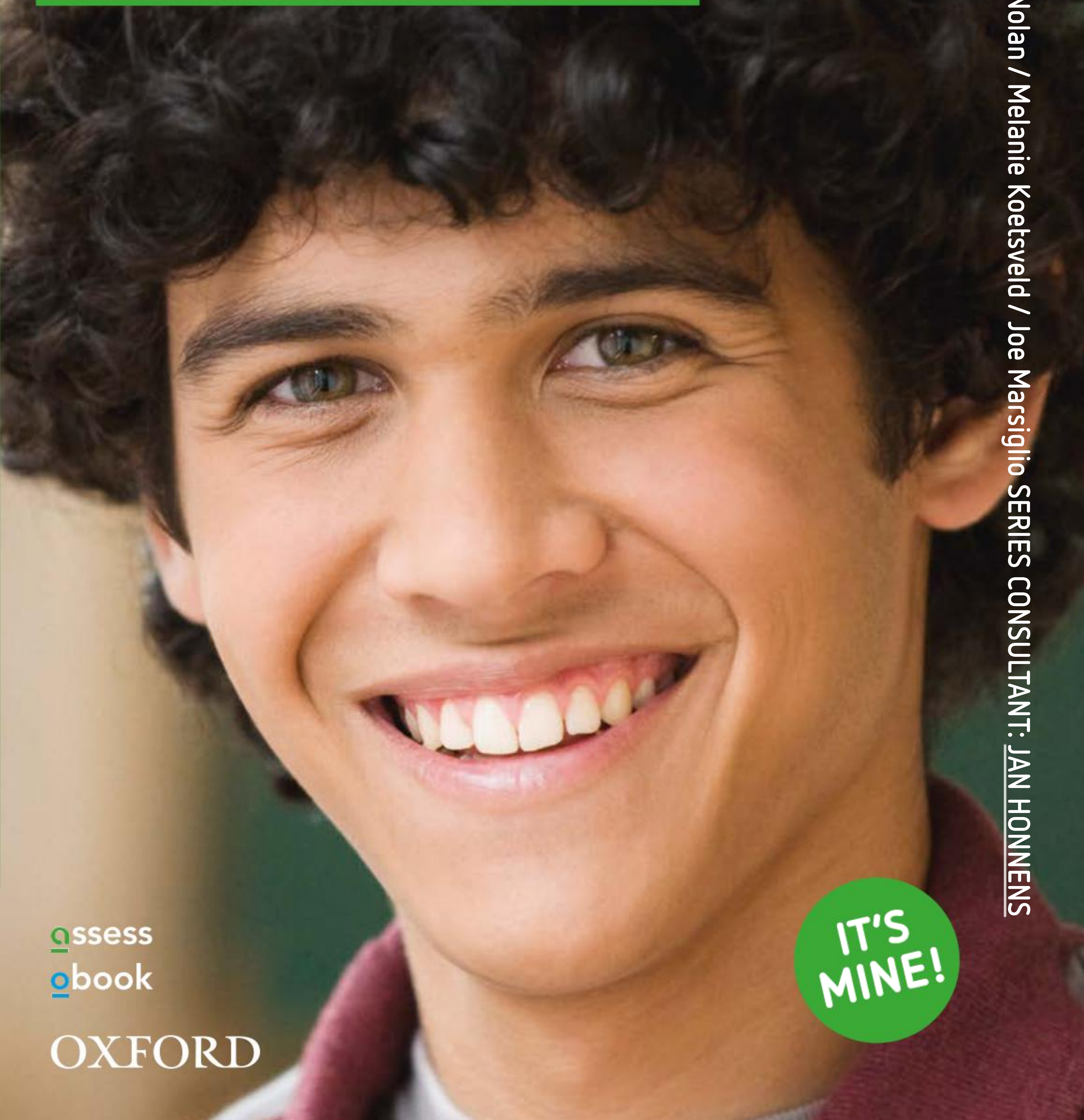
MY MATHS 9

WESTERN AUSTRALIAN CURRICULUM



AUTHORS: Jennifer Nolan / Melanie Koetsveld / Joe Marsiglio SERIES CONSULTANT: JAN HONNENS

IT'S MINE!

assess
obook

OXFORD

MY MATHS 9

AUTHORS: Jennifer Nolan / Melanie Koetsveld / Joe Marsiglio SERIES CONSULTANT: JAN HONNENS

OXFORD
UNIVERSITY PRESS
AUSTRALIA & NEW ZEALAND

OXFORD
UNIVERSITY PRESS

Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trademark of Oxford University Press in the UK and in certain other countries.

Published in Australia by
Oxford University Press
253 Normanby Road, South Melbourne, Victoria 3205, Australia

First published 2015

National Library of Australia Cataloguing-in-Publication data

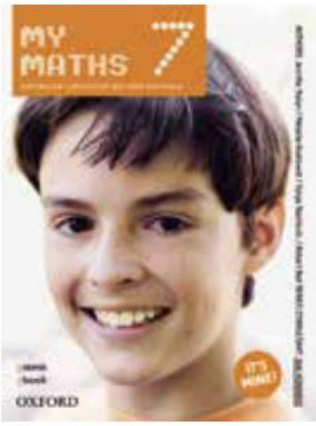
Nolan, Jennifer, author
MyMaths 9 Western Australian Curriculum / Jennifer Nolan, Melanie Koetsveld, Joe Marsiglio
9780195598117 (paperback)
Includes index.
For secondary school age.
Mathematics – Study and teaching (Secondary) – Western Australia.
Koetsveld, Melanie, author.
Marsiglio, Joe, author.

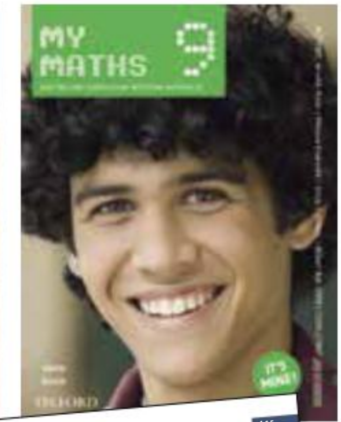
510

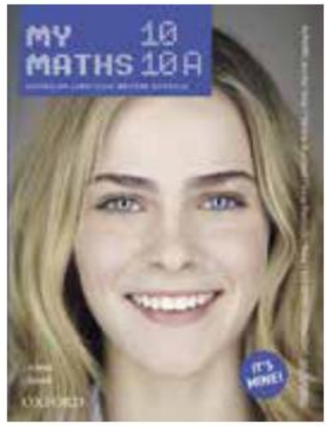
Text design by Ana Cosma
Cover design by Kim Ferguson
Edited by Jane Fitzpatrick
Typeset by Palmer Higgs
Indexed by Max McMaster, Master Indexing
Printed in Hong Kong by Sheck Wah Tong Printing Press Ltd

STATISTICS AND PROBABILITY

OXFORD MYMATHS FOR WESTERN AUSTRALIA







Oxford MyMaths for Western Australia has been specifically developed to support students wherever and whenever learning happens: in class, at home, with teacher direction or in independent study.

STUDENT BOOK + OBOOK/ASSESS

- Finely levelled exercises to ensure smooth progress
- Integrated worked examples – right where your students need them
- Learning organised around the 'big ideas' of mathematics
- Discovery, practice, thinking and problem-solving activities promote deep understanding
- A wealth of revision material to consolidate and prove learning
- Rich tasks to apply understanding
- Highly accessible and easy to navigate
- Comprehensive digital tutorials and guided examples to support independent progress

The student book accurately and carefully delivers the course.

Every question matched to the Australian Curriculum proficiencies.

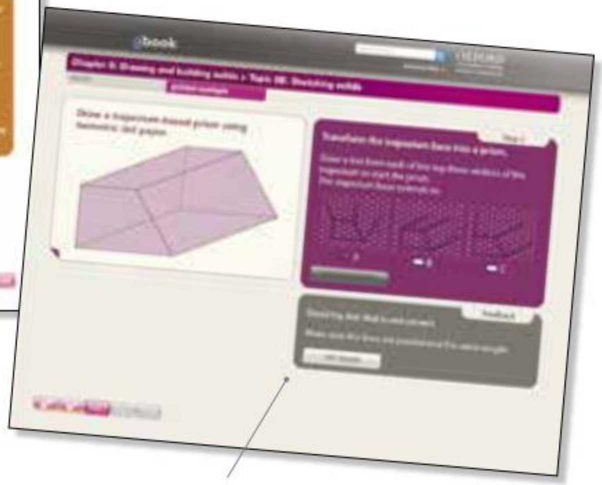
Worked examples are clearly laid out and located where students need them most.

24/7 LEARNING AND SUPPORT

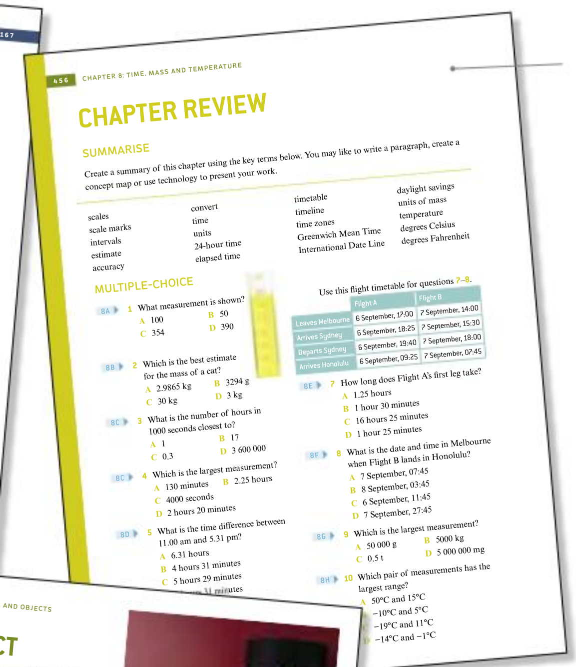
E-tutors scaffold understanding of key concepts and build confidence.

Self-discovery opportunities for students through guided exploration.

Guided examples support practice and fluency



Finely levelled content enables students to progress with ease



Ample revision to consolidate understanding and prove that learning has happened

Students receive feedback for incorrect responses

Optimise understanding and performance.

Personalised learning: tailor the very best learning experiences for all.

Intervention and extension worksheets supplied for every topic.

Rich tasks where students can demonstrate understanding

TEACHER OBOOK/ASSESS

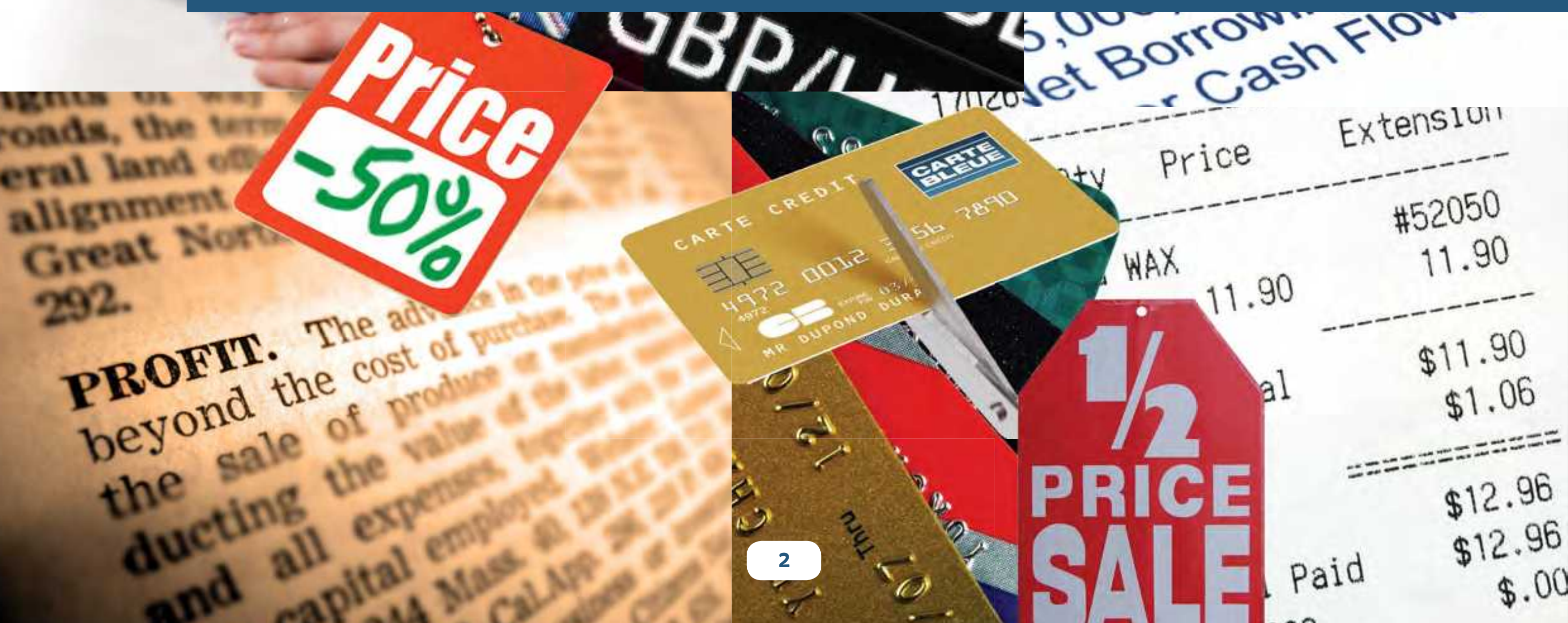
Practical classroom resources and tools:

- Manage student differentiation
- Correct common misconceptions
- Assign work
- Set tests
- Monitor results
- Any device, anytime, anywhere.

1 FINANCIAL MATHEMATICS

ESSENTIAL QUESTION

How can you make the best financial decisions?



Are you ready?

1A ▶ **1** Perform each calculation.

a 456 + 812 **b** 873 − 238

c 8721 × 6 **d** 864 ÷ 6

1A ▶ **2** What is 200 × 4000?

A 800 **B** 8000

C 80 000 **D** 800 000

1A ▶ **3** What is 9000 ÷ 30?

A 300 **B** 900

C 2700 **D** 270 000

1A ▶ **4** What is 358 × 24?

A 1432 **B** 2148 **C** 8592 **D** 9592

1B ▶ **5** The answer to 45.687 × 64.12 will have how many decimal places?

A 2 **B** 3 **C** 4 **D** 5

1B ▶ **6** Perform each calculation.

a 3.157 + 35.6214

b 68.501 − 26.627

c 4.851 ÷ 3

d 8.6147 × 9

1C ▶ **7** Gabe contributed $30 to the cost of a computer game and his mate Jake contributed $40. The ratio of Gabe's contribution to Jake's in simplest form is:

A 30:40 **B** 40:30 **C** 3:4 **D** 4:3

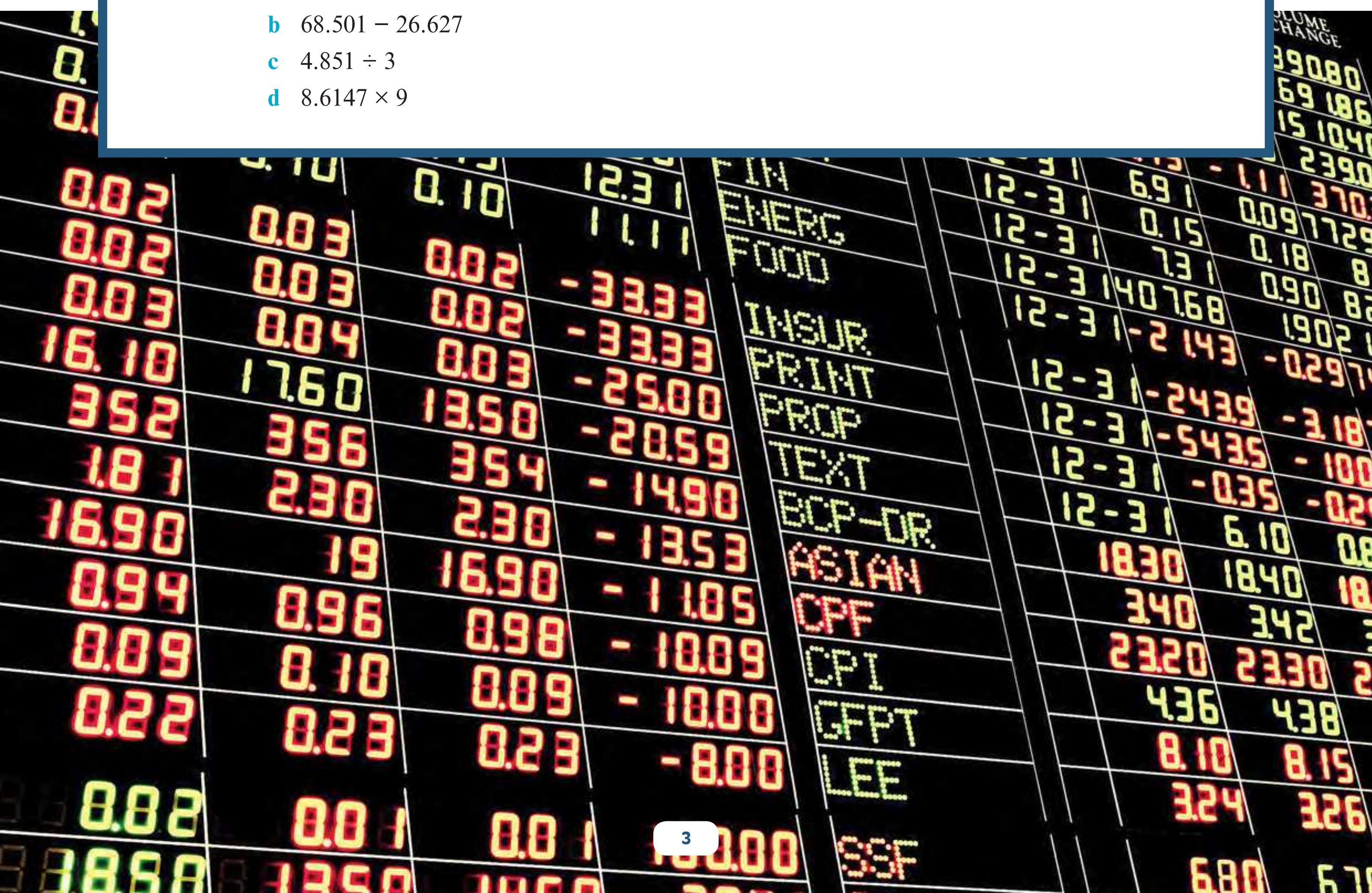
1D ▶ **8** Which of these represents the calculation of 20% of 150?

A 20 × 150 **B** $\frac{20}{150} \times 100$

C $\frac{20}{100} \times 150$ **D** $\frac{20}{100} \times \frac{2}{150}$

1F ▶ **9** How many years is 36 months equivalent to?

1G ▶ **10** What is the solution to the equation $x \times 5 = 50$?



1A Working with whole numbers

Start thinking!

The attendance figures at each match of the 2011 AFL final series is recorded in this table.

Adding all the values will provide the **exact value** for the attendance.

	Match 1	Match 2	Match 3	Match 4
Week 1 Qualifying and elimination finals	73 400	67 379	39 205	90 161
Week 2 Semi-finals	55 198	42 803		
Week 3 Preliminary finals	87 112	59 455		
Week 4 Grand final	99 537			

1 What was the exact attendance for the first week in the final series?

When an exact value is not required, and an **estimated value** is sufficient, then this value can be found by first **rounding** each number to the **leading digit**. Remember that the leading digit is the first digit in the number.

2 Make a list of the steps for rounding a number to its leading digit.

3 Round each of the attendance figures for each week of the finals to the leading digit and use these values to provide an estimated value for the attendance for each week.

4 Why do you think it may be appropriate to approximate calculations rather than calculating their exact value?

KEY IDEAS

- An approximate value for a calculation can be found by rounding each number to its leading digit before performing any operations.
- To round to the leading digit, consider the second digit in the number.
  - If the second digit is 0, 1, 2, 3 or 4, the first digit stays the same and the digits that follow are replaced by zero.
  - If the second digit is 5, 6, 7, 8 or 9, the first digit increases by one and the digits that follow are replaced by zero.
- Single-digit numbers are not changed when rounded.

EXERCISE 1A Working with whole numbers

1 Round each amount to its leading digit.

a \$480	**b** \$938	**c** \$2940	**d** \$1209
e \$97	**f** \$138	**g** \$83 017	**h** \$105 873
i \$592 084	**j** \$743 182	**k** \$981 984	**l** \$3 509 143

2 Perform each calculation.

a 10 000 + 9000	**b** 800 − 30	**c** 400 × 30	**d** 8000 ÷ 200
e 700 000 × 4000	**f** 30 000 ÷ 100	**g** 700 × 5000	**h** 60 000 ÷ 2000

EXAMPLE 1A-1 Estimating values for calculations using rounding

Estimate the result of each calculation by first rounding each number to its leading digit.

a \$45 708 + \$135 680 + \$269 358 **b** 129 394 × \$52 **c** \$845 032 ÷ 36

THINK	WRITE
a 1 Round each number to its leading digit.	**a** \$45 708 + \$135 680 + \$269 358 ≈ \$50 000 + \$100 000 + \$300 000
2 Perform the addition.	= \$450 000
3 Write your answer.	\$45 708 + \$135 680 + \$269 358 ≈ \$450 000
b 1 Round each number to its leading digit.	**b** 129 394 × \$52 ≈ 100 000 × \$50
2 Perform the multiplication.	= \$100 000 × 10 × 5 = \$5 000 000
3 Write your answer.	129 394 × \$52 ≈ \$5 000 000
c 1 Round each number to its leading digit.	**c** \$845 032 ÷ 36 ≈ \$800 000 ÷ 40
2 Perform the division.	= \$80 000 ÷ 4 = \$20 000
3 Write your answer.	\$845 032 ÷ 36 ≈ \$20 000

3 Estimate the result of each calculation by first rounding each number to its leading digit.

a \$26 358 + \$37 517 + \$42 012	**b** \$180 954 − \$39 648
c 436 027 × \$62	**d** \$936 038 ÷ 31
e \$42 658 + \$92 467 − \$38 513	**f** \$814 318 + \$103 687 − \$751 355
g 32 681 × \$125	**h** \$8 025 365 ÷ 390

EXAMPLE 1A-2 Finding the difference between exact and estimated values

Find the difference between the exact value and the estimated value for the calculation 89 034 × $57.

THINK	WRITE
1 Calculate the exact value by performing long multiplication or using a calculator.	89 034 × $57 = $5 074 938
2 To calculate the estimated value, first round each number to its leading digit.	89 034 × $57 ≈ 90 000 × $60
3 Perform the multiplication.	= 9 ×10 000 × $6 × 10 = $54 × 100 000 = $5 400 000
4 State the estimated value.	89 034 × $57 ≈ $5 400 000
5 Subtract the two results to find the difference. Remember to subtract the smaller number from the larger.	$5 400 000 − $5 074 938 = $325 062
6 Write the answer.	The difference between the exact value and the estimated value is $325 062.

UNDERSTANDING AND FLUENCY

4 Find the difference between the exact value and the estimated value for each calculation.

a $97 361 + $18 658
b $739 871 − $438 698
c $368 654 + $45 681 − $249 360
d 102 365 × $27
e 648 367 × $32
f $814 360 ÷ 40
g $351 × 1463
h $150 960 ÷ 20

5 Copy and complete the table below.

	Calculation	Each number rounded to its leading digit	Estimated answer	Exact answer	Difference between exact and estimated answers
a	$358 248 – $214 358				
b	$92 674 + $195 647 + $590 159				
c	924 328 × $37				
d	$4 258 935 ÷ 15				
e	$21 × 27 851				
f	$625 384 + $84 372 – $489 325				

UNDERSTANDING AND FLUENCY

6 At a budget meeting, a salesperson predicted the company would sell 26 laptops throughout the next month.

a Write an approximation for the number of laptops and the cost of the laptops by rounding each value to its leading digit.

b Estimate the total amount raised from the sale of the planned number of laptops.

7 Alice has a new part-time job working at a supermarket where she earns $16 per hour.

a Estimate her pay in a week in which she works 20 hours.

b Calculate the difference between the estimated pay in part **a** and Alice's exact pay for the week.

c Alice uses the estimate to budget her spending and savings. Is this the best strategy for Alice? Explain.

8 Carol works as a supervisor and has an annual salary of $60 424.

a Write a mathematical statement using values rounded to the leading digit that will give an estimate of Carol's weekly pay.

b Will Carol's actual weekly pay be higher or lower than the estimated value? Briefly explain your answer.

9 A group of eight people shared a major Lotto prize of $4 132 848.

a Round the amount to its leading digit and estimate the size of each person's share.

b Calculate the exact amount of each person's share.

c What is the difference between the estimated amount and the exact amount?

d Which value is more important for each person – the estimated value or the exact value?

PROBLEM SOLVING AND REASONING

10 A concert is attended by an audience of 12 947. Of these, 895 people paid the premium price of $185 per ticket. The remaining people each paid the standard $87 per ticket.

a What is the exact number of people who paid the standard ticket price?

b Round each of the ticket prices to its leading digit.

c Round the number of people who paid the premium ticket price to its leading digit and estimate the total amount made from the sale of the premium tickets.

d Round the number of people who paid the standard ticket price to its leading digit and estimate the total amount made from the sale of these tickets.

e What is the exact amount of money made from the sale of all tickets at this concert?

f What is the difference between the exact amount of ticket sales and the estimated amounts? (Hint: add the amounts from parts **c** and **d** to obtain the total estimated value.)

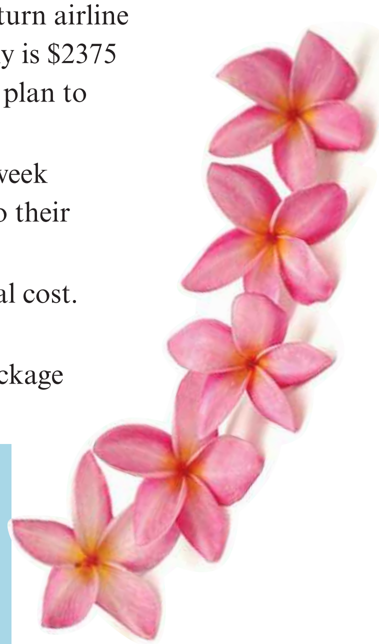
11 As an improving golfer, Stuart decided to buy himself a new set of golf clubs that would suit his game. He has been saving $90 per month for the past 18 months.

- **a** Estimate how much Stuart has saved by rounding the number of months to its leading digit.
- **b** Stuart decides on 13 golf clubs for his set, and these are priced at $169 per club. Round each of these values to its leading digit and estimate the cost of the new set of clubs.
- **c** Based on your estimated figures, has Stuart saved enough money to afford the new golf clubs?
- **d** If Stuart doesn't have enough money, estimate how many more months he needs to keep saving for.

While investigating the best club options, Stuart also decides on a new golf bag and golf buggy. The bag is priced at $229 and the buggy is priced at $184.

- **e** What is the total estimated price that Stuart needs to pay for the clubs, bag and buggy?
- **f** Perform the calculation to determine the exact price Stuart needs to pay for the clubs, bag and buggy.
- **g** What exact amount of money has Stuart saved in 18 months?
- **h** How much more money does Stuart need to save in order to complete the whole purchase?
- **i** Is it possible to perform an exact calculation to determine how much longer Stuart needs to continue saving for or is it best to estimate? Provide a brief reason to support your answer.

12 A family of six prepare for a holiday to Hawaii. The cost for a return airline ticket is $885 per person and accommodation for the whole family is $2375 for five nights and $315 per night for any additional nights. They plan to allow $2000 to cover the cost of all meals for a week.



- **a** Estimate the total cost of the flight, accommodation for one week and food allowance by first rounding the appropriate values to their leading digit.
- **b** Compare the estimated cost for the holiday with the exact total cost. State the difference in price.

Before finalising their plans, the family investigate some other package deals. Two offers are shown below.

> Option 1: Price includes airfare, seven-day accommodation and all meals: $1550 per person.
>
> Option 2: Price includes airfare, accommodation for five nights and all meals: $1435 per person. Extra nights: $85 per person per night.

PROBLEM SOLVING AND REASONING

- **c** Round each appropriate value to its leading digit and estimate the cost of both options.
- **d** According to the estimated costs, which option appears to be the best option?
- **e** Calculate the exact cost for each option.
- **f** Compare the exact cost for each option with the estimated costs found in part **c**. Does the best exact price match the best estimated price?
- **g** Why do you think there was such a difference between the estimated prices for both options.
- **h** Considering all pricing options, should the family stick with the original plan or take one of the package deals?

13 The **average** weekly income of Australian workers is considered to be $1323.

- **a** Round this amount to its leading digit.
- **b** Round the amount to its second digit; that is, to the nearest hundred.
- **c** Round the amount to its third digit; that is, to the nearest ten.

14 The Australian government's predicted future revenue is 405.2 billion dollars for the year. In the same year, they predict their expenses will be 399.0 billion dollars.

- **a** **i** Round each amount to its leading digit.
  - **ii** Round each amount to its second digit.
  - **iii** Round each amount to its third digit.
- **b** What degree of accuracy have the government figures been rounded to in part **a**? (Hint: have they been rounded to the nearest hundred, thousand, ten thousand etc.?)
- **c** Provide a list of reasons as to why you think government figures are not written as exact values.

15 Reconsider the calculations in the table in question **5**.

- **i** Round each value to its second digit and recalculate the estimates.
- **ii** Round each value to its third digit and recalculate the estimates.
- **iii** Compare each set of answers. Which method of estimating do you feel provided a result closest to the actual answers for each calculation?

CHALLENGE

16 Using the government figures in question **14**, write three examples of an amount that would:

- **a** round to the predicted expense value when rounded to its leading digit
- **b** round to the predicted revenue value when rounded to its second digit
- **c** round to the predicted expense value when rounded to its third digit.

17 When estimating the results to calculations after rounding values to the leading digit, how can you predict whether the estimate will be higher or lower than the exact result? You may wish to provide numerical values to help explain your answer.

Reflect

What advantages and disadvantages do you feel rounding can have on estimating financial calculations?

1B Working with decimals

Start thinking!

When using decimal numbers to represent money, the whole number part of the decimal represents the dollar amount and the decimal part of the number represents the number of cents.

1 Why do you think decimal numbers representing money are written to contain two **decimal places**?

A keen tennis player, Jake is researching which tennis balls offer him the best value for money. He knows the **best buy** will be the one where the price per tennis ball is the cheapest. Jake has decided on the two canisters shown in the photo.

The cost for each canister can be written as a **rate statement**. For the three-ball canister, the statement could be $7.99 for three balls or $7.99 per 3 balls.

2 Write the information provided with the second canister as a rate statement.

3 For a rate to be in simplest form, the second part of the statement (that is, the second of the two quantities being compared) must have a value of 1.

   a The rate for the three-ball canister is $7.99 per 3 balls. What operation needs to be performed to make the second quantity have a value of 1?

   b Copy and complete this calculation to write the rate in simplest form.
   $7.99 per 3 balls = $_________ per 1 ball

   c What is the cost of the ball to the nearest cent?

4 Complete question **3** for the four-ball canister.

5 Which option should Jake buy? Provide a brief explanation to help support your answer.

KEY IDEAS

- Decimals containing two decimal places can be used to represent money values.
- The rules for operations with decimals are also applied to calculations involving money.
- A rate compares two quantities that are of a different kind.
- For a rate to be in simplest form, the second of the two quantities being compared must have a value of 1.

EXERCISE 1B Working with decimals

UNDERSTANDING AND FLUENCY

1 Round each amount to the nearest five cents.

a $24.39 **b** $36.11 **c** $28.03
d $44.88 **e** $22.32 **f** $55.60
g $35.74 **h** $99.98 **i** $0.36
j $4.82 **k** $105.27 **l** $33.33

2 Perform each calculation.

a $37.84 + $156.32 **b** $352.36 − $87.84
c $523.68 + $364.62 + $92.65 **d** $17.80 × 8
e $110.40 ÷ 6 **f** $28.55 × 24
g 35 × $126.85 **h** $28.75 × 37.5
i $987.55 × 142.5 **j** $2045.68 ÷ 11.1

EXAMPLE 1B-1 Writing a rate statement

Write this statement as a rate with the appropriate unit.
$32.50 in each hour

THINK	WRITE
1 As two different quantities (dollars and time) are being compared, the statement can be written as a rate.	Rate is dollars per time.
2 Show the number of the first quantity (32.50) for one unit of the second quantity.	rate = $32.50 per 1 hour
3 The word ‘per’ can be replaced with the symbol /.	= $32.50/hour

3 Write each statement as a rate with the appropriate unit.

a $30 earned in each hour
b $1.35 for 1 L of petrol
c Hire cost of $55 for every hour
d Cost of $2.45 for every jar
e Call cost of 75 cents for every minute
f Cost of $12.99 for every kilogram
g Salary of $60 000 for every year
h Charge of $6.85 for each parcel mailed

EXAMPLE 1B-2 Writing a rate statement in simplest form

Write this statement as a rate in simplest form:
$196.65 for 9 hours work.

THINK	WRITE
1 Write the two quantities as a rate statement.	rate = \$196.65 per 9 hours
2 For the rate to be in simplest form, the second quantity needs to be 1. To achieve this, divide both quantities by 9.	$= \frac{\$196.65}{9}$ per $\frac{9 \text{ hours}}{9}$ = \$21.85 per 1 hour
3 Write your answer.	The rate is \$21.85/hour.

UNDERSTANDING AND FLUENCY

4 Write each statement as a rate in simplest form.
- **a** $42 for 8 hours
- **b** $22.35 for 15 L of petrol
- **c** $39.20 for 5 kg of apples
- **d** 50 mL bottle of perfume costs $180.00
- **e** $56.28 for 42 L of petrol
- **f** $24.36 for a 14 minute mobile phone call
- **g** 200 g bag of chips costs $3.20
- **h** $768.60 for 36 hours' work

5 Write each statement as a rate in simplest form. Where necessary, round each amount to the nearest cent.
- **a** $38.45 for 7 kg of oranges
- **b** $156.00 for 6.5 hours work
- **c** $22 collected in 60 minutes
- **d** 5.5 m length of timber costs $45.50

6 Rafael works as a courier delivering parcels around the city. He earns $18.50 per hour.
- **a** Write the information as a rate with the appropriate units.
- **b** Calculate Rafael's wage for a week in which he works 20 hours.
- **c** In one particular week, Rafael's wage totalled $684.50 before **deductions**. How many hours did Rafael work in this week?

UNDERSTANDING AND FLUENCY

7 Lina works in research and earns an annual **salary** of \$66 548.

a Write the information as a rate with the appropriate units.

b If Lina is paid monthly, write her monthly payment as a rate statement in simplest form.

c If Lina is paid fortnightly, write her fortnightly payment as a rate statement in simplest form.

8 A **wage** is a payment made to workers based on a fixed hourly rate. A salary is an annual amount of money that can be paid on a fortnightly or monthly basis. What other payment methods can be used?

9 A person's pay before any deductions are subtracted is referred to as **gross income**. Examples of deductions include **income tax**, superannuation, union fees, payments to health benefits, and so on. The amount of pay after **deductions** have been subtracted is referred to as **net income**. Calculate the net income for each of these.

a Gross income of \$498.95; income tax \$56.80; union fees \$9.45.

b Weekly wage: 36 hours at \$25.70 per hour; income tax \$187.50; health fund \$38.90.

c Annual salary: \$91 200 (paid monthly); monthly deductions: income tax \$1807.80, superannuation \$380 and health fund \$61.25.

d Weekly wage: 37.5 hours at \$18.50 per hour; income tax \$86.80; superannuation \$20.45.

PROBLEM SOLVING AND REASONING

10 Workers on a wage who work beyond the normal hours may be eligible for **overtime**, which means they receive a higher rate of pay for the extra hours worked. Common overtime rates used are **time-and-a-half** and **double time**.

Time-and-a-half means the worker is paid $1\frac{1}{2}$ times the normal hourly rate of pay.

Double time means the worker is paid twice the normal hourly rate of pay.

For each of these normal hourly rates, calculate:

i the time-and-a-half rate

ii the double time rate.

a \$18 **b** \$24

c \$18.80 **d** \$25.90

e \$32.60 **f** \$29.90

11 Ryan is a bricklayer and is paid a wage of \$28.90 per hour for a standard 36.5-hour week. The first 8 hours' overtime are paid at time-and-a-half and any additional hours are paid as double time.

a Calculate Ryan's gross income in a week in which he works 48.5 hours.

b Ryan's deductions for this week include income tax at \$372.40, union fees \$18.90 and superannuation \$82.60. Calculate his net income for the week.

12 Karen works as a casual barmaid and earns $22.50 per hour on weekdays. Time on Saturdays is paid at time-and-a-half until 9.00 pm and double time thereafter. Calculate Karen's gross income for the week she worked the hours shown.

Monday	9.00 am to 3.00 pm
Wednesday	12.00 pm to 4.30 pm
Thursday	11.00 am to 6.30 pm
Friday	4.00 pm to 11.00 pm
Saturday	10.00 am to 11.00 pm

13 For these calculations, round your answer to the nearest five cents.

a A bag of six cheese and bacon rolls is $5.94. What is the cost of one roll?

b 2.5 kg of pumpkin costs $11.90. What is the cost of 1 kg of pumpkin?

c A 24-can carton of soft drink sells for $14.88. What is the cost of one can?

d 300 g of shaved ham costs $7.98. What does it cost for 100 g?

14 April is a manager of a team of employees on a production line. Their hours worked are displayed in the table. The normal hourly rate of pay is $20.80.

	Total hours worked		
Employee	Normal rate	Time-and-a-half	Double time
Rodjay	36	0	0
Hansani	18	10	4
Anitya	28	0	12
Brendan	36	5	8

a Calculate the time-and-a-half rate of pay and the double time rate of pay.

b Use the information to determine each employee's gross income.

c Considering the total hours worked by each employee, how many hours at the normal rate is their total hours worked equivalent to?

15 Given the information in this table, calculate the net weekly income in each case.

	Normal rate of pay $	Hours worked			Deductions		
		Normal rate	Time-and-a-half	Double time	Income tax $	Superannuation $	Union fees $
a	12.40	20	0	0	19.90	0.00	0.00
b	25.00	35	6	8	326.00	35.00	24.50
c	35.60	28	5	0	255.10	30.00	0.00
d	19.90	10	1	3	34.90	0.00	8.75
e	26.80	36.5	3	4	269.90	78.25	21.80

PROBLEM SOLVING AND REASONING

16 Calculate the cost of the items listed in parts **a**–**d**. Write your answers correct to the nearest:

- **i** cent
- **ii** five cents.

a 5 kg of potatoes at $7.85 per kg

b 3.55 kg of apples at $4.90 per kg

c 0.825 kg of salad leaves at $7.20 per kg

d 2.6 kg of premium mince at $12.99 per kg

17 Consider the options for purchasing these strawberries.

a Represent each purchase option as a rate statement.

b In order to compare the two rates, the units must be the same. Convert the price of the punnet strawberries to an equivalent rate per kilogram and then compare. Which option represents the best buy?

c A friend tells you that the best buy is always the option with the lowest advertised price. Comment on the accuracy of this statement.

Option 1

Option 2

18 A trip to the supermarket offers many opportunities to investigate purchases that represent the best buy. Determine which of these represents the best buy.

a a 45-g bag of crisps for $1.40 or a 175-g bag of crisps for $3.24

b an 800-g box of cereal for $3.00 or a 500-g box for $1.90

c a pre-packed 750-g bag of salted peanuts for $16.90 or peanuts sold loose for $23.95 per kg

d a 425-g jar of pasta sauce for $2.80 or a 680-g jar for $4.00

e 1.7 kg of sausages costing $8.00 or 560 g of sausages costing $3.50

f a 2-L bottle of fruit juice for $6.94 or a 500-mL bottle for $3.57

19 A tennis club has two options to consider in determining the best value choice for their practice balls. They can purchase cases of 18 four-ball canisters for $198 per case or 50-ball boxes at $135 per box. Which option would you advise the tennis club to take?

CHALLENGE

20 Gary's net pay for a week was $1185.60. He had a deduction of $341.70 for income tax and $22.50 for union fees. He worked 30 hours at the normal rate, 8 hours at time-and-a-half and 6 hours at the double time rate. Calculate his normal hourly rate of pay.

Reflect

How do rates help to compare the prices of two items?

1C Working with ratios

Start thinking!

After contributing to their term charity, four students, Natalie, Daniel, Megan and Patrick discovered that they had contributed donations in the **ratio** 5:8:1:2.

1 There are four components in the given ratio, each representing the names listed in the given order. Is this ratio in simplest form or can it be simplified?

2 Adding all the numbers in the simplified ratio represents the total number of parts in the ratio. What is the total number of parts in this ratio?

3 The number of parts in the ratio that matches Natalie's contribution is 5. Natalie's contribution can be written as a fraction of the total number of parts in the ratio. Remember to always write the fraction in simplest form.

Write a fraction for each student's contribution to the charity.

Between them, the four students raised a total of $192 for the charity. Each person's actual contributions can now be determined by multiplying their fraction by the total amount raised.

4 Using the fractions from question **3**, calculate how much each person contributed.

KEY IDEAS

- When dividing a quantity in a given ratio, follow these steps.
  1. Find the total number of parts in the simplified ratio.
  2. Write each part of the ratio as a fraction of the total number of parts.
  3. Multiply each fraction by the quantity and simplify.
  4. Check your answer by adding the individual amounts and see that the result is the same as the original quantity.
- **Equivalent ratios** are formed by multiplying or dividing each part of a ratio by a whole number.
- Equivalent ratios can be used to find an unknown value.

EXERCISE 1C Working with ratios

UNDERSTANDING AND FLUENCY

1 Calculate the total number of parts for each ratio.

a 4:7	**b** 3:9	**c** 5:8	**d** 2:8
e 8:4:2	**f** 2:5:3	**g** 12:18	**h** 6:10:24
i 9:7:28	**j** 12:8:16	**k** 8:5:8	**l** 9:9:15

2 For each ratio in question **1**, write each part of the ratio as a fraction of the total number of parts in simplest form.

3 Identify the ratios from question **1** that are not in simplest form and write them in simplest form.

4 Complete these calculations.

a $\frac{1}{5} \times \$250$ **b** $\frac{3}{4} \times \$160$ **c** $\frac{5}{8} \times \$480$ **d** $\frac{2}{7} \times \$210$

EXAMPLE 1C-1 Dividing a quantity in a given ratio

Divide $3600 in the ratio 3:6:1.

THINK	WRITE
1 As the ratio is in simplest form, add the number of parts in the ratio.	$3 + 6 + 1 = 10$ parts
2 Write each part of the ratio as a fraction of the total number of parts in simplest form.	$\frac{3}{10}, \frac{6}{10} = \frac{3}{5}$ and $\frac{1}{10}$
3 Multiply each fraction by the quantity to be divided ($3600) and simplify.	$\frac{3}{10}$ of \$3600 $= 3 \times \$360 = \$1080$; $\frac{3}{5}$ of \$3600 $= 3 \times \$720 = \$2160$; $\frac{1}{10}$ of \$3600 $= 1 \times \$360 = \$360$
4 Answer the question, including the appropriate units. Remember to check your answer by adding the individual amounts. ($1080 + $2160 + $360 = $3600)	The ratio 3:6:1 divides $3600 into $1080, $2160 and $360.

5 Divide $7200 in each given ratio.

a 1:5 **b** 4:5 **c** 2:7 **d** 1:2 **e** 1:6:2 **f** 2:5:3

6 Divide $4800 in each given ratio.

a 4:6 **b** 7:5 **c** 4:8 **d** 3:6:3 **e** 4:6:2 **f** 7:5:4

7 Divide \$10 500 in each given ratio.

a 3:7 **b** 2:5:3 **c** 1:4 **d** 3:5 **e** 2:6:4 **f** 7:5:3

8 Divide each amount in the given ratio. Where necessary, round to the nearest cent.

a \$4500 (7:3) **b** \$2950 (3:5) **c** \$10 650 (2:3)
d \$8486 (3:2) **e** \$12 866 (6:1) **f** \$4628 (4:5)
g \$9837 (2:5) **h** \$41 982 (2:3:4) **i** \$5190 (2:7:1)
j \$2718 (2:4:5) **k** \$18 875 (4:6:5) **l** \$105 784 (3:5:4)

EXAMPLE 1C-2 Finding an unknown value in an equivalent ratio statement

Find the value of a in the equivalent ratio statement $3:7 = a:84$

THINK	WRITE
1 Equivalent ratios are formed by multiplying or dividing each part of the ratio by a whole number. The number to multiply by is 12, since 7 × 12 = 84.	$3:7 = a:84$ $7 \times 12 = 84$ $3 \times 12 = a$ $3:7 = 36:84$
2 Identify the value for a.	$a = 36$

UNDERSTANDING AND FLUENCY

9 Use your understanding of equivalent ratios to find the value of each **pronumeral**.

a $2:7 = 24:a$ **b** $7:5 = b:35$ **c** $c:96 = 4:8$ **d** $54:d = 9:12$
e $6:13 = 24:e$ **f** $5:f = 55:33$ **g** $12:15 = g:60$ **h** $h:8 = 56:64$

10 Three friends went into business together and contributed the amounts of \$10 000, \$12 500 and \$8000 to meet the initial costs, agreeing to divide all profits in the same ratio as their contributions. After their first six months, they made a profit of \$7564. Divide this amount into the three agreed shares.

PROBLEM SOLVING AND REASONING

11 Gabby and Jo have been investing in a savings plan. For every $4 that Gabby invested, Jo invested $7. At the end of a year, they had $2288 and planned to divide the money in the same ratio as their contribution.

- **a** Write the comparison of Gabby's contribution to Jo's as a ratio.
- **b** Write each part of the ratio as a fraction of the total number of parts and find the value of each share.
- **c** Gabby thought her final share should be $\frac{4}{7}$ of the total amount. What do you think she has done incorrectly?

12 For their major fundraiser, a basketball club is running a raffle. The major prize is $15 000 and each ticket costs $50 to buy. Connor, Luke, Jaymee and Maddie decide to pool their money to buy a ticket and share the winnings in the same ratio as their contribution. If they win, Connor will receive $4200, Luke will receive $2250, Jaymee will receive $4650 and Maddie will receive the rest.

- **a** How much money will Maddie receive if they win?
- **b** What is the ratio of their contributions in simplest form?
- **c** How much did each person contribute to the cost of one ticket?

13 Ratios must contain whole number parts. If they don't, then each part of the comparison needs to be multiplied by the same number to produce whole numbers and keep the comparison equivalent. The decimal parts of a comparison can be written as whole numbers by multiplying them by an appropriate power of 10 (10, 100, 1000 …).

Consider the comparison 10.4 to 4.2.

- **a** What power of 10 do both decimals in the comparison need to be multiplied by? Perform this multiplication.
- **b** Now that the equivalent comparison contains two whole numbers, write it as a ratio in simplest form.

14 When comparisons contain fractions, the fractions are best written in an equivalent form with common **denominators**. Multiplying each fraction by the common denominator will then result in whole number parts.

Consider the comparison $\frac{1}{10}$ to $\frac{3}{5}$.

- **a** Write each fraction in the comparison as an **equivalent fraction** with the same denominator.
- **b** Multiply each fraction by the common denominator value.
- **c** Now that the equivalent comparison contains two whole numbers, write it as a ratio in simplest form.
- **d** How would these steps change if one (or both) of the parts of the comparison is a mixed number?

EXAMPLE 1C-3 Writing a comparison as a ratio in simplest form

Write each comparison as a ratio in simplest form.

a 0.4 to 1.8 **b** $\frac{3}{4}$ to $\frac{3}{5}$

THINK	WRITE
a **1** Multiply each decimal by 10 to write the comparison with whole numbers.	**a** 0.4 to 1.8 = 4 to 18
2 Since the comparison now involves whole numbers, write as a ratio.	= 4:18
3 Simplify by dividing each part by the **highest common factor** (HCF) of 4 and 18. (HCF = 2.)	= 2:9
b **1** Write the comparison as equivalent fractions with a common denominator of 20.	**b** $\frac{3}{4}$ to $\frac{3}{5}$ $= \frac{15}{20}$ to $\frac{12}{20}$
2 Multiply each fraction by 20 to write the comparison with whole numbers.	= 15 to 12
3 Since the comparison now involves whole numbers, write as a ratio.	= 15:12
4 Simplify by dividing each part by the HCF of 15 and 12. (HCF = 3.)	= 5:4

PROBLEM SOLVING AND REASONING

15 Write each comparison as a ratio in simplest form.

a 3.2 to 2.1 **b** 5 to 2.5 **c** 3.22 to 2.44 **d** \$4.50 to \$3.00

e 5.6 to 2.92 **f** 1.84 to 6 **g** \$12.50 to \$3.70 **h** 17.1 to 14.2

i 4 to $\frac{1}{2}$ **j** $\frac{2}{3}$ to $\frac{1}{3}$ **k** $\frac{4}{5}$ to $\frac{1}{6}$ **l** $\frac{5}{7}$ to $\frac{2}{5}$

m $\frac{3}{4}$ to 6 **n** $\frac{1}{6}$ to $\frac{3}{4}$ **o** $\frac{1}{4}$ to $2\frac{1}{8}$ **p** $1\frac{2}{3}$ to $1\frac{3}{5}$

16 From her part-time job, Monika can earn a regular amount of \$120 per week. On average, she finds that she uses $\frac{3}{8}$ of her earnings on shopping, $\frac{2}{5}$ on iTunes, $\frac{1}{10}$ goes towards her mobile phone account and she saves the rest.

a What fraction of Monika's earnings can she save?

b On average, how much of Monika's weekly earnings goes towards each of the listed categories?

c Write a ratio for the average amount Monika contributes weekly to shopping, iTunes, mobile phone account and saving.

PROBLEM SOLVING AND REASONING

17 Jayden and Benjamin contribute to a computer game in the ratio 6:7. Jayden's actual amount is $24.

a How much is Benjamin's contribution?

b What is the total cost of the computer game?

18 Justin's weekly pocket money is one third of the amount his older brother Anthony receives.

a Write the comparison of Justin's weekly pocket money to his brother's as a ratio.

b Anthony's pocket money is $15 per week. How much money does Justin earn each week as pocket money?

c Now write the comparison of Anthony's weekly pocket money to Justin's as a ratio. Briefly explain how this is interpreted.

19 Consider the amount of $1000 and the ratio 8:5. To increase $1000 in the ratio 8:5, you can use an equivalent ratio statement. Since you want to increase the amount, match the known amount with the smaller number in the ratio. Using x to represent the increased amount, the equivalent ratio statement is: $x:1000 = 8:5$.

a Find the value of x.

b Check that x is larger than 1000 (as you are increasing the value) and that $x:1000$ simplifies to 8:5 when x is replaced with your calculated value.

c Now consider decreasing $1000 in the same ratio. This time, match the known amount with the larger number in the ratio. Using y to represent the decreased amount, the equivalent ratio statement would be $1000:y = 8:5$ (or $y:1000 = 5:8$). Find the value of y.

d Check that y is smaller than 1000 (as you are decreasing the value) and that $1000:y$ simplifies to 8:5 (or $y:1000$ simplifies to 5:8) when y is replaced with your calculated value.

CHALLENGE

20 Calculate each of these. (Hint: refer to the method shown in question **19**.)

a Increase $200 in the ratio 5:4.

b Decrease $5000 in the ratio 7:10.

c Decrease $150 in the ratio 2:3.

d Increase $4820 in the ratio 8:5.

e Increase $14 000 in the ratio 4:3.

f Decrease $7221 in the ratio 5:9.

21 As a shortcut, a student noticed that she could calculate the value for question **20a** by multiplying 200 by $\frac{5}{4}$.

a Check to see if this gives the same result.

b Using this shortcut method, what fraction would you multiply 5000 by to calculate the value for question **20b**? Check to see if this gives the same result.

c Use this method to calculate the results for parts **c–f** of question **20**. Do you obtain the same results as before?

d Why do both of these methods produce the same result?

22 To increase $1000 in the ratio 6:5, William wrote the equivalent ratio statement $1000:a = 6:5$. Is this correct? Explain.

Reflect

How do ratios ensure that amounts can be divided correctly?

1D Percentage of an amount

Start thinking!

While shopping at the end-of-year sales, Kaveri notices that any advertised **discount** is given as a **percentage of an amount**, not as a dollar amount. She knows that it is important to perform an appropriate percentage calculation in order to determine the correct **selling price** (or **retail price**).

Kaveri sees a shirt with an **original price** of \$84. A store is advertising it for sale with a discount of 20%.

1 Briefly explain what is meant by the term 'discount'.

2 **a** The discount amount is an example of a percentage of an amount calculation. Use the given figures to write an **expression** to calculate the discount.

b Calculate the discount on the shirt.

c Now Kaveri knows the discount amount, how would she find the selling price of the shirt?

d Calculate the selling price of the shirt.

Kaveri's father is a fabric importer and marks up the costs of all his materials by a percentage amount before he sells them to customers. The **wholesale price** for a roll of fabric is \$96 and he plans to offer it for sale with a **mark-up** of 70%.

3 Briefly explain what is meant by the term 'mark-up'.

4 **a** The mark-up amount is also an example of a percentage of an amount calculation. Use the given figures to write an expression to calculate the mark-up.

b Perform the calculation to determine how much mark-up is added to the cost.

c Now that mark-up amount is known, how would he set the selling price of the fabric?

d Calculate the selling price of the fabric.

KEY IDEAS

- To calculate a **percentage** of an amount, write the percentage as a fraction and multiply by the amount.

For example:

15% of $120 = \frac{15}{100} \times 120$

- The difference between the regular price and the lower price of an item is called a discount.
- The selling price following a percentage discount can be calculated by using the rule: selling price = (100 − percentage discount)% × original price.
- The amount added to the original price or wholesale price is called a mark-up.
- The selling price following a percentage mark-up can be calculated by using the rule: selling price = (100 + percentage mark-up)% × original price.
- The **unitary method** is used to find the original amount when a percentage of the original amount is known. It involves calculating 1% before finding 100% of the original amount.
- Besides wages and salaries, another form of payment is **commission** where a salesperson earns a percentage of the total amount of sales they make. Some sales people earn a fixed amount or **retainer** plus commission.

EXERCISE 1D Percentage of an amount

1 Calculate each of these.

- **a** 10% of $360
- **b** 25% of $4200
- **c** 20% of $550
- **d** 120% of $400
- **e** 190% of $850
- **f** 7% of $960
- **g** 32% of $729
- **h** 116% of $2950

EXAMPLE 1D-1 Calculating the selling price from a percentage discount

Calculate the selling price after a 45% discount is offered on a watch originally priced at $120.

THINK	WRITE
1 Write a calculation for the selling price. A percentage discount of 45% means you pay (100 − 45)% of the original price. A discount of 45% means you pay 45% less; that is, 55% of the original price.	selling price = (100 − 45)% of $120 = 55% of $120
2 Perform the calculation by writing the percentage as a fraction and multiplying by the amount.	$= \frac{55}{100} \times \$120$ = $66
3 State the selling price of the watch.	The selling price after a 45% discount is $66.

UNDERSTANDING AND FLUENCY

2 Calculate the selling price for each of these.

- **a** 20% discount on $150
- **b** 15% discount on $300
- **c** 25% discount on $840
- **d** 40% discount on $680
- **e** 50% discount on $1238
- **f** 12% discount on $460
- **g** 45% discount on $855
- **h** 30% discount on $124.50
- **i** 70% discount on $2075

EXAMPLE 1D-2 Calculating the selling price from a percentage mark-up

Calculate the selling price after a 60% mark-up on a pair of jeans originally priced at $42.

THINK	WRITE
1 Write a calculation for the selling price. A percentage mark-up of 60% means you pay (100 + 60)% of the original price. A mark-up of 60% means you pay 60% more. That is, 160% of the original price.	selling price = (100 + 60)% of \$42 = 160% of \$42
2 Perform the calculation by writing the percentage as a fraction and multiplying by the amount.	$= \frac{160}{100} \times \$42$ = \$66
3 State the selling price of the pair of jeans.	The selling price after a 60% mark-up is \$67.20.

UNDERSTANDING AND FLUENCY

3 Calculate the selling price for each of these.

- **a** 20% mark-up on \$420
- **b** 50% mark-up on \$668
- **c** 65% mark-up on \$120
- **d** 18% mark-up on \$924
- **e** 87% mark-up on \$1348
- **f** 120% mark-up on \$1600

4 For each of these, determine:

- **i** the selling price
- **ii** the mark-up or discount amount.

- **a** A camera is purchased for \$120 and sold later at a mark-up of 62%.
- **b** A laptop originally marked at \$1198 is offered for sale at a discount of 35%.
- **c** Work tools each marked at \$49.90 are offered for sale with a 15% discount.

5 Julia wishes to purchase a new pair of shoes at an end-of-year sale. She likes the pair shown which is originally priced at \$184.

- **a** Calculate the amount of the discount.
- **b** Calculate the amount Julia will pay for the shoes.

6 The selling price of an item is also known as the retail price. Michael plans to buy a new external hard drive to store his movies on. The hard drive has a retail price of $157.95, but Michael receives a 12.5% discount because he has a customer loyalty card.

- **a** If the discount is 12.5%, what percentage of the retail price will Michael pay?
- **b** Calculate the price Michael pays after the discount? Round your answer to the nearest five cents.

7 The following represent the original prices and the percentage discount amounts offered on some goods. In each case, calculate:

- **i** the selling price after the discount
- **ii** the discount amount.

Where appropriate, round answers to the nearest cent.

- **a** $500; 12% discount
- **b** $179.50; 15% discount
- **c** $249; 8% discount
- **d** $895.95; 4% discount
- **e** $624.60; 14% discount
- **f** $29 995; 5.5% discount
- **g** $12 680; 12.5% discount
- **h** $1495.99; 17.5% discount

8 Melinda makes her own jewellery and adds an 85% mark-up to her costs when determining her retail prices. One of her popular selling items is jewelled earrings. Each earring contains a metal hook, which cost Melinda $8.50 each, and three decorative stones, each costing $4.60.

- **a** How much does it cost Melinda to make each pair of these earrings?
- **b** What is the value of the mark-up?
- **c** How much would Melinda advertise these pairs of jewelled earrings for?

9 A manufacturer advertises their football boots for a wholesale price of $89.90. A sports store plans to sell these boots to the public at a mark-up of 110%.

- **a** If the mark-up is 110%, what percentage of the wholesale price will a member of the public pay for these boots?
- **b** Calculate the retail price for these boots to the nearest five cents.

10 The following represent the wholesale prices and the percentage mark-up amounts offered on some goods. In each case, calculate:

- **i** the retail price after the mark-up
- **ii** the mark-up amount.

Where appropriate, round answers to the nearest cent.

- **a** $620; 24% mark-up
- **b** $89.95; 45% mark-up
- **c** $1269; 80% mark-up
- **d** $450.50; 85.5% mark-up
- **e** $6250; 140% mark-up
- **f** $350.99; 125% mark-up
- **g** $14 625; 112.5% mark-up
- **h** $2295; 137.5% mark-up

11 A girl's bike is reduced to \$198 following a 20% discount. To determine the original price (that is, the price before the discount) Jane reckons you need to calculate 20% of \$198 and add the result to \$198. Tim thinks that Jane has it wrong and that the calculation is more complex. Which person do you think is correct? Show working to support your answer.

12 Calculations involving a percentage of an amount where the original amount is not known can be solved using the unitary method. Consider a television that has a retail price of \$765 after a discount of 15%.

- **a** A discount of 15% means you pay 85% of the original price. That is: 85% of the original price = \$765.
  The unitary method requires you to find how much 1% represents (one unit). Calculate 1% of the original price. (Hint: divide the amount for 85% by 85.)
- **b** The original price of the television represents the full amount, or 100%. Use your answer to part **a** to calculate 100% of the original price. (Hint: multiply the amount for 1% by 100.)
  100% of the original price = \$_______
- **c** What is the original price of the television?

13 The method outlined in question **12** can also be applied to calculate the original amount after a mark-up has been applied. Consider a different television that retails for \$1800 after an 80% mark-up. Calculate the wholesale price of the television. (Hint: a mark-up of 80% means you pay 180% of the original price.)

14 Reconsider the scenario in question **11**. What was the price of the bike before the discount?

15 Calculate the original price in each of these scenarios. Where necessary, round your answer to the nearest five cents.

- **a** A mobile phone sells for \$450 after a mark-up of 50%.
- **b** A pair of sports shorts sells for \$25 after a discount of 20%.
- **c** Eyeliner sells for \$11.85 following a 15% discount.
- **d** A hardware store sells an electric chain saw for \$169 after it is marked up by 95%.
- **e** A furniture store offers a leather lounge suite for sale for \$9995 after a discount of 12.5%.
- **f** Fitness equipment retails for \$1499 following a 140% mark-up.

16 Glenn sells cars and earns 2% commission on the total value of his sales. How much commission does he earn on the sale of a car that costs \$22 490?

17 If you were a salesperson and your income was commission based, what do you think could be an advantage and a disadvantage of this form of payment?

PROBLEM SOLVING AND REASONING

18 Some salespeople are paid a fixed amount, or retainer, plus their commission. This method of payment ensures that money is still earned even if no sales are made. Erica is paid a retainer of \$220 per week plus 5% commission on her sales. How much does Erica earn in a week in which the total value of her sales is \$7255?

19 Barry works as a real estate agent and earns commission on the sale of each house he sells. He earns 2% commission on the first \$300 000 and 1.75% on the amount greater than \$300 000. How much commission does Barry earn on a house that sells for \$485 000?

20 Maria and Paul plan to sell their house and are exploring which real estate agency to use. Their first agency charges a flat rate of 2.3% on the sale value of the house and a second agency charges 3.4% for the first \$200 000, 1.8% for values between \$200 000 and \$350 000 and 1.2% for the value greater than \$350 000. Which agency should they use if they plan to sell their house for \$590 000?

21 Charlotte earns a retainer of \$475 per week and 3.5% commission of the total value of her weekly sales. Calculate her earnings for a week with each of these total sales values.

a \$500 **b** \$8000 **c** \$0 **d** \$3029

e \$2397.50 **f** \$9480.95 **g** \$12 095 **h** \$25 800

22 Angelique is paid a commission of 2.5% of the total value of her sales. In one week, she earned \$375 in commission. What was the total value of her sales?

23 Mark earns a weekly retainer of \$325 plus 1.75% of all his sales. In one week, his earnings were \$937.50. What was the total value of his sales in this week?

CHALLENGE

24 How can percentages be applied to calculate income tax amounts? You may wish to investigate different methods used by the Australian Government and explore who these methods are applied to.

25 In an earlier exercise, superannuation was listed as a possible deduction when calculating an employee's net income. What is superannuation and how can percentages be applied to superannuation contributions and entitlements?

Reflect

What different situations can a percentage of an amount be applied to?

1E Writing one quantity as a percentage of another

Start thinking!

Alexandra is saving for a new model hand-held game console, like the one shown here. So far, she has saved a total of $110.

1 Write the amount that Alexandra has saved as a fraction of the total amount needed for the game console. Write this fraction in simplest form.

2 Explain how the fraction can be represented as a percentage.

3 Write the fraction as a percentage. This calculation is equivalent to writing one quantity ($110) as a percentage of another ($200).

4 How does the percentage amount you have calculated relate to Alexandra's saving progress towards her game?

KEY IDEAS

- To write one quantity as a percentage of another, write the first value as a fraction of the second and multiply the fraction by 100%.
- A **profit** occurs when the selling price is higher than the original price.
- A **loss** occurs when the selling price is lower than the original price.
- Percentage profit (or loss) on the original price = $\frac{\text{profit (or loss)}}{\text{original price}} \times 100\%$.
- Percentage profit (or loss) on the selling price = $\frac{\text{profit (or loss)}}{\text{selling price}} \times 100\%$.
- Percentage profit and loss calculations are generally written in relation to the original price unless directly specified that it is in relation to the selling price.

EXERCISE 1E Writing one quantity as a percentage of another

EXAMPLE 1E-1 Writing one amount as a percentage of another

Write \$65 as a percentage of \$150.

THINK	WRITE
1 Write the first quantity (\$65) as a fraction of the second (\$150).	$\frac{65}{150}$
2 Convert the fraction to a percentage by multiplying it by 100%.	$= \frac{65}{150} \times 100\%$
3 Cancel common factors to the **numerators** and denominators and simplify.	$= \frac{65}{\cancel{150}_3} \times \frac{\cancel{100}^2}{1}\%$ $= \frac{65}{3} \times \frac{2}{1}\%$ $= \frac{130}{3}\%$
4 Divide 130 by 3 and write the answer, rounding appropriately.	$= 43.3\%$

UNDERSTANDING AND FLUENCY

1 Write these amounts as percentages.

- **a** \$45 as a percentage of \$225
- **b** \$60 as a percentage of \$80
- **c** \$36 as a percentage of \$144
- **d** \$120 as a percentage of \$80
- **e** \$99 as a percentage of \$600
- **f** \$123 as a percentage of \$400
- **g** \$67 as a percentage of \$90
- **h** \$468 as a percentage of \$96
- **i** \$2460 as a percentage of \$480

2 Determine the profit or loss amount for each of these.

- **a** original price \$35, selling price \$45
- **b** original price \$82, selling price \$68
- **c** original price \$92.50, selling price \$87.95
- **d** original price \$299.98, selling price \$145.50

EXAMPLE 1E-2 Calculating a percentage profit or loss

A television initially bought for $800 is later sold for $950.

a State if a profit or loss has been made and determine the amount.
b Write the profit or loss amount as a percentage of the original price.

THINK	WRITE
a The selling price is more than the original price, so a profit has been made. Find the difference.	**a** profit = \$950 − \$800 = \$150
b **1** Write the profit amount as a fraction of the original price and multiply it by 100%.	**b** percentage profit $= \frac{150}{800} \times 100\%$ $= \frac{150}{800} \times \frac{100}{1}\%$ $= 18.75\%$
2 Write your final answer.	The television sold for an 18.75% profit.

UNDERSTANDING AND FLUENCY

3 For each scenario:
 i state if a profit or loss has been made and determine the amount
 ii write the profit or loss amount as a percentage of the original price, correct to two decimal places where appropriate.
 a Shoes are bought for $240 and later sold for $180.
 b A greengrocer buys cherries for $2.50 per kilogram and sells them for $9.80 per kilogram.
 c An investor buys shares for $5.20 and sells them for $4.80.
 d A car is purchased brand new for $24 640 and sold for $19 250.
 e Coins are purchased in a set for $120 and sold for $350.
 f A novel is purchased for $29.95 and sold for $8.

4 How do the percentage amounts calculated in question **3** change if the percentages are based on the selling price?

5 Calculate the percentage profit or loss on the original price for each part in question **2**.

6 Daniela pays $198 for her mp3 player and sells it to a friend for $150 when a new model comes out.
 a Did Daniela make a profit or a loss?
 b Write the amount in part **a** as a percentage of the original price.

UNDERSTANDING AND FLUENCY

7 As Benjamin became more successful at his BMX racing, he chose to sell his bike to buy a better model. The bike, which had cost him $240, was sold to a fellow competitor at a percentage profit of 5%.

- **a** How much did Benjamin sell the bike for?
- **b** The new bike Benjamin plans to buy will cost him $900. Write this as a percentage of the cost of his original bike.

8 For each of these:

- **i** state the value of the profit or loss
- **ii** write the profit or loss as a percentage of the original price (rounded to the nearest 1%).

- **a** original price $48, selling price $34
- **b** original price $112.50, selling price $240
- **c** original price $35.90, selling price $85.95
- **d** original price $1649, selling price $1238
- **e** original price $29 895, selling price $17 500
- **f** original price $156 985, selling price $425 850

9 John buys pears at the orchards for $2.95 per kilogram to sell at his market stall.

- **a** How much does John mark up the cost of the pears per kilogram (see photo)?
- **b** Write the mark-up amount as a percentage of the price John pays for the pears. State the mark-up as a percentage to the nearest 1%.

10 A wireless printer is initially priced at $249.95 and is offered for sale at a discounted price of $222.50.

- **a** State the amount of the discount.
- **b** Write the discount as a percentage of the initial price to the nearest 1%.

PROBLEM SOLVING AND REASONING

11 A goods and services tax (GST) is a 10% tax that is added to the cost of many goods and services. This means that prices are increased by 10%. Calculate the prices paid for these items after GST is added, rounding to the nearest cent where appropriate.

- **a** dining table and chairs $1285
- **b** services provided by a plumber $240
- **c** insurance purchased for a car $601.45
- **d** five 3-m lengths of timber at $6.50 per metre
- **e** electricity service and supply charge is $314.65
- **f** membership at a gymnasium at $72.95 per month

12
- **a** For each of the items in question **11**, multiply the given value by 1.1. Compare your answers to those you obtained in question **11**. What do you notice?
- **b** Explain why a 10% increase is the same as multiplying by 1.1.

13 The following items each include the GST charge in the price. Calculate the pre-GST price, rounding to the nearest cent where appropriate. (Hint: find 100% if the given amount is 110% of the pre-GST price.)

- **a** telephone and Internet services \$155.65
- **b** computer accessories purchased for \$235.95
- **c** garden maintenance provide for \$182
- **d** a necklace bought at a jewellery store for \$120.50

14 **a** For each of the items in question **13**, divide each of the given values by 1.1. Compare the answers with those you obtained in question **13**. What do you notice?

- **b** Why do you think the pre-GST price can be determined by dividing the final price by 1.1?

15 Joseph sells remote-controlled cars in his toy store. He knows that identical cars are being sold by a competitor for \$65. Joseph can purchase these cars from a wholesaler for \$28 per car.

- **a** Joseph aims to make a 150% profit on the sale of each car and must add 10% for GST. Do you think this profit margin is a suitable pricing strategy? Briefly explain.
- **b** Using whole number values, what is the maximum percentage increase Joseph should apply to the wholesale price? Remember to add the GST charge.
- **c** Why is it necessary to consider a maximum percentage increase rather than any percentage increase Joseph wishes to apply?

16 A small share portfolio was purchased at a price of \$1200 and sold 12 months later for \$1500.

- **a** Write the increase in price as a percentage of the original price.
- **b** Write the final selling price as a percentage of the original purchase price.
- **c** Compare the percentage increase in part **a** with the answer in part **b**. Briefly explain how they are related.

17 A car is bought for \$20 000 and sold six months later for \$16 000.

- **a** Write the decrease in price as a percentage of the original price paid for the car.
- **b** Write the final selling price as a percentage of the original purchase price.
- **c** Compare the percentage decrease in part **a** with the answer in part **b**. Briefly explain how they are related.

18 What percentage increases or decreases match the following profit/loss amounts?

- **a** doubling your money
- **b** tripling your money
- **c** breaking even
- **d** halving your money
- **e** quadrupling your money
- **f** losing all your money

PROBLEM SOLVING AND REASONING

19 The finance report on the nightly news displays the daily movement in the cost of some common commodities. If the values given in this table represent the end-of-day trading figures, what were the values at the start of the day's trading?

Commodity	Final price $	Movement %
Gold	1732.95	↓ 0.5
Silver	33.56	↓ 1.4
Oil	102.46	↑ 0.3
Copper	3.71	↑ 1.2

↓ represents a decrease in price
↑ represents an increase in price

20 Omar and his family purchase a large block of land and plan to build four townhouses on the block. The land costs \$645 000 and the cost for each house is \$230 000 (including plans, permits and other related charges). The project takes 2 years to complete and Omar is charged rates of \$2300 per year during this time. The amounts generated from the house sales were \$485 000, \$490 000, \$472 000 and \$461 000. The real estate agency earns a commission of 1.75% for the sale of each house.

a What were the total expenses accrued by Omar before the sale of the townhouses?

b From the total sales, how much of the money:

i goes to Omar and his family?

ii is earned as commission by the real estate agency?

c Does Omar make a profit or a loss?

d Write your answer from part **c** as a percentage of Omar's total expenses.

21 Mario runs a hairdressing business from his home and sells shampoos, hair treatment products and styling products to his customers. On all product sales, he plans to make a profit of 80% of the wholesale price he pays for the goods. On top of this, he knows he must add an additional 10% for GST. Mario believes he can determine the selling price by simply adding 90% to the wholesale price.

a A jar of styling gel has a wholesale price of \$8.50. What will the price be after Mario's profit mark-up?

b What will the selling price be after GST is added?

c Increase \$8.50 by 90% and compare your answer with the answer from part **b**. Is Mario's method of calculating the selling price correct? Why or why not?

CHALLENGE

22 **a** In question **21**, you learned that Mario likes to make a profit of 80% on his wholesale prices and then adds 10% for GST. What *single calculation* can Mario perform to determine his selling price for a jar of styling gel?

b A motorbike sells for \$1200 after a mark-up of 60% and then GST is added. What single calculation can be performed to determine the wholesale price of the motorbike?

c GST is added to a price and then the item is discounted by 25% to sell for \$400. What single calculation will determine the original price; that is, the pre-GST price?

Reflect

How important do you think it is to understand how percentages are applied to financial calculations?

1F Understanding simple interest

Start thinking!

When you borrow money from a bank, the total of your repayments is more than the amount borrowed. This additional repayment is known as **interest** and is charged by the banks for allowing you to have access to the money.

Alternatively, if you were to invest money with a bank rather than borrow it, interest can be paid to you on your **investment**. Banks do this for allowing them to have access to your money. (The banks use your money to lend to other customers.)

One form of interest calculation is known as **simple interest**. This calculation is based on the amount borrowed (for a **loan**) or the amount invested (for an investment). Simple interest is the most basic form of interest calculation and it forms the basis of a more advanced and more widely applied type of interest calculation, known as compound interest. (You will study compound interest next year.)

Consider an investment of $10 000 at an interest rate of 5% per annum (p.a.) for a period of one year.

1 The interest rate is stated as 5% per annum. Per annum means 'per year' and is often abbreviated to p.a. The interest earned on this investment is 5% of the amount invested. Calculate the interest earned in the year.

In interest calculations, the amount invested or borrowed is known as the **principal**. The interest rate is referred to as the **rate**, and **time** refers to the length of the investment or loan in years.

2 Use the example and the three key terms, principal, rate and time, to write a formula to calculate simple interest.

KEY IDEAS

- Interest can be an additional charge to a loan or a bonus payment to an investment.
- One type of interest calculation is called simple interest.
- Interest can be calculated using the formula: interest = principal × rate × time, or $I = P \times R \times T$, where
  - I = interest
  - P = principal (the amount borrowed or invested)
  - R = interest rate converted to a fraction or decimal. For example, 5% would be substituted as $\frac{5}{100}$ or 0.05.
  - T = time of the loan or investment in years.

EXERCISE 1F Understanding simple interest

1 Write these interest rates as:

- **i** a fraction in simplest form
- **ii** a decimal.

a 7% **b** 11% **c** 8% **d** 6% **e** 10% **f** 12%

EXAMPLE 1F-1 Calculating simple interest

For an investment of \$5200 at an interest rate of 6% p.a. for 4 years, calculate:

a the amount of simple interest **b** the value of the investment after 4 years.

THINK	WRITE
a **1** Write the formula to use and identify the key terms. The rate must be written as a fraction or a decimal.	**a** interest = principal × rate × time principal = \$5200 rate = 6% = $\frac{6}{100}$ time = 4 years
2 Substitute the values into the formula and calculate the result.	interest = \$5200 × $\frac{6}{100}$ × 4 = \$1248
3 Write the answer.	The simple interest earned in 4 years is \$1248.
b **1** The value of the investment after 4 years is the interest amount added to the principal.	**b** value = \$5200 + \$1248 = \$6448
2 Write your final answer.	The value of the investment after 4 years is \$6448.

UNDERSTANDING AND FLUENCY

2 For each investment, calculate:

- **i** the amount of simple interest
- **ii** the value of the investment after the term.

a an investment of \$5000 at an interest rate of 5% p.a. for 2 years

b an investment of \$4800 at an interest rate of 4% p.a. for 3 years

c an investment of \$12 500 at an interest rate of 8% p.a. for 5 years

3 For each loan, calculate:

- **i** the amount of simple interest
- **ii** the total amount to be repaid.

a a loan of \$7500 at an interest rate of 5% p.a. over 3 years

b a loan of \$10 800 at an interest rate of 12% p.a. over 5 years

c a loan of \$25 000 at an interest rate of 7% p.a. over 8 years

UNDERSTANDING AND FLUENCY

4 Calculate the simple interest given each of these.

a $P = \$4000$, $R = 6\%$, $T = 5$ years **b** $P = \$8650$, $R = 7\%$, $T = 4$ years

c $P = \$15\,000$, $R = 8\%$, $T = 10$ years **d** $P = \$9200$, $R = 4\%$, $T = 3$ years

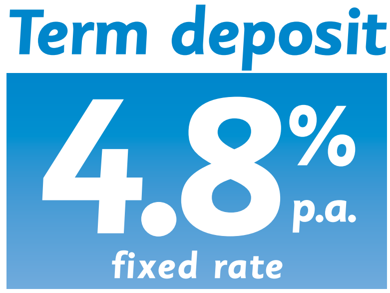
e $P = \$19\,999$, $R = 15\%$, $T = 6$ years **f** $P = \$20\,000$, $R = 20\%$, $T = 5$ years

5 Christian invests $3500 in a bank that offers the interest rate shown. He plans to leave the money invested for 2 years.

a Identify the values for P, R and T.

b How much simple interest does Christian earn?

c What is the total value of Christian's investment after 2 years?



6 Jenna plans to start her business in massage therapy and needs to borrow $44 000 to assist with her set-up costs. She obtains an agreement with her lender to repay the money over 5 years with interest charged at 9.5% p.a.

a Identify the values for P, R and T.

b How much simple interest is Jenna charged?

c What is the total amount that Jenna repays?

7 **a** Consider each of these situations. Calculate the amount of simple interest in each case.

i $5000 is invested at 4.75% p.a. for 3.5 years.

ii $5000 is borrowed at 4.75% for 3.5 years.

b Compare each of the answers in parts **a i** and **a ii**. Briefly explain how the simple interest formula is used for investment and loan situations.

c Given that the simple interest calculations involving loans and investments are identical, how are the calculations different when they are interpreted?

8 Convert each time to years. Where appropriate, write the fraction in simplest form.

a 11 months **b** 7 weeks **c** 26 weeks **d** 3 months

e 271 days **f** 155 days **g** 15 months **h** 48 months

i 84 weeks **j** 1241 days **k** 30 months **l** 286 weeks

9 An investment is made for 4 years and 3 months. Matthew thinks this is equivalent to 4.3 years while Lizzy is certain Matthew is wrong. How is 4 years and 3 months written as a decimal in years?

PROBLEM SOLVING AND REASONING

10 For the values given in the table at right, calculate:

i the amount of simple interest

ii the total amount at the end of the term.

	Principal $	Rate % p.a.	Time
a	9 000	6	3 years
b	10 500	4.5	6 months
c	7 500	9.8	130 weeks
d	29 000	3.2	90 days
e	8 600	6.4	35 days
f	155 570	12.5	11 months
g	19 999	19.9	25 weeks
h	45 950	14.05	2 years and 5 months
i	208 654	8.75	5 weeks and 4 days

11 Sade is investigating which is the best way to calculate her simple interest for a short-term investment. She invests $2400 for the month of June at an interest rate of 4.6% p.a.

a Calculate the simple interest amount after writing the time as a fraction of the total number of months in the year.

b Now calculate the simple interest amount after writing the number of days in June as a fraction of the total number of days in the year.

c Which method of calculation would Sade be hoping would be used? Briefly explain why.

d If the values given represented a short-term loan instead of an investment, which method of calculation would Sade prefer? Briefly explain why.

12 A bank is offering the interest rates advertised for its customers to invest in a term deposit. The interest is calculated at the end of the investment. Jasmine has $20 000 to invest and plans to invest it for 12 months.

a What interest rate will Jasmine receive for her investment?

b How much interest does she earn?

c Jasmine's brother informed her that she would have earned more interest if she invested the money for one day less than 12 months. Investigate whether this statement is true and show working to support your finding.

Term	Interest on investment amount $5000 to <$10 000 %	$10 000 to <$50 000 %	$50 000 to <$100 000 %
1 to <2 months	2.5	2.5	2.8
2 to <6 months	3.25	3.25	3.25
6 to <12 months	5.5	5.55	5.5
12 to <24 months	5.3	5.25	5.2

13 Banks vary in the ways in which they calculate interest on savings and transaction accounts. Some accounts earn no interest while others attract bonus interest rates if certain conditions are met. If an account provides interest, it is most likely calculated on the daily account balance. Consider the account balances for the month of February shown.

Date	Transaction	Amount $	Balance $
01/02	Opening balance		640.90
08/02	Withdrawal at Handybank	100.00	540.90
15/02	Deposit	240.00	780.90
24/02	EFTPOS Purchase	125.40	655.50
28/02	Interest		

a The opening balance of \$640.90 applies for the first seven days of the month as each new balance applies on the date the transaction is made. How many days does each balance on this account apply for?

b The account attracts interest at a rate of 2.1% p.a. For each new balance in the account, calculate the simple interest based on the number of days each balance applies.

c Add all the amounts from part **b** to calculate the total interest for the month.

d What is the account balance at the end of February, if the total interest is added to the account at the end of the last day of the month?

14 This bank statement shows the transactions made during the month of August. Interest is calculated daily at a rate of 1.8% p.a.

a How much interest is earned during the month?

b What is the final account balance?

Date	Transaction	Amount $	Balance $
01/08	Opening balance		345.50
09/08	ATM Withdrawal	50.00	295.50
14/08	Deposit – Pay	370.00	665.50
16/08	ATM Withdrawal	120.00	545.50
19/08	EFTPOS Purchase	85.95	459.55
28/08	Deposit – Pay	370.00	829.55
31/08	Interest		

15 A bank offers an interest rate of 1.5% p.a. on its savings accounts plus an extra 3.2% p.a. bonus rate if no more than one withdrawal is made in the month and the account balance has increased by at least \$200 for the month. Consider each of the account statements shown.

A

Date	Transaction	Amount $	Balance $
01/09	Opening balance		1200.85
15/09	Deposit – Pay	450.75	
24/09	Deposit – at branch	820.00	
29/09	EFTPOS Purchase	245.85	
30/09	Interest		

PROBLEM SOLVING AND REASONING

B

Date	Transaction	Amount $	Balance $
01/01	Opening balance		1548.90
08/01	EFTPOS Purchase	246.20	
15/01	Deposit – Pay	1920.00	
29/01	EFTPOS Purchase	85.94	
31/01	Interest		

a Will any of these accounts receive the bonus interest rate? Provide a reason to support your answer.

b Calculate the total interest earned on each account. You will need to determine the account balances following each transaction first.

c State the final account balance for each statement at the end of the month.

CHALLENGE

16 Joel plans to buy a second-hand car for $12 500. He has saved $2500 and plans to borrow the remaining money from his bank at an interest rate of 8.5% p.a. over 3 years.

a The seller asks for a deposit of 15% of the selling price. Is Joel's savings enough to cover the deposit? (Note that a deposit is the first part of a payment often used as a promise to pay.)

b How much does Joel borrow to buy the car?

c Calculate the total amount, including interest, that Joel pays for the car.

17 You have $2000 and wish to double this amount over 3 years. You plan to explore some different options to earn the most amount of interest possible.

a What is the annual simple interest rate that will enable this investment to double in 3 years?

b Explore how this rate changes if the time of the investment increases to:

i 4 years **ii** 5 years **iii** 6 years.

c Explore how this rate changes if the time of the investment decreases to:

i 2 years **ii** 1 year.

18 Provide three different annual interest rates and their corresponding times that would result in an investment of $5000 earning $1250 in simple interest.

Reflect

What is the difference between interest on an investment and interest on a loan?

1G Working with simple interest

Start thinking!

So far, the use of the simple interest formula has been limited to calculating the amount of interest, given values for the principal, rate and time. Now consider calculations where the interest amount is known and you are asked to find the value of one of the other **variables**; that is, P, R or T.

Tony borrows \$15 000 at an interest rate of 6% p.a. to buy a car. He needs to pay \$3600 in simple interest.

1 From the simple interest formula, which variable do you not know the value of?

2 What variable does each of the given values represent?

3 Substitute the values into the simple interest formula and show that it simplifies to $3600 = 900 \times T$.

4 Solve the **equation** formed to determine the value for T.

KEY IDEAS

- The simple interest formula is $I = P \times R \times T$.
- Strategies for solving equations can be used to find the value for P, R or T.

EXERCISE 1G Working with simple interest

UNDERSTANDING AND FLUENCY

1 Calculate the simple interest in each case.

a $P = \$2000$, $R = 7\%$, $T = 3$ years
b $P = \$250$, $R = 11\%$, $T = 1$ year
c $P = \$8500$, $R = 5\%$, $T = 4$ years
d $P = \$25\,000$, $R = 4\%$, $T = 5$ years
e $P = \$100\,000$, $R = 9.5\%$, $T = 4$ years
f $P = \$16\,000$, $R = 6\%$, $T = 2.5$ years

EXAMPLE 1G-1 Calculating time

How long will it take for an investment of $4000 at an interest rate of 4% p.a. to earn $800 in simple interest?

THINK	WRITE
1 Write the simple interest formula and identify the variables. R should be written as a fraction or a decimal.	$I = P \times R \times T$ $I = \$800$ $P = \$4000$ $R = 4\% = \frac{4}{100}$ $T = ?$
2 Substitute the values into the formula and simplify.	$800 = 4000 \times \frac{4}{100} \times T$ $800 = 160 \times T$
3 Use the balance method to find the value for T.	$\frac{800}{160} = \frac{160 \times T}{160}$ $T = 5$
4 Write the answer.	It will take 5 years for the investment to earn $800 in simple interest.

2 Find the value for T in each of these.

a How long will it take for an investment of $8000 at an interest rate of 3% p.a. to earn $1200 in simple interest?

b How long will it take for an investment of $1250 at an interest rate of 4% p.a. to earn $350 in simple interest?

c How long does a loan of $15 000 at an interest rate of 9% p.a. take to earn $5400 in simple interest?

d How long will it take for an investment of $5600 at an interest rate of 5% p.a. to earn $1120 in simple interest?

EXAMPLE 1G-2 Calculating the principal value

How much needs to be invested at an interest rate of 6% p.a. for 3 years to earn \$1440 in simple interest?

THINK	WRITE
1 Write the simple interest formula and identify the variables. R should be written as a fraction or a decimal.	$I = P \times R \times T$ $I = \$1440$ $R = 6\% = 0.06$ $T = 3$ years $P = ?$
2 Substitute the values into the formula and simplify.	$1440 = P \times 0.06 \times 3$ $1440 = P \times 0.18$
3 Use the balance method to find the value for P.	$\frac{1440}{0.18} = \frac{P \times 0.18}{0.18}$ $P = 8000$
4 Write the answer.	\$8000 needs to be invested to earn \$1440 in simple interest over 3 years.

UNDERSTANDING AND FLUENCY

3 Find the value for P in each of these.

- **a** How much needs to be invested at an interest rate of 8% p.a. for 5 years to earn \$2000 in simple interest?
- **b** How much is borrowed at an interest rate of 10% p.a. over 4 years to earn \$6000 in simple interest?
- **c** How much is borrowed at an interest rate of 9% p.a. over 5 years to earn \$1800 in simple interest?
- **d** How much needs to be invested at an interest rate of 6% p.a. for 2 years to earn \$576 in simple interest?

4 Find the unknown value in each of these.

- **a** $I = \$600$, $P = \$3000$, $R = 4\%$, $T = ?$
- **b** $I = \$1200$, $P = ?$, $R = 5\%$, $T = 4$ years
- **c** $I = \$450$, $P = ?$, $R = 9\%$, $T = 2$ years
- **d** $I = \$850$, $P = \$8500$, $R = 5\%$, $T = ?$
- **e** $I = \$1000$, $P = ?$, $R = 8\%$, $T = 4$ years
- **f** $I = \$5060$, $P = \$9200$, $R = 11\%$, $T = ?$

5 Jessica has invested \$4500 in a bank that offers simple interest of 5.0% p.a. She plans to earn \$675 in interest.

- **a** From the simple interest formula, which variable do you not know the value of?
- **b** What variable does each of the given values represent?
- **c** How long does the money need to be invested to earn \$675 in simple interest?
- **d** At a higher interest rate of 7.5% p.a., how much sooner can Jessica earn \$675 in simple interest?

6 Throughout the course of a simple interest investment, Stefan's money increased in value from $8400 to $8862. The interest was earned at a rate of 2.75% p.a.

a What is the total amount of interest earned on this investment?

b How many months was the initial amount of money invested for?

7 Up to this point, the simple interest formula has been used to calculate the amount of interest, or the principal amount, or the time period of the investment or loan. Now consider calculations requiring you to find the value of the interest rate.

Consider a loan of $6000 taken over 3 years. The amount of simple interest charged on the loan is $864.

a Substitute the values into the simple interest formula.

b Show that the formula simplifies to $864 = 18\,000 \times R$.

c Solve the equation in part **b** by dividing both sides of the equation by 18 000.

d The answer in part **c** represents the interest rate written as a decimal. What needs to be done to this decimal so that the value is written as a percentage amount?

e State the interest rate that was applied to this loan.

f Investigate what can be done to the simple interest formula so that, after substituting the values into the formula and solving, the rate will automatically be given as a percentage. Check that your modified formula produces the same answer you obtained in part **e**.

8 For the values given in the table, calculate the interest rate that applies.

	Simple interest $	Principal $	Time
a	1 640	8 200	4 years
b	420	3 500	2 years
c	985	9 850	48 months
d	1 680	12 000	30 months
e	3 936	18 000	3 years and 5 months
f	680	6 400	4 years and 3 months

9 Use the simple interest formula to determine the value for the missing amount in the table.

	Simple interest $	Principal $	Rate % p.a.	Time
a		3 700	5.6	4.5 years
b	234		4.8	13 months
c	42 532	70 000		6.2 years
d	3 549	19 500	5.2	

	Simple interest $	Principal $	Rate % p.a.	Time
e	1 711.00		14.5	48 months
f	2 631.60	15 480		130 weeks
g	56.88	948		1.2 years
h	1 534.40	13 700	6.4	

PROBLEM SOLVING AND REASONING

10 Craig borrowed a sum of money from his parents to help him buy his first car. They agreed to charge interest at a rate of 4% p.a. over a period of 3 years. The interest charge for the term of the loan is $1440.

- **a** From the simple interest formula, which variable do you not know the value of?
- **b** What variable does each of the given values represent?
- **c** How much money does Craig borrow from his parents?
- **d** Craig plans to pay his parents $350 each month for the 3 years and believes this will cover the agreed terms of their loan. Determine if Craig's plans are correct and show workings to support your finding.
- **e** What are the exact monthly payments Craig needs to make to repay his loan?

11 Although she has the savings to purchase the new iPad shown, Gabriella would rather let the interest earned from her investment cover the cost of the purchase.

- **a** One bank offers her a simple interest rate of 7.2% p.a. for her investment of $10 000. How long does this money need to be invested to earn enough money to pay for the iPad?
- **b** Gabriella decides on 12 months to reach her goal. At the same rate of interest, how much does she need to invest in order to fully pay for the iPad with the interest she earns?

12 Daniel has decided to learn the alto saxophone through his school music program. To encourage his development, his parents bought the saxophone shown through a purchase program arranged by his school. The repayment conditions involve quarterly payments over 3 years. The simple interest charged on the saxophone's cost is $162.

- **a** What is the annual interest rate charged?
- **b** What is the amount of each quarterly payment required?

13 An amount of $4000 is invested at 5.2% p.a. for a period of 3 years.

- **a** Calculate the amount of simple interest that is earned on this investment.
- **b** What is the value of the investment at the end of the 3-year term?

Investments involving simple interest result in the interest being passed on to the investor at maturity (at the end of the investment). Reconsider the investment of $4000 at 5.2% p.a. for 3 years, but now calculate interest during the investment period at yearly intervals and add these amounts to the principal.

- **c** How much interest is earned in the first year of the investment?
- **d** Add the interest amount from part **c** to the principal amount. This new amount is the principal for the second year of the investment.
- **e** Use the new principal value to calculate the interest earned in the second year of the investment.
- **f** Add the interest amount from part **e** to the principal amount for the second year. This new amount is the principal for the third year of the investment.
- **g** Use the new principal value to calculate the interest earned in the third (final) year of the investment.

PROBLEM SOLVING AND REASONING

h Add the interest amount from part g to the principal amount for the third year. This new amount is the final value of the investment.

i Compare your answer from part h with the answer you obtained in part b. Which method of calculation resulted in the higher value at the end of 3 years? Why do you think this is so?

14 The method of interest calculation you performed in question 13c–h is known as compound interest, and you will study it in further detail next year.
Calculate the final value of each of these investments by performing the interest calculations annually.

a an investment of $10 000 at 8% p.a. for 3 years
b an investment of $15 000 at 6.8% p.a. for 2 years
c an investment of $18 000 at 7.5% p.a. for 4 years
d an investment of $50 000 at 10% p.a. for 3 years

15 For each investment in question 14:

i determine the amount of interest earned over the investment term
ii calculate how much more was earned by using compound interest rather than simple interest.

CHALLENGE

16 This bank statement is linked to a savings account and shows the transactions made during the month of April.
What is the annual interest rate (% p.a.) that applies to this account? (Remember that each new balance applies from the day of the transaction.)

Date	Transaction	Amount $	Balance $
01/04	Opening balance		2 905.60
03/04	Deposit – Pay	1 230.75	4 136.35
08/04	ATM Withdrawal	250.00	3 886.35
15/04	EFTPOS Purchase	499.95	3 386.40
17/04	Deposit – Pay	1 230.75	4 617.15
	Monthly interest	7.54	4 624.69

17 The statement shown is linked to a credit card where interest is charged from the day of purchase. To avoid additional charges, the total amount spent, plus interest is to be paid each month.

Date	Description	Amount $
06/07	BPAY to Electricity provider	290.00
08/07	Gym membership	72.00
11/07	Petrol	45.00
20/07	AFL tickets	85.00
21/07	Clothing store	189.95
24/07	Petrol	52.87
	Interest charge for the month of July	6.31

a How much needs to be paid at the end of the month to avoid any additional charges?
b What is the annual interest rate (% p.a.) that is charged to this credit account?

Reflect

Why is it important for the interest rate to be written as a decimal or fraction rather than the given percentage value?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

exact value	overtime	original price	loss
estimated value	time-and-a-half	discount	percentage increase
rounding	double time	mark-up	percentage decrease
best buy	income tax	retail price	simple interest
rate statement	deductions	wholesale price	principal
wage	ratios	commission	rate
salary	equivalent ratio statement	retainer	time
gross income	percentage of an amount	interest	investment
net income	selling price	profit	loan

MULTIPLE-CHOICE

1A **1** After rounding each amount to its leading digit, the estimate for \$920 217 + \$384 521 − \$348 420 is:

A \$1 000 000 **B** \$956 318
C \$1 600 000 **D** \$900 000

1B **2** Which rate statement is in simplest form?

A earning \$631.90 for 35.5 hours work
B paying \$52.06 for 38 L of petrol
C being charged \$10.32 for a 12-minute mobile phone call
D driving at a speed of 100 km per hour

1C **3** When \$8400 is divided in the ratio 3:7, the size of the smallest share is:

A \$840 **B** \$2520
C \$3600 **D** \$5880

1D **4** A sport's store is selling children's tennis racquets at a discount of 20%. If the racquets are initially priced at \$49.50, what will their sale price will be?

A \$9.90 **B** \$29.50
C \$39.60 **D** \$59.40

1E **5** A bike rider paid \$240 for his bike and sold it 12 months later for \$180. Which statement is *not* correct?

A The sale represents a loss of \$60.
B The sale is a 25% loss on the original price.
C The sale is a 25% loss on the selling price.
D The selling price is 75% of the original price.

1F **6** \$12 000 is invested at 4.2% p.a. simple interest for 18 months. Which values should be substituted into the simple interest formula?

A $P = 12\ 000$, $R = 4.2$, $T = 18$
B $P = 12\ 000$, $R = 0.042$, $T = 1.5$
C $P = 12\ 000$, $R = 4.2$, $T = 1.5$
D $P = 12\ 000$, $R = 0.042$, $T = 18$

1G **7** A loan of \$4500 with simple interest 8.5% p.a. is charged \$1530 in interest. Which simple interest variable do you *not* know the value of?

A interest **B** principal
C rate **D** time

SHORT ANSWER

1A **1** Find the difference between the exact value and the estimated value for each calculation.

- **a** 368 983 × \$45
- **b** \$865 478 + \$921 854 − \$328 456
- **c** \$3 058 057 ÷ 98 647

1B **2** Write each statement as a rate in simplest form.

- **a** driving 185 km in 2 hours
- **b** earning \$193.80 for 8.5 hours work
- **c** a 275 mL can of drink costs \$2.50

1B **3** The hours worked by four employees are displayed in the table. The normal hourly rate of pay is \$22.50. Use the information to determine each employee's gross income.

Total hours worked		
Normal rate	Time-and-a-half	Double time
24	5	1
30	0	6
14	6	10
0	15	10

1C **4** Simplify each ratio.

- **a** 36:45
- **b** 9:27:18
- **c** 4:28:32
- **d** 12:45:33:21

1C **5** Write each comparison as a ratio in simplest form.

- **a** 15.1 to 11.3
- **b** \$10.50 to \$4.80
- **c** $\frac{1}{4}$ to $\frac{3}{4}$
- **d** $\frac{2}{3}$ to $5\frac{1}{3}$

1C **6** Divide \$5200 in each of the given ratios, rounding to the nearest cent where necessary.

- **a** 4:6
- **b** 7:3
- **c** 4:8:1
- **d** 2:3:5:3
- **e** 8:4
- **f** 7:5:3

1C **7** Two friends contributed donations of \$180 and \$240 respectively to their club.

- **a** What is the ratio of the donations in the given order in simplest form?
- **b** If next year's contribution is in the same ratio and the first friend contributes \$150, what is the amount of the second part of the ratio?

1D **8** Calculate the price to be paid after:

- **a** a 15% discount on \$758
- **b** a 22.5% discount on \$84
- **c** a 85% mark-up on \$140
- **d** a 155% mark-up on \$68.

1D **9** Calculate the original price for:

- **a** a mobile phone sold for \$225 after a discount of 20%
- **b** paint sold at \$49.95 per can after a mark-up of 80%.

1E **10** For each of these:

- **i** state if a profit or loss has been made and determine the amount
- **ii** write the profit or loss amount as a percentage of the original price, correct to two decimal places.

- **a** original price \$35, selling price \$50
- **b** original price \$104.50, selling price \$85.85
- **c** original price \$199.95, selling price \$245.65

1E **11** Write these amounts as percentages.

- **a** \$55 as a percentage of \$275
- **b** \$80 as a percentage of \$120
- **c** \$150 as a percentage of \$60
- **d** \$145 as a percentage of \$25

1F **12** Calculate the simple interest in each case.

- **a** P = \$3000, R = 5%, T = 4 years
- **b** P = \$6400, R = 2.5%, T = 3 years
- **c** P = \$35 000, R = 4.4%, T = 5 months

1G **13** Find the unknown value P, T or R when:

- **a** I = \$240, P = \$2000, R = 4%
- **b** I = \$854.40, R = 8.9%, T = 2 years
- **c** I = \$1400, P = \$16 000, R = 3.5%
- **d** I = \$630, P = \$3500, T = 2 years
- **e** I = \$1011.50, P = \$8500, T = 3.5 years

NAPLAN-STYLE PRACTICE

1 John's annual pay is \$55 827. Which statement is an estimate for his fortnightly pay?

- ⬭ \$55 827 ÷ 26
- ⬭ \$60 000 ÷ 26
- ⬭ \$60 000 ÷ 30
- ⬭ \$50 000 ÷ 30

2 You can buy a 5 kg bag of apples for \$14.50. Which rate statement is *not* true?

- ⬭ \$14.50 per 5 kg
- ⬭ \$2.90 per kg
- ⬭ \$9.50 per kg
- ⬭ \$7.25 per 2.5 kg

Questions **3** and **4** refer to this information.
Ainslee earns \$15.80 per hour. In one particular week, she worked 14 hours at the standard rate of pay, 5 hours at time-and-a-half and 3 hours at double time.

3 Which calculation would determine her gross income?

- ⬭ 14 × \$15.80 + 5 × \$15.80 + 3 × \$15.80
- ⬭ 14 × \$15.80 + 5 × 1.5 × \$15.80 + 3 × 2 × \$15.80
- ⬭ 14 × \$15.80 + 5 × 1.5 × \$15.80 + 3 × \$15.80
- ⬭ 14 × \$15.80 + 5 × \$23.07 + 3 × \$31.06

4 Ainslee has these items deducted from her pay in this week:
income tax: \$54.20, union fees: \$8.50, superannuation: \$15.75.
What is her net income?

☐

5 Three students contributed \$54, \$72 and \$36 to a fundraising charity. What is the ratio of their contributions in simplest form?

- ⬭ 54:72:36
- ⬭ 36:54:72
- ⬭ 3:4:2
- ⬭ 6:12:4

6 An amount of \$6400 is divided in the ratio 5:2:3. What is the size of the smallest share?

☐

7 What is the value of a in the equivalent ratio statement 5:12 = 45:a?

☐

8 Shorts originally priced at \$79.00 are offered for sale at a discount of 15%. Which represents the calculation for the discount amount?

- ⬭ 85% of \$79.00
- ⬭ 15% of \$79.00
- ⬭ 115% of \$79.00
- ⬭ \$79.00 − 15% of \$79.00

9 The wholesale price on a television is \$820. After a mark-up of 85%, what is the selling price of this television?

☐

10 A refrigerator originally marked at \$1245 is discounted by 17%. What is the selling price?

☐

11 A novel sells for \$15 after a discount of 20%. Which of these represents the original price before the discount was applied?

- ⬭ \$15 ÷ 80 × 100
- ⬭ \$15 + 20% of \$15
- ⬭ 80% of \$15
- ⬭ \$15 × 80 ÷ 100

12 A salesman receives 2.4% of the total sales made during a week. What is his pay in a week where his total sales are \$14 580?

☐

13 What is \$450 written as a percentage of \$550? Round your answer to two decimal places.

☐

Questions **14** and **15** refer to this information.
The original price for a coin set was \$40. When sold some time later, the selling price was \$150.

14 Which statement is correct?

- ⬭ The coins were sold at a loss of \$110.
- ⬭ The coins were sold at a profit of \$110.
- ⬭ The coins were sold at a profit of \$150.
- ⬭ The percentage profit is 375%.

15 What is the profit or loss amount as a percentage of the original price?

☐

Questions **16** and **17** refer to this information.
A loan of $12 500 is charged a simple interest rate of 8% p.a. for a period of 3 years.

16 How much interest is charged to the loan?

17 What is the total amount to be repaid?

Questions **18** and **19** relate to this savings account bank statement for the month of April.

Date	Transaction	Amount $	Balance $
01/04	Opening balance		825.00
05/04	Withdrawal at Handybank	100.00	925.00
13/04	Deposit – Pay	740.00	1 665.00
25/04	EFTPOS purchase	225.00	1 440.00
30/04	Interest		

18 How many days does the highest monthly balance apply to this account?

19 Interest for this account is calculated daily at a rate of 3.0% p.a. How much interest is earned during the month?

20 How much needs to be invested at an interest rate of 5% p.a. for 4 years to earn $1000 in simple interest?

21 How long does a loan of $25 000 at an interest rate of 8% p.a. take to earn $7000 simple interest?

ANALYSIS

Julie manages a clothing store. She earns an annual salary of $42 432 and the normal hourly rate of $18.80 applies to her casual staff, although the opportunity for overtime is available. The store's rent is $1800 per week and Julie allows an extra $200 per week to cover other costs.

a Each week, Julie's deductions include $120.80 in income tax, $24.50 in superannuation and $8.50 in union fees. What is her weekly net income?

b One week, Julie has three staff working. Simone works 24 hours at the normal hourly rate, Melanie works 15 hours at the normal rate, 3 hours at time-and-a-half and 5 hours at double time, and Tahlia works 30 hours at the normal rate and 4 hours at time-and-a-half. Calculate the gross weekly income for each employee.

c What is the minimum amount of money that Julie's store must make in sales each week to cover the cost of staff pay and store costs?

d The store rental is to increase by 40% per week. How much extra money does Julie need to make to cover the increase?

Julie buys dresses for $12 each and plans to sell them for $45 each.

e What is the percentage mark-up that Julie plans to make on the sale of each dress?

f Julie notices that a rival clothing store sells identical dresses for $34. She changes her pricing so that she beats her rival's price by 10%. What is the retail price of the dresses now?

g What is the current selling price as a percentage of the initial price paid?

h What is the new percentage mark-up and how does it compare with the original percentage mark-up in part **e**?

The owners receive a quote for $48 000 to re-fit the store. They have half of this amount in savings and plan to borrow the remaining amount.

i The bank lends the money at a simple interest rate of 8.2% p.a. over 3 years. What is the total amount of money that must be repaid?

j If the money is repaid in equal monthly instalments, what is the amount?

k In total, how much did the store makeover cost?

CONNECT

The best purchase option

As they complete the furnishings for their new house, Jose and his family plan to purchase a complete home-theatre system for their theatre room. They are aware of some different options available to finance the purchase and plan to investigate each option to decide which one best suits their financial needs.

Your task is to perform calculations to determine the selling price of the home-theatre system the family are interested in and investigate whether it would be better to purchase the system through a store's purchase plan or financed by a personal loan from a bank (details of each are shown opposite).

Your task

Follow these steps to complete this investigation:

- Work out the selling price of the home-theatre system following the advertised discount.
- Calculate the costs associated with the store's purchase plan.
- Calculate the costs associated with the loan offered by the bank.
- Compare the options available and explore conditions for which each option could be the best.
- Investigate purchase plans offered by stores and loans offered by various lending banks.

TODAY ONLY

12½%

off marked prices

In-store purchase plan*

- **10% deposit**
- **24 months interest-free**
- **Monthly repayments**

*Conditions Apply

BANK LOAN

Personal loan option

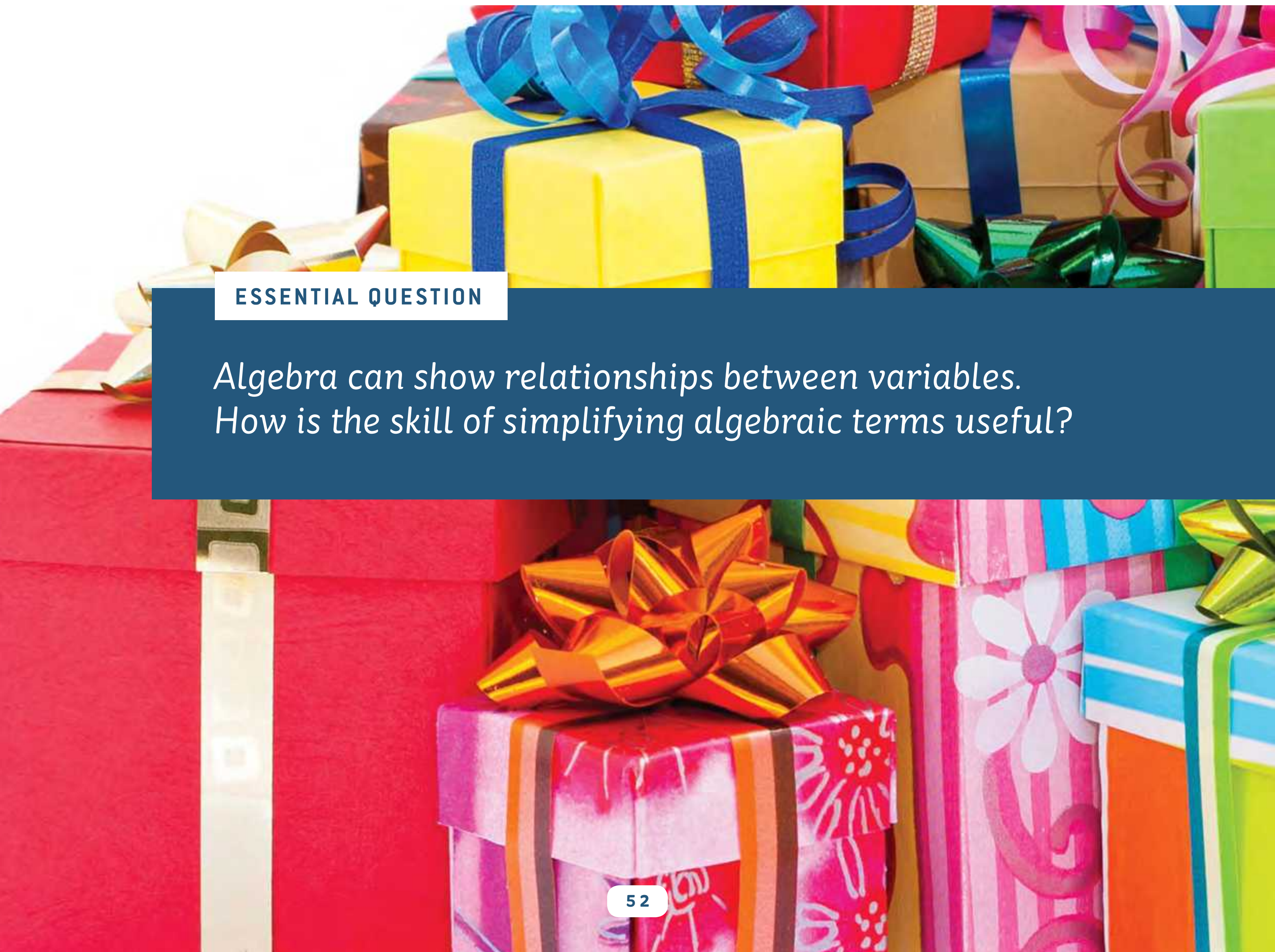
- **Simple interest rate 6.5% p.a.**
- **3-year term**
- **Monthly repayments**

You may like to present your findings as a report. Your report could be in the form of:

- an advertising brochure
- a PowerPoint presentation
- a technology demonstration
- other (check with your teacher).

NUMBER AND ALGEBRA

2 ALGEBRA



ESSENTIAL QUESTION

Algebra can show relationships between variables. How is the skill of simplifying algebraic terms useful?

2A 1 a Write each term in expanded form.
 i $4bcd$ ii $-9xy$
 iii m^2n iv $3ak^3p^2$
 b Write the coefficient of each term shown in part **a**.

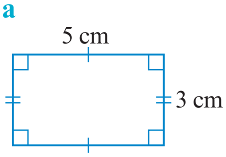
2A 2 Write two examples of like terms for each algebraic term shown.
 a $6d$ b $-2ab$
 c $5kmnp$ d x^2y

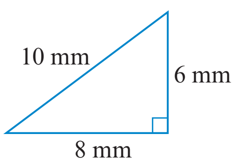
2A 3 Calculate:
 a $-3 + 8$ b $-7 - 6$
 c $2 - 9 + 4$ d $-15 + 4 - 1$
 e -10×-5 f 8×-11

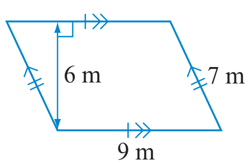
2A 4 a $5m + 3n + 2n + 8m$ simplifies to:
 A $18mn$ B $13m + 5n$
 C $8m + 10n$ D 18
 b $7a - 3a + 5a$ simplifies to:
 A $15a$ B 9 C $9a$ D 15

2A 5 Evaluate each expression after substituting $a = 3$ and $b = 7$.
 a $4a + 9b$ b $6a^2$
 c $3ab - 11$ d $8a + 5b^2$

2A 6 For each shape, find:
 i the perimeter ii the area.

a


b


c


2B 7 Consider 3^4.
 a Identify:
 i the base ii the index or power.
 b Write the number in expanded form as a repeated multiplication.
 c Perform the repeated multiplication to obtain the basic numeral.

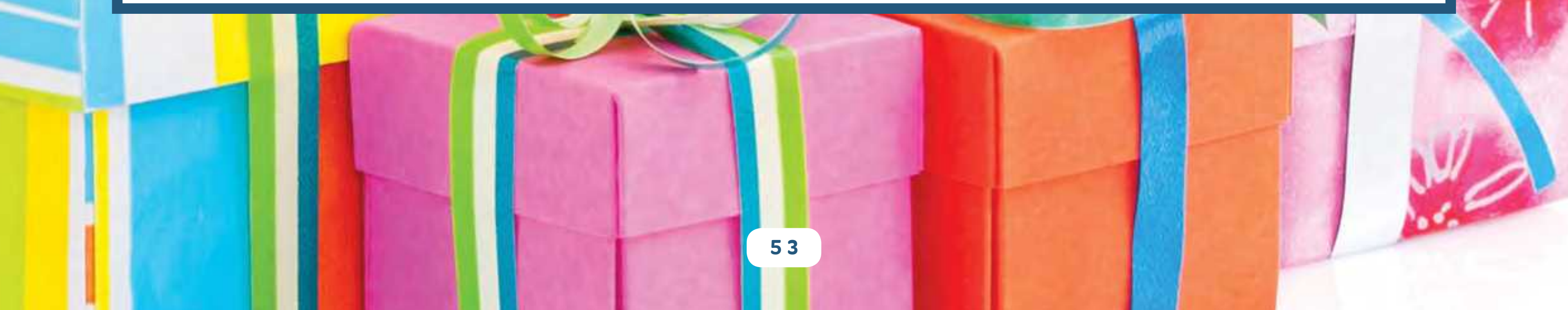
2B 8 Consider x^7.
 a Identify:
 i the base ii the index or power.
 b Write the term in expanded form.

2D 9 Find the result to each problem.
 a 7×100 b 3.25×1000
 c $63\,000 \div 10\,000$ d $0.49 \div 10$

2F 10 a Which list shows factors of 18?
 A 1, 2, 3, 6, 9, 18
 B 1, 3, 8, 9, 18
 C 2, 3, 6, 12, 18
 D 2, 3, 4, 6, 9, 18, 36
 b What is the highest common factor (HCF) of 18 and 24?

2F 11 a Which list shows factors of $6x$?
 A $1, 2, 3, 4, 6, x$
 B $x, 2x, 3x, 6x, 12x$
 C $1, 6, x, 6x, 12x, 18x$
 D $1, 2, 3, 6, x, 2x, 3x, 6x$
 b What is the HCF of 4 and $6x$?

2F 12 Fill in the gaps to make true statements.
 a $3a \times$ ____ $= 12a$
 b $7 \times$ ____ $= 35x$
 c $4k \times$ ____ $= 36km$
 d $5y \times$ ____ $= 40y^2$



2A Working with algebraic terms

Start thinking!

Operations such as addition, subtraction, multiplication and division can be applied to algebraic terms as well as numerical terms.

1 Which of these (A, B or C) is the expanded form of $5abc$?

A $5 + a + b + c$

B $5 \times a \times b \times c$

C $5 \times a + b + c$

2 Consider these terms: $3x$, $7xy$, $-x$, $2x^2$, xw, $20x$.

a Which are **like terms**?

b Explain how you can tell.

3 What is the **coefficient** of each term in question **2**?

4 In which situations can two algebraic terms be combined or simplified into one term? Think about each of the operations: addition, subtraction, multiplication and division. Discuss this with a classmate.

KEY IDEAS

- Terms containing exactly the same pronumerals are called like terms. The order of the pronumerals can be different.
- Like terms can be added (or subtracted) by adding (or subtracting) the coefficients of the terms. Terms can be multiplied or divided whether they are like terms or unlike terms.
- When multiplying algebraic terms, multiply the coefficients together first and simplify the products of any pronumerals that are the same using index notation. For example, $k \times k = k^2$.
- When dividing algebraic terms, the division problem should be written as a fraction. Writing algebraic terms in expanded form can help identify the common factors.

EXERCISE 2A Working with algebraic terms

1 Write each term in expanded form.

a $kmnp$ **b** $3xy$ **c** $-4gh$ **d** a^2c **e** $6xy^3w$ **f** $-2b^2c^4$

2 Write two examples of like terms for each algebraic term shown.

a $7f$ **b** $-3k$ **c** mn **d** $4a^2$ **e** $10xy^3$ **f** $-abcd$

EXAMPLE 2A-1 Adding and subtracting like terms

Simplify $10a - 6a + 5a$.

THINK	WRITE
1 Identify like terms.	$10a - 6a + 5a$
2 Simplify like terms by adding or subtracting the coefficients.	$= 4a + 5a$ $= 9a$

3 Simplify each expression.

a $6a - 4a + 8a$ **b** $4k - 5k - 7k$ **c** $x^2 + 3x^2 + 2x^2$

d $3cd + cd - 9cde$ **e** $3x + 4y + 9x + 2y$ **f** $7a + 5b - 3a + b$

g $m - 2p + 4p + 8m$ **h** $3 + 5k - 2 - 6k$ **i** $4xy + 3x^2 - xy + 2x^2$

j $d + de^2 + d - 5de^2$ **k** $5m^3 + 7 - m^3 - 5$ **l** $abc + ab + ac + 3ab$

EXAMPLE 2A-2 Adding and subtracting algebraic terms

Simplify $6xy - 3y - x + y^2 + y - 2yx$.

THINK	WRITE
1 Rearrange the expression so that like terms are grouped together. Check that the + or − sign in front of each term has moved with that term.	$6xy - 3y - x + y^2 + y - 2yx$ $= 6xy - 2yx - 3y + y - x + y^2$
2 Simplify like terms by adding or subtracting the coefficients.	$= 4xy - 2y - x + y^2$

4 Simplify each expression.

a $6x + 3y - x + 2y + 5x - 4y$ **b** $8ab - 4b - b + b^2 + a - 3ab$

c $2k + 3km - 6k + 4 + 4k - km$ **d** $4x^2 - 7x^2 - 3x + 5 + 6x - 9$

e $9a - 4a^2 + a^3 + 5a^2 - 3 - 7a$ **f** $m^2n + 3m^2 + 5nm^2 - 2n^2 + 4mn^2 - 3m^2$

EXAMPLE 2A-3 Multiplying algebraic terms

Simplify: **a** $4de \times 7ab$ **b** $5x^2y \times -2kwx$.

THINK	WRITE
a 1 Write in expanded form.	**a** $4de \times 7ab$ $= 4 \times d \times e \times 7 \times a \times b$
2 Multiply the coefficients together.	$= 28 \times d \times e \times a \times b$
3 Simplify by leaving out the multiplication signs. Write the pronumerals in alphabetical order.	$= 28abde$
b 1 Write in expanded form.	**b** $5x^2y \times -2kwx$ $= 5 \times x \times x \times y \times -2 \times k \times w \times x$
2 Multiply the coefficients together.	$= -10 \times x \times x \times y \times k \times w \times x$
3 Write the product of any pronumerals that are the same in index notation. ($x \times x \times x = x^3$)	$= -10 \times x^3 \times y \times k \times w$
4 Simplify by leaving out the multiplication signs.	$= -10kwx^3y$

5 Simplify each expression.

a $2ab \times 3cd$ **b** $-5xy \times 4mp$ **c** $9gh \times g$

d $4km \times -6kn$ **e** $7jp \times 8bpt$ **f** $-x^2y \times -ay$

g $6a^2b \times 3acd$ **h** $-10hk \times 2hkp$ **i** $3b \times -2b \times b$

j $m^2n \times 4n \times kn$ **k** $-5xy \times x^2 \times -3xy$ **l** $8abc \times 7a^3c \times b^2$

EXAMPLE 2A-4 Simplifying an algebraic fraction

Simplify $\dfrac{15ab}{10a}$.

THINK	WRITE
1 Write in expanded form.	$\dfrac{15ab}{10a}$ $= \dfrac{15 \times a \times b}{10 \times a}$
2 Cancel the coefficients (divide 15 and 10 by the HCF of 5). Cancel any common pronumerals from numerator and denominator.	$= \dfrac{{}^{3}\cancel{15} \times {}^{1}\cancel{a} \times b}{{}_{2}\cancel{10} \times {}_{1}\cancel{a}}$
3 Simplify the numerator and the denominator.	$= \dfrac{3b}{2}$

6 Simplify each expression.

a $\dfrac{abc}{ac}$ **b** $\dfrac{kmn}{kp}$ **c** $\dfrac{12cd}{3d}$ **d** $\dfrac{16w}{8xw}$ **e** $\dfrac{7ef}{14ef}$ **f** $\dfrac{5xy}{20x}$

g $\dfrac{18ab}{15ac}$ **h** $\dfrac{4mn}{22mn}$ **i** $\dfrac{18a^2}{3a}$ **j** $\dfrac{6x^2y}{2x}$ **l** $\dfrac{15mn^2}{9m}$ **m** $\dfrac{3a^2bc}{12ab}$

EXAMPLE 2A-5 Dividing algebraic terms

Simplify $8xy \div (2wx)$.

THINK	WRITE
1 First write the division problem as an algebraic fraction.	$8xy \div (2wx)$ $= \dfrac{8xy}{2wx}$
2 Cancel the coefficients (divide 8 and 2 by the HCF of 2). Cancel any common pronumerals from numerator and denominator.	$= \dfrac{{}^{4}\cancel{8} \times {}^{1}\cancel{x} \times y}{{}_{1}\cancel{2} \times w \times {}_{1}\cancel{x}}$
3 Simplify the numerator and the denominator.	$= \dfrac{4 \times 1 \times y}{1 \times w \times 1}$ $= \dfrac{4y}{w}$

UNDERSTANDING AND FLUENCY

7 Simplify each expression. (Hint: first write each as an algebraic fraction.)

a $mpq \div m$ **b** $2ade \div (ae)$ **c** $6ax \div (2ac)$

d $5km \div (10mp)$ **e** $7a^2bc \div (abd)$ **f** $3mn^2w \div (9nw)$

g $-12x^2y \div (8xyz)$ **h** $-abcd \div (-2abc^2)$ **i** $8km^2n \div (-12k^2mn)$

8 Answer true or false to each statement.

a Two like terms can be added to form one new term.

b Any term can be subtracted from another term to form one new term.

c Two terms can be multiplied to form one new term only if they are like terms.

d Any term can be divided by another term to form one new term.

9 Students in a class were asked to simplify three algebraic expressions. Three sets of working for each expression have been selected and shown below. One set is correct and the other two sets contain errors.

For each expression, choose which set of working is correct and identify the errors in the other two sets of working.

a **Expression 1:**
$4a - 3b + 2 + 2a + 8b - 7$

Set A
$4a - 3b + 2 + 2a + 8b - 7$
$= 4a + 2a + 2 + 7 + 3b + 8b$
$= 6a + 9 + 11b$

Set B
$4a - 3b + 2 + 2a + 8b - 7$
$= 4a + 2a + 2 - 7 - 3b + 8b$
$= 6a - 5 + 5b$

Set C
$4a - 3b + 2 + 2a + 8b - 7$
$= 4a - 2a + 2 - 7 - 3b + 8b$
$= 2a + 9 + 5b$

UNDERSTANDING AND FLUENCY

b **Expression 2:** $-3ab \times 4bc$

Set A

$-3ab \times 4bc$
$= -3 \times 4 \times a \times b \times b \times c$
$= -12 \times a \times b \times b \times c$
$= -12ab^2c$

Set B

$-3ab \times 4bc$
$= -3 \times 4 \times a \times b \times b \times c$
$= 12 \times a \times b \times b \times c$
$= 12abc$

Set C

$-3ab \times 4bc$
$= -3 \times 4 \times a \times b \times b \times c$
$= -12 \times a \times b \times 2 \times c$
$= -24abc$

c **Expression 3:** $4a^2bc \div (8abd)$

Set A

$4a^2bc \div (8abd)$
$= \frac{4a^2bc}{8abd}$
$= \frac{4 \times a \times a \times b \times c}{8 \times a \times b \times d}$
$= \frac{{}^{1}\cancel{4} \times {}^{1}\cancel{a} \times a \times {}^{1}\cancel{b} \times c}{{}_{2}\cancel{8} \times {}_{1}\cancel{a} \times {}_{1}\cancel{b} \times d}$
$= \frac{1 \times 1 \times a \times 1 \times c}{2 \times 1 \times 1 \times d}$
$= 2acd$

Set B

$4a^2bc \div (8abd)$
$= \frac{4a^2bc}{8abd}$
$= \frac{4 \times a \times a \times b \times c}{8 \times a \times b \times d}$
$= \frac{{}^{1}\cancel{4} \times {}^{1}\cancel{a} \times {}^{1}\cancel{a} \times {}^{1}\cancel{b} \times c}{{}_{2}\cancel{8} \times {}_{1}\cancel{a} \times {}_{1}\cancel{b} \times d}$
$= \frac{1 \times 1 \times 1 \times 1 \times c}{2 \times 1 \times 1 \times d}$
$= \frac{c}{2d}$

Set C

$4a^2bc \div (8abd)$
$= \frac{4a^2bc}{8abd}$
$= \frac{4 \times a \times a \times b \times c}{8 \times a \times b \times d}$
$= \frac{{}^{1}\cancel{4} \times {}^{1}\cancel{a} \times a \times {}^{1}\cancel{b} \times c}{{}_{2}\cancel{8} \times {}_{1}\cancel{a} \times {}_{1}\cancel{b} \times d}$
$= \frac{1 \times 1 \times a \times 1 \times c}{2 \times 1 \times 1 \times d}$
$= \frac{ac}{2d}$

10 If $x = 3$ and $y = -2$, evaluate each expression. To make it easier, simplify each expression first.

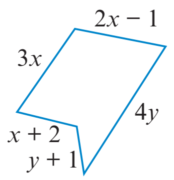
a $5x - 6y + 7y + 3x$ **b** $3xy - 8xy - xy$ **c** $5x \times 3y$
d $\frac{10x}{xy}$ **e** $xy \times xy^2$ **f** $6x^2y \div (2xy)$
g $x + y - 2xy + 5y - x$ **h** $x \times 3x - 2x^2 + 4x - y$

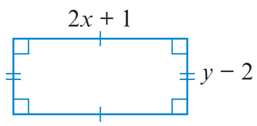
11 If $a = 2$, $b = -1$ and $c = 5$, evaluate each expression. Remember to simplify each expression first.

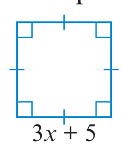
a $3a + 2b + 7c - a - 5c + b$ **b** $7ab + 4a - 5a + ab$
c $a^2b + ab^2 + ac - 3a^2b + 2ac$ **d** $2abc \times bc \times 5a$
e $18ab^2c \div (6bc)$ **f** $3ac^2 \times 4ab \div (9abc)$

PROBLEM SOLVING AND REASONING

12 Write the **perimeter** of each shape as an algebraic expression in simplest form.

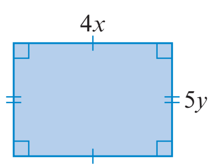
a


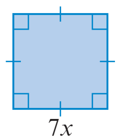
b


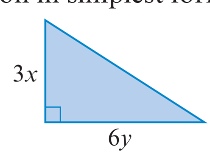
c


13 Calculate the perimeter of each shape in question **12** when $x = 4$ cm and $y = 5$ cm.

14 Write the area of each shaded shape as an algebraic expression in simplest form.

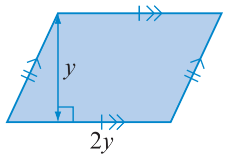
a


b


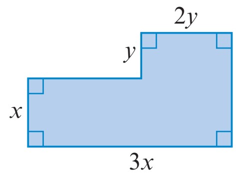
c


PROBLEM SOLVING AND REASONING

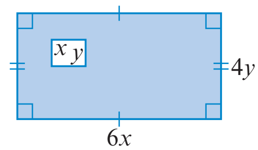
d



e



f



15 Calculate the area of each shape in question **14** when $x = 3$ m and $y = 2$ m.

16 The area of a rectangle is $16xy$.

a If the length is $8x$, write an expression for the width of the rectangle.

b If the length of the rectangle is 16 m and y is 5 m, calculate the width and the area of the rectangle.

17 The area of a right-angled triangle is $6x^2$.

a If the **base** length is $4x$, write an expression for the **height** of the triangle.

b If the height of the triangle is 12 cm, calculate the area and the base length of the triangle.

18 A rectangle has a width of k.

a If the length of the rectangle is twice the width, write an expression for:

i the perimeter of the rectangle **ii** the area of the rectangle.

b Calculate the perimeter and the area of the rectangle when $k = 5$ cm.

19 Lana plants a 1-metre wide flowerbed around a square section of lawn.

a If the lawn has a length of x metres, write an expression for:

i the perimeter of the lawn

ii the area of the lawn

iii the perimeter around the outer edge of the flowerbed

iv the area of the flowerbed given that the total area of the lawn and flowerbed is $(x^2 + 4x + 4)$ m^2.

b When $x = 8$, calculate:

i the area of the flowerbed

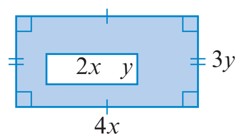
ii the length of edging needed around the inner edge of the flowerbed

iii the length of edging needed around the outer edge of the flowerbed

iv the area to be mown.

CHALLENGE

20 Consider the shaded region of this shape.



a Write an algebraic expression for the area of the shaded region.

b List three possible sets of values for x and y to give an area of 200 cm^2.

c Write an expression for the total length of the outer and inner edges of the shape.

d Use your sets of values from part **b** to calculate the total length of the outer and inner edges of the shape.

Reflect

Suggest at least one useful tip when working with algebraic terms that you can share with the rest of the class.

2B Index laws

Start thinking!

index or power

2^3 = and a^5

base

Numerical and algebraic terms can be written in **index form** (or **index notation**). Terms in index form may be multiplied or divided.

NOTE Each term is written in expanded form before simplifying.

1 **a** Copy and complete the following working to multiply terms in index form.

i $2^3 \times 2^4$
$= (2 \times 2 \times \_\_) \times (2 \times 2 \times \_\_ \times \_\_)$
$= 2^7$

ii $a^2 \times a^3$
$= (a \times \_\_) \times (a \times \_\_ \times \_\_)$
$= \_\_^5$

b Describe the pattern or shortcut you can see to go directly from the first line to the last line. (Hint: look at the **index**.) Will this shortcut work in all cases?

2 **a** Copy and complete the following working to divide terms in index form.

i $3^6 \div 3^2$

$= \frac{3^6}{3^2}$

$= \frac{3 \times \_\_ \times \_\_ \times \_\_ \times \_\_ \times 3}{3 \times 3}$

$= \frac{{}^1\cancel{3} \times {}^1\cancel{3} \times 3 \times 3 \times 3 \times 3}{{}_1\cancel{3} \times {}_1\cancel{3}}$

$= \frac{3^{-}}{1}$

$= 3^{-}$

ii $x^7 \div x^4$

$= \frac{x^7}{x^4}$

$= \frac{x \times \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_}{x \times \_\_ \times \_\_ \times x}$

$= \frac{{}^1\cancel{x} \times {}^1\cancel{x} \times {}^1\cancel{x} \times {}^1\cancel{x} \times x \times x \times x}{{}_1\cancel{x} \times {}_1\cancel{x} \times {}_1\cancel{x} \times {}_1\cancel{x}}$

$= \frac{x^{-}}{1}$

$= x^{-}$

b Describe the pattern or shortcut you can see. Will this shortcut work in all cases?

3 **a** Let's look at raising a term in index form to another power. Copy and complete the following working to simplify each expression.

i $(4^2)^3$
$= (4^2) \times (4^2) \times (4^2)$
$= (4 \times 4) \times (\_ \times \_) \times (\_ \times \_)$
$= 4^{-}$

ii $(k^3)^4$
$= (k^3) \times (k^3) \times (k^3) \times (k^3)$
$= (k \times k \times k) \times (\_ \times \_ \times \_) \times (k \times \_ \times \_) \times (\_ \times \_ \times \_)$
$= \_\_^{12}$

b Describe the pattern or shortcut you can see. Will this shortcut work in all cases?

The three different patterns you see in questions **1**, **2** and **3** are known as **index laws**.

KEY IDEAS

- The index laws can be used to simplify expressions involving terms written in index form.
- The index laws for multiplying or dividing terms in index form only apply when the bases are the same.
- The term x written in index form is x^1.
- Any **base** raised to the power of zero is equal to one; that is, $a^0 = 1$.

Index laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$(a \times b)^m = a^m \times b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

EXERCISE 2B Index laws

UNDERSTANDING AND FLUENCY

1 Copy and complete the following sentences using words from this list:
multiply, *divide*, *add*, *subtract*, *same*, *different*, *indices*, *powers*.

a When multiplying terms in index form with the ____ base, write the base and ____ the indices (or ______).

b When dividing terms in index form with the ____ base, write the base and ____ the _____ (or powers).

c When raising a term in index form to another power, write the base and ______ the indices (or ______).

2 a Use the shortcut described in question **1a** to simplify:

i $5^2 \times 5^6$ **ii** $y^9 \times y^7$ **iii** $x^4 \times x$.

b Use the shortcut described in question **1b** to simplify:

i $2^{11} \div 2^3$ **ii** $p^9 \div p^7$ **iii** $h^5 \div h^4$.

c Use the shortcut described in question **1c** to simplify:

i $(7^4)^5$ **ii** $(w^6)^2$ **iii** $(m^5)^5$.

EXAMPLE 2B-1 Simplifying numerical expressions using an index law

Use the appropriate index law to simplify each expression. (Leave each answer in index form.)

a $3^4 \times 3^2$ **b** $7^8 \div 7^5$ **c** $(2^3)^2$

THINK	WRITE
a Since the bases are the same, write the base and add the powers.	**a** $3^4 \times 3^2 = 3^{4+2} = 3^6$
b Since the bases are the same, write the base and subtract the powers.	**b** $7^8 \div 7^5 = 7^{8-5} = 7^3$
c Write the base and multiply the powers.	**c** $(2^3)^2 = 2^{3 \times 2} = 2^6$

3 Use the appropriate index law to simplify each expression. Leave each answer in index form.

a $3^5 \times 3^4$ **b** $7^8 \div 7^2$ **c** $(6^4)^3$ **d** $5^3 \div 5$

e $(2^6)^4$ **f** $10^2 \times 10^9$ **g** $3^6 \div 3^5$ **h** 6×6^2

i $\dfrac{4^7}{4^5}$ **j** $(3^2)^2$ **k** $\dfrac{13^9}{13^6}$ **l** $2^5 \times 2^2 \times 2^3$

4 Calculate the **basic numeral** for each result in question **3**.

EXAMPLE 2B-2 Simplifying algebraic expressions using an index law

Use the appropriate index law to simplify each expression.

a $x^6 \times x^3$ **b** $x^5 \div x^2$ **c** $(x^4)^3$

THINK	WRITE
a Since the bases are the same, write the base and add the powers.	**a** $x^6 \times x^3 = x^{6+3}$ $= x^9$
b Since the bases are the same, write the base and subtract the powers.	**b** $x^5 \div x^2 = x^{5-2}$ $= x^3$
c Write the base and multiply the powers.	**c** $(x^4)^3 = x^{4 \times 3}$ $= x^{12}$

5 Use the appropriate index law to simplify each expression.

a $a^6 \div a^4$ **b** $(b^5)^2$ **c** $c^{10} \times c^3$ **d** $d^7 \div d^6$

e $e^3 \times e^3$ **f** $(m^4)^2$ **g** $g^{11} \div g$ **h** $h^8 \times h$

i $\frac{x^9}{x^2}$ **j** $(j^5)^2$ **k** $\frac{k^7}{k}$ **l** $y^3 \times y \times y^4$

EXAMPLE 2B-3 Using index laws to simplify expressions

Use the index laws to simplify each expression.

a $2x^7 \times 3x^4$ **b** $8x^9 \div (12x^5)$ **c** $(x^3)^5 \times x^2$

THINK	WRITE
a **1** Rearrange the expression to group the coefficients together and the variables together.	**a** $2x^7 \times 3x^4$ $= 2 \times 3 \times x^7 \times x^4$
2 Multiply the coefficients.	$= 6 \times x^7 \times x^4$
3 Use an index law to multiply the terms with the same base. Write the base and add the indices. (7 + 4 = 11)	$= 6 \times x^{11}$ $= 6x^{11}$
b **1** Write the expression as a fraction and insert the multiplication sign between the coefficient and variable.	**b** $8x^9 \div (12x^5)$ $= \frac{8 \times x^9}{12 \times x^5}$
2 Divide the coefficients. Cancel by a common factor of 4.	$= \frac{2}{3} \times \frac{x^9}{x^5}$
3 Use an index law to divide the terms with the same base. Write the base and subtract the indices. (9 − 5 = 4)	$= \frac{2}{3} \times x^4$ $= \frac{2}{3}x^4$ or $\frac{2x^4}{3}$
c **1** Use an index law to simplify the first term. Write the base and multiply the indices. (3 × 5 = 15)	**c** $(x^3)^5 \times x^2$ $= x^{15} \times x^2$
2 Write the base and add the indices. (15 + 2 = 17)	$= x^{17}$

UNDERSTANDING AND FLUENCY

6 Use the index laws to simplify each expression.

a $3x^5 \times 4x^6$ **b** $5x^4 \times 2x^3$ **c** $8x^2 \times 3x^7$ **d** $6x^{10} \times 9x$

e $6x^7 \div (2x^3)$ **f** $20x^6 \div (5x^2)$ **g** $4x^8 \div (10x^7)$ **h** $15x^{12} \div (9x^4)$

i $(x^2)^4 \times x^5$ **j** $(x^5)^3 \times x^7$ **k** $x^3 \times (x^4)^6$ **l** $(x^3)^2 \times (x^7)^3$

7 **a** Copy and complete the following working to show two different methods of simplifying $2^4 \div 2^4$.

i $2^4 \div 2^4$
$= 2^{4-\_}$
$= 2^{\_}$

ii $2^4 \div 2^4$
$= \dfrac{2^4}{2^4}$
$= \dfrac{{}^1\!2 \times {}^1\!2 \times \_\_ \times \_\_}{{}_1\!2 \times {}_1\!2 \times \_\_ \times \_\_}$
$= 1$

b Repeat the steps of working from part **a** to simplify $x^5 \div x^5$.

8 Look at your answers to question **7**.

a What do you notice about the value of a term in index form with a power of zero?

b Write the result to each of these.

i 4^0 **ii** 25^0 **iii** k^0 **iv** $3^{11} \div 3^{11}$ **v** $a^7 \div a^7$

9 Use the property $a^0 = 1$ to simplify each expression.

a $2x^0$ **b** $(2x)^0$ **c** $7y^0$ **d** $(7y)^0$

e $(-3c)^0$ **f** $8^0 + 4^0$ **g** $5^0 - 3^0$ **h** $m^0 + m^0$

i $n^0 + p^0$ **j** $a^0 + b^0 + c^0$ **k** $(x + y)^0$ **l** $(a^0)^4$

m $(5^3)^0$ **n** $(-8)^0$ **o** -8^0 **p** $-(-3)^0$

10 Use the index laws to simplify each expression.

a $a^3 \div a^3$ **b** $7x^9 \div x^9$ **c** $(m^2)^3 \div m^6$

d $18(b^4)^5 \div [6(b^5)^4]$ **e** $y^7 \times y \div y^8$ **f** $(k^6)^0 \times k^2$

g $5g^4 \times 2(g^7)^0$ **h** $3(w^5)^2 \div (w^2)^5$ **i** $x^8 \times (x^2)^5 \div x^3$

j $4p^7 \times 3p^2 \div (6p^9)$ **k** $16(b^3)^3 \div [2(b^2)^4]$ **l** $4m^5 \times m \div [10(m^3)^2]$

EXAMPLE 2B-4 Combining index laws to simplify an expression

Use the index laws to simplify $\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3}$.

THINK	WRITE
1 Simplify the numerator and simplify the denominator.	$\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3} = \dfrac{12x^{13}}{2x^4 \times x^9}$ $= \dfrac{12x^{13}}{2x^{13}}$
2 Divide the numerator by the denominator. Divide the coefficients. Keep the base and subtract indices.	$= 6x^0$
3 Use the property $a^0 = 1$ to simplify further.	$= 6 \times 1$ $= 6$

UNDERSTANDING AND FLUENCY

11 Use the index laws to simplify each expression.

a $\frac{x^7 \times x^3}{x^4}$ **b** $\frac{2k^4 \times k^5}{k^6}$ **c** $\frac{4a^2 \times 3a^6}{2a^7}$

d $\frac{5m^2 \times 2x^4}{10x^6}$ **e** $\frac{x^4 \times (x^3)^5}{x^9}$ **f** $\frac{(w^2)^4 \times (w^5)^2}{(w^4)^3}$

g $\frac{6(b^4)^4 \times (b^3)^2}{18b^{21}}$ **h** $\frac{e^5 \times e^8}{e^3 \times e^4}$ **i** $\frac{(x^6)^2 \times x^3}{x^5 \times (x^2)^5}$

j $\frac{4a^6 \times 6(a^3)^4}{2a^4 \times 3a^5}$ **k** $\frac{5(n^7)^2 \times 6(n^2)^3}{15n^2 \times (n^3)^6}$ **l** $\frac{(k^8)^2 \times k \times k^9}{k^3 \times (k^4)^2 \times k^5}$

12 Simplify each expression.

a $a^3b^4 \times a^6b^2$ **b** $6m^5n^2 \times 3m^6n$ **c** $(x^4)^2y^7 \times x^3y^2$

d $\frac{c^2d^9}{d^7}$ **e** $\frac{k^3m^8}{km^5}$ **f** $\frac{a^5b^7 \times a^3b^6}{a^8b^{10}}$

g $\frac{6w^9x^6 \times 3w^4x^5}{9w^5x^4 \times w^6x^3}$ **h** $\frac{4(m^3)^4n^2 \times (m^2)^3}{8m^5n^6 \times mn}$ **i** $\frac{3h^7k^5 \times 2h^6(k^3)^2}{(h^2)^6k^3 \times 6h(k^4)^2}$

13 Copy and complete the following working.

a **i** $(3 \times 4)^2$
$= (12)^2$
$= 12 \times 12$
$=$ ___

ii $3^2 \times 4^2$
$= (3 \times \_\_) \times (4 \times \_\_)$
$=$ ___ $\times 16$
$=$ ___

b **i** $(5 \times 2)^4$
$= (\_\_)^4$
$= 10 \times 10 \times \_\_ \times \_\_$
$=$ ___

ii $5^4 \times 2^4$
$= (5 \times 5 \times \_\_ \times 5) \times (2 \times \_\_ \times 2 \times 2)$
$= 625 \times$ ___
$=$ ___

14 Look at your answers to question **13**.

a Describe the pattern you see. This is another index law.

b Use this index law to write each expression without brackets.

i $(x \times y)^6$ **ii** $(c \times d)^3$ **iii** $(5 \times k)^7$ **iv** $(9 \times p)^{10}$

v $(ab)^5$ **vi** $(gh)^2$ **vii** $(3m)^4$ **viii** $(2x)^8$

15 Copy and complete the following working.

a **i** $\left(\frac{3}{4}\right)^2$
$= \frac{3}{4} \times \frac{3}{4}$
$= \frac{9}{\_\_}$

ii $\frac{3^2}{4^2}$
$= \frac{3 \times 3}{4 \times \_\_}$
$= \frac{9}{\_\_}$

b **i** $\left(\frac{5}{2}\right)^4$
$= \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$
$= \frac{625}{\_\_}$

ii $\frac{5^4}{2^4}$
$= \frac{5 \times 5 \times \_\_ \times \_\_}{2 \times 2 \times \_\_ \times \_\_}$
$= \frac{\_\_}{16}$

More index laws

$(a \times b)^m = a^m \times b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

UNDERSTANDING AND FLUENCY

16 Look at your answers to question **15**.

a Describe the pattern you see. This is another index law.

b Use this index law to write each expression without brackets.

i $\left(\frac{x}{y}\right)^6$ **ii** $\left(\frac{k}{m}\right)^3$ **iii** $\left(\frac{d}{3}\right)^5$ **iv** $\left(\frac{8}{p}\right)^2$

17 Use the index laws to simplify each expression.

a $(xy)^3 \times x^6y^4$ **b** $(2k)^5 \times (7k)^2$ **c** $(3x^6)^4$

d $5(a^4b)^7$ **e** $\frac{x^4}{y^5} \times \left(\frac{x}{y}\right)^6$ **f** $\left(\frac{2m}{n}\right)^3$

g $\left(\frac{a^2}{b^5}\right)^4$ **h** $\left(\frac{w^5x^3}{y^4}\right)^2$ **i** $\left(\frac{k^3m}{n^2}\right)^5 \times \left(\frac{n^3}{k^2m}\right)^4$

j $\left(\frac{t^4}{r^2p^3}\right)^5 \times \frac{(4r^5)^2}{p^6t^7}$ **k** $\frac{(a^3b^2)^5 \times (ab^4)^6}{(a^5b)^4}$ **l** $\frac{(3e^4)^2(2h^6)^3}{(e^2h^3)^4}$

PROBLEM SOLVING AND REASONING

18 Use the index laws to decide whether each statement is true or false. Explain your reasoning. For each false statement, change the right side to make the statement true.

a $x^3 \times x^4 = x^{12}$ **b** $6 + k^0 = 7$ **c** $y^7 \div y = y^6$

d $a^5 \times a \times a^5 = a^{10}$ **e** $(3g)^4 = 3^4 \times g^4$ **f** $-8^0 = -1$

g $m^3n^5 \times m^2n^4 = m^{14}n^{14}$ **h** $100^9 \div 100^9 = 0$ **i** $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y}$

j $\frac{m^3 \times m^8}{m^{11}} = 1$ **k** $\frac{(k^3)^2 \times k^4}{k^2} = k^5$ **l** $\frac{a^5b^6}{a^2b^4} \times \frac{a^3b^5}{a^4b} = a^2$

19 Find the value of x that will make each statement true.

a $2^x = 2^7$ **b** $5^x \times 5^2 = 5^6$ **c** $4^x = 1$ **d** $7^x \div 7^3 = 7^5$

e $(9^x)^2 = 9^6$ **f** $\left(\frac{2}{3}\right)^x = \frac{32}{243}$ **g** $\frac{6^x \times 6^3}{6^5} = 6^5$ **h** $(3a^x)^4 = 81a^{20}$

20 Write three possible sets of x and y values to make each statement true.

a $a^x \times a^y = a^{12}$ **b** $k^x \div k^y = k^3$ **c** $(p^x)^y = p^{20}$ **d** $(5^x)^y = 1$

21 Do the index laws for multiplying and dividing terms in index form work when the terms have different bases? Explain, using $2^4 \times 3^2$ and $y^8 \div x^5$ as examples.

CHALLENGE

22 Write each number in index form with the base indicated.

a 8 (base 2) **b** 27 (base 3) **c** 25 (base 5) **d** 10 000 (base 10)

23 Write each number in index form with the base indicated. (Hint: use the index law $(a^m)^n = a^{mn}$.) For example, 16^3 written with a base of 2 is $16^3 = (2^4)^3 = 2^{12}$.

a 8^4 (base 2) **b** 27^5 (base 3) **c** 25^9 (base 5) **d** $10\,000^3$ (base 10)

e 16^7 (base 4) **f** 32^6 (base 2) **g** 216^2 (base 6) **h** 243^6 (base 3)

24 Explain why $x^m \times x^n$ does *not* equal x^{mn} for most values of x, m and n. Can you identify values for which the statement $x^m \times x^n = x^{mn}$ is true?

Reflect

Why do index laws apply to variables as well as numbers?

2C Negative indices

Start thinking!

So far you have worked with indices that are positive whole numbers or zero.
Can an index or power be negative?

1 You know that $3^1 = 3$. What do you think 3^{-1} means? Discuss this with a classmate.

2 **a** Copy and complete the following working that shows two different methods of simplifying $3^4 \div 3^5$.

i $3^4 \div 3^5$

$= 3^{4-\_}$ (use an index law)

$= 3^{\_}$

ii $3^4 \div 3^5$

$= \frac{3^4}{3^5}$ (write as a fraction)

$= \frac{3 \times \_ \times \_ \times \_}{3 \times \_ \times \_ \times \_ \times \_}$ (write in expanded form and cancel common factors)

$= \frac{{}^1\cancel{3} \times {}^1\cancel{3} \times {}^1\cancel{3} \times {}^1\cancel{3}}{{}_1\cancel{3} \times {}_1\cancel{3} \times {}_1\cancel{3} \times {}_1\cancel{3} \times 3}$

$= \frac{1}{\_}$

b Compare your answers to **2a** parts **i** and **ii**. What fraction is 3^{-1} equivalent to?

3 What do you think a^{-1} is equivalent to? Discuss this with a classmate.

4 Repeat question **2** but this time consider $a^2 \div a^3$. Show that $a^{-1} = \frac{1}{a}$.

5 Repeat question **2** but this time consider $3^4 \div 3^6$. Show that $3^{-2} = \frac{1}{3^2}$.

6 Repeat question **2** but this time consider $a^2 \div a^5$. Show that $a^{-3} = \frac{1}{a^3}$.

KEY IDEAS

- A negative index can be used to write a fraction in index form. For example, $\frac{1}{4} = 4^{-1}$.
- The index laws also apply to expressions containing terms with negative indices.
- The properties $a^{-1} = \frac{1}{a}$ and $a^{-m} = \frac{1}{a^m}$ can be used to write terms with positive indices only.

$a^{-1} = \frac{1}{a}$

$a^{-m} = \frac{1}{a^m}$

$\frac{1}{a^{-m}} = a^m$

EXERCISE 2C Negative indices

EXAMPLE 2C-1 Writing a term that has an index of −1 with a positive index

Write 2^{-1} with a positive index.

THINK	WRITE
Write as a fraction. The numerator will contain 1 and the denominator will contain the base of the original term with a positive power.	$2^{-1} = \frac{1}{2}$

UNDERSTANDING AND FLUENCY

1 Write each term with a positive index.

a 5^{-1} **b** 8^{-1} **c** 2^{-1} **d** 6^{-1} **e** 10^{-1} **f** 17^{-1}
g x^{-1} **h** p^{-1} **i** w^{-1} **j** b^{-1} **k** e^{-1} **l** n^{-1}

2 Explain how you could write any term with a negative index as a term with a positive index.

3 Write each fraction in index form with a negative index.

a $\frac{1}{7}$ **b** $\frac{1}{2}$ **c** $\frac{1}{10}$ **d** $\frac{1}{5}$ **e** $\frac{1}{13}$ **f** $\frac{1}{4}$
g $\frac{1}{d}$ **h** $\frac{1}{m}$ **i** $\frac{1}{y}$ **j** $\frac{1}{p}$ **k** $\frac{1}{c}$ **l** $\frac{1}{k}$

EXAMPLE 2C-2 Writing a term that has a negative index with a positive index

Write 7^{-4} with a positive index.

THINK	WRITE
Write as a fraction. The numerator will contain 1 and the denominator will contain the base of the original term with a positive power.	$7^{-4} = \frac{1}{7^4}$

4 Write each term with a positive index.

a 4^{-2} **b** 2^{-6} **c** 9^{-3} **d** 5^{-4} **e** 7^{-8} **f** 10^{-5}
g a^{-4} **h** x^{-7} **i** k^{-10} **j** m^{-2} **k** u^{-9} **l** g^{-11}

5 Write each fraction in index form with a negative index.

a $\frac{1}{3^4}$ **b** $\frac{1}{4^7}$ **c** $\frac{1}{6^5}$ **d** $\frac{1}{5^3}$ **e** $\frac{1}{9^2}$ **f** $\frac{1}{11^6}$
g $\frac{1}{n^2}$ **h** $\frac{1}{g^{11}}$ **i** $\frac{1}{x^8}$ **j** $\frac{1}{a^9}$ **k** $\frac{1}{p^4}$ **l** $\frac{1}{w^7}$

EXAMPLE 2C-3 Writing fractions with positive indices

Write each fraction in index form with positive indices. **a** $\frac{a^{-2}}{b^{-5}}$ **b** $\frac{5x^4}{y^{-7}}$

THINK

a 1 Write the fraction as a product of two factors.

2 Write each factor with a positive index.

3 Simplify.

b 1 Write the fraction as a product of factors.

2 Write each factor with a positive index.

3 Simplify.

WRITE

a $\frac{a^{-2}}{b^{-5}} = a^{-2} \times \frac{1}{b^{-5}}$

$= \frac{1}{a^2} \times b^5$

$= \frac{b^5}{a^2}$

b $\frac{5x^4}{y^{-7}} = 5 \times x^4 \times \frac{1}{y^{-7}}$

$= 5 \times x^4 \times y^7$

$= 5x^4y^7$

6 Write each fraction in index form with positive indices.

a $\frac{1}{2^{-3}}$ **b** $\frac{1}{5^{-6}}$ **c** $\frac{1}{8^{-4}}$ **d** $\frac{1}{3^{-9}}$ **e** $\frac{1}{7^{-5}}$ **f** $\frac{1}{4^{-2}}$

g $\frac{1}{x^{-7}}$ **h** $\frac{1}{y^{-3}}$ **i** $\frac{1}{c^{-4}}$ **j** $\frac{a^{-5}}{b^{-3}}$ **k** $\frac{k^{-6}}{p^{-2}}$ **l** $\frac{h^{-4}}{m^{-7}}$

m $\frac{e^8}{d^{-5}}$ **n** $\frac{u^3}{w^{-8}}$ **o** $\frac{3x^2}{y^{-4}}$ **p** $\frac{8h^5}{g^{-6}}$ **q** $\frac{6d^{-5}}{3c^{-9}}$ **r** $\frac{4m^{-7}}{10n^{-3}}$

EXAMPLE 2C-4 Writing terms with positive indices

Write each term with positive indices. **a** $x^{-5}y^3$ **b** $\frac{6a^7b^{-2}}{3c^{-4}}$

THINK

a 1 Write the term as a product of two factors.

2 Write the first factor with a positive index. You may like to write the second factor as a fraction (denominator is 1).

3 Simplify.

b 1 Cancel 6 and 3 by a common factor of 3 and simplify.

2 Write the term as a product of factors.

3 Write each base that has a negative index with a positive index.

4 Simplify.

WRITE

a $x^{-5}y^3 = x^{-5} \times y^3$

$= \frac{1}{x^5} \times \frac{y^3}{1}$

$= \frac{y^3}{x^5}$

b $\frac{6a^7b^{-2}}{3c^{-4}} = \frac{{}^{2}\cancel{6}a^7b^{-2}}{{}_{1}\cancel{3}c^{-4}}$

$= \frac{2a^7b^{-2}}{c^{-4}}$

$= 2 \times a^7 \times b^{-2} \times \frac{1}{c^{-4}}$

$= 2 \times a^7 \times \frac{1}{b^2} \times c^4$

$= \frac{2a^7c^4}{b^2}$

UNDERSTANDING AND FLUENCY

7 Write each term with positive indices.

a $x^{-2}y^3$ **b** m^6n^{-4} **c** $a^{-1}c^7$ **d** $2k^5p^{-3}$

e $5a^{-8}b^2$ **f** $4x^{-6}w^{-2}$ **g** $a^4b^{-5}c^7$ **h** $k^{-3}m^5n^{-8}$

i $7b^9c^{-6}d$ **j** $3x^{-2}y^{-7}z^{-4}$ **k** $\frac{m^{-3}n^4}{p^{-6}}$ **l** $\frac{8c^2d^{-5}}{2e^{-6}}$

m $\frac{4k^{-1}n^{-3}}{6p^{-4}}$ **n** $\frac{3^{-1}a^2c^{-5}}{b^{-3}d^4}$ **o** $\frac{m^{-6}n}{7^{-1}k^{-2}}$ **p** $\frac{5^{-1}x^3k^{-2}}{2u^{-5}w^8}$

8 **a** Copy and complete each statement using an index law.
Leave each answer in index form.

i $2^4 \times 2^{-7} = 2^{-}$ **ii** $5^{-6} \div 5^2 = 5^{-}$ **iii** $(3^{-3})^2 = 3^{-}$ **iv** $\left(\frac{4}{5}\right)^{-2} = \frac{4^{-}}{5^{-2}}$

b Write each result in part **a** with positive indices.

c Calculate the basic numeral for each result in part **b**. Write each answer as a whole number or fraction.

EXAMPLE 2C-5 Simplifying expressions involving negative indices using index laws

Use an appropriate index law to simplify each expression. Write your answers using positive indices only.

a $3^5 \times 3^{-7}$ **b** $2^4 \div 2^{-3}$ **c** $(5^{-6})^2 \times 5^3$

THINK	WRITE
a **1** Use an index law to multiply the terms. Write the base and add the indices. $(5 + (-7) = 5 - 7 = -2)$	**a** $3^5 \times 3^{-7}$ $= 3^{-2}$
2 Write 3^{-2} with a positive index.	$= \frac{1}{3^2}$
b Use an index law to divide the terms. Write the base and subtract the indices. $(4 - (-3) = 4 + 3 = 7)$	**b** $2^4 \div 2^{-3}$ $= 2^7$
c **1** Use an index law to simplify the first term. Write the base and multiply the indices. $(-6 \times 2 = -12)$	**c** $(5^{-6})^2 \times 5^3$ $= 5^{-12} \times 5^3$
2 Use an index law to multiply the terms. Write the base and add the indices. $(-12 + 3 = -9)$	$= 5^{-9}$
3 Write 5^{-9} with a positive index.	$= \frac{1}{5^9}$

9 Use an appropriate index law to simplify each expression. Write your answers in index form with positive indices.

a $4^{-5} \times 4^2$ **b** $7^3 \times 7^{-4}$ **c** $2^{-6} \times 2^8$ **d** $3^{-1} \times 3^{-5}$

e $5^7 \times 5^{-3}$ **f** $2^{-4} \div 2^3$ **g** $9^5 \div 9^7$ **h** $3^6 \div 3^{-2}$

i $4^{-1} \div 4^8$ **j** $10^{-7} \div 10^{-4}$ **k** $(5^{-3})^2$ **l** $(3^{-2})^4$

m $(2^{-4})^{-1}$ **n** $(3^{-1})^4 \times 3^2$ **o** $(6^{-5})^3 \times 6^{11}$ **p** $(4^{-2})^3 \times (4^{-5})^{-1}$

q $9^3 \times 9^{-6} \times 9^2$ **r** $\frac{5^4 \times 5^{-2}}{5^{-6}}$ **s** $\frac{7^{-5} \times 7^{-3}}{7^{-4} \times 7^{-7}}$ **t** $\frac{2^8 \times (2^{-2})^3}{2^5}$

UNDERSTANDING AND FLUENCY

10 Calculate the basic numeral for each result in question **9**. Write your answer as a whole number or fraction.

11 Use an appropriate index law to write each expression without brackets using positive indices only.

a $\left(\frac{4}{5}\right)^{-2}$ **b** $\left(\frac{7}{3}\right)^{-1}$ **c** $\left(\frac{3}{4}\right)^{-3}$ **d** $\left(\frac{a}{2}\right)^{-4}$ **e** $\left(\frac{6}{m}\right)^{-1}$

f $(a \times b)^{-7}$ **g** $(3 \times x)^{-5}$ **h** $(4 \times y)^{-1}$ **i** $(7k)^{-2}$ **j** $(2p)^{-3}$

12 Find the value of x that will make each statement true.

a $2^x = \frac{1}{2^3}$ **b** $5^x = \frac{1}{5^7}$ **c** $3^x = \frac{1}{3}$ **d** $6^x = \frac{1}{6^{-2}}$

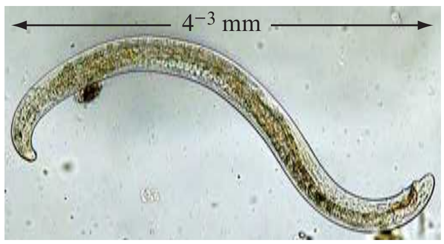
e $4^x = \frac{1}{16}$ **f** $3^x = \frac{1}{27}$ **g** $5^x = \frac{1}{25}$ **h** $10^x = \frac{1}{10\ 000}$

PROBLEM SOLVING AND REASONING

13 A microscopic worm is 4^{-3} mm in length. Write this length in millimetres:

a as a fraction

b as a decimal.



14 The time for light to travel 3 m is about 10^{-8} s. Write this time in seconds:

a as a fraction

b as a decimal.

15 The diameter of a strand of human hair is about 5^{-6} m. Write this measurement in metres:

a as a fraction

b as a decimal.

16 **a** Copy and complete this table.

Index form	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Basic numeral	32	16	8			1		$\frac{1}{4}$			

b Describe the pattern you can see in the table.

c Write 2^{-6} as a fraction.

d If 2^{10} is 1024, write the value of 2^{-10} as a fraction.

e If 2^{-7} is $\frac{1}{128}$, write the value of 2^7.

PROBLEM SOLVING AND REASONING

17 **a** Copy and complete this table.

Index form	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}	3^{-5}
Basic numeral		81	27			1		$\frac{1}{9}$			

b Describe the pattern you can see in the table.

c Write 3^{-6} as a fraction.

d If 3^8 is 6561, write the value of 3^{-8} as a fraction.

e If 3^{-7} is $\frac{1}{2187}$, write the value of 3^7.

18 **a** Copy and complete this table.

Index form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Basic numeral			100			$\frac{1}{10}$	$\frac{1}{100}$		

b Describe the pattern you can see in the table.

c Write the value of each term as a whole number.

i 10^5 **ii** 10^6 **iii** 10^7 **iv** 10^8 **v** 10^9

d Write the value of each term as a fraction.

i 10^{-5} **ii** 10^{-6} **iii** 10^{-7} **iv** 10^{-8} **v** 10^{-9}

e Write 10^{-1} as: **i** a fraction **ii** a decimal.

f Write each term as a decimal. (Hint: Use the matching fractions from your table.)

i 10^{-2} **ii** 10^{-3} **iii** 10^{-4}

g Write each fraction in part **d** as a decimal.

h Explain any shortcuts you have used to obtain your answers to parts **c**–**g**.

19 **a** Without using a calculator, find the whole number value of each of these. (Hint: what shortcut can you use when multiplying by a positive power of ten?)

i 2×10^4 **ii** 7×10^3 **iii** 3×10^5 **iv** 4×10^{11} **v** 9×10^7

b Write each expression as a fraction involving positive indices.

i 5×10^{-2} **ii** 8×10^{-5} **iii** 2×10^{-3} **iv** 7×10^{-4} **v** 6×10^{-9}

c Without using a calculator, find the decimal value of each result in part **b**. (Hint: what shortcut can you use when dividing by a positive power of ten?)

d Use your results from part **c** to describe a shortcut that can be used when multiplying by a negative power of ten.

CHALLENGE

20 Use an appropriate index law to simplify each expression. Write your answer using positive indices only.

a $x^4 \times x^{-6}$ **b** $x^{-3} \times x^{-1}$ **c** $4x^{-2} \times 2x^5$ **d** $5x^{-8} \times 6x^3$

e $3x^7 \times x^{-7}$ **f** $x^5 \div x^{-4}$ **g** $x^{-10} \div x^{-7}$ **h** $4x^3 \div (2x^{-2})$

i $6x^{-6} \div (18x^4)$ **j** $8x^7 \div (14x^{11})$ **k** $(x^{-2})^3 \times x^4$ **l** $(x^4)^5 \times x^{-9}$

m $(x^{-5})^3 \times 4x^2$ **n** $2x^{-3} \times (x^{-1})^5$ **o** $(x^{-4})^2 \times (x^{-3})^{-1}$ **p** $(xy)^{-7}$

q $(x^2y^{-4})^{-5}$ **r** $\left(\frac{x}{y}\right)^{-3}$

s $\left(\frac{x^{-2}}{y^3}\right)^{-1}$ **t** $\left(\frac{y^{-5}}{x^{-6}}\right)^4$

Reflect

What does a negative index or power mean?

2D Scientific notation

Start thinking!

Very large and very small numbers can be more efficiently written using indices. For example, the speed of light is approximately 300 000 000 m/s but it is often written in **scientific notation** (or **standard form**) as 3×10^8 m/s. This means that it is written as a number from 1 up to, but not including, 10 (in this case, 3) multiplied by a power of 10 (in this case, 10^8). That is, $300\,000\,000 = 3 \times 100\,000\,000 = 3 \times 10^8$.

1 Look at Table A. This shows large numbers in scientific notation.
   a Explain how the basic numeral is obtained.
   b Can you see a shortcut for changing a number in scientific notation to its equivalent basic numeral? How has the position of the decimal point changed? Compare this to the power of 10 in each case.

2 Look at Table B. This shows small numbers in scientific notation.
   a Explain how the basic numeral is obtained.
   b What shortcut can you use to change each number in scientific notation to its equivalent basic numeral?

Table A

	Scientific notation	Working	Basic numeral
i	4.1×10^3	4.1×1000 $= 4.1$	4100
ii	5.92×10^7	$5.92 \times 10\,000\,000$ $= 5.92$	59 200 000
iii	1.3058×10^6	$1.3058 \times 1\,000\,000$ $= 1.3058$	1 305 800

Table B

	Scientific notation	Working	Basic numeral
i	7.6×10^{-9}	$7.6 \times \frac{1}{10^9} = \frac{7.6}{1\,000\,000\,000}$ $= 7.6$	0.000 000 007 6
ii	2.54×10^{-2}	$2.54 \times \frac{1}{10^2} = \frac{2.54}{100}$ $= 2.54$	0.0254
iii	9.8103×10^{-5}	$9.8103 \times \frac{1}{10^5} = \frac{9.8103}{100\,000}$ $= 9.8103$	0.000 098 103

KEY IDEAS

- A number is written in scientific notation if it is the product of a number between 1 (inclusive) and 10 and a power of 10. That is, $a \times 10^m$ where $1 \le a < 10$ and m is an integer. If m is a positive integer, the number is larger than or equal to 10. If m is a negative integer, the number is between 0 and 1.
- A shortcut can be used to change a number in scientific notation to a basic numeral. The power indicates the number of places the decimal point is moved. If the power is positive, move the decimal point to the right. If the power is negative, move the decimal point to the left.
- To write a number in scientific notation, place the decimal point after the first non-zero digit and multiply by the appropriate power of 10. For example, $31\,500 = 3.15 \times 10^4$ and $0.042 = 4.2 \times 10^{-2}$.
- The number of significant figures in a measurement indicates the level of accuracy.
- All digits in numbers written in scientific form are significant.

EXERCISE 2D Scientific notation

1 Use a shortcut to calculate each of these. (Hint: move the decimal point an appropriate number of places.)

a 5.4×100 **b** $7.36 \times 10\,000$ **c** 1.8×1000

d $4.05 \times 100\,000$ **e** $2.753 \times 1\,000\,000$ **f** $\frac{6.1}{10}$

g $\frac{8.22}{1\,000\,000}$ **h** $\frac{9.76}{10\,000}$ **i** $\frac{7.003}{100\,000}$

2 Write each number as a power of 10.

a 100 **b** 1000 **c** 10 000 **d** 100 000 **e** 1 000 000

EXAMPLE 2D-1 Writing numbers in scientific notation as basic numerals

Write each number as a basic numeral.

a 7.2×10^6 **b** 3.4×10^{-8}

THINK	WRITE
a Use the shortcut of multiplying by 10^6 (or 1 000 000). Move the decimal point in 7.2 six places to the right and fill the 'empty' place values with zeros.	**a** 7.2×10^6 $= 7\,200\,000$
b Use the shortcut of multiplying by 10^{-8} (or dividing by 10^8). Since the power is negative, move the decimal point in 3.4 eight places to the left and fill the 'empty' place values with zeros.	**b** 3.4×10^{-8} $= 0.000\,000\,034$

UNDERSTANDING AND FLUENCY

3 Write each number as a basic numeral.

a 3.2×10^5 **b** 8.14×10^9 **c** 5.0×10^2

d 2.345×10^7 **e** 1.1×10^4 **f** 6.4×10^{-3}

g 7.28×10^{-6} **h** 9×10^{-7} **i** 3.02×10^{-5}

j 5.41×10^{-2} **k** 4.5×10^{11} **l** 6.12×10^{-9}

m 5.7×10^{-1} **n** 1.3068×10^3 **o** 2.7316×10^{-4}

4 **a** Use a calculator to find the value of each number in question **3**. Check with your teacher if you are not sure how to do this.

b Were there any numbers that you could not easily obtain on your calculator? Explain.

EXAMPLE 2D-2 Writing numbers in scientific notation

Write each number in scientific notation.

a 230 000 **b** 0.000 856

THINK	WRITE
a 1 Place the decimal point after the first non-zero digit (after 2) and multiply by the appropriate power of 10.	**a** 230 000
2 Count the number of places the decimal point in 2.3 would need to be moved to produce 230 000. The decimal point needs to be moved five places to the right to obtain the original number so the power is 5.	$= 2.3 \times 10^5$
b 1 Place the decimal point after the first non-zero digit (after 8) and multiply by the appropriate power of 10.	**b** 0.000 856
2 Count the number of places the decimal point in 8.56 would need to be moved to produce 0.000 856. The decimal point needs to be moved four places to the left to obtain the original number so the power is −4.	$= 8.56 \times 10^{-4}$

UNDERSTANDING AND FLUENCY

5 Write each number in scientific notation.

a 4500 **b** 7 320 000 **c** 200 000 **d** 190

e 3216 **f** 0.0063 **g** 0.000 000 18 **h** 0.05

i 0.000 070 2 **j** 0.427 **k** 11 220 **l** 0.000 004

m 568.2 **n** 0.000 249 **o** 679 300 **p** 0.0102

6 Without using a calculator, find the result of each problem. Write your answer in scientific notation.

a $(3.4 \times 10^2) + (7.3 \times 10^5)$ **b** $(8.52 \times 10^4) - (1.6 \times 10^3)$

c $(6.03 \times 10^{-3}) + (2.7 \times 10^{-4})$ **d** $(3.5 \times 10^{-2}) - (8.2 \times 10^{-3})$

e $(1.7 \times 10^5) \times (4 \times 10^2)$ **f** $(8 \times 10^7) \div (4 \times 10^5)$

g $(5 \times 10^{-5}) \times (9 \times 10^8)$ **h** $(6 \times 10^9) \div (1.5 \times 10^5)$

i $(4.1 \times 10^{-6}) \times (3 \times 10^4)$ **j** $(7.2 \times 10^{-2}) \div (2.4 \times 10^{-7})$

7 Use a calculator to check your answers to question **6**.

8 Measurements are recorded to varying degrees of accuracy depending on the measuring tools used. For example, the length of a piece of timber can be recorded to a different number of **significant figures**. A measurement of 410 cm has two significant figures, whereas a measurement of 412 cm has three significant figures. Give two examples of a measurement that has the following number of significant figures:

a 1 **b** 2 **c** 3 **d** 4.

EXAMPLE 2D-3 Identifying significant figures

How many significant figures are shown in each number?

a 5.42 b 20 803 c 6.200 d 4000 e 0.0082

THINK	WRITE
a All non-zero digits are significant.	a 5.42 has three significant figures.
b Zeros between non-zero digits are significant.	b 20 803 has five significant figures.
c Zeros at the end of a decimal number are significant.	c 6.200 has four significant figures.
d Zeros at the end of an integer are *not* significant.	d 4000 has one significant figure.
e Zeros to the left of the first non-zero digit in a decimal number are *not* significant.	e 0.0082 has two significant figures.

UNDERSTANDING AND FLUENCY

9 How many significant figures are shown in each number?

a 345 b 25 000 c 5072 d 400 e 809 f 0.59
g 0.003 h 1.472 i 48.062 j 7.300 k 36 020 l 0.009 04

10 How many significant figures are shown in each number?

a 2.4×10^3 b 5.06×10^{-4} c 1.900×10^7
d 8.0×10^5 e 3.206×10^{-9} f 7.00×10^5

11 Round each number to the number of significant figures indicated in brackets.

a 458 (2) b 73 051 (4) c 1279 (1) d 40 008 (3)
e 5.1437 (3) f 0.0349 (2) g 42.0607 (4) h 0.852 (1)
i 2.58×10^5 (2) j 5.037×10^4 (3) k 9.1042×10^6 (4) l 6.00×10^3 (2)

EXAMPLE 2D-4 Writing numbers in scientific notation using significant figures

Write each number in scientific notation with the number of significant figures indicated in brackets.

a 53 726 (2) b 0.084 03 (3)

THINK	WRITE
a 1 This number has five significant figures. Round to two significant figures (nearest thousand). Remember that zeros at the end of an integer are not significant.	a 53 726 ≈ 54 000
2 Write in scientific notation.	$= 5.4 \times 10^4$
b 1 This number has four significant figures. Round to three significant figures (nearest ten-thousandth).	b 0.084 03 ≈ 0.0840
2 Write in scientific notation. Remember that zeros at the end of a decimal are significant.	$= 8.40 \times 10^{-2}$

UNDERSTANDING AND FLUENCY

12 Write each number in scientific notation with the number of significant figures indicated in brackets.

a 327 (2) **b** 48 654 (3) **c** 190 760 (4)

d 2621 (1) **e** 0.4031 (3) **f** 0.0544 (2)

g 0.000 207 193 (4) **h** 0.008 327 (1) **i** 758.4 (2)

j 20 703.02 (4) **k** 40.155 (3) **l** 54 007.63 (5)

13 Consider the numbers A–H.

A 3.4×10^4 **B** 2.03×10^{-3} **C** 0.58×10^6 **D** 60.34×10^2

E 0.009 **F** 4.19×10^3 **G** 700×10^5 **H** 9×10^{-4}

a Which numbers are written in scientific notation?

b Which numbers are written with three significant figures?

c Which numbers are larger than 10?

d Which numbers are less than 1?

14 Write each approximate measurement in scientific notation.

a A medium-sized grain of sand has a length of 0.0005 m.

b The Maroondah Reservoir has a capacity of 22 000 ML.

c The thickness of the epidermal layer of skin on your eyelid is 0.048 mm.

d An estimate for the world's population in 2050 is 9 300 000 000.

15 Write each approximate measurement as a basic numeral.

a The number of times the wings of a hummingbird flap in a minute is 6.4×10^3.

b The diameter of a virus is 8×10^{-5} mm.

c The distance from the Sun to Earth is 1.496×10^8 km.

d The radius of an electron is 2.8×10^{-13} cm.

16 The core temperature of the Sun is 1.5×10^7°C and the surface temperature is about 6000°C. What is the difference between the two temperatures?

17 Light travels at a speed of 3×10^{10} cm/s. How many kilometres does it travel in 1 hour? Give your answer in scientific notation.

18 Earth revolves around the Sun at an average speed of 10^5 km/h.

a What distance does Earth travel in 1 day?

b How many days would it take Earth to travel 9.6×10^8 km?

PROBLEM SOLVING AND REASONING

19 The star Alpha Centauri is 4.1×10^{13} km from Earth, while the star Altair is 1.5×10^{14} km from Earth.

- **a** Which star is closer to Earth?
- **b** How much closer is this star?

20 The Australian \$1 coin has a mass of 9 g and a thickness of 3×10^{-1} cm.

- **a** Sarah has a pile of these coins on her desk. She stacks as many of them as she can on top of each other between two shelves in a bookcase. The distance separating the shelves is 26 cm.
  - **i** How many coins are in the stack?
  - **ii** What would be the mass of these coins?
- **b** Ben takes Sarah's stack of coins and places them end-to-end in a line. The line stretches to a length of 2.15 m. What is the diameter of a \$1 coin?

21 The Sun is 1.52×10^{8} km from Earth. Light from the Sun travels towards Earth at a speed of 3×10^{8} m/s. How long does it take this light to reach Earth? Give your answer to the nearest minute.

CHALLENGE

22 Sound travels at 330 m/s, while light travels at 3×10^{5} km/s.

- **a** Compare the speed of light and the speed of sound.

A timekeeper stands at the end of a 100 m straight running track. The starting gun at the beginning of the track goes off.

- **b** How long does it take:
  - **i** the sight of the smoke to reach the timekeeper?
  - **ii** the sound of the gun to reach the timekeeper?
- **c** What advice should you give the timekeeper in order to have an accurate recording of the time of the race?

23 The circumference of a hydrogen atom is 7.98×10^{-9} cm. How far would a line of 1 million hydrogen atoms stretch if placed next to each other?

24 Consider the multiplication problem $2^{350} \times 3^{2} \times 4^{3} \times 5^{355}$. Write the exact answer in scientific notation.

Reflect

Why is it useful to write some numbers in scientific notation?

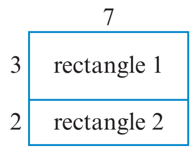
2E Expanding algebraic expressions

Start thinking!

In each problem below, you can write the area of the large rectangle in two ways: using the length and width, and adding separate areas.

1 **a** For each large rectangle below, copy and complete the given statements.

i

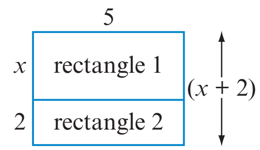


length × width

= area of rectangle 1 + area of rectangle 2

$7 \times (3 + 2) = 7 \times 3 + 7 \times$ __

ii

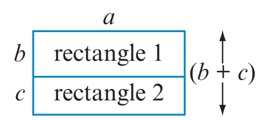


length × width

= area of rectangle 1 + area of rectangle 2

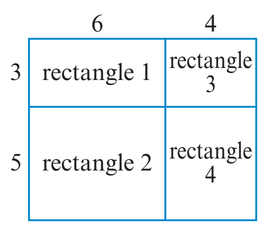
$5 \times (x + 2) = 5 \times$ __ $+ 5 \times$ __

b Use the rectangle on the right to explain how $a(b + c)$ can be expanded to obtain $a \times b + a \times c$ (or $ab + ac$).



2 **a** For each large rectangle below, copy and complete the given statements.

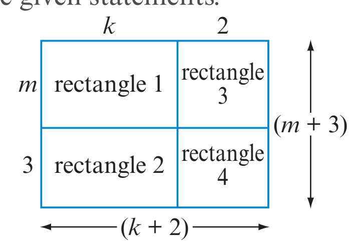
i



$(6 + 4) \times (3 + 5)$

$= 6 \times 3 + 6 \times$ __ $+ 4 \times 3 + 4 \times$ __

ii



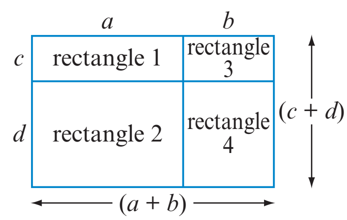
$(k + 2) \times (m + 3)$

$= k \times$ __ $+ k \times 3 + 2 \times m + 2 \times$ __

b Describe how the left side of each statement was formed.

c Describe how the right side of each statement was formed.

d Use this rectangle to explain how $(a + b)(c + d)$ can be expanded to obtain $a \times c + a \times d + b \times c + b \times d$ (or $ac + ad + bc + bd$).



KEY IDEAS

- The **distributive law** can be used when expanding expressions to remove brackets.
- For one pair of brackets, the distributive law is: $a(b + c) = ab + ac$.

$$a(b + c) = a \times b + a \times c$$

- For two pairs of brackets, the distributive law is: $(a + b)(c + d) = ac + ad + bc + bd$.

$$(a + b)(c + d)$$
$$= a \times c + a \times d + b \times c + b \times d$$

- Where possible, expanded expressions should be simplified.

EXERCISE 2E Expanding algebraic expressions

EXAMPLE 2E-1 Expanding expressions with one pair of brackets

Expand each algebraic expression to remove the brackets.

a $3(k + 2)$ b $4(x - 7)$ c $-5(3a + 8)$

THINK

a Use the distributive law. Multiply each term inside the brackets (k and 2) by the term in front of the brackets (3). Simplify each term. $3(k + 2)$

b Use the distributive law. Multiply each term inside the brackets (x and -7) by the term in front of the brackets (4). Simplify each term. $4(x - 7)$

c Use the distributive law. Multiply each term inside the brackets ($3a$ and 8) by the term in front of the brackets (-5). Take care with + and − signs when simplifying. $-5(3a + 8)$

WRITE

a $3(k + 2)$
$= 3 \times k + 3 \times 2$
$= 3k + 6$

b $4(x - 7)$
$= 4 \times x + 4 \times (-7)$
$= 4x - 28$

c $-5(3a + 8)$
$= (-5) \times 3a + (-5) \times 8$
$= -15a - 40$

UNDERSTANDING AND FLUENCY

1 Explain what it means to expand an expression.

2 Copy and complete to expand each algebraic expression.

a $2(a + 5)$
$= 2 \times \_\_ + 2 \times \_\_$
$= \_\_ + \_\_$

b $7(d - 4)$
$= 7 \times \_\_ + \_\_ \times (-4)$
$= \_\_ - \_\_$

c $-3(y + 6)$
$= (-3) \times \_\_ + (-3) \times \_\_$
$= \_\_ - \_\_$

d $4k(2m + 3)$
$= \_\_ \times 2m + \_\_ \times 3$
$= \_\_ + \_\_$

3 Expand each algebraic expression to remove the brackets.

a $4(a + 3)$ b $7(b + 5)$ c $3(c - 2)$ d $5(d - 1)$
e $6(4 + e)$ f $-2(f + 8)$ g $-3(g + 4)$ h $-8(h - 5)$
i $-4(x - 9)$ j $-5(2 - j)$ k $k(p + 6)$ l $2a(b - 4)$
m $6(3m + k)$ n $3n(2p + 4q)$ o $x(x - 7y)$ p $-k(5 + 3k)$

4 Expand and simplify each algebraic expression.

a $3(x + 2) + 8x$ b $5(p - 1) + 11$
c $a(b + 4) - 2a$ d $-7y(1 - y) + 4y + 3$
e $5k + 2 + 4(h - k)$ f $m(m - 6) - m^2$

5 Expand and simplify each algebraic expression.

a $2(x + 5) + 3(x - 6)$ **b** $8(k - 3) + 5(k + 4)$ **c** $3(p + 7) - 4(5 - p)$

d $x(x + 1) + 3(x + 4)$ **e** $m(m + 2) + 3(m + 2)$ **f** $y(y - 5) - 2(y - 5)$

EXAMPLE 2E-2 Expanding expressions with two pairs of brackets

Expand each algebraic expression to remove the brackets.

a $(a + 9)(b + 2)$ **b** $(x + 4)(x - 6)$

THINK	WRITE
a Use the distributive law. Multiply each term inside the second pair of brackets (b and 2) by the first term in the first pair of brackets (a) and then the second term in the first pair of brackets (9). $(a + 9)(b + 2)$ Simplify each term.	**a** $(a + 9)(b + 2)$ $= a(b + 2) + 9(b + 2)$ $= a \times b + a \times 2 + 9 \times b + 9 \times 2$ $= ab + 2a + 9b + 18$
b **1** Use the distributive law. Multiply each term inside the second pair of brackets (x and -6) by the first term in the first pair of brackets (x) and then the second term in the first pair of brackets (4). Simplify each term. $(x + 4)(x - 6)$	**b** $(x + 4)(x - 6)$ $= x(x - 6) + 4(x - 6)$ $= x \times x + x \times (-6) + 4 \times x + 4 \times (-6)$ $= x^2 - 6x + 4x - 24$
2 Simplify any like terms ($-6x + 4x = -2x$).	$= x^2 - 2x - 24$

UNDERSTANDING AND FLUENCY

6 Copy and complete to expand each algebraic expression.

a $(a + 2)(b + 7)$
$= a(b + 7) + 2(b + 7)$
$= a \times b + a \times \_\_ + 2 \times b + 2 \times \_\_$
$= \_\_ + 7a + \_\_ + \_\_$

b $(c - 3)(d + 5)$
$= c(d + 5) + (-3)(d + 5)$
$= c \times \_\_ + c \times \_\_ + (-3) \times \_\_ + (-3) \times \_\_$
$= \_\_ + \_\_ - 3d - \_\_$

c $(h + 4)(h + 1)$
$= h(h + \_\_) + 4(\_\_ + \_\_)$
$= h \times h + \_\_ \times \_\_ + 4 \times h + 4 \times \_\_$
$= \_\_ + \_\_ + \_\_ + 4$
$= \_\_ + \_\_ + 4$

7 Expand each algebraic expression to remove the brackets.

a $(a + 3)(b + 4)$ **b** $(c + 2)(d + 7)$ **c** $(m + 5)(n + 1)$

d $(x + 9)(y + 3)$ **e** $(k + 6)(p - 2)$ **f** $(f + 4)(g - 1)$

g $(a-5)(c+3)$ **h** $(w-7)(f+2)$ **i** $(x-4)(y-8)$
j $(j-9)(k-5)$ **k** $(2a+7)(b+3)$ **l** $(5c+2)(3d-4)$
m $(a+2)(a+3)$ **n** $(x+5)(x+10)$ **o** $(d+4)(d-6)$
p $(y+3)(y-8)$ **q** $(k-7)(k+9)$ **r** $(m-6)(m+3)$
s $(5e-2)(e-4)$ **t** $(7a-8)(3a-1)$ **u** $(3y-5)(2y-1)$

8 Expand each algebraic expression to remove the brackets.
a $(a+3)(a-3)$ **b** $(b+2)(b-2)$ **c** $(c+5)(c-5)$
d $(d+7)(d-7)$ **e** $(k-6)(k+6)$ **f** $(x-4)(x+4)$
g $(p-8)(p+8)$ **h** $(h-1)(h+1)$ **i** $(k+m)(k-m)$

9 Describe the pattern or shortcut you can see in question **8**. What is special about the two factors that are multiplied together?

10 The pattern you observed in question **8** is known as the **difference of two squares**. This rule can be written as: $(a+b)(a-b)=a^2-b^2$.
a Why do you think the rule is called the difference of two squares?
b Does it matter whether the product is $(a+b)(a-b)$ or $(a-b)(a+b)$? Explain.
c Use the rule (or shortcut) to expand each algebraic expression.
i $(x-2)(x+2)$ **ii** $(y+9)(y-9)$ **iii** $(m+6)(m-6)$
iv $(d-10)(d+10)$ **v** $(m+n)(m-n)$ **vi** $(3+x)(3-x)$
vii $(5-k)(5+k)$ **viii** $(1+a)(1-a)$ **ix** $(12-p)(12+p)$

11 Expand each algebraic expression to remove the brackets.
a $(a+2)(a+2)$ **b** $(b+7)(b+7)$ **c** $(c+4)(c+4)$
d $(d+9)(d+9)$ **e** $(k-3)(k-3)$ **f** $(x-6)(x-6)$
g $(p-5)(p-5)$ **h** $(h-1)(h-1)$ **i** $(m+n)(m+n)$

12 Describe the pattern or shortcut you can see in question **11**. What is special about the two factors that are multiplied together?

13 The pattern you observed in question **11** is known as the **expansion of a perfect square**. This rule can be written as: $(a+b)^2=a^2+2ab+b^2$.
a Why do you think the rule is called the expansion of a perfect square?
b Use the rule (or shortcut) to expand each algebraic expression.
i $(x+3)^2$ **ii** $(y+6)^2$ **iii** $(m+2)^2$
iv $(d+1)^2$ **v** $(b+11)^2$ **vi** $(m+n)^2$
vii $(5+x)^2$ **viii** $(8+k)^2$ **ix** $(1+p)^2$
c Why is $(a-b)^2=a^2-2ab+b^2$ also an expansion of a perfect square?
d Expand each algebraic expression.
i $(a-2)^2$ **ii** $(b-4)^2$ **iii** $(c-7)^2$
iv $(d-10)^2$ **v** $(w-6)^2$ **vi** $(k-p)^2$
vii $(3-x)^2$ **viii** $(9-y)^2$ **ix** $(1-w)^2$

PROBLEM SOLVING AND REASONING

14 A binomial is an expression with two terms.

a Write three examples of a binomial.

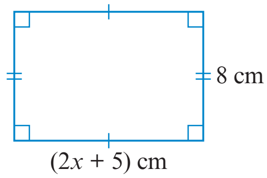
b $(x + 1)(x - 2)$ is an example of a binomial product. Explain what the term 'binomial product' means.

c How many terms do you think a trinomial has?

d Write three examples of a trinomial.

e Find out what a quadratic trinomial is and provide three examples.

15 **a** For the rectangle at right, write the area as a product of two factors (show brackets).



b Expand the expression to remove brackets.

c When $x = 3$, calculate the area using your answer to:

i part **a** **ii** part **b**.

d Compare your answers in part **c**. How does this show that you have expanded the area expression correctly?

16 For each shape:

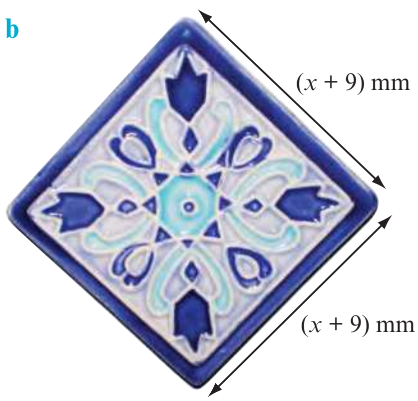
i write the area as a binomial product

ii expand the binomial product to remove brackets

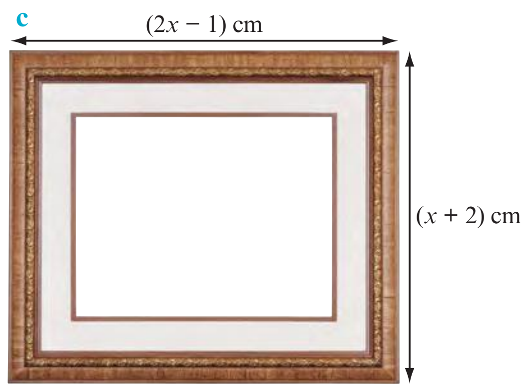
iii calculate the area when $x = 5$.

a

b



c



PROBLEM SOLVING AND REASONING

17 A rectangular trampoline is 6 m long and 4 m wide. Safety matting that is p metres wide is to be placed around the perimeter of the trampoline.

a Draw a labelled diagram of the top of the trampoline and the matting around it.

b Write an expression for:

i the total length of the trampoline and matting

ii the total width of the trampoline and matting.

c Write an expression for the total area taken up by the trampoline and matting. Simplify the expression by expanding to remove any brackets.

d Write an expression for the area of the matting only.

e If $p = 2$, calculate the area taken up by:

i the trampoline **ii** the trampoline and matting **iii** the matting.

18 **a** Use the difference of two squares expansion rule to expand $(100 + 3)(100 - 3)$.

b Use your answer to work out the result for 103×97 without using a calculator.

c Work out the result for each product without using a calculator.

i 102×98 **ii** 95×105 **iii** 1001×999 **iv** 994×1006

CHALLENGE

19 Expand and simplify each expression.

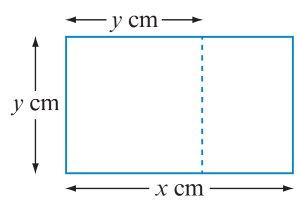
a $7(2x + y - 5)$ **b** $3x(2x + 5) - 1$ **c** $(4x + 3)(7x - 2)$

d $(10y - 7)^2$ **e** $x^4(x^3 - x^2)$ **f** $(x^2 - 5)(x^2 + 5)$

g $(y^6 + 3y)^2$ **h** $a^3(a^2 + 4) - a^2(a^3 + 9a)$ **i** $x^5y^2(xy^4 + 2w)$

20 Show that the expression $(a - b)^2 + (c - d)^2$ has the same value as $(b - a)^2 + (d - c)^2$.

21 A piece of paper is in the shape of a rectangle x cm long, and y cm wide. The paper is torn along a line parallel to its width, forming a square of side length y cm, and another rectangle.



a Write the **dimensions** of the newly-formed rectangle.

b Prove that the area of the original rectangle is the same as the total area of the two new shapes.

22 Paul has invented a new form of expansion as:

$a * b = b - a + ab$.

If, using this expansion, $3 * 8$ has the same value as $6 * x$, what is the value of x?

Reflect

How is the distributive law useful in algebra?

2F Factorising using common factors

Start thinking!

The expression $3(x + 4)$ is said to be in **factor form**, as it shows the product of factors; that is, $3 \times (x + 4)$. In this case, there are two factors: 3 and $(x + 4)$.

In the previous section, you used the distributive law to change an expression from factor form to **expanded form**.

factor form	⟶	expanded form
$3(x + 4)$	$=$	$3 \times x + 3 \times 4$
	$=$	$3x + 12$

You can also change an expression from expanded form to factor form by using the same law in reverse. This process is called factorising.

expanded form	⟶	factor form
$3x + 12$	$=$	$3(x + 4)$

1 Describe how you might **factorise** $3x + 12$ to obtain $3(x + 4)$.

2 Look at the terms $3x$ and 12. What is the highest common factor (HCF)?

3 Explain how finding the HCF of the terms helps you work out what term to place in front of the pair of brackets.

4 Show how finding the HCF can be used to help you factorise $5a + 30$.

KEY IDEAS

- The distributive law can be used to write expressions in factor form. One of the factors needs to be the highest common factor (HCF) of each term in the expression.
- To factorise an expression, you write the HCF of each term at the front of a pair of brackets with the other factor of each term inside the brackets.
- $a \times b + a \times c = a \times (b + c)$ or $ab + ac = a(b + c)$

EXERCISE 2F Factorising using common factors

1 Find the highest common factor (HCF) of each pair of terms.

a $4a$ and 28	**b** $6b$ and 10	**c** 15 and $35c$
d d^2 and $3d$	**e** $2e$ and $2k$	**f** $3f$ and -6
g $12g$ and -8	**h** $9h$ and $-15h^2$	**i** $24x^2$ and $36x$

2 Write each term in question **1** as a product of two factors where one of the factors is the HCF of the pair of terms. For example, for part **a**, $4a = 4 \times a$ and $28 = 4 \times 7$.

EXAMPLE 2F-1 Factorising simple expressions

Factorise each expression.

a $6a + 18$ **b** $20k - 8$ **c** $x^2 + 7x$

THINK	WRITE
a 1 Identify the HCF of $6a$ and 18. (HCF = 6.) Write each term as a product of the HCF and its other factor.	**a** $6a + 18$ $= 6 \times a + 6 \times 3$
2 Use the distributive law. Write the HCF at the front of a pair of brackets and the other factor for each term inside the brackets.	$= 6 \times (a + 3)$ $= 6(a + 3)$
b 1 Identify the HCF of $20k$ and -8. (HCF = 4.) Write each term as a product of the HCF and its other factor.	**b** $20k - 8$ $= 4 \times 5k + 4 \times -2$
2 Use the distributive law.	$= 4 \times (5k - 2)$ $= 4(5k - 2)$
c 1 Identify the HCF of x^2 and $7x$. (HCF = x.) Write each term as a product of the HCF and its other factor.	**c** $x^2 + 7x$ $= x \times x + x \times 7$
2 Use the distributive law.	$= x \times (x + 7)$ $= x(x + 7)$

3 Factorise each expression.

a $4a + 28$	**b** $6b + 10$	**c** $15 + 35c$
d $d^2 + 3d$	**e** $2e + 2k$	**f** $3f - 6$
g $12g - 8$	**h** $9h - 15h^2$	**i** $6k + 30$
j $10x + 5$	**k** $3y - 21$	**l** $8k + 12$
m $15 - 6d$	**n** $28x + 21$	**o** $20n - 50$
p $x^2 + 3x$	**q** $m^2 - 9m$	**r** $4a + a^2$

EXAMPLE 2F-2 Finding the highest common factor of a pair of terms

Find the HCF of $6ab$ and $15b$.

THINK	WRITE
1 Identify the HCF of the coefficients of the terms.	HCF of 6 and 15 is 3.
2 Identify any pronumerals that are common to both terms.	Common pronumeral is b.
3 Write the HCF of the pair of terms.	HCF of $6ab$ and $15b$ is $3b$.

4 Find the HCF of each pair of terms.

a bc and cd b $2xy$ and $2y$ c $18mn$ and $-9m$

d $abcd$ and bdf e $8xy$ and $28y$ f $6k^2$ and $-10k$

g p and $11p^2$ h $45ab$ and $-40cd$ i $3pq + 6p$

5 Write each term in question 4 as a product of two factors where one of the factors is the HCF of the pair of terms.

EXAMPLE 2F-3 Factorising more complex expressions

Factorise each expression.

a $abc + 4ac$ b $15km - 25mn$ c $8y - 12y^2$

THINK	WRITE
a 1 Identify the HCF of abc and $4ac$. (HCF = ac.) Write each term as a product of the HCF and its other factor.	a $abc + 4ac$ $= ac \times b + ac \times 4$
2 Use the distributive law. Write the HCF at the front of a pair of brackets and the other factor for each term inside the brackets.	$= ac \times (b + 4)$ $= ac(b + 4)$
b 1 Identify the HCF of $15km$ and $-25mn$. (HCF = $5m$.) Write each term as a product of the HCF and its other factor.	b $15km - 25mn$ $= 5m \times 3k + 5m \times (-5n)$
2 Use the distributive law.	$= 5m \times (3k - 5n)$ $= 5m(3k - 5n)$
c 1 Identify the HCF of $8y$ and $-12y^2$. (HCF = $4y$.) Write each term as a product of the HCF and its other factor.	c $8y - 12y^2$ $= 4y \times 2 + 4y \times (-3y)$
2 Use the distributive law.	$= 4y \times (2 - 3y)$ $= 4y(2 - 3y)$

UNDERSTANDING AND FLUENCY

6 Factorise each expression.

a $bc + cd$	**b** $2xy + 2y$	**c** $18mn - 9m$
d $abcd + bdf$	**e** $8xy + 28y$	**f** $6k^2 - 10k$
g $p + 11p^2$	**h** $45ab - 40cd$	**i** $3pq + 6p$
j $20ab - 5b$	**k** $8d + 8cde$	**l** $15x^2 + 10x$
m $4k^2 - 22k$	**n** $30n - 18n^2$	**o** $16a^2 + a$
p $2h^2 - 14h$	**q** $6p + 6p^2$	**r** $8xy^2 + 12xy$

7 Explain the difference between expanding an expression and factorising an expression.

8 Copy and complete the following to factorise each expression by removing a negative HCF.

a $-3ab - 6a$
$= (-3a) \times \_\_\_ + (-3a) \times 2$
$= -3a(\_\_\_ + 2)$

b $-8x^2 + 20x$
$= \_\_\_\_ \times 2x + \_\_\_\_ \times (-5)$
$= \_\_\_\_(2x - 5)$

9 Factorise each expression by removing a negative HCF.

a $-5mn - 10n$	**b** $-14xy - 7x$	**c** $-6c + 6cd$
d $-a^2 - 3a$	**e** $-4k^2 - 2k$	**f** $-8x^2 + 8x$
g $-12 - 3xy$	**h** $-16m - 10m^2$	**i** $-9x^2y + 18xy$

EXAMPLE 2F-4 Factorising using a HCF that is a binomial factor

Factorise each expression.

a $y(x + 3) + 7(x + 3)$ **b** $4k(2 - m) - 5(2 - m)$

THINK	WRITE
a 1 Identify the HCF. It is the binomial factor $(x + 3)$. Write each term as a product of the HCF and its other factor.	**a** $y(x + 3) + 7(x + 3)$ $= (x + 3) \times y + (x + 3) \times 7$
2 Use the distributive law. Write the binomial factor at the front of a pair of brackets and the other factor for each term inside the brackets.	$= (x + 3) \times (y + 7)$ $= (x + 3)(y + 7)$
b 1 Identify the HCF. It is $(2 - m)$. Write each term as a product of the HCF and its other factor.	**b** $4k(2 - m) - 5(2 - m)$ $= (2 - m) \times 4k + (2 - m) \times (-5)$
2 Use the distributive law. Write the binomial factor at the front of a pair of brackets and the other factor for each term inside the brackets.	$= (2 - m) \times (4k - 5)$ $= (2 - m)(4k - 5)$

10 Factorise each expression.

a $x(w + 4) + 2(w + 4)$	**b** $y(x - 1) + 7(x - 1)$	**c** $a(a + 6) - 3(a + 6)$
d $p(5 - n) + 8(5 - n)$	**e** $3k(4 - k) - 5(4 - k)$	**f** $2x(3x - 4) + 9(3x - 4)$

EXAMPLE 2F-5 Factorising by grouping terms

Factorise $xy + 2x + 3y + 6$ by grouping terms.

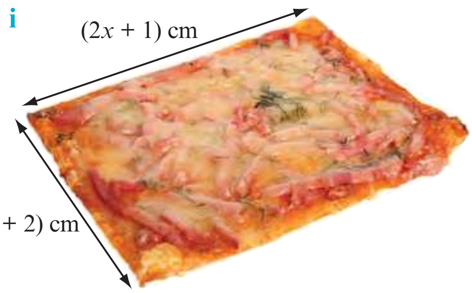
THINK	WRITE
1 Check for a HCF of all four terms (none). Group the terms 'two and two' and identify the HCF for each pair of terms. HCF of first pair is x and HCF of second pair is 3.	$xy + 2x + 3y + 6$ $= (xy + 2x) + (3y + 6)$
2 Use the distributive law to factorise each pair.	$= x(y + 2) + 3(y + 2)$
3 Write the binomial factor of $(y + 2)$ at the front of a pair of brackets and the other factor for each term inside the brackets.	$= (y + 2)(x + 3)$

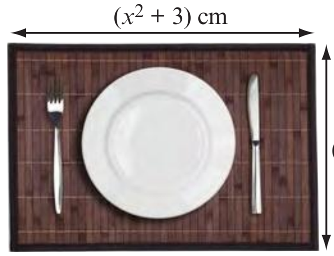
PROBLEM SOLVING AND REASONING

11 Factorise each expression by grouping terms.

a $ab + 5b + 4a + 20$ b $xy - 6x + 7y - 42$ c $mn + 4m - 2n - 8$
d $y^2 + 3y + 5y + 15$ e $k^2 - 7k + 2k - 14$ f $6x + 18 + x^2 + 3x$
g $a^2 - 7a - 2a + 14$ h $p^2 + 5p - 2p - 10$ i $6c^2 - 2c + 9c - 3$

12 a Write an expression for the perimeter of each rectangular item in factor form.

i


ii


b Calculate the perimeter of each item when $x = 5$.

13 Write an expression for the missing side length for each rectangular item.

a


Area of rug is $(8x + 20)$ m^2.

b


Area of stained glass panel is $(m^2 + 15m)$ cm^2.

PROBLEM SOLVING AND REASONING

14 Write an expression for the perimeter of each item in question **13** in factor form.

15 A rectangle has an area of $(10x - x^2)$ mm^2.

- **a** Write this in factor form.
- **b** List the expressions for the length and width of the rectangle.
- **c** Suggest values for the length and width of three different rectangles that would fit these expressions.

16 A triangle has an area of $(21x - 3x^2)$ cm^2.

- **a** List the expressions for the height and base length of the triangle.
- **b** Suggest values for the height and base length of three different triangles that would fit these expressions.

17 Consider the expression $x^2 + 3x - 4x - 12$.

- **a** Factorise the expression by grouping the first two terms together, and the last two terms together.
- **b** Perform the factorisation again by grouping the first and third terms together, and the second and fourth terms together.
- **c** Compare your answers to parts **a** and **b**.

18 Take a number represented by the pronumeral n.

- **a** Write down the next two consecutive numbers.
- **b** Write the sum of these three consecutive numbers.
- **c** Factorise your expression from part **b**. Explain the answer.
- **d** Investigate to see whether the same outcome results from the sum of three consecutive even numbers.

CHALLENGE

19 Factorise each expression by taking out a common factor.

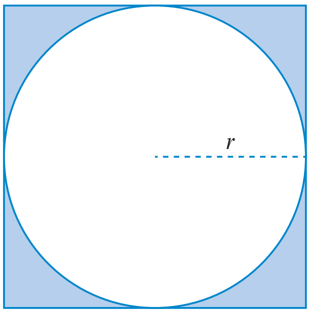
a $4x^2 - 8xy + 12xz$ **b** $\frac{1}{2}x - \frac{1}{4}y + \frac{1}{8}z$ **c** $-8x^2 - \frac{1}{4}x$

20 Completely factorise each expression.

a $x(y + 5) - (3y + 15)$ **b** $p(2q - 3) - 2q + 3$

c $2(a^2 - 3a) + (9 - 3a)$

21 A circle of radius r cm fits within a square as shown in the diagram. Write a factorised expression for the area of the square not covered by the circle. Leave your answer in terms of r and π.



22 In question **18**, the number of terms was odd. Investigate to see the outcome for an even number of consecutive terms. Write a factorised expression for the sum of 10 consecutive terms, where the first term is n.

Reflect

What does it mean to factorise an expression?

2G Factorising quadratic expressions

Start thinking!

In this section, you will look at a way of factorising expressions that are quadratic trinomials where the coefficient of the x^2 term is 1.

1 What is a quadratic trinomial? Use an example to help explain.

2 Consider the binomial product $(x + 2)(x + 3)$.

- **a** Show how $(x + 2)(x + 3)$ expands to $x^2 + 5x + 6$.
- **b** Which terms have been multiplied together to produce x^2?
- **c** Which terms have been multiplied together to produce 6?
- **d** Which terms have been multiplied together and the results added to produce $5x$?

3 Consider the quadratic trinomial $x^2 + 3x + 2$. In factor form, the expression is a binomial product of the form (__ + __)(__ + __).

- **a** To obtain x^2, what should the first term be in each pair of brackets? Show this in (_ + _)(_ + _).
- **b** To obtain 2, what should the second term be in each pair of brackets? Show this in $(x +$ _$)(x +$ _$)$. (Hint: what two whole numbers multiply to give 2?)
- **c** Expand the binomial product you have written in part **b**. Do you obtain $3x$ as the middle term?

4 Now look at factorising $x^2 + 8x + 12$.

- **a** To obtain x^2, what should the first term be in each pair of brackets? Show this in (_ + _)(_ + _).
- **b** To obtain 12, what could the second term be in each pair of brackets? List three different factor pairs of 12. Show each factor pair in the form $(x +$ _$)(x +$ _$)$.
- **c** Expand each of the three binomial products you wrote in part **b**. Which one produces the correct middle term of $8x$?
- **d** Explain why this quadratic trinomial was a little more difficult to factorise than the previous two.

5 Can you describe the pattern you are using to find the terms in the binomial product? Write a list of steps that you can follow.

KEY IDEAS

- Quadratic trinomials can be factorised to produce a binomial product.
- To factorise a quadratic trinomial of the form $x^2 + bx + c$, identify two numbers m and n that multiply to give c and add to give b. That is, $x^2 + bx + c = (x + m)(x + n)$.
- The difference of two squares rule can be used to factorise expressions of the form $a^2 - b^2$. The rule is $a^2 - b^2 = (a + b)(a - b)$.
- Always check whether you can take out a common factor first.
- You can check whether you have factorised correctly by expanding the result and comparing it to the original expression.

EXERCISE 2G Factorising quadratic expressions

1 Identify which two numbers multiply to give the first number and add to give the second number.

a 4, 5	**b** 8, 6	**c** 22, 13
d 20, 9	**e** 24, 10	**f** 12, 7
g 42, 13	**h** 35, 12	**i** 16, 8

2 Use your results from question **1** to factorise each of these quadratic trinomials.

a $x^2 + 5x + 4$	**b** $x^2 + 6x + 8$	**c** $x^2 + 13x + 22$
d $x^2 + 9x + 20$	**e** $x^2 + 10x + 24$	**f** $x^2 + 7x + 12$
g $x^2 + 13x + 42$	**h** $x^2 + 12x + 35$	**i** $x^2 + 8x + 16$

EXAMPLE 2G-1 Factorising simple quadratic trinomials

Factorise each quadratic trinomial. **a** $x^2 + 6x + 5$ **b** $a^2 + 11a + 18$ **c** $y^2 + 7y + 10$

THINK	WRITE
a **1** Show x as the first term in each set of brackets (so x^2 is obtained).	**a** $x^2 + 6x + 5$ $= (x \quad)(x \quad)$
2 Identify factor pairs of 5.	5: 1×5
3 Check that the factors add to 6.	$1 + 5 = 6$
4 Write the expression in factor form.	$x^2 + 6x + 5$ $= (x + 1)(x + 5)$
5 Check your result by expanding.	Check: $(x + 1)(x + 5)$ $= x^2 + 5x + x + 5$ $= x^2 + 6x + 5$
b **1** Show a as the first term in each set of brackets (so a^2 is obtained).	**b** $a^2 + 11a + 18$ $= (a \quad)(a \quad)$
2 Identify factor pairs of 18.	18: 1×18, 2×9, 3×6
3 Check which factors add to 11. (2 and 9)	$1 + 18 = 19$, $2 + 9 = 11$, $3 + 6 = 9$
4 Write the expression in factor form. (Check your result by expanding.)	$a^2 + 11a + 18$ $= (a + 2)(a + 9)$
c (Shorter method shown.) **1** Show y as the first term in each set of brackets.	**c** $y^2 + 7y + 10$
2 Identify a factor pair of 10 that adds to 7. (2 and 5)	$= (y + 2)(y + 5)$ $\quad 2 \times 5 = 10$ $2 + 5 = 7$
3 Write the expression in factor form. (Check your result by expanding.)	

UNDERSTANDING AND FLUENCY

3 Factorise each quadratic trinomial.

a $a^2 + 4a + 3$	**b** $b^2 + 9b + 14$	**c** $c^2 + 7c + 6$
d $d^2 + 10d + 21$	**e** $e^2 + 8e + 7$	**f** $f^2 + 8f + 15$
g $g^2 + 11g + 28$	**h** $h^2 + 13h + 36$	**i** $x^2 + 9x + 18$
j $j^2 + 14j + 45$	**k** $k^2 + 11k + 30$	**l** $y^2 + 13y + 40$

4 Identify which two numbers multiply to give the first number and add to give the second number.

a −8, 2	**b** −6, −1	**c** 12, −8
d −10, 3	**e** −9, −8	**f** 6, −5
g −6, 5	**h** −27, −6	**i** 11, −12

5 Use your results from question **4** to factorise each of these quadratic trinomials.

a $x^2 + 2x - 8$	**b** $x^2 - x - 6$	**c** $x^2 - 8x + 12$
d $x^2 + 3x - 10$	**e** $x^2 - 8x - 9$	**f** $x^2 - 5x + 6$
g $x^2 + 5x - 6$	**h** $x^2 - 6x - 27$	**i** $x^2 - 12x + 11$

EXAMPLE 2G-2 Factorising more complex quadratic trinomials (+ and –)

Factorise each quadratic trinomial.

a $m^2 + 2m - 3$ **b** $x^2 - 7x - 8$

THINK	WRITE
a **1** Show m as the first term in each set of brackets (so m^2 is obtained).	**a** $m^2 + 2m - 3$ $= (m \quad)(m \quad)$
2 Identify factor pairs of −3. One factor must be positive and the other negative.	-3: 1×-3, -1×3
3 Check which factors add to 2. (−1 and 3)	$1 + (-3) = -2$, $-1 + 3 = 2$
4 Write the expression in factor form.	$m^2 + 2m - 3$ $= (m - 1)(m + 3)$
5 Check your result by expanding.	Check: $(m - 1)(m + 3)$ $= m^2 + 3m - m - 3$ $= m^2 + 2m - 3$
b **1** Show x as the first term in each set of brackets (so x^2 is obtained).	**b** $x^2 - 7x - 8$ $= (x \quad)(x \quad)$
2 Identify factor pairs of −8.	-8: 1×-8, -1×8, 2×-4, -2×4
3 Check which factors add to −7. (1 and −8)	$1 + (-8) = -7$, $-1 + 8 = 7$, $2 + (-4) = -2$, $-2 + 4 = 2$
4 Write the expression in factor form. (Check your result by expanding.)	$x^2 - 7x - 8$ $= (x + 1)(x - 8)$

EXAMPLE 2G-3 Factorising more complex quadratic trinomials (– and –)

Factorise $d^2 - 5d + 6$.

THINK	WRITE
1 Show d as the first term in each set of brackets (so d^2 is obtained).	$d^2 - 5d + 6$ $= (d \quad)(d \quad)$
2 Identify factor pairs of 6. Since the middle term is negative, only look at the negative factor pairs.	$6: -1 \times -6, -2 \times -3$
3 Check which factors add to −5. (−2 and −3)	$-1 + (-6) = -7, -2 + (-3) = -5$
4 Write the expression in factor form. (Check your result by expanding.)	$d^2 - 5d + 6$ $= (d - 2)(d - 3)$

6 Factorise each quadratic trinomial.

a $a^2 + 2a - 3$ **b** $b^2 - 2b - 15$ **c** $c^2 - 5c + 4$
d $d^2 + 5d - 14$ **e** $e^2 - 10e + 24$ **f** $f^2 - 3f - 10$
g $g^2 + g - 12$ **h** $h^2 - 8h + 15$ **i** $x^2 - 5x - 24$
j $j^2 - 10j + 16$ **k** $k^2 + 3k - 18$ **l** $y^2 - y - 2$

EXAMPLE 2G-4 Factorising quadratic trinomials by first taking out a common factor

Factorise $2x^2 - 14x + 12$ by first taking out a common factor.

THINK	WRITE
1 Check whether the three terms have a common factor (yes). Take out the HCF of 2.	$2x^2 - 14x + 12$ $= 2(x^2 - 7x + 6)$
2 Now factorise $x^2 - 7x + 6$. Show x as the first term in each set of brackets (so x^2 is obtained).	$= 2(x \quad)(x \quad)$
3 Identify factor pairs of 6. Both factors must be negative to obtain a negative middle term.	$6: -2 \times -3, -1 \times -6$
4 Check which factors add to −7. (−1 and −6)	$-2 + (-3) = -5, -1 + (-6) = -7$
5 Write the expression in factor form. (Check your result by expanding.)	$2x^2 - 14x + 12$ $= 2(x - 1)(x - 6)$

7 Factorise each quadratic trinomial by first taking out a common factor.

a $3x^2 + 9x + 6$ **b** $2x^2 + 16x + 30$ **c** $5x^2 + 15x - 20$
d $-4x^2 - 20x - 24$ **e** $-6x^2 + 36x - 30$ **f** $-x^2 - 2x + 35$

8 **a** Expand $(x + 7)(x - 7)$ using the difference of two squares rule.
b Explain how you can factorise $x^2 - 49$.

EXAMPLE 2G-5 Factorising quadratic expressions using the difference of two squares rule

Factorise each quadratic expression.

a $x^2 - 25$ **b** $9y^2 - 16$ **c** $4k^2 - 36$ **d** $(m + 4)^2 - 1$

THINK	WRITE
a 1 Check for common factors (none). Write as a difference of two squares.	**a** $x^2 - 25$ $= x^2 - 5^2$
2 Factorise using the difference of two squares rule. $a^2 - b^2 = (a + b)(a - b)$ where $a = x$ and $b = 5$.	$= (x + 5)(x - 5)$
b 1 Check for common factors (none). Write as a difference of two squares.	**b** $9y^2 - 16$ $= (3y)^2 - 4^2$
2 Factorise using the difference of two squares rule.	$= (3y + 4)(3y - 4)$
c 1 Check for common factors (yes). Take out the HCF of 4.	**c** $4k^2 - 36$ $= 4(k^2 - 9)$
2 Write the expression in brackets as a difference of two squares.	$= 4(k^2 - 3^2)$
3 Factorise using the difference of two squares rule.	$= 4(k + 3)(k - 3)$
d 1 Look for common factors (none). Write as a difference of two squares.	**d** $(m + 4)^2 - 1$ $= (m + 4)^2 - 1^2$
2 Factorise using the difference of two squares rule.	$= (m + 4 + 1)(m + 4 - 1)$
3 Simplify the expression in each pair of brackets.	$= (m + 5)(m + 3)$

9 Factorise each quadratic expression.

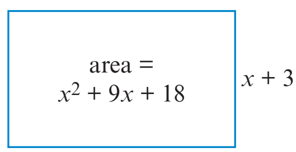
a $x^2 - 36$ **b** $a^2 - 100$ **c** $9 - y^2$

d $25m^2 - 4$ **e** $49 - 64c^2$ **f** $3x^2 - 12$

g $18p^2 - 50$ **h** $(ab)^2 - 9$ **i** $x^2y^2 - w^2$

j $(a + 2)^2 - 4$ **k** $(k - 3)^2 - 16$ **l** $(7 - m)^2 - 1$

10 Consider this rectangle, which has all length measurements in metres. The expression for the width is $x + 3$ and the expression for the area is $x^2 + 9x + 18$.



a Factorise the expression for the area of the rectangle.

b List the two factors that multiply to give the expression for the area.

c What is the expression for the length of this rectangle?

d Demonstrate that you have the correct expression for the length.

e If $x = 2$, calculate the area using:

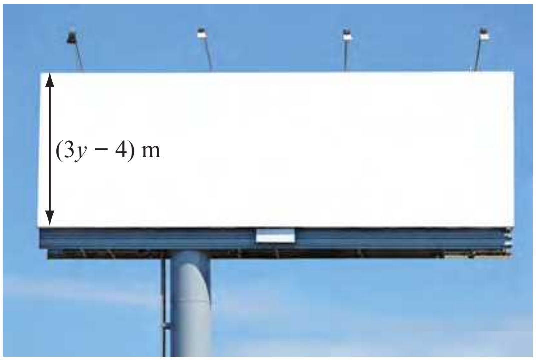
i values for length and width

ii the expression $x^2 + 9x + 18$.

PROBLEM SOLVING AND REASONING

11 **a** Write an expression for the missing side length for each rectangular item.

i area within frame is $(x^2 + 7x - 18)$ cm^2 **ii** area of billboard is $(9y^2 - 16)$ m^2



b Write three possible values for x and y in each case.

c Find the area of each rectangle using your answers to part **b**.

d Find the value of x that gives an area of 152 cm^2 for part **i**.

e Find the value of y that gives an area of 425 m^2 for part **ii**.

12 Use the fact that $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$ to completely factorise each quadratic trinomial.

a $x^2 + 8x + 16$ **b** $y^2 - 10y + 25$ **c** $4z^2 - 12z + 9$

13 Without using a calculator, work out the value of each problem.

a $63^2 - 37^2$ **b** $15.19^2 - 14.81^2$ **c** $\left(\frac{13}{25}\right)^2 - \left(\frac{12}{25}\right)^2$

14 Write a simplified expression to represent the difference between the squares of two consecutive numbers n and $(n + 1)$.

15 A square has side lengths of x cm. A second square has side lengths 2 cm longer than the first square. Write a factorised expression to represent the difference between the areas of the two squares.

CHALLENGE

16 Use the difference of two squares rule to fully factorise each expression.

a $x^4 - 16$ **b** $x^8 - y^8$

17 Show how $(x + 1)(x + y)^2 - (x + 1)(x - y)^2$ can be written in its simplest factorised form of $4xy(x + 1)$.

18 **a** Expand $(x + y)(x^2 - xy + y^2)$.

b Use your result in part **a** to factorise $x^3 + 8$.

c Predict the factorised form of $x^3 - 8$.

19 These are the known facts about two numbers:
The difference between the two numbers is 5, while the difference between the squares of the two numbers is 155. What is the sum of the two numbers?

Reflect

How can factorising an expression make some calculations easier?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

algebraic terms	index form	index laws	expansion of a perfect square
pronumeral	index notation	scientific notation	factor form
like terms	base	significant figures	expanded form
coefficient	power	expand	factorise
expression	basic numeral	distributive law	binomial product
equation	indices	difference of two squares	quadratic trinomial

MULTIPLE-CHOICE

2A ▶ **1** Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?

A $\frac{6b^2}{18a}$ **B** $\frac{ab^2c}{3a^2c}$

C $\frac{b^2}{3a}$ **D** $\frac{6ab^2}{18a^2}$

2B ▶ **2** Which statement does *not* correctly represent one of the index laws?

A $m^5 \times m^2 = m^{5+2}$

B $(p \times q)^8 = p^8 \times q^8$

C $w^7 \div w^5 = w^{7-5}$

D $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y}$

2B ▶ **3** Using the index laws, $\frac{5x^{13} \times 2x^4}{4x^8 \times x^0}$ simplifies to:

A $\frac{10x^9}{4x^8}$ **B** $\frac{5x^9}{2}$

C $\frac{5x^{17}}{2x^8}$ **D** $\frac{5}{2x^9}$

2C ▶ **4** Which statement is *false*?

A $\frac{1}{7} = 7^1$ **B** $4^{-2} = \frac{1}{16}$

C $\frac{1}{3^6} = 3^{-6}$ **D** $7^3 \times 7^{-5} = \frac{1}{49}$

2D ▶ **5** Which number is equivalent to 6.4724×10^2?

A 0.647 24 **B** 64.724

C 0.064 724 **D** 647.24

2E ▶ **6** Which expression cannot be expanded using the difference of two squares rule?

A $(x + 6)(x - 6)$

B $(7 - p)(7 + p)$

C $(2x - 7)(2x + 7)$

D $(d + 5)(d - 2)$

2E ▶ **7** Which statement is *incorrect*?

A $-3(b + 5) = -3b - 15$

B $(d + 3)(d - 7) = d^2 + 4d - 21$

C $(m - 4)(m + 4) = m^2 - 16$

D $(2b + 5)(3b - 2) = 6b^2 + 11b - 10$

2G ▶ **8** The expression $w^2 - 49$ factorises to:

A $(w - 7)^2$ **B** $(w + 7)^2$

C $(w + 7)(w - 7)$ **D** $(7 + w)(7 - w)$

2G ▶ **9** The expression $t^2 + 3t - 18$ factorises to:

A $(t + 6)(t - 3)$ **B** $(t - 6)(t + 3)$

C $(t + 9)(t - 2)$ **D** $(t - 9)(t + 2)$

SHORT ANSWER

2A **1** Simplify each expression.

a $15t - 7t + 8t$

b $a - 7p - 11p + 12a$

c $3k + 5km - 7k - 15 + 2k - 4km$

d $6m^2n - 2m^2 + 7nm^2 + 11n^2 - 4mn^2 - 3m^2$

2A **2** Simplify each expression.

a $4xy \times 11xyz$

b $9mnp \times 2m^3p \times 4n^2$

c $15de \div (18df)$

d $11klmn \div (-22klm^2)$

2B **3** Simplify each expression using the index laws.

a $a^{11} \times a^5$ b $b^9 \div b^8$

c $(c^8)^2$ d $18d^7 \div (54d^4)$

e $(e^5)^5 \times (e^{11})^2$ f $5a^0 + 3b^0 + 1c^0$

2B **4** Simplify each expression.

a $\dfrac{m^3n^4 \times m^9n^{11}}{m^7n^7}$

b $\dfrac{(3k^5l^2)^3 \times (2k^3l^3)^4}{(2k^3l^2)^3}$

2C **5** Write each term with a positive index.

a 4^{-1} b b^{-1}

c m^{-7} d $\dfrac{6x^3}{2y^{-3}}$

e $\dfrac{f^{-4}}{g^{-5}}$ f $\dfrac{a^{-3}b}{5^{-2}c^{-4}}$

g $(p^{-2})^5 \times p^{-5}$ h $\left(\dfrac{k}{l}\right)^{-7}$

2C **6** a If $4^8 = 65\,536$, write the value of 4^{-8} as a fraction.

b If $7^{-3} = \dfrac{1}{343}$, write the value of 7^3.

2D **7** Write each number as a basic numeral.

a 5.876×10^4 b 9.02×10^{-6}

2D **8** State the number of significant figures in each part of question 7.

2D **9** Write each number in scientific notation.

a 540 000 b 0.00076

2D **10** Round each of the following to the number of significant figures indicated.

a 879 (2) b 2.58×10^5 (1)

2D **11** A scientist estimates that there are 3.4×10^4 bacteria in one sample and 4.6×10^3 in the second. Write the total number of bacteria:

a as a basic numeral

b rounded to two significant figures

c in scientific notation.

2E **12** Expand each expression to remove brackets.

a $5(a + 2) - 3(7 - a)$

b $(b - 11)(b + 2)$

c $(3c - 2)(4c - 5)$

d $(d + w)(d - w)$

e $(6 + e)^2$

f $(f - 9)^2$

2F **13** Factorise each expression.

a $4a - 24$

b $b^2 + 11b$

c $36pq^2 + 144pq$

d $7d(8 - d) - 4(8 - d)$

e $5e + 15ef + 2 + 6f$

2G **14** Factorise each quadratic trinomial.

a $a^2 + 6a + 5$

b $b^2 - 7b + 12$

c $c^2 + 4c - 21$

d $d^2 - 16d - 36$

e $5e^2 - 45e + 70$

2G **15** Factorise each expression using the difference of two squares rule.

a $a^2 - 64$ b $121 - b^2$

c $36m^2 - 49n^2$ d $(p + 1)^2 - 4$

e $2e^2 - 32$ f $(f + 4)^2 - (f - 5)^2$

NAPLAN-STYLE PRACTICE

1 Which expression is equivalent to $7(2b + 4)$?

- $14b + 4$
- $14b + 28$
- $9b + 4$
- $18b$

2 In a set of three consecutive whole numbers, the value of the lowest number is w. What is the value of the highest number in the set?

- $3w$
- $w + 3$
- $3 - w$
- $w + 2$

3 What is the value of $4x^2$ when $x = -5$?

- 100
- −100
- 400
- 220

4 If $a = -3$ and $b = 3$, what is the value of $a^2 + b^2$?

5 What term makes this statement true for all values of x?

$7(x + 6) +$ ___ $(x - 2) = 10x + 36$

6 If $m = -7$, what is the value of $2m - 11$?

7 Which expression is equivalent to $5a + 6 + 7a - 11$?

- $12a - 5$
- $7a$
- $12a + 17$
- $11a - 4$

8 Which is equivalent to 4^{-6}?

- $-\frac{1}{4^{-6}}$
- -4^6
- $\frac{1}{4^6}$
- $-(-4)^6$

9 Simplify $x^6 \times x^4 \times x^{-11}$ using positive indices.

10 Simplify $\frac{w^{11}}{w^6}$.

11 Which expression shows $\frac{8r^{-2}p^{-6}}{12p^{-2}}$ in simplified form with positive indices only?

- $\frac{2r^2}{3p^4}$
- $\frac{8r^2}{12p^4}$
- $\frac{2}{3r^2p^4}$
- $\frac{2}{3r^2p^{-8}}$

12 Write 2.97×10^4 as a basic numeral.

13 Which sign makes the following statement true?

4.98×10^3 ___ 0.0498×10^6

- $>$
- $<$
- $=$
- $\leq$

14 What does $(5 \times 10^2) \times (4 \times 10^3)$ simplify to in scientific notation?

- 9×10^5
- 2×10^6
- 20×10^5
- 9×10^1

15 What is the highest common factor of $18x^2yz$ and $12xy^2z$?

- $6xyz$
- $9x^2yz$
- $6xy^2z$
- $6xy$

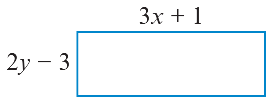
16 Expand $(a - 3)(a + 5)$.

17 Factorise $b^2 - 13b + 40$.

18 Factorise $m^2 + m - 12$.

19 Factorise $7d^2 - 14d + 9d - 18$.

Questions **20**–**23** refer to this rectangle.



20 Write the perimeter of this shape as an algebraic expression in simplest form.

21 Calculate the perimeter when $x = 5$ cm and $y = 2$ cm.

22 Write an algebraic expression for the area of the shape.

23 Calculate the area when $x = 12$ cm and $y = 8$ cm.

Questions **24** and **25** refer to a rectangle with an area of $(50m - 5m^2)$ cm^2.

24 Write the area in factor form.

25 List the expressions for the length and width of the rectangle.

ANALYSIS

In June 2013, the population of each Australian state was recorded. The figure for each state is shown in the table.

State	Population at 30 June 2013 ('000)
NSW	7407.7
VIC	5737.6
QLD	4658.6
SA	1670.8
WA	2517.2
TAS	513.0
NT	239.5
ACT	383.4

Source: http://www.abs.gov.au/ausstats/abs@.nsf/mf/3101.0/

a Which states and territories have a population listed to:
  i four significant figures?
  iii five significant figures?

b Copy the table and add three additional columns.

c In the first new column, write the population of each state and territory in full.

d Repeat part **a** but this time use your answers to part **c**. Do you obtain the same results? Explain.

e In the second new column, round each population to its leading digit.

f In the third new column, write each population in scientific notation to one significant figure.

g Use your answers from part **f** to work out the following. Write the values in scientific notation.
  i Which state or territory has the highest population?
  ii Which state or territory has the lowest population?
  iii Calculate the difference between the highest and the lowest population.
  iv Calculate the total population of SA, TAS, ACT and NT.
  v Calculate the total population of Australia.

h The actual total population value recorded at the end of June 2013 was 23 130 900. Calculate the difference between your answer to **g** part **v** and the actual value. Why is there a difference?

CONNECT

All about boxes!

Boxes come in all shapes and sizes. Some must meet certain conditions in terms of the amount of material used or the space inside.

For example, Tom needs to construct a rectangular box with a volume of 120 cm^3. What is the smallest amount of cardboard needed to make the box? Think about the process you will use to work this out. How will your working change if the box is open?

Algebra can be used to save time when working with a number of different-sized boxes.

Your next task is to investigate the dimensions of a rectangular gift box where the length is twice the height and the sum of the length and width is 96 cm. How much wrapping paper will be needed if the box is to have the largest volume possible?

Your task

Follow these steps to complete this investigation.

- Advise Tom on the dimensions to use for constructing his box so that it has a volume of 120 cm^3 but uses the minimum amount of cardboard. Start by considering all the whole number values in centimetres that could be used for the length, width and height of the box. You may like to use a spreadsheet to help you.
- Work out whether the dimensions would change for Tom's box if it has no top; that is, it is an open box.
- Use algebra to investigate the dimensions of a rectangular gift box so it has the largest volume possible when the length is twice the height and the sum of the length and width is 96 cm.
  - Assign a pronumeral for the height of the box and use this to write algebraic expressions for the length and the width of the box.
  - Write an expression for the volume and the surface area of the box. Show each in simplified factor form.
  - Find the dimensions of the box that fit the given conditions. You may like to use a spreadsheet to make the calculations easier.
  - Work out the amount of wrapping paper needed.

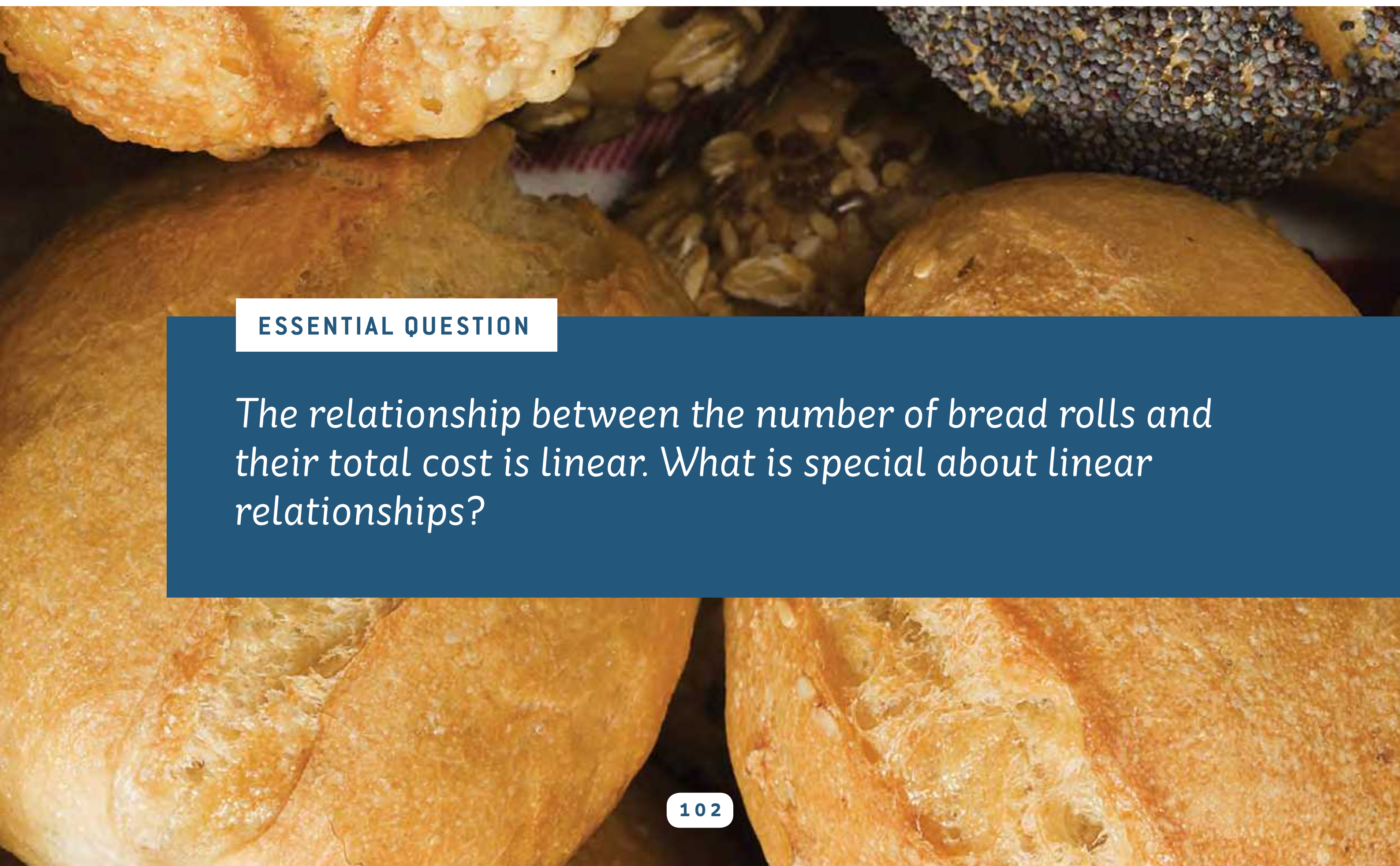
Include all necessary working to justify your answers.

As an extension, find the dimensions of the gift box that meets the conditions above and has the smallest surface area. What volume does this box have? Show all relevant diagrams and working.

You may like to present your findings as a report. Your report could be in the form of:

- a poster showing labelled diagrams and plans of your boxes
- models of your boxes
- a PowerPoint presentation
- a technology demonstration
- other (check with your teacher).

3 LINEAR RELATIONSHIPS



ESSENTIAL QUESTION

The relationship between the number of bread rolls and their total cost is linear. What is special about linear relationships?

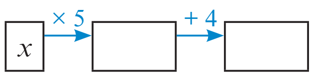
3A **1** Solve each equation to find the value of x.

a $7x = 42$ **b** $x - 15 = 8$

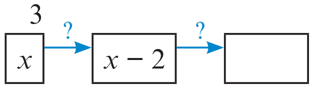
c $x + 9 = 26$ **d** $\frac{x}{3} = 12$

3A **2** Copy and complete each flowchart to build the required expression.

a $5x + 4$



b $\frac{x-2}{3}$



3A **3** Write each expression in expanded form without brackets.

a $3(x + 5)$

b $-5(2x - 1)$

3A **4** Simplify each expression.

a $4x + 1 + 7x$

b $5x - 9 - 3x + 2$

3C **5** Plot the points with coordinates shown in this table on a Cartesian plane.

x	−2	−1	0	1	2
y	−1	0	1	2	3

3D **6** Follow these instructions to find the position of a point on the Cartesian plane. Write the coordinates of the point.

a Start at (2, 1) and move 3 units right and 4 units up.

b Start at (−4, 3) and move 2 units right and 5 units down.

c Start at (0, 4) and move 1 unit right and 3 units up.

3G **7** Find the number that is halfway between each pair of numbers.

a 3 and 7

b −4 and 8

c 6 and 9

3G **8** Find the average of each pair of numbers.

a 5 and 11

b −6 and 2

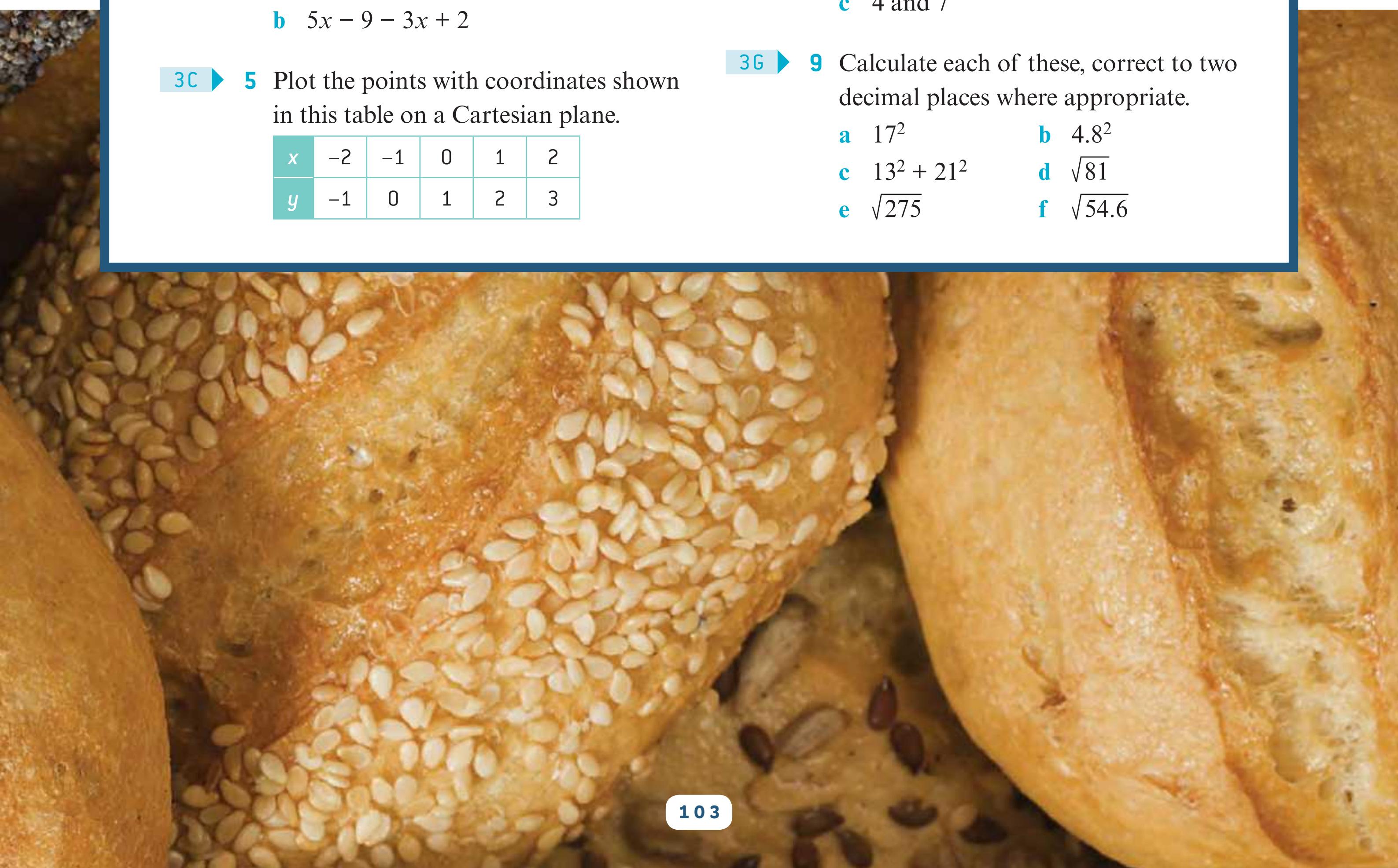
c 4 and 7

3G **9** Calculate each of these, correct to two decimal places where appropriate.

a 17^2 **b** 4.8^2

c $13^2 + 21^2$ **d** $\sqrt{81}$

e $\sqrt{275}$ **f** $\sqrt{54.6}$



3A Solving linear equations

Start thinking!

Tristan has saved \$75 to buy some retro vinyl records that cost \$11 each. There is a postage charge of \$9 to deliver the package. He wants to work out how many records he can buy.

1 Which **linear equation** would best match this scenario if n represents the number of vinyl records Tristan can buy?

A $11n - 9 = 75$ **B** $9n + 11 = 75$ **C** $11n + 9 = 75$ **D** $9n - 11 = 75$

2 Describe some possible methods that you could use to solve this equation to find the value of n. You may like to discuss this with a classmate.

The **balance method** can be used to solve both simple and more complex equations.

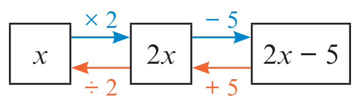
3 Solve $x + 7 = 13$ using the balance method. Copy and complete the steps shown to obtain an **equivalent equation** where x is by itself on one side of the equation.

$$x + 7 = 13$$
$$x + 7 - \_\_\_ = 13 - \_\_\_$$
$$x = \_\_\_$$

4 To solve the equation $2x - 5 = 3$, you will need to perform two operations to obtain x by itself on one side of the equation.

a Which operation should be performed: **i** first? **ii** second?

b How can this simple flowchart help you decide the order in which to perform the operations?



c Why is it important to perform the operations in the correct order? What would happen if you didn't?

d Copy and complete the steps shown at right to solve the equation $2x - 5 = 3$ using the balance method. Use your answers to parts **a** and **b** to help you.

$$2x - 5 = 3$$
$$2x - 5 + \_\_\_ = 3 + \_\_\_$$
$$2x = \_\_\_$$
$$\frac{2x}{2} = \frac{\_\_\_}{2}$$
$$x = \_\_\_$$

e How can you check that you have the correct value for the **solution**?

5 Now return to the scenario where Tristan is buying vinyl records.

a List the two operations that will need to be performed in their correct order to solve the equation in question **1**. You may like to draw a simple flowchart to help you.

b Use the balance method to solve the equation in question **1**.

c How many vinyl records can Tristan buy with his savings?

6 Why is the method used to solve equations in this task known as the balance method?

KEY IDEAS

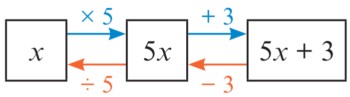
- To solve an equation using the balance method, perform operations on both sides of the equation to produce equivalent equations until you obtain the solution equation '$x = \ldots$'.
- An operation can be undone by performing the inverse operation. A simple flowchart can help you decide the order of the operations to undo.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

EXERCISE 3A Solving linear equations

EXAMPLE 3A-1 Solving an equation using the balance method (two steps)

Solve $5x + 3 = -12$.

THINK	WRITE
1 Consider how the expression on the left side was built (× 5 then + 3). A simplified flowchart can show you which operation to undo first. 	$5x + 3 = -12$
2 First undo '+ 3' by performing the inverse operation of '− 3' on both sides and simplify.	$5x + 3 - 3 = -12 - 3$ $5x = -15$
3 Undo '× 5' by performing the inverse operation of '÷ 5' on both sides and simplify to obtain the solution.	$\frac{5x}{5} = \frac{-15}{5}$ $x = -3$
4 Use substitution to check your solution.	$LS = 5 \times -3 + 3$ $= -15 + 3$ $= -12$ $= RS$

UNDERSTANDING AND FLUENCY

1 Copy and complete the steps shown to solve each equation by the balance method.

a
$$\frac{x}{6} - 4 = 1$$
$$\frac{x}{6} - 4 + \_\_\_ = 1 + \_\_\_$$
$$\frac{x}{6} = \_\_\_$$
$$\frac{x}{6} \times \_\_\_ = 5 \times \_\_\_$$
$$x = \_\_\_$$

b
$$\frac{x + 3}{4} = -2$$
$$\frac{x + 3}{4} \times \_\_\_ = -2 \times \_\_\_$$
$$x + 3 = \_\_\_$$
$$x + 3 - \_\_\_ = \_\_\_ - 3$$
$$x = \_\_\_$$

c
$$6(x + 9) = 12$$
$$\frac{6(x + 9)}{6} = \frac{12}{\_\_\_}$$
$$x + 9 = \_\_\_$$
$$x + 9 - \_\_\_ = \_\_\_ - \_\_\_$$
$$x = \_\_\_$$

2 Solve each equation. Show all steps of working and check your solutions.

a $4x + 5 = 29$ **b** $2x - 3 = 11$ **c** $\frac{x + 3}{2} = 4$

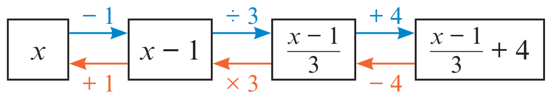
d $\frac{x}{4} - 2 = 7$ **e** $\frac{2x}{5} = 4$ **f** $2(x + 6) = 28$

g $3x - 5 = -17$ **h** $\frac{x + 8}{3} = 2$ **i** $\frac{x}{5} + 6 = 9$

j $7x + 9 = 2$ **k** $\frac{x - 4}{5} = 1$ **l** $4(x - 2) = -20$

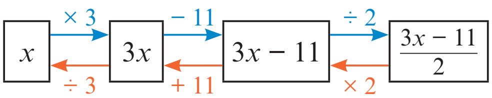
m $\frac{3x}{2} = -18$ **n** $5x + 1 = 16$ **o** $\frac{x}{2} + 7 = 4$

3 To solve $\frac{x-1}{3}+4=9$, you need to perform three operations to obtain x by itself on one side of the equation. List the three operations in their correct order. You may like to use this simple flowchart to help you.



EXAMPLE 3A-2 Solving an equation using the balance method (three steps)

Solve $\frac{3x-11}{2}=8$.

THINK	WRITE
	$\frac{3x-11}{2}=8$
1 Consider how the expression on the left side was built (× 3, −11 then ÷ 2). A simple flowchart can be used to help you decide which operation to undo first. 	
2 First undo '÷ 2' by performing the inverse operation of '× 2' on both sides and simplify.	$\frac{3x-11}{2}\times 2=8\times 2$ $3x-11=16$
3 Undo '− 11' by performing the inverse operation of '+ 11' on both sides and simplify.	$3x-11+11=16+11$ $3x=27$
4 Undo '× 3' by performing the inverse operation of '÷ 3' on both sides and simplify to obtain the solution.	$\frac{3x}{3}=\frac{27}{3}$ $x=9$
5 Use substitution to check your solution.	$\text{LS}=\frac{3\times 9-11}{2}$ $=8$ $=\text{RS}$

UNDERSTANDING AND FLUENCY

4 Solve the equation in question **3** using the balance method. Show all steps of your working and check your solution.

5 Solve each equation. Show all steps of working.

a $\frac{2x-5}{3}=1$ **b** $\frac{3x+4}{5}=2$ **c** $\frac{7x}{2}-9=12$

d $\frac{3x}{4}+1=7$ **e** $\frac{5(x-1)}{2}=20$ **f** $4(x+3)-2=30$

g $\frac{x-4}{3}+7=9$ **h** $6(2x+11)=18$ **i** $\frac{x-2}{6}-3=0$

j $\frac{11x}{4}+23=1$ **k** $\frac{7x+6}{2}=3$ **l** $\frac{x+1}{4}-3=2$

m $10(x-2)+11=1$ **n** $\frac{2(x+8)}{3}=2$ **o** $\frac{x+13}{6}-5=-4$

6 **a** To solve $-2x=6$, what operation should be performed on both sides?

A add −2 **B** multiply by −2 **C** divide by −2

b Solve $-2x=6$ using your answer to part **a**.

UNDERSTANDING AND FLUENCY

7 Solve each equation using the balance method. Show all steps of working.

a $-5x = 30$ b $-3x = -27$ c $-x = 11$ d $-x = -5$

e $-4x = 0$ f $\frac{x}{-4} = 8$ g $\frac{x}{-6} = 10$ h $\frac{x}{-8} = -1$

i $-\frac{x}{2} = 5$ j $-\frac{x}{3} = -7$ k $-6x = 42$ l $-\frac{x}{4} = 21$

8 Solve each equation. You can perform decimal operations.

a $x + 2.4 = 9$ b $x - 7.1 = 4.2$ c $4.5x = 18$

d $\frac{x}{3.6} = 2$ e $-1.2x = 6$ f $x - 5.1 = -3.2$

g $-2.3x = 4.6$ h $\frac{x}{0.25} = -12$ i $-x = -0.7$

j $-0.2x = -2.8$ k $2x + 5.4 = 8.6$ l $\frac{x - 2.9}{4} = 1$

m $\frac{x - 4}{3} + 11.2 = 9$ n $\frac{3x}{4} - 1.5 = 6.2$ o $\frac{5x + 2}{3} = -4.6$

9 Copy and complete the steps shown to solve each equation by the balance method.

a
$$-2x + 3 = 17$$
$$-2x + 3 - \_\_\_ = 17 - \_\_\_$$
$$-2x = \_\_\_$$
$$\frac{-2x}{-2} = \frac{\_\_\_}{-2}$$
$$x = \_\_\_$$

b
$$-4(x - 7) = -12$$
$$\frac{-4(x - 7)}{-4} = \frac{-12}{-4}$$
$$x - 7 = \_\_\_$$
$$x - 7 + \_\_\_ = 3 + \_\_\_$$
$$x = \_\_\_$$

c
$$5 - x = 11$$
$$-x + 5 - \_\_\_ = 11 - \_\_\_$$
$$-x = \_\_\_$$
$$\frac{-x}{\_\_\_} = \frac{\_\_\_}{-1}$$
$$x = \_\_\_$$

10 Solve each equation using the balance method.

a $-3x + 4 = 19$ b $-5x - 2 = 8$ c $1 - 2x = -7$

d $9 - 4x = -3$ e $-2(x + 4) = 14$ f $-7x + 3 = -4$

g $-x + 6 = 10$ h $8 - x = -2$ i $-x - 1 = 15$

j $-4(x - 5) = -16$ k $-5(x + 2) - 7 = 3$ l $\frac{-3x - 7}{4} = 5$

m $\frac{2 - 3x}{5} = -2$ n $\frac{-x - 4}{3} = 1$ o $\frac{-x}{2} - 4 = 6$

11 a Solve $5(x - 2) = 20$ by first dividing both sides by 5.

b Another way to solve this equation is first to expand the expression on the left side to remove the brackets. Try this method. Do you obtain the same solution?

c Solve $5(x - 2) = 18$ by first dividing both sides by 5.

d Solve $5(x - 2) = 18$ by first expanding the expression on the left side.

e Which method did you find easier to use when solving $5(x - 2) = 18$? Explain.

12 Solve each equation by first expanding the expression on the left to remove brackets. Where appropriate, write the solution as a fraction.

a $4(x - 1) = 8$ b $3(x + 7) = -6$ c $2(x - 3) = 5$

d $5(x + 4) = 8$ e $-4(x + 2) = -24$ f $-6(x - 2) = 1$

13 Solve each equation by expanding and simplifying the expression on the left side first.

a $3(x + 2) - 4 = 11$ **b** $2(x - 9) + 3 = 7$ **c** $5(x + 1) + 2x = 12$
d $4(x - 3) - x = -3$ **e** $7(x - 2) - 5 = 1$ **f** $3(x + 4) + 6 = 5$
g $2(6 - x) - 7 = 3$ **h** $4(1 - x) + 3x = 9$ **i** $3(2 - x) + 4 = -8$

14 **a** Solve the equation $\frac{12}{x} = 3$ by inspection. (Hint: $12 \div ? = 3$.)
b How might you solve this equation using the balance method?

EXAMPLE 3A-3 Solving an equation where the variable is in the denominator

Solve $\frac{12}{x} = 3$.

THINK	WRITE
1 Multiply both sides by x and simplify. (You may also like to swap the sides of the equation.)	$\frac{12}{x} = 3$ $\frac{12}{x} \times x = 3 \times x$ $12 = 3x$ $3x = 12$
2 Undo '× 3' by performing the inverse operation of '÷ 3' on both sides and simplify to obtain the solution.	$\frac{3x}{3} = \frac{12}{3}$ $x = 4$
3 Use substitution to check your solution.	$\text{LS} = \frac{12}{4} = 3 = \text{RS}$

UNDERSTANDING AND FLUENCY

15 Solve each equation using the balance method.

a $\frac{10}{x} = 2$ **b** $\frac{21}{x} = -7$ **c** $\frac{4}{x} = 1$ **d** $\frac{35}{x} = -5$
e $\frac{-18}{x} = 3$ **f** $\frac{-24}{x} = -6$ **g** $\frac{11}{x} = 2$ **h** $\frac{8}{x} = -5$
i $\frac{9.6}{x} = 2$ **j** $\frac{14.4}{x} = 1.2$ **k** $\frac{20}{x} = \frac{1}{2}$ **l** $-\frac{1}{x} = 3$

16 Some of the steps in the balance method can be done in your head. The example at right shows the setting out you can use. Try this setting out to solve each equation using the balance method. If you have difficulty, continue to include the extra steps that show the operations to be performed.
(Hint: not all solutions will be whole numbers.)

Example
$\frac{2x + 3}{5} = 7$
$2x + 3 = 35$
$2x = 32$
$x = 16$

a $4x - 1 = 23$ **b** $7x + 4 = 60$ **c** $\frac{x + 6}{2} = -5$
d $\frac{x}{5} + 9 = -2$ **e** $3x + 8 = 10$ **f** $-2x + 3 = 17$
g $13.5 - x = -8.2$ **h** $3(x - 7) = 8$ **i** $\frac{6}{x} = -3$
j $\frac{30}{x} + 10 = 50$ **k** $5(x + 2) - 1 = -6$ **l** $\frac{x - 3}{4} + 6 = 7$
m $4(3x + 7) = 19$ **n** $\frac{2 - x}{3} - 4 = 5$ **o** $\frac{9x}{5} + 27 = 0$

PROBLEM SOLVING AND REASONING

17 Trent was sharing a bag of jellybeans equally with three of his friends and found that there were two left over. How many did each person receive if there were a total of 34 jellybeans in the bag?

- **a** Represent the unknown quantity in the problem with a pronumeral.
- **b** Use this pronumeral to write an equation to represent the problem.
- **c** Solve the equation using the balance method.
- **d** Write your answer to the problem.

18 Lily is saving to buy a pair of shoes that cost \$395. She is able to save \$70 per month. If she currently has \$115, in how many months can she buy the shoes?

- **a** Represent the unknown quantity in the problem with a pronumeral.
- **b** Use this pronumeral to write an equation to represent the problem.
- **c** Solve the equation using the balance method.
- **d** Write your answer to the problem.

19 For each problem, set up an equation and solve it using the balance method.

- **a** Jack bought three model planes online for a total cost of \$590, which included the delivery charge of \$35. What is the cost of each model plane?
- **b** Andrew and Rob scored a total of 35 goals in a basketball match. Rob scored seven more goals than Andrew. How many goals did Andrew score?
- **c** The perimeter of a rectangular playing field is 100 m. If the length is 12 m longer than the width, what are the dimensions of the playing field?
- **d** The cost of hiring a party venue is \$500. There is a \$26 per person charge for food. If Casey has a budget of \$2700 for the party, what is the maximum number of people that can attend?

CHALLENGE

20 Violetta cooked sausages for the school sausage sizzle. Each sausage was placed in bread with tomato sauce. Twenty of these were sold with mustard. Half of those left were sold with fried onions. If there were 18 sausages sold with fried onions, how many sausages did Violetta cook?

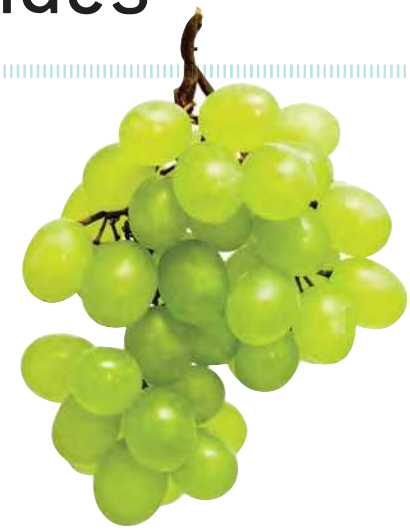
21 One angle in a triangle is 30°. The relationship between the other two angles is that one angle is twice the size of the other. Find the size of the largest angle. (Hint: What is the angle sum of a triangle?)

22 Investigate the 'solve' function of a calculator or other digital technology. Use this function to solve each equation in question **16** and compare your solutions.

Reflect

When solving an equation, how do you know the correct order to perform inverse operations?

3B Solving linear equations with the unknown on both sides



Start thinking!

Liam and Tanya have the same amount of money. Liam buys 5 kg of grapes and has \$2 left over. Tanya buys 3 kg of grapes and has \$8 left over.

1 If x represents the cost of 1 kg of grapes, which linear equation best suits this scenario?

A $5x + 8 = 3x + 2$ **B** $5x + 2 = 3x + 8$

C $8x + 3 = 2x + 5$ **D** $8x + 2 = 5x + 3$

2 What is different about this equation compared to the ones you worked with in the previous topic?

When there is a pronumeral term on both sides, the first step is to remove one of them by performing an inverse operation. You can then continue as before to find the solution.

3 Which pronumeral term do you think would be easier to remove?

4 Explain how you could remove this term by performing an inverse operation.

5 Copy and complete the first few steps shown at right when solving $5x + 2 = 3x + 8$.

$$5x + 2 = 3x + 8$$
$$5x + 2 - 3x = 3x + 8 - 3x$$
$$2x + \_\_ = 8$$

6 Continue on from the steps in question **5** to solve the equation.

7 What is the cost of 1 kg of grapes?

8 Write a sentence to explain what the first step should be when solving a linear equation that has a pronumeral term (or unknown) on both sides.

KEY IDEAS

- The first step in solving equations with a pronumeral term on each side is to remove the pronumeral term from one side. It can be removed by performing the inverse operation (adding or subtracting the term) on both sides so that an equivalent equation is formed.
- In some cases, it may be easier to swap the sides of an equation so that the pronumeral term with the larger coefficient is on the left side.
- A **literal equation** or **formula** contains two or more different pronumerals. The balance method can also be used to rearrange a literal equation or formula. Rearranging a formula is also known as **transforming a formula**.

EXERCISE 3B Solving linear equations with the unknown on both sides

EXAMPLE 3B-1 Solving an equation with an unknown on both sides

Solve each equation. **a** $8x - 9 = 5x + 6$ **b** $3x + 7 = -3 - 2x$

THINK	WRITE
a **1** Since there is a pronumeral term on each side, the first step is to remove one of them (choose $5x$ from the RS). Undo '+ $5x$' by performing the inverse operation of '− $5x$' on both sides and simplify.	**a** $8x - 9 = 5x + 6$ $8x - 9 - 5x = 5x + 6 - 5x$ $3x - 9 = 6$
2 Undo '− 9' by performing the inverse operation of '+ 9' on both sides and simplify.	$3x - 9 + 9 = 6 + 9$ $3x = 15$
3 Undo '× 3' by performing the inverse operation of '÷ 3' on both sides and simplify to obtain the solution.	$\frac{3x}{3} = \frac{15}{3}$ $x = 5$
4 Use substitution to check your solution.	LS = $8 \times 5 - 9 = 31$ RS = $5 \times 5 + 6 = 31$ So LS = RS.
b **1** Since there is a pronumeral term on each side, the first step is to remove one of them (choose $-2x$ from the RS). Undo '− $2x$' by performing the inverse operation of '+ $2x$' on both sides and simplify.	**b** $3x + 7 = -3 - 2x$ $3x + 7 + 2x = -3 - 2x + 2x$ $5x + 7 = -3$
2 Undo '+ 7' by performing the inverse operation of '− 7' on both sides and simplify.	$5x + 7 - 7 = -3 - 7$ $5x = -10$
3 Undo '× 5' by performing the inverse operation of '÷ 5' on both sides and simplify to obtain the solution.	$\frac{5x}{5} = \frac{-10}{5}$ $x = -2$
4 Use substitution to check your solution.	LS = $3 \times -2 + 7 = 1$ RS = $-3 - 2 \times -2 = 1$ So LS = RS.

1 Decide which operation should be performed first on both sides of the equation so that one of the pronumeral terms is removed.

a $8x + 10 = 3x + 5$ **b** $6x - 1 = 5x + 3$

c $4x - 7 = -2x + 11$ **d** $7x + 9 = -13 - 4x$

NOTE Discuss with a classmate whether you need to add or subtract a pronumeral term.

UNDERSTANDING AND FLUENCY

2 Copy and complete the steps shown to solve each equation. Remember to check your solutions.

a
$$
\begin{aligned}
4x + 1 &= x + 7\\
4x + 1 - \_\_\_ &= x + 7 - x\\
3x + 1 &= \_\_\_\\
3x + 1 - \_\_\_ &= \_\_\_ - \_\_\_\\
3x &= \_\_\_\\
\frac{3x}{\_\_\_} &= \frac{\_\_\_}{3}\\
x &= \_\_\_
\end{aligned}
$$

b
$$
\begin{aligned}
3x + 10 &= -2x - 5\\
3x + 10 + 2x &= -2x - 5 + \_\_\_\\
5x + 10 &= \_\_\_\\
5x + 10 - \_\_\_ &= -5 - \_\_\_\\
5x &= \_\_\_\\
\frac{5x}{\_\_\_} &= \frac{\_\_\_}{5}\\
x &= \_\_\_
\end{aligned}
$$

3 Solve each equation. Show all working.

a $6x + 5 = 4x + 9$ **b** $3x - 11 = x + 3$ **c** $8x + 10 = 3x + 5$
d $6x - 1 = 5x + 3$ **e** $3x + 4 = -2x + 9$ **f** $4x + 13 = -3x - 8$
g $7x - 6 = 6x - 1$ **h** $5x + 13 = 2x - 5$ **i** $5x - 7 = x + 5$
j $7x + 9 = -4x - 13$ **k** $2x + 5 = x - 3$ **l** $5x - 7 = 11 - x$

EXAMPLE 3B-2 Solving an equation containing brackets with an unknown on both sides

Solve $3(2x + 1) = -4x - 17$.

THINK	WRITE
1 Expand the expression on the left side to remove brackets.	$3(2x + 1) = -4x - 17$ $6x + 3 = -4x - 17$
2 Remove one of the pronumeral terms (choose $-4x$ from the RS). Perform the inverse operation of '+ $4x$' on both sides.	$6x + 3 + 4x = -4x - 17 + 4x$ $10x + 3 = -17$
3 Undo '+ 3' by performing '− 3' on both sides and simplify.	$10x + 3 - 3 = -17 - 3$ $10x = -20$
4 Undo '× 10' by performing '÷ 10' on both sides and simplify to obtain the solution.	$\frac{10x}{10} = \frac{-20}{10}$ $x = -2$
5 Check your solution.	LS $= 6 \times -2 + 3 = -9$ RS $= -4 \times -2 - 17 = -9$ So LS = RS.

4 Copy and complete the steps shown to solve each equation.

a
$$
\begin{aligned}
2(x + 4) &= x + 20\\
2x + \_\_\_ &= x + 20\\
2x + 8 - \_\_\_ &= x + 20 - x\\
x + 8 &= \_\_\_\\
x + 8 - \_\_\_ &= \_\_\_ - \_\_\_\\
x &= \_\_\_
\end{aligned}
$$

b
$$
\begin{aligned}
3(x - 1) &= 2(x + 1)\\
3x - 3 &= 2x + \_\_\_\\
3x - 3 - \_\_\_ &= 2x + 2 - 2x\\
x - 3 &= \_\_\_\\
x - 3 + \_\_\_ &= \_\_\_ + \_\_\_\\
x &= \_\_\_
\end{aligned}
$$

UNDERSTANDING AND FLUENCY

5 Solve each equation.

a $3(x-2)=2x-1$ **b** $7x+8=3(x-8)$ **c** $5x+17=4(x+5)$

d $4(x+3)=-2x-24$ **e** $2(3x-4)=5x-1$ **f** $11x+16=4(2x+7)$

g $8(x+2)=-3x+5$ **h** $3(2x-1)=-x+11$ **i** $4(x+5)=3(x+2)$

j $7(x-2)=2(x+3)$ **k** $6(x+1)=3(x-8)$ **l** $5(x+9)=-3(x-7)$

6 Solve each equation.

a $3x+7=6x-5$ **b** $-2x-9=4x+3$

c $9x-4=10x-11$ **d** $-x+10=7x-22$

e $2x-1=5(x-2)$ **f** $3(x-2)=8x-1$

g $4(x+3)=5(x+1)$ **h** $6(x+2)=11(x-3)$

> **NOTE** You may find it easier to swap the sides of the equation so that the larger pronumeral term is on the left side.

7 Consider the equation $\frac{2x+5}{3}=\frac{x-4}{3}$.

a How might you solve this equation using the balance method?

b Kristina suggests multiplying both sides by 3 as the first step. Try it. What equivalent equation do you obtain?

c Solve the equivalent equation obtained in part **b**. Use substitution to check that your solution is correct.

EXAMPLE 3B-3 Solving an equation with algebraic fractions on both sides (same denominators)

Solve $\frac{4x-3}{5}=\frac{2x+9}{5}$.

THINK	WRITE
1 Multiply both sides by 5 to obtain an equivalent equation without denominators.	$\frac{4x-3}{5}=\frac{2x+9}{5}$ $\frac{4x-3}{5}\times 5=\frac{2x+9}{5}\times 5$ $4x-3=2x+9$
2 Undo '+ 2*x*' by performing '− 2*x*' on both sides and simplify.	$4x-3-2x=2x+9-2x$ $2x-3=9$
3 Undo '− 3' by performing '+ 3' on both sides and simplify.	$2x-3+3=9+3$ $2x=12$
4 Undo '× 2' by performing '÷ 2' on both sides and simplify to obtain the solution.	$\frac{2x}{2}=\frac{12}{2}$ $x=6$
5 Check your solution.	$\text{LS}=\frac{4\times 6-3}{5}=\frac{21}{5}$ $\text{RS}=\frac{2\times 6+9}{5}=\frac{21}{5}$

8 Solve each equation.

a $\frac{3x-4}{7}=\frac{x+6}{7}$ **b** $\frac{5x+2}{4}=\frac{2x-7}{4}$ **c** $\frac{2x+3}{11}=\frac{15-4x}{11}$ **d** $\frac{4(2x+1)}{9}=\frac{x-17}{9}$

UNDERSTANDING AND FLUENCY

9 The balance method can also be used to rearrange a literal equation or formula that contains two or more different pronumerals.

a Copy and complete the steps shown to obtain x by itself on the left side of each formula. This is known as making x the subject of the formula.

i
$$\begin{aligned} x + b &= a \\ x + b - b &= a - \underline{\quad\quad} \\ x &= a - b \end{aligned}$$

ii
$$\begin{aligned} k &= 2x - m \\ 2x - m &= k \\ 2x - m + \underline{\quad\quad} &= k + \underline{\quad\quad} \\ 2x &= \underline{\quad\quad} \\ \frac{2x}{2} &= \frac{k+m}{\underline{\quad}} \\ x &= \frac{k+m}{2} \end{aligned}$$

iii
$$\begin{aligned} \frac{x+d}{g} &= f \\ \frac{x+d}{g} \times g &= f \times \underline{\quad\quad} \\ x + d &= \underline{\quad\quad} \\ x + d - \underline{\quad\quad} &= fg - \underline{\quad\quad} \\ x &= \underline{\quad\quad} \end{aligned}$$

b In your own words, describe what it means to make a variable (or pronumeral) the subject of a formula.

c What was the subject of each formula in part **a** before it was rearranged?

10 Rearrange each formula to make x the subject of the formula.

a $x + p = k$ **b** $d = x - c$ **c** $ax = b$

d $m + 2x = n$ **e** $y = 5x + 7$ **f** $\frac{x+a}{c} = d$

g $4x - 3y = 1$ **h** $\frac{x}{n} + m = k$ **i** $b = \frac{d}{x}$

j $5x - t = 2x + p$ **k** $3(x + a) = b$ **l** $\frac{x}{u} - y = t$

m $\frac{d}{x} + f = g$ **n** $m = \frac{nx + k}{3}$ **o** $x + p = k - 3x$

PROBLEM SOLVING AND REASONING

11 The number of mixed mini-chocolate bars in a bag is unknown. However, Lola can fill the family chocolate barrel with nine more than the contents of two bags, or four less than the contents of three bags.

a If there are n chocolate bars in a bag, which equation best suits this situation?

A $2n - 9 = 3n - 4$ **B** $2n - 9 = 3n + 4$ **C** $2n + 9 = 3n - 4$ **D** $2n + 9 = 3n + 4$

b Solve the equation to find the number of chocolate bars in a bag.

12 Sarah and Josh have the same amount of money. Sarah buys seven sushi rolls and has \$1.50 left over. Josh buys four sushi rolls and has \$12 left over.

a If x represents the cost of one sushi roll, which equation fits this situation?

A $7x + 150 = 4x + 12$ **B** $7x + 1.5 = 4x + 12$

C $7x - 1.5 = 4x - 12$ **D** $7x - 150 = 4x - 12$

b Solve the equation to find the cost of one sushi roll.

13 At the cinema, the cost of five boxes of popcorn and two choc-tops is the same as the cost of three boxes of popcorn and seven choc-tops. The cost of a choc-top is \$4.50 but you don't know the cost of the popcorn. Let p be the cost of a box of popcorn.

a Write an equation to represent this situation.

b Solve the equation to find the cost of a box of popcorn.

PROBLEM SOLVING AND REASONING

14 The formula $s = \frac{d}{t}$ links the average speed (s), the distance travelled (d) and the time taken (t).

a Substitute values for d and t into the formula to calculate the average speed if 20 m is travelled in 5 s.

b Calculate the distance travelled by a cyclist moving at an average speed of 10 m/s for 60 s. (Hint: substitute the values for s and t into the formula and find d.)

c Rearrange the formula to make d the subject.

d Repeat part **b** but this time use the formula obtained in part **c**.

e Which method (part **b** or part **d**) did you find quicker or easier to use to calculate the value for d? Would your answer be different if you were required to calculate d for five different sets of s and t values? Explain.

f Rearrange the formula to make t the subject.

g How long would it take each snail to complete a distance of 35 cm if they travelled at the average speeds given below? Write each answer to the nearest second.

i 5 cm/min

ii 7 cm/min

iii 6 cm/min

iv 6.5 cm/min

15 The formula $F = \frac{9C}{5} + 32$ can be used to calculate the temperature in °F (F) if you know the temperature in °C (C).

a What is the temperature in °F when it is 25°C? (Hint: substitute $C = 25$ into the formula to calculate F.)

b What is the temperature in °C when it is 88°F? (Hint: substitute $F = 88$ into the formula and solve the resulting equation to find C.)

c Rearrange the formula to make C the subject.

d Calculate C for each of these values of F.

i 104 **ii** 68 **iii** 113 **iv** 32

CHALLENGE

16 The sum of three consecutive integers is 13 more than the smallest of the three numbers. Write an equation to solve to find the three numbers. (Hint: let the smallest number be x.)

17 Transform each formula to make the pronumeral shown in brackets the subject of the formula.

a $A = lw \quad (w)$ **b** $P = 2l + 2w \quad (l)$

c $v = u + at \quad (u)$ **d** $A = \frac{bh}{2} \quad (h)$

e $v = u + at \quad (t)$ **f** $I = PRT \quad (R)$

g $P = 2\pi r \quad (r)$ **h** $A = \frac{h(a + b)}{2} \quad (a)$

Reflect

How are equivalent equations useful when working with equations?

3C Plotting linear graphs

Start thinking!

Relationships between two variables can be represented by a table of values. Consider the three relationships shown at right.

A

x	−3	−2	−1	0	1	2	3
y	4	3	2	1	0	−1	−2

B

x	−3	−2	−1	0	1	2	3
y	8	3	0	−1	0	3	8

C

x	−3	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1	2

1 What is the **independent variable** in each case? (Hint: the independent variable is generally shown first in a table of values.)

2 What is the **dependent variable** in each case?

3 For each table, list the coordinates of the seven points shown.

You can also graph these relationships on a **Cartesian plane**.

4 Which variable is shown along the horizontal axis? Is this the independent or dependent variable?

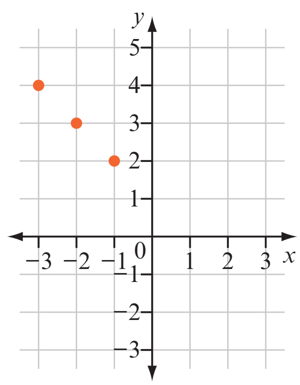
5 Plot the points for each relationship on a Cartesian plane. The first one has been started for you.

To indicate that there are many points that describe the relationship, you join the points with a smooth line.

6 Join the points you have plotted for each graph with a smooth line and describe the trend you see.

7 Use the trend to help you list the coordinates for another three points for each relationship.

If the line is straight, it shows a **linear relationship** between the variables. Otherwise, it is a **non-linear relationship**.



8 Which graph/s show:

a a linear relationship? **b** a non-linear relationship?

These relationships can also be written as a rule or formula.

9 Match each of these rules with the appropriate table of values. Write the appropriate rule with each graph.

a $y = x - 1$ **b** $y = 1 - x$ **c** $y = x^2 - 1$

10 Discuss with a classmate how you might recognise whether a relationship is linear by looking at the rule. (Hint: look at the power of each variable in the rule.)

KEY IDEAS

- The relationship between two variables can be graphed on a Cartesian plane.
- If the plotted points form a straight line, it shows a linear relationship.
- Creating a table of values helps work out the coordinates of points to be plotted.
- A linear graph has features such as **gradient**, ***x*-intercept** and ***y*-intercept**.
  - The gradient describes the slope of the graph.
  - The x-intercept is where the graph crosses the x-axis.
  - The y-intercept is where the graph crosses the y-axis.

EXERCISE 3C Plotting linear graphs

EXAMPLE 3C-1 Using a table of values to plot a graph

Use the table of values to plot the graph of the relationship between x and y.

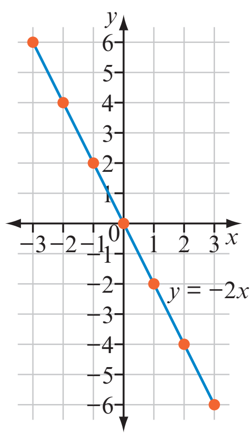
x	−3	−2	−1	0	1	2	3
y	6	4	2	0	−2	−4	−6

THINK

1 Use grid paper to draw a Cartesian plane with a scale from −3 to 3 on the horizontal axis and from at least −6 to 6 on the vertical axis. Plot each point.

2 Since there are many points that describe this relationship, join the points with a smooth line.

WRITE



UNDERSTANDING AND FLUENCY

1 a Use the table of values to plot the graph of each relationship between x and y.

i

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4

ii

x	−3	−2	−1	0	1	2	3
y	11	6	3	2	3	6	11

iii

x	−3	−2	−1	0	1	2	3
y	10	9	8	7	6	5	4

iv

x	−3	−2	−1	0	1	2	3
y	−28	−9	−2	−1	0	7	26

b Which of these graphs show a linear relationship?

2 For the linear relationship described by the rule $y = 3x - 5$:

a copy and complete the table

b plot the linear graph.

x	−1	0	1	2	3
y	−8			1	

EXAMPLE 3C-2 Plotting a linear graph

Plot the graph of $y = 2x - 1$ by first completing a table for x values from -3 to 3.

THINK

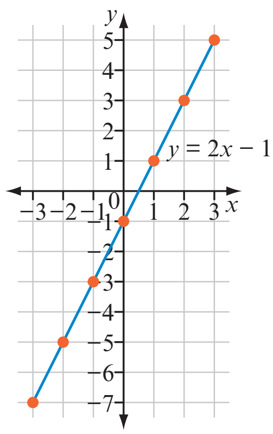
1 Create a table of values. Substitute each x value into the equation to find the corresponding y value. (For $x = -3$, $y = 2 \times -3 - 1 = -6 - 1 = -7$.)

2 Use grid paper to draw a Cartesian plane with a scale from -3 to 3 on the horizontal axis and from at least -7 to 5 on the vertical axis. Plot each point.

3 Since there are many points that describe this relationship, join the points with a smooth line.

WRITE

x	−3	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3	5



UNDERSTANDING AND FLUENCY

3 Plot the graph of each linear relationship by first completing a table for x values from -3 to 3.

a $y = x + 2$ **b** $y = x - 4$
c $y = 3 - x$ **d** $y = 2 - x$
e $y = -x - 3$ **f** $y = 4x$
g $y = 2x + 1$ **h** $y = 3x - 2$
i $y = 4 - 2x$ **j** $y = 8 - 3x$

4 Produce each graph in questions **2** and **3** using digital technology.

5 A feature of a linear graph is its slope or gradient. For example, the linear graph drawn in Example 3C-1 on page 117 has a negative gradient and the linear graph drawn in Example 3C-2 has a positive gradient. Copy and complete these statements using the words zero, positive or negative.

a A line has a ______________ gradient if it slopes upwards from left to right.
b A line has a ______________ gradient if it slopes downwards from left to right.
c A line has a ______________ gradient if it is horizontal.

6 Decide whether the gradient of each line you drew for question **3** is positive (P) or negative (N).

EXAMPLE 3C-3 Identifying the x-intercept and y-intercept from a linear graph

List the x-intercept and y-intercept for the linear graph in Example 3C-2 opposite.

THINK	WRITE
1 Locate the x-intercept. This is where the graph crosses the x-axis.	x-intercept is $\frac{1}{2}$. Coordinates of the x-intercept are $(\frac{1}{2}, 0)$.
2 Locate the y-intercept. This is where the graph crosses the y-axis.	y-intercept is -1. Coordinates of the y-intercept are $(0, -1)$.

UNDERSTANDING AND FLUENCY

7 List the x-intercept and the y-intercept for each graph you have drawn for question 3.

8 a Plot the graph of each relationship by using the table of values.

i $y = 3$

x	−2	−1	0	1	2
y	3	3	3	3	3

ii $y = -2$

x	−2	−1	0	1	2
y	−2	−2	−2	−2	−2

iii $y = 6$

x	−2	−1	0	1	2
y	6	6	6	6	6

b Are these relationships linear? Explain.
c What do you notice about the rule and its matching graph?
d Describe the gradient of each graph as positive, negative or zero.
e What is the y-intercept of each graph?
f Can you identify the x-intercept for each graph? Explain.

9 a Plot the graph of each relationship by using the table of values.

i $x = 2$

x	2	2	2	2	2
y	−2	−1	0	1	2

ii $x = -5$

x	−5	−5	−5	−5	−5
y	−2	−1	0	1	2

iii $x = 0$

x	0	0	0	0	0
y	−2	−1	0	1	2

b Are these relationships linear? Explain.
c What do you notice about the rule and its matching graph?
d Can you describe the gradient of each graph as positive, negative or zero? Discuss this with a classmate. Why do you think the gradient is said to be undefined?
e What is the x-intercept of each graph?
f Can you identify the y-intercept for each graph? Explain.

10 Without using a table of values, draw the graph of each relationship.

a $x = 4$
b $y = 1$
c $y = -5$
d $x = -3$
e $x = 7$
f $y = 0$

NOTE Use your answers to questions 8 and 9 to guide you.

11 **a** Plot the graph of each linear relationship on the same Cartesian plane. Produce a table of values for each rule to help you.

i $y = x$ **ii** $y = 2x$

iii $y = 3x$ **iv** $y = 4x$

b What is the same and what is different about each of the four graphs you plotted?

c Without using a table of values, predict where you would draw the graph of:

i $y = 5x$ **ii** $y = 3.5x$.

12 **a** Plot the graph of each linear relationship on the same Cartesian plane. Produce a table of values for each rule to help you.

i $y = -x$ **ii** $y = -2x$

iii $y = -3x$ **iv** $y = -4x$

b What is the same and what is different about each of the four graphs you plotted?

c Without using a table of values, predict where you would draw the graph of:

i $y = -5x$ **ii** $y = -1.5x$.

13 Look at the graphs you drew for questions **11a** and **12a**.

a Which graph has the steepest positive slope?

b Which graph has the steepest negative slope?

c Which graph would have the steeper slope:

i $y = 10x$ or $y = 20x$?

ii $y = -15x$ or $y = -25x$?

14 A bus is hired for a school trip to the snow. Each person is charged $40 towards the bus hire with the school contributing $250.

a Explain how you can obtain the rule $m = 40n + 250$ to represent this situation, where m is the total amount of money collected for n people going on the trip.

b Which is the independent variable? Which axis of a Cartesian plane will this variable be shown on?

PROBLEM SOLVING AND REASONING

c Plot a graph of this relationship. Show a scale from 0 to 30 along the horizontal axis.

d Is the relationship linear? Explain.

e Use the graph to find the total amount of money collected if 20 people go on the trip.

f Use the graph to determine how many people need to go on the trip to collect a total of \$850.

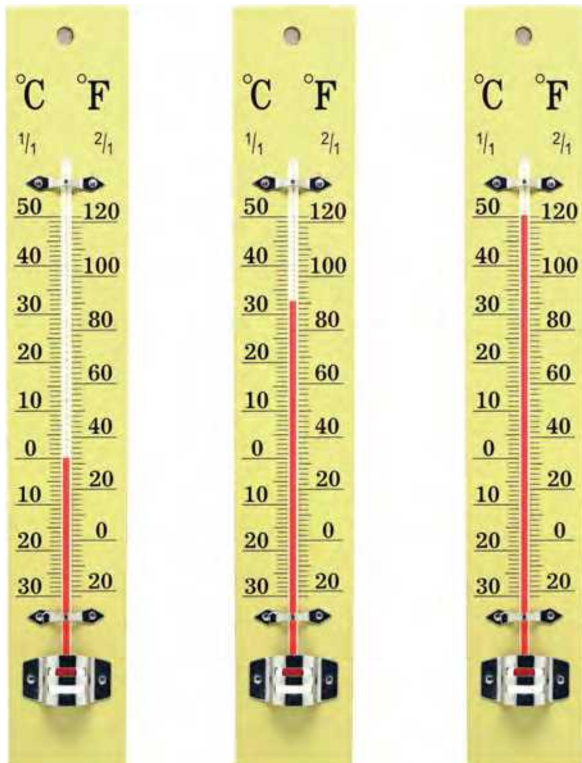
g The hire cost of the bus is \$1300. Use the graph to determine the minimum number of people who need to go on the trip to cover the hire cost.

15 The rule $F = 1.8C + 32$ describes the relationship between temperatures in °C (C) and temperatures in °F (F).

a Plot the graph of this relationship. Show a scale from −50 to 50 along the horizontal axis.

b Use the graph to find the temperature in °F for 30°C.

c Use the graph to find the temperature in °C for −22°F.



CHALLENGE

16 Use the graph in question **15** to find where the temperature in °C has the same numerical value as the matching temperature in °F. (Hint: locate where $C = F$ on the graph.)

17 Most of the linear graphs you have plotted have rules that start as $y = \ldots$ This makes it easy to produce a table of values by substituting values for x. This is not always the case. Consider the rule $4x + y = 6$.

a Rearrange the rule to make y the subject. (Hint: subtract $4x$ from both sides.)

b Complete a table for x values from −2 to 2.

c Plot the graph of $4x + y = 6$ and label with its rule.

18 Plot the graph of each linear relationship after first rearranging the rule to make y the subject and then completing a table of values.

a $x + y = 5$ **b** $x + y = -1$

c $2x + y = 3$ **d** $y - x = 4$

e $y - 3x = 1$ **f** $x + y + 2 = 0$

g $x - y = 3$ **h** $4x - y = -1$

i $3x - y = 0$ **j** $6x + 2y = 8$

Reflect

How can you identify if a graph is showing a linear relationship?

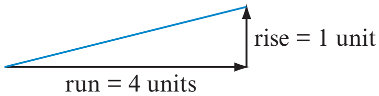
3D Gradient and intercepts

Start thinking!

You may have seen signs along the side of a road indicating a steep section of road.

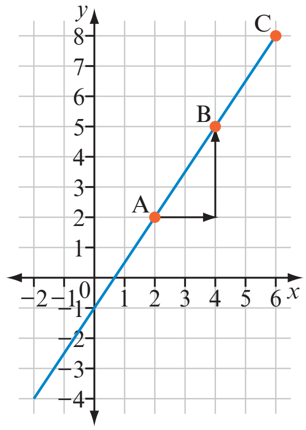
Sometimes, the signs indicate the gradient or slope of a road using numbers, such as a gradient of 1 in 4.

This means that the road rises vertically a distance of 1 m for every 4 m in horizontal distance, or has a vertical 'rise' of 1 m for a horizontal 'run' of 4 m.



Another way of describing this is as a gradient of $\frac{1}{4}$ (or 25%).

The gradient of a linear graph can also be described in this way.



1 Copy the linear graph shown at right on to grid or graph paper.

2 Does the line have a positive or negative gradient?

A right-angled triangle has been formed using two points on the line.

3 What are the coordinates of:

a point A? **b** point B?

4 Use the triangle and the grid to count the number of units that form the horizontal run. We always measure horizontally from left to right, so the run will always be a positive number.

5 Use the triangle and the grid to count the number of units that form the vertical rise. The distance is measured vertically as you move from left to right along the line segment. In this case you move up, so the rise is a positive number.

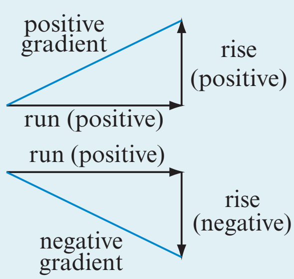
6 Copy and complete: The line segment between A and B rises ___ units for a run of ___ units.

7 Write the gradient of the line segment between points A and B as a fraction.

8 Is there a formula involving 'rise' and 'run' that you can use to calculate the gradient of any straight line? Discuss this with a classmate and write a possible formula.

KEY IDEAS

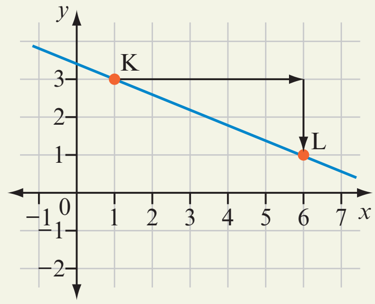
- The gradient is a numerical value that describes the slope of a graph.
- It can be found by comparing the vertical 'rise' with the horizontal 'run'. You can use the formula: gradient = $\frac{\text{rise}}{\text{run}}$.
- The formula for the gradient (m) of a line segment between two points (x_1, y_1) and (x_2, y_2) is: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- The x-intercept is where the graph crosses the x-axis.
- The y-intercept is where the graph crosses the y-axis.



EXERCISE 3D Gradient and intercepts

EXAMPLE 3D-1 Finding rise and run to calculate gradient

For the linear graph shown, find the rise and run and calculate the gradient.



THINK

1 Identify the horizontal run.

2 Identify the vertical rise. Moving down means that the rise is negative.

3 Use the rise and the run to calculate the gradient.

WRITE

run = 5

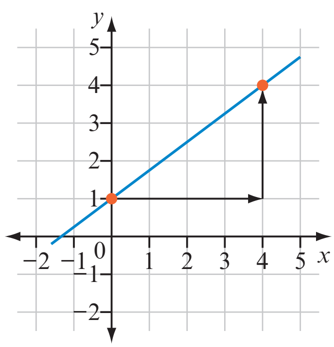
rise = −2

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{-2}{5} = -\frac{2}{5}$$

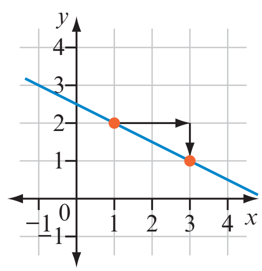
UNDERSTANDING AND FLUENCY

1 For each line shown, find the rise and run and calculate the gradient.

a



b



2 Consider points A and C on the line from 3D Start thinking! opposite. Form a right-angled triangle.

a What is the horizontal run for this line segment?

b What is the vertical rise for this line segment?

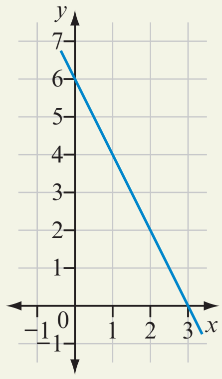
c Write the gradient of the line segment between points A and C as a fraction. Simplify if possible.

d Compare your answer to the gradient for the line segment between points A and B.

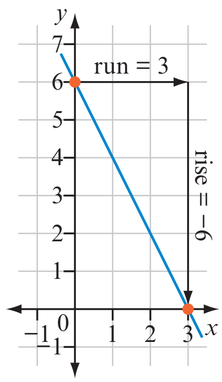
EXAMPLE 3D-2 Finding gradient, x-intercept and y-intercept from a linear graph

For the linear graph shown, find:

a the gradient
b the x-intercept
c the y-intercept.



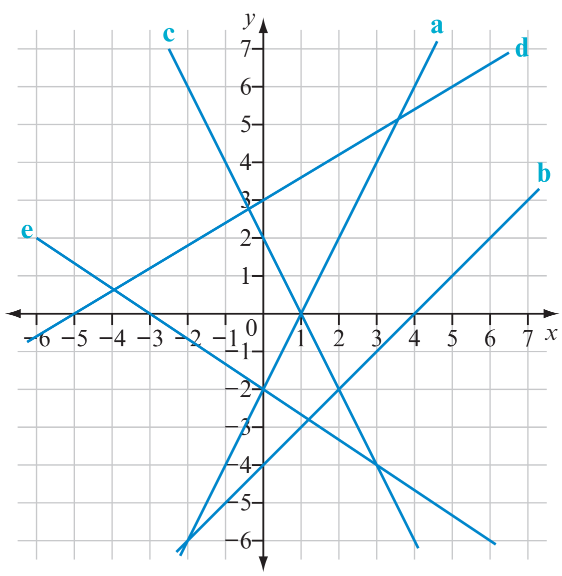
THINK	WRITE
a **1** Locate two points on the line that have whole number coordinates. Form a right-angled triangle using these two points as **vertices**.	**a** Choose (0, 6) and (3, 0).
2 Identify the horizontal run.	run = 3
3 Identify the vertical rise.	rise = −6
4 Calculate the gradient.	$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{-6}{3} = -2$
b Locate the x-intercept. This is where the line crosses the x-axis.	**b** x-intercept is 3.
c Locate the y-intercept. This is where the line crosses the y-axis.	**c** y-intercept is 6.



UNDERSTANDING AND FLUENCY

3 For the linear graphs shown, find:

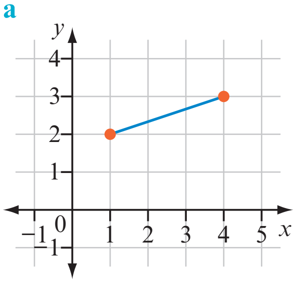
i the gradient
ii the x-intercept
iii the y-intercept.



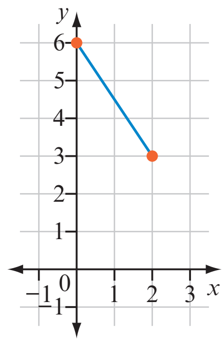
UNDERSTANDING AND FLUENCY

4 Find the gradient of each line segment.

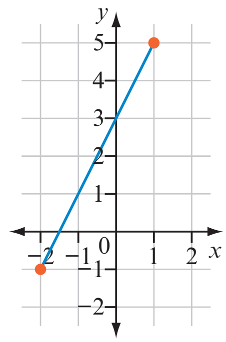
a



b



c



5 For each pair of points listed below:

- **i** plot the points on a Cartesian plane and join them with a straight line to form a line segment
- **ii** find the gradient of the line segment.

a (2, 3) and (6, 8) **b** (1, 2) and (3, 6)

c (3, 7) and (4, 4) **d** (−4, 5) and (2, −3)

6 Can you work out the gradient of a line segment directly from the coordinates of the end points without plotting them first? Discuss this with a classmate.

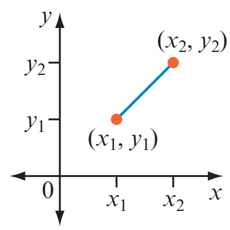
7 Consider a pair of points with coordinates (x_1, y_1) and (x_2, y_2).

a Explain why the horizontal run can be written as $x_2 - x_1$.

b Write an expression for the vertical rise using y_1 and y_2.

c Use your answers to write an expression for the gradient.

d Check that this expression works for each pair of points in question **5**.



EXAMPLE 3D-3 Using a formula to calculate the gradient of a line segment between two points

Use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to calculate the gradient of the line segment joining (3, 2) and (9, 5).

THINK	WRITE
1 Identify (x_1, y_1) and (x_2, y_2). It doesn't matter which point is matched to each one.	Let (x_1, y_1) be (3, 2) and (x_2, y_2) be (9, 5).
2 Substitute the x- and y-coordinates into the gradient formula.	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{5-2}{9-3}$
3 Calculate the gradient and simplify.	$= \dfrac{3}{6}$ $= \dfrac{1}{2}$

PROBLEM SOLVING AND REASONING

8 Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the gradient of the line segment joining each pair of points.

a (2, 4) and (5, 6) **b** (3, 1) and (7, 4)

c (1, 2) and (4, 8) **d** (4, 6) and (7, 5)

e (−2, 5) and (3, 7) **f** (0, 8) and (1, 5)

g (−1, −6) and (2, −1) **h** (0, 5) and (1, 0)

i (−4, −3) and (−1, 4) **j** (−3, 3) and (−1, −5)

k (−5, −6) and (−4, −8) **l** (−9, 3) and (−6, 3)

9 Kane calculated the gradient of the line segment between (2, 8) and (6, 5), as shown at right.

Explain why his answer is incorrect.

Work out the correct value for the gradient.

Kane

$$m = \frac{8-5}{6-2}$$
$$= \frac{3}{4}$$

10 Todd and Bridget calculated the gradient of the line segment between (−1, 9) and (3, −3) as shown at right.

Explain why both students have produced correct calculations.

Todd

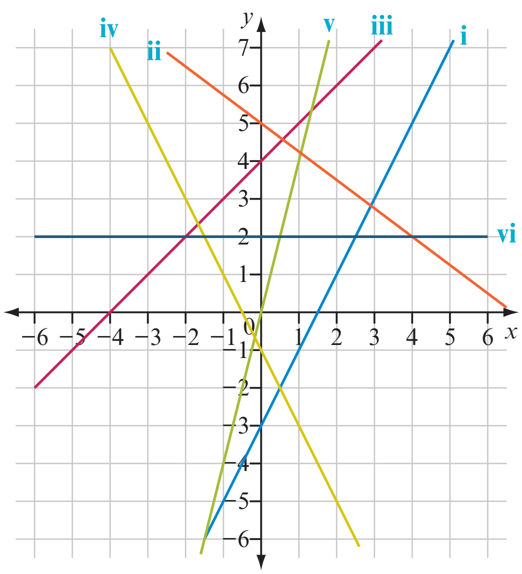
$$m = \frac{-3-9}{3-(-1)}$$
$$= \frac{-12}{4}$$
$$= -3$$

Bridget

$$m = \frac{9-(-3)}{-1-3}$$
$$= \frac{12}{-4}$$
$$= -3$$

11 When finding the gradient of the line segment between two points, does it matter which point is matched with (x_1, y_1) and which is matched with (x_2, y_2)? Use an example to help with your explanation.

12 Consider the linear graphs shown on this Cartesian plane.



a Copy and complete this table by calculating the gradient and identifying the y-intercept of each line.

	Rule	Gradient	y-intercept
i	$y = 2x - 3$		
ii	$y = -\frac{3}{4}x + 5$		
iii	$y = x + 4$		
iv	$y = -2x - 1$		
v	$y = 4x$		
vi	$y = 2$		

b Can you see a pattern in the table that helps identify the gradient and the y-intercept from the rule? Discuss with a classmate.

c Predict the gradient and y-intercept for a linear graph with the rule $y = 6x + 4$.

PROBLEM SOLVING AND REASONING

13 Gradient is often represented by m. Similarly, the y-intercept is represented by c. Use your results to question **12** to explain why the general form of the rule for a linear graph can be written as $y = mx + c$.

14 Use the general form of the rule for a linear graph ($y = mx + c$) to identify m and c for each of these rules.

a $y = 2x + 5$ **b** $y = 4x + 1$ **c** $y = -3x + 7$ **d** $y = -5x - 3$
e $y = x - 6$ **f** $y = 1 - x$ **g** $y = \frac{4}{3}x + 2$ **h** $y = \frac{1}{2}x - 8$
i $y = -\frac{4}{3}x + \frac{1}{4}$ **j** $y = 9 + 3x$ **k** $y = 2 - 7x$ **l** $y = 5 - \frac{2}{5}x$

15 You can identify the vertical rise and the horizontal run from the gradient if the value is written as a fraction. For example, a gradient of 3 can be written as $\frac{3}{1}$, so there is a rise of 3 for a run of 1. For negative gradients, you write the fraction with a negative numerator and a positive denominator. A gradient of $-\frac{2}{3}$ is written as $\frac{-2}{3}$, so the rise is -2 for a run of 3.

For each gradient:

i write the gradient as a fraction with a positive denominator
ii identify the rise
iii identify the run.

a 2 **b** 5 **c** -3 **d** 7 **e** -4 **f** -1
g $\frac{3}{2}$ **h** $\frac{5}{4}$ **i** $\frac{-3}{8}$ **j** $-\frac{2}{7}$ **k** $-\frac{4}{9}$ **l** $-\frac{1}{5}$

16 Why is the denominator always a positive number when writing a gradient in fractional form to identify the rise and the run?

17 Use your answers to question **14** to identify for each linear graph:

i the rise **ii** the run.

CHALLENGE

18 **a** Identify the coordinates of two points that can be joined with a straight line to form a line segment with a gradient of 4. Explain how you were able to do this.

b Repeat part **a** with these gradients.

i -2 **ii** $\frac{2}{3}$ **iii** $-\frac{5}{4}$

19 For each rule, identify:

i the gradient **ii** the y-intercept.

a $2x + y = 7$ **b** $-5x + y = 1$
c $3x + 2y = 6$ **d** $4x + 3y = -9$ **e** $-8x + 4y = 3$ **f** $6x - 2y = 10$
g $7x - y = 2$ **h** $-x - y = -7$ **i** $x + 2y + 4 = 0$ **j** $5y - 2x = -9$

> **NOTE** You will first need to rearrange the rule to make y the subject.

20 On a Cartesian plane, rule a linear graph with each of these gradients.

a 2 **b** $\frac{3}{2}$ **c** -5 **d** $-\frac{3}{4}$

> **Reflect**
>
> How can you identify the gradient and the y-intercept from the rule for a linear graph?

3E Sketching linear graphs using gradient and *y*-intercept

Start thinking!

Instead of plotting a linear relationship, you can produce a **sketch graph**. This means that you use your knowledge of what a linear graph will look like and given information to find two points. Once you plot these two points, you can rule a line through them to produce the required sketch of the linear graph.

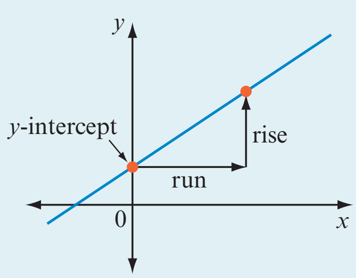
One way of sketching a linear graph is to use the **gradient–intercept method**. This provides a way of finding the two points you need to plot.

Consider $y = \frac{2}{3}x + 1$.

1 Compare this to $y = mx + c$ and identify m and c.
2 What are the coordinates of the y-intercept? This is our first point.
3 What is the gradient?
4 Use your answer to question **3** to identify the rise and the run.
5 Draw a Cartesian plane and plot your first point at the y-intercept.
6 Starting at your first point, use the rise and the run to locate the second point on the Cartesian plane. Explain how you were able to do this.
7 What are the coordinates of the second point you have located?
8 Rule a line through the two points you have plotted. Label your sketch graph with the rule.
9 In your own words, explain how to use the gradient–intercept method to sketch a linear graph.

KEY IDEAS

- To sketch a linear graph, you require a minimum of two points. Once these two points are plotted, you can rule a straight line through them.
- One way to identify these two points is to use the y-intercept and the gradient.
- In the gradient–intercept method, a point is plotted at the y-intercept and then the rise and run (identified from the gradient) is used to locate the second point.
- The general rule for a linear graph is $y = mx + c$, where m is the gradient and c is the y-intercept.



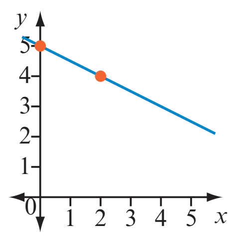
EXERCISE 3E Sketching linear graphs using gradient and y-intercept

EXAMPLE 3E-1 Sketching a linear graph given the y-intercept and gradient

A linear relationship has a y-intercept of 5 and a gradient of $-\frac{1}{2}$.

a Identify the rise and run from the gradient.

b Draw a sketch of the linear relationship using the gradient–intercept method.

THINK	WRITE
a 1 Write the gradient as a fraction with a positive denominator.	**a** $m = -\frac{1}{2}$ $= \frac{-1}{2}$
2 Write the rise (numerator of fraction) and the run (denominator of fraction).	rise $= -1$ run $= 2$
b 1 Write the coordinates of the y-intercept.	**b** y-intercept at (0, 5)
2 Draw a Cartesian plane and plot a point at the y-intercept. 3 From this point, use the rise and the run to locate a second point (move 1 unit down and 2 units right). The second point is at (2, 4). 4 Rule a straight line through the two plotted points.	

UNDERSTANDING AND FLUENCY

1 Use the information provided to:

a identify the rise and run from the gradient for each linear graph

b draw a sketch of each linear relationship using the gradient–intercept method.

i y-intercept is -1, gradient is $\frac{3}{2}$

ii y-intercept is 3, gradient is $-\frac{1}{4}$

iii y-intercept is 0, gradient is 2

2 List the coordinates of the two points you have plotted for each sketch graph in question **1**.

3 a Identify c (y-intercept) and m (gradient) for each linear rule.

i $y = \frac{3}{4}x + 1$ ii $y = 4x - 3$ iii $y = -\frac{1}{5}x - 1$

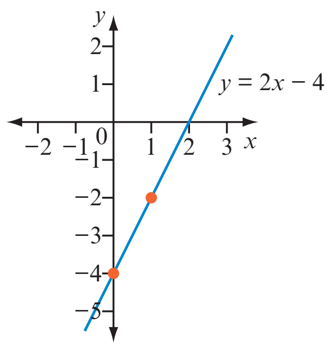
iv $y = -3x + 2$ v $y = 5x$ vi $y = -\frac{1}{2}x + 5$

b Use your answers to part **a** to identify the rise and run for each rule.

c Sketch the graph of each linear rule.

EXAMPLE 3E-2 Using the gradient–intercept method to sketch a linear graph

Sketch the graph of $y = 2x - 4$ using the gradient–intercept method.

THINK	WRITE
1 Compare the rule to the general formula for a linear graph, $y = mx + c$. Identify m (gradient) and c (y-intercept).	$y = 2x - 4$ $m = 2$, $c = -4$
2 Write the coordinates of the y-intercept.	y-intercept at $(0, -4)$.
3 Write the gradient as a fraction and identify the rise and the run.	gradient $= 2 = \frac{2}{1}$ rise $= 2$, run $= 1$
4 Plot a point at the y-intercept.	
5 From this point, use the rise and the run to locate a second point (move 2 units up and 1 unit right). The second point is at $(1, -2)$.	
6 Rule a straight line through the two plotted points and label with the rule.	

UNDERSTANDING AND FLUENCY

4 Sketch the graph of each linear rule using the gradient–intercept method.

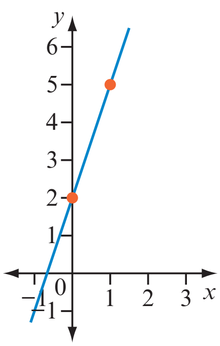
a $y = \frac{2}{3}x - 4$ **b** $y = \frac{1}{2}x + 3$ **c** $y = -\frac{1}{3}x + 5$

d $y = \frac{5}{4}x$ **e** $y = 3x + 1$ **f** $y = -2x - 2$

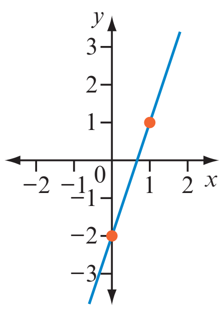
g $y = 5x - 3$ **h** $y = -4x + 6$ **i** $y = 6x - 4$

5 Which of these three options shows the correct sketch graph for $y = 3x - 2$? Explain why the other two options are incorrect, using your knowledge of the gradient–intercept method of sketching linear graphs.

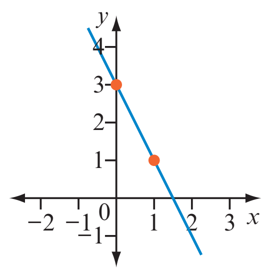
A



B



C

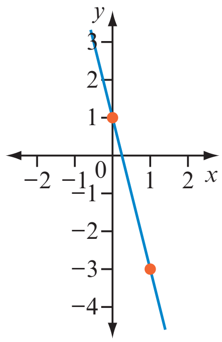


UNDERSTANDING AND FLUENCY

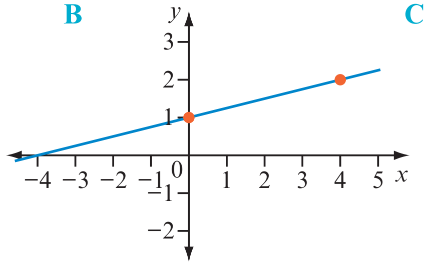
6 Match the correct sketch graph (A, B or C) to each rule shown below.

a $y = 4x + 1$ **b** $y = -4x + 1$ **c** $y = \frac{1}{4}x + 1$

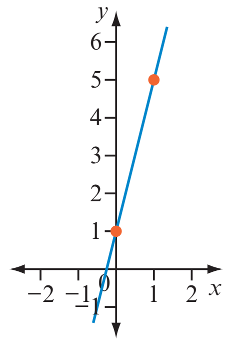
A



B



C



7 For each rule, identify:

i the gradient **ii** the y-intercept.

(Hint: you will first need to rearrange the rule so it is in the form $y = mx + c$.)

a $y = 2 + 3x$ **b** $y = 5 - 2x$ **c** $y = -1 - \frac{2}{3}x$

d $y = 2(x - 3)$ **e** $y = -3(2x - 1)$

8 Sketch the graph of each linear rule in question **7** using the gradient–intercept method.

9 For each rule, identify:

i the gradient **ii** the y-intercept.

(Hint: you will first need to rearrange the rule to make y the subject.)

a $2x + y = 7$ **b** $-5x + y = 1$ **c** $3x + 2y = 6$ **d** $-8x + 4y = 3$

e $6x - 2y = 10$ **f** $7x - y = 2$ **g** $x + 2y + 4 = 0$ **h** $5y - 2x = -9$

10 Sketch the graph of each linear rule in question **9** using the gradient–intercept method.

11 **a** Write $y = 2x$ in the form $y = mx + c$.

b What is the value of m?

c What is the value of c?

d Sketch the graph of $y = 2x$ using the gradient–intercept method.

12 Repeat question **11** for these linear rules.

i $y = 4x$ **ii** $y = -3x$ **iii** $y = -x$ **iv** $y = \frac{1}{2}x$ **v** $y = -\frac{5}{3}x$

13 **a** Write $y = 3$ in the form $y = mx + c$.

b What is the value of m?

c What is the value of c?

d Sketch the graph of $y = 3$ using the gradient–intercept method.

14 Repeat question **13** for these linear rules.

i $y = 8$ **ii** $y = -1$ **iii** $y = \frac{7}{5}$ **iv** $y = -1\frac{3}{4}$ **v** $y = -3.2$

15 Consider the linear graph with the rule $x = 2$. Can this rule be written in the form $y = mx + c$? Explain.

16 Explain how you can write the rule for a linear graph if you know the y-intercept and gradient.

EXAMPLE 3E-3 Writing the rule given the gradient and y-intercept

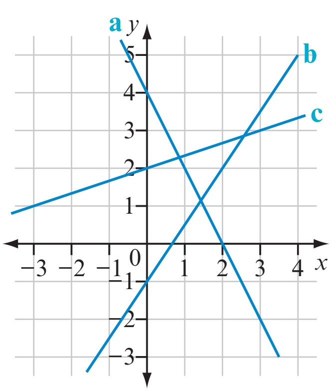
Write the rule for each linear graph with the given gradient and y-intercept.

a $m = 4, c = -3$ **b** $m = -\frac{1}{2}, c = 0.$

THINK	WRITE
a **1** Write the general formula for a linear graph.	**a** $y = mx + c$
2 Substitute for m and c.	$= 4x + (-3)$ $= 4x - 3$
3 Write the answer.	The rule is $y = 4x - 3$.
b **1** Write the general formula.	**b** $y = mx + c$
2 Substitute for m and c.	$= -\frac{1}{2}x + 0$ $= -\frac{1}{2}x$
3 Write the answer.	The rule is $y = -\frac{1}{2}x$.

17 Write the rule for each linear graph with the given gradient and y-intercept.

a $m = 5, c = 2$ **b** $m = 3, c = -7$
c $m = 1, c = 10$ **d** $m = -4, c = 0$
e $m = \frac{2}{9}, c = -1$ **f** $m = -\frac{5}{3}, c = \frac{2}{3}$



18 Write the rule for each linear graph shown in the Cartesian plane at right.

19 **a** Sketch the graph of each linear relationship on the same Cartesian plane, using the gradient–intercept method.

i $y = 2x - 3$ **ii** $y = 2x - 2$ **iii** $y = 2x - 1$ **iv** $y = 2x$
v $y = 2x + 1$ **vi** $y = 2x + 2$ **vii** $y = 2x + 3$ **viii** $y = 2x + \frac{1}{2}$

b What do you notice? What name is given to lines like these?

c Use the terms *gradient* or *y-intercept* to complete this sentence:
Parallel lines have the same _____________.

d What is the linear rule for a line that is parallel to those drawn in part **a** with a y-intercept of 5?

e Create your own linear rule for a line that is parallel to those drawn in part **a**.

PROBLEM SOLVING AND REASONING

20 **a** Sketch the graph of each linear relationship on the same Cartesian plane using the gradient–intercept method.

i $y = -3x - 3$ **ii** $y = -3x - 2$ **iii** $y = -3x - 1$ **iv** $y = -3x$

v $y = -3x + 1$ **vi** $y = -3x + 2$ **vii** $y = -3x + 3$ **viii** $y = -3x + \frac{7}{2}$

b What do you notice?

c What is the linear rule for a line that is parallel to those drawn in part **a** with a y-intercept of -6?

d Create your own linear rule for a line that is parallel to those drawn in part **a**.

21 Write the rule for a linear graph that has:

a a y-intercept of 3 and is parallel to the graph of $y = 4x + 1$

b a y-intercept of -2 and is parallel to the graph of $y = -5x - 7$.

22 A submarine moves upwards from a depth of 50 m below sea level, covering a vertical distance of 20 m over a horizontal distance of 100 m.

a Represent the original position of the submarine as a point on the y-axis of a Cartesian plane. (Hint: let the x-axis represent sea level.)

b Plot a second point to represent the new position of the submarine after the vertical and horizontal movement described. What are the coordinates of this point?

c Draw a straight line through the two plotted points to form a linear graph.

d Identify the gradient and y-intercept of this linear graph and hence write the rule.

e What do x and y represent in the rule for this scenario?

f Assuming the same speed and direction is maintained, use the graph to identify how far horizontally the submarine has travelled to reach the surface of the water from a depth of 50 m.

CHALLENGE

23 Draw a linear graph to represent each of these scenarios using the gradient–intercept method.

a A railway track rises 1 m vertically for every 100 m horizontally from an altitude of 20 m.

b A plane descends 2 km vertically for every 10 km horizontally from an altitude of 8 km.

c A road rises vertically a distance of 20 m for every 100 m horizontally.

24 Write a rule for each linear graph in question **23**.

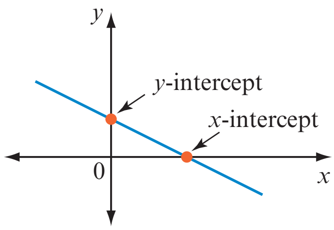
Reflect

How is a sketch graph different from a plotted graph?

3F Sketching linear graphs using *x*- and *y*-intercepts

Start thinking!

You have seen that one way of sketching a linear graph is to use the gradient–intercept method. This is a good approach to use when the linear rule is of the form $y = mx + c$, where m is the gradient and c is the y-intercept.



1 Explain why the gradient–intercept method is easy to use for $y = 2x + 3$ but not as easy for $2x + 3y = 12$.

Another way to identify two points to plot, is to find the x-intercept and the y-intercept.

2 Let's first consider finding the x-intercept for $2x + 3y = 12$ from its rule.
   - **a** What is the y-coordinate at the x-intercept of any graph?
   - **b** Substitute this value for y into the rule $2x + 3y = 12$.
   - **c** Solve the equation to find the value of x.
   - **d** What is the x-intercept?

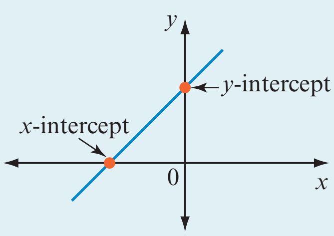
3 Now let's work out the y-intercept for $2x + 3y = 12$.
   - **a** What is the x-coordinate at the y-intercept of any graph?
   - **b** Substitute this value for x into the rule $2x + 3y = 12$.
   - **c** Solve the equation to find the value of y.
   - **d** What is the y-intercept?

4 List the coordinates of the two points you can use to help sketch the graph of $2x + 3y = 12$.

5 Plot the two points and rule a straight line through them to produce the sketch graph of $2x + 3y = 12$. Label your graph with its rule.

6 In your own words, explain how to use the **x- and y-intercept method** to sketch a linear graph.

KEY IDEAS



- ▶ One way of sketching a linear graph is to use the x- and y-intercept method. You identify two points to plot (x-intercept and y-intercept) so a straight line can be ruled through them.
- ▶ The x-intercept can be found by substituting $y = 0$ into the rule and solving for x.
- ▶ The y-intercept can be found by substituting $x = 0$ into the rule and solving for y.

EXERCISE 3F Sketching linear graphs using x- and y-intercepts

EXAMPLE 3F-1 Finding the x-intercept and the y-intercept

Find the x-intercept and the y-intercept for $x + 5y = 10$.

THINK	WRITE
1 Write the rule.	$x + 5y = 10$
2 Find the x-intercept by substituting $y = 0$ into the rule and solving for x.	x-intercept: when $y = 0$, $x + 5 \times 0 = 10$ $x + 0 = 10$ $x = 10$ x-intercept is 10. Coordinates of the x-intercept are (10, 0).
3 Find the y-intercept by substituting $x = 0$ into the rule and solving for y. (Divide both sides of the equation by 5.)	y-intercept: when $x = 0$, $0 + 5y = 10$ $5y = 10$ $y = 2$ y-intercept is 2. Coordinates of the y-intercept are (0, 2).

UNDERSTANDING AND FLUENCY

1 Copy and complete each set of working to find, for each linear rule:
 i the x-intercept ii the y-intercept.

a $3x + y = 15$

i x-intercept: when $y = 0$,
$3x +$ __ $= 15$
__ $= 15$
$x =$ __
x-intercept is __.
Coordinates of the x-intercept are (__, 0).

ii y-intercept: when $x = 0$,
$3 \times$ __ $+ y = 15$
__ $= 15$
y-intercept is __.
Coordinates of the y-intercept are (0, __).

b $2x - 3y = 6$

i x-intercept: when $y = 0$,
$2x -$ __ $= 6$
__ $= 6$
$x =$ __
x-intercept is __.
Coordinates of the x-intercept are (__, 0).

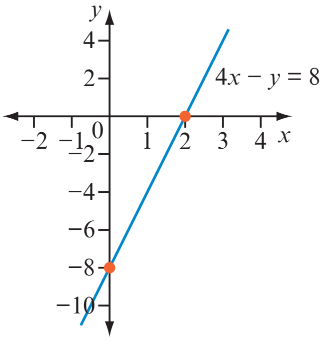
ii y-intercept: when $x = 0$,
$2 \times$ __ $- 3y = 6$
__ $= 6$
y-intercept is __.
Coordinates of the y-intercept are (0, __).

2 Find the x-intercept and the y-intercept for each of these.

a $2x + y = 6$ **b** $x + 4y = 12$
c $x - 2y = 8$ **d** $5x + y = -10$

EXAMPLE 3F-2 Using the x- and y-intercept method to sketch a linear graph

Sketch the graph of $4x - y = 8$ using the x- and y-intercept method.

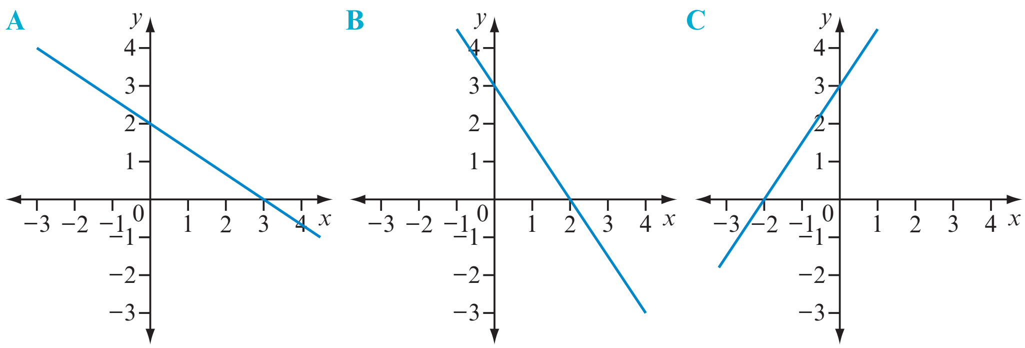
THINK	WRITE
1 Write the rule.	$4x - y = 8$
2 Find the x-intercept by substituting $y = 0$ into the rule and solving for x. (Divide both sides of the equation by 4.)	x-intercept: when $y = 0$, $4x - 0 = 8$ $4x = 8$ $x = 2$ x-intercept is 2. Coordinates of the x-intercept are (2, 0).
3 Find the y-intercept by substituting $x = 0$ into the rule and solving for y. (Divide both sides of the equation by −1.)	y-intercept: when $x = 0$, $4 \times 0 - y = 8$ $0 - y = 8$ $-y = 8$ $y = -8$ y-intercept is −8. Coordinates of the y-intercept are (0, −8).
4 Plot the points for the x-and y-intercepts on a Cartesian plane. **5** Rule a straight line through the points and label with the rule.	

3 Use your answers to question **2** to sketch the graph of each linear relationship.

4 Sketch the graph of each of these using the x- and y-intercept method.

a $4x + y = 4$ **b** $5x - y = 10$ **c** $x + 3y = 6$
d $3x + 4y = -12$ **e** $-2x + 3y = 6$ **f** $2x + 5y = -10$
g $7x + y = -7$ **h** $-x + 2y = 8$ **i** $3x - y = -3$
j $6x + 5y = 30$ **k** $-4x + 2y = 20$ **l** $3x + y = 2$

5 Which of these three options shows the correct sketch graph for $3x + 2y = 6$? Explain why the other two options are incorrect using your knowledge of the x- and y-intercept method of sketching linear graphs.



6 Which option in question **5** shows the correct sketch graph for:

a $2y - 3x = 6$? **b** $2x + 3y = 6$?

7 Consider the linear rule $y = 2x + 4$.

a Find the x-intercept by substituting $y = 0$ into the rule and then solving the resulting equation.

b Find the y-intercept by substituting $x = 0$ into the rule and simplifying.

c Use the x-intercept and y-intercept to sketch the graph of $y = 2x + 4$.

EXAMPLE 3F-3 Finding the x-intercept and the y-intercept given a rule in the form $y = mx + c$

Find the x-intercept and the y-intercept for $y = 3x + 12$.

THINK	WRITE
1 Write the rule.	$y = 3x + 12$
2 Find the x-intercept by substituting $y = 0$ into the rule and solving for x. (Subtract 12 from both sides of the equation and then divide both sides by 3.)	x-intercept: when $y = 0$, $0 = 3x + 12$ $3x + 12 = 0$ $3x = -12$ $x = -4$ x-intercept is -4. Coordinates of the x-intercept are $(-4, 0)$.
3 Find the y-intercept by substituting $x = 0$ into the rule and solving for y.	y-intercept: when $x = 0$, $y = 3 \times 0 + 12$ $= 0 + 12$ $= 12$ y-intercept is 12. Coordinates of the y-intercept are $(0, 12)$.

UNDERSTANDING AND FLUENCY

8 Find the x-intercept and the y-intercept for each of these.

a $y = 2x + 8$ **b** $y = 3x - 6$ **c** $y = x + 3$ **d** $y = x - 5$

e $y = -2x + 2$ **f** $y = -x + 7$ **g** $y = 2x - 3$ **h** $y = 4 - x$

9 Use your answers to question **8** to sketch the graph of each linear relationship.

10 Sketch the graph of each of these using the x- and y-intercept method.

a $2y = x + 4$ **b** $5y = 5x - 10$ **c** $6x + 3y + 12 = 0$ **d** $4x - 3y - 8 = 0$

11 Consider the linear rule $y = 3x$.

a Find:

i the x-intercept **ii** the y-intercept.

b Do you have enough information from part **a** to sketch the graph? Explain.

c What other information would you need? Discuss this with a classmate.

Another way of identifying a second point to plot is to find the corresponding y value for a chosen x value. Any x value can be used, but we often choose $x = 1$.

d Find the value of y when x is 1. (Hint: substitute $x = 1$ into the rule.)

e List the coordinates of the two points you can now use to sketch the graph of $y = 3x$.

f Plot the two points and rule a straight line through them to produce the sketch graph of $y = 3x$. Label your graph with its rule.

12 Use the approach seen in question **11** to sketch each of these linear graphs.

a $y = 5x$ **b** $y = -x$ **c** $y = 2x$ **d** $y = 4x$

13 **a** Decide whether each statement about the graph of $y = 3x + 6$ is true or false.

i The relationship is linear. **ii** The x-intercept is 2.

iii The y-intercept is 6. **iv** The point (1, 9) lies on the line.

v The line passes through the origin. **vi** The gradient of the line is 3.

b Sketch the graph of $y = 3x + 6$.

14 **a** Decide whether each statement about the graph of $y = -3x$ is true or false.

i The x-intercept is -3. **ii** The y-intercept is 0.

iii The gradient is negative. **iv** The point (1, 3) lies on the line.

v The line passes through the origin. **vi** The gradient of the line is $-\frac{1}{3}$.

b Sketch the graph of $y = -3x$.

15 So far you have worked with different methods for sketching graphs. Describe which method you think would be the best to use to sketch each of these.

a linear graphs with a rule in the general form $y = mx + c$; for example, $y = 2x + 5$.

b linear graphs with a rule in the general form $ax + by = d$; for example, $2x + 3y = 6$.

c linear graphs that pass through the **origin**; for example, $y = 7x$.

d linear graphs that are horizontal lines; for example, $y = 4$.

e linear graphs that are vertical lines; for example, $x = -3$.

PROBLEM SOLVING AND REASONING

16 Use the most appropriate method to sketch the graph of each linear relationship.

a $2x - 5y = 10$ **b** $y = 4x + 2$ **c** $x + y = 6$

d $y = -3x$ **e** $y = 7$ **f** $y = 6 - 3x$

g $x = 1$ **h** $y = 4(x - 1)$ **i** $x + 3y - 9 = 0$

j $y = x$ **k** $y = -3x + 5$ **l** $y = 1 - 4x$

17 Tony is buying a skateboard on a purchase plan where he makes a regular payment each week. He created the rule $y = 300 - 25x$ to describe the relationship between the amount still owed in dollars (y) after a number of weeks (x).

a Sketch the graph of this relationship using the x- and y-intercept method.

b What does the y-intercept represent on this graph?

c What does the x-intercept represent on this graph?

d Describe the purchase plan Tony is using. When will he be able to bring his skateboard home?

18 A rainwater tank has a capacity of 1500 L and feeds a drip system to water the garden. At the beginning of April, the tank is full but it is empty at the end of the last day of the month. Let x represent the number of days from the start of April and y represent the number of litres of water in the tank. Assume a constant rate of water use and no further rain during April.

a Could this relationship be represented by a linear graph? Explain.

b Write the coordinates of the x-intercept for this relationship. (Hint: what is the value of x when y is 0?)

c Write the coordinates of the y-intercept for this relationship.

d Use these intercepts to sketch a graph of the relationship.

e Use the graph to estimate the number of litres of water in the tank at the end of the day on 10 April.

f Use the graph to estimate when there is 600 L of water left in the tank.

CHALLENGE

19 Write the rule for the linear graph in question **18**. Use the rule to check your answers for parts **e** and **f**.

20 Create a rule for a linear relationship and define each variable. Write two questions for a classmate to answer after they sketch the graph of the relationship.

21 Use the given x- and y-intercepts to write the rule for each linear graph.

a x-intercept is 1, y-intercept is 4

b x-intercept is 7, y-intercept is 7

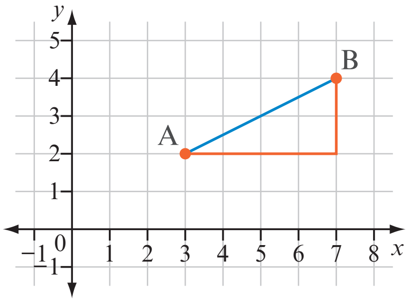
c x-intercept is -2, y-intercept is 3

Reflect

What is the common goal of each of the two methods used to sketch linear graphs?

3G Midpoint and length of line segments

Start thinking!



Consider the line segment (or interval) between the two points, A and B, shown on this Cartesian plane. A right-angled triangle has been drawn using these points as two of the vertices.

1 List the coordinates of points A and B.

2 What is the horizontal distance between points A and B?

3 What is the vertical distance between points A and B?

4 Copy the Cartesian plane and mark a point M halfway between points A and B. This is the **midpoint** of the line segment joining points A and B. Explain how you worked out where to place the midpoint.

5 What is the x-coordinate of point M?

6 Calculate the average of the x-coordinates for points A and B. (Hint: calculate $\frac{3+7}{2}$.)

7 Compare your answers to questions **5** and **6**. Explain how you can work out the x-coordinate for the midpoint if you know the x-coordinates of the endpoints of the line segment.

8 What is the y-coordinate of point M?

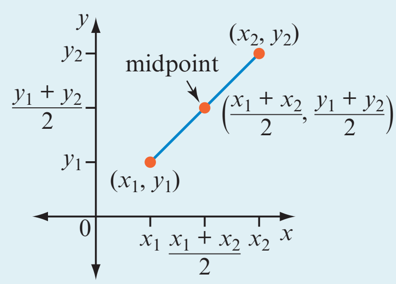
9 Calculate the average of the y-coordinates for points A and B.

10 Compare your answers to questions **8** and **9**. Explain how you can work out the y-coordinate for the midpoint if you know the y-coordinates of the endpoints of the line segment.

11 List the coordinates of the midpoint.

KEY IDEAS

- The midpoint of a line segment joining two points is halfway between the two points. One way to find the coordinates of the midpoint is to calculate the average of the x-coordinates and the average of the y-coordinates for the two endpoints.
- The distance between two points is the length of the line segment joining the two points.
- For the line segment joining two points (x_1, y_1) and (x_2, y_2):
  - the coordinates of the midpoint are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
  - the length of the line segment is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



EXERCISE 3G Midpoint and length of line segments

EXAMPLE 3G-1 Finding the coordinates of the midpoint of a line segment

Find the coordinates of the midpoint of the line segment joining (−1, 4) and (7, 9).

THINK	WRITE
1 Find the average of the x-coordinates and the average of the y-coordinates or use $\frac{x_1 + x_2}{2}$ and $\frac{y_1 + y_2}{2}$.	x-coordinate of midpoint $= \frac{-1+7}{2} = 3$ y-coordinate of midpoint $= \frac{4+9}{2} = 6.5$
2 Write the coordinates of the midpoint.	Coordinates of midpoint are (3, 6.5).

Where appropriate, write answers correct to one decimal place.

1 Find the coordinates of the midpoint of the line segment joining these pairs of points.

a (1, 4) and (3, 10)
b (2, 5) and (8, 3)
c (1, 2) and (5, 10)
d (3, 6) and (11, 4)
e (1, 0) and (5, 2)
f (0, 5) and (8, 9)
g (−1, 4) and (5, 2)
h (3, −4) and (7, 6)
i (2, −1) and (6, 7)
j (−4, −2) and (−2, 2)
k (3, 9) and (4, 8)
l (5, 0) and (8, 11)

EXAMPLE 3G-2 Finding the length of a line segment

Find the length of the line segment joining (−1, 4) and (7, 9), correct to one decimal place.

THINK	WRITE
1 Identify the coordinates of the two points.	Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (7, 9)$.
2 Substitute into the formula for the length of a line segment. Use a calculator to simplify your answer.	Length of line segment $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{[7 - (-1)]^2 + (9 - 4)^2}$ $= \sqrt{8^2 + 5^2}$ $= \sqrt{64 + 25}$ $= \sqrt{89}$ ≈ 9.4 units

2 Find the length of the line segment joining these pairs of points.

a (2, 5) and (3, 7) **b** (3, 4) and (5, 8) **c** (6, 2) and (9, 3)

d (1, 8) and (5, 5) **e** (−1, 2) and (2, 7) **f** (−3, 3) and (2, 4)

g (2, −4) and (4, 2) **h** (5, 0) and (8, −4) **i** (0, −1) and (1, −2)

j (4, −3) and (6, 0) **k** (−3, 6) and (−2, 2) **l** (−5, −4) and (−1, −2)

3 Find the distance between each pair of points in question **1**.

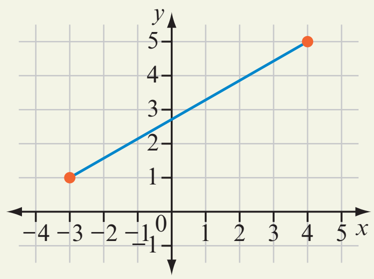
EXAMPLE 3G-3 Finding the midpoint, length and gradient of a line segment

For the line segment shown, find:

a the midpoint

b the length

c the gradient.



THINK

Identify the coordinates of the endpoints of the line segment, (x_1, y_1) and (x_2, y_2).

a Locate the midpoint of the line segment on the graph and write the coordinates. Alternatively, find the average of the x-coordinates and the y-coordinates of the endpoints.

b Substitute the coordinates of the endpoints into the formula for length of a line segment and calculate the result.

c Substitute into the formula for gradient of a line segment, $m = \frac{y_2 - y_1}{x_2 - x_1}$, and simplify.

WRITE

Endpoints are at (−3, 1) and (4, 5).

a Midpoint is at $\left(\frac{-3 + 4}{2}, \frac{1 + 5}{2}\right)$.

Coordinates of midpoint are $(\frac{1}{2}, 3)$.

b length of line segment

$= \sqrt{[4 - (-3)]^2 + (5 - 1)^2}$

$= \sqrt{7^2 + 4^2}$

$= \sqrt{65}$

≈ 8.1 units

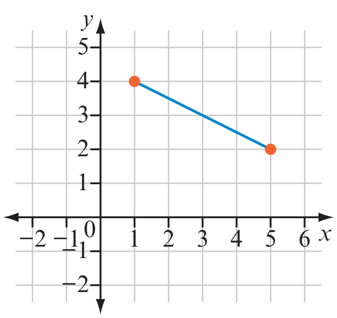
c gradient $= \frac{5 - 1}{4 - (-3)}$

$= \frac{4}{7}$

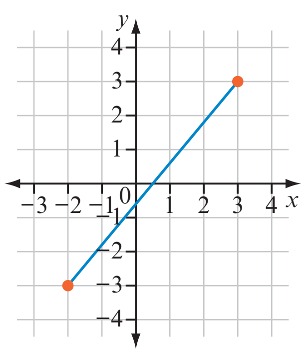
4 For each line segment, find:

i the midpoint **ii** the length **iii** the gradient.

a



b



PROBLEM SOLVING AND REASONING

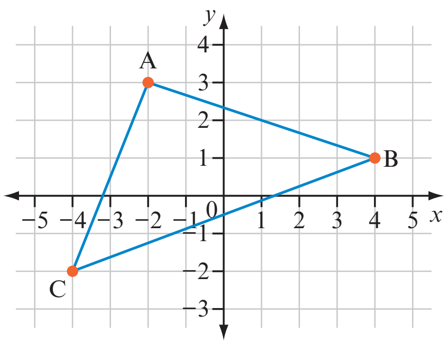
5 The midpoint of a line segment between points A and B has the coordinates (6, 4). If A has the coordinates (2, 3), find the coordinates of B.

6 The midpoint of a line segment between points C and D has the coordinates (−5, 1). If D has the coordinates (4, −7), find the coordinates of C.

7 **a** Write the coordinates of a pair of points that have a midpoint at (3, 8).
b Suggest another two pairs of points that have this midpoint.

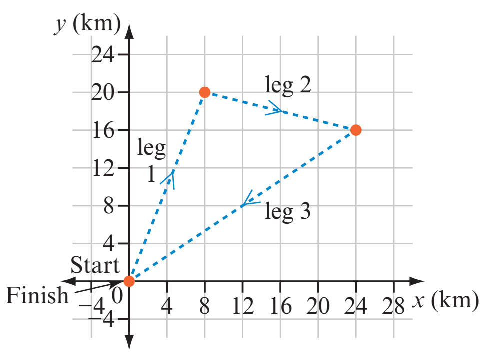
8 A point, A, on a circle has the coordinates (−1, −1).
a If the centre of the circle is at (2, 3), calculate the radius of the circle.
b Identify the coordinates of another point on the circle that forms a diameter with point A.

9 Find the perimeter of triangle ABC at right.



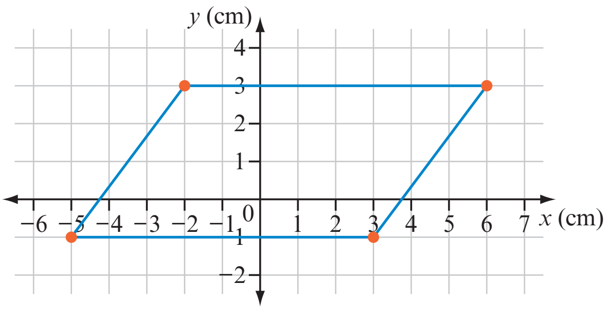
10 Find the perimeter of each shape.
a triangle with vertices at (−3, −2), (−2, 4) and (4, 2)
b square with vertices at (−1, 2), (2, 5), (5, 2) and (2, −1)
c rectangle with vertices at (−4, −2), (2, 4), (4, 2) and (−2, −4)
d trapezium with vertices at (−3, 3), (1, 5), (3, 3) and (2, −2)

11 A yacht race follows a triangular course that has been mapped on to a Cartesian plane. The scale on the axes indicates distances in kilometres. The race begins and ends at the origin.



a Calculate the length of each leg of the race.
b Calculate the total racing distance.
c If an observer's boat is located close to the midpoint of the second leg of the race, calculate the distance between this boat and the finishing point.

12 Consider the parallelogram drawn on this Cartesian plane. The scale on the axes shows distances in centimetres.



a List the coordinates of the vertices of the parallelogram.

b Find the coordinates of the midpoint of:

  i the longer diagonal

  ii the shorter diagonal.

c What do you notice about your answers to part **b**?

d Calculate the perimeter of the parallelogram.

e Find the difference in length of the two diagonals.

13 A shape (in blue) is drawn on to a Cartesian plane. A smaller shape (in orange) is then formed by joining the midpoints of the vertices of the original shape. The scale on the axes of each Cartesian plane indicates distances in centimetres.

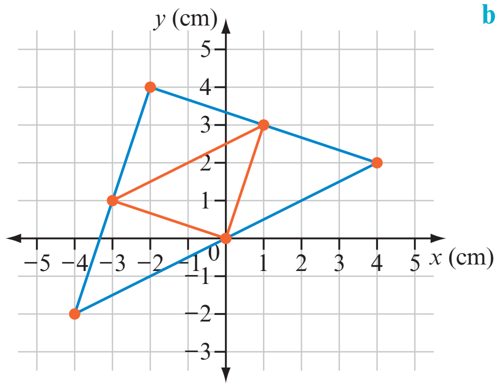
In each case, find the perimeter of:

  i the blue shape

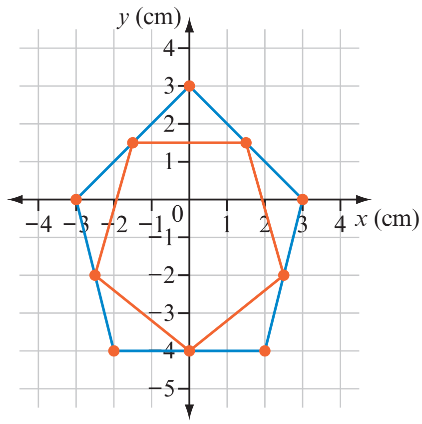
  ii the orange shape.

Compare your answers.

a



b



14 A **quadrilateral** ABCD has vertices at A(−4, 2), B(−1, 4), C(3, −2) and D(0, −4). The scale on the axes indicates distance in metres.

a Find the length of each side of the quadrilateral.

b Find the coordinates of the midpoint of each diagonal.

c Find the distance between each **vertex** and the midpoint.

d Hence prove that the quadrilateral is a rectangle.

CHALLENGE

15 Julia traced the outline of this starfish on to 1-cm grid paper and ruled axes to form a Cartesian plane.

a List the coordinates of the ten vertices A to J.

b Calculate the shortest distance between points A and C.

c Repeat part **b** to calculate the distance between points:

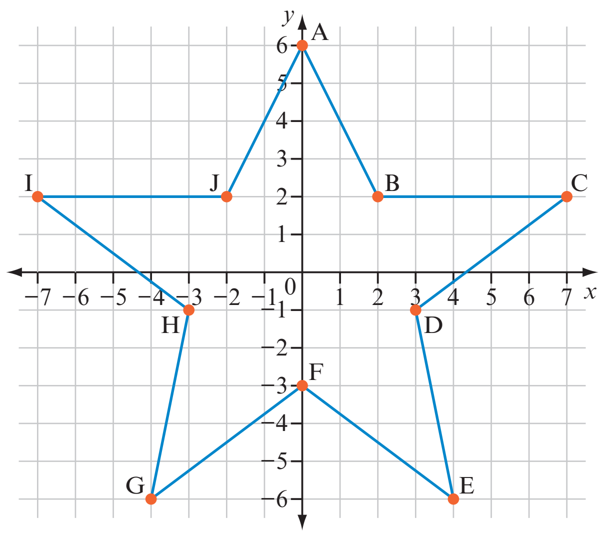
i C and E

ii E and G

iii G and I

iv I and A.

d Does the starfish outline on the Cartesian plane show any symmetry? (Hint: use your answers to parts **b** and **c** to help with your response.)



e Identify the coordinates of the midpoint, M, between points E and G.

f Find the distance between points M and F.

g Use your answers to parts **e** and **f** to calculate the area of triangle EFG.

h If the area of the pentagon ACEGI is 116 cm^2, use your answer to part **g** to help you estimate the area covered by the starfish.

i Calculate the perimeter of the starfish.

16 Prove that triangle ABC with vertices at A(3, 6), B(−1, −2) and C(−5, 2) is an **isosceles triangle**. Assume the scale on the axes shows distances in centimetres.

17 Calculate the area of the triangle in question **16** using your knowledge of the midpoint and length of a line segment.

18 Prove that quadrilateral ABCD with vertices at A(−5, 3), B(−1, 5), C(3, −2) and D(−1, −4) is a parallelogram.

Reflect

How can you remember each method for finding the midpoint and the length of a line segment?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

linear equation	equivalent equations	linear relationship	*y*-intercept
solve	balance method	non-linear relationship	sketch graph
solution	flowchart	Cartesian plane	line segment
solving by inspection	substitution	gradient	midpoint
inverse operations	variable	*x*-intercept	length of a line segment

MULTIPLE-CHOICE

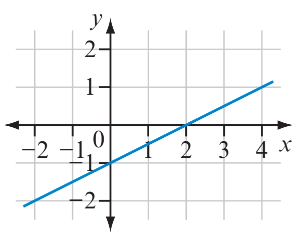
3A **1** The solution to $3x - 9 = 12$ is:
- **A** $x = 1$ **B** $x = 7$
- **C** $x = 13$ **D** $x = 63$

3A **2** After performing the first inverse operation on both sides of $\frac{x+3}{2} - 4 = 1$, the equivalent equation obtained is:
- **A** $(x + 3) - 4 = 2$ **B** $\frac{x+3}{2} = -3$
- **C** $\frac{x+3}{2} = 5$ **D** $\frac{x}{2} - 4 = -2$

3B **3** The solution to $5x - 2 = 3x + 7$ is:
- **A** $x = \frac{5}{8}$ **B** $x = 1\frac{1}{8}$
- **C** $x = 2\frac{1}{2}$ **D** $x = 4\frac{1}{2}$

3C **4** Which point does not lie on the graph of $y = 2x - 4$?
- **A** $(0, -4)$ **B** $(1, -2)$
- **C** $(4, 4)$ **D** $(-1, -2)$

3D **5** The gradient of the linear graph shown is:
- **A** -2
- **B** $-\frac{1}{2}$
- **C** $\frac{1}{2}$
- **D** 2



3D **6** The gradient of the line segment joining the points $(-2, 4)$ and $(7, -4)$ is:
- **A** $\frac{8}{9}$ **B** $-\frac{8}{9}$ **C** $\frac{9}{8}$ **D** $-\frac{9}{8}$

3D **7** A line with a gradient of 0 is:
- **A** vertical **B** horizontal
- **C** sloping upwards from left to right
- **D** sloping downwards from left to right

3E **8** In the general form of a linear equation, $y = mx + c$, the pronumeral c represents:
- **A** the *y*-intercept **B** the *x*-intercept
- **C** the rise **D** the gradient

3F **9** At the point where a line crosses the *x*-axis:
- **A** $x = 0$ **B** $y = 0$
- **C** $x = y$ **D** $y = -x$

3F **10** The *y*-intercept for the graph of $2y = 3x - 4$ is:
- **A** -4 **B** 4 **C** -2 **D** 2

3G **11** The coordinates of the midpoint of the line segment joining $(3, 5)$ and $(7, 9)$ are:
- **A** $(5, 7)$ **B** $(7, 5)$ **C** $(2, 2)$ **D** $(4, 8)$

3G **12** The length of the line segment joining $(3, 5)$ and $(7, 9)$ is closest to:
- **A** 4 **B** 5 **C** 6 **D** 7

SHORT ANSWER

3A 1 Solve each equation.

a $3x - 4 = 2$ b $-3x + 4 = 2$
c $\frac{x+2}{3} = 4$ d $\frac{x}{3} - 2 = -1$
e $\frac{x-2}{3} + 5 = 4$ f $\frac{4}{x} + 3 = -2$
g $4(x - 3) = 3$ h $2(3 - x) - 7 = -2$

3B 2 Check whether the value for x given in brackets is the solution to the equation.

a $3x - 2 = 5x + 6$ $(x = -4)$
b $\frac{2x-4}{3} = \frac{3x-2}{4}$ $(x = 10)$

3B 3 Solve each equation.

a $7x - 3 = 4x + 9$ b $x + 8 = 1 - 6x$

3B 4 Write an equation for each statement.

a The product of 4 and n is equal to the sum of 4 and n.
b When the sum of 4 and n is multiplied by 3, it is equal to twice the value of n.

3B 5 Solve each equation in question 4.

3B 6 Write x as the subject of each formula.

a $3x + 4 = a + 1$ b $ax + 3 = b$
c $4x - 2 = 7x + y$ d $y = mx + c$

3C 7 Complete a table of values from −2 to 2 for each of these and then plot each graph on the same Cartesian plane.

a $y = x + 5$ b $y = x - 5$
c $y = -x + 5$ d $y = -x - 5$

3C 8 Use your graphs from question 7 to:

a describe the gradient of each line
b find where each line crosses the axes.

3D 9 Find the gradient of the line segment joining each pair of points.

a (2, 3) and (−2, −3)
b (−3, −2) and (−2, −3)

3D 10 For each rule, identify the:

i gradient ii y-intercept.

a $y = 2x - 4$ b $y = 3x + 5$
c $y = 3 - 4x$ d $y = 4 - \frac{1}{2}x$

3D 11 For each rule, identify the:

i gradient ii y-intercept.

a $3x + 2y = 4$ b $4 - 2x = 3y$
c $5 = x - 3y$ d $2x + 3y + 6 = 0$

3D 12 Write the rule for each linear graph with the given gradient and y-intercept.

a gradient = 4, y-intercept = −2
b gradient = $\frac{1}{4}$, y-intercept = 0
c gradient = 0, y-intercept = $-\frac{1}{2}$

3E 13 Sketch each linear graph in question 10.

3E 14 Sketch each linear graph in question 11.

3E 15 Write the rule for a linear graph that has a y-intercept of 4 and is parallel to the graph of $y = -2x + 7$.

3F 16 For each of these rules, find the:

i x-intercept ii y-intercept.

a $2x + 3y = 18$ b $3x - y = 6$
c $y = 4x - 2$ d $2y = 5x - 3$
e $x - 2y = 4$ f $-y = 4 - x$

3F 17 Use your answers from question 16 to sketch each linear graph.

3F 18 Use the most appropriate method to sketch the graph of each linear relationship.

a $3x + 2y = 4$ b $4 - 2x = 3y$
c $y = \frac{1}{3}x$ d $y = 5 - \frac{2}{3}x$
e $y = 5$ f $2x + 3y + 6 = 0$
g $y = \frac{3}{4}x - 2$ h $x = -7$

3G 19 Find the coordinates of the midpoint for the line segment joining:

a (2, 3) and (8, 7)
b (−4, 6) and (−3, 5)
c (2, −4) and (−2, 4)
d (−3, −1) and (2, −9).

3G 20 Find the length of each line segment described in question 19.

NAPLAN-STYLE PRACTICE

1 Answer true or false to this statement:
The equation $4x - 3 = 2$ is a linear equation.

2 The solution to $\frac{2x-3}{4} = 3$ is:
- $x = 4\frac{1}{2}$
- $x = 7\frac{1}{2}$
- $x = 12$
- $x = 34$

3 Five is added to three times a number n. This is then divided by 4. The result is 6. The equation representing this process is:
- $5 + \frac{3n}{4} = 6$
- $\frac{3(5+n)}{4} = 6$
- $\frac{5(3 \times n)}{4} = 6$
- $\frac{3n+5}{4} = 6$

4 A number n is divided into 4, then 5 is added. The result is 10. Write this as an equation.

5 An equivalent equation to $2x + 3y - 4 = 0$ is:
- $3y = 2x + 4$
- $y = \frac{-2x+4}{3}$
- $x = \frac{3y+4}{2}$
- $2x = 3y + 4$

6 Write y as the subject of the formula $4 - 3y = 2x$.

7 Which equation does *not* have the solution $x = 3$?
- $\frac{4x-2}{5} = 2$
- $\frac{9-x}{6} = 1$
- $5 - 3x = 4$
- $4 - 2x = -2$

8 Consider an odd integer represented by n. The next odd integer would be represented by:
- $n + 1$
- $n + 2$
- $n - 1$
- $n - 2$

9 The sum of three consecutive odd numbers is 33. What is the smallest of the three numbers?

10 Write p as the subject of the literal equation $3p = \frac{4r}{t} - 2p$.

11 Which rule would *not* produce a linear graph?
- $y = \frac{x}{2} + 3$
- $y = 3x - \frac{5}{2}$
- $y = \frac{4}{x} + 5$
- $y = \frac{1}{2}x - 2$

12 Which point does *not* lie on the graph of $y = 2x - 4$?
- (1, −2)
- (2, 1)
- (3, 2)
- (4, 4)

13 Which point does *not* lie on the graph of $-x + 2y = 4$?
- (−1, 1.5)
- (2, 3)
- (−2, 1)
- (−3, 3.5)

14 The gradient of the graph of $y = x - 6$ is:
- positive
- negative
- zero
- undefined

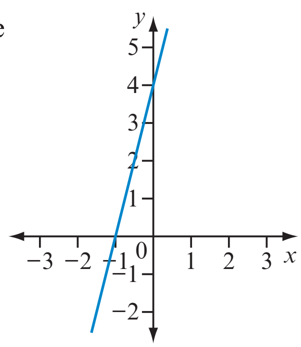
15 The gradient of the graph of $y = -4$ is:
- positive
- negative
- zero
- undefined

16 The gradient of the graph of $x = 10$ is:
- positive
- negative
- zero
- undefined

17 The gradient of the graph of $-x + 2y = 4$ is:
- −1
- 0.5
- 2
- 4

18 Compare the gradients of the graphs of $y = -x$ and $y = -4x$. Which would be the steeper line?

Questions **19–21** refer to the linear graph drawn on this Cartesian plane.



19 What is the x-intercept?
- −1
- 0
- 1
- 4

20 What is the y-intercept?
- −1
- 0
- 1
- 4

21 What is the gradient?
- −4
- $-\frac{1}{4}$
- $\frac{1}{4}$
- 4

Questions 22–24 refer to this information. Consider the linear relationship with the rule $2x - 3y = -6$.

22 What is the x-intercept of the graph?

23 What is the y-intercept of the graph?

24 What is the gradient of the graph?

Questions 25 and 26 refer to this information. A line segment AB has a midpoint X(2, 4). The coordinates of the point B are (5, 6).

25 What are the coordinates of the point A?

26 What is the length of the line segment AX, correct to one decimal place?

Questions 27–29 refer to a triangle PQR with vertices at P(−1, 3), Q(−1, −2) and R(4, −2).

27 Which line segment represents the longest side of the triangle?

PQ QR PR QP

28 Calculate the length of the longest side of the triangle, correct to one decimal place.

29 Find the coordinates of the midpoint of the line segment QR.

ANALYSIS

1 The cross-section of a roof is drawn on a Cartesian plane with the scale on the axes showing lengths in metres. The x-axis represents ground level.

- a On the same Cartesian plane, sketch the graph of:
  - i $4y - x = 12$ ii $y = 5 - \frac{1}{4}x$
- b To represent the cross-section of the roof, highlight the graph of $4y - x = 12$ between $x = 0$ and $x = 4$ and the graph of $y = 5 - \frac{1}{4}x$ between $x = 4$ and $x = 8$.
- c What is the height of the top of the roof?
- d What is the distance from the top of the roof to the lower edge of the roof, correct to one decimal place?
- e What is the gradient of the roof?
- f If a chimney is to be placed halfway along the slope of the roof, describe its position.

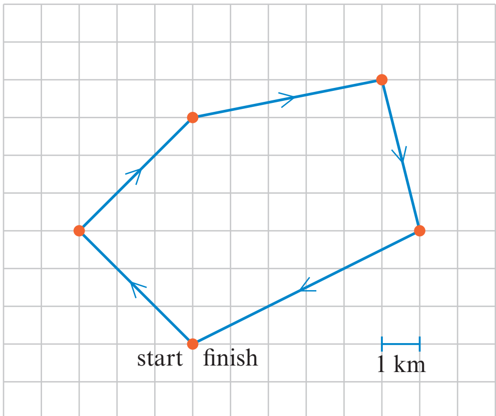
2 A rectangle DEFG has vertices at D(−2, −1), E(0, 1), F(3, −2) and G(1, −4).

- a Draw the rectangle on a Cartesian plane.
- b Identify the parallel sides.
- c Calculate the lengths of all the sides. Using these measurements, explain the shape.
- d Using your graph, identify the y-intercepts of the lines through:
  - i DE ii EF iii DG iv GF.
- e Find the gradients of the lines through:
  - i DE ii EF iii DG iv GF.
- f Use the gradient–intercept method to determine the equation of the lines through:
  - i DE ii EF iii DG iv GF.
- g The point P is the midpoint of DE, Q is the midpoint of EF, R is the midpoint of GF and S is the midpoint of DG. Find the coordinates of:
  - i P ii Q iii R iv S.
- h Describe the shape of the figure PQRS. Justify the statements you make.
- i If the original figure DEFG had been a square instead of a rectangle, explain whether the shape of PQRS would be any different. Support your answer with mathematical evidence.

CONNECT

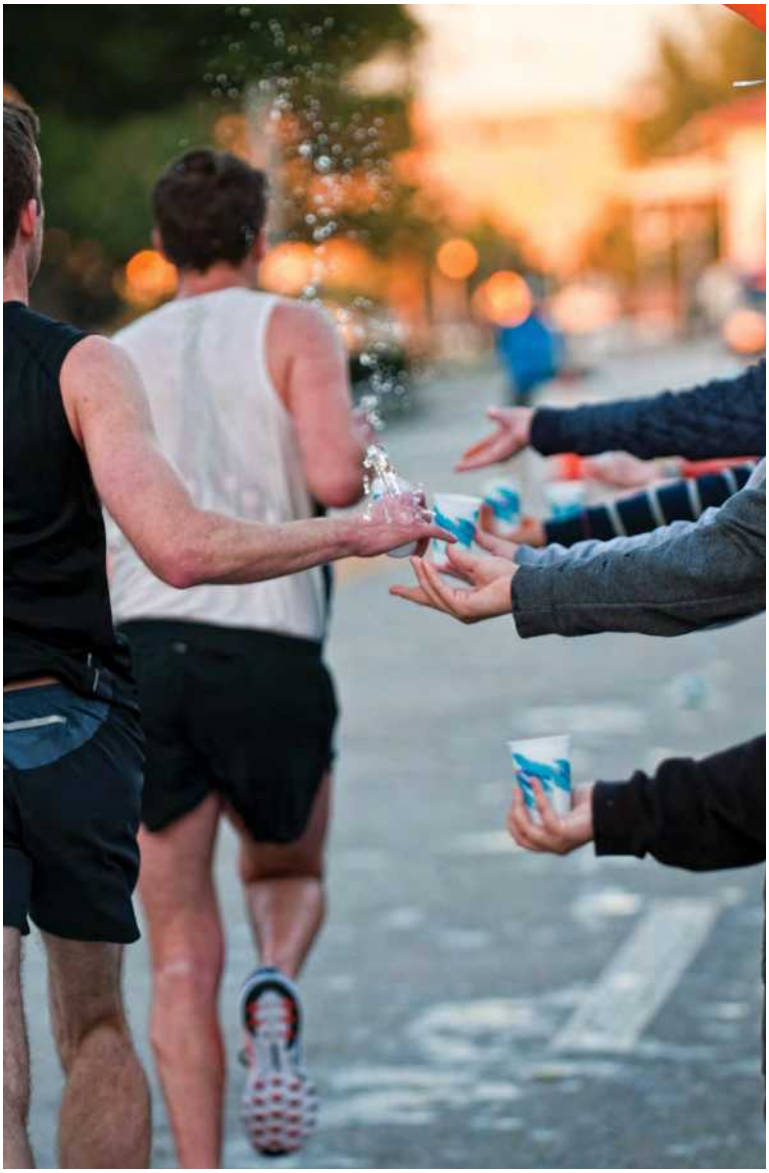
Fun run

A map of the course for a fun run has been drawn on grid paper. There are five legs to the course, with participants running up a hill in the third leg.



The organisers need to work out important information such as the length of each leg of the course and the position of stations for participants to collect water and sports drinks. They also want to inform participants in wheelchairs of how steep the third leg of the course is.

The position of the water station is halfway along the second leg of the course, while participants can collect sports drinks and more water from a station positioned halfway along the fourth leg.



Your task

You are to:

- reproduce the course on a Cartesian plane with the start/finish position located at the origin
- calculate the length of each leg of the course
- determine the total length of the fun run course
- locate the position of the first water station
- locate the position of the second station that has sports drinks and water
- determine the shortest distance from the start to each station
- describe the slope of the third leg of the run if the altitude at the bottom of the hill is 146 m and the altitude at the top of the hill is 150 m
- formulate a rule to describe each leg of the course as a linear graph on the Cartesian plane.

Include all necessary working to justify your answers.

As an extension, you may like to design the course for another fun run.
Repeat your calculations to work out relevant information for your participants.
Formulate a rule to describe each leg of your course as a linear graph on the Cartesian plane.

You may like to present your findings as a report.
Your report could be in the form of:

- a poster
- a PowerPoint presentation
- a technology demonstration
- other (check with your teacher).

4 NON-LINEAR RELATIONSHIPS

ESSENTIAL QUESTION

The path of a basketball in flight can be described by a non-linear relationship. What two variables could you relate in a rule for this relationship?

4A ▶ **1** Factorise each quadratic expression.

a $x^2 + 7x$ **b** $x^2 - 9$

c $x^2 + 5x + 6$ **d** $-x^2 - 3x$

4A ▶ **2** Substitute these values into each expression.

i $x = 2$

ii $x = -3$

iii $x = 0$

a $x^2 - 4x$ **b** $x^2 - 4$

c $x^2 - 2x + 1$ **d** $(x - 4)(x + 3)$

4A ▶ **3** Simplify each expression.

a $3x^2 + 4x + x^2 + 1$

b $x^2 - 7x + 2x - 5$

c $x - x^2 + 2 - 6x - 8$

d $2x^2 - 2 + 2x + 2 - 2x$

4B ▶ **4** **a** Copy and complete this table of values for each rule listed below.

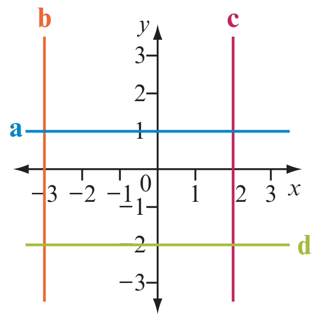
x	−3	−2	−1	0	1	2	3
y							

i $y = 2x$ **ii** $y = x^2$

b Plot the graph of each rule.

c Identify whether each graph shows a linear relationship.

4B ▶ **5** Write the rule for each of the linear graphs shown.



4C ▶ **6** Plot the point A(2, 3) on a Cartesian plane. Find the coordinates of a new point after each transformation has been performed on the point A.

a reflect in the x-axis

b translate 4 units right

c translate 5 units down

d translate 1 unit up

e translate 6 units left

f translate 2 units left and 3 units up

4E ▶ **7** Find the x- and y-intercepts of the graphs of each relationship.

a $y = x - 4$ **b** $y = 2x + 6$

4G ▶ **8** Find the gradient of the graph drawn in question **4ai** for $y = 2x$.

4A Solving quadratic equations

Start thinking!

1 What is a quadratic expression? Provide three examples.

2 You can also consider **quadratic equations**.

a What is the difference between a quadratic equation and a quadratic expression?

b Which of these are quadratic equations?

i $x^2 - 4 = 0$ ii $x - 4 = 0$ iii $x^2 + 7x = 0$ iv $x^2 + 3x - 4$

c Explain the difference between a quadratic equation and a linear equation.

3 One method to solve a quadratic equation is to use a guess, check and improve strategy (trial and error). With a partner, solve each equation. (Hint: each equation has two solutions.)

a $x^2 - 4 = 0$ b $(x - 3)(x - 5) = 0$ c $x^2 + 7x = 0$ d $x^2 + 3x - 4 = 0$

4 Another method involves the **Null Factor Law**.

a Calculate each of these.

i 5×0 ii 0×-4 iii 0×0 iv $x \times 0$ v $0 \times (x - 7)$ vi $(x + 2) \times 0$

b Use your answers to part **a** to help you work out the value of x in each equation.

i $2 \times x = 0$ ii $x \times 6 = 0$ iii $3 \times (x - 1) = 0$ iv $(x + 5) \times 8 = 0$

c Copy and complete this sentence: The Null Factor Law states that if the product of two factors equals ________ then one or both of the factors must equal ________.

5 Consider the Null Factor Law with $(x - 3)(x - 5) = 0$.

a What are the two factors that form the product on the left side of the equation?

b Since the right side of the equation equals 0, what do you know about $(x - 3)$ and $(x - 5)$?

c Copy and complete the workings on the right to solve $(x - 3)(x - 5) = 0$ using the Null Factor Law.

d Check your two solutions. Explain how you did this.

$(x - 3)(x - 5) = 0$
$x - 3 = 0$ or $x - 5 =$ ____
$x = 3$ or $x =$ ____

KEY IDEAS

- The general form of a quadratic equation is $ax^2 + bx + c = 0$ or $(x + m)(x + n) = 0$.
- The Null Factor Law states that if the product of two factors is 0, then one or both factors are 0. For example, if $a \times b = 0$ then $a = 0$ or $b = 0$ or both a and b are 0.
- A quadratic equation can be solved using the Null Factor Law if one side of the equation is in factor form and the other equals 0. Two linear equations are produced which are easy to solve. For example, $(x + m)(x + n) = 0$ has two solutions: $x = -m$ or $x = -n$.
- To obtain the product of two factors on one side of the equation, the quadratic expression needs to be factorised.

EXERCISE 4A Solving quadratic equations

1 Which of these are quadratic equations?

a $x^2 - 2 = 0$ **b** $3(x + 1) = 0$
c $x^2 + x = 5$ **d** $x^2 + 5x + 6$
e $2x^2 + x - 4 = 0$ **f** $x^3 + 8 = 0$
g $6x + 1 = 2x - 5$ **h** $x^2 + 7x = x - 3$

EXAMPLE 4A-1 Solving quadratic equations using the Null Factor Law

Solve each equation.

a $(x + 6)(x - 2) = 0$ **b** $x(x - 4) = 0$

THINK	WRITE
a 1 Write the equation. Check that the left side (LS) is in factor form (yes) and the right side (RS) equals 0 (yes).	**a** $(x + 6)(x - 2) = 0$
2 Apply the Null Factor Law.	$x + 6 = 0$ or $x - 2 = 0$
3 Solve each linear equation.	$x = -6$ or $x = 2$
b 1 Write the equation. Check that the LS is in factor form (yes) and the RS equals 0 (yes).	**b** $x(x - 4) = 0$
2 Apply the Null Factor Law.	$x = 0$ or $x - 4 = 0$
3 Solve each linear equation.	$x = 0$ or $x = 4$

UNDERSTANDING AND FLUENCY

2 Copy and complete the steps shown to solve each quadratic equation using the Null Factor Law.

a $(x + 7)(x - 4) = 0$
$x + 7 = 0$ or $x - 4 =$ __
$x =$ __ or $x =$ __

b $x(x - 2) = 0$
$x =$ __ or $x - 2 =$ __
$x =$ __ or $x =$ __

c $(x + 5)(x - 5) = 0$
$x + 5 =$ __ or $x - 5 =$ __
$x =$ __ or $x =$ __

3 Solve each equation. Show all steps of working.

a $(x + 2)(x - 3) = 0$ **b** $(x - 7)(x - 1) = 0$
c $(x + 4)(x - 4) = 0$ **d** $x(x - 6) = 0$
e $(x + 5)(x + 1) = 0$ **f** $x(x + 2) = 0$
g $(x - 8)(x + 8) = 0$ **h** $(x + 1)(x - 7) = 0$
i $x(x - 11) = 0$ **j** $(x + 3)(x - 5) = 0$
k $(x - 2)(x - 2) = 0$ **l** $(x + 5)(x + 5) = 0$

EXAMPLE 4A-2 Solving quadratic equations after first factorising

Solve each equation.

a $x^2 - 9 = 0$ b $x^2 - 3x - 10 = 0$

THINK	WRITE
a 1 Write the equation. Check that the LS is in factor form (no) and the right side equals 0 (yes).	a $x^2 - 9 = 0$
2 Factorise the LS of the equation (difference of two squares).	$(x + 3)(x - 3) = 0$
3 Apply the Null Factor Law.	$x + 3 = 0$ or $x - 3 = 0$
4 Solve each linear equation.	$x = -3$ or $x = 3$
b 1 Write the equation. Check that the LS is in factor form (no) and the RS equals 0 (yes).	b $x^2 - 3x - 10 = 0$
2 Factorise the LS of the equation.	$(x + 2)(x - 5) = 0$
3 Apply the Null Factor Law.	$x + 2 = 0$ or $x - 5 = 0$
4 Solve each linear equation.	$x = -2$ or $x = 5$

UNDERSTANDING AND FLUENCY

4 Solve each equation.

a $x^2 + 7x + 10 = 0$ b $x^2 - 3x + 2 = 0$ c $x^2 + 5x = 0$

d $x^2 - 3x = 0$ e $x^2 - 36 = 0$ f $x^2 + 10x + 21 = 0$

g $x^2 - 2x - 8 = 0$ h $x^2 - 1 = 0$ i $x^2 + 8x = 0$

j $x^2 - 4x + 3 = 0$ k $x^2 + 6x + 9 = 0$ l $x^2 - 2x + 1 = 0$

5 Use substitution to check that the solutions found in question **4** are correct.

6 For each quadratic equation:

i solve the equation

ii compare your solution to that obtained using the guess, check and improve strategy in question **3** of 4A Start thinking! on page 154.

a $x^2 - 4 = 0$ b $(x - 3)(x - 5) = 0$

c $x^2 + 7x = 0$ d $x^2 + 3x - 4 = 0$

7 Decide whether the value given in square brackets is a solution to the equation.

a $(x - 4)(x - 5) = 0$ [$x = 5$] b $(x + 2)(x - 8) = 0$ [$x = 2$]

c $x(x - 6) = 0$ [$x = 3$] d $x^2 + 8x + 7 = 0$ [$x = -1$]

e $x^2 - 4x + 4 = 0$ [$x = -2$] f $x^2 - 49 = 0$ [$x = 7$]

g $x^2 - 2x - 15 = 0$ [$x = -3$] h $x^2 - 8x + 12 = 0$ [$x = -4$]

8 a Why is $-x(x - 3) = 0$ equivalent to $x(x - 3) = 0$?

b Solve $x(x - 3) = 0$ and hence write the solutions for $-x(x - 3) = 0$.

c Use substitution to check that you have the correct solutions.

9 **a** Why is $-2(x + 4)(x - 5) = 0$ equivalent to $(x + 4)(x - 5) = 0$?
b Solve $(x + 4)(x - 5) = 0$ and hence write the solutions for $-2(x + 4)(x - 5) = 0$.
c Use substitution to check that you have the correct solutions.

EXAMPLE 4A-3 Solving quadratic equations after first dividing both sides by a negative number

Solve each quadratic equation.

a $-4(x - 3)(x - 5) = 0$ **b** $-x^2 - 11x - 18 = 0$

THINK	WRITE
a 1 Divide both sides of the equation by −4.	**a** $-4(x - 3)(x - 5) = 0$ $(x - 3)(x - 5) = 0$
2 Apply the Null Factor Law and solve each linear equation.	$x - 3 = 0$ or $x - 5 = 0$ $x = 3$ or $x = 5$
b 1 Take out −1 as a common factor on the LS.	**b** $-x^2 - 11x - 18 = 0$ $-(x^2 + 11x + 18) = 0$
2 Divide both sides of the equation by −1.	$x^2 + 11x + 18 = 0$
3 Factorise the LS of the equation.	$(x + 9)(x + 2) = 0$
4 Apply the Null Factor Law and solve each linear equation.	$x + 9 = 0$ or $x + 2 = 0$ $x = -9$ or $x = -2$

UNDERSTANDING AND FLUENCY

10 Solve each quadratic equation.

a $-x(x + 9) = 0$ **b** $-(x + 8)(x - 2) = 0$
c $-3(x - 1)(x - 4) = 0$ **d** $-7(x + 6)(x - 6) = 0$
e $-x^2 - 10x - 21 = 0$ **f** $-5x^2 - 5x + 10 = 0$

11 **a** How many solutions does $(x - 4)(x - 7) = 0$ have?
b How is this different from the number of solutions a linear equation has?
c Does every quadratic equation have two solutions? Discuss this with a classmate.
d How many solutions does $(x - 4)(x - 4) = 0$ have? Explain.
e How many solutions does $x^2 + 4 = 0$ have? Explain.
f Summarise your findings. How many solutions can a quadratic equation have?

12 Find the solution/s to each equation.

a $x^2 - 4 = 0$ **b** $x^2 + 3x - 10 = 0$
c $x^2 - 6x + 9 = 0$ **d** $x^2 + 1 = 0$
e $x^2 + 7x = 0$ **f** $x^2 + 12x + 32 = 0$
g $x^2 - x - 72 = 0$ **h** $x^2 + 2x + 5 = 0$

NOTE Some of these equations cannot be solved.

PROBLEM SOLVING AND REASONING

13 Solve each quadratic equation. (Hint: first rearrange each equation into the form $ax^2 + bx + c = 0$.)

a $x^2 + 2x = 3$	**b** $x^2 - 20 = x$	**c** $x^2 = 25$
d $x^2 + 4x + 11 = 7$	**e** $x^2 + 10x = 2x$	**f** $x^2 - 4x = 36 - 4x$
g $4x - x^2 = x$	**h** $x^2 + 41 = 12x + 5$	**i** $x^2 - x + 21 = -11x$
j $x(x - 7) = 8$	**k** $(x - 3)^2 = 1$	**l** $4x + 12 = x^2$

14 Use the 'solve' function of a calculator or other digital technology to solve each quadratic equation in question **13** and compare your solutions.

15 Louise borrowed some money from her brother Rick. This can be represented by the quadratic relationship $y = (x - 10)^2$, where y is the amount owed in dollars after x weeks.

a How much money did Louise borrow from Rick? (Hint: find y when $x = 0$.)

b How long did it take for Louise to repay the loan? (Hint: find x when $y = 0$.)

c Did Louise repay the same amount each week? Explain.

16 Alec throws a tennis ball back on to the court from the spectator stand. The height of the ball above the surface of the tennis court can be represented by the quadratic relationship $h = -(t + 2)(t - 4)$, where h is the height in metres after t seconds in the air.

a What is the height of the ball after:

i 1 second? **ii** 2 seconds?

b What is the height of the ball when Alec releases it from his hand?

c How long does it take for the ball to hit the tennis court after it is thrown? (Hint: what is the height of the ball when it hits the tennis court?)

d Explain why there is only one time value for your answer to part **c** even though you have solved a quadratic equation that has two solutions.

17 Frank keeps track of the number of goals he scores in each match of his club's football season. It follows the quadratic relationship $g = n^2 - 8n + 17$, where n is the number of the match from the start of the season and g is the number of goals he scored in that match.

a How many goals did Frank score in the first match of the season? (Hint: find g when $n = 1$.)

b How many goals did he score in the fifth match of the season?

c In which matches did he score exactly five goals? (Hint: find n when $g = 5$.)

d In which match did he score one goal only?

PROBLEM SOLVING AND REASONING

18 This rectangular mouse pad has the dimensions shown. Its length is 8 cm longer than its width.

- **a** Write an expression for the area of the mouse pad. (Hint: area of a rectangle is length × width.)
- **b** Write the expression in expanded form without brackets.
- **c** The area is estimated to be 560 cm^2. Write an equation for the area of the mouse pad.
- **d** Show how the equation can be written as $(x + 28)(x - 20) = 0$. (Hint: write the equation in the general form of $ax^2 + bx + c = 0$ before factorising.)
- **e** Solve the equation. Which value of x is a feasible solution in this scenario? Explain.
- **f** Write the dimensions of the mouse pad.

CHALLENGE

19 The area of a rectangular sand pit is 35 m^2. The length is 2 m longer than the width.

- **a** Write a quadratic equation to represent this scenario. (Hint: let x represent the width of the sand pit in metres.)
- **b** Solve the quadratic equation.
- **c** Write the dimensions of the sand pit.

20 The width of a laptop screen is 12 cm less than its length. Write an equation to represent the scenario where the area of the screen is 640 cm^2 and then solve it to find the dimensions of the screen.

21 Write a quadratic equation to match each set of solutions.

- **a** $x = 1$ and $x = 2$
- **b** $x = 0$ and $x = 10$
- **c** $x = -5$ and $x = 3$

22 Write another two quadratic equations for each set of solutions in question **21**.

23 Solve each equation.

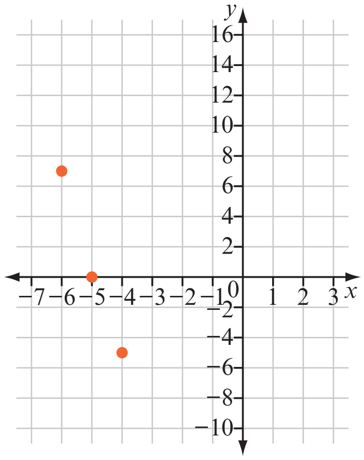
- **a** $-x^2 + 6x = 0$
- **b** $x - x^2 = 0$
- **c** $-x^2 - 2x + 3 = 0$
- **d** $-x^2 + 5x - 6 = 0$

Reflect

Why is the Null Factor Law useful when solving quadratic equations?

4B Plotting quadratic relationships

Start thinking!



1 Copy and complete this table of values using the rule for the quadratic relationship $y = x^2 + 4x - 5$. The first few y values have been calculated for you.

x	−6	−5	−4	−3	−2	−1	0	1	2	3
y	7	0	−5							

2 Plot the values for this relationship on a Cartesian plane. The first few points have been plotted for you.

3 Join the points you have plotted with a smooth line and describe the trend you see.

4 Does the graph show a linear relationship or a non-linear relationship between the variables? Explain.

The graph of a quadratic relationship is called a **parabola**.

5 A parabola changes direction at its **turning point**. If this is at its lowest point, the parabola has a **minimum turning point**. If this is at its highest point, the parabola has a **maximum turning point**.

   a What type of turning point does your graph have?

   b What are the coordinates of the turning point?

6 **a** Rule a vertical line on your graph through the turning point. Why do you think this is called the **axis of symmetry**?

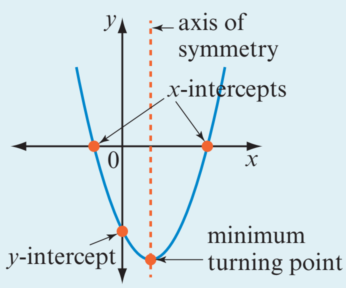
   b What is the equation of the axis of symmetry? (Hint: the equation or rule will be of the form $x = \ldots$)

7 **a** How many y-intercepts does your parabola have? List the coordinates of the y-intercept/s.

   b How many x-intercepts does your parabola have? List the coordinates of the x-intercept/s.

KEY IDEAS

- The graph of a quadratic relationship is a parabola.
- Creating a table of values helps you work out the coordinates of points to be plotted.
- These features can be identified from the graph:
  - type (or nature) of the turning point
  - coordinates of the turning point
  - equation of the axis of symmetry
  - x- and y-intercepts.



EXERCISE 4B Plotting quadratic relationships

EXAMPLE 4B-1 Plotting a parabola

Plot the graph of $y = -x^2 + 2x + 3$ after completing a table for x values from −2 to 4.

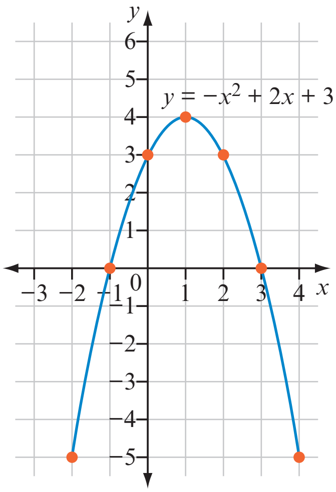
THINK

1 Create a table of values. Substitute each x value into the rule to find the corresponding y value.

2 Use grid paper to draw a Cartesian plane. Plot the points and join them with a smooth line. Label the graph with its rule.

WRITE

x	−2	−1	0	1	2	3	4
y	−5	0	3	4	3	0	−5



UNDERSTANDING AND FLUENCY

1 For each quadratic relationship:

i copy and complete the table of values

ii plot the graph.

a $y = x^2 + 2x - 8$

x	−5	−4	−3	−2	−1	0	1	2	3
y	7			−8		−8			

b $y = 9 - x^2$

x	−4	−3	−2	−1	0	1	2	3	4
y	−7			8					−7

2 Plot the graph of each quadratic relationship after completing a table of values. You may like to use the suggested x values in your table.

a $y = -x^2 - 6x - 5$ (x values from −6 to 0)

b $y = x^2 - 4$ (x values from −3 to 3)

c $y = x^2 - 2x - 15$ (x values from −4 to 6)

d $y = x^2 + 4x$ (x values from −5 to 1)

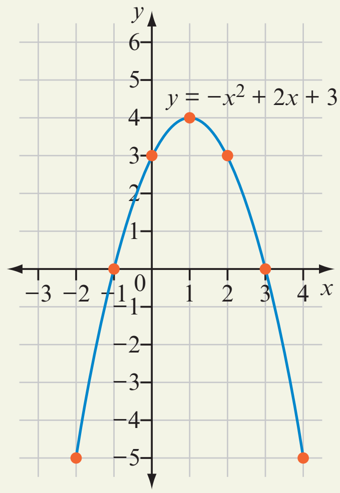
e $y = -x^2 + 2x$ (x values from −1 to 3)

f $y = x^2 - 6x + 9$ (x values from 0 to 6)

EXAMPLE 4B-2 Identifying features of a parabola

For the graph shown, identify:

a whether the parabola has a minimum or maximum turning point
b the coordinates of the turning point
c the equation of the axis of symmetry
d the y-intercept
e the x-intercepts.



THINK

a Locate the point where the graph changes direction. It is at the highest point on the parabola, so it is a maximum turning point.

b Write the x- and y-coordinates of the turning point.

c Locate the axis of symmetry. This is the vertical line that 'cuts' the parabola exactly in half.

d Locate the y-intercept. This is where the parabola crosses the y-axis.

e Locate the x-intercepts. This is where the parabola crosses the x-axis.

WRITE

a Parabola has a maximum turning point.

b Coordinates of turning point are (1, 4).

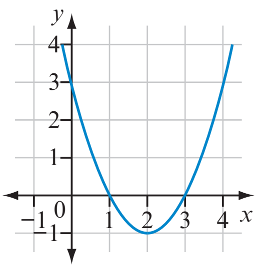
c The equation of the axis of symmetry is $x = 1$.

d y-intercept is 3.
The coordinates of the y-intercept are (0, 3).

e x-intercepts are -1 and 3.
The coordinates of the x-intercepts are $(-1, 0)$ and $(3, 0)$.

UNDERSTANDING AND FLUENCY

3 For the graph shown, identify:
a whether the parabola has a minimum or maximum turning point
b the coordinates of the turning point
c the equation of the axis of symmetry
d the y-intercept
e the x-intercepts.



4 For each graph drawn in question **1**, identify:
i whether the parabola has a minimum or maximum turning point
ii the coordinates of the turning point
iii the equation of the axis of symmetry
iv the y-intercept **v** the x-intercepts.

UNDERSTANDING AND FLUENCY

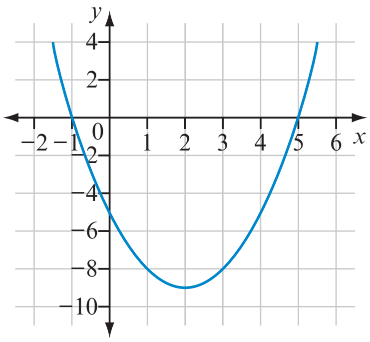
5 For each graph drawn in question **2**, identify:

- **i** whether the parabola has a minimum or maximum turning point
- **ii** the coordinates of the turning point
- **iii** the equation of the axis of symmetry
- **iv** the y-intercept
- **v** the x-intercepts.

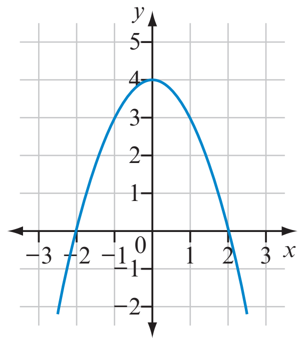
6 Produce each graph in questions **1** and **2** using digital technology.

7 Match each graph with its rule from the list below.

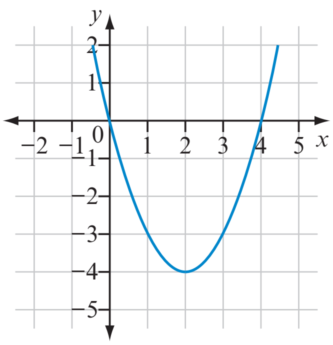
a



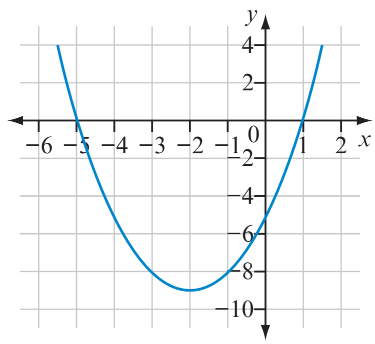
b



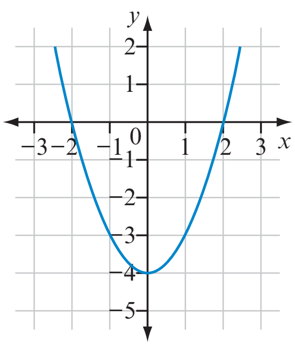
c



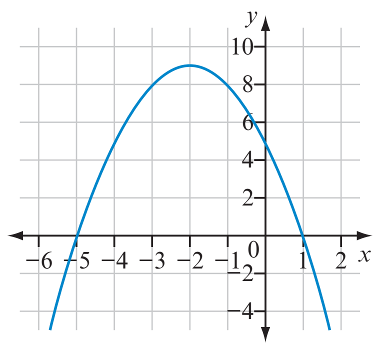
d



e



f



A $y = x^2 + 4x - 5$ **B** $y = x^2 - 4x$ **C** $y = x^2 - 4$

D $y = -x^2 - 4x + 5$ **E** $y = -x^2 + 4$ **F** $y = x^2 - 4x - 5$

8 A parabola with a minimum turning point is described as upright. Its shape is similar to the shape of $y = x^2$. A parabola with a maximum turning point is described as inverted. Its shape is upside down compared with that of $y = x^2$. Identify each parabola in question **7** as upright or inverted.

UNDERSTANDING AND FLUENCY

9 Plot the graph of each quadratic relationship after completing the table of values shown. Hence, identify:

- **i** whether the parabola is upright or inverted
- **ii** the type of turning point and its coordinates
- **iii** the y-intercept
- **iv** the x-intercepts.

a $y = 3x^2$

x	−2	−1	0	1	2
y					

b $y = 2x^2 - 2$

x	−2	−1	0	1	2
y					

c $y = -3x^2 - 6x$

x	−3	−2	−1	0	1
y					

d $y = 2x^2 - 4x - 6$

x	−3	−1	1	3	5
y					

e $y = -\frac{1}{2}x^2 + 4x - 6$

x	0	2	4	6	8
y					

f $y = 3x^2 + 12x + 12$

x	−3	−2	−1	0	1
y					

PROBLEM SOLVING AND REASONING

10 Lily's cap fell to the ground while she was on a roller coaster ride. The position of the cap during the time it was falling can be described by the relationship $h = 100 - 4t^2$ where h was the height of the cap above the ground in metres after t seconds.

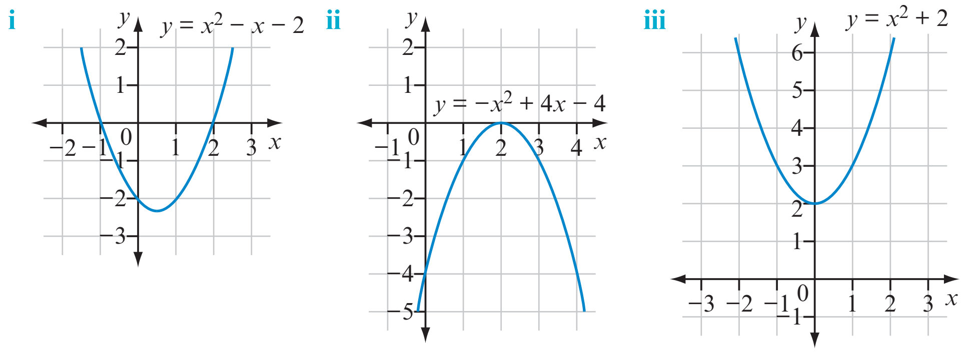
- **a** Plot the graph of the relationship for t values from 0 to 6.
- **b** Why didn't you draw the full parabola for this scenario?
- **c** What was the height of the cap after:
  - **i** 2 s? **ii** 3 s?
- **d** At what height off the ground did the cap start to fall?
- **e** How long did it take for the cap to hit the ground?

11 Kim throws a javelin. The position of the tip of the javelin can be represented by the quadratic relationship $y = -0.05x^2 + 1.5x + 1.55$, where y is the height above the ground and x is the horizontal distance from where the javelin was thrown. Both x and y are in metres.

- **a** Plot the graph of this relationship. Use 0, 5, 10, 15, …, 40 as the x values in the table.
- **b** What was the greatest height reached by the javelin?
- **c** At what height off the ground was the javelin thrown?
- **d** What horizontal distance did the javelin travel before hitting the ground?

PROBLEM SOLVING AND REASONING

12 Look at the graphs of these three quadratic relationships.



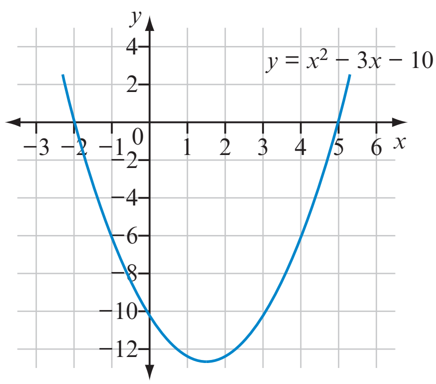
a How many x-intercepts does each parabola have?

b Can a parabola have more than two x-intercepts? Explain.

c How many y-intercepts does each parabola have?

d Can you draw a parabola with a different number of y-intercepts? Explain.

13 Consider the graph of $y = x^2 - 3x - 10$ shown at right.



a Identify the x-intercepts from the graph.

b Solve the quadratic equation $x^2 - 3x - 10 = 0$ by first factorising and then using the Null Factor Law.

c Compare your answers for parts **a** and **b**. What do you notice?

d Explain why you can use the graph to solve $x^2 - 3x - 10 = 0$. (Hint: what is the y value at each x-intercept?)

CHALLENGE

14 Solve each of these quadratic equations using the graphs in question **7**.

a $x^2 + 4x - 5 = 0$	**b** $x^2 - 4x = 0$	**c** $x^2 - 4 = 0$
d $-x^2 - 4x + 5 = 0$	**e** $-x^2 + 4 = 0$	**f** $x^2 - 4x - 5 = 0$

15 If a parabola has x-intercepts at (2, 0) and (8, 0), what is the x-coordinate of the turning point?

16 If a parabola has only one x-intercept, at (−4, 0), what is the x-coordinate of the turning point?

17 If an upright parabola has a turning point at (−3, 1), how many x-intercepts does it have? How many y-intercepts does it have?

18 If an inverted parabola has a turning point at (10, 0), how many x-intercepts does it have? How many y-intercepts does it have?

Reflect

How can you recognise a parabola?

4C Parabolas and transformations

Start thinking!

You can use the basic shape and features of a parabola to draw them without plotting points from a table. One way is to perform transformations such as dilation, reflection and translation on the basic parabola $y = x^2$ to produce the graph of another parabola.

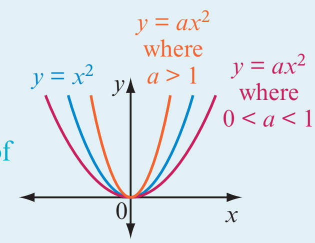
1 **a** Plot the graph of $y = x^2$ on a Cartesian plane for x values from −3 to 3.
  b Identify the features of this parabola (type and coordinates of the turning point, equation of the axis of symmetry, x- and y-intercepts).

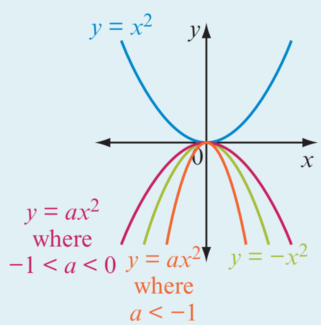
2 **a** On the same Cartesian plane as question **1**, plot the graphs of:
  i $y = 2x^2$ **ii** $y = 3x^2$ **iii** $y = 4x^2$ **iv** $y = \frac{1}{2}x^2$ **v** $y = \frac{1}{4}x^2$.
  b Compare with the graph of $y = x^2$. Which features are the same? What makes each one different?
  c Which graphs are: **i** narrower than $y = x^2$? **ii** wider than $y = x^2$?
  d Explain how the coefficient of the x^2 term affects each graph.
  e Each parabola you drew in part **a** can be produced by dilating the graph of $y = x^2$. For example, the graph of $y = 2x^2$ is produced by dilating the graph of $y = x^2$ by a factor of 2 (made narrower). Describe the dilation performed to produce each of the other graphs.

3 Describe how you can recognise from its rule whether a parabola will be narrower or wider than the graph of $y = x^2$.

KEY IDEAS

- The graph of $y = x^2$ is an upright parabola with a minimum turning point at (0, 0).
- The graph of $y = -x^2$ is an inverted parabola with a maximum turning point at (0, 0). This graph is the reflection of the graph of $y = x^2$ in the x-axis.
- Transformations such as dilation, reflection and translation of the graph of $y = x^2$ are often used to produce graphs of other quadratic relationships.
- For rules of the form $y = ax^2$ where a is positive, there is dilation only (dilation factor is a). For $0 < a < 1$ (coefficient between 0 and 1), the dilation produces a wider graph than $y = x^2$. For $a > 1$ (coefficient larger than 1), the dilation produces a narrower graph than $y = x^2$.
- For rules of the form $y = ax^2$ where a is negative, there is dilation and reflection. For $-1 < a < 0$ (coefficient is between −1 and 0), the dilation produces a wider graph than $y = x^2$ which is reflected in the x-axis. For $a < -1$ (coefficient is less than −1), the dilation produces a narrower graph than $y = x^2$, which is reflected in the x-axis.





EXERCISE 4C Parabolas and transformations

1 Consider the graphs of $y = x^2$, $y = 2x^2$ and $y = \frac{1}{2}x^2$. You may like to plot them on the same Cartesian plane.

a Which parabola is wider than the graph of $y = x^2$?

b Write the rule for another parabola that is wider than the graph of $y = x^2$.

c Which parabola is narrower than the graph of $y = x^2$?

d Write the rule for another parabola that is narrower than the graph of $y = x^2$.

EXAMPLE 4C-1 Describing a transformation used to produce a graph from the graph of $y = x^2$

For each rule below, describe the transformation needed to produce each graph from the graph of $y = x^2$.

a $y = 4x^2$ **b** $y = \frac{1}{3}x^2$

THINK	WRITE
a Since the coefficient of the x^2 term is 4, a dilation of factor 4 is needed. This will produce a narrower parabola.	**a** The graph of $y = x^2$ is dilated by a factor of 4 to produce $y = 4x^2$. The graph of $y = 4x^2$ will be narrower than the graph of $y = x^2$.
b Since the coefficient of the x^2 term is $\frac{1}{3}$, a dilation of factor $\frac{1}{3}$ is needed. This will produce a wider parabola.	**b** The graph of $y = x^2$ is dilated by a factor of $\frac{1}{3}$ to produce $y = \frac{1}{3}x^2$. The graph of $y = \frac{1}{3}x^2$ will be wider than the graph of $y = x^2$.

UNDERSTANDING AND FLUENCY

2 For each rule below, describe the transformation needed to produce each graph from the graph of $y = x^2$.

a $y = 2x^2$ **b** $y = \frac{1}{2}x^2$

c your rule from question **1b**

d your rule from question **1d**

> **NOTE** If you drew graphs for question **1**, you can use them to help you.

3 a Plot the graph of $y = -x^2$ on a Cartesian plane for x values from −3 to 3.

b Compare it to the graph of $y = x^2$. Has dilation been performed? Which transformation do you think has been performed?

c Has the graph of $y = x^2$ been reflected in the x-axis or the y-axis to produce $y = -x^2$? Explain.

4 a On the same Cartesian plane as question **3**, plot the graphs of:

i $y = -2x^2$ **ii** $y = -3x^2$ **iii** $y = -4x^2$ **iv** $y = -\frac{1}{2}x^2$ **v** $y = -\frac{1}{4}x^2$.

b Compare each graph with the graph of $y = -x^2$. Which features are the same? What makes each one different?

c Which graphs are: **i** narrower than $y = -x^2$? **ii** wider than $y = -x^2$?

EXAMPLE 4C-2 Describing a transformation used to produce a graph from the graph of $y = -x^2$

For each rule below, describe the transformation needed to produce each graph from the graph of $y = -x^2$.

a $y = -6x^2$ **b** $y = -\frac{1}{3}x^2$

THINK

a A dilation of factor 6 is needed. This will produce a narrower parabola.

b A dilation of factor $\frac{1}{3}$ is needed. This will produce a wider parabola.

WRITE

a The graph of $y = -x^2$ is dilated by a factor of 6 to produce $y = -6x^2$. The graph of $y = -6x^2$ will be narrower than the graph of $y = -x^2$.

b The graph of $y = -x^2$ is dilated by a factor of $\frac{1}{3}$ to produce $y = -\frac{1}{3}x^2$. The graph of $y = -\frac{1}{3}x^2$ will be wider than the graph of $y = -x^2$.

5 Describe the transformation needed to produce the graph for each rule below from the graph of $y = -x^2$.

a $y = -2x^2$

b $y = -\frac{1}{2}x^2$

c $y = -\frac{1}{4}x^2$

d $y = -4x^2$

NOTE Use your graph from question 4 to help you. Remember that you are comparing to the graph of $y = -x^2$.

EXAMPLE 4C-3 Identifying transformations to produce a graph from the graph of $y = x^2$

For each rule below, identify the transformation/s needed to produce each graph from the graph of $y = x^2$. Describe the effect of the transformations.

a $y = 3x^2$ **b** $y = -\frac{1}{5}x^2$

THINK

a Look at the coefficient of the x^2 term. Since it is 3, a dilation of factor 3 is needed. This will produce a narrower upright parabola.

b Look at the coefficient of the x^2 term. Since it is $-\frac{1}{5}$, a dilation of factor $\frac{1}{5}$ is needed and then a reflection in the x-axis. This will produce a wider parabola that is inverted (turned upside down).

WRITE

a Dilation is needed.
The graph of $y = x^2$ is dilated by a factor of 3 to produce $y = 3x^2$. The graph of $y = 3x^2$ will be narrower than the graph of $y = x^2$.

b Dilation and reflection are needed.
The graph of $y = x^2$ is dilated by a factor of $\frac{1}{5}$ to produce $y = \frac{1}{5}x^2$ and then reflected in the x-axis to produce $y = -\frac{1}{5}x^2$. The graph of $y = -\frac{1}{5}x^2$ will be wider than the graph of $y = x^2$ and reflected in the x-axis.

UNDERSTANDING AND FLUENCY

6 Identify the transformation/s needed to produce the graph of each rule below from the graph of $y = x^2$. Describe the effect of the transformations.

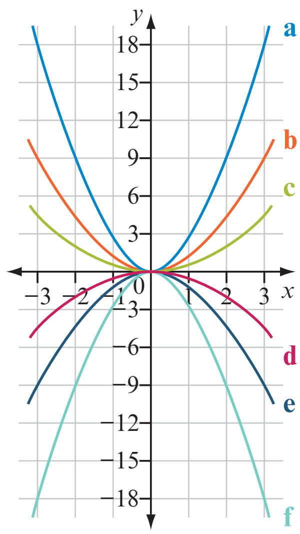
a $y = 5x^2$ **b** $y = -x^2$ **c** $y = -4x^2$ **d** $y = \frac{1}{4}x^2$

e $y = 10x^2$ **f** $y = -\frac{1}{7}x^2$ **g** $y = -8x^2$ **h** $y = -\frac{2}{3}x^2$

7 In question **4**, there was more than one transformation performed on the graph of $y = x^2$ to produce each graph.

a Which two transformations were used?

b Describe the transformations performed on the graph of $y = x^2$ to produce each parabola.



8 Match each graph drawn on this Cartesian plane with its rule from the list provided below.

A $y = x^2$ **B** $y = -2x^2$

C $y = -\frac{1}{2}x^2$ **D** $y = \frac{1}{2}x^2$

E $y = 2x^2$ **F** $y = -x^2$

9 **a** Plot the graph of $y = x^2$ on a Cartesian plane for x values from −3 to 3.

b On the same Cartesian plane, plot the graphs of:

i $y = x^2 + 1$ **ii** $y = x^2 + 2$

iii $y = x^2 + 3$ **iv** $y = x^2 + 4$

c Compare each graph with the graph of $y = x^2$.

i Which features are the same?

ii What makes each one different?

d Explain how the **constant term** added to x^2 affects each graph.

e Each parabola you drew in part **b** can be produced by translating the graph of $y = x^2$. For example, the graph of $y = x^2$ is translated 1 unit up to produce the graph of $y = x^2 + 1$. Describe the translation performed to produce each of the other graphs.

10 **a** On the same Cartesian plane as question **9**, plot the graphs of:

i $y = x^2 - 1$ **ii** $y = x^2 - 2$

iii $y = x^2 - 3$ **iv** $y = x^2 - 4$.

b Compare each graph with the graph of $y = x^2$.

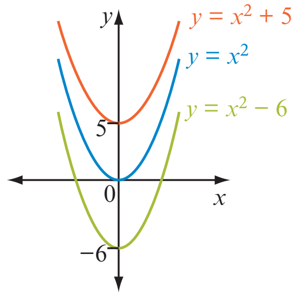
i Which features are the same?

ii What makes each one different?

c Explain how the constant term subtracted from x^2 affects each graph.

d Each parabola you drew in part **a** can be produced by translating the graph of $y = x^2$. For example, the graph of $y = x^2$ is translated 1 unit down to produce the graph of $y = x^2 - 1$. Describe the translation performed to produce each of the other graphs.

UNDERSTANDING AND FLUENCY



11 In questions **9** and **10**, you looked at graphs with rules of the form $y = x^2 + k$, where k could be any number.

a Describe how the value of k affects the graph of $y = x^2$.

b Explain how you know whether to move the graph of $y = x^2$ up or down.

c Copy and complete these sentences using the words *up* and *down*.

When k is positive, the graph of $y = x^2$ is moved ________.

When k is negative, the graph of $y = x^2$ is moved ________.

d What does it mean if k is 0?

12 For each rule below:

i identify the value of k, if the graph has the rule $y = x^2 + k$

ii describe the transformation needed to produce each graph from the graph of $y = x^2$.

a $y = x^2 + 6$ **b** $y = x^2 - 7$ **c** $y = x^2 - 5$ **d** $y = x^2 + 8$

e $y = x^2 + 9$ **f** $y = x^2 - 11$ **g** $y = x^2 + 1.5$ **h** $y = x^2 - 7.2$

13 **a** Plot the graph of $y = x^2$ on a Cartesian plane for x values from −6 to 6.

b On the same Cartesian plane, plot the graphs of:

i $y = (x - 1)^2$ **ii** $y = (x - 2)^2$

iii $y = (x - 3)^2$ **iv** $y = (x - 4)^2$

c Compare each graph with the graph of $y = x^2$.

i Which features are the same?

ii What makes each one different?

d Explain how the constant term subtracted from x before squaring affects each graph.

e Each parabola you drew in part **b** can be produced by translating the graph of $y = x^2$. For example, the graph of $y = x^2$ is translated 1 unit right to produce the graph of $y = (x - 1)^2$. Describe the translation performed to produce each of the other graphs.

14 **a** On the same Cartesian plane as question **13**, plot the graphs of:

i $y = (x + 1)^2$ **ii** $y = (x + 2)^2$

iii $y = (x + 3)^2$ **iv** $y = (x + 4)^2$

b Compare each graph with the graph of $y = x^2$.

i Which features are the same?

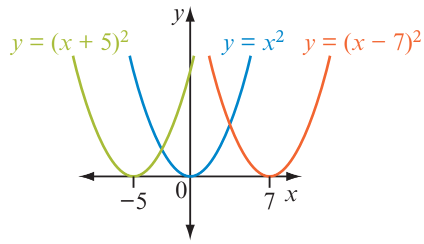
ii What makes each one different?

c Explain how the constant term added to x before squaring affects each graph.

d Each parabola you drew in part **a** can be produced by translating the graph of $y = x^2$. For example, the graph of $y = x^2$ is translated 1 unit left to produce the graph of $y = (x + 1)^2$. Describe the translation performed to produce each of the other graphs.

UNDERSTANDING AND FLUENCY

15 In questions **13** and **14**, you looked at graphs with rules of the form $y = (x - h)^2$, where h could be any number.



a Describe how the value of h affects the graph of $y = x^2$.

b Explain how you know whether to move the graph of $y = x^2$ left or right.

c Copy and complete these sentences using the words *left* and *right*.

When h is positive, the graph of $y = x^2$ is moved _______.

When h is negative, the graph of $y = x^2$ is moved _______.

d What does it mean if h is 0?

16 For each rule below:

i identify the value of h, if the graph has the rule $y = (x - h)^2$

ii describe the transformation needed to produce each graph from the graph of $y = x^2$.

a $y = (x - 5)^2$ **b** $y = (x + 7)^2$ **c** $y = (x - 6)^2$ **d** $y = (x + 9)^2$

e $y = (x + 8)^2$ **f** $y = (x - 12)^2$ **g** $y = (x - 2.5)^2$ **h** $y = (x + 6.7)^2$

PROBLEM SOLVING AND REASONING

17 You can now perform more than one transformation on the graph of $y = x^2$.

a Plot the graph of each of these rules.

i $y = -x^2 + 2$ **ii** $y = -x^2 - 3$ **iii** $y = -(x - 3)^2$

iv $y = -(x + 2)^2$ **v** $y = (x - 3)^2 + 2$ **vi** $y = -(x + 2)^2 - 3$

b List the coordinates of the turning point for each graph.

c Identify whether each graph is upright or inverted.

d Match each description with one of the rules listed in part **a**.

A The graph of $y = x^2$ is reflected in the x-axis and then translated 3 units right.

B The graph of $y = x^2$ is reflected in the x-axis and then translated 2 units up.

C The graph of $y = x^2$ is translated 3 units right and 2 units up.

D The graph of $y = x^2$ is reflected in the x-axis and then translated 2 units left.

E The graph of $y = x^2$ is reflected in the x-axis and then translated 2 units left and 3 units down.

F The graph of $y = x^2$ is reflected in the x-axis and then translated 3 units down.

CHALLENGE

18 One of the general forms of a quadratic relationship is $y = a(x - h)^2 + k$.

a Identify a, h and k for each rule in question **17a**.

b Explain how the coordinates of the turning point can be worked out from the values of h and k.

c Explain how the value of a affects whether the parabola is upright or inverted.

d Summarise your findings to explain how you think the values of a, h and k affect the graph of $y = x^2$. (Hint: describe the transformation that each value relates to.)

Reflect

Why is the graph of $y = x^2$ chosen as the basic parabola for transformations to be performed on to produce other parabolas?

4D Sketching parabolas using transformations

Start thinking!

Now that you can see the effect of transformations performed on the basic parabola of $y = x^2$, this knowledge can help you sketch a parabola without plotting points from a table.

1 Consider each quadratic relationship.

i $y = x^2 - 5$ **ii** $y = (x + 5)^2$ **iii** $y = -5x^2$

iv $y = -x^2 + 5$ **v** $y = -(x - 5)^2$

a Describe the transformation/s to be performed on the graph $y = x^2$ to produce the graph of each relationship.

b Will each parabola be upright or inverted? Explain how you can see this from the rule.

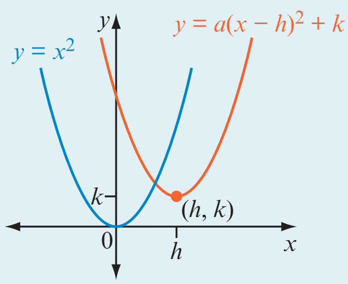
c Write the coordinates of the turning point of each parabola. Explain how you were able to do this.

d Use your answers from parts **a–c** to sketch the graph of each quadratic relationship.

e Which features did you use to sketch these graphs?

2 In your own words, explain how you can sketch a quadratic relationship without plotting points from a table.

KEY IDEAS



- Quadratic relationships can be written in the general form $y = a(x - h)^2 + k$. This is known as the turning point form, as the coordinates of the turning point of the parabola can be easily identified as (h, k).
- Transformations can be performed on the graph of $y = x^2$ to produce the sketch graph of $y = a(x - h)^2 + k$.

$$y = a(x - h)^2 + k$$

- a: dilation (narrower or wider). For $a > 0$, upright parabola. For $a < 0$, inverted parabola (reflection in x-axis).
- h: horizontal translation of h units. For $h > 0$, move right. For $h < 0$, move left.
- k: vertical translation of k units. For $k > 0$, move up. For $k < 0$, move down.

- You do not need to use grid paper to sketch a graph, as only the shape and important information is shown.

EXERCISE 4D Sketching parabolas using transformations

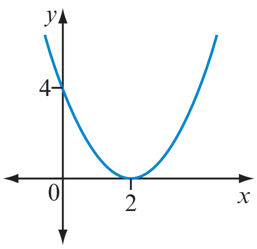
UNDERSTANDING AND FLUENCY

1 For each sketch graph shown below:

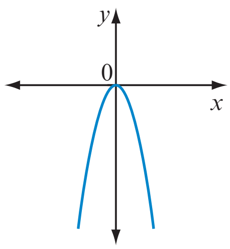
- **i** identify whether the parabola is upright or inverted
- **ii** write the coordinates of the turning point
- **iii** describe how the graph may have been produced from $y = x^2$
- **iv** match each graph with its rule from the list shown at right.

A $y = -2x^2$
B $y = x^2 + 2$
C $y = (x - 2)^2$

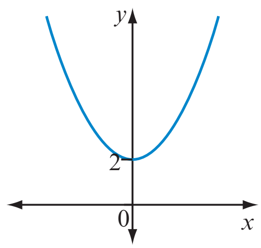
a



b

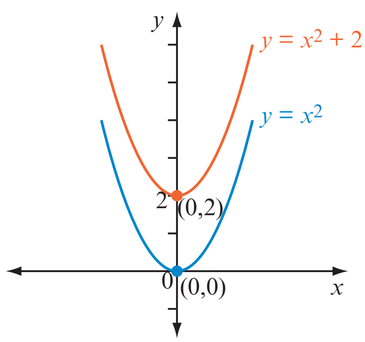


c



EXAMPLE 4D-1 Sketching a parabola by performing a vertical translation

Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a vertical translation to sketch the graph of $y = x^2 + 2$. Clearly show the coordinates of the turning point.

THINK	WRITE
1 Identify the transformation. Vertical translation of 2 units up. (No dilation or reflection.)	$y = x^2 + 2$ Graph of $y = x^2$ is translated 2 units up.
2 Sketch the graph of $y = x^2$ and locate its turning point. Translate this point 2 units up. This becomes the turning point for $y = x^2 + 2$.	
3 Use the position of the turning point at (0, 2) and the orientation of the parabola (upright) to sketch the graph.	

2 Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a vertical translation to sketch the graph of each quadratic relationship. Clearly show the coordinates of the turning point on each parabola.

a $y = x^2 + 3$ **b** $y = x^2 + 1$ **c** $y = x^2 - 2$
d $y = x^2 + 6$ **e** $y = x^2 - 4$

EXAMPLE 4D-2 Sketching a parabola by performing a horizontal translation

Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a horizontal translation to sketch the graph of $y = (x - 4)^2$. Clearly show the coordinates of the turning point.

THINK

1 Identify the transformation. Horizontal translation of 4 units right. (No dilation or reflection.)

WRITE

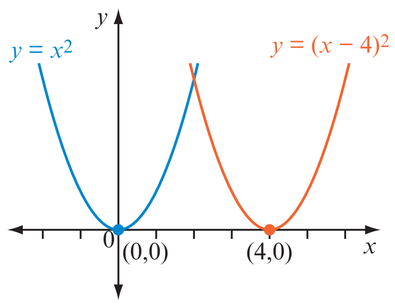
$y = (x - 4)^2$
Graph of $y = x^2$ is translated 4 units right.

THINK

2 Sketch the graph of $y = x^2$ and locate its turning point. Translate this point 4 units right. This becomes the turning point for $y = (x - 4)^2$.

3 Use the position of the turning point at (4, 0) and the orientation of the parabola (upright) to sketch the graph.

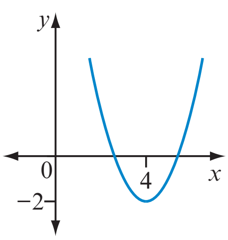
WRITE



3 Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a horizontal translation to sketch the graph of each quadratic relationship. Clearly show the coordinates of the turning point on each parabola.

a $y = (x - 3)^2$ **b** $y = (x - 1)^2$ **c** $y = (x + 2)^2$
d $y = (x + 4)^2$ **e** $y = (x - 5)^2$

4 Look at the graph shown at right.



a Is the parabola upright or inverted?
b Identify the coordinates of the turning point.
c Which of these rules would best match the graph? Explain.

A $y = (x - 2)^2 + 4$ **B** $y = -(x - 4)^2 - 2$
C $y = (x - 4)^2 - 2$ **D** $y = (x + 4)^2 - 2$

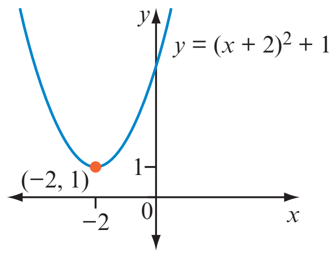
5 Compare $y = (x - 3)^2 + 4$ to the turning point form of a quadratic relationship.

a Identify a, h and k.
b What information can you identify from the values of a, h and k?
c Use (h, k) to write the coordinates of the turning point.
d Explain how (h, k) is related to the translations performed on $y = x^2$ to produce $y = (x - 3)^2 + 4$.

6 Repeat question **5** for the quadratic relationship you identified in question **4c**. Show that the information you obtain matches the graph provided in question **4**.

EXAMPLE 4D-3 Sketching a parabola by performing more than one transformation

Sketch the graph of $y = (x + 2)^2 + 1$ by performing transformations on $y = x^2$.
Clearly show the coordinates of the turning point.

THINK	WRITE
1 Identify the transformations. Horizontal translation of 2 units left and vertical translation of 1 unit up. (No dilation or reflection.) Alternatively, compare to the turning point form of a quadratic, $y = a(x - h)^2 + k$, to identify the transformations. $a = 1$ (upright parabola, same shape as $y = x^2$), $h = -2$ (move 2 units left) and $k = 1$ (move 1 unit up).	$y = (x + 2)^2 + 1$ $y = [x - (-2)]^2 + 1$ Graph of $y = x^2$ is translated 2 units left and 1 unit up.
2 Identify the type and position of the turning point. General coordinates are (h, k).	Minimum turning point at $(-2, 1)$.
3 Use the position of the turning point and the orientation of the parabola (upright) to sketch the graph.	

UNDERSTANDING AND FLUENCY

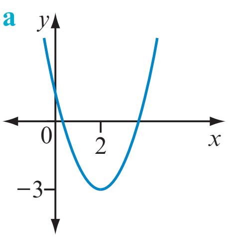
7 Sketch the graph of each quadratic relationship by performing transformations on $y = x^2$. Clearly show the coordinates of the turning point on each parabola.

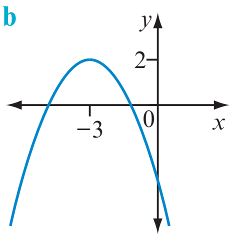
a $y = (x - 2)^2 + 3$ **b** $y = (x - 1)^2 - 2$ **c** $y = (x + 4)^2 + 6$
d $y = (x + 5)^2 - 4$ **e** $y = (x - 7)^2 - 5$

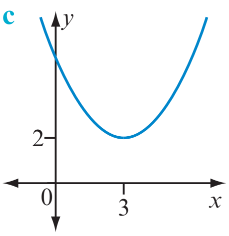
8 Perform a reflection and then a translation on $y = x^2$ to sketch the graph of each quadratic relationship. Show the coordinates of the turning point on each parabola.

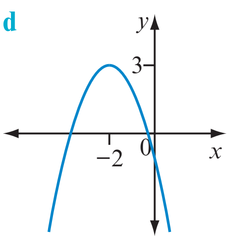
a $y = -(x - 2)^2$ **b** $y = -x^2 + 4$ **c** $y = -(x + 6)^2$ **d** $y = -x^2 - 3$ **e** $y = -(x - 1)^2$

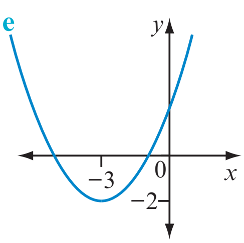
9 Match each graph with its rule from the list provided at right.

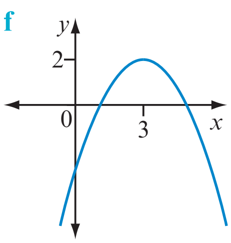
a


b


c


d


e


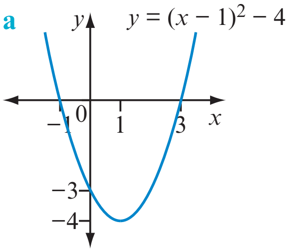
f


A $y = (x - 3)^2 + 2$
B $y = -(x - 3)^2 + 2$
C $y = (x + 3)^2 - 2$
D $y = -(x + 3)^2 + 2$
E $y = (x - 2)^2 - 3$
F $y = -(x + 2)^2 + 3$

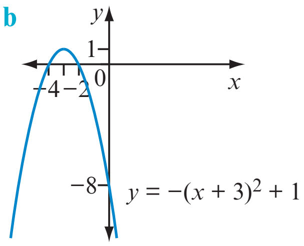
UNDERSTANDING AND FLUENCY

10 Answer the questions below for each parabola.

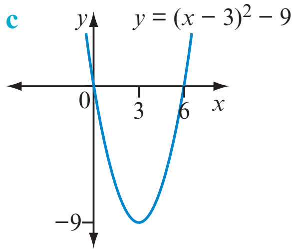
a



b



c



i Is the parabola upright or inverted?

ii What are the coordinates of the turning point?

iii What are the coordinates of the y-intercept and the x-intercepts?

11 Write the rule for each from the given information. Assume each parabola has the same shape as $y = x^2$; that is, no dilation has been performed.

a upright, turning point at (3, 7)
b upright, turning point at (−2, 5)
c inverted, turning point at (2, 4)
d inverted, turning point at (6, −1)
e upright, turning point at (9, 0)
f inverted, turning point at (0, 4)
g inverted, turning point at (−1, −2)
h upright, turning point at (0, −5)

12 Sketch each quadratic relationship after completing these steps:

i describe the transformation/s to be performed on the graph of $y = x^2$

ii identify whether the parabola will be upright or inverted

iii write the coordinates of the turning point.

a $y = x^2 + 4$
b $y = -(x - 8)^2$
c $y = (x - 3)^2 - 2$
d $y = -(x + 4)^2 + 5$
e $y = -4x^2$
f $y = -x^2 - 1$
g $y = (x + 7)^2$
h $y = (x + 1)^2 - 3$

PROBLEM SOLVING AND REASONING

13 A quadratic relationship has the rule $y = (x - 4)^2 + 5$.

a What are the coordinates of the turning point?

b What is the smallest y value that this relationship can have?

14 A quadratic relationship has the rule $y = -(x + 1)^2 + 2$. What is the largest y value that this relationship can have? Explain.

15 Jenna throws a basketball to a teammate. The height of the ball can be represented by the relationship $h = -(t - 2)^2 + 6$, where h is the height in metres after t seconds.

a What are the coordinates of the turning point of this relationship.

b What is the value of h when: **i** $t = 0$? **ii** $t = 4$?

c Sketch the graph of this relationship from $t = 0$ to $t = 4$.

d Use the graph to find:

i the height at which the ball left Jenna's hands

ii the maximum height of the ball during the pass to her teammate.

16 An amateur golfer hits a ball up into the air. The path of the ball follows the relationship $y = -(x - 10)^2 + 100$, where y is the height of the ball for a horizontal distance x from where the ball was hit. Both x and y are in metres.

a Sketch the graph of this relationship from when the ball was hit to when it landed. (Hint: use the fact that a parabola is symmetrical.)

b What was the maximum height of the golf ball?

c How far from the golfer did the ball land?

CHALLENGE

17 For each quadratic relationship, identify:

i whether the graph will be narrower or wider than the graph of $y = x^2$

ii whether the parabola will be upright or inverted

iii the coordinates of the turning point.

a $y = 2(x - 4)^2 - 3$ **b** $y = -3(x + 1)^2 + 5$ **c** $y = 4(x + 2)^2$

d $y = -5x^2 - 4$ **e** $y = -(x - 5)^2 + 4$ **f** $y = \frac{1}{2}(x + 2)^2 + 6$

g $y = -\frac{1}{3}(x - 3)^2 - 4$ **h** $y = \frac{1}{4}x^2 + 3$ **i** $y = -\frac{1}{2}(x - 7)^2$

18 Sketch the graph of each quadratic relationship on the same Cartesian plane. Clearly show the coordinates of the turning point.

a $y = 3(x - 2)^2 + 4$ **b** $y = \frac{1}{2}(x + 4)^2 + 1$ **c** $y = -2(x + 1)^2 - 3$

19 The height above the ground for a bungee jumper is measured from the start of the first downward movement to just before the start of the second downward movement. These measurements form the relationship $h = 5(t - 4)^2 + 10$, where h is the height in metres after time t seconds.

a Sketch a graph of the relationship.

b How high is the person off the ground at the start of the jump?

c What is the lowest height above the ground the person falls to in the first downward movement of the jump?

20 Explain why the graph of $y = -2x^2$ is a reflection in the x-axis of $y = 2x^2$.

21 Write the rule of the parabola that is the reflection in the x-axis of the graph with each rule below.

a $y = 4x^2$ **b** $y = -\frac{1}{3}x^2$ **c** $y = -(x + 2)^2$

d $y = x^2 + 5$ **e** $y = -(x + 1)^2 - 4$ **f** $y = 2(x - 5)^2 - 3$

22 Write a rule for the parabola produced after performing each set of transformations on the graph of $y = x^2$.

a dilation by a factor of 3 then a translation of 2 units right

b reflection in the x-axis then a translation of 4 units down and 5 units left

c dilation by a factor of $\frac{1}{2}$ and a reflection in the x-axis

d dilation by a factor of 4, reflection in the x-axis then a translation of 2 units left

Reflect

How is writing a quadratic relationship in turning point form useful when sketching its graph?

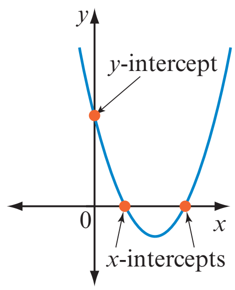
4E Sketching parabolas using intercepts

Start thinking!

1 Explain why transformations are easy to use for a quadratic rule like $y = (x + 3)^2 + 4$ but not as easy for one like $y = x^2 + 4x + 3$.

Another way to identify information to sketch a parabola is to find the x- and y-intercepts.

Consider sketching the graph of $y = x^2 + 4x + 3$.



2 **a** What is the x-coordinate at the y-intercept of any graph?
b Substitute this value for x into the rule $y = x^2 + 4x + 3$ and simplify.
c What is the y-intercept?

3 **a** What is the y-coordinate at the x-intercepts of any graph?
b Substitute this value for y into the rule $y = x^2 + 4x + 3$.
c Solve the equation to find the value of x.
d What are the x-intercepts?

4 List the coordinates of the three points you can use to help sketch the graph of $y = x^2 + 4x + 3$.

5 Look at the coefficient of the x^2 term. Will the parabola be upright or inverted?

6 Plot the three points and draw a parabola through them to produce a sketch graph of $y = x^2 + 4x + 3$. Label your graph with its rule.

7 How could you work out the coordinates of the turning point?

8 In your own words, explain how to sketch a quadratic relationship in the form $y = ax^2 + bx + c$ using intercepts.

KEY IDEAS

- One way of sketching a quadratic relationship is to use the x- and y-intercepts. The coordinates of the turning point and the orientation of the parabola (upright or inverted) can also be identified.
- The x-intercept/s are found by substituting $y = 0$ into the rule and solving for x. The equation may need to be factorised first so that the Null Factor Law can be used to solve the quadratic equation. A parabola can have two, one or no x-intercepts.
- The y-intercept is found by substituting $x = 0$ into the rule and simplifying.
- The axis of symmetry of a parabola is midway between the x-intercepts. Hence, the x-coordinate of the turning point is halfway between the x values at the x-intercepts. The y-coordinate of the turning point is found by substituting the x-coordinate into the rule and simplifying.

EXERCISE 4E Sketching parabolas using intercepts

EXAMPLE 4E-1 Finding coordinates of the x- and y-intercepts of a quadratic relationship

Find the coordinates of the x- and y-intercepts for $y = x^2 - 6x$.

THINK	WRITE
1 To find the x-intercept/s, substitute $y = 0$ into the rule. (You may like to swap the sides of the equation.)	$y = x^2 - 6x$ x-intercepts: when $y = 0$, $0 = x^2 - 6x$ $x^2 - 6x = 0$
2 Factorise the quadratic expression on the left side.	$x(x - 6) = 0$
3 Solve the equation using the Null Factor Law.	$x = 0$ or $x - 6 = 0$ $x = 0$ or $x = 6$
4 Write the coordinates of the x-intercepts.	Coordinates of the x-intercepts are (0, 0) and (6, 0).
5 To find the y-intercept, substitute $x = 0$ into the rule and simplify.	y-intercept: when $x = 0$, $y = 0 - 0$ $= 0$
6 Write the coordinates of the y-intercept.	Coordinates of the y-intercept are (0, 0).

UNDERSTANDING AND FLUENCY

1 Copy and complete the given working to find the coordinates of the x- and y-intercepts for each quadratic relationship.

a $y = x^2 - 2x - 15$

x-intercepts: when $y =$ __,

__ $= x^2 - 2x - 15$

$x^2 - 2x - 15 =$ __

$(x +$ __$)(x -$ __$) =$ __

$x +$ __ $= 0$ or $x -$ __ $= 0$

$x =$ __ or $x =$ __

Coordinates of the x-intercepts are (__, __) and (__, __).

y-intercepts: when $x =$ __,

$y =$ __ $-$ __ $-$ __

$=$ __

Coordinates of the y-intercept are (__, __).

b $y = x^2 - 1$

x-intercepts: when $y =$ __,

$0 = x^2 - 1$

$x^2 - 1 =$ __

$(x +$ __$)(x -$ __$) =$ __

$x +$ __ $= 0$ or $x -$ __ $= 0$

$x =$ __ or $x =$ __

Coordinates of the x-intercepts are (__, __) and (__, __).

y-intercepts: when $x =$ __,

$y =$ __ $-$ __

$=$ __

Coordinates of the y-intercept are (__, __).

2 For each quadratic relationship, find the coordinates of:

i the x-intercepts **ii** the y-intercept.

a $y = x^2 - 2x$ **b** $y = x^2 + 8x$ **c** $y = x^2 + 6x + 8$

d $y = x^2 - 8x + 12$ **e** $y = x^2 - 4x - 5$ **f** $y = x^2 - 9$

EXAMPLE 4E-2 Finding coordinates of the turning point using x-intercepts

Find the coordinates of the turning point for $y = x^2 - 6x$.

THINK	WRITE
1 Find the x-intercepts (see Example 4E-1).	$y = x^2 - 6x$ x-intercepts are 0 and 6.
2 Since a parabola is symmetrical, the x-coordinate of the turning point is halfway between the x-intercepts. Alternatively, find the average of the two x values.	Halfway between 0 and 6 is 3, or $x = \frac{0 + 6}{2} = 3$.
3 Find the y-coordinate of the turning point by substituting $x = 3$ into the rule and simplifying.	When $x = 3$, $y = 3^2 - 6 \times 3$ $= 9 - 18$ $= -9$
4 Write the coordinates of the turning point.	Coordinates of the turning point are $(3, -9)$.

UNDERSTANDING AND FLUENCY

3 Copy and complete the given working to find the coordinates of the turning point for each quadratic relationship. (Hint: refer to your answers for question **1**.)

a $y = x^2 - 2x - 15$

x-intercepts are -3 and __.

Halfway between

-3 and __ is __,

or $x = \frac{-3 + \_\_}{2} = \_\_$.

When $x = \_\_$,

$y = \_\_^2 - 2 \times \_\_ - 15$

$= \_\_ - \_\_ - 15$

$= \_\_$

Coordinates of the turning point are (__, __).

b $y = x^2 - 1$

x-intercepts are -1 and __.

Halfway between

-1 and __ is __,

or $x = \frac{-1 + \_\_}{2} = \_\_$.

When $x = \_\_$,

$y = \_\_^2 - \_\_$

$= \_\_ - \_\_$

$= \_\_$

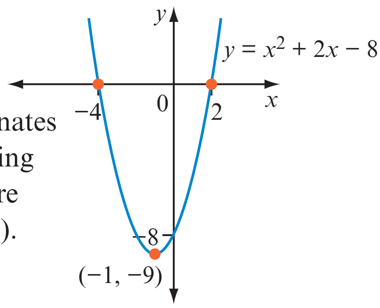
Coordinates of the turning point are (__, __).

4 Find the coordinates of the turning point for each quadratic relationship in question **2**.

5 Sketch the graph of each quadratic relationship in question **2** using your answers to questions **2** and **4**.

EXAMPLE 4E-3 Sketching a parabola using x- and y-intercepts

Sketch the graph of $y = x^2 + 2x - 8$ using intercepts. Label the turning point with its coordinates.

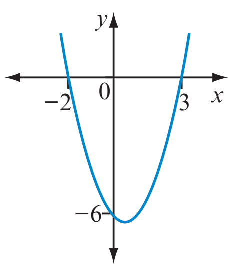
THINK	WRITE
1 Find the x-intercepts by substituting $y = 0$ into the rule and solving for x. Factorise so that the Null Factor Law can be used.	$y = x^2 + 2x - 8$ x-intercepts: when $y = 0$, $x^2 + 2x - 8 = 0$ $(x + 4)(x - 2) = 0$ $x + 4 = 0$ or $x - 2 = 0$ $x = -4$ or $x = 2$ x-intercepts are -4 and 2.
2 Find the y-intercept by substituting $x = 0$ into the rule and simplifying.	y-intercept: when $x = 0$, $y = 0^2 + 2 \times 0 - 8 = -8$ y-intercept is -8.
3 Find the coordinates of the turning point. The x-coordinate is halfway between the x-intercepts (or the average of the two x values).	At turning point, $x = \frac{-4 + 2}{2} = -1$ When $x = -1$, $y = (-1)^2 + 2 \times (-1) - 8 = -9$
4 Plot the points for the x- and y-intercepts and the turning point on a Cartesian plane. **5** Draw an upright parabola through the points and label with the rule. (The parabola is upright since the coefficient of the x^2 term is positive.)	Coordinates of turning point are $(-1, -9)$. 

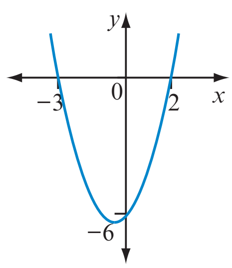
UNDERSTANDING AND FLUENCY

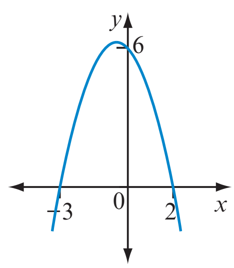
6 Sketch the graph of each quadratic relationship using intercepts. Label the turning point with its coordinates.

a $y = x^2 - 6x + 5$ **b** $y = x^2 + 4x - 12$ **c** $y = x^2 - 2x - 3$ **d** $y = x^2 - 4$
e $y = x^2 + 4x$ **f** $y = x^2 + 2x - 15$ **g** $y = x^2 - 6x - 7$ **h** $y = x^2 - 5x$

7 Match each graph with its rule from the list below.

a


b


c


A $y = x^2 + x - 6$ **B** $y = -x^2 - x + 6$ **C** $y = x^2 - x - 6$

8 Explain how you can tell whether a parabola will be upright or inverted from its rule. Use your answers to question **7** as examples in your explanation.

9 Consider the graphs of $y = (x + 3)(x - 2)$ and $y = -(x + 3)(x - 2)$.

a Find the x-intercepts for each graph.

b Explain why the graphs will be different even though each parabola has the same x-intercepts.

10 For each quadratic relationship:

i identify whether its graph will be an upright or inverted parabola

ii find the coordinates of the x- and y-intercepts

iii find the coordinates of the turning point

iv sketch its graph.

a $y = (x + 5)(x - 3)$ **b** $y = -(x + 5)(x - 3)$ **c** $y = -x(x + 4)$

d $y = x^2 + 4x$ **e** $y = x^2 + 8x + 12$ **f** $y = -x^2 - 8x - 12$

g $y = x^2 - 16$ **h** $y = 16 - x^2$ **i** $y = -x^2 + 6x + 7$

j $y = x^2 - 6x - 7$ **k** $y = x^2 + 3x + 2$ **l** $y = x^2 - x - 6$

11 For each quadratic relationship below:

i identify whether its graph will be an upright or inverted parabola

ii find the coordinates of the x- and y-intercepts

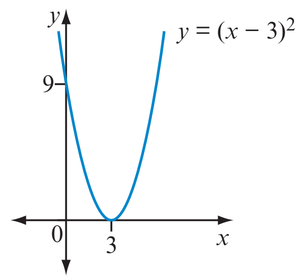
iii identify the coordinates of the turning point

iv sketch its graph.

a $y = (x - 2)^2 - 1$ **b** $y = -(x + 1)^2 + 9$

c $y = (x - 3)^2 - 4$ **d** $y = -(x + 4)^2 + 1$

12 Consider the graph shown at right.



a Is the parabola upright or inverted?

b How many y-intercepts does the parabola have? List the coordinates of the y-intercept/s.

c How many x-intercepts does the parabola have? List the coordinates of the x-intercept/s.

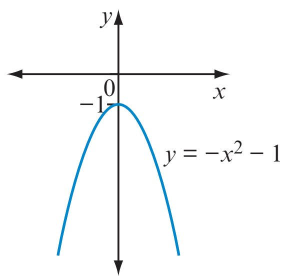
d What are the coordinates of the turning point?

13 Sketch the graph of each quadratic relationship using intercepts. Label the turning point with its coordinates.

a $y = (x - 1)^2$ **b** $y = -(x + 2)^2$ **c** $y = x^2 + 8x + 16$

d $y = x^2 - 4x + 4$ **e** $y = -x^2 - 6x - 9$

14 Consider the graph shown at right.



a Is the parabola upright or inverted?

b How many y-intercepts does the parabola have? List the coordinates of the y-intercept/s.

c How many x-intercepts does the parabola have? List the coordinates of the x-intercept/s.

d What are the coordinates of the turning point?

15 Sketch the graph of each quadratic relationship using intercepts. Label the turning point with its coordinates.

a $y = (x - 3)^2 + 1$ **b** $y = (x + 2)^2 + 3$ **c** $y = -(x - 1)^2 - 4$

d $y = (x + 4)^2 + 1$ **e** $y = -(x + 3)^2 - 2$

PROBLEM SOLVING AND REASONING

16 Rhys fires an arrow from a bow. The position of the arrow can be represented by the quadratic relationship $h = -0.1(d + 1)(d - 15)$ where h is the height above the ground and d is the horizontal distance from where the arrow was fired. Both h and d are in metres.

a Sketch the graph of this relationship by finding the intercepts.

b How high does the arrow reach?

c At what height off the ground was the arrow fired?

d What horizontal distance did the arrow fly before hitting the ground?

17 The amount of money in Teresa's bank account over a 3-week interval can be represented by the quadratic relationship $a = t^2 - 20t + 84$, where a is the account balance in dollars after t days.

a Sketch the graph of this relationship for the 3-week interval.

b How much money was in Teresa's account at the start of the 3 weeks?

c How much money was in her account after 2 days?

d When was Teresa's account first overdrawn?

e What was the highest amount that she owed the bank during the 3 weeks?

f When was her account balance back to zero?

g What was the highest amount in Teresa's account over the 3 weeks?

18 A soccer ball is kicked off the ground. Its path can be represented by the quadratic relationship $y = -0.2x^2 + 2.4x$, where x is the horizontal distance in metres and y is the vertical distance in metres.

a Sketch the graph of this relationship by finding the intercepts.

b What was the maximum height of the soccer ball?

c What horizontal distance had the soccer ball travelled when it was at its maximum height?

d What horizontal distance did the soccer ball travel before hitting the ground?

CHALLENGE

19 For each set of x-intercepts, write a rule for a parabola that would match.

a $x = 2$ and $x = 7$ **b** $x = 0$ and $x = 8$

c $x = -4$ and $x = 5$

20 Write another two quadratic rules for each set of x-intercepts in question **19**.

Reflect

How many x-intercepts and y-intercepts does a parabola have?

4F Circles and other non-linear relationships

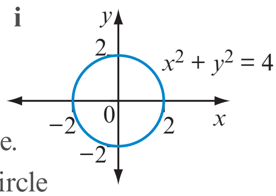
Start thinking!

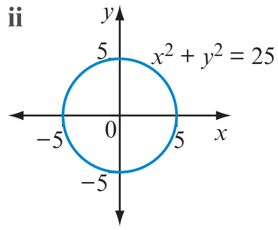
1 Look at the three circles shown.

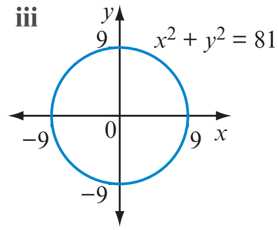
a Write the coordinates of the centre of each circle.

b Identify the radius of each circle.

c Compare the features of each circle with its rule. Can you see any patterns? Explain.

i


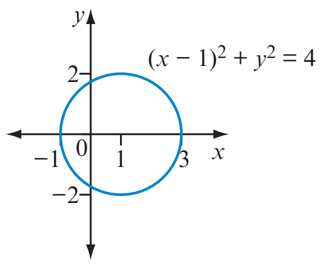
ii


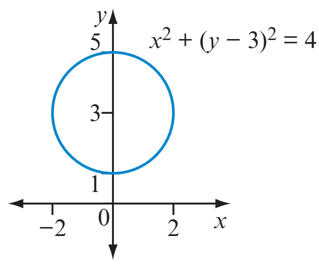
iii


2 Similar to parabolas, you can perform transformations on the basic graph of a circle with centre at (0, 0). Let's consider translations left or right and up or down. Look at each circle.

a Write the coordinates of the centre of each circle.

b By comparing the position of the centre of the circle, describe the translation that has been performed on the basic graph of $x^2 + y^2 = 4$ to produce each circle.

i


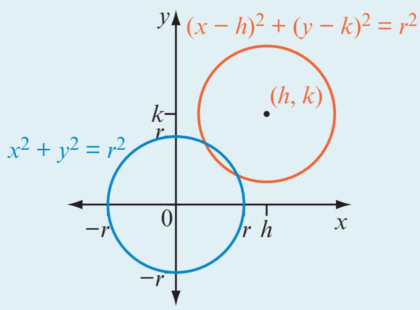
ii


c Compare the features of each circle with its rule. Can you see any patterns? Explain.

d Can you sketch the graph of $(x - 1)^2 + (y - 3)^2 = 4$? Try it. Explain your reasoning.

KEY IDEAS

- Relationships for circles can be written in the general form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) are the coordinates of the centre of the circle and r is the radius.
- The relationship for a circle with centre at (0, 0) and radius r is written as $x^2 + y^2 = r^2$.
- Translations can be performed on the graph of $x^2 + y^2 = r^2$ to produce the sketch graph of $(x - h)^2 + (y - k)^2 = r^2$.



$$(x - h)^2 + (y - k)^2 = r^2$$

horizontal translation of h units
For $h > 0$, move right.
For $h < 0$, move left.

vertical translation of k units
For $k > 0$, move up.
For $k < 0$, move down.

EXERCISE 4F Circles and other non-linear relationships

UNDERSTANDING AND FLUENCY

1 For each rule, identify the coordinates of the centre of the circle and the radius.

a $x^2 + y^2 = 36$ **b** $x^2 + y^2 = 64$ **c** $x^2 + y^2 = 1$ **d** $x^2 + y^2 = 100$

2 Consider the circle with the rule $x^2 + y^2 = 9$.

a What are the coordinates of the centre of this circle?

b What is the radius?

c To sketch this circle easily, you need to mark five points on the Cartesian plane. What do you think these five points will be? Discuss this with a classmate.

d On a Cartesian plane, mark a point for the centre of this circle. Use the radius to mark a point directly above, below, left and right of the centre. List the coordinates of the four points that sit on the circumference of the circle.

e Draw a circle through these four points. You may like to use a pair of compasses to help you. On the scale of the axes, indicate the highest and lowest values of x and y for the circle.

3 Sketch a circle on the Cartesian plane with centre at (0, 0) and radius of 4 units. Write the rule for this circle.

4 Write the rule for a circle with centre at (0, 0) and radius of 7 units.

5 Consider the circle with rule $(x - 2)^2 + (y - 5)^2 = 9$. Compare this to the general rule for a circle $(x - h)^2 + (y - k)^2 = r^2$.

a Identify h, k and r.

b Use your answers to part **a** to identify the coordinates of the centre of the circle.

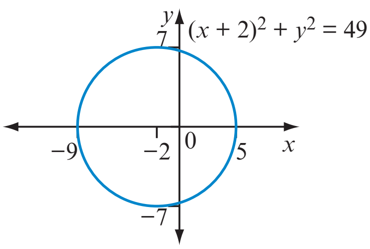
c What is the radius of the circle?

6 Repeat question **5** for each of these rules.

i $(x - 1)^2 + (y - 4)^2 = 16$ **ii** $(x - 6)^2 + (y + 3)^2 = 25$

iii $(x + 4)^2 + (y - 2)^2 = 1$ **iv** $(x + 7)^2 + (y + 2)^2 = 36$

7 Consider this circle drawn on the Cartesian plane.



a What are the coordinates of the centre of the circle?

b What is the radius of the circle?

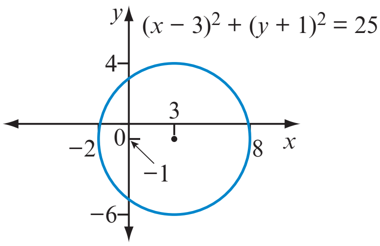
c Explain how the coordinates of the centre of the circle and the radius can be read from the rule if it is written as $(x + 2)^2 + (y - 0)^2 = 7^2$.

d Describe the translation/s that have been performed on the basic graph of $x^2 + y^2 = 49$ to produce the circle.

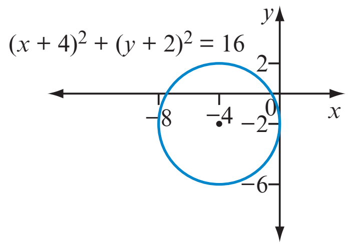
UNDERSTANDING AND FLUENCY

8 Consider each circle drawn on the Cartesian plane.

i



ii



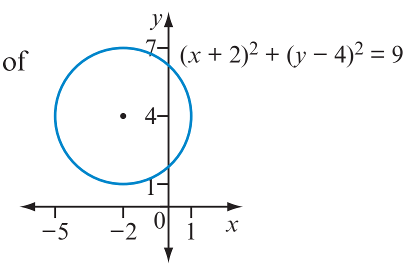
a What are the coordinates of the centre of each circle?

b What is the radius of each circle?

c Describe the translation/s that have been performed on the basic graph of $x^2 + y^2 = r^2$ to produce each circle.

EXAMPLE 4F-1 Sketching a circle from its rule

Identify the coordinates of the centre and the radius of the circle with the rule $(x + 2)^2 + (y - 4)^2 = 9$ and hence sketch its graph.

THINK	WRITE
1 Identify any translations of the graph of $x^2 + y^2 = 9$. Horizontal translation of 2 units left and vertical translation of 4 units up. Alternatively, compare to $(x - h)^2 + (y - k)^2 = r^2$, to identify the translations. $h = -2$ (move 2 units left) and $k = 4$ (move 4 units up).	$(x + 2)^2 + (y - 4)^2 = 9$ The graph of $x^2 + y^2 = 9$ is translated 2 units left and 4 units up.
2 Identify the coordinates of the centre. Perform translations on (0, 0) or use (h, k).	centre at (−2, 4)
3 Identify the radius: $r^2 = 9$ so $r = 3$.	radius of 3 units
4 Sketch the graph by first marking the centre of the circle at (−2, 4) and identifying four points that are 3 units above, below, left and right of the centre. These four points are: (−2, 7), (−2, 1), (−5, 4) and (1, 4).	

9 Identify the coordinates of the centre and the radius of each circle with these rules and hence sketch its graph.

a $(x - 2)^2 + (y - 3)^2 = 4$

b $(x - 1)^2 + (y - 5)^2 = 9$

c $(x + 3)^2 + (y - 2)^2 = 36$

d $(x - 4)^2 + (y + 3)^2 = 25$

e $x^2 + y^2 = 1$

f $(x - 6)^2 + y^2 = 4$

g $x^2 + (y + 4)^2 = 49$

h $(x + 5)^2 + (y + 1)^2 = 16$

10 Produce each graph in question **9** using digital technology. Compare your answers.

EXAMPLE 4F-2 Writing the rule for a circle

Write the rule for a circle with radius of 6 units and centre at (2, −5).

THINK	WRITE
1 Write the general rule for a circle.	Rule for circle with radius r and centre at (h, k) is $(x - h)^2 + (y - k)^2 = r^2$
2 Identify h, k, and r.	$h = 2$, $k = -5$ and $r = 6$
3 Substitute for h, k, and r in the general rule and simplify.	$(x - 2)^2 + [y - (-5)]^2 = 6^2$ $(x - 2)^2 + (y + 5)^2 = 36$

UNDERSTANDING AND FLUENCY

11 Write the rule for each of these circles using the information provided.

a circle with radius of 4 units and centre at (3, 5)

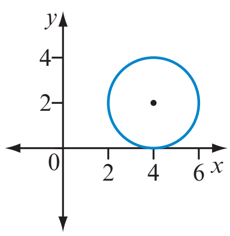
b circle with radius of 5 units and centre at (−2, 4)

c circle with radius of 9 units and centre at (−7, −6)

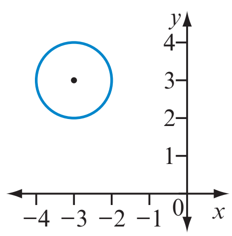
d circle with radius of 11 units and centre at (4, −8)

12 Identify the centre and the radius of each circle and hence write its rule.

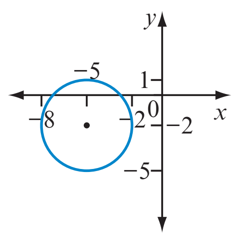
a



b



c



13 Sketch a circle on the Cartesian plane with centre at (2, 5) and radius of 4 units. Write the rule for this circle.

PROBLEM SOLVING AND REASONING

14 An unusual circular running track is mapped on to a Cartesian plane using the relationship $(x - 30)^2 + (y - 40)^2 = 2500$. All measurements are in metres.

a Sketch the graph of this relationship.

b Calculate the length of the running track to the nearest metre. (Hint: the formula for the circumference of a circle is $C = 2\pi r$.)

c The surface of the ground inside the running track is to be sown with grass seed. To the nearest square metre, what area is to be sown? (Hint: the formula for the area of a circle is $A = \pi r^2$.)

15 There are many other non-linear relationships. Let's look at a basic cubic relationship.

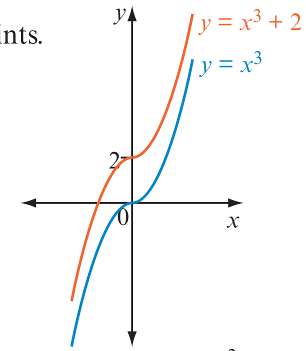
x	−3	−2	−1	0	1	2	3
y	−27					8	

a Plot the graph of $y = x^3$ after completing this table of values. Draw a smooth line through the points.

b Describe the shape of the graph.

c An important feature of the graph of $y = x^3$ is the **point of inflection** at (0, 0). Mark this on your graph.

d The sketch graphs of $y = x^3$ and $y = x^3 + 2$ are shown at right.



i Identify the coordinates of the point of inflection for $y = x^3 + 2$.

ii Describe how you could use a translation to produce the graph of $y = x^3 + 2$ from the graph of $y = x^3$.

e Use your understanding of translations to describe how the graphs of these cubic relationships can be produced from the graph of $y = x^3$.

i $y = x^3 + 1$ **ii** $y = x^3 - 3$ **iii** $y = (x - 2)^3$

iv $y = (x + 4)^3$ **v** $y = (x - 1)^3 + 2$

f Use your answers to part **e** to sketch each relationship. Clearly show the coordinates of the point of inflection on each graph.

g Use a calculator or other digital technology to produce the same graphs.

16 Consider a relationship involving the square root of x.

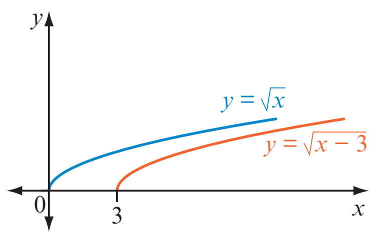
a Plot the graph of $y = \sqrt{x}$ after completing this table of values. Draw a smooth line through the points.

x	0	1	4	9	16	25
y	0			3		

b Describe the shape of the graph. Can you plot points for negative values of x? Explain.

c An important feature of the graph of $y = \sqrt{x}$ is the point on the graph where y is a minimum. The coordinates of this point are (0, 0). Mark this point on your graph.

d The sketch graphs of $y = \sqrt{x}$ and $y = \sqrt{x - 3}$ are shown at right.



i Identify the coordinates of the point on the graph of $y = \sqrt{x - 3}$ where y is a minimum.

ii Describe how you could use a translation to produce the graph of $y = \sqrt{x - 3}$ from the graph of $y = \sqrt{x}$.

e Use your understanding of translations to describe how the graphs of these relationships can be produced from the graph of $y = \sqrt{x}$.

i $y = \sqrt{x} + 2$ **ii** $y = \sqrt{x} - 1$ **iii** $y = \sqrt{x - 1}$ **iv** $y = \sqrt{x + 4}$ **v** $y = \sqrt{x - 2} + 3$

f Use your answers to part **e** to sketch each relationship. Clearly show the coordinates of the point on each graph where y is a minimum.

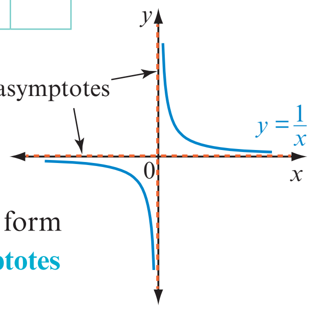
g Use a calculator or other digital technology to produce the same graphs.

CHALLENGE

17 Consider a relationship involving the reciprocal of x.

a Plot the graph of $y = \frac{1}{x}$ after completing this table of values. This graph is called a **hyperbola**.

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
y	$-\frac{1}{3}$			-2					2			

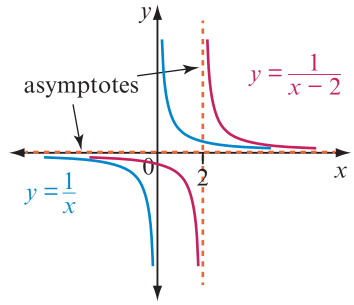


b You will obtain a graph similar to the sketch graph shown. Can you explain why you don't join the plotted points at $x = -\frac{1}{4}$ and $x = \frac{1}{4}$? (Hint: what is the y value when $x = 0$?)

c An important feature of the graph of $y = \frac{1}{x}$ is the lines that form boundaries for each part of the graph. These lines or **asymptotes** lie on the x- and y-axes.

Write the rule of the asymptote for $y = \frac{1}{x}$ that lies on the:

i x-axis **ii** y-axis.



d The sketch graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x - 2}$ are shown at right.

i Identify the rule of each asymptote (shown in red) for the graph of $y = \frac{1}{x - 2}$.

ii Describe how you could use a translation to produce the graph of $y = \frac{1}{x - 2}$ from the graph of $y = \frac{1}{x}$. (Hint: consider the position of the asymptotes.)

e Use your understanding of translations to describe how the graphs of these relationships can be produced from the graph of $y = \frac{1}{x}$.

i $y = \frac{1}{x} + 2$ **ii** $y = \frac{1}{x} - 3$ **iii** $y = \frac{1}{x - 4}$ **iv** $y = \frac{1}{x + 1}$ **v** $y = \frac{1}{x - 3} + 2$

f Use your answers to part **e** to sketch each relationship. Clearly show the asymptotes for each graph.

g Use a calculator or other digital technology to produce the same graphs.

18 Consider the relationships in questions **15–17**.

a Sketch the graphs of $y = -x^3$, $y = -x^3 + 4$ and $y = -(x - 3)^3$ on the same Cartesian plane. (Hint: first reflect the graph of $y = x^3$ in the x-axis.)

b Sketch the graphs of $y = -\sqrt{x}$, $y = -\sqrt{x} + 3$ and $y = -\sqrt{x - 4}$ on the same Cartesian plane. (Hint: first reflect the graph of $y = \sqrt{x}$ in the x-axis.)

c Sketch the graphs of $y = -\frac{1}{x}$, $y = -\frac{1}{x} + 4$ and $y = -\frac{1}{x - 5}$ on the same Cartesian plane. (Hint: first reflect the graph of $\frac{1}{x}$ in the x-axis.)

d Use a calculator or other digital technology to produce the same graphs.

Reflect

How can you identify the radius and the coordinates of the centre of a circle from its rule?

4G Relationships and direct proportion

Start thinking!

In some relationships between two variables, one variable will increase in direct proportion to the other.

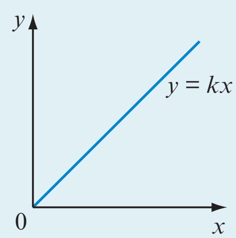
x	0	1	2	3	4
y	0	3	6	9	12

1 Consider the table of values on the right.
   - **a** Plot the points on a Cartesian plane and join them with a smooth line.
   - **b** What type of relationship do you see (linear or non-linear)?
   - **c** As x increases, does y increase or decrease? Is this change constant? Explain.
   - **d** One way to compare the **rate of change** for x and y is to divide each y value by its corresponding x value; that is, work out $\frac{y}{x}$. Calculate this value for each pair of coordinates except (0, 0). What do you notice?
   - **e** What is the gradient of this graph? What does this tell you about the rate of change for x and y?

2 This relationship is an example of direct proportion, as y increases at a constant rate with respect to x. This means that y is directly proportional to x, or $y \propto x$. What features do you see from the graph that show there is direct proportion? (Hint: what type of graph is it and where does the graph cross the axes?)

3 The general form of the rule for this type of graph is $y = mx$, where m is the gradient. When working with proportion, this is often written as $y = kx$, where k is the **constant of proportionality**. Use your answer to question **1e** to write the rule for the relationship above.

KEY IDEAS



- The relationship between x and y will show direct proportion if:
  - y increases as x increases
  - its graph is a straight line passing through (0, 0)
  - the rate of change (gradient or the value of $\frac{y}{x}$ for each coordinate pair) is constant.
- If y is directly proportional to x (or $y \propto x$), the rule for the relationship is $y = kx$, where k is the constant of proportionality and k = rate of change = gradient.
- If y is directly proportional to x^2 or x^3 or $\sqrt{x}$ or $\frac{1}{x}$, the rule will be $y = kx^2$ or $y = kx^3$ or $y = k\sqrt{x}$ or $y = \frac{k}{x}$.

EXERCISE 4G Relationships and direct proportion

EXAMPLE 4G-1 Identifying whether a relationship shows direct proportion

Identify whether each relationship shows direct proportion between x and y by calculating $\frac{y}{x}$ for each coordinate pair.

a

x	0	1	2	3	4
y	0	7	14	21	28

b

x	1	2	3	4	5
y	3	5	7	9	11

THINK	WRITE
a **1** To check whether there is a constant rate of change for x and y, calculate $\frac{y}{x}$ for each pair of coordinates except (0, 0).	**a** $\frac{y}{x} = \frac{7}{1} = 7$; $\frac{y}{x} = \frac{14}{2} = 7$; $\frac{y}{x} = \frac{21}{3} = 7$; $\frac{y}{x} = \frac{28}{4} = 7$
2 State whether there is direct proportion. A constant rate of change is required with the graph of the relationship forming a straight line through (0, 0).	As there is a constant rate of change starting from (0, 0), there is direct proportion between x and y.
b **1** Calculate $\frac{y}{x}$ for each pair of coordinates.	**b** $\frac{y}{x} = \frac{3}{1} = 3$; $\frac{y}{x} = \frac{5}{2} = 2\frac{1}{2}$; $\frac{y}{x} = \frac{7}{3} = 2\frac{1}{3}$; $\frac{y}{x} = \frac{9}{4} = 2\frac{1}{4}$; $\frac{y}{x} = \frac{11}{5} = 2\frac{1}{5}$
2 State whether there is direct proportion.	There is no constant rate of change so there is no direct proportion between x and y.

UNDERSTANDING AND FLUENCY

1 Identify whether each relationship shows direct proportion between x and y by calculating $\frac{y}{x}$ for each coordinate pair.

a

x	0	1	2	3	4
y	0	4	8	12	16

b

x	0	1	2	3	4
y	0	1	8	27	64

c

x	0	1	2	3	4
y	0	9	18	27	36

d

x	1	2	3	4	5
y	1	4	9	16	25

2 For each relationship in question **1** that shows direct proportion between x and y:

- **i** plot its graph
- **ii** find the gradient of the linear graph
- **iii** compare the gradient to the value of $\frac{y}{x}$ for each coordinate pair
- **iv** write its rule using $y = kx$, where k is the gradient of the linear graph.

EXAMPLE 4G-2 Finding the rule for a linear relationship using direct proportion

Find the rule for this relationship.

x	0	1	2	3	4
y	0	7	14	21	28

THINK

1 Calculate $\frac{y}{x}$ for each pair of coordinates except (0, 0). (Alternatively, plot the graph.)

2 State whether there is direct proportion between x and y.

3 Write the proportion statement and general form of the rule. Use the constant value of $\frac{y}{x}$ for k. (Alternatively, find the gradient of the linear graph.)

WRITE

1 $\frac{y}{x} = \frac{7}{1} = 7$; $\frac{y}{x} = \frac{14}{2} = 7$

$\frac{y}{x} = \frac{21}{3} = 7$; $\frac{y}{x} = \frac{28}{4} = 7$

2 As there is a constant rate of change, y is directly proportional to x.

3 $y \propto x$

$y = kx$ where k is 7.

Rule is $y = 7x$.

UNDERSTANDING AND FLUENCY

3 Find the rule for each relationship.

a

x	0	1	2	3	4
y	0	6	12	18	24

b

x	0	2	3	5	9
y	0	4	6	10	18

c

x	0	2	4	6	8
y	0	1	2	3	4

4 Consider this relationship.

x	0	1	2	3	4
y	0	2	8	18	32

a Plot the points on a Cartesian plane and join them with a smooth line.

b What type of relationship do you see? (Linear or non-linear?)

c As x increases, does y increase or decrease? Is this change constant? Explain.

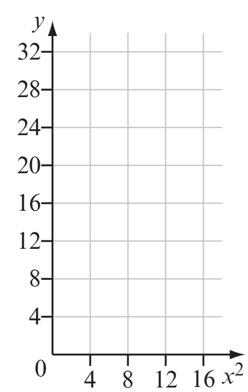
d Calculate the value of $\frac{y}{x}$ for each pair of coordinates. What do you notice?

e Is the relationship between x and y an example of direct proportion? Explain.

f Copy and complete this table of values for the relationship. Instead of x values, look at x^2 values.

x^2	0		4	9	
y	0	2	8		32

g Copy the Cartesian plane shown, then plot the points from the table in part **f** and join them with a smooth line.



h Is the relationship between x^2 and y an example of direct proportion? Explain.

i Suggest how you could use this graph to write the rule for the relationship. Discuss with a classmate.

j Since y is directly proportional to x^2, you can write $y \propto x^2$. So the rule will be of the form $y = kx^2$, where k is a constant value. Find the gradient of the linear graph and hence write the rule for this relationship.

k Check that your rule is correct by substituting a pair of x and y values, such as $x = 2$ and $y = 8$.

EXAMPLE 4G-3 Finding the rule for a non-linear relationship using direct proportion

Find the rule for this relationship.

x	0	1	2	3	4
y	0	5	20	45	80

THINK

1 Check whether there is a constant rate of change for x and y. Calculate $\frac{y}{x}$ for each pair of coordinates except (0, 0). (Alternatively, plot the graph.)

WRITE

$\frac{y}{x} = \frac{5}{1} = 5; \frac{y}{x} = \frac{20}{2} = 10$

$\frac{y}{x} = \frac{45}{3} = 15; \frac{y}{x} = \frac{80}{4} = 20$

THINK

2 State whether there is direct proportion between x and y.

WRITE

As there is not a constant rate of change, y is not directly proportional to x. Try x^2 and y.

THINK

3 Create a table of values for x^2 and y. Check whether there is a constant rate of change for x^2 and y. Calculate $\frac{y}{x^2}$ for each pair of coordinates except (0, 0). (Alternatively, plot the graph.)

WRITE

x^2	0	1	4	9	16
y	0	5	20	45	80

$\frac{y}{x^2} = \frac{5}{1} = 5; \frac{y}{x^2} = \frac{20}{4} = 5$

$\frac{y}{x^2} = \frac{45}{9} = 5; \frac{y}{x^2} = \frac{80}{16} = 5$

THINK

4 State whether there is direct proportion between x^2 and y.

WRITE

As there is a constant rate of change starting from (0, 0), y is directly proportional to x^2.

THINK

5 Write the proportion statement and general form of the rule. Use the constant value of $\frac{y}{x^2}$ for k. (Alternatively, find the gradient of the linear graph.)

WRITE

$y \propto x^2$

$y = kx^2$ where k is 5.

Rule is $y = 5x^2$.

UNDERSTANDING AND FLUENCY

5 Find the rule for each relationship.

a

x	0	1	2	3	4
y	0	3	12	27	48

b

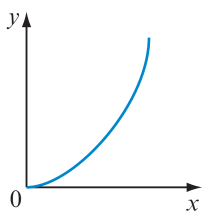
x	0	1	2	3	4
y	0	7	28	63	112

c

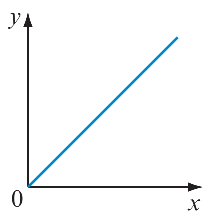
x	0	2	4	6	8
y	0	16	64	144	256

6 State whether each graph shows direct proportion. Provide a reason for your answer.

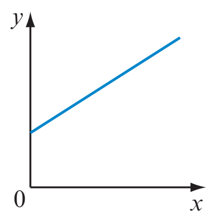
a



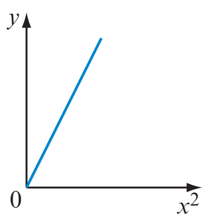
b



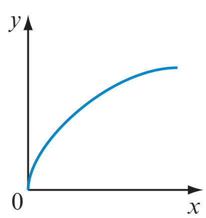
c



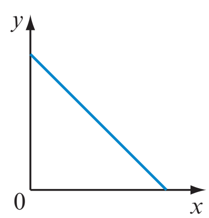
d



e



f



7 Write the constant of proportionality for each rule.

a $y = 4x^2$ **b** $h = 3.5t$ **c** $b = 6\sqrt{a}$ **d** $m = 10n^3$ **e** $y = \frac{2}{x}$

EXAMPLE 4G-4 Finding the constant of proportionality from given information

Find k, the constant of proportionality, using the given information in each case.

a $y = kx$ and $y = 18$ when $x = 3$ **b** $y = kx^2$ and $y = 36$ when $x = 2$

THINK	WRITE
a Substitute the known values for x and y into the rule and solve for k.	**a** $y = kx$ When $x = 3$, $y = 18$ so $18 = k \times 3$ $18 = 3k$ $k = 6$
b Substitute the known values for x and y into the rule and solve for k.	**b** $y = kx^2$ When $x = 2$, $y = 36$ so $36 = k \times 2^2$ $36 = 4k$ $k = 9$

UNDERSTANDING AND FLUENCY

8 Find k, the constant of proportionality, using the given information in each case.

a $y = kx$ and $y = 50$ when $x = 5$

b $y = kx^2$ and $y = 72$ when $x = 3$

c $y = kx^3$ and $y = 32$ when $x = 2$

d $y = k\sqrt{x}$ and $y = 21$ when $x = 9$

e $y = \frac{k}{x}$ and $y = 11$ when $x = 4$

f $y = kx^2$ and $y = 18$ when $x = 6$

9 Find the constant of proportionality using the given information in each case.

a $y \propto x$ and $y = 20$ when $x = 4$

b $p \propto q^2$ and $p = 20$ when $q = 4$

c $d \propto c^3$ and $d = 250$ when $c = 5$

d $h \propto \sqrt{g}$ and $h = 70$ when $g = 100$

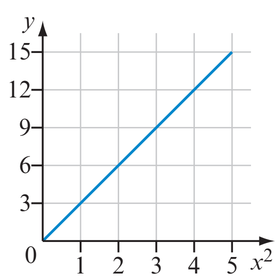
e $a \propto \frac{1}{m}$ and $a = 9$ when $m = 3$

f $w \propto v$ and $w = 15$ when $v = 6$

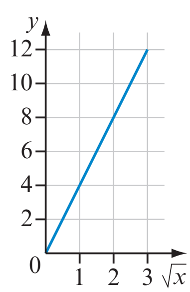
10 Write the rule for each relationship in question **9**.

11 Find the constant of proportionality in each case. (Hint: find the gradient of each straight line.)

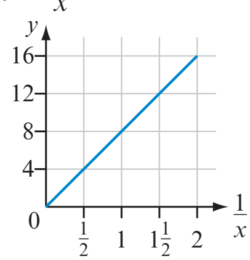
a $y \propto x^2$



b $y \propto \sqrt{x}$



c $y \propto \frac{1}{x}$



12 Write the rule for each relationship in question **11**.

PROBLEM SOLVING AND REASONING

13 Julia listed the cost in dollars (c) for different numbers of bread rolls (n) in a table.

n	0	1	2	3	4	5	6
c	0	0.5	1	1.5	2	2.5	3

a Plot the points on a Cartesian plane and join them with a smooth line.
b What type of relationship do you see?
c Is the relationship between n and c an example of direct proportion? Explain.
d Write the proportion statement and general form of the rule.
e Find the gradient of the linear graph and hence write the rule for this relationship.
f What is the cost of 20 bread rolls?

14 Tom is riding in a cycling event. His distance from the start line at given times is recorded. The table shows values for t (number of hours) and d (distance from the start line in km).

t	0	1	4	9	16
d	0	20	40	60	80

a Plot the points on a Cartesian plane and join them with a smooth line.
b What type of relationship do you see?
c Is the relationship between t and d an example of direct proportion? Explain.
d Copy and complete this table.

$\sqrt{t}$	0	1	2		4
d	0	20	40	60	

e Plot the points on a Cartesian plane and join them with a smooth line.
f Is the relationship between $\sqrt{t}$ and d an example of direct proportion? Explain.
g Write the proportion statement and general form of the rule.
h Find the gradient of the linear graph and hence write the rule for this relationship.
i What distance is Tom from the start line after 36 hours?

CHALLENGE

15 A bus is hired for students to attend a theatre night. The cost to each student depends on how many students agree to go. Some examples are shown in the table where the cost per student in dollars (c) is provided for different numbers of students (n).

n	1	2	4	8	10	20
c	200	100	50	25	20	10

a Plot the points on a Cartesian plane and join them with a smooth line.
b What type of relationship do you see?
c Is the relationship between n and c an example of direct proportion? Explain.
d Copy and complete this table.

$\frac{1}{n}$	1	$\frac{1}{2}$		$\frac{1}{8}$	$\frac{1}{10}$	
c	200	100	50			10

e Plot the points on a Cartesian plane and join them with a smooth line.
f Is the relationship between $\frac{1}{n}$ and c an example of direct proportion? Explain.
g Write the proportion statement and general form of the rule.
h Find the gradient of the linear graph and hence write the rule for this relationship.
i What is the cost of hiring the bus?
j What is the cost per student if 12 students wish to go on the bus?

Reflect

How can direct proportion be used to work out the rule for a non-linear relationship?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

quadratic equation	minimum turning point	dilation	centre of circle
Null Factor Law	maximum turning point	reflection	point of inflection
linear relationship	symmetrical	horizontal translation	hyperbola
non-linear relationship	axis of symmetry	vertical translation	asymptote
parabola	y-intercept	upright parabola	direct proportion
sketch	x-intercept	inverted parabola	rate of change
turning point	transformation	radius of circle	constant of proportionality

MULTIPLE-CHOICE

4A **1** Which is a quadratic expression?

A $y = x^2 + 2$ **B** $x = 34$
C $x^2 - 4x$ **D** $x = 12y$

4A **2** Which is a quadratic equation?

A $y = x^2 + 2$ **B** $x = 34$
C $x^2 - 4x$ **D** $x = 12y$

4B **3** Which of these is a non-linear relationship?

A $y = \frac{1}{3}x$ **B** $y = \frac{x}{5} - 3$
C $y = \frac{1}{3}x - 4$ **D** $y = x^2 - 4$

4C **4** Which rule would produce a graph of $y = x^2$ translated 5 units down?

A $y = x^2 + 5$ **B** $y = 5x^2$
C $y = x^2 - 5$ **D** $y = \frac{1}{5}x^2$

4C **5** The graph of $y = -4x^2 + 1$ is formed from the graph of $y = x^2$. Which statement is not true?

A $y = x^2$ has been dilated.
B $y = x^2$ has been reflected in the x-axis.
C $y = x^2$ has been translated vertically.
D $y = x^2$ has been translated horizontally.

4D **6** The coordinates of the turning point of the graph of $y = -(x - 4)^2$ are:

A (4, 0) **B** (−4, 0)
C (0, 4) **D** (0, −4)

4E **7** The coordinates of the x-intercepts of the graph of $y = x^2 - 4x - 12$ are:

A (0, −6), (0, 2) **B** (0, 6), (0, −2)
C (−6, 0), (2, 0) **D** (6, 0), (−2, 0)

4E **8** The y-intercept of the graph of $y = (x - 5)(x + 2)$ is:

A −2 **B** −10 **C** 5 **D** 10

Questions **9** and **10** refer to the graph of $(x - 2)^2 + (y + 4)^2 = 9$.

4F **9** The radius of the circle is:

A 2 units **B** 3 units
C 4 units **D** 9 units

4F **10** The centre of the circle is located at:

A (2, −4) **B** (−2, 4)
C (4, −2) **D** (−4, 2)

4G **11** If y is directly proportional to x, the graph of the relationship is a:

A parabola **B** circle
C straight line **D** hyperbola

SHORT ANSWER

4A 1 Find the solution/s to each equation. For any that do not have a solution, provide a reason.

a $x^2 - 5x + 6 = 0$ b $x^2 + x - 30 = 0$

c $x^2 + 9 = 0$ d $x^2 - 12x = 0$

4B 2 Plot each relationship and use your graph to identify:

i whether the parabola is upright or inverted

ii the type of turning point and its coordinates

iii the x- and y-intercepts.

a $y = 4x^2 - 4$ b $y = -4x^2 - 4x$

c $y = x^2 - 4x + 4$

Questions 3 and 4 refer to these quadratic relationships.

a $y = 5x^2$ b $y = \frac{1}{5}x^2$

c $y = -5x^2$ d $y = x^2 + 5$

e $y = -x^2 - 5$ f $y = -\frac{1}{5}x^2$

4C 3 Classify each graph as:

i upright or inverted

ii wider or narrower than the graph of $y = x^2$

iii having a minimum or maximum turning point.

4C 4 Describe the transformation/s needed to produce each graph from $y = x^2$.

4D 5 Describe the parabola produced by each rule. Identify whether the curve is upright or inverted, and list the transformations that have been performed on the graph of $y = x^2$.

a $y = -5(x + 2)^2 - 1$ b $y = 5x^2 + 4$

c $y = \frac{1}{4}(x - 5)^2$ d $y = -3x^2$

4D 6 Sketch the graph of each quadratic relationship, clearly showing the coordinates of the turning point.

a $y = (x + 1)^2 - 1$ b $y = -(x + 3)^2 - 3$

c $y = (x - 4)^2 + 4$ d $y = -(x - 2)^2 - 2$

4E 7 Find the coordinates of the x- and y-intercepts for the graph of each rule.

a $y = -x^2 - 4x$ b $y = x^2 - x - 12$

c $y = (x + 5)(x - 4)$

d $y = -(x + 2)(x + 1)$

4E 8 Sketch each graph described in question 7, showing the turning point.

4F 9 Sketch each circle on a Cartesian plane. Clearly identify the centre and radius.

a $(x + 4)^2 + y^2 = 1$

b $x^2 + (y + 3)^2 = 9$

c $(x - 4)^2 + (y - 3)^2 = 16$

d $(x + 3)^2 + (y + 5)^2 = 49$

4F 10 Use the graph of $y = x^3$, $y = \sqrt{x}$ or $y = \frac{1}{x}$ to describe and sketch the graph of each relationship.

a $y = x^3 + 4$ b $y = (x + 4)^3$

c $y = \sqrt{x} + 3$ d $y = \sqrt{x + 3}$

e $y = \frac{1}{x} - 2$ f $y = \frac{1}{x - 2}$

4G 11 Find the value of the constant of proportionality for each direct proportion relationship.

a $y \propto x^2$ and $y = 4$ when $x = 2$

b $y \propto x^3$ and $y = 2$ when $x = \frac{1}{2}$

c $y \propto \sqrt{x}$ and $y = 2$ when $x = 16$

d $y \propto \frac{1}{x}$ and $y = 8$ when $x = \frac{1}{2}$

4G 12 A direct proportion relationship exists in each of these. Find each rule.

a

x	0	1	2	3	4
y	0	5	10	15	20

b

x	0	1	2	3	4
y	0	0.5	2	4.5	8

c

x	0	1	2	3	4
y	0	3	24	81	192

d

x	0	1	4	25	36
y	0	3	6	15	18

NAPLAN-STYLE PRACTICE

1 If $(x - 6)(x + 1) = 0$, the two solutions for x are:

- ⬭ $x = -6$ or $x = 1$
- ⬭ $x = 6$ or $x = -1$
- ⬭ $x = -6$ or $x = -1$
- ⬭ $x = 6$ or $x = 1$

2 The expanded form of the equation $(x - 8)(x + 2) = 0$ is:

- ⬭ $x^2 + 6x - 16 = 0$
- ⬭ $x^2 - 6x - 16 = 0$
- ⬭ $x^2 - 10x - 16 = 0$
- ⬭ $x^2 + 10x - 16 = 0$

3 What is the solution to $x^2 - 11x + 10 = 0$?

4 For $y = x^2 + 3x - 2$, what is the value of y when $x = -1$?

5 Write the coordinates of the x-intercepts for the graph of $y = x^2 + x - 20$.

6 The axis of symmetry for the graph of $y = x^2 + 2x - 15$ has the rule:

- ⬭ $x = 1$
- ⬭ $x = -1$
- ⬭ $y = 1$
- ⬭ $y = -1$

7 Write the coordinates of the turning point for the graph of $y = x^2 - 8x + 15$.

8 What transformation has been performed on the graph of $y = x^2$ to produce the graph of $y = x^2 - 3$?

- ⬭ dilation by factor of 3
- ⬭ horizontal translation of 3 units
- ⬭ reflection in the x-axis
- ⬭ vertical translation of 3 units

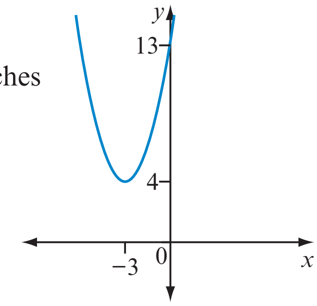
9 Which relationship would produce the narrowest parabola when graphed on the same Cartesian plane?

- ⬭ $y = x^2 + 4$
- ⬭ $y = -4x^2 - 5$
- ⬭ $y = \frac{1}{2}x^2 + 1$
- ⬭ $y = 2x^2 + 3$

10 What are the coordinates of the y-intercept for the graph of $y = -3(x - 2)^2 - 4$?

11 What are the coordinates of the turning point for the graph of $y = -3(x - 2)^2 - 4$?

12 Which rule best matches the graph shown?

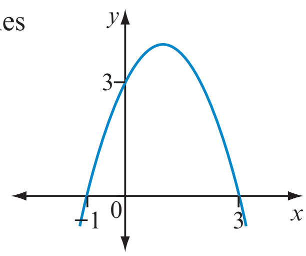


- ⬭ $y = (x - 3)^2 + 4$
- ⬭ $y = (x - 3)^2 - 4$
- ⬭ $y = (x + 3)^2 + 4$
- ⬭ $y = (x - 3)^2 + 4$

13 How many x-intercepts does the graph of $y = (x - 3)^2$ have?

- ⬭ 0
- ⬭ 1
- ⬭ 2
- ⬭ 3

14 Which rule best matches the graph shown?



- ⬭ $y = x^2 + 2x + 3$
- ⬭ $y = -x^2 + 2x + 3$
- ⬭ $y = x^2 - 2x - 3$
- ⬭ $y = -x^2 + 2x - 3$

15 A circle has its centre at $(2, -3)$ and a radius of 5 units. What is its rule?

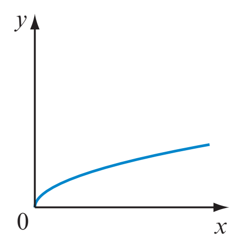
- ⬭ $(x - 2)^2 + (y + 3)^2 = 25$
- ⬭ $(x + 2)^2 + (y - 3)^2 = 25$
- ⬭ $(x - 2)^2 + (y - 3)^2 = 5$
- ⬭ $(x + 2)^2 + (y + 3)^2 = 5$

16 What are the coordinates of the centre and the radius of the circle with the rule $x^2 + (y - 8)^2 = 4$?

- ⬭ $(0, -8)$, 2 units
- ⬭ $(0, 8)$, 2 units
- ⬭ $(0, 8)$, 4 units
- ⬭ $(8, 0)$, 4 units

17 What is the area of the circle with the rule $(x - 2)^2 + (y + 5)^5 = 36$? Write your answer to the nearest square unit.

18 Which rule best matches the graph shown?



- ⬭ $y = x^2$
- ⬭ $y = \frac{1}{x}$
- ⬭ $y = x^3$
- ⬭ $y = \sqrt{x}$

19 If y is directly proportional to x and $y = 2$ when $x = 10$, what is the value of x when $y = 10$?

50	10	5	2
☐	☐	☐	☐

20 If y is directly proportional to x^2 and the constant of proportionality is $\frac{1}{4}$, what is the value of y when $x = 8$?

4	8	16	64
☐	☐	☐	☐

21 If $y \propto \sqrt{x}$ and $y = 15$ when $x = 25$, what is the value of the constant of proportionality?

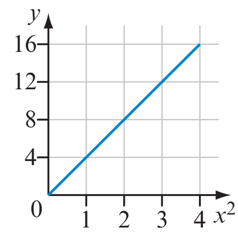
☐

22 Which of these statements is *not* correct? The graph of a direct proportion relationship:

- ☐ passes through the origin
- ☐ is linear
- ☐ has a constant gradient
- ☐ shows decreasing y values from left to right

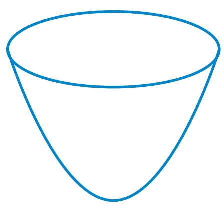
23 If $y \propto x^2$, use this graph to write the rule for the relationship between x and y.

☐



ANALYSIS

1 If a vertical cut is made through the centre of this bowl, the inside edge of the cross-section has the shape of a parabola. This cross-section can be modelled by the quadratic relationship $y = (x - 4)^2 + 1$, where y is the height of the cross-section in centimetres above the table at a horizontal distance of x cm from the left side of the cross-section.



- **a** On a Cartesian plane, sketch the graph of the quadratic relationship.
- **b** What are the coordinates of the two points representing the top rim of the bowl?
- **c** Determine the diameter of the upper rim of the bowl.
- **d** What is the thickness of the bowl where it is sitting on the table?

2 The outline of a bridge has an upper arch and a lower arch that can each be modelled by a quadratic relationship. If h is the height of the arch at a horizontal distance d from the left side of the bridge, the two rules are:

lower arch: $h = -\frac{1}{10}d^2 + 4d$

upper arch: $h = -\frac{1}{8}(d - 20)^2 + 65$.

All measurements are in metres.

- **a** Draw a sketch of the two arches on the same Cartesian plane.
- **b** Comment on the shape and features of the two parabolas.
- **c** What is the height of the upper arch above the lower arch at the ends of the bridge?
- **d** What is the span of the bridge? Show this:
  - **i** graphically, by referring to your graphs in part **a**
  - **ii** algebraically, by solving a quadratic equation.

The distance between the arches does not remain constant over the span of the bridge.

- **e** Find the height of the highest point of the:
  - **i** lower arch
  - **ii** upper arch.
- **f** How far apart are the two arches at their highest point?
- **g** How far apart are the two arches at a horizontal distance of 10 m from the left side of the bridge?
- **h** Describe the distance between the two arches over the span of the bridge.

CONNECT

Path of a soccer ball

To analyse the path of the ball during a soccer match, Lisa records some short pieces of video so measurements can be taken. The measurements (in metres) indicate the height of the ball (h) for a given horizontal distance (d) that the ball travels.

She chooses two plays where the ball follows a parabolic path after contact with a player. One is when the ball was kicked by David and the other when Nick hit the ball with his head. Each path can be modelled by a quadratic relationship involving d and h.

David: $h = -\frac{1}{100}d(d - 44)$

Nick: $h = -\frac{1}{20}(d - 8)^2 + 5$

Your task

You are to analyse each relationship to determine:

- the height of the ball when the player made contact with it
- the maximum height of the ball during the play
- the horizontal distance from where the player made contact to where the ball hits the ground (assuming no other player gets to it first).

Include all necessary graphs and working to justify your answers. Where appropriate, show how you can obtain your answers using both graphical and algebraic methods.

Compare your observations for both relationships to determine who kicked/hit the ball highest and furthest. Include an analysis of how far the ball travels if it is intercepted by another player when the ball is 1 m off the ground.

During training, each player dribbles the ball along the ground for the same distance from a marked point to the goal line. Lisa records measurements for the average speed of the ball in metres per second (s) and the time in seconds (t) for the ball to travel this distance (see table at right).

Use appropriate graphs and calculations to determine the rule for the relationship between t and s. Use this rule to work out the average speed of the ball if it takes 5 seconds to cover the distance. What distance has the ball travelled in each case?

t	2	3	4	6	8
s	24	16	12	8	6

As an extension, you may like to create a rule of your own that would describe the path of a soccer ball. Fully explain how you obtained the relationship.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a PowerPoint presentation
- a technology demonstration
- other (check with your teacher).

5 GEOMETRY

ESSENTIAL QUESTION

Geometry is the study of the size and position of shapes and objects. How is this relevant to you and your family?

5A ▶ **1** How many degrees are there in a right angle?

5B ▶ **2** Which statement is true?

A An equilateral triangle has four equal angles.
B An isosceles triangle has two equal angles.
C A scalene triangle has three equal sides.
D An equilateral triangle has two equal sides.

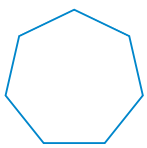
5B ▶ **3** Which statement is false?

A A kite has two pairs of equal sides.
B A parallelogram has two pairs of equal and parallel sides.
C A rhombus has four equal angles.
D A kite has a pair of equal angles.

5B ▶ **4** How many degrees in a quadrilateral?

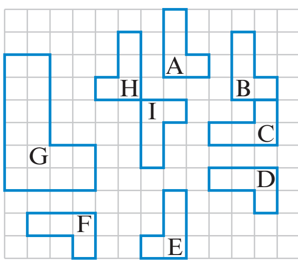
5B ▶ **5** **a** How many sides does an octagon have?

b This shape is:



A a regular heptagon
B an irregular hexagon
C an irregular heptagon
D a regular hexagon

5C ▶ **6** Refer to this grid of shapes.



a Which figure is a translation of H?

b Figure C can be described as a:

A 90° clockwise rotation from B
B vertical reflection of D
C translation of D
D 90° anticlockwise rotation from B.

c Which of these statements is false?

A G is an enlargement of B.
B D is a reflection of C.
C G is a reduction of A.
D I is a rotation of H.

5D ▶ **7** Congruent figures are:

A identical in shape but not size
B identical in both size and shape
C next to one another
D produced by dilation.

5E ▶ **8** Figure B has sides three times as long as figure A. Which statement is *not* true?

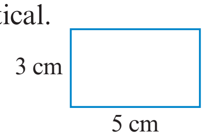
A A has been dilated by a scale factor of 3 to produce B.
B B is an enlargement of A.
C B has been dilated by a scale factor of $\frac{1}{3}$ to produce A.
D A is an enlargement of B.

5E ▶ **9** A triangle had a height of 5 cm. If this triangle was tripled in size, what would be the height of the new triangle?

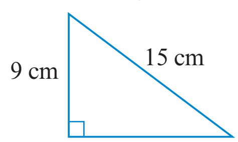
5E ▶ **10** Consider these figures. Which statement is false?

A They are the same shape but different size.
B Both are trapeziums.
C One has been dilated to produce the other.
D The shapes are identical.

5H ▶ **11** **a** What is the area of this rectangle?

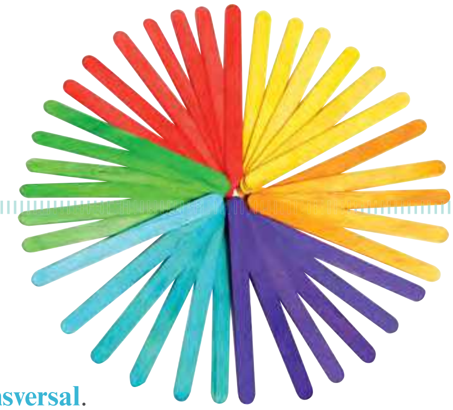


b What is the area of this triangle?



A 108 cm^2
B 54 cm^2
C 36 cm^2
D 1620 cm^2

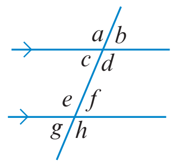
5A Angles and lines



Start thinking!

Whenever any lines intersect, angles are formed.

This figure shows a pair of **parallel** lines crossed by another line, called a **transversal**.



1 Draw your own pair of parallel lines intersected by a transversal on any angle you like.

2 Label each angle from *a* to *h*, as shown in the diagram.

3 Use a protractor to measure each angle. What do you find?

4 Angles *a* and *d* are called **vertically opposite angles**.

- **a** What do you notice about them?
- **b** Another pair of vertically opposite angles are *f* and *g*. What do you notice about them?
- **c** What can you say about vertically opposite angles? What other pairs are there?

5 Angles *d* and *e* are called **alternate angles**.

- **a** What can you say about angles *d* and *e*?
- **b** There is one other pair of alternate angles. List them and make a statement about alternate angles.

6 Angles *b* and *f* are called **corresponding angles**.

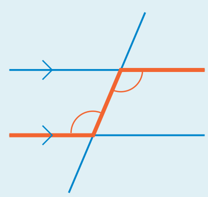
- **a** What can you say about them?
- **b** There are three other pairs of corresponding angles in your diagram. List them and make a statement about corresponding angles.

7 Angles *c* and *e* are called **co-interior angles**.

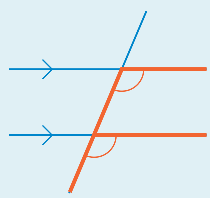
- **a** What can you say about them? (Hint: what do they add to?)
- **b** There is one other pair of co-interior angles. List them and make a statement about co-interior angles.

KEY IDEAS

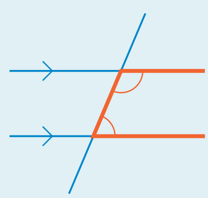
- **Complementary angles** add to 90°.
- **Supplementary angles** add to 180°.
- Angles around a point add to 360°.
- Vertically opposite angles are equal.
- When parallel lines are crossed by a transversal, a number of angles are formed:
  - alternate angles are equal
  - corresponding angles are equal
  - co-interior angles are supplementary.



alternate angles



corresponding angles



co-interior angles

EXERCISE 5A Angles and lines

UNDERSTANDING AND FLUENCY

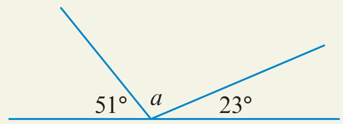
1 For each angle, find:

- **i** the complementary angle
- **ii** the supplementary angle. If it is not possible, write N/A.

a 23°	**b** 47°	**c** 176°	**d** 9°
e 97°	**f** 115°	**g** 17°	**h** 186°

EXAMPLE 5A-1 Finding angle size using complementary and supplementary angles

Find the size of the unknown angle.



THINK

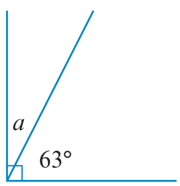
Angles around a straight line add to 180°. Subtract the known angles from 180° to find the value of a.

WRITE

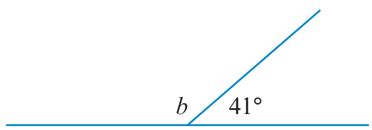
$a = 180° - 51° - 23°$
$= 106°$

2 Find the size of each unknown angle.

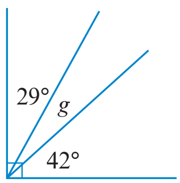
a



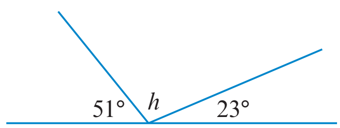
b



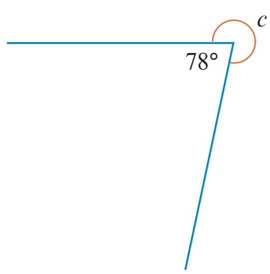
c



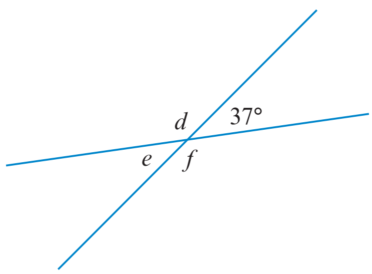
d



e

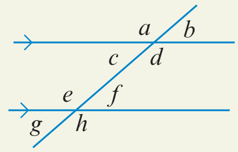


f



EXAMPLE 5A-2 Finding angles related to parallel lines

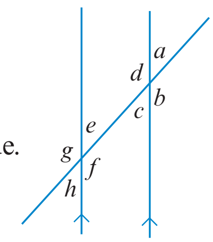
If angle c is equal to 41°, name its alternate angle and state its value.



THINK	WRITE
1 Alternate angles are on opposite sides of the transversal between the parallel lines – they form a 'Z' pattern.	The alternate angle to c is f.
2 Alternate angles are equal, so $f = c$.	$f = 41°$

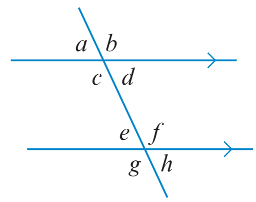
UNDERSTANDING AND FLUENCY

3 Use this figure to find the angles listed below.



- **a** If angle d is equal to 123°, name its alternate angle and state its value.
- **b** If angle a is equal to 46°, name its corresponding angle and state its value.
- **c** If angle h is equal to 37°, name its vertically opposite angle and state its value.
- **d** If angle f is equal to 98°, name its co-interior angle and state its value.
- **e** If angle b is equal to 138°, name its corresponding angle and state its value.
- **f** If angle e is equal to 53°, name its alternate angle and state its value.

4 For the figure shown, find the angle that is:



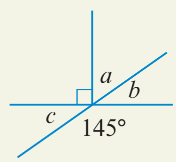
- **a** alternate to c **b** corresponding to g
- **c** co-interior to f **d** vertically opposite to d
- **e** alternate to e **f** co-interior to c
- **g** vertically opposite to a **h** corresponding to b.

5 Using the figure in question **4**, if a is equal to 78°, find the value of:

a b **b** c **c** d **d** e **e** f **f** g.

EXAMPLE 5A-3 Finding angles around a point

Find the size of the unknown angles.

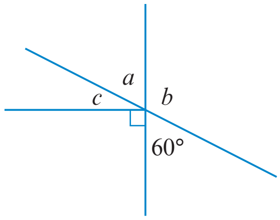


THINK	WRITE
1 Angle c is supplementary to 145°. Supplementary angles add to 180°.	$c + 145° = 180°$ $c = 35°$
2 Angle b is vertically opposite angle c. Vertically opposite angles are equal.	$b = c = 35°$
3 Angle a is complementary to angle b. Complementary angles add to 90°.	$a + b = 90°$ $a + 35° = 90°$ $a = 55°$

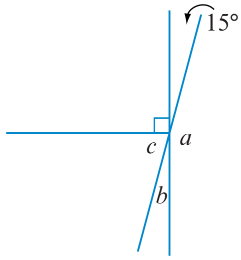
UNDERSTANDING AND FLUENCY

6 Find the size of the unknown angles.

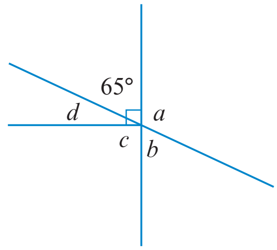
a



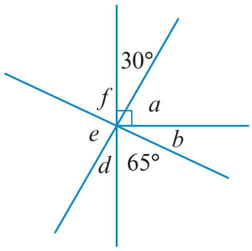
b



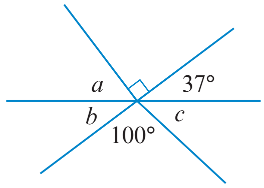
c



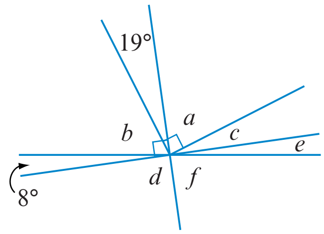
d



e

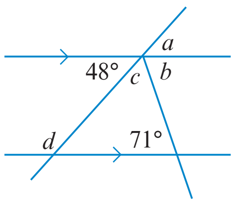


f

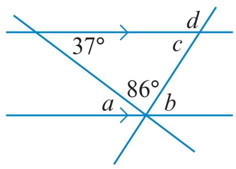


7 Find the size of the unknown angles.

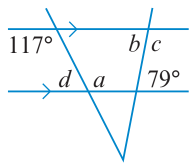
a



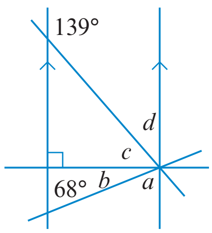
b



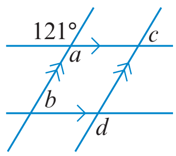
c



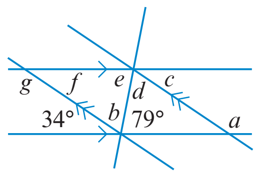
d



e



f



PROBLEM SOLVING AND REASONING

8 **a** Explain how you would find the angle between each blade of these windmills.

b Use your method to find the angle between the blades.

9 In Example 5A-3, one method for finding the three unknown angles was shown.

a State another two ways you could find the size of the unknown angles.

b Compare your way with a classmate's. What do you find?

c How many different angle definitions (for example, angles around a point add to 360°) can you use to do this?

10 Consider any pair of parallel lines cut by a transversal. Explain how, if you know the size of one angle, you are able to find the size of all remaining angles without using a protractor.

11 What could you do to determine if two lines are parallel?

PROBLEM SOLVING AND REASONING

12 **a** Copy and complete this table. The first column has been started for you.

	56°	78°	19°	146°	27°	103°
Complementary	34°					
Supplementary						
Vertically opposite	56°					
Co-interior						
Alternate						
Corresponding						
Around a point	304°					

b Why have two cells in the table been shaded out?

c Write a rule for each row so that somebody who doesn't know what the terms mean could complete the table.

13 If you shine light onto a mirror, it reflects off at the same angle, as shown in figure A.

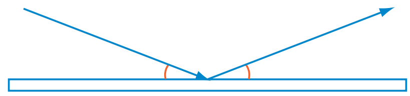


Figure A

a If you shine a light onto a mirror at 32°:

i what is the angle between the reflected ray of light and the mirror?

ii what is the angle between the two rays of light?

iii what is the angle between the ray of light and a line **perpendicular** to the mirror?

In physics, an imaginary line that is perpendicular to the mirror is known as the 'normal'. The law of reflection says that angle *a* (also called the angle of incidence) is equal to angle *b* (the angle of reflection), as shown in figure B.

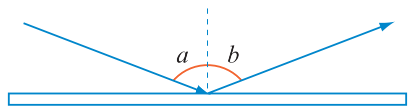


Figure B

b Explain how this is the same rule as you were using in part **a**.

c If *a* was equal to 54°, find the size of:

i angle *b* **ii** the angle the light ray makes with the mirror.

14 Angles are not limited to whole numbers. Rather than dividing them into decimals (such as 15.5°), angles are usually divided into minutes and seconds. This convention is known as **degrees-minutes-seconds (DMS)**.

a How many seconds in a minute?

b How many minutes do you think might be in a degree?

c If there are 60 minutes in a degree, how many minutes are there in:

i half a degree? **ii** quarter of a degree? **iii** a third of a degree?

Minutes are represented by a prime (′) and seconds are represented by a double prime (″). For example, 14°35′22″ represents 14 degrees, 35 minutes and 22 seconds.

d Is 14°35′22″ greater or less than 14.5°?

e Write each angle using the words degrees, minutes and seconds.

i 83°16′55″ **ii** 27°43′04″ **iii** 154°09′37″

f Write each angle using DMS conventions.

i 230 degrees, 29 minutes and 13 seconds

ii 67 degrees, 18 minutes and 2 seconds

iii 192 degrees, 56 minutes and 42 seconds

PROBLEM SOLVING AND REASONING

15 **a** Convert each angle into DMS conventions. (Hint: remember that there are 60 minutes in a degree.)

i 86.75° **ii** 113.8° **iii** 217.1° **iv** 9.65°

b Convert each DMS angle into degrees as a decimal number.

i 196°42′ **ii** 98°51′ **iii** 23°33′ **iv** 107°05′

16 Consider this analogue clock, which shows 6.00 pm.

a What is the angle between the two hands of the clock?

b What angle would there be between the hands of a clock displaying these times? (Start from 12 o'clock and move clockwise to the other hand.)

i 3.00 am **ii** 9.00 pm **iii** 5.00 pm **iv** 11.00 am **v** 8.00 pm

c What is the angle between two hour-marks?

d Using your answer to part **c**, what is:

i half this angle? **ii** one quarter of this angle?

iii three quarters of this angle?

e Use your answers to part **d** to find the smallest angle between the two hands of a clock displaying:

i 4.30 pm **ii** 7.15 am **iii** 9.45 pm **iv** 1.15 pm **v** 5.45 pm

CHALLENGE

17 Remembering that there are also 60 seconds in a minute, perform these conversions.

a 72°30′30″ into degrees as a decimal number **b** 46.3975° into DMS

c 89°36′18″ into degrees as a decimal number **d** 34.1575° into DMS

18 Find the size of the smallest angle between the two hands of a clock displaying these times.

a 11.25 am **b** 2.05 pm **c** 10.17 am **d** 6.32 am **e** 12.48 pm

19 Using the law of reflection and your understanding of complementary angles and angles in parallel lines, draw a diagram and label all angles to show what would happen if you shone a light at an angle of 50° to a mirror that was joined to another, perpendicular mirror. (Hint: use figure B in question **13** as a starting point and place a vertical mirror at its right edge.)

20 The law of reflection can also be helpful when you play billiards or pool. Copy this diagram, and measure and mark in the correct angle you should hit the white ball in order to hit the red ball into the top right pocket.

Reflect

In what situations do you think knowledge and understanding of angle properties might be useful?

5B Angles and polygons

Start thinking!

A **polygon** is a closed shape with sides that are all straight.

1 What is the simplest polygon you can make? (Hint: what is the minimum number of sides needed to make a closed shape?)

2 How many degrees in a **triangle**? If you are not sure, draw any triangle and use your protractor to measure the internal angles.

3 Triangles can be classified according to their side and angle properties. Brainstorm with a classmate and create a table showing the seven different types of triangles.

4 What is the next simplest polygon? How many sides does it have?

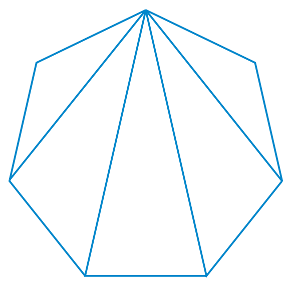
5 How many degrees in a quadrilateral? If you are not sure, draw any quadrilateral and use your protractor to measure the internal angles.

6 How does splitting a quadrilateral into two triangles show that its angle sum is 360°?

With the exception of the triangle and quadrilateral, polygons are usually named for their number of sides.

7 Pair up with a classmate and brainstorm the names for the polygons that have 5 to 10 sides.

8 You can use the method discussed in question **6** to work out the angle sum of any polygon. Consider this heptagon.



a How many sides does it have?

b How many triangles is it split into?

c What is the difference between these two numbers?

d Use your knowledge of the internal angle sum of a triangle and your answer to part **b** to calculate the internal angle sum for a heptagon.

e Draw two **irregular** heptagons to show that the angle sum for all heptagons is the same. Make sure that one of these heptagons is **convex** (all angles less than 180°) and one is **concave** (at least one reflex angle).

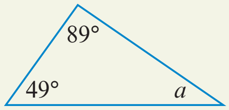
KEY IDEAS

- A polygon is a closed shape with straight sides.
- The internal angle sum of a triangle is 180°.
- The internal angle sum of a quadrilateral is 360°.
- The internal angle sum of a polygon is $(n - 2) \times 180°$, where n is the number of sides and $(n - 2)$ is the number of triangles into which it can split.

EXERCISE 5B Angles and polygons

EXAMPLE 5B-1 Finding the size of an angle in a triangle

Find the size of angle a.



THINK

Angles in a triangle add to 180°. Subtract the known angles from 180° to calculate a.

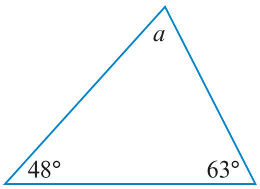
WRITE

$a = 180° - 49° - 89°$
$= 42°$

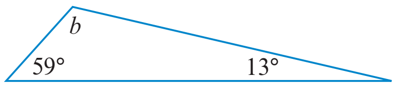
UNDERSTANDING AND FLUENCY

1 Find the size of each labelled unknown angle.

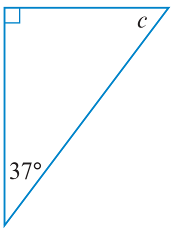
a



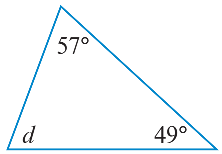
b



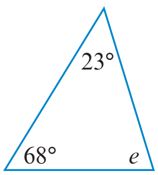
c



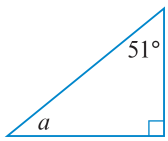
d



e

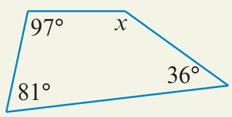


f



EXAMPLE 5B-2 Finding the size of an angle in a quadrilateral

Find the size of angle x.



THINK

Angles in a quadrilateral add to 360°. Subtract the known angles from 360° to calculate x.

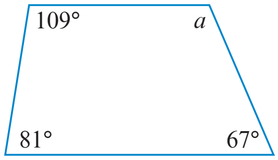
WRITE

$x = 360° - 97° - 81° - 36°$
$= 146°$

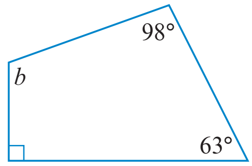
UNDERSTANDING AND FLUENCY

2 Find the size of each labelled unknown angle.

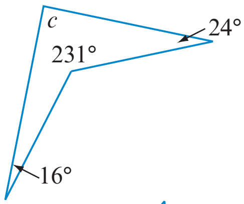
a



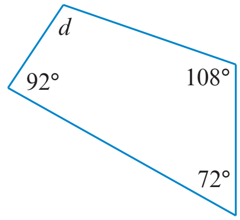
b



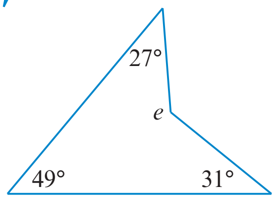
c



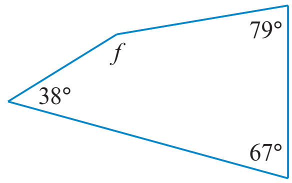
d



e

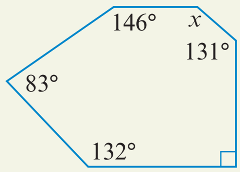


f



EXAMPLE 5B-3 Finding the size of an angle in a polygon

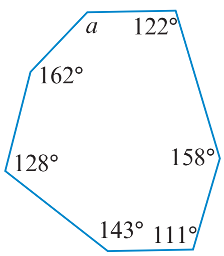
Find the size of angle x.



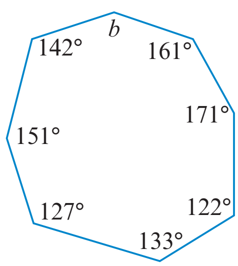
THINK	WRITE
1 Identify the polygon. How many sides does the shape have?	The shape has six sides, so it is an irregular hexagon.
2 Find the angle sum of a hexagon by using the formula.	internal angle sum $= (n - 2) \times 180°$ $= (6 - 2) \times 180°$ $= 4 \times 180°$ $= 720°$ There are 720° in a hexagon.
3 Calculate x by subtracting the known angles from 720°.	$x = 720° - 146° - 83° - 132° - 90° - 131°$ $= 720° - 582°$ $= 138°$

3 Find the size of each labelled unknown angle.

a

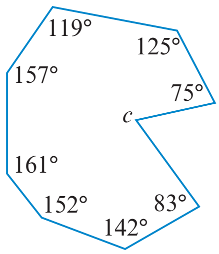


b

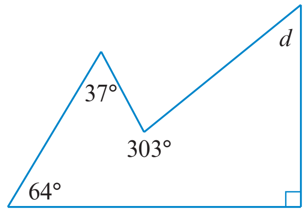


UNDERSTANDING AND FLUENCY

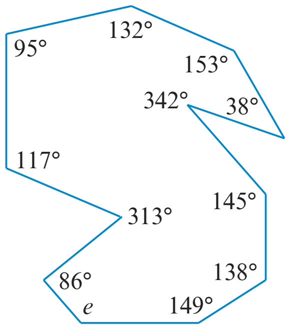
c



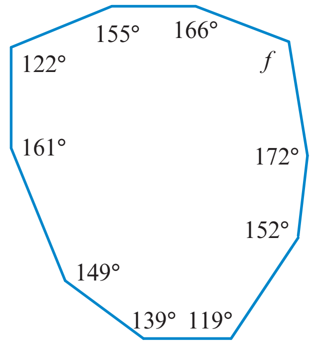
d



e



f



PROBLEM SOLVING AND REASONING

4 Why is a circle not a polygon?

5 **a** If a ladder makes an angle of 32° with the wall it is leaning on, what angle does it make with the ground?

b If a stepladder is opened up to make an angle of 43° at its top, what angle do its legs make with the ground?

c What is the obtuse angle that both sides of the ladder make with the ground – the angle on which you would climb it?

6 **a** How many degrees in a pentagon?

b What is the difference between a **regular** pentagon and an irregular pentagon?

c How do you know that every internal angle in a regular pentagon is equal to 108°?

d Explain why you can use the formula $x = \dfrac{180°(n-2)}{n}$ to find the size of one internal angle in any regular polygon.

e Sophia wanted to build this case to store her jewellery in. On what angle should she join together the sides?

7 Find the size of one internal angle in a regular:

a decagon **b** octagon

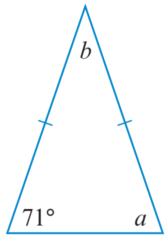
c dodecagon (12 sides) **d** icosagon (20 sides).

8 If one interior angle of a regular polygon is 168°, how many sides does it have?

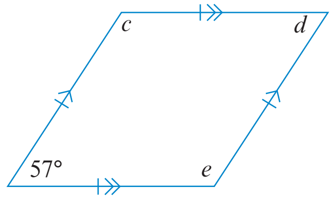
9 Does a regular polygon exist that has interior angles of 145° each? Explain.

10 Find the size of each labelled unknown angle.

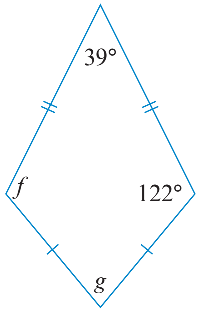
a



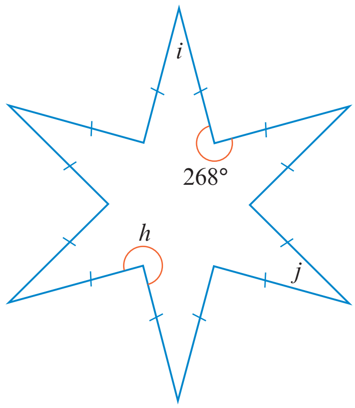
b



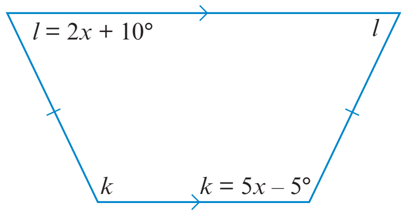
c



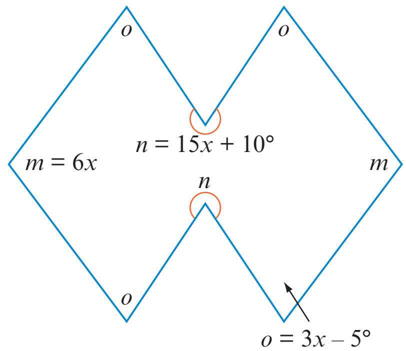
d



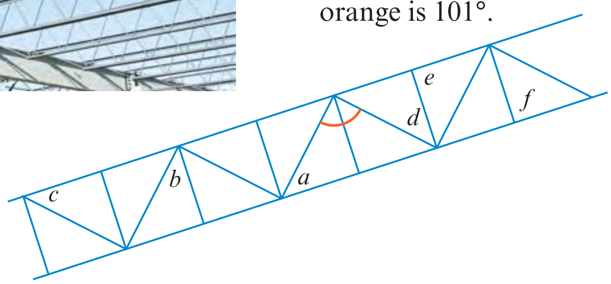
e



f



11 Use your understanding of angles and lines in polygons to find the size of the labelled angles in this diagram of a roof truss. Assume that all lines that look parallel are parallel and that the angle marked in orange is 101°.

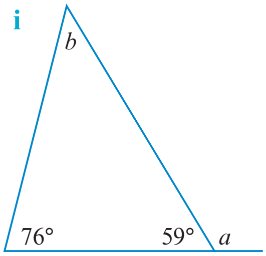


PROBLEM SOLVING AND REASONING

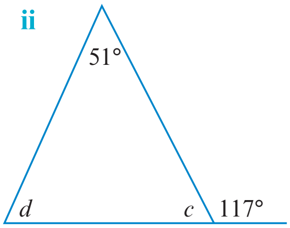
12 An **exterior angle** of a triangle can be formed by extending one of its sides.

a Use your understanding of angles within a triangle and supplementary angles to find the size of each labelled unknown angle.

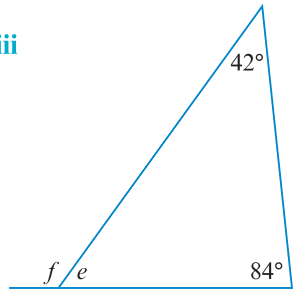
i



ii



iii



b Using your understanding of supplementary angles, explain why any exterior angle of a triangle is equal to the sum of the two opposite interior angles.

c Draw three different triangles and label all their exterior angles.

d Find the sum of the exterior angles for each triangle.

e What can you say about the sum of the exterior angles of a triangle?

13 An exterior angle of a quadrilateral can also be formed by extending any side.

a Draw three different quadrilaterals and label all their exterior angles.

b Find the sum of the exterior angles for each quadrilateral.

c What can you say about the sum of the exterior angles of a quadrilateral?

14 **a** Repeat the steps shown in question **13** for:

i a pentagon **ii** a hexagon

iii a heptagon **iv** an octagon.

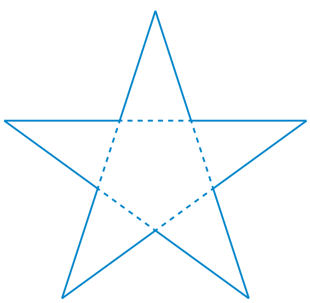
b What can you say about the exterior angle sum of any polygon?

CHALLENGE

15 Write a formula that will calculate one exterior angle in any regular polygon.

16 Logan realised that he could find the size of angle *j* from question **10d** by subtracting 268° from 300°. Use your understanding of the internal angle sum of a dodecagon to explain why this works.

17 Find the size of the angle in any point of this star, which is based on a regular pentagon.



Reflect

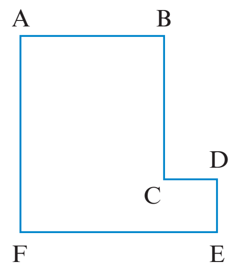
How does an understanding of angle and line properties help explain angles in polygons?

5C Transformations

Start thinking!

1 Use grid paper or draw a grid in your book. Make sure your grid has at least ten rows and columns and give it labelled axes like a Cartesian plane.

2 Draw and cut out three copies of a shape of your choice. Choose an irregular shape but one that is based on the squares in your grid, like this one.



3 Paste one copy of your shape somewhere on the grid (near the centre would be best) and label each corner of your shape with letters like the figure shown.

4 Translate (move) the second copy of your shape to anywhere on the grid and paste it into position, labelling the new letters of each corner. Remember when labelling an image you add a dash to the letter; for example, A′.

5 Using coordinates or other means, describe the translation (for example, 3 units right, 2 units down).

6 Alex described his translation from corner A to corner D′ as 2 units left, 1 unit up. Explain what he did wrong.

7 Draw a mirror line next to your original shape and then reflect the third copy of your shape across this mirror line. Paste it into place and label the corners with letters A′, etc.

8 Describe the reflection. Why is it important that each corresponding coordinate is the same distance from the mirror line (for example, if A is 2 units left of the mirror line, then A′ is 2 units right of the mirror line)?

9 You have demonstrated two of the four transformations: **translation**, **reflection**, **rotation** and **dilation**. Show how you could demonstrate the other two.

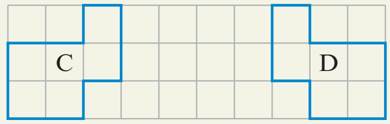
KEY IDEAS

- An **isometric transformation** is one that doesn't change the shape or size of an object.
- Translation, rotation and reflection are all isometric transformations.
- Dilation is a **non-isometric transformation** (it changes the size of an object).
- The transformation of a shape A is called the **image**, and is named A′.
- The number given to describe how many times bigger or smaller an image is than the original is called the **scale factor**.

EXERCISE 5C Transformations

EXAMPLE 5C-1 Describing transformations

Describe the transformation of C to D.



THINK

1 Identify whether C has been translated, rotated, reflected or dilated. (reflected)

2 Decide whether it has been reflected in a horizontal or vertical mirror line and where this line is located.

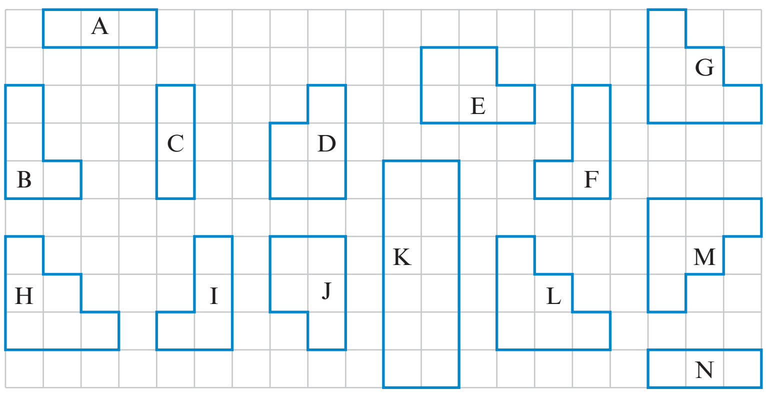
3 Write your answer.

WRITE

C has been reflected in a vertical mirror that is 2 units to the right of the shape.

UNDERSTANDING AND FLUENCY

For questions **1**–**3**, refer to this grid of shapes.



1 Describe each transformation as a translation, reflection, rotation or dilation.

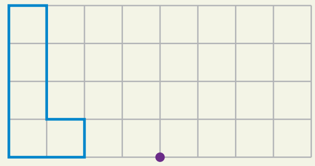
a E to D **b** L to H
c D to J **d** K to C
e B to F **f** N to A
g G to M **h** M to H

2 Describe each transformation.

a C to A **b** F to B
c L to M **d** I to F
e G to L **f** C to K
g J to D **h** H to G

EXAMPLE 5C-2 Performing transformations

Rotate this shape 90° clockwise around the point shown.



THINK

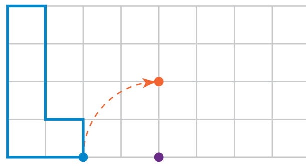
1 Select a vertex of the shape (blue dot). Rotate this vertex (and its accompanying edge) 90° clockwise around the point of rotation.

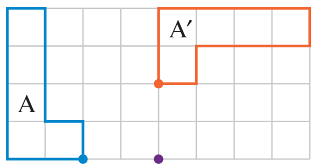
2 Check that the new vertex (orange dot) is the same distance from the point of rotation (purple dot) as the original vertex (blue dot).

3 Draw in the remainder of the image. Check that it is the same shape but rotated.

4 Erase the arrow and label the final image.

WRITE





PROBLEM SOLVING AND REASONING

3 Perform each transformation. You may need to draw each shape on its own grid first.

- **a** Translate G 5 units right, 2 units up.
- **b** Reflect J in a vertical mirror 3 units from its right.
- **c** Rotate F 90° anticlockwise around a point 2 units below its lower right vertex.
- **d** Dilate B to twice its size.
- **e** Reflect D in a horizontal mirror 2 units from its top.
- **f** Rotate M 90° clockwise around a point 1 unit above its upper right vertex.

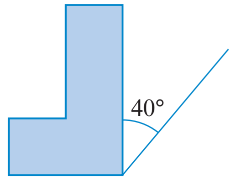
4 A series of translations were used in old and fictional treasure maps.

- **a** Draw a map of a deserted island on a grid and decide on a starting point and where to bury your treasure. Include landmarks (for example, a palm tree) on your island.
- **b** Write a series of at least six translations in order to move from the start to your buried treasure.
- **c** Write a short cut set of instructions that would help you find the treasure in the minimum number of movements possible. Don't forget that you may not be able to walk through landmarks.

5 Every piece on a chessboard can only move in certain ways. Write all the possible moves each piece can make as translations. You may wish to use the Internet or ask your teacher if you don't know how pieces move in chess. (Hint: describe moving diagonally as both its horizontal and vertical movements).

6 What is the difference between a rotation of 180° anti-clockwise and a rotation of 180° clockwise?

7 Rotations are not limited to multiples of 90°. They can be any angle at all. Consider this figure.

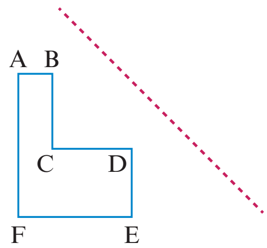


- **a** On what angle is this rotation? In what direction?
- **b** Copy and complete the rotation.
- **c** How would the rotation change if it was in an anti-clockwise direction?
- **d** How is the use of a protractor particularly valuable when performing rotations like this?

8 Rotate the original figure shown in question **7** by:

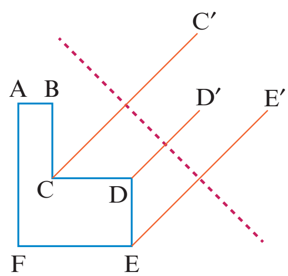
- **a** 20° clockwise
- **b** 45° anticlockwise
- **c** 160° clockwise
- **d** 295° anticlockwise.

9 Reflections don't have to be using just horizontal or vertical mirrors, they can use mirrors on any angle. You already know that the image must be the same distance from the mirror line as the original figure, just on the other side of the mirror.



- **a** Describe the angle and position of the mirror in this first figure.
- **b** How many vertices does the shape have?

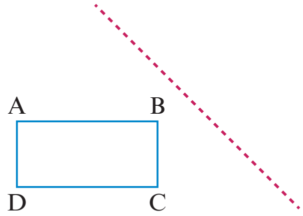
To make sure that the reflection is accurate, you draw lines from each vertex so that each line hits the mirror at a right angle, as shown in this figure.



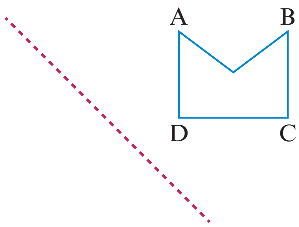
- **c** Copy this second figure and draw in the remaining perpendicular lines (for vertices A, B and F). Make sure that they extend past the mirror the same distance as before the mirror.
- **d** Connect your new vertices to complete your reflected figure.
- **e** Complete steps **a–d** with another copy of the figure, but with the mirror line on a different angle. What do you notice about the angle of the reflection as compared to the angle of the mirror?

10 Reflect these figures in the mirrors shown.

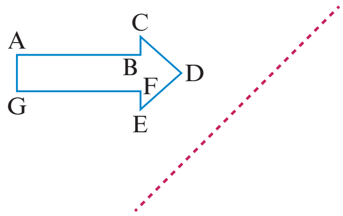
a



b



c



11 Consider figures A and B.

a Describe the dilation shown from figure A to figure B as an **enlargement** or a **reduction**.

To complete part **a**, figure A was the *original* figure and figure B was the image produced by the dilation.

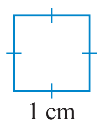


Figure A

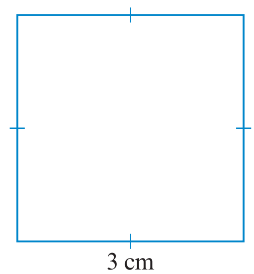


Figure B

b How would you describe the dilation if figure B was the original?

c How many times the size of figure A is figure B?

The number given to describe how many times as big or small an image is compared to the original is called the scale factor. The dilation from figure A to figure B has a scale factor of 3.

d If there was a figure C that was twice the size of figure B, what would be the scale factor from:

i figure B to figure C? **ii** figure A to figure C?

Describing the scale factor of a reduction is a little different.

e What is the opposite of doubling something in size?

f What is the opposite of tripling something in size?

g Use the answers to parts **e** and **f** to explain why the scale factor from figure B to figure A is $\frac{1}{3}$.

h What would be the scale factor from:

i figure C to figure B? **ii** figure C to figure A?

i Write a sentence or two summarising how to describe dilations.

12 Erik said that to dilate Figure A to Figure B you just have to add 2 cm to each side. Use a rectangle that has a width of 1 cm and a length of 3 cm to show why Erik is incorrect.

PROBLEM SOLVING AND REASONING

13 Julie has a dining table that fits two people along each of its long sides and one person at either end. If Julie bought a new dining table that was three times as big, how many more people could she fit onto the dining table?

14 How are transformations used in tessellations? Explain, with an example.

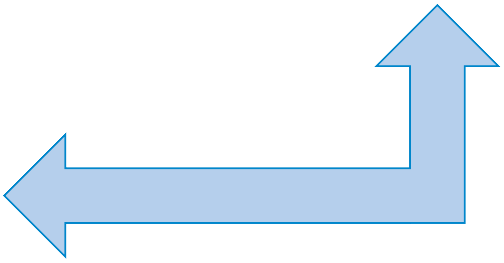
15 Kaleidoscopes are made using a series of mirrors (usually in the form of an **equilateral triangle**) and any number of colourful beads or even a colourful pattern. Can you explain how so many different images can be produced even if all the beads are the same shape?

16 Using three mirrors (or by constructing your own kaleidoscope) surrounding a number of colourful objects, investigate the effect of the position of the mirrors on the image produced. You may like to begin with a single object, rotating the mirrors around it to see how the reflections change.

CHALLENGE

17 Use a series of translations to move a knight around a chess board to end up at your starting point. Do it so that the only square you land on twice is the starting square – don't just retrace your steps backwards. What is the smallest number of moves you can do this in?

18 Using the figure shown, investigate the different shapes formed by placing a mirror in various positions on the figure. Draw at least three different reflections and write a sentence summarising what you find.



19 Using the figure from question **18**, investigate the effect of the place of the rotational point on the position of the images. Draw at least five images, where three points are on any vertex and two points are anywhere within the shape. Write a sentence or two about what you find.

Reflect

When working with transformations, why is it important that you indicate which figure is the original and which figure is the image?

5D Congruent figures

Start thinking!

Congruent figures are identical in size and shape.

Sometimes it can be easy to see if figures are congruent or not. Triangles can sometimes be difficult to judge, but there are conditions that you can look for to decide if two triangles are congruent.

1 Draw any triangle and measure its sides and angles.

2 Explain why any other triangle you draw with the same side lengths will be congruent.

3 Can you draw a non-congruent triangle with the same angles as your original triangle? Explain.

SSS (all three sides the same) is a condition for **congruence**, AAA (all three angles the same) is not. It is important to note that while AAA does not mean that two triangles are congruent, it also does not mean that they are *not* congruent. In these circumstances, more information is needed before making a decision.

4 Two other conditions for congruence are AAS (two angles and a side the same) and SAS (two sides and an angle between them the same). Can you explain why if two triangles meet either of these conditions then they must be congruent? You may like to draw some examples.

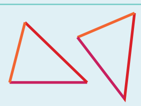
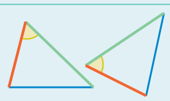
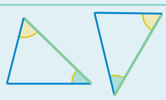
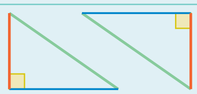
Another condition for congruence is only for a right-angled triangle, named **RHS**. This condition means that two right-angled triangles are congruent if their hypotenuses are the same length and also another pair of corresponding sides is the same length.

5 Can you explain why it only works on a right-angled triangle?

6 Write a summary of the conditions for congruency and swap it with a classmate. Discuss any differences.

KEY IDEAS

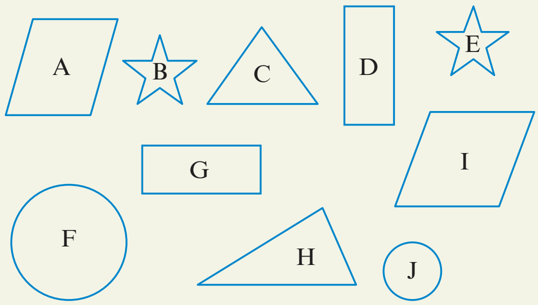
- Congruent figures are identical in shape and size but can be in any position or orientation.
- Two figures are congruent if their corresponding sides are all the same length and their corresponding angles all the same size.
- For triangles, there are four conditions for congruence (shown in the table).
- The specifications AAA and SSA do not necessarily mean congruence (more information is needed).

SSS	SAS	AAS	RHS
Three pairs of sides are equal in length.	Two pairs of sides are equal in length and the pair of angles in between is equal.	Two pairs of angles are equal and a corresponding pair of sides is equal in length.	The **hypotenuses** and a corresponding pair of sides are equal in length in a right-angled triangle.
			

EXERCISE 5D Congruent figures

EXAMPLE 5D-1 Identifying congruent figures

List any pairs of congruent figures from this selection.



THINK

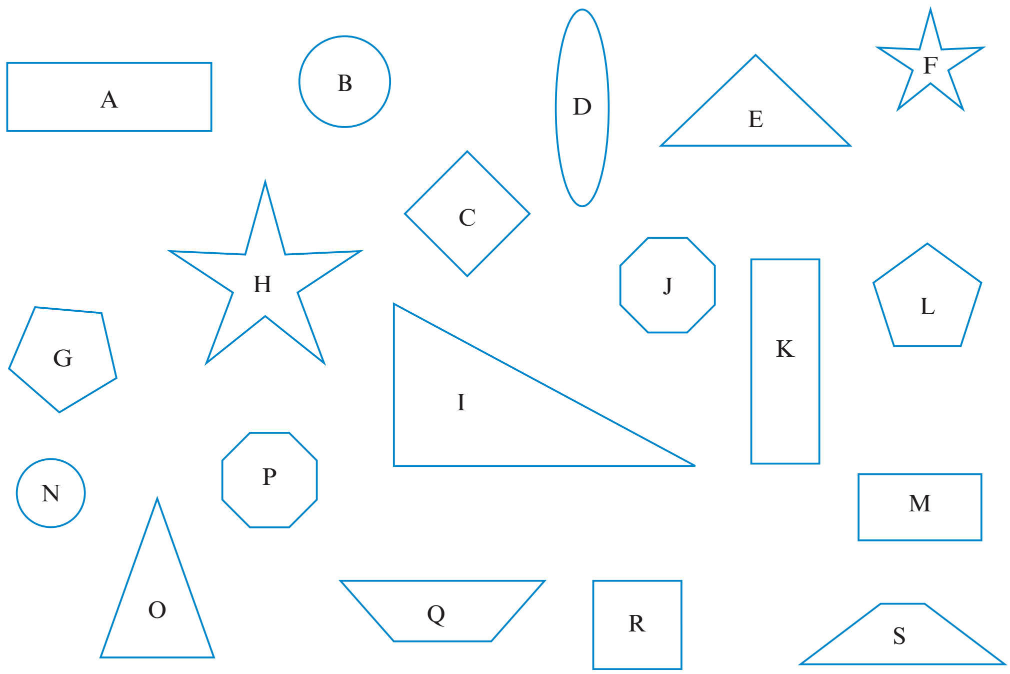
Check each figure to see if it is exactly the same shape and size to any other.
A and I are both parallelograms, but are different shapes.
B looks the same as E. Measure to check.
C and H are both triangles, but are different shapes.
D looks the same as G. Measure to check.
F and J are both circles, but are different sizes.

WRITE

B and E are congruent.
D and G are congruent.

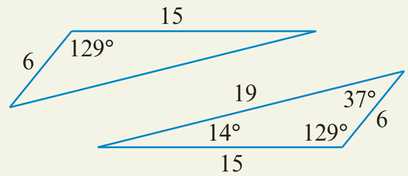
UNDERSTANDING AND FLUENCY

1 List any pairs of congruent figures from this selection.



EXAMPLE 5D-2 Identifying an appropriate congruence condition

Decide which congruence condition you would use to check if this pair of triangles is congruent.



THINK

The bottom triangle has all information given, but the top triangle has two sides and an angle. The angle is between the two sides, which matches the congruence condition SAS.

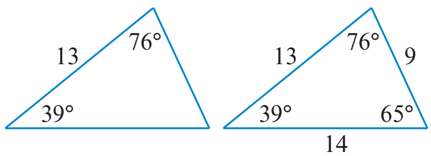
WRITE

SAS would be used to check if these triangles were congruent.

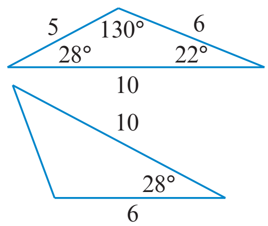
UNDERSTANDING AND FLUENCY

2 Decide which congruence condition you would use to check if each pair of triangles is congruent.

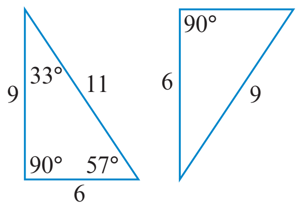
a



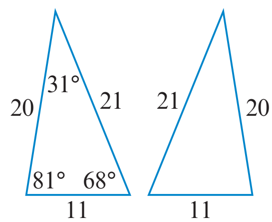
b



c

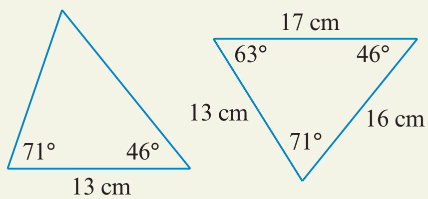


d



EXAMPLE 5D-3 Deciding if triangles are congruent

Decide if this pair of triangles is congruent, giving a reason for your answer.



THINK

1 Two angles and a side are given that are the same in both triangles. Check if this meets a condition for congruence.

2 In the first triangle the known side is between the two angles and has a length of 13 cm.

3 The corresponding side in the second triangle has a length of 16 cm. This means that these two triangles fail the AAS condition for congruence.

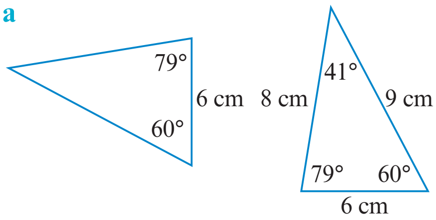
WRITE

The two triangles are not congruent as they fail the AAS condition for congruence.

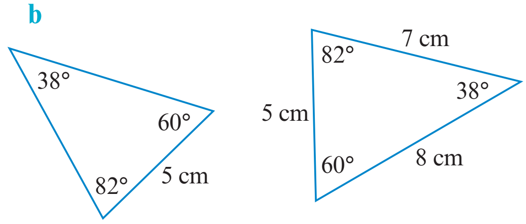
UNDERSTANDING AND FLUENCY

3 Decide if each pair of triangles is congruent, giving a reason for your answer.

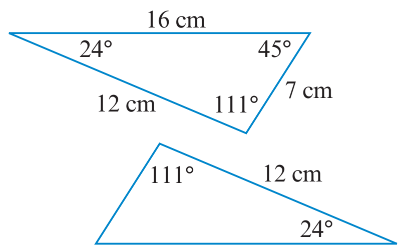
a



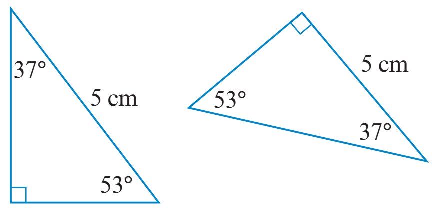
b



c



d



PROBLEM SOLVING AND REASONING

4 Decide if each pair of triangles in question **2** is congruent, giving a reason for your answer.

5 A copy is made of a key.

a Why is it important that the copy is congruent with the original?

b What other situations can you think of where it is essential that congruency is used?

6 If you had two figures in front of you that looked identical, what could you do to decide if they were congruent or not?

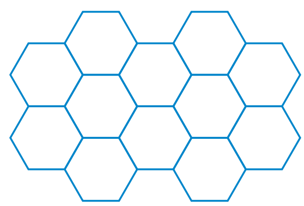
7 A complex figure and its image have all corresponding angles exactly the same. Explain why you need more information to determine if they are congruent or not.

8 A regular tessellation is a pattern of shapes that has no overlaps or gaps and uses only congruent, regular shapes.

a Why is it important that each hexagon in this tessellation be congruent?

b There are two other regular tessellations. Can you draw an example of each?

c Why must a tessellation that shows a repeating pattern make use of congruent shapes?



9 Are identical twins congruent? Explain.

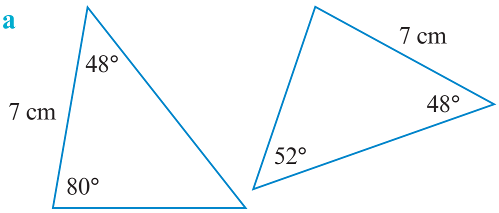
10 Imagine that you had a square and folded it in half along its diagonal.

a What shapes do you form?

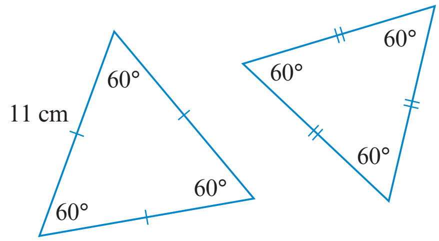
b Are these shapes congruent? Explain how you know. You may like to draw a diagram to aid your explanation.

11 Use your understanding of triangle properties to determine, if possible, whether each pair of triangles is congruent.

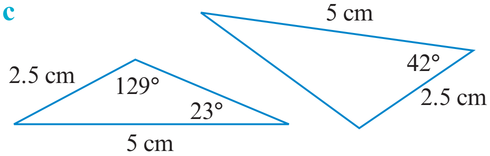
a



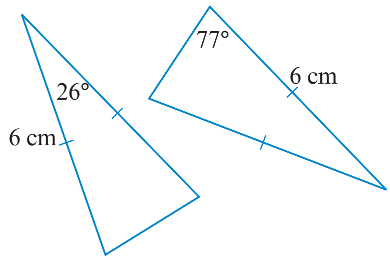
b



c

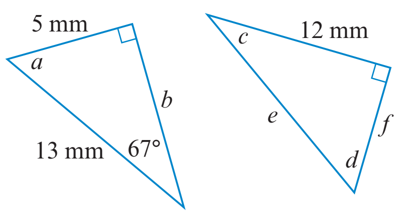


d

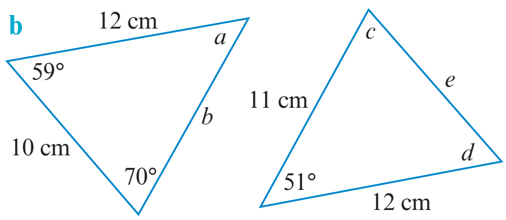


12 Find the unknown side lengths and angles in these triangles, given that each pair is congruent.

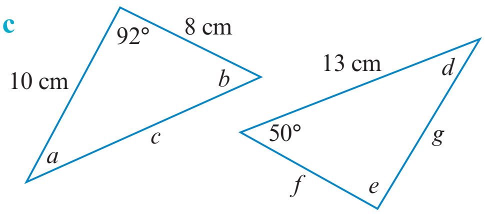
a



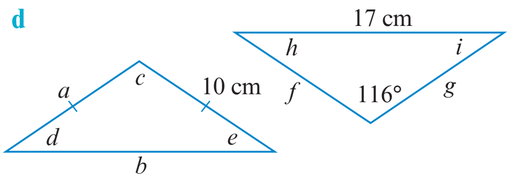
b



c



d



13 Some mathematicians consider a fifth condition for congruence. If any two angles and the side between them are the same in both triangles (ASA) then the triangles are congruent. Others believe that ASA and AAS are in fact the same test.

a Draw a pair of triangles that meet the AAS test.

b Use the angle sum of a triangle to show that the third angle pair is equal in your triangles.

c Show that your triangles now satisfy the ASA test.

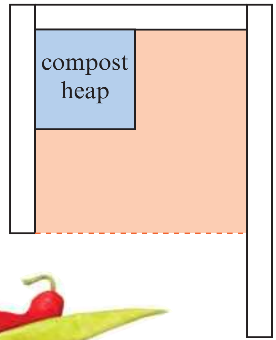
d Do you believe ASA and AAS are in fact the same congruency test? Explain your answer.

14 **a** Draw an equilateral triangle and split it into three congruent pieces.

b Draw another equilateral triangle and split it into four congruent pieces.

c In how many ways can you split a square into four congruent pieces?

15 Bella wants to turn the back corner of her backyard into a vegetable garden. The space available is a square with side lengths of 4 m. The very back left corner will be taken up with a compost heap covering a square of length 2 m, as shown in the diagram.

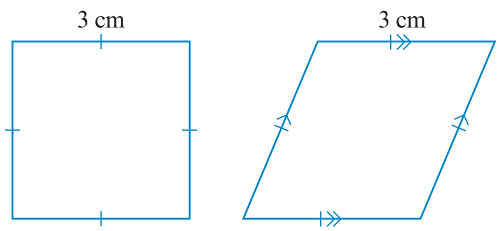


- **a** What is the entire area of the vegetable garden?
- **b** What area is left when you subtract the area of the compost heap?

Bella wants to plant four different vegetables in her garden, but she wants each type of vegetable to be in a congruent space.

- **c** How would you split the shape of the vegetable garden available for planting (the orange section) into four congruent sections? (Hint: use your answer to part **b** to first find out how much area each section should cover.)

16 Are the conditions for congruence in triangles conditions for congruence in other shapes? Let's investigate.



- **a** Use the figures of a square and a rhombus to explain why, even if two shapes have all corresponding sides the same length, it does not mean that they are congruent.
- **b** Use a rectangle and a parallelogram to give another example of two shapes with all corresponding sides the same length that are not congruent.
- **c** Draw a rectangle and a square and use them to explain why, if two shapes have all corresponding angles the same size, it does not mean that they are congruent (or even the same shape).
- **d** Use any other shapes (quadrilaterals are usually easiest) to draw pairs of shapes that explain why the remaining conditions for congruence for triangles (SAS and AAS) do not necessarily apply to other shapes.
- **e** Explain why, for shapes other than triangles, all corresponding sides must be equal *and* all corresponding angles must be equal for a shape to be deemed congruent.

17 Create your own puzzling problem like the one in question **15**.

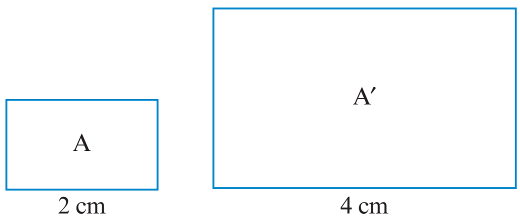
Reflect

How you would decide if two 3D objects are congruent?

5E Dilation and scale factor

Start thinking!

You can see that from A to A′ is an enlargement, but by how much? To determine this you would find the ratio or scale factor between corresponding lengths.



1 What is the length of: **a** A? **b** A′?

2 Divide the length of A′ by the length of A to find the scale factor from A to A′.

3 Why don't you divide the length of the original by the length of the image to find the scale factor from A to A′?

4 Copy and complete this sentence: A′ is _____ times as *small*/*large* as A.

This sentence can also be written in reverse to describe the reduction from A′ to A.

5 Write this reversed sentence.

Scale factors to represent a reduction are written as a fraction, where the denominator is the number that would be used to describe an enlargement. This can be summarised:

A is dilated by x to produce A′, and in reverse A′ is dilated by $\frac{1}{x}$ to produce A.

6 What is the scale factor from A′ to A?

7 Write statements like the one above for each situation.

a B′ is four times the size of B **b** C is three times the size of C′

c D′ is nine times the size of D **d** E is eight times the size of E′

KEY IDEAS

- Dilation is a transformation that does not produce congruent figures.
- Dilations are described using scale factor, found using the formula scale factor = $\frac{\text{image length}}{\text{original length}}$.
- A scale factor greater than 1 indicates an enlargement (the image is bigger).
- A scale factor between 0 and 1 indicates a reduction (the image is smaller).
- To calculate a side length of the image after dilation, multiply the original side length by the scale factor. For example, when dilating a shape with a side AB, use length of A′B′ = length of AB × scale factor.

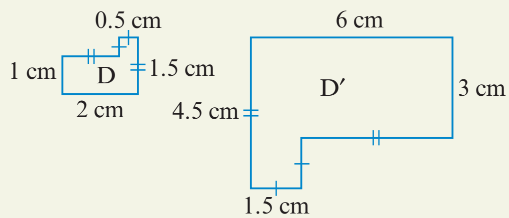
EXERCISE 5E Dilation and scale factor

1 Decide whether each scale factor produces an enlargement or a reduction.

a 4 **b** $\frac{1}{3}$ **c** 2 **d** 6 **e** $\frac{1}{10}$ **f** $\frac{1}{5}$

EXAMPLE 5E-1 Describing dilations

Describe the dilation shown from D to D′.

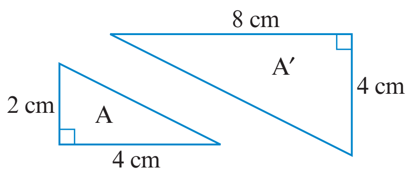


THINK	WRITE
1 D′ is larger than D, so the dilation is an enlargement and the scale factor will be more than 1.	$\text{scale factor} = \dfrac{\text{image length}}{\text{original length}}$
2 Compare the length of corresponding sides. The longest side on the original measures 2 cm and the longest side on the image measures 6 cm.	$= \dfrac{6}{2}$ $= 3$
3 Write your final answer.	The dilation from D to D′ is an enlargement with a scale factor of 3.

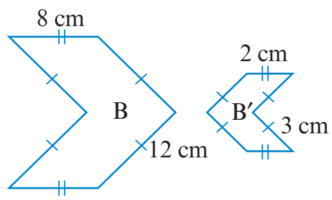
UNDERSTANDING AND FLUENCY

2 Describe each dilation.

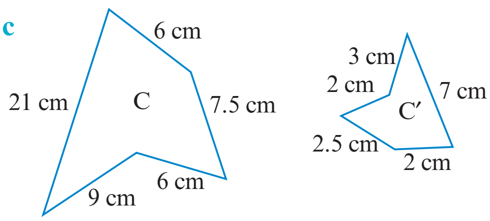
a



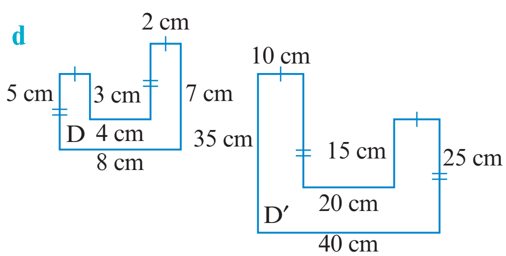
b



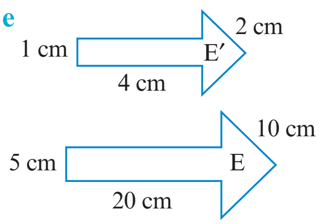
c



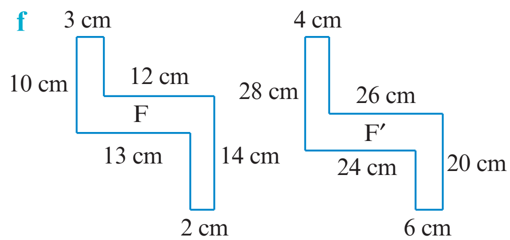
d



e

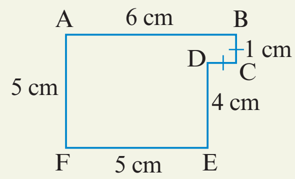


f



EXAMPLE 5E-2 Performing dilations

Dilate this figure by a scale factor of $\frac{1}{2}$.



THINK

1 The scale factor is $\frac{1}{2}$, which means that the side lengths of the image will be half the side lengths of the original.

2 Select any vertex of the shape (say vertex A). Two edges start from A: AB and AF. Apply the scale factor to these edges to find their lengths in the dilated image.

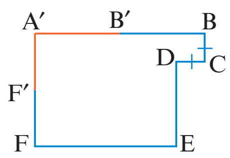
3 Draw these two edges in from the vertex A and mark in the new vertices B′ and F′.

4 For each remaining edge, find its new length and add it to the image of the figure. Label the image with vertices to complete it.

WRITE

length of A′B′ = length of AB × scale factor
$= 6 \text{ cm} \times \frac{1}{2}$
$= 3 \text{ cm}$

length of A′F′ = length of AF × scale factor
$= 5 \text{ cm} \times \frac{1}{2}$
$= 2.5 \text{ cm}$

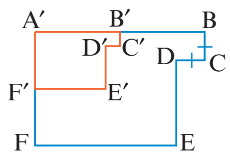


length of B′C′ $= 1 \text{ cm} \times \frac{1}{2} = 0.5 \text{ cm}$

length of C′D′ $= 1 \text{ cm} \times \frac{1}{2} = 0.5 \text{ cm}$

length of D′E′ $= 4 \text{ cm} \times \frac{1}{2} = 2 \text{ cm}$

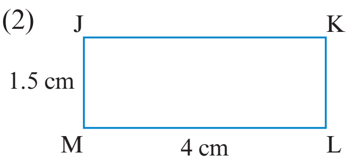
length of E′F′ $= 5 \text{ cm} \times \frac{1}{2} = 2.5 \text{ cm}$



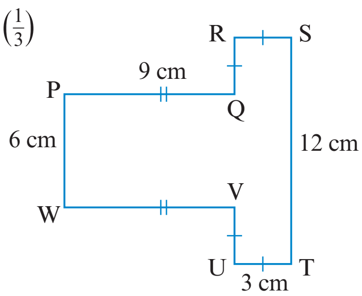
UNDERSTANDING AND FLUENCY

3 Dilate each figure by the scale factor shown in brackets.

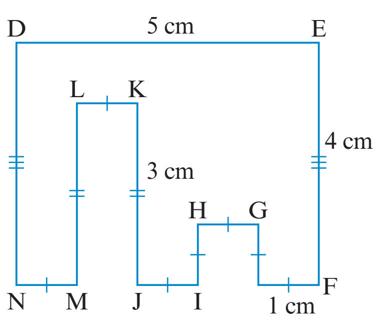
a (2)



b $\left(\frac{1}{3}\right)$



c (3)



PROBLEM SOLVING AND REASONING

4 Timothy got a model plane for his birthday, which had a scale factor of $\frac{1}{100}$.

a What does this scale factor mean?

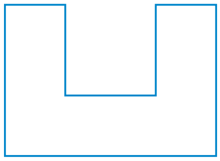
b If the real-life plane had a length of 34 m, what is the length of the model plane?

c If the model plane had a wingspan of 27 cm, what is the length of the wingspan on the real-life plane?

d Explain the difference between the processes you used to find the answers to parts **b** and **c**.

5 What does a scale factor of 1 mean?

6 Dilate this figure by each scale factor.



a $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** $\frac{2}{5}$

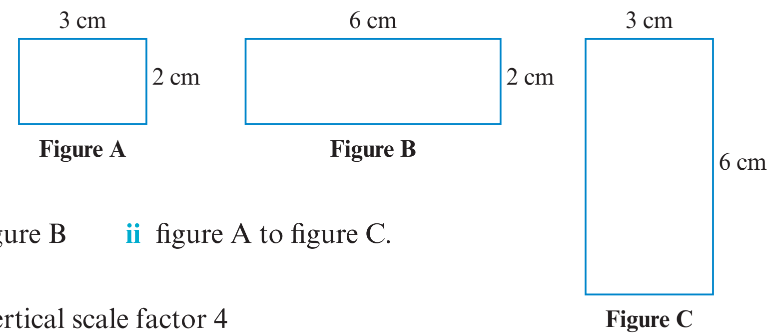
d $\frac{7}{10}$ **e** $\frac{6}{5}$ **f** $\frac{7}{3}$

7 Finn was explaining question **6** to a classmate and said that a dilation of $\frac{2}{3}$ was the same as dilating by a factor of 2 and then dilating by a factor of $\frac{1}{3}$. Can you explain why he is correct?

8 A microscope can be used to produce a dilated image of a very small object to enable you to see the details. A microscope was used to produce this image of a microorganism named *Paramecium*, which is only 0.15 mm long. Measure the length of the microorganism and find the scale factor at which the microscope was working.

9 A microscope was used to look at a bacterial cell. If, through the microscope, the cell looked 5.2 cm long, and the microscope was working on a scale factor of 20 000, find how long the bacterial cell actually is.

10 Dilations can also be done in a vertical direction only or a horizontal direction only. Consider these figures; B and C are separate dilations of figure A.



a By stating the direction of the dilation and the scale factor, describe the dilation from:

i figure A to figure B **ii** figure A to figure C.

b Dilate:

i figure A by vertical scale factor 4

ii figure A by horizontal scale factor 5

iii figure B by horizontal scale factor 3

iv figure C by vertical scale factor 2.

11 Example 5E-2 shows a method to dilate a figure from one of its vertices. The vertex that is picked is called the **centre of dilation**. Choosing a centre of dilation that is a vertex of the shape helps to guide the dilation by allowing you to follow along an edge. However, the centre of dilation does not need to be a vertex or even anywhere on the shape. Consider figure D. Dotted lines have been drawn from each vertex to the selected centre of dilation.

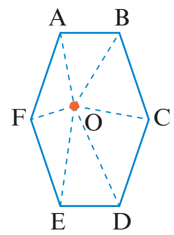


Figure D

a What is the centre of dilation in figure D?

b Measure the distance from each vertex to the centre of dilation.

Figure E shows the dotted lines extended so that they are now twice as long.

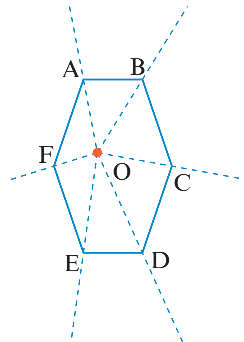


Figure E

c What scale factor is this?

d Join the ends of the dotted lines to complete the image, with sides twice the size of the original figure.

e How could you use the dotted lines to help you draw a reduction?

f Use the dotted lines to draw an image with a scale factor of $\frac{1}{2}$.

Consider figure F.

g Where is the centre of dilation?

The dotted lines can be extended or reduced in the same way as before to produce either an enlargement or reduction.

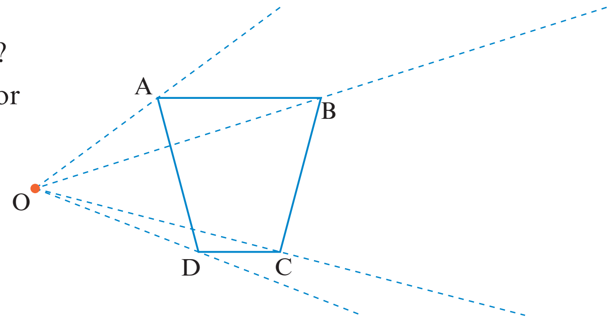


Figure F

h On which side of the original figure would a reduction be?

i On which side of the original figure would an enlargement be?

j Use the dotted lines to draw an image dilated by a scale factor of 2.

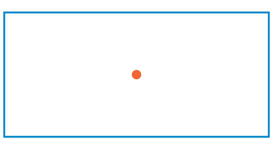
k Use the dotted lines to draw an image dilated by a scale factor of $\frac{1}{2}$.

l How is this similar to and how is this different from the process shown in parts **a–f**?

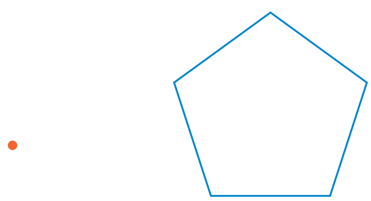
12 Use the methods shown in question **11** to dilate each figure by a scale factor of:

i 3 **ii** $\frac{1}{3}$.

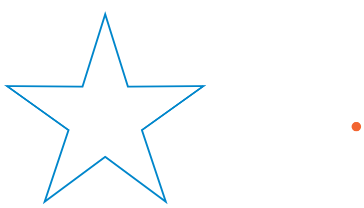
a



b

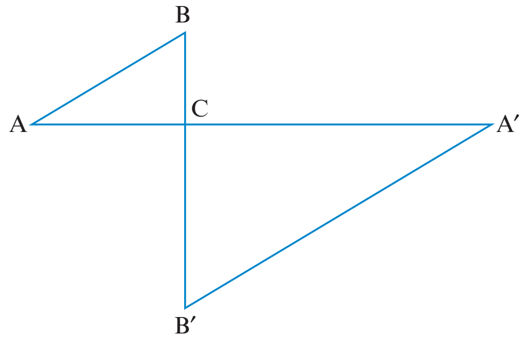


c



CHALLENGE

13 What does it mean if a scale factor is negative? Consider this figure.



a What is different about the image compared to what you have looked at so far?

The figure has been dilated by a scale factor of −2, which produces an **inverted image**.

b From which vertex has the original figure been dilated?

c Repeat the dilation of a scale factor of −2, but this time from the other vertices, A and B. What do you notice is different about these images?

d From a vertex or vertices of your choosing, dilate the figure by:

i −3 **ii** $-\frac{1}{2}$ **iii** −1.

14 Emily bought Timothy a model plane to go into a special glass case that allows for a maximum height of 15 cm.

a If the plane was 30 m tall in real life, what scale factor should be applied to the model plane?

When the plane arrives, Emily finds that the plane actually measures 16 cm in height.

b What scale factor has actually been applied to the model plane?

Assuming that all dimensions must stay in the same ratio, it is important that the plane fits into the glass case.

c What scale factor would the model need to be dilated by to fit into the space? (Hint: think carefully about which measurement would be the original and which measurement would be the image.)

d What scale factor would the space need to be dilated by to fit the current model?

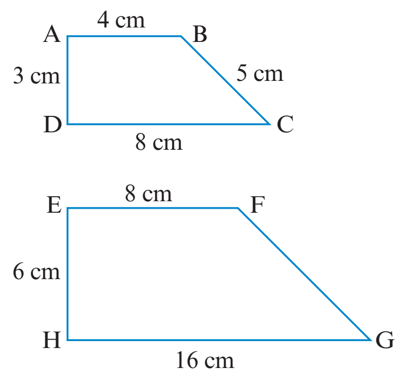
e How are the answers to parts **c** and **d** different and how are they similar?

Reflect

Why is scale factor important when working with dilations?

5F Similar figures

Start thinking!



Similar figures are exactly the same shape but can be different sizes.

1 Does dilation produce similar figures?

Consider these two trapeziums. EF is twice as long as AB.
If these figures are similar, FG must also be twice as long as BC.

2 If FG measured 9 cm, would these two shapes be similar? Why or why not?

3 What must FG measure if they are similar?

4 Explain how you found the answer to question **3**.

The formula: scale factor = $\frac{\text{image length}}{\text{original length}}$ can be rearranged to find an unknown length.

To find an unknown side length on an 'original' figure, the formula becomes:
original length = $\frac{\text{image length}}{\text{scale factor}}$.

To find an unknown side length on an 'image' figure, the formula becomes:
image length = original length × scale factor.

5 Explain why these three formulas are equivalent.

6 Which formula did you use without realising in question **3**?

If all angles are the same, figures are similar (and could even be congruent).

7 What could you do to check that they are similar but not congruent?

8 When checking if two figures are similar, why is it important that all angles and sides are checked?

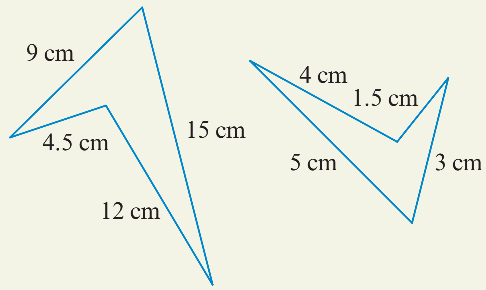
KEY IDEAS

- Similar figures are identical in shape but can be different in size.
- For two figures to be similar, all angles must be equal in size and all corresponding sides must be in the same ratio.
- If two figures are known to be similar, an unknown side length or angle can be found if the scale factor is known.
- Scale factor is calculated using the formula: scale factor = $\frac{\text{image length}}{\text{original length}}$.
- To find an unknown side length on an 'original' figure, use the formula: original length = $\frac{\text{image length}}{\text{scale factor}}$.
- To find an unknown side length on an 'image' figure, use the formula: image length = original length × scale factor.
- A scale factor can also be written as a **ratio scale**. For example, a scale factor of $\frac{1}{10}$ is the same as a ratio scale of 1:10.

EXERCISE 5F Similar figures

EXAMPLE 5F-1 Finding the scale factor

Find the scale factor for this pair of similar figures. Assume that the first figure is the original.

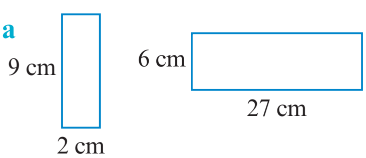


THINK	WRITE
1 Compare corresponding sides. The longest side of the original figure is 15 cm, and the longest side of the image is 5 cm.	Image length of 5 cm corresponds to original length of 15 cm.
2 Substitute these lengths into the formula: scale factor $= \dfrac{\text{image length}}{\text{original length}}$.	scale factor $= \dfrac{\text{image length}}{\text{original length}}$ $= \dfrac{5 \text{ cm}}{15 \text{ cm}}$ $= \dfrac{1}{3}$
3 Match the remaining corresponding sides and check that each side is in the same ratio.	scale factor $= \dfrac{4}{12} = \dfrac{3}{9} = \dfrac{1.5}{4.5} = \dfrac{1}{3}$
4 Check that the scale factor seems reasonable. (Number between 0 and 1 means a reduction.)	
5 Write your answer.	Scale factor is $\frac{1}{3}$.

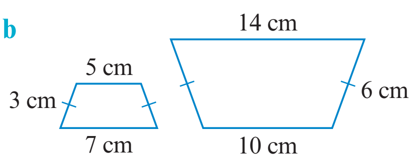
UNDERSTANDING AND FLUENCY

1 Find the scale factor for these similar figures. Assume that the first figure in each pair is the original.

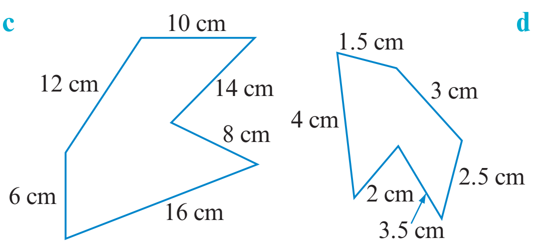
a



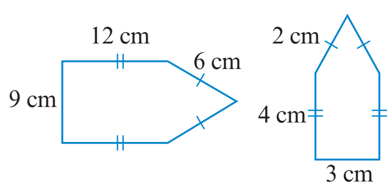
b



c



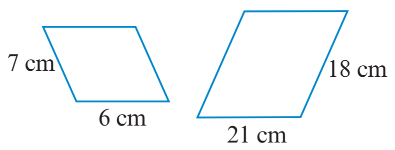
d



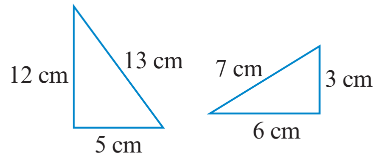
UNDERSTANDING AND FLUENCY

2 Decide if each pair of shapes is similar by calculating and comparing the scale factor for each pair of corresponding sides.

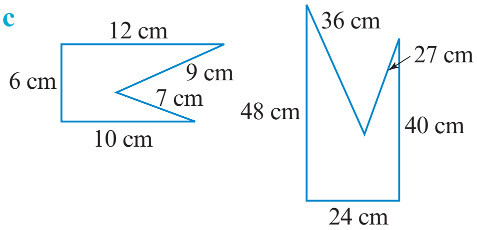
a



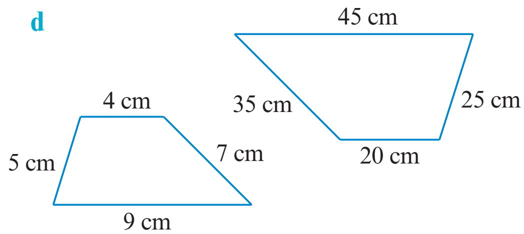
b



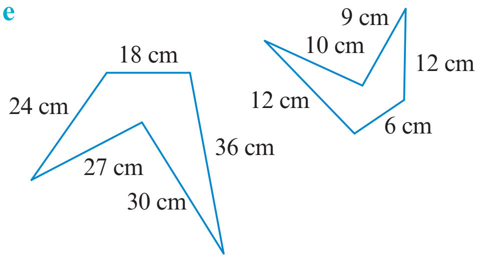
c



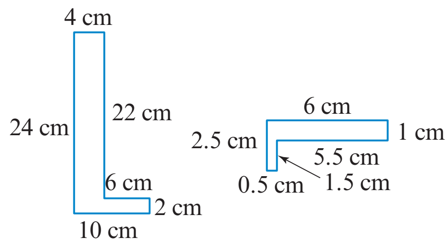
d



e

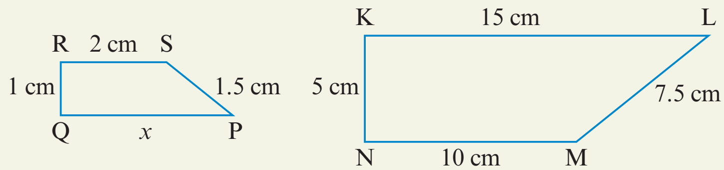


f



EXAMPLE 5F-2 Finding an unknown side length using similar figures

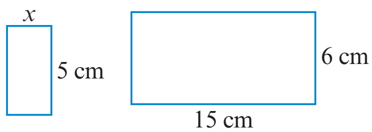
Given that these two figures are similar, find the value of x.



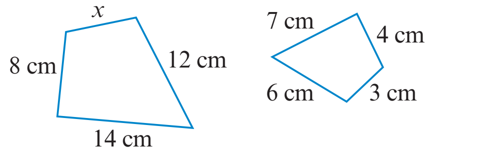
THINK	WRITE
1 Find a pair of corresponding sides and calculate the scale factor. Assume the first figure is the original. Use corresponding vertices to write the side names in the same order. As R corresponds to N and S corresponds to M, then side RS corresponds to side NM (not MN).	Sides RS and NM are corresponding. $\text{scale factor} = \dfrac{\text{image length}}{\text{original length}}$ $= \dfrac{NM}{RS}$ $= \dfrac{10\text{ cm}}{2\text{ cm}}$ $= 5$
2 Find a pair of corresponding sides that involve x.	Sides QP and KL are corresponding.
3 Choose the appropriate formula and substitute values.	$\text{original length} = \dfrac{\text{image length}}{\text{scale factor}}$ $x = \dfrac{15\text{ cm}}{5}$ $= 3\text{ cm}$

3 Given that these pairs of shapes are similar, find the value of x.

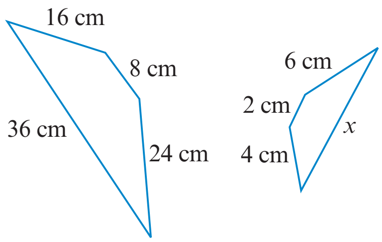
a



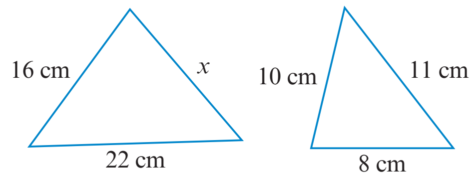
b



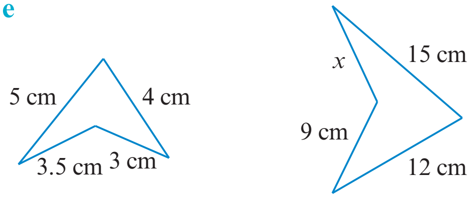
c



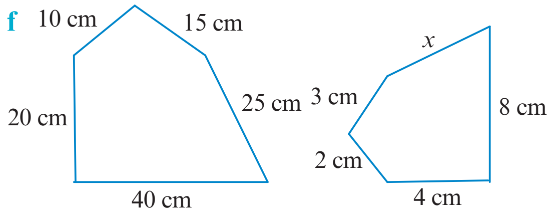
d



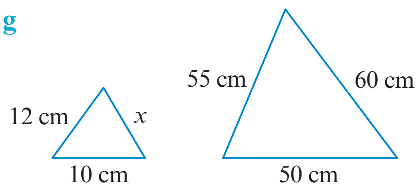
e



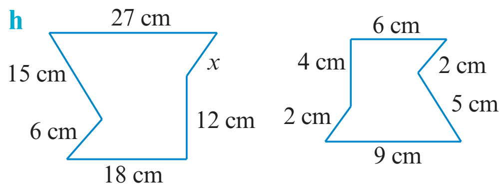
f



g



h



4 Explain the difference between congruent figures and similar figures.

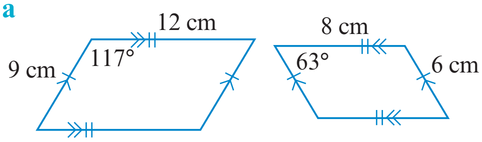
5 **a** If two figures are similar, does this mean that they are congruent?

b If two figures are congruent, does this mean that they are similar?

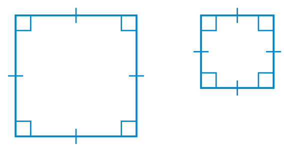
c Many mathematicians have different opinions about the answer to part **b**. Discuss with a classmate and then see what your class thinks.

6 Use your understanding of quadrilateral properties to decide whether each pair of shapes is similar.

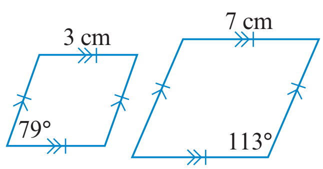
a



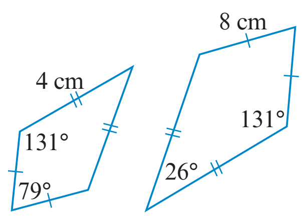
b



c



d



7 Using a square and a rhombus, explain why having all corresponding sides in the same ratio is not enough to make two figures similar.

PROBLEM SOLVING AND REASONING

8 Babushka or matryoshka dolls are a set of wooden dolls of decreasing size that can fit inside one another. Ignoring the painted designs, use your understanding of similar figures to decide if any or all of these dolls are similar figures.

9 Is a baby a similar figure to an adult? Think carefully about the proportion of different body parts.

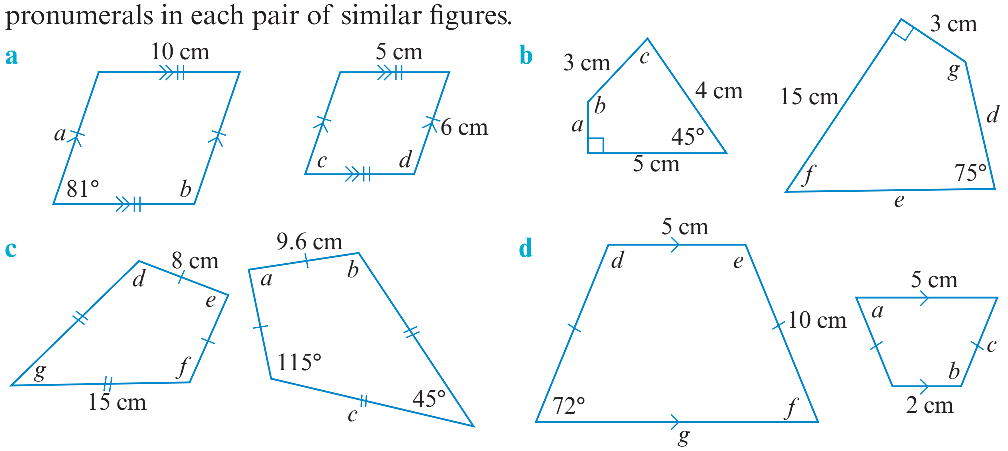
10 One standard print size for photographs is 10 cm × 15 cm. Alison wanted to enlarge a photograph to place it onto a canvas measuring 40 cm × 50 cm.

- **a** Find the vertical scale factor and the horizontal scale factor.
- **b** Are the photograph and its corresponding canvas art similar figures?
- **c** What will most likely happen to the photograph to fit it onto the canvas without distorting the picture?

11 There is a range of 'standard' print sizes for photographs. Using this list, compare and find which of these are similar figures.

- **A** 10 cm × 15 cm
- **B** 12.5 cm × 17.5 cm
- **C** 15 cm × 20 cm
- **D** 20 cm × 25 cm
- **E** 20 cm × 30 cm

12 Use your understanding of quadrilateral properties to find the value of the pronumerals in each pair of similar figures.



13 Elliot drew a regular hexagon with side lengths of 3 cm.

- **a** What is the difference between a regular and an irregular hexagon?
- **b** If Elliot drew another regular hexagon with different side lengths, explain how you know without measuring anything that the two hexagons must be similar.

14 Sometimes a ratio scale is used in place of scale factor. This usually happens when the similar figure is much smaller than the original figure (e.g. on maps). For example, a figure that is ten times as small as the original has a scale factor of $\frac{1}{10}$. As a ratio scale, this is written as 1:10.

- **a** How do you think a scale factor of $\frac{1}{2}$ would be written as a ratio scale?
- **b** Explain the relationship between the fraction and the ratio scale. (Hint: what parts of the ratio scale relate to the numerator and denominator?)

PROBLEM SOLVING AND REASONING

What about when the similar figure is an enlargement? Consider an enlargement with a scale factor of 2.

- **c** What is 2 written as a fraction?
- **d** Use your answer to part **c** to write a scale factor of 2 as a ratio scale.
- **e** Write these ratio scales as scale factors.
  - **i** 1:4 **ii** 1:100 **iii** 1:2500 **iv** 1:150 000 **v** 2:1 **vi** 100:1
- **f** Explain how you can tell if a similar figure is an enlargement or reduction by the order of the ratio scale. (Hint: what is the difference between a ratio scale of 1:4 and 4:1?)

15 A model car is constructed using the ratio scale of 1:40.

- **a** What is this as a scale factor?
- **b** If the model car had a length of 10 cm, what is the length of the real car?
- **c** If the real car had a height of 1.5 m, what is the height of the model car?
- **d** Do you think that a model car can be a true similar figure to a real car? Explain.

16 Muhammed was using a hiking map that had a ratio scale of 1:75 000.

- **a** How is this written as a scale factor?
- **b** Muhammad planned to walk the length of a 15 km track. What distance would represent this on the map?
- **c** At lunch, Muhammad figured out that he had walked 7 cm along the map. How far did he have left to walk?

CHALLENGE

17 This table shows the dimensions of standard paper sizes, rounded to the nearest millimetre. Use the table to decide whether all or any of these are similar figures. Compare:

- **a** sizes within the A group (e.g. A2, A4 and A7)
- **b** sizes within the B group (e.g. B0, B3 and B6)
- **c** sizes within the C group (e.g. C1, C5 and C8)
- **d** sizes between different groups (e.g. A0, B0 and C0).

Write a paragraph discussing what you find. What effect do you think rounding the measurements might have on the similarity of the paper sizes?

	A sizes (mm)	B sizes (mm)	C sizes (mm)
0	841 × 1189	1000 × 1414	917 × 1297
1	594 × 841	707 × 1000	648 × 917
2	420 × 594	500 × 707	458 × 648
3	297 × 420	353 × 500	324 × 458
4	210 × 297	250 × 353	229 × 324
5	148 × 210	176 × 250	162 × 229
6	105 × 148	125 × 176	114 × 162
7	74 × 105	88 × 125	81 × 114
8	52 × 74	62 × 88	57 × 81
9	37 × 52	44 × 62	40 × 57
10	26 × 37	31 × 44	28 × 40

18 Draw a map of your bedroom with an accurate scale. Write the scale as both a ratio scale and a scale factor.

Reflect

When might it be useful to determine if two figures are mathematically similar compared to the common meaning of similar?

5G Similar triangles

Start thinking!

For two figures to be classed as similar, all angles must be equal and all corresponding sides must be in the same ratio. This is true for all similar figures, including triangles.

Consider the conditions for congruence.

1 **a** What does SSS mean?
b How could you reword SSS so that it became a condition for similarity?

2 **a** What does SAS mean? Why does it guarantee congruence?
b How could you reword SAS to become a condition for similarity?

3 **a** What does RHS mean? Why does it guarantee congruence?
b How might you reword it to become a condition for similarity?

4 **a** What does AAA mean? Does it necessarily mean congruence?
b What can you say about two shapes that have all three corresponding pairs of angles the same size?
c Is AAA a condition for similarity?

5 **a** Another condition for congruence is AAS. Why is it hard to look at the ratio of corresponding sides in this case?
b You can still use AAS as a condition for similarity. Why does the length of the sides of two triangles that have two angles the same not matter?

6 Explain why replacing 'corresponding sides that are in the same ratio' with 'corresponding equal sides that are …' changes conditions for similarity into conditions for congruence.

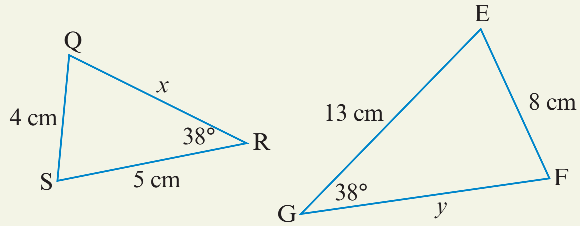
KEY IDEAS

- **Similar triangles** have corresponding angles that are equal and corresponding sides in the same ratio.
- For two triangles to be similar, they must meet one of four similarity conditions:
  - SSS: All three corresponding sides are in the same ratio.
  - SAS: Two corresponding sides are in the same ratio and the angles in between are equal.
  - AAA: All three corresponding angles are equal.
  - RHS: The hypotenuses are in the same ratio as another pair of sides in a right-angled triangle.

EXERCISE 5G Similar triangles

EXAMPLE 5G-1 Finding unknown side lengths using similar triangles

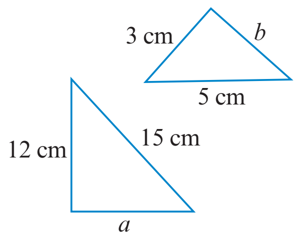
Find the value of the unknown side lengths in this pair of similar triangles.



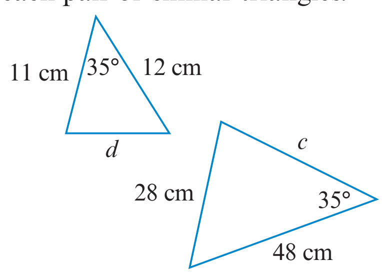
THINK	WRITE
1 Match corresponding sides.	Corresponding pairs of sides are: QR and EG; QS and EF; and RS and GF.
2 Using the pair of corresponding sides that have known measurements (QS and EF), find the scale factor from ΔQSR to ΔEFG. That is, consider ΔQSR as the original triangle and ΔEFG as the image.	$\text{scale factor} = \dfrac{\text{image length}}{\text{original length}}$ $= \dfrac{\text{EF}}{\text{QS}}$ $= \dfrac{8 \text{ cm}}{4 \text{ cm}}$ $= 2$
3 Use the appropriate formula to find x in ΔQSR.	$\text{original length} = \dfrac{\text{image length}}{\text{scale factor}}$ $x = \dfrac{13 \text{ cm}}{2}$ $= 6.5 \text{ cm}$
4 Use the appropriate formula to find y in ΔEFG.	image length = original length × scale factor $y = 5 \text{ cm} \times 2$ $= 10 \text{ cm}$
5 Write your final answer.	$x = 5$ cm and $y = 10$ cm

1 Find the value of the unknown side lengths in each pair of similar triangles.

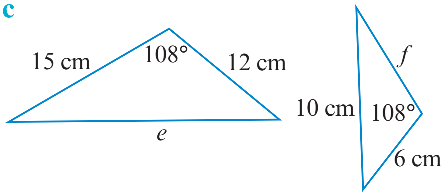
a



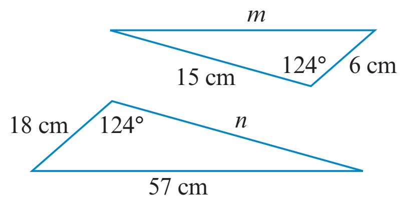
b



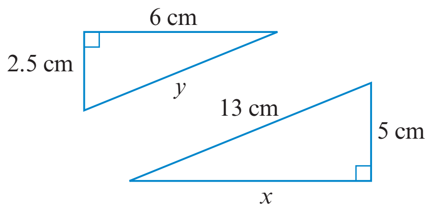
c



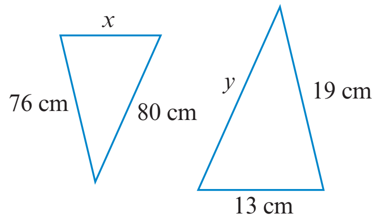
d



e

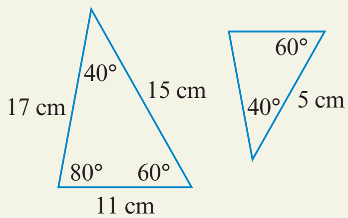


f



EXAMPLE 5G-2 Identifying an appropriate similarity condition

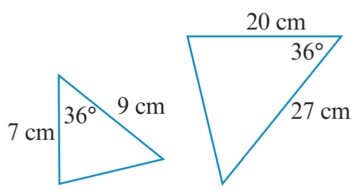
Which condition would you use to check to see if these triangles are similar?



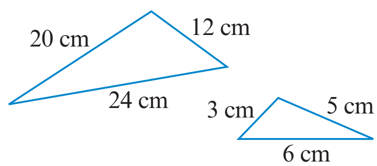
THINK	WRITE
1 Look at the triangles. The first triangle has all information given, but the second triangle has two angles and a side.	
2 As only one side is given on the second triangle, scale factor cannot be used to prove similarity. However, knowing two angles of a triangle means you can calculate the third angle, giving the similarity condition AAA.	You would use the similarity condition AAA to check if these triangles are similar.

2 Which condition would you use to check to see if these triangles are similar?

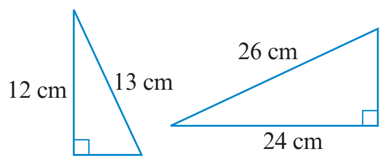
a



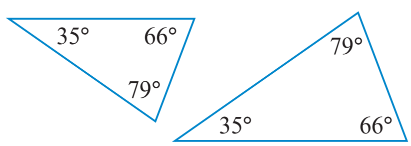
b



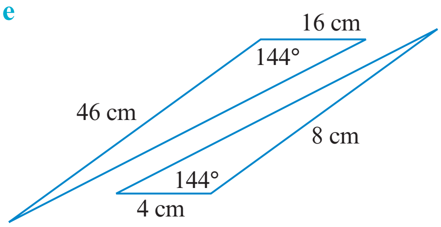
c



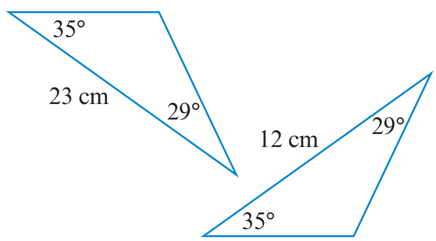
d



e

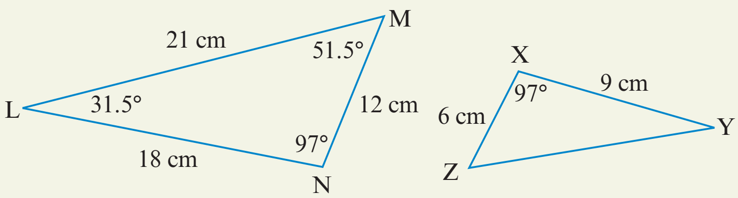


f



EXAMPLE 5G-3 Deciding if triangles are similar

Decide and explain if these two triangles are similar.

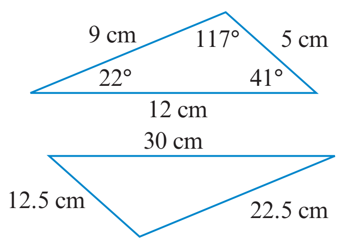


THINK	WRITE
1 ΔXYZ has two sides and an angle in between, so use the similarity condition SAS to compare. Match corresponding sides and angles. (≅ means 'corresponds to'.)	$YZ \cong LM$, $XY \cong NL$, $XZ \cong NM$, $\angle YXZ \cong \angle LNM$, $\angle XYZ \cong \angle NLM$, $\angle YZX \cong \angle LMN$
2 $\angle YXZ$ corresponds to $\angle LNM$. Are they equal?	$\angle YXZ = \angle LNM = 97°$
3 Side XY corresponds to side NL. Find the scale factor from ΔNLM to ΔXYZ.	$\text{scale factor} = \dfrac{\text{image length}}{\text{original length}} = \dfrac{XY}{NL} = \dfrac{9\text{ cm}}{18\text{ cm}} = \dfrac{1}{2}$
4 Side XZ corresponds to side NM. Find the scale factor.	$\text{scale factor} = \dfrac{\text{image length}}{\text{original length}} = \dfrac{XZ}{NM} = \dfrac{6\text{ cm}}{12\text{ cm}} = \dfrac{1}{2}$
5 Look at your results and write your final answer.	The two triangles are similar as they meet the similarity condition SAS.

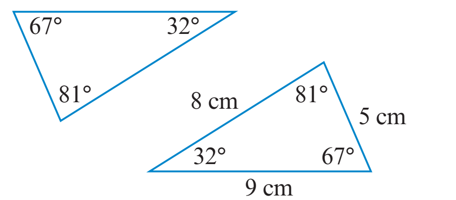
UNDERSTANDING AND FLUENCY

3 Decide and explain which of these pairs of triangles are similar.

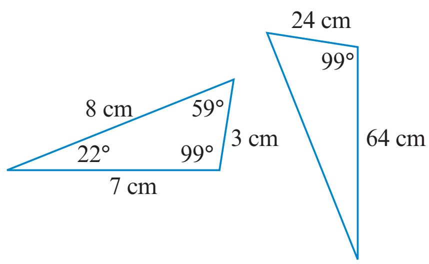
a



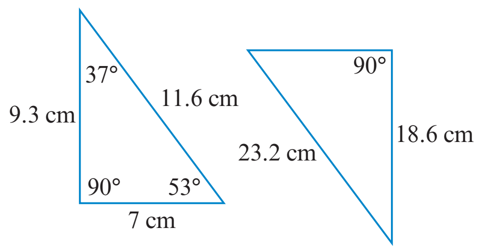
b



c

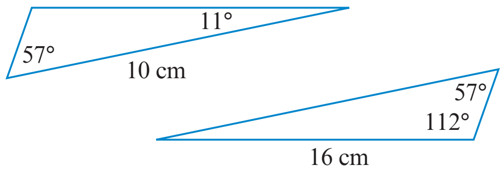


d

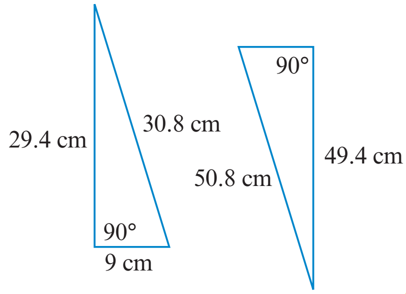


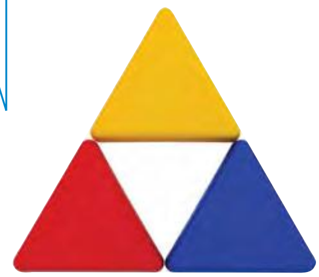
PROBLEM SOLVING AND REASONING

e



f

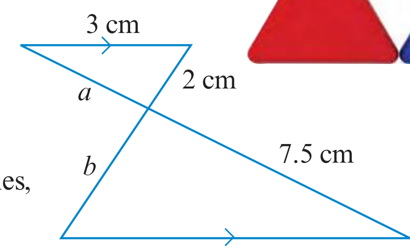




4 Decide and explain which of the pairs of triangles from question **2** are similar.

5 Why must all equilateral triangles be similar to one another?

6 Explain why, if you are using AAA as a test for similarity, only two angles need to be checked.



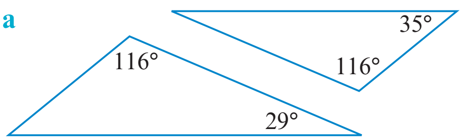
7 **a** Using an understanding of angles and parallel lines, explain why the triangles on the right are similar.

b Match corresponding sides and find the scale factor between the two triangles.

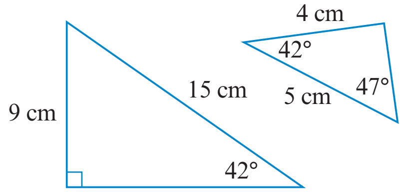
c Find the length of the unknown sides.

8 Use your understanding of triangle properties to determine if these pairs of triangles are similar.

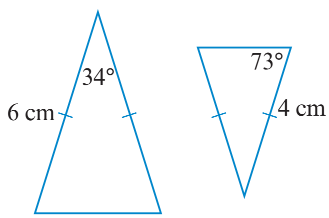
a



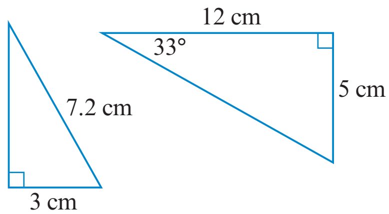
b



c



d

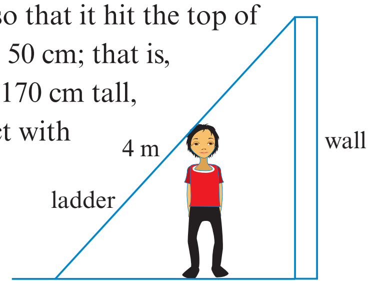


9 Similar triangles can be used to find the height or length of objects that are difficult to measure. Consider this diagram, showing the shadows cast by a person and a tree. The person is 1.76 m tall and casts a shadow of 2.1 m. If the tree casts a shadow of 7.8 m, how tall is the tree?

10 Luke wanted to know how tall the walls of his house were so that he could order some wall decorations. He placed his 4 m ladder on an angle so that it hit the top of the wall, as shown in the diagram. The ladder has rungs every 50 cm; that is, the first rung was at 50 cm, the second at 1 m, etc. If Luke, at 170 cm tall, stood directly under the ladder and his head came into contact with the fifth rung, find:

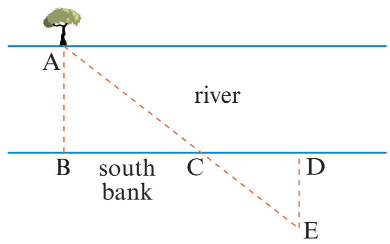
a the length of the ladder where it touched Luke's head

b the height of Luke's walls.

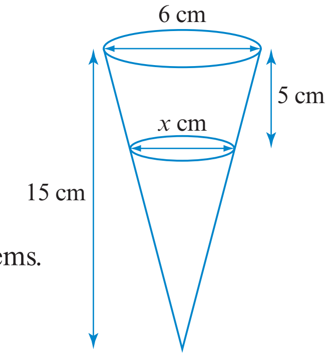


PROBLEM SOLVING AND REASONING

11 In order to find how wide a river is, Ahmed places three markers along its southern bank. Marker B is directly opposite a tree at position A. Marker C is 50 m east of marker B, and marker D is a further 40 m east. Ahmed then stands at point E where he is directly 30 m south of marker D and can see marker C in a direct line with the tree at point A. Mark in these measurements on the diagram and then use your understanding of similar triangles to find how wide this river is.



12 An ice-cream cone is 15 cm long and at its top has a diameter of 6 cm. If chocolate coats the inside of the cone from the bottom to 5 cm below its top, find the largest diameter of the chocolate coating inside the cone.



13 Draw diagrams to help you solve these similar triangle problems.

a Scott had a bunch of small, red triangular flags that were 10 cm high and 15 cm long. He wanted to create a larger version and found red material that was 2 m long. If he uses its full length, what height would this larger flag have?

b Christy bought a new netball ring but she wasn't sure that it was the correct, official height (3.05 m). She measured her shadow as 117 cm and the netball ring's shadow as 195 cm. If Christy was 1.8 m tall, is the netball ring the official height?

c Oscar wanted to know how tall a flag pole was. A guide wire positioned 3.5 m from its base reached to exactly half its height. Oscar stood exactly halfway between the flag pole and where the guide wire attached to the ground and the guide wire touched the top of his head. If Oscar is 1.75 m tall, how tall is the flag pole?

14 Sean and Tania wanted to know whose house was taller. One morning Sean measured his shadow to be 1.9 m and the shadow of his house to be 3.42 m. Later that day, Tania measured her shadow to be 2.3 m and the shadow of her house to be 3.8 m. If Sean is 172 cm tall and Tania is 165 cm tall, whose house is taller?

15 Explain why you cannot just compare the shadow lengths of the houses in question **14** to decide which house is taller.

16 Create your own similar triangle problem with a diagram and swap with a classmate to solve it.

CHALLENGE

17 Using examples, explain why the conditions for similar triangles (AAA, SSS, SAS and RHS) do not work for other polygons, for example quadrilaterals. You may wish to look at Exercise 5D question **16** on page 227 to help you.

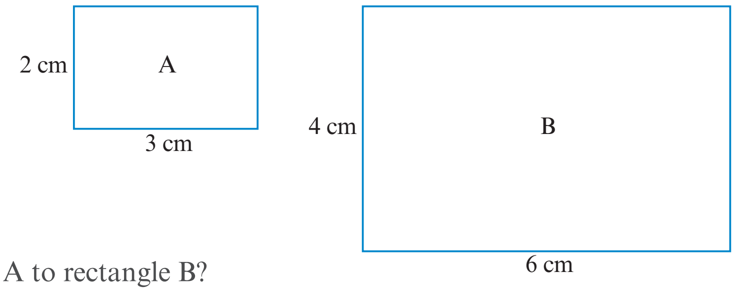
Reflect

How can an understanding of similar triangles be used to find lengths that are difficult or impossible to measure?

5H Scale factor and area

Start thinking!

When you enlarge a figure by a scale factor of 2, the lengths of its sides double in size. What about its area? Consider similar rectangles A and B.



1 What are the dimensions of rectangle:

i A? **ii** B?

2 What is the length scale factor from rectangle A to rectangle B?

3 How many square centimetres in rectangle:

i A? **ii** B?

4 How many times the area of rectangle A is the area of rectangle B?

5 Draw another pair of similar rectangles where one has sides that are twice the size of the other. Do you get the same results?

6 What is the **area scale factor** for a pair of figures that have a length scale factor of 2?

7 Copy and complete this table by drawing pairs of similar rectangles that have a length scale factor of 3, 4 and 5.

Length scale factor	Area scale factor
2	4
3	
4	
5	

8 What do you notice about the relationship between the length scale factor and the area scale factor?

9 How does this relate to the fact that area is always described in square units?

10 Predict the area scale factor for similar figures that have these length scale factors.

a 6 **b** 7 **c** 8 **d** 10

11 Write a sentence explaining why, if a figure is enlarged by a length scale factor of 4, it does not mean that the area of the image is four times the area of the original figure.

KEY IDEAS

- Area scale factor is equal to the square of the length scale factor.
- The formulas to use for area scale factor are the same as for length scale factor, but you must be careful to use area measurements, not length measurements.
- original area $= \frac{\text{image area}}{\text{area scale factor}}$.
- image area = original area × area scale factor.

EXERCISE 5H Scale factor and area

EXAMPLE 5H-1 Finding the area scale factor

A figure is dilated by a length scale factor of 3. What is its area scale factor?

THINK	WRITE
Area scale factor is equal to the square of the length scale factor.	area scale factor $= 3^2$ $= 9$

1 For each length scale factor, what is the area scale factor?

a 2 **b** 3 **c** 4

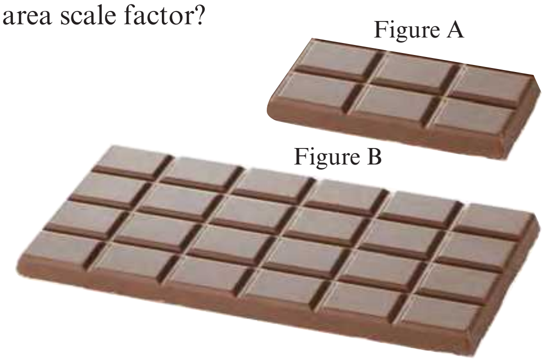
d $\frac{1}{2}$ **e** $\frac{1}{3}$ **f** $\frac{1}{4}$

2 Figure A has its side lengths doubled in size to produce Figure B.

a How many small blocks are there in:

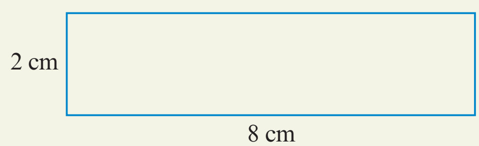
i Figure A? **ii** Figure B?

b Compare the area of each figure.



EXAMPLE 5H-2 Finding area using the area scale factor

If this rectangle has its side lengths doubled in size, what will be the area of the image?



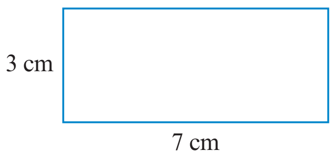
THINK	WRITE
1 Find the area of the original rectangle.	$A = lw$ $= 2 \times 8$ $= 16 \text{ cm}^2$
2 Find the area scale factor (the square of the length scale factor).	area scale factor $= 2^2 = 4$
3 Multiply the original area by the area scale factor to find the area of the image.	$16 \times 4 = 64 \text{ cm}^2$
4 Write your answer.	The area of the image will be 64 cm^2.

3 Copy and complete this table.

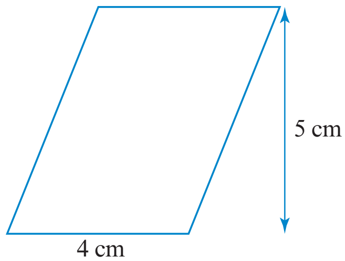
	Original area	Length scale factor	New area
a	12 cm^2	2	
b	5 cm^2	3	
c	10 cm^2	5	
d	40 cm^2	$\frac{1}{2}$	
e	27 cm^2	$\frac{1}{3}$	
f	100 cm^2	$\frac{1}{5}$	

4 For each figure, state the area of the image if the original figure is dilated by the length scale factor shown in brackets.

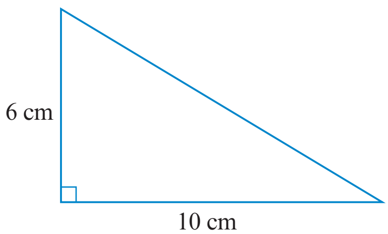
a (4)



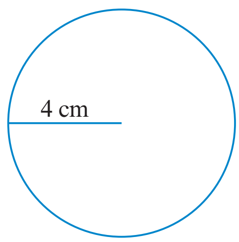
b (3)



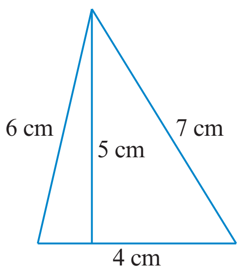
c (5)



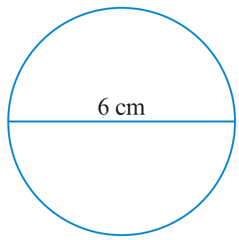
d $\left(\frac{1}{2}\right)$



e $\left(\frac{1}{3}\right)$



f (2)



5 Ethan needed to increase the size of the area of his working space to nine times its previous size in order to accommodate a new project. Explain why he does not need to increase the length measurements of his working space by nine.

6 Use your understanding of fraction multiplication to find the area scale factor for an image if the length scale factor is:

a $\frac{1}{2}$ **b** $\frac{1}{3}$

c $\frac{1}{4}$ **d** $\frac{1}{10}$

e $\frac{2}{3}$ **f** $\frac{3}{5}$

7 Explain how, if you know the area scale factor, you can find the length scale factor.

8 Calculate the length scale factor for these area scale factors.

a 49 **b** 25
c 100 **d** 64
e 4 **f** 400

PROBLEM SOLVING AND REASONING

9 In low light, the pupils of our eyes dilate to let in more light. In normal light, Chantelle's pupils had a radius of 1.2 mm. When she walked into a dark room, they dilated to have a radius of 3.6 mm.

a What is the length scale factor?
b What is the area scale factor?
c Using the formula $A = \pi r^2$ and your answer to part **b**, find these areas correct to two decimal places.
  i the original pupil area
  ii the dilated pupil area
  iii the difference in these two areas

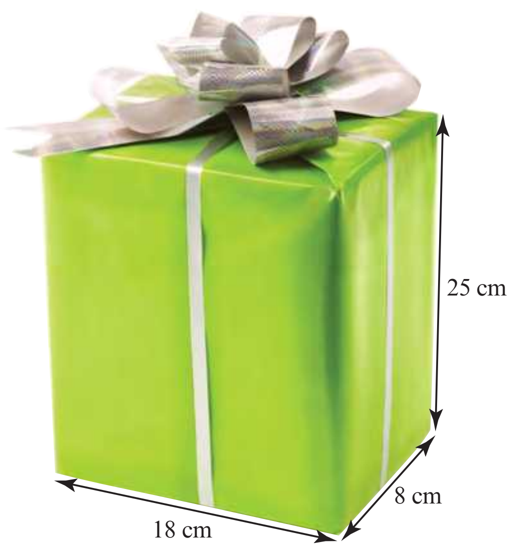
10 Sophie painted a square canvas with a length of 10 cm.

a What area did Sophie paint?

Sophie was asked to paint a replica, but increase its size to five-fold.

b What would be the new dimensions of this enlarged painting?
c What is the area of this enlarged painting?
d How much more paint will Sophie need to paint this painting in comparison to the original?

11 How much more wrapping paper will be needed to cover a gift box which has double the dimensions of the one shown?



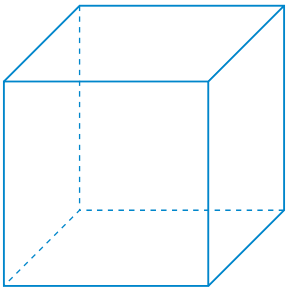
12 A farmer bought new land that was next to a paddock he already owned. If the area that he owned increased by a factor of 4, by what scale factor did the length of the land increase by?

13 A rectangular vegetable garden had its length (but not width) doubled. How would this affect its area?

14 A triangular flag needs its dimensions increased by a factor of 5. If the flag has a 30 cm base and a 45 cm height, find the difference in the area of material needed to make each flag.

15 A radar has a scanning radius of roughly 5 km. If the owner wanted to cover an area of 100 km^2, by how much would the radius of the radar need to increase?

16 If area scale factor is equal to the square of the length scale factor, what would volume scale factor be equal to? Consider this cube.



a What are the side lengths of this cube?

b What is the volume of this cube?

c What would be the volume of this cube if the side lengths were:

i doubled?

ii tripled?

iii quadrupled?

Create a table to show your answers.

d What pattern can you see between the length scale factor applied to the cube and the resulting increase in volume?

e Explain why the volume scale factor is equal to the cube of the length scale factor.

f Using your knowledge of how to calculate volume scale factor, find the volume of the cube if it was increased by a length scale factor of:

i 5 **ii** 6 **iii** 10.

17 A company wanted to create a new line of beach balls. A standard beach ball at this company had a volume of roughly 1000 cm^3.

a If they increased the radius of the standard beach ball by a length scale factor of 2, what would be the volume of the new beach ball?

b If they halved the radius of the standard beach ball by a length scale factor of 2, what would be the volume of the new beach ball?

c If they wanted to create a giant beach ball with 1000 times the volume of the standard beach ball, by what scale factor would they have to increase the radius?

18 A 30-cm deep sandpit had side lengths of 1 m.

- **a** If the sandpit was doubled in size (all side lengths), what would be its new dimensions?
- **b** By what factor would this increase the volume of the sandpit?
- **c** How much *more* sand would be required to fill this larger sandpit?

CHALLENGE

19 Most architects construct scale models of their building designs before the real one is built.

- **a** If a particular model for a building that will sit on a city block measuring 250 m × 300 m and the model has a base area of 4687.5 cm^2, what area scale factor was used to create the model?
- **b** What are the dimensions of the baseboard of the model?
- **c** If the stairwell window in the model measures 1.8 cm × 6.0 cm, what area of glass will be needed in the real building?
- **d** The stairwell will extend 20 m back from the window in the real building. What will the volume of the stairwell be in the real building? In the model?

20 An aquarium is in the form of a **rectangular prism**. If, to accommodate a new type of fish, it needed to have its height doubled, by what factor would this increase:

- **a** the area of glass needed for the sides of the tank?
- **b** the volume of the tank?

Reflect

How is knowledge of square numbers useful in determining area scale factor?

CHAPTER REVIEW

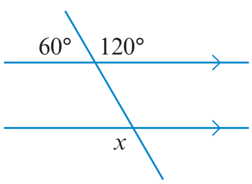
SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

parallel lines	complementary angles	reflection	scale factor
transversal	angles around a point	rotation	enlargement
vertically opposite angles	degrees-minutes-seconds	dilation	reduction
alternate angles	polygon	isometric transformation	similar figures
corresponding angles	convex	non-isometric transformation	ratio scale
co-interior angles	concave	image	similar triangles
supplementary angles	translation	congruence	area scale factor

MULTIPLE-CHOICE

5A **1** What is the size of angle x?

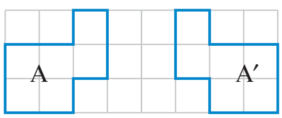


A $x = 60°$
B $x = 120°$
C $x = 180°$
D not enough information

5B **2** Which statement provides the internal angle sum of a hexagon?

A $180° \div 6$ **B** $6 \times 180°$
C $(6 - 2) \times 180°$ **D** $(6 + 2) \times 180°$

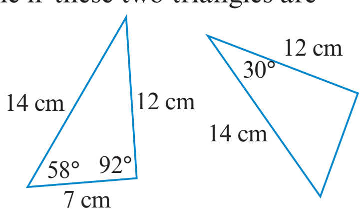
5C **3** Which statement best describes this transformation?



A a translation 2 units to the right
B a translation 2 units to the left
C a reflection in a vertical mirror that is 1 unit to right of original shape
D a reflection in a horizontal mirror that is 1 unit from original shape

5D **4** Which congruency test could you use to determine if these two triangles are congruent?

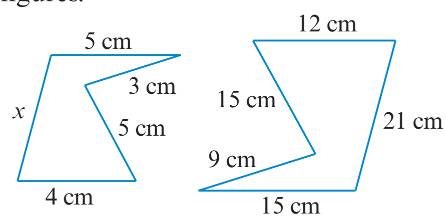
A SSS
B SAS
C AAS
D RHS



5E **5** Which scale factor will not result in an enlargement?

A $\frac{1}{3}$ **B** $\frac{5}{4}$ **C** 2 **D** 4

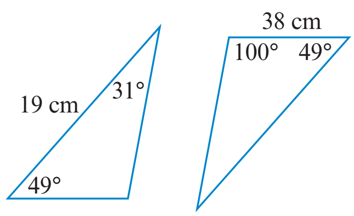
5F **6** Find the value of x for these similar figures.



A 21 cm **B** 63 cm
C 7 cm **D** 8.75 cm

5G **7** Which similarity condition would you use to decide if these two triangles are similar?

A AAA
B SSS
C SAS
D RHS



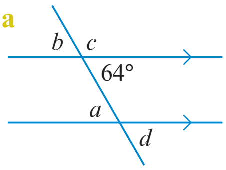
5H **8** All sides of a parallelogram are to be dilated by a scale factor of 3. What will the area scale factor be?

A $\frac{1}{3}$ **B** 3 **C** 6 **D** 9

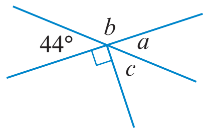
SHORT ANSWER

5A ▶ **1** Find the value of each pronumeral.

a

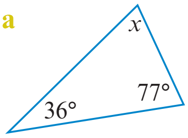


b

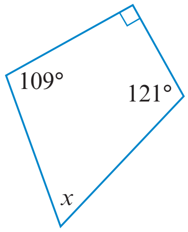


5B ▶ **2** Find the value of x in each shape.

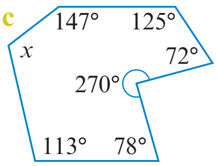
a



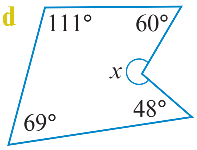
b



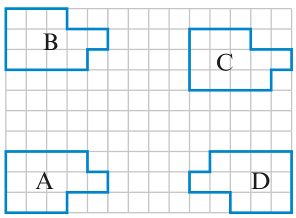
c



d



5C ▶ **3** Consider the shapes drawn on this grid.



a Describe each transformation.

i B to A **ii** C to B **iii** A to D

b Using B as the original shape, perform each transformation. Draw each shape on a new grid first.

i a translation 2 units right and 3 units down

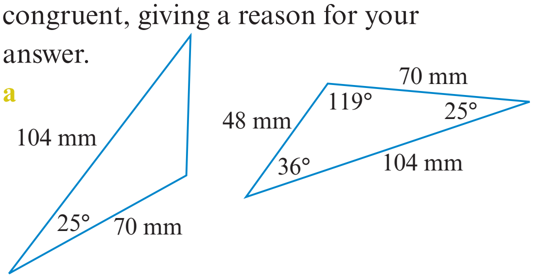
ii a reflection in a vertical mirror that is placed 2 units to the right

iii a rotation of 90° anticlockwise around a point that is 1 unit to the left of the top left vertex

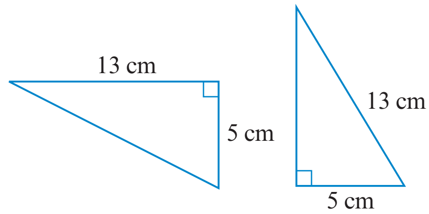
iv a dilation by a scale factor of 2

5D ▶ **4** Decide if each pair of triangles is congruent, giving a reason for your answer.

a

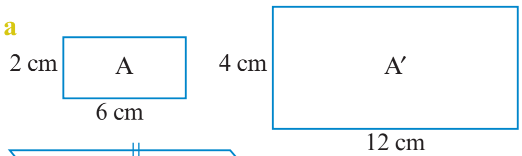


b

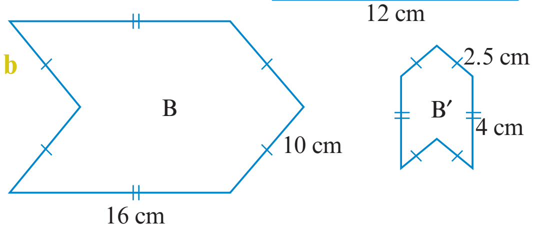


5E ▶ **5** Describe each transformation as a reduction or an enlargement and include the scale factor.

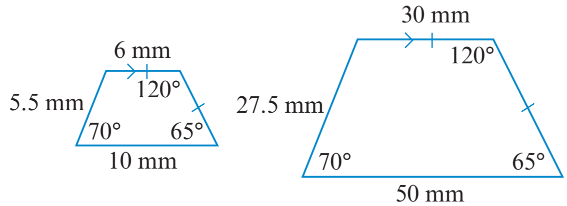
a



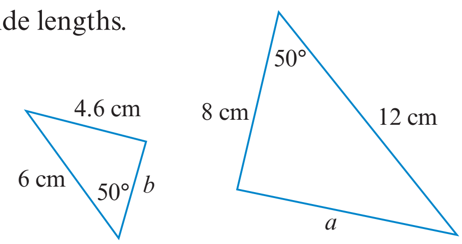
b



5F ▶ **6** Decide if this pair of shapes is similar, giving a reason for your answer.



5G ▶ **7** Given that these two triangles are similar, find the value of the unknown side lengths.



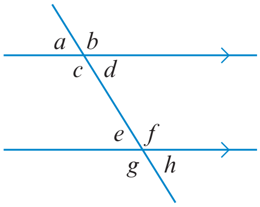
5H ▶ **8** State the area of the image if each figure is dilated by the length scale factor given.

a A 2 cm by 6 cm rectangle is dilated by a length scale factor of 4.

b A rectangle with width 6 cm and length 10 cm is dilated by a length scale factor of $\frac{1}{2}$.

NAPLAN-STYLE PRACTICE

Questions 1–3 refer to this diagram.



1 The angle that is alternate to c is:

☐ b ☐ f ☐ e ☐ g

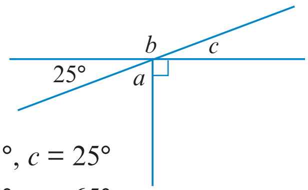
2 If b is equal to 125° what is the value of d?

☐

3 Which statement is *not* true?

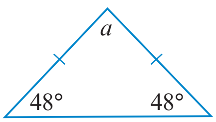
☐ Angles a and d are vertically opposite.
☐ Angles d and h are corresponding.
☐ Angle g is alternate to d.
☐ Angles d and f are co-interior.

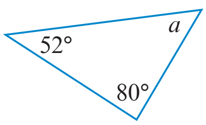
4 What are the values for the unknown angles?

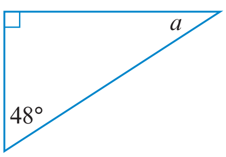


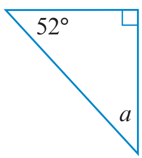
☐ $a = 65°, b = 155°, c = 25°$
☐ $a = 65°, b = 155°, c = 65°$
☐ $a = 65°, b = 65°, c = 25°$
☐ $a = 75°, b = 165°, c = 25°$

5 In which triangle is the value for a equal to 48°?

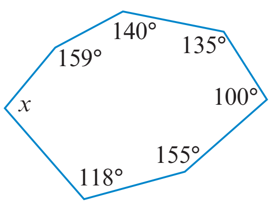
☐ 

☐ 

☐ 

☐ 

Questions 6 and 7 refer to this diagram.



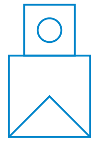
6 Which of these best describes the shape?

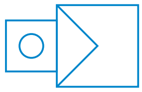
☐ a regular hexagon
☐ an irregular hexagon
☐ an irregular heptagon
☐ an irregular octagon

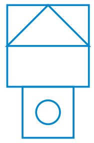
7 What is the value of x?

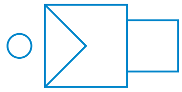
☐

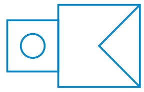
8 Which diagram could represent a rotation of this figure?



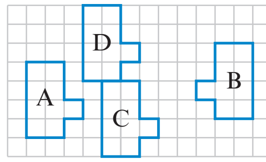
☐ 

☐ 

☐ 

☐ 

9 Between which two shapes is there a translation 4 units right and 1 unit down?

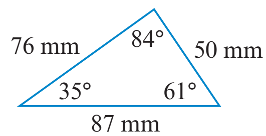


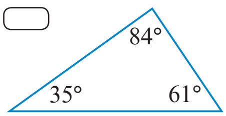
☐ A and C ☐ C and A
☐ A and B ☐ D and C

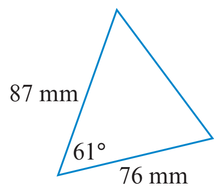
10 Two triangles contain two pairs of angles that are equal and a pair of corresponding sides that are also equal. Which of these is the relevant condition for congruence?

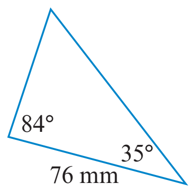
☐ AAA ☐ SSA
☐ SAS ☐ AAS

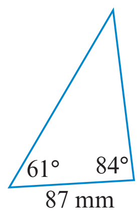
11 Which of the triangles below is congruent to this triangle?



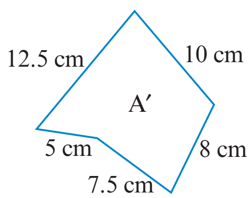
☐ 

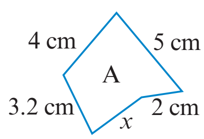
☐ 

☐ 

☐ 

Questions 12 and 13 refer to this diagram.





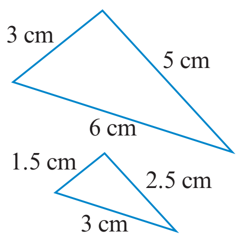
12 Describe the dilation from A to A′.

13 What is the value of x?

14 Which statement cannot be used to describe two triangles as similar?

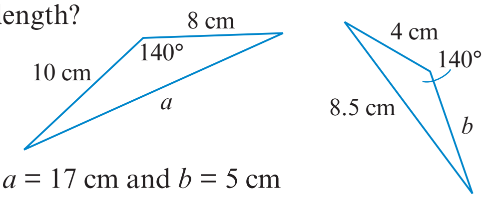
- ☐ All corresponding angles are equal and all corresponding sides are in the same ratio.
- ☐ All corresponding sides of the two triangles are in the same ratio.
- ☐ The two triangles contain a right-angle and the hypotenuse lengths are in the same ratio.
- ☐ All corresponding angles are equal in size.

15 Which condition would you use to check to see if these triangles are similar?



- ☐ AAA
- ☐ SSS
- ☐ SAS
- ☐ RHS

16 What is the value of each unknown side length?



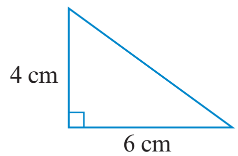
- ☐ a = 17 cm and b = 5 cm
- ☐ a = 4.25 cm and b = 5 cm
- ☐ a = 8.5 cm and b = 10 cm
- ☐ a = 17 cm and b = 4 cm

17 The sides of a rectangle were dilated by a scale factor of 4. What would the area scale factor be?

2	4	8	16
☐	☐	☐	☐

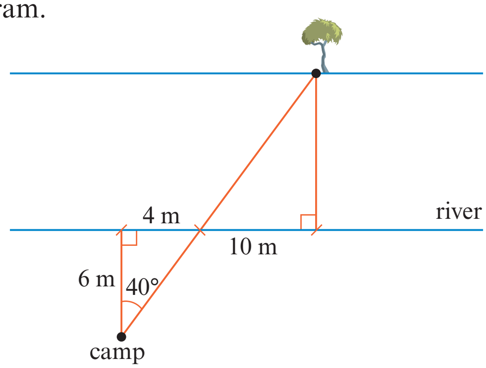
18 The triangle shown is dilated by a length scale factor of 2.

What will the area of the dilated image be?



ANALYSIS

While preparing to study a species of gum tree on a field trip, Stephanie set up a camp 6 m from a river's edge. From the camp, she measures the direct line of sight to a gum tree she wishes to get to and records various measurements, as shown in the diagram.



a Considering both of the triangles shown in Stephanie's diagram, what are the angle sizes in each triangle?

b Are the two triangles congruent or only similar? Provide a reason to support your answer. (Hint: which test could you use to prove congruence or similarity?)

c What is the scale factor linking the triangle attached to the camp site to the triangle attached to the gum tree?

d Briefly explain how the triangles can be used to determine the width of the river.

e Calculate the width of the river.

f Find the shortest and longest pathway Stephanie can take to get to the gum tree from her camp. (Hint: use the lengths of the triangles shown in the diagram and assume she has access to a boat.)

g What is the difference in length between your longest and shortest pathway in part **f**?

CONNECT

Mini-golf

Your family wants to open a mini-golf course, but lots of planning is needed. Different layouts for the different holes are required, and you want to have both a junior course and a senior course that is a larger version of the junior course. You also want to promote that it is possible to complete any hole in only one shot.

Your task

Use your understanding of geometry to create three layouts that will suit both the junior and senior courses. At least one of these layouts must include obstacles that are similar or congruent. For each layout, you will need to consider:

- the size and shape of the junior course
- the starting and finishing position
- the size, shape and position of any obstacles
- a 'cheat sheet' showing what angle to hit the ball (and the path it will take) in order to get a hole in one
- the difference in size between the junior and senior layout
- the area of felt needed to produce both the junior and senior layout.

You may like to present your findings as a report. Your report could be in the form of:

- a digital presentation
- a pamphlet
- a set of designs
- other (check with your teacher).

6 PYTHAGORAS' THEOREM AND TRIGONOMETRY

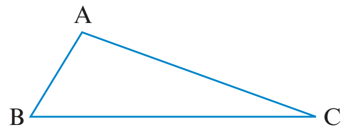
ESSENTIAL QUESTION

Why are right-angled triangles important to builders?

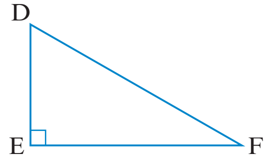
Are you ready?

6A ▶ **1** Consider these two triangles.

i



ii



a Use the given letters to name each triangle.

b Use the given letters to name the longest side in each triangle.

c Use the given letters to name the largest angle in each triangle.

d Which triangle is a right-angled triangle?

6A ▶ **2** Calculate each value.

a 7^2 **b** 25^2 **c** 3.8^2

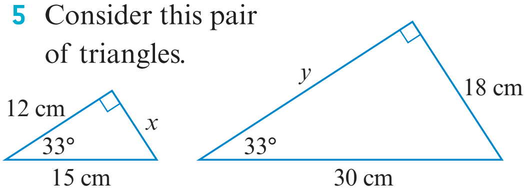
6A ▶ **3** Calculate each value. Where appropriate, write your answer correct to two decimal places.

a $\sqrt{81}$ **b** $\sqrt{140}$ **c** $\sqrt{5.82}$

6B ▶ **4** Solve each equation.

a $x^2 = 9$ **b** $x^2 + 3 = 19$

6D ▶ **5** Consider this pair of triangles.



a Explain why they are similar triangles.

b Find the value of each pronumeral.

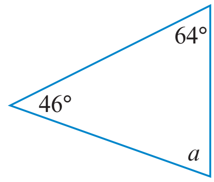
6E ▶ **6** Solve each equation.

a $\frac{x}{3} = 12$ **b** $\frac{35}{x} = 7$

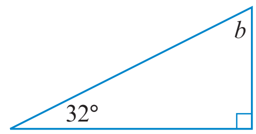
c $4 = \frac{9}{x}$ **d** $0.5 = \frac{11}{x}$

6F ▶ **7** Find the value of each unknown angle labelled with a pronumeral.

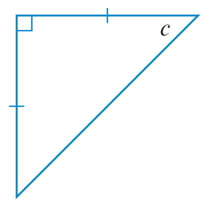
a



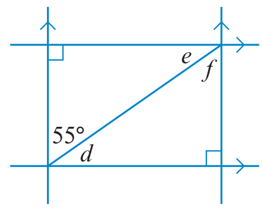
b



c



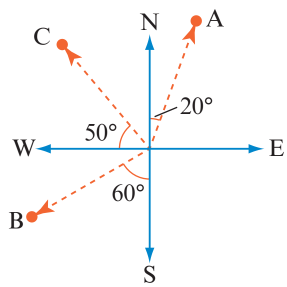
d



6G ▶ **8** Write each direction to the given point as:

i a compass bearing

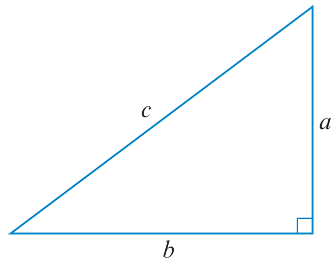
ii a true bearing.



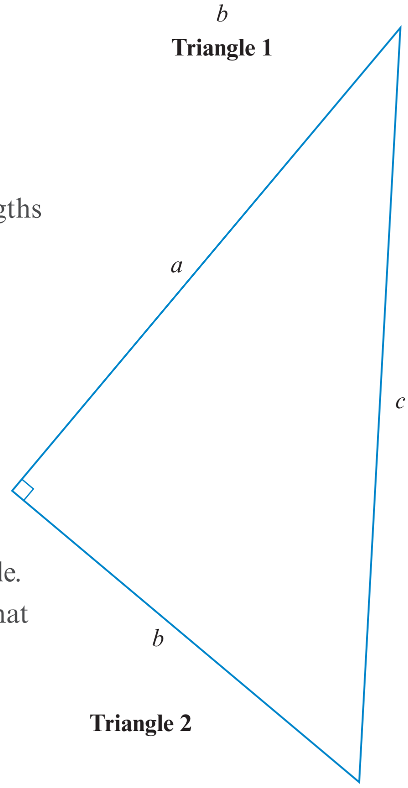
a A **b** B **c** C

6A Understanding Pythagoras' Theorem

Start thinking!



Triangle 1



Triangle 2

1 The longest side of a **right-angled** triangle is the **hypotenuse**. What pronumeral has been shown on the hypotenuse of each triangle? This pronumeral represents the length of the hypotenuse.

2 The hypotenuse is opposite a particular angle in a right-angled triangle. Which angle is it opposite?

3 One of the two shorter sides of each triangle has been labelled a to represent the length of that side. What pronumeral has been used to represent the length of the other shorter side?

4 Copy this table. Complete the columns for a, b and c by measuring the side lengths of each triangle to the nearest cm. Triangle 1 has been completed for you. Triangle 3 can be seen on the page opposite.

Triangle	a	b	c	a^2	b^2	c^2	$a^2 + b^2$
1	3	4	5				
2							
3							

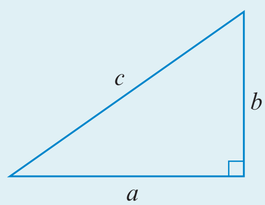
5 Complete the columns in the table for a^2, b^2 and c^2 for each triangle.

6 Calculate the value of $a^2 + b^2$ for each triangle and enter your results in the table.

7 Compare the values in the last two columns. Write a formula using a, b and c that shows the relationship between the three side lengths of each triangle.

8 The relationship you found is known as **Pythagoras' Theorem** and is named after the famous mathematician Pythagoras of Samos (c. 570–500 BCE). Does this theorem work for all triangles? Explain.

KEY IDEAS



- The hypotenuse is the longest side of a right-angled triangle.
- Pythagoras' Theorem states the relationship between the three side lengths of a right-angled triangle. The relationship for the triangle shown is $c^2 = a^2 + b^2$.
- A triangle is right-angled if it has side lengths that satisfy Pythagoras' Theorem.
- Any set of three whole numbers that satisfy Pythagoras' Theorem is called a **Pythagorean triad** (or **Pythagorean triple**).

EXERCISE 6A Understanding Pythagoras' Theorem

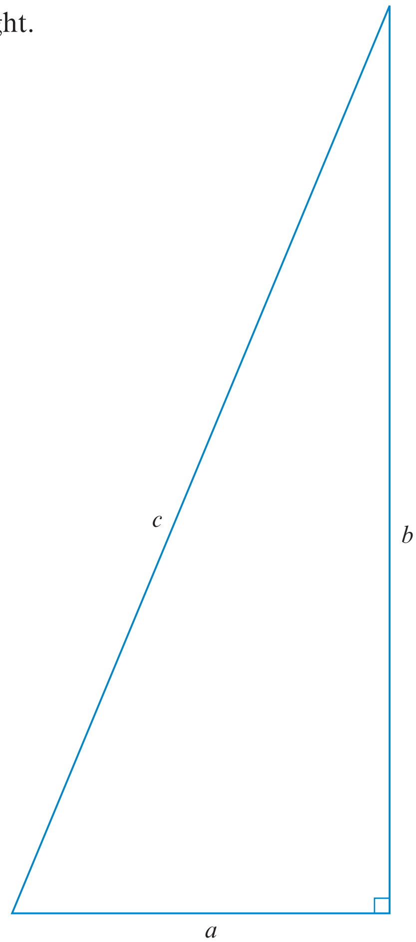
UNDERSTANDING AND FLUENCY

1 Pythagoras' Theorem works for the triangle shown at right. Check if it works for other triangles.

a Draw another three right-angled triangles and label the longest side as c and the two shorter sides as a and b. Carefully measure the length of each side to the nearest millimetre and record in a table.

b Do these triangles follow Pythagoras' Theorem? Explain.

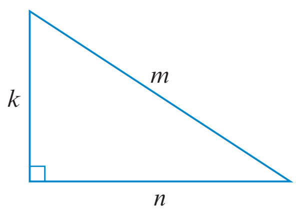
c Repeat parts **a** and **b** for three different triangles that are *not* right-angled.



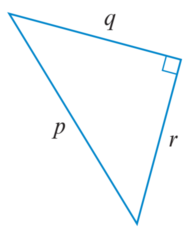
Triangle 3

2 Identify the pronumeral shown on the hypotenuse in each triangle.

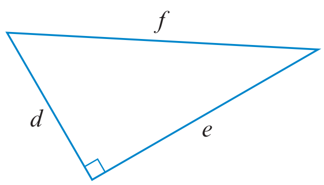
a



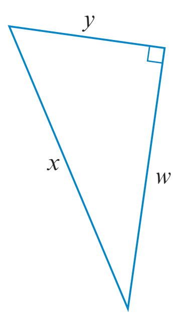
b



c



d



3 Using Pythagoras' Theorem, which statement (A, B, C or D) shows the relationship between the side lengths of the triangle in question **2a**?

A $k^2 = m^2 + n^2$ **B** $n^2 = k^2 + m^2$

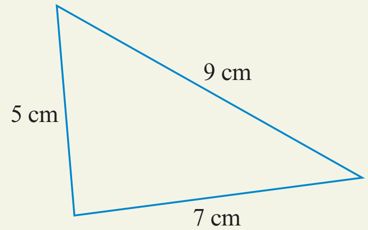
C $m^2 = k^2 + n^2$ **D** $m^2 = k^2 - n^2$

4 Use Pythagoras' Theorem to write a relationship between the side lengths of each triangle in questions **2b**, **c** and **d**.

5 If you know the three side lengths of a triangle, how could you use Pythagoras' Theorem to work out whether the triangle is right-angled? Discuss this with a classmate.

EXAMPLE 6A-1 Using Pythagoras' Theorem to identify whether a triangle is a right-angled

Use Pythagoras' Theorem to decide whether this triangle is a right-angled triangle.

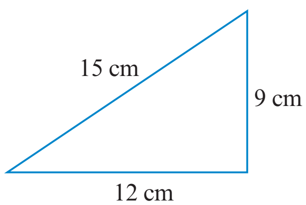


THINK	WRITE
1 Identify the length of the longest side.	Let c = 9 cm
2 Calculate the value of c^2.	$c^2 = 9^2 = 81$
3 Identify the lengths of the two shorter sides.	Let a = 5 cm and b = 7 cm.
4 Calculate the value of $a^2 + b^2$.	$a^2 + b^2 = 5^2 + 7^2$ $= 25 + 49$ $= 74$
5 Compare c^2 to $a^2 + b^2$. If the two values are equal, the triangle is right-angled. If the two values are not equal, it is not a right-angled triangle.	$c^2 \neq a^2 + b^2$ The triangle is not a right-angled triangle.

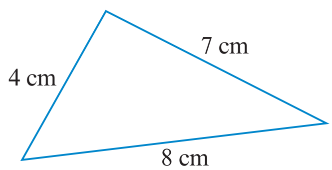
UNDERSTANDING AND FLUENCY

6 Use Pythagoras' Theorem to decide whether each triangle is a right-angled triangle.

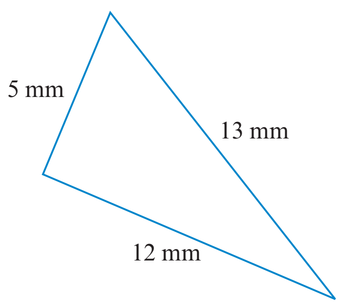
a



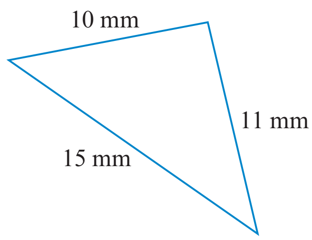
b



c



d



7 Decide whether each triangle with the given side lengths is right-angled. (Hint: which side length relates to the hypotenuse?)

a 6 mm, 8 mm, 10 mm
b 7 cm, 24 cm, 25 cm
c 8 cm, 10 cm, 14 cm
d 11 mm, 15 mm, 19 mm
e 20 cm, 21 cm, 29 cm
f 10 mm, 18 mm, 20 mm

PROBLEM SOLVING AND REASONING

8 In Pythagoras' Theorem, *a* and *b* refer to the lengths of the two shorter sides of a right-angled triangle. Does it matter which of the shorter sides is labelled *a* and which is labelled *b*? Explain.

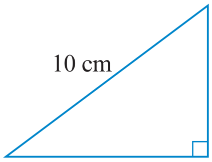
9 This window frame is constructed correctly if the timber meets at a right angle. The diagonal length of the glass in one section is measured to be 87 cm. Use Pythagoras' Theorem to check whether this window frame has been made correctly.

10 A rectangular picture frame measuring 70 cm by 24 cm is to be made. The picture framer checks that the frame is 'square' (right angled) by measuring the distance from the top left corner to the bottom right corner of the frame. The measured distance is 73 cm.

- **a** Draw a diagram of the frame and label it with all the known information.
- **b** Use Pythagoras' Theorem to check whether the frame is 'square'. If it is not, should the diagonal distance be longer or shorter?
- **c** Can you suggest what the diagonal distance should be?

11 Builders in ancient Egypt used a piece of rope with 12 knots equally spaced. How did this show that a corner was 'square'? Explain using a diagram.

12 Consider triangle A shown at right.



Triangle A

- **a** Use Pythagoras' Theorem to confirm that triangle A is right-angled.
- **b** Triangle B is formed by doubling the side lengths of triangle A. Does Pythagoras' Theorem still work for triangle B?
- **c** Triangle C is formed by tripling the side lengths of triangle A. Does Pythagoras' Theorem still work for triangle C?
- **d** Triangle D is formed by halving the side lengths of triangle A. Does Pythagoras' Theorem still work for triangle D?
- **e** Will Pythagoras' Theorem still work if the side lengths of triangle A are multiplied by 10? Explain.

13 Any set of three whole numbers that satisfy Pythagoras' Theorem is called a Pythagorean triad.

- **a** Does 6, 8 and 10 form a Pythagorean triad? (Hint: look at your answer to question **12a**.)
- **b** Use your answers to question **12** parts **b–e** to list four different Pythagorean triads.
- **c** Can you write another two Pythagorean triads? Discuss this with a classmate.

PROBLEM SOLVING AND REASONING

14 **a** Does 5, 12 and 13 form a Pythagorean triad? Explain.

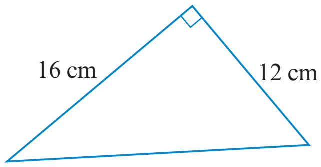
b Use your answer to part **a** to write another two Pythagorean triads.

15 **a** Does 8, 15 and 17 form a Pythagorean triad?

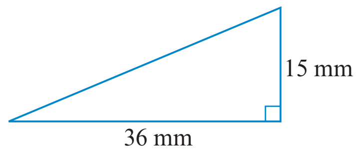
b Use your answer to part **a** to write another two Pythagorean triads.

16 Pythagorean triads can help you identify an unknown side length in a right-angled triangle if you know the lengths of the other two sides. Find the unknown side length in each right-angled triangle. (Hint: refer to your answers for questions **13**–**15**.)

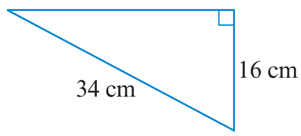
a



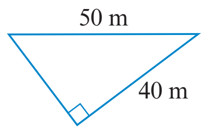
b



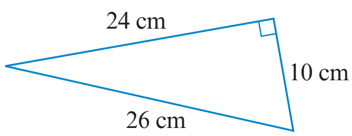
c



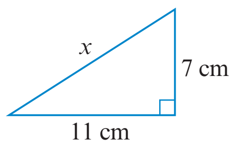
d



17 Consider triangle 1 and triangle 2 shown at right. You can use Pythagoras' Theorem to write a relationship for the side lengths of each right-angled triangle.



Triangle 1



Triangle 2

a Which relationship matches triangle 1?

A $10^2 = 24^2 + 26^2$ **B** $26^2 = 24^2 + 10^2$

C $24^2 = 10^2 + 26^2$ **D** $26^2 = 24^2 - 10^2$

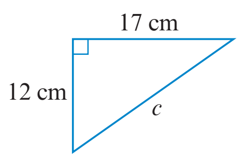
b Which relationship matches triangle 2?

A $x^2 = 7^2 + 11^2$ **B** $11^2 = x^2 + 7^2$

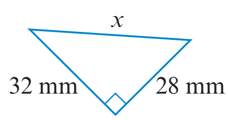
C $7^2 = 11^2 + x^2$ **D** $11 = x + 7$

18 Use Pythagoras' Theorem to write a relationship for each triangle.

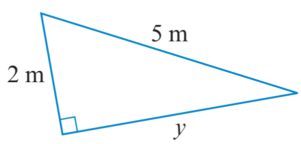
a



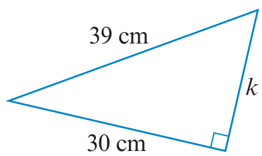
b



c

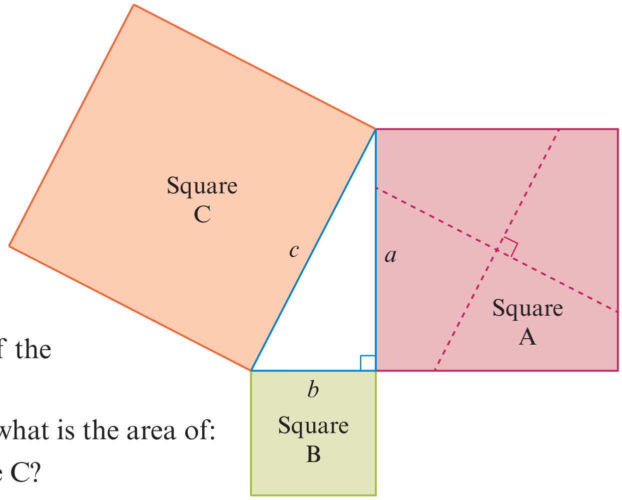


d



CHALLENGE

19 There are many ways to demonstrate that Pythagoras' Theorem is true. Here is one of them.



a Draw a right-angled triangle in the centre of a sheet of paper and label the side lengths as a, b and c.

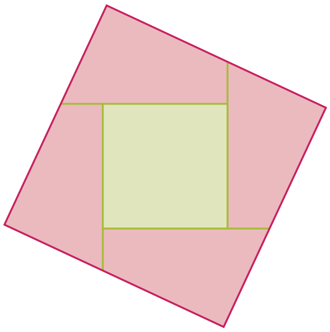
b Draw a square on each side of the triangle as shown.

c If the area of square A is a^2, what is the area of:

i square B? **ii** square C?

d Find the centre of the middle-sized square (square A) and rule a line through it that is **parallel** to the hypotenuse of the triangle. Rule another line through the centre that is **perpendicular** (at right angles) to the line you have just drawn.

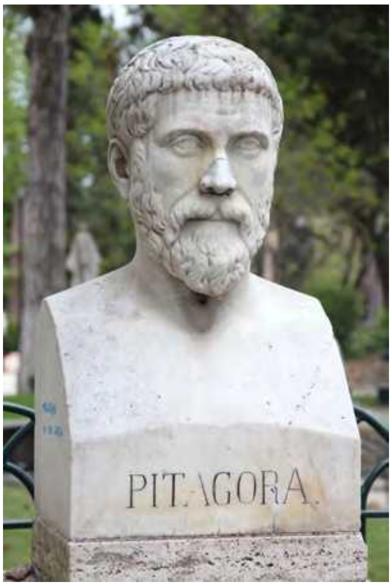
e Cut out all three squares and then cut square A into four pieces along the lines you have drawn.



f Arrange the four pieces of square A around square B to form another square as shown. What is the total area of this new square?

g Compare this new square to square C. Explain how this demonstrates Pythagoras' Theorem. (Hint: compare the areas.)

20 Pythagoras' Theorem is often described in words. Use your understanding of question **19** to copy and complete this description: Pythagoras' Theorem states that, in any right-angled _____, the area of the square on the ________ is equal to the sum of the areas of the squares on the other two sides.



21 Use the Internet or your school library to research information on the life of Pythagoras of Samos. Find out when he proved his famous theorem. Was he the first to discover this relationship for the side lengths of a right-angled triangle? What other mathematical proofs or discoveries did he make?

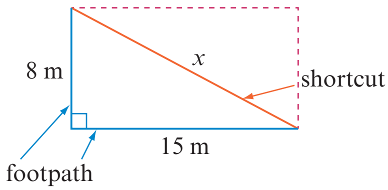
Reflect

How is Pythagoras' Theorem used to check whether a building frame is 'square'?

6B Using Pythagoras' Theorem to find the length of the hypotenuse

Start thinking!

Jethro wants to take a short cut across a vacant corner block of land instead of walking along the footpath. How much shorter is his path? This right-angled triangle shows the two options: walking along the footpath (in blue) and taking the short cut (in orange).



1 Which side of the triangle has an unknown length that is represented by x?

2 Using Pythagoras' Theorem, which is the correct statement (A, B or C) to describe the relationship between the three side lengths in this triangle? Explain your choice.

A $15^2 = x^2 + 8^2$ **B** $8^2 = x^2 + 15^2$ **C** $x^2 = 8^2 + 15^2$

3 Discuss with a classmate how you might solve the equation to find the value of x.

4 To solve for x, you first simplify the equation.
Copy and complete these steps of working on the right.

$$\begin{aligned} x^2 &= 8^2 + 15^2 \\ &= \_\_\_ + \_\_\_ \\ &= \_\_\_ \end{aligned}$$

5 Explain how you can obtain the value of x from x^2.

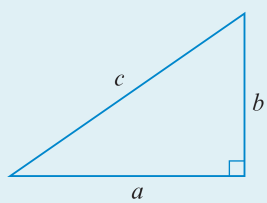
6 For $x^2 = 9$, there are two solutions for x. The solutions are $x = \sqrt{9}$ and $x = -\sqrt{9}$ or $x = 3$ and $x = -3$.
What are the two solutions to $x^2 = 289$?

7 In this case, since x represents the length of the hypotenuse, you only consider one of the solutions. Which solution do you write and why?

8 What is the length of the hypotenuse for this triangle? Remember to include the correct unit.

9 Calculate the length of the footpath around the edge of the vacant block and compare it to the length of Jethro's short cut across the land. How much shorter is Jethro's path?

KEY IDEAS



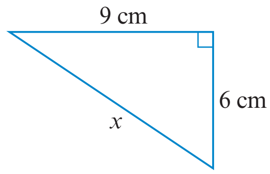
- Pythagoras' Theorem for a right-angled triangle with side lengths a, b and c, where c is the length of the hypotenuse, is $c^2 = a^2 + b^2$.
- Pythagoras' Theorem can be used to calculate an unknown side length if you know the lengths of the other two sides.
- To find the length of the hypotenuse, substitute values for a and b into $c^2 = a^2 + b^2$ and solve for c.
- The result for the length of the hypotenuse can be written as an exact value (whole number, terminating decimal or in surd form) or approximated to a specific number of decimal places.

EXERCISE 6B Using Pythagoras' Theorem to find the length of the hypotenuse

UNDERSTANDING AND FLUENCY

Where appropriate in the following questions, write answers correct to two decimal places.

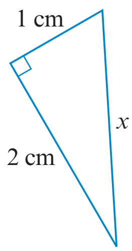
1 Using Pythagoras' Theorem, which equation (A, B or C) would you solve to find the length of the hypotenuse in this triangle?



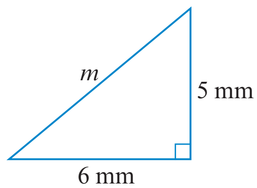
A $9^2 = x^2 + 6^2$ **B** $x^2 = 6^2 + 9^2$ **C** $6^2 = x^2 + 9^2$

2 For each triangle, use Pythagoras' Theorem to write the equation needed to find the length of the hypotenuse.

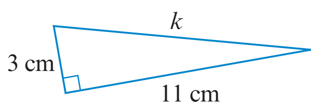
a



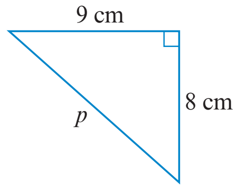
b



c

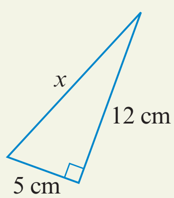


d



EXAMPLE 6B-1 Calculating the length of the hypotenuse (whole number)

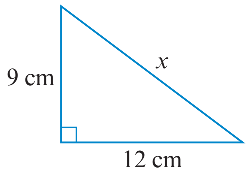
Use Pythagoras' Theorem to calculate the length of the hypotenuse of this triangle.



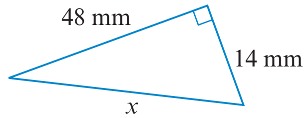
THINK	WRITE
1 Since the triangle is right-angled, state Pythagoras' Theorem and define a, b and c.	$c^2 = a^2 + b^2$ where $a = 5$ cm , $b = 12$ cm and $c = x$
2 Substitute the values for a, b and c and simplify the equation.	$x^2 = 5^2 + 12^2$ $= 25 + 144$ $= 169$
3 Solve for x by finding the square root of both sides of the equation. Write the positive solution only, since x represents a length.	$x = \sqrt{169}$ $= 13$ cm

3 Use Pythagoras' Theorem to calculate the length of the hypotenuse in each triangle.

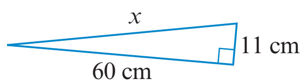
a



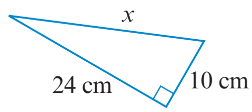
b



c



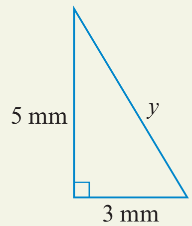
d



4 In your own words, explain how to calculate the length of the hypotenuse of a right-angled triangle when you know the lengths of the other two sides.

EXAMPLE 6B-2 Calculating the length of the hypotenuse (decimal value)

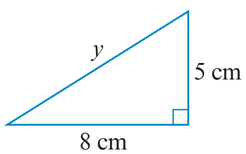
Calculate the length of the hypotenuse of this triangle, correct to two decimal places.



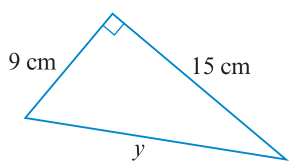
THINK	WRITE
1 Since the triangle is right-angled, state Pythagoras' Theorem and define a, b and c.	$c^2 = a^2 + b^2$ where $a = 3$ mm , $b = 5$ mm and $c = y$
2 Substitute the values for a, b and c and simplify the equation.	$y^2 = 3^2 + 5^2$ $= 9 + 25$ $= 34$
3 Solve for y by finding the square root of both sides of the equation.	$y = \sqrt{34}$
4 Use a calculator to find the approximate length, correct to two decimal places. (Note that $\sqrt{34}$ mm is the exact length of the hypotenuse.)	≈ 5.83 mm

5 Calculate the length of the hypotenuse in each triangle, correct to two decimal places.

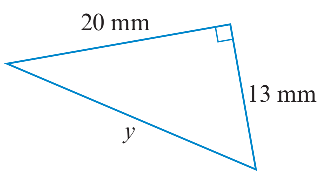
a



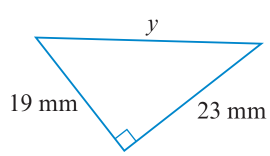
b



c



d

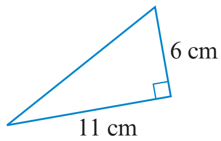


UNDERSTANDING AND FLUENCY

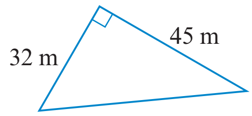
6 Solve the equations in questions **1** and **2** to find the length of the hypotenuse in each triangle. Leave each answer in exact surd form.

7 Find the unknown length in each right-angled triangle.

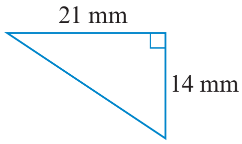
a



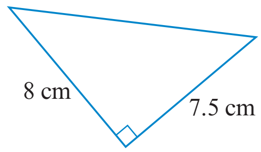
b



c

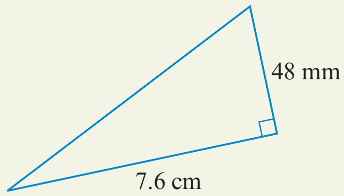


d



EXAMPLE 6B-3 Calculating the length of the hypotenuse after converting units

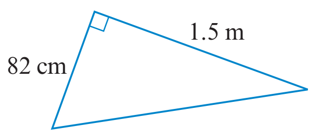
Find the length of the hypotenuse in this right-angled triangle, correct to the nearest centimetre.



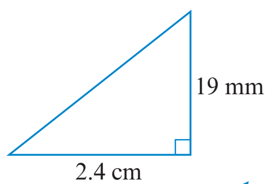
THINK	WRITE
1 State Pythagoras' Theorem and define a, b and c. Write each measurement in centimetres.	$c^2 = a^2 + b^2$ where $a = 48$ mm $= 4.8$ cm, $b = 7.6$ cm and $c = ?$
2 Substitute the values for a, b and c and simplify.	$c^2 = 4.8^2 + 7.6^2$ $= 80.8$
3 Find c using a calculator and round to the nearest centimetre.	$c = \sqrt{80.8}$ $= 8.988\ldots$ ≈ 9 cm

8 Find the length of the hypotenuse in each right-angled triangle, correct to the nearest centimetre. (Hint: convert all length measurements to centimetres first.)

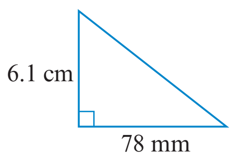
a



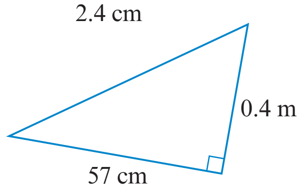
b



c

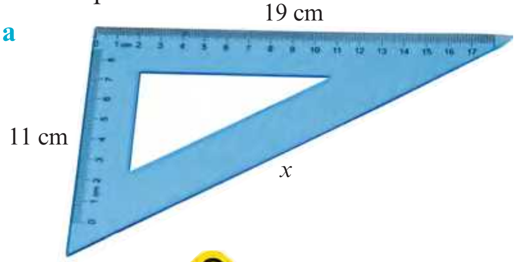


d

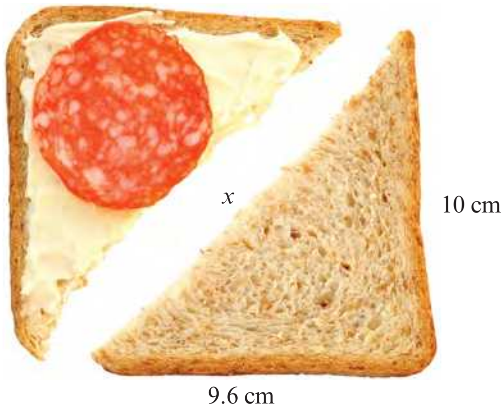


9 Use Pythagoras' Theorem to calculate the length measurement labelled as x in each photo.

a

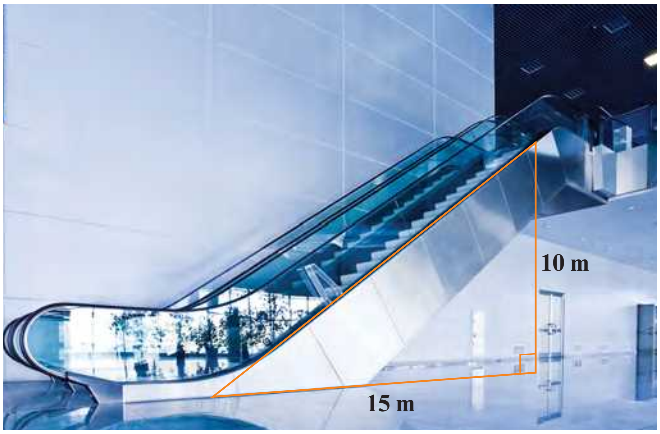


b



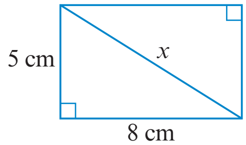
c

10 An escalator rises a vertical distance of 10 m for a horizontal distance of 15 m. What distance would you travel from the bottom to the top of the escalator?

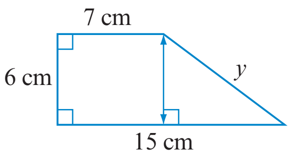


11 Use Pythagoras' Theorem to calculate the unknown length indicated by the pronumeral in each diagram.

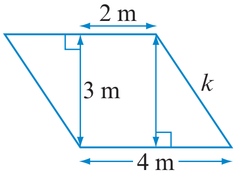
a



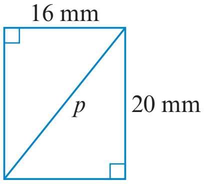
b



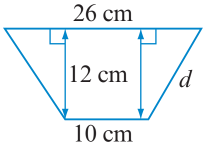
c



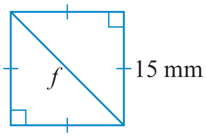
d



e

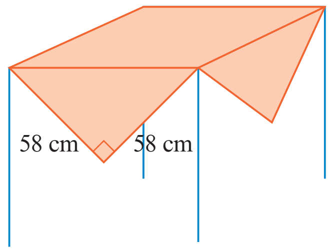


f

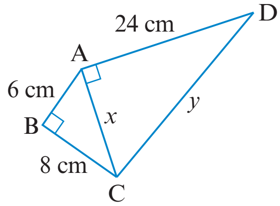


PROBLEM SOLVING AND REASONING

12 A table-cloth overlaps the top of a table as shown in the diagram. How wide is the table?



13 Consider the shape ABCD shown below.



a How many right-angled triangles can you see? Name them.

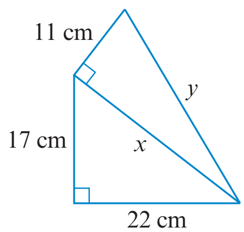
b Calculate the length of AC. Which side lengths did you use in your calculation?

c Calculate the length of CD. Explain how you were able to do this.

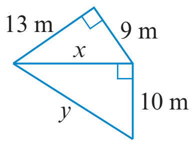
d Find the perimeter of the shape ABCD.

14 Find the lengths indicated by x and y in each diagram.

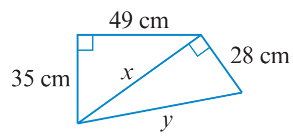
a



b

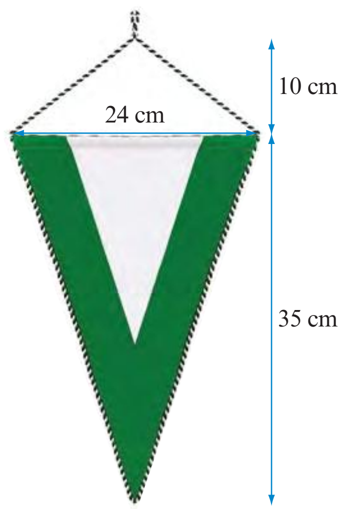


c



CHALLENGE

15 Striped cord is used to edge a flag and provide a way to hang it on the wall. Calculate the total length of the cord used for this flag.



16 **a** Besides using Pythagoras' Theorem (and knowledge of Pythagorean triads), is there another way to determine the length of the hypotenuse of a right-angled triangle? Discuss this with a classmate.

b Draw the right-angled triangle in question **7a** to its exact size using the two lengths shown. Use a ruler to measure the length of the hypotenuse. How does this value compare to the answer obtained using Pythagoras' Theorem?

c Draw the triangle in question **7b** as a scale drawing using the two lengths shown and a scale of 1 cm represents 5 m.
Use a ruler and the given scale to work out the length of the hypotenuse. How does this value compare to the answer obtained using Pythagoras' Theorem?

d Discuss the advantages and disadvantages of each method.

Reflect

What are some possible ways of finding the length of the hypotenuse in a right-angled triangle when the lengths of the other two sides are known?

6C Using Pythagoras' Theorem to find the length of a shorter side

Start thinking!

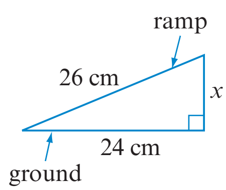
A ramp is used so a trolley can be wheeled on to a platform. The cross-section of the ramp is drawn as a right-angled triangle with the unknown height labelled as x.

1 Is the unknown length in this triangle (shown as x) the length of the hypotenuse? Explain.

2 Using Pythagoras' Theorem, which is the correct statement (A, B or C) that describes the relationship between the three side lengths in this triangle? Explain your choice.

A $26^2 = x^2 + 24^2$ **B** $24^2 = x^2 + 26^2$ **C** $x^2 = 24^2 + 26^2$

3 Discuss with a classmate how you might solve the equation to find the value of x.



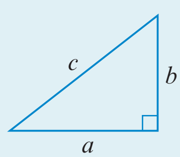
4 To solve for x, you first simplify the equation. Copy and complete these steps of working.

5 Continue the steps of working in question **4** to solve the equation for x.

$$\begin{aligned} 26^2 &= x^2 + 24^2 \\ x^2 + 24^2 &= 26^2 \\ x^2 + \_\_\_ &= \_\_\_ \\ x^2 + \_\_\_ - 576 &= \_\_\_ - \_\_\_ \\ x^2 &= \_\_\_ \end{aligned}$$

6 What is the height of the ramp? Remember to include the correct unit.

KEY IDEAS



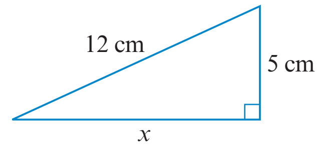
- Pythagoras' Theorem for a right-angled triangle with side lengths a, b and c, where c is the length of the hypotenuse, is $c^2 = a^2 + b^2$.
- Pythagoras' Theorem can be used to calculate an unknown side length if you know the lengths of the other two sides.
- To find the length of one of the shorter sides, substitute values for b and c into $c^2 = a^2 + b^2$ and solve for a.

EXERCISE 6C Using Pythagoras' Theorem to find the length of a shorter side

UNDERSTANDING AND FLUENCY

Where appropriate, write answers correct to two decimal places.

1 Using Pythagoras' Theorem, which equation (A, B or C) would you solve to find the unknown side length in this triangle?



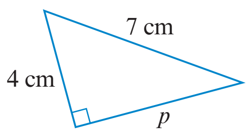
A $5^2 = x^2 + 12^2$

B $x^2 = 5^2 + 12^2$

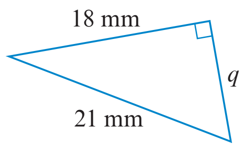
C $12^2 = x^2 + 5^2$

2 For each triangle, use Pythagoras' Theorem to write the equation needed to find the unknown side length.

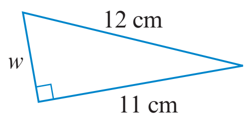
a



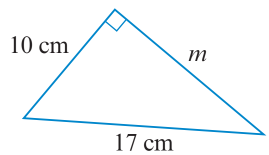
b



c

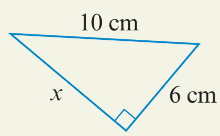


d



EXAMPLE 6C-1 Calculating the length of a shorter side (whole number)

Use Pythagoras' Theorem to calculate the unknown side length in this triangle.

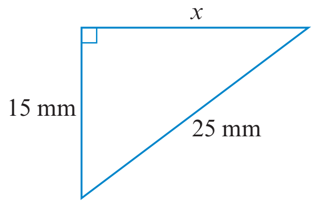


THINK	WRITE
1 Since the triangle is right-angled, state Pythagoras' Theorem and define a, b and c.	$c^2 = a^2 + b^2$ where $a = x$, $b = 6$ cm and $c = 10$ cm
2 Substitute the values for a, b and c and simplify the equation. It may be easier to swap the sides of the equation so x^2 is on the left side.	$10^2 = x^2 + 6^2$ $x^2 + 6^2 = 10^2$ $x^2 + 36 = 100$
3 Solve for x by first subtracting 36 from both sides of the equation and then finding the square root.	$x^2 = 64$ $x = \sqrt{64}$ $= 8$ cm

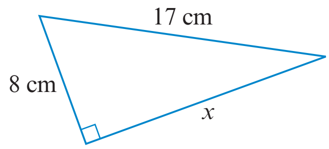
UNDERSTANDING AND FLUENCY

3 Use Pythagoras' Theorem to calculate the unknown side length in each triangle.

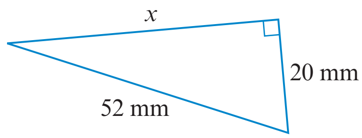
a



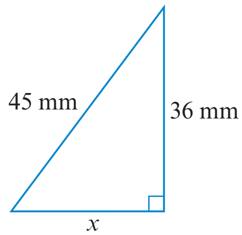
b



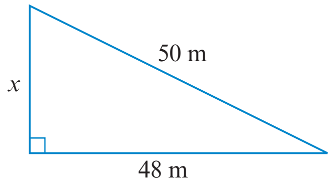
c



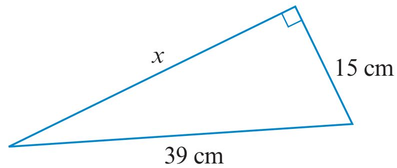
d



e



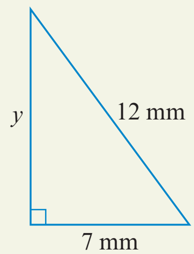
f



4 In your own words, explain how to calculate the length of a shorter side of a right-angled triangle when you know the length of the hypotenuse and the remaining side.

EXAMPLE 6C-2 Calculating the length of a shorter side (decimal value)

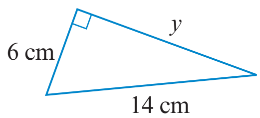
Calculate the unknown side length in this triangle, correct to two decimal places.



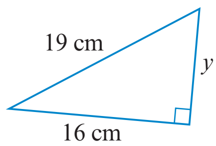
THINK	WRITE
1 Since the triangle is right-angled, state Pythagoras' Theorem and define a, b and c.	$c^2 = a^2 + b^2$ where $a = y$, $b = 7$ mm and $c = 12$ mm
2 Substitute the values for a, b and c and simplify the equation. It may be easier to swap the sides of the equation so y^2 is on the left side.	$12^2 = y^2 + 7^2$ $y^2 + 7^2 = 12^2$ $y^2 + 49 = 144$
3 Solve for y by first subtracting 49 from both sides of the equation and then finding the square root.	$y^2 = 95$ $y = \sqrt{95}$
4 Use a calculator to find the approximate length correct to two decimal places. (Note that $\sqrt{95}$ mm is the exact length.)	≈ 9.75 mm

5 Calculate the unknown side length in each triangle, correct to two decimal places.

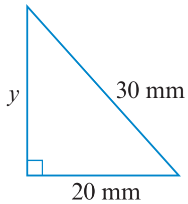
a



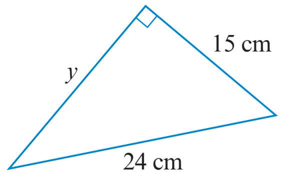
b



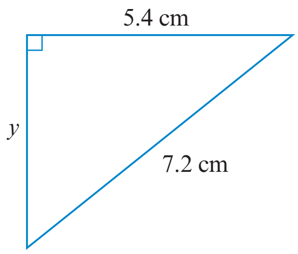
c



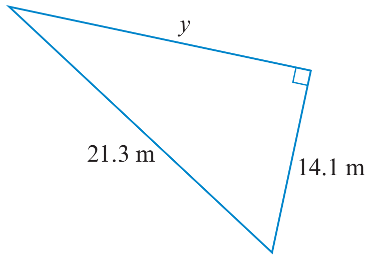
d



e



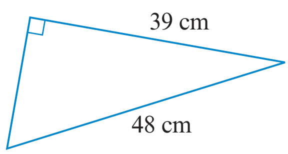
f



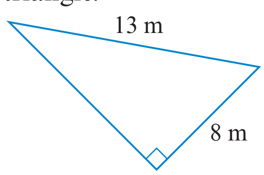
6 Solve the equations in questions **1** and **2** to find the unknown side length in each triangle. Leave each answer in exact surd form.

7 Find the unknown length in each right-angled triangle.

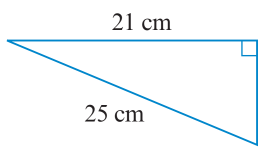
a



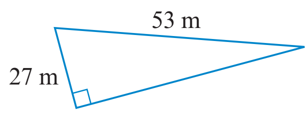
b



c

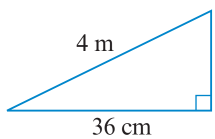


d

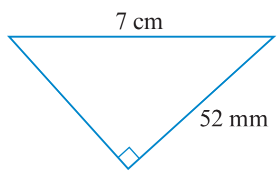


8 Find the unknown length in each right-angled triangle, correct to the nearest centimetre. (Hint: convert all length measurements to centimetres first.)

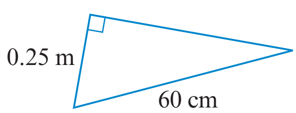
a



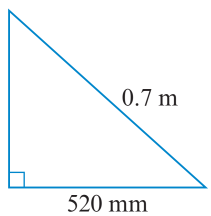
b



c

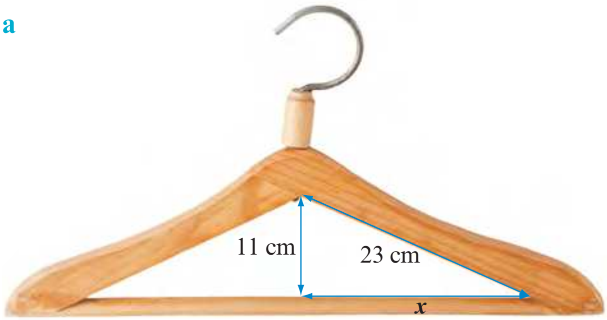


d

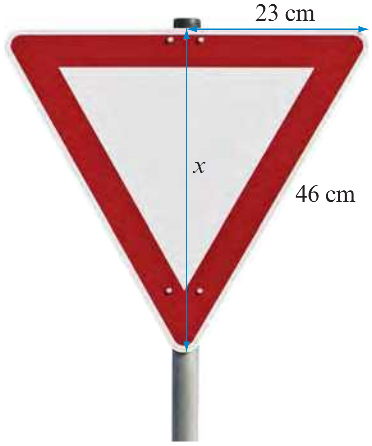


9 Use Pythagoras' Theorem to calculate the length labelled as x in each photo.

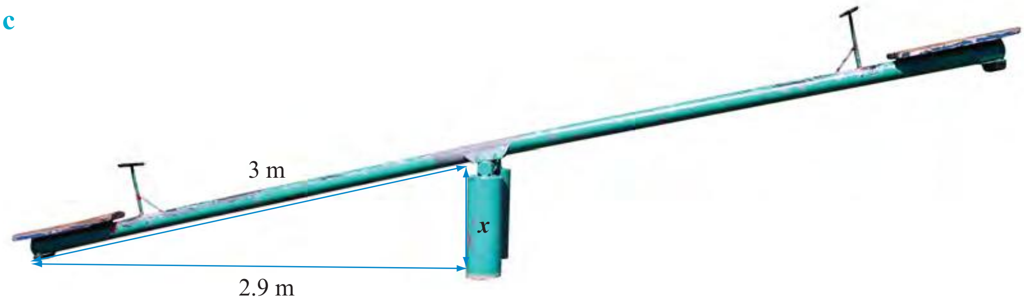
a



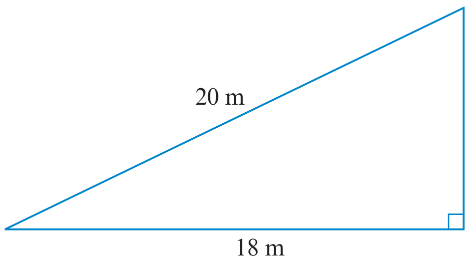
b



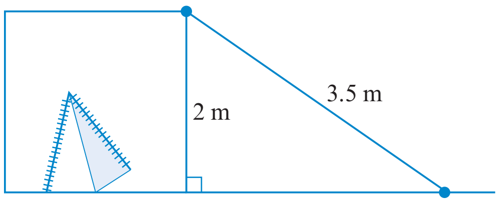
c



10 A BMX rider uses the ramp shown in this diagram to perform stunts. How high is the ramp?



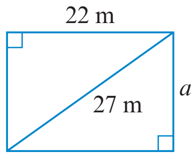
11 A guy rope on a tent is 3.5 m long and is attached to the tent at a height of 2 m. How far from the tent does the guy rope reach the ground?



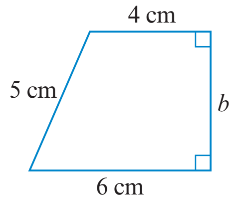
PROBLEM SOLVING AND REASONING

12 Use Pythagoras' Theorem to calculate the unknown length indicated by the pronumeral in each diagram.

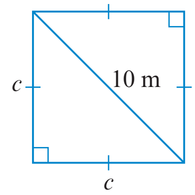
a



b

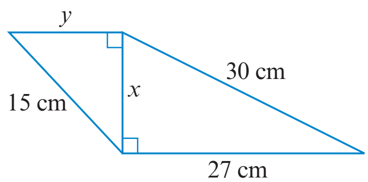


c

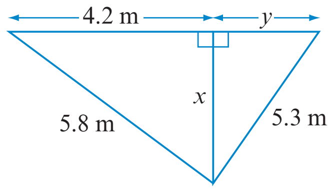


13 Find the lengths indicated by x and y in each diagram.

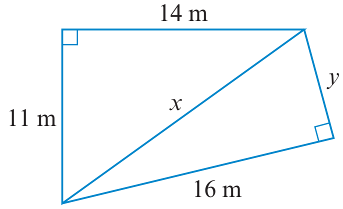
a



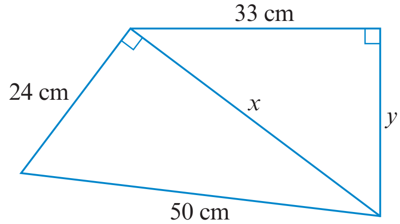
b



c



d



14 For each shape in question **13**, find:

i the perimeter **ii** the area.

15 The size of a television is described by the diagonal length of its screen. This 110 cm television has a height of 57 cm. How wide is the television screen? Give your answer to the nearest centimetre.

CHALLENGE

16 Draw the television screen described in question **15** as a scale drawing using the two given lengths and a scale of 1:10.
Use a ruler and the given scale to work out the width of the television screen from your diagram. How does this value compare to the answer obtained using Pythagoras' Theorem?

17 Suggest possible dimensions for a television screen with a diagonal length of 90 cm.

Reflect

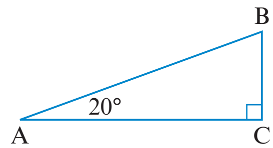
How is the method of finding an unknown side length different when you are considering one of the shorter sides of a triangle rather than the hypotenuse?

6D Understanding trigonometry

Start thinking!

Trigonometry is the study of relationships between angles and side lengths of a triangle. The word trigonometry comes from the Greek words 'trigonom' (triangle) and 'metron' (measurement). In this chapter, you will focus on right-angled triangles.

Consider the right-angled triangle ABC.



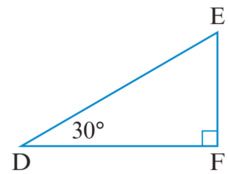
1 Which side is the hypotenuse: AB, AC or BC?

2 The other two sides are named according to a reference angle. The reference angle will be one of the two angles that is not the right angle. Let's consider the angle of 20° as the reference angle in this triangle.

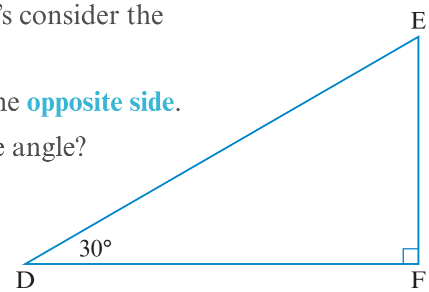
- **a** Which side is opposite the reference angle? This is known as the **opposite side**.
- **b** Which side (other than the hypotenuse) is next to the reference angle? This is known as the **adjacent side**.

3 Copy the triangle and highlight or circle the reference angle. With respect to the reference angle, label the sides with O for opposite side, A for adjacent side and H for hypotenuse.

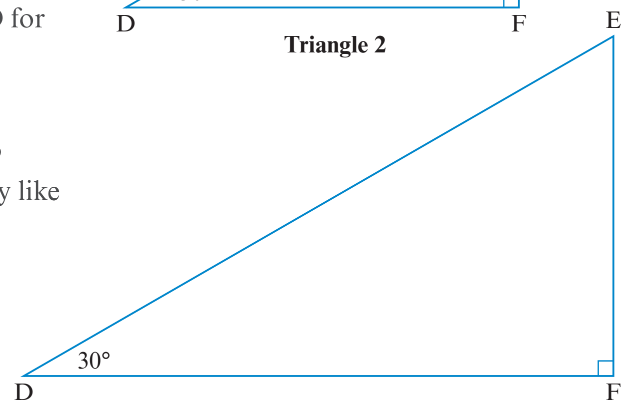
Consider triangles 1, 2 and 3 at right.



Triangle 1



Triangle 2



Triangle 3

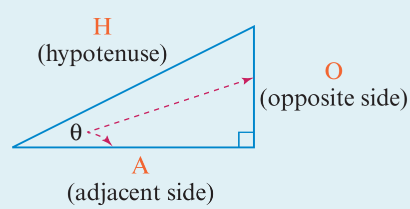
4 Triangles 1, 2 and 3 are similar triangles. List at least two different reasons why they are classed as similar. You may like to discuss this with a classmate.

5 If the reference angle is 30° in each triangle, name:

- **a** the opposite side
- **b** the adjacent side
- **c** the hypotenuse.

KEY IDEAS

- Trigonometry is about the relationships between angles and side lengths in a triangle.
- The sides of a right-angled triangle can be named O (opposite side), A (adjacent side) and H (hypotenuse) with respect to a reference angle. The symbol θ (Greek letter theta) is often used to represent the reference angle.

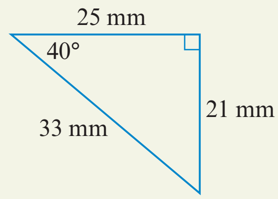


- sine of $\theta = \sin\theta = \frac{O}{H}$, cosine of $\theta = \cos\theta = \frac{A}{H}$, tangent of $\theta = \tan\theta = \frac{O}{A}$

EXERCISE 6D Understanding trigonometry

EXAMPLE 6D-1 Labelling the sides of a right-angled triangle with respect to a reference angle

Label the sides of this triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the angle of 40°.

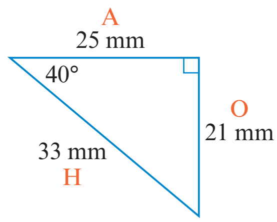


THINK

1 Label the hypotenuse with H. It is the longest side and is opposite the right angle.

2 Identify the opposite side and the adjacent side with respect to the reference angle of 40°. Label the side opposite 40° as O and the side next to 40° as A.

WRITE

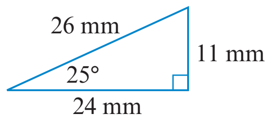


UNDERSTANDING AND FLUENCY

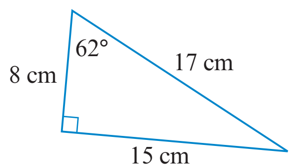
Where appropriate in the following questions, write angles correct to the nearest degree and trigonometric ratio values correct to two decimal places unless otherwise stated.

1 Label the sides of each triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the given reference angle.

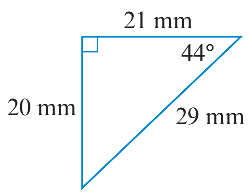
a



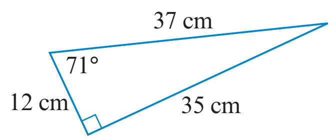
b



c



d



2 Refer to Triangles 1, 2 and 3 in Start thinking! opposite.

a Copy this table. Use a ruler to measure the side lengths of each triangle to the nearest millimetre. Complete the columns titled Reference angle, O (for opposite side), A (for adjacent side) and H (for hypotenuse) for each triangle. Triangle 1 has been completed for you.

Triangle	Reference angle	O (mm)	A (mm)	H (mm)	$\frac{O}{H}$	$\frac{A}{H}$	$\frac{O}{A}$
1	30°	15	26	30			
2							
3							

In previous work with similar triangles, you calculated the ratio of corresponding side lengths. That is, you divided the length of a side in one triangle by the corresponding side length of another triangle. Now consider a different set of ratios.

b Use your measurements related to O and H to calculate the ratio $\frac{\text{O}}{\text{H}}$ for each triangle, correct to two decimal places. What do you notice? This value is known as the **sine** of 30° and is abbreviated to sin 30°.

c Use your measurements related to A and H to calculate the ratio $\frac{\text{A}}{\text{H}}$ for each triangle, correct to two decimal places. What do you notice? This value is known as the **cosine** of 30° and is abbreviated to cos 30°.

d Use your measurements related to O and A to calculate the ratio $\frac{\text{O}}{\text{A}}$ for each triangle, correct to two decimal places. What do you notice? This value is known as the **tangent** of 30° and is abbreviated to tan 30°.

e Copy and complete these relationships:

$\sin 30° = \frac{\text{O}}{\text{H}} = 0.50$

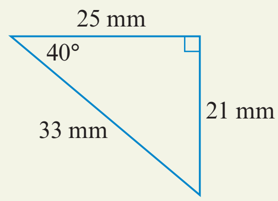
$\cos 30° = \frac{\text{A}}{\text{H}} \approx$ ___

$\tan 30° =$ ___ $\approx$ ___

f Explain why the sine, cosine or tangent of a given reference angle will always be the same value regardless of the size of the right-angled triangle.

EXAMPLE 6D-2 Using ratios of side lengths to calculate sine, cosine and tangent

Use the given side lengths of this triangle to calculate the values for sine, cosine and tangent of 40°, correct to two decimal places.



THINK	WRITE
1 List the measurements relating to O, A and H.	O = 21 mm, A = 25 mm, H = 33 mm
2 Calculate the ratio $\frac{\text{O}}{\text{H}}$ to find the sine of 40°. Since you are finding a ratio of numbers with the same unit, you can leave out the units in the calculation.	$\sin 40° = \frac{\text{O}}{\text{H}} = \frac{21}{33} \approx 0.64$
3 Calculate the ratio $\frac{\text{A}}{\text{H}}$ to find the cosine of 40°.	$\cos 40° = \frac{\text{A}}{\text{H}} = \frac{25}{33} \approx 0.76$
4 Calculate the ratio $\frac{\text{O}}{\text{A}}$ to find the tangent of 40°.	$\tan 40° = \frac{\text{O}}{\text{A}} = \frac{21}{25} \approx 0.84$

3 Use the given side lengths of each triangle in question **1** to calculate the values (correct to two decimal places) for sine, cosine and tangent of:

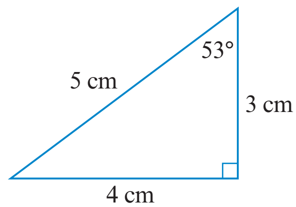
a 25° **b** 62° **c** 44° **d** 71°.

4 Use the measurements on the triangle at right to calculate each trigonometric ratio as:

i a fraction

ii a decimal, correct to two decimal places.

a sin 53° **b** cos 53° **c** tan 53°



5 Consider the pair of triangles shown at right.

a Measure the side lengths of triangle 1 to the nearest millimetre and hence calculate the value of:

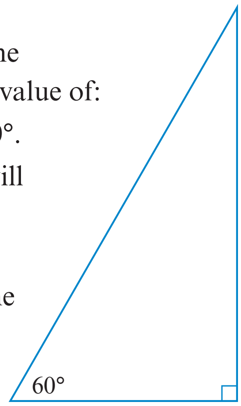
i sin 60° **ii** cos 60° **iii** tan 60°.

b What do you expect the value of cos 60° will be for triangle 2? Explain why.

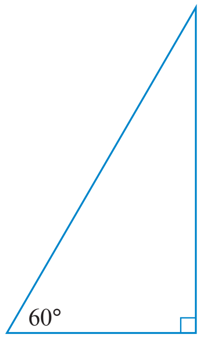
c Check your answer to part **b** by measuring the side lengths of the adjacent side and the hypotenuse and calculating the ratio.

d Without measuring the side lengths of triangle 2, write the value of:

i sin 60° **ii** tan 60°.



Triangle 1



Triangle 2

e Explain why the sine, cosine or tangent of 60° will always be the same value regardless of the size of the right-angled triangle.

6 Look at triangle 1 in question **5** again.

a What is the value of the third angle in this triangle?

b If this angle is considered to be the reference angle, list the lengths of the opposite and adjacent sides for this triangle.

c Use the side lengths of the triangle to calculate the value of:

i sin 30° **ii** cos 30° **iii** tan 30°.

d Compare your results to part **c** with your answer to part **e** of question **2**.

7 Use your trigonometric ratio values from questions **5** and **6** to look for patterns.

a What do you notice when you compare the values for:

i sin 30° and cos 60°?

ii sin 60° and cos 30°?

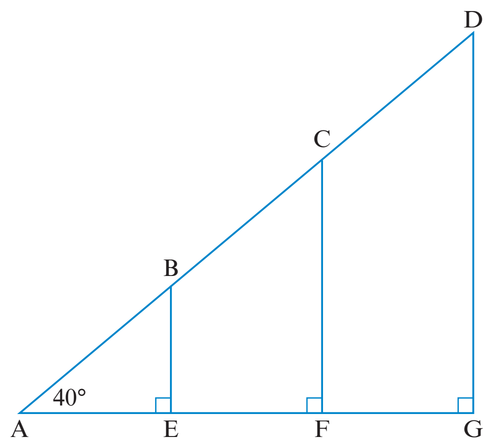
b Calculate $\frac{\sin 30°}{\cos 30°}$. (Hint: work out (sin 30°) ÷ (cos 30°).)

Compare your answer to the value for tan 30°. What do you notice?

c Calculate $\frac{\sin 60°}{\cos 60°}$. Compare your answer to the value for tan 60°.

What do you notice?

8 Consider the diagram shown at right.



- **a** Why are triangles ABE, ACF and ADG considered to be similar triangles?
- **b** Measure the lengths of AB, BE and AE and hence calculate:
  - **i** sin 40°
  - **ii** cos 40°
  - **iii** tan 40°.
- **c** Measure the lengths of AC, CF and AF and hence calculate:
  - **i** sin 40°
  - **ii** cos 40°
  - **iii** tan 40°.
- **d** The length of AG is 60 mm. Without measuring, what is the length of DG? Explain how you worked this out. (Hint: use the fact that the three triangles are similar.)
- **e** Measure the length of DG to check your answer to part **d**.

9 Explain how you can work out the values for sin 50°, cos 50° and tan 50° from the diagram in question **8**. Use the appropriate side lengths to calculate each value.

10 Use your trigonometric ratio values from questions **8** and **9** to look for patterns.

- **a** What do you notice when you compare the values for:
  - **i** sin 40° and cos 50°?
  - **ii** sin 50° and cos 40°?
- **b** Calculate $\frac{\sin 40°}{\cos 40°}$. Compare your answer to the value for tan 40°.

  What do you notice?
- **c** Calculate $\frac{\sin 50°}{\cos 50°}$. Compare your answer to the value for tan 50°.

  What do you notice?

11 In questions **7a** and **10a**, you noticed a pattern between sine and cosine.

- **a** Explain the pattern. What is special about each pair of angles?
- **b** A list of trigonometric ratio values (correct to two decimal places) is shown in the box at right. Use the pattern and this list to find the value of each trigonometric ratio below.

i sin 26°	**ii** cos 58°
iii sin 80°	**iv** cos 45°
v sin 58°	**vi** sin 15°
vii cos 26°	**viii** cos 80°

sin 45° = 0.71
cos 32° = 0.85
sin 10° = 0.17
cos 64° = 0.44
cos 10° = 0.98
sin 64° = 0.90
sin 32° = 0.53
cos 15° = 0.97
cos 75° = 0.26

PROBLEM SOLVING AND REASONING

12 In parts **b** and **c** of questions **7** and **10**, you noticed a pattern between sine, cosine and tangent of an angle.

a Explain the pattern. Use the ratio of side lengths for each to show why this works. (Hint: Show why $\frac{O}{H} \div \frac{A}{H} = \frac{O}{A}$.)

b Use this pattern and the information in question **11** to find the value of each tan ratio.

i tan 10° **ii** tan 64° **iii** tan 32°
iv tan 45° **v** tan 15° **vi** tan 75°

13 So far, you have been measuring two side lengths to calculate a trigonometric ratio. Conveniently, you can use a calculator as a short cut to find this value.

a Use a calculator to find the value of each of these trigonometric ratios.

i sin 60° **ii** cos 60° **iii** tan 60°

b Compare your answers for part **a** to those you obtained in question **5**.

c Use a calculator to find the value of each of these trigonometric ratios.

i sin 53° **ii** cos 53° **iii** tan 53°

d Compare your answers for part **c** to those you obtained in question **4**.

14 Use a calculator to check your answers to questions **11b** and **12b**. Explain why your answers might be slightly different.

15 Use a calculator to find the value for each of these trigonometric ratios.

a sin 30° **b** cos 25° **c** tan 71° **d** cos 44°
e sin 62° **f** tan 25° **g** cos 71° **h** sin 45°
i tan 62° **j** sin 25° **k** tan 45° **l** cos 30°

CHALLENGE

16 You can also find the angle that gives a particular trigonometric ratio value. Use your results from question **15** to find the angle θ in each case.

a $\sin\theta = 0.5$ **b** $\cos\theta = 0.91$ **c** $\tan\theta = 1.88$
d $\sin\theta = 0.88$ **e** $\tan\theta = 1$ **f** $\sin\theta = 0.42$
g $\cos\theta = 0.72$ **h** $\tan\theta = 2.90$ **i** $\cos\theta = 0.33$

17 Can you see how to use your calculator to obtain each value of θ in question **16**? You may like to discuss this with a classmate.

18 **a** Explain why tan 45° = 1. (Hint: what is special about the lengths of the opposite and adjacent sides?)

b For what angles is the tan value less than 1? Explain.

c For what angles is the tan value greater than 1? Explain.

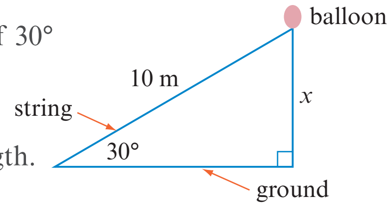
19 Why are sine and cosine values never greater than 1?

Reflect

How is SOH-CAH-TOA useful when working with trigonometry?

6E Using trigonometry to find lengths

Start thinking!



A balloon is attached to the ground by a string 10 m long, which makes an angle of 30° with the ground. Marnie wants to know how high the balloon is and draws this right-angled triangle with the unknown height labelled x.

Like Pythagoras' Theorem, you can use trigonometry to find an unknown side length.

1 What is the reference angle?

2 Copy the triangle and label the sides with O, A and H with respect to the reference angle.

3 Which side (O, A or H) is labelled with:

  a the unknown side length, x? Circle or highlight the letter of this side.

  b the known length of 10 m? Circle or highlight the letter of this side.

4 Refer to your answers for question **3**. Which trigonometric ratio relates to these two sides?
(Hint: $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$, $\tan\theta = \frac{O}{A}$.)

5 You can form an equation using the trigonometric ratio that matches the given information. Which equation (A, B or C) would match the information we have?

  A $\sin 30° = \frac{x}{10}$ **B** $\cos 30° = \frac{x}{10}$ **C** $\tan 30° = \frac{x}{10}$

6 Discuss with a classmate how you might solve the equation to find the value of x.

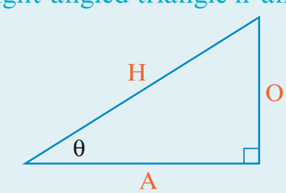
7 Copy and complete the steps of working on the right to solve the equation.

8 What is the height of the balloon?

$$\sin 30° = \frac{x}{10}$$
$$\frac{x}{10} = \sin 30°$$
$$\frac{x}{10} \times \_\_ = (\sin 30°) \times 10$$
$$x = \_\_\_\_\_\_ \times 10$$
$$x = 5 \text{ m}$$

KEY IDEAS

- A trigonometric ratio can be used to find an unknown side length in a right-angled triangle if an angle (other than the right angle) and one side length are known.
- For the reference angle θ: $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$, $\tan\theta = \frac{O}{A}$.
- SOH-CAH-TOA can help us decide which trigonometric ratio to use to form an equation to solve.

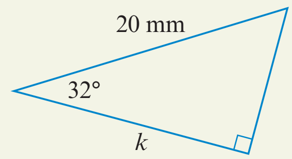


- Use a calculator to perform the final calculation and, where appropriate, round your answer. Always round at the end of the calculation to ensure the most accurate answer.

EXERCISE 6E Using trigonometry to find lengths

EXAMPLE 6E-1 Identifying which trigonometric ratio to use to write an equation

Decide which trigonometric ratio can be used with the information given for this triangle and hence write an equation that involves k.



THINK

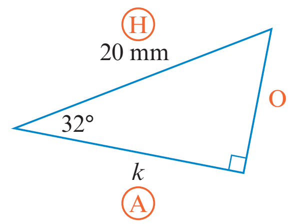
1 Label the sides of the triangle with O, A and H with respect to 32°.

2 Circle A (unknown length you want to find) and H (known length).

3 Decide which trigonometric ratio to use. Since A and H are involved, use cosine.

4 Substitute for θ, A and H to write an equation involving k.

WRITE



$$\cos \theta = \frac{A}{H}$$

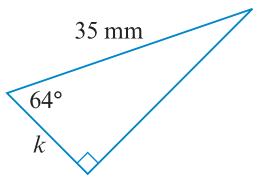
$$\cos 32° = \frac{k}{20}$$

UNDERSTANDING AND FLUENCY

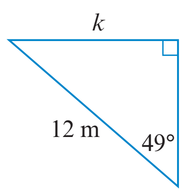
Where appropriate in the following questions, calculate each length correct to two decimal places.

1 Decide which trigonometric ratio can be used with the information given for each triangle and hence write an equation that involves k.

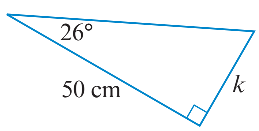
a



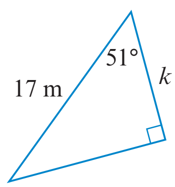
b



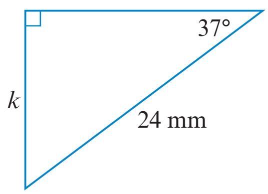
c



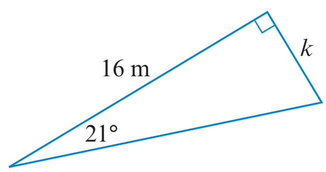
d



e

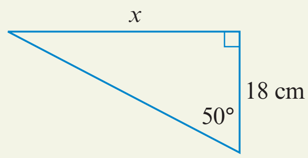


f



EXAMPLE 6E-2 Using trigonometry to find an unknown side length

Use trigonometry to find the side length x in this triangle, correct to two decimal places.



THINK

1 Label the sides of the triangle O, A and H with respect to 50°.

2 Circle O (unknown length you want to find) and A (known length).

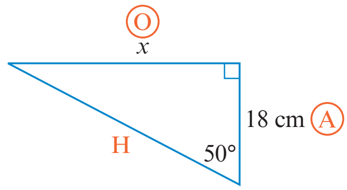
3 Decide which trigonometric ratio to use. Since O and A are involved, use tangent.

4 Substitute for θ, O and A.

5 Solve the equation for x. You may like to first swap the sides of the equation so x is on the left side. Multiply both sides of the equation by 18.

6 Use a calculator to multiply 18 by tan 50°. Round the value of x to two decimal places.

WRITE



$$\tan\theta = \frac{O}{A}$$

$$\tan 50° = \frac{x}{18}$$

$$\frac{x}{18} = \tan 50°$$

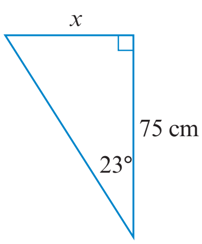
$$x = 18 \times \tan 50°$$

$$\approx 21.45 \text{ cm}$$

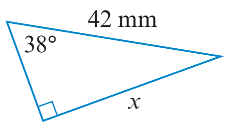
UNDERSTANDING AND FLUENCY

2 Use trigonometry to find the side length x in each triangle.

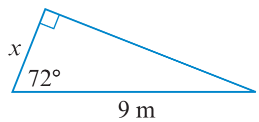
a



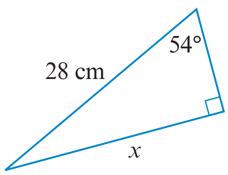
b



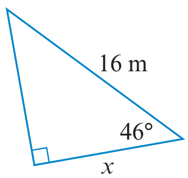
c



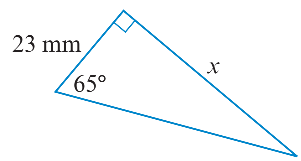
d



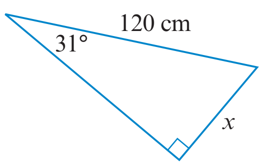
e



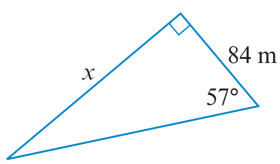
f



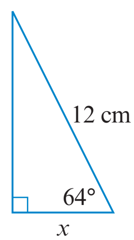
g



h



i

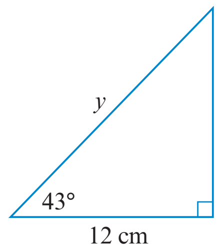


UNDERSTANDING AND FLUENCY

3 Solve each equation formed in question **1** to calculate the value of k.

4 How can remembering SOH-CAH-TOA help people to decide which trigonometric ratio to use when forming an equation?

5 Copy this right-angled triangle.



a Label the sides with O, A and H.

b Decide which trigonometric ratio would apply to the given information.

c Which equation (A, B or C) would you use to find y? Explain your choice.

A $\cos 43° = \frac{y}{12}$ **B** $\cos 43° = \frac{12}{y}$ **C** $\sin 43° = \frac{y}{12}$

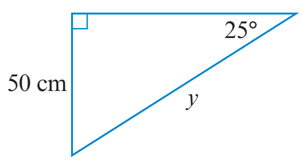
d This equation to different from others you have solved in this topic, as the unknown length is in the denominator of the fraction. Explain how you might solve the equation to find the value of y. Discuss this with a classmate.

e To find the solution to this equation, match each step **1–4** shown below with its corresponding line of working A–D in the calculation. Write the lines of working in the correct order in your workbook.

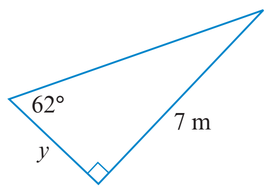
Steps	Lines of working
1 Form an equation.	**A** $y \times \cos 43° = 12$
2 Multiply both sides of the equation by y.	**B** $y \approx 16.41$
3 Divide both sides of the equation by cos 43°.	**C** $\cos 43° = \frac{12}{y}$
4 Use a calculator to work out the value of y to two decimal places.	**D** $y = \frac{12}{\cos 43°}$

6 Find the side length y in each triangle.

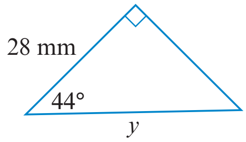
a



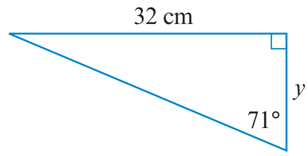
b



c



d



7 Consider each triangle in question **6**.

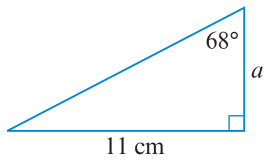
i Find the value of the third angle in each triangle.

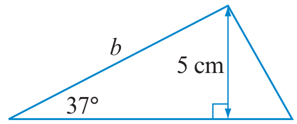
ii Use this angle to calculate y in each case.

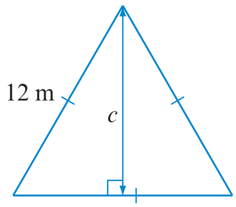
iii For each triangle, compare the equation you solved in question **6** to the equation you solved in question **7ii**. In which cases was it easier to solve the second equation? Explain.

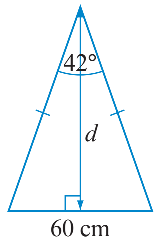
UNDERSTANDING AND FLUENCY

8 Find the value of the pronumeral in each triangle.

a


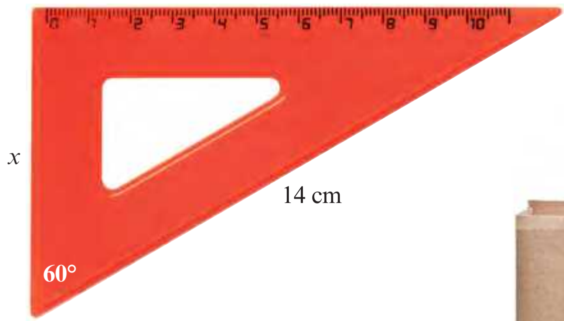
b


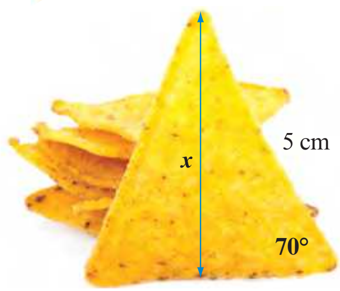
c


d


PROBLEM SOLVING AND REASONING

9 Use trigonometry to calculate the length labelled x in each photo.

a


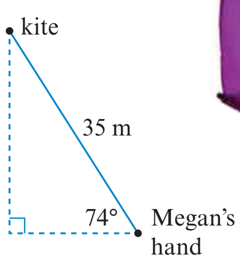
b


c


10 Megan holds the string attached to her kite at a height of 1 m above the ground. The 35-m long string makes an angle of 74° with the horizontal.

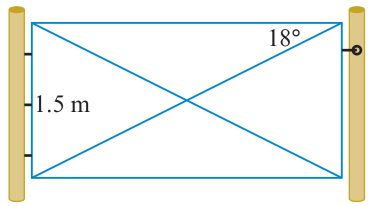
a Use trigonometry to calculate the vertical distance from one end of the string to the other.

b What is the height of the kite above the ground?



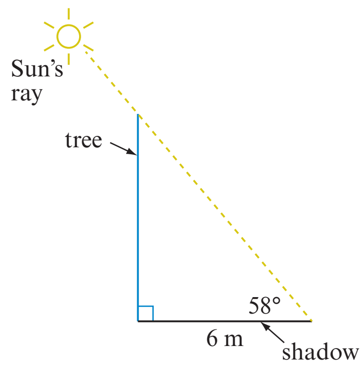
PROBLEM SOLVING AND REASONING

11 A farm gate has diagonal supporting braces that make an angle of 18° with the horizontal. How wide is the gate?



12 A shadow is formed when the Sun's rays are blocked by an object. The angle of the Sun's rays determines how long a shadow will be.

At a certain time of day, the Sun's rays make an angle of 58° with the ground and a tree forms a shadow that is 6 m long.



a Use the diagram to calculate the height of the tree.

b At the same time, another tree forms a shadow that is 10 m long. How high is this tree?

CHALLENGE

13 The two identical sides of this ladder meet at an angle of 50° and are 1.8 m apart where they touch the ground.

a How high is the top of the ladder above the ground?

b How long is each side of the ladder?

14 A ramp for wheelchair access to a building is to be built at an angle of 3° to the horizontal. The front door of the building is 45 cm above ground level. How long should the ramp be? Give your answer in metres.

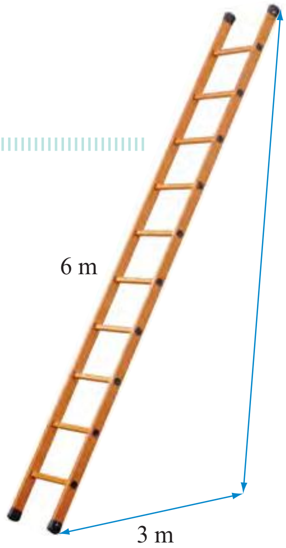
15 What is the minimum amount of information needed to find an unknown side length by using trigonometry?

Reflect

How do you know whether to use trigonometry or Pythagoras' Theorem to find an unknown side length?

6F Using trigonometry to find angles

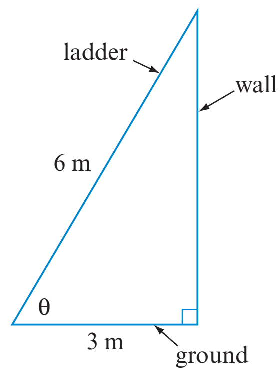
Start thinking!



A 6-m ladder leans against a wall. The bottom of the ladder sits on the ground at a distance of 3 m from the wall. Kate wants to know the angle that the ladder makes with the ground, so she draws a quick sketch of a right-angled triangle to represent this scenario. Let's look at how she can use trigonometry to find the size of the angle.

1 **a** If θ is the reference angle, which of O, A or H would relate to the side that is:
 i 3 m long? **ii** 6 m long?
 b Which trigonometric ratio relates to these two sides?

2 Form an equation using the trigonometric ratio that matches the given information. Which equation (A, B or C) matches the information you have?
 A $\sin\theta = \frac{3}{6}$ **B** $\cos\theta = \frac{3}{6}$ **C** $\tan\theta = \frac{3}{6}$



3 Discuss with a classmate how you might find the value of θ.

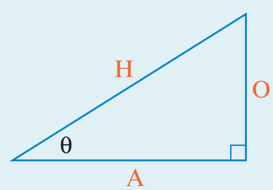
You can use a calculator to work 'backwards' to find the angle for a given trigonometric ratio value. This is known as finding inverse sine ($\sin^{-1}$), inverse cosine ($\cos^{-1}$) or inverse tangent ($\tan^{-1}$) of a value.
For example, if $\sin\theta = 0.5$ then $\theta = \sin^{-1}(0.5)$. This is read as:
if sine of the angle θ equals 0.5 then θ equals the inverse sine of 0.5.

4 **a** Use a calculator to find the inverse sine of 0.5. This is the value of θ. (Hint: check that your calculator is in degree mode.)
 b Use a calculator and your answer to part **a** to find the sine of θ. Do you obtain a value of 0.5? If not, compare the method you are using with a classmate's method.

5 Using your answer to question **3**, solve the equation to find the value of θ, correct to the nearest degree.

6 What angle does the ladder make with the ground?

KEY IDEAS



- A trigonometric ratio can be used to find an unknown angle in a right-angled triangle if two side lengths are known.
- For the reference angle θ: $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$, $\tan\theta = \frac{O}{A}$. (SOH-CAH-TOA)
- A calculator can be used to obtain the angle from its sine, cosine or tangent value. This is known as finding the inverse sine ($\sin^{-1}$), inverse cosine ($\cos^{-1}$) or inverse tangent ($\tan^{-1}$) of a value.

NOTE $\sin^{-1}$ does *not* mean sine to the power of −1.

EXERCISE 6F Using trigonometry to find angles

Where appropriate, calculate each angle correct to the nearest degree.

1 Calculate each of these.

a $\sin^{-1}(0.23)$ **b** $\cos^{-1}(0.72)$ **c** $\tan^{-1}(1.46)$

d $\sin^{-1}\left(\frac{2}{3}\right)$ **e** $\cos^{-1}\left(\frac{24}{25}\right)$ **f** $\tan^{-1}\left(\frac{39}{11}\right)$

EXAMPLE 6F-1 Solving an equation to find θ

Solve each equation to find the value of θ, correct to the nearest degree.

a $\tan\theta = 2.4$ **b** $\sin\theta = \frac{22}{35}$

THINK	WRITE
a 1 Rearrange to make θ the subject of the equation.	**a** $\tan\theta = 2.4$ $\theta = \tan^{-1}(2.4)$
2 Use a calculator to find the inverse tangent of 2.4. Round the value of θ to the nearest degree.	$\approx 67°$
b 1 Rearrange to make θ the subject of the equation.	**b** $\sin\theta = \frac{22}{35}$ $\theta = \sin^{-1}\left(\frac{22}{35}\right)$
2 Use a calculator to find the inverse sine of $\frac{22}{35}$. Round the value of θ to the nearest degree.	$\approx 39°$

UNDERSTANDING AND FLUENCY

2 Follow the steps below for each equation.

a $\sin\theta = 0.34$ **b** $\cos\theta = 0.81$ **c** $\tan\theta = 0.65$

i Write the equation with θ as the subject of the equation.

ii Use a calculator to find the value of θ, correct to the nearest degree.

iii Check your answer to part **ii** by calculating the value of the trigonometric ratio using a calculator and the value for θ. (Hint: since the value of θ has been rounded, will the trigonometric ratio value be *exactly* the same?)

3 Repeat question **2** for each of these equations.

a $\sin\theta = \frac{4}{5}$ **b** $\cos\theta = \frac{15}{19}$ **c** $\tan\theta = \frac{8}{21}$ **d** $\sin\theta = \frac{23}{26}$

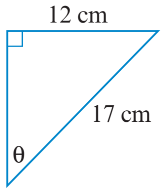
4 Solve each equation to find the value of θ.

a $\tan\theta = 0.47$ **b** $\sin\theta = \frac{9}{23}$ **c** $\cos\theta = \frac{17}{31}$ **d** $\tan\theta = \frac{26}{3}$

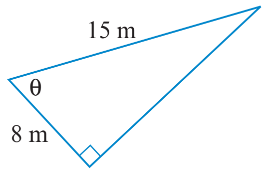
UNDERSTANDING AND FLUENCY

5 Decide which trigonometric ratio matches the given information in each triangle and hence write an equation that involves θ.

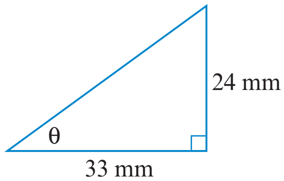
a



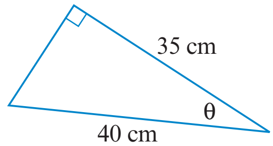
b



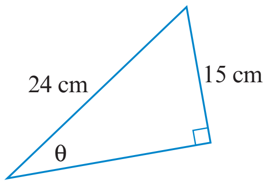
c



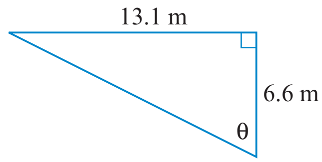
d



e

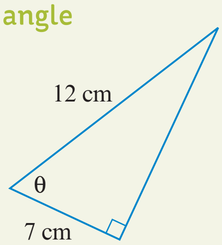


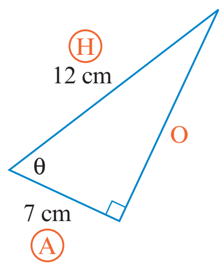
f



EXAMPLE 6F-2 Using trigonometry to find an unknown angle

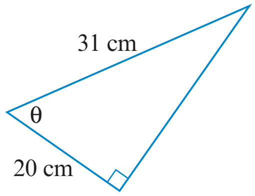
Use trigonometry to find the angle θ in this triangle, correct to the nearest degree.

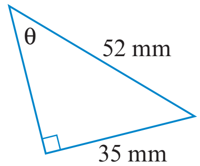


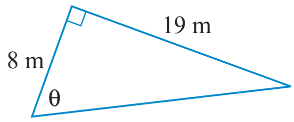
THINK	WRITE
1 Label the sides of the triangle with O, A and H with respect to θ. **2** Circle A (known length) and H (known length).	
3 Decide which trigonometric ratio to use. Since A and H are involved, use cosine.	$\cos\theta = \frac{A}{H}$
4 Substitute for A and H.	$\cos\theta = \frac{7}{12}$
5 Rearrange to make θ the subject of the equation.	$\theta = \cos^{-1}\left(\frac{7}{12}\right)$
6 Use a calculator to find the inverse cosine of $\frac{7}{12}$. Round the value of θ to the nearest degree.	$\approx 54°$

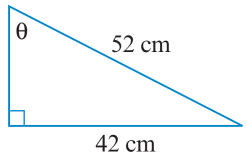
UNDERSTANDING AND FLUENCY

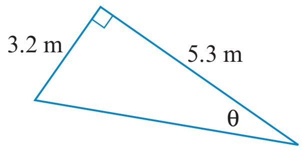
6 Use trigonometry to find the angle θ in each triangle.

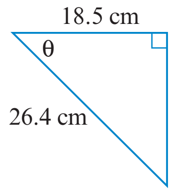
a


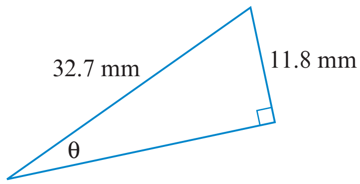
b


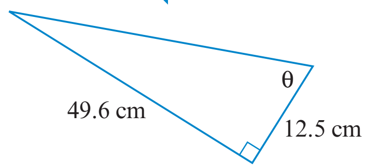
c


d


e


f


g


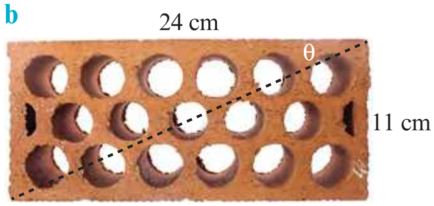
h


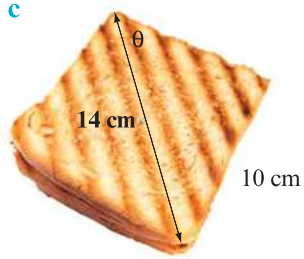
7 Solve each equation formed in question **5** to calculate the value of θ.

PROBLEM SOLVING AND REASONING

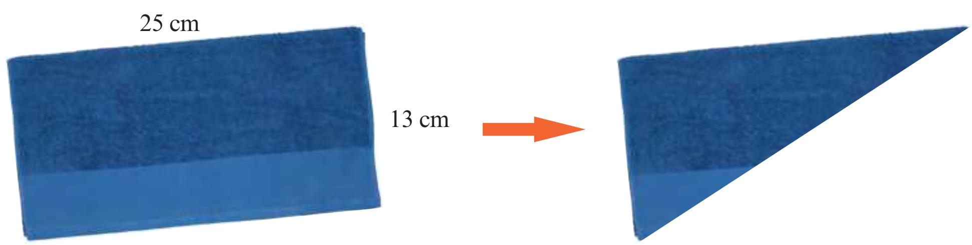
8 Use trigonometry to calculate the angle labelled θ in each photo.

a

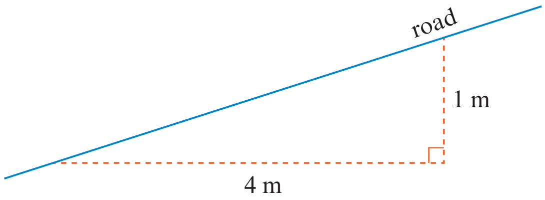

b


c


9 A rectangular towel is folded along a diagonal to form a right-angled triangle. What are the three angles in this triangle?



10 A road has a gradient of 1 in 4; that is, it rises 1 m vertically for every 4 m horizontally.



- **a** What angle does the road surface make with the horizontal?
- **b** How far have you travelled along the road if you are now 1.7 m higher than when you started?

11 Safety regulations require that the maximum incline of a ramp for wheelchair access is to be 1 in 14.

- **a** What does '1 in 14' mean?
- **b** What is the maximum angle of incline allowable for this type of ramp?
- **c** If a ramp is 4 m long to accommodate a 45 cm vertical rise, does it satisfy the safety regulations?

12 The roof of a holiday house has the dimensions shown.

- **a** Calculate the angle the roof makes with the horizontal.
- **b** Calculate the angle formed where the two sections of roof meet.

13 A ramp is used when removalists wish to move items in or out of a truck with a trolley.

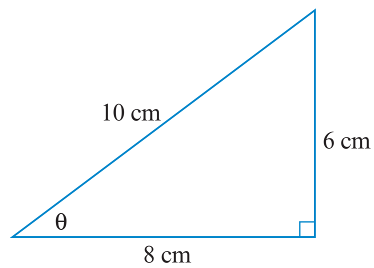
- **a** Use the dimensions shown to calculate the angle the ramp makes with the ground.
- **b** How long is the ramp?

PROBLEM SOLVING AND REASONING

14 An anchor holding a boat in position lies on the seabed at a depth of 7.5 m. It is attached to the boat by a chain that is 8.4 m long.

- **a** Draw a diagram of this scenario.
- **b** What angle does the chain make with the vertical?
- **c** If the chain was longer, would this angle be larger or smaller? Explain.

15 Consider the right-angled triangle shown.



- **a** Calculate the value of θ using a calculator and:
  - **i** the sine ratio
  - **ii** the cosine ratio
  - **iii** the tangent ratio.
- **b** Draw a scale diagram of the triangle and measure the angle θ with a protractor.
- **c** Compare your answers to parts **a** and **b**.
  Comment on the advantages and disadvantages of using:
  - **i** trigonometry
  - **ii** measurements from a scale diagram.

CHALLENGE

16 Nadia and Alex are set a problem-solving task by their teacher. Their challenge is to work out the height of a tree in their school yard without climbing it or using a ladder. The only allowable equipment is a 1-m ruler, a tape measure and a calculator.

Nadia and Alex discuss some possible ideas, take some measurements and draw these diagrams.

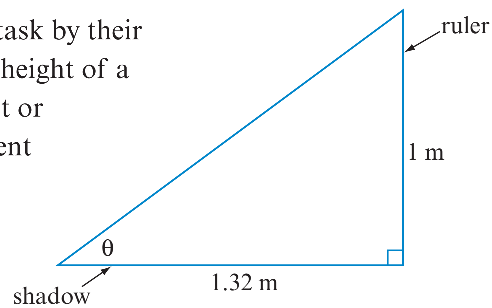


Diagram 1

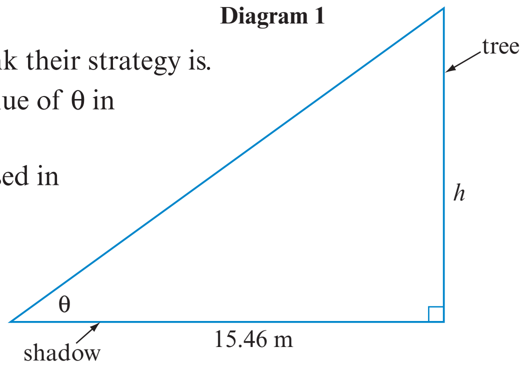


Diagram 2

- **a** With a partner, discuss what you think their strategy is.
- **b** Use trigonometry to calculate the value of θ in Diagram 1.
- **c** Explain why this value of θ can be used in Diagram 2.
- **d** Use trigonometry to calculate the height of the tree.
- **e** Instead of using trigonometry, Nadia and Alex could have used their knowledge of similar triangles.
  Explain how this strategy could be used to calculate the height of the tree.
- **f** Can you think of any other strategies that could have been used? Try them.

17 Use the strategies shown in question **16** to calculate the height of a tree or flagpole near your school using the same equipment.

Reflect

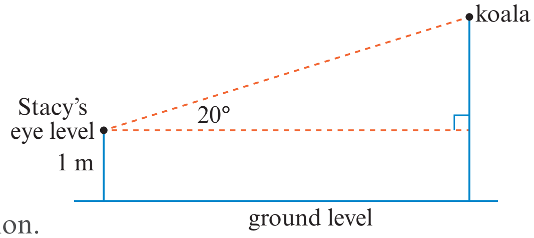
Explain what inverse sine, inverse cosine and inverse tangent mean.

6G Applications involving right-angled triangles

Start thinking!

Hayden and Stacy are sitting at their campsite. They see a koala in a tree and wonder how high it is above the ground. Stacy estimates that the **angle of elevation** from her horizontal line of sight to the koala is 20°.

1 Hayden draws a quick sketch. Is there enough information in Hayden's diagram to be able to work out how high the koala is above the ground? Explain.



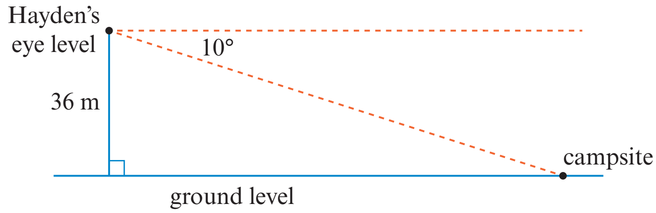
2 Stacy paces the distance from her chair to the base of the tree and estimates the horizontal distance to be 31 m.

- **a** Copy the diagram and label it with any additional information. Use x to represent the unknown height in the triangle.
- **b** Using trigonometry, write and solve an equation to find x, correct to the nearest metre.
- **c** Is the value of x the same as the height of the koala above the ground? Explain.

3 To find how high up the koala is, the vertical distance from the ground to Stacy's eye level needs to be taken into account. Find the height of the koala above the ground.

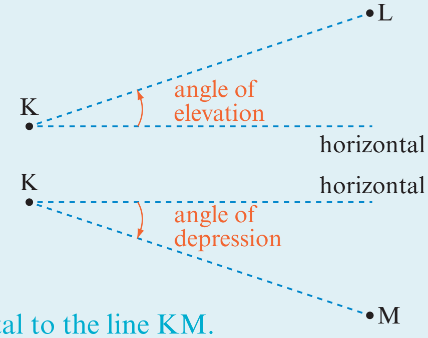
Hayden and Stacy then climb to the top of a lookout and see their campsite. They wonder how far the campsite is from the base of the lookout. Hayden estimates that the **angle of depression** from his horizontal line of sight to the campsite is 10°.

4 Stacy draws a quick sketch. She knows the height of the lookout and allows extra for the height to Hayden's eye level. She estimates the total height to be 36 m. Work out how far the campsite is from the base of the lookout.



KEY IDEAS

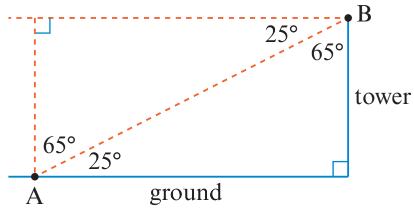
- Angles of elevation and depression are always measured from a horizontal line of sight.
- The angle of elevation from K to L is the angle formed when the line of sight moves upwards from the horizontal to the line KL.
- The angle of depression from K to M is the angle formed when the line of sight moves downwards from the horizontal to the line KM.



EXERCISE 6G Applications involving right-angled triangles

UNDERSTANDING AND FLUENCY

Where appropriate, calculate each angle correct to the nearest degree and each length to the nearest metre.



1 Consider the diagram shown at right, which shows point A on the ground and point B at the top of a tower.

  a What is the angle of elevation from A to B?

  b What is the angle of depression from B to A?

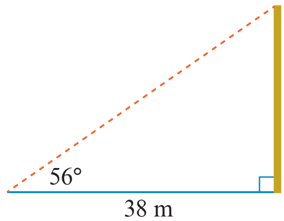
2 Angles of elevation and depression are always measured from a horizontal line of sight. Explain how an angle of depression is different from an angle of elevation.

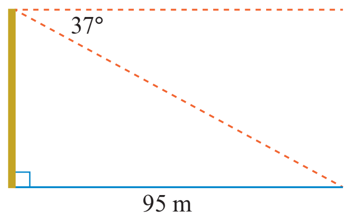
EXAMPLE 6G-1 Calculating height using angle of elevation or depression

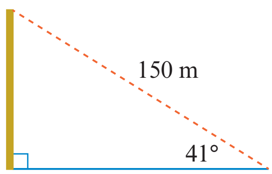
Find the height of the tower in question 1 if the distance along the ground from A to B is 70 m.

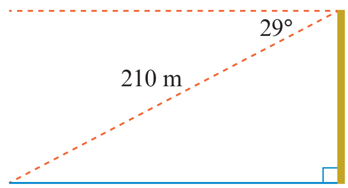
THINK	WRITE
1 Use the angle of elevation (25°) for θ so that the opposite side is the unknown length, h, and the adjacent side is 70 m.	Let h = height of tower. $\tan\theta = \frac{O}{A}$
2 Substitute for θ, O and A.	$\tan 25° = \frac{h}{70}$
3 Solve the equation to find h. Write the height to the nearest metre.	$h = 70 \times \tan 25°$ ≈ 33 m

3 Find the height of each building using the given angles of elevation or depression.

a


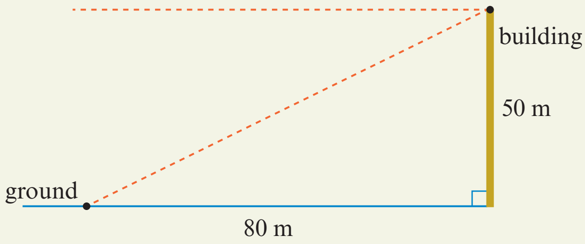
b


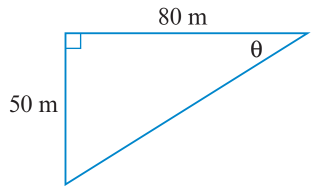
c


d


EXAMPLE 6G-2 Using trigonometry to find an angle of depression

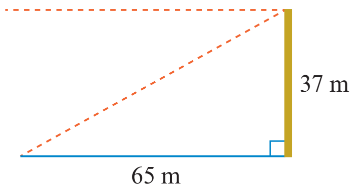
Find the angle of depression from the top of a building 50 m high to a point on the ground that is 80 m from the base of the building.



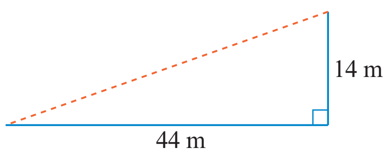
THINK	WRITE
1 Draw an appropriate right-angled triangle and label the angle of depression θ. (Note: there is more than one triangle you could use.)	
2 Use trigonometry to write an equation that involves θ.	$\tan\theta = \frac{50}{80}$
3 Rearrange to make θ the subject of the equation.	$\theta = \tan^{-1}\left(\frac{50}{80}\right)$
4 Use a calculator to find the inverse tangent of $\frac{50}{80}$. Round the value of θ to the nearest degree.	$\approx 32°$
5 Write the answer.	The angle of depression is 32°.

UNDERSTANDING AND FLUENCY

4 **a** Find the angle of depression from the top of a building 37 m high to a point on the ground that is 65 m from the base of the building.



b Find the angle of elevation from a point on the ground that is 44 m from the base of a lookout to the top of the lookout, which is 14 m high.



5 Look at the first diagram in 6G Start thinking! (page 296).

a Calculate the shortest distance from Stacy's eyes to the koala, correct to the nearest metre, using:

i Pythagoras' Theorem **ii** trigonometry.

b Is there more than one way of using trigonometry to calculate your answer to part **a**? Explain.

c Are there any other ways of calculating the answer to part **a**? Explain.

PROBLEM SOLVING AND REASONING

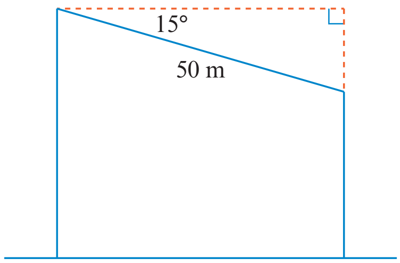
6 Both Pythagoras' Theorem and trigonometry can be used to solve problems involving right-angled triangles.

a When is it appropriate to use Pythagoras' Theorem? What is the minimum amount of information needed to use Pythagoras' Theorem to find an unknown amount?

b When is it appropriate to use trigonometry? What is the minimum amount of information needed to use trigonometry to find an unknown amount?

c Describe an example where either Pythagoras' Theorem or trigonometry could be used to find an unknown amount.

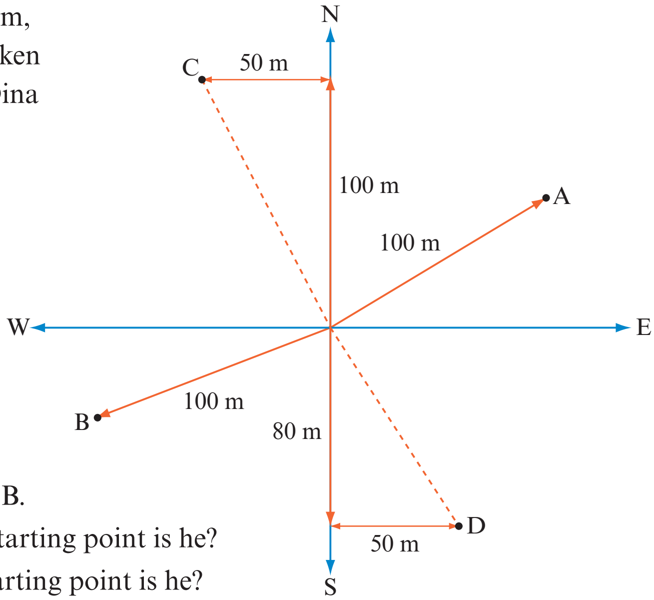
7 Lisa enjoys a flying fox ride along a 50 m cable. The cable makes an angle of 15° with the horizontal.



a What horizontal distance does Lisa travel during the ride?

b The start of the ride is 23 m above the ground. How high off the ground is the end of the ride?

8 Consider this compass diagram, which shows the directions taken by Anna, Bryan, Chris and Dina from the same starting point.



a Anna walks 100 m on a **bearing** of N58°E to point A.

i How far north of the starting point is she?

ii How far east of the starting point is she?

b Bryan walks 100 m on a bearing of 250°T to point B.

i How far south of the starting point is he?

ii How far west of the starting point is he?

c Chris walks 100 m north and then 50 m west.

i Describe the direction of his final position from the starting point as a **compass bearing**.

ii How far is he from the starting point?

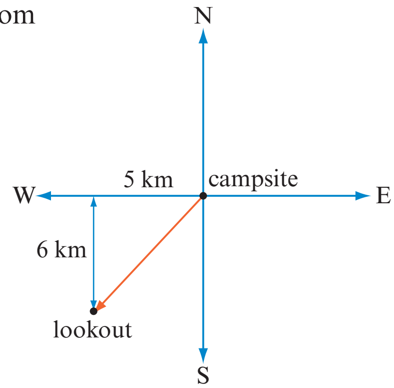
PROBLEM SOLVING AND REASONING

d Dina walks 80 m south and then 50 m east.

i How far is she from the starting point?

ii Describe the direction of her final position from the starting point as a **true bearing**.

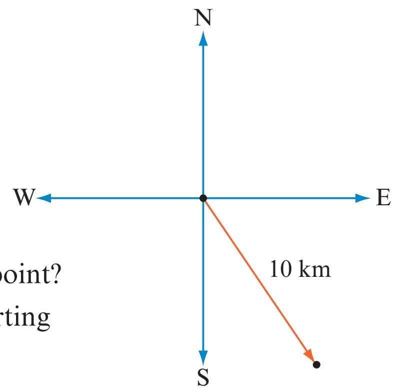
9 Ewan walks from his campsite directly to a lookout that is 5 km west and 6 km south of his starting point.



a What direction has he walked to reach the lookout?

b What is the distance between his campsite and the lookout?

10 Stephanie walks on a bearing of S40°E for 10 km.



a How far south of her starting point is she?

b How far east of her starting point is she?

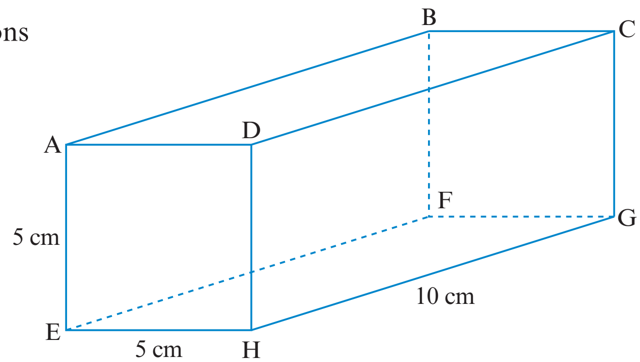
c From her current position, Stephanie walks to a point 8 km due east of her original starting point.

i What direction has she walked to reach this point?

ii If she now walks due west to her original starting point, what total distance has she walked?

CHALLENGE

11 A rectangular prism has the dimensions 5 cm × 5 cm × 10 cm, as shown at right.



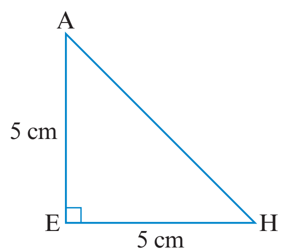
a To find the length of AH, a right-angled triangle can be formed. One of these is triangle AEH.

i Use Pythagoras' Theorem to calculate the length of AH.

ii Use trigonometry to calculate the length of AH. (Hint: what angle will you use?)

iii Do you obtain the same answer for parts **i** and **ii**?

iv Name another triangle that could have been used to find the length of AH.



b Find the length of each of these sides. (Hint: first identify a right-angled triangle that could be used.)

i AC **ii** DG **iii** BD

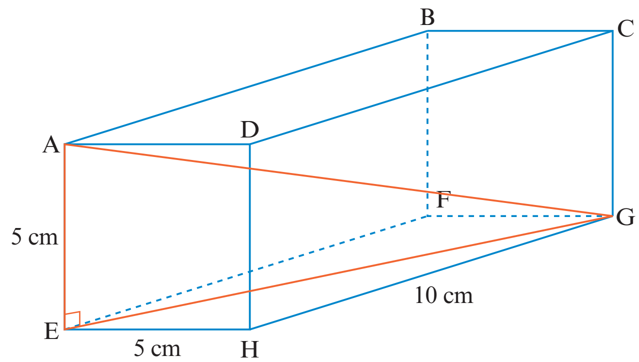
iv BE **v** CF **vi** EG

CHALLENGE

c Consider the right-angled triangle AEG formed in the rectangular prism.

  i Draw the right-angled triangle AEG and label the known side lengths.

  ii Use Pythagoras' Theorem to calculate the length of the line AG.

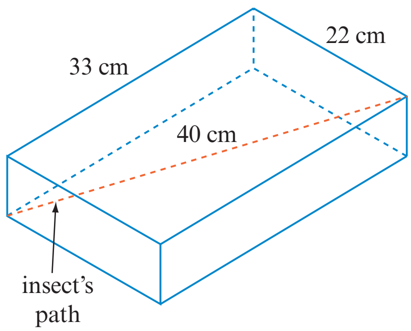


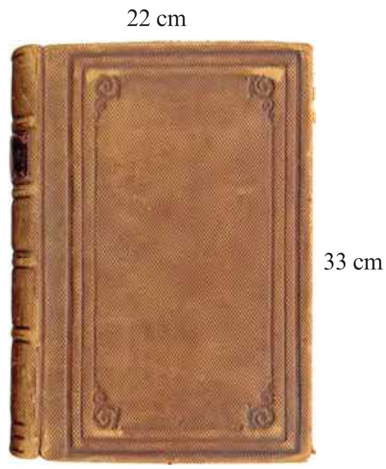
d Use triangle AEG to calculate the angle the line AG makes with the horizontal base of the rectangular prism.

12 Consider the book shown at right.

a Calculate the diagonal length of the front cover.

An insect burrows 40 cm through the pages of the book from the bottom left corner of the back cover to the top right corner of the front cover.





b Draw a right-angled triangle that shows the path of the insect, the diagonal length of the front cover and the thickness of the book. Label it with the two measurements you know.

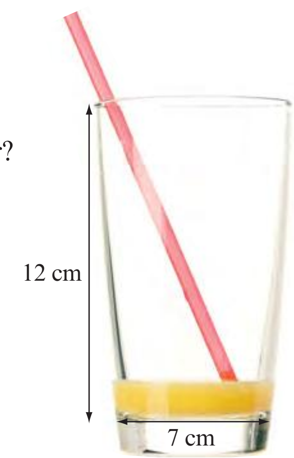
c Use your triangle to calculate the thickness of the book.

d What angle does the path of the insect make with the back cover?

13 A drinking straw extends 4 cm above the top of a glass.

a How long is the drinking straw? (Hint: first draw a labelled right-angled triangle.)

b What angle does the straw make with the base of the glass?



14 A thin glass tube is to be packed into a rectangular box of length 15 cm, width 8 cm and height 6 cm. What is the length of the longest glass tube that will fit inside the box?

Reflect

How is finding unknown angles and lengths in right-angled triangles useful?

CHAPTER REVIEW

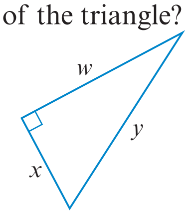
SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

right-angled triangle	perpendicular	cosine	inverse tangent
hypotenuse	opposite side length	tangent	angle of elevation
Pythagoras' Theorem	adjacent side length	theta	angle of depression
Pythagorean triad	trigonometric ratios	inverse sine	bearings
parallel	sine	inverse cosine	

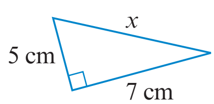
MULTIPLE-CHOICE

6A **1** Using Pythagoras' Theorem, which statement shows the relationship between the side lengths of the triangle?



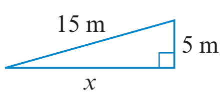
- A $w^2 = x^2 + y^2$
- B $x^2 = w^2 + y^2$
- C $y^2 = x^2 + w^2$
- D $y^2 = x^2 - w^2$

6B **2** Which equation can be used to find the length of the hypotenuse in this triangle?



- A $x = 7 + 5$
- B $7^2 = x^2 + 5^2$
- C $5^2 = x^2 + 7^2$
- D $x^2 = 5^2 + 7^2$

6C **3** The unknown side length x in this triangle, correct to the nearest m, is:

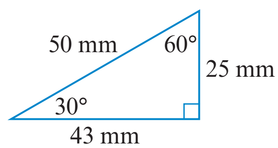


- A 10 m
- B 14 m
- C 16 m
- D 20 m

6D **4** Sine is the trigonometric ratio of which sides of a right-angled triangle?

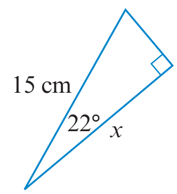
- A hypotenuse and opposite side
- B opposite side and adjacent side
- C adjacent side and hypotenuse
- D opposite side and hypotenuse

6D **5** Using the lengths on the triangle shown, the cosine of 30° is:



A $\frac{43}{50}$ B $\frac{25}{50}$ C $\frac{25}{43}$ D $\frac{50}{43}$

6E **6** Which equation can be used to find x in this triangle?

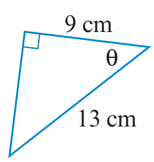


- A $\cos 22° = \frac{x}{15}$
- B $\tan 22° = \frac{x}{15}$
- C $\sin 22° = \frac{x}{15}$
- D $\cos 22° = \frac{15}{x}$

6F **7** If $\sin \theta = \frac{12}{15}$, then θ is equal to:

- A $\sin \frac{15}{12}$
- B $\sin^{-1}\left(\frac{12}{15}\right)$
- C $\sin^{-1}\left(\frac{15}{12}\right)$
- D $\sin\left(\frac{12}{15}\right)^{-1}$

6F **8** The angle θ in this triangle is closest to:

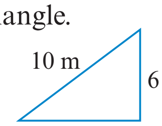


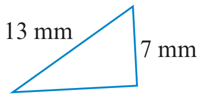
- A 35°
- B 44°
- C 46°
- D 55°

SHORT ANSWER

Where appropriate, calculate each angle correct to the nearest degree and each length correct to two decimal places, unless stated otherwise.

6A **1** Use Pythagoras' Theorem to decide whether each triangle is a right-angled triangle.

a


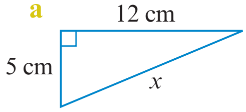
b


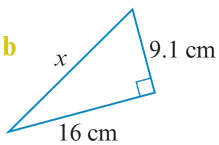
6A **2** A triangle has side lengths 30 mm, 40 mm and 50 mm.

a Show that the triangle is right-angled.

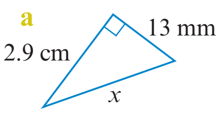
b Are the side lengths of the triangle an example of a Pythagorean triad? Briefly explain why or why not.

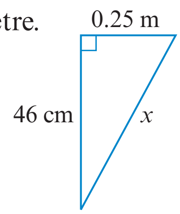
6B **3** Calculate the length of the hypotenuse in each triangle.

a


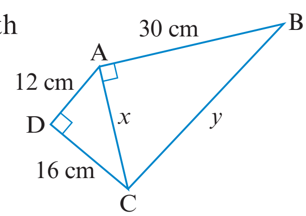
b


6B **4** Calculate the length of the hypotenuse in each triangle, correct to the nearest centimetre.

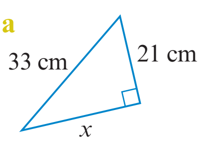
a


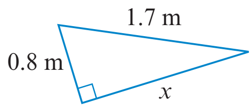
b


6B **5** Find the length of the sides represented by x and y in this diagram.

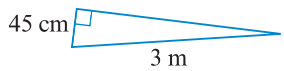


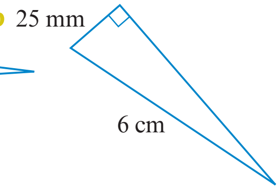
6C **6** Calculate the side length x in each triangle.

a


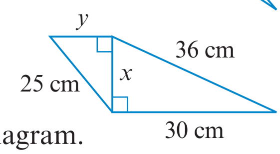
b


6C **7** Calculate the unknown length in each triangle, correct to the nearest centimetre.

a


b


6C **8** Find the length of the sides represented by x and y in this diagram.



6D **9** Consider the triangle shown in question **5** of the Multiple-choice section.

a What do you notice when you compare the values of each of these correct to two decimal places?

i sin 30° and cos 60°

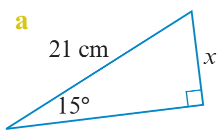
ii sin 60° and cos 30°

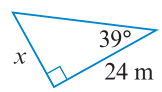
iii tan 30° and $\frac{\sin 30°}{\cos 30°}$

iv tan 60° and $\frac{\sin 60°}{\cos 60°}$

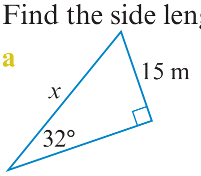
b Repeat part **a** using a calculator. What do you notice about your answers?

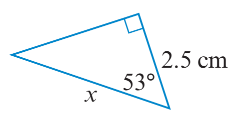
6E **10** Find the side length x in each triangle.

a


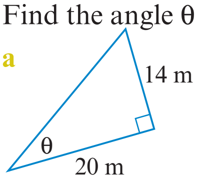
b


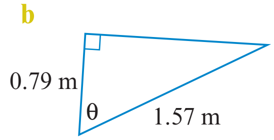
6E **11** Find the side length x in each triangle.

a


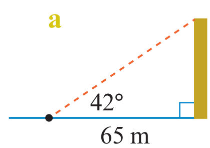
b


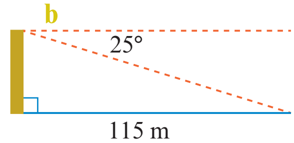
6F **12** Find the angle θ in each triangle.

a


b


6G **13** Find the height of each building given the angle of elevation or depression.

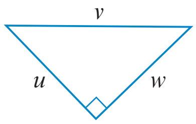
a


b


NAPLAN-STYLE PRACTICE

Where appropriate, calculate each angle correct to the nearest degree and each length correct to two decimal places unless stated otherwise.

Questions **1** and **2** refer to this triangle.



1 Which pronumeral represents the hypotenuse in the triangle?

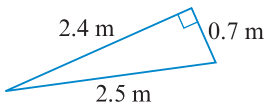
☐

2 Using Pythagoras' Theorem, which of these statements shows the relationship between the side lengths of the triangle?

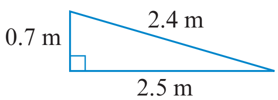
- ☐ $w^2 = u^2 + v^2$
- ☐ $v^2 = u^2 + w^2$
- ☐ $u^2 = v^2 + w^2$
- ☐ $v = u + w$

3 A right-angled triangle contains the side lengths 0.7 m, 2.4 m and 2.5 m. Which of these triangles best matches this description?

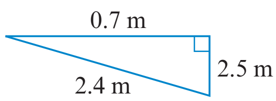
☐ Triangle A



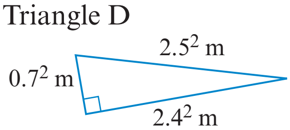
☐ Triangle B



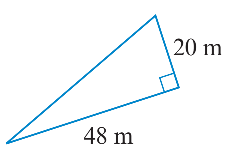
☐ Triangle C



☐ Triangle D



4 What is the length of the hypotenuse in this triangle?

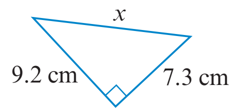


- ☐ 68 m
- ☐ 8.24 m
- ☐ 52 m
- ☐ 43.63 m

5 The two smaller sides of a right-angled triangle are 3 m and 4 m. What is the length of the third side?

☐

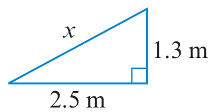
6 What is the value of x?



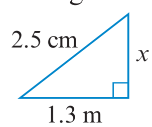
☐

7 A ladder leans against a vertical wall of a building so that the top of the ladder reaches 2.5 m up the wall. The foot of the ladder is 1.3 m from the base of the wall. Which of these triangles correctly displays this information, with the pronumeral representing the unknown side length?

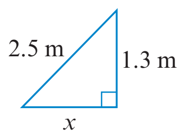
☐ Triangle A



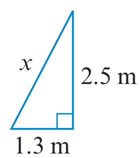
☐ Triangle B



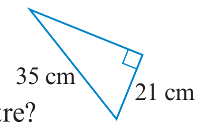
☐ Triangle C



☐ Triangle D

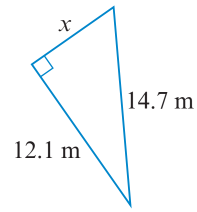


8 What is the length of the unknown side in this triangle, correct to the nearest centimetre?



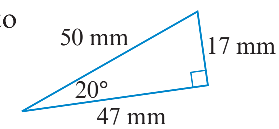
- ☐ 14 cm
- ☐ 28 cm
- ☐ 41 cm
- ☐ 784 cm

9 What is the value of x?



☐

Questions **10** and **11** refer to this triangle.



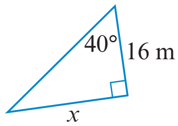
10 What is the value of cos 20° as a fraction?

- $\frac{47}{50}$ ☐
- $\frac{17}{50}$ ☐
- $\frac{17}{47}$ ☐
- $\frac{47}{17}$ ☐

11 What is the value of tan 20° as a decimal?

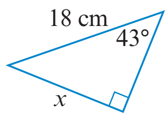
- 0.36
- 2.76
- 0.94
- 0.34

12 Which equation can be used to find the side length x in the triangle?



- $\tan 40° = \frac{16}{x}$
- $\tan 40° = \frac{x}{16}$
- $\cos 40° = \frac{x}{16}$
- $\sin 40° = \frac{x}{16}$

13 What is the side length x in this triangle?

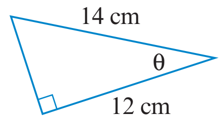


[ ]

14 If $\tan\theta = \frac{22}{25}$, then θ is equal to:

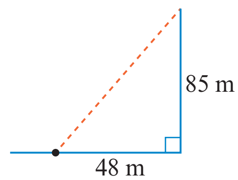
- $\tan^{-1}\left(\frac{22}{25}\right)$
- $\tan^{-1}\left(\frac{25}{22}\right)$
- $\tan\frac{22}{25}$
- $\tan\left(\frac{22}{25}\right)^{-1}$

15 What is the angle θ in this triangle?



[ ]

16 What is the angle of elevation from a point on the ground that is 48 m from the base of a building to the top of a building that is 85 m high?



- 29°
- 34°
- 56°
- 61°

ANALYSIS

From the top of a tower, Kim determines the angle of depression to her family's car in a nearby car park to be 27°. She knows that the car is 1.2 km from the base of the tower.

a Draw a diagram to display the information stated.

b Consider the information provided.

  i Is there enough information to use Pythagoras' Theorem to calculate the height of the tower? Explain why or why not.

  ii Is there enough information to use trigonometry to calculate the height of the tower? Explain why or why not.

c Using your answer from part **b**, draw a triangle and label the sides with the appropriate information.

d Calculate the height of the tower to the nearest metre.

e Show another two *different* triangles that also could have been used to calculate the height of the tower. Label each triangle with its relevant information and show that each triangle produces the same tower height as you calculated in part **d**.

Kim wonders whether she can use Pythagoras' Theorem to calculate the direct distance from her eye level to the car in the car park.

f Show that there is enough information for Kim to calculate the direct distance to the car using Pythagoras' Theorem. Write your answer to the nearest metre.

g Check your answer in part **f** using trigonometry. Show that both methods produce the same value.

CONNECT

How high is that?

Using Pythagoras' Theorem or trigonometry are just two ways to solve problems where you are working with right-angled triangles. Are there any other methods you can use?

You are to investigate different ways of working out the height of a tall object such as a tree or goal post without the use of a ladder. You will need a measuring tape or trundle wheel, a clinometer or other simple device for measuring angles, and a metre ruler or stick of known length.

Gareth and Rhonda use a clinometer to measure the angle of elevation to the top of a tree.

Nadia measures the length of the shadows formed by her chosen object and a vertical stick (such as a metre ruler) and works with similar triangles.

Tom also uses the length of shadows and draws scale diagrams.

Making a simple clinometer

Here is one way of making a simple device to measure an angle of elevation.

1. Tape a drinking straw along the base line of a protractor.
2. Attach string to the centre point of the base line.
3. Tie a weight (such as a washer) to the end of the string.
4. Look through the straw to the top of the object. Have a partner read the smaller angle measurement on the protractor between the straw and the string and subtract this from 90°.

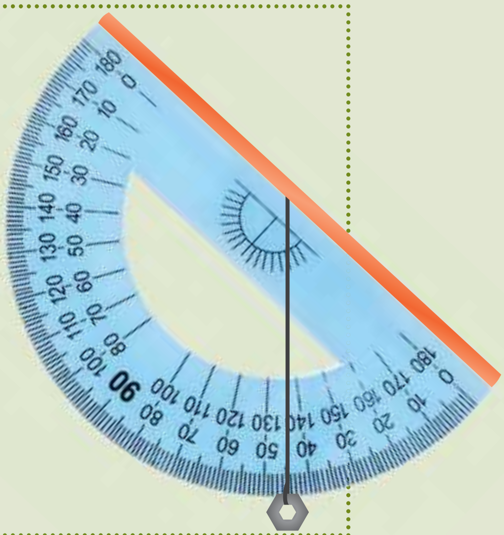
Your task

You are to:

- choose a tall object
- draw appropriate labelled diagrams showing all known information
- use at least two different methods to work out the height of the object
- provide a detailed description of how each method works
- calculate the distance along your line of sight to the top of the object.

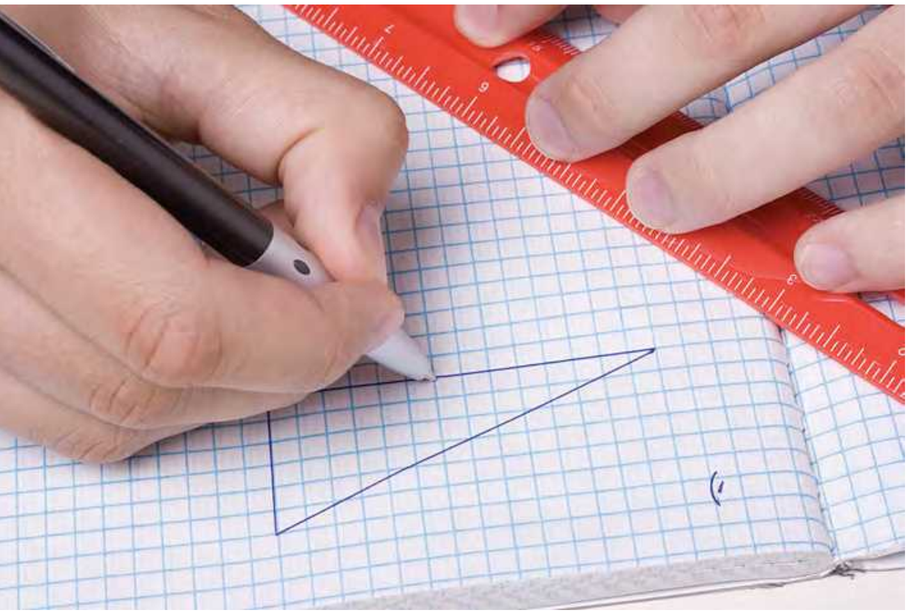
Include all necessary working to justify your answers.

As an extension, devise your own problem that requires Pythagoras' Theorem or trigonometry to solve it. Show all relevant diagrams and working.



You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a 'how to' booklet
- a PowerPoint presentation
- a technology demonstration
- other (check with your teacher).



7 MEASUREMENT

ESSENTIAL QUESTION

What kinds of things do you measure and how do you measure them?

7A **1** **a** How many metres in a kilometre?

b How many minutes in an hour?

c How many kilograms in a tonne?

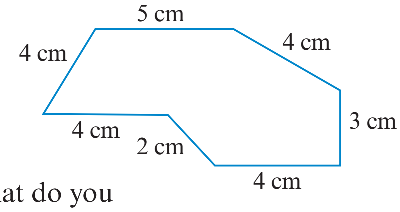
7A **2** Which option displays a number in scientific notation?

A 4.58 **B** 12 000 000

C 56.79×10^3 **D** 3.45×10^6

7A **3** How many decimal places in 0.000 894?

7B **4** **a** What is the perimeter of this shape?

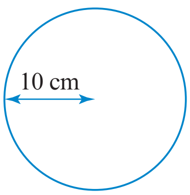


b What do you call the perimeter of a circle?

A around **B** radius

C diameter **D** circumference

c What is the diameter of this circle?



A 10 cm

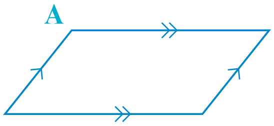
B 20 cm

C 5 cm

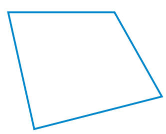
D 31.4 cm

7C **5** Which shape is a trapezium?

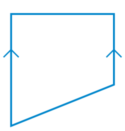
A



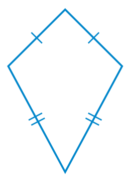
B



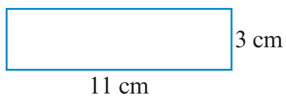
C



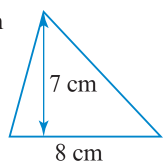
D



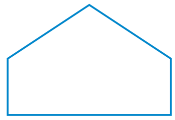
7C **6** **a** What is the area of this rectangle?



b What is the area of this triangle?



7D **7** What two shapes make up this figure?



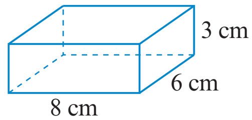
A rectangle and triangle

B square and rectangle

C trapezium and triangle

D triangle and square

7E **8** Look at this prism.



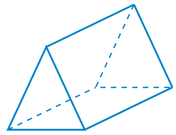
a How many faces does this prism have?

b What is the surface area of this prism?

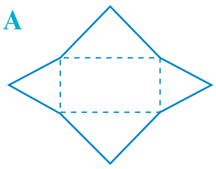
A $24\ cm^2$ **B** $144\ cm^2$

C $17\ cm^2$ **D** $180\ cm^2$

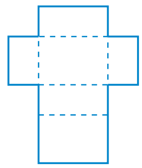
7E **9** Which option is the net that matches this figure?



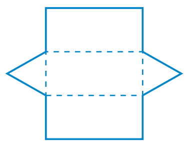
A



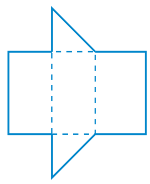
B



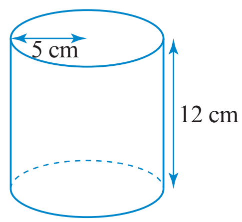
C



D



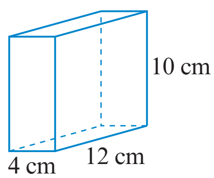
7F **10** Look at this cylinder.



a What is the radius?

b What is the height?

7G **11** What is the volume of this prism?



A $480\ cm^3$

B $26\ cm^3$

C $40\ cm^3$

D $416\ cm^3$

7A Understanding and representing measurement

Start thinking!

You use measurement every day to describe things such as length, mass and time.

1 Name three different units used to describe each of these terms: length, mass and time.

It is important that you are able to convert between different units. A conversion chart can help you with this. Look at the length conversion chart provided in Key ideas.

2 When you convert to a larger unit, do you multiply or divide?

3 What do the numbers on top and underneath the arrows represent?

4 Using the length conversion chart as a guide, create a conversion chart for:

a mass **b** time.

5 Why do you use different units when using measurement?
(Hint: which is more meaningful: 1 month or 2 678 400 seconds?)

Sometimes measurements can be extremely large or extremely small, and it is not enough to simply convert between units if we want to write numbers and units that are easy to read and understand. Scientific notation can help us to write these very large and very small numbers.

To write a number in scientific notation, write a number from 1 up to, but not including, 10 (any number of decimal places can be included), and multiply it by a power of 10.

Consider these two examples:

$1640 = 1.64 \times 10^3$

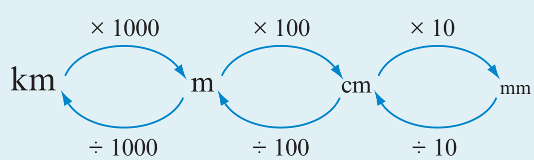
$0.000\,048\,9 = 4.89 \times 10^{-5}$

6 Explain why the power is positive in the first example but negative in the second example.

7 Why does the first example use 10^3 but the second example use 10^{-5}? (Hint: How many places does the decimal point move for each representation? In what direction?)

KEY IDEAS

- Different units are used to describe measurements to make them more meaningful.
- Conversion charts, such as this one shown for length, can be used to convert between these different units.



- Scientific notation is used to represent very large or very small measurements.
- The International System of Units uses a base unit such as metre or second and a prefix such as milli-, centi-, kilo-, micro-, etc. For example, 1 kilometre is 1×10^3 metres and 1 microsecond is 1×10^{-6} seconds.

EXERCISE 7A Understanding and representing measurement

EXAMPLE 7A-1 Converting length units (one step)

Convert 1.05 km to metres.

THINK	WRITE
1 The length is to be converted to a smaller unit, so multiply by the conversion factor.	1.05 km = (1.05 × 1000) m
2 The conversion factor from kilometres to metres is 1000. Multiply 1.05 by 1000.	= 1050 m

1 Perform these conversions.

a 43 cm into millimetres
b 187 m into kilometres
c 1200 s into minutes
d 2.5 h into minutes
e 4750 g into kilograms
f 9.45 t into kilograms
g 83 m into centimetres
h 210 min into hours
i 6.205 kg into grams
j 8 mm into centimetres

EXAMPLE 7A-2 Converting length units (two steps)

Convert 28 mm into metres.

THINK	WRITE
1 The length is to be converted to a larger unit, so divide by the conversion factor(s).	28 mm = (28 ÷ 10) cm
2 First convert to centimetres.	= 2.8 cm
3 Complete the conversion to metres.	= (2.8 ÷ 100) m = 0.028 m

2 Perform these conversions.

a 63 mm into metres
b 1.07 km into centimetres
c 475 g into kilograms
d 0.25 t into grams
e 2000 s into hours
f 3 days into seconds
g 3.25 h into seconds
h 1.77 m into millimetres
i 320 mg into kilograms
j 0.86 t into milligrams

EXAMPLE 7A-3 Writing numbers in scientific notation

One type of single bacterium has a mass of 0.000 000 000 000 19 g.
Write this in scientific notation so that it is easier to read.

THINK	WRITE
1 Use the non-zero digits to write the number between 1 and 10.	The first part of the measurement is 1.9.
2 Count the number of decimal places that the decimal point must move to create the original number.	The decimal point must move 13 places to the left.
3 Decide whether it will be a negative or positive power of 10 (negative). Write your answer.	0.000 000 000 000 19 g $= 1.9 \times 10^{-13}$ g

3 Write each measurement in scientific notation.

a 125 000 000 km
b 0.000 000 006 75 mm
c 13 750 000 000 years
d 0.000 002 s
e 5 946 000 000 000 t
f 0.000 000 000 000 000 000 25 g
g 875 500 000 000 min
h 0.000 000 076 kg
i 925 000 000 000 000 cm
j 0.000 000 000 000 001 7 h

EXAMPLE 7A-4 Comparing two measurements in scientific notation

Which measurement is bigger: 2.538×10^{-7} mm or 2.574×10^{-8} cm?

THINK	WRITE
1 Write each measurement as a basic numeral.	2.538×10^{-7} mm = 0.000 000 253 8 mm 2.574×10^{-8} cm = 0.000 000 025 74 cm
2 Convert the second measurement to millimetres so they are both in the same units.	= 0.000 000 257 4 mm
3 Compare the measurements.	2.574×10^{-8} cm is the bigger measurement.

4 Which measurement is bigger:

a 3.45×10^{6} g or 5.42×10^{5} g?
b 7.82×10^{12} kg or 9.42×10^{9} kg?
c 2.86×10^{-8} s or 1.99×10^{-9} s?
d 6.72×10^{-11} cm or 8.72×10^{-12} mm?
e 1.23×10^{6} kg or 3.57×10^{12} g?
f 9.67×10^{-19} g or 2.99×10^{-7} mg?
g 4.75×10^{18} mm or 8.19×10^{11} km?
h 5.08×10^{-3} L or 4.42×10^{-6} mL?

PROBLEM SOLVING AND REASONING

5 The Earth is estimated to be 4.54 billion years old.

a Write this:

i as a basic numeral

ii in scientific notation.

b The universe is estimated to be 13.75 billion years old.

i Write this as both a basic numeral and in scientific notation.

ii Find the difference in age between the Earth and the universe and write your final answer in scientific notation. Remember that you cannot add or subtract numbers in scientific notation unless they use the same power of 10.

6 Rather than always using scientific notation, scientists use a naming system called the International System of Units, often abbreviated to SI. It names measurements using a 'base' unit and a range of different prefixes (words placed at the start of other words, such as 'un' 'semi' and 're').

The table on the right shows the common SI prefixes. For example, one teragram is 1×10^{12} g (1 000 000 000 000 g), and one nanometre is 1×10^{-9} m (0.000 000 001 m).

Prefix	Symbol	Factor
yotta-	Y	10^{24}
zetta-	Z	10^{21}
exa-	E	10^{18}
peta-	P	10^{15}
tera-	T	10^{12}
giga-	G	10^{9}
mega-	M	10^{6}
kilo-	K	10^{3}
hecto-	h	10^{2}
deca-	da	10^{1}
–	–	1
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}
atto-	a	10^{-18}
zepto-	z	10^{-21}
yocto-	y	10^{-24}

a Write each measurement in both scientific notation and as a basic numeral.

i 1 nanogram **ii** 1 gigametre

iii 1 attosecond **iv** 8.1 petagrams

v 34 microseconds **vi** 17 picograms

b Write each measurement using the SI system.

i 756 000 000 000 000 m

ii 1 840 000 000 g

iii 0.000 000 000 023 s

iv 0.000 049 g

v 94 000 000 000 000 000 000 000 s

vi 0.000 000 000 000 000 028 m

c Write these common measurements using the SI system.

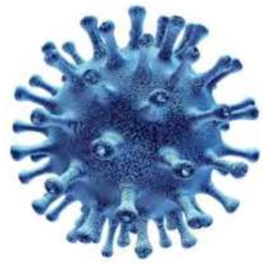
i a million metres **ii** a hundred grams

iii 10 seconds **iv** a trillion metres

v one thousandth of a second

vi a billionth of a gram

PROBLEM SOLVING AND REASONING



7 HIV is the very well-known virus that causes AIDS in humans. It is roughly spherical in shape, as shown in this image. It has a diameter of roughly 120 nanometres and an estimated mass of 2 femtograms.

a Write these measurements using scientific notation.

b How many individual viruses would make up 1 kg? Write this in scientific notation.

Viral load is the estimation of the number of virus particles, usually written as a number per millilitres of blood. The average adult has about 5 L of blood.

c Find the total number of virus particles and hence the approximate mass of virus in a person's bloodstream if they had:

i a low viral load of 40 **ii** a high viral load of 10 000

iii an extreme initial load of 2 500 000.

Write your answers using scientific notation and the SI system.

8 A light year is the distance that light can travel in a year. It is important to remember that it is a measure of distance, not time. Light moves at approximately 300 000 000 metres per second (m/s).

a Write this: **i** in scientific notation **ii** in km/h.

b What distance does light cover in:

i a minute? **ii** an hour? **iii** a day? **iv** a year?

c Use your answer to part **b iv** to write a light year in kilometres.

d Use scientific notation to write how long it takes light to cover:

i 1 km **ii** 1 m **iii** 1 cm **iv** 1 mm.

e Write your answers to parts **b** and **d** using the SI system. Which do you prefer to use?

f If the Earth is 150 million km from the Sun, how long does it take light from the Sun to reach Earth?

9 Although not as fast as light, sound also travels very fast, at about 343.2 m/s in air.

a Write this in km/h.

b What distance can sound cover in:

i a minute? **ii** an hour? **iii** a day? **iv** a year?

c Use scientific notation to write how long it takes sound to travel:

i 1 km **ii** 1 m **iii** 1 cm **iv** 1 mm.

d If sound moves at roughly 5.34×10^3 km/h in water, does sound move faster in air or water? By how much?

Sound can also be used as a measure of speed – the speed of sound can be called Mach 1, double the speed of sound can be called Mach 2 and so on.

e Write these speeds as a Mach number (correct to one decimal place).

i the world record for land speed (1228 km/h)

ii the world record for air speed (3530 km/h)

iii the speed a space shuttle must go to leave Earth (40 000 km/h)

iv the speed of the Earth as it moves around the Sun (107 280 km/h)

v the speed of light (1.08×10^9 km/h)

PROBLEM SOLVING AND REASONING

10 Films are produced by flashing many images per second onto a screen. The number of images produced per second is called frames per second (fps). Standard films use a rate of 24 fps.

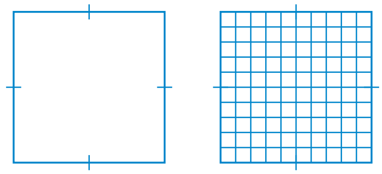
- **a** For how long is a single frame on the screen at:
  - **i** 24 fps? **ii** 72 fps?
- **b** How many frames should an animator create for a cartoon to run for 30 min at 24 fps?

High-speed cameras can be used to record from 1000 fps up to 1 trillion fps, and then slowed down to standard frame rates (for example, 24 fps) to produce slow-motion film.

- **c** For how long is a single frame on screen for these two speeds?
- **d** If 2 s of film was shot on a camera that shot at 1 trillion fps and it was replayed at 24 fps, how long would this play for?

CHALLENGE

11 One aspect of conversion where people make mistakes is when converting between units of area and units of volume.



Consider figures A and B.

- **a** How many square centimetres in each figure?
- **b** How many millimetres in a centimetre?
- **c** Explain why each figure has an area of 100 mm^2, not 10 mm^2.
- **d** Draw a square and label its sides as 1 m. Use it to explain why 1 m^2 is equal to 10 000 cm^2, not 100 cm^2.
- **e** Explain why, when converting between units of area, you square the conversion factor.

The same process can be performed with volume.

- **f** Draw a cube with 1 cm sides and use it to explain why 1 cm^3 = 1000 mm^3.
- **g** Draw a cube with 1 m sides and use it to explain why 1 m^3 = 1 000 000 cm^3.
- **h** Explain why, when converting between units of volume, you cube the conversion factor.

12 Convert these units of area.

- **a** 7 cm^2 into mm^2
- **b** 84 km^2 into m^2
- **c** 500 mm^2 into cm^2
- **d** 2000 cm^2 into m^2

13 Convert these units of volume.

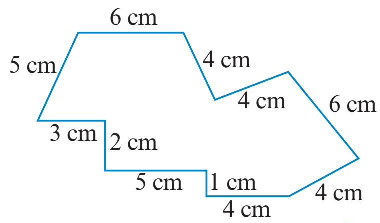
- **a** 17 m^3 into cm^3
- **b** 281 cm^3 into mm^3
- **c** 66 000 cm^3 into m^3
- **d** 600 mm^3 into cm^3

Reflect

What useful ways do you have to represent different measurements?

7B Perimeter

Start thinking!



Consider this irregular shape.

1 Describe how you would find the perimeter of this shape.

The irregular shape has some sides that are the same length. Often when you have sides the same length you draw marks on the equal side lengths rather than labelling the length of every side.

2 Redraw the figure with marks that show any sides that are equal in length.

Many familiar shapes have sides equal in length.

3 Draw an example of three different types of triangles according to their side lengths: equilateral, isosceles and scalene. Use marks to indicate any equal side lengths.

4 Draw examples of the six different types of quadrilaterals (**square**, **rectangle**, **parallelogram**, **rhombus**, **kite** and **trapezium**). Use marks to indicate any equal side lengths.

5 How does counting the number of sides a shape has help to make sure you find the entire perimeter?

The perimeter of a circle is called its **circumference** and has its own special formula, related to its **diameter**.

6 What is the diameter of a circle?

7 Draw five different circles and using a piece of string and a ruler, take the measurements of both the diameter and circumference. Write these in a table.

8 Add a column to your table and divide the circumference by the diameter. What do you find?

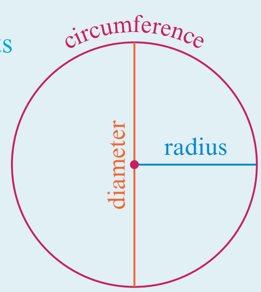
When you divide circumference by diameter you always get the number **pi**, π (roughly 3.14).

Written in mathematical terms, this is $\frac{C}{D} = \pi$.

9 Rewrite this formula in the form $C = \ldots$ to obtain the formula for the circumference of a circle.

10 What information do you need about a shape to calculate its perimeter?

KEY IDEAS



- Perimeter is the distance around the outside edge of a 2D shape.
- To calculate perimeter of a straight-edged shape, add the length measurements of each side together. Make sure that all measurements are in the same unit.
- The perimeter of a circle is called its circumference (C) and can be found using the formula $C = \pi D$ or $C = 2\pi r$ where D is the length of the diameter and r is the length of the radius.
- The perimeter of a **sector** can be found using $\frac{\theta}{360°} \times 2\pi r + 2r$, where θ represents the angle in degrees between the radii.

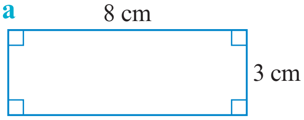
EXERCISE 7B Perimeter

UNDERSTANDING AND FLUENCY

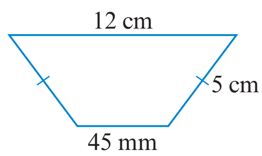
For any questions involving π in this topic, use the π button on your calculator and then round each answer to two decimal places.

1 Calculate the perimeter of each shape.

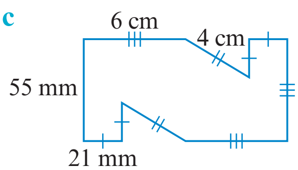
a



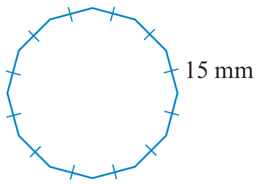
b



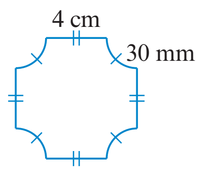
c



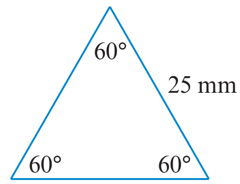
d



e

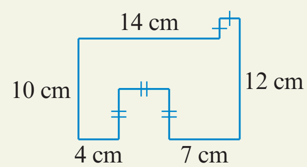


f



EXAMPLE 7B-1 Calculating perimeter when not all sides are given

Calculate the perimeter of this shape in centimetres.

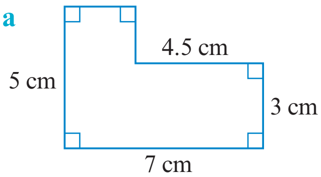


THINK	WRITE
1 Identify the ten lengths to be added.	
2 The vertical side with the single dash can be found by subtracting the vertical lengths. Subtract 10 cm from 12 cm.	length of side with single dash = 12 cm − 10 cm = 2 cm
3 The horizontal side with the double dash can be found by considering the horizontal sides. Find the horizontal length of the entire shape using the 14 cm and the length of the first unknown side.	horizontal length = 14 cm + 2 cm = 16 cm
4 Subtract the known horizontal lengths from 16 cm.	length of side with double dash = 16 cm − 4 cm − 7 cm = 5 cm
5 Write the expression for perimeter, ensuring you include all ten sides.	perimeter = 14 cm + 2 cm + 2 cm + 12 cm + 7 cm + 5 cm + 5 cm + 5 cm + 4 cm + 10 cm
6 Add all the lengths and include the correct unit.	= 66 cm

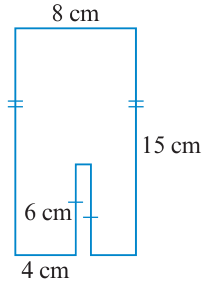
UNDERSTANDING AND FLUENCY

2 Calculate the perimeter of each shape.

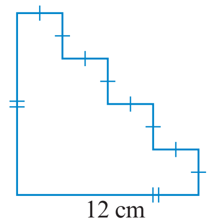
a



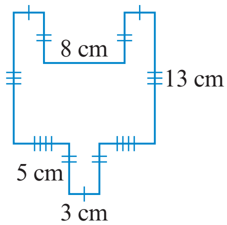
b



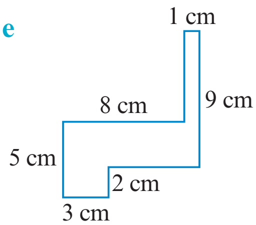
c



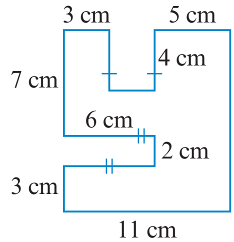
d



e

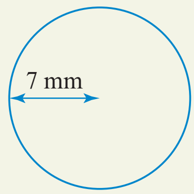


f



EXAMPLE 7B-2 Calculating circumference

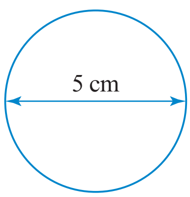
Calculate the circumference of this circle, correct to two decimal places.



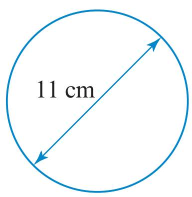
THINK	WRITE
1 Identify which formula to use. (Radius is given.)	$C = 2\pi r$
2 Identify the measurement for the radius and substitute it into the formula.	$= 2 \times \pi \times 7$
3 Calculate the result using the π button on your calculator, rounding to two decimal places. Remember to include the appropriate unit.	$= 43.982\,297\,15...$ ≈ 43.98 mm

3 Calculate the circumference of each circle.

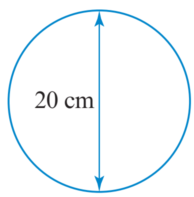
a



b

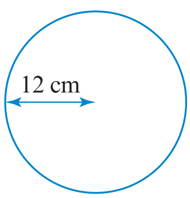


c

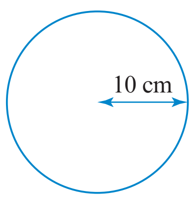


4 Calculate the circumference of each circle.

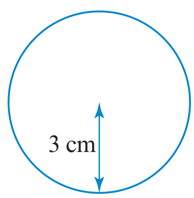
a



b



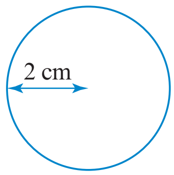
c



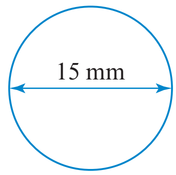
UNDERSTANDING AND FLUENCY

5 Calculate the circumference of each circle.

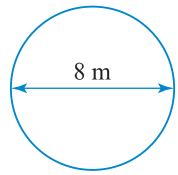
a



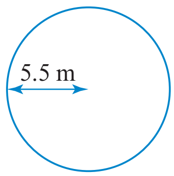
b



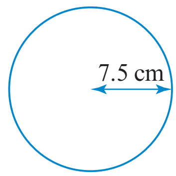
c



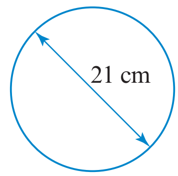
d



e



f



PROBLEM SOLVING AND REASONING

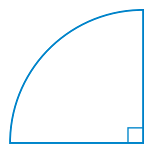
6 A sector can be thought of as a wedge of a circle, like this orange slice.

a What dimension of a circle is equal to the straight parts of a sector? (Hint: This dimension starts at the centre of the circle and ends at the circumference.)

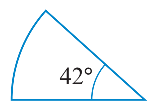
b The curved part of the sector is part of which dimension of the circle?

This curved part of the sector is called an **arc**. The length of the arc is directly linked to the angle between the two straight edges, also called radii (plural for **radius**). If you can calculate the length of the arc, you then just add the lengths of radii to find the total perimeter of the sector.

Consider these two figures.



2 cm



2 cm

c What is the angle contained within the first sector?

d How many degrees are in a circle in total?

e Use your answers to parts **c** and **d** to write the angle in the first sector as a fraction of the angle in a full circle.

f Explain why, when the fraction from part **e** is simplified, it becomes $\frac{1}{4}$.

This means that the length of the arc is equal to $\frac{1}{4}$ of the circumference of the full circle.

g Explain why the length of the arc for this sector can now be found using $\frac{1}{4} \times 2\pi r$.

h What is the length of the arc for the first sector?

i Add the lengths of the radii to your answer to part **h** to find the total perimeter of the sector.

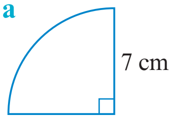
j Follow steps **c–d** to write the angle in the second sector as a fraction of the angle in a full circle.

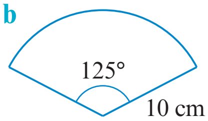
k Explain why the length of the arc for this sector can be found using $\frac{42°}{360°} \times 2\pi r$ or $\frac{7}{60} \times 2\pi r$.

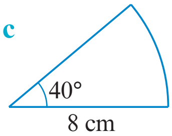
l Follow steps **h** and **i** to find the perimeter of this second sector.

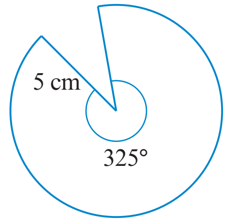
m Explain why the perimeter of any sector can be found using $\frac{\theta}{360°} \times 2\pi r + 2r$, where θ represents the angle in degrees between the radii.

7 Calculate the perimeter of each sector.

a


b


c


d


8 Some sectors are easier to calculate than others because they are simple fractions of a circle. In questions **6** and **7** you already met a **quadrant**.

a Why do you think a sector with an angle of 90° is called a quadrant?

You should also be familiar with a **semicircle**.

b What angle is 'between' the radii of a semicircle?

Two other sectors with special names are sextants (60°) and octants (45°).

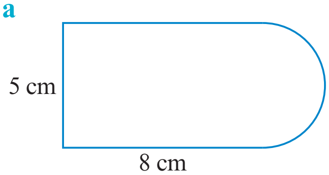
c How do you think they get their names? (Hint: what fraction are they of a complete circle?)

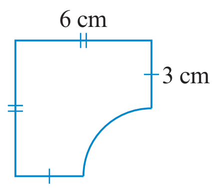
d If each sector had a radius of 5 cm, calculate the perimeter of:

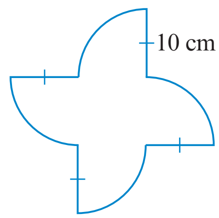
i an octant **ii** a sextant **iii** a quadrant **iv** a semicircle.

e What do you notice about your answers to part **d**?

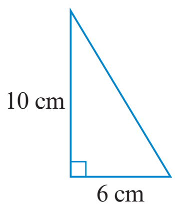
9 Calculate the perimeter of each composite shape.

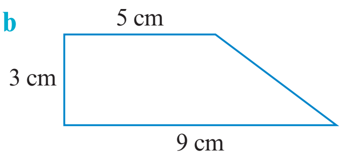
a


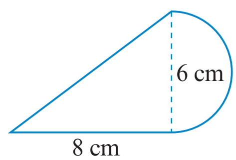
b


c


10 Use your understanding of Pythagoras' Theorem to calculate the perimeter of each shape.

a


b


c


11 The Earth is approximately 150 million km from the Sun.

a Write this measurement in scientific notation.

b If the Earth's orbit was a circle, what would this measurement represent?

c What is the approximate distance of the Earth's orbit around the Sun?

d If it takes the Earth roughly 365.25 days to orbit the Sun, what is its average speed? Write this in km/h.

12 a If a square has a perimeter of 20 cm, what is its side length?

b Write a formula that will give you the side length of a square if you know its perimeter. (Hint: it should start with $l = \ldots$)

c Could this formula be used to calculate the perimeter of a rhombus? Explain.

13 A rectangle has a perimeter of 50 cm.

a Write three possible sets of dimensions that it could have.

b Find its width if it has a length of:

i 10 cm **ii** 20 cm **iii** 3 cm.

c Write a formula that will give you the width of a rectangle if you know its perimeter and its length. (Hint: it should start with $w = \ldots$)

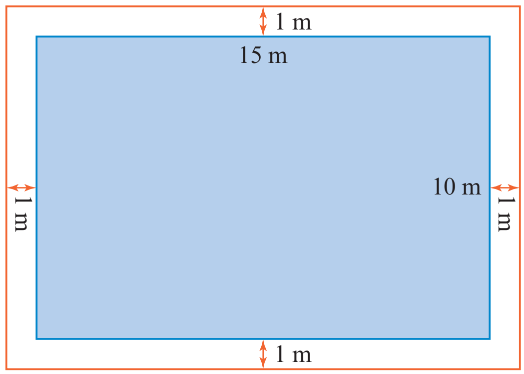
14 Repeat question **13** for both a kite and a parallelogram. Explain how the process is both different and similar for these shapes.

CHALLENGE

15 Josh is getting a pool in his backyard. His parents plan for it to be rectangular in shape, 15 m long and 10 m wide.

a What is the perimeter of the pool?

Josh's parents need to put a fence around the pool so that Josh's little sister Georgia cannot get to the pool without supervision. His parents plan to place the fence 1 m from the pool's edge around its perimeter, as shown in this diagram.



b How much fencing is required? (Hint: the length of the outer rectangle is *not* 16 m.)

c What is the difference between your answers to parts **a** and **b**?

d How much fencing would be required if Josh's parents decided they wanted 2 m between the pool's edge and the fence?

e What is the difference between your answers to parts **a** and **d**?

f How much fencing would be required if the fence was 3 m from the pool's edge? What is the difference between this answer and your answer to part **a**?

g What pattern do you notice in these differences? Use the pattern to write a formula that gives the perimeter of fencing required for this pool when the fence is placed x metres away from the pool's edge. (Hint: how and why is the number 8 important?)

h Use your formula to find the perimeter of fencing required for this pool when the distance from the fence to the pool's edge is:

i 0.5 m **ii** 1.5 m **iii** 2.5 m **iv** 5 m.

16 Use your formula from question **15** to investigate rectangles of different sizes. Does your formula still hold?

Reflect

How can you use properties of simple shapes to determine perimeter?

7C Area of simple shapes

Start thinking!

The **area** of a shape is the amount of space it encloses.

1 Draw a rectangle on a square centimetre grid and state its area in square centimetres.

For most basic shapes, you can use a formula to calculate the area if you know their dimensions.

2 For your rectangle, look at the number of columns and rows of square centimetres it has. How does this relate to the formula for the area of a rectangle, $A = lw$?

Other basic shapes can be linked to rectangles to determine their formulas.

3 Draw and cut out two copies of a parallelogram, a kite and a circle. Label these with their dimensions.

Follow these hints for each shape to find the area formula.

4 **a** What are the dimensions of the parallelogram?

b Use one copy and make a single vertical cut near its edge. Arrange the big piece and the smaller piece to form a rectangle. How does the base and height of a parallelogram relate to its area?

c Repeat the activity with your other copy, but this time make a horizontal cut. What do you find?

5 **a** What are the dimensions of the kite?

b Cut one copy of the kite along its diagonals. Rearrange these four pieces around the other copy to form a rectangle. How do the diagonals of a kite relate to its area?

6 **a** What are the dimensions of the circle?

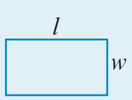
b Cut it into 16 sectors and arrange to form a rough parallelogram. How do the radius and the circumference of the circle relate to the dimensions of the parallelogram and hence its area?

c Repeat part **b** using a circle cut into 32 sectors. How does this improve the parallelogram?

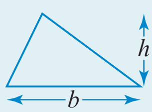
KEY IDEAS

- To calculate the area of any shape, identify the appropriate formula to use and substitute in the given values.

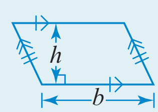
Rectangle
$A = lw$



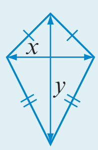
Triangle
$A = \frac{1}{2}bh$



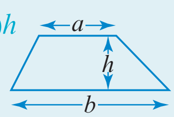
Parallelogram/rhombus
$A = bh$



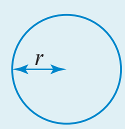
Kite/rhombus
$A = \frac{1}{2}xy$



Trapezium
$A = \frac{1}{2}(a + b)h$



Circle
$A = \pi r^2$

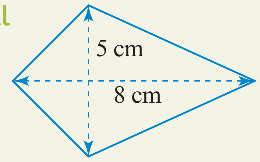


- The area of any sector can be found using $\frac{\theta}{360°} \times \pi r^2$, where θ is the size of the angle in degrees between the two radii.

EXERCISE 7C Area of simple shapes

EXAMPLE 7C-1 Calculating area of a quadrilateral

Calculate the area of this shape.



THINK

1 The shape is a kite; write the appropriate formula for its area.

2 Substitute the values into the formula and calculate its area. Remember to include the appropriate units.

WRITE

$A = \frac{1}{2}xy$

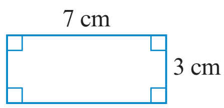
$A = \frac{1}{2} \times 5 \times 8$
$= 20\text{ cm}^2$

UNDERSTANDING AND FLUENCY

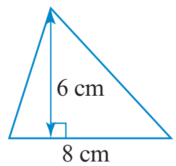
For any questions involving π in this topic, use the π button on your calculator and then round each answer to two decimal places.

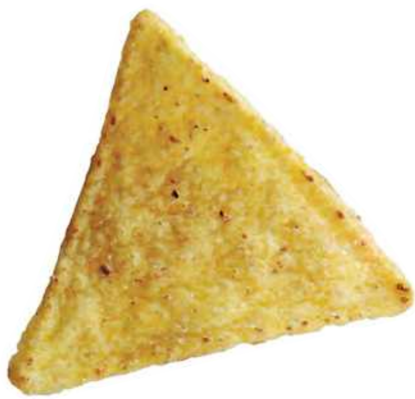
1 Calculate the area of each shape.

a

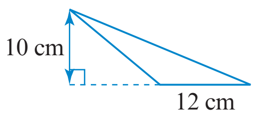


b

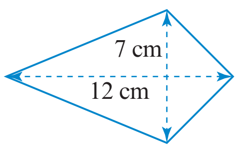




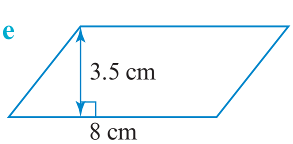
c



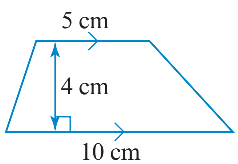
d



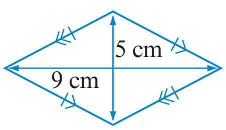
e



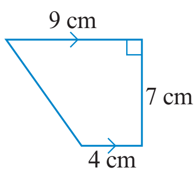
f



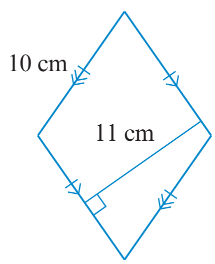
g



h

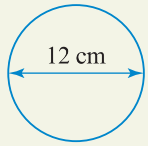


i



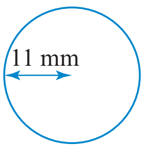
EXAMPLE 7C-2 Calculating area of a circle

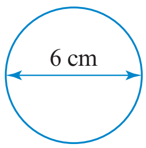
Calculate the area of this shape.

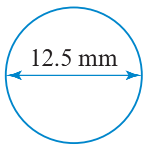


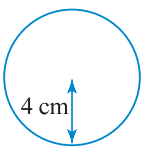
THINK	WRITE
1 Write the formula.	$A = \pi r^2$
2 Diameter is given so divide by 2 to find the radius.	$r = D \div 2$ $= 12 \div 2$ $= 6$ cm
3 Substitute the measurement for radius into the formula.	$A = \pi \times 6^2$
4 Calculate the result using the π button on your calculator, rounding to two decimal places. Remember to include the appropriate unit.	$= 113.097\,335\,5\ldots$ ≈ 113.10 cm^2

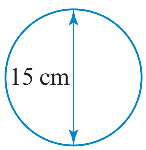
2 Calculate the area of each circle.

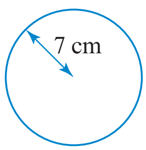
a


b


c


d


e


f


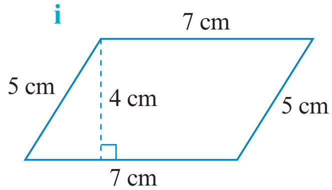
PROBLEM SOLVING AND REASONING

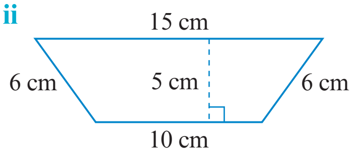
3 These blocks fit together into a square that has side lengths of 8 cm (measured inside the wooden holder). For each individual shape listed below, find:

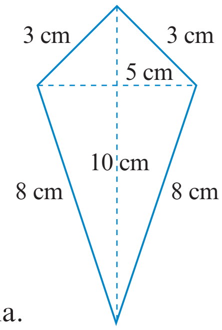
- **i** its dimensions
- **ii** its area.

a square
b isosceles triangle
c right-angled triangle
d trapezium

4 **a** Calculate the area of these quadrilaterals.

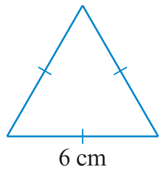
i


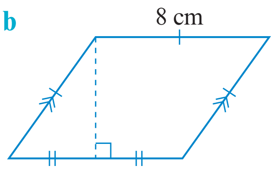
ii


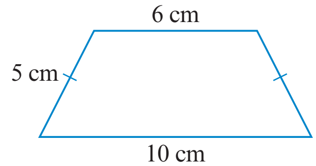
iii


b Explain why it is important to identify the dimensions of each quadrilateral before substituting into the correct formula.

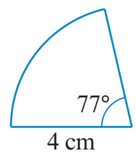
5 Use your understanding of Pythagoras' Theorem to find the area of each shape.

a


b


c


6 The area of any sector can be found in a similar way to finding its perimeter. Consider this sector.



a What is the length of its radius?

b What is the size of the angle between the two radii?

c How many degrees in a full circle?

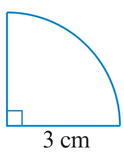
d Use your answers to parts **b** and **c** to write the angle in the sector as a fraction of the angle in a full circle.

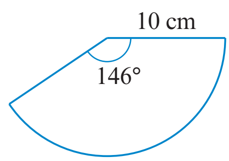
e Explain why the area of this sector can be calculated using $\frac{77°}{360°} \times \pi r^2$.

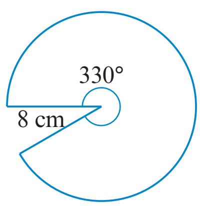
f What is the area of this sector?

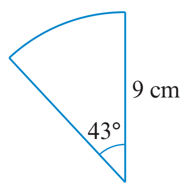
g Explain why the area of any sector can be found using $\frac{\theta}{360°} \times \pi r^2$, where θ is the size of the angle in degrees between the two radii.

7 Calculate the area of each sector.

a


b


c


d


8 **a** A square has an area of 49 cm^2. What is its length?

b Write a formula that gives the length of a square when you know its area.

9 Another commonly used unit of area is the hectare (ha). A hectare is 10 000 m^2.

a Draw a square that has an area of 10 000 m^2. What are its side lengths?

b Aaron's farm covered 5 km^2. What is this in hectares?

c If Aaron's parents bought a neighbouring farm that had an area of 10 ha, what is the total area of land now owned in:

i hectares?

ii square kilometres?

10 A rectangle has an area of 100 cm^2.

a Write three possible sets of dimensions that this rectangle could have.

b Find the length of this rectangle if its width is:

i 20 cm

ii 5 cm

iii 40 cm.

c Write a formula that gives you the length of a rectangle if you know its area and its width.

11 **a** A circle has an area of 250 cm^2. What is its radius?

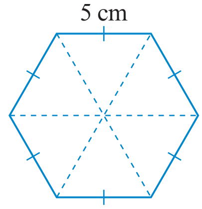
b Write a formula that gives the radius of a circle when you know its area.

12 Why does knowing the area of a circle or a square only give you one possibility for its dimensions?

13 If you know the area of a rhombus, is there only one possibility for the length of its sides? Explain.

14 A circle has a circumference of 10 cm. Calculate its area.

15 You can use your understanding of triangles and Pythagoras' Theorem to calculate the area of a regular hexagon.
Consider this regular hexagon with 5 cm sides.



a What shapes has this hexagon been split up into?

b Use your understanding of tessellations to explain why these triangles are equilateral triangles.

c How do you calculate the area of a triangle?

d Draw and label a diagram of one equilateral triangle split down its middle to show its base and height. (Hint: the height should split the base into two equal parts.)

e Use Pythagoras' Theorem to calculate the height of one of these triangles.

f Calculate the area of one of these triangles.

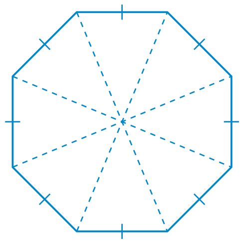
g Use your answer to part **f** to find the total area of this regular hexagon.

h Repeat parts **d–g** to find the area of a regular hexagon with side lengths of 12 cm.

i Why does this only work for regular hexagons and not other polygons? (Hint: the formula is based upon splitting the hexagon up into equilateral triangles.)

CHALLENGE

16 To calculate the area of any regular polygon, you can use a similar process to the one used in question **15**. However, because no other polygon splits into equilateral triangles, you need one more piece of additional information – the height of the polygon. Consider this regular octagon.



- **a** How many triangles is the octagon split up into?
- **b** If the octagon had a height of 12.07 cm, what is the height of an individual triangle?
- **c** Calculate the area of an individual triangle.
- **d** Calculate the total area in this regular octagon.
- **e** Follow the previous steps to calculate the area of these regular polygons with a side length of 5 cm. Include a diagram with your answers.
  - **i** pentagon (height = 6.88 cm)
  - **ii** heptagon (height = 10.38 cm)
  - **iii** decagon (height = 15.39 cm)

17 In Exercise 7A question **11** (page 315), you explored the process of converting between square units.

- **a** Explain why 1 cm^2 = 100 mm^2, not 10 mm^2. Draw a 1-cm square and split it into millimetres if you need help.
- **b** Explain why when you convert between square units, you multiply or divide by the square of the conversion factor.
- **c** Write your answers from question **4** in:
  - **i** square millimetres
  - **ii** square metres.
- **d** Why might it be easier to convert length measurements into the desired unit before calculating area?

18 Explain how you would find the area of an irregular polygon. What information would you need?

19 Investigate the maximum area you can enclose with 100 m of fencing if the area is:

- **a** a square
- **b** a circle
- **c** a rectangle
- **d** up against a wall (for example, if you made it rectangular, you would only need to form three of the four sides with the fencing).

Reflect

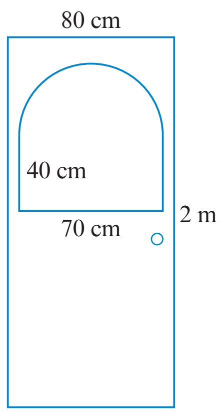
Why is it important to identify the correct dimensions of a shape before calculating area?

7D Area of composite shapes

Start thinking!

A **composite shape** is made up of a number of different shapes. To find the area of a composite shape, you split it into simpler, recognisable shapes such as triangles, quadrilaterals and circles for which an area formula can be used.

Consider this plan for a door and its window.



1 What shapes make up the window section of the door?

2 Redraw the window section of the door and use a dotted line to split it into its two known shapes.

3 For the window, what are the dimensions of:
  a the rectangular section?
  b the semicircular section?

4 Write the area formula for each window section.

5 Calculate the total area of the window.

6 What are the dimensions of the entire door?

7 What would be the area of the door if it had no window?

8 Calculate the amount of wood required to build the door.

9 Explain why it is easier to use subtraction rather than addition to work out the amount of wood for the door.

10 How would you decide whether to use addition or subtraction to calculate the area of a composite shape?

KEY IDEAS

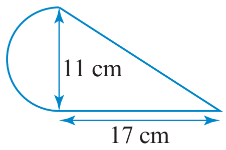
- To calculate the area of a composite shape, follow these steps.
  1 Split the shape into individual parts for which you can easily calculate the area using known formulas.
  2 Label the individual parts with any missing dimensions.
  3 Calculate the areas of the individual parts.
  4 Add or subtract the areas to calculate the total area.

EXERCISE 7D Area of composite shapes

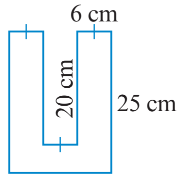
UNDERSTANDING AND FLUENCY

1 Identify the basic shapes within each composite shape.

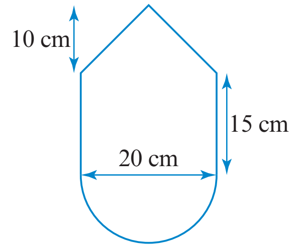
a



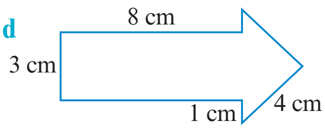
b



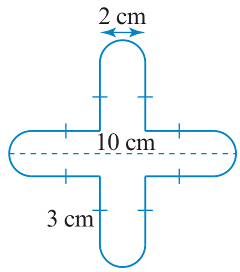
c



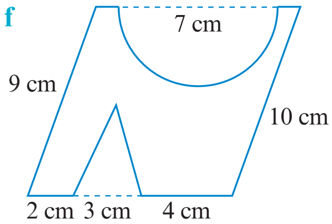
d



e

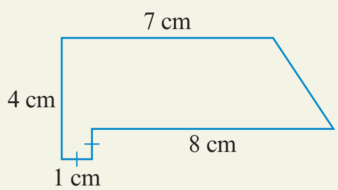


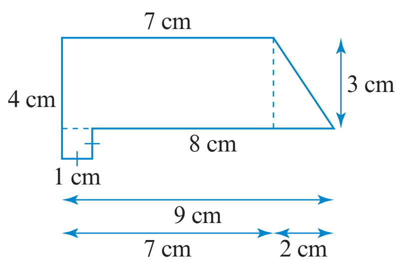
f



EXAMPLE 7D-1 Calculating area of a composite shape (addition method)

Calculate the area of this composite shape.

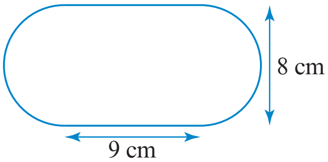


THINK	WRITE
1 Redraw the composite shape with dotted lines to split it into parts for which you can calculate the area easily. 2 Find any missing dimensions for each individual shape and label these on your figure.	
3 Calculate the areas of the individual shapes that make up the composite shape. $A_{rectangle} = lw$ $A_{square} = l^2$ $A_{triangle} = \frac{1}{2}bh$	$A_{rectangle} = 7 \times 3 = 21\text{ cm}^2$ $A_{square} = 1 \times 1 = 1\text{ cm}^2$ $A_{triangle} = \frac{1}{2} \times 3 \times 2 = 3\text{ cm}^2$
4 Add together the areas to find the total area of the composite shape. Remember to include the appropriate unit.	total area $= 21\text{ cm}^2 + 1\text{ cm}^2 + 3\text{ cm}^2 = 25\text{ cm}^2$

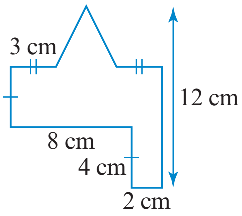
UNDERSTANDING AND FLUENCY

2 Calculate the area of each composite shape.

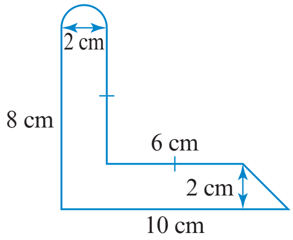
a



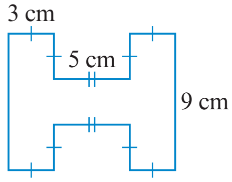
b



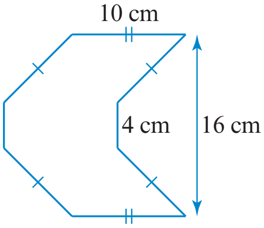
c



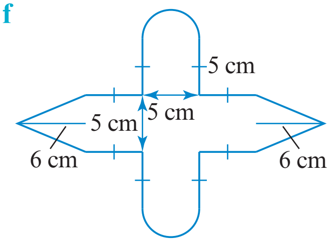
d



e

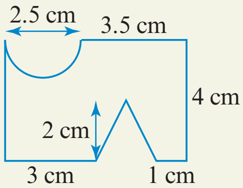


f



EXAMPLE 7D-2 Calculating area of a composite shape (subtraction method)

Calculate the area of this composite shape.



THINK

1 Redraw the composite shape with dotted lines to split it into parts for which you can calculate the area easily.

2 Find any missing dimensions for each individual shape and label these on your figure.
The rectangle has a length of:
2.5 cm + 3.5 cm = 6 cm.
The triangle therefore has a base of:
6 cm − 3 cm − 1 cm = 2 cm.

3 Calculate the areas of the individual shapes that make up the composite shape.

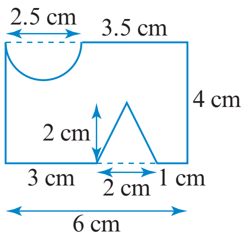
$A_{\text{rectangle}} = lw$

$A_{\text{triangle}} = \frac{1}{2}bh$

$A_{\text{semicircle}} = \frac{1}{2}\pi r^2$

4 Subtract the smaller areas from the main area to find the total area of the composite shape. Remember to include the appropriate unit.

WRITE



$A_{\text{rectangle}} = 6 \times 4$
$= 24 \text{ cm}^2$

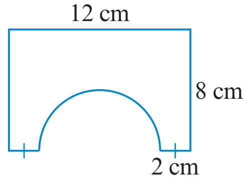
$A_{\text{triangle}} = \frac{1}{2} \times 2 \times 2$
$= 2 \text{ cm}^2$

$A_{\text{semicircle}} = \frac{1}{2} \times \pi \times 1.25^2$
$\approx 2.45 \text{ cm}^2$

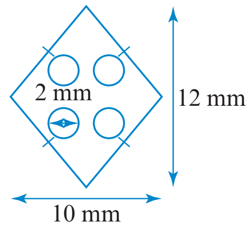
total area $= 24 \text{ cm}^2 - 2 \text{ cm}^2 - 2.45 \text{ cm}^2$
$= 19.55 \text{ cm}^2$

3 Calculate the area of each composite shape.

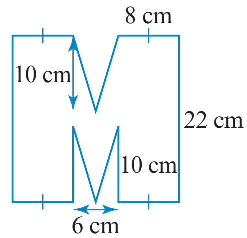
a



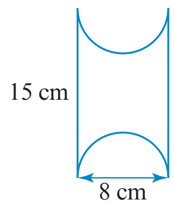
b



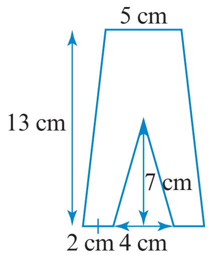
c



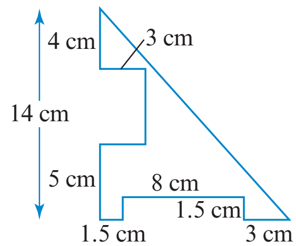
d



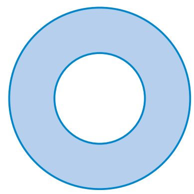
e



f



4 This figure is known as an **annulus** (plural annuli).



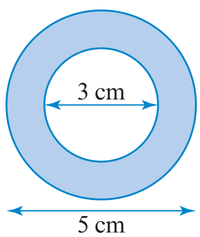
a What is an annulus?

b Explain how you find the area of this annulus.

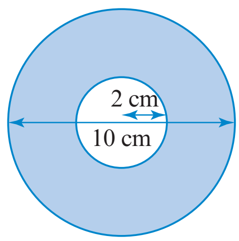
c Find the area of this annulus if it has an outer diameter of 16 cm and an inner diameter of 9 cm.

5 Calculate the area of each annulus.

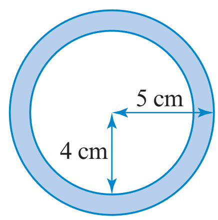
a



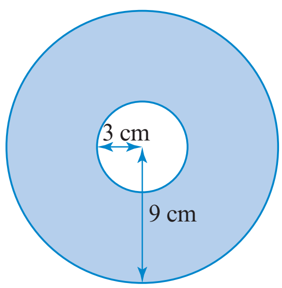
b



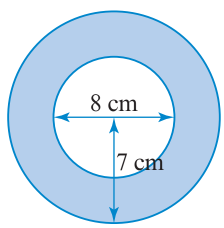
c



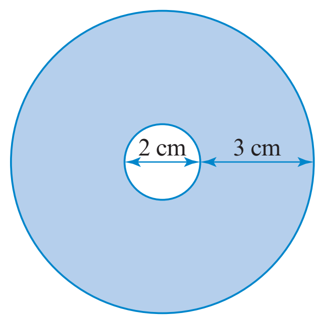
d



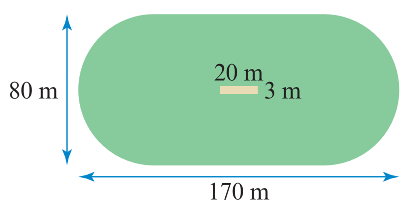
e



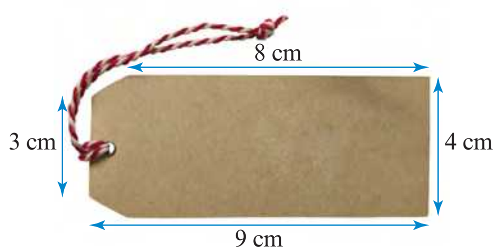
f



6 What area of grass needs to be mowed on this oval, assuming that the cricket pitch is artificial and does not require mowing?



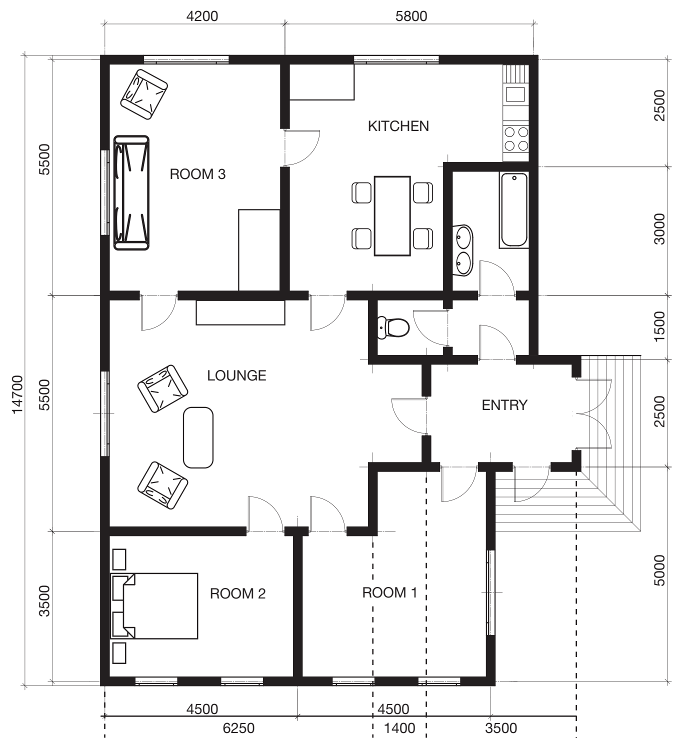
7 Emily makes and sells custom tags through the website Etsy.



a If she needs to make 550 for an order, what is the total area of cardboard required to make the tags if they have the dimensions shown?

b Why might Emily need to buy more than this amount of cardboard in order to make the tags?

8 Shannon wants to lay new carpet in his house. Consider the floor plan. All measurements are in millimetres.

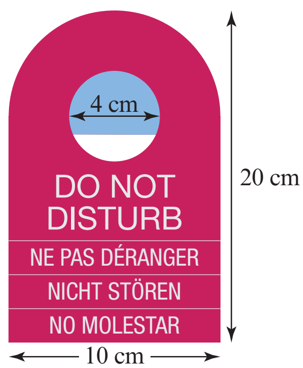


a What area of the house would he need to cover if everything except the bathroom, kitchen and toilet was to be carpeted? Write this in square metres. (Hint: you may find it easier to convert the measurements to metres before working with them.)

b How much would it cost him to get the house carpeted if the carpet he wanted cost $45/m^2?

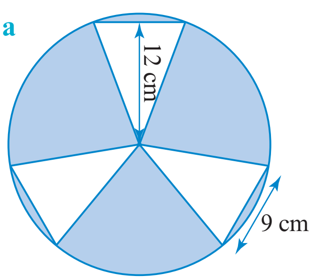
c How much money would he save if he instead chose a carpet worth $30/m^2?

9 Find the amount of cardboard required to make this 'Do not disturb' sign.

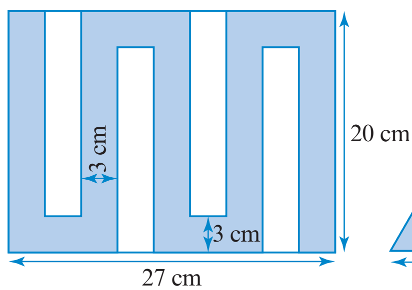


10 Calculate the shaded area of each shape.

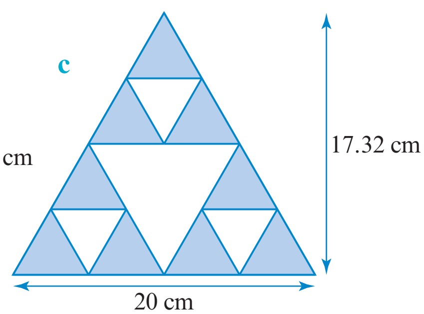
a



b

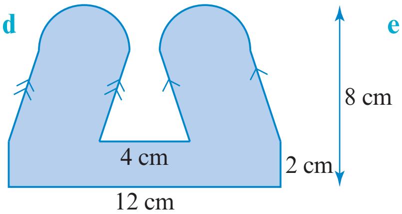


c

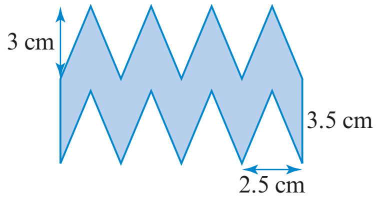


PROBLEM SOLVING AND REASONING

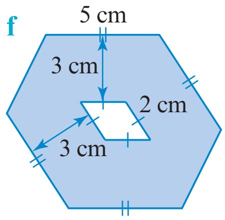
d



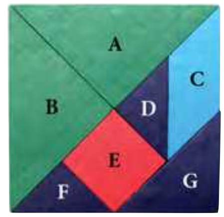
e



f

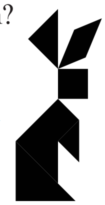


11 The tangram is an ancient Chinese puzzle that consists of seven pieces (usually stored in a square as shown) that can be rearranged to form a variety of shapes.

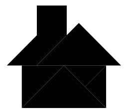


- **a** What is the area of each individual piece in this tangram square if the entire square has side lengths of 10 cm?
- **b** Use all seven pieces to form each figure. You may find it useful to draw and cut out the pieces to help you solve these puzzles.

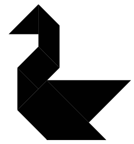
i



ii



iii



- **c** State the area of each figure you formed in part **b**.
- **d** Use the Internet or other means to find and solve at least five other tangram puzzles.

12 An arrow can be constructed using a rectangle and a triangle. Draw three different arrows that could have an area of 50 cm^2.

13 Create a composite shape with an area of 100 cm^2 using:

- **a** two different shapes
- **b** three different shapes
- **c** four shapes in total.

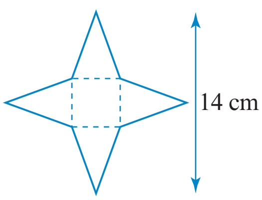
14 Find the area of an annulus with an outer circumference of 35 cm and an inner circumference of 25 cm.

15 A circular skirt is cut from a circle of material with a hole for the waist. If a particular circular skirt had an inner circumference of 70 cm and was 60 cm in length when it was worn, what was the area of material used to create the skirt?

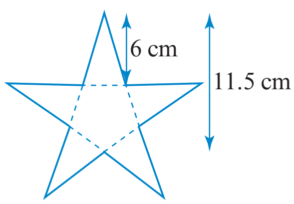
CHALLENGE

16 You can calculate the area of 'regular' stars using your understanding of composite shapes. If these stars are based on regular polygons with a side length of 4 cm, find the area contained within each star. (Hint: look at Exercise 7C questions **15** and **16** on pages 326–7 to help you find the area of the polygon that the stars are based on.)

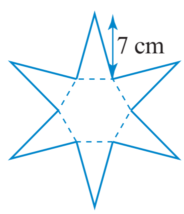
a



b



c



Reflect

What common composite shapes do you encounter in your everyday life?

7E Surface area

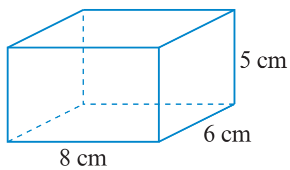
Start thinking!

1 What is the surface area of an object?

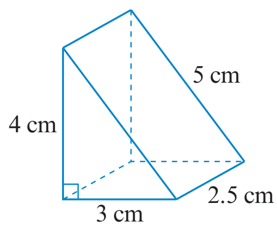
2 How do you find the surface area of an object?

3 Name these 3D objects.

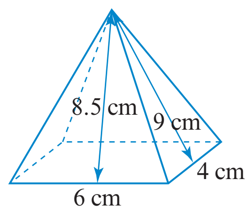
i



ii



iii



4 **Draw a net** for each 3D object, labelling each **face** with its dimensions.

5 For each 3D object:

- **a** state the number of faces it has
- **b** draw each face individually, labelling its dimensions
- **c** next to each face, write the formula used to find its area
- **d** calculate the area of each face
- **e** add all the areas to find the **total surface area** (TSA).

6 Can you see a shortcut to calculate the surface area of any or all of these objects? (Hint: are any faces the same?)

7 Write a formula to find the total surface area for each of these 3D objects.

8 Repeat the activity by first drawing another of each of these three 3D objects. Does your formula work for the objects you drew?

9 How does drawing a net help you to calculate the total surface area of an object?

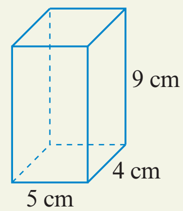
KEY IDEAS

- The total surface area (TSA) of a 3D object is the total area of the outer surface of the prism.
- The total surface area of **prisms** and **pyramids** is the sum of the areas of each face of the object.

EXERCISE 7E Surface area

EXAMPLE 7E-1 Calculating surface area of a rectangular prism

Calculate the surface area of this rectangular prism.



THINK

1 Identify the number of faces the rectangular prism has. It has six: three pairs of identical rectangular faces.

2 Calculate the area of each face.

3 Add the areas to find the total surface area. Include the appropriate unit.

WRITE

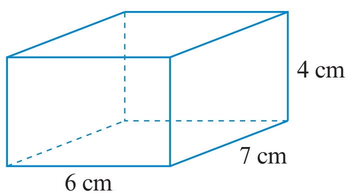
Area of:
front rectangular face = $5 \times 9 = 45$ cm^2
top rectangular face = $5 \times 4 = 20$ cm^2
side rectangular face = $4 \times 9 = 36$ cm^2

TSA = 2×45 cm^2 + 2×20 cm^2 + 2×36 cm^2
= 202 cm^2

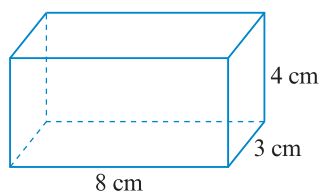
UNDERSTANDING AND FLUENCY

1 Calculate the surface area of each rectangular prism.

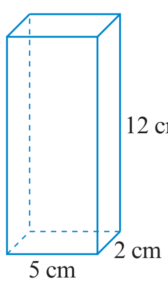
a



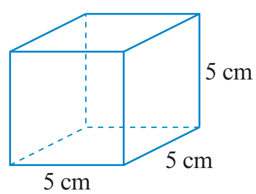
b



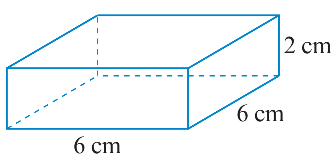
c



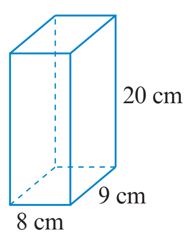
d



e

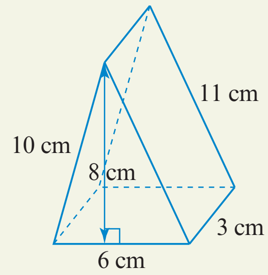


f



EXAMPLE 7E-2 Calculating surface area of a triangular prism

Calculate the surface area of this triangular prism.

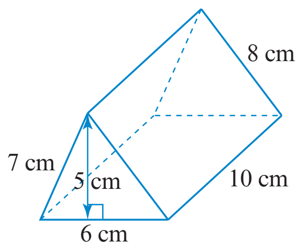


THINK	WRITE
1 Identify the number of faces the triangular prism has. It has five: three rectangular faces and one pair of identical triangular faces. **2** Calculate the area of each face.	Area of: bottom rectangular face = $6 \times 3 = 18\text{ cm}^2$ slanting rectangular face 1 = $11 \times 3 = 33\text{ cm}^2$ slanting rectangular face 2 = $10 \times 3 = 30\text{ cm}^2$ triangular face = $\frac{1}{2} \times 6 \times 8 = 24\text{ cm}^2$
3 Add the areas to find the total surface area. Include the appropriate unit.	TSA = $18\text{ cm}^2 + 33\text{ cm}^2 + 30\text{ cm}^2 + 2 \times 24\text{ cm}^2$ $= 129\text{ cm}^2$

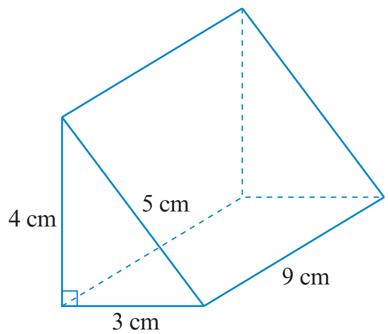
UNDERSTANDING AND FLUENCY

2 Calculate the surface area of each triangular prism.

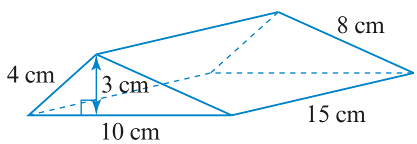
a



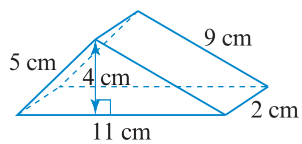
b



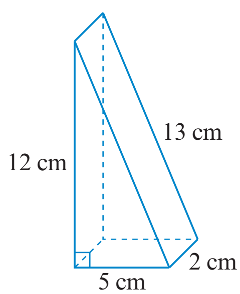
c



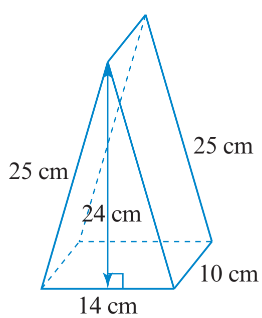
d



e

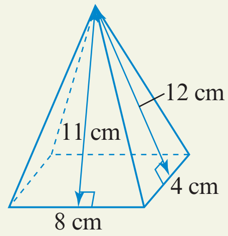


f



EXAMPLE 7E-3 Calculating surface area of a pyramid

Calculate the surface area of this pyramid.



THINK

1 Identify the number of faces the pyramid has. It has five: one rectangular face and two pairs of identical triangular faces.

2 Calculate the area of each face.

3 Add the areas to find the total surface area. Include the appropriate unit.

WRITE

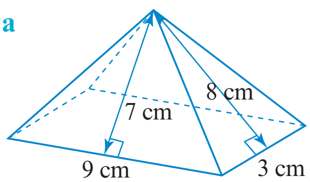
Area of:
rectangular face = $8 \times 4 = 32\ \text{cm}^2$
front triangular face = $\frac{1}{2} \times 11 \times 8 = 44\ \text{cm}^2$
side triangular face = $\frac{1}{2} \times 12 \times 4 = 24\ \text{cm}^2$

TSA = $32\ \text{cm}^2 + 2 \times 44\ \text{cm}^2 + 2 \times 24\ \text{cm}^2$
$= 168\ \text{cm}^2$

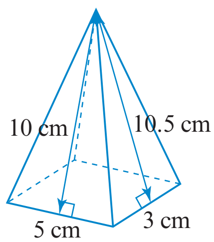
UNDERSTANDING AND FLUENCY

3 Calculate the surface area of each pyramid.

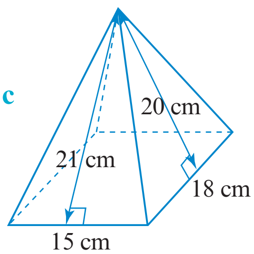
a



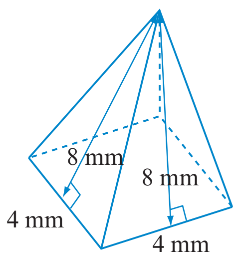
b



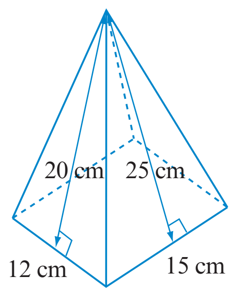
c



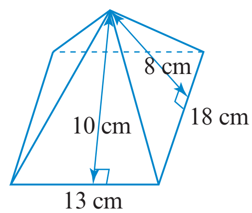
d



e



f



4 Find a cardboard box (in the form of a rectangular prism) in your classroom or at home.

a Measure its dimensions and draw a labelled diagram of it.

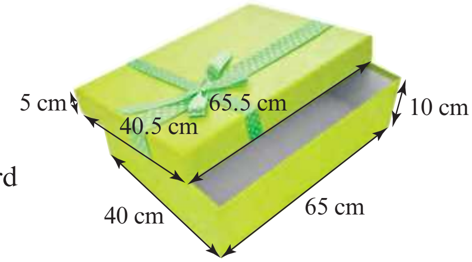
b Find its total surface area.

With permission, break the box down into its net.

c Draw the net and label its dimensions.

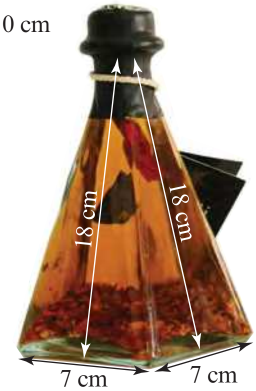
d Find its surface area.

e Do your answers to parts **b** and **d** match? Explain. Why do you think this happens?

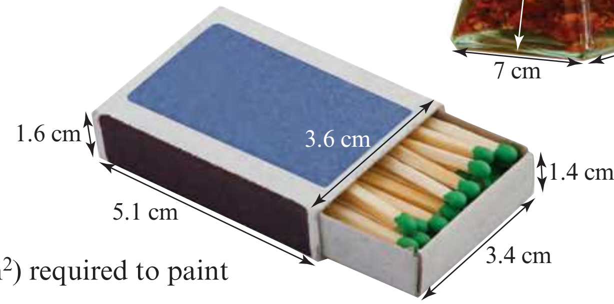


5 Find the total amount of cardboard required to create this box.

6 How much glass is required to create this container of chilli oil? Assume that it forms a perfect pyramid underneath its cap.



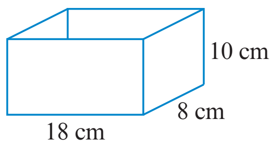
7 How much cardboard is used to make a matchbox?



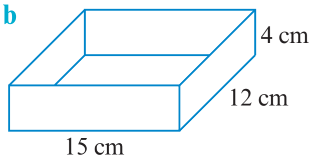
8 Find the surface area of these boxes if you were to consider:

- **i** the amount of material required to build the boxes
- **ii** the amount of paint (in cm^2) required to paint all surfaces of the boxes.

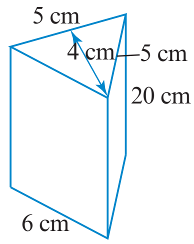
a



b



c



9 A block of butter is in the shape of a rectangular prism.

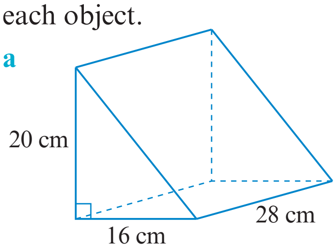
- **a** If the block was 7 cm wide, 16 cm long and 6 cm high, calculate its TSA.
- **b** If the block was cut in half (that is, to give two pieces 8 cm long), what is the surface area:
  - **i** of each piece? **ii** in total?
- **c** How is your answer to part **b ii** different from your answer to part **a**?
- **d** Imagine that you cut the block of butter into 1-cm cubes. What would be the total surface area of the butter now?
- **e** If somebody wanted to melt butter quickly, what would you recommend to them?

10 Billy is having his room painted. His room is 6 m long, 5 m wide and 2.5 m tall and has a large window on one wall that measures 150 cm × 95 cm. The walls are to be painted blue and the ceiling painted cream.

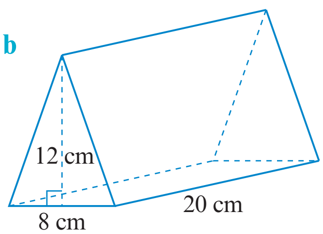
- **a** What area is to be painted blue and what area is to be painted cream?
- **b** How many litres of each colour paint is needed if 1 L of paint covers about 15 m^2 and the room will take two coats?

11 Use your understanding of Pythagoras' Theorem to calculate the surface area of each object.

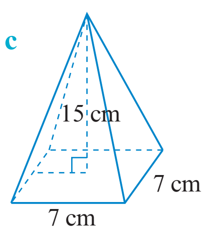
a



b



c



PROBLEM SOLVING AND REASONING

12 Photographs can be enlarged and put onto canvas. The majority of the photograph is displayed on the front face but the edges of the photograph are also on the edges of the canvas. If a canvas piece is 40 cm long, 50 cm wide and has edges 3.5 cm 'deep', what is the area of the photograph that is being displayed? (Hint: don't forget about the corners that have been folded down during production of the canvas.)

13 A cube has a surface area of 600 cm^2.

a What is the surface area of each face?

b What is the side length of the cube?

c Write a formula that will help you determine the side length of any cube if you know its surface area.

14 A rectangular prism has a surface area of 340 cm^2.

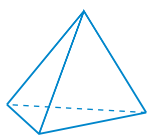
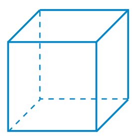
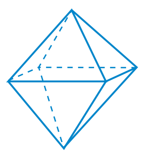
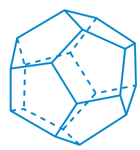
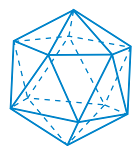
a Give a set of possible dimensions it could have.

b Explain why knowing the surface area of a cube is enough to determine its side lengths but knowing the surface area of a rectangular prism is not.

c If you know the length of the prism, is this enough for you to be able to determine the width and height of the prism? Explain.

CHALLENGE

15 A **polyhedron** (plural polyhedra) is a 3D object that contains only polygons. Regular polyhedra contain only one type of regular polygon. There are five regular polyhedra.

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
				

For each of these polyhedra, find:

a the number of faces

b which regular polygon it contains

c its total surface area, if its side lengths are 2 cm. (Hint: you may need to use Pythagoras' Theorem for some of these.) For the dodecahedron, assume that its polygon height is 2.75 cm. (See Exercise 7C question **15** on page 326 to see how to calculate the area of this polygon.)

16 A pyramid doesn't have to have a rectangular (or triangular) base. Its base can be any polygon. Use your understanding of Pythagoras' Theorem and polygon areas (see Exercise 7C question **15** on page 326) to find the surface area of these pyramids.

a a hexagonal pyramid with base side lengths of 6 cm and a pyramid height of 10 cm

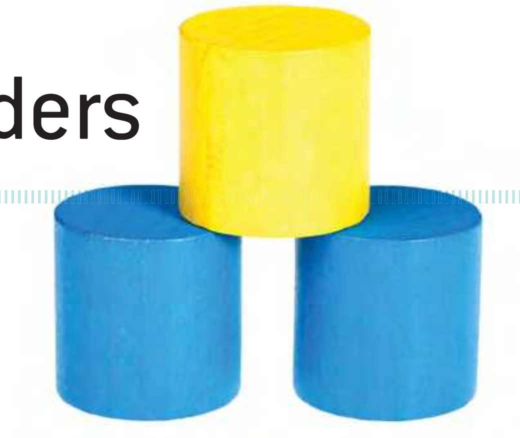
b an octagonal pyramid with base side lengths of 5 cm, base width of 12 cm and a pyramid height of 8 cm

c a nonagonal pyramid with base side lengths of 10 cm, base width of 27.5 cm and a pyramid height of 20 cm

Reflect

Why is it useful to draw or visualise the net of an object when finding its surface area?

7F Surface area of cylinders



Start thinking!

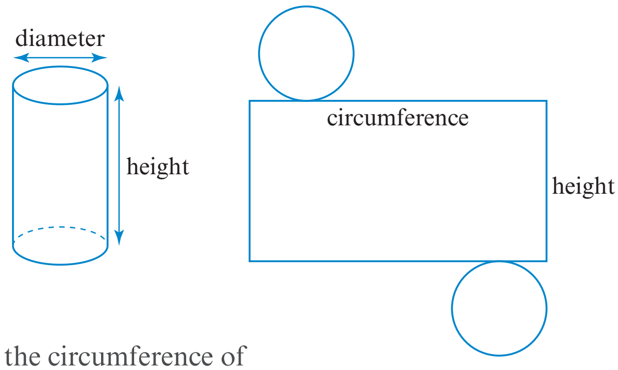
A **cylinder** is a 3D object that has a circular base or cross section.

1 How many faces does a cylinder have?

2 Two of these faces are identical. What shape are they?

3 The third face wraps around the outside of the circles. Imagine that you can 'unwrap' this third face. What shape is it? Roll up a piece of paper to help you if you can't visualise it.

Consider this cylinder and its net.



4 How would you calculate the area of its circular faces?

5 Explain why the width of the rectangle is equal to the height of the cylinder.

6 How does the length of the rectangle relate to the circles?

7 Explain why the length of the rectangle is equal to the circumference of the circles.

8 What is the formula for the circumference of a circle if you have the radius?

9 Explain why the area of the rectangular face is equal to $2\pi rh$.

10 Explain why the total surface area (TSA) of a cylinder is equal to $2\pi rh + 2\pi r^2$.

11 If the cylindrical blocks shown in the photograph had a radius of 2 cm and a height of 3 cm, what would be the total area necessary to paint these three blocks?

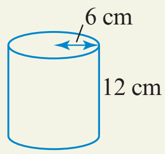
KEY IDEAS

- The surface area of a cylinder is formed by a rectangle and two identical circles.
- To calculate the surface area of a cylinder, use the formula TSA = $2\pi rh + 2\pi r^2$, where $2\pi rh$ relates to the area of the rectangle and $2\pi r^2$ relates to the area of the two circles.

EXERCISE 7F Surface area of cylinders

EXAMPLE 7F-1 Drawing the net of a cylinder

Draw the net of this cylinder, labelling dimensions accurately.

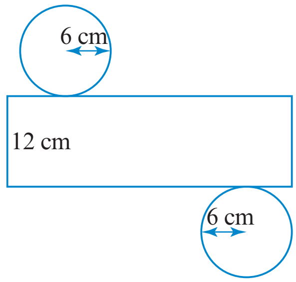


THINK

1 Draw the net of the cylinder, remembering that it consists of a rectangle and two circles.

2 Label the radius of the circles and the height of the rectangle.

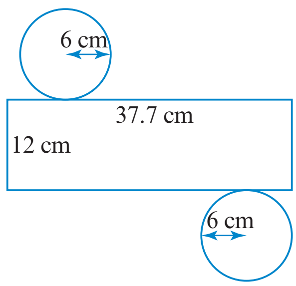
WRITE



3 To calculate the length of the rectangle, use the formula $C = 2\pi r$.

$$C = 2 \times \pi \times 6$$
$$= 12\pi$$
$$\approx 37.7$$

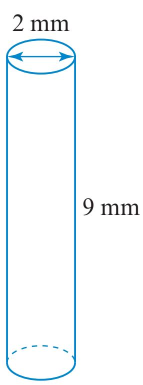
4 Label the length of the rectangle to complete your net.



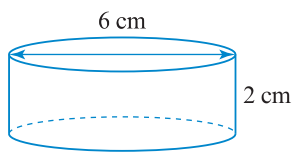
UNDERSTANDING AND FLUENCY

1 Draw a net for each cylinder, labelling the dimensions accurately.

a

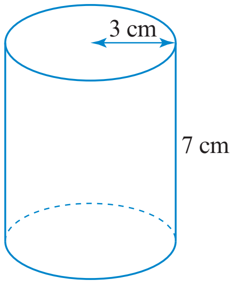


b

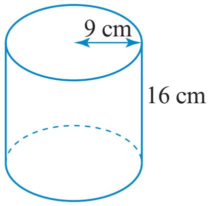


UNDERSTANDING AND FLUENCY

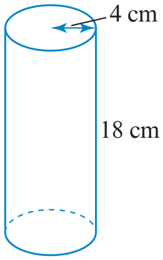
c



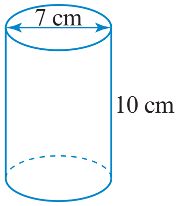
d



e

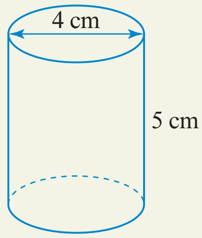


f



EXAMPLE 7F-2 Calculating surface area of a cylinder

Calculate the surface area of this cylinder.



THINK	WRITE
1 Write the formula for surface area of a cylinder.	$A = 2\pi rh + 2\pi r^2$
2 Identify the measurements for radius (r) and height (h) and substitute these into the formula.	$r = D \div 2 = 4 \div 2 = 2$ cm $h = 5$ cm So, $A = 2\pi \times 2 \times 5 + 2\pi \times 5^2$ $= 20\pi + 50\pi$ $= 70\pi$
3 Calculate the result using the π button on your calculator and round your answer to two decimal places. Remember to include the appropriate unit.	$= 219.911\,485\,8 \ldots$ ≈ 219.91 cm^2

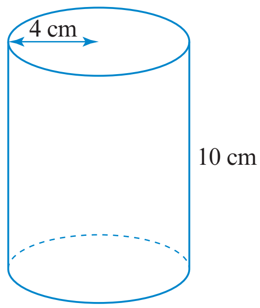
2 Consider this cylinder.

a Draw a net and label its dimensions.

b Calculate the area of the rectangular part of the net.

c Calculate the area of one circular part of the net.

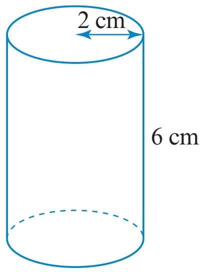
d Calculate its total surface area.



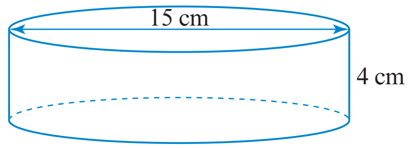
UNDERSTANDING AND FLUENCY

3 Calculate the total surface area for each cylinder.

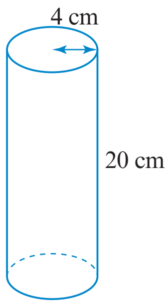
a



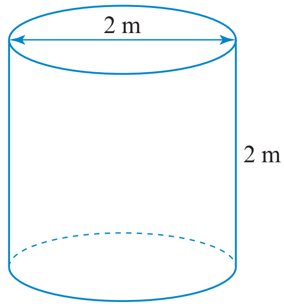
b



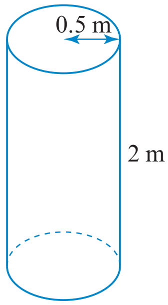
c



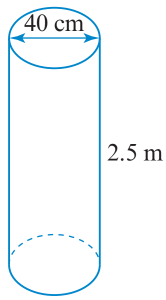
d



e



f

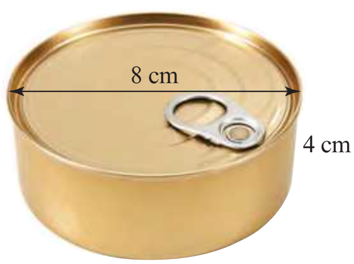


PROBLEM SOLVING AND REASONING

4 Calculate the area of paper required to label these containers.

a

b



c

5 Calculate the outer surface area of this box and its lid, if the box is 22 cm tall and has a radius of 15 cm and its lid is 4 cm tall and has a diameter of 31 cm.

6 A cylindrical pool is 2 m deep and has a radius of 6.5 m. How much would it cost to paint its interior if it needed two coats of paint and the special paint required cost $50 per litre? Assume that 1 L covers 15 m^2.

7 Maria has the choice of two paint rollers. One roller is 25 cm long and has a radius of 4 cm. The other roller is 30 cm long and has a diameter of 6 cm. Assuming they have the same absorbency, which roller would need to be re-dipped in paint the least often?

8 How much material is needed to make this lampshade if it is 25 cm tall with a 1 cm overlap on top and bottom, and has a diameter of 20 cm with an overlap of 3 cm on its circumference?

9 If you double the height of a cylinder, do you double its surface area? Explain, using an example.

10 a Explain why, if a cylinder has a height that is equal to the length of the radius, its surface area can be calculated using the formula TSA = $4\pi r^2$.

b Write a simplified formula to calculate the surface area of a cylinder if its height is:

i double the length of the radius

ii half the length of the radius

iii three times the length of the radius.

c Use the four formulas from parts **a** and **b** to calculate the TSA of the four cylinders described, if they each have a radius of 5 cm.

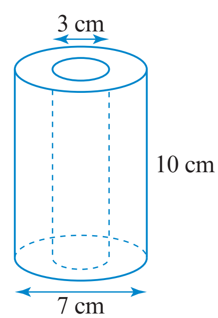
11 Write a formula that will calculate the outer surface area of an open cylinder.

12 A cylinder has a surface area of 200 cm^2. Give two possible sets of dimensions that it could have.

13 A cylinder has a radius of 4 cm and a surface area of 300 cm^2. What is its height to the nearest centimetre?

14 a What is the surface area of this tube?

b Lucian answered part **a** as 296.88 cm^2 and Curtis said it was 282.74 cm^2. Explain where they went wrong.



PROBLEM SOLVING AND REASONING

15 The surface area of a **sphere** can be calculated using the formula TSA = $4\pi r^2$. Calculate the surface area of a sphere that has:

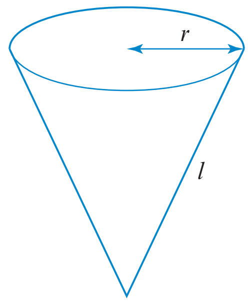
a radius 4 cm

b radius 7 cm

c diameter 10 cm

d circumference 20 cm.

16 The surface area of a **cone** can be calculated using the formula TSA = $\pi r(r + l)$ or $\pi r^2 + \pi rl$, where l is the slant length of the cone.



a Draw the net of this cone and label it with its dimensions. (Hint: the net will involve a large sector of a circle.)

b Calculate the surface area of a cone that has:

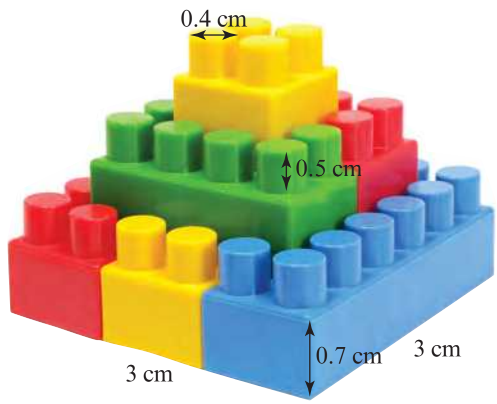
i radius 2 cm and slant length 9 cm

ii radius 6 cm and slant length 13 cm

iii diameter 10 cm and slant length 20 cm

iv circumference 20 cm and slant length 50 cm.

17 Use Pythagoras' Theorem to calculate the surface area of a cone that has a radius of 5 cm and a height of 15 cm.



CHALLENGE

18 Write a formula that will calculate the height of a cylinder given its surface area and radius.

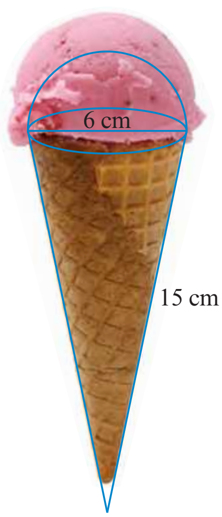
19 Calculate the surface area of the structure shown in the image on the right.

20 Calculate the surface area of the ice cream cone pictured, assuming that the ice cream forms a perfect hemisphere on top of the cone and both the ice cream and the cone share the same diameter.

i Calculate the surface area of the outside of the cone.

ii Calculate the surface area of the ice cream. (Hint: what fraction of a whole sphere is a hemisphere?)

iii Add the two areas to find the total surface area of the ice cream cone.



Reflect

How is the surface area of a cylinder related to its radius?

7G Volume

Start thinking!

The **volume** of an object is the amount of space it occupies.
One way to find volume is to split an object into cubes and then count them.

1 Draw a rectangular prism, label its dimensions and then split it into cubes.
2 State the volume of your rectangular prism by counting the total number of cubes it contains.
3 How did you actually count the number of cubes? Did you count every single one or did you use another method?

An easier way than counting every single cube is to think of a prism as containing layers.

4 How many cubes in the bottom layer of your prism?
5 How many layers high is your prism?
6 How can you use the information in questions **4** and **5** to find the volume of your prism in an easier way than by counting each individual cube?

Any prism can be thought of as containing layers. The general rule to find the volume of the prism is to find the area of its base (that is, the first layer) and then to multiply that by its height (that is, the number of layers). In mathematical terms, this is written as $V = AH$.

7 Write down what V, A and H represent in this formula.
8 What is the formula for the area of a rectangle?
9 Write the formula for the volume of a rectangular prism by substituting the formula you named in question **8** in place of A in $V = AH$.
10 What is the formula for the area of a triangle?
11 Use your answer from question **10** to write the formula for the volume of a triangular prism.

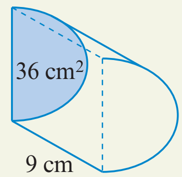
KEY IDEAS

- The volume of any prism can be found using the formula $V = AH$, where A is the area of the base and H is the height of the prism.
- The formula for an individual prism can be found by substituting in the formula for the area of its base. For example:
  - Volume of a rectangular prism $V = lwH$
  - Volume of a triangular prism $V = \frac{1}{2}bhH$
- A cylinder is not a prism (as it has curved edges) but the volume can be calculated using the formula for a prism and substituting $A = \pi r^2$ for the area of its base.
  - Volume of a cylinder $V = \pi r^2 H$
- **Capacity** is the amount of liquid a container can hold.

EXERCISE 7G Volume

EXAMPLE 7G-1 Calculating volume of a 3D object

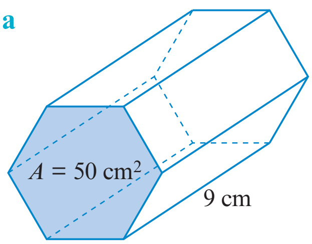
Calculate the volume of this 3D object.

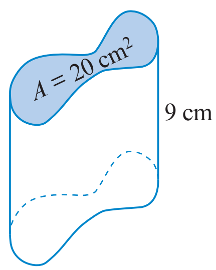


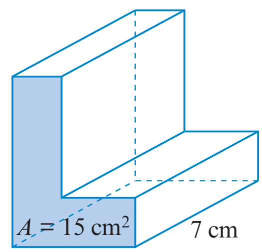
THINK	WRITE
1 Write the formula for the volume of a prism.	$V = AH$
2 Identify A (area of base is given).	$A = 36\text{ cm}^2$
3 Identify H and check that it is in centimetres.	$H = 9\text{ cm}$
4 Substitute the values for A and H into the formula and calculate the result. Remember to include the appropriate unit.	$V = 36 \times 9$ $= 324\text{ cm}^3$

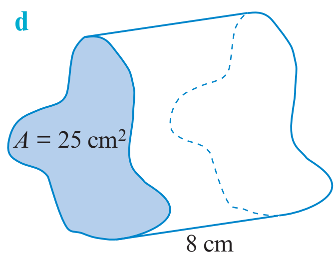
UNDERSTANDING AND FLUENCY

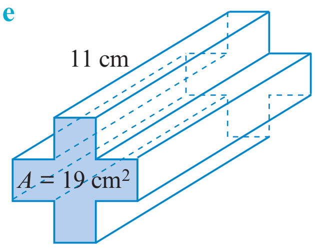
1 Calculate the volume of each 3D object.

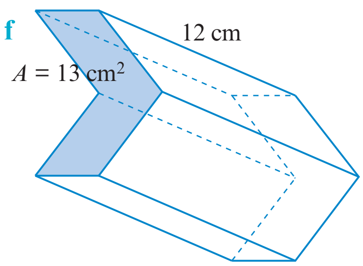
a


b


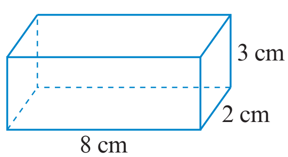
c


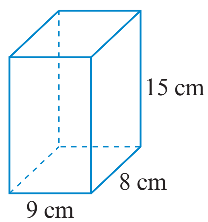
d


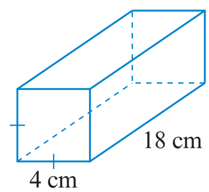
e


f


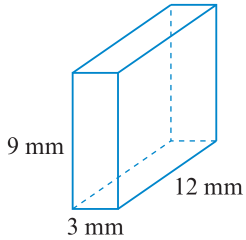
2 Calculate the volume of each rectangular prism.

a


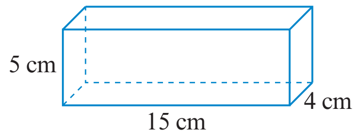
b


c


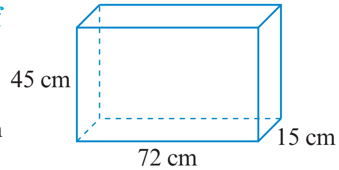
d



e

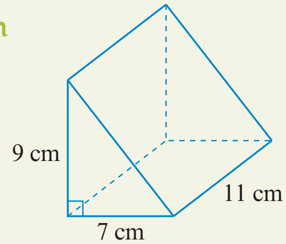


f



EXAMPLE 7G-2 Calculating volume of a triangular prism

Calculate the volume of this triangular prism.

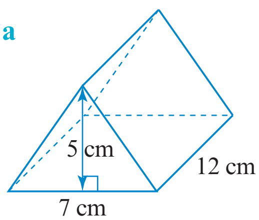


THINK	WRITE
1 Write the formula for the volume of a prism.	$V = AH$
2 Identify the shape of the base (a triangle) and write the appropriate formula.	$A = 36\text{ cm}^2$
3 Identify b, h and H. Check that they are all in the same units.	$b = 7$ cm, $h = 9$ cm and $H = 11$ cm
4 Substitute the values for b, h and H into the formula and calculate the result. Remember to include the appropriate unit.	$V = \frac{1}{2} \times 7 \times 9 \times 11$ $= 346.5\text{ cm}^3$

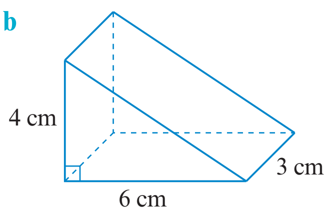
UNDERSTANDING AND FLUENCY

3 Calculate the volume of each triangular prism.

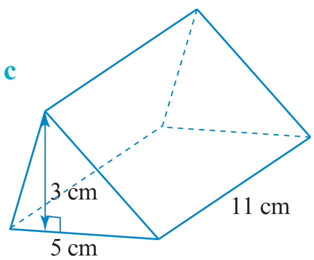
a



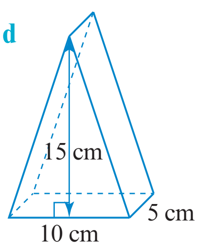
b



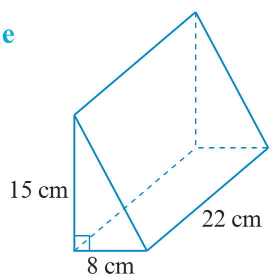
c



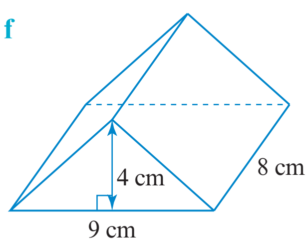
d



e

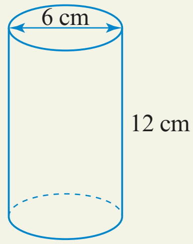


f



EXAMPLE 7G-3 Calculating volume of a cylinder

Calculate the volume of this cylinder.

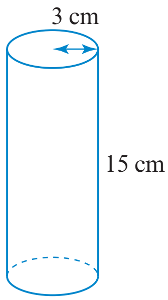


THINK	WRITE
1 Write the formula for the volume of a prism.	$V = AH$
2 Identify the shape of the base (a circle) and write the appropriate formula.	$V = \pi r^2 H$
3 Identify r and H and substitute them into the formula.	$r = D \div 2 = 6 \div 2 = 3$ cm $H = 12$ cm So, $V = \pi \times 3^2 \times 12$ $= 108\pi$
4 Calculate the result using the π button on your calculator and round your answer to two decimal places. Remember to include the appropriate unit.	$= 339.292\,006\,6\ldots$ ≈ 339.29 cm^3

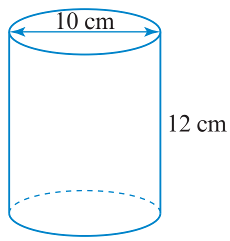
UNDERSTANDING AND FLUENCY

4 Calculate the volume of each cylinder.

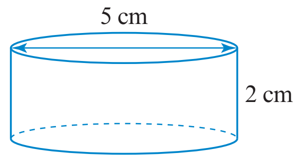
a



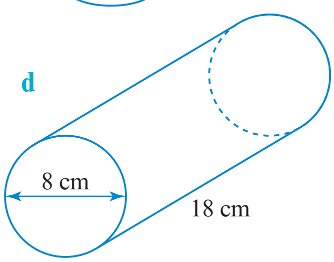
b



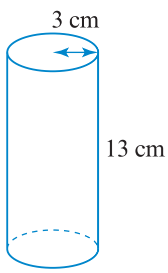
c



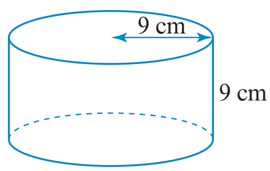
d



e

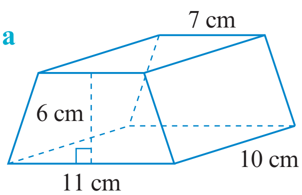


f

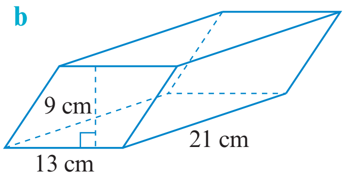


5 Calculate the volume of each prism.

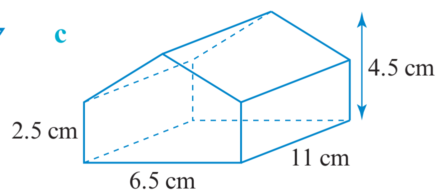
a



b



c



6 A tin of condensed milk had a diameter of 7 cm and a height of 12 cm.

- **a** What is the volume of condensed milk in the tin?
- **b** If condensed milk has a density of 1.3 g/cm^3, what is the net weight of this tin?

7 In Exercise 7A question **11** on page 315, you explored the process of converting between cubic units.

- **a** Explain why 1 cm^3 = 1000 mm^3, not 10 mm^3. Draw a 1-cm cube and split it into millimetres if it helps.
- **b** Explain why, when you convert between cubic units, you multiply or divide by the cube of the conversion factor.
- **c** Write your answers to question **5** in: **i** cubic millimetres **ii** cubic metres.
- **d** Why might it be easier to convert length measurements into the desired unit before calculating volume?

8 A \$1 coin has a diameter of 2.5 cm and a thickness of 3 mm.

- **a** How much metal is in a stack of 30 \$1 coins? Write this as both cubic millimetres and cubic centimetres.
- **b** How many coins could be made out of 1000 cm^3 of metal?
- **c** How much metal would be left over from part **b**?

9 A prism has a volume of 240 cm^3.

- **a** Give a possible set of dimensions for this prism, if it is:
  - **i** a rectangular prism **ii** a triangular prism.
- **b** If the prism is irregular and has a height of 8 cm, what is the area of its base?
- **c** If the prism is rectangular and has a base area of 60 cm^2:
  - **i** find its height **ii** give two possible sets of dimensions for it.

10 A cylinder has a volume of 150 cm^3.

- **a** Give two possible sets of dimensions that it could have.
- **b** If it has a radius of 6 cm, find its height.
- **c** If it has a height of 6 cm, find its radius.

11 Capacity is the amount of liquid that a container can hold.

- **a** How is this different from volume?
- **b** 1 cm^3 holds 1 mL. What is the volume of a container that holds 1 L?
- **c** What is the capacity of a container that is 1 m^3 in volume?
- **d** Write the amount of condensed milk in the tin from question **6** in millilitres.

> **NOTE** Be careful when converting units of volume!

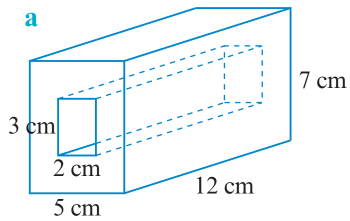
12 Oliver has two cylindrical glasses. The first glass has a diameter of 6.5 cm and a height of 22 cm. The second glass has a radius of 5 cm and a height of 10 cm. Which glass holds more, and by how much? Write your answer in millilitres.

13 How much water can a cylindrical bottle cap hold if it has a diameter of 2.8 cm and a height of 1.1 cm?

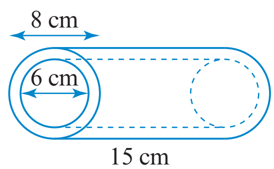
PROBLEM SOLVING AND REASONING

14 Find the volume of these hollow prisms.

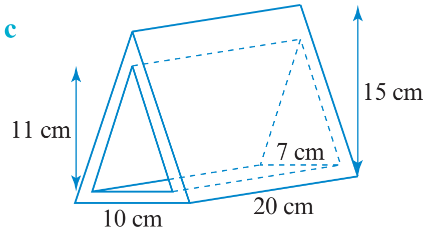
a



b



c



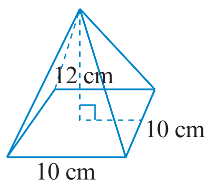
15 A candle has a diameter of 6 cm and a height of 16 cm. If the wick can be thought of as a cylinder with a diameter of 3 mm, what is the amount of wax contained within a single candle? Write your answer in both cubic centimetres and in millilitres.

16 The volume of any pyramid is equal to $\frac{1}{3}$ the volume of the corresponding prism. For example, the volume of a rectangular-based pyramid can be found using the formula $V = \frac{1}{3}lwH$. This can be written as $V = \frac{1}{3}AH$.

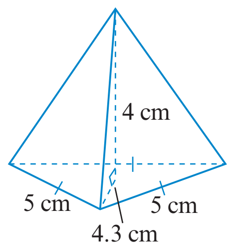
For each pyramid shown below:

i write the formula you would use to find its volume **ii** calculate its volume.

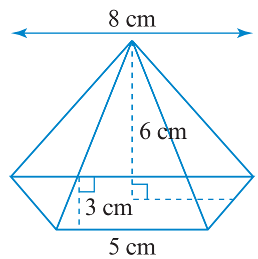
a



b



c



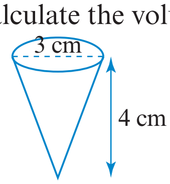
17 A cone can be thought of as the 'pyramid' that relates to the cylinder.

a Why is a cone not a true pyramid?

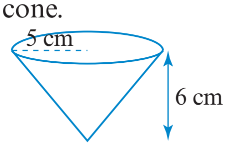
b Use the formula for the volume of a cylinder to write a formula that will calculate the volume of a cone.

c Calculate the volume of each cone.

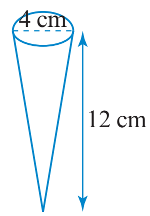
i



ii



iii



18 The volume of a sphere can be calculated using the formula $V = \frac{4}{3}\pi r^3$.

Calculate the volume of a sphere that has:

a radius 3 cm **b** radius 6 cm

c diameter 8 cm **d** circumference 21 cm.

CHALLENGE

19 How much ice-cream is held by a cone that has a diameter of 5 cm and height of 12 cm? Assume that the ice-cream fills the entire cone and has a perfect half sphere on its top. Write your answer in both cubic centimetres and in millilitres.

Reflect

How is the volume of an object related to its base?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

length	diameter	trapezium	pyramid
mass	formula	triangle	cylinder
time	arc	rhombus	sphere
conversion factor	sector	circle	cone
scientific notation	radii	Pythagoras' Theorem	volume
perimeter	rectangle	composite shapes	
circumference	parallelogram	surface area	
radius	kite	prism	

MULTIPLE-CHOICE

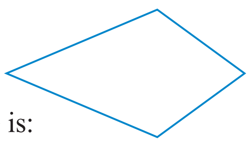
7A **1** Which measurement is the smallest?

A 2.76×10^2 g **B** 2.76×10^{-2} g
C 2.76 g **D** 2.76×10^1 g

7B **2** The diameter of a bicycle wheel is 715 mm. What is the circumference of the wheel to the nearest centimetre?

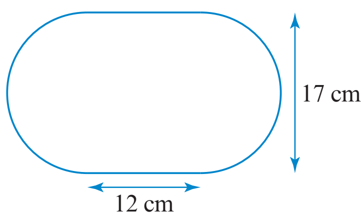
A 2246.24 m **B** 4.49 m
C 224.62 m **D** 2.25 m

7C **3** The formula used to calculate the area of this shape is:



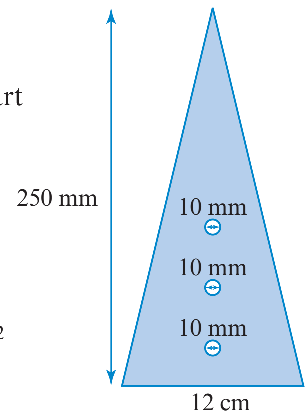
A $A = \frac{1}{2}xy$ **B** $A = \frac{1}{2}(a + b)h$
C $A = \frac{1}{2}bh$ **D** $A = bh$

7D **4** The area of this shape is closest to:



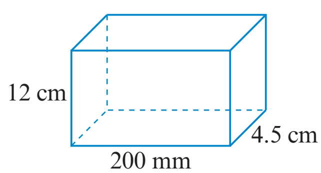
A 77 cm^2 **B** 204 cm^2
C 431 cm^2 **D** 1112 cm^2

7D **5** The shaded part of this figure has an area closest to:



A 140.58 cm^2
B 147.64 cm^2
C 149.21 cm^2
D 150 cm^2

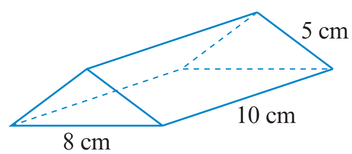
7E **6** The surface area of this rectangular prism is:



A 6708 cm^2 **B** 1080 cm^2
C 768 cm^2 **D** 678 cm^2

7G **7** The volume of this prism is:

A 200 cm^3
B 400 cm^3
C 240 cm^3
D 120 cm^3



SHORT ANSWER

Where appropriate, write your answers correct to two decimal places.

7A **1** Perform these conversions.

- **a** 35 mm into centimetres
- **b** 25 752 m into kilometres
- **c** 3.75 kg into grams
- **d** $4\frac{1}{2}$ h into minutes

7A **2** Perform these conversions.

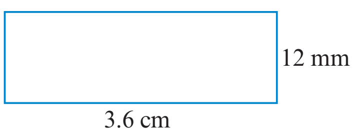
- **a** 55 mm into metres
- **b** 9.75 km into centimetres
- **c** 0.75 tonnes into grams
- **d** 3.75 h into seconds

7A **3** Arrange these measurements in ascending order.
5.75×10^5 g, 3.85×10^4 g, 6.75×10^3 g, 2.36×10^4 g, 9.12×10^2 g

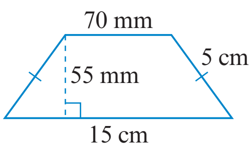
7A **4** Arrange these measurements in descending order.
3.85×10^{-8} g, 3.58×10^{-7} g, 5.38×10^{-9} g, 8.35×10^{-5} g, 5.83×10^{-6} g

7B **5** Calculate the perimeter of each shape in centimetres.

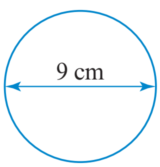
a



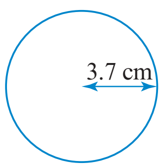
b



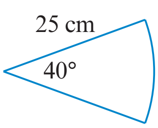
c



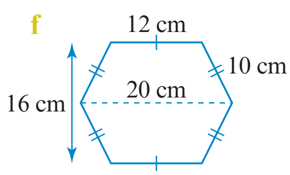
d



e



f

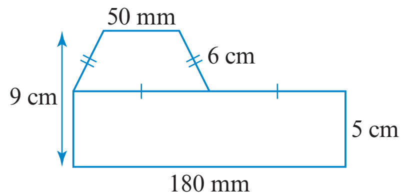


7C **6** Calculate the area of each shape in question **5**.

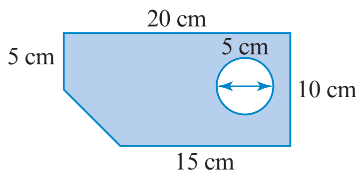
7C **7** The area of a circular dinner plate is 42 cm^2. Calculate:

- **a** its radius
- **b** its diameter.

7D **8** Calculate the area of this composite shape in square centimetres.

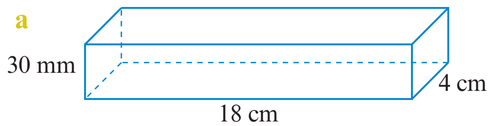


7D **9** Calculate the shaded area.

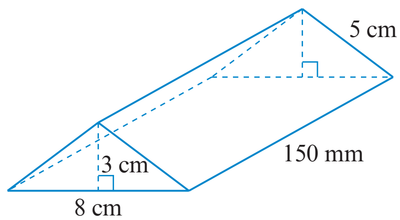


7E **10** Calculate the surface area of each prism.

a

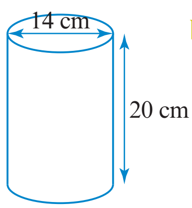


b

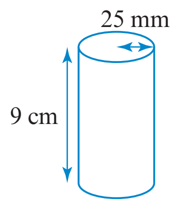


7F **11** Calculate the surface area of each cylinder.

a



b



7G **12** Calculate the volume of each prism in question **10**.

7G **13** Calculate the volume of each cylinder in question **11**.

7G **14** The volume of a water tank with radius 10 m is 4000 m^3. Calculate the height of the tank to the nearest metre.

NAPLAN-STYLE PRACTICE

1 What unit would best describe the volume of dirt in a tip truck?

- square centimetres
- square metres
- cubic centimetres
- cubic metres

2 Which of these is the longest distance?

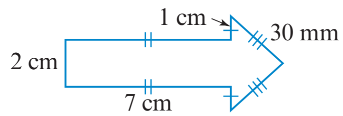
- 0.2305 km
- 235 m
- 2305 cm
- 23 500 mm

3 What is 0.000 000 000 000 000 000 000 000 029 in scientific notation?

4 A square field has a perimeter of 100 m. The length of one side is:

- 10 m
- 25 m
- 50 m
- 100 m

5 The perimeter of this shape is:



- 40 cm
- 13 cm
- 24 cm
- 78 cm

6 The radius of a circular table top is 2.5 m. What is its circumference, correct to two decimal places?

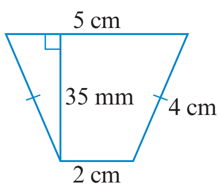
- 7.85 m
- 15.71 m
- 19.63 m
- 157.08 m

7 The area of the field in question **4** is:

8 The area of a rectangle is 96 mm^2. If the rectangle has a width of 8 mm, calculate the length.

- 12 mm
- 24 mm
- 48 mm
- 72 mm

Questions **9** and **10** refer to this shape.

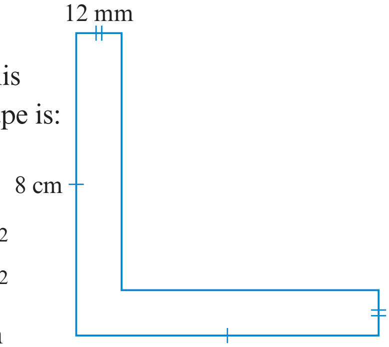


9 The correct formula to use when calculating the area of this shape is:

- $A = xy$
- $A = \frac{1}{2}bh$
- $A = \frac{1}{2}(a + b)h$
- $A = lw$

10 The area of the shape is:

11 The area of this composite shape is:

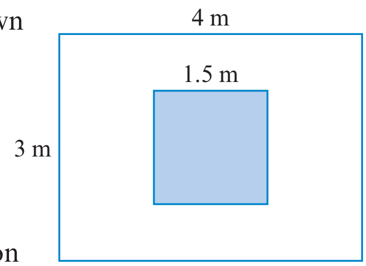


- 19.2 cm^2
- 96 cm^2
- 46.24 cm^2
- 17.76 cm^2

12 If the shape in question **5** has a total length of 92.5 mm, its area is:

- 18.5 cm^2
- 20 cm^2
- 26 cm^2
- 23 cm^2

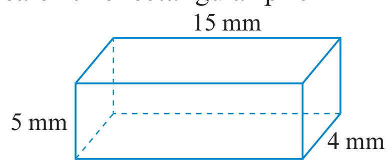
13 The square shown as the shaded portion of this diagram represents a garden, and the unshaded portion represents paving. What is the area of paving in this rectangular garden?



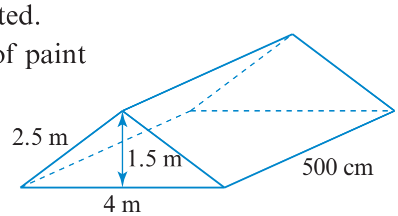
- 12 m^2
- 9.75 m^2
- 2.25 m^2
- 14.25 m^2

14 The surface area of a cube is 726 cm^2. How long is an edge of the cube?

15 The surface area of this rectangular prism in square centimetres is:

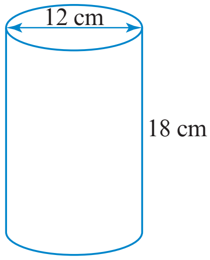


16 This solid triangular prism is to have all surfaces painted. If two coats of paint are required, what is the total area to be painted?



- 51 m^2
- 102 m^2
- 43.5 m^2
- 48 m^2

17 The surface area of this cylinder to the nearest centimetre is:



- 905 cm^2
- 2262 cm^2
- 904 cm^2
- 2261 cm^2

18 The volume of the rectangular prism from question 15 is (correct to one decimal place):

[ ] cm^3

19 The volume of the triangular prism from question 16 is (correct to one decimal place):

[ ] m^3

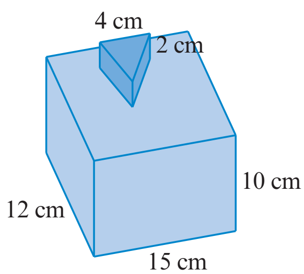
20 The volume of the cylinder from question 17 is (correct to two decimal places):

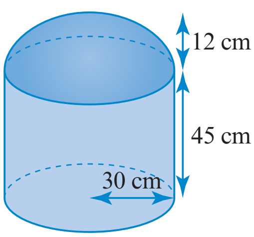
[ ] cm^3

21 Convert your answer from question 20 into cubic metres and write the answer in both decimal form and scientific notation.

[ ]

ANALYSIS





Angelique is making two sculptures. Her designs are shown.

a In the first sculpture, the rectangular part will be painted purple and the triangular part will be painted pink. The triangular prism has as its cross-section an equilateral triangle. Calculate the area to be painted each colour in square centimetres, assuming all sides will be painted before assembly.

b Purple paint comes in tins which have enough paint to cover 1 m^2. How many tins are required for one coat?

c Does Angelique have enough purple paint for two coats?

d Is there any purple paint remaining after two coats? If so, what area would the remaining paint cover?

e Pink paint comes in smaller tins which contain enough paint to cover 150 cm^2. What is the maximum number of coats possible from one tin?

f Is there any pink paint remaining? If so, what area would it cover?

g The cylinder part of the second sculpture is to be painted also but the dome will be clear perspex. The top of the cylinder must be painted also. Calculate the total surface area to be painted, correct to two decimal places.

h Angelique decides to paint the cylinder green. This paint comes in tins that cover 1 m^2. How many tins will be required?

i Is there any green paint remaining? If so, what area would it cover?

j In the first sculpture, Angelique decides to build the base of the sculpture as a mould and use concrete to form the rectangular prism. What volume of concrete (in m^3) will be required, correct to four decimal places?

k Represent this volume in scientific notation.

l Angelique decides to do the same for the cylindrical base of the second sculpture. What volume of concrete (in m^3) is required, correct to four decimal places?

m Represent this volume in scientific notation.

n Suggest dimensions for a gift box for the first sculpture, given that there must be 5 cm around the sculpture for packing material.

o Suggest dimensions for a gift box for the second sculpture, given the same restrictions.

CONNECT

Designing a playground

A new playground is to be built and the council is accepting suggestions from the community. What measurements are needed at the planning stage?

The council has set aside a rectangular block of land that is 40 m long and 60 m wide.

Your task

You are to design a playground set-up (including equipment) to submit to the council. This design must include measurements so that the council can cost your submission.

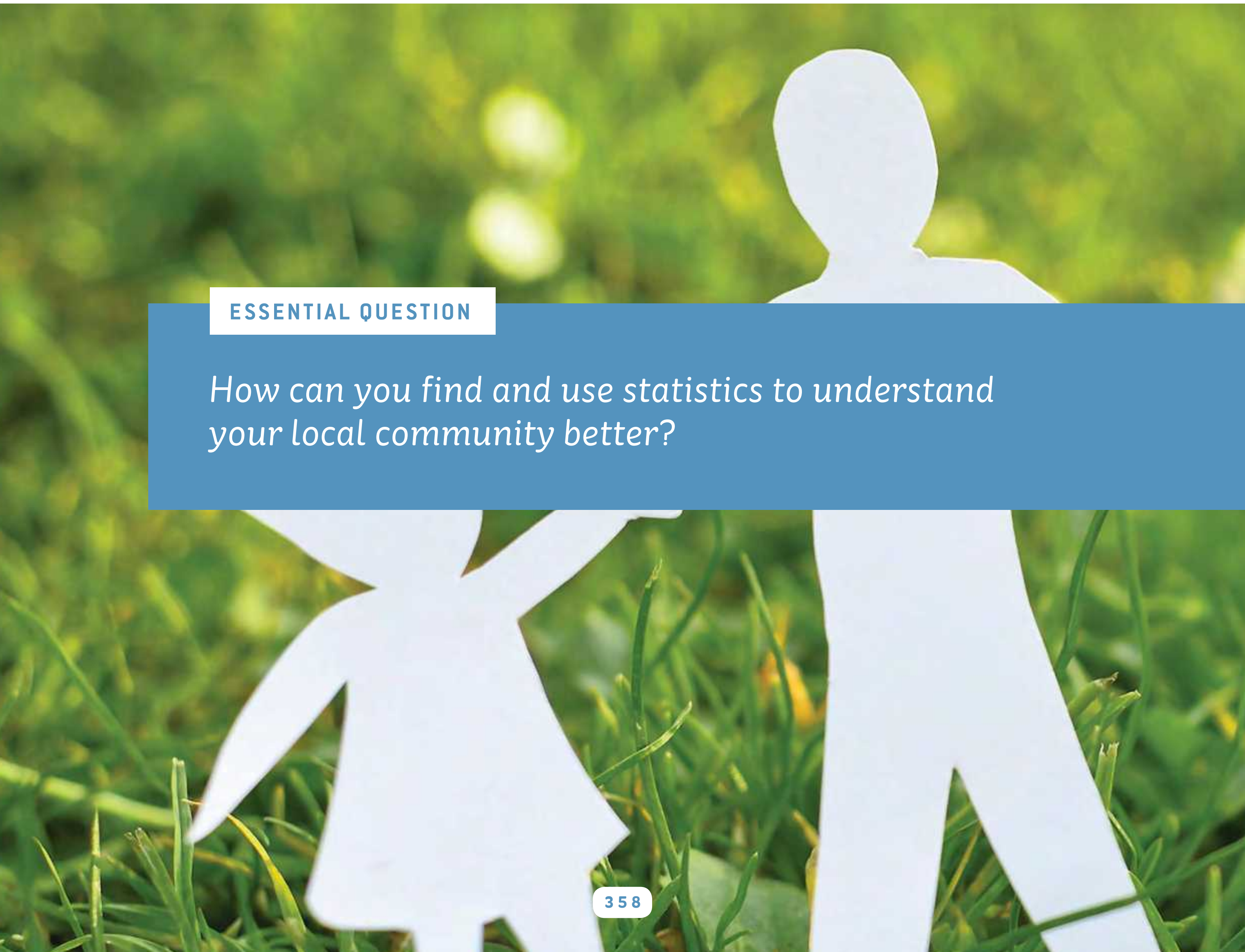
Things to consider:

- What age group are you designing your playground for?
- What type of equipment will you include in your playground? Think of the activities that the age group might enjoy. Equipment that is highly recommended includes:
  - a gym-like structure with slides, tubes, rings, steps and poles
  - a swing-set
  - small rocking structures or seesaws
  - a painted section where users can play ball games
  - a sandpit.
- How will you shield the users from the summer sun?
- What kind of surface would you use on the ground?
- What type of materials would the equipment be made from?
- Are there any areas that need fencing?
- Measurements for all areas and pieces of equipment will be required with the plans.

You may like to present your findings as a report. Your report could be in the form of:

- a pamphlet
- a digital presentation
- blueprints
- other (check with your teacher).

8 STATISTICS



ESSENTIAL QUESTION

How can you find and use statistics to understand your local community better?

Are you ready?

8A **1** Look at this table.

Colour	Frequency
Red	5
Yellow	4
Green	8
Blue	7
Purple	5

a Does it contain numerical or categorical data?

b How many people were surveyed?

8A **2** Match each type of graph with a figure below.

a pie graph **b** line graph

c scatterplot **d** column graph

e dot plot **f** bar graph

g stem-and-leaf plot

h histogram

Figure A

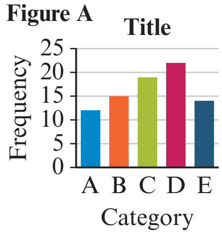


Figure B

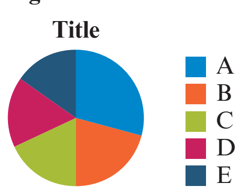


Figure C

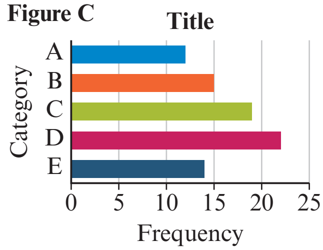


Figure D

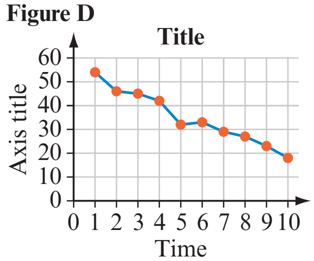
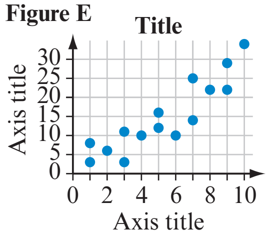
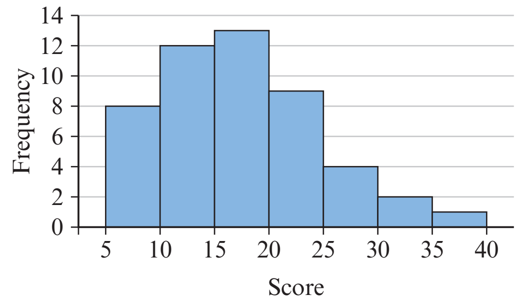


Figure E



8B **3** Look at this graph.



a What type of graph is this?

A column graph **B** bar graph

C line graph **D** histogram

b What is the size of the class intervals?

c How many people were surveyed?

8B **4** What is the most common score in this stem-and-leaf plot?

Key: 1 | 4 = 14

Stem	Leaf
0	4 7
1	3 6 6 7 8
2	0 1 2 5 5 7
3	7 9 9 9
4	0 3 5 7 7
5	7

8C **5** Consider this data set.

13 4 6 1 2 5 4 10 9

a What is the range of this data set?

b What are the mean, median and mode of this data set?

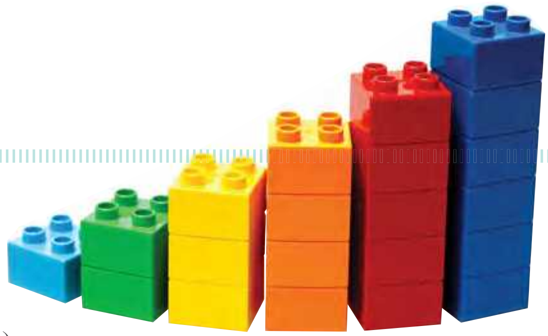
A mean = 6, median = 5, mode = 4

B mean = 46, median = 5, mode = 1

C mean = 6, median = 2, mode = 4

D mean = 46, median = 2, mode = 4

8A Understanding and representing data



Start thinking!

Data that can be counted or measured are called **numerical data**.

1 Name five different examples of numerical data (for example, height).

Data that can be put into categories or groups are called **categorical data**.

2 Name five different examples of categorical data (for example, hair colour).

Numerical data can be further split into two groups: **discrete data** can be counted (whole numbers only) and **continuous data** can be measured (includes decimal numbers).

3 Classify your examples from question **1** as either discrete or continuous.

Categorical data can be further split into two groups: **nominal data** can be arranged into unrelated groups and **ordinal data** can be arranged into groups that have an order.

4 Classify your examples from question **2** as either nominal or ordinal. If you did not include any ordinal data, think of at least two examples now.

5 Brainstorm with a classmate and list as many different types of graphs and visual displays as you can.

6 For each graph or visual display that you have listed, decide if it can be used to display numerical data, categorical data or both. Provide a reason for each decision.

7 Explain why **line graphs**, **scatterplots**, **histograms** and **stem-and-leaf plots** can only be used for numerical data.

8 Explain why **column graphs**, **bar graphs**, **pie graphs** and **dot plots** can be used to display both numerical and categorical data but are best suited to categorical data.

9 Why is it useful to collate data into a frequency table before drawing any graph?

KEY IDEAS

- Numerical data can be classified as either discrete (whole numbers only) or continuous (includes decimal numbers).
- Numerical data are best represented by visual displays such as frequency tables, histograms, stem-and-leaf plots, line graphs and scatterplots.
- Categorical data can be classified as either nominal (unrelated groups) or ordinal (groups that can be put in an order).
- Categorical data are best represented by visual displays such as frequency tables, column and bar graphs, dot plots and pie graphs.
- All graphs should include a title, clearly labelled axes with an even scale and a legend if necessary.

Check the glossary for definitions and examples of each of these graph types.

EXERCISE 8A Understanding and representing data

EXAMPLE 8A-1 Classifying data

Classify these data.

a how much people like chocolate

b number of people at a cinema

THINK	WRITE
a 1 Decide if the data is categorical or numerical.	a
2 Are the categories unrelated (nominal) or do they have an order (ordinal)?	How much people like chocolate is categorical, ordinal data.
b 1 Decide if the data is categorical or numerical.	b
2 Is the data in whole numbers (discrete) or decimal numbers (continuous)?	The number of people in a cinema is numerical, discrete data.

UNDERSTANDING AND FLUENCY

1 Classify these data.

a eye colour

b how much you like winter

c number of pets at home

d favourite movie type

e length of arm span

f distance between home and school

g shoe size

h number of girls in class

i mass of a car

j type of computer

k how fit somebody is

l number of planets in the solar system

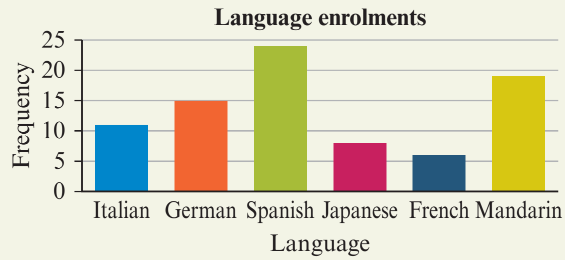
EXAMPLE 8A-2 Reading graphs

Consider this column graph.

a What is it showing?

b What is the least popular language?

c How many people were surveyed in total?



THINK	WRITE
a Look at the axes and the title of the graph.	**a** The graph is showing enrolment numbers for six languages.
b Look at the smallest column.	**b** The least popular language is French.
c Add all the frequencies together.	**c** Number of people surveyed $= 11 + 15 + 24 + 8 + 6 + 19 = 83$ 83 people were surveyed.

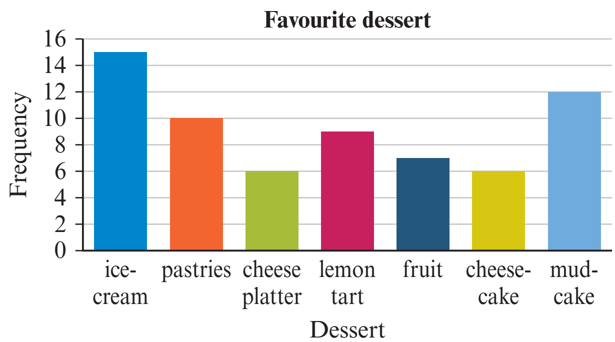
UNDERSTANDING AND FLUENCY

2 Consider this column graph.

a What is it showing?

b How many people were surveyed in total?

c If you were to offer three choices of dessert, which would you choose and why?



3 Consider this line graph.

a What is it showing?

b What time period does it cover?

c In what week is the maximum number of customers?

d In what week does the number of customers increase the fastest?

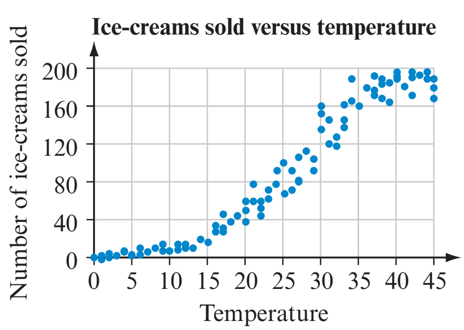
4 Consider this graph.

a What type of graph is it and what is it showing?

b How would you find how many people were surveyed?

c How might you describe the pattern that you see in the graph?

d If somebody asked what the average number of ice-creams sold on a day in April is, what would you tell them?



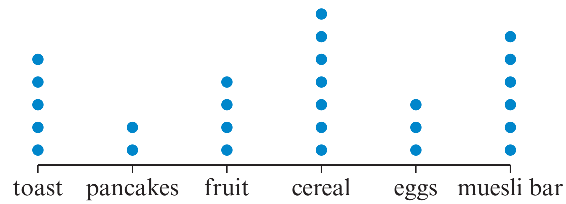
UNDERSTANDING AND FLUENCY

5 Consider this dot plot.

a How many people were surveyed?

b Briefly describe what the dot plot shows.

c Explain why dot plots should not be used when a large number of people are surveyed.



6 Consider this stem-and-leaf plot showing the ages of people in a cinema.

a How old is the youngest person at the cinema?

b How old is the oldest person at the cinema?

c What is the most common age bracket at the cinema?

d How does the key help you to read the stem-and-leaf plot?

Key: 2 | 1 = 21

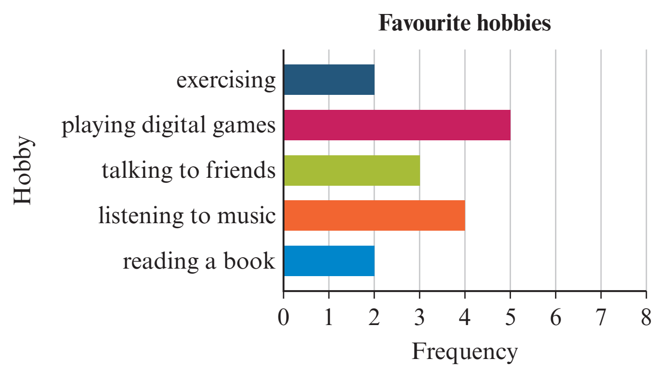
Stem	Leaf
1	5 9
2	1 3 4 9
3	0 1 3 5 5 7 8
4	1 2 2 2 3 7 8 9
5	0 6 8 8
6	
7	6

EXAMPLE 8A-3 Representing data

Gill collected this data on popular hobbies from a group of students.

reading a book, listening to music, talking to friends, playing digital games, listening to music, exercising, talking to friends, listening to music, playing digital games, playing digital games, listening to music, reading a book, playing digital games, talking to friends, playing digital games, exercising

Classify the data type and present it in an appropriate graph.

THINK	WRITE
1 This data can be arranged into unrelated groups, so it is categorical, nominal data.	It is categorical, nominal data.
2 Decide on an appropriate graph that suits categorical, nominal data. Suitable graph types are a column graph, bar graph, pie graph or a dot plot.	A bar graph would best suit this data because the category names are long and it shows the frequencies.
3 Draw your chosen graph, remembering to label both axes and include an appropriate title.	

UNDERSTANDING AND FLUENCY

7 Decide which type of data these graphs best represent.

a column graph **b** histogram **c** pie graph
d stem-and-leaf plot **e** line graph **f** dot plot

8 Joe collected this data on favourite colours.

blue, purple, green, blue, pink, pink, yellow, blue, green, red, pink, blue, red, purple, pink, green, blue, blue, yellow, purple, orange, blue, green, red, blue, red, purple, pink, blue, green, purple, purple, purple, blue, red, pink, green, blue, blue.

Classify the data and present it in appropriate graph.

9 Create an appropriate graph to represent the data in each frequency table.

a

Day	Mass (g)
1	15
2	20
3	22
4	28
5	37
6	45

b

Movie type	Frequency
Action	9
Comedy	14
Drama	7
Horror	4
Animated	10

c

Height (cm)	161	176	154	178	176	185	166	164	155	172	161	172	165	176	172	164
Weight (kg)	54	70	51	76	76	75	62	65	55	61	58	69	57	66	65	57

PROBLEM SOLVING AND REASONING

10 Create an appropriate graphical display to represent the objects shown in the photograph.

11 Create a table that shows which graphs can be used for each type of data. (Hint: the table should contain three columns with the headings 'Graph type', 'Numerical data' and 'Categorical data'.) Use ticks to complete the table.

12 For each type of graph listed in the Key ideas:

a give a definition/description of the graph
b draw an example
c explain when and for what type of data the graph is best used.

13 Explain how stem-and-leaf plots can represent numerical, continuous data.

14 Position on a sports ladder is often classified as the wrong data type.

a Find a current sports ladder (or create your own).
b Would you say that it is numerical or categorical data? Why?

PROBLEM SOLVING AND REASONING

c Can you measure or add together the numbers shown in a sports ladder? Explain.

d Use your answer to part c to explain why it cannot be numerical data.

e Is there an order to a sports ladder?

f What type of data is position on a sports ladder?

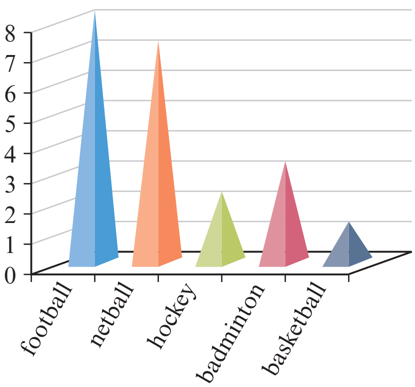
15 Explain why a pie graph is rarely the best graph to represent data, even though it is so commonly used.

16 Graphs can often be used in a misleading way in order to support a person's point of view. For each of these graphs:

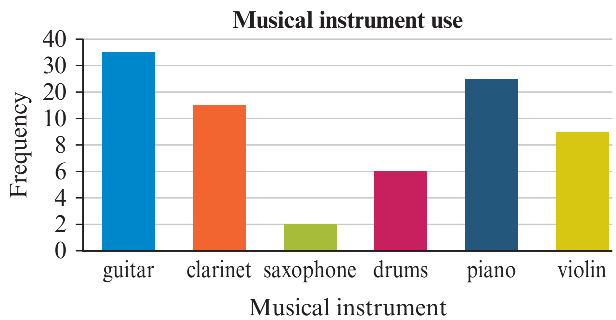
i describe how the graph is or could be misleading

ii describe what would need to be done in order to make the data in the graph clearer.

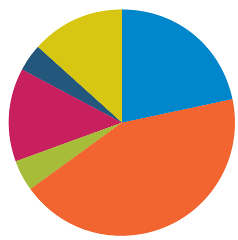
a



b



c



CHALLENGE

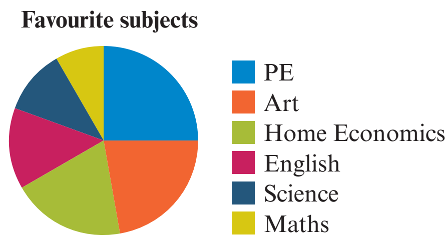
17 Consider this pie graph.

a What is it showing?

b What is the most popular subject?

c Explain why you cannot tell how many people chose Maths.

d If 36 people were surveyed, use your understanding of angles within a circle to find the number of people who chose each category. (Hint: you will need a protractor.)



Another 14 people were surveyed.
Three chose Maths, five chose Art, four chose Information technology and two chose PE.

e Redraw the pie graph to include these new people. Does this change the most popular subject? (Hint: use your answers from part d and the new information to first create a frequency table.)

Reflect

If all types of graphs can be used to represent numerical data, why are some better suited than others?

8B Grouped data

Start thinking!

Joseph wanted to collect data on the heights of the people in his class. Before drawing a **frequency table**, he measured the heights of the shortest person (145 cm) and the tallest person (183 cm) in the class.

1 How many rows would a simple frequency table require to cover this **range** of data?

When data covers a large range, you can group it into class intervals. Each table row contains a spread of data.

2 Explain why there should be no fewer than 5 groups and no more than 10 groups in a frequency table.

3 What **class intervals** should Joseph use for his frequency table?

Joseph collected this data:

145, 183, 167, 172, 161, 158, 153, 168, 165, 174, 157, 152, 173, 166, 158, 159, 160, 171, 171, 161, 165, 172, 165, 158, 154.

4 Arrange this data into a frequency table with your chosen class intervals from question **3**. Include a row that gives the total frequency.

When grouping data it is important to consider data type.

5 What are the two types of numerical data?

6 What type of data is height?

7 If you used class intervals such as 140–144, 145–149, where would you place 144.6 cm?

When grouping continuous data, use 'open' class intervals such as 140–<145.

8 Redraw your frequency table if necessary so that it uses open class intervals.

9 Why is it important to use open class intervals for this scenario?

KEY IDEAS

- Tables displaying grouped numerical data make use of class intervals to group the data.
- Class intervals should be chosen so that a table contains 5–10 groups.
- Identify the data type before constructing a table: continuous data need class intervals such as 0–<10; discrete data can be shown this way or in class intervals such as 0–9.
- A histogram can be used to represent grouped numerical data.
- There are no gaps between the columns of a histogram (but there is a small gap between the vertical axis and the first column) and category marks should be on the edges of the columns.

EXERCISE 8B Grouped data

EXAMPLE 8B-1 Using a frequency table to represent data

Draw an appropriate frequency table to represent this data.

4.5, 11.6, 67.3, 33.7, 28.1, 36.4, 22.6, 54.8, 1.4, 66.8, 36.4, 29.3, 37.8, 42.3, 52.1, 38.3

THINK

1 A frequency table should have 5–10 groups. The minimum **score** is 1.4 and the maximum score is 66.8. This gives a range of 65.4. Class intervals of 10 could be used, giving 7 groups.

2 This data is continuous, so the class intervals must in the form of 0–<10.

3 Draw the frequency table using the raw data. You may wish to include a tally column to ensure that you don't miss any scores.

WRITE

Class	Frequency
0–<10	2
10–<20	1
20–<30	3
30–<40	5
40–<50	1
50–<60	2
60–<70	2

UNDERSTANDING AND FLUENCY

1 Draw an appropriate frequency table to represent each data set.

a 5, 16, 28, 24, 31, 39, 3, 18, 13, 11, 25, 33, 8, 12, 19, 21, 31, 28

b 14.5, 73.2, 22.1, 43.9, 42.0, 58.4, 19.8, 37.6, 62.1, 29.4, 34.5, 72.1, 59.1, 52.3, 63.1, 26.3, 34.0, 41.9, 48.5, 16.4, 31.2, 52.9

c 1.2, 5.4, 9.3, 11.4, 3.3, 4.7, 3.3, 3.9, 4.8, 6.6, 2.9, 1.9, 10.6, 9.7, 10.8, 3.6, 4.8, 2.7, 2.1, 1.7, 1.9, 11.9, 6.7, 5.4, 5.1, 1.6, 1.8

d 42, 79, 56, 49, 77, 50, 51, 46, 48, 72, 61, 78, 63, 45, 58, 53, 73, 58, 49, 61, 68, 67, 43, 49, 75, 77, 58, 54, 67, 72, 51, 56, 53, 48, 76, 78, 72, 42, 48, 53

2 Use each table to draw a histogram.

a

Class	Frequency
5–<10	8
10–<15	6
15–<20	7
20–<25	2
25–<30	3
30–<35	1
35–<40	5
40–<45	6
45–<50	9

b

Class	Frequency
0–<20	14
20–<40	21
40–<60	18
60–<80	13
80–<100	8
100–<120	2

c

Class	Frequency
0–<10	5
10–<20	8
20–<30	12
30–<40	3
40–<50	11
50–<60	9
60–<70	6

EXAMPLE 8B-2 Drawing a histogram

Use this data to draw an appropriate histogram.

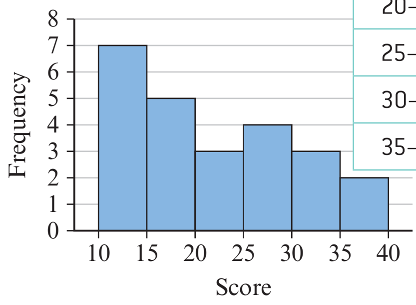
34, 22, 29, 16, 12, 16, 26, 32, 39, 20, 23, 19, 36, 25, 11, 16, 13, 13, 19, 28, 12, 14, 32, 10

THINK

1. Collate the data into a frequency table. Ensure that there are 5–10 class intervals.
2. Draw the axes with an even scale that allows the minimum and maximum values to be shown. Ensure that there is a half space between the vertical axis and first category mark.
3. Draw the histogram and remember to label both axes and give it a title.

WRITE

Class	Frequency
10–<15	7
15–<20	5
20–<25	3
25–<30	4
30–<35	3
35–<40	2



UNDERSTANDING AND FLUENCY

3 Use each data set to draw an appropriate histogram.

a 13, 46, 13, 17, 35, 9, 22, 15, 8, 2, 35, 42, 42, 17, 16, 22, 29, 31, 47, 29, 13, 20, 36, 47, 28, 23, 30, 38

b 18.1, 24.5, 32.1, 15.6, 22.5, 29.1, 34.6, 16.7, 19.4, 17.5, 21.8, 27.5, 29.2, 30.1, 20.0, 33.1, 32.8, 31.9, 33.8, 14.3

c 64, 18, 120, 7, 29, 40, 145, 38, 72, 38, 18, 29, 2, 56, 49, 87, 99, 104, 59, 5, 29, 112, 118, 34, 59, 29, 19, 13

d 125, 726, 632, 465, 428, 257, 283, 399, 619, 402, 132, 196, 183, 743, 120, 703, 336, 652, 349, 402, 560, 144, 759, 717, 588, 185, 464, 685, 268, 352, 310, 408, 114, 782, 189

e 1.25, 1.89, 1.09, 1.76, 1.15, 1.36, 1.55, 1.67, 1.99, 1.32, 1.08, 1.14, 1.17, 1.62, 1.88, 4.9, 1.68, 1.49, 1.08, 1.16, 1.24, 1.19, 1.26, 1.83, 1.52, 1.18, 1.07, 1.42, 1.01, 1.19

f 25, 58, 48, 33, 26, 53, 42, 49, 58, 53, 46, 24, 58, 53, 46, 41, 38, 47, 44, 58, 53, 57, 39, 21, 48, 46, 42, 58, 52, 43, 42, 37, 36, 27, 46, 42, 49, 53, 57, 59

4 Data was collected on the number of hours spent listening to music per week, as shown below.

7	10	2	4	24	3	7	9	5	15
17	16	19	20	5	3.5	5	10	14	7
7	5	9	10	17	7	4	10	11	16
4	5.5	12	14	6	7	12	14	16	3

a Create an appropriate frequency table to collate the data.

b Draw a histogram to represent this data.

EXAMPLE 8B-3 Drawing a stem-and-leaf plot

Draw a stem-and-leaf plot for this data set.
1.8, 2.6, 1.9, 3.4, 5.2, 1.8, 2.7, 4.2, 4.9, 5.1, 7.6, 3.1, 4.1,
3.0, 2.8, 2.1, 1.9, 1.3, 2.8, 2.9, 4.3, 4.9, 5.1, 3.3, 2.0.

THINK

1 The minimum value is 1.3 and the maximum value is 7.6, so show stems ranging from 1 to 7.

2 Place each piece of data into the plot. Start with 1.8, which has a stem of 1 and a leaf of 8. Write the digit 8 in the stem 1 row. Continue until all values have been considered.

3 Rearrange the leaves so that they are in order. Include a key.

WRITE

Key: 1 | 3 = 1.3

Stem	Leaf
1	3 8 8 9 9
2	0 1 6 7 8 8 9
3	0 1 3 4
4	1 2 3 9 9
5	1 1 2
6	
7	6

UNDERSTANDING AND FLUENCY

5 Consider this stem-and-leaf plot.

Key: 1 | 2 = 12

Stem	Leaf
1	2 6 7 8 8 9
2	0 1 5 6 7
3	3 6 8
4	0 2
5	1

a Which part of the plot represents the class intervals?
b What is the size of the class intervals?
c How many people were surveyed?
d What advantage does a stem-and-leaf plot have over a histogram?

6 Draw a stem-and-leaf plot to represent each data set.
a 16, 7, 36, 67, 14, 25, 42, 37, 19, 2, 46, 48, 51, 22, 18, 6, 17, 13, 13, 9, 11, 27, 31, 36, 42, 15, 23, 59, 33, 36, 99.
b 2.2, 2.7, 2.8, 1.6, 5.9, 3.4, 4.8, 6.2, 3.7, 2.8, 1.2, 4.2, 4.8, 5.1, 4.2, 5.3, 1.7, 1.9, 3.3, 2.2, 4.4, 4.8, 4.3, 1.8, 3.4.
c 56, 74, 36, 85, 22, 16, 48, 26, 95, 102, 16, 75, 59, 32, 15, 18, 68, 92, 43, 55, 12, 64, 66, 72, 42, 42, 18, 33, 81, 108, 111, 117, 19, 33, 36, 49, 43, 47, 52, 61, 77, 19, 8, 26, 22, 88, 46, 73, 42, 29.
d 112, 162, 124, 163, 177, 113, 134, 142, 165, 133, 119, 126, 142, 137, 153, 143, 166, 118, 121, 127, 132, 119, 144, 132, 119, 126, 172, 164, 134, 153, 142, 167, 146, 132, 119, 113, 127, 164, 138, 142, 165, 113.

7 Data was collected on the ages of customers in a clothes store in a day (see table on the right).
a Why is this table difficult to read?
b Redraw the table with larger class intervals so that it is easier to read.

Class	Frequency
<10	2
10–<12	6
12–<14	12
14–<16	16
16–<18	14
18–<20	13
20–<22	11
22–<24	9
24–<26	10
26–<28	8
28–<30	6
30–<32	4
32–<34	6
34–<36	5
36–<38	3
38–<40	2
40–<42	3
42–<44	1
44–<46	1
46–<48	0
48–<50	1
≥50	6

UNDERSTANDING AND FLUENCY

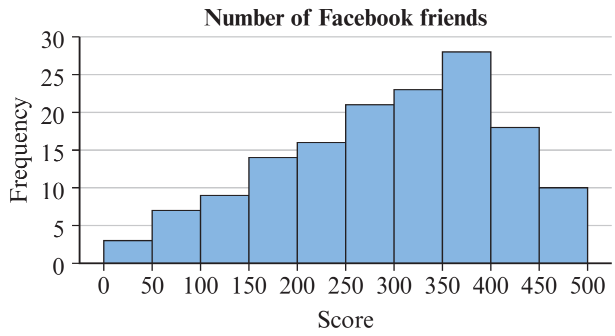
8 Use the stem-and-leaf plot on the right to draw a histogram. Why is this reasonably easy to do?

Key: 1 | 2 = 12

Stem	Leaf
1	0 0 1 1 3 4
1*	5 5 5 6 6 7 9
2	0 1 2 2 3 3 4 4 6
2*	5 6 6 7 8 8 9
3	0 1 1 2 2 3 4
3*	5 6 7 8

9 Explain why you can't accurately draw a stem-and-leaf plot from a histogram.

10 Consider this histogram.



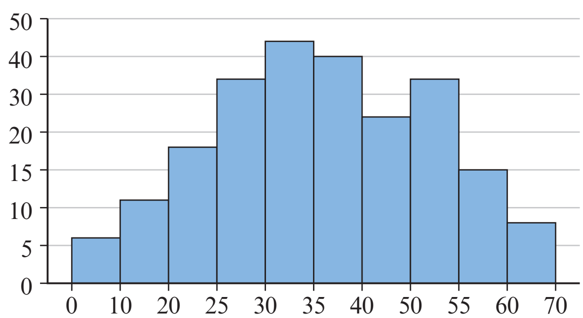
a What does it show?
b What is the size of its class intervals?
c State the most common class and its frequency.
d If you were to add another piece of data, such as 40, to the histogram, to which class interval would you add it? Explain.

PROBLEM SOLVING AND REASONING

11 Explain why you can't accurately decrease the size of the class intervals in this table.

Class	Frequency
0–<20	16
20–<40	48
40–<60	42

12 Consider this histogram.



a How would you improve this histogram?
b Redraw the histogram so that it is accurate.
c What is the most common class interval in this improved histogram? What is its frequency?
d How many people were surveyed for this histogram?

13 Stem-and-leaf plots are not limited to class intervals of 10. You can split the stems of plots so that they can be more easily read. Consider this split stem-and-leaf plot, showing the ages of people buying a cinema ticket for a particular film.

Key: 1 | 2 = 12

Stem	Leaf
1	0 1 2 4 4
1*	5 5 6 6 7 8
2	0 0 1 1 2 3 3 3
2*	6 6 6 6 7 7 8 8 9
3	1 2 2 3 4 4 4
3*	5 6 9

a Look at the values of the leaves. What is the size of the class intervals?
b What is the most common class interval?
c To which class interval would you add the value 25?

PROBLEM SOLVING AND REASONING

14 Use this data to draw a split stem-and-leaf plot and comment on what pattern you see.

8	24	18	17	2	13	22	8	9	11	16	10	7	16	12
13	13	22	19	6	5	9	7	14	12	11	10	20	16	13
11	20	17	19	8	9	12	13	7	14	16	19	21	9	12

15 How might you represent the stems of a stem-and-leaf plot that has class intervals of 2?

16 This stem-and-leaf plot is missing its key. For the possible keys below, state:

a the minimum score

b the maximum score

c the size of the class intervals

d the range of the plot.

i Key: 1|2 = 12 **ii** Key: 1|2 = 1.2

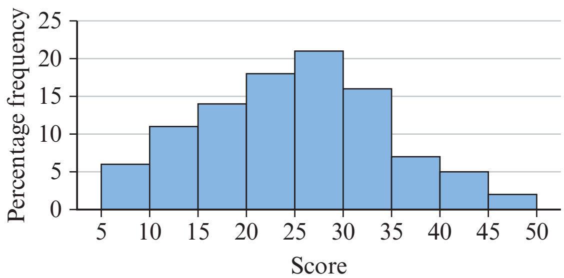
iii Key: 1|2 = 1200 **iv** Key: 1|2 = 0.12

Stem	Leaf
0	4 8 9
1	2 3 6 7 9
2	1 1 4 8
3	0 6 4 8 9 9
4	1 2 2 4
5	6 7

17 Draw a histogram to represent the heights of people in your class. Be sure to first draw a frequency table with appropriate class intervals. Which class interval do you belong in?

CHALLENGE

18 Consider this percentage frequency histogram.



a How is it different from a normal histogram?

b What percentage of scores are between 30 and 35?

c What percentage of scores are greater than 35?

d Without performing a calculation, state the sum of the percentage frequency columns. Explain how you know.

e If there were 400 scores in total, calculate:

i the number of scores between 15 and 20

ii the number of scores less than 25

iii the number of scores between 20 and 40.

19 Create a percentage frequency histogram to represent this data set.

Weights of newborn babies at a particular hospital in one week (in kg)

3.25, 4.15, 2.75, 3.60, 3.95, 3.05, 2.85, 4.20,
1.95, 3.50, 3.65, 3.15, 3.70, 3.95, 4.10, 4.85,
2.90, 3.10, 3.30, 3.25, 3.50, 4.05, 3.45, 3.85,
3.75, 3.15, 3.45, 3.20, 3.25, 4.25, 2.55, 2.95,
3.40, 3.85, 3.80, 3.55, 3.20, 3.00, 3.20, 3.75,
4.00, 4.15, 3.80, 3.75, 3.40, 3.25, 3.15, 3.05,
3.85, 2.95.

Reflect

In which situations would you use a histogram to represent data and in which would you use a stem-and-leaf plot? Why?

8C Summary statistics

Start thinking!

A numerical data set is usually summarised by its **centre** and **spread**.

Kylie collected data on ages of students in a canteen, shown below.

14 12 17 15 14 16 16 18 12 13 14 13 15

16 14 12 13 15 16 14 15 13 16 14 15

1 How many pieces of data (scores) are there?

2 Arrange the data in order. What is the most common number? This measure of centre is called the **mode**.

Another measure of centre is to find the exact middle of a data set, called the **median**.

3 Which score is the median of this data set?

4 Why is it important to ensure that the scores are ordered before finding the median?

The final measure of centre is the **mean** and is commonly known as the average. The mean is found by adding together all scores, then dividing by the number of scores.

5 What is the mean of this data set?

One measure of spread is the range; the difference between the minimum and maximum scores.

6 Find the range and explain why it is easier to do so when the data set is ordered.

KEY IDEAS

- A piece of data is often called a score rather than a number.
- There are three main measures of centre: the mode (most common number or numbers); the median (the middle of the ordered set) and the mean (the average of the set).
- One measure of spread is the range: the difference between the maximum and minimum scores.
- To calculate the mean from a table, add a column multiplying the score by the frequency. Divide the total of this new column by the total of the frequencies. In this table, the mean is $30 \div 20 = 1.5$.
- To calculate the median from a table include a column of cumulative frequency. For n scores, the row containing the $\frac{n+1}{2}$th score is then easily identified. In this table, $n = 20$ and the median is the 10.5th score, which is in the row '1', so the median is 1.
- An **outlier** is a piece of data that is very different from the rest of the data set.

Score (x)	Frequency (f)	Score × frequency ($x \times f$)	Cumulative frequency
0	3	$0 \times 3 = 0$	3
1	8	$1 \times 8 = 8$	11
2	7	$2 \times 7 = 14$	18
3	1	$3 \times 1 = 3$	19
4	0	$4 \times 0 = 0$	19
5	1	$5 \times 1 = 5$	20
Total	20	30	

EXERCISE 8C Summary statistics

EXAMPLE 8C-1 Finding summary statistics from raw data

Find the mean, median, mode and range for this data set.

5, 7, 8, 3, 4, 6, 2, 4, 9, 3

THINK	WRITE
1 To find the mean, add all the scores together and divide them by how many scores there are.	$\frac{5+7+8+3+4+6+2+4+9+3}{10} = 5.1$ The mean is 5.1.
2 To find the median, rearrange the scores in order and find the middle number. If the set contains an even number of scores, find the average of the two middle scores.	2, 3, 3, 4, 4, 5, 6, 7, 8, 9 The median is $\frac{4+5}{2}$ or 4.5.
3 To find the mode, look at the ordered number list and state the most common score/s.	The modes are 3 and 4.
4 To find the range, subtract the lowest number from the highest number.	$9 - 2 = 7$ The range is 7.

UNDERSTANDING AND FLUENCY

1 For each data set, find:

- **i** the mean
- **ii** the median
- **iii** the mode
- **iv** the range.

a 12, 4, 8, 2, 9, 5, 2

b 5, 8, 1, 4, 7, 10, 2, 5, 3

c 12, 16, 12, 7, 8, 11, 14, 6, 13, 18, 4

d 20, 21, 28, 15, 32, 19, 25, 38, 22

2 For each data set, find:

- **i** the mean
- **ii** the median
- **iii** the mode
- **iv** the range.

a 3, 11, 16, 8, 4, 7, 12, 9

b 2, 8, 5, 9, 7, 4, 3, 6, 2, 4

c 15, 12, 6, 35, 7, 8, 9, 10, 10, 8

d 100, 125, 148, 122, 76, 118, 142, 148, 109, 122

EXAMPLE 8C-2 Finding the mean from a table

Find the mean for the data shown in this table.

Score (x)	Frequency (f)
1	4
2	7
3	8
4	8
5	2
6	1

THINK

1 Add a 'score × frequency' column to the table and complete it.

2 Divide the two totals to find the mean.

WRITE

x	f	$x \times f$
1	4	4
2	7	14
3	8	24
4	8	32
5	2	10
6	1	6
Total	30	90

mean = 90 ÷ 30 = 3

UNDERSTANDING AND FLUENCY

3 Find the mean for the data shown in each table, correct to two decimal places.

a

Score (x)	Frequency (f)
1	6
2	7
3	5
4	3
5	1

b

Score (x)	Frequency (f)
10	8
20	6
30	8
40	2

c

Score (x)	Frequency (f)
13	3
14	4
15	8
16	11
17	12
18	4

d

Score (x)	Frequency (f)
0	11
1	13
2	6
3	3
4	1

e

Score (x)	Frequency (f)
15	29
20	41
25	58
30	72

f

Score (x)	Frequency (f)
1	6
2	11
3	9
4	4
5	3
19	1

EXAMPLE 8C-3 Finding the median from a table

Find the median for the data shown in Example 8C-2.

THINK

1 Add a **cumulative frequency** column to the table.

2 Find the $\frac{n+1}{2}$th score, where n is 30.
The 15.5th score will be in the row containing scores of 3.

WRITE

x	f	cf
1	4	4
2	7	11
3	8	19
4	8	27
5	2	29
6	1	30
Total	30	

median = 3

4 Find the median for the data shown in each table from question 3.

EXAMPLE 8C-4 Finding the mode and range from a table

Find the mode and range for the data shown in Example 8C-2.

THINK

1 The mode is the score with the highest frequency. The highest frequency in the table is 8. Which scores have this frequency?

2 The range is the difference between the highest score and the lowest score.

WRITE

modes = 3, 4

range = 6 – 1 = 5

UNDERSTANDING AND FLUENCY

5 Find the mode(s) and range for the data shown in each table from question 3.

6 Find the mean, median, mode and range for the data shown in each table.

a

Score (x)	Frequency (f)
1	3
2	5
3	8
4	7
5	2

b

Score (x)	Frequency (f)
5	8
6	6
7	4
10	2

c

Score (x)	Frequency (f)
10	2
11	4
12	7
13	8
14	3
15	1

7 A number of people were surveyed on how many pairs of shoes they bought in a year. Use the results shown in the table to find the mean, median, mode and range.

Score (x)	Frequency (f)
1	9
2	13
3	21
4	18
5	13
6	11
7	7
8	4
9	1
10	3

8 Explain why the only summary statistic that you can find for categorical data is the mode.

9 Consider this data obtained on ages of students in a sports club.

14 15 17 13 14 15 16 17 16 15 14 15 15
16 16 16 17 16 17 15 17 14 16 16 16

a Find the mean, median, mode and range for the data set.

One student included the age of the coach (62) in their data set.

b Recalculate the summary statistics for the data set to include the coach.

c How does the inclusion of this piece of data affect the statistics?

A piece of data that is very different from the rest of the data set is called an outlier.

d How does an outlier affect the mean? How does it affect the median?

e Which measure of centre (mean or median) would you use to describe this data set? Why?

10 Would you use the mean or median as the measure of centre for these data sets.

a 4, 8, 4, 2, 6, 9, 5, 23, 8, 5, 2, 6, 9, 4, 2, 6, 9, 5, 6, 3, 8, 6, 5, 7, 9, 7, 3, 5, 2, 4, 3

b 11, 14, 19, 29, 46, 23, 18, 8, 33, 38, 22, 27, 13, 16, 19, 37, 42, 49, 35, 28, 25

c 87, 99, 123, 145, 134, 98, 106, 114, 128, 32, 148, 133, 88, 107, 111, 135, 122

d

Score	Frequency
10	1
20	6
30	9
40	7
50	5

11 Summary statistics can also be calculated on tables that use class intervals. Consider this raw data list showing the ages of people at an all-ages festival.

16 48 22 28 31 27 19 18 17 20 24 23 35 42
16 18 18 21 22 19 27 26 19 17 18 21 22 38
19 18 31 24 16 18 34 27 21 20 18 17

a Use the raw list to calculate the mean, median, mode and range.

b Construct a frequency table with class intervals of 5 to represent the data.

c What is the modal class? How does this relate to the mode of the raw data?

d What is the median class? How does this relate to the median of the raw data?

e What is the range of class intervals? How does this relate to the range of the raw data?

PROBLEM SOLVING AND REASONING

To find the mean, use the midpoint of each class interval.

f Add a column to your frequency table that gives the midpoint (or halfway point) of each class interval.

g Add another column to your table and calculate midpoint × frequency for each class interval. (Hint: this is the same process as 'score × frequency' but using the midpoints instead.)

h Use the sum of the midpoint × frequency column and the total number of scores to find the mean.

i How does this mean compare to the mean you found using the raw data?

12 Dinith collected data on the number of ice-creams sold per day in January, shown below.

52	46	13	16	21	29	33	59	46
43	47	46	42	38	8	22	19	27
31	38	44	42	58	55	52	53	47
36	48	42	45					

a Calculate the mean, median, mode and range on the raw data.

b Construct a frequency table with class intervals of:

i 5 **ii** 10 **iii** 15.

c Calculate the mean, median, modal class and range for each table in part **b**.

d What can you say about the effect of class intervals (in particular the size of the class intervals) on the accuracy of summary statistics (in particular the mean)?

CHALLENGE

13 Another measure of spread is **standard deviation**. It measures the average spread from the mean. A small standard deviation means that most values are close to the mean and a large standard deviation means that the values are spread far from the mean. Standard deviation is best found using a calculator with the appropriate function. It can also be found (for a sample) using the formula

$$s = \sqrt{\frac{\Sigma(\bar{x} - x)^2}{n - 1}}$$

where s represents standard deviation, Σ means 'the sum of', $\bar{x}$ represents the mean, x represents an individual score and n represents the number of scores.

a Use your calculator or the standard deviation formula to find the standard deviation for each data set, correct to two decimal places.

i 4, 8, 2, 6, 4, 9, 7, 7, 4, 1, 2, 4, 6, 9, 7, 4, 6, 7, 8, 1, 2, 3, 1, 6, 2, 6

ii 22, 27, 35, 64, 12, 74, 37, 93, 27, 33, 11, 71, 64, 42, 81, 37, 13, 19, 88, 50

iii 103, 118, 109, 111, 117, 116, 117, 117, 105, 107, 116, 113, 108, 109, 112, 113

b Use your results to state if each data set from part **a** has a large or small spread from the mean.

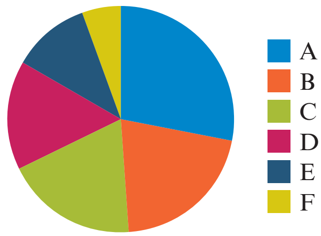
Reflect

Which summary statistics do you think are most useful to find and use? Explain.

8D Summary statistics from displays

Start thinking!

Consider these two graphical displays.



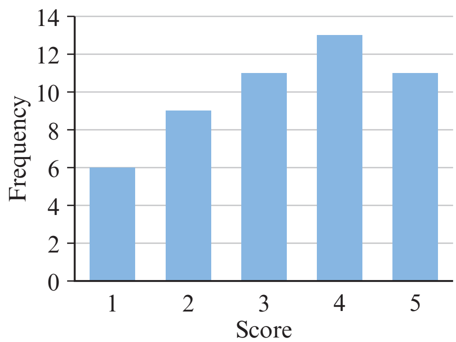
1 Explain why the mode is the only summary statistic that can be found using the pie graph.

2 How does this relate to the type of data usually displayed in a pie graph?

3 Which two summary statistics are easy to read from the column graph?

4 Find the summary statistics you identified in question **3** for the column graph.

5 How would you find the mean and median from this column graph?



To make finding summary statistics from a graph easier, create a table from the graph. This is not necessary, but it can make the process simpler and more obvious.

6 Create a frequency table from the column graph.

7 Add a score × frequency column to your table and calculate the mean.

8 Add a cumulative frequency column to your table and find the median.

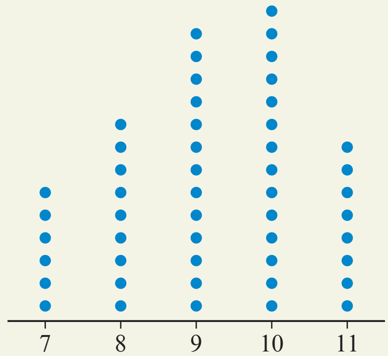
KEY IDEAS

- To calculate summary statistics from a display that individually lists scores (such as a stem-and-leaf plot), the data can be treated either as a raw list (most accurate) or as a table with class intervals (approximate).
- To calculate summary statistics from other displays (such as dot plots and column graphs), a table can be created to help with the calculations.
- The mode and range are usually very easily read from any display.
- If a display shows categorical data, only the mode can be found.

EXERCISE 8D Summary statistics from displays

EXAMPLE 8D-1 Finding summary statistics from a dot plot

Find the mean, median, mode and range for the data displayed in this dot plot.



THINK

WRITE

1 Create a frequency table by counting the number of dots in each column.

Score (x)	Frequency (f)
7	6
8	9
9	13
10	14
11	8

2 Add a total row, a 'score × frequency' column and a cumulative frequency column to the table.

x	f	$x \times f$	cf
7	6	42	6
8	9	72	15
9	13	117	28
10	14	140	42
11	8	88	50
Total	50	459	

3 Find the mean by dividing the $x \times f$ total by the f total.

$$\text{mean} = 459 \div 50 = 9.18$$

4 Find the median by locating the $\frac{n+1}{2}$th score, where $n = 50$. This is the 25.5th score.

median = 9

5 Find the mode by locating the score with the highest frequency.

mode = 10

6 Find the range by subtracting the minimum score from the maximum score.

$$\text{range} = 11 - 7 = 4$$

1 Find the mean, mode, median and range for the data displayed in each table.

a

Score (x)	Frequency (f)
1	3
2	6
3	8
4	2

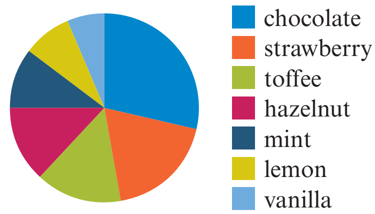
b

Score (x)	Frequency (f)
10	3
20	6
30	4
40	14
50	18

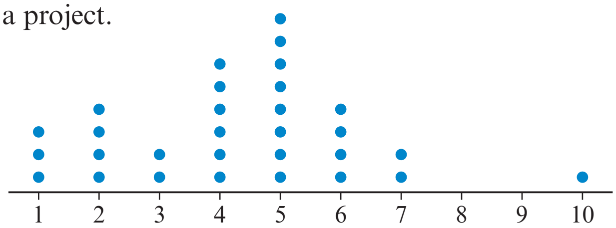
c

Score (x)	Frequency (f)
5	9
10	7
15	5
20	7
25	3

2 Find the mode of the data set from this pie graph.



3 This dot plot shows the time in hours to complete a project.



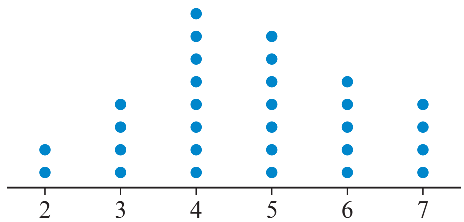
a How many people were surveyed?

b State the mode and range.

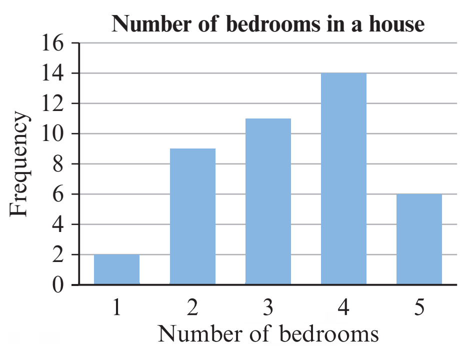
c Create a frequency table.

d Use the frequency table to calculate the median and mean of project completion time.

4 Find the mean, median, mode and range for the data displayed in this dot plot.



5 Data was collected on the number of bedrooms in a house and is shown in this column graph.



a What is the most common number of bedrooms?

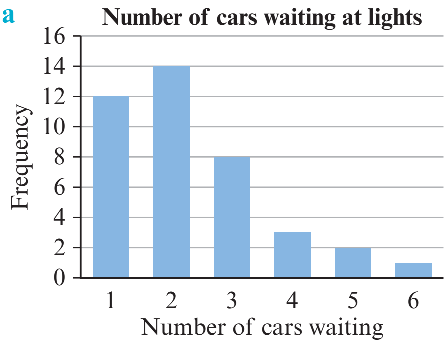
b What is the range of the number of bedrooms?

c Create a frequency table to represent the data shown in the column graph.

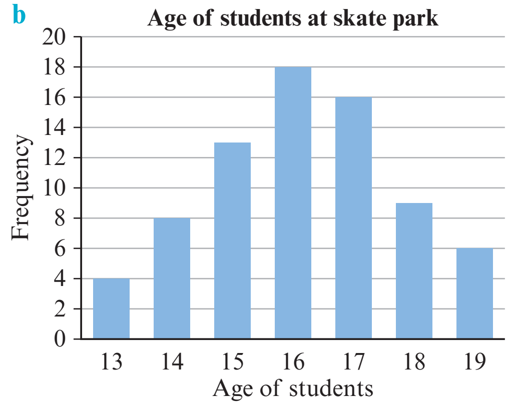
d Use the frequency table to calculate the median and the mean of the number of bedrooms.

6 Find the mean, median, mode and range for the data shown in each column graph.

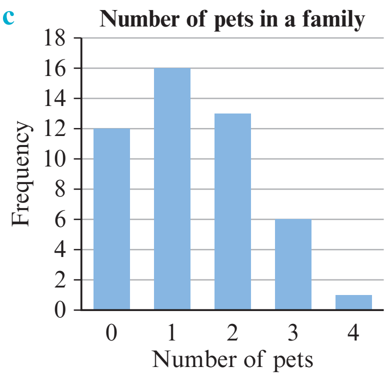
a



b



c



EXAMPLE 8D-2 Finding summary statistics from a stem-and-leaf plot

Find the mean, median, mode and range for the data displayed in this stem-and-leaf plot.

Key 2|1 = 21

Stem	Leaf
1	5 9
2	3 7 8 8 9
3	0 1 2 2 6 9 9
4	0 2 3 3 6 6 6 7 8
5	1 5

THINK	WRITE
1 Find the mean by dividing the sum of scores by the number of scores. There are 25 scores in this stem-and-leaf plot. Add these scores and divide by 25.	mean = 915 ÷ 25 = 36.6
2 Find the median by locating the $\frac{n+1}{2}$th score where $n = 25$. This is the 13th score.	median = 39
3 Find the mode by locating the most common score(s).	mode = 46
4 Find the range by calculating the difference between the minimum score (15) and the maximum score (55).	range = 55 – 15 = 40

PROBLEM SOLVING AND REASONING

7 Find the mean, median, mode and range for the data displayed in each stem-and-leaf plot.

a

Key 2|1 = 21

Stem	Leaf
3	2 4 7
4	2 2 2 6 9
5	3 6 8 9 9
6	1 1 4 6 7 7 8 8
7	0 4

b

Key 1|3 = 1.3

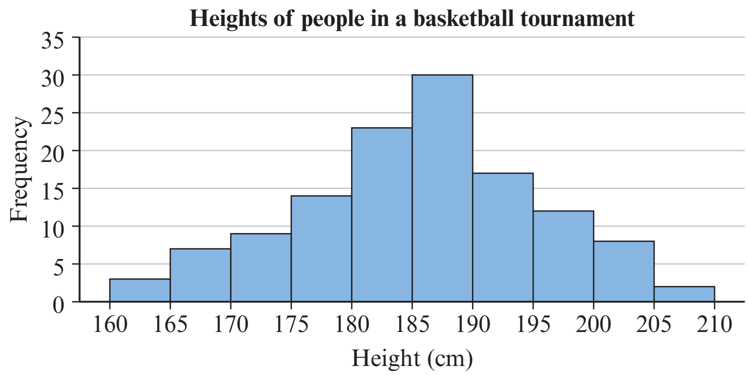
Stem	Leaf
1	2 3 8 8 8 9
2	0 0 1 2 3 6 7 8
3	2 3 7 9
4	4 6
5	2

c

Key 1|2 = 120

Stem	Leaf
1	0 1 2 4 5 6 7 7 9
2	2 3 4 4 6 8 8
3	0 1 1 1 3
4	3 7 9
5	1
6	
7	7

8 Consider this histogram.



a Create a frequency table with appropriate class intervals to represent the histogram.

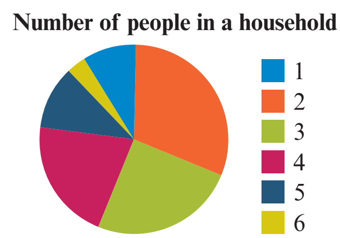
b Find the modal class, the median class and the range.

c Use the frequency table to find the mean.

9 **a** Why is calculating summary statistics from a histogram less accurate than calculating summary statistics from a column graph?

b When might it still be advantageous to use a histogram rather than a column graph?

10 This pie graph was produced by surveying 120 people on the number of people in their household.



a Create a frequency table to represent the pie graph. (Hint: you will need to use a protractor to find each sector size.)

b Find the mean, median, mode and range.

c Why do you think pie graphs are not generally used to represent numerical data?

11 Draw a better graphical display for the data from question **10**. How does this show the centre and spread of data better?

PROBLEM SOLVING AND REASONING

12 Match these summary statistics with these graphs.

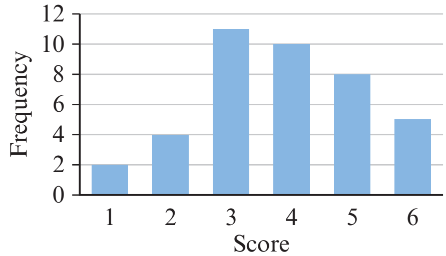
a mean = 3.3, median = 3, mode = 3, range = 5

b mean = 59.6, median = 65, mode = 75, range = 100

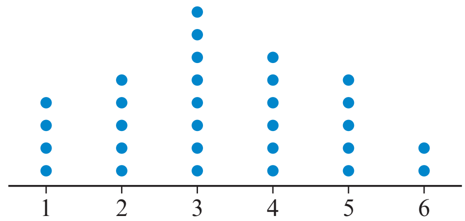
c mean = 3.825, median = 4, mode = 3, range = 5

d mean = 40.74, median = 36.5, mode = 74, range = 97

A



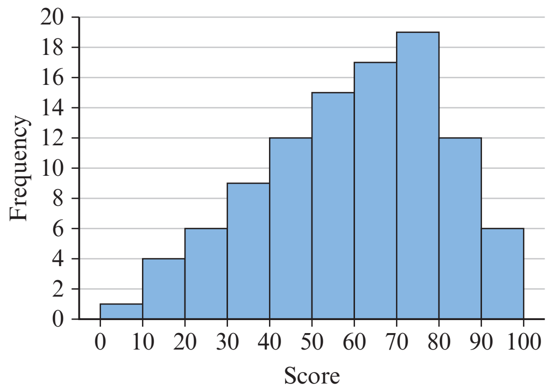
B



C Key 2|1 = 21

Stem	Leaf
0	1 2 6
1	2 6 7 7 8 9
2	0 1 1 2 4 6 7 7 8 9
3	0 3 4 4 5 6 7 9
4	0 1 2 2 5 9
5	0 1 2 3 5
6	0 1 4 8 9
7	4 4 4 4
8	1 9
9	8

D



CHALLENGE

13 Consider this stem-and-leaf plot.

a Find the mean, median, mode and range.

b Calculate the standard deviation.

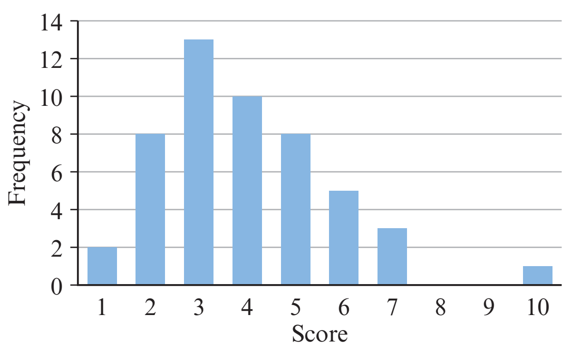
c Write a sentence comparing the measures of centre. Which would you choose to represent the data?

d Write a sentence comparing the measures of spread. Which would you choose to represent the data?

Key 1|4 = 14

Stem	Leaf
0	5 6
1	1 3 4 4 5 9
2	0 3 3 3 5 7 8
3	4 6 8 8
4	2 2
5	1

14 Use this column graph to calculate the measures of centre (mean, median and mode) and measures of spread (range and standard deviation). Which measures would you use to represent the centre and spread of the data? Why?



Reflect

How is calculating summary statistics from a graphical display different from calculating summary statistics from a raw list?

8E Collecting data

Start thinking!

Finn was investigating popular pets in his suburb and wrote down five questions on a survey sheet.

Q1. What pet do you have at home?
Q2. What do you think is the most popular pet?
Q3. What pet would you most like to have at home?
Q4. How many pets do you have at home?
Q5. What do you think is the average number of pets?

1 What type of data is collected for each question?

2 Why might it be important to consider what type of data you get for each response? (Hint: what summary statistics can you use for each data type? How can you display them?)

3 What is the difference between Finn's first and third questions?

4 Which question (**1** or **3**) do you think gives fairer results? That is, which question will tell you more about the most popular pet? Explain.

It is important that all questions in a survey give fair results that are useful to the investigator.

5 Decide and explain which of the five questions would give fair results in an investigation.

Once you have decided what questions to ask, how will you collect the information?
The first thing to decide is who to survey.
The **population** of an investigation is the entire group of people or objects under consideration.

6 What is the population of Finn's investigation?

Usually it is too time-consuming or difficult to survey the entire population (a **census**), so instead you take a **sample** by surveying only some of the population.

KEY IDEAS

- In statistics, a population is the entire group that is important to an investigation.
- A census is a survey of the entire population. Surveying only some of the population is called a sample.
- If a sample does not reflect the population it is said to be **biased**.
- Common sampling methods include **random sampling** (for example, pulling names out of a hat); systematic sampling (sampling at fixed intervals such as every fifth person); and **stratified sampling** (dividing the population into categories such as males and females and taking a random sample from each category that is proportional to its size).
- Using your own collected data is called using **primary data**. Using data that somebody else has collected is called using **secondary data**.

EXERCISE 8E Collecting data

EXAMPLE 8E-1 Classifying a survey as a sample or a census

Eden surveys everybody in town to find out the most popular sport in the district. Decide whether this is a census or sample.

THINK	WRITE
1 Identify the population.	The population is Eden's district.
2 Identify who is surveyed.	Eden's town has been surveyed.
3 A census is an entire population – is the survey taken the same as the population?	The survey taken is not the same as the population, so this is only a sample.

UNDERSTANDING AND FLUENCY

1 Classify each survey as a sample or a census.

- **a** Peter surveys everybody in his class to find out the favourite movie of the entire class.
- **b** Gaylia surveys 40 people at random from her year level to find out the favourite food of the year level.
- **c** Zoë surveys everybody in her class to find out the favourite song in the year level.
- **d** Matt surveys everybody in his family to find what should be the family pet.
- **e** Joel surveys the 25 people in his football club to find out the club's most popular fundraiser.
- **f** Silvia surveys everybody in her street to find out the town's average age.

2 For each situation:

- **i** identify the target population
- **ii** decide whether a census or sample would be more appropriate.

- **a** finding the opinion of the students in your school of a new school rule
- **b** deciding who will be the next Prime Minister of the country
- **c** cooking a meal for your friends and checking for allergies
- **d** finding the favourite music genre of teenagers in your town or suburb
- **e** finding the average electricity usage in Australian households
- **f** finding which local cinema is screening a movie at the best time

EXAMPLE 8E-2 Classifying sampling techniques

Lukas selects a sample of 100 people by surveying every 1000th person in the phone book. Classify this sampling technique as random, systematic or stratified.

THINK	WRITE
1 Look at the sampling definitions in the Key ideas. Which sampling method takes a sample at fixed intervals?	
2 Write your answer.	This sample is systematic.

UNDERSTANDING AND FLUENCY

3 Classify each sampling technique as random, stratified or systematic.

- **a** Renee asks every fourth person she sees at a shopping centre their opinion on a local issue.
- **b** Adrian asks 10 boys and 10 girls from town that he sees on a particular day their opinion on a new school uniform rule.
- **c** Oliver asks every fifth person on the school roll of Year 9s who should be the Year 9 school captain.
- **d** Luisa asks 12 boys and 13 girls from her school of 120 boys and 130 girls what they think the school canteen should offer at lunch times.
- **e** Bridget uses a ‘Lucky dip’ type system in her class to select people to survey.
- **f** Carlos surveys everybody he sees what their favourite movie is.

EXAMPLE 8E-3 Classifying results as biased or fair

Hayden was investigating the Australian public’s opinion on which was better out of AFL or rugby. He asked the question: ‘Which do you prefer: AFL or rugby?’ of every 10th person on the electoral roll from his hometown in Victoria. Classify the results he would obtain as biased or fair, providing a reason.

THINK	WRITE
1 Consider the question that he asks – would this provide fair or biased results? The question relates directly to the topic, so the results should be fair.	
2 Consider his sampling method – would this provide fair or biased results? He uses systematic sampling but the sample he takes only considers one town in Victoria, which may be biased towards one opinion.	The results that he would obtain would be biased because his sample is not representative of the entire population.

UNDERSTANDING AND FLUENCY

4 Classify each sample from questions **1** and **3** as either biased or fair, providing a reason for your answer.

5 For each of these questions, classify the results that would be obtained as biased or fair, providing a reason.

a To investigate average hourly wage rate for high school students in her area, Cynthia asks the question 'What are you paid per hour?' of 200 adults at random from her area.

b To investigate the most popular TV channel in his year level, Luke asks every 20th person on the school roll the question 'What TV channel do you watch most?'.

c To investigate the average number of people in households in his local area, Juan asks everybody he knows the question, 'How many people are in your household?'.

d To investigate the most popular movie genre in her year level, Jasmine asks everybody in the year level the question, 'Do you prefer action or comedy films?'.

6 For each of these scenarios, write three questions that would provide fair answers. Include at least one question that provides numerical data and one question that provides categorical data.

a investigating movie preferences in your school

b investigating opinions about graffiti in your community

c investigating family structure in your community

d investigating technology within the family home

7 For each scenario in question **6**, write a question that would provide biased answers. Include an explanation as to why the answers would be biased.

8 The size of a sample is important when conducting an investigation.

a If you wanted to sample from a group of 1000 people, would it be better to sample 10 people or 100 people? Explain.

b Which is a better sample: 10 people from a group of 50 or 20 people from a group of 200? Explain.

c Explain why the size of a population should be considered when deciding on a sample size.

There is no set minimum sample size in any study. It is obvious that the larger the sample taken, the more likely it is to reflect the population. However, it can be too time-consuming or difficult to always sample large numbers of people.

d For each scenario, state how trustworthy you think the results are.

i A survey finds that Channel 7 is the most popular channel in Tasmania, based on polling 190 homes.

ii A study finds that 86% of women see an improvement in wrinkles after using a moisturiser (36 women are surveyed).

iii Recent data shows that of 421 road deaths in NSW in 2010, speeding contributed to 160.

iv A study surveying 1000 couples worldwide finds that one in three marriages end in divorce.

e Why is it important to consider sample size when looking at the results of an investigation?

9 Explain why small sample sizes can often lead to biased results.

10 Rebecca said that collecting primary data was always better than using secondary data. Explain why it is more important that the sampling method is fair than where the data comes from.

11 A useful website to collect secondary data from is the Australian Government census website. Use the Internet to access http://www.abs.gov.au/websitedbs/censushome.nsf/home/Census for these questions.

a Enter your postcode into the 'QuickStatsSearch' box. This will give you data on your local area and how it compares to the rest of Australia.

b Choose one of these quick statistics and write down the data for your local area. You may wish to include a sentence on how it compares to other places in Australia.

To access more detailed data, click on 'Data' at the left-hand side. You can choose to find data by location or topic.

c Choose to search for data by topic. Use the latest census data.

d Leave the count method as 'Place of usual residence', as this sorts people based on where they usually live. Select a topic from the drop-down box for investigation.

e Select a more detailed topic from the list that appears. If the topics that appear seem confusing, select a different main topic from the drop-down box.

f Click 'Select location' and then enter in the name of your town or suburb. Select your town or suburb from the list that appears and click 'Select product'.

g Click to view the census table and download the Excel file from the 'Downloads' tab.

h Use the Excel document to collect the data and write a paragraph summarising the results found and how this compares to the rest of Australia.

CHALLENGE

12 Ellen wanted to collect some data from her school using stratified sampling. Her year level consists of 60 boys and 40 girls.

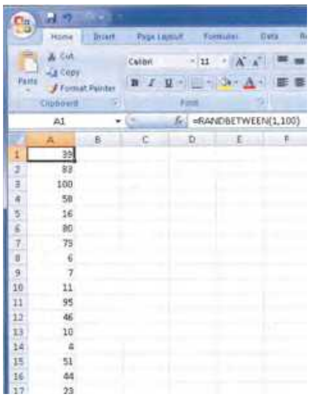
- **a** How many students in her year level in total?
- **b** What fraction of her year level are boys?
- **c** What fraction of her year level are girls?
- **d** If she wanted a sample of 10 people, how many boys and girls should she randomly select?
- **e** If she wanted a sample of 25 people, how many boys and girls should she randomly select?
- **f** Explain how you got your answers to parts **d** and **e**.
- **g** Explain why, if she wanted a sample of n people, that the stratified sample can be found using $n \times \frac{3}{5}$ boys and $n \times \frac{2}{5}$ girls.

13 Determine the structure of the stratified sample if a group of:

- **a** 10 people are to be chosen from 120 girls and 80 boys
- **b** 50 people are to be chosen from 350 adults and 150 children
- **c** 25 people are to be chosen from 180 Year 8s and 195 Year 9s
- **d** 9 animals are to be chosen from 38 birds, 57 cats and 76 dogs.

14 Random sampling often sounds like an easy way of carrying out fair sampling, but in practice it can lead to biased results because people are not good at choosing truly random samples. For small samples a process such as drawing names out of a hat can be used. But what about larger samples? To get unbiased results, random sampling works best when using a pre-gathered list and a random number generator. Microsoft Excel is one program that has a random number generator. Say you want to generate 20 random numbers from a list of 100.

- **a** Open a new Microsoft Excel document.
- **b** Type =RANDBETWEEN(1,100) into cell A1. What do you think the numbers 1 and 100 represent in this formula?
- **c** Either use the 'Fill down' function or copy and paste the cell contents of A1 down the column to fill the first 20 cells. This will generate 20 numbers for you.
- **d** Write down the 20 numbers that you generate.
- **e** How can this list of numbers be used to represent a list of 100 people?
- **f** What would you do if the same number was generated twice? Would you survey that same person twice?



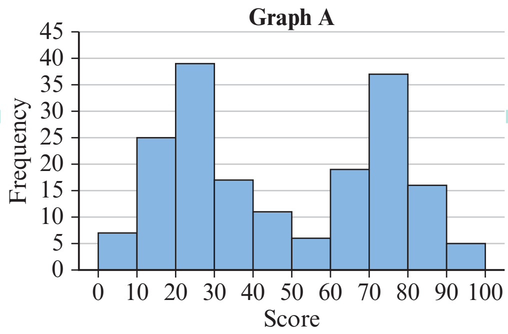
Most modern calculators also have a function that will generate random numbers.

- **g** Investigate your calculator and generate another 20 random numbers between 1 and 100. Remember to continue generating until you have 20 unique numbers. See your teacher if you need help.

Reflect

What needs to be considered in order to gain fair results from an investigation?

8F Describing data



Start thinking!

There are many ways to describe the distribution of data, such as using summary statistics to describe the data's centre and spread. A simpler way that provides a quick overview is to describe the shape of the data's distribution. Consider these three graphs.

1 Which of these graphs would you describe as roughly **symmetric**? Why?

2 Do you think it is important for a graph to be perfectly symmetric? Explain.

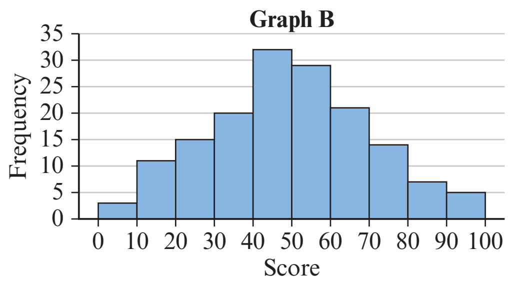
3 Which of these graphs would you describe as **skewed**? Explain.

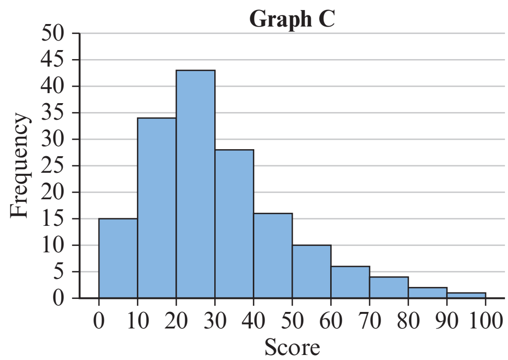
Skewed graphs can be further described as **positively skewed** or **negatively skewed**. A graph that is positively skewed is skewed towards the vertical axis. A graph that is negatively skewed is skewed away from the vertical axis.

4 Describe the skewed graph as either positively or negatively skewed.

The remaining graph can be described as **bimodal**.

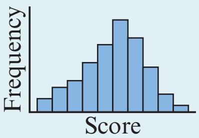
5 Explain why you think it gets this name.



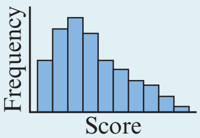


KEY IDEAS

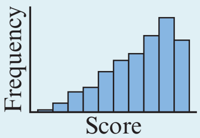
- Symmetric distributions have a middle peak and a roughly even spread on either side.



- Positively skewed distributions have a centre closer to the left of the distribution.



- Negatively skewed distributions have a centre closer to the right of the distribution.

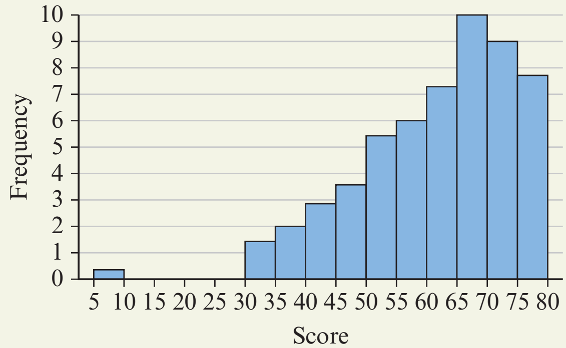


- An outlier is an unusual piece of data that is far away from the rest of the distribution.
- Any distribution that is skewed or has an outlier should have its centre described using the median rather than the mean.
- Bimodal distributions are more difficult to describe with statistics, and are best described using the mode of each peak.

EXERCISE 8F Describing data

EXAMPLE 8F-1 Describing the distribution

Describe the distribution of this histogram.



THINK

1 Is the distribution symmetric, skewed or bimodal?

2 Does the distribution have an outlier?

3 Write your answer.

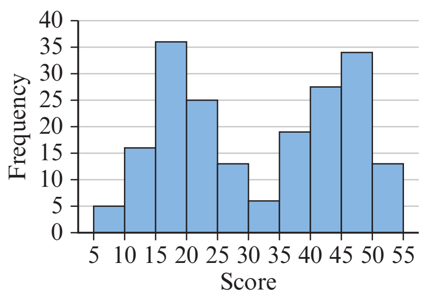
WRITE

The distribution is negatively skewed with an outlier.

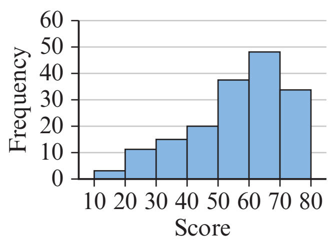
UNDERSTANDING AND FLUENCY

1 Describe the distribution of each histogram.

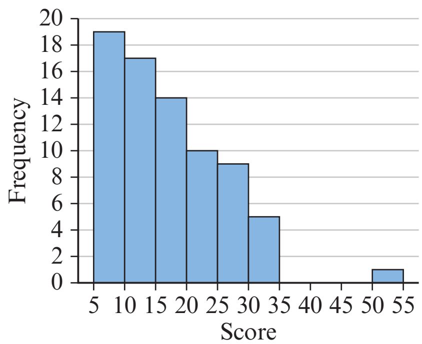
a



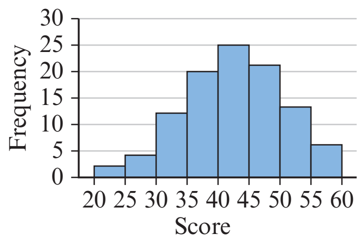
b



c



d



2 Describe the distribution of each stem-and-leaf plot.

a

Key 1|3 = 13

Stem	Leaf
1	0 2 5
2	1 1 4 5 6
3	0 4 5 6 8 8 8 9
4	4 5 9
5	3 3

b

Key 1|3 = 13

Stem	Leaf
0	1 4 6 8 8 9
1	3 4 5 5 6 6 7 7 8 9
2	1 2 4 4 5
3	4 8 9
4	3
5	1

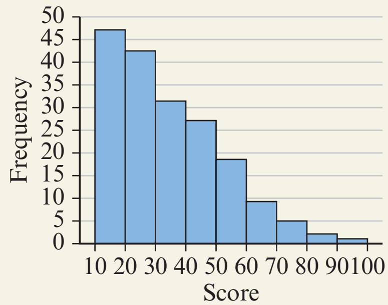
c

Key 1|3 = 13

Stem	Leaf
0	9
1	
2	
3	1 9
4	4 5 6 8 8
5	4 5 5 6 7 7 9
6	0 1 1 2 4 5 6 6 7 8

EXAMPLE 8F-2 Deciding which measure of centre best describes a distribution

Decide which measure of centre would best describe this distribution.



THINK

1 Describe the distribution.

2 The mean is affected by skew and outliers. In these cases it is better to use the median.

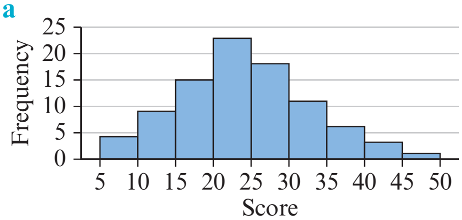
WRITE

The distribution is positively skewed.

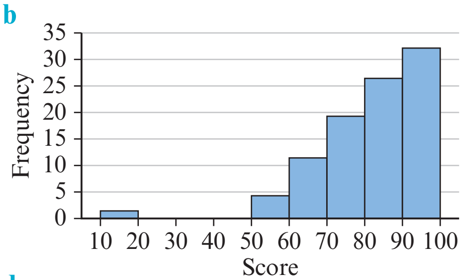
The centre of the distribution would be best described by the median, as it is skewed.

3 Decide which measure of centre would best describe each distribution.

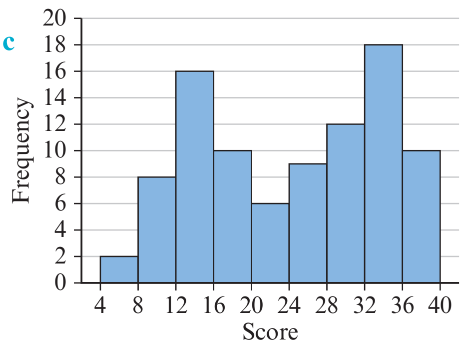
a



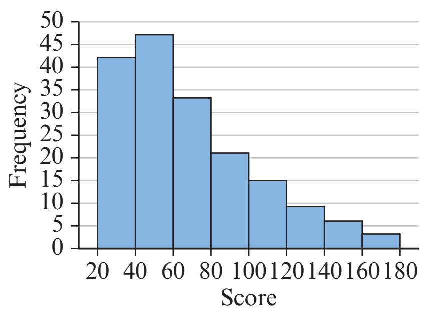
b



c



d



4 Decide which measure of centre would best describe each distribution in question **1**.

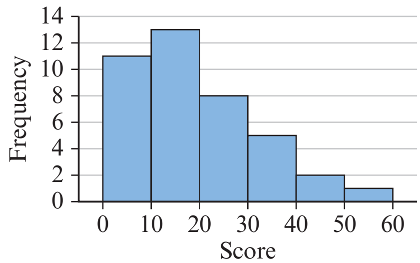
EXAMPLE 8F-3 Describing a histogram

Use this data to:

a draw a histogram b describe its distribution

c state the best measure of centre, providing a reason.

49, 3, 16, 12, 20, 49, 22, 37, 32, 18, 34, 13, 4, 7, 17, 9, 13, 59, 25, 1,
15, 30, 23, 27, 4, 17, 26, 3, 10, 5, 8, 2, 31, 11, 8, 20, 27, 13, 16, 11

THINK	WRITE
a Draw a histogram with an even scale and labels on both axes. The data has a range of 58, so class intervals of 10 would be appropriate.	a 
b Decide if the data is skewed.	b The distribution is positively skewed.
c Identify the best measure of centre. When the data is not symmetric, the median should always be used.	c The median should be used as the measure of centre because the distribution is skewed and therefore the mean may not be an accurate measure of centre.

UNDERSTANDING AND FLUENCY

5 For each data set:

i draw a histogram ii describe its distribution

iii state the best measure of centre, providing a reason.

a 35, 33, 42, 99, 54, 68, 4, 91, 97, 55, 99, 86, 40, 58, 41, 95, 38, 62, 35, 88, 82, 98, 77, 69, 78, 78, 82, 81, 98, 57, 88, 41, 60, 85, 82, 85, 91, 90, 80, 49, 58, 66, 97, 95, 82, 84, 78, 91, 62, 42

b 9, 2, 3, 9, 43, 8, 2, 15, 12, 17, 10, 10, 14, 9, 34, 7, 12, 18, 18, 47, 2, 12, 24, 34, 19, 1, 12, 18, 35, 47, 6, 14, 8, 35, 7, 9, 4, 17, 2, 20, 8, 12, 21, 24, 48, 6, 7, 8, 17, 41

c 49, 23, 45, 23, 31, 77, 62, 21, 52, 51, 60, 54, 46, 69, 27, 60, 142, 41, 32, 80, 52, 21, 80, 65, 37, 33, 74, 45, 48, 78, 70, 21, 55, 64, 33, 42, 59, 67, 32, 79, 30.

6 This stem-and-leaf plot does not seem to have much of a noticeable pattern to it.

Key 1|3 = 13

Stem	Leaf
1	0 0 0 1 2 2 2 2 2 3 3 5 5 5 5 5 6 6 7 8 9
2	0 0 1 2 2 2 3 3 3 9

a Redraw it as a split stem-and-leaf plot with:

i four stems ii ten stems.

b Describe the distributions that you see.

7 Ava collected data on the weight of dogs (in kilograms) in her community and put it into the table as shown at right.

a Draw a histogram to represent the data.

b What pattern can you see?

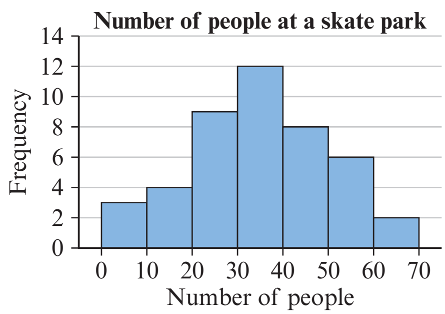
c Reorder the data into more appropriate class intervals and redraw your histogram.

d What pattern can you now see?

e Write a sentence describing the weight distribution of dogs as shown in your second histogram.

Class interval	Frequency
0–<2	1
2–<4	6
4–<6	3
6–<8	5
8–<10	6
10–<12	5
12–<14	8
14–<16	6
16–<18	7
18–<20	3
20–<22	7
22–<24	2
24–<26	5
26–<28	3
28–<30	4
30–<32	5
32–<34	4
34–<36	2
36–<38	1
38–<40	4

8 This histogram shows the number of people at a skate park recorded at various times.



a Describe the shape of the distribution.

b What measure of centre would be the most appropriate to use?

c Create a frequency table that represents the histogram.

d Use the frequency table to calculate the appropriate measure of centre.

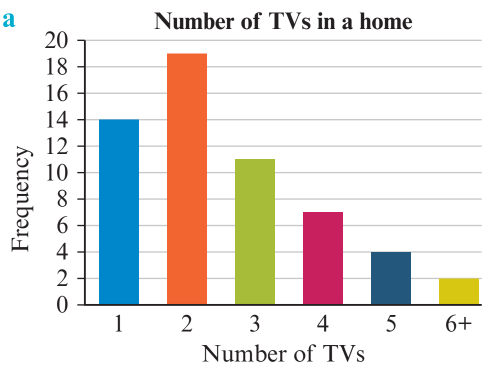
e Write a sentence that describes the distribution in terms of the number of people at the skate park.

9 For each data distribution:

i describe its shape **ii** calculate the appropriate measure of centre

iii write a sentence that summarises the graph in its context.

a

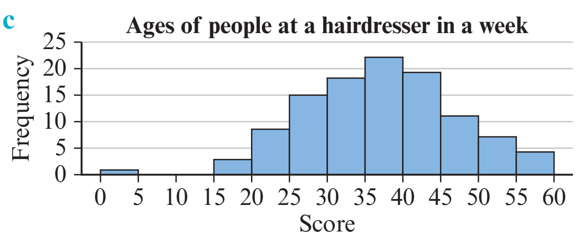


b **Length of hair in Year 9 students (cm)**

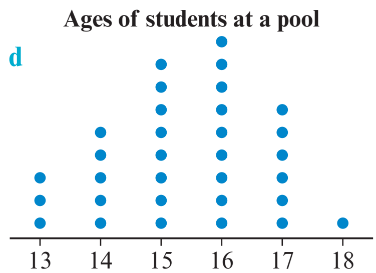
Key 1|3 = 13

Stem	Leaf
0	1 2 2 2 3 3 3 4 4 6 6 6 6 7 8 8 8 9
1	0 2 4 6 8
2	0 1 5 8 8 9 9
3	0 1 2 2 2 2 2 4 5 5 5 5 6 7 8 9 9
4	1 2 2 4 4 5 6 6 8 9
5	0 4 5 6 8 8 8
6	2 9

c



d

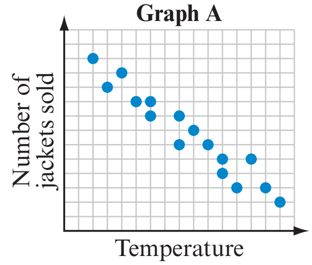


CHALLENGE

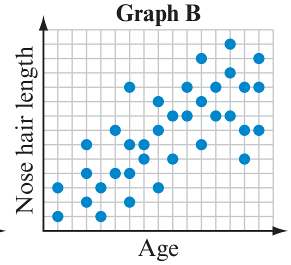
10 The relationships represented in scatterplots can be described in terms of their direction (positive or negative) and strength (strong, moderate, weak or none).

Consider these two graphs.

Graph A



Graph B



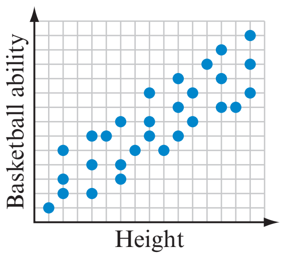
a Which would you consider to show a positive trend? Explain.

b Which would you consider to show a strong relationship? Explain.

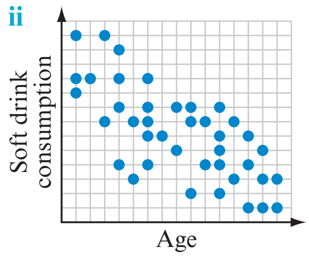
c Can you describe a graph that shows no relationship in terms of its direction? Explain.

d Describe each of these graphs in terms of both their direction and strength.

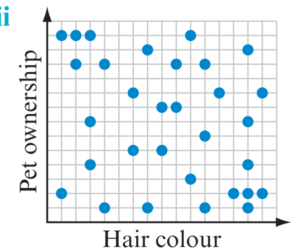
i



ii



iii



11 Describe each of the scatterplots from question **10** in context. For example, the first scatterplot could have the description 'the number of jackets sold decreases strongly as temperature increases'.

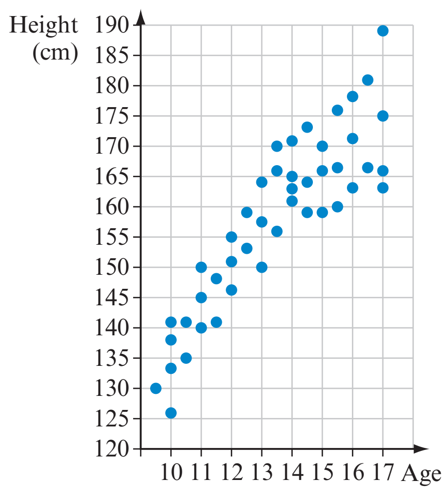
12 Scatterplots that have moderate to strong relationships can be used to make predictions.

a Consider Graph A shown in question **10**. If the temperature was cold, would you predict that you would sell many or few jackets?

b Consider the first scatterplot shown in question **10d**. If somebody was tall, what kind of basketball ability would you predict that they had?

c Consider your answer to part **b**. Does this mean that if somebody is tall that they will be good at basketball? Explain.

Consider the scatterplot on the right.



d Describe the scatterplot in context: what is it showing?

e A friend said that the graph must be wrong because it implies that a 30-year-old has a height of about 270 cm. Explain why their reasoning is incorrect.

f Use your answer to part **e** to explain why a graph should only be used to make predictions within the data range that it shows.

Reflect

In what ways can you describe data? How is this useful?

8G Comparing data

Start thinking!

When you want to compare two data sets, it is useful if they are in the same format so that the comparison is easier. One graphical display that can be used to easily compare two data sets is a back-to-back stem-and-leaf plot. Consider this plot, showing the ages of males and females in a gaming store.

Key: 1|3 = 13

Leaf Males	**Stem**	**Leaf Females**
	0	2 7 9
9 8 7 7 7 7	1	1 4 5 6 7 9
9 8 7 6 4 4 3 2	2	0 1 1 5 6 8 8
9 7 5 5 4 2 2	3	1 2 2 4 9 9 9
7 6 2 2	4	2 4 5 7
9 1	5	2 3
2	6	
	7	9

1. How is this plot different from and how is it the same as a standard stem-and-leaf plot?
2. What is the minimum and maximum age of females in the store?
3. What is the minimum age and the maximum age of males in the store?
4. What is the overall range of ages in the store?
5. How would you describe the distribution of the females in the store?
6. How is this different from the distribution of the males in the store?
7. Write a sentence comparing the ages of males and females in the store. Is one gender more likely to be older?
8. Why is it easier to place the two data sets into the same plot rather than drawing a separate plot?
9. For each data set, find the mean (to the nearest whole number), median and range and place them into a table.
10. Compare the ranges of the two data sets. Does this support your answer to question **7**?
11. Write a sentence comparing the centres of the two data sets. How does this compare to your answer to question **10**?

KEY IDEAS

- To briefly compare two data sets, it is easiest to place the data into a graphical display where the shape of the distributions can easily be compared.
- One way to do this is to construct a back-to-back stem-and-leaf plot and compare the distribution of the leaves.
- Other graphs, such as column graphs, histograms and dot plots can be used to do this as well.
- To make a more thorough comparison of two data sets, summary statistics should be calculated for each data set and the centre and spread of each set compared.

EXERCISE 8G Comparing data

EXAMPLE 8G-1 Comparing data sets by drawing a back-to-back stem-and-leaf plot

Use this data to draw a back-to-back stem-and-leaf plot and make a brief comparison of the two data sets.

Age of people at a pool:

Winter: 45, 23, 15, 36, 57, 31, 9, 38, 44, 56, 52, 13, 36, 27, 48, 44, 48, 14, 27, 45

Summer: 31, 16, 14, 15, 23, 56, 24, 18, 17, 8, 11, 13, 16, 21, 17, 36, 20, 17, 14, 15

THINK

1 Both sets of data share the same stem, so locate the minimum and maximum numbers across both sets.

2 The youngest age is 8 and the oldest age is 57, so you should have six stems: from 0 to 5.

3 Draw the back-to-back stem-and-leaf plot with these stems, placing the winter leaves on the left of the stems and the summer leaves on the right. Rearrange the leaves so that they are in order and include a key.

4 Look at the centre and spread of the two data sets. Where does the centre appear to be for each set? What does this mean when you think about ages?

WRITE

Key: 1|2 = 12

Leaf Winter	Stem	Leaf Summer
9	0	8
5 4 3	1	1 3 4 4 5 5 6 6 7 7 7 8
7 7 3	2	0 1 3 4
8 6 6 1	3	1 6
8 8 5 5 4 4	4	
7 6 2	5	6

The two data sets are spread roughly over the same age brackets, but the data sets are skewed in different directions. The pool seems to attract younger people in the summer and older people in the winter.

1 Use this back-to-back stem-and-leaf plot to answer these questions.

a What is the maximum score:

i in group A? ii in group B?

iii overall?

b What is the most common score:

i in group A? ii in group B?

iii overall?

c How would you describe the distribution of:

i group A? ii group B?

Key: 1|2 = 12

Leaf Group A	Stem	Leaf Group B
9 7 4 3 1	0	7 9
9 8 8 6 4 3 2 1 0	1	3 4 6 8
7 5 3 0	2	0 1 1 4 4 4 5 8 9
8 3	3	2 2 3 4 7 9 9
1	4	0 3 5

UNDERSTANDING AND FLUENCY

2 For each back-to-back stem-and-leaf plot, make a brief comparison of the two data sets.

a

Key: 1|2 = 12

Leaf Group A	Stem	Leaf Group B
1	1	0 1 3 5 8 8
8 7 2	2	2 4 5 6 7 8 9 9
9 7 6 3 1	3	4 6 7 8
9 8 7 7 6 5 3 1 1	4	0 1 2
9 7 6 4 3 1 0	5	1 3

b

Key: 3|2 = 3.2

Leaf Group A	Stem	Leaf Group B
	3	0 0 1 2 4 5 6
9 8 7 6 5 4 4 4	4	1 2 5 5 6 7 8 9 9
8 7 6 6 5 4 3 1 1 0	5	0 3 4 5 5 6 8
8 7 6 5 4 3 3 0	6	
	7	
	8	1

c

Key: 1|2 = 120

Leaf Group A	Stem	Leaf Group B
	0	2 3 4
9 7 6 5 5 5	1	1 7 8 9 9
9 8 7 7 6 5 4 4 3	2	0 2 3 5 5 7 9
9 7 6 5 4 4 4 3	3	0 4 6 7 8 8
3 2 2 1	4	0 1 5 6
	5	0 6

d

Key: 1|2 = 12

Leaf Group A	Stem	Leaf Group B
	5	1
9 8 7 1	6	
7 6 5 4 3 3 3 2 1	7	8 9 9
9 8 7 5 4 2 2 1 0 0	8	1 2 3 4 6 7 8 8
	9	0 0 5 6 7 8 8 8 9

3 For each of the following, draw a back-to-back stem-and-leaf plot and use it to make a brief comparison of the two data sets.

a Number of people at a cinema over 6 months of weekends:

Friday: 65, 48, 67, 55, 32, 92, 64, 51, 49, 57, 76, 61, 29, 46, 61, 59, 53, 67, 72, 88, 71, 58, 54, 57, 56, 42, 30

Saturday: 67, 78, 84, 26, 37, 42, 99, 84, 75, 68, 64, 55, 75, 85, 59, 66, 77, 78, 83, 81, 92, 94, 77, 76, 79, 89

b Daily maximum temperature in February

Darwin: 32.3, 32.1, 32.9, 33.2, 33.5, 33.8, 33.4, 32.8, 32.5, 33.3, 32.5, 29.5, 32.8, 33.4, 33.1, 32.7, 33.4, 31.4, 33.0, 31.3, 29.9, 30.8, 30.0, 32.2, 32.1, 32.6, 31.8, 30.4

Adelaide: 20.6, 23.7, 25.4, 29.4, 34.2, 38.4, 27.0, 28.3, 28.0, 26.3, 28.3, 31.2, 33.8, 34.3, 36.0, 37.1, 39.2, 40.5, 26.6, 28.4, 32.1, 33.8, 35.6, 38.2, 28.7, 33.4, 21.5, 22.7

c Mass of dogs of two breeds (to nearest hundred grams)

Boston Terriers: 5100, 6800, 7800, 10500, 9200, 5500, 4900, 11200, 8600, 9200, 10500, 4900, 5500, 7600, 8400, 6800, 9200, 8600, 7500, 4600, 8300, 10500, 11000, 8000, 7600, 5100, 6700, 8300, 9200, 10000, 4900, 7500, 7900, 6200, 8200, 4600, 7800, 8100, 6400, 9900.

French Bulldogs: 9800, 12900, 8600, 9900, 9800, 12500, 11200, 9100, 12900, 11200, 9500, 9200, 10500, 11900, 12500, 11800, 9400, 9800, 10800, 10600, 11500, 9600, 12300, 13000, 11000, 12000, 10600, 11100, 12700, 10900, 9500, 11500, 12900, 10100, 11100, 9200, 11500, 10900, 9800, 11300.

EXAMPLE 8G-2 Comparing data sets using summary statistics

Rachel was investigating the average age of customers in two different cafés in her local area. Use her results to make a comparison of the two different cafés.

Café	Mean	Median	Range
Gumtree	21	22	17
Jinx	27	21	45

THINK	WRITE
1 Compare the given measures of spread (the range).	Jinx covers a larger range of ages than Gumtree.
2 Compare the given measures of centre: the medians are similar but Jinx has a higher mean than Gumtree. This difference combined with Jinx's large range suggests that Jinx's distribution may be skewed or affected by an outlier. The median should be used as the measure of centre.	The average age for both cafés is similar, though Jinx may have a skewed distribution and possibly an outlier present.
3 Summarise your findings.	The average customer age at the cafés is similar but Jinx has a larger range of ages.

UNDERSTANDING AND FLUENCY

4 For each table, use the given statistics to make a comparison of the two data sets.

a

Heights in class	Mean	Median	Range
9A	172 cm	168 cm	46 cm
9B	166 cm	167 cm	22 cm

b

Average jeans price	Mean	Median	Range
Store A	$120	$122	$35
Store B	$124	$122	$30

c

Average Internet usage per week	Mean	Median	Range
Primary school	4.5 hours	4 hours	8 hours
Secondary school	11.2 hours	12 hours	17 hours

d

Average SD card capacity	Mean	Median	Range
Store A	20 GB	16 GB	60 GB
Store B	16 GB	16 GB	60 GB

e

Goals per game	Mean	Median	Range
Player A	4.2	4	9
Player B	4.8	4	7

5 Consider the column graph on the right.

a What is it showing?

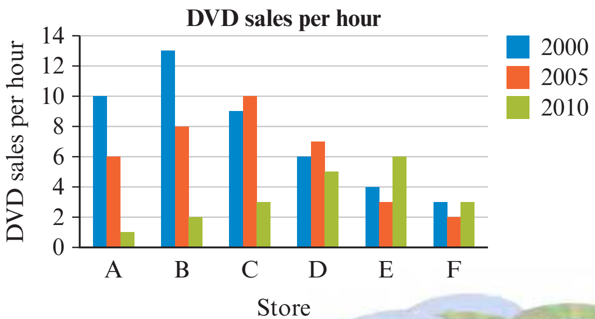
b How many data sets does it display?

c Describe the pattern shown in:

i 2000 **ii** 2005 **iii** 2010.

d What patterns can you see?

e Write a statement that summarises the graph in terms of the trends throughout each year and from 2000 to 2010.



PROBLEM SOLVING AND REASONING

6 Draw a back-to-back stem-and-leaf plot that you imagine would compare heights of Year 7s and Year 9s. Describe and explain the patterns that you choose to display.

7 Carly was investigating the average age of people visiting a new auction website. Using a fair sampling method, she took three separate samples (A, B and C, as shown). Explain which statistics Carly should use to summarise her findings.

Age of people visiting website			
	Sample A	Sample B	Sample C
Sample size	20	40	60
Mean	22	25	29
Median	20	26	25
Range	10	13	60

8 The local council was investigating how much people spend on computer games every year, and called for submissions. Only three submissions were made, shown in these tables.

Sample A	200 people at a gaming store
Mean	$450
Median	$395
Range	$200

Sample B	150 people from the state
Mean	$200
Median	$190
Range	$250

Sample C	50 local people
Mean	$210
Median	$165
Range	$400

Explain which sample and which statistic the council should use to predict the characteristics of the population.

9 Consider this back-to-back stem-and-leaf plot.

a Make a brief comparison of the two data sets, writing your answer in context.

b Calculate the mean, median and range for both data sets displayed in the stem-and-leaf plot.

Heights of Year 9 students (cm)

Key: 1|2 = 120

Leaf Boys	Stem	Leaf Girls
	14	8
9	15	6 7 9 9
9 9 8 7 5 4 3 1	16	0 0 1 1 2 2 2 3 4 6 7 8
9 8 8 7 6 5 5 4 3 3 3 1	17	0 1 2 3 4
7 3 2 1	18	2
2	19	

c Use these statistics to make a more detailed comparison of the two data sets.

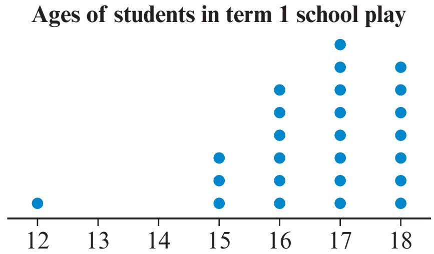
d Does your answer to part **c** support your answer to part **a**? Explain.

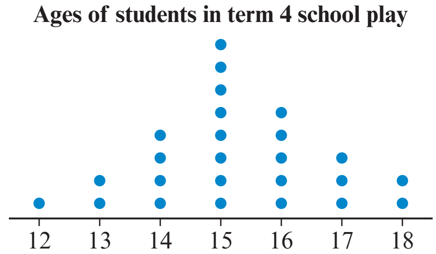
10 Use data from the latest census on the Australian Bureau of Statistics website to compare a characteristic of your choice between your local area and another area. See the process outlined in Exercise 8E question **11** (page 388) if you need help.

11 Reports in media will sometimes run data comparison on population means or medians from different countries. We have seen that a large range of data is available from the Australian Bureau of Statistics, but what about for other countries? Use the Internet or other means to investigate how reports and surveys find their data to estimate population characteristics.

PROBLEM SOLVING AND REASONING

12 **a** Compare the shape of these two dot plots.



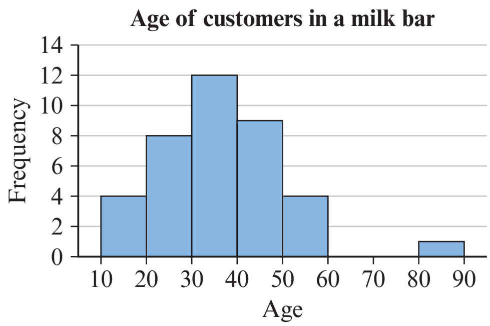


b Find the mean, median and range of each dot plot.

c Write a couple of sentences comparing the two data sets with reference to their summary statistics. Be sure to write in terms of what data is being presented.

d Can you think of a way to represent both dot plots using the same 'axis'?

13 **a** Calculate the mean, median and range for these two data displays.



Age of customers in a milk bar

Key 1|4 = 14

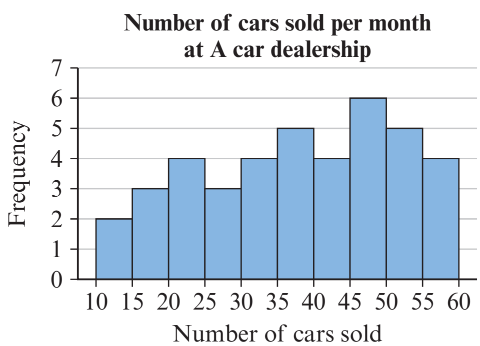
Stem	Leaf
1	0 1 2 5 5 5 5 6 7 8 9
2	0 0 1 2 2 2 3 4 5 7 8 8 9
3	0 1 2 2 3 4 5 7 7 8
4	2 2 4 5 5 6 9
5	4 8 9
6	3 6
7	2

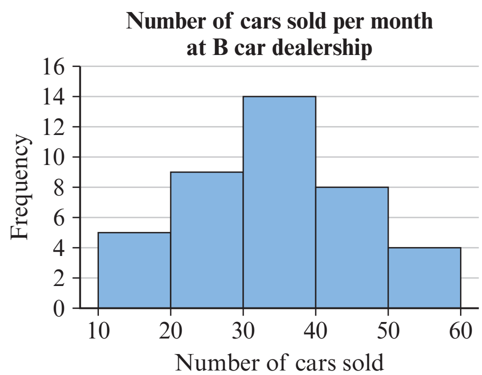
b Compare these two data sets using their shape and summary statistics.

c Explain why you can easily compare the sets even though the displays are different.

CHALLENGE

14 Consider these two histograms.





a Why is it difficult to make a quick comparison of the two histograms as they are?

b Redraw the first histogram so that its class intervals match those of the second.

c Use your answer to part **b** to make a comparison of the two data sets.

d Why can't you redraw the second histogram to match the first histogram?

14 Compare the data sets for the original histograms in question **14**. How does this compare to your answer to question **14c**?

Reflect

What needs to be considered when comparing two data sets?

CHAPTER REVIEW

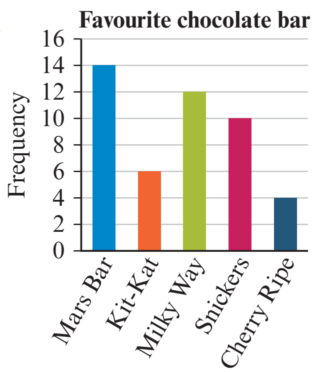
SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

numerical	stem-and-leaf plots	range	primary data
continuous	frequency table	standard deviation	secondary data
line graphs	class intervals	census	symmetric
scatterplots	histogram	sample	skewed
histograms	mean	fair	positively skewed
column graphs	mode	systematic sampling	negatively skewed
bar graphs	median	stratified sampling	bimodal
pie graphs	centre	random sampling	back-to-back stem-and-leaf plot
dot plots	spread	biased	

MULTIPLE-CHOICE

Questions **1** and **2** refer to this column graph.



8A **1** The number of people surveyed is:

A 46 **B** 5
C 14 **D** 6

8A **2** The most popular chocolate bar for the group surveyed is:

A Cherry Ripe **B** Milky Way
C Kit-Kat **D** Mars Bar

8C **3** If the score of 33 is removed from the data set below, which of these statements is not true?

12, 15, 12, 17, 19, 25, 15, 11, 13, 18, 12, 16, 12, 19, 33, 20

A The mode is unchanged.
B The mean and median are both reduced in value.
C The range is smaller.
D The measures of centre are unchanged.

8E **4** Which of these is an example of stratified sampling?

A pulling names out of a hat
B asking every 10th person in a crowd
C dividing a crowd of 100 into males and females, then surveying 10 people from each group
D interviewing your parents about their political views

8F **5** Which statement is true if describing distribution of data?

A Symmetric distributions have a centre closer to the right of the distribution, away from the y-axis.
B Any distribution that is skewed or has an outlier should have its centre described using the median rather than the mean.
C Positively skewed distributions have a middle peak and a roughly even spread on either side of this peak.
D Negatively skewed distributions have a centre closer to the left of the distribution, towards the y-axis.

SHORT ANSWER

8B **1** Data was collected on the number of hours Year 9 students spent using various forms of social media over the course of a week.

45 21 37 21 20 17 31 32 11 15 17 18
20 31 48 32 21 5 11 7 19 18 27 42
40 21 32 23 24 19 38 37 14 15 19 28
22 35 41 33 27 2 10 5 18 18 25 43

a Select an appropriate class interval and construct a frequency table.

b How many people were surveyed?

c What is the most common class interval?

d Draw a histogram to represent this data.

e Construct a stem-and-leaf plot to represent this data.

f Present this data in a stem-and-leaf plot using class intervals of 5.

g Which interval is the most common?

8C **2** Find the mean, median, mode and range for this data set (correct to two decimal places where appropriate).

Score	Frequency
3	10
4	12
5	23
6	5

8C **3** **a** Calculate the mean, median, mode and range (correct to one decimal place where appropriate) for the data set:

25.7 23.3 24.6 26.7 58.9 17.6
25.7 21.0 29.6 20.9 23.6

b Identify the outlier in this data set.

c If the outlier was removed from the data set, what changes do you predict will occur to the measures of centre?

d Remove the outlier. Recalculate the measures of centre to test your prediction.

8D **4** Calculate the mean, mode, median and range for the data represented in this stem-and-leaf plot (correct to two decimal places where appropriate).

Key 2|4 = 24

Stem	Leaf
0	2 4 5 7 7 7 7
1	5 8 8 8 9
2	1 1 3 4 7 9
3	7 8 8
4	1 2 5
5	8

8G **5** The results achieved in Maths tests for two different classes are recorded in a back-to-back stem-and-leaf plot. Find the mean, median, mode and range for each class. Compare the Maths results of the two classes.

Key: 4|3 = 43

Leaf Class 9A	Stem	Leaf Class 9B
8 5 5 3	4	6 7 8 8 9
9 7 3	5	1 3 6 7
7 6 5 4 3	6	3 4 7
0	7	2 4 8
9 7 5 3 1	8	2 5 6 7
0	9	8 9

8G **6** The following data comparing the number of hours spent at football training per fortnight were recorded for Year 9 boys from two different secondary schools.

School 1: 35 11 27 11 10 7 21 22 1 5 7 8 10 21 38 22 11 5 1 7 9 8 17 32

School 2: 12 14 16 17 20 11 22 11 11 16 19 14 2 4 10 19 17 6 4 6 2 7 16 31

a Collate the data in a back-to-back stem-and-leaf plot using class intervals of 5.

b Calculate the mean, median and range for each school (correct to one decimal place).

c Which measure of centre would you use to discuss the results for each school?

NAPLAN-STYLE PRACTICE

1 Which of these is classified as ordinal data?

- eye colour (hazel, blue, green)
- pizza sizes (small, medium, family)
- the number of SMS messages sent in a month
- the time taken to walk from home to school

2 Which of these would provide primary data for you to work with?

- visiting a website and using the data about television viewing habits
- surveying your class about their favourite television show
- using information from social media forums about television programs
- using your friend's survey results

Questions **3** and **4** refer to the following data, which is listed below and also displayed in the stem-and-leaf plot.

25, 27, 29, 41, 32, 35, 20, 27, 49

Stem	Leaf
2	5 7 7 9
3	2 5
4	1 9

3 Which score is missing in the stem-and-leaf plot?

4 What is an appropriate key for this stem-and-leaf plot?

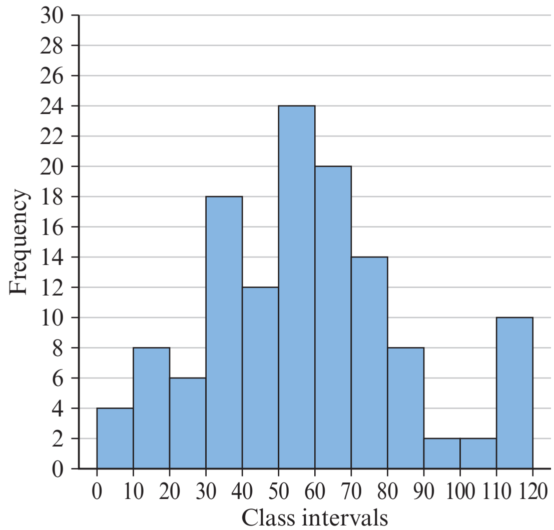
Questions **5** and **6** refer to this frequency table.

Score	Frequency
24	97
25	111
26	378
27	246
28	
29	301
Total	1325

5 What is the missing value in the frequency table?

6 Find the percentage of scores that are 25 or less (correct to one decimal place).

Questions **7** and **8** refer to the histogram below. The histogram displays the results of research where the heights of plants (in cm) were measured.



7 How many plants are less than 70 cm tall?

- 92
- 36
- 20
- 14

8 What percentage of plants (correct to two decimal places) are 100 cm or taller?

- 1.56%
- 12%
- 9.38%
- 90.62%

Questions **9** and **10** refer to the data set below.

15 17 18 45 13 15 15 16 15

9 Which value is closest to the mean of this data set?

- 18
- 15
- 32
- 19

10 A score was recorded incorrectly in the list of data. If the score of 45 should be 15, which statement is true?

- The range will be unchanged.
- The mean will be unchanged.
- The mode will be unchanged.
- The median will change.

11 A friend purchased 12 concert tickets for a total of $1188. What is the average price per ticket?

12 The heights of a group of Year 9 students were recorded, as follows:

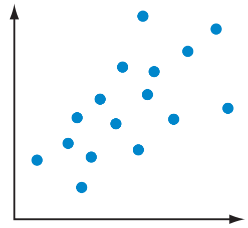
145 cm, 152 cm, 147 cm, 1.35 m, 165 cm, 170 cm.

Which statement is false?

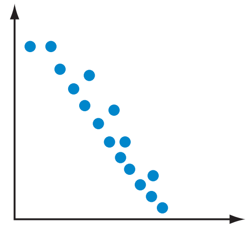
- ☐ The mean height is 1.52 m.
- ☐ The standard deviation is large, indicating a significant spread from the mean.
- ☐ The range is 35 cm.
- ☐ The median height is 152 cm.

13 Which graph shows a strong negative linear relationship?

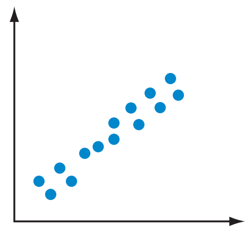
☐ Graph A



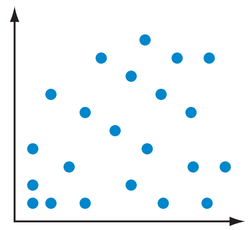
☐ Graph B



☐ Graph C



☐ Graph D



ANALYSIS

This stem-and-leaf plot shows the measurements of shrubs taken at different nurseries.

Key: 15|4 = 15.4 cm

Leaf Brisbane	Stem	Leaf Sydney
7 4 4 4	11	4 4 4 5 7 8 9
7 6 4 3 2 2	12	1 3 6 7
8 7 6 2 1	13	2 3 9
1 0	14	1 4 6 7 8 9
9 8 5 3 2	15	1 2 5 5

a Compare the number of shrubs measured at each location.

b Calculate the mean, median, mode and range of shrub heights at each location (correct to two decimal places).

c Calculate the standard deviation of shrub heights at each location.

d Write a short comparison of the height of shrubs at the two locations.

The manager of the nursery company wanted to collate the data for all of his businesses.

e Generate lists of raw data for Brisbane and Sydney. Using this data and the lists below, create a frequency table using suitable class intervals and collate all of the data about shrub height.

Victoria: 12.1, 11.7, 18.3, 11.4, 11.4, 14.5, 15.6, 17.1, 16.5, 18.6, 13.0, 12.6, 12.4, 10.9, 11.4, 14.0, 16.9, 17.1, 16.5, 18.6

Western Australia: 10.2, 10.6, 19.3, 11.4, 11.4, 15.9, 14.7, 18.3, 17.7, 13.4, 10.0, 12.5

f How many plants were measured in total?

g What is the modal class?

h Represent this data as a histogram and comment on the shape of the distribution.

i Calculate the mean shrub height for Victoria and Western Australia.

j Hence calculate the average height of shrub across all locations.

k Write a statement that could be used in a marketing campaign, which includes information about the smallest and largest shrubs and the average shrub height.

CONNECT

Investigating your local community

When a local council wants to make changes or seek input from the community, they will often survey a sample of the population in order to help them make their decisions. What issues do you think are important in your local community at the moment?

Your task

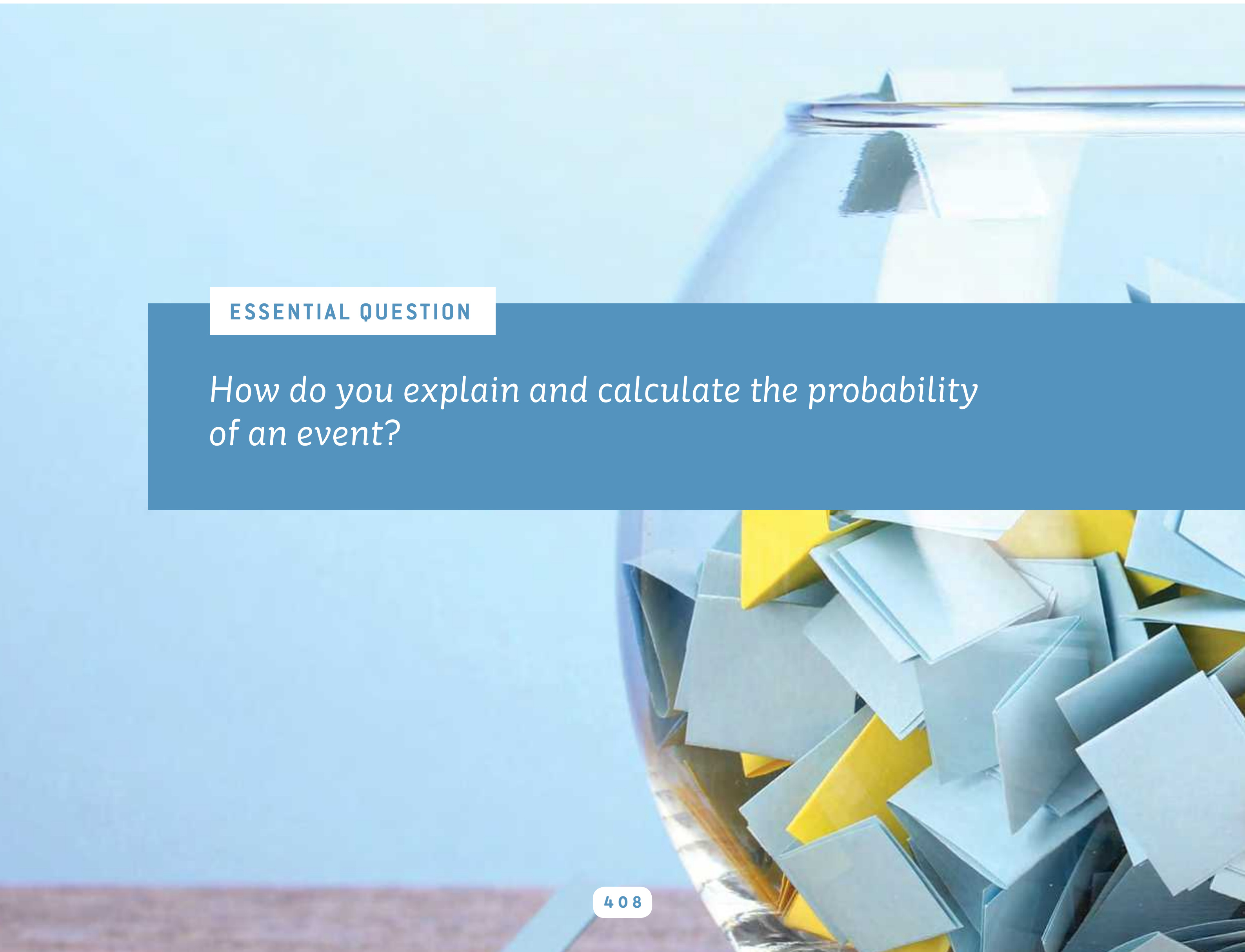
You need to:

- choose a topic relevant to your local community to investigate
- decide what information you want to discover
- formulate at least five questions that provide numerical and categorical data and that provide fair, unbiased results
- produce a survey sheet that is easy to fill out and provides information from which you can collate results into displays
- decide on a fair sampling method and the number of people you will survey
- perform your survey
- display your results in at least three forms (for example, table, histogram, stem-and-leaf plot)
- describe your numerical results according to its shape
- calculate summary statistics and use these to provide a more thorough analysis
- access a secondary source of data (for example, census data) in order to compare to your results for your local area and another community (another town, the state, the country)
- write a summary that describes your results and compares them to your second chosen locale.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a digital presentation
- an information pamphlet
- other (check with your teacher).

9 PROBABILITY



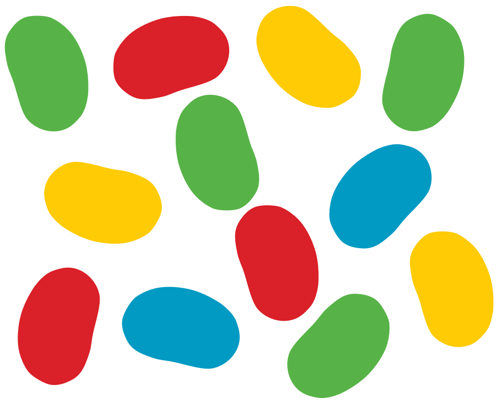
ESSENTIAL QUESTION

How do you explain and calculate the probability of an event?

9A **1** How would you describe the chance of an event occurring that has a probability of 0.2?

A impossible
B certain
C somewhat likely
D very unlikely

9A **2** Look at this figure.



a What is the theoretical probability of selecting a yellow jellybean?
A $\frac{1}{4}$ **B** 3 **C** $\frac{3}{11}$ **D** $\frac{3}{10}$

b What is the sample space of this figure?
A 11
B red, green, yellow, blue
C {3 red, 4 green, 3 yellow, 2 blue}
D {red, green, yellow, blue}

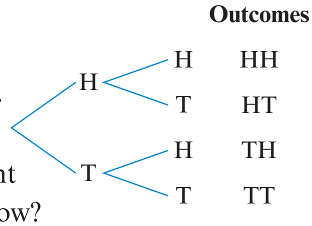
9A **3** Which of these experiments does not have equally likely outcomes?

A rolling a die
B selecting a letter from the word FLOWER
C drawing a card from a deck and recording its suit
D flipping two coins

9B **4** **a** If you flip a coin 10 times, how many heads would you expect to get?

b If a coin is flipped 20 times and 13 tails are obtained, what is the experimental probability of obtaining a tail?

9C **5** Look at this tree diagram.



a What experiment does it show?
A rolling a die
B flipping one coin twice
C flipping one coin three times
D flipping four coins

b How many outcomes in total are possible?

c What is the theoretical probability of flipping two tails?

9D **6** Look at this table.

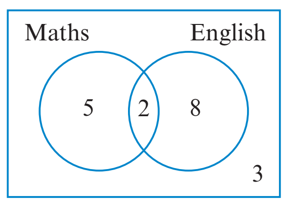
	Male	Female	Total
Dark hair	8	7	15
Fair hair	4	6	10
Total	12	13	25

a How many people were surveyed in total?

b How many females have fair hair?

c How many males were surveyed?

9E **7** Look at this figure.

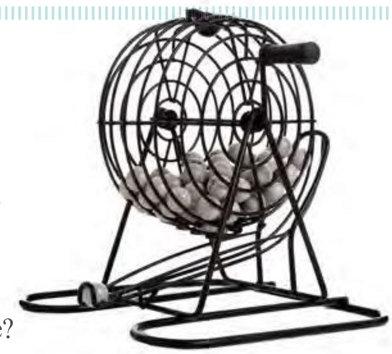


a How many people like both Maths and English?

b How many people don't like Maths or English?

c How many people were surveyed in total?

9A Theoretical probability



Start thinking!

The **probability** of something occurring is how likely it is to happen. To describe probability accurately, you calculate the **theoretical probability** of an **event** occurring. To find theoretical probability, you need to consider all possible outcomes.

Imagine a classmate sells you a raffle ticket for a local club.

1 If there is a total of 100 tickets, what is the probability that you will win the raffle?

2 How can you increase your chances of winning the raffle?

3 Explain why, if you bought seven tickets, you have 7 chances out of 100 to win the raffle.

The number of chances you have to win the raffle can also be called the number of **favourable outcomes**.

4 Explain why you can use the formula

$\text{Pr(event)} = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ to calculate theoretical probability.

5 How many tickets would you have to buy in order to have a 50% or 0.5 chance of winning the raffle?

6 The probability of winning a particular lottery is $\frac{1}{8\,145\,060}$. Explain what this means.

7 How many tickets would you have to buy in order to have a 50% chance of winning this lottery?

8 Why is this so many more than your answer to question **5**?

All these calculations have assumed that each ticket has an **equally likely** chance of being drawn.

9 Describe some circumstances where outcomes are not equally likely. Discuss with a classmate.

KEY IDEAS

- The probability of an event occurring can be described using words or numbers in the range 0 (impossible) to 1 (certain).

impossible	highly unlikely	very unlikely	unlikely	somewhat unlikely	even chance	somewhat likely	likely	very likely	highly likely	certain
0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

- To find the theoretical probability of an event occurring, use the formula:

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- The **sample space** of an **experiment** is a list of all the different outcomes possible and is written within curly brackets. It does not show whether each different outcome is equally likely to occur.
- The complement of an event A is the event where A does not occur. Event A and event ‘not A’ are **complementary events**. Pr(A) + Pr(not A) = 1.

EXERCISE 9A Theoretical probability

1 Describe the probability of each of these events occurring.

- **a** a random day in winter being cold
- **b** winning the lottery
- **c** a baby being born on a weekday
- **d** the sun rising in the north
- **e** flipping a coin and getting a tail
- **f** selecting a consonant from the word RHYTHM
- **g** selecting a picture card from a deck of cards
- **h** rolling a die and getting a number greater than 2

EXAMPLE 9A-1 Listing sample space

List the sample space for randomly selecting a letter from the word SIMULTANEOUS.

THINK	WRITE
List every different outcome within curly brackets.	{S, I, M, U, L, T, A, N, E, O}

2 List the sample space for each experiment.

- **a** rolling a die
- **b** randomly selecting a letter from the word TECHNOLOGY
- **c** drawing a card from a deck and recording its suit
- **d** spinning this spinner

- **e** randomly selecting a person and recording their birth day of week
- **f** drawing a card from a deck and recording if it is a picture card

EXAMPLE 9A-2 Identifying equally likely outcomes

State whether these experiments have equally likely outcomes.

a flipping a coin **b** selecting a letter at random from the word TELEPHONE

THINK	WRITE
a The possible outcomes are {head, tail}. No outcome is more likely to be selected than any other.	**a** Flipping a coin has two equally likely outcomes.
b The possible outcomes are {T, E, L, P, H, O, N}, where the E is three times as likely to be selected as any other letter.	**b** Selecting a letter at random from the word TELEPHONE does not have equally likely outcomes.

UNDERSTANDING AND FLUENCY

3 State whether these experiments have equally likely outcomes.
- **a** selecting a marble at random from a bag containing four blue, four yellow, four green and four red marbles
- **b** rolling a die and recording if the number is less than or greater than 3
- **c** selecting a letter at random from the word REGULAR
- **d** selecting a person at random from your class and recording gender
- **e** selecting a person at random and checking if they are left- or right-handed
- **f** selecting a letter at random from the word SUPERB

4 State whether the outcomes in the sample spaces for the experiments in question **2** are equally likely to occur or not.

EXAMPLE 9A-3 Calculating theoretical probability

Find the theoretical probability of rolling a die and obtaining a number greater than 4.

THINK	WRITE
1 Write the formula for theoretical probability.	$\text{Pr(event)} = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
2 Find the total number of possible outcomes.	total number of outcomes = 6
3 Find the number of favourable outcomes.	Favourable outcomes are rolling a 5 or a 6. number of favourable outcomes = 2.
4 Substitute these values into the formula and write the resulting fraction in simplest form.	$\text{Pr(event)} = \dfrac{2}{6} = \dfrac{1}{3}$

5 Find the theoretical probability of:
- **a** rolling a die and obtaining a 4
- **b** flipping a coin and obtaining a tail
- **c** randomly selecting a C from the word EXCLAIM
- **d** drawing a card from a deck and obtaining an ace
- **e** guessing the correct answer to a multiple-choice question with options A–D
- **f** randomly selecting an R from the word CHARGRILLED.

6 Find the theoretical probability of:

- **a** rolling a die and obtaining a number less than 4
- **b** selecting a picture card from a deck of cards
- **c** spinning green or blue on the spinner shown
- **d** randomly selecting a vowel from the word INTERACTIVE
- **e** rolling a die and obtaining any number except 1
- **f** randomly selecting an N, P or I from the word EMANCIPATION.

PROBLEM SOLVING AND REASONING

7 Use this spinner to calculate the probability of spinning:

- **a** red or blue
- **b** an odd number
- **c** 1 or 2
- **d** a 5 or green
- **e** yellow and 2
- **f** green but not 4.

8 For each of these spinners:

- **i** write the sample space
- **ii** state which colour you would bet on if the spinner was spun, giving a reason
- **iii** find the theoretical probability of spinning red.

a

b

c

9 Explain the difference between an outcome that has an even chance of occurring and outcomes that are equally likely.

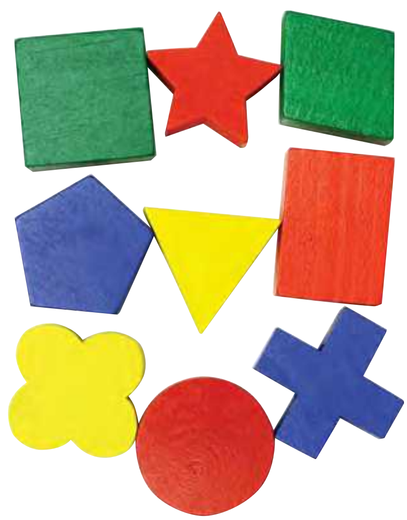
10 Provide an example of an experiment with:

- **a** outcomes that are equally likely, but do not have an even chance of occurring
- **b** outcomes that are equally likely and have an even chance of occurring
- **c** an outcome that has an even chance of occurring but is not equally likely to other outcomes.

11 For each of these experiments:

- **i** list a sample space where the outcomes are equally likely
- **ii** list a sample space where the outcomes are *not* equally likely
- **iii** choose a single outcome from part **i** and calculate its theoretical probability
- **iv** classify the outcome from part **iii** as belonging to the sample space from part **ii** and recalculate its theoretical probability.

- **a** drawing a card from a deck
- **b** selecting a letter from the word COMPUTER
- **c** selecting a coin from Australian currency
- **d** selecting a day of the week
- **e** selecting a shape from this photograph



12 A number of people made some probability statements that were not quite correct. Explain where each person went wrong and provide a better statement.

- **a** Adele said that the probability of spinning purple on the spinner from question **8a** was $\frac{1}{5}$.
- **b** Thanh flipped a coin five times and got tails each time. He then said that he was very unlikely to get another tail.
- **c** Ethan said that he had an even chance of rolling a 6 on a fair die.
- **d** Bianca said that she was highly likely to spin blue on the spinner from question **8c**.

13 Consider a lucky dip for gift vouchers: 5 are for \$50, 10 are for \$20 and 15 are for \$5.

- **a** List the three different outcomes in this lucky dip.
- **b** Explain why the chance of selecting a \$50 gift voucher is not $\frac{1}{3}$.

14 Consider this pile of lollies.

- **a** What is the probability of selecting a red lolly?
- **b** What is the probability of *not* selecting a red lolly?
- **c** What do these two probabilities add to?

Events that are 'opposite' to one another, such as selecting a red lolly and not selecting a red lolly, are complementary events.

- **d** Find the complementary event to:
  - **i** rolling a die and obtaining a 6
  - **ii** selecting a heart card from a deck of cards
  - **iii** flipping a coin and obtaining a tail
  - **iv** rolling a die and obtaining an even number
  - **v** selecting a picture card from a deck of cards
  - **vi** selecting a consonant from this sentence.
- **e** For each event listed in part **d**, find the probability of
  - **i** the event
  - **ii** its complementary event.
- **f** Find the sum of the probability of each event and its complementary event from part **d**. What do you find?
- **g** What can you say about the sum of the probability of complementary events?

PROBLEM SOLVING AND REASONING

15 In mathematical notation, the probability of an event can be written as Pr(x), where x can be substituted for any letter (usually a capital letter) or even words or a phrase. The probability of a complementary event is denoted using the symbol prime (′); for example the complementary event to A is A′.

a What is Pr(A) + Pr(A′)? (Hint: what do the probabilities of complementary events add to?)

b Find:

i Pr(E) when Pr(E′) = 0.7

ii Pr(W′) when Pr(W) = $\frac{1}{3}$

iii Pr(Y) when Pr(Y′) = $\frac{8}{9}$

iv Pr(M′) when Pr(M) = 0.16

16 Investigate how useful a **probability scale** is in real life.

a Draw a probability scale from 0 to 1, but rather than using decimal numbers (for example, 0.1), use fractions (for example, $\frac{1}{10}$).

b At each of the marks, place a word that describes each probability; for example, impossible, even chance, highly likely etc.

c Place each of these events onto the probability scale you have drawn and hence use a word or phrase to describe the probability of each event occurring.

i a baby being a girl

ii being born on a Monday

iii having at least one day in summer over 25°C

iv not being selected out of a group of three people

vi winning the lottery

d How do you think each event matches to its description and fractional probability? For example, winning the lottery would be placed as close to 0 as possible, which on this scale would give a probability of $\frac{1}{10}$ and a possible description of 'highly unlikely'. Do you think either of these descriptions accurately match the probability of winning the lottery?

e What might you be able to say about events that are close to the middle of the scale versus events that are closer to the ends of the scale?

f How might you improve the probability scale to allow it to better describe the probability of real-life events?

CHALLENGE

17 A spinner with four different colours has a $\frac{1}{5}$ chance of spinning blue, a $\frac{1}{3}$ chance of spinning green and a $\frac{1}{6}$ chance of spinning red.

a What is the probability of spinning the remaining colour (yellow)?

b Draw two examples of spinners that meet this description.

Reflect

Why is it important to establish if outcomes are equally likely before calculating theoretical probability?

9B Experimental probability and relative frequency

Start thinking!

Experimental probability (or **relative frequency**) is calculated using the results of an experiment rather than using theoretical probability. It is more commonly written as a decimal number rather than a fraction (for example, 0.1 rather than $\frac{1}{10}$) and can be found using the formula

$$\text{Pr(success)} = \frac{\text{number of successful trials}}{\text{total number of trials}}.$$

Consider a standard die.

1 List its sample space.

2 List the theoretical probability of each outcome.

3 If it was rolled 60 times, how many times would you expect to obtain a 6?

4 Explain how you got your answer to question **3**.

Before conducting an experiment, it can be useful to calculate the **expected number** of each outcome. This can help to determine if there is any bias in an experiment. To calculate expected number, multiply the theoretical probability by the number of trials in the experiment. This can be written as $E(x) = Pr(x) \times n$.

5 Explain how this is the same as your answer to question **4**.

6 Write down what $E(x)$, $Pr(x)$ and n represent in the expected number formula.

7 Calculate the expected number of each outcome if a die was rolled 30 times.

Imagine that a die was rolled 30 times and the results in the table were obtained.

Outcome	1	2	3	4	5	6
Frequency	2	6	4	5	3	10

8 How do these numbers differ from what you found in question **7**?

9 Copy the table and add two rows. In the first additional row, write the theoretical probability of each outcome as a decimal number. In the second additional row, calculate the relative frequency of each outcome.

KEY IDEAS

- To calculate experimental probability, use the formula $\text{Pr(success)} = \frac{\text{number of successful trials}}{\text{total number of trials}}$.
- Before conducting an experiment, calculate the expected number of successful outcomes.
- To calculate expected number, multiply theoretical probability by the number of trials. This can be written as $E(x) = Pr(x) \times n$.
- In an individual experiment, the experimental probability may not match the theoretical probability. However, as the number of trials increases, the experimental probability should get closer to the theoretical probability.

EXERCISE 9B Experimental probability and relative frequency

EXAMPLE 9B-1 Calculating experimental probability

Find the experimental probability of rolling a 6 if a die is rolled 80 times and 6 is obtained 18 times.

THINK	WRITE
1 Write the experimental probability formula.	$\Pr(\text{success}) = \dfrac{\text{number of successful trials}}{\text{total number of trials}}$
2 Identify the number of successful trials (18) and the total number of trials (80). Substitute into the formula and solve, simplifying if possible.	$= \dfrac{18}{80}$ $= \dfrac{9}{40}$
3 Convert to a decimal number and write your answer.	The experimental probability of obtaining a 6 in this experiment is 0.225.

1 Find the experimental probability for each of these.

a rolling a 4, if a die is rolled 180 times and a 4 is obtained 20 times

b drawing an ace, if a card is drawn from a deck and replaced 100 times and an ace is obtained 5 times

c flipping a tail, if a coin is flipped 36 times and a head is obtained 16 times

d rolling an odd number, if a die is rolled 200 and an odd number is obtained 87 times

e drawing a club, if card is drawn from a deck and replaced 250 times and a club is obtained 92 times

f guessing the correct answer, if 10 answers were guessed correctly out of 50

EXAMPLE 9B-2 Calculating expected number

Find the expected number of 5s or 6s if a die is rolled 90 times.

THINK	WRITE
1 Write the formula for expected number.	$E(x) = \Pr(x) \times n$
2 Identify the theoretical probability of rolling a 5 or a 6 ($\frac{2}{6}$) and the number of trials (90) and substitute into the formula.	$= \dfrac{2}{6} \times 90$ $= 30$

2 Find the expected number of:

a heads if a coin is flipped 250 times

b 1s or 2s if a die is rolled 120 times

c hearts if a card is drawn from a deck and replaced 100 times

d 6s if a die is rolled 30 times

e consonants if a letter is selected randomly from the alphabet 130 times

f picture cards if a card is drawn from a deck and replaced 260 times.

EXAMPLE 9B-3 Describing long-term probability

The experiment described in Example 9B-2 is performed and the results are shown in this table.

Outcome	1	2	3	4	5	6
Frequency	13	11	12	17	21	16

Find the experimental probability of rolling a 5 or a 6 and describe how you expect this to change in the long term.

THINK	WRITE
1 Write the formula for experimental probability.	$\Pr(\text{success}) = \dfrac{\text{number of successful trials}}{\text{total number of trials}}$
2 Identify the number of successful trials (21 fives and 16 sixes) out of the total number of trials (90) and substitute into the formula.	$\Pr(5 \text{ or } 6) = \dfrac{37}{90}$ ≈ 0.41
3 In the long term, experimental probability should approach theoretical probability ($\frac{2}{6} \approx 0.33$).	In the long term you would expect the experimental probability of rolling a die and obtaining a 5 or 6 to decrease as it approaches theoretical probability.

UNDERSTANDING AND FLUENCY

3 A die is rolled 150 times and the results are shown in this table.

Outcome	1	2	3	4	5	6
Frequency	38	32	19	24	21	16

a Find the experimental probability of rolling a number greater than 2.

b Describe how you expect the experimental probability of rolling a number greater than 2 to change in the long term.

4 A card is drawn from a deck and replaced, and this is repeated 130 times.

a What is the expected number of picture cards?

b If seven picture cards were obtained, find the experimental probability of drawing a picture card.

c Describe how you expect the experimental probability of drawing a picture card to change in the long term.

5 Three coins are flipped, and this is repeated 200 times. If three tails appear 22 times, describe how you expect the experimental probability of flipping three tails to change in the long term.

6 A magician uses a number of props in his show, but you aren't sure that they are fair. You watch and record his movements over several shows, and results are shown in the tables below. For each experiment:

- **i** find the total number of trials
- **ii** state the theoretical probability of each outcome
- **iii** calculate the expected number of each outcome
- **iv** calculate the relative frequency of each outcome
- **v** state if you think the prop used is fair, biased, or if there are not enough trials to make a firm decision
- **vi** give a reason to support your answer to part **v**.

a

Outcome	Heads	Tails
Frequency	8	2

b

Outcome	1	2	3	4	5	6
Frequency	966	971	1036	994	1031	1002

c

Outcome	Hearts	Diamonds	Clubs	Spades
Frequency	38	32	19	24

7 If you flip three coins (5c, 10c and 20c) at the same time, how often would you expect to get a triple heads or tails?

- **a** Make a list of all the possible outcomes. (Hint: there are eight.) How many of these are 'triples'?
- **b** What is the theoretical probability of flipping a 'triple'?
- **c** How many 'triples' would you expect to get if you performed 40 trials?
- **d** Perform 40 trials of the experiment and record your results.
- **e** Does the relative frequency of a 'triple' match the theoretical probability?
- **f** Describe how you would expect this relative frequency to change if you performed 4000 trials.

8 If you roll two dice (one red and one blue), how often would expect to get a 'double' number? Follow the steps shown in question **7** and perform at least 30 trials of the experiment. Discuss your results. (Hint: there are 36 outcomes.)

9 A number of experiments were performed and their results recorded below. For each experiment, find the number of times each outcome occurred.

a total number of trials = 40

Outcome	Heads	Tails
Relative requency	0.625	0.375

b total number of trials = 120

Outcome	1	2	3	4	5	6
Relative requency	0.15	0.2	0.175	0.1	0.125	0.25

c total number of trials = 60

Outcome	Hearts	Diamonds	Clubs	Spades
Relative requency	0.2	0.3	0.35	0.15

10 In real life, it can be difficult to perform a large number of trials in an experiment. **Simulations** can be used to generate results when these experiments are impractical. A simulation makes use of a simple random device, such as a coin, die or spinner, or digital technology that generates random outcomes. When planning to perform a simulation, it is important that all outcomes listed are equally likely and each outcome of a device is matched to each outcome of the experiment.

a List the number of outcomes in these real-life situations.

- **i** a baby's gender at birth
- **ii** guessing the answer to a multiple choice question with answers A, B, C or D
- **iii** selecting a prize from a lucky dip with three different prizes that come in two different colours each
- **iv** selecting your favourite flavour Clinker from a bag (from pink, green and yellow)

b For each situation in part **a**, list:

- **i** a device with the same number of outcomes that could be used to simulate the situation
- **ii** at least two limitations of using the device in order to simulate the situation.

11 Sometimes chocolate companies have promotions where one in six chocolate bars wins a free bar.

a Explain why, even though there are only two outcomes (winning and not winning), you can treat this problem like it has six outcomes.

b Select a device to simulate winning a free chocolate bar and perform as many trials as necessary in order to simulate winning a free bar.

c Repeat part **b** another 19 times and hence state the average number of bars you would have to buy in order to win a free bar.

CHALLENGE

12 A classic probability problem that confuses many people is the Monty Hall problem. The problem is as follows:

Imagine that you are a contestant on a game show. You are shown three doors and told that behind one door is a car, and behind the other two doors are goats.
If you correctly select the right door you win the car. After selecting one of the doors (say door 2), the host opens up another door (say door 1) to show a goat. The host then asks you if you want to stay with door 2 or switch to door 3. Should you switch? Is it an advantage, a disadvantage, or does it not matter if you switch?

a Decide if you would switch or stay.

b What is the probability that you correctly select the car (say, door 2)?

c What is the probability that you select a goat?

d Does showing you what is behind another door (say, door 1) change the probability of your initial selection?

e Explain why there is still a $\frac{1}{3}$ chance that the car is behind door 2 and therefore a $\frac{2}{3}$ chance that the car is not behind door 2.

f Use your answer to part **e** and the fact that there is a goat behind door 1 to explain why there is a $\frac{2}{3}$ chance that the car is behind door 3 and hence it is better to switch.

Many people believe that after seeing that there is a goat behind door 1 that there is now a 50% chance that you selected correctly. Sometimes, some perspective can help. Imagine now that rather being offered to pick one door out of three, you were offered to pick one door out of 1 000 000.

g What is the probability you would correctly select the car now?

Imagine that all doors except the one you picked and one other were opened.

h Would you switch or stay? Why does this seem more obvious than the original problem?

Still, some people can confuse the initial probability of guessing correctly with having only two options left. Use a simulation to explore this.

i With a classmate, obtain materials to simulate this problem. It may be as simple as writing ‘car’, ‘goat’, ‘goat’ on three pieces of paper. Set up the experiment so that one person is the host and the other is the contestant.

j The contestant should decide on a strategy: to switch or stay; and do this consistently for 20 trials.

k Perform 20 trials and record your results. What is the relative frequency of your chosen strategy?

l Switch who is the host and who is the contestant, and now perform 20 trials of the other strategy, recording your results. What is the relative frequency of this other strategy?

m Your results should have roughly given a relative frequency of 0.67 for switching and 0.33 for staying. Did your results reflect this? What do you think would happen to your results if you performed 2000 trials?

Reflect

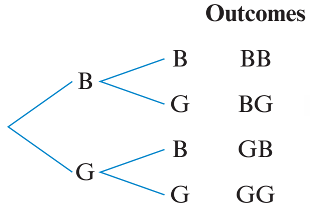
How is experimental probability related to theoretical probability?

9C Tree diagrams

Start thinking!

When performing multi-step experiments (or tracking multi-step events), you can use a **tree diagram** to display the possible outcomes.

Consider this tree diagram.



1 What experiment is it showing?

2 How many steps are there in this tree diagram?

3 How many possible outcomes are there? List each one.

4 How many outcomes involve at least one boy?

5 If a family has two children, what is the probability that at least one of the children is a boy?

6 Copy the tree diagram and add another branch to represent the family having a third child.

7 How many outcomes now involve at least one boy?

8 If a family has three children, what is the probability that at least one of the children is a boy?

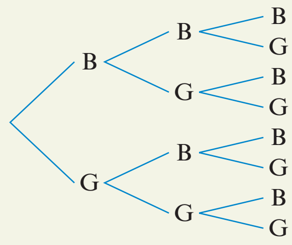
9 How does drawing a tree diagram help to calculate probabilities in multi-step experiments and events?

KEY IDEAS

- Tree diagrams display the outcomes of multi-step experiments.
- The possibilities for each step of the experiment are represented by a number of branches.
- The final outcomes are listed at the end of the branches.
- This list of final outcomes can be used to calculate the probability of an outcome occurring.

EXERCISE 9C Tree diagrams

EXAMPLE 9C-1 Understanding tree diagrams



Use this tree diagram to find:

a the total number of outcomes

b the number of outcomes containing at least one girl

c the probability of a family of three children containing at least one girl.

THINK	WRITE
a Count the number of final outcomes at the right end of the tree diagram.	**a** There are eight possible outcomes.
b Trace the branches carefully and count the number that contain a girl.	**b** Seven outcomes contain a girl.
c Consider the number of favourable outcomes (7) out of the total number of possible outcomes (8).	**c** Pr(at least one girl) = $\frac{7}{8}$

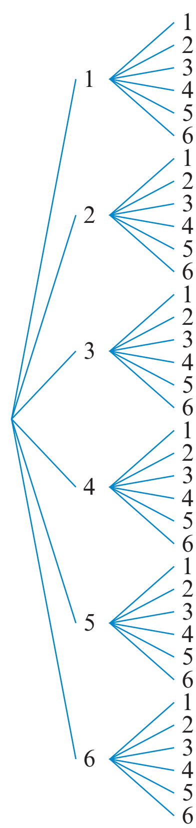
UNDERSTANDING AND FLUENCY

1 Consider the tree diagram at far right.

a How many possible outcomes are there?

b How many of these outcomes contain a 6?

c How many of these outcomes contain a double number?

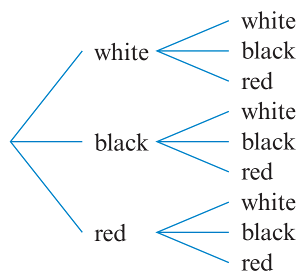


2 Consider this tree diagram.

a How many possible outcomes are there?

b How many of these outcomes contain at least one red?

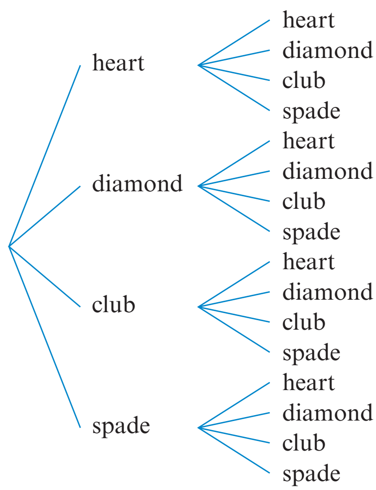
c What is the probability of drawing at least one red?



UNDERSTANDING AND FLUENCY

3 Use this tree diagram to find the probability of:

a drawing at least one diamond
b drawing a club and a heart
c drawing two spades
d not drawing a heart
e drawing a diamond or a spade
f drawing a spade then a club.



4 Use the tree diagram from question **1** to find the probability of:

a rolling a double 6
b rolling at least one even number
c rolling two odd numbers
d rolling at least one 4
e rolling a double
f rolling a total of 6.

EXAMPLE 9C-2 Calculating probability using a tree diagram

Use a tree diagram to calculate the probability of flipping at least three tails when flipping a coin four times.

THINK

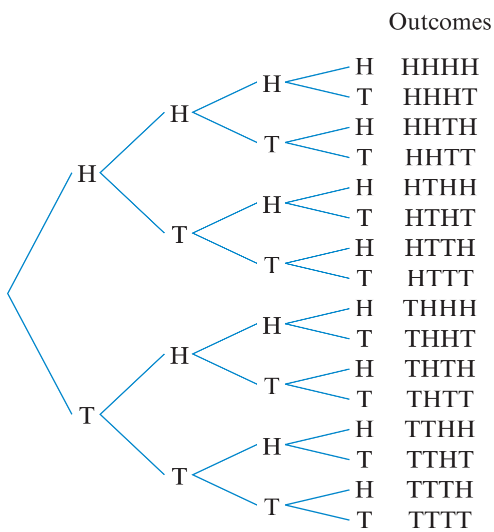
1 Draw the first two branches to represent the first coin flip. Label the end of these branches with H and T to represent the two different outcomes.

2 From each branch, draw another two branches to represent the next coin flip and label them appropriately. Repeat this twice more so that you are representing the four coin flips.

3 At the end of each of the 16 branches, write the final outcome to complete the tree diagram.

4 There are five outcomes that contain at least three tails (three tails or four tails) out of a possible 16 outcomes.

WRITE



$$\text{Pr(at least three tails)} = \frac{5}{16} = 0.3125$$

UNDERSTANDING AND FLUENCY

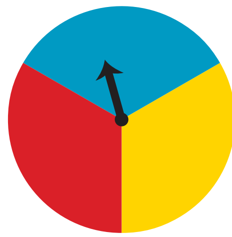
5 Use a tree diagram to find the probability of:

- **a** exactly three tails when flipping a coin four times
- **b** no more than one head when flipping a coin four times
- **c** flipping a coin four times and getting the same outcome each time.

6 A coin was flipped three times. Use a tree diagram to find the probability of:

- **a** at least two heads
- **b** no tails
- **c** exactly two tails
- **d** only one head.

7 This spinner was spun three times.
Use a tree diagram to find the probability of:



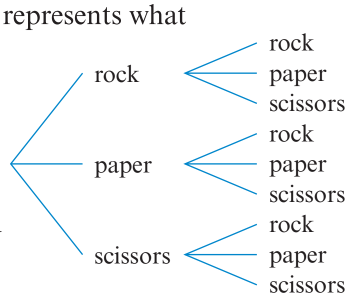
- **a** three different results
- **b** spinning blue at least once
- **c** spinning the same colour each time
- **d** spinning red each time.

8 A coin was flipped five times. Use a tree diagram to find the probability of:

- **a** no heads
- **b** at least one tail
- **c** exactly three tails
- **d** at least three tails
- **e** more than one head
- **f** less than two tails.

PROBLEM SOLVING AND REASONING

9 This tree diagram represents a single round of a game of rock, paper, scissors. The first set of branches represents what you choose and the second set of branches represents what your opponent chooses.



- **a** List the outcomes that result in you
  - **i** winning
  - **ii** losing
  - **iii** drawing the game.
- **b** Hence calculate the probability that you win a game of rock, paper, scissors.

10 Tree diagrams aren't limited to repeated trials of the same experiment. They can also be used to display unrelated events. Imagine that you flip a coin and then roll a die.

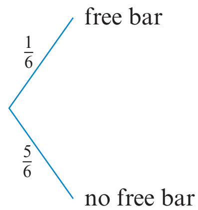
- **a** Draw a tree diagram to represent this multi-step experiment.
- **b** How many outcomes are there?
- **c** What is the probability that:
  - **i** you flip a tail and roll a six?
  - **ii** you flip a tail and roll a number less than four?
  - **iii** you flip a tail or roll a six?
  - **iv** you flip a tail or roll a number less than four?
- **d** Explain why the answer to part **c iii** is not $\frac{8}{12}$.

11 Use a tree diagram to find the probability in a family of four children that:

- **a** all are girls
- **b** at least one is a boy
- **c** the first child is a boy
- **d** two are boys
- **e** more than one is a girl
- **f** at least two are girls.

PROBLEM SOLVING AND REASONING

12 All the tree diagrams you have looked at so far assume that each outcome is equally as likely as any other. However, this is not always the case. Consider the situation mentioned in Exercise 9B question **11** on page 420, where one in every six chocolate bars wins a free bar.



a Explain how this tree diagram represents the probabilities of winning a free chocolate bar when you buy a single chocolate bar.

b Extend this tree diagram so that it represents buying three chocolate bars.

c Write down the final outcomes at the ends of the third set of branches.

To find the probability of each final outcome, you simply multiply together the probabilities of the branches that you move across. For example, the probability of winning a free bar with the first purchase, but then not again (FNN) is $\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{25}{216} \approx 0.12$.

d Find the probability of each of the final outcomes, expressing them both as fractions and as decimals rounded to three decimal places.

e Add together these eight probabilities. What do you find?

f When purchasing three chocolate bars, what is the probability of:

i not winning a free bar? **ii** winning a free bar each time?

iii winning a free bar with your second but not first or third purchase?

Each of the parts in part **f** are single final outcomes. To calculate the probability of an event that involves more than one final outcome, you need to add the probabilities of each favourable final outcome.

g Which final outcomes involve winning one free bar? Add together the probabilities of these final outcomes to find the probability of winning one free bar.

h Find the probability of winning:

i at least one free bar **ii** winning a free bar with your first purchase

iii two free bars **iv** more than one bar.

i How does a tree diagram help to find the probability when outcomes are not equally likely?

13 Consider sitting a quiz consisting of five multiple-choice questions, with answers A–D.

a What is the probability of correctly guessing a problem with four possibilities?

b Draw a tree diagram complete with probabilities to represent guessing the answers to these five questions.

c Use your tree diagram to calculate the probability of correctly guessing:

i all five questions **ii** no questions

iii two questions **iv** at least one question

v less than four questions **vi** at least three questions.

14 A tree diagram is helpful but not necessary when calculating the probabilities of outcomes in multi-step experiments. Rather than drawing a tree diagram, you can just write a list of the outcomes. But how do you know how many outcomes to write down?

CHALLENGE

Situation	Number of branches at each trial	Number of trials	Number of outcomes
Flipping a coin four times	2	4	
Selecting a card from a deck three times and noting its suit	4		
Rolling a die twice		2	
Recording the gender of three children at birth		3	
Recording two rounds of rock, paper, scissors	3		

a Copy and complete this table.

b Can you see a pattern between the number of outcomes and the other two numbers in the table? (Hint: it involves powers of numbers (repeated multiplication).)

c Explain how the number of outcomes for flipping a coin five times can be calculated using 2^5. How many outcomes is this?

d Describe how to find the total number of outcomes for any experiment with repeated trials.

e When writing down the outcomes for experiments without a tree diagram, what strategy might you use to ensure that you don't leave out any outcomes?

15 Consider a similar situation to question **12**; however, this time one in every five chocolate bars wins. Imagine that you bought four chocolate bars.

a Using the strategy from question **14**, state how many outcomes there could be and list them.

b What is the probability of winning a free bar (F)?

c What is the probability of not winning a free bar (N)?

To find the probability of each outcome, simply multiply together the probabilities like you did in question **12**. For example, the outcome FNNF would have the probability $\frac{1}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{1}{5} = \frac{16}{625} \approx 0.0256$.

d Calculate the probability of each outcome from your list in part **a**. Write each probability as both a fraction and a decimal (to four decimal places). Check that they all add to 1 to be sure you calculated correctly.

e Use these probabilities to find the probability of:

i not winning a free bar **ii** winning four free bars

iii winning a free bar with your first purchase but then not again

iv winning at least one free bar **v** winning two free bars

vi winning a free bar with your last purchase.

16 The game of Yahtzee involves five dice. To get a 'Yahtzee', you need to roll the same value on all five dice. Use a tree diagram or the strategy shown in questions **14** and **15** to find the probability of rolling a Yahtzee.

Reflect

How do tree diagrams help you calculate probability in multi-step experiments?

9D Two-way tables

Start thinking!

Two-way tables can also be used to display outcomes for a two-step experiment.

	Boy	Girl
Boy	B, B	B, G
Girl	G, B	G, G

Consider a family having two children.

1 How does a two-way table display the four possible outcomes?

A two-way table is more commonly used to display the relationship between different sets of data. Consider this two-way table.

	Dark hair	Light hair	Total
Dark eyes	16	4	20
Pale eyes	8	12	20
Total	24	16	40

2 What data is it showing?

3 Of the people surveyed, 24 had dark hair.
How many people with dark hair also had dark eyes?

4 Twelve people had light hair and pale eyes. How many people had pale eyes in total?

5 How many people were surveyed in total?

6 Use your answer to question **5** to help you calculate the probability of a person selected randomly having:

a dark hair **b** light hair and pale eyes

c dark eyes **d** dark hair and dark eyes.

7 Alice said that the probability of a person selected at random having dark hair and dark eyes was $\frac{16}{24}$. Explain where she went wrong.

8 Explain why, even though there are four different outcomes, the probability of each outcome is not $\frac{1}{4}$.

9 How does a two-way table assist in calculating probabilities?

KEY IDEAS

- A two-way table is another way to display the outcomes of an experiment or survey.
- You can use two-way tables to calculate the probabilities using outcome results and totals.
- **Conditional probability** is the probability of an outcome, given conditions. 'Calculate the probability that a randomly selected card is an ace given that it is a red card' is an example of conditional probability.

EXERCISE 9D Two-way tables

UNDERSTANDING AND FLUENCY

1 Consider this two-way table.

	Male	Female	Total
Prefer sweet food	23	32	55
Prefer savoury food	29	16	45
Total	52	48	100

a What are the four different outcomes?

b How many people were surveyed in total?

c How many males prefer savoury food?

d How many females were surveyed?

e What does the number 32 represent?

f What does the number 45 represent?

EXAMPLE 9D-1 Understanding a two-way table

Consider this two-way table.

	Year 8	Year 9	Total
Jam	28	38	66
Vegemite	25	34	59
Total	53	72	125

a How many students were surveyed in total?

b How many students in Year 9 prefer Vegemite?

c What is the probability that a student chosen randomly from the group in Year 9 prefers Vegemite?

THINK

a Check the bottom right corner cell for the total number of people surveyed.

b Find the cell that is in the 'Year 9' column and the 'Vegemite' row.

c Consider the number of favourable outcomes (34) out of the total number of possible outcomes (125) in this table.

WRITE

a 125 students were surveyed in total.

b 34 students in Year 9 prefer vegemite on their toast.

c Pr(Year 9 student who prefers Vegemite)

$= \frac{34}{125}$

$= 0.272$

2 Consider this two-way table.

	Primary school	High school	Total
Watch sport	8	7	15
Play sport	22	18	40
Total	30	25	55

a How many students were surveyed in total?

b How many students in high school prefer to watch sport?

c What is the probability of a student chosen randomly from the group being in high school and preferring to watch sport?

3 Consider this two-way table.

	Fair	Dark	Total
Blue/Green	23	11	34
Brown	6	35	41
Total	29	46	75

a How many people were surveyed in total?

b How many people with dark hair have blue eyes?

c What is the probability of a person chosen randomly from the group having dark hair and blue eyes?

EXAMPLE 9D-2 Calculating probability using a two-way table

Use this two-way table to find the probability of a person chosen randomly from the group being a male who does not have a pet.

	Male	Female	Total
Owns pet	11	14	25
Does not own pet	6	4	10
Total	17	18	35

THINK	WRITE
1 Locate the cell that shows the number of favourable outcomes: males who do not own a pet.	Six males do not own a pet.
2 Locate the cell that shows the total number of people surveyed.	35 people were surveyed in total.
3 Write this as a fraction and simplify if possible. You may also like to express your answer as a decimal number.	$\Pr(x) = \frac{6}{35}$ ≈ 0.17

UNDERSTANDING AND FLUENCY

4 Use this two-way table to find the probability of a person chosen randomly from the group:

	Year 8	Year 9	Total
Alternative	14	23	37
Mainstream	29	19	48
Total	43	42	85

a being a Year 8 student who prefers mainstream music

b preferring alternative music

c being a Year 9 student

d being a Year 9 student who prefers alternative music.

5 Use this two-way table to find the probability that a person chosen randomly from the group:

	Short	Tall	Total
Light hair	16	18	34
Dark hair	21	23	44
Total	37	41	78

a is short with light hair

b is tall

c has dark hair

d is tall with dark hair.

UNDERSTANDING AND FLUENCY

6 Use this two-way table to find the probability that a person chosen randomly from the group:

	City	Country	Total
Automatic	67	24	91
Manual	15	44	59
Total	82	68	150

- **a** is from the country and drives a manual car
- **b** drives an automatic car
- **c** is from the city
- **d** is from the city and drives an automatic car.

7 Copy and complete the two-way table below using the buttons in this photograph.

	Green	Not green	Total
Two holes			
Four holes			
Total			

8 Use this two-way table to find the probability that a person chosen randomly from this group:

	Year 7	Year 8	Year 9	Total
Tennis	23	26	37	86
Basketball	19	42	34	95
Hockey	31	13	25	69
Total	73	81	96	250

- **a** is in Year 9
- **b** plays basketball
- **c** is in Year 8 and plays tennis
- **d** plays hockey or tennis **e** does not play hockey
- **f** is in Year 7 and doesn't play basketball.

PROBLEM SOLVING AND REASONING

9 **a** Copy and complete this two-way table.

	Cinema	Home	Total
Action	22		
Comedy		14	33
Total			85

b Use it to calculate the probability that a person chosen randomly from the group is someone who on the weekend:

- **i** went to the cinema **ii** watched a comedy film at home
- **iii** watched an action film.

10 Two dice are rolled and the numbers that are uppermost are added together to give the final outcome.

- **a** Create a two-way table that lists all the outcomes. (Hint: List the possibilities for die 1 across the top row and the possibilities for die 2 down the first column.)
- **b** How many different outcomes are there?
- **c** What is the most likely outcome? What is the probability of this occurring?
- **d** What is/are the least likely outcome(s)? What is the probability of this/these occurring?
- **e** State the probability of rolling two dice and obtaining a sum of:
  - **i** four **ii** greater than 10 **iii** an odd number **iv** less than seven.

PROBLEM SOLVING AND REASONING

11 Consider this two-way table.

	Year 8	Year 9	Total
Maths		11	
English	10		
Total	22		50

- **a** Copy and complete the table.
- **b** Use it to find the probability that a person selected randomly from the group:
  - **i** likes Maths
  - **ii** is in Year 9
  - **iii** is in Year 9 and likes Maths.
- **c** A person is selected randomly from the group. You know that they are in Year 9.
  - **i** How many people are in Year 9?
  - **ii** How many people in Year 9 like Maths?
  - **iii** What is the probability that somebody in Year 9 likes Maths?
- **d** What is the difference between parts **b iii** and **c iii**?

The problem represented in part **c iii** is an example of conditional probability. It looks at the probability of an outcome given certain conditions. In part **c iii**, you are looking for the probability that somebody likes Maths given that they are in Year 9. This means that you consider only the limited group of the condition rather than the entire population.

- **e** Use the following steps to calculate the probability of somebody being in Year 8 given that they like English.
  - **i** What is the condition? How many people in this group?
  - **ii** What is the specific group you are after? (Hint: it is not just somebody in Year 8.)
  - **iii** How many people are in this specific group?
  - **iv** Use your answers to parts **i** and **iii** to calculate the probability of selecting somebody being in Year 8 given that they like English.

12 Use the two-way table in question **8** to calculate the probability of randomly selecting a person from the group that:

- **a** plays hockey given that they are in Year 8
- **b** plays tennis given that they are in Year 9
- **c** is in Year 7 given that they play basketball
- **d** is in Year 8 given that they play tennis
- **e** is in Year 9 given that they *don't* play hockey
- **f** is not in Year 7 given that they play basketball.

13 A group of 200 people with a single pet were surveyed on their pets. Of the 113 who owned cats, 29 had a specific breed. This gives a total of 104 pet owners who owned a specific breed. Create a two-way table showing this information and use it to calculate the probability that a person chosen at random is an owner of a cat of a specific breed.

CHALLENGE

14 Elsa recorded the make and colour of cars that went past her house over a week and recorded her results as shown. Add totals to her results and then find the probability that a car going past her house is:

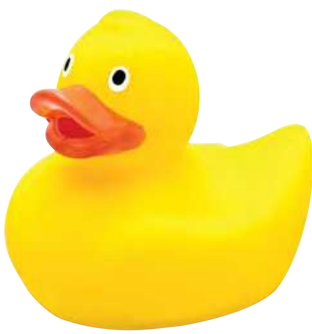
a white
b a Holden
c a white Holden
d white or a Holden
e white given that is a Holden
f a Holden given that it is white
g not a Holden
h not white
i neither white nor a Holden
j white but not a Holden
k a Holden but not white
l not white given that is a Holden.

	Ford	Hyundai	Holden	Mitsubishi	Mazda	Toyota
Silver	11	9	17	8	6	12
White	15	12	16	11	8	10
Red	12	10	8	6	13	14
Blue	8	15	12	7	9	11
Black	13	8	10	10	7	8

15 What is the difference between parts **k** and **l** in question **14**?

16 Use the two-way table shown in question **14** and create four probability questions. Swap these with a classmate and discuss any differences in answers.

17 A group of people were surveyed on their bathing habits. Sixty per cent of women surveyed said they preferred a bath over a shower, whereas 80% of men said they preferred to have a shower rather than a bath. Fifty-five per cent of the group was female.



a Create a two-way table showing these percentages as relative frequencies. Remember that each row and column should add correctly to their totals. (Hint: the statement '60% of women prefer a bath' refers to 60% of the proportion of women, not the total.)

b If a person was randomly selected from a group of 500 people, find the probability that they are:

i a male who prefers to shower
ii someone who prefers a bath
iii a female who prefers to shower.

c Of a group of 500 people, find the number of people who:

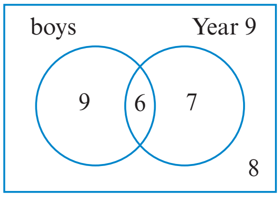
i prefer a bath to a shower
ii are female
iii are male and prefer a bath.

Reflect

How do two-way tables allow you to calculate probabilities when examining the relationship of two different things?

9E Venn diagrams

Start thinking!

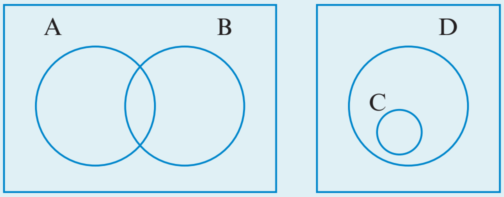


Venn diagrams display the relationship between different sets of data. They consist of a number of circles within a rectangle. Consider this Venn diagram.

1 What are the two sets of data that it is showing?
2 The first circle represents the boys in the group. Which two numbers are in this first circle?
3 Use your answer to question **2** to state how many boys in total are in the group.
4 The second circle represents Year 9 students in the group. Which two numbers are in this second circle?
5 Use your answer to question **4** to state how many Year 9 students in total are in the group.
6 The crossover of the two circles represents people who are in both sets. How many boys are in Year 9?
7 How many girls are in Year 9? (Hint: they belong to the second set but not the first.)
8 The number outside the circles but in the rectangle represents people that don't belong to either set. Describe this set of people.
9 Add all the numbers in the Venn diagram to find the total number of people in the group.
10 Use your answers to calculate the probability of selecting a:
a boy in Year 9 **b** boy **c** Year 9 student
d boy not in Year 9 **e** girl in Year 9 **f** girl not in Year 9.
11 The same information can be displayed in a two-way table. Discuss with a classmate the advantages and disadvantages of each display. Which do you prefer?

KEY IDEAS

- A Venn diagram is used to display the relationship between different sets of data. It consists of a number of circles contained within a rectangle.
- Numbers are placed within each section to show how many elements or individuals are in each group and can be used to calculate the probability of elements belonging to different sets.
- In the Venn diagrams shown:

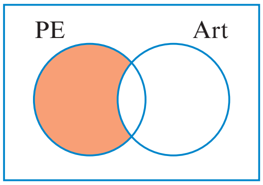


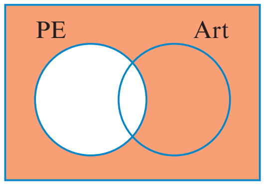
  - $A \cap B$ means A and B, or the **intersection** of sets A and B, and includes the elements in common with both sets.
  - $A \cup B$ means A or B, or the **union** of sets A and B, and includes the elements in either A or B or both.
  - A' means the **complement** of A and includes the elements not in A.
  - If the set C is contained totally in set D, then $C \subset D$ means C is a **subset** of D.

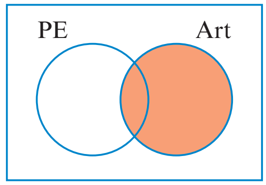
EXERCISE 9E Venn diagrams

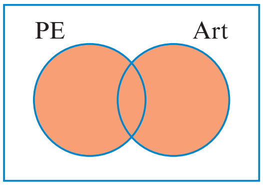
UNDERSTANDING AND FLUENCY

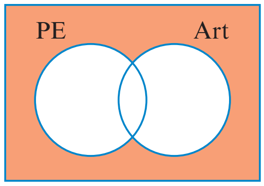
1 These Venn diagrams represent people who like PE and art. Write what the shaded section in each diagram represents.

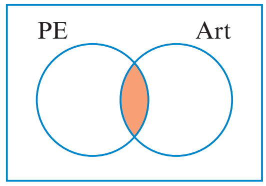
a


b


c


d


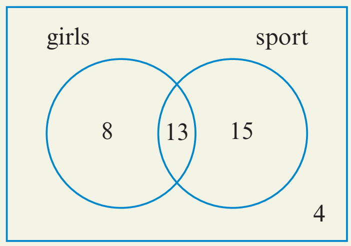
e


f


EXAMPLE 9E-1 Understanding a Venn diagram

This Venn diagram shows people who play sport.

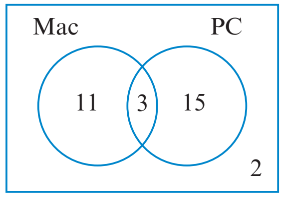
a How many people are girls who play sport?
b How many people play sport?
c How many people are not girls?
d How many people were surveyed in total?



THINK	WRITE
a Look for the section that represents people who are girls and who play sport. This is the middle section where the two circles overlap – the intersection of both sets.	**a** 13 people are girls who play sport.
b To find the number of people who play sport, add all the numbers that are within the 'sport' circle.	**b** 13 + 15 = 28 people play sport.
c To find the number of people who are not girls, add all the numbers not inside the 'girls' circle.	**c** 15 + 4 = 19 people are not girls.
d Add all the numbers in the Venn diagram.	**d** 8 + 13 + 15 + 4 = 40 people were surveyed in total.

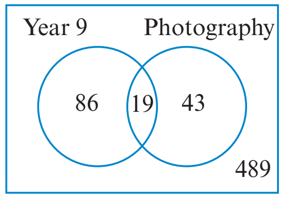
UNDERSTANDING AND FLUENCY

2 Consider the Venn diagram at right.



- **a** How many people own a Mac?
- **b** How many people own a PC but not a Mac?
- **c** How many people don't own either a Mac or a PC?
- **d** How many people were surveyed in total?

3 Consider the Venn diagram at right.



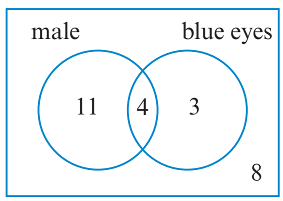
- **a** How many people take Photography?
- **b** How many people are in Year 9 that take Photography?
- **c** How many people are not in Year 9?
- **d** How many people were surveyed in total?

EXAMPLE 9E-2 Calculating probability using a Venn diagram

Use the Venn diagram from Example 9E-1 to find the probability that a person chosen randomly from the group plays sport.

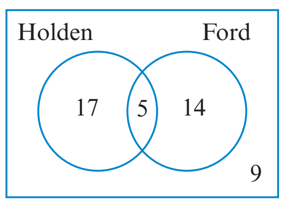
THINK	WRITE
1 Find the section(s) that represent people who play sport (all the numbers within the sport circle). Add these.	$13 + 15 = 28$ 28 people play sport.
2 Find the total number of people that were surveyed. Add together all the numbers in the Venn diagram (don't forget the number outside the circles).	$8 + 13 + 15 + 4 = 40$ 40 people were surveyed.
3 Write these two numbers as a probability fraction, simplifying if possible.	$\text{Pr(person who plays sport)} = \frac{28}{40}$ $= \frac{7}{10}$

4 Use this Venn diagram to find the probability of a person chosen randomly:



- **a** being male with blue eyes
- **b** having blue eyes
- **c** being male with eyes not blue
- **d** not having blue eyes
- **e** being female with blue eyes
- **f** being female.

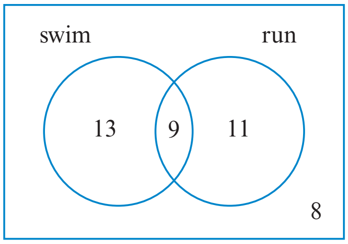
5 Use this Venn diagram to find the probability of a person chosen randomly:



- **a** liking Holden cars only
- **b** liking both Holden and Ford cars
- **c** liking Ford cars
- **d** liking neither
- **e** not liking Holden cars
- **f** liking Holden or Ford (but not both).

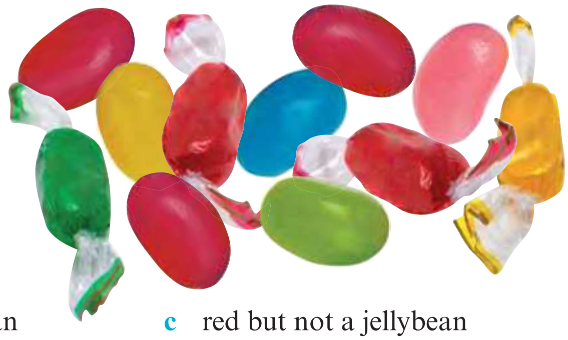
PROBLEM SOLVING AND REASONING

6 Use this Venn diagram to find the probability of a person chosen randomly who:



a swims and runs **b** does not run
c runs but does not swim **d** does not swim or run
e swims **f** swims or runs.

7 Construct a Venn diagram for the objects in this photograph, using the categories ‘jellybeans’ and ‘red’.



8 Use the Venn diagram you constructed in question **7** to calculate the probability that a randomly chosen lolly is:

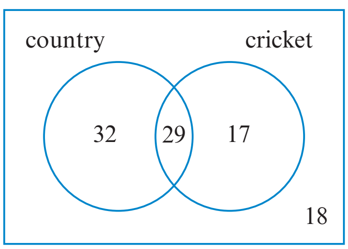
a a jellybean **b** a red jellybean **c** red but not a jellybean
d not a jellybean **e** a jellybean but not red **f** neither red nor a jellybean.

9 Consider this statement. In a group of 40 people, 27 have a brother and 29 have a sister. There are four people who do not have either a brother or a sister.

a Explain why there must be some people who have both a brother and a sister. (Hint: What do the last three numbers add to?)
b Draw a Venn diagram with two circles that overlap. Label one ‘Brother’ and the other ‘Sister’.
c Place the number of people who don’t have either a brother or a sister in the rectangle outside the circles. How many people are left to fill the circles?
d The entire ‘Brother’ circle represents people who have a brother. How many people is this?
e Use your answers to parts **c** and **d** to find how many people must have only a sister and write this into the correct section.
f The entire ‘Sister’ circle must contain 29 people. How many people does this mean must have a brother *and* a sister? Write this in the overlap section.
g Complete the Venn diagram by finding the number of people who have only a brother. Check that all the numbers in your Venn diagram add to 40.

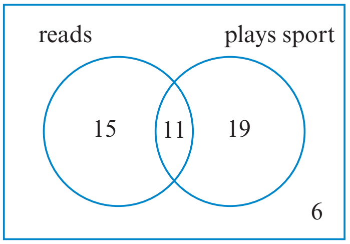
10 In a group of 50 students, 24 are in Year 9, 19 walk to school and 16 are not in Year 9 and do not walk to school. Draw a Venn diagram to represent this situation.

11 Remember that conditional probability examines the probability of an outcome given a condition (page 428). Use this Venn diagram to calculate the probability that a person chosen randomly:



a likes cricket given that they are from the country
b is from the country given that they like cricket
c does not like cricket given that they are from the country
d is not from the country given that they do not like cricket.

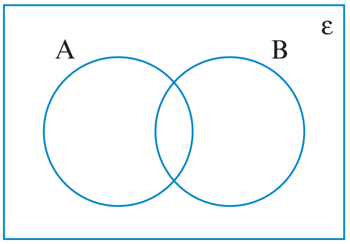
12 Use this Venn diagram to calculate the probability that a person chosen randomly:



- **a** reads given that they play sport
- **b** plays sport given that they read
- **c** does not play sport given that they read
- **d** does not read given that they do not play sport.

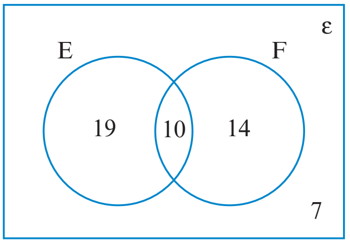
13 When you discuss sets in mathematics, you usually use set notation. Venn diagrams are a useful way to visually display the relationship between sets and understand what set notation represents.

Consider this Venn diagram, showing the relationship between two sets of data, A and B.



- **a** Everything contained within the rectangle is said to belong to the **universal set**. What symbol is used to represent the universal set?
- **b** Copy the Venn diagram into your book five times.
- **c** On your first diagram, shade the overlapping section of sets A and B. This is the intersection of sets A and B and is written as $A \cap B$.
- **d** On your second diagram, shade everything within sets A and B. This is the union of sets A and B and is written as $A \cup B$.
- **e** On your third diagram shade everything that does not belong to set A. This is the complement of set A and is written as A'.
- **f** On your fourth diagram, shade the complement of the union of sets A and B. Label this as $(A \cup B)'$. (Hint: This is related to your second diagram.)
- **g** On your fifth diagram, shade the union of set A and the complement of set B. Label this as $A \cup B'$. (Hint: Remember that union is a combination of sets, not an intersection.)
- **h** Use your diagrams to parts **f** and **g** to explain the difference that brackets can make in set notation.

14 Consider this Venn diagram.



- **a** Given that n(F) means the number of elements in set F, find:

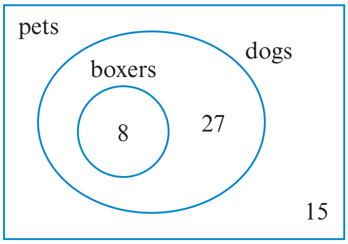
i $n(E)$	**ii** $n(E \cup F)$
iii $n(F')$	**iv** $n(E \cap F)$
v $n(\varepsilon)$	**vi** $n(E' \cap F)$.

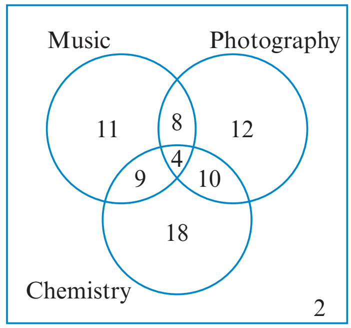
- **b** Given that Pr(E) means the probability of selecting an element from set E, find:

i $\Pr(F)$	**ii** $\Pr(E')$
iii $\Pr(E \cap F)$	**iv** $\Pr(E \cup F)'$
v $\Pr(E' \cap F')$	**vi** $\Pr(F \cup E')$.

PROBLEM SOLVING AND REASONING

15 Venn diagrams are not restricted to displaying the relationship between two sets of data. They can show the relationship between three or more sets and also include subsets. Consider these two Venn diagrams.





a Which Venn diagram shows a subset of data? Explain the relationship shown in this Venn diagram.

b Copy this Venn diagram three times and on separate copies shade the section that represents:

i pets that are not dogs **ii** dogs that are not boxers

iii all dogs.

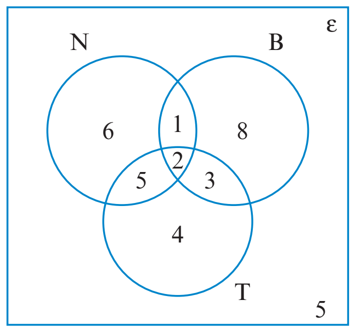
c The other Venn diagram shows the relationship between three different sets. Copy this Venn diagram five times and shade the section that represents students:

i taking all three elective subjects **ii** taking only Photography

iii taking Music or Chemistry **iv** taking Music

v not taking any of these three electives.

16 Consider this Venn diagram. Find the probability that a person chosen randomly from the group:



a plays netball but not tennis

b plays basketball only

c plays all three sports

d doesn't play any of these sports

e plays basketball or tennis

f plays tennis and netball

g plays netball given that they play basketball.

CHALLENGE

17 Draw a Venn diagram that represents the relationship between four different sets of data. There should be a total of 16 sections (including the section outside the sets) with no repeated sections. (Hint: Use ovals rather than circles.)

18 Use the following paragraph to draw a Venn diagram that shows the relationship between three different sets of data.

In a group of 100 people surveyed, 35 liked western films, 45 liked romance films and 46 liked horror films. Nineteen people did not like any of these three types. Fifteen people liked both western and romance, 16 only liked horror and 55 did not like romance films. Five people liked all three types and 18 liked both horror and romance.

Reflect

What mistakes do you think people might make when identifying sections of Venn diagrams?

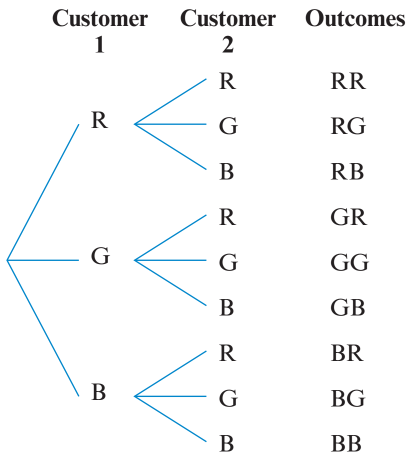
9F Experiments with replacement

Start thinking!

A store has a 'lucky dip' sale, where you get a discount based upon the colour of a ball you draw out of a box. If you draw a red ball you get 10% off, if you draw a green ball you get 25% off and if you draw a blue ball you get 50% off. There are 10 balls of each colour in the box.

1 What is the probability of drawing a blue ball?

After you draw a ball for your discount, you must place it back into the box so that the next customer also has an equal chance of drawing a blue ball. This, and other experiments you have seen in this unit, is called experiment with replacement.



Consider the tree diagram on the right.

2 What is it showing?

3 How many outcomes are possible when looking at the results of the first two customers?

4 Use the tree diagram to explain why, when looking at the first two customers, the probability of:

a both customers drawing a blue ball is $\frac{1}{9}$

b both customers drawing a red ball is $\frac{1}{9}$

c both customers drawing a green ball is $\frac{1}{9}$

d the first customer drawing a red ball and the second customer drawing a green ball is $\frac{1}{9}$.

5 Why are these probabilities the same?

When outcomes are simple and equally likely, drawing a tree diagram is not necessary and you may find it easier to just make a list of the outcomes. When outcomes are not equally likely, a tree diagram becomes more helpful. Look back at the tree diagram.

6 What is the probability of any branch in this tree diagram? (Hint: what is the probability of drawing a blue ball? A red ball? A green ball?)

KEY IDEAS

- Experiments with replacement involve selecting or drawing an item, recording the results, and replacing the item before performing another selection.
- A tree diagram or list of outcomes can help you to find the probabilities of individual outcomes or events involving more than one outcome.
- When outcomes are not equally likely, a tree diagram with probabilities written on the branches is useful in determining the probability of each final outcome.
- The probabilities of the final outcomes will always add to 1.

EXERCISE 9F Experiments with replacement

1 For each experiment, state the theoretical probability of the outcome in brackets in any given trial.

  a drawing a card and recording its suit (drawing a club)

  b selecting a marble and recording its colour out of a bag containing 10 blue, 5 red, 10 yellow and 5 green marbles (selecting a green marble)

  c drawing a card and recording if it is a number or picture card (drawing a picture card)

  d rolling a die and recording the number on top (rolling a 5)

EXAMPLE 9F-1 Representing experiments with equally likely outcomes

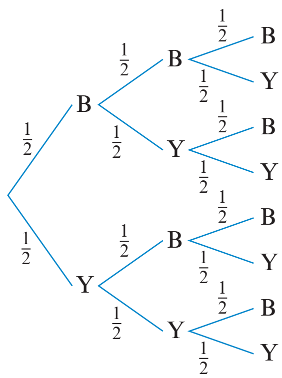
A box contains equal numbers of blue activity cards and yellow activity cards. A card is drawn, its colour recorded, then replaced. This is repeated two more times. Draw a tree diagram to represent this situation, complete with probabilities on each branch and for each final outcome.

THINK

1 Start by drawing a tree diagram for this three-step experiment. To calculate the probability of each final outcome, you need to know the probability of an individual outcome at each step.

2 You have a $\frac{1}{2}$ chance of drawing a blue activity card, and a $\frac{1}{2}$ chance of drawing a yellow activity card. Include these probabilities on the branches and calculate the probability of each final outcome.

WRITE



Outcomes	Probability
BBB	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
BBY	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
BYB	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
BYY	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
YBB	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
YBY	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
YYB	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
YYY	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

UNDERSTANDING AND FLUENCY

2 Draw a tree diagram with probabilities on the branches for each of these experiments.

  a A pencil case contains equal numbers of red and blue pens. A pen is drawn, its ink colour recorded, then replaced. This is repeated one more time.

  b A box contains equal numbers of $5, $20 and $75 vouchers. A voucher is drawn, its value recorded, then replaced. This is repeated one more time.

  c A box contains 10 cards, numbered 1–10. A card is drawn, it is recorded whether it shows an even or odd number, then it is replaced. This is repeated another two times.

  d A ball-pit contains equal numbers of blue, red, yellow and green balls. A ball is drawn, its colour recorded, then replaced. This is repeated one more time.

EXAMPLE 9F-2 Calculating probability for experiments with equally likely outcomes

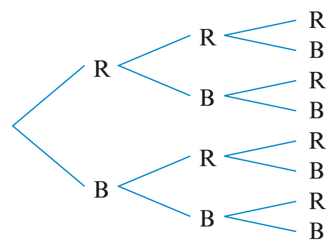
Use the tree diagram from Example 9F-1 to find the probability that:

a three blue activity cards are selected **b** a yellow activity card is selected first

THINK	WRITE
a Locate the outcome(s) where all selections produce a blue activity card (BBB).	**a** Pr(three blue) = Pr(BBB) $= \frac{1}{8}$
b Locate the outcome(s) where a yellow activity card is selected first.	**b** Pr(yellow first) = Pr(YBB, YBY, YYB, YYY) $= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

UNDERSTANDING AND FLUENCY

3 A bag contains six red counters and six black counters. A counter is drawn and replaced three times. This tree diagram shows the possibilities of the three counter draws.



a In any given trial, what is the probability of drawing a black counter?

b How many outcomes are there in total?

c What is the probability of drawing:

i three black counters? **ii** exactly one black counter?

iii at least two red counters? **iv** more than one black counter?

4 A card is drawn from a deck, its suit recorded, then replaced. This is then repeated. What is the probability that:

a both cards are hearts? **b** both cards are spades?

c the first card is a diamond and the second is a club?

d at least one card is a spade?

5 A box contains milk, dark and white chocolates in equal numbers. A chocolate is selected from the box, its flavour recorded, then replaced. This is then repeated. What is the probability that:

a both chocolates are white? **b** at least one chocolate is dark?

c the first chocolate is white and the second chocolate is milk?

d one chocolate is white and one chocolate is milk?

6 A card is drawn from a deck, its suit recorded, then replaced. This is repeated twice more. What is the probability of drawing:

a three hearts? **b** at least two diamonds?

c exactly two clubs? **d** no spades?

e at least one diamond or spade? **f** at least one heart and at least one club?

EXAMPLE 9F-3 Representing experiments with outcomes that are not equally likely

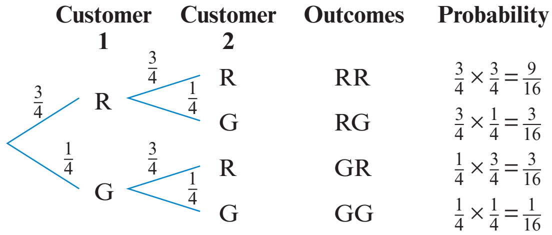
A lucky dip contains five green gift vouchers for \$50 and 15 red gift vouchers for \$5. A gift voucher is drawn, its value recorded and it is then replaced. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment.

THINK

1 Start by drawing a tree diagram for this two-step experiment. To calculate the probability of each final outcome, you need to know the probability of an individual outcome at each step.

2 You have a $\frac{15}{20}$ or $\frac{3}{4}$ chance of drawing a red gift voucher, and a $\frac{5}{20}$ or $\frac{1}{4}$ chance of drawing a green gift voucher. Include these probabilities on the branches and calculate the probability of each final outcome.

WRITE



7 Draw a tree diagram with probabilities on the branches for each experiment.

a A pencil case contains five blue pens and two red pens. A pen is drawn, its ink colour is recorded and then it is replaced. This is repeated one more time.

b A box contains 15 \$5 vouchers, 10 \$20 vouchers and 5 \$75 vouchers. A voucher is drawn, its value recorded is and then it is replaced. This is repeated one more time.

c A box contains 15 cards, numbered 1–15. A card is drawn, it is recorded whether it shows an even or odd number, then replaced. This is repeated another two times.

d A ball-pit contains five blue, four red, three yellow and two green balls. A ball is drawn, its colour recorded, then replaced. This is repeated one more time.

EXAMPLE 9F-4 Calculating probability for experiments with outcomes that are not equally likely

Use the tree diagram from Example 9F-3 to find the probability that:

a a \$50 voucher is selected twice

b a \$50 voucher then a \$5 voucher is selected.

THINK

a Locate the outcome where both customers select a \$50 voucher (GG).

b Locate the outcome where the first customer selects a \$50 voucher and the second customer selects a \$5 voucher (GR).

WRITE

a $\Pr(\text{both select } \$50) = \Pr(GG)$
$= \frac{1}{16}$

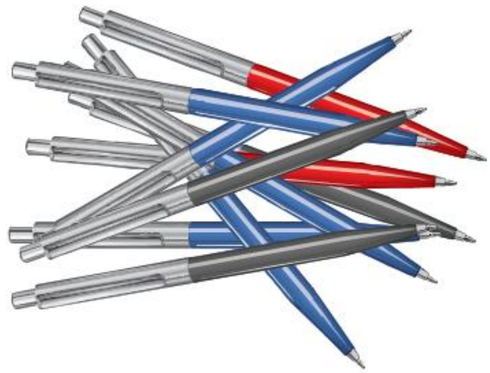
b $\Pr(\$50 \text{ then } \$5) = \Pr(GR)$
$= \frac{3}{16}$

UNDERSTANDING AND FLUENCY

8 A lucky dip contains 10 pink gift vouchers for \$100 and 40 green gift vouchers for \$10. A voucher is drawn, its value recorded and then it is replaced. If this was repeated, find the probability that:

- **a** a \$100 voucher was selected twice
- **b** a \$100 voucher was not selected at all
- **c** a \$100 voucher was selected first, and a \$10 voucher selected second.

9 The contents of a pencil case are shown here. The owner of the pencil case takes out a pen for each lesson.



- **a** Use the photo to draw a tree diagram to represent the pens drawn for the first two lessons of the day. Remember to include the probabilities along each branch and for the final outcomes.
- **b** Find the probability that the owner draws:
  - **i** a blue pen each time
  - **ii** a red pen each time
  - **iii** a black pen each time
  - **iv** a blue pen, then a black pen
  - **v** a blue pen, then a red pen
  - **vi** a red pen, then a black pen.

PROBLEM SOLVING AND REASONING

10 A card was drawn from a deck of cards; it was recorded if it was a picture card or not, then the card was replaced. This was repeated twice more.

- **a** Draw a tree diagram to represent this situation. Remember to include probabilities on the branches and calculate the final probability of each outcome. It is probably easiest to write these final probabilities as a decimal number rounded to four decimal places.
- **b** Find the probability of drawing:
  - **i** exactly one picture card
  - **ii** at least one picture card
  - **iii** less than two picture cards
  - **iv** at least two picture cards
  - **v** no picture cards
  - **vi** exactly two picture cards.

11 Use your tree diagram from question **9** to find the probability that in the first three lessons of the day, the owner of the pencil case selects:

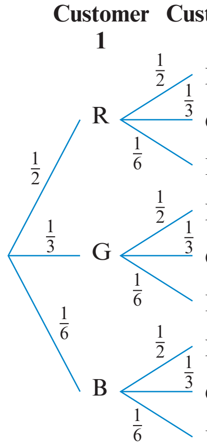
- **a** at least one blue pen
- **b** exactly two black pens
- **c** no more than one red pen
- **d** no blue pens.

12 Consider the pencil case from question **9**. Use a tree diagram or the strategy to find outcomes discussed in Exercise 9C questions **14** and **15** (page 426–7) to find the probability that, over the three double lessons of the day, the owner of the pencil case selects:

- **a** all black pens
- **b** a blue pen, then a red pen, then a black pen
- **c** at least one blue pen
- **d** no red pens
- **e** at least two black pens.

PROBLEM SOLVING AND REASONING

13 How do you calculate the probability of a final outcome in a multi-step experiment when the individual outcomes at each step are not equally likely? Consider the experiment from 9F Start thinking! where the box now contains 15 red balls, 10 green balls and 5 blue balls.



Outcomes	Probability
RR	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
RG	$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$
RB	$\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$
GR	$\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$
GG	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
GB	$\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}$
BR	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
BG	$\frac{1}{6}\times\frac{1}{3}=\frac{1}{18}$
BB	$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

a State the theoretical probability of drawing each colour ball.

To calculate the probability of a particular event that involves more than one final outcome, you need to add the probabilities of each favourable final outcome.

b Find the probability of exactly one customer drawing a blue ball by:

i listing the final outcomes where exactly one customer draws a blue ball

ii adding together these probabilities.

c Explain why the probability in part **b** is $\frac{10}{36}$ and not $\frac{5}{9}$.

d Calculate the probability of drawing:

i at least one green ball **ii** at least one red ball **iii** at least one blue ball

iv exactly one red ball **v** exactly one green ball **vi** a blue ball and a green ball.

14 Explain why you can use the process described in question **13** (adding together the probabilities of each favourable final outcome) to calculate the probability of final outcomes when individual outcomes are equally likely, but it is not necessary to do so.

15 Imagine that you are a customer in the experiment with equally likely outcomes described in 9F Start thinking! on page 440. The sales assistant is in a good mood and will give you a second chance if you don't draw a blue ball out of the box first up, as long as you put the first ball that you draw back into the box.
Use a tree diagram or other means to show that you have a $\frac{5}{9}$ chance of drawing a blue ball from the box.

CHALLENGE

16 A sock drawer contains 10 socks; some are black and some are white. You need to figure out how many of each colour are in the drawer, but you can only select one sock at a time and place it back.

a If you selected with replacement 10 times and selected 3 black socks and 7 white socks, does this mean that there are 3 black and 7 white socks in the drawer? Explain.

b If you selected with replacement 50 times, selecting 21 black socks and 29 white socks, how many socks of each colour would you estimate are in the drawer?

c If you selected with replacement 80 times, selecting 34 black socks and 46 white socks, does this support your previous estimate?

d State how many socks of each colour you believe to be in the drawer.

Reflect

Why is it important to consider all the possible outcomes when calculating probability?

9G Experiments without replacement

Start thinking!

In real life, games and experiments rarely involve an item being replaced. Rather, once an item is selected, it is usually kept. This affects the probability of every other item remaining. Consider the experiment described in the 9F Start thinking! on page 440.

A store has a 'lucky dip' sale, where you get a discount based upon the colour of a ball you draw out of a box. If you draw a red ball you get 10% off, if you draw a green ball you get 25% off and if you draw a blue ball you get 50% off. There are 10 balls of each colour in the box.

1 What is the probability of drawing a blue ball?

2 Draw the tree diagram that represents two customers selecting a ball from the box.
Do not include any probabilities at this stage, but write the final outcomes at the end of the branches.

Imagine that the first customer draws a blue ball out of the box and keeps it.

3 How many balls are left in total in the box?

4 How many of these balls are:

a blue? **b** green? **c** red?

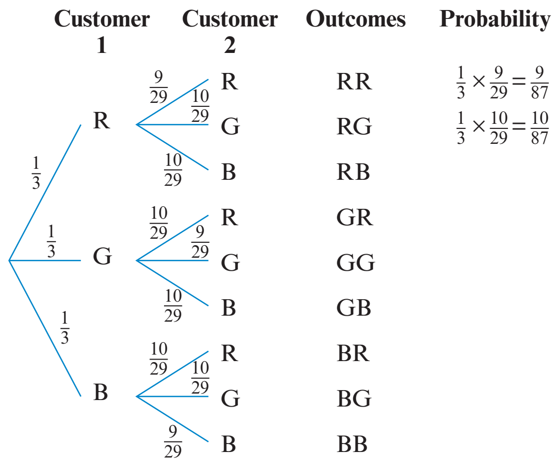
5 Imagine that you are the second customer to come along.
What is the probability of you selecting a ball that is:

a blue? **b** green? **c** red?

6 Explain why you have a different probability of drawing a blue ball from the first customer.

These probabilities have now been added to the branches of the tree diagram, as well as the probabilities that result if the first customer drew a green or red ball.

7 Copy and complete the tree diagram.



KEY IDEAS

- Experiments without replacement involve selecting or drawing an item, recording the results, and not replacing the item before performing another selection.
- Because each item that is selected is not replaced, this changes the probability of selection of the remaining items.
- A tree diagram or list of outcomes can help to find the probabilities of individual outcomes or events involving more than one outcome.

EXERCISE 9G Experiments without replacement

UNDERSTANDING AND FLUENCY

1 A bag contains five red counters and five black counters. A red counter is drawn.
  a How many of the counters that remain are:
    i red? ii black?
  b What is the probability that the next counter will be:
    i red? ii black?

2 A bag contains eight blue counters and eight green counters. A blue counter is drawn.
  a How many of the counters that remain are:
    i blue? ii green?
  b What is the probability that the next counter will be:
    i blue? ii green?

EXAMPLE 9G-1 Representing experiments without replacement

A lucky dip contains five red gift vouchers for \$50 and five green gift vouchers for \$5. A gift voucher is drawn and the customer keeps it. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment.

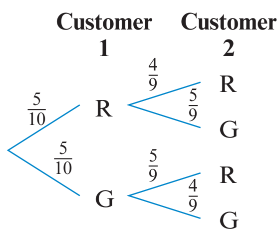
THINK

1 Start by drawing a tree diagram for this two-step experiment. To calculate the probability of each final outcome, you need to know the probability of an individual outcome at each step.

2 Initially you have a $\frac{5}{10}$ chance of drawing a red gift voucher, and a $\frac{5}{10}$ chance of drawing a green gift voucher. Write these on the first branches.

3 If you draw a red gift voucher, that leaves four red and five green vouchers in the box, but if you draw a green gift voucher, that leaves five red and four green gift vouchers in the box. Write these probabilities on the second lot of branches and calculate the probability of each final outcome.

WRITE



Outcomes	Probability
RR	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} \approx 0.22$
RG	$\frac{5}{10} \times \frac{5}{9} = \frac{25}{90} \approx 0.28$
GR	$\frac{5}{10} \times \frac{5}{9} = \frac{25}{90} \approx 0.28$
GG	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} \approx 0.22$

UNDERSTANDING AND FLUENCY

3 Draw a tree diagram with probabilities on the branches for each of these experiments.

- **a** A drawer contains five black socks and five white socks. A sock is drawn and its colour recorded. This is repeated two more times.
- **b** An esky contains six cans of Coke and six cans of Pepsi. A can is drawn and its type recorded. This is repeated two more times.
- **c** A box contains five 16 GB SD cards, five 32 GB SD cards and five 64 GB SD cards. A card is drawn and its capacity recorded. This is repeated one more time.
- **d** A bowl contains 10 Smarties and 10 M&Ms. A chocolate is drawn and its type recorded. This is repeated two more times.
- **e** A small ball-pit contains 10 blue, 10 red, 10 yellow and 10 green balls. A ball is drawn and its colour recorded. This is repeated one more time.
- **f** A box contains 10 names from 9D, 10 names from 9E and 10 names from 9F. A name is drawn and it is recorded which class the name is from. This is repeated two more times.

EXAMPLE 9G-2 Calculating probability for experiments without replacement

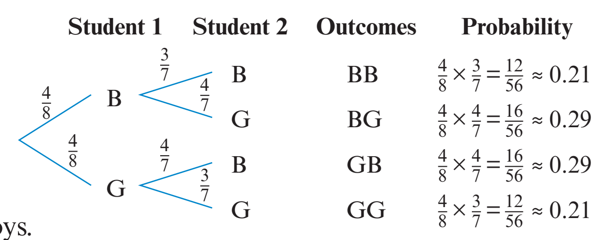
Use the tree diagram from Example 9G-1 to find the probability that:

- **a** both customers select a $50 voucher
- **b** the first customer draws a $50 voucher and the second customer draws a $5 voucher.

THINK	WRITE
a Locate the outcome where both customers select a $50 voucher (GG).	**a** Pr(both select $50) = Pr(GG) $= \frac{20}{90} \approx 0.22$
b Locate the outcome where the first customer selects a $50 voucher and the second customer selects a $5 voucher (GR).	**b** Pr($50 then $5) = Pr(GR) $= \frac{25}{90} \approx 0.28$

4 This tree diagram represents selecting two students from a group of four boys and four girls. Find the probability of selecting:

- **a** two boys
- **b** a boy then a girl
- **c** no boys.



5 A lucky dip contains four purple gift vouchers for $100 and four yellow gift vouchers for $10. A gift voucher is drawn and the customer keeps it. If this was repeated for a second customer, find the probability that:

- **a** both customers select a $100 voucher
- **b** the first customer selects a $100 voucher and the second customer selects a $10 voucher.

UNDERSTANDING AND FLUENCY

6 A lucky dip contains five purple gift vouchers for $100 and five yellow gift vouchers for $10. A gift voucher is drawn and the customer keeps it. If this was repeated for a second customer, find the probability that:

a both customers select a $100 voucher

b the first customer selects a $100 voucher and the second customer selects a $10 voucher.

7 Each bonbon in a pack of 12 contains one toy, and there are three different kinds of toy: a whistle, a yo-yo and a bouncy ball. In total, the pack contains four of each kind of toy. You and two friends select a bonbon from the pack.

a Draw a tree diagram to represent this situation. Be sure to include probabilities on each branch and the final probabilities for each outcome.

b Find the probability that:

  i all three of you select a bonbon with a whistle

  ii the first two bonbons have a yo-yo and the third has a bouncy ball

  iii the first bonbon has a whistle, the second a bouncy ball and the third a yo-yo.

PROBLEM SOLVING AND REASONING

8 When calculating the probability of an event involving more than one final outcome in a multi-step experiment, remember that you need to add together the probabilities of each favourable final outcome. Consider the situation from question **7**.

a Find the probability of at least one person selecting a bonbon with a whistle by:

  i listing the outcomes where at least one person selects a bonbon with a whistle

  ii adding together these probabilities.

b Find the probability that:

  i the three of you select a bonbon with a different toy

  ii two of you select a bonbon with a yo-yo

  iii at least one of you selects a bonbon with a bouncy ball

  iv a bonbon with a whistle is not drawn.

9 Consider a situation where a bag contains six red counters and six black counters.

a Draw a tree diagram to represent selecting three counters without replacement.

b Use the tree diagram to find the probability of drawing:

  i three black counters

  ii exactly one black counter

  iii at least two red counters

  iv more than one black counter

  v less than two red counters.

c Compare these answers to the experiment with replacement (Exercise 9F question **3**, page 442). What do you notice?

10 A card is drawn from a deck, and its suit is recorded. This is then repeated twice more.

a Draw a tree diagram to help you. Remember that there are 52 cards in a pack.

b What is the probability of drawing:

i three hearts? **ii** at least two diamonds?

iii exactly two clubs? **iv** no spades?

v at least one heart and at least one club? **vi** at least one diamond or spade?

c Compare these answers to the experiment with replacement (9F question **6**, page 442). What do you notice?

11 A drawer contains two pink socks, two purple socks and two green socks. Use a tree diagram or other means to calculate the probability that you draw out a pair when you select two socks from the drawer.

12 Imagine instead that the drawer from question **11** contained six of each colour sock. How does this change the probability of drawing a pair when you select two socks from the drawer?

13 A sports team needs to select a captain and a vice-captain.
Five people have put their names forward: Adrian, Chantelle, Katie, Ben and Sam.

a Draw a tree diagram to represent the selection (start with 'captain' branches).

b How does this tree diagram differ from the other ones you have done beforehand? (Hint: Does the second set of branches contain the same number as the first set of branches?)

c How many different combinations of captains and vice-captains are there? Remember that order is important!

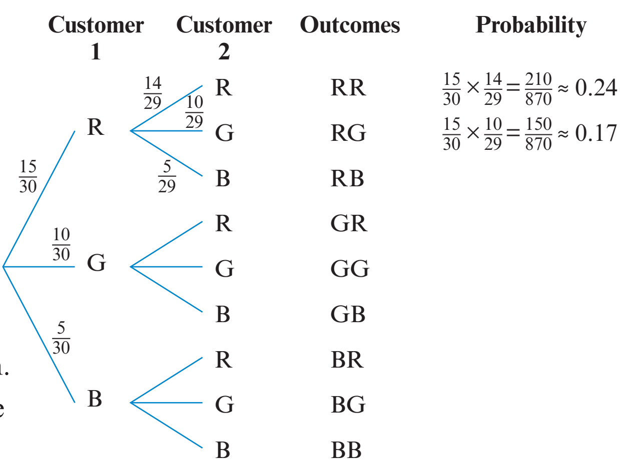
d Find the probability that:

i Katie is selected captain **ii** Sam is selected either captain or vice-captain

iii Adrian is captain and Chantelle is vice-captain

iv Ben does not get a position **v** Katie and Sam both get a position.

14 Experiments without replacement can also start with unequally likely outcomes. Consider the situation in the 9F question **13** (page 445), where the lucky dip contains 15 red balls, 10 green and 5 blue balls, but each ball is not replaced after it has been drawn.



a Copy and complete the tree diagram.

b Use the tree diagram to calculate the probability of:

i both customers drawing a blue ball

ii both customers drawing a green ball

PROBLEM SOLVING AND REASONING

iii both customers drawing a red ball

iv the first customer drawing a red ball and the second customer drawing a green ball.

15 Use the tree diagram from question **14** to calculate the probability of selecting:

a at least one green ball
b at least one red ball
c at least one blue ball
d exactly one red ball
e exactly one green ball
f a blue ball and a green ball.

16 If a drawer contained five red socks, four black socks and three white socks, find the probability that the first two socks selected from the draw form a pair.

17 A box of chocolates contains four milk chocolates, three white chocolates and two dark chocolates. Three chocolates are selected from the box. Find the probability of selecting:

a all three white chocolates
b no dark chocolates
c one of each chocolate type
d at least one white chocolate
e at least one milk chocolate
f all the dark chocolates.

18 A card was drawn from a deck of cards and it was recorded if it was a picture card or not. This was repeated twice more.

a Draw a tree diagram to represent this situation. Remember to include probabilities on the branches and calculate the final probability of each outcome. It is probably easiest to express these final probabilities as a decimal number rounded to four decimal places.

b Find the probability of drawing:

i exactly one picture card
ii at least one picture card
iii less than two picture cards
iv at least two picture cards
v no picture cards
vi exactly two picture cards.

c Compare your answers to that in Exercise 9F question **10** (page 444). How do the probabilities change when the cards are not replaced?

CHALLENGE

19 What is the probability of drawing a 21 in blackjack with the first two cards of the deck?

20 A common lottery consists of 45 numbered balls, of which six winning balls are drawn and two supplementary balls are drawn. To win the first division prize, you must pick all six winning numbers. What is the probability of winning the first division prize in this lottery?

Reflect

How does experiment without replacement differ from experiment with replacement?

CHAPTER REVIEW

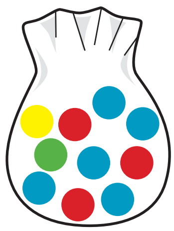
SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

certain	favourable outcome	trials	set notation
impossible	equally likely	expected number	universal set
even chance	sample space	tree diagrams	union
probability scale	complementary events	multi-step experiment	elements
event	experimental probability	conditional probability	subsets
outcome	relative frequency	two-way table	replacement
theoretical probability		Venn diagram	without replacement

MULTIPLE-CHOICE

9A **1** The theoretical probability of selecting a blue marble from the bag of marbles shown at right is:



A $\frac{3}{10}$ **B** 5 **C** $\frac{1}{2}$ **D** $\frac{1}{10}$

9B **2** Consider this table. Which of these statements is true?

Number on die	Frequency
1	45
2	35
3	27
4	30
5	32
6	31

A The number of trials performed was 200 rolls.

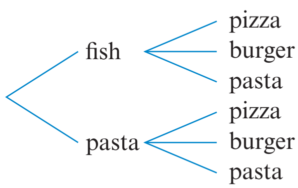
B The experimental probability of rolling an even number is $\frac{13}{25}$.

C The number rolled the least frequently was 1.

D The experimental probability of rolling a 6 is $\frac{1}{6}$.

9C **3** The probability that a person orders pasta for both entrée and main is:

Meal choices at a local café



A $\frac{1}{3}$ **B** $\frac{2}{3}$ **C** $\frac{1}{6}$ **D** $\frac{1}{2}$

9D **4** The probability of a person randomly selected who prefers Holden is:

	Male	Female	Total
Prefer Holden	176	98	
Prefer Ford	101	25	126
Total	277		400

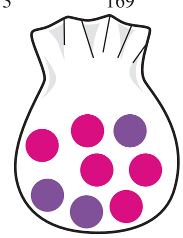
A $\frac{137}{200}$ **B** $\frac{49}{200}$ **C** $\frac{11}{25}$ **D** $\frac{101}{400}$

9F **5** A card is selected from a playing deck and an ace is drawn. The card is replaced into the deck. What is the probability of drawing an ace on the second selection?

A $\frac{1}{13}$ **B** $\frac{1}{169}$ **C** $\frac{12}{13}$ **D** $\frac{144}{169}$

Questions **6** and **7** refer to diagram at right.

A marble is selected and not replaced and then a second selection is made.



9G **6** The probability of selecting a pink marble on the second selection, given that the first marble was pink, is:

A $\frac{4}{7}$ **B** $\frac{3}{7}$ **C** $\frac{5}{7}$ **D** $\frac{2}{7}$

9G **7** The probability of selecting two purple marbles is:

A $\frac{3}{8}$ **B** $\frac{2}{7}$ **C** $\frac{5}{14}$ **D** $\frac{3}{28}$

SHORT ANSWER

9A 1 Consider rolling a 12-sided die. State the:

i probability of the listed event

ii probability of the complementary event.

a rolling a 6

b rolling an even number

c rolling a number less than 10

9C 2 A company runs a competition where one out of every four purchases contains the winning bar code for an eBook.

a What is the probability of not winning an eBook?

b Complete a tree diagram showing all probabilities on the branches for three steps.

c Calculate the probability of:

i winning three times in a row

ii not winning three times in a row

iii not winning on the first two tries and then winning on the third try.

9D 3 Refer to this two-way table for Years 8, 9 and 10 girls and their favourite brands of make-up.

	Year 8	Year 9	Year 10	Total
Napoleon Perdis	32	25	45	
Rimmel	22		22	54
Covergirl	17	31	13	
Maybelline		29		
Total	120		95	310

a Copy and complete the table.

b Calculate the probability that a girl chosen at random from the group:

i prefers Maybelline

ii is in Year 8 and prefers Rimmel

iii is in Year 9 and prefers Napoleon Perdis

c Calculate the probability that a girl chosen from the Year 9 girls:

i prefers Napoleon Perdis

ii prefers Covergirl or Maybelline.

9E 4 Counters numbered 1 to 15 are placed in a bag. One is drawn and recorded if it fits any of the following events.

Event 1: {numbers ≤ 5}

Event 2: {multiples of 2}

Event 3: {odd numbers}

a List the sample space for each of the events in this experiment.

b Draw a Venn diagram to represent this experiment.

c Calculate the following:

i Pr(Event 1) ii Pr(Event 2)

iii Pr(Event 2 or 3)

iv Pr(neither event 1, 2 nor 3).

d Explain why there is no number placed in the intersection of all three circles.

9F 5 A bag contains six chocolates. Two have orange wrappers, one has a green wrapper and three have pink wrappers. A chocolate is drawn, the colour recorded, the chocolate replaced and another selected.

a Show all possible outcomes on a tree diagram.

b Calculate the probability that:

i both wrappers are orange

ii both wrappers are green

iii both wrappers are pink.

c Calculate the probability that the first wrapper is green.

d Calculate the probability that the second wrapper is pink.

9G 6 Two cards are drawn from a deck of 52 and the suit(s) noted. Assuming selection without replacement, calculate:

a Pr(two spades)

b Pr(heart, then spade).

A third card is also drawn. Calculate:

c Pr(spade, heart, heart)

d Pr(spade, heart, diamond).

NAPLAN-STYLE PRACTICE

1 The theoretical probability of rolling a number greater than 4 on a standard die is:

$\frac{1}{6}$ ☐ $\frac{1}{3}$ ☐ $\frac{1}{2}$ ☐ 0 ☐

2 The results of an experiment are recorded in this table.

Option	Frequency
A	400
B	250
C	360
D	375
E	198
F	220
G	315
H	

Which of these statements is *not* correct?

- ☐ If 2500 trials were performed, the frequency for outcome H was 382.
- ☐ If 2500 trials were performed, the experimental probability for option A is $\frac{4}{25}$.
- ☐ A spinner with six equal segments may have been used in this simulation.
- ☐ If an additional 2500 trials were performed and the options were equally likely, the experimental probability for each option would theoretically get closer to $\frac{1}{8}$.

3 In a coin-flipping experiment, this result was recorded.

Outcome	Heads	Tails
Relative frequency		0.465

Which of these statements is false?

- ☐ The relative frequency of heads for this experiment is 0.535.
- ☐ If 3000 trials were performed, 1605 heads were recorded.
- ☐ If 5000 trials were performed, in theory 2500 tails would be expected.
- ☐ The relative frequency of heads for this experiment is 0.465.

4 Two spinners are spun at the same time. The first spinner has four equally-likely outcomes and the second has five equally-likely outcomes. The total number of possible outcomes to be represented on a tree diagram is:

4 ☐ 5 ☐ 10 ☐ 20 ☐

Questions **5** and **6** refer to the two-way table below.

A survey of 700 students was conducted relating to student enjoyment in different subjects at primary, secondary and tertiary levels.

	Primary	Secondary	Tertiary	Total
Maths	100		50	
English		69	122	
Sport	75	96		
Total	250	250		700

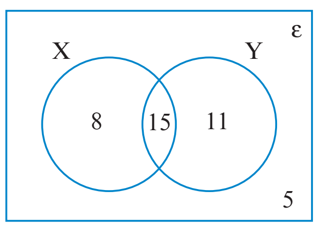
5 The probability that a student chosen at random from the group was a secondary student who enjoyed Maths is:

$\frac{1}{7}$ ☐ $\frac{1}{14}$ ☐ $\frac{17}{140}$ ☐ $\frac{47}{140}$ ☐

6 The probability that a student chosen from the primary group of the survey group enjoyed English is:

$\frac{2}{5}$ ☐ $\frac{3}{10}$ ☐ $\frac{69}{250}$ ☐ $\frac{3}{28}$ ☐

Questions **7–9** refer to this Venn diagram. The number of elements in sets X and Y is shown in the Venn diagram.



7 n(Y) is:

8 ☐ 11 ☐ 23 ☐ 26 ☐

8 $n(X \cup Y)$ is:

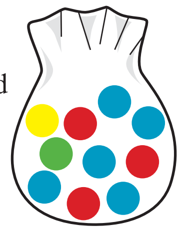
23 ☐ 26 ☐ 34 ☐ 39 ☐

9 $n(X \cap Y)$ is:

8 ☐ 15 ☐ 11 ☐ 5 ☐

Questions **10**–**13** refer to the diagram at right.

A bag contains a number of coloured marbles. A marble is drawn, the colour recorded, the marble replaced and another selected.



10 The probability of selecting two blue marbles is:

$\frac{1}{2}$ ☐ $\frac{1}{4}$ ☐ $\frac{9}{100}$ ☐ $\frac{1}{100}$ ☐

11 The probability of selecting a red marble first is:

$\frac{3}{20}$ ☐ $\frac{3}{100}$ ☐ $\frac{9}{100}$ ☐ $\frac{3}{10}$ ☐

Assume instead that marbles are selected without replacement.

12 The probability of selecting a blue marble on the second try, given that the first marble was not blue, is:

$\frac{5}{9}$ ☐ $\frac{1}{2}$ ☐ $\frac{3}{9}$ ☐ $\frac{1}{9}$ ☐

13 The probability of selecting three red marbles, if a third trial is performed, is:

$\frac{2}{9}$ ☐ $\frac{1}{120}$ ☐ $\frac{2}{72}$ ☐ 0 ☐

ANALYSIS

When moving from Year 9 into Year 10, students undertake core studies such as English and Mathematics, but are able to select from a range of electives to complete their study program.

A student is considering four subject choices: Art, Health and Human Development (HHD), Italian and Food Technology. Three subjects must be selected from this offering. He decides to write each subject onto a card and places them into a hat.

a Would this selection be with or without replacement? Explain your reasoning.

b i Draw a tree diagram to represent the possible subject combinations.

ii How many combinations are possible?

c Find the probability of selecting:

i Art, HHD and Food Technology

ii Italian, Art and HHD.

d The year level was surveyed about the Food Technology and Art electives offered. Twelve per cent of girls said that they preferred Art to Food Technology; 28% of boys said that they preferred Food Technology to Art. The year level is 52% boys.

i Construct a two-way table showing these percentages as relative frequencies.

Based on these survey results, if a person was randomly selected from the year level group of 150 students, find the probability that they are:

ii a girl who prefers Food Technology

iii a boy who prefers Art

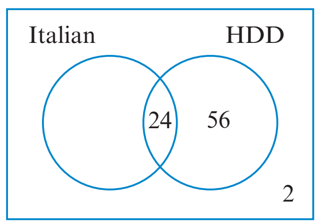
iv a student who prefers Art.

Of these 150 students, find the number of people who:

v are boys and prefer Food Technology

vi prefer Food Technology.

e An analysis of numbers of students studying Italian and/or HHD was also undertaken and the results collated into this Venn diagram.



i How many of the 150 students study both Italian and HHD?

ii How many students study neither subject?

iii How many students study Italian only?

iv If selecting a student at random from the cohort, what is the probability that they study Italian only?

v If selecting a student at random from the cohort, what is the probability that they study HHD only?

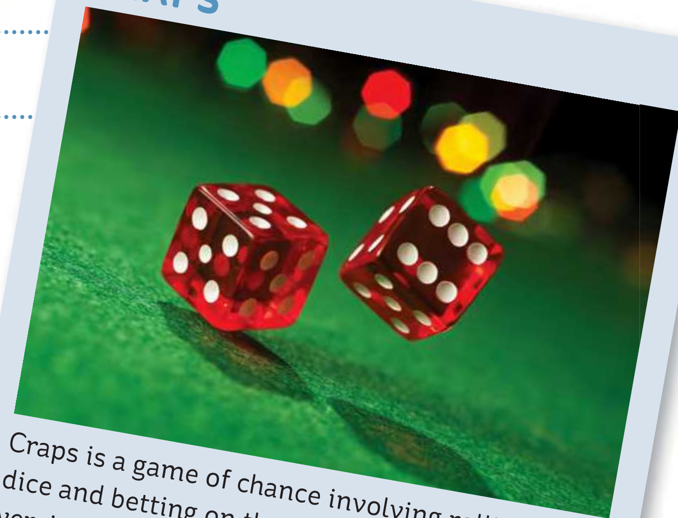
CONNECT

The house always wins

Casinos take advantage of long-term trends in experimental probability to make millions of dollars. Casinos use probabilities and payouts that slightly favour themselves (also known as 'the house'), and the mathematics always ensures that the casinos make money.

You are to investigate the probabilities and payouts of at least two simple games of chance and use this knowledge to construct a lottery-style game that will benefit the casino but that gamblers will still play. Within your investigation you should also address and explain the 'Gambler's fallacy'. Two simple games are listed on this page and you can use the Internet to find many others.

CRAPS



Craps is a game of chance involving rolling two dice and betting on the outcome. In a simplified version of craps, a player bets $1 and if they roll a sum of 2, 3, 4, 10, 11 or 12, they win $2, but if they roll a 5, 6, 7, 8 or 9, they lose.

Your task

For each game that you investigate, you will need to:

- calculate the theoretical probability of a player winning a game
- research and find a common 'payout' for a bet in the game
- calculate the expected winnings/loss for the player for 10 rounds of the game
- perform an experiment or simulation for 10 rounds of the game
- compare your theoretical probability and expected winnings/loss to the experimental probability
- alter the payout so that it pays the player more and recalculate expected winnings/loss and perform another experiment/simulation
- alter the payout so that it pays the player less and recalculate expected winnings/loss and perform another experiment/simulation
- discuss your findings and how the player is disadvantaged in a casino with a 'standard' payout.
- research the 'Gambler's fallacy' and explain how it applies to the game and why it is an incorrect line of thinking.

When constructing the lottery-style game, you will need to:

- decide how many numbered balls will be in the lottery
- decide how many numbered balls will be drawn from the lottery
- decide how many numbers that players must guess
- decide on payouts for the players based on the number of numbers they correctly guess
- perform a simulation to see if your payouts favour the casino
- alter your variables if necessary to ensure that the casino will make money in the long term.

You will need:

- access to the Internet
- access to equipment such as dice and cards.

You may like to present your findings as a report. Your report could be in the form of:

- a digital presentation
- a poster
- an information brochure
- other (check with your teacher).

ROULETTE

Roulette is a game of chance involving betting on the outcome of a spinning wheel. A standard roulette wheel has 37 numbers: 18 are red, 18 are black, and the number 0 is green. In a simplified version of roulette, a player bets $1 on red or black and they win $2 if they guess correctly.

Answers

CHAPTER 1 FINANCIAL MATHEMATICS

1 Are you ready?

1 a 1268 b 635 c 52 326 d 144
2 D 3 A 4 C 5 D
6 a 38.7784 b 41.874 c 1.617
d 77.5323
7 C 8 C 9 3 years 10 $x = 10$

1A Working with whole numbers

1A Start thinking!

1 270 145
2 If second digit is 0, 1, 2, 3 or 4, first digit stays the same and digits that follow are replaced with zero. If second digit is 5, 6, 7, 8 or 9, first digit increases by one and digits that follow are replaced with zero.
3 Week 1: 270 000; Week 2: 100 000; Week 3: 150 000; Week 4: 100 000
4 Easier to perform calculations when numbers are very large. Provides a quick overview of a situation; in this case, attendance for finals series.

Exercise 1A Working with whole numbers

1 a $500 b $900 c $3000
d $1000 e $100 f $100
g $80 000 h $100 000 i $600 000
j $700 000 k $1 000 000 l $3 000 000
2 a 19 000 b 770 c 12 000
d 40 e 2 800 000 000
f 300 g 3 500 000 h 30
3 a $110 000 b $160 000 c $24 000 000
d $30 000 e $90 000 f $100 000
g $3 000 000 h $20 000
4 a $3981 b $1173 c $85 025
d $236 145 e $2 747 744 f $359
g $113 513 h $2452
5

	Each number rounded to its leading digit	Estimated answer $	Exact answer $	Difference between exact and estimated answers $
a	400 000 – 200 000	200 000	143 890	56 110
b	90 000 + 200 000 + 600 000	890 000	878 480	11 520
c	900 000 × 40	36 000 000	34 200 136	1 799 864
d	4 000 000 ÷ 20	200 000	283 929	83 929
e	20 × 30 000	600 000	584 871	15 129
f	600 000 + 80 000 – 500 000	180 000	220 431	40 431

6 a laptops: 30, cost $800 b $24 000
7 a $400 b $80
c Possibly not the best strategy. Her estimate gives an extra $80 and she will think she earns more income than she actually does.
8 a $60 000 ÷ 50
b Alice's actual weekly pay will be lower. Her estimate is $1200 and her actual weekly pay is $1162. The number of weeks is rounded down, and dividing by a smaller number will give a larger answer for her weekly estimate.
9 a $4 000 000; $500 000 for each person
b $516 606 c $16 606
d Exact value, as it tells them exactly how much they will receive.
10 a 12 052 people
b premium: $200, standard: $90
c 900 people, total sales: $180 000
d 12 000 people, total sales: $1 080 000
e $1 214 099 f $45 901
11 a $1800 b $2000
c No, he is $200 short.
d 3 more months e $2400
f $2610 g $1620 h $990
i It is best to perform an exact calculation. He can save $90 per month, which divides exactly into $990. It will take him 11 months to save the money.
12 a $10 000
b Exact cost: $10 315. Exact cost is greater than estimate; difference is $315.
c Option 1: $2000 per person, total cost ≈ $12 000
Option 2: $1000 per person and $90 extra night charge, total cost ≈ $7080
d Option 2
e Option 1: $9300; Option 2: $9630
f Option 1: Estimate $2700 more than exact cost. Option 2: Estimate $2550 less than exact cost. No, best exact price does not match best estimated price.
g Possible answers are: costs per person for each option are close in price but Option 1 is rounded up and Option 2 is rounded down. Hence, the difference in their estimates will be greater.
h Family should take Option 1 as it is the cheapest.
13 a $1000 b $1300 c $1320

14 a i revenue: \$400 billion; expenses: \$400 billion
ii revenue: \$410 billion; expenses: \$400 billion
iii revenue: \$405 billion; expenses: \$399 billion
b i hundred billion ii ten billion
iii billion
c Possible answers are: numbers are too large to write and work with; it takes too long to count billions of dollars and so estimates must be relied upon; it is too difficult to count income and expenditure in every government department.

15 i

	Calculation	Each number rounded to its second digit	Estimated answer \$
a	\$358 248 – \$214 358	360 000 – 210 000	150 000
b	\$92 674 + \$195 647 + \$590 159	93 000 + 200 000 + 590 000	883 000
c	924 328 × \$37	920 000 × 37	34 040 000
d	\$4 258 935 ÷ 15	4 300 000 ÷ 15	286 666.67
e	\$21 × 27 851	21 × 28 000	588 000
f	\$625 384 + \$84 372 – \$489 325	630 000 + 84 000 – 490 000	224 000

ii

	Calculation	Each number rounded to its third digit	Estimated answer \$
a	\$358 248 – \$214 358	358 000 – 214 000	144 000
b	\$92 674 + \$195 647 + \$590 159	92 700 + 196 000 + 590 000	878 700
c	924 328 × \$37	924 000 × 37	34 188 000
d	\$4 258 935 ÷ 15	4 260 000 ÷ 15	284 000
e	\$21 × 27 851	21 × 27 900	585 900
f	\$625 384 + \$84 372 – \$489 325	625 000 + 84 400 – 489 000	220 400

iii Rounding to the third digit.

16 a Possible answers are: \$375.2 billion, \$418.9 billion, \$444.4 billion
b \$406.2 billion, \$411.5 billion, \$409.9 billion
c \$398.6 billion, \$399.2 billion, \$398.9 billion

17 If both numbers are rounded up or both are rounded down estimate will be greater than exact value. For example, 4827 + 15 342 ≈ 5000 + 20 000 = 25 000. Exact value is 20 169.
If one number is rounded down and other is rounded up, estimate will be less than exact value. For example,
211 254 − 56 314 ≈ 200 000 − 60 000
= 140 000. Exact value is 154 940.

1B Working with decimals

1B Start thinking!

1 There are 100 cents in \$1, so one cent is one-hundredth of a dollar or \$0.01. Hence, money is written in dollars to two decimal places.
2 \$9.99 per four balls
3 a division b \$7.99 per three balls

$= \frac{\$7.99}{3} \text{ per } \frac{3 \text{ balls}}{3}$

= \$2.6633… per 1 ball
= \$2.66 per ball

c \$2.66
4 \$9.99 per four balls

$= \frac{\$9.99}{4} \text{ per } \frac{4 \text{ balls}}{4}$

= \$2.4975 per 1 ball
= \$2.50 per ball
5 Jake should buy the four-ball canister for \$9.99; the price per ball is \$2.50 compared with \$2.66 per ball for other canister.

Exercise 1B Working with decimals

1 a \$24.40 b \$36.10 c \$28.05 d \$44.90
e \$22.30 f \$55.60 g \$35.75 h \$100.00
i \$0.35 j \$4.80 k \$105.25 l \$33.35
2 a \$194.16 b \$264.52 c \$980.95
d \$142.40 e \$18.40 f \$685.20
g \$4439.75 h \$1078.13 i \$140 725.88
j \$184.30
3 a \$30/h b \$1.35/L c \$55/h
d \$2.45/jar e \$0.75/min f \$12.99/kg
g \$60 000/year h \$6.85/parcel mailed
4 a \$5.25/h b \$1.49/L c \$7.84/kg
d \$3.60/mL e \$1.34/L f \$1.74/min
g \$0.02/g or \$1.60/100 g h \$21.35/h
5 a \$5.49/kg b \$24/h c \$0.37/min
d \$8.27/m
6 a \$18.50/h b \$370 c 37 h
7 a \$66 548/year b \$5545.67/month
c \$2559.54/fortnight
8 Possible answers are: piece work, commission, royalties.
9 a \$432.70 b \$698.80 c \$5350.95
d \$586.50
10 a i \$27 ii \$36 b i \$36 ii \$48
c i \$28.20 ii \$37.60 d i \$38.85 ii \$51.80
e i \$48.90 ii \$65.20 f i \$44.85 ii \$59.80
11 a \$1632.85 b \$1158.95
12 \$1023.75
13 a \$1.00 b \$4.75 c \$0.60 d \$2.65
14 a time-and-a-half: \$31.20/h; double time: \$41.60/h
b, c

Employee	Gross income \$	Hours at normal rate
Rodjay	748.80	36
Hansani	852.80	41
Anitya	1 081.60	52
Brendan	1 237.60	59.5

15 a \$228.10 b \$1114.50 c \$978.70
d \$304.60 e \$943.25
16 a i \$39.25 ii \$39.25 b i \$17.40 ii \$17.40
c i \$5.94 ii \$5.95 d i \$33.77 ii \$33.75
17 a Option 1: \$1.99/100g or \$0.2/g; Option 2: \$15.92/kg

b Option 1: \$19.92/kg; Option 2: 15.92/kg. Best buy is option 2.
c The statement is not accurate. The best buy is the option that has the lowest price per unit or lowest price when written as rates with the same units.

18 a 175-g bag for \$3.24 b 800-g box for \$3.00
c 750-g bag for \$16.90 d 680-g jar for \$4.00
e 1.7 kg for \$8.00 f 2-L bottle for \$6.94

19 Purchase the 50-ball box at \$135 per box. Each ball costs \$2.70 compared to \$2.75 per ball for four-ball canisters.

20 \$28.70/h

1C Working with ratios

1C Start thinking!

1 Ratio is in simplest form. 2 16
3 Natalie: $\frac{5}{16}$; Daniel: $\frac{1}{2}$; Megan: $\frac{1}{16}$; Patrick: $\frac{1}{8}$
4 Natalie: \$60; Daniel: \$96; Megan: \$12; Patrick: \$24

Exercise 1C Working with ratios

1 a 11 b 12 c 13 d 10 e 14 f 10
g 30 h 40 i 44 j 36 k 21 l 33

2 a $\frac{4}{11}, \frac{7}{11}$ b $\frac{1}{4}, \frac{3}{4}$ c $\frac{5}{13}, \frac{8}{13}$ d $\frac{1}{5}, \frac{4}{5}$
e $\frac{4}{7}, \frac{2}{7}, \frac{1}{7}$ f $\frac{1}{5}, \frac{1}{2}, \frac{3}{10}$ g $\frac{2}{5}, \frac{3}{5}$ h $\frac{3}{20}, \frac{1}{4}, \frac{3}{5}$
i $\frac{9}{44}, \frac{7}{44}, \frac{28}{44}$ j $\frac{1}{3}, \frac{2}{9}, \frac{4}{9}$ k $\frac{8}{21}, \frac{5}{21}, \frac{8}{21}$ l $\frac{3}{11}, \frac{3}{11}, \frac{5}{11}$

3 b 1:3 d 1:4 e 4:2:1 g 2:3
h 3:5:12 j 3:2:4 l 3:3:5

4 a \$50 b \$120 c \$300 d \$60

5 a \$1200 and \$6000 b \$3200 and \$4000
c \$1600 and \$5600 d \$2400 and \$4800
e \$800, \$4800 and \$1600
f \$1440, \$3600 and \$2160

6 a \$1920 and \$2880 b \$2800 and \$2000
c \$1600 and \$3200
d \$1200, \$2400 and \$1200
e \$1600, \$2400 and \$800
f \$2100, \$1500 and \$1200

7 a \$3150 and \$7350
b \$2100, \$5250 and \$3150
c \$2100 and \$8400
d \$3937.50 and \$6562.50
e \$1750, \$5250 and \$3500
f \$4900, \$3500 and \$2100

8 a \$3150 and \$1350
b \$1106.25 and \$1843.75
c \$4260 and \$6390
d \$5091.60 and \$3394.40
e \$11 028 and \$1838
f \$2056.89 and \$2571.11
g \$2810.57 and \$7026.43
h \$9329.33, \$13 994 and \$18 658.67
i \$1038, \$3633 and \$519
j \$494.18, \$988.36 and \$1235.45
k \$5033.33, \$7550 and \$6291.67
l \$26 446, \$44 076.67 and \$35 261.33

9 a $a = 84$ b $b = 49$ c $c = 48$ d $d = 72$
e $e = 52$ f $f = 3$ g $g = 48$ h $h = 7$

10 \$2480, \$3100 and \$1984

11 a 4:7 b Gabby: $\frac{4}{11}$, \$832; Jo: $\frac{7}{11}$, \$1456
c Gabby needed to find the total number of parts and use this figure in the denominator. She has used the number of parts belonging to Jo in the denominator.

12 a \$3900 b 28:15:31:26
c Connor: \$14; Luke: \$7.50; Jaymee: \$15.50; Maddie: \$13

13 a 1, $10.4 \times 10 = 104$ and $4.2 \times 10 = 42$
b 52:21

14 a $\frac{1}{10}, \frac{6}{10}$ b $\frac{1}{10} \times 10 = 1, \frac{6}{10} \times 10 = 6$
c 1:6
d Convert the mixed number to an improper fraction after writing each fraction as an equivalent fraction with the same denominator.

15 a 32:21 b 2:1 c 161:122 d 3:2
e 140:73 f 23:75 g 125:37 h 171:142
i 8:1 j 2:1 k 24:5 l 25:14
m 1:8 n 2:9 o 2:17 p 25:24

16 a $\frac{1}{8}$
b shopping: \$45; iTunes: \$48;
mobile phone account: \$12; savings: \$15
c 15:16:4:5

17 a \$28 b \$52

18 a 1:3 b \$5
c 3:1; Anthony gets three times what Justin receives.

19 a $x = 1600$ c $y = 625$

20 a \$250 b \$3500 c \$100
d \$7712 e \$18 666.67 f \$4011.67

21 a Answers should be the same. b $\frac{7}{10}$

22 William is incorrect. He needs to match a known amount with a smaller number in ratio in order to increase amount.

1D Percentage of an amount

1D Start thinking!

1 A discount is an amount by which an item's selling price is reduced.
2 a 20% of \$84 = $\frac{20}{100} \times 84$ b \$16.80
c Subtract discount from original price.
d \$67.20
3 Mark-up is amount added to cost of item before it is sold to the public.
4 a mark-up = 70% of \$96 = $\frac{70}{100} \times 96$
b \$67.20
c Add mark-up to cost price.
d \$163.20

Exercise 1D Percentage of an amount

1 a $36 b $1050 c $110 d $480
e $1615 f $67.20 g $233.28 h $3422
2 a $120 b $255 c $630 d $408
e $619 f $404.80 g $470.25 h $87.15
i $622.50
3 a $504 b $1002 c $198
d $1090.32 e $2520.76 f $3520
4 a i $194.40 ii $74.40
b i $778.70 ii $419.30
c i $42.42 ii $7.48
5 a $64.40 b $119.60
6 a 87.5% b $138.20
7 a i $440 ii $60
b i $152.58 ii $26.92
c i $229.08 ii $19.92
d i $860.11 ii $35.84
e i $537.16 ii $87.44
f i $28 345.28 ii $1649.72
g i $11 095 ii $1585
h i $1234.19 ii $261.80
8 a $44.60 b $37.91 c $82.51
9 a 210% b $188.80
10 a i $768.80 ii $148.80
b i $130.43 ii $40.48
c i $2284.20 ii $1015.20
d i $835.68 ii $385.15
e i $15 000 ii $8750
f i $789.73 ii $438.74
g i $31 078.13 ii $16 453.13
h i $5450.63 ii $3155.63
11 Jane is finding 20% of discount price, not original price. Her calculations will give: $\frac{20}{100} \times 198 = 39.60$
original price = $198 + $39.60 = $237.60
Find 20% of $237.60 and see if it gives a reduced price of $198. A 20% discount means you pay 80% of original price of bike; 80% of $237.60 = 0.8 × $237.60 = $190.08. Hence, Tim is correct.
12 a $9 b 900 c $900
13 $1000 14 $247.50
15 a $300 b $31.25 c $13.95
d $86.65 e $11 422.85 f $1070.70
16 $449.80
17 Possible answers are: an advantage is the more you sell the more you earn; a disadvantage is you will not earn much money if your sales are low. You will feel the pressure of needing to make sales at all times.
18 $582.75 19 $9237.50
20 They should choose the second agency, which charges $12 380 commission, as compared to the $13 570 commission charged by the first agency.
21 a $492.50 b $755 c $475 d $581.02
e $558.91 f 806.83 g $898.33 h $1378
22 $15 000 23 $35 000
24 Income is taxed at different percentages depending how much you earn.
25 Superannuation is a type of investment towards retirement. Employers contribute a certain percentage of an employee's salary to a fund (this percentage can change over time) and employees can contribute extra amounts.

1E Writing one quantity as a percentage of another

1E Start thinking!

1 $\frac{11}{20}$
2 Multiply the fraction by 100, simplify and add a % sign.
3 55%
4 Alexandra has saved just over half the amount.

Exercise 1E Writing one quantity as a percentage of another

1 a 20% b 75% c 25% d 150%
e 16.5% f 30.75% g 74.44% h 487.5%
i 512.5%
2 a profit of $10 b loss of $14
c loss of $4.55 d loss of $154.48
3 a i loss of $60 ii 25%
b i profit of $7.30 ii 292%
c i loss of $0.40 ii 7.69%
d i loss of $5390 ii 21.88%
e i profit of $230 ii 191.67%
f i loss of $21.95 ii 73.29%
4 a 33.33% b 74.49% c 8.33% d 28%
e 65.71% f 274.38%
5 a 28.57% b 17.07% c 4.92% d 51.50%
6 a loss of $48 b 24.24%
7 a $252 b 375%
8 a i loss of $14 ii 29%
b i profit of $127.50 ii 113%
c i profit of $50.05 ii 139%
d i loss of $411 ii 25%
e i loss of $12 395 ii 41%
f i profit of $268 865 ii 171%
9 a $2.55 per kg b 86%
10 a $27.45 b 11%
11 a $1413.50 b $264 c $661.60
d $7.15 e $346.12 f $80.25
12 a Answers are identical.
b When multiplying by 1.1, the 1 in the unit column is equivalent to 100% or a whole, and the 1 in the tenths column is equivalent to $\frac{1}{10}$ or 10%.
13 a $141.50 b $214.50 c $165.45 d $109.55
14 a Answers are identical.
b Dividing by 1.1 is equivalent to dividing by 110%. The 1 in the unit column is equivalent to 100% or a whole, and the 1 in the tenths column is equivalent to $\frac{1}{10}$ or 10%.
15 a No, profit margin is not a suitable pricing strategy as it makes Joseph's cars more expensive than his competitors.

b 111% profit
c A maximum percentage increase enables Joseph to keep his selling price for cars below selling price of his competitor's cars.

16 a 25% b 125%
c 125% = 100% (the purchase price) + 25% (the price increase)

17 a 20% b 80%
c 20% + 80% = 100%; if percentage decrease is subtracted from 100%, remaining percentage value gives percentage of original price car will be sold for.

18 a 100% increase b 200% increase
c 0% increase or decrease
d 50% decrease e 300% increase
f 100% decrease

19

Commodity	Starting price $	Movement %	Final price $
Gold	1 741.66	↓0.5	1 732.95
Silver	34.04	↓1.4	33.56
Oil	102.15	↑0.3	102.46
Copper	3.67	↑1.2	3.71

20 a $1 569 600
b i $1 874 610 ii $33 390
c profit of $305 010 d 19.43%

21 a $15.30 b $16.83
c $16.15; less than price in part b, because it doesn't take into account that GST is 10% of the marked-up price, not the wholesale price.

22 a selling price = wholesale price × 1.8 × 1.1
b $1200 = wholesale price × 1.6 × 1.1, so wholesale price = $1200 ÷ 1.6 ÷ 1.1 = $681.82
c $400 = original price × 1.1 × 0.75, so original price = $400 ÷ 1.1 ÷ 0.75 = $484.85

1F Understanding simple interest

1F Start thinking!

1 $500
2 interest earned = principal × rate × time

Exercise 1F Understanding simple interest

1 a i $\frac{7}{100}$ ii 0.07 b i $\frac{11}{100}$ ii 0.11
c i $\frac{2}{25}$ ii 0.08 d i $\frac{3}{50}$ ii 0.06
e i $\frac{1}{10}$ ii 0.1 f i $\frac{3}{25}$ ii 0.12

2 a i $500 ii $5500 b i $576 ii $5376
c i $5000 ii $17 500

3 a i $1125 ii $8625 b i $6480 ii $17 280
c i $14 000 ii $39 000

4 a $1200 b $2422 c $12 000
d $1104 e $17 999.10 f $20 000

5 a P = $3500, R = 4.8%, T = 2 years
b $336 c $3836

6 a P = $44 000, R = 9.5%, T = 5 years
b $20 900 c $64 900

7 a i $831.25 ii $831.25
b Answers to parts a i and ii are identical. Simple interest is calculated in same manner for investments and loans. In each case, simple interest is found by multiplying principal, rate (as a fraction or decimal) and time.
c Interest on an investment is a bonus payment and on a loan is an additional charge.

8 a $\frac{11}{12}$ b $\frac{7}{52}$ c $\frac{1}{2}$ d $\frac{1}{4}$ e $\frac{271}{365}$ f $\frac{31}{73}$
g $1\frac{1}{4}$ h 4 i $1\frac{8}{13}$ j $3\frac{2}{5}$ k $2\frac{1}{2}$ l $5\frac{1}{2}$

9 4.25 years

10 a i $1620 ii $10 620
b i $236.25 ii $10 736.25
c i $1837.50 ii $9337.50
d i $228.82 ii $29 228.82
e i $52.78 ii $8652.78
f i $17 825.73 ii $173 395.73
g i $1913.37 ii $21 912.37
h i $15 601.94 ii $61 551.94
i i $1950.77 ii $210 604.77

11 a $9.20 b $9.07
c Sade would hope that method which writes time as fraction of total number of months in year was used. She earns more interest with this method.
d Sade would prefer method that uses number of days in June as a fraction of total number of days in year. She would pay less interest on her loan using this method.

12 a 5.25% b $1050
c Statement is true. If Jasmine invests her money for 364 days, the calculations are:
$$I = \frac{\$20\,000 \times 0.0555 \times 364}{365} = \$1106.96$$
She receives slightly more interest.

13 a, b

Balance $	Days applied for	Interest earned $
640.90	7 days	0.26
540.90	7 days	0.22
780.90	9 days	0.40
655.50	5 days	0.19

c $1.07 d $656.57

14 a $0.72 b $830.27

15 a Yes, account A will receive bonus interest rate. They have only made one withdrawal for the month and the account balance has increased by more than $200.
b Account A: $6.23, Account B: $3.06
c Account A: $2231.98, Account B: $3139.82

16 a Deposit is $1875, so Joel's savings are enough to cover it.
b $10 000 c $15 050

17 a $33\frac{1}{3}$% p.a.
b i 25% p.a. ii 20% p.a. iii $16\frac{2}{3}$% p.a.
c i 50% p.a. ii 100% p.a.

18 Possible answer is:
5% p.a. for 5 years, 2.5% for 10 years, 10% for 2.5 years.

1G Working with simple interest

1G Start thinking!

1 time (T)
2 $P = \$15\,000$, $R = 6\%$, $I = \$3600$
3 $3600 = 15\,000 \times 0.06 \times T$
$3600 = 900T$
4 $T = 4$

Exercise 1G Working with simple interest

1 a $420 b $27.50 c $1700
d $5000 e $38 000 f $2400
2 a $T = 5$ years b $T = 7$ years c $T = 4$ years
d $T = 4$ years
3 a $P = \$5000$ b $P = \$15\,000$ c $P = \$4000$
d $P = \$4800$
4 a $T = 5$ years b $P = \$6000$ c $P = \$2500$
d $T = 2$ years e $P = \$3125$ f $T = 5$ years
5 a T, number of years
b $P = \$4500$, $R = 5.0\%$ p.a., $I = \$675$
c 3 years
d One year sooner ($T = 2$ years compared to $T = 3$ years)
6 a $462 b 24 months
7 a $864 = 6000 \times R \times 3$
b $6000 \times 3 = 18\,000$, hence $864 = 18\,000 \times R$
c 0.048 d multiply by 100 e 4.8%
f $I = \dfrac{P \times R \times T}{100}$, check result with teacher.
8 a 5.0% p.a. b 6.0% p.a. c 2.5% p.a.
d 5.6% p.a. e 6.4% p.a. f 2.5% p.a.
9 a $932.40 b $4500 c 9.8%
d 3.5 years or 3 years and 6 months
e $2950 f 6.8% g 5.0%
h 1.75 years or 1 year and 9 months
10 a P, amount borrowed
b $R = 4.0\%$ p.a., $T = 3$ years, $I = \$1440$
c $12 000
d total amount to be repaid = $12 000 + $1440 = $13 400; repayments = $350 × 36 = $12 600 (3 years = 36 months). Craig's repayments of $350 per month will not be enough to repay the loan over 3 years.
e $372.22 (will be 8 cents short at the end, but can make this adjustment later).
11 a 1 year, 3 months b $12 125
12 a 4.5% b $113.50
13 a $624 b $4624 c $208
d $4208 e $218.82 f $4426.82
g $230.19 h $4657.01
i Calculating interest at yearly intervals gives a higher value at the end of three years. $4657.01 is more than $4624. Interest is being added at the end of each year, so principal is greater each time interest is calculated and results in greater interest payments.
14 a $12 597.12 b $17 109.36 c $24 038.44
d $66 550
15 a i $2597.12 ii $197.12
b i $2109.36 ii $69.36
c i $6038.44 ii $638.44
d i $16 550 ii $1550
16 2.2%
17 a $741.13
b 16.8% (rounded to 1 dec. place)

1 Chapter review

MULTIPLE-CHOICE

1 A 2 D 3 B 4 C
5 C 6 B 7 D

SHORT ANSWER

1 a $3 395 765 b $41 124 c $1
2 a 92.5 km/h b $22.80/h
d $0.01/mL or $0.91/100 mL
3 $753.75, $945.00, $967.50, $956.25
4 a 4:5 b 1:3:2 c 1:7:8 d 4:15:11:7
5 a 151:113 b 35:16 c 1:3 d 1:8
6 a $2080 and $3120 b $3640 and $1560
c $1600, $3200 and $400
d $800, $1200, $2000 and $1200
e $3466.67 and $1733.33
f $2426.67, $1733.33 and $1040
7 a 3:4 b $200
8 a $644.30 b $65.10 c $259 d $173.40
9 a $281.25 b $27.75
10 a i profit of $15 ii 42.86%
b i loss of $18.65 ii 17.85%
c i profit of $45.70 ii 22.86%
11 a 20% b 66.67% c 250% d 580%
12 a $600 b $480 c $641.67
13 a $T = 3$ years b $P = \$4800$
c $T = 2.5$ years d $R = 9\%$ p.a.
e $R = 3.4\%$ p.a.

NAPLAN-STYLE PRACTICE

1 $60 000 ÷ 30 2 $9.50 per kg
3 14 × $15.80 + 5 × 1.5 × $15.80 + 3 × 2 × $15.80
4 $356.05 5 3:4:2 6 $1280 7 $a = 108$
8 15% of $79.00 9 $1517
10 $1033.35 11 $15 ÷ 80 × 100
12 $349.92 13 81.82%
14 Coins sold at a profit of $110.
15 275% 16 $3000 17 $15 500
18 12 days 19 $3.23 20 $5000
21 3.5 years

ANALYSIS

a $662.20
b Melanie: $554.60; Tahlia: $676.80; Simone: $451.20
c $4498.60 per week d $720
e 275% f $30.60 g 255%
h The new percentage mark-up is 155%, which is 120% less than the percentage mark-up in part e.
i $29 904 j $830.67 k $53 904

1 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 2 ALGEBRA

2 Are you ready?

1 a i $4 \times b \times c \times d$ ii $-9 \times x \times y$
iii $m \times m \times n$ iv $3 \times a \times k \times k \times k \times p \times p$
b i 4 ii −9 iii 1 iv 3
2 Some possible answers are given.
a $4d, -9d$ b $8ab, -3ab$
c $7kmnp, -kmnp$ d $2x^2y, -x^2y$
3 a 5 b −13 c −3 d −12 e 50 f −88
4 a B b C
5 a 75 b 54 c 52 d 269
6 a i 16 cm ii 15 cm^2
b i 24 mm ii 24 mm^2
c i 32 m ii 54 m^2
7 a i 3 ii 4
b $3 \times 3 \times 3 \times 3$ c 81
8 a i x ii 7
b $x \times x \times x \times x \times x \times x \times x$
9 a 700 b 3250 c 6.3 d 0.049
10 a A b 6 11 a D b 2
12 a 4 b $5x$ c $9m$ d $8y$

2A Working with algebraic terms

2A Start thinking!

1 B
2 a $3x, -x, 20x$
b Like terms have exactly the same pronumerals. The coefficient can vary.
3 3, 7, −1, 2, 1, 20
4 Only like terms can be added or subtracted to produce one term. Any terms can be multiplied and divided.

Exercise 2A Working with algebraic terms

1 a $k \times m \times n \times p$ b $3 \times x \times y$
c $-4 \times g \times h$ d $a \times a \times c$
e $6 \times x \times y \times y \times y \times w$
f $-2 \times b \times b \times c \times c \times c \times c$
2 Some possible answers are given.
a $8f, -4f$ b $2k, -7k$
c $9mn, -10mn$ d $5a^2, -11a^2$
e $xy^3, -10xy^3$ f $-2abcd, 7abcd$
3 a $10a$ b $-8k$ c $6x^2$
d $4cd - 9cde$ e $12x + 6y$ f $4a + 6b$
g $9m + 2p$ h $1 - k$ i $3xy + 5x^2$
j $2d - 4de^2$ k $4m^3 + 2$ l $abc + 4ab + ac$
4 a $10x + y$ b $5ab - 5b + b^2 + a$
c $2km + 4$ d $-3x^2 + 3x - 4$
e $2a + a^2 + a^3 - 3$ f $6m^2n + 4mn^2 - 2n^2$
5 a $6abcd$ b $-20mpxy$ c $9g^2h$
d $-24k^2mn$ e $56bjp^2t$ f ax^2y^2
g $18a^3bcd$ h $-20h^2k^2p$ i $-6b^3$
j $4km^2n^3$ k $15x^4y^2$ l $56a^4b^3c^2$
6 a b b $\frac{mn}{p}$ c $4c$ d $\frac{2}{x}$ e $\frac{1}{2}$ f $\frac{y}{4}$
g $\frac{6b}{5c}$ h $\frac{2}{11}$ i $6a$ j $3xy$ l $\frac{5n^2}{3}$ m $\frac{ac}{4}$
7 a pq b $2d$ c $\frac{3x}{c}$ d $\frac{k}{2p}$ e $\frac{7ac}{d}$ f $\frac{mn}{3}$
g $-\frac{3x}{2z}$ h $\frac{d}{2c}$ i $-\frac{2m}{3k}$
8 a true b false c false d true
9 a Set B correct. Set A: all signs changed to + when expression rearranged. Set C: sign in front of $2a$ term changed from + to − and 2 − 7 should be −5 not 9.
b Set A correct. Set B: -3×4 should be −12 not 12 and b should be squared in last step. Set C: $b \times b$ (or b^2) has been written as $b \times 2$.
c Set C correct. Set A: terms not in their appropriate location in fraction in last step. Set B: error in cancelling 'a' terms.
10 a $8x + y = 22$ b $-6xy = 36$ c $15xy = -90$
d $\frac{10}{y} = -5$ e $x^2y^3 = -72$ f $3x = 9$
g $6y - 2xy = 0$ h $x^2 + 4x - y = 23$
11 a $2a + 3b + 2c = 11$ b $8ab - a = -18$
c $-2a^2b + ab^2 + 3ac = 40$
d $10a^2b^2c^2 = 1000$ e $3ab = -6$
f $\frac{4ac}{3} = \frac{40}{3} = 13\frac{1}{3}$
12 a $6x + 5y + 2$ b $4x + 2y - 2$ c $12x + 20$
13 a 51 cm b 24 cm c 68 cm
14 a $20xy$ b $49x^2$ c $9xy$
d $2y^2$ e $3x^2 + 2y^2$ f $23xy$
15 a 120 m^2 b 441 m^2 c 54 m^2
d 8 m^2 e 35 m^2 f 138 m^2
16 a $2y$ b 10 m, 160 m^2
17 a $3x$ b 96 cm^2, 16 cm
18 a i 6k ii $2k^2$ b i 30 cm ii 50 cm^2
19 a i $4x$ m ii x^2 m^2
iii $(4x + 8)$ m iv $(4x + 4)$ m^2
b i 36 m^2 ii 32 m iii 40 m iv 64 m^2
20 a $10xy$
b Some possible answers are: $x = 4$ cm and $y = 5$ cm, $x = 2$ cm and $y = 10$ cm, $x = 1$ cm and $y = 20$ cm.
c $12x + 8y$ d 88 cm, 104 cm, 172 cm

2B Index laws

2A Start thinking!

1 a i $2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^7$
ii $a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^5$
b When multiplying terms with the same base, add indices.
Some possible examples are:
$x^2 \times x^2 = x^4$, $y^7 \times y^1 = y^8$, $a^4 \times a^6 = a^{10}$.

2 a i $3^6 \div 3^2 = \frac{3^6}{3^2}$
$= \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$
$= \frac{{}^1\cancel{3} \times {}^1\cancel{3} \times 3 \times 3 \times 3 \times 3}{{}_1\cancel{3} \times {}_1\cancel{3}}$
$= \frac{3^4}{1}$
$= 3^4$

ii $x^7 \div x^4 = \frac{x^7}{x^4}$
$= \frac{x \times x \times x \times x \times x \times x \times x}{x \times x \times x \times x}$
$= \frac{{}^1\cancel{x} \times {}^1\cancel{x} \times {}^1\cancel{x} \times {}^1\cancel{x} \times x \times x \times x}{{}_1\cancel{x} \times {}_1\cancel{x} \times {}_1\cancel{x} \times {}_1\cancel{x}}$
$= \frac{x^3}{1}$
$= x^3$

b When dividing terms with same base, subtract indices.
Some possible examples are:
$x^5 \div x^2 = x^3$, $y^7 \div y^1 = y^6$, $a^9 \div a^6 = a^3$

3 a i $(4^2)^3 = (4^2) \times (4^2) \times (4^2)$
$= (4 \times 4) \times (4 \times 4) \times (4 \times 4)$
$= 4^6$

ii $(k^3)^4 = (k^3) \times (k^3) \times (k^3) \times (k^3)$
$= (k \times k \times k) \times (k \times k \times k) \times (k \times k \times k) \times (k \times k \times k)$
$= k^{12}$

b When a term in index form is raised to another power, multiply the index inside the brackets with the one outside the brackets.
Some possible examples are:
$(a^2)^4 = a^8$, $(b^3)^4 = b^{12}$, $(c^4)^4 = c^{16}$.

Exercise 2B Index laws

1 a same, add, powers
b same, subtract, indices **c** multiply, powers

2 a i 5^8 **ii** y^{16} **iii** x^5
b i 2^8 **ii** p^2 **iii** h^1 or h
c i 7^{20} **ii** w^{12} **iii** m^{25}

3 a 3^9 **b** 7^6 **c** 6^{12} **d** 5^2
e 2^{24} **f** 10^{11} **g** 3^1 or 3 **h** 6^3
i 4^2 **j** 3^4 **k** 13^3 **l** 2^{10}

4 a 19 683 **b** 117 649 **c** 2 176 782 336
d 25 **e** 16 777 216
f 100 000 000 000 **g** 3 **h** 216
i 16 **j** 81 **k** 2197 **l** 1024

5 a a^2 **b** b^{10} **c** c^{13} **d** d^1 or d
e e^6 **f** m^8 **g** g^{10} **h** h^9
i x^7 **j** j^{10} **k** k^6 **l** y^8

6 a $12x^{11}$ **b** $10x^7$ **c** $24x^9$ **d** $54x^{11}$
e $3x^4$ **f** $4x^4$ **g** $\frac{2x}{5}$ **h** $\frac{5x^8}{3}$
i x^{13} **j** x^{22} **k** x^{27} **l** x^{27}

7 a i $2^4 \div 2^4 = 2^{4-4}$
$= 2^0$

ii $2^4 \div 2^4 = \frac{2^4}{2^4}$
$= \frac{{}^1\cancel{2} \times {}^1\cancel{2} \times {}^1\cancel{2} \times {}^1\cancel{2}}{{}_1\cancel{2} \times {}_1\cancel{2} \times {}_1\cancel{2} \times {}_1\cancel{2}}$
$= 1$

b i $x^5 \div x^5 = x^{5-5}$
$= x^0$

ii $x^5 \div x^5 = \frac{x^5}{x^5}$
$= \frac{{}^1\cancel{x} \times {}^1\cancel{x} \times {}^1\cancel{x} \times {}^1\cancel{x} \times {}^1\cancel{x}}{{}_1\cancel{x} \times {}_1\cancel{x} \times {}_1\cancel{x} \times {}_1\cancel{x} \times {}_1\cancel{x}}$
$= 1$

8 a A term with an index of zero always equals 1.
b i 1 **ii** 1 **iii** 1 **iv** 1 **v** 1

9 a 2 **b** 1 **c** 7 **d** 1 **e** 1 **f** 2
g 0 **h** 2 **i** 2 **j** 3 **k** 1 **l** 1
m 1 **n** 1 **o** −1 **p** −1

10 a 1 **b** 7 **c** 1 **d** 3 **e** 1 **f** k^2
g $10g^4$ **h** 3 **i** x^{15} **j** 2 **k** $8b$ **l** $\frac{2}{5}$

11 a x^6 **b** $2k^3$ **c** $6a$ **d** $\frac{m^2}{x^2}$ **e** x^{10} **f** w^6
g $\frac{b}{3}$ **h** e^6 **i** 1 **j** $4a^9$ **k** 2 **l** k^{10}

12 a a^9b^6 **b** $18m^{11}n^3$ **c** $x^{11}y^9$
d c^2d^2 **e** k^2m^3 **f** b^3
g $2w^2x^4$ **h** $\frac{m^{12}}{2n^5}$ **i** 1

13 a i $(3 \times 4)^2$
$= (12)^2$
$= 12 \times 12$
$= 144$

ii $3^2 \times 4^2$
$= (3 \times 3) \times (4 \times 4)$
$= 9 \times 16$
$= 144$

b i $(5 \times 2)^4$
$= (10)^4$
$= 10 \times 10 \times 10 \times 10$
$= 10\,000$

ii $5^4 \times 2^4$
$= (5 \times 5 \times 5 \times 5) \times (2 \times 2 \times 2 \times 2)$
$= 625 \times 16$
$= 10\,000$

14 a The product of two numbers within brackets raised to a power is the same as the product of each number raised to that power. Alternatively, when the powers are the same but the bases are different, the bases can be multiplied together and shown with the same index.
b i x^6y^6 **ii** c^3d^3 **iii** $78\,125k^7$
iv $3\,486\,784\,401p^{10}$ **v** a^5b^5
vi g^2h^2 **vii** $81m^4$ **viii** $256x^8$

15 a i $\left(\frac{3}{4}\right)^2$
$= \frac{3}{4} \times \frac{3}{4}$
$= \frac{9}{16}$

ii $\frac{3^2}{4^2}$
$= \frac{3 \times 3}{4 \times 4}$
$= \frac{9}{16}$

b **i** $\left(\frac{5}{2}\right)^4$
$= \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$
$= \frac{625}{16}$

ii $\frac{5^4}{2^4}$
$= \frac{5 \times 5 \times 5 \times 5}{2 \times 2 \times 2 \times 2}$
$= \frac{625}{16}$

16 **a** The division of terms in a set of brackets raised to a power is the same as the division of each term raised to that power. Alternatively, when the powers are the same but the bases are different, the bases can be divided and shown with the same index.

b **i** $\frac{x^6}{y^6}$ **ii** $\frac{k^3}{m^3}$ **iii** $\frac{d^5}{243}$ **iv** $\frac{64}{p^2}$

17 **a** x^9y^7 **b** $1568k^7$ **c** $81x^{24}$ **d** $5a^{28}b^7$
e $\frac{x^{10}}{y^{11}}$ **f** $\frac{8m^3}{n^3}$ **g** $\frac{a^8}{b^{20}}$ **h** $\frac{w^{10}x^6}{y^8}$
i k^7mn^2 **j** $\frac{16t^{13}}{p^{21}}$ **k** ab^{30} **l** $72h^6$

18 **a** false, $x^3 \times x^4 = x^7$ **b** true **c** true
d false, $a^5 \times a \times a^5 = a^{11}$ **e** true
f true **g** false, $m^3n^5 \times m^2n^4 = m^5n^9$
h false, $100^9 \div 100^9 = 1$
i false, $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y^6}$ **j** true
k false, $\frac{(k^3)^2 \times k^4}{k^2} = k^8$
l false, $\frac{a^5b^6}{a^2b^4} \times \frac{a^3b^5}{a^4b} = a^2b^6$

19 **a** 7 **b** 4 **c** 0 **d** 8 **e** 3 **f** 5
g 7 **h** 5

20 **a** Some possible answers are: $x = 1$ and $y = 11$, $x = 2$ and $y = 10$, $x = 3$ and $y = 9$.
b Some possible answers are: $x = 8$ and $y = 5$, $x = 10$ and $y = 7$, $x = 12$ and $y = 9$.
c Some possible answers are: $x = 1$ and $y = 20$, $x = 2$ and $y = 10$, $x = 4$ and $y = 5$.
d Some possible answers are: $x = 0$ and $y = 1$, $x = 0$ and $y = 2$, $x = 0$, $y = 3$.

21 no

22 **a** 2^3 **b** 3^3 **c** 5^2 **d** 10^4

23 **a** $(2^3)^4 = 2^{12}$ **b** $(3^3)^5 = 3^{15}$ **c** $(5^2)^9 = 5^{18}$
d $(10^4)^3 = 10^{12}$ **e** $(4^2)^7 = 4^{14}$ **f** $(2^5)^6 = 2^{30}$
g $(6^3)^2 = 6^6$ **h** $(3^5)^6 = 3^{30}$

24 Using an index law, $x^m \times x^n = x^{m+n}$. The statement is true when $x^{mn} = x^{m+n}$; that is, when $x = 1$ or $m = n = 0$ or $m = n = 2$.

2C Negative indices

2A Start thinking!

2 **a** **i** $3^4 \div 3^5$
$= 3^{4-5}$
$= 3^{-1}$

ii $3^4 \div 3^5$
$= \frac{3^4}{3^5}$
$= \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3}$
$= \frac{{}^1\cancel{3} \times {}^1\cancel{3} \times {}^1\cancel{3} \times {}^1\cancel{3}}{{}_1\cancel{3} \times {}_1\cancel{3} \times {}_1\cancel{3} \times {}_1\cancel{3} \times 3}$
$= \frac{1}{3}$

b $\frac{1}{3}$

3 $\frac{1}{a}$

4 $\frac{a^2}{a^3}$
$= \frac{a \times a}{a \times a \times a}$
$= \frac{{}^1\cancel{a} \times {}^1\cancel{a}}{{}_1\cancel{a} \times {}_1\cancel{a} \times a}$
$= \frac{1}{a}$

5 $\frac{3^4}{3^6}$
$= \frac{{}^1\cancel{3} \times {}^1\cancel{3} \times {}^1\cancel{3} \times {}^1\cancel{3}}{{}_1\cancel{3} \times {}_1\cancel{3} \times {}_1\cancel{3} \times {}_1\cancel{3} \times 3 \times 3}$
$= \frac{1}{3^2}$

6 $\frac{a^2}{a^5}$
$= \frac{{}^1\cancel{a} \times {}^1\cancel{a}}{{}_1\cancel{a} \times {}_1\cancel{a} \times a \times a \times a}$
$= \frac{1}{a^3}$

Exercise 2C Negative indices

1 **a** $\frac{1}{5}$ **b** $\frac{1}{8}$ **c** $\frac{1}{2}$ **d** $\frac{1}{6}$ **e** $\frac{1}{10}$ **f** $\frac{1}{17}$
g $\frac{1}{x}$ **h** $\frac{1}{p}$ **i** $\frac{1}{w}$ **j** $\frac{1}{b}$ **k** $\frac{1}{e}$ **l** $\frac{1}{n}$

2 Write the term as the denominator of a fraction, with a numerator of 1.

3 **a** 7^{-1} **b** 2^{-1} **c** 10^{-1} **d** 5^{-1}
e 13^{-1} **f** 4^{-1} **g** d^{-1} **h** m^{-1}
i y^{-1} **j** p^{-1} **k** c^{-1} **l** k^{-1}

4 **a** $\frac{1}{4^2}$ **b** $\frac{1}{2^6}$ **c** $\frac{1}{9^3}$ **d** $\frac{1}{5^4}$ **e** $\frac{1}{7^8}$ **f** $\frac{1}{10^5}$
g $\frac{1}{a^4}$ **h** $\frac{1}{x^7}$ **i** $\frac{1}{k^{10}}$ **j** $\frac{1}{m^2}$ **k** $\frac{1}{u^9}$ **l** $\frac{1}{g^{11}}$

5 **a** 3^{-4} **b** 4^{-7} **c** 6^{-5} **d** 5^{-3} **e** 9^{-2} **f** 11^{-6}
g n^{-2} **h** g^{-11} **i** x^{-8} **j** a^{-9} **k** p^{-4} **l** w^{-7}

6 **a** 2^3 **b** 5^6 **c** 8^4 **d** 3^9 **e** 7^5
f 4^2 **g** x^7 **h** y^3 **i** c^4 **j** $\frac{b^3}{a^5}$
k $\frac{p^2}{k^6}$ **l** $\frac{m^7}{h^4}$ **m** d^5e^8 **n** u^3w^8
o $3x^2y^4$ **p** $8g^6h^5$ **q** $\frac{2c^9}{d^4}$ **r** $\frac{2n^3}{5m^7}$

7 **a** $\frac{y^3}{x^2}$ **b** $\frac{m^6}{n^4}$ **c** $\frac{c^7}{a}$ **d** $\frac{2k^5}{p^3}$
e $\frac{5b^2}{a^8}$ **f** $\frac{4}{w^2x^6}$ **g** $\frac{a^4c^7}{b^5}$ **h** $\frac{m^5}{k^3m^8}$

i $\frac{7b^9d}{c^6}$ j $\frac{3}{x^2y^7z^4}$ k $\frac{n^4p^6}{m^3}$ l $\frac{4c^2e^6}{d^5}$

m $\frac{2p^4}{3kn^3}$ n $\frac{a^2b^3}{3c^5d^4}$ o $\frac{7nk^2}{m^6}$ p $\frac{x^3u^5}{10k^2w^8}$

8 a i $2^4 \times 2^{-7} = 2^{-3}$ ii $5^{-6} \div 5^2 = 5^{-8}$

iii $(3^{-3})^2 = 3^{-6}$ iv $\left(\frac{4}{5}\right)^{-2} = \frac{4^{-2}}{5^{-2}}$

b i $\frac{1}{2^3}$ ii $\frac{1}{5^8}$ iii $\frac{1}{3^6}$ iv $\frac{5^2}{4^2}$

c i $\frac{1}{8}$ ii $\frac{1}{390\,625}$ iii $\frac{1}{729}$ iv $\frac{25}{16}$

9 a $\frac{1}{4^3}$ b $\frac{1}{7}$ c 2^2 d $\frac{1}{3^6}$ e 5^4

f $\frac{1}{2^7}$ g $\frac{1}{9^2}$ h 3^8 i $\frac{1}{4^9}$ j $\frac{1}{10^3}$

k $\frac{1}{5^6}$ l $\frac{1}{3^8}$ m 2^4 n $\frac{1}{3^2}$ o $\frac{1}{6^4}$

p $\frac{1}{4}$ q $\frac{1}{9}$ r 5^8 s 7^3 t $\frac{1}{2^3}$

10 a $\frac{1}{64}$ b $\frac{1}{7}$ c 4 d $\frac{1}{729}$

e 625 f $\frac{1}{128}$ g $\frac{1}{81}$ h 6561

i $\frac{1}{262\,144}$ j $\frac{1}{1000}$ k $\frac{1}{15\,625}$ l $\frac{1}{6561}$

m 16 n $\frac{1}{9}$ o $\frac{1}{1296}$ p $\frac{1}{4}$

q $\frac{1}{9}$ r 390 625 s 343 t $\frac{1}{8}$

11 a $\frac{5^2}{4^2}$ b $\frac{3}{7}$ c $\frac{4^3}{3^3}$ d $\frac{2^4}{a^4}$ e $\frac{m}{6}$ f $\frac{1}{a^7b^7}$

g $\frac{1}{3^5x^5} = \frac{1}{243x^5}$ h $\frac{1}{4y}$

i $\frac{1}{7^2k^2} = \frac{1}{49k^2}$ j $\frac{1}{2^3p^3} = \frac{1}{8p^3}$

12 a -3 b -7 c -1 d 2

e -2 f -3 g -2 h -4

13 a $\frac{1}{64}$ b 0.015 625

14 a $\frac{1}{100\,000\,000}$ b 0.000 000 01

15 a $\frac{1}{15\,625}$ b 0.000 064

16 a

Index form	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Basic numeral	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

b The basic numeral of a term with base 2 and a negative index is the reciprocal of the value of the term with the matching positive index.

c $\frac{1}{64}$ d $\frac{1}{1024}$ e 128

17 a

Index form	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}	3^{-5}
Basic numeral	243	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

b The basic numeral of a term with base 3 and a negative index is the reciprocal of the value of the term with the matching positive index.

c $\frac{1}{729}$ d $\frac{1}{6561}$ e 2187

18 a

Index form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Basic numeral	10 000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$

b The basic numeral of a term with base 10 and a negative index is the reciprocal of the value of the term with the matching positive index.

c i 100 000 ii 1 000 000

iii 10 000 000 iv 100 000 000

v 1 000 000 000

d i $\frac{1}{100\,000}$ ii $\frac{1}{1\,000\,000}$

iii $\frac{1}{10\,000\,000}$ iv $\frac{1}{100\,000\,000}$

v $\frac{1}{1\,000\,000\,000}$

e i $\frac{1}{10}$ ii 0.1

f i $\frac{1}{100} = 0.01$ ii $\frac{1}{1000} = 0.001$

iii $\frac{1}{10\,000} = 0.0001$

g i $\frac{1}{100\,000} = 0.000\,01$

ii $\frac{1}{1\,000\,000} = 0.000\,001$

iii $\frac{1}{10\,000\,000} = 0.000\,000\,1$

iv $\frac{1}{100\,000\,000} = 0.000\,000\,01$

v $\frac{1}{1\,000\,000\,000} = 0.000\,000\,001$

h The number of decimal places in the decimal matches the number of zeros in the denominator of the fraction.

19 a i 20 000 ii 7000

iii 300 000 iv 400 000 000 000

v 90 000 000

b i $\frac{5}{100}$ ii $\frac{8}{100\,000}$ iii $\frac{2}{1000}$

iv $\frac{7}{10\,000}$ v $\frac{6}{1\,000\,000\,000}$

c i 0.05 ii 0.000 08 iii 0.002

iv 0.0007 v 0.000 000 006

d When multiplying a decimal by a negative power of ten, move the decimal point one place to the left for each zero you see.

20 a $\frac{1}{x^2}$ b $\frac{1}{x^4}$ c $8x^3$ d $\frac{30}{x^5}$ e 3

f x^9 g $\frac{1}{x^3}$ h $2x^5$ i $\frac{1}{3x^{10}}$ j $\frac{4}{7x^4}$

k $\frac{1}{x^2}$ l x^{11} m $\frac{4}{x^{13}}$ n $\frac{2}{x^8}$ o $\frac{1}{x^5}$

p $\frac{1}{x^7y^7}$ q $\frac{y^{20}}{x^{10}}$ r $\frac{y^3}{x^3}$ s x^2y^3 t $\frac{x^{24}}{y^{20}}$

2D Scientific notation

2A Start thinking!

1 a Multiplying by the given power of 10.
b A short cut is to move the decimal point the same number of places to the right as the power of 10. That is, for 8.9×10^4, move the decimal point 4 places to the right.

2 a Multiplying by the given negative power of 10, which is the same as dividing by the matching positive power of 10.
b A short cut is to move the decimal point the same number of places to the left as the power of 10. That is, for 8.9×10^{-4} which is $8.9 \div 10^4$, move the decimal point 4 places to the left.

Exercise 2D Scientific notation

1 a 540 b 73 600 c 1800
d 405 000 e 2 753 000 f 0.61
g 0.000 008 22 h 0.000 976
i 0.000 070 03

2 a 10^2 b 10^3 c 10^4 d 10^5 e 10^6

3, 4 a 320 000 b 8 140 000 000
c 500 d 23 450 000
e 11 000 f 0.0064
g 0.000 007 28 h 0.000 000 9
i 0.000 030 2 j 0.0541
k 450 000 000 000 l 0.000 000 006 12
m 0.57 n 1306.8
o 0.000 273 16

4 b Some of the very large and very small numbers are difficult to see on the calculator due to limited screen size.

5 a 4.5×10^3 b 7.32×10^6 c 2.0×10^5
d 1.9×10^2 e 3.216×10^3 f 6.3×10^{-3}
g 1.8×10^{-7} h 5.0×10^{-2} i 7.02×10^{-5}
j 4.27×10^{-1} k 1.122×10^4 l 4.0×10^{-6}
m 5.682×10^2 n 2.49×10^{-4} o 6.793×10^5
p 1.02×10^{-2}

6, 7 a 7.3034×10^5 b 8.36×10^4
c 6.3×10^{-3} d 2.68×10^{-2}
e 6.8×10^7 f 2.0×10^2
g 4.5×10^4 h 4.0×10^4
i 1.23×10^{-1} j 3.0×10^5

8 One possible answer is given.
a 400 cm b 450 cm c 455 cm d 4556 cm

9 a 3 b 2 c 4 d 1 e 3 f 2
g 1 h 4 i 5 j 4 k 4 l 3

10 a 2 b 3 c 4 d 2 e 4 f 3

11 a 460 b 73 050 c 1000
d 40 000 e 5.14 f 0.035
g 42.06 h 0.9 i 2.6×10^5
j 5.04×10^4 k 9.104×10^6 l 6.0×10^3

12 a 3.3×10^2 b 4.87×10^4 c 1.908×10^5
d 3×10^3 e 4.03×10^{-1} f 5.4×10^{-2}
g 2.072×10^{-4} h 8×10^{-3} i 7.6×10^2
j 2.070×10^4 k 4.02×10^1 l 5.4008×10^4

13 a A, B, F, H b B, F
c A, C, D, F, G d B, E, H

14 a 5×10^{-4} m b 2.2×10^4 ML
c 4.8×10^{-2} mm d 9.3×10^9 people

15 a 6400 times in a minute
b 0.000 08 mm c 149 600 000 km
d 0.000 000 000 000 28 cm

16 1.4994×10^7°C 17 1.08×10^9 km/h

18 a 2.4×10^6 km b 400 days

19 a Alpha Centauri b 1.09×10^{14} km

20 a i 86 coins ii 774 g
b 0.025 m = 2.5 cm

21 Approximately 506 seconds or about 8 minutes

22 a Light is faster than sound.
b i 3×10^{-7} seconds
ii about 0.3 seconds
c Watch for the smoke rather than listen for the shot.

23 diameter of one atom = 2.54×10^{-9} cm
The line will be 2.54×10^{-3} cm long.

24 1.8×10^{356}

2E Expanding algebraic expressions

2A Start thinking!

1 a i $7 \times (3 + 2) = 7 \times 3 + 7 \times 2$
ii $5 \times (x + 2) = 5 \times x + 5 \times 2$
b Area of large rectangle is length × width or $a \times (b + c)$. The total area of the two smaller rectangles is $a \times b + a \times c$. As the two area calculations are equivalent, $a(b + c) = ab + ac$.

2 a i $(6 + 4) \times (3 + 5)$
$= 6 \times 3 + 6 \times 5 + 4 \times 3 + 4 \times 5$
ii $(k + 2) \times (m + 3)$
$= k \times m + k \times 3 + 2 \times m + 2 \times 3$.
b i Area of large rectangle is length × width or $(6 + 4) \times (3 + 5)$.
ii Area of large rectangle is length × width or $(k + 2) \times (m + 3)$.
c i Total area of four smaller rectangles is $6 \times 3 + 6 \times 5 + 4 \times 3 + 4 \times 5$.
ii Total area of four smaller rectangles is $k \times m + k \times 3 + 2 \times m + 2 \times 3$.
d Area of large rectangle is length × width or $(a + b) \times (c + d)$. Total area of four smaller rectangles is $a \times c + a \times d + b \times c + b \times d$. As the two area calculations are equivalent, $(a + b)(c + d) = ac + ad + bc + bd$.

Exercise 2E Expanding algebraic expressions

1 To expand means to remove the brackets and simplify the expression.

2 a $2(a + 5)$
$= 2 \times a + 2 \times 5$
$= 2a + 10$
b $7(d - 4)$
$= 7 \times d + 7 \times (-4)$
$= 7d - 28$
c $-3(y + 6)$
$= (-3) \times y + (-3) \times 6$
$= -3y - 18$
d $4k(2m + 3)$
$= 4k \times 2m + 4k \times 3$
$= 8km + 12k$

3 a $4a + 12$ b $7b + 35$ c $3c - 6$
d $5d - 5$ e $24 + 6e$ f $-2f - 16$

g $-3g - 12$ **h** $-8h + 40$ **i** $-4x + 36$
j $-10 + 5j$ **k** $kp + 6k$ **l** $2ab - 8a$
m $18m + 6k$ **n** $6np + 12nq$ **o** $x^2 - 7xy$
p $-5k - 3k^2$

4 **a** $11x + 6$ **b** $5p + 6$ **c** $ab + 2a$
d $7y^2 - 3y + 3$ **e** $4h + k + 2$ **f** $-6m$

5 **a** $5x - 8$ **b** $13k - 4$ **c** $7p + 1$
d $x^2 + 4x + 12$ **e** $m^2 + 5m + 6$ **f** $y^2 - 7y + 10$

6 **a** $(a + 2)(b + 7)$
$= a(b + 7) + 2(b + 7)$
$= a \times b + a \times 7 + 2 \times b + 2 \times 7$
$= ab + 7a + 2b + 14$

b $(c - 3)(d + 5)$
$= c(d + 5) + (-3)(d + 5)$
$= c \times d + c \times 5 + (-3) \times d + (-3) \times 5$
$= cd + 5c - 3d - 15$

c $(h + 4)(h + 1)$
$= h(h + 1) + 4(h + 1)$
$= h \times h + h \times 1 + 4 \times h + 4 \times 1$
$= h^2 + h + 4h + 4$
$= h^2 + 5h + 4$

7 **a** $ab + 4a + 3b + 12$ **b** $cd + 7c + 2d + 14$
c $mn + m + 5n + 5$ **d** $xy + 3x + 9y + 27$
e $kp - 2k + 6p - 12$ **f** $fg - f + 4g - 4$
g $ac + 3a - 5c - 15$ **h** $wf + 2w - 7f - 14$
i $xy - 8x - 4y + 32$ **j** $jk - 5j - 9k + 45$
k $2ab + 6a + 7b + 21$ **l** $15cd - 20c + 6d - 8$
m $a^2 + 5a + 6$ **n** $x^2 + 15x + 50$
o $d^2 - 2d - 24$ **p** $y^2 - 5y - 24$
q $k^2 + 2k - 63$ **r** $m^2 - 3m - 18$
s $5e^2 - 22e + 8$ **t** $21a^2 - 31a + 8$
u $6y^2 - 13y + 5$

8 **a** $a^2 - 9$ **b** $b^2 - 4$ **c** $c^2 - 25$
d $d^2 - 49$ **e** $k^2 - 36$ **f** $x^2 - 16$
g $p^2 - 64$ **h** $h^2 - 1$ **i** $k^2 - m^2$

9 Middle two terms always combine to give zero, so you are left with first term squared minus second term squared. The two factors are almost the same with one set of brackets containing a sum of two terms and the other containing a difference of the same terms.

10 **a** It is the first term squared minus the second term squared.
b No, as you can multiply two factors in any order.
c **i** $x^2 - 4$ **ii** $y^2 - 81$ **iii** $m^2 - 36$
iv $d^2 - 100$ **v** $m^2 - n^2$ **vi** $9 - x^2$
vii $25 - k^2$ **viii** $1 - a^2$ **ix** $144 - p^2$

11 **a** $a^2 + 4a + 4$ **b** $b^2 + 14b + 49$
c $c^2 + 8c + 16$ **d** $d^2 + 18d + 81$
e $k^2 - 6k + 9$ **f** $x^2 - 12x + 36$
g $p^2 - 10p + 25$ **h** $h^2 - 2h + 1$
i $m^2 + 2mn + n^2$

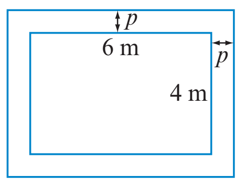
12 Result is always first term squared + (or −) two times product of both terms + second term squared. The two factors are exactly the same.

13 **a** $(a + b)^2$ is a perfect square.
b **i** $x^2 + 6x + 9$ **ii** $y^2 + 12y + 36$
iii $m^2 + 4m + 4$ **iv** $d^2 + 2d + 1$
v $b^2 + 22b + 121$ **vi** $m^2 + 2mn + n^2$
vii $25 + 10x + x^2$ **viii** $64 + 16k + k^2$
ix $1 + 2p + p^2$
c $(a - b)^2$ is also a perfect square. The rule or pattern holds true. The negative sign in the factor $(a - b)$ means that the middle term is $-2ab$ rather than $2ab$.
d **i** $a^2 - 2a + 4$ **ii** $b^2 - 8b + 16$
iii $c^2 - 14c + 49$ **iv** $d^2 - 20d + 100$
v $w^2 - 12w + 36$ **vi** $k^2 - 2kp + p^2$
vii $9 - 6x + x^2$ **viii** $81 - 18y + y^2$
ix $1 - 2w + w^2$

14 **a** Some possible answers are: $2x + 5$, $9p - 7$ and $6c^2 - 14$.
b A binomial product is where two terms are multiplied by two terms.
c three terms
d Some possible answers are: $x + 6y + 9$, $a - 2d + 4$ and $81 - 18z + y$.
e A quadratic trinomial is an expression with three terms, where the highest power of a variable is 2. For example: $p^2 - 10p + 25$, $b^2 + 22b + 121$ and $9 - 6x + x^2$.

15 **a** $(8)(2x + 5)$ **b** $16x + 40$
c **i** 88 cm² **ii** 88 cm²
d Answers should be the same if you have expanded correctly.

16 **a** **i** $(x + 7)(x + 3)$ **ii** $x^2 + 10x + 21$
iii 96 m²
b **i** $(x + 9)(x + 9)$ **ii** $x^2 + 18x + 81$
iii 196 mm²
c **i** $(2x - 1)(x + 2)$ **ii** $2x^2 + 3x - 2$
iii 63 cm²

17 **a**


b **i** $(2p + 6)$ m
ii $(2p + 4)$ m
c $(2p + 6)(2p + 4)$
$= 4p^2 + 20p + 24$
d $(4p^2 + 20p + 24) - (6 \times 4) = 4p^2 + 20p$
e **i** 24 m² **ii** 80 m² **iii** 56 m²

18 **a** $100^2 - 3^2 = 10\,000 - 9 = 9991$
b $(100 + 3)(100 - 3)$ is the same as 103×97 so the result is 9991.
c **i** 9996 **ii** 9975 **iii** 999 999
iv 999 964

19 **a** $14x + 7y - 35$ **b** $6x^2 + 15x - 1$
c $28x^2 + 13x - 6$ **d** $100y^2 - 140y + 49$
e $x^7 - x^6$ **f** $x^4 - 25$
g $y^{12} + 6y^7 + 9y^2$ **h** $-5a^3$
i $x^6y^6 + 2x^5y^2w$

20 $(a-b)^2+(c-d)^2$
$=(a^2-2ab+b^2)+(c^2-2cd+d^2)$
$=a^2+b^2+c^2+d^2-2ab-2cd$
$(b-a)^2+(d-c)^2$
$=(b^2-2ab+a^2)+(d^2-2cd+c^2)$
$=a^2+b^2+c^2+d^2-2ab-2cd$

21 **a** y by $(x-y)$
b $(y)(x-y)+y^2=xy-y^2+y^2=xy$
$=$ area of the original rectangle

22 $x=5$

2F Factorising using common factors

2A Start thinking!

1 As $3x+12=3\times x+3\times 4$, the HCF of 3 is written in front of a pair of brackets and the remaining factors of x and 4 are shown inside the brackets.
2 3
3 The HCF is common to both terms and is written in front of the pair of brackets.
4 As $5a+30=5\times a+5\times 6$, the HCF of 5 is written in front of a pair of brackets and the remaining factors of a and 6 are written inside the brackets. So $5a+30=5(a+6)$. This can be checked by expanding $5(a+6)$ to see if you get the original expression.

Exercise 2F Factorising using common factors

1 **a** 4 **b** 2 **c** 5 **d** d **e** 2 **f** 3 **g** 4 **h** $3h$ **i** $12x$

2 **a** $4\times a$ and 4×7 **b** $2\times 3b$ and 2×5
c 5×3 and $5\times 7c$ **d** $d\times d$ and $d\times 3$
e $2\times e$ and $2\times k$ **f** $3\times f$ and 3×-2
g $4\times 3g$ and 4×-2 **h** $3h\times 3$ and $3h\times -5h$
i $12x\times 2x$ and $12x\times 3$

3 **a** $4(a+7)$ **b** $2(3b+5)$ **c** $5(3+7c)$
d $d(d+3)$ **e** $2(e+k)$ **f** $3(f-2)$
g $4(3g-2)$ **h** $3h(3-5h)$ **i** $6(k+5)$
j $5(2x+1)$ **k** $3(y-7)$ **l** $4(2k+3)$
m $3(5-2d)$ **n** $7(4x+3)$ **o** $10(2n-5)$
p $x(x+3)$ **q** $m(m-9)$ **r** $a(4+a)$

4 **a** c **b** $2y$ **c** $9m$ **d** bd **e** $4y$ **f** $2k$ **g** p **h** 5 **i** $3p$

5 **a** $c\times b$ and $c\times d$ **b** $2y\times x$ and $2y\times 1$
c $9m\times 2n$ and $9m\times -1$
d $bd\times ac$ and $bd\times f$ **e** $4y\times 2x$ and $4y\times 7$
f $2k\times 3k$ and $2k\times -5$
g $p\times 1$ and $p\times 11p$
h $5\times 9ab$ and $5\times -8cd$
i $3p\times q+3p\times 2$

6 **a** $c(b+d)$ **b** $2y(x+1)$ **c** $9m(2n-1)$
d $bd(ac+f)$ **e** $4y(2x+7)$ **f** $2k(3k-5)$
g $p(1+11p)$ **h** $5(9ab-8cd)$ **i** $3p(q+2)$
j $5b(4a-1)$ **k** $8d(1+ce)$ **l** $5x(3x+2)$
m $2k(2k-11)$ **n** $6n(5-3n)$ **o** $a(16a+1)$
p $2h(h-7)$ **q** $6p(1+p)$ **r** $4xy(2y+3)$

7 Expanding means to write an expression without brackets. Factorising means to write an expression in factor form and generally involves using brackets. Factorising and expanding are opposite processes.

8 **a** $-3ab-6a=(-3a)\times b+(-3a)\times 2$
$=-3a(b+2)$
b $-8x^2+20x=-4x\times 2x+(-4x)\times(-5)$
$=-4x(2x-5)$

9 **a** $-5n(m+2)$ **b** $-7x(2y+1)$
c $-6c(1-d)$ **d** $-a(a+3)$
e $-2k(2k+1)$ **f** $-8x(x-1)$
g $-3(4+xy)$ **h** $-2m(8+5m)$
i $-9xy(y-2)$

10 **a** $(w+4)(x+2)$ **b** $(x-1)(y+7)$
c $(a+6)(a-3)$ **d** $(5-n)(p+8)$
e $(4-k)(3k-5)$ **f** $(3x-4)(2x+9)$

11 **a** $(a+5)(b+4)$ **b** $(y-6)(x+7)$
c $(n+4)(m-2)$ **d** $(y+3)(y+5)$
e $(k-7)(k+2)$ **f** $(x+3)(6+x)$
g $(a-7)(a-2)$ **h** $(p+5)(p-2)$
i $(3c-1)(2c+3)$

12 **a** **i** $6(x+1)$ cm **ii** $2x(x+4)$ cm
b **i** 36 cm **ii** 90 cm

13 **a** $(2x+5)$ m **b** $(m+15)$ cm

14 **a** $2(2x+9)$ m **b** $2(2m+15)$ cm

15 **a** $x(10-x)$ mm^2
b Length and width are x mm and $(10-x)$ mm.
c Some possible answers are: 6 mm by 4 mm, 7 mm by 3 mm, 9 mm by 1 mm.

16 **a** height $=(7-x)$ cm, base length $=6x$ cm
b Some possible answers are:
height = 3 cm, base length = 24 cm;
height = 6 cm, base length = 6 cm;
height = 2 cm, base length = 30 cm.

17 **a** $(x+3)(x-4)$ **b** $(x-4)(x+3)$
c They are equivalent.

18 **a** $n+1, n+2$ **b** $3n+3$
c $3(n+1)$; this is three times the middle number.
d $n+(n+2)+(n+4)=3n+6=3(n+2)$; this is also three times the middle number.

19 **a** $4x(x-2y+3z)$
b $\frac{1}{2}(x-\frac{1}{2}y+\frac{1}{4}z)$ or $\frac{1}{8}(4x-2y+z)$
c $-\frac{1}{4}x(32x+1)$

20 **a** $(y+5)(x-3)$ **b** $(2q-3)(p-1)$
c $(a-3)(2a-3)$

21 area of square $=l\times w=2r\times 2r=4r^2$,
area of circle $=\pi r^2$
area of shaded section $=4r^2-\pi r^2=r^2(4-\pi)$

22 For four consecutive terms, the sum is four times the average of the two middle terms. For six consecutive terms, the sum is six times the average of the two middle terms.
For 10 consecutive terms, the sum is 10 times the average of the two middle terms.
That is, $10\times\left[\frac{(n+4)+(n+5)}{2}\right]$
$=10\times\left(\frac{2n+9}{2}\right)=5(2n+9)$.

2G Factorising quadratic expressions

2A Start thinking!

1 A quadratic trinomial is an expression with three terms where the highest power of the variable is 2; for example, $x^2 + 4x + 3$.

2 a $(x + 2)(x + 3)$
$= x(x + 3) + 2(x + 3)$
$= x \times x + 3 \times x + 2 \times x + 2 \times 3$
$= x^2 + 3x + 2x + 6$
$= x^2 + 5x + 6$

b x and x c 2 and 3 d 3 and x, 2 and x

3 a $(x + \_\_)(x + \_\_)$ b $(x + 2)(x + 1)$
c $x^2 + 1x + 2x + 2 = x^2 + 3x + 2$

4 a $(x + \_\_)(x + \_\_)$
b $(x + 6)(x + 2)$ or $(x + 3)(x + 4)$ or $(x + 12)(x + 1)$
c $(x + 6)(x + 2) = x^2 + 2x + 6x + 12$
$= x^2 + 8x + 12$
d Because there is more than one factor pair.

5 You need to find factor pairs that add to give the term in the middle of the quadratic trinomial and multiply to give you the last term. For example, the factor pair which 'works' for $x^2 + 8x + 12$ is 6 and 2; that is, $6 + 2 = 8$ and $6 \times 2 = 12$.

Exercise 2G Factorising quadratic expressions

1 a 1 and 4 b 2 and 4 c 2 and 11
d 4 and 5 e 4 and 6 f 3 and 4
g 6 and 7 h 5 and 7 i 4 and 4

2 a $(x + 1)(x + 4)$ b $(x + 2)(x + 4)$
c $(x + 2)(x + 11)$ d $(x + 4)(x + 5)$
e $(x + 4)(x + 6)$ f $(x + 3)(x + 4)$
g $(x + 6)(x + 7)$ h $(x + 5)(x + 7)$
i $(x + 4)(x + 4)$

3 a $(a + 3)(a + 1)$ b $(b + 2)(b + 7)$
c $(c + 1)(c + 6)$ d $(d + 3)(d + 7)$
e $(e + 1)(e + 7)$ f $(f + 3)(f + 5)$
g $(g + 4)(g + 7)$ h $(h + 4)(h + 9)$
i $(x + 3)(x + 6)$ j $(j + 5)(j + 9)$
k $(k + 5)(k + 6)$ l $(y + 5)(y + 8)$

4 a −2 and 4 b −3 and 2 c −6 and −2
d −2 and 5 e −9 and 1 f −3 and −2
g −1 and 6 h −9 and 3 i −11 and −1

5 a $(x - 2)(x + 4)$ b $(x - 3)(x + 2)$
c $(x - 6)(x - 2)$ d $(x - 2)(x + 5)$
e $(x - 9)(x + 1)$ f $(x - 3)(x - 2)$
g $(x - 1)(x + 6)$ h $(x - 9)(x + 3)$
i $(x - 11)(x - 1)$

6 a $(a - 1)(a + 3)$ b $(b - 5)(b + 3)$
c $(c - 4)(c - 1)$ d $(d - 2)(d + 7)$
e $(e - 6)(e - 4)$ f $(f - 5)(f + 2)$
g $(g - 3)(g + 4)$ h $(h - 5)(h - 3)$
i $(x - 8)(x + 3)$ j $(j - 8)(j - 2)$
k $(k - 3)(k + 6)$ l $(y - 2)(y + 1)$

7 a $3(x + 1)(x + 2)$ b $2(x + 3)(x + 5)$
c $5(x - 1)(x + 4)$ d $-4(x + 2)(x + 3)$
e $-6(x - 5)(x - 1)$ f $-(x - 5)(x + 7)$

8 a $x^2 - 49$
b Show each term as a square (x^2 and 7^2) and then write the base of each term in two pairs of brackets. One pair of brackets will contain the sum of the two bases and the other the difference.

9 a $(x + 6)(x - 6)$ b $(a + 10)(a - 10)$
c $(3 + y)(3 - y)$ d $(5m + 2)(5m - 2)$
e $(7 + 8c)(7 - 8c)$ f $3(x + 2)(x - 2)$
g $2(3p + 5)(3p - 5)$ h $(ab + 3)(ab - 3)$
i $(xy + w)(xy - w)$ j $a(a + 4)$
k $(k + 1)(k - 7)$ l $(8 - m)(6 - m)$

10 a $(x + 3)(x + 6)$ b $(x + 3)$ and $(x + 6)$
c $(x + 6)$
d area = length × width = $(x + 6)(x + 3)$
$= x^2 + 3x + 6x + 18 = x^2 + 9x + 18$
e i length = 8 m, width = 5 m,
area = $8 \times 5 = 40$ m^2
ii area = $2^2 + 9 \times 2 + 18 = 40$ m^2

11 a i $(x + 9)$ cm ii $(3y + 4)$ m
b i Some possible answers are: $x = 3$, 4 or 5.
ii Some possible answers are: $y = 2$, 3 or 4.
c i for $x = 3$, area = 12 cm^2
for $x = 4$, area = 26 cm^2
for $x = 5$, area = 42 cm^2
ii for $y = 2$, area = 20 m^2
for $y = 3$, area = 65 m^2
for $y = 4$, area = 128 m^2
d 10 e 7

12 a $(x + 4)(x + 4) = (x + 4)^2$
b $(y - 5)(y - 5) = (y - 5)^2$
c $(2z - 3)(2z - 3) = (2z - 3)^2$

13 a 2600 b 11.4 c $\frac{1}{25}$

14 $2n + 1$ 15 $4(x + 1)$

16 a $(x^2 + 4)(x + 2)(x - 2)$
b $(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

17 $(x + 1)(x + y)^2 - (x + 1)(x - y)^2$
$= (x + 1)[(x + y)^2 - (x - y)^2]$
$= (x + 1)[(x + y) + (x - y)][(x + y) - (x - y)]$
$= (x + 1)(x + y + x - y)(x + y - x + y)$
$= (x + 1)(2x)(2y)$
$= 4xy(x + 1)$

18 a $x^3 + y^3$ b $(x + 2)(x^2 - 2x + 4)$
c $(x - 2)(x^2 + 2x + 4)$

19 31

2 Chapter review

MULTIPLE-CHOICE

1 C 2 D 3 B 4 A 5 D
6 D 7 B 8 C 9 A

SHORT ANSWER

1 a $16t$ b $13a - 18p$ c $-2k + km - 15$
d $13m^2n - 5m^2 + 11n^2 - 4mn^2$

2 a $44x^2y^2z$ b $72m^4n^3p^2$ c $\frac{5e}{6f}$ d $-\frac{n}{2m}$

3 a a^{16} b b c c^{16} d $\frac{d^3}{3}$
e e^{47} f 9

4 a m^5n^8 b $54k^{18}l^{12}$

5 a $\frac{1}{4}$ b $\frac{1}{b}$ c $\frac{1}{m^7}$ d $3x^3y^3$
e $\frac{g^5}{f^4}$ f $\frac{25c^4b}{a^3}$ g $\frac{1}{p^{15}}$ h $\frac{l^7}{k^7}$

6 a $\frac{1}{65\,536}$ b 343

7 a 58 760 b 0.000 009 02

8 a 4 b 3

9 a 5.4×10^5 b 7.6×10^{-4}

10 a 880 b 300 000

11 a 38 600 b 39 000 c 3.9×10^4

12 a $8a - 11$ b $b^2 - 9b - 22$
c $12c^2 - 23c + 10$ d $d^2 - w^2$
e $36 + 12e + e^2$ f $f^2 - 18f + 81$

13 a $4(a - 6)$ b $b(b + 11)$ c $36pq(q + 4)$
d $(8 - d)(7d - 4)$ e $(1 + 3f)(5e + 2)$

14 a $(a + 2)(a + 3)$ b $(b - 4)(b - 3)$
c $(c - 3)(c + 7)$ d $(d - 18)(d + 2)$
e $5(e - 7)(e - 2)$

15 a $(a + 8)(a - 8)$ b $(11 + b)(11 - b)$
c $(6m + 7n)(6m - 7n)$ d $(p + 3)(p - 1)$
e $2(e + 4)(e - 4)$ f $9(2f - 1)$

NAPLAN-STYLE PRACTICE

1 $14b + 28$ 2 $w + 2$ 3 100 4 18
5 3 6 −25 7 $12a - 5$ 8 $\frac{1}{4^6}$
9 $\frac{1}{x}$ 10 w^5 11 $\frac{2}{3r^2p^4}$ 12 29 700
13 < 14 2×10^6 15 $6xyz$
16 $a^2 + 2a - 15$ 17 $(b - 5)(b - 8)$
18 $(m - 3)(m + 4)$ 19 $(d - 2)(7d + 9)$
20 $6x + 4y - 4$ 21 34 cm
22 $6xy + 2y - 9x - 3$ 23 481 cm^2
24 $5m(10 - m)$
25 length = $5m$ cm and width = $(10 - m)$ cm

ANALYSIS

a i TAS, NT, ACT
ii NSW, VIC, QLD, SA, WA

b, c, e, f

State	Population at end June 2013 ('000)	Population written in full	Population rounded to leading digit	Population written in scientific notation
NSW	7407.7	7 407 700	7 000 000	7×10^6
VIC	5737.6	5 737 600	6 000 000	6×10^6
QLD	4658.6	4 658 600	5 000 000	5×10^6
SA	1670.8	1 670 800	2 000 000	2×10^6
WA	2517.2	2 517 200	3 000 000	3×10^6
TAS	513.0	513 000	500 000	5×10^5
NT	239.5	239 500	200 000	2×10^5
ACT	383.4	383 400	400 000	4×10^5

d All the same except for Tas which is now shown with three significant figures.

g i NSW ii NT iii 6.8×10^6
iv 3.1×10^6 v 2.41×10^7

h 9.691×10^5 or 969 100. Difference is due to rounding.

2 Connect

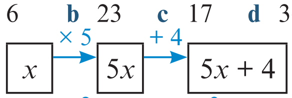
For feedback on this open-ended task, see your teacher.

CHAPTER 3 LINEAR RELATIONSHIPS

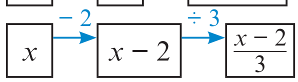
3 Are you ready?

1 a 6 b 23 c 17 d 36

2 a



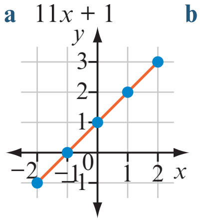
b



3 a $3x + 15$ b $-10x + 5$

4 a $11x + 1$ b $2x - 7$

5



6 a (5, 5) b (−2, −2) c (1, 7)

7 a 5 b 2 c 7.5

8 a 8 b −2 c 5.5

9 a 289 b 23.04 c 610
d 9 e 16.58 f 7.39

3A Solving linear equations

3A Start thinking!

1 C

2 Some possible answers are: trial and error/guess, check and improve; backtracking with a flowchart; undo each operation using the balance method.

3 $x + 7 - 7 = 13 - 7,\ x = 6$

4 a i add 5 (inverse of subtract 5)
ii divide by 2 (inverse of multiply by 2)
b Flowcharts show how an expression is built one operation at a time.
c Performing operations in wrong order would probably give you a wrong answer.
d $2x - 5 + 5 = 3 + 5,\ 2x = 8,\ \frac{2x}{2} = \frac{8}{2},\ x = 4$
e Substitute solution back into original expression and see if answer is still true.

5 a − 9, ÷ 11 b $x = 6$ c 6

6 Because same operation is applied to both sides of equation.

Exercise 3A Solving linear equations

1 a
$$\frac{x}{6} - 4 + 4 = 1 + 4$$
$$\frac{x}{6} = 5$$
$$\frac{x}{6} \times 6 = 5 \times 6$$
$$x = 30$$

b
$$\frac{x + 3}{4} \times 4 = -2 \times 4$$
$$x + 3 = -8$$
$$x + 3 - 3 = -8 - 3$$
$$x = -11$$

c $\frac{6(x+9)}{6} = \frac{12}{6}$
$x + 9 = 2$
$x + 9 - 9 = 2 - 9$
$x = -7$

2 a $x = 6$ b $x = 7$ c $x = 5$ d $x = 36$
e $x = 10$ f $x = 8$ g $x = -4$ h $x = -2$
i $x = 15$ j $x = -1$ k $x = 9$ l $x = -3$
m $x = -12$ n $x = 3$ o $x = -6$

3 -4, $\times 3$, $+1$ 4 $x = 16$

5 a $x = 4$ b $x = 2$ c $x = 6$ d $x = 8$
e $x = 9$ f $x = 5$ g $x = 10$ h $x = -4$
i $x = 20$ j $x = -8$ k $x = 0$ l $x = 19$
m $x = 1$ n $x = -5$ o $x = -7$

6 a C b $x = -3$

7 a $x = -6$ b $x = 9$ c $x = -11$ d $x = 5$
e $x = 0$ f $x = -32$ g $x = -60$ h $x = 8$
i $x = -10$ j $x = 21$ k $x = -7$ l $x = -84$

8 a $x = 6.6$ b $x = 11.3$ c $x = 4$
d $x = 7.2$ e $x = -5$ f $x = 1.9$
g $x = -2$ h $x = -3$ i $x = 0.7$
j $x = 14$ k $x = 1.6$ l $x = 6.9$
m $x = -2.6$ n $x = 10.27$ o $x = -3.16$

9 a $-2x + 3 - 3 = 17 - 3$
$-2x = 14$
$\frac{-2x}{-2} = \frac{14}{-2}$
$x = -7$

b $\frac{-4(x-7)}{-4} = \frac{-12}{4}$
$x - 7 = 3$
$x - 7 + 7 = 3 + 7$
$x = 10$

c $-x + 5 = 11$
$-x + 5 - 5 = 11 - 5$
$-x = 6$
$\frac{-x}{-1} = \frac{6}{-1}$
$x = -6$

10 a $x = -5$ b $x = -2$ c $x = 4$ d $x = 3$
e $x = -11$ f $x = 1$ g $x = -4$ h $x = 10$
i $x = -16$ j $x = 9$ k $x = -4$ l $x = -9$
m $x = 4$ n $x = -7$ o $x = -20$

11 a $x = 6$ b $x = 6$ c $x = 5.6$ d $x = 5.6$
e Expanding brackets first means you are not dealing with decimals or fractions until the very last step.

12 a $x = 3$ b $x = -9$ c $x = 5\frac{1}{2}$
d $x = -2\frac{2}{5}$ e $x = 4$ f $x = 1\frac{5}{6}$

13 a $x = 3$ b $x = 11$ c $x = 1$
d $x = 3$ e $x = 2\frac{6}{7}$ f $x = -4\frac{1}{3}$
g $x = 1$ h $x = -5$ i $x = 6$

14 a $x = 4$ b multiply both sides by x

15 a $x = 5$ b $x = -3$ c $x = 4$ d $x = -7$
e $x = -6$ f $x = 4$ g $x = 5\frac{1}{2}$ h $x = -1\frac{3}{5}$
i $x = 4.8$ j $x = 12$ k $x = 40$ l $x = -\frac{1}{3}$

16 a $x = 6$ b $x = 8$ c $x = -16$ d $x = -55$
e $x = \frac{2}{3}$ f $x = -7$ g $x = 21.7$ h $x = 9\frac{2}{3}$
i $x = -2$ j $x = \frac{3}{4}$ k $x = -3$ l $x = 7$
m $x = -\frac{3}{4}$ n $x = -25$ o $x = -15$

17 a x = number of jelly beans b $4x + 2 = 34$
c $x = 8$ d each person gets 8 jelly beans

18 a x = number of months
b $70x + 115 = 395$ c $x = 4$
d 4 months

19 a $185 b 14 goals
c 31 m × 19 m d 84 people

20 56 sausages 21 100°

3B Solving linear equations with the unknown on both sides

3B Start thinking!

1 B
2 There is an unknown on both sides of equation.
3 $3x$
4 Subtract $3x$ from both sides of equation.
5 $5x + 2 - 3x = 3x + 8 - 3x$
$2x + 2 = 8$
6 $x = 3$ 7 $3
8 Remove an unknown from one side of equation by finding the pronumeral with the smallest coefficient and performing its inverse operation to both sides of equation.

Exercise 3B Solving linear equations with the unknown on both sides

1 a $-3x$ b $-5x$ c $+2x$ d $+4x$

2 a $4x + 1 - x = x + 7 - x$
$3x + 1 = 7$
$3x + 1 - 1 = 7 - 1$
$3x = 6$
$\frac{3x}{3} = \frac{6}{3}$
$x = 2$

b $3x + 10 + 2x = -2x - 5 + 2x$
$5x + 10 = -5$
$5x + 10 - 10 = -5 - 10$
$5x = -15$
$\frac{5x}{5} = \frac{-15}{5}$
$x = -3$

3 a $x = 2$ b $x = 7$ c $x = -1$ d $x = 4$
e $x = 1$ f $x = -3$ g $x = 5$ h $x = -6$
i $x = 3$ j $x = -2$ k $x = -8$ l $x = 3$

4 a $2x + 8 = x + 20$
$2x + 8 - x = x + 20 - x$
$x + 8 = 20$
$x + 8 - 8 = 20 - 8$
$x = 12$

b $3x - 3 = 2x + 2$
$3x - 3 - 2x = 2x + 2 - 2x$
$x - 3 = 2$
$x - 3 + 3 = 2 + 3$
$x = 5$

5 a $x = 5$ b $x = -8$ c $x = 3$ d $x = -6$
e $x = 7$ f $x = 4$ g $x = -1$ h $x = 2$
i $x = -14$ j $x = 4$ k $x = -10$ l $x = -3$

6 **a** $x = 4$ **b** $x = -2$ **c** $x = 7$ **d** $x = 4$
e $x = 3$ **f** $x = -1$ **g** $x = 7$ **h** $x = 9$

7 **a** $\times 3, -x, -5$ **b** $2x + 5 = x - 4$
c $x = -9$

8 **a** $x = 5$ **b** $x = -3$ **c** $x = 2$ **d** $x = -3$

9 **a** **i** $x + b - b = a - b$
$x = a - b$

ii $2x - m = k$
$2x - m + m = k + m$
$2x = k + m$
$\frac{2x}{2} = \frac{k - m}{2}$
$x = \frac{k - m}{2}$

iii $\frac{x + d}{g} \times g = f \times g$
$x + d = fg$
$x + d - d = fg - d$
$x = fg - d$

b Subject of formula is the variable that is on its own on one side of equation.

c **i** a **ii** k **iii** f

10 **a** $x = k - p$ **b** $x = d + c$ **c** $x = \frac{b}{a}$
d $x = \frac{n - m}{2}$ **e** $x = \frac{y - 7}{5}$ **f** $x = cd - a$
g $x = \frac{3y + 1}{4}$ **h** $x = n(k - m)$ **i** $x = \frac{d}{b}$
j $x = \frac{p + t}{3}$ **k** $x = \frac{b}{3} - a$ **l** $x = u(t + y)$
m $x = \frac{d}{g - f}$ **n** $x = \frac{3m - k}{n}$ **o** $x = \frac{k - p}{4}$

11 **a** C **b** 13 bars **12** **a** B **b** \$3.50

13 **a** $5p + 9 = 3p + 31.5$ **b** \$11.25

14 **a** $s = 4$ m/s **b** $d = 600$ m
c $d = st$ **d** $d = 600$ m
e Having unknown variable as subject of formula is easier to calculate.
f $t = \frac{d}{s}$
g **i** $t = 7$ min **ii** $t = 5$ min
iii $t = 5$ min 50 s **iv** $t = 5$ min 23 s

15 **a** $F = 77°$ **b** $C = 31.11°$ **c** $C = \frac{5(F - 32)}{9}$
d **i** $C = 40°$ **ii** $C = 20°$
iii $C = 45°$ **iv** $C = 0°$

16 $3x + 3 = x + 13$; $x = 5$. Numbers are 5, 6 and 7.

17 **a** $w = \frac{A}{l}$ **b** $l = \frac{p - 2w}{2}$ **c** $u = v - at$
d $h = \frac{2A}{b}$ **e** $t = \frac{v - u}{a}$ **f** $R = \frac{I}{PT}$
g $r = \frac{P}{2\pi}$ **h** $a = \frac{2A}{h} - b$

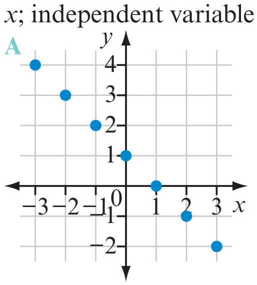
3C Plotting linear graphs

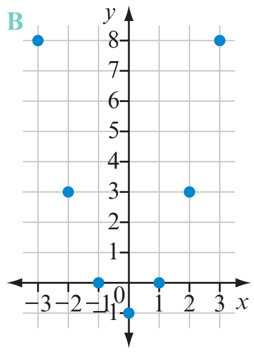
3C Start thinking!

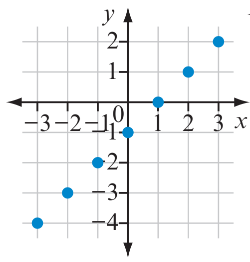
1 x **2** y

3 A $(-3, 4), (-2, 3), (-1, 2), (0, 1), (1, 0), (2, -1), (3, -2)$
B $(-3, 8), (-2, 3), (-1, 0), (0, -1), (1, 0), (2, 3), (3, 8)$
C $(-3, -4), (-2, -3), (-1, -2), (0, -1), (1, 0), (2, 1), (3, 2)$

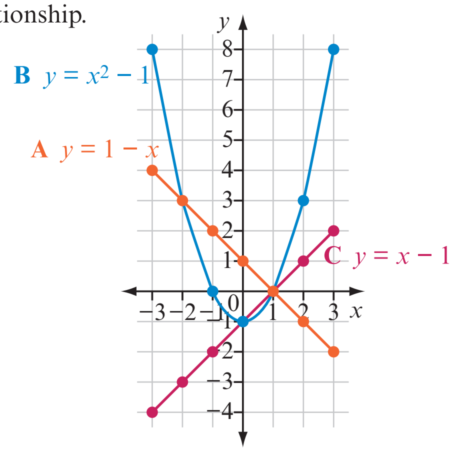
4 x; independent variable

5 A


B


C


6 **A** As x increases, y decreases in a linear relationship.



B As x increases, y first decreases, then increases again in a non-linear relationship.
C As x decreases, y increases in a linear relationship.

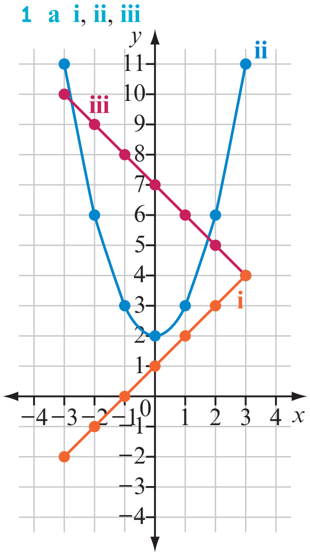
8 **a** A and C **b** B

9 **a** C **b** A **c** B

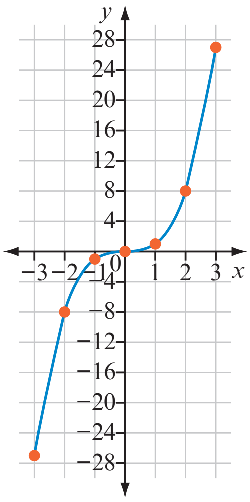
10 When all the variables in the rule have a maximum power of 1, they produce a linear graph.

Exercise 3C Plotting linear graphs

1 a i, ii, iii



iv

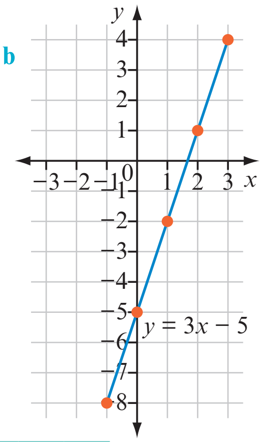


b i and iii

2 a

x	−1	0	1	2	3
y	−8	−5	−2	1	4

b



3 a

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

b

x	−3	−2	−1	0	1	2	3
y	−7	−6	−5	−4	−3	−2	−1

c

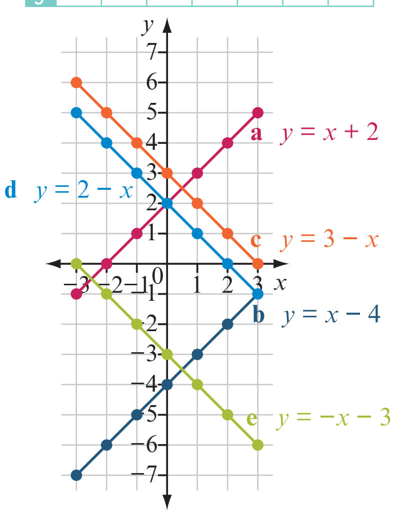
x	−3	−2	−1	0	1	2	3
y	6	5	4	3	2	1	0

d

x	−3	−2	−1	0	1	2	3
y	5	4	3	2	1	0	−1

e

x	−3	−2	−1	0	1	2	3
y	0	−1	−2	−3	−4	−5	−6

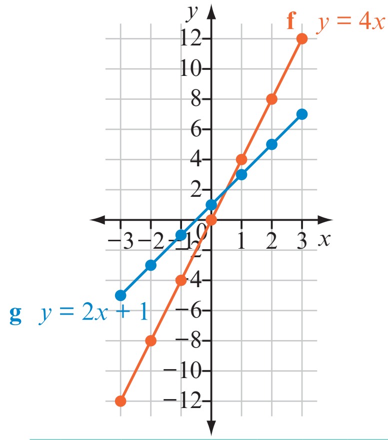


f

x	−3	−2	−1	0	1	2	3
y	−12	−8	−4	0	4	8	12

g

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7



h

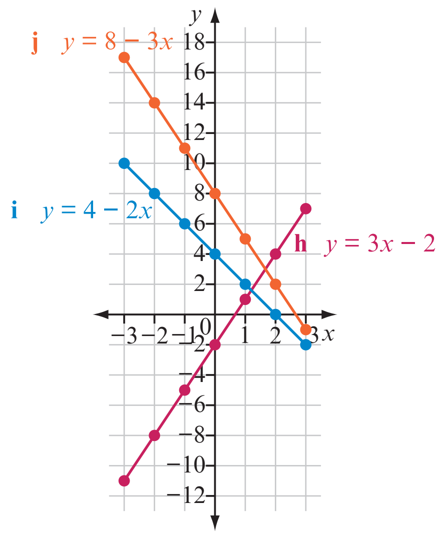
x	−3	−2	−1	0	1	2	3
y	−11	−8	−5	−2	1	4	7

i

x	−3	−2	−1	0	1	2	3
y	10	8	6	4	2	0	−2

j

x	−3	−2	−1	0	1	2	3
y	17	14	11	8	5	2	−1

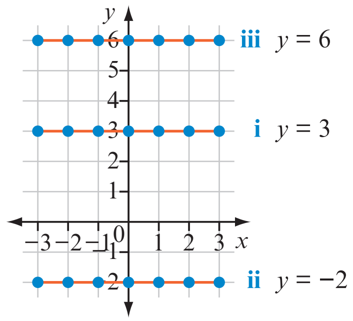


5 a positive **b** negative **c** zero

6 a P **b** P **c** N **d** N **e** N
f P **g** P **h** P **i** N **j** N

7 a x-intercept $(-2, 0)$, y-intercept $(0, 2)$
b x-intercept $(4, 0)$, y-intercept $(0, -4)$
c x-intercept $(3, 0)$, y-intercept $(0, 3)$
d x-intercept $(2, 0)$, y-intercept $(0, 2)$
e x-intercept $(-3, 0)$, y-intercept $(0, -3)$
f x-intercept $(0, 0)$, y-intercept $(0, 0)$
g x-intercept $(-0.5, 0)$, y-intercept $(0, 1)$
h x-intercept $(\frac{2}{3}, 0)$, y-intercept $(0, -2)$
i x-intercept $(2, 0)$, y-intercept $(0, 4)$
j x-intercept $(2\frac{2}{3}, 0)$, y-intercept $(0, 8)$

8 a



b Yes: they form a straight line.

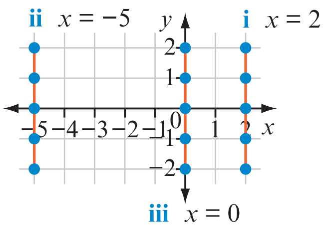
c Rule only has one variable, y. So y-coordinate for every possible point is the same and graph forms a horizontal line.

d gradients all zero

e **i** (0, 3) **ii** (0, −2) **iii** (0, 6)

f None of these graphs have an x-intercept because they form horizontal lines that run parallel to and will never cross the x-axis.

9 a



b Yes: they form a straight line.

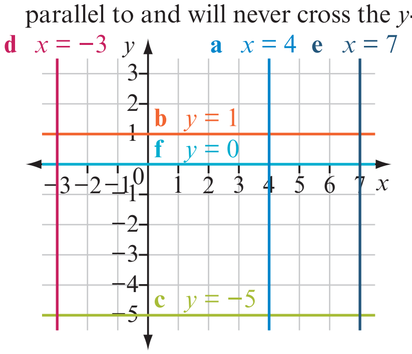
c The rule only has one variable, x. So the x-coordinate for every possible point is the same and forms a vertical line.

d The gradients are undefined (cannot be calculated) because the rise is infinite and the run is zero.

e **i** (2, 0) **ii** (−5, 0) **iii** (0, 0)

f None of these graphs have a y-intercept because they form vertical lines that run parallel to and will never cross the y-axis.

10



11 a i $y = x$

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

ii $y = 2x$

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

iii $y = 3x$

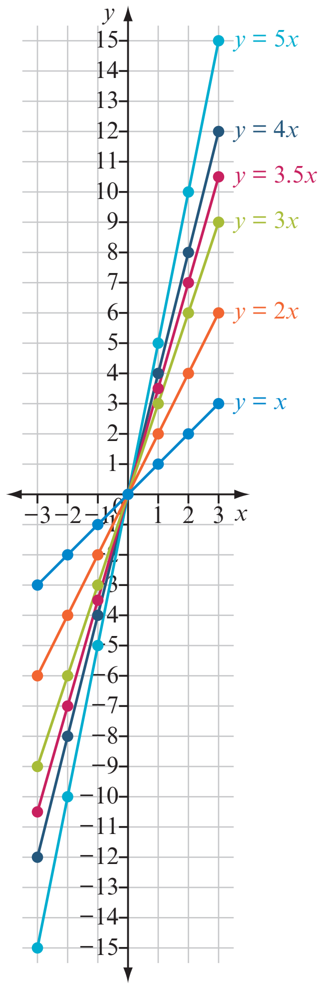
x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9

iv $y = 4x$

x	−3	−2	−1	0	1	2	3
y	−12	−8	−4	0	4	8	12

b All four graphs are linear with positive gradients. However, the gradients increase as the coefficient of x increases.

c



12 a i $y = -x$

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3

ii $y = -2x$

x	−3	−2	−1	0	1	2	3
y	6	4	2	0	−2	−4	−6

iii $y = -3x$

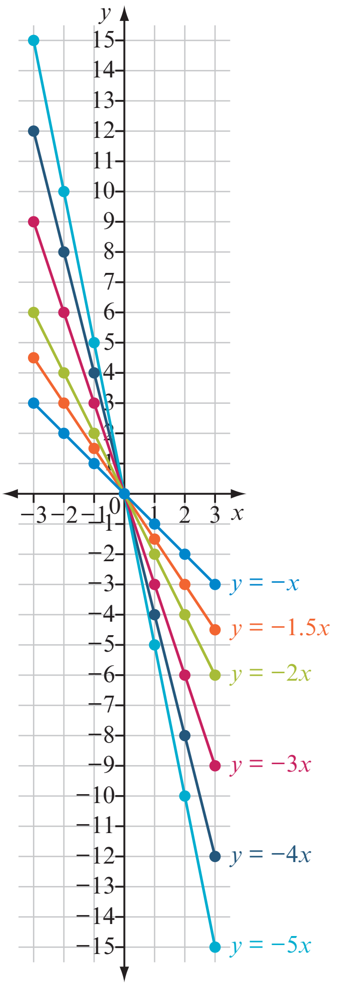
x	−3	−2	−1	0	1	2	3
y	9	6	3	0	−3	−6	−9

iv $y = -4x$

x	−3	−2	−1	0	1	2	3
y	12	8	4	0	−4	−8	−12

b All four graphs are linear with negative gradients. However, the gradients become steeper as the coefficient of x decreases.

c



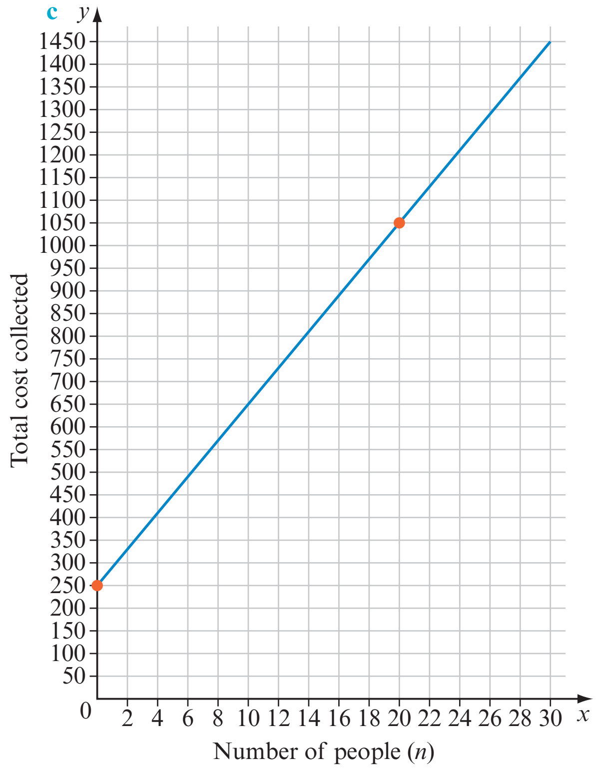
13 **a** $y = 4x$ **b** $y = -4x$

c **i** $y = 20x$ **ii** $y = -25x$

14 **a** Total amount collected is $40 per person, which will change depending on number of people who go, plus the $250 that school pays regardless of how many people go.

b The number of people (n) is the independent variable. This would be shown on x-axis.

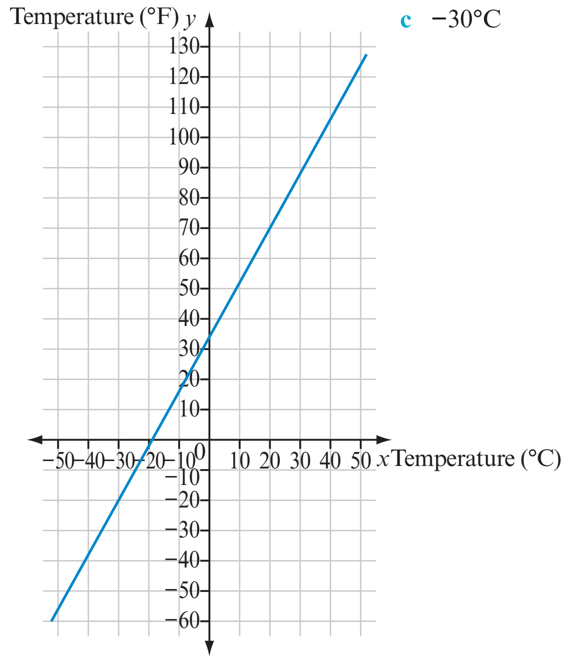
c



d Relationship is linear because it forms a straight line.

e $1050 **f** 15 **g** 27 people

15 **a**



b 86°F

c −30°C

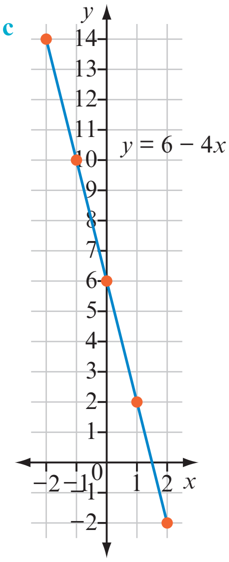
16 −40°F = −40°C

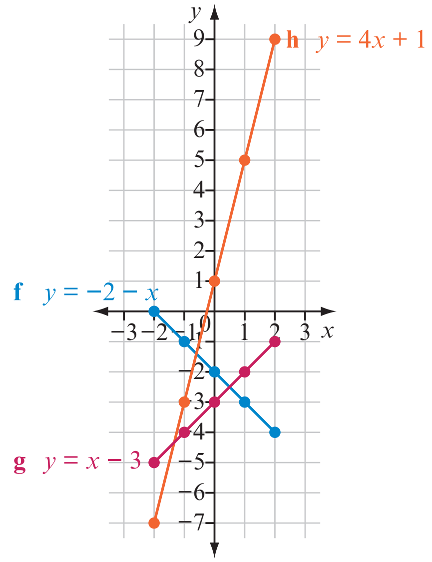
17 **a** $y = 6 - 4x$

b

x	−2	−1	0	1	2
y	14	10	6	2	−2

c





18 **a** $y = 5 - x$

x	−2	−1	0	1	2
y	7	6	5	4	3

b $y = -1 - x$

x	−2	−1	0	1	2
y	1	0	−1	−2	−3

c $y = 3 - 2x$

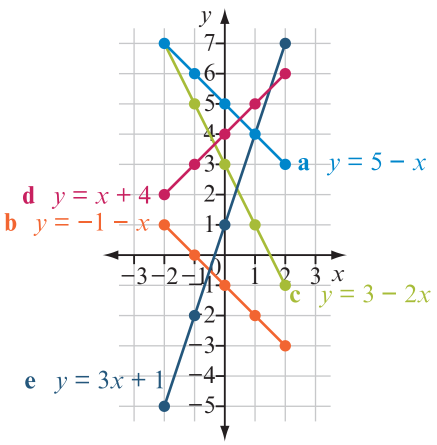
x	−2	−1	0	1	2
y	7	5	3	1	−1

d $y = x + 4$

x	−2	−1	0	1	2
y	2	3	4	5	6

e $y = 3x + 1$

x	−2	−1	0	1	2
y	−5	−2	1	4	7



f $y = -2 - x$

x	−2	−1	0	1	2
y	0	−1	−2	−3	−4

g $y = x - 3$

x	−2	−1	0	1	2
y	−5	−4	−3	−2	−1

h $y = 4x + 1$

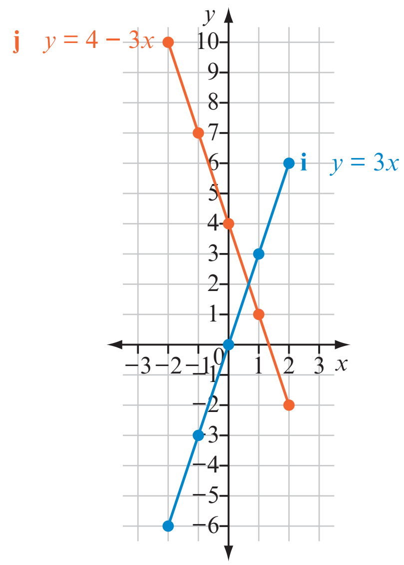
x	−2	−1	0	1	2
y	−7	−3	1	5	9

i $y = 3x$

x	−2	−1	0	1	2
y	−6	−3	0	3	6

j $y = 4 - 3x$

x	−2	−1	0	1	2
y	10	7	4	1	−2



3D Gradient and intercepts

3D Start thinking!

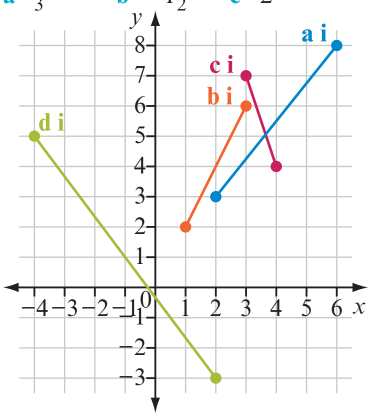
2 positive gradient
3 **a** (2, 2) **b** (4, 5)
4 run = 2 **5** rise = +3 **6** 3; 2
7 $\frac{3}{2}$ **8** gradient = $\frac{\text{rise}}{\text{run}}$

Exercise 3D Gradient and intercepts

1 **a** 3, 4, $\frac{3}{4}$ **b** −1, 2, $-\frac{1}{2}$
2 **a** run = 4 **b** rise = 6 **c** $\frac{6}{4} = \frac{3}{2}$
d Gradient is the same regardless of where measured along the line.
3 **a** **i** 2 **ii** 1 **iii** −2
b **i** 1 **ii** 4 **iii** −4
c **i** −2 **ii** 1 **iii** 2
d **i** $\frac{3}{5}$ **ii** −5 **iii** 3
e **i** $-\frac{2}{3}$ **ii** −3 **iii** −2

4 **a** $\frac{1}{3}$ **b** $-1\frac{1}{2}$ **c** 2

5



a **ii** $1\frac{1}{4}$
b **ii** 2
c **ii** −3
d **ii** $-1\frac{1}{3}$

6 Find difference between two x-coordinates (run) and difference between two y-coordinates (rise).

7 **a** x-coordinate of second point is to right of first point, so difference between two x-coordinates will be number of units graph travels to right.
b $y_2 - y_1$ **c** $\frac{y_2 - y_1}{x_2 - x_1}$

8 **a** $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** 2 **d** $-\frac{1}{3}$
e $\frac{2}{5}$ **f** −3 **g** $\frac{5}{3}$ or $1\frac{2}{3}$ **h** −5
i $\frac{7}{3}$ or $2\frac{1}{3}$ **j** −4 **k** −2 **l** 0

9 Kane subtracted second y-coordinate from first, not other way around; gradient = $-\frac{3}{4}$.

10 Negative number in numerator or denominator results in negative gradient overall.

11 No, the same answer results.

12 **a**

	Rule	Gradient	y-intercept
i	$y = 2x - 3$	2	(0, −3)
ii	$y = -\frac{3}{4}x + 5$	$-\frac{3}{4}$	(0, 5)
iii	$y = x + 4$	1	(0, 4)
iv	$y = -2x - 1$	−2	(0, −1)
v	$y = 4x$	4	(0, 0)
vi	$y = 2$	0	(0, 2)

b gradient is coefficient of x; y-intercept is the constant
c gradient = 6, y-intercept = (0, 4)

13 m is gradient, which is coefficient of x; c is y-intercept

14 **a** $m = 2, c = 5$ **b** $m = 4, c = 1$
c $m = -3, c = 7$ **d** $m = -5, c = -3$
e $m = \frac{1}{4}, c = -6$ **f** $m = -1, c = 1$
g $m = \frac{4}{3}, c = 2$ **h** $m = \frac{1}{2}, c = -8$
i $m = -\frac{4}{3}, c = \frac{1}{4}$ **j** $m = 3, c = 9$
k $m = -7, c = 2$ **l** $m = -\frac{2}{5}, c = 5$

15 **a** **i** $\frac{2}{1}$ **ii** 2 **iii** 1
b **i** $\frac{5}{1}$ **ii** 5 **iii** 1
c **i** $\frac{-3}{1}$ **ii** −3 **iii** 1
d **i** $\frac{7}{1}$ **ii** 7 **iii** 1
e **i** $\frac{-4}{1}$ **ii** −4 **iii** 1
f **i** $\frac{-1}{1}$ **ii** −1 **iii** 1
g **i** $\frac{3}{2}$ **ii** 3 **iii** 2
h **i** $\frac{5}{4}$ **ii** 5 **iii** 4
i **i** $\frac{-3}{8}$ **ii** −3 **iii** 8
j **i** $\frac{-2}{7}$ **ii** −2 **iii** 7
k **i** $\frac{-4}{9}$ **ii** −4 **iii** 9
l **i** $\frac{-1}{5}$ **ii** −1 **iii** 5

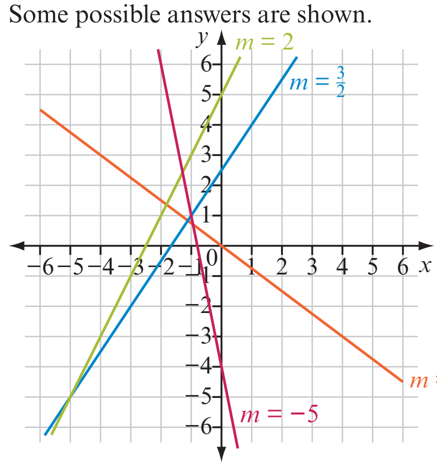
16 Denominator represents change in x-coordinates, which is always from left to right and therefore always positive.

17 **a** **i** 2 **ii** 1 **b** **i** 4 **ii** 1
c **i** −3 **ii** 1 **d** **i** −5 **ii** 1
e **i** 1 **ii** 1 **f** **i** −1 **ii** 1
g **i** 4 **ii** 3 **h** **i** 1 **ii** 2
i **i** −4 **ii** 3 **j** **i** 3 **ii** 1
k **i** −7 **ii** 1 **l** **i** −2 **ii** 5

18 **a** Pick any point and add value of gradient (4) to y-coordinate (rise). Then add 1 to x-coordinate (run) to find coordinates of second point. Or substitute coordinates of any point and value of gradient into formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.
One possible answer is: (3, 4) and (4, 8).
b One possible answer is given.
i (6, 5) and (5, 3) **ii** (1, 3) and (4, 5)
iii (6, 8) and (10, 3)

19 **a** **i** −2 **ii** 7 **b** **i** 5 **ii** 1
c **i** $-\frac{3}{2}$ **ii** 3 **d** **i** $-\frac{4}{3}$ **ii** −3
e **i** 2 **ii** $\frac{3}{4}$ **f** **i** 3 **ii** −5
g **i** 7 **ii** −2 **h** **i** −1 **ii** 7
i **i** $-\frac{1}{2}$ **ii** −2 **j** **i** $\frac{2}{5}$ **ii** $-\frac{9}{5}$

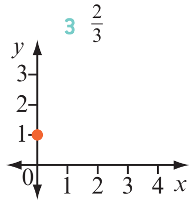
20 Some possible answers are shown.



3E Sketching linear graphs using gradient and y-intercept

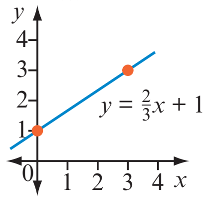
3E Start thinking!

1 $m = \frac{2}{3}, c = 1$ **2** (0, 1) **3** $\frac{2}{3}$
4 rise = 2, run = 3
5



6 Starting at the original point, move up the graph (rise) 2 units and right (run) 3 units.

7 (3, 3)

8 

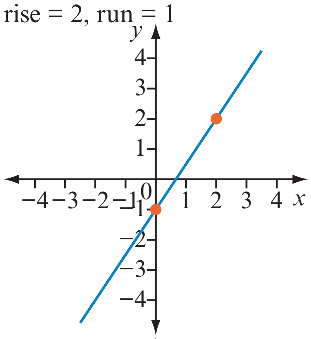
9 Using formula $y = mx + c$, where m = gradient and c = y-intercept, first plot a point at y-intercept $(0, c)$.

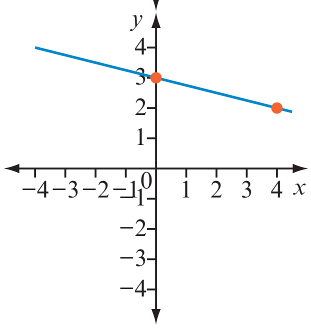
Next convert gradient into fraction $\frac{\text{rise}}{\text{run}}$.

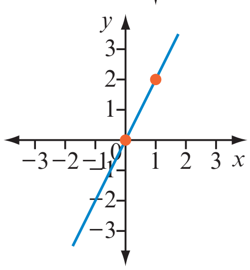
From y-intercept, count across 'run' units and up (positive) or down (negative) 'rise' units and plot second point. Join points with ruled line.

Exercise 3E Sketching linear graphs using gradient and y-intercept

1 a i rise = 3, run = 2 ii rise = −1, run = 4
iii rise = 2, run = 1

b i 

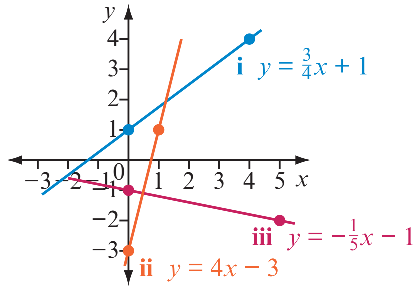
ii 

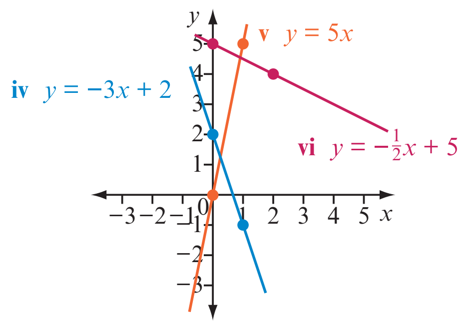
iii 

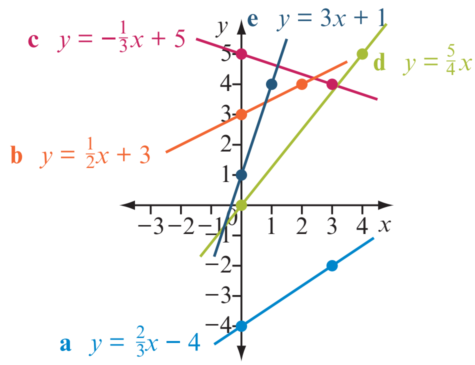
2 i (0, −1) and (2, 2) ii (0, 3) and (4, 2)
iii (0, 0) and (1, 2)

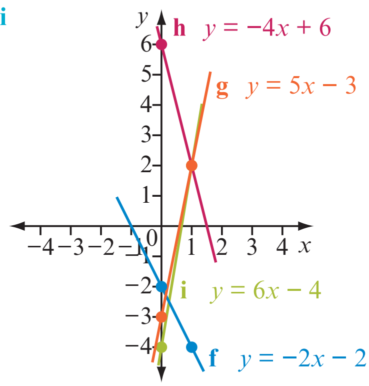
3 a i $c = 1, m = \frac{3}{4}$ ii $c = -3, m = 4$
iii $c = -1, m = -\frac{1}{5}$ iv $c = 2, m = -3$
v $c = 0, m = 5$ vi $c = 5, m = -\frac{1}{2}$

b i rise = 3, run = 4 ii rise = 4, run = 1
iii rise = −1, run = 5 iv rise = −3, run = 1
v rise = 5, run = 1 vi rise = −1, run = 2

c 



4 a–e 

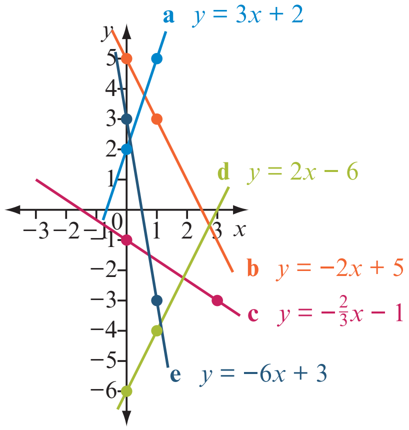
f–i 

5 A incorrect, y-intercept at +2 instead of −2
B correct
C incorrect, gradient and y-intercept reversed

6 a C b A c B

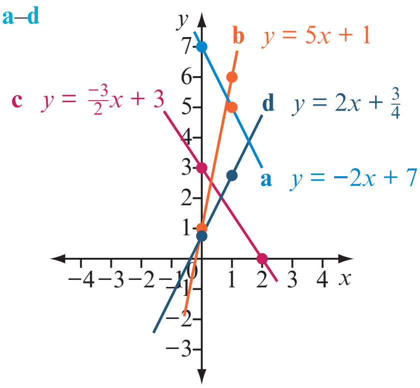
7 a i $m = 3$ ii $c = 2$
b i $m = -2$ ii $c = 5$
c i $m = -\frac{2}{3}$ ii $c = -1$
d i $m = 2$ ii $c = -6$
e i $m = -6$ ii $c = 3$

8

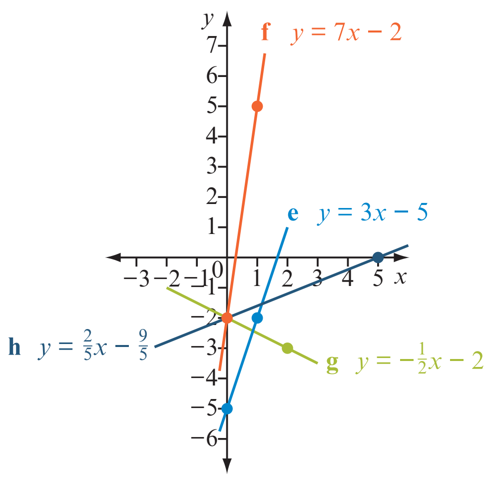


9 **a** **i** $m = -2$ **ii** $c = 7$
b **i** $m = 5$ **ii** $c = 1$
c **i** $m = -\frac{3}{2}$ **ii** $c = \frac{3}{3}$
d **i** $m = 2$ **ii** $c = \frac{3}{4}$
e **i** $m = 3$ **ii** $c = -5$
f **i** $m = 7$ **ii** $c = -2$
g **i** $m = -\frac{1}{2}$ **ii** $c = -2$
h **i** $m = \frac{2}{5}$ **ii** $c = -\frac{9}{5}$

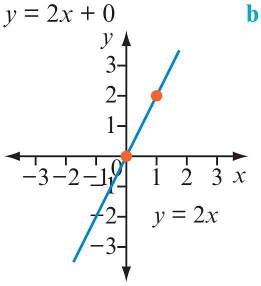
10 **a–d**



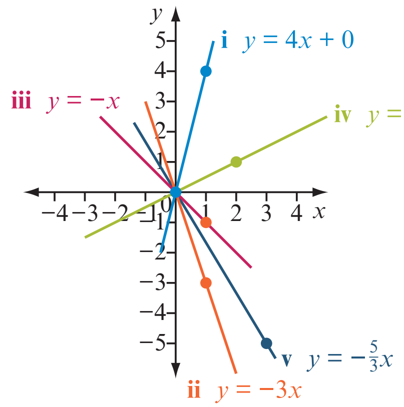
e–h



11 **a** $y = 2x + 0$ **b** 2 **c** 0
d

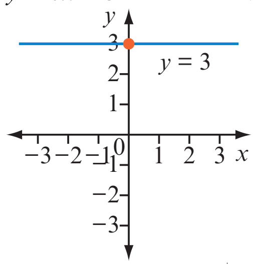


12 **a, d**

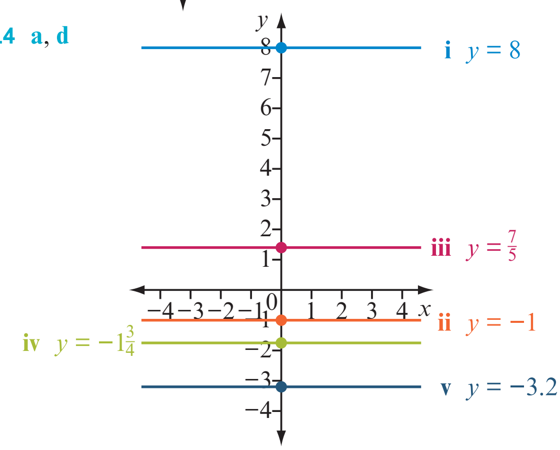


b, c **i** $m = 4, c = 0$ **ii** $m = -3, c = 0$
iii $m = -1, c = 0$ **iv** $m = \frac{1}{2}, c = 0$
v $m = \frac{-5}{3}, c = 0$

13 **a** $y = 0x + 3$ **b** 0 **c** 3
d



14 **a, d**



b, c **i** $m = 0, c = 8$ **ii** $m = 0, c = -1$
iii $m = 0, c = \frac{7}{5}$ **iv** $m = 0, c = 1\frac{3}{4}$
v $m = 0, c = -3.2$

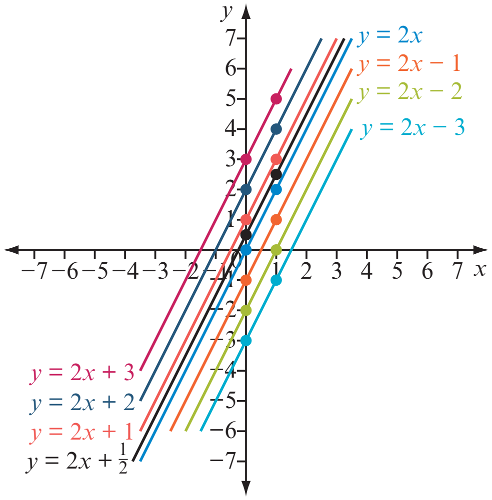
15 No. Without the variable y, the general rule cannot be applied to $x = 2$.

16 A linear rule can be written by inserting the gradient as the coefficient for x, and then adding the y-intercept.

17 **a** $y = 5x + 2$ **b** $y = 3x - 7$ **c** $y = x + 10$
d $y = -4x$ **e** $y = \frac{2}{9}x - 1$ **f** $y = -\frac{5}{3}x + \frac{2}{3}$

18 **a** $y = 4 - 2x$ **b** $y = \frac{3}{2}x - 1$ **c** $y = \frac{1}{3}x + 2$

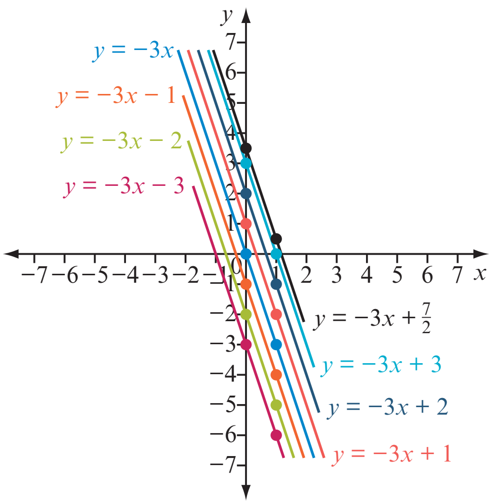
19 a



b All the lines have same positive gradient and are parallel, they are just shifted up or down the y-axis.

c gradient **d** $y = 2x + 5$

20 a



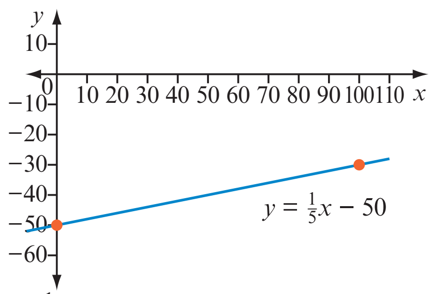
b All the lines have same negative gradient and are parallel, they are just shifted up or down the y-axis.

c $y = -3x - 6$

21 a $y = 4x + 3$ **b** $y = -5x - 2$

22 a $(0, -50)$ **b** $(100, -30)$

c

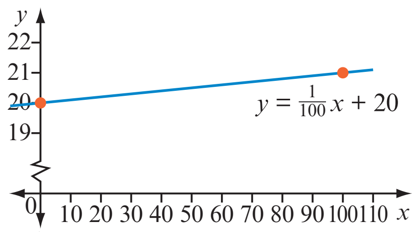


d $y = \frac{1}{5}x - 50$

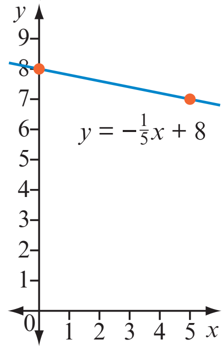
e x = horizontal distance covered in metres, y = depth underwater in metres

f 250 m

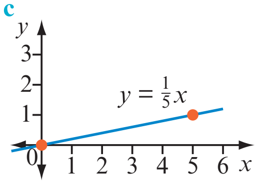
23 a



b



c



24 a $y = \frac{1}{100}x + 20$ **b** $y = -\frac{1}{5}x + 8$ **c** $y = \frac{1}{5}x$

3F Sketching linear graphs using *x*- and *y*-intercepts

3F Start thinking!

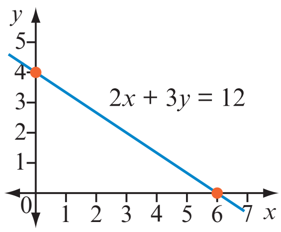
1 If rule is in form of $y = mx + c$ both gradient and y-intercept can be read straight from rule without any need for calculation. If rule is in a different form, calculation is required.

2 **a** 0 **b** $2x + 3 \times 0 = 12$ **c** 6 **d** 6

3 **a** 0 **b** $2 \times 0 + 3y = 12$ **c** 4 **d** 4

4 (6, 0) and (0, 4)

5



6 Substitute $y = 0$ into rule and solve for x to find coordinates of x-intercept. Next, substitute $x = 0$ into rule and solve for y to find coordinates of y-intercept. Then join the two points with a straight line.

Exercise 3F Sketching linear graphs using *x*- and *y*-intercepts

1 a i x-intercept is 5. Coordinates are (5, 0)

ii y-intercept is 15. Coordinates are (0, 15)

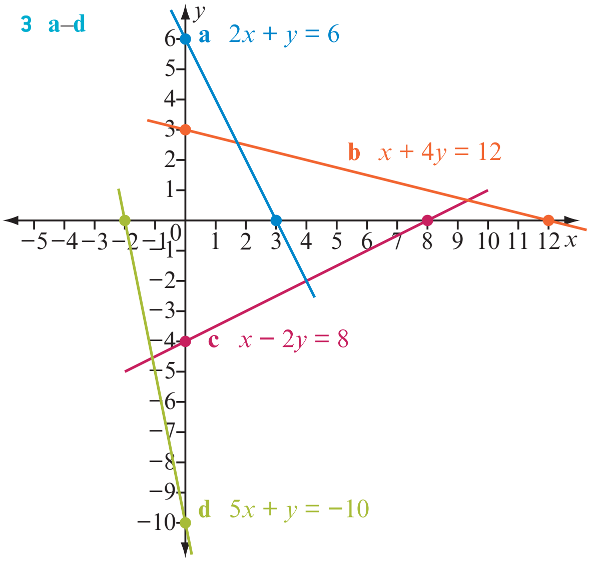
b i x-intercept is 3. Coordinates are (3, 0)

ii y-intercept is −2. Coordinates are (0, −2)

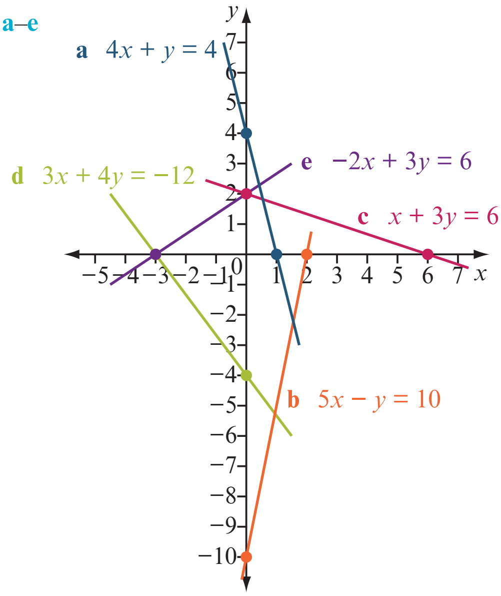
2 a $x = 3, y = 6$ **b** $x = 12, y = 3$

c $x = 8, y = -4$ **d** $x = -2, y = -10$

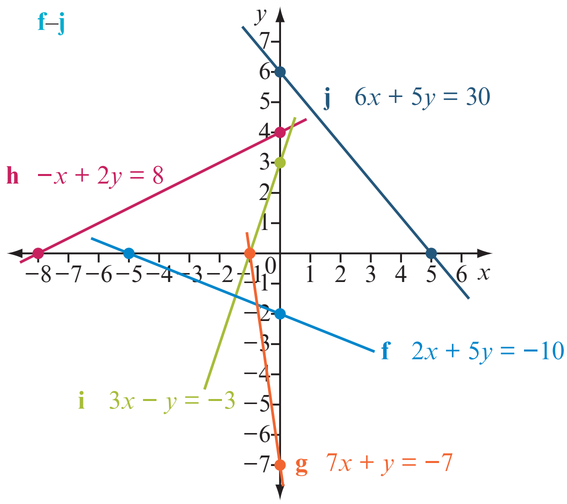
3 a–d



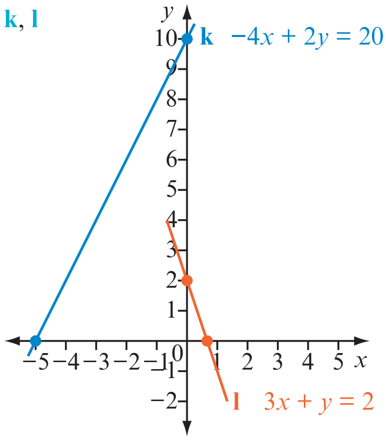
4 a–e



f–j



k, l



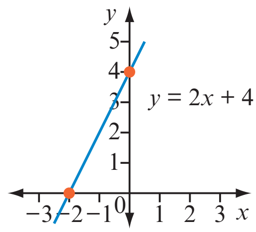
5 **A** incorrect: coefficients of variables have been sketched as intercepts rather than substituting $x = 0$ and $y = 0$ to calculate intercepts

B correct

C incorrect: x-intercept is at -2 instead of $+2$

6 **a** C **b** A

7 **a** -2 **b** 4 **c**



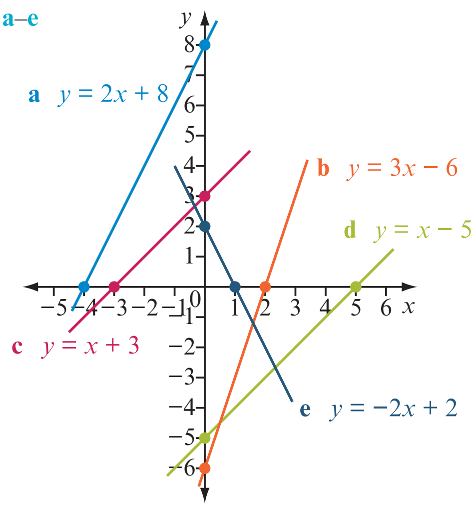
8 **a** $x = -4, y = 8$ **b** $x = 2, y = -6$

c $x = -3, y = 3$ **d** $x = 5, y = -5$

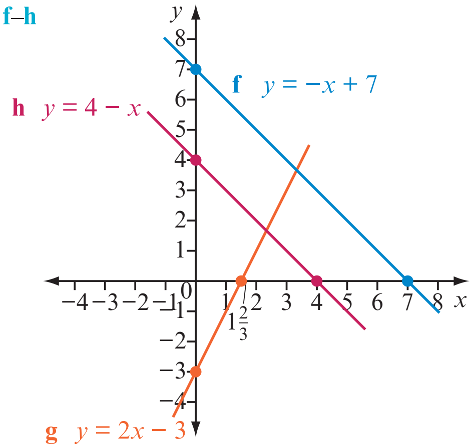
e $x = 1, y = 2$ **f** $x = 7, y = 7$

g $x = \frac{3}{2}, y = -3$ **h** $x = 4, y = 4$

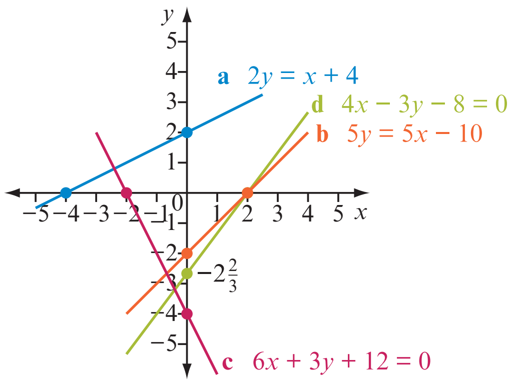
9 a–e



f–h



10



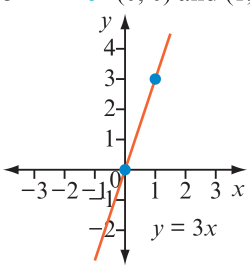
11 **a** **i** 0 **ii** 0

b Cannot sketch graph because both intercepts are at (0, 0) and one point is not enough information to produce graph.

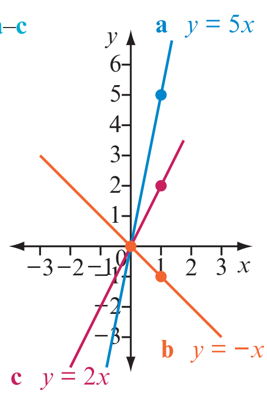
c Need coordinates of at least one other point.

d 3 **e** (0, 0) and (1, 3)

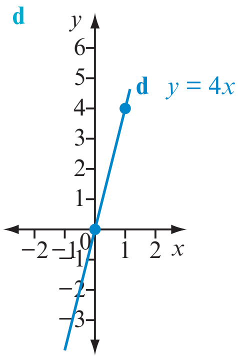
f



12 **a–c**

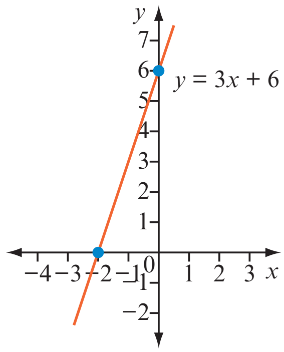


d



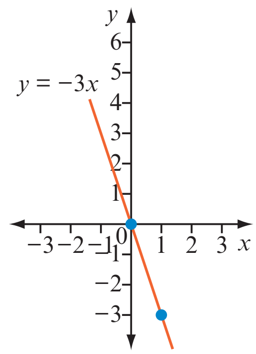
13 **a** **i** true **ii** false **iii** true
iv true **v** false **vi** true

b



14 **a** **i** false **ii** true **iii** true
iv false **v** true **vi** false

b



15 **a** gradient–intercept method

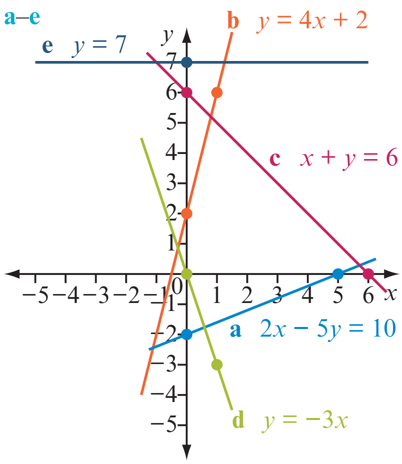
b x- and y-intercept method

c Substitute $x = 1$ into rule to determine second point to plot.

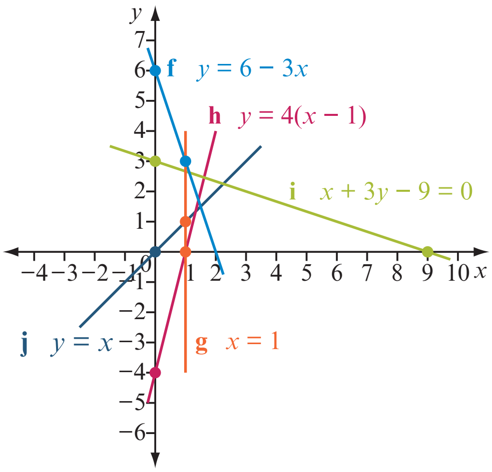
d Rule a horizontal line that crosses y-axis at given value.

e Rule a vertical line that crosses x-axis at given value.

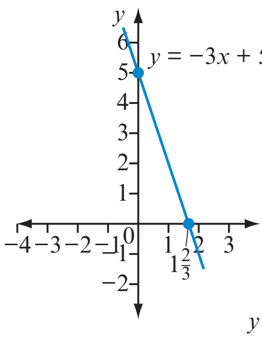
16 **a–e**



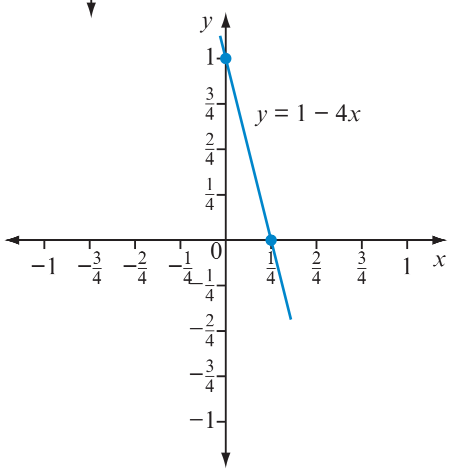
f–j



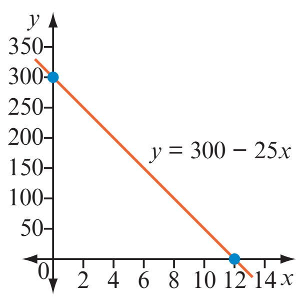
k



l



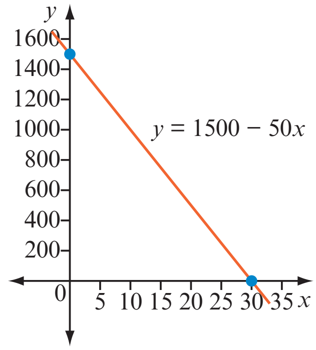
17 a



b original cost of skateboard
c number of weeks needed to pay off skateboard
d Tony is paying $25 a week towards the skateboard. He will pay it off in 12 weeks.

18 a Constant rate of water use and no additions to tank suggest a linear relationship between time and amount of water.
b (30, 0) **c** (0, 1500)

d



e 1000 L **f** end of day 18

19 $y = 1500 - 50x$

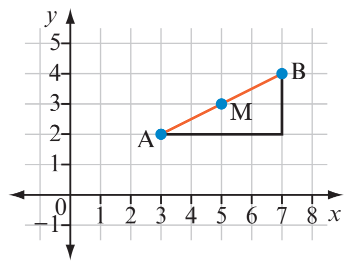
21 a $y = -4x + 4$ **b** $y = -x + 7$ **c** $y = \frac{3}{2}x + 3$

3G Midpoint and length of line segments

3G Start thinking!

1 A = (3, 2), B = (7, 4) **2** 4 **3** 2

4



Half of 4 is 2 units across, and half of 2 is 1 unit up.

5 5 **6** 5

7 x-coordinate of midpoint found by averaging x-coordinates of end points of line segment

8 3 **9** 3

10 y-coordinate of midpoint found by averaging y-coordinates of end points of line segment

11 (5, 3)

Exercise 3G Midpoint and length of line segments

1 a (2, 7) **b** (5, 4) **c** (3, 6)
d (7, 5) **e** (3, 1) **f** (4, 7)
g (2, 3) **h** (5, 1) **i** (4, 3)
j (−3, 0) **k** (3.5, 8.5) **l** (6.5, 5.5)

2 a 2.2 units **b** 4.5 units **c** 3.2 units **d** 5 units
e 5.8 units **f** 5.1 units **g** 6.3 units **h** 5 units
i 1.4 units **j** 3.6 units **k** 4.1 units **l** 4.5 units

3 a 6.3 units **b** 6.3 units **c** 8.9 units **d** 8.2 units
e 4.5 units **f** 8.9 units **g** 6.3 units **h** 10.8 units
i 8.9 units **j** 4.5 units **k** 1.4 units **l** 11.4 units

4 a i (3, 3) **ii** 4.5 units **iii** $-\frac{1}{2}$
b i (0.5, 0) **ii** 7.8 units **iii** $1\frac{1}{5}$

5 (10, 5) **6** (−14, 9)

7 Some possible answers are: (2, 6) and (4, 10); (0, 3) and (6, 13); (−3, −1) and (9, 17).

8 a 5 units **b** (5, 7) **9** 20.3 units

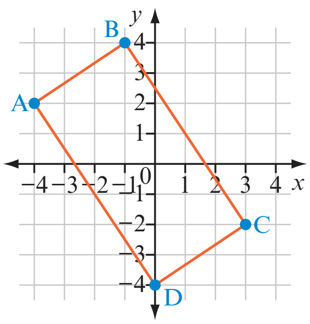
10 a 20.5 units **b** 17.0 units **c** 22.6 units **d** 19.5 units

11 a leg 1: 21.5 km; leg 2: 16.5 km; leg 3: 28.8 km
b 66.8 km **c** 24.1 km

12 **a** (−5, −1), (−2, 3), (6, 3) and (3, −1)
b **i** (0.5, 1) **ii** (0.5, 1)
c midpoints of both diagonals have same coordinates
d 26 cm **e** 5.3 cm

13 **a** **i** 21.6 cm
ii 10.8 cm, half the perimeter of blue shape
b **i** 20.7 cm **ii** 16.7 cm, $\frac{4}{5}$ of blue shape

14 **a** AB 3.6 m, BC 7.2 m, CD 3.6 m and AD 7.2 m
b midpoint for both diagonals (−0.5, 0)
c A 4.0 m, B 4.0 m, C 4.0 m, D 4.0 m
d two pairs of sides of equal length, diagonals equal in length and bisect each other, with each vertex same distance from midpoint

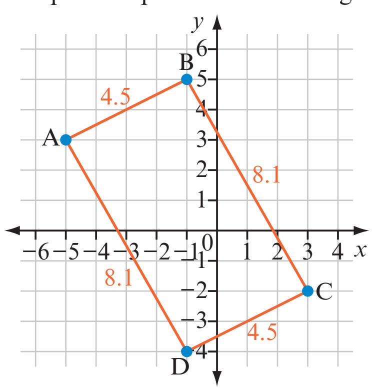


15 **a** A (0, 6), B (2, 2), C (7, 2), D (3, −1), E (4, −6), F (0, −3), G (−4, −6), H (−3, −1), I (−7, 2), J (−2, 2)
b 8.1 cm
c **i** 8.5 cm **ii** 8 cm **iii** 8.5 cm
iv 8.1 cm
d It is symmetrical across the y-axis.
e (0, −6) **f** 3 cm **g** 12 cm^2
h 56 cm^2 **i** 49.2 cm

16 two sides of 8.9 cm and one of 5.7 cm

17 24.2 cm^2

18 two pairs of side lengths 4.5 units and 8.1 units; two pairs of parallel sides with gradients $\frac{1}{2}$ and $-\frac{7}{4}$



3 Chapter review

MULTIPLE-CHOICE

1 B **2** C **3** D **4** D
5 C **6** B **7** B **8** A
9 B **10** C **11** A **12** C

SHORT ANSWER

1 **a** $x = 2$ **b** $x = \frac{2}{3}$ **c** $x = 10$ **d** $x = 3$
e $x = -1$ **f** $x = -\frac{4}{5}$ **g** $x = 3\frac{3}{4}$ **h** $x = \frac{1}{2}$

2 **a** true **b** false, $x = -10$

3 **a** $x = 4$ **b** $x = -1$

4 **a** $4n = 4 + n$ **b** $3(4 + n) = 2n$

5 **a** $n = 1\frac{1}{3}$ **b** $n = -12$

6 **a** $x = \frac{a}{3} - 1$ **b** $x = \frac{b - 3}{a}$ **c** $x = \frac{-y - 2}{3}$
d $x = \frac{y - c}{m}$

7 **a**

x	−2	−1	0	1	2
y	3	4	5	6	7

b

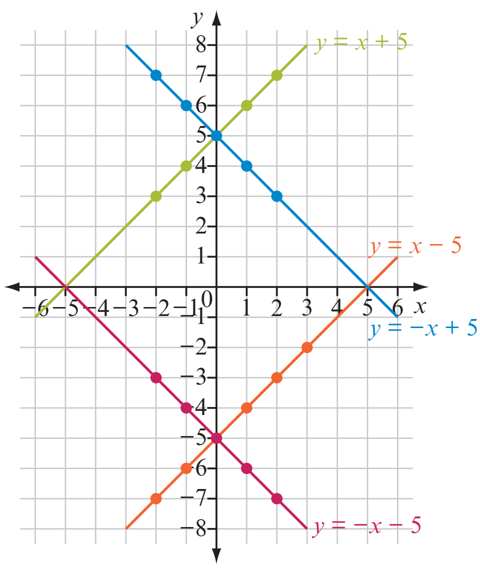
x	−2	−1	0	1	2
y	−7	−6	−5	−4	−3

c

x	−2	−1	0	1	2
y	7	6	5	4	3

d

x	−2	−1	0	1	2
y	−3	−4	−5	−6	−7



8 **a** $y = x + 5$ and $y = x - 5$ have gradient +1; $y = -x + 5$ and $y = -x - 5$ have gradient of −1
b $y = x + 5$ and $y = -x - 5$ cross x-axis at −5
$y = x - 5$ and $y = -x + 5$ cross x-axis at +5
$y = x + 5$ and $y = -x + 5$ cross y-axis at +5
$y = x - 5$ and $y = -x - 5$ cross y-axis at −5

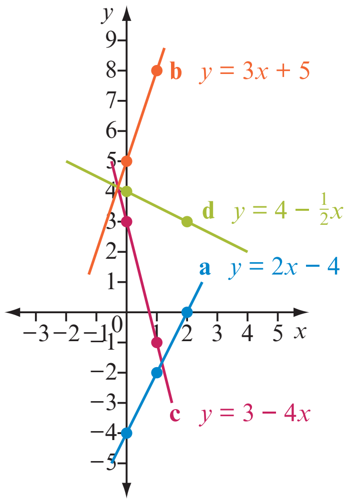
9 **a** $1\frac{1}{2}$ **b** −1

10 **a** **i** 2 **ii** −4 **b** **i** 3 **ii** 5
c **i** −4 **ii** 3 **d** **i** $-\frac{1}{2}$ **ii** 4

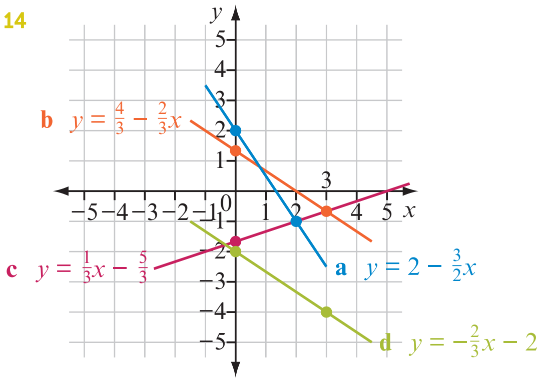
11 **a** **i** $-\frac{3}{2}$ **ii** 2 **b** **i** $-\frac{2}{3}$ **ii** $\frac{4}{3}$
c **i** $\frac{1}{3}$ **ii** $-\frac{5}{3}$ **d** **i** $-\frac{2}{3}$ **ii** −2

12 **a** $y = 4x - 2$ **b** $y = \frac{1}{4}x$ **c** $y = -\frac{1}{2}$

13



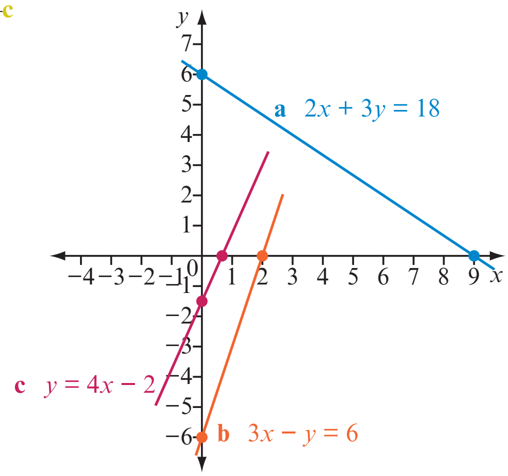
14



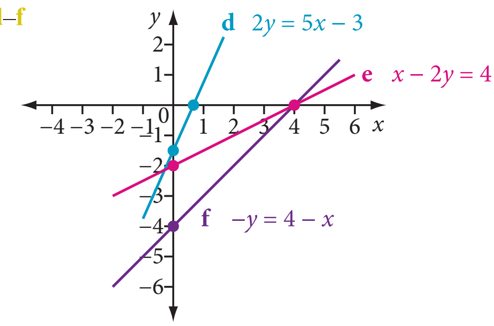
15 $y = -2x + 4$

16 **a** **i** 9 **ii** 6 **b** **i** 2 **ii** −6
c **i** $\frac{1}{2}$ **ii** −2 **d** **i** $\frac{3}{5}$ **ii** $-\frac{3}{2}$
e **i** 4 **ii** −2 **f** **i** 4 **ii** −4

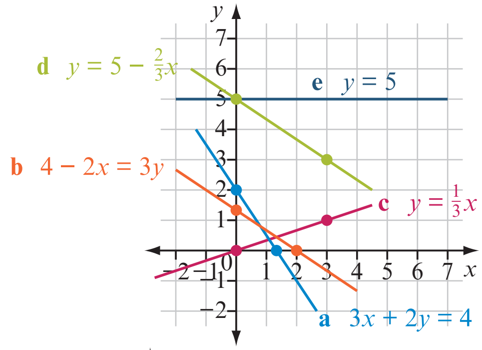
17 **a–c**



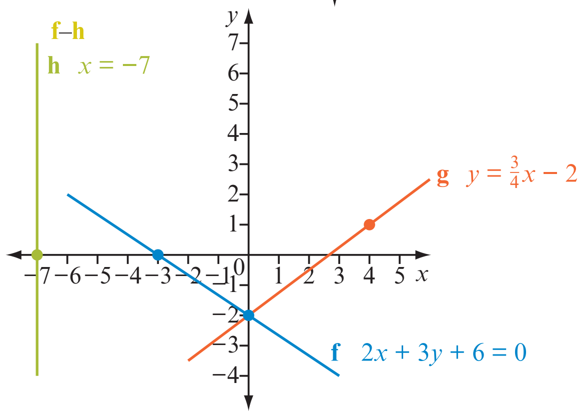
d–f



18 **a–e**



f–h



19 **a** (5, 5) **b** (−3.5, 5.5) **c** (0, 0)
d (−0.5, −5)

20 **a** 7.2 units **b** 1.4 units **c** 8.9 units **d** 9.4 units

NAPLAN-STYLE PRACTICE

1 true **2** $7\frac{1}{2}$ **3** $\frac{3n + 5}{4} = 6$ **4** $\frac{4}{n} + 5 = 10$

5 $y = \frac{-2x + 4}{3}$ **6** $y = \frac{4 - 2x}{3}$ **7** $5 - 3x = 4$

8 $n + 2$ **9** 9 **10** $p = \frac{4r}{5t}$

11 $y = \frac{4}{x} + 5$ **12** (2, 1) **13** (−3, 3.5)

14 positive **15** zero **16** undefined

17 0.5 **18** $y = -4x$ **19** −1 **20** 4

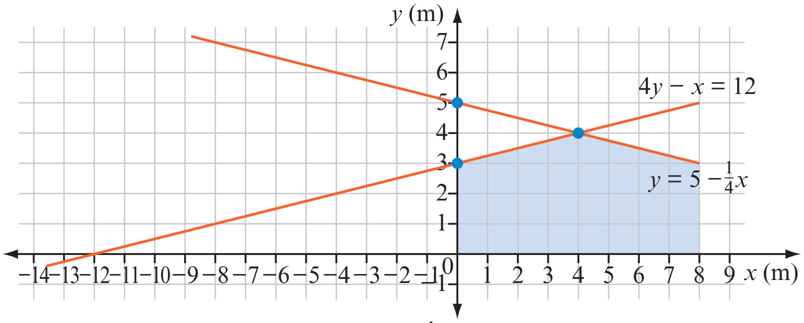
21 4 **22** −3 **23** 2 **24** $\frac{2}{3}$

25 (−1, 2) **26** 3.6 units **27** PR

28 7.1 units **29** (1.5, −2)

ANALYSIS

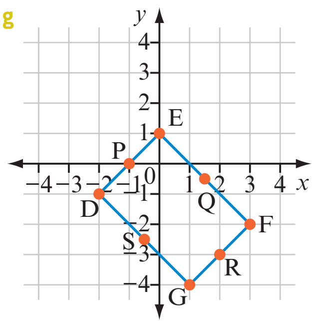
1 a, b



c 4 m **d** 4.1 m **e** $\frac{1}{4}$

f chimney will be at (2, 3.5) or (6, 3.5)

2 a, g



b DE parallel with FG, EF parallel with DG.

c DE and FG are parallel and 2.8 units long, EF and DG are parallel and 4.2 units long.

d **i** (0, 1) **ii** (0, 1) **iii** (0, −3) **iv** (0, −5)

e **i** 1 **ii** −1 **iii** −1 **iv** 1

f **i** $y = x + 1$ **ii** $y = -x + 1$ **iii** $y = -x - 3$ **iv** $y = x - 5$

g **i** (−1, 0) **ii** (1.5, −0.5) **iii** (2, −3) **iv** (−0.5, −2.5)

h PQRS is a parallelogram with two pairs of parallel sides (PQ and RS, SP and QR).

i If DEFG was square, then PQRS would also have been square because midpoints of each side would have been same distance from midpoints of diagonals.

3 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 4 NON-LINEAR RELATIONSHIPS

4 Are you ready?

1 **a** $x(x + 7)$ **b** $(x - 3)(x + 3)$ **c** $(x + 3)(x + 2)$ **d** $-x(x + 3)$

2 **a** **i** −4 **ii** 21 **iii** 0

b **i** 0 **ii** 5 **iii** −4

c **i** 1 **ii** 16 **iii** 1

d **i** −10 **ii** 0 **iii** −12

3 **a** $4x^2 + 4x + 1$ **b** $x^2 - 5x - 5$ **c** $-x^2 - 5x - 6$ **d** $2x^2$

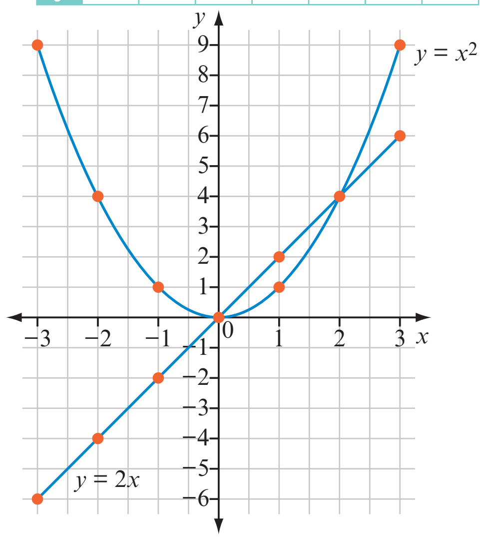
4 **a** **i**

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

ii

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

b



c **i** linear **ii** non-linear

5 **a** $y = 1$ **b** $x = -3$ **c** $x = 2$ **d** $y = -2$

6 **a** (2, −3) **b** (6, 3) **c** (2, −2) **d** (2, 4) **e** (−4, 3) **f** (0, 6)

7 **a** $x = 4$; $y = -4$ **b** $x = -3$; $y = 6$

8 gradient = 2

4A Solving quadratic equations

4A Start thinking!

1 A quadratic expression has 2 as the highest power of the variable, e.g. $x^2 + 3x$, $3x^2$, $5 - x^2$.

2 **a** A quadratic equation has an equals sign, quadratic expression does not.

b i, iii

c Highest power in a quadratic equation is 2; highest power in a linear equation is 1.

3 **a** $x = -2$ or $x = 2$ **b** $x = 3$ or $x = 5$ **c** $x = 0$ or $x = -7$ **d** $x = -4$ or $x = 1$

4 **a** **i** 0 **ii** 0 **iii** 0 **iv** 0 **v** 0 **vi** 0

b **i** $x = 0$ **ii** $x = 0$ **iii** $x = 1$ **iv** $x = -5$

c zero; zero

5 **a** $x - 3$ and $x - 5$

b One factor or the other must equal zero.

c $(x-3)(x-5)=0$
$x-3=0$ or $x-5=0$
$x=3$ or $x=5$

d Substitute each x value into the original equation $(x-3)(x-5)=0$ and show that each value makes the equation a true statement.

Exercise 4A Solving quadratic equations

1 a, c, e and h

2 a $(x+7)(x-4)=0$
$x+7=0$ or $x-4=0$
$x=-7$ or $x=4$
b $x(x-2)=0$
$x=0$ or $x-2=0$
$x=0$ or $x=2$
c $(x+5)(x-5)=0$
$x+5=0$ or $x-5=0$
$x=-5$ or $x=5$

3 a $x=-2$ or $x=3$ b $x=1$ or $x=7$
c $x=-4$ or $x=4$ d $x=0$ or $x=6$
e $x=-5$ or $x=-1$ f $x=-2$ or $x=0$
g $x=-8$ or $x=8$ h $x=-1$ or $x=7$
i $x=0$ or $x=11$ j $x=-3$ or $x=5$
k $x=2$ l $x=-5$

4 a $x=-2$ or $x=5$ b $x=1$ or $x=2$
c $x=0$ or $x=-5$ d $x=0$ or $x=3$
e $x=-6$ or $x=6$ f $x=-7$ or $x=-3$
g $x=-2$ or $x=4$ h $x=-1$ or $x=1$
i $x=0$ or $x=-8$ j $x=1$ or $x=3$
k $x=-3$ l $x=1$

6 a $x=-2$ or $x=2$ b $x=3$ or $x=5$
c $x=0$ or $x=-7$ d $x=-4$ or $x=1$

7 a yes b no c no d yes
e no f yes g yes h no

8 a Dividing both sides of equation by −1 gives an identical equation.
b $x=0$ or $x=3$

9 a Dividing both sides of equation by −2 gives an identical equation.
b $x=-4$ or $x=5$

10 a $x=-9$ or $x=0$ b $x=-8$ or $x=2$
c $x=1$ or $x=4$ d $x=-6$ or $x=6$
e $x=-7$ or $x=-3$ f $x=-2$ or $x=1$

11 a two b linear equation has one solution
c no
d one; the two factors produce the same solution.
e In part d the two factors are the same, whereas in part a they are different.
f Zero; not possible to factorise x^2+4.
g A quadratic equation can have zero, one or two solutions.

12 a $x=-2$ or $x=2$ b $x=-5$ or $x=2$
c $x=3$ d no solutions
e $x=-7$ or $x=0$ f $x=-8$ or $x=-4$
g $x=-8$ or $x=9$ h no solutions

13 a $x=-3$ or $x=1$ b $x=-4$ or $x=5$
c $x=-5$ or $x=5$ d $x=-2$
e $x=-8$ or $x=0$ f $x=-6$ or $x=6$
g $x=0$ or $x=3$ h $x=6$
i $x=-7$ or $x=-3$ j $x=-1$ or $x=8$
k $x=2$ or $x=4$ l $x=-2$ or $x=6$

15 a \$100 b 10 weeks
c No; if amounts were the same, relationship would be linear

16 a i 9 m ii 8 m b 8 m c 4 s
d Not possible to have negative time values.

17 a 10 b 2 c 2 and 6 d 4

18 a $x(x+8)$ cm^2 b x^2+8x cm^2
c $x^2+8x=560$ d $x^2+8x-560=0$
$(x+28)(x-20)=0$
e $x=20$ or $x=-28$. $x=20$ is the feasible solution and $x=-28$ is not, as it is not possible to have a negative length.
f width = 20 cm and length = 28 cm

19 a $x^2+2x=35$ b $x=-7$ or $x=5$
c length = 7 m, width = 5 m

20 $x(x-12)=640$
length = 32 cm, width = 20 cm

21 a $x^2-3x+2=0$ b $x^2-10x=0$
c $x^2+2x-15=0$

22 Multiplying an equation by any constant factor will result in the same solution. For example, the solution $x=1$ or $x=2$ matches the equation $(x-1)(x-2)=0$, which is equivalent to $x^2-3x+2=0$.
Multiplying by a constant (say, 3) results in an equation $3(x^2-3x+2=0)$ or $3x^2-9x+6=0$, which still has the same solutions, $x=1$ or $x=2$.

23 a $x=0$ or $x=6$ b $x=0$ or $x=1$
c $x=-3$ or $x=1$ d $x=2$ or $x=3$

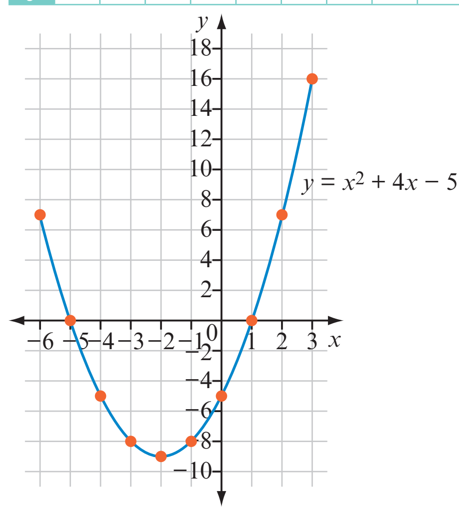
4B Plotting quadratic relationships

4B Start thinking!

1

x	−6	−5	−4	−3	−2	−1	0	1	2	3
y	7	0	−5	−8	−9	−8	−5	0	7	16

2



3 The points form a symmetrical curve that changes direction at the point (−2, −9).

4 Non-linear relationship, as the points do not form a straight line.

5 a minimum b (−2, −9)

6 a It acts like a mirror line so that the left side is symmetrical to the right side.

b $x = -2$

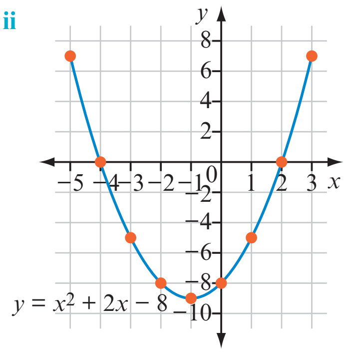
7 a one; (0, −5) b two; (−5, 0) and (1, 0)

Exercise 4B Plotting quadratic relationships

1 a i

x	−5	−4	−3	−2	−1	0	1	2	3
y	7	0	−5	−8	−9	−8	−5	0	7

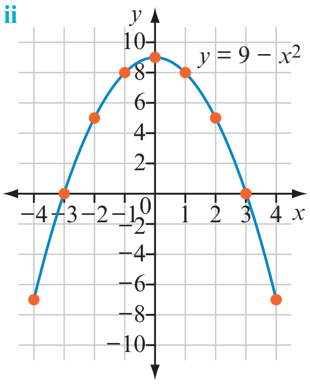
ii



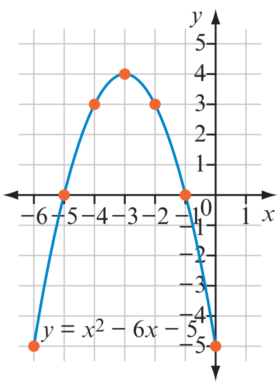
b i

x	−4	−3	−2	−1	0	1	2	3	4
y	−7	0	5	8	9	8	5	0	−7

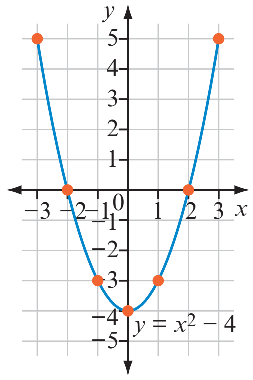
ii



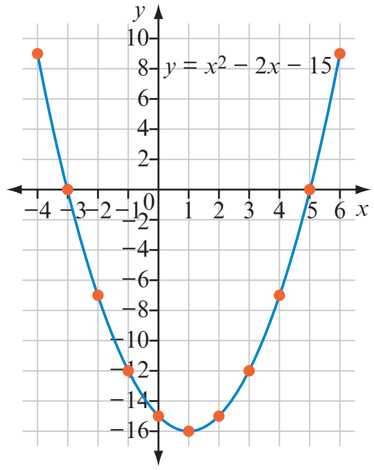
2 a



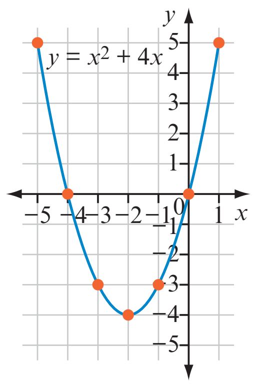
b



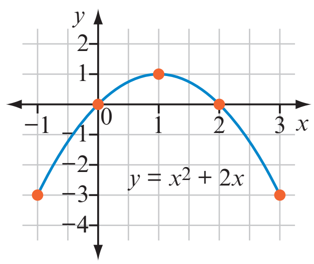
c



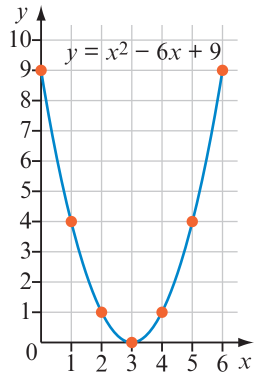
d



e



f



3 **a** minimum **b** (2, −1) **c** $x = 2$
d (0, 3) **e** (1, 0) and (3, 0)

4 **a** **i** minimum **ii** (−1, −9) **iii** $x = -1$
iv −8 **v** −4 and 2
b **i** maximum **ii** (0, 9) **iii** $x = 0$
iv 9 **v** −3 and 3

5 **a** **i** maximum **ii** (−3, 4) **iii** $x = -3$
iv −5 **v** −5 and −1
b **i** minimum **ii** (0, −4) **iii** $x = 0$
iv −4 **v** −2 and 2
c **i** minimum **ii** (1, −16) **iii** $x = 1$
iv −15 **v** −3 and 5
d **i** minimum **ii** (−2, −4) **iii** $x = -2$
iv 0 **v** −4 and 0
e **i** maximum **ii** (1, 1) **iii** $x = 1$
iv 0 **v** 0 and 2
f **i** minimum **ii** (3, 0) **iii** $x = 3$
iv 9 **v** 3

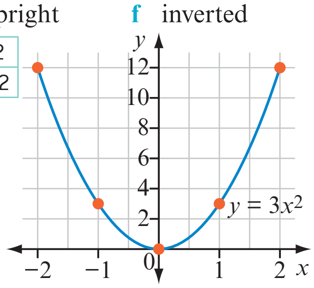
7 **a** F **b** E **c** B **d** A **e** C **f** D

8 **a** upright **b** inverted **c** upright
d upright **e** upright **f** inverted

9 **a**

x	−2	−1	0	1	2
y	12	3	0	3	12

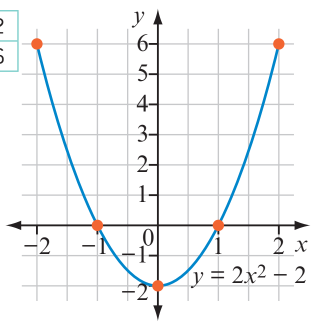
i upright
ii minimum; (0, 0)
iii 0
iv 0



b

x	−2	−1	0	1	2
y	6	0	−2	0	6

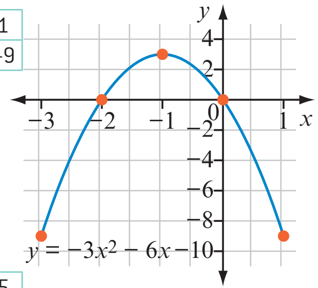
i upright
ii minimum; (0, −2)
iii −2
iv −1 and 1



c

x	−3	−2	−1	0	1
y	−9	0	3	0	−9

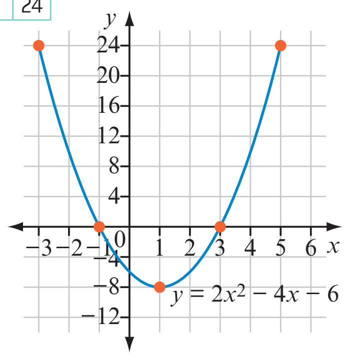
i inverted
ii maximum; (−1, 3)
iii 0
iv −2 and 0



d

x	−3	−1	1	3	5
y	24	0	−8	0	24

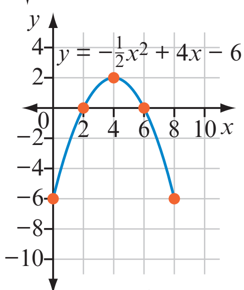
i upright
ii minimum; (1, −8)
iii −6
iv −1 and 3



e

x	0	2	4	6	8
y	−6	0	2	0	−6

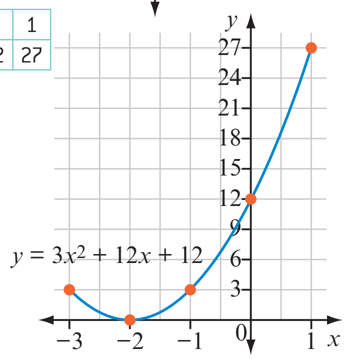
i inverted
ii maximum; (4, 2)
iii −6
iv 2 and 6



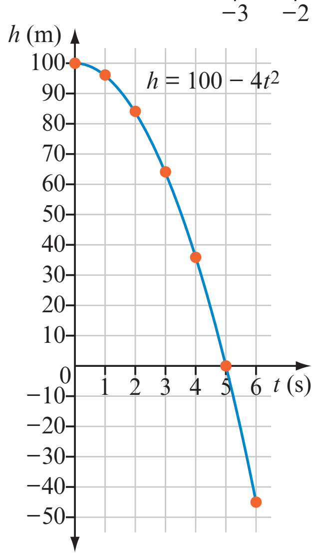
f

x	−3	−2	−1	0	1
y	3	0	3	12	27

i upright
ii minimum; (−2, 0)
iii 12
iv −2



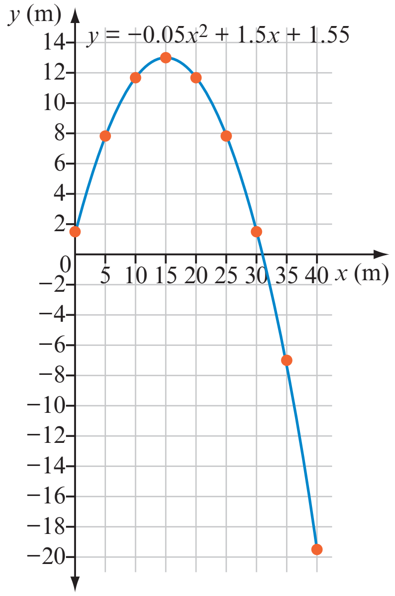
10 **a**



b not possible to have negative time values

c **i** 84 m **ii** 64 m **d** 100 m **e** 5 s

11 **a**



b 12.8 m **c** 1.55 m **d** 31 m

12 **a** **i** two **ii** one **iii** zero

b A parabola changes direction only once, so there can only be a maximum of two x-intercepts.

c one

d No; a parabola only intersects the y-axis once.

13 **a** −2 and 5 **b** $x = -2$ or $x = 5$

c They are the same.

d The x-intercepts of a parabola represent the solutions to a quadratic equation.

14 **a** $x = -5$ or $x = 1$ **b** $x = 0$ or $x = 4$

c $x = -2$ or $x = 2$ **d** $x = -5$ or $x = 1$

e $x = -2$ or $x = 2$ **f** $x = -1$ or $x = 5$

15 $x = 5$ **14** $x = -4$

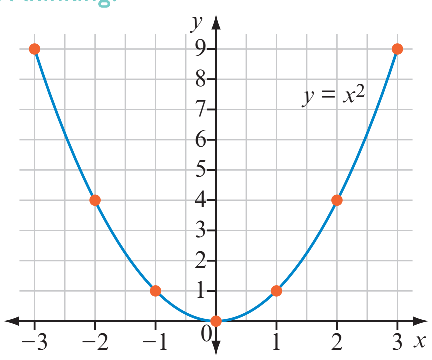
17 no x-intercepts, one y-intercept

18 one x-intercept and one y-intercept

4C Parabolas and transformations

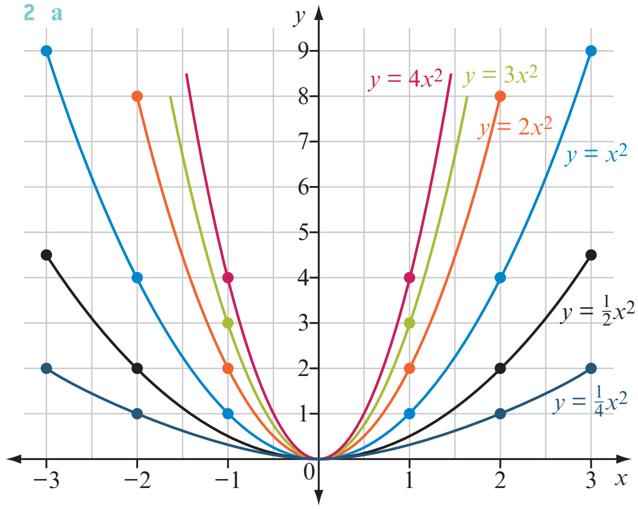
4C Start thinking!

1 **a**



b minimum turning point at (0, 0), x-intercept at 0 and y-intercept at 0, axis of symmetry $x = 0$

2 **a**



b All graphs have same minimum turning point, axis of symmetry and x- and y-intercepts but different shapes (some wider than $y = x^2$ and some narrower).

c **i** $y = 2x^2$, $y = 3x^2$ and $y = 4x^2$

ii $y = \frac{1}{2}x^2$ and $y = \frac{1}{4}x^2$

d For rules of the form $y = ax^2$ where a is positive, there is dilation only (dilation factor is a). For $0 < a < 1$, dilation produces a wider graph than $y = x^2$.
For $a > 1$, dilation produces a narrower graph than $y = x^2$.

e **ii** dilated by factor of 3, narrower

iii dilated by factor of 4, narrower

iv dilated by factor of $\frac{1}{2}$, wider

v dilated by factor of $\frac{1}{4}$, wider

3 For rules of the form $y = ax^2$ where a is positive, there is dilation only (dilation factor is a). For $0 < a < 1$, dilation produces a wider graph than $y = x^2$. For $a > 1$, dilation produces a narrower graph than $y = x^2$.

Exercise 4C Parabolas and transformations

1 **a** $y = \frac{1}{2}x^2$

b One possible answer is $y = \frac{1}{4}x^2$.

c $y = 2x^2$ **d** One possible answer is $y = 4x^2$.

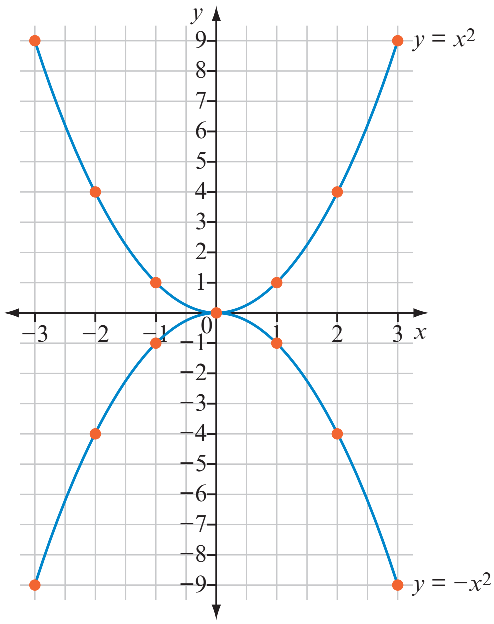
2 **a** dilate by factor of 2, narrower

b dilate by factor of $\frac{1}{2}$, wider

c dilate by factor of $\frac{1}{4}$, wider

d dilate by factor of 4, narrower

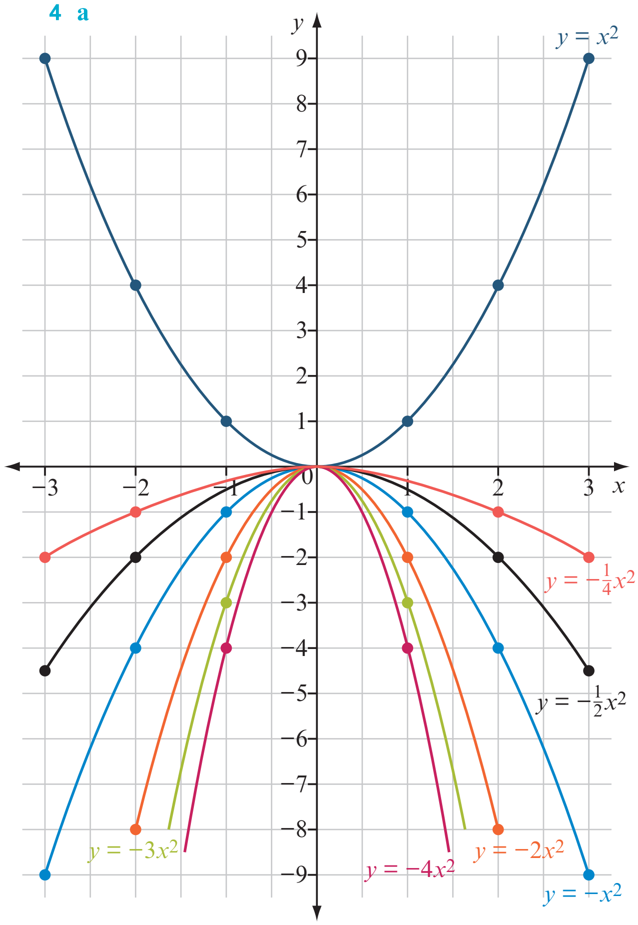
3 **a**



b Mirror image of $y = x^2$ (reflected in x-axis); no dilation; reflection has been performed.

c x-axis; exact image appears beneath x-axis, which acts as mirror line.

4 **a**



b All graphs have same maximum turning point, axis of symmetry and x- and y-intercepts, but different shapes (some wider than $y = -x^2$ and some narrower)

c **i** $y = -2x^2$, $y = -3x^2$ and $y = -4x^2$

ii $y = -\frac{1}{2}x^2$ and $y = -\frac{1}{4}x^2$

5 **a** dilate by factor of 2 (narrower)

b dilate by factor of $\frac{1}{2}$ (wider)

c dilate by factor of $\frac{1}{4}$ (wider)

d dilate by factor of 4 (narrower)

6 **a** dilate by factor of 5 (narrower)

b reflect in x-axis

c dilate by factor of 4 (narrower) and reflect in x-axis

d dilate by factor of $\frac{1}{4}$ (wider)

e dilate by factor of 10 (narrower)

f dilate by factor of $\frac{1}{7}$ (wider) and reflect in x-axis

g dilate by factor of 8 (narrower) and reflect in x-axis

h dilate by factor of $\frac{2}{3}$ (wider) and reflect in x-axis

7 **a** dilation and reflection

b **i** $y = -2x^2$: dilate by factor of 2 (narrower) and reflect in x-axis

ii $y = -3x^2$: dilate by factor of 3 (narrower) and reflect in x-axis

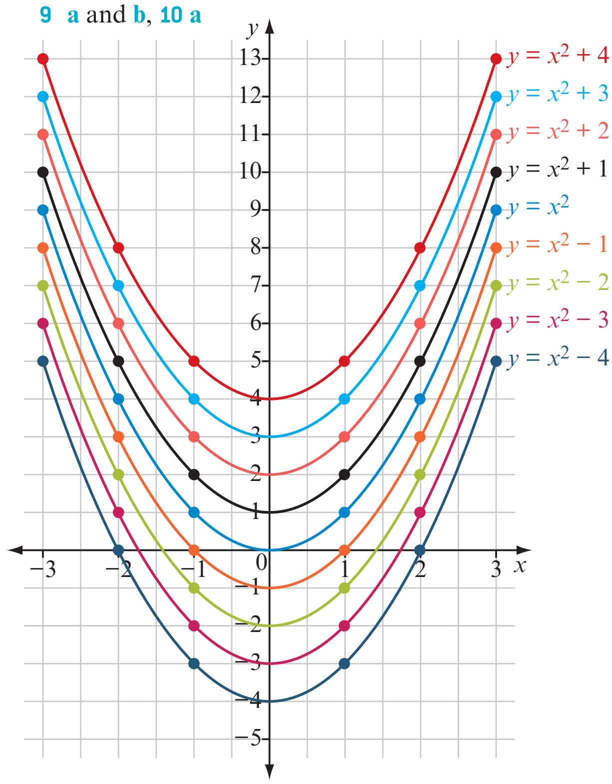
iii $y = -4x^2$: dilate by factor of 4 (narrower) and reflect in x-axis

iv $y = -\frac{1}{2}x^2$: dilate by factor of $\frac{1}{2}$ (wider) and reflect in x-axis

v $y = -\frac{1}{4}x^2$: dilate by factor of $\frac{1}{4}$ (wider) and reflect in x-axis

8 **a** E **b** A **c** D **d** C **e** F **f** B

9 **a** and **b**, **10** **a**



9 **c** **i** All graphs have same shape as $y = x^2$, a minimum turning point and same axis of symmetry.

ii Graphs have different turning point coordinates and different y-intercepts; graphs have no x-intercepts.

d Graph shifts up by same number of units as constant term.

e **i** translated 1 unit up
ii translated 2 units up
iii translated 3 units up
iv translated 4 units up

10 **a** see above

b **i** All graphs have same shape as $y = x^2$, a minimum turning point and same axis of symmetry.
ii Graphs have different turning point coordinates and different y-intercepts. Graphs all have two x-intercepts.

c Graph shifts down by same number of units as constant term.

d **i** translated 1 unit down
ii translated 2 units down
iii translated 3 units down
iv translated 4 units down

11 **a** shifts the graph vertically (translation)

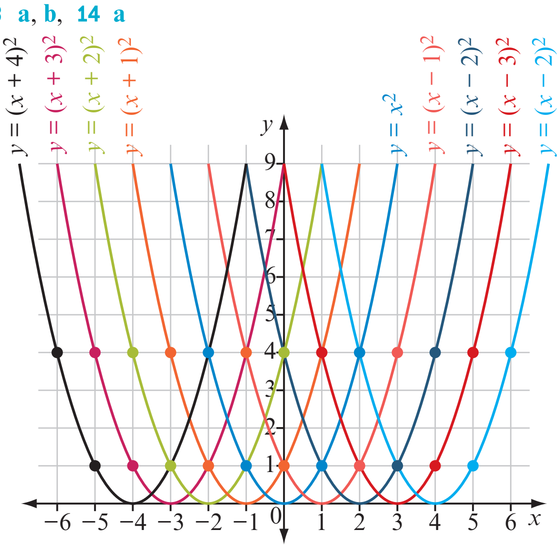
b If k is positive, graph shifts up and if k is negative, graph shifts down.

c up, down

d Graph has no vertical movement.

12 **a** **i** $k = 6$ **ii** translated 6 units up
b **i** $k = -7$ **ii** translated 7 units down
c **i** $k = -5$ **ii** translated 5 units down
d **i** $k = 8$ **ii** translated 8 units up
e **i** $k = 9$ **ii** translated 9 units up
f **i** $k = -11$ **ii** translated 11 units down
g **i** $k = 1.5$ **ii** translated 1.5 units up
h **i** $k = -7.2$ **ii** translated 7.2 units down

13 **a**, **b**, **14** **a**



13 **c** **i** All graphs have the same shape as $y = x^2$ and a minimum turning point.
ii Graphs have different turning point coordinates, axes of symmetry and y-intercepts. Graphs all have one x-intercept (they sit on the x-axis).

d Shifts graph horizontally to right.

e **i** translated 1 unit right
ii translated 2 units right
iii translated 3 units right
iv translated 4 units right

14 **a** see above

b **i** All graphs have same shape as $y = x^2$, and a minimum turning point.
ii Graphs have different turning point coordinates, axes of symmetry and y-intercepts. Graphs all have one x-intercept (they sit on the x-axis).

c Shifts graph horizontally to left.

d **i** translated 1 unit left
ii translated 2 units left
iii translated 3 units left
iv translated 4 units left

15 **a** Shifts graph horizontally (translation).

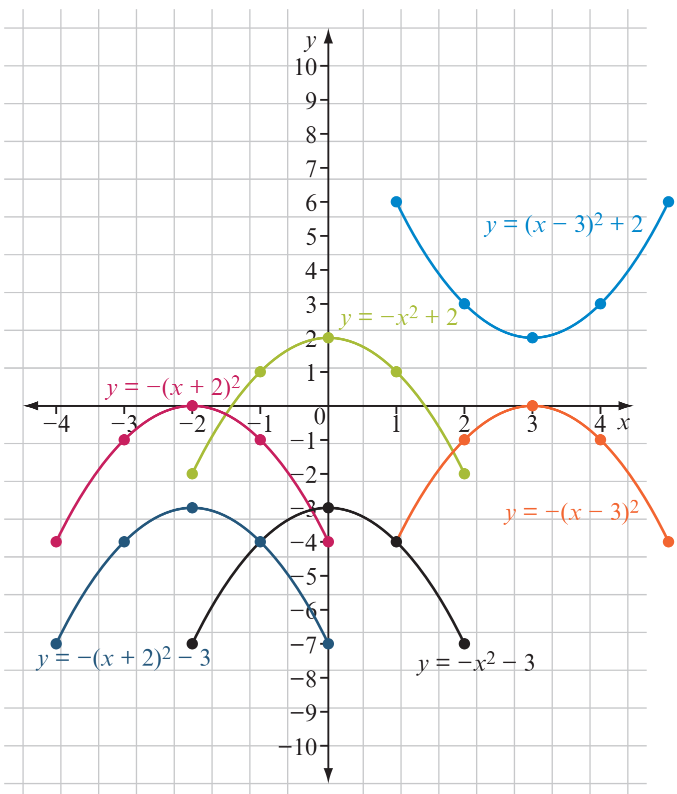
b If h positive, graph moves to right; if h negative, graph moves to left.

c right, left

d Graph has no horizontal movement.

16 **a** **i** $h = 5$ **ii** translated 5 units right
b **i** $h = -7$ **ii** translated 7 units left
c **i** $h = 6$ **ii** translated 6 units right
d **i** $h = -9$ **ii** translated 9 units left
e **i** $h = -8$ **ii** translated 8 units left
f **i** $h = 12$ **ii** translated 12 units right
g **i** $h = 2.5$ **ii** translated 2.5 units right
h **i** $h = -6.7$ **ii** translated 6.7 units left

17 **a**



b **i** (0, 2) **ii** (0, −3) **iii** (3, 0)
iv (−2, 0) **v** (3, 2) **vi** (−2, −3)

c **i** inverted **ii** inverted **iii** inverted
iv inverted **v** upright **vi** inverted

d A iii, B i, C v, D iv, E vi, F ii

18 a **i** $a = -1; h = 0; k = 2$
ii $a = -1; h = 0; k = -3$
iii $a = -1; h = 3; k = 0$
iv $a = -1; h = -2; k = 0$
v $a = 1; h = 3; k = 2$
vi $a = -1; h = -2; k = -3$

b Turning point of $y = x^2$ (0, 0) shifts horizontally h units and vertically k units and end result is coordinates (h, k).

c If a greater than 0, parabola will be upright and if a less than 0, parabola will be inverted.

d Dilation: narrower when $a < -1$ or $a > 1$ and wider when $-1 < a < 0$ or $0 < a < 1$. For $a > 0$, upright parabola and for $a < 0$, inverted parabola (reflection in x-axis). Horizontal translation (h units). For $h > 0$, move right and for $h < 0$, move left. Vertical translation (k units). For $k > 0$, move up and for $k < 0$, move down.

4D Sketching parabolas using transformations

4D Start thinking!

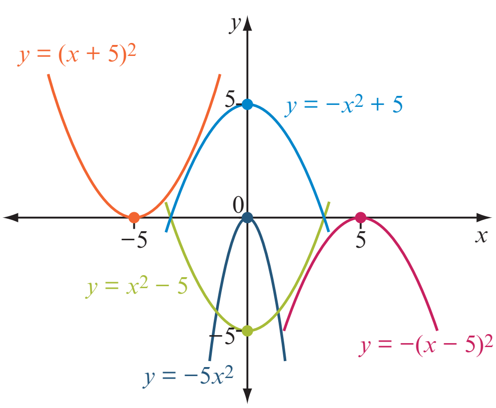
1 a **i** translate 5 units down
ii translate 5 units left
iii dilate by factor of 5 and reflect in x-axis
iv reflect in x-axis and translate 5 units up
v reflect in x-axis and translate 5 units right

b **i** upright **ii** upright **iii** inverted
iv inverted **v** inverted

Compare the rule to $y = a(x - h)^2 + k$. Look at sign of a. If negative, graph is inverted and if positive, graph is upright.

c Use (h, k) values.
i (0, −5) **ii** (−5, 0) **iii** (0, 0)
iv (0, 5) **v** (5, 0)

d



e Use values for a, h and k.

2 Use value of a to determine if there is dilation (narrower or wider) or reflection.
For $a > 0$, upright parabola and for $a < 0$, inverted parabola (reflection in the x-axis). Horizontal translation (h units). For $h > 0$, move right and for $h < 0$, move left. Vertical translation (k units). For $k > 0$, move up and for $k < 0$, move down.

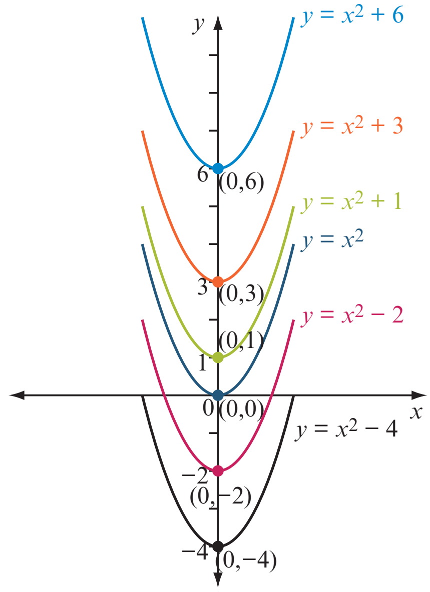
Exercise 4D Sketching parabolas using transformations

1 a **i** upright **ii** (2, 0)
iii translated 2 units right **iv** C

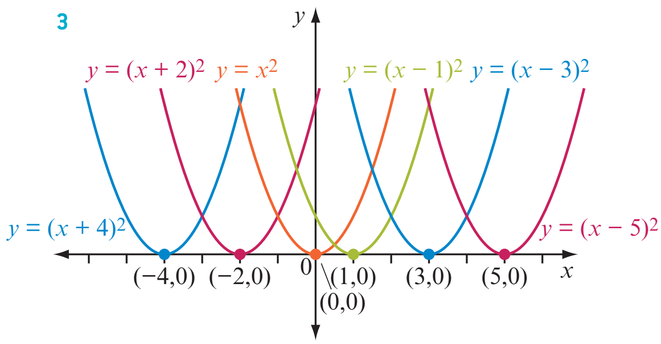
b i inverted **ii** (0, 0)
iii dilation by factor of 2, reflection in x-axis
iv A

c **i** upright **ii** (0, 2)
iii translated 2 units up **iv** B

2



3



4 a upright **b** (4, −2)

c C. Turning point has moved right 4 units ($h = 4$) and down 2 units ($k = -2$).

5 a $a = 1$, $h = 3$ and $k = 4$

b no dilation or reflection; horizontal translation (3 units right) and vertical translation (4 units up)

c (3, 4)

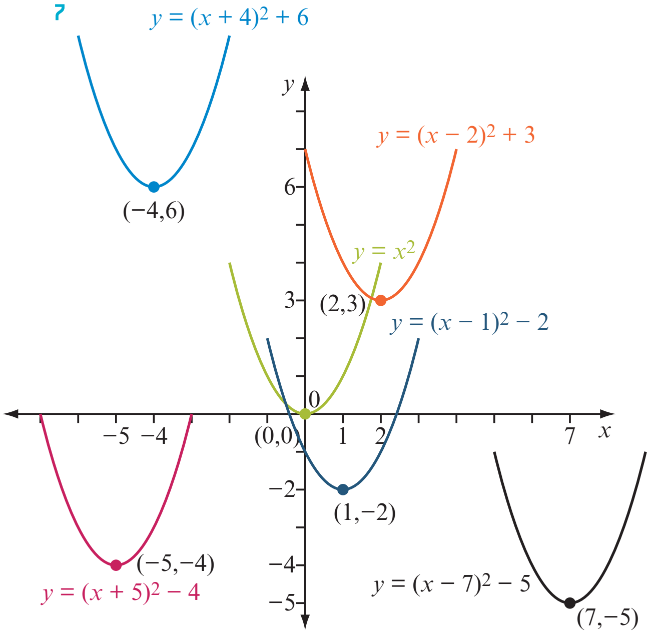
d Graph shifts right 3 units and up 4 units, resulting in turning point coordinates (3, 4)

6 a $a = 1$, $h = 4$ and $k = -2$

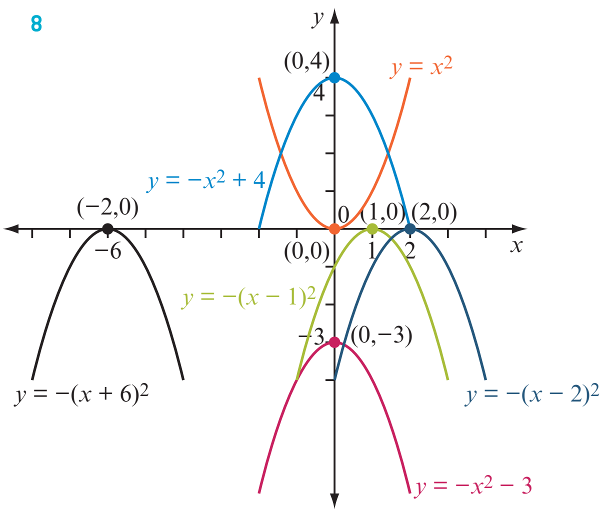
b no dilation or reflection; horizontal translation (4 units right) and vertical translation (2 units down)

c $(4, -2)$

7



8



9 a E **b** D **c** A **d** F **e** C **f** B

10 a i upright **ii** $(1, -4)$

iii y-intercept: $(0, -3)$;
x-intercepts: $(-1, 0)$ and $(3, 0)$

b i inverted **ii** $(-3, 1)$

iii y-intercept: $(0, -8)$;
x-intercepts: $(-4, 0)$ and $(-2, 0)$

c i upright **ii** $(3, -9)$

iii y-intercept: $(0, 0)$;
x-intercepts: $(0, 0)$ and $(6, 0)$

11 a $y = (x - 3)^2 + 7$ **b** $y = (x + 2)^2 + 5$

c $y = -(x - 2)^2 + 4$ **d** $y = -(x - 6)^2 - 1$

e $y = (x - 9)^2$ **f** $y = -x^2 + 4$

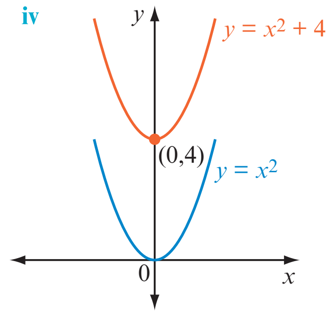
g $y = -(x + 1)^2 - 2$ **h** $y = x^2 - 5$

12 a i translate 4 units up

ii upright

iii $(0, 4)$

iv

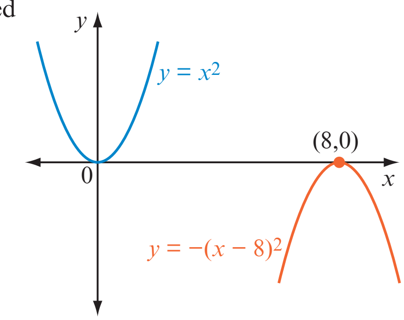


b i reflect in x-axis and translate 8 units right

ii inverted

iii $(8, 0)$

iv

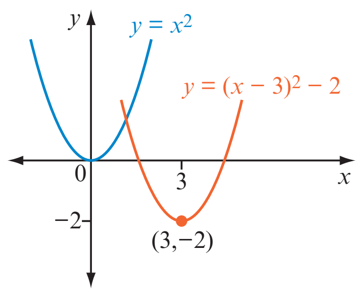


c i translate 3 units right and 2 units down

ii upright

iii $(3, -2)$

iv

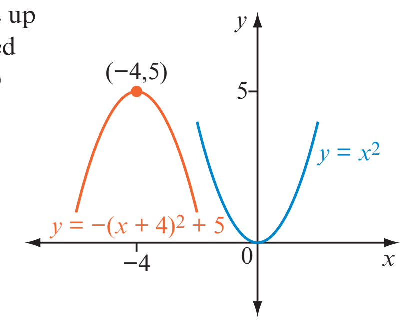


d i reflect in x-axis and translate 4 units left and 5 units up

ii inverted

iii $(-4, 5)$

iv

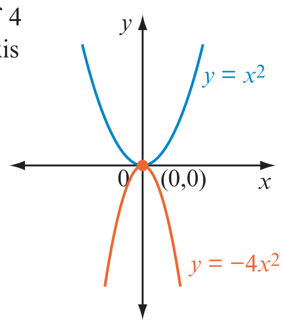


e i dilate by factor of 4 and reflect in x-axis

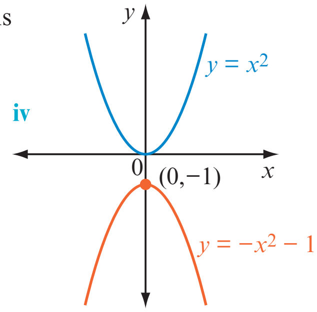
ii inverted

iii $(0, 0)$

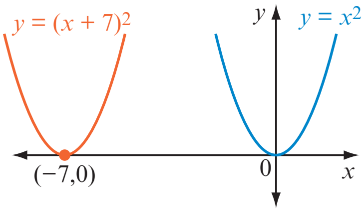
iv



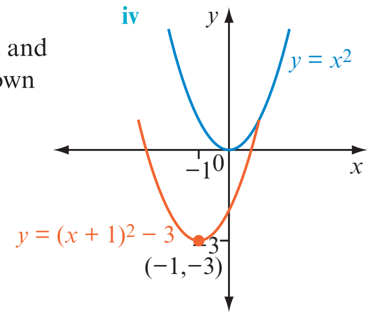
f **i** reflect in x-axis and translate 1 unit down
ii inverted
iii $(0, -1)$
iv



g **i** translate 7 units left
ii upright
iii $(-7, 0)$
iv



h **i** translate 1 unit left and 3 units down
ii upright
iii $(-1, -3)$
iv

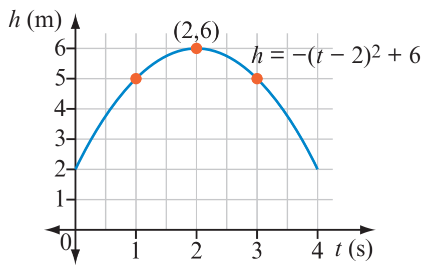


13 **a** $(4, 5)$ **b** 5

14 2. Graph is inverted so has maximum turning point. No possible larger y value than the y coordinate of the turning point, 2.

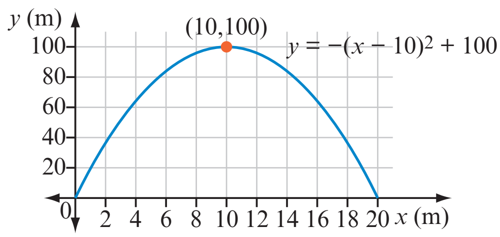
15 **a** $(2, 6)$ **b** **i** 2 m **ii** 2 m

c



d **i** 2 m **ii** 6 m

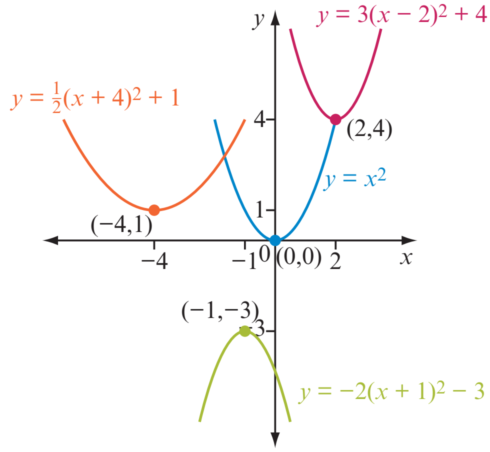
16 **a**



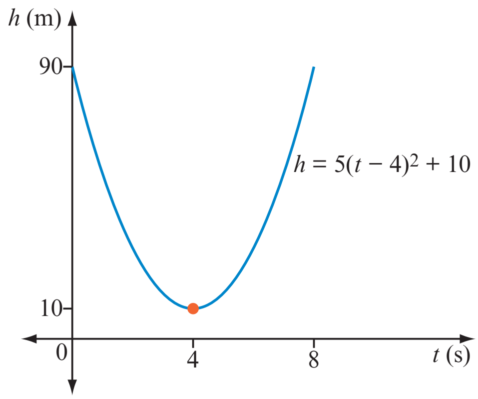
b 100 m **c** 20 m

17 **a** **i** narrower **ii** upright **iii** $(4, -3)$
b **i** narrower **ii** inverted **iii** $(-1, 5)$
c **i** narrower **ii** upright **iii** $(-2, 0)$
d **i** narrower **ii** inverted **iii** $(0, -4)$
e **i** the same **ii** inverted **iii** $(5, 4)$
f **i** wider **ii** upright **iii** $(-2, 6)$
g **i** wider **ii** inverted **iii** $(3, -4)$
h **i** wider **ii** upright **iii** $(0, 3)$
i **i** wider **ii** inverted **iii** $(7, 0)$

18



19 **a**



b 90 m **c** 10 m

20 The graph of $y = 2x^2$ is the result of multiplying each y value in the graph of $y = x^2$ by a factor of 2. The graph of $y = -2x^2$ is the result of multiplying each y value in the graph of $y = 2x^2$ by a factor of -1. This results in an 'upside down' or inverted graph. An inverted graph is also a reflection in the x-axis.

21 **a** $y = -4x^2$ **b** $y = \frac{1}{3}x^2$
c $y = (x + 2)^2$ **d** $y = -x^2 - 5$
e $y = (x + 1)^2 + 4$ **f** $y = -2(x - 5)^2 + 3$

22 **a** $y = 3(x - 2)^2$ **b** $y = -(x + 5)^2 - 4$
c $y = -\frac{1}{2}x^2$ **d** $y = -4(x + 2)^2$

4E Sketching parabolas using intercepts

4E Start thinking!

1 When a quadratic relationship is in turning point form, it is easy to identify the transformations applied to $y = x^2$ to result in the given relationship. The general form of a quadratic, that is, $y = ax^2 + bx + c$, does not follow the same rules as turning point form.

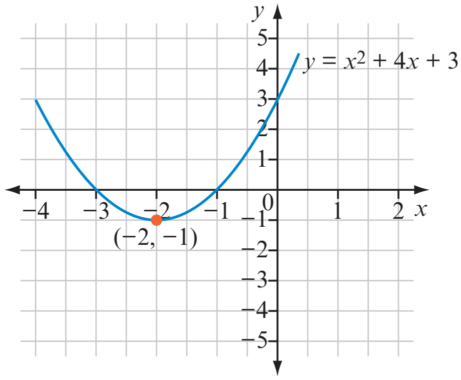
2 **a** $x = 0$ **b** $y = 3$ **c** 3

3 **a** $y = 0$ **b** $0 = x^2 + 4x + 3$
c $x = -3$ or $x = -1$ **d** -3 and -1

4 $(0, 3)$, $(-3, 0)$ and $(-1, 0)$

5 upright

6



7 Axis of symmetry of a parabola is halfway between x-intercepts. Hence, x-coordinate of turning point is halfway between x values at x-intercepts. y-coordinate of turning point is found by substituting x-coordinate into rule and simplifying.

8 Can sketch quadratic relationships using x- and y-intercepts. x-intercept/s are found by substituting $y = 0$ into rule and solving for x. Equation may need to be factorised first so that Null Factor Law can be used. A parabola can have two, one or no x-intercepts. y-intercept is found by substituting $x = 0$ into rule and simplifying. The x-coordinate of turning point is halfway between x values at x-intercepts. y-coordinate of turning point is found by substituting x-coordinate into rule and simplifying. Orientation of parabola (upright or inverted) can be identified from coefficient of x^2 term: if $a > 0$, parabola is upright and if $a < 0$, parabola is inverted.

Exercise 4E Sketching parabolas using intercepts

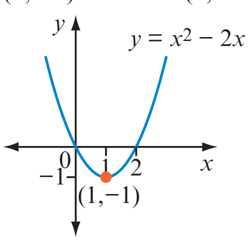
1 a coordinates of x-intercepts (–3, 0) and (5, 0)
coordinates of y-intercept (0, –15)
c coordinates of x-intercepts (–1, 0) and (1, 0)
coordinates of y-intercepts (0, –1)

2 a i (0, 0) and (2, 0) ii (0, 0)
b i (−8, 0) and (0, 0) ii (0, 0)
c i (−4, 0) and (−2, 0) ii (0, 8)
d i (2, 0) and (6, 0) ii (0, 12)
e i (−1, 0) and (5, 0) ii (0, −5)
f i (−3, 0) and (3, 0) ii (0, −9)

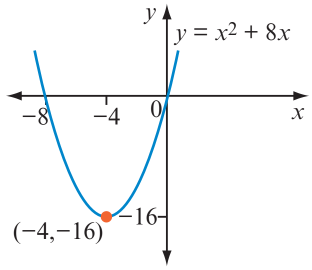
3 a coordinates of turning point (1, –16).
b coordinates of turning point (0, –1).

4 a (1, −1) b (−4, −16) c (−3, −1)
d (4, −4) e (2, −9) f (0, −9)

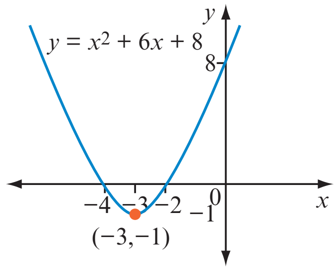
5 a



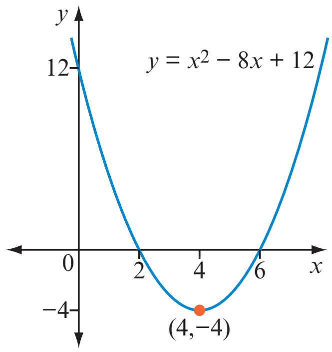
b



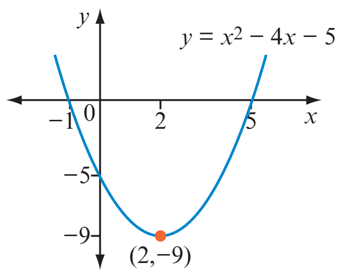
c



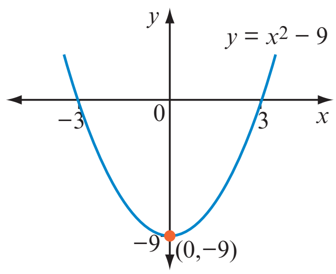
d



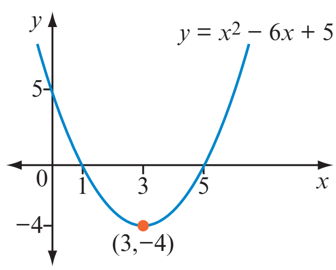
e



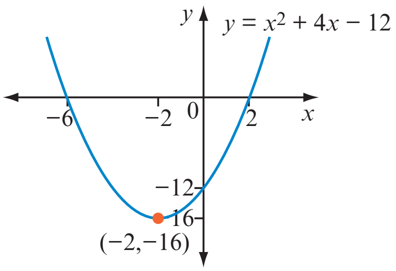
f



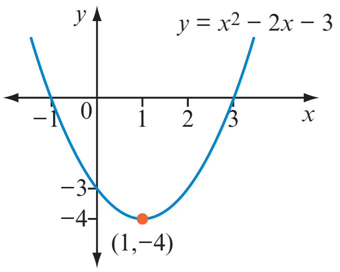
6 a



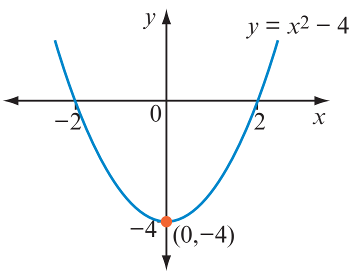
b



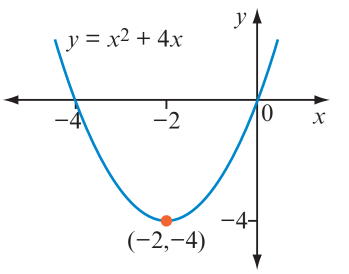
c



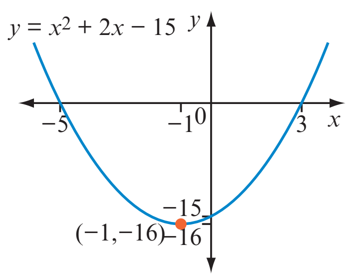
d



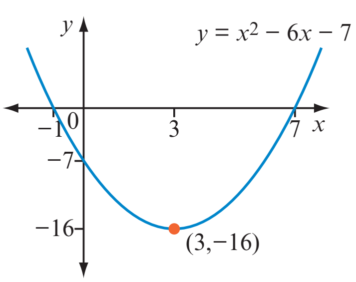
e



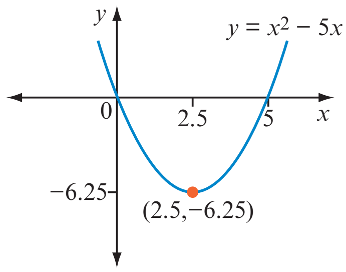
f



g



h



7 a C **b** A **c** B

8 Look at coefficient of x^2 term (a). If $a > 0$, parabola is upright and if $a < 0$, parabola is inverted.

9 a Both have x-intercepts of -3 and 2.

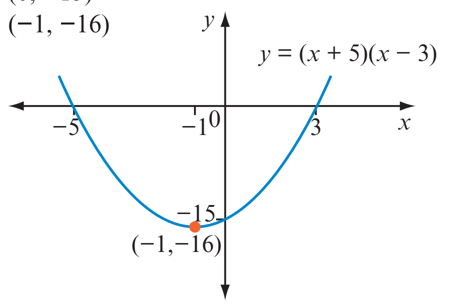
b One graph has negative x^2 coefficient and other has positive x^2 coefficient, so one will be inverted and the other upright.

10 a i upright

ii x-intercepts: $(-5, 0)$ and $(3, 0)$; y-intercept: $(0, -15)$

iii $(-1, -16)$

iv

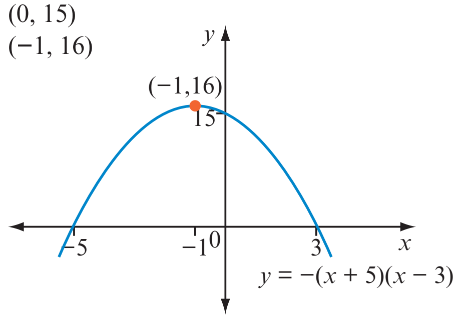


b i inverted

ii x-intercepts: $(-5, 0)$ and $(3, 0)$; y-intercept: $(0, 15)$

iii $(-1, 16)$

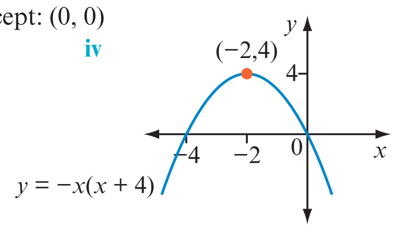
iv



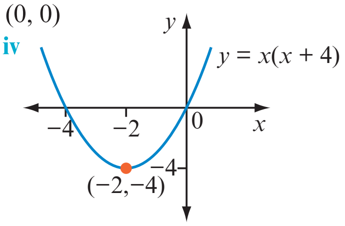
c i inverted

ii x-intercepts: $(-4, 0)$ and $(0, 0)$; y-intercept: $(0, 0)$

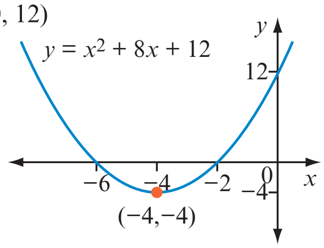
iii $(-2, 4)$ **iv**



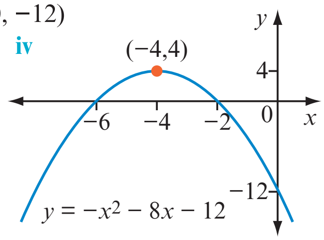
d **i** upright
ii x-intercepts: (−4, 0) and (0, 0);
y-intercept: (0, 0)
iii (−2, −4) **iv**



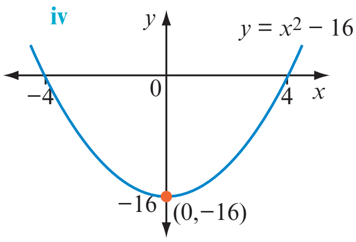
e **i** upright
ii x-intercepts: (−6, 0) and (−2, 0);
y-intercept: (0, 12)
iii (−4, −4) **iv**



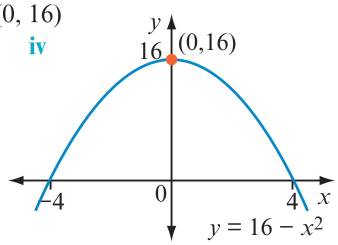
f **i** inverted
ii x-intercepts: (−6, 0) and (−2, 0);
y-intercept: (0, −12)
iii (−4, 4) **iv**



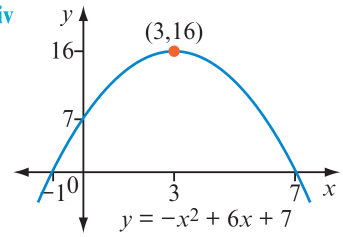
g **i** upright
ii x-intercepts: (−4, 0) and (4, 0);
y-intercept: (0, −16)
iii (0, −16) **iv**



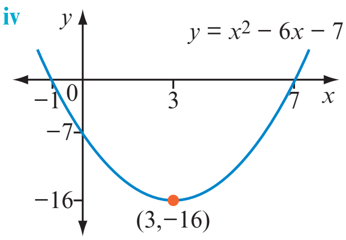
h **i** inverted
ii x-intercepts: (−4, 0) and (4, 0);
y-intercept: (0, 16)
iii (0, 16) **iv**



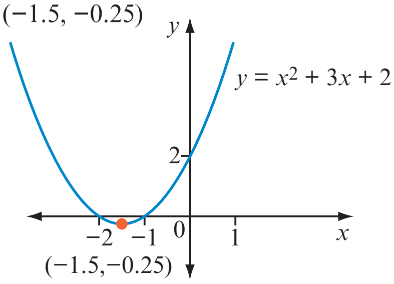
i **i** inverted
ii x-intercepts: (−1, 0) and (7, 0);
y-intercept: (0, 7)
iii (3, 16) **iv**



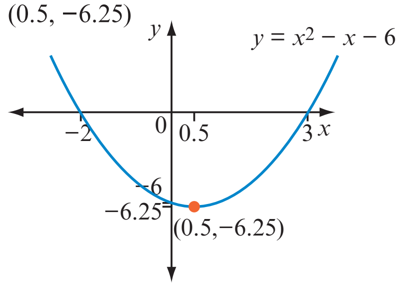
j **i** upright
ii x-intercepts: (−1, 0) and (7, 0);
y-intercept: (0, −7)
iii (3, −16) **iv**



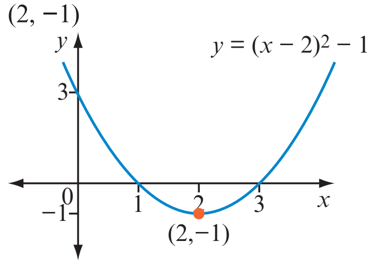
k **i** upright
ii x-intercepts: (−2, 0) and (−1, 0);
y-intercept: (0, 2)
iii (−1.5, −0.25)
iv



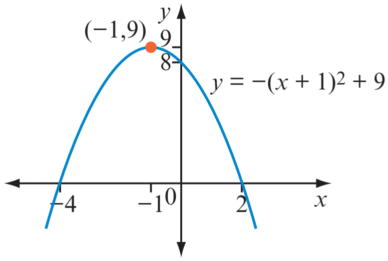
l **i** upright
ii x-intercepts: (−2, 0) and (3, 0);
y-intercept: (0, −6)
iii (0.5, −6.25)
iv



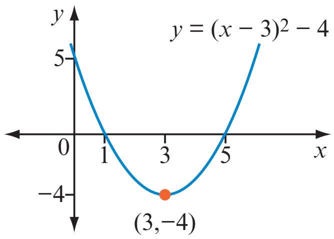
11 **a** **i** upright
ii x-intercepts: (1, 0) and (3, 0);
y-intercept: (0, 3)
iii (2, −1)
iv



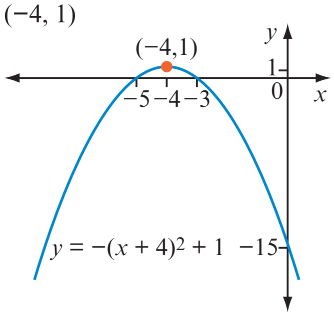
b **i** inverted
ii x-intercepts: (−4, 0) and (2, 0); y-intercept: (0, 8)
iii (−1, 9)
iv



c **i** upright
ii x-intercepts: (1, 0) and (5, 0); y-intercept: (0, 5)
iii (3, −4)
iv

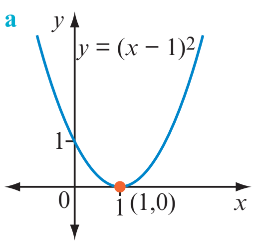


d **i** inverted
ii x-intercepts: (−5, 0) and (−3, 0); y-intercept: (0, −15)
iii (−4, 1)
iv

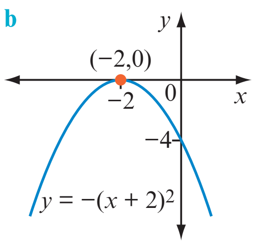


12 **a** upright **b** one (0, 9) **c** one (3, 0)
d (3, 0)

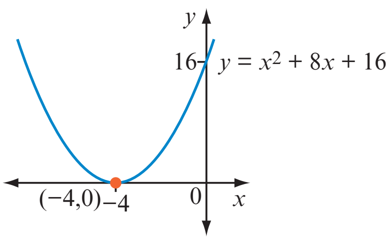
13 **a**



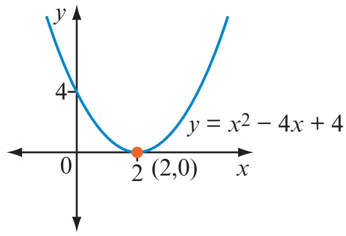
b



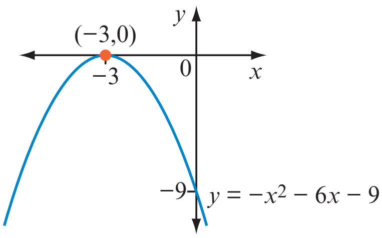
c



d

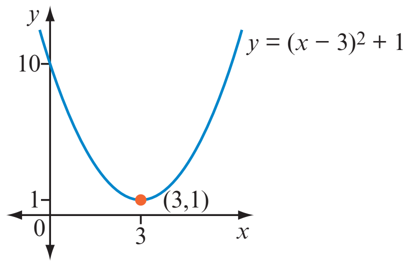


e

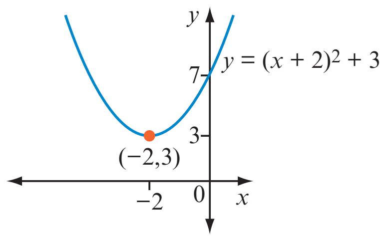


14 **a** inverted **b** one (0, −1) **c** none
d (0, −1)

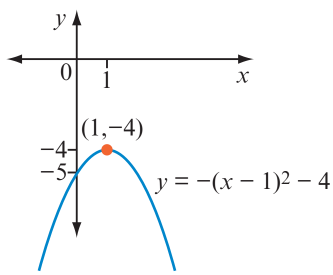
15 **a**



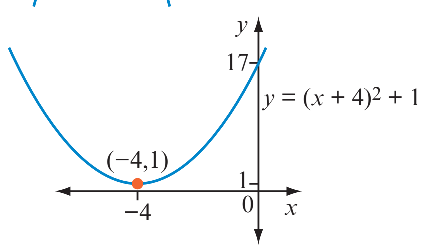
b



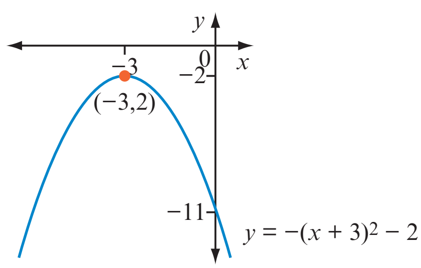
c



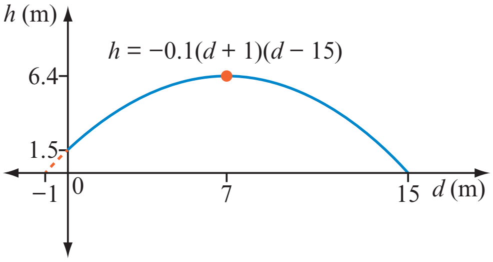
d



e

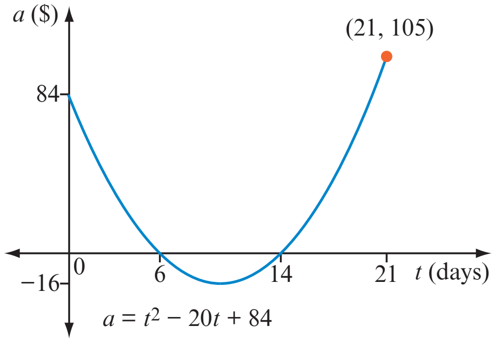


16 a



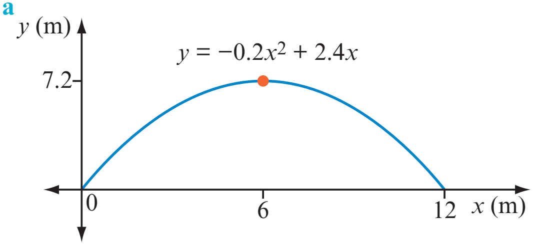
b 6.4 m **c** 1.5 m **d** 15 m

17 a



b \$84 **c** \$48 **d** just after sixth day
e \$16 **f** after 14th day
g \$105 (at end of the three weeks)

18 a



b 7.2 m **c** 6 m **d** 12 m

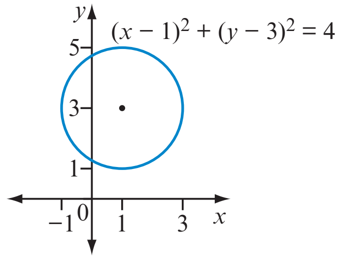
19, 20 Some possible answers are given.
a $y = (x - 7)(x - 2)$ or $y = -(x - 7)(x - 2)$ or $y = -6(x - 7)(x - 2)$
b $y = x^2 - 8x$ or $y = -x(x - 8)$ or $y = -4x(x - 8)$
c $y = (x + 4)(x - 5)$ or $y = -(x + 4)(x - 5)$ or $y = -3(x + 4)(x - 5)$

4F Circles and other non-linear relationships

4F Start thinking!

1 a i (0, 0) ii (0, 0) iii (0, 0)
b i 2 units ii 5 units iii 9 units
c Each has centre at (0, 0) and $x^2 + y^2$ written on left side of rule. Radius is equal to square root of number on right side of rule.

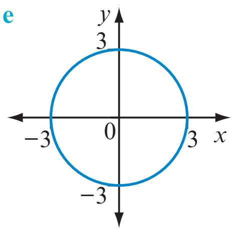
2 a i (1, 0) ii (0, 3)
b i translated 1 unit right
ii translated 3 units up
c Each circle has radius 2 units. Values that describe the translations help to determine coordinates of centre of circle.
d Graph of $x^2 + y^2 = 4$ translated 1 unit right and 3 units up.



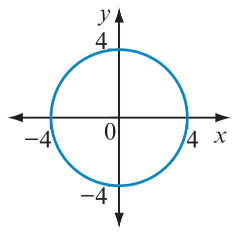
Exercise 4F Circles and other non-linear relationships

1 a centre (0, 0), radius = 6 units
b centre (0, 0), radius = 8 units
c centre (0, 0), radius = 1 unit
d centre (0, 0), radius = 10 units

2 a (0, 0) **b** 3 units
c centre and points on circle directly above, below, left and right of centre
d, e



3 $x^2 + y^2 = 16$



4 $x^2 + y^2 = 7^2$ or $x^2 + y^2 = 49$

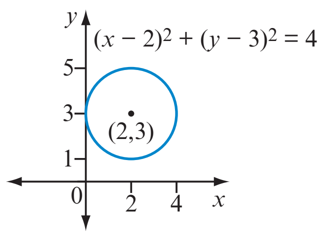
5 a $h = 2, k = 5, r = 3$
b (2, 5) **c** 3 units

6 i a $h = 1, k = 4, r = 4$
b (1, 4) **c** 4 units
ii a $h = 6, k = -3, r = 5$
b (6, −3) **c** 5 units
iii a $h = -4, k = 2, r = 1$
b (−4, 2) **c** 1 unit
iv a $h = -7, k = -2, r = 6$
b (−7, −2) **c** 6 units

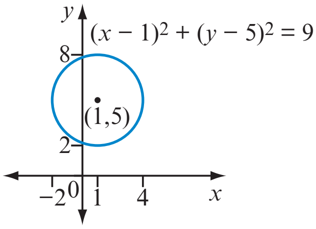
7 a (−2, 0) **b** 7 units
c Since $h = -2$ and $k = 0$, centre coordinates (h, k) are (−2, 0). Since $r = 7$, the radius is 7 units.
d translated 2 units left

8 a i (3, −1) ii (−4, −2)
b i 5 units ii 4 units
c i translated 3 units right and 1 unit down
ii translated 4 units left and 2 units down

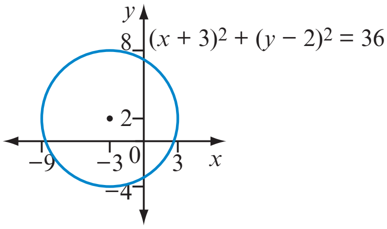
9 a centre (2, 3); radius = 2 units



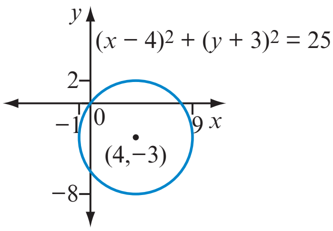
b centre (1, 5); radius = 3 units



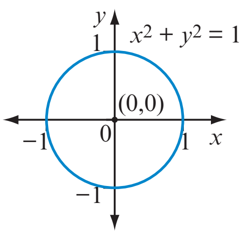
c centre (−3, 2); radius = 6 units



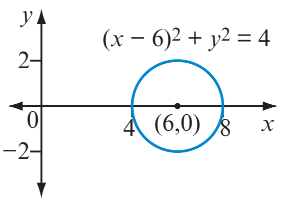
d centre (4, −3); radius = 5 units



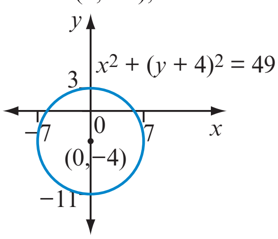
e centre (0, 0); radius = 1 units



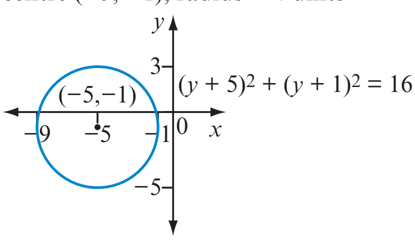
f centre (6, 0); radius = 2 units



g centre (0, −4); radius = 7 units



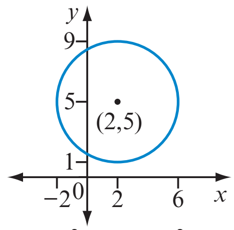
h centre (−5, −1); radius = 4 units



11 **a** $(x-3)^2+(y-5)^2=16$
b $(x+2)^2+(y-4)^2=25$
c $(x+7)^2+(y+6)^2=81$
d $(x-4)^2+(y+8)^2=121$

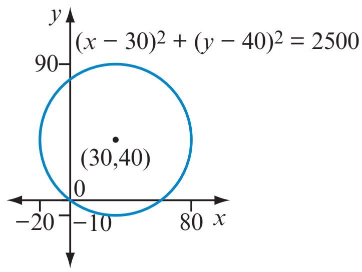
12 **a** centre (4, 2); radius = 2 units
$(x-4)^2+(y-2)^2=4$
b centre (−3, 3); radius = 1 units
$(x+3)^2+(y-3)^2=1$
c centre (−5, −2); radius = 3 units
$(x+5)^2+(y+2)^2=9$

13



$(x-2)^2+(y-5)^2=16$

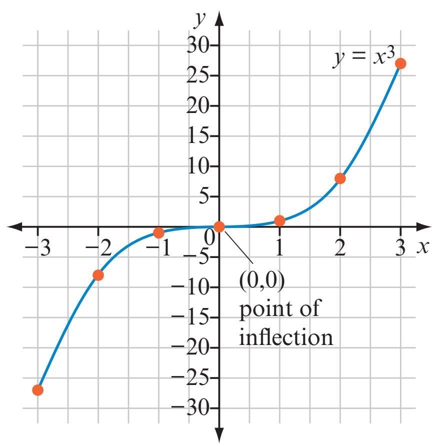
14 **a**



b 314 m **c** 7854 m^2

15 **a, c**

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

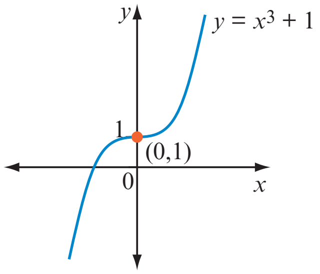


b As x increases, y increases. Graph appears to level out at $x = 0$ before rising more steeply again.

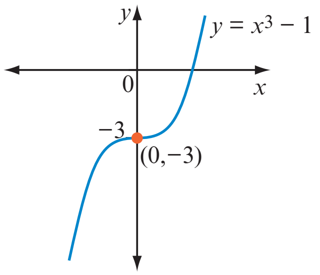
d **i** (0, 2) **ii** translate graph 2 units up

e **i** translate 1 unit up
ii translate 3 units down
iii translate 2 units right
iv translate 4 units left
v translate 1 unit right and 2 units up

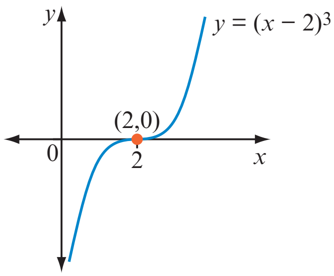
f **i**



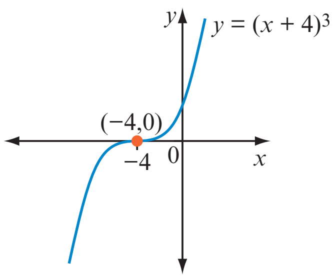
ii



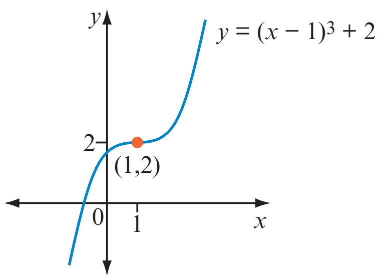
iii



iv

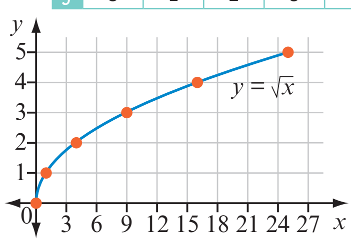


v



16 **a, c**

x	0	1	4	9	16	25
y	0	1	2	3	4	5

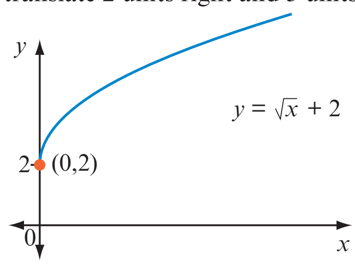


b No; not possible to take square root of negative number.

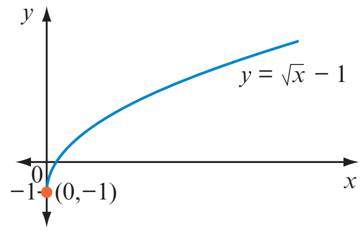
d **i** (3, 0) **ii** translate 3 units to right

e **i** translate 2 units up
ii translate 1 unit down
iii translate 1 unit right
iv translate 4 units left
v translate 2 units right and 3 units up

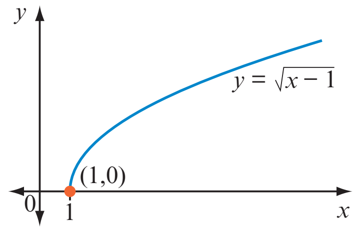
f **i**



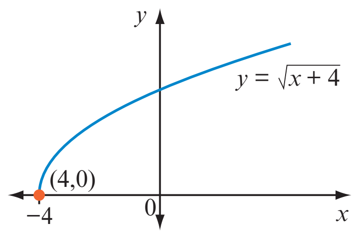
ii



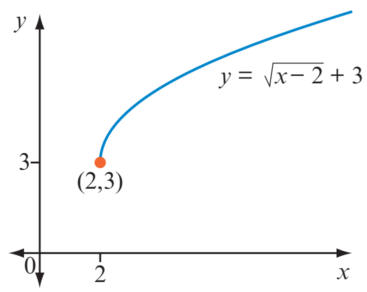
iii



iv

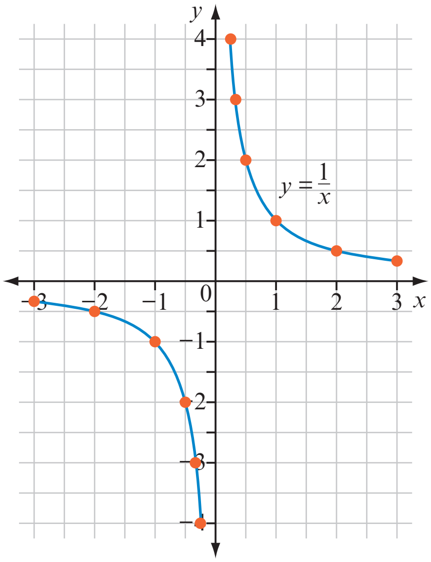


v



17 a

x	−3	−2	−1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	−1	−2	−3	−4	4	3	2	1	$\frac{1}{2}$	$\frac{1}{3}$



b y is undefined when $x = 0$. As x approaches 0 from left, graph approaches $-\infty$ (very small y values) and as x approaches 0 from right, graph approaches $+\infty$ (very large y values).

c **i** $y = 0$ **ii** $x = 0$

d **i** $y = 0$ (horizontal) and $x = 2$ (vertical)

ii translate 2 units right

e **i** translate horizontal asymptote 2 units up

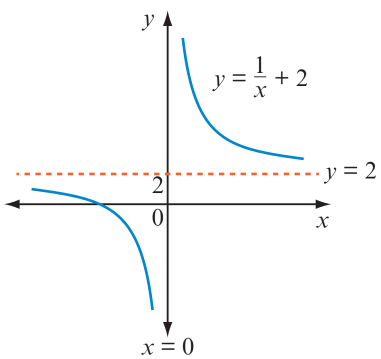
ii translate horizontal asymptote 3 units down

iii translate vertical asymptote 4 units right

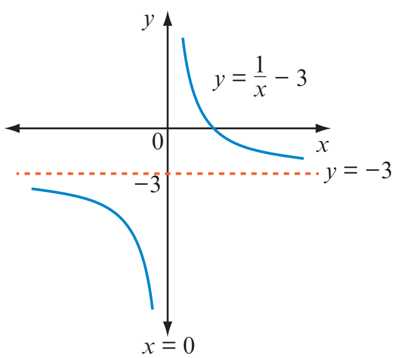
iv translate vertical asymptote 1 unit left

v translate vertical asymptote 3 units right and horizontal asymptote 2 units up

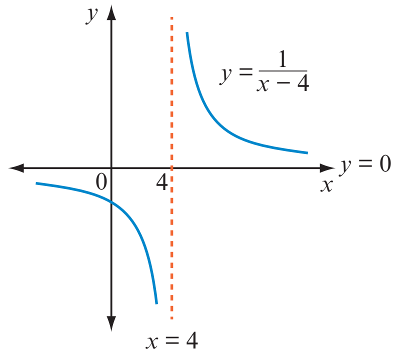
f **i**



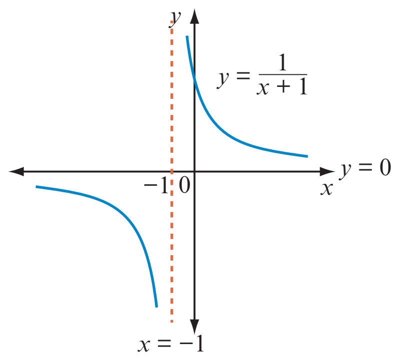
ii



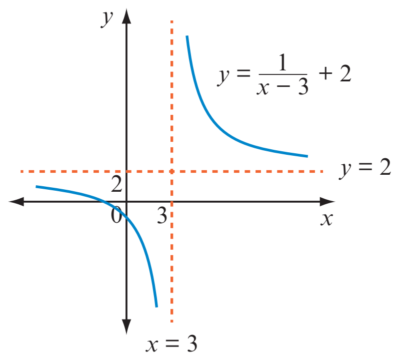
iii



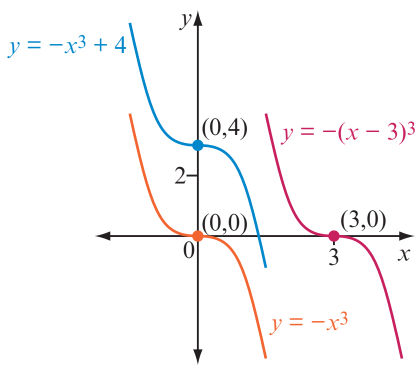
iv



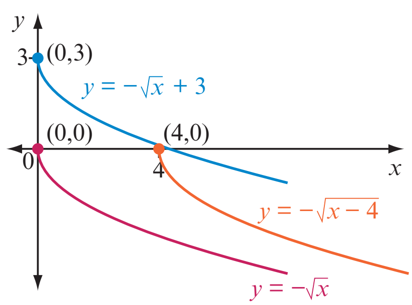
v



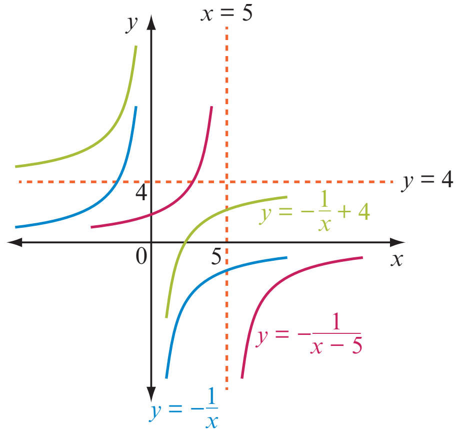
18 a



b



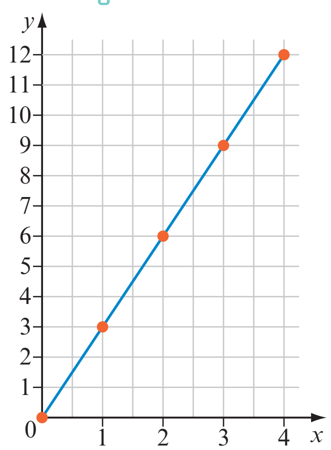
c



4G Relationships and direct proportion

4G Start thinking!

1 a



b linear

c Increase; as increase is by same amount each time, change is constant.

d $\frac{y}{x} = 3$

e Gradient is 3. Rate of change is same as gradient and is constant for a linear relationship.

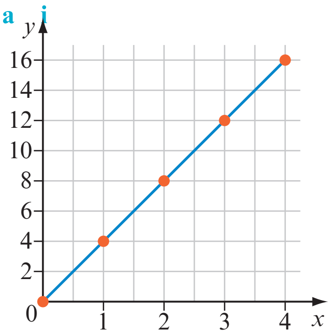
2 Graph is linear and passes through the origin (0, 0).

3 $y = 3x$

Exercise 4G Relationships and direct proportion

1 a and c show direct proportion

2 a i

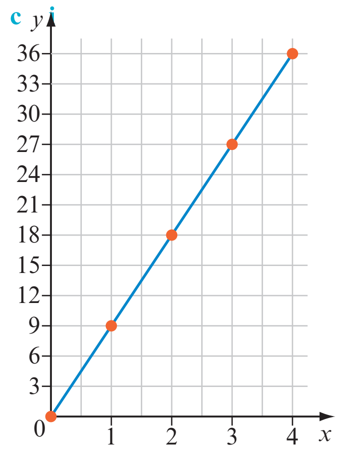


ii 4

iii They are the same.

iv $y = 4x$

c i



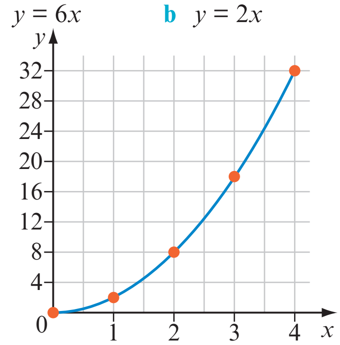
ii 9

iii They are the same.

iv $y = 9x$

3 a $y = 6x$ **b** $y = 2x$ **c** $y = \frac{1}{2}x$

4 a



b non-linear

c Increase; as increase is not by same amount each time, change is not constant.

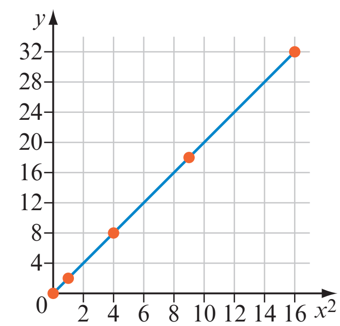
d 2, 4, 6, 8. Value for $\frac{y}{x}$ is different for each set of values.

e Not in direct proportion as $\frac{y}{x}$ is not constant.

f

x^2	0	1	4	9	16
y	0	2	8	18	32

g



h Yes, as it forms a straight line passing through (0, 0).

i–k $y = 2x^2$

5 a $y = 3x^2$ **b** $y = 7x^2$ **c** $y = 4x^2$

6 a no direct proportion (not a straight line passing through (0, 0))

b direct proportion (straight line passing through (0, 0))

c no direct proportion (straight line but does not pass through (0, 0))

d direct proportion (straight line passing through (0, 0))

e no direct proportion (not a straight line passing through (0, 0))

f no direct proportion (straight line but does not pass through (0, 0); also, y decreases as x increases)

7 a 4 **b** 3.5 **c** 6 **d** 10 **e** 2

8 a 10 **b** 8 **c** 4 **d** 7 **e** 44 **f** $\frac{1}{2}$

9 a 5 **b** 1.25 **c** 2 **d** 7 **e** 27 **f** 2.5

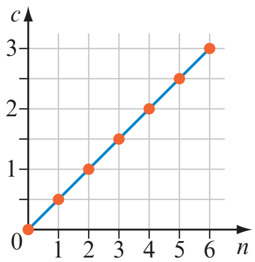
10 a $y = 5x$ **b** $p = 1.25q^2$ **c** $d = 2c^3$

d $h = 7\sqrt{g}$ **e** $a = \dfrac{27}{m}$ **f** $w = 2.5v$

11 a 3 **b** 4 **c** 8

12 a $y = 3x^2$ **b** $y = 4\sqrt{x}$ **c** $y = \dfrac{8}{x}$

13 a



b linear

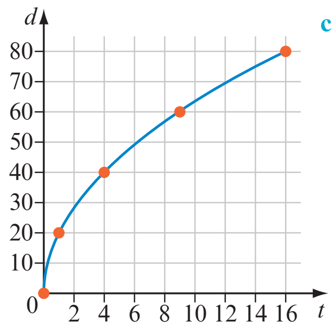
c yes, as forms a straight-line graph that passes through (0, 0)

d $c \propto n$; $c = kn$

e gradient $= \frac{1}{2}$, $c = \frac{1}{2}n$

f $10

14 a



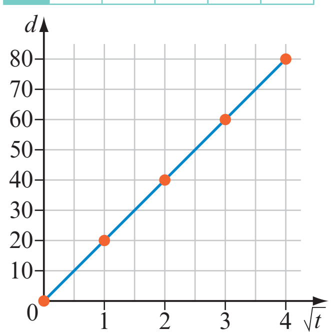
b non-linear

c no, as graph is not a straight line passing through (0, 0)

d

$\sqrt{t}$	0	1	2	3	4
d	0	20	40	60	80

e

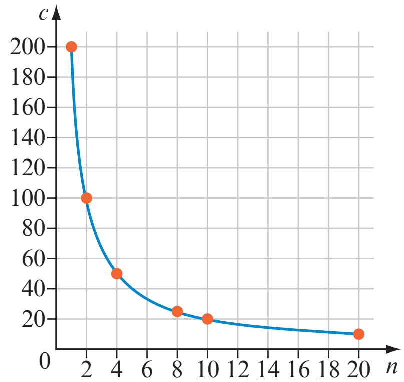


f Yes; it forms a straight-line graph that passes through (0, 0).

g $d \propto \sqrt{t}$; $d = k\sqrt{t}$

h gradient = 20, $d = 20\sqrt{t}$ **i** 120 km

15 a



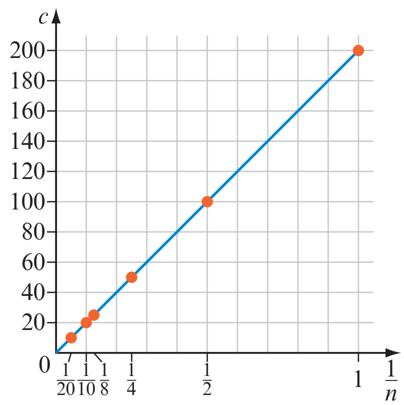
b non-linear

c No; graph is not a straight line passing through (0, 0).

d

$\frac{1}{n}$	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{20}$
c	200	100	50	25	20	10

e



f Yes; it forms a straight-line graph that passes through (0, 0).

g $c \propto \dfrac{1}{n}$; $c = \dfrac{k}{n}$

h gradient = 200, $c = \dfrac{200}{n}$

i $200 **j** $16.67

4 Chapter review

MULTIPLE-CHOICE

1 C **2** A **3** D **4** C **5** D **6** A
7 D **8** B **9** B **10** A **11** C

SHORT ANSWER

1 a $x = 2$ or $x = 3$ **b** $x = -6$ or $x = 5$

c No solutions; expression cannot be factorised.

d $x = 0$ or $x = 12$

2 a i upright **ii** minimum (0, −4)

iii x-intercepts: −1 and 1, y-intercept: −4

b i inverted **ii** maximum (−0.5, 1)

iii x-intercepts: −1 and 0, y-intercept: 0

c i upright **ii** minimum (2, 0)

iii x-intercept: 2, y-intercept: 4

3 a i upright **ii** narrower **iii** minimum

b i upright **ii** wider **iii** minimum

c i inverted **ii** narrower **iii** maximum

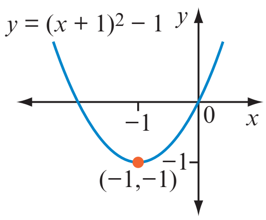
d i upright **ii** same width **iii** minimum

- **e** i inverted ii same width iii maximum
- **f** i inverted ii wider iii maximum

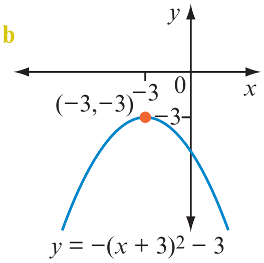
4 **a** Dilate by factor of 5.
b Dilate by factor of $\frac{1}{5}$.
c Dilate by factor of 5 and reflect in x-axis.
d Translate 5 units up.
e Reflect in x-axis and translate 5 units down.
f Dilate by factor of $\frac{1}{5}$ and reflect in x-axis.

5 **a** Inverted; dilate by factor of 5, reflect in x-axis and translate 2 units left and 1 unit down.
b Upright; dilate by factor of 5 and translate 4 units up.
c Upright; dilate by factor of $\frac{1}{4}$ and translate 5 units right.
d Inverted; dilate by factor of 3 and reflect in x-axis.

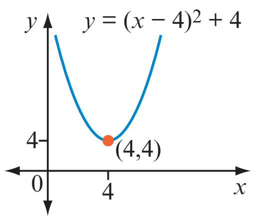
6 **a**



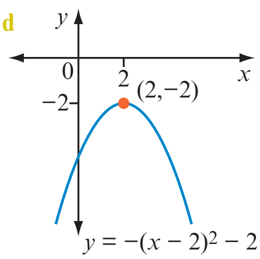
b



c

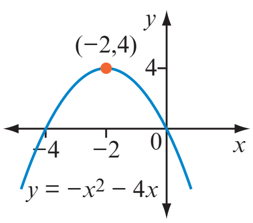


d

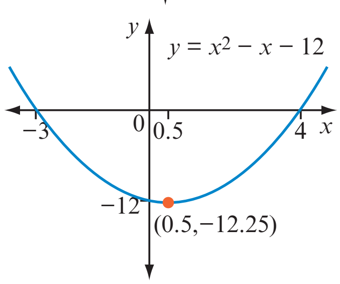


7 **a** x-intercepts: (−4, 0) and (0, 0); y-intercept: (0, 0)
b x-intercepts: (−3, 0) and (4, 0); y-intercept: (0, −12)
c x-intercepts: (−5, 0) and (4, 0); y-intercept: (0, −20)
d x-intercepts: (−2, 0) and (−1, 0); y-intercept: (0, −2)

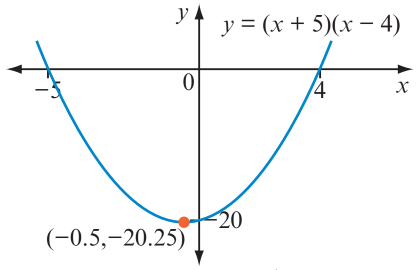
8 **a**



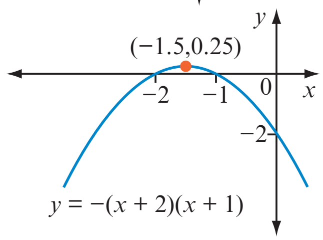
b



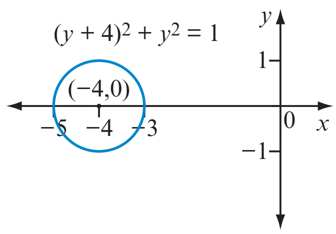
c



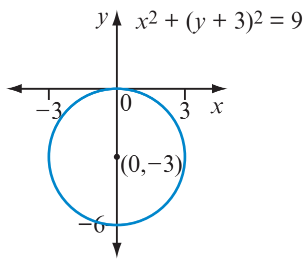
d



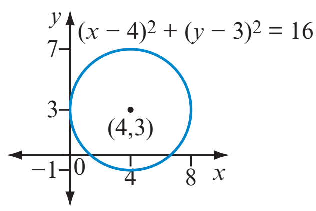
9 **a** centre (−4, 0); radius = 1 unit



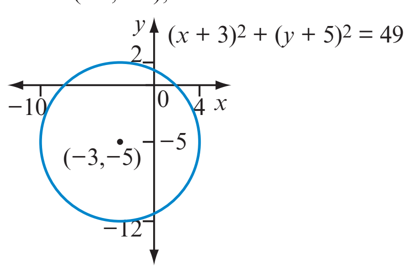
b centre (0, −3); radius = 3 units



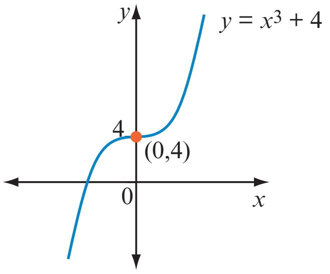
c centre (4, 3); radius = 4 units



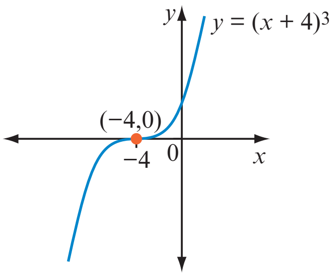
d centre (−3, −5); radius = 7 units



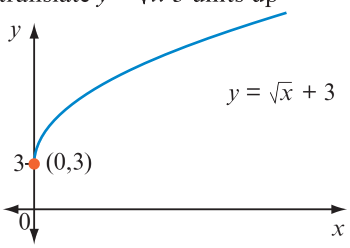
10 a translate $y = x^3$ 4 units up



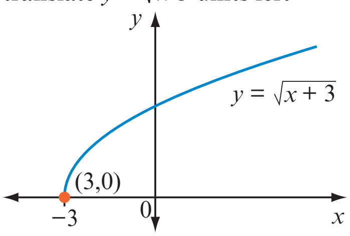
b translate $y = x^3$ 4 units left



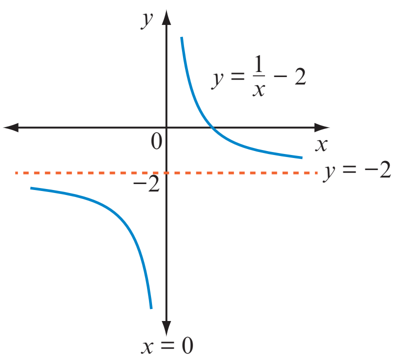
c translate $y = \sqrt{x}$ 3 units up



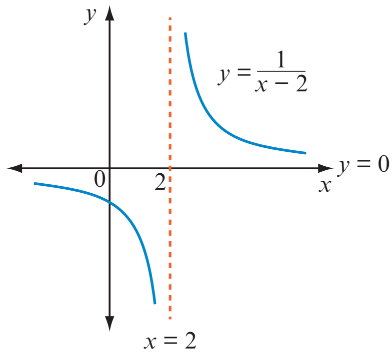
d translate $y = \sqrt{x}$ 3 units left



e translate $y = \frac{1}{x}$ 2 units down



f translate $y = \frac{1}{x}$ 2 units right



11 a 1 **b** 16 **c** $\frac{1}{2}$ **d** 4

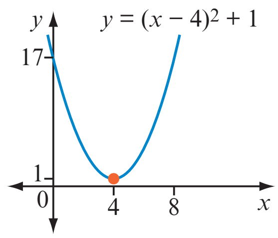
12 a $y = 5x$ **b** $y = \frac{1}{2}x^2$ **c** $y = 3x^3$ **d** $y = 3\sqrt{x}$

NAPLAN-STYLE PRACTICE

1 $x = 6$ or $x = -1$ **2** $x^2 - 6x - 16 = 0$
3 $x = 1$ or $x = 10$ **4** -4
5 $(-5, 0)$ and $(4, 0)$ **6** $x = -1$ **7** $(4, -1)$
8 vertical translation of 3 units
9 $y = -4x^2 - 5$ **10** $(0, -16)$
11 $(2, -4)$ **12** $y = (x + 3)^2 + 4$
13 1 **14** $y = -x^2 + 2x + 3$
15 $(x - 2)^2 + (y + 3)^2 = 25$
16 (0, 8), 2 units **17** 113 square units
18 $y = \sqrt{x}$ **19** 50 **20** 16 **21** 3
22 shows decreasing y values from left to right
23 $y = 4x^2$

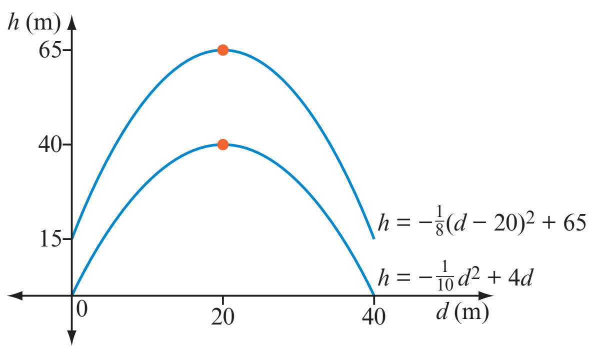
ANALYSIS

1 a



b (0, 17) and (8, 17) **c** 8 cm **d** 1 cm

2 a



b Both have same axis of symmetry ($d = 20$) and are inverted. Lower arch has maximum turning point at (20, 40) and upper arch has maximum turning point at (20, 65). Lower arch has wider shape than upper arch. End points of upper arch are 15 m above end points of lower arch.

c 15 m d 40 m
e i 40 m ii 65 m
f 25 m g 22.5 m
h The two arches are 15 m apart at the start. Approaching the highest points on both arches, the distance between the two arches increases until it is a maximum 25 m apart at the top of the arch. Approaching the far right side of the bridge, the distance between the two arches decreases until they are a distance of 15 m apart at the end of the bridge.

4 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 5 LINEAR RELATIONSHIPS

5 Are you ready?

1 90° 2 B 3 C 4 360°
5 a 8 b A
6 a E b D c C
7 B 8 D 9 15 cm 10 D
11 a 15 cm^2 b B

5A Angles and lines

5A Start thinking!

1 There are only two different sizes of angles (which are supplementary).
4 a Angles *a* and *d* are equal.
b Angles *f* and *g* are also equal, but are different from *a* and *d*.
c Vertically opposite angles are equal. Pairs: *a* and *d*, *b* and *c*, *e* and *h*, *f* and *g*
5 a Angles *d* and *e* are equal. They are on opposite sides of the transversal.
b Angles *c* and *f* are alternate angles. Alternate angles are equal.
6 a Angles *b* and *f* are equal. They are in corresponding places.
b Pairs: *a* and *e*, *c* and *g*, *d* and *h*. Corresponding angles are equal.
7 a Angles *c* and *e* add to 180°. They are both on the inside of the parallel lines.
b Angles *d* and *f* are co-interior. Co-interior angles add to 180°.

Exercise 5A Angles and lines

1 a i 67° ii 157° b i 43° ii 133°
c i N/A ii 4° d i 81° ii 171°
e i N/A ii 83° f i N/A ii 65°
g i 73° ii 163° h i N/A ii N/A
2 a $a = 27°$ b $b = 139°$ c $g = 19°$ d $h = 106°$
e $c = 282°$ f $d = 143°, e = 37°, f = 143°$
3 a $f = 123°$ b $e = 46°$ c $e = 37°$ d $c = 82°$
e $f = 138°$ f $c = 53°$
4 a *f* b *c* c *d* d *a* e *d* f *e*
g *d* h *f*
5 a $b = 102°$ b $c = 102°$ c $d = 78°$
d $e = 78°$ e $f = 102°$ f $g = 102°$
6 a $a = 60°, b = 120°, c = 30°$
b $a = 165°, b = 15°, c = 75°$
c $a = 115°, b = 65°, c = 90°, d = 25°$
d $a = 60°, b = 25°, d = 30°, e = 85°, f = 65°$
e $a = 53°, b = 37°, c = 43°$
f $a = 71°, b = 63°, c = 19°, d = 90°, e = 8°, f = 82°$
7 a $a = 48°, b = 71°, c = 61°, d = 132°$
b $a = 37°, b = 57°, c = 57°, d = 123°$
c $a = 117°, b = 79°, c = 101°, d = 63°$
d $a = 68°, b = 22°, c = 49°, d = 41°$
e $a = 121°, b = 59°, c = 59°, d = 121°$
f $a = 146°, b = 67°, c = 34°, d = 67°, e = 79°, f = 34°, g = 146°$
8 a Divide 360° by the number of blades in each windmill (18 in the first, 3 in the second).
b 20° and 120°
9 a i Find angle *b* first: 180° − 145° = 35°, *c* is vertically opposite *b*, so *c* = 35°, *a* = 180° − 90° − 35° = 55°.
ii Find angle *c* first: 180° − 145° = 35°, *a* is supplementary to angle *c* and the right angle, so is equal to 180° − 90° − 35° = 55°; *b* is vertically opposite *c*, so *b* = 35°.
c Complementary angles add to 90°. Supplementary angles add to 180°. A right angle measures 90° exactly. A straight angle measures 180° exactly. Angles around a point add to 360°. Vertically opposite angles are equal.
10 There are only two different-sized (and supplementary) angles formed in this scenario that can be found using angle facts.
11 Check that, when the two lines are cut by a transversal, vertically opposite, corresponding, and alternate angles are equal. Also check that co-interior angles add to 180°.
12 a

	56°	78°	19°	146°	27°	103°
Complementary	34°	12°	71°		63°	
Supplementary	124°	102°	161°	34°	153°	77°
Vertically opposite	56°	78°	19°	146°	27°	103°
Co-interior	124°	102°	161°	34°	153°	77°
Alternate	56°	78°	19°	146°	27°	103°
Corresponding	56°	78°	19°	146°	27°	103°
Around a point	304°	282°	341°	214°	333°	257°

b It is not possible to make complementary angles where one angle is larger than 90°.
c Vertically opposite, alternate and corresponding angles are equal. Co-interior angles add to 180°. Complementary angles add to 90°. Supplementary angles add to 180°. A right angle measures 90° exactly. A straight angle measures 180° exactly. Angles around a point add to 360°.

13 a i 32° ii 116° iii 58°
b Angles a and b are equal, as are the angles between the light rays and the mirror.
c i 54° ii 36°

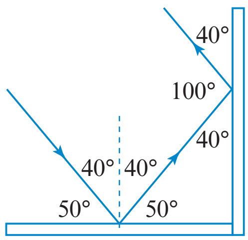
14 a 60 seconds b 60 minutes
c i 30 minutes ii 15 minutes iii 20 minutes
d greater than 14.5°
e i eighty-three degrees, sixteen minutes and fifty-five seconds
ii twenty-seven degrees, forty-three minutes and four seconds
iii one hundred and fifty-four degrees, nine minutes and thirty-seven seconds
f i 230°29′13″ ii 67°18′2″
iii 192°56′42″

15 a i 86°45′ ii 113°48′ iii 217°6′ iv 9°39′
b i 196.7° ii 98.85° iii 23.55°
iv 107.08°

16 a 180°
b i 90° ii 270° iii 150° iv 330°
v 240°
c 30°
d i 15° ii 7.5° iii 22.5°
e i 45° ii 127.5° iii 22.5° iv 52.5°
v 97.5°

17 a 72.51° b 46°23′51″ c 89.61°
d 34°9′27″

18 a 192.5° b 32.5° c 153.5° d 4° e 96°

19


20


5B Angles and polygons

5B Start thinking!

1 triangle (three sides) 2 180°

3 Acute-angled scalene triangle: no equal sides or angles, all angles less than 90°
Obtuse-angled scalene triangle: no equal sides or angles, one angle greater than 90° but less than 180°
Right-angled scalene triangle: no equal sides or angles, one angle equal to 90°
Acute-angled isosceles triangle: two equal sides and two equal angles, all angles less than 90°
Obtuse-angled isosceles triangle: two equal sides and two equal angles, one angle greater than 90° but less than 180°
Right-angled isosceles triangle: two equal sides and two equal angles, one angle equal to 90°
Equilateral: three equal sides and three equal angles (60°)

4 quadrilateral (four sides)

5 360° 6 180° + 180° = 360°

7 five sides = pentagon; six sides = hexagon; seven sides = heptagon; eight sides = octagon; nine sides = nonagon; ten sides = decagon

8 a seven sides b five triangles
c 2 d 900°
e Diagrams will vary. Internal angle sum for all heptagons = 900°

Exercise 5B Angles and polygons

1 a 69° b 108° c 53° d 74°
e 89° f 39°

2 a 103° b 109° c 89° d 88°
e 253° f 176°

3 a 76° b 73° c 246° d 46°
e 92° f 105°

4 A circle is not a polygon, as it has curved sides. A polygon has only straight sides.

5 a 58° b 68.5° c 111.5°

6 a 540°
b A regular pentagon has all sides and angles equal. An irregular pentagon can have angles and sides that are different.
c Because all angles in a regular shape are equal, 540° ÷ 5 = 108°.
d First find the number of degrees in the entire shape using the formula $180(n-2)$. Then, divide this by the number of sides to find the size of an individual angle. This only works for regular polygons.
e hexagon, interior angle sum = 720°, each angle = 120°

7 a 144° b 135° c 150° d 162°

8 30 sides

9 No. Rearranging the formula to find a regular polygon with interior angles of 145° would mean the shape has 10.28 sides, which is not possible.

10 a $a = 71°$, $b = 38°$
b $c = 123°$, $d = 57°$, $e = 123°$
c $f = 122°$, $g = 77°$
d $h = 268°$, $i = 32°$, $j = 32°$
e $k = 120°$, $l = 60°$
f $m = 120°$, $n = 310°$, $o = 55°$

11 $a = 39.5°$, $b = 50.5°$, $c = 39.5°$, $d = 50.5°$, $e = 90°$, $f = 90°$

12 a i $a = 121°$, $b = 45°$ ii $c = 63°$, $d = 66°$
iii $e = 54°$, $f = 126°$

b Supplementary angles add to 180°, so an interior and its exterior angle add to 180°. The sum of that interior angle and the other two opposite interior angles is also 180°, so the exterior angle must be equal to the sum of the opposite interior angles.
d 360°
e sum of exterior angles of a triangle = 360°

13 b 360°
c sum of exterior angles of a quadrilateral = 360°

14 a **i** 360° **ii** 360° **iii** 360° **iv** 360°
b sum of exterior angles of a polygon = 360°

15 single exterior angle of a polygon = 360° ÷ n, where n = number of sides

16 The star has 12 sides so has an internal angle sum of 1800°. Although it is not regular, it has six equal pairs of angles (an acute and reflex angle) because all side lengths are the same. These angle pairs sum to 300° (1800° ÷ 6). By subtracting 268° from 300° Logan can find the size of the acute angle.

17 36°

5C Transformations

5B Start thinking!

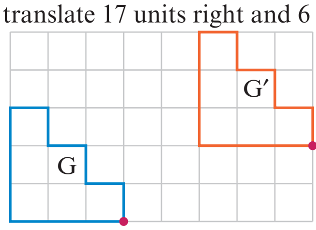
6 Translations should always be described from the same vertex. Alex should have described the translations from corner A to corner A′.

Exercise 5C Transformations

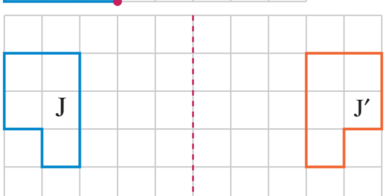
1 a rotation **b** translation **c** reflection
d dilation (reduction) **e** reflection
f translation **g** reflection **h** rotation

2 a rotate 90° clockwise around point 1 unit up from top right vertex of C
b reflect in vertical mirror line 6 units left of F
c rotate 90° clockwise around point 1 unit right from bottom right vertex
d translate 10 units right and 4 units up
e translate 4 units left and 6 units down
f enlarged by a factor of 2
g reflect in horizontal mirror line half a unit over J
h translate 17 units right and 6 units up

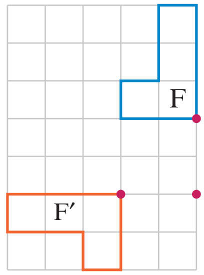
3 a



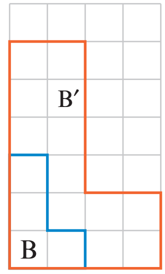
b



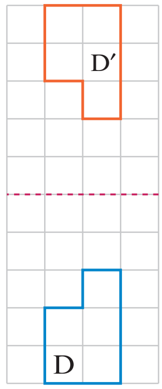
c



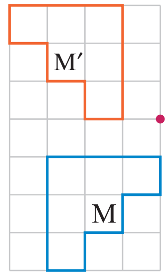
d



e



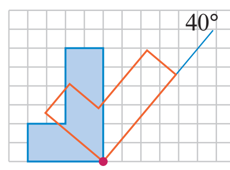
f



5 Rook (castle) – moves any number of squares in any horizontal or vertical direction; e.g. 5 units up; 3 units left etc.
Bishop – moves any number of squares diagonally; e.g. 3 units left and 3 units up; 6 units right and 6 units down etc.
Queen – moves any number of squares in any direction; e.g. 6 units left; 2 units down; 4 units left and 4 units up etc.
King – moves only one square at a time in any direction; e.g. 1 unit left; 1 unit down; 1 unit right and 1 unit up etc.
Knight – moves in an L shape only; e.g. 2 units left and 1 unit down; 1 unit right and 2 units up etc.
Pawn – Initially it may move 2 units up, then only 1 unit up, except when attacking it can move diagonally: 1 unit left and 1 unit up or 1 unit right and 1 unit up. It may never move down.

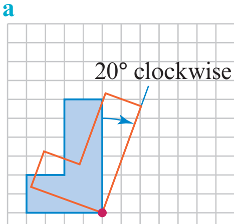
6 no difference

7 a 40° clockwise
b

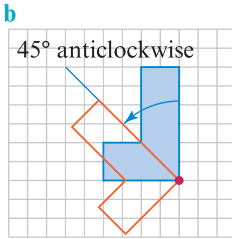


c the shape will tip the other way
d to measure angles accurately

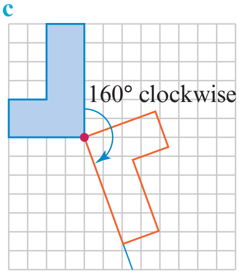
8 a



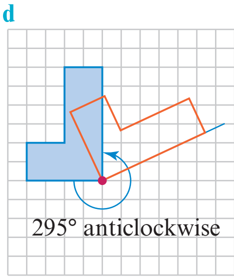
b



c



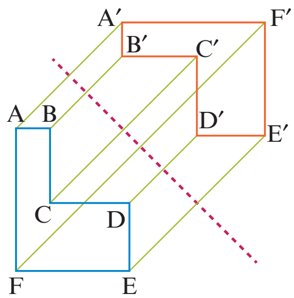
d



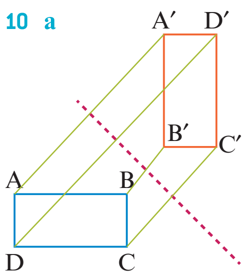
9 a Mirror is placed diagonally on the shape's top right.

b six vertices

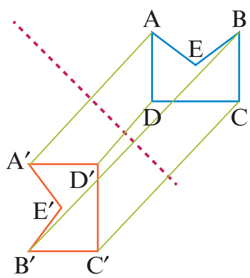
c, d



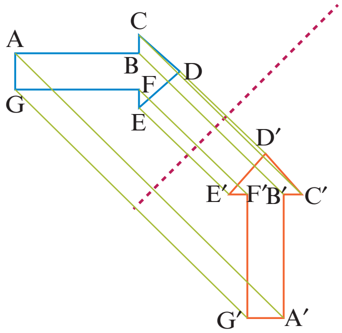
10 a



b



c



11 a enlargement **b** reduction

c 3 times **d i** 2 **ii** 6

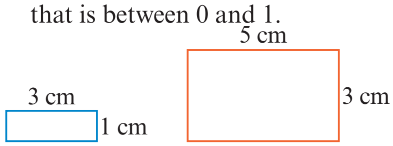
e halving **f** dividing into thirds

g dividing by 3 is the same as multiplying by $\frac{1}{3}$

h i $\frac{1}{2}$ **ii** $\frac{1}{6}$

i Enlargements have a scale factor which is greater than 1. Reductions have a scale factor that is between 0 and 1.

12



Scale factor length = 1.67; scale factor width = 3; scale factor must be the same for all sides.

13 12 more

14 Tessellations are patterns constructed from repeated congruent shapes, with no gaps and no overlapping. Shapes are rotated, reflected and translated to produce the patterns.

15 A kaleidoscope is a tube of mirrors. Viewer looks in one end and light enters the other, reflecting off mirrors. Different images are produced by reflections on mirrors at different angles and reflections bouncing off more than one mirror.

17 Four moves (if you don't just perform the same translation in reverse).

18 Mirror will only reflect in one direction so shape of reflection produced relies heavily on placement of mirror – it could reflect only a very small part of shape or almost all of it.

19 Position of rotational point affects position of image. The further to the edge a rotational point is, the more the image will seem to 'swing' in its rotation.

5D Congruent figures

5D Start thinking!

2 If side lengths are the same, angles will be also.

3 You can draw triangles that have the same angles but different size, so they are not congruent.

4 SAS is a condition for congruence because if the angle between the two pairs of equal corresponding sides is the same, it means the third pair of corresponding sides must be the same length. AAS is a condition for congruence because the two side lengths that are not given must be drawn at the angles given, and so will meet at the same angle in both triangles (180° − the other two angles). This means that these pairs of corresponding sides must be equal.

5 In a right-angled triangle, when you know the length of two sides you can calculate the other using Pythagoras' Theorem because of the right angle. In other triangles, two sides can be known but the third cannot be calculated because of the varying angles.

6 SSS – three pairs of sides are equal.
AAS – two pairs of angles are equal and a corresponding pair of sides are equal in length.
SAS – two pairs of sides are equal in length and the pair of angles in between are equal in size.
RHS – in a right-angled triangle, the hypotenuses and one other pair of sides are equal in length.

Exercise 5D Congruent figures

1 congruent figures: A and K, C and R, G and L, J and P

2 a AAS **b** SAS **c** RHS **d** SSS

3 a congruent (AAS) **b** congruent (AAS)

c congruent (AAS) **d** not congruent

4 a congruent (AAS)

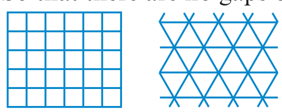
b not congruent (SAS)

c not congruent (RHS) **d** congruent (SSS)

5 a key must be identical in size and shape to fit the lock
b Some possible answers are: braces in building construction, pins on an electric cord, computer cables.

6 Cut out shapes and place on top of one another to see if they are identical, or measure their side lengths and angles to see if all pairs are identical.

7 Two shapes can have the same angles but be different in size.

8 a So that there are no gaps or overlapping.
b 
c So that shapes fit together, with no gaps or overlaps.

9 Identical twins would need to be identical in every aspect of size and shape to be considered congruent. This is not usually the case because, despite having identical DNA, environmental factors can make them slightly different.

10 a triangles
b These are congruent; they will be identical halves of the square.

11 a congruent, AAS b need more information
c not congruent d congruent, SAS

12 a $a = 23°$, $b = 12$ mm, $c = 67°$, $d = 23°$, $e = 13$ mm, $f = 5$ mm
b $a = 51°$, $b = 11$ cm, $c = 70°$, $d = 59°$, $e = 10$ cm
c $a = 38°$, $b = 50°$, $c = 13$ cm, $d = 38°$, $e = 92°$, $f = 8$ cm, $g = 10$ cm
d $a = 10$ cm, $b = 17$ cm, $c = 116°$, $d = 32°$, $e = 32°$, $f = 10$ cm, $g = 10$ cm, $h = 32°$, $i = 32°$

13 b If two angles are equal, third will also be equal because all triangles have interior angle sum 180°.
c Side length given will now be between two angles, giving congruence condition ASA. Note: only works if side lengths are corresponding.
d Yes, ASA is a special case of AAS.

14 c 4

15 a 16 m^2 b 12 m^2
c Use four congruent L-shapes each with an area of 3 m^2 (to give the total of 12 m^2).

16 a, b Corresponding sides may be equal, but angles are not.
c Corresponding angles may be equal, but sides are not.
e Because you can have shapes that have corresponding equal sides but not corresponding equal angles and vice versa.

5E Dilations and scale factor

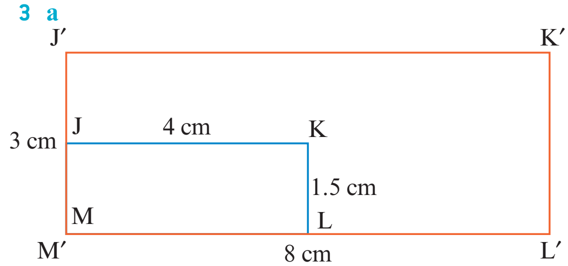
5E Start thinking!

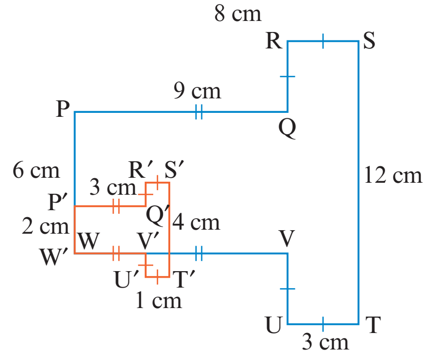
1 a 2 cm b 4 cm
2 2
3 Create image from original by multiplying by scale factor.
4 A′ is two times as large as A.
5 A is two times as small as A′.
6 $\frac{1}{2}$
7 a B is dilated by 4 to produce B′; B′ is dilated by $\frac{1}{4}$ to produce B.
b C is dilated by $\frac{1}{3}$ to produce C′; C′ is dilated by 3 to produce C.
c D is dilated by 9 to produce D′, D′ is dilated by $\frac{1}{9}$ to produce D.
d E is dilated by $\frac{1}{8}$ to produce E′; E′ is dilated by 8 to produce E.

Exercise 5E Dilations and scale factor

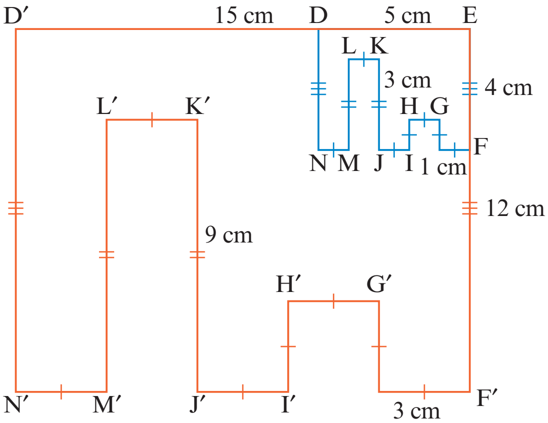
1 a enlargement b reduction c enlargement
d enlargement e reduction f reduction

2 a A has been dilated (enlarged) by 2 to produce A′.
b B has been dilated (reduced) by $\frac{1}{4}$ to produce B′.
c C has been dilated (reduced) by $\frac{1}{3}$ to produce C′.
d D has been dilated (enlarged) by 5 to produce D′.
e E has been dilated (reduced) by $\frac{1}{5}$ to produce E′.
f F has been dilated (enlarged) by 2 to produce F′.

3 a 

b 

c



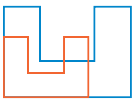
4 a Size of plane reduced by scale factor 100.

b 34 cm **c** 27 m

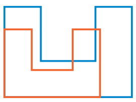
d To find length of model you multiply by scale factor; to find length of real-life plane you divide by scale factor.

5 Image is same size as original.

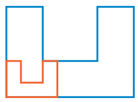
6 a



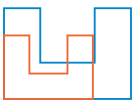
b



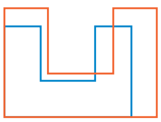
c



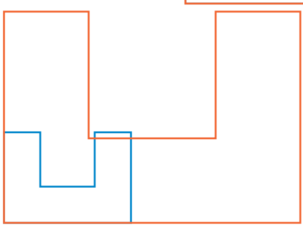
d



e



f

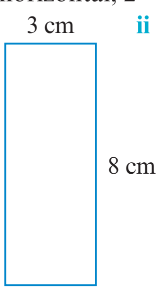


7 because $\frac{2}{3}$ is same as $2 \times \frac{1}{3}$

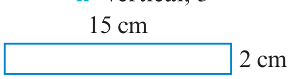
8 220 **9** 0.0026 mm

10 a i horizontal, 2 **ii** vertical, 3

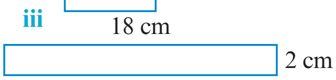
b i



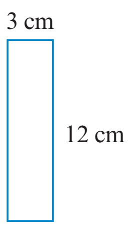
ii



iii



iv



11 a O

b OA = 1.0 cm, OB = 1.1 cm, OC = 1.0 cm, OD = 1.4 cm, OE = 1.3 cm, OF = 0.6 cm

c 2

d

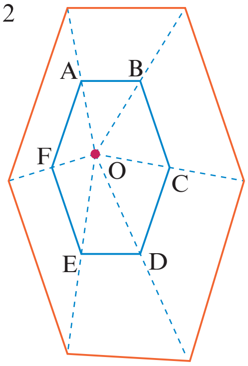


Figure E

e Erase an equal length from each dotted line then join ends to form smaller image.

f

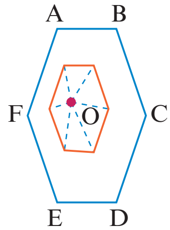


Figure D

g O

h left

i right

j

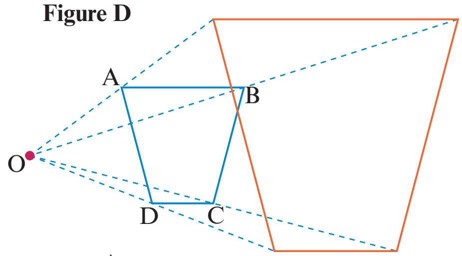


Figure F

k

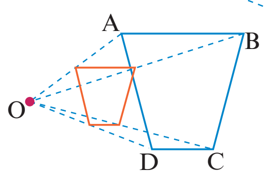
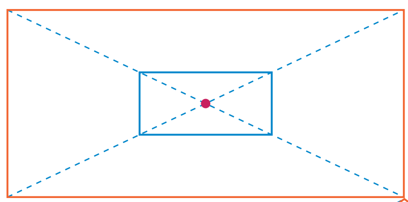
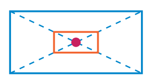


Figure F

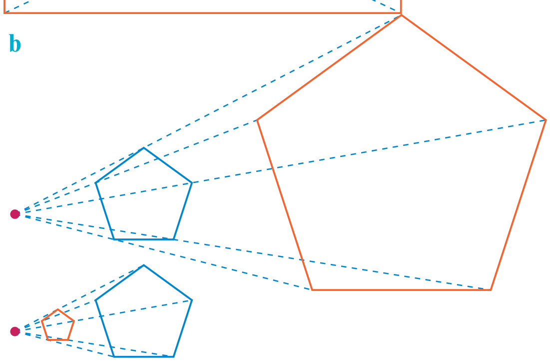
l Both use dotted lines, but figure F has centre of dilation outside shape.

12 a

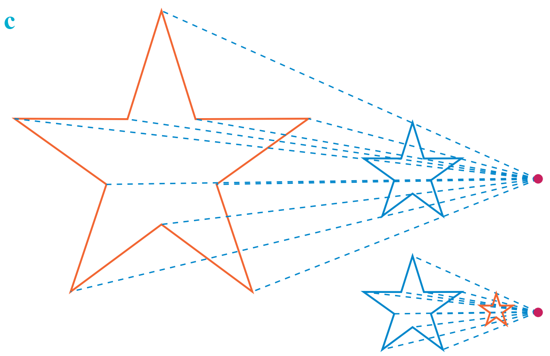




b

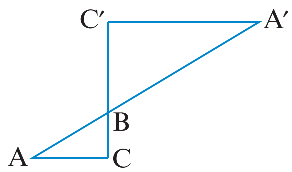


c

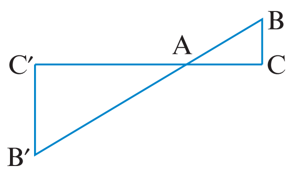


13 a The image is upside down and back to front.
b C
c Position of image is different.

Vertex B:

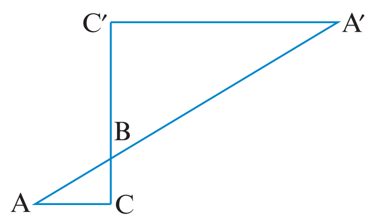


Vertex A:

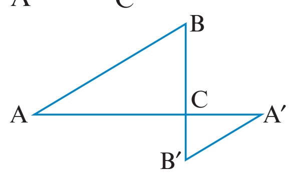


d Some possible answers are:

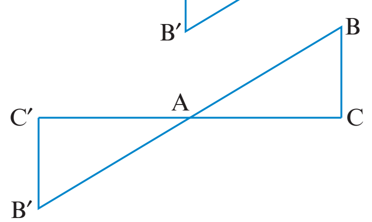
i



ii



iii



14 a $\frac{1}{200}$ b $\frac{2}{375}$ c $\frac{15}{16}$ d $\frac{16}{15}$
e Numerators and denominators are swapped because in part c the plane is the original, but in part d the case is the original.

5F Similar figures

5F Start thinking!

1 yes
2 No, because FG and BC would not be in same ratio as all other pairs of corresponding sides.
3 10 cm
4 EF is twice as big as AB, so FG must be twice as big as BC; $5 \times 2 = 10$ cm.
5 All are different arrangements of same equation.
6 image length = original length × scale factor
7 Check all corresponding pairs of sides are in same ratio.
8 To make sure that they are mathematically similar and do not just look similar.

Exercise 5F Similar figures

1 a 3 b 2 c $\frac{1}{4}$ d $\frac{1}{3}$
2 a similar b not similar c not similar
d similar e not similar f similar
3 a $x = 2$ cm b $x = 6$ cm c $x = 9$ cm
d $x = 20$ cm e $x = 10.5$ cm f $x = 5$ cm
g $x = 11$ cm g $x = 6$ cm
4 Congruent figures are exactly same size and shape. Similar figures are same shape, but not necessarily same size.
5 a not necessarily; they may be a different size
b Depends on definitions used; most commonly similarity is defined to include congruence.
6 a yes, similar b yes, similar
c not similar d not similar
7 Because angles also need to be equal in size, angles are different in a square and a rhombus.
8 not similar; not all aspects reduced in same ratio
9 No, a baby is a very different shape from an adult, with the head, body and limbs in a different ratio from those of an adult.
10 a vertical scale factor = 4,
horizontal scale factor = $3\frac{1}{3}$
b no
c enlarged four times, then cropped to fit
11 20 cm × 30 cm is similar to 10 cm × 15 cm (scale factor = 2)
12 a $a = 12$ cm, $b = 99°$, $c = 81°$, $d = 99°$
b $a = 1$ cm, $b = 150°$, $c = 75°$, $d = 9$ cm, $e = 12$ cm, $f = 45°$, $g = 150°$
c $a = 85°$, $b = 115°$, $c = 18$ cm, $d = 115°$, $e = 85°$, $f = 115°$, $g = 45°$
d $a = 72°$, $b = 108°$, $c = 4$ cm, $d = 108°$, $e = 108°$, $f = 72°$, $g = 12.5$ cm
13 a Regular hexagon has all side lengths equal and all angles equal (120°), irregular hexagon does not.
b Because all angles will be same (they are for every regular hexagon) and all side lengths must be equal, any regular hexagon is similar to any other regular hexagon.
14 a 1:2
b First part of ratio is numerator and second is denominator.
c $\frac{2}{1}$ d 2:1
e i $\frac{1}{4}$ ii $\frac{1}{100}$ iii $\frac{1}{2500}$ iv $\frac{1}{150\,000}$ v 2 vi 100
f If first part of ratio is smaller than second part, it is a reduction. If first part of ratio is larger than second part, it is an enlargement.
15 a $\frac{1}{40}$ b 4 m c 3.75 cm
d if all proportions are in same ratio, yes; however, would be very difficult to manufacture a model car with every aspect in same ratio

16 **a** $\frac{1}{75\,000}$ **b** 20 cm **c** 9750 m

17 **a** Looking at all paper sizes (from A0 to A8), the sizes are almost all similar (scale factor 0.71), though rounding limits exact dilations to even numbers and odd numbers.
b similar relationship in B sized paper
c similar relationship in C sized paper
d B series covers sizes which are not found in the A series. The C series envelopes are made to fit A-sized paper perfectly.

5G Similar triangles

5G Start thinking!

1 **a** All sides are equal.
b All three pairs of corresponding sides are in the same ratio.

2 **a** Two sides and the angle between them are equal. For two sides of known length and a known angle between, the third side can only be one size. The third pair of sides will correspond and be equal, giving three pairs of equal corresponding sides (SSS).
b Two pairs of corresponding sides are in the same ratio and the pair of angles in between them are equal.

3 **a** RHS means that two right-angled triangles have equal hypotenuses, and also one other pair of equal corresponding sides. Because you can use Pythagoras' Theorem to calculate the length of the third side (which can only be one size), it means that again you have three pairs of corresponding equal sides (SSS).
b In two right-angled triangles, the hypotenuses are in same ratio as another pair of corresponding sides.

4 **a** AAA means that three pairs of equal angles.
b If two triangles have corresponding equal angles also need to check side lengths. If side lengths are equal then triangle is congruent; if not equal, then triangle is similar.
c yes

5 **a** Because only one pair of sides is given.
b If two angles are equal, third will also be equal, giving similarity condition AAA.

6 In congruent triangles, sides must be same length. In similar triangles, sides must have increased or decreased in same ratio.

Exercise 5G Similar triangles

1 **a** $a = 9$ cm, $b = 4$ cm **b** $c = 44$ cm, $d = 7$ cm
c $e = 20$ cm, $f = 7.5$ cm
d $m = 19$ cm, $n = 45$ cm
e $x = 12$ cm, $y = 6.5$ cm
f $x = 52$ cm, $y = 20$ cm

2 **a** SAS **b** SSS **c** RHS
d AAA **e** SAS **f** AAA

3 **a** similar **b** similar **c** not similar
d similar **e** similar **f** not similar

4 **a** not similar **b** similar **c** similar
d similar **e** not similar **f** similar

5 All equilateral triangles have three equal angles (60°), meeting similarity condition AAA.

6 Because third angle will be equal if other two angles are equal.

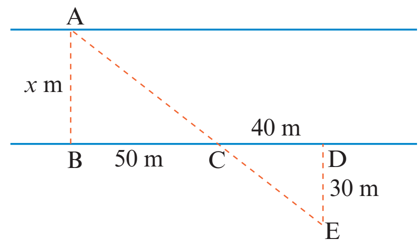
7 **a** Using knowledge of alternate angles within parallel lines (∠ABC = ∠DEC and ∠BAC = ∠EDC) and vertically opposite angles (∠ACB = ∠DCE), these triangles have three corresponding equal pairs of angles and hence are similar.
b scale factor = 3 **c** $a = 2.5$ cm, $b = 6$ cm

8 **a** similar **b** not similar **c** similar
d not similar

9 $x = 6.54$ m

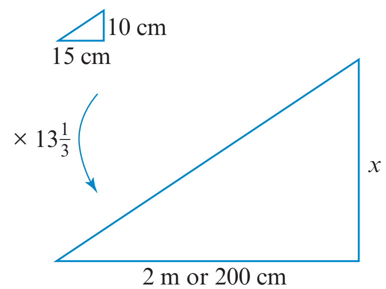
10 **a** 2.5 m **b** 2.72 m tall

11 $x = 37.5$ m

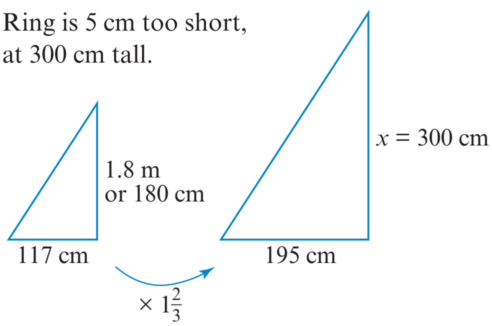


12 4 cm

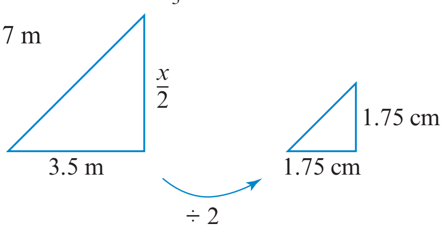
13 **a** 133.33 cm



b Ring is 5 cm too short, at 300 cm tall.



c 7 m



14 Sean's house is 3.1 m tall; Tania's house is 2.7 m tall. Sean's house is taller.

15 You need to have the second similar triangle to compare house heights and shadows because measurements were taken at different times of day so length of shadows would change.

17 Some possible answers include: compare a square and a rhombus to see that having all corresponding sides in the same ratio does not make two figures similar; compare a square and a rectangle to see that all corresponding angles being equal does not make two figures similar; draw any two quadrilaterals that have two corresponding pairs of sides in the same ratio and an angle in between the same size to show that it does not make two figures similar; quadrilaterals do not have an hypotenuse so they can't meet RHS.

5H Scale factor and area

5H Start thinking!

1 **i** length = 3 cm, width = 2 cm
ii length = 6 cm, width = 4 cm
2 length scale factor = 2
3 **i** 6 cm^2 **ii** 24 cm^2 **4** 4
5 Area of larger figure is four times as big.
6 4 **7** 9, 16, 25
8 area scale factor = (length scale factor)^2
9 Both are squared.
10 **a** $6^2 = 36$ **b** $7^2 = 49$ **c** $8^2 = 64$
d $10^2 = 100$
11 If figure is enlarged by length scale factor 4, area of image is 4^2 (16) times the size of the area of original.

Exercise 5H Scale factor and area

1 **a** $2^2 = 4$ **b** $3^2 = 9$ **c** $4^2 = 16$
d $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ **e** $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ **f** $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$
2 **a** **i** 6 **ii** 24
b When the side lengths are doubled, the area is quadrupled. Area scale factor is $2^2 = 4$.
3 **a** 48 cm^2 **b** 45 cm^2 **c** 250 cm^2
d 10 cm^2 **e** 3 cm^2 **f** 4 cm^2
4 **a** area of original shape = 21 cm^2; area of image = 336 cm^2
b area of original shape = 20 cm^2; area of image = 180 cm^2
c area of original shape = 30 cm^2; area of image = 750 cm^2
d area of original shape = 50.27 cm^2; area of image = 12.57 cm^2
e area of original shape = 10 cm^2; area of image = $\frac{10}{9}$ cm^2
f area of original shape = 28.27 cm^2; area of image = 113.1 cm^2
5 To increase area by scale factor 9, increase length by scale factor $\sqrt{9} = 3$.
6 **a** $\frac{1}{4}$ **b** $\frac{1}{9}$ **c** $\frac{1}{16}$ **d** $\frac{1}{100}$ **e** $\frac{4}{9}$ **f** $\frac{9}{25}$
7 If area scale factor = (length scale factor)^2, then length scale factor = $\sqrt{\text{area scale factor}}$.
8 **a** 7 **b** 5 **c** 10 **d** 8 **e** 2 **f** 20
9 **a** length scale factor = 3
b area scale factor = 9
c **i** 4.52 mm^2 **ii** 40.72 mm^2 **iii** 36.19 mm^2
10 **a** 100 cm^2 **b** 50 cm by 50 cm
c 2500 cm^2 **d** 25 times as much paint
11 4764 cm^2 more paper
12 2 **13** Area is doubled.
14 16 200 cm^2 **15** 0.64 km
16 **a** 2 cm **b** 8 cm^3
c **i** 64 cm^3 **ii** 216 cm^3 **iii** 512 cm^3
d volume increase equal to cube of length increase
e Because you increase length in three dimensions, multiplying by length scale factor three times (in other words, by cube of length scale factor).
f **i** 125 **ii** 216 **iii** 1000
17 **a** 8000 cm^3 **b** 125 cm^3 **c** 10
18 **a** new dimensions: 2 m × 2 m × 60 cm
b eight times **c** 2.1 m^3 more sand required
19 **a** area scale factor = 0.000 006 25
b 62.5 cm × 75 cm **c** 172.8 m^2
d volume of stairwell in real building = 3456 m^3; in model = 54 cm^3
20 **a** Area is doubled. **b** Volume is doubled.

5 Chapter review

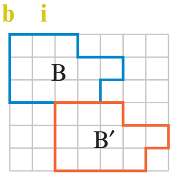
MULTIPLE-CHOICE

1 B **2** C **3** C **4** B
5 A **6** C **7** A **8** D

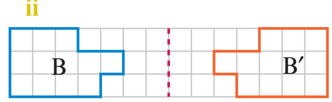
SHORT ANSWER

1 **a** $a = 64°$, $b = 64°$, $c = 116°$ and $d = 64°$
b $a = 44°$, $b = 136°$ and $c = 46°$
2 **a** 67° **b** 40° **c** 95° **d** 252°
3 **a** **i** reflection in horizontal mirror 2 units from base of B
ii translation 9 units to left and 1 unit up
iii reflection in vertical mirror 2 units to right of A

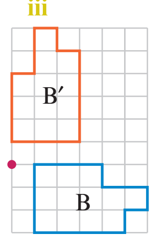
b **i**



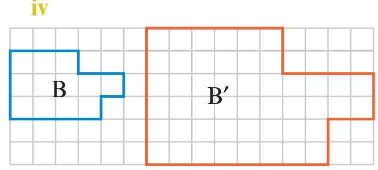
ii



iii



iv



4 a Congruent; triangles satisfy SAS.
b Not congruent; triangles do not satisfy RHS.
5 a enlargement; scale factor = 2
b reduction; scale factor = $\frac{1}{4}$
6 Shape is similar as all sides are in same ratio and corresponding angles are same.
7 $a = 9.2$ cm and $b = 4$ cm
8 a 192 cm^2 b 15 cm^2

NAPLAN-STYLE PRACTICE

1 f 2 55° 3 g is alternate to d
4 $a = 65°$, $b = 155°$, $c = 25°$
5 second triangle 6 irregular heptagon
7 93° 8 fourth diagram 9 A to C
10 AAS 11 third triangle
12 enlargement with scale factor 2.5
13 $x = 3$ cm
14 The two triangles contain right angle and hypotenuse lengths are in same ratio.
15 SSS 16 $a = 17$ cm, $b = 5$ cm
17 16 18 48 cm^2

ANALYSIS

a camp triangle: 40°, 90°, 50°; gum-tree triangle: 40° (angle at the gum tree), 90°, 50°
b Corresponding angles are same; triangles are similar.
c 2.5
d Side length of 6 m in ‘camp triangle’ corresponds to side in ‘gum triangle’ that represents width of river. To determine width of the river, multiply 6 m by scale factor (6 × 2.5).
e 15 m
f Longest route (along given lines) is straight to river (6 m), then along river edge until level with gum tree (4 m + 10 m) and across the river (15 m). This route is 35 m long.
Shortest route is to travel directly from camp to gum tree (follow the straight line). This route is 25.24 m long. Other routes are possible in between these distances.
g 9.76 m

5 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 6 PYTHAGORAS’ THEOREM AND TRIGONOMETRY

6 Are you ready?

1 a i scalene triangle ABC
ii right-angled triangle DEF
b i BC ii DF
c i ∠BAC ii ∠DEF
d triangle DEF
2 a 49 b 625 c 14.44
3 a 9 b 11.83 c 2.41
4 a $x = -3$ or $x = 3$ b $x = -4$ or $x = 4$
5 a Angles in corresponding positions are the same.
b $x = 9$ and $y = 24$
6 a $x = 36$ b $x = 5$ c $x = \frac{9}{4}$ d $x = 22$
7 a 70° b 58° c 45°
d $d = 35°$, $e = 35°$, $f = 55°$
8 a i N20°E ii 020°T
b i S60°W ii 240°T
c i N40°W ii 320°T

6A Understanding Pythagoras’ Theorem

6A Start thinking!

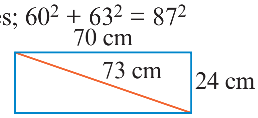
1 c 2 right angle 3 b
4–6

Triangle	a	b	c	a^2	b^2	c^2	$a^2 + b^2$
1	3	4	5	9	16	25	25
2	8	6	10	64	36	100	100
3	5	12	13	25	144	169	169

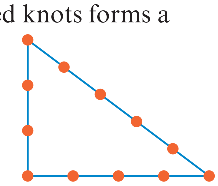
7 $c^2 = a^2 + b^2$
8 Pythagoras’ Theorem works for right-angled triangles only.

Exercise 6A Understanding Pythagoras’ Theorem

1 a, b yes. There may be errors due to measurement.
c no
2 a m b p c f d x
3 C
4 b $p^2 = q^2 + r^2$ c $f^2 = d^2 + e^2$
d $x^2 = w^2 + y^2$
5 If the sum of the squares of the two shorter sides is equal to the square of the hypotenuse length, then the triangle is right-angled.
6 a right-angled b not right-angled
c right-angled d not right-angled
7 a right-angled b right-angled
c not right-angled d not right-angled
e right-angled f not right-angled
8 a No, because $a^2 + b^2$ is the same as $b^2 + a^2$.
9 yes; $60^2 + 63^2 = 87^2$
10 a



b The frame is not square. The diagonal distance should be longer.
c 74 cm
11 The rope with 12 equally spaced knots forms a right-angled triangle with sides of 3 intervals, 4 intervals and 5 intervals. Pythagoras’ Theorem applies ($3^2 + 4^2 = 5^2$).



12 a $10^2 = 8^2 + 6^2$, $100 = 64 + 36$
b yes; $20^2 = 12^2 + 16^2$
c yes; $30^2 = 18^2 + 24^2$
d yes; $5^2 = 3^2 + 4^2$
e yes; $100^2 = 60^2 + 80^2$

13 **a** yes
b 12, 16 and 20; 18, 24 and 30; 3, 4 and 5; 60, 80 and 100
c Multiply the values of a Pythagorean triad by a whole number and the resulting values will also be a Pythagorean triad.
14 **a** yes; $13^2 = 5^2 + 12^2$
b Some examples include 10, 24 and 26; 15, 36 and 39.
15 **a** yes; $17^2 = 8^2 + 15^2$
b Some examples include 16, 30 and 34; 24, 45 and 51.
16 **a** 20 cm **b** 39 mm **c** 30 cm **d** 30 m
17 **a** B **b** A
18 **a** $c^2 = 12^2 + 17^2$ **b** $x^2 = 28^2 + 32^2$
c $5^2 = y^2 + 2^2$ **d** $39^2 = k^2 + 30^2$
19 **c** **i** b^2 **ii** c^2 **f** c^2 **g** $c^2 = a^2 + b^2$
20 triangle, hypotenuse

6B Using Pythagoras' Theorem to find the length of the hypotenuse

6B Start thinking!

1 hypotenuse
2 C; c represents the hypotenuse and a and b represent the two smaller sides.
3, 4
$$\begin{aligned} x^2 &= 8^2 + 15^2 \\ &= 64 + 225 \\ &= 289 \end{aligned}$$
5 Take the square root of both sides of the equation.
6 $x = -17$ and $x = +17$
7 $x = 17$. Do not accept $x = -17$ as it is not possible to have a negative value for a length.
8 17 m
9 distance along footpath = 8 m + 15 m = 23 m
distance using shortcut = 17 m (which is 6 m shorter)

Exercise 6B Using Pythagoras' Theorem to find the length of the hypotenuse

1 B
2 **a** $x^2 = 1^2 + 2^2$ **b** $m^2 = 6^2 + 5^2$
c $k^2 = 3^2 + 11^2$ **d** $p^2 = 8^2 + 9^2$
3 **a** 15 cm **b** 50 mm **c** 61 cm **d** 26 cm
4 Substitute values for a and b into Pythagoras' Theorem $c^2 = a^2 + b^2$ and solve for c.
5 **a** 9.43 cm **b** 17.49 cm **c** 23.85 mm
d 29.83 mm
6 From question 1: $\sqrt{117}$ cm
From question 2:
a $\sqrt{5}$ cm **b** $\sqrt{61}$ mm **c** $\sqrt{130}$ cm **d** $\sqrt{145}$ cm
7 **a** 12.53 cm **b** 55.22 m **c** 25.24 mm
d 10.97 cm
8 **a** 171 cm **b** 3 cm **c** 10 cm **d** 70 cm
9 **a** 21.95 cm **b** 13.86 cm **c** 42.43 cm
10 18.03 m
11 **a** 9.43 cm **b** 10 cm **c** 3.61 m
d 25.61 mm **e** 14.42 cm **f** 21.21 mm
12 82.02 cm
13 **a** triangle ABC and triangle ACD
b AC = 10 cm; AB = 6 cm and BC = 8 cm
c CD = 26 cm; the hypotenuse in triangle ABC becomes one of the smaller sides in triangle ACD
d 64 cm
14 **a** $x = 27.80$ cm, $y = 29.90$ cm
b $x = 15.81$ m, $y = 18.71$ m
c $x = 60.22$ cm, $y = 66.41$ cm
15 105.24 cm
16 Possible answers include drawing the triangles to scale or using trigonometry.
Results should be similar. Difficult to obtain an answer correct to two decimal places when measuring with a ruler.

6C Using Pythagoras' Theorem to find the length of the shorter side

6C Start thinking!

1 The unknown side is one of the shorter sides. The hypotenuse is marked 26 cm.
2 A; c represents the hypotenuse and a and b represent the two smaller sides.
3, 4
$$\begin{aligned} x^2 + 24^2 &= 26^2 \\ x^2 + 576 &= 676 \\ x^2 + 576 - 576 &= 676 - 576 \\ x^2 &= 100 \end{aligned}$$
5 10 6 10 cm

Exercise 6C Using Pythagoras' Theorem to find the length of the shorter side

1 C
2 **a** $7^2 = p^2 + 4^2$ **b** $21^2 = q^2 + 18^2$
c $12^2 = w^2 + 11^2$ **d** $17^2 = m^2 + 10^2$
3 **a** 20 mm **b** 15 cm **c** 48 mm **d** 27 mm
e 14 m **f** 36 cm
4 Substitute the value for b and c into $c^2 = a^2 + b^2$ and solve for a.
5 **a** 12.65 cm **b** 10.25 cm **c** 22.36 mm
d 18.73 cm **e** 4.76 cm **f** 15.96 m
6 From question 1: $\sqrt{119}$ cm
From question 2:
a $\sqrt{33}$ cm **b** $\sqrt{117}$ mm **c** $\sqrt{23}$ cm **d** $\sqrt{189}$ cm
7 **a** 27.98 cm **b** 10.25 m **c** 13.56 cm
d 45.61 m
8 **a** 398 cm **b** 5 cm **c** 55 cm **d** 47 cm
9 **a** 20.20 cm **b** 39.84 cm **c** 0.77 m
10 8.72 m **11** 2.87 m
12 **a** 15.65 m **b** 4.58 cm **c** 7.07 m
13 **a** $x = 13.08$ cm, $y = 7.35$ cm
b $x = 4$ m, $y = 3.48$ m
c $x = 17.80$ m, $y = 7.81$ m
d $x = 43.86$ cm, $y = 29.09$ cm
14 **a** **i** 79.35 cm **ii** 224.65 cm^2
b **i** 18.78 m **ii** 15.36 m^2
c **i** 48.81 m **ii** 139.48 m^2
d **i** 136.09 cm **ii** 1006.31 cm^2
15 94 cm

16 The results should be similar. Difficult to obtain an answer correct to two decimal places when measuring with a ruler.

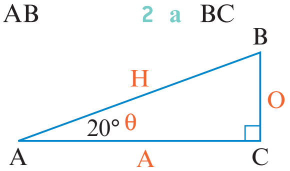
17 Select any two values whose sum of squares is the same as 90^2 (or 8100).

6D Understanding trigonometry

6D Start thinking!

1 AB 2 a BC b AC

3

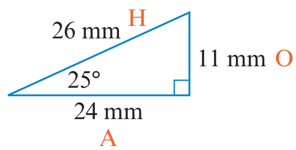


4 Corresponding angles are the same and corresponding sides have all been multiplied by the same scale factor.

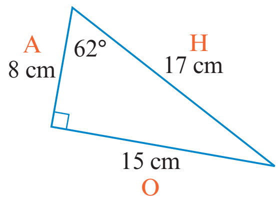
5 a EF b DF c DE

Exercise 6D Understanding trigonometry

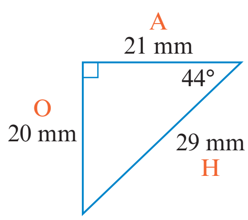
1 a



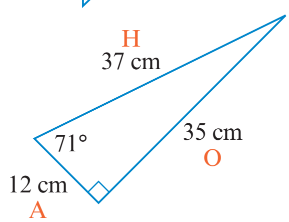
b



c



d



2 a

Triangle	Reference angle	O (mm)	A (mm)	H (mm)	$\frac{O}{H}$	$\frac{A}{H}$	$\frac{O}{A}$
1	30°	15	26	30	0.5	0.87	0.58
2	30°	30	52	60	0.5	0.87	0.58
3	30°	45	78	90	0.5	0.87	0.58

b See table. The ratios are all the same (0.5).

c See table. The ratios are all the same (0.87).

d See table. The ratios are all the same (0.58).

e $\sin 30° = \frac{O}{H} = 0.50$, $\cos 30° = \frac{A}{H} \approx 0.87$,

$\tan 30° = \frac{O}{A} \approx 0.58$

f The corresponding sides in similar triangles are all in the same ratio, so when determining the sine, cosine or tangent of a given reference angle in similar triangles, the value will always be the same.

3 a $\sin 25° = 0.42$, $\cos 25° = 0.92$, $\tan 25° = 0.46$

b $\sin 62° = 0.88$, $\cos 62° = 0.47$, $\tan 62° = 1.88$

c $\sin 44° = 0.69$, $\cos 44° = 0.72$, $\tan 44° = 0.95$

d $\sin 71° = 0.95$, $\cos 71° = 0.32$, $\tan 71° = 2.92$

4 a i $\frac{4}{5}$ ii 0.80 b i $\frac{3}{5}$ ii 0.60

c i $\frac{4}{3}$ ii 1.33

5 a i 0.87 ii 0.5 iii 1.73

b 0.5; because the adjacent and hypotenuse sides must be in the same ratio

c 0.5 d i 0.87 ii 1.73

e The corresponding sides in similar triangles are all in the same ratio, so when determining the sine, cosine or tangent of a given reference angle in similar triangles, the value will always be the same.

6 a 30°

b opposite = 30 mm and adjacent = 52 mm

c i 0.5 ii 0.87 iii 0.58

d They are the same.

7 a i They are the same (0.5).

ii They are the same (0.87).

b They are the same (0.58).

c They are the same (1.73).

8 a Corresponding angles are the same.

b i 0.64 ii 0.77 iii 0.84

c i 0.64 ii 0.77 iii 0.84

d 50 mm; multiply the length of BE by 3

e 50 mm

9 $\sin 50° = \frac{AG}{AD} = 0.77$; $\cos 50° = \frac{DG}{AD} = 0.64$;

$\tan 50° = \frac{AG}{DG} = 1.19$

10 a They are the same.

b, c $\frac{\sin\theta}{\cos\theta} = \tan\theta$

11 a $\cos\theta = \sin(90° - \theta)$

b i 0.44 ii 0.53 iii 0.98 iv 0.71

v 0.85 vi 0.26 vii 0.90 viii 0.17

12 a $\frac{\sin\theta}{\cos\theta} = \frac{\frac{O}{H}}{\frac{A}{H}} = \frac{O}{H} \div \frac{A}{H} = \frac{O}{H} \times \frac{H}{A} = \frac{O}{A} = \tan\theta$

b i 0.17 ii 2.05 iii 0.62 iv 1

v 0.27 vi 3.73

13 a i 0.87 ii 0.5 iii 1.73

b They are the same.

c i 0.80 ii 0.60 iii 1.33

d They are the same.

14 For 11b: i 0.44 ii 0.53 iii 0.98 iv 0.71 v 0.85 vi 0.26 vii 0.90 viii 0.17
For 12b: i 0.18 ii 2.05 iii 0.62 iv 1 v 0.27 vi 3.73
Differences due to using rounded values provided in table.

15 a 0.5 b 0.91 c 2.90 d 0.72 e 0.88 f 0.47 g 0.33 h 0.71 i 1.88 j 0.42 k 1 l 0.87

16 a 30° b 25° c 62° d 62° e 45° f 25° g 44° h 71° i 71°

17 Use the inverse function (for example, $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$)

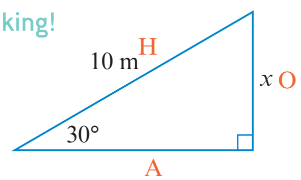
18 a In a 45° right-angled triangle, the opposite and adjacent sides are the same length, so the $\frac{O}{A}$ ratio will always be 1.
b angles less than 45°
c angles more than 45°

19 Because their ratios ($\frac{O}{H}$ and $\frac{A}{H}$) are always less than 1. As the hypotenuse is the longest side in the triangle, the denominator in the ratios will always be larger than the numerator, resulting in a value less than 1.

6E Using trigonometry to find lengths

6E Start thinking!

1 30° 2



3 a O b H 4 $\sin\theta = \frac{O}{H}$
5 A
6 Multiply sin 30° by 10.
7 $\frac{x}{10} = \sin 30°$ 8 5 m

$\frac{x}{10} \times 10 = (\sin 30°) \times 10$

$x = 0.5 \times 10$

$x = 5$ m

Exercise 6E Using trigonometry to find lengths

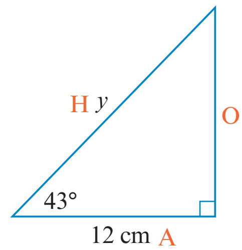
1 a $\cos\theta = \frac{A}{H}$, $\cos 64° = \frac{k}{35}$
b $\sin\theta = \frac{O}{H}$, $\sin 49° = \frac{k}{12}$
c $\tan\theta = \frac{O}{A}$, $\tan 26° = \frac{k}{50}$
d $\cos\theta = \frac{A}{H}$, $\cos 51° = \frac{k}{17}$
e $\sin\theta = \frac{O}{H}$, $\sin 37° = \frac{k}{24}$
f $\tan\theta = \frac{O}{A}$, $\tan 21° = \frac{k}{16}$

2 a 31.84 cm b 25.86 mm c 2.78 m d 22.65 cm e 11.11 m f 49.32 mm g 61.80 cm h 129.35 m i 5.26 cm

3 a 15.34 mm b 9.06 m c 24.39 cm d 10.70 m e 14.44 mm f 6.14 m

4 SOH represents the sine ratio $\sin\theta = \frac{O}{H}$;
CAH represents the cosine ratio $\cos\theta = \frac{A}{H}$;
TOA represents the tangent ratio $\tan\theta = \frac{O}{A}$;

5 a



b cosine
c B
d, e 1, C; 2, A; 3, D; 4, B

6 a 118.31 cm b 3.72 m c 38.92 mm d 11.02 cm

7 i a 65° b 28° c 46° d 19°
ii a 118.31 cm b 3.72 m c 38.92 mm d 11.02 cm
iii They are the same; b and d appeared easier as the pronumeral is in the numerator after substitution.

8 a 4.44 cm b 8.31 cm c 10.39 m d 78.15 cm

9 a 7 cm b 4.70 cm c 46.08 cm

10 a 33.64 m b 34.64 m

11 4.62 m

12 a 9.60 m b 16.00 m

13 a 1.93 m b 2.13 m

14 8.60 m

15 One angle size (other than the right angle) and one side length

6F Using trigonometry to find angles

6F Start thinking!

1 a i A ii H b $\cos\theta = \frac{A}{H}$
2 B
3 One possible method is to use a calculator.
4 a 30° b 0.5 5 60° 6 60°

Exercise 6F Using trigonometry to find angles

1 a 13° b 44° c 56° d 42° e 16° f 74°

2 a i $\theta = \sin^{-1}(0.34)$ ii 20° iii 0.34
b i $\theta = \cos^{-1}(0.81)$ ii 36° iii 0.81
c i $\theta = \tan^{-1}(0.65)$ ii 33° iii 0.65
The trigonometric ratio value is very close, although not *exactly* the same due to rounding.

3 a i $\theta = \sin^{-1}\left(\frac{4}{5}\right)$ ii 53° iii 0.80
b i $\theta = \cos^{-1}\left(\frac{15}{19}\right)$ ii 38° iii 0.79
c i $\theta = \tan^{-1}\left(\frac{8}{21}\right)$ ii 21° iii 0.38
d i $\theta = \sin^{-1}\left(\frac{23}{26}\right)$ ii 62° iii 0.88

4 a 25° b 23° c 57° d 83°

5 a $\sin\theta = \frac{O}{H}, \sin\theta = \frac{12}{17}$
b $\cos\theta = \frac{A}{H}, \cos\theta = \frac{8}{15}$
c $\tan\theta = \frac{O}{A}, \tan\theta = \frac{24}{33}$
d $\cos\theta = \frac{A}{H}, \cos\theta = \frac{35}{40}$
e $\sin\theta = \frac{O}{H}, \sin\theta = \frac{15}{24}$
f $\tan\theta = \frac{O}{A}, \tan\theta = \frac{13.1}{6.6}$

6 a 50° b 42° c 67° d 54° e 31° f 46° g 21° h 76°

7 a 45° b 58° c 36° d 29° e 39° f 63°

8 a 53° b 25° c 46°

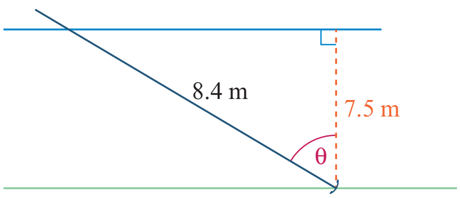
9 27°, 63° and 90°

10 a 14° b 7.03 m

11 a The ramp rises 1 unit vertically for every 14 units horizontally.
b 4°
c 6°; as it is greater than the maximum angle size, the ramp does not satisfy the safety regulations.

12 a 47° b 86° 13 a 25° b 1.77 m

14 a



b 27°
c Larger. If the chain is longer, the angle with the vertical would be greater. If the chain is shorter, the angle is closer to the vertical, hence making less angle size with the vertical.

15 a i 37° ii 37° iii 37° b θ = 37°
c i Using trigonometry can be accurate to a required degree of accuracy but is difficult to perform without a calculator.
ii It is an advantage to use a scale diagram to check the results from trigonometric calculations, but it is difficult to record measurements with sufficient accuracy

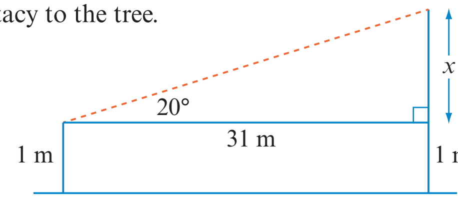
16 a First, calculate the angle of the Sun's rays at the time of measurement by using the length of a metre ruler's shadow. Then use the length of the tree's shadow as a measurement to calculate the height of the tree.
b 37°
c It refers to the angle of the Sun's rays at the time of measurement.
d 11.65 m
e By forming an equivalent ratio statement using the corresponding sides.
x:1 m = 15.46 m:1.32 m
$$x = \frac{1 \times 15.46}{1.32}$$
$$= 11.71 \text{ m}$$
f You could draw the diagram to scale.

6G Applications involving right-angled triangles

6G Start thinking!

1 No; a side length of the triangle is needed. The easiest one to find is the horizontal distance from Stacy to the tree.

2 a



b $\tan 20° = \frac{x}{31}$, $x = 11.28$ m
c No. The measurement for angle of elevation is taken from Stacy's eye level; that is, 1 m above the ground. The koala is 11.28 m + 1 m above the ground.

3 12.28 m 4 204 m

Exercise 6G Applications involving right-angled triangles

1 a 25° b 25°

2 An angle of depression measures downwards from a horizontal line of sight and an angle of elevation measures upwards from a horizontal line of sight.

3 a 56 m b 72 m c 98 m d 102 m

4 a 30° b 18°

5 a i 32.99 m ii 32.99 m
b yes, by using the sine or the cosine ratio
c yes, by drawing a right-angled triangle to scale

6 a When the length of two sides are known and you need to find the length of the third side.
b When you know one angle (other than the 90° angle) and one side length, to find another side length. If finding an angle, the minimum information required is two known side lengths.
c When you know one angle (other than the 90° angle) and the length of two sides.

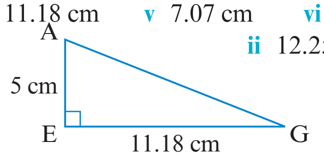
7 a 48.30 m b 10.05 m

8 a i 52.99 m ii 84.80 m
b i 34.20 m ii 93.97 m
c i N27°W ii 111.80 m
d i 94.34 m ii 148° true

9 a S40°W or 220°T b 7.81 km

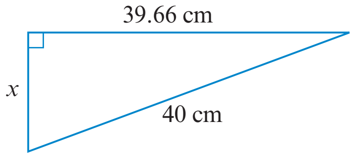
10 a 7.66 km b 6.43 km
c i N12°E or 012°T ii 25.82 km

11 a i 7.07 cm ii 7.07 cm iii yes
iv ADH
b i 11.18 cm ii 11.18 cm iii 11.18 cm
iv 11.18 cm v 7.07 cm vi 11.18 cm
c i



ii 12.25 cm
d 24°

12 **a** 39.66 cm **b**



c 5.20 cm
d 7°

13 **a** 17.89 cm **b** 60° **14** 18.03 cm

6 Chapter review

MULTIPLE-CHOICE

1 C **2** D **3** B **4** D **5** A
6 A **7** B **8** C

SHORT ANSWER

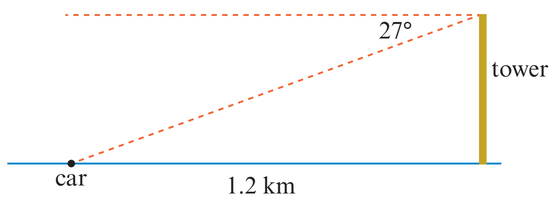
1 **a** right-angled **b** not right-angled
2 **a** $a^2 + b^2 = 30^2 + 40^2 = 2500$, $c^2 = 50^2 = 2500$
b Yes, as they are three whole numbers that satisfy Pythagoras' Theorem.
3 **a** 13 cm **b** 18.41 cm
4 **a** 3 cm **b** 52 cm
5 **a** $x = 20$ cm, $y = 36.06$ cm
6 **a** 25.46 cm **b** 1.5 m
7 **a** 297 cm **b** 5 cm
8 $x = 19.90$ cm, $y = 15.13$ cm
9 **a** **i** both equal $\frac{25}{50}$ or 0.5
ii both equal $\frac{43}{50}$ or 0.86
iii both equal $\frac{25}{43}$ or 0.58
iv both equal $\frac{43}{25}$ or 1.72
b In each case, the answers are the same.
10 **a** 5.44 cm **b** 19.43 m
11 **a** 28.31 m **b** 4.15 cm
12 **a** 35° **b** 60°
13 **a** 58.53 m **b** 53.63 m

NAPLAN-STYLE PRACTICE

1 v **2** $v^2 = u^2 + w^2$ **3** Triangle A
4 52 m **5** 5 m **6** 11.74 cm
7 Triangle D **8** 28 cm **9** 8.35 m
10 $\frac{47}{50}$ **11** 0.36 **12** $\tan 40° = \frac{x}{16}$
13 12.28 cm **14** $\tan^{-1}\left(\frac{22}{25}\right)$ **15** 31°
16 61°

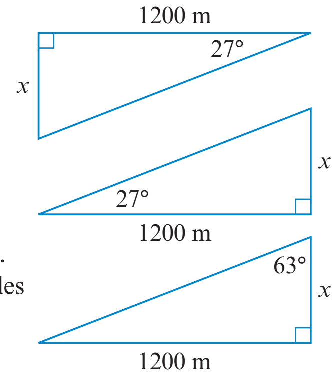
ANALYSIS

a



b **i** No; need value of two side lengths to find value of third side length.
ii Yes; one angle and value of one side length has been given.

c In total, there are three figures that can be used to determine the height of the tower.



d 611 m
e See figures in part c. Note that all triangles result in the same length; that is, $x = 611$ m.
f 1347 m **g** $\frac{1200}{\cos 27°} \approx 1347$ m

6 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 7 MEASUREMENT

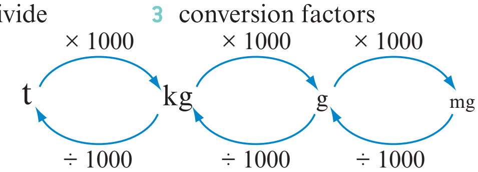
7 Are you ready?

1 **a** 1000 m **b** 60 min **c** 1000 kg
2 D **3** six decimal places
4 **a** 26 cm **b** D **c** B
5 C
6 **a** 33 cm^2 **b** 28 cm^2
7 A
8 **a** 6 **b** D
9 C
10 **a** 5 cm **b** 12 cm
11 A

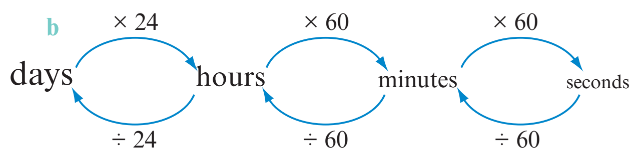
7A Understanding and representing measurement

7A Start thinking!

1 Any combination of the following: length: millimetres, centimetres, metres, kilometres; mass: milligrams, grams, kilograms, tonnes; time: seconds, minutes, hours, days, weeks, years
2 divide **3** conversion factors
4 **a**



b



5 One possible answer is: some units are too large or too small to work with, or are difficult to envisage. For example, 1 month is easier to imagine and work with than 2 678 400 s.
6 First example has positive power because it needs to be increased to return to the basic numeral; second example has negative power because it needs to be reduced back to the basic numeral.

7 1.64×10^3 uses 10^3 because decimal point moves 3 places to right to return to basic numeral.
4.89×10^{-5} uses 10^{-5} because decimal point moves 5 places to left to return to basic numeral.

Exercise 7A Understanding and representing measurement

1 a 430 mm b 0.187 km c 20 min
d 150 min e 4.75 kg f 9450 kg
g 8300 cm h 3.5 hours i 6205 g
j 0.8 cm

2 a 0.063 m b 107 000 cm c 0.475 kg
d 250 000 g e 0.56 h f 259 200 s
g 11 700 s h 1770 mm i 0.00032 kg
j 860 000 000 mg

3 a 1.25×10^8 km b 6.75×10^{-9} mm
c 1.375×10^{10} years d 2.0×10^{-6} s
e 5.946×10^{12} t f 2.5×10^{-19} g
g 8.755×10^{11} mins h 7.6×10^{-8} kg
i 9.25×10^{14} cm j 1.7×10^{-15} h

4 a 3.45×10^6 g b 7.82×10^{12} kg
c 2.86×10^{-8} s d 6.72×10^{-11} cm
e 3.57×10^{12} g f 2.99×10^{-7} mg
g 4.75×10^{18} mm h 5.08×10^{-3} L

5 a i 4 540 000 000 ii 4.54×10^9
b i 13 750 000 000 or 1.375×10^{10}
ii 9.21×10^9

6 a i 1.0×10^{-9} g, 0.000 000 001 g
ii 1.0×10^9 m, 1 000 000 000 m
iii 1.0×10^{-18} s, 0.000 000 000 000 000 001 s
iv 8.1×10^{15} g, 8 100 000 000 000 000 g
v 3.4×10^{-5} s, 0.000 34 s
vi 1.7×10^{-11} g, 0.000 000 000 017 g
b i 756 terametres ii 1.84 gigagrams
iii 23 picoseconds iv 49 micrograms
v 94 zettaseconds vi 28 attometres
c i 1 megametre ii 1 hectogram
iii 1 decasecond iv 1 terametre
v 1 millisecond vi 1 nanogram

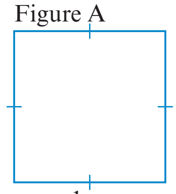
7 a mass: 2.0×10^{-15} g, diameter: 1.2×10^{-7} m
b 5.0×10^{17}
c i number of viruses: 2.0×10^5, mass: 4.0×10^{-10} g or 400 picograms
ii number of viruses: 5.0×10^7, mass: 1.0×10^{-9} g or 1 nanogram
iii number of viruses: 1.25×10^{10}, mass: 2.5×10^{-5} g or 25 micrograms

8 a i 3.0×10^8 m/s ii 1.08×10^9 km/h
b i 1.8×10^{10} m/min, 18 gigametres/min
ii 1.08×10^{12} m/h, 1.08 terametres/h
iii 2.592×10^{13} m/day, 25.92 terametres/day
iv 9.4608×10^{15} m/year, 9.4608 petametres/year
c 9.4608×10^{12} km/year
d i 3.333×10^{-6} s, 3.333 microseconds
ii 3.333×10^{-9} s, 3.333 nanoseconds
iii 3.333×10^{-11} s, 33.33 picoseconds
iv 3.333×10^{-12} s, 3.333 picoseconds
e See parts b and d above.
f 0.1389 h or 8.33 min (approximately 8 min and 20 s)

9 a 1235.52 km/h
b i 20.592 km ii 123.52 km
iii 29 652.48 km iv 10 823 155.2 km
c i 2.914×10^0 s ii 2.914×10^{-3} s
iii 2.914×10^{-5} s iv 2.914×10^{-6} s
d Sound moves about 4.3 times as fast through water as it does through air.
e i Mach 1 ii Mach 2.9 iii Mach 32.4
iv Mach 86.8 v Mach 874 125.9

10 a i 0.042 s ii 0.014 s
b 43 200 frames
c 1000 fps: 0.001 s, 1 trillion fps: 1.0×10^{-12} s
d 23 148 148.15 h, about 2642.5 years

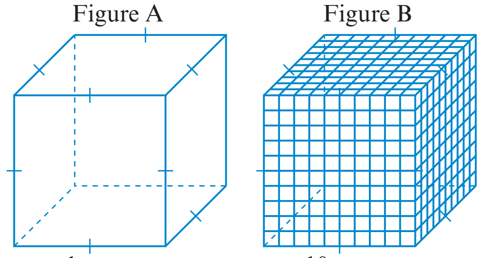
11 a 1 cm² b 10 mm
c $1\text{ cm}^2 = 1\text{ cm} \times 1\text{ cm}$
$= 10\text{ mm} \times 10\text{ mm}$
$= 100\text{ mm}^2$
d Figure A



1 m

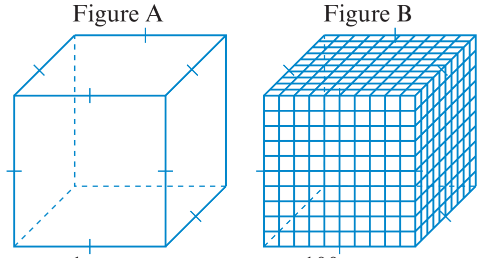
Figure B – check with your teacher

A 1-m square will be divided into 100 by 100 cm squares;
$100\text{ cm} \times 100\text{ cm} = 10\ 000\text{ cm}^2$.
e To find the area of a square you square the length of one side, therefore you also need to square the conversion factor.
f Figure A Figure B



1 cm 10 mm

1 cm = 10 mm. A cube with 1 cm sides can have each side divided into 10 mm,
$10\text{ mm} \times 10\text{ mm} \times 10\text{ mm} = 10\ 000\text{ cm}^3$.
g Figure A Figure B



1 m 100 cm

1 m = 100 cm. A cube with 1 m sides can have each side divided into 100 cm;
$100\text{ cm} \times 100\text{ cm} \times 100\text{ cm} = 1\ 000\ 000\text{ cm}^3$.

h To find the volume of a cube you cube the length of one side, therefore you also need to cube the conversion factor.

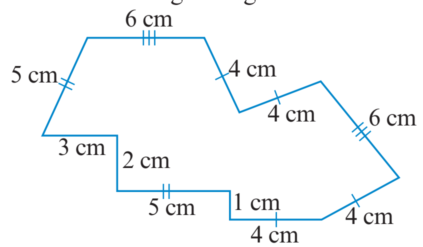
12 **a** 700 mm^2 **b** 84 000 000 m^2
c 5 cm^2 **d** 0.2 m^2

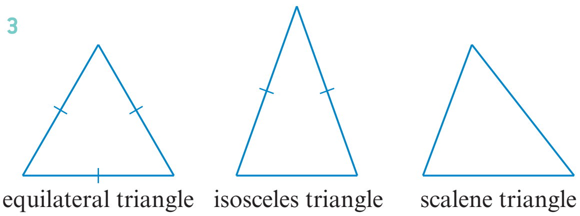
13 **a** 17 000 000 cm^3 **b** 281 000 mm^3
c 0.066 m^3 **d** 0.6 cm^3

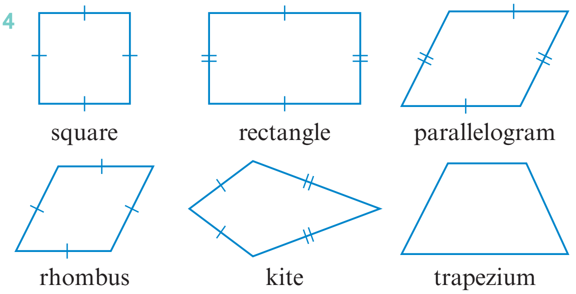
7B Perimeter

7B Start thinking!

1 Add all side lengths together.

2


3


4


5 Counting number of sides of a shape is a way of checking that a side that does not have a side length marked is not missed when calculating perimeter.

6 Diameter of a circle is the length of a straight line passing from one point on the circumference through the centre of the circle to another point on the circumference.

8 Answers should be close to 3.14.

9 $C = \pi D$

10 Side lengths for shapes other than a circle. For a circle, you need diameter or radius.

Exercise 7B Perimeter

1 **a** 22 cm **b** 26.5 cm **c** 39.9 cm
d 180 mm **e** 28 cm **f** 75 mm

2 **a** 24 cm **b** 58 cm **c** 48 cm
d 74 cm **e** 40 cm **f** 66 cm

3 **a** 15.71 cm **b** 34.56 cm **c** 62.83 cm

4 **a** 75.40 cm **b** 62.83 cm **c** 18.85 cm

5 **a** 12.57 cm **b** 47.12 mm **c** 25.13 m
d 34.56 m **e** 47.12 cm **f** 65.97 cm

6 **a** radius **b** circumference
c 90° **d** 360° **e** $\frac{1}{4}$
f 90° is a quarter of 360°
g circumference of full circle = $2\pi r$. Arc is $\frac{1}{4}$ of full circle, so length of arc = $\frac{1}{4} \times 2\pi r$.
h 3.14 cm **i** 7.14 cm **j** $\frac{42}{360} = \frac{7}{60}$
k $\frac{42}{360}$ gives the fraction of the circumference required; hence, length of arc = $\frac{42}{360} \times 2\pi r$.
l 5.47 cm
m $\frac{\theta}{360°}$ gives the fraction or part of circumference required. Multiply by $2\pi r$ to find length of arc then add 2 × radius (which form the sides of the sector) to find perimeter of sector.

7 **a** 25 cm **b** 41.82 cm **c** 21.59 cm
d 38.36 cm

8 **a** It is a quarter of a circle. **b** 180°
c $\frac{60}{360} = \frac{1}{6}$, hence sextant = $\frac{1}{6}$ of circle; $\frac{45}{360} = \frac{1}{8}$, hence octant = $\frac{1}{8}$ of circle
d **i** 13.93 cm **ii** 15.24 cm **iii** 17.85 cm
iv 25.71 cm
e All lengths are less than circumference of full circle.

9 **a** 28.85 cm **b** 22.71 cm **c** 102.83 cm

10 **a** 27.66 cm **b** 22 cm **c** 27.42 cm

11 **a** 1.5×10^8 km
b radius of circle formed by Earth's orbit with Sun as centre.
c 942 477 796 km **d** 107 515.149 km/h

12 **a** 5 cm **b** $l = \frac{P}{4}$
c Yes, as all four sides of rhombus are equal in length.

13 **a** Some possible answers are: l = 13 cm and w = 12 cm; l = 15 cm and w = 10 cm; l = 20 cm and w = 5 cm.
b **i** 15 cm **ii** 5 cm **iii** 22 cm
c $w = \frac{P}{2} - l$

14 **a–c**: Same as answers for question 13a–c. They are similar as they all have two pairs of equal sides, but parallelogram and rectangle have opposite sides equal, whereas kite has adjacent sides equal.

15 **a** 50 m **b** 58 m **c** 8 m **d** 66 m **e** 16 m
f 74 m, difference of 24 m
g Differences are first three multiples of 8. $P = 50 + 8x$
h **i** 54 m **ii** 62 m **iii** 70 m **iv** 90 m

16 Answers should be such that formula still holds.

7C Area of simple shapes

7C Start thinking!

2 l = number of columns, w = number of rows, A = total number of cm^2 inside the rectangle (i.e. $l \times w$)

4 **a** base and height

b base = length of rectangle and height = width of rectangle formed. $A = bh$

c Same result obtained for different shaped rectangle.

5 a length of one diagonal (x) and length of other diagonal (y)

b Area is half the product of the lengths of the diagonals; $A = \frac{1}{2}xy$.

6 a radius, diameter and circumference

b radius equivalent to height of parallelogram, half the circumference equivalent to base of parallelogram

area of circle ≈ area of parallelogram

$= \text{base} \times \text{height}$

$= \frac{1}{2} \times 2\pi r \times r$

$= \pi r^2$

c parallelogram should look more like a rectangle

Exercise 7C Area of simple shapes

1 a 21 cm^2 b 24 cm^2 c 60 cm^2
d 42 cm^2 e 28 cm^2 f 30 cm^2
g 22.5 cm^2 h 45.5 cm^2 i 110 cm^2

2 a 380.13 mm^2 b 28.27 cm^2 c 122.72 mm^2
d 50.27 cm^2 e 176.71 cm^2 f 153.94 mm^2

3 a i $l = 2$ cm ii $A = 4$ cm^2
b i $b = 2$ cm, $h = 1$ cm ii $A = 1$ cm^2
c i $b = 2$ cm, $h = 2$ cm ii $A = 2$ cm^2
d i $a = 2$ cm, $b = 3$ cm, $h = 1$ cm
ii $A = 2.5$ cm^2

4 a i 28 cm^2 ii 62.5 cm^2 iii 25 cm^2
b To ensure correct values are substituted into formula for area.

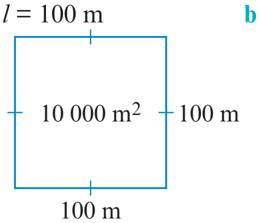
5 a 15.6 cm^2 b 55.44 cm^2 c 36.64 cm^2

6 a 4 cm b 77° c 360° d $\frac{77}{360}$
e $\frac{77}{360}$ gives fraction of circle for which area is required and πr^2 is formula for area of a circle.
f 10.75 cm^2
g $\frac{\theta}{360°}$ gives fraction of circle for which area is required and πr^2 is formula for area of a circle.

7 a 7.07 cm^2 b 127.41 cm^2 c 184.31 cm^2
d 30.39 cm^2

8 a 7 cm b $l = \sqrt{A}$

9 a $l = 100$ m b 500 ha



c i 510 ha ii 5.1 km^2

10 a Some possible answers are: $l = 20$ cm, $w = 5$ cm; $l = 25$ cm, $w = 4$ cm; $l = 50$ cm, $w = 2$ cm.
b i 5 cm ii 20 cm iii 2.5 cm
c $l = \frac{A}{w}$

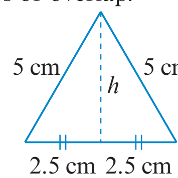
11 a 8.92 cm b $r = \sqrt{\frac{A}{\pi}}$

12 There is only one variable; r for a circle and l for a square.

13 Yes, as all sides of a rhombus are equal in length.

14 7.96 cm^2

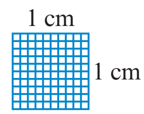
15 a triangles
b They are equilateral triangles because they fit together without any gaps or overlap.
c $A = \frac{1}{2}bh$
d



e 4.33 cm
f 10.83 cm^2
g 64.98 cm^2
h 374.04 cm^2
i Regular hexagons have equal sides and diagonals which are equal in length and meet at the centre of the hexagon. This enables all sides of the triangles to be equal in length.

16 a eight triangles b 6.035 cm
c 15.09 cm^2 d 120.72 cm^2
e i 43 cm^2 ii 90.825 cm^2 iii 192.375 cm^2

17 a



$1\ cm^2 = 1\ cm \times 1\ cm$
$= 10\ mm \times 10\ mm$
$= 100\ mm^2$

b One possible answer is: to calculate area of a square you must square side lengths, hence you also need to square units it is measured in. Therefore you must multiply or divide by conversion factor squared when converting between square units.

c i 28 cm^2 = 2800 mm^2 = 0.0028 m^2
ii 62.5 cm^2 = 6250 mm^2 = 0.00625 m^2
iii 25 cm^2 = 2500 mm^2 = 0.0025 m^2

d Easier to convert length measurements by multiplying/dividing by 10, 100, 1000, etc. than by using squared units which you need to be multiplied/divided by 10^2, 100^2, 1000^2, etc.

18 To find area of irregular polygon you need to know area of each triangle within polygon. To find area, find sum of areas of each triangle within polygon.

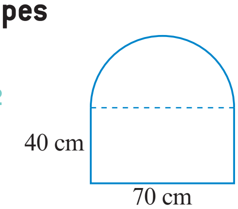
19 a 625 m^2 b 796.23 m^2 c 625 m^2
d 1250 m^2

7D Area of composite shapes

7D Start thinking!

1 rectangle and a semicircle

2



3 a $l = 70$ cm, $w = 40$ cm
b $D = 70$ cm or $r = 35$ cm

4 rectangle: $A = lw$; semicircle: $A = \frac{1}{2}\pi r^2$

5 4724.23 cm^2

6 $l = 80$ cm, $h = 2$ m or 200 cm

7 16 000 cm^2 or 1.6 m^2

8 11 275.77 cm^2 or 1.13 m^2 of timber

9 Subtracting area of window from area of door is easier as you only need to find area of a rectangle and one composite shape. If addition was used, you would have to find a larger number of areas to add together.

10 Some possible answers are: use whichever method allows figure to be split into least number of areas to be calculated. Use addition if one or more shapes are added onto a simpler shape. Use subtraction if a shape is removed from within a simpler shape.

Exercise 7D Area of composite shapes

1 a triangle, semicircle b three rectangles
c triangle, rectangle, semicircle
d triangle, rectangle
e rectangles, semicircles
f parallelogram, triangle, semicircle

2 a 122.27 cm^2 b 56 cm^2 c 31.57 cm^2
d 69 cm^2 e 160 cm^2 f 174.64 cm^2

3 a 70.87 cm^2 b 47.43 mm^2 c 424 cm^2
d 69.73 cm^2 e 70.5 cm^2 f 60.5 cm^2

4 a An annulus is the shape formed by two different size circles with a common centre. It is the area between the two circles.
b area of outer circle – area of inner circle
c 137.44 cm^2

5 a 12.56 cm^2 b 65.97 cm^2 c 28.27 cm^2
d 226.20 cm^2 e 103.67 cm^2 f 47.13 cm^2

6 12 166.55 m^2

7 a 19 525 cm^2 or 1.9525 m^2
b Some possible answers are: to cater for wastage between tags when they are cut out; in case any mistakes are made in production of tags.

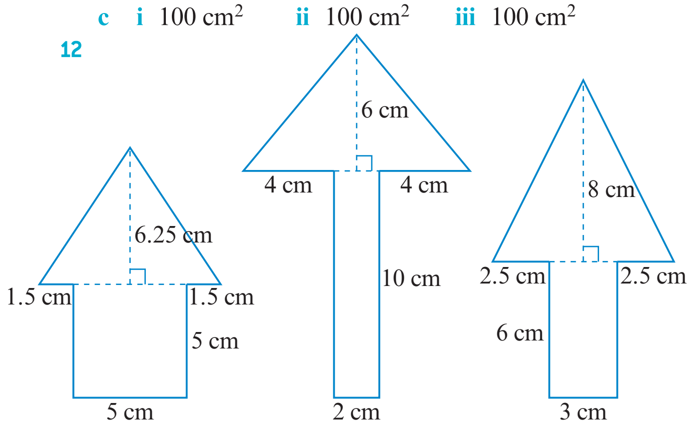
8 a 105.35 m^2 b $4740.75 c $1580.25

9 176.7 cm^2

10 a 354.33 cm^2 b 336 cm^2 c 97.425 cm^2
d 68.57 cm^2 e 35 cm^2 f 51 cm^2

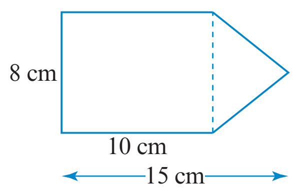
11 a large triangle: $A = 25$ cm^2;
small triangle: $A = 6.25$ cm^2;
medium triangle: $A = 12.5$ cm^2;
square/rhombus: $A = 12.5$ cm^2;
trapezium: $A = 12.5$ cm^2
c i 100 cm^2 ii 100 cm^2 iii 100 cm^2

12

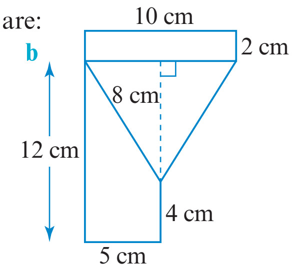


13 Some possible answers are:

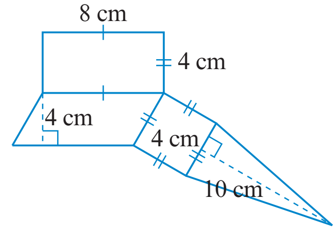
a



b



c



14 47.71 cm^2 15 15 509.42 cm^2 ≈ 1.55 m^2

16 a 56 cm^2 b 87.5 cm^2 c 125.52 cm^2

7E Surface area

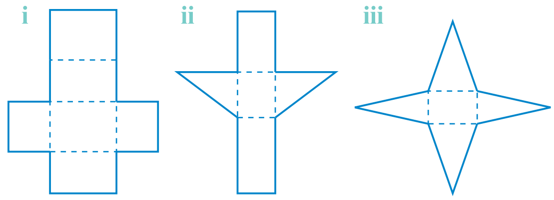
7E Start thinking!

1 Surface area of a 3D object is the area of its outside surfaces.

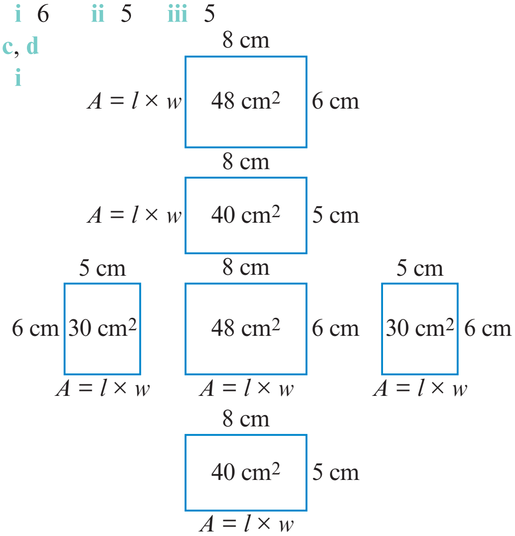
2 Find area of each outside surface using appropriate area formula. Add all areas together.

3 i rectangular prism ii triangular prism
iii rectangular pyramid

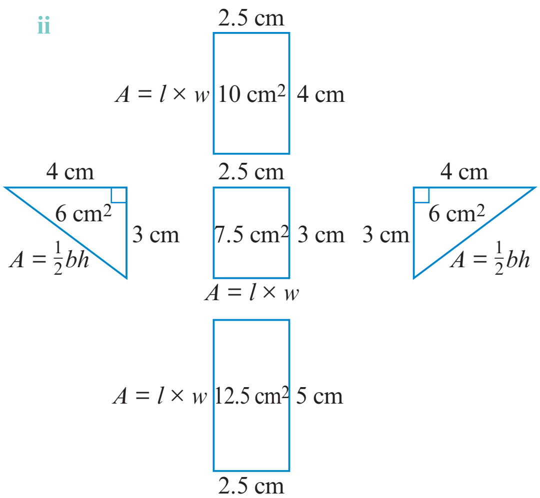
4 See question 5b for dimensions.



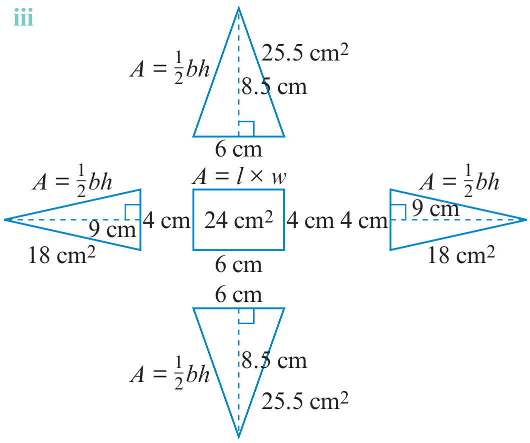
5 a i 6 ii 5 iii 5
b, c, d
i



ii



iii



e i 236 cm² ii 42 cm² iii 111 cm²

6 i Three pairs of rectangles have same area; add areas of different rectangles and double the answer.
ii The two triangles have same area. Find area of one triangle and double the answer. Then add areas of the three rectangles.
iii Two pairs of triangles have same area. Find area of each different triangle once and double the answer. Then add area of rectangle to area of triangles.

7 i TSA = $2(lw + lh + wh)$
ii TSA = $l_1w_1 + l_2w_2 + l_3w_3 + 2(\frac{1}{2}bh)$
iii TSA = $2(\frac{1}{2}b_1h_1) + 2(\frac{1}{2}b_2h_2) + lw$

9 Drawing a net allows you to see all shapes that make up faces of 3D object. Correct area formula can be used to find area of each face and, hence, total surface area of object.

Exercise 7E Surface area

1 a 188 cm² b 136 cm² c 188 cm²
d 150 cm² e 120 cm² f 824 cm²

2 a 240 cm² b 120 cm² c 360 cm²
d 94 cm² e 120 cm² f 976 cm²

3 a 114 cm² b 96.5 cm² c 945 cm²
d 80 mm² e 795 cm² f 508 cm²

5 8412.75 cm² 6 301 cm² 7 94.18 cm²

8 a i 664 cm² ii 1328 cm²
b i 396 cm² ii 792 cm²
c i 330 cm² ii 660 cm²

9 a 500 cm² b i 292 cm² ii 584 cm²
c Answer to b ii is 84 cm² greater, because two more faces (7 cm by 6 cm) created when the block of butter is cut in half.
d 4032 cm²
e One possible comment is: cutting butter into smaller pieces increases the amount of surface exposed to the heat, so butter will melt quicker.

10 a cream: 30 m², blue: 53.575 m²
b cream: 4 L, blue: 7.14 L

11 a 2045.08 cm² b 762 cm² c 264.6 cm²

12 2630 cm²

13 a 100 cm² b 10 cm c $l = \sqrt{\frac{\text{TSA}}{6}}$

14 a Possible set of dimensions is: $l = 10$ cm, $w = 8$ cm, $h = 5$ cm
b Cube has six identical faces in shape of square. Hence, all sides are equal in length. Length of one face will also determine length of remaining sides. A rectangular prism could have a different length, width and height.
c No, because width and height of prism could still vary.

15 a tetrahedron: 4; cube: 6; octahedron: 8; dodecahedron: 12; icosahedron: 20
b tetrahedron: triangle; cube: square; octahedron: triangle; dodecahedron: pentagon; icosahedron: triangle
c tetrahedron: 6.93 cm²; cube: 24 cm²; octahedron: 13.86 cm²; dodecahedron: 82.5 cm²; icosahedron: 34.64 cm²

16 a 296.46 cm² b 320 cm² c 1710.9 cm²

7F Surface area of cylinders

7F Start thinking!

1 three faces 2 circle 3 rectangle

4 $A = \pi r^2$; requires the radius (r) or diameter (d)

5 When third face is unwrapped and laid flat, height of cylinder forms width of rectangle.

6 Length of rectangle is equal to circumference of circles.

7 Length of rectangle wraps exactly around circumference of circles to form body of cylinder.

8 $C = 2\pi r$

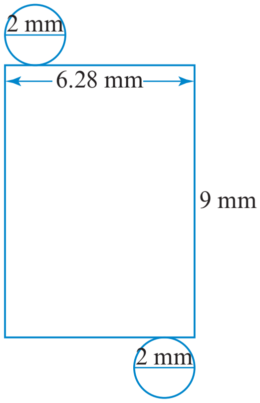
9 area of rectangle = lw, where $l = 2\pi r$ (circumference of the circle) and $w = h$ (height of cylinder)

10 total surface area = area of rectangular face + 2 × area of circle = $2\pi rh + 2\pi r^2$

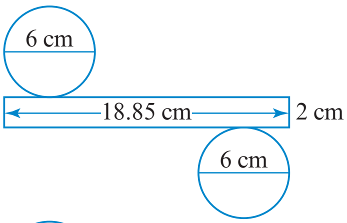
11 188.5 cm²

Exercise 7F Surface area of cylinders

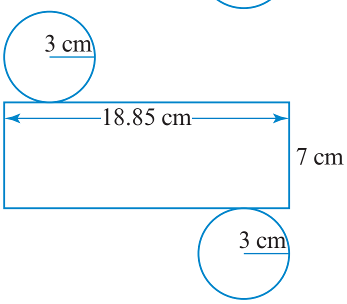
1 **a**



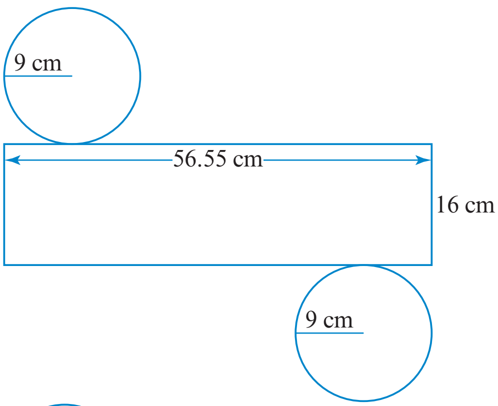
b



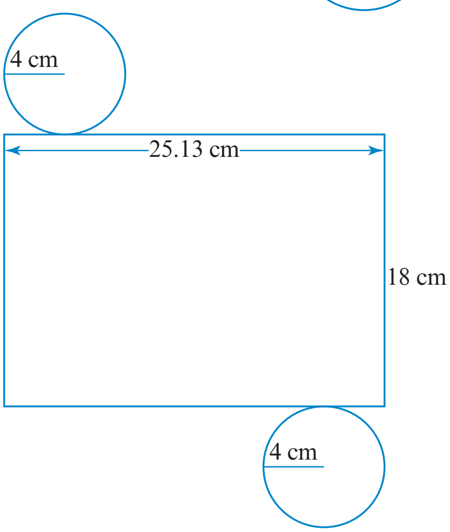
c



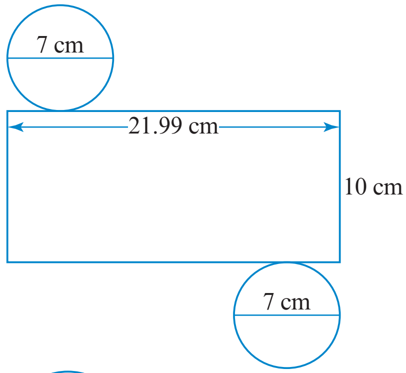
d



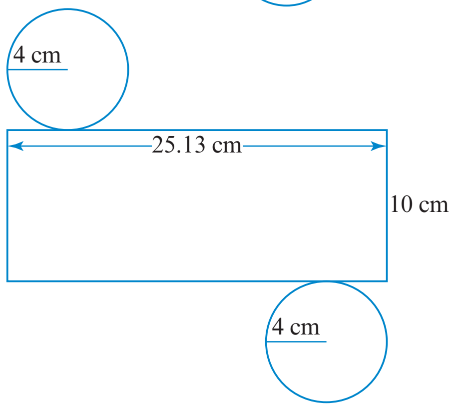
e



f



2 **a**



b 251.3 cm^2 **c** 50.27 cm^2 **d** 351.84 cm^2

3 **a** 100.53 cm^2 **b** 541.92 cm^2 **c** 603.19 cm^2
d 18.85 m^2 **e** 7.85 m^2 **f** 3.39 m^2

4 **a** 248.81 cm^2 **b** 100.53 cm^2 **c** 424.12 cm^2

5 3924.64 cm^2

6 \$1429.40 (\$1450 if rounded to needing 29 L of paint)

7 first roller with radius 4 cm and length 25 cm

8 1777.46 cm^2

9 No, surface area will not double if you double height of cylinder.

10 **a** If $h = r$, $SA = 2\pi rh + 2\pi r^2 = 2\pi r \times r + 2\pi r^2 = 2\pi r^2 + 2\pi r^2 = 4\pi r^2$
b **i** $SA = 6\pi r^2$ **ii** $SA = 3\pi r^2$ **iii** $SA = 8\pi r^2$
c **a** SA = 314.16 cm^2
b **i** SA = 471.24 cm^2 **ii** SA = 235.61 cm^2
iii SA = 628.32 cm^2

11 $A = 2\pi rh$

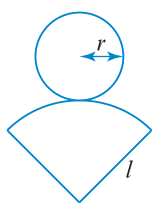
12 Possible answers are: $r = 5$ cm, $h = 1.366$ cm; $r = 3$ cm, $h = 7.61$ cm

13 8 cm

14 **a** 376.99 cm^2
b Lucian has calculated surface area of whole cylinder (ignoring hollowed-out section) and Curtis has forgotten to include area of inside surface.

15 **a** 201.06 cm^2
b 615.75 cm^2
c 314.16 cm^2
d 127.32 cm^2

16 **a**



b **i** 69.12 cm^2 **ii** 358.14 cm^2
iii 392.7 cm^2 **iv** 531.28 cm^2

17 326.88 cm^2

18 $h = \frac{SA - 2\pi r^2}{2\pi r} = \frac{SA}{2\pi r} - r$

19 57.42 cm^2
20 i 141.372 cm^2 ii 56.549 cm^2 iii 197.92 cm^2

7G Volume

7G Start thinking!

3 Count number of cubes in one layer and multiply by number of layers in rectangular prism.
6 Multiply answers from questions 4 and 5 together; that is, multiply number of cubes in bottom layer by number of layers in prism.
7 V = volume of prism, A = area of base of prism, H = height of prism
8 $A = lw$ 9 $V = lwH$
10 $A = \frac{1}{2}bh$ 11 $V = \frac{1}{2}bhH$

Exercise 7G Volume

1 a 450 cm^3 b 180 cm^3 c 105 cm^3
d 200 cm^3 e 209 cm^3 f 156 cm^3
2 a 48 cm^3 b 1080 cm^3 c 288 cm^3
d 324 mm^3 e 300 cm^3 f 48 600 cm^3
3 a 210 cm^3 b 36 cm^3 c 82.5 cm^3
d 375 cm^3 e 1320 cm^3 f 144 cm^3
4 a 424.12 cm^3 b 942.48 cm^3 c 39.27 cm^3
d 904.78 cm^3 e 367.57 cm^3 f 2290.22 cm^3
5 a 540 cm^3 b 2457 cm^3 c 250.25 cm^3
6 a 461.81 cm^3 b 600.35 g ≈ 600 g
7 a $1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$
$= 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$
$= 1000 \text{ mm}^3$
b To find volume of a cube you cube the length of one side and units are also cubed. Since units for volume are cubed, you need to multiply or divide by cube of conversion factor.
c a i 540 000 mm^3 ii 0.000 54 m^3
b i 2 457 000 mm^3 ii 0.002 457 m^3
c i 250 250 mm^3 ii 0.000 250 25 m^3
d It is easier to convert length measurements by multiplying or dividing by 10, 100, 1000 etc. rather than cubed units for which you need to multiply or divide by 10^3, 100^3, 1000^3 etc.
8 a 44178.65 mm^3 or 44.18 cm^3
b 680 coins c 0.4 cm^3
9 a i One possible answer is l = 12 cm, w = 2 cm, H = 10 cm
ii One possible answer is b = 8 cm, h = 5 cm, H = 12 cm
b 30 cm^2
c i 4 cm
ii Possible answers are: l = 12 cm, w = 5 cm, H = 4 cm; l = 10 cm, w = 6 cm, H = 4 cm
10 a Possible answers are r = 2.185 cm, H = 10 cm; r = 3.09 cm, H = 5 cm
b 1.33 cm c 2.82 cm
11 a Volume is amount of space taken up by 3D object. Capacity is amount of liquid a container can hold.
b 1000 cm^3 c 1 000 000 mL = 1000 L
d 461.81 mL
12 Second glass holds 55.32 mL more.
13 6.77 mL
14 a 348 cm^3 b 329.86 cm^3 c 730 cm^3
15 451.26 cm^3 or 451.26 mL
16 a i $V = \frac{1}{3}l^2H$ ii 400 cm^3
b i $V = \frac{1}{3}(\frac{1}{2}bhH) = \frac{1}{6}bhH$ ii 14.33 cm^3
c i $V = \frac{1}{3}(\frac{1}{2}(a+b)h)H = \frac{1}{6}(a+b)hH$
ii 39 cm^3
17 a A cone is not a true pyramid because it has a curved edge, i.e. a circular base.
b $V = \frac{1}{3}\pi r^2 H$
c i 9.42 cm^3 ii 157.08 cm^3
iii 50.27 cm^3
18 a 113.1 cm^3 b 904.78 cm^3 c 268.08 cm^3
d 156.07 cm^3
19 111.26 cm^3 or 111.26 mL

7 Chapter review

MULTIPLE-CHOICE

1 B 2 D 3 A 4 C 5 B
6 C 7 D

SHORT ANSWER

1 a 3.5 cm b 25.752 km c 3750 g
d 270 min
2 a 0.055 m b 975 000 cm c 750 000 g
d 13 500 s
3 9.12×10^2 g, 6.75×10^3 g, 2.36×10^4 g, 3.85×10^4 g, 5.75×10^5 g
4 8.35×10^{-5} g, 5.83×10^{-6} g, 3.58×10^{-7} g, 3.85×10^{-8} g, 5.38×10^{-9} g
5 a 9.6 cm b 32 cm c 28.27 cm
d 23.25 cm e 67.45 cm f 64 cm
6 a 4.32 cm^2 b 60.5 cm^2 c 63.62 cm^2
d 43.01 cm^2 e 218.17 cm^2 f 256 cm^2
7 a 3.66 cm b 7.31 cm
8 118 cm^2 9 167.87 cm^2
10 a 276 cm^2 b 294 cm^2
11 a 1187.52 cm^2 b 180.64 cm^2
12 a 216 cm^3 b 180 cm^3
13 a 3078.76 cm^3 b 176.71 cm^3
14 13 m

NAPLAN-STYLE PRACTICE

1 cubic metres 2 235 m 3 2.9×10^{-26}
4 25 m 5 24 cm 6 15.71 m
7 625 m^2 8 12 mm 9 $A = \frac{1}{2}(a+b)h$
10 12.25 cm^2 11 17.76 cm^2 12 18.5 cm^2
13 9.75 m^2 14 11 cm 15 3.1 cm^2
16 102 m^2 17 905 cm^2 18 0.3 cm^3
19 15 m^3 20 2035.75 cm^3
21 0.00204 m^3, 2.04×10^{-3} m^3

ANALYSIS

a purple area = 900 cm^2, pink area ≈ 37.86 cm^2 ≈ 38 cm^2
b 1 tin   c yes
d 8200 cm^2 or 0.82 m^2
e three coats   f yes, 36 cm^2
g 14 137.17 cm^2 or 1.41 m^2
h two tins   i yes, 5862.83 cm^2 or 0.59 m^2
j 0.0018 m^3 k 1.8×10^{-3} m^3
l 0.1272 m^3 m 1.272×10^{-1} m^3
n 25 cm by 22 cm by 22 cm
o 70 cm by 70 cm by 67 cm

7 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 8 STATISTICS

8 Are you ready?

1 a categorical data   b 29
2 a B b D c E d A e C
3 a D b 5 c 49
4 39
5 a 12 b A

8A Understanding and representing data

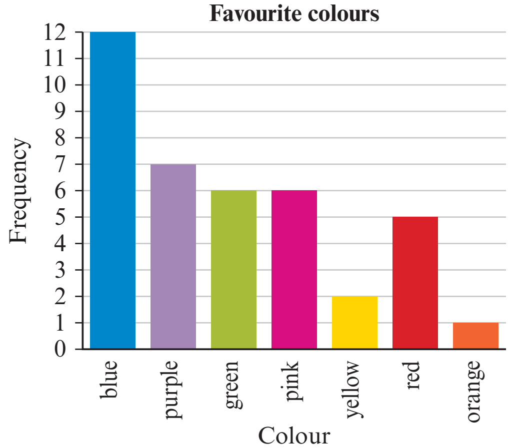
8A Start thinking!

1 Some possible answers are: ages (in years), shoe size, temperature, number of children in a family, weight.
2 Some possible answers are: gender of newborn babies, makes of car, subjects studied in Year 9, top five favourite movies, sporting activities played by Year 9 students.
3 discrete: ages (in years), shoe size, number of children in a family; continuous: temperature, weight
4 nominal: gender of newborn babies, makes of car, subjects studied in Year 9, sporting activities played by Year 9 students; ordinal: top five favourite movies
5 Possible answers are: line graph, histogram, column graph, bar graph, pie graph, scatterplot, box plot, stem-and-leaf plot, dot plot, picture graph, histogram.
6 numerical data: stem-and-leaf plot, line graph, scatterplot, histogram; categorical data: picture graph; both: bar graph, column graph, dot plot, pie graph, box plot
7 Possible comments are: stem-and-leaf plots, line graphs, scatterplots, histograms can show exact values and data which is changing continuously. They can also display large data sets easily.
8 Possible comments are: column graphs, bar graphs, pie graphs and dot plots can represent both numerical and categorical data but are better suited to categorical data because they are visually more appealing and easier to read. It is difficult to represent large amounts of data accurately on these graphs. Bar and column graphs can be difficult to read accurately. Dot plots have difficulty recording large amounts of numerical data. Sectors in pie graphs can be difficult to draw accurately and are best used for displaying up to 5–7 categories.
9 Possible comments are: frequency tables summarise data. Managing and operating data in a table is easier than working with raw data, especially if there is a very large amount of data.

Exercise 8A Understanding and representing data

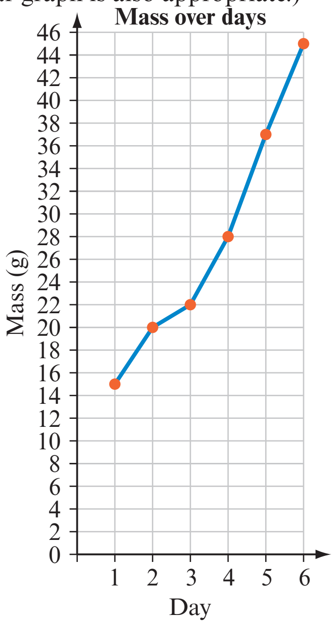
1 a categorical, nominal
b categorical, ordinal
c numerical, discrete   d categorical, nominal
e numerical, continuous
f numerical, continuous
g numerical, discrete   h numerical, discrete
i numerical, continuous
j categorical, nominal
k categorical, ordinal   l numerical, discrete
2 a favourite desserts   b 65 people
c One possible answer is: ice-cream – most people chose this dessert so it is possibly the tastiest. Students could also choose whichever dessert they prefer.
3 a number of customers over time (weeks)
b 10 weeks   c week 6   d week 3
4 a scatterplot; ice-creams sold versus temperature
b Count or estimate number of dots.
c positive correlation – number of ice creams sold increases as temperature increases
d 40 (April temperatures would usually range between 15 and 20°C. Scatterplot shows approx. 40 ice-creams were sold when these temperatures occurred.)
5 a 27   b favourite breakfast foods
c It is too difficult to record large number of data; dot plot would become too cluttered, may not be enough space to record all dots.
6 a 15   b 76
c most common age bracket: 40–49; most common age: 42
d Key tells you how to read the numbers in stem-and-leaf plot. The number on left is the stem and the first digit in the number. The number on right is the leaf, which forms the rest of the number.
7 a categorical   b numerical   c categorical
d numerical   e numerical   f categorical

8 categorical and nominal data

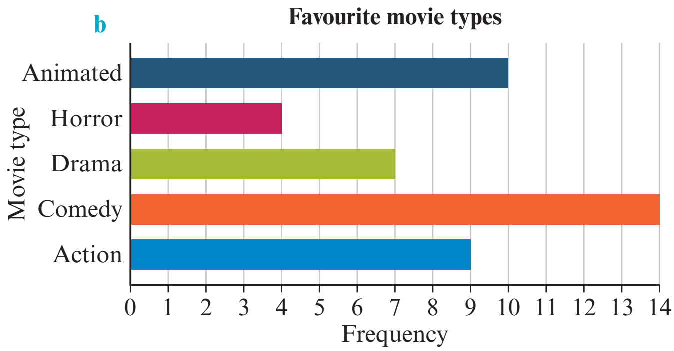


(Bar graph is also appropriate.)

9 a

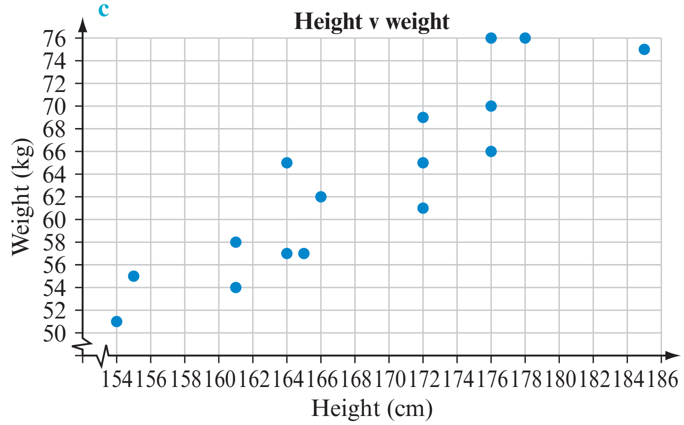


b

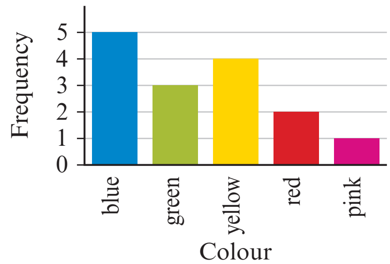


(Column graph is also appropriate.)

c



10 One possible answer is given; a dot plot could also be used.



11

Graph type	Numerical data	Categorical data
stem-and-leaf plot	✓	
line graph	✓	
scatterplot	✓	
histogram	✓	
picture graph		✓
column graph		✓
bar graph		✓
dot plot		✓
pie graph		✓

12 a histogram: graphical display of data using columns of different height. No gaps between columns except for small gap between vertical axis and first column.

stem-and-leaf plot: contains two columns separated by a vertical line. Left column contains the stems (all the tens, hundreds etc. digits) and right-hand column contains the leaves (all the ones); has a key to show how the numbers are to be read.

line graph: shows information that is connected in some way. Points are plotted and connected by a line.

scatterplot: similar to a line graph. Uses a horizontal axis and a vertical axis to plot data points to show relationship between two variables.

column or bar graph: graphical representation of data using columns or bars of different heights. Equal spacing between columns/bars, and columns/bars are same width.

dot plot: consists of a horizontal axis labelled with an even scale. Dots are used to represent each piece of data.

pie graph: uses sector within a circle to show the relative sizes of data.

c histogram: numerical, continuous data. Useful when recording frequency of an event occurring.
stem-and-leaf plot: numerical, discrete data. Used when data sets are not too small or too large and need to see the distribution of data.
line graph: numerical, continuous data. Usually compares changes over time.
scatterplot: numerical, discrete data. Useful when comparing the relationship between two sets of data.
column or bar graph: categorical and nominal or ordinal data. Useful when showing relative size of data.
dot plot: numerical, discrete data. Helps to give quick overview of the size of data and its spread.
pie graph: categorical, nominal data. Useful when displaying and comparing different groups of data that make up a whole event.

13 Numerical, continuous data needs to be rounded before it can be placed in a stem-and-leaf plot. Last digit of rounded number will become the leaf and remaining digits will form the stem.

14 b categorical data, as the ladder lists names of sporting teams
c No point measuring or adding together numbers in sports ladder as they represent statistics for each individual team and are only used to order teams according to their wins and losses.
d Cannot be numerical data as it lists names of teams in order from first to last according to number of wins they have.
e Yes, teams are listed according to number of wins they have. Order shows team with most wins at top and team with the least wins at bottom.
f categorical, ordinal data

15 Difficult to represent large number of data on pie graphs; best for data with no more than five to seven categories. Difficult to draw all sectors accurately. Unless total is provided, difficult to analyse data and to compare two sets of data. Pie graphs better suited to categorical data than numerical data.

16 a i Difficult to compare popularity of each sport when graph is drawn 3D with pyramids rather than columns.
ii Draw graph 2D and use columns rather than pyramids.
b i Not possible to compare size of columns as they are not drawn to scale.
ii Use better scale on vertical axis for frequency. Scale should increase in equal intervals.
c i No information provided to explain what pie graph and each of sectors represent.
ii Pie graph should include heading, description of each sector and percentage (or number) to indicate size of each sector so can read and analyse the graph.

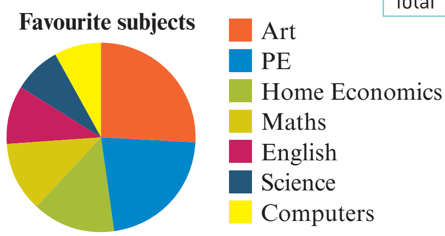
17 a favourite subjects b PE
c Do not know total number of people surveyed to calculate number whose favourite subject was Maths.

d

Subject	Frequency
English	5
Maths	3
Science	4
PE	9
Home Economics	7
Art	8
Total	36

e

Subject	Frequency
English	5
Maths	6
Science	4
PE	11
Home Economics	7
Art	13
Computers	4
Total	50



Most popular subject is now Art.

8B Grouped data

8B Start thinking!

1 39 rows (without the titles)
2 5–10 groups in a frequency table allows it to be manageable and easily read.
3 class intervals 5 cm
4 Students may also use class intervals of 10 cm for table.

Class	Frequency
145–<150	1
150–<155	3
155–<160	5
160–<165	3
165–<170	6
170–<175	6
175–<180	0
180–<185	1
Total	25

5 continuous data and discrete data
6 numerical, continuous data
7 Not possible to place 144.6 cm into a class interval if intervals were written 140–144; 145–149 etc.
9 Data is continuous. Open class intervals allow all measurements to be placed in a class interval.

Exercise 8B Grouped data

1 a

Class	Frequency
0–<5	1
5–<10	2
10–<15	3
15–<20	3
20–<25	2
25–<30	3
30–<35	3
35–<40	1

b

Class	Frequency
10–<20	3
20–<30	3
30–<40	4
40–<50	4
50–<60	4
60–<70	2
70–<80	2

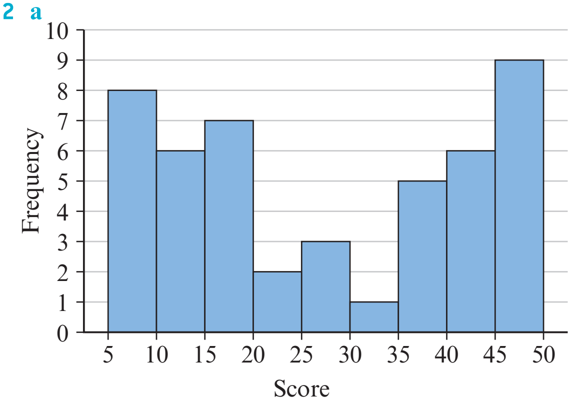
c

Class	Frequency
0–<2	6
2–<4	7
4–<6	6
6–<8	2
8–<10	2
10–<12	4

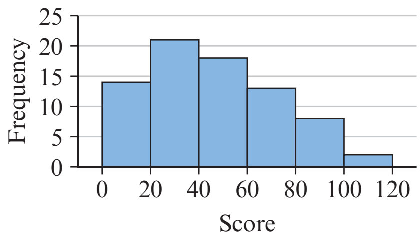
d

Class	Frequency
40–<45	3
45–<50	8
50–<55	7
55–<60	5
60–<65	3
65–<70	3
70–<75	4
75–<80	7

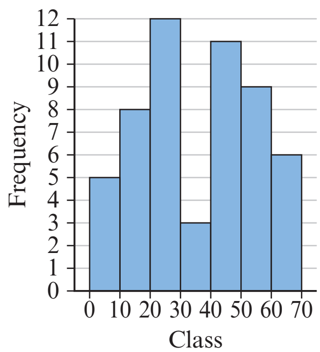
2 a



b

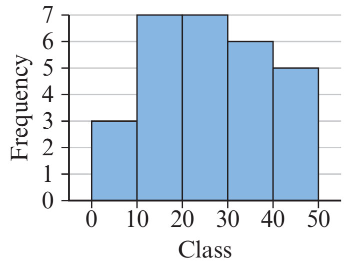


c



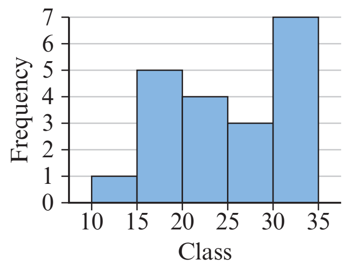
3 a

Class	Frequency
0–9	3
10–19	7
20–29	7
30–39	6
40–49	5



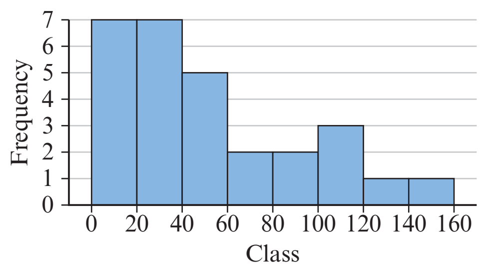
b

Class	Frequency
10–<15	1
15–<20	5
20–<25	4
25–<30	3
30–<35	7



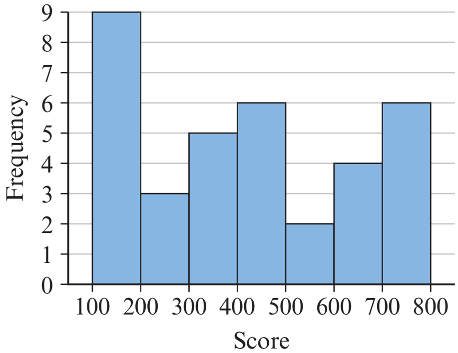
c

Class	Frequency
0–19	7
20–39	7
40–59	5
60–79	2
80–99	2
100–119	3
120–139	1
140–159	1



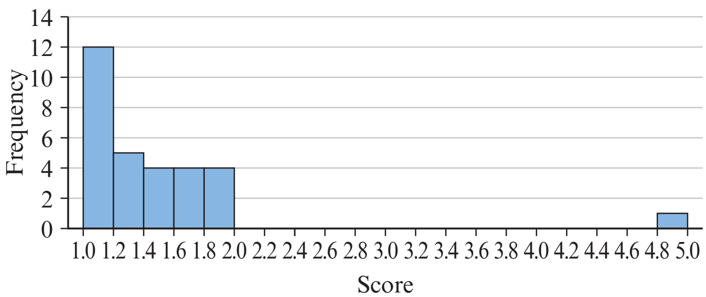
d

Class	Frequency
100–<200	9
200–<300	3
300–<400	5
400–<500	6
500–<600	2
600–<700	4
700–<800	6



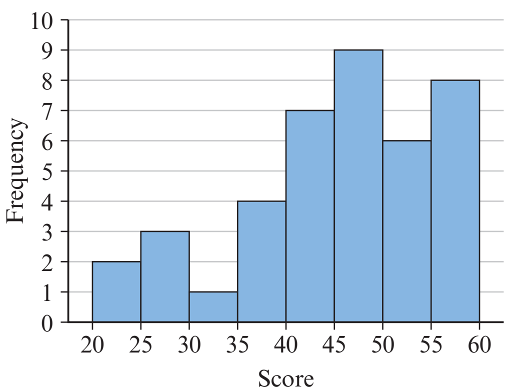
e

Class	Frequency
1.0–<1.2	12
1.2–<1.4	5
1.4–<1.6	4
1.6–<1.8	4
1.8–<2.0	4
4.8–<5.0	1



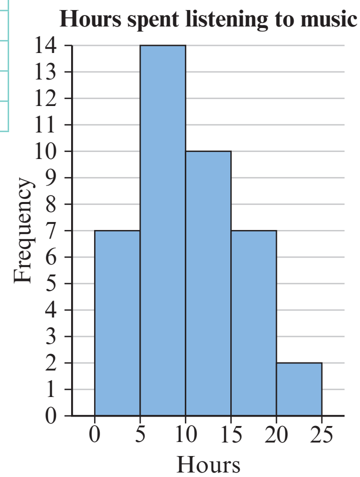
f

Class	Frequency
20–<25	2
25–<30	3
30–<35	1
35–<40	4
40–<45	7
45–<50	9
50–<55	6
55–<60	8



4

Hours	Frequency
0–4	7
5–9	14
10–14	10
15–19	7
20–24	2



5 **a** The stem represents the class intervals.
The stem represents the tens in the data.

b 10; e.g. 10–19, 20–29 etc. **c** 17

d In stem-and-leaf plot, can view each individual piece of data as well as being able to see spread of data.

6 **a** Key: 1 | 2 = 12

Stem	Leaf
0	2 6 7 9
1	1 3 3 4 5 6 7 8 9
2	2 3 5 7
3	1 3 6 6 6 7
4	2 2 6 8
5	1 9
6	7
7	
8	
9	9

b Key: 1 | 2 = 1.2

Stem	Leaf
1	2 6 7 8 9
2	2 2 7 8 8
3	3 4 4 7
4	2 2 3 4 8 8 8
5	1 3 9
6	2

c
Key: 1 | 2 = 12

Stem	Leaf
0	8
1	2 5 6 6 8 8 9 9
2	2 2 6 6 9
3	2 3 3 6 6
4	2 2 2 3 3 6 7 8 9
5	2 5 6 9
6	1 4 6 8
7	2 3 4 5 7
8	1 5 8
9	2 5
10	2 8
11	1 7

d
Key: 11 | 2 = 112

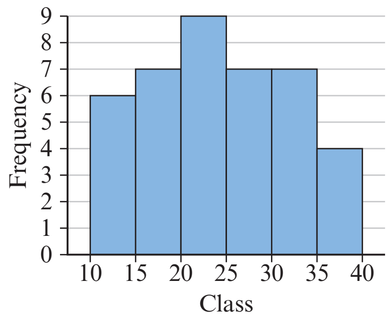
Stem	Leaf
11	2 3 3 3 8 9 9 9 9
12	1 4 6 6 7 7
13	2 2 2 3 4 4 7 8
14	2 2 2 2 3 4 6
15	3 3
16	2 3 4 4 5 5 6 7
17	2 7

7 **a** too many class intervals; table is too long and difficult to read

b

Ages of customers	Frequency
0–<10	2
10–<20	61
20–<30	44
30–<40	20
40–<50	6
≥50	6

8 A stem-and-leaf plot is a histogram on its side. The data has been organised into class intervals and the frequency can be easily calculated.

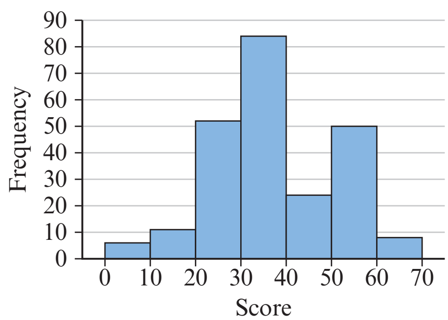


9 Histogram does not provide specific data within each class interval; only provides frequency of data for each class interval.

10 **a** number of Facebook friends
b 50 **c** 350–400; 28
d 0–50; score 40 lies between 0 and 50, the first class interval

11 Cannot accurately decrease size of class intervals, because without raw data it is not possible to determine number in each of smaller class intervals.

12 **a** Label axes; include title for histogram; use correct scales on both axes.
b



c 30 –<40, frequency = 84 **d** 235

13 **a** 5 **b** 25–29 **c** 2*

14 Key: 1 | 2 = 12

Stem	Leaf
0	2
0*	5 6 7 7 7 8 8 8 9 9 9 9
1	0 0 1 1 1 2 2 2 2 3 3 3 3 3 4 4
1*	6 6 6 6 7 7 8 9 9 9
2	0 0 1 2 2 4

The majority of the data falls within the range 5 to 20. The most popular class interval is 10–14.

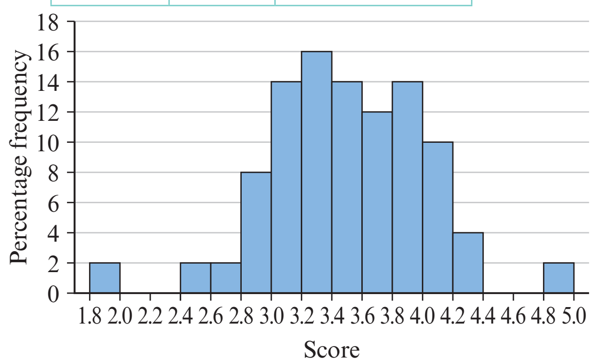
15 One possible answer is: Use four different symbols to split the classes into intervals of 2; e.g. the stems could be 6, 6*, 6+, 6^, 6′.

16 **a** i 4 ii 0.4 iii 400 iv 0.04
b i 57 ii 5.7 iii 5700 iv 0.57
c i 10 ii 1 iii 1000 iv 0.1
d i 53 ii 5.3 iii 5300 iv 0.53

18 **a** The vertical axis represents percentage frequency rather than frequency.
b 16% **c** 14%
d 100%; because percentages of a whole add to 100%
e i 56 ii 196 iii 248

19

Weight	Frequency	Percentage frequency
1.8–<2.0	1	2
2.0–<2.2		
2.2–<2.4		
2.4–<2.6	1	2
2.6–<2.8	1	2
2.8–<3.0	4	8
3.0–<3.2	7	14
3.2–<3.4	8	16
3.4–<3.6	7	14
3.6–<3.8	6	12
3.8–<4.0	7	14
4.0–<4.2	5	10
4.2–<4.4	2	4
4.4–<4.6		
4.6–<4.8		
4.8–<5.0	1	2



8C Summary statistics

8C Start thinking!

1. 25
2. 12, 12, 12, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 15, 15, 15, 15, 15, 16, 16, 16, 16, 16, 17, 18; mode = 14
3. median = 14
4. Scores must be ordered to find the score in the centre of all scores.
5. 14.48
6. 6; range = highest score − lowest score. Easier to find lowest and highest scores when they are written in numerical order.

Exercise 8C Summary statistics

1 **a** i 6 ii 5 iii 2 iv 10
b i 5 ii 5 iii 5 iv 9
c i 11 ii 12 iii 12 iv 14
d i 24.4 ii 22 iii no mode
iv 23

2 **a** **i** 8.75 **ii** 8.5 **iii** no mode
iv 13
b **i** 5 **ii** 4.5 **iii** 2 and 4 **iv** 7
c **i** 12 **ii** 9.5 **iii** 8 and 10
iv 29
d **i** 121 **ii** 122 **iii** 122 and 148
iv 72

3 **a** 2.36 **b** 21.67 **c** 15.88
d 1.12 **e** 24.33 **f** 3.09

4 **a** 2 **b** 20 **c** 16 **d** 1 **e** 25 **f** 2

5 **a** 2; 4 **b** 10, 30; 30 **c** 17; 5
d 1; 4 **e** 30; 15 **f** 2; 18

6 **a** mean = 3, median = 3, mode = 3, range = 4
b mean = 6.2, median = 6, mode = 5, range = 5
c mean = 12.36, median = 12, mode = 13, range = 5

7 mean = 4.21, median = 4, mode = 3, range = 9

8 Categorical data does not contain any numerical scores. You can only find the most popular/common category that is the mode.

9 **a** mean = 15.52, median = 16, mode = 16, range = 4
b mean = 17.31, median = 16, mode = 16, range = 49
c It increases mean slightly and increases range greatly.
d An outlier will either decrease or increase the mean, depending on whether it is a much smaller or much larger score. Median usually not affected by outlier.
e median, as it is not affected by outlier

10 **a** median **b** mean **c** median **d** mean

11 **a** mean = 23.375, median = 21, mode = 18, range = 32

b

Ages	Frequency
15–19	17
20–24	11
25–29	5
30–34	3
35–39	2
40–44	1
45–49	1
Total	40

c 15–19; mode for raw data (18) falls within the modal class.
d 20–24; median for raw data (21) falls within the median class.
e 34; it is very close to the range for the raw data.

f, g

Ages	Frequency	Midpoint	Midpoint × frequency
15–19	17	17	289
20–24	11	22	242
25 –29	5	27	135
30–34	3	32	96
35–39	2	37	74
40–44	1	42	42
45–49	1	47	47
Total	40		925

h mean = $\frac{925}{40}$ = 23.125
i This mean is very close to mean found using raw data, only a difference of tenths.

12 **a** mean = 38.65, median = 42, mode = 46, range = 51

b **i**

Number of ice-creams	Frequency	Midpoint	Midpoint × frequency
5–9	1	7	7
10–14	1	12	12
15–19	2	17	34
20–24	2	22	44
25–29	2	27	54
30–34	2	32	64
35–39	3	37	111
40–44	5	42	210
45–49	7	47	329
50–54	3	52	156
55–59	3	57	171
Total	31		1 192

ii

Number of ice-creams	Frequency	Midpoint	Midpoint × frequency
0–9	1	4.5	4.5
10–19	3	14.5	43.5
20–29	4	24.5	98
30–39	5	34.5	172.5
40–49	12	44.5	534
50–59	6	54.5	327
Total	31		1 179.5

iii

Number of ice-creams	Frequency	Midpoint	Midpoint × frequency
0–14	2	7	14
15–29	6	22	132
30–44	10	37	370
45–59	13	52	676
Total	31		1192

c **i** mean = 38.45, median = 40–44, modal class = 45–49, range = 54
ii mean = 38.05, median = 40–49, modal class = 40–49, range = 59
iii mean = 38.45, median = 30–44, modal class = 45–59, range = 59

d Comments could include: Mean varies slightly; summary statistics may not be as exact; median and mode might not fall within correct class interval; the table with class interval of 10 was the most accurate.

13 a **i** 2.56 **ii** 26.37 **iii** 4.72

b The second set of data has a large spread of data from the mean (standard deviation = 26.37), but the other two data sets have a small spread of data from the mean (standard deviations = 2.56 and 4.72 respectively).

8D Summary statistics on displays

8D Start thinking!

1 No numerical values to help calculate mean, median, range or interquartile range. Can only determine mode by looking at size of sectors.

2 Pie graph usually displays categorical data. Categorical data does not contain numerical values, which can be used to find mean, median, range or interquartile range. Can only determine mode by looking at size of sectors; the larger the sector, the more popular the category.

3 mode and range

4 mode = 4, range = 4

5 First create a table that lists each score and its frequency. For mean, multiply each score by its frequency, add results and divide by total number of scores. For median, add cumulative frequency column and then locate the $\frac{n+1}{2}$ score using this column.

6, 7, 8

Score	Frequency	Score × frequency	Cumulative frequency
1	6	6	6
2	9	18	15
3	11	33	26
4	13	52	39
5	11	55	50
Total	50	164	

mean = 3.28, median = 3

Exercise 8D Summary statistics on displays

1 a mean = 2.47, mode = 3, median = 3, range = 3

b mean = 38.44, mode = 50, median = 40, range = 40

c mean = 13.06, mode = 5, median = 10, range = 20

2 chocolate

3 a 30 people **b** mode = 5, range = 9

c

Hours	Frequency
1	3
2	4
3	2
4	6
5	8
6	4
7	2
8	0
9	0
10	1
Total	30

mean = 4.3, median = 4.5

4 mean = 4.7, median = 5, mode = 4, range = 5

5 a four bedrooms **b** 4

c

Number of bedrooms	Frequency
1	2
2	9
3	11
4	14
5	6
Total	42

mean = 3.31, median = 3

6 a mean = 2.3, mode = 2, median = 2, range = 5

b mean = 16.15, mode = 16, median = 16, range = 6

c mean = 1.33, mode = 1, median = 1, range = 4

7 a mean = 55.43, median = 59, mode = 42, range = 42

b mean = 2.7, median = 2.3, mode = 1.8, range = 4

c mean = 280.39, median = 250, mode = 310, range = 670

8 a

Heights	Frequency
160–<165	3
165–<170	7
170–<175	9
175–<180	14
180–<185	23
185–<190	30
190–<195	17
195–<200	12
200–<205	8
205–<210	2
Total	125

b modal class = 185–<190 cm, median class = 185–<190 cm, range = 50 cm

c mean = 185.58 cm

9 a A histogram has data grouped into class intervals and uses the midpoint of class interval, rather than individual scores, to find mean. Median and mode (modal class) are given as class intervals rather than as specific values and range is estimated by using end points. Column graphs usually display the data individually.

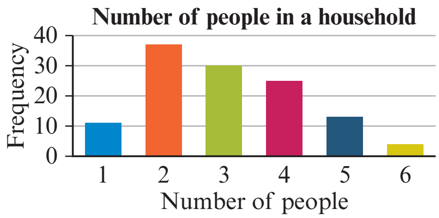
b One possible answer is: when there is a large amount of continuous, numerical data.

10 a

Number of people	Frequency
1	11
2	37
3	30
4	25
5	13
6	4
Total	120

b mean = 3.03; median = 3; mode = 2; range = 6 − 1 = 5

c One possible answer is: too difficult to read or too time consuming to calculate numerical data.

11 

12 a B b D c A d C

13 a mean = 24.86, median = 23, mode = 23, range = 46
b 12.58
c Mean, median and mode are all close in value with median and mode being equal. You can choose median or mean to represent data.
d Measures of spread vary in value. Standard deviation is quite high.

14 mean = 3.96, median = 4, mode = 3, range = 9, s = 1.77. Use mean to represent measures of centre for the data. There are no outliers and scores are clustered together, so mean would be an accurate representation of the data. Can use standard deviation to represent spread of data. It indicates that scores have a small spread and are clustered around measures of centre.

8E Collecting data

8E Start thinking!

1 1: categorical, 2: categorical, 3: categorical, 4: numerical, 5: numerical
2 Data type determines which summary statistics and which type of graphical displays can be used to analyse the data. Easier to find summary statistics for numerical data and there are more options for graphical displays. Categorical data only allows for mode to be found and a limited number of graphical displays.
3 Question 1 is asking for a factual response whereas Question 3 is asking for an opinion or preference.
4 Question 1 tells you which pets people actually have at home, therefore it is a more factual response. Question 3 asks for a preference and therefore will not provide accurate responses for the most popular pet.
5 Question 1 or Question 4; both are asking for a factual response.
6 All the people living in his suburb.

Exercise 8E Collecting data

1 a census b sample c sample
d census e census f sample

2 a i all students who attend the school
ii sample or census (depending on size of school)
b i Australian population ii sample
c i your friends ii census
d i teenagers in your town/suburb
ii sample
e i Australian households ii sample
f i local cinemas ii census

3 a systematic b random c systematic
d stratified e random f random

4 Biased: 1c (her class may not be fair representation of entire year level), 1f (sample may be too small and may not represent entire population), 3a (may not represent entire population as there are some people in local community who may not visit this shopping centre), 3b (sample may to be too small and may not represent the population in Adrian's survey), 3f (sample may be too small and may not represent entire population)
Fair: 1a (entire population has been surveyed), 1b (smaller number of people who represent population have been selected at random), 1d (every member of population has been surveyed), 1e (entire population has been surveyed), 3c (systematic approach should enable a fair cross-section of population), 3d (sample chosen which is in proportion to size of population), 3e (everybody in population has an equal chance of being selected)

5 a biased; should sample a representative number of high school students (not adults)
b fair; has used appropriate sampling method
c biased; sample may be too small and may not represent entire population
d biased; question does not provide for adequate answers.

6 Sample answers are given.
a Name your favourite movie currently playing at cinemas. Provide a list of five movies and have students rate them from 1 to 5 in order of preference. How often have you visited a cinema in the last 3 months?
b Has your property ever been affected by graffiti: yes or no?; Which of the following age groups do you think participates in graffiti: Under 12, 13–20, 21–40, over 40?; Is graffiti a valid art form: yes, no, unsure?
c How many members in your family?; How many children in your family?; How many males and females in your family?
d How many pieces of technology do you have in your home?; Name the most commonly used piece of technology used within your family home?; How many hours a day do you or other family members use a computer at home?

7 Sample answers are given.
a What do you think is the most popular movie at the cinemas this week?
b Do you like graffiti?
c What do you think is the average number of children per family in your community?
d What type of technology do you like using the most?

8 a 100 people; the larger the sample, the more likely it will reflect opinion of population
b 10 people from a group of 50. This represents 20% of the population. 20 people from a group of 200 represents 10% of the population.
c The larger the sample size, the more likely you will get answers which truly reflect the population.
d **i** not very trustworthy, sample very small
ii not very trustworthy, sample very small
iii trustworthy, all deaths considered
iv not very trustworthy, sample very small.
e An investigation that uses a large sample size is more likely to reflect the population than one that uses a small sample size.

9 Small sample sizes mean that data might not represent point of view or responses of population.

10 Sampling method should be fair to get a good representation of population so that results reflect views of entire population.

12 a 100 **b** $\frac{60}{100} = \frac{3}{5}$ **c** $\frac{40}{100} = \frac{2}{5}$
d 6 boys, 4 girls e 15 boys, 10 girls
f for boys, multiply sample size by $\frac{3}{5}$; for girls, multiply sample size by $\frac{2}{5}$
g The year level comprises of $\frac{3}{5}$ boys and $\frac{2}{5}$ girls. Multiply the sample size (n) by each fraction to find the number of boys and girls respectively to be surveyed.

13 a 6 girls, 4 boys **b** 35 adults, 15 children
c 12 Year 8s, 13 Year 9s
d 2 birds, 3 cats, 4 dogs

8F Describing data

8F Start thinking!

1 Graph B. Data peaks in centre and columns on both sides of centre column trail off in height. Left and right sides of middle column are roughly mirror images of each other.
2 No, not always possible to have exactly same amount of data, and hence same size columns, on either side of centre column. As long as graph is roughly symmetrical, central scores can be used to analyse data.
3 Graph C. Data peaks to left of the graph, close to y-axis, and then trails off to right.
4 positively skewed
5 Graph A said to be bi-modal because it has two peaks, which indicates two modes for this set of data.

Exercise 8F Describing data

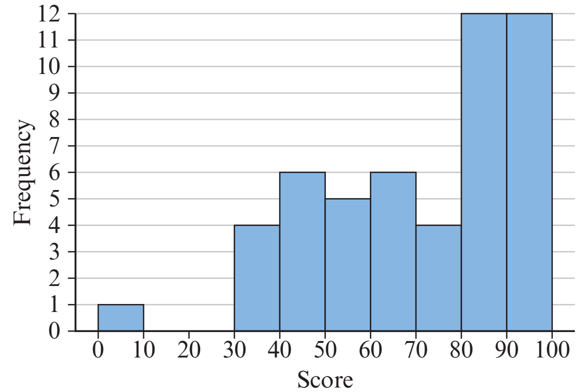
1 a bi-modal **b** negatively skewed
c positively skewed with an outlier
d symmetric

2 a symmetric **b** positively skewed
c negatively skewed with an outlier

3 a mean or median **b** median
c mode of each peak **d** median

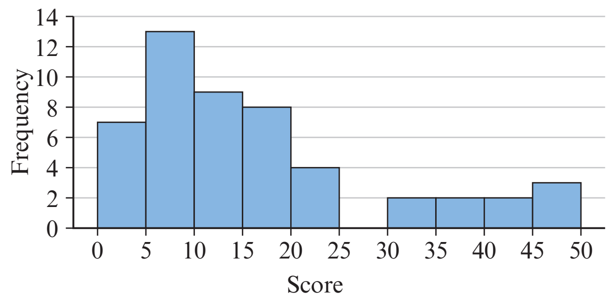
4 a mode of each peak **b** median
c median **d** mean or median

5 a i



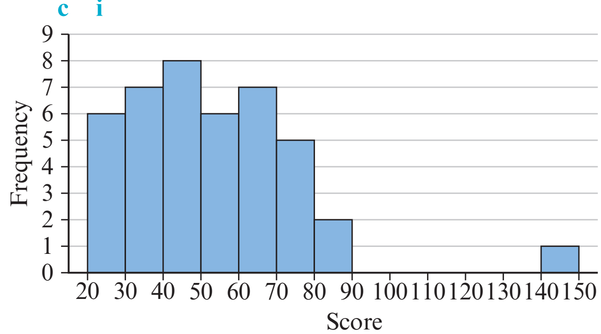
ii negatively skewed
iii median. Graph is skewed and there is an outlier present, therefore the mean may not be an accurate measure of centre.

b i



ii positively skewed
iii median. Graph is skewed, therefore the mean may not be an accurate measure of centre.

c i



ii symmetric with an outlier
iii median. An outlier is present, therefore the mean may not be an accurate measure of centre.

6 a i Key: 1 | 3 = 13

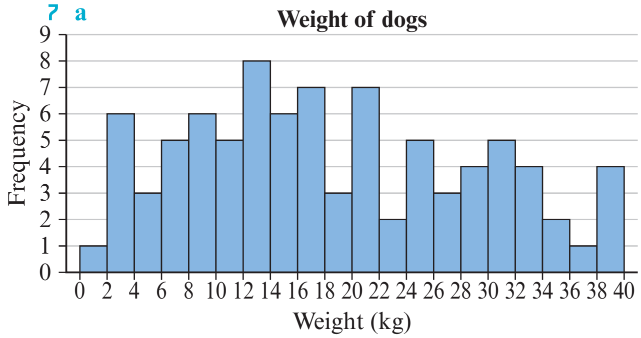
Stem	Leaf
1	0 0 0 1 2 2 2 2 2 2 3 3
1*	5 5 5 5 5 6 6 7 8 9
2	0 0 1 2 2 2 3 3 3
2*	9

ii Key: 1 | 3 = 13

Stem	Leaf
1	0 0 0 1
1*	2 2 2 2 2 3 3
1^	5 5 5 5 5
1#	6 6 7
1+	8 9
2	0 0 1
2*	2 2 2 3 3 3
2^	
2#	
2+	9

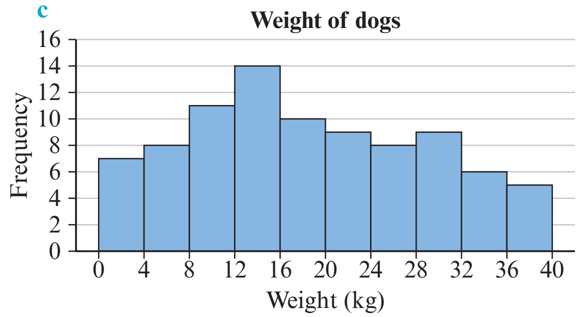
b **i** positively skewed **ii** bi-modal

7 **a**



b no pattern visible

c



d very slightly positively skewed

e Weight of dogs ranges from below 2 kg up to 40 kg, with a centre around 12–16 kg.

8 **a** symmetric **b** mean

c

Number of people	Frequency
0–<10	3
10–<20	4
20–<30	9
30–<40	12
40–<50	8
50–<60	6
60–<70	2
Total	44

d mean = 35

e One possible answer is: the majority of the time there are between 30 and 40 people at the skate park.

9 **a** **i** positively skewed **ii** median = 2

iii The majority of homes surveyed have one or two TVs with the most common number of TVs in each home being two.

b **i** bi-modal

ii mode of each peak = 6 cm and 32 cm

iii Year 9 students surveyed have either short hair ranging from 1 to 10 cm in length or longer hair ranging from 30 to 40 cm in length.

c **i** symmetrical with an outlier

ii median is 35–<40

iii Ages of people at a hairdressers in a week usually ranges from 15 to 60 years, with median age being between 35 and 40 years.

d **i** symmetrical

ii mean = 15.4, median = 15.5

iii Ages of students at a pool ranges from 13 to 18 years, with median age 15.5 years and mean age 15.4 years.

10 **a** Graph B: as age increases so does nose hair length. Data points form a path that rises from left to right.

b Graph A: data points are clustered close to a straight line.

c Data points do not display any type of pattern. Not possible to determine if points are rising or falling and there is no evidence of a straight line.

d **i** positive direction, moderate

ii negative direction, weak

iii no direction, no strength

11 Graph A: Number of jackets sold decreases strongly as temperature increases.
Graph B: Number of nose hairs increases slightly as age increases.

d **i** Ability to play basketball increases moderately as height increases.

ii Soft drink consumption decreases slightly as age increases.

iii no correlation or relationship between hair colour and pet ownership

12 **a** Many jackets would be sold.

b good basketball ability

c Probably, but not always. Only a moderate relationship between height and basketball ability, therefore cannot be certain a tall person will be good at basketball.

d Height versus age. Height increases as age of a person increases.

e They are assuming that height will continue to increase at same rate after 17 years of age. People stop growing once they reach adulthood.

f Not always possible to predict if trend displayed will continue outside data range.

8G Comparing data

8G Start thinking!

1 two 'leaf' parts instead of one; one 'stem'
2 minimum = 2, maximum = 79
3 minimum = 17, maximum = 62

4 females: 77 years; males: 45 years
5 symmetrical with an outlier
6 Distribution of the males in store is more positively skewed; that is, there are more scores towards top of stem-and-leaf plot.
7 Some possible answers are: the ages of males and females are spread roughly over same age brackets. Both males and females between 10 and 40 years of age are most frequent visitors to gaming store. Male visitors to gaming store tend to be older than female visitors.
8 Easier to compare data if placed in same plot, e.g. range, age group most likely to visit gaming store, overall spread of data.
9

Gaming store visitors	Mean	Median	Range
Males	32	30.5	45
Females	30	28	77

10 Range for females (51) is greater than range for males (45). Does not necessarily prove that males visiting gaming store are more likely to be older, only shows that spread of ages of male is less than ages of females. Other measures must be examined to determine if males are likely to be older than those of females.
11 Mean and median for males is greater than mean and median for females. This indicates that males visiting gaming store are slightly older than females visiting the store.

Exercise 8G Comparing data

1 a i 41 ii 45 iii 45
b i 18 ii 24 iii 18 and 24
c i positively skewed ii symmetrical
2 a Group A and group B have a similar spread of data but are skewed in different directions. Group A has a larger centre than group B.
b Group A has a smaller range and a symmetric distribution, whereas group B has a larger range due to the presence of an outlier (and is otherwise symmetric). The median should be used as a measure of centre for group B, which has a slightly smaller centre than group A.
c Both groups have a symmetric distribution and have a similar centre, but group B is spread over a larger range.
d Both groups are negatively skewed, but group A has a smaller centre than group B, even though group B has an outlier present and a larger range.

3 a Key: 2|9 = 29

Leaf Friday	Stem	Leaf Saturday
9	2	6
2 0	3	7
9 8 6 2	4	2
9 8 7 7 6 5 4 3 1	5	5 9
7 7 5 4 1 1	6	4 6 7 8
6 2 1	7	5 5 6 7 7 8 8 9
8	8	1 3 4 4 5 9
2	9	2 4 9

Data sets are spread roughly over same range but data set for Friday night is symmetrical, whereas data set for Saturday night is negatively skewed. More people seem to attend cinema on Saturday night than on Friday night.

b Key: 32|9 = 32.9

Leaf Adelaide	Stem	Leaf Darwin
6	20	
5	21	
7	22	
7	23	
	24	
4	25	
6 3	26	
0	27	
7 4 3 3 0	28	
4	29	5 9
	30	0 4 8
2	31	3 4 8
1	32	1 1 2 3 5 5 6 7 8 8 9
8 8 4	33	0 1 2 3 4 4 4 5 8
3 2	34	
6	35	
0	36	
1	37	
4 2	38	
2	39	
5	40	

Adelaide has a much larger range of temperatures than Darwin. Adelaide seems to have a roughly symmetric distribution with a lower average temperature than Darwin. Darwin's temperatures cover a small range and appear to be negatively skewed, with a higher average temperature than Adelaide.

c Key: 10|9 = 10.9 kg

Leaf Boston Terriers	Stem	Leaf French Bulldogs
9 9 9 6 6	4	
5 5 1 1	5	
8 8 7 4 2	6	
9 8 8 6 6 5 5	7	
6 6 4 3 3 2 1 0	8	6
9 2 2 2 2	9	1 2 2 4 5 5 6 8 8 8 8 9
5 5 5 0	10	1 5 6 6 8 9 9
2 0	11	0 1 1 2 2 3 5 5 5 8 9
	12	0 3 5 5 7 9 9 9
	13	0

Both dog breeds appear to have roughly symmetric distributions, with Boston Terriers covering a larger range of masses. French Bulldogs appear to be heavier on average than Boston Terriers.

4 a Average heights of students in both classes is similar but 9A has a larger range of heights. The larger range suggests that 9A heights may be affected by presence of an outlier. Median should be used as measure of centre.

b Average price of jeans and range of prices in both stores are similar. Distributions seem to be symmetrical. Mean should be used as measure of centre.

c Average use of Internet in secondary school is greater than average use in primary school. Range of hours is also greater in secondary school. Distribution for primary school may be positively skewed and distribution for secondary school may be negatively skewed. Median should be used as measure of centre.

d Both stores have the same range and median of storage capacity in their cards, however store A has a larger mean than than store B (and larger than its median), which suggests that store A has a skewed distribution and the median is a better representation of centre.

e Players A and B have the same median but player B has a larger mean. Combined with the smaller range, it appears that player B is a more consistent goal kicker with a negatively skewed distribution, which would make him a better goal kicker.

5 a DVD sales per hour in six stores for years 2000, 2005 and 2010

b three sets

c i positively skewed ii symmetrical
iii negatively skewed

d DVD sales per hour have decreased over the years.

e 2000: Stores A, B, C sold most DVDs per hour, with store A making most sales per hour. 2005: Stores A, B, C again sold most DVDs per hour, with store C making most sales per hour. 2010: Sales per hour are fewer, with stores D, E, F making most sales per hour, with store E having the most sales of DVDs per hour.

6 One possible example is:
Key: 14|2 = 142 cm

Leaf Year 7	Stem	Leaf Year 9
9	12	
4 2	13	
9 8 6 5 4 3 2 2 1	14	6
9 8 7 4 3 1	15	3
7 5 1 1	16	1 2 2 3 4 5 7 9
2 1	17	1 4 5 5 6 7 7 7 8 8 8
5	18	1 3 4
	19	4

Heights of Year 7 and Year 9 students have similar ranges, but distributions are skewed in different directions. Year 7 student heights are positively skewed, whereas Year 9 student heights are negatively skewed. Overall, Year 9 students are taller than Year 7 students.

7 Sample C is a reasonable size. Mean and median are close in value and range is large enough to include a good representation of all age groups. Other two samples have smaller sample size and smaller range; hence may not be enough data to make a sound analysis.

8 Sample B is a reasonable size and is taken from a wider cross-section of population. Mean and median are close in value and spread of data is not too great.

9 a The range of heights is similar for boys and girls but boys' heights are slightly skewed in negative direction. Boys seem to be taller overall.

b

Class heights	Mean (cm)	Median (cm)	Range (cm)
Boys	174	173.5	33
Girls	164	162	34

c Range of heights for Year 9 boys and girls is almost identical. Measures of centre are close but mean and median for boys is slightly higher, indicating the boys are slightly taller than the girls.

d Yes; mean and median indicate that the boys are slightly taller than the girls and ranges of heights for boys and girls are almost identical.

12 a dot plot 1: negatively skewed; dot plot 2: symmetrical

b dot plot 1: mean = 16.6, median = 17, range = 6
dot plot 2: mean = 15.2, median = 15, range = 6

c Scores in both data sets are spread over same range, 6. Data set in first dot plot has higher mean, median and mode than data set in second dot plot. Therefore, students participating in school play in term 1 tended to be older than students who participated in school play in term 4.

d Plot one set of data above line and second set of data below line.

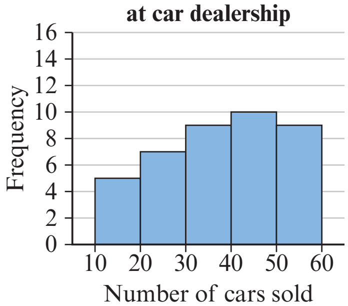
13 a histogram: mean = 36.58, median class = 30–<40, range = 80; stem-and-leaf plot: mean = 31.87, median = 29, range = 62

b Data represented in histogram is symmetrical but there is an outlier present. Data shown in stem-and-leaf plot is positively skewed. First set of data has greater spread and its mean is higher than that of second set of data. Could be due to presence of outlier in first data set. Median class and median are close in value, therefore median would be best measure of centre to use to compare these two sets of data and to determine average age of customers in a milk bar.

c Mean, median, mode, range and other statistical values can be calculated using given data, and can then be used to compare sets of data which are displayed differently.

14 a Class intervals used are not same size.

b **Number of cars sold per month at car dealership**



c Both histograms display data spread over same range. First histogram displays data that is negatively skewed. Car dealership represented in first histogram sold average of 30–40 cars a month, with most sold in any month being 40–60 cars. Second histogram displays data that is symmetrical. Car dealership represented by second histogram also sold average of 30–40 cars, which is also the modal class for this data.

d We do not know how the data is spread over the class intervals to enable us to draw the histogram with smaller class intervals.

15 histogram 1: mean = 37.88, median class = 35–<40, modal class = 45–<50
histogram 2: mean = 34.25, median class = 30–<40, modal class = 30–<40
Data from first histogram has slightly higher mean and median class. Modal class of first histogram is significantly greater than that of the second histogram. Dealership in first histogram sells more cars per month, on average, than dealership represented by second histogram. First dealership was also able to sell most number of cars in one given month.

8 Chapter review

MULTIPLE-CHOICE

1 A **2** D **3** D **4** C **5** B

SHORT ANSWER

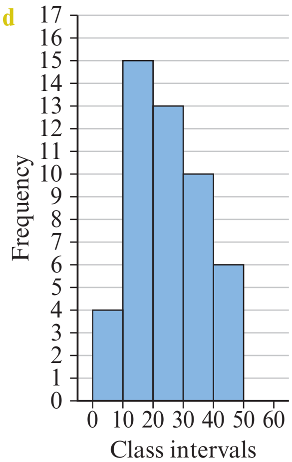
1 a

Class interval	Frequency
0–<10	4
10–<20	15
20–<30	13
30–<40	10
40–< 50	6
Total	48

b 48

c 10–<20

d



e Key: 2|0 = 20

Stem	Leaf
0	2 5 5 7
1	0 1 1 4 5 5 7 7 8 8 8 8 9 9 9
2	0 0 1 1 1 1 2 3 4 5 7 7 8
3	1 1 2 2 2 3 5 7 7 8
4	0 1 2 3 5 8

f Key: 2|0 = 20

Stem	Leaf
0	2
0*	5 5 7
1	0 1 1 4
1*	5 5 7 7 8 8 8 8 9 9 9
2	0 0 1 1 1 1 2 3 4
2*	5 7 7 8
3	1 1 2 2 2 3
3*	5 7 7 8
4	0 1 2 3
4*	5 8

g 15–19

2 mean = 4.46, median = 5, mode = 5, range = 3

3 a mean = 27.1, median = 24.6, mode = 25.7, range = 41.3

b 58.9

c Expect mean to change and median to remain same.

d Both change: mean = 23.9, median = 24.1

4 mean = 22.84, median = 21, mode = 7, range = 56

5 Class 9A: mean = 66.32, median = 65, mode = 45, range = 47
Class 9B: mean = 67.14, median = 64, mode = 48, range = 53

6 a Key: 1|1 = 11

Leaf School 1	Stem	Leaf School 2
1 1	0	2 2 4 4
9 8 8 7 7 7 5 5	0*	6 6 7
1 1 1 0 0	1	0 1 1 1 2 4 4
7	1*	6 6 6 7 7 9 9
2 2 1 1	2	0 2
7	2*	
2	3	1
8 5	3*	

b school 1: mean = 14.4, median = 10.5, range = 37; school 2: mean = 12.8, median = 13, range = 29

c In discussing the results, median will give best measure of centre, as in school 1 mean is affected by inclusion of two outliers.

NAPLAN-STYLE PRACTICE

1 pizza sizes
2 surveying your class about their favourite television show
3 20 4 One possible answer is: Key: 2|7 = 27
5 192 6 15.7% 7 92 8 9.38%
9 19 10 Mode will be unchanged. 11 $99
12 The median height is 152 cm.
13 Graph B

ANALYSIS

a Brisbane: 22 shrubs were measured, Sydney: 24 shrubs were measured

b Brisbane: mean = 13.34, median = 13.15, mode = 11.4, range = 4.5
Sydney: mean = 13.33, median = 13.25, mode = 11.4, range = 4.1

c Brisbane: Sx = 1.47; Sydney: Sx = 1.49

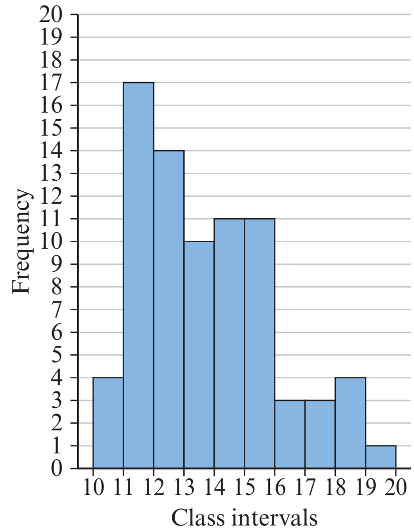
d Data for locations is very similar: 22 shrubs measured in Brisbane and 24 shrubs measured in Sydney, mean and mode for locations almost identical. Median slightly higher for Sydney. The standard deviations are very close, and low in value, indicating that spread of results from mean is small in both locations. Could be said with confidence that average shrub height was 13.3 cm.

e

Class interval	Frequency
10–<11	4
11–<12	17
12–<13	14
13–<14	10
14–<15	11
15–<16	11
16–<17	3
17–<18	3
18–<19	4
19–<20	1
Total	78

f 78 plants
g 11–<12

h



Data displayed in histogram is positively skewed.

i Victoria average = 14.53, WA average = 13.78

j average = 13.75 cm

k If purchasing a shrub from this nursery chain, it will most likely be in height range of 10–20 cm, with average shrub height being 13.75 cm.

8 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 9 PROBABILITY

9 Are you ready?

1 D 2 a A b D 3 D
4 a 5 b $\frac{13}{20}$
5 a B b 4 c $\frac{1}{4}$
6 a 25 b 6 c 12
7 a 2 b 3 c 18

9A Theoretical probability

9A Start thinking!

1 $\frac{1}{100}$
2 Purchase more raffle tickets. The greater the number of raffle tickets purchased, the greater the chance one of your tickets will be drawn.

3 If you purchase seven tickets, you have 7 out of the possible 100 tickets which could win the raffle: in other words, 7 out of 100 chances of winning.

4 Number of favourable outcomes indicates how many tickets you have and total number of outcomes indicates how many tickets are in the raffle. Formula gives probability (or chance) we expect to have in winning the raffle.

5 50 tickets

6 You have 1 out of 8 145 060 chances of winning the raffle.

7 4 072 530 tickets

8 There are a greater number of tickets to choose from in the raffle. In question 5 there were 100 tickets in the raffle, but in question 7 there were 8 145 060 tickets in the raffle.

Exercise 9A Theoretical probability

1 **a** very likely **b** highly unlikely **c** likely **d** impossible **e** even chance **f** certain **g** unlikely **h** likely

2 **a** {1, 2, 3, 4, 5, 6}
b { T, E, C, H, N, O, L, G, Y}
c {clubs, spades, hearts, diamonds}
d {yellow, green, red, blue, pink, purple}
e {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
f {picture, non-picture}

3 **a** equally likely **b** not equally likely **c** not equally likely **d** equally likely **e** not equally likely **f** equally likely

4 **a** equally likely **b** not equally likely **c** equally likely **d** not equally likely **e** equally likely **f** not equally likely

5 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{7}$ **d** $\frac{1}{13}$ **e** $\frac{1}{4}$ **f** $\frac{2}{11}$

6 **a** $\frac{1}{2}$ **b** $\frac{3}{13}$ **c** $\frac{4}{9}$ **d** $\frac{5}{11}$ **e** $\frac{5}{6}$ **f** $\frac{5}{12}$

7 **a** $\frac{1}{3}$ **b** $\frac{4}{9}$ **c** $\frac{5}{9}$ **d** $\frac{4}{9}$ **e** $\frac{1}{9}$ **f** $\frac{2}{9}$

8 **a** **i** {red, blue, yellow, purple, green}
ii blue; as there are more blue sectors on the spinner
iii $\frac{1}{9}$
b **i** {red, blue, yellow, purple, green}
ii Red; the red sector occupies a larger section of the circle than the other colours.
iii $\frac{3}{8}$
c **i** {red, blue, yellow, purple, green}
ii Blue; the blue sector occupies a larger section of the circle than the other colours.
iii $\frac{1}{4}$

9 Some possible answers are: 'even chance' means each outcome has probability of $\frac{1}{2}$, or a 50-50 chance of occurring. 'Equally likely' means each outcome has same chance of occurring, so each probability is same.

10 **a** Rolling a die; each outcome (1, 2, 3, 4, 5, 6) is equally likely, with probability of $\frac{1}{6}$.
b Gender of newborn baby; each outcome (boy, girl) is equally likely, with probability of $\frac{1}{2}$.
c Spinning a spinner with half coloured blue, a quarter coloured red and a quarter coloured yellow; probability that arrow lands on blue is $\frac{1}{2}$ but not all outcomes are equally likely to occur.

11 Some possible answers are given.
a **i** {black card, red card}
ii {picture card, non-picture card}
iii Pr(drawing a black card) = $\frac{1}{2}$
iv Pr(drawing a black picture card) = $\frac{3}{26}$
b **i** {C, O, M, P, U, T, E, R}
ii {vowel, consonant}
iii Pr(selecting C) = $\frac{1}{8}$
iv Pr(selecting a vowel) = $\frac{3}{8}$
c **i** {5c, 10c, 20c, 50c, \$1, \$2}
ii {silver coin, gold coin}
iii Pr(selecting a 5c coin) = $\frac{1}{6}$
iv Pr(selecting a gold coin) = $\frac{1}{3}$
d **i** {Mon, Tue, Wed, Thu, Fri, Sat, Sun}
ii {week day, weekend day}
iii Pr(selecting Monday) = $\frac{1}{7}$
iv Pr(selecting a week day) = $\frac{5}{7}$
e **i** {square, star, rhombus, pentagon, triangle, rectangle, clover shape, circle, cross}
ii {red shape, green shape, blue shape, yellow shape}
iii Pr(selecting a triangle) = $\frac{1}{9}$
iv Pr(selecting a red shape) = $\frac{1}{3}$

12 **a** Adele used number of colours as total outcomes rather than number of sectors in circle. Correct statement: Adele said that the probability of spinning purple on the spinner from question 8a was $\frac{1}{9}$.
b Thanh is assuming there is not an equal chance of flipping a head or tail because he has already flipped a tail five times. Correct statement: Thanh flipped a coin five times and got tails each time. He then said he had an equal chance of getting another tail.
c Ethan assumed that 'equal chance' and 'even chance' meant the same probability. Correct statement: Ethan said that he had an equal chance, or $\frac{1}{6}$ chance, of rolling a 6 on a fair die.
d Bianca assumed landing the spinner on blue was highly likely because blue is largest sector. However, blue occupies just less than half the circle. Correct statement: Bianca said that she had a less than even chance of spinning blue on the spinner from question 8c.

13 **a** \$50 gift voucher, \$20 gift voucher, \$5 gift voucher

b There are five \$50 gift vouchers out of total of 30 gift vouchers. Hence, chance of selecting a \$50 gift voucher is $\frac{5}{30}$, not $\frac{1}{3}$.

14 **a** $\frac{6}{13}$ **b** $\frac{7}{13}$ **c** 1

d **i** rolling a die and not obtaining a 6 (that is, obtaining a 1, 2, 3, 4, or 5)

ii not selecting a heart card from a deck of cards (i.e. selecting a club, spade or diamond card)

iii flipping a coin and obtaining a head

iv rolling a die and obtaining an odd number

v not selecting a picture card from a deck of cards (i.e. selecting a number card from a deck of cards)

vi selecting a vowel from this sentence

e **i** Pr(the event) = $\frac{1}{6}$, Pr(complementary event) = $\frac{5}{6}$

ii Pr(the event) = $\frac{1}{4}$, Pr(complementary event) = $\frac{3}{4}$

iii Pr(the event) = $\frac{1}{2}$, Pr(complementary event) = $\frac{1}{2}$

iv Pr(the event) = $\frac{1}{2}$, Pr(complementary event) = $\frac{1}{2}$

v Pr(the event) = $\frac{3}{13}$, Pr(complementary event) = $\frac{10}{13}$

vi Pr(the event) = $\frac{23}{35}$, Pr(complementary event) = $\frac{12}{35}$

f The sum of the probability of each event and its complement equals 1 in every case.

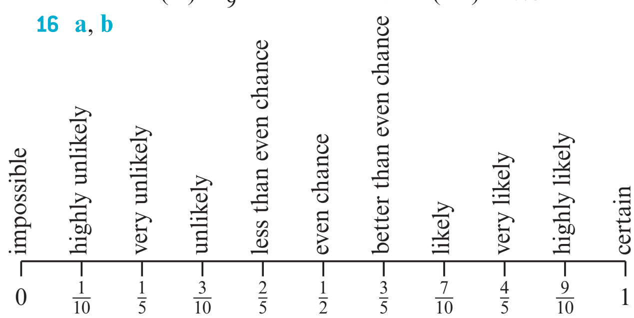
g The sum of the probability of complementary events always equals 1.

15 **a** Pr(A) + Pr(A′) = 1

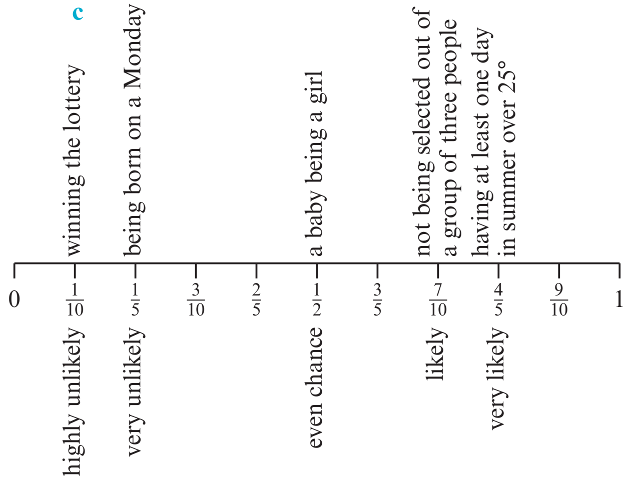
b **i** Pr(E) = 0.3 **ii** Pr(W′) = $\frac{2}{3}$

iii Pr(Y) = $\frac{1}{9}$ **iv** Pr(M′) = 0.84

16 **a**, **b**



c

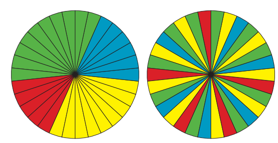


d Descriptors do match probability of winning lottery. There are usually a large number of tickets in a lottery and hence a very low chance that one particular ticket will be drawn.

e One possible answer is: events that are close to middle of the scale have a closer to equal chance of occurring than the events that are closer to ends of scale.

f One possible answer is: use smaller intervals on the number line so there is more choice for probabilities.

17 **a** $\frac{3}{10}$ **b**



9B Experimental probability and relative frequency

9B Start thinking!

1 {1, 2, 3, 4, 5, 6}

2 Pr(1) = $\frac{1}{6}$, Pr(2) = $\frac{1}{6}$, Pr(3) = $\frac{1}{6}$, Pr(4) = $\frac{1}{6}$, Pr(5) = $\frac{1}{6}$, Pr(6) = $\frac{1}{6}$

3 10

4 Multiplied number of rolls by probability of rolling a 6, that is, $60 \times \frac{1}{6}$.

5 Pr(x) is the probability of rolling a 6 = $\frac{1}{6}$ and n is the number of rolls = 60.
Hence, $E(x) = Pr(x) \times n = \frac{1}{6} \times 60 = 10$

6 E(x) represents the expected number of times the required outcome will occur; Pr(x) represents the probability of the required outcome; n represents the number of trials.

7 E(1) = E(2) = E(3) = E(4) = E(5) = E(6) = 5

8 The frequencies are not all equal to the expected number in question 7. The only outcome that has the same frequency as its expected number is 4. The others vary slightly, except for 6, which has a frequency double its expected number.

9

Outcome	1	2	3	4	5	6
Frequency	2	6	4	5	3	10
Theoretical probability	$\frac{1}{6} \approx 0.17$	$\frac{1}{6} \approx 0.17$	$\frac{1}{6} \approx 0.17$	$\frac{1}{6} \approx 0.17$	$\frac{1}{6} \approx 0.17$	$\frac{1}{6} \approx 0.17$
Relative frequency	$\frac{1}{15} \approx 0.07$	$\frac{1}{5} \approx 0.2$	$\frac{2}{15} \approx 0.13$	$\frac{1}{6} \approx 0.17$	$\frac{1}{10} \approx 0.1$	$\frac{1}{3} \approx 0.33$

Exercise 9B Experimental probability and relative frequency

1 **a** 0.1 **b** 0.05 **c** 0.56 **d** 0.435
e 0.368 **f** 0.2

2 **a** 125 **b** 40 **c** 25 **d** 5 **e** 105 **f** 60

3 **a** $\frac{8}{15} \approx 0.53$
b In the long term we would expect experimental probability of rolling a die and obtaining a number greater than 2 to increase as it approaches theoretical probability of 0.67.

4 **a** 30 **b** 0.05
c In the long term we would expect experimental probability of drawing a picture card to increase as it approaches theoretical probability of 0.23.

5 In the long term we would expect experimental probability of flipping three coins and obtaining three tails to increase (from 0.11) as it approaches the theoretical probability of 0.125.

6 **a** **i** 10
ii Pr(head) = $\frac{1}{2}$, Pr(tail) = $\frac{1}{2}$
iii E(head) = 5, E(tail) = 5
iv Pr(head) = 0.8, Pr(tail) = 0.2
v not enough trials
vi There are only 10 trials; too few to tell if prop is biased.
b **i** 6000
ii Pr(1) = Pr(2) = Pr(3) = Pr(4) = Pr(5) = Pr(6) $= \frac{1}{6} \approx 0.17$
iii E(1) = E(2) = E(3) = E(4) = E(5) = E(6) = 1000
iv Pr(1) = 0.16, Pr(2) = 0.16, Pr(3) = 0.17, Pr(4) = 0.17, Pr(5) = 0.17, Pr(6) = 0.17
v Prop is fair.
vi Frequency for each outcome is very close to expected number of each outcome. Relative frequency for each outcome is approximately equal to its theoretical probability after large number of trials.
c **i** 112
ii Pr(hearts) = Pr(diamonds) = Pr(clubs) = Pr(spades) = $\frac{1}{4}$
iii E(hearts) = E(diamonds) = E(clubs) = E(spades) = 28
iv Pr(hearts) = 0.34, Pr(diamonds) = 0.28, Pr(clubs) = 0.17, Pr(spades) = 0.21
v Prop is biased.
vi Not all frequencies are close to expected numbers of each outcome. Hearts and diamonds are much more and clubs and spades are less than expected. Relative frequencies for all outcomes are not approximately equal to their theoretical probabilities after large number of trials.

7 **a** {HHH, HHT, HTH, HTT, THH, TTH, THT, TTT}; 2 **b** $\frac{1}{4}$ **c** 10
e Answers will vary but relative frequencies should be approaching theoretical probabilities.
f Relative frequency should get closer to theoretical probability after 4000 trials.

8 Experiment has 36 outcomes of which 6 are doubles, i.e. {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}. Theoretical probability of rolling a ‘double’ number is $\frac{1}{6}$. After 30 trials you would expect 5 ‘double’ numbers to be rolled.

9 **a** heads: 25; tails: 15
b one: 18; two: 24; three: 21; four: 12; five: 15; six: 30
c hearts: 12; diamonds: 18; clubs: 21; spades: 9

10 **a** **i** 2 **ii** 4 **iii** 6 **iv** 3
b **i** coin; spinner; die; spinner
ii Some possible answers are given.
i Coin might not be tossed same way each time. Coin might not be completely fair, i.e. might not have equal chance of coming up heads or tails.
ii Spinner might not be spun with same strength each time. Spinners sectors might not be exactly same size.
iii Die might not be rolled in same manner each time. Die might not be fair.
iv Spinner might not be spun with same strength each time. Spinner’s sectors might not be exactly same size.

11 **a** Five will be losing chocolate bars and only one will be the winning chocolate bar, hence there are six outcomes: five losses and one win.
b Device selected will be either a die or a spinner with six equal sectors.

12 **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** yes
e At the start you are choosing one door out of three, therefore there are two out of three $\left(\frac{2}{3}\right)$ chances of choosing incorrectly. Once one door is revealed to show a goat behind it, it leaves two doors to choose from.
f If you stay with the original door chosen your chance of being correct remains the same $\left(\frac{1}{3}\right)$ and the other unopened door now has the rest of the probability $\left(\frac{2}{3}\right)$ of being correct. Hence, it is better to switch.
g $\frac{1}{1\,000\,000}$

h Switch; choosing correct door when choice is 1 out of 999 999 $\left(\frac{1}{999\,999}\right)$ is better than staying with original choice, which gives you 1 chance out of 1 000 000 $\left(\frac{1}{1\,000\,000}\right)$.

m After 2000 trials results should better reflect relative frequency of 0.67 for switching and 0.33 for staying.

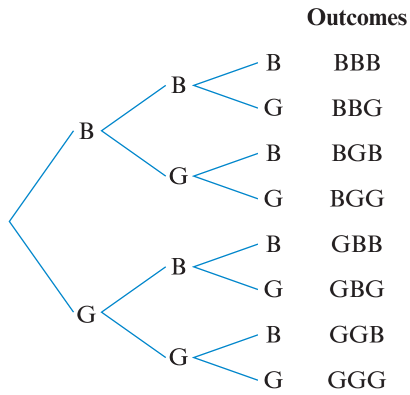
9C Tree diagrams

9C Start thinking!

1 possible gender of two children born in a family

2 2 **3** 4; {BB, BG, GB, GG} **4** 3 **5** $\frac{3}{4}$

6



7 7 **8** $\frac{7}{8}$

9 Possibilities for each step are represented by branches. Final outcomes are listed at ends of branches.

Exercise 9C Tree diagrams

1 a 36 **b** 11 **c** 6

2 a 9 **b** 5 **c** $\frac{5}{9}$

3 a $\frac{7}{16}$ **b** $\frac{1}{8}$ **c** $\frac{1}{16}$ **d** $\frac{9}{16}$ **e** $\frac{3}{4}$ **f** $\frac{1}{16}$

4 a $\frac{1}{36}$ **b** $\frac{3}{4}$ **c** $\frac{1}{4}$ **d** $\frac{11}{36}$ **e** $\frac{1}{6}$ **f** $\frac{5}{36}$

5 a $\frac{1}{4}$ **b** $\frac{5}{16}$ **c** $\frac{1}{8}$

6 a $\frac{1}{2}$ **b** $\frac{1}{8}$ **c** $\frac{3}{8}$ **d** $\frac{3}{8}$

7 a $\frac{2}{9}$ **b** $\frac{19}{27}$ **c** $\frac{1}{9}$ **d** $\frac{1}{27}$

8 a $\frac{1}{32}$ **b** $\frac{31}{32}$ **c** $\frac{10}{32} = \frac{5}{16}$

d $\frac{16}{32} = \frac{1}{2}$ **e** $\frac{26}{32} = \frac{13}{16}$ **f** $\frac{6}{32} = \frac{3}{16}$

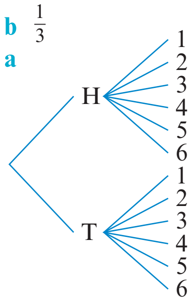
9 a i rock, scissors; paper, rock; scissors, paper

ii rock, paper; paper, scissors; scissors, rock

iii rock, rock; paper, paper, scissors, scissors

b $\frac{1}{3}$

10 a



b 12

c i $\frac{1}{12}$ **ii** $\frac{1}{4}$

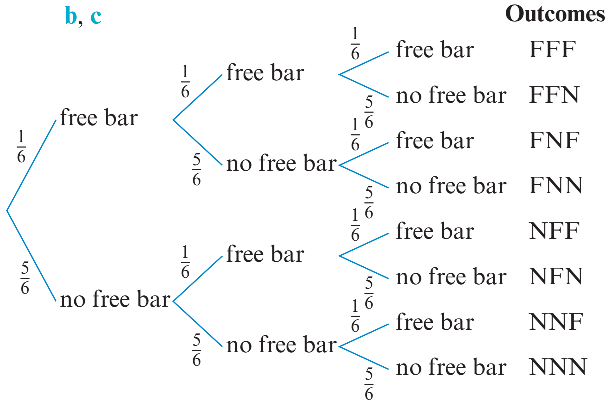
iii $\frac{7}{12}$ **iv** $\frac{3}{4}$

d Flipping a tail and rolling a six is counted as one outcome, not two.

11 a $\frac{1}{16}$ **b** $\frac{15}{16}$ **c** $\frac{1}{2}$ **d** $\frac{3}{8}$ **e** $\frac{11}{16}$ **f** $\frac{11}{16}$

12 a Probabilities are written on branches. Probability of winning free chocolate bar is written on branch leading to 'Free bar'.

b, c



d $\Pr(FFF) = \frac{1}{216} \approx 0.005$,

$\Pr(FFN) = \Pr(FNF) = \Pr(NFF) = \frac{5}{216} \approx 0.023$,

$\Pr(FNN) = \Pr(NFN) = \Pr(NNF) = \frac{25}{216} \approx 0.116$,

$\Pr(NNN) = \frac{125}{216} \approx 0.579$

e total = 1

f i $\frac{125}{216} \approx 0.579$ **ii** $\frac{1}{216} \approx 0.005$ **iii** $\frac{25}{216} \approx 0.116$

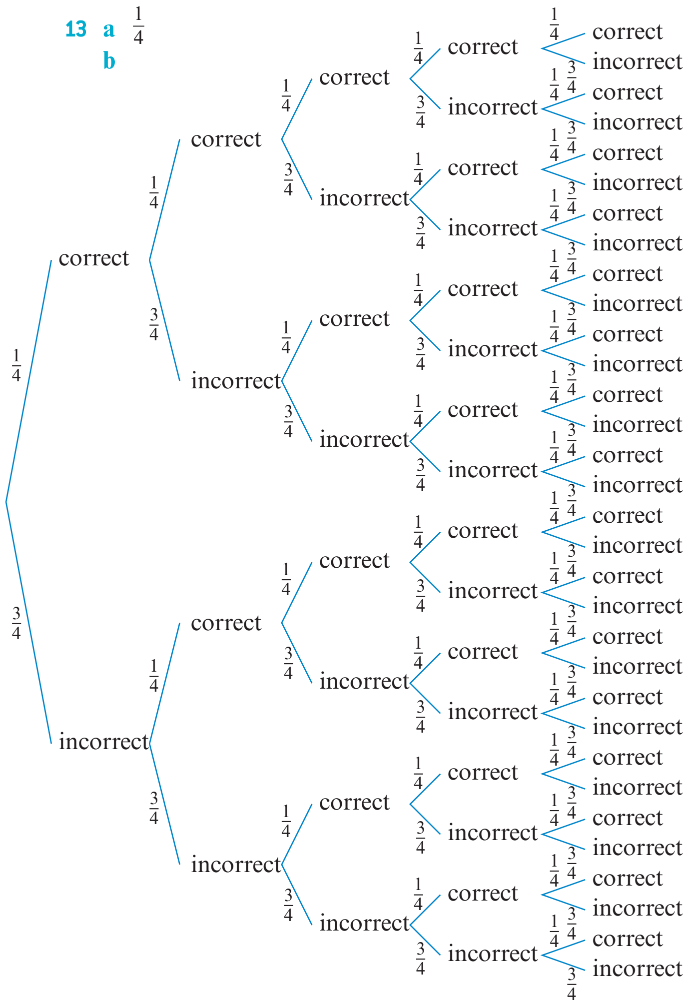
g FNN, NFN, NNF; $\frac{25}{72}$

h i $\frac{91}{216}$ **ii** $\frac{1}{6}$ **iii** $\frac{5}{72}$ **iv** $\frac{2}{27}$

i Follow the branches and multiply the probabilities listed on the required branches together.

13 **a** $\frac{1}{4}$

b



c **i** $\frac{1}{1024}$ **ii** $\frac{243}{1024}$ **iii** $\frac{135}{512}$ **iv** $\frac{781}{1024}$ **v** $\frac{63}{64}$ **vi** $\frac{53}{512}$

14 **a**

Scenario	Number of branches at each trial	Number of trials	Number of outcomes
Flipping a coin four times	2	4	16
Selecting a card from a deck three times and noting its suit	4	3	64
Rolling a die twice	6	2	36
Recording the gender of three children at birth	2	3	8
Recording two 'throws' in rock-paper-scissors	3	2	9

b Number of outcomes equals number of branches at each trial raised to the power of number of trials conducted. In other words, repeat multiplication of number of branches according to number of trials conducted.

c Flipping a coin has two outcomes, a head or a tail. Flipping coin five times gives the number of outcomes as:
$2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$.

d Use number of branches as base and number of trials as power and then calculate.

e Calculate number of expected outcomes using pattern found above. Write down outcomes and then count how many you have to see if number of outcomes agrees with expected number of outcomes you found at start.

15 **a** 16; FFFF, FFFN, FFNF, FFNN, FNFF, FNFN, FNNF, FNNN, NFFF, NFFN, NFNF, NFNN, NNFF, NNFN, NNNF, NNNN

b $\frac{1}{5} = 0.2$ **c** $\frac{4}{5} = 0.8$

d $\Pr(\text{FFFF}) = \frac{1}{625} \approx 0.0016$

$\Pr(\text{FFFN}) = \Pr(\text{FFNF}) = \Pr(\text{FNFF}) =$
$\Pr(\text{NFFF}) = \frac{4}{625} \approx 0.0064$

$\Pr(\text{FFNN}) = \Pr(\text{FNFN}) = \Pr(\text{FNNF}) =$
$\Pr(\text{NFFN}) = \Pr(\text{NFNF})\ \Pr(\text{NNFF}) =$
$\frac{16}{625} \approx 0.0256$

$\Pr(\text{FNNN}) = \Pr(\text{NFNN}) = \Pr(\text{NNFN}) =$
$\Pr(\text{NNNF}) = \frac{64}{625} \approx 0.1024$,

$\Pr(\text{NNNN}) = \frac{256}{625} \approx 0.4096$

e **i** $\frac{256}{625} \approx 0.4096$ **ii** $\frac{1}{625} \approx 0.0016$

iii $\frac{64}{625} \approx 0.1024$ **iv** $\frac{369}{625} \approx 0.5901$

v $\frac{96}{625} \approx 0.1536$ **vi** $\frac{1}{5} = 0.2$

16 $\frac{1}{1296}$

9D Two-way tables

9D Start thinking!

1. First row shows outcomes for first child. Second row shows outcomes for second child. All four possible combination or pairs are then listed in table.
2. hair colour and eye colour
3. 16 **4** 20 **5** 40
6. **a** $\frac{3}{5}$ **b** $\frac{3}{10}$ **c** $\frac{1}{2}$ **d** $\frac{2}{5}$
7. Alice used total number of people with dark hair in denominator of probability fraction. She should have used total number of people surveyed (40) in denominator to get an answer of $\frac{16}{40} = \frac{2}{5}$
8. Each outcome does not have an equal chance of occurring as there is a different number of people in each category.
9. A two-way table displays outcomes clearly in a grid and helps list all possible outcomes so that no outcomes are missed. Number for each possible outcome or combination of outcomes can be read quickly and easily.

Exercise 9D Two-way tables

1 **a** males who prefer sweet food; males who prefer savoury food; females who prefer sweet food; females who prefer savoury food

b 100 **c** 29 **d** 48

e females who prefer sweet food

f total number of people who prefer savoury food

2 **a** 55 **b** 7 **c** $\frac{7}{55}$

3 **a** 75 **b** 11 **c** $\frac{11}{75}$

4 **a** $\frac{29}{85}$ **b** $\frac{37}{85}$ **c** $\frac{42}{85}$ **d** $\frac{23}{85}$

5 **a** $\frac{8}{39}$ **b** $\frac{41}{78}$ **c** $\frac{22}{39}$ **d** $\frac{23}{78}$

6 **a** $\frac{22}{75}$ **b** $\frac{91}{150}$ **c** $\frac{41}{75}$ **d** $\frac{67}{150}$

7

	Green	Not green	Total
Two holes	3	6	9
Four holes	1	6	7
Total	4	12	16

8 **a** $\frac{48}{125}$ **b** $\frac{19}{50}$ **c** $\frac{13}{125}$ **d** $\frac{31}{50}$ **e** $\frac{181}{250}$ **f** $\frac{27}{125}$

9

	Cinema	Home	Total
Action	22	30	52
Comedy	19	14	33
Total	41	44	85

b **i** $\frac{41}{85}$ **ii** $\frac{14}{85}$ **iii** $\frac{52}{85}$

10 **a**

		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b 11 **c** 7; $\Pr(7) = \frac{1}{6}$

d 2 and 12; $\Pr(2) = \Pr(12) = \frac{1}{36}$

e **i** $\frac{1}{12}$ **ii** $\frac{1}{12}$ **iii** $\frac{1}{2}$ **iv** $\frac{5}{12}$

11 **a**

	Year 8	Year 9	Total
Maths	12	11	23
English	10	17	27
Total	22	28	50

b **i** $\frac{23}{50}$ **ii** $\frac{14}{25}$ **iii** $\frac{11}{50}$

c **i** 28 **ii** 11 **iii** $\frac{11}{28}$

d Part c iii says student is in Year 9, hence total number of possible outcomes restricted to total number of Year 9 students (28). In part b iii you are selecting from total number of students, as you need to select a student in Year 9 who likes Maths.

e **i** likes English; 27

ii Year 8 students who like English

iii 10 **iv** $\frac{10}{27}$

12 **a** $\frac{13}{81}$ **b** $\frac{37}{96}$ **c** $\frac{1}{5}$ **d** $\frac{13}{43}$ **e** $\frac{71}{181}$ **f** $\frac{4}{5}$

13

	Cat	No cat	Total
Specific breed	29	75	104
No specific breed	84	12	96
Total	113	87	200

$\Pr(\text{specific breed cat}) = \frac{29}{200}$

14 **a** $\frac{18}{79}$ **b** $\frac{63}{316}$ **c** $\frac{4}{79}$ **d** $\frac{119}{316}$ **e** $\frac{16}{63}$ **f** $\frac{2}{9}$
g $\frac{253}{316}$ **h** $\frac{61}{79}$ **i** $\frac{197}{316}$ **j** $\frac{14}{79}$ **k** $\frac{47}{316}$ **l** $\frac{47}{63}$

15 Part k considers entire population (all cars) when finding probability of event. Part l looks at probability of outcome given a condition: i.e. car is white given that car is a Holden.

17 **a**

	Women	Men	Total
Bath	0.33	0.09	0.42
Shower	0.22	0.36	0.58
Total	0.55	0.45	1.00

b **i** $\frac{9}{25}$ **ii** $\frac{21}{50}$ **iii** $\frac{11}{50}$

c **i** 210 **ii** 275 **iii** 45

9E Venn diagrams

9E Start thinking!

1 'boy' and 'Year 9' **2** 9 and 6 **3** 15
4 6 and 7 **5** 13 **6** 6 **7** 7
8 not a boy and not in Year 9 **9** 30

10 **a** $\frac{1}{5}$ **b** $\frac{1}{2}$ **c** $\frac{13}{30}$ **d** $\frac{3}{10}$ **e** $\frac{7}{30}$ **f** $\frac{4}{15}$

11 Some possible answers are: Venn diagrams show relationship between different sets of data more clearly. It is easy to read numbers in a Venn diagram but errors can be made when calculating totals because of the intersecting part of the circles.

Two-way tables show numbers clearly and there is less chance of miscalculating probabilities. However, they can only be used for two events and there are more calculations to perform when finding probabilities.

Exercise 9E Venn diagrams

1 **a** people who like PE only
b people who do not like PE
c people who like Art
d people who like Art or PE
e people who do not like Art or PE
f people who like Art and PE

2 **a** 14 **b** 15 **c** 2 **d** 31

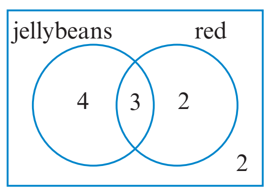
3 **a** 62 **b** 19 **c** 532 **d** 637

4 **a** $\frac{2}{13}$ **b** $\frac{7}{26}$ **c** $\frac{11}{26}$ **d** $\frac{19}{26}$ **e** $\frac{3}{26}$ **f** $\frac{11}{26}$

5 **a** $\frac{17}{45}$ **b** $\frac{1}{9}$ **c** $\frac{19}{45}$ **d** $\frac{1}{5}$ **e** $\frac{23}{45}$ **f** $\frac{31}{45}$

6 **a** $\frac{9}{41}$ **b** $\frac{21}{41}$ **c** $\frac{11}{41}$ **d** $\frac{8}{41}$ **e** $\frac{22}{41}$ **f** $\frac{33}{41}$

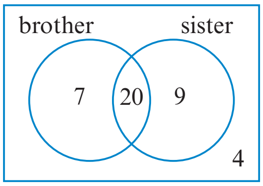
7



8 **a** $\frac{7}{11}$ **b** $\frac{3}{11}$ **c** $\frac{2}{11}$ **d** $\frac{4}{11}$ **e** $\frac{4}{11}$ **f** $\frac{2}{11}$

9 a 27 + 29 + 4 = 60 but there are only 40 people in the group. Hence, there must be some overlap: some people have been counted twice as they have both a brother and a sister.

b

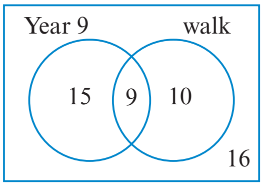


c 36 **d** 27

e 9 **f** 20

g 7 (for parts c–g see figure in part b.)

10 a

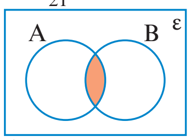


11 a $\frac{29}{61}$ **b** $\frac{29}{46}$ **c** $\frac{32}{61}$ **d** $\frac{9}{25}$

12 a $\frac{11}{30}$ **b** $\frac{11}{26}$ **c** $\frac{15}{26}$ **d** $\frac{6}{21}$

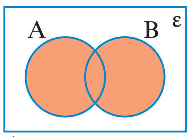
13 a ε

c



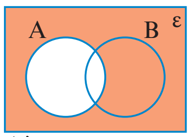
A ∩ B

d



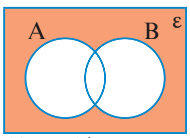
A ∪ B

e



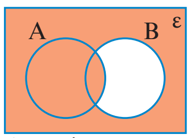
A′

f



(A ∪ B)′

g



A ∪ B′

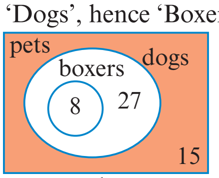
h Part f: brackets mean first find A ∪ B, then find and shade complement of A ∪ B.
Part g: no brackets, so first locate complement of B, then shade union of A and B′.

14 a **i** 29 **ii** 43 **iii** 26 **iv** 10 **v** 50 **vi** 14

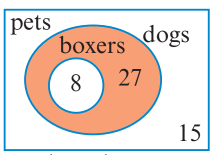
b **i** $\frac{12}{25}$ **ii** $\frac{21}{50}$ **iii** $\frac{1}{5}$ **iv** $\frac{7}{50}$ **v** $\frac{7}{50}$ **vi** $\frac{31}{50}$

15 a The first Venn diagram shows a subset of data. All the set of 'Boxers' belongs to the set of 'Dogs', hence 'Boxers' is a subset of 'Dogs'.

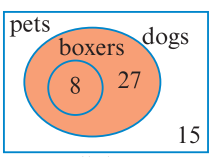
b



pets that are not dogs

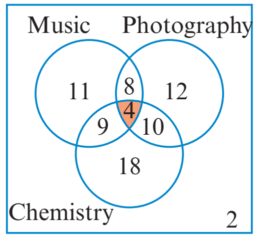


dogs that are not boxers

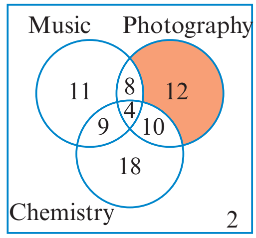


all dogs

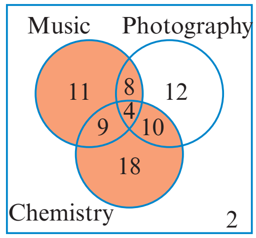
c **i**



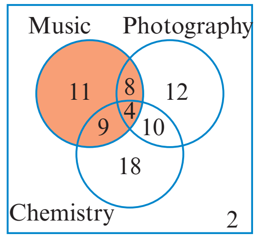
ii



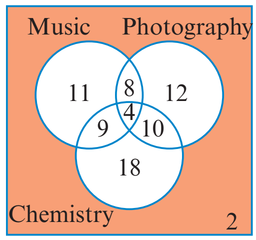
iii



iv



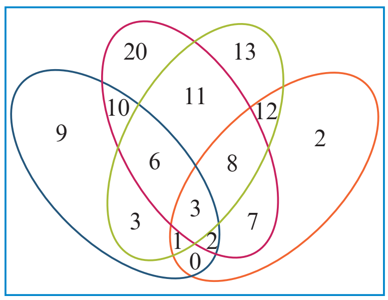
v



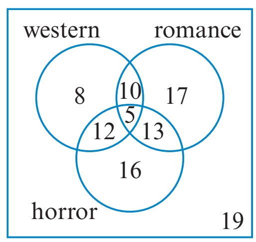
16 a $\frac{7}{34}$ **b** $\frac{4}{17}$ **c** $\frac{1}{17}$ **d** $\frac{5}{34}$ **e** $\frac{23}{34}$

f $\frac{7}{34}$ **g** $\frac{3}{14}$

17 One possible answer is:



18 One possible answer is:



9F Experiments with replacement

9F Start thinking!

1 $\frac{1}{3}$

2 Combination of ball colours that can be drawn by first two customers.

3 9

4 a Probability that first customer draws blue ball is 1 out of 3. Ball is replaced; therefore probability that second customer also draws a blue ball is 1 out of 3. Hence, probability that both customers draw blue ball is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

b Probability that first customer draws red ball is 1 out of 3. Ball is replaced; therefore probability that second customer also draws a red ball is 1 out of 3. Hence, probability that both customers draw a red ball is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

c Probability that first customer draws green ball is 1 out of 3. Ball is replaced; therefore probability that second customer also draws green ball is 1 out of 3. Hence, probability that both customers draw green balls is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

d Probability that first customer draws red ball is 1 out of 3. Ball is replaced; therefore probability that second customer draws green ball is 1 out of 3. Hence, probability that first customer draws red and second customer draws green is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

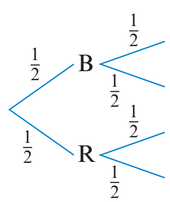
5 There is same number of blue, red and green balls in box. Balls are replaced after each draw.

6 $\frac{1}{3}$

Exercise 9F Experiments with replacement

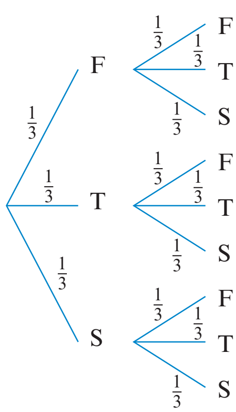
1 a $\frac{1}{4}$ b $\frac{1}{6}$ c $\frac{3}{13}$ d $\frac{1}{6}$

2 a



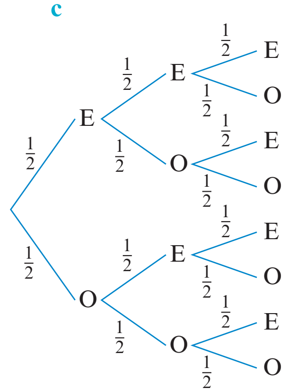
Outcomes	
BB	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
BR	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
RB	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
RR	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

b



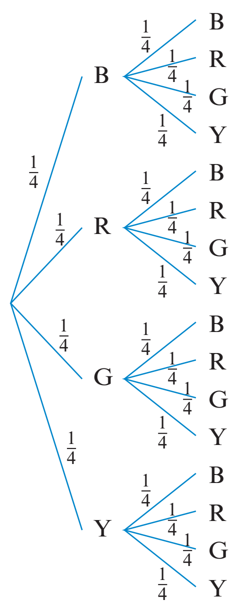
Outcomes	Probability
FF	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
FT	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
FS	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
TF	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
TT	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
TS	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
SF	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
ST	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
SS	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

c



Outcomes	Probability
EEE	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
EEO	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
EOE	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
EOO	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
OEE	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
OEO	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
OOE	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
OOO	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

d



Outcomes	Probability
BB	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
BR	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
BG	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
BY	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
RB	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
RR	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
RG	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
RY	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
GB	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
GR	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
GG	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
GY	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
YB	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
YR	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
YG	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
YY	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

3 a $\frac{1}{2}$ b 8

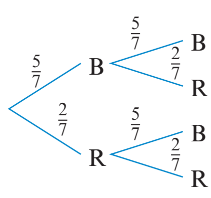
c i $\frac{1}{8}$ ii $\frac{3}{8}$ iii $\frac{1}{2}$ iv $\frac{1}{2}$

4 a $\frac{1}{16}$ b $\frac{1}{16}$ c $\frac{1}{16}$ d $\frac{7}{16}$

5 a $\frac{1}{9}$ b $\frac{5}{9}$ c $\frac{1}{9}$ d $\frac{2}{9}$

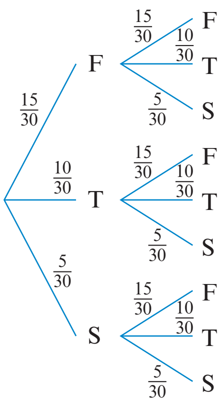
6 a $\frac{1}{64}$ b $\frac{5}{32}$ c $\frac{9}{64}$ d $\frac{27}{64}$ e $\frac{9}{32}$ f $\frac{7}{8}$

7 a



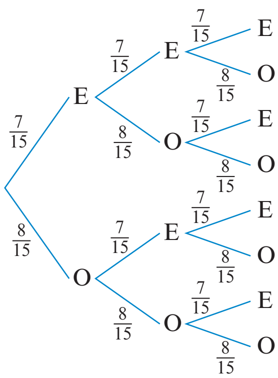
Outcomes	
BB	$\frac{5}{7} \times \frac{5}{7} = \frac{25}{49}$
BR	$\frac{5}{7} \times \frac{2}{7} = \frac{10}{49}$
RB	$\frac{2}{7} \times \frac{5}{7} = \frac{10}{49}$
RR	$\frac{2}{7} \times \frac{2}{7} = \frac{4}{49}$

b



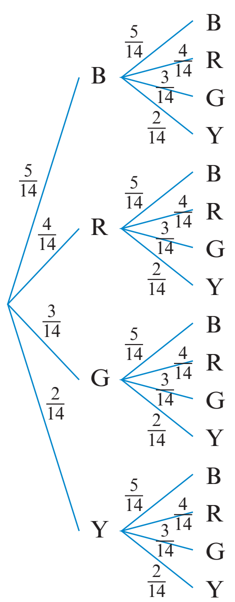
Outcomes	Probability
FF	$\frac{15}{30}\times\frac{15}{30}=\frac{225}{900}$
FT	$\frac{15}{30}\times\frac{10}{30}=\frac{150}{900}$
FS	$\frac{15}{30}\times\frac{5}{30}=\frac{75}{900}$
TF	$\frac{10}{30}\times\frac{15}{30}=\frac{150}{900}$
TT	$\frac{10}{30}\times\frac{10}{30}=\frac{100}{900}$
TS	$\frac{10}{30}\times\frac{5}{30}=\frac{50}{900}$
SF	$\frac{5}{30}\times\frac{15}{30}=\frac{75}{900}$
ST	$\frac{5}{30}\times\frac{10}{30}=\frac{50}{900}$
SS	$\frac{5}{30}\times\frac{5}{30}=\frac{25}{900}$

c



Outcomes	Probability
EEE	$\frac{7}{15}\times\frac{7}{15}\times\frac{7}{15}=\frac{343}{3375}$
EEO	$\frac{7}{15}\times\frac{7}{15}\times\frac{8}{15}=\frac{392}{3375}$
EOE	$\frac{7}{15}\times\frac{8}{15}\times\frac{7}{15}=\frac{392}{3375}$
EOO	$\frac{7}{15}\times\frac{8}{15}\times\frac{8}{15}=\frac{448}{3375}$
OEE	$\frac{8}{15}\times\frac{7}{15}\times\frac{7}{15}=\frac{392}{3375}$
OEO	$\frac{8}{15}\times\frac{7}{15}\times\frac{8}{15}=\frac{448}{3375}$
OOE	$\frac{8}{15}\times\frac{8}{15}\times\frac{7}{15}=\frac{448}{3375}$
OOO	$\frac{8}{15}\times\frac{8}{15}\times\frac{8}{15}=\frac{512}{3375}$

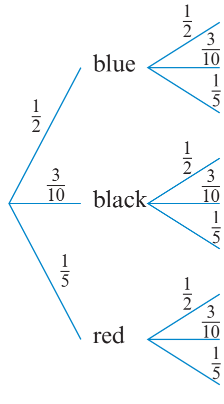
d



Outcomes	Probability
BB	$\frac{5}{14}\times\frac{5}{14}=\frac{25}{196}$
BR	$\frac{5}{14}\times\frac{4}{14}=\frac{20}{196}$
BG	$\frac{5}{14}\times\frac{3}{14}=\frac{15}{196}$
BY	$\frac{5}{14}\times\frac{2}{14}=\frac{10}{196}$
RB	$\frac{4}{14}\times\frac{5}{14}=\frac{20}{196}$
RR	$\frac{4}{14}\times\frac{4}{14}=\frac{16}{196}$
RG	$\frac{4}{14}\times\frac{3}{14}=\frac{12}{196}$
RY	$\frac{4}{14}\times\frac{2}{14}=\frac{8}{196}$
GB	$\frac{3}{14}\times\frac{5}{14}=\frac{15}{196}$
GR	$\frac{3}{14}\times\frac{4}{14}=\frac{12}{196}$
GG	$\frac{3}{14}\times\frac{3}{14}=\frac{9}{196}$
GY	$\frac{3}{14}\times\frac{2}{14}=\frac{6}{196}$
YB	$\frac{2}{14}\times\frac{5}{14}=\frac{10}{196}$
YR	$\frac{2}{14}\times\frac{4}{14}=\frac{8}{196}$
YG	$\frac{2}{14}\times\frac{3}{14}=\frac{6}{196}$
YY	$\frac{2}{14}\times\frac{2}{14}=\frac{4}{196}$

8 a $\frac{1}{25}$ **b** $\frac{16}{25}$ **c** $\frac{4}{25}$

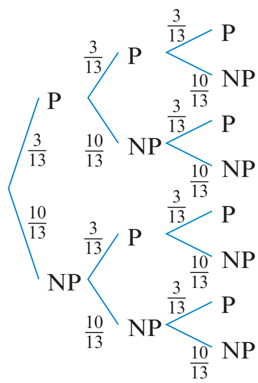
9 a



Outcomes	Probabilities
blue, blue	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
blue, black	$\frac{1}{2}\times\frac{3}{10}=\frac{3}{20}$
blue, red	$\frac{1}{2}\times\frac{1}{5}=\frac{1}{10}$
black, blue	$\frac{3}{10}\times\frac{1}{2}=\frac{3}{20}$
black, black	$\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$
black, red	$\frac{3}{10}\times\frac{1}{5}=\frac{3}{50}$
red, blue	$\frac{1}{5}\times\frac{1}{2}=\frac{1}{10}$
red, black	$\frac{1}{5}\times\frac{3}{10}=\frac{3}{50}$
red, red	$\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$

b i $\frac{1}{4}$ **ii** $\frac{1}{25}$ **iii** $\frac{9}{100}$ **iv** $\frac{3}{20}$ **v** $\frac{1}{10}$ **vi** $\frac{3}{50}$

10 a



Outcomes	Probabilities
P, P, P	$\frac{3}{13}\times\frac{3}{13}\times\frac{3}{13}=\frac{27}{2197}\approx 0.0123$
P, P, NP	$\frac{3}{13}\times\frac{3}{13}\times\frac{10}{13}=\frac{90}{2197}\approx 0.0410$
P, NP, P	$\frac{3}{13}\times\frac{10}{13}\times\frac{3}{13}=\frac{90}{2197}\approx 0.0410$
P, NP, NP	$\frac{3}{13}\times\frac{10}{13}\times\frac{10}{13}=\frac{300}{2197}\approx 0.1365$
NP, P, P	$\frac{10}{13}\times\frac{3}{13}\times\frac{3}{13}=\frac{90}{2197}\approx 0.0410$
NP, P, NP	$\frac{10}{13}\times\frac{3}{13}\times\frac{10}{13}=\frac{300}{2197}\approx 0.1365$
NP, NP, P	$\frac{10}{13}\times\frac{10}{13}\times\frac{3}{13}=\frac{300}{2197}\approx 0.1365$
NP, NP, NP	$\frac{10}{13}\times\frac{10}{13}\times\frac{10}{13}=\frac{1000}{2197}\approx 0.4552$

b i 0.4095 **ii** 0.5448 **iii** 0.8647
iv 0.1353 **v** 0.4552 **vi** 0.1230

11 a $\frac{7}{8}$ **b** $\frac{189}{1000}$ **c** $\frac{112}{125}$ **d** $\frac{1}{8}$

12 a $\frac{27}{1000}$ **b** $\frac{3}{100}$ **c** $\frac{7}{8}$ **d** $\frac{64}{125}$ **e** $\frac{27}{125}$

13 a red: $\frac{1}{2}$, green: $\frac{1}{3}$, blue: $\frac{1}{6}$

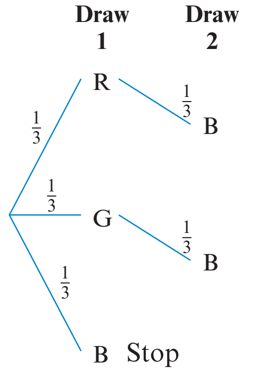
b i RB, GB, BR, BG **ii** $\frac{10}{36}=\frac{5}{18}$

c One possible answer is: each ball is not equally likely to be drawn. You need to follow the branches that lead to the desired outcome and multiply the probabilities together. Then add these individual probabilities to get an answer of $\frac{10}{36}$. The probability is not the number of outcomes that contain blue balls over the total number of outcomes $\left(\frac{5}{9}\right)$.

d i $\frac{5}{9}$ **ii** $\frac{3}{4}$ **iii** $\frac{11}{36}$ **iv** $\frac{1}{2}$ **v** $\frac{4}{9}$ **vi** $\frac{1}{9}$

14 In equally likely outcomes probabilities will be same for each outcome. You can still follow branches and multiply probabilities together and then add individual probabilities. However, when you find one probability the other outcomes will have the same probability. You can now multiply probability found by number of possible outcomes rather than adding each individual probability.

15



Outcomes	Probabilities
RB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
GB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$

Pr(one blue ball) $= \frac{1}{3}+\frac{1}{9}+\frac{1}{9}=\frac{5}{9}$

Pr(blue ball) = Pr(blue ball on first draw) or

Pr(not blue, then blue) $= \frac{1}{3}+(2\times\frac{1}{3}\times\frac{1}{3})=\frac{5}{9}$

16 a One possible answer is: not necessarily. Since socks are replaced each time, it is difficult to say that this is the exact number of white and black socks. However, number of white and black socks in drawer is not equal as there is a tendency to pull out more white than black socks.

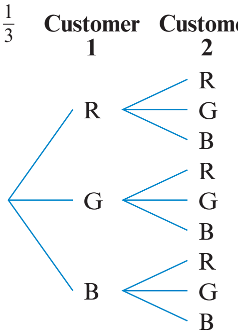
d Answers may vary but most likely to be four black and six white socks.

9G Experiments without replacement

9G Start thinking!

1 $\frac{1}{3}$

2



Outcomes
RR
RG
RB
GR
GG
GB
BR
BG
BB

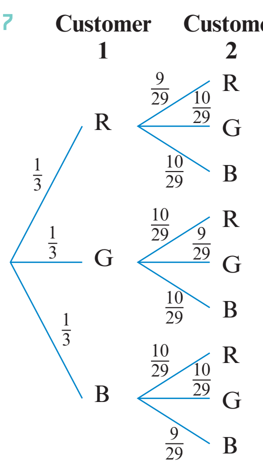
3 29

4 a 9 **b** 10 **c** 10

5 a $\frac{9}{29}$ **b** $\frac{10}{29}$ **c** $\frac{10}{29}$

6 One less blue ball in box, hence only nine blue balls to draw from total of 29 balls.

7



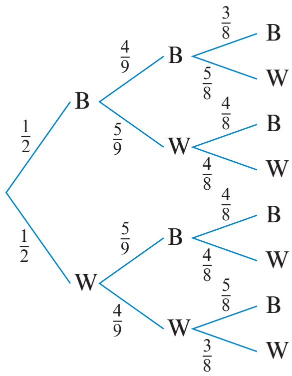
Outcomes	Probability
RR	$\frac{1}{3}\times\frac{9}{29}=\frac{9}{87}$
RG	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
RB	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
GR	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
GG	$\frac{1}{3}\times\frac{9}{29}=\frac{9}{87}$
GB	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
BR	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
BG	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
BB	$\frac{1}{3}\times\frac{9}{29}=\frac{9}{87}$

Exercise 9G Experiments without replacement

1 a i 4 **ii** 5 **b i** $\frac{4}{9}$ **ii** $\frac{5}{9}$

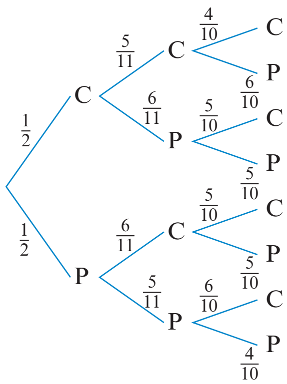
2 a i 7 **ii** 8 **b i** $\frac{7}{15}$ **ii** $\frac{8}{15}$

3 a



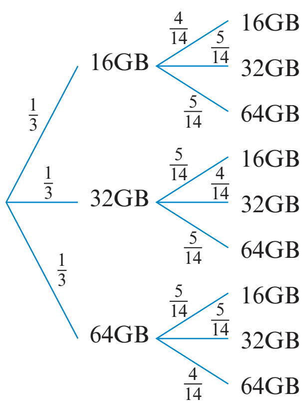
Outcomes	Probability
BBB	$\frac{1}{2}\times\frac{4}{9}\times\frac{3}{8}=\frac{12}{144}$
BBW	$\frac{1}{2}\times\frac{4}{9}\times\frac{5}{8}=\frac{20}{144}$
BWB	$\frac{1}{2}\times\frac{5}{9}\times\frac{4}{8}=\frac{20}{144}$
BWW	$\frac{1}{2}\times\frac{5}{9}\times\frac{4}{8}=\frac{20}{144}$
WBB	$\frac{1}{2}\times\frac{5}{9}\times\frac{4}{8}=\frac{20}{144}$
WBW	$\frac{1}{2}\times\frac{5}{9}\times\frac{4}{8}=\frac{20}{144}$
WWB	$\frac{1}{2}\times\frac{4}{9}\times\frac{5}{8}=\frac{20}{144}$
WWW	$\frac{1}{2}\times\frac{4}{9}\times\frac{3}{8}=\frac{12}{144}$

b



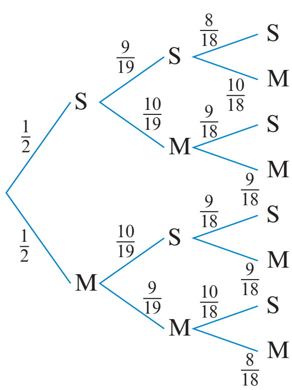
Outcomes	Probability
CCC	$\frac{1}{2}\times\frac{5}{11}\times\frac{4}{10}=\frac{20}{220}$
CCP	$\frac{1}{2}\times\frac{5}{11}\times\frac{6}{10}=\frac{30}{220}$
CPC	$\frac{1}{2}\times\frac{6}{11}\times\frac{5}{10}=\frac{30}{220}$
CPP	$\frac{1}{2}\times\frac{6}{11}\times\frac{5}{10}=\frac{30}{220}$
PCC	$\frac{1}{2}\times\frac{6}{11}\times\frac{5}{10}=\frac{30}{220}$
PCP	$\frac{1}{2}\times\frac{6}{11}\times\frac{5}{10}=\frac{30}{220}$
PPC	$\frac{1}{2}\times\frac{5}{11}\times\frac{6}{10}=\frac{30}{220}$
PPP	$\frac{1}{2}\times\frac{5}{11}\times\frac{4}{10}=\frac{20}{220}$

c



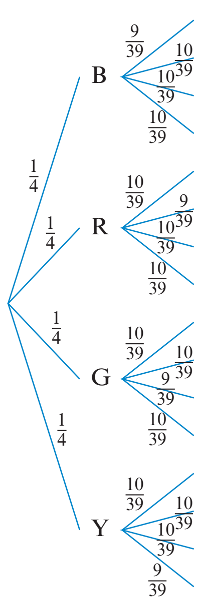
Outcomes	Probability
16, 16	$\frac{1}{3}\times\frac{4}{14}=\frac{4}{42}$
16, 32	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
16, 64	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
32, 16	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
32, 32	$\frac{1}{3}\times\frac{4}{14}=\frac{4}{42}$
32, 64	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
64, 16	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
64, 32	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
64, 64	$\frac{1}{3}\times\frac{4}{14}=\frac{4}{42}$

d



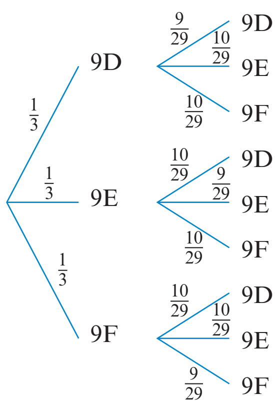
Outcomes	Mrobability
SSS	$\frac{1}{2}\times\frac{9}{19}\times\frac{8}{18}=\frac{72}{684}$
SSM	$\frac{1}{2}\times\frac{9}{19}\times\frac{10}{18}=\frac{90}{684}$
SMS	$\frac{1}{2}\times\frac{10}{19}\times\frac{9}{18}=\frac{90}{684}$
SMM	$\frac{1}{2}\times\frac{10}{19}\times\frac{9}{18}=\frac{90}{684}$
MSS	$\frac{1}{2}\times\frac{10}{19}\times\frac{9}{18}=\frac{90}{684}$
MSM	$\frac{1}{2}\times\frac{10}{19}\times\frac{9}{18}=\frac{90}{684}$
MMS	$\frac{1}{2}\times\frac{9}{19}\times\frac{10}{18}=\frac{90}{684}$
MMM	$\frac{1}{2}\times\frac{9}{19}\times\frac{8}{18}=\frac{72}{684}$

e



Outcomes	Probability
BB	$\frac{1}{4}\times\frac{9}{39}=\frac{9}{156}$
BR	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
BG	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
BY	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
RB	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
RR	$\frac{1}{4}\times\frac{9}{39}=\frac{9}{156}$
RG	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
RY	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
GB	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
GR	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
GG	$\frac{1}{4}\times\frac{9}{39}=\frac{9}{156}$
GY	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
YB	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
YR	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
YG	$\frac{1}{4}\times\frac{10}{39}=\frac{10}{156}$
YY	$\frac{1}{4}\times\frac{9}{39}=\frac{9}{156}$

f



Outcomes	Probability
9D, 9D	$\frac{1}{3}\times\frac{9}{29}=\frac{9}{87}$
9D, 9E	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
9D, 9F	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
9E, 9D	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
9E, 9E	$\frac{1}{3}\times\frac{9}{29}=\frac{9}{87}$
9E, 9F	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
9F, 9D	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
9F, 9E	$\frac{1}{3}\times\frac{10}{29}=\frac{10}{87}$
9F, 9F	$\frac{1}{3}\times\frac{9}{29}=\frac{9}{87}$

4 a 0.21 **b** 0.29 **c** 0.21

5 a $\frac{3}{14}$ **b** $\frac{2}{7}$ **6 a** $\frac{2}{9}$ **b** $\frac{5}{18}$

7 a

Outcomes	Probability
WWW	$\frac{1}{3}\times\frac{3}{11}\times\frac{1}{5}=\frac{1}{55}$
WWY	$\frac{1}{3}\times\frac{3}{11}\times\frac{2}{5}=\frac{2}{55}$
WWB	$\frac{1}{3}\times\frac{3}{11}\times\frac{2}{5}=\frac{2}{55}$
WYW	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
WYY	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
WYB	$\frac{1}{3}\times\frac{4}{11}\times\frac{2}{5}=\frac{8}{165}$
WBW	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
WBY	$\frac{1}{3}\times\frac{4}{11}\times\frac{2}{5}=\frac{8}{165}$
WBB	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
YWW	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
YWY	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
YWB	$\frac{1}{3}\times\frac{4}{11}\times\frac{2}{5}=\frac{8}{165}$
YYW	$\frac{1}{3}\times\frac{3}{11}\times\frac{2}{5}=\frac{2}{55}$
YYY	$\frac{1}{3}\times\frac{3}{11}\times\frac{1}{5}=\frac{1}{55}$
YYB	$\frac{1}{3}\times\frac{3}{11}\times\frac{2}{5}=\frac{2}{55}$
YBW	$\frac{1}{3}\times\frac{4}{11}\times\frac{2}{5}=\frac{8}{165}$
YBY	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
YBB	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
BWW	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
BWY	$\frac{1}{3}\times\frac{4}{11}\times\frac{2}{5}=\frac{8}{165}$
BWB	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
BYW	$\frac{1}{3}\times\frac{4}{11}\times\frac{2}{5}=\frac{8}{165}$
BYY	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
BYB	$\frac{1}{3}\times\frac{4}{11}\times\frac{3}{10}=\frac{2}{55}$
BBW	$\frac{1}{3}\times\frac{3}{11}\times\frac{2}{5}=\frac{2}{55}$
BBY	$\frac{1}{3}\times\frac{3}{11}\times\frac{2}{5}=\frac{2}{55}$
BBB	$\frac{1}{3}\times\frac{3}{11}\times\frac{1}{5}=\frac{1}{55}$

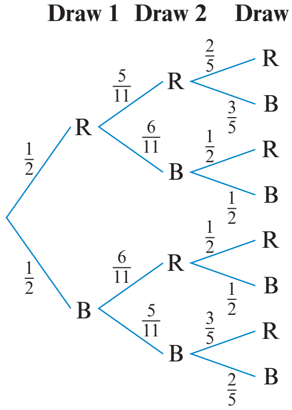
b i $\frac{1}{55}$ **ii** $\frac{2}{55}$ **iii** $\frac{8}{165}$

8 a i WWW WWY WWB WYW WYY
WYB WBW WBY WBB YWW
YWY YWB YYW YBW BWW
BWY BWB BYW BBW

ii $\frac{41}{55}$

b i $\frac{16}{55}$ **ii** $\frac{12}{55}$ **iii** $\frac{41}{55}$ **iv** $\frac{14}{55}$

9 a



Outcomes	Probabilities
RRR	$\frac{1}{2}\times\frac{5}{11}\times\frac{2}{5}=\frac{1}{11}$
RRB	$\frac{1}{2}\times\frac{5}{11}\times\frac{3}{5}=\frac{3}{22}$
RBR	$\frac{1}{2}\times\frac{6}{11}\times\frac{1}{2}=\frac{3}{22}$
RBB	$\frac{1}{2}\times\frac{6}{11}\times\frac{1}{2}=\frac{3}{22}$
BRR	$\frac{1}{2}\times\frac{6}{11}\times\frac{1}{2}=\frac{3}{22}$
BRB	$\frac{1}{2}\times\frac{6}{11}\times\frac{1}{2}=\frac{3}{22}$
BBR	$\frac{1}{2}\times\frac{5}{11}\times\frac{3}{5}=\frac{3}{22}$
BBB	$\frac{1}{2}\times\frac{5}{11}\times\frac{2}{5}=\frac{1}{11}$

b i $\frac{1}{11}$ **ii** $\frac{9}{22}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{2}$ **v** $\frac{1}{2}$

c Answers to parts iii and iv are the same. Probabilities for events in parts i and ii are different from those in experiment with replacement.

10 a Tree diagram shows three-step experiment with four branches (H, D, C, S) at each step.

b **i** $\frac{11}{850}$ **ii** $\frac{64}{425}$ **iii** $\frac{117}{850}$ **iv** $\frac{703}{1700}$ **v** $\frac{247}{850}$ **vi** $\frac{15}{17}$

c All probabilities are different.

11 $\frac{1}{5} = 0.2$

12 Pr(drawing a pair) $= \frac{5}{17} \approx 0.2941$; probability of drawing a pair increases slightly.

13 a

Captain	Vice-captain	Outcomes
Adrian	Chantelle	AC
	Katie	AK
	Ben	AB
	Sam	AS
Chantelle	Adrian	CA
	Katie	CK
	Ben	CB
	Sam	CS
Katie	Adrian	KA
	Chantelle	KC
	Ben	KB
	Sam	KS
Ben	Adrian	BA
	Chantelle	BC
	Katie	BK
	Sam	BS
Sam	Adrian	SA
	Chantelle	SC
	Katie	SK
	Ben	SB

b Second set of branches has one fewer branch. Once captain has been selected there is one fewer people to choose from for vice-captain's position.

c 20

d **i** $\frac{1}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{1}{20}$ **iv** $\frac{3}{5}$ **v** $\frac{1}{10}$

14 a

Customer 1	Customer 2	Outcomes	Probability
$\frac{15}{30}$ R	$\frac{14}{29}$ R	RR	$\frac{15}{30} \times \frac{14}{29} = \frac{210}{870} \approx 0.24$
	$\frac{10}{29}$ G	RG	$\frac{15}{30} \times \frac{10}{29} = \frac{150}{870} \approx 0.17$
	$\frac{5}{29}$ B	RB	$\frac{15}{30} \times \frac{5}{29} = \frac{75}{870} \approx 0.09$
$\frac{10}{30}$ G	$\frac{15}{29}$ R	GR	$\frac{10}{30} \times \frac{15}{29} = \frac{150}{870} \approx 0.17$
	$\frac{9}{29}$ G	GG	$\frac{10}{30} \times \frac{9}{29} = \frac{90}{870} \approx 0.10$
	$\frac{5}{29}$ B	GB	$\frac{10}{30} \times \frac{5}{29} = \frac{50}{870} \approx 0.06$
$\frac{5}{30}$ B	$\frac{15}{29}$ R	BR	$\frac{5}{30} \times \frac{15}{29} = \frac{75}{870} \approx 0.09$
	$\frac{10}{29}$ G	BG	$\frac{5}{30} \times \frac{10}{29} = \frac{50}{870} \approx 0.06$
	$\frac{4}{29}$ B	BB	$\frac{5}{30} \times \frac{4}{29} = \frac{20}{870} \approx 0.02$

b **i** $\frac{20}{870} = \frac{2}{87} \approx 0.0230$ **ii** $\frac{90}{870} = \frac{3}{29} \approx 0.1034$

iii $\frac{210}{870} = \frac{7}{29} \approx 0.2414$ **iv** $\frac{15}{870} = \frac{1}{58} \approx 0.1724$

15 a $\frac{490}{870} = \frac{49}{87} \approx 0.5632$ **b** $\frac{660}{870} = \frac{22}{29} \approx 0.7586$

c $\frac{270}{870} = \frac{9}{29} \approx 0.3103$ **d** $\frac{450}{870} = \frac{15}{29} \approx 0.5172$

e $\frac{400}{870} = \frac{40}{87} \approx 0.4598$ **f** $\frac{100}{870} = \frac{10}{87} \approx 0.1149$

16 $\frac{38}{132} = \frac{19}{66} \approx 0.2879$

17 a $\frac{6}{504} = \frac{1}{84} \approx 0.0119$ **b** $\frac{210}{504} = \frac{5}{12} \approx 0.4167$

c $\frac{114}{504} = \frac{2}{7} \approx 0.2857$ **d** $\frac{384}{504} = \frac{16}{21} \approx 0.7619$

e $\frac{444}{504} = \frac{37}{42} \approx 0.8810$ **f** 0

18 a

Card 1	Card 2	Card 3	Outcomes	Probabilities
$\frac{12}{52}$ P	$\frac{11}{51}$ P	$\frac{10}{50}$ P	P, P, P	$\frac{12}{52} \times \frac{11}{51} \times \frac{10}{50} = \frac{33}{3315} \approx 0.0100$
		$\frac{40}{50}$ NP	P, P, NP	$\frac{12}{52} \times \frac{11}{51} \times \frac{40}{50} = \frac{132}{3315} \approx 0.0398$
	$\frac{40}{51}$ NP	$\frac{11}{50}$ P	P, NP, P	$\frac{12}{52} \times \frac{40}{51} \times \frac{11}{50} = \frac{132}{3315} \approx 0.0398$
		$\frac{39}{50}$ NP	P, NP, NP	$\frac{12}{52} \times \frac{40}{51} \times \frac{39}{50} = \frac{12}{85} \approx 0.1412$
$\frac{40}{52}$ NP	$\frac{12}{51}$ P	$\frac{11}{50}$ P	NP, P, P	$\frac{40}{52} \times \frac{12}{51} \times \frac{11}{50} = \frac{132}{3315} \approx 0.0398$
		$\frac{39}{50}$ NP	NP, P, NP	$\frac{40}{52} \times \frac{12}{51} \times \frac{39}{50} = \frac{12}{85} \approx 0.1412$
	$\frac{39}{51}$ NP	$\frac{12}{50}$ P	NP, NP, P	$\frac{40}{52} \times \frac{39}{51} \times \frac{12}{50} = \frac{12}{85} \approx 0.1412$
		$\frac{38}{50}$ NP	NP, NP, NP	$\frac{40}{52} \times \frac{39}{51} \times \frac{38}{50} = \frac{38}{85} \approx 0.4471$

b **i** $\frac{36}{85} \approx 0.4235$ **ii** $\frac{47}{85} \approx 0.5529$

iii $\frac{74}{85} \approx 0.8706$ **iv** $\frac{429}{3315} \approx 0.1294$

v $\frac{38}{85} \approx 0.4471$ **vi** $\frac{396}{3315} \approx 0.1195$

c The probabilities without replacement are slightly greater for most events. The only events where probabilities are slightly less is for 'drawing at least two picture cards' or 'drawing no picture cards'.

19 $\frac{32}{663} \approx 0.0483$ **20** $\frac{1}{8\,145\,060} \approx 0.000\,000\,123$

9 Chapter review

MULTIPLE-CHOICE

1 C **2** A **3** C **4** A **5** B
6 A **7** D

SHORT ANSWER

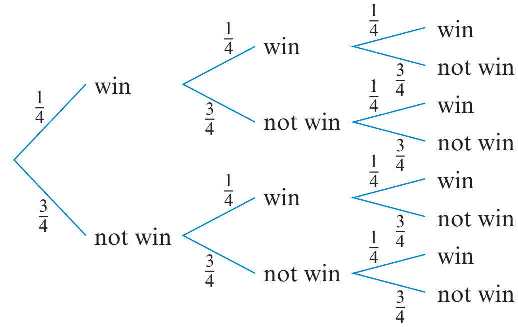
1 a **i** $\frac{1}{12}$ **ii** $\frac{11}{12}$ **b** **i** $\frac{6}{12} = \frac{1}{2}$ **ii** $\frac{6}{12} = \frac{1}{2}$

c **i** $\frac{9}{12} = \frac{3}{4}$ **ii** $\frac{3}{12} = \frac{1}{4}$

2 a $\frac{3}{4}$

b

Chance of winning an eBook



c i $\frac{1}{64}$ ii $\frac{27}{64}$ iii $\frac{9}{64}$

3 a

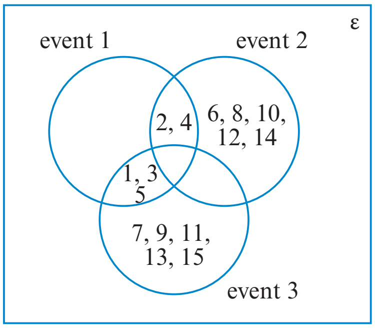
	Year 8	Year 9	Year 10	Total
Napoleon Perdis	32	25	45	102
Rimmel	22	10	22	54
Covergirl	17	31	13	61
Maybelline	49	29	15	93
Total	120	95	95	310

b i $\frac{3}{10}$ ii $\frac{11}{155}$ iii $\frac{5}{62}$

c i $\frac{5}{19}$ ii $\frac{12}{19}$

4 a Event 1: {1, 2, 3, 4, 5};
Event 2: {2, 4, 6, 8, 10, 12, 14};
Event 3: {1, 3, 5, 7, 9, 11, 13, 15}

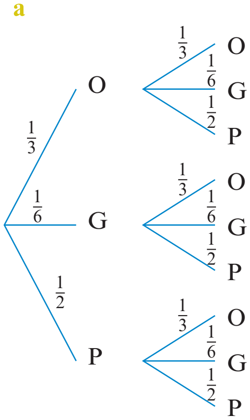
b



c i $\frac{5}{15}=\frac{1}{3}$ ii $\frac{7}{15}$ iii 1 iv 0

d There are no numbers that are ≤5 and both odd and even.

5 a



Outcomes	Probabilities
OO	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
OG	$\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}$
OP	$\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$
GO	$\frac{1}{6}\times\frac{1}{3}=\frac{1}{18}$
GG	$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$
GP	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
PO	$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$
PG	$\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$
PP	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

b i $\frac{1}{9}$ ii $\frac{1}{36}$ iii $\frac{1}{4}$

c $\frac{1}{6}$ d $\frac{1}{2}$

6 a $\frac{1}{17}$ b $\frac{13}{204}$ c $\frac{13}{850}$ d $\frac{169}{10\,200}$

NAPLAN-STYLE PRACTICE

1 $\frac{1}{3}$

2 A spinner with six equal segments may have been used in this simulation.

3 Relative frequency of heads for this experiment is 0.465.

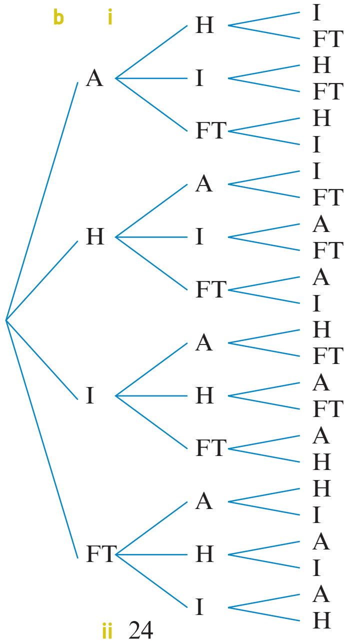
4 20 5 $\frac{17}{140}$ 6 $\frac{3}{10}$ 7 26 8 34

9 15 10 $\frac{1}{4}$ 11 $\frac{3}{10}$ 12 $\frac{5}{9}$ 13 $\frac{1}{120}$

ANALYSIS

a Without replacement, as same subject cannot be selected twice.

b i



A, H, I
A, H, FT
A, I, H
A, I, FT
A, FT, H
A, FT, I
H, A, I
H, A, FT
H, I, A
H, I, FT
H, FT, A
H, FT, I
I, A, H
I, A, FT
I, H, A
I, H, FT
I, FT, A
I, FT, H
FT, A, H
FT, A, I
FT, H, A
FT, H, I
FT, I, A
FT, I, H

Key:
A = Art
H = Health and Human Development
I = Italian
FT = Food Technology

ii 24

c i $\frac{1}{4}$ ii $\frac{1}{4}$

d i

	Male	Female	Total
Art	0.24	0.12	0.36
Food Technology	0.28	0.36	0.64
Total	0.52	0.48	1.00

ii $\frac{9}{25}$ iii $\frac{6}{25}$ iv $\frac{9}{25}$ v 42 vi 96

e i 24 ii 2 iii 68 iv $\frac{34}{75}$ v $\frac{28}{75}$

9 Connect

For feedback on this open-ended task, see your teacher.

Glossary

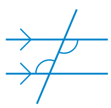
A

AAA abbreviation for angle-angle-angle. It describes the information needed to draw a triangle and also relates to a condition for similarity. Triangles are similar if the three angles they contain are the same. The condition AAA does not necessarily mean the triangles are congruent (more information is needed).

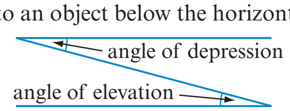
AAS abbreviation for angle-angle-side. It describes the information needed to draw a triangle and also relates to a condition for congruence. Triangles are congruent if two corresponding angles and a corresponding side are the same.

adjacent side (trigonometry) in a right-angled triangle, the side next to the reference angle that is not the hypotenuse.

alternate angles pair of angles not adjacent to (next to) each other on opposite sides of the transversal between two lines. If the two lines are parallel, the alternate angles are equal.

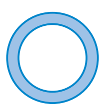


angle of depression angle between a horizontal line and the line of sight to an object below the horizontal.



angle of elevation angle between a horizontal line and the line of sight to an object above the horizontal.

annulus region formed between two circles of different radius but with the same centre.



arc the curved part of a sector.

area amount of space enclosed by a two-dimensional (2D) shape. Common units are mm^2, cm^2, m^2, km^2, ha.

area scale factor factor by which the area of a shape changes when its dimensions are increased or decreased by a factor.

asymptotes lines which a graph approaches but never reaches.

average for a list of scores, it is the value obtained by adding all of the scores together and dividing by the number of scores.

axis of symmetry imaginary line that divides a symmetrical shape or graph so that one side is a reflection of the other.

B

back-to-back stem and leaf plot stem-and-leaf plot that displays two sets of data using the one set of stems.

balance method strategy for solving equations by using inverse operations to make equivalent equations. An equation remains balanced (or equivalent) by performing the same operation on both sides of the equation.

bar graph graph where the frequency of categorical data is presented in bars or columns. Category names are placed underneath the bars if they are drawn vertically or beside the bars if they are drawn horizontally.

base **1** for a value expressed in index form, the base is the number which is repeatedly multiplied. For example, 2^4 has a base of 2 and can be written as $2 \times 2 \times 2 \times 2$. **2** in a 2D shape, it is a nominated side of the shape. The base and height are perpendicular to each other. *See* height for diagram. **3** in a 3D object, it is a nominated face of the object. The base and height are perpendicular to each other.

basic numeral result of a repeated multiplication. For example, $2^4 = 16$ where 16 is the basic numeral.

bearing direction from one position to another.

best buy found by comparing a number of purchase options to find the one that is the best value for money.

biased a biased sample is one where the method of collecting data produces a sample that does not accurately reflect the population.

bimodal a set of data that has two scores that are more frequent than the other scores.

C

capacity amount of liquid that a three-dimensional object (container) can hold. Common units are mL, L, kL, ML.

Cartesian plane number plane or region formed by a pair of horizontal and vertical axes that allows any point to be described.

categorical data data that can be put into categories.

census survey of an entire population.

centre in statistics, measures of centre include the mode, median or mean.

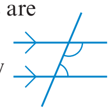
centre of dilation a point in a shape from which a dilation is made; often one of the vertices of the shape.

circumference perimeter of a circle.

class intervals groupings of data into equal-sized classes.

coefficient number that appears in front of a variable showing that the number and the pronumeral are to be multiplied together. For example, the coefficient of $5m$ is 5.

co-interior angle pair of angles on the same side of the transversal between two lines. If the two lines are parallel, the co-interior angles are supplementary (add to 180°).



column graph graph where the frequency of categorical data is presented in columns.

commission payment to an employee calculated by finding a percentage of sales made.

compass bearing direction from one position to another and described by the number of degrees from north or south towards east or west. For example, S43°E represents a direction that is 43° east of due south.

complement the complement of a set is everything in the universal set that is not in the given set. For example, if the universal set contains whole numbers from 1 to 5 ($\varepsilon = \{1, 2, 3, 4, 5\}$) and set A contains even numbers ($A = \{2, 4\}$) then the complement of A contains odd numbers ($A' = \{1, 3, 5\}$).

complementary angles angles that add to make a right angle (90°).

complementary events two probability events are complementary when their probabilities add to 1.

composite shapes shapes made up of more than one type of shape.

concave type of polygon with at least one interior angle that is greater than 180°.

conditional probability the probability of an outcome given certain conditions.

cone 3D object that tapers from a circular base to a point.

congruence occurs when figures are identical in shape and size but can be in any position or orientation.

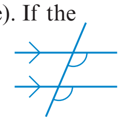
constant of proportionality if two quantities x and y are related so that $y = kx$, k is the constant of proportionality.

constant term numerical value in an algebraic expression or equation. It is a term without any pronumeral component. For example, the constant in the expression $3b - 4c + 10$ is 10.

continuous data type of numerical data that can be measured; for example, height.

convex type of polygon with interior angles that are all less than 180°.

corresponding angles pair of angles on the same side of the transversal and in corresponding positions on the two lines (both above a line or both below a line). If the two lines are parallel, the corresponding angles are equal.



cosine (cos) in trigonometry, the cosine of an angle in a right-angled triangle equals the ratio of the length of the adjacent side to the length of the hypotenuse; that is, the length of the adjacent side divided by the length of the hypotenuse.

cumulative frequency running total of frequencies in a frequency table.

cylinder 3D object with a uniform circular cross-section.

D

decimal place position of a digit after the decimal point. Each decimal place represents a different fractional value. The first decimal place is the first digit to the right of the decimal point. The number of digits after the decimal point is the number of decimal places. For example, 5.249 has three decimal places.

deductions amounts of money, such as payments for tax and superannuation, which are taken from gross income to leave net income.

degrees–minutes–seconds (DMS) the convention of describing an angle using the three units of degrees, minutes and seconds. Each degree can be divided into 60 minutes and each minute can be divided into 60 seconds.

denominator bottom number of a fraction. For example, the denominator of $\frac{2}{3}$ is 3.

dependent variable variable that depends on another variable (independent variable) in a relationship. On a Cartesian plane, the dependent variable is shown along the vertical axis.

diameter line segment from one side of a circle to the other through the centre of the circle. Also defined as the length of this line segment.

difference of two squares a pattern observable when expanding a factorised expression, represented as $(a + b)(a - b) = a^2 - b^2$.

dilation transformation that does not produce an identical figure. A dilation produces an enlargement or a reduction of the original figure.

dimensions measurements on a shape or object. For example, the length and width of a rectangle are the dimensions of the rectangle.

discount an amount off the selling price. New selling price = original selling price − discount.

discrete data type of numerical data that can be counted; for example, number of siblings.

distributive law same answer is obtained whether you add the numbers inside a pair of brackets first before multiplying by another number or multiply each number inside the brackets by the number outside the brackets before adding. For example, $3(2 + 5) = 3 \times 7 = 21$ and $3(2 + 5) = 3 \times 2 + 3 \times 5 = 6 + 15 = 21$. In algebraic terms, the distributive law can be written as $a(b + c) = ab + ac$.

dot plot graph that presents every piece of data in the data set as a dot above a matching number or category on a horizontal scale. Suitable for discrete numerical data and categorical data.

double time pay rate twice the normal hourly rate, paid for non-standard hours of work.

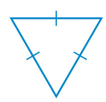
E

enlargement larger image produced after a figure has been dilated.

equally likely when the outcomes of a event have the same probability of occurring; there could be two or more possible outcomes.

equation collection of two or more algebraic terms separated by mathematical operation symbols and containing an equals sign; for example, $2d + 5 = 10$.

equilateral triangle triangle with all three sides equal in length. All three angles are also equal in size.



equivalent equations equations that have the same solution.

equivalent fractions different fractions that have the same value. An equivalent fraction is formed by multiplying or dividing the numerator and the denominator of a fraction by the same number. For example, $\frac{24}{36} = \frac{4}{6}$.

equivalent ratio a ratio that can be written as an equivalent fraction when compared to another ratio. An equivalent ratio can be produced by multiplying or dividing all numbers in a ratio by the same value.

estimated value value obtained by calculation that uses rounded values.

event a specified result that may be obtained in a probability experiment.

exact value value exactly as measured or recorded, not rounded off.

expanded form **1** written as a repeated multiplication. For example, 4^3 written in expanded form is $4 \times 4 \times 4$. **2** an algebraic expression written without brackets (not in factor form).

expansion of a perfect square a pattern observable when expanding a factorised expression, represented as $(a + b)(a + b) = a^2 + 2ab + b^2$.

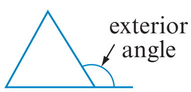
expected number number of favourable outcomes expected in a probability experiment. It is calculated from theoretical probability; that is, expected number = theoretical probability × number of trials.

experiment (probability) where trials are performed to obtain data to predict the chances of an event occurring.

experimental probability probability of an event based upon the results of a probability experiment. Also known as relative frequency.

expression collection of two or more algebraic terms separated by mathematical operation symbols like +, −, × and ÷. An expression does not have an equals sign. For example, $3b - 4c + 10$ is an expression.

exterior angle angle formed outside a triangle where a side has been extended. The exterior angle and the interior angle directly adjacent to it are supplementary (add to 180°).



F

faces flat surfaces of a polyhedron. Each face is a polygon. *See* polyhedron.

factor form expression made up of a product of factors.

factorise to write an expression in factor form. For example, $6x + 3$ factorises to $3(2x + 1)$ where the two factors are 3 and $2x + 1$.

favourable outcome outcome within the sample space that we want to occur.

formula relationship or rule between two or more variables. A formula contains an equals sign; for example, $y = 3x + 7$.

frequency table table organised to show data by recording the frequency of each value in the data set.

G

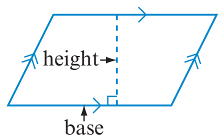
gradient the slope of a graph.

gradient–intercept method used for drawing a line graph, in which the gradient is used to find a second point from the y-intercept.

gross income amount earned before any deductions are made.

H

height length measurement from the base to the top or end of a shape or object. The height is perpendicular to the base.



highest common factor (HCF) largest factor that is common to two or more given numbers. For example, factors of 8 are 1, 2, 4, 8 and factors of 12 are 1, 2, 3, 4, 6, 12 so HCF of 8 and 12 is 4.

histogram graph used to display the frequency of grouped continuous data where the horizontal scale boundaries are located at the edges of the columns, rather than the centre.

hypotenuse the longest side of a right-angled triangle; it is the side opposite the right angle.

I

Image shape produced after a transformation.

income tax a percentage of a person's pay taken by the government to pay for services such as healthcare, schools, roads, etc.

independent variable one of two variables in a relationship. On a Cartesian plane, the independent variable is shown along the horizontal axis. It is listed first in a table of values and in pairs of coordinates.

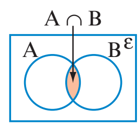
index (power) for a value expressed in index form, the index (or power) indicates the number of times the base is written as a repeated multiplication. For example, 2^4 has an index of 4 and can be written as 2 . 2 . 2 . 2. (Plural of index is indices.)

index form (index notation) shorter form of writing a repeated multiplication. A number in index form is written with a base and an index (power). For example, 2^4 is written in index form.

index laws set of laws that make calculations of numbers (or pronumerals) in index form easier to perform.

interest extra payment made for a loan or paid on an investment.

intersection the intersection of two sets ($A \cap B$) on a Venn diagram is where the two sets overlap.

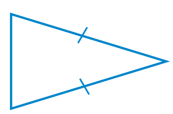


investment money put into a bank, financial institution or other business for which interest is paid to the investor.

irregular describes a polygon that is not regular. It does not have all sides equal in length or all angles equal in size.

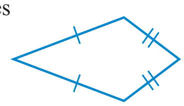
isometric transformation type of transformation (such as translation, rotation or reflection) applied to a shape or object that does not change its shape or size.

isosceles triangle triangle with two sides equal in length. The angles opposite the equal sides are also equal.



K

kite quadrilateral (four-sided shape) with two pairs of adjacent sides equal in length, one pair of opposite angles equal in size and no parallel sides.



L

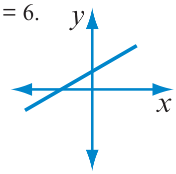
leading digit first non-zero digit of a number. For example, the leading digit of 328 is 3.

like terms algebraic terms that contain exactly the same pronumerals.

line graph graph that displays the relationship between two variables where points are plotted and joined by a line. Often used to show data collected over time (horizontal axis labelled as time).

linear equation an equation that contain a pronumeral term or terms where the highest power is one. For example, $2x + 3 = 6$.

linear graph graph of linear relationship. The graph is a straight line.



linear relationship relationship between two variables where the coordinate points describing this relationship lie in a straight line when plotted on the Cartesian plane.

literal equation *see* formula.

loan amount of money given by a bank or other institution to a person for a limited time and on which they must pay interest.

loss the amount lost when the selling price is less than the original price.

lowest common denominator (LCD) lowest common multiple of the denominators in two or more fractions. For example, $\frac{1}{3}$ and $\frac{1}{2}$ have a LCD of 6.

M

mark-up amount added to an original price. For example, an original price has a 25% mark-up added to obtain the selling price.

maximum turning point a point at which a parabola changes direction and at which it has its maximum y value.

mean a measure of the centre of a data set. It is calculated by adding all of the data values together and dividing by the number of values. Also known as the average.

median a measure of the centre of a data set. For data placed in ascending order (smallest to largest), it is the middle number of the data set if there is an odd number of values, and the average of the two middle numbers if there is an even number of values.

midpoint half way point or middle point in an interval.

minimum turning point a point at which a parabola changes direction and at which it has its minimum y value.

mode a measure of the centre of a data set. It is the value that occurs most frequently in the data set. There can be more than one mode or no mode.

N

negatively skewed describes a distribution that has its centre to the right.

net two-dimensional plan that can be folded to form a three-dimensional object.

net income income that remains after deductions are taken from gross income.

nominal data type of categorical data where the categories are unrelated; for example, eye colour.

non-isometric transformation a transformation that changes the size of a shape.

non-linear relationship relationship between two variables where the coordinate points describing this relationship do not lie in a straight line when plotted on the Cartesian plane.

Null Factor Law states that if the product of two factors is zero, then one or both of the factors must equal zero.

numerator top number of a fraction. For example, the numerator of $\frac{2}{3}$ is 2.

numerical data data that can be counted or measured.

O

opposite side (trigonometry) in a right-angled triangle, the side opposite the reference angle.

ordinal data type of categorical data that can be placed in categories in a specific order; for example, a rating system from 1–5.

origin point where the x- and y-axes cross on a Cartesian plane. Coordinates are (0, 0).

original price the price of an item before a discount is applied or a mark-up is added.

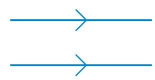
outlier extreme piece of data that is much higher or lower than the rest of the data in the data set.

overtime higher rate of pay given for extra time worked in addition to normal work hours.

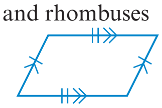
P

parabola the graph of a quadratic relationship.

parallel lines (or rays or segments) that are drawn side by side and are the same distance apart so they never touch are parallel. Matching arrowheads are used to indicate that lines are parallel.



parallelogram quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel and equal in length, and two pairs of opposite angles that are equal in size. Rectangles, squares and rhombuses are also classified as parallelograms.

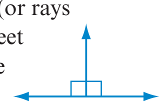


percentage a number expressed as parts out of one hundred.

percentage of an amount when an amount is represented as a percentage of another amount.

perimeter distance around the outside edge or boundary of a two-dimensional (2D) shape. Common units are mm, cm, m, km.

perpendicular two lines (or rays or segments) that meet to form a right angle are perpendicular.



pi irrational number (3.141 592 653 …) used in calculations involving circumference and area of a circle. It is the ratio of the circumference of a circle to its diameter. The symbol for pi is π.

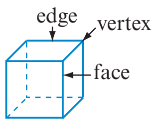
pie graph graph where the size of each sector in a circle represents the frequency of each data category as a fraction of the total number of pieces of data in the data set.

point of inflection point on a graph where the size of the gradient decreases to zero before increasing again.

polygon general name for a two-dimensional (2D) shape with straight sides.

polyhedra plural of polyhedron. *See* polyhedron.

polyhedron object made up of flat surfaces or faces (each face is a polygon) with vertices and straight edges. For example, a cube is a polyhedron but a cylinder is not.



population complete set of individuals or things that we are seeking information about.

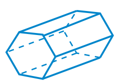
positively skewed describes a distribution that has its centre to the left.

power (index) *see* index.

primary data data collected first hand, e.g. by survey or questionnaire.

principal sum of money invested or amount borrowed for a loan.

prism object with two ends that are identical polygons and joined by straight edges.



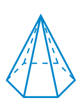
probability chance of an event occurring. The event can be very likely and have a high probability or not likely and have a low probability. Words or numbers can be used to describe the chance of an event occurring.

probability scale number line from 0 to 1 that can be used to indicate the probability of an event. An impossible event has a probability of 0. A certain event has a probability of 1.

profit amount made when the selling price is greater than the original price.

pronumeral letter or symbol that takes the place of a number.

pyramid object with an end that is a polygon and triangular sides (faces) that meet at a point (vertex).



Pythagoras' theorem states that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the two other sides. For a right-angled triangle with hypotenuse of length c and other side lengths a and b, Pythagoras' Theorem states that $c^2 = a^2 + b^2$.

Pythagorean triad (Pythagorean triple) any set of three whole numbers that satisfy Pythagoras' theorem.

Q

quadrant a sector with angle of 90°.

quadratic equation an equation that contains a pronumeral term or terms where the highest power is 2. For example, $x^2 + 3x - 2 = 10$.

quadrilateral two-dimensional shape with four straight sides.

R

radius line segment from the centre of a circle to the edge of the circle. Also defined as the length of this line segment (and half the diameter of the circle).

random sampling selecting a sample randomly (such as pulling names out of a hat).

range difference between the highest and lowest value in a data set.

rate comparison of the change in one quantity with respect to another. For example, speed is a rate as it compares the distance over time.

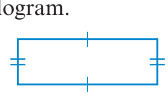
rate of change the change in one variable compared to the change in another variable; often represented as $\frac{y}{x}$.

rate statement representing a value as a rate per unit, e.g. cost per number of items or wage per hour.

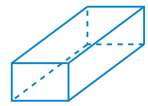
ratio comparison of two or more quantities of the same kind. For example, when comparing the number of red jellybeans to blue jellybeans, the ratio is 2:3.

ratio scale scale factor written as a ratio. For example, scale factor of $\frac{1}{10}$ is a ratio scale of 1:10.

rectangle quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel and equal in length, and four angles of 90°. A rectangle is also classified as a parallelogram.



rectangular prism prism with three pairs of identical faces (total of six faces). A cube is also a rectangular prism.



reduction smaller image produced after a figure has been dilated.

reflection transformation where a shape or object is reflected (flipped) in a mirror line to produce its exact mirror image.

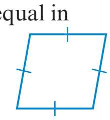
regular describes a polygon or shape with all sides equal in length and all angles equal in size.

relative frequency *see* experimental probability.

retail price *see* selling price.

retainer a fixed amount paid to a person who also earns commissions on sales.

rhombus quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel, all four sides equal in length and two pairs of opposite angles that are equal in size. A rhombus can also be classified as a parallelogram.



RHS abbreviation for right-angled triangle-hypotenuse-side. It describes the information needed to draw a triangle and also relates to a condition for congruence. Right-angled triangles are congruent if their hypotenuses are the same length and one other corresponding pair of sides has the same length.

right-angled describes a triangle where the size of one interior angle is 90°.

rotation transformation where a shape or object is rotated (turned) around a point. Movement is described as an angle of rotation in either a clockwise or anticlockwise direction from a given point.

rounding writing a number as an approximate value by rounding to the leading digit or a given number of decimal places. If the digit to the right of the leading digit or given decimal place is 0, 1, 2, 3 or 4, round down. If it is 5, 6, 7, 8 or 9, round up.

S

salary annual payment for a job, not a wage paid by the hour.

sample small selection from a population.

sample space list of all the different outcomes possible for a probability experiment. For example, sample space when rolling a die is $\{1, 2, 3, 4, 5, 6\}$.

SAS abbreviation for side-angle-side. It describes the information needed to draw a triangle and also relates to a condition for congruence. Triangles are congruent if two corresponding sides are equal in length and the angle between them is the same.

scale factor indicates how many times larger or smaller an image is after a figure has been dilated. A scale factor larger than 1 produces an enlargement and a scale factor between 0 and 1 produces a reduction.

scatterplot graph that displays a relationship between two variables where points are plotted but not joined.

scientific notation a value written as a number from 1 up to, but not including, 10 (with any number of decimal places) multiplied by a power of 10.

scores pieces of data.

secondary data data that has been collected by someone else.

sector region of a circle between two radii. (Plural of radius is radii.)

selling price price for which an item is sold.

semicircle a half circle.

set a group of data. For example, those in your class that have brown hair could be described as belonging to the set of people with brown hair.

significant figures the number of digits in a number that contribute to its accuracy.

similar figure an image produced after dilation. The size of the figure has changed but not the shape.

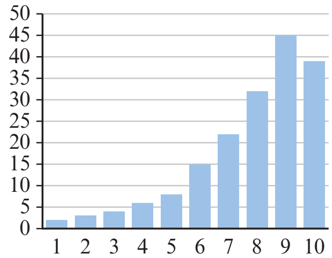
simple interest an amount (usually a percentage) paid on a loan or investment.

simulation using technology or simple devices like coins or dice to simulate real-life events. Makes performing a large number of trials in a probability experiment easier.

sine (sin) the sine of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the hypotenuse; that is, the length of the opposite side divided by the length of the hypotenuse.

sketch graph a simple graph, not on graph paper, which has main features such as intercepts and turning points labelled.

skewed description of a data distribution where there is a higher proportion of the data displayed to the right or to the left of a graph.

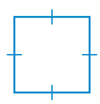


solution value of the pronumeral that makes an equation a true statement.

sphere 3D object shaped like a ball.

spread describes how widely distributed the data in a set is.

square **1** quadrilateral (four-sided shape) with all four sides equal in length, two pairs of opposite sides that are parallel and four angles of 90°. A square is also classified as a rectangle.



2 to raise to the power of 2; for example, $3^2 = 9$.

SSS abbreviation for side-side-side. It describes the information needed to draw a triangle and also relates to a condition for congruence. Triangles are congruent if all three corresponding sides are equal in length.

standard deviation a measure of spread of a data set, describing how much data differs from the mean.

standard form *see* scientific notation.

stem-and-leaf plot (stem plot) a display of data where each piece of data is split into two parts: the last digit becoming the leaf and the other digits becoming the stem. The stem is written once in the left column of the plot and the leaves are listed in numerical order beside the appropriate stem.

stratified sampling dividing the population into separate categories (such as gender or age) and then taking a random sample from each category that is proportional to its size.

subset a set that lies entirely within another set.

substitution replacement of a pronumeral with a given number. For example, substituting $x = 2$ into $3x + 4$ gives $3 \times 2 + 4$.

summary statistics information such as the mean, median, mode and range that provide a summary of the data set.

supplementary angles angles that add to make a straight angle (180°).

symmetric (symmetrical) of graphs, having a similar distribution of frequencies either side of a central value.

systematic sampling selecting a sample at fixed intervals (such as every third person). The starting point should be random.

T

tangent (tan) in trigonometry, the tangent of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the adjacent side; that is, the length of the opposite side divided by the length of the adjacent side.

theoretical probability probability calculated by considering the number of favourable outcomes compared to the total number of outcomes; that is, $\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$. Each outcome must be equally likely to occur.

time in interest calculations, the period of the loan or investment.

time-and-a-half a pay rate of one and a half times the normal hourly rate, paid for non-standard hours of work.

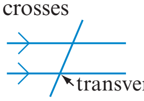
total surface area (TSA) total area of the surface of an object. Surface area of a prism is the sum of the areas of each face of the prism. Common units are mm^2, cm^2, m^2.

transformations general name for translations, reflections, rotations and dilations.

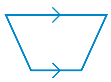
transforming a formula rearranging a formula to make a different variable the subject.

translation transformation where a shape or object is translated (moved) in a straight line without turning or changing size. Movement is often described as the number of units up or down and left or right.

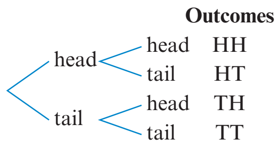
transversal line that crosses (intersects) a pair or set of lines.



trapezium quadrilateral (four-sided shape) with one pair of opposite sides that are parallel.



tree diagram display of outcomes for a multi-step probability experiment. For example, a tree diagram can be used to display the outcomes of flipping two coins.



triangle two-dimensional shape with three straight sides.

trigonometry the study of relationships between angles and side lengths of a triangle.

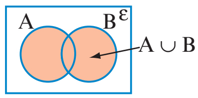
true bearing direction from one position to another and described by the number of degrees from north in a clockwise direction. The angle is written as three digits followed by T (or true). For example, 090°T is due east, 180°T is due south, 270°T is due west.

turning point the point where a parabola changes direction.

two-way table display of outcomes for a two-step probability experiment in a table.

U

union the union of two sets ($A \cup B$) on a Venn diagram is everything contained in both sets.



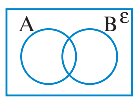
unitary method a method of finding an original amount when the percentage of the original amount is known. We find 1% (one unit) and then multiply the result by 100 to find 100% (100 units or the original amount).

universal set set of all elements. Everything inside the rectangle of a Venn diagram belongs to the universal set. Symbol for universal set is ε.

V

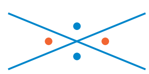
variables quantities that can have varying or different values. A variable can be represented with words or a symbol.

Venn diagram consists of a number of circles contained in a rectangle to display the relationship between different sets of data. The rectangle represents the universal set, ε, and each set of data within the universal set is represented by a circle.



vertex corner point where three or more edges of a polyhedron meet. *See* polyhedron.

vertically opposite angles pair of equal angles not adjacent to (next to) each other, formed where two lines intersect.



vertices plural of vertex. *See* vertex.

volume amount of space that a three-dimensional (3D) object occupies. Common units are mm^3, cm^3, m^3.

W

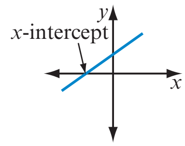
wage money paid to an employee for work performed, usually as an hourly rate.

wholesale price original price.

X

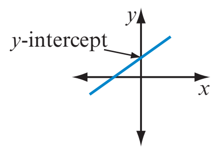
x- and y-intercept method used for drawing a straight line graph, where the x- and y-intercepts are found and a straight line is drawn through them.

x-intercept point where a line crosses the x-axis of a Cartesian plane. For example, the x-intercept is −3 or the coordinates of the x-intercept are (−3, 0).



Y

y-intercept point where a line crosses the y-axis of a Cartesian plane. For example, the y-intercept is 2 or the coordinates of the y-intercept are (0, 2).



Index

Acknowledgements

The author and the publisher wish to thank the following copyright holders for reproduction of their material.

istockPhoto/Sklep Spozywezy, 30; **Science Source**/Biophoto Associates, 70 (bottom); **Shutterstock**, 3, 40, 50, 51, 139 (top), 151 (bottom right), 152-3, 164 (bottom), 195, 200 (top), 338 (top), 345 (bottom), 364 (top), 450, /54613, 427, /aastock, 457 (bottom), /Africa Studio, 34, 227, 249 (middle), 408-9, /Ilya Akinshin, 319, /Potapov Alexander, 307 (bottom left), / Alis Photo, 217, /Aaron Amat, 293 (top middle), 376, /amgun, 332 (middle), /Bruce Amos, 301 (top), /and4me, 11 (right), /Andre R, 101 (bottom), /ARENA Creative, 381, /Artistic Photo, 307 (bottom right), /artist wongpracha, 306 (middle), / Sue Ashe, 258-9, /Stefan Ataman, 346, /auremar, 120, /axz700, 289 (top), /Denis Babenko, 270 (bottom), /Kitch Bain, 449 (top), /Hamiza Bakirci, 15 (top), /Bikeworldtravel, 407 (top), / bmaki, 343 (right), /Joe Belanger, 19, /Ali Ender Birel, 51, /Max Blain, 4, / BonD80, 396, /Maaike Boot, 23, /Sylvie Bouchard, 406 (right), /Javier Brosch, 406 (bottom), / Fernando Blanco Calzada, 368, /Konstantin Chagin, 18, /Norman Chan, 109 (top), /Karyna Che, 256 (right), 257 (top), / Stephen Clarke, 207 (bottom), /Stephen Coburn, 256 (bottom), /Cosma, 288 (top), /covenant, 296, /Ivan Covlad, 88 (top left), /CREATISTA, 385, /Curioso, 31, /Linn Currie, 256 (left), / Vesna Cvorovic, 332 (top), /de2marco, 345 (top), /Gilles DeCruyenaere, 356 (top), /Design56, 10 (left), 101 (top), 213 (bottom), /Dimedroi68, 145, /Claudio Divizia, 333, /Dja 65, 32, /Le Do, 250, /Donatas 1205, 101 (left), /dotshock, 176, /Danilo Ducak, 133, /elnavegante, 122, /Elnur, 450, /elxeneize, 244, /Alex Emanuel Koch, 83, /ET1972, 251 (bottom), /EtiAmmos, 377, /exopixel, 306 (middle), /Marie C Field, 431, / Firenze, 28, /FixMik, 308-9, /Mike Flippo, 364 (bottom), 410, /Pashin Georgly, 451, /Lukas Gojda, 372, /Nicole Gordine, 109 (bottom), 233, /Jaroslaw Grudzinski, 323, /Levente Gyori, 425 (bottom), /Martin Haas, 183 (top), /Peter Hansen, 263, /homeydesign, 247, /Margo Harrison, 276 (middle), 307 (top), 357 (bottom), /Haslam Photography, 276 (bottom), / Chris Hill, 306 (top), /Oliver Hoffman, 102-3, /Horlyan, 44 (bottom), /Feng Hu, 288 (right), /ifong, 7, /liplk, 457 (top right), /Illke, 239, /infografick, 52-3, /Tischenko Irinia, 357 (top), /Rafa Irusta, 11 (bottom), 15 (bottom), /Jagodka, 432, / Jaros, 270 (top right), /jcjphotography, 456 (bottom), /Ben Jeayes, 356 (bottom), /jocic, 407 (bottom left), /Vava Vladimir Jovanovic, 419 (top), /Dmitry Kalinovsky, 13, /John Kasawa, 324, /Kaspri, 240 (right), 332 (bottom), /Kbiros, 221 (top), / ketsu, 238, /Alexia Khruscheva, 394, /Soo Hee Kim, 100 (bottom), /Kokhanchikov, 315, 344 (top), 361 (top), /Kongsak, 114, /Daniel Korzeniewski, 150, 151 (bottom left)/Patryk Kosmider, 82 (top), /Kostov, 11 (left), /Veniamin Kraskov, 213 (top), /Volodymyr Krasyuk, 228, /D.Kucharski & K.Kucharska, 70 (top), /Jess Kunerth, 407 (bottom right), /Kuzma, 95 (right), /Andrey Kuzmin, 88 (top right), /Igor Lateci, 159 (bottom), /Laurelie, 82 (left), /Katrina Leigh, 449 (bottom), / leisuretime70, 121, /Lightspring, 314, /Logutenko, 399, /Loskutnikov, 360, /Svetiana Lukienko, 414 (bottom), /lunatic67, 218, /Robyn Mackenzie, 77 (top), /majeczka, 357 (middle), /Mama Mia, 82 (right), /manasapat, 356 (left), /mangostock, 12, /Gill Mar, 95 (left), /Oleksly Mark, 272, /Maques, 433, /marpan, 76 (top), /maryo, 313, 388, /mates, 293 (top left), /Mavrick, 151 (top), /Stephen Mcsweeny, 187, /Christopher Meade, 231 (bottom), /Phillip Meyer, 88 (bottom right), /mezzotint, 77 (bottom), /Biehler Michael, 276 (top right), /Dmitri Miktenko, 344 (bottom), /MilanB, 334, /Aleksandar Miljatovic, 249 (top), /Sergey Mironov, 456 (top), /Miaden Mitrinovic, 159 (top), /Monkey Business Images, 14, 27, /motorolka, 288 (left), /Myimagine, 158 (bottom), /mypokcik, 8 (bottom), /Denis Nata, 301 (bottom), /Nathaol HPS, 204, /Nejat, 104, / nightcat, 139 (bottom), /Ninell, 343 (middle), /Neveshkin Nikolay, 276 (top left), /Oksana2010, 128, 270 (top left), /Tyler Olson, 20, /Pakhnyushchy, 277, /Palabra, 8 (top), /Ksenia Palimski, 88 (bottom left), /Lena Pan, 366, /Constantine Pankin, 307 (middle left), /Jim Parkin, 214, /Varina and Jan Patel, 406 (left), /Edyta Pawlowska, 39, /Shawn Pecor, 200 (bottom), / Dasha Petrenko, 361(bottom), / Peterfz30, 37, /Richard Peterson, 115, /peterwort, 101 (right), /James Marvin Phelps, 164 (top), /Phiseksit, 310 (bottom), /photomatz, 76 (bottom), /Photastic, 225, /Tatiana Popova Vakeri Potapova, 240 (left), 457 (top left), / Kristina Postnilkova, 356 (middle), /Poznyakov, 36 (bottom), /Pressmaster, 249 (bottom), /Lev Radin, 299, / ravi, 15 (middle), /Peeradach Rattanakoses, 358-9, /Sergei Razvodovskij, 59, 327, /Phil Reid, 370, /Christina Richards, 294 (bottom), /ronstik, 414 (top), /Room27, 380, /sagir, 293 (bottom), /saiko3p, 343 (left), /Sailorr, 110, /Daniel Sainthorant, 143, /Natalia Sannikova, 284, / Jochen Schoenfeld, 351, /Galushko Sergey, 316, 340, / Rusian Semichev, 338 (left), / sergign, 221 (bottom), /Shout it Out Design, 257 (bottom), /Silhouette Lover, 270 (bottom left), /Skobrik, 310 (top), / Ljupco Smokovski, 216, /Becky Stares, 210, /Sandra van der Steen, 307 (middle right), /stenic56, 288 (bottom), /StudioVin, 22, /Gladkova Svetlana, 219, /tarog, 422, /Tashatuvango, 44 (top), /THPStock, 16, /Tish1, 100 (top), /Tupungato, 265, / vidguten, 293 (top right), /Kulish Viktoria, 24, /viphotos, 289 (bottom), 290, /vision77, 419 (bottom), /Krivosheev Vitaly, 183 (bottom), /Mr Vitkin, 271, /Matka_Wariatka, 328, /Edward Westmacott, 38 (bottom), /wheatley, 338 (right), /Jan de Wild, 26, /Jan Martin Will, 207 (right), /Kelvin Wong, 350, /Kalabi Yau, 202-3, /Andrey Yurlov, 201 (top), 201 (bottom), / yykkaa, 251 (top), /Nadia Zagainova, 294 (top), /Zzvet, 231 (top); cover image Getty Images/KidStock

Every effort has been made to trace the original source of copyright material contained in this book. The publisher will be pleased to hear from copyright holders to rectify any errors or omissions.

OXFORD
UNIVERSITY PRESS
AUSTRALIA & NEW ZEALAND

ISBN 978-0-19-559811-7

9 780195 598117

visit us at: **oup.com.au** or
contact customer service: **cs.au@oup.com**