 2 metres. What is the difference in area of the surface of the water between the current water level and when the dam is full? Use a spreadsheet to help you.



4.8 Review

4.8.1 Summary

In this topic you have learnt:

- how to calculate the perimeter of common shapes
- how to calculate the circumference of a circle
- how to calculate an arc length
- how to calculate the perimeter of composite shapes
- how to calculate the area of common shapes
- how to calculate the area of a sector
- how to calculate the area of composite shapes
- how to use Pythagoras' theorem to solve problems
- how to use the trapezoidal rule
- how to solve practical problems involving perimeter and area.

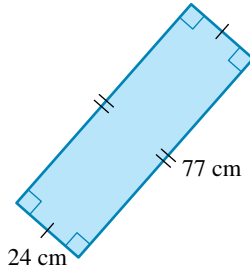
Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 4.8 Review

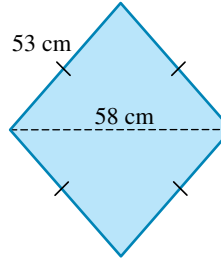
Understanding, fluency and communicating

1. Calculate the perimeter of each of the following shapes.

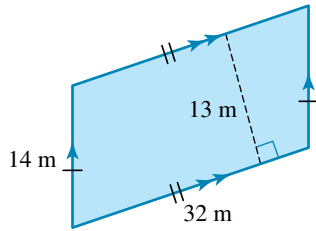
a.



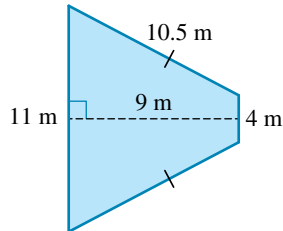
b.



c.



d.



2. If a triangle has a perimeter of 27 cm and two side lengths of 12 cm and 7 cm, how long is the other side?

3. **MC** Which of the following formulae can be used to calculate the perimeter of a rectangle?

(Note: There may be more than one correct answer.)

A. $P = 2(l + w)$

B. $P = l + l + w + w$

C. $P = (w + l) \times 2$

D. $P = w + w + 2l$

E. $P = 2l + 2w$

4. **MC** The circumference of a circle is 75.43 mm. The radius of the circle correct to the nearest whole number is:

A. 12 mm

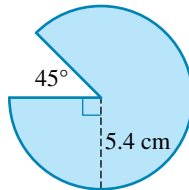
B. 24 mm

C. 10 mm

D. 38 mm

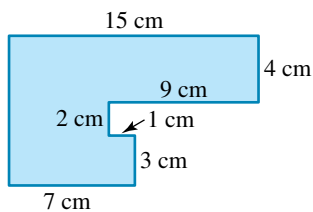
E. 48 mm

5. Calculate the perimeter of this shape, correct to 2 decimal places.

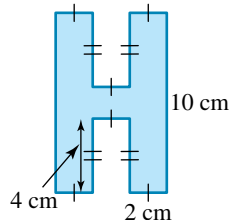


6. Calculate the perimeter of each of the following shapes, correct to 1 decimal place where necessary.

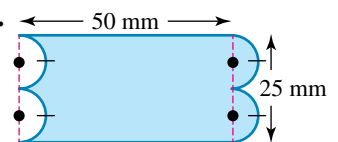
a.



b.

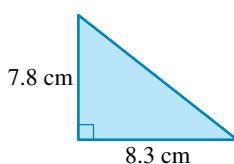


c.

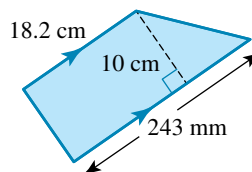


7. Calculate the areas of each of the following shapes.

a.



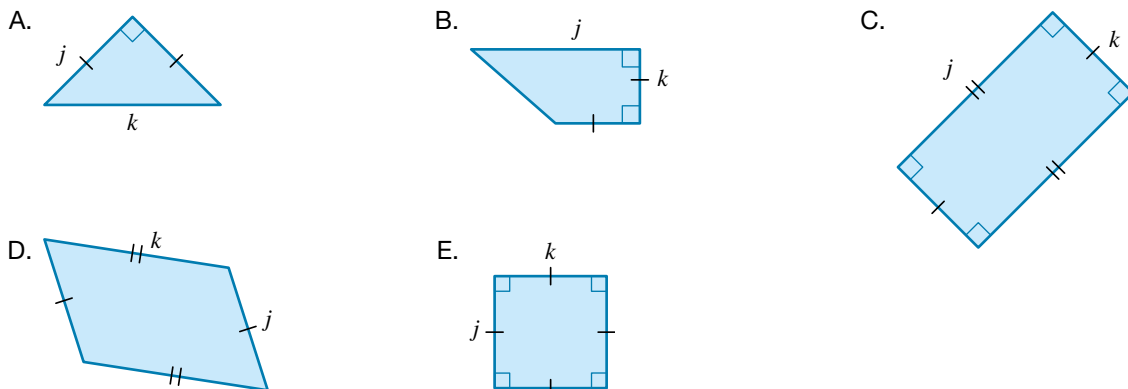
b.



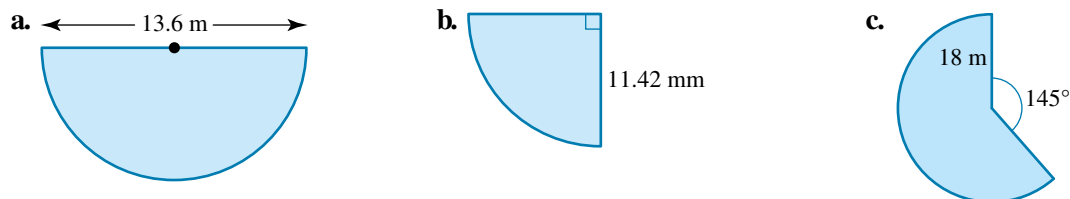
8. The area of a rectangle is 333.935 cm^2 . If its length is 23.5 cm, what is its width?

9. **MC** The area of a circle is equal to 415.64 mm^2 . The diameter of the circle is closest to:
A. 11.5 mm **B.** 23 mm **C.** 132.25 mm **D.** 17490 mm **E.** 12 cm

10. For which of the following shapes can the formula $A = j \times k$ be used to determine area? (Note: There may be more than one correct answer.)



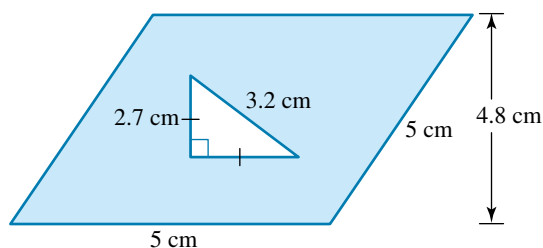
11. Calculate the area of the following shapes, correct to 2 decimal places.



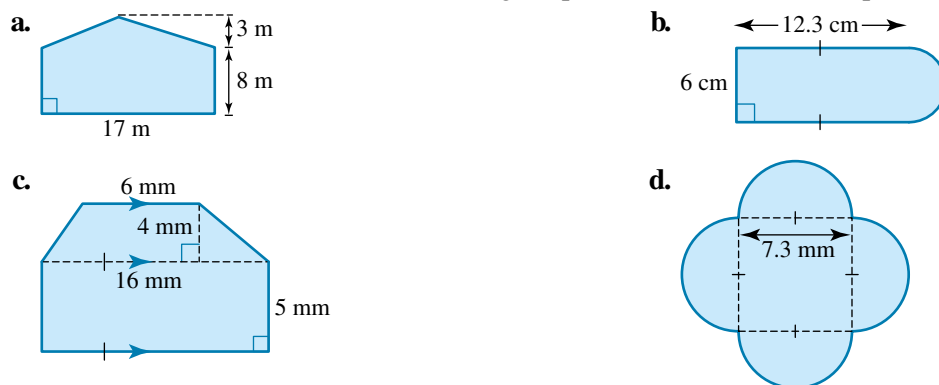
12. Find the perimeter and area of each of the following shapes.



13. Find the shaded area of the following parallelogram.



14. Calculate the area of each of the following shapes, correct to 1 decimal place where necessary.



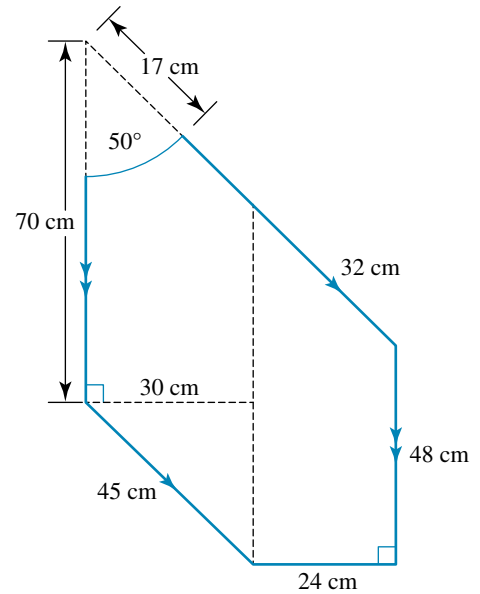
15. Identify the mistakes in the steps to calculate the area of the shape at right.

$$\begin{aligned} 1 \quad A_{\text{parallelogram}} &= b \times h \\ &= 70 \times 45 \\ &= 3150 \end{aligned}$$

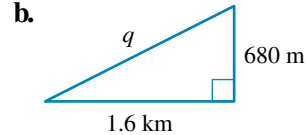
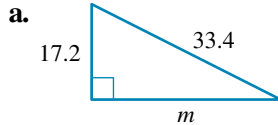
$$\begin{aligned} 2 \quad A_{\text{trapezium}} &= \frac{1}{2}(a \times b)h \\ &= \frac{1}{2} \times (70 \times 48) \times 24 \\ &= \frac{1}{2} \times 3360 \times 24 \\ &= 40\,320 \end{aligned}$$

$$\begin{aligned} 3 \quad A_{\text{sector}} &= \pi r^2 \times \frac{1}{\theta} \\ &= \pi \times 17^2 \times \frac{1}{50} \\ &= \pi \times 289 \times \frac{1}{50} \\ &= 18.16 \end{aligned}$$

$$\begin{aligned} 4 \quad \text{Total area} &= 3150 + 40\,320 - 18.16 \\ &= 43\,470 - 18.16 \\ &= 43\,451.84 \text{ cm} \end{aligned}$$



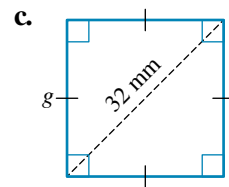
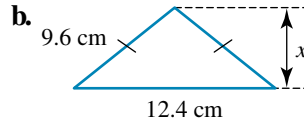
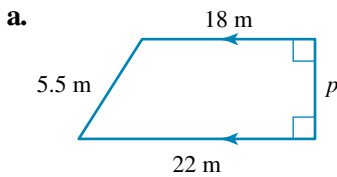
16. Calculate the value of the missing side in each of the following shapes, correct to 2 decimal places.



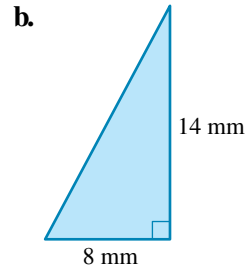
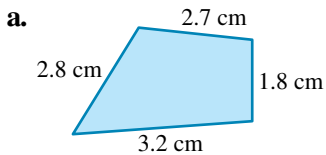
c. A right-angled triangle with sides 6.5 cm and 9.4 cm adjacent to the right angle

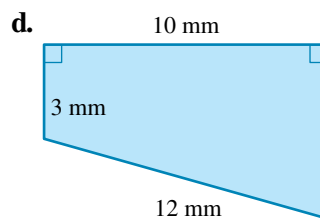
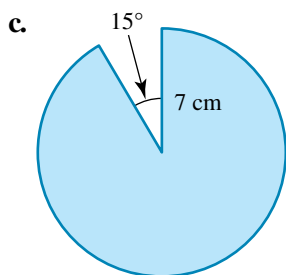
d. A right-angled triangle with a hypotenuse of 68.7 km and another side of 22.3 km

17. Calculate the values of the pronumerals in each of the following shapes, correct to 1 decimal place.



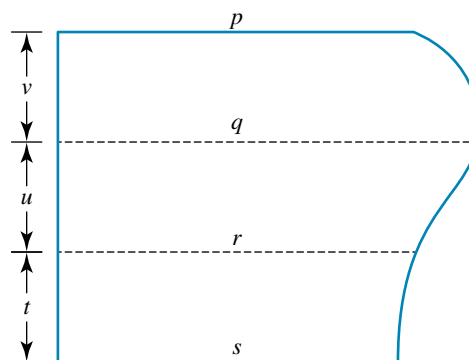
18. Calculate the perimeters of the following shapes correct to 2 decimal places where necessary.



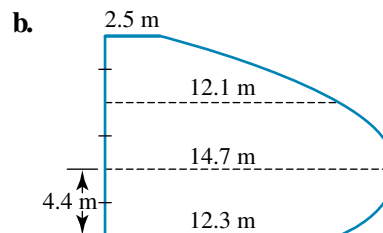
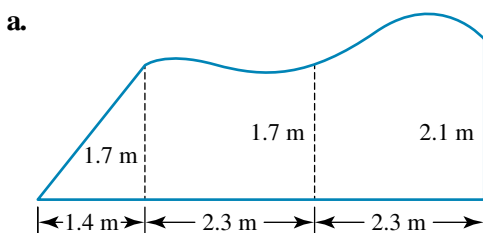


19. **MC** Which formula can be used to calculate the approximate area of the shape at right?

- A. $A \approx \frac{v + u + t}{2} \times (p + q + r + s)$
 B. $A \approx v(p + q) + u(q + r) + t(r + s)$
 C. $A \approx (v \times q) + (u \times r) + (t \times s)$
 D. $A \approx (p + s) \times (t + u + v)$
 E. $A \approx \frac{v}{2}(p + q) + \frac{u}{2}(q + r) + \frac{t}{2}(r + s)$



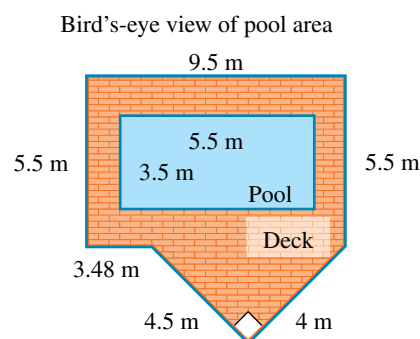
20. Calculate the approximate area of the following shapes.



Problem solving, reasoning and justification

21. A new pool is being installed in your friend's backyard. A diagram for the new pool and surrounding area is as shown.

- a. Calculate the length of safety fence required to go around the outside of the deck area.
 b. Pool mats stop leaves falling into the pool and stop water loss by evaporation. Calculate the area of pool mat required to cover the top of the pool.
 c. The deck area is to be tiled. Calculate the area of the deck.



22. A DVD has a data area as shown. The outer radius is 6 cm and the inner radius is 2.5 cm. The capacity of this DVD is 4.7 gigabytes and one gigabyte is equal to 1000 megabytes.

Assuming that each megabyte occupies the same area on the disc, calculate the area of 1 megabyte to the nearest mm^2 .

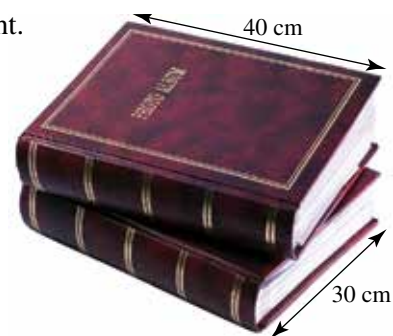
23. A football coach asks his team to warm up by completing 3 km running around the football oval.

- a. If the running track has straight lengths of 60 m and the end semicircles have a radius of 15 m, how many laps need to be completed to run 3 km?
 b. If you run 800 m on the inside track, how much further than your starting line will you finish, rounded up to the nearest whole metre?



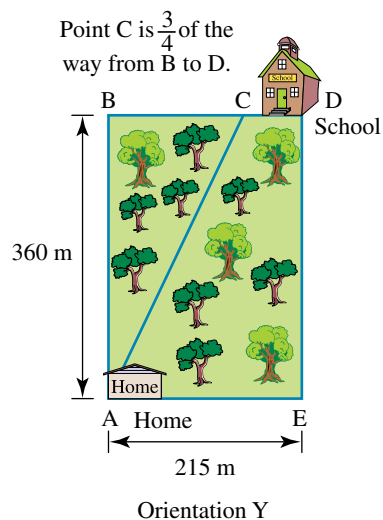
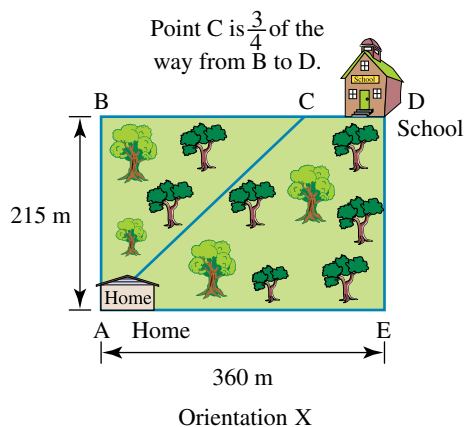
24. A particular photo album has pages with the dimensions shown at right.

- What is the maximum number of 6 cm by 8 cm photos you could fit onto one page so that no photos overlap? How much space would you have left over?
- What is the maximum number of 10 cm by 15 cm photos you could fit onto one page so that no photos overlap? How much space would you have left over?



25. Two classmates investigate the best cycling route to school; along the footpath around a park or through the park on the gravel path. The park is rectangular in shape. One of the classmates has drawn a simple diagram (Orientation X) showing the two routes.

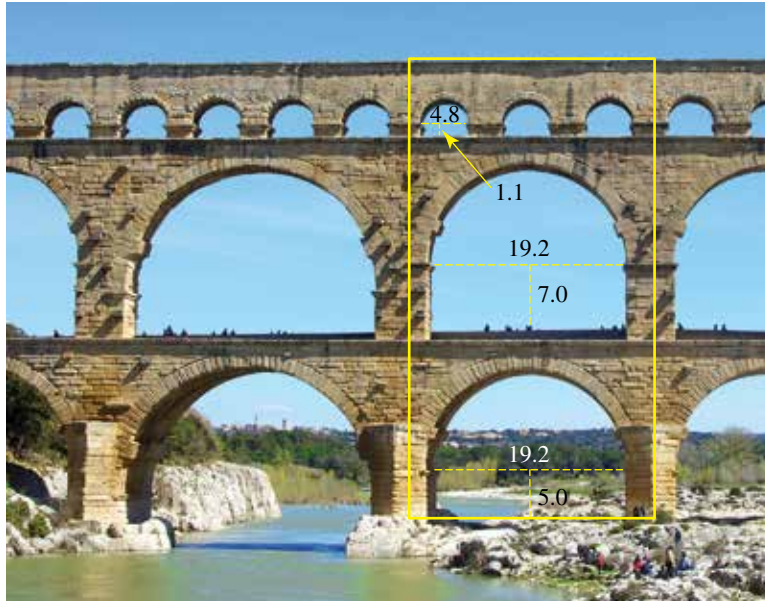
- Calculate the distance from home to school via Route I (A to B to C to D).
- Calculate the distance from home to school via Route II (A to C to D), correct to the nearest whole metre.
- The classmate's partner claims that the diagram is incorrect and redraws the diagram (Orientation Y). They disagree on how this might affect their calculations. One classmate says that, since the dimensions of the park are the same, it will make no difference. Use calculations to investigate whether this does make a difference.



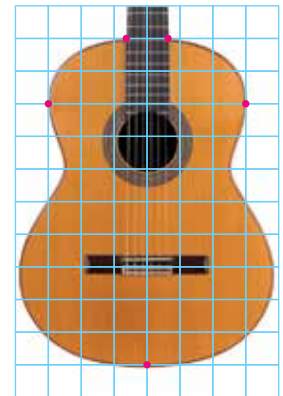
26. Two friends are looking at a photo of a block of land for sale. They are trying to determine whether the block is truly a square, and each is arguing differently. Prove mathematically whether the land is square or not.



27. The Pont du Gard is a Roman aqueduct built in France around 2000 years ago. The section shown in yellow is 25.0 metres wide and 44.9 metres high and consists of three levels of arches. All dimensions in the photo are in metres. Calculate the area of the stone face of the section marked in yellow, correct to 1 decimal place.



28. In the picture at right, each grid square represents an area 5 cm by 5 cm. Use a spreadsheet to help you calculate the area of the guitar body to the nearest whole centimetre. Do not include the hole in the top half or the part of the neck of the guitar.
29. Look at the photo of the roundabout shown.



- Describe two ways of calculating the total area of grass in the middle of the roundabout.
- Use one of your methods to calculate the approximate total area of grass in the middle of the roundabout to the nearest whole metre. Use the following measurements to help you.
 - The inner diameter of the roundabout is 80 metres.
 - The longer edge of the paved section is approximately the same length as the inner diameter and touches the centre of the circle.
 - The width of the paved section cutting across the grass is 22.4 metres.
 - The shorter edge of the paved section is 63.6 metres.
- A ride-on mower can mow at 8 km/h with a cutting width of 1.5 metres. How long to the nearest minute will it take to mow the grass at that rate?

- 30.** A chicken farmer wants to have a stocking density of 2000 birds per hectare on her farm. This means that in a 10 000 m² outside area she would have 2000 chickens free to range. The farmer has 800 chickens so she won't actually need the whole 10 000 m² available.
- a.** What is the actual area needed for 800 chickens at a stocking density of 2000 birds per hectare?
 - b.** What are four of the many possible whole-number dimensions of a rectangular ranging area for the 800 chickens?
 - c.** What are two of the many possible whole-number dimensions (base and height) of a triangular ranging area for the 800 chickens? (*Note: All dimensions must be at least 20 m.*)
 - d.** Imagine that the triangular areas in part **c** are right-angled. If the farmer only has 500 metres of fencing wire, which of the areas can be made with the wire?



Answers

Exercise 4.2 Perimeter 1

1. B, D
2. 37 cm
3. a. 11.4 m b. 120 cm c. 96 cm d. 34.5 cm
4. a. $x = 16$ cm b. $x = 7.2$ mm
5. 152 cm
6. 50 cm
7. 27 mm
8. A, C
9. E
10. a. True b. True c. True d. False e. True
11. a. 43.98 mm b. 29.85 mm c. 8.17 m
12. B
13. B

14.

Width	Length
1	11
2	10
3	9
4	8
5	7
6	6
7	5
8	4
9	3
10	2
11	1

15. $P = a + b + c + d$ or $P = a + b + 2c$
16. 15.92 cm
17. 5305
18. 12 cm
19. a. $P = 2a + 2b$ b. A, B, D
20. The maximum whole number of times the cable can be wrapped is 38. It is unlikely to be that number of times because if the winch works like the one shown in the picture, then the cable will wrap over itself. This will cause the second layer of cable to wrap around a thicker diameter which means the number of times the cable will wrap will be less than 38.

Exercise 4.3 Perimeter 2

1. a. 11.00 cm b. 5.03 cm c. 52.36 m
2. a. 22.98 cm b. 217.72 m c. 53.70 cm
3. a. 680 mm b. 140 cm c. 225 cm
4. a. 40.3 m b. 81.7 cm c. 106.7 m
5. B
6. 9.48 m
7. 24.00 m
8. a. True b. False c. False d. True e. True
9. A, D, E
10. 5.2 cm
11. 691.2 cm
12. 121°
13. 26.2 m

14. 5 laps and 172 m

15. a. 43 cm

b. 37.152 km

Exercise 4.4 Area 1

1. A, C, D

2. a. 15 cm

b. 2 m

3. A, B, D

4. a. 40.5 cm^2

b. 8 cm^2

c. 3.9 cm^2

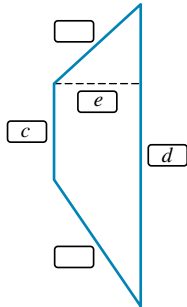
d. 810 mm^2

e. 487.5 cm^2

5. a. 275 mm^2

b. 2400 mm^2

6.



Note: c and d can also swap positions.

7. a. 23.04 cm^2

b. 52.5 m^2

c. 3.04 cm^2

d. 11.1 m^2

8. a. False

b. True

c. True

d. True

e. True

9. a. 29.4 cm^2

b. 1.96 cm^2

c. 23.97 m^2

d. 13.0625 cm^2

e. 75.6 cm^2

f. 16.38 km

10. a. 336 mm^2

b. $17\,537 \text{ cm}^2$

c. 2400 mm^2

11. 22.05 m^2

12. 9.5 cm

13. a. 452.39 mm^2

b. 907.92 cm^2

14. B

15. 24 cm

16. 67% of the paint will be left.

17. a. 15 cm^2

b. 10 cm by 6 cm

c. 60 cm^2

d. If the side lengths of the rectangle are doubled its area is four times larger, or quadrupled.

18. a. 31.15 m^2

b. 6 with a remainder, so 7 bags are needed.

c. \$62.65

19. Answers will vary.

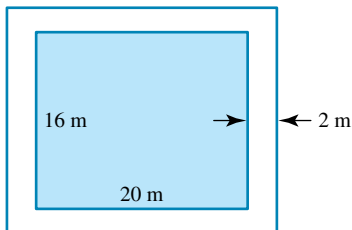
Possible answers include:

A square with side lengths of 8 cm.

A rectangle with length of 32 cm and width of 2 cm.

A triangle with base length of 64 cm and height of 2 cm.

20. a.



b. 320 m^2

c. 160 m^2

d. \$800

21. a. 261.4 m

b. 3220 m^2

22. It doesn't matter which side is considered the length and which the width, because the formula only involves multiplying two factors. With multiplication, it doesn't matter which order the factors are multiplied in because the product will be the same.

Therefore, either side can be labelled length or width as long as the sides are adjacent and not opposite each other.

23. a. 79 cm^2 , 177 cm^2 , 315 cm^2

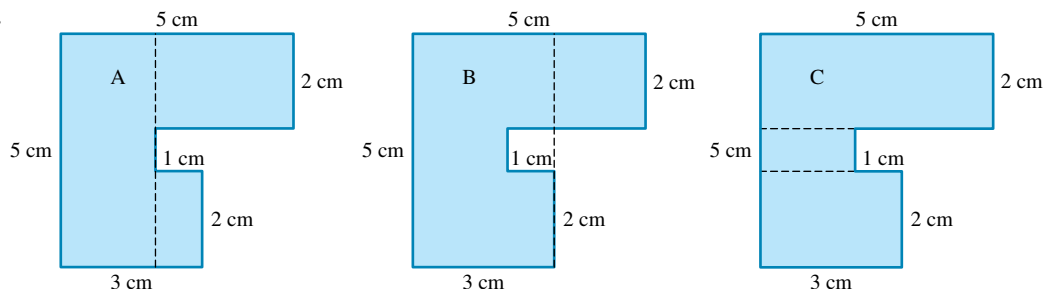
b. $28\,550 \text{ cm}^2$

24. 0.137 km^2

Exercise 4.5 Area 2

- C
- The area of a sector is one-third the area of the circle it sits inside. Therefore, the [angle] of the sector at the centre of the circle will be one-third of [360°].
- a. 84.83 cm^2 b. 351.85 mm^2 c. 63.05 cm^2
- a. 0.38 b. 136.8°
- a. 16 cm^2 b. 56 cm^2 c. 9600 mm^2 d. 222.75 cm^2 e. 97.04 m^2
- B, D, E
- a. 686 mm^2 b. 72 cm^2 c. 135 mm^2
- a. True b. True c. True d. False e. False
- a. 756 m^2 b. 157.08 cm^2 c. 31.49 cm^2 d. 0.86 m^2 e. 40.18 mm^2 f. 132.72 cm^2
- A

11. a.



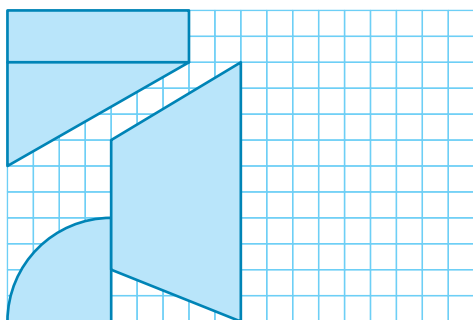
b. 18 cm^2

12. a. 325 m^2

b. \$8125

13. a. 36%

b. Answers will vary. A possible arrangement is:



14. a. 0.46 m^2

b. 3.66 m^2

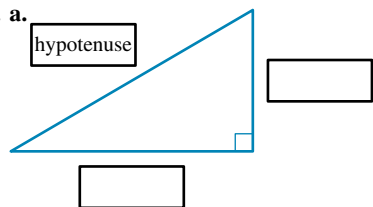
15. a. 2025 cm^2

b. 943.41 cm^2

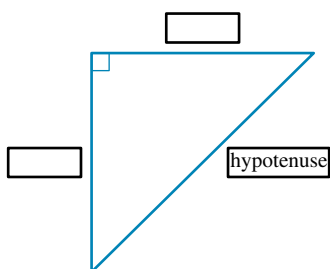
c. 6031.59 cm^2

Exercise 4.6 Applications of Pythagoras' theorem

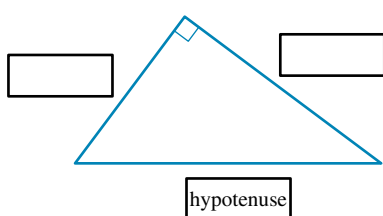
1. a.



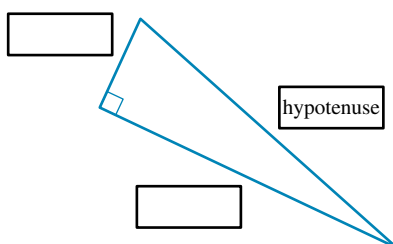
b.



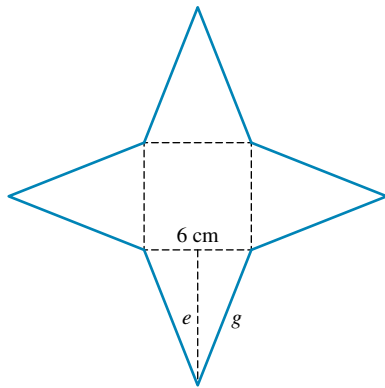
c.



d.



2. a. 5 cm b. 10.15 cm c. 4.25 m d. 1.44 m
 3. a. 65 b. 185 c. 305
 4. a. Right-angled triangle b. Right-angled triangle c. Not right-angled
 5. a. False b. True c. True
 6. a. 5.5 cm b. 67 cm c. r
 d. 22 cm e. 55 mm f. h
 7. a. 4.47 cm b. 66.39 cm c. 3.50 m
 8. a. II b. III c. III
 9. a. $x = 5.18$ cm b. $t = 8.20$ cm c. $h = 49.75$ cm
 10. a. $x = 3.74$ b. $x = 6.93$
 $y = 9.17$ $y = 5.80$
 11. a. 15 cm b. 300 cm^2
 12. a. 32.89 mm b. 15 cm c. 41.21 mm d. 44 cm e. 106 m
 13. 0.53
 14. a. 0.47 m b. 0.63 m^2 c. 12.6 m^2
 15. 2.62 m
 16. a. 18% b. 1558.65 m
 17. a.

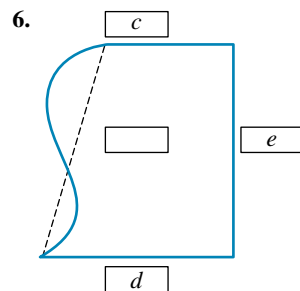


- b. The dimensions of the triangle should be a 6 cm base, sides of 10 cm and a perpendicular height of 9.5 cm.
 c. 150 cm^2
 18. a. 541 m
 b. 11 minutes
 c. Walking will be faster by 2 minutes.

Exercise 4.7 Irregularly shaped blocks of land

Note: Answers to some questions may vary according to the trapezia chosen.

1. a. 15.99 m^2 b. 67.86 m^2 c. 2.34 km^2
 2. a. 694 m b. 28 m c. 7.6 m
 3. a. 2016 cm^2 b. 51 m^2
 4. a. 73.5 m b. 159 m
 5. a. 69.2 m^2 b. 181.30 m^2



Note: c and d can also swap positions.

7. a. True b. False c. False

8. a. Overestimate b. Overestimate c. Underestimate d. Underestimate
9. B
10. a. 1105.0 m^2 b. 150.6 m^2 c. 49.7 m^2 d. 3.8 m^2
11. It is a very close estimate. The overestimate in the left-most trapezium is balanced by the underestimate in the middle trapezium and the right-most trapezium is very close to the shape of the curve.
12. a. The base area of the pool is approximately 379 m^2 .
b. 568.5 m^2
13. a. The total area is approximately 154.8 m^2 .
b. 194 hours
14. a. 9.8 m^2 b. 980 g
15. 91.5 m^2

Exercise 4.8 Review

1. a. 202 cm b. 212 cm c. 92 m d. 36 m
2. 8 cm
3. A, B, C, D, E
4. A
5. 40.49 cm
6. a. 84 cm b. 48 cm c. 178.5 mm
7. a. 32.37 cm^2 b. 212.5 cm^2
8. 14.21 cm
9. B
10. C, E
11. a. 72.63 m^2 b. 102.43 mm^2 c. 607.90 m^2
12. a. 19.33 cm, 9.6 cm^2 b. 33.6 cm, 47.15 cm^2
13. 20.355 cm^2
14. a. 161.5 m^2 b. 87.9 cm^2 c. 124 mm^2 d. 137.4 mm^2
15. There are four mistakes.
In calculating the area of the parallelogram, the side length (45 cm) was used instead of height (30 cm).
In calculating the area of the trapezium the sides a and b were multiplied instead of summed.
In calculating the area of the sector, the angle of 50° is treated as a unit fraction $\left(\frac{1}{50}\right)$ instead of as a fraction out of $360^\circ \left(\frac{50}{360}\right)$.
In calculating the total area, the unit was recorded as cm instead of cm^2 .
16. a. 28.63 b. 1.74 km c. 11.43 cm d. 64.98 km
17. a. 3.8 m b. 7.3 cm c. 22.6 mm
18. a. 10.5 cm b. 38.12 cm c. 56.15 cm^2 d. 34.63 mm
19. E
20. a. 9.47 m^2 b. 150.48 m^2
21. a. 32.48 m^2 b. 19.25 m^2 c. 42 m^2
22. 2 mm^2
23. a. 14 laps b. 157 m
24. a. 25 photos with no space left over
b. 8 photos in total are possible with no space left over.
25. a. 575 m
b. 435 m
c. There is a difference in distance for Route II, so orientation matters.
26. The shape is not square.
27. 559.6 m^2
28. 1204 cm^2

- 29. a. i.** Calculate the area of the circle. The paved section is approximately a trapezium so calculate the area using that formula. Then subtract the area of the trapezium from the area of the circle.
- ii.** Use the trapezoidal rule to calculate the area of both grassed sections.
- b.** 3419 m^2
- c.** 17 minutes

30. a. 4000 m^2

b.

Length	Width
20	200
25	160
40	100
50	80

c.

Base	Height
20	400
25	320
40	200
50	160

d. Area 3, Area 4

TOPIC 5

Surface area and volume

5.1 Overview

5.1.1 Introduction

If we're able to calculate the surface area of shapes, we're able to know the amount of fabric we need to make a tent or how much paint we need to paint a new living room. Likewise, by knowing how to calculate volume and capacity, we can work out how much water is needed to fill a swimming pool, or even the Grand Canyon!



DISCUSSION BOX

How many rugby balls do you think can fit into an Olympic-sized swimming pool?

LEARNING SEQUENCE

- 5.1** Overview
- 5.2** Surface area 1
- 5.3** Surface area 2
- 5.4** Volume 1
- 5.5** Volume 2
- 5.6** Review

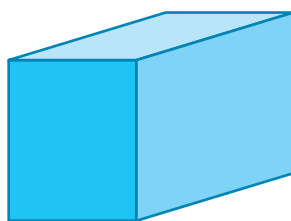
CURRICULUM CONTENT

- solve problems involving surface area of solids including but not limited to prisms, cylinders, spheres and composite solids
- solve problems involving volume and capacity of solids including but not limited to prisms, cylinders, spheres and composite solids
- solve problems involving perimeters, area, surface area, volumes and capacity in a variety of contexts **AAM**

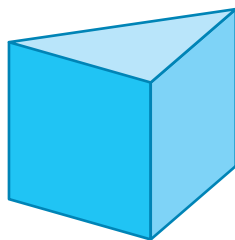
5.2 Surface area 1

5.2.1 Total surface area of a right prism

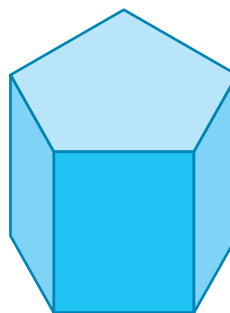
- The **total surface area** (TSA) of a three-dimensional object is the total area of all of the outside faces (sides).
- A **prism** is a three-dimensional shape with identical opposite ends joined by straight edges. It can be sliced into identical slices called **cross-sections**.
- A prism is named according to the shape of its base.



Rectangular prism



Triangular prism



Pentagonal prism

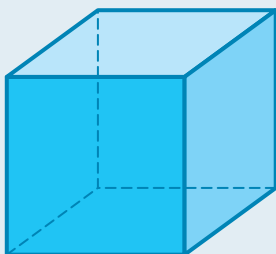


- Objects with a uniform cross-section with ends that are not polygons cannot be classified as prisms. For example, a cylinder is not a prism, even though it has a uniform cross-section.

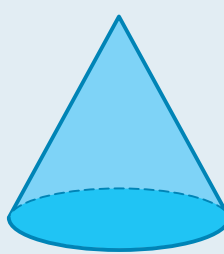
WORKED EXAMPLE 1

Which of the following shapes are prisms? *Note: There may be more than one correct answer.*

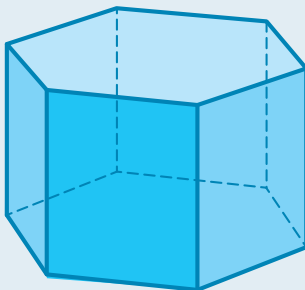
a



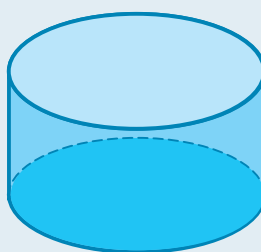
b



c



d



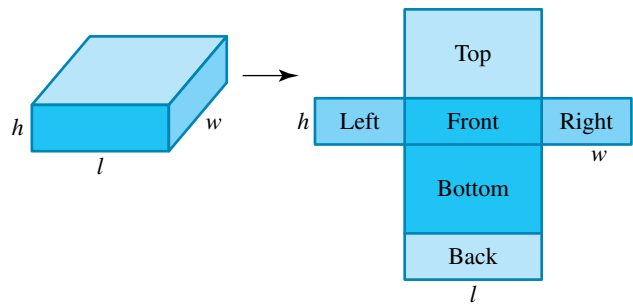
THINK

A prism can be sliced into identical cross-sections.
Therefore, B is not a prism.
A prism must have a polygon for the base.
Therefore, D is not a prism.

WRITE

The prisms are A and C.

- A **net** is often used to visualise the outside faces that make up a prism.
- A rectangular prism and its net are shown at right.
- These rectangles are actually pairs. The rectangles that form the top and the bottom are the same, the rectangles that form the front and the back are the same and the rectangles that form the left and right ends are the same.



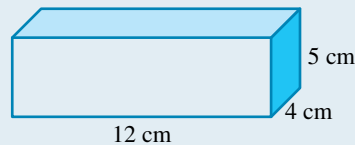
- To find the total area of the top and the bottom, $A = 2 \times (l \times w)$ or $A = 2lw$.
- To find the total area of the front and the back, $A = 2 \times (l \times h)$ or $A = 2lh$.
- To find the total area of both ends, $A = 2 \times (h \times w)$ or $A = 2hw$.

- The total surface area is the sum of the area of all six rectangles. This can be found using the formula $TSA = 2lw + 2lh + 2hw$. Using your knowledge of algebra, this can be simplified to:

$$TSA_{\text{rectangular prism}} = 2(lw + lh + hw)$$

WORKED EXAMPLE 2

Calculate the total surface area of the following rectangular prism.



THINK

- 1 The rectangular prism is made up of 6 rectangular faces. The area of each of these faces can be calculated by multiplying the length by the width.
- 2 Identify the pronumerals and substitute them into the formula to calculate the TSA.
- 3 Include the appropriate units in your final answer.

WRITE

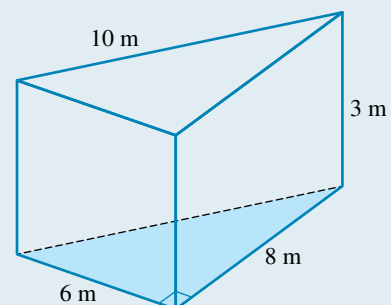
$$TSA_{\text{rectangular prism}} = 2(lw + wh + lh)$$

$$\begin{aligned} TSA_{\text{rectangular prism}} &= 2(5 \times 4 + 4 \times 12 + 5 \times 12) \\ &= 2(20 + 48 + 60) \\ &= 256 \end{aligned}$$

$$TSA = 256 \text{ cm}^2$$

WORKED EXAMPLE 3

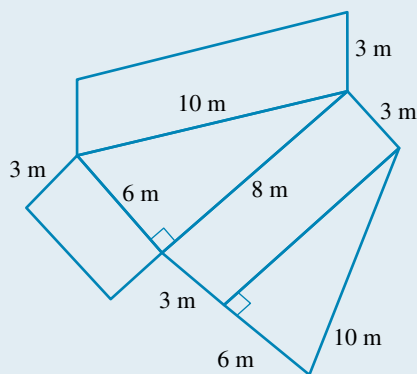
- a Draw the net for the triangular prism shown.
- b Find the total surface area of the prism.



THINK

- a** The net will consist of two triangles and three rectangles. Draw the net, marking the lengths of all sides.

WRITE



- b 1** The triangular prism is made up of two identical triangles and three different rectangles. The triangles are right-angled triangles, so the area of each triangle is found by multiplying the perpendicular sides together and halving the results.
The area of each rectangle is found by multiplying the length and width of the rectangle.

$$TSA_{\text{triangular prism}} = 2A_{\text{triangle}} + A_{\text{rectangle 1}} + A_{\text{rectangle 2}} + A_{\text{rectangle 3}}$$

- 2** Identify the required side lengths and substitute them into the formula.

$$\begin{aligned} TSA_{\text{triangular prism}} &= 2 \times \frac{6 \times 8}{2} + 8 \times 3 + 6 \times 3 + 10 \times 3 \\ &= 48 + 24 + 18 + 30 \\ &= 120 \end{aligned}$$

- 3** Include the appropriate units in your answer.

$$TSA = 120 \text{ m}^2$$

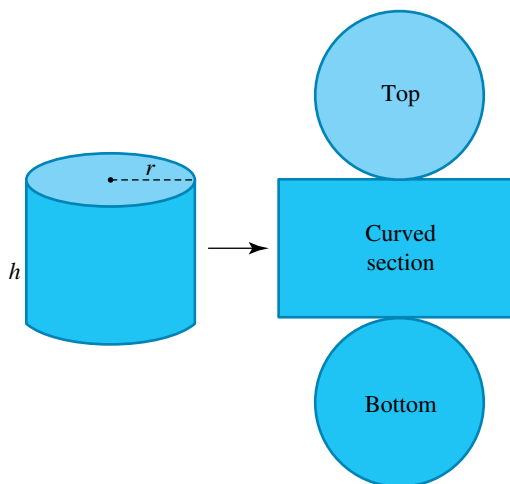
5.2.2 Total surface area of a cylinder

- If a cylinder is flattened into its net, it becomes two circles of equal size and a rectangle.
- The area of each of the circles can be found using the area formula:

$$A = \pi r^2$$

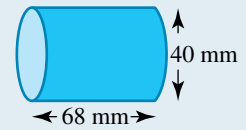
- The circumference of the circle gives us the length of the rectangle. The height of the cylinder is the width of the rectangle. The formula to find the area of the rectangle is $A = (2 \times \pi \times r) \times h$ or $A = 2\pi rh$.
- The total surface area is the sum of the areas of the two circles and the rectangle. This can be found using the formula $TSA = 2\pi r^2 + 2\pi rh$. Using your knowledge of algebra, this can be simplified to:

$$TSA_{\text{cylinder}} = 2\pi r(r + h)$$



WORKED EXAMPLE 4

Calculate the total surface area of this cylinder. Give your answer correct to 2 decimal places.



THINK

- 1 Write the formula for the total surface area of a cylinder.
- 2 To find the radius of the cylinder, halve the value of the diameter that is given.
- 3 Identify the values of the pronumerals. Substitute the values of the pronumerals into the formula and evaluate. Remember to include units in your answer.

WRITE

$$TSA_{\text{cylinder}} = 2\pi r(r + h)$$

$$r = \frac{D}{2} = \frac{40}{2} = 20 \text{ mm}$$

$$r = 20 \text{ mm}; h = 68 \text{ mm}$$

$$\begin{aligned} TSA &= 2 \times \pi \times 20 \times (20 + 68) \\ &= 40\pi \times 88 \\ &= 3520\pi \\ &\approx 11\,058.41 \text{ mm}^2 \end{aligned}$$

The total surface area of the cylinder is 11 058.41 mm².

eBookplus RESOURCES

eLesson: Total surface area of prisms (eles-1909)

Interactivity: Surface area (int-6477)

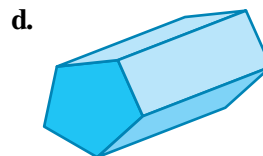
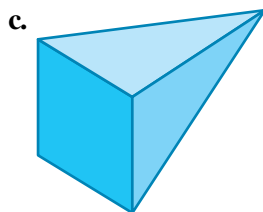
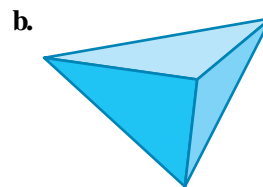
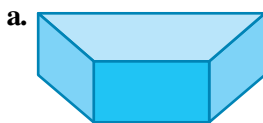
Interactivity: Cross-sections (int-6078)

Interactivity: Prisms? (int-3792)

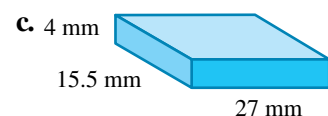
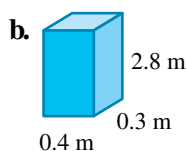
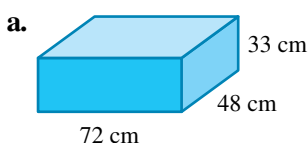
Exercise 5.2 Surface area 1

Understanding, fluency and communicating

1. **WE1** Which of the shapes below are prisms?



2. **WE2** Calculate the total surface area of each of the following rectangular prisms.



3. Calculate the total surface area of a cube with each of the following side lengths.

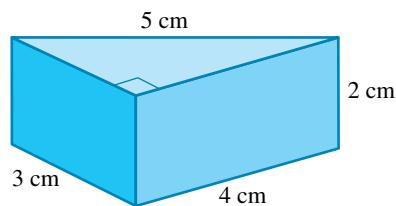
a. 21 mm

b. 0.42 m

c. $3\frac{3}{4}$ cm

4. **WE3** a. Draw the net for the triangular prism.

b. Find the total surface area of the prism.

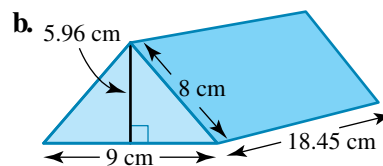
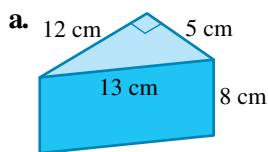


5. For each of the following triangular prisms, calculate:

i. the area of the triangular face

ii. the area of each of the three rectangular faces

iii. the total surface area.

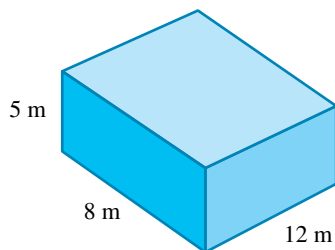


6. For each of the following prisms:

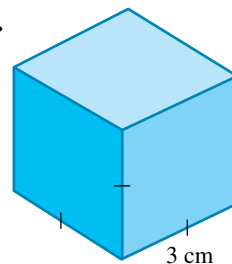
i. draw the net of the prism

ii. calculate the total surface area of the prism.

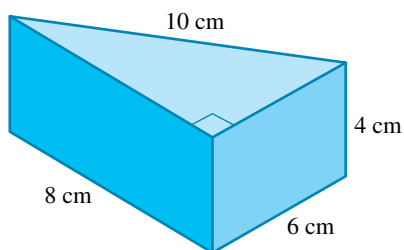
a.



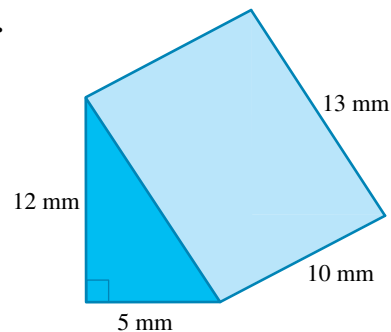
b.



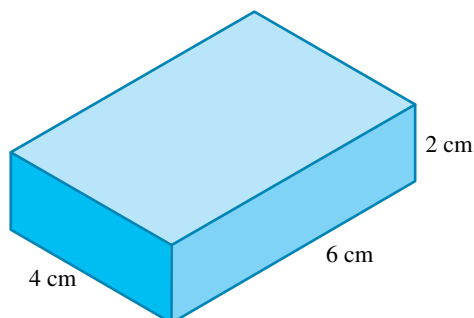
c.



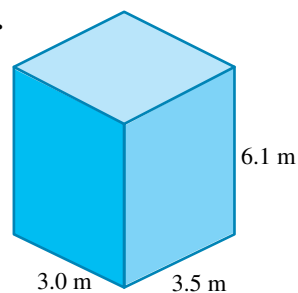
d.



e.



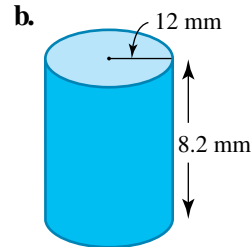
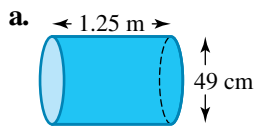
f.



7. Calculate the total area of glass needed to make the open-topped fish tank shown.



8. **WE4** Calculate the total surface area, to the nearest whole number, of each of the following cylinders.

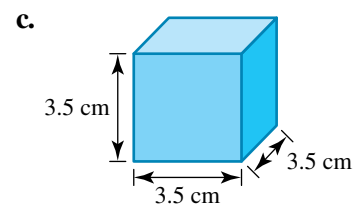
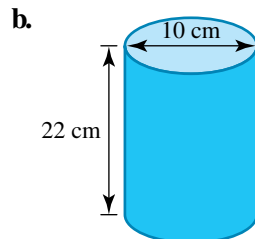
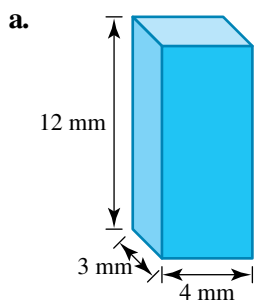


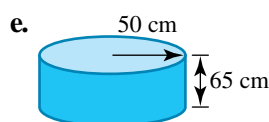
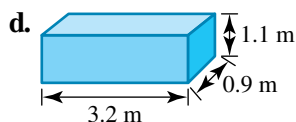
9. A can of salmon is in the shape of a cylinder with the dimensions as shown. A paper label runs all the way around the curved surface of the can without overlapping itself.

- Calculate the area of the label in mm^2 to the nearest whole number.
- Calculate the total area of aluminium needed to make the can. Give your answer in mm^2 to the nearest whole number.



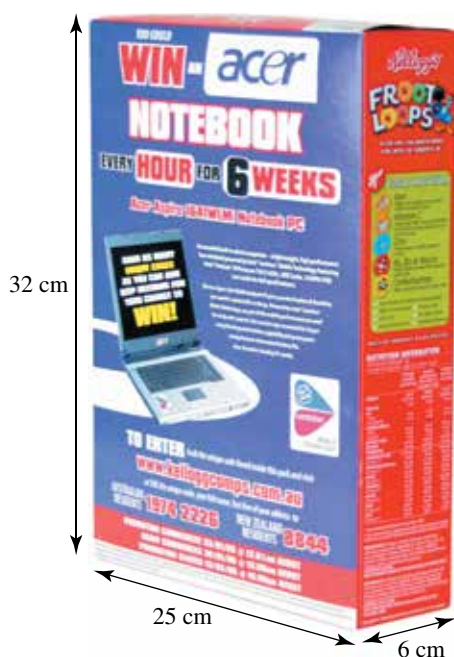
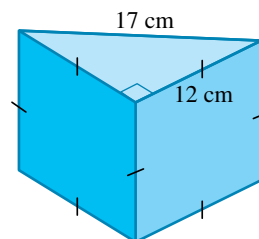
10. Calculate the total surface area of each of the following objects. Give your answers to 2 decimal places where necessary.





Problem solving, reasoning and justification

11. a. Design, draw and label a cube that has a total surface area of 150 cm^2 .
 b. Calculate the side length of a cube with a total surface area of 238.14 cm^2 .
12. Consider an open cardboard box in the shape of a cube with one face missing. The cardboard used to make the box has a surface area of 328.05 m^2 (ignoring overlap). Calculate the side length of the box.
13. A cube with all side lengths equal to 12 cm is cut in half diagonally to form a triangular prism, as shown at right.
 - a. Calculate the total surface area of a cube with all side lengths equal to 12 cm .
 - b. Calculate the total surface area of the triangular prism shown.
 - c. If you cut a cube in half, does this halve its surface area? Use your results from parts a and b to help explain your answer.
14. A cube has side lengths of 1 cm .
 - a. If each side is doubled in length, how does this affect the total surface area?
 - b. Investigate this further by finding how the surface area changes if each side length is:
 - i. tripled
 - ii. quadrupled.
 - c. Describe any pattern in your results.
15. A magazine holder can be made from a cereal box measuring $25 \text{ cm} \times 32 \text{ cm} \times 6 \text{ cm}$ by cutting the top section off as shown.



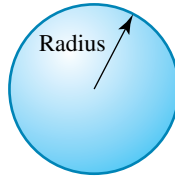
- a. Calculate the total area of the cardboard used in the cereal box (assuming no overlaps).
- b. Calculate the area of cardboard that remains after the top section is cut off (that is, the TSA of the outside of the magazine holder).

5.3 Surface area 2

5.3.1 Total surface area of a sphere

- The total surface area of a sphere can be calculated by using the formula:

$$\text{TSA}_{\text{sphere}} = 4 \times \pi \times r^2$$



$$\text{TSA}_{\text{sphere}} = 4\pi r^2$$

WORKED EXAMPLE 5

Calculate the total surface area of a soccer ball with a radius of 10 cm. Give your answer to 2 decimal places.



THINK

- The total surface area of the sphere can be calculated by using the formula $\text{TSA}_{\text{sphere}} = 4 \times \pi \times r^2$.
- Substitute the radius value into the formula.
- Calculate the total surface area of the sphere.
- Include the appropriate units in your final answer.

WRITE

$$\text{TSA}_{\text{sphere}} = 4 \times \pi \times r^2$$

$$\text{TSA}_{\text{sphere}} = 4 \times \pi \times 10^2$$

$$\begin{aligned}\text{TSA}_{\text{sphere}} &= 4 \times \pi \times 100 \\ &= 400\pi\end{aligned}$$

$$\text{TSA}_{\text{sphere}} \approx 1256.64 \text{ cm}^2$$

5.3.2 Total surface area of composite shapes

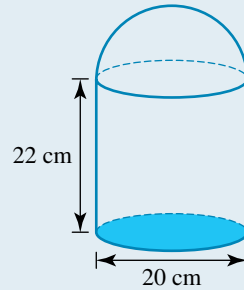
- The total surface area of a composite object can be calculated by finding the total surface area of each individual shape of its faces.
- If part of a sphere is used in a composite object, the total surface area of the object can be found by calculating the appropriate fraction of the TSA of a sphere. For example, if half a sphere is used, the TSA can be found by applying the formula:

$$\begin{aligned}\text{TSA}_{\text{half a sphere}} &= \frac{1}{2} \times 4 \times \pi \times r^2 \\ &= 2\pi r^2\end{aligned}$$

Note that this formula finds the area of the curved surface only. For example, to find the TSA of a closed hemisphere, you would need to add the surface area of half a sphere to the surface area of the circular base.

WORKED EXAMPLE 6

Calculate the total surface area of the following object. Give your answer correct to 2 decimal places.



THINK

- 1 The composite object is made up of a cylinder and half a sphere. The TSA can be calculated by finding the TSA of each and adding them together.
- 2 The TSA of a cylinder can be found by using the formula $2\pi rh + 2\pi r^2$. But we need to find only the TSA of one end (circle).
- 3 Substitute the values of the pronumerals into the formula and calculate.
- 4 The TSA of the sphere can be found by using the formula $A = 4\pi r^2$; but we have only half a sphere.
- 5 Substitute the values of the pronumerals into the formula and calculate.
- 6 Add together the TSA of the cylinder and the sphere to find the TSA of the composite object.
- 7 Include the appropriate units in your final answer.

WRITE

$$TSA = TSA_{\text{cylinder}} + TSA_{\text{half a sphere}}$$

$$TSA_{\text{cylinder}} = 2\pi rh + \pi r^2$$

$$\begin{aligned} TSA_{\text{cylinder}} &= 2\pi \times 10 \times 22 + \pi \times 10^2 \\ &= 440\pi + 100\pi \\ &= 540\pi \end{aligned}$$

$$TSA_{\text{half a sphere}} = \frac{1}{2} \times 4 \times \pi \times r^2$$

$$\begin{aligned} TSA &= 2 \times \pi \times 10^2 \\ &= 200\pi \end{aligned}$$

$$\begin{aligned} TSA_{\text{composite object}} &= 540\pi + 200\pi \\ &= 740\pi \end{aligned}$$

$$TSA_{\text{composite object}} \approx 2324.78 \text{ cm}^2$$

Exercise 5.3 Surface area 2

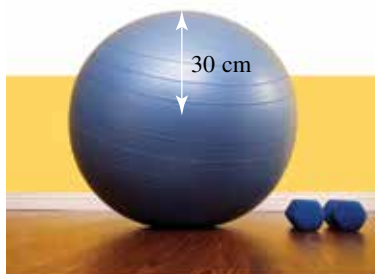
Understanding, fluency and communicating

Unless otherwise stated, give answers correct to the nearest whole number.

1. **WE5** a. What is the surface area of the balance ball shown below (assume it is a sphere)?

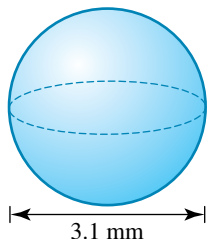


b. What is the surface area of the balance ball shown below to 1 decimal place?

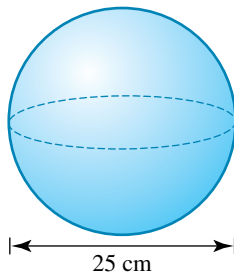


2. Calculate the total surface area of each of the following objects.

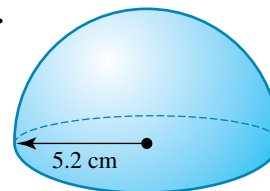
a.



b.

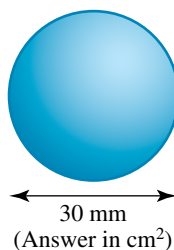


c.

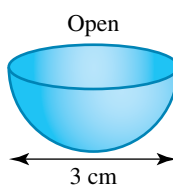


3. Find the total surface areas of the following spheres and hemispheres. Give your answers correct to 2 decimal places. (For part b, find the TSA of the outer surface.)

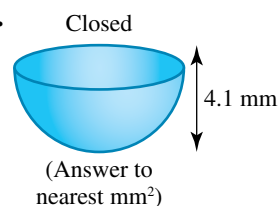
a.



b.



c.



4. **MC** A sphere has a total surface area of 125 mm^2 . The radius of the sphere correct to 2 decimal places is:

A. 3.15 mm

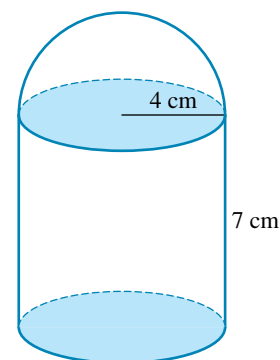
B. 9.94 mm

C. 31.5 mm

D. 31.25 mm

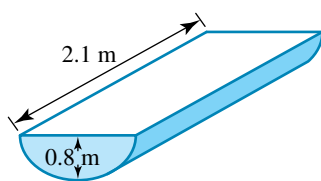
E. 15.71 mm

5. **WE6** Calculate the total surface area of the following object, correct to 1 decimal place.

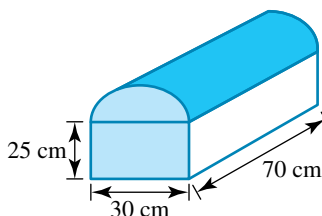


6. Calculate the total surface area of the following objects.

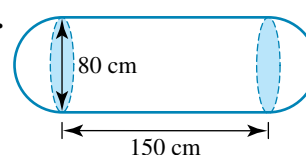
a.



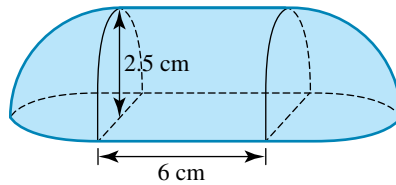
b.



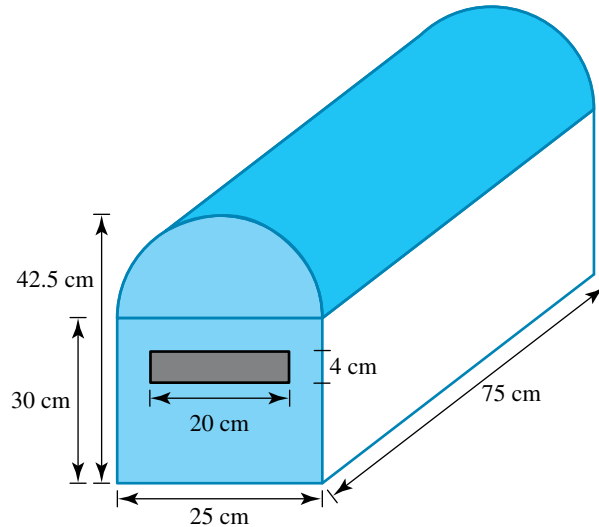
c.



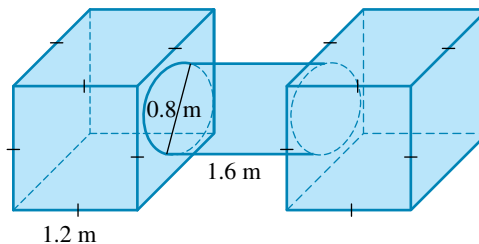
7. Calculate the surface area of the following solid.



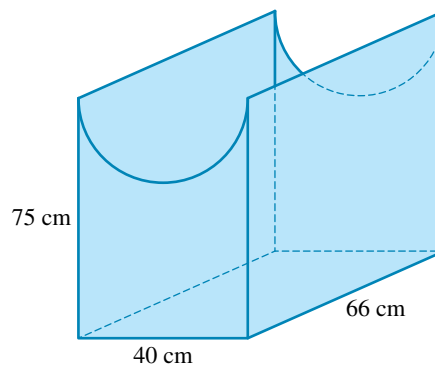
8. The following letterbox is to be spray-painted on the outside. What is the total area to be spray-painted?



9. The drawing shown is of part of a child's playground toy. Calculate the TSA, giving the answer to 2 decimal places.

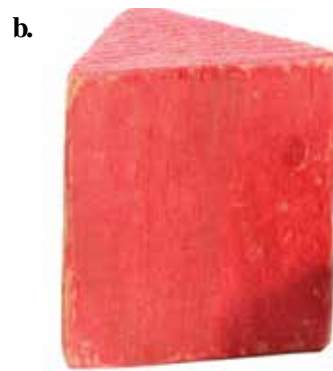
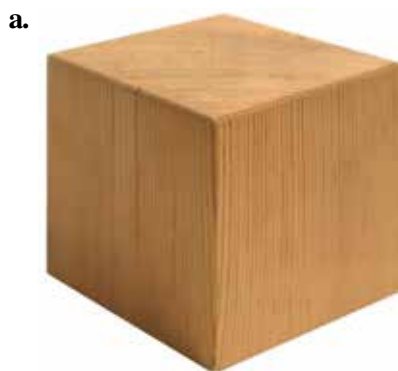


10. Calculate the TSA of the following shape correct to 2 decimal places, assuming the shape is solid.

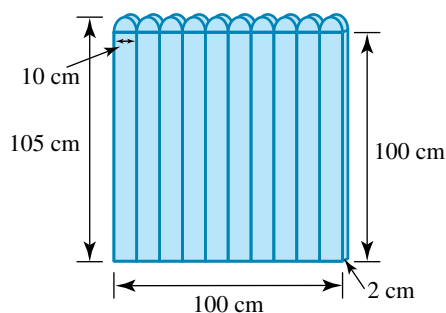


Problem solving, reasoning and justification

11. The following objects are to be used by an artist for a sculpture. Before they can be used, the artist would like them painted completely on the outside. Explain how you would calculate how much paint is needed to complete the job for each of the objects. There are no measurements so you do not need to complete the calculation.



12. a. Calculate the total surface area of spheres with radius 5 cm, 10 cm, 15 cm, 20 cm, 25 cm and 30 cm respectively, giving your answers correct to 2 decimal places. Complete a table to show your results.
- b. Describe what happens to the surface area of a sphere when the radius changes.
13. A timber fence is designed as shown. How many square metres of paint are required to completely paint the fence front and back with 2 coats of paint? Assume each paling is 2 cm in thickness. Give your answer in m^2 , correct to 2 decimal places.

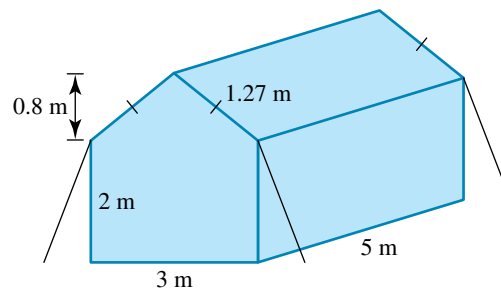


14. The outside walls of the beach hut shown at right are to be painted in a different colour.

- a. Calculate the total surface area to be painted correct to 1 decimal place. (Hint: The front and back walls are composite shapes.)
- b. If 1 litre of paint covers 20m^2 and the new paint colour will have to be applied twice to the whole area, how many whole litres of paint will need to be purchased to complete the painting job?



15. What is the total area of canvas needed for the tent (excluding the base) shown below? Give your answer correct to 1 decimal place.

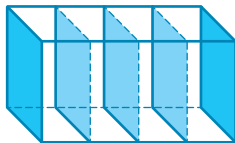


5.4 Volume 1

5.4.1 Volume of a right prism

- The **volume** of a prism can be calculated by the formula $V = A \times H$, where A is the area of the cross-section of the prism and H is the dimension that is at right angles to the cross-section, often referred to as the height.

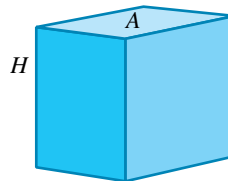
Rectangular prism



Two identical rectangular ends

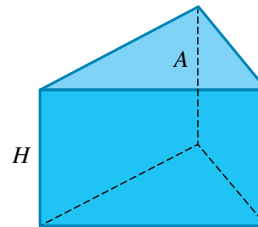
All cross-sections parallel to the base of the prism are the same shape as the base.

$$V_{\text{prism}} = A_{\text{base}} \times H$$



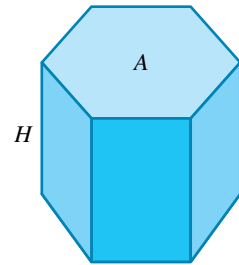
$$V = A_{\text{rectangle}} \times H$$

Rectangular prism



$$V = A_{\text{triangle}} \times H$$

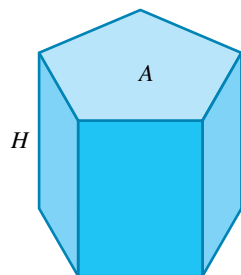
Triangular prism



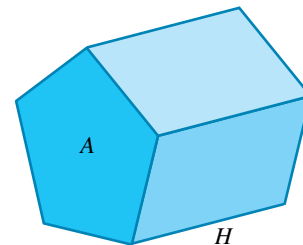
$$V = A_{\text{hexagon}} \times H$$

Hexagonal prism

- If a prism is standing on a side that is the same shape as its cross-section, then H literally represents its height; otherwise, it represents the length of the side perpendicular to the base.



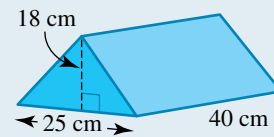
This pentagonal prism is standing on its cross-section, so H represents the height of the prism.



This is the same prism, but it is lying on its side. In this diagram, H represents the prism's length.

WORKED EXAMPLE 7

Calculate the volume of the following 3-dimensional shape.



THINK

- 1 Write the formula for the volume of a prism.
- 2 This object has a triangular cross-section, so A represents the area of the triangle. Write the formula for the area of the triangle.
- 3 Identify the values of the pronumerals. (*Note:* Here, h is the height of the triangle, not of the prism.) Substitute the values of the pronumerals into the formula and evaluate.
- 4 State the value of H . Since the prism is not standing on its triangular end, H represents the prism's length.
- 5 Substitute the values of A and H into the volume formula and evaluate. Remember to include units in your answer.

WRITE

$$V = A \times H$$

$$A_{\text{triangle}} = \frac{1}{2} \times b \times h$$

$$b = 25 \text{ cm}; h = 18 \text{ cm}$$

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2} \times 25 \times 18 \\ &= 225 \text{ cm}^2 \end{aligned}$$

$$H = 40 \text{ cm}$$

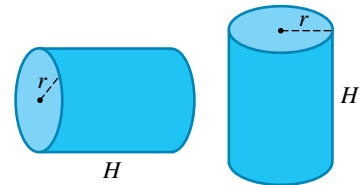
$$\begin{aligned} V &= 225 \times 40 \\ &= 9000 \text{ cm}^3 \end{aligned}$$

The volume of the prism is 9000 cm^3 .

5.4.2 Volume of a cylinder

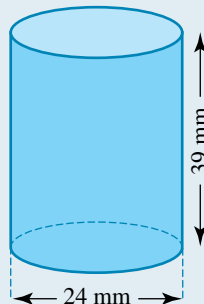
- The volume of a cylinder can be calculated using your knowledge of circles.
- First calculate the area of the circle at the base of the cylinder using the formula $A = \pi r^2$, where r is the radius. Then multiply the area by the height (or length) of the cylinder.
- The formula used to calculate the volume of a cylinder is:

$$V_{\text{cylinder}} = \pi r^2 H$$



WORKED EXAMPLE 8

Calculate the volume of the cylinder correct to 2 decimal places.



THINK

- 1 Write the formula for the volume of a cylinder.
- 2 State the values of the pronumerals.
(The value of r can be found by halving the given diameter.)
- 3 Substitute the values of the pronumerals into the formula and evaluate. Remember to include units in your answer.

WRITE

$$V_{\text{cylinder}} = \pi r^2 H$$

$$r = \frac{D}{2} = \frac{24}{2} = 12 \text{ mm}$$

$$H = 39 \text{ mm}$$

$$\begin{aligned} V &= \pi \times 12^2 \times 39 \\ &= \pi \times 5616 \\ &= 17643.18 \text{ mm}^3 \end{aligned}$$

The volume of the cylinder is 17643.18 mm^3 .

5.4.3 Capacity

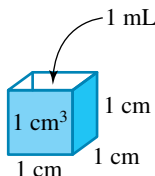
- The **capacity** of a container refers to the amount of liquid that the container can hold. It is a measure of the internal volume.
- Units used to describe capacity include the millilitre (mL), litre (L) and kilolitre (kL).

$$1000 \text{ mL} = 1 \text{ L}$$

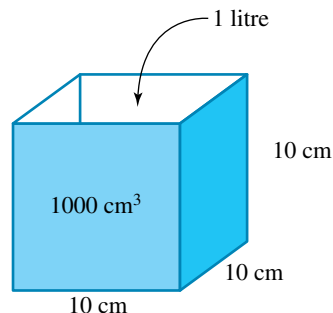
$$1000 \text{ L} = 1 \text{ kL}$$

- A container, such as a jar, has both volume and capacity, as it occupies space and can hold, contain or absorb something other than what it is made of. A solid block of wood has volume but not capacity, because it cannot hold or absorb anything.
- The metric volume unit, 1 cm^3 , was defined as having a capacity of 1 mL. Therefore, volume and capacity units are related as shown below.

$$1 \text{ cm}^3 = 1 \text{ mL}$$



$$\begin{aligned} 1000 \text{ cm}^3 &= 1000 \text{ mL} \\ &= 1 \text{ L} \end{aligned}$$



WORKED EXAMPLE 9

Complete the following unit conversions.

a $50 \text{ mL} = \underline{\hspace{1cm}} \text{ cm}^3$

b $150 \text{ mL} = \underline{\hspace{1cm}} \text{ L}$

c $0.35 \text{ L} = \underline{\hspace{1cm}} \text{ cm}^3$

THINK

a 1 cm^3 holds 1 mL.

b There are 1000 mL in 1 L. So, to convert from millilitres to litres, divide by 1000.

c **1** First, convert 0.35 L to mL. There are 1000 mL in 1 L. So, to convert from litres to millilitres, multiply by 1000.

WRITE

$$1 \text{ mL} = 1 \text{ cm}^3, \text{ so } 50 \text{ mL} = 50 \text{ cm}^3$$

$$\begin{aligned} 150 \text{ mL} &= (150 \div 1000) \text{ L} \\ &= 0.15 \text{ L} \end{aligned}$$

$$\begin{aligned} 0.35 \text{ L} &= (0.35 \times 1000) \text{ mL} \\ &= 350 \text{ mL} \end{aligned}$$

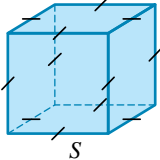
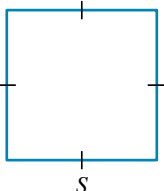
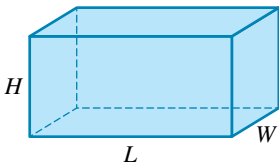
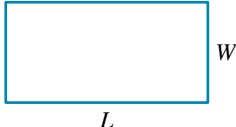
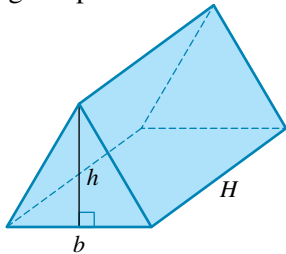
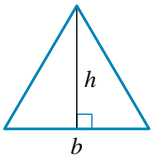
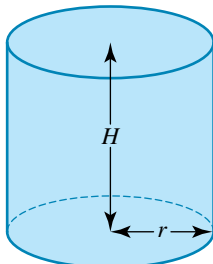
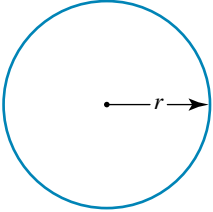
2 Use the relationship $1 \text{ cm}^3 = 1 \text{ mL}$ to complete the conversion.

$$350 \text{ mL} = 350 \text{ cm}^3$$

$$0.35 \text{ L} = 350 \text{ cm}^3$$

5.4.4 Useful formulas

The following table contains some useful formulas that can be used to find the volume and capacity of three-dimensional shapes.

Shape	Cross-sectional area	Volume
Cube 	 $A = s^2$	$V = s^3$
Rectangular prism 	 $A = LW$	$V = LWH$
Triangular prism 	 $A = \frac{1}{2}bh$	$V = \frac{1}{2}bhH$ <i>Note: The lower case h represents the height of the triangle.</i>
Cylinder 	 $A = \pi r^2$	$V = \pi r^2 H$

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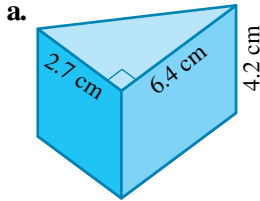
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Interactivity: Capacity (int-4024)

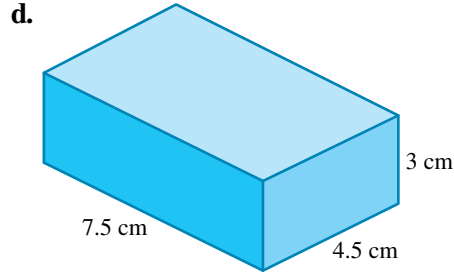
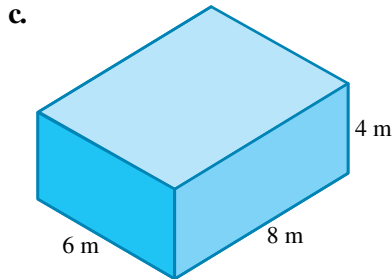
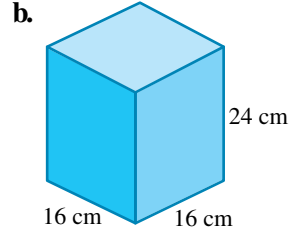
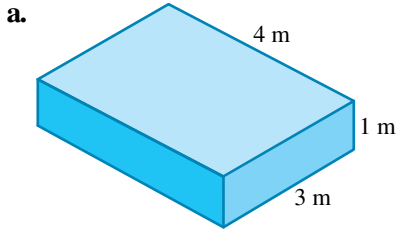
Exercise 5.4 Volume 1

Understanding, fluency and communicating

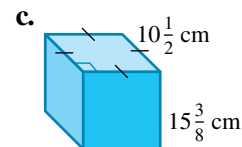
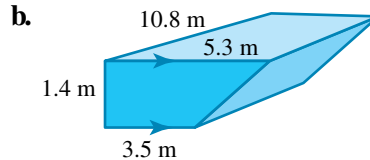
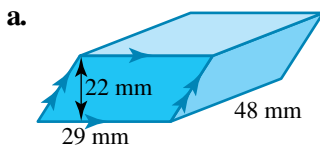
1. **WE7** Calculate the volume of the following three-dimensional shape.



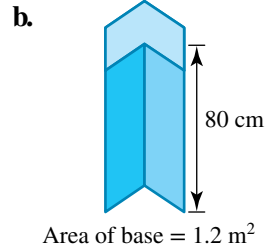
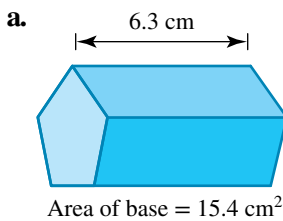
2. Calculate the volume of each of the following rectangular prisms.



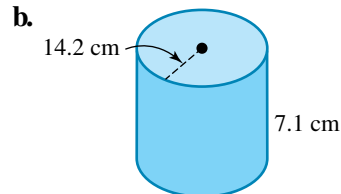
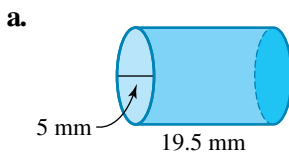
3. Calculate the volume of each of the following prisms.



4. Calculate the volume of each of the following objects.



5. **WE8** Calculate the volumes of the following shapes, correct to 2 decimal places.



6. A diving pool in the shape of a rectangular prism has a length of 16 m, a width of 12 m and a depth of 4 m. Calculate the volume of the pool.

7. **WE9** Complete the following unit conversions.

a. $3 \text{ L} = \underline{\hspace{2cm}} \text{ mL}$

b. $0.524 \text{ L} = \underline{\hspace{2cm}} \text{ mL}$

c. $425 \text{ mL} = \underline{\hspace{2cm}} \text{ L}$

d. $0.04 \text{ L} = \underline{\hspace{2cm}} \text{ mL}$

e. $2500 \text{ mL} = \underline{\hspace{2cm}} \text{ L}$

f. $1.3 \text{ L} = \underline{\hspace{2cm}} \text{ mL}$

8. Complete the following unit conversions.

a. $165 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mL} = \underline{\hspace{2cm}} \text{ L}$

b. $6350 \text{ L} = \underline{\hspace{2cm}} \text{ mL} = \underline{\hspace{2cm}} \text{ cm}^3$

c. $4.2 \text{ L} = \underline{\hspace{2cm}} \text{ mL} = \underline{\hspace{2cm}} \text{ cm}^3$

9. a. Calculate the volume of the fish tank shown below in cm^3 .

b. If 1 cm^3 is equivalent to 1 mL , what is the capacity of the tank in litres?



10. **MC** A triangular prism has a volume of 225 cm^3 . If the triangular base has an area of 25 cm^2 , the height is equal to:

A. 10 cm

B. 9 cm

C. 19 cm

D. 1 cm

E. 90 cm

11. **MC** The volume of a cylinder is 64 cm^3 . If the height is equal to 8 cm, the radius of the cylinder correct to 2 decimal places is:

A. 2.82 cm

B. 1.60 cm

C. 8 cm

D. 3.20 cm

E. 2.56 cm

12. A skateboard is packaged in a rectangular box that is 0.8 m long, 30 cm wide and 20 cm high.

a. Calculate the volume of the box in cubic centimetres.

b. If the skateboard takes up 7200 cm^3 of the space in the box, how much of the space in the box is not occupied by the skateboard?

Problem solving, reasoning and justification

13. Standard measurements for cooking are shown at right.

In cooking quantities:

a. how many teaspoons are there in a tablespoon

b. how many tablespoons in a cup

c. how could you measure half a cup if you had only teaspoons and tablespoons

d. how could you measure a quarter of a cup if you had only teaspoons and tablespoons?

1 cup = 250 mL
1 tablespoon = 20 mL
1 teaspoon = 5 mL

14. Consider a set of three food containers of different sizes each of which is in the shape of a rectangular prism.

The smallest container is 6 cm long, 4 cm wide and 3 cm high. All the dimensions of the medium container are double those of the smallest one; all the dimensions of the largest container are triple those of the smallest one.

a. Calculate the volume of the smallest container.

- b. State the dimensions and hence calculate the volume of:
- the medium container
 - the largest container.

c. Calculate the ratio of the volumes of:

- the smallest container to the medium container
- the smallest container to the largest container.

d. Consider your answers to part c and use them to copy and complete the following:

If all dimensions of a rectangular prism are increased by a factor of n , the volume of the prism is increased by a factor of _____.



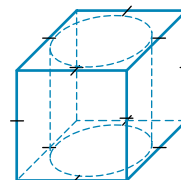
15. a. Design, draw and label a prism that has a volume of 16 m^3 .

b. Draw and label a second prism that also has a volume of 16 m^3 .

16. A cylinder fits exactly into a cube of side 8 cm.

a. Determine the volume of the cylinder, correct to 2 decimal places.

b. Determine the percentage of space inside the cube that is not occupied by the cylinder, correct to 2 decimal places.



17. A block of cheese is in the shape of a triangular prism. The area of the triangle is 2.3 cm^2 and the length of the prism is 9.7 cm. If the price of cheese is \$0.41 per cm^3 , calculate the price of the block.

18. A water tank is in the shape of a cylinder whose diameter is twice its height. If the tank holds 1000 litres, what are the dimensions of the tank, correct to 1 decimal place?

5.5 Volume 2

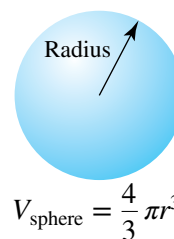
5.5.1 Volume of a sphere

- The volume of a sphere can be calculated by using the formula:

$$V_{\text{sphere}} = \frac{4}{3} \times \pi \times r^3$$

where r = radius

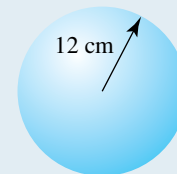
- The volume of a sphere is measured using cubic units appropriate to the task (mm^3 , cm^3 , m^3 or km^3).



$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

WORKED EXAMPLE 10

Calculate the volume of the sphere at right correct to 2 decimal places.



THINK

- Write out the formula for calculating the volume of a sphere.
- Substitute the value of the radius into the equation and calculate.
- Include the appropriate units in your answer.

WRITE

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$$V_{\text{sphere}} = \frac{4}{3} \times \pi \times 12^3$$

$$\approx 7238.23$$

$$V_{\text{sphere}} = 7238.23 \text{ cm}^3$$

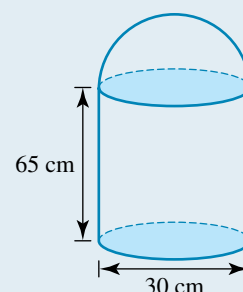
5.5.2 Volume of composite shapes

- The total volume of a **composite solid** can be calculated by finding the volume of each individual solid that it contains and adding them together to find the total volume.
- Some examples include:



WORKED EXAMPLE 11

Calculate the volume of the following object, correct to 2 decimal places.



THINK

- The composite object is made up of a cylinder and half a sphere.
- Write out the formula for calculating the volume of half a sphere and substitute the value of the radius into the formula. Calculate.
- Write out the formula for calculating the volume of a cylinder and substitute the values of the height and radius into the equation. Calculate.
- Add the volumes of the hemisphere and the cylinder.
- Include appropriate units in your answer.

WRITE

$$\begin{aligned} V_{\text{half a sphere}} &= \frac{1}{2} \times \frac{4}{3} \times \pi \times r^3 \\ &= \frac{1}{2} \times \frac{4}{3} \times \pi \times 15^3 \\ &= 2250\pi \end{aligned}$$

$$\begin{aligned} V_{\text{cylinder}} &= \pi \times r^2 \times h \\ &= \pi \times 15^2 \times 65 \\ &= 14625\pi \end{aligned}$$

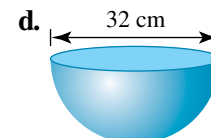
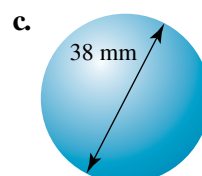
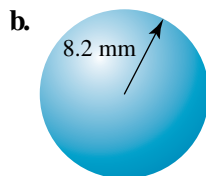
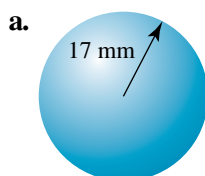
$$\begin{aligned} \text{Total volume} &= 2250\pi + 14625\pi \\ &= 16875\pi \\ &\approx 53\,014.38 \end{aligned}$$

$$V_{\text{total}} \approx 53\,014.38 \text{ cm}^3$$

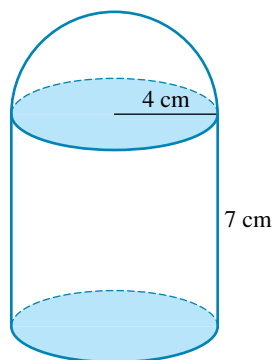
Exercise 5.5 Volume 2

Understanding, fluency and communicating

- WE10** Calculate the volumes of the following shapes. Give your answer correct to the nearest whole number.

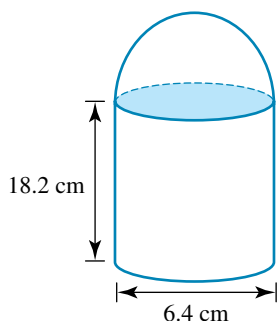


2. A sphere has a volume of 523.81 cm^3 . What is the radius of the sphere, correct to the nearest whole number?
3. The radius of the Earth is 6380 km. Assuming the Earth is a sphere, calculate its volume correct to 2 significant figures.
4. **MC** A hemispherical bowl has a volume of 2145.52 cm^3 . The diameter of the bowl is closest to:
A. 20 cm **B.** 8.0 cm **C.** 16 cm **D.** 512 cm **E.** 10 cm
5. **WE11** Calculate the volume of the solid below, correct to the nearest cm^3 .

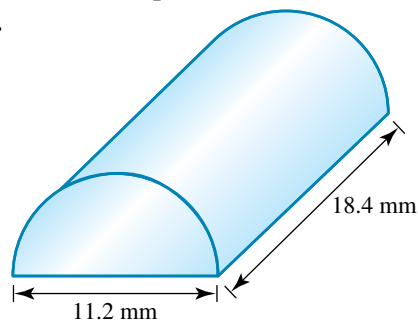


6. Calculate the volumes of the following objects, correct to 2 decimal places.

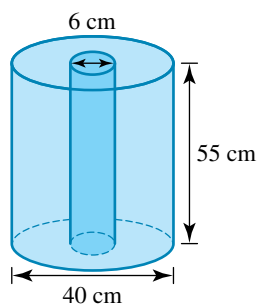
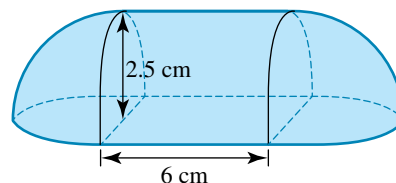
a.



b.



7. Calculate the capacity of the object at right in cm^3 . State the result to 4 significant digits.
8. Calculate the volume of water that can completely fill the inside of this washing machine. Give your answer in litres, correct to 2 decimal places.

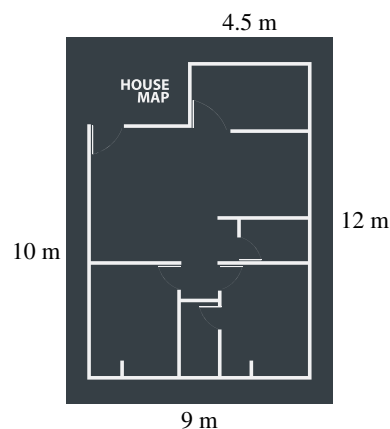


9. This cylindrical steel water tank has a diameter of 2 m and a length of 3 m. Ignoring the thickness of the material, calculate:
 - a. the volume of the tank, correct to 3 decimal places.
 - b. the capacity of the tank, to the nearest litre.
 $(1 \text{ m}^3 = 1000 \text{ L})$



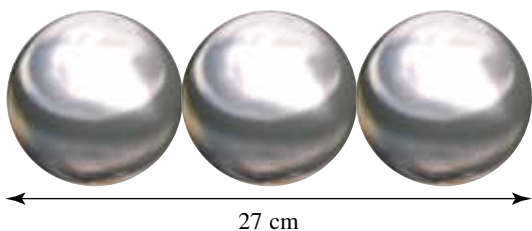
10. A house plan is drawn at right. The house is going to be built on a concrete slab.

- Calculate the area of the slab.
- The slab is to be 15 cm thick. Calculate the volume of concrete needed for the slab. (*Hint:* Write 15 cm as 0.15 m.)
- Concrete costs \$180/m³ to pour. Calculate the cost of this slab.



Problem solving, reasoning and justification

- Give some real-life examples of when it would be important to know how to calculate the volume of a sphere.
- A spherical container with a radius of 15 cm³ is filled with water. A tap at the bottom of the container allows the water to be emptied at a rate of 80 cm³/s. How long would it take to completely empty the container? Give your answer in seconds correct to 1 decimal place.
- The sculpture shown below is to be packaged into a rectangular or cylindrical cardboard box.



- Assuming each sphere touches the one next to it, calculate correct to 2 decimal places (where necessary) the:
 - volume of the smallest rectangular box that completely contains the spheres
 - volume of the smallest cylindrical box that completely contains the spheres
 - volume of space in your boxes not occupied by the spheres.
 - Use your calculations to justify which box you would choose.
14. A sphere is enclosed inside a cube of side length equal to the diameter of the sphere. What percentage of the volume of the cube is taken up by the sphere? Give your answer correct to 1 decimal place.
15. a. Calculate the surface area and volume of:
 - a cube of side length 1
 - a sphere of radius 1 (correct to 2 decimal places).
- Which of the objects described above has the larger volume?
 - Which object contains a larger volume per unit of surface area?

5.6 Review

5.6.1 Summary

In this topic you have learnt:

- how to calculate the total surface area (TSA) of a three-dimensional object
- how to identify a prism
- how to name different prisms
- which objects with a uniform cross-section cannot be classified as prisms
- how to visualise the outside faces that make up a prism using a net
- the formula for calculating the TSA of a rectangular prism
- the formula for calculating the TSA of a cylinder
- the formula for calculating the TSA of a sphere
- how to find the TSA of composite objects
- how to calculate the volume of a prism

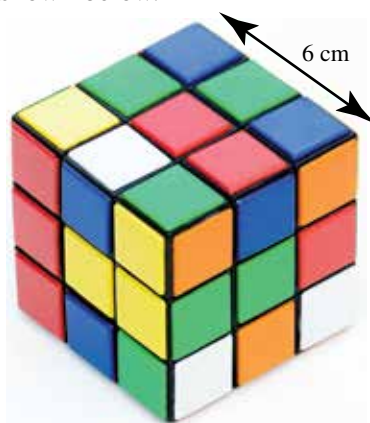
- how to calculate the volume of a cylinder
- units of capacity
- how to convert between units of capacity
- how to convert between units of volume and units of capacity
- how to calculate the volume of a sphere
- how to calculate the volume of a composite solid.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 5.6 Review

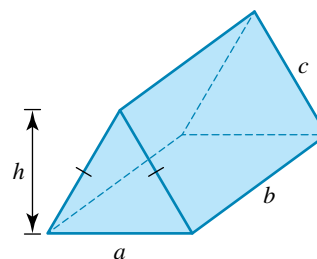
Understanding, fluency and communicating

1. Calculate the total surface area of a cube with side lengths of $5\frac{3}{4}$ mm.
2. Find the surface area of the cube shown below.



3. Find the surface area of a rectangular prism with the length of 10 cm, width of 5 cm and height of 7 cm.
4. **MC** The formula for the total surface area for the object shown is:

- A.** $\frac{1}{2}abh$
- B.** $\frac{1}{2}abc$
- C.** $ah + 2bc + ab$
- D.** $\frac{1}{2}ah + 2bc + ab$
- E.** $ah + ab + ac$



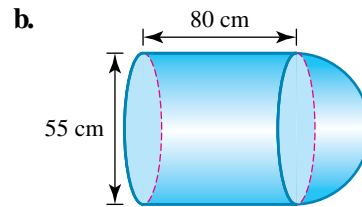
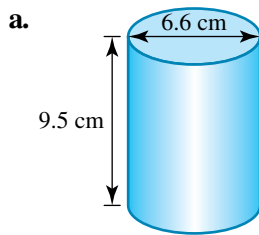
5. Convert the following units as indicated.

a. $300\text{ cm}^3 = \underline{\hspace{1cm}} \text{ mL}$	b. $750\text{ cm}^3 = \underline{\hspace{1cm}} \text{ L}$
c. $3300\text{ cm}^3 = \underline{\hspace{1cm}} \text{ L}$	d. $25\,200\text{ cm}^3 = \underline{\hspace{1cm}} \text{ mL}$
e. $5\,000\,000\text{ cm}^3 = \underline{\hspace{1cm}} \text{ mL} = \underline{\hspace{1cm}} \text{ L}$	
6. Convert the following units as indicated.

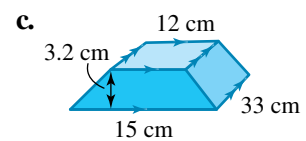
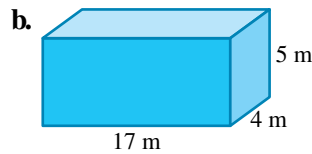
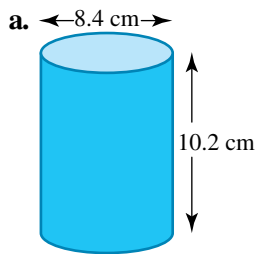
a. $66\text{ mL} = \underline{\hspace{1cm}} \text{ cm}^3$	b. $2.5\text{ mL} = \underline{\hspace{1cm}} \text{ cm}^3$
c. $3.2\text{ L} = \underline{\hspace{1cm}} \text{ cm}^3$	d. $15\text{ L} = \underline{\hspace{1cm}} \text{ mL} = \underline{\hspace{1cm}} \text{ cm}^3$
7. For a rectangular prism with dimensions of 12 cm by 8 cm by 6 cm, calculate:
 - a. the total surface area of the prism
 - b. the volume of the prism
 - c. the capacity of the prism in mL.
8. **MC** A sphere has a total surface area of 85 cm^2 . The radius of the sphere correct to 2 decimal places is:

A. 21.25 cm	B. 5.2 cm	C. 45.7 cm	D. 6.76 cm	E. 2.60 cm

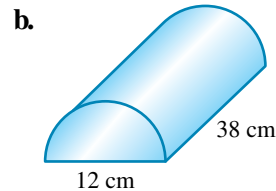
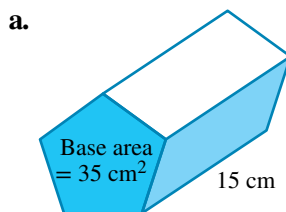
9. Calculate the total surface area of a cylinder with a radius of 6.32 cm and a height of 7.98 cm. Give your answer correct to 2 decimal places.
10. Calculate the total surface areas of the following objects. Give your answer correct to 1 decimal place.



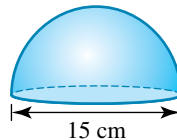
11. Calculate the volume of each of the following objects. Give your answers correct to 2 decimal places where necessary.



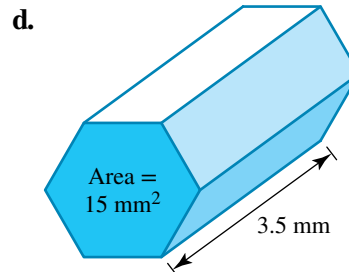
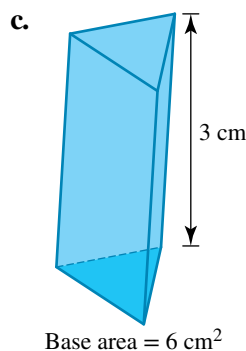
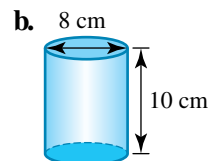
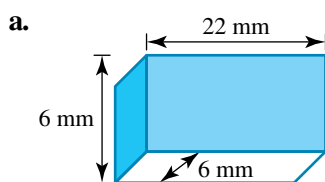
12. Calculate the volumes of the following objects. Give your answers correct to 2 decimal places where necessary.



13. Calculate the volume of the following object correct to 1 decimal place.

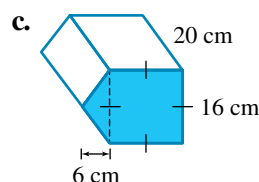
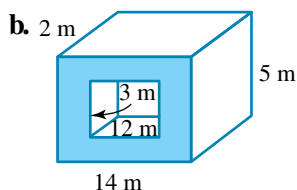
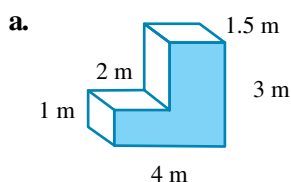


14. Calculate the volume of each of the following objects. Give your answers correct to 2 decimal places where necessary.

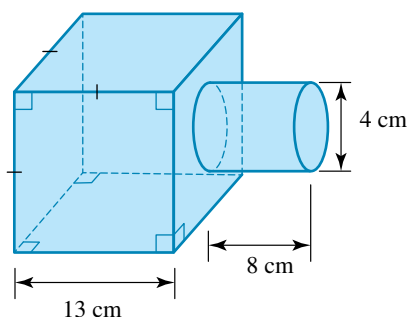


15. In each of the following prisms, the front face is made up of a composite figure. Find:

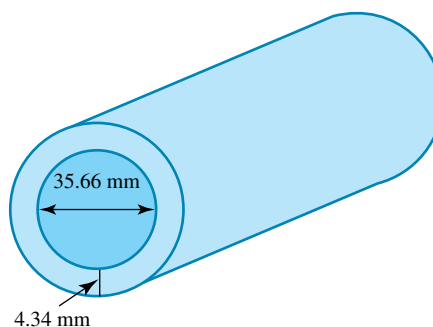
- i. the area of the front face
- ii. the volume of the prism.



16. Find the capacity of the object shown (to the nearest litre).

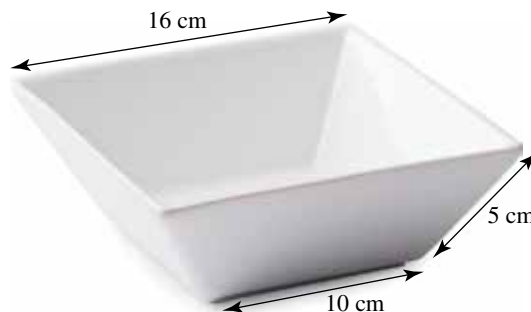


17. A car's engine has 4 cylinders, each having a diameter of 8.5 cm and a height of 11.2 cm. Calculate the capacity of the engine in cubic centimetres. Give your answer correct to 1 decimal place.
18. A fish tank has the dimensions $90\text{ cm} \times 40\text{ cm} \times 60\text{ cm}$. How much water would be needed to fill three-quarters of the tank?
19. A new (unsharpened) pencil is in the shape of a hexagonal prism. If the area of the hexagonal end is 80 mm^2 and the volume of the whole pencil is 14.4 cm^3 , calculate the length of the pencil.
20. A copper pipe has an internal diameter of 35.66 mm and a thickness of copper of 4.34 mm. What volume of copper is used per metre in making this pipe? Give your answer in cm^3 , correct to the nearest whole number.

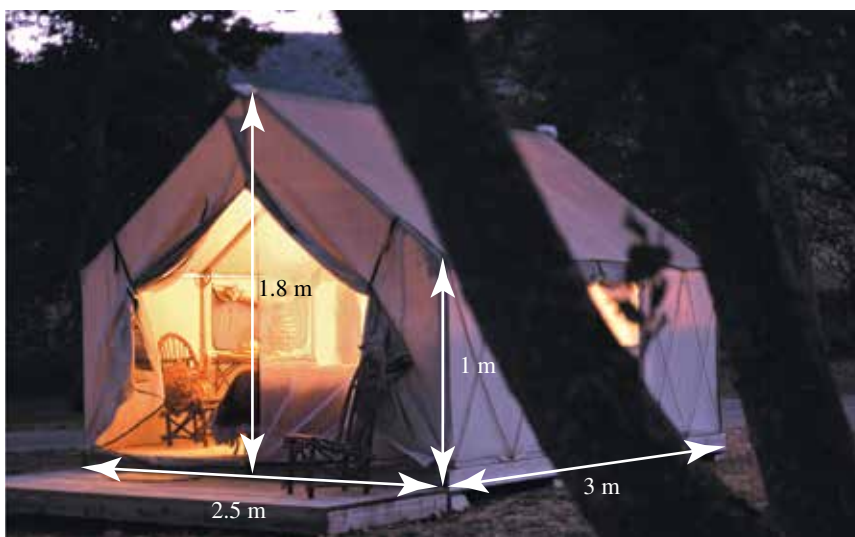
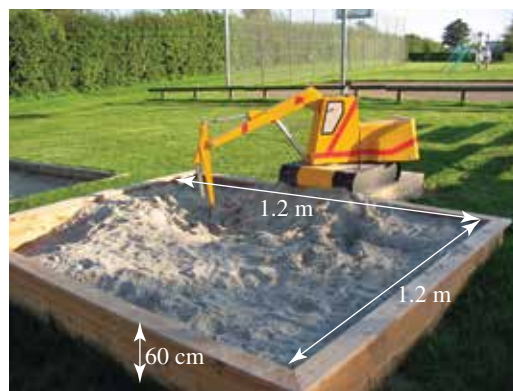


Problem solving, reasoning and justification

21. A swimming pool has a surface length of 50 m and a width of 28 m. The shallow end of the pool has a depth of 0.8 m, which increases steadily to 3.8 m at the deep end.
 - a. Before filling the pool needs painting. Calculate the area that will be painted.
 - b. If the pool is to be filled to the top, how much water will be needed?
22. Heather is planning on painting her bedroom. The floor is $3.5\text{ m} \times 4\text{ m}$ and the walls are 2.5 m high. One litre of paint can cover 11 m^2 . She is planning on using 2 coats of paint. She can only buy paint by the litre.
 - a. How many litres of ceiling paint should she buy?
 - b. How many whole litres of paint will she need to paint the walls? (Ignore doorways and windows.)
23. A baker is considering changing the shape of his loaves of bread. The smaller the surface area, the longer the loaf takes to go stale. He is considering two options, both 20 cm long. One loaf has a square end with a side of 7 cm; the other has a circular end with a diameter of 7 cm. Which loaf should he make to stay fresh for longer?
24. Levi has a new design for a square-based bowl with dimensions as shown. If the surface of the outside of the bowl (including the base) is greater than 300 cm^2 , the cost of materials for the bowl will be too expensive to make. Can Levi afford to make the bowl?



25. The radius of the Earth is 6380 km.
- Calculate the TSA of the Earth in km^2 , correct to the nearest whole number. (Assume the Earth is a sphere.)
 - If the area of Australia is $7\,692\,024\text{ km}^2$, what percentage of the Earth's surface does this represent? Give your answer correct to 1 decimal place.
 - Calculate the volume of the Earth in km^3 . Give your answer correct to 3 significant figures.
26. The radius of Jupiter is 71 500 km.
- Calculate the TSA and volume of Jupiter. Give your answers correct to 3 significant figures. (Assume Jupiter is a sphere.)
 - How many times larger is the TSA and volume of Jupiter than those of Earth? Give your answers correct to the nearest whole number.
27. A water tank designer has been asked to make a water tank that holds 2000 litres ($2\,000\,000\text{ cm}^3$) of water. Design three possible shapes that would hold 2000 L. Each water tank must be a different shape.
28. A sandpit has the dimensions shown at right.
- How much sand is needed to completely fill it?
 - A cover is to be made for the sandpit. If the cover is to overlap by 50cm on each side, what size would it need to be?
29. A cylindrical can holds 4 tennis balls stacked one on top of another, packed tightly. If the diameter of a tennis ball is 8.8cm, determine the volume of the can correct to 1 decimal place.
30. A large tourist company owns some camping grounds, equipped with permanent tents. The dimensions of each tent are shown.



- Calculate the area of material required to make each tent, including the floor.
 - Calculate the amount of space inside the tent.
31. A rectangular roof is 15m long and 12m wide. When it rains, water from the roof is collected in a cylindrical tank. One night, 20 mm of rain falls.
- Calculate the volume of water that falls on the roof.
 - How many litres of water fall on the roof?
 - If the tank is 2.27 m high and 7.06 m in diameter, calculate the volume of water in litres that the tank could hold. Give your answer to the nearest whole number.
 - After the rainfall, by how many centimetres does the depth of water in the tank increase? Give your answer correct to 1 decimal place.

Answers

Topic 5 Answers

Exercise 5.2 Surface area 1

1. a and d

2. a. $14\,832\text{ cm}^2$

b. 4.16 m^2

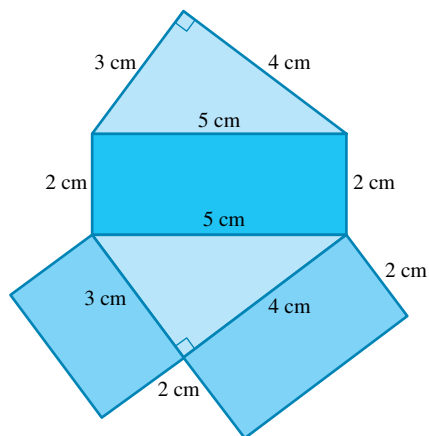
c. 1177 mm^2

3. a. 2646 mm^2

b. 1.0584 m^2

c. $84\frac{3}{8}\text{ cm}^2$

4. a.



b. 36 cm^2

5. a. i. 30 cm^2

ii. $A_{R1} = 104\text{ cm}^2$, $A_{R2} = 40\text{ cm}^2$, $A_{R3} = 96\text{ cm}^2$

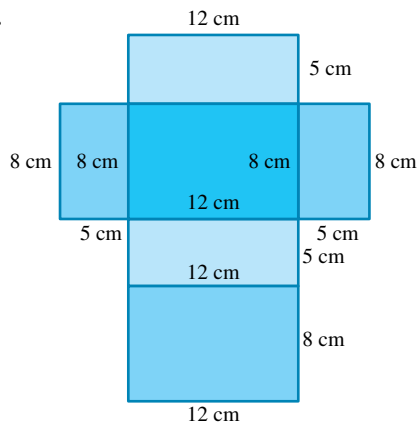
iii. $\text{TSA} = 300\text{ cm}^2$

b. i. 26.82 cm^2

ii. A_{R1} and $A_{R2} = 147.6\text{ cm}^2$, $A_{R3} = 166.05\text{ cm}^2$

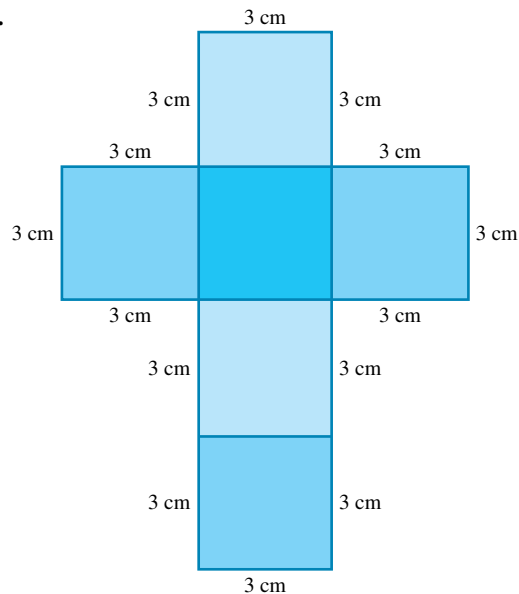
iii. $\text{TSA} = 514.89\text{ cm}^2$

6. a. i.



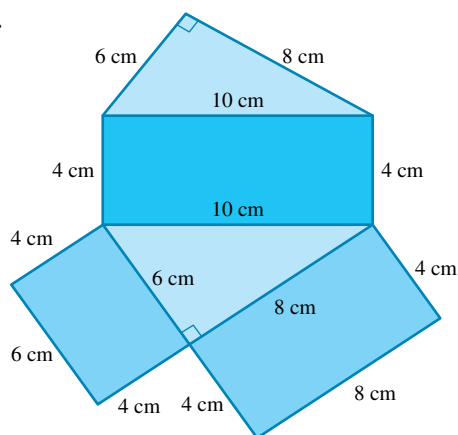
ii. 392 m^2

b. i.



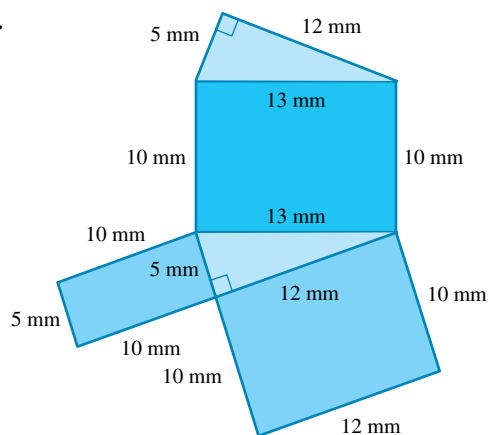
ii. 54 cm^2

c. i.



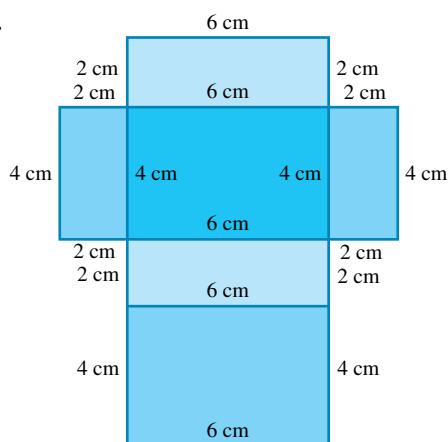
ii. 144 cm^2

d. i.



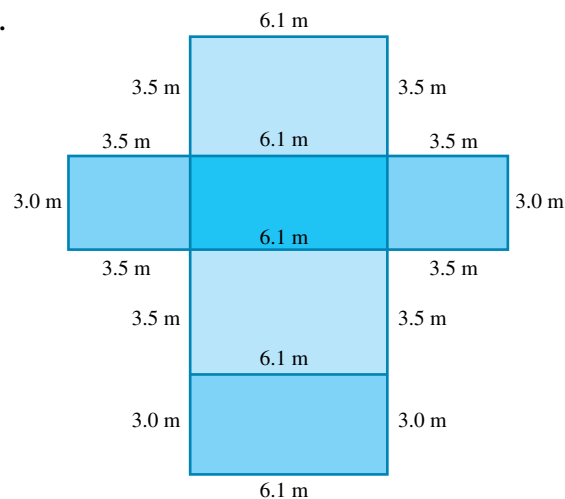
ii. 360 mm^2

e. i.



ii. 88 cm^2

f. i.



ii. 100.3 m^2

7. 7664 cm^2

8. a. $23\,014 \text{ cm}^2$

b. 1523 mm^2

9. a. $10\,537 \text{ mm}^2$

b. $25\,397 \text{ mm}^2$

10. a. 192 mm^2

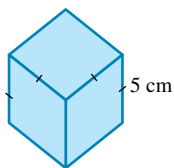
b. 848.23 cm^2

c. 73.5 cm^2

d. 14.78 m^2

e. $18\,064.16 \text{ cm}^2$

11. a.



b. 6.3 cm

12. 8.1 m

13. a. 864 cm^2

b. 636 cm^2

c. No

14. a. Doubling the side length causes the surface area to increase by a factor of 4.

b. i. Tripling the side length causes the surface area to increase by a factor of 9.

ii. Quadrupling the side length causes the surface area to increase by a factor of 16.

c. If you increase the side length by a factor of r , the surface area increases by a factor of r^2 .

15. a. 2284 cm^2

b. 1111 cm^2

Exercise 5.3 Surface area 2

1. a. $20\,106 \text{ cm}^2$

b. $11\,310 \text{ cm}^2$

2. a. 30 mm^2

b. 1963 cm^2

c. 255 cm^2

3. a. 28.27 cm^2 b. 14.14 cm^2 c. 158 mm^2
4. A
5. 326.7 cm^2
6. a. 10.65 m^2 b. $11\,106 \text{ cm}^2$ c. $57\,805 \text{ cm}^2$
7. 136 cm^2
8. $11\,231 \text{ cm}^2$
9. 20.30 m^2
10. $21\,430.27 \text{ cm}^2$
11. a. The object is a cube. Measure the length of one side (l). $\text{TSA} = 6l^2$
- b. The object is a triangle-based pyramid. Measure one side (a) and the perpendicular height of the triangle (h). Measure the other 2 sides of the triangle (s_1, s_2) and the height (H) of the prism. $\text{TSA} = ah + (a + s_1 + s_2)H$
- c. The object is a cylinder. Measure the height of the cylinder (H). To find the radius of the cylinder (r), measure the diameter and halve it. $\text{TSA} = 2\pi r(r + H)$
- d. The object is a sphere. To find the radius (r) of the sphere, measure the diameter and halve it. $\text{TSA} = 4\pi r^2$

12. a.

$r \text{ (cm)}$	$\text{TSA} = 4\pi r^2$	$\text{TSA (cm}^2\text{)}$
5	$\text{TSA} = 4\pi \times 5^2$ $= 100\pi$	314.16
10	$\text{TSA} = 4\pi \times 10^2$ $= 400\pi$	1256.64
15	$\text{TSA} = 4\pi \times 15^2$ $= 900\pi$	2827.43
20	$\text{TSA} = 4\pi \times 20^2$ $= 1600\pi$	5026.55
25	$\text{TSA} = 4\pi \times 25^2$ $= 2500\pi$	7853.98
30	$\text{TSA} = 4\pi \times 30^2$ $= 3600\pi$	11309.73

- b. When the radius increases by a factor of a , the surface area increases by a factor of a^2 .
13. 4.30 m^2
14. a. 17.4 m^2 b. 2 L
15. 47.1 m^2

Exercise 5.4 Volume 1

1. 36.288 cm^3
2. a. 12 m^3 b. 6144 cm^3 c. 192 m^3 d. 101.25 cm^3
3. a. $30\,624 \text{ mm}^3$ b. 66.528 m^3 c. $1695\frac{3}{32} \text{ cm}^3$
4. a. 97.02 cm^3 b. 0.96 m^3 (or $960\,000 \text{ cm}^3$)
5. a. 382.88 mm^3 b. 4497.64 cm^3
6. 768 m^3
7. a. 3000 mL b. 524 mL c. 0.425 mL
d. 40 mL e. 2.5 L f. 1300 mL
8. a. $165 \text{ mL} = 0.165 \text{ L}$
b. $6\,350\,000 \text{ mL} = 6\,350\,000 \text{ cm}^3$
c. $4200 \text{ mL} = 4200 \text{ cm}^3$
9. a. $112\,500 \text{ cm}^3$ b. 112.5 L
10. B
11. B
12. a. $48\,000 \text{ cm}^3$ b. $40\,800 \text{ cm}^3$
13. a. 4 teaspoons

- b. 12.5 tablespoons
 - c. 6 tablespoons and 1 teaspoon
 - d. 3 tablespoons and $\frac{1}{2}$ teaspoon
14. a. 72 cm^3
- b. i. 576 cm^3 ii. 1944 cm^3
 - c. i. 1 : 8 ii. 1 : 27
 - d. n^3
15. Multiple solutions are possible.
16. a. 402.12 cm^3 b. 21.46%
17. \$9.15
18. The tank has a height of 68.3 cm and a diameter of 136.6 cm.

Exercise 5.5 Volume 2

- 1. a. $20\,580 \text{ mm}^3$ b. 2310 mm^3 c. $28\,731 \text{ mm}^3$ d. 8579 cm^3
- 2. 5 cm
- 3. $1.1 \times 10^{12} \text{ km}^3$
- 4. A
- 5. 486 cm^3
- 6. a. 654.12 cm^3 b. 906.39 mm^3
- 7. 91.63 cm^3
- 8. 67.56 L
- 9. a. 9.425 m^3 b. 9425 L
- 10. a. 99 m^2 b. 14.85 m^3 c. \$2673
- 11. Answers will vary.
- 12. 176.7 s
- 13. a. i. 2187 cm^3
 ii. 1717.67 cm^3
 iii. Empty space in the rectangular prism: 1041.89 cm^3
 Empty space in the cylinder: 572.56 cm^3
- b. Cylinder
- 14. 52.4%
- 15. a. i. $\text{TSA} = 6, V = 1$
 ii. $\text{TSA} = 12.57, V = 4.19$
- b. Sphere
- c. Sphere

Exercise 5.6 Review

- 1. $198\frac{3}{8} \text{ mm}^2$
- 2. 216 cm^2
- 3. 310 cm^2
- 4. C
- 5. a. 300 mL b. 0.75 L c. 3.3 L
 d. 25 200 mL e. 5 000 000 mL = 5000 L
- 6. a. 66 cm^3 b. 2.5 cm^3 c. 3200 cm^3 d. 15 000 mL = 15 000 cm^3
- 7. a. 432 cm^2 b. 576 cm^3 c. 576 mL
- 8. E
- 9. 567.85 cm^2
- 10. a. 265.4 cm^2 b. $20\,950.5 \text{ cm}^2$
- 11. a. 565.26 cm^3 b. 340 m^3 c. 1425.6 cm^3

12. a. 525 cm^3 b. 2148.85 cm^3
13. 883.6 cm^3
14. a. 792 mm^3 b. 502.65 cm^3 c. 18 cm^3 d. 52.5 mm^3
15. a. i. 8 m^2 ii. 12 m^3
 b. i. 34 m^2 ii. 68 m^3
 c. i. 304 cm^2 ii. 6080 cm^3
16. 2 L
17. 2542.2 cm^3
18. 162 L
19. 18 cm
20. 545 cm^3
21. a. 1761 m^2 b. 3220 m^3
22. a. 3 L b. 7 L
23. The circular loaf
24. No, the surface area is 308 cm^2 .
25. a. $511\,506\,576 \text{ km}^2$ b. 1.5% c. $1.09 \times 10^{12} \text{ km}^3$
26. a. The TSA of Jupiter is $6.42 \times 10^{10} \text{ km}^2$. The volume of Jupiter is $1.53 \times 10^{15} \text{ km}^3$.
 b. The TSA of Jupiter is 126 times larger than Earth. The volume of Jupiter is 1406 times larger than Earth.
27. Answers will vary.
28. a. 0.864 m^3 b. 4.84 m^2
29. 2140.9 cm^3
30. a. 29.4 m^2 b. 10.5 m^3
31. a. 3.6 m^3 b. 3600 L c. $88\,864 \text{ L}$ d. 9.2 cm

TOPIC 6

Units of energy and mass

6.1 Overview

6.1.1 Introduction

The energy we intake from food and drink gives us the ability to undertake physical and mental tasks. By understanding energy intake, we can ensure that we're properly fuelled for the day ahead, whilst not having an excess of energy which can turn to fat.

Electrical appliances also require energy and have changed the way we use energy in our homes. Purchasing energy-efficient appliances and using them efficiently can save energy and lower energy costs.



DISCUSSION

How many electrical appliances do you use each week? Come up with a list and compare with your classmates.

LEARNING SEQUENCE

- 6.1 Overview
- 6.2 Units of mass and energy
- 6.3 Food and nutrition
- 6.4 Energy expended in activities
- 6.5 Energy consumption
- 6.6 Review

CURRICULUM CONTENT

- review the use of metric units of mass in solving problems, including grams, kilograms and tonnes, their abbreviations and how to convert between them (ACMEM025) 
- use metric units of energy to solve problems, including calories, kilocalories, joules and kilojoules, their abbreviations and how to convert between them (ACMEM034)
- use units of energy and mass to solve problems related to food and nutrition, including calories (ACMEM032) 
- use units of energy to solve problems involving the amount of energy expended in activities, for example, kilojoules (ACMEM033) 
- use units of energy to solve problems involving the consumption of electricity, for example, kilowatt hours, and investigate common appliances in terms of their energy consumption (ACMEM031) **AAM**  

6.2 Units of mass and energy

6.2.1 Common prefixes of the metric system

- The International System of Units (SI) prefixes are symbols used to denote powers of ten of very large or very small numeric values.
- The prefix attaches directly to the name of a unit, and a prefix symbol attaches directly to the symbol for a unit.

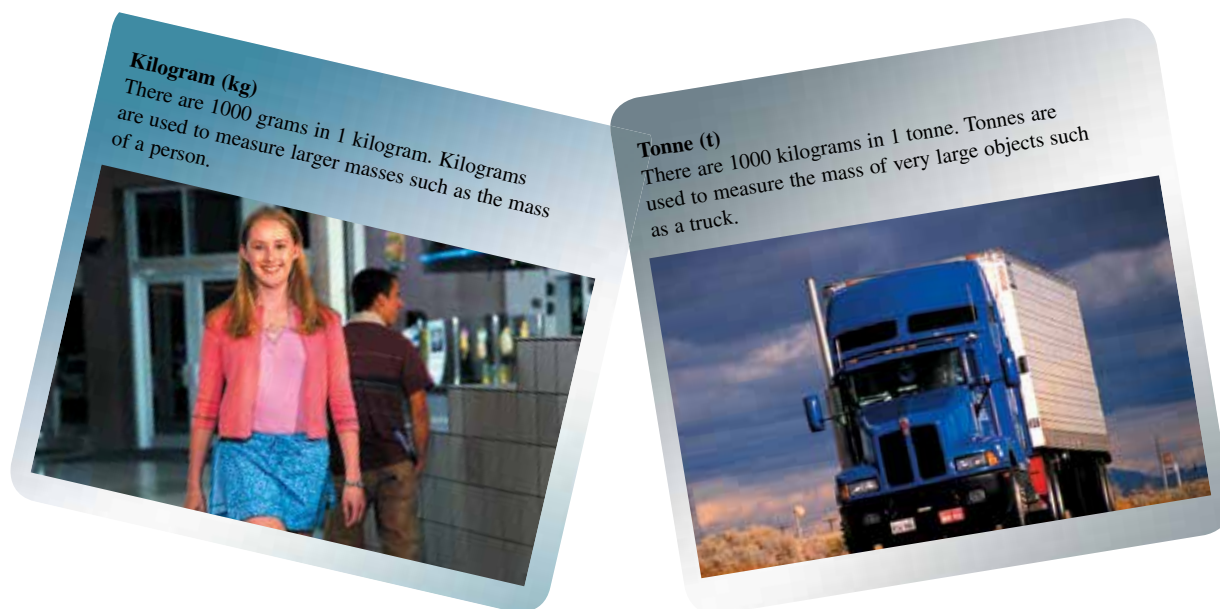
Some common SI prefixes are listed in the table below.

Factor	Name	Symbol
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^1	deca	da
10^3	kilo	k
10^6	mega	M
10^9	giga	G

6.2.2 Metric units of mass

- **Mass** and **weight** are different.
- Mass describes how much matter makes up an object, and is measured in kilograms (kg).
- Weight describes the gravitational force acting on an object, and is measured in newtons (N).
- When people talk about the weight of an object, what they are really referring to is its mass.
- The mass of an object can be measured in milligrams (mg), grams (g), kilograms (kg) and tonnes (t).



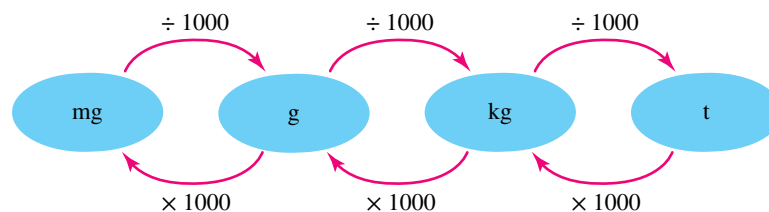


6.2.3 Converting between units of mass

- The relationships between the units of mass can be used to change a measurement from one unit to another.

$$\begin{aligned} 1000 \text{ mg} &= 1 \text{ g} \\ 1000 \text{ g} &= 1 \text{ kg} \\ 1000 \text{ kg} &= 1 \text{ t} \end{aligned}$$

You may find the following conversion diagram useful when converting masses.



WORKED EXAMPLE 1

Change the following mass measurements to the units specified.

- a** 4.7 kg to grams
b 15 000 mg to kilograms

THINK

- a** There are 1000 g in every 1 kg. 4.7 kg is 4.7 lots of 1000 g; to change from kilograms to grams, multiply by 1000.
- b 1** To change from milligrams to kilograms, first change from milligrams to grams, and then change from grams to kilograms.
- 2** There are 1000 mg in every 1 g; to change from milligrams to grams, group the milligrams into groups of 1000.
- 3** There are 1000 g in every 1 kg; to change from grams to kilograms, group the milligrams into groups of 1000. Write the answer as a decimal.

WRITE

$$\begin{aligned} 4.7 \text{ kg} &= (4.7 \times 1000) \text{ g} \\ &= 4700 \text{ g} \end{aligned}$$

$$\begin{aligned} 15\,000 \text{ mg} &= \frac{15\,000}{1000} \text{ g} \\ &= 15 \text{ g} \\ &= \frac{15}{1000} \text{ kg} \\ &= 0.015 \text{ kg} \end{aligned}$$

6.2.4 Metric units of energy

- Energy is the ability to do work. There are many different forms of energy: food energy, kinetic energy, heat energy, light energy, sound energy and electrical energy.
- **Joules** (J) and **kilojoules** (kJ) are the common measurement of energy. A joule is the standard metric unit of energy used in Australia, although as the joule is a small unit, the kilojoule is more commonly used. Most Australian websites and books use kilojoules.
- Food energy is commonly measured in kilojoules (kJ). Larger amounts of energy such as electrical energy are measured in megajoules (MJ).

$$1 \text{ kJ} = 1000 \text{ J}$$

$$1 \text{ MJ} = 1\,000\,000 \text{ J}$$

- **Calories** are also units of energy. Calories are the older (imperial) style, non-SI unit of measurement of energy, whereas kilojoules are the modern style of measurement used more commonly today.
- The calorie (cal) is the approximate amount of energy needed to raise the temperature of 1 kilogram of water from 0 to 1 degree Celsius.
- The **kilocalorie** (kcal) is equal to 1000 calories.

$$1 \text{ kcal} = 1000 \text{ cal}$$



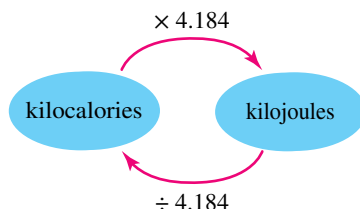
6.2.5 Converting between units of energy

- The relationships between the units of energy can be used to change a measurement from one unit to another.

$$4.184 \text{ kilojoules} = 4184 \text{ joules} = 1 \text{ kilocalorie} = 1000 \text{ calories}$$

- To convert from kilojoules to kilocalories, divide the number of kilojoules by 4.184.
- To convert from kilocalories to kilojoules, multiply the number of kilocalories by 4.184.

You may find the conversion diagram useful when converting from calories to kilojoules.



WORKED EXAMPLE 2

Change the following energy measurements to the units specified. Give your answers correct to 2 decimal places where necessary.

- a 500 kilocalories to kilojoules
b 10 850 joules to kilocalories

THINK

- a To convert from kilocalories to kilojoules, multiply the number of kilocalories by 4.184.
- b 1 To convert from joules to kilojoules, divide by 1000, since 1000 joules = 1 kilojoule.

WRITE

$$500 \text{ kcal} = (500 \times 4.184) \text{ kJ} \\ = 2092 \text{ kJ}$$

$$10\,850 \text{ J} = \frac{10\,850}{1000} \text{ kJ} \\ = 10.85 \text{ kJ}$$

- 2 Then to convert from kilojoules to kilocalories, divide the number of kilojoules by 4.184.

$$10.85 \text{ kJ} = \frac{10.85}{4.184} \text{ kcal} \\ \approx 2.59 \text{ kcal}$$

- 3 Alternatively, to convert from joules to kilocalories, divide the number of joules by 4184.

$$10\,850 \text{ J} = \frac{10\,850}{4184} \text{ kcal} \\ \approx 2.59 \text{ kcal}$$

eBookplus RESOURCES

Interactivity: Converting between units of mass (int-6907)

Interactivity: Converting between units of energy (int-6908)

Exercise 6.2 Units of mass and energy

Understanding, fluency and communicating

- WE1** Change the following mass measurements to the units specified.
 - 2 g to milligrams
 - 15.1 t to kilograms
 - 1200 mg to grams
 - 22 g to kilograms
 - 0.04 kg to grams
 - 13 000 mg to kilograms
- Change the following mass measurements to the units specified.
 - $\frac{1}{2}$ kg to milligrams
 - 20 kg to tonnes
 - 109.02 kg to grams
 - 3.8 t to kilograms
 - $\frac{4}{5}$ g to milligrams
 - 5 000 000 mg to kilograms
- The luggage limit on many domestic plane flights is 20 kg. If the luggage that you check in for a flight to Sydney has a mass of 18.2 kg, what extra mass are you entitled to take in your luggage?



- Determine whether each of the following is True or False.
 - 345 grams = 0.345 mg
 - 42 g = 0.042 kg
- Calculate how many joules of energy are in:
 - 2 kJ
 - 3.5 MJ.
- Calculate how many megajoules of energy are contained in:
 - 52 000 000 J
 - 370 000 000 000 J.
- WE2** Convert the following to kJ, giving your answer correct to 1 decimal place.
 - 100 kilocalories
 - 2200 kilocalories
- MC** If x kilocalories is equivalent to k kilojoules, which best represents the relationship between x and k ?
 - $k = 0.4184x$
 - $k = 4.184x$
 - $x = 4.184k$
 - $xk = 4.184$
 - $x^2 = 4.184k$

9. A pregnant woman has a craving for sweet food. If she requires 10 150 kJ each day and eats 5 chocolate bars each worth 1332 kJ, how many more kilocalories does she require that day? Give your answer to the nearest whole number.
10. a. The treadmill at the gym shows that Alecia has used 440 kilocalories. How many kilojoules is this? Give your answer to the nearest whole number.
- b. Alecia wants to use 2800 kJ on the machine. How many more kilocalories does she have to burn? Give your answer to the nearest whole number.



Problem solving, reasoning and justification

11. Chips are sold in packets weighing 50 g, 100 g, 200 g and 250 g. These are packed in boxes of 200 packets. Each empty box weighs 4.5 kg. A supermarket orders three boxes of 50 g, four boxes of 100 g and 200 g and ten boxes of 250 g. What is the total mass of the supermarket order?
12. A shipping-container company has four different shipping containers to choose from. Each container has a maximum weight. Container 1 has a maximum cargo weight of 21 460 kg, container 2 has 26 500 kg, container 3 has 26 330 kg and container 4 has 20 770 kg. The company uses eight containers to export goods overseas. They fill to maximum cargo weight two lots of container 4, one lot each of container 3 and 2, three lots of container 1, and they half-fill container 3. What is the total mass of all the containers in tonnes?
13. Consider the following nutritional information.

Product	Energy per serve (kJ)
Margarine (10 g)	240
White bread (1 slice)	280
Strawberry jam (15 g)	144
Muesli (50 g)	950
Full-cream milk (1 cup)	556
Skim milk (1 cup)	438

- a. How many kilojoules are there in 2 slices of white bread spread with 5 g of margarine and 15 g of strawberry jam?
- b. How many kilojoules are there in 100 g of muesli and $1\frac{1}{2}$ cups of full-cream milk?
14. a. Given that 1 kilocalorie is equal to 4.184 kilojoules, complete the following table listing some standard drink types.

Food item	Kilocalories	Kilojoules
Can of Coke	154	
Can of Fanta	195	
Apple juice	102	
Glass of milk	164	
Cappuccino	53	

- b. Paula has a diet which limits her daily intake of energy to 8200 kJ. If she drinks three cans of Coke, what will her remaining food intake (to the nearest kJ) be limited to?
- c. What percentage of her daily intake is drinking three cans of Coke? Give your answer correct to 2 decimal places.
15. As a consumer, list three examples of situations in which it is useful to have an understanding of the units of mass. For each example, state why it is useful.
16. Harry lives in the city. He is driven to school every day. In the evenings, he watches television or plays his computer after doing his homework. He doesn't like sport and he prefers to play games on his computer at weekends. In a day he usually eats foods such as burgers (2050 kJ), chips (1000 kJ), 4 biscuits (400 kJ), 2 cans of soft drink (400 kJ) and 2 apple pies (2000 kJ). His total energy consumption each day is 12 300 kJ.



Ben lives in the country. He walks to school, which is about 2.5 kilometres away. Once a week he is lucky and gets a ride to school. After school, he often helps on his parents' farm and has hardly any spare time. The little spare time he has he plays footy or basketball. In a day he usually consumes milk (370 kJ), 3 vegetables (450 kJ), water (0 kJ), sausages and mashed potatoes (1300 kJ). His total energy consumption each day is 8270 kJ.

Both Harry and Ben are the same age and in Year 9. Taking into consideration both Harry's and Ben's diets and energy needs, give a recommendation for changes that they could make to their diets.

6.3 Food and nutrition

6.3.1 Energy in food

- Energy is supplied from the food we eat and what we drink. We need energy for our bodies and for physical activity.
- Energy content of food in Australia is measured in kilojoules (kJ). Kilojoules (kJ) can be found on food labels.
- Some examples of the amount of energy stored in some normal-sized serves of food are:
 - 1 medium apple – 308 kJ
 - 2 eggs – 760 kJ
 - 1 slice of bread – 353 kJ



WORKED EXAMPLE 3

Consider the following nutritional information.

Food item	Energy per serve
1 cheese-and-bacon roll (75 g)	221 kcal
1 mixed-grain roll (70 g)	721 kJ
1 sourdough roll (55 g)	148 kcal
1 chocolate-filled croissant (125 g)	2263 kJ
1 white bagel (30 g)	78 kcal
1 toasted crumpet (50 g)	418.4 kJ

How many calories are there in 1 cheese-and-bacon roll, 1 chocolate-filled croissant and 2 toasted crumpets?



THINK

- 1 Look up the energy content for each food item from the table.
- 2 Convert all food items to kilocalories. To convert to kilocalories from kilojoules, divide by 4.184.
- 3 Calculate the total amount of calories in the food items.

WRITE

$$1 \text{ cheese-and-bacon roll} = 221 \text{ kcal}$$

$$1 \text{ chocolate-filled croissant} = 2263 \text{ kJ}$$

$$1 \text{ toasted crumpet} = 418.4 \text{ kJ}$$

$$1 \text{ cheese-and-bacon roll} = 221 \text{ kcal}$$

$$\begin{aligned} 1 \text{ chocolate-filled croissant} &= \frac{2263}{4.184} \\ &= 540.8... \\ &\approx 541 \text{ kcal} \end{aligned}$$

$$\begin{aligned} 1 \text{ toasted crumpet} &= \frac{418.4}{4.184} \\ &= 100 \text{ kcal} \end{aligned}$$

$$\begin{aligned} \text{Total energy} &= 1 \text{ cheese-and-bacon roll} + \\ &1 \text{ chocolate-filled croissant} + 2 \text{ toasted crumpets} \\ \text{Total energy} &= 221 + 541 + 2 \times 100 \\ &= 962 \text{ kcal} \end{aligned}$$

In total there are 962 kilocalories in 1 cheese-and-bacon roll, 1 chocolate-filled croissant and 2 toasted crumpets.

6.3.2 Nutritional labels

- Nutritional labelling appears on prepared and packaged foods to assist people in making informed decisions about what they eat. Nutritional labels are often displayed as a panel or grid on the back or side of packaging. A nutritional label includes information on energy, fat, saturates, carbohydrates, sugars, protein and salt. It may also provide additional information on certain nutrients, such as fibre. All nutritional information is provided per 100 g, and additionally sometimes per portion/serve. There are no standards for serving sizes and they are decided by the manufacturer.
- The nutritional information label on a packaged food item provides the number of kJ per serve or per 100 g.
- From the label, we can determine the number of kJ we consume by either:
 - determining how many of the manufacturer's serves you have consumed and multiplying this by the number of kJ per serve, or
 - dividing the weight of the amount you have consumed (in g) by 100 and multiplying this by the number of kJ in 100 g.



WORKED EXAMPLE 4

You eat 150 g of baked beans from a 210 g can.



How many kilojoules of baked beans have you consumed if the nutrition information providing the energy content from the label of a can of baked beans shows that:

- a a serving size of 210 g beans is 810 kJ
- b a 100 g serve of beans is 385 kJ.

THINK

- a 1 Calculate the number of manufacturer's serves.
- 2 The kJ consumed is the number of serves multiplied by the number of kJ per serve.
- b 1 Divide the weight of the amount you have consumed by 100.
- 2 Multiply the number of kJ by the number of serves per 100 g.

WRITE

Number of serves is $\frac{150}{210} = 0.714$.

$0.714 \times 810 = 578.6$
578.6 kJ is consumed from eating 150 g of baked beans.

Number of serves per 100g is $\frac{150}{100} = 1.5$.

$1.5 \times 385 = 577.5$
577.5 kJ is consumed from eating 150 g of baked beans.

Note: The 1.07 kJ difference between the two calculations is most likely due to the manufacturer rounding the values in either the per-serve or the 100 g value.

6.3.3 Energy content in food

- Most of the energy in food comes from fat, protein and carbohydrates. Protein, fats and carbohydrates are converted into energy in different quantities. Based on the Atwater general system:
 - 1 gram of carbohydrates = 17 kJ
 - 1 gram of protein = 17 kJ
 - 1 gram of fat = 37 kJ
 - 1 gram of dietary fibre = 8 kJ
 - 1 gram of alcohol = 29 kJ
- To calculate the energy available from a food, multiply the number of grams of carbohydrate, protein and fat by 17, 17 and 37, respectively. Then add the results together.

- The formula for calculating the energy content of food from the amount of carbohydrate, protein, and fat can be summarised as:

$$E = 17c + 17p + 37f$$

where p is the protein in grams, f is the fat in grams and c is the carbohydrates in grams.

WORKED EXAMPLE 5

Calculate the energy available, in both calories and kilojoules, from a slice of bread with a tablespoon of peanut butter on it containing 16g carbohydrates, 7g protein, and 9g fat.

THINK

- 1 Define the number of carbohydrates, protein and fat in grams.
- 2 To calculate the energy content, multiply the number of grams of carbohydrate, protein and fat by 17, 17 and 37, respectively. Then add the results together using the formula.
- 3 To calculate the energy content in kilocalories, divide the amount in kilojoules by 4.184.

WRITE

$$c = 16$$

$$p = 7$$

$$f = 9$$

$$\begin{aligned} E &= 17 \times 16 + 17 \times 7 + 37 \times 9 \\ &= 272 + 119 + 333 \\ &= 724 \end{aligned}$$

The energy available from a slice of bread with a tablespoon of peanut butter on it containing 16 g carbohydrates, 7 g protein and 9 g fat is 724 kJ.

$$\frac{724}{4.184} \approx 173.0$$

The energy available from a slice of bread with a tablespoon of peanut butter on it containing 16g carbohydrates, 7g protein and 9g fat is approximately 173 kilocalories.

6.3.4 Recommended daily energy needs

- The average daily energy requirement for an adult is approximately 8700 kJ.
- The recommended daily intake of fat for an average adult is 70 g.
- Knowing the number of kilojoules in the food you eat can help you ensure you get the right amount of energy for your needs. Maintaining a healthy weight means balancing the kilojoules you take in from food and drink against your individual daily needs.
- The average office worker aged 31–50 years old, weighing 60–70 kilograms, with no medical conditions, who sits at their desk all day and does little strenuous physical activity only needs to eat 8000–9900 kJ a day to maintain their energy balance.



WORKED EXAMPLE 6

A 3-year-old girl has daily energy needs of 4900 kJ. Calculate the recommended daily intake, to 2 decimal places, of fat for a child with daily energy needs of 4900 kJ.

THINK

- 1 Find the ratio of the child's daily energy needs against the recommended daily intake of energy for an average adult.
- 2 To calculate the recommended daily intake of fat for the child, multiply the recommended daily intake of fat of an average adult by the ratio of the child's energy needs against an adult's energy needs.

WRITE

$$\frac{4900}{8700} \approx 0.563$$

$$\frac{4900}{8700} \times 70 \approx 39.43$$

The recommended daily intake of fat for a child with daily energy needs of 4900 kJ is 39.43 g.

Exercise 6.3 Food and nutrition

Understanding, fluency and communicating

1. For breakfast, Tyler ate an egg muffin (2200 kJ) and an orange juice (525 kJ). How many kilojoules did Tyler consume?
2. Given that 1 kilocalorie is equal to 4.184 kilojoules, complete the following table listing some different types of snacks. Give the values in kilocalories to the nearest whole number.

Food item	Kilocalories	Kilojoules
265 g T-bone steak grilled		2001
140 g chopped grilled chicken		966
100 g lamb ribs		669
425 g tuna in brine		1849
100 g crumbed flounder		940



3. **WE3** Consider the following nutritional information.

Food item	Energy per serve
2 slices supreme pan pizza	2474 kJ
1 hamburger	563 kcal
6 sticks chicken satay	216 kcal
1 wedge of fruit cake	743 kJ
1 iced doughnut	306 kcal
1 slice of chocolate cake	1054 kJ

- a. How many kilocalories, to the nearest whole number, are there in 1 slice of pizza and 2 iced doughnuts?
 - b. How many kilocalories, to the nearest whole number, are there in 12 sticks of chicken satay and 1 wedge of fruit cake?
4. **MC** Mark consumed 2 macaroons (848 kJ) and a milkshake (1187 kJ) for morning tea. The energy content of the meal is:

A. 1696 kJ

B. 2035 kJ

C. 2883 kJ

D. 3222 kJ

E. 1611 kJ

5. A large birthday cream cake contains 6112 kJ. Jo cuts the cake into 8 equal slices. How many kilojoules are in each slice of cake?
6. **WE4** You eat 125 g of yoghurt from a 175 g tub. How many kilojoules of yoghurt have you consumed, to the nearest whole number, if the nutrition information providing the energy content from the label shows that 1 serving size of 175 g yoghurt is 659 kJ?
7. You eat 2 chocolate Easter eggs from a pack of 10. How many kilojoules of chocolate have you consumed if the nutrition information from the label shows that the energy content for the pack of 10 Easter eggs is 2240 kJ?



8. The average serving of 100 g of chicken and noodles is 622 kJ. Oiyee ate 350 g of chicken and noodles. What is the energy content of the meal in kilocalories to the nearest whole number?
9. **WE5** Calculate the energy available, in kilojoules, from a slice of sourdough bread which contains 13 g carbohydrates, 2.2 g protein, and 0.8 g fat.
10. Calculate the energy available, in kilojoules, from a 20 g cookie which contains 11.8 g carbohydrates, 1.8 g protein, and 4.8 g fat.

11. A suggested breakdown of our daily energy intake is:

Breakfast	20%
Morning snack	10%
Lunch	20%
Afternoon snack	10%
Dinner	40%

An inactive female requires 8000 kJ energy during the day. How much energy should be in each of her meals?



12. The percentage daily intake (%DI) is based on the recommended amounts of energy and nutrients needed to meet daily nutritional needs of an average adult male. The percentages are based on the values below.

Nutrient	Energy	Protein	Fat	Carbohydrates
Reference value used in %DI	8700 kJ	50 g	70 g	310

- a. Calculate the recommended daily amount of fat for a person with daily energy needs of 7200 kJ.
- b. Calculate the recommended daily amount of protein for a person with daily energy needs of 9300 kJ. Give your answers correct to 2 decimal places.

13. The following information is printed on the takeaway container of a 200 g deluxe hamburger.

	Energy	Protein	Fat	Carbohydrate
Adult quantity per day	8700 kJ	50 g	70 g	310 g
Hamburger	2010 kJ	25 g	25.5 g	35 g
%DI				

Calculate each %DI nutrient value for the hamburger based on a daily energy level of 8700 kJ. Give your answers correct to 1 decimal place where necessary.

14. **WE6** The ideal daily fat intake is 20%. Since 1 gram of fat is equal to 37 kJ, calculate the ideal fat intake (in grams) for Heidi and her diet of 7400 kJ a day.

Problem solving, reasoning and justification

15. Bridgette is 17 years old. On a typical day she eats:

Breakfast	Lunch	Dinner
1 bowl cereal 480 kJ 1 glass milk 400 kJ 1 slice of toast 447 kJ	1 banana 374 kJ 1 chicken-and-mayo sandwich 1672 kJ 1 bottle of water 0 kJ	1 soft drink 600 kJ 1 grilled chicken breast 1264 kJ 1 serving of potatoes 365 kJ 1 serving of carrots 135 kJ

- Calculate the total number of kilojoules Bridgette eats in a typical day.
- An average 16–18-year-old girl should intake 9200 kJ per day. What effect will Bridgette’s typical daily food have on her bodyweight?
- A suggested breakdown of our daily energy intake for the main meals is:

Breakfast	20%
Lunch	20%
Dinner	40%

An average 16–18-year-old girl requires 9200 kJ of energy during the day. Is Bridgette’s energy intake for the main meals enough?

16. At a 10-year-old boy’s birthday, each child eats:

- serving of popcorn 1351 kJ
- chocolate bar 1332 kJ
- ice-cream 633 kJ
- cheeseburger 2209 kJ
- serving of French fries 2579 kJ
- soft drink 600 kJ.

- Calculate the energy content of each child’s birthday party meal in calories.
- An average 10–12-year-old boy should intake 8800 kJ each day. What effect will each birthday meal have on a child’s energy intake for the day?



17. At a childcare centre at snack time, the 4-year-old children can choose from a platter of various fruits.

- 100 g serving of apple 202 kJ
- 100 g serving of blueberries 239 kJ
- 100 g serving of grapes 290 kJ
- 100 g serving of raisins 1256 kJ
- 100 g serving of watermelon 126 kJ

A child (4–6 years old) requires 6300 kJ each day.

- Ruby eats a 50 g serving of grapes and a 20 g serving of raisins.
 - How many kilojoules were in Ruby’s snack?
 - What percentage of Ruby’s daily energy requirement, to 2 decimal places, does she have at snack time?
- Dean is hungry and eats a 100 g serving of blueberries, a 200 g serving of watermelon and a 25 g serving of apple.
 - How many kilojoules were in Dean’s snack?
 - What percentage of Dean’s daily energy requirement, to 2 decimal places, does he have at snack time?



18. The table shown contains incomplete data for the number of grams of fat, protein and carbohydrates and the number of kilojoules for some selected foods.

Food	Number of grams of fat	Number of grams of protein	Number of grams of carbohydrates	Number of kilojoules
1 hot cross bun	3	3		485
100 grams of chocolate cake	16	4	56	
100 grams of roast chicken	14	26	0	
70 grams of bacon	8	21	0	655
2 grilled sausages		13	15	1113
1 piece of fish (flake), no batter	1	21	0	
1 banana	0	1	20	
1 apple	0	0	17	
250 mL of milk	10	8	12	
210 grams of tinned tomato soup	0.8		14.9	324

- a. Calculate the number of grams of:
- carbohydrates in a hot cross bun
 - fat in 2 grilled sausages
 - protein in 210 grams of tinned tomato soup.
- b. Which food in the list has the highest number of kilojoules? Is this what you expected?
- c. According to the Australia New Zealand Food Standards Code, a balanced diet for an average adult is made up of the following nutrients each day:

Nutrient	Energy	Protein	Fat	Carbohydrate
Quantity per day	8700 kJ	50 g	70 g	310 g

- If 1 piece of fish (flake) with no batter is consumed, what percentage of protein has been eaten for the day?
- If 100 grams of chocolate cake is consumed, what percentage of fat has been eaten for the day?
Give your answer to 1 decimal place.

19. A balanced diet for an average adult is made up of the following nutrients each day:

Nutrient	Energy	Protein	Fat	Carbohydrate
Quantity per day	8700 kJ	50 g	70 g	310 g

Determine the difference in maximum carbohydrate intake between a 7200 kJ daily diet and a 9800 kJ daily diet. Give your answer to 1 decimal place.

20. A balanced diet for an average adult is made up of the following nutrients each day:

Nutrient	Energy	Protein	Fat	Carbohydrate
Quantity per day	8700 kJ	50 g	70 g	310 g

Determine the difference in maximum protein intake between a 7000 kJ daily diet and a 9000 kJ daily diet. Give your answer to 1 decimal place.

21. Below is a list of food items and their energy values.

Food item	Energy (kJ)	Food item	Energy (kJ)
White bread (slice)	306	Hamburger	1800
Wholegrain bread (slice)	252	Sushi	155
Vegemite (1 tsp)	30	White rice (1 serve)	930
Egg	330	Chicken breast	1040
Bacon (2 rashers)	1060	Steak (200g)	1560
Cornflakes (bowl)	497	Sausages (2)	1400
Coco Pops (bowl)	728	Mixed vegetables (1 serve)	330
Apple	284	Garden salad (1 serve)	300
Banana	432	Chocolate mousse (1 serve)	770
Doughnut	770	Ice-cream (1 serve)	450
Yoghurt (1 serve)	596	Tuna sandwich	350

- Choose your favourite five food items from the list and calculate the total energy intake in calories.
- Plan a breakfast meal. Calculate the total energy intake.



6.4 Energy expended in activities

6.4.1 Energy inputs and outputs

- The human body gets energy from food it consumes. Maintaining a healthy weight is important.
- If you eat more than you actually need, the excess kilojoules will be stored in your body as fat and you will put on weight.

$$1 \text{ kg of fat} = 37\,000 \text{ kJ stored}$$

- When you are active, your body burns more energy.

$$\text{Weight change} = \text{energy input} - \text{energy output}$$

- If you use more energy than you input, you lose body weight.
 - If you input more energy than you use, you increase body weight.
- People's daily energy requirements differ based on their gender, how much exercise they do, their height and weight, and whether or not they suffer from particular illnesses or disorders. The table on the next page shows the estimated amount of energy the human body of an average body weight requires each day to maintain a healthy weight, based on age group, gender and level of activity.

Age (years)	Males (kJ)	Females (kJ)
1–3	5200	4900
4–6	6300	6300
7–9	7500	7500
10–14	9300	7800
15–18	11 600	8900
18–70		
Light work	10 000	8000
Medium work	12 500	9200
Heavy work	14 250	10 750
Pregnant women		10 150
Breastfeeding women		11 800

- A way to burn up kilojoules is to increase the time spent on an activity.
- A cappuccino and muffin have a total energy content of 1745 kJ.
- The equivalent activity of 1745 kJ is to walk for 81 minutes, jog for 35 minutes or run for 20 minutes.
- Below is a table of activities and the energy burnt per hour for an average person weighing 70 kg.

Activity	Kilojoules burnt per hour
Computing	500
Walking at 3.5 km/h	1000
Aerobics	7000
Sitting on a train	500
Watching TV	350
Reading	500
Sleeping	250
Housework	600
Running	5300
Swimming	1100



WORKED EXAMPLE 7

Jo, aged 45, eats 8200 kJ per day. Every morning she walks on her treadmill for 30 minutes. She burns 1000 kJ/h. She catches the train to and from work for 45 minutes each way and burns 500 kJ/h. She works as an administrator and does computing work for 6 hours a day burning 500 kJ/h. Calculate the amount of kilojoules she has left to burn.

THINK

- 1 Calculate the kilojoules burnt for each activity.

WRITE

Walking on treadmill = 1000 kJ for 1 hour
= 500 kJ for 30 mins

Catching train = 500 kJ for 1 hour
= 750 kJ for 1.5 hours

Computer work = 500 kJ for 1 hour
= 3000 kJ for 6 hours

- 2 Total the energy output, kilojoules burnt for activities.

Energy output = 500 + 750 + 3000
= 4250 kJ

3 Calculate the energy difference.

$$\begin{aligned}\text{Energy difference} &= \text{energy input} - \text{energy output} \\ &= 8200 - 4250 \\ &= 3950 \text{ kJ}\end{aligned}$$

Jo still needs to burn 3950 kJ.

6.4.2 Basal energy expenditure

- You can now calculate the food energy in kilojoules, but how much energy do you need to meet the demands of your **Basal Metabolic Rate (BMR)** and **Physical Activity Level (PAL)**?
- The Basal Metabolic Rate (BMR) or Basal Energy Expenditure is the calculation of how much energy it takes to maintain basic body functions on a daily basis, regardless of activity level. The amount of energy that the body needs to function while resting for 24 hours is known as BMR. This number of kilojoules reflects how much energy your body requires to support vital bodily functions. The BMR is the minimum amount of energy that a body requires to fuel its normal metabolic activity at rest.
- BMR can be calculated using the Mifflin St. Jeor formulae:

Male: $\text{BMR} = 10 \times \text{weight (kg)} + 6.25 \times \text{height (cm)} - 5 \times \text{age (years)} + 5$

Female: $\text{BMR} = 10 \times \text{weight (kg)} + 6.25 \times \text{height (cm)} - 5 \times \text{age (years)} - 161$

This gives a result in kilocalories, so to convert to kilojoules multiply by 4.184.

WORKED EXAMPLE 8

A 16-year-old boy weighs 60 kg and is 180 cm tall. Calculate his BMR to the nearest whole number.

THINK

1 List the information (age, height and weight) for the boy.

2 Use the Male BMR equation to calculate the BMR in kilocalories.

3 Convert BMR from kilocalories to kilojoules.

WRITE

Weight = 60 kg

Height = 180 cm

Age = 16 years

$$\begin{aligned}\text{BMR} &= (10 \times \text{weight}) + (6.25 \times \text{height}) - (5 \times \text{age}) + 5 \\ &= (10 \times 60) + (6.25 \times 180) - (5 \times 16) + 5 \\ &= 600 + 1125 - 80 + 5 \\ &= 1650 \text{ kcal}\end{aligned}$$

$$\begin{aligned}\text{BMR} &= 1650 \times 4.184 \\ &\approx 6931 \text{ kJ}\end{aligned}$$

A 16-year-old boy who weighs 60 kg and is 180 cm tall has a BMR of 6931 kJ.

6.4.3 Physical activity level

- The physical activity level (PAL) is a way to express a person's daily physical activity as a number, and is used to estimate a person's total energy expenditure in combination with the basal metabolic rate. It can be used to compute the amount of food energy a person needs to consume in order to maintain a particular lifestyle.

$$\text{PAL} = \frac{\text{total energy expenditure}}{\text{BMR}}$$

- The physical activity level can be estimated using a list of the physical activities a person performs within a 24-hour period and the amount of time spent on each activity, e.g. walking to work, light housework, swimming, carrying bricks at work, or whatever applies to an individual person. PAL is estimated at 1.4 for inactive men and women, 1.6 for moderately active women and 1.7 for moderately active men.

- There are five activity ‘levels’ ranging from sedentary to athletic. These factors are based most often upon the rigor of your lifestyle and exercise routine.

Activity	Level description	Activity factor
Sedentary	Little or no exercise/desk job	1.2
Lightly active	Light exercise/sports 1–3 days per week	1.375
Moderately active	Moderate exercise/sports 3–5 days per week	1.55
Very active	Hard exercise/sports 3–5 days per week	1.725
Extremely active/athletic	Hard exercise/sports daily and/or a physical job	1.9

- To calculate your total energy expenditure in a 24-hour period, use the formula:

$$\text{Total energy expenditure} = \text{PAL} \times \text{BMR}$$



WORKED EXAMPLE 9

A 15-year-old girl weighs 60 kg, is 165 cm tall and has an activity level which is classified as lightly active. Calculate her daily energy needs to the nearest whole number.

THINK

- List the information (age, height and weight) for the girl.

WRITE

Weight = 60 kg

Height = 165 cm

Age = 15 years

- Use the female BMR equation to calculate the BMR in kilocalories.

$$\begin{aligned} \text{BMR} &= (10 \times \text{weight}) + (6.25 \times \text{height}) - (5 \times \text{age}) - 161 \\ &= (10 \times 60) + (6.25 \times 165) - (5 \times 15) - 161 \\ &= 600 + 1031.25 - 75 - 161 \\ &= 1395.25 \text{ kcal} \end{aligned}$$

- Convert BMR from kilocalories to kilojoules.

$$\begin{aligned} \text{BMR} &= 1395.25 \times 4.184 \\ &= 5837.726 \text{ kJ} \end{aligned}$$

- Determine the activity factor (PAL).

For lightly active: activity factor is 1.375.

5 Calculate the daily energy needs.

$$\begin{aligned}\text{Total energy expenditure} &= \text{PAL} \times \text{BMR} \\ &= 1.375 \times 5837.726 \\ &\approx 8027\end{aligned}$$

A 15-year-old girl who weighs 60 kg and is 165 cm tall has daily energy needs of 8027 kJ.

eBookplus RESOURCES

Interactivity: Basal metabolic rate (BMR) (int-6909)

Exercise 6.4 Energy expended in activities

Understanding, fluency and communicating

- WE8** Calculate the BMR for the following people.
 - A 128 cm, 8-year-old girl weighing 28 kg
 - A 97 cm, 3-year-old boy weighing 12 kg
- The BMR for a 180 cm 20-year-old woman weighing 55 kg is 1414 kJ. True or False?
- WE9** Calculate the daily energy needs to the nearest whole number for:
 - a 165 cm, sedentary 68-year-old woman weighing 88 kg
 - a 178 cm, very active 17-year-old boy weighing 68 kg.
- Megan has a BMR of 6400 kJ. Her body's energy expenditure breakdown is:

Energy expenditure breakdown	
Liver	27%
Brain	19%
Other organs	19%
Skeletal muscle	18%
Kidneys	10%
Heart	7%

How much energy does her brain use of her BMR?

- An increase in age lowers the BMR of both males and females. True or False?
- Show that a 165 cm, 15-year-old male weighing 60 kg has an approximate BMR of 1561.25 kilocalories.
- Luke is 24 years old, works in an office and requires 12500 kJ for his active lifestyle. Every day on his way to work he stops to buy a bacon-and-egg roll that contains 2194 kJ for breakfast. What percentage of his daily energy needs does Luke have in the morning for breakfast? Give your answer to 1 decimal place.
- WE7** Emily, aged 25, eats 7600 kJ per day. Each day she walks to and from work for 20 minutes (per direction), burning 600 kJ/h. At work she serves customers for 7 hours, burning 500 kJ/h. Then when she gets home she dances for 30 minutes, burning 1200 kJ/h. Calculate the amount of kilojoules she has left to burn.
- Jordan has a very active lifestyle as a 20-year-old bricklayer who requires 14800 kJ per day. For lunch Jordan eats 2 chicken-and-mayo sandwiches, each containing 1672 kJ. What percentage of his daily energy requirement does Jordan have for lunch? Give your answer to 1 decimal place.



10. Melinda is 50 years old. During the week she is inactive and only requires 8000 kJ; during the weekend she is very active, requiring 10 400 kJ per day. How many more kJ does she require per day on the weekend than during the week?
11. The more you weigh the more kilojoules you will burn doing different activities. True or False?
12. Avril ate a bucket of hot chips with an energy content of 5700 kJ. How long will it take her to burn the energy from the hot chips if she runs at rate of 75 kJ/min?
13. **MC** The number of kJ an average person uses when gardening for 1.2 hours if they use 23 kJ/min is:
 A. 19.2 kJ B. 63.6 kJ
 C. 1656 kJ D. 1840 kJ
 E. 27.6 kJ
14. Calculate the number of kJ an average person uses when circuit-training for 30 minutes at 53 kJ/min.
15. Calculate the number of kJ an average person uses when ironing for 10 minutes at 17 kJ/min.
16. Calculate the number of kJ an average person uses when playing tennis for two and a half hours at 31 kJ/min.



Problem solving, reasoning and justification

17. Every afternoon, Eddie works out at the gym for 2 hours, burning 4000 kJ/h. He lives a very active lifestyle and requires 13 500 kJ per day. How much energy does he have left for the remaining hours in the day? Is this enough for someone who lives a very active lifestyle?
18. Michelle is a sales representative and drives her car two hours a day. If she uses 6.4 kJ per hour per kg of body weight and she weighs 78 kg, how many kilojoules does she burn driving?
19. Sarah swims every morning as a member of a swim squad. She uses 29 kJ per hour per kg of her body weight. She swims for 90 minutes and uses approximately 2500 kJ. What is Sarah's approximate body weight correct to 1 decimal place?
20. Matt weighs 65 kg and plays basketball every Saturday. He uses 40 kJ per hour per kg of his body weight. His game lasts for 40 minutes. How many kilojoules, to the nearest whole number, does he burn playing basketball?
21. John weighs 75 kg and plays lawn bowls every weekday. He uses 5 kJ per 30 minutes per kg of his body weight. His lawn bowl games last for 3 hours each day. How many kilojoules does John burn each week?



22. The table below shows the average length of time it takes to burn 1000 kJ for different activities.

Activity	Time required to use 1000 kJ
Computing	2 hours
Walking at 3.5 km/h	1 hour
Aerobics	10 minutes
Sitting on a train	2 hours
Watching TV	2 hours and 30 minutes
Bike riding	50 minutes
Working in class, studying	2 hours and 30 minutes
Eating	3 hours
Sleeping	4 hours

- How long will it take to use 250 kJ while sleeping?
- How many kilojoules are used to work on a computer for 1 hour?
- James, aged 18, leads an active life. He requires 12 500 kJ per day. On an average day, he spends 6 hours in class studying, 8 hours sleeping, 1 hour watching TV and 30 minutes walking to and from school, and the rest of his time he spends playing sport. Does James eat enough to meet his energy requirements so that he can play sport?



6.5 Energy consumption

6.5.1 Electrical appliances

- Electrical appliances require energy. The wattage is the number of watts an appliance uses when it is working. A watt (W) is a unit of measurement used for calculating energy use. It is the unit of power. One watt is defined as the energy consumption rate of one joule per second. $1 \text{ W} = 1 \text{ J/s}$
- The faster you use energy, the more watts you use.
A 1500 W hair dryer uses energy 60 times faster than a 25 W globe.
- The kilowatt hour or kWh is not a unit of power but of energy. To calculate the energy used in one kilowatt hour (kWh), the watts are divided by 1000.

$$\text{kilowatt hour} = \frac{\text{watt}}{1000}$$

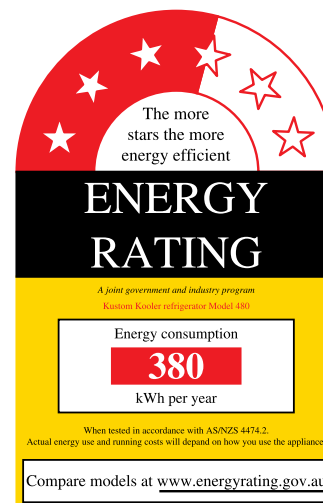
- If a hair dryer uses 1500 W consistently for one hour, then the energy used is 1.5 kWh.
- The energy in milliwatts (mW) is equal to the energy in watts times 1000.

$$\text{milliwatt} = \text{watt} \times 1000$$
- The wattage of an item can be found on the manufacturer's label. The watts needed by an appliance can be quite large or small, so metric prefixes are used to make it easier.
- Energy rating labels provide consumers with information on the energy efficiency of a product. The label shows the energy performance of particular appliances and equipment. It allows consumers to understand how much a particular model will cost to run, and also how energy efficient it might be in comparison to similar models.
- Energy-supply companies charge you for the wattage of the appliance that you are using at one time and multiply it by the number of hours that it is used.
- A 100 W ceiling fan that is used for 1 hour is equal to:

$$100 \times 1 = 100 \text{ watts/hour}$$
- If the ceiling fan was used for half an hour, it uses:

$$100 \times 0.5 = 50 \text{ watts/hour}$$
- Most energy rating labels provide the energy consumption value in kilowatts per hour. A kilowatt hour is equal to 1 kW delivered continuously for 1 hour (3600 sec).
- We can calculate the wattage (kWh) being used by an electrical appliance over time using the formula:

$$\begin{aligned}\text{Energy consumption} &= \text{time} \times \text{kilowatt hour} \\ &= \text{hour} \times \frac{\text{watt}}{1000}\end{aligned}$$



WORKED EXAMPLE 10

An oven uses 11 000 W. Convert this to:

a kilowatt hours

b milliwatts.

THINK

a To convert from W to kWh, divide by 1000.

WRITE

$$\frac{11\,000}{1000} = 11$$

A 11 000 W oven is equal to 11 kWh.

b To convert from W to mW, multiply by 1000.

$$11\,000 \times 1000 = 11\,000\,000$$

A 11 000 W oven is equal to 11 000 000 mW.

- We can calculate joules from watts and seconds.
- The energy in joules is equal to the power in watts multiplied by the time period in seconds.

$$E = P \times t, \text{ so that joules} = \text{watts} \times \text{seconds}$$
- It can be seen that 3600 joules (J) is equal to 1 watt hour (Wh).

$$\begin{aligned}\text{Joules} &= \text{watts} \times \text{seconds} \\ &= 1 \times (60 \times 60) \\ &= 3600\end{aligned}$$

$$3600 \text{ joules (J)} = 1 \text{ watt hour (Wh)}$$

$$3600 \text{ kilojoules (kJ)} = 1 \text{ kilowatt hour (kWh)}$$

WORKED EXAMPLE 11

What is the energy consumption of a 11 000 W hairdryer for the time duration of 30 seconds?



THINK

- 1 Define the power in watts and the time in seconds.
- 2 To calculate energy consumption, use the formula
 $E = P \times t$.
- 3 Write the answer.

WRITE

$$P = 11\,000, t = 30$$

$$\begin{aligned} E &= P \times t \\ &= 11\,000 \times 30 \\ &= 330\,000 \end{aligned}$$

The energy consumption is 330 000 J.

- The power in watts is equal to the energy in joules divided by the time in seconds.

$$P = \frac{E}{t}, \text{ so watts} = \frac{\text{joules}}{\text{seconds}} \text{ or } W = \frac{J}{s}.$$

WORKED EXAMPLE 12

What is the power consumption of a light globe that has an energy consumption of 90 J for the time duration of 3 seconds?

THINK

- 1 Define the energy in joules and time in seconds.
- 2 To calculate power consumption, use the formula
 $P = \frac{E}{t}$.
- 3 Write the answer.

WRITE

$$E = 90, t = 3$$

$$P = \frac{90}{3} = 30$$

The power consumption of the light globe is 30 W.

- Some appliances do not give the wattage, but only the values for current and voltage.
- To calculate the wattage, multiply the voltage by the current.

$$\text{Power} = \text{voltage} \times \text{current}$$

where power is in watts (W), voltage is in volts (V) and current is in amps (A).

WORKED EXAMPLE 13

A clock radio has 240 V and 25 mA. Calculate the clock radio's wattage.

THINK

- 1 Convert the current mA to A, by dividing by 1000.
- 2 Define the voltage in volts and the current in amps.
- 3 To calculate the wattage, use the formula
power = voltage $\times$ current.

WRITE

$$25 \text{ mA} = \frac{25}{1000} \text{ A} \\ = 0.025 \text{ A}$$

$$240 \text{ V}, 0.025 \text{ A}$$

$$\text{Power} = \text{voltage} \times \text{current} \\ P = 240 \times 0.025 \\ = 6$$

The clock radio has a wattage of 6 W.

Exercise 6.5 Energy consumption

Understanding, fluency and communicating

1. **WE10** A computer uses 350 W. Convert the measurement to kilowatt hour.
2. A 400 000 000 W generator is the same as a 400 MW generator.
True or False?
3. A light sensor uses 0.002 W. Convert the measurement to milliwatts.
4. **WE11** Calculate the energy consumption of a 250 W games console for the time duration of 1 hour.
5. **WE12** Calculate the power consumption of a microwave that has an energy consumption of 144 000 J for the time duration of 2 minutes.
6. **WE13** A toaster has 240 V and 7 A. Calculate the toaster's wattage.
7. A female's average energy requirement is 8000 kJ. How many watt hours, to 1 decimal place, is this?
8. How long will it take to power a 2400 W clothes dryer to be the same as an adult using 12 000 kJ?
9. The average energy used by an adult in a day (12 000 kJ) is the same as powering a 1.5 W mobile phone charger for how long?
10. How much energy, in kJ, does a 75-watt light bulb use when it is turned on for 25 minutes?



Problem solving, reasoning and justification

11. Assume your electrical bill shows that you used 1250 kWh over a 30-day period.
 - a. Find the energy used in kJ for the 30-day period.
 - b. Find the energy in joules per day.
 - c. At the rate of \$0.12/kWh, what will your electricity bill be for the month?
12. A 100-watt light bulb is 20% efficient (the bulb converts 20% of electrical energy into light and 80% is wasted by being transformed into heat).
 - a. How much energy does it use in 12 hours of operation?
 - b. How much energy does the bulb convert to light during 12 hours?
 - c. Convert the total energy use to kWh.



- 13.** An electric clothes dryer has a power rating of 3000 W. A family does 5 loads of laundry each week for 4 weeks. It takes the dryer 1 hour to complete each load.
- Find the energy used in kWh.
 - The electricity company charges \$0.08/kWh. Find the operating cost for 4 weeks.
- 14.** A single-door, manual-defrost refrigerator uses 650 kWh/year. A large two-door automatic-defrost refrigerator uses 1870 kWh/year. If $1 \text{ kWh} = 860 \text{ kcal}$, how many kcal/year does each refrigerator use?
- 15.** An air conditioner is used for a total of 125 days, 24 hours per day at a rate of 7.25 kWh per hour. The cost per kWh is \$0.0875.
- Calculate the total number of kWh used per year.
 - Determine the cost of air-conditioning for one year.
 - How many kcal are used per year?
($1 \text{ kWh} = 860 \text{ kcal}$)
- 16.** The electric lights in a household use 400 watts per hour and average four hours per day, every day for one year.
- How many kWh per year does this represent?
 - If replacing the lights with a fluorescent bulb would save 60 W per night, what savings in kWh does this represent in one year?
 - The cost per kWh is \$0.0825. If the fluorescent bulb costs \$18 but lasts for 10 years, would you consider it a wise investment over incandescent bulbs? Explain.



6.6 Review

6.6.1 Summary

In this topic you have learnt:

- common prefixes of the metric system
- the metric units of mass and converting between different units of mass
- the units of energy and converting between different units of energy
- the average daily energy requirement for an adult (8700 kJ)
- how to calculate the number of kilojoules consumed from a nutritional label
- how to calculate the energy content of food
- how to calculate weight change
- how to calculate the Basal Metabolic Rate (BMR) for males and females
- how to calculate the Physical Activity Level (PAL) for a person
- how to calculate the total energy expenditure in activities in a 24-hour period
- the definition of a watt
- how to calculate the energy used in one kilowatt hour (kWh)
- how to calculate the energy consumption of an electrical appliance
- how to calculate the wattage (kWh) being used by an electrical appliance over time
- how to calculate the wattage given current and voltage.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 6.6 Review

Understanding, fluency and communicating

1. The luggage limit on an international flight is 2 suitcases of 25 kg each. A passenger checks in for a flight to Singapore and her luggage has a mass of 42.3 kg. What extra mass is she entitled to take in her luggage?
2. Given that 1 kilocalorie is equal to 4.184 kilojoules, complete the following table listing some different types of snacks.

Food item (serving size is 75 g)	Kilocalories	Kilojoules
Marshmallows	239	
Crisps	365	
Pretzels	256	
Nuts	434	
Chocolate bar	317	

3. Given the nutritional information:

Food item	Energy per serve
2 slices supreme pan pizza	2474 kJ
1 hamburger	563 kcal
6 sticks chicken satay	216 kcal
1 wedge of fruit cake	743 kJ
1 iced doughnut	306 kcal
1 slice of chocolate cake	1054 kJ

- a. How many kilojoules are there in 2 slices of pizza and 1 iced doughnut?
 - b. How many kilojoules are there in 1 hamburger and 1 slice of chocolate cake? Give your answer correct to 1 decimal place.
4. You eat 125 g of yoghurt from a 175 g tub of yoghurt. How many kilojoules of yoghurt have you consumed if the nutrition information providing the energy content from the label shows that a 100 g serve of yoghurt is 376 kJ?
 5. You eat 2 chocolate Easter eggs from a pack of 10. How many kilojoules of chocolate have you consumed if the nutrition information providing the energy content from the label shows that 1 serving size of 17 g (1 egg) is 381 kJ?
 6. The rowing machine at the gym shows that James has used 520 kcal.
 - a. How many kilojoules is this?
 - b. James wants to use 3500 kJ on the machine. How many more calories does he have to burn correct to 2 decimal places?

7. Calculate the energy available from a 30 g serving of porridge which contains 23.1 g carbohydrates, 2.7 g protein, and 1.5 g fat.
8. Calculate the energy available in kilocalories from a 75 g serving of lemon meringue pie which contains 29.25 g carbohydrates, 3 g protein, and 9.8 g fat. Give your answer to 1 decimal place.
9. The percentage daily intake (% DI) is based on the recommended amounts of energy and nutrients needed for an average adult male to meet his daily nutritional needs. The percentages are based on the values below.



Nutrient	Energy	Protein	Fat	Carbohydrate
Reference value used in %DI	8700 kJ	50 g	70 g	310 g

Calculate the recommended daily amount of fat for a person with daily energy needs of 8300 kJ. Give your answer to 1 decimal place.

10. The BMR for a 85 cm 2-year-old girl weighing 12 kg is 646.25 kJ. True or False?
11. Calculate the BMR for a 155 cm, 13-year-old boy weighing 43.5 kg.
12. Calculate the daily energy, to the nearest whole number, needs for:
 - a. a 171 cm, lightly active 78-year-old man weighing 72 kg
 - b. a 165 cm, moderately active 14-year-old girl weighing 45 kg.
13. Calculate the number of kJ an average person uses when swimming breast stroke for 30 minutes at 7.2 kcal/min. Give your answer to 2 decimal places.
14. Calculate the number of kJ an average person uses when mopping a floor for 18 minutes at 2.79 kcal/minute. Give your answer to 2 decimal places.
15. Kimberley is 42 years old, works in a shop and requires 8500 kJ for her lifestyle. Every day on her way to work she stops to buy a cafe latte and croissant that contain 1286 kJ. What percentage of her daily energy, to 1 decimal place, needs does Kimberley have in the morning?
16. A coffee machine uses 1500 W. Convert the measurement to kilowatt hours.
17. A 900 W blender is the same as 0.9 mW blender. True or False?
18. Calculate the energy consumption of a 3500 W oven for 20 minutes.
19. A juice extractor has 230 V and 25 A. Calculate its wattage.
20. How long will it take to power a 5000 W central heating to be the same as an adult using 8800 kJ?



Problem solving, reasoning and justification

21. Chips are sold in packets weighing 50 g, 100 g, 200 g and 250 g. These are packed in boxes of 200 packets. Each empty box weighs 4.5 kg. A supermarket orders two boxes of 50 g, five boxes of 100 g and 200 g and twelve boxes of 250 g. What is the total mass of the supermarket order?



22. The table shown contains data for the number of grams of fat, protein and carbohydrates for some selected foods.

Food (100 g serving)	Number of grams of carbohydrates	Number of grams of protein	Number of grams of fat
Bananas	23	1	0
Raspberries	12	1	1
Strawberries	8	1	0
Grapes	18	1	0
Pears	15	0	0
Raisins	79	3	0
Apricots	11	1	0
Cheese spread	4	7	29
Cheddar cheese	1	25	33
Cottage cheese	3	12	5

- a. Calculate the energy content of 100 g of raspberries, 100 g of strawberries, 50 g of raisins and 50 g of the cheese spread.
- b. According to the Australia New Zealand Food Standards Code, a balanced diet for an average adult is made up of the following nutrients each day:

Nutrient	Energy	Protein	Fat	Carbohydrate
Quantity per day	8700 kJ	50 g	70 g	310 g

- i. If 50 g of grapes is consumed, what percentage of carbohydrates has been eaten for the day?
- ii. If 200 grams of cheddar cheese is consumed, what percentage of fat has been eaten for the day?
- Give your answers to 1 decimal place.

23. The table below shows the calories per kilo per hour used for different activities lasting for an hour.

Activity (1 hour)	Calories per kilo per hour
Surfing	3
Playing golf	5
Playing drums	4
Playing rugby	10
Ballet	6
Cycling	7
Waterskiing	6
Playing squash	12
Water aerobics	4

- a. How many kilojoules, to the nearest whole number, does a person weighing 68 kg use to play squash for 20 minutes?
- b. How many kilojoules, to the nearest whole number, does a person weighing 55 kg use to do a ballet class for 2 hours?

24. Storm weighs 63 kg and teaches karate every Saturday. She uses 10 kcal per kilo per hour of her body weight. Her lessons last for 45 minutes. How many kilojoules does she burn in a karate lesson?



25. A balanced diet for an average adult is made up of the following nutrients each day:

Nutrient	Energy	Protein	Fat	Carbohydrate
Quantity per day	8700 kJ	50 g	70 g	310 g

Determine the difference in maximum protein intake between a 6500 kJ daily diet and a 10500 kJ daily diet. Give your answer to the nearest whole number.

26. A family eat Mexican food for lunch. Each family member eats:
- 100 g beef mince 1033 KJ
 - 50 g baked taco shells 982 KJ
 - 50 g tortillas 655 KJ
 - 50 g cheese 844 KJ
 - 200 g chili con carne 848 KJ
 - 100 g tomato 97 KJ
 - 100 g onions 185 kJ
- a. Calculate the energy content of each family member's meal.
- b. An average person should intake 8700 kJ each day. If 20% of the daily energy intake should be for lunch, what effect will the lunch meal have on a family member's energy intake for the day?
27. Your electrical bill shows that you used 4520 kWh over a 30-day period.
- a. Find the energy used in kJ for the 30-day period.
- b. Find the energy in kJ per day.
- c. At the rate of \$0.16/kWh, what will your electricity bill be for the month?
28. A CFL 100-watt light bulb is 30% efficient (the bulb converts 30% of electrical energy into light and 70% is wasted by being transformed into heat).
- a. How much energy does it use in 8 hours of operation?
- b. How much energy does the bulb convert to light during 8 hours?
- c. Convert the total energy use to kWh.
29. A washing machine has a power rating of 500 W. A family does 8 loads of laundry each week for 4 weeks. It takes the washing machine 50 minutes to complete each load.
- a. Find the energy used in J and kWh. Give your answers to 1 decimal place.
- b. The electricity company charges \$0.11/kWh. Find the operating cost for 4 weeks.
30. A 42-inch plasma television is used for a total of 82 days, 8 hours per day at a rate of 0.346 kWh per hour. The cost per kWh is \$0.0925.
- a. Calculate the total number of kWh used for the 82 days.
- b. Determine the cost of the operating television for the 82 days.
- c. How many kcal are used per year? (1 kWh = 860 kcal)
- d. When the 42-inch plasma television is not used, it is on standby at a rate of 0.021 kWh per hour. What is the total cost (both operational and standby) of using the television for the 82 days?

Answers

Exercise 6.2 Units of mass and energy

1. a. 2000 mg b. 15 100 kg c. 1.2 g
 d. 0.022 kg e. 40 g f. 0.013 kg
2. a. 500 000 mg b. 0.02 t c. 109 020 g
 d. 3800 kg e. 800 mg f. 5 kg
3. 1.8 kg
4. a. False b. True
5. a. 2000 J b. 3 500 000 J
6. a. 52 MJ b. 370 000 MJ
7. a. 418.4 kJ b. 9204.8 kJ
8. B
9. 831 kcal
10. a. 1841 kJ b. 229 kcal
11. 864.5 kg
12. 171.915 t
13. a. 824 kJ b. 2734 kJ

14. a.

Food item	Kilocalories	Kilojoules
Can of Coke	154	$154 \times 4.184 = 644.336$ kJ
Can of Fanta	195	$195 \times 4.184 = 815.88$ kJ
Apple juice	102	$102 \times 4.184 = 426.768$ kJ
Glass of milk	164	$164 \times 4.184 = 686.176$ kJ
Cappuccino	53	$53 \times 4.184 = 221.752$ kJ

- b. 6267 kJ
 c. 23.57%
15. Answers will vary.
16. Since Harry prefers not to exercise and eats a lot of snacks and fast foods, he is eating more than he is burning and will likely gain weight as a result. It would be a good recommendation for Harry to eat healthier food with a lower calorie count. Ben is a healthy eater and very active, although he probably could eat a bit more, since he is potentially burning more energy than he is consuming through his sport and work.

Exercise 6.3 Food and nutrition

1. 2725 kJ

2.

Food item	Kilocalories	Kilojoules
265 g T-bone steak grilled	478	2001
140 g chopped grilled chicken	231	966
100 g lamb ribs	160	669
425 g tuna in brine	442	1849
100 g crumbed flounder	225	940

3. a. 908 kcal b. 610 kcal
4. B
5. 764 kJ
6. 471 kJ
7. 448 kJ
8. 520 kcal
9. 288 kJ
10. 408.8 kJ

11. Breakfast: 1600 kJ
 Morning snack: 800 kJ
 Lunch: 1600 kJ
 Afternoon snack: 800 kJ
 Dinner: 3200 kJ

12. a. 57.93 g
 b. 53.45 g

13.

	Energy	Protein	Fat	Carbohydrate
Adult quantity per day	8700 kJ	50 g	70 g	310 g
Hamburger	2010 kJ	25 g	25.5 g	35 g
%DI	23.1%	50%	36.4%	11.3%

14. 40 g

15. a. 5737 kJ

- b. Bridgette will lose weight.
 c. Bridgette is not eating enough for breakfast or dinner, but she is eating more for lunch.

16. a. 8704 kJ

- b. Nearly all the children's recommended energy intake is consumed in 1 meal. The children have not eaten balanced meals throughout the day and have consumed a lot of unhealthy foods.

17. a. i. 396.2 kJ ii. 6.29%

- b. i. 541.5 kJ ii. 8.60%

18. a. i. 19 g ii. 17.2 g iii. 2.4 g

- b. The 100 g piece of chocolate cake has the highest kilojoules. This is to be expected as it is high in fat and carbohydrates.

- c. i. 42% ii. 22.9%

19. 92.6 g

20. 11.5 g

21. a. Answers will vary.

- b. Answers will vary.

Exercise 6.4 Energy expended in activities

1. a. 879 kcal; 3677 kJ b. 716.25 kcal; 2997 kJ

2. False

3. a. 7080 kJ b. 12360 kJ

4. 1216 kJ

5. True

6. Answers will vary.

7. 17.6%

8. 3100 kJ

9. 22.6%

10. 2400 kJ

11. True

12. 76 minutes

13. C

14. 1590 kJ

15. 170 kJ

16. 4650 kJ

17. 5500 kJ

- This is probably not enough for someone who lives a very active lifestyle.

18. 998.4 kJ

19. 57.5 kg

20. 1733 kJ
 21. 11 250 kJ
 22. a. 1 hour
 b. 500 kJ
 c. James has 7200 kJ to play sport. This should be sufficient.

Exercise 6.5 Energy consumption

1. 0.35 kW
 2. True
 3. 2 mW
 4. 15 000 J
 5. 1200 W
 6. 1680 W
 7. 2222.2 Wh
 8. $83\frac{1}{3}$ minutes
 9. 92.6 days
 10. 112.5 kJ
 11. a. 4 500 000 kJ b. 1.5×10^8 J/day c. \$150
 12. a. 4.32×10^6 J b. 864 000 J c. 1.2 kWh
 13. a. 60 kWh b. \$4.80
 14. Single-door: 559 000 kcal/year
 Two-door: 1 608 200 kcal/year
 15. a. 21 750 kWh/year b. \$1903.13 c. 18 705 000 kcal/year
 16. a. 584 kWh/year
 b. 87.6 kWh/year
 c. Yes, the bulb would be paid for in just under 2.5 years.

Exercise 6.6 Review

1. 7.7 kg
 2.
Food item (serving size is 75 g)	Kilocalories	Kilojoules
Marshmallows	239	999.976
Crisps	365	1527.16
Pretzels	256	1071.104
Nuts	434	1815.856
Chocolate bar	317	1326.328
3. a. 3754.3 kJ b. 3409.6 kJ
 4. 470 kJ
 5. 762 kJ
 6. a. 2175.68 kJ b. 316.52 kcal
 7. 494.1 kJ
 8. 217.7 kcal
 9. 66.8 g
 10. False
 11. 5622 kJ
 12. a. 8075 kJ b. 8108 kJ
 13. 903.74 kJ
 14. 210.12 kJ
 15. 15.1%
 16. 1.5 kWh

17. False
18. 4 200 000 J
19. 5750 W
20. 29 minutes 20 seconds
21. 1028 kg
22. a. 1738 kJ
b. i. 2.9% ii. 94.3%
23. a. 1138 kJ b. 2761 kJ
24. 1976.94 kJ
25. 23 g
26. a. 4644 kJ
b. Each family member is consuming approximately 53.4% of their recommended daily intake, which is way above the 20% recommended for lunch.
27. a. 16 272 000 kJ b. 542 400 kJ/day c. \$723.20
28. a. 2 880 000 J b. 864 000 J c. 0.8 kWh
29. a. 48 000 000 J, 13.3 kWh b. \$1.47
30. a. 227 kWh b. \$21.00 c. 195 220 kcal d. \$23.55

TOPIC 7

Working with time

7.1 Overview

7.1.1 Introduction

Time is an ever-present factor in our lives. To make the best use of the time that we have for any given task, it's important to understand the fundamentals of time. Being able to plan the time it will take to undertake a journey, as well as knowing the different time zones in the world, can help you to save time and to plan better for the present and the future.



DISCUSSION

How much time each day do you allocate to different tasks? Do you think you could use your time more efficiently?

LEARNING SEQUENCE

- 7.1 Overview
- 7.2 Time and timetables
- 7.3 Positions on the Earth's surface
- 7.4 Time zones
- 7.5 Review

CURRICULUM CONTENT

- calculate times and time differences around the world **AAM** ◇
- indicate positions on the Earth's surface
- calculate times and time differences around the world **AAM** ◇

7.2 Time and timetables

7.2.1 Time

- **Time** is a measurement that we all use every day.
- Time is divided into units, including seconds, minutes, hours, days, weeks, months and years.
- A day can be divided into two 12-hour periods. The time interval from midnight to midday is indicated by the abbreviation **am** (from *ante meridiem*, meaning ‘before noon’). The time interval from midday to midnight is indicated by the abbreviation **pm** (from *post meridiem*, meaning ‘after noon’).
- Time can also represent one instant in time rather than a period or length of time. An instant in time can be shown in 12-hour time or 24-hour time on a clock or in writing.
- Most analog clocks show 12-hour time. They do not specify whether the time is am or pm.

There are:

60 seconds in 1 minute
60 minutes in 1 hour
24 hours in 1 day
7 days in 1 week
14 days in a fortnight
about 4 weeks in 1 month
12 months in 1 year
about 365 days in 1 year
about 366 days in 1 leap year
10 years in 1 decade
100 years in 1 century
1000 years in 1 millennium.

7.2.2 Converting between units of time

- To convert from minutes to hours and minutes, divide by 60. The whole number part is the number of hours and the remainder is the number of excess minutes.

Example: convert 160 minutes to hours and minutes.

$$\begin{aligned}\frac{160}{60} &= 2 \text{ remainder } 40 \\ &= 2 \text{ hours, } 40 \text{ minutes}\end{aligned}$$

- To convert from hours to hours and minutes, multiply the decimal part by 60.

Example: convert 3.1 hours to hours and minutes.

$$\begin{aligned}0.1 \times 60 &= 6 \\ 3.1 \text{ hours} &= 3 \text{ hours, } 6 \text{ minutes}\end{aligned}$$

- To convert from minutes to minutes and seconds, multiply the decimal part by 60.

Example: convert 6.6 minutes to minutes and seconds.

$$\begin{aligned}0.6 \times 60 &= 36 \\ 6.6 \text{ minutes} &= 6 \text{ minutes, } 36 \text{ seconds}\end{aligned}$$

WORKED EXAMPLE 1

A movie runs for 131 minutes. How long is this in hours and minutes?



THINK

- 1 There are 60 minutes in 1 hour, so divide 131 minutes into groups of 60 minutes.

WRITE

$$131 \div 60 = 2.18 \text{ hours}$$

- 2 2 hours is 120 minutes $131 - 120 = 11$
 $(2 \times 60 = 120)$. Subtract 120 minutes
 from 131 minutes to find the number
 of minutes more than 2 hours.
- 3 Write the answer. 131 minutes is 2 hours and 11 minutes.

WORKED EXAMPLE 2

Write the following lengths of time in hours, minutes and seconds.

a 4.30 hours

b 12.156 hours

THINK

a 1 There are 4 whole hours and 0.3
 of an hour in 4.30 hours.

2 There are 60 minutes in 1 hour, so
 0.3 of an hour is the same as 0.3
 of 60 minutes.

b 1 There are 12 whole hours and
 0.156 of an hour in 12.156 hours.

2 There are 60 minutes in 1 hour so
 0.156 of an hour is the same as
 0.156 of 60 minutes.

3 There are 60 seconds in 1 minute,
 so 0.36 of a minute is the same as
 0.36 of 60 seconds.

WRITE

$$4.30 \text{ hours} = 4 \text{ hours} + 0.3 \text{ of an hour} \\ = 4 \text{ hours} + 0.3 \times 1 \text{ hour}$$

$$= 4 \text{ hours} + 0.3 \times 60 \text{ minutes} \\ = 4 \text{ hours} + 18 \text{ minutes}$$

$$12.156 \text{ hours} = 12 \text{ hours} + 0.156 \text{ of an hour} \\ = 12 \text{ hours} + 0.156 \times 1 \text{ hour}$$

$$= 12 \text{ hours} + 0.156 \times 60 \text{ minutes} \\ = 12 \text{ hours} + 9.36 \text{ minutes} \\ = 12 \text{ hours} + 9 \text{ minutes} + 0.36 \text{ of a minute}$$

$$= 12 \text{ hours} + 9 \text{ minutes} + 0.36 \times 60 \text{ seconds} \\ = 12 \text{ hours} + 9 \text{ minutes} + 21.6 \text{ seconds}$$

7.2.3 12- and 24-hour clocks

- Time is usually told using 12-hour clocks, for example 'I will be going to the cinema at 7 o'clock tonight'. However, in the military and in hospitals, 24-hour clocks are used. When using 24-hour clocks we don't need to specify whether it is am or pm, thus eliminating the possibility of mistakes arising from this.
- To convert from 12-hour to 24-hour time:
 - add 1200 to pm times and omit the colon.
 - for times in am, omit the colon and insert 0 if the time is before 10 am.
 - 12:00 noon is 1200 hours.
 - 12:00 midnight is 0000 hours or 2400 hours.

12-hour time	24-hour time
8:30 am	0830
8:30 pm	2030

- To convert from 24-hour to 12-hour time:
 - when the time is between 1200 and 2400, subtract 1200 and add a colon between the hours and minutes. These times are pm.
 - when the time is between 0000 and 1200, simply add a colon between the hours and minutes. These times are am.
 - 1200 is 12:00 noon.
 - 0000 and 2400 are 12:00 midnight.

24-hour time	12-hour
1352	1:52 pm
0630	6:30 am

- The following chart shows corresponding times in 12- and 24-hour clocks.

12-hour time	24-hour time
12:00 midnight	0000 or 2400
1:00 am	0100
2:00 am	0200
3:00 am	0300
4:00 am	0400
5:00 am	0500
6:00 am	0600
7:00 am	0700
8:00 am	0800
9:00 am	0900
10:00 am	1000
11:00 am	1100

12-hour time	24-hour time
12:00 noon	1200
1:00 pm	1300
2:00 pm	1400
3:00 pm	1500
4:00 pm	1600
5:00 pm	1700
6:00 pm	1800
7:00 pm	1900
8:00 pm	2000
9:00 pm	2100
10:00 pm	2200
11:00 pm	2300

WORKED EXAMPLE 3

Tina was sailing from Sydney to Tahiti when the global positioning system indicated a change of direction at 1525 hours.

- What is the corresponding 12-hour time for the change of direction?
- If Tina receives a phone call at 11:05 pm from her friend, what is the corresponding 24-hour time?

THINK

- To convert 1525 to 12-hour time, subtract 1200. The time is after midday (pm).
- To convert 11:05 pm to 24-hour time, add 1200.

WRITE

- $1500 - 1200 = 325$
The change of direction occurred at 3:25 pm.
- $1105 + 1200 = 2305$
The phone call was at 2305 hours.

7.2.4 Time intervals

- We can calculate how much time has passed between two given times either by subtracting one time from another, or by working our way up from one time to another in time intervals.

WORKED EXAMPLE 4

A navy officer is on duty between 0750 hours and 2225 hours. How many hours does she complete in her shift?

THINK

- 1 Draw two columns with the headings 'Hours' and 'Minutes'. Write the times in the columns as you would set out a subtraction problem, with the highest hour value at the top.
- 2 • You cannot subtract 50 minutes from 25 minutes, so convert 1 hour to 60 minutes, as shown in pink. This makes a total of 85 minutes.
• Subtract the minutes, and then subtract the hours.
- 3 Write the answer.

WRITE

Hours	Minutes
22	25
7	50
Hours	Minutes
21	85
7	50
14	35

Her shift is 14 hours and 35 minutes long.

WORKED EXAMPLE 5

The woman shown is flying overseas. How long does her flight last?

THINK

- 1 Find the number of minutes up to the next hour (1:00 am).
- 2 Find the number of hours from 1:00 am to 7:00 pm.
- 3 Find the number of minutes from 7:00 pm to the end of the time period.
- 4 • Add the minutes, as shown in blue; 80 minutes equal 1 hour and 20 minutes.
• Add the hours.
- 5 Write the answer.

WRITE

12:30 am to 1:00 am = 30 minutes

1:00 am to 7:00 pm = 18 hours

7:00 pm to 7:50 pm = 50 minutes

30 minutes + 18 hours + 50 minutes
= 80 minutes + 18 hours
= 1 hour + 20 minutes + 18 hours
= 19 hours and 20 minutes

There are 19 hours and 20 minutes between 12:30 am and 7:50 pm.

I left at 12:30 am Sydney time and will arrive at 7:50 pm Sydney time.



7.2.5 Timetables

- Information is usually easiest to read when it is presented in a table.
- A **timetable** is a list of events that are scheduled to happen.
- Timetables are useful because they allow you to quickly see a lot of information.
- It is important to check the title and key of a timetable to ensure that you are reading it correctly.
- Timetables are commonly used for public transport schedules, such as trains and buses.



WORKED EXAMPLE 6

Sydney (Central) – Dubbo – Lightning Ridge – Coolabah – Brewarrina – Bourke – Broken Hill						
	T	T	C	C	C	C
Destination	Broken Hill	Dubbo	Broken Hill	Bourke	Brewarrina	Lightning Ridge
Days of Operation	Mon	Daily	Daily	Tue/Thu/ Fri/Sun	Tue/Thu/ Fri/Sun	Daily
Service No.	445	427	589	511	517	513
Sydney (Central)	06:18	07:18				
Strathfield	u06:30	u07:30				
Parramatta	u06:41	u07:43				
Blacktown	...	u07:53				
Penrith	u07:04	u08:06				
Katoomba	u07:59	u08:59				
Lithgow	u08:39	09:40				
Rydal	...	a10:00				
Tarana	...	a10:18				
Bathurst	09:47	10:52				
Blayney	10:35	11:38				
Orange	10:59	12:02				
Parkes	12:48	...				
Condobolin	14:00	...				
Stuart Town	...	12:42				
Wellington	...	13:04				
Geurie	...	13:22				
Dubbo arr	...	13:45				
Connection	...		↓	↓		↓
Dubbo dep	...		14:15	14:16		14:20
Eumungerie	...		...	...		14:48
Gilgandra	...		...	...		15:10
Gulargambone	...		...	...		15:45
Coonamble arr	...		...	...		16:15
Coonamble dep	...		...	...		16:45
Walgett	...		...	...		18:00
Lightning Ridge	...		...	...		18:50
Narromine	...		14:48	14:51		
Trangie	...		15:12	15:16		
Nevertire	...		15:36	15:38		
Nyngan	...		16:15	16:13		
Girilambone	...		...	16:44		
Coolabah arr	...		...	17:09		
Connection	...		...	...	↓	
Coolabah dep	...		...	...	17:24	
Gongolgon	...		...	...	18:30	
Brewarrina	...		...	...	19:00	
Byrock	...		...	17:54		
Bourke	...		...	18:46		
Euabalong West	14:45		...	...		
Ivanhoe	16:31		...	...		
Darnick	a17:16		...	...		
Hermidale	...		16:45	...		
Boppy Mountain	...		17:15	...		
Cobar arr	...		17:45	...		
Cobar dep	...		...	18:15		
Emmdale	...		...	20:05		
Wilcannia	...		...	20:55		
Menindee	18:22		...	...		
Broken Hill (CST)	19:10		22:45	...		

CST – Central Standard Time

Explanation of definitions and symbols

- a Stops to pick up and set down booked customers only when required.
- b Stops to set down booked customers only when required.
- c Stops to pick up booked customers only when required.
- d Stops to set down booked customers only.
- u Stops to pick up booked customers only.
- ★ Daylight Savings NSW. During this time services arrive or depart Queensland locations one (1) hour earlier than shown (QLD time).
- Refreshment break—usually 30 minutes.
Length of break may change when required.
- Shows train/coach connection.
- 09:00 Local travel under 40 km not permitted where times are shown in this colour, except where there is no other local coach operator on that route.
- NSW TrainLink Regional train service.
- NSW TrainLink Intercity train service.
- NSW TrainLink coach service.
- NSW TrainLink Taxi service.

Refer to the timetable above. If your sister needs to be in Lightning Ridge at 7:00 pm on Wednesday, what is the latest time she can depart Sydney (Central)?

THINK

- 1 Convert 7:00 pm to 24-hour time.
- 2 Find the closest arrival time at Lightning Ridge before 1900 hours. Now work backwards to find the departure time from Sydney (Central). Find the time that the coach departs from Dubbo. Then go to the top of this column to find the required departure time from Sydney (Central).
- 3 Write the answer.

WRITE

$$700 + 1200 = 1900 \text{ hours}$$

The arrival time in Lightning Ridge is 1850 (6:50 pm).

The coach departs from Dubbo at 1420 (2:20 pm).

The required departure time from Sydney (Central) is 0718 (7:18 am).

The latest time your sister can depart Sydney (Central) is 0718 (7:18 am).

eBookplus RESOURCES

Interactivity: Converting between units of time (int-6910)

Exercise 7.2 Time and timetables

Understanding, fluency and communicating

1. **WE1** A dance competition lasts for 235 minutes. How long is this in hours and minutes?
2. **WE2** Convert each of the following time periods to minutes.
 - a. 2 hours
 - b. $3\frac{1}{2}$ hours
 - c. 1 day
 - d. $\frac{1}{2}$ hours
 - e. 4 hours and 25 minutes
3. Convert each of the following time periods to minutes.
 - a. $6\frac{1}{4}$ hours
 - b. 2 hours and 10 minutes
 - c. 7 hours and 35 minutes
 - d. $\frac{3}{4}$ hour
4. You are allowed to stay up until 10:30 pm on Saturday night. What time will you need to start a movie if you watch:
 - a. *Finding Nemo* (97 minutes)
 - b. *Aladdin* (86 minutes)
 - c. *Star Wars* (124 minutes)?
5. Use the time units to help you answer each of the following. (Note: Assume that there are 365 days in 1 year.) How many:
 - a. seconds are in 1 hour
 - b. seconds are in 1 day
 - c. minutes are in 1 day
 - d. hours are in 1 year?
6. If you spend half an hour doing homework each weeknight (Monday to Friday), how many hours of homework will you have done after 5 weeks?
7. **WE3a** Convert the following times to 12-hour time.
 - a. 1200 hours
 - b. 1800 hours
 - c. 0058 hours
 - d. 2400 hours
 - e. 0700 hours
 - f. 2300 hours
8. **WE3b** Convert the following times to 24-hour time.
 - a. 12:00 noon
 - b. 12:00 midnight
 - c. 3:00 am
 - d. 4:00 pm
 - e. 12:15 am
9. **WE4** Find the difference between each of the following times.
 - a. 0125 hours and 2345 hours
 - b. 1000 hours and 1237 hours
 - c. 1430 hours and 1745 hours
 - d. 0715 hours and 1550 hours



10. **WE5** Find the time difference between:
- 4:15 pm and 5:20 pm
 - 3:16 pm and 5:09 pm
 - 9:21 am and 1:06 pm
 - 6:30 pm and 2:45 am the following day
 - 7:20 am on Monday and 6:30 pm the following day
 - 1:20 pm on Wednesday and 9:09 am the following Friday.
11. What is the time:
- 1 hour after 12 noon
 - 2 hours before 6:15 am
 - 1 hour and 20 minutes after 10:30 am
 - 2 hours and 30 minutes before 7:45 pm
 - 4 hours and 27 minutes after 1:12 pm
 - 3 hours and 45 minutes before 3:30 pm?
12. A friend uses 24-hour time and writes you a message: ‘Can you call me at 2000 hours to talk about our secret meeting at 1235 hours tomorrow, because my phone will be off after 0915 hours tomorrow?’ Convert the times she has used to 12-hour time.
13. A train left the station at 0920 and arrived at its destination at 1305. How long did the trip take?
14. You have a training schedule for tennis. If you follow the schedule, how long will you train each week?



Tennis training
Monday: 0630–0745
Friday: 1650–1825

Use the train and coach timetable on page 234 to answer questions 15–20.

15. Does the train travel directly from Sydney (Central) to Lightning Ridge? Explain your answer.
16. If you are planning to travel from Strathfield to Broken Hill, what are you required to do before your departure date?
17. **WE6** If you arrive at Coolabah at 17:25 on a Tuesday, what time is the next bus to Bourke?
18. Is this the correct timetable to find times for travelling from Lightning Ridge to Dubbo? Why or why not?
19. How long does it take to travel:
- from Sydney (Central) to Broken Hill on a Monday
 - from Bathurst to Bourke on a Sunday
 - from Narromine to Brewarrina on Tuesday?
20. If you are travelling from Orange to Bourke on a Tuesday, how many stops are there on:
- the train portion of the trip
 - the coach portion of the trip?

Tide tables are used by fisherman, sailors, and surfers to determine the time of high or low tides on any specific day. Use the tide table below to answer questions 21–29.

SYDNEY (FORT DENISON)

LAT 30°51' S LONG 151°14' E

TIMES AND HEIGHTS OF HIGH AND LOW WATERS

Times are in local standard time (UTC + 10:00) or daylight-saving time (UTC + 11:00) when in effect.

MARCH 2017

Time	m	Time	m	Time	m
1 0451 0.33		12 0250 0.40		23 0522 1.49	
WE 1108 1.77		SU 0907 1.78		TH 1208 0.58	
1725 0.26		1530 0.31		1813 1.27	
2337 1.66		2136 1.58		2353 0.69	
2 0541 0.35		13 0333 0.39		24 0617 1.56	
TH 1154 1.68		MO 0947 1.73		FR 1256 0.50	
1807 0.32		1605 0.33		1900 1.36	
		2214 1.60			
3 0024 1.67		14 0414 0.41		25 0045 0.60	
FR 0635 0.40		TU 1026 1.67		SA 0704 1.65	
1245 1.57		1638 0.38		1337 0.41	
1853 0.40		2249 1.60		1942 1.47	
4 0115 1.65		15 0454 0.44		26 0131 0.50	
SA 0735 0.46		WE 1102 1.59		SU 0748 1.72	
1340 1.44		1709 0.44		1415 0.34	
1944 0.49		2324 1.59		2021 1.57	
5 0213 1.62		16 0533 0.49		27 0216 0.41	
SU 0845 0.51		TH 1139 1.50		MO 0832 1.78	
1445 1.33		1739 0.50		1454 0.28	
2044 0.56		2359 1.57		2102 1.67	
6 0319 1.61		17 0615 0.54		28 0302 0.33	
MO 1002 0.52		FR 1216 1.41		TU 0916 1.80	
1604 1.27		1812 0.56		1533 0.25	
2155 0.60				2144 1.76	
7 0430 1.62		18 0036 1.54		29 0350 0.28	
TU 1119 0.50		SA 0700 0.59		WE 1003 1.78	
1725 1.28		1257 1.33		1615 0.26	
2309 0.60		1847 0.63		2228 1.82	
8 0541 1.66		19 0118 1.50		30 0440 0.27	
WE 1227 0.44		SU 0751 0.64		TH 1052 1.73	
1832 1.34		1345 1.25		1657 0.30	
		1931 0.69		2315 1.84	
9 0015 0.55		20 0208 1.46		31 0533 0.29	
TH 0642 1.72		MO 0852 0.68		FR 1143 1.63	
1324 0.37		1445 1.19		1742 0.37	
1928 1.41		2029 0.75			
10 0113 0.48		21 0309 1.43			
FR 0735 1.77		TU 1001 0.68			
1411 0.32		1558 1.17			
2015 1.49		2139 0.77			
11 0203 0.43		22 0417 1.44			
SA 0823 1.79		WE 1111 0.65			
1453 0.30		1713 1.20			
2057 1.54		2251 0.75			

APRIL 2017

Time	m	Time	m	Time	m
1 0003 1.83		12 0255 0.45		23 0526 1.63	
SA 0630 0.34		WE 0858 1.56		SU 1152 0.42	
1237 1.52		1458 0.47		1806 1.57	
1830 0.47		2116 1.70			
2 0056 1.79		13 0332 0.46		24 0005 0.53	
SU 0632 0.40		TH 0934 1.51		MO 0615 1.70	
1237 1.41		1528 0.51		1234 0.36	
1825 0.57		2148 1.68		1850 1.70	
3 0055 1.72		14 0411 0.49		25 0055 0.42	
MO 0742 0.46		FR 1012 1.45		TU 0703 1.74	
1345 1.32		1559 0.56		1316 0.31	
1930 0.65		2222 1.68		1933 1.82	
4 0201 1.66		15 0451 0.52		26 0145 0.32	
TU 0855 0.49		SA 1050 1.39		WE 0753 1.75	
1503 1.29		1631 0.62		1400 0.29	
2045 0.69		2259 1.64		2018 1.92	
5 0315 1.63		16 0533 0.57		27 0237 0.26	
WE 1005 0.49		SU 1132 1.33		TH 0844 1.72	
1618 1.33		1709 0.68		1445 0.31	
2200 0.67		2338 1.60		2105 1.98	
6 0424 1.64		17 0622 0.61		28 0330 0.24	
TH 1107 0.46		MO 1219 1.27		FR 0937 1.66	
1719 1.40		1752 0.74		1531 0.36	
2306 0.62				2154 2.00	
7 0524 1.66		18 0025 1.55		29 0426 0.25	
FR 1159 0.43		TU 0717 0.65		SA 1032 1.58	
1810 1.48		1315 1.23		1621 0.44	
		1846 0.79		2245 1.96	
8 0003 0.55		19 0120 1.50		30 0524 0.30	
SA 0615 1.67		WE 0819 0.66		SU 1130 1.49	
1242 0.41		1421 1.22		1714 0.53	
1853 1.56		1953 0.82		2340 1.89	
9 0052 0.50		20 0225 1.49			
SU 0700 1.67		TH 0922 0.63			
1321 0.40		1531 1.26			
1932 1.62		2107 0.80			
10 0136 0.46		21 0332 1.51			
MO 0742 1.65		FR 1018 0.58			
1355 0.41		1631 1.34			
2009 1.66		2215 0.74			
11 0216 0.45		22 0432 1.56			
TU 0821 1.61		SA 1107 0.50			
1427 0.44		1722 1.45			
2043 1.68		2313 0.64			

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Indicates high tide of 1.7 m or more Indicates low tide of 0.3 m or less

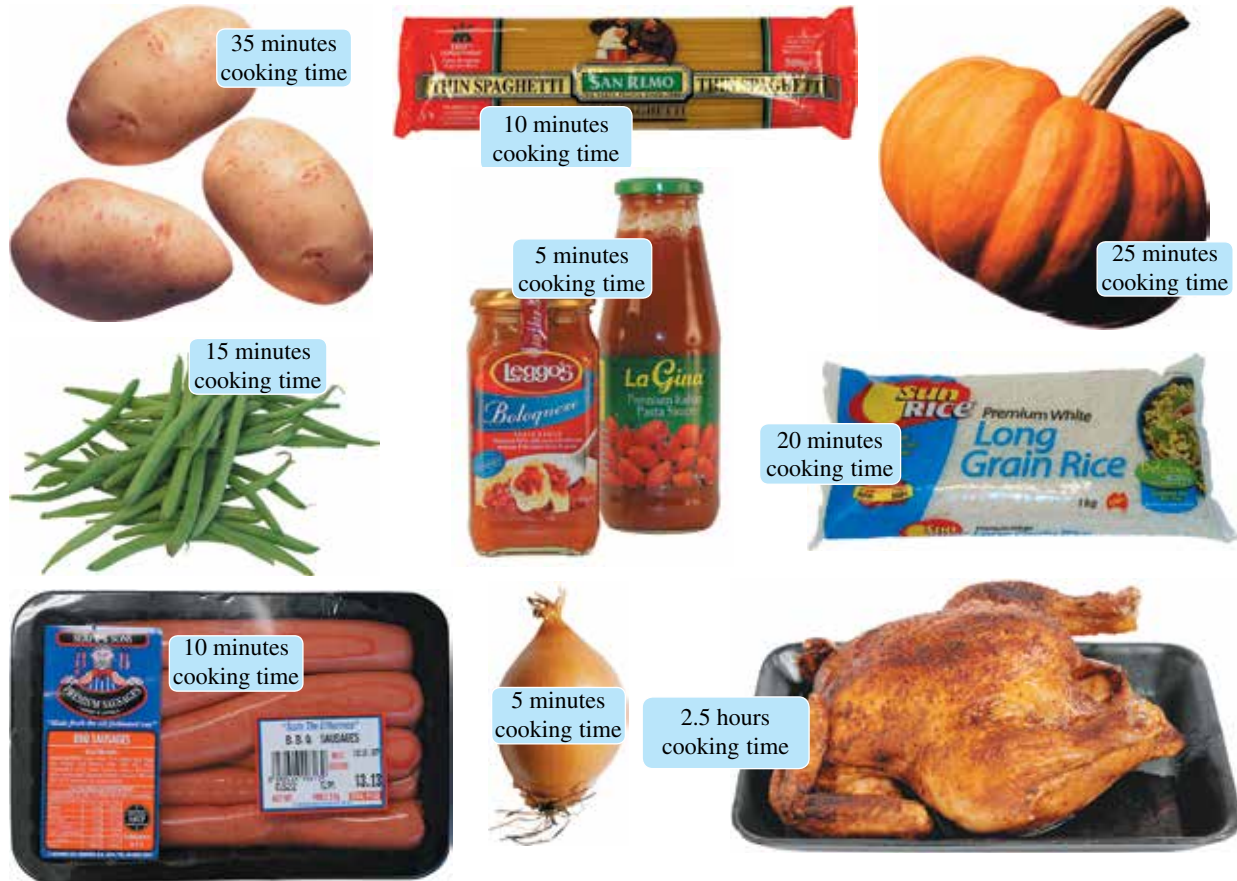
New Moon ● First Quarter ● Full Moon ○ Last Quarter ●

21. What do the blue and green shadings indicate on the tide table?
22. How can you distinguish if a time indicates a high tide or a low tide?
23. a. How many high tides occur on 8 March? State the high tide times in 12-hour time.
b. How many high tides occur on 30 March? State the high tide times in 12-hour time.
24. a. How many low tides occur on 31 March? State the low tide times in 12-hour time.
b. How many low tides occur on 10 March? State the low tide times in 12-hour time.
25. Why are there days with only three tides?
26. a. What day in April has the lowest tide? What time is it?
b. What day in April has the highest tide? What time is it?
27. a. Do the highest and lowest tides occur on the same day in March?
b. Do the highest and lowest tides occur on the same day in April?
28. How much time elapsed between the first low tide and the last high tide on April 26?

29. Is the time difference between:
- low tides the same every day
 - high tides the same every day?

Problem solving, reasoning and justification

30. Many jobs and everyday situations involve the use of 24-hour time. Explain why it is important that:
- the armed services use 24-hour time
 - arrival and departure times at airports are in 24-hour time.
 - Suggest other situations or jobs in which 24-hour time is used.
31. A classmate attempted to calculate some time differences. Explain where he may have gone wrong in his calculations and then find the correct answers.
- | | |
|---|--|
| <p>a. $\begin{array}{r} 22\ 40 \\ -20\ 05 \\ \hline 2\ 35\ \text{min} \end{array}$</p> | <p>b. $\begin{array}{r} 9:22 \\ -7:38 \\ \hline 1:84 \end{array}$</p> |
|---|--|
32. Transport timetables are useful when planning a trip. Why is it important to determine departure and arrival times when planning a trip using public transport?
33. From the following ingredients, design a meal for your family and create a timetable for the cooking times so that your meal can be served at 7:00 pm. (Note: You do not have to use all of the ingredients.)



34. Every time the minute hand completes a full rotation of the clock, the hour hand moves $\frac{1}{12}$ of the distance around the clock.
- When it is 3:00 pm, the minute hand is at a right angle to the hour hand. At 3:15 pm what is the position of the hour hand compared to its original position at 3:00 pm?
 - When it is 3:20 pm and the second hand makes two complete rotations, how far has the hour hand moved compared to its original position at 3:20 pm?

7.3 Positions on the Earth's surface

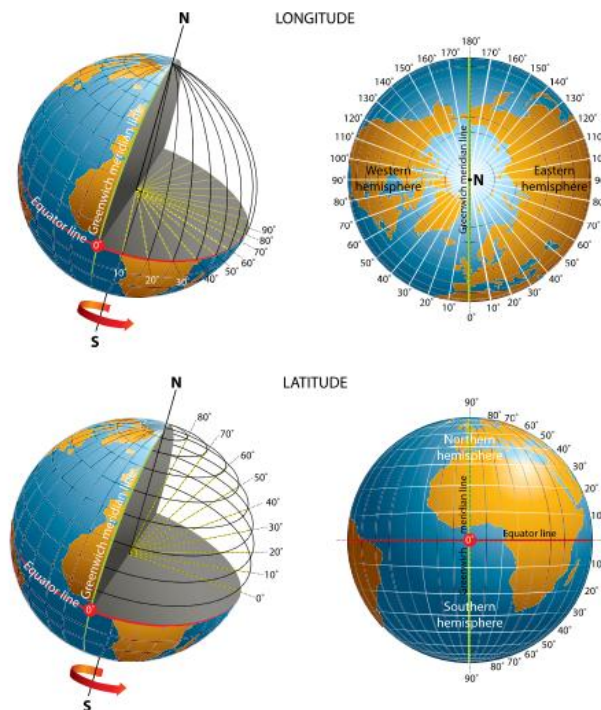
7.3.1 The lines of latitude and longitude

- In order to read geographical coordinates on a map or globe, we must first understand latitude and longitude.
- The **lines of latitude** are lines that run around the globe horizontally from east to west. They begin at the equator, which is 0 degrees. (The equator is the longest line of latitude.) Going north (up) from the equator we come to the North Pole, and going south (down) from the equator we come to the South Pole. There are 90 lines of latitude ascending north from the equator, and 90 lines descending south from the equator, with each line representing 1 degree. The North and South Poles are represented by dots and are 90 degrees north and 90 degrees south respectively.
- All lines of latitude north of the equator are written as degrees from the equator followed by N, for example 15 °N. All lines of latitude south of the equator are written as the degrees from the equator followed by S, for example 15 °S.

Latitude facts

1 degree of latitude $\approx$ 111 km
1 minute of latitude = 1.85 km
1 second of latitude = 0.0322 km

- The **lines of longitude** are vertical lines that run from the North Pole to the South Pole beginning at the prime meridian, or 0 degrees. One line of longitude east of the prime meridian represents 1 degree east, and one line of longitude west of the prime meridian indicates 1 degree west. Unlike latitude, which has a maximum of 90 degrees, longitude has a maximum of 180 degrees to the east and west. Easterly lines of longitude are written as the degrees followed by E, for example 110 °E. Westerly lines of longitude are written as the degrees followed by W, for example 110 °W.
- Points on the Earth's surface are written with the latitude first and the longitude second. For example, if you were sailing to Suva in Fiji, the latitude would be represented first and the longitude second: 18°10' S 178°27' E. Your GPS might not indicate which is which.



- Usually a specific point does not lie directly on the lines of latitude and longitude, so instead we can use one of three methods to represent smaller increments of distance.

Degrees, minutes and seconds (DDD, MM, SS)

- Degrees can be split up into minutes and seconds using the following scale:

$$1^{\circ} = 60 \text{ min}$$

$$1 \text{ min} = 60 \text{ sec}$$

- When we write coordinates using degrees, minutes and seconds, the minutes are written with one dash and the seconds are written with two dashes.

For example, $58^{\circ}24'05''$ N represents 58 degrees, 24 minutes and 5 seconds north of the equator. As minutes and seconds are smaller than degrees, the latitude for this falls between 58 and 59 degrees north. The seconds can also be represented more precisely using decimals.

Degrees and minutes (DDD, MM)

- This is the same scale as above, but instead of splitting the minutes into seconds the minutes are represented more precisely using decimals.

For example, if a point is located at $18^{\circ}28.03'S$, $160^{\circ}49.21'E$, the latitude falls between 18 and 19 degrees south, and the longitude falls between 160 and 161 degrees east.

Signed degrees (DDD.dddd)

- The signed degrees format uses decimal values instead of degrees and minutes. In this format latitude and longitude can be detailed by up to 8 decimal values.
- In signed degrees, instead of indicating the direction by using N, S, E and W, positive and negative values are used. Northern latitudes are given as positive values and southern latitudes are given as negative values. Likewise, eastern longitudes are given positive values and western longitudes are given negative values.

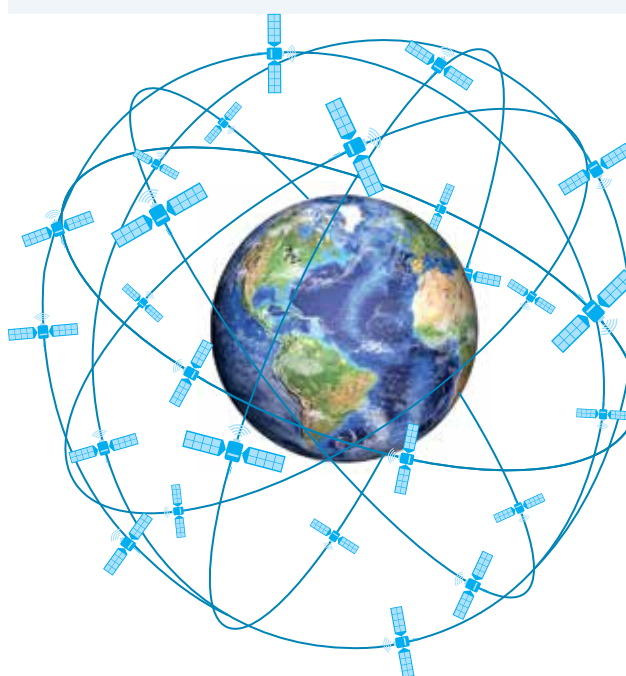
For example, the Sydney Opera House is located at -33.856159° , 151.215256° . This means that the latitude is south of the equator and the longitude is east of the Prime Meridian.



7.3.2 Global Positioning Systems (GPS)

- The **global positioning system (GPS)** is used across the world in many different products for many different purposes. It was first developed by the United States Department of Defense for military purposes; the United States called this system Navstar. Other countries have developed their own global positioning systems, such as Russia's GLONASS-K, China's BDS and the European Union's Galileo.
- The Navstar system operates using 31 satellites orbiting the earth at an altitude of approximately 20200 km. These satellites orbit the earth at approximately 9000 kilometres per hour, two times a day. At any place on earth, the signals of approximately

25 GPS satellites in orbit



9 satellites can be accessed at any time. The satellites transmit radio signals at regular intervals, and these signals send location and time data to a GPS device, which only has the ability to receive the information. These radio signals are able to pass through anything except buildings and mountains.

- The accuracy of GPS devices is based upon how many satellites they can receive simultaneous signals from at any specific location. All GPS devices are able to lock into at least 4 satellite signals at any one time. However, the accuracy of the location is higher if the device can obtain signals from more satellites.
- All locations on GPS devices are described by a corresponding longitude and latitude measurement. For example, if your mobile phone is trying to lock in your location on the Sydney Harbour Bridge, the GPS device is in constant contact with the satellites and hones in on your location according to where the latitude and longitude lines cross.
- If a GPS device can detect a signal from only 1 satellite, it cannot determine its location. The device can determine the distance to the satellite, but there are other points on the Earth's surface that will also be that distance from the satellite.
- Likewise, a location can't be determined when signals from 2 satellites are detected. However, when a third satellite is added, an exact location is possible. As more satellites become visible from the GPS device, the accuracy of the location becomes more precise. This process of using 3 satellites and their distance to detect the location of the GPS device is called trilateration.
- There are many different types of GPS devices, including car navigation devices; portable GPS devices used for hiking, cycling, running and surfing; golf GPS devices (which check the distance to a hole); and devices for boating and aviation. Cameras, mobile phones and watches can also contain GPS receivers.

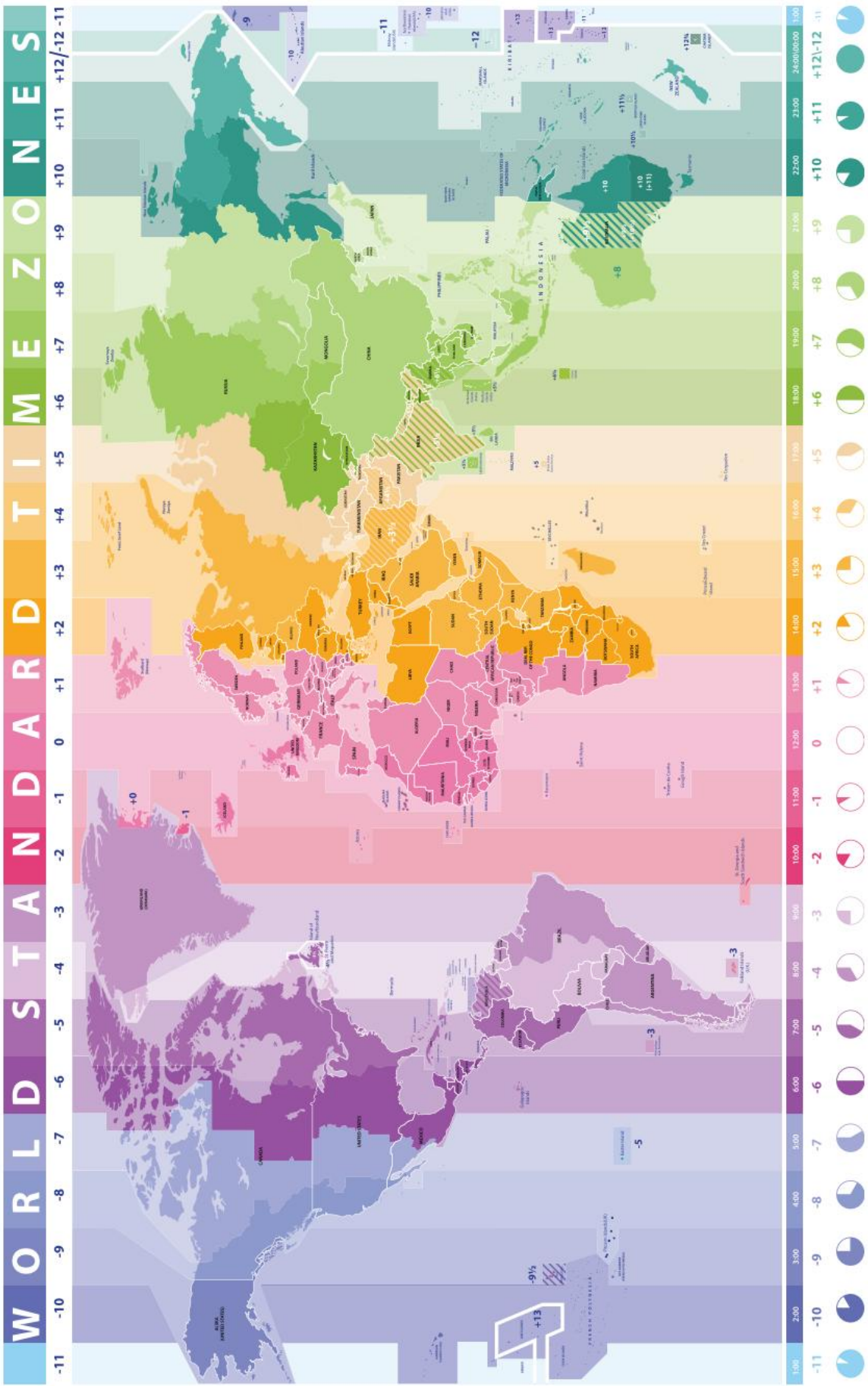
Note: There are no questions for this section, but students should be able to locate points on Earth's surface using latitude, longitude or position coordinates, with a globe, an atlas or GPS device.

7.4 Time zones

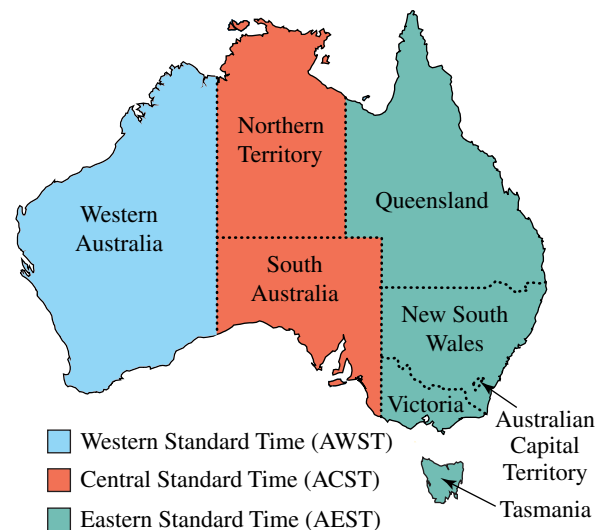
7.4.1 Time zones

- The world is divided into 24 different **time zones**. These zones are approximately bound by lines of longitude.
- As discussed in Section 7.3, lines of longitude are lines that run from the North Pole to the South Pole of the Earth. They are measured in degrees either east or west of the prime meridian, which passes through Greenwich in England.
- The time in England used to be called **Greenwich Mean Time (GMT)**; it is now known as **Coordinated Universal Time (UTC)**. It is referred to as Zulu time in the military.
- UTC is the reference point for every time zone in the world. Time zones are either ahead of, equal to or behind UTC.
- The lines of longitude cover 360° (a full turn). Therefore, each time zone represents a turn of $\frac{360}{24} = 15^\circ$.
- Be careful when evaluating time zones, because not all time zones follow longitude lines exactly. For example, Mexico City should be in the 7th time zone from UTC according to its position on Earth, but is actually in the 6th time zone from UTC.
- The time in New South Wales is UTC + 10. This means that the time in Sydney is 10 hours ahead of the time in London.

Time zones of the world



- If you move from one time zone to another in an easterly direction, you need to set your watch forward. If you move from one time zone to another in a westerly direction, you need to set your watch back.
- Australia is divided into three different standard time zones: Western Standard Time (AWST), Central Standard Time (ACST) and Eastern Standard Time (AEST).
- Western Australia is 2 hours behind Eastern Standard Time.
- The Northern Territory and South Australia are half an hour behind Eastern Standard Time.
- Broken Hill, in the far west of New South Wales, follows Central Standard Time.
- To calculate the time difference between two locations, you must determine the difference in time zones between the two places.



WORKED EXAMPLE 7

If the time in Sydney is 10:30 pm AEST, what time is it in Perth?

THINK

- 1 Perth is 2 hours behind Sydney. Subtract 2 hours from the Sydney time.
- 2 Calculate the time in Perth.

WRITE

The time in Sydney is 10:30 pm.
 The time in Perth is 10:30 pm – 2 hours.
 It is 8:30 pm in Perth.

WORKED EXAMPLE 8

What time is it in Sydney if the time in London is 9:00 am UTC?

THINK

- 1 Sydney is 10 hours ahead of London. Add 10 hours to the London time.
- 2 Calculate the time in Sydney.

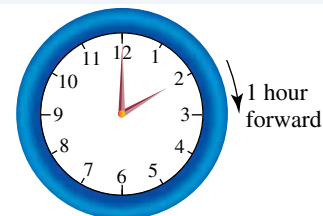
WRITE

The time in London is 9:00 am.
 The time in Sydney is 9:00 am + 10 hours.
 It is 7:00 pm in Sydney.

7.4.2 Daylight saving

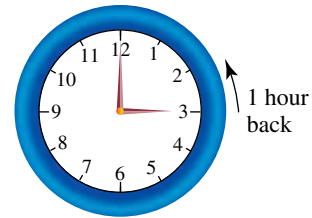
- Many countries around the world have daylight-saving time during summer so that people can make the most of the warmer weather.
- When daylight-saving time begins, clocks are turned forward by 1 hour at 2:00 am.
- At the end of daylight-saving time, clocks are then turned back by 1 hour at 3:00 am.
- Daylight-saving time starts and finishes on the same date in all participating states.
 - Daylight-saving time starts on the first Sunday in October, when clocks go forward 1 hour.

When daylight-saving time starts, 2:00 am becomes 3:00 am; the night is 1 hour shorter.



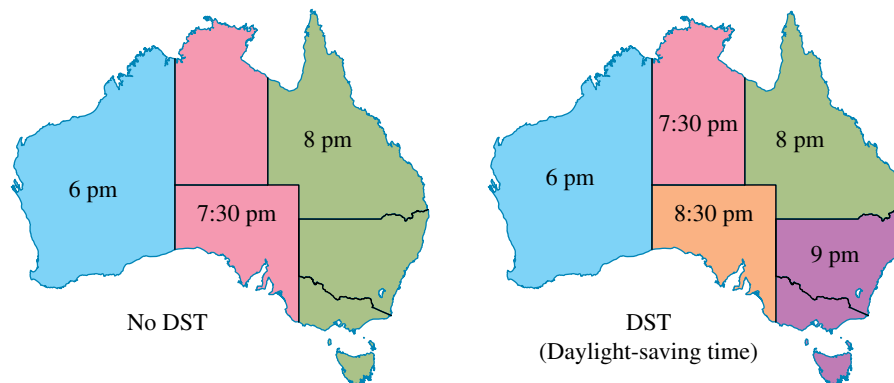
- Daylight-saving time finishes on the first Sunday in April, when clocks are turned back 1 hour.
- Queensland, the Northern Territory and Western Australia do not observe daylight-saving time.
- When you are calculating the time difference between two locations, you must take into account whether one or both of the locations are observing daylight-saving time.

When daylight-saving time finishes, 3:00 am becomes 2:00 am; the night is 1 hour longer.



7.4.3 Time differences across Australia

- The time differences between the states and territories in Australia are as shown below.



WORKED EXAMPLE 9

You want to call your friend in Darwin to wish her a happy New Year. If it is 8:00 pm in Sydney, what time is it in Darwin?

THINK

- 1 There is normally half an hour between the time in Darwin and the time in Sydney.
- 2 On New Year's Day, Sydney observes daylight-saving time because it is summer. This means Sydney is another hour ahead.
- 3 Write the answer.

WRITE

$$8:00 \text{ pm} - 0.5 \text{ hours} = 7:30 \text{ pm in Darwin}$$

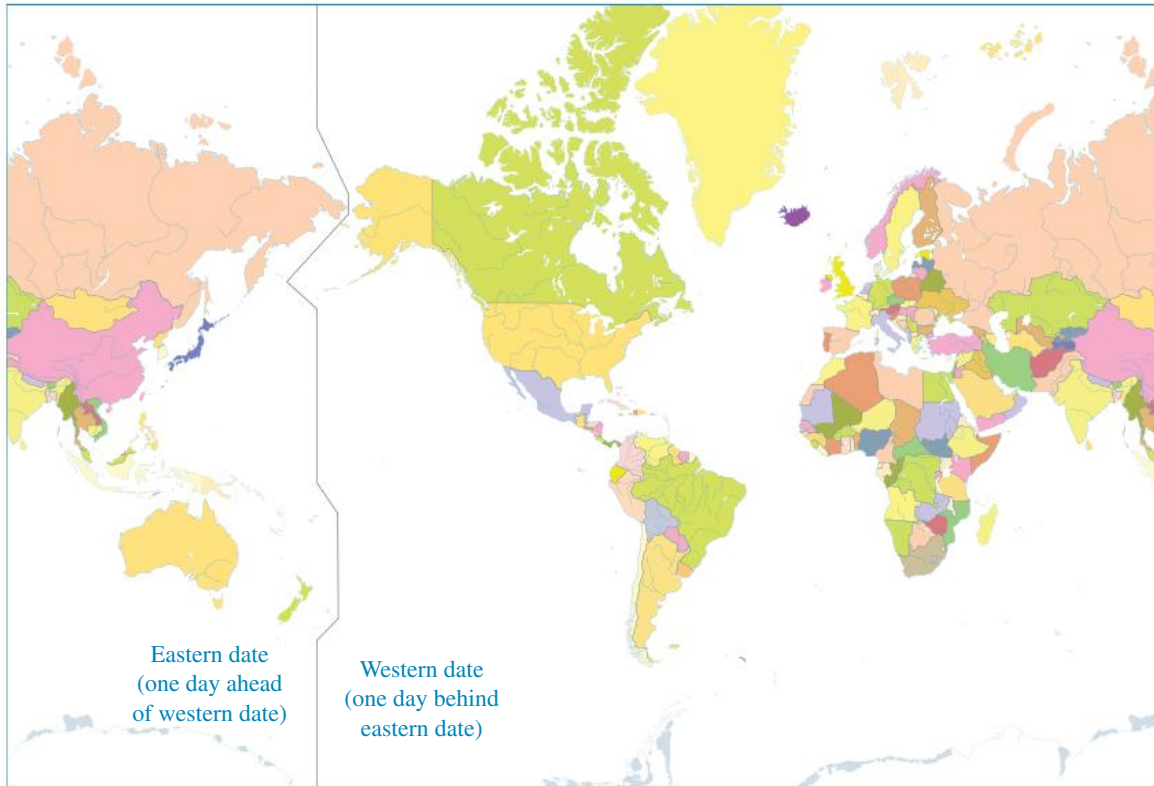
$$7:30 \text{ pm} - 1 \text{ hour} = 6:30 \text{ pm}$$

It is 6:30 pm in Darwin.

7.4.4 The International Date Line (IDL)

- The **International Date Line (IDL)** is the line on Earth where a calendar day officially starts and ends. If you cross the International Date Line in an easterly direction, your calendar goes back a day. If you cross the International Date Line in a westerly direction, your calendar goes forward one day.
- The IDL is located 180 degrees from the Prime Meridian. The IDL is not a straight line to ensure that nations lie on either side of the date line, and that their time is not split over two days.

The International Date Line



WORKED EXAMPLE 10

Dallas left Sydney on a plane at 0900 hours on 11/04/2017 (Tuesday).

- If Dallas travelled east to Mexico City from Sydney, how many time zones did he cross?
- Did he pass over the International Date Line?
- What is the time difference between Sydney and Mexico City?
- Is Sydney time ahead or behind Mexico City time?
- If he arrives in Mexico City after flying for 20 hours, what is the date and time in Mexico City?

THINK

- Count the number of time zones between Sydney and Mexico City.
- Did Dallas pass over the International Date Line?
- Calculate the difference between Sydney time (UTC + 10) and Mexico City time (UTC – 5).
- Places further to the west of the IDL are behind in time.
- 1 Calculate the time in Sydney by adding the length of the flight to the time that Dallas left Sydney.
2 Subtract the time difference between Sydney and Mexico city to calculate the time in Mexico City.
3 Write the answer.

WRITE

Dallas crossed 9 time zones.

Yes, Dallas passed over the International Date Line.

$$10 - -5 = 15 \text{ hours}$$

Sydney is ahead of Mexico City time.

$0900 + 20 \text{ hours} = 0500$ on the following day (Wednesday)

$$0500 - 15 \text{ hours} = 1400 \text{ on Tuesday}$$

Dallas arrived at 1400 in Mexico City on 11/04/2017 (Tuesday).

Exercise 7.4 Time zones

Understanding, fluency and communicating

Note: For the following questions, assume DST is not in effect unless the dates in the question indicate otherwise.

1. **WE7** Complete the following table to show the times in different cities in June. (*Note:* Auckland is 2 hours ahead of AEST.)

Time in Perth	Time in Adelaide	Time in Melbourne	Time in Auckland
2:00 pm		4:00 pm	
			9:00 pm
	8:00 am		
10:30 pm			

2. **WE8** What is the time in Sydney if the time in London (UTC) is:
 a. 11:00 am b. 12:30 pm c. noon?
3. What is the time in London (UTC) if the time in Sydney is:
 a. 3:30 am b. 5:50 pm c. noon?
4. Your friend in New York (UTC – 5) wishes to call you. If you wish to receive the telephone call at 6:00 pm AEST, at what time in New York should your friend make the call?
5. The 2010 FIFA World Cup soccer tournament was held in South Africa (UTC + 2). If the final started at 1:00 pm in July in Johannesburg, what time did the live television broadcast start in:
 a. Melbourne b. Sydney c. Brisbane?
6. The 2010 Winter Olympic Games were held in February in Vancouver, Canada (UTC – 8). If the opening ceremony began at 7:00 pm in Vancouver, what time did the live television broadcast start in:
 a. Victoria b. New South Wales c. Queensland?

(*Note:* In February, parts of Australia observe DST.)

7. Your friend's relatives live in Greece (UTC + 2), and each New Year's Eve they ring at exactly midnight in Greece to wish your friend happy New Year. What time does the telephone ring in your friend's house in Sydney?
8. A flight from Sydney to Perth can take 4 hours. If your flight departs Sydney airport at 2:10 pm (AEST), at what time will you arrive in Perth (AWST)?

A flight from Sydney to Perth takes about 4 hours.



9. A plane flew a direct route from Sydney to London. Some of the details of the trip are listed below.
- Departure date: Friday 5 August
 - Departure time: 8:00 am
 - Flight time: 25 hours

(*Note:* In August, London is observing DST.)

- a. What was the time and date in Sydney when the plane arrived in London?
 b. What was the local time when the plane arrived?

10. **MC** Which of the following is true outside DST? (*Note:* There may be more than one correct answer.)
- A.** Perth is 2 hours ahead of Melbourne.
 - B.** Broome (WA) is in the same time zone as Adelaide.
 - C.** London's time zone is called Coordinated Universal Time.
 - D.** Alice Springs (NT) is an hour and a half ahead of Fremantle (WA).
 - E.** Adelaide is 30 minutes ahead of Hobart

11. Match the following times in Australian places with the equivalent time in another Australian place in the blue box.

a	Perth (WA), 3:00 pm	Port Augusta (SA), 7:30 pm
b	Darwin (NT), 7:00 am	Hobart (Tas.), 1730 hours
c	Ballarat (Vic.), 2000 hours	Fremantle (WA), 9:30 am
d	Cairns (Qld), 0300 hours	Alice Springs (NT), 2:30 am
e	Coober Pedy (SA), 11:00 am	Broome (WA), 3:00 pm
f	Parkes (NSW), 5:30 pm	Canberra (ACT), 0730 hours

12. Change each of the following Eastern Standard Times (AEST) to Eastern Daylight-Saving Times (AEDT).

a. 7:00 am **b.** 10:00 am **c.** Noon **d.** 1:30 pm **e.** 3:45 pm

13. Change each of the following Eastern Standard Times (AEST) to Eastern Daylight-Saving Times (AEDT).

a. 10:15 pm **b.** 0630 **c.** 1500 **d.** 2118

14. Change each of the following Eastern Daylight-Saving Times (AEDT) to Eastern Standard Times (AEST).

a. 5:00 am **b.** 11:00 am **c.** Midnight **d.** 2:30 pm **e.** 5:45 pm

15. Change each of the following Eastern Daylight-Saving Times (AEDT) to Eastern Standard Times (AEST).

a. 9:50 pm **b.** 0430 **c.** 1300 **d.** 2255

16. **WE9** You would like to call your friend in Queensland on 3 February to wish him a happy birthday. If it is 8:00 am in Sydney, what time is it in Queensland?

(Note: In February, parts of Australia observe DST.)

17. During summer, what time is it in Sydney if it is:

a. noon in the Australian Capital Territory
b. 10:00 am in Melbourne
c. 7:45 am in Hobart
d. 5:15 pm in Darwin
e. 8:30 pm in Perth?



18. If it is 3:00 pm in Sydney during summer, what time is it in:

a. Melbourne **b.** Canberra **c.** Brisbane **d.** Perth **e.** Adelaide?

19. Consider workers who live in northern New South Wales and drive for 1 hour to the Gold Coast in Queensland each day for work during daylight-saving time.

a. What time do they need to leave home to be at work by 8:30 am?
b. If they leave work at 5:00 pm, what time will it be when they arrive home?

20. Match each of the following dates and times in Australian places with the equivalent time in another Australian place in the blue box.

a.	Swan Hill (Vic.), 3:00 pm, 17 February	Launceston (Tas.), 1:30 pm
b.	Geelong (Vic.), 7:30 am, 21 July	Townsville (Qld), 2:00 pm
c.	Sydney (NSW), 2130 hours, 31 October	Bendigo (Vic.), 11:00 am
d.	Newcastle (NSW), 11:00 am, 28 January	Port Hedland (WA), 0530 hours
e.	Brisbane (Qld), 12:30 pm, 19 November	Darwin (NT), 8:00 pm
f.	Fremantle (WA), 1400 hours, 15 August	Adelaide (SA), 3:30 pm

21. **WE10** Lillian left San Francisco on a plane at 2300 hours on Monday 12 June 2017. She flew west to Sydney. San Francisco's time zone is UTC – 7 in June.

a. Did Lillian pass over the International Date Line?
b. What is the time difference between San Francisco and Sydney?
c. Is Sydney ahead or behind San Francisco time?
d. If Lillian arrived in Sydney after flying for 15 hours, what is the date and time in Sydney?

Problem solving, reasoning and justification

22. Why are time zones necessary?
23. a. How does the local time change as you move around the world in a westerly direction?
b. How does the local time change as you move around the world in an easterly direction?
24. What change would you need to make to your watch on each of the following flights?
a. Melbourne to Perth
b. Darwin to Hobart
c. Melbourne to Los Angeles (UTC – 8)
25. You are visiting a friend in Victoria, and at 8:00 pm you realise that you have forgotten to watch the latest episode of your favourite television show, which starts at 7.30 pm. If you know people who live in the other Australian time zones, what could you do?
26. In an Australian reality television show a few years ago, a contestant from Perth was voted off the show as a result of a 20-minute voting period during the 1-hour show. The show was telecast live in the eastern states but delayed to the corresponding time in all other states. Use your knowledge of time zones to explain why her family may have been upset.

7.5 Review

7.5.1 Summary

In this topic you have learnt:

- the units into which time is divided
- how to convert between units of time
- how to convert from 12-hour time to 24-hour time (and vice versa)
- why time can be expressed in either 12-hour or 24-hour time and how to convert between them
- how to calculate how much time has elapsed between two clock readings in both 12-hour and 24-hour time
- how to interpret timetables and use them to solve problems
- how positions on the Earth's surface can be determined
- how to locate positions on the Earth's surface using latitude and longitude
- about the 24 time zones
- about the International Date Line (IDL)
- how to calculate times and time differences around the world
- how to make allowances for time differences due to daylight-saving time
- how to solve practical problems such as travelling east and west across time zones.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 7.5 Review

Understanding, fluency and communicating

1. Match the following units of time with a time in the blue box.

a	135 minutes	3.5 hours
b	2 hours 45 minutes	45 minutes
c	0.75 hours	2 hours 15 minutes
d	210 minutes	165 minutes

2. Calculate the time elapsed between the following times.
- a. 2:35 pm and 11:14 pm
- b. 14 November 2006 and 6 February 2008

3. Copy and complete the following table.

Time written in 12-hour time	Time written in 24-hour time
	0845
	2320
7:30 pm	

4. Change the following 12-hour times to 24-hour times.

- a. 3:58 am b. 12:00 noon c. 8:30 am
d. 3:15 pm e. 1:03 am f. 7:00 pm

5. Change the following 24-hour times to 12-hour times.

- a. 0010 hours b. 1548 hours c. 2300 hours
d. 1700 hours e. 2400 hours f. 0000 hours

6. How many time zones are there in Australia:

- a. when DST is not in effect b. when DST is in effect?

Refer to the ferry timetable below for questions 7 to 10.

City Loop Daily – Circular Quay – Luna Park – Darling Harbour – Luna Park – Circular Quay

	AM	AM	AM	AM	PM	PM	PM	PM	PM	PM	PM	PM	PM
Circular Quay – Harbour Master Steps		9.55	10.40	11.25	12.10	12.55	1.40	2.25	3.10 [#]	3.55	4.40 [#]	5.25	6.10 [^]
Luna Park		10.00	10.45	11.30	12.15	1.00	1.45	2.30	3.15 [#]	4.00	4.45 [#]	5.30 [^]	6.15 [^]
Darling Harbour – Harbourside		10.10	10.55	11.40	12.25	1.10	1.55	2.40	3.25 [#]	4.10	4.55 [#]	5.40 [^]	6.25 [^]
Darling Harbour – Pier 26	9.30	10.15	11.00	11.45	12.30	1.15	2.00	2.45	3.30 [#]	4.15	5.00 [#]	5.45	6.30 [^]
Circular Quay – No. 6 Jetty	*9.45	*10.30	*11.15	*12.00	*12.45	*1.30	*2.15	*3.00	*3.45 [#]	*4.30	*5.15 [#]	*6.00 [^]	

* Set down only.

[#] Services which operate on weekends only (year round) and during peak periods subject to demand

[^] Services which operate during peak periods subject to demand

7. If you leave Circular Quay at 9:55 am on Monday morning, what time do you arrive at Darling Harbour Pier 26?
8. If you catch the next ferry leaving Darling Harbour, when do you return to Circular Quay No. 6 Jetty?
9. Is it possible to leave Circular Quay No. 6 Jetty at noon? Explain your answer.
10. Why do you think most of the ferry departure times between 3:00 pm and 6:30 pm, shaded in green, have a greater chance of being crowded Monday through Friday?
11. Calculate the time in:
- a. Sydney when it is 9:00 pm in Perth on 23 July
- b. Perth when it is 1430 hours in Darwin on 1 November.
12. If the time in Sydney is 10:00 pm on April 20, will the time in the Northern Territory be before 10:00 pm, after 10:00 pm or exactly 10:00 pm?
13. If Maddy leaves Sydney on a plane at 1400 hours on January 15, travelling to Perth, is it possible for her to be in Perth by 1500 hours (Perth time) on the same day if the flight takes 5 hours? Explain your answer.
14. **MC** If the Australian Eastern Daylight time (AEDT) is 6:54 pm, what is the corresponding Australian Eastern Standard time (AEST)?
- A. 7:54 pm B. 3:54 pm C. 0554 hours
D. 5:54 pm E. None of the above

15. If it is 11:45 am in the Northern Territory in summer, what time is it in:
- | | | |
|--------------|-------------|--------------|
| a. Melbourne | b. Adelaide | c. Perth |
| d. Sydney | e. Brisbane | f. Canberra? |
16. If it is 8:00 pm in Perth in July, what time is it in:
- | | | |
|--------------|-------------|--------------|
| a. Melbourne | b. Adelaide | c. Darwin |
| d. Sydney | e. Brisbane | f. Canberra? |
17. Dave looked at the flight times in Sydney airport and saw that his flight to Buenos Aires, Argentina (UTC – 3) had been delayed 45 minutes. If the flight was originally due to leave Sydney at 10:45 am on 1 July, how would this effect his scheduled arrival time in Argentina?
18. Geoff's plane left Darwin airport at 0945 on 20 February, and it took 23 hours and 5 minutes to fly to New York City (UTC – 5).
- What is Geoff's arrival time in New York City?
 - What is the corresponding time in Sydney?
 - Would it be appropriate for Geoff to call his sons in Sydney when he arrives in New York City? Why or why not?
19. Glen wants to talk to his girlfriend in Sydney while he is on a surf trip in Papeete, Tahiti (UTC – 10). His girlfriend works from 9 am to 5 pm on Tuesday 5 February, and she goes to bed at 10:00 pm. On what day(s) and times in Tahiti can he call her without disturbing her at work or waking her up? (Note: Tahiti does not use daylight-saving time.)



20. Ritchie went on a cruise from Sydney to Hawaii (UTC – 10). How many time zones did he cross?

Problem solving, reasoning and justification

21. Work out the 12-hour time from the following information.
- The time is between 3:25 pm and 6:05 pm.
 - If you calculate the time difference between 6:05 pm and the mystery time, and the time difference between 3:25 pm and the mystery time, there is a 40-minute gap between the lower and upper time differences.
 - 30% of the time difference between 6:05 pm and the mystery time is equal to 50% of the difference between the mystery time and 3:25 pm.
22. On a 12-hour clock, how many times between 12:00 pm and 8:00 pm have consecutive number combinations? For example, 12:34 pm is a consecutive number combination consisting of 1, 2, 3, 4, but 1:34 pm is not a consecutive number combination because all the numbers in the time have to be consecutive, and 1, 3, 4 are not consecutive.

- 23.** On a 24-hour clock, how many times between 1200 hours and 2400 hours have consecutive number combinations? (All numbers included in the time have to be consecutive.)
- 24.** Are high-tide and low-tide times in a country affected by the daylight-saving time in that country? Explain your answer.
- 25.** Your friend from London is coming to visit you. She tells you that she is leaving London at 12:30 pm (UTC) on 15 January.
- What will be the time in Sydney when your friend departs London?
 - If she is scheduled to arrive in Sydney at 10:00 pm on 16 January, how long will she have been travelling?
 - Her flight from London is delayed by 115 minutes. What time will she actually walk off the plane in Sydney?
- 26.** Why do some countries have daylight-saving time but other countries do not?
- 27.** Why is Queensland time (AEST) only half an hour ahead of Northern Territory time rather than one hour ahead?
- 28.** The International Date Line (IDL) is not a straight line. Which country and which state of the USA does the IDL separate? Why do you think the IDL is designed to go between the two countries rather than cut through one of them?
- 29.** If you crossed the IDL on a Tuesday at 12:00 noon going from east to west, and 12.5 hours later you crossed back over the IDL, what day would it be?
- 30.** If you leave Sydney, Australia on a plane at 9:00 pm on December 31, 2017, and it takes approximately 4.5 hours flying time to reach the IDL:
- will you celebrate New Year's Eve west of the IDL
 - will you celebrate New Year's Eve east of the IDL
 - can you celebrate New Year's Eve twice? Explain your answer.



Topic 7 Answers

Exercise 7.2 Time and timetables

1. 3 hours 55 minutes
2. a. 120 minutes b. 210 minutes c. 1440 minutes d. 30 minutes e. 265 minutes
3. a. 375 minutes b. 130 minutes c. 455 minutes d. 45 minutes
4. a. 8:53 pm b. 9:04 pm c. 8:26 pm
5. a. 3600 seconds b. 86400 seconds c. 1440 minutes d. 8760 hours
6. 12.5 hours
7. a. 12:00 noon b. 6:00 pm c. 12:58 am
d. 12:00 midnight e. 7:00 am f. 11:00 pm
8. a. 1200 hours b. 0000 hours or 2400 hours c. 0300 hours
d. 1600 hours e. 0015 hours
9. a. 22 hours 20 minutes b. 2 hours 37 minutes
c. 3 hours 15 minutes d. 8 hours 35 minutes
10. a. 1 hour 5 minutes b. 1 hour 53 minutes c. 3 hours 45 minutes
d. 8 hours 15 minutes e. 35 hours 10 minutes f. 43 hours 49 minutes
11. a. 1:00 pm b. 4:15 am c. 11:50 am
d. 5:15 pm e. 5:39 pm f. 11:45 am
12. a. 8:00 pm b. 12:35 pm c. 9:15 am
13. 3 hours 45 minutes
14. 2 hours 50 minutes
15. No; you must change to a coach at Dubbo station.
16. You must book before departure from Strathfield.
17. Thursday at 1724 hours
18. No, it is the timetable listed as Dubbo to Lightning Ridge.
19. a. 12 hours 52 minutes b. 7 hours 54 minutes c. 4 hours 9 minutes
20. a. 4 b. 8
21. Blue indicates a high tide of 1.7 metres or more.
Green indicates a low tide of 0.3 metres or less.
22. Refer to the height measurement of the tide in column 'm', and look for the one or two highest numbers, or the one or two lowest numbers, on any given day.
23. a. 5:41 am and 6:32 pm b. 10:52 am and 11:15 pm
24. a. 5:33 am and 5:42 pm b. 1:13 am and 2:11 pm
25. The times between high and low tide are approximately 6 hours. Therefore, there is an 18-hour gap between the first tide and last of the day. If the first tide of the day is less than 18 hours away from the end of the day, then the 4th tide will be the first tide of the next day. See 2 March as an example.
26. a. 28 April, 3:30 am b. 28 April, 9:45 pm
27. a. No b. Yes
28. 18 hours 33 minutes
29. a. No b. No
30. a. The armed forces use 24-hour time in order to prevent confusion between am and pm in high-pressure situations.
b. Airlines use 24-hour time so that there is never any confusion about when planes are taking off or landing.
c. Answers will vary but could include schools or hospitals.
31. a. The calculation is correct but the interpretation of the answer is incorrect. It should be 2 hours 35 minutes rather than 235 minutes.
b. The calculation is incorrect; the times have been subtracted as if they were decimals. The correct answer is 1 hour 44 minutes.
32. Knowing departure and arrival times allows you to calculate the duration of the trip. Also, a different departure time will cause the arrival time and possibly the duration time to be different.
33. Answers will vary. An example meal is provided below.

For all food items to be ready for dinner at 7:00 pm, the following timetable has been designed.

Ingredients for roast chicken and vegetables	Time to cook ingredient	Start cooking	Ingredient ready
Chicken	2.5 hours	4:30 pm	7:00 pm
Potatoes	35 minutes	6:25 pm	7:00 pm
Pumpkin	25 minutes	6:35 pm	7:00 pm
Beans	15 minutes	6:45 pm	7:00 pm

34. a. $\frac{1}{4}$ of the distance between 3 and 4 on the clock.
 b. The hour hand has moved $\frac{1}{30}$ ahead of its previous position.

Exercise 7.4 Time zones

Time in Perth	Time in Adelaide	Time in Melbourne	Time in Auckland
2:00 pm	3:30 pm	4:00 pm	6:00 pm
5:00 pm	6:30 pm	7:00 pm	9:00 pm
6:30 pm	8:00 am	8:30 am	10:30 am
10:30 pm	12:00 midnight	12:30 am	2:30 am
- 9:00 pm
 - 10:30 pm
 - 10:00 pm
- 5:30 pm
 - 7:50 am
 - 2:00 am
- 3:00 am
- 9:00 pm
 - 9:00 pm
 - 9:00 pm
- 2:00 pm
 - 2:00 pm
 - 1:00 pm
- 9:00 am
- 4:10 pm
- 9:00 am Saturday 6 August
 - Midnight Saturday 6 August
- A, C, D, E
- Broome
 - Canberra
 - Port Augusta
 - Alice Springs
 - Fremantle
 - Hobart
- 8:00 am
 - 11:00 am
 - 1:00 pm
 - 2:30 pm
 - 4:45 pm
- 11:15 pm
 - 0730 hours
 - 1600 hours
 - 2218 hours
- 4:00 am
 - 10:00 am
 - 11:00 pm
 - 1:30 pm
 - 4:45 pm
- 8:50 pm
 - 0330 hours
 - 1200 hours
 - 2155 hours
- 7:00 am
- Noon
 - 10:00 am
 - 7:45 am
 - 6:45 pm
 - 11:30 pm
- 3:00 pm
 - 3:00 pm
 - 2:00 pm
 - Noon
 - 2:30 pm
- 8:30 am
 - 7:00 pm
- Townsville
 - Port Hedland
 - Darwin
 - Bendigo
 - Launceston
 - Adelaide
- Yes
 - 17 hours
- Ahead
 - 0700 hours on Wednesday 14 June
- Time zones are necessary to account for the rotation of the Earth. If there were only one time zone, midnight would be the correct time for some places on earth but in the middle of the day for others. The time zones were created in the nineteenth century to account for these differences.
- You lose time.
 - You gain time.
- 2 hours
 - + 0.5 hours
 - 18 hours
- Ask anyone in South Australia, Northern Territory or Western Australia to record the show.

26. Assuming the contestant's family lived in Perth, they would have already been informed by their daughter that she was off the show before it aired in Perth. Perth time is 2 hours behind the Eastern States when DST is not in effect, and 3 hours behind when DST is in effect.

Exercise 7.5 Review

1. a. 2 hours 15 minutes b. 165 minutes c. 45 minutes d. 3.5 hours
2. a. 8 hours 39 minutes b. 449 days
3.

Time written in 12-hour time	Time written in 24-hour time
8:45 am	0845
11:20 pm	2320
7:30 pm	1930
4. a. 0358 b. 1200 c. 0830 d. 1515 e. 0103 f. 1900
5. a. 12:10 am b. 3:48 pm c. 11:00 pm
d. 5:00 pm e. 12:00 midnight f. 12:00 midnight
6. a. 3 b. 5
7. 10:15 am
8. 11:15 am
9. No, as it is a drop-off-only stop.
10. This is considered to be peak hour for commuters leaving work.
11. a. 11:00 pm b. 1300 hours
12. Before 10:00 pm
13. No. If Maddy leaves Sydney during DST, then she arrives in Perth at 1900 hours Sydney time, which is 1600 hours Perth time. Perth has a 3-hour difference from Sydney during DST, as Western Australia does not follow DST.
14. D
15. a. 1:15 pm b. 12:45 pm c. 10:15 am d. 1:15 pm e. 12:15 pm f. 1:15 pm
16. a. 10:00 pm b. 9:30 pm c. 9:30 pm d. 10:00 pm e. 10:00 pm f. 10:00 pm
17. The 45-minute late departure would delay the arrival time by 45 minutes.
18. a. 6:20 pm on 20 February
b. 10:20 am on 21 February
c. Answers will vary. Geoff's sons might be at school or work at that time, but it is not an unreasonable time to call.
19. Between the hours of 8:00 pm on February 4 and 1:00 am on 5 February
20. At least 4 time zones will be crossed.
21. 4:25 pm
22. 5 times
23. 2 times
24. Yes and no. For example, in NSW, you have to adjust the tide table when daylight saving is in effect because the tide table is based upon Eastern Standard Time. The tides themselves are not affected, but the times at which people observe the tides are.
25. a. 11:30 pm b. She has been travelling for 22.5 hours.
c. 11:55 pm
26. Answers will vary. Daylight-saving time (DST) was originally created to save electricity. Many countries do not observe DST because some citizens do not like to get home from work when it is dark. Other countries do not like their school children waking up in the dark, while some citizens find DST confusing.
27. Answers will vary. There is no solid explanation for this anomaly. The Northern Territory and South Australia sit on almost 135 degrees west, so they probably should be on the UTC + 9 time zone.
28. Russia and Alaska. The Russians and Americans didn't want two parts of their countries to be on different sides of the IDL.
29. Thursday at 12:30 am on the west side and Wednesday at 12:30 am on the east side.
30. a. Yes b. Yes
a. Yes. If you go from east to west you can, or if you cross the IDL from the east.

TOPIC 8

Interest and depreciation

8.1 Overview

8.1.1 Introduction

Did you know that \$1 today is worth more than \$1 next year? Due to inflation the value of the dollar gradually decreases year on year, but this effect is usually more noticeable when looking at longer periods of time.

However there are ways that we can increase the amount of money we have without having to work. By putting money in a bank or savings account you will earn interest on your money, which means the amount in the account will steadily increase over time.

Knowing how the interest on the balance in your bank account, or on the mortgage of your house, is calculated can help you to make smarter choices with your finances. Learning about interest and depreciation can really pay off in the long term!



DISCUSSION

What strategies could you employ to ensure that you save more money?

LEARNING SEQUENCE

- 8.1 Overview
- 8.2 Percentage increase or decrease
- 8.3 Simple interest
- 8.4 Applications of simple interest
- 8.5 Review

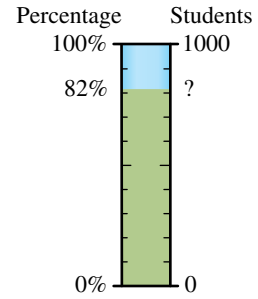
CURRICULUM CONTENT

- calculate simple interest for different rates and periods (ACMEM064) ◇ 
- calculate the depreciation of an asset using the straight-line method as an application of the simple interest formula **AAM** ◇
- use a spreadsheet to calculate and graph compound interest as a recurrence relation involving repeated applications of simple interest (ACMEM169) **AAM** ◇ 

8.2 Percentage increase or decrease

8.2.1 Finding the percentage of an amount

- Quantities are often expressed as the percentage of an amount. For example, 82% of 1000 Year 8 students believe that they get too much homework. This statement gives an indication of the proportion or amount of the 1000 Year 8 population that feels this way.
- Calculations involving a percentage of an amount can be performed by changing the percentage to a fraction and finding this fraction of the amount.



WORKED EXAMPLE 1

Calculate 15% of \$140.

THINK

- Convert 15% to a fraction and change 'of' to $\times$.
- Simplify by cancelling common factors and then multiply the fractions (highest common factor (HCF) of 15 and 100 is 5).
- Simplify the answer if possible.
- Answer the question.

WRITE

$$\begin{aligned}
 15\% \text{ of } \$140 &= \frac{15}{100} \times \frac{140}{1} \\
 &= \frac{3\cancel{15}}{20\cancel{100}} \times \frac{140}{1} \\
 &= \frac{3}{20} \times \frac{140}{1} \\
 &= \frac{420}{20} = 21
 \end{aligned}$$

15% of \$140 is \$21.

8.2.2 Percentage increases

- A percentage increase is the percentage by which an amount is increased.
- Calculations involving percentage increases can be found by finding the percentage of an amount and adding it to that amount.

WORKED EXAMPLE 2

A shop attendant has to label items with price tags. If the shop normally increases prices by 15%, what price should he put on the label of an item that the shop bought for \$55?

THINK

- Calculate 15% of \$55 by first expressing both as fractions.
- Simplify and then multiply the two fractions. Simplify the answer.
- Add this answer to the original price.
- Answer the question.

WRITE

$$\begin{aligned}
 15\% \text{ of } 55 &= \frac{15}{100} \times \frac{55}{1} \\
 &= \frac{3\cancel{15}}{20\cancel{100}} \times \frac{55}{1} \\
 &= \frac{3}{20} \times \frac{55}{1} \\
 &= \frac{165}{20} \\
 &= \$8.25
 \end{aligned}$$

$$\$55 + \$8.25 = \$63.25$$

He should label the item at \$63.25.

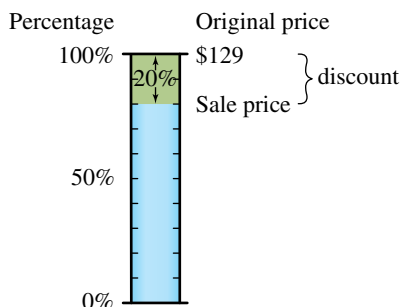
8.2.3 Discounts

- A **discount** is a price reduction on an item. It is usually expressed as a percentage or fraction of the original price of the item or as a dollars and cents amount. The amount of money by which a price has been reduced will depend on the percentage reduction and the original price of the item.



- The **sale price** is the result of subtracting the discount from the original price.

$$\text{Sale price} = \text{original price} - \text{discount}$$



- The **percentage discount** is the percentage of the original price that is subtracted to give the reduced price. For example, a pair of shoes may be offered at a 20% discount.

WORKED EXAMPLE 3

A pair of running shoes normally sells for \$129, but this week is being offered by the local shoe shop at a 20% discount. What is the sale price?

THINK

- Convert 20% to a decimal.
- The discount is 20% of \$129. Calculate 20% of \$129 by multiplying the original price by 0.2.
- To calculate the sale price, subtract the discount of \$25.80 from the original price of \$129.
- Answer the question.

WRITE

$$20\% = \frac{20}{100} = 0.2$$

$$0.2 \times 129 = 25.8$$

This is the discount.

$$129 - 25.8 = \$103.20$$

The sale price is \$103.20.

8.2.4 Goods and services tax (GST)

- The **goods and services tax (GST)** is a tax that is added to the cost of most goods and services in Australia. The GST is currently 10%, which means that 10% is added to the cost of most goods and services.
- The prices of most goods and services bought in Australia include GST. To determine the price of the goods and services before GST was applied, the selling price is divided by 1.1 (100% + 10% GST).

WORKED EXAMPLE 4

The price of a football before GST is added is \$25.50. Determine the price of the football after GST is added.

THINK

- 1 Convert 10% into a decimal.
- 2 Calculate 10% of the football by multiplying 0.1 by \$25.50.
- 3 To calculate the price of the football, add the GST amount of \$2.55 to the price of the football before GST.
- 4 Answer the question.

WRITE

$$\frac{10}{100} = 0.1$$

$$0.1 \times 25.50 = 2.55$$

$$2.55 + 25.50 = 28.05$$

The price of the football after GST is added is \$28.05.

WORKED EXAMPLE 5

The selling price (including GST) of a pair of runners is \$130. Calculate the price of the runners before GST was added.



THINK

- 1 Write the selling price (including GST) of the runners.
- 2 Since this price includes GST, divide it by 1.1 (100% + 10% (GST)) to obtain the price before GST was added.
- 3 Round the answer to 2 decimal places (cents).
- 4 Answer the question.

WRITE

$$\text{\$130}$$

$$\frac{130}{1.1} = \$118.1818$$

$$\approx \$118.18$$

The price of the runners before GST was added is \$118.18.

8.2.5 Profit or loss

- **Profit** is the difference between the amount the seller pays for the goods or services (the cost) and the amount for which the seller sells the same goods or services (the price).
A **net profit** occurs if the price is greater than the cost.
A **loss** occurs if the price is less than the cost.
- Profit and loss can be calculated in both absolute and percentage terms.
Absolute profit and **absolute loss** finds the difference between the selling and buying (original) prices.

$$\text{Absolute profit} = \text{selling price} - \text{original price}$$

$$\text{Absolute loss} = \text{original price} - \text{selling price}$$

Note: For an absolute loss, the selling price must be less than the original price.

- To calculate the profit or loss in percentage terms, express the profit/loss as a fraction of the original price, then convert the fraction to a percentage.

$$\text{Percentage profit/loss} = \frac{\text{absolute profit/loss}}{\text{original price}} \times 100$$

WORKED EXAMPLE 6

A coin collector bought a rare coin for \$560 and sold it 2 years later for \$680.

- Calculate the absolute profit or loss.
- Calculate the percentage profit or loss. Write your answer correct to 2 decimal places.



THINK

- Find the difference between the sold price and the coin collector's bought price.
- If the amount sold is greater than the amount it was bought for, then it is a profit.
- Find the difference between the sold price and the coin collector's bought price.
- Write this difference as a fraction of the bought price (original price).
- Convert the fraction to a percentage by multiplying it by 100.
- Round the answer to 2 decimal places.
- Answer the question.

WRITE

$$680 - 560 = \$120$$

The absolute profit made by the coin collector is \$120.

$$680 - 560 = \$120$$

$$\frac{120}{560}$$

$$\frac{120}{560} \times 100 = 21.428\,57\%$$

$$\approx 21.43\%$$

The coin collector made a 21.43% profit.

eBookplus RESOURCES

eLesson: Percentages of an amount (eles-1882)

Interactivity: Percentage of an amount (int-3743)

Interactivity: Selling price (int-3745)

Interactivity: Profit and loss (int-3746)

Exercise 8.2 Percentage increase or decrease

Understanding, fluency and communicating

- WE1** Calculate the value of each of the following by converting the percentage to a fraction and simplifying.

a. 20% of \$50

b. 80% of \$70

c. 5% of \$40

d. 95% of \$150

e. $\frac{3}{4}\%$ of \$1600

2. Calculate the value of each of the following.

a. 17.5% of \$275

b. $2\frac{1}{2}\%$ of \$15 400

c. 47% of \$978

d. $33\frac{1}{3}\%$ of \$72

e. 110% of \$25 325

3. **WE3** Calculate the sale price for each of the following items.

a. A skirt originally priced at \$64 is discounted by 25%.

b. A CD valued at \$29 is discounted by 10%.

c. Roller blades originally priced at \$149 are discounted by 15%.

d. A digital camera with a marked price of \$99 is discounted by 40%.

4. **WE2** A shop attendant at a sports store has to label items with price tags. If the store normally increases prices by 35%, what price should be put on the tennis racquet that the store bought for \$250?

5. A personal assistant currently makes \$30 per hour and is given an increase of 3.75%. What is his new hourly rate?

6. An office manager earns an annual salary of \$55 000. She is given a salary increase of 1% for one year, and then another 3% for the subsequent year. What is her new salary after 2 years?

7. **MC** The sale price of the car is:

A. \$25 000

B. \$27 000

C. \$28 000

D. \$28 500

E. \$29 850



8. The following table shows the buying price and selling price for five items. Copy and complete the table.

	Buying price	Selling price	Absolute	
			Profit	Loss
a	1500	1895		
b		342	129	
c	6410			2122
d	3200		460	
e		1157		260

9. **WE6** Calculate the percentage profit or loss for the following sales. Give your answers to 2 decimal places.

a. An antique lamp is bought for \$325 and sold for \$415.

b. The selling price of a baseball mitt at a garage sale was \$16.50. The baseball mitt was originally bought for \$25.95.

c. A car was bought for \$13 950 and sold for \$5800 after 8 years.

d. The purchase price of a gold necklace was \$950. It was sold for \$1080.

10. **WE4 MC** An electrician charged \$450 plus GST. The total charges are:
A. \$45 **B.** \$405 **C.** \$409 **D.** \$460 **E.** \$495
11. **WE5 MC** The price of a packet of cereal is \$5.45 including GST. What was the price of the packet of cereal before GST was added?
A. \$0.50 **B.** \$0.55 **C.** \$4.91 **D.** \$4.95 **E.** \$6.00
12. **MC** Which one of the following is the greatest percentage profit?
A. A speedboat is bought for \$25 400 and sold for \$20 350.
B. Two baseball cards are purchased for \$1.85 and sold for \$3.65.
C. A vintage toy car is purchased for \$35 and sold for \$45.
D. Three items of costume jewellery are bought for \$45 and sold in a garage sale for \$25.
E. An original oil painting is purchased for \$950 and sold for \$1350.



Problem solving, reasoning and justification

13. In your own words, explain two different methods that can be used to calculate the sale price of an item when you are given the original price and the percentage discount.
14. **a.** If you increase \$100 by 30% and then decrease the new amount by 30%, will you end up with more than \$100, less than \$100 or exactly \$100?
b. Explain your answer to part **a**, using mathematics to support your answer.
c. If you decrease \$100 by 30% and then increase the new amount by 30%, will you end up with more than \$100, less than \$100 or exactly \$100?
d. Explain your answer to part **c**, using mathematics to support your answer.
15. A salesperson's salary is \$67 800, and she is offered an increase of 5% for 2 years, or 2% for the first year and 3% for the second year. Which option will give her the higher salary after 2 years? Justify your answer using mathematical calculations.
16. A boat is bought for \$4500 and sold 3 years later for a 45% profit. Determine the selling price of the boat. Hence, explain the method to determine the selling price of an item when given the percentage profit or loss.
17. To determine the price of an item before GST was added, the shop owner calculates 10% of the total cost including GST and then subtracts this amount from the selling price. Explain, using mathematical calculations, why this method will not find the price of items before GST is added.



8.3 Simple interest

8.3.1 Principal, interest and time period

- When you put money in a financial institution such as a bank or credit union, the amount of money you start with is called the **principal**.
- People who place money in a bank or financial institution (investors) receive a payment called **interest** from the financial institution in return for leaving their money with it.
- The amount of interest is determined by the **interest rate**.

- An interest rate is quoted as a percentage for a given time period, usually a year. For example, a bank might offer 5.8% per year interest on its savings accounts. This is also written as 5.8% per annum, or 5.8% p.a.

8.3.2 Simple interest calculations

- Simple interest** is the interest paid on the principal amount of an investment.
- Since the amount of interest paid each time period is based on the principal, the amount of interest is constant.
 - For example, \$500 is placed in an account that earns 10% p.a. simple interest every year.

Time period (years)	Amount of money at the start of the year	Interest for the year
1	\$500	\$50
2	\$550	\$50
3	\$600	\$50
4	\$650	\$50

- The total interest paid is $4 \times 50 = \$200$, so the value of the investment after 4 years is $500 + 200 = \$700$.
- Simple interest is calculated using the formula below.

$$I = \frac{P \times R \times T}{100}$$

where I = interest

P = principal

R = interest rate (as a percentage)

T = the number of time periods for which the money is invested.

- If R is given as a percentage per annum, then the time, T , must be given in years. If R is given as a percentage per month, then T must be given in months.
- The total amount of money is known as the **value** of the investment.

$$\text{Value} = \text{principal } (P) + \text{interest } (I)$$

WORKED EXAMPLE 7

A real-estate developer offers investors a chance to invest in their latest development. If \$10000 is invested, the developer will pay 11.5% simple interest per year for 5 years. Calculate the value of the investment at the end of 5 years.

THINK

- Identify the known quantities:
 - principal (P): amount of money invested
 - interest rate (R)
 - number of time periods (T) for which the money is invested.

- Calculate the interest (I) using the formula $I = \frac{P \times R \times T}{100}$.

- The value of the investment is the sum of the interest (\$5750) and the principal (\$10000).

- Answer the question.

WRITE

$$P = 10000$$

$$R = 11.5$$

$$T = 5$$

$$\begin{aligned} \text{Interest} &= \frac{10000 \times 11.5 \times 5}{100} \\ &= 5750 \end{aligned}$$

$$\begin{aligned} \text{Value} &= 10000 + 5750 \\ &= 15750 \end{aligned}$$

The value of the investment after 5 years is \$15 750.

- In some cases, the time period for which the money is invested is not a multiple of the time quantity that the interest rate is quoted as; for example, if the interest rate is given as a per annum rate and the money is invested for 3 months. In these instances it is necessary to calculate the equivalent interest rate for the time units for which the money is invested.

WORKED EXAMPLE 8

A bank offers interest on its savings account at 4% per annum. If a Year 9 boy opens an account with \$600 and leaves his money there for 4 months, how much interest does he earn?

THINK

- 1 • The interest rate period is per annum and the amount of time the money is invested for is in months. Calculate an equivalent monthly interest rate.
 - Since there are 12 months in a year, divide the annual interest rate by 12 to get a monthly interest rate.
- 2 Identify the known quantities. Note that we are now dealing in months, rather than years.
 - Principal (P)
 - Interest rate (R) per month
 - Time period (T), number of months

- 3 Calculate the interest (I) using the formula $I = \frac{P \times R \times T}{100}$.

- 4 Answer the question.

WRITE

$$\frac{4}{12} = \frac{1}{3}$$

4% p.a. is $\frac{1}{3}\%$ per month.

$$P = 600$$

$$R = \frac{1}{3}$$

$$T = 4$$

$$\text{Interest} = \frac{600 \times \frac{1}{3} \times 4}{100}$$

$$= 8$$

The total interest is \$8.

8.3.3 Simple interest graphs

- A simple interest graph shows the growth of the invested amount over a time interval. The simple interest graph will form a straight line because the amount the investment is growing by each year is the same (constant). The amount, in dollars, of growth (interest earned) will be the gradient of the line.
- Constructing a simple interest graph requires the calculation of simple interest at various times, and then plotting these values against time as coordinates on a set of axes.

WORKED EXAMPLE 9

A sum of \$1200 is invested in a savings account that attracts 5% per year simple interest. Construct a graph that shows the amount the investment is worth over five years.

THINK

- 1 Find the principal, P , and rate, R , of the investment.

WRITE

$$P = \$1200$$

$$R = 5$$



- 2 Calculate the amount of interest earned each year.

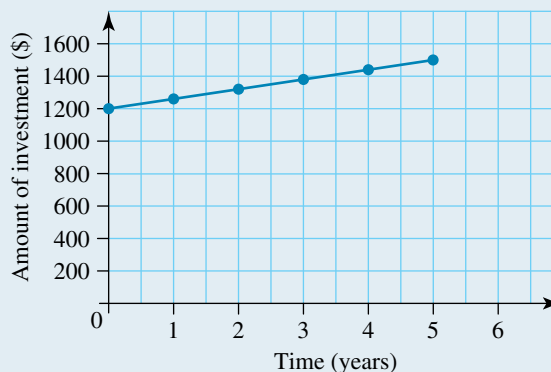
$$\begin{aligned}T &= 1 \\I &= \frac{PRT}{100} \\&= \frac{1200 \times 5 \times 1}{100} \\&= 60\end{aligned}$$

\$60 interest is earned each year.

- 3 Calculate the amount the investment is worth over the five years.

Year	Investment amount (\$)
0	1200
1	$1200 + 60 = 1260$
2	$1260 + 60 = 1320$
3	$1320 + 60 = 1380$
4	$1380 + 60 = 1440$
5	$1440 + 60 = 1500$

- 4 Plot the points on a set of axes.



eBookplus RESOURCES

Interactivity: Simple interest (int-6074)

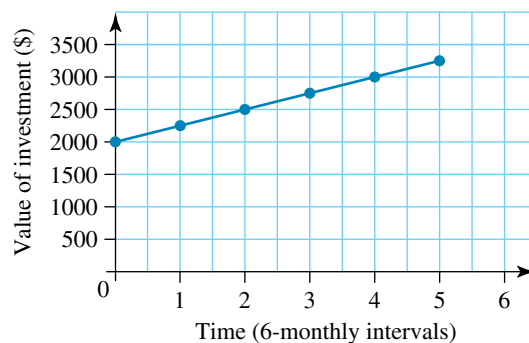
Exercise 8.3 Simple interest

Understanding, fluency and communicating

1. **WE7** A film producer offers investors the chance to invest in his latest movie. If \$20 000 is invested, the producer will pay 22.3% simple interest per year for 2 years. Determine the value of the investment at the end of the 2 years.



2. **MC** If an investment of \$400 pays 8% simple interest per year, then the value of the investment at the end of 3 years is:
A. \$32 **B.** \$96 **C.** \$432 **D.** \$496 **E.** \$1296
3. Calculate the simple interest paid on the following investments.
a. \$500 at 6.7% per year for 2 years
b. \$500 at 6.7% per year for 4 years
c. \$1000 at 6.7% per year for 4 years
4. Calculate the amount of interest paid on a \$1000 investment at 5% per year for 5 years.
5. **WE9** Construct simple interest graphs for the following investments, where T = time in years.
a. $P = \$500$, $R = 3\%$ p.a., $T = 4$
b. $P = \$1500$, $R = 7\%$ p.a., $T = 5$
c. \$12 000 invested at 6.5% p.a. for 6 years.
6. **MC** For the simple interest graph shown, the total amount of interest earned on the investment in 1 year is:



- A.** \$250 **B.** \$500 **C.** \$1250 **D.** \$2000 **E.** \$3250
7. **WE8** A bank offers interest on its savings account of 6% p.a. If a Year 9 girl opens an account with \$750 and leaves her money there for 5 months, how much interest does she earn?
8. **MC** If the annual interest rate is 8%, then the monthly interest rate is:
A. 0.8% **B.** 0.77% **C.** 0.67%
D. 0.6% **E.** 0.4%
9. Calculate the interest paid on the following investments.
a. \$500 invested at 8% per annum for 1 month
b. \$500 invested at 8% per annum for 3 months
c. \$500 invested at 8% per annum for 6 months
10. A bank offers investors an annual interest rate of 9% if they buy a term deposit. If a customer has \$5600 and leaves the money in the term deposit for 2.5 years, what is the value of the investment at the end of the 2.5 years?



Problem solving, reasoning and justification

11. In your own words, explain the difference in meaning in the following pairs of terms:
a. principal and value of the investment
b. amount of interest and interest rate.
12. Bank A offers an interest rate of 7.8% on investments, while Bank B offers an interest rate of 7.4% in the first year and 7.9% in subsequent years. If a customer has \$20 000 to invest for 3 years, which is the better investment?

13. Your parents decide to borrow money to improve their boat, but cannot agree which loan is the better value. They would like to borrow \$2550. Your mother goes to Bank A and finds that they will lend the money at $11\frac{1}{3}\%$ p.a. simple interest for 3 years. Your father finds that Bank B will lend the \$2550 at 1% per month simple interest.

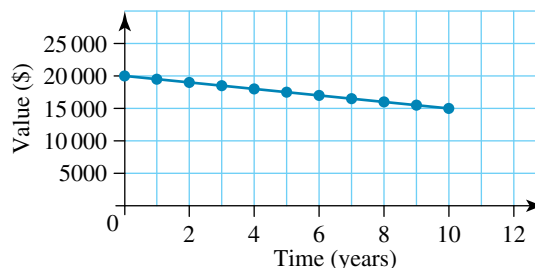


- a. Which bank offers the better rate over the three years?
 - b. Provide reasons for your answer to part a.
14. A loan is an investment in reverse; you *borrow* money from a bank and are *charged* interest. The value of a loan becomes its total cost.
- a. A worker wishes to borrow \$10000 from a bank, which charges 11.5% interest per year. If the loan is over 2 years:
 - i. calculate the total interest paid
 - ii. calculate the total cost of the loan.
 - b. Most loans require a monthly payment. The monthly payment of a simple interest loan is calculated by dividing the total cost by the number of payments made during the term of the loan. Determine the monthly payment for the loan in part a.
15. A student wishes to invest money to buy a car. She chooses a savings account that pays 7.5% p.a. simple interest for 5 years. She plans on buying a car and will need to earn \$3750 in interest over the 5 years.
- a. How much does she need to invest so that she will have enough money to buy the car after 5 years?
She invests the amount found in part a and after 2 years, she is able to reinvest the original principal plus interest into another account that pays 8.25% p.a.
 - b. How much will the investment be worth after the end of the next 3 years?
 - c. How much more interest, in dollars, does she earn by changing to the other account?
After the 5 years, she sees a car to buy which is priced at \$18900.
 - d. If she can negotiate a price, what percentage discount will she need to ask for so that she can afford to buy the car? Write your answer to the nearest whole percent.

8.4 Applications of simple interest

8.4.1 Straight-line depreciation

- Some items we purchase decrease in value over time due to wear and tear, and age. This is known as **depreciation**. When an item depreciates by a set amount in dollars each year, it is known as **straight-line depreciation**.
- Calculating the value of an item using straight-line depreciation involves finding the percentage amount of the initial value of the item and then subtracting that amount each year. Items depreciate on a yearly basis, so the time and rate (%) of depreciation is in years or per annum.



$$\text{Value} = \text{amount} - \frac{\text{amount} \times R \times T}{100}$$

WORKED EXAMPLE 10

A car's value depreciates by straight-line depreciation of 8%. If the car's initial value is \$12500, calculate its value after 5 years.



THINK

- 1 Convert 8% to a decimal.
- 2 Find 8% of \$12500.
- 3 Subtract the amount in step 2 from the initial value of the car. Continue to subtract \$1000 from the car's value at the end of each year.
- 4 Answer the question.

WRITE

$$\frac{8}{100} = 0.08$$

$$0.08 \times 12\,500 = 1000$$

The car decreases by \$1000 each year.

Time	Value
0 (initially)	12 500
1	$12\,500 - 1000 = 11\,500$
2	$11\,500 - 1000 = 10\,500$
3	$10\,500 - 1000 = 9\,500$
4	$9\,500 - 1000 = 8\,500$
5	$8\,500 - 1000 = 7\,500$

The car's value after 5 years is \$7500.

8.4.2 Compound interest

- With **compound interest**, investors get interest on the current total in the account. This includes both the principal and the interest that has already been paid.
- Investments accumulate more money and grow faster with compound interest than with simple interest.
- The **compounding period** is the time after which interest is calculated and added to the principal.
- This 'new principal' (interest + principal) earns interest after a second compounding period elapses. This is repeated for the total time of the investment. This is also known as 'earning interest on interest'.
- One method of calculating compound interest is to use a table.
 - Step 1: Identify the principal, interest rate and compounding period.
 - Step 2: Calculate the interest for the first compounding period.
 - Step 3: Add the interest to the principal; this becomes the 'new principal'.
 - Repeat steps 2 and 3 for as many compounding periods as are required.

WORKED EXAMPLE 11

A bank offers interest at 8% per year, compounded each year. If a customer puts \$500 in the bank, how much will be in the bank after 3 years?

THINK

- 1 Calculate the interest on \$500 at 8% for 1 year.

WRITE

$$\text{Interest} = \frac{PRT}{100} = \frac{500 \times 8 \times 1}{100} = 40$$

- 2 Add the interest of \$40 to the principal of \$500 to obtain the new principal. Use this new principal for the second time period.

$$500 + 40 = 540$$

- 3 Calculate the interest on \$540 at 8% for the second year.

$$\text{Interest} = \frac{PRT}{100} = \frac{540 \times 8 \times 1}{100} = 43.20$$

- 4 Add the interest of \$43.20 to the principal of \$540 to obtain the new principal. Use this new principal for the third time period.

$$540 + 43.20 = 583.20$$

- 5 Calculate the interest on \$583.20 at 8% for the third year.

$$\text{Interest} = \frac{PRT}{100} = \frac{583.20 \times 8 \times 1}{100} = 46.66 \text{ (rounded to 2 d.p.)}$$

- 6 Add the interest of \$46.66 to the principal of \$583.20 to obtain the new principal.

$$583.20 + 46.66 = 629.86$$

- 7 Answer the question.

After 3 years there will be \$629.86 in the bank.

Time period	Current principal	Interest paid	New principal
1 year	500	$\text{Interest} = \frac{PRT}{100} = \frac{500 \times 8 \times 1}{100} = 40$	$500 + 40 = 540$
2 years	540	$\text{Interest} = \frac{PRT}{100} = \frac{540 \times 8 \times 1}{100} = 43.20$	$540 + 43.20 = 583.20$
3 years	583.20	$\text{Interest} = \frac{PRT}{100} = \frac{583.20 \times 8 \times 1}{100} = 46.66$ (rounded to 2 d.p.)	$583.20 + 46.66 = 629.86$

- If the compounding period is not the same as the interest rate period, it is necessary to calculate the interest rate for the compounding period.
 - For example, a bank may offer an interest rate of 8% per annum, but the compounding period could be quarterly. This means that the interest will compound at the rate of 2% quarterly.

WORKED EXAMPLE 12

A bank offers a term deposit for 2 years, at an interest rate of 12% per year with a compounding period of 6 months. What would be the end value of a \$10 000 investment under these conditions?

THINK

- 1 • Convert the interest rate to that of the compounding period.
Since the interest rate is quoted for 1 year and the compounding period is 6 months, divide the interest rate by 2.
 - Calculate the number of compounding periods in the investment period. There are 2 six-month periods per year. Multiply the number of years by 2.
- 2 Calculate the interest on \$10 000 at 6% for 6 months.
- 3 Add the interest of \$600 to the principal of \$10 000 to obtain the new principal. Use this new principal for the second time period (12 months).
- 4 Calculate the interest on \$10 600 at 6% for 6 months.
- 5 Add the interest of \$636 to the principal of \$10 600 to obtain the new principal. Use this new principal for the third time period (18 months).
- 6 Calculate the interest on \$11 236 at 6% for 6 months.
- 7 Add the interest of \$674.16 to the principal of \$11 236 to obtain the new principal. Use this new principal for the fourth time period (24 months).
- 8 Calculate the interest on \$11 910.16 at 6% for 6 months.
- 9 Add the interest of \$714.61 to the principal of \$11 910.16 to obtain the new principal.
- 10 Answer the question.

WRITE

$$I = 12\% \text{ p.a.}$$

The 6-monthly interest rate is 6%.

$$\begin{aligned} 2 \text{ years} &= (2 \times 2) \text{ six-month periods} \\ &= 4 \text{ six-month periods} \end{aligned}$$

$$\begin{aligned} \text{Interest} &= \frac{PRT}{100} \\ &= \frac{10\,000 \times 6 \times 1}{100} \\ &= 600 \end{aligned}$$

$$10\,000 + 600 = 10\,600$$

$$\begin{aligned} \text{Interest} &= \frac{PRT}{100} \\ &= \frac{10\,600 \times 6 \times 1}{100} \\ &= 636 \end{aligned}$$

$$10\,600 + 636 = 11\,236$$

$$\begin{aligned} \text{Interest} &= \frac{PRT}{100} \\ &= \frac{11\,236 \times 6 \times 1}{100} \\ &= 674.16 \end{aligned}$$

$$11\,236 + 674.16 = 11\,910.16$$

$$\begin{aligned} \text{Interest} &= \frac{PRT}{100} \\ &= \frac{11\,910.16 \times 6 \times 1}{100} \\ &= 714.61 \text{ (rounded to 2 d.p.)} \end{aligned}$$

$$11\,910.16 + 714.61 = 12\,624.77$$

The value of the investment after 2 years is \$12 624.77.

Time period	Current principal	Interest paid	New principal
1 (6 months)	10000	Interest $= \frac{PRT}{100}$ $= \frac{10000 \times 6 \times 1}{100}$ $= 600$	10000 + 600 $= 10600$
2 (12 months)	10600	Interest $= \frac{PRT}{100}$ $= \frac{10600 \times 6 \times 1}{100}$ $= 636$	10600 + 636 $= 11236$
3 (18 months)	11236	Interest $= \frac{PRT}{100}$ $= \frac{11236 \times 6 \times 1}{100}$ $= 674.16$	11236 + 674.16 $= 11910.16$
4 (24 months; 2 years)	11910.16	Interest $= \frac{PRT}{100}$ $= \frac{11910.16 \times 6 \times 1}{100}$ $= 714.61$ (rounded to 2 d.p.)	11910.16 + 714.61 $= 12624.77$

8.4.3 Compound interest graphs

- Compound interest graphs show the value of an investment at regular time intervals.
- Unlike simple interest graphs, which are straight lines, compound interest graphs are exponentials (curves), because the investment is growing by a set percentage instead of a set dollar amount.



WORKED EXAMPLE 13

Construct a compound interest graph for the investment in Worked example 10.

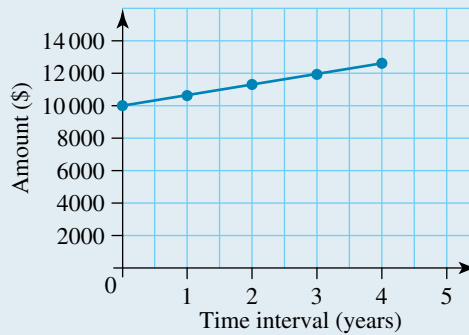
THINK

- Create a table of values.

WRITE

Time interval (6-monthly)	Amount (\$)
0 (initial investment)	10000
1	10600
2	11236
3	11910.16
4	12624.77

- 2 Draw a set of axes and plot the points on the graph. Label the vertical axis 'Amount (\$)' and horizontal axis 'Time interval (years)', and draw a curve through the points.



8.4.4 Using a spreadsheet to calculate compound interest

Calculating compound interest over a long period of time requires numerous repetitive calculations. Spreadsheets are a useful tool to perform these calculations.

WORKED EXAMPLE 14

A bank offers 7.5% per annum compounded yearly on investments. \$5000 is invested for 10 years. Using a spreadsheet, determine the value of the investment after the 10 years.

THINK

- 1 Open a spreadsheet labelling the columns 't, year' and 'value, \$'. Enter the values 0 and 5000 into the first row as shown.
- 2 Write the calculation that will determine the value of the investment after 1 year.
 $R = 7.5$, $P = 5000$, $T = 1$
- 3 Rewrite this calculation as a spreadsheet formula calculation; replace 5000 with the cell number from step 1, B2.
- 4 Insert this calculation into the spreadsheet in cell B3.
- 5 Copy the calculation for the next 8 years by using the fill-down function (or drag the bottom right corner down the cells). Complete column A to show 10 years.

WRITE

	A	B
1	t, year	value, \$
2	0	5000
3	1	
4	2	

$$V = P + I$$

$$= 5000 + \frac{5000 \times 7.5 \times 1}{100}$$

$$V = B2 + \frac{B2 \times 7.5 \times 1}{100}$$

	A	B
1	t, year	value, \$
2	0	5,000
3	1	= B2 + (B2 * 7.5 * 1)/100
4	2	

	A	B
1	t, year	value, \$
2	0	5000
3	1	5375
4	2	5778.125
5	3	6211.484
6	4	6677.346
7	5	7178.147
8	6	7716.508
9	7	8295.246
10	8	8917.389
11	9	9586.193
12	10	10305.16

6 Answer the question.

After 10 years, the value of the investment is \$10305.16.

eBookplus RESOURCES

Interactivity: Compound interest (int-6075)

Exercise 8.4 Applications of simple interest

Understanding, fluency and communicating

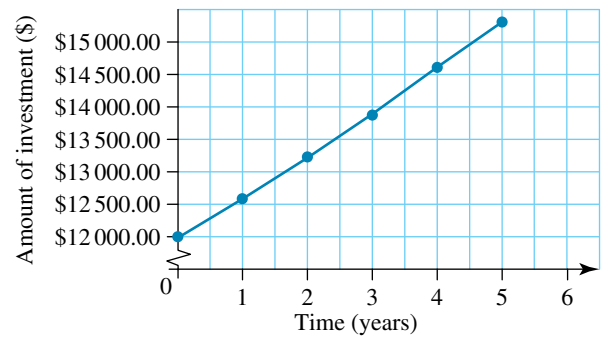
- WE10** A car's value depreciates by straight-line depreciation of 9%. If the car's initial value is \$19850, calculate its value after 7 years.
- Office furniture depreciates by straight-line depreciation. The initial value of the furniture was \$35 000 and after two years the value of the furniture had decreased by \$8400. Determine the straight-line depreciation percentage.
- WE11** **a.** A bank offers interest at 7% per year, compounded each year. If a customer puts \$900 in the bank, how much will be there after 3 years?
b. Another bank offers interest at 8% per year, compounded each year. If a customer puts \$14 000 in the bank, how much will be there after 4 years?
- WE12** A bank offers a term deposit for 3 years at an interest rate of 8% p.a. with a compounding period of 6 months. What would be the end value of a \$5000 investment under these conditions?
- WE13** Construct a compound interest graph for the investment in question 4.
- WE14** A bank offers 8% per annum compounded yearly on investments. \$2000 is invested for 5 years. Using a spreadsheet determine the value of the investment after the 5 years.
- A bank offers compound interest on its savings account of 4% per year. A boy opens an account with \$1125.
 - Using a spreadsheet, calculate the value of the boy's investment for the first four years.
 - Using your values from part **a**, construct a compound interest graph.
- A sum of \$1000 is invested at 10.5% interest. Using a spreadsheet, find the value of the investment for the following years and types of interest.



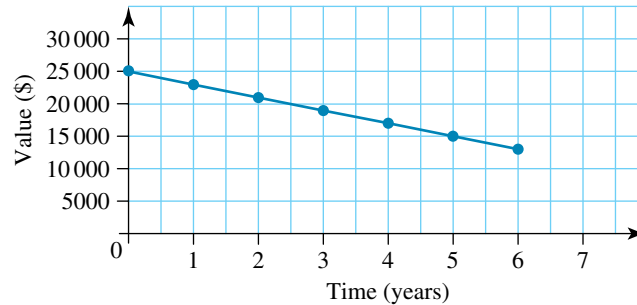
	Principal	Annual interest rate	Time (years)	Simple interest	Compound interest
a.	\$1000	10.5%	3		
b.	\$1000	10.5%	6		
c.	\$1000	10.5%	9		
d.	\$1000	10.5%	12		

9. A sum is invested into a bank account that is compounded annually. The graph shows the value of the investment over 5 years.

- What was the sum of money invested?
- Calculate the compound interest for this investment.



10. The graph shows the value of a car that depreciates with straight-line depreciation.



- What was the initial value of the car?
- What was the value of the car after 5 years?
- Calculate the straight-line depreciation percentage.

Problem solving, reasoning and justification

- In your own words, explain the difference between simple and compound interest.
- One aspect of compound interest is of great importance to investors: how long does it take to double my money? Consider a principal of \$100 and an annual interest rate of 10% (annual compounding).
 - How long does it take for this investment to be worth \$200?
 - How long would it take for the investment to be worth \$400 (a second doubling)?
- Using a spreadsheet, determine how long it would take to 'double your money' with compound interest at the following annual rates.

a. 5%	b. 7%	c. 10%	d. 12%

State your answers in years. Is there a 'pattern' to your answers?

14. There are two options available for a \$4500 investment:

Option 1: simple interest at 6.25% per year

Option 2: compound interest 6% per year, compounded half-yearly.

To calculate the interest on the investment for Option 2 after the first year, a student performed the following calculation:

$$\text{Option 2: investment value} = 4500 + \frac{4500 \times 6 \times 1}{100}$$

- Explain why the rate, R , should be 3%.

Using a spreadsheet:

- determine the value of the investment for both Option 1 and Option 2 after three years.
 - determine how many years it will take for Option 2 to be the better option.
15. Items can also depreciate by a set percentage each year. For example, if office equipment depreciates by 5% each year, it means that each year the value of the item is 5% less than the previous year.
- Write a calculation that would determine the value of a car with initial value of \$13 800 that depreciates 8% each year at the end of the first year.
 - Explain how this is both similar and different to calculating compound interest for investments.

- c. Using a spreadsheet, find the value of a car with an original value of \$13 800 that depreciates by 7% each year.
- d. After how many years will the car's value be half of its original value?

8.5 Review

Summary

In this topic you have learnt:

- how to increase and decrease amounts using percentages
- the difference between absolute and percentage profit and loss
- the difference between simple and compound interest
- how to calculate simple and compound interest for investments
- how to use spreadsheets to calculate compound interest
- how to create simple and compound interest graphs.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

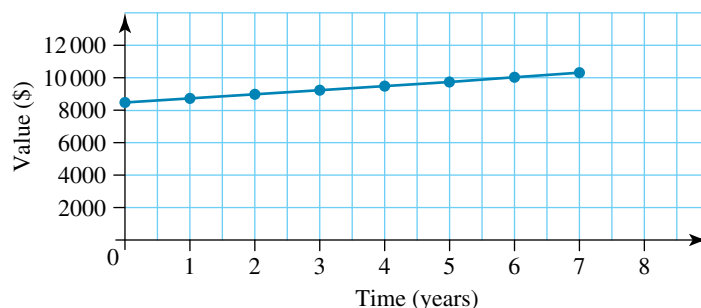
Exercise 8.5 Review of Interest and depreciation

Understanding, fluency and communicating

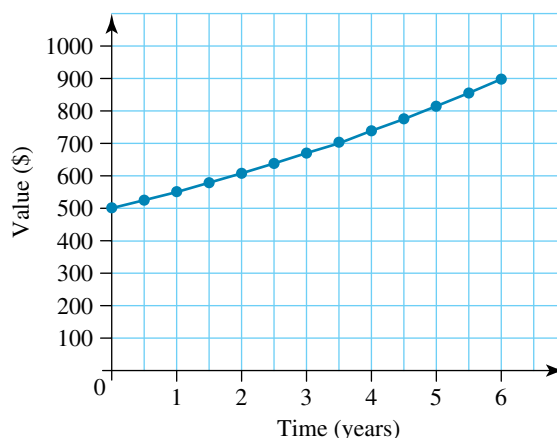
1. Calculate:
 - a. 43% of \$230
 - b. 110% of \$80.
2. **MC** A store is having a 25% off sale. An item originally priced at \$116 is now worth:
 - A. \$29
 - B. \$87
 - C. \$91
 - D. \$145
 - E. \$155
3. For the following, find the:
 - i. absolute profit or loss
 - ii. percentage profit or loss (to 2 decimal places).
 - a. A bicycle bought for \$450 and sold for \$350
 - b. A gold ring bought for \$550 and sold for \$615
4. The percentage loss on a boat purchased for \$17 500 was 28%. Determine the boat's selling price in dollars.
5. **MC** \$15 000 is invested in an account that attracts simple interest of 5.5% per annum. What is the value of the investment after 6 years?
 - A. \$825
 - B. \$900
 - C. \$4950
 - D. \$15 825
 - E. \$19 950
6. Determine the value of an investment of \$3500 at simple interest of 8% p.a. after 5 years.
7. Calculate the amount, in dollars, of GST at 10% that needs to be added to the following goods and services.
 - a. A T-shirt priced before GST at \$15.00
 - b. Hiring a plumber for a cost of \$550
8. A TV is bought for \$2999, including GST at 10%. Determine the amount, in dollars, of GST.
9. The original value of a yacht is \$23 000. If it depreciates at straight-line depreciation at 8%, determine the value of the yacht after 5 years.



10. An office photocopier depreciates at straight-line depreciation. The original value of the photocopier was \$3575; after three years its value had decreased to \$2975. Determine the straight-line depreciation percentage to 2 decimal places.
11. Calculate the compound interest on an investment of \$10600 at an interest rate of 7.55% p.a. over 15 years.
12. Calculate the percentage pay increase (to 2 decimal places) for the following cases.
 - a. Annual salary increases from \$45 000 to \$48 500.
 - b. Hourly wage increases from \$15.50 to \$16.25.
 - c. The rate of pay for making running shoes (piecework) increases from \$4.50 to \$4.65 per shoe.
13. Calculate the sale price of the following items. Round your answers to the nearest 5 cents.
 - a. A \$50 lamp at a 15% discount
 - b. A \$100 desk at a 30% discount
 - c. A \$200 dresser at a 22.5% discount
14. Using a spreadsheet, determine the value of the following investments.
 - a. \$1000 at 8% interest per year for 4 years
 - b. \$2000 at 9.2% interest per year for 5 years
 - c. \$3000 at 1.1% interest per month for 11 months
 - d. \$4000 at 9.18% interest per year for 3 months
15. The graph shows \$8500 invested in a savings account at simple interest.



- a. The investment is worth \$10 285 after 7 years. By how much, in dollars, does the investment grow each year?
 - b. At the end of the second year the investment was \$9010. Determine the simple interest percentage.
 - c. After how many years will the investment's value first exceed \$10 000?
16. A personal assistant currently makes \$30 per hour and is given an increase of 3.75%. What is his new hourly rate?
17. **MC** The graph shows the value of a sum of money over 5 years. Which one of the following statements about the investment is true?
 - A. The money was invested at simple interest.
 - B. The money was invested at compound interest, compounded yearly.
 - C. The money was invested at compound interest, compounded monthly.
 - D. The money was invested at compound interest, compounded half-yearly.
 - E. The money was invested at compound interest, compounded quarterly.



18. Before a pay rise, a salesperson's annual salary is \$55 600. After her pay rise, her salary is \$57 824. Calculate the:
- amount, in dollars, of her pay rise
 - the percentage increase of her pay.
19. A real-estate developer needs money very quickly for a short time. She offers 1.5% simple interest per month to people who invest \$10 000. How much interest does she have to pay in 3 months for a \$10 000 principal?
20. **MC** A boy buys a skateboard for \$32.35. Which one of the following does *not* represent this situation, correctly rounded to the nearest whole percent?
- The skateboard is originally priced at \$39.80 and discounted by 23%.
 - The skateboard is originally priced at \$42.00 and discounted by 23%.
 - The skateboard is originally bought for \$29.95 and marked up by 8%.
 - The skateboard is originally bought for \$26.30 and marked up by 23%.
 - The skateboard is originally bought for \$35.16 and sold for a percentage loss of 8%.



Problem solving, reasoning and justification

21. A new dining table retails for \$1500 but is not selling, so the shop decides to discount the table each week, by 5% of the last week's price, for up to 6 weeks, until it is sold. What is the total discount on the original price after 6 weeks?
22. The size of the percentage profit changes as the cost of an item decreases.
- Calculate the profit as a percentage of the cost for each of the following cases.
 - A profit of \$10, cost of \$1, percentage profit = _____%
 - A profit of \$10, cost of \$0.20, percentage profit = _____%
 - A profit of \$10, cost of \$0.05, percentage profit = _____%
 - Repeat parts **i** to **iii**, calculating profit as a percentage of price this time. Give your answers to 2 decimal places.
 - Which calculations best express profit when the cost is small?

23. A sales assistant saves \$2000 each year and puts it into a special savings account that pays 8% interest on the amount of money in the account at the end of the year. If the assistant contributes \$2000 for 10 years at the end of each year, use a spreadsheet to work out how much money will be in the account at the end of the 10 years.
24. Using a spreadsheet, calculate the value of each of the following investments if the principal is \$1000.
- Interest rate = 8% p.a., compounding period = 1 year, time = 2 years
 - Interest rate = 8% p.a., compounding period = 6 months, time = 2 years
 - Interest rate = 8% p.a., compounding period = 3 months, time = 2 years
25. A building society offers term deposits at 9%, compounded annually. A credit union offers term deposits at 10% but with simple interest only.
- After 2 years, which has the larger value?
 - After 3 years, which has the larger value?
 - How many years before the compound interest offer has the greater value?
26. Using a spreadsheet, complete the following table of simple and compound interest investments.



	Principal	Annual interest rate	Time (years)	Simple interest	Compound interest
a.	\$1000	4%	7		
b.	\$1000	6%	7		
c.	\$1000	8%	7		
d.	\$1000	10%	7		

27. An online bank has a special offer on term deposits as shown below.

Term Deposit

Special Offers:

4.50% p.a.* for 5 months

4.35% p.a.* for 6 months

4.50% p.a.* for 24 months

*Interest is calculated daily and paid at maturity.

- a. For each of the offers, use technology to determine the total amount of the term deposit at the end of the term.

The fine print at the bottom of the web page contains the following information:

Rates shown above are the nominal interest rates for a term deposit of \$10000. Interest is payable at maturity and if you choose a 24-month deposit you can request interest to be transferred to you every month, once a quarter, at six months or at maturity.

- b.** Calculate the final amount in a 24-month term deposit that was opened with \$10000 if the interest is calculated on the *new* total when the interest is added to the account:
- annually
 - at the end of every month
 - at the end of every quarter
 - at the end of each six-month period.
- c.** Which of the interest payment options from part **b** would you choose? Why?
- 28.** In a new three-year enterprise bargaining agreement (EBA), the union representatives are requesting a 3% pay rise for their workers each year for 3 years. The employers have offered a once-only 9% pay rise for the 3 years. Which deal will give workers the greater pay rise at the end of the 3-year period? Justify your answer using calculations. Explain why this is the case.
- 29.** The cost of some goods increases due to inflation. The price of a loaf of bread last year was \$3.00 and this year the price rose to \$3.10.
- Determine the percentage increase for a loaf of bread from last year to this year. Give your answers to 2 decimal places.
 - By what percentage will the bread need to be discounted so that the price is the same as last year?



- 30.** A shop owner buys goods for his shop. He needs to add 10% GST and then mark up each item by 15%. Instead of performing the two calculations, he decides to take a shortcut and increase the items by 25% as a one-off calculation. Show, using calculations, that these two methods give different results.

Answers

Exercise 8.2 Percentage increase or decrease

1. a. \$10 b. \$56 c. \$2 d. \$142.50 e. \$12
2. a. \$48.13 b. \$385 c. \$459.66 d. \$24 e. \$27857.50
3. a. \$48 b. \$26.10 c. \$126.65 d. \$59.40
4. \$337.50
5. \$31.13
6. \$57216.50
7. D

8.

	Buying price	Selling price	Absolute	
			Profit	Loss
a	1500	1895	395	
b	213	342	129	
c	6410	4288		2122
d	3200	3640	460	
e	1417	1157		260

9. a. 27.69% profit b. 36.42% loss c. 58.42% loss d. 13.68% profit
10. E
11. D
12. B

13. Method 1: Find the percentage (discount) of the sale item and then subtract from the original price.

Method 2: Find the percentage of the amount paid for the sale item (e.g. if the discount is 15% you will pay $100 - 15 = 85\%$ of the item), then find this percentage of the original price.

14. a. Less than \$100

b. 30% of 100

$$0.3 \times 100 = 30$$

$$\text{Add } \$30 \text{ to } \$100 = \$130.$$

Find 30% of \$130.

$$0.3 \times 130 = \$39$$

$$\text{Subtract } \$39 \text{ from } \$130 = \$91.$$

c. Less than \$100

d. Find 30% of \$100.

$$0.3 \times 100 = \$30$$

$$\text{Subtract } \$30 \text{ from } \$100 = \$70.$$

Find 30% of \$70.

$$0.3 \times 70 = \$21$$

$$\text{Add to } \$70 = \$91.$$

15. Option 2 salary after two years is \$71 230.68.

Option 1 salary after two years is \$71 190.

Therefore Option 2 is the better option.

16. \$6525

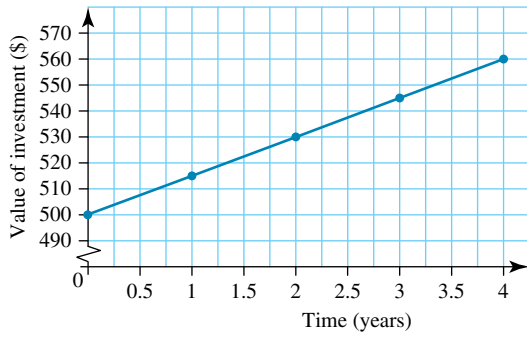
17. Price including GST is 10% more than the original price (i.e. 110%). Finding 10% of the selling price gives 10% of $110 = 11$.

Subtracting 11% from the selling price gives $110 - 11 = 99\%$, which is less than the original price.

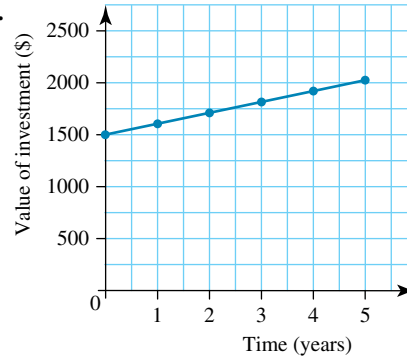
Exercise 8.3 Simple interest

1. \$28920
2. D
3. a. \$67 b. \$134 c. \$268
4. \$250

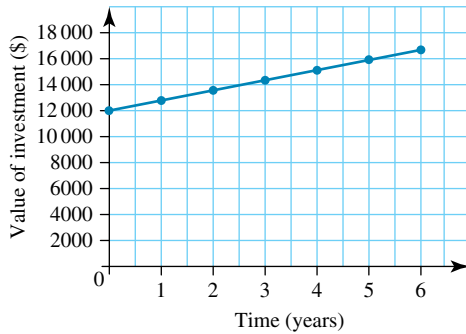
5. a.



b.



c.



6. A

7. \$18.75

8. C

9. a. \$3.33

b. \$10

c. \$20

10. \$6860

11. a. Principal is the amount that is invested initially (time = 0). The value of the investment is the principal plus the interest earned over time.

b. The amount of interest is the amount by which the investment has increased (or grown) in dollars. The interest rate is the percentage increase of the investment.

12. Bank A: interest earned after 3 years is \$40 more than with Bank B.

13. Bank A has the better rate over 3 years: $R = 11\frac{1}{3}\%$ p.a.

Bank B: rate per month = 1%, over a year = 12% p.a.

14. a. i. \$2300

ii. \$12 300

b. \$512.50

15. a. \$10 000

b. \$14 346.25

c. \$596.25

d. 24%

Exercise 8.4 Applications of simple interest

1. \$7344.50

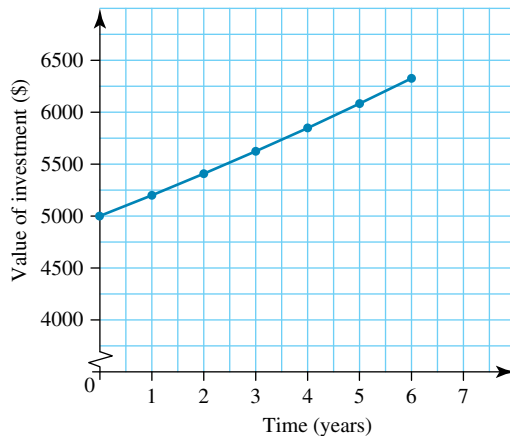
2. 12%

3. a. \$1102.54

b. \$19046.85

4. \$6326.59

5.



6.

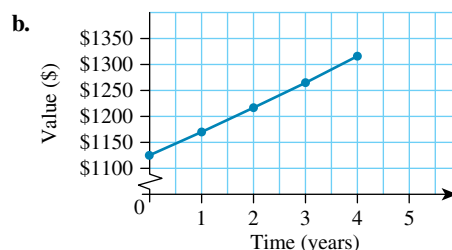
	A	B	C
1	t	value	
2	0	\$2000.00	
3	1	\$2160.00	$= (B2 * 0.08) + B2$
4	2	\$2332.80	
5	3	\$2519.42	
6	4	\$2720.98	
7	5	\$2938.66	

After 5 years the investment is worth \$2938.66.

7. a.

	A	B	C
1	t	value	
2	0	\$1125.00	
3	1	\$1170.00	$= (B2 * 0.04) + B2$
4	2	\$1216.80	
5	3	\$1265.47	
6	4	\$1316.09	

After 4 years the investment is worth \$1316.09.



8.

Principal	Annual interest rate	Time (years)	Simple interest	Compound interest
\$1000	10.5%	3	\$315.00	\$349.23
\$1000	10.5%	6	\$630.00	\$820.43
\$1000	10.5%	9	\$945.00	\$1456.18
\$1000	10.5%	12	\$1260.00	\$2313.96

9. a. \$12000

b. 5% p.a.

10. a. \$25000

b. \$15000

c. 8%

11. Simple interest is the interest paid on the original amount invested. It is found by finding the percentage of the principal (amount invested) and multiplying by the number of years the amount is invested for.

Compound interest is the interest paid on the growing investment. The amount of interest, in dollars, increases over the length of the investment because it calculates interest on interest already earned.

12. a. 8 years

b. 15 years

13.

	A	B	C	D	E
1	t, years	value, \$			
2		5%	7%	10%	12%
3	0	\$100.00	\$100.00	\$100.00	\$100.00
4	1	\$105.00	\$107.00	\$110.00	\$113.00
5	2	\$110.25	\$114.49	\$121.00	\$127.69
6	3	\$115.76	\$122.50	\$133.10	\$144.29
7	4	\$121.55	\$131.08	\$146.41	\$163.05
8	5	\$127.63	\$140.26	\$161.05	\$184.24
9	6	\$134.01	\$150.07	\$177.16	\$208.20
10	7	\$140.71	\$160.58	\$194.87	\$235.26
11	8	\$147.75	\$171.82	\$214.36	\$265.84
12	9	\$155.13	\$183.85	\$235.79	\$300.40
13	10	\$162.89	\$196.72	\$259.37	\$339.46
14	11	\$171.03	\$210.49	\$285.31	\$383.59
15	12	\$179.59	\$225.22	\$313.84	\$433.45
16	13	\$188.56	\$240.98	\$345.23	\$489.80
17	14	\$197.99	\$257.85	\$379.75	\$553.48
18	15	\$207.89	\$275.90	\$417.72	\$625.43
19	Formulas	$= (b3 * 0.05) + B3$	$= (c3 * 0.07) + c3$	$= (d3 * 0.1) + d3$	$= (e3 * 0.13) + e3$

As the rate of interest increases, the length of time taken to double the investment decreases.

14. a. Investment is compounded half-yearly. 6% is for the full year, therefore R is half that value: $\frac{6}{2} = 3$.

b. Option 1: \$5343.75

c. Option 2: \$5373.24

d. After 2 years, Option 2 will be the better option with a value of \$5064.79, which is more than Option 1 (\$5062.50).

24.

	A	B	C	D
1	t	yearly = $(B3 * 0.08) + B3$	6 month = $(C3 * 0.04) + C3$	3 month = $(D3 * 0.02) + D3$
2		$R = 8\%$	$R = 4\%$	$R = 2\%$
3	0	\$1000.00	\$1000.00	\$1000.00
4	1	\$1080.00	\$1040.00	\$1020.00
5	2	\$1166.40	\$1081.60	\$1040.40
6	3		\$1124.86	\$1061.21
7	4		\$1169.86	\$1082.43
8	5			\$1104.08
9	6			\$1126.16
10	7			\$1148.69
11	8			\$1171.66

25. a. Credit union

b. Credit union

c. 4 years

26.

	Principal	Annual interest rate	Time (years)	Simple interest	Compound interest
a.	1000	4%	7	\$1280.00	\$1315.93
b.	1000	6%	7	\$1420.00	\$1503.63
c.	1000	8%	7	\$1560.00	\$1713.82
d.	1000	10%	7	\$1700.00	\$1948.72

27. a.

t	4.5	4.35	4.5
Value	10187.50	10217.50	10900.00
	5 months	6 months	24 months

b. i. \$10920.25

ii. \$10939.90

iii. \$10936.25

iv. \$10930.83

c. Monthly will result in the greater investment at the end of 24 months.

28. 3% pay rise each year for 3 years is the better deal.

Each year the wage increases by 1.03; over 3 years: $1.03 \times 1.03 \times 1.03 = 1.0927$

Wage increase over the 3 years is 1.09.

$1.0927 > 1.09$

29. a. 3.33 %

b. 3.23%

30.

10% GST & 15% mark-up	25% increase
Item = \$100	Item = \$100
10% of \$100 $0.1 \times 100 = 10$ $\$100 + 10 = \$110$	25% of \$100 $0.25 \times 100 = 25$ Selling price = \$125
15% of \$110 $0.15 \times 110 = 16.50$ Selling price = $\$110 + 16.50$ = \$126.50	Price of item increase 1.25 from original price.
Price of item increase 1.1 (GST), then 1.15 (15% mark-up) = 1.265 increase from original price	

TOPIC 9

Earning and managing money

9.1 Overview

9.1.1 Introduction

Most people would like to earn more money, but just as important is learning how to manage that money once you've earned it. The first step on that path is knowing where the money you've earned goes – how much of it goes towards tax and superannuation, and how much you're left with afterwards.



DISCUSSION

How much money do you earn each week, and where does that money go?

LEARNING SEQUENCE

- 9.1 Overview
- 9.2 Getting paid
- 9.3 Commission, piecework and royalties
- 9.4 Taxation and deductions
- 9.5 Performing financial calculations
- 9.6 Review

CURRICULUM CONTENT

- calculate monthly, fortnightly, weekly, daily or hourly pay rates from a given salary, wages involving hourly rates and penalty rates, including situations involving overtime and other special allowances, and earnings based on commission (including commission based on a sliding scale), piecework or royalties (ACMGM002)    
- calculate income tax 
- use digital technology to perform financial computations, for example calculating percentage change, calculating tax payable and preparing a wage-sheet (ACMGM009)   

9.2 Getting paid

9.2.1 Salaries, wages and penalty rates

Salaries

- A person on a **salary** gets a fixed amount of money for a year, but is usually paid a portion of this each fortnight (2 weeks) or month.
- Each payment is an equal amount and is calculated by dividing the annual amount by the number of payment periods per year.

Hourly wages

- A person on an **hourly wage** gets paid a set amount for each hour worked.
- If a person works more than a certain number of hours per week, any additional hours are called **overtime**, and are paid at a higher rate.
- The overtime hourly rate is usually a multiple of the regular hourly rate:
 - $1.5 \times$ regular hourly wage (time-and-a-half)
 - $2 \times$ regular hourly wage (double-time), and so on.

Penalty rates

- Workers may receive a higher pay rate when they are required to work on weekends and public holidays, do shift work (early mornings or late nights), and work overtime. The pay rate paid is called a **penalty rate**.
- Penalty rates will vary according to the working agreements and will be a certain percentage more than the normal pay rate.



WORKED EXAMPLE 1

- a** A bank employee earns a salary of \$61 000 per year. How much does he get paid each fortnight?
b A fast-food employee is paid \$15.50 per hour. If she worked 72 hours last fortnight, how much did she get paid?

THINK

- a 1** Determine the number of fortnights in a year by dividing the number of days in a year by the number of days in a fortnight.
Note that the result will be different in a leap year (366 days).

- 2** Divide the annual salary by 26.07 and round to the nearest cent.

- 3** Answer the question.

- b 1** The worker is paid \$15.50 for each hour they work, so multiply this amount by the number of hours worked.

- 2** Answer the question.

WRITE

$$\frac{365}{14} \approx 26.07$$

(correct to 2 decimal places)

$$61\,000 \div 26.07 \approx 2339.85$$

The bank employee's fortnightly pay is \$2339.85.

$$15.50 \times 72 = 1116.00$$

Last fortnight the fast-food employee was paid \$1116.00.

Note: We will use the approximations of 52.14 weeks and 26.07 fortnights in a year in this text.

WORKED EXAMPLE 2

A hospitality worker is paid time-and-a-half for working on Saturday nights and double-time for working on Sundays or public holidays. The normal pay rate is \$15.50. If the worker worked 5 hours on Saturday and 6 hours on Sunday, calculate the amount of money she earned over the weekend.

THINK

- 1 Calculate the pay for Saturday. Time-and-a-half means $\times 1.5$.
- 2 Calculate the amount paid for Saturday.
- 3 Calculate the pay for Sunday. Double-time means $\times 2$.
- 4 Calculate the amount paid for Sunday.
- 5 Answer the question. *Note:* This is the amount she earned before tax was taken out.

WRITE

$$15.50 \times 1.5 = \$23.25 \text{ per hour}$$

$$23.25 \times 5 = \$116.25$$

$$15.50 \times 2 = \$31$$

$$6 \times 31 = \$186$$

The amount earned for the weekend is

$$\$116.25 + 186 = \$302.25.$$

9.2.2 Other allowances

- Employees may receive an **allowance** to complete certain tasks, such as to use their own tools at work, or to carry work in certain conditions. Common allowances include uniforms, tools and equipment, travel costs, car and phone costs, and first-aid training.
- Allowances are included in wages and are documented separately on pay sheets.

9.2.3 Superannuation

- All workers in Australia get an additional sum of money for their retirement. This sum is called **superannuation**. As of 2017 the law requires employers to pay an additional 9.5% of annual salary into a recognised superannuation fund. The amount, paid in dollars, in a superannuation fund is based on usual earnings before tax and is calculated each pay cycle.
- Some workers choose to contribute additional funds from their wages to increase their superannuation. There are tax incentives, such as paying lower tax rates on superannuation lump sums, to encourage workers to save for their retirement.
- Additional government financial support, known as the **pension**, is also available for some people who have retired. The amount a retired person receives in the pension depends on their personal financial security and wealth.



WORKED EXAMPLE 3

A worker's hourly rate is \$28.75 and he works 38 hours each week. The worker is paid fortnightly. Calculate the amount of superannuation the employer pays on his behalf each fortnight.

THINK

- 1 Calculate the fortnightly wage.
- 2 Calculate 9.5% of the fortnightly wage.
Remember to convert 9.5% to a decimal.
- 3 Answer the question.

WRITE

$$76 \times 28.75 = \$2185.00$$

$$9.5\% \times 2185 = 0.095 \times 2185 \\ \approx \$207.58$$

The employer pays \$207.58 into the superannuation fund each fortnight.

9.2.4 Annual leave loading

- Some workers will receive an extra payment on top of the 4-week annual-leave pay. It is usually 17.5% of their normal pay for 4 weeks. It will depend on the working agreement workers have with their employers if they receive an **annual leave loading**.
- The purpose of annual leave loading is to compensate workers who are unable to earn additional money through overtime while on leave, and to help cover the costs associated with taking holidays.

WORKED EXAMPLE 4

A worker's annual salary is \$58 056. Calculate the amount, in dollars, paid for annual leave loading before tax.

THINK

- 1 Calculate the weekly salary by dividing the annual salary by 52.14.
- 2 Calculate the wage for 4 weeks.
- 3 Calculate 17.5% of the 4-week wage.
Remember to convert 17.5% to a decimal.
- 4 Answer the question.

WRITE

$$\frac{58\,056}{52.14} \approx \$1113.46$$

$$\$1113.46 \times 4 = \$4453.84$$

$$17.5\% \times 4453.84 = 0.175 \times 4453.84 \\ \approx 779.42$$

The amount paid for annual leave loading, before tax, is \$779.42.

eBookplus RESOURCES

Interactivity: Special rates (int-6068)

Exercise 9.2 Getting paid

Understanding, fluency and communicating

Note: Assume 52.14 weeks and 26.07 fortnights in a year.

1. A lawyer is offered a job with a salary of \$74 000 per year, or \$40 per hour. Assuming that she works 80 hours every fortnight, which is the greater pay?
2. **MC** An hourly wage of \$32.32 for 77.5 hours results in a fortnightly pay of:
A. \$1252.40 **B.** \$2504.80 **C.** \$842.58 **D.** \$1292.80 **E.** \$2585.60
3. **WE1** Calculate the wage for the following hourly rates and hours worked.
a. \$19.75 for 74.75 hours **b.** \$24.85 for 45.75 hours **c.** \$45.30 for 35.25 hours
4. A worker earns \$20.40 an hour. He needs to earn a minimum of \$700 each week before tax, to buy food, pay rent and bills and have some money for entertainment. Calculate the minimum number of hours he has to work. Give your answer correct to 2 decimal places.

5. **WE2** A retail worker is paid time-and-a-half for working on Saturday and double-time for working on Sundays or public holidays. The normal pay rate is \$18.75. The worker worked over a long weekend, with the Monday being a public holiday. The hours worked for Saturday, Sunday, and Monday (public holiday) were 8, 5 and 6 hours respectively. Calculate the amount of money she earned over the weekend.
6. Over the last four weeks, a woman has worked 35, 36, 34 and 41 hours. If she earns \$24.45 per hour, how much did she earn for each of the two fortnights?
7. **WE3** A worker's hourly rate is \$29.45 and she works 25.75 hours each week. The worker is paid fortnightly. Calculate the amount of superannuation the employer pays on her behalf each fortnight.
8. A school principal is on an annual salary of \$155 750.
 - a. How much does she earn each month?
 - b. What is her superannuation fund payment, if the fund is paid 9.5% of her annual salary?
9. A salary earner makes \$62 000 per year.
 - a. How much does he earn each month?
 - b. What is his superannuation fund payment each month if he receives 9.5% superannuation?
 - c. What is the total amount deposited into the fund in a year?
10. **WE4** A worker's annual salary is \$85 980. Calculate the amount, in dollars, paid for annual leave loading before tax.



Problem solving, reasoning and justification

11. Some employers offer a superannuation bonus scheme. They pay you 9.5%, then for every additional 1% of your salary that you contribute, they match it with a further 1%. For example, if you contribute 2% of your salary to superannuation, you will receive a total of $9.5\% + 2\% + 2\% = 13.5\%$ put into your fund. A software saleswoman makes \$70 000 per year and decides to contribute 3% of her salary into superannuation.
 - a. How much does the saleswoman contribute from her own salary into superannuation each year?
 - b. In total, how much does she receive into her superannuation fund each year?
12. There is a proposal to have the same penalty rates for both Saturday and Sunday of time-and-a-half. Currently workers receive time-and-a-half to work Saturdays and double-time to work Sundays.
A waiter receives penalty rates and her hourly rate is \$17.75.
 - a. By what percentage, to the nearest whole number, will she be worse off if the penalty rates change?
 - b. By what percentage, to 2 decimal places, will her hourly rate need to increase to ensure she receives the same amount she currently does for working both Saturday and Sundays?
13. A factory worker receives an hourly rate of \$28.40 to work a standard 38-hour-week. She receives overtime of time-and-a-half for any hour worked above 38 hours.
 - a. On average, she works 42.5 hours each week. Calculate her weekly wage before tax.
 - b. The worker is offered to go on a salary of \$68 000. Will she be better off remaining on a wage or taking the salary offer? Justify your answer using calculations.
14. An employer is proposing a new working agreement: the removal of the 17.5% annual leave loading, an increase of 5% in annual wages and 10.5% superannuation contributions. Employees are considering whether to accept the agreement. Should they accept the agreement? Justify your answer using calculations.



15. A secretary's current annual salary is \$56890. She is offered two packages:
 Package 1: pay increase of 1.5% each year for 3 years
 Package 2: superannuation contribution of 10.75% for 5 years.

By calculating her salary each year for 3 years and the additional amount of superannuation contributions that will be deposited into her superannuation fund, which package should she choose?



9.3 Commission, piecework and royalties

9.3.1 Commission

- A **commission** is paid to a person when an item is sold. For example, if a real-estate agent sells a house, then he or she is paid a commission.
- Commissions are calculated as a percentage of the sale price. If no sales are made, no commission is received.
- A commission table is often used to determine the value of a commission.

WORKED EXAMPLE 5

When selling real estate, an agent is paid according to the table below.

Sale price	Commission	Plus
Between \$0 and \$80 000	2% of sale price	0
Between \$80 001 and \$140 000	1.5% of amount over \$80 000	\$1600
\$140 000 and over	1.1% of amount over \$140 000	\$2500

If a house is sold for \$200 000, what is the commission paid to the real-estate agent?

THINK

- Since the amount of commission varies according to the sale price, determine which of the 3 ranges the sale falls into. $200\ 000 > 140\ 000$, so the price falls into range 3 (\$140 000 and over).
- The commission for this range is based on the amount of the sale *over* \$140 000. Calculate the amount over \$140 000.

WRITE

$$\begin{array}{r} 200\ 000 \\ -140\ 000 \\ \hline 60\ 000 \end{array}$$

- The commission is 1.1% of 60 000; write 1.1% as a decimal.

$$1.1\% = \frac{1.1}{100} = 0.011$$

- Multiply 0.011 by the amount over \$140 000, that is \$60 000.

$$1.1\% \text{ of } 60\ 000 = 0.011 \times 60\ 000 = 660$$

- The table also specifies an additional commission of \$2500, based on the amount of the sale *under* \$140 000. Calculate the total of the commission.

$$660 + 2500 = 3160$$

- Answer the question.

The total commission is \$3160.

9.3.2 Salary and commissions

- In some industries the rate of sales varies widely, so it is not practical to live on commissions only. For example, car salespeople are given a base salary in addition to a commission.



WORKED EXAMPLE 6

A car salesperson earns a salary of \$1200 per month plus a commission of 3% of the total sales that she makes that month. In February, she sold cars worth \$198 500. What is her total wage (salary + commission) for that month?

THINK

- 1 Her commission is 3% of sales. Write 3% as a decimal.
- 2 Calculate 3% of the total sales in order to determine the commission.
- 3 Her total wage is the commission plus her base salary. Add the commission of \$5955 to the base salary of \$1200.
- 4 Answer the question.

WRITE

$$3\% = \frac{3}{100} = 0.03$$

$$3\% \text{ of } 198\,500 = 0.03 \times 198\,500 \\ = 5955$$

$$5955 + 1200 = 7155$$

The total wage paid in February is \$7155.

9.3.3 Piecework

- A person paid by **piecework** is paid a fixed amount for each item produced. Often the work is done at home.
- There may be bonuses for faster workers.

WORKED EXAMPLE 7

A seamstress gets paid \$4.75 for every dress she sews. If she sewed 124 dresses last week, how much money did she earn?

THINK

- 1 The seamstress gets \$4.75 for each dress, so multiply this amount by the number of dresses sewn.
- 2 Answer the question.

WRITE

$$4.75 \times 124 = 589$$

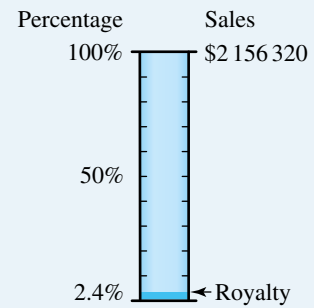
Last week the seamstress was paid \$589.00.

9.3.4 Royalties

- A **royalty** is a payment made to an author, composer or creator for each copy of the work or invention sold. For example, if a pop star sold \$2 million worth of music last year, she is entitled to some of the profits; other people such as managers also get a share.
- Royalties are usually calculated as a percentage of the total sales (not total profit).

WORKED EXAMPLE 8

Last year, a new rock star sold music worth \$2 156 320. His recording contract specifies that he receives 2.4% royalty on the total sales. How much money did he earn last year?



THINK

- 1 He earned 2.4% of the sales (\$2 156 320). Write 2.4% as a decimal.
- 2 Calculate 2.4% of the total sales; multiply 0.024 by the total sales.
- 3 Answer the question.

WRITE

$$2.4\% = \frac{2.4}{100} = 0.024$$

$$2.4\% \text{ of } 2\,156\,320 = 0.024 \times 2\,156\,320 = 51\,758.68$$

The rock star earned \$51 758.68 in royalties.

eBookplus RESOURCES

Interactivity: Piecework (int-6069)

Interactivity: Commission and royalties (int-6070)

Exercise 9.3 Commission, piecework and royalties

Understanding, fluency and communicating

1. **WE5** Using the commission table below, calculate the commission on the sale of a house for \$945 000.



Sale price	Commission	Plus
Between \$0 and \$400 000	0.5% of sale price	0
Between \$400 001 and \$700 000	0.3% of amount over \$400 000	\$2000 (0.5% of \$400 000)
\$700 001 and over	0.2% of amount over \$700 000	\$2900 (0.5% of \$400 000 + 0.3% of \$300 000)

2. Using the commission table from question 1, calculate the commission for the following sales.
 - a. \$330 000
 - b. \$525 000
 - c. \$710 000
 - d. \$1 330 000
3. **MC** Using the commission table from question 1, the commission on the sale of a \$500 000 house would be:
 - A. \$1000
 - B. \$2000
 - C. \$2300
 - D. \$2500
 - E. \$2900

4. **WE6** A car salesperson earns a salary of \$1400 per month plus a commission of 3.5% of the total sales he makes that month. In April, he sold cars worth \$155 000. What is his total wage (salary + commission) for that month?
5. **WE7** A shoemaker is paid \$5.95 for each pair of running shoes he can make. If the shoemaker made 235 pairs of shoes last week, what was the amount paid?
6. **MC** If a software engineer gets paid \$3.40 for every line of computer code she writes, then how much will she make if she writes 865 lines in a fortnight?
A. \$29 410.00 **B.** \$2941.00 **C.** \$294.10
D. \$2329.00 **E.** \$2076.00
7. A seamstress gets paid per item she sews. If she received \$545 for sewing 45 items, how much did she get paid per item?
8. **WE8** Last year, a pop star sold \$5 342 562 worth of music on her website and via CD sales. Her contract calls for a 3.5% royalty on all sales. How much did she earn last year?
9. **MC** If an author earned \$25 560 on the basis of sales of \$568 000, then the royalty payment:
A. is 4.5%
B. is 45%
C. is 0.045%
D. is 0.45%
E. cannot be determined from the given information.
10. An actress is paid a 2.5% royalty plus \$300 000 cash to act in the latest blockbuster movie.
a. Complete the following table of royalties.

Time	Jan–Mar	Apr–Jun	Jul–Aug	Sep–Dec
Sales	\$123 400	\$2 403 556	\$432 442	\$84 562
Royalty payment				

- b.** What is the total amount the actress received, including the cash?
11. To keep matters simple, the winner of a television talent show receives a royalty of 1% on sales of their first album. If this year's winner sells \$4563 453 worth of music, what is the royalty payment?

Problem solving, reasoning and justification

12. A new children's book author is offered a choice of \$100 000 cash or a 4% royalty on sales. What would the sales need to be to match the cash offer?
13. A hot-shot used-car salesman earns \$1500 per month plus 6.2% commission on all sales. If he sold \$243 540 worth of cars last month, what is his total wage?
14. A songwriter receives royalties on the sales of her songs. She received \$11 473.75 in royalties for the sale of \$458 950 in songs. Explain how the royalty percentage she receives can be determined, and hence find the royalty percentage she receives.



15. A shoe manufacturer decides to offer his shoemakers two pay options:
 Option 1: \$8.25 per pair of shoes made
 Option 2: receive a 3.5% commission on the shoes sold.
 A shoemaker can make 15 pairs of shoes each day and each pair of shoes sells for \$75.95. In one week the manufacturer will sell 250 pairs of shoes. Which option will earn the shoemaker the most money? Justify your answer using calculations.
16. A telecommunications salesperson receives a base wage of \$450 per week plus 2.5% commission on all new telephone plans sold during the week. In one week, his earnings (before tax) were \$502.25.
- What was the total amount (in dollars) of telephone plans he sold during the week?
 - If each plan was \$95, how many plans did he sell?
 - The salesperson is offered the choice to remain on a weekly base wage of \$530 with an increase of 3.5% in commission on all sales, or to receive an annual salary of \$37 500. Which option would you recommend he choose? Justify your answers using calculations.

9.4 Taxation and deductions

9.4.1 Income tax

- Income tax** is a tax levied on the financial income of people.
- Income tax is deducted from each fortnightly or monthly pay.
- The amount of income tax is based upon **total income** and **tax deductions**, which determines a worker's **taxable income**.

$$\text{Taxable income} = \text{total income} - \text{tax deductions}$$

- The calculation of income tax is based upon an 'income tax table'.
 The current income tax table is:

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$37 000	19 c for each \$1 over \$18 200
\$37 001–\$87 000	\$3572 plus 32.5 c for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37 c for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45 c for each \$1 over \$180 000

9.4.2 Tax deductions

- Workers who spend their own money for work-related expenses are entitled to claim the amount spent as tax deductions.
- The tax deductions are recorded in the end-of-the-financial-year tax return.
- The deductions are subtracted from the taxable income, which lowers the amount of money earned and hence reduces the amount of tax to be paid.
- What can be claimed as tax deductions is determined by the Australian Taxation Office. Some examples of deductions which can be claimed include using your car to travel to work-related events, purchasing materials, union fees, donations made to charities, and using a home office.
- Deductions must be work-related expenses and evidence, such as receipts, must be provided.

WORKED EXAMPLE 9

Calculate the income tax payable by the teacher who earns a salary of \$67 400. Tax deductions for the teacher are \$4240 and the teacher earned \$1240 in bank interest.

THINK

- 1 Determine the total income by adding the salary and bank interest together.
- 2 Subtract the tax deductions from the total income.
- 3 Determine the tax bracket based on the taxable income of \$64 400.
- 4 Determine the amount of tax to be paid.
- 5 Determine the amount of taxable income over \$37 000 by subtracting \$37 000 from the teacher's taxable income.
- 6 Determine the tax rate amount as 32.5% of \$27 400. Remember to convert 32.5% to a decimal.
- 7 Add the 'plus amount' to find the payable tax.
- 8 Answer the question.

WRITE

$67\,400 + 1\,240 = \$68\,640$

Taxable income = $\$68\,640 - 4\,240$
= $\$64\,400$

Taxable income is between \$37 001 and \$87 000.

\$3572 plus 32.5 c for every \$1 over \$37 000

$64\,400 - 37\,000 = \$27\,400$

$0.325 \times 27\,400 = \$8905$

$\$3572 + 8905 = \$12\,477$

The total tax payable by the teacher is \$12 477.

9.4.3 The Medicare levy

- Australian residents have access to healthcare through Medicare, which is partly funded by taxpayers through the payment of a **Medicare levy**.
- The current Medicare levy is 2% of taxable income, and may be reduced if the taxable income is below a certain amount.
- In addition to the Medicare levy, residents who do not have adequate private healthcare may also be required to pay a Medicare levy surcharge.

WORKED EXAMPLE 10

A plumber's taxable income for the financial year is \$65 850. He claims \$5680 in work-related expenses (deductions) and he also has private health insurance. Determine the amount he has to pay for the Medicare levy.



THINK

- 1 Calculate the taxable income.
- 2 Calculate 2% of taxable income. Remember to convert 2% to a decimal.
- 3 Answer the question.

WRITE

Taxable income = $\$65\,850 - \$5680$
= $\$60\,170$

$2\% \times 60\,170 = 0.02 \times 60\,170$
= $\$1203.40$

He will pay \$1203.40 in Medicare levy.

9.4.4 Pay As You Go (PAYG) tax

- The Australian Taxation Office administers a **Pay As You Go (PAYG) tax**, which is a withholding tax system. It requires employers to calculate the amount of income tax to withhold from employees. The amount withheld is determined by taxation tables provided by the tax office.
- This amount is withheld from the employee's regular earnings (gross pay) and contributes towards the employee's tax to be paid at the end of the financial year.

$$\text{Net pay} = \text{gross pay} - \text{tax withheld}$$

- At the end of the financial year, workers submit a tax return which lists their deductions and all money earned.
 - If the total amount of tax paid is less than is required, the worker will have to pay the difference.
 - If the amount of tax paid over the year is more than is required, the worker receives a refund.
- Australian workers who provide their tax file number and do not work another job claim the tax-free threshold. This means that they do not pay tax on the first \$18 200 earned in the financial year.

WORKED EXAMPLE 11

A factory worker is paid \$19.94 per hour. Each week she works 38 hours, with 4 hours overtime paid at time-and-a-half. Using the taxation table, calculate her net pay (the amount she receives).

Weekly earnings (\$)	Tax withheld (\$)
856.00	133.00
857.00	133.00
858.00	133.00
859.00	134.00
860.00	134.00
861.00	134.00
862.00	135.00
863.00	135.00
864.00	135.00
865.00	136.00
866.00	136.00
867.00	136.00
868.00	137.00
869.00	137.00
870.00	137.00
871.00	138.00
872.00	138.00
873.00	138.00
874.00	139.00
875.00	139.00
876.00	139.00
877.00	140.00
878.00	140.00

Weekly earnings (\$)	Tax withheld (\$)
879.00	141.00
880.00	141.00
881.00	141.00
882.00	142.00
883.00	142.00
884.00	142.00
885.00	143.00
886.00	143.00
887.00	143.00
888.00	144.00
889.00	144.00
890.00	144.00
891.00	145.00
892.00	145.00
893.00	145.00
894.00	146.00
895.00	146.00
896.00	146.00
897.00	147.00
898.00	147.00
899.00	147.00
900.00	148.00

THINK

- Calculate the weekly wage for normal hours.
- Calculate the overtime.
- Calculate the total weekly wage.

WRITE

$$38 \times 19.94 = \$757.72$$

$$19.94 \times 1.5 \times 4 = \$119.64$$

$$\$757.72 + 119.64 = \$877.36$$

- 4 Using the table, locate the weekly wage and read the amount of tax withheld.
- 5 Calculate the net pay by subtracting the tax withheld from the total wages.

Weekly wage \$877, tax withheld \$140

$$\text{Weekly income} = 877.36 - 140 = \$737.36$$

Exercise 9.4 Taxation and deductions

Understanding, fluency and communicating

1. **WE9** Calculate the income tax payable by a baker whose salary is \$51 260. Tax deductions of the baker are \$2120, and the baker earned \$1850 in bank interest. Refer to the tax table provided on page 294.
2. **WE10** Calculate the Medicare levy for the following taxable incomes.
 - a. \$60400
 - b. \$77300
 - c. \$89400
 - d. \$108423
3. **MC** A shop worker earns \$18.95 per hour. She works 52 hours in a fortnight. The amount of Medicare levy she will be expected to pay at the end of the financial year is closest to? *Note:* Assume there are 26.07 fortnights in a year.
 - A. \$20
 - B. \$51
 - C. \$200
 - D. \$514
 - E. \$520
4. **WE11** A garage mechanic is paid by the hour.
 - a. If she works 84 hours in a fortnight and is paid \$22.50 an hour, what is her gross pay (amount before tax)?
 - b. Using the taxation table shown, calculate her net pay.

Fortnightly earnings (\$)	Tax withheld (\$)
1832.00	306.00
1834.00	308.00
1836.00	308.00
1838.00	308.00
1840.00	310.00
1842.00	310.00
1844.00	310.00
1846.00	312.00
1848.00	312.00
1850.00	314.00
1852.00	314.00
1854.00	314.00
1856.00	316.00
1858.00	316.00
1860.00	316.00
1862.00	318.00
1864.00	318.00
1866.00	318.00
1868.00	320.00
1870.00	320.00
1872.00	320.00
1874.00	322.00
1876.00	322.00
1878.00	322.00
1880.00	324.00



Fortnightly earnings (\$)	Tax withheld (\$)
1882.00	324.00
1884.00	324.00
1886.00	326.00
1888.00	326.00
1890.00	326.00
1892.00	328.00
1894.00	328.00
1896.00	330.00
1898.00	330.00
1900.00	330.00
1902.00	332.00
1904.00	332.00
1906.00	332.00
1908.00	334.00
1910.00	334.00
1912.00	334.00
1914.00	336.00
1916.00	336.00
1918.00	336.00
1920.00	338.00

5. For the worker in question 4, any hours worked over 76 hours per fortnight are paid overtime at a rate of \$45 an hour. Adjust her gross and net pay accordingly.
6. Using the taxation table provided:
- write down the percentage tax payable for the following annual salaries.
 - hence, calculate the amount of tax payable.
- a. \$37 500 b. \$15 879 c. \$85 670 d. \$112 500

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$37 000	19c for each \$1 over \$18 200
\$37 001–\$87 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37c for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45c for each \$1 over \$180 000

7. The table shows the tax payable for fortnightly wages. Using the table, calculate the tax payable on the following fortnightly wages by workers who claim the tax-free threshold.

Fortnightly income (\$)	Tax to be withheld		Fortnightly income (\$)	Tax to be withheld	
	With tax-free threshold (\$)	No tax-free threshold (\$)		With tax-free threshold (\$)	No tax-free threshold (\$)
1000	74	262	2000	366	608
1100	96	296	2200	434	676
1200	118	330	2500	540	780
1500	192	434	3000	712	968
1800	296	538	4000	1086	1358

- a. \$1500 b. \$2000 c. \$2500 d. \$4000
8. Calculate the percentage of tax payable for the incomes from question 7 by workers who don't claim the tax-free threshold. Give your answers to 2 decimal places where necessary.
9. An apprentice electrician is paid \$17.14 per hour. She is paid the normal rate for the first 38 hours worked in any week and then the overtime rate of \$25 for hours worked over 38 hours. In one week, she worked 45 hours. Using the taxation table, calculate her wage for the week (the amount she receives).

Weekly earnings (\$)	Amount to be withheld (\$)
816.00	119.00
817.00	119.00
818.00	119.00
819.00	120.00
820.00	120.00
821.00	120.00
822.00	121.00
823.00	121.00
824.00	121.00
825.00	122.00
826.00	122.00
827.00	122.00
828.00	123.00
829.00	123.00
830.00	123.00

Weekly earnings (\$)	Amount to be withheld (\$)
831.00	124.00
832.00	124.00
833.00	125.00
834.00	125.00
835.00	125.00
836.00	126.00
837.00	126.00
838.00	126.00
839.00	127.00
840.00	127.00
841.00	127.00
842.00	128.00
843.00	128.00
844.00	128.00
845.00	129.00

10. A nurse's gross annual salary is \$58 284 and he is paid monthly.
- Calculate his gross monthly salary.
 - Using the tax table shown, determine the amount of tax withheld from his monthly salary.

Monthly earnings (\$)	Tax withheld (\$)
4792.67	949.00
4797.00	953.00
4801.33	953.00
4805.67	958.00
4810.00	958.00
4814.33	958.00
4818.67	962.00
4823.00	962.00
4827.33	962.00
4831.67	966.00
4836.00	966.00
4840.33	966.00
4844.67	971.00
4849.00	971.00
4853.33	971.00
4857.67	975.00
4862.00	975.00
4866.33	975.00

Monthly earnings (\$)	Tax withheld (\$)
4870.67	979.00
4875.00	979.00
4879.33	979.00
4883.67	984.00
4888.00	984.00
4892.33	984.00
4896.67	988.00
4901.00	988.00
4905.33	988.00
4909.67	992.00
4914.00	992.00
4918.33	997.00
4922.67	997.00
4927.00	997.00
4931.33	1001.00
4935.67	1001.00
4940.00	1001.00

- Determine the percentage of tax, to 2 decimal places, he pays each month.
 - Calculate the Medicare levy he will pay.
11. An accountant pays \$1719.40 in Medicare levy.
- If she has private health insurance, calculate her taxable income for the year.
 - Calculate the tax payable for the financial year. Use the tax table provided on page 294.
 - How much superannuation should her employer pay into her superannuation fund for the year?

Problem solving, reasoning and justification

12. A truck driver earns \$24.07 per hour for working a 76-hour fortnight. Overtime is paid for additional hours worked over the 76 hours during Monday to Friday, and double-time is paid for hours worked on weekends (Saturday and Sunday). Assume 52 weeks in 1 year.
- Over a fortnight, the truck driver worked 80 hours Monday–Friday and 15 hours over the weekend.
- Calculate the gross fortnightly wage for the truck driver.

Fortnightly earnings (\$)	Tax withheld (\$)
2682.00	602.00
2684.00	602.00
2686.00	604.00
2688.00	604.00
2690.00	604.00
2692.00	606.00
2694.00	606.00
2696.00	606.00
2698.00	608.00
2700.00	608.00
2702.00	608.00
2704.00	610.00
2706.00	610.00
2708.00	610.00
2710.00	612.00

Fortnightly earnings (\$)	Tax withheld (\$)
2712.00	612.00
2714.00	614.00
2716.00	614.00
2718.00	614.00
2720.00	616.00
2722.00	616.00
2724.00	616.00
2726.00	618.00
2728.00	618.00
2730.00	618.00
2732.00	620.00
2734.00	620.00
2736.00	620.00
2738.00	622.00
2740.00	622.00

- b. Show that the truck driver's net fortnightly pay was \$2089.84 by using the PAYG tax table at the bottom of the previous page.
- c. If the truck driver claims \$2486 in deductions, calculate his taxable income, and hence calculate the amount of payable tax for the year by using the tax table provided on page 294.
- d. How much Medicare levy should the driver expect to pay, given that he has private health insurance?
13. A computer technician's annual salary is \$67374. He claims \$1580 in deductions and receives \$225 in dividends from shares.
- a. The technician is paid monthly. Calculate his gross monthly salary.
- b. His employer withholds \$1200 PAYG tax each month. Calculate the total amount of tax withheld for the year.
- c. Determine the tax payable by the computer technician for the financial year. Hence, determine whether he receives a tax refund or is required to pay more tax, and state the amount.
14. A hairdresser receives a weekly wage of \$930.60, which includes 38 hours of normal pay plus 4 hours of overtime, paid at time-and-a-half.
- a. Explain how his hourly rate can be determined, and hence find his hourly rate.
- b. Determine his annual salary if he works an average of 40 hours a week (38 hours at normal pay and 2 hours overtime). Assume 52 weeks in 1 year.
- c. Using your answer from part b, determine the amount of tax payable at the end of the financial year.
- d. The hairdresser forgot to claim \$1980 in deductions and is looking forward to receiving \$1980 in a tax refund. By recalculating the amount of tax payable at the end of the financial year based on his new taxable income, explain why he won't receive \$1980 in a tax refund.



15. Taxable income	Tax on this income
0–\$18200	Nil
\$18201–\$37000	19c for each \$1 over \$18200
\$37001–\$87000	\$3572 plus 32.5c for each \$1 over \$37000
\$87001–\$180000	\$19822 plus 37c for each \$1 over \$87000
\$180001 and over	\$54232 plus 45c for each \$1 over \$180000

The Australian tax system is a tiered system, as shown in the table.

A dentist's annual salary is \$97065.

- a. Explain why the tax payable is not found by calculating 37% of \$97065.
- b. Explain how the additional 'plus amount' of \$19822 is calculated.
16. A mechanical engineer pays \$14812 in tax.
- a. Explain how her taxable income can be determined.
- b. Determine her taxable income.
- c. After the financial year she realises that she forgot to claim \$985 in tax deductions. Determine her taxable income and the tax refund she should expect.

9.5 Performing financial calculations

9.5.1 Calculating percentage change

$$\text{Percentage change} = \frac{\text{difference between two numbers}}{\text{original number}} \times 100$$

- A spreadsheet can be used to find the percentage change by inserting a formula that calculates the difference between the cell numbers, and then divides by the original number and multiplies by 100.

WORKED EXAMPLE 12

A shop assistant receives a pay increase. Her original weekly pay was \$545 and her new pay is \$567. Using a spreadsheet, calculate the percentage change of her wage. Write your answer to the nearest whole percent.

THINK

- 1 Set up a spreadsheet with headings 'original pay', 'new pay' and 'percentage change'.

WRITE

	A	B	C
1	original pay	new pay	percentage change
2			
3			
4			
5			
6			

- 2 Enter her original pay (\$545) in cell A2 and her new pay (\$567) in cell B2.

	A	B	C
1	original pay	new pay	percentage change
2	545	567	
3			
4			
5			
6			

- 3 In Cell C1, enter the formula that calculates the difference between cells A2 and B2: ' $(B2-A2)$ ' and then divides by the original pay: ' $(B2-A2)/A2$ ', and finally converts to a percentage by multiplying by 100: ' $(B2-A2)/A2*100$ '.

	A	B	C
1	original pay	new pay	percentage change
2	545	567	$= (B2-A2)/A2*100$
3			
4			
5			
6			

- 4 Answer the question.

The percentage change of her wage is 4%
(a 4% pay rise).

9.5.2 Calculating tax payable

- Calculating the tax payable on gross incomes can be found using a spreadsheet and inserting formulas that determine the amount of tax payable for each tax tier.

WORKED EXAMPLE 13

A law clerk's annual salary is \$45 675. She claims \$450 in tax deductions and receives \$250 in interest from investments. Using a spreadsheet, calculate the amount of tax payable for the financial year.

THINK

- 1 Create a spreadsheet with headings 'gross income', 'interest', 'deductions' and 'taxable income'. Enter the values.

WRITE

	A	B	C	D
1	gross income	interest	deductions	taxable income
2				
3				
4				
5				
6				

- 2 In cell D2 calculate the taxable income: add the interest (\$250) to the annual salary (\$45 675) and subtract the deductions (\$450) by writing the formula '=A2+B2-C2'.

D2		fx		=A2+B2-C2
	A	B	C	D
1	gross income	interest	deductions	taxable income
2	45675	250	450	45475
3				
4				
5				
6				

- 3 Insert a column to represent the tax bracket that the law clerk falls into.

	A	B	C	D	E
1	gross income	interest	deductions	taxable income	tax tier (37001–87000)
2	45675	250	450	45475	
3					
4					
5					
6					

- 4 Insert a formula to calculate the tax payable for this tax tier.

Type the following formula into G2:
'=3575 + (D2-37000)*0.325'

- 5 Calculate the tax payable.

	A	B	C	D	E
1	gross income	interest	deductions	taxable income	tax tier (37001–87000)
2	45675	250	450	45475	6329.375
3					
4					
5					
6					

- 6 Answer the question, giving the answer to the nearest cent.

The tax payable by the law clerk is \$6329.38.

9.5.3 Preparing a wage sheet

- A wage sheet shows a list of workers and details of their earnings. These include net wages, pay rates, gross deductions and allowances. A wage sheet may look similar to the one shown on the next page.

Employee	Pay rate	Normal hours worked	Overtime	Penalty rate		Allowance	Gross pay	Tax withheld	Net pay
			1.5	1.5	2				
Dave	19.5	38	2	5		25.5	945.75	163.00	782.75
Jose	21.8	32	4	3.5	5		1253.50	271.00	982.50
Neeve	25.7	25		4		15	1002.30	183.00	819.30
Niha	22.45	38				30	853.10	131.00	722.10
Yen	29.85	36			6		1074.60	208.00	866.60

- A pay sheet (or payslip) shows individual workers' details, including the hours worked, pay rate, net wage, tax deduction, leave days and superannuation paid.
- A wage sheet can be set up in a spreadsheet containing formulas that perform the necessary calculations.

WORKED EXAMPLE 14

Using a spreadsheet, complete the following wage sheet.

Employee	Pay rate	Normal hours worked	Overtime	Penalty rate		Allowance	Gross pay	Tax withheld	Net pay
			1.5	1.5	2				
Honura	38.95	38	4		6	15		620.00	
Skye	28.40	38	2	5				289.00	
Beau	19.15	35		5				234.00	

THINK

- 1 Set up a spreadsheet.

WRITE

	A	B	C	D	E	F	G	H	I	J
1			Normal hours worked	Overtime	Penalty rate				Tax withheld	
2				1.5	1.5	2	Allowance	Gross pay		Net pay
3	Employee	Pay rate	worked	1.5	1.5	2	Allowance	Gross pay	withheld	Net pay
4	Honura	38.95	38	4		6	15			
5	Skye	28.4	38	2	5					
6	Beau	19.15	35		5					

- 2 Calculate pay for hours worked:
Pay for normal hours formula
(e.g. 38×38.95)
Overtime pay calculation
(e.g. $4 \times 1.5 \times 38.95$)
Pay for working on penalty rates
(e.g. $0 \times 1.5 \times 38.95 + 6 \times 2 \times 38.95$)
- 3 Calculate gross pay (= hours worked plus allowances).
- 4 Complete the table by copying the formula in column H down for all workers.

$$B4 \times C4$$

$$D4 \times D3 \times B4$$

$$E4 \times E3 \times B4 + F4 \times F3 \times C4$$

$$H4 = B4 \times C4 + D4 \times D3 \times B4 + E4 \times E3 \times B4 + F4 \times F3 \times C4 + G4$$

	A	B	C	D	E	F	G	H	I	J
1			Normal hours worked	Overtime	Penalty rate				Tax withheld	
2				1.5	1.5	2	Allowance	Gross pay		Net pay
3	Employee	Pay rate	worked	1.5	1.5	2	Allowance	Gross pay	withheld	Net pay
4	Honura	38.95	38	4		6	15	2184.8	620	
5	Skye	28.4	38	2	5			1306.4	289	
6	Beau	19.15	35		5			1149	234	

- 5 Calculate the net pay by deducting tax withheld. $J4=H4-I4$
- 6 Answer the question.

	A	B	C	D	E	F	G	H	I	J
1			Normal							
2			hours	Overtime	Penalty rate				Tax	
3	Employee	Pay rate	worked	1.5	1.5	2	Allowance	Gross pay	withheld	Net pay
4	Honura	38.95	38	4		6	15	2184.8	620	1564.8
5	Skye	28.4	38	2	5			1306.4	289	1017.4
6	Beau	19.15	35		5			1149	234	915

Exercise 9.5 Performing financial computations

Understanding, fluency and communicating

- WE12** Calculate the percentage change for the following using a spreadsheet. Give your answers correct to 2 decimal places.
 - Original tax paid is \$450 and the new tax paid is \$435.
 - A person receives a \$45 pay rise. The fortnightly pay was originally \$1680.
 - The original price of an item was \$1580 and after two years it is worth \$1375.
 - After a \$115 pay increase, a person's new monthly pay is \$5890.

- A bus driver receives a pay increase. Her original weekly pay was \$875 and her new pay is \$897. Using a spreadsheet, calculate the percentage change of her wage. Give your answer correct to 2 decimal places.



- A steel worker receives a pay increase of 5.5%. If her original weekly pay (before tax) is \$1023.45, calculate her new weekly pay.

- WE13** A fast-food manager's annual salary is \$48 131. He claims \$980 in tax deductions and receives \$500 in interest from investments.

Using a spreadsheet, calculate the amount of tax payable for the financial year by using the tax table provided on page 294.

- Using a spreadsheet, calculate the tax payable by the following workers.

Employee	Gross salary	Interest	Deductions	Taxable income	Tax free (0–18 200) 0c	Tax tier (18 201–37 000) 19c	Tax tier (37 001–87 000) 32.5c	Tax tier (87 001–180 000) 37c	Tax tier (180 000+) 45c
Stan	52 895	350							
Bett	65 845		650						
Xiao	36 080	125							
Mohamed	75 040	870	1500						

- Workers in a textile factory are paid per item. The number of items 4 employees made for the week are shown.

	Rate/item	Monday	Tuesday	Wednesday	Thursday	Friday
Harry	\$4.75	25	26	24	30	20
Theo	\$6.50	19	18	16	20	15
Maria	\$3.50	35	40	32	29	32
Marcia	\$7.95	22	18	25	21	15

Using a spreadsheet, calculate each employee's weekly earnings, before tax.

7. **WE14** Using a spreadsheet, complete the following wage sheet.

Employee	Pay rate	Normal hours worked	Overtime	Penalty rate		Allowance	Gross pay	Tax withheld	Net pay
			1.5	1.5	2				
Rex	15.85	28		7	7			124	
Tank	22.15	35		5		15		167	
Gert	30.10	32	1.5		8	19		367	

8. A pay sheet for an individual employee is shown. Complete the pay sheet using a spreadsheet, and hence state the employee's net weekly pay and amount of superannuation paid into her superannuation fund.

Entitlements	Unit	Rate	Total
Wages for ordinary hours worked	30 hours	\$35.05	\$
TOTAL ORDINARY HOURS = 78 hours			
Penalty (double-time)	5 hours	\$	\$
Gross payment			\$

Deductions	
Taxation	\$325
Total deductions	
Net payment	

Employer superannuation contribution	
Contribution	\$

9. A worker receives a 4.5% pay increase. Their new weekly pay after the pay rise is \$1095.
- Calculate their annual salary, assuming 52.14 weeks in a year.
 - Calculate their annual salary before the pay rise.
10. A small business owner employs a manager who receives a monthly salary, 2 office workers who are paid an hourly rate and 3 other workers who are paid per item constructed. The following table shows the work information of the 6 employees for one week.

Employee	Hours worked	Hourly rate	Items	Rate/item	Annual salary
Sara (manager)	40 hours	—	—	—	\$55 850
Troy (office worker)	25	\$16.85	—	—	
Helga	30	\$22.50	—	—	
Nina	20	—	19	\$4.25	
Bill	15	—	15	\$5.75	
Max	35	—	28	\$4.95	

Using a spreadsheet, construct a weekly wage sheet that shows the earnings, before tax, for the 6 employees.

Problem solving, reasoning and justification

11. The taxation table is shown.

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$37 000	19 c for each \$1 over \$18 200
\$37 001–\$87 000	\$3572 plus 32.5 c for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37 c for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45 c for each \$1 over \$180 000

Using a spreadsheet, explain how the ‘plus’ calculations of \$3572, \$19 822 and \$54 232 are found for the different tax levels in the table.

12. A waiter is paid an hourly rate of \$21.50. He receives penalty rates of time-and-a-half for working Saturday and double-time for working Sundays and public holidays. Over the Christmas period, he worked 5 hours on Friday night, 8 hours on Saturday, 9 hours on Sunday and 7.5 hours on Boxing Day (public holiday). His employer withholds \$185 in tax. Prepare a pay sheet using a spreadsheet to represent his weekly pay.
13. A teacher’s annual salary is \$94 961. He decides to reduce his time to 0.8 (this means he receives 0.8 of his salary and works 4 days out of 5), claiming that he will be on the lower tax level and pay less in tax overall, so his net fortnightly pay will not be that much less. By calculating the tax payable for both the full-time salary and the 0.8 salary, determine how accurate his claim is.
14. Each year, an IT technician moves up one level on a pay scale. The current pay scale is shown.



Level	Annual salary
1–1	\$51 758
1–2	\$53 258
1–3	\$55 333
1–4	\$57 508
1–5	\$60 088

The technician is currently on level 1–3. At the start of the following year she moves up the pay scale to level 1–4.

- a. Determine the percentage change in her wage correct to 2 decimal places.
- A new working agreement is reached which increases the pay level of technicians on levels 1 and 2 by 3.75%.
- b. Determine the percentage change in the technician’s wage from her original 1–3 wage correct to 2 decimal places.
- c. After the pay increase of 3.75%, another technician moves to level 2–3 with an annual salary of \$65 880. The overall percentage change in his wage is 8.5%. Determine the salary for level 2–2 before and after the pay increase.
15. A new taxation system is being proposed.

Taxable income	Tax on this income
0–\$19 500	Nil
\$19 501–\$65 000	21 c for each \$1 over \$19 500
\$65 001–\$125 000	A plus 35.5 c for each \$1 over \$65 000
\$125 001–\$180 000	B plus 37 c for each \$1 over \$12 500
\$180 001 and over	C plus 45 c for each \$1 over \$180 000

- Calculate the values for **A**, **B** and **C** using a spreadsheet.
- Write down the formula that would calculate the tax payable for a taxable income within the tax level \$65 001–\$125 000.
- By comparing this proposed taxation table with the current table, would this be a fairer taxation system? Justify your answer using calculations.

9.6 Review

9.6.1 Summary

In this topic you have learnt:

- about the difference between salary and wages
- how workers earn money
- how to calculate commissions, piecework and royalties
- about leave loading and allowances
- about taxation and deductions
- how to calculate taxable income and use the taxation table to determine the tax payable
- why workers pay the Medicare levy and how to calculate the amount required
- about superannuation and how employers calculate the amount
- how to do financial computations using a spreadsheet, such as finding the tax payable and completing wage sheets.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 9.6 Review

Understanding, fluency and communicating

- A nurse is paid \$28.50 per hour for an 80-hour fortnight. Assuming 26.07 fortnights per year, calculate:
 - the nurse's annual salary
 - the nurse's annual superannuation (assuming 9.5% superannuation)
 - the nurse's weekly pay.



- MC** Which term represents getting paid for every item constructed?
 - Wages
 - Piecework
 - Commission
 - Superannuation
 - Salary

- A novelist gets a royalty payment of 2.9% of gross sales on her book. Complete the following table.

	January	February	March	April
Gross sales	\$45 000	\$125 000	\$320 000	
Royalty				\$1508

- Convert the following annual salaries to fortnightly pay. (Assume 26.07 fortnights per year.)
 - \$67 899
 - \$98 765
 - \$101 010
 - \$123 456
- Calculate the amount of superannuation, each fortnight, that is paid (assume 9.5% superannuation) for the following annual salaries:
 - \$67 899
 - \$98 765
 - \$101 010
 - \$123 456.
- An architect makes a salary of \$85 500 per year and is given 9.5% extra in the form of superannuation.
 - How much superannuation does she get each year?
 - In the following year, the architect is not given a pay increase, but instead her superannuation increases to 14% of her salary. Calculate the amount of the increase in superannuation.
- A first-year out teacher makes \$49 200 per year.
 - How much do they get paid each fortnight (assuming 26.07 fortnights in a year)?
 - How much additional money is paid into their superannuation fund (assuming 9.5% contribution) each fortnight?

- d. \$112570**

- 

Calculate the earnings before tax for each employee.

- 

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Fortnightly earnings (\$)	Tax withheld (\$)
1442.00	172.00
1444.00	172.00
1446.00	172.00
1448.00	174.00
1450.00	174.00
1512.00	196.00
1514.00	196.00
1516.00	196.00
1518.00	198.00
1520.00	198.00
1612.00	230.00
1614.00	230.00
1616.00	232.00

Fortnightly earnings (\$)	Tax withheld (\$)
1618.00	232.00
1620.00	234.00
1682.00	254.00
1684.00	256.00
1686.00	256.00
1688.00	256.00
1690.00	258.00
2502.00	540.00
2504.00	540.00
2506.00	542.00
2508.00	542.00
2510.00	542.00

19. **MC** A worker receives 17.5% leave loading. If the worker's annual salary is \$57 840, the amount of leave loading received will be closest to:
A. \$195 **B.** \$409 **C.** \$777 **D.** \$844 **E.** \$10 122
20. **MC** A dressmaker receives \$5.75 for each item sewn. In one working day (7.5 hours) she earns \$517.50. The average number of items she sews in one hour is:
A. 2 **B.** 12 **C.** 13 **D.** 69 **E.** 90

Problem solving, reasoning and justification

21. An up-and-coming movie star is given the following two options for pay on her next movie.
- Plan A: A flat fee of \$100 000
 - Plan B: A royalty structure which gives her:
 - 1% of the sales under \$5 000 000
 - 0.5% of the sales between \$5 000 000 and \$10 000 000
 - 0.25% of the sales over \$10 000 000
- What do the sales have to be to make Plan B the better option?
22. A factory worker's hourly rate is \$21.50. He works 38 hours a week with 4 hours overtime paid at time-and-a-half. The tax withheld each week is \$195. He claims \$25 a week for the uniform and earns \$120 in interest for the year. (Assume 52.14 weeks in the year.)
- Calculate the taxable income and Medicare levy.
 - Explain why the worker should expect to receive a tax refund. Support your explanation with calculations.
23. To reduce the amount of tax payable, an investment banker decides to contribute more into his superannuation fund, which is taken out of his monthly pay before tax. He deposits \$25 000 into his superannuation fund, which will be taxed at 15% after he retires and starts to access the fund. His annual salary is \$98 475.
- Calculate the amount of tax payable on his annual salary of \$98 475.
 - Calculate the amount of tax payable after he contributes \$25 000 to his superannuation fund.
 - Taking into account that he will pay 15% tax after he retires on the money he contributes to his superannuation fund, determine if he will pay less in tax.

24. The fortnightly pay slip for an employee is shown.

Entitlements	Unit	Rate	Total
Wages for ordinary hours worked	76 hours	\$20.95	\$
TOTAL ORDINARY HOURS = 76 hours			
Overtime	15 hours	\$31.40	\$
Travel allowance	350 km	\$0.66/km	\$
Gross payment			\$

Deductions	
Taxation	\$430
Total deductions	
Net payment	

- a. Using a spreadsheet, calculate the fortnightly pay.
- b. Assuming she receives the same fortnightly pay for the year, calculate her annual taxable income. Assume 26.07 fortnights in the year.
- c. Determine if the employee will receive a tax refund or have to pay more in tax.
25. A new working agreement is proposed for workers at a local abattoir. Penalty rates for working on weekends and overtime will be standardised to a rate of 1.25 hourly rate with a pay rise of 4.5%. The current penalty rates for working Saturday and Sunday are time-and-a-half and double-time respectively, and working overtime is paid at time-and-a-half. If the average hourly rate for a worker at the abattoir is \$19.78, will workers be better off under the new working agreement? Justify your answer using calculations.
26. A window cleaner has the opportunity to move from casual to part-time employment. Workers employed on a casual basis do not receive annual or sick leave. All employees in the cleaning business receive 9.5% superannuation. The window cleaner's current casual hourly rate is \$25.76 and the part-time hourly rate is \$23.70. Should he move to part-time employment? Justify your answer by calculating his annual salary based on working 38 hours each week at the normal hourly rate and his annual leave loading.
27. A worker receives 17.5% annual loading just before his four-week annual leave. He is paid an hourly rate of \$20.50 for working a 38-hour week. He does not receive overtime or penalty rates.
- a. Calculate the amount of annual loading he will receive for the four weeks he takes his leave.
- b. Complete his pay slip for the month he receives his annual leave loading.



Entitlements	Unit	Rate	Total
Wages for ordinary hours worked	38 hours	\$20.50	\$
Annual leave loading (17.5%)			\$
TOTAL ORDINARY HOURS = 38 hours			
Gross payment			\$

Deductions	
Taxation	\$295
Total deductions	
Net payment	

The worker complains that he has paid too much in tax in the month he received his annual leave loading, and asks to have his leave loading paid over a 6-month period (26 weeks). His weekly PAYG tax is \$106.

c. Calculate his weekly wage if his annual leave loading is paid over 26 weeks.

The PAYG tax if he is paid his annual leave loading over 26 weeks is \$113.

d. Determine if his claim of paying too much tax is valid. Justify by using your answers from the previous parts.

- 28.** A baker is paid an hourly rate of \$25.27 and overtime rate of \$35. In a usual working week she works 38 hours at normal pay and 6 hours overtime.

She receives her annual leave loading of \$672.18 and superannuation contributions of \$91.22 for the week.

She complains that she has been underpaid for both her annual loading and superannuation contributions. Explain how her annual leave loading and superannuation contributions were calculated. Should she have received more money?

- 29.** Business groups are seeking a change to the taxation system. Their proposal is shown:

Taxable income	Tax on this income
0–\$29 000	Nil
\$29 001–\$85 000	35c for each \$1 over \$29 000
\$85 001–\$175 000	A plus 40c for each \$1 over \$85 000
\$175 001	B plus 58c for each \$1 over \$175 000

a. Calculate the values of A and B.

b. Compare the tax payable for the following annual salaries under the new proposed system and the current taxation table.

i. \$25 890

ii. \$45 870

iii. \$67 950

iv. \$81 940

v. \$195 870

c. Based on your calculations from part **b**, how will workers be better or worse off under the proposed new taxation system?

- 30.** A physiotherapist's annual salary is \$63 821. Her annual travel claim is 950 km and she is allowed to claim 66c for each km travelled as a tax deduction. She receives a uniform allowance of \$15 each week. She is paid monthly and her employer withholds \$1120 in tax. Assume 52.14 weeks in the year.

a. Using a spreadsheet, calculate her taxable income, tax payable, Medicare levy, and employer superannuation contribution.

b. Explain why she will have to pay tax. Justify your answer using calculations.



Answers

Exercise 9.2 Getting paid

1. Wage (hourly rate) is the greater pay (\$3200 fortnight compared to \$2838.51).
2. B
3. a. \$1476.31 b. \$1136.89 c. \$1596.83
4. 34.31 hours
5. \$637.50
6. First fortnight: \$1735.95
Second fortnight: \$1833.75
7. \$144.08
8. a. \$12 979.17 b. \$14 796.25 annually, \$1233.02 monthly
9. a. \$5166.67 b. \$490.83 c. \$5890
10. \$1154.31
11. a. \$2100 b. \$10 850
12. a. She will be worse off by 25% for every hour she works on Sunday, but no worse off for working on Saturdays.
b. 16.65%
13. a. \$1270.90
b. Her weekly salary would be \$1304.18, which is higher than her hourly rate calculation (\$1270.90). She should take the salary offer.
14. The employees should accept the new agreement.
The new agreement is 15.5% more than the base salary compared to 10.8% more under the current agreement.
15. He will receive the most amount over 5 years by choosing package 1 (\$322 836.55 compared to \$315 028.38).

Exercise 9.3 Commission, piecework and royalties

1. \$3390
2. a. \$1650 b. \$2375 c. \$2920 d. \$4160
3. C
4. \$6825
5. \$1398.25
6. B
7. \$12.11
8. \$186 989.67
9. A
10. a.

Time	Jan–Mar	Apr–Jun	Jul–Aug	Sep–Dec
Sales	\$123 400	\$2 403 556	\$432 442	\$84 562
Royalty payment	\$3085.00	\$60 088.90	\$10 811.05	\$2114.05

b. \$376 099
11. \$45 634.53
12. \$2 500 000
13. \$16 599.48
14. Royalty percentage can be found by dividing the royalty amount by the total sale amount and then multiplying by 100. The songwriter's royalty percentage is 2.5%.
15. Option 2
16. a. \$2090 b. 22
c. If he sells 22 new telephone plans each week, taking the salary option of \$37 500 per annum would give him a higher weekly wage.

Exercise 9.4 Taxation and deductions

1. \$8118.75
2. a. \$1208 b. \$1546 c. \$1788 d. \$2168.46

4. a. \$1890 **b.** \$1564
5. \$1578.50

6. a. i. 32.5% on amounts over \$37 000 + \$3572
 ii. \$3734.50
c. i. 32.5% for amounts over \$37 000 + \$3572
 ii. \$19 389.75

7.

a. \$192	b. \$366
c. \$384	d. \$768

8.

a. 28.93%	b. 30.4%
c. 31.86%	d. 33.33%

9. \$704.32

10. a. \$4857 **b.** \$975
11. a. \$85 970 **b.** \$19 487.25

12. a. \$2695.84
b. $\$2695.84 - \$606 = \$2089.84$
c. \$13 518.89
d. \$1352.12

- 13. a.** \$5614.50
- b.** \$14 400
- c.** He will receive a refund of \$1396.82.

Working 4 hours overtime equates to the same pay as working 6 hours normal time ($4 \times 1.5 = 6$). Calculate the number of 'normal' hours worked ($38 + 6 = 44$) and divide into wage.

d. \$5558.34

He won't receive \$1980 as a tax refund because the tax payable is a percentage of his taxable income. His taxable income was reduced by \$1980 but the tax payable was reduced by 32.5% of the difference ($0.325 \times (8091.80 - 1980)$).

15. a. When calculating tax using this tiered system, the taxable salary of \$97 605 is split into the different tax-bracket amounts, which are found by the difference between the lower and upper limits, e.g. $37\,000 - 18\,200 = \$18\,800$.

$\$18\,200 + \$18\,800 + \$50\,000 + \$10\,605 = \$97\,605$, and each amount is multiplied by different percentages, as shown.

This is much lower than $37\% \times \$97\,605 = \$36\,113.85$.

b. The dentist's taxable salary falls into the fourth tax bracket, 37% for amounts over \$87 000 plus \$19 822. The \$19 822 is the tax amount from the previous tax brackets ($19\% \times \$18\,800 + 32.5\% \times \$50\,000$), as shown.

16. a. Determine the tax bracket her taxable income falls within. Since she paid more than \$3572 and less than \$19 822, her taxable income is within the third tax bracket: $\$3572 + 32.5\% \times (\text{taxable income} - \$37\,000)$.

b. \$71 584.62 **c. Refund: \$320.13**

1. a. 3.33% b. 2.68% c. 12.97% d. 1.99%

2. 2.51%

3. \$1079.74

4. \$7033.58

5. Stan = \$8851.63, Bett = \$12 735.38, Xiao = \$3420.95, Mohamed = \$15 730.25

TOPIC 9 Earning and managing money **313**

Employee	Pay rate	Normal hours worked	Overtime	Penalty rate		Allowance	Gross pay	Tax withheld	Net pay
			1.5	1.5	2				
Rex	15.85	28		7	7		832.13	124	708.13
Tank	22.15	35		5		15	956.38	167	789.38
Gert	30.10	32	1.5		8	19	1531.53	367	1164.53

Entitlements	Unit	Rate	Total
Wages for ordinary hours worked	30 hours	\$35.05	\$1051.50
TOTAL ORDINARY HOURS = 78 hours			
Penalty (double time)	5 hours	\$70.10	\$350.50
Gross payment			\$1402.00

Deductions	
Taxation	\$325
Total deductions	\$325
Net payment	\$1077.00

Employer superannuation contribution	
Contribution	\$133.19

9. a. \$57 093.30

b. \$54 634.74

10. Sara = \$1074.04, Troy = \$421.25, Helga = \$675.00, Nina = \$80.75, Bill = \$86.25, Max = \$138.60

11. The 'plus' calculations are found by adding the tax payable in the previous brackets.

Hourly rate	Normal hours	Saturday	Sunday	Public holiday	Gross pay	PAYG tax	Net pay
		1.5	2	2			
21.5	5	8	9	7.5	\$1,075.00	\$185.00	\$890.00

13. The claim is incorrect. While the teacher will pay \$6530.71 less in tax, reducing his salary by 20% has an overall reduction of \$18 922.20 in pay, so he will have \$12 461.49 less pay.

14. a. 3.93%

b. 7.83%

c. Before pay increase = \$60 718.89

After pay increase = \$62 995.85

15. a. A = \$9555, B = \$30 855, C = \$51 205

b. $= 9555 + 0.355 \times (\text{taxable salary} - 65\,000)$

c. Workers who previously earned between \$37 000 and \$65 000 will pay less tax in the new system; however, workers previously earning between \$65 000 and \$87 000 will pay 3% more in tax. Workers who previously earned between \$18 200 and \$37 000 will pay 1% more in tax, while workers earning more than \$180 000 will pay less. So on balance, the middle earners will pay less but those who are low-income earners will pay more and those on high wages pay less; hence it does not seem to be a fairer system.

Exercise 9.6 Review

1. a. \$59 439.60

b. \$5646.76

c. \$1140

2. B

	January	February	March	April
Gross sales	\$45 000	\$125 000	\$320 000	\$52 000
Royalty	\$1305	\$3625	\$9280	\$1508

4. a. \$2604.49

b. \$3788.45

c. \$3874.57

d. \$4735.56

5. a. \$247.43

b. \$359.90

c. \$368.08

d. \$449.88

6. a. \$8122.50

b. \$11 970

Superannuation increase is \$3847.50.

7. a. \$1887.23

b. \$179.29

8. C

9. a. \$970.02

b. \$651.40

c. \$1317.80

d. \$2251.40

10.

Employee	Saturday	Sunday	Monday (public holiday)	Gross wage
Rate	\$27.75	\$37	\$37	
Tran	5	8		\$434.75
Gus		6		\$222
Meg	3.5		7	\$356.13
Warren	4		6	\$333

11. \$56 880

12. D

13. a. i. \$23 100.20

b. i. \$9682

ii. \$9106.70

ii. \$5301

c. i. \$7455.75

d. i. \$1263.50

ii. \$4650.25

ii. \$2360.75

14. A

15. \$38 491

Lower limit: \$38 491, tax payable: \$4056.58

Upper limit: \$84 767, tax payable: \$19 096.28

16. \$14 812.50

17. 3.57%

18.

Employee	Pay rate	Normal hours worked	Overtime	Penalty rate		Allowance	Gross pay	Tax withheld	Net pay
			1.5	1.5	2				
Eric	\$18.75	76	3			10	\$1519.38	198	\$1321.38
Mara	\$22.85	74					\$1690.90	258	\$1432.90
Kip	\$20.00	56		5	8	24	\$1614.00	230	\$1384.00
Nuang	\$28.75	65		4	6	120	\$2506.25	542	\$1964.25
Lex	\$17.20	72		8			\$1444.80	172	\$1272.80

19. C

20. B

21. Sales would have to be at least \$20 million to make the Plan B better than Plan A.

22. a. Taxable income: \$48 144.44

Medicare levy: \$962.89

b. He has paid \$2010.47 more in tax than is required, so he will receive a tax refund.

23. a. \$24 067.75

b. \$15 426.38

c. He will pay less in tax ($\$24\,067.75 - \$19\,176.37 = \$4891.38$) because the \$25 000 is taxed at a lower rate (15% compared to 37%) and his overall annual salary has been taxed at a lower rate (32.5% compared to 37%).

24. a. \$1864.20

b. \$59 809.79

c. A tax return of \$224.92

25. For workers who only work standard hours (38 hours per week), the new agreement will result in more money (an increase of 4.5% more). However, workers who work on weekends or overtime will be worse off under the new agreement.

26. Casual employment will give him more money overall (\$52 048.64 compared to \$51 599.99 including annual leave loading) based on working 38 hours each week for 52 weeks. He will not have sick or annual leave. Considering taking annual leave, working part-time would be the better deal. He will have sick leave and job security.

27. a. \$545.30

b.

Entitlements	Unit	Rate	Total
Wages for ordinary hours worked	38 hours	\$20.50	\$779.00
Annual leave loading (17.5%)			\$545.30
TOTAL ORDINARY HOURS = 38 hours			
Gross payment			\$1324.30

Deductions	
Taxation	\$295
Total deductions	\$295
Net payment	\$1029.30

c. \$686.97

d. Taking the annual leave over 26 weeks reduces his PAYG tax for the year by \$7.00. So his claim is correct.

28. Average working week: $38 \times \$25.27 = \$960.26$

Annual leave loading is 17.5% of 4 weeks' pay: $0.175 \times \$960.25 \times 4 = \$672.18$

Superannuation is 9.5% of normal pay: $0.095 \times \$960.25 = \$91.22$

Both annual leave loading and superannuation are calculated on the average working week paid at normal rates. Additional pay from working overtime is not included in the calculation. Therefore both calculations for the baker are correct and she should not receive more money.

29. a. A: \$19 600

B: \$55 600

b.

	Taxable income	Tax payable current	Tax payable proposed
i	\$25 890	\$1461.10	\$0
ii	\$45 870	\$6454.75	\$5904.50
iii	\$67 950	\$13 630.75	\$13 632.50
iv	\$81 940	\$18 177.50	\$18 529.00
v	\$195 870	\$61 373.50	\$67 704.60

c. Under the proposed taxation system, lower income earners will pay less in tax due to an increase in the upper limit within the tax-free threshold from \$18 200 to \$25 890. Higher income earners pay more (\$67 704.60 compared to \$61 373.50) due to a higher percentage tax rate (58% compared to 45% currently). Middle income workers will pay slightly more (\$18 529 compared to \$18 177.50) under the proposed system due to an increase in the tax rate (35% compared to 32.5% under the current system).

30. a. Taxable income = \$65 230.10

Tax payable = \$12 746.78

Medicare levy = \$1304.60

Superannuation contribution = \$6063.00

b. Her PAYG tax is \$13 440 and amount-payable tax based on her taxable income (\$65 230.10) is \$12 746.78, not including the 2% Medicare levy (\$1304.60). Including the Medicare levy, the total amount of tax she needs to pay is \$14 051.38, which is \$611.38 more than she paid, so she will have to pay the difference.

TOPIC 10

Budgeting and household expenses

10.1 Overview

10.1.1 Introduction

Before you take the first steps to becoming an independent adult it's a good idea to be aware of some of the costs you'll be expected to pay.

Basic household expenses, such as electricity and water bills, can't be avoided so should be budgeted for. Many companies offer discounts for paying these bills early or on time, so it's good to be prepared for these costs.

Purchasing your first car is one of the biggest steps you can take, but as well as the initial cost you also need to take into account the cost to insure and run the vehicle.

By budgeting you can have an understanding of your income and expenses and can make good decisions in the present and looking forward.



DISCUSSION

How much do you think you will need to save to purchase your first car?

LEARNING SEQUENCE

- 10.1** Overview
- 10.2** Household expenses
- 10.3** Purchasing, running and maintaining a vehicle
- 10.4** Budgeting
- 10.5** Review

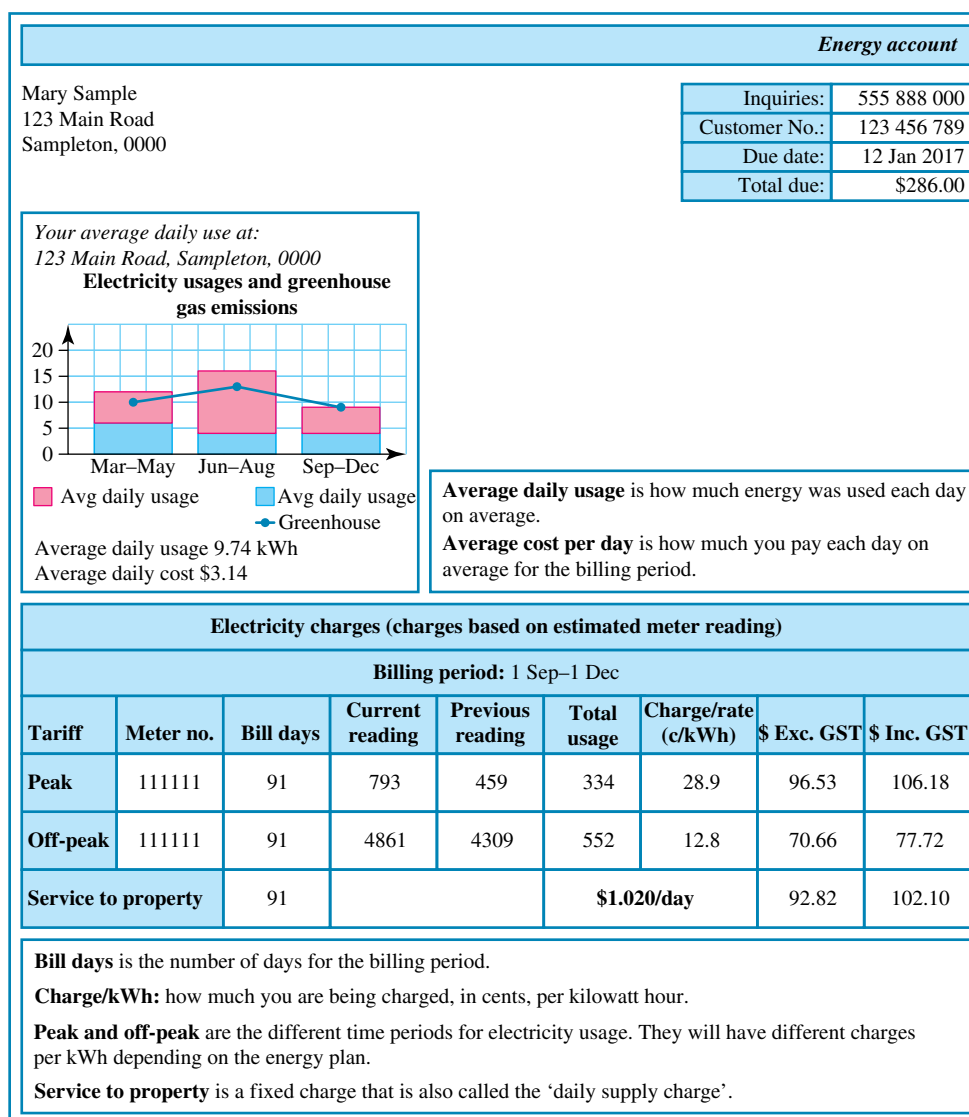
CURRICULUM CONTENT

- interpret and use information about a household's electricity, water or gas usage and related charges and costs from household bills **AAM** ◇ 🌿 🇦🇺
- plan for the purchase of a car **AAM** ◇ ⚙️ 👤
- plan for the running and maintenance of a car **AAM** ◇ ⚙️ 👤
- prepare a personal budget for a given income, taking into account fixed and discretionary spending (ACMGM004) **AAM** ◇ ⚙️ 💻 🇦🇺

10.2 Household expenses

10.2.1 Understanding household electricity and gas bills

- Electricity and gas are supplied to our homes to help us keep warm and cool, to allow us to cook, and to provide lighting.
- There are three main parts to our electricity and gas supply systems: wholesalers (or generators), distributors and retailers.
- Wholesalers produce electricity and extract gas.
- Distributors own and maintain infrastructure such as power poles, wiring and gas lines. When there are disruptions to the electricity or gas supply, it is the distributors who are contacted to fix the problem.
- Retailers purchase electricity and gas from wholesalers and then sell them to customers. There are many electricity and gas options available to customers, and customers can select whichever deal best suits their needs. Therefore, it is important to know how much you will be charged per unit of energy used and what discounts are available before you sign any contracts.
- The standard measurement of electricity consumption is the **kilowatt hours** (kWh). This is the average amount of electricity (in kilowatts) used per hour (1000 watts make up a kilowatt). Gas consumption is measured in **megajoules** (MJ).
- To understand electricity and gas bills, it is important to know some of the terms that are used.



WORKED EXAMPLE 1

The meter readings and charge/rates for a household are as shown.

Electricity charges (charges based on meter reading)								
Billing period: 1 Jan–31 March								
Tariff	Meter no.	Bill days	Current reading	Previous reading	Total usage	Charge/rate (c/kWh)	\$ Exc. GST	\$ Inc. GST
Peak		90	12 791	12 353	438	29.85		
Off-peak		90	2587	1883	704	13.87		
Service to property		90				\$1.254/day		

Calculate the total bill amount.

THINK

- 1 Calculate the electricity usage by subtracting the previous reading from the current reading for both peak and off-peak.
- 2 Calculate the cost for each time period by multiplying the usage by the charge/rate.
- 3 Add GST by adding 10% of each amount.
- 4 Calculate the costs for service to property by multiplying the cost per day by the number of billing days.
- 5 Add 10% GST.
- 6 Find the total cost by adding the usage cost to the service cost.
- 7 Answer the question.

WRITE

Peak: $12\,791 - 12\,353 = 438$
 Off-peak: $2587 - 1883 = 704$
 Peak: $438 \times 0.2985 \approx \$130.74$
 Off-peak: $704 \times 0.1337 \approx \$94.12$
 Peak: $10\% \times \$130.74 = \$13.074$
 $\$130.74 + \$13.074 = \$143.81$
 Off-peak: $10\% \times \$94.12 = \$9.412$
 $\$94.12 + \$9.412 = \$103.53$
 Total for usage = $\$143.81 + \$103.53 = \$247.34$
 Service costs = $1.254 \times 90 = \$112.86$
 $10\% \times \$112.86 = \$11.286$
 Service costs = $\$112.86 + \$11.286 = \$124.15$
 Total cost = $\$247.34 + \$124.15 = \$371.49$
 The bill total is \$371.49.

10.2.2 Understanding household water bills

- Water is provided to homes and businesses through a series of water pipes that are managed and serviced by providers.
- Home owners are charged a fixed fee to provide water and remove water wastage, known as sewerage.
- The amount of water used in the home over a fixed period, usually 3 months, is measured through a water meter fixed to the main water supply to the home. Home owners are charged for their water usage at a rate, in dollars, per kilolitre (kL).

1000 litres = 1 kilolitre

- *Note:* GST does not apply to water bills.

WORKED EXAMPLE 2

Calculate the water rates for a home with the following usage and fixed charges:

Bill details as of 31 March		Value	Price
Water service charge	1 Jan to 31 Mar	22.51	
Sewerage service charge	1 Jan to 31 Mar	145.90	
Storm water	1 Jan to 31 Mar	18.70	
	Volume	Charge/kL	
Water usage	55	\$2.00	
Recycled water usage	35	\$1.79	

THINK

- 1 Calculate the cost of the water usage by multiplying the volume by the cost.
- 2 Calculate the cost of the recycled water usage by multiplying the volume by the cost.
- 3 Calculate the fixed service cost.
- 4 Add the fixed service cost to the water usage cost.
- 5 Answer the question.

WRITE

$$\begin{aligned}\text{Water usage} &= 55 \times 2 \\ &= \$110\end{aligned}$$

$$\begin{aligned}\text{Recycled water} &= 35 \times 1.79 \\ &= \$62.65\end{aligned}$$

$$22.51 + 145.90 + 18.70 = 187.11$$

$$187.11 + 110 + 62.65 = \$359.76$$

The water rates cost \$359.76.

10.2.3 Understanding council rates

- Councils provide services and infrastructure to people living in communities, such as parklands, libraries, rubbish collection and road maintenance. To support councils in providing these services, home owners pay **council rates**.
- Council rates are calculated as a percentage on the capital improved value of the land and buildings within the council municipality. The rate at which rates are charged — the **rate in the dollar** — is determined by finding how much revenue the council plans to raise and dividing this by the amount of capital improvement of land and buildings within the council municipality.
- For example, if the council plans to raise \$15 million for the annual budget and there is \$2 billion in capital improved value within the municipality, then the rate in the dollar is $\frac{15\,000\,000}{2\,000\,000\,000} = 0.0075$.
If the capital improvement on a rate payer's house and land is \$4 500 000, then the payable council rates for that property are $0.0075 \times 4\,500\,000 = \$3375$.
- *Note:* GST does not apply to council rates.



WORKED EXAMPLE 3

A council municipality plans to raise \$15 million, and there is \$3.25 billion in capital improved value in the municipality. Calculate the payable council rates for a property with \$275 000 capital improved value. Write your answer correct to the nearest cent.

THINK

- Find the rate in the dollar. Divide the revenue to be raised by the capital improved value.
- Multiply the rate in the dollar by the capital improved value.
- Answer the question.

WRITE

$$\frac{15\,000\,000}{3\,250\,000\,000} = \frac{15}{3250} = 0.004\,62\dots$$

$$0.004\,62\dots \times 275\,000 = \$1269.23$$

The council rates payable are \$1269.23.

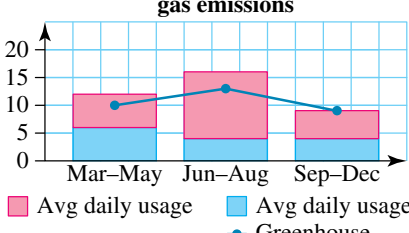
eBookplus RESOURCES

Interactivity: Electricity bills (int-6912)

Exercise 10.2 Household expenses

Understanding, fluency and communicating

- WE1** The meter readings and charge/rates for a household are as shown. Calculate the total bill amount. (Remember to include GST.)

<i>Energy account</i>																
J Citizen 111 High Street City Town, 0000						<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">Inquiries:</td> <td style="padding: 2px 5px;">555 888 000</td> </tr> <tr> <td style="padding: 2px 5px;">Customer No.:</td> <td style="padding: 2px 5px;">258 794 360</td> </tr> <tr> <td style="padding: 2px 5px;">Due date:</td> <td style="padding: 2px 5px;">15 July 2017</td> </tr> <tr> <td style="padding: 2px 5px;">Total due:</td> <td style="padding: 2px 5px;"></td> </tr> </table>			Inquiries:	555 888 000	Customer No.:	258 794 360	Due date:	15 July 2017	Total due:	
Inquiries:	555 888 000															
Customer No.:	258 794 360															
Due date:	15 July 2017															
Total due:																
<i>Your average daily use at:</i> 111 High Street, City Town, 0000 Electricity usages and greenhouse gas emissions  Avg daily usage Avg daily usage —•— Greenhouse Average daily usage Average daily cost																
Electricity charges (charges based on meter reading)																
Billing period: 1 April–30 June																
Tariff	Meter no.	Bill days	Current reading	Previous reading	Total usage	Charge/rate (c/kWh)	\$ Exc. GST	\$ Inc. GST								
Peak	20205	91	5897	5490		25.84										
Off-peak	20205	91	12370	11841		12.95										
Service to property		91				\$1.105/day										

2. Calculate the energy charges (excluding GST) for the following, giving your answers correct to the nearest cent.

	Energy usage	Charge/rate
a.	563 kWh	10.56 c/kWh
b.	1070 MJ	2.629 c/MJ
c.	895 kWh	14.64 c/kWh
d.	6208 MJ	1.827 c/MJ

3. The usage component of a family's energy bill for a quarter is \$545 excluding GST. The charge/rate is 11.43 c/kWh. Calculate their energy usage for the quarter. Give your answer to the nearest whole number.
4. Part of an electricity bill is shown below.

Electricity charges (charges based on meter reading)								
Billing period: 1 Sep–31 Dec								
Tariff	Meter no.	Bill days	Current reading	Previous reading	Total usage	Charge/rate (c/kWh)	\$ Exc. GST	\$ Inc. GST
All day	28 710	91	47 890	46 995		30.54		
Service to property		91			\$1.017/day			

- a. Write down:
- the charge/rate
 - the daily service to property fee
 - the usage amount in kWh.
- b. Calculate:
- the amount charged for electricity usage excluding GST
 - the total amount for the bill, including GST.
5. There is a 15% discount for customers who pay their energy bill before the due date. The energy usage of a customer is:
- electricity: 456 kWh at 12.46 c/kWh
 - gas: 967 MJ at 1.956 c/MJ
 - \$95 fixed fee for service to the property.
- If the customer pays before the due date, calculate the amount paid including GST.
6. **MC** A family's electricity usage is shown in the following table.

Tariff	Bill days	Current reading	Previous reading	Total usage	Charge/rate c/kWh
Peak	90	58 973	58 615		29.05
Off-peak	90	8569	8028		15.59
Service to property		90			\$1.08/day

The total cost for their electricity usage, including GST and correct to the nearest 5 cents, is:

- A. \$188.35 B. \$269.00 C. \$285.55
D. \$314.10 E. \$366.20

7. **WE2** Calculate the water rates for a home with the following usage and fixed charges.

Bill details as of 31 March		Value	Price
Water service charge	1 Jan to 31 Mar	22.51	
Sewerage service charge	1 Jan to 31 Mar	145.90	
Storm water	1 Jan to 31 Mar	18.70	
	Volume (L)	Charge/kL	
Water usage	58 000	\$2.00/kL	
Recycled water usage	39 000	\$1.79/kL	

8. A customer pays \$245.85 for their water bill for 3 months. They are charged a fixed service fee of \$180.30. The charge/kL is \$1.95. The customer does not have recycled water service. Calculate the amount of water used by the customer for the 3 months. Give your answer correct to 2 decimal places.
9. **WE3** A council municipality plans to raise \$12 million, and there is \$1.8 billion in capital improved value in the municipality. Calculate the payable council rates for a property with \$275 000 capital improved value.
10. The council rates for a home that has a capital improved value of \$325 000 are \$1950.
- Calculate the rate in the dollar charged by the council.
 - Calculate the amount of revenue the council intends to collect if the total amount of capital improved value for the municipality is \$2.67 billion.

Problem solving, reasoning and justification

11. A customer has a choice between two electricity plans, A and B. The two plans are shown below.

Plan A		Plan B	
Charge/rate		Charge/rate	
All day	12.03 c/kWh	Off-peak (11 pm–7 am)	9.08 c/kWh
		Peak	13.01 c/kWh
Service to property	\$1.01/day	Service to property	\$0.99/day
Pay-on-time discount	15%	Pay-on-time discount	12%

The customer's energy usage over 90 billing days is 534 (peak time) and 757 (off-peak time).

Which plan would be the more cost-effective for the customer? Justify your answer by calculating the electricity costs for each plan for if the customer pays on time and if he doesn't pay on time.



12. Here is an incomplete water bill received by a household.

Bill details as of 30 June		Value	Price
Water service charge	1 Apr to 30 Jun	23.80	
Sewerage service charge	1 Apr to 30 Jun	150.20	
Storm water	1 Apr to 30 Jun	17.95	
	Volume (L)	Charge/kL	
Water usage	64 000		130.56
Recycled water usage	38 000		
		Total:	\$393.57

- a. Complete the water bill by filling in the missing values.

As of 1 July, the charges per kL will increase by 4%, the water service charge will increase by \$2.50, and the sewerage service charge will increase by \$1.50.

- b. Calculate the new charges for water usage.
- c. Determine the percentage increase for water and sewerage service charges. Give your answers correct to 2 decimal places.
- d. By what percentage will the average water bill increase (assuming water usage remains the same)? Give your answer correct to 2 decimal places.
13. A family has both electricity and gas connected to their house. Their average electricity usage for 90 billing days is 987 kWh and their average gas usage is 1650 MJ per month (30 days). Service to property charges are the same for both providers. The family decides to change providers. They have the following choices:

Choice 1: Separate energy providers

- Electricity: all-day rate 10.03 c/kWh
- Pay-on-time discount (electricity only): 12.5%
- Gas: all-day rate 1.89 c/MJ

Choice 2: Bundle both electricity and gas with one provider; receive the following discounts

- 17% discount on gas
- 10% discount on electricity
- Electricity daily rate: 12.01 c/kWh
- Gas daily rate: 2.03 c/MJ

Which choice will be the most cost-effective for the family, assuming they always pay on time?

14. A council plans to raise \$7.5 million in revenue. The capital improved value of home owners in the council area is \$2.5 billion. To calculate the rate in the dollar, the council performs the following calculation: $\frac{7.5}{2.5} = 3$.

The capital improved value on a particular home is \$175 000.

To calculate the council rates owing for that home, a council worker performs the following calculation: $\frac{3}{100} \times 175\,000 = \$5250$.

The home owner complains that the rates are far too high. Is there an error in the calculation performed by the council and/or by the council worker? Justify your answer by calculating the council rates for the rate payer's property.

15. A family household's bills for 3 months (1 quarter) are as shown.

Electricity bill

Tariff	Bill days	Current reading	Previous reading	Total usage	Charge/rate (c/kWh)
Peak	90	12 589	11 902		31.08
Off-peak	90	2058	1511		14.05
Service to property	90				\$1.075/day

Water bill

Bill details as of 31 March		Value	Price
Water service charge	1 Jan to 31 Mar	22.51	
Sewerage service charge	1 Jan to 31 Mar	145.90	
Storm water	1 Jan to 31 Mar	18.70	
	Volume (kL)	Charge/kL	
Water usage	55	\$2.00/kL	
Recycled water usage	35	\$1.79/kL	

Council rates

Capital improved value \$280 000, rate in the dollar 0.008

- Show that the quarterly council rate instalment is \$560.
- Calculate the amount that the family spends on electricity, water and council rates for the quarter. (Remember to include GST where relevant.)
- The family's monthly net income is \$6019.35 (after tax). Calculate the percentage of their income that goes towards paying the household bills. Give your answer correct to 2 decimal places.
- If the family wishes to reduce their household bills, what advice would you give?

10.3 Purchasing, running and maintaining a vehicle

10.3.1 Planning for the purchase of a vehicle

- Buying a vehicle for the first time can be very exciting. However, it pays to do research and know exactly which type of vehicle will best suit one's needs. For example, do you need a small, medium-sized or large vehicle? How many seats, and how big an engine?
- There are many costs associated with buying a vehicle, but the primary cost is the purchase price or sale price.
- The advertised price for the vehicle is not always the sale price. Many vehicle dealerships are willing to negotiate on price or match other competitive prices for a similar or same-model vehicle. It is important to know how much the vehicle is worth and then shop around for the best price.



Paying for the vehicle

- There are many financial institutions that offer loans to purchase vehicles. It is also possible to obtain finance through vehicle dealerships. As with all loans, it is important to know exactly how much interest is charged and what additional costs apply, such as administration fees and loan servicing fees. It is worth noting that if the customer does not meet the loan repayments, then the vehicle may be repossessed by the dealership or financial institution.
- A **comparison rate** gives a more accurate indication of the interest rate for the loan. It includes all the additional fees in the calculation.
- At times, 0% finance is advertised for the purchase of a vehicle. This means that there is no interest being charged on the loan. However, this type of finance may only apply to certain models and the sale price may not be negotiable.

WORKED EXAMPLE 4

A customer takes out a loan to purchase a car for \$15 090. The monthly repayments are \$294.26 and the loan is for 5 years. The customer pays a 10% deposit. Calculate:

- the deposit paid
- the total amount paid for the car
- the interest rate charged for the loan.



THINK

a 1 Find 10% of \$15 090.

2 Answer the question.

b 1 Find the total amount paid in repayments.

2 Calculate the amount paid for the car by adding the deposit paid to the total amount in monthly repayments.

3 Answer the question.

c 1 Find the principal by subtracting the deposit from the amount borrowed.

2 Find the interest paid for the car by subtracting the principal from the total amount in monthly repayments.

3 Use the formula $I = \frac{PRT}{100}$ to find the rate.

$$P = \$13\,581, I = \$4074.30, T = 5$$

4 Answer the question.

WRITE

$$\frac{10}{100} \times 15\,090 = 1509$$

The deposit paid is \$1509.

$$\begin{aligned} \text{Monthly repayments} &= \$294.26 \text{ for 5 years} \\ \$294.26 \times 12 \times 5 &= \$17\,655.30 \end{aligned}$$

$$\begin{aligned} \text{Amount paid for the car} &= \$1509 + \$17\,655.30 \\ &= \$19\,164.30 \end{aligned}$$

The total amount paid for the car is \$19 164.30.

$$\$15\,090 - \$1509 = \$13\,581$$

$$\begin{aligned} \text{Interest} &= \$17\,655.30 - \$13\,581 \\ &= \$4074.30 \end{aligned}$$

$$4074.30 = \frac{13\,581 \times R \times 5}{100}$$

$$407430 = 13\,581 \times R \times 5$$

$$\begin{aligned} \frac{407430}{13\,581 \times 5} &= R \\ R &= 6\% \end{aligned}$$

The interest rate charged is 6% per year.

10.3.2 On-road costs for new and used vehicles

- A **registration fee** is a combination of administration fees, taxes and charges paid to legally drive a vehicle on the roads. Registered vehicles have number plates that help emergency services identify to whom the vehicle belongs and help transport authorities keep track of vehicle ownership.
- Registration costs depend on the type of vehicle and the location.
- **Vehicle insurance** covers the costs associated with vehicle accidents, such as repairs, replacement vehicles and medical expenses. The different types of vehicle insurance available are:
 - **compulsory third-party (CTP)** — mandatory insurance to cover personal injuries in motor vehicle accidents. This is required in all states and territories of Australia. It covers any person whom you might injure while you are driving. In New South Wales, CTP is required before a vehicle can be registered.
 - **comprehensive** — covers damage to your own vehicle and other people's property, as well as theft and some other risks, plus legal costs
 - **third-party property** — covers damage to other people's property and legal costs, but not damage to your own vehicle
 - **third-party fire and theft** — third-party property with some add-on features that cover your vehicle.
- The cost of vehicle insurance depends on the driver's experience, the type of vehicle, the age of the car, the address where the vehicle is kept and the type of insurance required.

- For comprehensive insurance, vehicles can be insured at the market value (the vehicle's current value) or the agreed value (a value that is agreed on by the owner and insurer).
- There is an excess cost for insurance claims. This is the cost the insured driver needs to pay before the claim is processed. Drivers under a certain age usually have to pay higher excess costs for insurance claims.
- **Stamp duty** is a tax levied by all Australian states and territories on purchases of property such as homes, land and vehicles. The tax rate for stamp duty differs between states. In New South Wales, the stamp duty payable on vehicles is currently:



\$44 999 or less — 3 per cent (\$3 per \$100)
 \$45 000 or more — \$1350, plus \$5 per \$100 over \$45 000.

WORKED EXAMPLE 5

Calculate the amount of stamp duty to be paid for the following vehicle purchases.

a \$14 560

b \$54 990

THINK

- a 1** Determine the percentage to be paid.
- 2** Calculate 3% of the vehicle purchase price.
- 3** Answer the question.

WRITE

The vehicle costs less than \$44 999.
 Therefore, the percentage is 3%.

$$\frac{3}{100} \times 14\,560 = \$436.80$$

The stamp duty to be paid on a vehicle purchased for \$14 560 is \$436.80.

- b 1** Determine the percentage to be paid.
- 2** Calculate the amount of money over \$45 000 to be charged.
- 3** Calculate 5% of \$9990.
- 4** Add \$1350 to the value calculated in step 3.
- 5** Answer the question.

The vehicle costs more than \$45 000.
 Therefore, the percentage is 5%.

$$54\,990 - 45\,000 = \$9990$$

$$0.05 \times 9990 = \$499.50$$

$$\$499.50 + 1350 = \$1849.50$$

The stamp duty to be paid on a vehicle purchased for \$54 990 is \$1849.50.

10.3.3 Sustainability, fuel consumption rates, servicing and tyres

- The running costs of a vehicle include the fuel consumption and maintenance. The latter includes the cost to service the vehicle and replace the tyres.

- **Fuel consumption rates** show the amount of fuel (petrol or diesel) used per 100 km. For example, 7.8 L/100 km means that the vehicle uses an average of 7.8 litres of fuel for every 100 km travelled.
- Fuel consumption rates will vary depending on where you drive. Driving in city areas usually uses more fuel because of the constant stopping and starting. Driving on freeways, where the speed travelled is more constant, generally has a lower fuel consumption rate.

WORKED EXAMPLE 6

A vehicle travels 580 km and uses 62 litres of fuel. Calculate the fuel consumption rate in L/100 km. Give your answer correct to 2 decimal places.

THINK

- 1 Divide the total distance travelled by 100 km (this gives the distance as 100 km lots).
- 2 Divide the amount of fuel in litres by the answer in step 1. Round to 2 decimal places.
- 3 Answer the question.

WRITE

$$\frac{580}{100} = 5.8$$

$$\frac{62}{5.8} = 10.6896$$

$$\approx 10.69$$

The fuel consumption rate is 10.69 L/100 km.

- Vehicles require servicing by experienced mechanics to maintain performance. Serviced vehicles generally perform more efficiently and therefore cost less to run.
- The cost of servicing a vehicle varies according to the distance travelled and general wear on moving parts such as brakes. Services are generally required every 10 000–15 000 km, depending on the manufacturer's guidelines.
- The service cost will include labour plus the cost to replace parts.

WORKED EXAMPLE 7

A driver travels on average 450 km every week.

a If the vehicle requires servicing every 15 000 km, how often will this vehicle need servicing over 3 years?

b At the 30 000 km service, the vehicle needs the following parts. (The prices for the parts include GST.)

- Oil filter \$30
- Fuel filter \$25
- Oil \$35
- Windscreen wiper blades \$15

The service will take 3 hours of labour at \$105 excluding GST.

Calculate the cost for the 30 000 km service.

THINK

- 1 Calculate the number of weeks the driver takes to travel 15 000 km.
- 2 Calculate the number of weeks in 3 years.

WRITE

$$\frac{15000}{450} = 33.33$$

$$52 \times 3 = 156$$

- 3 Divide the number in step 2 by the number in step 1.

$$\frac{156}{33.33} = 4.68$$

- 4 Answer the question.

The number of 15 000 km services in 3 years is 4.

- b** 1 Calculate the labour costs.

$$3 \times \$105 = \$315$$

Add 10% GST:

$$10\% \text{ of } \$315 = \$31.50$$

$$\begin{aligned}\text{Labour costs} &= \$315 + \$31.50 \\ &= \$346.50\end{aligned}$$

- 2 Calculate the cost of parts.

$$\$30 + 25 + 35 + 15 = \$105$$

- 3 Add the labour costs to the cost of the parts.

$$\$105 + \$346.50 = \$451.50$$

- 4 Answer the question.

The total cost for the 30 000 km service is \$451.50.

- Tyres help keep a vehicle on the road and are subject to road-worthy inspection by police. Vehicles without the regulated amount of tyre tread are deemed unsafe, and the owner is required to have the tyres replaced.
- The wear on tyres depends on the driving conditions, such as the road surface, vehicle performance and style of driving.
- The tyres are usually checked during routine vehicle servicing to ensure that they are safe to drive on and wearing evenly. If there is an issue with the vehicle's steering alignment, the tyres may wear more on one side.
- There are many different qualities and styles of tyres available on the market at varying costs.



WORKED EXAMPLE 8

For a particular brand of tyres, the manufacturer recommends replacement after 40 000 km. A driver using these tyres travels an average of 550 km each week. How many times should she expect to replace her tyres in 5 years?

THINK

- 1 Find the number of weeks it takes to travel 40 000 km. Divide 40 000 km by the average weekly distance.
- 2 Find the number of years by dividing the answer by 52.
- 3 Find the number of times in 5 years by dividing 5 by the answer from step 2.
- 4 Answer the question.

WRITE

$$\frac{40\,000}{550} = 72.73$$

$$\frac{72.73}{52} = 1.398$$

$$\frac{5}{1.398} = 3.576$$

In 5 years she should expect to replace her tyres 3 times.

10.3.4 Calculate and compare the cost of purchasing different vehicles using a spreadsheet

- A spreadsheet can be used to calculate and compare the cost of purchasing different vehicles.
- There are many available resources that compare vehicle performance and costs.
- To compare vehicles it is important to compare similar vehicles and common elements, such as fuel consumption and servicing costs.
- Registration and insurance costs will vary due to other factors, such as location of the vehicle and state of registration.

WORKED EXAMPLE 9

Using a spreadsheet, which one of the following three cars is the most cost-effective to buy? Assume that the average distance travelled each year is 15 000 km, the average price of fuel per litre is \$1.20, and all of the cars have the same safety rating.

Four-door sedan 1.6 litre engine				
Model	Sale price	Fuel consumption (L/100 km)	Servicing (km)	Average service costs
Car A	\$15 480	8.2	15 000	\$450
Car B	\$16 250	7.5	20 000	\$350
Car C	\$14 999	9.8	10 000	\$475

THINK

- 1 Create a spreadsheet.

WRITE

	A	B	C	D	E
1	Vehicle	Sales price	Fuel consumption (L/100 km)	Servicing (km)	Average service costs
2	Car A	\$15 480.00	8.2	15 000	\$450.00
3	Car B	\$16 250.00	7.5	20 000	\$350.00
4	Car C	\$14 999.00	9.8	10 000	\$475.00

- 2 Calculate the number of litres used (on average) for one year of driving.

Average distance travelled = 15 000 km

Number of 100 km units: $\frac{15\,000}{100} = 150$

Insert the formula '=150*C2' into cell F2.

Copy the formula into cells F3 and F4.

- 3 Calculate the cost to fuel the car for one year at \$1.20/L.

Enter the formula '=F2*1.2' into cell G2.

Copy the formula into cells G3 and G4.

	A	B	C	D	E	F	G
1	Vehicle	Sales price	Fuel consumption (L/100 km)	Servicing (km)	Average service costs	Litres/ years	Fuel cost
2	Car A	\$15 480.00	8.2	15 000	\$450.00	1230	\$1476.00
3	Car B	\$16 250.00	7.5	20 000	\$350.00	1125	\$1350.00
4	Car C	\$14 999.00	9.8	10 000	\$475.00	1470	\$1764.00

- 4 Calculate the servicing costs. To compare the costs, we need to find the proportional service costs by dividing the average distance travelled (15 000 km) by the service distance for each car, then multiply by the average service costs.

Enter the formula '=15 000/D2*E2' into cell H2.
Copy the formula into cells H3 and H4.

	A	B	C	D	E	F	G	H
	Vehicle	Sales price	Fuel consumption (L/100 km)	Servicing (km)	Average service costs	Litres/years	Fuel cost	Service cost
1	Car A	\$15 480.00	8.2	15 000	\$450.00	1230	\$1476.00	\$450.00
2	Car B	\$16 250.00	7.5	20 000	\$350.00	1125	\$1350.00	\$262.00
3	Car C	\$14 999.00	9.8	10 000	\$475.00	1470	\$1764.00	\$712.00

- 5 Calculate the total cost for each car over the year.

Insert the formula '=B2+G2+H2' into cell I2.
Copy the formula into cells I3 and I4.

	A	B	C	D	E	F	G	H	I
	Vehicle	Sales price	Fuel consumption (L/100 km)	Servicing (km)	Average service costs	Litres/years	Fuel cost	Service cost	Total cost
1	Car A	\$15 480.00	8.2	15 000	\$450.00	1230	\$1476.00	\$450.00	\$17 406.00
2	Car B	\$16 250.00	7.5	20 000	\$350.00	1125	\$1350.00	\$262.00	\$17 862.50
3	Car C	\$14 999.00	9.8	10 000	\$475.00	1470	\$1764.00	\$712.00	\$17 475.50

- 6 Answer the question.

The most cost-effective car to buy is Car A, with an overall annual cost in the first year of \$17 406.

eBookplus RESOURCES

Interactivity: Car loans (int-6913)

Exercise 10.3 Purchasing, running and maintaining a vehicle

Understanding, fluency and communicating

- WE4** A customer takes out a loan to purchase a car for \$17 995. The monthly repayments are \$313.38 and the loan is for 5 years. The customer pays a 10% deposit. Calculate:
 - the deposit paid
 - the total amount paid for the car
 - the interest rate charged for the loan, correct to 2 decimal places.
- A \$24 995 car is purchased through finance of 5.99% per annum. A 10% deposit is paid plus monthly repayments for 5 years. Calculate:
 - the amount borrowed for the car (principal)
 - the monthly repayments for the 5 years
 - the total amount paid for the car.
- WE5** Stamp duty payable on the purchase of vehicles is:

\$44 999 or less — 3 percent (\$3 per \$100)

\$45 000 or more — \$1350, plus \$5 per \$100 over \$45 000.

Calculate the amount of stamp duty to be paid for the following vehicle purchases.

 - \$22 995
 - \$65 890

4. The stamp duty paid for a vehicle was \$250. Determine the price paid for the vehicle.
5. **WE6** A vehicle travels 640 km and uses 58 litres of fuel. Calculate the fuel consumption rate in L/100 km. Give your answer correct to 2 decimal places.
6. A vehicle's fuel consumption rate for a journey was calculated at 8.9 L/100 km. If the vehicle used a total of 71 L of fuel for the journey, how far in km did it travel? Give your answer to the nearest whole number.
7. A family is on a road trip. The vehicle they are travelling in averages 765 km for 99 L of fuel.



- a. Calculate the fuel consumption rate correct to 2 decimal places.
- b. If the average fuel price is \$1.34/L, calculate the cost per 100 km.
8. **WE7** A driver travels on average 350 km every week.
- a. If the vehicle requires servicing every 10 000 km, how often will his vehicle need servicing over 2 years?
- b. At the 30 000 km service, the vehicle needs the following parts and labour. (The prices for the parts include GST.)
- Oil filter \$20
 - Fuel filter \$35
 - Oil \$40
 - Windscreen wiper blades \$10
 - 2.5 hours labour at \$85 excluding GST
- Calculate the cost for the 30 000 km service.
9. **WE8** For a particular brand of tyres, the manufacturer recommends replacement after 35 000 km. A driver using these tyres travels an average of 2050 km each month. How many times should she expect to replace her tyres in 10 years?
10. **MC** A vehicle's average fuel consumption is 9.8 L/100 km. The driver fills the tank with 65 litres of fuel. The average number of kilometres she can travel before the tank is empty is:
- A.** 6.63 **B.** 65 **C.** 151 **D.** 637 **E.** 663

Problem solving, reasoning and justification

11. **WE9** Using a spreadsheet, work out which one of the following three cars is the most cost-effective to buy. Assume that the average distance travelled each year is 15 000 km and the cost of fuel is \$1.15.

Four-door sedan (medium size car)				
Model	Sale price	Fuel consumption (L/100 km)	Servicing (km)	Average service costs
Car A	\$37990	12.7	15000	\$255
Car B	\$36748	8.6	20000	\$595
Car C	\$38490	10.5	10000	\$235

12. A female 19-year-old driver has three options to purchase comprehensive car insurance for a small car that will be parked in a locked garage in a suburban property. All options include replacing her car, providing transportation if car is unable to be driven, and costs for injuries for all passengers.

Option A: \$1143, \$850 excess, 10% online discount

Option B: \$1149, \$850 excess

Option C: \$1155, \$800 excess, 15% online discount

The excess fee is the amount that is paid to the insurer if a claim is made.

Which option is the most cost-effective?

13. The fuel consumption for a vehicle travelling in city traffic is recorded as 9.8 L/100 km. When travelling along highways, the fuel consumption is recorded as 7.5 L/100 km.
- Show that the average fuel consumption is 8.65 L/100 km.
 - The vehicle travels 126 km along city roads and 458 km along the highway. Calculate the total number of litres of fuel used, correct to 1 decimal place.
 - The cost of fuel is \$1.25 per litre. Calculate the actual cost of fuel, correct to the nearest cent.
 - When calculating the fuel costs using the average fuel consumption, the value is different to the actual cost. Explain why.
14. A vehicle is purchased for \$24 980 through finance that attracts 6% interest for 5 years. The owner pays a 10% deposit and then pays \$496.28 monthly. This value for the monthly repayments includes a \$250 one-off administration fee to set up the loan and a monthly \$5 loan-servicing fee. Show that the comparison rate for the loan is 6.489% per annum.
15. A person is looking to buy a new car. The person drives to work in the city, a return distance of 45 km, Monday to Friday. On the weekend, the person drives along a major highway to visit a relative who lives in a regional town 112.5 km from the city.



Car	Fuel consumption for city driving (L/100 km)	Fuel consumption for highway driving (L/100 km)	Average fuel consumption (L/100 km)
A	12.5	7.8	
B	11.2	8.5	

- Calculate the average fuel consumption for both cars.
- Based on the weekly distances driven by the person, which car would be most cost-effective to purchase? Justify your answer using calculations.

10.4 Budgeting

10.4.1 Monthly income

- A **budget** is a list of all planned income and costs. A budget can be prepared for long periods of time, but for individuals it is usually a monthly plan.
- In order to create a monthly budget, monthly income and expenses need to be known.
- Monthly income** is $\frac{1}{12}$ of total annual income.

WORKED EXAMPLE 10

Calculate the monthly income of a worker who has an annual salary of \$71 400 and earns \$1590 each year in bank interest.

THINK

- Find the total annual income after tax as the sum of salary and bank interest.
- Find the total monthly income after tax by dividing the answer in step 1 by 12.
- Answer the question.

WRITE

$$\begin{aligned}\text{Total annual income after tax} &= 71\,400 + 1590 \\ &= 72\,990\end{aligned}$$

$$\begin{aligned}\text{Total monthly income after tax} &= \frac{72\,990}{12} \\ &= 6082.50\end{aligned}$$

The worker's monthly income after tax is \$6082.50.

10.4.2 Fixed and discretionary spending

- Fixed spending** is spending that occurs every month (or week, fortnight, year or other period) of the same amount. Examples of fixed spending are rent, school fees, health insurance and vehicle insurance.
- Discretionary spending** is spending that occurs from time to time and is not a fixed amount. Examples of discretionary spending are food, clothing, entertainment and medical bills.

WORKED EXAMPLE 11

Determine the total monthly spending from the following list of fixed and discretionary spending. The discretionary spending represents the amounts spent in a month.

Fixed spending			Discretionary spending	
Item	Frequency	Amount	Item	Amount
Rent	Weekly	\$230	Food	\$423
Health insurance	Monthly	\$78	Clothing	\$107
Vehicle registration	Yearly	\$620	Entertainment	\$85
Vehicle insurance	Yearly	\$389	Car repairs	\$325

THINK

- Convert any weekly spending to monthly spending by multiplying by $\frac{52}{12}$. The only weekly cost is rent of \$230.
- Add up the annual spending. The annual spending are vehicle registration and insurance ($620 + 389$).
 - Convert the annual spending to monthly spending by dividing by 12.

WRITE

$$\begin{aligned}\text{Monthly rent} &= 230 \times \frac{52}{12} \\ &= 996.67 \text{ (correct to 2 decimal places)}\end{aligned}$$

$$\begin{aligned}\text{Annual spending} &= 620 + 389 \\ &= 1009\end{aligned}$$

$$\begin{aligned}\text{Monthly cost} &= \frac{1009}{12} \\ &= 84.08 \text{ (correct to 2 decimal places)}\end{aligned}$$

- 3 Find the total fixed spending by adding the numbers from steps 1 and 2 to the monthly fixed cost of health insurance (\$78).

$$\begin{aligned}\text{Total monthly fixed spending} &= 996.67 + 84.08 + 78 \\ &= 1158.75\end{aligned}$$

- 4 Find the total discretionary spending by adding up the numbers in the last column of the table.

$$\begin{aligned}\text{Total monthly discretionary spending} &= 423 + 107 + 85 + 325 \\ &= 940\end{aligned}$$

- 5 To find the total monthly spending, add the total fixed spending to the total discretionary spending obtained in step 4.

$$\begin{aligned}\text{Total monthly spending} &= \text{fixed spending} + \text{discretionary spending} \\ &= 940 + 1158.75 \\ &= 2098.75\end{aligned}$$

- 6 Answer the question.

Total spending is \$2098.75 per month.

10.4.3 Preparing a personal budget

- Personal budgets can be prepared using spreadsheets to calculate the total fixed and discretionary spending for each month and subtract this from the net monthly income.
- A profit is made when the net monthly income is greater (more) than the total monthly spending.
- A loss is made when the net monthly income is less than the total monthly spending.



WORKED EXAMPLE 12

A nurse's net annual salary is \$59 500. Her spending is shown in the following table.

Spending description	Amount
Rent (monthly)	\$658
Electricity (quarterly)	\$295
Water usage (quarterly)	\$150
Health insurance (monthly)	\$205
Car finance (fortnightly)	\$172.50
Vehicle registration (yearly)	\$595
Vehicle insurance (half-yearly)	\$680
Food (weekly)	\$75
Entertainment (monthly)	\$200
Fuel (weekly)	\$68



Using a spreadsheet, determine whether the nurse has a profit or loss at the end of each month.

THINK

- 1 Calculate the monthly costs.
For yearly, divide by 12.
For half-yearly, divide by 6.
For weekly, multiply by 52, then divide by 12.
For quarterly, divide by 3.
For fortnightly, multiply by 26, then divide by 12.

WRITE

Insert the formulas:

'=(cell in column B)/12'

'=(cell in column B)/6

'=(cell in column B)*52/12'

'=(cell in column B)/3

'=(cell in column B)*26/12'.

	A	B	C
1	Spending description	Amount	Monthly expenses
2	Rent (monthly)	\$658.00	\$658.00
3	Electricity (quarterly)	\$295.00	\$98.33
4	Water usage (quarterly)	\$150.00	\$50.00
5	Health insurance (monthly)	\$205.00	\$205.00
6	Car finance (fortnightly)	\$172.50	\$373.75
7	Vehicle registration (yearly)	\$595.00	\$49.58
8	Vehicle insurance (half yearly)	\$680.00	\$113.33
9	Food (weekly)	\$75.00	\$325.00
10	Entertainment (monthly)	\$200.00	\$200.00
11	Fuel (weekly)	\$68.00	\$294.67

- 2 Calculate the total monthly spending.

Create new row: Monthly expenses

Insert '=sum(C2:C11)' into cell C12.

- 3 Calculate the net annual income.

Create new row: Net monthly income

Insert '\$9500' into B13.

Insert '=B13/12' into C13.

- 4 Calculate the profit/loss for the month.

Create new row: Profit/loss

Insert '=C13-C12'.

	A	B	C
1	Spending description	Amount	Monthly expenses
2	Rent (monthly)	\$658.00	\$658.00
3	Electricity (quarterly)	\$295.00	\$98.33
4	Water usage (quarterly)	\$150.00	\$50.00
5	Health insurance (monthly)	\$205.00	\$205.00
6	Car finance (fortnightly)	\$172.00	\$373.75
7	Vehicle registration (yearly)	\$595.00	\$49.58
8	Vehicle insurance (half yearly)	\$680.00	\$113.33
9	Food (weekly)	\$75.00	\$325.00
10	Entertainment (monthly)	\$200.00	\$200.00
11	Fuel (weekly)	\$68.00	\$294.67
12	Monthly expenses		\$2 367.67
13	Net monthly income	\$59 500.00	\$4 958.33
14	Profit/loss		\$2 590.67

- 5 Answer the question.

She makes a profit of \$2590.67.

Exercise 10.4 Budgeting

Understanding, fluency and communicating

- WE10** Calculate the net monthly income of a worker who has a net annual salary of \$63 200 and earns \$3745 each year in bank interest.
- MC** A person earns \$74600 net annual salary and also receives a bonus payment from their employer of \$3500. They have a fortnightly salary of:
A. \$1501.92 **B.** \$3003.85 **C.** \$3124.00 **D.** \$6369.23 **E.** \$6484.00
- A bank teller earns \$1100 per week (after tax) plus \$2200 per year in bank interest (after tax). Find her net monthly income.
- Complete the following table.

	Net annual salary	Total net annual investment income	Total net monthly income
a.	\$70 500	\$4812	
b.	\$81 234	\$10 387	
c.	\$90 349	\$9885	
d.	\$24 500	\$42 500	

- WE11** Determine the total monthly cost from the following list of fixed and discretionary spending. The discretionary spending represent the amounts spent in a month.

Fixed spending			Discretionary spending	
Item	Frequency	Amount	Item	Amount
Rent	Weekly	\$205	Food	\$494
Health insurance	Monthly	\$89	Clothing	\$205
Vehicle registration	Yearly	\$540	Entertainment	\$123
Vehicle insurance	Yearly	\$499	Car repairs	\$72

- A baker earns \$18.50 an hour after tax. She works an average of 38 hours a week plus 7 hours overtime, paid at time and a half. Her monthly spending is 47% of her net monthly income. Calculate how much she spends.
- Determine the total monthly cost from the following list of fixed and discretionary spending. Discretionary spending represents the amounts spent in a month.



Fixed spending			Discretionary spending	
Item	Frequency	Amount	Item	Amount
Home mortgage	Monthly	\$1254	Food	\$563
Health insurance	Monthly	\$124	Clothing	\$321
Vehicle registration	Yearly	\$702	Entertainment	\$389
Vehicle insurance	Yearly	\$899	Car repairs	\$396
School fees	Half-yearly	\$5300	Travel	\$1379

8. **MC** Which of the following can be considered as fixed costs?
- A. Daughter's speeding tickets B. Son's soccer club membership
C. Wife's shoe purchases D. Winter holiday cost
E. Father's new dentures
9. Complete this table of income and costs for a family of five over 4 months.

	Net annual salary	Net monthly bank interest	Monthly fixed spending	Monthly discretionary spending	Monthly profit
a.	\$73 700	\$784	\$3623	\$2563	
b.	\$73 700	\$792	\$3623	\$639	
c.	\$73 700	\$804	\$3623	\$4456	
d.	\$73 700	\$823	\$3623	\$1065	

10. **WE12** The net annual salary for a childcare worker is \$43 470. Her spending is shown in the following table.

Spending description	Amount
Rent (weekly)	\$225
Electricity (quarterly)	\$150
Water usage (quarterly)	\$75
Travel to work (weekly)	\$60
Food (weekly)	\$45
Entertainment (monthly)	\$250

Using a spreadsheet, determine how much profit or loss the childcare worker has at the end of each month.

Problem solving, reasoning and justification

11. The following information shows the spending and income for a couple.

Spending	Mortgage	\$672 per fortnight
	Food	\$123 per week
	Clothing	\$78 per week
	Transport	\$35 per day
	Entertainment	\$120 per week
	Health insurance	\$200 per month
	Car (insurance, registration, fuel, servicing)	\$600 per month
Net income	Partner A	\$1680 per fortnight
	Partner B	\$1100 per week

- a. Using a spreadsheet, calculate the total monthly spending.
b. Show that the couple make a monthly profit.

The couple would like to save to buy a house for sale for \$350 000. To buy the house, they will need to make a 10% deposit.

- c. If the couple's spending does not change, how many months will it take to save the 10% deposit? Give your answer correct to 1 decimal place.
- d. The couple's financial adviser recommends they reduce their spending to be less than 50% of their net monthly income. Where should they adjust their spending? Justify your answer by using calculations.
12. The following shows a personal budget prepared by a graduate IT technician.

Item	Frequency	Amount	Monthly
Rent	Weekly	\$275	\$1100
Bills	Monthly	\$145	\$145
Travel	Monthly	\$350	\$350
Entertainment	Monthly	\$330	\$330
Clothes	Fortnightly	\$220	\$440
Insurance	Yearly	\$1150	\$95.83
Monthly spending			\$2460.83
Monthly net income			
Profit/loss			\$2355.92

- a. Determine his monthly net income, and hence calculate his annual net income.
- b. Find the percentage amount he spends of his net income. Give your answer correct to 2 decimal places.
- c. He would like to save up for an overseas holiday and has budgeted \$5500 for his trip. If he can save 5% of his monthly net income, how many months will it take for him to have saved enough money for his trip?
13. A family's net annual income is \$69480. Their spending is shown in the following table.

Item	Frequency	Amount
Holiday/entertainment	Yearly	\$4500
Mortgage	Weekly	\$245
Insurance	Yearly	\$1980
Car costs	Monthly	\$950
Food	Weekly	\$145
Electricity, water	Quarterly	\$580
School fees	Yearly	\$8500
Health insurance	Monthly	\$368

- a. Prepare a monthly budget and show whether the family is living within their means (i.e. making a profit).
- b. The family needs to adjust their spending. By identifying the fixed and discretionary spending items, explain which items they should consider reducing.
14. A person's monthly spending is \$1850. His current spending is 55% of his net monthly income.
- a. Determine his net monthly income and net annual salary.
- b. If he gets paid an hourly rate of \$22.50, how many hours a week does he work? Give your answer correct to 1 decimal place.
- c. He gets a job that pays 4% more than he is currently earning. If his spending does not change, what percentage will his spending be of his net monthly income? Give your answer correct to 2 decimal places.

15. A family of four are going on an overseas holiday for 6 weeks. They have saved \$20 000 for the trip and recorded the costs for the essentials but don't know how much to spend on food.

Item	Cost
Passports	\$832
Flights	\$5668
Travel insurance	\$658
Accommodation	\$245 per night
Food	



- They estimate food costs to be about \$45 per day. How much money do they have remaining from the \$20 000 to spend on their trip?
- The family reassess and decide to adjust their spending to save more money for their trip. Their monthly spending is \$2585 and net monthly income is \$4580. If they can save half of their monthly profit for 4 months, how much more money will they have for their overseas trip?

10.5 Review

10.5.1 Summary

In this chapter you have learnt:

- how to interpret and use information about a household's electricity, water or gas usage and council rates
- how to calculate bills
- how to prepare and plan for purchasing and running a vehicle
- about the different costs associated with running a vehicle, such as insurance, tyres and servicing
- how to calculate financial costs associated with purchasing a vehicle
- how to calculate stamp duty and fuel consumption for vehicles
- how to use spreadsheets to compare similar vehicles
- the definitions of fixed and discretionary spending
- how to use spreadsheets to prepare a personal budget and determine whether a person has a profit or loss.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 10.5 Review

Understanding, fluency and communicating

- MC** The energy usage and rates for a household are as shown. There is a 25% discount for paying the bill before the due date. If the bill was paid before the due date, what was the amount paid? (Remember to include GST.)

Electricity

Tariff	Bill days	Current reading	Previous reading	Total usage	Charge/rate (c/kWh)
All day	90	78250	77682		25.08
Service to property	90			\$1.105/day	

Gas

Tariff	Bill days	Current reading	Previous reading	Total usage	Charge/rate (c/MJ)
All day	90	8920	8138		2.452
Service to property	90			\$0.985/day	

- A.** \$107.82 **B.** \$241.90 **C.** \$262.29 **D.** \$349.73 **E.** \$384.70
2. A household's daily water usage is 1700 litres. The total water bill for 60 billing days is \$392.12, which includes \$187.10 for water services. Calculate the charge per kilolitre.
3. A worker's net annual income is \$34 105. The worker makes a monthly loss of \$145. Calculate the monthly spending.
4. A vehicle is purchased for \$18 570 on finance of 5.5% per annum for 3 years. A 10% deposit is paid and there are no other administration costs. Stamp duty payable on the purchase of vehicles is:
- \$44 999 or less — 3 per cent (\$3 per \$100)
 \$45 000 or more — \$1350, plus \$5 per \$100 over \$45 000.

Calculate, correct to the nearest cent:

- a. the monthly instalments cost
 b. the stamp duty.
5. **MC** A car can travel 485 km on 68 litres of fuel. The fuel consumption in L/100km for the car is:
- A.** 0.14 **B.** 7.01 **C.** 7.13 **D.** 13.60 **E.** 14.02
6. Complete the following table.

	Energy usage	Charge/rate	Cost
a.	987 kWh	19.02 c/kWh	
b.	2058 MJ		\$53.28
c.		15.48 c/kWh	\$98.61
d.	5439 MJ	1.534 c/MJ	

7. According to the manufacturer's guidelines, a vehicle has its four tyres replaced 3 times in 9 years. If the vehicle travels an average of 17 500 km each year, how many kilometres (on average) does the driver get out of a set of tyres?
8. **MC** Which one of the following is not the responsibility of energy distributors?
- A.** Repairing and maintaining infrastructure
B. Ensuring supply is maintained
C. Ownership of energy infrastructure
D. Fixing issues of energy supply
E. Collecting payment for energy usage
9. The quarterly council rates for a property with capital improved value of \$385 000 are \$721.88.
- a. Calculate the amount in the dollar charged annually by the council.
 b. If the total capital improved value for all properties within the council municipality is \$1.5 billion, calculate the amount of revenue the council plans to raise this year.
10. **MC** In determining the budget for a small business, which of the following can be considered a discretionary cost?
- A.** Rent **B.** Employee's wages
C. Public liability insurance **D.** Fire insurance
E. An electricity bill



11. **MC** In its annual budget, the local council plans to raise \$25 million in revenue to repair local infrastructure. The capital improvement for the area is valued at \$1.5 billion. The council rate quarterly instalment for a rate payer whose house has a capital improved value of \$475 000 is:
A. \$712.50 **B.** \$791.67 **C.** \$1979.17 **D.** \$2850.00 **E.** \$7916.67
12. The monthly repayments for a car are \$396.25 for 48 months.
a. Calculate the total repayment amount.
b. The interest charged was \$3170 and the owner paid \$2000 deposit. How much was the car purchased for?
13. Stamp duty payable on the purchase of vehicles is:
 \$44 999 or less — 3 per cent (\$3 per \$100)
 \$45 000 or more — \$1350, plus \$5 per \$100 over \$45 000.
 Calculate the stamp duty for each of the following.
a. A four-door sedan with a selling price of \$24 580
b. A 4WD with a selling price of \$54 950
c. A luxury sports car purchased for \$62 985
d. A small four-door car purchased for \$14 750
14. Calculate the monthly net income for each of the following.
a. A net annual income of \$63 480
b. A weekly net income of \$945.35
c. A fortnightly net income of \$1450
15. The following shows the net annual incomes and monthly spending of 4 people. Calculate the profit or loss for each person.

Net annual income	Monthly spending	Profit/loss
\$51 890	\$3950	
\$84 755	\$8640	
\$75 120	\$6580	
\$62 308	\$4980	

16. Calculate, correct to 1 decimal place, the interest rates (% p.a.) charged on the following vehicle purchases.
a. Principal borrowed: \$14 805; 48 monthly instalments of \$363.96
b. Interest of \$4568 on principal of \$12 181, borrowed for 5 years
17. There is a 12% discount for customers who pay their energy bill before the due date. The energy usage of a customer is:
 • electricity: 875 kWh at 19.02 c/kWh
 • gas: 720 MJ at 2.034 c/MJ.
 The customer is also charged a \$112 fixed fee for service to the property.
 If the customer pays before the due date, calculate the amount they paid, including GST.
18. Calculate the monthly running costs for the following vehicles. (Assume fuel costs \$1.00/L.)

Vehicle	Fuel consumption (L/100 km)	Tyres (every 30 000 km)	Servicing (every 15 000 km)	Insurance (yearly)	Registration (yearly)	Average distance travelled in 1 year
Van	13.5	\$550	\$220	\$558	\$666.40	28 500
SUV	11.8	\$870	\$390	\$680	\$735.70	19 400

19. **MC** A person spends 65% of their net annual income. If their net annual income is \$45 890, the amount of monthly profit they make is closest to:
A. \$1338 **B.** \$2486 **C.** \$16 061 **D.** \$29 828 **E.** \$45 825
20. The sale price of a car is \$22 580 (excluding on-road costs). The buyer will need to pay stamp duty (3% of the sale price), a car dealership fee of \$325 and a registration fee of \$815.40. Calculate the total cost of the car.

Problem solving, reasoning and justification

21. A council raises its council rates by 5% to meet the growing costs associated with improving infrastructure. Last year the council raised \$19 million in revenue with total capital improved value of \$1.6 billion.
- Calculate the rate in the dollar for last year's revenue.
 - Determine the amount of revenue the council plans to raise this year based on the 5% increase.
 - A rate payer's council rates last year were \$4135.25. Calculate this year's council rates for this rate payer after the 5% increase.

Over the year, the capital improved value for properties in the municipality increased by 2.5% due to the property market.

- Show that the rate in the dollar charged by the council for this year is approximately 0.012.
 - Explain why the overall increase in the rate in the dollar is closer to 2.5% than the 5% stated by the council. Use calculations to support your explanation.
22. A finance consultant receives 0.75% commission (after tax) on all new loans, in addition to an annual net salary of \$65 940. Over 3 months the consultant signs up \$857 000 of new loans.
- Calculate her monthly net income over the three months.
 - In one month she spends 73% of her spending on fixed items, so she has 27% of her spending available for discretionary items such as entertainment and clothing. If she makes a profit of \$747.50, determine how much she spends on fixed and discretionary items.
23. The monthly credit card spending of a family of four is \$8650, which includes all bills and household expenses. Partner A's annual net salary is \$74486 and Partner B's fortnightly wage is \$1685.
- Calculate the partners' net monthly salaries.
 - Show that the family spends approximately 88% of their net monthly income.
24. A person is saving for a new car priced at \$31 910 (on-road costs). Their net annual income is \$52 680 and their monthly spending is shown in the table.
- Show that the person has a monthly profit.
The person pays \$2000 deposit for the car and will borrow the remainder at 6.5% per annum for 4 years with monthly repayments.
 - Calculate the principal borrowed.
 - Show that the monthly instalment is \$785.14.
 - The annual registration and insurance costs will be \$1625. Fuel costs will replace the person's current spending on travel to work. Will they be able to afford to pay for the car? Justify your answer using calculations.



Item	Amount
Mortgage	\$880
Electricity bills	\$158
Council rates	\$185
Travel to work	\$250
Food	\$120
Health insurance	\$225
Entertainment	\$250

25. A family of four are going on a holiday. They have a choice of driving or flying. They can average 650 km on 55 litres of fuel and the average cost of fuel is \$1.41. The distance to their destination is 1716.1 km. The cheapest one-way flight to their destination costs \$219 per person.



- Which option (driving or flying) is the more cost-effective for the family?
 - Which option would you advise them to choose? Support your advice by providing a positive and negative for each option.
26. In deciding on buying a new vehicle, a couple have narrowed down their selection to the following two vehicles. They decide they will base their decision on which vehicle will be the more cost-effective, based on 15 000 kilometres per year. (Assume fuel costs \$1.25/L.)

Vehicle	Cost	Stamp duty	Insurance	Fuel consumption (L/100 km)	Servicing	Registration
Vehicle A	\$28 840		\$880.97	9.8	\$219 every 15 000 km	\$789.40
Vehicle B	\$27 490		\$953.59	10.1	\$290 every 10 000 km	\$789.40

- Explain why including registration does not change the overall cost-effectiveness of the vehicles.
 - Stamp duty payable on the purchase of vehicles is:
\$44 999 or less — 3 per cent (\$3 per \$100)
\$45 000 or more — \$1350, plus \$5 per \$100 over \$45 000.
Calculate the value of the stamp duty for each vehicle.
 - Use a spreadsheet to determine which vehicle they should choose.
27. A household has two options for energy providers:

Option A	Charge/rate (c/kWh)	Option B	Charge/rate (c/kWh)
Peak	31.91	Peak	36.66
Off-peak	14.080	Off-peak	19.43
Service to property	\$1.075/day	Service to property	\$1.259/day
Pay-on-time discount	12%	Pay-on-time discount	17.5%

Which option is the more cost-effective for the household? Justify your answer by calculating the costs for both options.

28. A retail worker receives an hourly rate of \$22.05. In a fortnight, she works 74 hours at normal rate and 6 hours at penalty rate of 1.5. She pays \$306 in tax.
- Calculate her net monthly income.
 - After spending, the worker makes a 35% profit. Show that her monthly spending is \$2146.51.
 - Of her spending, she spends 45% on fixed items and 55% on discretionary items. Calculate the total amount she spent on fixed items.
29. An electricity provider has informed customers that all rates will increase by 3% due to rising costs associated with the distributor improving infrastructure. The new and previous rates are shown.

Electricity charges (charges based on meter reading)								
Billing period: 1 Apr–30 Jun								
Tariff	Meter no.	Bill days	Current reading	Previous reading	Total usage	Charge/rate (c/kWh)	\$ exc. GST	\$ inc. GST
Peak	1287	60	87950	87545		25.80		
		30	88177	87950		27.09		
Off-peak	1287	60	14780	14395		11.40		
		30	14970	14780		11.97		
Service to property		60			\$1.017/day			
		30			\$1.068/day			

- Calculate the total amount paid for the bill, including GST.
 - Calculate the daily energy usage (correct to 2 decimal places) and daily electricity cost.
 - Show that the provider's claims of increasing rates by 3% are incorrect.
30. A family of 6 lives in a 5-bedroom house in the outer suburbs of a major city. They own two cars. Partner A uses one car to drive to the local train station; Partner B uses the other car to drive their children to school, sports training and holidays.



They spend \$165 on food each week. Their health insurance costs \$368 per month, their house insurance costs \$1580 a year, and their annual council rates are \$3580. The children's school fees total \$2850 a year.

Partner A's net annual salary is \$74805.

Partner B works part-time Monday–Friday as an administrator at a local law firm. Her hourly rate is \$27.50 and her hours are 9 am–3 pm with a 45-minute lunch break. Her employer withholds \$85 in PAYG tax each week.

Their bills and other expenses are shown below.

Electricity bill

Billing period: 1 Jul–30 Sep					
Tariff	Bill days	Current reading	Previous reading	Total usage	Charge/rate (c/kWh)
Peak	91	23890	23150		33.58
Off-peak	91	48901	48334		15.60
Service to property	91			\$1.148/day	

Water bill

Bill details as of 30 September		Value	Price
Water service charge	1 Jul to 30 Sep	25.75	
Sewerage service charge	1 Jul to 30 Sep	149.80	
Storm water	1 Jul to 30 Sep	22.05	
	Volume (kL)	Charge/kL	
Water usage	65	\$2.05/kL	
Recycled water usage	45	\$1.82/kL	

Cars

Average fuel price: \$1.31/litre

Vehicle	Fuel consumption (L/100 km)	Registration (annually)	Insurance (6-monthly)	Servicing (every 15 000 km)	Average distance travelled each year
Partner A	8.8	\$568	\$302	\$250	12 500
Partner B	10.2	\$568	\$257	\$320	17 500

- Calculate their joint net monthly income.
- Prepare a budget for the family.
- To be solvent means to be able to pay one's debts (expenses, bills). Are the family solvent? Justify your answer using your calculations.

Answers

Exercise 10.2 Household expenses

- 1.** \$301.67
- 2. a.** \$59.45 **b.** \$28.13 **c.** \$131.03 **d.** \$113.42
- 3.** 4768 kWh
- 4. a. i.** 30.54 c/kWh **ii.** \$1.017 **iii.** 895 kWh
 b. i. \$273.33 **ii.** \$402.47
- 5.** \$159.63
- 6.** D **7.** \$372.92 **8.** 33.62 kL **9.** \$1833.33
- 10. a.** 0.006 **b.** \$16020000
- 11.** Plan B is the better option for both when he pays on time (\$220.04 compared to \$230.21) and when he does not (\$250.04 compared to \$270.83).

12. a.	Bill details as of 30 June		Value	Price
	Water service charge	1 Apr to 30 Jun	23.80	23.80
	Sewerage service charge	1 Apr to 30 Jun	150.20	150.20
	Storm water	1 Apr to 30 Jun	17.95	17.95
		Volume (L)	Charge/kL	
	Water usage	64 000	\$2.04	130.56
	Recycled water usage	38 000	\$1.87	71.06
			Total:	\$393.57

- b. Water usage: \$2.12; recycled water usage: \$1.94
- c. Water service charge increase: 10.50%; sewerage service charge increase: 1.00%
- d. Average percentage increase: 2.99%
13. Choice 1
14. The council rate is incorrect. The council made an error in the first calculation for the rate in the dollar: $\frac{7.5}{2.5} = 3$ should be $\frac{7.5}{2500} = 0.003$. The rates for this property should be \$525.
15. a. Council rate: $280\,000 \times 0.008 = \$2240$
- Quarterly instalment: $\frac{2240}{4} = \$560$
- b. \$1345.60
- c. 22.35%
- d. Answers will vary but could include the following.
- The family could reduce their electricity bill by installing energy-saving light bulbs and turning off lights and power appliances at the power point. They could reduce their water usage by ensuring there are no leaking taps, reducing time spent in the shower, washing full loads of washing, and ensuring the dishwasher is full before turning it on.
- Council rates are set and will not be reduced by changing household behaviours.

Exercise 10.3 Purchasing, running and maintaining a vehicle

1. a. \$1799.50	b. \$20 602.30	c. 3.22% pa
2. a. \$22 495.50	b. \$487.22	c. \$31 732.40
3. a. \$689.85	b. \$2394.50	
4. \$8333.33	5. 9.06 L/100 km	6. 798 km
7. a. 12.94 L/100 km	b. \$17.34	
8. a. 3 times	b. \$338.75	
9. 7 times	10. E	11. Car B
13. a. $\frac{9.8 + 7.5}{2} = 8.65$ L/100 km		12. Option C
b. 46.7 L		
c. \$58.38		

- d. The average fuel consumption is a value that assumes equal distance (1 : 1) of driving under the two conditions (city and highway). In this case, the ratio of city to highway driving was 126 : 458 (1 : 3.6). This means the overall fuel consumption is lower than the average value would predict, because 3.6 times more driving took place at the lower fuel consumption.

14. Total paid for the car: $496.28 \times 60 = \$29\,776.80$

Deposit: $10\% \times 24\,980 = \$2498$

Principal borrowed: $24\,980 - 2498 = \$22\,482$

Interest paid: $29\,776.80 - 22\,482 = \$7294.80$

$$I = \frac{PRT}{100}$$

$$7294.80 = \frac{22\,482 \times R \times 5}{100}$$

$$\frac{7294.80}{22\,482 \times 5} = R$$

$$R \approx 6.489\%$$

15. a. Car A: 10.15 L/100 km

Car B: 9.85 L/100 km

- b. Car B has a lower average fuel consumption (9.85 L/100 km) and will therefore be more cost-effective. (The average is valid here because the distances of city and highway driving come to the same value, 225 km.)

Exercise 10.4 Budgeting

1. \$5578.88

2. B

3. \$4950.00

4.

Net annual salary	Total net annual investment income	Total net monthly income
\$70 500	\$4812	\$6276.00
\$81 234	\$10 387	\$7635.08
\$90 349	\$9 885	\$8352.83
\$24 500	\$42 500	\$5583.33

5. \$1957.91

6. \$1827.40

7. \$5442.75

8. B

9.

	Net annual salary	Net monthly bank interest	Monthly fixed spending	Monthly discretionary spending	Monthly profit
a.	\$73 700	\$784	\$3623	\$2563	\$739.67
b.	\$73 700	\$792	\$3623	\$639	\$2671.67
c.	\$73 700	\$804	\$3623	\$4456	-\$1133.33
d.	\$73 700	\$823	\$3623	\$1065	\$2276.67

10. A profit of \$1867.50

11. a. \$4711.58

- b. The net monthly income of \$8406.67 exceeds the monthly spending, for a monthly profit of \$3695.09.

c. 9.5 months

- d. Areas in which spending can be reduced (adjusted) include clothing, entertainment and food. Mortgage, health insurance and car registration/insurance are fixed spending.

12. a. Monthly net income = \$4816.75; annual net income = \$57 801

b. 51.09%

c. 23 months

13. a.

Item	Frequency	Amount	Monthly cost
Holiday/entertainment	Yearly	\$4500	\$375.00
Mortgage	Weekly	\$245	\$1061.67
Insurance	Yearly	\$1980	\$165.00
Car costs	Monthly	\$950	\$950.00
Food	Weekly	\$145	\$628.33
Electricity, water	Quarterly	\$580	\$193.33
School fees	Yearly	\$8500	\$708.33
Health insurance	Monthly	\$368	\$368.00
		Total	\$4449.67
Income	Yearly	\$69 480	\$5790.00
		Profit	\$1340.33

b.

Fixed	Discretionary
Mortgage	Holiday/entertainment
Insurance	Car costs
Health insurance	Food
School fees	Electricity, water

The items they should consider reducing are the discretionary items, particularly the non-essential items: holidays and entertainment. These items do not have fixed costs, which means that the costs can be adjusted based on usage or need.

14. a. Net monthly income: \$3363.64; net annual salary: \$40 363.68

b. 34.4 hours

c. 52.88%

15. a. \$662

b. \$3990

Exercise 10.5 Review

1. C

2. 2.01 c/kL

3. \$2987.08

4. a. \$540.85

b. \$557.10

5. E

6.

	Energy usage	Charge/rate	Cost
a.	987 kWh	19.02 c/kWh	\$187.73
b.	2058 MJ	2.589 c/MJ	\$53.28
c.	637 kWh	15.48 c/kWh	\$98.61
d.	5439 MJ	1.534 c/MJ	\$83.43

7. 52500km

8. E

9. a. 0.0075

b. \$11 250 077.92. The council plans to raise approximately \$11 million in revenue.

10. E

11. C

12. a. \$19 020

b. \$17 850

13. a. \$737.40

b. \$1847.50

c. \$2249.25

d. \$442.50

14. a. \$5290

b. \$4096.52

c. \$3141.67

15.

Net annual income	Monthly spending	Profit/loss
\$51 890	\$3950	\$374.17 profit
\$84755	\$8640	−\$1577.08 loss
\$75 120	\$6580	−\$320 loss
\$62308	\$4980	\$212.33 profit

30. a. Net monthly income: \$8993.56

b.

Item	Cost	Frequency	Monthly cost
Food	\$165.00	Weekly	\$715.00
Health insurance	\$368.00	Monthly	\$368.00
House insurance	\$1580.00	Yearly	\$131.67
School fees	\$2850.00	Yearly	\$237.50
Council rates	\$895.00	Quarterly	\$298.33
Electricity	\$485.56	Quarterly	\$161.85
Water	\$412.75	Quarterly	\$137.58
Car insurance	\$559.00	Half-yearly	\$93.17
Car registration	\$1136.00	Yearly	\$94.67
Fuel	\$3779.35	Yearly	\$314.95
Servicing	\$581.67	Yearly	\$48.47
Total			\$2601.19
Net monthly income			\$8993.56
Profit			\$6392.37

c. Yes, the family are solvent. Their net monthly income is greater than their spending.

TOPIC 11

Data analysis: classifying and representing data

11.1 Overview

11.1.1 Introduction

The world we live in produces an ever-increasing amount of data, and being able to understand and interpret these large sets of data is a valuable skill in the 21st century.

Data analysis allows us to effectively categorise, organise, display and compare data, which can give us a greater understanding of how the world around us works.



DISCUSSION

Think of the digital footprint that you leave by using your mobile phone and the internet each day. How much data do you produce?

LEARNING SEQUENCE

- 11.1** Overview
- 11.2** Data collection methods
- 11.3** Classifying data and displaying categorical data
- 11.4** Organising and displaying data
- 11.5** Comparing data
- 11.6** Review

CURRICULUM CONTENT

Students:

- describe and use appropriate data collection methods for samples and population ◇
- classify data relating to a single random variable ◇
- review how to organise and display data into appropriate tabular and/or graphical representations

AAM ◇  

- interpret and compare data by considering it in tabular and/or graphical representations **AAM** ◇  

11.2 Data collection methods

11.2.1 Population and samples

- In statistics, a **population** refers to all the members of a particular group being considered in a research study. That is, a population is the entire set about which we want to draw conclusions. A **sample** is a subset or group of members selected from the population. This sample information is used to make inferences about the population.
- When data is collected for analysis, consideration needs to be given to whether the data represents the population or a sample.
- If data is collected about the number of students at a school who ate at the canteen on a particular day, the *population* would be every student at the school and a *sample* could be one class or one year level.
- When a sample of data is collected, it is important to ensure that the sample is indicative of the population and is selected at random.
- If the sample is not large enough and is not selected at random, it may provide data that is **biased** towards one particular group of people.

WORKED EXAMPLE 1

A school with 750 students is surveying 25 randomly selected students to find which sport is most popular.

a Who makes up the population and how many people are in it?

b Who makes up the sample and how many people are in it?

THINK

- a** The population is made up of everyone who could possibly be asked this question. That would be every student at the school.
- b** The sample is made up of people randomly selected to take part in the survey.

WRITE

The population is made up of every student at this school, so that is 750 people.

The sample is the number of students selected, so that is 25 people.

11.2.2 Sampling methods

- Usually populations are too large for researchers to attempt to survey all of their members. **Sampling methods** are methods used to select members from the population to be in a statistical study.
- It is important to have a group of people who will participate in a survey and be able to represent the whole target population. This group is called a sample. Determining the right kind and number of participants to be in a sample group is one of the first steps in collecting data.
- Before you begin to select a sample, you first need to define your target population. For example, if your goal is to know the effectiveness of a product or service, then the target population should be the customers who have utilised it.



- There are numerous ways of obtaining a sample, with sampling methods classifiable as either probability or non-probability methods.
 - In probability samples, each member of the population has a known non-zero probability of being selected. The key benefit of probability sampling methods is that they guarantee that the sample chosen is representative of the population. This helps to ensure that the statistical conclusions drawn are valid.
 - In non-probability samples, members are selected from the population in a non-random manner. Non-probability sampling methods offer two potential advantages: convenience and cost.
- Only probability sampling methods allow you to estimate the extent to which **sample statistics** are likely to differ from **population parameters**.
- In this course we will look at three different types of probability sampling methods (**simple random sampling**, **stratified sampling** and **systematic sampling**) and one non-probability sampling method (**self-selected sampling**, also known as voluntary sampling).

Sampling method	Description	Examples
Simple random sampling	Each member of the population has an equal chance of selection. This is a simple method and is easy to apply when small populations are involved. It is free of bias.	A Tattslotto draw — a sample of 6 numbers is randomly generated from a population of 45, with each number having an equal chance of being selected.
Systematic sampling	This technique requires the first member to be selected at random as a starting point. There is then a gap or interval between each further selection. A sampling interval can be calculated using $I = \frac{N}{n}$, where N is the population size and n is the sample size. This method is only practical when the population of interest is small and accessible enough for any member to be selected. A potential problem is that the period of the sampling may exaggerate or hide a periodic pattern in the population.	Every 20th item on a production line is tested for defects and quality. The starting point is item number 5, so the sample selected would be the 5th item, the 25th, the 45th, ... Every 10th person who enters a particular store is selected, after a person has been selected at random as the starting point. Occupants in every 5th house in a street are selected, after a house has been selected at random as a starting point.
Stratified sampling	The population is divided into groups called strata, based on chosen characteristics, and samples are selected from each group. Examples of strata are states, ages, sex, religion, marital status and academic ability. An advantage is that information can be obtained on each stratum as well as the population as a whole.	A national survey is conducted. The population is divided into groups based on geography — north, east, south and west. Within each stratum respondents are randomly selected.
Self-selected sampling	A voluntary sample is made up of people who self-select into the survey. The sample can often be biased, as the people who volunteer tend to have a strong interest in the main topic of the survey. The sample tends to over-represent individuals who have strong opinions.	A news channel on TV asks viewers to participate in an online poll. The sample is chosen by the viewers.

11.2.3 Bias

- Generalising from a sample that is too small may lead to conclusions about a larger population that lack credibility. However, there is no need to sample every element in a population to make credible, reliable conclusions. Providing that a sufficiently large sample size has been drawn (as discussed below), a sample can provide a clear and accurate picture of a data set. However, it is important to try to eliminate bias when choosing your sampling method.
- Bias can be introduced in sampling by:
 - selecting a sample that is too small and not representative of the bigger population
 - relying on samples made up of volunteer respondents
 - sampling from select groups within a population, without including the same proportion from all the groups in the population
 - sampling from what is readily available
 - selecting a sample that is not generated randomly.
- A good sample is representative. If a sample isn't randomly selected, it will be biased in some way and the data may not be representative of the entire population. The bias that results from an unrepresentative sample is called **selection bias**. Some common examples of selection bias are:
 - **undercoverage** — this occurs when some members of the population are inadequately represented in the sample.
 - **non-response bias** — individuals chosen for the sample are unwilling or unable to participate in the survey. Non-response bias typically relates to questionnaire or survey studies. It occurs when the group of study participants that responds to a survey is different in some way from the group that does not respond to the survey. This difference leads to survey sample results being skewed away from the true population result.
 - **voluntary response bias** — this occurs when sample members are self-selected volunteers.

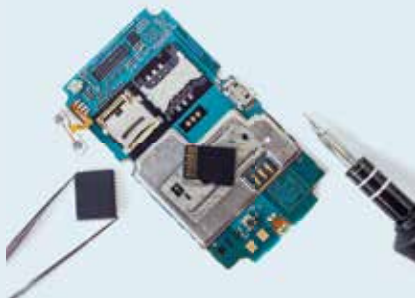
11.2.4 Determining the sample size

- Once you have identified the target population, you have to decide the number of participants in the sample. This is called the **sample size**.
- A sample size must be sufficiently large. As a general rule, the sample size should be at least $\sqrt{N}$, where N is the size of the population.
- If a sample size is too small, the data obtained is likely to be less reliable than that obtained from larger samples.

WORKED EXAMPLE 2

A factory produces 5000 mobile phones per week. Phones are randomly checked for defects and quality.

- What type of probability sampling method would be used?
- What size sample would be appropriate?
- Calculate the sampling interval.



THINK

- Think about how the data is collected. It would be easy to pick out every 50th member of the population.

WRITE

- Systematic sampling would be the most appropriate probability sampling method.

b The sample size should be at least $n = \sqrt{N}$, where N is the size of a population.

$$\begin{aligned}\mathbf{b} \quad N &= 5000 \\ n &= \sqrt{N} \\ &= \sqrt{5000} \\ &\approx 70.71\end{aligned}$$

A sample size of 71 would be appropriate.

c To calculate the sampling interval, use $I = \frac{N}{n}$.

$$\begin{aligned}\mathbf{c} \quad I &= \frac{N}{n} \\ &= \frac{5000}{71} \\ &= 70.42 \\ &\approx 70\end{aligned}$$

Every 70th mobile phone should be selected.

11.2.5 Potential flaws in data collection

- To derive conclusions from data, we need to know how the data was collected. There are four methods of data collection.

Census

- Sometimes the entire population will be sufficiently small, and the researcher can include the entire population in the study. This type of research is called a **census**, because the data is gathered from every member of the population. For most studies a census is not practical because of the cost and time required.

Sample survey

- A sample survey is a study that obtains data from a subset of a population in order to estimate population attributes. When writing survey questions, care must be taken to avoid language and phrases that may introduce bias. Biased questions are sometimes referred to as ‘leading questions’ because they are more likely to lead to particular responses.
- Bias can be introduced in a survey by:
 - selecting questions that include several unpopular choices along with one favoured choice
 - phrasing questions positively or negatively
 - limiting the number of options provided when respondents have to make a choice
 - including closed questions without giving respondents the opportunity to give a reason for a particular response
 - relying on a sample of one that reflects a personal opinion, which is often based on limited experiences.

Experiment

- An **experiment** is a controlled study in which the researcher attempts to understand the cause-and-effect relationships. It is a method of applying treatments to a group and recording the effects. A good group experiment will have two basic elements: a control and a treatment. The control group remains untreated throughout the duration of an experiment. The study group is controlled as the researcher controls how subjects are assigned to groups and which treatment each group receives.

Observation

- An **observation** study is a study in which researchers simply collect data based on what is seen and heard. Researchers then make inferences based on the data collected. Researchers should not interfere with the subjects or variables in any way. They can’t add in any extra information. All of the information must be evidence in the observational study.

11.2.6 Misunderstanding samples and sampling

- Each day we are bombarded with numbers, facts and figures in the media and news. It is interesting to look at how newspapers from different regions can put a different perspective on the same facts. Sometimes data is misunderstood by the media. Media reports often focus on one person's opinion and do not include data to support a claim. These reports should require further investigations to determine if a larger or more appropriate sample reflects the same results.
- Factual information must have integrity, objectivity and accuracy. It is important to recognise that information can be misinterpreted by personal bias, inaccurate statistics, and even by the addition of fictional data.
- Some people and organisations do manipulate information for their own uses. For this reason, always be critical about information that is provided to you. Make certain you know where the information is coming from and find out whether or not the source is credible. Also, try to find out what sampling processes and methods were used to collect the data.

WORKED EXAMPLE 3

As part of a Year 11 research project, students need to collect data. A group of students put out a message on social media asking for responses to their survey.

a What type of sampling method is used by the students?

b Why is the sampling method used probably biased?

THINK

a Deduce what type of sampling method was used from the various sampling methods.

b Explain the potential issues with the sampling method.

WRITE

a The sampling method used was self-selected sampling.

b Self-selected sampling is a non-probability sampling method.
The survey was made up of people who were willing to volunteer to answer the questions. The sample was not randomly generated and is probably not representative of the population.

eBookplus RESOURCES

Interactivity: Selecting samples (int-3811)

Exercise 11.2 Data collection methods

Understanding, fluency and communicating

- WE1** A company with 1200 employees and offices all over the world conducts a survey to see how happy their employees are with their work environment. They survey people from offices in London (120 employees), Sydney (180 employees), Milan (45 employees) and Japan (75 employees).
 - Who makes up the population and how many people are in it?
 - Who makes up the sample and how many people are in it?
- A university has 55 000 student enrolments. The university conducts a survey about online access for students. They survey students from the city campus (250 students) and the country campus (45 students).
 - Who makes up the population and how many people are in it?
 - Who makes up the sample and how many people are in it?

15. When would it be essential to survey the entire population and not just take a sample?
16. Briefly explain the difference between a census and a sample survey.
17. What is one main disadvantage of a telephone survey?
18. What research strategy is being used in each of the following situations?
 - a. To determine the effect of a new fertiliser on productivity of tomato plants, one group of plants is treated with the new fertiliser while a second group is grown without the treatment.
 - b. A sociologist joins a group of homeless people to study their way of life.
 - c. A company sends a satisfaction questionnaire to its current customers at the end of the year.
19. For a political survey, 1470 householders were selected at random from the electoral roll and asked whether they would vote for the currently elected political party. In the survey, 520 householders answered 'Yes' to voting for the currently elected political party.
 - a. If there are 17 million people in Australia over the age of 18, estimate how many would vote for 'No'. Give your answer to the nearest million.
 - b. What percentage of Australians over 18, to the nearest whole number, would vote 'Yes' in your estimation?
20. Do you agree or disagree with the following statement? Explain.
'I don't trust telephone surveys anymore. More and more individuals — particularly young individuals — do not have a landline. Moreover, these individuals are likely to differ from older individuals on key issues. If we are missing these younger individuals, our survey estimates will be biased.'
21. Some distance education students are enrolled in an online course. Depending on the location of the students, they are allocated to a region. There are 20 regions. In 10 of these regions, students are allocated to one of three tutors; in 7 of these regions students are allocated to one of two tutors; and in the remaining 3 regions, there is a single tutor. There are 10–15 students in each tutor's tutorial group. The distance education centre is planning a survey of the students to find out their opinion on the course. Suggest a way of selecting a sample of regions using the stratified sampling method.
22. A hotel manager is undecided about ways of administering a questionnaire. In particular, he is unsure whether to leave questionnaires in the hotel rooms or post them to clients' home addresses, and whether to select clients who book in during a 2-month period or select a proportion of clients who book in during a full year. Discuss which approach you would use and why.
23. An insurance company wishes to obtain customers' views on their satisfaction with the service they received. The company decides to survey callers who telephone its call centre to obtain their views. The call centre receives approximately 400 calls a day. If systematic sampling is used to select a sample of 100 callers over a six-day period from Monday to Saturday, estimate n where n represents every n th caller to be selected.

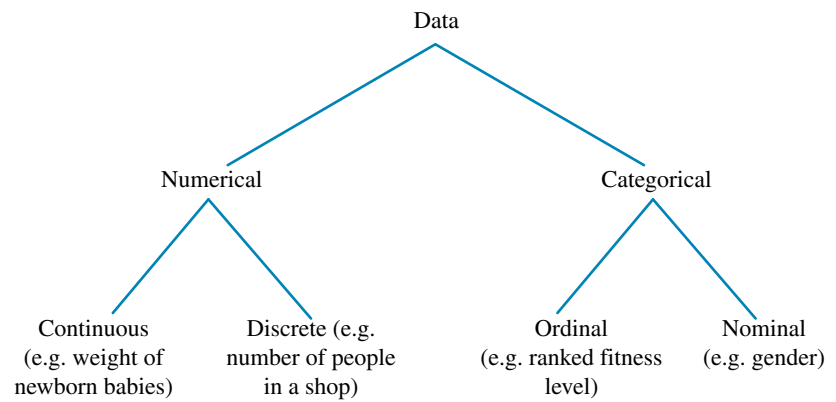


11.3 Classifying data and displaying categorical data

11.3.1 Classifying data

- Data is information that has been collected for the purpose of analysis. Data can either be **categorical** (able to be placed into categories) or **numerical** (able to be counted or measured).
- Categorical data can be:
 - **nominal** — arranged in categories, for example by colour or age
 - **ordinal** — arranged in categories that have an order; for example, karate ability ranges from white belt (beginner) to black belt (accomplished).

- Numerical data can be:
 - discrete** — data can have only particular values; for example, amounts of money, shoe sizes
 - continuous** — data can have *any* value within a range; for example, the time it takes a person to run around an oval.



WORKED EXAMPLE 4

The people in your class all share a mega-sized pizza. Students are then asked to rate the quality of the pizza as poor, average, above average or excellent. What type of data is being collected?



THINK

- This data does not involve numbers. Therefore the data is categorical.
- Categorical data can be broken into nominal and ordinal data. There is an order implied in the ranking.

WRITE

Categorical data

The data is ordinal, as the pizza could be rated from poor to excellent.

11.3.2 Frequency tables

- Data collected through surveys and experiments can be organised into **frequency tables**.
- When summarising data in a frequency table, it is important to work systematically to ensure that no value is missed or counted twice.
 - Work through the data from start to finish, in order, one value at a time.
 - Place your finger on the first number and write a tally mark for that number in the appropriate place on the frequency table; this will help ensure that no value is missed.
- Tally marks are collected into groups of five to make counting easier. Groups of five can appear in different arrangements; a group of five in the table below is shown as IIII.
- Data can be sorted into ranges of values, called **class intervals**. These class intervals must be the same size and must be set so that each value belongs to one interval only.
- Class intervals can be represented as follows:
 - 0–4, 5–9 etc. represents discrete data from and including 0 up to and including 4, then from and including 5 up to and including 9.
 - 0–<5, 5–<10 etc. represents continuous data ranging from and including 0 up to less than 5, then from and including 5 up to less than 10.
- Consider the frequency table at right displaying the number of goals scored in hockey matches.

Number of goals	Tally	Frequency
0–4	II	2
5–9	IIII	4
10–14	IIII II	7
15–19	IIII	4
20–24	III	3
Total		20

WORKED EXAMPLE 5

The continuous data below shows the number of kilometres that members of the cross-country running team ran over the last month.

20, 31, 42, 49, 46, 36, 42, 25, 28, 37, 48, 49, 45, 35, 25, 42, 30, 23, 25, 26, 29, 31, 46, 25, 40, 30, 31, 49, 38, 41, 23, 46, 29, 38, 22, 26, 31, 33, 34, 32, 41, 23, 29, 30, 29, 28, 48, 49, 31, 49, 48, 37, 38, 47, 25, 43, 38, 48, 37, 20, 38, 22, 21, 33, 35, 27, 38, 31, 22, 28, 20, 30, 41, 49, 41, 32, 43, 28, 21, 27, 20, 39, 40, 27, 26, 36, 36, 41, 46, 28, 32, 33, 25, 31, 33, 25, 36, 41, 28, 33

Present the data in a frequency table.

THINK

- 1 Choose a suitable class interval. The smallest value is 20 and the largest value is 49. Class intervals of 5 would be appropriate. Draw up a frequency table using the class intervals in the first column. The class interval 20–<25 includes kilometres ranging from and including 20 to less than 25.

WRITE

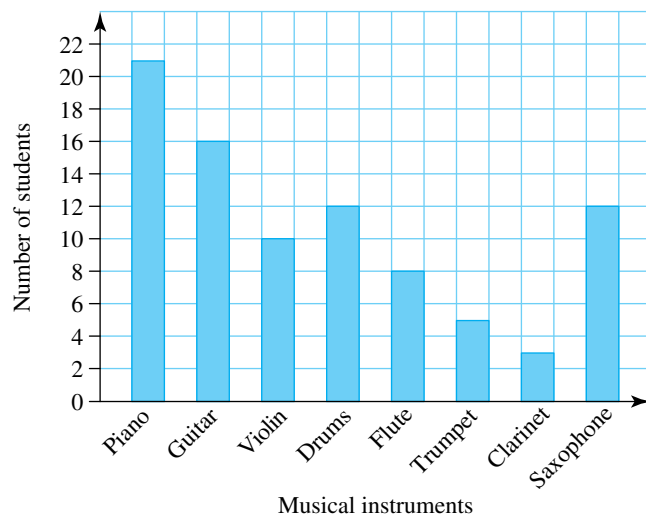
Number of kilometres	Tally	Frequency (<i>f</i>)
20–<25		12
25–<30		23
30–<35		20
35–<40	I	16
40–<45		13
45–<50	I	16
Total		100

- 2 Go through the list systematically and complete the tally column. Determine the frequency of each class interval.
- 3 Calculate the total of the frequency column.

11.3.3 Bar charts

- **Bar charts** can be used to display categorical and discrete numerical data.
- In a bar chart, each category has its own bar (or column). The height or length of each column in the graph is determined by the frequency for the category.
- One axis of the graph represents the categories; the other axis is scaled and represents the frequencies.
- Bars must be the same width, and an equal gap must be used between bars. A space should be left before the first bar.
- Bar charts can be displayed either vertically or horizontally, as shown at right.

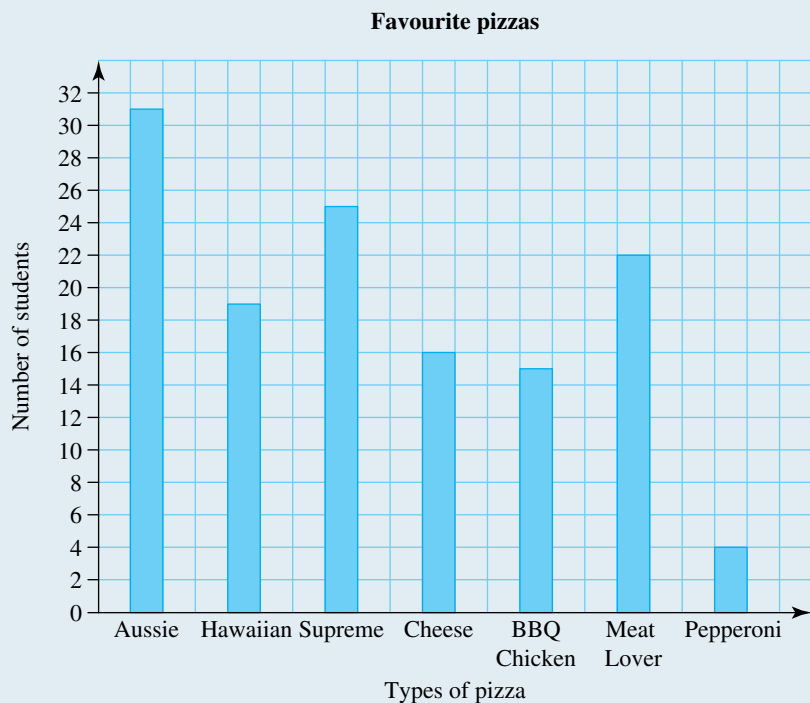
Year 11 students who learn a musical instrument



WORKED EXAMPLE 6

Year 11 students from a particular school in regional New South Wales were asked about their favourite pizzas. The results are shown in the bar chart.

- What is the sample size?
- Find the mode of the data.
- If a pizza franchise in Sydney is trying to determine the buying patterns of Year 11 students, does this sample represent all Year 11 students in New South Wales? Explain your answer.



THINK

- To calculate the sample size, add the values of each column together.
- The mode of the data is the highest frequency.
- Consider the sample and what type of sampling method was used to represent the population.

WRITE

- $\text{Sample size} = 31 + 19 + 25 + 16 + 13 + 22 + 4$
 $= 130$
130 students were surveyed.
- The highest value was 31.
Aussie pizzas were the most popular among the students.
- The sample was only taken from one particular school. This could produce a biased sample that may not be representative of all Year 11 students in New South Wales.

11.3.4 Pareto charts

- The objective of a **Pareto chart** is to highlight the most important factors that affect a variable. A Pareto chart contains both vertical bars and a line graph. It is a bar chart in which the categories are arranged in order of their frequencies, from the most frequent to the least frequent. This allows readers to see clearly what the most important factors are in a given situation.
- A Pareto chart can also include a **cumulative percentage graph**. For each category, this shows the total percentage contribution of that category and all preceding categories. Since the factors are always represented in decreasing order, the line graph is always concave.
- In a Pareto chart:
 - the horizontal axis shows the categories of the nominal data
 - the heights of the bars represent frequencies. The bars are arranged or ranked in order, with the highest or most frequent frequency on the left and the lowest frequency on the right.
 - the left side of the vertical axis is frequency
 - the right side of the vertical axis is the cumulative percentage of the total.
- For example, a survey was conducted in Term 1 from a random sample of 50 students for the main reasons of lateness to school. The data is shown on the next page.

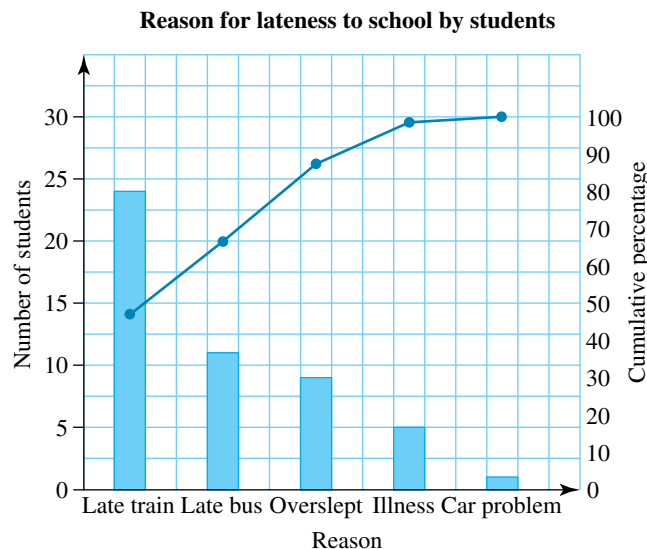
- The **cumulative frequency** is obtained by adding up the frequencies as you go along, to give a ‘running total’. The cumulative frequency is calculated by adding each frequency to the sum of its predecessors. The last value will always be equal to the total for all observations, since all frequencies have already been added to the previous total.

Reason for lateness	Number of students	Cumulative frequency	Cumulative percentage frequency
Late train	24	24	48
Late bus	11	35	70
Overslept	9	44	88
Illness	5	49	98
Car problem	1	50	100

- The **cumulative percentage frequency** is calculated using the formula:

$$\text{Cumulative frequency percentage} = \frac{\text{cumulative frequency}}{\text{total frequency}} \times 100\%$$

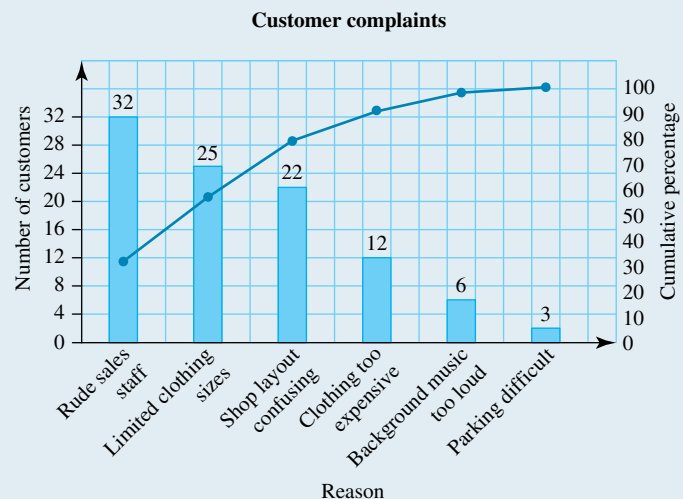
- The Pareto chart below shows the results.



WORKED EXAMPLE 7

The manager of a clothing store observed a decline in sales. The manager assumed that customer dissatisfaction was the reason behind the decline in sales, so she conducted a customer survey. At right is a Pareto chart displaying the reasons for customer dissatisfaction with the clothing store.

- How many customers were surveyed?
- Which customer dissatisfaction is the biggest concern?
- What is the cumulative percentage frequency value for the customer reason of finding the shop layout confusing?



THINK

- a** The totals of the heights of the column bars represent the frequency.
- b** The highest frequency in a Pareto chart can always be located on the left side of the chart, as the column bars are arranged in order from the highest frequency to the lowest frequency.
- c** Calculate the cumulative frequency and cumulative percentage frequency.

WRITE

- a** $32 + 25 + 22 + 12 + 6 + 3 = 100$
The total number of surveyed customers is 100.
- b** The highest frequency is 32.
This corresponds to rude sales staff.
The biggest concern for customer dissatisfaction is rude sales staff.

- c** The cumulative frequency is equal to $32 + 25 + 22 + 12 + 6 + 3 = 100$.
- Cumulative frequency percentage = $\frac{\text{cumulative frequency}}{\text{total frequency}} \times 100\%$
- For example, if the cumulative frequency = 79
- Cumulative frequency percentage = $\frac{79}{100} \times 100\% = 79\%$

Customer dissatisfaction reasons	Frequency	Cumulative frequency	Cumulative percentage frequency
Rude sales staff	32	32	32
Limited clothing sizes	25	57	57
Shop layout confusing	22	79	79
Clothing too expensive	12	91	91
Background music too loud	6	97	97
Parking difficult	3	100	100

Alternatively, you could estimate the cumulative percentage frequency value from the line graph.

eBookplus RESOURCES

Interactivity: Types of data (int-6086)

Interactivity: Frequency tables (int-3816)

Interactivity: Create a bar chart (int-6493)

Exercise 11.3 Classifying data and displaying categorical data

Understanding, fluency and communicating

1. State whether each of the following represents categorical or numerical data.
 - a. The heights in centimetres of a group of Year 11 students
 - b. The type of transport that students in Year 11 take to school
 - c. The blood groups of students in Year 11
 - d. The number of visitors to the library each day
2. **WE4** State whether each of the following represents nominal, ordinal, discrete or continuous data.
 - a. The NRL ladder at the end of each round
 - b. The time spent watching TV
 - c. The types of cars in the teacher car park
 - d. The number of children in the families in your suburb
3. At a hospital nursing station, the following information is available about a patient.
Temperature: 30.2°C
Blood type: A
Response to treatment: Excellent
Which of the data groups is ordinal?
4. At a used car lot, the following information is obtained about one of the cars on the lot.
Make: Honda
Model year: 2015
Petrol consumption (per 100km): 9.8 litres
Which of the data groups is discrete?
5. A classmate attempted to classify data as either nominal, ordinal, discrete or continuous. Correct her work, shown in the table below, and reclassify any data that fits better elsewhere, explaining why.

Nominal	Ordinal	Discrete	Continuous
Gender	Ability to play basketball	Finish position in a race	Height
Eye colour	Number of students in your class	Time taken to walk to school	Number of songs on an MP3 player

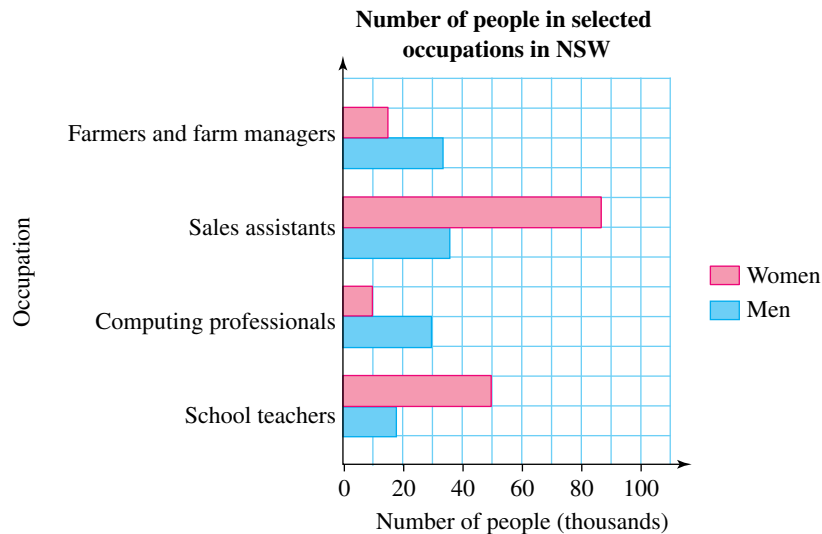
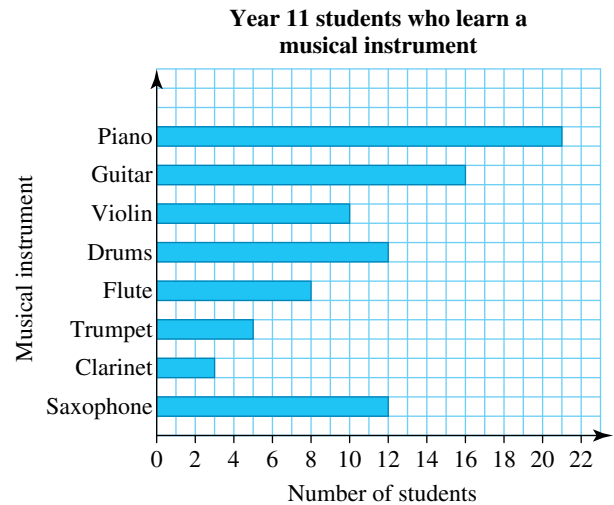
6. When reading the menu at the local Chinese restaurant, you notice that the dishes are divided into sections. The sections are labelled chicken, beef, duck, vegetarian and seafood.
 - a. What type of data is this?
 - b. What is the best way to represent this data?
7. **MC** The Bureau of Meteorology collects data on rainfall over catchment areas. The rainfall is measured in millimetres. What type of data is being collected?
 - A. Nominal
 - B. Ordinal
 - C. Discrete
 - D. Continuous
 - E. None of the above
8. **MC** The number of white, blue, red and silver cars in the car park at the SCG on a Saturday is counted. What type of data is being collected?
 - A. Nominal
 - B. Ordinal
 - C. Discrete
 - D. Continuous
 - E. None of the above



9. One hundred teenagers were surveyed about their favourite type of music genre. The data was organised into a frequency table.

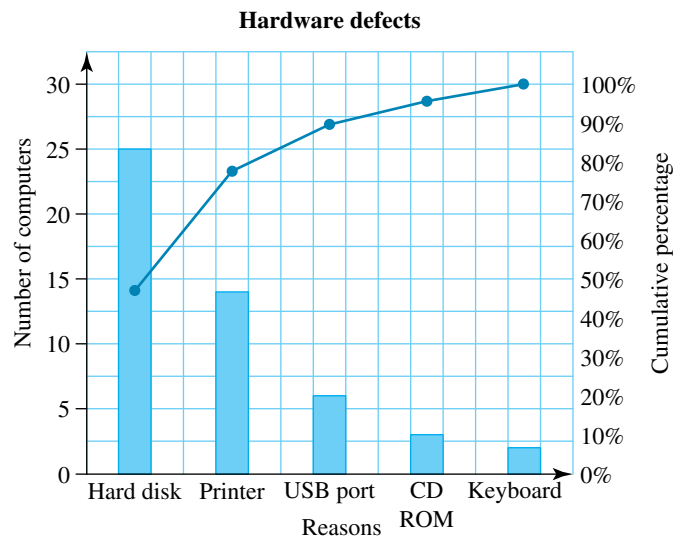
Music genre	Frequency
Hip hop/rap	28
Pop/R&B/soul	27
Rock	26
Country	3
Blues/jazz	4
Classical	2
Alternative	10

- a. Is the data numerical or categorical?
b. What type of data is 'music genre'?
c. What percentage of teenagers preferred rock music?
10. **WE6** Look at the bar chart of musical instruments played by Year 11 students.
- a. What is the sample size?
b. Find the mode of the data.
c. A large music supply store in Sydney is trying to determine which instruments to stock. Does this sample represent all Year 11 students in New South Wales? Explain your answer.
11. The following graph shows the number of men and women in selected occupations in NSW.
- a. Use the graph to estimate the number of female sales assistants in NSW at the time the data was collected.
b. Estimate the number of school teachers in NSW.
c. In which occupation are there about 30 000 male workers?
d. In which occupation is the total number of workers about 40 000?



12. **MC** A small computer shop recorded for the year the number of computers it had to return to a computer manufacturer for replacement due to hardware defects. The Pareto chart at right shows the hardware defects for the returned computers. Which hardware defect reason is of least concern for the computer manufacturer?

- A. Hard disk
- B. Printer
- C. USB port
- D. CD-ROM
- E. Keyboard



Problem solving, reasoning and justification

13. A survey was conducted asking members of the public to nominate their preferred sport. The results recorded were: football, cricket, cricket, tennis, basketball, netball, tennis, netball, swimming, netball, tennis, football, cricket, basketball, lawn bowls, football, swimming, netball, tennis, netball, cricket, tennis, football, basketball, swimming, lawn bowls, swimming, swimming, netball, netball, tennis, golf, football, football, basketball, swimming, golf, football, netball, swimming, basketball, basketball, golf, tennis, cricket, cricket, football, basketball, netball, golf.

- a. Is the data collected in this survey an example of numerical or categorical data?
- b. What problems arise when the results of surveys are presented as they are above instead of in table form?
- c. Present the data in a frequency table and use the table to write a description summarising the survey results.



14. Draw a bar chart to represent the information in each of the following tables.

a.

Main method of travel to school this morning	
Car	20
Bus	15
Train	10
Bicycle	12
Walk	15

b.

Favourite ice-cream flavour	
Chocolate	18
Strawberry	15
Vanilla	13
Banana	5
Peppermint	7

15. **a.** List two examples of your own that would be considered discrete data.
b. List two examples of your own that would be considered continuous data.
16. **WE5** Students from a Year 11 class were asked about their favourite subjects and the following data was recorded.

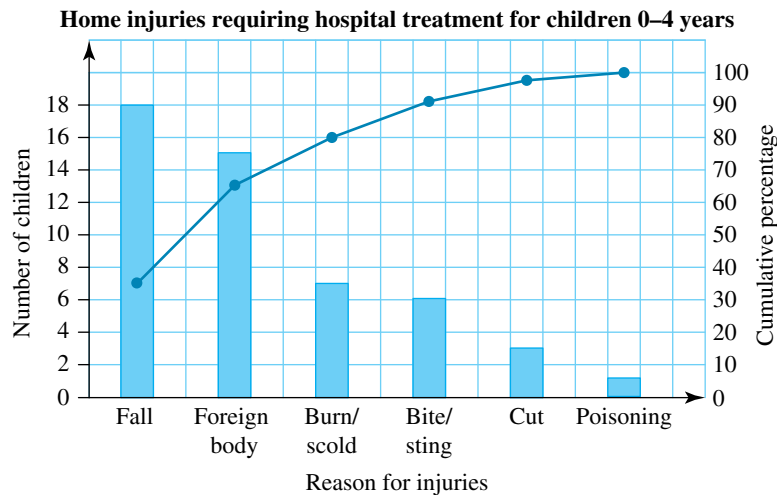
Maths	English	PE	Science	PE
Art	Maths	Science	English	Science
Cooking	PE	English	Cooking	Maths
PE	Art	Science	Maths	Art
Science	Cooking	Art	PE	PE

- a.** Put the data in a frequency table and record the number of students who were surveyed.
b. What percentage of students preferred Maths?
c. What was the most popular subject? What percentage of students preferred this?
d. What type of data is this?
17. A manager of a restaurant wants to survey a sample of customers after they have had a meal. He has decided to use a paper-based questionnaire to be given out with a free coffee after the meal and to be handed in at the reception desk. He asks the following questions in his questionnaire:
- Q1 How satisfied were you with the range of choice on the menu?
 Very dissatisfied Dissatisfied Satisfied Very satisfied
- Q2 How satisfied were you with the quality of service?
 Very dissatisfied Dissatisfied Satisfied Very satisfied
- Q3 What is your opinion about the value for money of your meal?
 Very poor value for money
 Poor value for money
 Good value for money
 Very good value for money
- a.** What type of data is being collected?
b. Write a suitable question for this questionnaire that would provide numerical data.
c. What type of sampling method could the restaurant manager use to select the sample of customers?
18. A study of money and casual work of New South Wales senior secondary students (Years 9–12) is to be conducted. Data will be collected using a questionnaire. The questionnaire begins with the following four questions.
- Q1. Are you: Male ☐ Female ☐
 Q2. Do you have a part-time job? Yes ☐ No ☐
 Q3. Which company do you work for? _____
 Q4. How do you get to work?
 Public transport ☐ Car ☐ Bike ☐ Walk ☐
- a.** Classify the type of categorical data that will be collected in Q3 of the questionnaire.
b. Write a suitable question for this questionnaire that would provide numerical data.
c. The study is to be conducted using a stratified sample. How could a representative stratified sample be obtained?
19. A study of mobile phone usage of New South Wales secondary students is to be conducted. Data will be collected using a questionnaire. The questionnaire begins with the following three questions.
- Q1. Do you own a mobile phone? Yes ☐ No ☐
 Q2. Which phone carrier do you use? _____
 Q3. Do you use prepaid or a plan? Prepaid ☐ Plan ☐
 Q4. How do you rate your mobile phone plan?
 Poor Good Very good Excellent

- a. Classify the type of categorical data that will be collected in Q4 of the questionnaire.
- b. Write a suitable question for this questionnaire that would provide numerical data.
- c. The study is to be conducted using a stratified sample. How could a representative stratified sample be obtained?

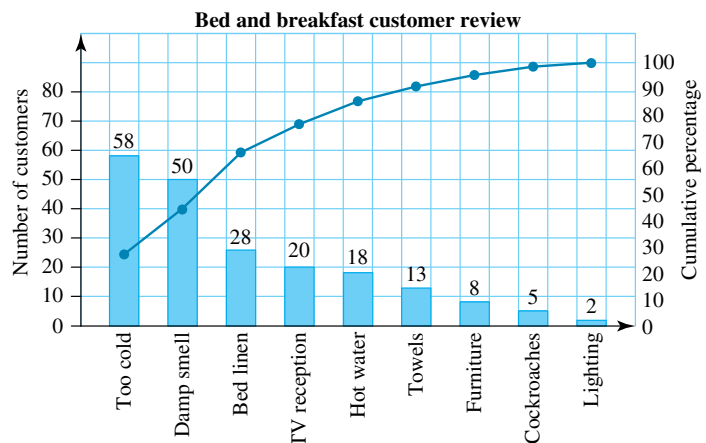
20. **WE7** The following Pareto chart shows the causes of home injuries for which children aged 0 to 4 years required hospital treatment in April 2017.

- a. How many children required hospital treatment in April 2017 for home injuries?
- b. Which reason for home injury of children 0 to 4 years is the biggest concern?
- c. What is the cumulative percentage frequency value for burn/scold injuries?



21. An online bed-and-breakfast company gives customers the opportunity to review online the bed-and-breakfast places that they stayed in. The table and Pareto chart below show the complaints of customers staying at a particular bed and breakfast located in an alpine resort area.

Customer reasons	Frequency
Too cold	58
Damp smell	50
Old bed linen	26
Poor TV reception	20
Insufficient hot water	18
Towels too small/thin	13
Dated furniture	8
Cockroaches	5
Inadequate lighting	2



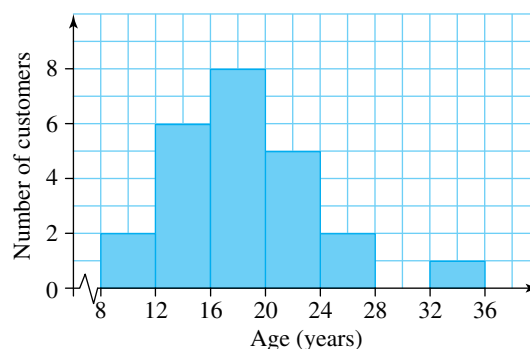
- a. How many customers reviewed the bed and breakfast?
- b. Which reason for customer complaints is the biggest concern for the bed and breakfast?
- c. Is this sampling method used by the online bed-and-breakfast company biased? Explain.

11.4 Organising and displaying data

11.4.1 Histograms

- **Histograms** are used for displaying grouped numerical data.
- Histograms can be used to highlight trends and distributions.
- Histograms display data that has been summarised in a frequency table using class intervals. The data in the frequency table shown is displayed as a histogram.
- The columns on the histogram represent the different class intervals over which they sit.
- When drawing a histogram:
 - the columns must be the same width and have no gaps.
 - the first number of the class interval should be at the left-hand side of the column.
 - the height of each column is determined by the frequency of the category.
- An **axis break** is a gap inserted into an axis to indicate that a range of values has been skipped. An axis break is shown as a zigzag on the axis. The histogram above right has an axis break on the horizontal axis.

Ages of customers in a music shop on a Saturday morning



Age of customers	Number of customers
8–11	2
12–15	6
16–19	8
20–23	5
24–28	2
28–32	0
32–36	1

WORKED EXAMPLE 8

The maximum temperatures, in degrees Celsius, for the month of February 2008 in Pascoe Vale are shown below.

24.6, 26.2, 26.8, 32.8, 25.2, 26.1, 19.9, 19.5, 19.8, 21.8, 26.9, 23.0, 19.4, 22.1, 25.5, 29.2, 34.4, 33.6, 35.1, 19.6, 27.6, 26.0, 21.2, 20.8, 21.2, 22.9, 22.9, 18.1, 19.5

Use a histogram to display the data. Classify the data as ‘Mild’ (15–<20), ‘Warm’ (20–<25), ‘Warm to hot’ (25–<30), ‘Hot’ (30–<35) or ‘Very hot’ (35–<40).

THINK

- 1 Draw a frequency table and classify the data. For example, the class interval 15–<20° contains temperatures of 15° to less than 20°.

WRITE

Temperature (°C)	Frequency
Mild (15–<20)	7
Warm (20–<25)	9
Warm to hot (25–<30)	9
Hot (30–<35)	3
Very hot (35–<40)	1

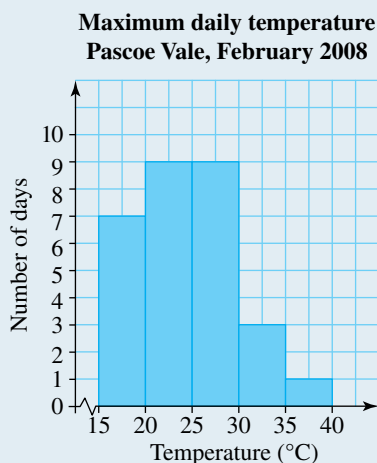
2 Draw a set of axes.

- The vertical axis will show the frequency (number of days) and needs to go up to 9 as that is the maximum number of days in any category.
- The horizontal axis needs to cover values from 15 to 40, which is the range of temperatures included in the data. Use an axis break to start the horizontal axis at 15.

3 • The column for the first category ($15 < 20$) has its left edge at 15 and its right edge at 20. It is 5 units high as this is the frequency of the category.

- Draw columns for the rest of the categories.

4 Label each of the axes and include a title.



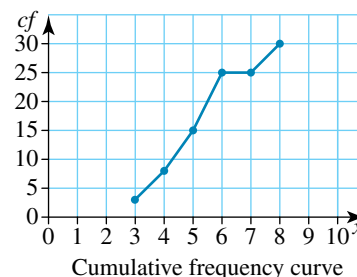
11.4.2 Cumulative frequency tables

- As previously discussed, the cumulative frequency of a data value is the number of observations that are above or below the particular value.
- Cumulative frequency is recorded in a **cumulative frequency table**.
- The final value in a cumulative frequency table will always equal the total number of observations in the data set. The cumulative frequency table at right shows the number of movies watched in the last month by a group of thirty Year 11 students.

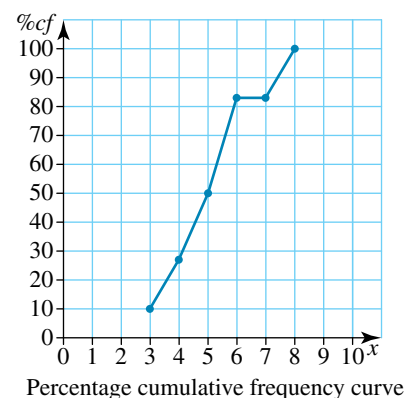
Data (x)	Frequency (f)	Cumulative frequency (cf)
3	3	3
4	5	$3 + 5 = 8$
5	7	$8 + 7 = 15$
6	10	$15 + 10 = 25$
7	0	$25 + 0 = 25$
8	5	$25 + 5 = 30$

11.4.3 Ogives

- Data from a cumulative frequency table can be plotted to form a **cumulative frequency curve**, which is also called an **ogive** (pronounced 'oh-jive').
- To plot an ogive for data that is in class intervals, the maximum value for the class interval is used as the value against which the cumulative frequency is plotted.
- Percentiles** divide a data set into 100 equal-sized parts.
- A percentile is named after the percentage of data that lies at or below that value. For example, 60% of the data values lie at or below the 60th percentile.
- Percentiles can be read off a percentage cumulative frequency curve.
- A percentage cumulative frequency curve is created by:
 - writing the cumulative frequencies as a percentage of the total number of data values
 - plotting the percentage cumulative frequencies against the maximum value for each interval.



Data (x)	Frequency (f)	Cumulative frequency (cf)	Percentage cumulative frequency (%cf)
3	3	3	$\frac{3}{30} \times \frac{100\%}{1} = 10\%$
4	5	8	$\frac{8}{30} \times \frac{100\%}{1} = 27\%$
5	7	15	$\frac{15}{30} \times \frac{100\%}{1} = 50\%$
6	10	25	$\frac{25}{30} \times \frac{100\%}{1} = 83\%$
7	0	25	$\frac{25}{30} \times \frac{100\%}{1} = 83\%$
8	5	30	$\frac{30}{30} \times \frac{100\%}{1} = 100\%$



WORKED EXAMPLE 9

The mass of eggs in three egg cartons ranges between 55 and 65 grams, as shown in the table at right.

- Draw a percentage cumulative frequency table for the data.
- Draw the ogive.

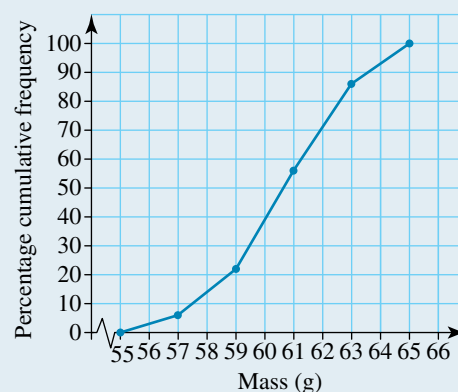
Mass (g)	Frequency
55–<57	2
57–<59	6
59–<61	12
61–<63	11
63–<65	5

THINK

- Construct the cumulative frequency table by calculating the cumulative frequency for each class interval, as shown in black.
 - Calculate the percentage cumulative frequency for each interval by dividing the cumulative frequency for each interval by the total cumulative frequency, as shown in pink.
 - For the first interval: (55–<57),
 $\%cf = \frac{2}{36} = 0.06 = 6\%$.
- Plot the percentage cumulative frequency curve.
 - For the first interval (55–<57), plot the minimum value for the interval (55) against 0%.
 - Plot the maximum value for each interval against the percentage cumulative frequency for the interval.
 - For the first interval, plot (57, 6%).
 - For the second interval, plot (59, 22%).

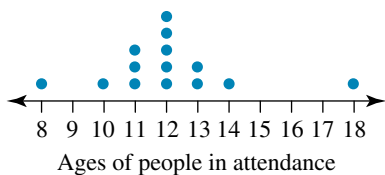
WRITE

Mass (g)	Frequency (f)	Cumulative frequency (cf)	Percentage cumulative frequency (%cf)
55–<57	2	2	6%
57–<59	6	2 + 6 = 8	22%
59–<61	12	8 + 12 = 20	56%
61–<63	11	20 + 11 = 31	86%
63–<65	5	31 + 5 = 36	100%



11.4.4 Dot plots

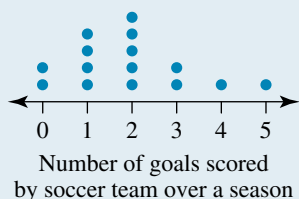
- Dot plots** display discrete numerical data by using dots on a number line to represent the individual data values. The dot plot below displays the ages of people in attendance at a dancing lesson. Each dot represents the age of one person.



- Where a value appears more than once, the dots are stacked vertically. In the example above, there were five people aged 12 at the dancing lesson.
- Dot plots give a quick overview of the distribution of data values. They show **clustering** (groups of dots) and **outliers** (extreme values), and help to determine whether data should be grouped.

WORKED EXAMPLE 10

The dot plot below shows the number of goals scored by a soccer team over a season.



- How many matches were played over the season?
- What was the greatest number of goals scored in a match?
- What was the most common number of goals scored in a match over the season?
- In how many matches did the team score 0 goals?

THINK

- The score for each match is represented by a dot. There are 15 dots so there were 15 matches.
- The greatest score shown on the number line with a dot is 5.
- The most common number of goals is seen where there are the most dots for a given score. There were five matches with a score of 2 goals.
- There are two dots over the score of 0.

WRITE

There were 15 matches played during the season.

The greatest number of goals scored in a match was 5.

The most common number of goals scored in a match over the season was 2.

There were two matches where the team scored 0 goals.

11.4.5 Stem-and-leaf plots

- Stem-and-leaf plots** are a type of table used to organise and display data.
- Each piece of data in a stem-and-leaf plot is made up of two parts: a stem and a leaf.

- When arranging numbers into a stem-and-leaf plot, the last digit of each number becomes the leaf part and the other digits form the stem. For example, the numbers 125, 127 and 133 can be arranged as shown at right. When numbers have the same stem part, the leaf parts are added to the same row.
- A **key** must accompany a stem-and-leaf plot.
- The leaves must be carefully spaced in rows and columns so that the distribution of the data can be seen clearly.
- A stem-and-leaf plot retains all the original data values.
- An ordered stem-and-leaf plot has the leaf parts written in ascending order from left to right.
- There is one leaf for every data value.

Key: $12|5 = 125$

Stem	Leaf
12	5 7
13	3

WORKED EXAMPLE 11

The ages of people in a yoga class are displayed in the stem-and-leaf plot shown below.

Key: $2|5 = 25$ years

Stem	Leaf
1	4 8 8 9
2	0 1 3 5 7
3	2 6 8
4	1 5
5	0

- How many people are in the class?
- What is the age of the youngest person in the class?
- What is the age of the oldest person in the class?
- Are there any people of the same age in the class? If so, how old are they?



THINK

- The last digit of each person's age is a leaf in the stem-and-leaf plot. The number of people in the class can be found by counting the number of leaf parts. There are 15 leaf parts.
- The stem-and-leaf plot is ordered so the numbers run from smallest to largest. The smallest number has a stem of 1 and a leaf of 4. By using the key, this represents an age of 14 years.
- The largest number has a stem of 5 and a leaf of 0. This represents an age of 50 years.
- Look for numbers that are the same. For the stem of 1 there are two leaf parts that are 8. This means that two people in the class are aged 18 years.

WRITE

There are 15 people in the class.

The youngest person in the class is 14 years old.

The oldest person in the class is 50 years old.

There are two people in the class of the same age. They are 18 years old.

eBookplus RESOURCES

Interactivity: Create a histogram (int-6494)

Interactivity: Dot plots, frequency tables and histograms, and bar charts (int-6243)

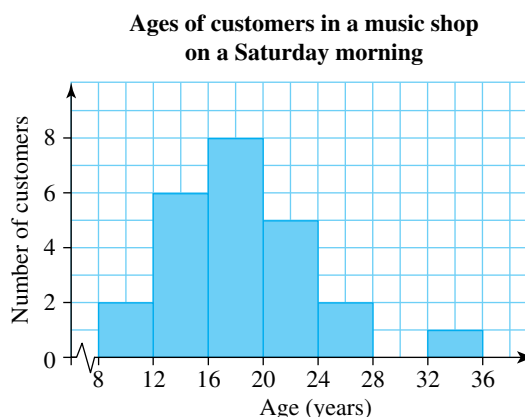
Interactivity: Stem plots (int-6242)

Interactivity: Create stem plots (int-6495)

Exercise 11.4 Organising and displaying data

Understanding, fluency and communicating

- Refer to the histogram at right.
 - How many customers were aged between 8 and 12?
 - What was the most common age of the customers on this Saturday morning?
 - How many 30-year-old customers were in the shop on this Saturday morning?
 - How many customers were in the shop on this Saturday morning? How could this be calculated if you had only the histogram?



- WE8, 9** The frequency table at right shows the number of hours of television that a group of Year 11 students watched last week.

- Draw a histogram to represent the data.
- Draw a cumulative frequency curve.

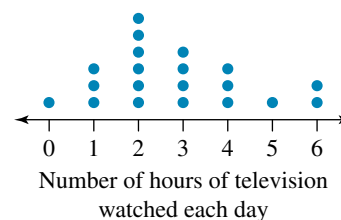
- A group of Year 11 students were asked how many movies they had watched in the last 6 months. The results were as follows:

2, 11, 8, 6, 5, 11, 6, 6, 12, 10, 14, 9, 14, 19, 6, 14, 3, 7, 10, 16, 16, 4, 3, 16, 16, 17, 9, 3, 10, 13, 4, 8, 6, 17, 20, 10, 4, 11, 8, 1, 15, 5, 3, 7, 20, 20, 14, 9, 4, 1, 3, 2, 14, 15, 6, 13, 3, 15, 18, 9, 19, 9, 10, 16, 4, 2, 9, 14, 18, 14, 8, 2, 4, 19, 6, 13, 14, 2, 7, 9, 18, 6, 9, 4, 6, 16, 18, 16, 17, 16, 7, 6, 2, 16, 3, 4, 17.

- Draw a histogram to represent the data.
- Draw a cumulative frequency curve.

- WE10** The dot plot at right shows the number of hours of television watched each day by a group of people.

- How many people were surveyed?
- What is the greatest number of hours of television watched by a person in this group each day?
- What is the most common number of hours of television watched by a person in this group each day?
- How many people in this group watched 3 hours of television each day?
- How many people in this group watched more than 3 hours of television each day?



- WE11** The stem-and-leaf plot below displays the mass of each bag that is being packed on a bus for a school camp.

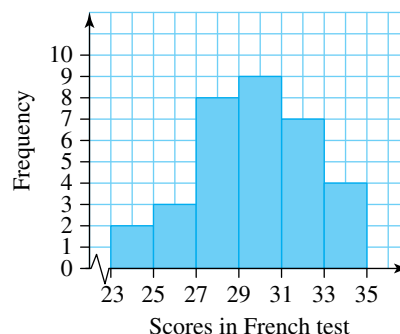
Key: 2|0 = 20 kg

Stem	Leaf
0	5 5 9
1	4 5 7 8
2	3 4 8 9
3	2 3 8
4	1

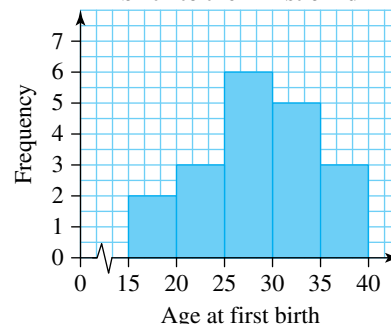


- a. What is the mass of the lightest bag being taken to the camp?
 - b. How many bags have a mass greater than 30 kg?
 - c. Are there any bags that have the same mass? If so, how can you tell?
 - d. How can you calculate the total number of bags being taken to the camp?
6. The histogram at right represents the scores achieved by a Year 11 class in a French test that was marked out of 35.
- a. What was the most common result for the test?
 - b. What was the least common result?
 - c. How many students completed the French test?
 - d. How many students achieved a result of 31 or greater?
 - e. Which results occurred eight times?
7. At right is a histogram showing the ages of women when they gave birth to their first child.
- a. What type of data is this?
 - b. What is the most common age bracket when the women first gave birth?
 - c. How many women first gave birth between 30 and 35?
 - d. How many women were surveyed?
8. Over a 2-week period, the number of students absent from a group of 1000 Year 11 students was recorded as follows:
15, 17, 20, 10, 14, 16, 14, 12, 5, 14.
Draw a dot plot to display the data.
9. Prepare an ordered stem-and-leaf plot for each of the following data sets.
- a. 132, 117, 108, 129, 165, 172, 145, 189, 137, 116, 152, 164, 118, 131, 173, 152, 146, 150, 171, 130
 - b. 12, 40, 31, 33, 2, 16, 19, 12, 0, 25, 22, 28, 10, 3, 15, 31, 8, 12, 7, 32, 16, 27, 12, 6, 1, 17, 14, 21, 18, 32
10. Prepare an ordered stem-and-leaf plot for each of the following data sets. (Note: Each whole-number part will form a stem and each decimal part will form a leaf; that is, $14|5 = 14.5$.)
- a. 14.8, 15.2, 13.8, 13.0, 14.5, 16.2, 15.7, 14.7, 15.1, 15.9, 13.9, 14.5
 - b. 2.8, 2.7, 5.2, 6.2, 6.6, 2.9, 1.8, 5.7, 3.5, 2.5, 4.1

Marks achieved in a Year 11 French test

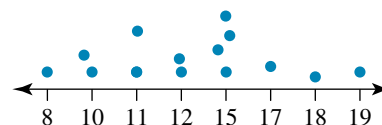


Ages of women when they gave birth to their first child



Problem solving, reasoning and justification

11. The diagram at right is the result of a student's attempt to draw a dot plot to display the values
8, 10, 10, 11, 11, 12, 12, 15, 15, 15, 15, 17, 18, 19.
- a. List the mistakes that the student made in drawing the dot plot.
 - b. Draw the correct dot plot.
12. The maximum daily temperatures ($^{\circ}\text{C}$) in Sydney for the month of February were recorded as follows:
28, 24, 29, 37, 35, 30, 31, 27, 34, 29, 25, 36, 36, 37, 36, 30, 42, 41, 42, 41, 38, 37, 33, 27, 26, 36, 27, 26
- a. Draw a dot plot to display the data.
 - b. Comment on the distribution of the data.



- 13.** At the same time as the data from question **12** was recorded, the maximum temperatures ($^{\circ}\text{C}$) on the Gold Coast were recorded as follows: 31, 27, 28, 36, 30, 30, 28, 30, 34, 30, 30, 30, 29, 29, 28, 29, 28, 30, 28, 28, 30, 32, 34, 36, 30, 28, 27, 26.

- Draw a dot plot to display the data.
- Compare this distribution with that of the dot plot from question **12**.
- Comment on the similarities and differences in the temperatures experienced in Sydney and on the Gold Coast.

- 14.** Train timetables at railway stations are often presented in a format similar to stem-and-leaf plots. A section of a timetable is displayed at right.

- What does this timetable use as the stem and the leaf?
- If you arrive at the station at 8:20 am, what train will you take?
- What benefit does a timetable in this form have compared with a timetable that lists the full time for each departure?

Departure times from Central Station	
am	
6	03 23 43
7	03 18 33 48
8	03 10 18 25 33 43 50
9	03 18 33 48

- 15. a.** For the stem-and-leaf plot shown at right:

- what is the minimum value
- what is the maximum value
- what does the blank line in the leaf section represent
- what type of data is represented?

- What other type of data can be represented on a stem-and-leaf plot?

Key: 2 | 1 = 21

Stem	Leaf
0	1 3 3 5
1	
2	1 4 9
3	2 7
4	1

- 16.** The amount of pocket money (\$) given to a group of 45 Year 11 students is shown below.

35, 28, 48, 30, 20, 36, 41, 26, 32, 42, 21, 31, 45, 42, 50, 30, 32, 40, 25, 44, 21, 27, 18, 10, 55, 42, 32, 43, 19, 38, 28, 12, 24, 50, 40, 35, 26, 28, 16, 30, 25, 19, 20, 50

- Using appropriate class intervals, present this data in a frequency distribution table.
- Draw a histogram on grid paper to represent this data.
- Write a paragraph to describe this data set.



- 17.** Data was collected on the number of times people go to the cinema in a month.

- Put the data in a frequency table.
- Draw a histogram to represent the data.
- How many people go to the cinema less than three times a month?
- How many people go to the cinema at least three times a month?

4, 5, 7, 9, 1, 2, 5, 2, 4, 8, 3, 6,
2, 3, 8, 1, 1, 4, 5, 3, 3, 6, 1, 2,
7, 1, 3, 2, 2, 4, 10, 0, 1, 3, 4, 6



18. The frequency table at right represents the number of times people go to the local swimming pool during summer.

- Draw a histogram to represent this data.
- Is this data discrete? Explain.

19. Sometimes when you go through a fast-food drive-through, they don't have your order ready and you have to wait in the waiting bay for it to be brought out to you. The waiting times of 50 customers were recorded in minutes and seconds, and are shown below.

Number of times	Frequency
0–5	4
6–10	8
11–15	15
16–20	20
21–25	12
26–30	6

5:45, 3:21, 6:34, 8:23, 4:18, 3:22, 4:08, 7:12, 3:37, 3:40, 3:17, 4:55, 4:39, 5:08, 2:16, 3:41, 4:09, 4:55, 3:27, 2:48, 6:45, 3:34, 4:20, 3:44, 8:11, 7:37, 3:19, 4:56, 5:55, 3:20, 4:30, 2:06, 4:27, 5:44, 3:05, 5:23, 4:46, 3:49, 7:23, 5:39, 4:55, 2:12, 3:09, 3:58, 4:07, 4:24, 3:46, 2:36, 4:19, 4:00

- Construct a frequency table with class intervals of 30 seconds.
- Draw a percentage cumulative frequency table.
- Construct a percentage cumulative frequency curve.

11.5 Comparing data

11.5.1 Back-to-back stem-and-leaf plots

- Back-to-back stem-and-leaf plots** allow us to compare two sets of data on the same graph.
- A back-to-back stem-and-leaf plot has a central stem with leaves on either side. The stem-and-leaf plot at right gives the ages of members of two teams competing in a bowling tournament.

Key: 5|5 = 55

Team A	Stem	Team B
1	4	
	5	5789
	6	02357
6543	7	012
8654321	8	8
	9	0

WORKED EXAMPLE 12

The following data was collected from two Year 11 Maths classes who completed the same test. The total mark available was 100.

Class 1	84	90	86	95	92	81	83	97	88	99	79	100	85	82	97
Class 2	90	55	48	62	70	58	63	67	72	59	60	88	57	65	71

Draw a back-to-back stem-and-leaf plot.

THINK

- To create a back-to-back stem-and-leaf plot, determine the highest and lowest values of each set of data to help you decide on a suitable scale for the stems.
- Put the data for each class in order from lowest to highest value.

WRITE

The highest value is 100 and lowest value is 48, so the stems should be 4, 5, 6, 7, 8, 9, 10.

Class 1: 79, 81, 82, 83, 84, 85, 86, 88, 90, 92, 95, 97, 97, 99, 100

Class 2: 48, 55, 57, 58, 59, 60, 62, 63, 65, 67, 70, 71, 72, 88, 90

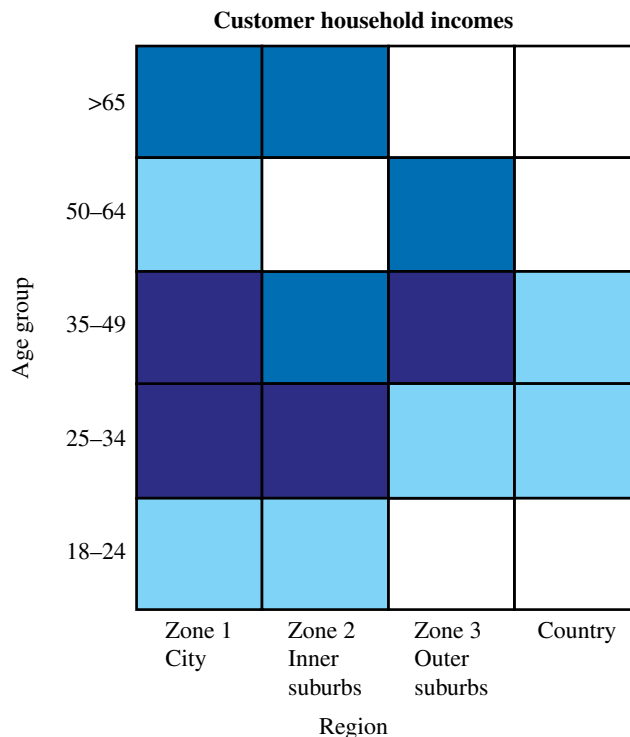
3 Create a back-to-back stem-and-leaf plot.

Key: $4|8 = 48$

Leaf (Class 1)	Stem	Leaf (Class 2)
	4	8
	5	5 7 8 9
	6	0 2 3 5 7
9	7	0 1 2
8 6 5 4 3 2 1	8	8
9 7 7 5 2 0	9	0
0	10	

11.5.2 Heat maps

- A **heat map** chart represents data in a tabular format with user-defined colour ranges indicating the level of activity instead of numbers. The condensed colour-coded format of a heat map chart makes the data easy to understand.
- For example, a car dealership collects data about its customers to assist with the price range of cars that customers purchase. The heat map shown represents the mean household income for the dealership's customers in different age groups and from different regions.



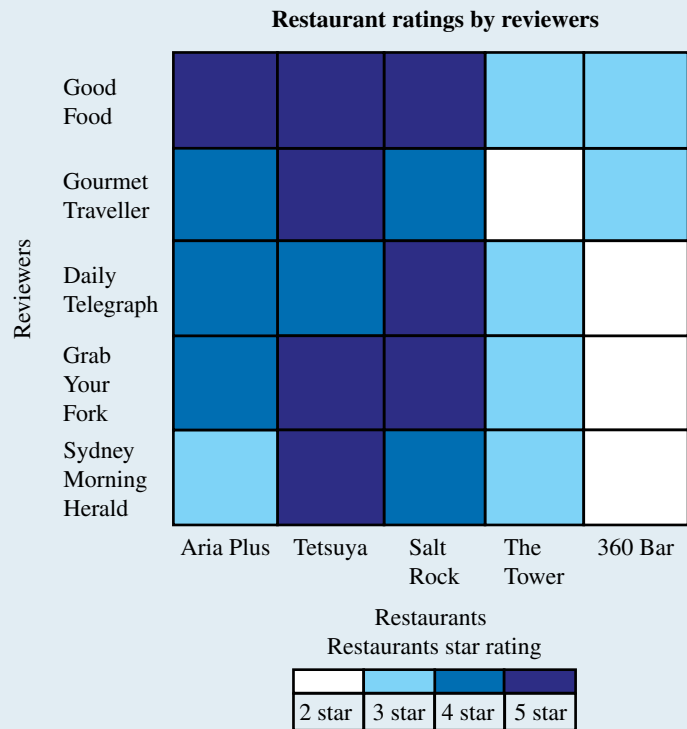
Mean household income

	Light Blue	Dark Blue	Dark Blue
0–\$30 000	\$30 001– \$60 000	\$60 001– \$90 000	\$90 001– \$120 000

WORKED EXAMPLE 13

A businesswoman wants to take her clients to an expensive restaurant for dinner to negotiate a contract. At right is a restaurant ratings heat map based on reviews by selected food reviewers.

- Which restaurant received the best reviews?
- How many restaurants received 3-star ratings?
- Which reviewer only gave two types of ratings?



THINK

- Look at the colour-coded legend defining the ratings and compare this to the map.
- Count the number of rectangles that are shaded with the corresponding colour for 3-star ratings.
- For two types of ratings, the reviewer's row should only have two different colours.

WRITE

A 5-star review is the best rating.
 The restaurant Tetsuya received four 5-star ratings and one 4-star rating.
 Aria Plus received one 3-star rating.
 The Tower received four 3-star ratings.
 360 Bar received three 2-star ratings.
 Three restaurants received 3-star ratings.
 Good Food allocated only two types of ratings.
 Gourmet Traveller, Daily Telegraph, Grab Your Fork and Sydney Morning Herald all used all four of the different star ratings.

11.5.3 Choosing appropriate data representations

- Examining multiple data sets is a common practice in statistics. To display similar sets of data, appropriate tabular and/or graphical representations are selected to enable comparisons of similarities and differences.
- Side-by-side bar charts** or back-to-back bar charts are used when comparing multiple sets of categorical data.
- Back-to-back stem-and-leaf plots and dot plots are used to compare distributions of numerical data.

WORKED EXAMPLE 14

A sample of New South Wales residents were asked to state the environmental issue that was most important to them. The results were sorted by the age of the people surveyed. The results were as follows.

Age	Environmental issue		
	Reducing pollution	Conserving water	Recycling rubbish
Under 30	52	63	41
Over 30	32	45	23

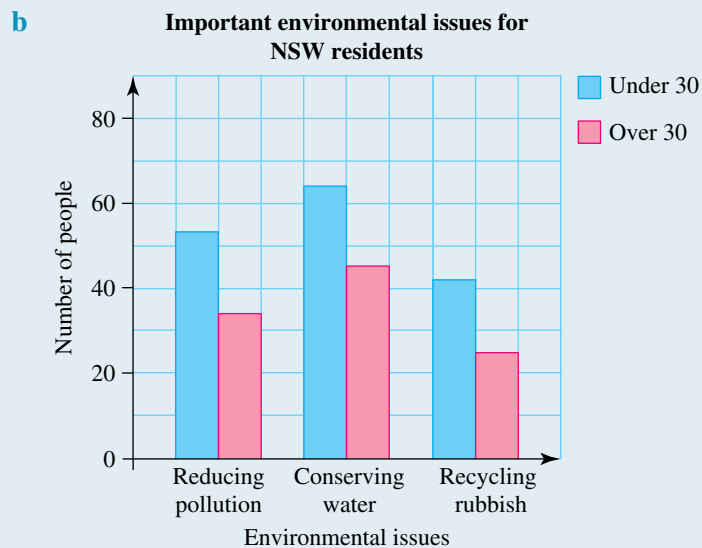
- What type of data display would you use to compare the data sets?
- Construct a side-by-side bar chart.
- How many people surveyed were over 30?
- What percentage of under-30s surveyed listed conserving water as the most important environmental issue?

THINK

- What type of data is being compared?
- Construct a side-by-side column graph for the data.

WRITE

- Categorical data is being compared. A side-by-side bar chart or back-to-back bar chart would be the most suitable data display.



- Total the over-30s' responses.

$$32 + 45 + 23 = 100$$
- For over-30s there were 32 responses for reducing pollution, 45 for conserving water and 23 for recycling rubbish. The total number of people over 30 who were surveyed is $32 + 45 + 23 = 100$.
- To calculate the percentage of under-30s who listed conserving water as an important issue, use

$$\text{percentage} = \frac{\text{frequency}}{\text{total frequency}} \times 100\%$$

$$\text{percentage} = \frac{63}{52 + 63 + 41} \times 100\%$$

$$= \frac{63}{156} \times 100\%$$

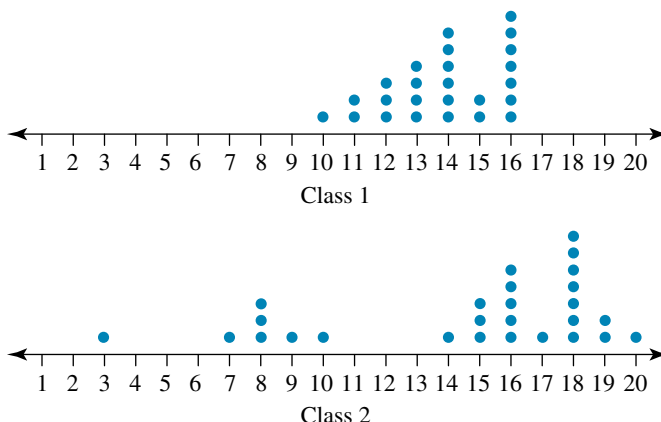
$$= 40.4\%$$

Approximately 40% of residents under 30 listed conserving water as the most important environmental issue.

Exercise 11.5 Comparing data

Understanding, fluency and communicating

1. The two dot plots below display the latest Maths test results for two Year 11 classes. The results show the marks out of 20.



- How many students are in each class?
 - For each class, how many students scored 15 out of 20 for the test?
 - For each class, how many students scored more than 10 for the test?
 - Use the dot plots to describe the performance of each class on the test.
2. **WE12** The following data shows the ages of male and female players at a ten-pin bowling centre. Draw a back-to-back stem-and-leaf plot of the data.
- Male: 20, 36, 16, 38, 32, 18, 19, 21, 25, 45, 29, 60, 31, 21, 16, 38, 52, 43, 17, 28, 23, 23, 43, 17, 22, 23, 32, 34
- Female: 21, 23, 30, 16, 31, 46, 15, 17, 22, 17, 50, 34, 65, 25, 27, 19, 15, 43, 22, 17, 22, 16, 48, 57, 54, 23, 16, 30, 18, 21, 28, 35

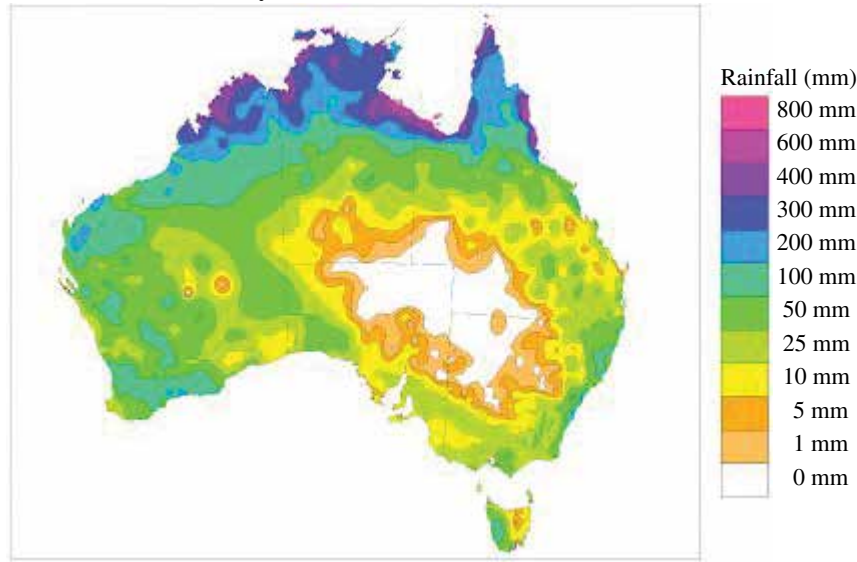


Key: 6|1 = 61 hours

Brand A	Stem	Brand B
8 8 7 5	0	7
9 7 4 1 0	1	0 5 5 5 7 9
2 2 2 1	2	0 2 2 6 7
8 6 4 2 0	3	0 2 4 6 8
	4	
	5	6
1	6	
	7	5

4. **MC** A geographic heat map shows the total rainfall in Australia for February 2017.

Total rainfall in February 2017



Source: Bureau of Meteorology

For Victoria, the rainfall in February was mostly in the range:

- A.** 0–10 mm **C.** 100–200 mm **E.** 400–800 mm
B. 10–100 mm **D.** 200–400 mm

Use the following information to answer questions 5 and 6.

An online tourism business summarises customers' feedback on visiting different tourist attractions in New South Wales.

At right is a heat map based on reviews by different bus tour groups.

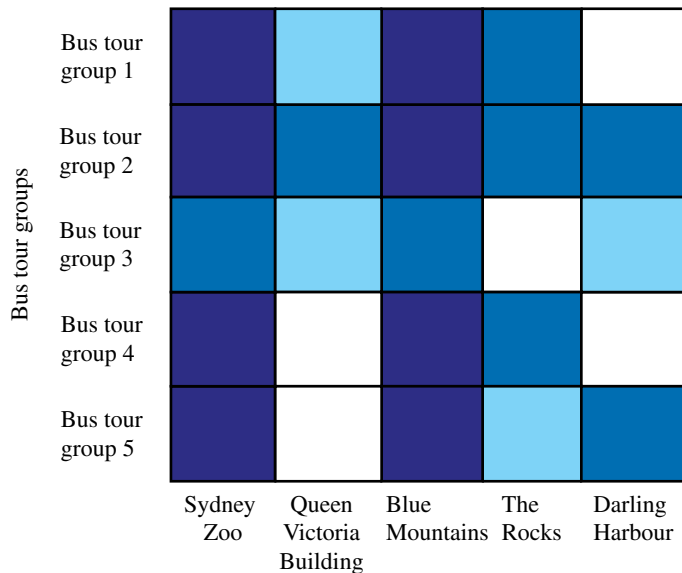
5. **MC WE13** The tourist attraction that received the worst reviews was:

- A.** Sydney Zoo
B. Queen Victoria Building
C. Blue Mountains
D. The Rocks
E. Darling Harbour

6. **MC** Which bus tour group did not give a rating of 'bad'?

- A.** Bus tour group 1
B. Bus tour group 2
C. Bus tour group 3
D. Bus tour group 4
E. Bus tour group 5

Tourists attractions ratings by bus tour groups



Tourist attractions

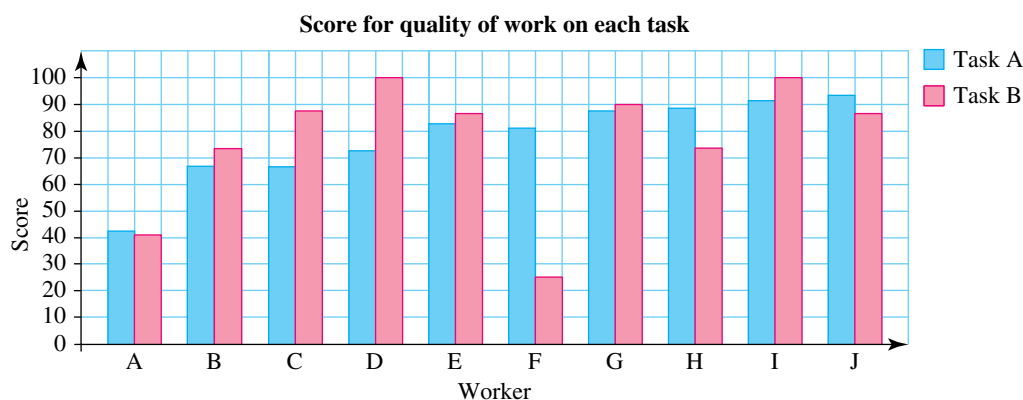
Tourist attraction rating

Bad	Average	Good	Best
White	Light Blue	Dark Blue	Dark Blue

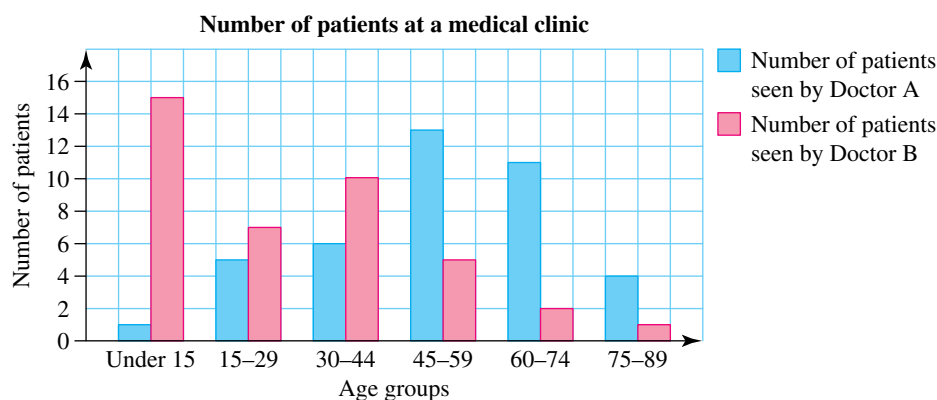
7. **WE14** A city newspaper surveyed a sample of New South Wales residents about an upcoming state election on the issues most important to each age group. The results were as follows:

Age	Election issue					
	Marriage equality	Education	Refugees and immigration	Tax and superannuation	Health	Housing affordability
18–40	18	35	38	43	22	47
Over 40	12	38	19	61	41	32

- What type of data display would you use to compare the data sets?
 - How many people surveyed were over 40?
 - What issue is most important to the people aged over 40?
 - What percentage of under-40s surveyed listed marriage equality as the most important election issue? Give your answer to the nearest whole number.
8. Ten workers were required to complete two tasks. Their supervisor observed the workers and gave them a score for the quality of their work on each task, where higher scores indicated better-quality work. The results are indicated in the following side-by-side bar chart.



- Which worker had the largest difference between scores for the two tasks?
 - How many workers received a lower score for task B than task A?
9. The following graphical display summarises the ages at the last birthday of patients seen by two doctors in a medical surgery during one particular day.
- How many more patients aged under 15 did Doctor B consult compared to Doctor A?
 - Doctor A tends to consult patients aged over 45, whereas Doctor B tends to consult patients aged under 45. True or False?



10. Two households estimate the electricity consumed by different household appliances and devices as follows.

Appliance or device	Household 1 electricity consumption (%)	Household 2 electricity consumption (%)
Water heater	29	32
Refrigerator	18	21
Stove/cooktop	21	24
Washing machine	6	10
Lighting	8	7
Computer	7	2
Audiovisual equipment	2	2
Air-conditioner	6	2
Heating	3	3

Assuming that both households use the same amount of electricity overall, answer True or False to the following statements.

- Household 1's computer uses more electricity than household 2's computer.
- Household 2 uses less electricity in heating and air-conditioning than household 1.
- Household 2 watches less TV than household 1.

Problem solving, reasoning and justification

11. The daily number of hits a fashion blogger gets on her new website over 3 weeks are:

126	356	408	404	420	425	176
167	398	433	446	419	431	189
120	431	390	495	454	215	117

At the same time, the daily number of hits a healthy lifestyle blogger gets on his new website over 3 weeks are:

240	156	462	510	420	474	520
225	402	426	563	621	339	195
320	621	340	495	700	415	371

- Compare the two data sets using an appropriate graphical display.
- Comment on the two data sets.

- 12.** The winning times in seconds for the women's and men's 100-metre sprint in the Olympics are shown below.

Year	Women's 100m sprint	Men's 100m sprint
1928	12.2	10.8
1932	11.9	10.3
1936	11.5	10.3
1948	11.9	10.3
1952	11.5	10.4
1956	11.5	10.5
1960	11.0	10.2
1964	11.4	10.0
1968	11.0	9.9
1972	11.07	10.14
1976	11.08	10.06
1980	11.60	10.25
1984	10.97	9.99
1988	10.54	9.92
1992	10.82	9.96
1996	10.94	9.84
2000	10.75	9.87
2004	10.93	9.85
2008	10.78	9.69



- a.** Display the winning times for women and men using a stem-and-leaf plot.
b. Is there a large difference in winning times? Explain your answer.
- 13.** The following data sets show the rental price (in \$) of two-bedroom apartments in two different suburbs of Wollongong.

Suburb A

215 225 211 235 244 210 215 210 256 207
 200 200 242 225 231 205 240 205 235 200

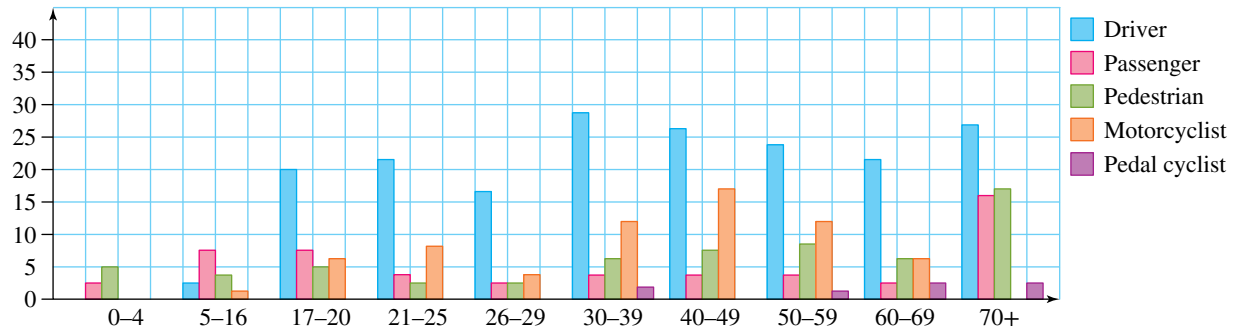
Suburb B

235 245 231 232 240 280 280 270 255 275
 275 285 245 265 270 255 260 258 251 285

- a.** Draw a back-to-back stem-and-leaf plot to compare the data sets.
b. Compare and contrast the rental price in the two suburbs.

14. A side-by-side bar chart shows the distribution of New South Wales road fatalities from March 2016 to March 2017.

Distribution of fatalities for 12 months ending March 2017, by age and road user class

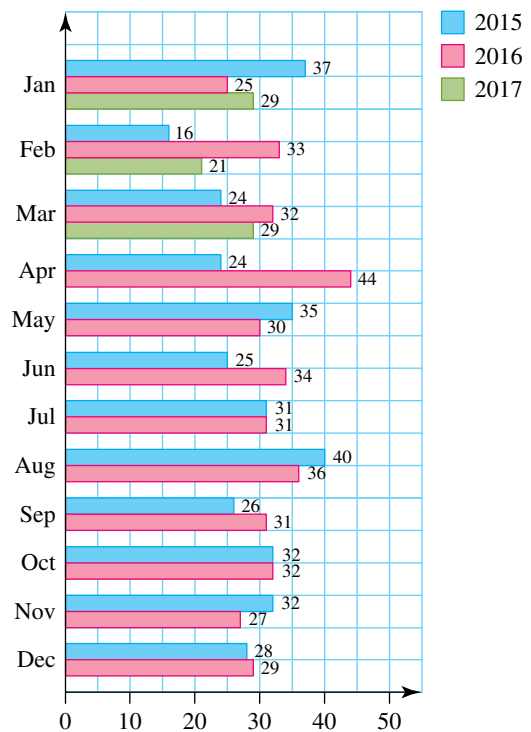


Source: Transport for NSW, Centre for Road Safety

- a. Which age group had the most passenger and pedestrian fatalities? Give a reason why this might be.
- b. The New South Wales government wants to introduce a road campaign. Which age groups should the government focus on? Explain your answer.
15. The horizontal side-by-side bar chart at right shows a monthly comparison of the New South Wales road fatalities.
- a. Using the data from 2015 and 2016, which year had the most fatalities on New South Wales roads?
- b. Which year has had the most fatalities from January to March?
- c. Why do you think there were 44 fatalities in the month of April 2016?
16. A coffee bar serves either skim, reduced-fat or whole milk in coffees. The coffees sold on a particular day are shown in the table below, sorted by the type of milk and the genders of the customers.

Type of milk	Gender	
	Male	Female
Skim	87	124
Reduced fat	55	73
Whole	112	49

Monthly comparison of New South Wales road fatalities



Source: Transport for NSW, Centre for Road Safety

- a. How many coffees were sold on this day?
- b. Represent this data in a graphical display.
- c. What percentage of males used skim milk for their coffees? Give your answer to the nearest whole number.
- d. What percentage of coffees sold contained reduced-fat milk? Give your answer to the nearest whole number.
- e. If this was the daily trend of sales for the coffee bar, what percentage of the coffee bar's customers would you expect to be female? Give your answer to the nearest whole number.

11.6 Review

11.6.1 Summary

In this topic you have learnt:

- the difference between a population and a sample
- the difference between probability and non-probability sampling methods
- about four different types of sampling methods
- how bias can be introduced in sampling
- how to determine the minimum required sample size
- about potential flaws in data collection
- the difference between categorical and numerical data
- how to organise data into frequency tables
- how to display data in bar charts
- how to display data in Pareto charts
- how cumulative percentage frequency is calculated
- how to display data in histograms
- how to plot a cumulative frequency curve
- how to divide a data set into percentiles
- how to create dot plots
- how to create stem-and-leaf plots
- how to compare two sets of data using back-to-back stem-and-leaf plots
- how to interpret data from a heat map
- the most appropriate data representations for displaying similar sets of data.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 11.6 Review

Understanding, fluency and communicating

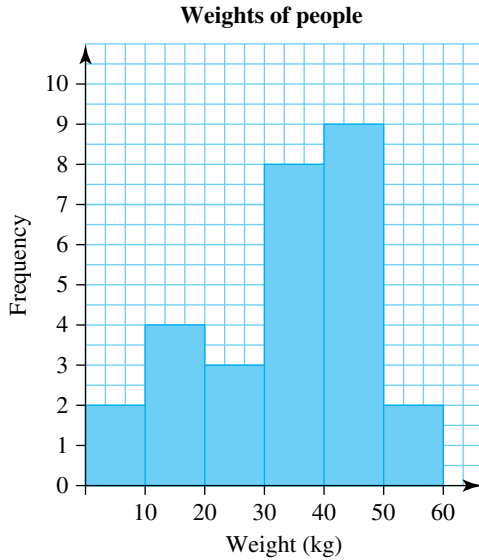
1. Classify the following data sets as:
 - i. categorical or numerical
 - ii. nominal, ordinal, discrete or continuous.
 - a. Weight of laptop computers
 - b. Hair colour
 - c. Number of movies you watch each month
 - d. Length of a piece of string
 - e. Level of achievement in a competition
 - f. Type of car
2. **MC** The Australian Bureau of Statistics collects data on the price of unleaded petrol. What type of data is being collected?
A. Nominal **B.** Ordinal **C.** Discrete
D. Continuous **E.** None of the above
3. A university survey was given that included data on each student's field of interest, age in years and number of languages spoken. Which of the data collected is/are categorical?
4. An airline company wants to survey its customers one day, so they randomly select 5 flights that day and survey every passenger on those flights. This type of sampling is stratified sampling. True or False?

5. **MC** When every member of the accessible population has an equal chance of being selected to participate in a study, the researcher is using:
- A. simple random sampling B. stratified sampling C. self-selected sampling
D. systematic sampling E. none of the above.
6. **MC** A teacher randomly selects 20 student names from a hat. What type of sampling is this?
- A. Simple random sampling B. Systematic sampling C. Stratified sampling
D. Self-selected sampling E. None of the above
7. **MC** A principal selects 3 classrooms and surveys every student in those classrooms. What type of sampling is this?
- A. Simple random sampling B. Systematic sampling C. Stratified sampling
D. Self-selected sampling E. None of the above
8. A student council surveys 150 students by getting random samples of 25 Year 7 students, 25 Year 8 students, 25 Year 9 students, 25 Year 10 students, 25 Year 11 students and 25 Year 12 students. The method used is stratified random sampling as it guarantees that members from each year level will be represented in the sample. True or False?
9. How many replies are required from a survey for a sample to be representative?
10. Determine if the following statements are True or False.
- An advantage of using interviews as a data collection method is that interviewers can repeat or explain the meaning of questions to respondents face to face if necessary, minimising errors in the data collection and ensuring there are no incomplete responses.
 - Disadvantages of postal questionnaires include that it can take a long time to collect data and that we cannot be sure that the target respondent is filling out the questionnaire.
 - Advantages of internet surveys include that they are cost effective; large numbers of respondents can be reached quickly; and data can be stored electronically, which minimises errors associated with entering data. A disadvantage is that not everyone in the target population may have access to the internet, which can lead to sample bias.
11. The ages (in years) of a group of people are shown below: 25, 18, 51, 29, 70, 38, 49, 33, 56, 68, 17, 83, 37, 40, 56, 21, 35, 45, 24, 31, 47, 62, 15, 26, 32, 39, 29, 74, 23, 43, 30, 52, 66, 36, 22, 35, 46, 57. Represent the data as a frequency table using a class interval of 10 years.
12. Arrange the following data into a frequency table:
- without class intervals
 - with class intervals.
- 8, 9, 4, 10, 17, 19, 7, 3, 18, 21, 4, 2, 10, 19, 23, 21, 21, 5, 9, 10, 13, 24, 1, 6, 8, 11, 15, 19, 20, 16, 22, 21, 17, 19, 7, 9
13. Represent the data in question 12b in a histogram.
14. The prices in cents of regular unleaded petrol in New South Wales are listed below.

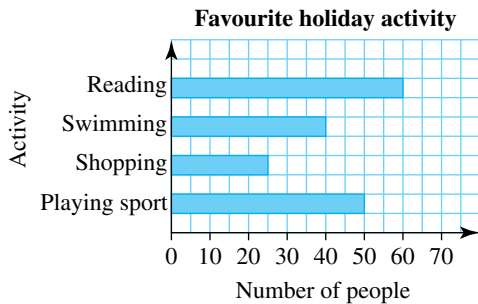
Region	Average price for week ending 9 April 2017	Average price for week ending 2 April 2017
Sydney	132.4	112.7
Canberra	129.1	129.8
Central Coast	121.7	121.6
Newcastle	120.5	122.5
Wollongong	126.1	117.0
Tweed Heads South	129.4	132.1

Which region in New South Wales had the biggest difference in petrol prices between the two weeks?

15. Draw a frequency table using the histogram below.

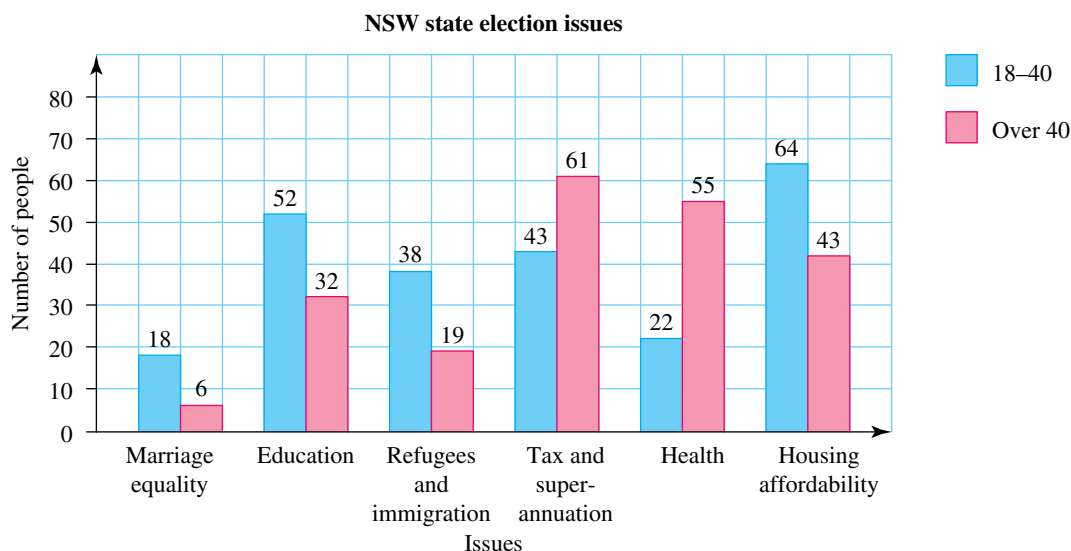


16. The results of a survey are displayed in the graph below.

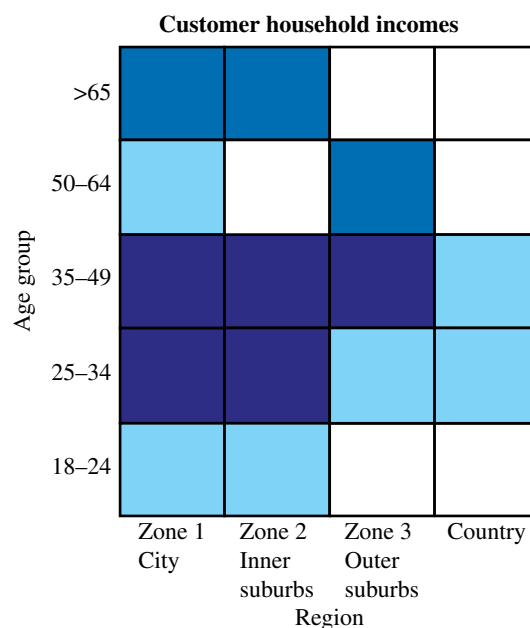


- What type of graph has been used to display the data?
 - Which activity is the most popular?
 - How many people were surveyed?
17. Twenty people were asked about the number of times they had received a haircut in the past 6 months. The results of the survey were: 4, 5, 1, 3, 6, 0, 6, 9, 5, 4, 3, 3, 5, 0, 5, 5, 4, 3, 5, 4.
- Present the data as a dot plot.
 - What was the most common number of haircuts received?
 - Comment on the distribution of the data.
18. Surfers like big waves! The data at right indicates heights of waves and the number of days in a month that they occurred.
- Draw a cumulative frequency curve for this data.
 - On how many days were the waves less than 6m tall?
 - Draw a percentage cumulative frequency curve for this data.
- | Wave height (m) | Days |
|-----------------|------|
| 0-<2 | 11 |
| 2-<4 | 8 |
| 4-<6 | 5 |
| 6-<8 | 4 |
| 8-<10 | 2 |
19. Prepare an ordered stem-and-leaf plot for each of the following sets of data.
- 29, 37, 25, 62, 73, 41, 58, 62, 73, 67, 47, 21, 33, 71, 92, 41, 62, 54, 31, 82, 93, 28, 31, 67, 29, 53, 62, 21, 78, 81, 51, 25, 93, 68, 72, 46, 53, 39
 - 132, 117, 108, 129, 165, 172, 145, 189, 137, 116, 152, 164, 78
 - 14.8, 15.2, 13.8, 13.0, 14.5, 16.2, 15.7, 14.7, 14.3, 15.6, 14.6, 13.9, 14.7, 15.1, 15.9, 13.9, 14.5

20. A city newspaper surveyed a sample of New South Wales residents on the issues most important to each age group for an upcoming state election.



- How many residents surveyed were between the ages of 18 and 40?
 - What percentage of over-40s surveyed listed tax and superannuation as the most important election issue? Give your answer to the nearest whole number.
 - What percentage of New South Wales residents thought health was the most important election issue? Give your answer to the nearest whole number.
21. A car dealership collects data about its customers to assist with the price range of cars that customers purchase. At right is a heat map representing the mean household income for the dealership's customers in different age groups and in different regions.
- What is the maximum mean household income for customers who live in the country?
 - What is the most common age group of people with a mean household income of \$60 001–\$90 000?



Mean household income

	Light Blue	Dark Blue	Dark Blue
0–\$30 000	\$30 001–\$60 000	\$60 001–\$90 000	\$90 000–\$120 000

Problem solving, reasoning and justification

22. It is suggested that people often do not like self-completion questionnaires, so clients should be interviewed either face to face or by telephone. What would be your advice? Discuss the advantages and disadvantages.
23. A factory produces 6400 mobile phones per week. Phones are randomly checked for defects and quality.
- What type of probability sampling method would be used?
 - What size sample would be appropriate?
 - Calculate the sampling interval.

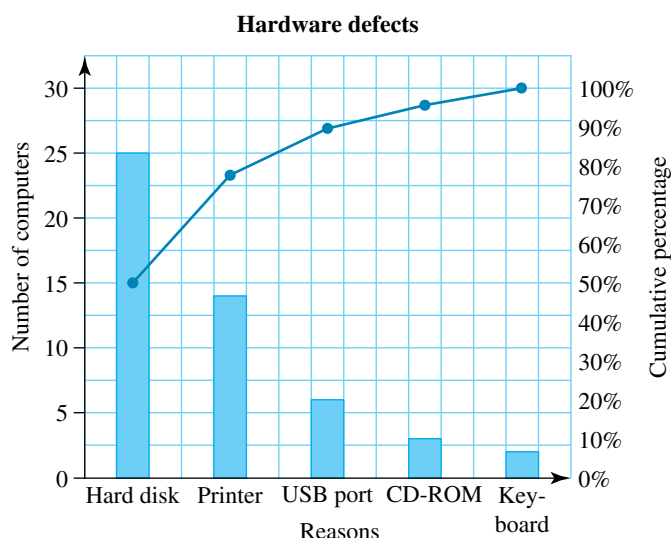
- 24.** Two market research companies have run similar surveys on comparable populations. They have used the same questionnaire.
 Company A sent out 1300 questionnaires and received 650 responses.
 Company B sent out 600 questionnaires and received 400 responses.
 Company A claims that their survey is better because they have a larger number of responses, thereby enabling them to calculate better estimates from the survey data. Comment on whether you think this claim is justified, giving your reasons.
- 25.** A database consists of 1000 records of three different types with the prefix A, B or C. The records are labelled as follows.
- A1–A500
 - B1–B100
 - C1–C400
- A sample of 100 records is required.
- a.** Explain why a simple random sample is not appropriate in this situation.
- b.** The following sample is drawn.
 A10 A20 A30 A40 A50 ... A480 A490 A500 (sample of size 50)
 B10 B20 B30 B40 B50 ... B80 B90 B100 (sample of size 10)
 C10 C20 C30 C40 C50 ... C380 C390 C400 (sample size of 40)
 How does this sample appear to be obtained? Are there any problems with this method?
- c.** How would you select an appropriate sample of 100 records?
- 26.** A study of technology consumption of New South Wales senior secondary students (Years 9–12) is to be conducted. Data will be collected using a questionnaire.
 The questionnaire begins with the following four questions.
- Q1. Are you: Male ☐ Female ☐
- Q2. Are you interested in current trends in technology?
 Yes ☐ No ☐
- Q3. What was the latest technology device that you purchased? _____
- Q4. What is your favourite technology brand?
 Apple ☐ HP ☐ Lenovo ☐ Acer ☐ Dell ☐
- a.** Classify the type of categorical data that will be collected in Q3 of the questionnaire.
- b.** Write a suitable question for this questionnaire that would provide numerical data.
- c.** The study is to be conducted using a stratified sample. How could a representative stratified sample be obtained?



27. A small computer shop recorded for the year the number of computers it had to return to a computer manufacturer for replacement due to hardware defects. The Pareto chart at right shows the hardware defects for the returned computers.

- How many computers were returned to the manufacturer for replacement?
- Which reason for hardware defects is the biggest concern?
- What is the cumulative percentage frequency value for printer defects?

28. The following data was collected from a group of ten Year 11 students about how each travelled to school.



Gender	Male	Male	Female	Female	Male	Female	Male	Female	Female	Female
Transport method	Car	Bus	Walk	Car	Walk	Bus	Bus	Car	Bus	Car

- Construct a frequency table for the data.
 - Select and draw a graphical display that is best suited to compare the data.
 - What percentage of Year 11 students were female and travelled to school by bus?
29. A juice bar offers the choice of soy milk and dairy milk in their smoothies. The smoothies sold on a particular day are shown in the table below, sorted by customer gender.

Type of milk	Gender	
	Male	Female
Dairy milk	54	78
Soy milk	4	16

- How many smoothies were sold on this day?
- Represent this data in a graphical display.
- What percentage of males chose soy milk for their smoothies? Give your answer to the nearest whole number.
- What percentage of smoothies sold contained dairy milk? Give your answer to the nearest whole number.
- If this was the daily trend of sales for the juice bar, what proportion of the juice bar's customers would you expect to be male? Give your answer to 2 decimal places.



30. A Morgan Gallup survey was conducted to investigate community views about the teaching of religion in New South Wales government schools. The question asked ‘Do you think religion should be taught once a week in government schools?’ The responses are summarised in the table below.

Survey response	Highest education level of participant		
	Primary level	Secondary level	Tertiary level
Agree	80	75	70
Disagree	16	24	25
Undecided	4	1	5

- a. Represent this data in an appropriate graphical display.
b. Comment on the survey responses.
31. The following data sets show the rental prices (in \$) of one-bedroom apartments in two different suburbs of Sydney.

Suburb A

275	275	281	285	284	310	315	310	296	307
300	300	242	295	281	305	290	305	295	300

Suburb B

235	225	231	232	240	280	300	310	295	275
275	285	245	305	270	255	270	228	241	285

- a. Draw a back-to-back stem-and-leaf plot to compare the data sets.
b. Compare and contrast the rental prices in the two suburbs.
The rental prices in a third suburb, suburb C, were also analysed.

Suburb C

335	325	351	335	340	360	300	390	395	285
357	385	354	305	375	345	270	358	340	365

- c. Compare the rents in the third suburb with the rents in the other two suburbs.

Answers

Exercise 11.2 Data collection methods

1. **a.** The population is the company's 1200 employees.
b. The sample consists of 120 employees from London, 180 employees from Melbourne, 45 employees from Milan and 75 employees from Japan. The total sample is 420 employees.
2. **a.** The population is the university's total student enrolment, which is 55 000.
b. The sample is made up of 250 city campus students and 45 country campus students. The total sample is 295 students.
3. **a.** 1240 **b.** 50
4. **a.** Systematic sampling **b.** 45 **c.** 45 **d.** 222
5. C 6. C 7. C 8. Strategy 1 9. C 10. A
11. **a.** Self-selected sampling
b. Answers will vary. The sample is made up of self-selection with people volunteering to respond. People who volunteer to respond will tend to have strong opinions. This can mean over-representation and may cause bias.
12. B
13. **a.** Random sampling **b.** Self-selected sampling
14. Answers will vary. A sample size should be sufficiently large and random. It should not be biased. The sample should be representative of the population. A sample size that is too small is less reliable.
15. Answers will vary but could include when the population is sufficiently small and all the members can be included, or when a census is conducted.
16. Answers will vary. In a census, everyone in the population is intended to be included. In a sample survey, only a subset of the population is included.
17. Answers will vary. The person surveyed could respond multiple times, and you don't know whether the person responding fits the criteria of the survey.
18. **a.** Experiment **b.** Observation **c.** Sample survey
19. **a.** 11 million **b.** 35%
20. Agree. Answers will vary. Most households do not have landlines. Younger individuals only have mobile phones. If younger people are not well represented in the surveys, the survey samples will be biased.
21. Answers will vary but could include:
Stratified sampling — the students can be divided into 20 groups based on the 20 regions. Students will be randomly selected from each group.
22. Answers will vary. A sample answer is given below.
It would be best for the hotel manager to post surveys to clients' homes and select a sample based on clients who book during a full year. This will give better representation and randomness, as leaving a questionnaire in a room will allow for self-selection. Also, clients who have grievances and want their opinions considered will be the majority who complete on site; this will cause bias. A two-month period will not give a complete representation of the hotel experience compared to a full year, in which clients are staying in the hotel in different seasons.
23. 24

Exercise 11.3 Classifying data and displaying categorical data

1. **a.** Numerical **b.** Categorical **c.** Categorical **d.** Numerical
2. **a.** Ordinal **b.** Continuous **c.** Nominal **d.** Discrete
3. Response to treatment
4. Model year
- 5.

Nominal	Ordinal	Discrete	Continuous
- Gender - Eye colour	Ability to play basketball	- Number of students in class (this data is numerical rather than ordered in groups) - Finish position in a race - Number of songs on an MP3 player (data cannot have an infinite number of values)	- Time taken to walk to school (data can have an infinite number of values) - Height

6. **a.** Categorical/nominal **b.** Column graph or pie chart

7. D

8. A

9. a. Categorical

b. Categorical, nominal

c. 26%

10. a. 87

b. Piano

c. The sample only contained 87 students. This could produce a biased sample and is unlikely to be representative of all Year 11 students in New South Wales.

11. a. 87 000

b. 68 000

c. Computing professionals

d. Computing professionals

12. E

13. a. Categorical

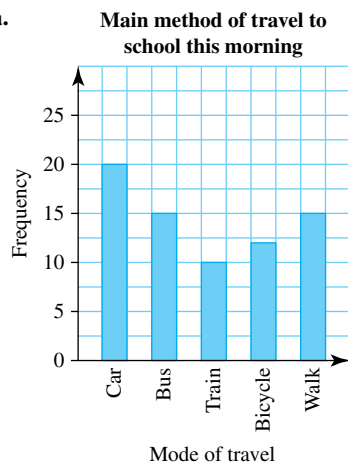
b. Very difficult to accurately analyse the result

c.

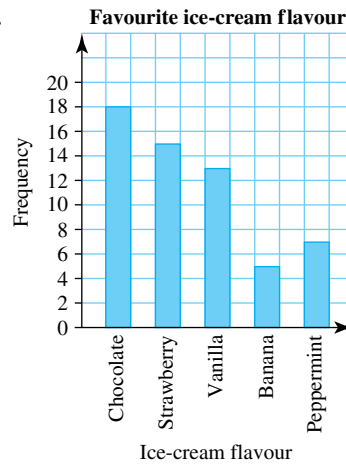
Sport	Frequency
Netball	9
Football	8
Cricket	6
Tennis	7
Basketball	7
Swimming	7
Golf	4
Lawn bowls	2

The popularity of most sports was evenly spread, with the exception of the low-scoring lawn bowls and golf.

14. a.



b.



15. Answers will vary. Examples are given.

a. Number of people in a cinema; number of security boxes at a bank

b. Weights of students in your class; distance each student lives from school

16. a.

Subject	Tally	Frequency (<i>f</i>)
Maths		4
Art		4
Cooking		3
PE		6
Science		5
English		3
Total		25

b. 16%

c. PE; 24%

d. Nominal (categorical)

17. a. Ordinal

b. Answers will vary. One possible answer is: How much money would you spend on a meal?

c. Self-selected sampling

18. a. Nominal

b. Answers will vary. One possible answer is: How many hours do you work a week?

c. Answers will vary. Students could be divided into four year levels to represent Year 9, Year 10, Year 11 and Year 12. Students could then be selected randomly from each of the four groups representing each year level.

19. a. Ordinal

b. Answer will vary. How much a month do you spend on your monthly mobile plan?

c. The students could be divided into six year levels to represent Year 7, Year 8, Year 9, Year 10, Year 11 and Year 12. Students could then be selected randomly from each of the six groups representing each year level.

20. a. 50

b. Fall

c. 80%

21. a. 200

b. Room temperature too cold

a. Answers will vary. The sampling method used by the bed and breakfast is biased as the online reviews are voluntary. Guests can submit multiple reviews, and dissatisfied guests are more likely to write a review and give their opinion.

Exercise 11.4 Organising and displaying data

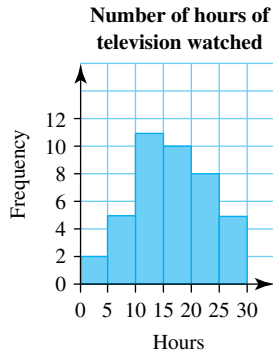
1. a. 2

b. 16–20

c. 0

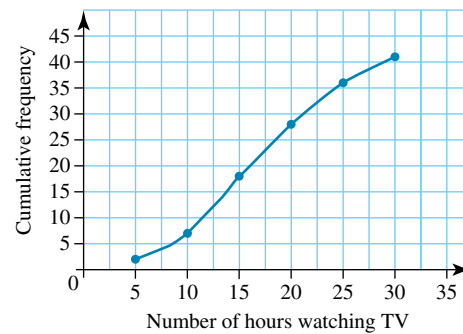
d. 24; find the height of each column and add the heights together.

2. a.

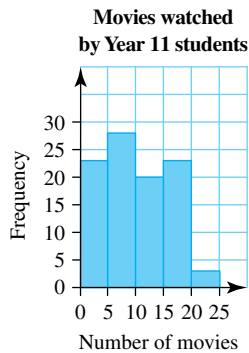


b.

Hours	Frequency	Cumulative frequency
0–<5	2	2
5–<10	5	7
10–<15	11	18
15–<20	10	28
20–<25	8	36
25–<30	5	41

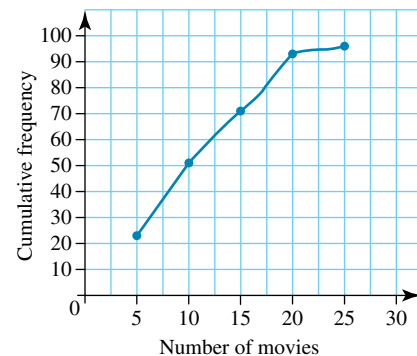


3. a.



b.

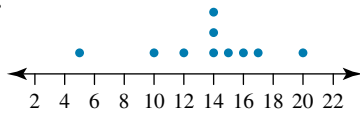
Number of movies	Frequency	Cumulative frequency
0–<5	23	23
5–<10	28	51
10–<15	20	71
15–<20	22	93
20–<25	3	96



4. a. 20 b. 6 c. 2 d. 4 e. 6
 5. a. 5 kg b. 4
 c. Yes. The same leaf appears more than once on the same row (stem).
 d. Count the number of leaves.

6. a. The most common result was $29 - < 31$. b. The least common result was $23 - < 25$.
 c. 33 d. 11 e. $27 - < 29$
 7. a. Continuous (numerical) b. The most common age bracket is $25 - < 30$.
 c. 5 d. 19

8.



9. a. Key: $12|9 = 129$

Stem	Leaf
10	8
11	6 7 8
12	9
13	0 1 2 7
14	5 6
15	0 2 2
16	4 5
17	1 2 3
18	9

b. Key: $1|2 = 12$

Stem	Leaf
0	0 1 2 3 6 4 7 8
1	0 2 2 2 2 4 5 6 6 7 8 9
2	1 2 5 7 8
3	1 1 2 2 3
4	0

10. a. Key: $27|9 = 27.9$

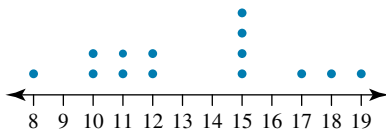
Stem	Leaf
13	0 8 9
14	5 5 7 8
15	1 2 7 9
16	2

b. Key: $7|9 = 7.9$

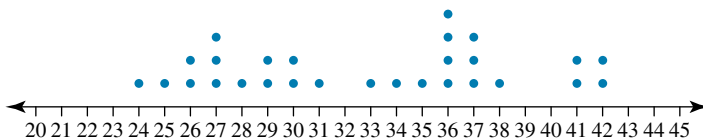
Stem	Leaf
1	8
2	5 7 8 9
3	5
4	1
5	2 7
6	2 6

11. a. Dots are not aligned vertically. The horizontal scale is not equally spaced.

b.

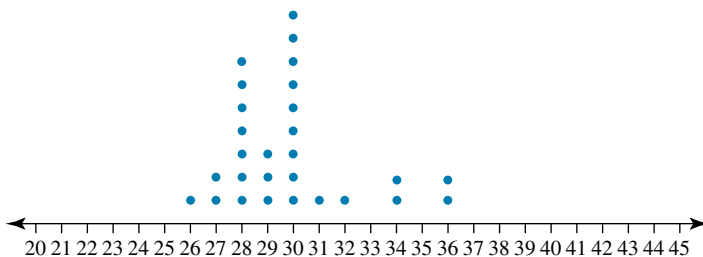


12. a.



b. There is a wide spread of temperatures.

13. a.



b. The data is clustered between 28°C and 30°C for the Gold Coast, compared to a large range of temperatures in Sydney.

- c. Sydney's temperatures were generally higher and with a wider range, but the Gold Coast recorded more consistent temperatures.

14. a. The stem represents the hour and the leaves represent the minutes.

b. 8:25 am

c. This type of timetable enables the viewer to see times at a glance, and it saves space.

15. a. i. 1 ii. 41

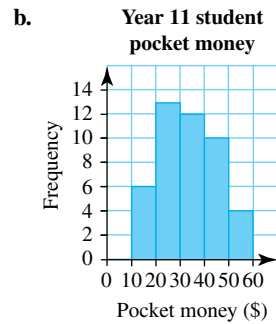
iii. There are no numbers between 10 and 19 in the data set.

iv. Discrete (numerical) — whole numbers

b. Continuous (numerical)

16. a.

Pocket money	Tally	Frequency (<i>f</i>)
0–<10		0
10–<20		6
20–<30		13
30–<40		12
40–<50		10
50–<60		4
Total		45

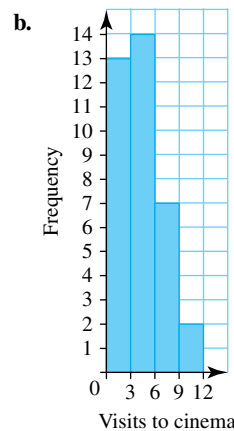


c. Most students had between \$20 and \$50, with none over \$50, and none less than \$10.

17. a. Answers will depend on class intervals chosen.

An example is given.

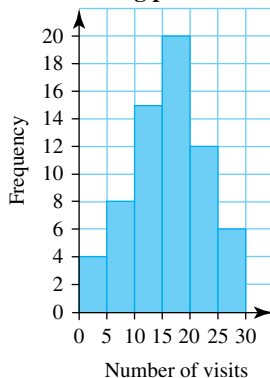
Number of visits to the cinema	Tally	Frequency (<i>f</i>)
0–2		13
3–5		14
6–8		7
9–11		2
Total		36



c. 13

d. 23

18. a. Number of visits to the local swimming pool in summer



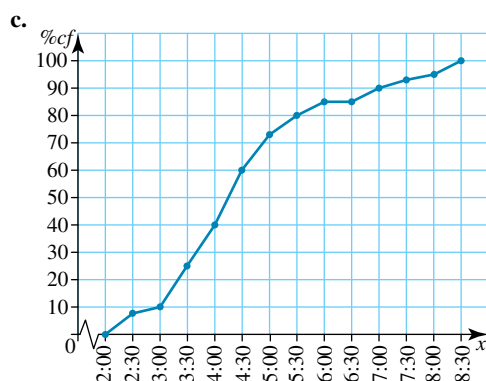
b. This data is discrete because it is made up of numerical values that can be only whole numbers.

19. a.

Class intervals	Frequency
2:00–<2:30	3
2:30–<3:00	2
3:00–<3:30	8
3:30–<4:00	8
4:00–<4:30	9
4:30–<5:00	7
5:00–<5:30	2
5:30–<6:00	4
6:00–<6:30	0
6:30–<7:00	2
7:00–<7:30	2
7:30–<8:00	1

b.

Class intervals	Frequency	Cumulative frequency (<i>cf</i>)	Percentage cumulative frequency (% <i>cf</i>)
2:00–<2:30	3	3	6%
2:30–<3:00	2	5	10%
3:00–<3:30	8	13	26%
3:30–<4:00	8	21	42%
4:00–<4:30	9	30	60%
4:30–<5:00	7	37	74%
5:00–<5:30	2	39	78%
5:30–<6:00	4	43	86%
6:00–<6:30	0	43	86%
6:30–<7:00	2	45	90%
7:00–<7:30	2	47	94%
7:30–<8:00	1	48	96%
8:00–<8:30	2	50	100%



Exercise 11.5 Comparing data

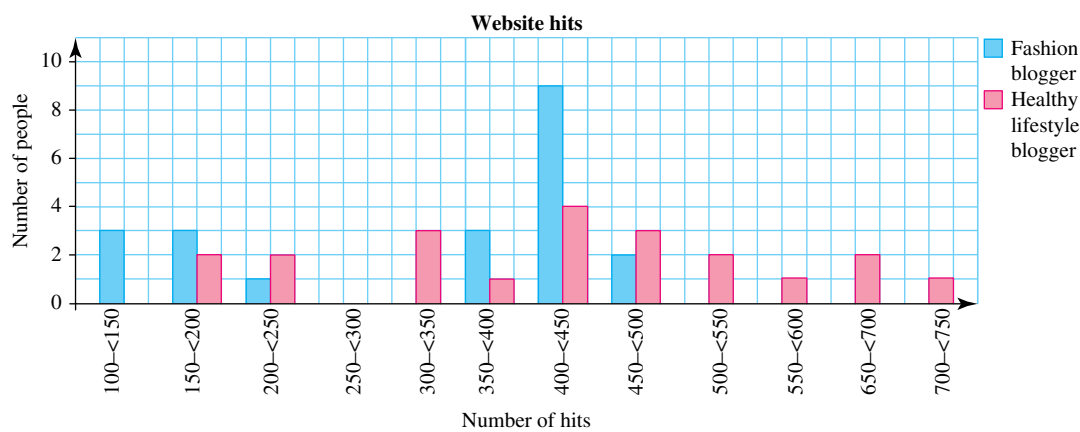
1. a. Class 1: 25; Class 2: 27 b. Class 1: 2; Class 2: 3 c. Class 1: 24; Class 2: 20
 d. Eleven students in class 2 scored better than class 1 but 6 students scored worse than class 1. Class 2's results are more widely spread than class 1's results.
2. Key: 1|6 = 16 years

Leaf (female)	Stem	Leaf (male)
987766	1	5566677789
9853332110	2	1122233578
8864221	3	00145
553	4	368
2	5	047
0	6	5

3. Brand B seems to have a better battery life, as Brand A has more batteries that have a battery life of less than 10 hours. Brand B has a battery life of up to 75 hours, which is higher than Brand A's battery life maximum of 61 hours.
4. B
5. B
6. B
7. a. Side-by-side bar chart b. 203 c. Tax and superannuation d. 9%
8. a. Worker F b. 4
9. a. 14 b. True
10. a. True b. True c. False

11. a.

Number of hits	Fashion blogger	Healthy lifestyle blogger
100–<150	3	0
150–<200	3	2
200–<250	1	2
250–<300	0	0
300–<350	0	3
350–<400	3	1
400–<450	9	4
450–<500	2	3
500–<550	0	2
550–<600	0	1
600–<650	0	2
650–<700	0	0
700–<750	0	1



b. Answers will vary. Example answer:

The fashion blogger had 400–<450 hits on her website on 9 days in the 3-week period. The healthy lifestyle blogger got a wider spread of hits. He received over 400 hits for 11 days in the 3-week period. His website seems to be gaining in popularity as its spread falls over a higher maximum and minimum than the spread of the fashion blog.

12. a. Key: $9|69 = 9.69$

Leaf: Female	Stem	Leaf: Male
	9	69
	9	84 85 87 90 92 96 99
	10	00 06 14 20 25 30 30 30 40
97 94 93 82 75 54	10	50 80
40 08 07 00 00	11	
90 90 60 50 50 50	11	
20	12	

b. Answers will vary. There is not a large difference within each gender, but there is a large difference in time between the two genders.

13. a. Key: $23|1 = \$231$

Leaf: Suburb A	Stem	Leaf: Suburb B
7 5 5 0 0 0	20	
5 5 1 0 0	21	
5 5	22	
5 5 1	23	1 2 5
4 2 0	24	0 5 5
6	25	1 5 5 8
	26	0 5
	27	0 0 5 5
	28	0 0 5 5

b. Answers will vary. Suburb A has lower rent than Suburb B. Suburb B is a more expensive suburb to rent an apartment.

14. a. 70+ age group. Answers about the reason will vary. Elderly people don't drive as much and tend to walk or be passengers.

b. Answers will vary. The road campaign could focus on:

- elderly people as pedestrians — being alert when crossing a road
- motorcyclists in the 30–49 age group, not on probationary licence — risk taking
- driving for all age groups — awareness and safety for all drivers.

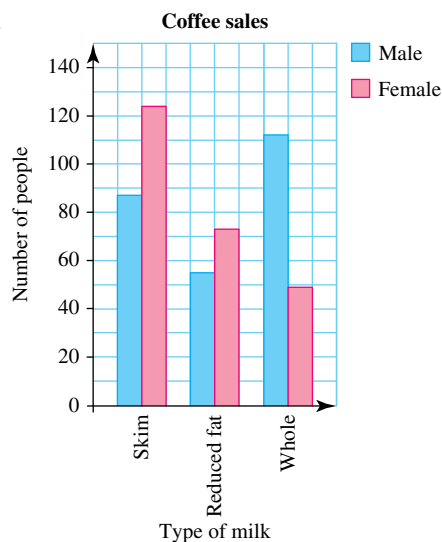
15. a. 2016

b. 2016

c. Answers will vary but could include that more people are on the roads during the Easter break and school holidays.

16. a. 500

b.



c. 34%

d. 26%

e. 49%

Exercise 11.6 Review

1. a. i. Numerical ii. Continuous

b. i. Categorical ii. Nominal

c. i. Numerical ii. Discrete

d. i. Numerical ii. Continuous

e. i. Categorical ii. Ordinal

f. i. Categorical ii. Nominal

2. C

3. Age and number of languages spoken

4. False

5. A

6. A

7. E

8. True

9. Sample size = $\sqrt{N}$, where N is the size of the population.

10. a. True

b. True

c. True

11.

Years	Frequency
0–9	0
10–19	3
20–29	8
30–39	10
40–49	6
50–59	5
60–69	3
70–79	2
80–89	1

12. a.

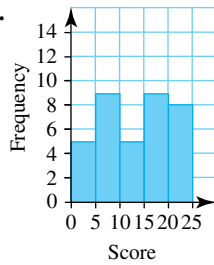
Score (x)	Frequency (f)
1	1
2	1
3	1
4	2
5	1
6	1
7	2
8	2
9	3
10	3
11	1
12	0
13	1

Score (x)	Frequency (f)
14	0
15	1
16	1
17	2
18	1
19	4
20	1
21	4
22	1
23	1
24	1
Total	36

b.

Score (x)	Frequency (f)
0–<5	5
5–<10	9
10–<15	5
15–<20	9
20–<25	8
Total	36

13.



14. Sydney. Difference: $132.4 - 112.7 = 19.7$

15.

Weights of people (kg)	Frequency (f)
0–<10	2
10–<20	4
20–<30	3
30–<40	8
40–<50	9
50–<60	2
Total	28

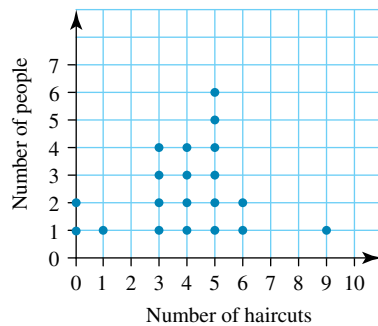
16. a. Bar graph

b. Reading

c. 175

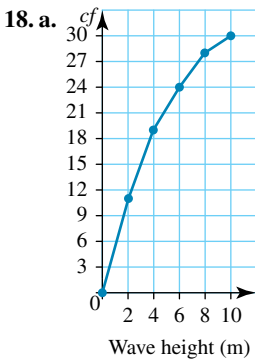
17. a.

Number of haircuts in the past 6 months

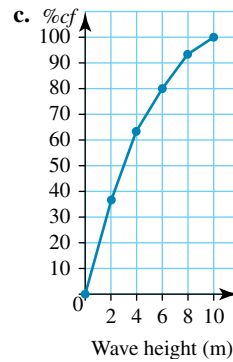


b. 5

c. Centred around 3 to 6 haircuts, with a few outliers



b. 24



19. a. Key: 2|1 = 21

Stem	Leaf
2	1 1 5 5 8 9 9
3	1 1 3 7 9
4	1 1 6 7
5	1 3 3 4 8
6	2 2 2 2 7 7 8
7	1 2 3 3 8
8	1 2
9	2 3 3

b. Key: 7|8 = 78

Stem	Leaf
7	8
8	
9	
10	8
11	6 7
12	9
13	2 7
14	5
15	2
16	4 5
17	2
18	9

c. Key: 13|8 = 13.8

Stem	Leaf
13	0 8 9 9
14	3 5 5 6 7 7 8
15	1 2 6 7 9
16	2

20. a. 237

b. 28%

c. 17%

21. a. \$60 000

b. >65 years

22. Answers will vary. Example answer:

An advantage of using interviews as a data collection method is that interviewers can repeat or explain the meaning of questions to respondents face to face if necessary, minimising errors in the data collection or incomplete responses.

A disadvantage is that some respondents may feel intimidated and under pressure to answer quickly. They may give biased results based on the tone or the behaviour of the researcher. Respondents can pace themselves when completing a questionnaire and will have time to think and reflect about their responses.

23. a. Systematic sampling

b. 80 mobile phones

c. 80

24. Answers will vary. Example answer:

Company A received $\frac{650}{1300} = 50\%$ responses.

Company B received $\frac{400}{600} \approx 67\%$ responses.

Company B received responses from approximately 67% of the population they surveyed, whereas Company A only received responses from 50% of their population. Company B will have better estimates of their survey to represent their population.

25. a. Answers will vary. The records are already classified into three groups. A random sample of the entire population may not represent all three groups.

b. Answers will vary. The records are already classified into three groups. 10% of each group has been sampled. The sampling interval is 10. The starting point of each group has not been randomly selected; each group starts at the same point (the tenth record in that letter category).

c. Answers will vary. Since the records are already classified into three groups, stratified sampling would be the best process. To select a sample from each group, randomly select 50 from record A, 10 from record B and 40 from record C. The number of selections is based on the proportional size of each group.

26. a. Nominal

b. Answers will vary. One possible answer is: How much would you spend on a device?

c. Answers will vary. Students could be divided into four year levels to represent Year 9, Year 10, Year 11 and Year 12. Students could then be selected randomly from each of the four groups representing each year level.

27. a. 50

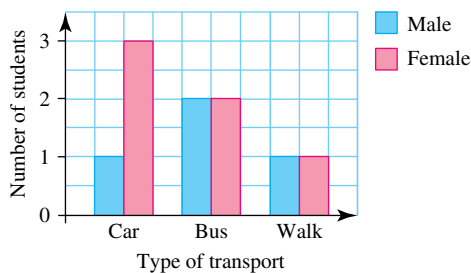
b. Hard disk

c. 78%

28. a.

Transport	Gender	
	Male	Female
Car	1	3
Bus	2	2
Walk	1	1

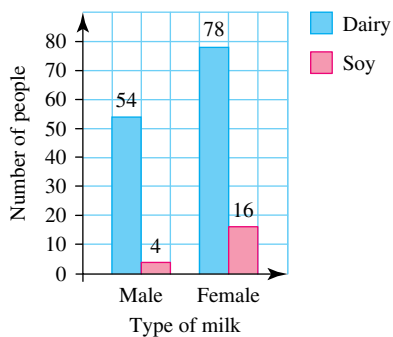
b.



c. 20%

29. a. 152

b.

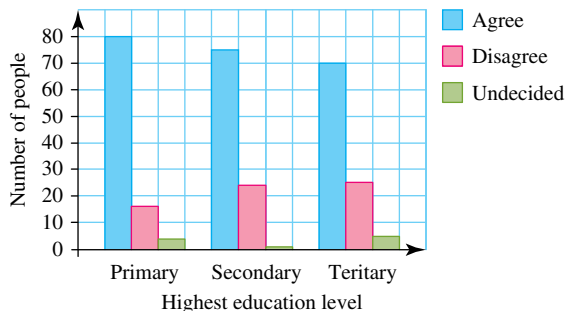


c. 7%

d. 87%

e. 0.38

30. a.



b. Answers will vary. The majority of people surveyed agree with religion being taught. The proportions of responses is similar in each highest education level group.

31. a. Key: 22|5 = 225

Suburb A	Stem	Suburb B
	22	5 8
	23	1 2 5
2	24	0 1 5
	25	5
	26	
5 5	27	0 0 5 5
5 4 1 1	28	0 5 5
6 5 5 0	29	5
7 5 5 0 0 0	30	0 5
5 0 0	31	0

b. Answers will vary. Suburb A has higher rent than in suburb B.

c. Answers will vary. Suburb C is higher than suburb A, which is higher than suburb B. The most expensive suburb to rent is suburb C. The least expensive suburb to rent a one-bedroom unit is suburb B.

TOPIC 12

Data analysis: univariate data

12.1 Overview

12.1.1 Introduction

Data collection and analysis has become a major focus of governments and businesses in the 21st century. How to effectively handle the data collected is one of the biggest challenges to overcome in the modern world.

Analysing and comparing data allows us to better understand the data, and with that to better understand the world around us. By analysing data we can improve our knowledge of effective medicine, the weather in the world, and the way artificial intelligence works.



DISCUSSION

How do you think artificial intelligence will affect the way we live in the 21st century?

LEARNING SEQUENCE

- 12.1** Overview
- 12.2** Measures of central tendency
- 12.3** Measures of spread
- 12.4** Comparing graphical distributions
- 12.5** Parallel boxplots
- 12.6** Review

CURRICULUM CONTENT

Students:

- describe the distinguishing features of a population and sample ◇
- summarise and interpret grouped and ungrouped data through appropriate graphs and summary statistics (ACMGM030) **AAM** ◇
- investigate and describe the effect of outliers on summary statistics ◇
- investigate real-world examples from the media illustrating appropriate and inappropriate use or misuse of measures of central tendency and spread (ACMEM056) **AAM**
- describe, compare and interpret the distributions of graphical displays and/or numerical datasets and report findings in a systematic and concise manner (ACMGM029, ACMGM032) **AAM** ◇ ⚙️ 📊 📖
- construct and compare parallel box-plots (ACMEM059) **AAM** 📊

12.2 Measures of central tendency

12.2.1 Summary statistics

- In statistics, a population is a large group of subjects or objects being studied or investigated.
- A sample is a smaller selection of subjects or objects taken from the population.
- Statistical analysis on a sample can often be used to make generalisations about the population, as long as the subjects or objects in the sample are selected at random.
- Values associated with populations are called **parameters**, whereas those associated with samples are called **summary statistics**.

The mean

- The **mean** of a population is a theoretical measure of the centre of the entire population. It is not always a value in a data set; for example, the mean number of dogs in an Australian household is 1.5.
- Obtaining the mean of a population is not always practical, so the mean of a sample is often calculated instead.
- The mean of a population, μ , and the mean of a sample, $\bar{x}$, are calculated using the same formula:

$\mu = \bar{x} = \frac{\sum x}{n}$, where x is a data value, n is the total number of data values and $\sum$ is the summation symbol.

- The summation symbol, $\sum$ (the Greek capital letter sigma), means to add all the occurrences of the formula to the right of the summation symbol.

The median

- The **median** is the middle value of the data set by position. It is the value that half the observations are less than and half are greater than.
- The median is also a *theoretical measure* of the centre of the set of data. It is not always a value in the set of data.
- To calculate the median, arrange the values in the set of data in *ascending* order.
 - For an *odd* number of values, the median is the *middle* value. For example, if the data is 6, 7, 9, 10 and 15, then the median is 9.
 - For an *even* number of values, the median is the *mean* of the two middle values. In this case the median may not be an actual data value. For example, if the data is 3, 5, 7, 9, 11 and 17, the median is 8.
- If the set of data contains n values, the position of the median when the values are arranged in numerical order can be calculated using the formula:

$$\text{median position} = \frac{n + 1}{2}.$$

$$\begin{aligned} \text{If } n = 6, \text{ then } \frac{n + 1}{2} &= \frac{6 + 1}{2} \\ &= \frac{7}{2} \\ &= 3.5 \end{aligned}$$

The median is between the third and fourth values.

$$\begin{aligned} \text{If } n = 5, \text{ then } \frac{n + 1}{2} &= \frac{5 + 1}{2} \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

The median is the third value.

6, 7, 9, 10, 15
 ↑
 Median = 9
 3, 5, 7, 9, 11, 17

$$\begin{aligned} &\uparrow \\ \text{Median} &= \frac{7 + 9}{2} = 8 \end{aligned}$$

The mode

- The **mode** is the value that occurs most often. It is often described as the typical value of the observations.
- The mode can be found for both numerical and categorical data.

- Some sets of data have no mode at all. If all of the values occur only once, there is no mode.
- It is possible for a data set to have more than one mode. A data set with two modes is referred to as bimodal. A data set with three modes is referred to as trimodal. A data set with more than three modes is referred to as multimodal.

WORKED EXAMPLE 1

Ten Year 11 students were asked how many hours they spent competing in sport out of school hours. The results of the survey are: 2, 4, 5, 6, 7, 2, 2, 0, 5, 1. Calculate the values of:

- a the mean b the median c the mode
and explain what they tell us about the data.

THINK

- a 1 Share the value of the data equally. Find the sum of all the values.
- 2 Count the number of values (the 0 must be included).
- 3 The mean is the sum of all the values divided by the number of values.

WRITE

Sum of all the values
 $= 2 + 4 + 5 + 6 + 7 + 2 + 2 + 0 + 5 + 1$
 $= 34$

Number of values = 10

$$\text{Mean } (\bar{x}) = \frac{\text{sum of all the values}}{\text{number of values}}$$

$$= \frac{34}{10} = 3.4$$

If the total number of hours spent competing in sport was shared equally between all the students, each student would spend 3.4 hours competing in sport.

- b 1 The median is the middle value. Arrange the values in ascending order.
- 2 There are 10 values in the set of data so the median is the $\frac{10 + 1}{2} = 5.5$ th value (mean of the 5th and 6th values).

0, 1, 2, 2, 2, 4, 5, 5, 6, 7

0, 1, 2, 2, ②, | ④, 5, 5, 6, 7

$$\text{Median} = \frac{2 + 4}{2} = \frac{6}{2} = 3$$

Half of the students do more than 3 hours and half of the students do less than 3 hours.

- c 1 The mode is the most common value in the set of data.

②, 4, 5, 6, 7, ②, ②, 0, 5, 1

Mode = 2

The most common amount of time that students spend competing in sport is 2 hours.

12.2.2 Measures of central tendency in real-world contexts

- The mean is often called the average in real-world contexts. It is used in scientific research to describe many different phenomena, from weather patterns to sports statistics. The mean is also used to analyse the results of students in schools and universities. Examples include: average number of children per household in Australia, average daily rainfall, average number of goals scored in a game, and average mark on a mathematics exam.
- The median is the middle value of a data set by position, and is often used when the information being investigated has some extremely high or extremely low values in the data set. The median is not greatly affected by



these extreme values, as it looks at the number of data points, not their values. For this reason, the median is often used in economics. Examples include median household income and median house price.

- The mode is less commonly used in real-world contexts, but it can still be a practical and useful way of describing information. For example, clothing shop owners will look at the highest-selling clothing size to make sure they have the most floor stock in that particular size.
- Always remember to relate summary statistics back to real-world contexts, ensuring they have relevance and meaning. For example, a shoe-shop owner might work out that she sells a mean of 26.7 pairs of shoes a day. Obviously there cannot be 0.7 of a shoe, so the shop owner would round this up to 27 pairs of shoes a day.

12.2.3 Grouped data

- For large data sets, it is necessary to group data into smaller sets of data called **class intervals**. These class intervals must be the same size and must be set so that each value belongs to one interval only.
- Class intervals are written differently for different types of data. A class interval for:
 - discrete data is written showing the minimum and maximum values for the interval. For example, 10–20 includes the values between and including 10 and 20, for example 12 or 16.
 - continuous data is written like an inequation. For example, for values of 10 to less than 20 ($10 \leq x < 20$), the class interval is written 10–<20.
- A frequency table shows the frequency of values appearing in each class interval.
- The class interval with the highest frequency is the mode of the grouped data and is called the **modal class**.
- The **midpoint** of the class interval is the average of the maximum and minimum values for the class interval.
- The midpoint is used as the representative value for the class interval. It is assumed that half of the data values will be greater than the midpoint and half will be less than the midpoint.
- The mean of grouped data is calculated using the midpoint of each class interval to represent the data values in the interval.

$$\bar{x} = \frac{\sum xf}{n}$$

where:
 x is the midpoint of the class interval
 f is the frequency of the class interval
 n is the total number of data values
 $\sum$ is the summation symbol.

WORKED EXAMPLE 2

A takeaway food shop is trying to improve its ordering system. The manager takes a survey of the number of hamburgers made each day for a month and collects the data shown in the frequency table. Calculate the mean number of hamburgers made each day.

Class interval	Frequency (f)
0–4	6
5–9	8
10–14	2
15–19	3
20–24	5

THINK

- To calculate the mean of grouped data, a single value is needed to represent the class interval.
 - To calculate the midpoint of the class intervals, calculate the mean of the maximum and minimum values for each class interval.
 - For the first class interval 0–4, $x = \frac{0 + 4}{2} = 2$, as shown in black.
 - For the second class interval 5–9, $x = \frac{5 + 9}{2} = 7$, as shown in red.
- The midpoint will be the representative value for the interval.
 - Add a new column, xf , and multiply the midpoint by the frequency to calculate a value to represent the sum of all values in that class interval.
 - For the first class interval 0–4, $xf = 6 \times 2 = 12$, as shown in green.
 - Calculate the total sum of all the data values by totalling the xf column, as shown in pink.
 - Calculate the total number of data values by totalling the frequency column, as shown in purple.
- Substitute the values found into the formula for the mean of grouped data.
- Since the mean is a theoretical measure, it is necessary to check whether the answer makes sense. It is not possible to make 10.54 hamburgers each day, so the realistic answer would be the rounded value of 11 hamburgers.

WRITE

Class interval	Frequency (f)	Midpoint (x)
0–4	6	2
5–9	8	7
10–14	2	12
15–19	3	17
20–24	5	22

Class interval	Frequency (f)	Midpoint (x)	xf
0–4	6	2	12
5–9	8	7	56
10–14	2	12	24
15–19	3	17	51
20–24	5	22	110
Totals	24		253

$$\begin{aligned}
 \bar{x} &= \frac{\sum xf}{n} \\
 &= \frac{253}{24} \\
 &\approx 10.54 \text{ (2 decimal places)}
 \end{aligned}$$

It is not possible to make 10.54, so the mean number of hamburgers made each day was 11.

12.2.4 Outliers

- Outliers are extreme values on either end of a data set that appear very different from the rest of the data.
- Outliers can affect the summary statistics for a sample.
- The median is calculated using the number of data points, not the value of each data point. Therefore, outliers do not have much of an influence on the median.
- The mean involves a summation of the values of the data points, so extreme high or low values can greatly affect the mean.
- If outliers are present in a data set, the median is a better measure of central tendency to use, as it is less affected by outliers.

WORKED EXAMPLE 3

The number of times employees in a small company took sick days per year was recorded.

The results were as follows:

5, 2, 4, 3, 26, 1, 1, 6, 4, 29, 0, 2, 1

a Which, if any, of these values could be outliers?

b For the data set, calculate:

- i the mean ii the median.

c Exclude the outlier(s) in the data set and calculate:

- i the mean ii the median.

d Comment on the difference in results between parts b and c.

THINK

a 1 Arrange the values in ascending order.

2 Outliers are extreme values on either end of the data set.

b i 1 Find the sum of all the values.

2 Count the number of values.

3 The mean is the sum of all the values divided by the number of values.

ii 1 Arrange the values in ascending order. The median is the middle value.

2 There are 13 values, so the median is the $\frac{13+1}{2} = 7$ th value.

c i 1 Write out the data set excluding the outliers.

Find the sum of all the values.

2 Count the number of values.

3 The mean is the sum of all the values divided by the number of values.

ii 1 Arrange the values in ascending order. The median is the middle value.

2 There are 11 values, so the median is the $\frac{11+1}{2} = 6$ th value.

d 1 Comment on the difference between the mean in parts b and c.

WRITE

a 0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6, 26, 29

0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6, 26, 29
26 and 29 are possible outliers.

$$5 + 2 + 4 + 3 + 26 + 1 + 1 + 6 + 4 + 29 + 0 + 2 + 1 = 84$$

Number of values = 13

$$\bar{x} = \frac{84}{13} = 6.5$$

The mean number of sick days employees took in a year was 6.5.

0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6, 26, 29

0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6, 26, 29

The median number of sick days employees took in a year was 3.

0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6

$$5 + 2 + 4 + 3 + 1 + 1 + 6 + 4 + 0 + 2 + 1 = 29$$

Number of values = 11

$$\bar{x} = \frac{29}{11} = 2.6$$

The mean number of sick days employees took in a year, when excluding outliers, was 2.6.

0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6

0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6

The median number of sick days employees took in a year, when excluding outliers, was 2.

The mean sick days per year taken by employees reduced from 6.5 days to 2.6 days when outliers were excluded. Excluding the large outliers significantly decreased the mean.

2 Comment on the difference between the median in parts **b** and **c**.

The median sick days per year taken by employees changed from 3 days to 2 days when outliers were excluded.

3 Make a conclusion about the effect of outliers on the mean and median.

The outliers had a greater effect on the mean than on the median.

eBookplus RESOURCES

eLesson: Mean and median (eles-1905)

Exercise 12.2 Measures of central tendency

Understanding, fluency and communicating

Note: Where necessary give answers correct to 1 decimal place.

1. **WE1** For each of the following data sets, find the:

i. mean

ii. median

iii. mode.

a. 3, 3, 4, 5, 5, 6, 6, 7, 8, 8, 8, 9

b. 12, 18, 4, 17, 5, 12, 0, 10, 12

c. 42, 29, 11, 28, 21

d. 8, 2, 5, 6, 9, 9, 7, 3, 2, 9, 3, 7, 6, 8

e. 5, 5, 6, 4, 8, 3, 4

f. 3.7, 3.5, 3.8, 3.8, 3.5

2. At a preview cinema session, the ages of the viewers were recorded and displayed in a stem-and-leaf plot (as shown). For this data, find the:

a. mean

b. median

c. mode.

Key: 1|6 = 16 years

Stem	Leaf
1	5 6 7 7 8 9 9
2	1 2 4 8 8
3	0 1 1 1 5
4	2 3
5	3

3. The police conducted a survey of the speed of cars down a highway. The lowest and highest speeds recorded were 91 km/h and 154 km/h respectively. The average speed was 104 km/h, and the police found that the speed most commonly recorded was 101 km/h. Half of the cars were also found to be travelling under 102 km/h. State the value of the mean, median and mode.

4. The number of students standing in line at the canteen 5 minutes after the start of lunch was recorded over a 2-week period. The results were as follows: 52, 45, 41, 42, 53, 45, 47, 32, 52, 56. What was the mean number of students standing in line at this time for this 2-week period? Round your answer to the nearest whole number.

5. For each of the following data sets, calculate the mean, median and mode.

a. 10, 12, 21, 23, 23, 25, 44

b. 7, 8, 10, 6, 9, 11, 4, 12, 2

c. 50, 44, 50, 46, 50, 48

d. 2.5, 1.4, 1.7, 2.1, 1.4, 1.8, 1.6, 1.7, 2.9



6. Calculate the mean, median and modal scores in each of the following frequency tables.

a.

Score (x)	Frequency (f)
1	12
2	10
3	8
4	7
5	2

b.

Score (x)	Frequency (f)
25	1
26	15
27	11
28	7
29	3

c.

Score (x)	Frequency (f)
1.5	2
2.0	9
2.5	7
3.0	11
3.5	4

7. **WE2** Calculate the mean number of DVDs presented in this frequency table, using the median of each interval as the x -value for the interval.

Number of DVDs (x)	Frequency (f)
1–15	3
16–30	9
31–45	8
46–60	11
61–75	10
76–90	14
91–105	15
106–120	18

8. Calculate the mean number of calls made on mobile phones in the month shown in the graph at right.

9. **MC** Which of the following are the mean, median and mode, respectively, of this data set?

1024, 1032, 1067, 1112, 1112, 1178, 1236, 1269, 1290, 1301
1345, 1357, 1365, 1377, 1400

A. 1269, 1112, 1231

B. 1231, 1269, 1112

C. 1112, 1231, 1269

D. 1231, 1112, 1269

E. 1024, 1127, 1400

10. **WE3** Below is the data collected during a survey of the number of pets people own.

2, 1, 0, 3, 1, 1, 2, 1, 2, 3, 3, 2, 0, 2, 1, 2, 3, 4, 1, 2, 1, 1, 3, 3, 1, 3, 2, 1, 2, 2, 2, 3, 21, 1, 2, 3

a. Calculate the mean and median.

b. Are there any outliers?

c. Remove any outliers and recalculate the mean and median.

d. Comment on the difference in means and medians in parts a and c.

Problem solving, reasoning and justification

11. a. Write the values that are represented by the dot plot at right.

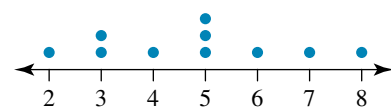
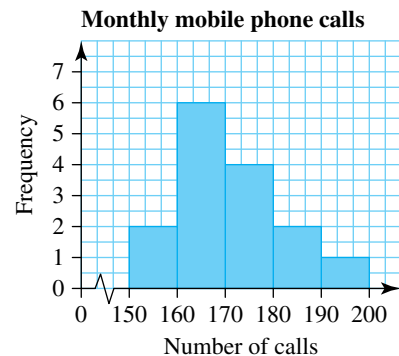
b. Use your data set to find the value of:

i. the mean

ii. the median

iii. the mode.

c. Explain how you could calculate summary statistics directly from a dot plot.



12. The number of goals a netballer scored in the 12 games of a season was as follows:
1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 8, 12.
A local newspaper reporter asked the netballer what his average was for the season.
- Which measure of centre (mean, median or mode) should the netballer give the reporter as his 'average' so that the value of the average is as high as possible?
 - Which measure of centre would *you* choose to best describe the 'average' number of goals the netballer scored each game? Why?
13. Create a data set that fits each of the following descriptions.
- Five data values with a mean of 3 and a mode of 3
 - Five data values with a mean of 3 and a mode of 4
 - Five data values with a mean of 3 and a median of 2
14. The mean length of three pieces of string is 145 cm. If they are joined together from end to end, what will their total length be?
15. The median mark for a Science test was 45. No student actually achieved this result. Explain how this is possible.
16. The number of M&M's in 20 packets was recorded by a group of students.
- Calculate the mean number of M&M's per packet.
 - Calculate the median number of M&M's per packet.
 - Calculate the modal number of M&M's per packet.
 - Based on your calculations, how many M&M's would you expect to find in a packet?
 - If you were advertising M&M's, which measure of centre would you use? Explain.
17. Explain how the mean, median and mode for both ungrouped and grouped data are similar and different.
18. a. Calculate the mean and median of the two data sets below.
Data set A: 20, 24, 29, 33, 37, 42, 51, 53, 96
Data set B: 20, 24, 29, 33, 37, 42, 51, 53, 66
- Comment on the means and medians of the data sets and explain any similarities and differences you see.
 - Write a short statement to explain when to use the mean as a measure of central tendency and when to use the median.
19. **MC** The mean of a data set was calculated to be 117. The median was calculated to be 107. The data set has two outliers of 178 and 190. If the outliers were replaced with values of 118 and 120:
- the mean and median would remain the same.
 - the median would increase, but the mean would stay the same.
 - the mean would increase, but the median would remain the same.
 - the median would decrease, but the mean would stay the same.
 - the mean would decrease, but the median would stay the same.
20. The Mathematics test results for two students over a year were recorded as follows.
Student A: 49, 52, 51, 50, 54, 49, 100
Student B: 65, 71, 64, 63, 60, 81, 0
- Calculate the mean and median for the test results for each student.
 - Student B received a mark of 0 as they were away for one of the tests. Explain how this affects the mean and median for Student B.
 - Who do you think is the better mathematics student? Justify your answer.

30, 32, 33, 35, 37, 37, 38,
38, 39, 39, 40, 40, 40, 41,
41, 41, 41, 41, 41, 50



12.3 Measures of spread

12.3.1 The range, quantiles and interquartile range (IQR)

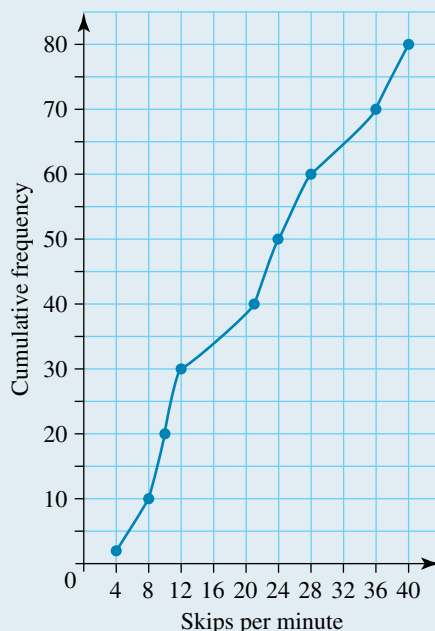
- The **spread** of a set of data indicates how far the data values are spread from the centre or from each other. This is also known as the distribution.
- The range is the difference between the highest and lowest values of the data set.
- Data sets can be split up into any given number of equal parts called **quantiles**.
- Quantiles are named after the number of parts that the data is divided into.
- **Deciles** divide the data into 10 equal-sized parts.
- Percentiles divide the data into 100 equal-sized parts.
- **Quartiles** divide the data into 4 equal-sized parts. For example, 25% of the data values lie at or below the first quartile.

Percentile	Quartile and symbol	Common name
25th percentile	First quartile, Q_1	Lower quartile
50th percentile	Second quartile, Q_2	Median
75th percentile	Third quartile, Q_3	Upper quartile
100th percentile	Fourth quartile, Q_4	Maximum

- A percentile is named after the percentage of data that lies at or below that value. For example, 60% of the data values lie at or below the 60th percentile.
- Percentiles can be read off a percentage cumulative frequency curve.

WORKED EXAMPLE 4

The following cumulative frequency graph shows the cumulative frequency versus the number of skips with a skipping rope per minute for Year 11 students at a school.



Calculate:

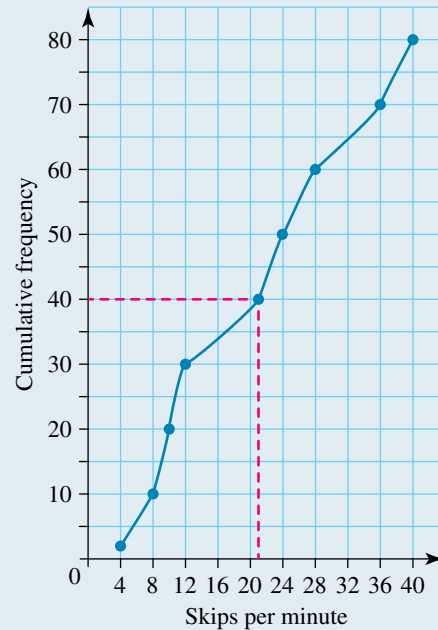
- a the median b Q_1 c Q_3
d the number of skips per minute for the 40th percentile.

THINK

- a** 1 Determine the total number of subjects. This is the highest vertical value on the cumulative frequency curve.
- 2 The median occurs at the middle of the total cumulative frequency, so divide the total frequency by 2.

WRITE

a 80 students

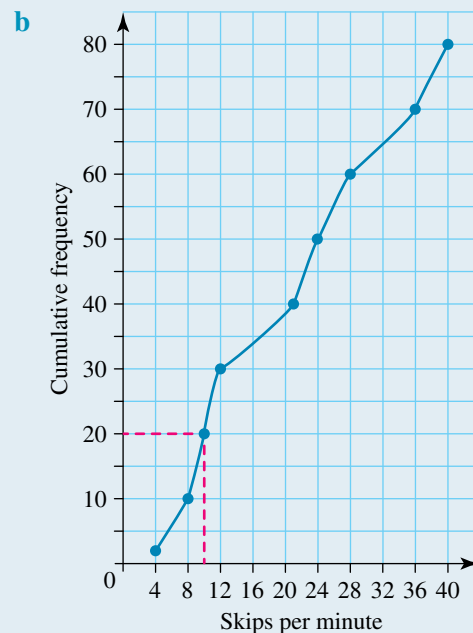


3 The median occurs at the $\frac{80}{2} = 40$ th person.

4 Draw a line from 40 on the vertical axis until it hits the curve. Read down to the corresponding skips per minute on the horizontal axis.

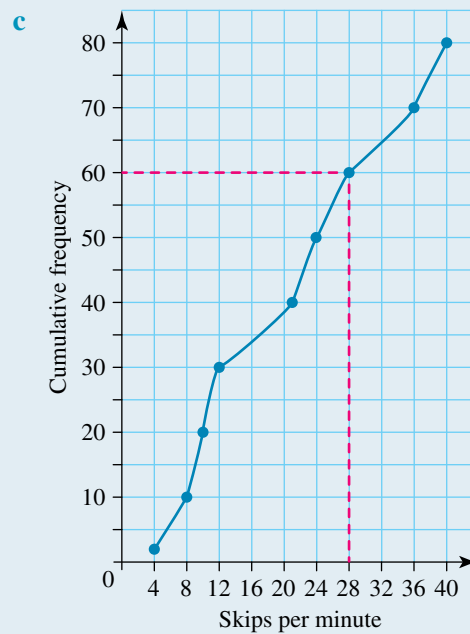
The median number of skips per minute is 21.

- b** • Q_1 occurs at a quarter of the total cumulative frequency, so divide the total frequency by 4.
- Q_1 occurs at the $\frac{80}{4} = 20$ th person.
 - Draw a line from 20 on the vertical axis until it hits the curve. Read down to the corresponding skips per minute on the horizontal axis.



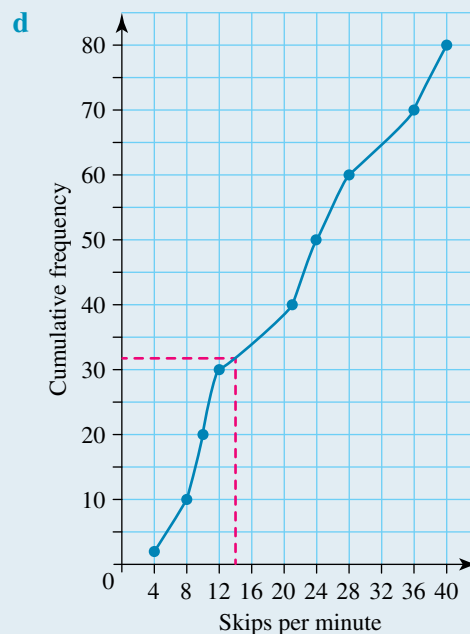
Q_1 is at 10 skips per minute.

- c • Q_3 occurs at three-quarters of the total cumulative frequency, so divide the total cumulative frequency by 4 and multiply the result by 3.
- Q_3 occurs at the $\frac{80}{4} \times 3 = 60$ th person.
 - Draw a line from 60 on the vertical axis until it hits the curve. Read down to the corresponding skips per minute on the horizontal axis.



Q_3 is at 28 skips per minute.

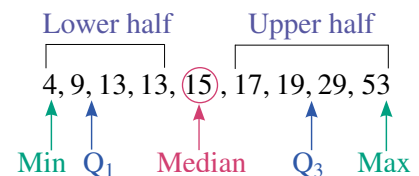
- d • Find the 40th percentile of the cumulative frequency total.
- $\frac{40}{100} \times 80 = 32$ nd person
 - Draw a line from 32 on the vertical axis until it hits the curve. Read down to the corresponding skips per minute on the horizontal axis.



The 40th percentile is at 14 skips per minute.

12.3.2 The interquartile range

- The **interquartile range (IQR)** is the range of the middle 50% of the data set. It measures the spread of the middle 50% of data about the median. The IQR is found by subtracting the lower quartile from the upper quartile.
- The lower quartile (Q_1) is the median of the bottom half of the data and the upper quartile (Q_3) is the median of the top half of the data.
- If the median is one of the actual data values, it is not considered to be in either the upper or lower half of the data. For example:



$$IQR = Q_3 - Q_1$$

WORKED EXAMPLE 5

Calculate the IQR for the following data.

26, 32, 15, 12, 35, 27, 22, 31, 38, 20, 41, 26, 17, 29

THINK

- Put the data in ascending order.
- The median value will split the data into the top and bottom half.
There are 14 values, so the median is at the $\frac{14 + 1}{2} = 7.5$ th value.
- List the bottom half of data.
Find the median of the bottom half of the data.
This is Q_1 .
There are 7 values, so the median is at the $\frac{7 + 1}{2} = 4$ th value.
- List the top half of data.
Find the median of the top half of the data.
This is Q_3 .
There are 7 values, so the median is at the $\frac{7 + 1}{2} = 4$ th value.
- Calculate the IQR.

WRITE

12, 15, 17, 20, 22, 26, 26, 27, 29, 31, 32, 35, 38, 41

12, 15, 17, 20, 22, 26, 26, 27, 29, 31, 32, 35, 38, 41

$$\text{Median} = \frac{26 + 27}{2} = 26.5$$

12, 15, 17, 20, 22, 26, 26

12, 15, 17, 20, 22, 26, 26

$$Q_1 = 20$$

27, 29, 31, 32, 35, 38, 41

27, 29, 31, 32, 35, 38, 41

$$Q_3 = 32$$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 32 - 20 \\ &= 12 \end{aligned}$$

12.3.3 The standard deviation

- The **standard deviation**, like the IQR, is a measure of the spread of the data. However, the standard deviation measures the spread of data about the mean.
- The standard deviation of a population and the standard deviation of a sample are calculated differently:

$$\text{Population standard deviation: } \sigma = \sqrt{\frac{\sum (x - \mu)^2}{n}}$$

where:

σ = population standard deviation

x = a data value

μ = the population mean

n = the total number of population data values

$$\text{Sample standard deviation: } s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

where:

s = population standard deviation

x = a data value

$\bar{x}$ = the sample mean

n = the total number of sample data values

- These formulas use the square of the distance between data points and the mean as part of the calculation. Therefore, a small standard deviation shows that the data is clustered closer to the mean, whereas a large standard deviation indicates that the distribution is more spread out about the mean.

The sample standard deviation is more commonly used because questions usually involve samples instead of whole populations.

- In this course, calculations for standard deviation can be completed using different digital technologies, such as graphical calculators, statistics programs or spreadsheets.

WORKED EXAMPLE 6

The number of chocolate drops in different bags was recorded. These values were: 271, 211, 221, 288, 209, 285, 230, 220, 296, 216.

Calculate the standard deviation, correct to 1 decimal place, of the number of chocolate drops in a bag.

THINK

- 1 Enter the data into the digital technology of your choice. This example will be done in Excel.

WRITE

	A12	↕	✕	✓	fx
	A		B		C
1	Chocolate drops				
2	271				
3	211				
4	221				
5	288				
6	209				
7	285				
8	230				
9	220				
10	296				
11	216				
12					

- 2 To calculate the sample standard deviation, enter the formula '=STDEV()' and select the values to be included in the calculation.

	SUM	↕	✕	✓
	A		B	
1	Chocolate drops			
2	271			
3	211			
4	221			
5	288			
6	209			
7	285			
8	230			
9	220			
10	296			
11	216			
12				
13				
14	=STDEV(A2:A11)			
15				

- 3 Press Enter to calculate the sample standard deviation.

A15		↕	✕	✓
	A		B	
1	Chocolate drops			
2	271			
3	211			
4	221			
5	288			
6	209			
7	285			
8	230			
9	220			
10	296			
11	216			
12				
13				
14	35.65903345			
15				

- 4 Write the answer correct to 1 decimal place.

The standard deviation is 35.7.

12.3.4 Calculating outliers

- As previously stated, outliers are extreme values on either end of a data set that appear very different from the rest of the data.
- Outliers can be calculated by considering the distance a data point is from the mean or median compared to the rest of the data. If the value of a data point is clearly far away from the mean or median, as well as from other data points, it may be considered an outlier. For example, in the data set 2, 2, 4, 6, 7, 7, 100, it is reasonable to conclude that 100 is an outlier, as it is much larger than the rest of the data and is far away from the median of 5.
- It is not always obvious if a data point is an outlier.
- The IQR can be used in a calculation to determine if a data point is far away enough from the median to be considered an outlier.
- To determine if a data point is an outlier, first calculate the upper and lower fences:
 $\text{Lower fence} = Q_1 - 1.5 \times \text{IQR}$
 $\text{Upper fence} = Q_3 + 1.5 \times \text{IQR}$
 where Q_1 is the lower quartile, Q_3 is the upper quartile and IQR is the interquartile range.
- Any data point below the **lower fence** is an outlier.
- Any data point above the **upper fence** is an outlier.

WORKED EXAMPLE 7

Consider the following stem-and-leaf plot.

Key: 1|4 = 14

Stem	Leaf
0	13
1	00134446899
2	2255678889
3	4577
4	01129
5	
6	
7	1

Calculate the IQR and use this to determine if there are any outliers.

THINK

- 1 To calculate the IQR, you must first calculate Q_1 and Q_3 .
 Q_1 is at a quarter of the data. There are 33 data values, so Q_1 occurs at the $\frac{33 + 1}{4} = 8.5$ th value.

Q_3 is at three-quarters of the data. There are 33 data values, so Q_3 occurs at the $\frac{33 + 1}{4} \times 3 = 25.5$ th value.
 Calculate the IQR.

- 2 Calculate the lower fence.

Are there any values below the lower fence?

Calculate the upper fence.

Are there any values below the upper fence?

WRITE

Stem	Leaf
0	1 3
1	0 0 1 3 4 4 4 6 8 9 9
2	2 2 5 5 6 7 8 8 8 9
3	4 5 7 7
4	0 1 1 2 9
5	
6	
7	1

$$Q_1 = 14$$

Stem	Leaf
0	1 3
1	0 0 1 3 4 4 4 6 8 9 9
2	2 2 5 5 6 7 8 8 8 9
3	4 5 7 7
4	0 1 1 2 9
5	
6	
7	1

$$Q_3 = 36$$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 36 - 14 \\ &= 22 \end{aligned}$$

$$\begin{aligned} \text{Lower fence} &= Q_1 - 1.5 \times \text{IQR} \\ &= 14 - 1.5 \times 22 \\ &= -19 \end{aligned}$$

No values are below this point.

$$\begin{aligned} \text{Upper fence} &= Q_3 + 1.5 \times \text{IQR} \\ &= 36 + 1.5 \times 22 \\ &= 69 \end{aligned}$$

There is a value above this point, therefore 71 is an outlier.

eBookplus RESOURCES

Interactivity: Mean, median, mode and quartiles (int-6496)

Interactivity: The median, the interquartile range, the range and the mode (int-6244)

Interactivity: The mean and the standard deviation (int-6246)

Exercise 12.3 Measures of spread

Understanding, fluency and communicating

1. Calculate the range of each of the following sets of data.

a. 4, 6, 8, 11, 15

b. 1.7, 1.9, 2.5, 0.5, 3.1, 1.9, 1.7, 1.6, 1.2

c.

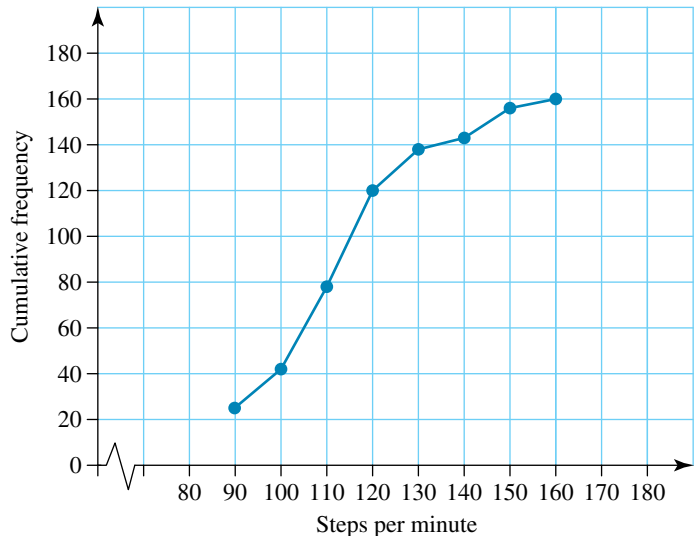
Score (x)	Frequency (f)
110	12
111	9
112	18
113	27
114	5

2. a. Calculate the maximum value of a data set if its minimum value is 23 and its range is 134.
 b. Calculate the minimum value of a data set if its range is 32 and its maximum value is 101.
3. Put these expressions in order from the one with the smallest value to the one with the largest value: upper quartile; minimum; median; maximum; lower quartile.
4. Aptitude tests are often used by companies to help them decide who to employ. An employer gave 30 potential employees an aptitude test with a total of 90 marks. The scores achieved are shown below.
 67, 67, 68, 68, 68, 69, 69, 72, 72, 73, 73, 74, 74, 75, 75, 77, 78, 78, 78, 79, 79, 79, 81, 81, 81, 82, 83, 83, 83, 86

Only applicants who score above the 80th percentile receive an interview. Use your knowledge of percentiles to work out how many interviews the employer will have to run.

5. **WE4** The number of steps per minute runners took at a local park run event was recorded. The cumulative frequency graph is as shown.

- a. How many people participated in the park run?
 b. Calculate:
 i. the median
 ii. Q_1
 iii. Q_3 .
 c. A local running group invited all of the people over the 65th percentile to join the group. How many steps per minute would a runner need to take to be invited to join the running group?



Use the following data set to answer questions 6–8:

171, 122, 182, 153, 167, 184, 171, 177,
 189, 175, 128, 190, 135, 147, 171

6. **MC** The range of the data is:

A. 67 B. 68 C. 69 D. 70 E. 71

7. **WE5 MC** The IQR of the data is:

A. 32 B. 33 C. 34 D. 35 E. 36

8. **WE6 MC** The standard deviation of the data (correct to 1 decimal place) is:

A. 19.5 B. 21.3 C. 22.0 D. 24.3 E. 25.0

9. The speed of 20 cars (in km/h) is monitored along a stretch of road that is a designated 80km/h zone. Find the range and IQR of the data.

80, 82, 77, 75, 80, 80, 81, 78, 79, 78, 80, 80, 85, 70, 79, 81, 81, 80, 80, 80

10. 30 pens are randomly selected off the conveyor belt at the factory and are tested to see how long they will last (values given in hours). Find the range and IQR of the data.
20, 32, 38, 22, 25, 34, 47, 31, 26, 29, 30, 36, 28, 40, 31, 26, 37, 38, 32, 36, 35, 25, 29, 30, 40, 35, 38, 39, 37, 30

11. The ages of patients (in years) that came into a hospital emergency room during an afternoon were recorded. Using a digital technology of your choice, calculate the standard deviation of the data.
14, 1, 3, 87, 27, 42, 19, 91, 17, 73, 68, 83, 62, 29, 32, 2

12. **WE7** Consider the following stem plot.

Key: 1|6 = 16

Stem	Leaf
0	1
1	5
2	
3	
4	2
5	6 7
6	0 1 4 5 6
7	1 1 2 3 5 7 9
8	2 2 4 4 4 8 8
9	3 5

Calculate:

- the range
 - the standard deviation
 - the IQR, and use this to determine if there are any outliers.
13. The number of swimmers at a pool were recorded over a fortnight.
The number of swimmers were as follows.
56, 69, 59, 113, 9, 100, 80, 111, 94, 77, 57, 166, 101, 96
Determine if any outliers exist and justify your answer with a calculation.



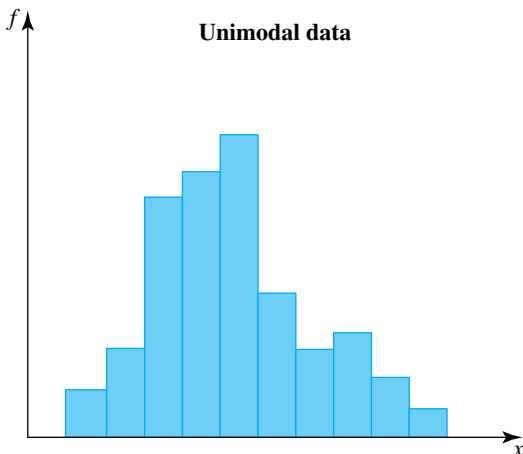
Problem solving, reasoning and justification

14. Create a data set that fits the following descriptions.
- 8 data values with a range of 43
 - 6 data values with a lower quartile of 5 and an upper quartile of 12
 - 9 data values with a lower quartile of 7 and an upper quartile of 13
15. Explain how the standard deviation can be calculated for grouped data.
16. A Mathematics teacher wanted to design a question for her class. She wanted the question to have 10 data points, a lower quartile of 14 and an IQR of 20. State a data set that could be used in her question.
17. The number of trees in several different parks was recorded, and the mean number of trees was calculated to be 15. The council wanted to determine whether the majority of parks contained close to 15 trees, or whether there was a large difference in the number of trees. Describe how the council could use a measure of spread to investigate this problem.
18. A statistician investigated the monthly household sales in a suburb over a year. The Q_1 , median and Q_3 were calculated to be 18, 27, and 35 respectively.
- Prove that 64 houses per month is an outlier.
 - The statistician realised he made a mistake and the outlier of 64 was recorded incorrectly. The actual value was 46 houses per month. Using your answer from **a** to help you, explain what would happen to the values of Q_1 , Q_3 and the IQR if the data point was changed from 64 to 46 houses per month.

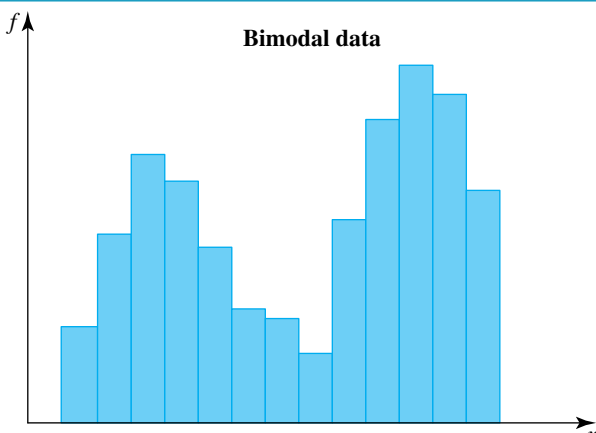
12.4 Comparing graphical distributions

12.4.1 Modality

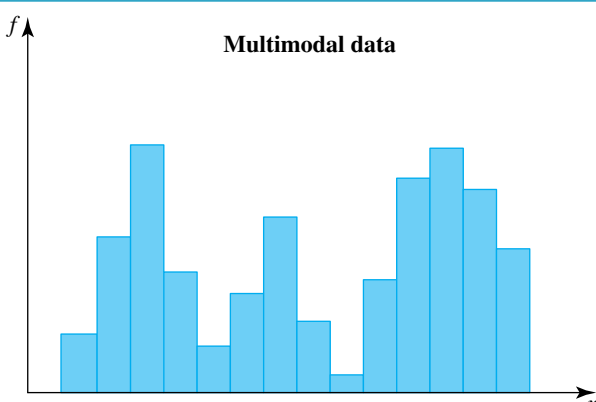
- **Modality** is a measure of the number of obvious peaks in a data set. The modality of a data set can be described as unimodal, bimodal or multimodal.
- Data that is **unimodal** will have one obvious peak.

Histogram	Stem plot																				
Unimodal data  A histogram with 10 bars. The distribution is unimodal, starting with a low frequency on the left, rising to a single peak at the 5th bar, and then gradually decreasing towards the right.	<table> <tr> <th>Stem</th><th>Leaf</th></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>0 3</td></tr> <tr><td>2</td><td>2 4 9</td></tr> <tr><td>3</td><td>1 2 2 6 7</td></tr> <tr><td>4</td><td>0 1 4 7 7 8 9</td></tr> <tr><td>5</td><td>4 6 6 7 8</td></tr> <tr><td>6</td><td>0 2 4 6</td></tr> <tr><td>7</td><td>3 7</td></tr> <tr><td>8</td><td>5</td></tr> </table>	Stem	Leaf	0	1	1	0 3	2	2 4 9	3	1 2 2 6 7	4	0 1 4 7 7 8 9	5	4 6 6 7 8	6	0 2 4 6	7	3 7	8	5
Stem	Leaf																				
0	1																				
1	0 3																				
2	2 4 9																				
3	1 2 2 6 7																				
4	0 1 4 7 7 8 9																				
5	4 6 6 7 8																				
6	0 2 4 6																				
7	3 7																				
8	5																				

- Data that is **bimodal** will have two obvious peaks.

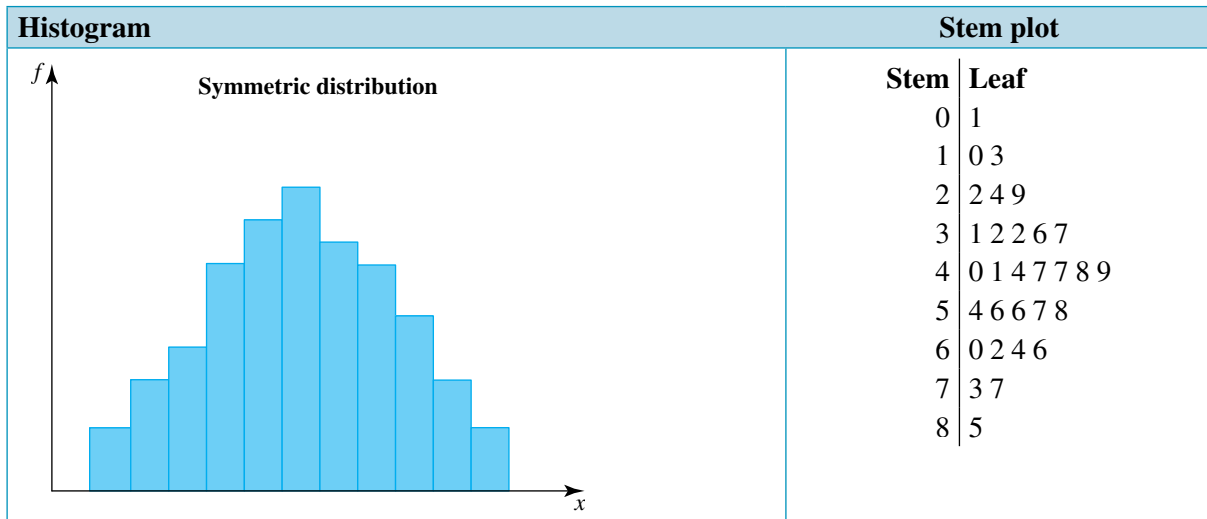
Histogram	Stem plot																				
Bimodal data  A histogram with 15 bars. The distribution is bimodal, with a smaller peak at the 4th bar and a larger, more prominent peak at the 11th bar. There is a noticeable dip in frequency between the two peaks.	<table> <tr> <th>Stem</th><th>Leaf</th></tr> <tr><td>0</td><td>0 9</td></tr> <tr><td>1</td><td>8 8 9</td></tr> <tr><td>2</td><td>0 1 3 4 4 7 8 9 9</td></tr> <tr><td>3</td><td>2 4 5 5 6</td></tr> <tr><td>4</td><td>1 3 7</td></tr> <tr><td>5</td><td>6</td></tr> <tr><td>6</td><td>0 0 0 1</td></tr> <tr><td>7</td><td>1 2 2 4 5 6 9</td></tr> <tr><td>8</td><td>3 7</td></tr> </table>	Stem	Leaf	0	0 9	1	8 8 9	2	0 1 3 4 4 7 8 9 9	3	2 4 5 5 6	4	1 3 7	5	6	6	0 0 0 1	7	1 2 2 4 5 6 9	8	3 7
Stem	Leaf																				
0	0 9																				
1	8 8 9																				
2	0 1 3 4 4 7 8 9 9																				
3	2 4 5 5 6																				
4	1 3 7																				
5	6																				
6	0 0 0 1																				
7	1 2 2 4 5 6 9																				
8	3 7																				

- Data that is **multimodal** will have three or more obvious peaks.

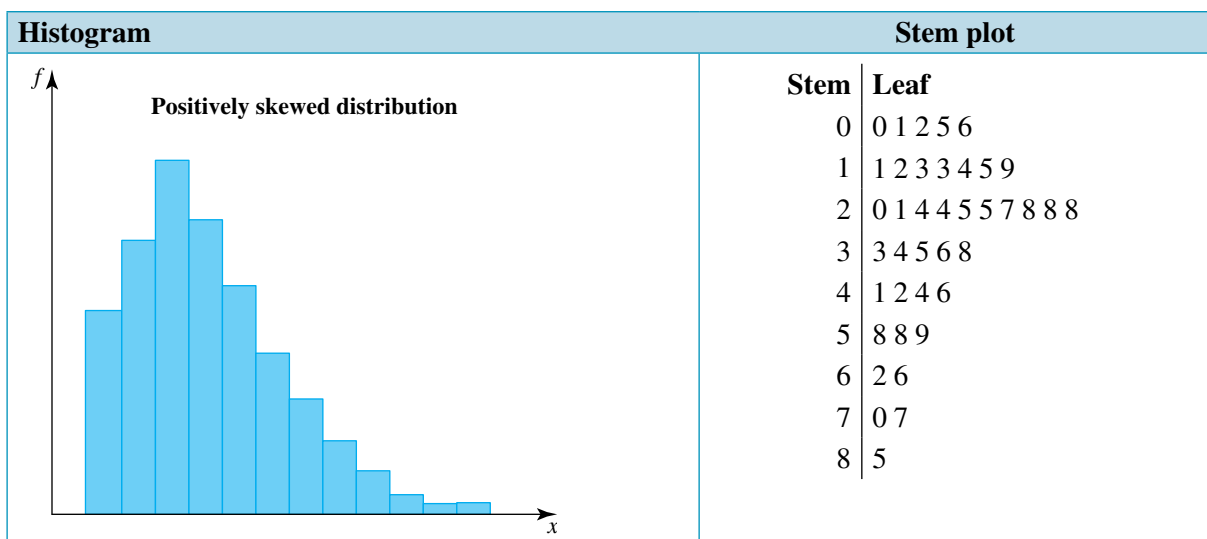
Histogram	Stem plot																				
Multimodal data  A histogram with 15 bars. The distribution is multimodal, showing three distinct peaks at the 4th, 8th, and 11th bars. The frequencies rise and fall multiple times across the range of the data.	<table> <tr> <th>Stem</th><th>Leaf</th></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>6 6 7 8 8 8 9</td></tr> <tr><td>2</td><td>5 6 7</td></tr> <tr><td>3</td><td></td></tr> <tr><td>4</td><td>1 2 2 3 4 4 4 5</td></tr> <tr><td>5</td><td>6</td></tr> <tr><td>6</td><td>4 7 8</td></tr> <tr><td>7</td><td>0 0 1 2 3 6 7 9</td></tr> <tr><td>8</td><td>1 2 4 6 8</td></tr> </table>	Stem	Leaf	0	1	1	6 6 7 8 8 8 9	2	5 6 7	3		4	1 2 2 3 4 4 4 5	5	6	6	4 7 8	7	0 0 1 2 3 6 7 9	8	1 2 4 6 8
Stem	Leaf																				
0	1																				
1	6 6 7 8 8 8 9																				
2	5 6 7																				
3																					
4	1 2 2 3 4 4 4 5																				
5	6																				
6	4 7 8																				
7	0 0 1 2 3 6 7 9																				
8	1 2 4 6 8																				

12.4.2 Shape

- The **shape** of a data distribution can be described as **symmetric** or skewed (asymmetric). If skewed, the distribution can either be positively or negatively skewed.
- Distributions are symmetric if the two halves of the distribution are roughly equal about the mean. The distributions will tail off evenly to the left and right of the peak, with the bulk of the data generally close to the centre of the distribution. If symmetry exists, the median and mean will occur at approximately the same position. An example of a symmetric distribution might be the examination results for students taking the Mathematics Standard course.

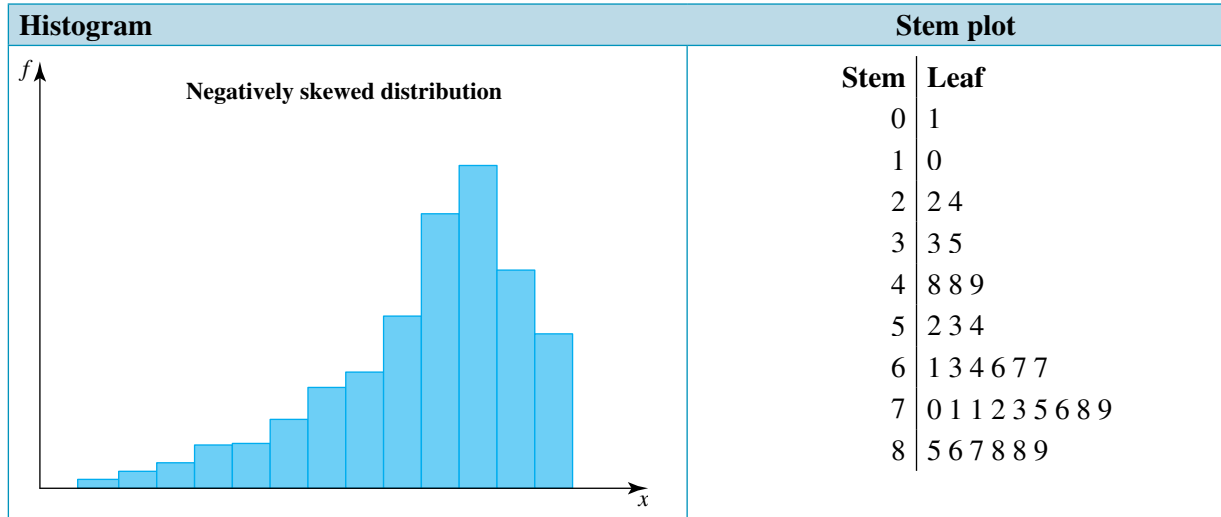


- A distribution is **positively skewed** if it tapers off in a long tail to the right-hand side of the peak of the distribution. The bulk of the data is usually around the peak of the distribution, which is on the left-hand side of the distribution.
- For positively skewed distributions, the median is usually smaller than the mean (the mean will be on the same side of the median as the tail or skew). An example of a positively skewed distribution would be the number of hours spent playing sports versus age, as the number of hours spent playing sport is likely to reduce with age.



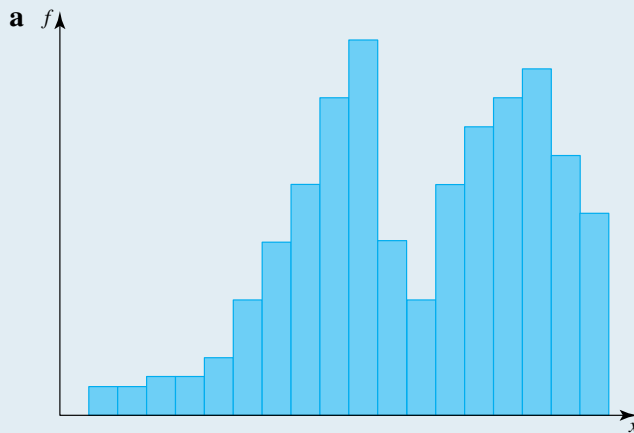
- A distribution is **negatively skewed** if it tapers off in a long tail to the left-hand side of the peak of the distribution. The bulk of the data is usually around the peak of the distribution, which is on the right-hand side of the distribution.

- For negatively skewed distributions, the median is usually greater than the mean. An example of a negatively skewed distribution would be the distribution of the number of heart attacks with age, as the number of people having heart attacks would increase with age.



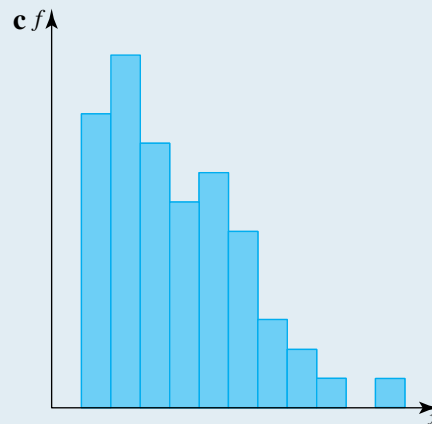
WORKED EXAMPLE 8

Identify the modality and shape of each of the following diagrams.



b

Stem	Leaf
0	1 3
1	2 2 4 6 7
2	1 6 7 9
3	0 8
4	
5	1 3
6	4 5 7 8
7	0 1 3
8	8 9
9	



THINK

a 1 Look at the modality. Are there one, two or three or more peaks?

WRITE

The data is bimodal.

2 Does the data appear asymmetric or symmetric?

3 If the data is asymmetric, does the distribution tail off to the left or right?

The data is negatively skewed.

b 1 Look at the modality. Are there one, two or three or more peaks?

The data is bimodal.

2 Does the data appear asymmetric or symmetric?

The data is symmetric.

c 1 Look at the modality. Are there one, two or three or more peaks?

The data is unimodal.

2 Does the data appear asymmetric or symmetric?

3 If the data is asymmetric, does the distribution tail off to the left or right?

The data is positively skewed.

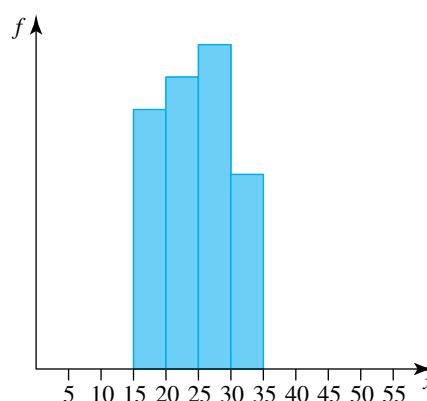
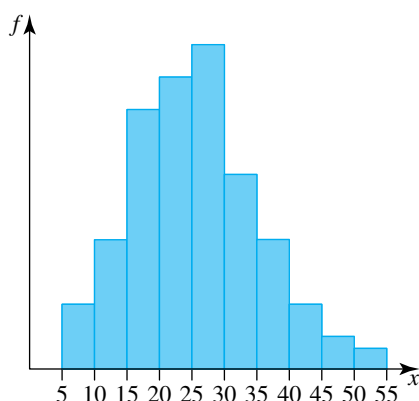
12.4.3 Central tendency, spread and outliers

Measures of central tendency

- A histogram is a graphical representation of group data.
- Measures of central tendency (mean, median and mode) can be calculated from a histogram.
- The mode is the easiest value to determine, as it is the class interval that has the highest frequency.
- The mean can be calculated by using the midpoint of each class interval as the representative value for each class interval on the histogram.
- The median of a histogram can be written as a class interval. It occurs at the middle data value on the histogram. The median can be calculated using the following steps:
 1. Add the frequencies of each class interval of the histogram. This will give the total number of data values.
 2. Find the position of the median.
$$\text{Median position} = \frac{n + 1}{2}$$
 3. Determine the class interval that contains the position of the median by looking at the cumulative frequency of the histogram. This class interval will be the median.

Spread

- The spread of a histogram examines how closely packed together the data is. If the bulk of the data is close together, the histogram has a small spread. If the bulk of the data is far apart, the histogram has a large spread. In the following diagrams, the histogram on the left has a larger spread than the histogram on the right.

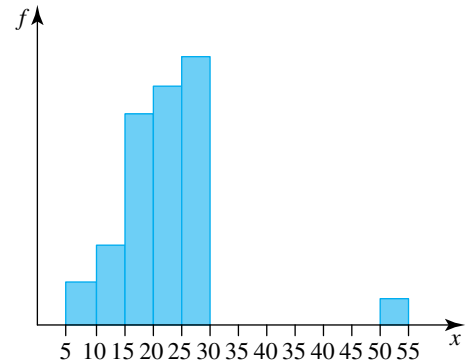


- The measure most commonly used to discuss the spread of a histogram is the range.
- The range = maximum value – minimum value

- The lower and upper quartiles of a histogram can be written as class intervals. The IQR is more complicated to calculate, but it will usually only need to be determined if a list of data values are given. Q_1 will occur at the $\frac{n+1}{4}$ th value and Q_3 will occur at the $\frac{n+1}{4} \times 3$ th value.
- The standard deviation can be calculated from a histogram using digital technologies that allow the input of class intervals. The standard deviation will usually only need to be calculated if the data values of a histogram are provided.

Outliers

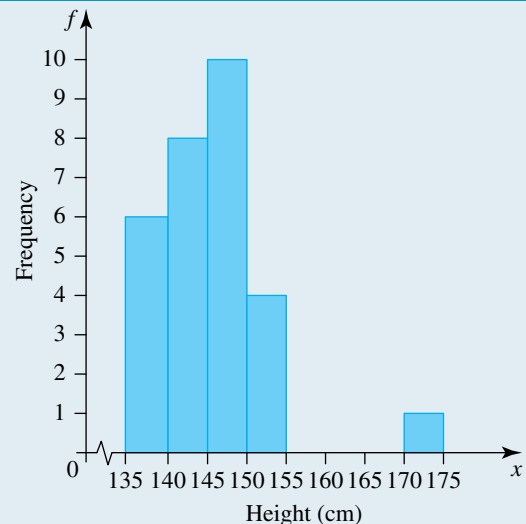
- Spaces between bars on a histogram represent class intervals that have a frequency of 0. An outlier on a histogram will appear as a low frequency class interval, with several 0-frequency class intervals separating it from the main cluster of data.
- In this course, no calculations are made directly from a histogram that will confirm whether a data point is an outlier. Therefore, all outliers read from a histogram (where no other data is provided) must be written as 'possible outliers'. The class interval 50–55 on the histogram at right represents a possible outlier.



WORKED EXAMPLE 9

The histogram at right shows the heights of students in a class.

- a Using the histogram, calculate the following statistics for heights of students.
- Mean
 - Median
 - Mode
 - Range
 - Q_1 and Q_3
- b Are there any possible outliers?



THINK

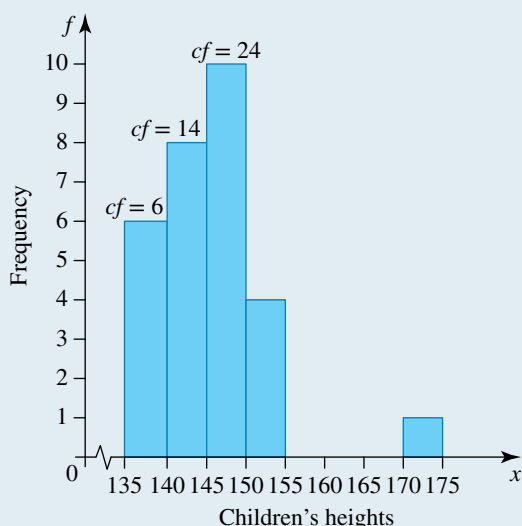
- a i 1 The mean can be calculated using the formula for the mean of grouped data. Use the midpoint to calculate the mean as in Worked example 2.

WRITE

Class interval	Frequency (f)	Midpoint (x)	xf
135–140	6	137.5	825
140–145	8	142.5	1140
145–150	10	147.5	1475
150–155	4	152.5	610
155–160	0	157.5	0
160–165	0	162.5	0
165–170	0	167.5	0
170–175	1	172.5	172.5
Totals	29		4222.5

$$\begin{aligned}
 \bar{x} &= \frac{\sum xf}{n} \\
 &= \frac{4222.5}{29} \\
 &= 145.6
 \end{aligned}$$

- ii To calculate the median, count the total number of data values by adding the frequency of each class interval on the histogram, then divide the total by 2.
- The median occurs at the $\frac{29 + 1}{2} = 15$ th value.
 - Determine which class interval contains the 15th value by calculating the cumulative frequency. The 15th value is contained in the 145–150 cm class interval, which starts at the 15th value and finishes at the 24th data value.



- iii The mode is the class interval that has the highest frequency.
- iv Calculate the range by subtracting the minimum class interval value from the maximum class interval value.
- v 1 Q_1 occurs at the $\frac{29 + 1}{4} = 7.5$ th value.
Determine which class interval contains the 7.5th value by calculating the cumulative frequency. The 7.5th value is contained in the 140–145 cm class interval.
- 2 Q_3 occurs at the $\frac{29 + 1}{4} \times 3 = 22.5$ th value.
Determine which class interval contains the 22.5th value by calculating the cumulative frequency. The 22.5th value is contained in the 145–150 cm class interval.
- b Look at the graph to see if any extreme values occur.
Make sure to state that it is a possible outlier as no calculations have been made to prove whether it is or isn't an outlier.

$$\begin{aligned} \text{Total number of data values} &= 6 + 8 + 10 + 4 + 1 \\ &= 29 \end{aligned}$$

$$\text{The median occurs at the } \frac{29 + 1}{2} = 15\text{th value}$$

The median occurs at the 15th value.

The median occurs in the 145–150 cm class interval.

The mode is 145–150 cm.

$$\begin{aligned} \text{Range} &= 175 - 135 \\ &= 40 \text{ cm} \end{aligned}$$

$$Q_1 \text{ occurs at the } \frac{29 + 1}{4} = 7.5\text{th value.}$$

Q_1 occurs in the 140–145 cm class interval.

$$Q_3 \text{ occurs at the } \frac{29 + 1}{4} \times 3 = 22.5\text{th value.}$$

Q_3 occurs in the 145–150 cm class interval.

A possible outlier occurs at the height of 170–175 cm.

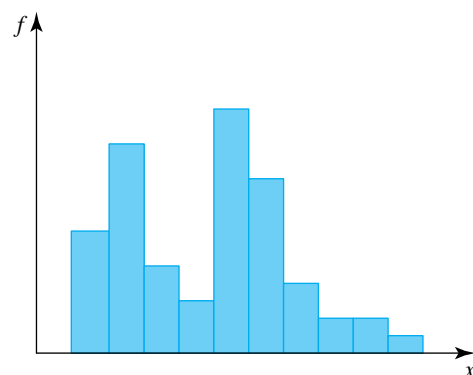
Exercise 12.4 Comparing graphical distributions

Understanding, fluency and communicating

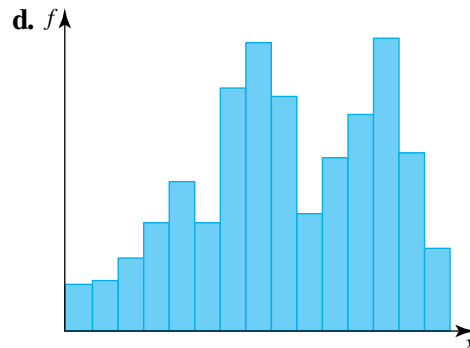
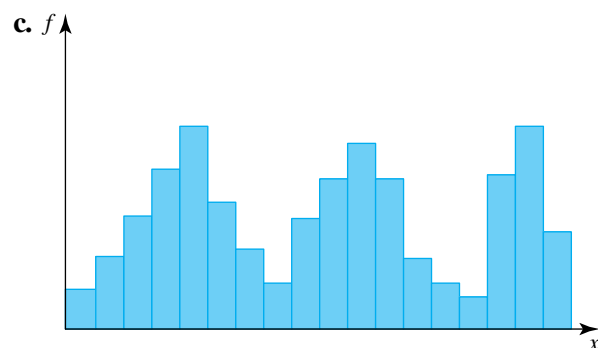
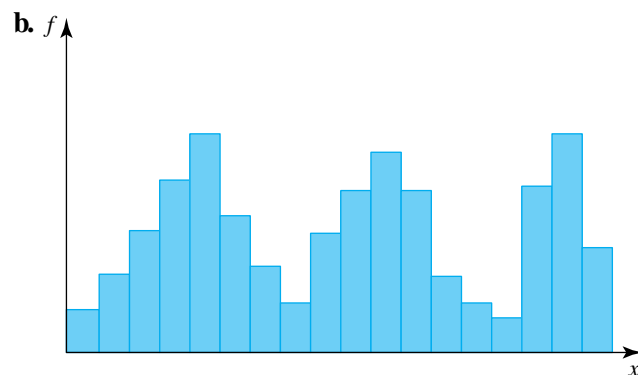
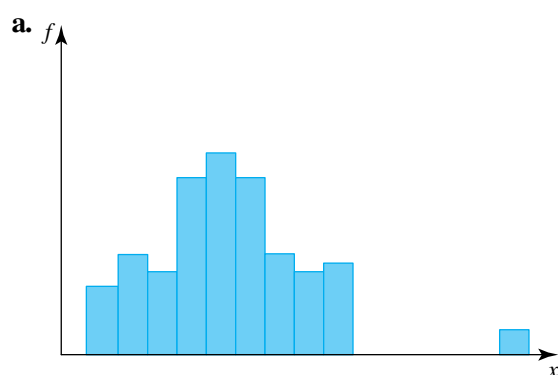
Note: Where necessary give answers correct to 1 decimal place.

1. **MC** The histogram at right is:

- A. unimodal and positively skewed
- B. bimodal and positively skewed
- C. unimodal and symmetric
- D. unimodal and negatively skewed
- E. bimodal and negatively skewed



2. Identify the modality of each of the following histograms.



3. What is the shape of each of the histograms in question 2?

4. **WE8** Describe the modality and shape of the following stem plot, and state whether there are any possible outliers.

Key: 4|3 = 43

Stem	Leaf
0	7
1	1
2	01
3	48
4	5679
5	23459
6	012223478
7	455678
8	
9	0

Use the histogram at right to answer questions 5–7.

5. **MC** The range of student weights is:

- A. 20 kg B. 18 kg
C. 36 kg D. 38 kg
E. 80 kg

6. **MC** The mean weight of students is closest to:

- A. 53.0 kg B. 54.6 kg
C. 54.7 kg D. 52.8 kg
E. 52.9 kg

7. **MC** The median weight falls into which class interval?

- A. 50–52 kg B. 52–54 kg C. 54–56 kg D. 56–58 kg E. 58–60 kg

8. Calculate the mean, median and mode for the data represented by the histogram at right. Compare these values.

9. A florist recorded the number of bouquets of flowers sold daily over a three-week period. The results are recorded below.



2, 14, 15, 1, 14, 17, 8, 19, 8, 5, 6, 16, 12, 9, 4, 14, 2, 3, 9, 19, 16

a. Calculate:

- i. the mean ii. the median iii. the IQR
iv. the standard deviation.

b. Draw a histogram showing the number of bouquets of flowers sold.

c. Mark the median, mean and IQR on the histogram.

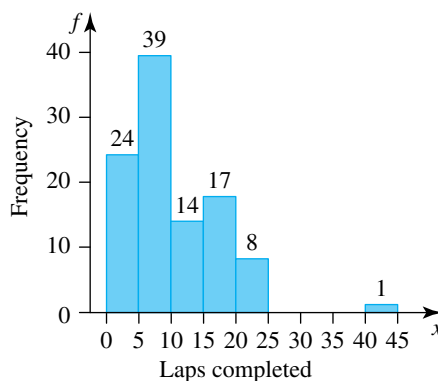
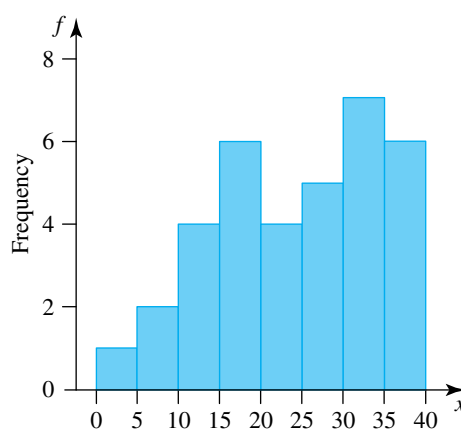
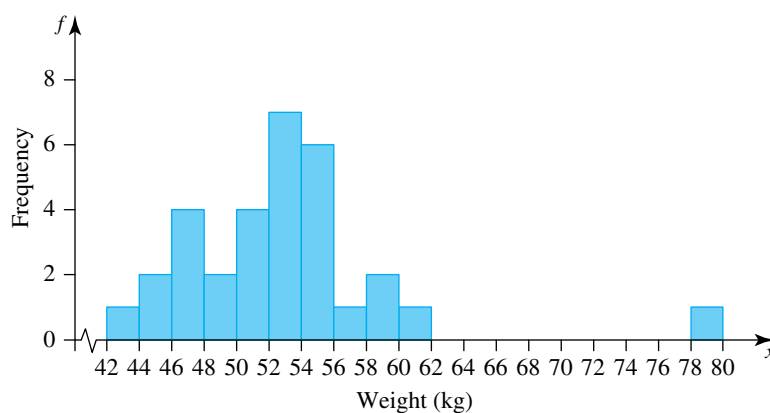
10. **WE9** The histogram at right shows the number of 50 m laps completed by swimmers at a local pool.

a. Using the histogram, calculate the following measures of central tendency for laps completed.

- i. Mean ii. Median
iii. Mode iv. Range
v. Q_1 and Q_3

b. Are there any possible outliers?

11. Describe how an outlier can be identified from a histogram.



Problem solving, reasoning and justification

12. The mean of a histogram is 22.5 cm and the median is in the class interval 30–35 cm. What does this suggest about the shape of the histogram?

13. Give a possible real-life example of:

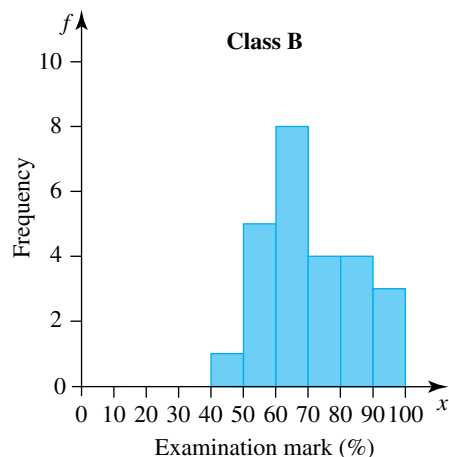
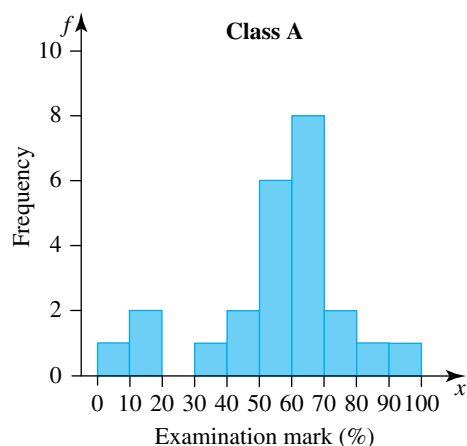
- a. bimodal distribution b. negatively skewed distribution.

14. Darwyn and Morgan were discussing the following stem plot. Darwyn said that the plot was symmetric but Morgan thought it was negatively skewed. Who is correct and why?

Key: 1|2 = 12

Stem	Leaf
0	1
1	0
2	
3	
4	
5	
6	1 2 4
7	2 5 7 7
8	0 3 3 5 8 9
9	0 1 2 2 2 6 7 8 8
10	4 4 5 6 7 8 9
11	1 2 3 4 5
12	1 8 9

15. The following histograms show the Mathematics examination results for two different classes. Write a report comparing the results of the two classes in terms of central tendency and spread (range only).

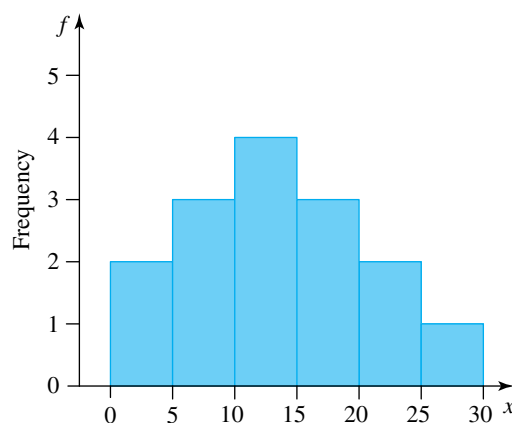


16. The median of the histogram at right occurs at the 8th data value. State a possible exact value of the median and justify your answer.

12.5 Parallel boxplots

12.5.1 Boxplots

- Boxplots are used to show the spread or distribution of the data values.
- Boxplots are drawn after the data has been summarised using a **five-figure summary**.
- A five-figure summary of a data set consists of the following statistics:
 - minimum value
 - lower quartile (quartile 1 or Q_1)
 - median (quartile 2 or Q_2)
 - upper quartile (quartile 3 or Q_3)
 - maximum value.



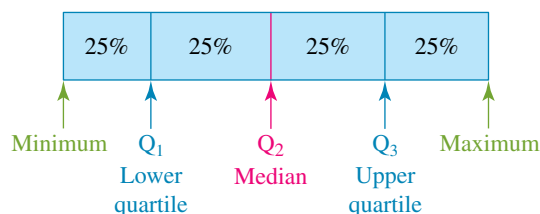
For example:



Five-figure summary:

Minimum	2
Lower quartile (Q ₁)	3
Median (Q ₂)	$\frac{5 + 5}{2} = 5$
Upper quartile (Q ₃)	8
Maximum	12

- The five-figure summary provides a concise data summary of both centre and spread.
- The five-figure summary divides the data set into quarters, as shown at right.



WORKED EXAMPLE 10

Calculate the five-figure summary for the following set of data.

12, 4, 22, 31, 33, 2, 8, 27, 38, 5

THINK

- Arrange the values in ascending order.
- Determine the minimum value.
- Find the median.

There are 10 values so the median is at the $\frac{10 + 1}{2} = 5.5$ th value.

- Calculate the lower quartile.
List the bottom half of the data.

There are 5 values, so Q₁ is at the $\frac{5 + 1}{2} = 3$ rd value.

- Calculate the upper quartile.
List the upper half of the data.

There are 5 values, so Q₃ is at the $\frac{5 + 1}{2} = 3$ rd value.

- Determine the maximum value.
- List the five-figure summary.

WRITE

2, 4, 5, 8, 12, 22, 27, 31, 33, 38

Minimum = 2

2, 4, 5, 8, 12, | 22, 27, 31, 33, 38

Median = $\frac{12 + 22}{2}$
= 17

2, 4, 5, 8, 12

2, 4, 5, 8, 12

Q₁ = 5

22, 27, 31, 33, 38

22, 27, 31, 33, 38

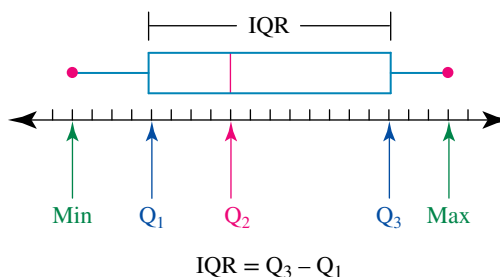
Q₃ = 31

Maximum = 38

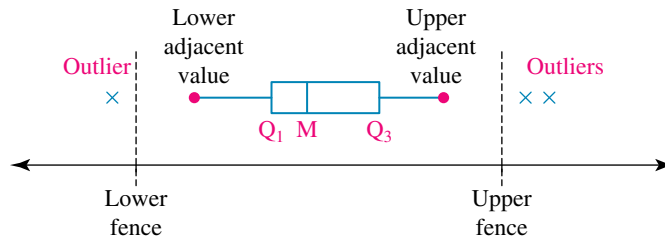
The five-figure summary is
2, 5, 17, 31, 38.

12.5.2 Drawing boxplots

- Boxplots** are a visual representation of the five-figure summary for a data set.
- A boxplot is drawn above a scale.
- The vertical lines on a boxplot show the positions of the statistics from the five-figure summary on the scale below the boxplot.
- The interquartile range (IQR) is the range of the middle 50% of the data set. It is found by subtracting the lower quartile from the upper quartile.



- Always check for outliers before drawing a boxplot. A boxplot with outliers is drawn as shown.



- *Note:* The upper and lower fences are not usually drawn on a boxplot; they appear here for illustrative purposes.
 - Outliers are marked with an $\times$ on a boxplot.
 - Any data point below the lower fence is an outlier.
Lower fence = $Q_1 - 1.5 \times \text{IQR}$
 - Any data point above the upper fence is an outlier.
Upper fence = $Q_3 + 1.5 \times \text{IQR}$
 - The **lower adjacent value** is the first value above the lower fence.
 - The **upper adjacent value** is the last value below the upper fence.
 - If outliers are present on a boxplot, they must be included when calculating the minimum and maximum values, and the range.

WORKED EXAMPLE 11

Draw a boxplot using the data from Worked example 10.

THINK

1 Write down the five-figure summary.

2 Determine if there are any outliers.
Calculate the IQR.

Calculate the lower fence.

Calculate the upper fence.

Determine if there are any outliers.

3 Using a ruler, draw a scale long enough to include the minimum and maximum values. The scale must go from at least 2 to 38.

4 Draw a vertical line above the scale at Q_1 and Q_3 , then connect these to form a box.

WRITE

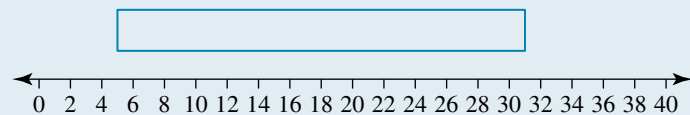
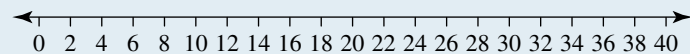
The five-figure summary is 2, 5, 17, 31, 38.

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 31 - 5 \\ &= 26\end{aligned}$$

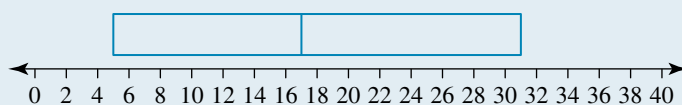
$$\begin{aligned}\text{Lower fence} &= 5 - 1.5 \times 26 \\ &= -34\end{aligned}$$

$$\begin{aligned}\text{Upper fence} &= 31 + 1.5 \times 26 \\ &= 70\end{aligned}$$

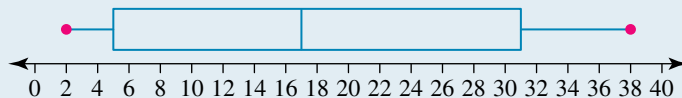
As there are no values below the lower fence or above the upper fence, there are no outliers.



- 5 Draw a vertical line in the box at the value of the median.

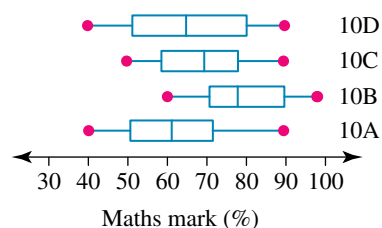


- 6 To complete the boxplot, draw the tails: one from the minimum value to Q_1 and one from Q_3 to the maximum value.



12.5.3 Parallel boxplots

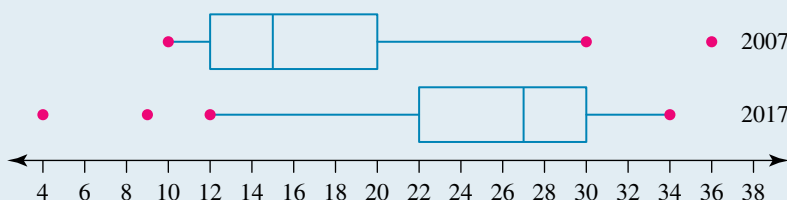
- Parallel boxplots are drawn one above the other, using the same scale. The set of parallel boxplots on the right provides a means of comparing the results of four classes in a Maths test.
- Reporting on a parallel boxplot involves comparing the individual boxplot features, including:
 - central tendency or median
 - spread (both IQR and range)
 - outliers.
- Comparisons made on parallel boxplots should always be in context. For example, to compare the above parallel boxplots, comment on the median, spread and outliers in terms of Maths marks for the different classes.
- Values of the median, IQR, range and/or outliers should always be included in a comparison of parallel boxplots.



WORKED EXAMPLE 12

The following parallel boxplots show the number of hats produced in a factory per hour over a two-day period in 2007 and 2017.

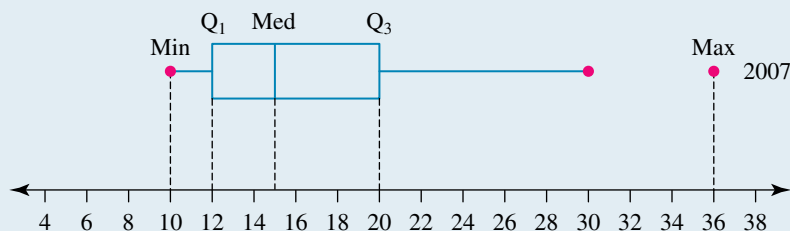
- Write down the five-figure summary for the hourly hat production in both 2007 and 2017.
- The top 25% of hats produced in 2017 were above which value?
 - The middle 50% of hats produced in 2007 were between which values?
- Compare the hourly hat production for the two different years.



THINK

- Look at the boxplot for 2007. Using the scale and the boxplot, determine the values for the five-figure summary.

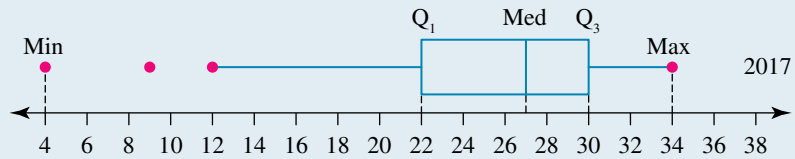
WRITE



Five-figure summary for 2007:

10, 12, 15, 20, 36

- 2 Look at the boxplot for 2017. Using the scale and the boxplot, determine the values for the five-figure summary.



Five-figure summary for 2017:

4, 22, 27, 30, 34

- b i The top 25% on a boxplot occurs above the third quartile, Q_3 .

The top 25% of hourly hats produced in 2017 occurred above 30 hats per hour.

- ii The middle 50% on a boxplot is represented by the IQR.

The middle 50% of hourly hats produced in 2007 occurred between 12 and 20 hats per hour.

- c 1 Write a sentence, comparing the two boxplots for the median, IQR, range and outliers.

The parallel boxplots show that median hourly hat production in 2007 (15 hats/hour) was lower than in 2017 (27 hats/hour).

The IQR was greater in 2017 (12 hats/hour) than in 2007 (8 hats/hour).

The range was slightly larger in 2017 (30 hats/hour) than in 2007 (26 hats/hour).

Both boxplots have outliers. These were 36 hats/hour in 2007 and 4 and 9 hats/hour in 2017.

- 2 Write a sentence summarising the comparison.

Overall, the hourly hat production over each two-day period in 2017 was generally greater than it was in 2007.

eBookplus RESOURCES

Interactivity: Boxplots (int-6245)

Interactivity: Parallel boxplots (int-6248)

Exercise 12.5 Parallel boxplots

Understanding, fluency and communicating

- WE10** A trumpet teacher recorded the number of lessons she taught each day for 2 weeks. The data is shown at right.
Find the five-figure summary for the data set.
- A gardener recorded how long it took to mow each lawn on the weekly route (in minutes). The data is shown below.
32, 45, 56, 28, 19, 38, 26, 47, 54, 21, 33, 40, 17, 58, 21
Find the five-figure summary for the data set.
- MC** The five-figure summary values for the following data set of ages for a family with 6 members, 45, 42, 12, 9, 7, 4, in order of minimum, Q_1 , median, Q_3 and maximum are:

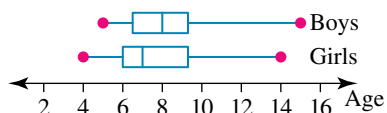
A. 4, 7, 10.5, 42, 45	B. 4, 9, 12, 44, 45	C. 4, 7, 12, 42, 45
D. 4, 9, 10.5, 42, 45	E. 4, 7, 9, 42, 45	



4. **WE11** The weights of 14 boxes (in kilograms) being moved from one house to another are as follows.
3, 6, 13, 15, 15, 15, 17, 17, 18, 20, 20, 22, 26, 30
Draw a boxplot to display this data.
5. The prices of 10 mobile phones are viewed online and are as follows.
\$349, \$469, \$265, \$497, \$159, \$52, \$999, \$489, \$599, \$577
Draw a boxplot to represent this data.
6. The speeds of 15 cars in a school zone road were as follows.
39, 51, 60, 42, 44, 38, 75, 45, 40, 52, 41, 42, 46, 41, 39
Draw a boxplot to represent this data.
7. A restaurant trialled three different menus for a month each to work out which type of cuisine its patrons enjoy the most. The restaurant has asked for some help analysing data that they have collected.
- Draw a boxplot for each month of data on the same scale.
 - Write a comparison of the boxplots to help the restaurant decide which menu will be the most successful.

	Week 1	Week 2	Week 3	Week 4
Month 1: Greek	75	62	48	50
Month 2: Italian	48	17	9	68
Month 3: Spanish	56	43	37	28

The following parallel boxplots, showing the ages at which boys and girls learn to ride a two-wheel bike, should be used to answer questions **8–10**.



Girls	6	7	5	6	8	10	14	9	11	7	6	4
Boys	7	6	8	10	8	9	7	5	9	6	15	11



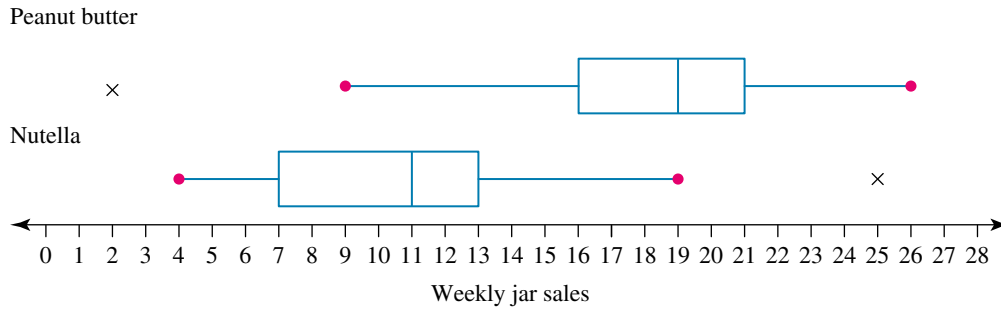
Who learns to ride bikes earlier:
girls or boys?

8. **MC** Looking at the parallel boxplots, at what age did the youngest girl and the oldest boy learn to ride a two-wheel bike?
- 15 and 4
 - 4 and 15
 - 5 and 14
 - 14 and 5
 - 5 and 15
9. **MC** Which value is the same for both boxplots?
- Q_1
 - Q_3
 - IQR
 - Range
 - Median
10. **MC** What is the difference between the medians of the boxplots?
- 1
 - 2
 - 3
 - 4
 - 5

Problem solving, reasoning and justification

11. **a.** What does it mean if the median on a boxplot is not placed exactly halfway between the upper and lower quartiles?
- b.** What does it mean if the median on a boxplot is closer to the upper quartile than the lower quartile?
- c.** Write a statement explaining what the position of the median can tell you about a data set.
12. **WE12** The parallel boxplots below show the weekly sales for jars of peanut butter and Nutella in a supermarket.
- Write down the five-figure summary for the weekly sales of peanut butter and Nutella.
 - The lowest 25% of weekly peanut butter jar sales were below which value?
 - The middle 50% of Nutella weekly jar sales were between which values?

- c. Compare the weekly sales for the two different products.



13. The following data was collected from a company that is comparing two different compounds for sunscreen to see how long (measured in minutes) they remain effective in water before they need to be reapplied.

Compound 1	176	156	123	169	149	133	164	127	139	121	172	168
Compound 2	130	125	128	121	118	120	122	127	130	122	131	126

- a. Draw parallel boxplots for this data.
b. Compare the two compounds.
14. A comparison of Year 11 students' achievements (measured as a number out of 100) in History and English was recorded and the results below were obtained.

History	75	78	42	92	59	67	78	82	84	64	77	98
English	78	80	57	96	58	71	74	87	79	62	75	100

- a. Draw parallel boxplots for this data.
b. What do the parallel boxplots tell you about Year 11 students' achievements in History and English?
15. The heights of Year 11 and Year 12 students (to the nearest centimetre) are being investigated. The results of some sample data are below.

Year 11	160	154	157	170	167	164	172	158	177	180	175	168	159	155	163	163	169	173	172	170
Year 12	160	172	185	163	177	190	183	181	176	188	168	167	166	177	173	172	179	175	174	180

- a. Draw parallel boxplots for this data.
b. Comment on what the plots tell you about the heights of Year 11 and Year 12 students.

12.6 Review

12.6.1 Summary

In this chapter you have learnt:

- the difference between populations and samples
- measures of central tendency (mean, median and mode) and how to calculate and interpret them from grouped and ungrouped data and graphs
- measures of spread (range, interquartile range, and standard deviation) and how to calculate and interpret them from grouped and ungrouped data and graphs
- the definition of outliers and how to determine if a point is an outlier
- the effect of outliers on summary statistics
- how to calculate percentiles
- different ways to describe graphical distributions, including modality and spread
- how to calculate and interpret the five-figure summary
- how to construct and interpret boxplots, with and without outliers
- how to compare different sets of data using parallel boxplots.

Note: A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

Exercise 12.6 Review

Understanding, fluency and communicating

Note: Where necessary give answers correct to 1 decimal place.

1. Calculate the mean, median and mode of each of the following data sets.

a. 4, 7, 2, 6, 9, 3, 6

b. 11, 63, 24, 36, 25, 61, 29, 42

c.

Score	Frequency
6	2
8	4
10	10

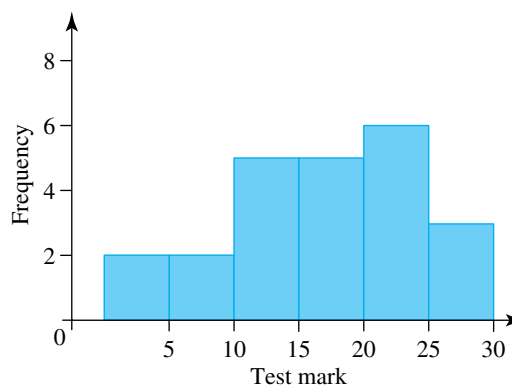
2. The following data was collected on the number of days people go away for during the holidays.

2, 10, 5, 7, 9, 14, 2, 0, 6, 7, 0, 7, 14, 7, 8, 10, 12,
5, 2, 1, 16, 12, 10

a. What is the range of this data?

b. What are the mean and median?

The test marks for a class of students are shown in the histogram at right. Use this information to answer questions 3–6.



3. **MC** The median test mark for the class falls into which category?

A. 5–10

B. 10–15

C. 15–20

D. 20–25

E. 25–30

4. **MC** The mean of this histogram, to 1 decimal place, is:

A. 14.3

B. 16.8

C. 22.5

D. 19.3

E. 16.2

5. **MC** The mode of this histogram is:

A. 5–10

B. 10–15

C. 15–20

D. 20–25

E. 20–25

6. **MC** The range of this histogram is:

A. 5

B. 15

C. 20

D. 25

E. 30

7. The screen time per day for a group of 0–14-year-olds is listed in the table below. Calculate the mean number of minutes of screen time for 0–14-year-olds.

Age (years)	Screen time/day (minutes)
0–2	45
3–5	73
6–8	98
9–11	124
12–14	142

8. Match the following terms with the descriptions below.

Mean Range Median IQR Standard deviation Mode

a. A measure of spread of the distribution that uses the highest and lowest value only

b. A measure of central tendency that uses the middle value of the data set by position

c. A measure of spread that contains exactly 50% of the distribution

d. A measure of central tendency that uses the sum of all data points divided by the number of data points

e. A measure of spread that uses the distance from the mean in the calculation

f. The measure of central tendency that uses the most common value

9. Below is a data set showing the number of doors people have in their houses.
18, 11, 9, 14, 11, 16, 14, 16, 6, 18, 19, 9, 10, 17, 36
- State the minimum and maximum number of doors, and hence calculate the range.
 - Calculate the mean and median.
 - Are there any outliers? If yes, remove these and recalculate the mean and median.
 - Compare your answers to parts **a** and **c**.

10. The number of books borrowed by 20 students over the period of a month is monitored. Find the mean, median, mode, range and IQR of the data.

12, 8, 6, 10, 4, 5, 2, 7, 8, 0, 6, 4, 8, 13, 5, 3, 0, 8, 7, 9

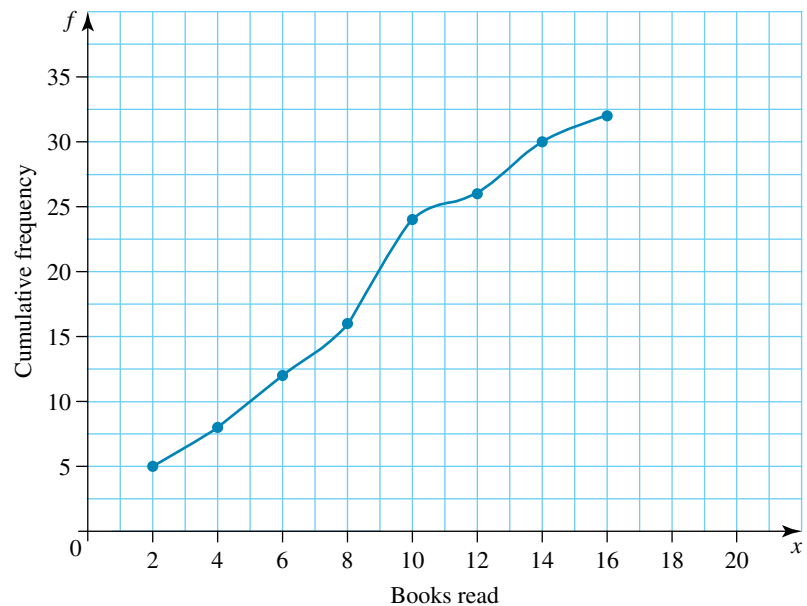
11. Find the mean (to 2 decimal places), median, mode, range and IQR of the following data collected when the temperature of the soil around 25 germinating seedlings was recorded.

28.9, 27.4, 23.6, 25.6, 21.1, 22.9, 29.6, 25.7, 27.4, 23.6, 22.4, 24.6, 21.8, 26.4, 24.9, 25.0, 23.5, 26.1, 23.6, 25.3, 29.5, 23.5, 22.0, 27.9, 23.6



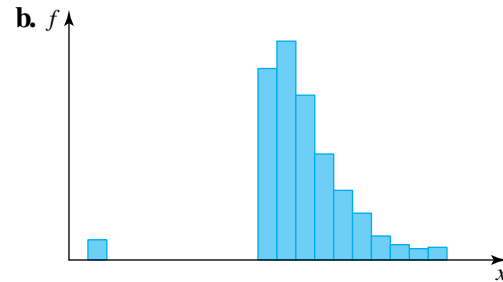
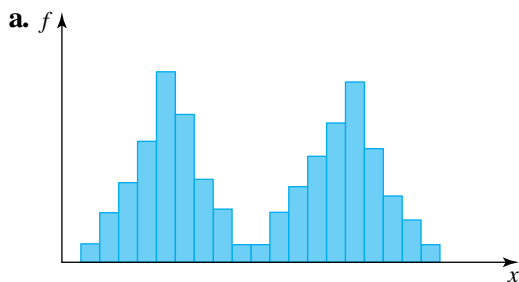
12. A class participated in a read-a-thon. The number of books read in a week by each student was recorded. The cumulative frequency graph of the data is as shown.

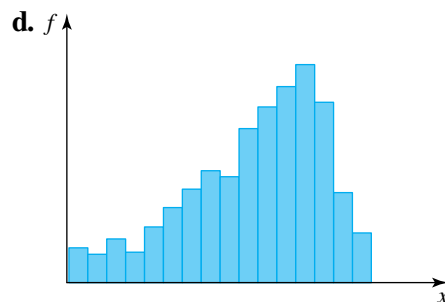
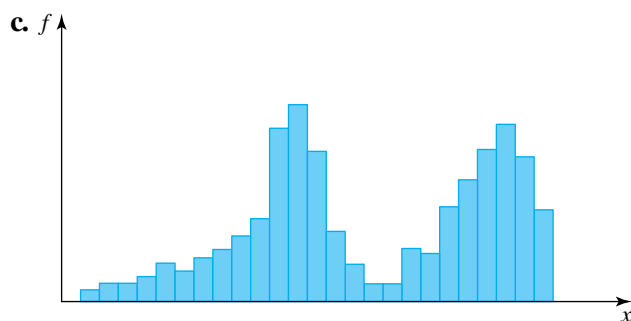
- How many students were in the class?
- Calculate:
 - the median
 - Q_1
 - Q_3 .
- The top 20% of children in the competition were given a prize. How many books would a child need to read in order to be given a prize?



13. The following data shows the number of ice creams sold nightly over two weeks.
4, 6, 48, 29, 39, 48, 44, 45, 39, 47, 48, 32, 31, 51
Determine if there are any outliers.

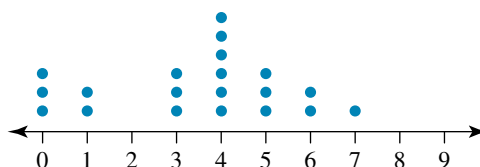
14. Identify the modality and shape of each of the following graphs.





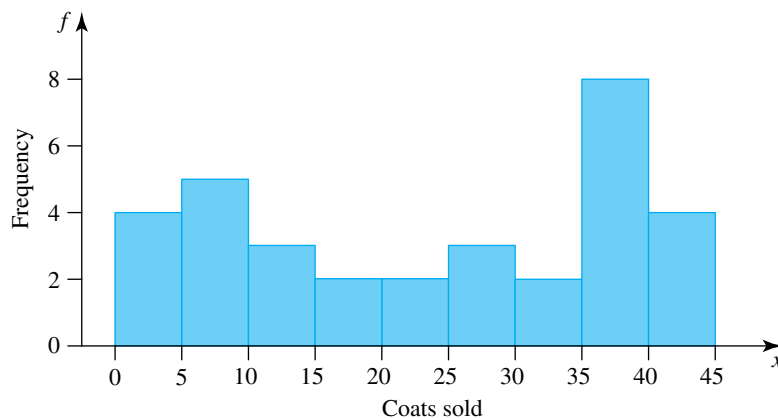
15. For the dot plot at right, calculate:

- the mean (to 2 decimal places)
- the median
- the IQR
- the standard deviation
- the range.



16. The histogram at right shows the number of coats sold in a department store. Using the histogram, calculate the following for coats sold.

- Mean
- Median
- Mode
- Range



17. Consider the following stem plot.
Key: 11|2 = 112

Stem	Leaf
5	0 1 2
6	4 5 8 9
7	0 7
8	0 3 3
9	5 5 5 7 8
10	1 1 2 3 4 6 6
11	0 1 3 3 4 5 5 5 7 9
12	1 1 2 2 3 3 4 7
13	2 6 9

Calculate:

- the mean
- the median
- the IQR
- the standard deviation.

18. a. Using the data from question 17, construct a histogram using a class interval of 10.
b. Show the median, mean and IQR on the histogram.
c. Describe the modality and shape of the histogram.

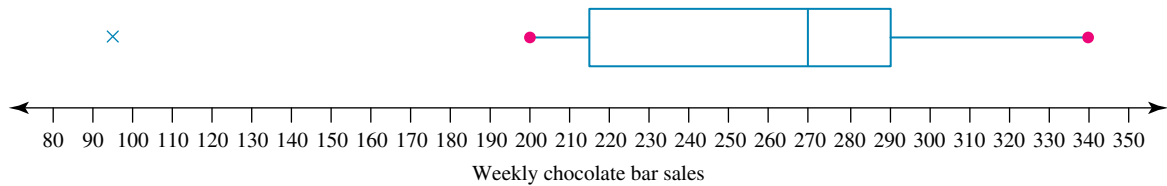
19. Below is a frequency table showing the number of glasses of water people drink in one day. Calculate the five-figure summary from this data.

Number of glasses of water consumed	Frequency
1	1
2	2
3	3
4	4
5	3
6	2
7	5
8	2
15	1

20. The price of a return plane ticket from Melbourne to London from nine different airlines is as follows:

\$3400, \$2800, \$3500, \$4100, \$2900, \$5200, \$3900, \$4000, \$4575

- Calculate the five-figure summary for the data.
 - Calculate the interquartile range.
 - Draw the boxplot to represent this data.
21. The monthly sales of a particular chocolate bar were recorded over the period of a year. A boxplot representing these sales is as shown.

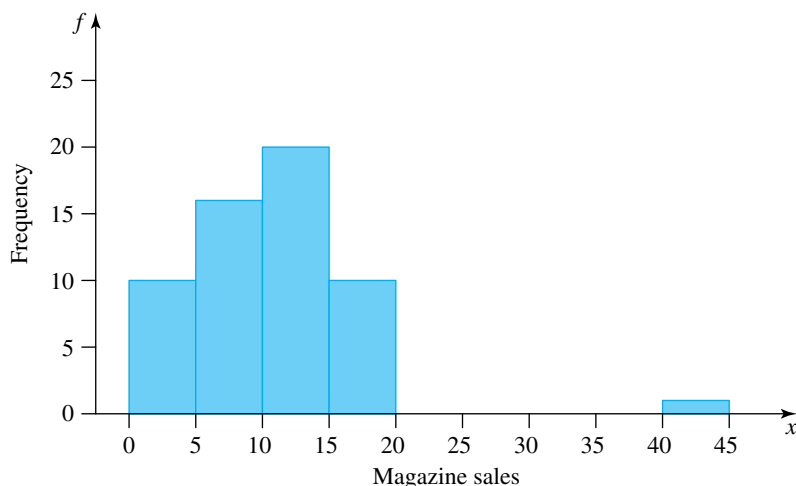


- State the five-figure summary for this data.
 - What is the IQR?
 - Are there any outliers?
22. Use the following two data sets to construct parallel boxplots.
 Data set A: 2, 3, 6, 7, 9, 10, 1, 7, 7, 19
 Data set B: 1, 2, 3, 3, 4, 1, 2, 5, 3, 6
23. Referring to the information and parallel boxplots in question 22, complete the following paragraph.
 The median of data set A was _____ compared to _____ for data set B. For data set A, the middle 50% of the data was between _____ and _____, whereas for data set B it was between _____ and _____. The range of data set A was _____ compared to a range of _____ for data set B. Data set A had _____ outlier(s), whereas data set B had _____ outlier(s).

Problem solving, reasoning and justification

24. A survey of Year 9 students was conducted to find out the average travel time to school. The results (in minutes) were as follows:
 12, 18, 17, 15, 15, 12, 14, 14, 14, 17, 10, 14, 11, 16, 14, 14, 11, 15, 10, 9, 13, 20
- Calculate the mean, median and mode of this data set.
 - Which measure of centre would you choose as the average travel time? Why?

- 25.** A data set contains the numbers 5, 6, 6 and 10. A fifth number is added to the set. If this fifth number is a whole number, determine the possible values of this fifth number if:
- the mean is now equal to the median
 - the mean is now greater than the mode
 - the mean is now greater than the range.
- 26.** Kris and Loren were discussing the following histogram showing the number of magazines sold in a convenience store. Kris said that the best way to describe the central tendency for magazine sales would be to look at the median. Loren disagreed and said to use the mean. Who is correct and why?



- 27.** The data set used to generate the histogram relating to magazine sales in question **26** was as follows.
 0, 1, 1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10, 10, 10, 10, 10, 11,
 11, 11, 11, 11, 11, 11, 12, 12, 12, 12, 13, 13, 14, 14, 15, 15, 15, 15, 16, 17, 18, 18, 18, 19, 44
 Is the 44 magazine sales in a day an outlier? Justify your answer using calculations.
- 28.** Create a data set that has 11 data values with a median of 23, a range of 40, and an IQR of 20.
- 29.** The New South Wales government records data relating to road toll deaths. Is the distribution of male deaths on New South Wales roads with age likely to be symmetric, positively skewed or negatively skewed? Justify your response.
- 30.** The time it took to complete surgeries in a particular hospital was recorded over a one-week period. The times, in minutes, were as follows.
 15, 65, 124, 45, 37, 34, 54, 109, 29, 126, 129, 21, 63, 55,
 22, 99, 55, 98, 114, 107, 71, 128, 101, 41, 59
- Calculate:
 - the mean (to 2 decimal places)
 - the median
 - the IQR.
 - Construct a histogram from the data set using a class interval of 10 minutes.
 - Show the mean, median and IQR on the histogram.
 - Use your answer to part **c** to discuss the modality and shape of the surgical time data.



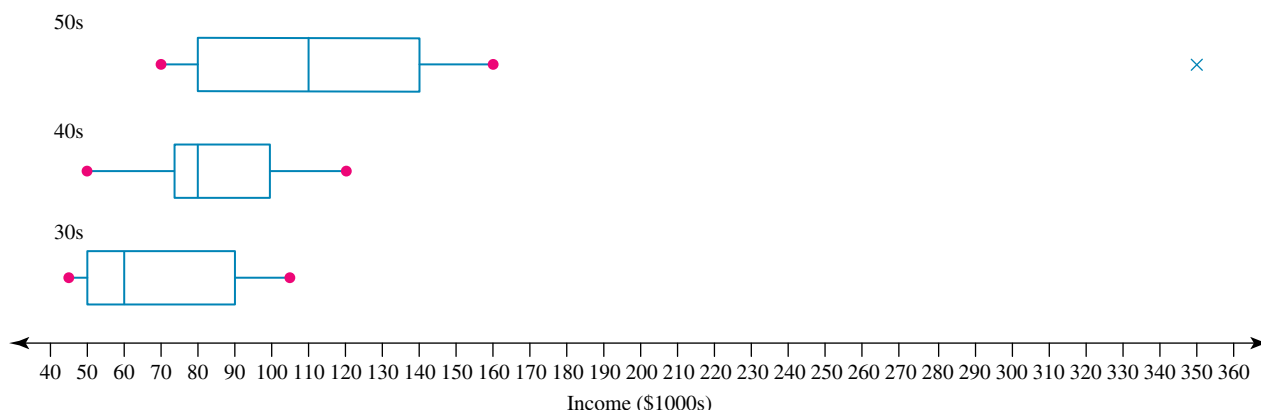
31. The mean of a histogram is 17.5 and the median is 8. The mode is at both 6 and 12. What does this suggest about the data distribution?
32. The number of runs scored per match by two players of a cricket team during 10 matches is recorded below.

Player 1	65	72	78	63	58	74	66	70	57	71
Player 2	42	87	90	96	53	40	44	46	67	51

- a. Draw parallel boxplots for this data.
- b. Comment on what the plots tell you about the number of runs scored by each player.
33. The number of goals scored by two members of a basketball team during 10 matches is recorded below.

Player 1	6	8	12	9	5	15	2	1	10	24
Player 2	6	8	7	8	7	5	8	6	7	7

- a. Draw parallel boxplots for this data.
- b. Which player would you rather have on your team and why?
34. A large company completed a survey of the incomes of their employees. The incomes were recorded for people in their 30s, 40s and 50s. The results are summarised in the parallel boxplots below.
Note: The incomes are written in \$1000s; for example, 110 = \$110 000 and 75 = \$75 000.



- a. **MC** The highest 25% of people in their 30s have an income between:
- A. \$60 000 and \$105 000
 - B. \$50 000 and \$105 000
 - C. \$100 000 and \$120 000
 - D. \$60 000 and \$90 000
 - E. \$90 000 and \$105 000
- b. The middle 50% of people in their 40s earn between what incomes?
- c. How are age and income at this company related?
- d. One of the employees surveyed at the company earns \$46 000. Which age category do they belong to?
- e. The person in their 50s who earns \$350 000 was demoted and now earns \$190 000. Is his income still an outlier? Justify your response.

Answers

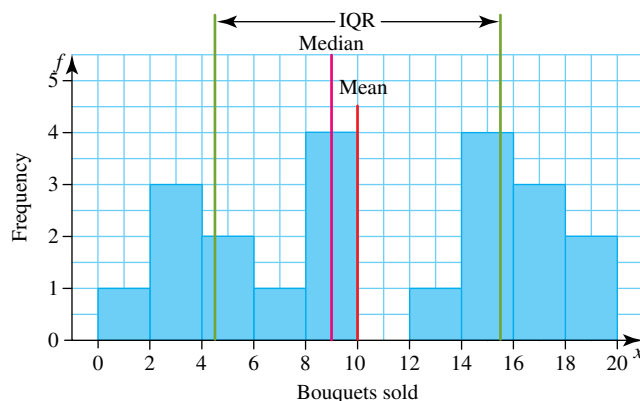
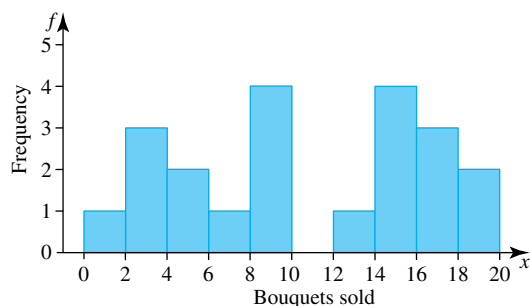
Exercise 12.2 Measures of central tendency

1. a. i. 6 ii. 6 iii. 8
 b. i. 10 ii. 12 iii. 12
 c. i. 26.2 ii. 28 iii. 11, 21, 28, 29, 42
 d. i. 6 ii. 6.5 iii. 9
 e. i. 5 ii. 5 iii. 4, 5
 f. i. 3.66 ii. 3.7 iii. 3.5, 3.8
2. a. 27 b. 26 c. 31
3. Mean: 104 km/h; median: 102 km/h; mode: 101 km/h
4. 47
5. a. Mean: 22.6; median: 23; mode: 23 b. Mean: 7.7; median: 8; mode: 2, 4, 6, 7, 8, 9, 10, 11, 12
 c. Mean: 48; median: 49; mode: 50 d. Mean: 1.9; median: 1.7; mode: 1.4, 1.7
6. a. Mean: 2.4; median: 2; mode: 1 b. Mean: 26.9; median: 27; mode: 26
 c. Mean: 2.6; median: 2.5; mode: 3.0
7. 73.5
8. 171
9. B
10. a. Mean: 2.4; median: 2 b. 21
 c. Mean: 1.9; median: 2 d. Mean value lowered, median remained unchanged
11. a. 2, 3, 3, 4, 5, 5, 5, 6, 7, 8 ii. 5 iii. 5
 b. i. 4.8 ii. 5 iii. 5
12. a. Mean
 b. Median. There are two possible outliers of 8 and 12 goals, which were not typical goal numbers for the netballer. The median is less affected than the mean by these two atypically high scores, so it is the best measure to use.
13. Answers will vary. Example answers are shown.
 a. 2, 3, 3, 3, 4. (Several different data sets are possible.)
 b. 1, 2, 4, 4, 4. (Several different data sets are possible.)
 c. 2, 2, 2, 4, 5. (Several different data sets are possible, but the number in bold must be present in the same position.)
14. 435 cm
15. There would have to be an even number of students so the median would be the average between two test scores.
16. a. 39 b. 40 c. 41 d. 40
 e. The mode, as it gives the highest number of M&M's in a packet
17. The mean of grouped data is calculated using the midpoint of the class interval, making it an approximation. The mean of ungrouped data will give an exact value that is not an approximation. Using both grouped and ungrouped data, a single value can be obtained for the mean.
 The median for grouped data will be a class interval, rather than an exact value as for ungrouped data.
 The mode is calculated the same way for grouped and ungrouped data: the value with the greatest frequency.
18. a. Data set A: mean: 42.8, median: 37; data set B: mean: 39.4, median: 37
 b. The mean of data set A is larger than the mean of data set B. This is due to the last value being 96 in data set A, compared to 66 in data set B. The medians of the two data sets are the same as each data set has the same number of data values and all data values are the same, except the last value.
 c. The mean or median can be used as good measures of central tendency when there are no outliers or extreme values. The median is best to use when there are outliers or extreme values in the data.
19. E
20. a. Student A: mean: 57.9, median: 51; student B: mean: 57.7, median: 64
 b. A mark of 0 brings the mean down but does not affect the median.
 c. Student B. Student B's mean test mark was roughly the same as that of Student A, but Student B's median test mark was higher.

- a. 11
 - b. 2
 - c. 4
- a. 157
 - b. 69
3. Minimum, lower quartile, median, upper quartile, maximum
4. 5
- a. 160
 - i. 110
 - ii. 99
 - iii. 120
 - c. 117 steps/minute
- a. B
 - b. D
 - c. C
9. 15 km/h, 2 km/h
10. Range = 27, IQR = 8
11. 32.0 years
- a. 94
 - b. 21.9
 - c. 23; outliers are 1 and 15
13. 166 is an outlier.
14. Answers will vary. Example answers are shown.
 - a. 2, 5, 17, 21, 29, 35, 39, 45. (Several different data sets are possible.)
 - b. 1, **5**, 6, 8, **12**, 14. (Several different data sets are possible, but the numbers in bold must be present in the same position.)
 - c. 5, 6, 8, 9, 10, 11, 12, 14, 15. (Several different data sets are possible.)
15. The midpoint of the class interval could be used as the data value.
16. 5, 10, **14**, 21, 22, 23, 31, **34**, 35, 40. (Several different data sets are possible, but the numbers in bold must be present in the same position.)
17. Calculate the standard deviation. If the standard deviation is small, then the majority of parks will contain close to 15 trees
18.
 - a. $IQR = 35 - 18 = 17$
Upper fence = $Q_3 + 1.5 \times IQR = 35 + 1.5 \times 17 = 60.5$
Any value above 60.5 will be an outlier, so 64 is an outlier.
 - b. Q_1 , Q_3 , and IQR would remain unchanged.

Exercise 12.4 Comparing graphical distributions

1. B			
2. a. Unimodal	b. Multimodal	c. Multimodal	d. Bimodal
3. a. Symmetric	b. Symmetric	c. Positively skewed	d. Negatively skewed
4. Unimodal, negatively skewed with no outliers			
5. D	6. E	7. B	
8. Mean: 24.4, median: 25–30, mode: 30–35			
9. a. i. 10.1	ii. 9	iii. 11	iv. 5.9
b. The histogram may vary due to the class intervals used			



10. a. i. 10.2 laps

ii. 5–10 laps

iii. 5–10 laps

iv. 45

v. Q_1 is in the 5–10 laps class interval; Q_3 is in the 15–20 laps class interval.

b. The data point in the 40–45 class interval is a possible outlier.

11. Outliers, on a histogram, will usually have a low frequency, and there will be several class intervals with zero data separating the outlier from the main cluster of data.

12. The histogram is likely to be negatively skewed.

13. Answers will vary. Example answers are shown.

a. Peak hours in a coffee shop

b. The age at which people retire

14. Darwyn is correct. The stem plot is roughly symmetric about the 90–99 class interval with two possible outliers of 1 and 10.

15. The two histograms show that Class A had a larger range of examination marks (100%) than Class B (60%). The median and modal examination mark for both classes was in the 60–70% class interval. However, the mean examination mark for Class A (56%) was lower than for Class B (71%). Overall, Class B achieved better examination results than Class A.

16. The median could be 13.75. The 8th value lies in the 10–15 class interval, which contains 4 data values; the 8th value will be the 3rd value in that group of 4 data values.

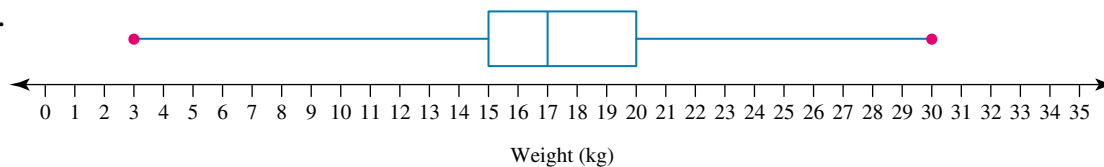
Exercise 12.5 Parallel boxplots

1. 3, 8, 9, 10, 12

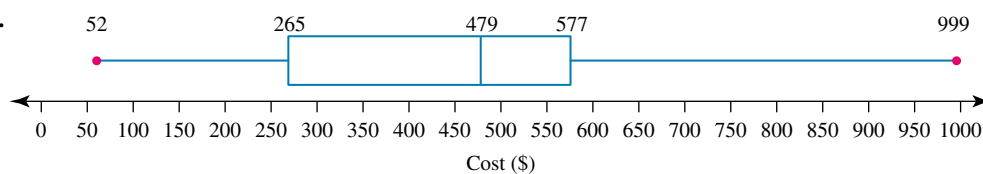
2. 17, 21, 33, 47, 58

3. A

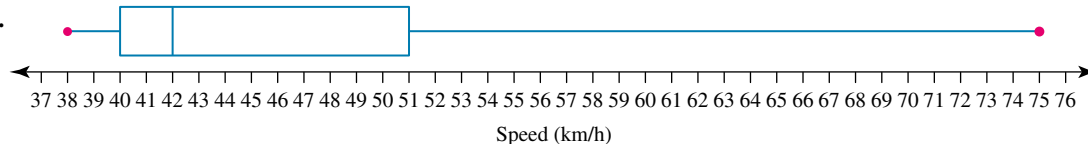
4.



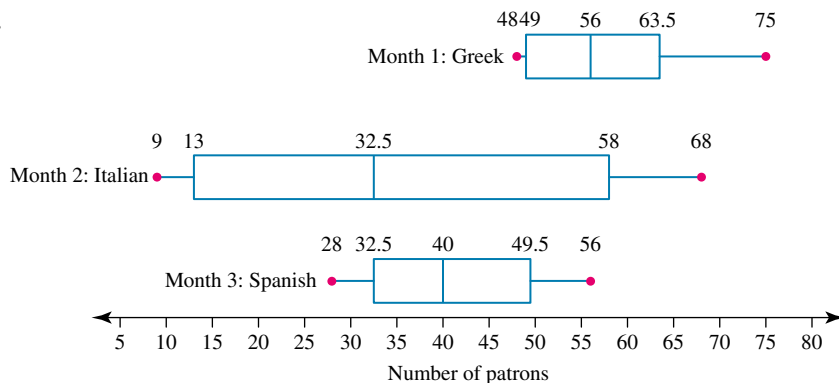
5.



6.



7. a.



b. The parallel boxplots show that the median number of patrons that preferred the Greek cuisine in month 1 (56) was higher than the median of those that preferred Italian in month 2 (32.5) and Spanish in month 3 (40).

The IQR was smallest in month 1 (14.5) and largest in month 2 (45). The range was also smallest in month 1 (27) compared with the range in month 2 (59) and month 3 (28).

Overall, patrons preferred Greek food over Italian and Spanish food, as month 1 Greek had a larger median and smaller measures of spread compared to the other two months. The Greek menu will therefore be the most successful.

8. B

9. B

10. A

11. a. The data set is likely to be asymmetric.

b. The data are more clustered between the median and upper quartile (less spread) than between the median and the lower quartile.

c. If the median is halfway between the upper and lower quartiles, the data set has a symmetric distribution; if it is not, then the data set is asymmetric.

12. a. Peanut butter: 2, 16, 19, 21, 26; Nutella: 4, 7, 11, 13, 25

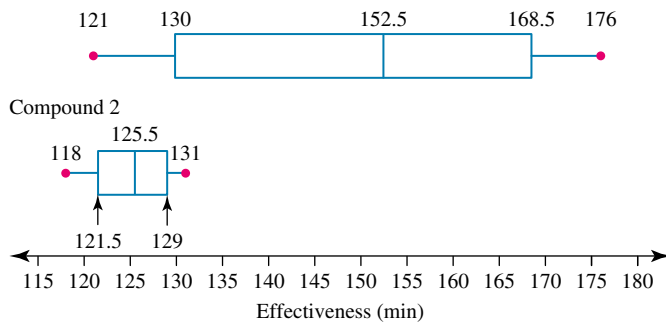
b. i. 16

ii. 7 and 13

c. The parallel boxplots show that the median weekly jar sales were much higher for peanut butter (19 jars) than for Nutella (11 jars). The IQR was approximately the same for both peanut butter and Nutella (5 and 6 jars, respectively), whereas the range was larger for peanut butter (24 jars) than for Nutella (21 jars). Both boxplots have outliers (2 jars/week for peanut butter and 25 jars/week for Nutella). Overall, the weekly jar sales for peanut butter were greater than for Nutella.

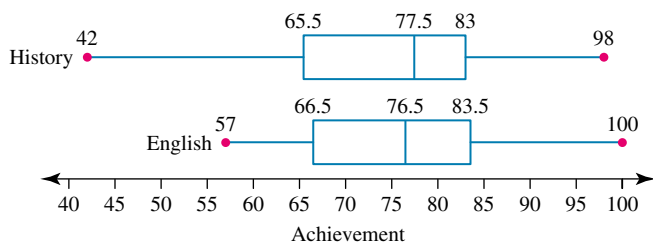
13. a.

Compound 1



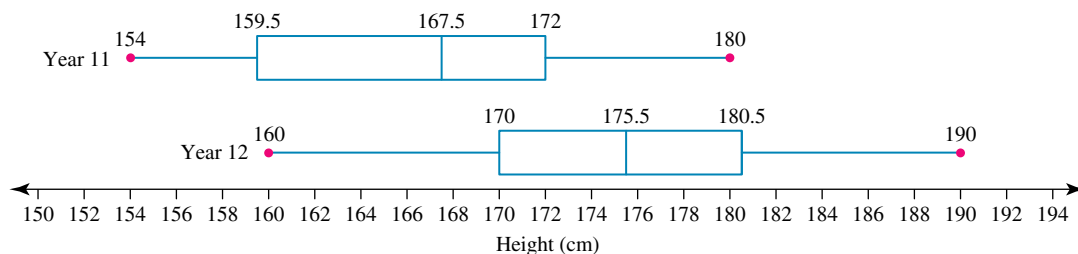
b. The parallel boxplots show that the median minutes of effectiveness of the compounds in water was much higher for compound 1 (152.5 min) than for compound 2 (125.5 min). The IQR and ranges were also much higher for compound 1 (38.5 min and 55 min, respectively), showing more variation in minutes of effectiveness than compound 2 (7.5 min and 13 min, respectively). Neither boxplots have outliers. Overall, compound 1 was more effective in water before needing to be reapplied than compound 2.

14. a.



b. The results for Year 10 students' achievements in History and English are fairly similar, with the range being the only statistic showing an obvious difference. The range for History was 56; for English it was 43.

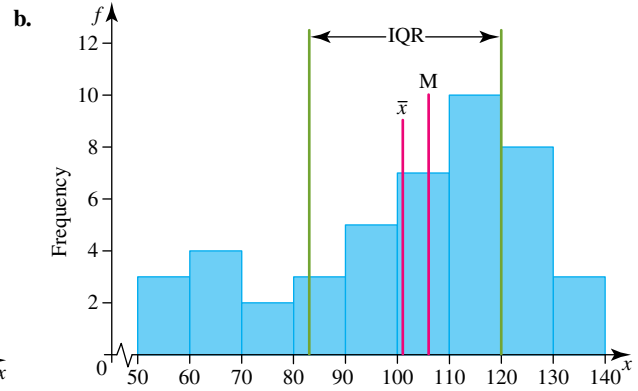
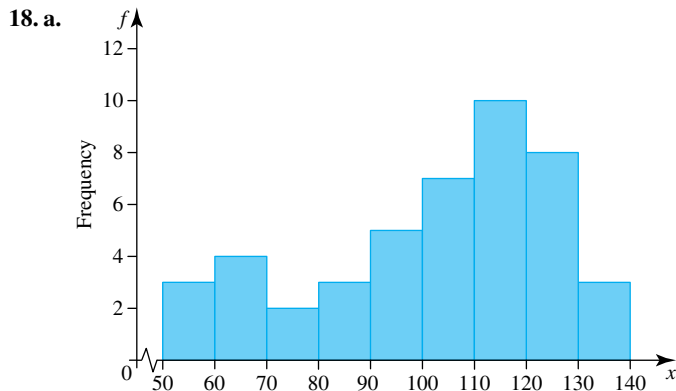
15. a.



b. The parallel boxplots show that the median student height was greater for Year 12 students (175.5 cm) than for Year 11 students (167.5 cm). The IQR and range for both Year 11 (12.5 cm and 26 cm, respectively) and Year 12 (10.5 cm and 30 cm, respectively) students was approximately the same, showing that the spread of heights in a year level are roughly equal. Neither boxplots have outliers. Overall, height increased with year level.

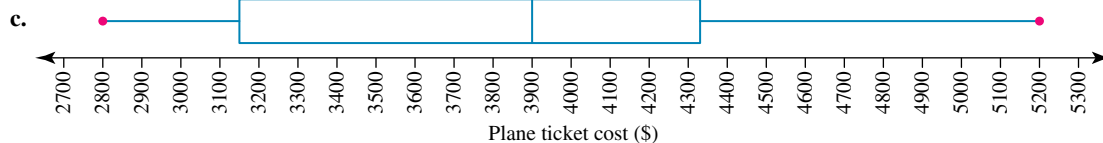
Exercise 12.6 Review

1. a. Mean: 5.3; median: 6; mode: 6
b. Mean: 36.4; median: 32.5; mode: 11, 24, 25, 29, 36, 42, 61, 63
c. Mean: 9; median: 10; mode: 10
2. a. 16 days b. Mean: 7.2; median: 7
3. C 4. B 5. D 6. D
7. 96.4 minutes
8. a. Range b. Median c. IQR
d. Mean e. Standard deviation f. Mode
9. a. Minimum: 6; maximum: 36; range: 30
b. Mean: 14.9; median: 14
c. Possible outlier 36; mean: 13.4; median: 14
d. The median was unchanged by removing outlier, whereas the mean decreased.
10. Mean: 6.25; median: 6.5; mode: 8; range: 13; IQR: 4
11. Mean: 25.0; median: 24.9; mode: 23.6; range: 8.5; IQR: 3.4
12. a. 32
a. i. 8 ii. 4 iii. 10
c. 12
13. 4 is an outlier.
14. a. Bimodal, symmetric b. Unimodal, positively skewed
c. Bimodal, negatively skewed d. Unimodal, negatively skewed
15. a. 3.45 b. 4 c. 3 d. 2.1 e. 7
16. a. 22.8 b. 20–25 class interval c. 35–40 class interval d. 45
17. a. 101.0 b. 106 c. 37 d. 23.5

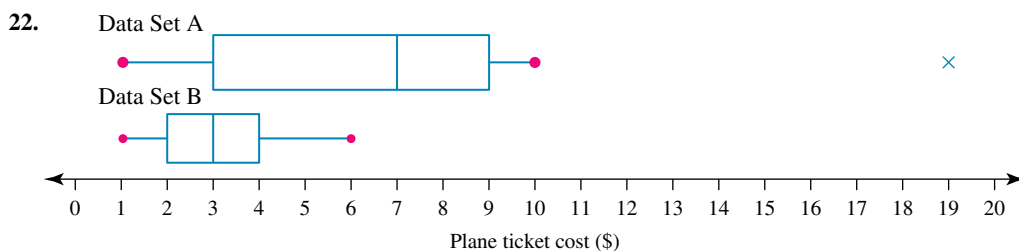


- c. Unimodal, negatively skewed
19. 1, 3, 5, 7, 15
20. a. 2800, 3150, 3900, 4337.5, 5200

b. 1187.5



21. a. 95, 215, 270, 290, 340 b. 75 c. Yes, 95



23. The median of data set A was 7 compared to 3 for data set B. For data set A, the middle 50% of the data was between 3 and 9, whereas for data set B it was between 2 and 4. The range of data set A was 18 compared to a range of 5 for data set B. Data set A had one outlier, whereas data set B had no outliers.

24. a. Mean: 13.9; median: 14; mode: 14

b. Any measure (mean, median, or mode) could be used, as they all give similar results. However, average usually refers to the mean.

25. a. 3

b. Any number greater than 3

c. Any number from 4 to 12

26. Kris. There is a possible outlier at 40–45, so the median is a better measure of central tendency, as it is less affected by outliers than the mean.

27. 40–45 is above the upper fence, so the data value in this class interval (44) is an outlier.

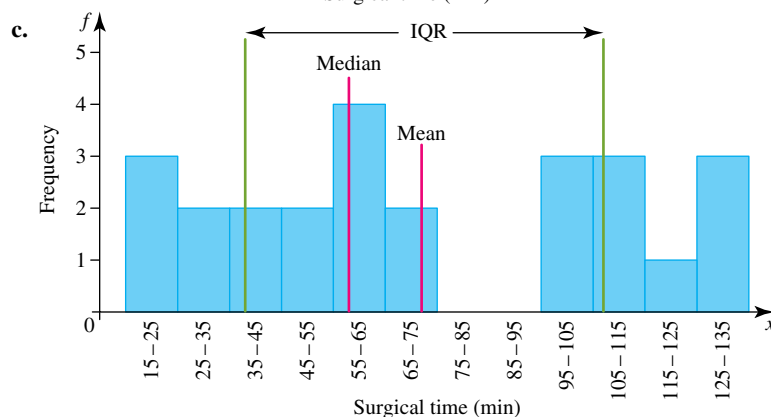
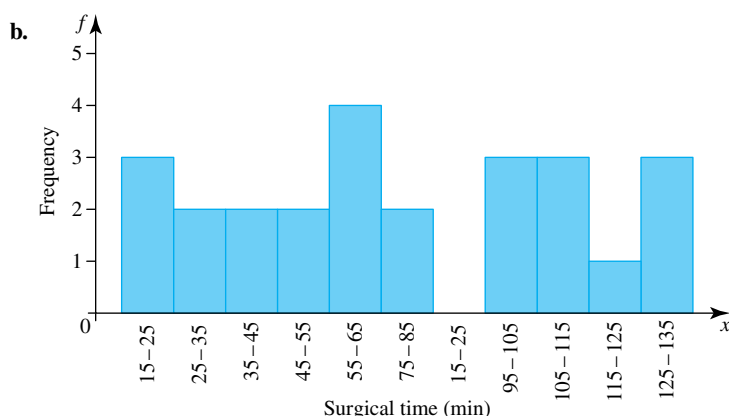
28. Answers will vary. Possible answer: 2, 3, 10, 12, 15, 23, 24, 25, 30, 36, 42

29. Positively skewed. Male deaths on roads are likely to peak around 18–30-year-olds and then decrease with increasing age.

30. a. i. 72.04 min

ii. 63 min

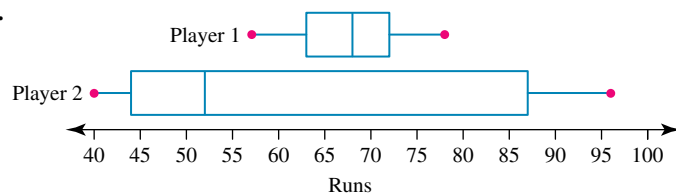
iii. 69



d. Unimodal, roughly symmetric

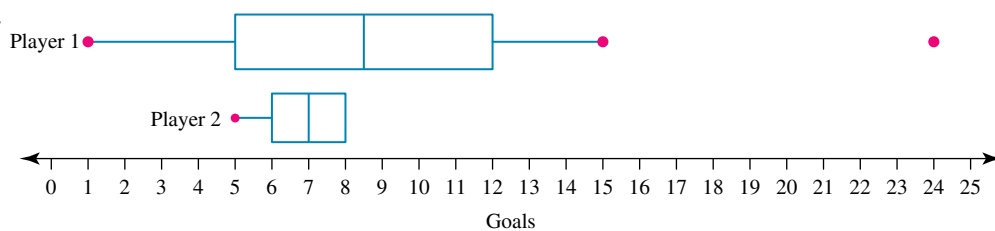
31. Bimodal, positively skewed

32. a.



b. The parallel boxplots show that the median number of runs scored per match for Player 1 (68) was much higher than for Player 2 (52). Player 2 had a much larger spread (IQR = 43, range = 56) in number of runs scored per match than Player 1 (IQR = 9, range = 21).

33. a.



b. Player 1. Higher median number of goals kicked (8.5) than Player 2 (7). Also, over 50% of the number of goals kicked by Player 1 across the ten matches lie above the maximum number of goals kicked by Player 2 across the 10 matches.

34. a. E

b. \$75 000 and \$100 000

c. Generally, salary increases with age.

d. 30s

e. \$190 000 is lower than the upper fence, so it is no longer an outlier.

TOPIC 13

Relative frequency and probability

13.1 Overview

13.1.1 Introduction

We come across probabilities in the media all the time; for example, ‘There is a 50% chance it will rain tomorrow’ and ‘The Sea Eagles have a $\frac{1}{10}$ chance to win the next Premiership’.

Being able to calculate the probability of an event allows us to determine how likely it is to occur. Probabilities are used in weather forecasting, in calculating the price of insurance premiums and in determining the likelihood of winning games. It pays to know your probability!



DISCUSSION

Are you able to calculate a theoretical or experimental probability for any event?

LEARNING SEQUENCE

- 13.1** Overview
- 13.2** Theoretical probability
- 13.3** Multi-stage experiments
- 13.4** Experimental probability
- 13.5** Review

CURRICULUM CONTENT

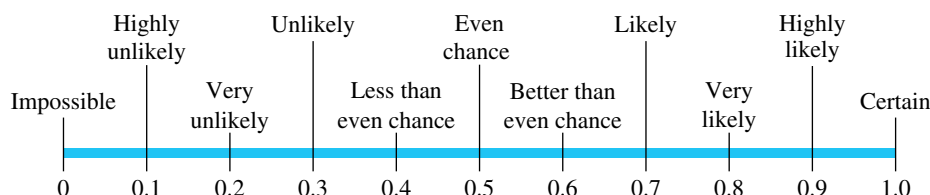
Students:

- review, understand and use the language associated with theoretical probability and relative frequency ◇ 🎓
- determine the probabilities associated with simple games and experiments (ACMEM157) ◇
- use arrays and tree diagrams to determine the outcomes and probabilities for multi-stage experiments (ACMEM156) **AAM**
- solve problems involving simulations or trials of experiments in a variety of contexts **AAM** ◇
- solve problems involving probability and/or relative frequency in a variety of contexts **AAM** 📱

13.2 Theoretical probability

13.2.1 The probability scale

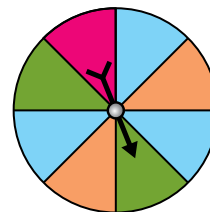
- Probability is a measure of the likely occurrence of an event.
- The probability of an event occurring is measured with a scale ranging from and including 0 (**impossible** event) to 1 (**certain** event).



- A probability can be written as a decimal number or as a fraction in simplest form.
- The probability of an event (A) occurring can be denoted by $P(A)$.
- This means that $0 \leq P(A) \leq 1$, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty.

13.2.2 Sample spaces

- An **outcome** is a particular result of an experiment.
- A **sample space** is a list of all possible outcomes.
- If an outcome appears more than once, it is counted only once and written once in the sample space.
- A sample space is often listed as a set of values. For example, the sample space for the spinner at right is {pink, blue, green, orange}; the sample space includes all of the colours on the spinner.



WORKED EXAMPLE 1

For each of the following experiments, list the sample space to show the total possible outcomes.

a A coin is flipped.

b An arrow is shot at a target where there is a blue area, a white area, a red area and a black area.

c A circular spinner with seven sectors that are labelled 1 to 7 is spun.

THINK

- a** A coin has two sides: Head and Tail. List each possible outcome.
- b** There are four possible areas that the arrow could hit on the target: blue, white, red and black. List each possible outcome.
- c** There are seven different sectors on the spinner so there are seven possible outcomes. List each possible outcome.

WRITE

When a coin is flipped, the two possible outcomes are Head and Tail.

When an arrow is shot at the target, the four possible outcomes are blue, white, red and black.

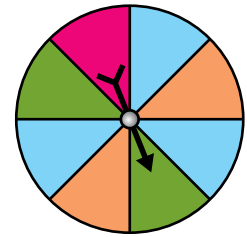
When the circular spinner is spun, the seven possible outcomes are 1, 2, 3, 4, 5, 6 and 7.

13.2.3 Calculating theoretical probability

- A **favourable outcome** is an outcome that you want or are looking for.
- The theoretical probability of an event can be calculated using the following probability formula.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- Probabilities can be expressed as fractions, decimals or percentages. In this topic, unless otherwise specified, we will express probabilities as fractions.
- **Equally likely outcomes** have the same chance of occurring.
 - For the spinner at right, $P(\text{blue}) = \frac{3}{8}$ because there are three favourable outcomes (blue sectors) out of eight possible outcomes (total number of equal sectors on the spinner).
 - Orange and green are equally likely outcomes as they both occupy two of the eight sectors on the spinner. $P(\text{orange}) = P(\text{green}) = \frac{2}{8}$ or $\frac{1}{4}$.
 - Pink is the least likely outcome as it occupies only one of the eight sectors on the spinner. $P(\text{pink}) = \frac{1}{8}$. Pink has the smallest value for its theoretical probability.



WORKED EXAMPLE 2

A standard six-sided die is rolled.

What is the probability of rolling:

a a 3

b an odd number?



THINK

- a
- 1 The number 3 occurs once on a die.
 - 2 The possible outcomes are 1, 2, 3, 4, 5 and 6.
 - 3 Calculate the probability of rolling a 3.

WRITE

Number of favourable outcomes = 1

Number of outcomes = 6

$$P(3) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ = \frac{1}{6}$$

- b
- 1 The odd numbers are 1, 3 and 5. There are six numbers on a die.
 - 2 Calculate the probability of rolling an odd number. Write the fraction in simplest form.

Number of favourable outcomes = 3
Number of outcomes = 6

$$P(\text{odd number}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ = \frac{3}{6}$$

The probability of rolling an odd number is $\frac{1}{2}$.

13.2.4 The complement of an event

- The probability of the **complement** of an event is the probability that the event discussed will *not* happen. For example, if an event is 'it will rain today' with a probability of 70%, then the complement of this is 'it will not rain today' with a probability of 30%. The other way to interpret the complement is to recognise it as the opposite of the event.
- If the probability of an event is denoted by $P(R)$, then the probability of the complement is denoted by $P(R')$.
- In order to calculate $P(R')$ it is necessary to understand that the sum of the probability of an event and its complement is equal to 1.

$$P(R) + P(R') = 1$$

This equation can be manipulated so it is expressed in terms of the complement:

$$P(R') = 1 - P(R)$$

- For example, if the probability of rain today is 0.7, then:
 $P(R') = 1 - 0.7$
 $= 0.3$
 This means there is a 30% chance that it will not rain.

WORKED EXAMPLE 3

If you randomly choose one number from the set $L = \{1, 2, 3, 4, 5, 6, 7\}$, what is the probability of not picking an even number?

THINK

- There are 3 even numbers (2, 4 and 6) out of 7 possible outcomes.
- Use the formula to find the complement.

WRITE

$$P(\text{even}) = \frac{3}{7}$$

$$\begin{aligned} P(\text{not even}) &= 1 - P(\text{even}) \\ &= 1 - \frac{3}{7} \\ &= \frac{4}{7} \end{aligned}$$

eBookplus RESOURCES

Interactivity: Probability scale (int-3824)

Interactivity: Sample spaces and theoretical probability (int-3826)

Interactivity: Theoretical probability (int-6081)

Exercise 13.2 Theoretical probability

Note: Unless otherwise stated, express probabilities as fractions.

Understanding, fluency and communicating

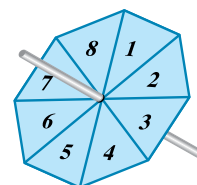
- Match the words below with one of the numbers between 0 and 1 that are given. Choose the number depending on what sort of chance the word indicates, between *impossible* and *certain*. You may use a number more than once. If you are unsure, discuss your choice with a classmate. The numbers to choose from are 1, 0.75, 0.5, 0.25 and 0.

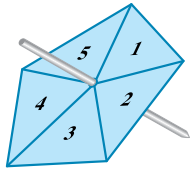
a. certain	b. likely	c. unlikely	d. probable
e. improbable	f. definite	g. impossible	h. slim chance
i. sure thing	j. doubtful	k. fifty-fifty	l. more than likely
- WE1** For each of the following experiments, list the sample space to show the total possible outcomes.
 - A standard six-sided die is rolled.
 - A marble is randomly selected from a bag containing 4 green, 2 yellow and 3 blue marbles.
 - A letter is selected from the letters of the alphabet.
- WE2** A standard 6-sided die is rolled. What is the probability of rolling:

a. a 5	b. a number less than 3	c. an even number?
- The letters of the word *MATHEMATICS* are each written on a small piece of card and placed in a bag. If one card is selected from the bag, what is the probability that it is:

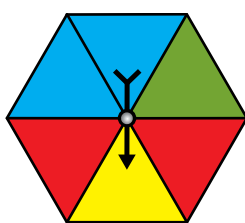
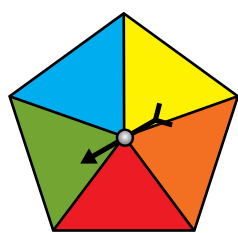
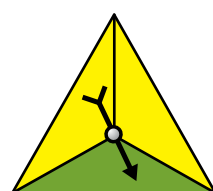
a. a vowel	b. a consonant	c. the letter M	d. the letter C?
- A spinner of 8 equally likely numbers is spun. The spinner is numbered 1–8.
 - List the sample space.
 - What is the probability, expressed as a percentage, of obtaining:

i. an odd number	ii. a number that is less than 6
iii. the number 9	iv. a number that is at most 8?



6. A normal pack of 52 playing cards is well shuffled and one card is drawn.
- List the sample space.
 - What is the probability of being dealt:
 - a heart
 - a red card
 - an ace of diamonds
 - a picture card (jack, queen or king)?
7. **MC** The probability of obtaining a number less than 5 on the spinner at right is:
- 1
 - 0
 - $\frac{1}{5}$
 - $\frac{4}{5}$
 - None of these
- 
8. What is the complement of each of these events?
- Owning a dog
 - Flipping a coin and getting a Tail
 - Sleeping
 - Attending school
 - Passing your learner knowledge test
 - Owning a mobile phone
9. **WE3** Wayne's Physics teacher told him that he had a 20% chance of passing the final exam, based on Wayne's prior exam scores.
- What is the complement of this event?
 - What is the probability of the complement of this event? Give your answer as a percentage.
10. If two dice are rolled, what are the probabilities that the sum of the uppermost numbers is:
- 5
 - not 5
 - 6 or 7
 - not 6 or 7
 - 2
 - not 2?
11. There are three different colours of flowers in a basket: red, yellow, and purple. Assuming that you pick one flower from the basket, the complement of picking a red flower is picking a purple flower. True or False? Explain your answer.
- 

Problem solving, reasoning and justification

12. List two events that have a probability of:
- 0
 - 0.5
 - 1
 - 0.75
 - 0.25.
13. Answer these questions for each of the following spinners.
- Is there an equal chance of landing on each colour? Explain.
 - List all the possible outcomes.
 - Find the probability of each outcome.
- 
 - 
 - 
14. All the jelly beans in the photograph at right are placed in a bag for a simple probability experiment.
- Which colour jelly bean is most likely to be selected from the bag? Explain.
 - Which colour jelly bean is least likely to be selected from the bag? Explain.
 - Find the probability of selecting each coloured jelly bean from the bag.
- 

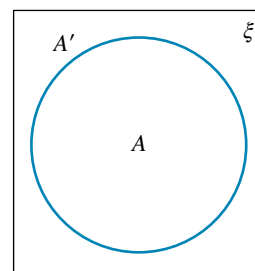
15. Draw spinners with the following probabilities.

a. $P(\text{blue}) = \frac{1}{3}$ and $P(\text{white}) = \frac{2}{3}$

b. $P(\text{blue}) = \frac{1}{2}$, $P(\text{white}) = \frac{1}{4}$, $P(\text{green}) = \frac{1}{8}$ and $P(\text{red}) = \frac{1}{8}$

c. $P(\text{blue}) = 0.75$ and $P(\text{white}) = 0.25$

16. Refer to the Venn diagram at right. If set $A = \{1, 2, 8, 9, 15\}$ and the universal set $\xi = \{1, 2, 3, 4, 5, 8, 9, 10, 12, 13, 15\}$, what is A' (the complement of A)?



13.3 Multi-stage experiments

13.3.1 Arrays

- When two events occur to form a single result, an **array** can be used to display the sample space.
- Consider an experiment in which a red die and a blue die are rolled, and the two numbers that land uppermost are recorded. The array below shows the sample space for that experiment. There are 36 possible outcomes in this sample space.

		Blue die					
		1	2	3	4	5	6
Red die	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

- An array is also useful when there are two events but there are a different number of outcomes for each event. For example, the array below shows the sample space for an experiment where a coin is flipped and a die is rolled.

		Die					
		1	2	3	4	5	6
Coin	H	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
	T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

WORKED EXAMPLE 4

Two dice are rolled and the numbers that appear uppermost are recorded. What is the probability of rolling at least one 1?

THINK

- 1 • Draw the array for the sample space.
- Highlight or circle the outcomes that have at least one 1, as shown in green.
 - There are 11 outcomes with at least one 1.

WRITE

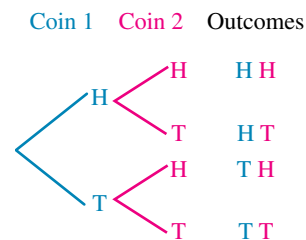
		Die 2					
		1	2	3	4	5	6
Die 1	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

- 2 Calculate the probability using the formula. There are 36 outcomes in the sample space.

$$P(\text{at least one 1}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} = \frac{11}{36}$$

13.3.2 Tree diagrams

- Tree diagrams** are used to list all possible outcomes of two or more events. The branches show the possible links between one outcome and the next outcome. The tree diagram at right represents the sample space for flipping two coins.
- When all outcomes are equally likely, probabilities can be calculated from tree diagrams. For example, in the tree diagram shown, the probability of obtaining one Tail and one Head can be calculated:



$$P(\text{one H and one T}) = \frac{\text{number of outcomes with one H and one T}}{\text{total number of outcomes}} = \frac{2}{4} = \frac{1}{2}$$

- Each event is treated as if it were a separate event, even if events occur at the same time. For example, the tree diagram above applies to experiments where:
 - two coins are flipped together
 - the same coin is flipped twice.

WORKED EXAMPLE 5

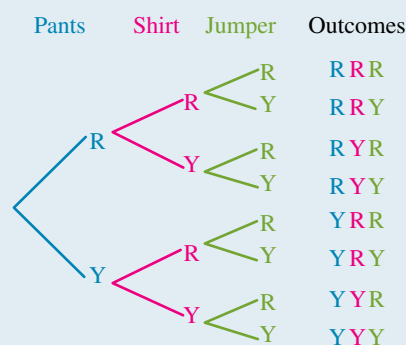
The uniform committee at school is deciding on a new colour combination for the school uniform. The two colour choices for the school pants, shirt and jumper are red and yellow.

- Use a tree diagram to show all the possible combinations.
- Calculate the probability that the uniform will consist of only one colour.
- Calculate the probability that a red jumper will be part of the uniform.

THINK

- The three events are the colour of the pants, the colour of the shirt and the colour of the jumper. There are two choices for each of these.

WRITE



- For the uniform to consist of only one colour, all pieces need to be red or all pieces need to be yellow. There is one outcome with all red and one outcome with all yellow. There are eight outcomes altogether.
- Four of the outcomes have a red jumper.

$$P(\text{all one colour}) = \frac{2}{8} = \frac{1}{4}$$

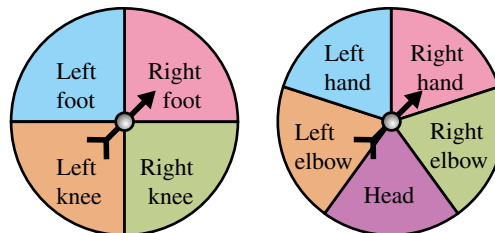
$$P(\text{red jumper}) = \frac{4}{8} = \frac{1}{2}$$

Exercise 13.3 Multi-stage experiments

Note: Unless otherwise stated, express probabilities as fractions.

Understanding, fluency and communicating

1. **WE4** In the game of Pretzel, the two spinners shown are spun and each contestant places the two body parts spun in contact and holds them there until the next two body parts are spun. If you fall over, you are out. The winner is the last person standing.



When the spinners are spun, what is the probability, expressed as a percentage, that the contestants will need to place:

- their right foot on their head
 - their left elbow on a knee
 - their head on a foot
 - their left hand on a body part lower than their belly button?
2. **WE5** A restaurant has the menu shown.
- Use a tree diagram to show how many different combinations are possible for a meal with one selection at each course.
 - If a person selects a dinner at random, what is the probability that:
 - they have soup for entrée
 - they have spring rolls and curry
 - they don't have ice-cream for dessert
 - they have soup and pasta but not ice-cream?
3. For a fundraiser, your class is selling single-scoop ice-cream cones at lunchtime. There is a choice of vanilla and chocolate ice-cream, topped with chocolate curls, sprinkles or melted chocolate or no topping at all.
- Draw a tree diagram to show all the possible combinations of ice-cream and topping that you could have from the stall.
 - If a person chooses an ice-cream at random, calculate the probability that:
 - both the ice-cream and the topping are chocolate
 - it is a vanilla ice-cream with sprinkles
 - there is no topping on the ice-cream.
4. In your drawers at home, there are two white T-shirts, a green T-shirt and a red T-shirt. There is also a pair of black pants and a pair of khaki pants.
- Draw a tree diagram to show all the possible combinations of T-shirts and pants that you could wear.
 - If you get dressed in the dark and put on one T-shirt and one pair of pants, calculate the probability that you put on the red T-shirt and khaki pants.

Entrée

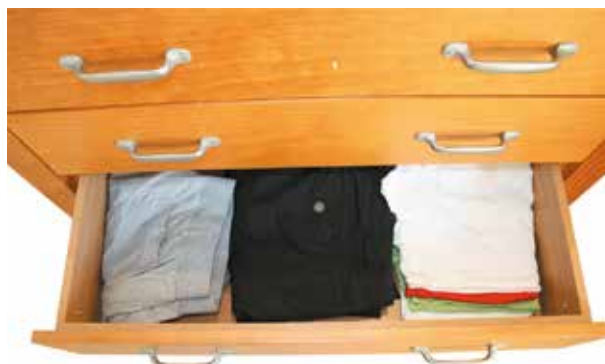
Pumpkin soup
Mini spring rolls

Main course

Pasta bolognaise
Grilled fish
Beef curry

Dessert

Lemon pie
Ice-cream



5. In the last Science test, your friend guessed the answers to three True/False questions.
- Use a tree diagram to show all the different answer combinations for the three questions.
 - Calculate the probability that your friend:
 - got all three answers correct
 - got two correct answers
 - got no correct answers.
6. Assuming that the chance of a baby being a boy or a girl is the same:
- Calculate the probability that a family with three children has:
 - all boys
 - two girls and one boy
 - three children of the same gender
 - at least one girl
 - two children of the same gender.
 - If the family was expecting another baby, what is the probability that the new baby will be a boy?
 - If the family already has three boys, what is the probability that the new baby will be a boy?
 - If the family has three girls, what is the probability that the new baby will be a boy?
 - How likely is the combination of children shown in the photo at right?
7. A spinner is divided into 5 sections (red, blue, green, yellow and orange). A six-sided die is also rolled. List the sample space for one spin of the spinner and one roll of the die.
8. **MC** A six-sided die and a coin are tossed at the same time. The sample space is:
- { 1, 2, 3, 4, 5, 6, 7, 8 }
 - { 1, 2, 3, 4, 5, 6, H, T }
 - { (1, H), (2, H), (3, H), (4, H), (5, H), (6, H) }
 - { (1, H), (2, H), (3, H), (4, H), (5, H), (6, H), (1, T), (2, T), (3, T), (4, T), (5, T), (6, T) }
 - None of the above
9. How many *final branches* in a tree diagram would there be in an experiment of:
- flipping 5 coins
 - flipping 6 coins
 - rolling 4 dice?
10. A fair coin is flipped 3 times. Calculate the probability of obtaining:
- at least two H or at least two T
 - exactly two T.
11. **MC** A fair coin is flipped 3 times. The probability of obtaining at most 1 Tail is:
- $\frac{1}{3}$
 - $\frac{1}{2}$
 - $\frac{3}{8}$
 - $\frac{1}{8}$
 - $\frac{2}{3}$
12. A spinner of 7 equally likely numbers (numbered 1–7) is spun twice and the two numbers are added. Calculate the probability of obtaining a total of:
- 2 or 14
 - 9
 - at least 12.
13. Two dice are rolled and the product of the two numbers is found. Calculate the probability that the product of the two numbers is:
- an odd number
 - a prime number
 - more than 1
 - at most 36.
14. A fair die is rolled and a fair coin flipped. Calculate the probability of obtaining:
- an even number and a Head
 - a Tail from the coin
 - a prime number from the die
 - a number less than 5 and a Head.



Problem solving, reasoning and justification

15. a. Use a table to display the sample space for an experiment where two dice are rolled and the sum of the two numbers appearing uppermost is found.
 b. List any patterns you found in the sample space.
 c. Copy and complete the following table.

Sum of two dice	2	3	4	5	6	7	8	9	10	11	12
Probability											

- d. List any patterns that you notice in the probabilities you found in the table above.
16. In a new game, two dice are rolled and the difference between the two dice is noted. If the difference is larger than 2, you win; otherwise you lose.
 a. Use a table to find the sample space for the game.
 b. Calculate the probability of winning (having a difference greater than 2).
 c. Calculate the probability of losing (having a difference of 2 or less).
 d. A fair game is one in which the chances of winning are the same as the chances of losing. Is this game fair?
17. The game rock, paper, scissors is sometimes used to make a decision between two people. It uses the three different hand signs shown below. On the count of 3, each player displays one of the hand signs. If both players display the same hand sign then the game is a tie. The photos below right show all other possible results.
 a. Draw up a table like the one below to show all the possible combinations for a single game of rock, paper, scissors.

		Person 2		
		Rock	Paper	Scissors
Person 1	Scissors			
	Paper			
	Rock			



Rock breaks scissors

Rock wins



Paper covers rock

Paper wins



Scissors cut paper

Scissors wins

- b. Use the rules given to circle the winner for each combination in the table.
 c. Calculate the probability that:
 i. rock wins
 ii. paper wins
 iii. scissors wins.
 d. Is rock, paper, scissors a fair game?
 e. A friend suggested that you add dynamite to the game with the following rules.
 • Dynamite blows up the rock and wins.
 • Dynamite has its fuse cut by scissors and loses.
 • Dynamite sets the paper on fire and wins.
 i. Modify your table in part a to include dynamite.
 ii. Does each hand sign have an equal chance of winning?
 iii. Support your answer to part ii using the table and probabilities.

-
- ```
graph LR; School[School] -- A --> Shops(Shops); School -- B --> Shops; School -- C --> Shops; Shops -- D --> Library[Library]; Shops -- E --> Library; Library -- F --> Home[Home];
```

**a.** Use a tree diagram to calculate the number of different routes you could use on your journey.

represent the different routes from school to the shops and D and E to represent the different routes from the shops to the library. Use F to represent the route from the library to home.)

- TOPIC 13**
- Relative frequency and probability
- 463**

# 13.4 Experimental probability

## 13.4.1 Relative frequency

- Often in real life it is not possible to calculate a theoretical probability for an event. In these situations, an experiment is carried out.
- An experiment that is repeated a number of times is called a **trial** and becomes part of a larger experiment. For example, a die is rolled and the number uppermost is recorded. This is an experiment. If the experiment is repeated another 49 times, the first roll of the die is now referred to as the first trial in an experiment where the die is rolled 50 times.
- A **successful trial** is a trial where the result is the outcome that you wanted.
- The **relative frequency**, sometimes called the empirical or experimental probability, of an event is calculated using the following formula:

$$\text{relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

- The relative frequency of an event for a very large number of trials gives an indication of the value of the theoretical probability.
- As the number of trials increases, the relative frequency of an event will gradually become closer in value to the theoretical probability. This is known as the 'law of large numbers'.
- If the number of trials is large enough, the relative frequency can be used as an estimate of probability.

## 13.4.2 Simulations

- Sometimes it is not possible to conduct trials of a real experiment because it is too expensive, too difficult or impractical. In situations like this, outcomes of events can be modelled using devices such as spinners, dice or coins to simulate or represent what happens in real life. These simulated experiments are called **simulations**.

### WORKED EXAMPLE 6

To simulate whether a baby is born male or female, a coin is flipped. If the coin lands Heads up, the baby is a boy. If the coin lands Tails up, the baby is a girl. Student 1 flips a coin 10 times and obtains 3 Heads and 7 Tails, while Student 2 flips a coin 100 times and obtains 43 Heads and 57 Tails.

- a Determine the relative frequency of females in both cases. Give your answers as decimals.  
b Compare the results from the two simulations and determine which is a better estimate of the true probability.

#### THINK

- a 1 A baby born female is a successful trial.
- For Student 1: in the simulation, females are Tails. There were 7 Tails, so 7 females were born and there were 7 successful trials out of 10 trials.
  - Write the formula for relative frequency and substitute in the results.
- 2 For Student 2: there were 57 Tails, so 57 females were born and there were 57 successful trials out of 100 trials. Write the formula for relative frequency and substitute in the results.

#### WRITE

Student 1:

$$\begin{aligned}\text{Relative frequency} &= \frac{\text{number of successful trials}}{\text{total number of trials}} \\ &= \frac{7}{10} \\ &= 0.7\end{aligned}$$

Student 2:

$$\begin{aligned}\text{Relative frequency} &= \frac{\text{number of successful trials}}{\text{total number of trials}} \\ &= \frac{57}{100} \\ &= 0.57\end{aligned}$$

- b 1** Compare the results of the two simulations.  
There are an almost equal number of males and females born, so we would expect the results to be close to 0.5.

| Student | Relative frequency |
|---------|--------------------|
| 1       | 0.7                |
| 2       | 0.43               |

- 2** Give your answer.

The true probability is likely to be close to 0.5, so Student 2 achieved the better estimate of probability.

- As you can see from Worked example 6, the greater the number of trials, the better the estimate of probability achieved.

### 13.4.3 Using technology to perform simulations

- Simulating experiments using manual devices such as dice and spinners can take a lot of time. A more efficient method of collecting results is to use a list of randomly generated numbers.
- Random number generators** can generate a series of numbers between two given values, for example decimals between 0 and 0.9999. The following table shows some Excel formulae that generate random numbers.

| Formula           | Output                                                                                   |
|-------------------|------------------------------------------------------------------------------------------|
| =RAND()           | A random decimal from 0 to 0.9999                                                        |
| =INT(6*RAND()+1)  | Integers between 1 and 6 that can be used to simulate rolling a die.                     |
| =RANDBETWEEN(1,6) | Both of these formulae can be modified to generate the type of numbers that you require. |

#### WORKED EXAMPLE 7

- a** Use a random number generator to simulate the number of chocolate chips in 50 biscuits, with a maximum of 80 chocolate chips in each.
- b** Calculate the probability that there will be more than 30 chocolate chips in a randomly chosen biscuit.

##### THINK

- Use a random number generator to generate 50 numbers between 0 and 80.
- Count the number of biscuits with more than 30 chocolate chips. There are 34 in the example shown.

##### WRITE

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 15 | 7  | 64 | 73 | 47 | 12 | 74 | 74 | 16 | 42 |
| 41 | 22 | 50 | 33 | 68 | 72 | 64 | 16 | 6  | 72 |
| 8  | 55 | 8  | 62 | 80 | 50 | 42 | 80 | 57 | 39 |
| 70 | 72 | 52 | 48 | 14 | 22 | 59 | 48 | 65 | 34 |
| 67 | 62 | 72 | 59 | 10 | 30 | 13 | 7  | 40 | 18 |

- The probability is the number of successful outcomes (34) divided by the number of trials (50).

$$P(\text{more than 30 chocolate chips}) = \frac{34}{50}$$

### 13.4.4 Factors that could complicate simulations

- Simulations are used to reproduce real-life situations when it is not feasible to implement the actual test.
- The reasons for not conducting an actual test can include financial or safety constraints, or just the overall complexity of running the test.

- Most simulations are conducted using computers and various other mechanical devices.
- For example, airbag manufacturers run airbag tests using cars, but instead of using real humans they use human mannequins. These simulations are fairly accurate, but they have their limitations. One problem with the airbag simulation is the difficulty of modelling the interaction of the airbag with the seatbelt.
- There are numerous other factors that can affect the success of the airbag in an accident. Some of these factors are easy to simulate, but others are not yet known or are too difficult to simulate.



## WORKED EXAMPLE 8

**Six runners are to compete in a race.**

**a Explain how simulation could be used to determine the winner.**

**b Are there any limitations in your answer?**

### THINK

- a** There are six runners, so six numbers are needed to represent them. A die is the obvious choice. Assign numbers 1 to 6 to each runner.
- b** In this simulation, each runner had an equally likely chance of winning each game.

### WRITE

Each runner could be assigned a number on a die, which could be rolled to determine the winner of the race. When the die is rolled, it simulates a race. The number that lands uppermost is the winner of the race.

The answer depends on each runner having an equally likely chance of winning the race. This is rarely the case.

## 13.4.5 Using relative frequencies

- Statisticians sometimes want to compare the attitudes of certain groups, with frequency tables used to visually show the statistical differences in the groups' attitudes. Frequency tables typically show results obtained from taking a random sample from a large population, or from gathering all the data from an entire population when the population is small.
- When calculating relative frequencies, complete the following steps:
  1. Gather the required statistics.
  2. Determine the total number of people polled.
  3. Determine each group's attitude in relation to the total number of people polled and write in fractional form, e.g.  

$$100 \text{ people polled, } 30 \text{ liked the caravan park} = \frac{30}{100}$$
  4. Convert the fraction to a decimal.  

$$\frac{30}{100} = 0.3$$
  5. Convert the decimal to a percentage.  

$$\frac{30}{100} = 0.3$$

$$= 30\%$$
  6. Check that all relative frequencies add to 100%.

*Note:* When the decimals or percentages are rounded off, the figure may be slightly below or above 100%.

### WORKED EXAMPLE 9

In a recent election, there were 4 candidates. During a telephone poll conducted a week before the election, 500 randomly selected voters were asked to indicate their preferences for the 4 candidates. The results are shown in the table below.

| Candidate                                     | 1   | 2   | 3   | 4  |
|-----------------------------------------------|-----|-----|-----|----|
| Number of voters who preferred this candidate | 115 | 168 | 145 | 72 |

Estimate each candidate's probability of winning the election, giving your answers as decimals.

#### THINK

- 1 The probability of a candidate winning is the number of votes they received divided by the total number of votes.

#### WRITE

$$P(\text{candidate 1 wins}) = \frac{115}{500} = 0.230$$

$$P(\text{candidate 2 wins}) = \frac{168}{500} = 0.336$$

$$P(\text{candidate 3 wins}) = \frac{145}{500} = 0.290$$

$$P(\text{candidate 4 wins}) = \frac{72}{500} = 0.144$$

- 2 The closer the relative frequency is to 1, the higher the probability of success. Candidate 2 is the favourite to win.

### WORKED EXAMPLE 10

Use the table to answer the following questions.

Percentage of US women who develop breast cancer over 10-, 20- and 30-year intervals according to their current age, 2010–12

| Current age | 10 years | 20 years | 30 years |
|-------------|----------|----------|----------|
| 30          | 0.44     | 1.87     | 4.05     |
| 40          | 1.44     | 3.65     | 6.80     |
| 50          | 2.28     | 5.53     | 8.75     |
| 60          | 3.46     | 6.89     | 8.89     |
| 70          | 3.89     | 6.16     | N/A      |

Source: The Surveillance, Epidemiology, and End Results (SEER) program of the National Cancer Institute (NCI)

If 3500 women whose current age is 30 are questioned every year for the next 10 years about whether or not they have contracted breast cancer:

- a approximately how many will answer yes  
b approximately how many will answer no?

#### THINK

- a 1 Look on the chart and find the row with current age 30 and the corresponding column that indicates 10 years. The percent in this box is 0.44%.

- 2 Convert 0.44% to a decimal.

#### WRITE

$$\frac{0.44}{100} = 0.0044$$

3 Multiply 3500 by the result from step 2. The answer represents the number of women from this group who will answer yes in the next 10 years.

$$0.0044 \times 3500 = 15.4$$

4 Since a fractional result is not valid in this context, round to the nearest whole number.

Approximately 15 women will answer yes.

b 1 The number of women who contract cancer and the number of women who don't contract cancer is 3500. Subtract the answer to part a from 3500.

$$3500 - 15 = 3485$$

2 Write the answer.

Approximately 3485 women will answer no.

### eBookplus RESOURCES

Interactivity: Experimental probability (int-3825)

## Exercise 13.4 Experimental probability

*Note:* Unless otherwise stated, express probabilities as fractions.

### Understanding, fluency and communicating

- WE6** To simulate whether or not the weather will be suitable for sailing, a coin is flipped. If the coin lands Heads up, the weather is perfect. If the coin lands Tails up, the weather is not suitable.  
Mandy flips a coin 20 times and obtains 13 Heads and 7 Tails. Sophia flips a coin 100 times and obtains 47 Heads and 53 Tails.
  - Determine the relative frequency of perfect weather days in both cases.
  - Compare the results from the two simulations and determine which is a better estimate of the true probability of the experiment.
- A box of 50 batteries was tested and three were found to be flat.
  - What was the relative frequency of a flat battery?
  - In a box of 500 batteries, how many batteries would you expect to be flat?
- The results of a class experiment that involved rolling a standard six-sided die 300 times are shown in the table below.

| Number on die          | 1  | 2  | 3  | 4  | 5  | 6  |
|------------------------|----|----|----|----|----|----|
| Number of times rolled | 42 | 50 | 61 | 37 | 52 | 58 |

Use the results to calculate the experimental probability of rolling:

- the number 5
  - an odd number
  - an even number.
- In her last 20 games, a basketball player sank 17 free throws and missed 11. Estimate the probability of sinking her next free throw.
  - MC** A student rolls a pair of dice 10 times and records the number of times the total is 7. He repeats the experiment numerous times and his results are as follows.  
1, 5, 3, 2, 1, 0, 2, 3, 4, 3, 2, 1, 0, 1, 2, 6, 2, 0, 1, 2  
The estimate of the probability of getting a total of 7 when 2 dice are tossed is:

A.  $\frac{7}{12}$

B.  $\frac{41}{100}$

C.  $\frac{41}{200}$

D.  $\frac{41}{400}$

E. None of the above



6. **WE7** a. Use a random number generator to simulate the number of walnuts in a carrot cake, with a maximum of 60 in each cake.  
 b. Calculate the probability that there will be more than 40 walnuts in a randomly chosen cake.
7. The gender of babies in a set of triplets is simulated by flipping 3 coins. If a coin lands Tails up, the baby is a boy. If a coin lands Heads up, the baby is a girl. In the simulation, the trial is repeated 40 times and the following results show the number of Heads obtained in each trial:  
 0, 3, 2, 1, 1, 0, 1, 2, 1, 0, 1, 0, 2, 0, 1, 0, 1, 2, 3, 2, 1, 3, 0, 2, 1, 2, 0, 3, 1, 3, 0, 1, 0, 1, 3, 2, 2, 1, 2, 1.
- a. Calculate the probability, as a percentage, that exactly one of the babies in a set of triplets is female.  
 b. Calculate the probability, as a percentage, that more than one of the babies in the set of triplets is female.
8. **WE8** There are 24 drivers lining up at the start of a Formula One race.  
 a. Explain how simulation could be used to determine the winner.  
 b. Are there any limitations in your answer?
9. **WE9** A survey of the favourite foods of Year 11 students is recorded, with the following results.

| Meal                | Tally |
|---------------------|-------|
| Hamburger           | 45    |
| Fish and chips      | 31    |
| Macaroni and cheese | 30    |
| Lamb souvlaki       | 25    |
| BBQ pork ribs       | 21    |
| Cornflakes          | 17    |
| T-bone steak        | 14    |
| Banana split        | 12    |
| Corn-on-the-cob     | 9     |
| Beef hot dogs       | 8     |
| Garden salad        | 8     |
| Veggie burger       | 7     |
| Smoked salmon       | 6     |
| Muesli              | 5     |
| Fruit salad         | 3     |



- a. Estimate the probability that macaroni and cheese is the favourite food among Year 11 students.  
 b. Estimate the probability that a vegetarian dish is the favourite food.  
 c. Estimate the probability that a beef dish is the favourite food.
10. If a computer manufacturer wanted to simulate the probability of their laptops having a fault, would it be appropriate to use a random number generator to select random computers on the assembly line? Why or why not?
11. A random number generator was used to select how many students will receive a free lunch on any given day. The maximum number of students who can receive a free lunch is 10 and the minimum is 0.
- a. If the test is run for 30 days, what is the theoretical probability that:
- 8 or more students are selected on any given day
  - 0 students are selected on any given day
  - 10 students are selected on any given day?

- b. Use a random number generator to simulate the free lunch program over 30 days, and compare the results to those from part a.
- c. Are the results from the random number generator the same as your predictions? Why or why not?
12. Determine if the following simulations are easy or difficult to undertake. If your response is 'difficult', explain why.
- Predicting how a helicopter pilot will react in an emergency
  - Estimating how many times you will win when spinning a roulette wheel
  - Predicting how many kilometres a new car will travel before any major mechanical problems
  - Estimating how many car accidents you will have in your lifetime
  - Estimating the number of defective mobile phones that one manufacturer will produce over two months
13. **WE10** a. Five thousand new domestic university students are selected to take part in a study. Use the data in the chart below to predict how many students will not come back after their first year of university for the 4-year period of 2015–18.
- b. Based on the domestic trend for students not coming back after the first year, do you believe that your estimate in part a will be higher or lower than the actual percentage? Explain your answer.

#### Completion rates of higher education students – cohort analysis, 2005–2014

|                                       | Year | Completed<br>(in any year) | Still enrolled<br>at the end of<br>the 4-year<br>cohort period | Re-enrolled,<br>but dropped<br>out | Never came<br>back after<br>the first year |
|---------------------------------------|------|----------------------------|----------------------------------------------------------------|------------------------------------|--------------------------------------------|
| National total<br>(domestic students) | 2005 | 47.3%                      | 30.2%                                                          | 11.5%                              | 11.0%                                      |
|                                       | 2006 | 46.7%                      | 32.7%                                                          | 10.6%                              | 10.0%                                      |
|                                       | 2007 | 45.9%                      | 33.8%                                                          | 10.4%                              | 9.9%                                       |
|                                       | 2008 | 46.7%                      | 33.5%                                                          | 10.9%                              | 8.9%                                       |
|                                       | 2009 | 46.1%                      | 34.3%                                                          | 11.1%                              | 8.5%                                       |
|                                       | 2010 | 45.1%                      | 34.7%                                                          | 11.2%                              | 9.0%                                       |
|                                       | 2011 | 45.0%                      | 34.5%                                                          | 11.7%                              | 8.8%                                       |

*Source:* Department of Education and Training, *Completion Rates of Higher Education Students — Cohort Analysis, 2005–2014*

#### Problem solving, reasoning and justification

14. The results of a coin-flipping experiment are shown.
- What are the relative frequencies, expressed as a percentage, of:
    - Heads
    - Tails?
  - If the experiment was repeated, would you expect the same results? Explain your answer.
  - If the experiment was repeated 100 times, what do you expect to happen to the relative frequencies of the outcomes?
15. (Note: You will need a six-sided die for this question.)

| Outcome | Frequency |
|---------|-----------|
| Heads   | 38        |
| Tails   | 62        |
| Total   | 100       |

The latest promotion by a chocolate bar manufacturer claims 'Every third one is free'. When you get the special message inside the wrapper, you get a refund on the purchase price. The manufacturer has put the special message in every third wrapper. When the chocolate bars are packaged into boxes, they are mixed randomly. Use the numbers 3 and 6 on a die to represent getting a free chocolate bar.

- a. Carry out 50 trial purchases of chocolate bars.
  - b. Calculate the relative frequency of a free chocolate bar.
  - c. Compare your results with the students around you.
  - d. Did other students get different results? Explain why.
  - e. In 100 trials, how many free chocolate bars would you expect to receive?
16. a. How many times, to the nearest whole number, would you expect to roll a 4 if you rolled a fair die:
- i. 10 times
  - ii. 60 times
  - iii. 1000 times?
- b. If you actually carried out these experiments, would you get the exact number that you predicted? Explain your answer.

17. The mouse in the maze is trying to reach the cheese. Use simulation to help the mouse to decide to go left, right or straight on when a new choice is available. If you come to a dead end, turn around and go back to where you have a choice to go left or right. There are two possible outcomes for the mouse:
- escaping the maze
  - reaching the cheese.

Use simulation to determine the relative frequencies of each of the outcomes.

18. A simulation of the ages of a class of Year 11 students using random numbers is shown.

|    |    |    |    |    |    |
|----|----|----|----|----|----|
| 16 | 17 | 16 | 17 | 18 | 18 |
| 16 | 17 | 16 | 16 | 16 | 17 |
| 16 | 17 | 17 | 17 | 17 | 18 |
| 18 | 17 | 18 | 17 | 16 | 17 |

- a. Carry out the same simulation using your random number generator.
  - b. Complete a frequency table for each simulation.
  - c. Compare the results of the two simulations and comment on any differences or similarities.
19. A popular pokies game requires you to spin up the same picture in all five slots to win. There are six different pictures possible in each of the five positions.
- a. Use a random number generator to generate 100 sets of five random numbers between 1 and 6.
  - b. Calculate the probability of winning (having the same picture in each of the five slots).
  - c. What other device could be used to perform this simulation?
  - d. List the benefits and the disadvantages of each method of generating random results.
20. A player suspects that a die is loaded (unfair), so she rolls the die 600 times to find out. The results of the experiment are shown in the table below.

| Number on die          | 1  | 2  | 3  | 4   | 5  | 6   |
|------------------------|----|----|----|-----|----|-----|
| Number of times rolled | 63 | 90 | 96 | 118 | 95 | 138 |

- a. Predict the number of times each number should appear if the die were a fair die.
  - b. Explain whether you think the die is loaded.
21. a. The possible outcomes for a tennis match are win or lose. Does this mean that you will have a probability of  $\frac{1}{2}$  (or 0.5) of beating your friend in a match?
- b. What other factors can be involved in determining the probability you have of winning a tennis match? (Assume that you and your friend regularly play each other.)
  - c. How could you determine the experimental probability that you will beat your friend in a tennis match?



# 13.5 Review

## 13.5.1 Summary

In this topic you have learnt:

- about the probability scale
- how to list the sample space to show the total possible outcomes
- how to calculate theoretical probability
- about the complement of an event
- how to calculate the probability of the complement of an event occurring
- how to use an array to display sample spaces where two events occur
- how to use tree diagrams to show all possible combinations
- how to calculate relative frequency
- about simulations and how to perform them using technology
- about factors that could complicate simulations
- how to use relative frequencies.

*Note:* A comprehensive summary of key learning points can be found in the resources section of your eBookPLUS.

## Exercise 13.5 Review

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*Note:* Unless otherwise stated, express probabilities as fractions.

### Understanding, fluency and communicating

1. What is the sample space for the following scenarios?
  - a. Rolling an 8-sided die numbered 1–8
  - b. Drawing an 'a' from a bag containing the letters of the alphabet
  - c. Drawing the winning ticket in a raffle with 100 tickets sold
  - d. A random number generator selecting an even number from 0 to 50
2. Choose which of the following events are 'most likely' and 'very unlikely'.
  - a. A cyclone in Victoria
  - b. The temperature rising above 30 degrees Celsius on a summer day in Darwin
  - c. Snow in Cairns
  - d. Getting stung by an Irukandji jellyfish in Tasmania
3. Create three events that meet the following criteria.
  - a. Less than even chance of occurring
  - b. Greater than an even chance of occurring
4. Two fair dice are rolled and the product of the uppermost faces is recorded.
  - a. List the sample space for the experiment.
  - b. What is the most likely outcome?
  - c. What is the least likely outcome?
5. Two dice are rolled and the difference of the two uppermost faces is recorded.
  - a. Construct a table to illustrate the sample space.
  - b. What is the most likely outcome?
  - c. What is the least likely outcome?
6.
  - a. What is the probability of rolling a 3 on a 6-sided die?
  - b. What is the complement of the event in part a?
  - c. What is the probability of the complement occurring?
7.
  - a. If you toss two fair dice, what is the probability that the two numbers you roll are the same?
  - b. What is the probability of the complement of this event?
  - c. Would it be easier to calculate the probability of the complement before calculating the original event? Explain your answer.

8. The following table shows what people do with their old mobile phones in Australia. What is the probability of NOT selling or NOT losing your mobile phone in Australia? Give your answer as a percentage.

| Saved it | Shared it | Binned it (recycled) | Sold it | Lost it |
|----------|-----------|----------------------|---------|---------|
| 48%      | 27%       | 15%                  | 8%      | 2%      |

**Source:** © 2015 Deloitte Touche Tohmatsu, *Mobile Consumer Survey – The Australian Cut*

9. In the game of backgammon, two dice are rolled together. The resulting numbers can be used separately in two separate moves, or their totals can be combined for one move.
- How many possible combinations are there?
  - Calculate the probability of ‘getting a 4 on a die or a total of 4 on both dice’.
  - Calculate the probability of NOT ‘getting a 4 on a die or a total of 4 on both dice’.
  - Calculate the probability of ‘getting a 4 or getting a 5 on a die or a total of 5 on both dice’.

10. A soft-drink chemist is trying to create a new drink with three different ingredients. The chemist wants to experiment with the mixing order of the three key ingredients. Draw a tree diagram to show the sample space and determine how many combinations are possible.

11. A car dealer offers these package options on one of their new models.

- Interior: leather or suede
  - Transmission: manual or automatic
  - Front seats: bucket, captain or regular
- How many different configurations of options are available?
  - What is the theoretical probability a random customer chooses a bucket seat in their package?

12. A fair coin is flipped and then an 8-sided die is rolled.

- List all the possible outcomes in a chart.
- What is the probability of getting a Head and an 8?
- Would you bet on any of the outcomes? Why or why not?

13. What are the three most probable results in a game where two 6-sided dice are rolled and the results of the two dice are added?

14. If a random card is chosen from a deck of 52 cards, what is the experimental probability of:

- selecting an ace
- selecting an ace or a king
- not selecting a 2
- not selecting any face card?

15. A car manufacturer conducted a survey to see which were the most popular colours for 4-door sedans currently being driven. The results were as follows:

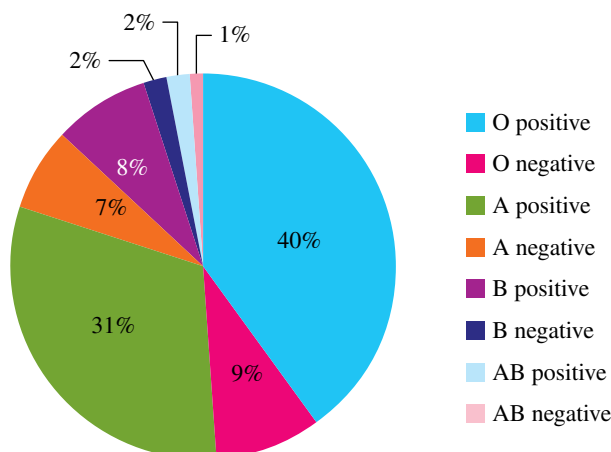
| Colour | Red | Silver | White | Blue |
|--------|-----|--------|-------|------|
| Number | 134 | 167    | 109   | 89   |

- What is the relative frequency of owners who had blue sedans?
  - What is the relative frequency of owners who had sedans that were not red or blue?
- Give your answers as percentages correct to 2 decimal places.





16. According to the Australian Red Cross Blood Service, the percentage of blood group frequency in Australia is as shown.



**Source:** Data from Australian Red Cross Blood Service

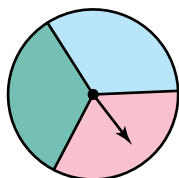
- a. What is the sum of the percentages of all the blood types?
- b. If you were to select 1000 random people, how many would you expect to have AB negative blood?
- c. Is the result from part **b** guaranteed? Why or why not?
- d. If you tested 1000 random people, could the number of O positive people be 50% rather than 40%? Why or why not?
17. A biologist was trying to calculate the probability of being attacked by a shark in Australian waters when surfing.
- a. What data would be necessary to increase the accuracy of his prediction?
- b. What data in part **a** would be difficult to obtain? What data would be easier to obtain?
- c. Would you accept the biologist's probability results? Why or why not?
18. Is it possible to flip a fair coin 10 times and obtain a result closer to the theoretical probability than if you flip a fair coin 50000 times? Explain your answer.
19. Over a three-year period, a Mathematics teacher recorded the following scores on exams.
- a. How many As should she expect on her next exam if she has 20 students?
- b. How many Fs should she expect on her next exam if she has 20 students?
20. If a basketball player is making 80% of free throws for the season, and has missed all of three free throws in regulation time for this game, would you expect him to make a free throw for the game-winning point in overtime? Explain your answer.



| Grade | Frequency | Percentage |
|-------|-----------|------------|
| A     | 500       | 10%        |
| B     | 800       | 16%        |
| C     | 2000      | 40%        |
| D     | 1000      | 20%        |
| F     | 700       | 14%        |

### Problem solving, reasoning and justification

21. In a simple experiment, a coin was flipped 50 times, resulting in 28 Heads and 22 Tails.
- Calculate the experimental probability of flipping a Head with the coin. Give your answer as a decimal.
  - Compare the experimental probability with the theoretical probability of flipping a Head with the coin. Explain why the two probabilities are different in this case.
22. a. Use a table to show the sample space for an experiment where a coin is flipped and a die is rolled.  
b. Calculate the probability of obtaining a Tail and a 5.
23. At a fair, people were asked to taste and state their preference for two new brands of soft drink: brand A and brand B.
- List all the possible outcomes using a tree diagram.
  - What is the probability, expressed as a percentage, that a randomly selected person liked:
    - brand A but not brand B
    - brand A and brand B
    - brand A?
24. a. How many times, to the nearest whole number, would you expect to roll a 6 if you rolled a fair die:
  - 10 times
  - 150 times
  - 5000 times?
- b. If you actually did the experiments in part a, would you get the exact number that you predicted? Explain your answer.
25. You are about to sit a Mathematics examination that contains 40 multiple-choice questions. You didn't have time to study so you are going to choose the answers completely randomly. There are five choices for each question.  
Explain how you could use random numbers to select your answers for you.
26. The spinner shown below was spun 50 times and the outcome each time was recorded in the table below.



| Segment | Blue | Green | Pink |
|---------|------|-------|------|
| Tally   | 17   | 23    | 10   |

- List the sample space.
  - Given the experimental results, determine the relative frequency for each segment.
  - The theoretical probability of the spinner landing on any particular segment with one spin is  $\frac{1}{3}$ . How could the experiment be changed to give a better estimate of the true probabilities?
27. Jose has to call his girlfriend, but he cannot remember her full number. He knows the first eight digits, but can't recall the final two digits.
- What is the probability that he correctly guesses either one of the two final digits (but not both)?
  - What is the probability that he correctly guesses both of the two final digits?
  - What is the probability that he incorrectly guesses both of the final two digits?
28. A friend offers you a chance to roll a single die three times, and if you roll a 3 on any one of the rolls he will pay for your lunch. Otherwise, you will pay for his lunch. Should you play his game? Explain your answer.
29. There are five different possible hair colours: red, brunette, brown, grey and blonde. Assume that each hair colour is equally likely. If there are 5 people in a room, what is the probability that at least 2 have the same coloured hair? Give your answer as a percentage correct to 2 decimal places. (*Hint: Use the complement to solve this problem.*)
30. A group of young entrepreneurs invented a new method for making frozen chocolate-covered bananas. They want to test how long it would take for the product to melt at certain temperatures.
- If you were hired as a consultant, do you think that you could produce a viable simulation? Why or why not?
  - What would your simulation entail?
  - What factors would be easy to test?
  - What factors would be difficult to test?



# Answers

## Exercise 13.2 Theoretical probability

1. a. 1                      b. 0.75                      c. 0.25                      d. 0.75  
     e. 0.25                      f. 1                      g. 0                      h. 0.25  
     i. 1                      j. 0.25                      k. 0.5                      l. 0.75
2. a. 1, 2, 3, 4, 5, 6  
     b. Green, yellow, blue  
     c. a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z
3. a.  $\frac{1}{6}$                       b.  $\frac{1}{3}$                       c.  $\frac{1}{2}$
4. a.  $\frac{4}{11}$                       b.  $\frac{7}{11}$                       c.  $\frac{2}{11}$                       d.  $\frac{1}{11}$
5. a. 1, 2, 3, 4, 5, 6, 7, 8  
     b. i. 50%                      ii. 62.5%                      iii. 0%                      iv. 100%

6. a.

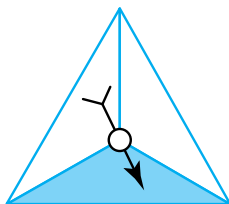
|   | A  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  | J  | Q  | K  |
|---|----|----|----|----|----|----|----|----|----|-----|----|----|----|
| ♥ | A♥ | 2♥ | 3♥ | 4♥ | 5♥ | 6♥ | 7♥ | 8♥ | 9♥ | 10♥ | J♥ | Q♥ | K♥ |
| ♦ | A♦ | 2♦ | 3♦ | 4♦ | 5♦ | 6♦ | 7♦ | 8♦ | 9♦ | 10♦ | J♦ | Q♦ | K♦ |
| ♠ | A♠ | 2♠ | 3♠ | 4♠ | 5♠ | 6♠ | 7♠ | 8♠ | 9♠ | 10♠ | J♠ | Q♠ | K♠ |
| ♣ | A♣ | 2♣ | 3♣ | 4♣ | 5♣ | 6♣ | 7♣ | 8♣ | 9♣ | 10♣ | J♣ | Q♣ | K♣ |

- b. i.  $\frac{1}{4}$                       ii.  $\frac{1}{2}$                       iii.  $\frac{1}{52}$                       iv.  $\frac{12}{52} = \frac{3}{13}$
7. D
8. a. Not owning a dog                      b. Flipping a coin and getting a Head  
     c. Being awake                      d. Truancy  
     e. Failing your learner knowledge test                      f. Not owning a mobile phone
9. a. Failing the final exam                      b. 80%
10. a.  $\frac{4}{36} = \frac{1}{9}$                       b.  $\frac{32}{36} = \frac{8}{9}$                       c.  $\frac{11}{36}$                       d.  $\frac{25}{36}$                       e.  $\frac{1}{36}$                       f.  $\frac{35}{36}$
11. False. Assuming that you pick one flower from the basket, the complement of picking a red flower is picking a purple or yellow flower.
12. Answers will vary.

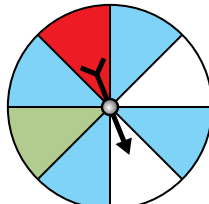
13. a. i. No  
     ii. Blue, green, yellow, red  
     iii.  $P(\text{blue}) = \frac{1}{3}$ ,  $P(\text{green}) = \frac{1}{6}$ ,  $P(\text{red}) = \frac{1}{3}$ ,  $P(\text{yellow}) = \frac{1}{6}$   
     b. i. Yes  
     ii. Blue, green, yellow, red, orange  
     iii.  $P(\text{blue}) = \frac{1}{5}$ ,  $P(\text{green}) = \frac{1}{5}$ ,  $P(\text{red}) = \frac{1}{5}$ ,  $P(\text{yellow}) = \frac{1}{5}$ ,  $P(\text{orange}) = \frac{1}{5}$   
     c. i. No  
     ii. Green, yellow  
     iii.  $P(\text{green}) = \frac{1}{3}$ ,  $P(\text{yellow}) = \frac{2}{3}$

14. a. Blue. There are more blue jelly beans (6) than beans of any other colour.  
     a. Yellow. There are fewer yellow jelly beans (2) than beans of any other colour.  
     b.  $P(\text{blue}) = \frac{6}{17}$ ,  $P(\text{green}) = \frac{5}{17}$ ,  $P(\text{red}) = \frac{4}{17}$ ,  $P(\text{yellow}) = \frac{2}{17}$

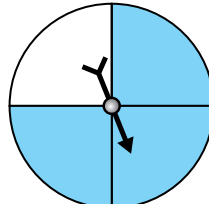
15. a.



b.



c.



16. {3, 4, 5, 10, 12, 13}

### Exercise 13.3 Multi-stage experiments

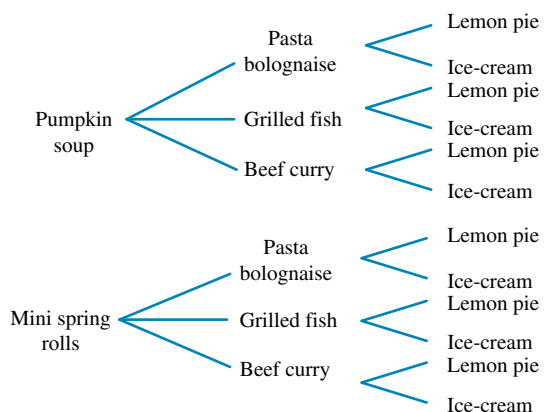
1. a. 5%

b. 10%

c. 10%

d. 20%

2. a.



b. i.  $\frac{1}{2}$

ii.  $\frac{1}{6}$

iii.  $\frac{1}{2}$

iv.  $\frac{1}{12}$

3. a.

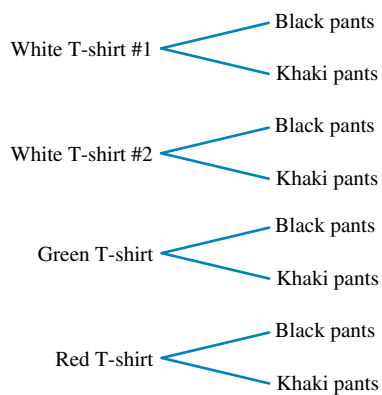


b. i.  $\frac{1}{4}$

ii.  $\frac{1}{8}$

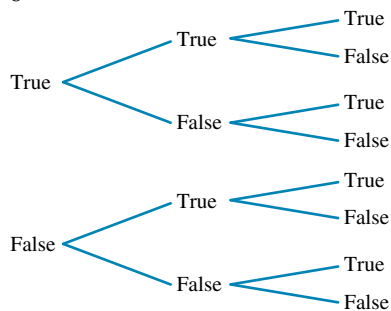
iii.  $\frac{1}{4}$

4. a.



b.  $\frac{1}{8}$

5. a.



b. i.  $\frac{1}{8}$

ii.  $\frac{3}{8}$

iii.  $\frac{1}{8}$

6. a. i.  $\frac{1}{8}$

ii.  $\frac{3}{8}$

iii.  $\frac{1}{4}$

iv.  $\frac{7}{8}$

v.  $\frac{3}{4}$

b.  $\frac{1}{2}$

c.  $\frac{1}{2}$

d.  $\frac{1}{2}$

e.  $\frac{1}{8}$

7.

|        | 1  | 2  | 3  | 4  | 5  | 6  |
|--------|----|----|----|----|----|----|
| Red    | R1 | R2 | R3 | R4 | R5 | R6 |
| Blue   | B1 | B2 | B3 | B4 | B5 | B6 |
| Green  | G1 | G2 | G3 | G4 | G5 | G6 |
| Yellow | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| Orange | O1 | O2 | O3 | O4 | O5 | O6 |

8. D

9. a. 32

b. 64

c. 1296

10. a. 1

b.  $\frac{3}{8}$

11. B

12. a.  $\frac{2}{49}$

b.  $\frac{6}{49}$

c.  $\frac{6}{49}$

13. a.  $\frac{9}{36} = \frac{1}{4}$

b.  $\frac{6}{36} = \frac{1}{6}$

c.  $\frac{35}{36}$

d.  $\frac{36}{36} = 1$

14. a.  $\frac{3}{12} = \frac{1}{4}$

b.  $\frac{1}{2}$

c.  $\frac{6}{12} = \frac{1}{2}$

d.  $\frac{4}{12} = \frac{1}{3}$

15. a.

| + | 1 | 2 | 3 | 4  | 5  | 6  |
|---|---|---|---|----|----|----|
| 1 | 2 | 3 | 4 | 5  | 6  | 7  |
| 2 | 3 | 4 | 5 | 6  | 7  | 8  |
| 3 | 4 | 5 | 6 | 7  | 8  | 9  |
| 4 | 5 | 6 | 7 | 8  | 9  | 10 |
| 5 | 6 | 7 | 8 | 9  | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

b. The diagonals in the sample space are repeating numbers. Also, 7 is the outcome that shows up the most, and each outcome further away from 7 has a reduced chance of occurring.

c.

| Sum of two dice | 2              | 3              | 4              | 5             | 6              | 7             | 8              | 9             | 10             | 11             | 12             |
|-----------------|----------------|----------------|----------------|---------------|----------------|---------------|----------------|---------------|----------------|----------------|----------------|
| Probability     | $\frac{1}{36}$ | $\frac{1}{18}$ | $\frac{1}{12}$ | $\frac{1}{9}$ | $\frac{5}{36}$ | $\frac{1}{6}$ | $\frac{5}{36}$ | $\frac{1}{9}$ | $\frac{1}{12}$ | $\frac{1}{18}$ | $\frac{1}{36}$ |

d. The probabilities on either side of the 7 decrease at the same rate.

16. a.

| – | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 2 | 3 | 4 | 5 |
| 2 | 1 | 0 | 1 | 2 | 3 | 4 |
| 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| 4 | 3 | 2 | 1 | 0 | 1 | 2 |
| 5 | 4 | 3 | 2 | 1 | 0 | 1 |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 |

b.  $\frac{12}{36} = \frac{1}{3}$

c.  $\frac{24}{36} = \frac{2}{3}$

d. No

17. a, b.

|          |          | Person 2      |               |               |
|----------|----------|---------------|---------------|---------------|
|          |          | Rock          | Paper         | Scissors      |
| Person 1 | Scissors | S( <u>R</u> ) | ( <u>S</u> )P | SS            |
|          | Paper    | ( <u>P</u> )R | PP            | P( <u>S</u> ) |
|          | Rock     | RR            | R( <u>P</u> ) | ( <u>R</u> )S |

c. i.  $\frac{2}{9}$

ii.  $\frac{2}{9}$

iii.  $\frac{2}{9}$

d. Yes

e. i.

|          |          | Person 2      |               |               |               |
|----------|----------|---------------|---------------|---------------|---------------|
|          |          | Rock          | Paper         | Scissors      | Dynamite      |
| Person 1 | Scissors | S( <u>R</u> ) | ( <u>S</u> )P | SS            | ( <u>S</u> )D |
|          | Paper    | ( <u>P</u> )R | PP            | P( <u>S</u> ) | P( <u>D</u> ) |
|          | Rock     | RR            | R( <u>P</u> ) | ( <u>R</u> )S | R( <u>D</u> ) |
|          | Dynamite | ( <u>D</u> )R | ( <u>D</u> )P | D( <u>S</u> ) | DD            |

ii. No

iii.  $P(\text{Rock wins}) = \frac{2}{16} = \frac{1}{8}$

$P(\text{Paper wins}) = \frac{2}{16} = \frac{1}{8}$

$P(\text{Scissors win}) = \frac{4}{16} = \frac{1}{4}$

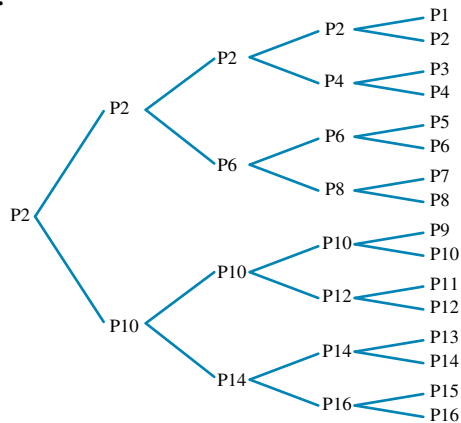
$P(\text{Dynamite wins}) = \frac{4}{16} = \frac{1}{4}$

Scissors and dynamite are twice as likely to win as rock and paper.

18. a. 6

b. No. There are still six paths, even if we omit home from the journey. Since there is only one choice to go from the library to home, the outcomes do not change.

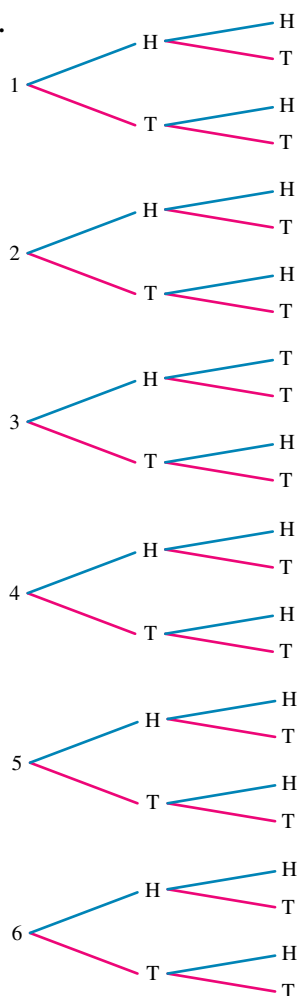
19. a.



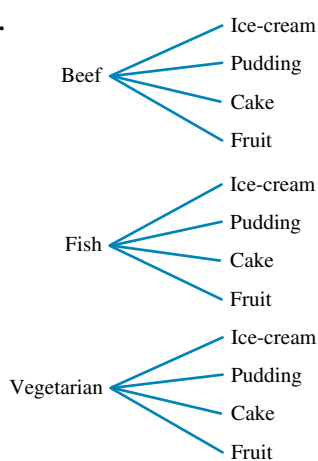
b. You will need to look at their past competition history, injuries and many other factors to determine the probability of who will have the best chance of winning the tournament. Even if you have done extensive analysis, you have a low probability of determining the winner.

c. No. Tennis players have different physical and mental skill sets, experience, fitness, and injuries, which affect every performance. Every day the probability of who will win changes.

20.



21.

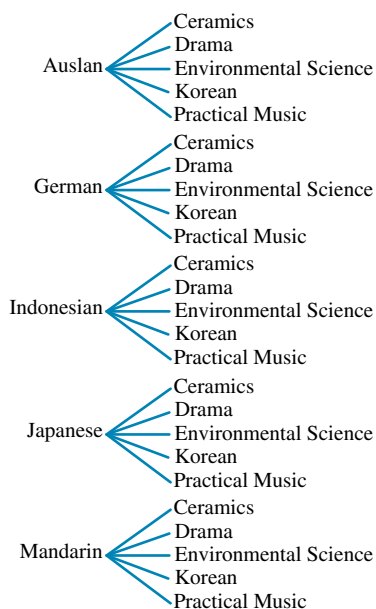


22. a.

| + | 1 | 2 | 3 | 4  | 5  | 6  |
|---|---|---|---|----|----|----|
| 1 | 2 | 3 | 4 | 5  | 6  | 7  |
| 2 | 3 | 4 | 5 | 6  | 7  | 8  |
| 3 | 4 | 5 | 6 | 7  | 8  | 9  |
| 4 | 5 | 6 | 7 | 8  | 9  | 10 |
| 5 | 6 | 7 | 8 | 9  | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

b. i.  $\frac{10}{36} = \frac{5}{18}$   
 ii.  $\frac{9}{36} = \frac{1}{4}$

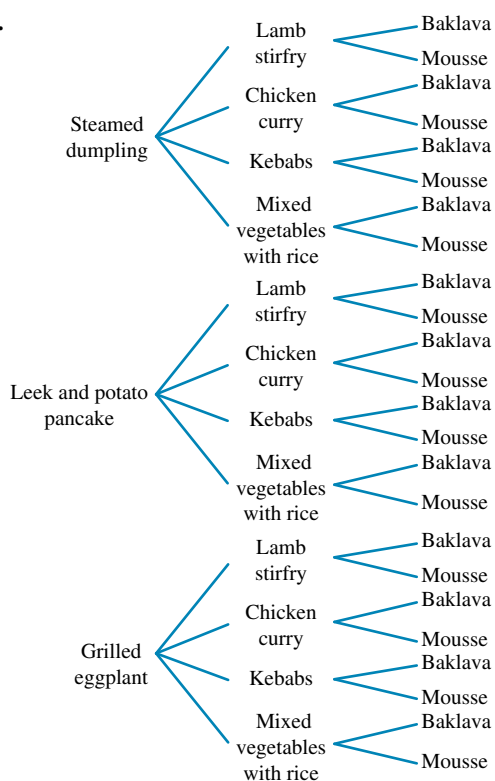
23. a.



b. i.  $\frac{5}{25} = \frac{1}{5}$

ii.  $\frac{1}{25}$

24. a.



b.  $\frac{1}{24}$

25. a.  $\frac{21}{36} = \frac{7}{12}$

b.  $\frac{21}{36} = \frac{7}{12}$

### Exercise 13.4 Experimental probability

1. a. Mandy's simulation:  $\frac{13}{20}$

Sophia's simulation:  $\frac{47}{100}$

b. Sophia's simulation is likely to be more reliable as there is a higher number of coin tosses.

2. a.  $\frac{3}{50}$

b. 30 batteries

3. a.  $\frac{52}{300} = \frac{13}{75}$

b.  $\frac{155}{300} = \frac{31}{60}$

c.  $\frac{145}{300} = \frac{29}{60}$

4.  $\frac{17}{20}$

- ### Exercise 13.5 Review

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2. a. Very unlikely

b. Most likely

c. Very unlikely

d. Very unlikely

3. Answers will vary.

4. a.

| $\times$ | 1 | 2  | 3  | 4  | 5  | 6  |
|----------|---|----|----|----|----|----|
| 1        | 1 | 2  | 3  | 4  | 5  | 6  |
| 2        | 2 | 4  | 6  | 8  | 10 | 12 |
| 3        | 3 | 6  | 9  | 12 | 15 | 18 |
| 4        | 4 | 8  | 12 | 16 | 20 | 24 |
| 5        | 5 | 10 | 15 | 20 | 25 | 30 |
| 6        | 6 | 12 | 18 | 24 | 30 | 36 |

b. 6 and 12

c. 1, 4, 9, 16, 25, 36

5. a.

| — | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 2 | 3 | 4 | 5 |
| 2 | 1 | 0 | 1 | 2 | 3 | 4 |
| 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| 4 | 3 | 2 | 1 | 0 | 1 | 2 |
| 5 | 4 | 3 | 2 | 1 | 0 | 1 |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 |

b. 1

c. 5 and 6

6. a.  $\frac{1}{6}$

b. The complement is the set  $\{1, 2, 4, 5, 6\}$ .

c.  $\frac{5}{6}$

7. a.  $\frac{6}{36} = \frac{1}{6}$

b.  $\frac{30}{36} = \frac{5}{6}$

c. No. The probability of the original event is a smaller probability, and thus requires less counting.

8. 90%

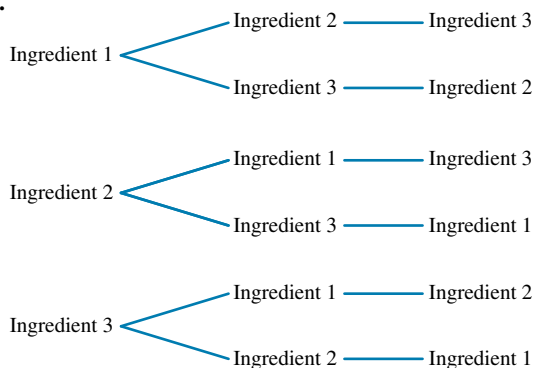
9. a. 72

b.  $\frac{14}{36} = \frac{7}{18}$

c.  $\frac{22}{36} = \frac{11}{18}$

d.  $\frac{24}{36} = \frac{2}{3}$

10.



11. a. 12

b.  $\frac{1}{3}$

12. a.

|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|---|----|----|----|----|----|----|----|----|
| H | H1 | H2 | H3 | H4 | H5 | H6 | H7 | H8 |
| T | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 |

a.  $\frac{1}{16}$

b. No. They all only have a 6.25% chance of success.

13. The three most probable results are 6, 7 and 8.

14. a.  $\frac{4}{52} = \frac{1}{13}$

b.  $\frac{8}{52} = \frac{2}{13}$

c.  $\frac{48}{52} = \frac{12}{13}$

d.  $\frac{40}{52} = \frac{10}{13}$

15. a. 17.84%                                      b. 55.31%
16. a. 100%
- b. 10 people
- c. No. This is the estimated number of people and is not a guaranteed amount. It is likely that the amount will be slightly above or below this.
- d. Probably not. A sample of 1000 is relatively large, and you would expect that the experimental probabilities at this scale would be approaching the theoretical probabilities.
17. a. Answers will vary but could include:
- the latest shark attack data in Australia
  - the number of surfers in Australia
  - the surfing habits of surfers in Australia: every day, once a week, etc.
  - the prevalence of baitfish
  - the surf break's vicinity to shark breeding grounds.
- b. Easy: i, vi
- c. Difficult: ii, iii, v
- d. No. There are too many variables. The variables that can be obtained may not be accurate. There may be some variables that science is not aware of.
18. According to the law of large numbers, this result is not likely; however, there is a possibility it could occur.
19. a. 2 As                                      b. 3 Fs
20. Yes. Each free throw is an independent event. Therefore, the player still has an 80% chance of making the basket. However, there is 20% chance that the player could miss.
21. a. 0.56
- b. The theoretical probability is 0.5 for a head. The coin has only been tossed 50 times, but as the coin is tossed more times the result will close in on the theoretical probability.
22. a.
- |   | 1  | 2  | 3  | 4  | 5  | 6  |
|---|----|----|----|----|----|----|
| H | H1 | H2 | H3 | H4 | H5 | H6 |
| T | T1 | T2 | T3 | T4 | T5 | T6 |
- b.  $\frac{1}{12}$
23. a.
- ```

      Like brand A — Like brand B
                    — Dislike brand B

      Dislike brand B — Like brand B
                     — Dislike brand B
  
```
- b. i. 25%
- ii. 25%
- iii. 50%
24. a. i. 1 ii. 25 iii. 833
- b. No, these are estimated numbers calculated from the theoretical probabilities. As the experiment is carried out more times, the experimental probabilities will approach the theoretical probabilities. There is a slight chance in any experiment that you may obtain the theoretical probability, but it is unlikely.
25. You could run your random number generator using the numbers 1–5. These numbers would represent A, B, C, D, E respectively in the multiple-choice questions. You would run this simulation 40 times.
26. a. Blue, green, pink
- b. $P(B) = \frac{17}{50}$
- $P(G) = \frac{23}{50}$
- $P(P) = \frac{10}{50} = \frac{1}{5}$
- c. Run the experiment more times (e.g. 10 000 times).
27. a. $\frac{18}{100} = \frac{9}{50}$ b. $\frac{1}{100}$ c. $\frac{81}{100}$
28. No. The theoretical probability of rolling a 3 in three rolls is only $\frac{91}{216}$.
29. 96.16%
30. Answers will vary.

GLOSSARY

Absolute error: the value of an error without a positive or negative sign

Absolute loss: if the selling price is less than the original price, the difference between the two values

Absolute percentage error: the absolute relative error converted to a percentage

Absolute profit: if the selling price is more than the original price, the difference between the two values

Absolute relative error: to compare an estimated value with an actual value.

Accuracy: how closely a measured value agrees with an actual value

Allowance: an additional amount of money given to workers to allow them to complete certain tasks

am: the time interval from midnight to midday

Annual leave loading: an additional payment received on top of the 4-week annual-leave pay

Approximation: estimation of a value

Arc: a portion of the circumference of a circle

Arc length: the length of a portion of a circle's circumference

Area: the amount of flat space surface enclosed by a two-dimensional shape

Array: a table that systematically displays all the outcomes of an experiment

Axis break: a gap inserted into the axis to indicate that a range of values has been skipped

Back-to-back stem-and-leaf plot: a stem plot used to compare two different sets of data. Back-to-back stem plots share the same stem, with one data set appearing on the left of the stem and the other data set appearing on the right

Bar chart: a display with the categories of data on one axis (usually the horizontal axis) and the frequency of the data on the other (usually the vertical axis)

Basal Metabolic Rate (BMR): calculation of how much energy is required to maintain basic body functions on a daily basis

Biased: when data favours a particular group as the sample is not large enough or is not selected at random

Bimodal: data with two obvious peaks in a graph

Blood alcohol content (BAC): the measure of alcohol in the bloodstream at a given time, measured in grams of alcohol per 100 millilitres of blood

Boxplots: a visual representation of the five-figure summary for a data set

Budget: a list of all planned income and costs

Calorie (cal): imperial style for measuring energy, used for food energy

Capacity: the maximum amount of fluid that can be contained in an object

Categorical: data that can be organised into groups or categories

Census: when data is collected from the entire population

Certain: in probability, an outcome that will definitely occur

Circumference: the distance around the outside of a circle

Clark's formula: a method of estimating the dosage of medication for a child

Class interval: ranges of values that data can be sorted into

Clustering: groups of data in a distribution

Commission: a percentage of the sale price given to a salesperson when a sale is made

Comparison rate: a more accurate indication of the true interest rate of a loan, including all additional fees in the calculation

Complement: in probability, the complement of a set, A, written A' , is the set of elements that are in ξ but not in A

Composite solid: a solid object composed of a number of other simple solid objects

Compound interest: interest earned on the current total in a bank account

Compounding period: the time after which interest is calculated and added to the principal when compound interest is applied

Comprehensive: motor vehicle insurance that covers damage to your own vehicle and other people's property, as well as theft and some other risks, plus legal costs

Compulsory third-party (CTP): mandatory motor vehicle accident personal injuries insurance, required by each state and territory. CTP covers any person that you might injure while you are driving

Constant of variation: the value of m in the equation $y = mx$

Continuous: numerical data that can take any value that lies within an interval

Convert: to change from one unit to another

Council rates: fee home owners pay to support councils in providing park lands, libraries, rubbish collection and road maintenance

Cross-section: identical slices produced when cuts are made across a prism parallel to its ends

Cumulative frequency: the total of all frequencies up to and including the frequency for a particular score in a distribution

Cumulative frequency curve (ogive): a line graph that is formed when the cumulative frequencies of a set of data are plotted against the end points of their respective class intervals and then joined by straight-line segments

Cumulative frequency table: a table showing the cumulative frequency of all of the scores of a distribution

Cumulative percentage frequency: the total percentage of all frequencies up to and including the frequency for a particular score in a distribution

Cumulative percentage graph: a graph showing the total percentage contribution of a category and all preceding categories

Deciles: divide a data set into 10 equal-sized parts

Depreciation: the decrease in value of an item over time

Diameter: the straight-line distance across a circle through its centre

Direct variation: a particular relationship between two variables or quantities; as one variable increases, so does the other variable

Discount: a price reduction on an item

Discrete: numerical data that is counted in exact values, with the values often being whole numbers

Discretionary spending: spending that occurs over a given time interval that is not a fixed amount

Dot plot: a plot in which every data value is represented by a dot

Equally likely outcomes: outcomes that have the same probability of occurring in an experiment

Equation: mathematical statement containing a left-hand side, a right-hand side and an equals sign between them

Error: the difference between an estimated value and actual value

Evaluate: to determine a value for an expression

Exact: a precise measurement

Experiment: the process of performing repeated trials of an activity for the purpose of obtaining data in order to be able to predict the chances of certain things happening

Expression: mathematical statement made up of pronumerals (letters) and numbers. Expressions do not contain an equals sign.

Favourable outcome: any outcome of a trial that belongs to an event

Five-figure summary: consists of the minimum value, lower quartile, median, upper quartile and maximum value of a data set

Fixed spending: spending that occurs over a given time interval that is fixed amount

Formula: a rule that is written using pronumerals

Frequency table: a display that tabulates data according to the frequencies of predetermined groupings

Fried's formula: a method of estimating the dosage of medication for an infant under 2 years old

Fuel consumption rates: the amount of fuel (petrol/diesel) used per 100 km

Global positioning system (GPS): a tool used to location any position on the Earth's surface by using satellites

Goods and services tax (GST): a 10% tax that is added to the cost of most goods and services in Australia

Gradient: a measure of the slope of a line at any point

Gram: the standard unit of weight in the metric system

Greenwich Mean Time (GMT): *see* Universal Time (UTC)

Heat map: data presented in a tabular-format with user-defined colour ranges like low, average and high indicating the level of activity instead of numbers

Histogram: a display of continuous numerical data similar to a bar chart, in which the width of each column represents a range of data values and the height of the column represents that range's frequency

Hourly wage: a set amount of money earned each hour

Hypotenuse: the longest side of a right-angled triangle that is situated opposite the right angle

Impossible: in probability, an outcome that can never happen

Income tax: a tax levied on people's financial income and based on an 'income tax table'

Interest: payment earned for having money stored in a bank or financial institution

Interest rate: the percentage of the principal that is paid out in a given time period as interest

International Date Line (IDL): an imaginary line on Earth located 180 degrees from the Prime Meridian that defines the boundary from one day to the next

Interquartile range (IQR): the difference between the upper (or third) quartile and the lower (or first) quartile

Inverse operation: operations that undo a previous operation

Joules (J): the standard metric measurement of energy

Key: an information box outside of a graph that indicates details about the components in the graph

Kilocalorie (kcal): 1000 calories, used for food energy

Kilojoules (kJ): 1000 joules, the commonly used metric measurement of energy in Australia

Kilowatt hour (kWh): a standard measurement of electricity consumption per hour

Limit of accuracy: half of the smallest unit marked on a measuring tool

Linear equation: an equation in which the variable has an index (power) of 1

Linear function: a function that is a straight line when drawn

Linear pattern: a pattern that has a constant difference between each term

Linear relationship: a relationship between two variables, where as one variable changes uniformly, so does the other

Lines of latitude: lines that flow around the Earth from east to west and begin at the equator

Lines of longitude: lines that flow from the North Pole to South Pole

Litre: the standard unit of volume in the metric system

Loss: what occurs if an item's selling price is less than the cost price

Lower adjacent value: the first value above the lower fence in a data set

Lower fence: a low limit of a data set with any data lying below this this considered an outlier

Mass: describes how much matter makes up an object. The base unit of mass is grams

Mean: the sum of all the values (scores) divided by the number of values

Median: the middle value(s) of a data set when the values are placed in numerical order

Medicare levy: a portion of taxpayer's funds used to pay for Medicare (healthcare for Australian residents)

Megajoules (MJ): a standard measurement for gas consumption

Metre: the standard unit of length in the metric system

Midpoint: the centre score of each class interval in grouped data

Modal class: the class interval with the highest frequency

Modality: a measure of the number of obvious peaks in a data set

Mode: the value in a data set that occurs most often

Modulus: a calculation done inside vertical bars, which means you can ignore the sign of the number

Monthly income: the amount of income that a person receives in one month

Multimodal: data with three or more obvious peaks in a graph

Negatively skewed: a distribution that has larger amounts of data as the values of the data increase

Net: a two-dimensional representation of the surfaces of the three-dimensional shape

Net profit: the profit made if an item's price is greater than the cost

Nominal: categorical data that has no natural order or ranking

Non-response bias: when individuals chosen for a sample are unwilling or unable to participate in a survey, leading to survey sample results being skewed away from the true population result

Numerical: data that can be counted or measured

Observation: a study where researchers collect data based on what is seen and heard and make inferences based on the data collected

Ogive: *see* cumulative frequency curve

Ordinal: categorical data that can be placed into a natural order or ranking

Outcome: the result obtained from a probability experiment

Outlier: an extreme value or unusual reading in the data set, generally considered to be any value beyond the lower or upper fences

Overtime: any additional hours a person works after a certain number of hours a week. This is usually paid at a higher rate than the normal hourly wage

Parameter: a value associated with a population

Pareto chart: a chart that consists of both vertical bars and a line graph

Pay As You Go (PAYG) tax: a withholding tax system administered by the Australian Taxation Office

Penalty rate: the pay rate paid when workers in some jobs are required to work on weekends and public holiday

Pension: government financial support given to some people after they have retired

Percentage discount: the percentage of the original price subtracted to get the reduced price

Percentile: the value below which a given percentage of all scores lie

Perimeter: the distance around the outside (border) of a shape

Physical Activity Level (PAL): the expression of a person's daily physical activity, used to estimate a person's total energy expenditure in combination with the BMR

Piecework: a fixed method of payment typically for production of individual items

pm: the time interval from midday to midnight

Population: the whole group from which a sample is drawn

Population parameters: the statistical values calculated from a sample, e.g. the mean and standard deviation

Positively skewed: a distribution that has larger amounts of data as the values of the data decrease

Precision: how closely individual measurements agree with each other

Principal: the initial amount of money you start with in a financial institution, such as a bank or credit union

Prism: a solid shape with identical opposite ends joined by straight edges

Profit: the difference between the amount the seller buys an item for and sells the item for

Pronumeral: a letter used in place of a number; another name for a variable

Pythagoras' theorem: in any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

Quantiles: data sets split up into any given number of equal parts

Quartiles: values that divide an ordered data set into four equal parts

Radius (plural 'radii'): the distance from the centre of a circle to its circumference

Random number generator: a device or program that generates random numbers between two given values

Range: difference between the highest and lowest scores in a set of data.

Rate in the dollar: the rate at which council taxes are charged

Registration fee: a combination of administration fees, taxes and charges paid to legally drive a vehicle on the road

Relative frequency: the frequency of a particular score divided by the total sum of the frequencies

Rise: the change in the y -value over a defined period

Royalty: a payment made to authors, composers or creators for each copy of the work or invention sold. Royalties are typically calculated as a percentage of the total sales

Run: the increase in the x -value over a defined period

Salary: a fixed amount of money earned in one year, but usually paid a portion of this each fortnight or month

Sale price: the price of an item after subtracting the discount

Sample: part of a population chosen so as to represent the population as a whole

Sample size: the number of participants in a sample

Sample space: a list of all the possible outcomes obtained from a probability experiment. It is written as ξ or S , and the list is enclosed in a pair of curled brackets $\{ \}$.

Sample statistics: the statistical values calculated from a sample, e.g. the mean and standard deviation

Sampling methods: different ways of selecting a data sample for a study

Scientific notation: used to express very large or very small numbers. To express a number in standard form, write it as a number between 1 and 10 multiplied by a power of 10

Sector: a portion of a circle bounded by two radii and an arc

Selection bias: bias that results from an unrepresentative sample

Shape: the form or distribution of a data set

Side-by-side bar chart: bar charts that contain multiple sets of categorical data presented as multiple bars in the same chart

Significant figures: the number of digits in a whole number or decimal that are reliably known

Simple interest: interest paid on the principal amount of an investment

Simulation: a model of an experiment using devices such as spinners, dice or coins to represent a real life scenario

Spread (distribution): how far data values are spread from the centre or from each other

Stamp duty: a tax levied by all Australian territories and states on property purchases, such as homes, land, and vehicles

Standard deviation: a measure of the variability of spread of a data set

Standard form: *see* scientific notation

Standard unit: a unit used most commonly for measurement

Stem-and-leaf plot: an arrangement used for numerical data in which data points are grouped according to their numerical place values (the ‘stem’) and then displayed horizontally as single digits (the ‘leaf’)

Straight-line depreciation: depreciation that happens by a fixed amount of dollars each time interval

Substitute: to replace a variable in a formula with a number

Successful trial: when a trial is the result that is wanted

Summary statistic: a value associated with a sample

Superannuation: A percentage of annual salary that is set aside for retirement

Symmetric: when the data distribution has two halves roughly equal about the mean

Tax deductions: work related expenses that are subtracted from taxable income, which lowers the amount of money earned and amount of tax paid

Taxable income: the amount of income remaining after tax deductions have been subtracted from the total income

Third-party fire and theft: third party property insurance with some add-on features that cover your vehicle

Third-party property: motor vehicle insurance that covers damage to other people's property and legal costs, but not damage to your own vehicle

Time: measurement used to work out how long we have been doing things or how long something has been happening

Time zones: regions on the Earth's surface that run on the same time

Timetable: a list of events that are scheduled to occur

Total income: the sum of all money earned by an individual

Total surface area (TSA): the total area of all outside faces of a three-dimensional object

Transposing: rearranging a formula to make a particular pronumeral the subject

Trapezoidal rule: splitting shapes into trapezia to calculate perimeter or area of an irregular shape

Tree diagram: a diagram used to list all possible outcomes of two or more events

Trial: an experiment that is repeated a number of times and becomes part of a larger experiment

Undercoverage: when some members of the population are inadequately represented in a sample

Unimodal: data with one obvious peak in a graph

Universal Time (UTC): the reference point for every time zone in the world. All other time zones are either ahead of, or behind, UTC

Upper adjacent value: the last value below the upper fence in a data set

Upper fence: a high limit of a data set with any data lying above this this considered an outlier

Value: the total amount of money after the interest has been added to the principal

Vehicle insurance: covers the costs associated with vehicle accidents, such as repairs, replacement vehicle, injuries to people

Volume: the amount of space a three-dimensional object occupies

Voluntary response bias: when bias occurs through sample members being self-selected volunteers

Weight: the gravitational force acting on an object

y-intercept: The point at which a line crosses the y-axis

Young's formula: A method of estimating the dosage of medication for a child

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