VICTORIAN CURRICULUM VERSION 2.0

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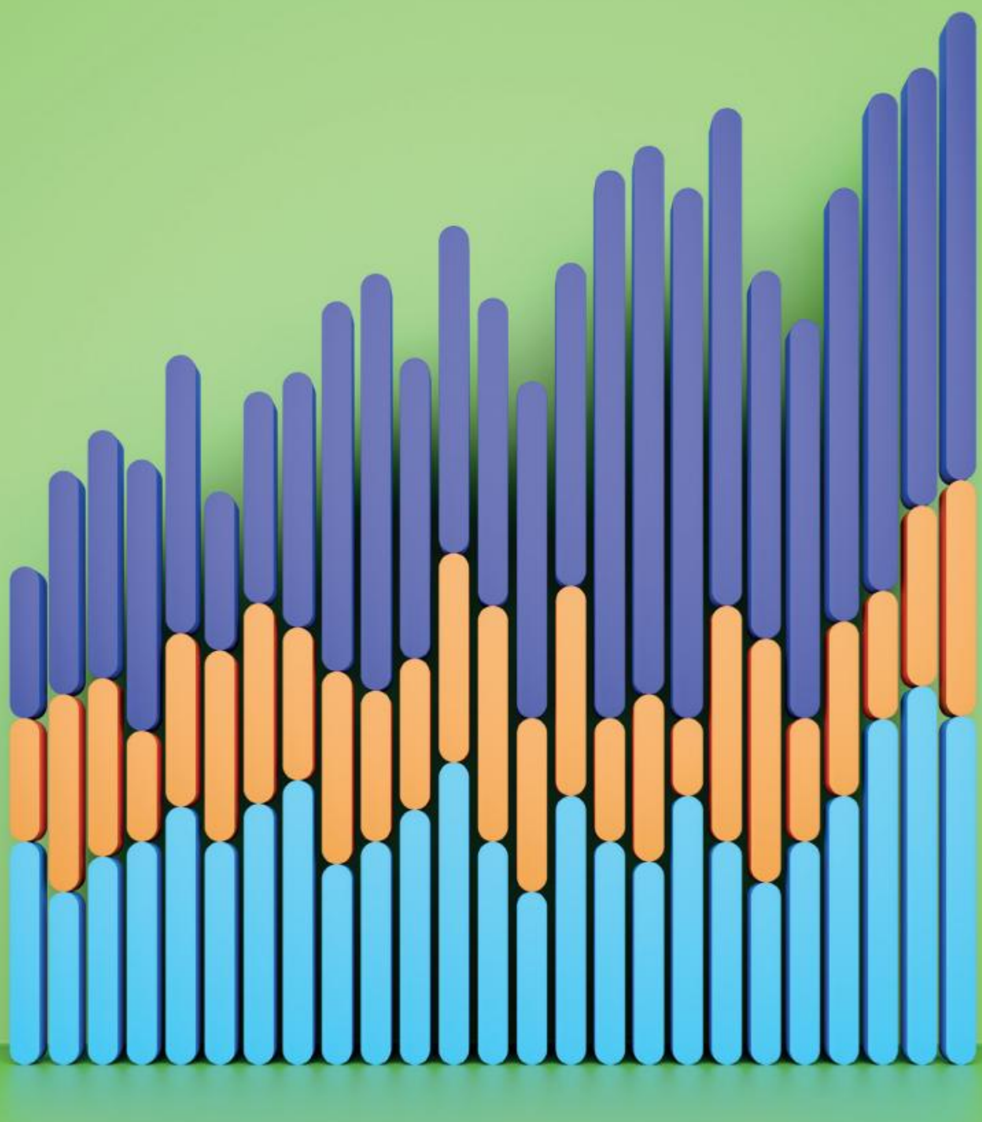
OXFORD MATHS 9

SECOND EDITION



Series Consultant: Thomas Christiansen

Alexander Blanksby   Morgan Levick

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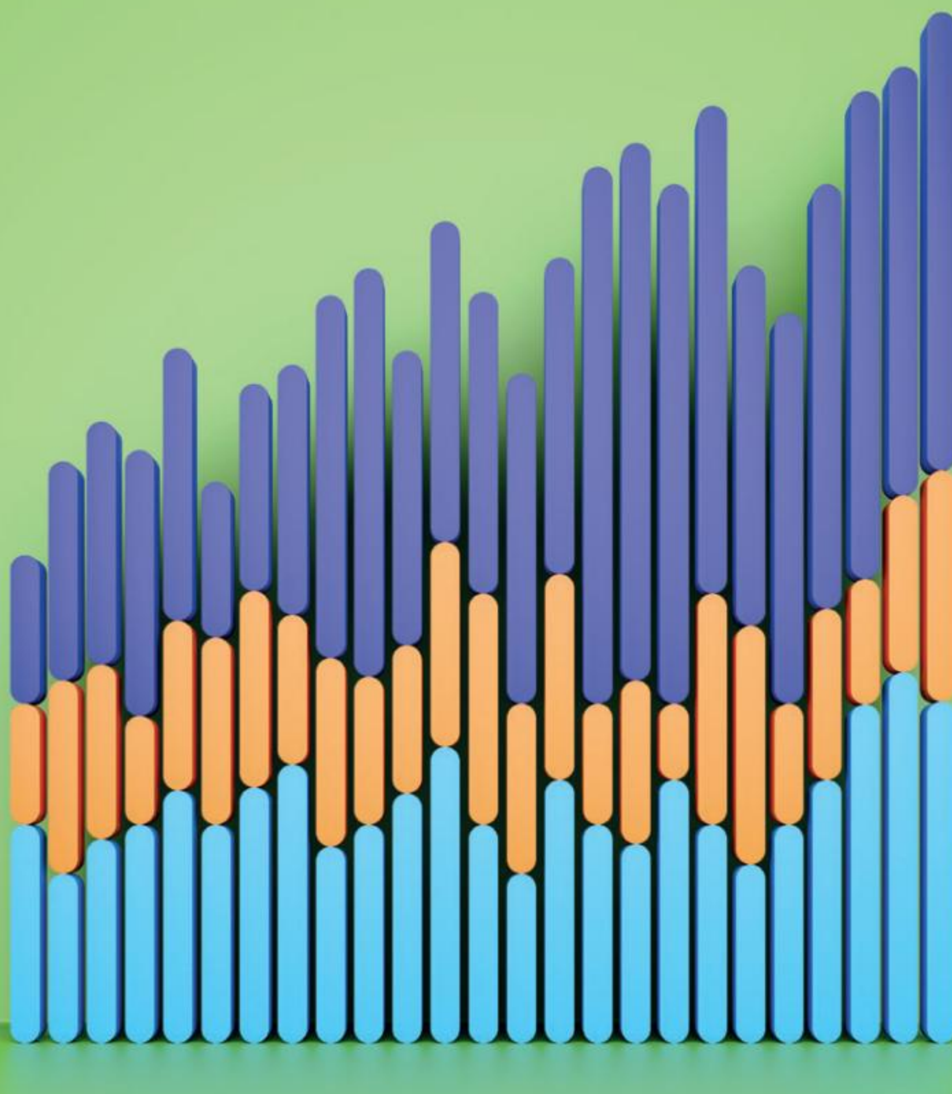

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Aboriginal and Torres Strait Islander peoples are advised that this publication may include images or names of people now deceased.

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Oxford Maths 7–10 Victorian Curriculum utilises an innovative suite of print and digital resources to guide students on a focused mathematics journey. The series makes maths accessible to students with differing levels of understanding, increasing engagement by giving learners the opportunity to achieve success at their own skill level while also providing comprehensive syllabus coverage.

Key features of Student Books

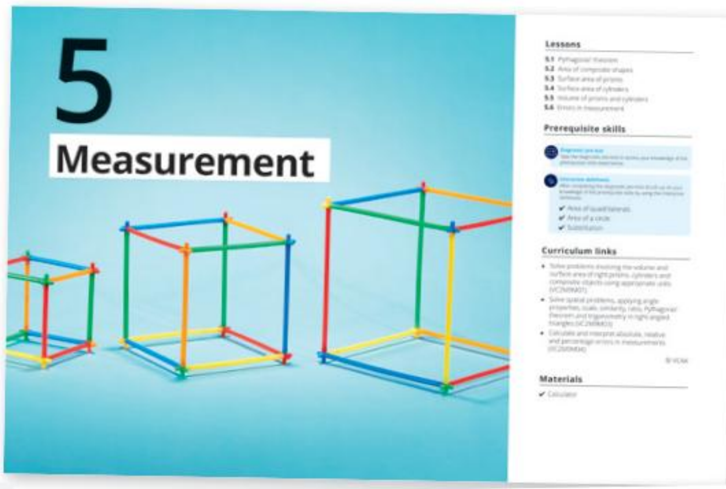
> Complete access to all digital resources.

> Australian Maths Trust (AMT) spreads offer unique questions designed to challenge students and build engagement.

> STEAM projects encourage inter-disciplinary thinking.

> Semester reviews provide an opportunity to revise key concepts from each semester.

> NAPLAN practice allows students to revise numeracy skills for the National Assessment Program.

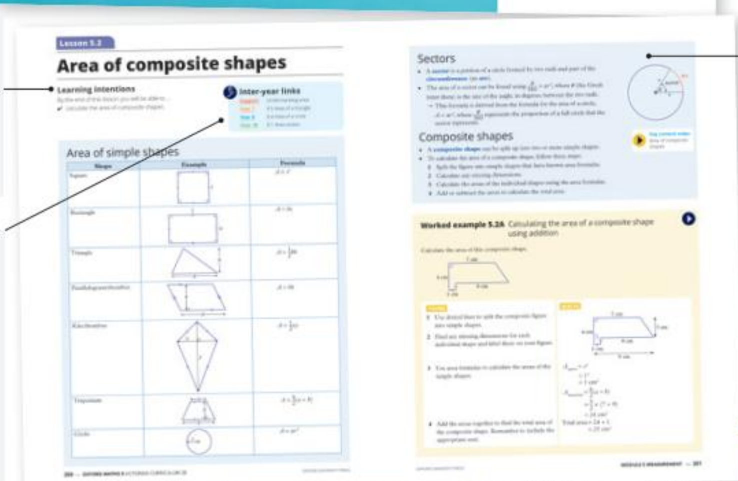


Each module opens with:

- **Prerequisite skills** with reference to an online diagnostic pre-test and interactive skillsheets.
- **Curriculum links** to all relevant content descriptions in the Victorian Curriculum.
- **Materials** used to complete the exercises.

Learning intentions

- Signpost the foundational skills being developed in each lesson.

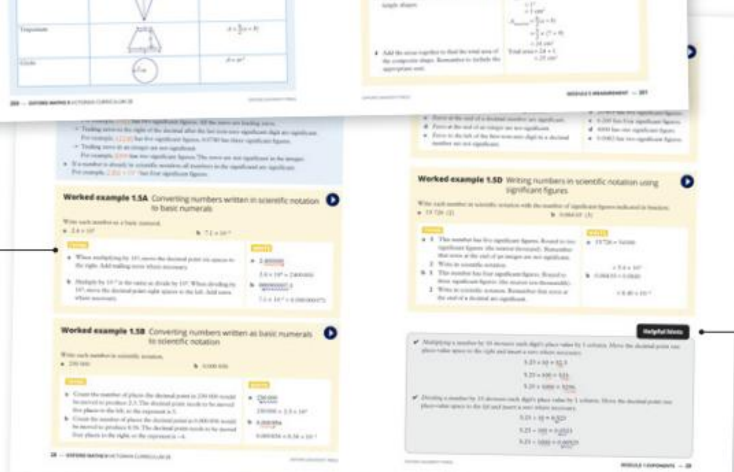


Inter-year links

- Provide easy access to support and extension material from each of the 7–10 Student Books as students build knowledge year on year.

New theory

- Backed by the latest pedagogical research to promote engagement with the material.
- Filled with precise diagrams that bring key concepts to life, and aid understanding.



Worked examples

- Outline a step-by-step thought process for solving essential questions with direct reference to the exercises.

Helpful hints

- Provide additional strategies for tackling problems.
- Highlight important elements of the theory.
- Point out common misconceptions.

Understanding and fluency

- Basic exercises dedicated to practising key concepts.

Problem solving and reasoning

- Comprehensive exercises bring together new ideas and provide engaging contexts from real-world problems.

Challenge

- Advanced exercises designed to build engagment and anticipate future learning outcomes.

Module summary

- Condenses all the theory from each section into one accessible revision page.

Differentiated learning pathways

- Each exercise is separated into three pathways, tailoring for students of all skill levels.
- Each pathway can be assigned based on results of the diagnostic pre-tests that are recommended at the beginning of every module.

Checkpoint

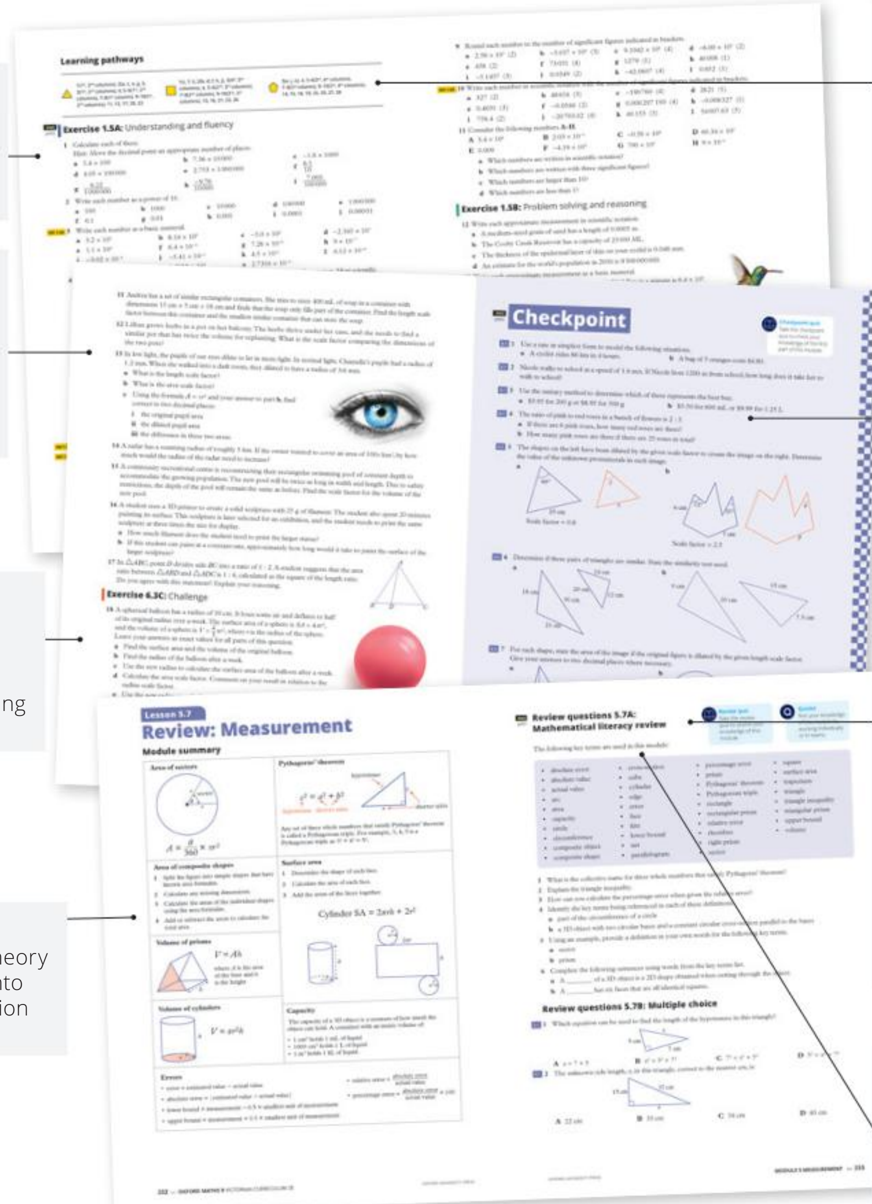
- A section in the middle of each module dedicated to summarising key skills and encouraging memory retention.
- Reference to an online checkpoint quiz to gauge student progress.

Module review

- Additional practice questions to further consolidate understanding at the end of each module.
- Reference to an online module review quiz to track results.
- Reference to Quizlet test to revise new terminology.

Mathematical literacy reviews

- A comprehensive literacy review assesses understanding of key terms.



Integrated STEAM projects

- Take the hard work out of cross-curricular learning with engaging STEAM projects. Two fully integrated projects are included at the end of each book in the series, and are scaffolded and mapped to the Science, Maths and Humanities curricula. The same projects also feature in the corresponding Oxford Humanities and Oxford Science series to assist cross-curricular learning.

Problem solving through design thinking

- Each STEAM project investigates a real-world problem that students are encouraged to problem-solve using design thinking.

Full digital support

- Each STEAM project is supported by a wealth of digital resources, including student booklets (to scaffold students through the design-thinking process of each project), videos to support key concepts and skills, and implementation and assessment advice for teachers.

oxforddigital

This Student Book is supported by a range of digital resources and features for students and teachers on Oxford Digital.

Key features for students

Student digital access allows you to:

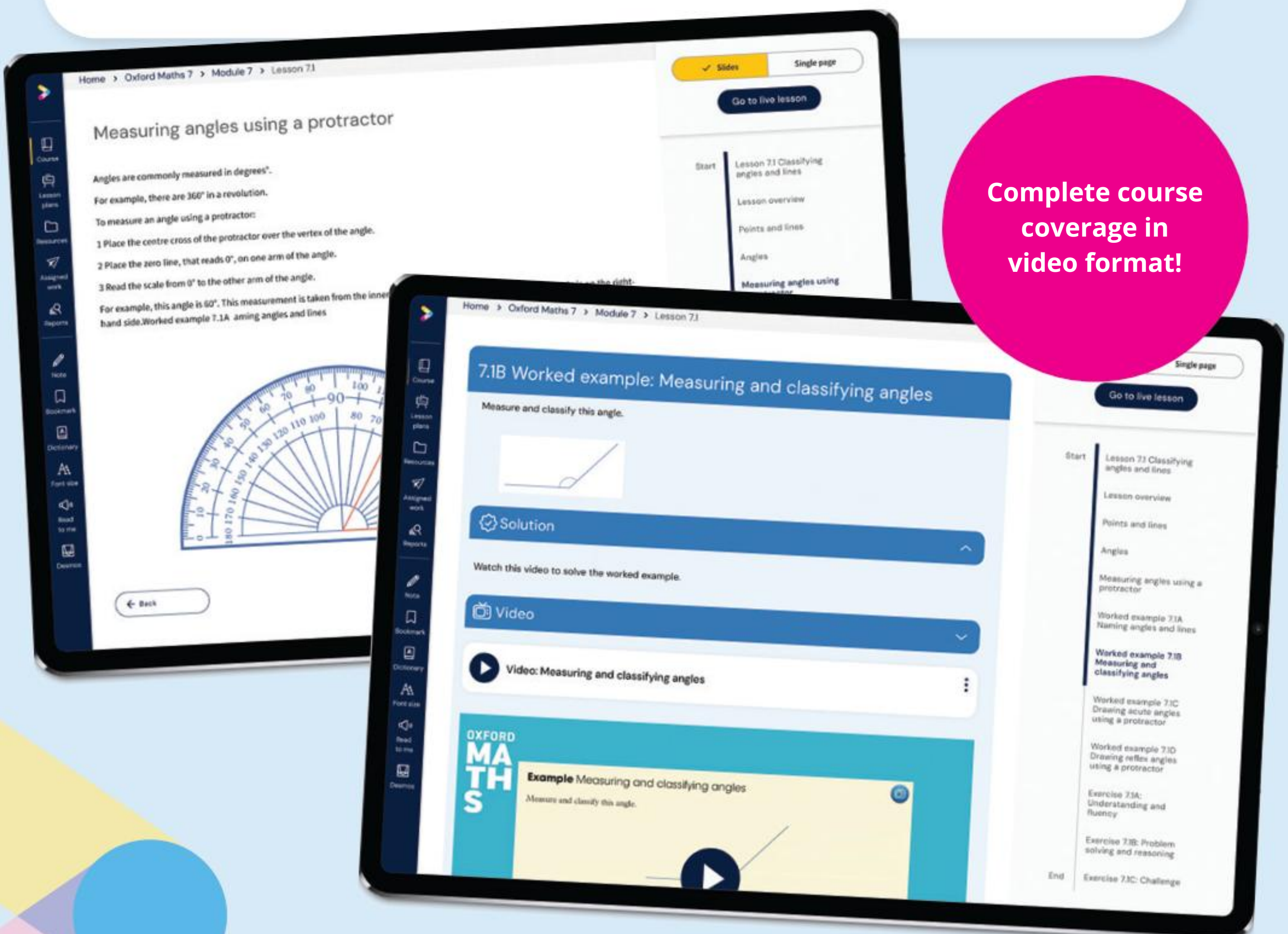
> **view** all the Student Book content in a fully accessible, reflowable format that's delivered in bite-sized chunks so you can work at you own pace

> **highlight**, **take notes**, **bookmark** pages, or define words with the built-in **Australian Oxford Dictionary**

> **watch** hundreds of short **content videos** and **worked example videos** to help you revise anything you don't understand, catch up on things you've missed or complete your homework

> **complete** thousands of interactive questions and quizzes as you work through the content and get the answers and results sent to you instantly

> **use** a suite of **Desmos calculators and graphing tools** to help you complete your work.



Complete course coverage in video format!

Key features for teachers

Teacher digital access allows you to:

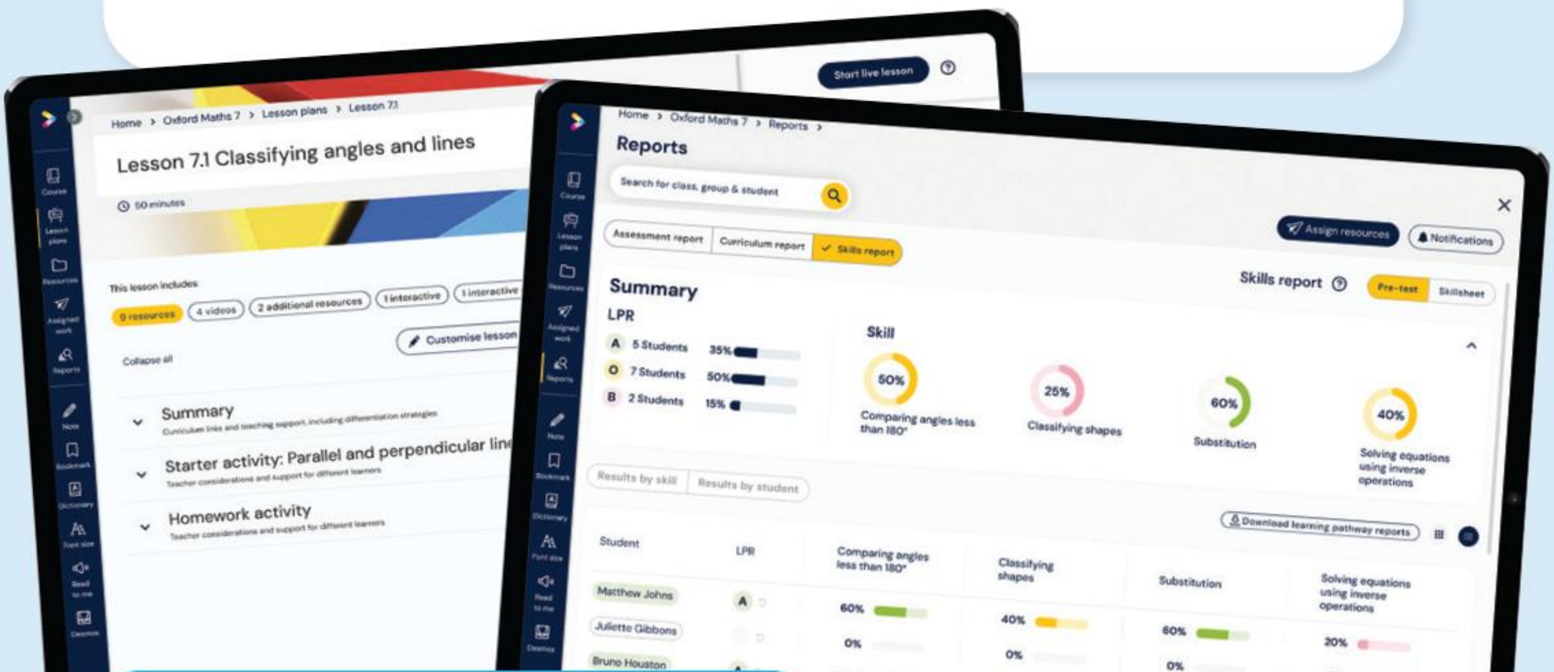
> **elevate** your teaching and reduce planning and preparation time with **Live Lesson mode**. This Premium feature is an Australian first that lets you upgrade from traditional print-based lesson plans to **fully integrated, perfectly sequenced and timed interactive lessons complete with classroom activities** that drive engagement and participation

> **personalise** learning for every student and **differentiate** content based on student strengths and weaknesses. Assign support or extension resources to any student in any year level using **inter-year links**

> **identify gaps in understanding** with diagnostic quizzes that generate personalised learning pathway reports for every student based on ability. Learning pathway reports provide clear learning goals and exercises for every lesson including interactive resources designed to target misconceptions and give each student what they need to address them!

> **begin every lesson** with ready-made learning intentions and success criteria

> **revolutionise** your planning, marking and reporting with powerful analytics on student performance and progress:
- **Assessment report** shows how students are performing in each online interactive assessment, providing instant feedback for teachers about areas of understanding
- **Curriculum report** summarises student performance against specific curriculum content descriptors and curriculum codes
- **Skills report** displays students' understanding of prerequisite skills in mathematics.



We partner with the best to bring you the best

ClickView | desmos studio PBC | AMT AUSTRALIAN MATHS TRUST | Quizlet

NAPLAN-style practice assessments

NAPLAN (National Assessment Program – Literacy and Numeracy) is an annual assessment for students in Years 3, 5, 7 and 9. NAPLAN is made up of tests in the areas of:

> reading

> writing

> conventions of language (spelling, grammar and punctuation)

> numeracy.

From 2022, all NAPLAN tests are being conducted online. OUP has developed a number of NAPLAN-style practice assessments* for this title, which can be accessed on Oxford Digital.

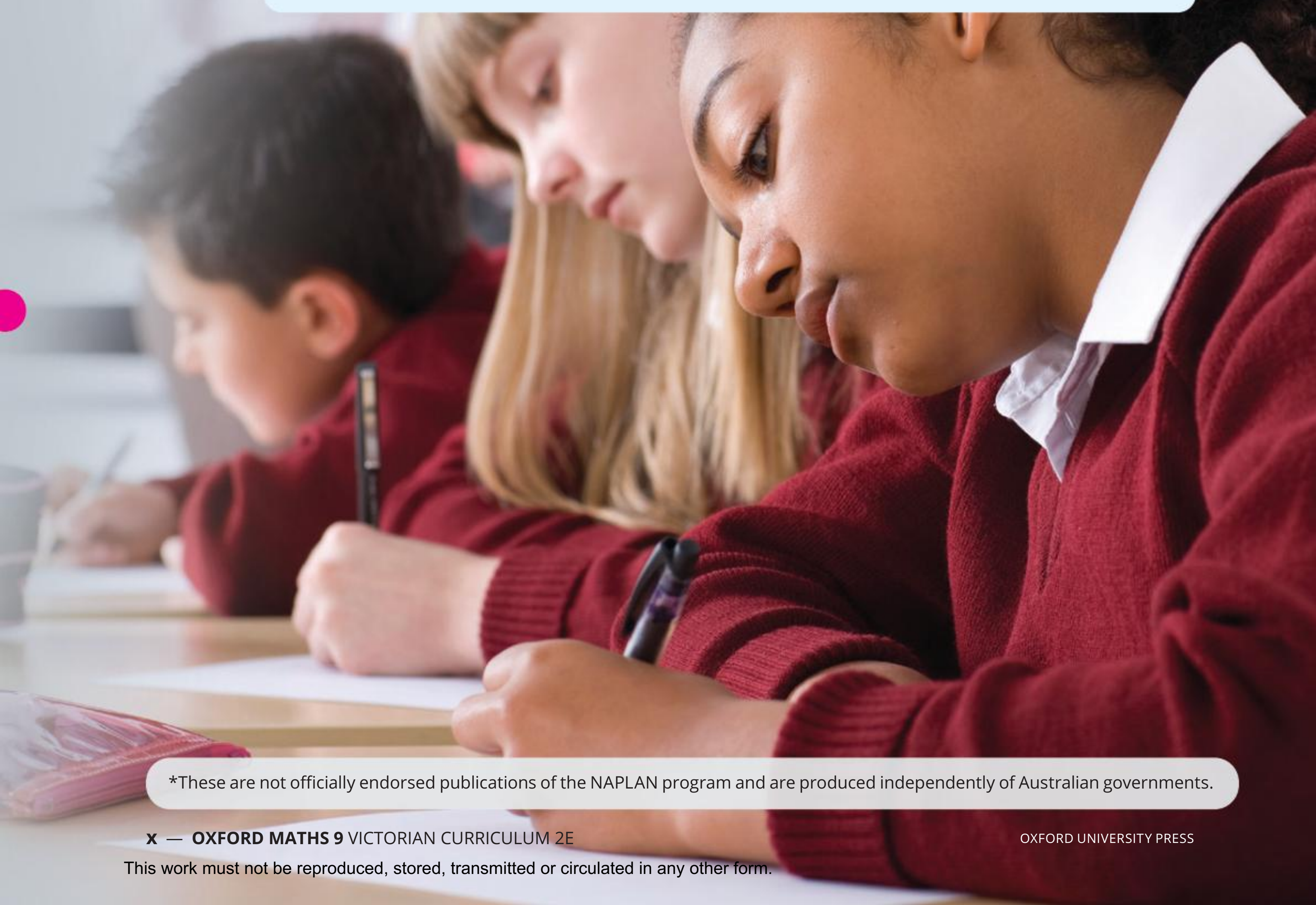
Access the practice tests on oxforddigital

Practice test 1
Non-calculator

Practice test 1
Calculator

Practice test 2
Non-calculator

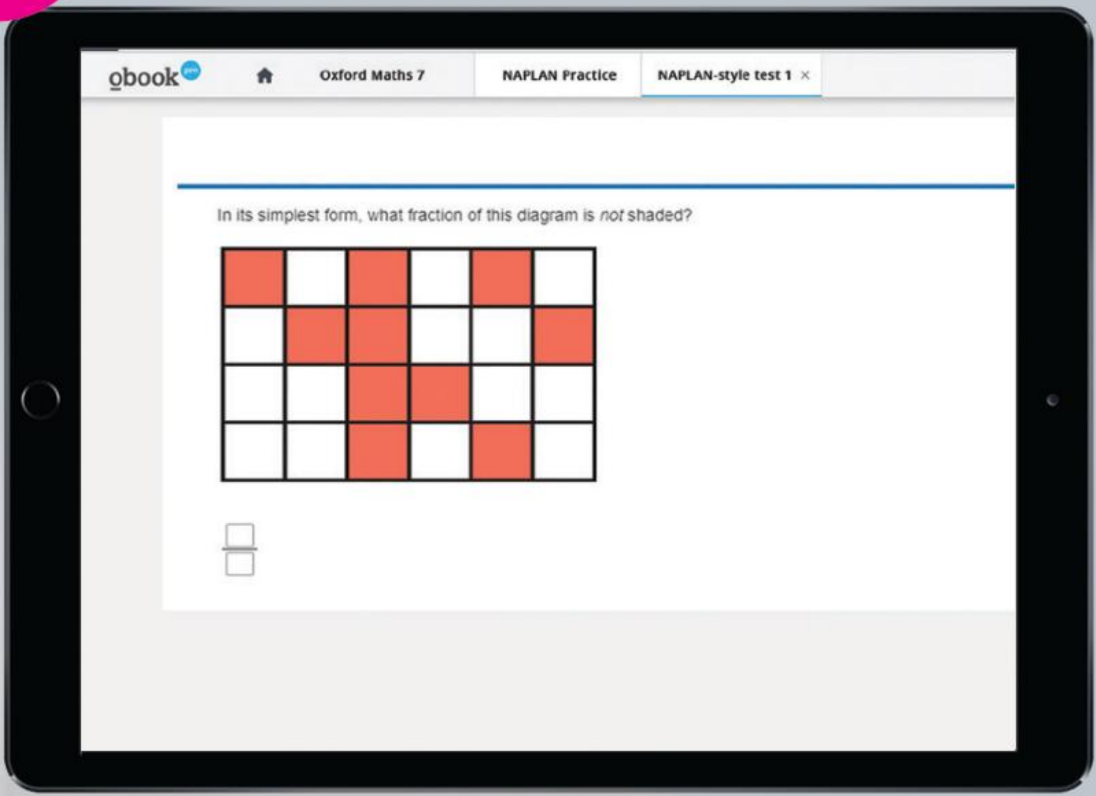
Practice test 2
Calculator



*These are not officially endorsed publications of the NAPLAN program and are produced independently of Australian governments.

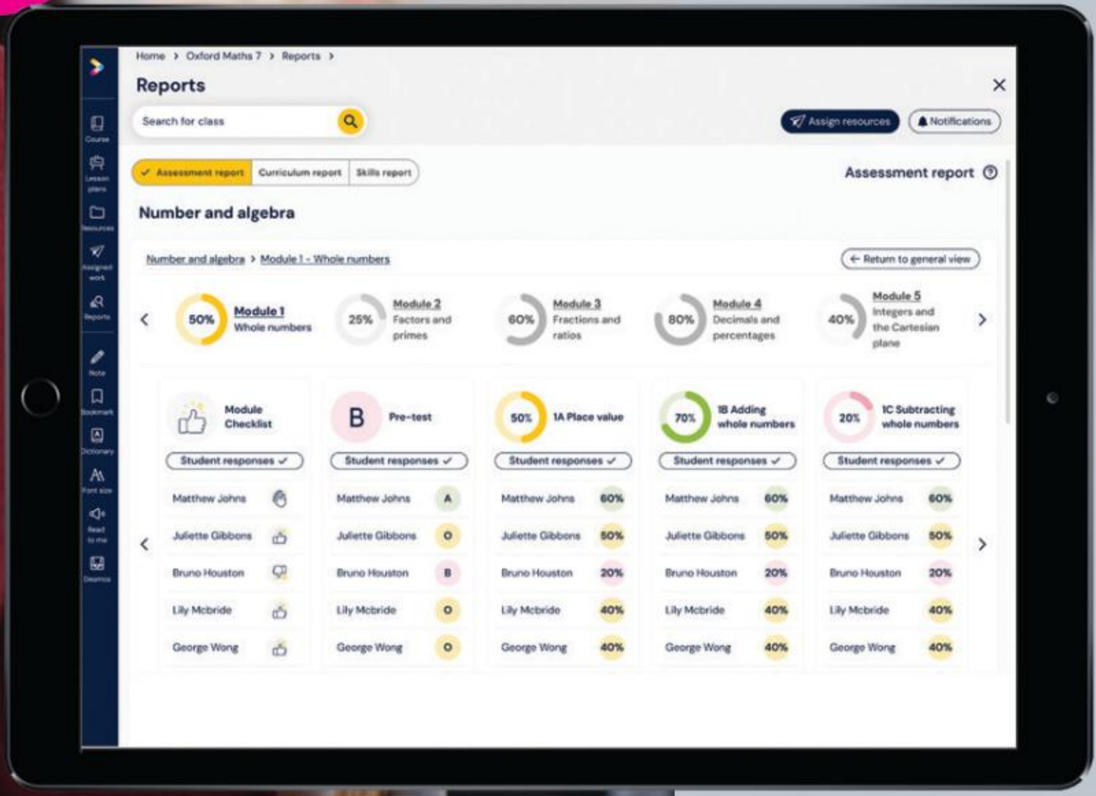
Online practice

There are four online practice assessments available on Oxford Digital.



Reporting

The results from your practice assessments can be found under the Reports tab on Oxford Digital.



1 Exponents

Lessons

1.1 Exponents
1.2 Products and quotients of powers
1.3 Raising exponents and the zero exponent
1.4 Negative exponents
1.5 Scientific notation
1.6 Rational and irrational numbers

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Order of operations
- ✔ Highest common factor
- ✔ Multiplying and dividing fractions
- ✔ Equivalent fractions
- ✔ Squares and square roots

Curriculum links

- Recognise that the real number system includes the rational numbers and the irrational numbers, and solve problems involving real numbers with and without using digital tools (VC2M9N01)
- Apply the exponent laws to numerical expressions with integer exponents and the zero exponent, and extend to variables (VC2M9A01)
- Solve problems involving very small and very large measurements, timescales and intervals expressed in scientific notation (VC2M9M02)

Materials

✔ Calculator

Exponents

Learning intentions

By the end of this lesson you will be able to ...

✔ evaluate numbers expressed in exponent form and use exponent form for prime factorisation.

Inter-year links

Year 7	1.7 Exponents and square roots
Year 8	4.1 Exponents
Year 10	1.1 Exponents

Exponent form

Key content video
Exponents

- **Exponent form** (or exponent notation) is used to represent repeated multiplication.
  → 3^4 is read as '3 to the power of 4'.
  → a^3 is read as 'a to the power of 3'.
- The **base** is the number or variable that is multiplied repeatedly.
- The **exponent** (or **index**) indicates the number of times the base is multiplied.
- If no exponent is indicated, then the base has an exponent of 1.

$$2 = 2^1$$
$$x = x^1$$

exponent

base → 3^4 exponent form

$= 3 \times 3 \times 3 \times 3$ expanded form

$= 81$ basic numeral

exponent

base → $a^3 = a \times a \times a$

exponent form expanded form

- Exponent form can also be used to represent powers of negative numbers.

exponent

base → $(-2)^3$ exponent form

$= (-2) \times (-2) \times (-2)$ expanded form

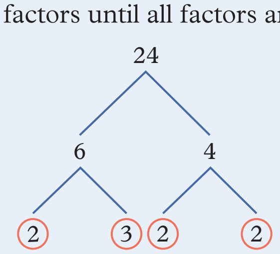
$= -8$ basic numeral

→ If the base is negative and the exponent is an even number, the basic numeral is positive.
→ If the base is negative and the exponent is an odd number, the basic numeral is negative.
→ If the base is positive, regardless if the exponent is an odd or even number, the basic numeral is positive.

Prime factorisation

Key content video
Prime factorisation

- The **prime factorisation** of a positive integer is the product of all prime factors of that integer.
  → Prime factorisation is often expressed in exponent form with the bases listed in ascending order.
  For example, the prime factorisation of 24 is: $24 = 2 \times 2 \times 2 \times 3$
  $= 2^3 \times 3$
- The prime factorisation of a positive integer can be found using **factor trees**. In factor trees, composite numbers are broken down into pairs of factors until all factors are prime numbers.



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Worked example 1.1A Calculating the value of a number in exponent form

Write the following in expanded form and evaluate.

a 2^5 **b** $(-4)^3$ **c** $\left(\frac{2}{5}\right)^4$

THINK	WRITE
a **1** Identify the base and the exponent. The base is 2 and the exponent is 5, so 2 is multiplied by itself 5 times.	**a** $2^5 = 2 \times 2 \times 2 \times 2 \times 2$
2 Perform the multiplications.	$= 4 \times 2 \times 2 \times 2$ $= 8 \times 2 \times 2$ $= 16 \times 2$ $= 32$
b **1** Identify the base and the exponent. The base is -4 and the exponent is 3, so -4 is multiplied by itself 3 times.	**b** $(-4)^3 = (-4) \times (-4) \times (-4)$
2 Perform the multiplications. Recall that if the base is negative and the exponent is an odd number, then the basic numeral is negative.	$= 16 \times (-4)$ $= -64$
c **1** Identify the base and the exponent. The base is $\frac{2}{5}$ and the exponent is 4, so $\frac{2}{5}$ is multiplied by itself 4 times.	**c** $\left(\frac{2}{5}\right)^4 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$
2 Perform the multiplications. Recall that to multiply fractions, you multiply the numerators together and the denominators together.	$= \frac{2 \times 2 \times 2 \times 2}{5 \times 5 \times 5 \times 5}$ $= \frac{16}{625}$

Worked example 1.1B Writing expressions in expanded form

Write the following in expanded form.

a x^4 **b** $(-ab)^3$ **c** $2xy^2z$

THINK	WRITE
a Identify the base and the exponent. The base is x and the exponent is 4, so x is multiplied by itself 4 times.	**a** $x^4 = x \times x \times x \times x$
b Identify the base and the exponent. The base is $-ab$ and the exponent is 3, so $-ab$ is multiplied by itself 3 times.	**b** $(-ab)^3 = (-ab) \times (-ab) \times (-ab)$
c There are four bases in this term. Identify the bases and the matching exponent. Recall that if a base doesn't have an indicated exponent, then the exponent is 1. Therefore, 2, x and z each have an exponent of 1, and y has an exponent of 2.	**c** $2xy^2z = 2 \times x \times y \times y \times z$

Worked example 1.1C Prime factorisation using factor trees

Use a factor tree to express each number as the product of its prime factors. Write your answer in exponent form.

a 20 **b** 315

THINK

1 Identify a factor pair by dividing the composite number by its smallest prime factor. The smallest prime factor of an even number is always 2. Remember that if the sum of all the digits in a number is divisible by 3, then that number is also divisible by 3, and that any number ending in 0 or 5 is divisible by 5.

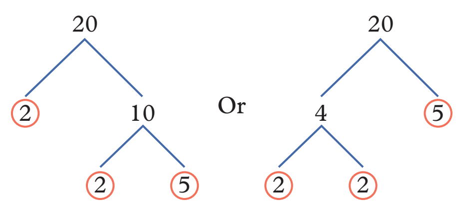
2 Continue to split factors into factor pairs until all remaining factors are prime.

3 Write the composite number as a product of its prime factors. Write the answer in exponent form and list the bases in ascending order.

WRITE

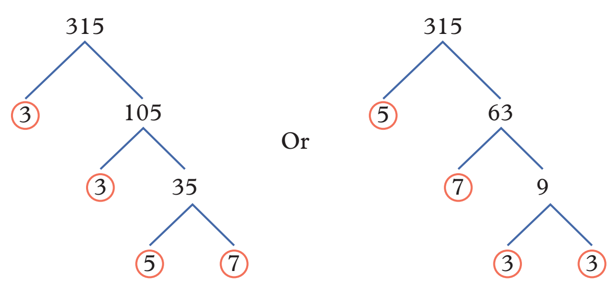
a $20 = 2 \times 2 \times 5$

$= 2^2 \times 5$



b $315 = 3 \times 3 \times 5 \times 7$

$= 3^2 \times 5 \times 7$



Helpful hints

- ✔ Remember that raising a number to an exponent and multiplying are two different operations. For example: $2^4 \neq 2 \times 4$; $2^4 = 2 \times 2 \times 2 \times 2$.
- ✔ Take care when writing exponents – they should be a smaller size than the base and sit high up on the shoulder of the base to avoid confusion between 3^4 and 34.
- ✔ When creating factor trees, remember that a branch always ends on a prime number. If the number is composite, keep dividing! Use the divisibility tests to ensure the numbers at the end of the tree are prime numbers.
- ✔ The first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Learning pathways

△ 1–8, 9(a, c, e, g), 10, 11, 13, 14, 17(a, b)	□ 1–3(b, d, f, h), 4, 6–8, 9(b, d, f, h), 12, 13, 15, 17, 19	⬠ 1–3(g, h), 4, 6(e–h), 8(e, f), 12–14, 16, 17(c, d), 18–21

ANS p448

Exercise 1.1A: Understanding and fluency

WE 1.1A **1** Write the following in expanded form and evaluate.

a 6^4 **b** 8^3 **c** $(-2)^5$ **d** $(-3)^6$

e $\left(\frac{5}{4}\right)^3$ **f** $\left(\frac{1}{2}\right)^7$ **g** $\left(-\frac{2}{3}\right)^4$ **h** $\left(-\frac{3}{5}\right)^5$

WE 1.1B **2** Write the following in expanded form.

a b^6 **b** $(-n)^5$ **c** $(-cd)^2$ **d** $(2pq)^4$

e $2pq^4$ **f** $-4a^2b^3c$ **g** $(3m^2)^5$ **h** $3(m^2)^5$

3 Write the following in exponent form.

a $5 \times 5 \times 5 \times 5$

b $a \times a \times a \times a$

c $v \times k \times k \times v \times k \times v \times v \times 7$

d $qu \times qu \times qu \times qu \times qu$

e $(-h) \times (-h) \times (-h)$

f $-(h \times h \times h)$

g $n^3 \times n^3 \times n^3 \times n^3 \times n^3 \times n^3$

h $5b^3d^4 \times 5b^3d^4$

WE 1.1C **4** Express each number as the product of its prime factors. Write your answers in exponent form.

a 50 **b** 72 **c** 135 **d** 378

e 152 **f** 812 **g** 550 **h** 1665

5 Evaluate the following.

a $(0.2)^2$ **b** $(-0.2)^2$ **c** $(0.02)^2$

d $(0.2)^3$ **e** $(-0.2)^3$ **f** $(0.02)^3$

g $(0.2)^4$ **h** $(-0.2)^4$ **i** $(0.02)^4$

6 Write the following in exponent form without brackets.

a $(-5)^4$ **b** $(-5)^3$ **c** $(ab)^4$ **d** $(5xy)^8$

e $5(xy)^8$ **f** $\left(\frac{11}{2}\right)^6$ **g** $(-3abc)^5$ **h** $(-3abc)^8$

7 Substitute in the given values and evaluate the expressions.

a x^3, where $x = 7$

b $6a^4b^2$, where $a = -2$ and $b = \frac{1}{4}$

c $\frac{p^4}{qr^3}$, where $p = 3$, $q = 5$ and $r = -4$

d $2x^3 + 8x^2 + x + 7$, where $x = 10$

8 Write the following in exponent form.

a $2 \times 2 \times 2 \times 3 \times 3$ **b** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 6$

c $13 \times 13 \times 13 \times 13 \times 17 \times 17 \times 17 \times 17 \times 17$ **d** $101 \times 101 \times 103 \times 103 \times 103 \times 103 \times 103$

e $4 \times 4 \times 4 \times x \times x \times x \times x$ **f** $7 \times 7 \times xy \times xy \times xy \times xy \times xy \times xy \times xy$

9 Use the fact that $300 = 2^2 \times 3 \times 5^2$ to help find the prime factors of each of the following numbers and then write in exponent form.

a 600 **b** 150 **c** 900 **d** 1500

e 3000 **f** 1800 **g** 2100 **h** 2400

10 Express each of the following in exponent form.

a $xxxxxxx$ **b** $aaabb$ **c** $3rsstt$ **d** $4eeeeeeeff$

Exercise 1.1B: Problem solving and reasoning

11 Explain the mistake in each of the following. Then change the right-hand side so that the equation is correct.

a $tk^5 = t \times k \times t \times k \times t \times k \times t \times k \times t \times k$

b $(2rw)^4 = 2 \times r \times w \times w \times w \times w$

c $-3 \times (-2)^4 = 6^4$

12 Substitute in the given values and evaluate each expression.

a $(2x + 3)^8$, where $x = -2$

b $\left(\frac{y}{3}\right)^3 + 4\sqrt{y}$, where $y = 9$

c $ab^3 - ba^2$, where $a = 5$ and $b = -3$

d $2r^3 + 8r^2 - 3r$, where $r = -\frac{3}{2}$

13 When evaluating a negative number raised to an exponent, the sign of the basic numeral is related to whether the exponent is odd or even.

a Evaluate each of the following.

i $(-1)^2$ **ii** $(-1)^3$ **iii** $(-1)^4$ **iv** $(-1)^5$ **v** $(-1)^6$

vi $(-1)^7$ **vii** $(-1)^8$ **viii** $(-1)^9$ **ix** $(-1)^{10}$ **x** $(-1)^{11}$

b Complete the following sentences.

i When the exponent n is odd, the basic numeral of $(-1)^n$ is __________.

ii When the exponent n is even, the basic numeral of $(-1)^n$ is __________.

c Decide for each of the following whether the basic numeral is positive or negative. Do not evaluate.

i $(-2)^{15}$ **ii** $(-4)^{27}$

iii $(-24)^{30}$ **iv** $(-17)^{198}$

v $(-16)^7 \times (-34)^{11}$ **vi** $(-8)^{14} \times (-5)^{27}$

vii $(-78)^{99} \times (-81)^{45} \times (-21)^{68}$ **viii** $\left(-\frac{77}{101}\right)^{108} \times \left(-\frac{301}{22}\right)^{404}$

14 Consider each pair of numbers written in exponent form.

i Using a calculator, evaluate each pair.

ii Describe how the two numbers are similar and how they are different in their exponent form and as a basic numeral.

a $(0.7)^3$ and $(0.07)^3$

b $(-0.4)^3$ and $-(0.4)^3$

c $(-1.2)^3$ and $(-1.2)^4$

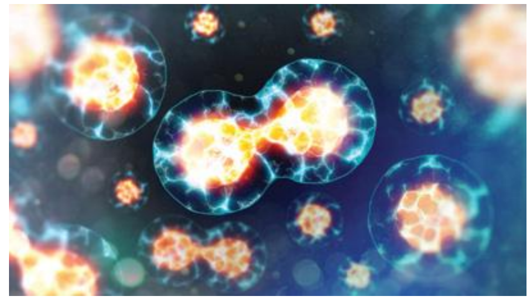
d $(2.1)^3$ and $(2.01)^3$

15 A farmer's herd of cattle grows by approximately 20% each year. In 2023, the farmer had 20 cows.

a By what number can the number of cows be multiplied by to increase it by 20%?

b Predict the size of the farmer's herd in 2024, 2025 and 2028. Round to the nearest whole number.

16 Three different groups of bacteria, Bacteria A, Bacteria B and Bacteria C, reproduce at different rates.



a Bacteria A splits into two bacteria every day.
How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in exponent form.

b Bacteria B splits into two bacteria twice each day.
How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in exponent form.

c Bacteria C splits into two bacteria once every two days.
How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in exponent form.

d Populations of Bacteria A, B and C each have 3 bacteria initially.
Determine the size of each bacteria population after 3 days.

17 The lowest common multiple is the product of the largest exponent of each prime factor or pronumeral in each term. The highest common factor is the product of the smallest exponent of each prime factor or pronumeral in each term. Find the lowest common multiple and highest common factor of each pair of terms. Write your answers in exponent form.

a $2^8 \times 3^5 \times 5^2 \times 7$ and $2^4 \times 3^{15} \times 5^2 \times 7^4$

b $a^8b^5c^2d$ and $a^4b^{15}c^2d^4$

c $pq^5r^7s^2$ and $pq^3r^{10}s^4$

d $8x^3y^9z^4$ and $12xy^3z^4$

Exercise 1.1C: Challenge

18 Find the smallest positive integer that the following numbers can be multiplied by to get a perfect square.

a 3 **b** 4 **c** 8 **d** 12 **e** 60

19 For each of the following, state how many different sequences of answers there are. Write your answers in exponent form.

a A quiz that has 10 true or false questions.

b A quiz that has 10 multiple choice questions each with options A, B, C, D, E.

c A quiz that has 12 true or false questions and 8 multiple choice questions with options A, B, C, D, E.

20 Fill in the base and exponent using positive and negative whole numbers (integers). Find the number of different exponent expressions that will equal 64.

$$\square^{\square} = 64$$

21 Evaluate each of the following.

a $\dfrac{ab^2}{c^3}$, where $a = 6$, $b = \dfrac{1}{3}$, and $c = -2$

b $\dfrac{p^3}{q^2}$, where $p = \dfrac{3}{2}$ and $q = \dfrac{2}{3}$

c $\dfrac{r^4}{(mn)^3}$, where $m = -0.5$, $n = 0.2$, and $r = -0.7$

Online resources:

- **Interactive skillsheet** Exponents
- **Worksheet** Writing terms in expanded form
- **Investigation** Using exponents to determine how card tricks work
- **CAS instructions** CAS basics
- **Quick quiz** 1.1

Lesson 1.2

Products and quotients of powers

Learning intentions

By the end of this lesson you will be able to ...

✔ simplify products and quotients of terms in exponent form with the same base.

Inter-year links

Year 7	3.2 Equivalent fractions
Year 8	4.2 Products and quotients of powers
Year 10	1.1 Exponents

Key content video
Simplifying expressions using products and quotients of powers

Product of powers law

- When multiplying terms in exponent form with the same base, add the exponents and write the result with the same base. This could be verified by writing both numbers or terms in expanded form and simplifying.
  For example, $2^3 \times 2^5 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ is the same as $2^3 \times 2^5 = 2^{3+5}$
  $= 2^8$ $\qquad$ $= 2^8$

$$\begin{aligned} 2^3 \times 2^5 &= 2^{3+5} \\ &= 2^8 \end{aligned} \qquad \begin{aligned} a^3 \times a^5 &= a^{3+5} \\ &= a^8 \end{aligned}$$

- To multiply terms where variables have exponents and coefficients:
  1. Multiply the coefficients of each term.
  2. Apply the product of powers law to add the exponents of any common bases.
  3. Write the coefficient first, followed by the variables listed in alphabetical order.

Quotient of powers law

- When dividing terms in exponent form with the same base, subtract the second exponent from the first exponent and write the result with the same base.

$$\begin{aligned} 2^5 \div 2^3 &= 2^{5-3} \\ &= 2^2 \end{aligned} \qquad \begin{aligned} a^5 \div a^3 &= a^{5-3} \\ &= a^2 \end{aligned}$$

- Remember that quotients can be written as fractions. When simplifying fractional quotients, subtract the exponent of the term in the denominator from the exponent of the term in the numerator.

$$\begin{aligned} \frac{2^5}{2^3} &= 2^{5-3} \\ &= 2^2 \end{aligned} \qquad \begin{aligned} \frac{a^5}{a^3} &= a^{5-3} \\ &= a^2 \end{aligned}$$

- To divide terms where the variables have exponents and coefficients:
  1. Divide the coefficients by their highest common factor.
  2. Apply the quotient of powers law to subtract the exponents of any common bases.
  3. Write the simplified coefficient first, followed by the variables listed in alphabetical order.

Worked example 1.2A Simplifying numerical expressions using an exponent law

Use the appropriate exponent law to simplify each expression. Leave each answer in exponent form.

a $3^4 \times 3^2$ **b** $\frac{8^5}{8^2}$

THINK	WRITE
a Product of powers law: Add exponents with a common base, 3.	**a** $3^4 \times 3^2 = 3^{4+2}$ $= 3^6$
b Remember that fractions can be written as division problems. Use the quotient of powers law to subtract exponents with a common base, 8.	**b** $\frac{8^5}{8^2} = 8^{5-2}$ $= 8^3$

Worked example 1.2B Simplifying algebraic expressions using the product of powers law

Using the product of powers law, simplify each expression.

a $2x^7 \times 3x^4$ **b** $a^3b^2 \times ab^{10}$

THINK	WRITE
a 1 Multiply the coefficients of each term. 2 Use the product of powers law to add exponents with a common base, x. 3 Write the coefficient first, followed by the variable.	**a** $2x^7 \times 3x^4 = (2 \times 3) \times x^7 \times x^4$ $= 6 \times x^{7+4}$ $= 6 \times x^{11}$ $= 6x^{11}$
b 1 Group the terms with common bases. Remember that $a = a^1$. 2 Use the product of powers law to add exponents with common bases, a and b. 3 Write the variables in alphabetical order.	**b** $a^3b^2 \times ab^{10} = a^3 \times b^2 \times a^1 \times b^{10}$ $= a^{3+1} \times b^{2+10}$ $= a^4b^{12}$

Worked example 1.2C Simplifying algebraic expressions using the quotient of powers law

Using the quotient of powers law, simplify each expression.

a $x^5 \div x^2$ **b** $a^5b^3 \div a^2b$

THINK	WRITE
a Use the quotient of powers law to subtract exponents with a common base, x.	**a** $x^5 \div x^2 = x^{5-2}$ $= x^3$
b 1 Use the quotient of powers law to subtract exponents with common bases, a and b. Remember that $b = b^1$. 2 Write the variables in alphabetical order.	**b** $a^5b^3 \div a^2b = a^{5-2}b^{3-1}$ $= a^3b^2$

Helpful hints

- ✔ 'Simplify' and 'evaluate' are different commands:
  - → To simplify in this module, use exponent laws to combine the terms and hence reduce the complexity of the expression.
  - → To evaluate or 'find the value' of a calculation or numerical expression, convert the expression from exponent form to a basic numeral.
- ✔ Exponents only apply to the number or pronumeral immediately to the left of the exponent. For example, in the term $4gh^3$, the exponent of 3 only applies to the variable h, so $4gh^3 = 4 \times g \times h \times h \times h$.
- ✔ When an algebraic term immediately follows a division symbol, the entire term is the divisor in the division. For example, $x^2 \div 2x = \frac{x^2}{2x} = \frac{x}{2}$.

Learning pathways

△ 1–5, 6–8(1st, 3rd columns), 9, 11(a, b), 14

□ 1(f, h, j, l), 3–9(2nd, 4th columns), 10, 11(b, c), 12(a, b), 14, 15(a–d)

⬠ 5–9(2nd, 4th columns), 10, 11(c, d), 12(b, c), 13, 15, 16

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Exercise 1.2A: Understanding and fluency

WE 1.2A **1** Use the appropriate exponent law to simplify each expression. Leave each answer in exponent form.

a $3^5 \times 3^4$ **b** $7^8 \div 7^2$ **c** $(-2)^7 \times (-2)^5$ **d** 6×6^2

e $(-8)^{13} \div (-8)^6$ **f** $10^2 \times 10^9$ **g** $3^6 \div 3^5$ **h** $5^3 \div 5$

i $\frac{4^7}{4^5}$ **j** $\frac{(-9)^{18}}{(-9)^9}$ **k** $\frac{13^9}{13^6}$ **l** $2^5 \times 2^2 \times 2^3$

2 Using a calculator, calculate the basic numeral for question **1** parts **a** to **d**.

WE 1.2B **3** Using the product of powers law, simplify each expression.

a $3y^3 \times y^6$ **b** $g^2 \times 7g^5$ **c** $2b^8 \times 3b^3$ **d** $-6k^5 \times 2k^8$

e $-2b^8 \times (-3b^3)$ **f** $-5g^5 \times (-2g) \times (-8g^5)$ **g** $3c \times 3c^7 \times 3c^6$ **h** $p^6 \times (-3p^2) \times (-5p^2)$

WE 1.2C **4** Using the quotient of powers law, simplify each expression.

a $a^6 \div a^4$ **b** $d^7 \div d^6$ **c** $g^{11} \div g$ **d** $p^{10} \div p^7$

e $a^8 \div a^3$ **f** $n^{14} \div n^{11}$ **g** $r^9 \div r$ **h** $8x^{17} \div x^6$

5 Use the product and quotient of powers laws to simplify each expression.

a $3x^5 \times 4x^6$ **b** $5x^4 \times 2x^3$ **c** $-8x^2 \times 3x^7$ **d** $-6x^{10} \times (-9x)$

e $6x^7 \div 2x^3$ **f** $-20x^6 \div (-5x^2)$ **g** $4x^8 \div 10x^7$ **h** $15x^{12} \div 9x^4$

i $\frac{-24t^{18}}{3t^6}$ **j** $\frac{-20r^8}{-32r^2}$ **k** $\frac{10c^7}{2c^3}$ **l** $\frac{15y^{12}}{6y^5}$

6 Use the exponent laws to simplify:

a $\frac{ax^{13}}{ax^4}$ **b** $\frac{a^2b}{a^3}$ **c** $\frac{m^5n}{m^5}$ **d** $\frac{b^{20}d}{b^{12}d}$

e $\frac{x^4 \times x^3}{x^2}$ **f** $\frac{m^7 \times m^6}{m^9}$ **g** $\frac{6a^2 \times a^8}{a^4}$ **h** $\frac{n^5 \times n^7}{n^3 \times n^4}$

i $\frac{5d^6 \times d^3}{d^9}$ **j** $\frac{8t^2 \times t^3}{-2t^3}$ **k** $\frac{-4k \times 3k^9}{-6k^{10}}$ **l** $\frac{15e^{13}}{3e^8 \times 5e^5}$

7 Simplify each expression.

a $a^3b^4 \times a^6b^2$ **b** $6m^5n^2 \times (-3m^6n)$ **c** $\frac{c^2d^9}{d^7}$ **d** $\frac{k^3m^8}{km^5}$

e $x^2 \times y^5 \times x^6 \times y^2$ **f** $3g^4 \times 5h^3 \times 2g^6$ **g** $a^5b^4 \times a^3b^2$ **h** $5x^6y^5 \times 3x^2y^5$

i $9w^4x^8 \times 6x^5y^4$ **j** $\frac{1}{tu^3} \times \frac{tu^7}{5}$ **k** $\frac{-6e^5f^{11}}{8e^4f}$ **l** $-4v^9 \times (-9y^3) \times (-3v^8y^7)$

8 Use the exponent laws to simplify each expression.

a $\dfrac{x^7 \times x^3}{x^4}$ **b** $\dfrac{2k^4 \times k^5}{k^6}$ **c** $\dfrac{4a^2 \times 3a^6}{2a^7}$ **d** $\dfrac{5m^2 \times 2x^4}{10x^6}$

e $\dfrac{a^5b^7 \times a^3b^6}{a^8b^{10}}$ **f** $\dfrac{n^{17}p^{13}}{n^3p^2 \times np^8}$ **g** $\dfrac{-6jq^5 \times 5j^7q^2}{15j^3q}$ **h** $\dfrac{6w^9x^6 \times 3w^4x^5}{9w^5x^4 \times w^6x^3}$

Exercise 1.2B: Problem solving and reasoning

9 Use the exponent laws to determine whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

a $x^3 \times x^4 = x^{12}$ **b** $k^3 + k^3 = k^6$ **c** $y^7 \div y = y^6$ **d** $a^5 \times a \times a^5 = a^{10}$

e $m^3n^5 \times m^2n^4 = m^{14}n^{14}$ **f** $100^8 \div 100^2 = 100^4$ **g** $\dfrac{m^3 \times m^7}{m^{11}} = \dfrac{1}{m}$ **h** $\dfrac{a^5b^6}{a^2b^4} \times \dfrac{a^3b^5}{a^4b} = a^2$

10 When simplifying a fraction with both the denominator and numerator written in exponent form, a variation of the quotient of powers law can be used to simplify the fraction. For example:
$\dfrac{2^3}{2^5} = \dfrac{1}{2^{5-3}} = \dfrac{1}{2^2}$

a Complete the following by cancelling common factors.

i $\dfrac{2^3}{2^5} = \dfrac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \dfrac{1}{\square \times \square} = \dfrac{1}{2^{\square}} = \dfrac{1}{2^{\square - \square}}$

ii $\dfrac{2^4}{2^8} = \dfrac{\square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square \times \square} = \dfrac{\square}{\square \times \square \times \square \times \square} = \dfrac{\square}{2^{\square}} = \dfrac{\square}{2^{\square - \square}}$

iii $\dfrac{5^6}{5^7} = \dfrac{\square \times \square \times \square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square} = \dfrac{\square}{\square} = \dfrac{\square}{\square^{\square}} = \dfrac{\square}{\square^{\square - \square}}$

b Simplify the following. Write your answers in exponent form.

i $\dfrac{3^4}{3^{10}}$ **ii** $\dfrac{5^2}{5^8}$ **iii** $\dfrac{2^5 \times 3^2}{2^9 \times 3^7}$ **iv** $\dfrac{2^5 \times 3^7}{2^9 \times 3^2}$ **v** $\dfrac{2^9 \times 3^2}{2^5 \times 3^7}$

11 Write the following products in exponent form with prime number bases.

a $2 \times 4 \times 8 \times 16 \times 32$ **b** $3 \times 9 \times 27 \times 81 \times 243$ **c** $6 \times 36 \times 216$ **d** $4 \times 16 \times 64 \times 256 \times 1024$

12 Determine the values of the unknowns.

a $2^5 \times 3^{4x} \times 5^{12} \times 7^{z+3} = 2^w \times 3^{12} \times 5^{6y} \times 7^{11}$

b $(5^x \times 7^{4y} \times 11^z) \times (3^9 \times 5^6 \times 11) = 3^9 \times 5^{15} \times 7^{24} \times 11^5$

c $\dfrac{11^a \times 13^b \times 17^{2c} \times 19^8}{11^5 \times 13^6 \times 17^3 \times 19^d} = 11^{11} \times 13^3 \times 17^5 \times 19^2$

13 Do the exponent laws for multiplying and dividing terms in exponent form work when the terms have different bases? Explain, using $2^4 \times 3^2$ and $y^8 \div x^5$ as examples.

Exercise 1.2C: Challenge

14 Fill in the box to make the statement true.

a $2^{\square} = 8$ **b** $3^{\square} = 27$ **c** $5^{\square} = 25$ **d** $10^{\square} = 10\,000$

15 Fill in the box to make the statement true. Start by writing the base on the right as a power of the base on the left. For example: $8^4 = (2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{12}$

a $2^{\square} = 8^4$ **b** $3^{\square} = 27^5$ **c** $5^{\square} = 25^9$ **d** $10^{\square} = 10\,000^3$

e $4^{\square} = 16^7$ **f** $2^{\square} = 32^6$ **g** $6^{\square} = 216^2$ **h** $3^{\square} = 243^6$

16 Simplify each of the following expressions.

a $a^mb^x \times a^nb^y$ **b** $a^mb^x \div (a^nb^y)$

Online resources:

Interactive skillsheet Products and quotients of powers

Worksheet Multiplying and dividing using exponent form

Quick quiz 1.2

Lesson 1.3

Raising exponents and the zero exponent

Learning intentions

By the end of this lesson you will be able to ...

✔ simplify expressions using the power of a power law and the zero exponent.

Inter-year links

Year 8	4.3 Raising exponents and the zero exponent
Year 10	1.1 Exponents

Key content video
Simplifying expressions using powers of powers and zero exponents

Power of a power law

- When raising a power to another power, multiply the exponents. This could be verified by writing the term in expanded form and applying the product of powers law to simplify. For example, $(2^3)^5 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$ $= 2^{15}$ is the same as $(2^3)^5 = 2^{3 \times 5}$ $= 2^{15}$

$$\begin{aligned}(2^3)^5 &= 2^{3 \times 5} \\ &= 2^{15}\end{aligned} \qquad \begin{aligned}(a^3)^5 &= a^{3 \times 5} \\ &= a^{15}\end{aligned}$$

- To raise an exponent by another exponent:
  1. Multiply the exponent of every base inside the brackets by the exponent outside the brackets. If there is no indicated exponent for a term, the exponent is 1 and still must be multiplied.
  2. Write the coefficient first, followed by the variables listed in alphabetical order.
- Every term inside the brackets should have its exponent multiplied by the exponent outside the brackets.

Example	Algebraic form
$(2 \times 3)^5 = 2^5 \times 3^5$	$(a \times b)^n = a^n \times b^n$
$(2^4 \times 3^5)^6 = 2^{4 \times 6} \times 3^{5 \times 6}$	$(a^m \times b^n)^p = a^{m \times p} \times b^{n \times p}$
$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
$\left(\frac{2^4}{3^5}\right)^6 = \frac{2^{4 \times 6}}{3^{5 \times 6}}$	$\left(\frac{a^m}{b^n}\right)^p = \frac{a^{m \times p}}{b^{n \times p}}$

The zero exponent

- Excluding 0, any term with an exponent of 0 is equal to 1. This is because for every non-zero base, the exponent indicates the number of times we multiply 1 by the base. If we multiply 1 by the base zero times, we haven't performed any multiplications and are left with 1.

  For example, $1 = \frac{a^m}{a^m}$
  $= a^{(m - m)}$
  $= a^0$
  Therefore, $a^0 = 1$

$$2^0 = 1 \qquad k^0 = 1$$

- The order of operations also applies to simplification. Calculations in grouping symbols should be simplified first. Remember **BIDMAS**: Brackets, Indices (exponents), Division and Multiplication, Addition and Subtraction.

Worked example 1.3A Simplifying numerical expressions using the power of a power law

Use the power of a power law to simplify the following expressions. Give your answer in exponent form.

a $(3^4)^5$ **b** $(4^3 \times 7)^2$ **c** $\left(\frac{5}{2}\right)^4$ **d** $3(2^2)^4$

THINK	WRITE
a Multiply the exponent of the base by the exponent outside the brackets.	**a** $(3^4)^5 = 3^{4\times5}$ $= 3^{20}$
b Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Remember that base numbers that do not have an indicated exponent have an exponent of 1, so $7 = 7^1$.	**b** $(4^3 \times 7)^2 = 4^{3\times2} \times 7^{1\times2}$ $= 4^6 \times 7^2$
c Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Remember that $5 = 5^1$.	**c** $\left(\frac{5}{2}\right)^4 = \frac{5^{1\times4}}{2^{1\times4}}$ $= \frac{5^4}{2^4}$
d Multiply the exponent of 2 by the exponent outside the brackets. The exponent outside the brackets only applies to the term inside the brackets.	**d** $3(2^2)^4 = 3 \times 2^{2\times4}$ $= 3 \times 2^8$

Worked example 1.3B Simplifying algebraic expressions using the power of a power law

Using the power of a power law, simplify each expression.

a $(2x^4)^3$ **b** $(3a^2b^3)^2$ **c** $\left(\frac{-x}{y^2}\right)^3$ **d** $4(a^2b)^4$

THINK	WRITE
a **1** Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Remember that $2 = 2^1$.	**a** $(2x^4)^3 = 2^{1\times3}x^{4\times3}$
2 Write the coefficient first, followed by the variable.	$= 2^3x^{12}$ or $8x^{12}$
b **1** Multiply the exponent of every base inside the brackets by the exponent outside the brackets.	**b** $(3a^2b^3)^2 = 3^{1\times2}a^{2\times2}b^{3\times2}$
2 Write the coefficient first, followed by the variables listed in alphabetical order.	$= 3^2a^4b^6$ $= 9a^4b^6$
c Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Recall that if the base is negative and the exponent is an odd number, then the basic numeral is negative.	**c** $\left(\frac{-x}{y^2}\right)^3 = \frac{(-x)^{1\times3}}{y^{2\times3}}$ $= \frac{-x^3}{y^6}$ $= -\frac{x^3}{y^6}$
d **1** Multiply the exponent of every base inside the brackets by the exponent outside the brackets. The exponent only applies to the terms inside the brackets.	**d** $4(a^2b)^4 = 4 \times a^{2\times4}b^{1\times4}$
2 Write the coefficient first, followed by the variables listed in alphabetical order.	$= 4a^8b^4$

Worked example 1.3C Simplifying expressions using the zero exponent

Use the property $a^0 = 1$ to simplify each expression.

a 23^0 **b** $3x^0$ **c** $(x^2)^0$

THINK	WRITE
a Any number, excluding 0, raised to an exponent of 0 is equal to 1.	**a** $23^0 = 1$
b Any variable raised to an exponent of 0 is equal to 1. Recall that an exponent only applies to the term immediately to its left.	**b** $3x^0 = 3 \times x^0$ $= 3 \times 1$ $= 3$
c **1** Multiply the exponent of every base inside the brackets by the exponent outside the brackets.	**c** $(x^2)^0 = x^{2\times 0}$ $= x^0$
2 Any variable to the power of 0 is equal to 1.	$= 1$

Worked example 1.3D Simplifying expressions using multiple exponent laws

Use the exponent laws to simplify each expression.

a $(x^3)^5 \times x^2$ **b** $\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3}$

THINK	WRITE
a **1** Use the power of a power law to simplify the first term. Multiply the exponent of the base inside the brackets by the exponent outside the brackets.	**a** $(x^3)^5 \times x^2 = x^{3\times 5} \times x^2$ $= x^{15} \times x^2$
2 Apply the product of powers law and add the exponents of the common base, x.	$= x^{15+2}$ $= x^{17}$
b **1** Simplify the brackets using the power of a power law. Remember BIDMAS.	**b** $\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3} = \dfrac{4x^8 \times 3x^5}{2x^4 \times x^9}$
2 Simplify the numerator and simplify the denominator.	$= \dfrac{12x^{8+5}}{2x^{4+9}}$
3 Divide the numerator by the denominator. Divide the coefficients. Keep the base and subtract the exponent of the denominator from the exponent of the numerator.	$= \dfrac{12x^{13}}{2x^{13}}$ $= 6x^0$
4 Use the property $a^0 = 1$ to simplify further.	$= 6 \times 1$ $= 6$

Helpful hints

✔ Take care not to mix up the exponent laws.
- → Across a multiplication sign, add exponents. $a^5 \times a^3 = a^{5+3}$
- → Across a division sign, subtract exponents. $a^5 \div a^3 = a^{5-3}$
- → Across brackets, multiply exponents. $(a^5)^3 = a^{5\times 3}$ $(ab)^3 = a^3b^3$ $\left(\dfrac{a}{b}\right)^3 = \dfrac{a^3}{b^3}$

✔ Remember that $2^0 = 1$, not 0. $a^0 = 1$

Learning pathways

- 1–10(1st, 3rd columns), 11, 12(a–e), 14(a, b)
- 1–8(2nd, 4th columns), 9, 10(c, h, j, l), 12–14, 15(a, b), 16
- 2–4(e, f) 6–8(2nd, 4th columns), 9, 10, 12, 14, 15(c, d), 17–19

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Exercise 1.3A: Understanding and fluency

WE 1.3A **1** Use the power of a power law to simplify the following. Give your answer in exponent form.

a $(6^4)^3$ **b** $(3^2)^2$ **c** $(3^3 \times 4)^2$ **d** $(2^6)^4$
e $(5 \times 2^7)^4$ **f** $\left(\frac{3}{4}\right)^2$ **g** $\left(\frac{5^3}{2^2}\right)^4$ **h** $\left(\frac{1}{8^5}\right)^5$
i $(-3^4)^7$ **j** $(-3^5)^4$ **k** $(-7^4 \times -11^3)^7$ **l** $\left(\frac{13^8}{-17^4}\right)^6$

WE 1.3B **2** Using the power of a power law, simplify each expression.

a $(b^5)^2$ **b** $(m^4)^2$ **c** $(j^5)^2$ **d** $(j^2)^5$ **e** $(n^{10})^8$ **f** $(p^{11})^9$

3 Using the power of a power law, simplify each expression. Give your answer in exponent form.

a $(xy)^6$ **b** $(2d)^3$ **c** $(-5k)^7$ **d** $(9p)^{10}$
e $(-3m)^4$ **f** $\left(\frac{8}{p}\right)^2$ **g** $\left(\frac{x}{y}\right)^6$ **h** $(gh)^2$
i $(ab)^5$ **j** $\left(\frac{k}{m}\right)^3$ **k** $(-2x)^8$ **l** $\left(-\frac{d}{3}\right)^5$

4 Using the power of a power law, simplify each expression. Give your answer in exponent form.

a $(3x^6)^4$ **b** $5(a^4b)^7$ **c** $\left(\frac{2m}{n}\right)^3$ **d** $\left(\frac{a^2}{b^5}\right)^4$
e $-2(u^3)^4$ **f** $\frac{4}{3}(v^7w^3)^{10}$ **g** $\frac{7}{9}\left(\frac{p}{q^6}\right)^7$ **h** $\frac{1}{2^3}(3^2r^9)^5$
i $8\left(\frac{1}{5^6t^{11}}\right)^4$ **j** $7(3i^{17})^2$ **k** $2^3(2^4c^5)^8$ **l** $\frac{5}{7}\left(\frac{5x^{25}}{7y^{30}}\right)^3$

WE 1.3C **5** Use the property $a^0 = 1$ to simplify each expression.

a 34^0 **b** $(18)^0$ **c** y^0 **d** $(7a)^0$

6 Use the property $a^0 = 1$ to simplify each expression.

a $2x^0$ **b** $(2x)^0$ **c** $-7y^0$ **d** $(-7y)^0$
e $-(-3c)^0$ **f** $8^0 + 4^0$ **g** $2 \times 5^0 - 3^0$ **h** $m^0 + m^0$
i $n^0 + p^0$ **j** $a^0 + b^0 + c^0$ **k** $(x + y)^0$ **l** $(-a^0)^4$
m $(5^3)^0$ **n** $(-8)^0$ **o** -8^0 **p** $-(-3)^0$

WE 1.3D **7** Use the exponent laws to simplify each expression.

a $(x^2)^4 \times x^5$ **b** $(x^5)^3 \times x^7$ **c** $x^3 \times (x^4)^6$ **d** $(x^3)^2 \times (x^7)^3$
e $\frac{x^4 \times (x^3)^5}{x^9}$ **f** $\frac{(w^2)^4 \times (w^5)^2}{(w^4)^3}$ **g** $\frac{6(b^4)^4 \times (b^3)^2}{18b^{21}}$ **h** $\frac{e^5 \times e^8}{e^3 \times e^4}$
i $\frac{(x^6)^2 \times x^3}{x^5 \times (x^2)^5}$ **j** $\frac{4a^6 \times 6(a^3)^4}{2a^4 \times 3a^5}$ **k** $\frac{t^8}{(t^2)^5} \times \frac{(t^6)^7}{t^{15}}$ **l** $(f^6)^9 \times \left(\frac{f^7}{f^2}\right)^{11}$

8 Use the exponent laws to simplify each expression.

a $a^3 \div a^3$ **b** $-7x^9 \div x^9$ **c** $(m^2)^3 \div m^6$ **d** $-18(b^4)^5 \div -6(b^5)^4$
e $y^7 \times y \div y^8$ **f** $(k^6)^0 \times k^2$ **g** $5g^4 \times 2(-g^7)^0$ **h** $3(w^5)^2 \div (w^2)^5$
i $x^8 \times (x^2)^5 \div x^3$ **j** $4p^7 \times 3p^2 \div (6p^9)$ **k** $16(b^3)^3 \div -2(b^2)^4$ **l** $4m^5 \times m \div 10(m^3)^2$

9 Use the exponent laws to simplify each expression.

a $\frac{5(n^7)^2 \times [-6(n^2)^3]}{15n^2 \times (n^3)^6}$ **b** $\frac{(k^8)^2 \times k \times k^9}{k^3 \times (k^4)^2 \times k^5}$ **c** $(x^4)^2y^7 \times x^3y^2$
d $\frac{4(m^3)^4n^2 \times (m^2)^3}{8m^5n^6 \times mn}$ **e** $\frac{-3h^7k^5 \times 2h^6(k^3)^2}{(h^2)^6k^3 \times -6h(k^4)^2}$ **f** $\frac{(b^2)^2 \times ac}{a^2b \times c^3} \times bc \div a$

10 Use the exponent laws to simplify each expression.

a $(xy)^3 \times x^6y^4$ **b** $(2k)^5 \times (7k)^2$ **c** $(-3x^6)^4$ **d** $-5(a^4b)^7$

e $\dfrac{x^4}{y^5} \times \left(\dfrac{x}{y}\right)^6$ **f** $\left(\dfrac{2m}{n}\right)^3$ **g** $\left(\dfrac{a^2}{b^5}\right)^4$ **h** $\left(\dfrac{w^5x^3}{y^4}\right)^2$

i $\left(\dfrac{k^3m}{n^2}\right)^5 \times \left(\dfrac{n^3}{k^2m}\right)^4$ **j** $\left(\dfrac{t^4}{r^2p^3}\right)^5 \times \dfrac{(4r^5)^2}{p^6t^7}$ **k** $\dfrac{(a^3b^2)^5 \times (ab^4)^6}{(a^5b)^4}$ **l** $\dfrac{(3e^4)^2(2h^6)^3}{(e^2h^3)^4}$

Exercise 1.3B: Problem solving and reasoning

11 Use the exponent laws to decide whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

a $(3g)^4 = 3^4 \times g^4$ **b** $-8^0 = -1$ **c** $\left(\dfrac{x}{y}\right)^6 = \dfrac{x^6}{y}$ **d** $\dfrac{(k^3)^2 \times k^4}{k^2} = k^5$

e $6 + k^0 = 7$ **f** $100^9 \div 100^9 = 0$ **g** $\dfrac{m^3 \times m^8}{m^{11}} = 1$ **h** $a^0 \div b^0 = 0$

12 Find the value of x that makes each statement true.

a $2^x = 2^7$ **b** $5^x \times 5^2 = 5^6$ **c** $4^x = 1$ **d** $7^x \div 7^3 = 7^5$

e $(9^x)^2 = 9^6$ **f** $\left(\dfrac{2}{3}\right)^x = \dfrac{32}{243}$ **g** $\dfrac{6^x \times 6^3}{6^5} = 6^5$ **h** $(3a^x)^4 = 81a^{20}$

13 Eden simplified $3^4 \times (3^5)^3$ as $(3^9)^3 = 3^{27}$. Explain and correct her mistake.

14 A teacher tries to explain the power of a power law, and the products of power law. Complete their working out for the following.

a $(2^3)^5 = (2^3) \times (2^3) \times \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_$
$= 2^{3+3+\square+\square+\square}$
$= 2^{3\times\square}$

b $(x^7)^4 = (x^7) \times \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_$
$= x^{7+\square+\square+\square}$
$= x^{7\times\square}$

c $(2 \times 3)^4 = (2 \times 3) \times \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_$
$= 2 \times \_\_ \times \_\_ \times \_\_ \times 3 \times \_\_ \times \_\_ \times \_\_$
$= 2^{\square} \times 3^{\square}$

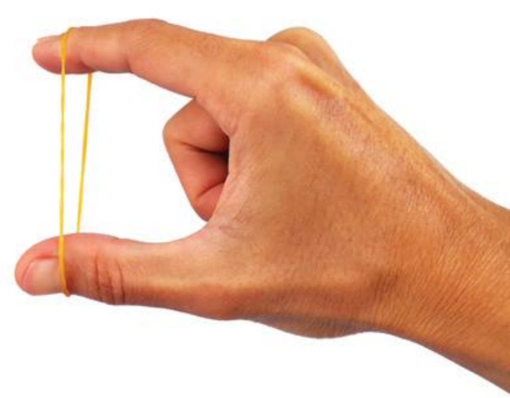
d $\left(\dfrac{2}{3}\right)^6 = \dfrac{2}{3} \times \dfrac{\square}{\square} \times \dfrac{\square}{\square} \times \dfrac{\square}{\square} \times \dfrac{\square}{\square} \times \dfrac{\square}{\square}$
$= \dfrac{2^{\square}}{3^{\square}}$

15 We can describe multiplication in terms of repeated addition, $2 \times 3 = 2 + 2 + 2$, and raising to the power of an exponent in terms of repeated multiplication, $2^3 = 2 \times 2 \times 2$. However, repeatedly raising a number to the same exponent does not require a new operation as it can be simplified to exponent form.
For example: $(((2^5)^5)^5) = 2^{5\times5\times5} = 2^{5^3} = 2^{125}$
Write the following in exponent form with a single exponent.

a $(((((3^2)^2)^2)^2)^2)^2$ **b** $(((5^3)^3)^3)^3$ **c** $((((7^4)^4)^4)^4)^4$ **d** $((((10^5)^5)^5)^5)^5$

16 A cube has side lengths of 8^5 cm. What is the volume of the cube in cm^3? Write your answer in exponent form.

17 A rubber band is stretched to $\frac{4}{3}$ of its current length, and this is repeated another four times until it snaps. How many times longer was the rubber band when it snapped than it was originally?



Exercise 1.3C: Challenge

18 Use the power of a power to show that $(a^m)^n = (a^n)^m$.

19 Solve the following equation for x.

$$\frac{12x^6y^7}{7x^2y^{10}z^4} \times \frac{35y^3z^4}{3x^3} = 3$$

Online resources:

Interactive skillsheet
Raising exponents and the zero exponent

Quick quiz
1.3

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

1.1 **1** Write the following in expanded form then evaluate.

a 2^6 **b** $(-3)^4$ **c** $(-4)^3$ **d** $\left(\frac{5}{6}\right)^3$

1.1 **2** Write the following in expanded form.

a a^6 **b** $(-b)^4$ **c** $(3y)^5$ **d** $3(xy)^5$

1.1 **3** Write the following in exponent form.

a $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$ **b** $u \times u \times u \times u$

c $4b \times 4b \times 4b \times 4b \times 4b$ **d** $-7 \times k \times k \times k \times h \times h \times h \times h \times h$

1.1 **4** Write the following numbers as a product of their prime factors. Express your answer in exponent form.

a 28 **b** 72 **c** 484 **d** 270

1.2 **5** Use the exponent laws to simplify each expression. Express your answers in exponent form.

a $8^5 \times 8^6$

b $5^7 \times 7^4 \times 5^3 \times 7^8$

c $\dfrac{6^8}{6^3}$

d $\dfrac{3^{14} \times 10^{12}}{3^6 \times 10^5}$

1.2 **6** Use the exponent laws to simplify each expression.

a $a^3 \times a^9$

b $4b^{11}c^8 \times (-3b^7c^{13})$

c $\dfrac{u^{14}}{u^9}$

d $\dfrac{-15p^{17}q^{21}}{-21p^3q^4}$

1.2 **7** Use the exponent laws to simplify each expression. Write your answer in exponent form.

a $\dfrac{3^7 \times 3^{12}}{3^9}$

b $\dfrac{k^{23}}{k^7 \times k^8}$

c $\dfrac{c^3t^8}{c^{14}t^7}$

d $\dfrac{9d^7w^4}{10d^{12}w^7} \times \left(-\dfrac{25d^{17}w^{12}}{6dw^2}\right)$

1.3 **8** Use the exponent laws to simplify each expression.

a 87^0 **b** $t^5 \div t^5$ **c** $-(4g)^0$ **d** $7a^0 + (8b)^0$

1.3 **9** Use the exponent laws to simplify each expression. Write your answers in exponent form.

a $(3^4)^6$ **b** $(j^5)^9$ **c** $(-5a^3b^7)^6$ **d** $-\left(\dfrac{3p^5}{2q^7}\right)^8$

1.3 **10** Use the exponent laws to simplify each expression. Write your answers in exponent form.

a $\dfrac{t^4 \times (t^2)^3}{t^{10}}$

b $\dfrac{3(g^2)^8 \times (3g^5)^3}{(3g)^{11}}$

c $\dfrac{(5m^{11}n^{10})^8 \times (5mn^6)^6}{(5m^9n^7)^2 \times (5m^2n^3)^5}$

d $\dfrac{(8j^5p)^0 \times 6(j^0p^4)^3}{(j^7p^2)^6}$

Negative exponents

Learning intentions

By the end of this lesson you will be able to ...

✔ evaluate or simplify expressions with negative exponents.

Inter-year links

Year 8	2.3 Multiplying and dividing fractions
Year 10	1.2 Negative exponents

Negative exponents

Key content video
Negative exponents

- When a number is multiplied by its **reciprocal** (or multiplicative inverse), the result is 1, the multiplicative identity. That is, for non-zero a and b:
  - → a and $\frac{1}{a}$ are reciprocals as $a \times \frac{1}{a} = 1$.
  - → $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals as $\frac{a}{b} \times \frac{b}{a} = 1$.
- The reciprocal of a fraction is found by swapping the numerator and denominator.
- A power with an exponent of −1 is the reciprocal of the base.

$$2^{-1} = \frac{1}{2} \qquad a^{-1} = \frac{1}{a} \qquad \left(\frac{n}{m}\right)^{-1} = \frac{m}{n}$$

- A power with a negative exponent is the reciprocal of the base with the positive exponent.
- For any non-zero number a, and positive integer m:

$$2^{-3} = \frac{1}{2^3} \qquad a^{-m} = \frac{1}{a^m}$$
$$\frac{1}{2^{-3}} = 2^3 \qquad \frac{1}{a^{-m}} = a^m$$

- The exponent laws apply to expressions containing terms with negative exponents.

Worked example 1.4A Determining the reciprocal

Determine the reciprocal of each of the following.

a $\frac{3}{2}$ **b** $\frac{1}{4}$ **c** 3 **d** $2x$ **e** $\frac{1}{2x}$

THINK

1 To find the reciprocal of a term, apply an exponent of −1.

2 Write the base with an exponent of −1 as a fraction if it is not already.

3 Find the reciprocal of the fraction. Swap the numerator and denominator and remove the −1 exponent.

4 Simplify the result.

WRITE

a $\left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$

b $\left(\frac{1}{4}\right)^{-1} = \frac{4}{1}$
$= 4$

c $3^{-1} = \left(\frac{3}{1}\right)^{-1}$
$= \frac{1}{3}$

d $(2x)^{-1} = \left(\frac{2x}{1}\right)^{-1}$
$= \frac{1}{2x}$

e $\left(\frac{1}{2x}\right)^{-1} = \frac{2x}{1}$
$= 2x$

Worked example 1.4B Writing simple terms with positive exponents

Write each term with a positive exponent.

a 3^{-3} **b** 7^{-4} **c** x^{-3} **d** $(2x)^{-2}$

THINK

1 Write the base with a negative exponent as a fraction if it is not already.

2 Find the reciprocal of the fraction and remove the minus sign in the exponent.

3 Use the power of a power law to remove the brackets.

4 Simplify the result.

WRITE

a $3^{-3} = \left(\frac{3}{1}\right)^{-3}$
$= \left(\frac{1}{3}\right)^{3}$
$= \frac{1}{3^3}$

b $7^{-4} = \left(\frac{7}{1}\right)^{-4}$
$= \left(\frac{1}{7}\right)^{4}$
$= \frac{1}{7^4}$

c $x^{-3} = \left(\frac{x}{1}\right)^{-3}$
$= \left(\frac{1}{x}\right)^{3}$
$= \frac{1}{x^3}$

d $(2x)^{-2} = \left(\frac{2x}{1}\right)^{-2}$
$= \left(\frac{1}{2x}\right)^{2}$
$= \frac{1}{(2x)^2}$
$= \frac{1}{4x^2}$

Worked example 1.4C Writing fractions with positive exponents

Write each fraction in exponent form with a positive exponent.

a $\frac{1}{3^{-2}}$ **b** $\frac{1}{(2x)^{-1}}$ **c** $\left(\frac{2}{5}\right)^{-3}$ **d** $\left(\frac{3x}{y}\right)^{-2}$

THINK

1 Write the base with a negative exponent as a fraction if it is not already.

2 Find the reciprocal of the fraction and remove the minus sign in the exponent.

3 Use the power of a power to remove the brackets.

4 Simplify the calculation. Dividing by a fraction is the same as multiplying by the reciprocal.

5 Simplify the result.

WRITE

a $\frac{1}{3^{-2}} = \frac{1}{\left(\frac{3}{1}\right)^{-2}}$
$= \frac{1}{\left(\frac{1}{3}\right)^{2}}$
$= 1 \times \frac{3^2}{1}$
$= 3^2$

b $\frac{1}{(2x)^{-1}} = \frac{1}{\left(\frac{2x}{1}\right)^{-1}}$
$= \frac{1}{\left(\frac{1}{2x}\right)^{1}}$
$= 1 \times \frac{2x}{1}$
$= 2x$

c $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3}$
$= \frac{5^3}{2^3}$

d $\left(\frac{3x}{y}\right)^{-2} = \left(\frac{y}{3x}\right)^{2}$
$= \frac{y^2}{3^2x^2}$

Worked example 1.4D Writing terms with positive exponents

Write each term with positive exponents.

a $x^{-5}y^3$ **b** $\dfrac{5x^{-4}}{y^{-7}}$ **c** $\dfrac{6a^7b^{-2}}{3c^{-4}}$

THINK

a 1 Write the term as a product of two factors and write the base with a negative exponent as a fraction.
2 Find the reciprocal of the fraction and write the exponent as a positive number.
3 Simplify the result.

b 1 Write the term as a product of three factors and write the base with a negative exponent as a fraction.
2 Find the reciprocal of the fraction and write the exponent as a positive number.
3 Simplify the result.

c 1 Cancel 6 and 3 by a common factor of 3.
2 Write the term as a product of four factors and write the base with a negative exponent as a fraction.
3 Find the reciprocal of the fraction and write the exponent as a positive number.
4 Simplify the result.

WRITE

a
$$\begin{aligned} x^{-5}y^3 &= \left(\frac{x}{1}\right)^{-5} \times y^3 \\ &= \frac{1}{x^5} \times y^3 \\ &= \frac{y^3}{x^5} \end{aligned}$$

b
$$\begin{aligned} \frac{5x^{-4}}{y^{-7}} &= 5 \times \left(\frac{x}{1}\right)^{-4} \times \frac{1}{\left(\frac{y}{1}\right)^{-7}} \\ &= 5 \times \frac{1}{x^4} \times 1 \times \frac{y^7}{1} \\ &= \frac{5y^7}{x^4} \end{aligned}$$

c
$$\begin{aligned} \frac{6a^7b^{-2}}{3c^{-4}} &= \frac{\cancel{6}^{2}a^7b^{-2}}{\cancel{3}^{1}c^{-4}} \\ &= \frac{2a^7b^{-2}}{c^{-4}} \\ &= 2 \times a^7 \times \left(\frac{b}{1}\right)^{-2} \times \frac{1}{\left(\frac{c}{1}\right)^{-4}} \\ &= 2 \times a^7 \times \frac{1}{b^2} \times c^4 \\ &= \frac{2a^7c^4}{b^2} \end{aligned}$$

Worked example 1.4E Simplifying expressions with negative exponents using exponents laws

Use an appropriate exponent law to simplify each expression. Write your answers using positive exponents.

a $3^5 \times 3^{-7}$ **b** $2^4 \div 2^{-3}$ **c** $(5^{-6})^2 \times 5^3$

THINK

a 1 Apply the product of powers law to multiply the terms. Write the base and add the exponents.
2 Find the reciprocal of the fraction and write the exponent as a positive number.

b Apply the quotient of powers law to divide the terms. Write the base and subtract the exponents.

c 1 Apply the power of a power to simplify the first term. Multiply the exponent of every base inside the brackets by the exponent outside the brackets.
2 Apply the product of powers law to multiply the terms. Write the base and add the exponents.
3 Write the term with a positive exponent.

WRITE

a
$$\begin{aligned} 3^5 \times 3^{-7} &= 3^{5+(-7)} \\ &= 3^{-2} \\ &= \frac{1}{3^2} \end{aligned}$$

b
$$\begin{aligned} 2^4 \div 2^{-3} &= 2^{4-(-3)} \\ &= 2^7 \end{aligned}$$

c
$$\begin{aligned} (5^{-6})^2 \times 5^3 &= 5^{-6\times 2} \times 5^3 \\ &= 5^{-12} \times 5^3 \\ &= 5^{-12+3} \\ &= 5^{-9} \\ &= \frac{1}{5^9} \end{aligned}$$

Helpful hints

- ✔ Keep the exponent laws and the rules for negative exponents close by until they become second nature.

$$a^{-1} = \frac{1}{a} \qquad a^{-m} = \frac{1}{a^m} \qquad \frac{1}{a^{-m}} = a^m$$

- ✔ If you want to move a term from the numerator to the denominator, remember that a 1 is left in its place, not zero.

For example: $\frac{a}{b} = \frac{a}{1} \times \frac{1}{b}$

$= \frac{1}{a^{-1}} \times \frac{1}{b}$

$= \frac{1}{a^{-1}b}$

- ✔ Don't confuse negative exponents with negative numbers. For example: $2^{-3} = \frac{1}{2^3}$ and $2^{-3} \neq -(2^3)$

Learning pathways

△ 1–12(1st, 2nd columns), 14, 15(a–d), 16, 19, 21, 22	□ 1(c, d, g, h), 2(c, f), 3–12(3rd, 4th columns), 14, 15(e–h), 17, 20–23, 25	⬠ 1(c, f, h), 6–12(4th column), 13, 14, 15(e–h), 18, 22, 24–28

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Exercise 1.4A: Understanding and fluency

WE 1.4A **1** Determine the reciprocal of each of the following.

a $\frac{8}{7}$ **b** $\frac{1}{2}$ **c** -3 **d** $3w$

e $\frac{1}{d}$ **f** $\frac{r}{m}$ **g** $\frac{1}{5y}$ **h** u^2

WE 1.4B **2** Write each term with a positive exponent.

a 5^{-1} **b** 8^{-1} **c** $(-2)^{-1}$

d x^{-1} **e** p^{-1} **f** $(3w)^{-1}$

3 Write each of the following in exponent form with a negative exponent.

a $\frac{1}{5}$ **b** $-\frac{1}{13}$ **c** 5 **d** -8

e $\frac{1}{m}$ **f** x **g** $\frac{1}{5y}$ **h** $3w$

i $\frac{4}{3}$ **j** $\frac{5}{6}$ **k** $\frac{w}{a}$ **l** v^3

4 Write each term with a positive exponent.

a 4^{-2} **b** 2^{-6} **c** $(-9)^{-3}$ **d** $(-5)^{-4}$

e -7^{-8} **f** 10^{-5} **g** a^{-4} **h** x^{-7}

i k^{-10} **j** m^{-2} **k** u^{-9} **l** g^{-11}

5 Write each fraction in exponent form with a negative exponent.

a $\frac{1}{3^4}$ **b** $\frac{1}{4^7}$ **c** $\frac{1}{6^5}$ **d** $\frac{1}{(-5)^3}$

e $\frac{1}{(-9)^2}$ **f** $-\frac{1}{11^6}$ **g** $\frac{1}{n^2}$ **h** $\frac{1}{g^{11}}$

i $\frac{1}{x^8}$ **j** $\frac{1}{a^9}$ **k** $\frac{1}{p^4}$ **l** $\frac{1}{w^7}$

WE 1.4C **6** Write each fraction in exponent form with a positive exponent.

a $\frac{1}{2^{-3}}$ **b** $\frac{1}{5^{-6}}$ **c** $\frac{1}{(-8)^{-4}}$ **d** $\frac{1}{3^{-9}}$

e $\frac{1}{(-7)^{-5}}$ **f** $-\frac{1}{4^{-2}}$ **g** $\frac{1}{x^{-7}}$ **h** $\frac{1}{y^{-3}}$

i $\frac{1}{c^{-4}}$ **j** $\frac{1}{z^{-35}}$ **k** $\frac{1}{(5t)^{-4}}$ **l** $\frac{1}{(uv)^{-15}}$

7 Write each fraction in exponent form with a positive exponent.

a $\left(\frac{4}{5}\right)^{-2}$ **b** $\left(\frac{7}{3}\right)^{-1}$ **c** $\left(-\frac{3}{4}\right)^{-3}$ **d** $\left(-\frac{a}{2}\right)^{-4}$

e $\left(\frac{6}{m}\right)^{-1}$ **f** $\left(\frac{144}{u}\right)^{-1}$ **g** $\left(\frac{x}{y}\right)^{-6}$ **h** $\left(-\frac{9}{7}\right)^{-11}$

i $\left(-\frac{13}{17}\right)^{-14}$ **j** $\left(\frac{500}{43}\right)^{-11}$ **k** $\left(\frac{u}{97}\right)^{-198}$ **l** $\left(\frac{123}{g}\right)^{-654}$

WE 1.4D **8** Write each term with positive exponents.

a $x^{-2}y^3$ **b** m^6n^{-4} **c** $a^{-1}c^7$ **d** $2k^5p^{-3}$

e $5a^{-8}b^2$ **f** $-4x^{-6}w^{-2}$ **g** $a^4b^{-5}c^7$ **h** $k^{-3}m^5n^{-8}$

i $7b^9c^{-6}d$ **j** $3x^{-2}y^{-7}z^{-4}$ **k** $-12p^{17}q^{-18}u^{-21}$ **l** $-34j^{-65}b^{-78}$

9 Write each term with positive exponents.

a $\frac{a^{-5}}{b^{-3}}$ **b** $\frac{k^{-6}}{p^{-2}}$ **c** $\frac{h^{-4}}{m^{-7}}$ **d** $\frac{e^8}{d^{-5}}$

e $\frac{u^3}{w^{-8}}$ **f** $\frac{3x^2}{y^{-4}}$ **g** $\frac{8h^5}{g^{-6}}$ **h** $\frac{-6d^{-5}}{3c^{-9}}$

i $\frac{-4m^{-7}}{-10n^{-3}}$ **j** $\frac{y^{-8}}{6e^5}$ **k** $\frac{6g^{-99}}{-12h^{11}}$ **l** $\frac{14r^{-72}}{35c^{-101}}$

10 Write each term with positive exponents.

a $\frac{m^{-3}n^4}{p^{-6}}$ **b** $\frac{8c^2d^{-5}}{2e^{-6}}$ **c** $\frac{4k^{-1}n^{-3}}{6p^{-4}}$ **d** $\frac{3^{-1}a^2c^{-5}}{b^{-3}d^4}$

e $\frac{m^{-6}n}{7^{-1}k^{-2}}$ **f** $\frac{5^{-1}k^{-2}x^3}{2u^{-5}w^8}$ **g** $\frac{-5q^4r^{-4}}{p^{-1}s^3}$ **h** $\frac{7b^{14}u^{10}}{3^{-2}g^{-15}k^{-17}}$

i $\frac{t^{-4}u^{-3}}{5v^7w^{12}}$ **j** $\frac{-4^{-3}l^{-88}n^{41}}{7^{-2}m^{-59}v}$ **k** $\frac{a^{97}b^{-105}c^{153}}{d^{-167}e^{111}f^{-89}}$ **l** $\frac{k^{402}l^{812}}{m^{-999}n^{-571}}$

WE 1.4E **11** Use an appropriate exponent law to simplify each expression. Write your answers in exponent form with positive exponents.

a $4^{-5} \times 4^2$ **b** $7^3 \times 7^{-4}$ **c** $2^{-6} \times 2^8$ **d** $(-3)^{-1} \times (-3)^{-5}$

e $5^7 \times 5^{-3}$ **f** $(-2)^{-4} \div (-2)^3$ **g** $9^5 \div 9^7$ **h** $3^6 \div 3^{-2}$

i $4^{-1} \div 4^8$ **j** $10^{-7} \div 10^{-4}$ **k** $211^{-9} \times 211^{-5}$ **l** $13^{-87} \div 13^{13}$

12 Use an appropriate exponent law to simplify each expression. Write your answers in exponent form with positive exponents.

a $(5^{-3})^2$ **b** $(3^{-2})^4$ **c** $(-2^{-4})^{-1}$ **d** $(3^{-1})^4 \times 3^2$

e $(6^{-5})^3 \times 6^{11}$ **f** $(4^{-2})^3 \times (4^{-5})^{-1}$ **g** $9^3 \times 9^{-6} \times 9^2$ **h** $\frac{5^4 \times 5^{-2}}{5^{-6}}$

i $\frac{7^{-5} \times 7^{-3}}{7^{-4} \times 7^{-7}}$ **j** $\frac{2^8 \times (2^{-2})^3}{2^5}$ **k** $\frac{(99^{-12})^{-6} \times 99^{15}}{(99^8)^{-5}}$ **l** $\frac{(15^{-9})^8 \times (15^7)^6}{(15^{11})^{12}}$

13 Using a calculator, calculate the basic numeral for parts **a** to **f** in question **11**. Write your answers as integers or fractions.

14 Use an appropriate exponent law to write each expression without brackets using positive exponents only.

a $(a \times b)^{-7}$ **b** $(3 \times x)^{-5}$ **c** $(4 \times y)^{-1}$

d $(7k)^{-2}$ **e** $(2p)^{-3}$ **f** $(mk)^{-11}$

Exercise 1.4B: Problem solving and reasoning

15 Find the value of x that makes each statement true.

a $2^x = \frac{1}{2^3}$ **b** $5^x = \frac{1}{5^7}$ **c** $3^x = \frac{1}{3}$ **d** $6^x = \frac{1}{6^{-2}}$

e $4^x = \frac{1}{16}$ **f** $3^x = \frac{1}{27}$ **g** $5^x = \frac{1}{25}$ **h** $10^x = \frac{1}{10\,000}$

16 A microscopic worm is 4^{-3} mm in length. Using a calculator, write this length in millimetres:

a as a fraction

b as a decimal.

17 The time it takes for light to travel 3 m is about 10^{-8} s. Using a calculator, write this time in seconds:

a as a fraction

b as a decimal.

18 The diameter of a strand of human hair is about 5^{-6} m. Using a calculator, write this measurement in metres:

a as a fraction

b as a decimal.

19 Investigate the powers of base 2.

a Complete this table.

Exponent form	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Basic numeral	32	16	8			1		$\frac{1}{4}$			

b Describe the pattern you can see in the table.

c Following the pattern, write 2^{-6} as a fraction.

d Given 2^{10} is 1024, write the value of 2^{-10} as a fraction.

e Given 2^{-7} is $\frac{1}{128}$, write the value of 2^7.

20 Investigate the powers of base 3.

a Complete this table.

Exponent form	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}	3^{-5}
Basic numeral		81	27			1		$\frac{1}{9}$			

b Describe the pattern you can see in the table.

c Following the pattern, write 3^{-6} as a fraction.

d Given that 3^8 is 6561, write the value of 3^{-8} as a fraction.

e Given that 3^{-7} is $\frac{1}{2187}$, write the value of 3^7.

21 Investigate the powers of base 10

a Complete this table.

Exponent form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Basic numeral			100			$\frac{1}{10}$	$\frac{1}{100}$		

b Describe the pattern you can see in the table.

c Write the value of each of the following terms as a whole number.

i 10^5 **ii** 10^6 **iii** 10^7 **iv** 10^8 **v** 10^9

d Write the value of each of the following terms as a fraction.

i 10^{-5} **ii** 10^{-6} **iii** 10^{-7} **iv** 10^{-8} **v** 10^{-9}

e Write 10^{-1} as:

i a fraction **ii** a decimal.

f Write each of the following terms as a decimal. Hint: Use the matching fractions from your table.

i 10^{-2} **ii** 10^{-3} **iii** 10^{-4}

g Write each fraction in part **d** as a decimal.

h Explain any shortcuts you used to obtain your answers to parts **c** to **g**.

22 Investigate a shortcut to perform calculations involving powers of 10.

a Without using a calculator, find the whole number value of each of the following.
Hint: What shortcut can you use when multiplying by a positive power of 10?

i 2×10^4 **ii** 7×10^3 **iii** 3×10^5
iv 4×10^{11} **v** 9×10^7

b Write each expression as a fraction involving positive exponents.

i 5×10^{-2} **ii** 8×10^{-5} **iii** 2×10^{-3}
iv 7×10^{-4} **v** 6×10^{-9}

c Without using a calculator, find the decimal value of each result in part **b**.
Hint: What shortcut can you use when dividing by a positive power of 10?

d Use your results from part **c** to describe a shortcut that can be used when multiplying by a negative power of 10.

23 Consider the relationship between multiplication, division and reciprocals.

a Complete the following by writing the missing numerals and operation.

i $2 \times 3^{-1} = 2 \times \frac{\square}{\square} = \frac{2}{\square} = 2\square 3$ **ii** $2 \div 3^{-1} = \frac{2}{\square} = 2 \times \frac{\square}{\square} = 2\square 3$

b Explain the connection between multiplication, division and reciprocals.

24 Investigate the effect of taking the reciprocal of a reciprocal.

a Evaluate the following.

i $(3^{-1})^{-1}$ **ii** $(5^{-1})^{-1}$ **iii** $\left(\left(\frac{5}{4}\right)^{-1}\right)^{-1}$

b Use exponent laws to explain why $(a^{-1})^{-1} = a$.

c Explain what $(a^{-1})^{-1} = a$ means in terms of reciprocals.

25 Write the following as products without fractions by using negative exponents.

For example: $\frac{9x^3}{y^2} = 9x^3y^{-2}$

a $\frac{2}{a^2b}$ **b** $-\frac{3t^2}{v^3}$ **c** $\frac{x^4}{5y^4}$ **d** $\frac{1}{pq^5r^{-2}}$

Exercise 1.4C: Challenge

26 Use an appropriate exponent law to simplify each expression. Write your answer using positive exponents only.

a $x^4 \times x^{-6}$ **b** $x^{-3} \times x^{-1}$ **c** $4x^{-2} \times 2x^5$ **d** $5x^{-8} \times 6x^3$
e $3x^7 \times x^{-7}$ **f** $x^5 \div x^{-4}$ **g** $x^{-10} \div x^{-7}$ **h** $4x^3 \div (2x^{-2})$
i $6x^{-6} \div (18x^4)$ **j** $8x^7 \div (14x^{11})$ **k** $(x^{-2})^3 \times x^4$ **l** $(x^4)^5 \times x^{-9}$
m $(x^{-5})^3 \times 4x^2$ **n** $2x^{-3} \times (x^{-1})^5$ **o** $(x^{-4})^2 \times (x^{-3})^{-1}$ **p** $(xy)^{-7}$

27 Write all answers from question **26** that have a positive exponent using a negative exponent.

28 Write $a^m \div a^n$ in each of the following different forms.

a $a^{\square} \times a^{\square} = a^{\square + \square}$ **b** $\frac{1}{a^{\square} \times a^{\square}} = \frac{1}{a^{\square + \square}}$

c $a^m \div a^n = a^{m-n} = a^{\square} \div a^{\square} = a^{\square - \square}$

Online resources:

Interactive skillsheet
Negative exponents

Quick quiz
1.4

Lesson 1.5

Scientific notation

Learning intentions

By the end of this lesson you will be able to ...

✔ convert between scientific notation and basic numerals, and identify significant figures in a number.

Inter-year links

Support	Place value
Year 7	1.1 Place value
Year 8	1.1 Rounding and estimating
Year 10	8.4 Logarithmic scales

Scientific notation

Key content video
Scientific notation

- **Scientific notation** (or **standard form**) is a simple way of writing very large and very small numbers.
- A number is written in scientific notation if it is the product of a number, the significand, a, between 1 (inclusive) and 10 (exclusive) or -1 and -10, and a power of 10, written in exponent form.
  - → That is, $a \times 10^m$, where $1 \le a < 10$ or $-10 < a \le -1$ and m is an integer.
- When the value of a is positive:
  - → If m is a positive integer, the number is larger than or equal to 10.
  - → If m is a negative integer, the number is between 0 and 1.
  - → If m is zero, the number is between 1 and 10.

significand

$$\underbrace{1230}_{\text{basic numeral}} = \underbrace{\overbrace{1.23}^{\text{significand}} \times 10^3}_{\text{scientific notation}}$$

$$\underbrace{0.00\,123}_{\text{basic numeral}} = \underbrace{\overbrace{1.23}^{\text{significand}} \times 10^{-3}}_{\text{scientific notation}}$$

- The exponent laws can be used to perform operations on numbers in scientific notation.
- To convert a number in scientific notation to a basic numeral, the exponent indicates the number of places the decimal point is moved.
  - → If the exponent is positive, move the decimal point to the right.
  - → If the exponent is negative, move the decimal point to the left.
- To write a number in scientific notation, place the decimal point after the first non-zero digit and multiply by the appropriate power of 10.

exponent of 4

$$3.1500 \times 10^4 = 31\,500$$

move 4 spaces to the right

exponent of 4

$$31500 = 3.15 \times 10^4$$

move 4 spaces to the left

exponent of -2

$$004.2 \times 10^{-2} = 0.042$$

move 2 spaces to the left

exponent of -2

$$0.042 = 4.2 \times 10^{-2}$$

move 2 spaces to the right

- While we can write the expanded notation of a basic numeral using powers of 10, scientific notation uses only the highest power of 10 from the expansion.

highest power of 10

Expanded notation: $315.042 = 3 \times 10^2 + 1 \times 10^1 + 5 \times 10^0 + 4 \times 10^{-2} + 2 \times 10^{-3}$

highest power of 10

Scientific notation: $315.042 = 3.15042 \times 10^2$

Significant figures

Key content video
Significant figures

- **Significant figures** are the number of digits in a number that contribute to the level of **accuracy**.
- When counting significant figures, count the first non-zero digit from left to right.
  - → All non-zero digits are significant.
    For example, 7.789 has four significant figures as all are non-zero.
  - → Zeros between two non-zero digits are significant.
    For example, 4056 has four significant figures including the zero between 4 and 5.
  - → Leading zeros are not significant.
    For example, 0.051 has two significant figures. All the zeros are leading zeros.
  - → Trailing zeros to the right of the decimal after the last non-zero significant digit are significant.
    For example, 112.00 has five significant figures. 0.0780 has three significant figures.
  - → Trailing zeros in an integer are not significant.
    For example, 8300 has two significant figures. The zeros are not significant in the integer.
- If a number is already in scientific notation, all numbers in the significand are significant.
  For example, 2.301×10^{-2} has four significant figures.

Worked example 1.5A Converting numbers written in scientific notation to basic numerals

Write each number as a basic numeral.

a 2.4×10^6 **b** 7.1×10^{-8}

THINK	WRITE
a When multiplying by 10^6, move the decimal point six spaces to the right. Add trailing zeros where necessary.	**a** 2.400000 $2.4 \times 10^6 = 2\,400\,000$
b Multiply by 10^{-8} is the same as divide by 10^8. When dividing by 10^8, move the decimal point eight spaces to the left. Add zeros where necessary.	**b** 000000007.1 $7.1 \times 10^{-8} = 0.000\,000\,071$

Worked example 1.5B Converting numbers written as basic numerals to scientific notation

Write each number in scientific notation.

a 230 000 **b** 0.000 856

THINK	WRITE
a Count the number of places the decimal point in 230 000 would be moved to produce 2.3. The decimal point needs to be moved five places to the left, so the exponent is 5.	**a** 230 000 $230\,000 = 2.3 \times 10^5$
b Count the number of places the decimal point in 0.000 856 would be moved to produce 8.56. The decimal point needs to be moved four places to the right, so the exponent is −4.	**b** 0.000 856 $0.000\,856 = 8.56 \times 10^{-4}$

Worked example 1.5C Identifying significant figures

How many significant figures are shown in each number?

a 5.42 **b** 20 803 **c** 6.200 **d** 4000 **e** 0.0082

THINK	WRITE
a All non-zero digits are significant.	**a** 5.42 has three significant figures.
b Zeros between non-zero digits are significant.	**b** 20 803 has five significant figures.
c Zeros at the end of a decimal number are significant.	**c** 6.200 has four significant figures.
d Zeros at the end of an integer are not significant.	**d** 4000 has one significant figure.
e Zeros to the left of the first non-zero digit in a decimal number are not significant.	**e** 0.0082 has two significant figures.

Worked example 1.5D Writing numbers in scientific notation using significant figures

Write each number in scientific notation with the number of significant figures indicated in brackets.

a 53 726 (2) **b** 0.084 03 (3)

THINK	WRITE
a **1** This number has five significant figures. Round to two significant figures (the nearest thousand). Remember that zeros at the end of an integer are not significant.	**a** $53\,726 \approx 54\,000$
2 Write in scientific notation.	$= 5.4 \times 10^4$
b **1** This number has four significant figures. Round to three significant figures (the nearest ten-thousandth).	**b** $0.084\,03 \approx 0.0840$
2 Write in scientific notation. Remember that zeros at the end of a decimal are significant.	$= 8.40 \times 10^{-2}$

Helpful hints

✔ *Multiplying* a number by 10 *increases* each digit's place value by 1 column. Move the decimal point one place-value space to the *right* and insert a zero where necessary.

$$5.23 \times 10 = 52.3$$

$$5.23 \times 100 = 523.$$

$$5.23 \times 1000 = 5230.$$

✔ *Dividing* a number by 10 *decreases* each digit's place value by 1 column. Move the decimal point one place-value space to the *left* and insert a zero where necessary.

$$5.23 \div 10 = 0.523$$

$$5.23 \div 100 = 0.0523$$

$$5.23 \div 1000 = 0.00523$$

Learning pathways

△ 1(1st, 2nd columns), 2(a, c, e, g, i), 3(1st, 2nd columns), 4, 5–6(1st, 2nd columns), 7–8(1st column), 9–10(1st, 2nd columns), 11, 12, 17, 20, 22

□ 1(c, f, i), 2(b, d, f, h, j), 3(4th, 5th columns), 4, 5–6(2nd, 3rd columns), 7–8(2nd column), 9–10(2nd, 3rd columns), 13, 16, 21, 23, 26

⬠ 3(e, j, o), 4, 5–6(3rd, 4th columns), 7–8(3rd column), 9–10(3rd, 4th columns), 14, 15, 18, 19, 24, 25, 27, 28

ANS p452

Exercise 1.5A: Understanding and fluency

1 Calculate each of these.
Hint: Move the decimal point an appropriate number of places.

a 5.4×100 **b** $7.36 \times 10\,000$ **c** -1.8×1000
d $4.05 \times 100\,000$ **e** $2.753 \times 1\,000\,000$ **f** $\frac{6.1}{10}$
g $\frac{8.22}{1\,000\,000}$ **h** $\frac{-9.76}{10\,000}$ **i** $\frac{7.003}{100\,000}$

2 Write each number as a power of 10.

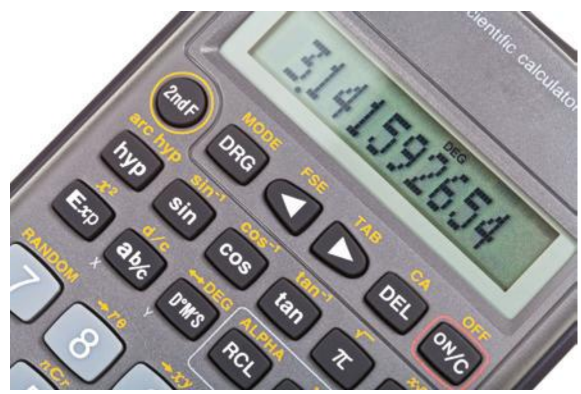
a 100 **b** 1000 **c** 10 000 **d** 100 000 **e** 1 000 000
f 0.1 **g** 0.01 **h** 0.001 **i** 0.0001 **j** 0.000 01

WE 1.5A **3** Write each number as a basic numeral.

a 3.2×10^5 **b** 8.14×10^9 **c** -5.0×10^2 **d** -2.345×10^7
e 1.1×10^4 **f** 6.4×10^{-3} **g** 7.28×10^{-6} **h** 9×10^{-7}
i -3.02×10^{-5} **j** -5.41×10^{-2} **k** 4.5×10^{11} **l** 6.12×10^{-9}
m 5.7×10^{-1} **n** 1.3068×10^3 **o** 2.7316×10^{-4}

4 Calculators use different methods for entering in and displaying numbers in scientific notation. Most scientific calculators have a button for entering numbers in scientific notation quickly. It is usually labelled with a bold **E**, Exp, $\times\ 10^x$ or $\times\ 10^n$. Check with your teacher if you cannot find this button. To use the button, type the significand, press the scientific notation button, and then type the exponent of 10.

a Use a calculator to verify each number in question **3**.
b Were there any numbers that could not easily be displayed on your calculator? Explain.



5 Basic numerals can be written in expanded notation with powers of 10 to show the place value of each digit. For example, $23.41 = 2 \times 10^1 + 3 \times 10^0 + 4 \times 10^{-1} + 1 \times 10^{-2}$. Write the following numbers in expanded notation.

a 4500 **b** 7 320 000 **c** 200 000 **d** −190
e 3216 **f** 0.0063 **g** 0.000 000 18 **h** 0.05
i −0.000 0702 **j** 0.427 **k** 11 220 **l** 0.000 004
m −568.2 **n** 0.000 249 **o** 679 300 **p** −0.0102

WE 1.5B **6** Write each number in question **5** in scientific notation.

WE 1.5C **7** How many significant figures are shown in each number?

a 345 **b** 25 000 **c** 5072 **d** 400
e −809 **f** 0.59 **g** −0.003 **h** 1.472
i 48.062 **j** −7.300 **k** 36 020 **l** 0.009 04

8 How many significant figures are shown in each number?

a 2.4×10^3 **c** 1.900×10^7 **e** -3.206×10^{-9}
b 5.06×10^{-4} **d** 8.0×10^5 **f** 7.00×10^5

9 Round each number to the number of significant figures indicated in brackets.

a 2.58×10^5 (2) **b** -5.037×10^4 (3) **c** 9.1042×10^6 (4) **d** -6.00×10^3 (2)

e 458 (2) **f** 73 051 (4) **g** 1279 (1) **h** 40 008 (1)

i −5.1437 (3) **j** 0.0349 (2) **k** −42.0607 (4) **l** 0.852 (1)

WE 1.5D 10 Write each number in scientific notation with the number of significant figures indicated in brackets.

a 327 (2) **b** 48 654 (3) **c** −190 760 (4) **d** 2621 (1)

e 0.4031 (3) **f** −0.0544 (2) **g** 0.000 207 193 (4) **h** −0.008 327 (1)

i 758.4 (2) **j** −20 703.02 (4) **k** 40.155 (3) **l** 54 007.63 (5)

11 Consider the following numbers **A–H**.

A 3.4×10^4 **B** 2.03×10^{-3} **C** -0.58×10^6 **D** 60.34×10^2

E 0.009 **F** -4.19×10^3 **G** 700×10^5 **H** 9×10^{-4}

- **a** Which numbers are written in scientific notation?
- **b** Which numbers are written with three significant figures?
- **c** Which numbers are larger than 10?
- **d** Which numbers are less than 1?

Exercise 1.5B: Problem solving and reasoning

12 Write each approximate measurement in scientific notation.

- **a** A medium-sized grain of sand has a length of 0.0005 m.
- **b** The Cooby Creek Reservoir has a capacity of 23 000 ML.
- **c** The thickness of the epidermal layer of skin on your eyelid is 0.048 mm.
- **d** An estimate for the world's population in 2050 is 9 300 000 000.

13 Write each approximate measurement as a basic numeral.

- **a** The number of times the wings of a hummingbird flap in a minute is 6.4×10^3.
- **b** The diameter of a virus is 8×10^{-5} mm.
- **c** The distance from the Sun to Earth is 1.496×10^8 km.
- **d** The radius of an electron is 2.8×10^{-13} cm.

14 Complete the table below by writing the numbers as a product with each of the powers of 10. Underline the answers that are in scientific notation. The first row has been completed for you.

	1234.56	4.0191	0.0492	0.007 40
$\times 10^3$	$1.234\,56 \times 10^3$	$0.004\,0191 \times 10^3$	$0.000\,0492 \times 10^3$	$0.000\,007\,40 \times 10^3$
$\times 10^2$				
$\times 10^1$				
$\times 10^0$				
$\times 10^{-1}$				
$\times 10^{-2}$				
$\times 10^{-3}$				

15 We can perform arithmetic operations in scientific notation. Multiplication and division can be performed by multiplying or dividing the significands and then using the product and quotient of powers laws to multiply or divide the powers of 10. For example:

$$\begin{aligned}&(2.1 \times 10^7) \times (8.4 \times 10^3)\\ &= (2.1 \times 8.4) \times (10^7 \times 10^3)\\ &= 17.64 \times 10^{10}\\ &= (17.64 \div 10) \times 10^{10+1}\\ &= 1.764 \times 10^{11}\end{aligned}$$

$$\begin{aligned}&(2.1 \times 10^7) \div (8.4 \times 10^3)\\ &= (2.1 \div 8.4) \times (10^7 \div 10^3)\\ &= 0.25 \times 10^4\\ &= (0.25 \times 10) \times 10^{4-1}\\ &= 2.5 \times 10^3\end{aligned}$$

Evaluate the following products and quotients. Write your answer in scientific notation.

a $(1.7 \times 10^5) \times (4 \times 10^2)$ **b** $(8 \times 10^7) \div (4 \times 10^5)$ **c** $(-5 \times 10^{-5}) \times (-9 \times 10^8)$

d $(-6 \times 10^9) \div (1.5 \times 10^5)$ **e** $(4.1 \times 10^{-6}) \times (-3 \times 10^4)$ **f** $(7.2 \times 10^{-2}) \div (2.4 \times 10^{-7})$

16 Addition and subtraction require digits with the same place value to be added together. Therefore, in scientific notation, both numbers must be written using the same power of 10 so that the digits in the significands have the same place value relative to the decimal point. For example:

$$\begin{aligned} &(2.1 \times 10^5) + (8.4 \times 10^3) \\ &= (2.1 \times 10^5) + (0.084 \times 10^5) \\ &= (2.1 + 0.084) \times 10^5 \\ &= 2.184 \times 10^5 \end{aligned} \qquad \begin{aligned} &(2.1 \times 10^{-2}) - (8.4 \times 10^{-3}) \\ &= (2.1 \times 10^{-2}) - (0.84 \times 10^{-2}) \\ &= (2.1 - 0.84) \times 10^{-2} \\ &= 1.26 \times 10^{-2} \end{aligned}$$

Evaluate the following sums and differences. Write your answer in scientific notation.

a $(3.4 \times 10^2) + (7.3 \times 10^5)$ **b** $(8.52 \times 10^4) - (1.6 \times 10^3)$ **c** $(6.03 \times 10^{-3}) + (2.7 \times 10^{-4})$

d $(8.2 \times 10^{-3}) - (3.5 \times 10^{-2})$ **e** $(-9.8 \times 10^3) + (-7.7 \times 10^2)$ **f** $(1.01 \times 10^5) - (7.5 \times 10^3)$

17 Light travels at a speed of approximately 3.00×10^{10} cm/s. How many kilometres does it travel in 1 hour? Give your answer in scientific notation.

18 Earth revolves around the Sun at an average speed of 10^5 km/h.

a What distance does Earth travel in 1 day?

b How many days would it take Earth to travel 9.6×10^8 km?

19 The Australian $1 coin has a mass of 9 g and a thickness of 3×10^{-1} cm.

a Sarah has a pile of these coins on her desk. She stacks as many of them as she can on top of each other between two shelves in a bookcase. The distance separating the shelves is 26 cm.

i How many coins are in the stack?

ii What would be the mass of these coins?

b Ben takes Sarah's stack of coins and places them end-to-end in a line. The line stretches to a length of 2.15 m. What is the diameter of a $1 coin?

20 The Sun is 1.52×10^8 km from Earth. Light from the Sun travels towards Earth at a speed of 3×10^8 m/s. How long does it take this light to reach Earth? Give your answer to the nearest minute.

21 Numbers can be rounded to a certain number of significant figures.

a Round each of the following to one, two and three significant figures.

i 1.901 **ii** 1.994 **iii** 1.997 **iv** 2.003 **v** 2.006 **vi** 2.098

b Explain how the significant trailing zeros are important in determining to how many significant figures a number is rounded.

22 Consider 0.41, 0.000 000 000 0012 and −0.000 034.

a Round each number to one significant figure.

b Assume, only for this question, that we included leading zeros as significant. Round each number to one significant figure.

c Explain why not including leading zeros as significant is more useful than including them. Consider how including them is similar to rounding to a place value or number of decimal places.

23 Explain the mistake each student made.

a Jane rounded 4.1025 to three significant figures as 4.103.

b Kaleb rounded 0.0432 to three significant figures as 0.04.

c Lisa rounded 102 948.3618 to three significant figures as 102 900.

d Marius rounded 102 948.3618 to three significant figures as 103.

24 Scientific notation is very useful when converting between different units of time.

a Write the following in seconds in scientific notation.

i 47 minutes **ii** 14 days **iii** 40 weeks **iv** 1 year (not a leap year)

b Write the following in the unit in brackets, correct to three significant figures.

i 10 000 seconds (hours)
ii 1 000 000 seconds (days)
iii 1 000 000 000 seconds (years)
iv 1 000 000 000 000 seconds (millennia)

25 We use metric prefixes for very large and very small measurements. These are closely related to engineering notation or engineering form, which is similar to scientific notation.

A number written in engineering notation is the product of a number, the significand, a, between positive or negative 1 (inclusive) and positive or negative 1000 (exclusive) and a power of 1000 written as a power of 10 in exponent form. That is, $a \times 10^{3m}$, where $1 \le a < 1000$ or $-1000 < a \le -1$ and m is an integer.

For example, 3.456×10^9, 34.56×10^6, 345.6×10^{-6} are in engineering form, while 0.3456×10^9, 3456×10^6, 345.6×10^{-5} are not in engineering form.

Prefix	Abbreviation	Meaning	Power of 10
tera	T	one trillion of the unit	10^{12}
giga	G	one billion of the unit	10^9
mega	M	one million of the unit	10^6
kilo	k	one thousand of the unit	10^3
milli	m	one-thousandth of the unit	10^{-3}
micro	μ	one-millionth of the unit	10^{-6}
nano	n	one-billionth of the unit	10^{-9}
pico	p	one-trillionth of the unit	10^{-12}

a Write the following in seconds in engineering form.

i 7.3 kiloseconds (7.3 ks)
ii 9.1 microseconds (9.1 μs)
iii 54 nanoseconds (54 ns)
iv 82 teraseconds (82 Ts)
v 129 megaseconds (129 Ms)
vi 974 picoseconds (974 ps)

b Write the following times in engineering form using the appropriate prefix.

i 5.601×10^7 seconds
ii 9.2×10^5 seconds
iii 4.31×10^{-5} seconds
iv 7.88×10^{-7} seconds
v 8×10^{-2} seconds
vi 1.0356×10^{14} seconds

Exercise 1.5C: Challenge

26 Sound travels at 330 m/s, whereas light travels at 3×10^5 km/s. A timekeeper stands at the end of a 100 m straight running track. The starting gun at the beginning of the track goes off.

a How long does it take:

i for the sight of the smoke to reach the timekeeper
ii for the sound of the gun to reach the timekeeper?

b What advice should you give the timekeeper in order to have an accurate recording of the time of the race?

27 The circumference of a hydrogen atom is 7.98×10^{-9} cm. How far would a line of 1 million hydrogen atoms stretch if placed next to each other?

28 Consider the multiplication problem $2^{350} \times 3^2 \times 4^3 \times 5^{355}$. Write the exact answer in scientific notation.

Online resources:

Interactive skillsheet Scientific notation
Investigation Measuring large units in our solar system
Investigation Accuracy in science experiments
CAS instructions Scientific notation
Quick quiz 1.5

Rational and irrational numbers

Learning intentions

By the end of this lesson you will be able to ...

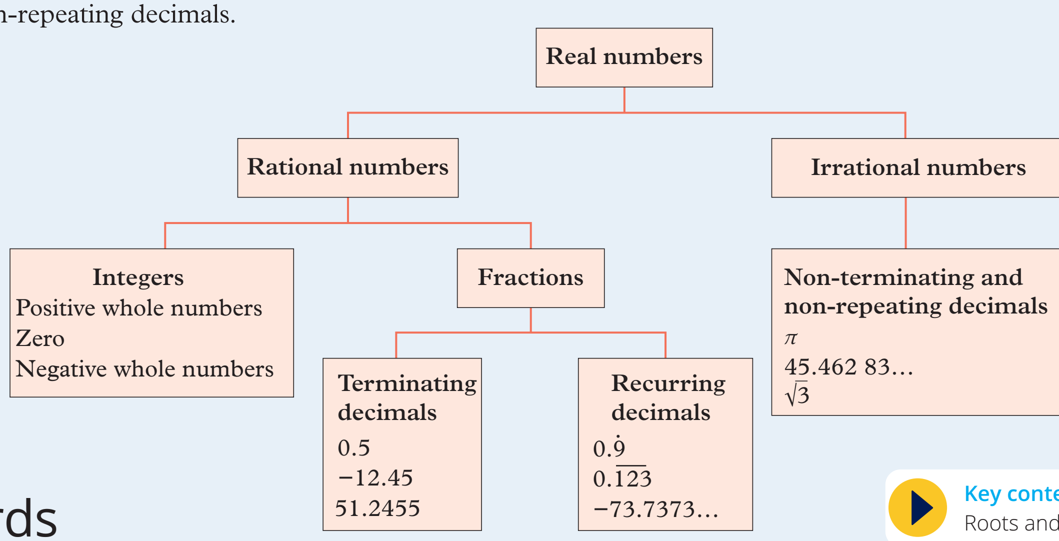
✔ differentiate between rational and irrational numbers, and simplify surds.

Inter-year links

Year 7	1.7 Exponents and square roots
Year 8	4.5 Roots
Year 10	2.1 Surds (10A)

Rational and irrational numbers

- A **real number** is a number that can be represented by a point on a number line.
- The real number system is made up of **rational numbers** and **irrational numbers**.
- Rational numbers are numbers that can be written as a fraction where both the numerator and denominator are integers. The set of rational numbers includes integers, fractions, terminating decimals and recurring decimals.
- Irrational numbers are numbers that cannot be written as fractions with both the numerator and denominator being integers. The set of irrational numbers includes **surds** and other non-terminating, non-repeating decimals.



Key content video
Roots and surds

Surds

- A **perfect square** is the square of an integer. For example, 1, 4, 9, 16, 25, ... are perfect squares.
- A surd is an irrational number expressed as a root that cannot be simplified.
  - → $\sqrt{2}$ cannot be written as a fraction, integer or recurring or terminating decimal, so it is a surd.
  - → $\sqrt{4}$ can be simplified to 2, so it is not a surd.
- The roots of fractions and decimals may be either rational or irrational numbers.
  - → $\sqrt{\frac{4}{9}}$ can be simplified to $\frac{2}{3}$, so $\sqrt{\frac{4}{9}}$ is a rational number.
  - → $\sqrt{\frac{3}{5}}$ is an irrational root that cannot be simplified, so it is a surd.
  - → $\sqrt{1.44}$ can be simplified to 1.2, so $\sqrt{1.44}$ is a rational number.
  - → $\sqrt{0.9}$ is an irrational root that cannot be simplified, so it is a surd.
- Exact value refers to a value given precisely as measured or recorded, not rounded off. Surds are a way to represent irrational numbers as exact values. They can be rounded off to designated significant figures to give approximate values.
- Surds can represent many types of roots, such as the square root, $\sqrt{2}$, the cube root, $\sqrt[3]{2}$, or the fourth root, $\sqrt[4]{2}$.

$\sqrt{1} = 1$
$\sqrt{2} \approx 1.414\,214...$
$\sqrt{3} \approx 1.732\,051...$
$\sqrt{4} = 2$

Basic rules for square roots

- The inverse operation of taking a square root of a number is raising the number to an exponent of 2. $\left(\sqrt{2}\right)^2 = 2$ $\quad \left(\sqrt{a}\right)^2 = a$
- The inverse operation of raising a number to an exponent of 2 is taking the square root of the number. $\sqrt{2^2} = 2$ $\quad \sqrt{a^2} = a$
- A square root can be written as the product of the square roots of its positive factors. $\sqrt{6} = \sqrt{2} \times \sqrt{3}$ $\quad \sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- A square root divided by a square root can be written as the square root of the quotient. $\dfrac{\sqrt{2}}{\sqrt{3}} = \sqrt{\dfrac{2}{3}}$ $\quad \dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

Simplifying square roots to surds

Key content video
Simplifying surds

- To simplify a square root with a perfect square factor:
  1. Rewrite the number as a product of its factors, including the square number.
  2. Write the square root as a product of the square root of the factors.
  3. Simplify the perfect square roots.
  4. Multiply the remaining square roots and simplify the expression.

$$\begin{aligned}\sqrt{12} &= \sqrt{4 \times 3} \\ &= \sqrt{4} \times \sqrt{3} \\ &= 2 \times \sqrt{3} \\ &= 2\sqrt{3}\end{aligned}$$

Worked example 1.6A Identifying rational and irrational numbers

Determine whether each of the following numbers is rational or irrational.

a 0.26 **b** $-\dfrac{2}{3}$ **c** $-\sqrt{11}$ **d** $\sqrt{16}$

e $2.3\overline{54}$ **f** 6.584 381 257… **g** $\sqrt[3]{9}$

THINK	WRITE
a 0.26 is a terminating decimal. A terminating decimal can be written exactly as a fraction: $0.26 = \frac{26}{100}$. So 0.26 is rational.	**a** 0.26 is a rational number.
b Fractions are rational numbers, so $-\frac{2}{3}$ is rational.	**b** $-\frac{2}{3}$ is a rational number.
c $-\sqrt{11}$ is approximately equal to −3.316 624 79… This is a non-terminating and non-recurring decimal, which cannot be written as a fraction, so $-\sqrt{11}$ is irrational.	**c** $-\sqrt{11}$ is an irrational number.
d $\sqrt{16}$ simplifies to 4. Integers are rational numbers, so $\sqrt{16}$ is rational.	**d** $\sqrt{16}$ is a rational number.
e $2.3\overline{54}$ represents the recurring decimal 2.354 545 454… Recurring decimals are rational numbers, so $2.3\overline{54}$ is rational.	**e** $2.3\overline{54}$ is a rational number.
f 6.584 381 257… is a non-terminating decimal number. Its decimal place values do not have a pattern of repeating digits, so 6.584 381… is irrational.	**f** 6.584 381 257… is an irrational number.
g $\sqrt[3]{9}$ is approximately equal to 2.080 083 823… This is a non-terminating and non-recurring decimal, which cannot be written as a fraction, so $\sqrt[3]{9}$ is irrational.	**g** $\sqrt[3]{9}$ is irrational.

Worked example 1.6B Identifying surds

Which of the following numbers are surds?

a $\sqrt{9}$ **b** $2\sqrt{6}$ **c** $4\sqrt{25}$

THINK

a $\sqrt{9}$ can be simplified to 3, so $\sqrt{9}$ is not a surd.

b $2\sqrt{6} = 2 \times \sqrt{6}$

$\sqrt{6} = 2.449\,489\,743\ldots$, so $2\sqrt{6}$ is a surd.

c $4\sqrt{25} = 4 \times \sqrt{25}$

$\sqrt{25}$ can be simplified to 5.

$4\sqrt{25} = 4 \times 5 = 20$, which is rational.

So $4\sqrt{25}$ is not a surd.

WRITE

a $\sqrt{9}$ is not a surd.

b $2\sqrt{6}$ is a surd.

c $4\sqrt{25}$ is not a surd.

Worked example 1.6C Simplifying surds

Simplify the following surds.

a $\sqrt{8}$ **b** $\sqrt{22}$ **c** $\sqrt{108}$

THINK

a
1. Rewrite the number as a product of its factors, including the square number 4.
2. Write the square root as a product of the square roots of the factors using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.
3. Simplify the perfect square roots.
4. Multiply the remaining square roots and simplify the expression.

b Rewrite the number as a product of its factors. There are no square number factors so $\sqrt{22}$ is already in its simplest form.

c
1. Rewrite the number as a product of its factors, including the square numbers 4 and 9.
2. Write the square root as a product of the square roots of the factors using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.
3. Simplify the perfect square roots and multiply together.
4. Multiply the remaining square roots and simplify the expression.

WRITE

a $\sqrt{8} = \sqrt{4 \times 2}$

$= \sqrt{4} \times \sqrt{2}$

$= 2 \times \sqrt{2}$

$= 2\sqrt{2}$

b $\sqrt{22} = \sqrt{2 \times 11}$

$= \sqrt{2} \times \sqrt{11}$

Therefore, $\sqrt{22}$ is already in its simplest form.

c $\sqrt{108} = \sqrt{4 \times 9 \times 3}$

$= \sqrt{4} \times \sqrt{9} \times \sqrt{3}$

$= 2 \times 3 \times \sqrt{3}$

$= 6 \times \sqrt{3}$

$= 6\sqrt{3}$

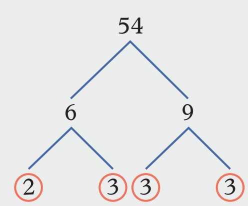
Worked example 1.6D Writing simplified surds as square roots

Write the simplified surd $4\sqrt{3}$ as the square root of a single number.

THINK	WRITE
1 Write the simplified surd as a product of the integer and square root.	$4\sqrt{3} = 4 \times \sqrt{3}$
2 Rewrite the integer as a square root using $a = \sqrt{a^2}$.	$= \sqrt{16} \times \sqrt{3}$
3 Multiply the square roots together using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.	$= \sqrt{16 \times 3}$ $= \sqrt{48}$

Helpful hints

- ✔ Some people think that decimals are more 'exact' than fractions. However, mathematicians prefer fractions because it is never possible to tell if a decimal has been rounded, truncated or if it has only been measured to a certain number of decimal places. This uncertainty is not an issue with a fraction which has integers for both its numerator and denominator.
- ✔ Remember that factor trees can help to determine square number factors as a number that is multiplied by itself to give a square number. For example, $3 \times 3 = 3^2 = 9$ is a perfect square factor of 54.



- ✔ If there are no repeated prime factors in a surd, then the surd cannot be simplified.
- ✔ Simplifying surds is an important skill for Year 11 and 12 Maths.

Learning pathways

1–10, 13, 16, 17   2, 3, 5–11, 14, 17, 19, 20   2, 3, 6, 7–9, 11, 12, 15, 18, 20–22

ANS p453

Exercise 1.6A: Understanding and fluency

1 Match each number below with the correct term from the list **I–VI**.

a $\frac{3}{4}$

b 45.710 326 58…

c –9

d $\sqrt{23}$

e 0.154 715 471 547…

f 6.213 764 29

I recurring decimal

II integer

III terminating decimal

IV fraction

V surd

VI non-terminating and non-recurring decimal

WE 1.6A **2** Determine whether each of the following numbers is rational or irrational.

a $\frac{4}{5}$ **b** -5 **c** $\sqrt{3}$ **d** $\sqrt{64}$ **e** 2.548 247 001…

f 5.684 **g** $-\frac{4}{9}$ **h** $-3\sqrt{5}$ **i** $5\frac{1}{6}$ **j** -1.5

k $\sqrt{9}$ **l** $\sqrt{8}$ **m** $\sqrt[3]{125}$ **n** 0.001 002 142… **o** $\sqrt[4]{32}$

WE 1.6B **3** Which of the following numbers are surds?

a $\sqrt{21}$ **b** $-\sqrt{16}$ **c** $\sqrt[4]{64}$ **d** $-\sqrt{6}$

e $\frac{1}{2}\sqrt{9}$ **f** $5\sqrt{5}$ **g** $-3\sqrt{27}$ **h** $-\sqrt[3]{27}$

i $\sqrt[3]{3}$ **j** $3\sqrt{3}$ **k** $\frac{\sqrt{100}}{5}$ **l** $\frac{\sqrt{6}}{5}$

4 Use your calculator to help you write a decimal approximation, rounded to four decimal places, for each of the surds in question **3**.

5 Which of the irrational numbers in question **2** are surds?

WE 1.6C **6** Simplify the following surds.

a $\sqrt{20}$ **b** $\sqrt{27}$ **c** $\sqrt{28}$ **d** $\sqrt{50}$

e $\sqrt{72}$ **f** $\sqrt{800}$ **g** $\sqrt{147}$ **h** $\sqrt{384}$

i $\sqrt{432}$ **j** $\sqrt{847}$ **k** $\sqrt{486}$ **l** $\sqrt{891}$

WE 1.6D **7** Write each of the following simplified surds as the square root of a single number.

a $3\sqrt{5}$ **b** $4\sqrt{6}$ **c** $2\sqrt{3}$ **d** $2\sqrt{11}$

e $5\sqrt{10}$ **f** $8\sqrt{7}$ **g** $13\sqrt{3}$ **h** $12\sqrt{5}$

8 The coefficient of a surd represents the number of surds that have been added together. This is similar to the coefficient of a variable.

For example:

$\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} = 5 \times \sqrt{2} = 5\sqrt{2}$

a Write each of the following sums in the form $a\sqrt{b}$.

i $\sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$ **ii** $\sqrt{5} + \sqrt{5} + \sqrt{5}$ **iii** $\sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6}$ **iv** $\sqrt{10} + \sqrt{10}$

b Write the following as repeated sums.

i $5\sqrt{3}$ **ii** $2\sqrt{6}$ **iii** $8\sqrt{2}$ **iv** $3\sqrt{11}$

9 State whether each of the following numbers is rational or irrational.

a $\sqrt{\frac{4}{16}}$ **b** $\sqrt{\frac{1}{12}}$ **c** $\frac{\sqrt{16}}{\sqrt{5}}$ **d** $\sqrt{\frac{1}{16}}$

e $\frac{2\sqrt{6}}{\sqrt{4}}$ **f** $\sqrt{\frac{1}{2}}$ **g** $\frac{\sqrt{6}}{\sqrt{25}}$ **h** $\sqrt{\frac{7}{49}}$

10 The Greek letter π (pi) is used as the symbol for a number commonly used in mathematics (equal to 3.141 592…). Is π a rational or irrational number? Provide a reason for your answer.

11 When simplifying a surd, write the number as a product of its prime factors to help you find the square factors. You can do this by pairing repeated prime factors. For example:

$$\begin{aligned}&\sqrt{3 \times 7 \times 7 \times 7 \times 7 \times 11 \times 11 \times 11}\\&= \sqrt{3 \times (7 \times 7) \times (7 \times 7) \times (11 \times 11) \times 11}\\&= \sqrt{3} \times \sqrt{7 \times 7} \times \sqrt{7 \times 7} \times \sqrt{11 \times 11} \times \sqrt{11}\\&= 7 \times 7 \times 11 \times \sqrt{3 \times 11}\\&= 539\sqrt{33}\end{aligned}$$

Simplify the following by pairing repeated prime factors.

a $\sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 5}$ **b** $\sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 7 \times 11 \times 11}$

c $\sqrt{2^4 \times 3^2 \times 3 \times 5 \times 7}$ **d** $\sqrt{2^3 \times 3 \times 5^5}$

Exercise 1.6B: Problem solving and reasoning

12 Consider the two fractions $\sqrt{\frac{5}{8}}$ and $\frac{\sqrt{5}}{\sqrt{8}}$.

- **a** Use your calculator to help you determine the approximate value for each of $\sqrt{\frac{5}{8}}$ and $\frac{\sqrt{5}}{\sqrt{8}}$, correct to six decimal places.
- **b** Are the two fractions equivalent? Explain.
- **c** Use your results to comment on the relationship between fractions written in the forms $\sqrt{\frac{a}{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}}$.
- **d** Is $\sqrt{\frac{5}{8}}$ a rational number or an irrational number? Provide a reason for your answer.
- **e** Write each of these fractions in the form $\sqrt{\frac{a}{b}}$ to help you decide if it is a rational or irrational number.
  - **i** $\frac{\sqrt{32}}{\sqrt{2}}$ **ii** $\frac{\sqrt{15}}{\sqrt{3}}$ **iii** $\frac{\sqrt{7}}{\sqrt{28}}$

13 Perfect squares can help us estimate the value of a surd under the square root.

- **a** Calculate:
  - **i** $\sqrt{4}$ **ii** $\sqrt{9}$
- **b** Between which two consecutive integers does $\sqrt{5}$ lie?
- **c** How does knowledge of perfect squares help you decide which two consecutive integers a square root lies between?
- **d** Between which two consecutive integer values do each of these square roots lie?
  - **i** $\sqrt{11}$ **ii** $-\sqrt{20}$ **iii** $\sqrt{131}$ **iv** $-\sqrt{48}$ **v** $\sqrt{180}$ **vi** $\sqrt{300}$

14 Prime factorisation can be used to simplify surds.

- **a** Factorise the following numbers by writing them as a product of their prime factors.
  - **i** 288 **ii** 270 **iii** 980 **iv** 1125
- **b** Hence, simplify the following surds.
  - **i** $\sqrt{288}$ **ii** $\sqrt{270}$ **iii** $\sqrt{980}$ **iv** $\sqrt{1125}$

15 When simplifying products of surds, writing the numbers as products of their prime factors before multiplying can help you to find the square factors. Pair repeated factors from both sets of products. For example:

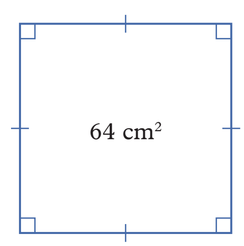
$$\begin{aligned}\sqrt{15} \times \sqrt{6} &= \sqrt{3 \times 5} \times \sqrt{2 \times 3} \\ &= \sqrt{2 \times 3 \times 3 \times 5} \\ &= \sqrt{9 \times 2 \times 5} \\ &= \sqrt{9} \times \sqrt{10} \\ &= 3\sqrt{10}\end{aligned}$$

Simplify the following by first writing the numbers as a product of their prime factors.

- **a** $\sqrt{21} \times \sqrt{14}$ **b** $\sqrt{27} \times \sqrt{33}$ **c** $\sqrt{30} \times \sqrt{18}$ **d** $\sqrt{42} \times \sqrt{231}$

16 A square has an area of 64 cm².

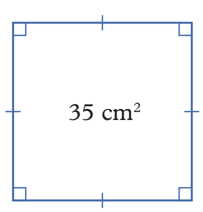
- **a** What is the length of each side of the square?
- **b** Calculate the length of the diagonal of the square:
  - **i** as an exact value **ii** rounded to two decimal places.



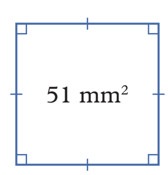
17 Find the side length of each of the following squares:

- **i** as an exact value
- **ii** rounded to two decimal places.

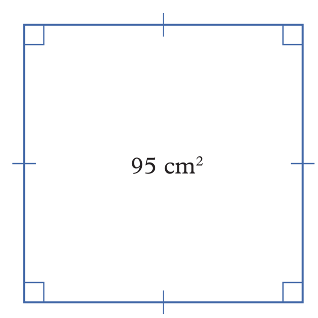
a



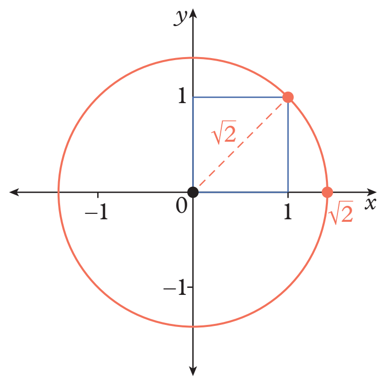
b



c



18 Geometric constructions can be used to locate surds on the number line. For example, draw a square on the Cartesian plane with side length equal to 1 and the bottom left corner at the origin as shown. Pythagoras' theorem tells us that the length of the diagonal is equal to:
$\sqrt{(1^2 + 1^2)} = \sqrt{(1 + 1)} = \sqrt{2}$.
Sketch the circle with radius equal to $\sqrt{2}$ centred at the origin. The point of intersection between the circle and the positive half of the x-axis shows the location of $\sqrt{2}$ on the number line.



a Copy the given construction to locate $\sqrt{2}$ on the number line.

b Use a square with side length equal to 2 to locate $\sqrt{8}$ on the same number line.

c Use a square with side length equal to 3 to locate $\sqrt{18}$ on the same number line.

d Simplify $\sqrt{8}$ and $\sqrt{18}$.

e How does your answer to part **d** relate to the locations of $\sqrt{2}$, $\sqrt{8}$ and $\sqrt{18}$ on the number line?

19 Given a surd in the form $\sqrt[n]{x}$, explain why the value of x must be non-negative when the value of n is even, but the value of x can be negative when the value of n is odd.

Exercise 1.6C: Challenge

20 Complete the working below to write $0.\overline{36}$ as a fraction.

a
$$\begin{aligned}
\text{Let: } x &= 0.363\,636\ldots \\
100x &= \underline{\hspace{3cm}} \\
100x - x &= \underline{\hspace{3cm}} - 0.363\,636\ldots \\
99x &= \underline{\hspace{3cm}} \\
x &= \frac{\square}{99} \\
\text{So:}\quad 0.\overline{36} &= \frac{\square}{11}
\end{aligned}$$

b Explain why multiplying x by 100 was helpful in this case.

21 Write each of these recurring decimals as a fraction.

a $0.\overline{14}$ **b** $0.\dot{2}\dot{1}$ **c** $0.\overline{48}$ **d** $1.\dot{7}\dot{3}$

22 Write each of these recurring decimals as a fraction. Show all your working.

a $0.333\,333\,333\ldots$ **b** $0.\overline{24}$ **c** $0.\overline{785}$ **d** $0.\dot{4}\dot{1}$

e $0.5\dot{6}$ **f** $3.\overline{18}$ **g** $4.2\dot{5}$ **h** $3.5\overline{126}$

Online resources:

Interactive skillsheet Terminating, non-terminating and recurring decimals

Interactive skillsheet Simplifying surds

CAS instructions Simplifying surds

Quick quiz 1.6

Lesson 1.7

Review: Exponents

Module summary

Exponents

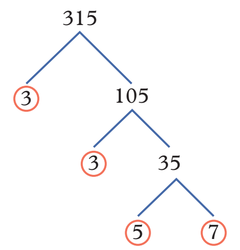
exponent

base → 3^4 exponent form

$= 3 \times 3 \times 3 \times 3$ expanded form

$= 81$ basic numeral

Prime factorisation



$315 = 3 \times 3 \times 5 \times 7$

$= 3^2 \times 5 \times 7$

Product of powers law

$2^3 \times 2^5 = 2^{3+5}$ $a^3 \times a^5 = a^{3+5}$

$= 2^8$ $= a^8$

Negative exponents

$2^{-1} = \left(\frac{2}{1}\right)^{-1} = \frac{1}{2}$ $a^{-1} = \left(\frac{a}{1}\right)^{-1} = \frac{1}{a}$

$\frac{1}{2^{-3}} = \frac{1}{\left(\frac{2}{1}\right)^{-3}}$ $\frac{1}{a^{-m}} = \frac{1}{\left(\frac{a}{1}\right)^{-m}}$

$= \frac{1}{\left(\frac{1}{2}\right)^{3}}$ $= \frac{1}{\left(\frac{1}{a}\right)^{m}}$

$= 1 \times \frac{2^3}{1}$ $= 1 \times \frac{a^m}{1}$

$= 2^3$ $= a^m$

Quotient of powers law

$2^5 \div 2^3 = 2^{5-3}$ $a^5 \div a^3 = a^{5-3}$

$= 2^2$ $= a^2$

Power of a power law

$(2^3)^5 = 2^{3\times5}$ $(a^2)^3 = a^{2\times3}$

$= 2^{15}$ $= a^6$

The zero exponent law

$2^0 = 1$ $a^0 = 1$

Scientific notation

$1230 = \underbrace{1.23}_{\text{significand}} \times 10^3$

basic numeral scientific notation

$3.1500 \times 10^4 = 31\,500$ (exponent of 4)

move 4 spaces to the right

$31500 = 3.15 \times 10^4$ (exponent of 4)

move 4 spaces to the left

$0.00123 = \underbrace{1.23}_{\text{significand}} \times 10^{-3}$

basic numeral scientific notation

$004.2 \times 10^{-2} = 0.042$ (exponent of -2)

move 2 spaces to the left

$0.042 = 4.2 \times 10^{-2}$ (exponent of -2)

move 2 spaces to the right

Significant figures

7.789 has four significant figures

4056 has four significant figures

0.051 has two significant figures

112.00 has five significant figures

8300 has two significant figures

2.301×10^{-2} has four significant figures

Surds

- Irrational numbers are non-recurring and non-terminating decimals
- Surds cannot be simplified to remove the root
- Surds are exact values
- Raising a number to the exponent of 2 and taking the square root of a number are inverse operations

$(\sqrt{2})^2 = 2$ $(\sqrt{a})^2 = a$ $\sqrt{2^2} = 2$ $\sqrt{a^2} = a$

ANS p454

Review questions 1.7A: Mathematical literacy review

Review quiz Take the review quiz to assess your knowledge of this module.

Quizlet Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- accuracy
- base
- basic numeral
- BIDMAS
- decimal point
- expanded form
- exponent
- exponent form
- factor
- factor tree
- index
- integer
- irrational number
- leading zero
- metric prefix
- negative exponent
- perfect square
- place value
- positive exponent
- power
- prime factor
- prime factorisation
- prime number
- product
- quotient
- rational number
- real numbers
- reciprocal
- rounding
- scientific notation
- significant figure
- standard form
- surd
- trailing zero
- zero exponent

1 Which key term can be used to help find the prime factorisation of a number?

2 Which description best explains the term prime factorisation of a number?

A the sum of two or more prime factors that equals the given number

B the product of two or more integers that equals the given number

C the product of two or more prime factors that equals the given number

D the product of two or more composite numbers that equals the given number

3 Use one of the numbers 7, 12 or 15 to clearly explain the difference between factors and prime factors.

4 Show how 24 can be written in expanded form and exponent form.

5 Complete the following sentences using words from the key terms list.

a The ________ of a value written in ________ is the number or variable that is multiplied repeatedly.

b Raising a number to a negative exponent is the same as raising the ________ of the base to a ________ .

6 Determine if the following statements about significant figures are true or false. If they are false, give an example to support your claim.

a All non-zero digits are significant.

b All zeros are not significant.

c All leading zeros are not significant.

d All trailing zeros are significant.

Review questions 1.7B: Multiple choice

1.1 **1** Which of the following is not equivalent to $9(xy)^4$?

A $9 \times xy \times xy \times xy \times xy$ **B** $-3^2x^4y^4$ **C** $9x^4y^4$ **D** $9xxxxyyyy$

1.1 **2** Which of the following is the prime factorisation of 360?

A $6^2 \times 10$ **B** $4 \times 9 \times 10$ **C** $2^3 \times 3^2 \times 5$ **D** $2^2 \times 3^2 \times 10$

1.2 **3** Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?

A $\frac{6ab^2}{18a}$ **B** $\frac{ab^2c}{3a^2c}$ **C** $\frac{b^2}{3a}$ **D** $\frac{6ab^2}{18a^2}$

1.2 **4** Which statement does not correctly represent one of the exponent laws?

A $m^5 \times m^2 = m^{5+2}$ **B** $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y}$ **C** $w^7 \div w^5 = w^{7-5}$ **D** $a^5 \times a = a^6$

1.3 **5** Using the exponent laws, $\frac{5x^{13} \times 2x^4}{4x^8 \times x^0}$ fully simplifies to:

A $\frac{10x^9}{4x^8}$ **B** $\frac{5x^9}{2}$ **C** $\frac{5x^{17}}{2x^8}$ **D** $\frac{5}{2x^9}$

1.4 **6** Which of the following is not the reciprocal of $\frac{4}{3}$?

A $\frac{3}{4}$ **B** 3×4^{-1} **C** $\left(\frac{4}{3}\right)^{-1}$ **D** $\frac{1}{3 \times 4}$

1.4 **7** Which statement is false?

A $\frac{1}{7} = 7^1$ **B** $4^{-2} = \frac{1}{16}$ **C** $\frac{1}{3^6} = 3^{-6}$ **D** $7^3 \times 7^{-5} = \frac{1}{49}$

1.5 **8** Which number is equivalent to 6.4724×10^2?

A 0.647 24 **B** 647.24 **C** 0.064 724 **D** 64 724

1.6 **9** What is the value of 9.1517×10^{-2} when rounded to three significant figures?

A 9.151×10^{-2} **B** 9.15×10^{-2} **C** 9.1517×10^1 **D** 9.1517×10^{-5}

1.6 **10** Which of the following numbers is irrational?

A $\sqrt{0.064}$ **B** $\sqrt{\frac{1}{64}}$ **C** $\sqrt{64}$ **D** $\sqrt[3]{64}$

Review questions 1.7C: Short answer

1.1 **1** Evaluate the following.

a 3^4 **b** $(-5)^3$ **c** -4^3 **d** $\left(\frac{3}{2}\right)^5$ **e** $(0.6)^3$ **f** $(1.2)^4$

1.2 **2** Write the following in exponent form.

a $17 \times 17 \times 17 \times 17 \times 17 \times 17$ **b** $-5b^2 \times -5b^2 \times -5b^2 \times -5b^2 \times -5b^2$

c $-10 \times f \times f \times f \times v \times v \times v \times v \times v \times v \times v$ **d** $\frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3}$

1.3 **3** Simplify each expression using the exponent laws.

a $a^{11} \times a^5$ **b** $b^9 \div b^8$ **c** $(c^8)^2$

d $18d^7 \div (54d^4)$ **e** $(e^5)^5 \times (e^{11})^2$ **f** $5a^0 + 3b^0 + 1c^0$

1.3 **4** Simplify each expression.

a $\frac{m^3n^4 \times m^9n^{11}}{m^7n^7}$ **b** $\frac{(3k^5l^2)^3 \times (2k^3l^3)^4}{(2k^3l^2)^3}$

1.4 **5** Write each term with a positive exponent.

a 5^{-5} **b** $(-11)^{-3}$ **c** $\frac{1}{4^{-4}}$ **d** $\left(-\frac{4}{5}\right)^{-3}$

1.4 **6** Consider the properties of negative exponents to answer the following questions.

a Using the fact that $4^8 = 65\,536$, write the value of 4^{-8} as a fraction.

b Using the fact that $7^{-3} = \frac{1}{343}$, write the value of 7^3.

1.5 **7** Write each number as a basic numeral.

a 5.876×10^4 **b** 9.02×10^{-6}

1.5 **8** Write each number in scientific notation.

a 540 000 **b** 0.000 76

1.5 **9** A scientist estimates that there are 3.40×10^4 bacteria in one sample and 4.6×10^3 in a second sample. Write the total number of bacteria:

a as a basic numeral **b** in scientific notation.

1.5 **10** State the number of significant figures in each part of question **7**.

1.6 **11** Simplify the following surds.

a $\sqrt{128}$ **b** $\sqrt{1089}$

Review questions 1.7D: Mathematical modelling

1 Thy and Asha are playing a game. They are using dice rolls and coin flips to generate the product of three numbers in exponent form per round. They each roll the dice to determine the value of the bases and exponents and flip the coin to determine if each exponent is positive or negative.

The products generated after each round are multiplied together and simplified with the goal to end up with the least number of remaining factors after three rounds.

The table below shows the numbers Thy and Asha got in their three rounds.

	Thy	Asha
Round 1	$2^4 \times 4^3 \times 5^{-3}$	$1^3 \times 3^6 \times 5^{-5}$
Round 2	$3^2 \times 5^6 \times 6^{-3}$	$2^{-3} \times 3^1 \times 6^4$
Round 3	$2^{-6} \times 2^3 \times 3^4$	$3^{-5} \times 4^2 \times 5^3$

a Use the facts that $4 = 2^2$ and $6 = 2 \times 3$ to write Thy and Asha's round 1, 2 and 3 numbers in exponent form with positive exponents using only the bases 2, 3 and 5.

b Determine Thy and Asha's final number for their game by multiplying their round 1, 2 and 3 numbers together and simplifying the products in exponent form with positive exponents.

c Who won the game with the least number of factors? Hint: Find the sum of the positive exponents.

d Did the winner have the smaller value? Explain.

e Thy and Asha decide to play one more round of the game.
What products do Thy and Asha each need to generate to end up with a total product of 1 respectively?

f Thy and Asha decide to change the rules so that they can choose which base gets which exponent. Their products from the first two rounds are given in the table below.

	Thy	Asha
Round 1	$4^{-3} \times 5^4 \times 6^5$	$2^{-3} \times 5^{-2} \times 6^4$
Round 2	$1^5 \times 2^3 \times 3^{-4}$	$3^{-3} \times 4^4 \times 5^2$

Determine the product of round 1 and 2 for Thy and Asha respectively. Write the products in exponent form.

g For round 3:

- Thy gets the bases 2, 4 and 6 and the exponents -6, -4 and 1.
- Asha gets the bases 1, 5 and 5 and the exponents -4, 3 and 6.

i Determine which exponent should go with which base so that Thy and Asha each get the minimum number of factors remaining for the game.

ii Who wins this game and by how many factors?

2 Consider powers with prime number bases.

a Complete the following table.

Base (n)	n^2	n^3	n^4	n^5	n^6	n^7
3						
5						
7						
11						
13						
17						
19						

b Describe the patterns in the final digits for the different powers of each prime number in part **a**.

c Predict the last digit of n^{100} for each of the prime numbers in part **a**.

d Use the patterns you observed in part **a** to predict the last digit of $(a \times b)^{100}$ for the following products.

i 3×5 **ii** 3×7 **iii** 3×11 **iv** 7×19

v 13×17 **vi** $3^2 \times 7$ **vii** $13^2 \times 17$

3 In June 2022, the population of each Australian state was recorded. The figure for each state is shown in the table.

State	Population at 30 June 2022 ('000)
NSW	8153.6
Vic	6613.7
Qld	5322.1
SA	1820.5
WA	2785.3
Tas	571.5
NT	250.6
ACT	456.7

Source: ABS

a Which states and territories have a population listed to:
  i four significant figures
  ii five significant figures?

b Copy the table and add three additional columns.

c In the first new column, write the population of each state and territory in full.

d In the second new column, round each population to its leading digit.

e In the third new column, write each population in scientific notation to one significant figure.

f Use your answers from part **d** to determine the following. Write the values in scientific notation.
  i Which state or territory has the highest population?
  ii Which state or territory has the lowest population?
  iii Calculate the difference between the highest and the lowest population.
  iv Calculate the total population of SA, Tas, ACT and NT combined.
  v Calculate the total population of Australia.

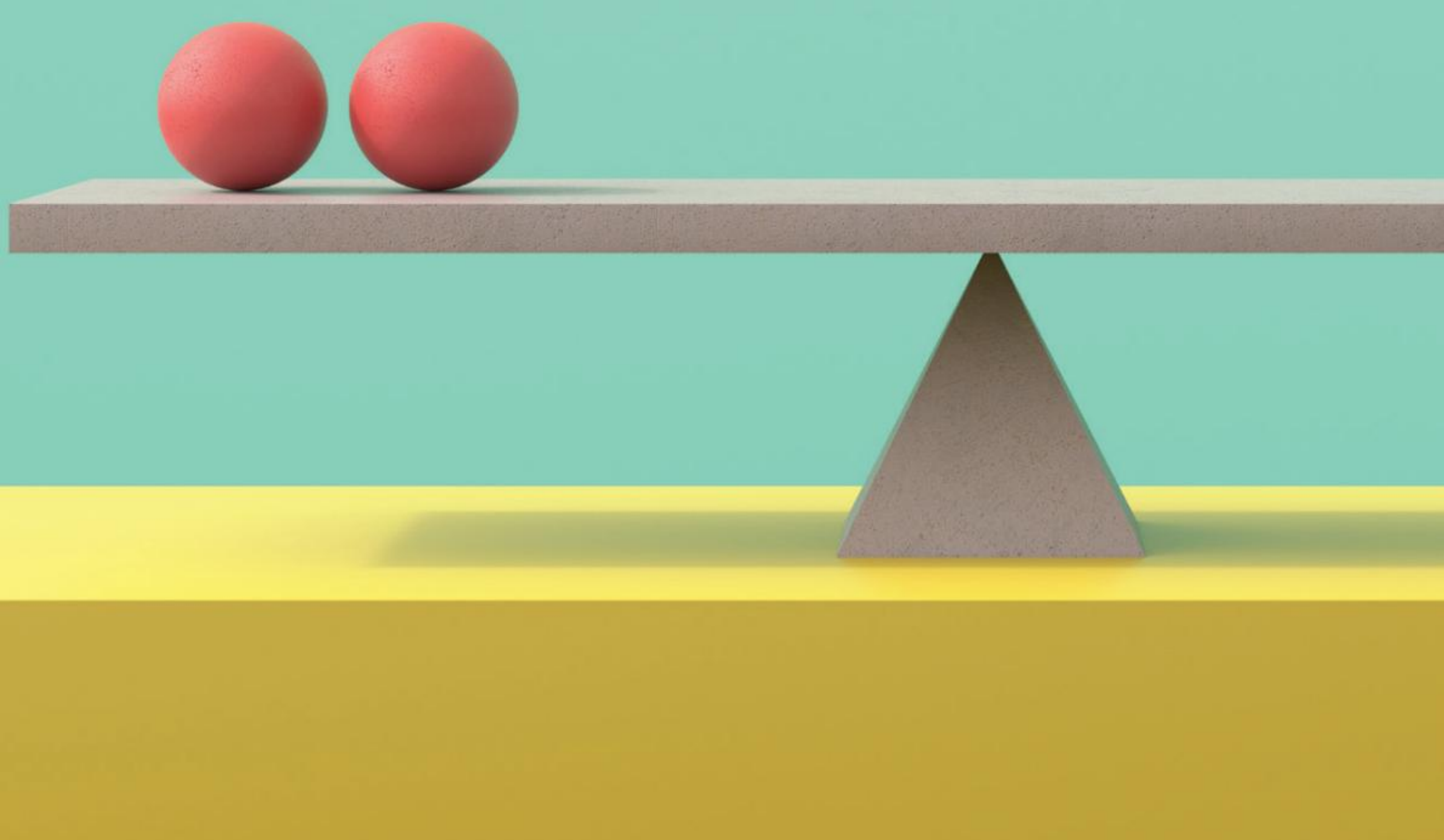
g The actual total population value recorded at the end of June 2022 was 25 978 935. Calculate the difference between your answer to **f** part **v** and the actual value. Why is there a difference?

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Convert terms between exponent and expanded form ☐ Evaluate numbers in exponent form ☐ Express integers as a product of prime factors	☐ Go back to Lesson 1.1 Exponents
☐ Simplify products of terms in exponent form with the same base ☐ Simplify quotients of terms in exponent form with the same base	☐ Go back to Lesson 1.2 Products and quotients of powers
☐ Raise a term in exponent form to another exponent ☐ Simplify expressions and evaluate calculations involving the zero exponent	☐ Go back to Lesson 1.3 Raising exponents and the zero exponent
☐ Write a term with a negative exponent as a term with a positive exponent ☐ Write a term with a positive exponent as a term with a negative exponent ☐ Apply exponent laws to simplify and evaluate numerical and algebraic expressions with negative exponents	☐ Go back to Lesson 1.4 Negative exponents
☐ Convert between scientific notation and basic numerals ☐ Identify significant figures in a number ☐ Round a number to a given number of significant figures	☐ Go back to Lesson 1.5 Scientific notation
☐ Determine whether a number is rational or irrational ☐ Determine whether a root is a surd or not ☐ Simplify surds ☐ Write simplified surds as square roots	☐ Go back to Lesson 1.6 Rational and irrational numbers

2 Algebra



Lessons

2.1 Simplifying
2.2 Expanding
2.3 Factorising using the HCF
2.4 Factorising the difference of two squares
2.5 Factorising quadratic expressions

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Equivalent fractions
- ✔ Exponent laws
- ✔ Highest common factor
- ✔ Expressions and equations

Curriculum links

- Simplify algebraic expressions, apply the distributive law to expand algebraic expressions including binomial products, and factorise monic quadratic expressions (VC2M9A02)

Materials

- ✔ Calculator

Lesson 2.1

Simplifying

Learning intentions

By the end of this lesson you will be able to ...

✔ simplify algebraic expressions.

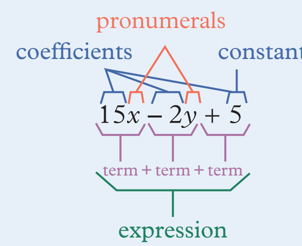
Inter-year links

Year 8	5.3 Adding and subtracting algebraic terms
Year 10	1.3 Simplifying

Terms and expressions

Key content video
Terms, expressions and equations

- A **pronumeral** is a letter or symbol that is used in place of a number. Pronumerals can be used to represent an **unknown** or a **variable**.
- An **expression** is a quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations.
- An **equation** consists of a pair of equivalent expressions, one on either side of an equal sign.
  For example, $15x - 2y + 5 = 0$ is an equation.
- A **term** is a single pronumeral, number or the product or quotient of numbers and pronumerals.
- A **coefficient** is the number multiplying any pronumerals in an algebraic term and is usually written first.
  → A term is said to be positive or negative depending on whether its coefficient is positive or negative.
- A **constant** is a term without any pronumerals. Constants can be considered as coefficients of a term where the pronumeral has a zero exponent.



Adding and subtracting algebraic terms

Key content video
Adding and subtracting algebraic terms

- **Like terms** contain the same pronumerals in **expanded form**.
  → The order of pronumerals can be different in two like terms.
  For example, xyz, $4yxz$, and $7zxy$ are all like terms.
  → You can write terms containing exponents in expanded form to determine whether they are like terms. For example, $a^2b = a \times a \times b$ and $ab^2 = a \times b \times b$, so a^2b and ab^2 are not like terms.a

$$\underbrace{3a^2b}_{\text{exponent form}} = \underbrace{3 \times a \times a \times b}_{\text{expanded form}}$$

- Like terms can be added or subtracted by adding or subtracting the coefficients of the terms.
  For example, $a^2b + 2a^2b = 1a^2b + 2a^2b = 3a^2b$.

Multiplying algebraic terms

Key content video
Multiplying algebraic terms

- To multiply algebraic terms:
  1. Write the coefficients and pronumerals for each term in expanded form, without expanding exponent form.
  2. Multiply the coefficients together.
  3. Apply the product of powers law to multiply the pronumerals in exponent form. Keep the base and add the exponents.
  4. Simplify by leaving out the multiplication signs. Write the coefficient first, followed by the pronumerals listed in alphabetical order.

$$\begin{aligned} 3a^3b \times (-2a^2b^2) &= 3 \times (-2) \times a^{3+2} \times b^{1+2} \\ &= -6a^5b^3 \end{aligned}$$

Dividing algebraic terms

Key content video
Dividing algebraic terms

- Remember that quotients can be expressed as fractions.
  For example, $x \div 7 = \frac{x}{7}$
- To divide algebraic terms:
  1. Write the coefficients and pronumerals for each term in expanded form, without expanding exponent form.
  2. Divide the coefficients by the HCF.
  3. Apply the quotient of powers law for the division of pronumerals in exponent form. Keep the base and subtract the exponents.
  4. Write the coefficient first, followed by the pronumerals listed in alphabetical order.

$$\frac{6a^5b^3}{3a^3b} = \frac{\cancel{6}^2 \times a^5 \times b^3}{\cancel{3}^1 \times a^3 \times b}$$
$$= 2 \times a^{5-3} \times b^{3-1}$$
$$= 2a^2b^2$$

Worked example 2.1A Adding and subtracting algebraic terms

Simplify each expression where possible.

a $10a - 6a + a$

b $4xy + 2x - 5xy$

c $-5bc + 2b - 3cb - \frac{b}{2}$

d $6x^2y - 3y^3 - 2y^2x + y^3$

THINK	WRITE
a Identify like terms and simplify by adding and subtracting the coefficients.	**a** $10a - 6a + a = (10 - 6 + 1)a$ $= 5a$
b **1** Rearrange the expression so that like terms are grouped together. **2** Simplify by adding and subtracting the coefficients.	**b** $4xy + 2x - 5xy = 2x + 4xy - 5xy$ $= 2x + (4 - 5)xy$ $= 2x - xy$
c **1** Rearrange the expression so that like terms are grouped together. Check that the + or − sign in front of each term has moved with that term. **2** Simplify by adding and subtracting the coefficients. Remember that fractional coefficients can be written in two ways, so $\frac{b}{2} = \frac{1}{2}b$.	**c** $-5bc + 2b - 3cb - \frac{b}{2} = -5bc - 3cb + 2b - \frac{b}{2}$ $= (-5 - 3)bc + \left(2 - \frac{1}{2}\right)b$ $= -8bc + \frac{3}{2}b$
d **1** Rearrange the expression so that like terms are grouped together. Check that the + or − sign in front of each term has moved with that term. Note that xy^2 and x^2y are not like terms. **2** Simplify by adding and subtracting the coefficients. Rearrange pronumerals in alphabetical order.	**d** $6x^2y - 3y^3 - 2y^2x + y^3 = 6x^2y - 3y^3 + y^3 - 2y^2x$ $= 6x^2y + (-3 + 1)y^3 - 2xy^2$ $= 6x^2y - 2y^3 - 2xy^2$

Worked example 2.1B Multiplying algebraic terms

Simplify the following products.

a $4de \times 7ab$ **b** $5x^2y^5 \times (-2kwxy^3)$

THINK	WRITE
a 1 Write in expanded form.	**a** $4de \times 7ab = 4 \times d \times e \times 7 \times a \times b$
2 Multiply the coefficients together.	$= 28 \times d \times e \times a \times b$
3 Simplify by leaving out the multiplication signs. Write the pronumerals in alphabetical order.	$= 28abde$
b 1 Write the coefficients and pronumerals for each term in expanded form, without expanding exponent form.	**b** $5x^2y^5 \times (-2kwxy^3)$ $= 5 \times x^2 \times y^5 \times (-2) \times k \times w \times x \times y^3$
2 Multiply the coefficients together.	$= -10 \times x^{2+1} \times y^{5+3} \times k \times w$
3 Apply the product of powers law to simplify pronumeral products by adding exponents with the same base, so $x^2 \times x = x^{2+1}$ and $y^5 \times y^3 = y^{5+3}$.	$= -10 \times x^3 \times y^8 \times k \times w$
4 Simplify by leaving out the multiplication signs.	$= -10kwx^3y^8$

Worked example 2.1C Dividing algebraic terms

Simplify the following quotients.

a $\dfrac{-15xy}{10x}$ **b** $\dfrac{8a^3b}{2a^2b^2}$

THINK	WRITE
a 1 Write in expanded form.	**a** $\dfrac{-15xy}{10x} = \dfrac{-15 \times x \times y}{10 \times x}$
2 Cancel the coefficients, and divide −15 and 10 by the HCF of 5. Cancel any common pronumerals from the numerator and denominator.	$= \dfrac{-\cancel{15}^{3} \times x^{1} \times y}{\cancel{10}^{2} \times x^{1}}$
3 Simplify the numerator and the denominator.	$= -\dfrac{3y}{2}$
b 1 Write the coefficients and pronumerals for each term in expanded form, without expanding exponent form.	**b** $\dfrac{8a^3b}{2a^2b^2} = \dfrac{8 \times a^3 \times b}{2 \times a^2 \times b^2}$
2 Divide each coefficient by the HCF. Write the rest of the expression as a product of fractions, each with only one pronumeral.	$= \dfrac{8^{4}}{2^{1}} \times \dfrac{a^3}{a^2} \times \dfrac{b}{b^2}$
3 Apply the quotient of powers law to simplify pronumeral quotients. Keep the base and subtract the exponents.	$= 4 \times a^{3-2} \times b^{1-2}$ $= 4ab^{-1}$
4 Use $a^{-m} = \dfrac{1}{a^m}$ to write the pronumeral as a fraction with a positive exponent.	$= \dfrac{4a}{b}$

Helpful hints

- ✔ Recall the rules for writing algebraic notation.
  - → Products are simplified by leaving out the multiplication sign and placing the coefficient first. For example: $7 \times x = 7x$ and $7 \times (x + 2) = 7(x + 2)$
  - → When a pronumeral is multiplied by 1, the 1 is not shown. For example: $1 \times x = x$
  - → Quotients are represented by fractions. For example: $x \div 7 = \frac{x}{7}$ and $(x + 2) \div 7 = \frac{x + 2}{7}$
  - → Terms with fractional coefficients can be written in two ways. For example: $\frac{x}{7} = \frac{1}{7}x$
- ✔ Write pronumerals in a term in alphabetical order to help identify like terms correctly. For example: $ba^2c = a^2bc$
- ✔ Recall the rules for multiplying and dividing positive and negative numbers.

$$+ \times - = - \qquad + \times + = +$$
$$- \times + = - \qquad - \times - = +$$

- ✔ Take care not to mix up the exponent laws and definitions.
  - → Across a multiplication sign, add exponents.
  - → Across a division sign, subtract exponents.
  - → Across brackets, multiply exponents.
  - → Zero exponent: $a^0 = 1$
  - → Negative exponent: $a^{-1} = \frac{1}{a}$

Exponent law	Example
Product of powers law	$a^5 \times a^3 = a^{5+3}$
Quotient of powers law	$a^5 \div a^3 = a^{5-3}$
Power of a power law	$(a^5)^3 = a^{5 \times 3}$

Learning pathways

△ 1, 2, 3–4(1st column), 5, 6–8(1st column), 9–12, 13(1st column), 14–16, 21

□ 3–4(2nd column), 5, 6–8(2nd column), 9, 10, 13(2nd column), 14(a, c, f), 17–20, 22, 23

⬠ 5, 6(3rd column), 7, 8(3rd column), 9, 10, 13(3rd column), 14(a, c, f), 17–20, 22–24

ANS p456

Exercise 2.1A: Understanding and fluency

1 Consider these terms: $3x$, $7xy$, $-x$, $2x^2$, xw, $20x$

- **a** Which are like terms?
- **b** Explain how you can tell.
- **c** What is the coefficient of each term?

2 List each group of like terms from these terms: $2ba^2$, $3a$, $2b^2a$, $6a^3$, aaa, $6a$, $3aab$, $6ab^2$, $6a^2a$, $6a^2b$, $3abb$

WE 2.1A 3 Simplify each expression where possible.

- **a** $6a - 4a + 8a$
- **b** $4k - 5k - 7k$
- **c** $x^2 + 3x^2 + 2x^2$
- **d** $3cd + cd - 9cde$
- **e** $3x + 4y + 9x + 2y$
- **f** $7a + 5b - 3a + b$
- **g** $m - 2p + 4p + 8m$
- **h** $3 + 5k - 2 - 6k$
- **i** $4xy + 3x^2 - xy + 2x^2$
- **j** $d + de^2 + d - 5de^2$
- **k** $5m^3 + 7 - m^3 - 5$
- **l** $abc + ab + ac + 3ab$

4 Simplify each expression.

- **a** $6x + 3y - x + 2y + 5x - 4y$
- **b** $8ab - 4b - b + b^2 + a - 3ab$
- **c** $2k + 3km - 6k + 4 + 4k - km$
- **d** $4x^2 - 7x^2 - 3x + 5 + 6x - 9$
- **e** $9a - 4a^2 + a^3 + 5a^2 - 3 - 7a$
- **f** $m^2n + 3m^2 + 5nm^2 - 2n^2 + 4mn^2 - 3m^2$

5 Simplify each expression by first using exponent laws.

a $7a^2 + 5(3a)^2 + (-5a)^2 - 9aa$

b $-5bbb + 40b^2b - (2b)^3 + 2(-3b)^3$

c $9c^2d^2 - (6cd)^2 + 15(cd)^2 + 9d^2c^2$

d $r(pq)^2 + \frac{q^2}{(pr)^{-1}} + ppqqr + p^2q^2r + pq^2r$

e $\frac{2}{f^{-1}} + \frac{3}{e^{-1}} + \frac{4}{(ef)^0} + \frac{5}{e^{-1}} + \frac{6}{f^{-1}}$

f $\frac{4y}{x^{-2}} + \frac{1}{(3xy)^{-1}} - \frac{2x}{y^{-1}} + \frac{1}{2x^{-1}y^{-1}} + \frac{3x^2}{y^{-1}}$

WE 2.1B 6 Simplify each expression.

a $2ab \times 3cd$

b $-5xy \times 4mp$

c $9gh \times g$

d $4km \times (-6kn)$

e $7jp \times 8bpt$

f $-x^2y \times (-ay)$

g $6a^2b \times 3acd$

h $-10hk \times 2hkp$

i $3b \times (-2b) \times b$

j $m^2n \times 4n \times kn$

k $-5xy \times x^2 \times (-3xy)$

l $8abc \times 7a^3c \times b^2$

7 Simplify each expression by first using exponent laws.

a $(5a^3z^8)^2 \times 3(a^3z^5)^4$

b $(-3by^4)^3 \times (-2yb^7)^4$

c $5(cmx)^5 \times c(-2mx)^3 \times mx(-2c)^2$

d $4(d^3n^{-2}w)^5 \times 11(d^{-2}n^5w^{-3})^4$

e $(5e^3p^{12}t^5)^{-1} \times (e^{10}p^4t^2)^0 \times (-3e^4p^6t^7)^{-2}$

f $-6(g^{-2}q^5u^3)^5 \times 4(-g^4q^{-4}u^5)^{-3} \times -7(g^{-2}qu^7)^{-4}$

WE 2.1C 8 Simplify each expression.

a $\frac{abc}{ac}$

b $\frac{kmn}{kp}$

c $\frac{12cd}{3d}$

d $\frac{-16w}{8xw}$

e $\frac{7ef}{-14ef}$

f $\frac{-5xy}{-20x}$

g $\frac{18ab}{15ac}$

h $\frac{4mn}{22mn}$

i $\frac{18a^2}{3a}$

j $\frac{6x^2y}{-2x}$

k $\frac{15mn^2}{9m}$

l $\frac{-3a^2bc}{12ab}$

9 Simplify each expression using exponent laws.

a $\left(\frac{12d^8}{48d^3}\right)^{-1}$

b $\left(\frac{c^3d^{-4}}{c^{-5}d^{-9}}\right)^{-3}$

c $\frac{(-7u^6r^{-5})^{-2}}{(-5u^5r^{-2})^{-3}}$

d $\frac{(2k^{-3}p^4)^5}{18(k^{-3}p^6)^{-3}}$

e $\frac{1.6xy^{-4}z^0}{0.8(x^2z^3)^{-4}(y^{-2}z^3)^5}$

f $\frac{4.4f^{-8}}{1.1(f^5)^2} \times \frac{0.35(f^{-4})^{-3}}{0.07f^{-7}}$

10 Simplify each expression. Hint: First write each as an algebraic fraction.

a $m^4p^3q \div m^4$

b $-2a^8de^7 \div (a^8e^7)$

c $6ax \div (-2ac)$

d $-5km \div (-10mp)$

e $7a^2bc \div (abd)^2$

f $3mn^2w^2 \div (9n^3w)^2$

g $-12x^2y \div [8(xyz)^3]$

h $-ab^3cd \div (-2abc^2)^{-4}$

i $8km^2n \div [-12(k^2mn)^{-2}]$

11 Determine whether each statement is true or false.

a Two like terms can be added to form one new term.

b Any term can be subtracted from another term to form one new term.

c Two terms can be multiplied to form one new term only if they are like terms.

d Any term can be divided by another term to form one new term.

Exercise 2.1B: Problem solving and reasoning

12 Students in a class were asked to simplify three algebraic expressions. Three sets of working for each expression are shown below. One set is correct and the other two sets contain errors.

For each expression, choose the correct set and then identify the errors in the other two sets of working.

a **Expression 1:** $4a - 3b + 2 + 2a + 8b - 7$

Set A	**Set B**	**Set C**
$4a - 3b + 2 + 2a + 8b - 7$	$4a - 3b + 2 + 2a + 8b - 7$	$4a - 3b + 2 + 2a + 8b - 7$
$= 4a + 2a + 2 + 7 + 3b + 8b$	$= 4a + 2a + 2 - 7 - 3b + 8b$	$= 4a - 2a + 2 - 7 - 3b + 8b$
$= 6a + 9 + 11b$	$= 6a - 5 + 5b$	$= 2a + 9 + 5b$

b **Expression 2:** $-3ab \times 4bc$

Set A	Set B	Set C
$-3ab \times 4bc$ $= -3 \times 4 \times a \times b^{1+1} \times c$ $= -12ab^2c$	$-3ab \times 4bc$ $= -3 \times 4 \times a \times b^{1+1} \times c$ $= 12 \times a \times b^2 \times c$ $= 12abc$	$-3ab \times 4bc$ $= -3 \times 4 \times a \times b^{1+1} \times c$ $= -12 \times a \times b \times 2 \times c$ $= -24abc$

c **Expression 3:** $4a^2bc \div (8abd)$

Set A	Set B	Set C
$4a^2bc \div (8abd)$ $= \frac{4a^2bc}{8abd}$ $= \frac{4 \times a^2 \times b \times c}{8 \times a \times b \times d}$ $= \frac{4^{\color{red}1}}{8^{\color{red}2}} \times \frac{a^2}{a} \times \frac{b}{b} \times c \times \frac{1}{d}$ $= \frac{1}{2} \times a^{2-1} \times 1 \times c \times \frac{1}{d}$ $= 2acd$	$4a^2bc \div (8abd)$ $= \frac{4a^2bc}{8abd}$ $= \frac{4 \times a^2 \times b \times c}{8 \times a \times b \times d}$ $= \frac{4^{\color{red}1}}{8^{\color{red}2}} \times \frac{a^2}{a} \times \frac{b}{b} \times c \times \frac{1}{d}$ $= \frac{1}{2} \times 1 \times 1 \times c \times \frac{1}{d}$ $= \frac{c}{2d}$	$4a^2bc \div (8abd)$ $= \frac{4a^2bc}{8abd}$ $= \frac{4 \times a^2 \times b \times c}{8 \times a \times b \times d}$ $= \frac{4^{\color{red}1}}{8^{\color{red}2}} \times \frac{a^2}{a} \times \frac{b}{b} \times c \times \frac{1}{d}$ $= \frac{1}{2} \times a^{2-1} \times 1 \times c \times \frac{1}{d}$ $= \frac{ac}{2d}$

13 Simplify and then evaluate each expression using $x = 3$ and $y = 2$.

a $5x - 6y + 7y + 3x$ **b** $3xy - 8xy - xy$ **c** $5x \times 3y$

d $\frac{10x}{xy}$ **e** $xy \times xy^2$ **f** $6x^2y \div (2xy)$

g $x + y - 2xy + 5y - x$ **h** $x \times 3x - 2x^2 + 4x - y$ **i** $xy^{12} \div (x^4y^8)$

14 Simplify and then evaluate each expression using $a = 2$, $b = -1$ and $c = 5$.

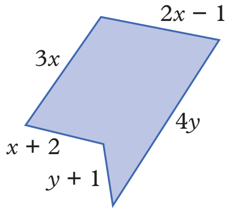
a $3a + 2b + 7c - a - 5c + b$ **b** $7ab + 4a - 5a + ab$

c $a^2b + ab^2 + ac - 3a^2b + 2ac$ **d** $2abc \times bc \times 5a$

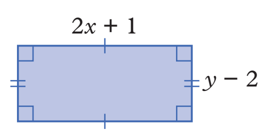
e $18ab^2c \div (6bc)$ **f** $3ac^2 \times 4ab \div (9abc)$

15 Write the perimeter of each shape as an algebraic expression in simplest form.

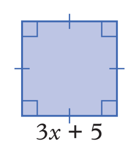
a



b



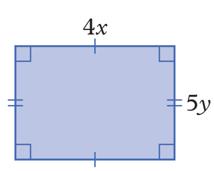
c



16 Calculate the perimeter of each shape in question **15** for $x = 4$ cm and $y = 5$ cm.

17 Write the shaded area in each shape as an algebraic expression in simplest form.

a



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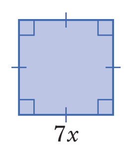

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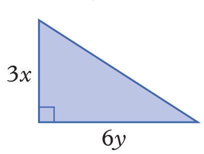
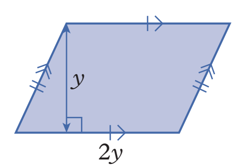


d



18 Calculate the area shaded in each shape in question **17** for $x = 3$ m and $y = 2$ m.

19 The area of a rectangle is $16xy$.

a If the length is $8x$, write an expression for the width of the rectangle.

b If the length of the rectangle is 16 m and y is 5 m, calculate the width and the area of the rectangle using the result from part **a**.

20 The area of a right-angled triangle is $6x^2$.

a If the base length is $4x$, write an expression for the height of the triangle.

b If the height of the triangle is 12 cm, calculate the area and the base length of the triangle using the result from part **a**.

21 A rectangle has a width of k.

a If the length of the rectangle is twice the width, write an expression for:

i the perimeter of the rectangle

ii the area of the rectangle.

b Calculate the perimeter and the area of the rectangle when $k = 5$ cm.

22 Lana plants a 1-metre-wide flowerbed around a square section of lawn.

a If the lawn has a length of x metres, write an expression for:

i the perimeter of the lawn

ii the area of the lawn

iii the perimeter around the outer edge of the flowerbed

iv the area of the flowerbed, given that the total area of the lawn and flowerbed is $(x^2 + 4x + 4)$ m^2.

b When $x = 8$, calculate:

i the area of the flowerbed

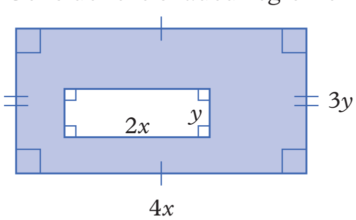
ii the length of edging needed around the inner edge of the flowerbed

iii the length of edging needed around the outer edge of the flowerbed

iv the area to be mown.

Exercise 2.1C: Challenge

23 Consider the shaded region of this shape.



a Write an algebraic expression for the area of the shaded region.

b If $x = 4$ cm and $y = 5$ cm, calculate the area of the shaded region.

c Write an expression for the total length of the outer and inner edges of the shape.

d Use the values from part **b** to calculate the total length of the outer and inner edges of the shape.

e If $x = 12$ cm and the area of the shaded region is 210 cm^2, determine the total length of the outer and inner edges of the shape.

24 Write $((2x)^7 \times (4x)^{-3} \times (8x)^3 \div (16x)^4)^8$ in the form $2^{\square}x^{\square}$.

Online resources:

Interactive skillsheet Like terms	Interactive skillsheet Multiplying terms	Interactive skillsheet Dividing terms	Worksheet Simplifying algebraic expressions	Quick quiz 2.1

Lesson 2.2

Expanding

Learning intentions

By the end of this lesson you will be able to ...

✔ use the distributive law to expand algebraic expressions.

Inter-year links

Year 8 5.6 Expanding
Year 10 1.4 Expanding

The distributive law

Key content video
Expanding algebraic expressions

Key content video
Expanding binomial products

- A **binomial** is an expression containing two terms that are either added or subtracted.
  For example, $x - 7$, $2x + y$ and $x^2 + y^2$ are all binomials.
- A **trinomial** is an expression containing three terms that are either added or subtracted. For example, $x - 7 + y$, $2x^2 + 2y + y$ and $x^2 + 2xy + y^2$ are all trinomials.
- The **distributive law** is used to distribute or **expand** products over addition and subtraction.

$$a(b + c) = ab + ac$$

$$a(b + c + d) = ab + ac + ad$$

  Note: The distributive law only applies if the expression inside the brackets is not raised to the power of any exponent (other than 1). For example, in $a(b + c)^2$, $(b + c)^2$ needs to be expanded first to remove the exponent before being multiplied by a.
- To expand algebraic expressions with one pair of brackets:
  1. Multiply each term inside the brackets by the term outside the brackets.
  2. Simplify the results by performing any multiplication, addition and subtraction.
- The distributive law for expanding over a **binomial product** is: $(a + b)(c + d) = ac + ad + bc + bd$

$$(a + b)(c + d) = ac + ad + bc + bd$$

- To expand binomial products:
  1. Multiply each term inside the second pair of brackets, c and d, by each term in the first pair of brackets, a and b.
  2. Simplify the results by performing any multiplication, addition and subtraction.
- Where possible, expanded expressions should be simplified.
- The distributive law for expanding a binomial product can be derived by treating the first binomial as a single term and then applying the distributive law for expanding over one pair of brackets.

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= c(a + b) + d(a + b) \\ &= ca + cb + da + db \\ &= ac + ad + bc + bd\end{aligned}$$

Worked example 2.2A Expanding over one pair of brackets

Expand and simplify each expression using the distributive law.

a $-5(3a + 8)$ **b** $-3a(5a - 4b + 3c)$

THINK	WRITE
a 1 Multiply each term inside the brackets by the term outside the brackets.	**a** $-5(3a + 8) = -5 \times 3a + (-5) \times 8$
2 Simplify the results by performing the multiplications. Take care with + and − signs when simplifying.	$= -15a - 40$
b 1 Multiply each term inside the brackets by the term outside the brackets. Enclose negative terms in brackets to avoid errors.	**b** $-3a(5a - 4b + 3c) = -3a \times 5a + (-3a) \times (-4b) + (-3a) \times (3c)$
2 Simplify the results by performing the multiplications. Remember the product of two negative numbers gives a positive number.	$= -15a^2 + 12ab - 9ac$

Worked example 2.2B Expanding binomial products

Expand each algebraic expression to remove the brackets.

a $(a + 9)(b + 2)$ **b** $(x + 4)(x - 6)$

THINK	WRITE
a 1 Multiply each term inside the second pair of brackets, b and 2, by the first term in the first pair of brackets, a, and then the second term in the first pair of brackets, 9.	**a** $(a + 9)(b + 2) = a \times b + a \times 2 + 9 \times b + 9 \times 2$
2 Simplify each term.	$= ab + 2a + 9b + 18$
b 1 Multiply each term inside the second pair of brackets, x and -6, by the first term in the first pair of brackets, x, and then the second term in the first pair of brackets, 4.	**b** $(x + 4)(x - 6) = x \times x + x \times (-6) + 4 \times x + 4 \times (-6)$
2 Simplify each term.	$= x^2 - 6x + 4x - 24$
3 Simplify the resulting expression.	$= x^2 - 2x - 24$

Helpful hints

- ✔ When multiplying by a negative term, use brackets for clarity.
  For example: $-7p(3q - 4) = -7p \times 3q + (-7p) \times (-4)$
- ✔ Show all your working to avoid arithmetical errors and to ensure all signs are correct.
- ✔ After expanding expressions containing brackets, always simplify your results by looking for like terms. Simplify, simplify, simplify!
- ✔ When expanding a binomial, remember to multiply every term in one bracket by every term in the other bracket.

$$(a + b)(c + d) = ac + ad + bc + bd$$

Learning pathways

△ 1–8(1st column), 9–15, 17, 19, 21	□ 1–8(2nd column), 11–16, 18, 20, 22, 24	⬠ 1–8(3rd column), 11, 15, 16, 19–25

ANS p456

Exercise 2.2A: Understanding and fluency

WE 2.2A **1** Expand each algebraic expression to remove the brackets.

a $4(a + 3)$ **b** $7(b + 5)$ **c** $3(c - 2)$
d $5(d - 1)$ **e** $6(4 + e)$ **f** $-2(f + 8)$
g $-3(g + 4)$ **h** $-8(h - 5)$ **i** $-4(x - 9)$
j $-5(2 - j)$ **k** $k(p + 6)$ **l** $a(b - 4)$
m $6(3m + k)$ **n** $n(2p + 4q)$ **o** $x(x - 7y)$

2 Expand and simplify each algebraic expression.

a $2(2ab + 4bc + 6ac)$ **b** $-3(xy + x + y)$ **c** $-(m^2 - n^2 + 2m^2n^2)$
d $a^2(a^2 - b^2 + 2ab)$ **e** $-\frac{1}{4}\left(4pq - 16p + 32q\right)$ **f** $\dfrac{4abc(4b + 2a + 8c)}{8}$

3 Expand and simplify each algebraic expression.

a $3(x + 2) + 8x$ **b** $11 + 5(p - 1)$ **c** $a(b + 4) - 2a$
d $-7(1 - y) + 4y + 3$ **e** $5k + 2 + 4(h - k)$ **f** $m(m - 6) - m^2$

4 Expand and simplify each algebraic expression.

a $2(x + 5) + 3(x - 6)$ **b** $8(k - 3) + 5(k + 4)$ **c** $3(p + 7) - 4(5 - p)$
d $x(x + 1) + 3(x + 4)$ **e** $m(m + 2) + 3(m + 2)$ **f** $y(y - 5) - 2(y - 5)$

5 Expand and simplify each expression.

a $3a(4z + 5)$ **b** $8b(7 + 5y)$ **c** $2c(7 - 3x)$
d $-5d(4w + 5)$ **e** $-6e(8v - 9)$ **f** $-10e(4t + 3u)$
g $4g(2g - 7t)$ **h** $-6h(5r - 7h)$ **i** $3ij(4ik + 5jk)$

WE 2.2B **6** Expand each algebraic expression to remove the brackets.

a $(a + 3)(b + 4)$ **b** $(c + 2)(d + 7)$ **c** $(m + 5)(n + 1)$
d $(j - 9)(k - 5)$ **e** $(2a + 7)(b + 3)$ **f** $(5c + 2)(3d - 4)$

7 Expand each algebraic expression to remove the brackets.

a $(a + 2)(a + 3)$	**b** $(x + 5)(x + 10)$	**c** $(d + 4)(d - 6)$
d $(y + 3)(y - 8)$	**e** $(k - 7)(k + 9)$	**f** $(m - 6)(m + 3)$
g $(5e - 2)(e - 4)$	**h** $(7a - 8)(3a - 1)$	**i** $(3y - 5)(2y - 1)$

8 Expand each algebraic expression to remove the brackets.

a $(5 - a)(b + 4)$	**b** $(x + 6)(3 - x)$	**c** $(2c + 3)(5 - 4c)$
d $(11d - 9e)(4d - 6e)$	**e** $(6p + 5q)(8q - 7p)$	**f** $(x + 2)(x - 3) + 3x^2 - 8x + 5$
g $(x^3 + x^2)(x^5 + x)$	**h** $(4x^2 + 7)(12 - 5x^2)$	**i** $(x + 2)(x + 3) + (x + 1)(x + 5)$

Exercise 2.2B: Problem solving and reasoning

9 Expand each algebraic expression to remove the brackets.

a $(a + 3)(a - 3)$	**b** $(b + 2)(b - 2)$	**c** $(c + 5)(c - 5)$
d $(3p - 8)(3p + 8)$	**e** $(h - 1)(h + 1)$	**f** $(k + m)(k - m)$

10 Describe the pattern or shortcut you can see in question **9**. What is special about the two binomial factors that are multiplied together?

11 The pattern you observed in question **10** is known as the difference of two squares. This rule can be written as: $(a + b)(a - b) = a^2 - b^2$

a Why do you think the rule is called the difference of two squares?

b Does it matter whether the product is $(a + b)(a - b)$ or $(a - b)(a + b)$? Explain.

c Use the rule (or shortcut) to expand each algebraic expression.

i $(x - 2)(x + 2)$	**ii** $(y + 9)(y - 9)$	**iii** $(m + 6)(m - 6)$
iv $(3d - 10)(3d + 10)$	**v** $(m + n)(m - n)$	**vi** $(3 + x)(3 - x)$

12 The difference of two squares rule can be used to simplify numerical expressions.

a Use the difference of two squares rule to expand $(100 + 3)(100 - 3)$.

b Use your answer to work out the result for 103×97.

c Work out the result for each product without using a calculator.

i 102×98	**ii** 95×105
iii 1001×999	**iv** 994×1006

13 Expand each algebraic expression to remove the brackets and simplify.

a $(a + 2)(a + 2)$	**b** $(b + 7)(b + 7)$	**c** $(c + 4)(c + 4)$
d $(3p - 5)(3p - 5)$	**e** $(h - 1)(h - 1)$	**f** $(m + n)(m + n)$

14 Describe the pattern or shortcut you can see in question **13**. What is special about the two binomial factors that are multiplied together?

15 The pattern you observed in question **14** is known as the expansion of a perfect square. This rule can be written as: $(a + b)^2 = a^2 + 2ab + b^2$.

a Why do you think the rule is called the expansion of a perfect square?

b Use the rule (or shortcut) to expand each algebraic expression.

i $(x + 3)^2$	**ii** $(y + 6)^2$	**iii** $(m + 2)^2$
iv $(4b + 11)^2$	**v** $(m + n)^2$	**vi** $(5 + x)^2$

c Is $(a - b)^2 = a^2 - 2ab + b^2$ also an expansion of a perfect square? Explain.

d Expand each algebraic expression.

i $(a - 2)^2$	**ii** $(b - 4)^2$	**iii** $(c - 7)^2$
iv $(5w - 6)^2$	**v** $(k - p)^2$	**vi** $(3 - x)^2$

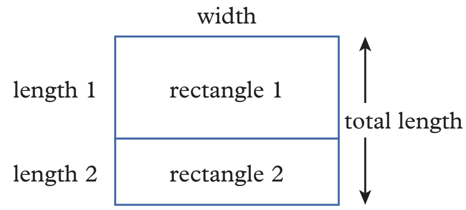
16 Expand each algebraic expression.

a $2x(3w - 5x + 7y - 4z)$	**b** $(3a + 5b - 6)(2x + 3y)$
c $(x + 3y + 6)(2x + y + 3)$	**d** $(x + 2)(x^2 + 6x + 7)$

17 The area of the large rectangle in this diagram can be determined in two ways:

Area of large rectangle = total length × width

Area of large rectangle = area of rectangle 1 + area of rectangle 2
= length 1 × width + length 2 × width



a For Diagram 1 and Diagram 2 below:

i write an expression for the area of the large rectangle by multiplying the total length by the width

ii write an expression for the area of the large rectangle by adding the areas of rectangle 1 and rectangle 2.

Diagram 1

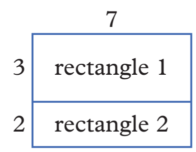
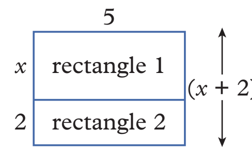
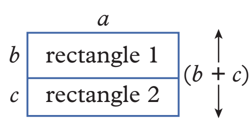


Diagram 2



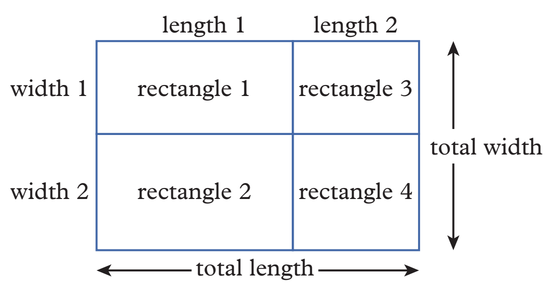
b Use the rectangle below to explain how $a(b + c)$ can be expanded to obtain $ab + ac$.



18 The area of the largest rectangle in this diagram can be determined in two ways:

Area of largest rectangle = total length × total width

Area of largest rectangle = area of rectangle 1 + area of rectangle 2 + area of rectangle 3 + area of rectangle 4
= width 1 × length 1 + width 2 × length 1 + width 1 × length 2 + width 2 × length 2



a For Diagram 1 and Diagram 2 below:

i write an expression for the area of the large rectangle by multiplying the total length by the total width

ii write an expression for the area of the large rectangle by adding the areas of rectangle 1, rectangle 2, rectangle 3 and rectangle 4.

Diagram 1

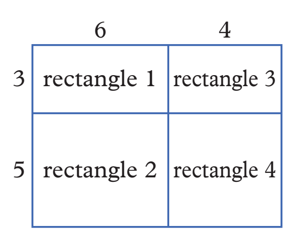
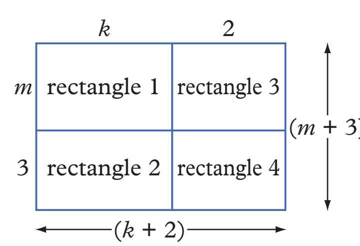
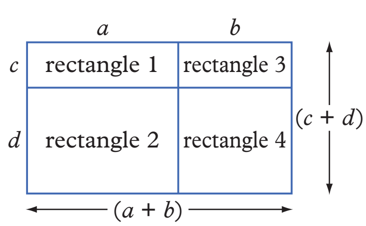


Diagram 2



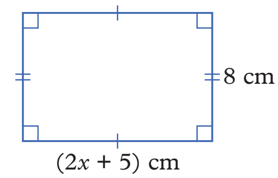
b Use this rectangle to explain how $(a + b)(c + d)$ can be expanded to obtain $ac + ad + bc + bd$.



19 Which of the following is not equivalent to $3(x + 2)(x + 8)$?

A $(3x + 6)(x + 8)$ **B** $(x + 2)(3x + 24)$ **C** $3(x^2 + 10x + 16)$ **D** $(3x + 6)(3x + 24)$

20 Consider the rectangle on the right.



a Write the area of the rectangle as a product of two factors.

b Expand the expression to remove brackets.

c When $x = 3$, calculate the area using your answer to:

i part **a** **ii** part **b**.

d Compare your answers in part **c**. How does this show that you have expanded the area expression correctly?

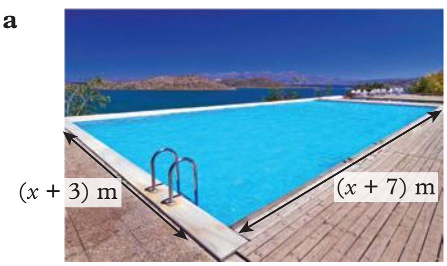
21 For each of the following rectangles:

i write the area as a binomial product

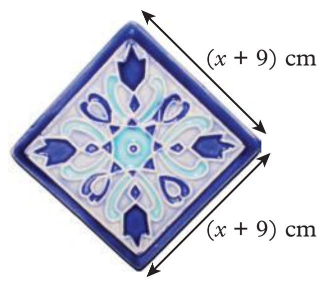
ii expand the binomial product to remove brackets

iii calculate the area when $x = 5$.

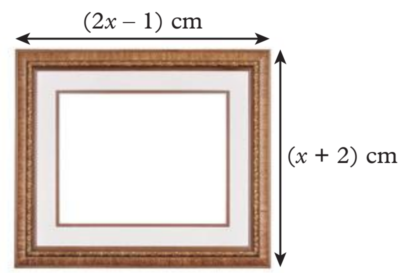
a



b



c



22 A rectangular trampoline is 6 m long and 4 m wide. Safety matting that is p metres wide is to be placed around the perimeter of the trampoline.

a Draw a labelled diagram of the top of the trampoline and the matting around it.

b Write an expression for:

i the total length of the trampoline and matting **ii** the total width of the trampoline and matting.

c Write an expression for the total area taken up by the trampoline and matting. Simplify the expression by expanding to remove any brackets.

d Write an expression for the area of the matting only.

e If $p = 2$, calculate the area taken up by:

i the trampoline **ii** the trampoline and matting **iii** the matting.

Exercise 2.2C: Challenge

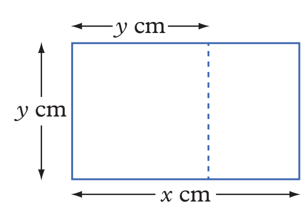
23 Expand and simplify each expression.

a $(10y - 7)^2$ **b** $x^4(x^3 - x^2)$ **c** $(x^2 - 5)(x^2 + 5)$

d $(y^6 + 3y)^2$ **e** $a^3(a^2 + 4) - a^2(a^3 + 9a)$ **f** $x^5y^2(xy^4 + 2w)$

24 Show that the expression $(a - b)^2 + (c - d)^2$ is equivalent to $(b - a)^2 + (d - c)^2$.

25 A rectangular piece of paper is x cm long and y cm wide. The paper is torn along a line parallel to its width, forming a square of side length y cm and another rectangle.



a Write the dimensions of the newly formed square and rectangle.

b Prove that the area of the original rectangle is the same as the total area of the two new shapes.

Online resources:

Interactive skillsheet Expanding over one pair of brackets

Interactive skillsheet Expanding binomial products

Worksheet Expanding

Investigation It's a mystery

CAS instructions Expanding

Quick quiz 2.2

Lesson 2.3

Factorising using the HCF

Learning intentions

By the end of this lesson you will be able to ...

✔ factorise algebraic expressions using a variety of techniques.

Inter-year links

Year 7	2.4 Factors and the highest common factor
Year 8	5.7 Factorising
Year 10	1.6 Factorising

The highest common factor (HCF) of an expression

Key content video
Factorising using the HCF

- The **highest common factor (HCF)** of two or more algebraic terms is the 'highest' or 'largest' combination of coefficients and pronumerals that are factors of every term.
- The HCF of two or more algebraic expressions can be another algebraic expression, like a binomial.
- To identify the HCF of an expression:
  1 Find the HCF of the coefficients.
  2 Find the HCF of the pronumerals.
  3 Multiply the factors from steps 1 and 2, and then simplify.

Algebraic expressions	HCF
$x + 7x$	x
$6m + 12mn + 8mp$	$2m$
$15a^3bc + 9a^2c$	$3a^2c$
$x(x - 1) + 2(x - 1)$	$(x - 1)$

Factorising algebraic expressions

- **Factorising** an algebraic expression is the process of writing an algebraic expression as a product of its factors. It is the opposite to expanding an algebraic expression.
- An expression is in fully **factorised form** if the HCF of the factors is 1.
- An expression is in expanded form if it has no brackets and is simplified.
- To factorise an expression:
  1 Identify the HCF of the expression.
  2 Write the HCF in front of the brackets. Inside the brackets, divide each term in the original expression by the HCF.
  3 Simplify the bracketed expression. Both the HCF and the bracketed expression are factors of the original expression.

$$7(a + 2) \underset{\text{factorising}}{\overset{\text{expanding}}{\rightleftarrows}} 7a + 14$$

$$\underbrace{2x^2 - 6x}_{\text{expanded form}} = \underbrace{2x(x - 3)}_{\text{factorised form}}$$

$$\begin{aligned} 7a + 14 &= 7\left(\frac{\cancel{7}^1 a}{\cancel{7}^1} + \frac{\cancel{14}^2}{\cancel{7}^1}\right) \\ &= 7(a + 2) \end{aligned}$$

Factorising by grouping terms

Key content video
Factorising by grouping terms

- To factorise an expression with four terms by grouping:
  1 Identify pairs of terms with common factors. Group the pairs of terms, remembering to move the positive or negative sign with the term.
  2 Factorise each pair of terms by dividing out the HCF.
  3 Factorise using the binomial factor and simplify the expression. Both bracketed expressions are factors of the original expression.

$$\begin{aligned} x^2 + 2x - 3x - 6 &= x(x + 2) - 3(x + 2) \\ &= (x + 2)(x - 3) \end{aligned}$$

Worked example 2.3A Identifying the HCF

Find the highest common factor (HCF) of each pair of terms.

a 24 and 84 **b** a^2b and b^2ac **c** $12x^2y^3$ and $-18x^3y$

THINK	WRITE
a 1 Write the prime factorisation for each number.	**a** $24 = 2 \times 2 \times 2 \times 3$ $84 = 2 \times 2 \times 3 \times 7$
2 Multiply the common factors to find the HCF.	$\text{HCF} = 2 \times 2 \times 3$ $= 12$
b 1 Write each term in expanded form.	**b** $a^2b = a \times a \times b$ $b^2ac = b \times b \times a \times c$
2 Multiply the common factors to find the HCF.	$\text{HCF} = a \times b$ $= ab$
c 1 Find the HCF of the coefficients. The negative sign can be represented using the factor '-1'.	**c** The HCF of 12 and -18 is 6.
2 Find the HCF of the pronumerals.	The HCF of x^2y^3 and x^3y is x^2y.
3 Multiply the common factors to find the HCF. Simplify the product.	The HCF of $12x^2y^3$ and $-18x^3y$ is $6x^2y$.

Worked example 2.3B Factorising algebraic expressions

Factorise each expression.

a $6a + 18$ **b** $x^2 - 7x$ **c** $10k^3m^2 + 15k^2m$

THINK	WRITE
a 1 Identify the HCF.	**a** $\text{HCF} = 6$
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$6a + 18 = 6\left(\frac{\cancel{6}^1 a}{\cancel{6}^1} + \frac{\cancel{18}^3}{\cancel{6}^1}\right)$
3 Simplify the bracketed expression.	$= 6(a + 3)$
b 1 Identify the HCF.	**b** $\text{HCF} = x$
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$x^2 - 7x = x\left(\frac{x^2}{x} - \frac{7x}{x}\right)$
3 Simplify the bracketed expression. Use the exponent laws to help you.	$= x(x^{2-1} - 7x^{1-1})$ $= x(x - 7)$
c 1 Identify the HCF.	**c** $\text{HCF} = 5k^2m$
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$10k^3m^2 + 15k^2m$ $= 5k^2m\left(\frac{\cancel{10}^2 k^3m^2}{\cancel{5}^1 k^2m} + \frac{\cancel{15}^3 k^2m}{\cancel{5}^1 k^2m}\right)$
3 Simplify the bracketed expression. Use the exponent laws to help you.	$= 5k^2m(2k^{3-2}m^{2-1} + 3k^{2-2}m^{1-1})$ $= 5k^2m(2km + 3)$

Worked example 2.3C Factorising using an HCF that is a binomial factor

Factorise each expression.

a $y(x+3)+7(x+3)$

b $4k(2-m)-5(2-m)$

THINK	WRITE
a 1 Identify the HCF. It is the binomial factor $(x+3)$.	**a** HCF $=(x+3)$
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$y(x+3)+7(x+3)=(x+3)\left(\frac{y\cancel{(x+3)}^{1}}{\cancel{(x+3)}^{1}}+\frac{7\cancel{(x+3)}^{1}}{\cancel{(x+3)}^{1}}\right)$
3 Simplify the bracketed expression.	$=(x+3)(y+7)$
b 1 Identify the HCF. It is the binomial factor $(2-m)$.	**b** HCF $=(2-m)$
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$4k(2-m)-5(2-m)=(2-m)\left(\frac{4k\cancel{(2-m)}^{1}}{\cancel{(2-m)}^{1}}-\frac{5\cancel{(2-m)}^{1}}{\cancel{(2-m)}^{1}}\right)$
3 Simplify the bracketed expression.	$=(2-m)(4k-5)$

Worked example 2.3D Factorising by grouping terms

Factorise $xy+2x+3y+6$ by grouping terms.

THINK	WRITE
1 Check for an HCF of all four terms. Group the terms in pairs and identify the HCF for each pair. Make sure to write the plus sign between the two pairs of grouped terms.	$xy+2x+3y+6=(xy+2x)+(3y+6)$ HCF of xy and $2x$ is x. HCF of $3y$ and 6 is 3.
2 Factorise each pair of terms by dividing out the HCF.	$xy+2x+3y+6=x(y+2)+3(y+2)$
3 Factorise using the binomial factor $(y+2)$. Simplify the expression.	$=(y+2)(x+3)$

Helpful hints

- ✔ When looking for the highest common factor, remember to consider any coefficients and all pronumerals.
- ✔ The divisibility rules can help you to find the HCF. Recall that a number is:
  - → divisible by 2 if the number is even
  - → divisible by 3 if the number's digits add to a multiple of 3
  - → divisible by 5 if the number ends in 0 or 5.
- ✔ The great thing about factorising is that you can always check your answer by expanding the brackets!

$$2x-16=2(x-8)=2x-16$$

Learning pathways

△ 1–4(1st, 2nd columns), 5, 6, 7–10(1st, 2nd columns), 11, 12, 15, 16(a, b), 17	□ 2–4(3rd column), 5–6(e–h), 7–10(2nd, 3rd columns), 13–16, 18, 20, 21, 24	⬠ 4(g–i), 7–10(2nd, 3rd columns), 15, 16, 19–25

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Exercise 2.3A: Understanding and fluency

WE 2.3A **1** Find the highest common factor (HCF) of each pair of terms.

a 4 and 28 **b** 6 and 10 **c** 15 and 35
d d^2 and $3d$ **e** $2e$ and $2k$ **f** 3 and −6
g 12 and −8 **h** $9h$ and $-15h^2$ **i** $24x^2$ and $36x$

2 Find the HCF of each pair of terms.

a bc and cd **b** $2xy$ and $2y$ **c** $18mn$ and $-9m$
d $abcd$ and bdf **e** $8xy$ and $28y$ **f** $6k^2$ and $-10k$
g p and $11p^2$ **h** $45ab$ and $-40cd$ **i** $3pq$ and $6p$

3 Factorise each expression using the HCFs from question **2**.

a $bc + cd$ **b** $2xy + 2y$ **c** $18mn - 9m$
d $abcd + bdf$ **e** $8xy + 28y$ **f** $6k^2 - 10k$
g $p + 11p^2$ **h** $45ab - 40cd$ **i** $3pq + 6p$

4 Find the HCF of each pair of terms.

a a^4 and a^7 **b** $5b^6$ and $3b^2$ **c** $16c^5$ and $-2c^3$
d $6d^4z^7$ and $6d^3z^5$ **e** $7e^6y^5$ and $14e^7y^8$ **f** $-20fx^{12}$ and $6f^3x^9$
g $-2g^2$ and $-2gh^5$ **h** $12i^6k^5$ and $-18j^3k^8$ **i** $x^9y^{11}z^{13}$ and $-x^{11}y^7z^{16}$

WE 2.3B **5** Factorise each expression.

a $10x + 5$ **b** $3y - 21$ **c** $8k + 12$ **d** $15 - 6d$
e $28x + 21y$ **f** $20n - 50$ **g** $x^2y + 3xy^4$ **h** $n^3m^2 - 9n^2m$

6 Factorise each expression.

a $20ab - 5b$ **b** $8d + 8cde$ **c** $15x^2 + 10x$ **d** $4k^2 - 22k$
e $30n - 18n^2$ **f** $16a^2 + a$ **g** $2h^2 - 14h$ **h** $6p + 6p^2$

7 Factorise each expression. Remember to use the exponent laws to help you simplify.

a $24m^6 + 16m^4$ **b** $21q^5r^2 + 35q^3r^6$ **c** $15t^4u^2 - 5t^8u^7$
d $12c^9d^5 + 6c^4d^4$ **e** $7e^{11}f^3 - 7e^5f^3$ **f** $3i^5j^2k^6 + 27ij^7k$
g $a^7b^{12}c^7 + b^3c^9d^4$ **h** $5mpq - 3np^{10}q^5$ **i** $u^4w^5y^2z^3 + v^4w^5x^2z^3$

8 Factorise each expression by factoring out a negative HCF. Remember that $-5mn = -1 \times 5 \times m \times n$.

a $-5mn - 10n$ **b** $-14xy - 7x$ **c** $-6c + 6cd$
d $-a^2 - 3a$ **e** $-4k^2 - 2k$ **f** $-8x^2 + 8x$
g $-12 - 3xy$ **h** $-16m - 10m^2$ **i** $-9x^2y + 18xy$

WE 2.3C **9** Factorise each expression.

a $x(w + 4) + 2(w + 4)$ **b** $y(x - 1) + 7(x - 1)$ **c** $a(a + 6) - 3(a + 6)$
d $p(5 - n) + 8(5 - n)$ **e** $3k(4 - k) - 5(4 - k)$ **f** $2x(3x - 4) + 9(3x - 4)$
g $4g(2g + 1) + (2g + 1)$ **h** $2h(8n - d) - (8n - d)$ **i** $3a(7x + 6y) + 2b(7x + 6y)$

WE 2.3D **10** Factorise each expression by grouping terms.

a $ab + 5b + 4a + 20$ **b** $xy - 6x + 7y - 42$ **c** $mn + 4m - 2n - 8$
d $y^2 + 3y + 5y + 15$ **e** $k^2 - 7k + 2k - 14$ **f** $6x + 18 + x^2 + 3x$
g $a^2 - 7a - 2a + 14$ **h** $p^2 + 5p - 2p - 10$ **i** $6c^2 - 2c + 9c - 3$

Exercise 2.3B: Problem solving and reasoning

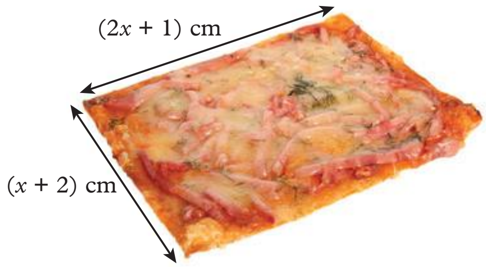
11 Explain the difference between expanding an expression and factorising an expression.

12 For each of the rectangular shapes below:

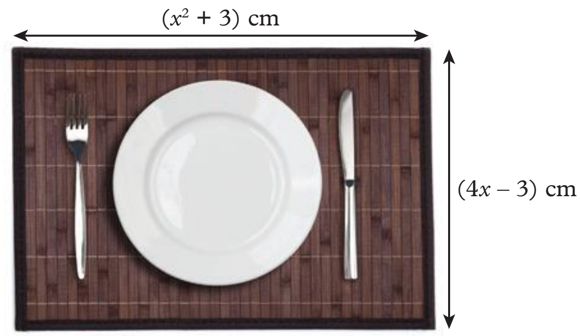
i write an expression in simplest factorised form for the perimeter

ii calculate the perimeter when $x = 5$.

a



b



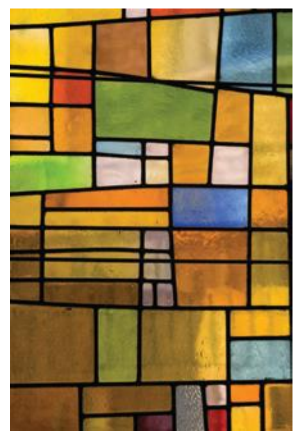
13 Write an expression for the missing side length of each rectangular object, given the area shown.

a area of rug is $(8x + 20)$ m^2

4 m

b area of stained glass panel is $(m^2 + 15m)$ cm^2

m cm



14 Write an expression for the perimeter of each item in question **13** in factorised form.

15 Expressions with more than two terms can be factorised in the same way as those with two terms. For example, to factorise $8a + 12b - 6c$, write the HCF, 2, in front of the brackets and divide each term inside the brackets by the HCF. So, $2\left(\frac{8a}{2} + \frac{12b}{2} - \frac{6c}{2}\right) = 2(4a + 6b - 3c)$. Factorise each expression.

a $27x - 9y + 15z$

b $45p - 50q - 5$

c $4 - 20i + 40j - 60k$

d $a^5 + a^3 + a^2$

e $18b^3c^5 - 36b^4c^4 + 24b^8c^5$

f $84r^{12}t^8 + 7r^5t^7 + 49r^8t^{14} + 14r^9t^8$

16 We can take fractions out as a factor so that all terms in the brackets no longer involve fractions. For example:

$$\begin{aligned}\frac{5}{3}a + \frac{5}{6}b &= \frac{10}{6}a + \frac{5}{6}b \\ &= \frac{1}{6}\left(\frac{10}{6}a \times 6 + \frac{5}{6}b \times 6\right) \\ &= \frac{1}{6}(10a + 5b) \\ &= \frac{5}{6}\left(\frac{\cancel{10}^{2}a}{\cancel{5}} + \frac{\cancel{5}^{1}b}{\cancel{5}}\right) \\ &= \frac{5}{6}(2a + b).\end{aligned}$$

Factorise each of the following expressions..

a $\frac{5}{2}x + \frac{3}{2}y$

b $\frac{9}{5}g + \frac{3}{5}h$

c $\frac{5}{2}p + \frac{15}{4}q$

d $\frac{7}{3}m - \frac{5}{4}n$

17 A rectangle has an area of $(10x - x^2)$ mm^2.

a Write the area in factorised form.

b Use your answer to part **a** to suggest possible expressions for the length and width of the rectangle.

c Suggest values for the length and width of three different rectangles that have the given area.

18 A triangle has an area of $(21x - 3x^2)$ cm^2.

a List possible expressions for the height and base length of the triangle.

b Suggest numerical values for the height and base length of three different triangles that have the given area.

19 A square-based prism has a volume of $(9x^3 + 45x^2)$ cm^3 and a height of $(x + 5)$ cm. Determine the length of the square base of the prism in terms of x.

20 Consider the expression $x^2 + 3x - 4x - 12$.

a Factorise the expression by grouping the first two terms together and the last two terms together.

b Perform the factorisation again by grouping the first and third terms together, and the second and fourth terms together.

c Compare your answers to parts **a** and **b**.

21 A number is represented by the pronumeral n.

a Write down the next two consecutive numbers in terms of n.

b Write the sum of these three consecutive numbers.

c Factorise your expression from part **b**. Explain the answer.

d Investigate to see whether the same outcome results from the sum of three consecutive even numbers.

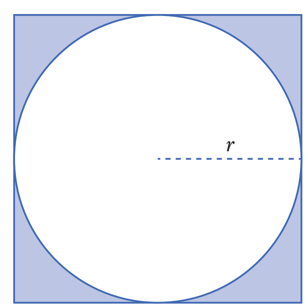
Exercise 2.3C: Challenge

22 In question **21**, an odd number (three) of consecutive numbers were added together. Investigate to see the outcome when an even number of consecutive numbers are added together. Write a factorised expression for the sum of 10 consecutive numbers, where the first number is n.

23 Fully factorise each of the following expressions.

a $x(y + 5) - (3y + 15)$ **b** $p(2q - 3) - 2q + 3$ **c** $2(a^2 - 3a) + (9 - 3a)$

24 A circle of radius r cm fits within a square as shown in the diagram. Write a factorised expression for the area of the square not covered by the circle. Leave your answer in terms of r and π.



25 It is possible to take any term or expression as a factor of another expression by dividing it out. For example, to take 7 out as a factor of $3x + 5$, write the 7 in front of the brackets; inside the brackets, divide each term by 7.

So, $3x + 5 = 7\left(\frac{3x}{7} + \frac{5}{7}\right)$.

a Take 5 out as a factor of $9x + 17$.

b Take 12 out as a factor of $12x^2 + 3x + 6$.

c Take x out as a factor of $3x^2 + 5xy + y^2$.

d Take $x + 2$ out as a factor of $7(x + 2)^2 + 5(x + 2) + 9$.

Online resources:

- **Interactive skillsheet** Factorising using the HCF
- **Interactive skillsheet** Factorising by grouping terms
- **Worksheet** Factorising
- **Investigation** Surface area of a soft drink can
- **CAS instructions** Factorising
- **Quick quiz** 2.3

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

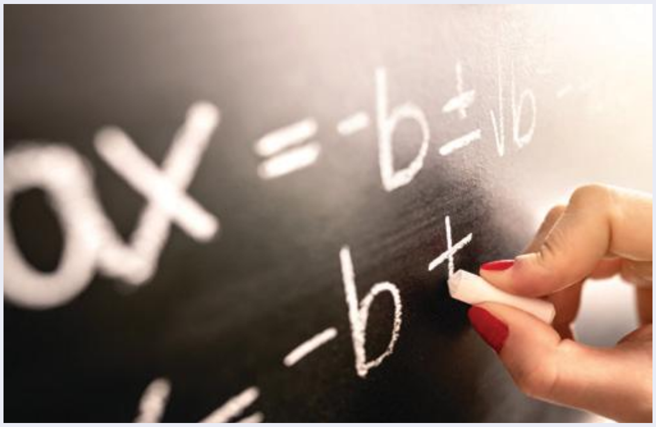
2.1 **1** Simplify the expressions.

a $-3a + 6b - 5a - 8b + 12a - 5$

b $9t - t^2 + 4t - 6 + 5t^2 + 2 - t$

c $7x^3 - 4x^2 - 3x + 2 + 8x - 3x^2 - 5x^3$

d $7cd^2 - 7d^2 + 2c - 8d^2 - 3d^2c + 5dd$



2.1 **2** Simplify the expressions.

a $5a \times (-7b)$

b $-8ac \times 3a \times (-2bc)$

c $\dfrac{45gh}{20g}$

d $\dfrac{18t^2u}{54tu^2}$

2.1 **3** Simplify the expressions.

a $(-5a^3b^6)^2 \times (-2a^2b^4)^5$

b $\left(-\dfrac{72k^4p^3}{16k^5p}\right)^3$

c $(8r^{-3}t^4)^{-1} \times \left(\dfrac{1}{3}r^5t^{-4}\right)^{-2}$

d $-42(x^3y^{-5}z^4)^{-4} \div [-63(x^{-5}y^3z^{-2})^8]$

2.2 **4** Expand the following.

a $8(a + 3)$ **b** $-4(2b - 5)$

c $3c(7 - y)$ **d** $9d(2d - 1)$

2.2 **5** Expand and simplify the following.

a $5(w + 7) - 2w + 6$ **b** $8 - 3(x - 9) + 2x$

c $10y - (12 - 5y) + 3$ **d** $z(z + 2) + 3(z - 5)$

2.2 **6** Expand and simplify the following.

a $(p + 2)(v + 5)$ **b** $(4 - q)(u + 3)$

c $(r + 6)(r - 2)$ **d** $(2t - 5)(3t - 7)$

2.2 **7** Expand and simplify the following.

a $(a^4 + 2a^2)(a^3 + 5)$ **b** $(b^2 - 3b)(b + 8) + 2b^2 - 4b + 1$

c $(c + 2)(c - 9) - 4(c - 2)$ **d** $(5 - d)(4 - d) + (2d + 1)(3d - 1)$

2.3 **8** Factorise the following.

a $6a + 9$ **b** $12b - 36$

c $-28c - 7$ **d** $8d^2 + 36d$

2.3 **9** Factorise the following.

a $2m(3g + 2) + 5(3g + 2)$ **b** $8(h - 3) - n(h - 3)$

c $p(q + 7r) - (q + 7r)$ **d** $2x(x + 3) - 5(x + 3)$

2.3 **10** Factorise the following by grouping terms.

a $ab + 5a + 4b + 20$ **b** $12cd - 8c - 21d + 14$

c $15ef - 5e + 3f - 1$ **d** $x^2 + 6x + 5x + 30$

Lesson 2.4

Factorising the difference of two squares

Learning intentions

By the end of this lesson you will be able to ...

✔ factorise the difference of two squares.

Inter-year links

Year 8 5.7 Factorising

Year 10 1.6 Factorising

The difference of two squares

- Binomials that have the same terms but one different sign, such as $a + b$ and $a - b$, are **conjugates** of each other.
- If a pair of conjugates are multiplied together, $(a + b)(a - b)$, then the middle terms add to zero and the solution is the difference of two squares:

$$\begin{aligned}(a + b)(a - b) &= a^2 + ab - ab - b^2 \\ &= a^2 - b^2\end{aligned}$$

Key content video
Factorising the difference of two squares

Factorising the difference of two squares

- The **difference of two squares** rule can be used to factorise expressions of the form $a^2 - b^2$:

$$a^2 - b^2 = (a - b)(a + b)$$

1 Check for common factors. Write the factor in front of the brackets. Inside the brackets, divide each term by the factor. $3x^2 - 12 = 3(x^2 - 4)$

2 Write the expressions in the brackets as the difference of two squares. $= 3(x^2 - 2^2)$

3 Factorise using the difference of two squares rule. $= 3(x - 2)(x + 2)$

Worked example 2.4A Factorising simple expressions using the difference of two squares rule

Factorise the following expressions.

a $x^2 - 25$ **b** $36 - y^2$

THINK	WRITE
a 1 There are no common factors. Write the expressions as the difference of two squares.	**a** $x^2 - 25 = x^2 - 5^2$
2 Factorise using the difference of two squares rule with $a = x$ and $b = 5$.	$= (x - 5)(x + 5)$
b 1 There are no common factors. Write the expressions as the difference of two squares.	**b** $36 - y^2 = 6^2 - y^2$
2 Factorise using the difference of two squares rule with $a = 6$ and $b = y$.	$= (6 - y)(6 + y)$

Worked example 2.4B Factorising expressions using the difference of two squares rule

Factorise each quadratic expression.

a $9y^2 - 16$ **b** $4k^2 - 9m^2$

THINK	WRITE
a 1 There are no common factors. Write the expressions as a difference of two squares. Use brackets to show the square of the term with a coefficient and a pronumeral.	**a** $9y^2 - 16 = (3y)^2 - 4^2$
2 Factorise using the difference of two squares rule with $a = 3y$ and $b = 4$.	$= (3y - 4)(3y + 4)$
b 1 There are no common factors. Write the expressions as a difference of two squares. Use brackets to show the squares of the terms with a coefficient and a pronumeral.	**b** $4k^2 - 9m^2 = (2k)^2 - (3m)^2$
2 Factorise using the difference of two squares rule with $a = 2k$ and $b = 3m$.	$= (2k - 3m)(2k + 3m)$

Worked example 2.4C Factorising complex expressions using the difference of two squares rule

Factorise each quadratic expression.

a $2x^2 - 8y^2$ **b** $(m + 4)^2 - 1$

THINK	WRITE
a 1 Check for common factors: 2. Write the factor in front of the brackets. Inside the brackets, divide each term by the factor.	**a** $2x^2 - 8y^2 = 2(x^2 - 4y^2)$
2 Write the expressions as a difference of two squares.	$= 2(x^2 - (2y)^2)$
3 Factorise using the difference of two squares rule with $a = x$ and $b = 2y$. Write the factor of 2 before the brackets.	$= 2(x - 2y)(x + 2y)$
b 1 There are no common factors. Write the expressions as a difference of two squares. Use brackets to show the square of the term with a coefficient and a pronumeral.	**b** $(m + 4)^2 - 1 = (m + 4)^2 - 1^2$
2 Factorise using the difference of two squares rule with $a = m + 4$ and $b = 1$.	$= (m + 4 - 1)(m + 4 + 1)$
3 Simplify the expression in each pair of brackets.	$= (m + 3)(m + 5)$

Helpful hints

- ✔ Memorise the first 16 square numbers to help you with factorising and mental arithmetic.
- ✔ You can check your factorisation by expanding the brackets!
- ✔ Simplify the expression inside the brackets before you give your final answer.
- ✔ The difference of two squares rule only applies to two square terms that are subtracted, not added together. For example, $x^2 - 4$ can be factorised using the rule but $x^2 + 4$ cannot.

$$\begin{aligned} a^2 - 9^2 &= (a - 9)(a + 9) \\ &= a^2 - 9a + 9a - 9^2 \\ &= a^2 - 81 \end{aligned}$$

Learning pathways

△ 1, 2, 3(1st, 2nd columns), 4–5(1st column), 6(a, b, d, e), 7–9(a, c, e), 10, 13–15	□ 3(2nd, 3rd columns), 4–5(2nd column), 6(b, c, e, f), 7–9(b, d, f), 12, 15, 16, 18, 19(a, c, e, g), 20(a, c), 21	⬠ 4–5(2nd column), 6(c, f), 7–9(b, d, f), 11, 15–18, 19(b, d, f, h), 20(b, d), 21–23

ANS p459

Exercise 2.4A: Understanding and fluency

1 Consider the expression $(3x + 7)(3x - 7)$.

a Expand $(3x + 7)(3x - 7)$ using the difference of two squares rule.

b Explain how to factorise $9x^2 - 49$.

WE 2.4A 2 Factorise each expression.

a $x^2 - 36$ **b** $a^2 - 100$ **c** $9 - y^2$

d $49 - x^2$ **e** $81 - a^2$ **f** $16 - y^2$

WE 2.4B 3 Factorise each expression using the difference of two squares rule.

a $25m^2 - 4$ **b** $49 - 64c^2$ **c** $(ab)^2 - 9$

d $x^2y^2 - w^2$ **e** $p^2 - q^2r^2$ **f** $b^2n^2 - d^2m^2$

g $(5jk)^2 - (6mn)^2$ **h** $81a^2b^2 - 49c^2d^2$ **i** $144p^2q^2 - 121r^2t^2$

WE 2.4C 4 Factorise each expression.

a $3x^2 - 12$ **b** $18p^2 - 50$

c $500 - 245g^2$ **d** $7 - 28u^2$

e $36r^2 - 100$ **f** $6400 - 12100z^2$

g $6x^2 - 6y^2$ **h** $pq^2 - pr^2$

i $u^2t^2 - d^2u^2$ **j** $36dk^2 - 16dj^2$

k $162a^2b^2 - 242a^2d^2$ **l** $9x - 81x^3$

5 Factorise each expression.

a $(k - 3)^2 - 16$ **b** $(7 - m)^2 - 1$

c $9 - (y + 2)^2$ **d** $4 - (a + 2)^2$

e $(y + 5)^2 - x^2$ **f** $u^2 - (t - 9)^2$

g $(2p + 4)^2 - p^2$ **h** $(9 - 4m)^2 - m^2$

i $(x + 3)^2 - x^2$ **j** $(q + 6)^2 - (r - 5)^2$

k $(7x - 4)^2 - (3x - 5)^2$ **l** $(z + 1)^2 - (z - 1)^2$

6 Factorise each expression.

a $-x^2 + 36$ **b** $-16 + b^2$ **c** $-25 + 9h^2$

d $-49d^2 + 64n^2$ **e** $-(rp)^2 + 1$ **f** $-t^2u^2 + w^2x^2$

7 Factorise each expression.

a $t^2 - \frac{4}{9}$
b $r^2 - \frac{25}{49}$
c $\frac{81}{16} - u^2$
d $\frac{9}{100}d^2 - \frac{121}{144}$
e $\frac{1}{64}k^2 - \frac{1}{25}$
f $\frac{w^2}{169} - \frac{x^2}{196}$

8 Factorise each expression. Hint: It might help to convert each decimal to a fraction first.

a $z^2 - 0.16$
b $y^2 - 1.21$
c $0.04 - x^2$
d $c^2 - 0.0025$
e $0.49g^2 - 0.01h^2$
f $0.0144 - 0.0121k^2$

9 Factorise each expression as the difference of two squares. Hint: Use the exponent laws.

a $x^6 - 9$
b $36 - y^{10}$
c $z^{22} - 49$
d $4p^{14} - 25$
e $\frac{q^{18}}{16} - \frac{81}{121}$
f $-r^{30}t^{50} + 1$

Exercise 2.4B: Problem solving and reasoning

10 The difference of two squares rule can be applied to numerical expressions.

a Write each of the following differences as a product.

i $14^2 - 4^2$
ii $10^2 - 15^2$
iii $99^2 - 98^2$
iv $16 - 64$
v $169 - 9$
vi $225 - 196$

b Evaluate each difference in part **a** using the product.

c Write the following products as a difference of two square numbers.

i $(8 + 5)(8 - 5)$
ii $(12 + 18)(12 - 18)$
iii $(30 + 1)(30 - 1)$
iv 12×6
v $40 \times (-20)$
vi 13×15

d Evaluate each product in part **c** using the difference.

11 Without using a calculator, work out the value of each problem.

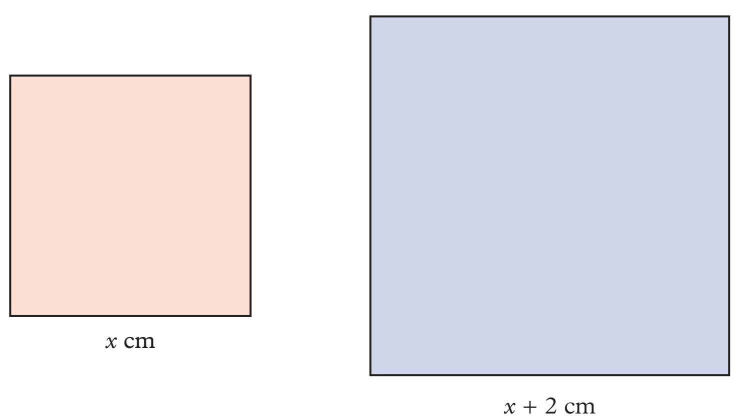
a $63^2 - 37^2$
b $15.19^2 - 14.81^2$
c $\left(\frac{13}{25}\right)^2 - \left(\frac{12}{25}\right)^2$

12 Consider two consecutive numbers, n and $(n + 1)$.

a Write a simplified expression to represent the difference between the squares of the two numbers.

b If n is an integer, what type of number is the difference of the squares of two consecutive numbers?

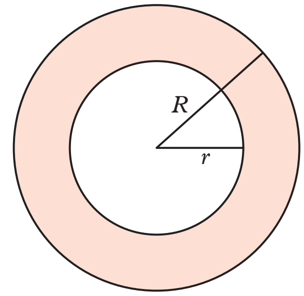
13 A square has side lengths of x cm. A second square has side lengths 2 cm longer than the first square. Write a factorised expression to represent the difference between the areas of the two perfect squares.



14 Use the difference of two squares rule to write each expression in its simplest factorised form.

a $x^4 - 16$
b $x^8 - y^8$

15 An annulus (plural annuli) is a 2D shape that looks like a donut, with a circle cut out of the centre of another circle. The radius of the larger circle is labelled as R and the radius of the smaller circle is labelled as r.

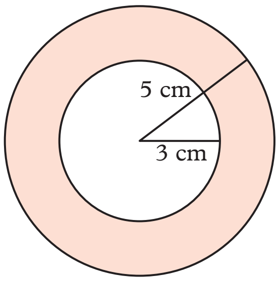


a Write an expression, in terms of r and R, for the area of the annulus. Remember, the area of a circle with radius of r units is πr^2.

b Factorise the expression from part **a**.

c Calculate the area of the following annuli. Write your answers correct to two decimal places.

i



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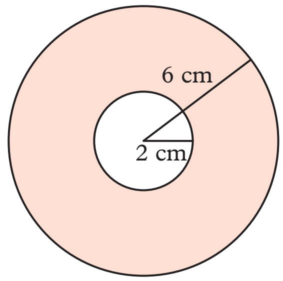

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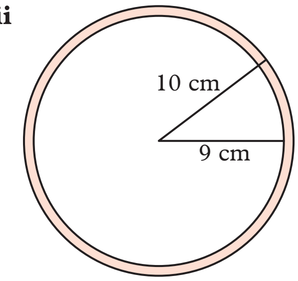
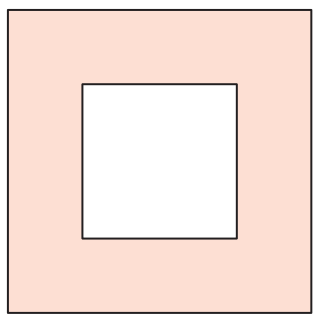


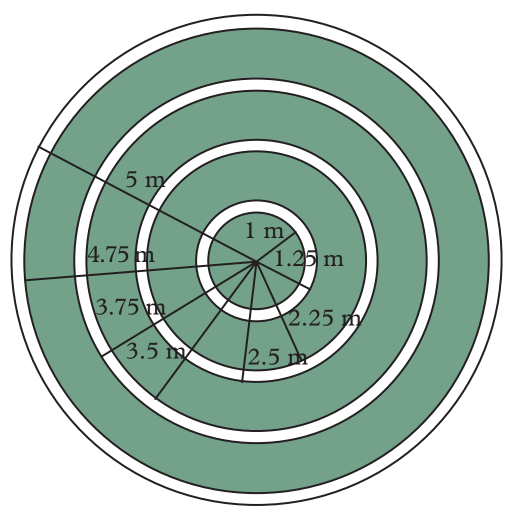
16 A square annulus is similar to an annulus but uses squares instead of circles.
The square annulus shown has integer side lengths where the larger square has a side length 2 cm greater than the smaller square. If the shaded area of the square annulus is 24 cm^2, determine the side lengths of the two squares.



17 A landscaper has designed a concentric circle pattern for the lawn at a university. The innermost circle of lawn has a diameter of 1 m and is surrounded by a concrete ring that is 0.25 m wide. The concrete ring is surrounded by a 1 m-wide ring of lawn. Three more rings of concrete and two more rings of lawn of the same width surround the original circle.

a Write a calculation using the radii lengths for the area of concrete needed.

b By factorising, show that the area of concrete needed is given by $0.25\pi(5 + 4.75 + 3.75 + 3.5 + 2.5 + 2.25 + 1.25 + 1)$ m^2.



18 Show how $(x + 1)(x + y)^2 - (x + 1)(x - y)^2$ can be written in its simplest factorised form of $4xy(x + 1)$.

19 We can factorise the difference of any two terms by using square roots. For example, $x - y$ can be factorised to $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$. Factorise the following to the product of conjugates. Remember to evaluate square roots and simplify surds where possible.

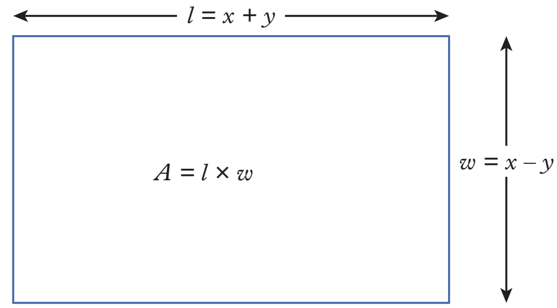
a $a - 5$ **b** $b^2 - 7$ **c** $c - 8$ **d** $3d - 16$

e $4e - 45$ **f** $2f^2 - 75$ **g** $x^3 - y^5$ **h** $8x^2 - 12y^2$

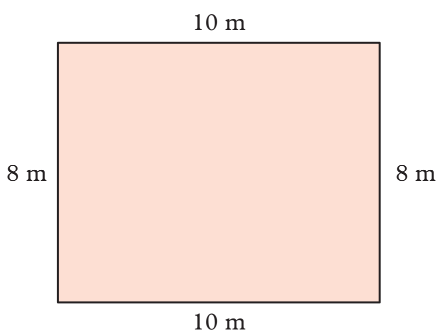
Exercise 2.4C: Challenge

20 If the length of a rectangle is given by $l = x + y$ and the width is given by $w = x - y$, then the area of any rectangle, $A = l \times w$, can be expressed as the difference of two squares $A = x^2 - y^2$.

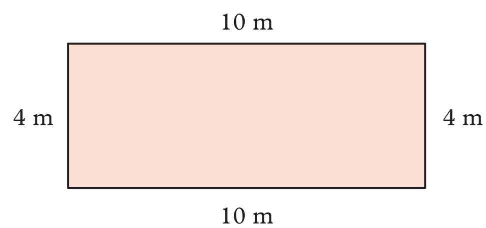
For each of the following rectangles, state the side lengths (x and y) of two squares, where the difference in the areas of the squares is the same as the area of the rectangle.



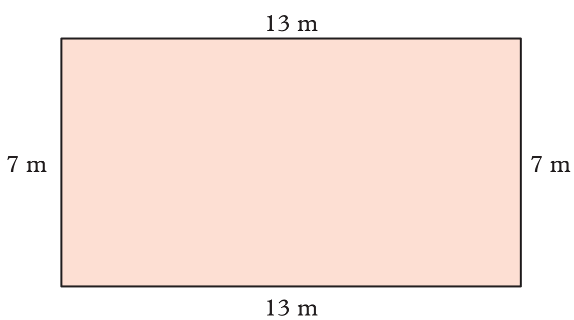
a



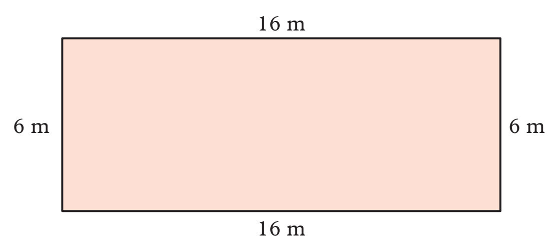
b



c



d



21 A rectangle has a length 6 cm longer than its width. The rectangle's area is $y^2 - x^2$. What is the value of x^2 in square centimetres?

22 There are other rules we can use to expand and factorise algebraic expressions.

a Expand $(x + y)(x^2 - xy + y^2)$.

b Use your result in part **a** to factorise $x^3 + 8$.

c Predict the factorised form of $x^3 - 8$.

d Use expansion to verify that your prediction in part **c** is correct.

23 These are the known facts about two numbers: the difference between two numbers is 5 and the difference between the squares of the two numbers is 155. What is the sum of the two numbers?

Online resources:

Interactive skillsheet Factorising the difference of two squares

CAS instructions Factorising

Quick quiz 2.4

Factorising quadratic expressions

Learning intentions

By the end of this lesson you will be able to ...

✔ factorise monic quadratic trinomials.

Inter-year links

Support	Multiples and factors
Year 7	2.4 Factors and the highest common factor
Year 8	5.7 Factorising
Year 10	1.7 Factorising quadratic expressions

Quadratic trinomials

- A **quadratic expression** is an algebraic expression that contains a squared pronumeral, with no exponents greater than 2 in the expression.
  For example, $6t^2$, $x^2 + 5$, $k^2 + 14k + 30$ and $4b^2 - a^2$ are all quadratic expressions.
- An expression with three terms is called a trinomial.
- A **quadratic trinomial** is an expression of the form $ax^2 + bx + c$, where a and b are coefficients and c is a constant.

$$ax^2 + bx + c$$

→ a is the **leading coefficient**

→ b is the coefficient of the **linear term**

→ c is the constant term

- Many quadratic trinomials can be factorised to produce a binomial product.

expanding ←

$$x^2 + 4x + 3 = (x + 1)(x + 3)$$

quadratic trinomial expanded form — binomial product factorised form

factorising →

Factorising monic quadratic trinomials

Key content video
Factorising monic quadratic trinomials

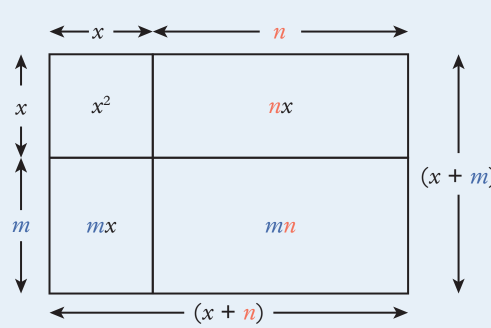
- A **monic quadratic trinomial** is a quadratic trinomial in which the leading coefficient is equal to 1.

$$x^2 + bx + c$$

- Expanding a binomial product of the form $(x + m)(x + n)$, where m and n are constants, gives a quadratic trinomial of the form $x^2 + bx + c$.

$$\begin{aligned}(x + m)(x + n) &= x(x + m) + n(x + m)\\ &= x^2 + mx + nx + mn\\ &= x^2 + (m + n)x + mn\\ &= x^2 + bx + c\end{aligned}$$

where $b = m + n$ and $c = m \times n$



Area $= x^2 + (m + n)x + mn$

- The process can be reversed to factorise quadratic trinomials of the form $x^2 + bx + c$, by finding two numbers (m and n) that add to give b and multiply to give c.

For example, $x^2 + 4x + 3 = x^2 + (3 + 1)x + (3 \times 1)$
$= (x + 3)(x + 1)$

Worked example 2.5A Finding the numbers for a given product and sum

Identify which two numbers add to give the first number and multiply to give the second number.

a sum: 5, product: 6 **b** sum: −2, product: −8 **c** sum: 1, product: −6

THINK	WRITE
a **1** List the factor pairs of 6. Remember that if the positive/negative sign is changed for both factors then the product is the same, so for each factor pair, consider both combinations of positive and negative values.	**a** $1 \times 6 = 6 \rightarrow 1 + 6 = 7$ $-1 \times (-6) = 6 \rightarrow -1 + (-6) = -7$ $2 \times 3 = 6 \rightarrow 2 + 3 = 5$ $-2 \times (-3) = 6 \rightarrow -2 + (-3) = -5$
2 Add the factor pairs together and determine which pair adds to 5.	The numbers are 2 and 3.
b **1** List the factor pairs of −8. Remember that a negative multiplied by a positive is a negative, so for each factor pair, consider both combinations of positive and negative values.	**b** $-1 \times 8 = -8 \rightarrow -1 + 8 = 7$ $1 \times (-8) = -8 \rightarrow 1 + (-8) = -7$ $-2 \times 4 = -8 \rightarrow -2 + 4 = 2$ $2 \times (-4) = -8 \rightarrow 2 + (-4) = -2$
2 Add the factor pairs together and determine which pair adds to −2.	The numbers are 2 and −4.
c **1** List the factor pairs of −6. Remember that a negative multiplied by a positive is a negative, for each factor pair, consider both combinations of positive and negative values.	**c** $-1 \times 6 = -6 \rightarrow -1 + 6 = 5$ $1 \times (-6) = -6 \rightarrow 1 + (-6) = -5$ $-3 \times 2 = -6 \rightarrow -3 + 2 = -1$ $3 \times (-2) = -6 \rightarrow 3 + (-2) = 1$
2 Add the factor pairs together and determine which pair adds to 1.	The numbers are 3 and −2.

Worked example 2.5B Factorising simple quadratic trinomials

Factorise the quadratic trinomial $x^2 + 7x + 10$.

THINK	WRITE
1 List the factor pairs of the constant term, 10. Remember that if the positive/negative sign is changed for both factors then the product is the same, so for each factor pair, consider both combinations of positive and negative values.	$1 \times 10 = 10 \rightarrow 1 + 10 = 11$ $-1 \times (-10) = 10 \rightarrow -1 + (-10) = -11$ $2 \times 5 = 10 \rightarrow 2 + 5 = 7$ $-2 \times (-5) = 10 \rightarrow -2 + (-5) = -7$
2 Add the factor pairs together and identify which pair adds to the linear coefficient, 7.	The numbers are 2 and 5.
3 Write the expression in factorised form.	$x^2 + 7x + 10 = (x + 2)(x + 5)$
4 Check your result by expanding.	Check: $(x + 2)(x + 5) = x^2 + 2x + 5x + 10$ $= x^2 + 7x + 10$

Worked example 2.5C Factorising more complex quadratic trinomials (+ and −)

Factorise each quadratic trinomial.

a $m^2 + 2m - 3$

b $x^2 - 7x - 8$

THINK	WRITE
1 List the factor pairs of the constant term. Remember that if the positive/negative sign is changed for both factors then the product is the same, so for each factor pair, consider both combinations of positive and negative values. **2** Add the factor pairs together and identify which pair adds to the linear term. **3** Write the expression in factorised form. **4** Check your result by expanding.	**a** $1 \times (-3) = -3 \rightarrow 1 + (-3) = -2$ $-1 \times 3 = -3 \rightarrow \quad -1 + 3 = 2$ $m^2 + 2m - 3 = (m + 3)(m - 1)$ Check: $(m + 3)(m - 1) = m^2 - m + 3m - 3$ $= m^2 + 2m - 3$ **b** $1 \times (-8) = -8 \rightarrow 1 + (-8) = -7$ $-1 \times 8 = -8 \rightarrow \quad -1 + 8 = 7$ $2 \times (-4) = -8 \rightarrow 2 + (-4) = -2$ $-2 \times 4 = -8 \rightarrow -2 + (4) = 2$ $x^2 - 7x - 8 = (x - 8)(x + 1)$ Check: $(x - 8)(x + 1) = x^2 + x - 8x - 8$ $= x^2 - 7x - 8$

Worked example 2.5D Factorising quadratic trinomials by first taking out a common factor

Factorise by first taking out a common factor.

a $2x^2 - 14x + 12$

b $-x^2 + 4x - 3$

THINK	WRITE
1 If the coefficient of x^2 is not 1, check for a common factor of the three terms. Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF. A common factor can be −1. **2** Factorise the expression inside the brackets. List the factor pairs of the constant term. **3** Add the factor pairs together and identify which pair adds to the linear term. **4** Write the expression in factorised form. **5** Check your result by expanding.	**a** $2x^2 - 14x + 12 = 2(x^2 - 7x + 6)$ $1 \times 6 = 6 \rightarrow \quad 1 + 6 = 7$ $-1 \times (-6) = 6 \rightarrow -1 + (-6) = -7$ $2 \times 3 = 6 \rightarrow \quad 2 + 3 = 5$ $-2 \times (-3) = 6 \rightarrow -2 + (-3) = -5$ $2(x^2 - 7x + 6) = 2(x - 6)(x - 1)$ Check: $2(x - 6)(x - 1) = 2(x^2 - x - 6x + 6)$ $= 2(x^2 - 7x + 6)$ $= 2x^2 - 14x + 12$ **b** $-x^2 + 4x - 3 = -(x^2 - 4x + 3)$ $1 \times 3 = 3 \rightarrow \quad 1 + 3 = 4$ $-1 \times (-3) = 3 \rightarrow -1 + (-3) = -4$ $-(x^2 - 4x + 3) = -(x - 3)(x - 1)$ Check: $-(x - 3)(x - 1) = -(x^2 - x - 3x + 3)$ $= -(x^2 - 4x + 3)$ $= -x^2 + 4x - 3$

Helpful hints

- ✔ The order of the binomial products doesn't matter. This is because of the commutative law for multiplication. For example: $2 \times 3 = 3 \times 2$ and $(x - 2)(x + 3) = (x + 3)(x - 2)$
- ✔ The order of the terms in a quadratic trinomial does not change the value of the expression. This is because of the commutative law for addition. For example: $x^2 + 6x + 8 = 6x + 8 + x^2 = 8 + x^2 + 6x$
- ✔ Always look for the HCF of the three terms before factorising. Simpler expressions are usually easier to factorise!
- ✔ You can check your factorisation of the binomial products by expanding the brackets!

$$x^2 + 4x + 3 = (x + 1)(x + 3)$$
$$= x^2 + 3x + x + 3$$
$$= x^2 + 4x + 3$$

If the constant term, c, is positive, then both factors have the same sign. So:

→ if b is *positive*, then both factors are *positive*

→ if b is *negative*, then both factors are *negative*.

If the constant term, c, is negative, then one factor is positive and the other is negative.

Learning pathways

△ 1–3(1st, 2nd columns), 4, 5–6(1st, 2nd columns), 7–13, 16(a–d), 18	□ 1–3(3rd column), 6–11, 13, 14, 16(b, e, f, h, i), 18, 19, 21(a, c, e)	⬠ 4(c, f, i, l), 5(c, f, i), 6(c, f, i, l), 7–11, 14, 15, 16(d, g, h, i), 17, 20, 21(b, d, f), 22, 23

ANS p460

Exercise 2.5A: Understanding and fluency

WE 2.5A **1** Identify which two numbers add to give the first number and multiply to give the second number.

a sum: 5, product: 4 **b** sum: 6, product: 8 **c** sum: 13, product: 22

d sum: 9, product: 20 **e** sum: 10, product: 24 **f** sum: 7, product: 12

g sum: 13, product: 42 **h** sum: 12, product: 35 **i** sum: 8, product: 16

2 Identify which two numbers add to give the first number and multiply to give the second number.

a sum: 2, product: −8 **b** sum: −1, product: −6 **c** sum: −8, product: 12

d sum: 3, product: −10 **e** sum: −8, product: −9 **f** sum: −5, product: 6

g sum: 5, product: −6 **h** sum: −6, product: −27 **i** sum: −12, product: 11

3 Use your results from question **1** to factorise each of these quadratic trinomials.

a $x^2 + 5x + 4$ **b** $x^2 + 6x + 8$ **c** $x^2 + 13x + 22$

d $x^2 + 9x + 20$ **e** $x^2 + 10x + 24$ **f** $x^2 + 7x + 12$

g $x^2 + 13x + 42$ **h** $x^2 + 12x + 35$ **i** $x^2 + 8x + 16$

WE 2.5B **4** Factorise each quadratic trinomial.

a $a^2 + 4a + 3$ **b** $b^2 + 9b + 14$ **c** $c^2 + 7c + 6$

d $d^2 + 10d + 21$ **e** $e^2 + 8e + 7$ **f** $f^2 + 8f + 15$

g $g^2 + 11g + 28$ **h** $h^2 + 13h + 36$ **i** $x^2 + 9x + 18$

j $j^2 + 14j + 45$ **k** $k^2 + 11k + 30$ **l** $y^2 + 13y + 40$

5 Use your results from question **2** to factorise each of these quadratic trinomials.

a $x^2 + 2x - 8$ **b** $x^2 - x - 6$ **c** $x^2 - 8x + 12$

d $x^2 + 3x - 10$ **e** $x^2 - 8x - 9$ **f** $x^2 - 5x + 6$

g $x^2 + 5x - 6$ **h** $x^2 - 6x - 27$ **i** $x^2 - 12x + 11$

WE 2.5C **6** Factorise each quadratic trinomial.

a $a^2 + 2a - 3$ **b** $b^2 - 2b - 15$ **c** $c^2 - 5c + 4$

d $d^2 + 5d - 14$ **e** $e^2 - 10e + 24$ **f** $f^2 - 3f - 10$

g $g^2 + g - 12$ **h** $h^2 - 8h + 15$ **i** $x^2 - 5x - 24$

j $j^2 - 10j + 16$ **k** $k^2 + 3k - 18$ **l** $y^2 - y - 2$

WE 2.5D **7** Factorise each quadratic trinomial by first taking out a common factor.

a $3x^2 + 9x + 6$ **b** $2x^2 + 16x + 30$

c $5x^2 + 15x - 20$ **d** $-4x^2 - 20x - 24$

e $-6x^2 + 36x - 30$ **f** $-x^2 - 2x + 35$

8 Factorise each quadratic trinomial by first taking out a common factor.

a $ax^2 + 8ax + 12a$ **b** $bx^2 - 11bx + 28b$

c $10cx^2 - 10cx - 300c$ **d** $-2dx^2 + 34dx - 120d$

e $p^2x^2 + 13p^2x + 30p^2$ **f** $q^2x^2 + 15qx^2 + 44x^2$

9 Factorise each quadratic expression.

a $x^2 - 6x + 9$ **b** $x^2 + 0x - 9$ **c** $x^2 - 6x + 0$

10 Factorise each quadratic expression.

a $x^2 - 25$ **b** $x^2 - 10x + 25$

c $x^2 - 25x$ **d** $x^2 + 14x + 49$

e $x^2 - 49x$ **f** $x^2 - 49$

11 Factorise each quadratic by first reordering and simplifying the terms of each expression.

a $21 + 10a + a^2$ **b** $2b - 35 + b^2$

c $3c + c^2 - 18$ **d** $48 + 2d - d^2$

e $-24 - 10e - e^2$ **f** $10f - f^2 - 16$

g $10g + 70 + g^2 + 7g + 2$ **h** $-2h + 5 + h^2 - 19 - 3h$

i $50 - 20x - x^2 + 7x - 86$

Exercise 2.5B: Problem solving and reasoning

12 Consider a quadratic expression of the form $x^2 + 6x + c$.

a Determine all positive integer or zero values of c such that the expression can be factorised. Write both the expanded and factorised forms for each value of c.

b Determine all negative integer values of c greater than or equal to -40 such that the expression can be factorised. Write both the expanded and factorised forms for each value of c.

c When the coefficient of x is positive, how can you tell if the factors are both positive or if one is positive and one negative?

13 Consider a quadratic expression of the form $x^2 - 6x + c$.

a Determine all positive integer or zero values of c such that the expression can be factorised. Write both the expanded and factorised forms for each value of c.

b Determine all negative integer values of c greater than or equal to -40 such that the expression can be factorised. Write both the expanded and factorised forms for each value of c.

c When the coefficient of x is negative, how can you tell if the factors are both positive or if one is positive and one negative?

14 Consider a quadratic expression of the form $x^2 + bx + 64$.

a Determine all positive integer values of b such that the expression can be factorised. Write both the expanded and factorised forms for each possible value.

b Determine all negative integer values of b such that the expression can be factorised. Write both the expanded and factorised forms for each possible value.

c When the constant term is positive:

i how can you tell if the factors need to be positive or negative

ii is the coefficient of x the sum or difference of the positive values of the factors?

15 Consider a quadratic expression of the form $x^2 - bx - 64$.

a Determine all positive integer or zero values of b such that the expression can be factorised. Write both the expanded and factorised forms for each possible value.

b Determine all the negative integer or zero values of b such that the expression can be factorised. Write both the expanded and factorised forms for each possible value.

c When the constant term is negative:

i how can you tell which factor must be positive and which must be negative

ii is the coefficient of x the sum or difference of the positive values of the factors?

16 Use the expansion of a perfect square rule, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$, to completely factorise each quadratic trinomial as a perfect square.

a $x^2 + 8x + 16$ **b** $y^2 - 10y + 25$ **c** $v^2 + 2v + 1$

d $(2z)^2 - 12z + 9$ **e** $9w^2 + 42w + 49$ **f** $3h^2 - 12h + 12$

g $-k^2 - 18k - 81$ **h** $f^2 + 2fg + g^2$ **i** $(5p)^2 - 40pq + (4q)^2$

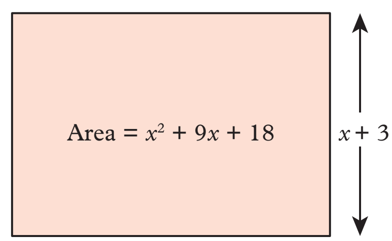
17 Consider quadratic trinomials of the form $ax^2 + bx + c$.

a Describe how you can check if a quadratic trinomial is a perfect square.

b The following quadratical trinomials cannot be factorised as a perfect square. Explain why.

i $x^2 + 6x - 9$ **ii** $-x^2 + 6x + 9$ **iii** $x^2 + 3x + 9$ **iv** $x^2 + 6x + 3$

18 A rectangle has an area of $(x^2 + 9x + 18)\,\text{m}^2$ and a width of $(x + 3)\,\text{m}$.



a Determine the expression of the length, in m, of the rectangle.

b Write the area, in m^2, of the rectangle as a product of its length and width.

c If $x = 2$ m, calculate the area of the rectangle, in m^2, using:

i the product of the length and width

ii the expression $x^2 + 9x + 18$.

19 Consider the two rectangular objects below.

a Write an expression for the missing side length for each object.

i area within frame is $(x^2 + 7x - 18)$ cm^2

ii area of billboard is $(9y^2 - 16)$ m^2

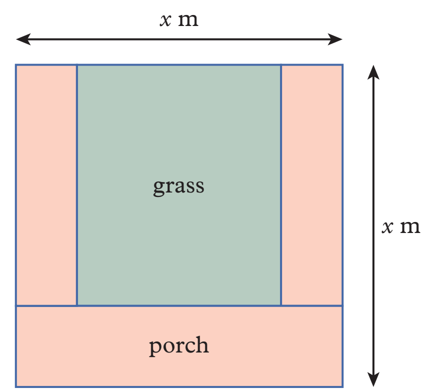
b Find the value of x that gives an area of 152 cm^2 for part **i**.

c Find the value of y that gives an area of 425 m^2 for part **ii**.

d Determine the positive values of x and y such that the areas of the rectangular objects is zero.

e Substitute $x = 1$ and $y = 1$ into the length, width and area expression, and state why they are not appropriate for the items.

20 Two friends, Melissa and Lena, want to renovate their square-shaped backyards by adding a full-length rectangular porch and planting a rectangular section of grass with garden beds on either side. Both friends want to have the same width for their porches and their garden beds but have different-sized backyards, so they decide to use algebra to determine the area of grass that they need for both. They've already decided on the width of their porches, and have narrowed down the area of grass to four options:



Option 1: $(x^2 - 5x + 6)$ m^2

Option 2: $(x^2 - 7x + 10)$ m^2

Option 3: $(x^2 - 4x + 4)$ m^2

Option 4: $(x^2 - 6x + 8)$ m^2

a Factorise the quadratic expression for each of the four options.

b Consider the factors of each equation and relate them to the possible side lengths of the grass. Judging from Melissa and Lena's four options, what is the width of Melissa and Lena's porches?

c Lena also suggested the grass areas $(x^2 - x - 6)$ m^2 and $(x^2 + x - 6)$ m^2. Explain why Melissa said these areas wouldn't be possible.

d Melissa and Lena decide to have 2 m of space either side of the lawn for their garden beds. Which quadratic expression did they decide on?

e If Melissa's backyard is 6 m by 6 m, determine what area of grass she will plant.

f If Lena's backyard is 7.5 m by 7.5 m, determine what area of grass she will plant.

g Determine how much area Melissa and Lena will each have for planting flowers.

21 We can factorise quadratic trinomials by splitting the linear term once we have determined the two factors and then using grouping. For example, two numbers that add to 7 and multiply to 12 are 3 and 4 so:

$$\begin{aligned} x^2 + 7x + 12 &= x^2 + 3x + 4x + 12 \\ &= x(x + 3) + 4(x + 3) \\ &= (x + 3)(x + 4) \end{aligned}$$

Factorise each of the following by splitting the linear term and using grouping.

a $x^2 + 5x + 4$ **b** $x^2 + 6x + 8$ **c** $x^2 + 13x + 22$

d $x^2 + 3x - 10$ **e** $x^2 - 8x - 9$ **f** $x^2 - 5x + 6$

Exercise 2.5C: Challenge

22 Factorise each quadratic expression.

a $(3x)^2 + 5(3x) - 14$ **b** $(11x)^2 + 2(11x) - 80$ **c** $(5x)^2 - 12(5x) + 32$

d $(x + 2)^2 + 3(x + 2) - 4$ **e** $(x - 4)^2 - 11(x - 4) + 30$ **f** $(x - 9)^2 + 14(x - 9) + 45$

g $(3x)^2 - (3x) - 42$ **h** $(2x)^2 + 7(2x) - 30$ **i** $(8x)^2 + 32(2x) - 48$

23 Write each expression in its simplest factorised form.

a $(x^2)^2 - 13x^2 + 36$ **b** $(x^2)^2 - 12x^2 - 64$ **c** $(x^2)^2 + 24x^2 - 25$

Online resources:

Interactive skillsheet Factorising quadratic expressions

Worksheet Factorising quadratic expressions

Quick quiz 2.5

Lesson 2.6

Review: Algebra

Module summary

Terms and expressions

pronumerals

coefficients constant

$$15x - 2y + 5$$

term + term + term

expression

Multiplying algebraic terms

$$\begin{aligned} 3a^3b \times (-2a^2b^2) &= 3 \times (-2) \times a^{3+2} \times b^{1+2} \\ &= -6a^5b^3 \end{aligned}$$

Dividing algebraic terms

$$\begin{aligned} \frac{6a^5b^3}{3a^3b} &= \frac{\cancel{6}^{2} \times a^5 \times b^3}{\cancel{3}^{1} \times a^3 \times b} \\ &= 2 \times a^{5-3} \times b^{3-1} \\ &= 2a^2b^2 \end{aligned}$$

Expanding

Use the distributive law to expand brackets.

$$a(b + c) = ab + ac$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

Difference of two squares

$$(a + b)(a - b) \underset{\text{factorising}}{\overset{\text{expanding}}{\rightleftarrows}} a^2 - b^2$$

Expansion of a perfect square

$$(a + b)^2 \underset{\text{factorising}}{\overset{\text{expanding}}{\rightleftarrows}} a^2 + 2ab + b^2$$

Adding and subtracting like terms

- Like terms contain the exact same pronumerals with the same exponents.

  Like terms: xyz, $4yzx$, $7zxy$
  Not like terms: ab^2, a^2b

- Like terms can be added and subtracted by adding and subtracting their coefficients.

$$a^2b + 2a^2b = 1a^2b + 2a^2b = 3a^2b$$

Factorising using the HCF

$$2x^2 - 6x \xrightarrow{\text{factorising}} 2x(x - 3)$$

HCF of $2x^2$ and $6x$

Factorising by grouping terms

$$\begin{aligned} x^2 + 2x - 3x - 6 &= x(x + 2) - 3(x + 2) \\ &= (x + 2)(x - 3) \end{aligned}$$

Factorising monic trinomials

$$x^2 + bx + c$$

- Find two numbers that add to give b and multiply to give c.

$$\begin{aligned} (x + m)(x + n) &= x(x + m) + n(x + m) \\ &= x^2 + mx + nx + mn \\ &= x^2 + (m + n)x + mn \\ &= x^2 + bx + c \end{aligned}$$

where $b = m + n$ and $c = m \times n$

- Substitute those two numbers into the binomial product factorised form.

$$x^2 + 4x + 3 \underset{\text{expanding}}{\overset{\text{factorising}}{\rightleftarrows}} (x + 1)(x + 3)$$

ANS
p461

Review questions 2.6A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- binomial
- binomial product
- coefficient
- conjugates
- constant
- constant term
- difference of two squares
- distributive law
- expand
- expanded form
- exponent form
- expression
- factorise
- highest common factor (HCF)
- leading coefficient
- like terms
- linear term
- monic quadratic trinomial
- pronumeral
- quadratic expression
- quadratic trinomial
- term
- trinomial
- unknown
- variable

1 Complete the following sentences using words from the list of key terms.

a An algebraic expression contains one or more ________. A ________ has only two terms and a ________ has three terms.

b A ________ has three terms and the leading coefficient of the x^2 term is 1.

c In a quadratic trinomial, the ________ is the term with x and the term with no pronumeral is called the ________.

d ________ is the opposite process to expanding. The first step is to identify if there is a ________.

2 For the expression $9x^2 - 6x + 12$, match the following in the left-hand column with a key term in the right-hand column.

a	12	**I**	leading coefficient
b	$9 \times x \times x$	**II**	constant term
c	-6	**III**	linear term
d	x	**IV**	quadratic expression
e	9	**V**	highest common factor
f	$9x^2 - 6x + 12$	**VI**	expanded form
g	$-6x$	**VII**	variable/pronumeral
h	3	**VIII**	coefficient

Review questions 2.6B: Multiple choice

2.1 **1** Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?

A $\frac{6ab^2}{18a}$ **B** $\frac{ab^2c}{3a^2c}$ **C** $\frac{b^2}{12a}$ **D** $\frac{b^2}{3a}$

2.1 **2** Which is not a like term to $4ab^3$?

A $\frac{ab^3}{7}$ **B** $4ba^3$ **C** $-4b^3a$ **D** $2bab^2$

2.1 **3** If $x = -3$ and $y = 2$, then $-5y^3x^2$ is equal to:

A -360 **B** 360 **C** -540 **D** 540

2.2 **4** Which expression is equivalent to $8 - 2(3 - 5g)$?

A $18 - 30g$ **B** $10g + 2$ **C** $10g + 10$ **D** $2 - 10g$

2.2 **5** Which expression is not equivalent to $(3x + 4)(2y - 5)$?

A $2y(3x + 4) - 5(3x + 4)$ **B** $6xy - 20$

C $6xy - 15x + 8y - 20$ **D** $3x(2y - 5) + 4(2y - 5)$

2.2 **6** Which expression cannot be expanded using the difference of two squares rule?

A $(x + 6)(x - 6)$ **B** $(7 - p)(7 + p)$

C $(d + 5)(d - 2)$ **D** $(q + 9)(-q + 9)$

2.2 **7** Which statement is incorrect?

A $(d + 3)(d - 7) = d^2 + 4d - 21$ **B** $(e - 6)^2 = e^2 - 12e + 36$

C $(m - 4)(m + 4) = m^2 - 16$ **D** $(2b + 5)(3b - 2) = 6b^2 + 11b - 10$

2.3 **8** The simplest factorised form of the expression $9x^2 - 3x^3$ is:

A $3(3x^2 - x^3)$ **B** $3x^2(3 - x)$

C $x^2(9 - 3x)$ **D** $3x(3x - x^2)$

2.3 **9** Which of the following is not a factor of $12x^2y - 24xy^2$?

A x^2 **B** $12xy$

C 12 **D** $6y$

2.3 **10** The expression $3x - 2$ is not a factor of which expression?

A $24xy + 15x - 18y - 10$ **B** $6(3x - 2) - y(3x - 2)$

C $3x(5y + 9) - 2(5y + 9)$ **D** $21x - 14$

2.4 **11** The expression $w^2 - 49$ factorises to:

A $(w - 7)^2$ **B** $(w - 7)(7 - w)$

C $(w + 7)(w - 7)$ **D** $(7 + w)(7 - w)$

2.4 **12** The simplest factorised form of the expression $36x^2z - 100y^2z$ is:

A $4z(5y + 3x)(5y - 3x)$ **B** $z(6x + 10y)(6x - 10y)$

C $4z(6x - 10y)(6x - 10y)$ **D** $4z(3x + 5y)(3x - 5y)$

2.4 **13** The expression $(a - 3)^2 - 9$ factorises to:

A $(a + 6)(a - 12)$ **B** $(a - 12)(a + 6)$

C $a^2 - 6a$ **D** $a(a - 6)$

2.5 **14** The expression $t^2 + 3t - 18$ factorises to:

A $(t + 9)(t - 2)$ **B** $(t - 6)(t + 3)$

C $(t + 6)(t - 3)$ **D** $(t - 9)(t + 2)$

2.5 **15** The simplest factorised form of the expression $3y^2 - 33y + 84$ is:

A $3(y - 7)(y - 4)$ **B** $(3y - 7)(3y - 4)$

C $(3y - 21)(y - 4)$ **D** $3(y + 7)(y + 4)$

2.5 **16** Which of the following is not equivalent to $b^2 - 8b + 16$?

A $(b - 4)^2$ **B** $(4 - b)^2$

C $(b - 4)(4 - b)$ **D** $(4 - b)(4 - b)$

Review questions 2.6C: Short answer

2.1 **1** Simplify each expression.

a $15t - 7t + 8t$ **b** $a - 7p - 11p + 12a$

c $3k + 5km - 7k - 15 + 2k - 4km$ **d** $6m^2n - 2m^2 + 7nm^2 + 11n^2 - 4mn^2 - 3m^2$

2.1 **2** Simplify each expression.

a $4xy \times 11xyz$ **b** $9mnp \times 2m^3p \times 4n^2$

c $15de \div (18df)$ **d** $11klmn \div (-22klm^2)$

2.2 **3** Expand each product.

a $4(z - 7)$ **b** $-8(5 - 3y)$ **c** $5x(7 - 6w)$

d $(u + 3)(t - 4)$ **e** $(5r + 6)(8 - 3v)$ **f** $(9p + 11q)(7m - 3n)$

2.2 **4** Expand and simplify each expression to remove the brackets.

a $5(a + 2) - 3(7 - a)$ **b** $(b - 11)(b + 2)$ **c** $(3c - 2)(4c - 5)$

d $(d + w)(d - w)$ **e** $(6 + e)^2$ **f** $(f - 9)^2$

2.2 **5** Expand and simplify each product.

a $x^4(x^3 - x^2)$ **b** $y^3z^4(y^9 - z^2)$ **c** $(x^5 - y^3)(x^6 + y^4)$

2.3 **6** Factorise each expression.

a $4a - 24$ **b** $36pq^2 + 144pq$ **c** $7d(8 - d) - 4(8 - d)$ **d** $5e + 15ef + 2 + 6f$

2.3 **7** Factorise each expression.

a $6z^5 - 5z^4$ **b** $9d^2y + 15dx^2$ **c** $-36r^{10}t^8u^6 - 96r^7t^8u^3$ **d** $a^5b^2c^7d^8 + a^6b^5c^9d^5 + a^4b^2c^8d^7$

2.3 **8** Factorise each expression.

a $24x^2(x^2 + 2) - 8x(x^2 + 2)$ **b** $(r + q)(t + 5) + (r + q)(p - 3)$ **c** $4x^2 + 16x - 6x - 24$

2.4 **9** Factorise each expression using the difference of two squares rule.

a $a^2 - 64$ **b** $121 - b^2$ **c** $36m^2 - 49n^2$

d $(p + 1)^2 - 4$ **e** $2e^2 - 32$ **f** $(f + 4)^2 - (f - 5)^2$

2.4 **10** Factorise each expression.

a $y^{10} - z^{10}$ **b** $q^2 - (q + 4)^2$ **c** $-r^2 + 100$ **d** $-64 + 81j^2$

2.4 **11** Factorise each expression.

a $\frac{196}{25}x^2 - \frac{169}{4}y^2$ **b** $\frac{144}{c^2} - \frac{225}{d^2}$ **c** $-\frac{u^2}{16} + \frac{36}{49}$ **d** $0.04v^2 - 1.21$

2.5 **12** Factorise each quadratic trinomial.

a $a^2 + 6a + 5$ **b** $b^2 - 7b + 12$ **c** $c^2 + 4c - 21$ **d** $d^2 - 16d - 36$

2.5 **13** Factorise each quadratic trinomial.

a $15 + 8x + x^2$ **b** $60 + 20x - 5x^2$ **c** $32x - 2x^2 - 120$ **d** $240 - 6x - 3x^2$

2.5 **14** Factorise each quadratic expression.

a $98 - 2x^2$ **b** $16y^2 + 64x^2$ **c** $x^2 - 10x + 25$ **d** $9x^2 - 36x$

Review questions 2.6D: Mathematical modelling

1 Expressions in the form $(x + a)^2 - b^2$ can be factorised using the difference of two squares rule.

a Factorise each of the following using the difference of two squares rule.

i $(x + 8)^2 - 9$ **ii** $(x - 10)^2 - 16$ **iii** $(x + 11)^2 - 25$

b Describe the connection between the numbers in the brackets in factor form and the values of a and b in $(x + a)^2 - b^2$ form.

c Expand each of the following.

i $(x + 8)^2 - 9$ **ii** $(x - 10)^2 - 16$ **iii** $(x + 11)^2 - 25$

d Describe the connection between the coefficient of x in the expanded form and the value of a in $(x + a)^2 - b^2$ form.

e Describe the connection between the constant in the expanded form and the values of a and b in $(x + a)^2 - b^2$ form.

f Fill in the spaces so that the expressions are equivalent.

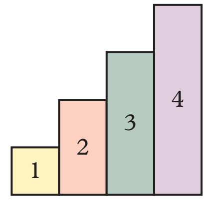
i $(x + \square)^2 - 4 = x^2 + 6x + 5 = (x + \square)(x + \square)$

ii $(x - 5)^2 - \square = x^2 - \square x + \square = (x - 9)(x - 1)$

iii $(x + \square)^2 - \square = x^2 + 14x - 51 = (x - \square)(x + \square)$

2 Malak and Samara are investigating how to quickly add the positive integers from 1 to n:
$1, 2, 3, 4, 5, \ldots, n$

Malak finds the expression $\frac{1}{2}n(n + 1)$ and Samara finds the expression $\frac{(n + 1)^2 - (n + 1)}{2}$, which will give the sum of the integers from 1 to 4, if $n = 4$. Visually, the first four positive integers can be represented using bars.



a Expand both of Malak's and Samara's expressions to show that they are equivalent.

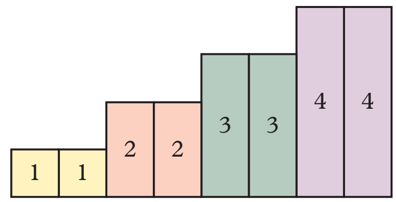
b Calculate the sum of the first five positive integers:

i by manually computing the sum $1 + 2 + 3 + 4 + 5$ **ii** by using one of the expressions.

c Calculate the sum of the first 100 positive integers.

d Malak decides to investigate the sum of the first n positive even numbers: $2, 4, 6, 8, 10, \ldots, n$

Malak finds the expression $n^2 + n$, which will give the sum of the positive even numbers from 2 to 8, if $n = 4$. Visually, the first four even numbers can be represented using double bars.



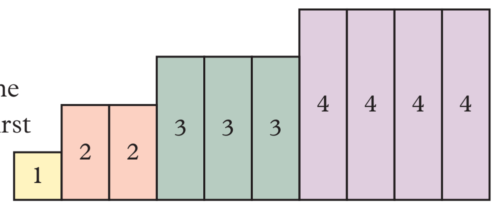
i Factorise $n^2 + n$.

ii Calculate the sum of the first 100 positive even numbers.

iii Describe the connection between the sum of the first n positive even numbers and the sum of the first n positive integers.

e Samara decides to investigate the sum of the first n square numbers: $1, 4, 9, 16, 25, \ldots, n$.

Samara finds the expression $\frac{1}{6}(n^2 + n)(2n + 1)$, which will give the sum of the square numbers from 1 to 16, if $n = 4$. Visually, the first four square numbers can be represented using squares of bars.



i Expand $\frac{1}{6}(n^2 + n)(2n + 1)$.

ii Calculate the sum of the first five square numbers using Method 1 and Method 2:

Method 1: manually computing the sum $1 + 4 + 9 + 16 + 25$

Method 2: using one of the expressions

iii Calculate the sum of the first 100 square numbers.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Simplify algebraic expressions involving addition and subtraction ☐ Simplify algebraic expressions involving multiplication and division	☐ Go back to Lesson 2.1 Simplifying
☐ Expand algebraic expressions of the form $a(b + c)$ ☐ Expand algebraic expressions of the form $(a + b)(c + d)$	☐ Go back to Lesson 2.2 Expanding
☐ Find the HCF of two or more algebraic terms ☐ Factorise algebraic expressions by identifying and dividing by the HCF ☐ Factorise algebraic expressions by grouping terms	☐ Go back to Lesson 2.3 Factorising using the HCF
☐ Factorise expressions in the form $a^2 - b^2$ ☐ Factorise complex expressions using the difference of two squares rule	☐ Go back to Lesson 2.4 Factorising the difference of two squares
☐ Factorise monic quadratic trinomials by finding the numbers required for a given product and sum ☐ Factorise monic quadratic trinomials by first taking out a common factor	☐ Go back to Lesson 2.5 Factorising quadratic expressions

3

Linear relationships

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

✔ Substitution
✔ Order of operations
✔ Expanding over one pair of brackets
✔ The Cartesian plane
✔ Finding the length of the hypotenuse

Curriculum links

- Recognise that the real number system includes the rational numbers and the irrational numbers... (VC2M9N01)
- Sketch linear graphs of equations in various algebraic forms, using the coordinates of 2 points, and solve linear equations (VC2M9A03)
- Find the gradient of a line segment, the midpoint of the line interval and the distance between 2 distinct points on the Cartesian plane (VC2M9A04)
- Use mathematical modelling to solve applied problems involving change, including financial contexts involving simple interest; formulate problems, choosing to use either linear or quadratic functions... (VC2M9A06)
- Experiment with the effects of the variation of parameters on graphs of related functions, using digital tools, making connections between graphical and algebraic representations, and generalising emerging patterns (VC2M9A07)

Solving linear equations

Learning intentions

By the end of this lesson you will be able to …

✔ solve linear equations using inverse operations.

Inter-year links

Year 7 6.6 Solving equations using inverse operations

Year 8 6.3 Solving equations with the unknown on both sides

Year 10 3.1 Solving linear equations

Linear equations

- An equation is a mathematical statement that shows equivalence between the expression on the left-hand side (LHS) and the right-hand side (RHS) of the equation.
- An equation may contain one or more pronumerals (such as x, y, a or b) that represent values, sometimes referred to as unknowns. If the pronumeral represents an unknown that can have more than one value, then it is called a variable.

$$\underbrace{3n + 10}_{\text{left-hand side of equation (LHS)}} \overset{\text{equals symbol}}{=} \underbrace{40}_{\text{right-hand side of equation (RHS)}}$$

- A **linear equation** is an equation containing only pronumerals that are raised to a power of 1 and no two such pronumerals are multiplied together.

Linear equations	Non-linear equations
$y = x$	$y = x^2$
$\frac{n}{4} = 2$	$\frac{mn}{4} = 2$
$2a - \frac{3}{5}b = 10$	$a^3 + 6b = 0$

Key content video
Solving linear equations

Solving linear equations using inverse operations

- A **solution** is a value for an unknown that makes the equation a true statement.
- To check whether a value is a solution to an equation, substitute that value into the equation to see whether it makes a true statement.
- To solve equations using **inverse operations**, identify and apply the inverse operation(s) required to reverse the operation(s) and isolate the unknown on the LHS of the equation ($x = \ldots$).
  → For equations involving more than one operation, inverse operations must be performed in the reverse order to BIDMAS.
  → A useful shorthand is to put the inverse operation in brackets to the right of the equation for each line of working out.
  → To solve an equation in which the unknown appears on both sides of the equation, use inverse operations to eliminate the pronumeral term from one side of the equation, then solve the equation using inverse operations.

Operation	Inverse operation
$+3$	-3
-3	$+3$
$\times 3$	$\div 3$
$\div 3$	$\times 3$

BIDMAS

For example,
$$\begin{aligned} 4x - 2 &= 3x + 1 \quad (-3x) \\ x - 2 &= 1 \quad (+2) \\ x &= 3 \end{aligned}$$

Worked example 3.1A Solving two-step equations using inverse operations

Solve the following equations using inverse operations.

a $\frac{x}{3} - 5 = 7$

b $10 = -2(x + 6)$

THINK	WRITE
a Identify and apply the inverse operations to both sides of the equation in the reverse order of BIDMAS, and then write the solution to the equation.	**a** $\frac{x}{3} - 5 = 7 \quad (+\,5)$ $\frac{x}{3} = 12 \quad (\times\,3)$ $x = 36$
b Divide both sides by -2, remembering that the sign changes when you multiply or divide by a negative number. Subtract 6 from both sides. Note that $-11 = x$ is the same as $x = -11$.	**b** $10 = -2(x + 6) \quad (\div\,(-2))$ $-5 = x + 6 \quad (-\,6)$ $x = -11$

Worked example 3.1B Solving three-step equations using inverse operations

Solve the following equations using inverse operations.

a $12 = \frac{7x}{2} - 9$

b $\frac{-x - 4}{3} = 1$

THINK	WRITE
a Identify and apply the inverse operations to both sides of the equation in the reverse order to BIDMAS, and then write the solution to the equation.	**a** $12 = \frac{7x}{2} - 9 \quad (+\,9)$ $21 = \frac{7x}{2} \quad (\times\,2)$ $42 = 7x \quad (\div\,7)$ $x = 6$
b Identify and apply the inverse operations to both sides of the equation in the reverse order to BIDMAS, then write the solution to the equation. Note that $-x = 7$ is not the solution, as the $-x$ has a coefficient of -1.	**b** $\frac{-x - 4}{3} = 1 \quad (\times\,3)$ $-x - 4 = 3 \quad (+\,4)$ $-x = 7 \quad (\div\,(-1))$ $x = -7$

Worked example 3.1C Solving equations with the unknown on both sides

Solve each equation for x.

a $4x + 7 = 2x - 3$

b $3(2x + 1) = -17 - 4x$

THINK	WRITE
a 1 Eliminate the pronumeral term from one side of the equation by subtracting the pronumeral with the smaller coefficient, $2x$, from both sides of the equation.	**a** $4x + 7 = 2x - 3 \quad (-\,2x)$
2 Solve the equation using inverse operations.	$2x + 7 = -3 \quad (-\,7)$ $2x = -10 \quad (\div\,2)$ $x = -5$

b **1** Remove the brackets by expanding the expression on the left-hand side of the equation. **2** Eliminate the negative pronumeral term by adding $4x$ to both sides of the equation. **3** Solve the equation using inverse operations.	**b** $3(2x + 1) = -17 - 4x$ $6x + 3 = -17 - 4x \quad (+\,4x)$ $10x + 3 = -17 \quad (-\,3)$ $10x = -20 \quad (\div\,10)$ $x = -2$

Helpful hints

- ✔ Writing the inverse operation beside the appropriate line of working out is a great way to keep track of your calculations! $\frac{x}{3} - 5 = 7 \quad (+\,5)$
- ✔ Remember that you have to apply the inverse operation to both sides of the equation.
- ✔ Remember to define your pronumerals before writing equations to represent the variables in worded problems. For example, let n = number of eggs in a carton or let w = weight of eggs (grams).
  Note: 'n = eggs in a carton' would not be correct, as a pronumeral must always represent a quantity. Similarly, 'w = weight of eggs' would not be correct because it doesn't specify a unit of measurement.

Learning pathways

△ 1–6, 7(a, c, f, g), 8–14, 16(a, b, c), 17, 21

□ 2–13, 15, 16, 18, 19, 23(a, b)

⬠ 2–5, 7(e–h), 9, 11–13, 16(c, d), 20–23

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Exercise 3.1A: Understanding and fluency

WE 3.1A **1** Solve the following equations using inverse operations.

a $4x + 5 = 29$ **b** $\frac{x+3}{2} = 4$ **c** $\frac{x}{4} - 2 = 7$ **d** $\frac{2x}{5} = 4$

e $-2(x + 6) = 28$ **f** $-17 = -3x - 5$ **g** $9 = \frac{x}{5} + 6$ **h** $\frac{x-4}{5} = -1$

i $-20 = 4(x - 2)$ **j** $-5x + 1 = 16$ **k** $\frac{-3x}{2} = -18$ **l** $\frac{x}{2} + 7 = 4$

WE 3.1B **2** Solve the following equations using inverse operations.

a $\frac{3x+4}{5} = 2$ **b** $\frac{-3x}{4} + 1 = 7$ **c** $\frac{5(x-1)}{2} = 20$ **d** $4(x + 3) - 2 = 30$

e $\frac{x-2}{6} - 3 = 0$ **f** $1 = \frac{11x}{4} + 23$ **g** $3 = \frac{-7x+6}{2}$ **h** $-5(x + 2) - 7 = 3$

i $2 = \frac{2(x+8)}{3}$ **j** $-16 = 4(5 - x) + 4$ **k** $6 = \frac{-x}{2} - 4$ **l** $\frac{2-3x}{5} = -2$

3 Solve:

a $\frac{x-2.9}{4} = 1$ **b** $\frac{x-4}{3} + 11.2 = 9$ **c** $\frac{3x}{4} - 1.9 = 6.2$ **d** $\frac{5x+2}{3} = -4.6$

4 Linear equations with brackets can be solved using two different methods.

a Solve $5(x - 2) = 20$ by first dividing both sides by 5.

b Another way to solve this equation is first to expand the expression on the left-hand side. Try this method. Do you obtain the same solution?

c Solve $5(x - 2) = 18$ by first dividing both sides by 5.

d Solve $5(x - 2) = 18$ by first expanding the expression on the left side.

e Which method did you find easier to use when solving $5(x - 2) = 18$? Explain.

5 Solve the following equations using the appropriate method from question **4**. Where relevant, write the solution as an improper fraction.

a $4(x-1)=8$ **b** $3(x+7)=-6$ **c** $2(x-3)=5$

d $5(x+4)=8$ **e** $-4(x+2)=-24$ **f** $-6(x-2)=1$

6 Consider a linear equation with the unknown in the denominator.

a What value of x would make $\frac{12}{x}=3$ a true statement?

b What is the first step to solving this equation using inverse operations?

c Solve $\frac{12}{x}=3$ using inverse operations. Use substitution to check that your solution is correct.

7 Solve the following equations using inverse operations. Use substitution to check that your solution is correct.

a $\frac{10}{x}=2$ **b** $\frac{21}{x}=-7$ **c** $\frac{-18}{x}=3$ **d** $\frac{-24}{x}=-6$

e $\frac{11}{x}=2$ **f** $\frac{8}{x}=-5$ **g** $\frac{9.6}{x}=2$ **h** $-\frac{1}{x}=3$

WE 3.1C **8** Solve each equation for x.

a $6x+5=4x+9$ **b** $3x-11=x+3$ **c** $3x-8=6x-5$ **d** $-15-2x=4x+3$

e $3x+7=-3-2x$ **f** $9x-4=10x-11$ **g** $10-x=7x-22$ **h** $-5x-5=11-x$

9 Solve each equation for x. Use substitution to check that your solution is correct.

a $3(x-2)=8x-1$ **b** $2x-1=5(x-2)$ **c** $2(3x-4)=5x-1$ **d** $-3(-2x-1)=-18-x$

e $4(x+3)=5(x+1)$ **f** $5(x+9)=-3(x-7)$ **g** $-6(1-x)=3(x-8)$ **h** $-6(x+1)=-10(x-3)$

10 Consider the equation $\frac{2x+5}{3}=\frac{x-4}{3}$.

a The first step to solving this equation using inverse operations is to multiply both sides by 3. Multiply both sides of the equation by 3 to obtain an equivalent equation.

b Solve the equation obtained in part **a**. Use substitution to check that your solution is correct.

11 Solve:

a $\frac{3x-4}{7}=\frac{x+6}{7}$ **b** $\frac{5x+2}{4}=\frac{2x-7}{4}$ **c** $\frac{2x+3}{11}=\frac{15-4x}{11}$ **d** $\frac{4(2x+1)}{9}=\frac{x-17}{9}$

12 Consider the equation $\frac{5x-1}{2}=\frac{x+5}{3}$.

a The first step to solving this equation using inverse operations is to find a common denominator. Write an equivalent equation where the fractions have a common denominator.

$$\frac{\_\_(5x-1)}{\_\_}=\frac{\_\_(x+5)}{\_\_}$$

b Solve the equation obtained in part **a**, using the method from question **10**. Use substitution to check that your solution is correct.

13 Use the method in question **12** to solve the following equations. Use substitution to check that your solution is correct.

a $\frac{4x+1}{5}=\frac{3x+2}{4}$ **b** $\frac{13x-8}{10}=\frac{4x-2}{3}$ **c** $\frac{2x-1}{3}=\frac{4x+11}{9}$ **d** $\frac{x+15}{4}=\frac{x+9}{6}$

Exercise 3.1B: Problem solving and reasoning

14 Trent is sharing a bag of jellybeans equally with three of his friends and finds that there are two left over. Consider the number of jellybeans that each person receives, including Trent, if there were 34 jellybeans in the bag.

a Define a pronumeral to represent the unknown quantity in this problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation using inverse operations.

d How many jellybeans did each person receive?

15 Lily is saving to buy a pair of sneakers that cost \$395. She is able to save \$70 per month. If she currently has \$115, consider the number of months it will take for Lily to buy the shoes.

a Define a pronumeral to represent the unknown quantity in the problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation using inverse operations. **d** In how many months can Lily buy the shoes?

16 For each problem, set up an equation and solve it using inverse operations.

a Darnell buys three model planes online for a total cost of \$590, which includes the delivery charge of \$35. What is the cost of each model plane?

b Emma and Maggie score a total of 35 goals in a basketball match. Maggie scores seven more goals than Emma. How many goals did Emma score?

c The perimeter of a rectangular playing field is 100 m. If the length is 12 m longer than the width, what are the dimensions of the playing field?

d The cost of hiring a party venue is \$500. There is a \$26 per person charge for food. If Kasey has a budget of \$2700 for the party, what is the maximum number of people that can attend?

17 Nylah and Josh have the same amount of money. Nylah buys seven sushi rolls and has \$1.50 left over. Josh buys four sushi rolls and has \$12 left over.

a If x represents the cost of one sushi roll (\$), which equation fits this situation?

A $7x + 150 = 4x + 12$ **B** $7x + 1.5 = 4x + 12$

C $7x - 1.5 = 4x - 12$ **D** $7x - 150 = 4x - 12$

b Solve the equation to find the cost of one sushi roll.

18 The length, l, of a rectangle is twice its width, w.

a Write an expression for the perimeter, P, of the rectangle in terms of l.

b If the perimeter of the rectangle is 60 m, find the width of the rectangle.

19 Violetta cooked sausages for the school sausage sizzle. Each sausage was placed in bread with tomato sauce. Twenty of these were sold with mustard. Half of those left were sold with fried onions. If there were 18 sausages sold with fried onions, how many sausages did Violetta cook?

20 One angle in a triangle is 30°. The second angle is twice the size of the third angle in the triangle. Find the size of the largest angle.

21 The sum of three consecutive integers is 13 more than the smallest of the three numbers. Identify the three numbers using algebra.

Exercise 3.1C: Challenge

22 The linear equations you have dealt with have sometimes been equal. That is, for a particular value of the pronumeral, the left- and right-hand sides are equal. The linear equations $2x + 5 = 2x + 5$, $3x + 5 = 3x + 5$ and $3x + 6 = 3x + 6$ are equal for all values of x. That is, regardless of the value of x, the equations are always equal. The linear equations $2x + 5 = 2x + 6$, $3x + 5 = 3x + 6$ and $3x + 4 = 3x + 6$ are equal for no value of x. That is, regardless of the value of x, the equations are never equal.

a For each of the following, determine whether the equation is always, sometimes or never equal.

i $2x + 5 + 3x = 1 + 5x + 4$ **ii** $7x - 2 + 4 = 3x + 4x - 6$ **iii** $2x + 3x - 5 = 6x - 9$

iv $15 - 12x = -3(4x + 5)$ **v** $4(6x + 10) = 8(3x + 5)$ **vi** $4(3 - 4x) + x = 4 - 3(5x - 3)$

b When are the two sides of a linear equation with one pronumeral equal?

c In which cases are the two sides of a linear equation with one pronumeral never equal?

23 Solve the following equations for x. Use substitution to check your solution.

a $\frac{3x+2}{2} + \frac{5-x}{3} = \frac{5x+1}{4} + \frac{7x-2}{6}$ **b** $\frac{5x-1}{2} = \frac{x+5}{3} + 1$ **c** $\frac{5x-1}{2} = \frac{x+5}{3} + x$

Online resources:

Interactive skillsheet — Solving equations using inverse operations

Interactive skillsheet — Solving equations with the unknown on both sides

Worksheet — Solving linear equations

CAS instructions — Solving equations

Quick quiz — 3.1

Lesson 3.2

Solving linear inequalities

Learning intentions

By the end of this lesson you will be able to ...

✔ solve and represent linear inequalities using a number line.

Inter-year links

Year 7 6.6 Solving equations using inverse operations

Year 8 6.9 Solving linear inequalities

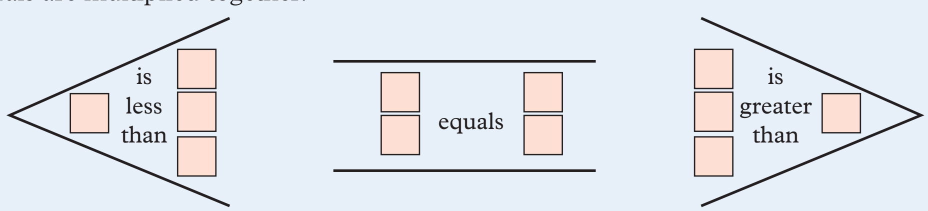
Year 10 3.7 Solving linear inequalities

Linear inequalities

- An **inequality** is a mathematical statement that compares the values of two unequal expressions. Inequalities are written using inequality signs, like the ones shown in the table below.

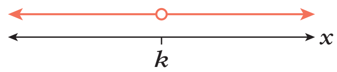
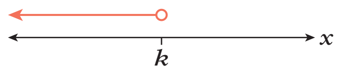
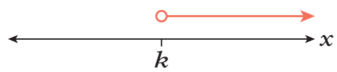
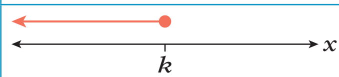
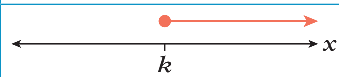
Inequality sign	Meaning	Example	Meaning of example
$\neq$	'is *not* equal to'	$x \neq 2$	x is *not* equal to 2
$<$	'is less than'	$x < 2$	x is less than 2
$>$	'is greater than'	$x > 2$	x is greater than 2
$\leq$	'is less than or equal to'	$x \leq 2$	x is less than or equal to 2
$\geq$	'is greater than or equal to'	$x \geq 2$	x is greater than or equal to 2

- A **linear inequality** is an inequality containing pronumerals only raised to the power of 1 and no two pronumerals are multiplied together.



Representing linear inequalities on a number line

- Inequalities involving a single variable can be represented on a number line, using rays and line segments to represent all of the values that satisfy the inequality (see the table on the next page). On a number line:
  → a line or line segment indicates that all the values the line passes through are included in the possible range of values for the variable
  → a ray (arrowed line) indicates that the possible values for the variable have no limit and continue to positive or negative infinity in the direction indicated
  → a closed circle (solid dot ●) indicates that the endpoint value is included, meaning the value is either 'less than or equal to' ($\leq$) or 'greater than or equal to' ($\geq$) the value represented by that point on the number line
  → an open circle (hollow dot ○) indicates that the value is not included, meaning the value is either 'less than' ($<$) or 'greater than' ($>$) the value represented by that point on the number line.

Inequality	Number line representation	Meaning
$x \neq k$		x is not equal to k
$x < k$		x is less than k
$x > k$		x is greater than k
$x \leq k$		x is less than or equal to k
$x \geq k$		x is greater than or equal to k

Solving linear inequalities

Key content video
Solving linear inequalities

- In most cases, if the same operation is performed on both sides of an inequality sign, the result is an equivalent linear inequality of a different form and the possible values of the unknown remain the same.
- Inverse operations can be used to solve linear inequalities. However, the inequality sign is reversed when both sides of the inequality are multiplied or divided by a negative number.

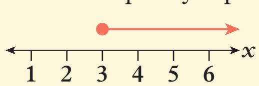
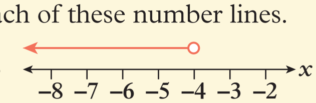
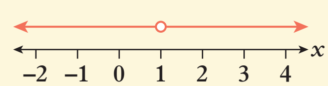
For example, $5 - 2x \leq 1 \quad (-5)$

$$-2x \leq -4 \quad (\div (-2))$$

$$x \geq 2$$

Worked example 3.2A Representing linear inequalities on a number line

Write the inequality represented by each of these number lines.

a 
b 
c 

THINK

a The line has a closed circle at 3, indicating that x can be equal to 3. The ray points to the right, indicating that x can be any value greater than 3. So x is greater than or equal to 3.

b The line has an open circle at –4, indicating that x cannot be equal to –4. The ray points to the left, indicating that x can be any value less than –4. So x is less than –4.

c The line has an open circle at 1, indicating that x cannot be equal to 1. A ray continues on in both directions, indicating that x can be any other value. So x is not equal to 1.

WRITE

a $x \geq 3$

b $x < -4$

c $x \neq 1$

Worked example 3.2B Solving linear inequalities

Solve each of these inequalities.

a $\frac{x}{4} - 8 \le 0$ **b** $-2 > 10x + 3$ **c** $3 > -1 - \frac{x}{7}$

THINK	WRITE
a 1 Apply the inverse operations to both sides in the correct order.	**a** $\frac{x}{4} - 8 \le 0$ $(+\,8)$ $\frac{x}{4} \le 8$ $(\times\,4)$ $x \le 32$
2 Check your solution. Substitute $x = 32$ into the LHS of the inequality. The result should equal the right-hand side (RHS). Then substitute any value for x that is less than 32 into the original inequality to check the inequality sign. The result should be less than or equal to the RHS.	Check by substituting $x = 32$ into $\frac{x}{4} - 8 \le 0$. LHS $= \frac{32}{4} - 8$ $= 0$ $=$ RHS (For $x = 32$, LHS = RHS as required.) Check by substituting a value less than 32 (for example, $x = 8$). LHS $= \frac{8}{4} - 8$ $= -6$ $-6 < 0$ (For $x < 32$, LHS < RHS as required.)
b 1 Apply the inverse operations to both sides in the correct order.	**b** $-2 > 10x + 3$ $(-\,3)$ $-5 > 10x$ $(\div\,10)$ $-\frac{1}{2} > x$
2 Write the solution with x on the LHS, reverse the inequality sign if needed.	$x < -\frac{1}{2}$
3 Check your solution. Substitute $x = -\frac{1}{2}$ into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is less than $-\frac{1}{2}$ into the original inequality to check the inequality sign. The result should be less than the LHS.	Check by substituting $x = -\frac{1}{2}$ into $-2 > 10x + 3$. RHS $= 10(-\frac{1}{2}) + 3$ $= -2$ $=$ LHS (For $x = -\frac{1}{2}$, LHS = RHS as required.) Check by substituting a value less than $-\frac{1}{2}$ (for example, $x = -2$). RHS $= 10(-2) + 3$ $= -17$ $-2 > -17$ (For $x < -\frac{1}{2}$, LHS > RHS as required.)
c 1 Apply the inverse operations to both sides in the correct order.	**c** $3 > -1 - \frac{x}{7}$ $(+\,1)$ $4 > -\frac{x}{7}$ $(\times\,(-7))$ $-28 < x$
2 Write the solution with x on the LHS, reverse the inequality sign if needed.	$x > -28$
3 Check your solution. Substitute $x = -28$ into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is greater than -28 into the original inequality to check the inequality sign. The result should be less than the LHS.	Check by substituting $x = -28$ into $3 > -1 - \frac{x}{7}$. RHS $= -1 - \frac{-28}{7}$ $= -1 + 4$ $= 3$ $=$ LHS (For $x = -28$, LHS = RHS as required.) Check by substituting a value greater than -28 (for example, $x = 0$). RHS $= -1 - \frac{0}{7}$ $= -1$ $3 > -1$ (For $x > -28$, LHS > RHS as required.)

Worked example 3.2C Solving inequalities with the unknown on both sides

Solve each of these inequalities.

a $7x - 2 \geq 3x - 10$ **b** $4 - 5x < 3 - 8x$

THINK

1. Use inverse operations to remove the pronumeral on the side where the coefficient is a number less than the coefficient on the other side.
2. Identify and apply the correct inverse operations, in order, to both sides of the inequality.
3. Check your solution by substituting appropriate values for x into the original inequality to see if true inequalities result.

WRITE

a

$7x - 2 \geq 3x - 10 \quad (-\,3x)$

$4x - 2 \geq -10 \quad (+\,2)$

$4x \geq -8 \quad (\div\,4)$

$x \geq -2$

Check by substituting $x = -2$ into $7x - 2 \geq 3x - 10$.

$\text{LHS} = 7(-2) - 2$

$= -16$

$\text{RHS} = 3(-2) - 10$

$= -16$ (For $x = -2$, LHS = RHS as required.)

Check by substituting a value greater than -2 (for example, $x = 0$).

$\text{LHS} = 7(0) - 2$

$= -2$

$\text{RHS} = 3(0) - 10$

$= -10$

$-2 > -10$ (For $x > -2$, LHS > RHS as required.)

b

$4 - 5x < 3 - 8x \quad (+\,8x)$

$4 + 3x < 3 \quad (-\,4)$

$3x < -1 \quad (\div\,3)$

$x < -\frac{1}{3}$

Check by substituting $x = -\frac{1}{3}$ into $4 - 5x < 3 - 8x$.

$\text{LHS} = 4 - 5\left(-\frac{1}{3}\right)$

$= \frac{17}{3}$

$\text{RHS} = 3 - 8\left(-\frac{1}{3}\right)$

$= \frac{17}{3}$ (For $x = -\frac{1}{3}$, LHS = RHS as required.)

Check by substituting a value less than $-\frac{1}{3}$ (for example, $x = -1$).

$\text{LHS} = 4 - 5(-1)$

$= 9$

$\text{RHS} = 3 - 8(-1)$

$= 11$

$9 < 11$ (For $x < -\frac{1}{3}$, LHS < RHS as required.)

Helpful hints

- ✔ Be careful when multiplying and dividing inequalities by negative numbers.
- ✔ If you need to swap the LHS and RHS of an inequality, remember to reverse the inequality sign so that it is still pointing towards the lesser expression. For example, if $5 \leq x$, then $x \geq 5$.
- ✔ When solving linear inequalities with the unknown on both sides, always remove the pronumeral on the side where the coefficient is less. That way you will always have a positive value for the remaining coefficient. Remember, $4 > 2$ but $-4 < -2$.

Learning pathways

1-4, 5(1st, 2nd columns), 6, 7, 8(a–e), 9, 10(a–c), 11, 13	2–3(2nd column), 5–7(3rd, 4th columns), 8(e–i), 9–13, 14	2–3(2nd column), 6–7(e–h), 8(e–i), 9, 10, 12, 14–18

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Exercise 3.2A: Understanding and fluency

1 Write each of the following statements as an inequality.

a Two is less than three.

b Three is greater than two.

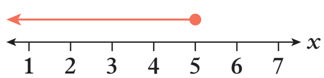
c Three is greater than or equal to two.

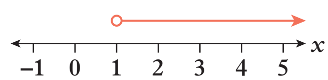
d Negative three is less than or equal to negative two.

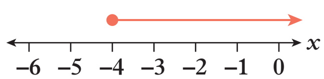
e Three is not equal to negative three.

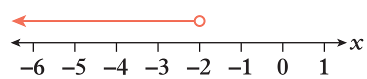
f Negative three is not equal to three.

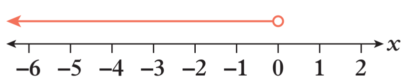
WE 3.2A **2** Write the inequality represented on each of these number lines.

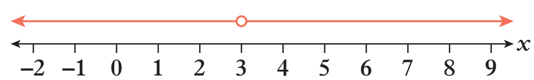
a 

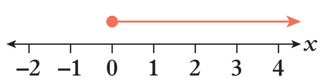
b 

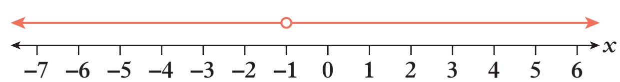
c 

d 

e 

f 

g 

h 

3 For each of these inequalities, show the possible values of x on a number line.

a $x > 4$ **b** $x \le 3$

c $x \ge -2$ **d** $x < 0$

e $x \ne 2$ **f** $x \ne -2$

4 Apply operations **i–viii** to each side of the following inequalities. Ensure the correct inequality symbol is written between the two numbers.

a $-4 < 9$ **b** $x \ge 6$

i Add five **ii** Subtract nine

iii Multiply by three **iv** Multiply by one-half

v Multiply by negative three **vi** Multiply by negative one-half

vii Divide by three **viii** Divide by negative three

WE 3.2B **5** Solve each of these inequalities.

a $x + 5 > 19$ **b** $\frac{x}{10} \le 30$ **c** $-20 > 4x$ **d** $-3x \ge -15$

e $7 < -\frac{x}{10}$ **f** $\frac{x}{5} + 10 \le 11$ **g** $2x - 9 \ge -11$ **h** $26 > 14 - 4x$

i $-x + 3 \ge 12$ **j** $5(x + 3) \le 50$ **k** $\frac{x-8}{15} > -3$ **l** $1 < 10x + 14$

m $12 - 14x > 14$ **n** $-17 \le -6x + 12$ **o** $-6(x - 4) < 13$ **p** $-11(3x + 5) \ge 40$

6 Solve each of these inequalities and show the solution on a number line each time.

a $3x + 2 \ge -13$ **b** $1 - 7x < -6$ **c** $5 - \frac{x}{6} \le 2$ **d** $\frac{x-4}{2} < 5$

e $\frac{7-x}{3} \ge -1$ **f** $\frac{2+x}{5} > 3$ **g** $\frac{3x}{2} - 5 \le 4$ **h** $\frac{8-x}{7} - 3 < -2$

WE 3.2C **7** Solve each of these inequalities. Use substitution to check your solutions.

a $4x - 3 > 2x + 5$ **b** $7x + 1 \leq 3x - 7$ **c** $5x - 9 \geq 9 - x$ **d** $2x + 4 < 5x - 8$

e $3 - x > 4x - 2$ **f** $x + 11 \geq 7 - 3x$ **g** $8 - 3x \leq 18 - 5x$ **h** $1 - 6x > -3x - 5$

8 Solve each of these inequalities by first expanding to remove brackets.

a $2(x - 3) > x + 5$ **b** $4x - 7 \leq 3(x + 2)$ **c** $5(x + 1) \geq -3x - 11$

d $3(2x - 5) < 7x + 4$ **e** $9(x + 5) < 10(x + 6)$ **f** $3(3x - 1) \leq -2(x - 4)$

g $4(x + 3) \geq 2(5x - 3)$ **h** $2(2 - x) > 3(1 - 2x)$ **i** $5(3 - 2x) \leq -4(2x - 7)$

9 Solve each of these inequalities. Use substitution to check your solutions.

a $\frac{3x + 2}{7} \leq \frac{2x - 5}{7}$ **b** $\frac{2 - x}{3} < \frac{6x - 5}{3}$ **c** $\frac{1 - 4x}{2} \geq \frac{11 - 5x}{4}$

d $\frac{x + 4}{5} > \frac{x + 2}{3}$ **e** $\frac{x - 4}{7} \geq \frac{4 - x}{2}$ **f** $\frac{4x - 1}{6} < \frac{3x + 1}{4}$

10 Solve the following inequalities.

a $1.2x - 5.4 > 10.8$ **b** $3.2 - 4.2x < -5.2$ **c** $2x - 3.3 \geq -0.3$

d $-6.2x + 5.8 > -12.8$ **e** $20.1 - 2x \leq 17.6 + 3x$ **f** $32.9x + 14.2 < 25.3x - 12.4$

Exercise 3.2B: Problem solving and reasoning

11 Write an inequality statement to represent each of the following situations. Use a pronumeral for the unknown quantity each time.

a A boat is sold for at least \$750 000. What could the boat have sold for?

b To fit in an economy class seat on an aeroplane, a person must be less than 200 cm tall. What could be the height of a person in such a seat?

c A Vespa cannot travel faster than 55 km/h. At what speeds could a Vespa travel?

d To fit in a new sleeping bag, a person must be less than 196 cm tall. How tall could a person be to fit in the sleeping bag?

12 Emily and Klaus are selling watermelons at a market. They start with 20 melons and agree to share any that are left at the end of the day.

a Write an expression for the number of melons they each will take home if they sell x watermelons.

b Emily and Klaus aim to take home no more than three watermelons each. Use your expression from part **a** to write an inequality for this situation and then solve it.

c How many watermelons can they sell to meet their goal?

13 Ella runs a business selling candles where customers can pay for delivery or pick them up from her store. She sells each candle for \$2.50 and can mail a package containing one candle for \$3, two candles for \$5.50 and four candles for \$10.

A customer has \$26 to spend on candles. They live too far away to visit the store, so they must pay the delivery fee.

a How many candles could they buy if each candle is packaged individually?

b If the candles in part **a** are packaged in groups of two, can the customer afford an extra candle and the delivery fee?

c The customer decides to prioritise buying packs of four candles instead. If they have enough money left over, they will buy a pack of two candles or a single candle. How many candles can the customer afford now? Make sure to include the delivery fee in your calculations.

14 Hassan is deciding how many packs of playing cards he should buy. Each pack costs \$3 and he has \$25 in his wallet.

a Write an inequality to represent this situation and then solve it.

b List the number of packs of playing cards Hassan could possibly buy.

15 When performing the same operations on both sides of an inequality, the direction of the inequality sign sometimes needs to be changed.

a Square all parts of the following inequalities. Ensure the correct inequality symbols are written between the numbers.

i $2 < 3$ **ii** $-2 > -3$ **iii** $2 > -3$ **iv** $-2 < 3$

v $\frac{1}{2} > \frac{1}{3}$ **vi** $-\frac{1}{2} < \frac{1}{3}$ **vii** $-2 \neq 2$

b Explain why we need to be careful when squaring both sides of an inequality.

c Apply the reciprocal to all parts of the following inequalities. Ensure the correct inequality symbols are written between the numbers.

i $3 > \frac{1}{2}$ **ii** $-3 < \frac{1}{2}$

iii $-3 < -\frac{1}{2}$ **iv** $3 > -\frac{1}{2}$

v $\frac{1}{3} < \frac{1}{2}$ **vi** $-\frac{1}{3} < \frac{1}{2}$

d Explain why we need to be careful when applying the reciprocal to both sides of an inequality.

Exercise 3.2C: Challenge

16 A company makes and sells two products: X and Y. It costs the company \$4 for each of product X to be made and \$5 for each of product Y. The company needs the cost of producing the two products to be no more than \$100 per day. Let x be the number of product X the company makes and let y be the number of product Y the company makes.

a Write an inequality describing the cost of producing products X and Y for one day.

b State two other inequalities that must be true given that x and y represent the number of each product.

c Determine the greatest number of product X that could be produced in one day.

d Determine the greatest number of product Y that could be produced in one day.

e Determine the greatest number of product X that could be produced in one day if 8 of product Y are produced.

17 Solve each of these inequalities.

a $\frac{2(4-x)}{5} - 3 \leq 3$ **b** $\frac{3(7x-4)}{2} - 8 < 6x + 4$

c $\frac{5x-1}{3} + \frac{x-2}{4} \geq 3$ **d** $\frac{x-4}{8} - \frac{x+3}{2} > 1$

e $\frac{4(2x+1)}{5} + \frac{7-2x}{3} \geq 2x + 1$ **f** $\frac{3x+4}{2} - \frac{x+4}{6} \leq \frac{x-5}{3}$

18 Solve each of these inequalities.

a $-3 \leq 5 - 2x \leq 7$ **b** $15 < \frac{5(1-2x)}{2} < 25$

c $-2 < 4 - \frac{x}{2} < 8$ **d** $1 \leq \frac{5}{2} - \frac{5x}{3} < 10$

Online resources:

- **Interactive skillsheet** Representing linear inequalities on a number line
- **Interactive skillsheet** Solving linear inequalities
- **Worksheet** Solving linear inequalities
- **Desmos activity** Point collector
- **Quick quiz** 3.2

Plotting linear relationships

Learning intentions

By the end of this lesson you will be able to ...

✔ plot linear relationships using tables of values and equations.

Inter-year links

Support	The Cartesian plane
Year 7	5.4 The Cartesian plane
Year 8	6.5 Plotting linear relationships
Year 10	3.3 Sketching linear graphs

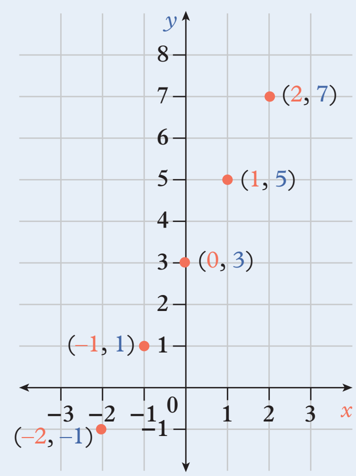
Linear relationships

Key content video
Independent and dependent variables

- The relationship between two variables can be represented by an algebraic equation, a table of values, a set of **coordinate points**, or a graph.
- A **linear relationship** is a relationship between two variables, the independent variable and the dependent variable, which produces a linear graph.
- A **linear graph** is a straight line on a **Cartesian plane**.
- The **independent variable** is the quantity that does not depend on the other variable.
- The value of the **dependent variable** depends on the value of the independent variable.
- It is conventional for the independent variable to be:
  → listed in the top row of a table of values

x	−2	−1	0	1	2
y	−1	1	3	5	7

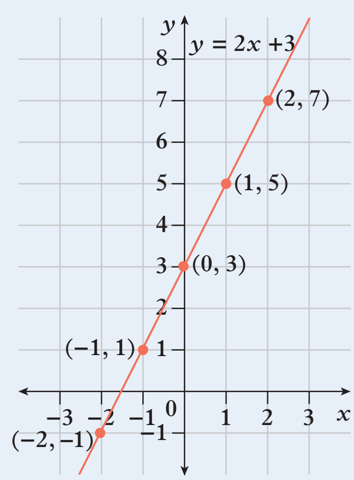
  → listed first in a pair of coordinates
  (−2, −1), (−1, 1), (0, 3), (1, 5), (2, 7)
  → shown on the horizontal axis.
  Independent variable: x
  Dependent variable: y



Plotting linear relationships

Key content video
Plotting linear relationships

- A **plot** is composed of individual coordinate points.
  A linear graph is a continuous line made up of an infinite number of coordinate points.
- To sketch a graph from an equation, follow these steps:
  1. Construct a table of values by selecting values for x, then substituting each value of x into the equation to find the corresponding value of y.
  2. Write out the coordinate points listed in the table.
  3. Plot the coordinate points on the Cartesian plane.
  4. Join the points using a straight line.



Worked example 3.3A Plotting linear relationships from a table of values

Use the table of values to construct a plot of the relationship between x and y. Is the relationship linear or non-linear?

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

THINK

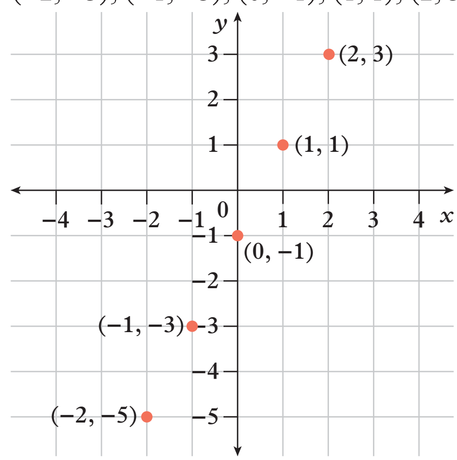
1 Write out the coordinate points listed in the table.

2 Plot the points on the Cartesian plane.

3 Consider whether the points form a straight line. As the points form a straight line, the relationship is linear.

WRITE

(−2, −5), (−1, −3), (0, −1), (1, 1), (2, 3)



The relationship is linear.

Worked example 3.3B Graphing linear relationships from an equation

Plot a graph of $y = -x - 3$ by first completing a table of values for x from −3 to 2.

THINK

1 Construct a table of values for x from −3 to 2. Substitute each value of x into the equation to find the corresponding value of y.

2 Write out the coordinate points listed in the table.

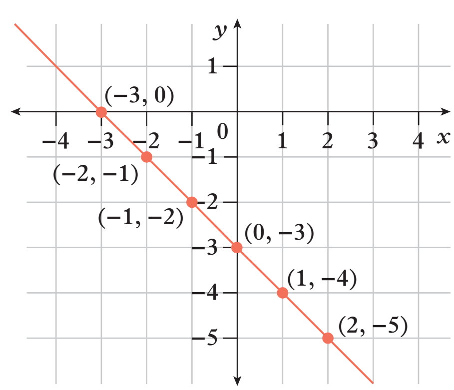
3 Plot the points on the Cartesian plane.

4 Join the points with a straight line.

WRITE

x	−3	−2	−1	0	1	2
y	0	−1	−2	−3	−4	−5

(−3, 0), (−2, −1), (−1, −2), (0, −3), (1, −4), (2, −5)



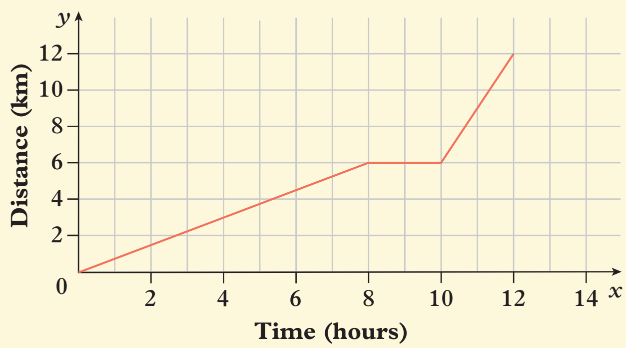
Worked example 3.3C Identifying independent and dependent variables

Identify the independent and dependent variables in the following relationships. Explain why for each case.

a

Age (years)	10	11	12	13
Height (cm)	140	148	154	160

b



c The number of oranges you put in a juicer and the volume of orange juice produced by the juicer

THINK

a In a table of values, the values of the independent variable are listed in the first row and the values of the dependent variable are listed in the second row. To explain why height depends on age, think about which variable cannot change without the other.

b In a graph, the independent variable is shown on the horizontal axis and the dependent variable is shown on the vertical axis. To explain why distance depends on time, think about which variable cannot change without the other.

c The volume of orange juice *depends* on the number of oranges you put in a juicer. To explain why, think about which one is required to make the other.

WRITE

a Independent variable: *Age* because a person ages constantly, but their height might not change.
Dependent variable: *Height* because as a person ages their height changes.

b Independent variable: *Time* because time passes regardless of an object's movement.
Dependent variable: *Distance* because something cannot move without time passing.

c Independent variable: *Number of oranges* because oranges cannot be made from orange juice.
Dependent variable: *Volume of orange juice* because the amount of orange juice depends on the number of oranges put in the juicer.

Helpful hints

- ✔ Remember that in Cartesian coordinates, the x-coordinate is always listed first, followed by the y-coordinate.
- ✔ When constructing plots and sketches, always label your x- and y-axes and label your graph with the equation of the graph.

$(-4, 1)$

x-coordinate: horizontal distance from the origin

y-coordinate: vertical distance from the origin

Learning pathways

△ 1, 2(a–e), 3–9, 12 ▢ 2(f–i), 3–10, 12–14 ⬠ 2(h, i), 3, 5–11, 13–15

ANS p464

Exercise 3.3A: Understanding and fluency

WE 3.3A **1** Consider the following table of values.

a Construct a plot of each relationship between x and y.

i

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4

ii

x	−3	−2	−1	0	1	2	3
y	11	6	3	2	3	6	11

iii

x	−3	−2	−1	0	1	2	3
y	10	9	8	7	6	5	4

iv

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

b Classify each relationship as linear or non-linear.

WE 3.3B **2** For each of the following linear relationships, construct a table of values for x from −3 to 3 and then write out the coordinate points listed in the table.

a $y = x + 2$ **b** $y = x - 4$ **c** $y = 3 - x$

d $y = 2 - x$ **e** $y = -x - 3$ **f** $y = 4x$

g $y = 2x + 1$ **h** $y = 3x - 2$ **i** $y = 4 - 2x$

WE 3.3C **3** Identify the independent and dependent variables in the following relationships. Explain why for each case.

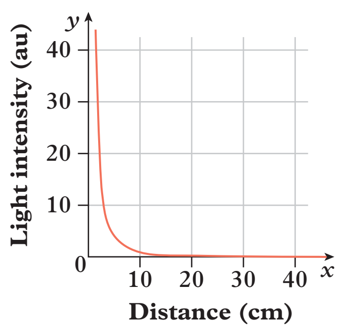
a

Radius (cm)	1	2	3	4
Area (cm²)	3.14	12.57	28.27	50.27

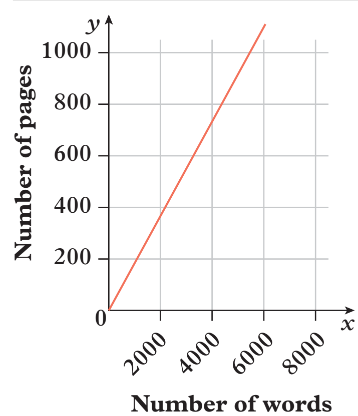
b

Number sold	20	40	60	80
Revenue ($)	100	200	300	400

c



d



e The *amount of breath* used to play a note on a flute and the *volume* of the note produced

f The *chance* of drawing the ace of spades from a standard deck after removing cards that are not the ace of spades and the *number of cards* remaining in the deck

4 Consider each of the following relationships.

a Complete the following tables of values using the equations provided.

i $x + y = 24$

x	0	3	6	9	12	15	18
y							

ii $xy = 24$

x	−12	−6	−2	2	6	12
y						

iii $y = x(x + 2)$

x	−3	−2	−1	0	1	2	3
y							

iv $x = 4y - 2$

x							
y	−3	−2	−1	0	1	2	3

b Plot the coordinates from the tables of values and classify each relationship as linear or non-linear. Do not attempt to join the points with curves or lines.

Exercise 3.3B: Problem solving and reasoning

5 Explore the similarities and differences of the following linear relationships.

a Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = x$ **ii** $y = 2x$ **iii** $y = 3x$ **iv** $y = 4x$

b Describe the similarities and differences between each of the four graphs.

c Describe the relationship between the coefficient of x and the steepness of the corresponding graph.

d Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

i $y = 5x$ **ii** $y = 3.5x$

e Use your answers to parts **b** and **c** to describe the graph of $y = 0.5x$.

6 Explore the similarities and differences of the following linear relationships.

a Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = -x$ **ii** $y = -2x$ **iii** $y = -3x$ **iv** $y = -4x$

b How do these graphs differ from those sketched in question **5**?

c Describe the relationship between the coefficient of x and the feature of the graph identified in part **b**.

d Without using a table of values, sketch the graphs of the following relationships on the same set of axes.

i $y = -5x$ **ii** $y = -1.5x$

7 Explore the similarities and differences of the following linear relationships.

a Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = x$ **ii** $y = x + 1$ **iii** $y = x + 2$ **iv** $y = x + 3$

b Describe the similarities and differences between each of the four graphs.

c Describe the relationship between the equations and corresponding graphs.

d Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

i $y = x + 4$ **ii** $y = x + 1.5$

8 Explore the similarities and differences of the following linear relationships.

a Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = x$ **ii** $y = x - 1$ **iii** $y = x - 2$ **iv** $y = x - 3$

b Describe the similarities and differences between each of the four graphs.

c Describe the relationship between the equations and corresponding graphs.

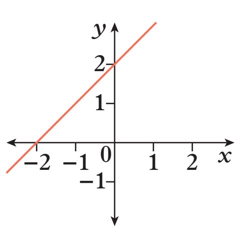
d Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

i $y = x - 4$ **ii** $y = x - 5$

9 Use your answers to questions **5** to **8** to match each equation with its corresponding graph.

a $y = x + 2$ **b** $y = 2x$ **c** $y = -x - 1$ **d** $y = -0.5x$

A



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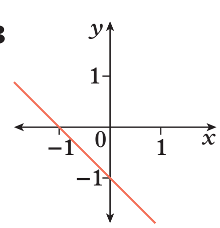
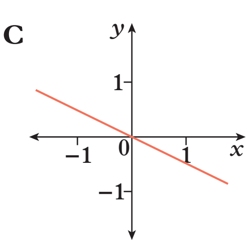
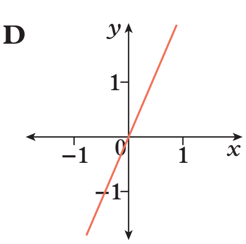


C



D



10 Use your answers to questions **5** to **8** to sketch the graphs of the following linear relationships. In each part, sketch the two graphs on the same set of axes.

a the graphs of **i** $y = 2x$, and **ii** $y = 2x + 1$

b the graphs of **i** $y = -x$, and **ii** $y = -x - 1$

c Describe the relationship between the first and second graph in parts **a** and **b**.

11 Javier is training for the cross-country race. He runs laps of the oval every other day and records the number of laps he runs.

Day	1	3	5	7	9	11	13
Number of laps	3	3.5	4	4.5	5	5.5	6

a Identify the independent variable and dependent variable.

b The table above can be used to sketch a graph on a Cartesian plane.

i What feature(s) of the table of values indicates that the graph is likely to be linear?

ii Hence, what is the minimum number of points you need to plot to sketch the line?

c Sketch the relationship on a Cartesian plane and extend the line to day 25.

d Use the graph to determine:

i the number of laps Javier should run on day 17 **ii** the day Javier should run 8 laps.

12 A bus is hired for a school trip to the snow. The school pays \$250 towards the bus hire and charges each student an additional \$40.

a If m is the total amount of money collected for bus hire and n students go on the trip, write an equation for the relationship between m and n.

b Identify the independent and dependent variables in the relationship.

c Calculate how much money would be collected for bus hire if:

i 0 students go on the trip **ii** 20 students go on the trip.

d Use your answer to part **c** to sketch a graph of this relationship using a scale from 0 to 30 along the horizontal axis.

e Is the relationship linear? Explain.

f Use the graph to find the total amount of money collected if 30 students go on the trip.

g Use the graph to determine how many students need to go on the trip to collect a total of \$850 for bus hire.

h The hire cost of the bus is \$1300. Use the graph to determine the minimum number of students who need to go on the trip to cover the hire cost.

13 The formula $F = 1.8C + 32$ describes the relationship between temperatures in degrees Celsius, C, and temperatures in degrees Fahrenheit, F.

a Plot the graph of this relationship. Show a scale from −50 to 50 along the horizontal axis.

b Use the graph to find the temperature in °F for 30°C.

c Use the graph to find the temperature in °C for −22°F.

Exercise 3.3C: Challenge

14 Use the graph in question **13** to find where the temperature in °C has the same numerical value as the matching temperature in °F.

15 For each of the following pairs of linear relationships, plot two graphs on the same Cartesian plane to determine the coordinates where the two lines intersect.

a $y = x + 1$ and $y = 2x$ **b** $y = 5x$ and $y = \frac{1}{3}x$

c $y = \frac{1}{2}x - 1$ and $y = -x + 2$ **d** $y + 2x = 12$ and $3y + x = 16$

Online resources:

- **Interactive skillsheet** Plotting linear relationships
- **Interactive skillsheet** Classifying variables
- **Worksheet** Plotting graphs of linear relationship
- **Investigation** How accurate is a 'rule of thumb' temperature formula?
- **CAS instructions** Graphing functions
- **Quick quiz** 3.3

Gradient and intercepts

Learning intentions

By the end of this lesson you will be able to ...

✔ identify the gradient and the x- and y-intercepts of a linear graph.

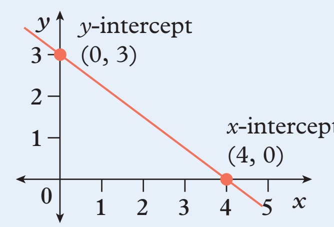
Inter-year links

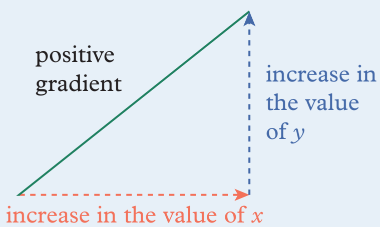
Support	The Cartesian plane
Year 7	5.4 The Cartesian plane
Year 8	6.8 Finding linear equations
Year 10	3.3 Sketching linear graphs

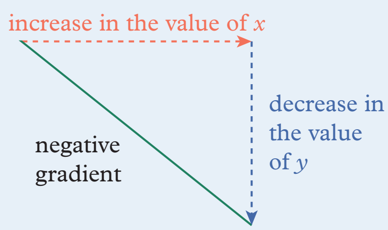
Features of linear graphs

Key content video
Gradient and intercepts

- The features of a linear graph include its x-intercept, y-intercept and gradient.
- The **x-intercept** is the point where the graph crosses the x-axis.
- The **y-intercept** is the point where the graph crosses the y-axis.
- The **gradient** is a numerical measure of the slope of the graph. It is also referred to as the rate of change between the two variables.

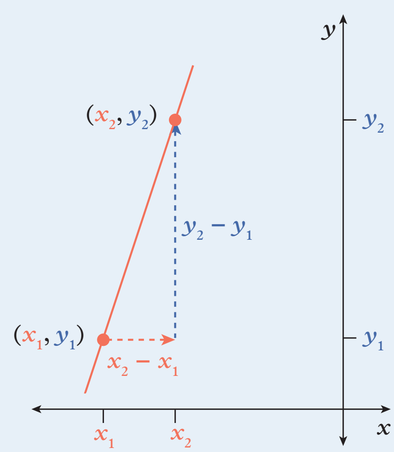




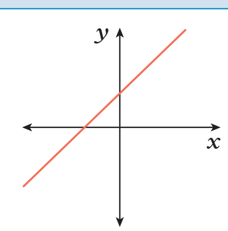
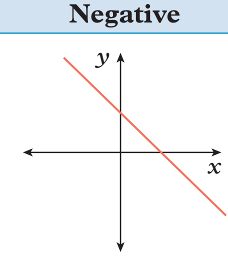
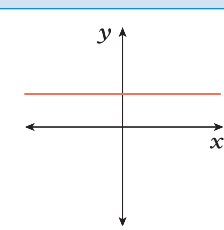
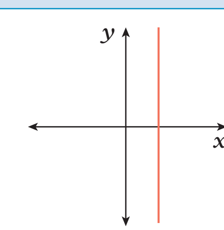


- The gradient of a linear graph is constant.
- The value of the gradient is the number of units that the graph increases in the y-direction for every one unit that it increases in the x-direction.
- The formula for the gradient, m, between any two points, (x_1, y_1) and (x_2, y_2), is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



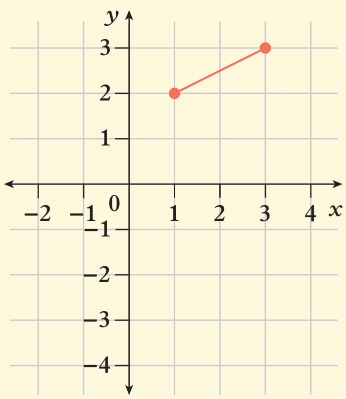
- Gradients can be positive, negative, zero or undefined.

Positive	Negative	Zero	Undefined
			

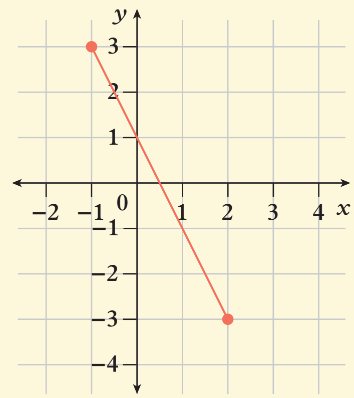
Worked example 3.4A Determine the gradient of a line segment

Find the gradient of each of the following line segments.

a



b



c the line segment between the points (3, 2) and (−2, 5)

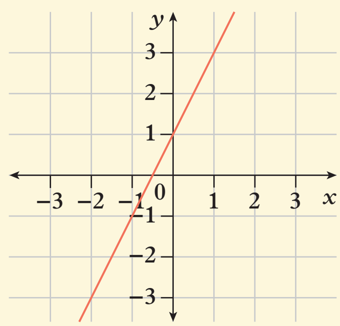
THINK	WRITE
a **1** Identify the coordinates of the endpoints of the line segment and label one as (x_1, y_1) and the other as (x_2, y_2).	**a** $(x_1, y_1) = (1, 2), (x_2, y_2) = (3, 3)$
2 Substitute the points into the gradient formula and simplify the result where possible.	$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-2}{3-1} = \dfrac{1}{2}$
b **1** Identify the coordinates of the endpoints of the line segment and label one as (x_1, y_1) and the other as (x_2, y_2).	**b** $(x_1, y_1) = (-1, 3), (x_2, y_2) = (2, -3)$
2 Substitute the points into the gradient formula and simplify the result where possible.	$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3-3}{2-(-1)} = \dfrac{-6}{3} = -2$
c **1** Identify the coordinates of the endpoints of the line segment and label one as (x_1, y_1) and the other as (x_2, y_2).	**c** $(x_1, y_1) = (3, 2), (x_2, y_2) = (-2, 5)$
2 Substitute the points into the gradient formula and simplify the result where possible.	$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-2}{-2-3} = \dfrac{3}{-5} = -\dfrac{3}{5}$

Worked example 3.4B Determining the gradient, x-intercept and y-intercept of a graph

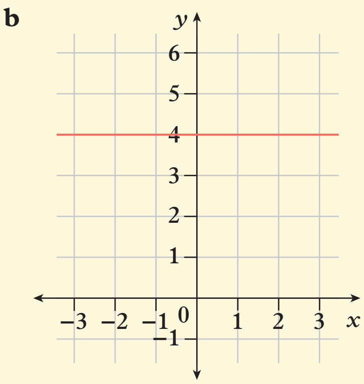
For each of the linear graphs shown, determine the:

i gradient **ii** x-intercept **iii** y-intercept.

a



b



THINK	WRITE
a i Identify any two points on the line and label one as (x_1, y_1) and the other as (x_2, y_2). Substitute the points into the gradient formula and simplify the result where possible.	**a i** $(x_1, y_1) = (-1, -1), (x_2, y_2) = (1, 3)$ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{3 - (-1)}{1 - (-1)}$ $= \dfrac{4}{2}$ $= 2$
ii State the coordinates of the point at which the graph crosses the x-axis.	**ii** x-intercept: $\left(-\frac{1}{2}, 0\right)$
iii State the coordinates of the point at which the graph crosses the y-axis.	**iii** y-intercept: $(0, 1)$
b i The graph is a horizontal line, so it has a zero gradient.	**b i** Gradient = 0
ii The graph does not cross the x-axis, so it does not have an x-intercept.	**ii** no x-intercept
iii State the coordinates of the point at which the graph crosses the y-axis.	**iii** y-intercept: $(0, 4)$

Helpful hints

✔ The order in which you substitute points into the formula for the gradient of a line won't affect your final value. Just make sure the x- and y-coordinates of a given point match up vertically!

point 2 point 1

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Learning pathways

1, 2(1st, 2nd columns), 3–9, 11, 13	2(2nd, 3rd columns), 3–5, 7–10, 12, 13, 14(a–c)	2(3rd column), 3, 4(c, e), 6, 8, 11–15

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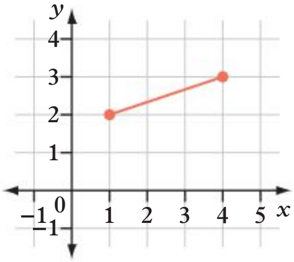
Exercise 3.4A: Understanding and fluency

WE 3.4A **1** Find the gradient of each line segment.

a


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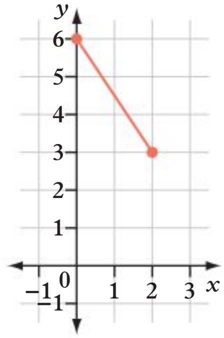
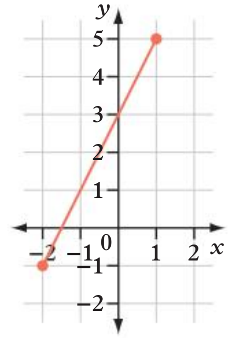


c


2 Use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to calculate the gradient of the line segment joining each pair of points.

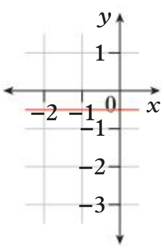
a (2, 4) and (5, 6) **b** (3, 1) and (7, 4) **c** (1, 2) and (4, 8)

d (4, 6) and (7, 5) **e** (−2, 5) and (3, 7) **f** (0, 8) and (1, 5)

g (−1, −6) and (−1, −1) **h** (0, 5) and (1, 0) **i** (−4, −3) and (−1, 4)

j (−3, 3) and (−3, −5) **k** (−5, −6) and (−4, −8) **l** (−9, 3) and (−6, 3)

3 Classify the gradients of the following lines as positive, negative, zero or undefined.

a


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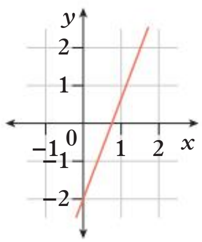
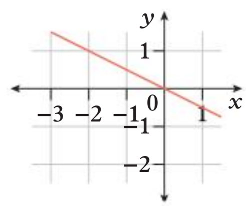
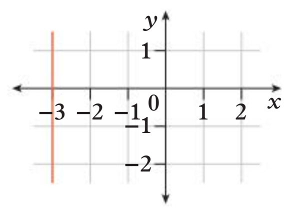
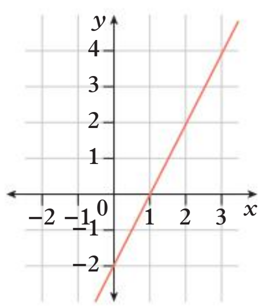


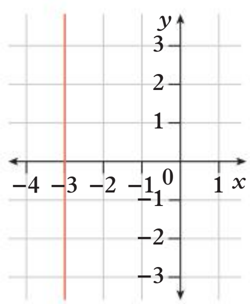
c


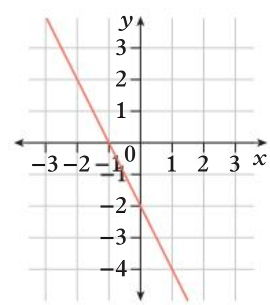
d


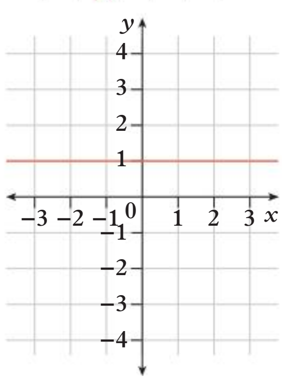
WE 3.4B **4** For the linear graphs shown, determine the:

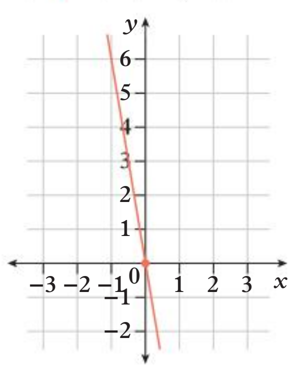
i gradient **ii** x-intercept **iii** y-intercept.

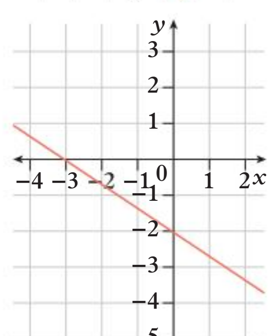
a


b


c


d


e


f


5 Consider the following pairs of points.

i Plot each pair of points on the Cartesian plane and join them with a straight line to form a line segment.

ii Determine the gradient of the line segment.

a (2, 3) and (6, 8) **b** (1, 2) and (3, 6) **c** (3, 7) and (4, 4) **d** (−4, 5) and (2, −3)

6 Consider the following pairs of points.

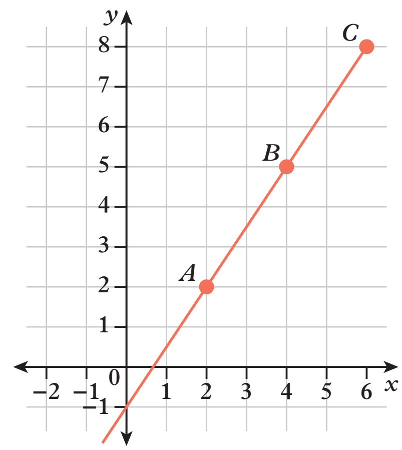
i Plot each pair on the Cartesian plane and then join them with a straight line. Ensure your lines cross both axes.

ii Determine the x- and y-intercepts of the line, if they exist. Write the intercepts as coordinates.

a $(2, 2)$ and $(5, 8)$ **b** $(1, 2)$ and $(3, 6)$ **c** $(-6, 6)$ and $(-4, 3)$ **d** $(-2, 1)$ and $(-2, -3)$

Exercise 3.4B: Problem solving and reasoning

7 Consider the graph on the right.



a Determine the gradient of the following line segments:

i AB **ii** AC.

b Compare your answers to part **a** and describe the relationship between the gradient of a line and the gradient of line segments on that line.

8 There are two types of special lines.

a Consider the line from question **3** part **d**.

i Find the gradient of the line.

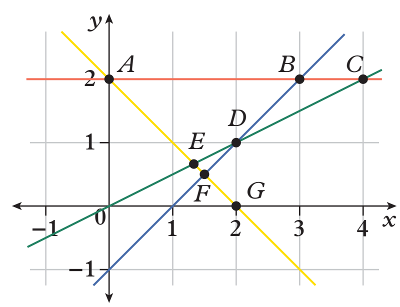
ii Explain why the gradient of a vertical line is undefined.

b Consider the line from question **3** part **a**.

i Find the gradient of the line.

ii Explain why the gradient of a horizontal line is zero.

9 Consider the four linear graphs on the Cartesian plane on the right.



a Calculate the gradient for the following line segments.

i AB **ii** BD **iii** CD **iv** AE

b Use your answers from part **a** and your observations in question **9** to determine the gradient for the following line segments.

i DE **ii** BC **iii** DF **iv** FG

10 Kane calculates the gradient of the line segment joining $(2, 8)$ and $(6, 5)$, as shown below. Identify Kane's error and calculate the correct value for the gradient of the line segment.

Kane

$$\begin{aligned} m &= \frac{8-5}{6-2} \\ &= \frac{3}{4} \end{aligned}$$

11 Todd and Bridget calculated the gradient of the line segment joining $(-1, 9)$ and $(3, -3)$, as shown below. Explain why both students have produced correct calculations.

Todd

$$\begin{aligned} m &= \frac{-3-9}{3-(-1)} \\ &= \frac{-12}{4} \\ &= -3 \end{aligned}$$

Bridget

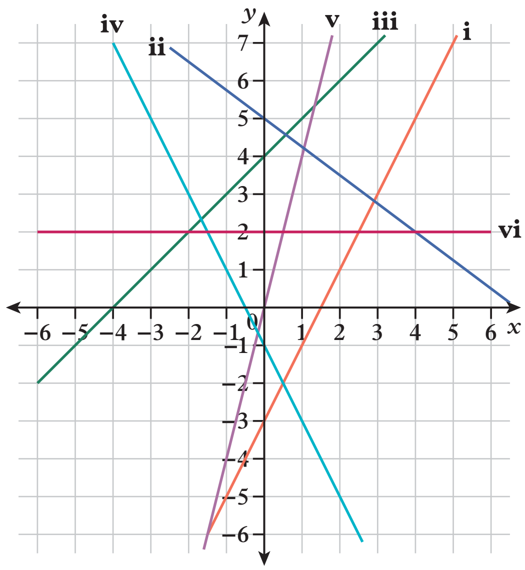
$$\begin{aligned} m &= \frac{9-(-3)}{-1-3} \\ &= \frac{12}{-4} \\ &= -3 \end{aligned}$$

12 Complete the table below.

$x_2 - x_1$	8	−20			4	
$y_2 - y_1$			−4	−16		30
Gradient	$\frac{3}{2}$	$-\frac{4}{5}$	$\frac{1}{7}$	$-\frac{2}{5}$	7	−5

13 Consider these linear graphs.

a Complete the following table by calculating the gradient and identifying the coordinates of the y-intercept of each graph.



	Equation	Gradient	y-intercept
i	$y = 2x - 3$		
ii	$y = -\frac{3}{4}x + 5$		
iii	$y = x + 4$		
iv	$y = -2x - 1$		
v	$y = 4x$		
vi	$y = 2$		

b What pattern can you see in the table that allows you to identify the gradient and the y-intercept from the equation for the graph?

c Determine the gradient and y-intercept of the linear graphs with the following rules:

i $y = 6x + 4$ **ii** $y = x - 5$ **iii** $y = -3x$

Exercise 3.4C: Challenge

14 Determine the value of the unknown in each of the following.

a A line with a gradient of 3 passes through the points $(1, 4)$ and $(x, 10)$.

b A line with a gradient of -2 passes through the points $(1, 4)$ and $(3, y)$.

c A line with a gradient of 2 passes through the points $(1, 4)$ and $(3, y)$.

d A line with a gradient of 2 passes through the points $(0, 4)$ and $(3, y)$.

e A line with a gradient of $-\frac{1}{2}$ passes through the points $(1, 4)$ and $(3, y)$.

f A line with a gradient of $\frac{3}{2}$ passes through the points $(-1, -4)$ and $(x, 8)$.

15 The gradient of a line is 2 and it passes through the points $(0, 4)$ and (x, y).

a Write an equation for the gradient of this line.

b Rearrange the equation in part **a** to solve for y in terms of x.

c The gradient of another line is 2 and it passes through the points $(1, 4)$ and (x, y).

i Write an equation for the gradient of this line.

ii Rearrange the equation in part **c** to solve for y in terms of x.

Online resources:

- **Interactive skillsheet** Gradients
- **Worksheet** Finding the gradient of a linear graph
- **Investigation** Rise and run
- **Quick quiz** 3.4
- **Interactive skillsheet** Intercepts
- **Worksheet** Identifying features of a linear graph
- **Desmos activity** Which is steepest?

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

3.1 **1** Solve the following equations for x.

a $\frac{x}{3} + 2 = -7$ **b** $4(x + 2) = 28$

c $\frac{4 - x}{9} = -2$ **d** $5(2x - 3) - 8x = -1$

3.1 **2** Solve the following equations for x.

a $\frac{24}{x} = 4$ **b** $\frac{3}{x} = 18$ **c** $\frac{5}{x} = 0.8$ **d** $\frac{10}{x} = 0.25$

3.1 **3** Solve the following equations for x (variables on both sides).

a $5x + 7 = 2x - 5$ **b** $7 - 4x = 6x - 13$ **c** $7(x - 3) = -56 - x$ **d** $8(2x + 7) = 4(3x + 11)$

3.1 **4** Jamie orders five equally priced video games online for a total cost of \$409.70, which includes the delivery charge of \$9.95.

a Define a pronumeral to represent the unknown quantity in this problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation using inverse operations.

d What is the cost of each game?

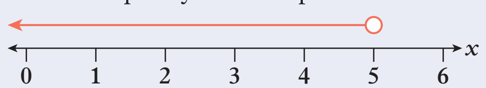
3.2 **5** Represent the values of x for each inequality on a number line.

a $x \le 5$ **b** $x > -3$

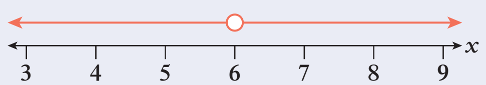
c $x \ne 2$ **d** $x \ge 4$

3.2 **6** Write the inequality that is represented on each of these number lines.

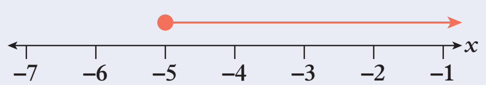
a



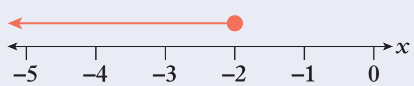
b



c



d



3.2 **7** Solve the following inequalities.

a $6x > 18$ **b** $-4x \ge 20$

c $-13 < \frac{x}{5} - 8$ **d** $7 - 3x < 4$

e $4x + 7 \ne 6x + 13$ **f** $4(3x - 4) \le 3(3x + 2)$

3.3 **8** Consider the following tables of coordinate points.

i Plot the points from the following tables.

ii State whether the graph is linear or non-linear.

a

x	−3	−2	−1	0	1	2	3
y	−10	−8	−6	−4	−2	0	2

b

x	−3	−2	−1	0	1	2	3
y	4.5	4	3.5	3	2.5	2	1.5

c

x	−3	−2	−1	0	1	2	3
y	5	2.25	0	−1.75	−3	−3.75	−4

d

x	−3	−2	−1	0	1	2	3
y	−2	0	2	4	6	8	12

3.3 **9** Sketch a graph of each of the following linear relationships by first completing a table of values for x from −3 to 3.

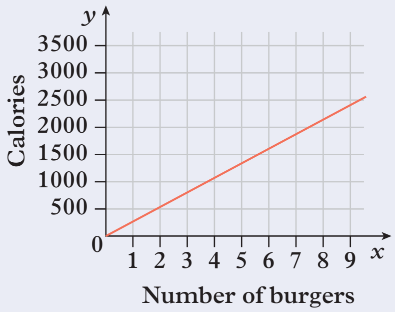
a $y = 2x - 5$ **b** $y = 5 - x$ **c** $y = -3x + 1$ **d** $y = -2x - 3$

3.3 **10** Identify the independent and dependent variables in the following relationships.

a

Length (cm)	5	10	15	20
Volume (cm^3)	125	1000	3375	8000

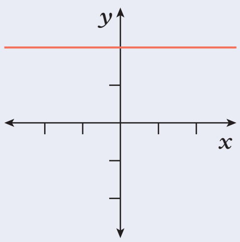
b



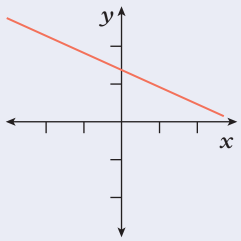
c Time spent playing a video game and the number of achievements unlocked

3.4 **11** State whether the following lines have a gradient that is positive, negative, zero or undefined.

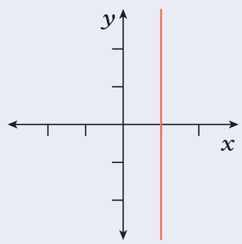
a



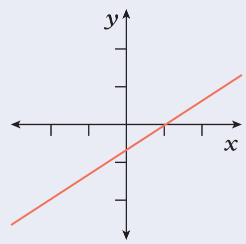
b



c



d



3.4 **12** Determine the gradients of the lines that pass through the following pairs of points.

a (1, 5) and (3, 11)

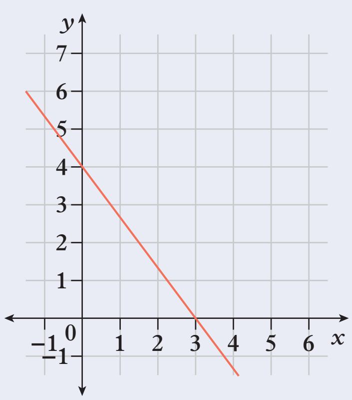
b (−1, 3) and (2, −6)

c (0, 5) and (3, 0)

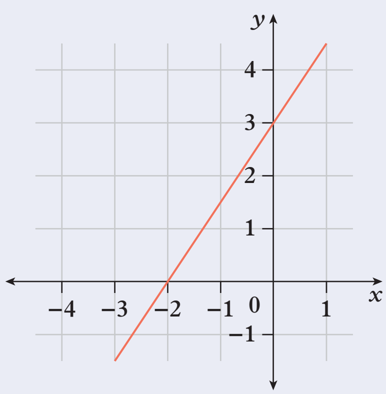
d (−6, −2) and (−1, −8)

3.4 **13** State the x- and y-intercepts of the following lines as coordinates.

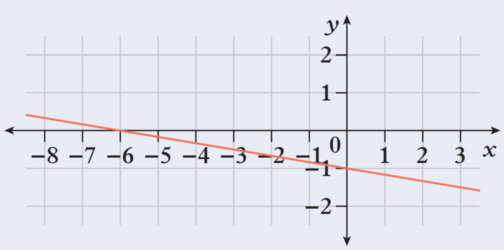
a



b



c



Lesson 3.5

Sketching linear graphs

Learning intentions

By the end of this lesson you will be able to ...

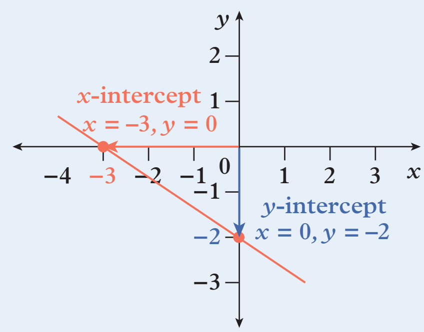
✔ sketch linear graphs using a variety of methods.

Inter-year links

Support	The Cartesian plane
Year 7	5.4 The Cartesian plane
Year 8	6.5 Plotting linear relationships
Year 10	3.3 Sketching linear graphs

Sketching linear graphs with two intercepts

Key content video
Sketching linear graphs using intercepts



- A minimum of two coordinate points are required to **sketch** a linear graph.
- The x-intercept is the point where the linear graph crosses the x-axis and $y = 0$.
- The y-intercept is the point where the linear graph crosses the y-axis and $x = 0$.
- Sketch a linear graph with two intercepts by first finding the x- and y-intercepts:
  1. Determine the x-intercept by substituting $y = 0$ into the equation of the line and solving for x.
  2. Determine the y-intercept by substituting $x = 0$ into the equation of the line and solving for y.
  3. Plot and label the x- and y-intercepts on the Cartesian plane.
  4. Draw a straight line through the two points.

  For example, to sketch the graph of $y = 2x + 3$:

  For x-intercept, let $y = 0$:

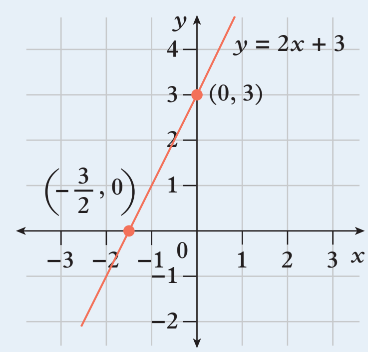
  $$0 = 2x + 3$$
  $$-3 = 2x$$
  $$x = -\frac{3}{2}$$

  x-intercept: $\left(-\frac{3}{2}, 0\right)$

  For y-intercept, let $x = 0$:

  $$y = 2 \times 0 + 3$$
  $$y = 0 + 3$$
  $$y = 3$$

  y-intercept: $(0, 3)$



Sketching linear graphs with one intercept

- There are three cases in which a linear graph has only one intercept:

Description	General equation	Intercept	Gradient
Vertical lines	$x = a$	x-intercept: $(a, 0)$	undefined
Horizontal lines	$y = b$	y-intercept: $(0, b)$	0
Lines that pass through the origin	$y = mx$	origin: $(0, 0)$	m

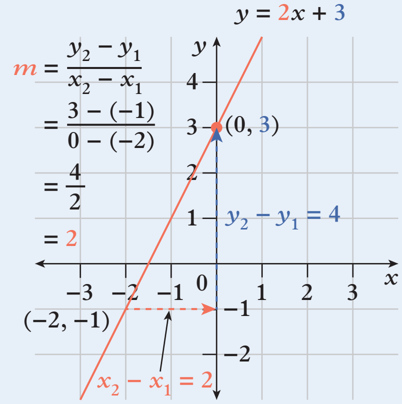
  where a, b and m are constants.
- To sketch a linear graph that passes through the origin $(0, 0)$, determine a second point on the graph by substituting any value of x into the equation and then solve for y.

The gradient–intercept method

- The equation for all linear relationships can be expressed in **gradient–intercept form** shown below, where:
  - → m is the gradient of the line
  - → c is the y-coordinate of the y-intercept.

$$y = \underbrace{m}_{\text{gradient}}x + \underbrace{c}_{y\text{-intercept}}$$

- Gradient–intercept form can be used to identify the gradient and y-intercept, enabling us to sketch the graph.
  For example, the equation of the linear graph on the right is $y = 2x + 3$. It has a gradient of 2 and y-intercept at (0, 3) so $m = 2$ and $c = 3$.



Worked example 3.5A Calculating x- and y-intercepts

Determine the coordinates of the x- and y-intercepts of the graphs of the linear relationships:

a $x + 5y = 10$ **b** $y = 3x - 4$

THINK	WRITE
a **1** To determine the x-coordinate of the x-intercept, substitute $y = 0$ into the equation and solve for x. The coordinates of the x-intercept have the form $(x, 0)$.	**a** For x-intercept, let $y = 0$: $x + 5(0) = 10$; $x + 0 = 10$; $x = 10$. x-intercept: (10, 0)
2 To determine the y-coordinate of the y-intercept, substitute $x = 0$ into the equation and solve for y. The coordinates of the y-intercept have the form $(0, y)$.	For y-intercept, let $x = 0$: $0 + 5y = 10$; $5y = 10$ $(\div 5)$; $y = 2$. y-intercept: (0, 2)
b **1** To determine the x-coordinate of the x-intercept, substitute $y = 0$ into the equation and solve for x. The coordinates of the x-intercept have the form $(x, 0)$.	**b** For x-intercept, let $y = 0$: $0 = 3x - 4$ $(+ 4)$; $4 = 3x$ $(\div 3)$; $x = \frac{4}{3}$. x-intercept: $\left(\frac{4}{3}, 0\right)$
2 To determine the y-coordinate of the y-intercept, substitute $x = 0$ into the equation and solve for y. The coordinates of the y-intercept have the form $(0, y)$.	For y-intercept, let $x = 0$: $y = 3(0) - 4$; $= 0 - 4$; $= -4$. y-intercept: (0, −4)

Worked example 3.5B Sketching linear graphs with two intercepts

Sketch a graph of $4x - y = 8$ by first finding the x- and y-intercepts.

THINK

1 Determine the x-intercept by substituting $y = 0$ into the equation and solving for x. Determine the y-intercept by substituting $x = 0$ into the equation and solving for y.

2 Plot and label the x- and y-intercepts on the Cartesian plane.

3 Rule a straight line through the points. Label the graph with its equation.

WRITE

For x-intercept, let $y = 0$:

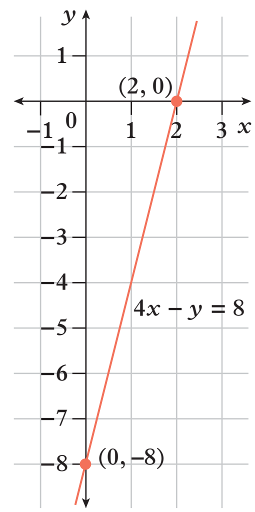
$$\begin{aligned} 4x - 0 &= 8 \\ 4x &= 8 \qquad (\div 4) \\ x &= 2 \end{aligned}$$

x-intercept: $(2, 0)$

For y-intercept, let $x = 0$:

$$\begin{aligned} 4(0) - y &= 8 \\ 0 - y &= 8 \\ -y &= 8 \qquad (\div (-1)) \\ y &= -8 \end{aligned}$$

y-intercept: $(0, -8)$



Worked example 3.5C Sketching linear graphs using the gradient–intercept method

Sketch the graph of $y = -\frac{1}{2}x + 3$.

THINK

1 Identify the gradient, m, and the y-intercept, c, by comparing the equation to the general gradient–intercept form $y = mx + c$.

2 Compare the gradient with the gradient formula. Identify values of $y_2 - y_1$ and $x_2 - x_1$ that result in the gradient. Make sure $x_2 - x_1$ is a positive value.

3 Plot a point at the y-intercept and label the coordinates $(0, 3)$.

4 Identify a second point. The numerator of the gradient tells us to move down by 1 unit. The denominator of the gradient tells us to move to the right by 2 units.

5 Rule a straight line through the points. Label the graph with its equation.

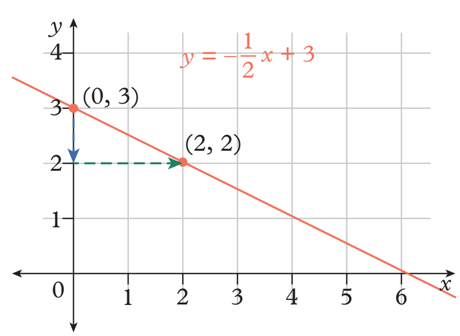
WRITE

$y = -\frac{1}{2}x + 3$

$m = -\frac{1}{2}$, $c = 3$

$m = \frac{y_2 - y_1}{x_2 - x_1} = -\frac{1}{2}$

Let $y_2 - y_1 = -1$ and $x_2 - x_1 = 2$



Worked example 3.5D Sketching linear graphs that pass through the origin

Sketch the graph of $y = -3x$.

THINK

1 The x- and y-intercepts are both (0, 0) so the graph passes through the origin (0, 0). Determine a second point on the graph by substituting any value of x into the equation and solving for y.

2 Plot and label the two points on the Cartesian plane.

3 Rule a straight line through the points and label the graph with its equation.

WRITE

For x-intercept, let $y = 0$:

$0 = -3x \; (\div (-3))$

$x = 0$

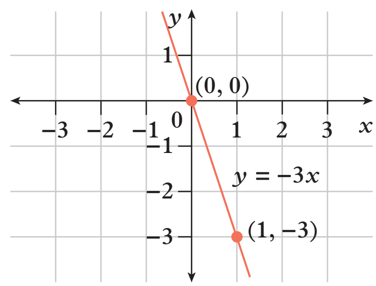
x- and y-intercept: (0, 0)

For a second point, let $x = 1$:

$y = -3(1)$

$\quad = -3$

Second point: (1, −3)



Worked example 3.5E Sketching vertical and horizontal lines

Sketch a graph of the following linear relationships.

a $x = 4$ **b** $y = -2$

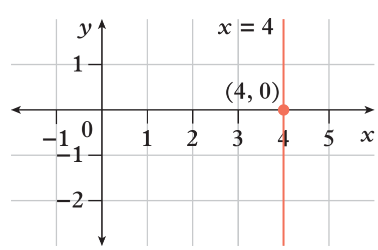
THINK

a The graph of $x = 4$ is a vertical line that passes through the point (4, 0). Rule a vertical line through (4, 0) and label the x-intercept. Label the graph with its equation.

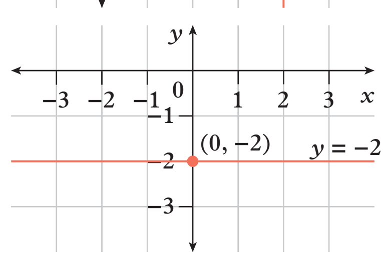
b The graph of $y = -2$ is a horizontal line that passes through the point (0, −2). Rule a horizontal line through (0, −2) and label the y-intercept. Label the graph with its equation.

WRITE

a



b



Helpful hints

- ✔ To find the x-intercept of a linear relationship, set $y = 0$ and solve for x.
- ✔ To find the y-intercept of a linear relationship, set $x = 0$ and solve for y.
- ✔ Find the x- and y-intercepts before you start sketching, so that you can plan the scale on the axes of your Cartesian plane.
- ✔ Always label your graph with the following information:
  - → the equation of the graph
  - → any x- and y-intercepts.
- ✔ A linear graph can be sketched using any two distinct points on the graph.

Learning pathways

△ 1–7, 9, 11, 13, 14   □ 3–8, 10, 12–14, 16, 18   ⬠ 3, 6, 7, 12–19

ANS p471

Exercise 3.5A: Understanding and fluency

WE 3.5A **1** Determine the coordinates of the x- and y-intercepts of the graphs of the following linear relationships.

a $x + 4y = 12$ **b** $y = x + 4$ **c** $2x + y = 6$
d $y = x - 5$ **e** $y = -2x + 8$ **f** $y = 3x - 6$
g $5x + y = -10$ **h** $y = -x + 7$ **i** $y = 4 - x$

2 Sketch the linear graphs that have the following x- and y-intercepts.

a x-intercept: $(2, 0)$, y-intercept: $(0, 1)$ **b** x-intercept: $(-3, 0)$, y-intercept: $(0, 5)$
c x-intercept: $(-1, 0)$, y-intercept: $(0, -1)$ **d** x-intercept: $\left(\frac{1}{2}, 0\right)$, y-intercept: $\left(0, -\frac{5}{2}\right)$

WE 3.5B **3** Sketch a graph of each of the following linear relationships by first finding the x- and y-intercepts.

a $4x + y = 4$ **b** $y = x - 2$ **c** $-2x + 3y = 6$
d $y = -3x + 3$ **e** $5x + y = -5$ **f** $y = 2 - x$
g $2y = 2 - 4x$ **h** $3y - 9x + 12 = 0$ **i** $4x - 3y - 8 = 0$

4 Identify the gradient, m, and the coordinates of the y-intercept of the linear graphs with the following equations.

a $y = 2x + 5$ **b** $y = 4x + 1$ **c** $y = -3x + 7$ **d** $y = -5x - 3$
e $y = x - 6$ **f** $y = 1 - x$ **g** $y = \frac{4}{3}x + 2$ **h** $y = \frac{x}{2} - 8$
i $y = -\frac{4x}{3} + \frac{1}{4}$ **j** $y = 9$ **k** $y = -7x$ **l** $y = 5 - \frac{2}{5}x$

WE 3.5C **5** Sketch the graph of each linear relationship using the gradient–intercept method.

a $y = 2x - 4$ **b** $y = \frac{2}{3}x + 1$ **c** $y = -3x + 5$
d $y = \frac{5}{4}x - 2$ **e** $y = -5x - 1$ **f** $y = -\frac{1}{2}x + 3$
g $y = 3x - 4$ **h** $y = x + 6$ **i** $y = -x - 3$

WE 3.5D **6** Sketch the graphs of the following linear relationships.

a $y = 3x$ **b** $y = -2x$ **c** $y = 6x$ **d** $y = -x$

WE 3.5E **7** Sketch the graphs of the following linear relationships.

a $x = 1$ **b** $y = 4$ **c** $x = -3$ **d** $y = 0$

8 Use the most appropriate method to sketch the graph of each linear relationship.

a $2x - 5y = 10$ **b** $y = 4x + 2$ **c** $x + y = 6$ **d** $y = -3x$
e $y = 7$ **f** $y = 6 - 3x$ **g** $x = 1$ **h** $y = 4x - 4$
i $x + 3y - 9 = 0$ **j** $y = x$ **k** $y = -3x + 5$ **l** $y = -7$

Exercise 3.5B: Problem solving and reasoning

9 Without sketching, determine how many axis intercepts the graph of each relationship has.

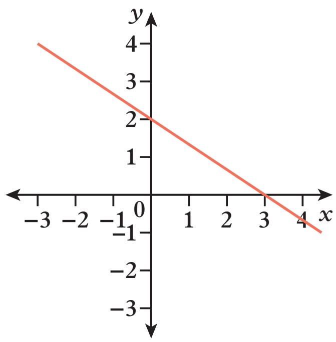
a $y = 10$　　**b** $y = 2x + 1$　　**c** $x = -12$

d $y = 15x$　　**e** $5x - y = 25$　　**f** $x + y = 1$

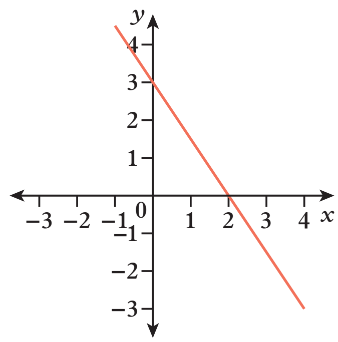
g $y = x$　　**h** $x = 1$　　**i** $-2y = -7x$

10 Which of these three graphs is a correct sketch of $3x + 2y = 6$? Identify the errors in the other two options.

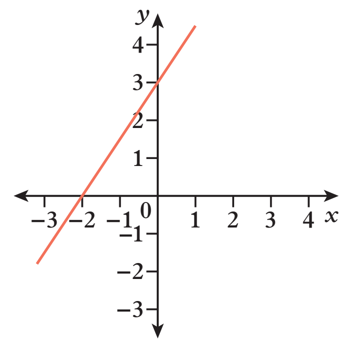
A



B



C



11 Decide whether each statement about the graph of $y = 3x + 6$ is true or false. Correct each false statement.

a The y-intercept is 6.　　**b** The x-intercept is 2.

c The relationship is linear.　　**d** The point $(1, 9)$ lies on the line.

e The line passes through the origin.　　**f** The gradient of the line is 3.

12 Decide whether each statement about the graph of $y = -4x$ is true or false. Correct each false statement.

a The x-intercept is -4.　　**b** The y-intercept is 0.

c The line passes through the origin.　　**d** The point $(1, 4)$ lies on the line.

e The gradient is negative.　　**f** The gradient of the line is $-\frac{1}{4}$.

13 Explore the similarities and differences of the following linear relationships.

a Sketch the following lines on the same set of axes:

$2x + 3y = 12$, $2x + 3y = 18$, $2x + 3y = 0$ and $2x + 3y = -12$

b State the similarities between the equations and their graphs.

c Describe the impact of the differences between the equations and their graphs.

14 Explore the similarities and differences of the following linear relationships.

a Sketch the following lines on the same set of axes:

$2x + 3y = 12$, $2x - 3y = 12$, $-2x + 3y = 12$ and $-2x - 3y = 12$

b State the similarities between the equations and their graphs.

c Describe the impact of the differences between the equations and their graphs.

15 Determine the gradient of the following lines by finding and using the x- and y-intercepts.

a $3x - 5y = 30$　　**b** $7x + 6y + 9 = 0$　　**c** $10y = 3 - 5x$　　**d** $8x = 4y - 3$

16 Tony is buying a skateboard on a purchase plan where he makes a regular payment each week. He creates the formula $y = 300 - 25x$ to describe the relationship between the amount still owed in dollars (y) after a number of weeks (x).

a Explain why Tony has chosen to solve for y (amount still owed in dollars) in terms of x (number of weeks), rather than the other way around.
Hint: Think about the relationship between the variables.

b Sketch the graph of this relationship by first determining the coordinates of the x- and y-intercepts.

c What does the y-intercept represent on this graph?

d What does the x-intercept represent on this graph?

e Describe the purchase plan Tony is using. When will he be able to bring his skateboard home?

17 A rainwater tank has a capacity of 1500 L and feeds a drip system to water the garden. At the beginning of April, the tank is full, but it is empty at the end of the last day of the month. Let x represent the number of days from the start of April and y represent the number of litres of water in the tank. Assume a constant rate of water use and no further rain during April.

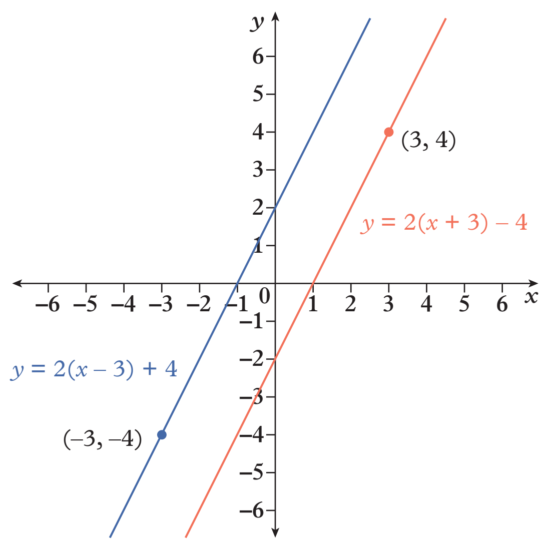
- **a** Explain why this relationship can be represented by a linear graph.
- **b** Determine the coordinates of the x-intercept for this relationship.
- **c** Determine the coordinates of the y-intercept for this relationship.
- **d** Use an appropriate method to sketch the graph of the relationship.
- **e** Use the graph to estimate the number of litres of water in the tank at the end of the day on 10 April.
- **f** Use the graph to estimate when there is 600 L of water left in the tank.

Exercise 3.5C: Challenge

18 Consider a linear relationship where the sum of ax and by is a constant.

- **a** Explain why the reciprocals of a and b are respectively the x- and y-intercepts of $ax + by = 1$.
- **b** Explain how you could find the x- and y-intercepts of linear equations in the form $ax + by = d$.
- **c** For the equation $ax + by = d$, write an expression for d in terms of a and b such that the x-intercept is equal to b and the y-intercept is equal to a.

19 The gradient–intercept form $y = mx + c$ is a specific case of the gradient–point form $y = m(x - h) + k$, where (h, k) is a point on the line. For example, $y = 2(x - 3) + 4$ passes through the point $(3, 4)$ and $y = 2(x + 3) - 4$ passes through the point $(-3, -4)$.



- **a** Substitute $(h, k) = (0, c)$ and show that the equation $y = m(x - h) + k$ becomes $y = mx + c$.
- **b** Sketch the following lines by plotting the point (h, k) and then use the gradient to plot another point in the same way you would using the gradient–intercept method.
  - **i** $y = -3(x - 2) + 6$ **ii** $y = 3(x - 2) + 6$
  - **iii** $y = \frac{5}{2}\left(x + 3\right) - 4$ **iv** $y = -\frac{3}{4}\left(x + 7\right) + 10$
- **c** Determine the coordinates for the x- and y-intercepts of $y = m(x - h) + k$ in terms of m, h and k.
- **d** Follow the steps below to determine a new general form for writing a linear relation in terms of the x-intercept.
  - **i** Substitute $(h, k) = (b, 0)$.
  - **ii** Explain what you can do to the equation $y = mx + c$ to write it in this form.
  - **iii** Explain how you can use this form to determine the x-intercept.
- **e** Use your method from part **d** to write the following equations in the new general form and determine their x-intercepts.
  - **i** $y = 3x - 12$ **ii** $y = 2x - 5$ **iii** $y = -3x - 12$ **iv** $y = 2x + 5$

Online resources:

- **Interactive skillsheet** Sketching linear relationships using intercepts
- **Investigation** Making a stained glass window
- **Desmos activity** Marble slides: Lines
- **Quick quiz** 3.5

Lesson 3.6

Determining linear equations

Learning intentions

By the end of this lesson you will be able to …

✔ determine the equation of a linear graph.

Inter-year links

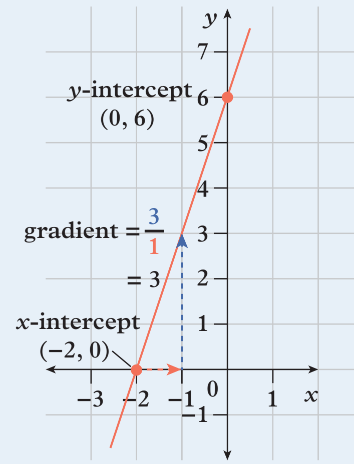
Year 8 6.8 Finding linear equations

Year 10 3.3 Determining linear equations

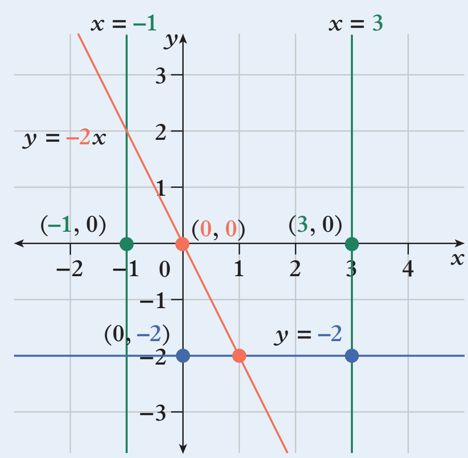
Determining the equation of a linear graph

Key content video
Determining linear equations

- To find the equation of a linear graph in the form of $y = mx + c$, determine the value of the:
  → gradient, m, for any two points (x_1, y_1) and (x_2, y_2): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
  → y-coordinate of the y-intercept: $(0, c)$
  For example, this linear graph has a gradient of 3 and a y-intercept at $(0, 6)$.
  $m = 3$ and $c = 6$ so the equation of the graph is $y = 3x + 6$.



- For lines with only one intercept, the equations can be determined using the following general equations where a, b and m are constants.
  → A vertical line with an x-intercept at $(a, 0)$ is given by $x = a$.
  For example, the graph of the equation $x = -1$ is a vertical line with an x-intercept at $(-1, 0)$.
  → A horizontal line with a y-intercept at $(0, b)$ is given by $y = b$.
  For example, the graph of the equation $y = -2$ is a horizontal line with a y-intercept at $(0, -2)$.
  → A line that passes through the origin $(0, 0)$ with gradient m is given by $y = mx$.
  For example, the graph of the equation $y = -2x$ passes through the origin and has a gradient of -2.



Worked example 3.6A Determining the equation of a linear graph given the gradient and the y-intercept

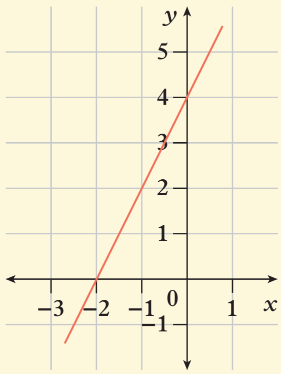
Determine the equation of a linear graph with a gradient of -3 and a y-intercept of $(0, 7)$.

THINK	WRITE
Identify the values of gradient, m, and the y-coordinate of the y-intercept, c, and then substitute into the gradient-intercept form, $y = mx + c$.	$m = -3$ The y-intercept is $(0, c)$. $c = 7$ $y = -3x + 7$

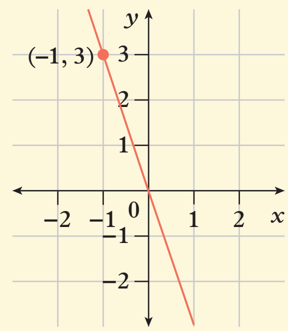
Worked example 3.6B Determining the equation of a linear graph

Determine the equation of each of the following linear graphs.

a



b



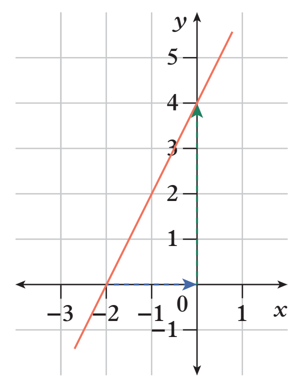
THINK

a **1** Determine the value of the gradient, m, by identifying two points on the graph and substituting them into the gradient formula.

WRITE

a $(x_1, y_1) = (-2, 0)$,
$(x_2, y_2) = (0, 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4 - 0}{0 - (-2)}$$
$$= 2$$



2 Determine the value of the constant, c, by identifying the y-coordinate of the y-intercept.

y-intercept: $(0, 4)$

$c = 4$

3 Substitute the values of m and c into the general equation for a straight line.

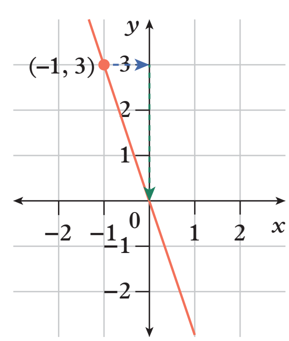
$y = mx + c$

$y = 2x + 4$

b **1** Determine the value of the gradient, m, by identifying points on the graph and substituting them into the gradient formula.

b $(x_1, y_1) = (-1, 3)$,
$(x_2, y_2) = (0, 0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - 3}{0 - (-1)}$$
$$= -3$$



2 Determine the value of the constant, c, by identifying the y-coordinate of the y-intercept.

y-intercept: $(0, 0)$

$c = 0$

3 Substitute the values of m and c into the general equation for a straight line.

$y = mx + c$

$y = -3x + 0$

$y = -3x$

Worked example 3.6C Determining the equation of a linear graph given two points

Find the equation of the linear graph that passes through the points (6, 5) and (2, −3).

THINK	WRITE
1 Calculate the value of the gradient, m.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - (-3)}{6 - 2}$ $= \frac{8}{4}$ $= 2$
2 Substitute the value of the gradient, m, into the general equation for a straight line, $y = mx + c$.	$y = 2x + c$
3 Select one of the two coordinates, substitute the values of x and y into the partially completed equation, and solve for c.	From the point (6, 5): Let $x = 6$ and $y = 5$: $5 = 2(6) + c$ $5 = 12 + c \quad (-12)$ $c = -7$
4 State the equation of the line.	$y = 2x - 7$

Helpful hints

✔ The different methods for finding the equation of a line can be overwhelming, so remember that you just need to find the gradient, m, and the y-intercept $(0, c)$, which you can then substitute into the formula for a linear graph, $y = mx + c$.

Learning pathways

△ 1(1st, 2nd columns), 2, 3, 4(a, c, e, g), 5, 6(a–e), 7(a, c, e), 8, 11(a, b), 13	□ 1(2nd, 3rd columns), 3(b, e, g, h), 4(b, d, f, h), 5, 6(e–h), 7(b, d, f), 9, 10, 11(a, b, d), 13, 14(a, b)	⬠ 1(3rd, 4th columns), 3(f, h), 4(b, d, f, h), 5, 6(e–h), 7(b, d, f), 10, 11(c, d), 12–15

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Exercise 3.6A: Understanding and fluency

1 Identify the gradient, m, and the coordinates of the y-intercept of the linear graphs with the following equations.

a $y = 2x + 5$ **b** $y = 4x + 1$ **c** $y = -3x + 7$ **d** $y = -5x - 3$

e $y = x - 6$ **f** $y = 1 - x$ **g** $y = \frac{4}{3}x + 2$ **h** $y = \frac{x}{2} - 8$

i $y = -\frac{4x}{3} + \frac{1}{4}$ **j** $y = 9$ **k** $y = -7x$ **l** $y = 5 - \frac{2}{5}x$

WE 3.6A 2 Determine the equations of the lines in the following graphs in the form $y = mx + c$.

a gradient: 3, y-intercept: (0, 4) **b** gradient: −2, y-intercept: (0, 10)

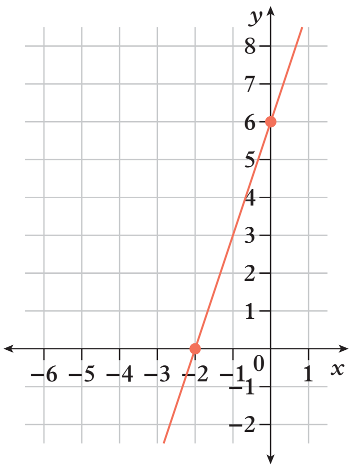
c gradient: 1, y-intercept: (0, −7) **d** gradient: −12, y-intercept: (0, −1)

e gradient: −1, y-intercept: (0, 20) **f** gradient: 5, y-intercept: (0, 0)

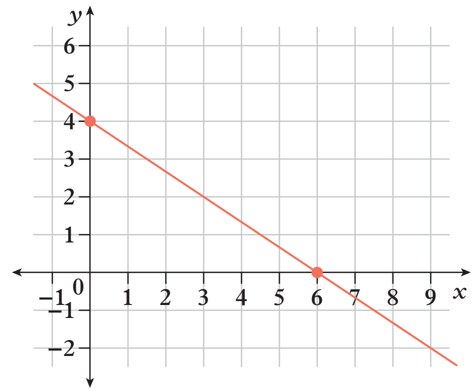
g gradient: $\frac{1}{3}$, y-intercept: $\left(0, \frac{4}{5}\right)$ **h** gradient: $-\frac{3}{4}$, y-intercept: (0, 0)

WE 3.6B **3** Determine the equation of each of the following linear graphs.

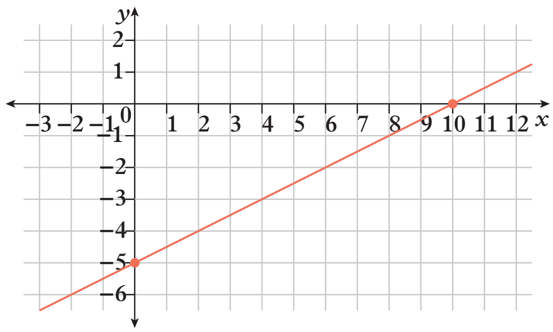
a



b



c



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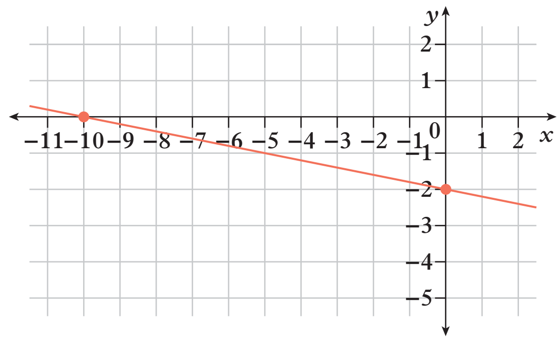

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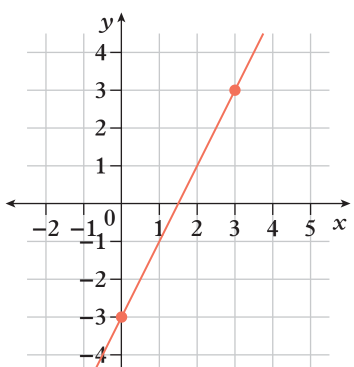

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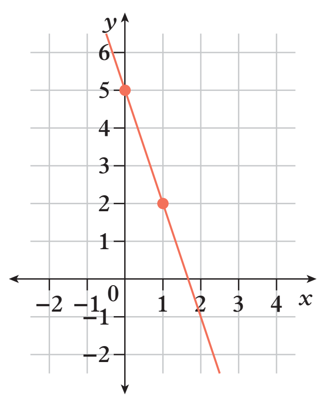
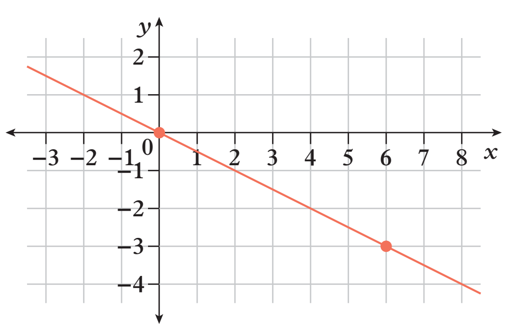
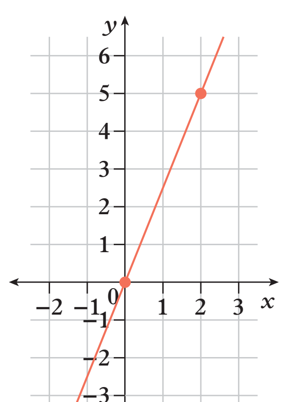


g



h

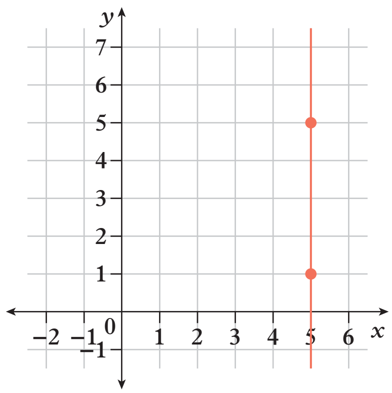


WE 3.6C **4** Find the equations of the linear graphs that pass through each pair of points.

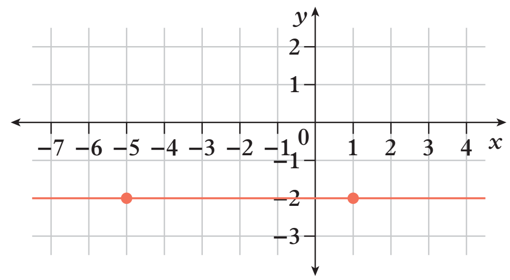
a $(1, -20)$ and $(11, 0)$
b $(2, 4)$ and $(3, 2)$
c $(-1, 6)$ and $(3, 2)$
d $(-4, -12)$ and $(-2, -6)$
e $(2, 5)$ and $(1, 3)$
f $(9, -4)$ and $(11, 6)$
g $(-1, -2)$ and $(5, -5)$
h $(6, -3)$ and $(9, 1)$

5 Find the equation of the linear graphs below. Recall that the general equation for a vertical line is $x = a$ and the general equation for a horizontal line is $y = b$, where a and b are constants.

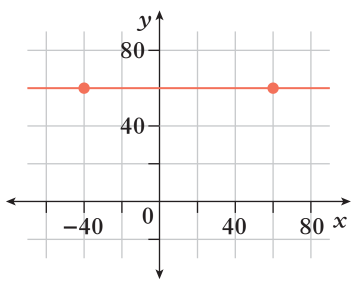
a



b



c



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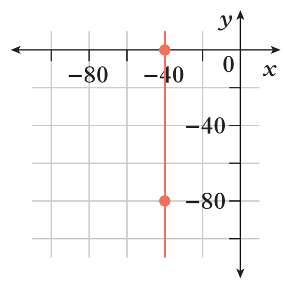

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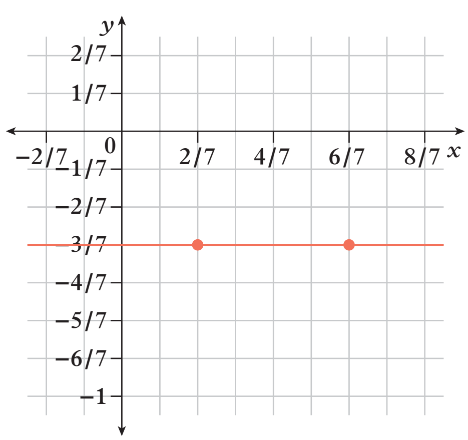
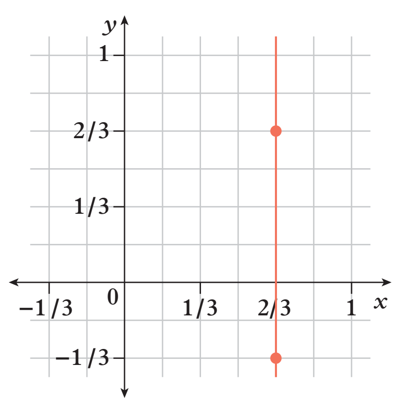


f

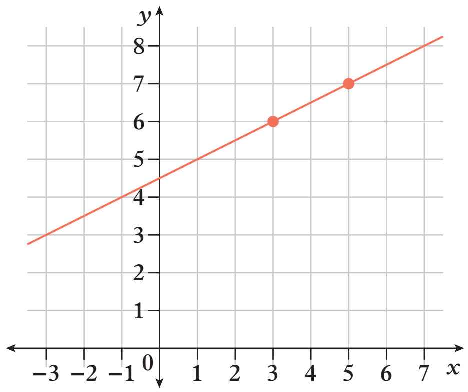


6 Determine the equations of the linear graphs that have the following intercepts.

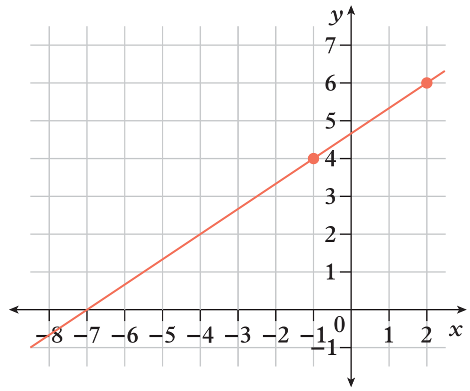
a x-intercept: $(4, 0)$, y-intercept: $(0, 12)$
b x-intercept: $(-5, 0)$, y-intercept: $(0, 10)$
c x-intercept: $(3, 0)$, y-intercept: $(0, -21)$
d x-intercept: $(-6, 0)$, y-intercept: $(0, -24)$
e x-intercept: $(-1, 0)$, y-intercept: $(0, 10)$
f x-intercept: $(18, 0)$, y-intercept: $(0, 6)$
g x-intercept: $(5, 0)$, y-intercept: $(0, -2)$
h x-intercept: $(-6, 0)$, y-intercept: $(0, 9)$

7 Identify two integer coordinates on each linear graph and use these to determine their equations.

a



b



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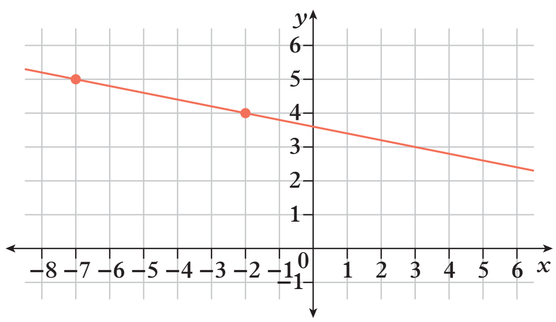

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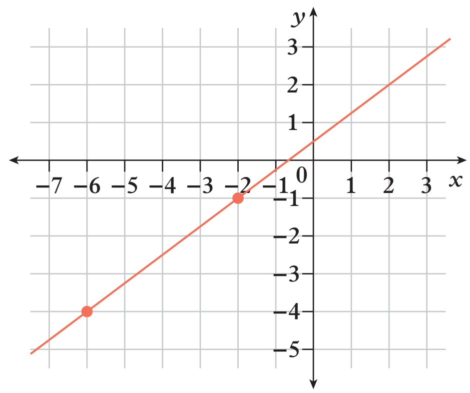

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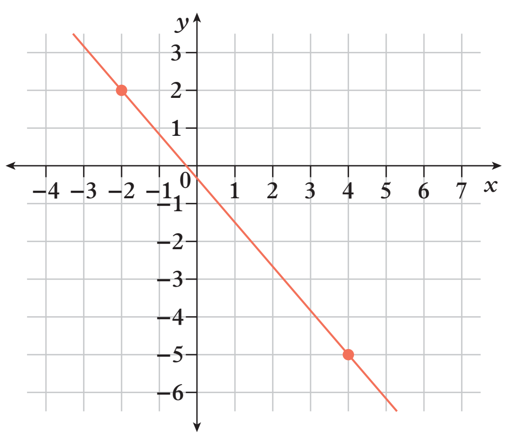
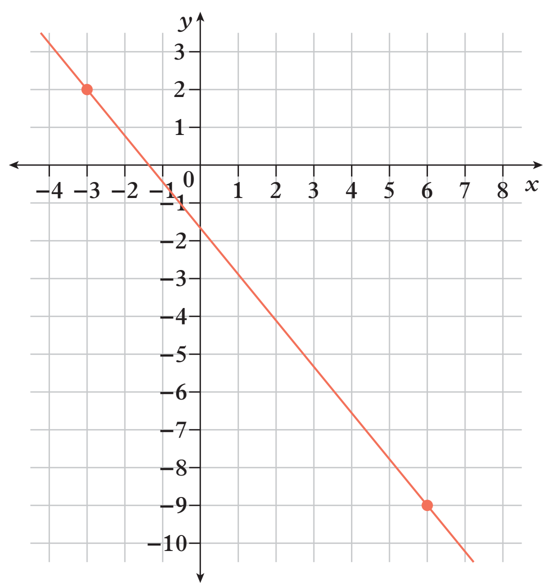


f



Exercise 3.6B: Problem solving and reasoning

8 Azami invests some money into a simple interest account. After 2 years the account has \$2500 and after 5 years the account has \$3400.

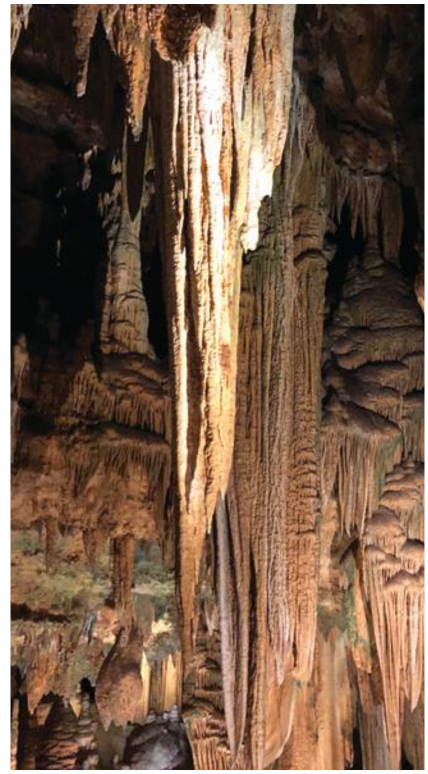
  a Identify the independent variable and the dependent variable.

  b Determine the equation that gives the amount in Azami's account, \$$A$, after n years.

9 Roland recently started a business and realised his profits were increasing at a constant rate. After 7 days of being open, he had made a profit of \$450, and after 14 days, he had made \$870 in profit.

  a Write a linear equation that gives the amount of profit, \$$P$, that Roland earns after n days.

  b Being overly optimistic about his launch, Roland uses his model to predict the profit he will earn 1 year (365 days) after opening. What is the predicted profit Roland will have earned exactly one year (365 days) after opening?



10 As water drips from the ceiling of a cave it creates a long crystalline formation that hangs from the ceiling called a stalactite. A specific stalactite is measured to be 85 centimetres long in 2010 and then 10 years later it is measured to be 87 centimetres long.

- **a** Identify the independent variable and dependent variable.
- **b** Determine a linear equation for the growth of the stalactite, g, in centimetres, n years after 2010.
- **c** Use your equation to estimate the age of the stalactite in 2010. What year did it begin to form?

11 Determine the equation of the lines with the following tables of values.

a

x	−3	−2	−1	0	1	2	3
y	−32	−23	−14	−5	4	13	22

b

x	−3	−1	1	3	5	7	9
y	−2	1	4	7	10	13	16

c

x	10	20	30	40	50	60	70
y	48	51	54	57	60	63	66

d

x	1	4	9	16	25	36	49
y	28	22	12	−2	−20	−42	−68

12 A carpenter sells his wares based on the cost of the materials and the time spent crafting. A particular collection all cost the same in materials but take different amounts of time to craft. A piece that takes 16 hours to craft costs \$845 and a piece that takes 28 hours to craft costs \$1385. The carpenter then marks up the total cost by 120% to sell.

- **a** Write the equation that gives the total cost, \$$C$, for a piece in this collection that takes n hours to craft.
- **b** Write the equation that gives the revenue, \$$R$, for a piece in this collection that takes n hours to craft.
- **c** Write the equation that gives the profit, \$$P$, for a piece in this collection that takes n hours to craft.

13 Consider each of the linear relationships, labelled **i–vi**.

- **a** Factorise the right-hand side of each of the equations.
- **b** Find the x-intercept of each of the equations.
- **c** Describe the connection between the factor form in part **a** and the x-intercept in part **b**.

i $y = 3x + 12$ **ii** $y = -5x - 15$ **iii** $y = 2x - 10$

iv $y = 6x - 8$ **v** $y = 21 - 7x$ **vi** $y = 16x - 40$

Exercise 3.6C: Challenge

14 Find the equations of the linear graphs that pass through each pair of points.

a $\left(\frac{2}{3}, 0\right)$ and $\left(0, \frac{7}{5}\right)$ **b** $\left(-\frac{2}{5}, \frac{3}{4}\right)$ and $\left(\frac{1}{5}, -\frac{1}{4}\right)$ **c** $\left(-\frac{9}{2}, -\frac{7}{4}\right)$ and $\left(-\frac{9}{8}, -\frac{5}{6}\right)$

15 The product of the gradient, m, and y-intercept, c, for a particular line is 99. The sum of the gradient and y-intercept is 20.

- **a** Write the two possible equations for the line.
- **b** Find the coordinates of the point where the graphs of the two equations intersect.

Online resources:

- **Interactive skillsheet** – Determining linear equations
- **Investigation** – The human body
- **Quick quiz** – 3.6
- **Worksheet** – Writing the rule for horizontal and vertical linear graphs
- **Investigation** – Parallel and perpendicular lines
- **Desmos activity** – Land the plane

Lesson 3.7

Midpoint and length of a line segment

Learning intentions

By the end of this lesson you will be able to …

✔ determine the midpoint and the length of a line segment on the Cartesian plane.

Inter-year links

Year 7	1.7 Exponents and square roots
Year 8	7.3 Pythagoras' theorem

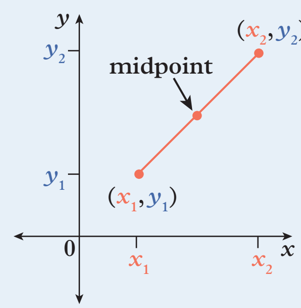
Midpoint of a line segment

Key content video
Midpoint and length of a line segment

- The **midpoint** of a line segment is the point located exactly halfway between the endpoints of the line segment. The coordinates of two endpoints of a line segment (x_1, y_1) and (x_2, y_2) can be used to find the midpoint.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

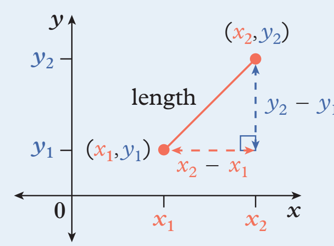
  → The x-coordinate of the midpoint is the average of the x-coordinates of the endpoints.
  → The y-coordinate of the midpoint is the average of the y-coordinates of the endpoints.



Length of a line segment

- The formula for the length of a line segment can be determined by drawing an appropriate right-angled triangle and using Pythagoras' theorem. If the coordinates of two endpoints are (x_1, y_1) and (x_2, y_2), then the two shorter side lengths of the triangle are $a = x_2 - x_1$ and $b = y_2 - y_1$. The distance, c, between the two points is the length of the hypotenuse, and can be found by taking the positive solution to Pythagoras' theorem.

$$c^2 = a^2 + b^2$$



Worked example 3.7A Determining the midpoint of a line segment

Find the coordinates of the midpoint of the line segment joining $(-1, 4)$ and $(7, 9)$.

THINK	WRITE
1 Calculate the x-coordinate of the midpoint by averaging the x-coordinates of the endpoints.	x-coordinate of the midpoint $= \frac{-1 + 7}{2} = 3$
2 Calculate the y-coordinate of the midpoint by averaging the y-coordinates of the endpoints.	y-coordinate of the midpoint $= \frac{4 + 9}{2} = \frac{13}{2}$
3 Write the coordinates of the midpoint.	Coordinates of the midpoint: $\left(3, \frac{13}{2}\right)$

Worked example 3.7B Calculating the length of a line segment

Calculate the length of the line segment joining (−1, 4) and (7, 9), correct to one decimal place.

THINK	WRITE
1 Define the points (x_1, y_1) and (x_2, y_2). The order of the points does not affect the result.	Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (7, 9)$
2 Determine the lengths of the two shorter sides of the corresponding right-angled triangle.	$a = x_2 - x_1 = 7 - (-1) = 8$ $b = y_2 - y_1 = 9 - 4 = 5$
3 Substitute the shorter side lengths into Pythagoras' theorem and solve for c.	$c^2 = a^2 + b^2$ $c^2 = 8^2 + 5^2 = 64 + 25 = 89$
4 Use a calculator to evaluate the root and round the answer to one decimal place as specified by the question.	$c = \sqrt{89}$ $c \approx 9.4$ units

Helpful hints

- ✔ Take care when finding the horizontal or vertical distances between two points – watch carefully for any changes in sign!
- ✔ When finding the distance between two points, it may help to plot the two points on the Cartesian plane and identify a suitable right-angled triangle before applying Pythagoras' theorem.

Learning pathways

- △ 1, 2–3(1st column), 4–7, 9, 10(a, b), 11
- □ 2–3(2nd column), 4, 8, 9, 10(b, c), 12, 13(a), 15
- ⬠ 2–3(g–l), 4, 8, 10(c, d), 12, 13(b), 14, 16–18

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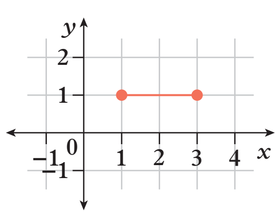
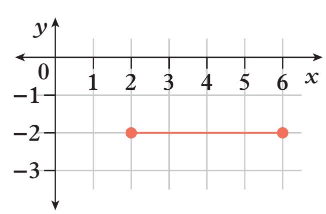
Exercise 3.7A: Understanding and fluency

1 State the coordinates of the midpoint of the following line segments.

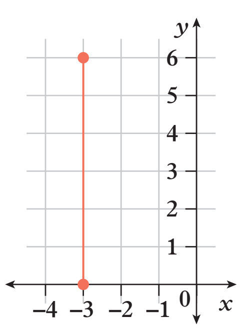
a



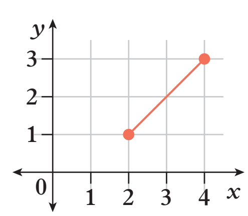
b



c



d



WE 3.7A **2** Find the coordinates of the midpoint of the line segment joining each pair of points.

a (1, 4) and (3, 10)
b (2, 5) and (8, 3)
c (1, 0) and (5, 2)
d (2 , 6) and (2 , 10)
e (0, 5) and (8, 9)
f (3, −4) and (7, 6)
g (2, −1) and (6, 7)
h (−3, −4) and (5, −4)
i (−4, −2) and (−2, 2)
j (3, 9) and (4, 8)
k (5, 0) and (8, 11)
l (−5, 7) and (5, −7)

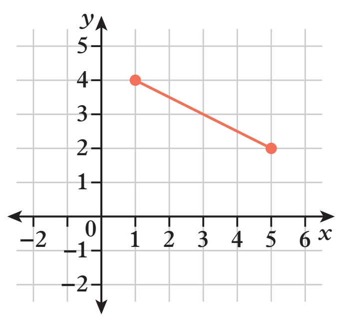
WE 3.7B **3** Calculate the length of the line segment joining each pair of points, correct to one decimal place.

a (2, 5) and (3, 7)
b (3, 4) and (5, 8)
c (6, 2) and (9, 3)
d (−4, 5) and (−4, 9)
e (2, −4) and (4, 2)
f (5, 0) and (8, −4)
g (0, −1) and (1, −2)
h (7 , 8) and (−7 , 8)
i (4, −3) and (6, 0)
j (−3, 6) and (−2, 2)
k (−5, −4) and (−1, −2)
l (6, −5) and (−6 , 5)

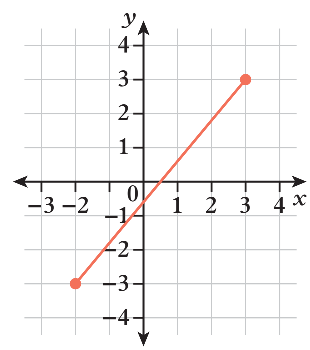
4 For each line segment, find:

i the midpoint **ii** the length (to one decimal place) **iii** the gradient.

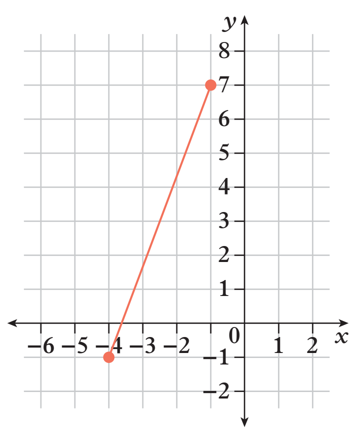
a



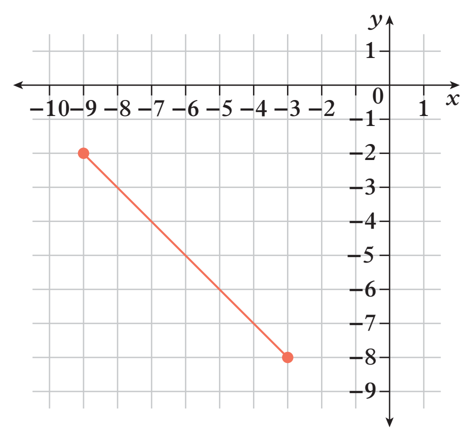
b



c



d



5 Calculate the distance between the midpoint and one of the endpoints of each of the line segments in question **3**.

6 The midpoint of a line segment *AB* has the coordinates (6, 4). If point *A* has the coordinates (2, 3), find the coordinates of point *B*.

7 The midpoint of a line segment *CD* has the coordinates (−5, 1). If point *D* has the coordinates (4, −7), find the coordinates of point *C*.

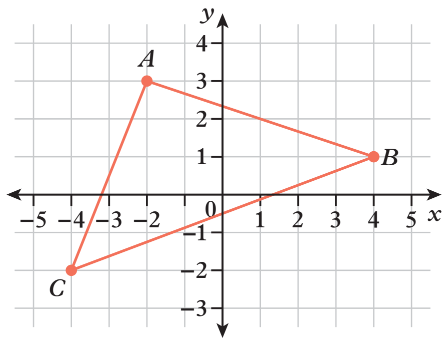
Exercise 3.7B: Problem solving and reasoning

8 A point, A, on a circle has the coordinates $(-1, -1)$.

a If the centre of the circle is at $(2, 3)$, calculate the radius of the circle.

b Identify the coordinates of another point on the circle that forms a diameter with point A.

9 Calculate the perimeter of $\triangle ABC$, correct to one decimal place.



10 Calculate the perimeter of each shape, correct to one decimal place.

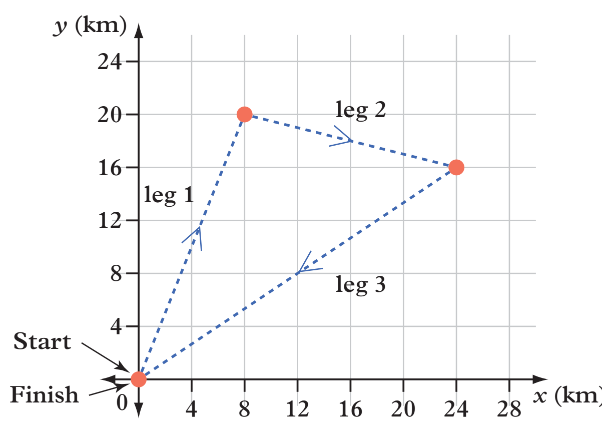
a triangle with vertices at $(-3, -2)$, $(-2, 4)$ and $(4, 2)$

b square with vertices at $(-1, 2)$, $(2, 5)$, $(5, 2)$ and $(2, -1)$

c rectangle with vertices at $(-4, -2)$, $(2, 4)$, $(4, 2)$ and $(-2, -4)$

d trapezium with vertices at $(-3, 3)$, $(1, 5)$, $(3, 3)$ and $(2, -2)$

11 A yacht race follows a triangular course that has been mapped onto a Cartesian plane. The scales on the axes represent distances in kilometres. The race begins and ends at the origin.



a Calculate the length of each leg of the race, correct to one decimal place.

b Calculate the total distance covered during the race, correct to one decimal place.

c An observer's boat is located close to the midpoint of the second leg of the race. Determine the distance between the observer's boat and the finishing point, correct to one decimal place.

12 Consider this parallelogram drawn on a Cartesian plane.

a List the coordinates of the vertices of the parallelogram.

b Find the coordinates of the midpoint of:

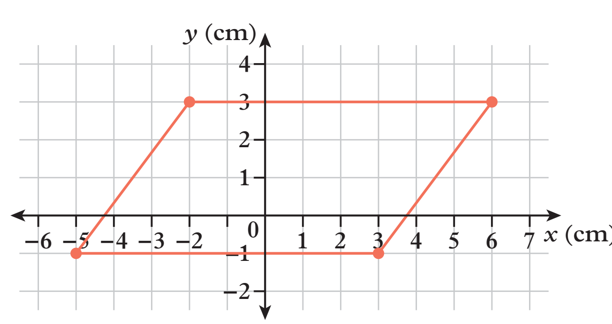
i the longer diagonal

ii the shorter diagonal.

c What do you notice about your answers to part **b**?

d Calculate the perimeter of the parallelogram.

e Find the difference in length of the two diagonals, correct to one decimal place.



13 A shape (in blue) is drawn on a Cartesian plane. A smaller shape (in orange) is then formed by joining the midpoints of the vertices of the original shape.

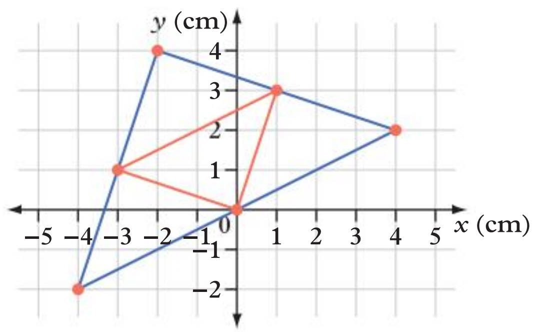
In each case:

i find the perimeter of the blue shape (to one decimal place)

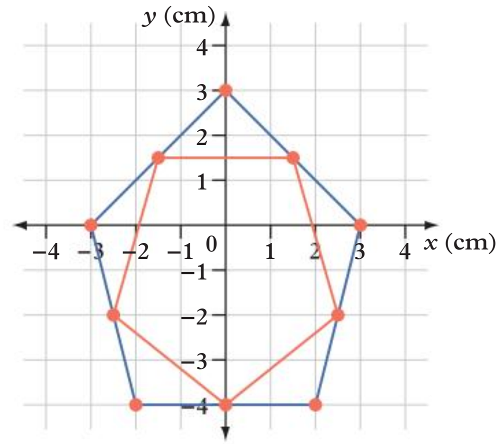
ii find the perimeter of the orange shape (to one decimal place)

iii compare the perimeter of the orange shape to the perimeter of the blue shape.

a



b



14 A quadrilateral, $ABCD$, has vertices at $A(-4, 2)$, $B(-1, 4)$, $C(3, -2)$ and $D(0, -4)$. The scales on the axes represent distances in metres.

a Determine the length of each side of the quadrilateral, correct to one decimal place.

b Determine the coordinates of the midpoint of each diagonal.

c Determine the distance between each vertex and the midpoint, correct to one decimal place.

d Use your answers for parts **a–c** to identify the shape of the quadrilateral $ABCD$. Explain your reasoning.

e Hence use the correct formula to determine the area of the quadrilateral, correct to one decimal place.

Exercise 3.7C: Challenge

15 Consider a triangle, ABC, with vertices at $A(3, 6)$, $B(-1, -2)$ and $C(-5, 2)$.

a Prove that triangle ABC is an isosceles triangle. State any dimensions in simplified surd form.

b Calculate the area of the triangle using your knowledge of the midpoint and length of a line segment.

16 Prove that quadrilateral $ABCD$ with vertices at $A(-5, 3)$, $B(-1, 5)$, $C(3, -2)$ and $D(-1, -4)$ is a parallelogram. State any dimensions in simplified surd form.

17 A line segment has endpoints at $A(-1, -1)$ and $B(2, 5)$. If point C lies between A and B such that AC is $\frac{1}{3}$ the length of AB, determine the exact coordinates of C.

18 A line segment has a midpoint at the origin, a length of 10 units and a gradient of 2. Determine the coordinates of the two endpoints. Write your answer in simplified surd form.

Hint: Sketch a diagram and then consider only one half of the line segment.

Online resources:

- **Interactive skillsheet** Midpoint of a line segment
- **Interactive skillsheet** Length of a line segment
- **Investigation** From square to triangle
- **Quick quiz** 3.7

Direct proportion

Learning intentions

By the end of this lesson you will be able to ...

✔ identify and solve problems related to direct proportion.

Inter-year links

Support	Multiplying and dividing whole numbers
Year 8	3.7 Rates
Year 10	4.6 Direct and inverse proportion

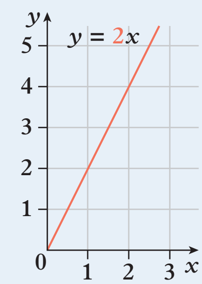
Direct proportion

Key content video
Direct proportion

$$y = kx$$

k = constant of proportionality
= rate of change
= gradient

- y is said to be **directly proportional** to x if:
  - → when $x = 0$, $y = 0$
  - → the **rate of change** of y with respect to x is constant.
- Direct proportionality is denoted using the symbol $\propto$.
- If $y \propto x$, then the equation for the relationship between x and y is $y = kx$, where k is the rate of change of y with respect to x and is called the **constant of proportionality**.
- The graph of $y = kx$ is a straight line that passes through the origin (0, 0) and has a gradient of k.
  For example, in the relationship represented by the graph to the right, the constant of proportionality is 2.



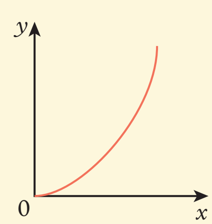
Worked example 3.8A Identifying direct proportion

Determine whether y is directly proportional to x in each of the following relationships. If the relationship is not directly proportional, provide a reason for your answer.

a

x	0	1	2	3	4
y	0	−7	−14	−21	−28

b



THINK

a 1 Check that $x = 0$ when $y = 0$.

2 Check whether the rate of change is constant. To determine the rate of change from a table of values, calculate $\frac{y}{x}$ for each pair of coordinates excluding (0, 0).

3 State whether or not $y \propto x$.

b 1 Check that $x = 0$ when $y = 0$.

2 Identify whether the gradient of the graph is constant.

3 State whether or not $y \propto x$.

WRITE

a When $x = 0$, $y = 0$.

$\frac{y}{x} = \frac{-7}{1} = -7$ $\qquad \frac{y}{x} = \frac{-21}{3} = -7$

$\frac{y}{x} = \frac{-14}{2} = -7$ $\qquad \frac{y}{x} = \frac{-28}{4} = -7$

y is directly proportional to x.

b The graph passes through the origin (0, 0).

The graph of y versus x is non-linear and, therefore, does not have a constant gradient.

y is not directly proportional to x.

Worked example 3.8B Finding the constant of proportionality

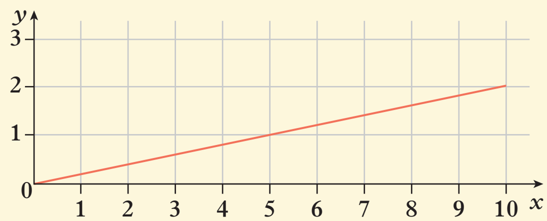
Find the constant of proportionality if $y \propto x$ using the given information, then write the equation for each directly proportional relationship.

a $y = 18$ when $x = 3$

b

x	0	2	4	6	8
y	0	24	48	72	96

c



THINK

a **1** If $y \propto x$, $y = kx$. Substitute the given values into $y = kx$ and solve for k.

2 Substitute into the general equation $y = kx$.

b **1** Select any pair of values in the table and substitute the corresponding values of x and y into $y = kx$, then solve for k.

2 Substitute into the general equation $y = kx$.

c **1** As the graph passes through the origin, (0, 0), to find the gradient, k, select any point on the graph, substitute the corresponding values of x and y into $y = kx$, then solve for k.

2 Substitute into the general equation $y = kx$.

WRITE

a $y = kx$

Let $x = 3$ and $y = 18$:

$18 = k \times 3$
$18 = 3k \quad (\div 3)$
$k = 6$

$y = 6x$

b

x	0	2	4	6	8
y	0	24	48	72	96

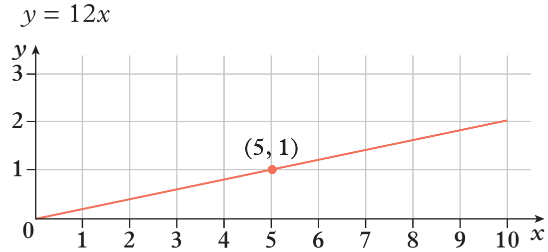
$y = kx$

Let $x = 2$ and $y = 24$:

$24 = k \times 2$
$24 = 2k \quad (\div 2)$
$k = 12$

$y = 12x$

c



$y = kx$

Let $x = 5$ and $y = 1$:

$1 = k \times 5$
$1 = 5k \quad (\div 5)$
$k = \frac{1}{5}$

$y = \frac{1}{5}x$

Worked example 3.8C Solving simple rate problems

Jamie completes a 200 metre sprint in 40 seconds.

a Assuming Jamie runs at a constant speed, sketch a graph of Jamie's distance, d, from the starting line against the time, t, for the duration of the sprint.

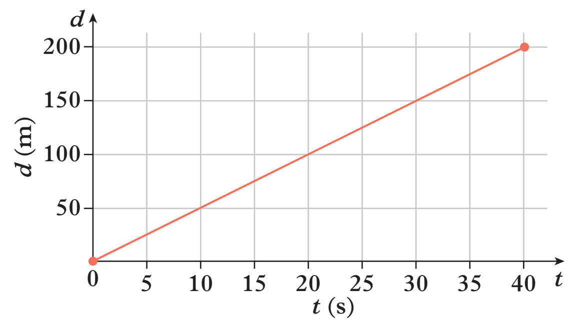
b From the graph of Jamie's run:

i determine the gradient of the graph

ii express Jamie's average speed as a rate.

c Determine the equation for the relationship between d and t.

d How far had Jamie run after 10 seconds?

THINK	WRITE
a Plot the endpoints: Start $t = 0$ s, Jamie is $d = 0$ m from the starting line: (0, 0). End $t = 40$ s, Jamie is $d = 200$ m from the starting line: (40, 200).	**a** 
b **i** To find the gradient, select the points (0, 0) and (40, 200), and calculate $m = \frac{\text{rise}}{\text{run}}$.	**b** **i** $m = \frac{\text{rise}}{\text{run}}$ $= \frac{200}{40}$ $= 5$
ii The rate of change of d with respect to t is the same as the gradient of the graph of d vs. t. Jamie runs 5 metres per second.	**ii** Speed = 5 m/s
c The equation for direct proportion is $y = kx$, where k is the constant of proportionality.	**c** $d = 5t$
d Substitute time, $t = 10$ seconds, into the equation and solve for the distance, d.	**d** Let $t = 10$: $d = 5(10)$ $= 50$ m

Helpful hints

- ✔ Direct proportion is an example of linear algebra and linear graphs. Remember that for the relationship $y = kx$, where k = constant of proportionality = rate of change = gradient.
- ✔ For a relationship to be directly proportional, the constant of proportionality, k, must be a constant, but it can still be fractional, irrational or negative!

Learning pathways

△ 1–4, 6, 7 □ 1–3, 5, 6, 8, 9(a–e) ⬠ 1, 3, 5, 6, 8–10

ANS p476

Exercise 3.8A: Understanding and fluency

WE 3.8A **1** Determine whether y is proportional to x for each of the following relationships. If the relationship is not directly proportional, provide a reason for your answer.

a

x	0	1	2	3	4
y	0	4	8	12	16

b

x	0	1	2	3	4
y	0	1	8	27	64

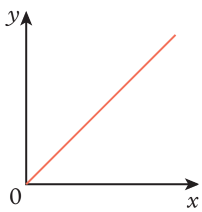
c

x	0	1	2	3	4
y	0	−9	−18	−27	−36

d

x	0	1	2	3	4
y	1	2	4	6	8

e



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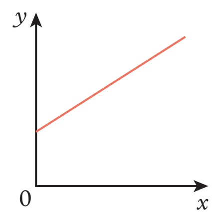

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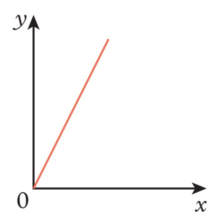
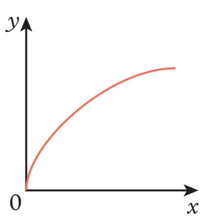


i



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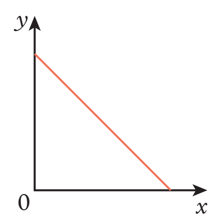


2 Identify and state the constant of proportionality for each relationship.

a $y = 4x$

b $m = -10n$

c $h = 3.5t$

d $y = \frac{x}{3}$

WE 3.8B **3** Find the constant of proportionality using the given information, then write the equation for each directly proportional relationship.

a $y = 50$ for $x = 5$

b $y = 63$ for $x = -7$

c $y = 16$ for $x = 64$

d $y = -12$ for $x = 18$

e

x	0	1	2	3	4
y	0	−6	−12	−18	−24

f

x	0	5	10	15	20
y	0	25	50	75	100

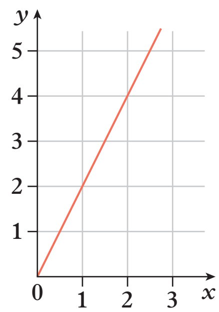
g

x	0	10	20	30	40
y	0	−5	−10	−15	−20

h

x	0	2	4	6	8
y	0	10	20	30	40

i



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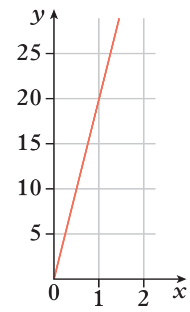

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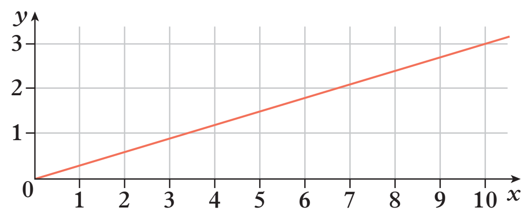

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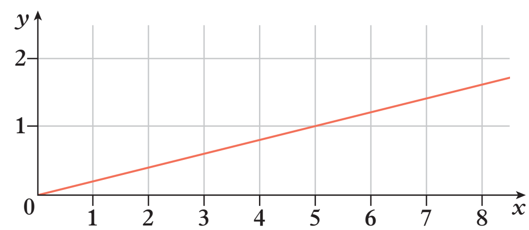


WE 3.8C 4 Julian earns $24 an hour working at the local deli.

a Let c represent Julian's earnings (in $) and t the number of hours he works during an 8-hour shift.

i Sketch a graph of c against t.

ii Determine the gradient of the graph.

b Express Julian's hourly wage as a rate.

c Determine the equation for the relationship between c and t.

d If Julian has to leave halfway through the 8-hour shift, how much will he earn in total?

Exercise 3.8B: Problem solving and reasoning

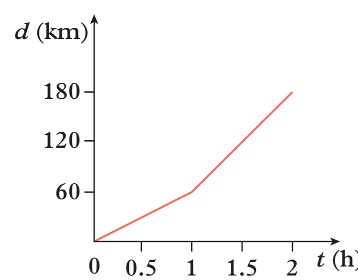
5 The graph on the right shows the distance travelled by a car over time.

a Describe the car's journey, including the speed at which the car is travelling at different times in the journey.

b Write an equation for the relationship between d and t during the first hour of the journey.

c Write an equation for the relationship between d and t during the second hour of the journey.

d Is $d \propto t$? Why or why not?



6 The cost, C dollars, of building a house is directly proportional to the area, A, of the floor space in square metres. It costs $90 000 to build a house with a floor space of 150 m^2.

a Given $C \propto A$, determine the constant of proportionality.

b Write an equation for the relationship between C and A.

c Use your answer from part **b** to calculate the floor space of a house that you could build with a budget of $126 000.

7 A physiotherapist sees 160 patients every week.

a If the physiotherapist works 40 hours each week, on average, how many patients does the physiotherapist see per hour?

b On average, how many minutes does the physiotherapist spend with each patient?

c If the physiotherapist wishes to earn at least \$10 000 every week, what is the minimum he must charge each patient?

8 Jasmine lives 1200 m from school. She walks to school each day at a speed of 1 m/s. Nelson lives 4.5 km from school. He rides in a car to school each day at a speed of 15 m/s.

a If both Jasmine and Nelson arrive at school at 8.30 am, what time did each student leave home?

b Jasmine and Nelson live 4.8 km apart. They plan to meet each other one morning and both leave their homes at 9.00 am. At what time will they meet?

Exercise 3.8C: Challenge

9 Tom is riding in a cycling event. His distance from the start line at given times is recorded. The table shows values for t (number of hours) and d (distance from the start line in km).

t	0	1	4	9	16
d	0	20	40	60	80

a Plot the points on a Cartesian plane and then join them with a smooth line.

b Is the relationship between t and d an example of direct proportion? Explain.

c Complete this table.

$\sqrt{t}$					
d	0	20	40	60	80

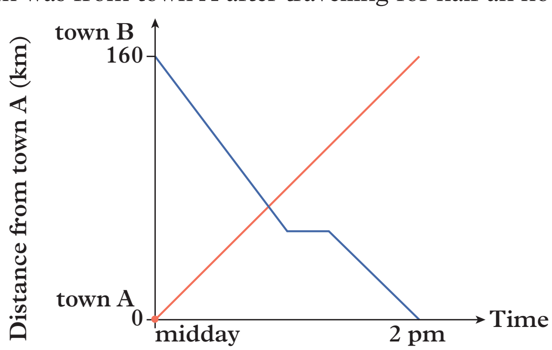
d Plot the points on a Cartesian plane and join them with a smooth line.

e Is the relationship between $\sqrt{t}$ and d an example of direct proportion? Explain.

f Determine the constant of proportionality for the relationship between $\sqrt{t}$ and d and hence write an equation for the relationship.

g What distance is Tom from the start line after 36 hours?

10 The graph below shows the journeys made by two friends between towns A and B.
Both friends leave their homes at midday and arrive at their destinations 2 hours later.
Helena travels at a constant speed from town A to town B. Julia travels at a constant speed for the first hour, then rests for 15 minutes before continuing on to town A at the same speed travelled as Helena.
Determine how far each of them was from town A after travelling for half an hour.



Online resources:

Interactive skillsheet Direct proportion

Investigation How much gold is actually in a piece of gold jewellery?

Desmos activity Marcellus the Giant

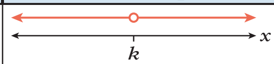
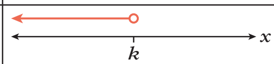
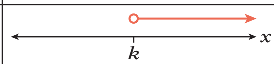
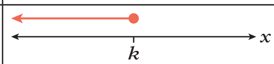
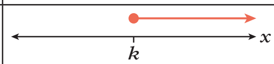
Quick quiz 3.8

Lesson 3.9

Review: Linear relationships

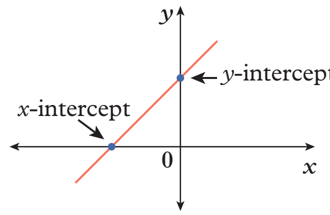
Module summary

Linear inequalities

Inequality	Number line representation	Meaning
$x \neq k$		x is not equal to k
$x < k$		x is less than k
$x > k$		x is greater than k
$x \leq k$		x is less than or equal to k
$x \geq k$		x is greater than or equal to k

Intercepts

- To find the y-intercept, let $x = 0$.
- To find the x-intercept, let $y = 0$.



Gradient-intercept form

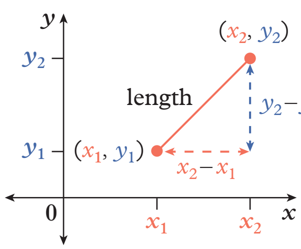
$$y = mx + c$$

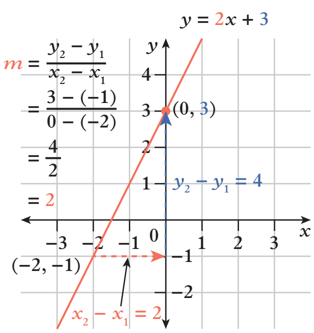
gradient

y-coordinate of y-intercept

Length

$$\text{Length} = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$$

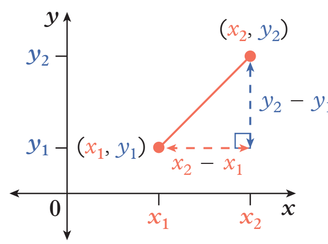




$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-1)}{0 - (-2)} = \frac{4}{2} = 2$$

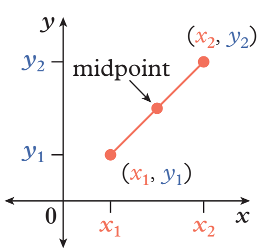
Gradient

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$



Midpoint

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Direct proportion

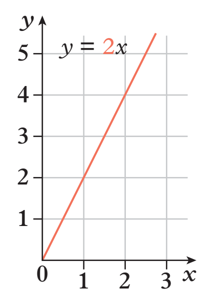
- Two variables, x and y, are in direction proportion when:

→ $x = 0, y = 0$

→ the rate of change of y with respect to x is constant

$$y \propto x \longrightarrow y = kx$$

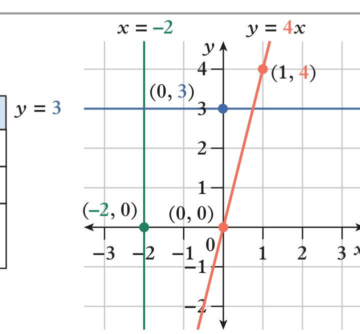
k = rate of change
= gradient
= constant of proportionality



Linear graphs with one intercept

Description	General equation	Intercept	Gradient
Vertical lines	$x = a$	x-intercept: $(a, 0)$	undefined
Horizontal lines	$y = b$	x-intercept: $(0, b)$	0
Lines that pass through the origin	$y = mx$	origin: $(0, 0)$	m

where a, b and m are constants.



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Review questions 3.9A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- Cartesian plane
- constant
- constant of proportionality
- coordinate points
- dependent variable
- direct proportion
- gradient
- gradient–intercept form
- independent variable
- inequality
- inverse operations
- linear equation
- linear graph
- linear inequality
- linear relationship
- midpoint
- number line
- parallel
- perpendicular
- rate of change
- variable
- x-intercept
- y-intercept

1 Complete the following sentences using words from the list of key terms.

a A linear ________ is a mathematical statement that compares two unequal expressions. The result can be plotted on a ________.

b A ________ is a relationship between two variables that produces a straight line when drawn on the ________.

c A ________ is a mathematical statement that compares two equal expressions. It contains a ________ that has a power of 1 and is not multiplied by another variable.

2 Given two linear relationships in gradient–intercept form, $y = mx + b$, what do the values of m in each equation tell us? State true or false for the following statements.

a They indicate if the variables are directly proportional.

b They indicate if the line is horizontal.

c They indicate what the y-intercepts are.

d They indicate what the x-intercepts are.

e They indicate the rate of change of each relationship.

3 Explain the difference between the following terms using one or both of the following equations as an example.

Equation 1: $y = 3x + 1$ Equation 2 involving cost and time: $C = 6t$

a The *constant* in the gradient–intercept form of a linear equation and the *constant of proportionality*.

b A *linear relationship* and a *directly proportional relationship*.

Review questions 3.9B: Multiple choice

3.1 **1** The solution to the equation $3x - 9 = 12$ is:

A $x = 1$ **B** $x = 7$ **C** $x = 12$ **D** $x = 13$

3.1 **2** The value of x in the equation $5x - 2 = 3x + 7$ is:

A $\frac{5}{8}$ **B** $1\frac{1}{8}$ **C** $2\frac{1}{2}$ **D** $4\frac{1}{2}$

3.2 **3** What is the solution to $2x - 1 < 5$?

A $x > 3$ **B** $x < 3$ **C** $x > 2$ **D** $x < 2$

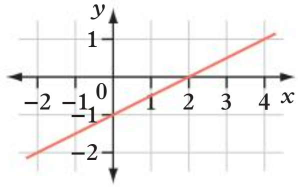
3.2 **4** Which of the following values of x satisfies the inequality $60 < 2x + 12$?

A 36 **B** 24 **C** 12 **D** 6

3.3 **5** Which point does *not* lie on the graph of $y = 2x - 4$?

A $(0, -4)$ **B** $(1, -2)$ **C** $(4, 4)$ **D** $(-1, -2)$

3.4 **6** The gradient of the linear graph shown below is:



A -2 **B** $-\frac{1}{2}$ **C** $\frac{1}{2}$ **D** 1

3.4 **7** The gradient of the line segment joining the points $(-2, 4)$ and $(7, -4)$ is:

A $\frac{8}{9}$ **B** $-\frac{8}{9}$ **C** $\frac{9}{8}$ **D** $-\frac{9}{8}$

3.4 **8** A line with a gradient of 0:

A is vertical **B** is horizontal

C is undefined **D** increases from left to right

3.5 **9** At the point where a line crosses the x-axis:

A $x = y$ **B** $y = -x$ **C** $y = 0$ **D** $x = 0$

3.5 **10** The y-intercept for the graph of $2y = 3x - 4$ is:

A $(0, -4)$ **B** $(0, -2)$ **C** $(0, 0)$ **D** $(0, 2)$

3.5 **11** In a linear graph, $y = mx + c$, the pronumeral c represents the:

A y-intercept **B** x-intercept **C** gradient **D** origin

3.6 **12** A linear graph has a gradient of 3 and a y-intercept of $(0, -2)$. The equation of the graph is:

A $y = -2x + 3$ **B** $y = \frac{1}{3}x - 2$ **C** $y = 3x - 2$ **D** $y = -3x + 2$

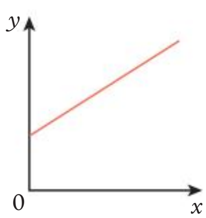
3.7 **13** The length of the line segment joining $(3, 5)$ and $(7, 9)$ is closest to:

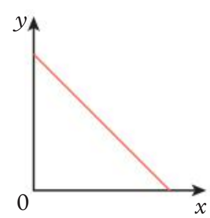
A 4 **B** 5 **C** 6 **D** 7

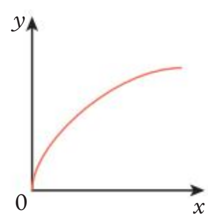
3.7 **14** The coordinates of the midpoint of the line segment joining $(3, 5)$ and $(7, 9)$ are:

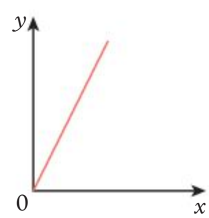
A $(5, 7)$ **B** $(7, 5)$ **C** $(2, 2)$ **D** $(4, 8)$

3.8 **15** For which graph is $y \propto x$?

A


B


C


D


Review questions 3.9C: Short answer

3.1 **1** Solve each equation using inverse operations.

a $3x - 4 = 2$ **b** $2 = 4 - 3x$ **c** $4(x - 3) = 3$ **d** $\frac{x}{3} - 2 = -1$

3.1 **2** Solve each equation for x.

a $7x - 3 = 4x + 9$ **b** $x + 8 = 1 - 6x$

3.1 **3** One more than three-fifths of the class is 16 people.

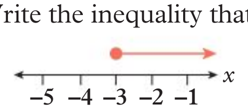
a Define a pronumeral to represent the unknown quantity in this problem.

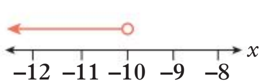
b Use this pronumeral to write an equation to represent the problem.

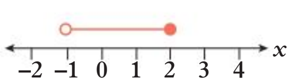
c Solve the equation.

d How many people are in the class?

3.2 **4** Write the inequality that is represented on each of these number lines.

a


b


c


3.2 **5** Solve each of these inequalities.

a $5 - 2x < 7$ **b** $2 - x > 10 + 2x$

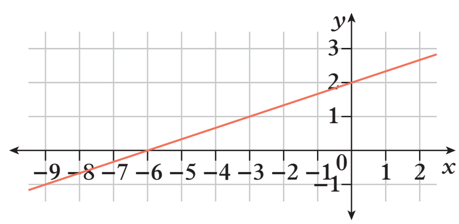
3.3 **6** Complete a table of values from $x = -2$ to $x = 2$ for each linear relationship, and then plot the relationship between x and y.

a $y = x + 5$ **b** $y = x - 5$ **c** $y = -x + 5$ **d** $y = -x - 5$

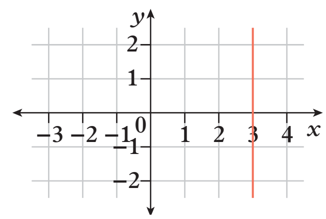
3.4 **7** For each linear graph shown, determine the:

i gradient **ii** x-intercept **iii** y-intercept.

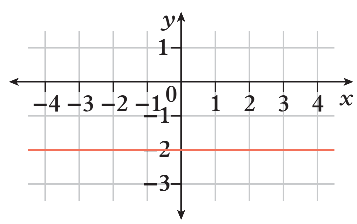
a



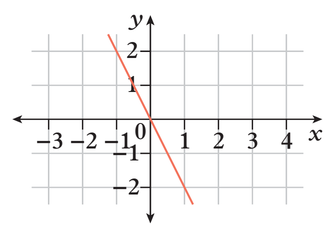
b



c



d



3.4 **8** Find the gradient of the line segment joining each pair of points.

a (2, 3) and (−2, −3) **b** (−3, −2) and (−2, −3)

3.5 **9** For the graph of each linear relationship below, determine the coordinates of the:

i x-intercept **ii** y-intercept.

a $2x + 3y = 18$ **b** $3x - y = 6$ **c** $y = 4x - 2$

d $2y = 5x - 3$ **e** $x - 2y = 4$ **f** $-y = 4 - x$

3.5 **10** Use your answers from question **9** to sketch each linear graph.

3.5 **11** Use the most appropriate method to sketch the graph of each linear relationship.

a $3x + 2y = 4$ **b** $4 - 2x = 3y$ **c** $y = \frac{1}{3}x$ **d** $y = 5 - \frac{2}{3}x$

e $y = 5$ **f** $2x + 3y + 6 = 0$ **g** $y = \frac{3}{4}x - 2$ **h** $x = -7$

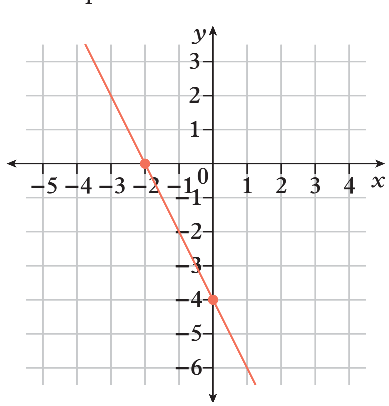
3.6 **12** Write the equations of the linear graphs with the following properties.

a gradient: 4, y-intercept: (0, −2) **b** gradient: $\frac{1}{4}$, y-intercept: (0, 0)

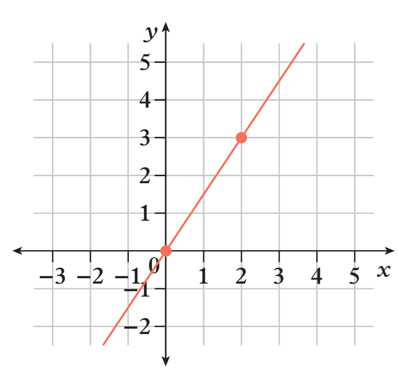
c gradient: 0, y-intercept: $\left(0, -\frac{1}{2}\right)$

3.6 **13** Find the equation of each of the following linear graphs.

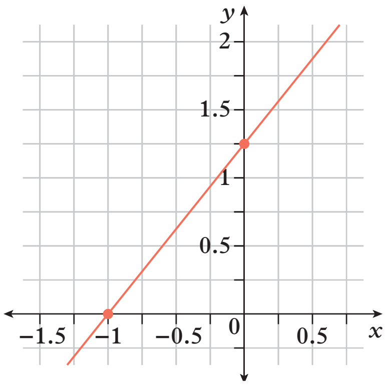
a



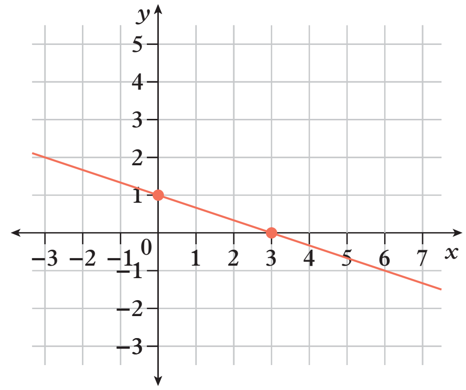
b



c



d



3.7 **14** Determine the equations of the linear graphs that pass through each pair of points.

a (−5, 2) and (1, −4)
b (100, 39) and (−47, 39)
c (−3, 4) and (6, −8)
d (7, 12) and (−3, 4)

3.7 **15** For the line segment that joins each of the following pairs of points, find the:

i midpoint
ii length (correct to one decimal place).

a (2, 3) and (8, 7)
b (−4, 6) and (−3, 5)
c (2, −4) and (−2, 4)
d (−3, −1) and (2, −9)

3.8 **16** Determine whether x is directly proportional to y in each of the following relationships. If the relationship is not directly proportional, provide a reason for your answer.

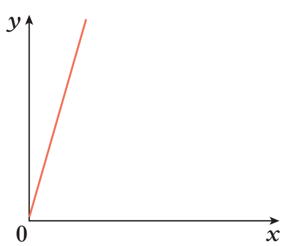
a

x	0	1	2	3	4
y	0	1	4	9	16

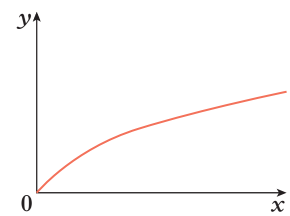
b

x	0	3	6	9	12
y	0	−1.5	−3	−4.5	− 6

c

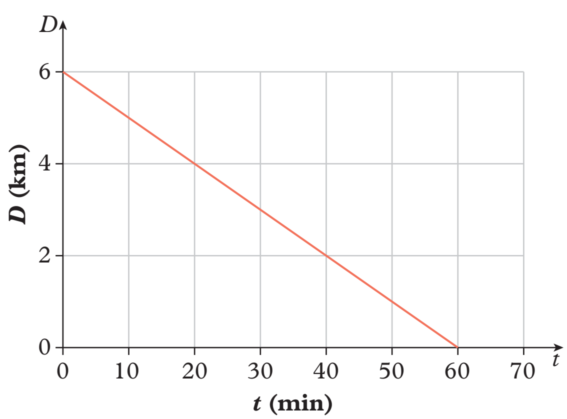


d



Review questions 3.9D: Mathematical modelling

1 David decides to visit his friend Steve who lives 6 km away. It takes him 60 min to get there if he walks quickly. This can be shown in the graph of distance, D, in kilometres, from Steve's house and time, t, in minutes.



a Determine the gradient of the graph.

b Express David's average speed for walking as a rate in km per minute.

c Determine the equation for David's journey in gradient–intercept form using the variables D and t.

d Steve decides to meet David halfway. What is this distance from his home?

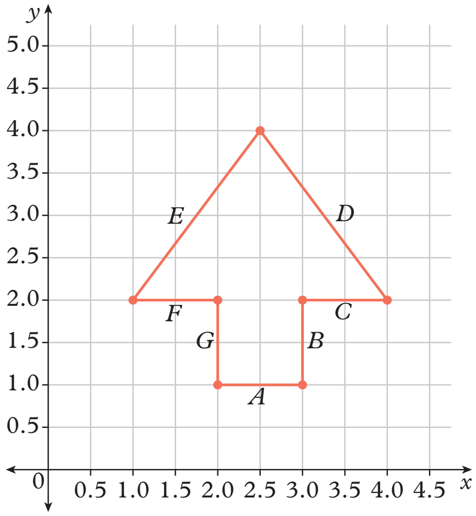
e Steve leaves home on a skateboard at the same time as David leaves and gets to the halfway point after 20 minutes. Plot this point on the graph and draw in line segments to show Steve's journey and him waiting to meet David.

f How long did Steve have to wait to meet David?

g Express Steve's average speed when riding his skateboard as a rate in km per minute.

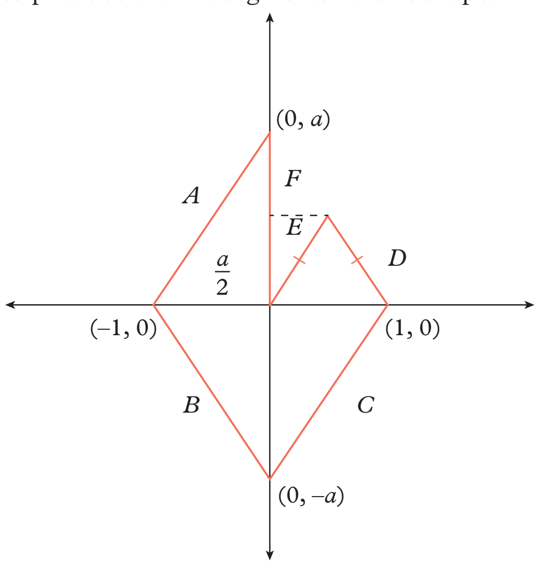
h Determine the equation for Steve's journey in gradient–intercept form using the variables D and t.

2 **a** The shape below is made from seven line segments. Determine the linear equation and an inequality showing the values of x or y involved to produce each line segment in the following table.



Line segment	Linear equation	x- or y-values involved
A	$y = 1$	$2 \le x \le 3$
B		$1 \le y \le$ ___
C		
D		$2.5 \le x \le 4$
E		$1 \le x \le 2.5$
F		
G		

b Complete the table of linear equations and corresponding inequalities showing the values of x or y involved to produce the line segments for this shape.



Line segment	Linear equation	x- or y-values involved
A		
B		
C		
D		
E		
F		

3 A rectangle, $DEFG$, has vertices at $D(-2, -1)$, $E(0, 1)$, $F(3, -2)$ and $G(1, -4)$.

a Draw the rectangle on the Cartesian plane.

b Calculate the lengths of all the sides of the rectangle, correct to one decimal place.

c Using your sketch, identify the coordinates of the y-intercepts of the line segments:

i $\overline{DE}$ **ii** $\overline{EF}$ **iii** $\overline{DG}$ **iv** $\overline{FG}$.

d Find the gradients of the lines through:

i $\overline{DE}$ **ii** $\overline{EF}$ **iii** $\overline{DG}$ **iv** $\overline{FG}$.

e Point P is the midpoint of $\overline{DE}$, point Q is the midpoint of $\overline{EF}$, point R is the midpoint of $\overline{FG}$ and point S is the midpoint of $\overline{DG}$. Find the coordinates of:

i P **ii** Q **iii** R **iv** S.

f Describe the shape of the figure $PQRS$. Justify the statements you make.

4 The cross-section of a building is drawn on a Cartesian plane with the scale on the axes showing length in metres. The x-axis represents ground level.

a On the same Cartesian plane, sketch the graph of:

i $4y - x = 12$ **ii** $y = 5 - \frac{1}{4}x$.

b To represent the cross-section of the building, shade the area between the graph of $4y - x = 12$ and the x-axis from $x = 0$ to $x = 4$, as well as the area between the graph of $y = 5 - \frac{1}{4}x$ and the x-axis between $x = 4$ and $x = 8$.

c The tallest point of the building is 4 m.

i What is the distance from the top of the roof to the lower edge of the roof, correct to one decimal place?

ii What is the positive gradient of the roof?

iii If a chimney is to be placed halfway along the slope of the roof on the side with the positive gradient, describe its position on the Cartesian plane.

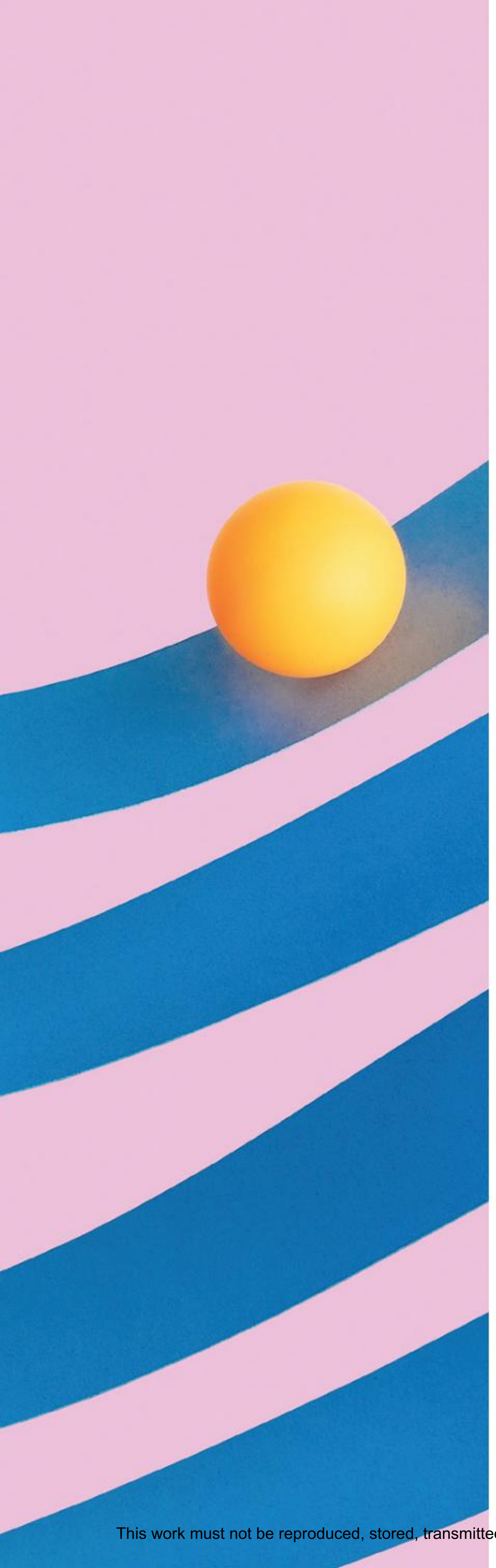
Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Solve linear equations using inverse operations ☐ Solve linear equations with the unknown on both sides using inverse operations	☐ Go back to Lesson 3.1 Solving linear equations
☐ Represent linear inequalities using a number line ☐ Solve linear inequalities using inverse operations	☐ Go back to Lesson 3.2 Solving linear inequalities
☐ Plot linear relationships using tables of values and equations ☐ Identify independent and dependent variables	☐ Go back to Lesson 3.3 Plotting linear relationships
☐ Identify the x- and y-intercepts of a linear graph ☐ Determine the gradient of a line segment from a graph or by using two given coordinate points	☐ Go back to Lesson 3.4 Gradient and intercepts
☐ Determine the x- and y-intercepts of a linear graph from its equation ☐ Sketch linear graphs with intercepts ☐ Sketch vertical and horizontal lines ☐ Sketch linear graphs that pass through the origin ☐ Sketch linear graphs using the gradient-intercept method	☐ Go back to Lesson 3.5 Sketching linear graphs
☐ Determine the equation of a linear equation when given its graph ☐ Determine the equation of a linear relationship when given two points	☐ Go back to Lesson 3.6 Determining linear equations
☐ Determine the midpoint of a line segment on the Cartesian plane ☐ Calculate the length of a line segment on the Cartesian plane	☐ Go back to Lesson 3.7 Midpoint and length of a line segment
☐ Identify when two variables are directly proportional to each other ☐ Find the constant of proportionality between two quantities ☐ Solve simple rate problems	☐ Go back to Lesson 3.8 Direct proportion

4

Non-linear relationships



Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Solving equations using inverse operations
- ✔ Factorising quadratic expressions
- ✔ Factorising the difference of two squares
- ✔ The Cartesian plane
- ✔ Translations

Curriculum links

- Identify and graph quadratic functions, solve quadratic equations graphically and numerically, and use null factor law to solve monic quadratic equations with integer roots algebraically, using graphing software and digital tools as appropriate (VC2M9A05)
- Use mathematical modelling to solve applied problems involving change, including financial contexts involving simple interest; formulate problems, choosing to use either linear or quadratic functions or other simple variations; interpret solutions in terms of the context; evaluate the model and report methods and findings (VC2M9A06)
- Experiment with the effects of the variation of parameters on graphs of related functions, using digital tools, making connections between graphical and algebraic representations, and generalising emerging patterns (VC2M9A07)

Materials

- ✔ Calculator
- ✔ Graph paper

Lesson 4.1

Solving quadratic equations

Learning intentions

By the end of this lesson you will be able to ...

✔ solve simple quadratic equations using the Null Factor Law.

Inter-year links

Year 7	6.2 Terms, expressions and equations
Year 8	6.4 Solving simple quadratic equations
Year 10	4.1 Solving quadratic equations

Quadratic equations

- A single variable **quadratic equation** is an equation that, when expanded, has its variable raised to an exponent no more than 2.
- The **general form** of a quadratic equation is: $ax^2 + bx + c = 0$ where $a \neq 0$.
- A **monic quadratic equation** is a quadratic equation in which the leading coefficient, a, is 1. The general form of a monic quadratic equation is $x^2 + bx + c = 0$.

The Null Factor Law

- The **Null Factor Law** states that if the product of factors is 0, then at least one of the factors must be 0. Given the factors a and b, if $a \times b = 0$, then $a = 0$ or $b = 0$.
- Applying the Null Factor Law to quadratic equations in factorised form, $(x + p)(x + q) = 0$: if $(x + p)(x + q) = 0$, then $x + p = 0$ or $x + q = 0$.

Solving quadratic equations

Key content video
Solving quadratic equations

- To solve a factorisable quadratic equation in general form:
  1 Factorise the quadratic expression.
  2 Apply the Null Factor Law by equating each factor to 0.
  3 Solve each linear equation.

  For example, $x^2 + 2x - 3 = 0$

  $$(x + 3)(x - 1) = 0$$
  $$x + 3 = 0 \text{ or } x - 1 = 0$$
  $$x = -3 \text{ or } x = 1$$

- Quadratic equations can have 0, 1 or 2 real solutions.

Type of quadratic equation	Example	Number of real solutions
Factorised form is $(x + a)(x + b) = 0$, where $a \neq b$	$x^2 + 3x + 2 = 0$ $(x + 1)(x + 2) = 0$ $x + 1 = 0$ or $x + 2 = 0$ $x = -1$ or $x = -2$	2
Factorised form is a perfect square: $(x + a)^2 = 0$	$x^2 + 2x + 1 = 0$ $(x + 1)^2 = 0$ $x + 1 = 0$ $x = -1$	1
$ax^2 + bx + c = 0$ cannot be factorised for some values of a, b and c	$x^2 + x + 1 = 0$	0

Worked example 4.1A Solving factorised quadratic equations

Solve each quadratic equation.

a $(x-6)(x+2)=0$ **b** $x(x-4)=0$ **c** $(2x+1)(4x-3)=0$

THINK	WRITE
1 Check that the quadratic equation is in factorised form and is equal to 0.	**a** $(x-6)(x+2)=0$
2 Apply the Null Factor Law by equating each factor to 0.	$x-6=0$ or $x+2=0$
3 Solve each linear equation.	$x=6$ or $x=-2$
	b $x(x-4)=0$
	$x=0$ or $x-4=0$
	$x=0$ or $x=4$
	c $(2x+1)(4x-3)=0$
	$2x+1=0$ or $4x-3=0$
	$2x=-1$ or $4x=3$
	$x=-\frac{1}{2}$ or $x=\frac{3}{4}$

Worked example 4.1B Factorising and solving quadratic equations

Solve the quadratic equation $x^2-3x-10=0$.

THINK	WRITE
1 Factorise the quadratic equation.	$x^2-3x-10=0$ $(x+2)(x-5)=0$
2 Apply the Null Factor Law by equating each factor to 0.	$x+2=0$ or $x-5=0$
3 Solve each linear equation.	$x=-2$ or $x=5$

Worked example 4.1C Rearranging quadratic equations before solving

Solve $x^2+12=8x$.

THINK	WRITE
1 Rearrange the equation so that one side of the equation is equal to 0, and the x^2 term is positive.	$x^2+12=8x$ $x^2-8x+12=0$ $(-8x)$
2 Factorise the quadratic equation.	$(x-2)(x-6)=0$
3 Apply the Null Factor Law by equating each factor to 0.	$x-2=0$ or $x-6=0$
4 Solve each linear equation.	$x=2$ or $x=6$

Worked example 4.1D Solving quadratic equations with common factors

Solve each quadratic equation.

a $2x^2 + 22x + 36 = 0$ **b** $-x^2 + 10x = 25$

THINK	WRITE
a 1 Take the highest common factor (HCF) out of the quadratic expression.	**a** $2x^2 + 22x + 36 = 0$ $2(x^2 + 11x + 18) = 0$
2 Divide both sides by the HCF. Note that you can never divide both sides by the variable, x, in case $x = 0$.	$\frac{2(x^2 + 11x + 18)}{2} = \frac{0}{2}$
3 Factorise the quadratic equation.	$x^2 + 11x + 18 = 0$ $(x + 2)(x + 9) = 0$
4 Apply the Null Factor Law by equating the factors to 0.	$x + 2 = 0$ or $x + 9 = 0$
5 Solve each linear equation.	$x = -2$ or $x = -9$
b 1 Rearrange so that the right-hand side is 0.	**b** $-x^2 + 10x = 25$ $-x^2 + 10x - 25 = 0$
2 Take out the HCF. Remember, the HCF can be −1.	$-1(x^2 - 10x + 25) = 0$
3 Divide both sides by the HCF.	$\frac{\cancel{-1}(x^2 - 10x + 25)}{\cancel{-1}} = \frac{0}{-1}$
4 Factorise and apply the Null Factor Law. Remember, there is only one solution when the factorised form is a perfect square.	$x^2 - 10x + 25 = 0$ $(x - 5)^2 = 0$ $x - 5 = 0$
5 Solve the linear equation.	$x = 5$

Helpful hints

✔ Watch for changes in sign when solving quadratic equations for x.

$$(x + 6)(x - 2) = 0 \quad \rightarrow \quad x = -6 \text{ or } x = 2$$

✔ Although it is possible to add and subtract multiples of x, you must *not* divide both sides of an equation by x in case $x = 0$, since dividing by 0 is *undefined*.
For example,

$x^2 = 5x$ $(-5x)$
$x^2 - 5x = 0$
$x(x - 5) = 0$
$x = 0$ or $x = 5$ ✔

$x^2 = 5x$ $(\div x)$
$x = 5$
Solution $x = 0$ is missing! ✖

✔ Always check the coefficients for a highest common factor (HCF). You can factor out the HCF from any quadratic equation, $ax^2 + bx + c = 0$, by dividing on both sides. This is because $\frac{0}{\text{HCF}} = 0$ for any HCF.
For example,

$$3x^2 - 15x + 12x = 0 \quad \rightarrow \quad x^2 - 5x + 4 = 0$$

Learning pathways

△ 1, 2(a, c, e, g), 3–11, 13, 16 □ 1, 2(b, d, f, h), 4–10, 12–15, 17 ⬠ 1, 5–7, 9, 10, 12, 14–18

ANS p479

Exercise 4.1A: Understanding and fluency

1 Which of the following are quadratic equations?

a $x^2 - 2 = 0$ **b** $3(x + 1) = 0$ **c** $x^2 + x = 5$ **d** $x^2 + 5x + 6$

e $2x^2 + x - 4 = 0$ **f** $x^3 + 8 = 0$ **g** $6x + 1 = 2x - 5$ **h** $x^2 + 7x = x - 3$

2 Use substitution to check whether the value in brackets is a solution to the given quadratic equation.

a $(x - 4)(x - 5) = 0 \quad (x = 5)$ **b** $(x + 2)(x - 8) = 0 \quad (x = 2)$

c $x(x - 6) = 0 \quad (x = 3)$ **d** $x^2 + 8x + 7 = 0 \quad (x = -1)$

e $x^2 - 4x + 4 = 0 \quad (x = -2)$ **f** $x^2 - 49 = 0 \quad (x = 7)$

g $x^2 - 2x - 15 = 0 \quad (x = -3)$ **h** $x^2 - 8x + 12 = 0 \quad (x = -4)$

3 Solve each of the following linear equations.

a $x + 3 = 0$ **b** $x - 6 = 0$ **c** $4x - 8 = 0$ **d** $3x + 1 = 0$

WE 4.1A **4** Solve each of the following quadratic equations.

a $(x + 2)(x - 3) = 0$ **b** $(x - 7)(x - 1) = 0$ **c** $(x + 4)(x - 4) = 0$

d $x(x - 6) = 0$ **e** $(x + 5)(x + 1) = 0$ **f** $x(x + 2) = 0$

g $(x - 8)(x + 8) = 0$ **h** $(2x + 2)(x - 7) = 0$ **i** $x(x - 11) = 0$

j $(2x + 3)(x - 5) = 0$ **k** $(4x - 2)(4x - 2) = 0$ **l** $(3x + 5)(2x + 3) = 0$

WE 4.1B **5** Factorise and then solve each quadratic equation.

a $x^2 + 7x + 10 = 0$ **b** $x^2 - 3x + 2 = 0$ **c** $x^2 + 5x = 0$

d $x^2 - 3x = 0$ **e** $x^2 - 36 = 0$ **f** $x^2 + 10x + 21 = 0$

WE 4.1C **6** Rearrange, factorise and then solve each quadratic equation.

a $x^2 - 2x = 8$ **b** $x^2 = 1$ **c** $x^2 = -8x$

d $x^2 + 3 = 4x$ **e** $6x = -x^2 - 9$ **f** $1 = 2x - x^2$

7 Use substitution to check that your solutions for questions **5** and **6** are correct.

8 Solve each quadratic equation.

a $2(x + 8)(x - 2) = 0$ **b** $-3(x - 1)(x - 4) = 0$

c $-7(x + 6)(x - 6) = 0$ **d** $-5x(x + 9) = 0$

WE 4.1D **9** Solve each quadratic equation by first dividing both sides by the highest common factor (HCF).

a $3x^2 + 9x + 6 = 0$ **b** $2x^2 + 8x = 0$ **c** $2x^2 - 4x - 16 = 0$

d $3x^2 - 12x + 12 = 0$ **e** $-5x^2 - 5x + 10 = 0$ **f** $-4x^2 + 8x = 0$

g $-x^2 - 10x - 21 = 0$ **h** $-3x^2 - 24x - 48 = 0$ **i** $-2x^2 + 32 = 0$

10 Solve each equation.

a $x^2 + 14x + 48 = 0$ **b** $3x^2 - 12x = 0$ **c** $x - x^2 = 0$

d $x^2 - 64 = 0$ **e** $-x^2 - 2x + 3 = 0$ **f** $-x^2 + 8x - 16 = 0$

g $-x^2 + 5x = 6$ **h** $2x^2 + 50 = -20x$ **i** $20 = 5x^2$

11 Quadratic equations may have 0, 1 or 2 real solutions.

a Determine how many real solutions each of the following quadratic equations has.

i $(x - 4)(x - 7) = 0$ **ii** $(x - 4)(x - 4) = 0$ **iii** $x^2 + 4 = 0$

b Identify and describe the feature(s) of each equation in part **a** that result(s) in the number of real solutions that the equation has.

Exercise 4.1B: Problem solving and reasoning

12 Determine the number of real solutions to each of the following equations.

a $x^2 + 3x - 10 = 0$ **b** $x^2 - 4 = 0$ **c** $x^2 - 6x + 9 = 0$ **d** $x^2 + 1 = 0$

e $x^2 + 7x = 0$ **f** $x^2 + 12x + 32 = 0$ **g** $x^2 - x - 72 = 0$ **h** $x^2 + 2x + 5 = 0$

13 The length of a rectangular mouse pad is 8 cm longer than its width (w).

a Write an expression for the area of the mouse pad in terms of w.

b Expand the expression.

c The area of the mouse pad is estimated to be 560 cm^2. Write an equation for the area of the mouse pad.

d Factorise and solve the equation. Which value of w is a feasible solution in this scenario? Explain.

e State the dimensions of the mouse pad.

14 The area of a rectangular sand pit is 35 m^2. The length is 2 m longer than the width (w).

a Write a quadratic equation to represent this scenario.

b Solve the quadratic equation.

c State the dimensions of the sand pit.

15 The width of a laptop screen is 12 cm less than its length. If the area of the screen is 640 cm^2, use algebra to determine the dimensions of the screen.

16 Alec throws a tennis ball back on to the court from the spectator stand. The height of the ball above the surface of the tennis court can be represented by the quadratic relationship $h = -4(t + 1)(t - 2)$, where h is the height in metres after t seconds in the air.

a What is the height of the ball after:

i 1 second **ii** 2 seconds?

b What is the height of the ball when Alec releases it from his hand?

c How long does it take for the ball to hit the tennis court after it is thrown?

d Explain why there is only one time value for your answer to part **c** even though you have solved a quadratic equation that has two solutions.

17 The height above the ground, h, of a firework rocket, x seconds after it is launched, is given by the equation $h = -5x^2 + 40x$.

a When is the rocket at a height of 60 m above the ground?

b Why are there two solutions? Explain the significance of each solution.

Exercise 4.1C: Challenge

18 An equation containing a pronumeral that is taken to the power of three (and no higher) is called a cubic equation. Use the Null Factor Law to solve the following cubic equations.

a $(x - 1)(x - 3)(x - 5) = 0$ **b** $x(x + 6)(x - 6) = 0$

c $(x + 11)(x - 4)^2 = 0$ **d** $(2x - 4)(3x + 1)(4x - 6) = 0$

Online resources:

Interactive skillsheet Solving quadratic equations

Worksheet Solving quadratic equations

CAS instructions Solving equations

Quick quiz 4.1

Lesson 4.2

Plotting quadratic relationships

Learning intentions

By the end of this lesson you will be able to ...

✔ identify the key features and plot graphs of quadratic relationships.

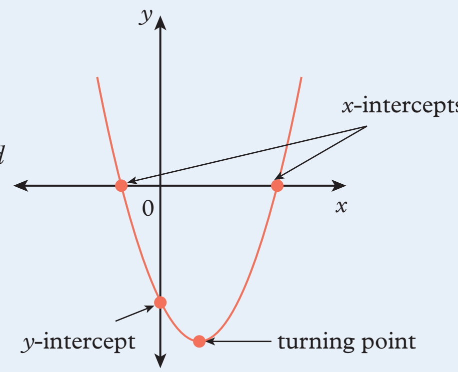
Inter-year links

Support	The Cartesian plane
Year 7	5.4 The Cartesian plane
Year 8	6.5 Plotting linear relationships

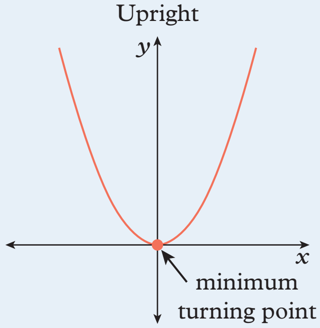
Key features of parabolas

Key content video
Plotting quadratic relationships

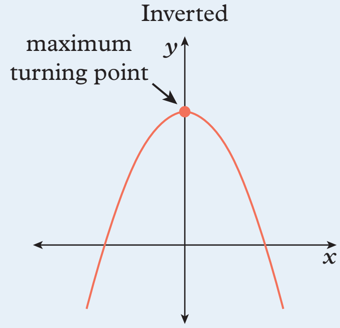
- The graph of a quadratic relationship is called a **parabola**.
- The key features of a parabola include:
  → a **turning point** (or **vertex**)
    - A *positive* leading coefficient gives the parabola an *upright* shape and a **minimum turning point**.
    - A *negative* leading coefficient gives the parabola an *inverted* shape and a **maximum turning point**.



Upright

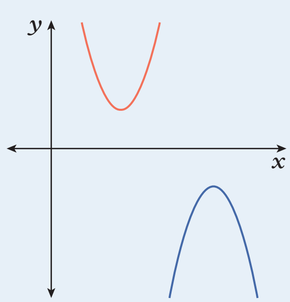


Inverted

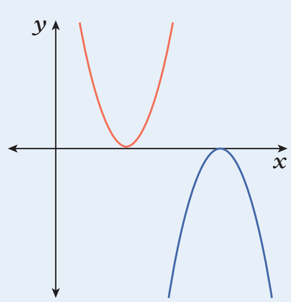


  → x- and y-intercepts
    - All parabolas have one y-intercept.
    - Parabolas can have zero, one or two x-intercepts.

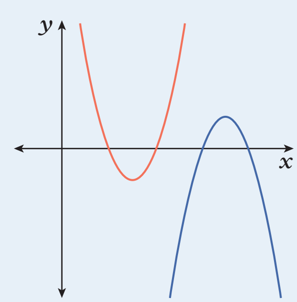
no x-intercepts



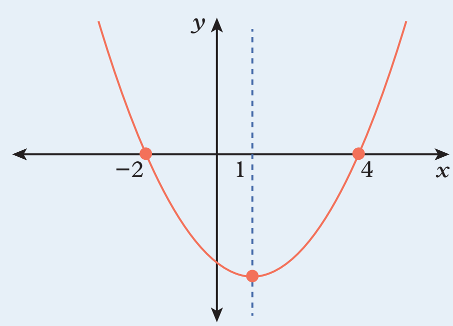
1 x-intercept



2 x-intercepts



  → a vertical **axis of symmetry**, which passes through the x-coordinate of the turning point.
    - When a parabola has two x-intercepts, the axis of symmetry passes through their midpoint.
      For example, the parabola on the right has x-intercepts at -2 and 4. So the axis of symmetry is $x = 1$, and the x-coordinate of the turning point is 1.

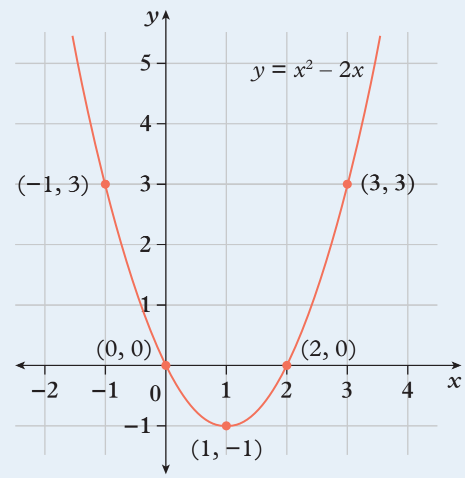


Plotting quadratic relationships

- To plot the graph of a quadratic relationship, create a table of coordinate points and join the points on a Cartesian plane with a smooth curve.
  1. Construct a table of x- and y-values by selecting values for x, then substituting each value of x into the relationship to find the corresponding value of y.
  2. Write the coordinate points from the table.
  3. Plot the coordinate points on a Cartesian plane.
  4. Join the points using a smooth curve to form a parabola and label the graph with its equation.

x	−1	0	1	2	3
$y = x^2 - 2x$	3	0	−1	0	3

(−1, 3), (0, 0), (1, −1), (2, 0), (3, 3)



Worked example 4.2A Identifying the key features of a quadratic graph

For the quadratic graph shown, identify:

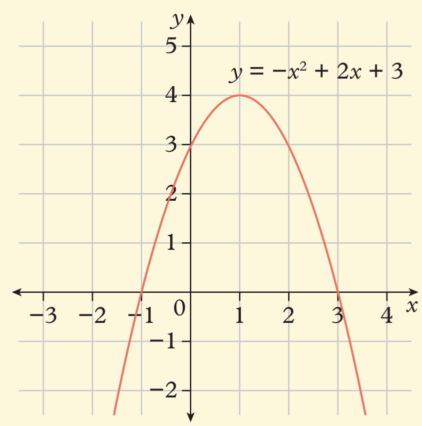
a the coordinates of the x-intercepts

b the coordinates of the y-intercept

c the coordinates of the turning point

d whether the turning point is a maximum or minimum

e the equation of the axis of symmetry.



THINK	WRITE
a State the coordinates of the points where the graph crosses the x-axis.	**a** x-intercepts: (−1, 0) and (3, 0)
b State the coordinates of the point where the graph crosses the y-axis.	**b** y-intercept: (0, 3)
c State the coordinates of the point where the graph changes direction.	**c** turning point: (1, 4)
d The turning point is the highest point on the parabola, so it is a maximum turning point.	**d** maximum turning point
e State the equation of the axis of symmetry. It is the vertical line which passes through the turning point.	**e** axis of symmetry: $x = 1$

Worked example 4.2B Plotting quadratic relationships

Sketch a graph of $y = x^2 - 2x - 1$ by first completing a table of values for x from -1 to 3.

THINK

1 Construct a table of values for x from -1 to 3. Substitute each value of x into the equation to find the corresponding value of y.

2 List the coordinate points.

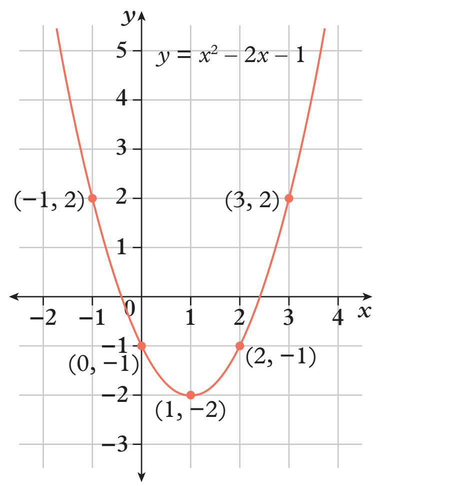
3 Plot the points on a Cartesian plane.

4 Join the points with a smooth curve. Label the graph with its equation.

WRITE

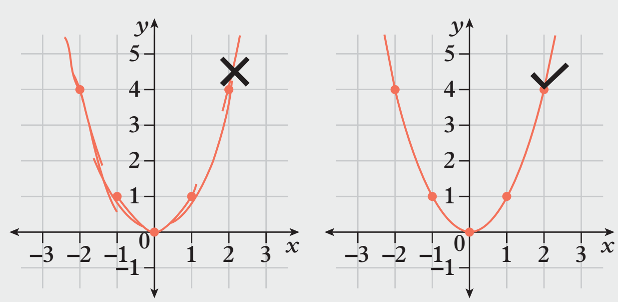
x	-1	0	1	2	3
y	2	-1	-2	-1	2

$(-1, 2), (0, -1), (1, -2), (2, -1), (3, 2)$

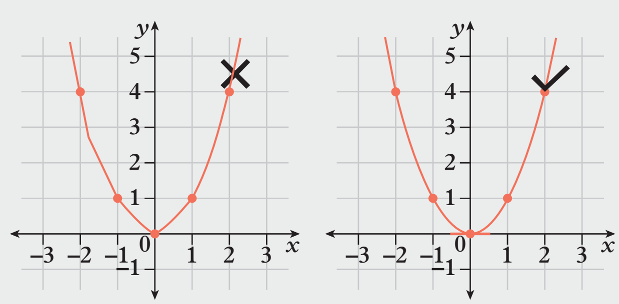


Helpful hints

✔ Rather than drawing 'hairy' parabolas, try starting at the turning point and then drawing two smooth symmetrical curves through the other plotted points.



✔ Draw a short horizontal dash where the turning point is to ensure the parabola does not end up with a pointy turning point.



✔ Take care when reading a question. If you are asked to find the x- and y-intercepts, then you must give your answer as coordinates. So write the coordinates of the intercepts as $(5, 0)$ and $(0, -1)$, instead of $x = 5$ and $y = -1$.

Learning pathways

△ 1(a, c, e, g, i), 2–4, 7, 9, 10, 12

□ 1(b, d, f, h), 2, 3, 5, 6, 8, 9,11–14

⬠ 1(d, f, h), 2, 3, 6, 8, 9, 11–15

ANS p480

Exercise 4.2A: Understanding and fluency

WE 4.2A **1** For each quadratic graph, identify:

i the coordinates of the x-intercepts, if any

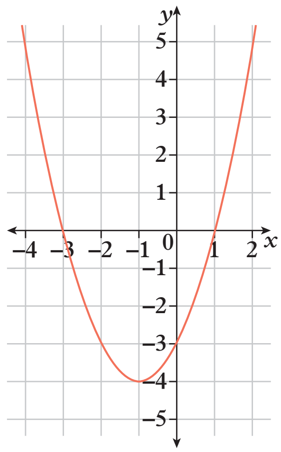
ii the coordinates of the y-intercept

iii the coordinates of the turning point

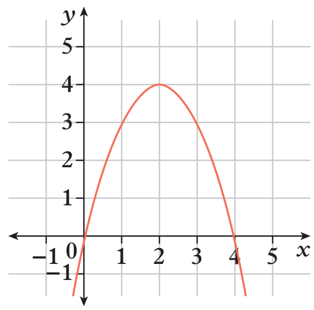
iv whether the turning point is a maximum or minimum

v the equation of the axis of symmetry.

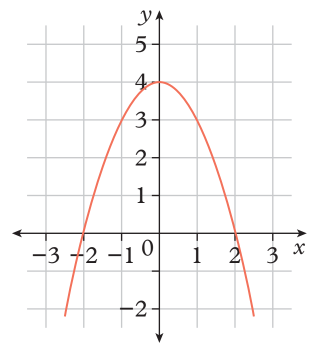
a



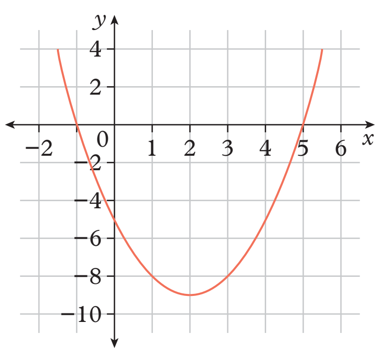
b



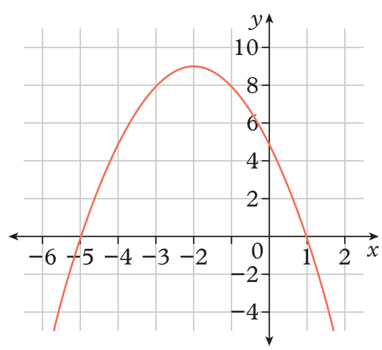
c



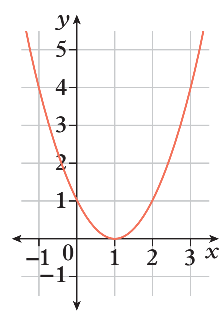
d



e



f



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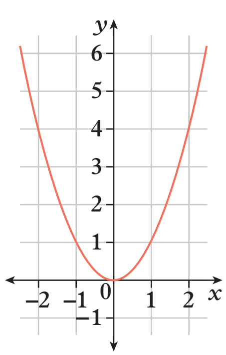
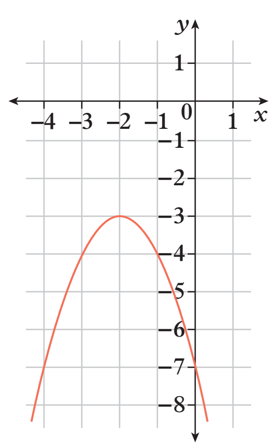
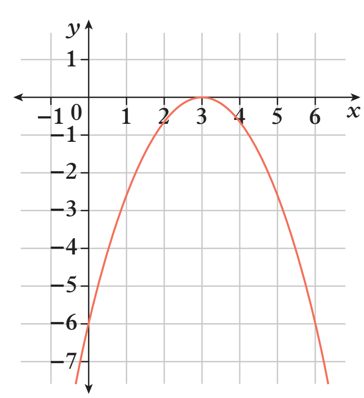


h



i



2 For each quadratic relationship:
  i complete the table of values
  ii plot the graph
  iii identify the axis intercepts and the turning point, and then state the equation of the axis of symmetry
  iv state whether the turning point is a maximum or minimum.
    a $y = x^2 + 2x - 8$

x	−5	−4	−3	−2	−1	0	1	2	3
y									

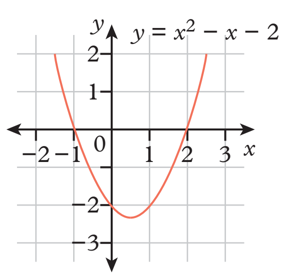
    b $y = 9 - x^2$

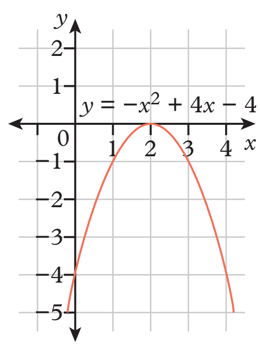
x	−4	−3	−2	−1	0	1	2	3	4
y									

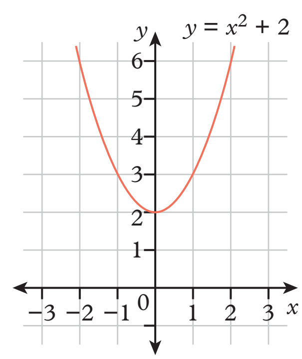
WE 4.2B 3 For each quadratic relationship:
  i plot its graph by first completing a table of coordinate points, using the x-values given in brackets
  ii identify the coordinates of the x- and y-intercepts and the turning point, and state the equation of the axis of symmetry.
    a $y = -x^2 - 6x - 5$ (−6 to 0)
    b $y = x^2 - 4$ (−3 to 3)
    c $y = x^2 - 2x - 15$ (−4 to 6)
    d $y = x^2 + 4x$ (−5 to 1)
    e $y = -x^2 + 2x$ (−1 to 3)
    f $y = x^2 - 6x + 9$ (0 to 6)

Exercise 4.2B: Problem solving and reasoning

4 Consider the x- and y-intercepts of a parabola.
  a How many x-intercepts do each of the following parabolas have?

i


ii


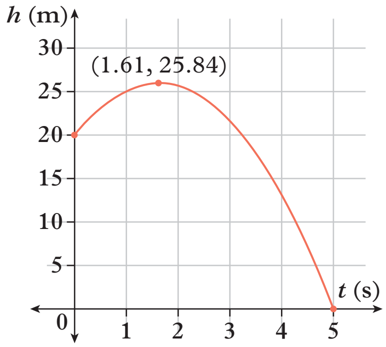
iii


  b Can a parabola have more than two x-intercepts? Explain.
  c How many y-intercepts does each parabola in part **a** have?
  d Describe the link between the y-intercept of a parabola and the general form $y = ax^2 + bx + c$.

5 For each of the following parabolas state the number of:
  i x-intercepts
  ii y-intercepts.
    a Parabola with a minimum turning point at (−3, 1)
    b Parabola with a maximum turning point at (10, 0)

6 Find the equation of the axis of symmetry if the x-intercepts of a quadratic graph are:
  a (−3, 0) and (5, 0)
  b (5, 0) and (6, 0)
  c (−2, 0) and (1, 0)
  d (−2, 0) and (−1, 0).

7 The graph below shows the height, h, (in metres) of a firework rocket t seconds after it is launched.



a From what height is the rocket launched?

b What is the maximum height of the rocket to the nearest metre?

c For how long is the rocket in the air?

8 Alani's hair clip falls to the ground while she is on a roller coaster. The position of the clip as it falls can be described by the relationship $h = 100 - 4t^2$, where h is the height of the clip above the ground in metres after t seconds.

a Plot the graph of the relationship for t-values from 0 to 5.

b Why should the parabola only be drawn for t-values between 0 and 5 (not values less than 0, or values greater than 5)?

c What is the height of the clip above the ground after:

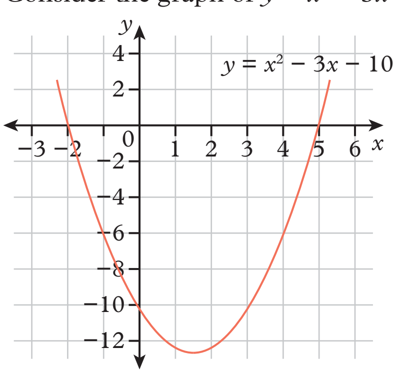
i 2 s

ii 3 s?

d From what height above the ground did the hair clip start to fall?

e How long did it take for the hair clip to hit the ground?

9 Consider the graph of $y = x^2 - 3x - 10$ shown below.



a Identify the x-coordinates of the x-intercepts from the graph.

b Solve the quadratic equation $x^2 - 3x - 10 = 0$ by first factorising the quadratic expression and then applying the Null Factor Law.

c Compare your answers for parts **a** and **b**. What do you notice?

d Explain how you can use the graph of $y = x^2 - 3x - 10$ to solve $x^2 - 3x - 10 = 0$.

10 Determine the equation for the axis of symmetry of a parabola that has:

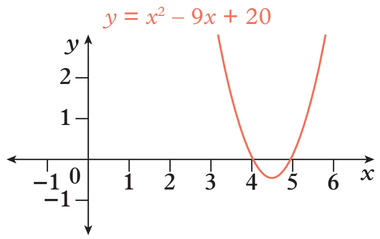
a x-intercepts at (2, 0) and (8, 0)

b x-intercepts at (−3, 0) and (3, 0)

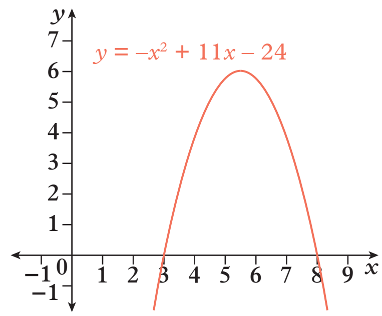
c one x-intercept at (−4, 0). Hint: Draw a rough sketch of the parabola and consider the midpoint.

11 Solve the following quadratic equations using the provided graphs.

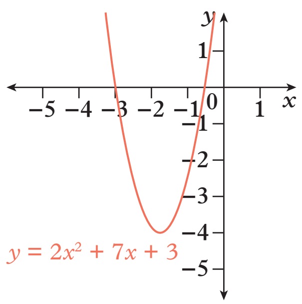
a $x^2 - 9x + 20 = 0$



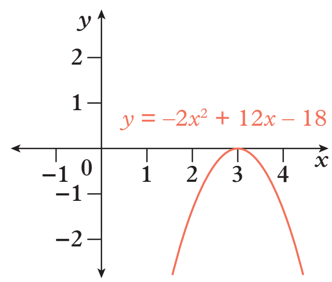
b $-x^2 + 11x - 24 = 0$



c $2x^2 + 7x + 3 = 0$



d $-2x^2 + 12x - 18 = 0$



12 The axis of symmetry of a quadratic graph is $x = -4$. If the graph passes through the x-axis at $(-8, 0)$, determine the coordinates of the second x-intercept.

13 Use the graph of $y = x^2 - 3x - 10$ in question **9** to write an inequality to show the values of x which satisfy $x^2 - 3x - 10 \le 0$.

Exercise 4.2C: Challenge

14 The equation for a graph with x-intercepts at $(-7, 0)$ and $(5, 0)$ is written as $y = (x - p)(x - q)$, where p and q are constants.

a Determine the x-intercepts of $y = (x - p)(x - q)$ in terms of p and q.

b Hence identify appropriate values for p and q and write the equation for the graph.

c Determine the minimum value of y.

d Determine the equation for a graph with x-intercepts at $(-7, 0)$ and $(5, 0)$ and a maximum value of 36.

15 Kim throws a javelin. The position of the tip of the javelin can be represented by the quadratic relationship $y = -0.05x^2 + 1.5x + 1.55$, where y is the height above the ground and x is the horizontal distance from where the javelin was thrown. Both x and y are in metres.

a Plot the graph of this relationship. Use 0, 5, 10, 15, …, 30 as the x-values in the table.

b At what height off the ground was the javelin thrown?

c Let $y = 0$ and solve the quadratic equation for x.
Hint: Start by multiplying both sides of the equation by -20.

d What horizontal distance did the javelin travel before hitting the ground?

e What was the maximum height reached by the javelin?

Online resources:

- **Interactive skillsheet** Plotting quadratic relationships
- **Investigation** Calculating the second difference
- **CAS instructions** Graphing functions
- **Quick quiz** 4.2

Lesson 4.3

Sketching parabolas using intercepts

Learning intentions

By the end of this lesson you will be able to ...

✔ sketch a parabola using its x- and y-intercepts.

Inter-year links

Year 7	6.4 Substitution
Year 8	6.2 Solving linear equations
Year 10	4.3 Sketching parabolas

Sketching parabolas using intercepts

Key content video
Sketching parabolas using intercepts

- To sketch a graph means to identify the shape and key features of the graph and use them to draw the graph.
- The axis intercepts of some quadratic relationships can be used to sketch the corresponding graph. Find the coordinates of all axis intercepts and the turning point and plot them on the Cartesian plane before joining them with a smooth curve.
- To find the x-intercepts of $y = ax^2 + bx + c$:
  1. Substitute $y = 0$ into the quadratic relationship.
  2. Factorise the quadratic equation.
  3. Solve the equation by applying the Null Factor Law.

$y = x^2 + 2x - 3$

Let $y = 0$:

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x + 3 = 0$ or $x - 1 = 0$

$x = -3$ or $x = 1$

x-intercepts: $(-3, 0)$ and $(1, 0)$

- To find the y-intercept, substitute $x = 0$ into the quadratic relationship.
  → When a quadratic relationship is in the general form $y = ax^2 + bx + c$, the y-intercept is $(0, c)$.

Let $x = 0$:

$y = (0)^2 + 2(0) - 3$

$y = -3$

y-intercept: $(0, -3)$

- The turning point is located on the axis of symmetry. When there are two x-intercepts, the axis of symmetry passes through their midpoint. To determine the turning point of such a parabola:
  1. Calculate the x-coordinate by finding the midpoint of the two x-intercepts.
  2. Calculate the y-coordinate by substituting the x-coordinate into the quadratic relationship.

$x = \frac{-3 + 1}{2}$

$= -1$

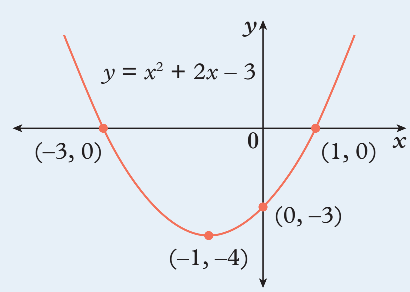
Let $x = -1$:

$y = (-1)^2 + 2(-1) - 3$

$= -4$

Turning point: $(-1, -4)$

- Plot the axis intercepts and turning point on the Cartesian plane and join them together with a smooth curve in the shape of a parabola. Label the graph with its equation.



Worked example 4.3A Finding the coordinates of the x- and y-intercepts

For each quadratic relationship, find the coordinates of the:

i x-intercepts **ii** y-intercept.

a $y = x^2 - 6x$ **b** $y = x^2 + 7x + 10$

THINK

i **1** To find the x-coordinate of the x-intercept, substitute $y = 0$ into the quadratic relationship.

2 Factorise the quadratic equation.

3 Apply the Null Factor Law to solve the equation.

ii To find the y-coordinate of the y-intercept, substitute $x = 0$ into the quadratic relationship and solve for y.

WRITE

a **i** $y = x^2 - 6x$

For x-intercepts, let $y = 0$:

$$x^2 - 6x = 0$$
$$x(x - 6) = 0$$
$$x = 0 \text{ or } x - 6 = 0$$
$$x = 0 \text{ or } x = 6$$

x-intercepts:
(0, 0) and (6, 0)

ii $y = x^2 - 6x$

For y-intercept, let $x = 0$:

$$\begin{aligned} y &= (0)^2 - 6(0) \\ &= 0 \end{aligned}$$

y-intercept: (0, 0)

b **i** $y = x^2 + 7x + 10$

For x-intercepts, let $y = 0$:

$$x^2 + 7x + 10 = 0$$
$$(x + 2)(x + 5) = 0$$
$$x + 2 = 0 \text{ or } x + 5 = 0$$
$$x = -2 \text{ or } x = -5$$

x-intercepts:
(−2, 0) and (−5, 0)

ii $y = x^2 + 7x + 10$

For y-intercept, let $x = 0$:

$$\begin{aligned} y &= (0)^2 + 7(0) + 10 \\ &= 10 \end{aligned}$$

y-intercept: (0, 10)

Worked example 4.3B Finding the coordinates of the turning point using the x-intercepts

Find the coordinates of the turning point of $y = (x + 3)(x - 9)$.

THINK

1 Identify the coordinates for the x-intercepts.

2 Calculate the x-coordinate of the turning point by finding the midpoint of the x-intercepts.

3 Calculate the y-coordinate of the turning point by substituting the x-coordinate into the quadratic relationship.

WRITE

x-intercepts: (−3, 0) and (9, 0)

$$\begin{aligned} x &= \frac{-3 + 9}{2} \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

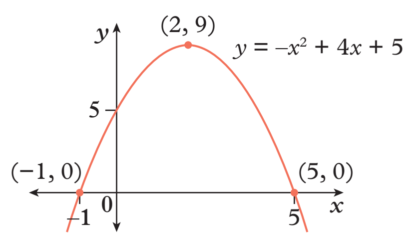
Let $x = 3$:

$$\begin{aligned} y &= (x + 3)(x - 9) \\ &= (3 + 3)(3 - 9) \\ &= 6 \times (-6) \\ &= -36 \end{aligned}$$

Turning point: (3, −36)

Worked example 4.3C Sketching parabolas using the intercepts

Sketch the graph of $y = -x^2 + 4x + 5$ by first finding the x- and y-intercepts. Label the turning point with its coordinates.

THINK	WRITE
1 Find the x-intercepts by substituting $y = 0$ into the relationship.	$y = -x^2 + 4x + 5$ For the x-intercepts, let $y = 0$:
2 Divide both sides of the equation by the coefficient of x^2, which is -1, so that the equation is of the form $x^2 + bx + c = 0$.	$-x^2 + 4x + 5 = 0$ $-(x^2 - 4x - 5) = 0$ $x^2 - 4x - 5 = 0$
3 Factorise the quadratic equation.	$(x + 1)(x - 5) = 0$
4 Apply the Null Factor Law to solve each linear equation for x.	$x + 1 = 0$ or $x - 5 = 0$ $x = -1$ or $x = 5$ x-intercepts: $(-1, 0)$ and $(5, 0)$
5 Find the y-intercept by substituting $x = 0$ into the relationship and solving for y.	For the y-intercept, let $x = 0$: $y = -(0)^2 + 4(0) + 5$ $= 5$ y-intercept: $(0, 5)$
6 Find the coordinates of the turning point.	For the turning point: $x = \frac{-1 + 5}{2}$ $= \frac{4}{2}$ $= 2$ Let $x = 2$: $y = -(2)^2 + 4(2) + 5$ $= -4 + 8 + 5$ $= 9$ Turning point: $(2, 9)$
7 Mark the x- and y-intercepts on the axes, plot the turning point, then draw the parabola. Label the equation of the graph and the coordinates of the turning point.	

Helpful hints

- ✔ Label your working out so that it is clear that you are calculating the x-intercepts, the y-intercept and the turning point. This can make it easier to check your calculations on a test.
- ✔ Always label the key features of a graph such as the x-intercepts, the y-intercept and the turning point when sketching parabolas.
- ✔ For any given quadratic relationship, $y = ax^2 + bx + c$:
  - if $a > 0$, then the parabola has a minimum turning point
  - if $a < 0$, then the parabola has a maximum turning point.

Learning pathways

△ 1, 2, 3(a, b, d, i), 4, 5, 6(1st, 2nd columns), 12, 14	□ 2–5, 6(2nd, 3rd columns), 8–11, 13, 15	⬠ 4, 5, 6(3rd, 4th columns), 8, 10, 11, 13, 15, 16, 17

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Exercise 4.3A: Understanding and fluency

1 For each quadratic relationship, find the coordinates of the:

i x-intercepts ii y-intercept.

a $y = (x + 7)(x - 5)$ b $y = 3(x - 2)(x - 6)$ c $y = (3x + 12)(4x + 8)$

WE 4.3A 2 For each quadratic relationship, find the coordinates of the:

i x-intercepts ii y-intercept.

a $y = x^2 - 2x$ b $y = x^2 + 8x$ c $y = x^2 + 6x + 8$

d $y = x^2 - 8x + 12$ e $y = x^2 - 4x - 5$ f $y = x^2 - 9$

WE 4.3B 3 Find the coordinates of the turning point for each quadratic relationship.

a $y = x(x - 4)$ b $y = (x - 3)(x - 5)$ c $y = (x + 4)(x - 6)$

d $y = -(x + 1)(x + 3)$ e $y = (x - 2)(x + 2)$ f $y = -(x + 2)(x - 4)$

g $y = x(x + 3)$ h $y = -x(x + 7)$ i $y = 2(x - 2)(x - 8)$

4 Sketch the graph of each quadratic relationship by first finding the x- and y-intercepts. Label the turning point with its coordinates.

a $y = (x + 2)(x + 4)$ b $y = (x + 3)(x - 3)$ c $y = x(x - 2)$

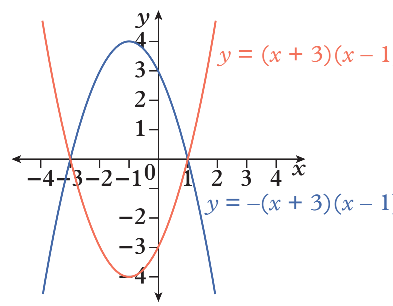
d $y = -(x - 2)(x - 6)$ e $y = (x - 4)^2$ f $y = (2x - 4)(x - 6)$

5 Consider the graphs of $y = (x + 3)(x - 1)$ and $y = -(x + 3)(x - 1)$.

a Identify the x-intercepts for each graph.

b Find the turning point of each graph.

c Describe the similarities and differences between the two graphs.



WE 4.3C 6 Sketch the graph of each quadratic relationship by first finding the x- and y-intercepts. Label the turning point with its coordinates.

a $y = x^2 - 6x + 5$ b $y = x^2 + 4x - 12$ c $y = -x^2 + 6x + 7$ d $y = x^2 + 4x$

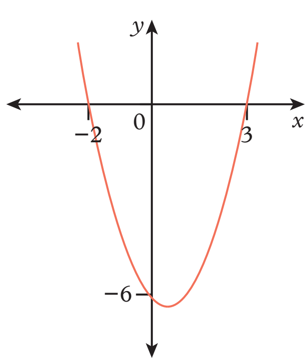
e $y = -x^2 - 8x - 12$ f $y = x^2 - 4$ g $y = x^2 + 2x - 15$ h $y = 10x - x^2$

i $y = x^2 - 6x - 7$ j $y = x^2 - 5x$ k $y = 16 - x^2$ l $y = -x^2 + x$

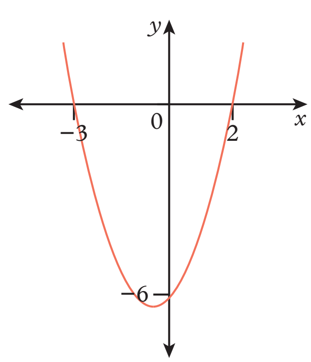
Exercise 4.3B: Problem solving and reasoning

7 Match each graph with its rule from the options below.

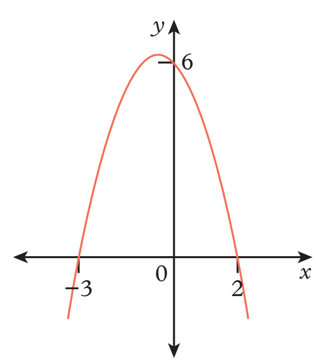
a



b



c



A $y = x^2 + x - 6$ **B** $y = -x^2 - x + 6$ **C** $y = x^2 - x - 6$

8 A parabola with a maximum turning point is called an inverted parabola. Explain how you can tell whether a parabola is upright or inverted from its equation. You may refer to your answers to question **6** as examples in your explanation.

9 Without sketching the graph, decide whether these equations describe upright or inverted parabolas.

a $y = x^2 - 2x - 3$ **b** $y = x^2 - 100$ **c** $y = -x^2 - 8x + 15$

d $y = -x^2 + 6x$ **e** $y = 9x^2 - 10x$ **f** $y = 81 - 4x^2$

10 Consider the graph shown to the right.

a Is the parabola upright or inverted?

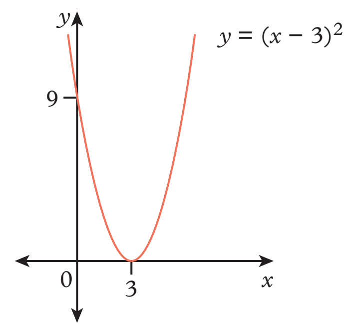
b How many y-intercepts does the parabola have? List the coordinates of the y-intercept(s).

c How many x-intercepts does the parabola have? List the coordinates of the x-intercept(s).

d What are the coordinates of the turning point?

e Describe the relationship between the x-intercept(s) and the turning point.

f Describe the relationship between equations of the form $y = a(x - p)^2$ (where a and p are constants) and the coordinates of the turning point.



11 Sketch the graph of each quadratic relationship. Label the turning point with its coordinates.

a $y = (x - 1)^2$

b $y = -(x + 2)^2$

c $y = x^2 + 8x + 16$

d $y = x^2 - 4x + 4$

e $y = -x^2 - 6x - 9$

12 Consider the graph shown to the right.

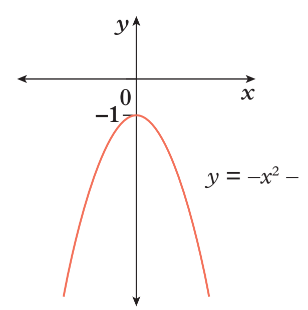
a Is the parabola upright or inverted?

b How many y-intercepts does the parabola have? List the coordinates of the y-intercept(s).

c How many x-intercepts does the parabola have? List the coordinates of the x-intercept(s).

d What are the coordinates of the turning point?

e Describe the relationship between the equations of the form $y = ax^2 + c$ (where a and c are constants) and the coordinates of the turning point.



13 Rhys fires an arrow from a bow. The path of the arrow can be represented by the quadratic relationship $h = -0.1(d + 1)(d - 15)$, where h is the height above the ground and d is the horizontal distance from where the arrow was fired. Both h and d are in metres.

a Sketch the graph of this relationship by first finding the intercepts and turning point.

b Between what two values of d does the graph represent the path of the arrow?

c How high does the arrow reach?

d At what height above the ground was the arrow fired?

e What horizontal distance did the arrow fly before hitting the ground?

14 A soccer ball is kicked off the ground. The path of the ball can be represented by the quadratic relationship $y = -0.2x^2 + 2.4x$, where x is the horizontal and y is the vertical distance from where the ball was kicked (in metres).

a Sketch the graph of this relationship by first finding the intercepts and turning point.

b What was the maximum height of the soccer ball?

c What horizontal distance had the soccer ball travelled when it was at its maximum height?

d What horizontal distance did the soccer ball travel before hitting the ground?

15 During a game of netball, Ayca throws the ball to her teammate Siegrid. The ball's position is described by the equation $h = -4t^2 + 2t + 2$, where h is the height of the ball above the ground, in metres, and t is the time the ball spends in the air in seconds.

a If Siegrid catches the ball 0.5 seconds after Ayca throws it to her, from what height does Siegrid catch the ball?

b List the coordinates that represent the point at which the ball is thrown and the point at which it is caught. What do you notice about the height of the ball in each case?

c Use your observations from part **b** and the symmetry of the parabola to determine the coordinates of the turning point.

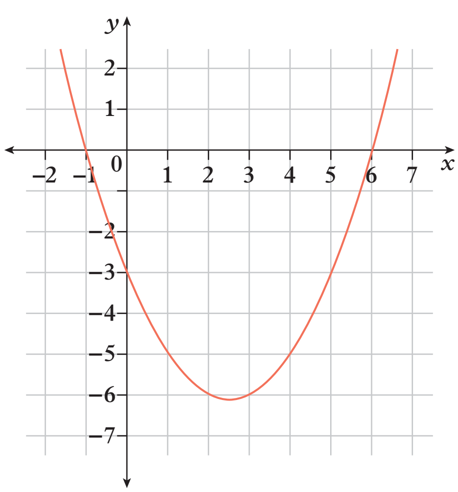
d Hence, sketch the graph from $t = 0$ seconds to $t = 0.5$ seconds.

e What is the maximum height of the netball?

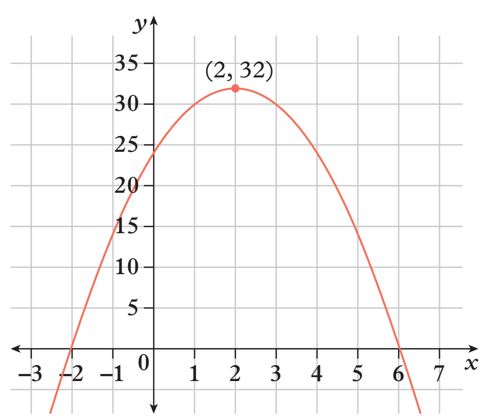
Exercise 4.3C: Challenge

16 The equation for a quadratic graph with at least one x-intercept can be expressed in factorised form:
$y = a(x - p)(x - q)$.
The values of p and q are equal to the x-coordinates of the x-intercepts. The value of a can be determined by substituting the coordinates of any other point on the graph into the values for x and y. Determine the equation of each of the following quadratic graphs in factorised form.

a



b



17 Find the equation of the parabola that has only one x-intercept at $(3, 0)$ and passes through the point $(-1, 6)$.

Online resources:

Interactive skillsheet Sketching parabolas using intercepts

Investigation Fencing the chicken run

Quick quiz 4.3

ANS p485

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

4.1 **1** Solve each quadratic equation using the Null Factor Law.

a $(x-4)(x-3)=0$ **b** $(x-2)(x+2)=0$

c $(x+9)(x+9)=0$ **d** $x(x+4)=0$

4.1 **2** Solve the following quadratic equations.

a $x^2-12x+35=0$ **b** $x^2+3x-28=0$

c $x^2-12x+36=0$ **d** $x^2-81=0$

4.1 **3** Solve the following quadratic equations by first identifying the HCF.

a $2x^2-32=0$ **b** $8x^2-16x-64=0$

c $-2x^2-12x-10=0$ **d** $45x-3x^2=0$

4.2 **4** For the quadratic graph shown on the right, identify:

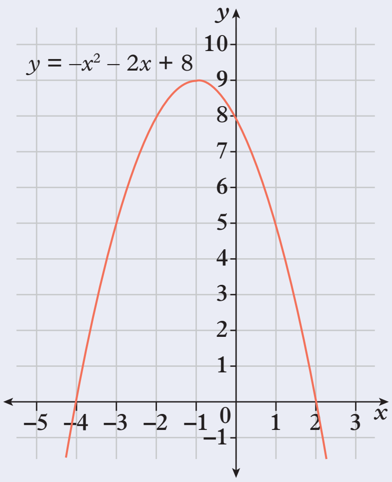
a the coordinates of the x-intercepts

b the coordinates of the y-intercept

c the coordinates of the turning point

d whether it is a maximum or minimum turning point

e the equation of the axis of symmetry.



4.2 **5** Sketch a graph of $y=2x^2-8$ by first completing a table of values for x from -3 to 3.

4.2 **6** A quadratic graph has a minimum turning point at (5, 0).

a What is the equation of the axis of symmetry?

b What are the coordinates of the x-intercept(s)?

4.3 **7** For each quadratic relationship, find the coordinates of the:

i x-intercept(s)

ii y-intercept.

a $y=x^2+3x$

b $y=2x^2-50$

c $y=x^2+2x-8$

d $y=x^2-16x+64$

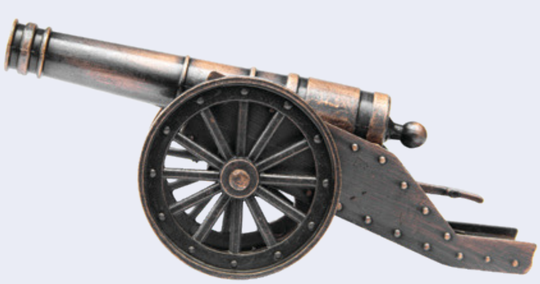
4.3 **8** Find the coordinates of the turning point for each quadratic relationship.

a $y=(x-2)(x+4)$ **b** $y=x(x-4)$

c $y=(x-6)(x+6)$ **d** $y=-(x-8)(x-12)$

4.3 **9** Sketch the graph of $y=x^2-4x+3$ by first finding the x- and y-intercepts and the turning point. Label the turning point with its coordinates.

4.3 **10** A cannon is fired up into the air as an experiment. The height, h, in metres, of the cannonball above the ground is given by $h=15d-d^2$, where d is the horizontal distance, in metres, from where the cannonball is fired.



a Sketch the graph of this relationship by first finding the intercepts. Label the turning point with its coordinates.

b How far from the cannon does the cannonball land?

c What is the maximum height reached by the cannonball?

Lesson 4.4

Sketching parabolas using transformations

Learning intentions

By the end of this lesson you will be able to ...

✔ sketch quadratic relationships in turning point form using transformations.

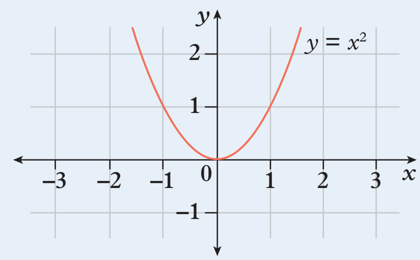
Inter-year links

Year 7	8.3 Translations
Year 8	9.1 Transformations
Year 10	4.3 Sketching parabolas

Sketching parabolas using transformations

Key content video
Sketching parabolas using transformations

- The graph of $y = x^2$ is an upright parabola with a minimum turning point at the origin, (0, 0).
- Quadratic relationships can be written in **turning point form**: $y = a(x - h)^2 + k$.
  → The coordinates of the turning point are (h, k).
- **Transformations** can be performed on the graph of $y = x^2$ to sketch a graph of $y = a(x - h)^2 + k$.
- A **dilation** by a factor of a in the y-direction:
  → enlarges the parabola in the y-direction if $a > 1$
  → compresses the parabola in the y-direction if $0 < a < 1$.
- A **reflection** in the x-axis takes place if $a = -1$.
  → If $a < 0$ and $a \neq -1$, then both a dilation and reflection take place.
- A **horizontal translation** moves the parabola h units:
  → to the right if $h > 0$
  → to the left if $h < 0$.
- A **vertical translation** moves the parabola k units:
  → up if $k > 0$
  → down if $k < 0$.
- A sequence of transformations can be applied to the graph of $y = x^2$.
  → Dilations and reflections should be applied before translations.

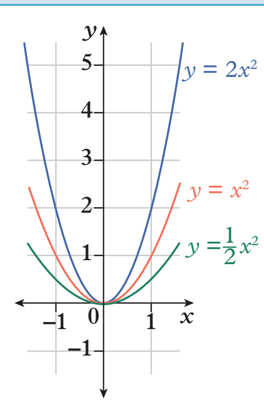
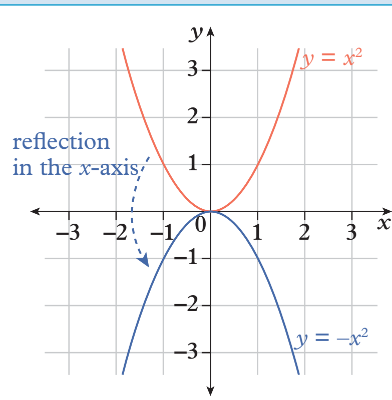
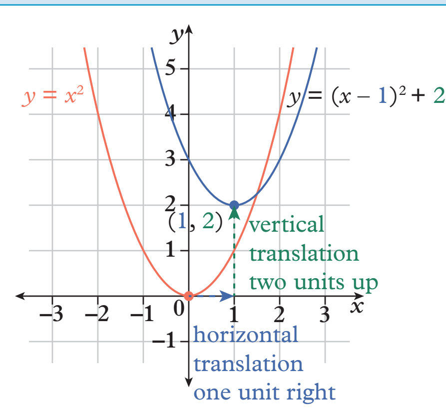


vertical translation of k units
$k > 0$ up
$k < 0$ down

$$y = a(x - h)^2 + k$$

dilation
$a > 0$ upright
$a < 0$ inverted

horizontal translation of h units
$h > 0$ right
$h < 0$ left

Dilations	Reflections	Translations
		

Worked example 4.4A Sketching a parabola using one transformation

Sketch the graph of each quadratic relationship by first sketching the graph of $y = x^2$ on a Cartesian plane and then performing the transformation. Label the turning point on each graph with its coordinates.

a $y = x^2 - 3$ **b** $y = 2x^2$ **c** $y = -x^2$

THINK

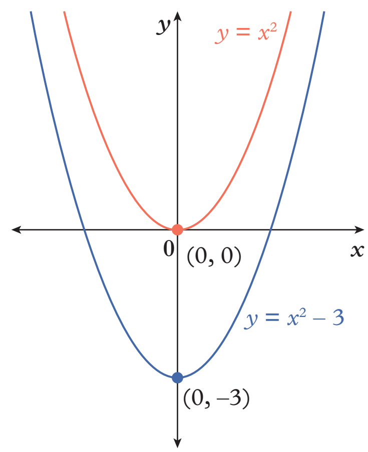
a 1 Identify the transformation. The relationship is in the form $y = x^2 + k$, where $k = -3$, so the graph of $y = x^2$ undergoes a vertical translation of 3 units down.

2 Sketch the graph of $y = x^2$ and label its turning point, (0, 0). Translating this point down 3 units gives the turning point of $y = x^2 - 3$ as $(0, -3)$.

3 Sketch the graph of $y = x^2 - 3$. It has the same shape as $y = x^2$, with its turning point at $(0, -3)$.

WRITE

a



THINK

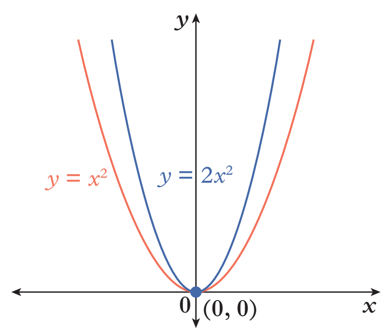
b 1 Identify the transformation. The relationship is in the form $y = ax^2$ where $a = 2$, so the graph of $y = x^2$ is dilated by a factor of 2 in the y-direction.

2 Sketch the graph of $y = x^2$ and label its turning point, (0, 0). Stretch this curve in the y-direction by a factor of 2. This means that for any given x-coordinate, the point on $y = 2x^2$ is twice as tall as the point on $y = x^2$.

3 Identify the turning point. Doubling the y-coordinate of (0, 0) leaves the point unchanged, so the turning point is (0, 0).

WRITE

b



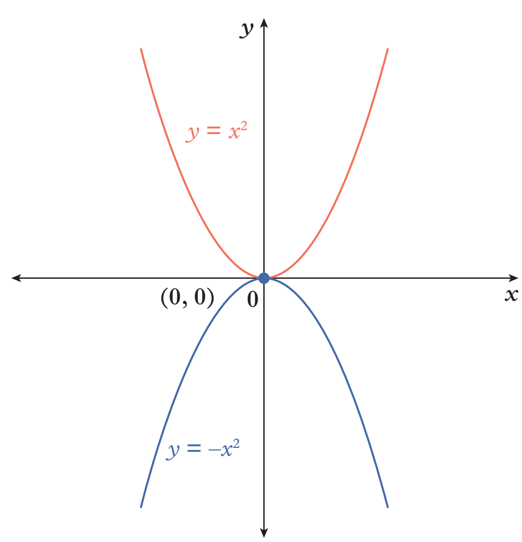
THINK

c 1 Identify the transformation. The relationship is in the form $y = ax^2$ where $a = -1$, so the graph of $y = x^2$ is reflected across the x-axis.

2 Sketch the graph of $y = x^2$ and label its turning point, (0, 0). Reflect the curve across the x-axis. Observe that the minimum turning point of $y = x^2$ at (0, 0) becomes the maximum turning point of $y = -x^2$.

WRITE

c



Worked example 4.4B Sketching a parabola using multiple transformations

Sketch the graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the y-intercept and label the turning point with its coordinates.

a $y = -(x + 3)^2$ **b** $y = (x - 2)^2 - 1$

THINK

a **1** Find the y-coordinate of the y-intercept and mark it on the y-axis.

2 Identify the transformations. The relationship is in the form $a(x - h)^2$ so the graph of $y = x^2$:

- is inverted as $a = -1$
- is translated 3 units to the left as $h = -3$.

The turning point (h, k) is $(-3, 0)$.

$$y = -(x + 3)^2$$

inverted

horizontal translation of 3 units to the left

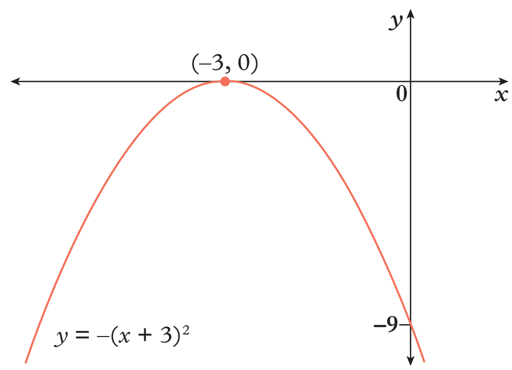
3 Sketch the graph.

WRITE

a $y = -(x + 3)^2$

Let $x = 0$:

$y = -(0 + 3)^2$
$= -9$



THINK

b **1** Find the y-coordinate of the y-intercept and mark it on the y-axis.

2 Identify the transformations. The relationship is in the form $(x - h)^2 + k$ so the graph of $y = x^2$:

- is translated 2 units to the right as $h = 2$
- is translated 1 unit down as $k = -1$.

The turning point (h, k) is $(2, -1)$.

$$y = (x - 2)^2 - 1$$

horizontal translation of 2 units to the right

vertical translation of 1 unit down

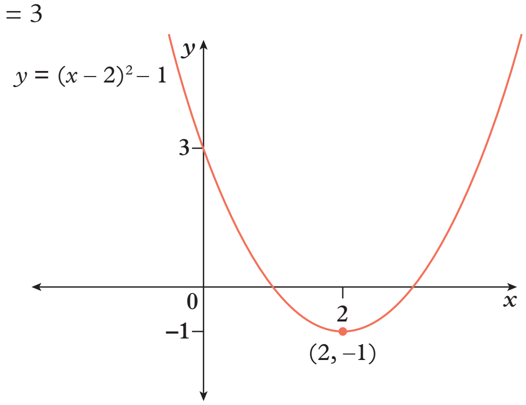
3 Sketch the graph.

WRITE

b $y = (x - 2)^2 - 1$

Let $x = 0$:

$y = (0 - 2)^2 - 1$
$= 4 - 1$
$= 3$



Helpful hints

- ✔ In turning point form:
  - → vertical translations are determined by the constant *outside* the brackets
  - → horizontal translations are determined by the constant *inside* the brackets with x.

$$y = a(x - h)^2 + k$$

horizontal translation of h units   vertical translation of k units

- ✔ It is always helpful to label the key features of your graph before sketching. The turning point, y-intercept, and a dashed line for the axis of symmetry will help you sketch an accurate parabola.
- ✔ Remember that dilations and reflections must be performed before any translations.

Learning pathways

1–9, 10(a–d), 11(a, c, e, f), 12, 14, 16	2–8, 10, 11(b, d, f, h), 12, 15–17, 18(a–e), 19(a–c)	7, 8, 10(f–i), 11(b, d, f, h), 13, 15, 16, 18–20

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Exercise 4.4A: Understanding and fluency

1 Complete the descriptions of transformations below.

a To sketch the graph of $y = x^2 - 10$, the graph of $y = x^2$ is translated __________ units __________.

b To sketch the graph of $y = (x - 5)^2$, the graph of $y = x^2$ is translated __________ units __________.

c To sketch the graph of $y = (x + 12)^2$, the graph of $y = x^2$ is translated __________ units __________.

d To sketch the graph of $y = x^2 + 8$, the graph of $y = x^2$ is translated __________ units __________.

WE 4.4A 2 Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a vertical translation to sketch the graph of each quadratic relationship on the same set of axes. Label the turning point on each parabola with its coordinates.

a $y = x^2 + 3$ b $y = x^2 + 1$ c $y = x^2 - 2$

d $y = x^2 + 6$ e $y = x^2 - 4$ f $y = x^2 + 9$

3 Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a horizontal translation to sketch the graph of each quadratic relationship on the same set of axes. Label the turning point on each parabola with its coordinates.

a $y = (x - 3)^2$ b $y = (x - 1)^2$ c $y = (x + 2)^2$

d $y = (x + 4)^2$ e $y = (x - 5)^2$ f $y = (x + 7)^2$

4 Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a dilation to sketch the graph of each quadratic relationship on the same set of axes. Label the turning point on each graph with its coordinates.

a $y = 2x^2$ b $y = \frac{1}{2}x^2$ c $y = 3x^2$

5 Add the graph of $y = -x^2$ to the Cartesian plane from question **4**. Sketch the following graphs by reflecting your answers to question **4** across the x-axis.

a $y = -2x^2$ b $y = -\frac{1}{2}x^2$ c $y = -3x^2$

6 Complete the following descriptions of transformations.

a To sketch the graph of $y = (x + 2)^2 - 3$, the graph of $y = x^2$ is translated __________ units __________ and __________ units __________.

b To sketch the graph of $y = -x^2 + 4$, the graph of $y = x^2$ is __________ across the x-axis and translated __________ units __________.

WE 4.4B 7 Sketch the graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the turning point on each parabola with its coordinates.

a $y = -(x - 2)^2$ b $y = (x - 2)^2 + 3$

c $y = -(x + 6)^2$ d $y = -x^2 + 4$

e $y = (x - 1)^2 - 2$ f $y = -x^2 - 3$

g $y = -(x + 4)^2 + 6$ h $y = -(x - 7)^2 - 5$

8 For each quadratic relationship:

i identify whether its graph is an upright or inverted parabola

ii find the coordinates of the x- and y-intercepts

iii write the coordinates of the turning point

iv sketch the graph of the relationship.

a $y = (x - 2)^2 - 1$ b $y = -(x + 1)^2 + 9$

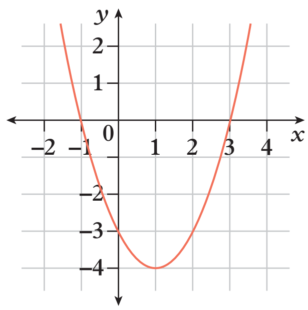
c $y = (x - 3)^2 - 4$ d $y = -(x + 4)^2 + 1$

Exercise 4.4B: Problem solving and reasoning

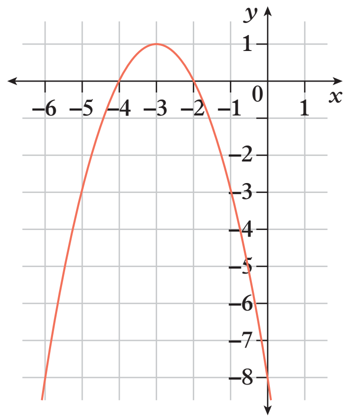
9 Consider the quadratic graphs below, each sketched using the rule $y = a(x - h)^2 + k$. For each graph:.

i State whether the parabola is upright or inverted.
ii Identify the coordinates of the turning point.
iii Decide whether a is positive or negative.
iv Determine the values of h and k.

a



b



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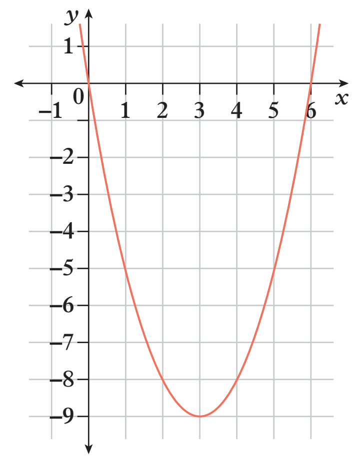
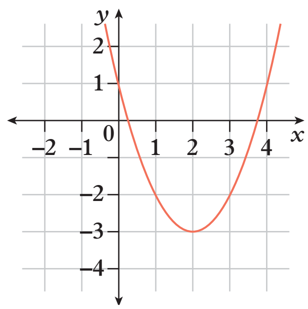
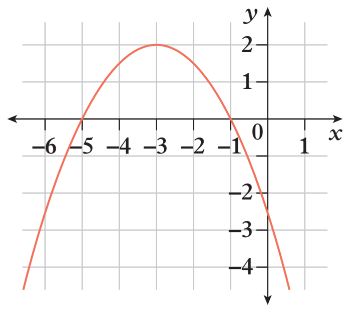


10 Match each graph with its equation from the list provided below.

a



b



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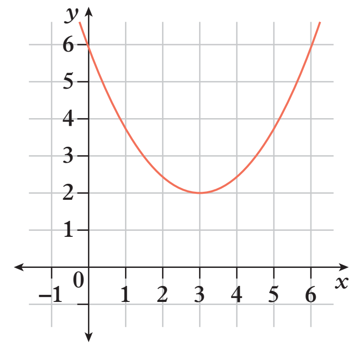
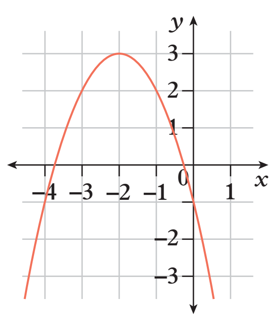
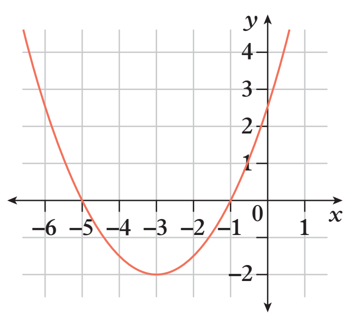


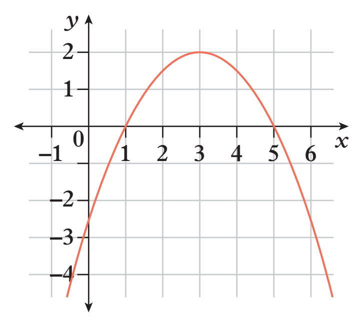
d



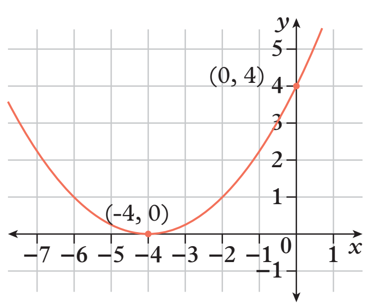
e



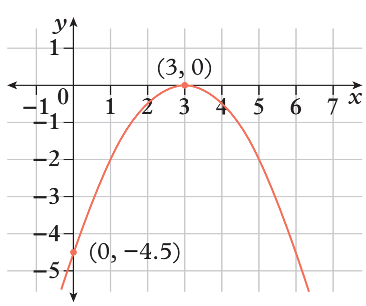
f



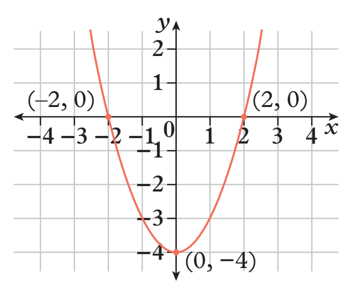
g



h



i



A $y = (x - 3)^2 + 2$
B $y = -(x - 3)^2 + 2$
C $y = (x + 3)^2 - 2$
D $y = -(x + 3)^2 + 2$
E $y = (x - 2)^2 - 3$
F $y = -(x + 2)^2 + 3$
G $y = x^2 - 4$
H $y = \frac{1}{4}(x + 4)^2$
I $y = -\frac{1}{2}(x - 3)^2$

11 Write the equation for each of the quadratic graphs described below. Assume each parabola has the same shape as $y = x^2$; that is, the dilation factor is $+1$ or -1.

a upright, turning point at $(3, 7)$
b upright, turning point at $(-2, 5)$
c inverted, turning point at $(2, 4)$
d inverted, turning point at $(6, -1)$
e upright, turning point at $(9, 0)$
f inverted, turning point at $(0, 4)$
g inverted, turning point at $(-1, -2)$
h upright, turning point at $(0, -5)$

12 Consider the quadratic relationship $y = (x - 4)^2 + 5$.

a What are the coordinates of the turning point?
b What is the smallest y-value that this relationship can have?
c Is there a maximum value for y? Explain your answer.

13 Consider the quadratic relationship $y = -(x + 1)^2 + 2$. What is the maximum y-value in this relationship? Explain.

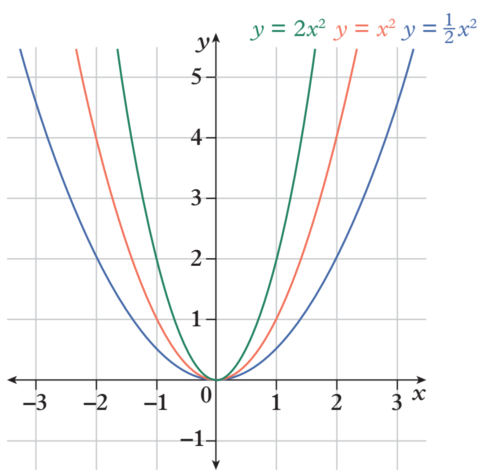
14 Jenna throws a netball to a teammate. The height of the ball can be represented by the relationship $h = -(d - 2)^2 + 6$, where h is the height in metres and d is the horizontal distance from where Jenna throws the ball.

a What are the coordinates of the turning point of this relationship?
b What is the value of h when:
 i $d = 0$
 ii $d = 4$?
c Sketch the graph of this relationship from $d = 0$ to $d = 4$.
d Use the graph to find:
 i the height at which the ball left Jenna's hands
 ii the maximum height of the ball during the pass to her teammate.

15 The path of a firework follows the relationship $y = -(x - 10)^2 + 100$, where y is the height of the firework for a horizontal distance, x, from where it was launched. Both x and y are in metres.

a Sketch the graph of this relationship from when the firework was launched to when it landed on the ground after exploding. Hint: Use the fact that a parabola is symmetrical.
b What was the maximum height of the firework?
c How far from the launch pad did the firework land?

16 Consider these graphs of the form $y = ax^2$.

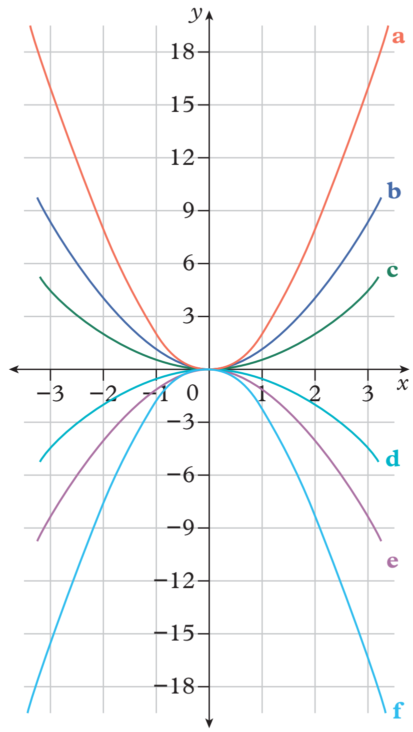


a What is the value of the dilation factor, a, for $y = x^2$?
b Which graph is narrower than the graph of $y = x^2$? State the value of the graph's dilation factor.
c Which graph is wider than the graph of $y = x^2$?
d Describe how the value of the dilation factor affects the width of the corresponding parabola compared to the parabola $y = x^2$.

17 Match each graph on the Cartesian plane to the right with its equation from the list provided below.

A $y = x^2$ **B** $y = -2x^2$ **C** $y = -\frac{1}{2}x^2$

D $y = \frac{1}{2}x^2$ **E** $y = 2x^2$ **F** $y = -x^2$



18 For each quadratic relationship, identify:

i whether the graph has wider, narrower or equal width to the graph of $y = x^2$

ii whether the parabola is upright or inverted

iii the coordinates of the turning point.

a $y = 2(x - 4)^2 - 3$ **b** $y = \frac{1}{2}(x + 2)^2 + 6$

c $y = 4(x + 2)^2$ **d** $y = -5x^2 - 4$

e $y = -\frac{1}{2}(x - 7)^2$ **f** $y = -3(x + 1)^2 + 5$

g $y = -\frac{1}{3}(x - 3)^2 - 4$ **h** $y = \frac{1}{4}x^2 + 3$

i $y = -(x - 5)^2 + 4$ **j** $y = 4(x + 3)^2 - 7$

Exercise 4.4C: Challenge

19 Write an equation for the parabola produced after performing each sequence of transformations on the graph of $y = x^2$.

a Dilation by a factor of 3, then a translation of 2 units right

b Reflection in the x-axis, then a translation of 4 units down and 5 units left

c Dilation by a factor of $\frac{1}{2}$, then a reflection in the x-axis

d Dilation by a factor of 4, reflection in the x-axis, then a translation of 2 units left

20 A ball is kicked during a competition to see who can kick it the highest. The height of the ball, h, in metres, over a horizontal distance, d, in metres, is given by $h = -(d - 3)^2 + 9$.

a Sketch a graph showing the path of the ball. Label the turning point with its coordinates.

b What is the maximum height reached by the ball?

c When the ball lands, how far away is it from where it was kicked?

d A second ball is kicked from the same point. It travels 4 metres further than the first ball.

i Assuming the second ball also follows a parabolic pathway with a dilation factor of 1, determine the equation for the path of the second soccer ball.

ii On the same set of axes used in part **a**, sketch the path of the second ball. Label the turning point with its coordinates.

iii How much higher does the second ball reach than the first ball?

Online resources:

Interactive skillsheet Sketching parabolas using transformations

Worksheet Quadratic relationships

Desmos activity Marble slides: Parabolas

Quick quiz 4.4

Lesson 4.5

Other non-linear relationships

Learning intentions

By the end of this lesson you will be able to ...

✔ sketch non-linear relationships using transformations.

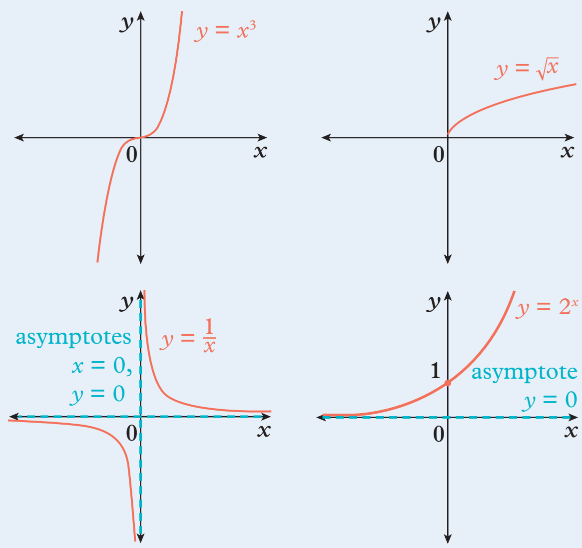
Inter-year links

Year 7	8.5 Combined transformations
Year 8	9.1 Transformations
Year 10	4.4 Sketching circles
Year 10	4.5 Sketching exponential relationships

Key content video
Graphs of other non-linear relationships

Non-linear relationships

- Linear relationships produce straight-line graphs. **Non-linear relationships** produce graphs which are not straight lines.



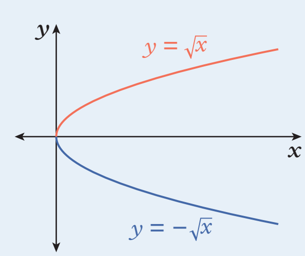
- For all linear and non-linear relationships:
  → The x-intercept(s) are the point(s) where the graph crosses the x-axis and $y = 0$.
  → The y-intercept(s) are the point(s) where the graph crosses the y-axis and $x = 0$.

Square roots

- Many **non-linear equations** are solved by taking the square root of a number.
- For $a > 0$, the equation $x^2 = a$ has two solutions: $x = \sqrt{a}$ and $x = -\sqrt{a}$.
  For example, $2 \times 2 = 4$ and $-2 \times (-2) = 4$, so the equation $x^2 = 4$ has two solutions: $x = 2$ and $x = -2$.
- The **plus-minus symbol**, $\pm$, is used as shorthand to write both results in an efficient way.
  For example, $x^2 = 4$

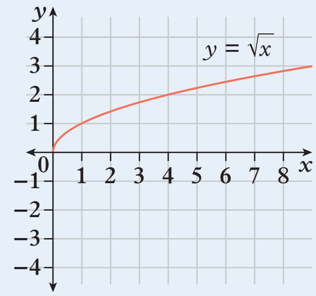
$$\begin{aligned} x &= \pm\sqrt{4} \\ &= \pm 2 \end{aligned}$$

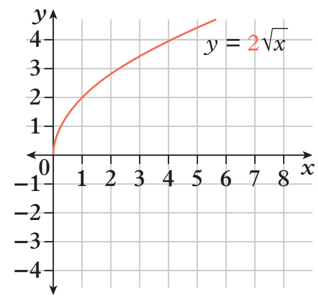
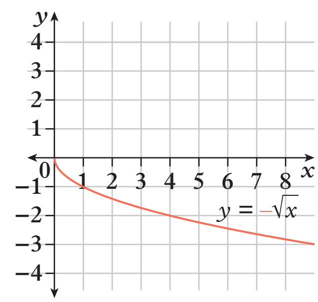
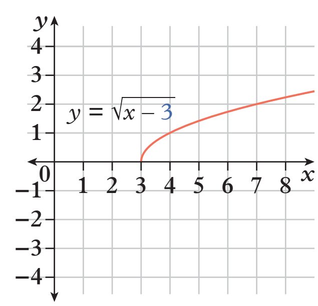
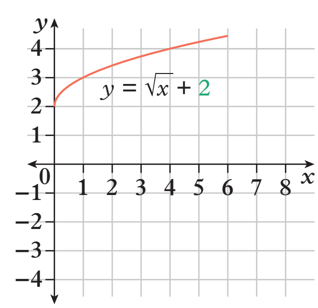
- The graph of $y^2 = x$ is a sideways parabola. It can be split into two graphs:
  → $y = \sqrt{x}$ represents its *upper* half, with *positive* y-values
  → $y = -\sqrt{x}$ represents its *lower* half, with *negative* y-values.



General transformations

- Transformations can be performed on any non-linear relationship of the form $y = \ldots$, much like the transformations applied to the turning point form of a quadratic graph.



Transformation on equation	Transformation on graph	Example
Multiply the RHS by a constant, a.	Dilate by a factor of a in the y-direction.	
Multiply the RHS by -1.	Reflect in the x-axis.	
Substitute $x - h$ for x.	Translate h units in the x-direction.	
Add k to the RHS.	Translate k units in the y-direction.	

Worked example 4.5A Finding the coordinates of the *x*- and *y*-intercepts

For each non-linear relationship, find the coordinates of the:

i x-intercept **ii** y-intercept.

a $y = 2x^3$ **b** $y = 2 + \sqrt{x + 3}$

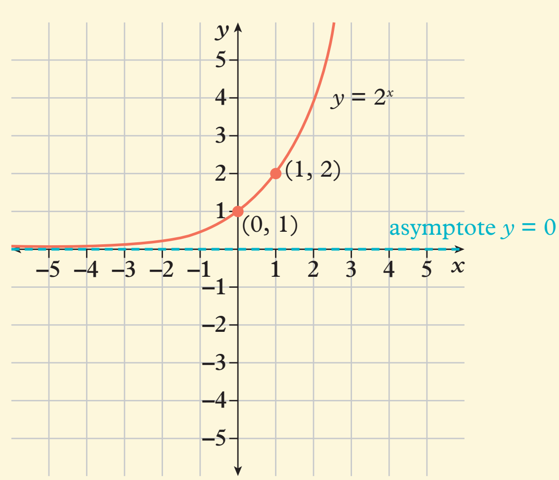
THINK	WRITE
a i To find the x-coordinate of the x-intercept, substitute $y = 0$ into the given equation and solve for x.	**a i** For x-intercept, let $y = 0$: $2x^3 = 0$ $x^3 = 0$ $x = 0$ x-intercept: $(0, 0)$
ii To find the y-coordinate of the y-intercept, substitute $x = 0$ into the given equation and solve for y.	**ii** For y-intercept, let $x = 0$: $y = 2(0)^3$ $y = 0$ y-intercept: $(0, 0)$
b i To find the x-coordinate of the x-intercept, substitute $y = 0$ into the given equation and solve for x. The LHS of the equation is a non-negative number, which means the equation has no solution.	**b i** For x-intercept, let $y = 0$: $2 + \sqrt{x + 3} = 0$ $\sqrt{x + 3} = -2$ There is no real solution for x. $y = 2 + \sqrt{x + 3}$ does not have any x-intercept.
ii To find the y-coordinate of the y-intercept, substitute $x = 0$ into the given equation and solve for y.	**ii** For y-intercept, let $x = 0$: $y = 2 + \sqrt{(0 + 3)}$ $y = 2 + \sqrt{3}$ y-intercept: $(0, 2 + \sqrt{3})$

Worked example 4.5B Sketching graphs using transformations

Sketch the following non-linear relationships by transforming the graph of $y = 2^x$.

a $y = 1 + 2^x$

b $y = -2^x$



THINK

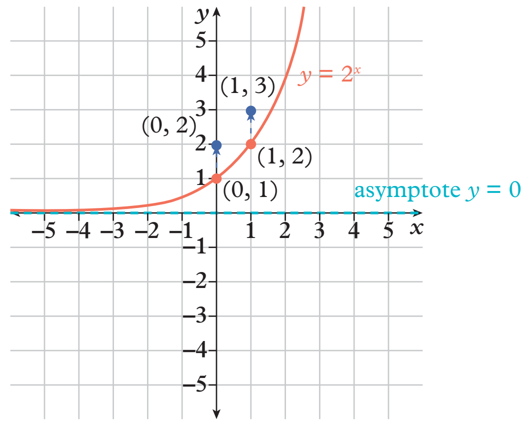
a **1** Identify the transformations. 1 is added to the right-hand side of the equation, so the graph undergoes a vertical translation of 1 unit up.

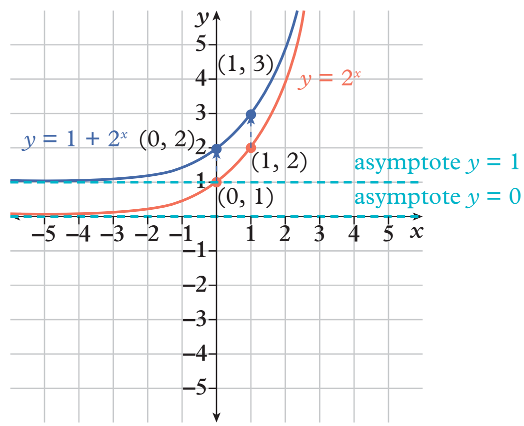
2 Identify the corresponding coordinates of the key points. The whole graph has been moved up by 1 unit, so for the same x-coordinate, add 1 unit to the y-coordinate. The corresponding coordinates for (0, 1) and (1, 2) on the graph of $y = 1 + 2^x$ are (0, 2) and (1, 3). The asymptote is also translated 1 unit up.

3 Sketch the new graph following the shape of $y = 2^x$, ensure the graph passes through the points (0, 2) and (1, 3) and label the points and the translated asymptote.

WRITE

a



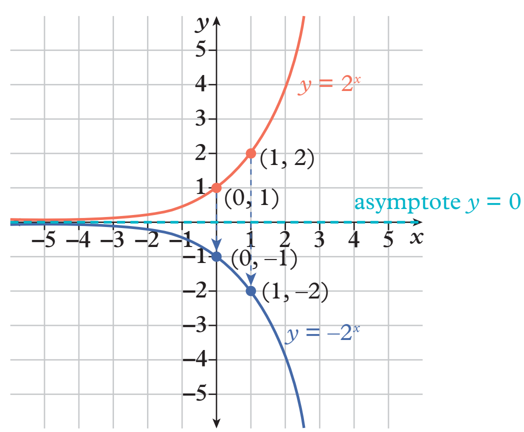


b **1** Identify the transformations. The right-hand side of the equation is multiplied by −1, so the graph is reflected in the x-axis.

2 Identify the corresponding coordinates of the key points. The whole graph has been reflected in the x-axis, so for the same x-coordinate, the y-coordinate is multiplied by −1. The corresponding coordinates for (0, 1) and (1, 2) on the graph of $y = -2^x$ are (0, −1) and (1, −2). The asymptote, $y = 0$, is not changed after reflecting in the x-axis.

3 Reflect the graph of $y = 2^x$ in the x-axis, ensuring the graph passes through the points (0, −1) and (1, −2).

b



Helpful hints

- ✔ Remember that the value of h is the opposite sign to how it appears in the equation. For example, in the equation $y = \sqrt{x + 2} + 4$, $h = -2$ but $k = +4$.
- ✔ Non-linear relationships may have 0, 1 or multiple x-intercepts and y-intercepts.

Learning pathways

△ 1–6, 8, 9 ◻ 2–5, 8–10 ⬠ 3–5, 7–9, 11–13

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Exercise 4.5A: Understanding and fluency

1 Match the table of values with the correct graph.

a

x	-2	-1	0	1	2
y	undefined	undefined	0	1	$\sqrt{2}$

b

x	-2	-1	0	1	2
y	-8	-1	0	1	8

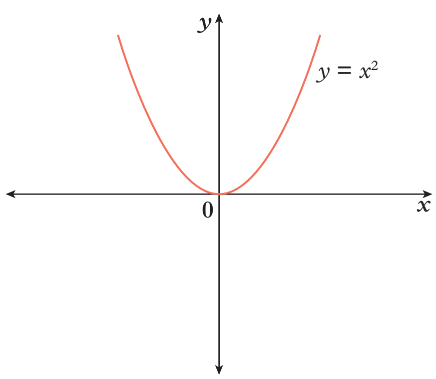
c

x	-2	-1	0	1	2
y	4	1	0	1	4

d

x	-2	-1	0	1	2
y	-0.5	-1	undefined	1	0.5

A



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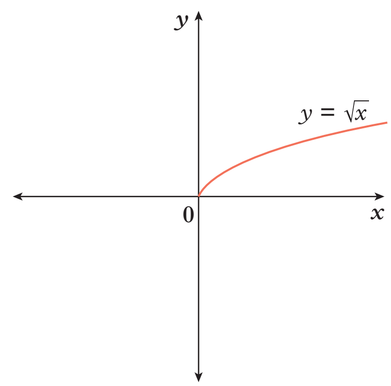

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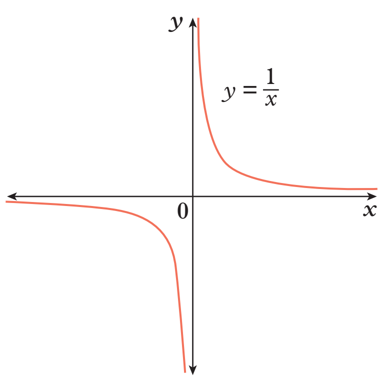
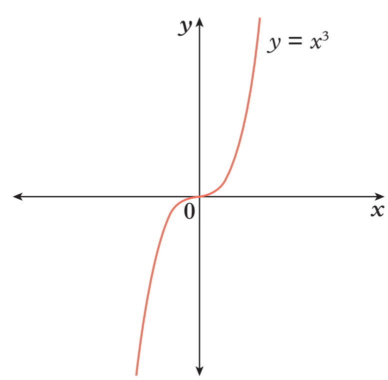


D



2 Use the corresponding graph and table of values from question **1** to write the coordinates of the x- and y-intercept(s) of the following non-linear relationships (if there are any).

a $y = x^2$ **b** $y = \sqrt{x}$ **c** $y = \frac{1}{x}$ **d** $y = x^3$

WE 4.5A **3** For each non-linear relationship, find the coordinates of the:

i x-intercept **ii** y-intercept.

a $y = 3x^4$ **b** $y = \sqrt{x + 5}$ **c** $y = -2x^3$ **d** $y = \sqrt{2x}$

4 Perform each of the following transformations on the points you found in question **3**. Do not perform the transformations as a sequence; perform each one individually.

i Reflect in the x-axis

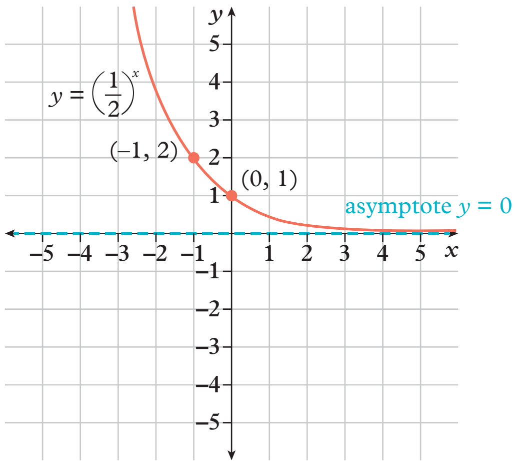
ii Translate 2 units in the positive x-direction

iii Translate 5 units in the negative y-direction

iv Translate 3 units in the positive y-direction first, then 3 units in the positive x-direction

WE 4.5B **5** Sketch the following non-linear relationships by transforming the graph of $y = \left(\frac{1}{2}\right)^x$.

a $y = -\left(\frac{1}{2}\right)^x$ **b** $y = \left(\frac{1}{2}\right)^x + 1$ **c** $y = \left(\frac{1}{2}\right)^{x+1}$



Exercise 4.5B: Problem solving and reasoning

6 Complete the descriptions of transformations below.

a To sketch the graph of $y = \frac{1}{x} + 20$, the graph of $y = \frac{1}{x}$ is translated ________ units ________.

b To sketch the graph of $y = \frac{1}{x-15}$, the graph of $y = \frac{1}{x}$ is translated ________ units ________.

c To sketch the graph of $y = -1 + \sqrt[3]{x}$, the graph of $y = \sqrt[3]{x}$ is translated ________ units ________.

d To sketch the graph of $y = \sqrt[3]{2+x}$, the graph of $y = \sqrt[3]{x}$ is translated ________ units ________.

7 Write the equation of the new non-linear relationship if the following transformations are performed on $y = 5^x$.

a The graph is stretched by a factor of 3 in the y-direction.

b The graph is reflected in the x-axis.

c The graph is translated 3 units in the x-direction.

d The graph is translated −2 units in the y-direction.

8 Consider the non-linear relationship $y = \sqrt{x}$, where only the positive roots of x are taken.

x	0	1	4	9	16	25
y	0					

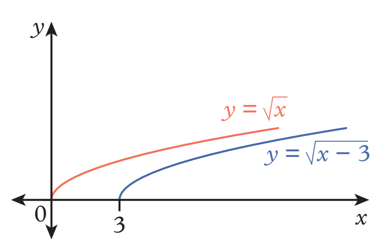
a Complete the table of values for the relationship $y = \sqrt{x}$.

b Use the table of values to construct a plot of $y = \sqrt{x}$. Join the plotted points with a smooth curve.

c Describe the shape of the graph.

d An important feature of the graph of $y = \sqrt{x}$ is the point on the graph where y is a minimum. Label this point with its coordinates on your graph.

e The graphs of $y = \sqrt{x}$ and $y = \sqrt{x-3}$ are shown to the right.



i Identify the coordinates of the point on the graph of $y = \sqrt{x-3}$ where y is a minimum.

ii Describe the translation required to produce the graph of $y = \sqrt{x-3}$ from the graph of $y = \sqrt{x}$.

f Use your understanding of translations to describe how the graphs of these relationships can be produced from the graph of $y = \sqrt{x}$.

i $y = \sqrt{x} + 2$ **ii** $y = \sqrt{x} - 1$ **iii** $y = \sqrt{x-1}$ **iv** $y = \sqrt{x+4}$ **v** $y = \sqrt{x-2} + 3$

g Use your answers to part **f** to sketch each relationship. Label the point on each graph where y is a minimum with its coordinates on each graph.

9 Consider the basic cubic relationship, $y = x^3$.

x	−3	−2	−1	0	1	2	3
y	−27						

a Complete the table of values for the relationship $y = x^3$.

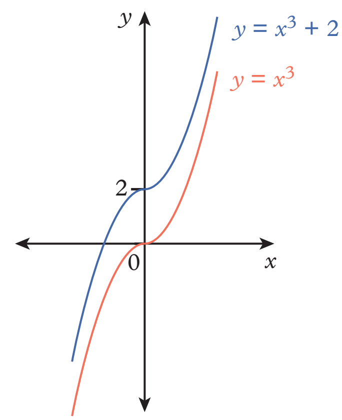
b An important feature of the graph of $y = x^3$ is the stationary point at (0, 0), where the slope of the graph is flat (gradient = 0) at a single point. This point is known as the point of inflection. Use the table of values to construct a plot of $y = x^3$. Label the point of inflection with its coordinates on your graph and join the plotted points with a smooth curve.

c Describe the shape of the graph.

d The graphs of $y = x^3$ and $y = x^3 + 2$ are shown on the right.

i Identify the coordinates of the point of inflection for $y = x^3 + 2$.

ii Describe the translation required to produce the graph of $y = x^3 + 2$ from the graph of $y = x^3$.



e Use your understanding of translations to describe how the graphs of these cubic relationships can be produced from the graph of $y = x^3$.

i $y = x^3 + 1$ **ii** $y = x^3 - 3$ **iii** $y = (x - 2)^3$ **iv** $y = (x + 4)^3$ **v** $y = (x - 1)^3 + 2$

f Use your answers to part **e** to sketch each relationship. Label the point of inflection with its coordinates on each graph.

10 Consider the relationships in questions **8–9**.

a Sketch the graphs of $y = -x^3$, $y = -x^3 + 4$ and $y = -(x - 3)^3$ on the same Cartesian plane.

b Sketch the graphs of $y = -\sqrt{x}$, $y = -\sqrt{x} + 3$ and $y = -\sqrt{x - 4}$ on the same Cartesian plane.

11 Use your knowledge of transformations to sketch the graph of each of the following relationships by first finding any x- and y-intercepts. Label all intercepts with their coordinates.

a $y = 3\sqrt{x + 1} - 6$ **b** $y = (x + 2)^3 - 1$

Exercise 4.5C: Challenge

12 The graphs of $y = \pm\sqrt{x}$ and $y = x^2$ are related.

a Sketch a graph of $y = \sqrt{x}$ and $y = -\sqrt{x}$ on the same Cartesian plane.

b Write one equation to describe all the points on both graphs. What is the shape of the graph for this equation?

c Add the graph of $y = x^2$ to your answer in part **b**. Label the coordinates where the graphs intersect each other.

d Draw a dotted line that passes through the point where the two graphs intersect each other. Write an equation for this line.

e Describe the transformation required to turn the graph of $y = x^2$ into the graph in part **b**.

13 Use your observations from question **12** to sketch the graph for $x = y^3$.

Online resources:

- **Interactive skillsheet** Non-linear relationships
- **Worksheet** Non-linear relationships
- **Desmos activity** Marble slides: Exponentials
- **Desmos activity** Marble slides: Hyperbolas
- **Quick quiz** 4.5

Lesson 4.6

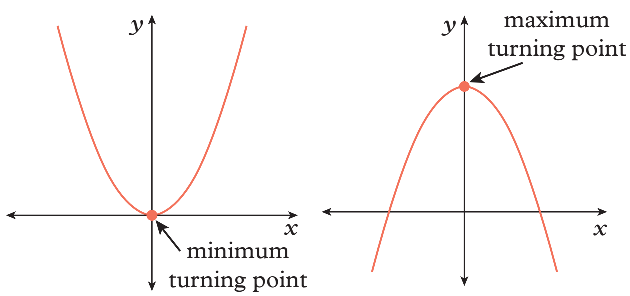
Review: Non-linear relationships

Module summary

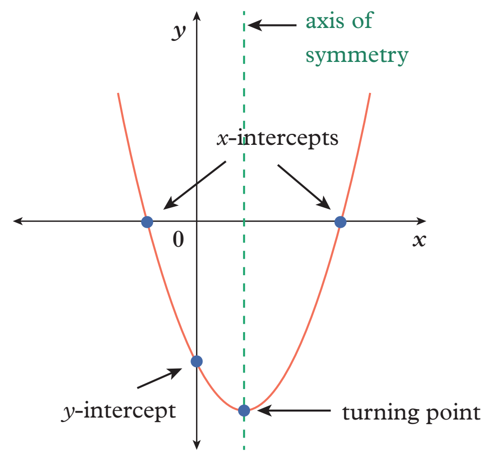
The Null Factor Law

- If $a \times b = 0$, then $a = 0$ or $b = 0$.
- If $(x + p)(x + q) = 0$, then $x + p = 0$ or $x + q = 0$.

Turning points



Parabolas



Turning point form

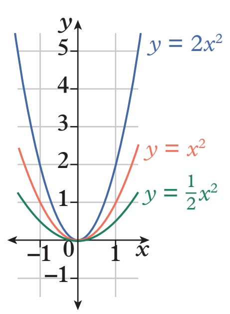
$$y = a(x - h)^2 + k$$

vertical translation of k units
If $k > 0$, up
If $k < 0$, down

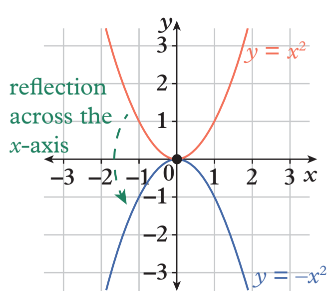
horizontal translation of h units
If $h > 0$, right
If $h < 0$, left

$a > 0$ upright
$a < 0$ inverted

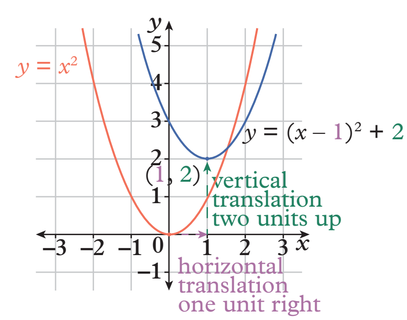
Dilations in the y-direction



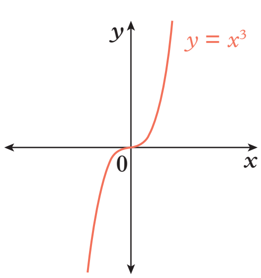
Reflections

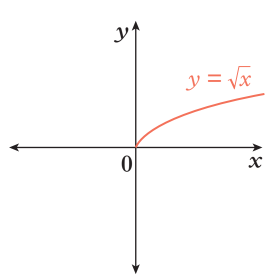


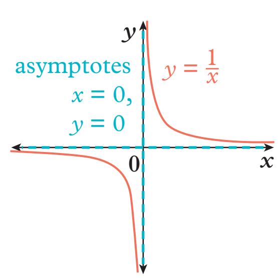
Translations

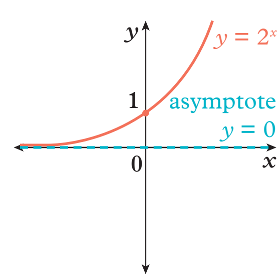


Non-linear relationships









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Review questions 4.6A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- asymptote
- axis of symmetry
- dilation
- factor
- factorised form
- general form
- highest common factor
- leading coefficient
- maximum turning point
- minimum turning point
- monic quadratic equation
- non-linear equation
- non-linear relationship
- Null Factor Law
- parabola
- plot
- plus-minus symbol
- quadratic equation
- reflection
- sketch
- symmetry
- transformation
- translation
- turning point
- turning point form
- vertex
- x-intercept
- y-intercept

1 Complete the following sentences using words from the list of key terms.
Solving a quadratic equation involves the following steps:

1 Write all the terms on one side so the equation equals zero.
2 Identify and divide by the ________ if it is not equal to 1.
3 ________ the quadratic equation into the product of two binomial expressions.
4 Apply the ________ by equating each ________ to 0.
5 Solve any resulting linear equations.

2 Give an example of a quadratic relationship in:
a general form
b factorised form
c turning point form.

3 Match the following descriptions in the left-hand column with a term in the right-hand column for quadratic relationships.

a	a quadratic relationship	**I**	parabola
b	how to find the y-intercept	**II**	leading coefficient
c	the shape of a quadratic graph	**III**	turning point or vertex
d	the maximum or minimum point on the graph	**IV**	$y = ax^2 + bx + c$
e	a vertical line of reflection for the graph	**V**	0, 1 or 2
f	the possible number of x-intercepts	**VI**	the x-intercepts
g	determines if the curve opens upwards or downwards	**VII**	substitute $x = 0$
h	the solutions to the factored form of a quadratic equation	**VIII**	axis of symmetry

4 State true or false for the following statements about the transformation of a quadratic graph $y = ax^2$. If it is false, change the statement to make it true.
a If $a > 1$, then the graph is dilated and enlarged.
b If $a < -1$, then the graph is dilated and compressed.
c If $a = -1$, then the graph is dilated.
d If $0 < a < 1$, then the graph is compressed.

e A constant, k, where $k \neq 0$, is added to the equation to give $y = ax^2 + k$.

i The graph is translated vertically.

ii The graph passes through the origin.

iii The coordinates of the vertex are $(k, 0)$.

Review questions 4.6B: Multiple choice

4.1 **1** The solution(s) to the quadratic equation $x(x + 1) = 0$ are:

A $x = -1$ **B** $x = 1$ **C** $x = -1$ or $x = 1$ **D** $x = 0$ or $x = -1$

4.1 **2** In the quadratic equation $x^2 - 13x - 48 = 0$, the solutions are $x =$

A -3 or 16 **B** -16 or 3 **C** -12 or 4 **D** -6 or 8

4.2 **3** If a parabola intersects the x-axis when $x = -1$ and -5, the turning point could be:

A $(-3, 7)$ **B** $(-2, 2)$ **C** $(0, -3)$ **D** $(2, -6)$

4.3 **4** The coordinates of the x-intercepts of the graph of $y = x^2 - 4x - 12$ are:

A $(0, -6), (0, 2)$ **B** $(0, 6), (0, -2)$ **C** $(-6, 0), (2, 0)$ **D** $(6, 0), (-2, 0)$

4.3 **5** The y-coordinate of the y-intercept of the graph of $y = (x - 5)(x + 2)$ is:

A 10 **B** -2 **C** -10 **D** 5

4.4 **6** The graph of $y = x^2$ is translated 5 units down. The equation of the translated graph is:

A $y = x^2 + 5$ **B** $y = 5x^2$ **C** $y = x^2 - 5$ **D** $y = (x + 5)^2$

4.4 **7** Which of the following transformations is *not* required to sketch the graph of $y = -4(x + 2)^2 + 1$?

A $y = x^2$ must be dilated.

B $y = x^2$ must be reflected in the x-axis.

C $y = x^2$ must be translated up.

D $y = x^2$ must be translated to the right.

4.4 **8** The coordinates of the turning point of the graph of $y = -(x - 4)^2$ are:

A $(0, 0)$ **B** $(4, 0)$ **C** $(-4, 0)$ **D** $(0, 4)$

4.5 **9** The y-intercept of the relationship $y = 4^x$ is $(0, 1)$. What is the y-intercept of $y = 2 + 4^x$?

A $(0, 1)$ **B** $(0, 2)$ **C** $(0, 3)$ **D** $(0, -1)$

4.5 **10** What is the y-intercept of $y = \dfrac{1}{(x-3)}$?

A $(0, 3)$ **B** $(3, 0)$ **C** $(0, \frac{1}{3})$ **D** $(0, -\frac{1}{3})$

Review questions 4.6C: Short answer

4.1 **1** Solve the following quadratic equations.

a $x^2 - 5x + 6 = 0$ **b** $x^2 + x - 30 = 0$ **c** $x^2 + 9 = 0$ **d** $3x^2 - 36x = 0$

4.2 **2** Sketch a graph of the following relationships by first completing a table of values for x from -3 to 3. Use your graph to identify:

i the coordinates of the x- and y-intercepts

ii the coordinates of the turning point

iii whether there is a maximum or a minimum turning point

iv the equation of the axis of symmetry.

a $y = 4x^2 - 4$ **b** $y = -4x^2 - 4x$ **c** $y = x^2 - 4x + 4$

4.3 **3** Complete the following for each quadratic relationship below.

i Find the coordinates of the x- and y-intercepts.

ii Find the coordinates of the turning point.

iii Sketch the graph.

a $y = -x^2 - 4x$ **b** $y = x^2 - x - 12$ **c** $y = (x + 5)(x - 4)$ **d** $y = -(x + 2)(x + 1)$

4.4 **4** Sketch a graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the turning point with its coordinates.

a $y = x^2 + 3$ **b** $y = (x - 1)^2$ **c** $y = x^2 - 5$ **d** $y = (x + 4)^2$

4.4 **5** Sketch the graph by performing transformations on the graph of $y = x^2$. Label the turning point on each parabola with its coordinates.

a $y = (x + 1)^2 - 1$ **b** $y = -(x + 3)^2 - 3$ **c** $y = (x - 4)^2 + 4$ **d** $y = -(x - 2)^2 - 2$

4.5 **6** For each of the relationships below, describe the transformation performed on $y = 3^x$.

a $y = 3 + 3^x$ **b** $y = -2 \times 3^x$

4.5 **7** For each of the relationships below, describe the transformation performed on $y = \frac{2}{x}$.

a $y = -\frac{2}{x}$ **b** $y = 2 + \frac{2}{x}$

Review questions 4.6D: Mathematical modelling

1 A parabolic through-arch bridge has an upper arch and a lower arch that are closely modelled by the following quadratic relationships:

$$\text{Lower arch: } h = -\frac{1}{120}d^2 + d$$

$$\text{Upper arch: } h = -\frac{1}{180}(d - 60)^2 + 40$$

where h is the height of the arch at a horizontal distance of d from the left side of the bridge. Both h and d are expressed in metres.

a Sketch the two arches on the same Cartesian plane from $d = 0$ m to $d = 120$ m.

b For each parabola, determine:

i the coordinates of the y-intercepts

ii the coordinates of the turning points

iii the equation of the axis of symmetry.

c What is the height of the upper arch above the lower arch at each end of the bridge?

d What is the span of the bridge? Show this:

i graphically, by referring to your graphs in part **a**

ii algebraically, by solving a quadratic equation.

e The distance between the arches does not remain constant over the span of the bridge. Find the height of the highest point of the:

i lower arch **ii** upper arch.

f How far apart are the two arches at their highest point?

g How far apart are the two arches at a horizontal distance of 30 m from the left side of the bridge?

h Describe the distance between the two arches over the span of the bridge.

2 Buzz can hit a golf ball that lands 100 metres away after reaching a height of 80 metres. Assume the course is on level ground and ignore any additional bouncing and rolling. The ball's path can be modelled by the equation $h = a(d^2 - 100d)$, where h is the height in metres and d is the horizontal distance, in metres, from where the ball is hit. The value a is a constant.

- **a** Factorise the given equation.
- **b** Determine the d-axis intercepts and describe what they represent.
- **c** Write down the coordinates for the maximum height of the ball.
- **d** Sketch a graph showing the path of the ball.
- **e** Use the coordinates for the maximum height to determine the value of a.
- **f** Calculate the height of the ball after it has travelled 20 metres horizontally. Round to one decimal place.
- **g** Find the other value of h where the ball is at the same height as in part **f**.

3 Exponential relationships can be written in different forms.

- **a** Complete the following table of values for equations A and B.

Equation	x	-2	-1	0	1	2
A	$y = 2^x$					
B	$y = 2 \times 2^x$					
C						

- **b** Describe how Equation A has been transformed to form Equation B.
- **c** Complete the table for Equation C, $y = 2^{x+1}$. What do you notice?
- **d** Write two equations which represent dilating $y = 2^x$ by a factor of 4 in the y-direction.
- **e** For the equation $y = a \times 2^x$, what values of a follow a similar pattern to what you have noticed?
- **f** Show your answer to part **e** is correct using the exponent laws.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Solve quadratic equations using the Null Factor Law ☐ Solve quadratic equations with common factors	☐ Go back to Lesson 4.1 Solving quadratic equations
☐ Identify the key features of the graph of a quadratic relationship ☐ Plot quadratic relationships from tables of values	☐ Go back to Lesson 4.2 Plotting quadratic relationships
☐ Find the x- and y-intercepts of a parabola from its equation ☐ Find the coordinates of the turning point of a parabola given the two x-intercepts and the equation of the graph ☐ Sketch quadratic graphs using the x- and y-intercepts and the turning point	☐ Go back to Lesson 4.3 Sketching parabolas using intercepts
☐ Describe transformations applied to quadratic relationships in turning point form ☐ Sketch quadratic relationships in turning point form using dilations, reflections and translations	☐ Go back to Lesson 4.4 Sketching parabolas using transformations
☐ Identify and apply transformations to non-linear relationships ☐ Find the x- and y-intercepts of non-linear graphs ☐ Sketch non-linear relationships using transformations	☐ Go back to Lesson 4.5 Other non-linear relationships

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Semester 1 review

Short answer

1 Round the following numbers as specified in the brackets.

a 20.040 194 (three decimal places)
b 20.040 194 (three significant figures)
c 0.000 782 718 (four decimal places)
d 0.000 782 718 (four significant figures)

2 State the number of significant figures in the following numbers.

a 946 025 **b** 120 000 000 **c** 0.002 520 20 **d** 630.00

3 Write the following numbers in scientific notation.

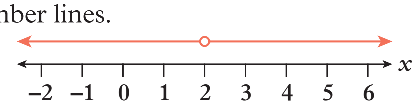
a 751 425 963, correct to four significant figures
b 25.360 141, correct to five significant figures
c 0.000 085 945 6, correct to two significant figures
d 75 000, correct to three significant figures

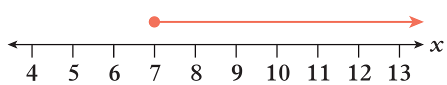
4 Simplify the following.

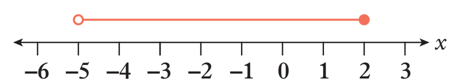
a $\sqrt{32}$ **b** $\sqrt{180}$ **c** $\sqrt{84}$ **d** $\sqrt{1008}$

5 State the inequality represented on each of the following number lines.

a


b


c


d


6 Solve each of the following inequalities. Show the solution on a number line.

a $4x + 5 \neq 17$
b $-6x + 4 > 28$
c $8 - 4x \leq 5$
d $5(2x - 3) \leq 4x$

7 Write the following numbers in exponent form (as the product of powers of prime factors).

a 81 **b** 500 **c** 216 **d** 2700

8 Write the following as basic numerals.

a 7^3 **b** 3×5^3 **c** 8.213×10^4
d 1.0530×10^{-2} **e** $2^{-3} \times 3^3$ **f** $\left(\frac{5}{7}\right)^{-2}$

9 Write the following expressions in exponent form with positive exponents.

a $(2^3 \times 7^2)^4 \times (2^9 \times 7^5)^2$
b $\dfrac{11^{12} \times 13^9 \times 17^6}{11^4 \times 13^9 \times 17^{15}}$
c $101^{-5} \times 97^5 \times 101^{-7} \times 97^{-9}$
d $\left(\dfrac{41^{-3}}{57^5}\right)^{-8} \times \left(\dfrac{57^{-4}}{41^{-7}}\right)^6$

10 Simplify the following expressions. Write your answer using positive exponents.

a $(ab^3)^4 \times (ba^6)^2$
b $\dfrac{c^2de^8}{c^{12}de^7}$
c $f^{-4}g^6 \times f^3g^{-2}$
d $\dfrac{(p^8q^{-4})^3 \times (p^{-5}q^3)^8}{(p^{-3}q^{-6})^2 \times (p^{-2}q^{-9})^3}$

11 Expand the following products.

a $8(7 - 2x)$ **b** $-2x^3(3x^2 - 8)$ **c** $(x + 2)(x - 7)$
d $(5x - 2)(2y + 3)$ **e** $5 - 4(3x + 7)$ **f** $2x(x + 9) + 3(x^2 - 4x + 2)$

12 Factorise the following expressions.

a $12a + 18b^2 - 6cd$ **b** $21g^2 - 7g$ **c** $a^2 - 25$
d $8m^2 - 18n^2$ **e** $(t + 2)^2 - 49$ **f** $r^2 + 5r - 36$

13 Solve the following equations for x.

a $12x - 7 = 4$

b $9 - 2x = 15$

c $4x + 9 = 6x - 7$

d $\frac{21}{x} = 3$

e $\frac{4}{x} = 5$

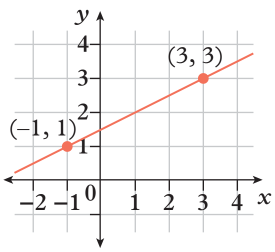
f $(x + 3)(x - 7) = 0$

g $(9x - 1)(5x + 3) = 0$

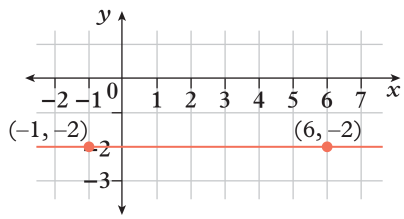
h $x^2 + 12x + 35 = 0$

14 Calculate the gradient of the following lines.

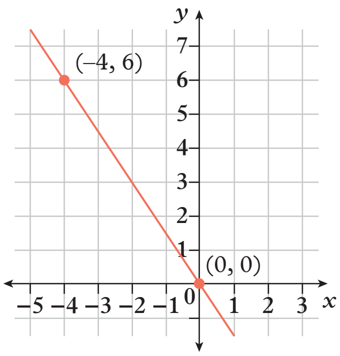
a



b



c



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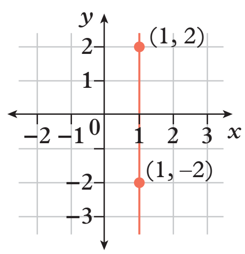


15 Plot the graphs of the following equations by completing each table of values and plotting the coordinates.

a $y = 3x - 4$

x	0	1	2	3	4
y					

b $y = -\frac{5}{2}x$

x	−4	−2	0	2	4
y					

c $y = (x + 1)^2 - 5$

x	−4	−3	−2	−1	0	1	2
y							

d $y = -x^2 + 6x - 2$

x	0	1	2	3	4	5	6
y							

16 Determine the x- and y-intercepts for the graphs of each of the following equations.

a $y = 3x - 6$

b $5x + 8y = -60$

c $y = (x - 5)(x + 9)$

d $y = x^2 - 6x - 40$

17 Sketch the graphs of the following equations.

a $2x + 3y = 12$

b $y = 2$

c $y = 2x$

d $y = x^2$

e $y = (x - 2)(x + 3)$

f $y = x^2 - 5x - 6$

18 Find the constant of variation using the given information, then write the equation for each directly proportional relationship.

a $y = 5$ for $x = 20$

b $y = -12$ for $x = 4$

c

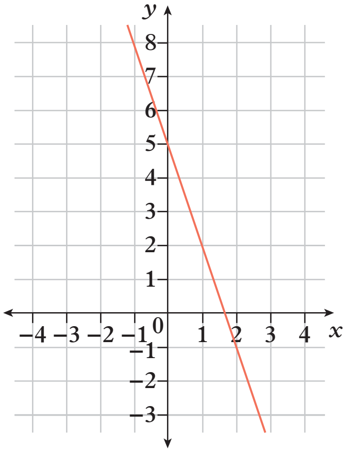
x	0	1	2	3	4
y	0	−5	−10	−15	−20

d

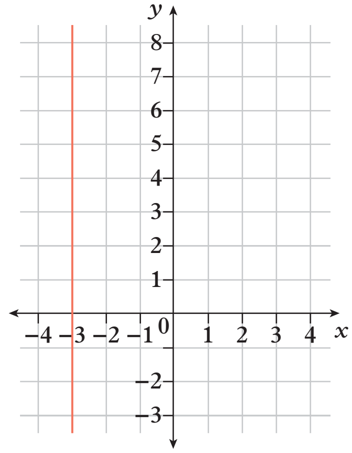
x	0	4	8	12	16
y	0	−1	−2	−3	−4

19 Determine the equations of the following graphs.

a

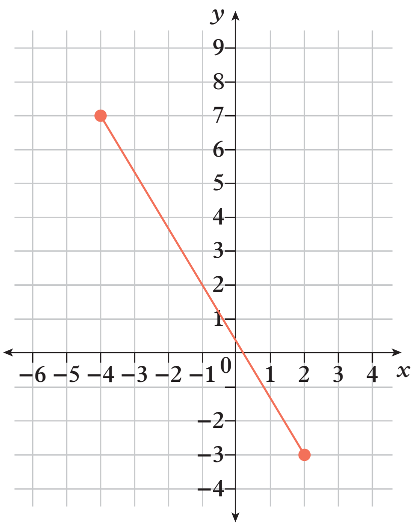


b

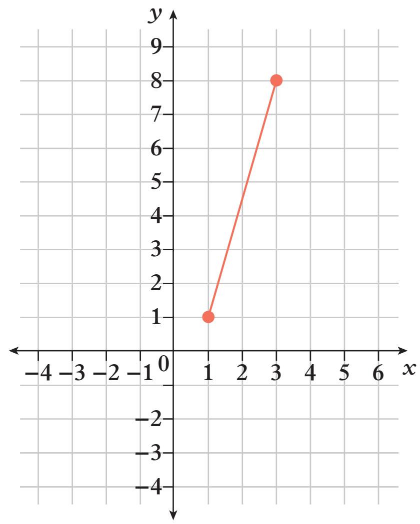


20 Find the midpoint of the following straight-line segments.

a

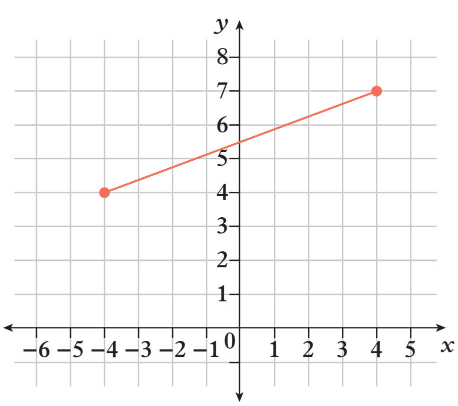


b

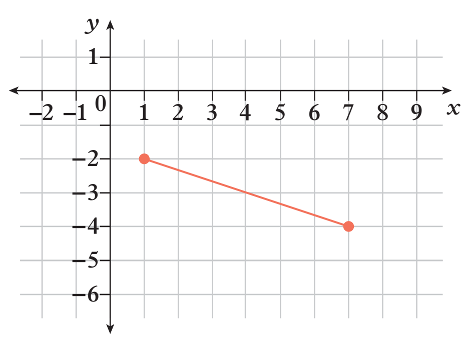


21 Determine the length of the following line segments, correct to two decimal places.

a



b



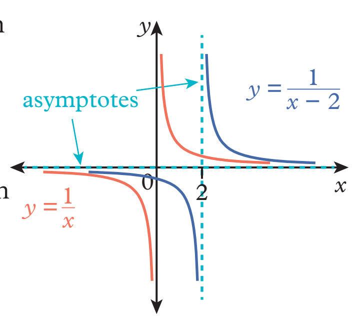
22 Write the equation for each of the quadratic graphs described below. Assume each parabola has the same shape as $y = x^2$; that is, the dilation factor is +1 or −1.

a upright, turning point at (7, 3)

b upright, turning point at (−12, 5)

c inverted, turning point at (4, 3)

d inverted, turning point at (3, −2)



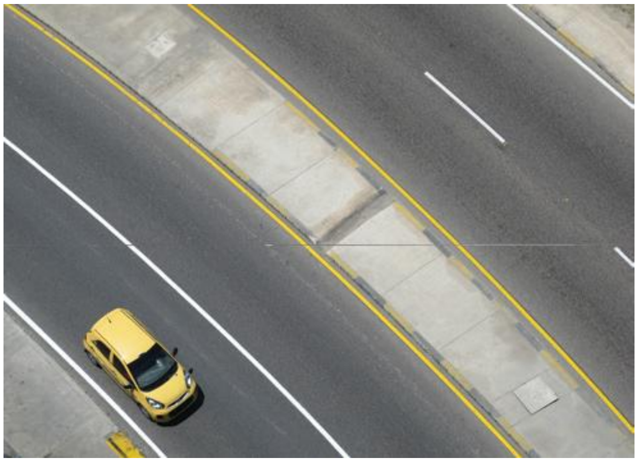
23 The graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x-2}$ are shown on the right. Describe the translation required to produce the following graphs from the graph of $y = \frac{1}{x}$.

a $y = \dfrac{1}{x-2}$

b $y = \dfrac{1}{x+5}$

c $y = \dfrac{1}{x+3} + 1$

Mathematical modelling



1 Jane is driving home from work, which is 30 km from where she lives. It takes her 36 minutes to drive home. Assume Jane drives at a constant speed throughout her journey. Jane's distance from home, d kilometres, at time t minutes after leaving work can be described using a linear equation.

a Which two features of the graph of d are given in the information above?

b Determine Jane's speed throughout her journey in kilometres per minute.

c How does the gradient of d relate to your answer to part **b**?

d Write the equation of d in gradient–intercept form.

e Sketch the graph of d and label the axis intercepts.

f Jane's daughter Vanessa attends a school located at the exact midpoint of Jane's journey home. Occasionally, Jane picks up Vanessa on her way home. It takes three minutes for Vanessa to get into the car and for Jane to be on her way again at the same constant speed as before.

i Find the coordinates of the point on your graph that represents Jane stopping to pick up Vanessa.

ii How should you represent the time spent picking up Vanessa from school on your graph? How does this affect the key features of the graph of d?

iii Sketch the graph representing Jane's journey home on a day when she picks up Vanessa. Label any key coordinates.

iv Your sketch from part **iii** can be broken into three linear graph segments. Find the equation of each segment in terms of d and t.

2 To analyse the path of the ball during a soccer match, Lisa records some short pieces of video so measurements can be taken. The measurements (in metres) indicate the height of the ball (h) for a given horizontal distance (d) that the ball travels.

She chooses two plays where the ball follows a parabolic path after contact with a player. One is when the ball was kicked by David and the other when Nick hit the ball with his head. Each path can be modelled by a quadratic relationship involving d and h.

David: $h = -\frac{1}{100}d(d - 44)$

Nick: $h = -\frac{1}{20}(d - 8)^2 + 5$

Answer each of the questions below for:

i David **ii** Nick.

a What is the height of the ball when the player made contact with it?

b What is the maximum height of the ball during the play?

c What is the horizontal distance from where the player made contact to where the ball hits the ground?

d Show the trajectory of the ball graphically.

e During training, each player dribbles the ball along the ground for the same distance from a marked point to the goal line. Lisa records measurements for the average speed of the ball in metres per second (s) and the time in seconds (t) for the ball to travel this distance (see table below).

Time (t)	2	3	4	6	8
Speed (s)	24	16	12	8	6

Use the table of values to plot the relationship between t and s with t on the horizontal axis and s on the vertical axis. Is the relationship linear or non-linear?

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EXPLORATIONS 1

1 Overtake

A car, a van, a truck and a bicycle are all travelling in the same direction on the same road, each at its own constant speed. At 10 am, the car overtakes the van; at noon, it overtakes the truck; at 2 pm it overtakes the bicycle. At 4 pm, the truck overtakes the bicycle. At 6 pm, the van overtakes the truck.

a Suppose the speed of the car is 120 km/h and the speed of the truck is 80 km/h.

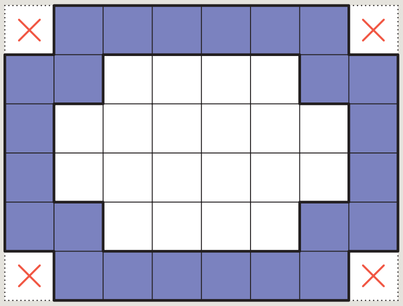
i Find the speeds of the van and the bicycle. Then find the time at which the van overtakes the bicycle.

ii A motorbike overtakes the van at 10 am and the bicycle at 3 pm. What is its speed?

iii A semi-trailer travelling at 80 km/h in the opposite direction passes the van at 6 pm. At what time does it pass the bicycle?

b Let c and t represent the speeds, in km/h, of the car and truck, respectively. Find the speeds of the van and the bicycle in terms of c and t. Then show that the time at which the van overtakes the bicycle is always the same, regardless of the speeds of the car and the truck.

2 Borders



The design shown is formed by removing one square from each corner of a rectangular grid and shading a border of thickness 1 unit. In this 6×8 example, there are 24 shaded squares in the border and 20 unshaded squares in the interior.

a Ignoring rotations, we want to find the number of such designs that have the same numbers of border and interior squares. Follow these steps:

i Let the dimensions of the original rectangular grid be $x \times y$. Find expressions, in terms of x and y, for the number of border squares and the number of interior squares.

ii Setting the expressions equal, rearrange the equation to get 0 on the right-hand side.

iii The left-hand side can almost be fully factorised as a binomial product of the form $(x - \_\_\_)(y - \_\_\_)$. What number do you need to add to both sides of the equation to make this work?

iv Remember that x and y are positive integers, so the factorised expression on the left-hand side of your equation should match up with a factorisation of the number on the right-hand side. Deduce that there are six possible pairs of positive integer solutions for x and y.

v Why does this mean that there are only three possible designs?

b Adapt the above methods to find, ignoring rotations, the number of such designs in which there are:

i twice as many interior squares as border squares

ii twice as many border squares as interior squares.

3 Absurd surds

What does the infinite surd expression $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$ equal? Does it even equal anything? Maybe as you add more and more sixes, the result gets larger and larger without ever settling down.

a One way to get an idea of what's going on is to use the 'answer' button on your scientific calculator, as follows. The instructions for your model might be slightly different.

- Enter √ 6 = – this calculates the first approximation of $\sqrt{6}$ and stores it as the most recent answer.
- Enter √ 6 + Ans = – this calculates the next approximation $\sqrt{6 + \text{Ans}}$ based on the previous answer.
- Now repeatedly enter = – this repeats the previous calculation as often as you like.
- What number do the approximations appear to get closer to?

b Now let's prove that your observation in part **a** is correct. Let x be the final answer, then notice that $x = \sqrt{6 + x}$. Square both sides and solve the quadratic equation. Which solution can you ignore and why?

c **i** Adapt the method in part **b** to calculate the exact values of $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}$ and $\sqrt{2 - \sqrt{2 - \sqrt{2 - \ldots}}}$

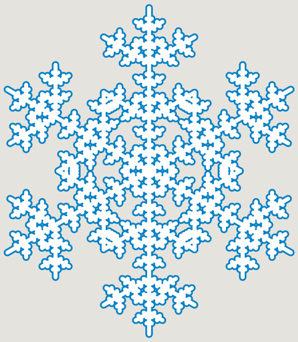
ii Find two different infinite surd expressions that are equal to 5.

4 Snowflakes

There is a famous conjecture in mathematics about so-called *hailstone* numbers. Starting with any positive integer, we halve the number if it is even, otherwise we triple it and add 1. Now repeat this process to form a sequence of numbers called hailstones, because of the way they rise and fall like real hailstones forming in a cloud. The conjecture states that every number will eventually 'fall to the ground' by reaching the number 1. Even though this idea dates back to 1937, when it was first introduced by Lothar Collatz, mathematicians still do not know whether this will always happen. You might like to check what happens when you start with 27; it takes a surprisingly long time for the hailstones to fall!

Let's look at a gentler version of this problem: *snowflake* numbers. Again, if a number is even we halve it, otherwise we triple it and *subtract* 1. For example, the snowflakes of 11 are 32, 16, 8, 4, 2, 1, 2, 1 . . . and so on. The sequence starting with 11 has a cycle of length 2 that repeats indefinitely, which we can represent as follows:

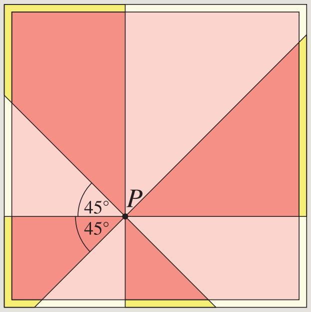
$$11 \longrightarrow 32 \longrightarrow 16 \longrightarrow 8 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1 \curvearrowright 2$$



We call them snowflakes because, being lighter than hailstones, they might never fall to the ground!

- **a** Show that the snowflakes of 12 also result in a cycle of length 2.
- **b** Show that the snowflakes of 13 result in a cycle of length 5.
- **c** Find a cycle of snowflakes with length greater than 5.
- **d** Explain why no cycle of snowflakes can contain a multiple of 3.
- **e** Find all cycles of length 5.
- **f** Find an odd number whose snowflakes are alternately even and odd, which do not form a cycle for at least 1000 terms.

5 Square pizzas



Angelo likes square pizzas because they fit inside square boxes more efficiently than round pizzas do. However, not having as many symmetries, he worries that it might not be possible to share square pizzas as fairly as round ones. By 'sharing fairly', Angelo means that everyone gets the same amount of topping (total area of their slices) and the same amount of crust (total length of boundary shared with the original square).

Angelo makes a cut parallel to one pair of sides, and then makes a second cut parallel to the other pair of sides. Through the point P where these two cuts meet, he makes two more cuts at 45° to the others, as shown.

- **a** A pizza has side length 20 cm and the point P is 8 cm from the left edge and 6 cm from the bottom edge. Find the length of each piece of crust (ignoring its thickness) and the area of each slice (including the crust).
- **b** Verify that the pizza in part **a** will be shared fairly among two people if they take alternate slices, shaded lighter and darker in the diagram.
- **c** Show that, regardless of the size of the pizza and the location of the point P, the method above will always share a pizza fairly among two people. Hint: Use coordinates $(0, 0)$, $(c, 0)$ and (a, b) for the bottom-left corner, bottom-right corner and intersection point P, respectively. What if P is on a diagonal? What if it isn't?
- **d** Next, Angelo is interested in sharing a square pizza fairly among three people. He wants to do this by making three straight cuts from a common point somewhere inside the square. Is this possible?
- **e** For what values of n is it possible to share a square pizza fairly among n people by making n straight cuts from a common point?

Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

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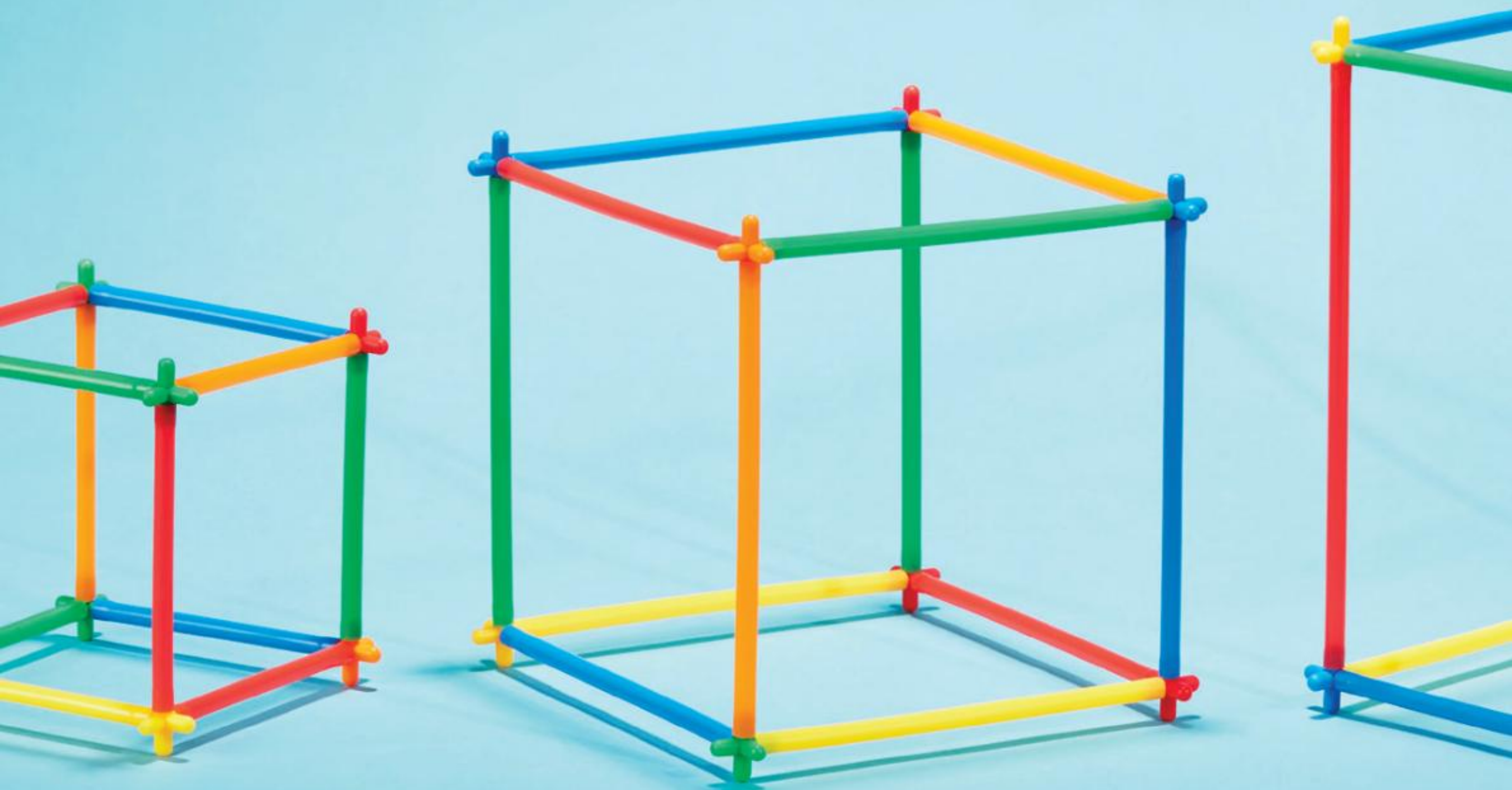
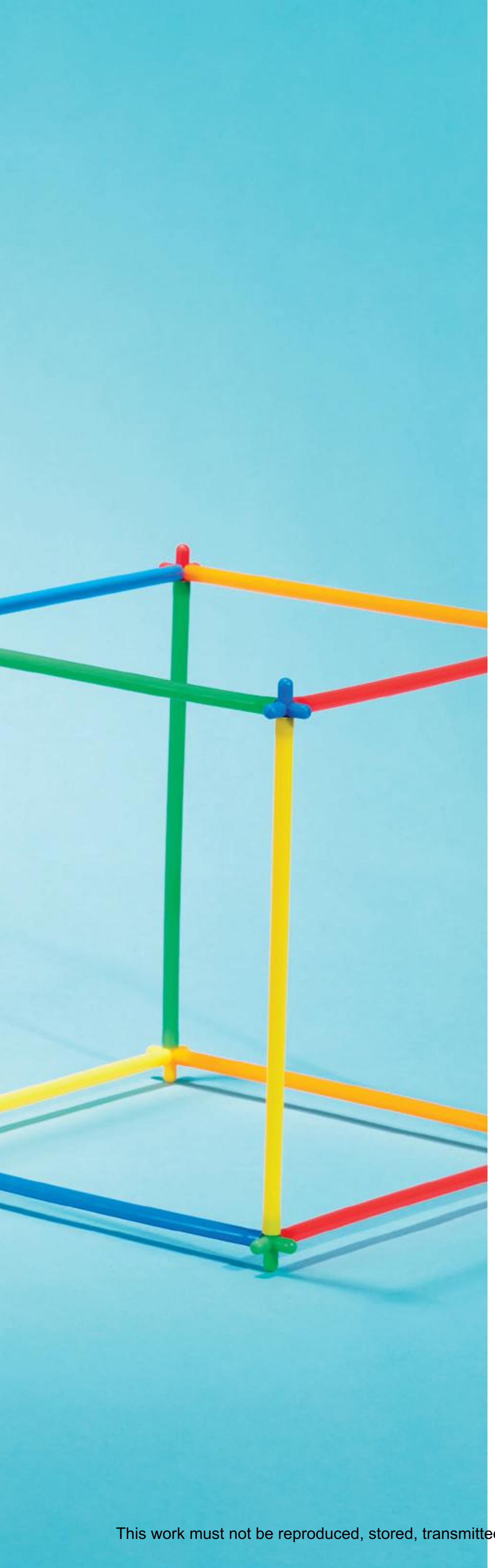
5 Measurement





Lessons

5.1 Pythagoras' theorem
5.2 Area of composite shapes
5.3 Surface area of prisms
5.4 Surface area of cylinders
5.5 Volume of prisms and cylinders
5.6 Errors in measurement

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

✔ Area of quadrilaterals
✔ Area of a circle
✔ Substitution

Curriculum links

- Solve problems involving the volume and surface area of right prisms, cylinders and composite objects using appropriate units (VC2M9M01)
- Solve spatial problems, applying angle properties, scale, similarity, ratio, Pythagoras' theorem and trigonometry in right-angled triangles (VC2M9M03)
- Calculate and interpret absolute, relative and percentage errors in measurements (VC2M9M04)

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Materials

✔ Calculator

Lesson 5.1

Pythagoras’ theorem

Learning intentions

By the end of this lesson you will be able to ...

✔ use Pythagoras’ theorem to solve problems involving right-angled triangles.

Inter-year links

Year 7	1.7 Exponents and square roots
Year 8	7.3 Pythagoras’ theorem
Year 10	7.1 Pythagoras’ theorem

Pythagoras’ theorem

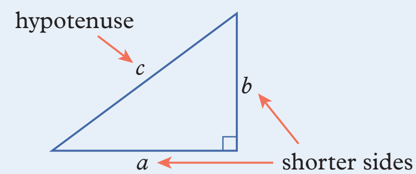
Key content video
Pythagoras' theorem

Key content video
Finding the length of a shorter side

- The longest side of a right-angled triangle is called the hypotenuse. It is always opposite the right angle.
- **Pythagoras’ theorem** states the relationship between the three side lengths of a right-angled triangle: the sum of the squares of the shorter sides that meet at the right angle is equal to the square of the hypotenuse. The relationship for the triangle shown below is:

$$c^2 = a^2 + b^2$$

hypotenuse shorter sides



→ The converse of Pythagoras’ theorem states that if three side lengths of a triangle satisfy Pythagoras’ theorem then the triangle is right-angled.

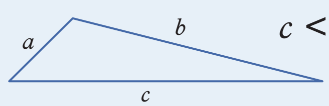
- Any set of three whole numbers that satisfies Pythagoras’ theorem is called a **Pythagorean triple** (or Pythagorean triad). For example, 3, 4, 5 is a Pythagorean triple because $5^2 = 3^2 + 4^2$.
- If the length of the unknown side of a right-angled triangle is an irrational number, the solution can be written as either an approximate decimal value or an exact value in simplified surd form.

→ An approximate decimal value should only be given if it is asked for in a question.

The triangle inequality

Key content video
Triangle inequalities

- In every triangle the length of any one side must be less than the sum of the lengths of the other two sides.

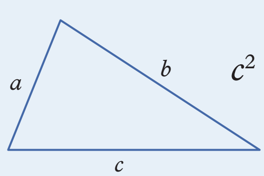


$$c < a + b$$

→ If the length of one side of a triangle was greater than the sum of the lengths of the other two sides, then the other two sides would not meet, as each end of a side must be attached to one of the other sides.

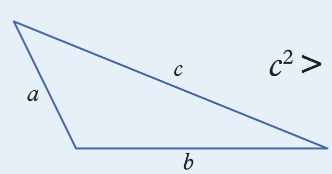
Inequalities for acute and obtuse triangles

- In an acute triangle, the square of the longest side is less than the sum of the squares of the other two sides.



$$c^2 < a^2 + b^2$$

- In an obtuse triangle, the square of the longest side is greater than the sum of the squares of the other two sides.



$$c^2 > a^2 + b^2$$

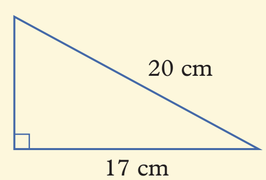
Worked example 5.1A Using Pythagoras' theorem to determine whether a triangle is right-angled

Determine whether a triangle with side lengths of 10 cm, 15 cm and 12 cm is right-angled.

THINK	WRITE
1 Identify the length of the longest side and call it c.	$c = 15$ cm
2 Calculate the value of c^2.	$c^2 = (15)^2$ $= 225$
3 Identify the lengths of the two shorter sides and call them a and b.	$a = 10$ cm and $b = 12$ cm
4 Calculate the value of $a^2 + b^2$.	$a^2 + b^2 = (10)^2 + (12)^2$ $= 100 + 144$ $= 244$
5 Compare c^2 to $a^2 + b^2$. If these two values are equal, the triangle is right-angled. If the two values are not equal, the triangle is not a right-angled triangle.	$c^2 \neq a^2 + b^2$ The triangle is not a right-angled triangle.

Worked example 5.1B Using Pythagoras' theorem to calculate an unknown side (decimal value)

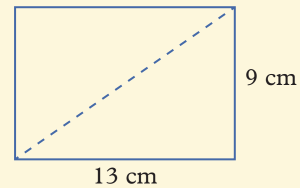
Calculate the length of the unknown side of this triangle, correct to one decimal place.



THINK	WRITE
1 Since the triangle is right-angled, state Pythagoras' theorem and define a, b and c. Remember that c is the hypotenuse and use b for the unknown side.	$c^2 = a^2 + b^2$, where $c = 20$ cm and $a = 17$ cm
2 Substitute the values for a and c into the equation for Pythagoras' theorem and simplify the equation.	$(20)^2 = (17)^2 + b^2$ $400 = 289 + b^2$
3 Solve the equation for b using inverse operations.	$b^2 = 400 - 289$ $= 111$ $b = \sqrt{111}$
4 Use a calculator to find the approximate length of the unknown side. Round to one decimal place and include the units.	$b \approx 10.5$ cm

Worked example 5.1C Using Pythagoras' theorem to calculate an unknown length (exact value)

Calculate the length of the diagonal of the rectangle on the right. Give your answer as an exact value in simplest form.



THINK	WRITE
1 Because the shape is a rectangle, the corners are right angles. State Pythagoras' theorem and define *a*, *b* and *c*. Remember that the diagonal of a rectangle is the hypotenuse of the triangle including the two side lengths.	$c^2 = a^2 + b^2$, where $a = 13$ cm and $b = 9$ cm
2 Substitute the values for *a* and *b* into the equation for Pythagoras' theorem. Let *c* be the unknown side and solve the equation for *c*.	$c^2 = (13)^2 + (9)^2$ $= 169 + 81$ $= 250$ $c = \sqrt{250}$
3 Simplify the surd by identifying any factors of 250 that are perfect squares. Include the units.	$= \sqrt{25 \times 10}$ $= \sqrt{25} \times \sqrt{10}$ $= 5\sqrt{10}$ cm

Helpful hints

- ✔ Make sure you correctly identify the hypotenuse. It is always the longest side and is opposite the right angle on the triangle. It may not be obvious, so always look at the numbers carefully.
- ✔ It doesn't matter which of the shorter sides you assign to be *a* or *b*, but the hypotenuse must always be *c*.
- ✔ Pythagoras' theorem is an excellent formula to memorise. You will use it many times during your mathematics studies.
- ✔ Remember to always give your answer with the correct units.
- ✔ The longest side in an obtuse triangle is the side opposite the obtuse angle.

Learning pathways

△ 1, 3–7, 9(a, c, f), 10, 12, 13

□ 1, 2, 3(b, d, f, h), 4(d–f), 5, 8, 9, 11, 14, 15, 17

⬠ 1, 2, 3(e, g, i), 6(b, d), 9(c, d, f), 11, 15, 16, 18

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Exercise 5.1A: Understanding and fluency

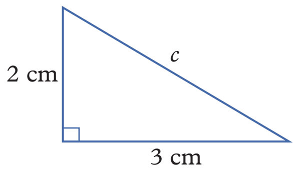
WE 5.1A **1** Determine whether each set of side lengths form a right-angled triangle.

a 26 cm, 24 cm, 10 cm **b** 100 cm, 96 cm, 28 cm **c** 3.5 cm, 8.5 cm, 9.3 cm

d 1 cm, 2.4 cm, 2.6 cm **e** 7.5 cm, 4.2 cm, 8.5 cm **f** 5 cm, 7 cm, 6 cm

2 For the triangles in question **1** which do not form a right-angled triangle, determine if they form an acute or an obtuse triangle.

WE 5.1B **3** Calculate the length of the unknown side in each of these triangles, correct to one decimal place.

a


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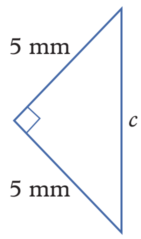
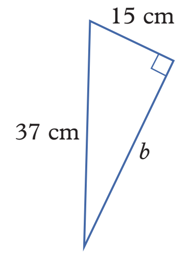


c


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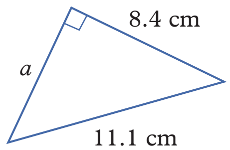
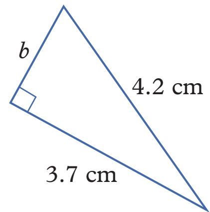


e


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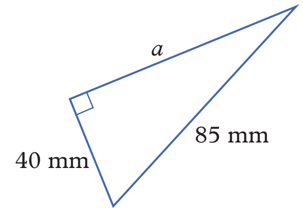

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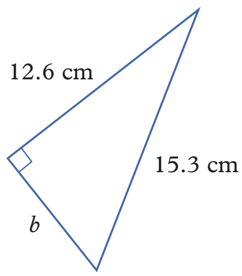

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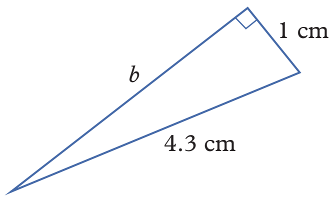

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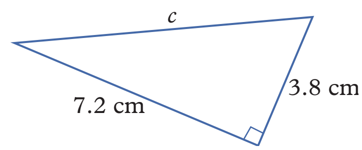
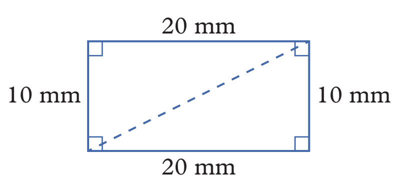


WE 5.1C **4** Calculate the length of the diagonal in each of these rectangles. Give your answer as an exact value in simplest form.

a


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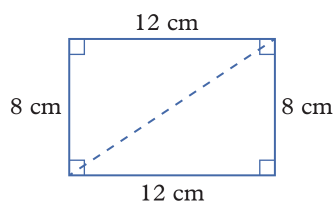

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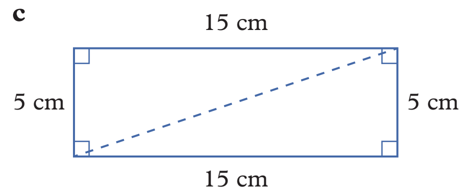

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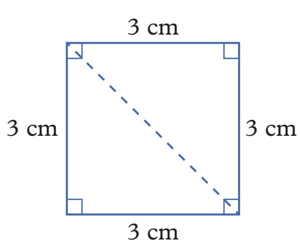

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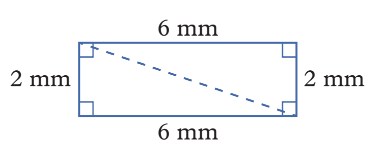

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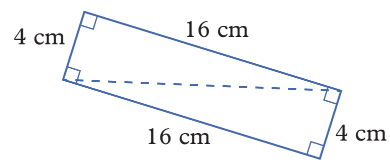


5 Rewrite the lengths of the diagonals of the rectangles in question **4**, correct to one decimal place.

6 Calculate the length of the diagonal of a square with each of these side lengths. Give your answer as an exact value in simplest form.

a 5 cm **b** 20 mm **c** 8 cm **d** 25 cm

7 Rewrite the lengths of the diagonals of the squares in question **6**, correct to one decimal place.

8 An equilateral triangle has side lengths of 8 cm.

a Draw a sketch of the triangle with two vertices forming the base of the triangle. From the top vertex, draw a line that is perpendicular (at right angles) to the base. Label the lengths of all sides of the figure.

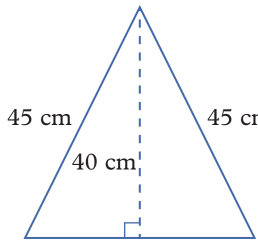
b Use Pythagoras' theorem to calculate the height of the triangle. Give your answer as:

i an exact value **ii** an approximate value, correct to one decimal place.

9 Calculate the length of the unknown side for each of these triangles, using the given height of each triangle to help you. Give your answers as:

i exact values **ii** approximate values, correct to one decimal place.

a



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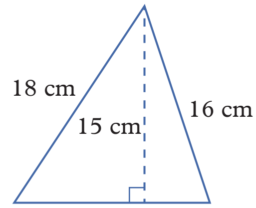

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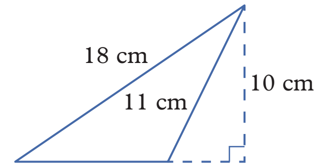

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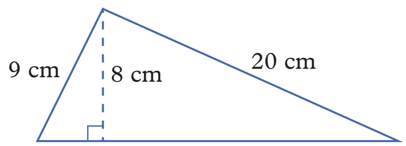

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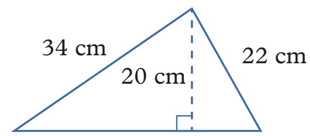

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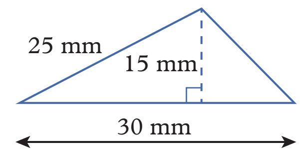


Exercise 5.1B: Problem solving and reasoning

10 A squaring tool is used when making picture frames, to ensure that the corners of the frames are true right angles.

Imagine you are constructing a square picture frame with side lengths of 54 cm.

a One of the diagonals of the frame measures 78 cm. Explain how you can be sure that the frame is not truly square.

b The other diagonal measures 76 cm. Draw a labelled sketch of the frame, showing its true shape.

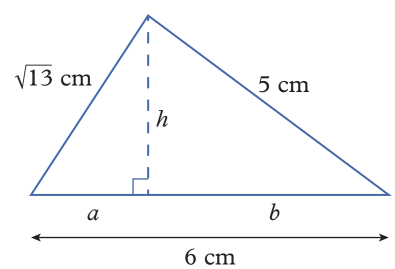
11 A sheet of writing paper is 21 cm wide and 30 cm long. The envelope used for this writing paper has a length of 22 cm and a diagonal of 24.6 cm.

a What is the width of the envelope?

b The writing paper is folded into equal-sized strips along its longer side. What is the minimum number of folds necessary to ensure the paper will fit in the envelope?

c If the paper can be folded along its shorter side as well as its longer side, what would be the minimum number of folds needed for it to fit in the envelope?

12 Afra wanted to determine the perpendicular height to the 6 cm edge of the triangle on the right. She was able to write the following three equations based on the information in the triangle.



$a^2 + h^2 = 13$ (1)

$b^2 + h^2 = 25$ (2)

$a + b = 6$ (3)

a Explain how Afra was able to write the three equations.

b Rearrange:

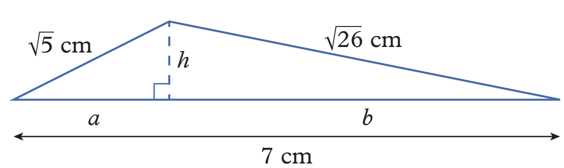
i equation (1) to make h^2 the subject **ii** equation (3) to make b the subject.

c Use your answers from part **b** to show that equation (2) can be written as $49 - 12a = 25$.

d Solve the equation in part **c**.

e Hence, determine the values of b and h.

f Repeat this process to find the exact perpendicular height to the 7 cm edge of the triangle shown on the right.



13 We know that a triangle with side lengths 3 cm, 4 cm and 5 cm is right-angled. The numbers 3, 4, 5 are one example of a Pythagorean triple. There are many more.

a To investigate other triples, we can start by finding out if multiples of the numbers 3 : 4 : 5 also satisfy Pythagoras' theorem. Do each of the following sets of numbers satisfy Pythagoras' theorem?

i 30 : 40 : 50 **ii** 6 : 8 : 10 **iii** 12 : 16 : 20

b We can use the Pythagorean triple 3, 4, 5 to check whether sets of side lengths that are not whole numbers are also Pythagorean triples. Use the 3, 4, 5 triple to show that triangles with the following side lengths are indeed right-angled triangles.

i 1.5 cm, 2 cm, 2.5 cm **ii** 0.6 cm, 0.8 cm, 1 cm **iii** 1.2 cm, 1.6 cm, 2 cm

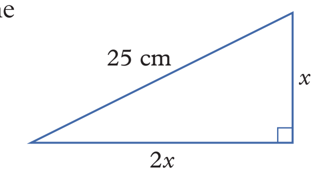
c What do you conclude from your investigations in parts **a** and **b**?

d All the triangles in parts **a** and **b** are similar triangles. Explain why this is so.

14 The two sides of a right-angled triangle, other than the hypotenuse, are such that one of those sides is twice as long as the other. The hypotenuse is 25 cm long. What are the lengths of the two shorter sides? Write your answer:

a as a simplified exact value

b correct to one decimal place.



15 Explain why it is impossible to draw a triangle where the length of one side is equal to the sum of the lengths of the other two sides.

16 Romesh has an extension ladder that is 9.8 m long. He needs to clean the windows on the outside of a building. The lower part of the windows are 9.8 m from the base of the building, and the windows are 1.2 m tall. Unfortunately, bushes mean that Romesh cannot place the base of the ladder any closer than 3 m from the base of the building. Romesh is 1.7 m tall.

a Draw a diagram to show the situation.

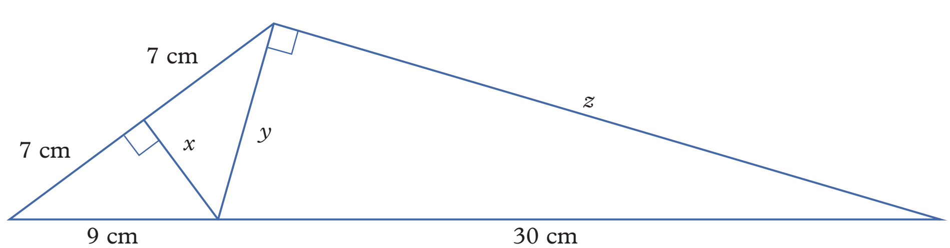
b Calculate the height the ladder reaches up the building, correct to one decimal place.

c Explain whether Romesh will be able to clean the windows.

17 A cubby house is built on two levels in a tree. The lower level is 10 m above the ground, and the upper level is a further 5 m higher vertically. The ladder from the ground to the lower level is 12 m long. Sam wants to construct another ladder to go from the ground, 1 m further along the ground from the base of the tree than the first ladder, to the top level. How long should he make this ladder?

Exercise 5.1C: Challenge

18 Determine the exact simplified values of x, y, and z.



Online resources:

- **Interactive skillsheet** Pythagoras' theorem
- **Interactive skillsheet** Finding the length of the hypotenuse
- **Interactive skillsheet** Finding the length of a shorter side
- **Quick quiz** 5.1

Lesson 5.2

Area of composite shapes

Learning intentions

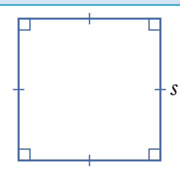
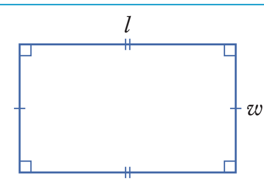
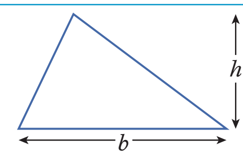
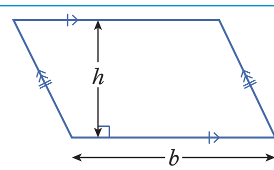
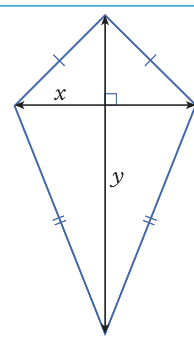
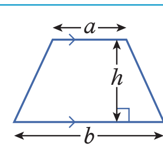
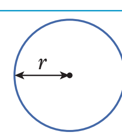
By the end of this lesson you will be able to ...

✔ calculate the area of composite shapes.

Inter-year links

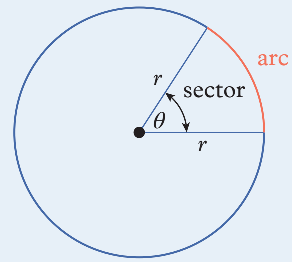
Support	Understanding area
Year 7	9.5 Area of a triangle
Year 8	8.4 Area of a circle
Year 10	8.1 Area review

Area of simple shapes

Shape	Example	Formula
Square		$A = s^2$
Rectangle		$A = lw$
Triangle		$A = \frac{1}{2}bh$
Parallelogram/rhombus		$A = bh$
Kite/rhombus		$A = \frac{1}{2}xy$
Trapezium		$A = \frac{h}{2}(a + b)$
Circle		$A = \pi r^2$

Sectors

- A **sector** is a portion of a circle formed by two radii and part of the **circumference** (an **arc**).
- The area of a sector can be found using $\frac{\theta}{360} \times \pi r^2$, where θ (the Greek letter theta) is the size of the angle, in degrees, between the two radii.
  → This formula is derived from the formula for the area of a circle, $A = \pi r^2$, where $\frac{\theta}{360}$ represents the proportion of a full circle that the sector represents.



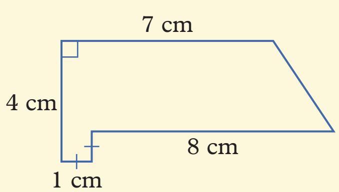
Composite shapes

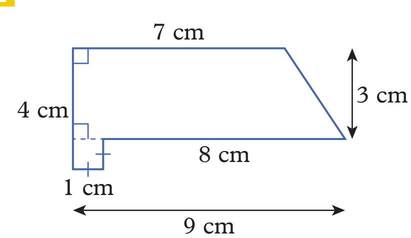
Key content video
Area of composite shapes

- A **composite shape** can be split up into two or more simple shapes.
- To calculate the area of a composite shape, follow these steps:
  1 Split the figure into simple shapes that have known area formulas.
  2 Calculate any missing dimensions.
  3 Calculate the areas of the individual shapes using the area formulas.
  4 Add or subtract the areas to calculate the total area.

Worked example 5.2A Calculating the area of a composite shape using addition

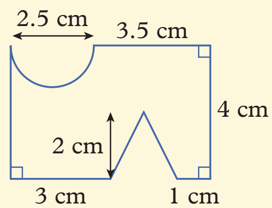
Calculate the area of this composite shape.



THINK	WRITE
1 Use dotted lines to split the composite figure into simple shapes. **2** Find any missing dimensions for each individual shape and label these on your figure.	
3 Use area formulas to calculate the areas of the simple shapes.	$A_{\text{square}} = s^2$ $= 1^2$ $= 1\text{ cm}^2$ $A_{\text{trapezium}} = \frac{h}{2}(a + b)$ $= \frac{3}{2} \times (7 + 9)$ $= 24\text{ cm}^2$
4 Add the areas together to find the total area of the composite shape. Remember to include the appropriate unit.	Total area $= 24 + 1$ $= 25\text{ cm}^2$

Worked example 5.2B Calculating the area of a composite shape using subtraction

Calculate the area of this composite shape, correct to one decimal place.

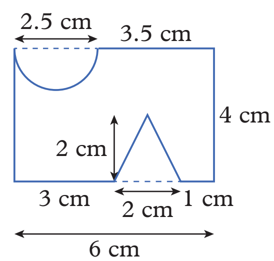


THINK

1 Use dotted lines to split the composite figure into simple shapes.

2 Find any missing dimensions for each individual shape and label these on your figure.

WRITE



THINK

3 Use area formulas to calculate the areas of the simple shapes.

WRITE

$$\begin{aligned} A_{\text{rectangle}} &= lw \\ &= 6 \times 4 \\ &= 24 \text{ cm}^2 \\ A_{\text{triangle}} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 2 \times 2 \\ &= 2 \text{ cm}^2 \\ A_{\text{semicircle}} &= \frac{1}{2}\pi r^2 \\ &= \frac{1}{2} \times \pi \times 1.25^2 \\ &= 2.45\ldots \text{ cm}^2 \end{aligned}$$

THINK

4 Subtract the smaller areas from the main area to find the total area of the composite shape. Remember to round your answer and include the appropriate unit.

WRITE

$$\begin{aligned} \text{Total area} &= 24 - 2 - 2.45\ldots \\ &= 19.54\ldots \\ &\approx 19.5 \text{ cm}^2 \end{aligned}$$

Helpful hints

- ✔ When calculating the areas of composite shapes, a useful technique can be to number the different parts, for example, A_1 = Area of shape 1. This is useful when there is more than one of a particular type of simple shape in the composite figure.
- ✔ Many composite shapes can be split up in more than one way. There is not a correct way to split up a composite shape, but you should look for the easiest method. However the shape is split up, the total area should be the same!
- ✔ Remember that area is a measure of two-dimensional space so the units of measurement must be squared: mm^2, cm^2, m^2, km^2.

Learning pathways

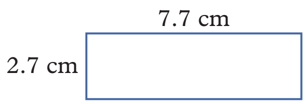
△ 1–3, 4(a, b, c, e), 5–7, 8(a, c, d, f), 9, 12, 13(a, b)	□ 1, 2, 4, 5, 7, 8(b, d, e, f), 10, 12, 13(c, d, f), 14, 16, 19(a, b)	⬠ 2, 4, 5, 8(b, e, f), 10, 11, 13(d, e, f), 14, 15, 17, 18, 19(b, c)

ANS p497

Exercise 5.2A: Understanding and fluency

1 Calculate the area of these simple shapes. Give your answers to two decimal places where necessary.

a



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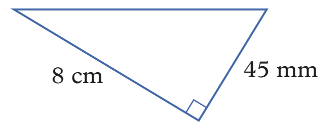

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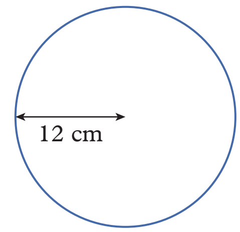

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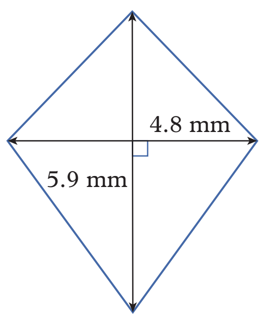

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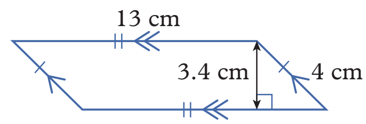

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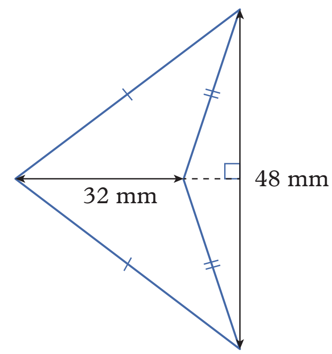
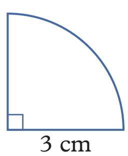


2 Calculate the area of each sector, correct to two decimal places.

a



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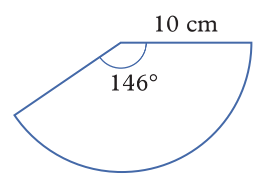

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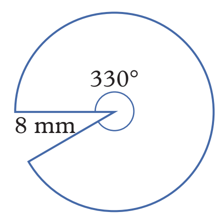
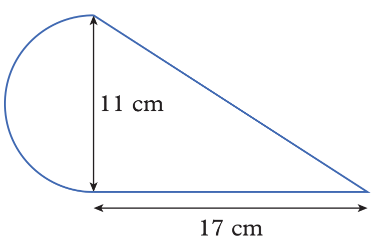


3 Identify the basic shapes within each composite shape.

a



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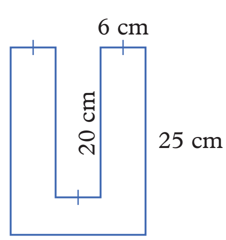

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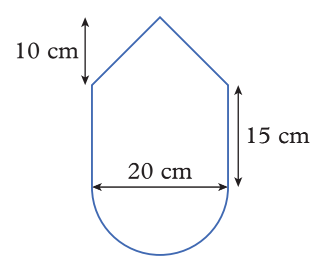

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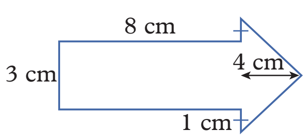

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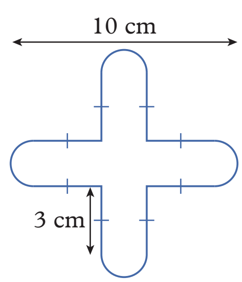

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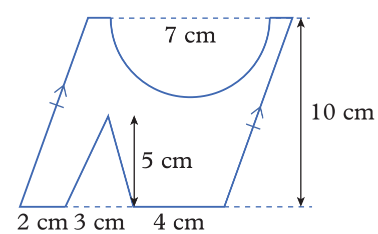
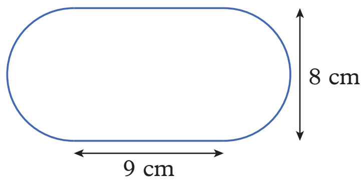
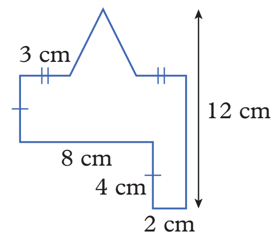


WE 5.2A **4** Calculate the area of each composite shape. Give your answers to two decimal places where necessary.

a



b



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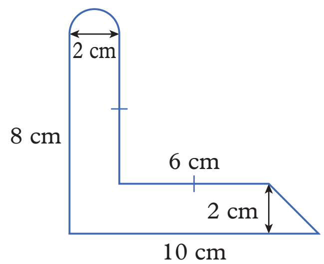

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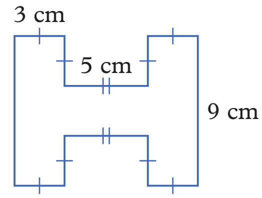

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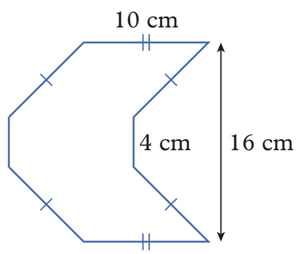

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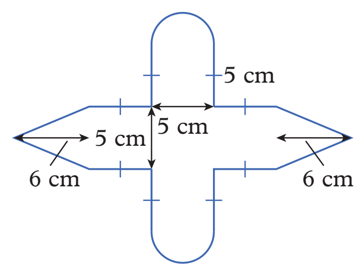
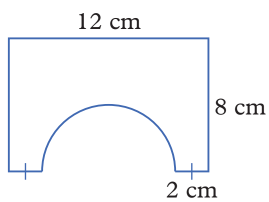


WE 5.2B **5** Calculate the area of each composite shape. Give your answers to two decimal places where necessary.

a



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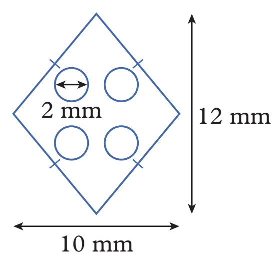

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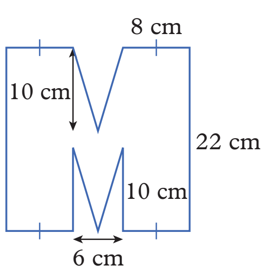

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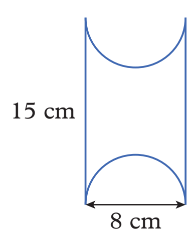

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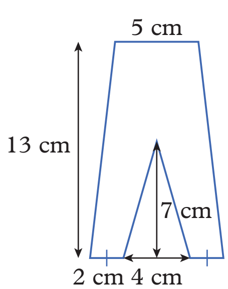

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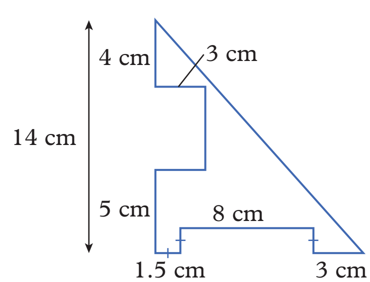
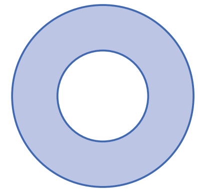


6 Calculate the area of each composite shape in question **3**. Give your answers to two decimal places where necessary.

Exercise 5.2B: Problem solving and reasoning

7 The figure on the right is known as an annulus (plural: annuli).



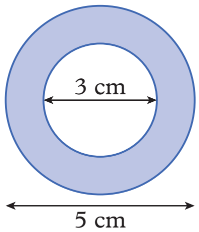
a What is an annulus?

b Explain how you can find the area of this annulus using a formula.

c Find the area of this annulus if it has an outer diameter of 16 cm and an inner diameter of 9 cm. Give your answer to two decimal places.

8 Calculate the area of each annulus, correct to two decimal places.

a



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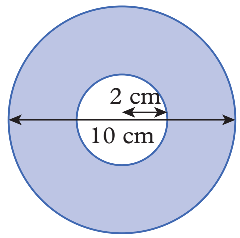

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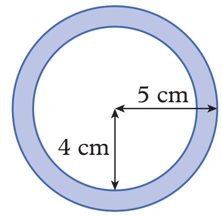

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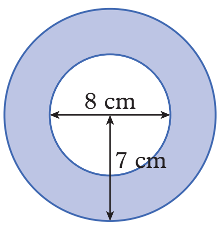

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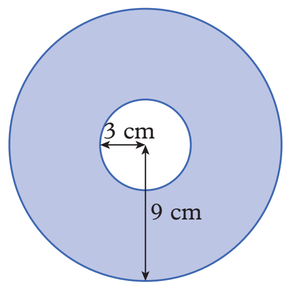

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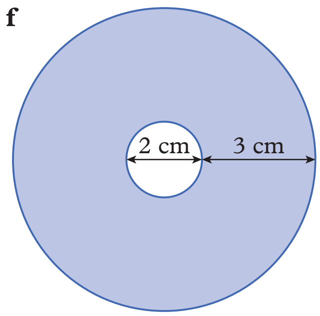
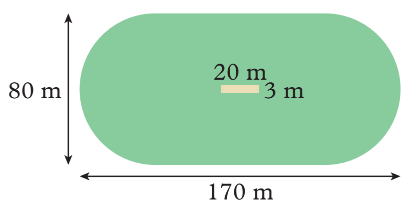


9 What area of grass needs to be mowed on this sports oval, assuming that the cricket pitch in the middle is artificial and does not require mowing? Give your answer to two decimal places.

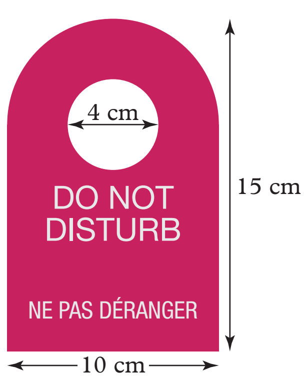


10 Emily makes and sells custom tags through the website *Etsy*.

a What is the area of cardboard in a tag with the dimensions shown?

b What is the total area of cardboard required to make 550 tags with the dimensions shown? Explain why Emily might need to buy more than this amount of cardboard in order to make the tags.

11 Find the amount of cardboard in this DO NOT DISTURB sign, correct to two decimal places. Note: Image is not drawn to scale.

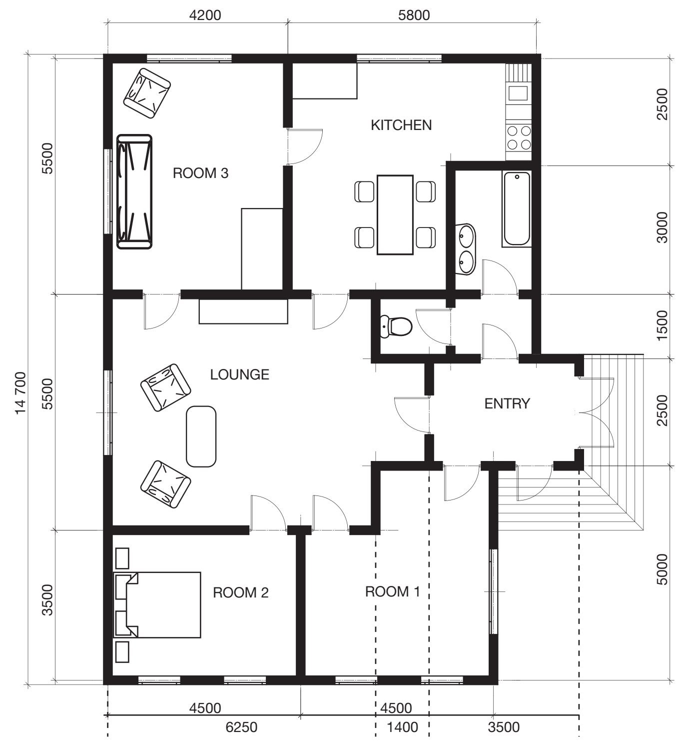


12 Shannon wants to lay new carpet in his house. Consider the floor plan on the right with measurements in millimetres.

a What area of the house would he need to cover if everything except the bathroom, kitchen and toilet was to be carpeted? Write this in square metres. Hint: You may find it easier to convert the measurements to metres.

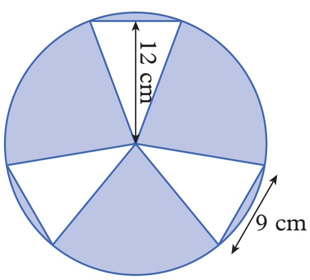
b How much would it cost if the carpet is priced at $45/m²?

c How much money would he save if he chose a cheaper carpet at $30/m²?



13 Calculate the shaded area of each shape, correct to two decimal places where necessary.

a



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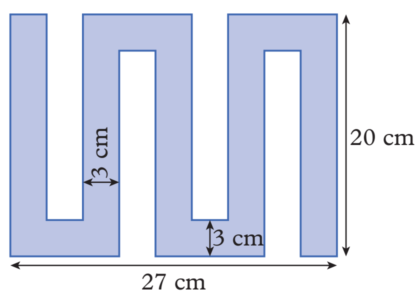

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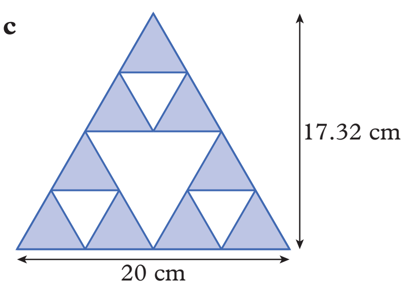

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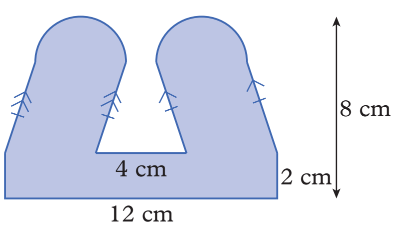

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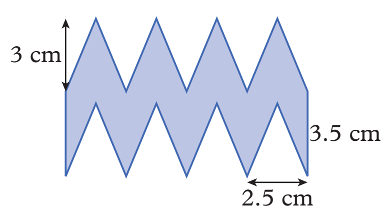

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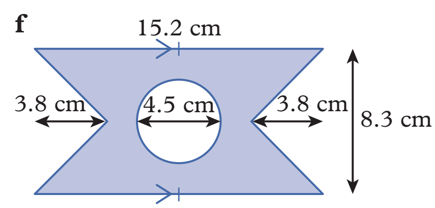
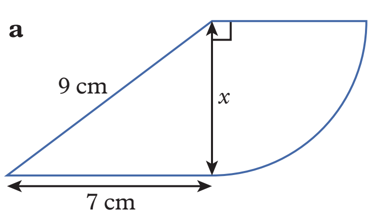


14 For each of the following composite shapes:

i use Pythagoras' theorem to find the length of x as an exact value in simplest form

ii use your answer to part **i** to calculate the area, correct to two decimal places.

a



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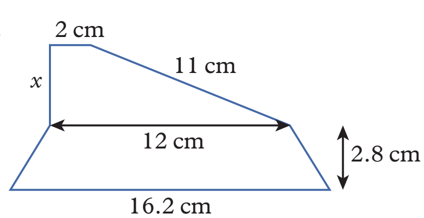

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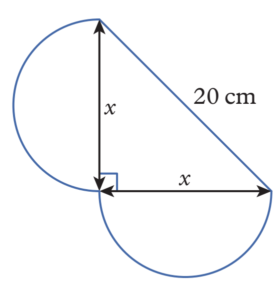
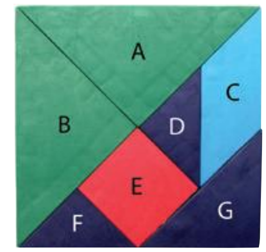


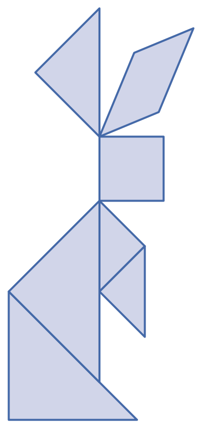
15 The tangram is an ancient Chinese puzzle that consists of seven pieces (usually placed in a square as shown) that can be rearranged to form a variety of shapes.



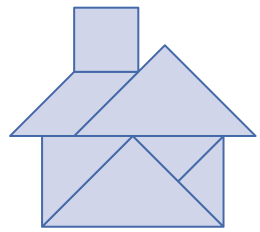
a What is the area of each individual piece in this tangram square if the entire square has side lengths of 10 cm?

b What is the total area of each of these shapes that can be made from the seven pieces of the tangram?

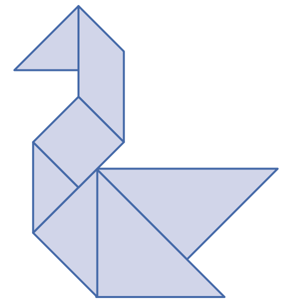
i



ii



iii

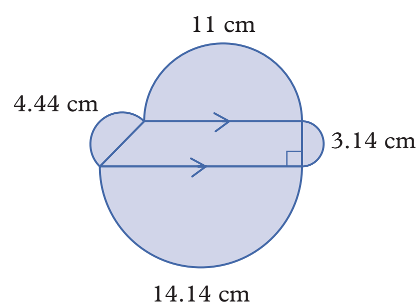


16 Find the area of an annulus with an outer circumference of 35 cm and an inner circumference of 25 cm. Give your answer to two decimal places.

Exercise 5.2C: Challenge

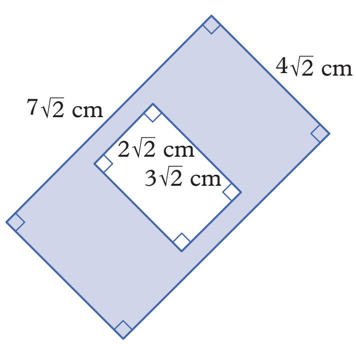
17 A circular skirt, without hems, is cut from a circle of material with a hole for the waist. If a particular circular skirt had an inner circumference of 70 cm and was 60 cm in length when worn, what was the area of material used to create the skirt? Give your answer to two decimal places.

18 Calculate the shaded area, correct to two decimal places, given the number shown on each arc is the length of the corresponding semicircle.

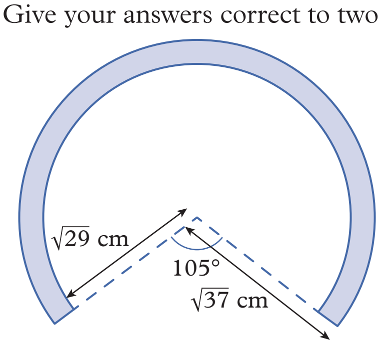


19 Calculate the shaded area of each shape. Give your answers correct to two decimal places where necessary.

a



b



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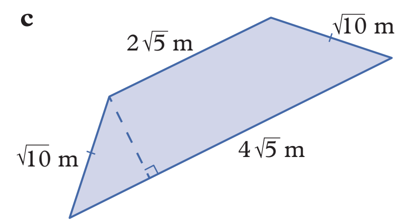


Online resources:

Interactive skillsheet Area of composite shapes

Worksheet Identifying simple shapes within composite shapes

Worksheet Area of shapes

Quick quiz 5.2

Surface area of prisms

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the surface area of right prisms.

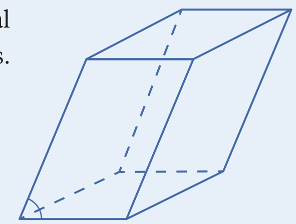
Inter-year links

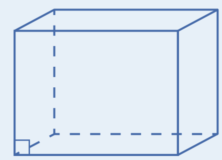
Year 7	9.3 Area of a rectangle
Year 8	8.3 Area of quadrilaterals
Year 10	8.2 Surface area

Prisms

Key content video
Surface area of prisms

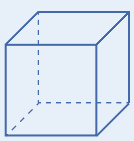
- A **prism** is a 3D object with straight edges, two identical bases and a constant **cross-section** parallel to the bases.
  - → A **face** is a flat surface on a three-dimensional (3D) object.
  - → An **edge** is a line segment that joins two faces.
  - → A cross-section of a 3D object is a 2D shape obtained when cutting through the object.


Non-right prism

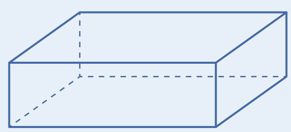

Right prism

- A **right prism** has right angles between the base and the sides.
  - → All the sides (non-bases) of a right prism are rectangles.

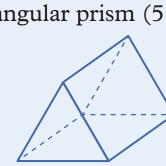
Cube (6 faces)



Rectangular prism (6 faces)



Triangular prism (5 faces)



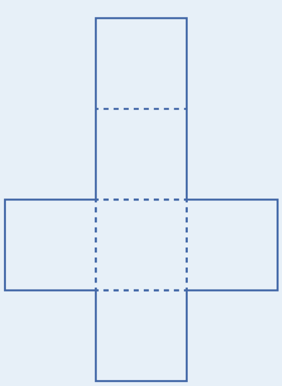
- A **cube** has six faces that are all identical squares.
- A **rectangular prism** has six faces. There are three pairs of identical rectangular faces.
- A **triangular prism** has five faces. The two ends of a triangular prism are identical triangular faces.

Nets

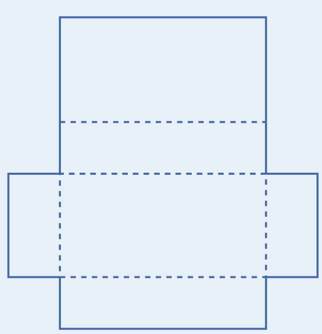
Key content video
Nets

- A **net** is a 2D plan that can be folded to form a 3D object.
- Nets can be used to show all the faces of 3D objects and hence to help calculate the surface areas of those objects.

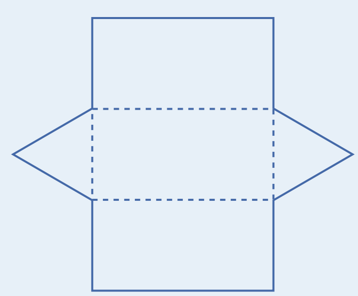
Net of a cube



Net of a rectangular prism



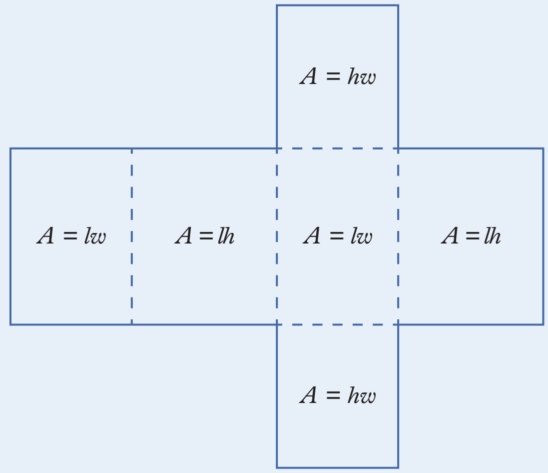
Net of a triangular prism



Surface area

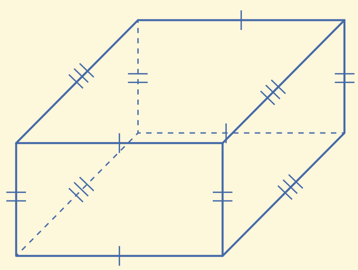
- The **surface area** of a three-dimensional (3D) object is the total area of the outer surface of that object.
- To calculate the surface area of a 3D object:
  1. Determine the shape of each face.
  2. Calculate the area of each face.
  3. Add the areas of the faces together.

  → The surface area of a rectangular prism can be calculated using the formula: SA = $2lw + 2lh + 2hw$, where l is the length of the base rectangle, w is the width of the base rectangle and h is the height of the prism.



Worked example 5.3A Drawing the net of a right prism

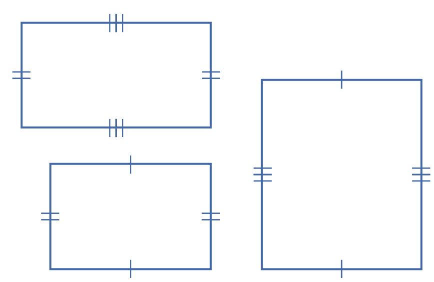
Draw a net for this 3D object.

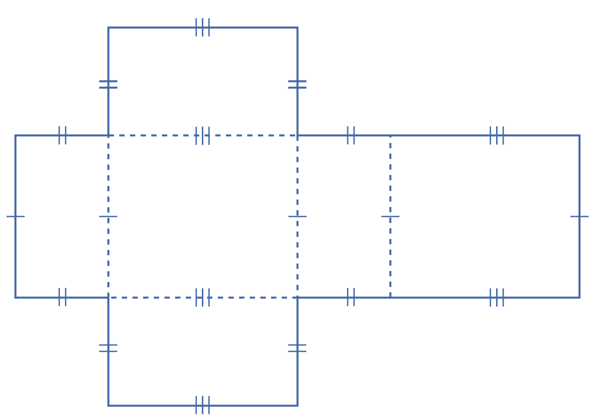


THINK

1. A rectangular prism has six faces, made up of three different kinds. Draw each different one.
2. Try to visualise the rectangular prism unfolding – which face is on the bottom? This will form the centre of the net.
3. Arrange the remaining faces in a hopscotch-style net. Look carefully at which faces are touching the centre face.
4. Add tabs if you wish to construct the object.

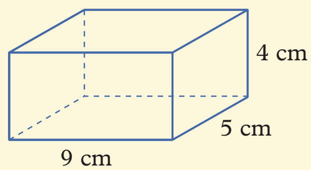
WRITE





Worked example 5.3B Calculating the surface area of a rectangular prism

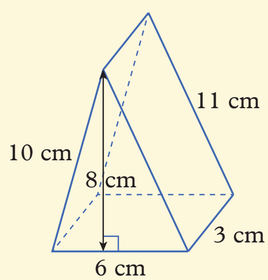
Calculate the surface area of this rectangular prism.



THINK	WRITE
1 Determine the shape of each face. A rectangular prism has six rectangular faces; three pairs of identical faces.	Area of front and back faces $= 9 \times 4$ $= 36\ \text{cm}^2$
2 Calculate the area of each face.	Area of side faces $= 5 \times 4$ $= 20\ \text{cm}^2$ Area of top and bottom faces $= 9 \times 5$ $= 45\ \text{cm}^2$
3 Add the areas of all the faces together. Check that all six faces have been included in the sum.	Surface area $= 2 \times 36 + 2 \times 20 + 2 \times 45$ $= 202\ \text{cm}^2$

Worked example 5.3C Calculating the surface area of a triangular prism

Calculate the surface area of this triangular prism.



THINK	WRITE
1 Determine the shape of each face. A right triangular prism has two identical triangular faces and three rectangular faces.	Area of triangular face $= \frac{1}{2} \times b \times h$ $= \frac{1}{2} \times 6 \times 8$ $= 24\ \text{cm}^2$
2 Calculate the area of each face.	Area of bottom rectangular face $= 6 \times 3$ $= 18\ \text{cm}^2$ Area of left-hand rectangular face $= 10 \times 3$ $= 30\ \text{cm}^2$ Area of right-hand rectangular face $= 11 \times 3$ $= 33\ \text{cm}^2$
3 Add the areas of the faces together. Include the appropriate unit.	Surface area $= 2 \times 24 + 18 + 30 + 33$ $= 129\ \text{cm}^2$

Helpful hints

- ✔ All of the non-matching end faces in a right prism are rectangles.
- ✔ A prism can be orientated in any way. Identify the matching ends to determine the shape of the base of a prism.

Learning pathways

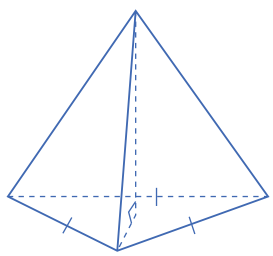
△ 1, 2, 3(a, b), 4(a–c), 5–8, 10, 13, 15 □ 3(c, d), 4(d–f), 5, 6, 8, 12, 14–17 ⬠ 6, 8–14, 17–19

ANS p497

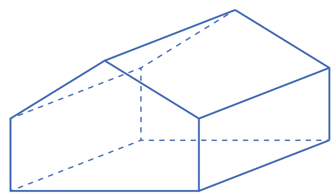
Exercise 5.3A: Understanding and fluency

1 Which of the following objects are right prisms?

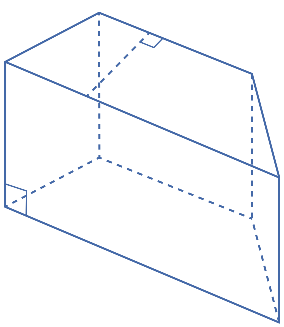
a



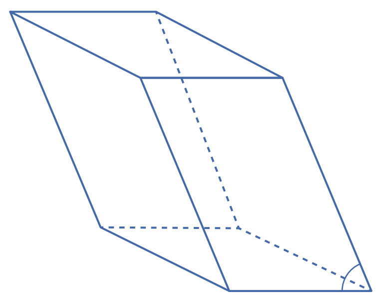
b



c



d

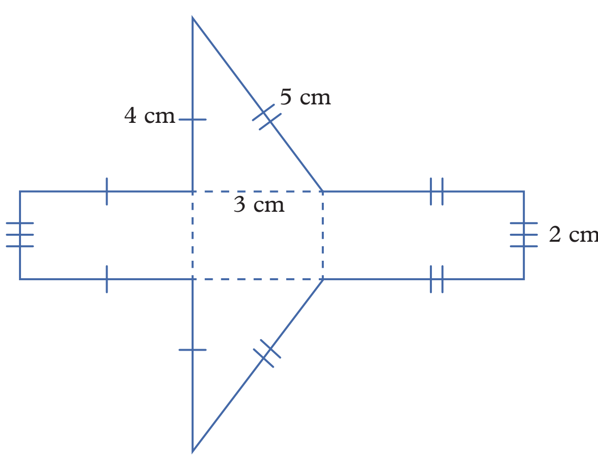


2 For each of the following nets, determine:

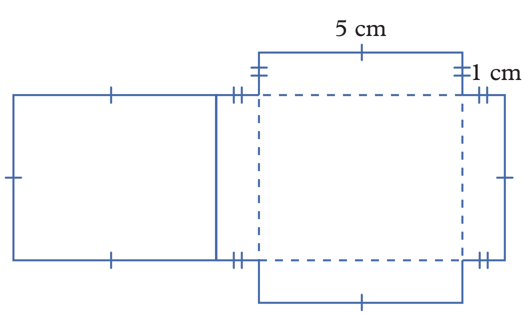
i the name of the 3D object it folds up to

ii the surface area of the 3D object.

a

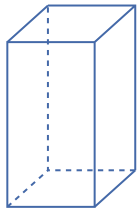


b

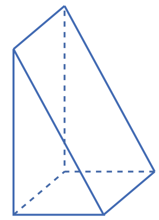


WE 5.3A **3** Draw a net for each 3D object.

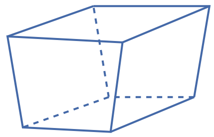
a



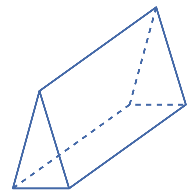
b



c

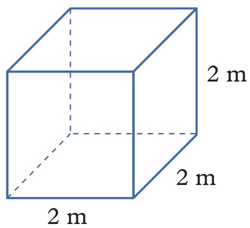


d

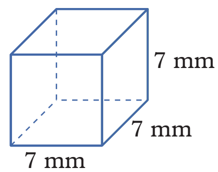


4 Calculate the surface area of each cube.

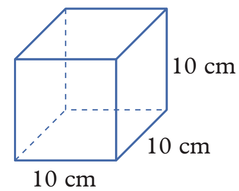
a



b



c



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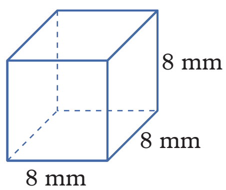
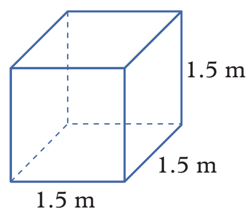
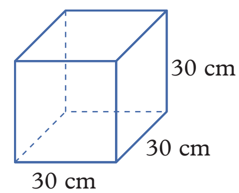


e



f

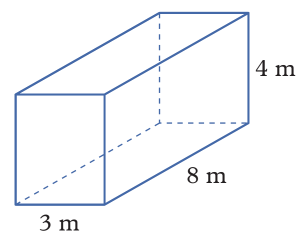


5 Consider the rectangular prism on the right.

a Draw a net for the rectangular prism.

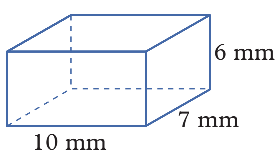
b Calculate the area of each pair of identical faces of the prism.

c Use your answer from part **b** to calculate the surface area of the prism.

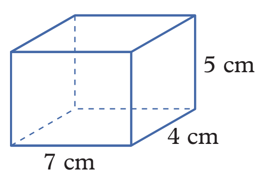


WE 5.3B **6** Calculate the surface area of each of these rectangular prisms.

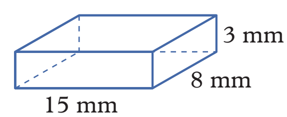
a



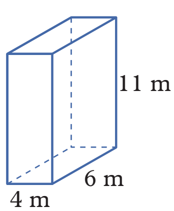
b



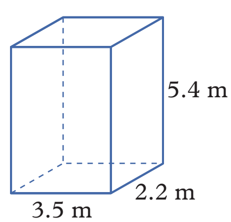
c



d



e



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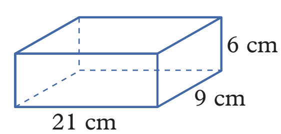
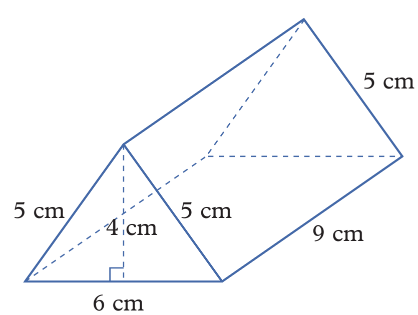


7 Consider the triangular prism on the right.

a Draw a net for the triangular prism.

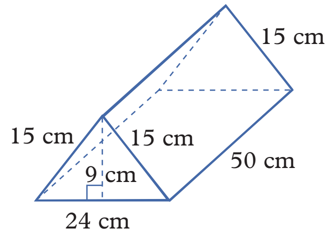
b Calculate the area of each face.

c Use your answer from part **b** to calculate the surface area of the triangular prism.

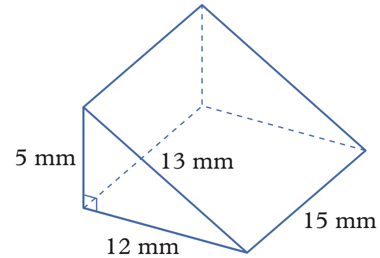


WE 5.3C **8** Calculate the surface area of each triangular prism.

a



b



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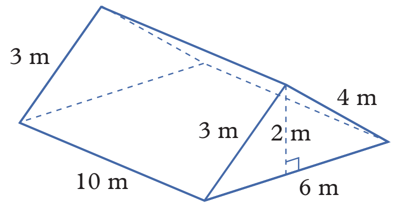
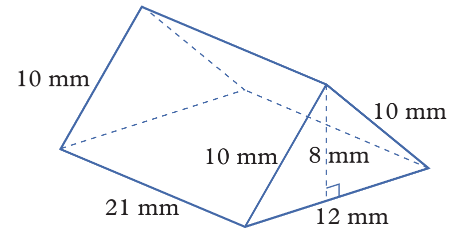
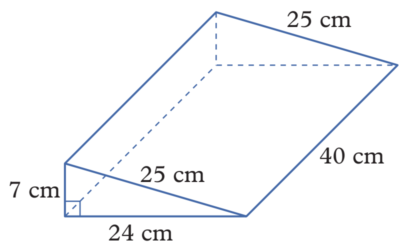


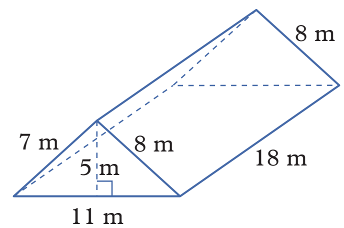
d



e



f

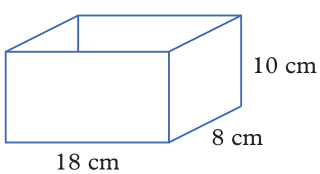


9 Find the surface area of these boxes if you were to consider:

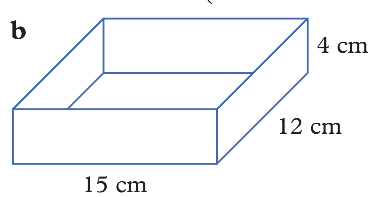
i the amount of material required to build the boxes (note: there is no top of the box)

ii the amount of paint (in cm^2) required to paint all surfaces of the boxes (inside and outside).

a

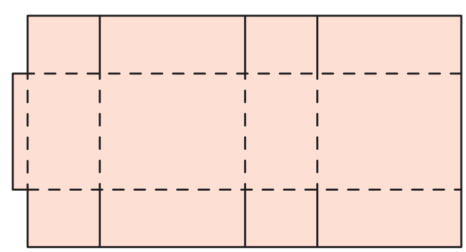


b

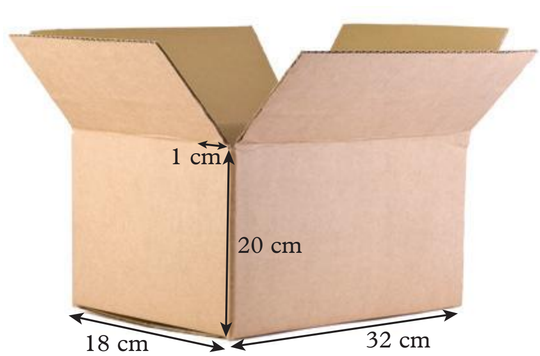


Exercise 5.3B: Problem solving and reasoning

10 The net of a basic cardboard box is shown. The dashed lines represent folds, and the solid lines represent cuts. The tab on the left is used to join the central section of the box, and the flaps at the top and bottom are all half of the breadth of the box. Calculate the amount of cardboard required to make these boxes.



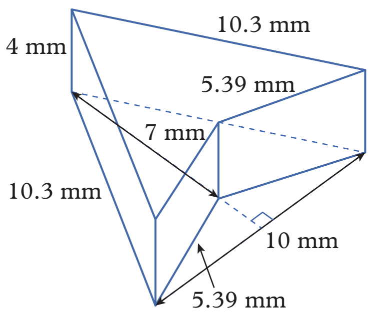
a



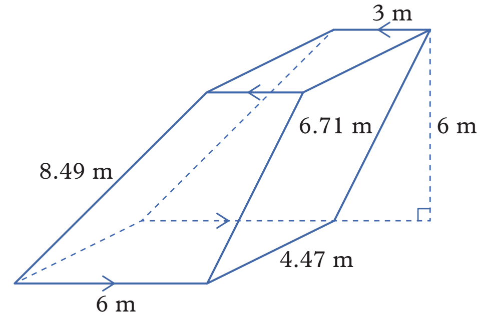
b

11 Calculate the surface area of these right prisms.

a



b



12 A block of butter is in the shape of a rectangular prism.

a If the block was 7 cm wide, 16 cm long and 6 cm high, calculate its surface area.

b If the block was cut halfway along the length (that is, to give two pieces 8 cm long), what is the surface area:

i of each piece **ii** in total?

c How is your answer to part **b ii** different from your answer to part **a**?

d Imagine that you cut the block of butter into 1-cm cubes. What would be the surface area of the butter now?

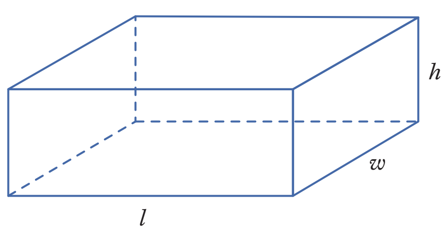
e If somebody wanted to melt butter quickly, what would you recommend to them? Why?

13 Billy is having his room painted. His room is 6 m long, 5 m wide and 2.5 m high and has a large window on one wall that measures 150 cm × 95 cm. The walls are to be painted blue and the ceiling is to be painted cream.

a What area is to be painted blue and what area is to be painted cream?

b How many litres of each colour paint is needed if 1 L of paint covers about 15 m^2 and the room will take two coats?

14 Consider the following rectangular prism.



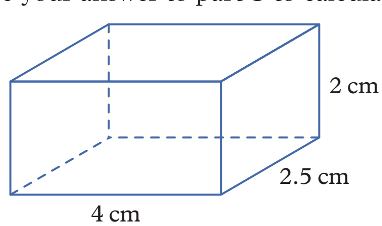
a Draw a net for this rectangular prism.

b What is the surface area of this rectangular prism in terms of l, w and h? Give your answer as a simplified algebraic expression.

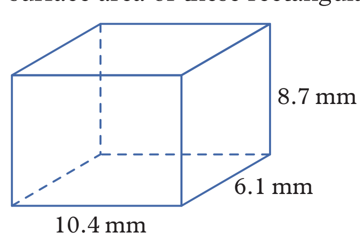
c Explain why your answer to part **b** can be used to calculate the surface area of any rectangular prism.

d Use your answer to part **b** to calculate the surface area of these rectangular prisms.

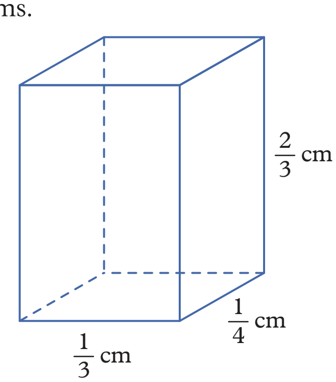
i



ii



iii



15 A cube has a surface area of 150 cm².

a What is the surface area of each face?

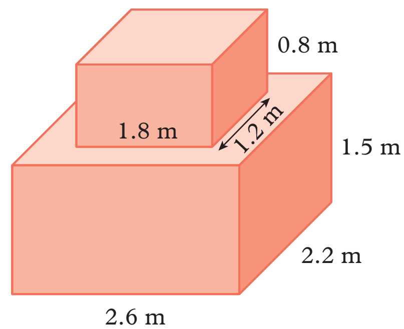
b What is the side length of the cube?

c Write a formula that will help you determine the side length of any cube if you know its surface area.

16 Explain why knowing the surface area of a cube is enough to determine its side lengths but knowing the surface area of a rectangular prism is not.

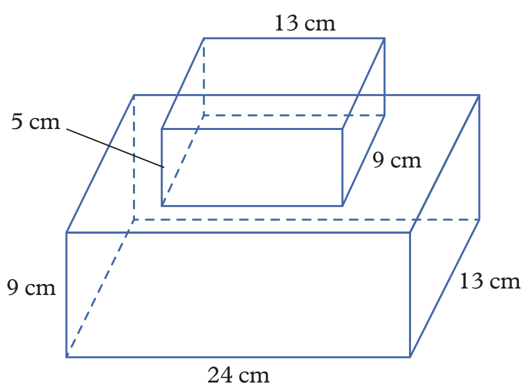
Exercise 5.3C: Challenge

17 A sculpture is constructed in the shape of a rectangular box with a smaller box placed on top. The dimensions of the boxes are shown in the diagram. To finish the sculpture, the artist must work out the surface area to be painted. Calculate the surface area of the sculpture, excluding the base which will not be visible to the public. Note: The overlapping parts of the boxes will also not be painted.

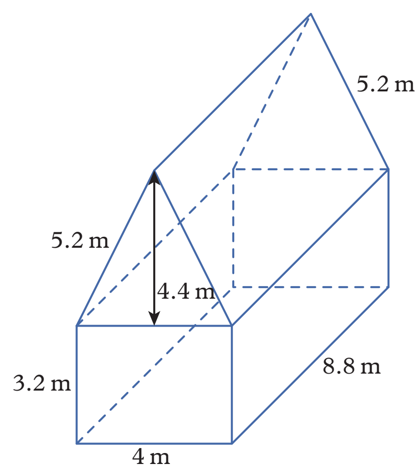


18 Calculate the surface area of these composite objects. Hint: Do not include the area where the shapes overlap.

a

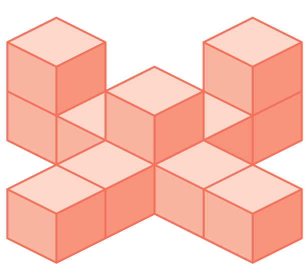


b

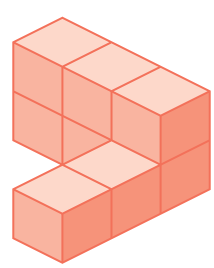


19 These objects are made of 1 cm cubic blocks. Calculate their surface area (including the base).

a



b



Online resources:

Interactive skillsheet Surface area of prisms	**Worksheet** Surface area of prisms	**Investigation** Dissecting cubes	**Quick quiz** 5.3

ANS p498

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

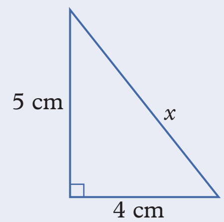
5.1 **1** Determine whether the following lengths can form a right-angled triangle.

a 9 cm, 40 cm, 41 cm
b 39 cm, 89 cm, 80 cm
c 137 cm, 104 cm, 89 cm
d 21 cm, 221 cm, 220 cm

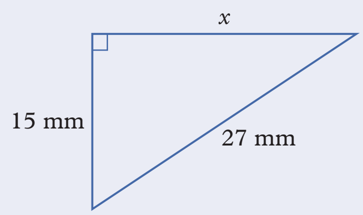
5.1 **2** Determine the value of x in each of the following:

i as a simplified exact value
ii correct to two decimal places.

a

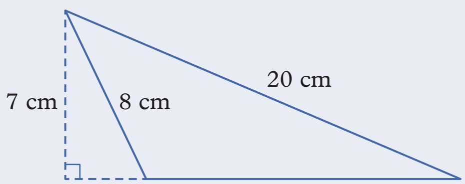


b

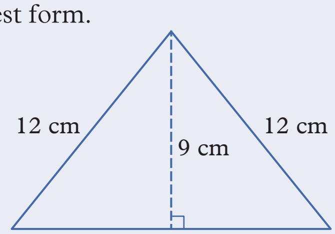


5.1 **3** Calculate the length of the unknown side for each of these triangles, using the given height of each triangle to help you. Give your answer as an exact value in simplest form.

a

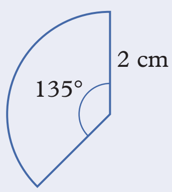


b

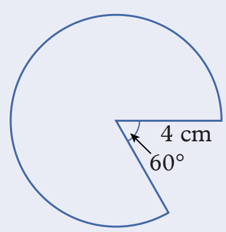


5.2 **4** Calculate the area of these sectors, correct to two decimal places.

a

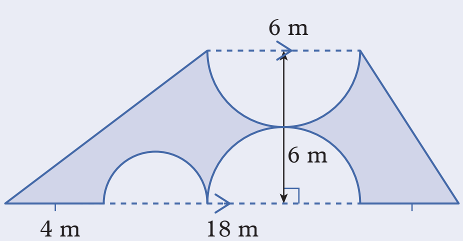


b

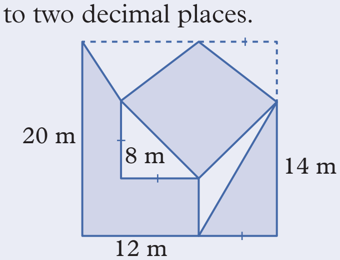


5.2 **5** Calculate the shaded area of these composite shapes, correct to two decimal places.

a

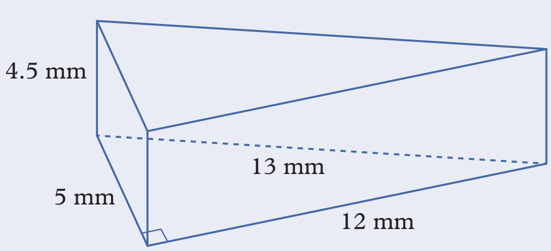


b

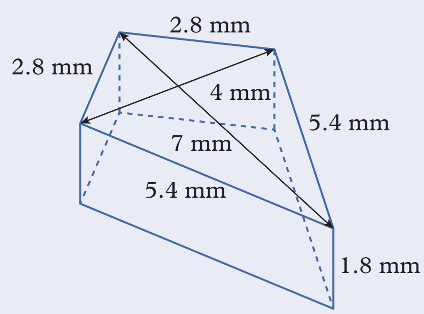


5.3 **6** Calculate the surface area of these prisms.

a



b



Lesson 5.4

Surface area of cylinders

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the surface area of cylinders.

Inter-year links

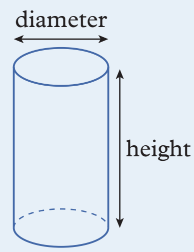
Year 8 8.4 Area of a circle

Year 10 8.2 Surface area

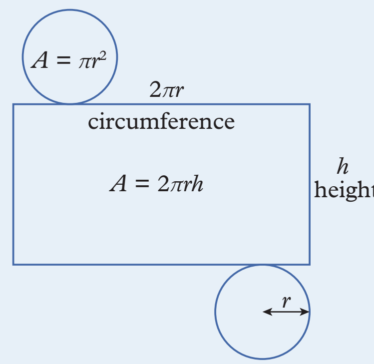
Cylinders

Key content video
Surface area of cylinders

- A **cylinder** is a 3D object with two circular bases and a constant circular cross-section parallel to the bases.
  → The net of a cylinder is formed by a rectangle and two identical circular bases.
  → The surface area of a cylinder can be calculated using the formula SA = $2\pi r^2 + 2\pi rh$, where $2\pi r^2$ is the area of the two circles, and $2\pi rh$ is the area of the rectangle.

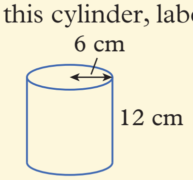


Cylinder



Worked example 5.4A Drawing the net of a cylinder

Draw the net of this cylinder, labelling dimensions accurately.



THINK

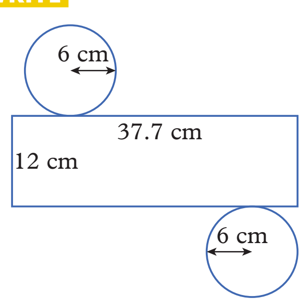
1 Draw a net of a cylinder, remembering that it consists of a rectangle and two circles.

2 Label the radius of the circles and the height of the rectangle. Include appropriate units.

3 The length of the rectangle is the circumference of the circular ends. Use the formula $C = 2\pi r$ to calculate this length, correct to one decimal place.

4 Label the length of the rectangle to complete your net. Include appropriate units.

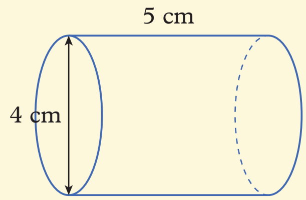
WRITE



$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \times 6 \\ &\approx 37.7 \text{ cm} \end{aligned}$$

Worked example 5.4B Calculating the surface area of a cylinder

Calculate the surface area of this cylinder, correct to two decimal places.



THINK

1 Identify the measurements for the radius (r) and the height (h) and substitute these into the formula.

2 Write the formula for surface area of a cylinder.

3 Substitute the values for r and h into the formula and calculate the result, correct to two decimal places. Include the appropriate unit.

WRITE

$$\begin{aligned} r &= d \div 2 \\ &= 4 \div 2 \\ &= 2 \text{ cm} \\ h &= 5 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{SA} &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times 2^2 + 2 \times \pi \times 2 \times 5 \\ &= 8\pi + 20\pi \\ &= 28\pi \\ &= 87.964... \\ &\approx 87.96 \text{ cm}^2 \end{aligned}$$

Helpful hints

✔ The formula for the surface area of a cylinder, $\text{SA} = 2\pi r^2 + 2\pi rh$, can be factorised. Therefore, another way to find the surface area of a cylinder is:

$$\text{SA} = 2\pi r(r + h)$$

Learning pathways

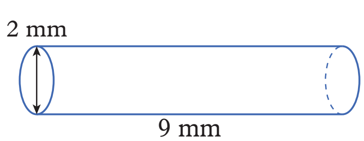
△ 1–5(a, e, f), 7, 9 ■ 5, 6, 8, 11, 14, 17 ⬠ 5, 8, 10, 12–17

ANS p499

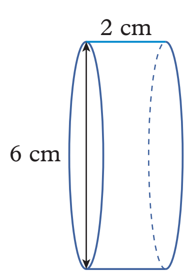
Exercise 5.4A: Understanding and fluency

WE 5.4A **1** Draw a net for each cylinder, labelling the dimensions, correct to two decimal places where appropriate.

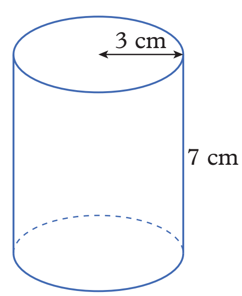
a



b



c



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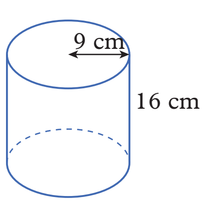

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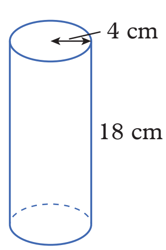

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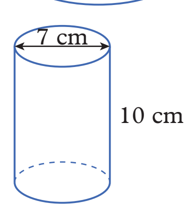
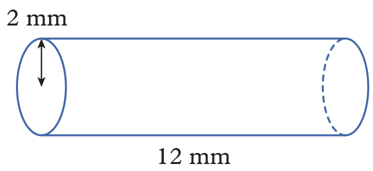


WE 5.4B **2** Calculate the surface area of each cylinder in question **1**. Give your answers to two decimal places.

3 Draw a net for each cylinder, labelling the dimensions, correct to two decimal places where appropriate.

a


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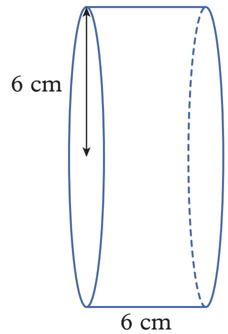

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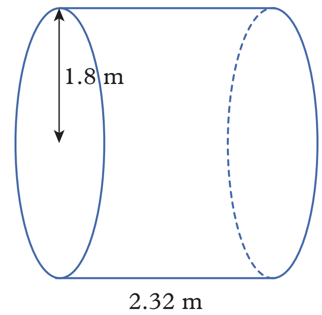

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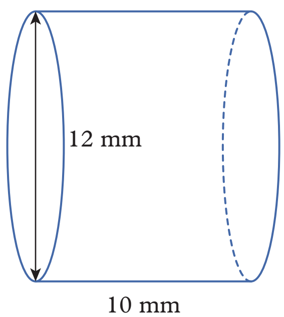

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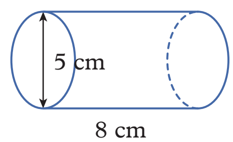

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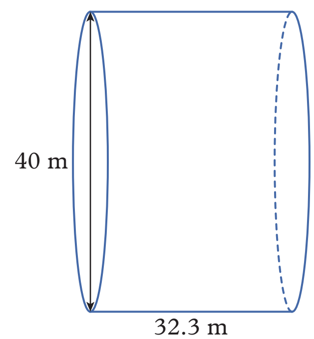
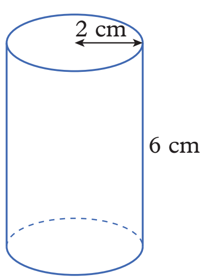


4 Calculate the surface area of each cylinder in question **3**. Give your answers to two decimal places.

5 Calculate the total surface area for each cylinder, correct to two decimal places.

a


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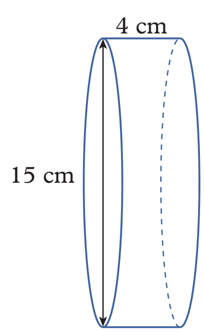

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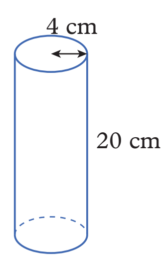

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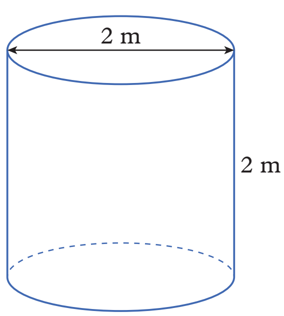

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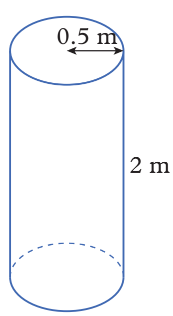

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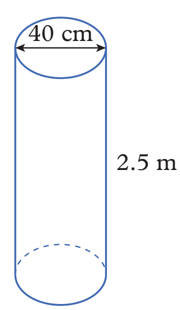


6 Calculate the total surface area for each cylinder, correct to two decimal places.

a $h = 8$ cm, $r = 10$ cm

b $h = 2$ cm, $d = 3.1$ cm

c $h = 2.1$ m, $r = 3$ m

d $h = 3.2$ m, $d = 7.3$ m

e $h = \frac{1}{2}$ mm, $r = 1\frac{1}{3}$ mm

f $h = \frac{3}{2}$ mm, $d = \frac{4}{3}$ mm

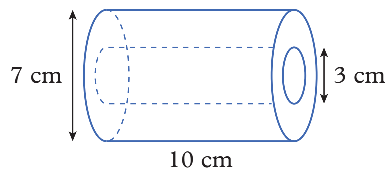
7 Calculate the outer surface area of this box and its lid if the box is 22 cm tall and has a radius of 15 cm. The lid is 4 cm tall and has a diameter of 31 cm. Give your answer to two decimal places.

Exercise 5.4B: Problem solving and reasoning

8 A cylindrical pool is 2 m deep and has a radius of 6.5 m. How much would it cost, to the nearest dollar, to paint its interior if it needs two coats of paint that costs \$50 per litre? Assume that 1 L covers 15 m^2.

9 Maria has the choice of two paint rollers. One roller is 25 cm long and has a radius of 4 cm. The other roller is 30 cm long and has a diameter of 6 cm. Assuming they have the same absorbency, which roller would need to be re-dipped in paint the least often?

10 Consider the following tube.



a What is the surface area of the tube, including the internal exposed surface? Give your answer correct to two decimal places.

b Lucian answered part **a** as 296.88 cm^2 and Curtis said it was 282.74 cm^2. Explain where they went wrong.

11 A cylinder has a radius of 4 cm and a surface area of 300 cm^2. What is its height to the nearest centimetre?

12 The surface area for any prism can be calculated using the formula:

$$\text{SA} = 2 \times \text{area of base} + \text{perimeter of base} \times \text{height}.$$

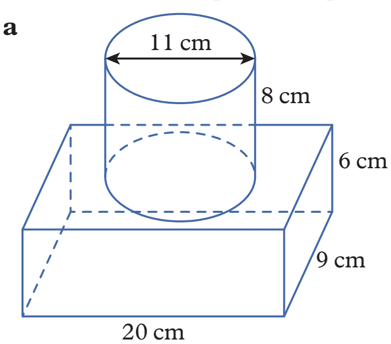
Explain why this formula works.

13 If you double the height of a cylinder, does its surface area also double? Explain, using an example.

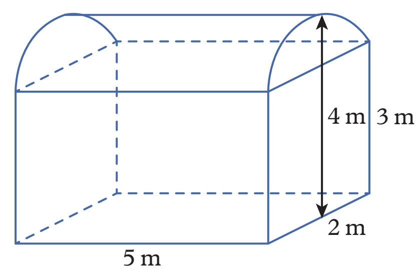
Exercise 5.4C: Challenge

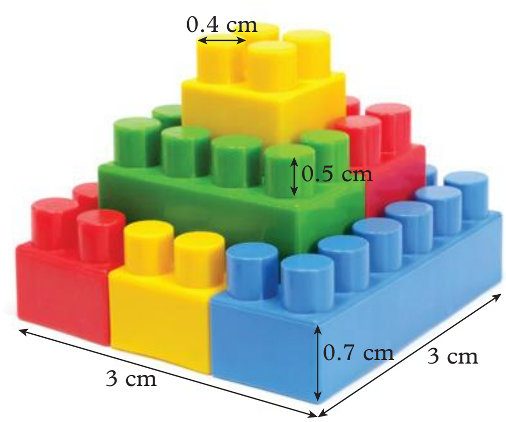
14 Calculate the surface area of these composite objects, correct to two decimal places. Hint: do not include the area where the shapes overlap.

a



b





15 Write a formula that calculates the height of a cylinder, given its surface area and radius.

16 Calculate the exterior surface area, correct to two decimal places, of the structure shown in the image on the right (do not include the base).

17 The surface area of a sphere can be calculated using the formula SA = $4\pi r^2$. Calculate the surface area of a sphere, correct to two decimal places, that has:

a radius 4 cm **b** radius 7 mm

c diameter 10 cm **d** circumference 20 cm.

Online resources:

Interactive skillsheet Surface area of cylinders

Investigation Making can-coolers

Quick quiz 5.4

Lesson 5.5

Volume of prisms and cylinders

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the volume and capacity of prisms, cylinders and composite objects.

Inter-year links

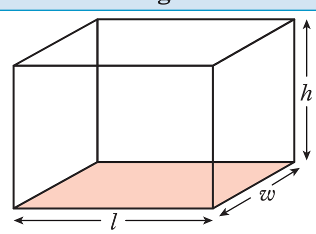
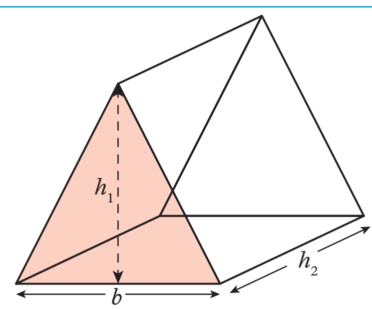
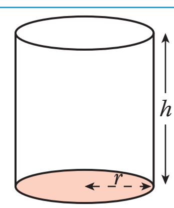
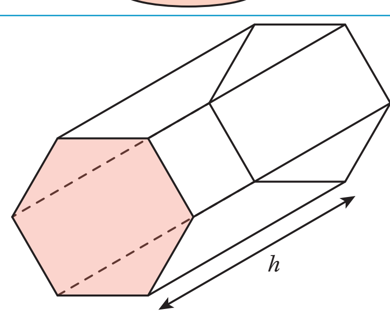
Support	Volume and capacity
Year 7	9.6 Volume of right prisms
Year 8	8.5 Volume and capacity
Year 10	8.3 Volume

Volume of prisms and cylinders

Key content video
Volume of prisms

Key content video
Volume of cylinders

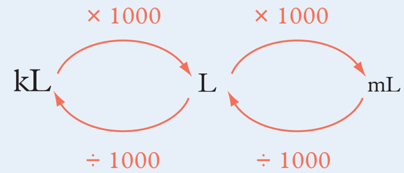
- The **volume** of a 3D object is the amount of three-dimensional space that it occupies.
- The volume of a right prism or a cylinder can be found by multiplying the base area by the height. $V = Ah$, where A is the area of the base and h is the height of the prism.

Shape	Diagram	Formula
Rectangular prism		$V =$ (area of rectangle) $\times h$ $= lwh$
Triangular prism		$V =$ (area of triangle) $\times h$ $= \frac{1}{2}bh_1h_2$
Cylinder		$V =$ (area of circle) $\times h$ $= \pi r^2 h$
Right prism		$V =$ (area of base) $\times h$

Capacity

Key content video
Volume and capacity

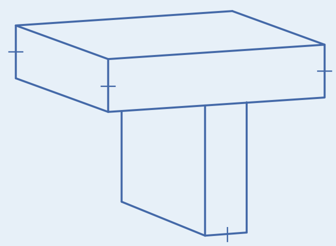
The **capacity** of a 3D object is a measure of how much the object can hold.
A container with an inside volume of:
→ 1 cm^3 holds 1 mL of liquid
→ 1000 cm^3 holds 1 L of liquid
→ 1 m^3 holds 1 kL of liquid.

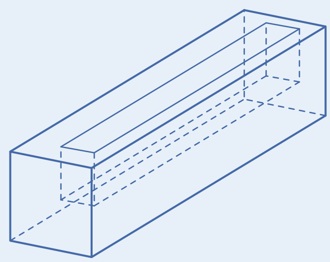


Volume of composite objects

Key content video
Volume of composite solids

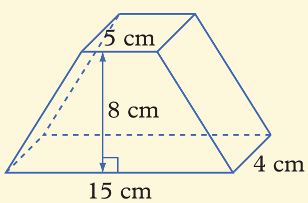
- A **composite object** consists of two or more simple objects.
- To calculate the volume of a composite object:
  1. Split the object into individual parts.
  2. Calculate any missing dimensions.
  3. Calculate the volume of each individual part.
  4. Add or subtract the part volumes to find the total volume.





Worked example 5.5A Calculating the volume of a prism

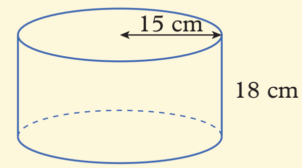
Calculate the volume of this prism.



THINK	WRITE
1 Identify the shape of the base of the right prism and write the appropriate formula for the area: $A = \frac{h_1}{2}(a + b)$.	$A = \frac{h_1}{2}(a + b)$ $= \frac{8}{2} \times (5 + 15)$ $= 80\text{ cm}^2$
2 Substitute the values for the base area (A) and the height of the prism (h_2) into the formula for the volume of the prism: $V = Ah_2$.	$V = Ah_2$ $= 80 \times 4$
3 Calculate the result. Remember to include the appropriate unit.	$= 320\text{ cm}^3$

Worked example 5.5B Calculating the volume and capacity of a cylinder

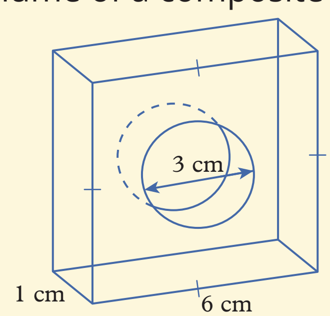
Calculate the volume (correct to two decimal places) and capacity (correct to the nearest litre) of the cylinder on the right.



THINK	WRITE
1 Identify the values for the radius of the base (r) and the height of the cylinder (h).	$r = 15$ cm $h = 18$ cm
2 Calculate the volume of the cylinder, using the π button on your calculator.	$V = \pi r^2 h$ $= \pi \times 15^2 \times 18$ $= 4050\pi$ $\approx 12\,723.45$ cm^3
3 Convert the volume into capacity by using an appropriate conversion factor.	Capacity $= (12\,723.45 \div 1000)$ L ≈ 13 L

Worked example 5.5C Calculating the volume of a composite object

Calculate the volume of the composite object on the right, correct to two decimal places.



THINK	WRITE
1 Split the composite object into a prism and cylinder. List the dimensions of the prism and the cylinder.	The length, width and height of the rectangular prism are 6 cm, 6 cm and 1 cm. The diameter and height of the cylinder are 3 cm and 1 cm.
2 Use volume formulas to calculate the volume of the prism and cylinder.	$V_{\text{prism}} = lwh$ $= 6 \times 6 \times 1$ $= 36$ cm^3 $V_{\text{cylinder}} = \pi\left(\frac{d}{2}\right)^2 h$ $= \pi \times \frac{3^2}{4} \times 1$ $= \frac{9}{4}\pi$ cm^3
3 Subtract the volume of the cylinder from the volume of the rectangular prism. Use exact values of π until the last step of your calculation, then round your answer to two decimal places.	Total volume $= V_{\text{prism}} - V_{\text{cylinder}}$ $= 36 - \frac{9}{4}\pi$ ≈ 28.93 cm^3

Helpful hints

- ✔ For rectangular prisms, any of the faces can be designated as the base, but remember that the height must be perpendicular (at right angles) to the base.
- ✔ Remember that volume and capacity are not the same thing! Capacity is a measure of how much an object can hold (mL, L, kL), whereas volume is a measure of how much space an object occupies (mm^3, cm^3, m^3).

Learning pathways

△ 1–3, 4(a, b), 5–7, 9, 13

□ 1(d, f, g), 2(d–f), 3(d–f), 4(b, c), 6–9, 11, 14

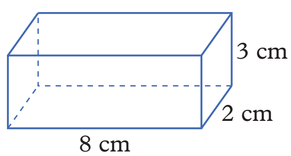
⬠ 1(g–i), 2(e, f), 3(e, f), 4(c), 8, 10, 12–15

ANS p500

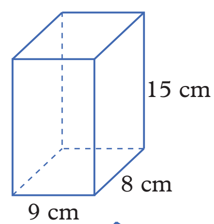
Exercise 5.5A: Understanding and fluency

WE 5.5A **1** Calculate the volume of each prism.

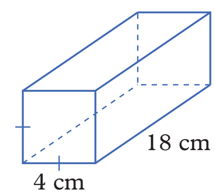
a



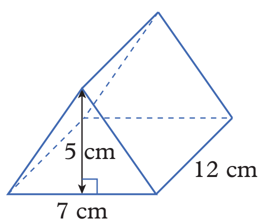
b



c



d



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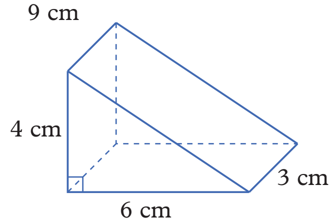

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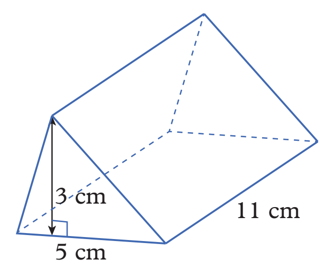
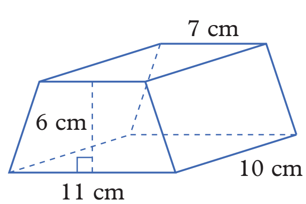
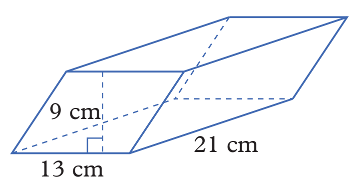


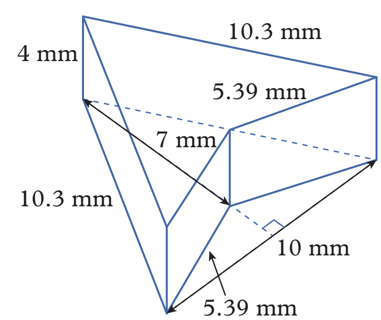
g



h

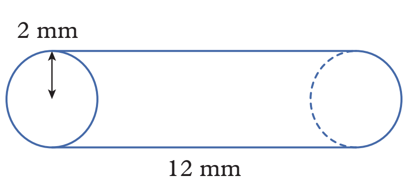


i

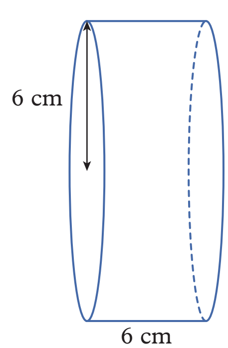


WE 5.5B **2** Calculate the volume of each cylinder. Give your answers to two decimal places.

a



b



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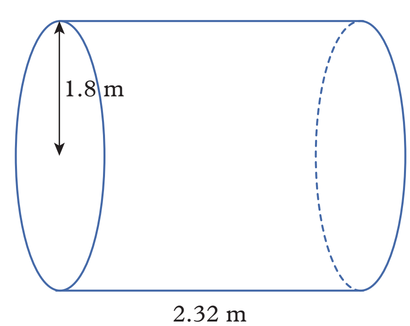

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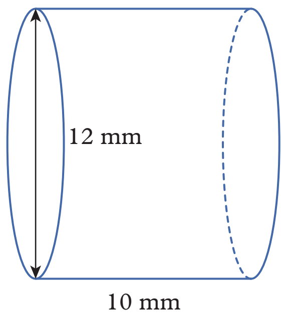
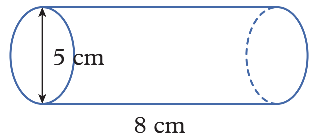
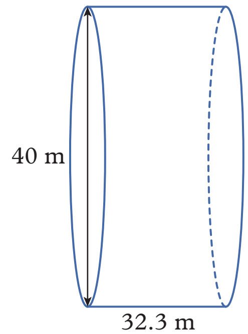


e



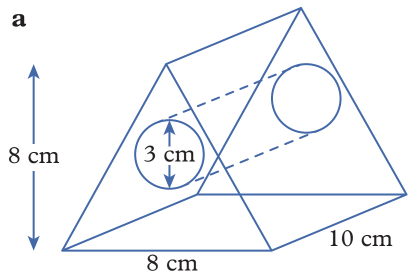
f



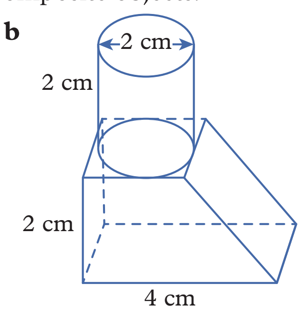
3 Determine the capacity of each cylinder in question **2**.

WE 5.5C 4 Calculate the volume of the following composite objects.

a



b



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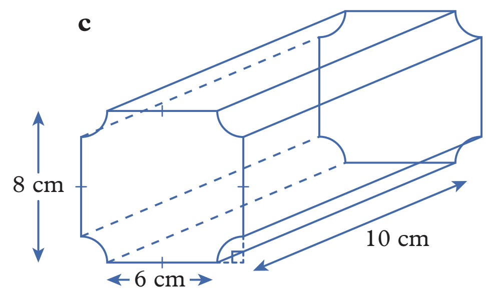


5 A prism has a volume of 240 cm^3.

a If the prism has a height of 8 cm, what is the area of its base?

b If the prism has a base area of 60 cm^2, find its height.

6 A prism has a volume of 360 cm^3.

a If the prism has a height of 60 mm, what is the area of its base in cm^2?

b If the prism has a base area of 90 cm^2, find its height in mm.

Exercise 5.5B: Problem solving and reasoning

7 The height of a cube of side length x is increased by 4 cm. The surface area of the new rectangular prism formed is 80 cm^2 more than the surface area of the cube.

a Write an expression to represent the surface area of the cube.

b Write an expression to represent the volume of the cube.

c What are the dimensions of the rectangular prism?

d What is the surface area of the rectangular prism?

e What is the value of x?

f What is the volume of the rectangular prism?

8 The height of a cube is increased by 6 cm and the surface area of the new rectangular prism formed is 96 cm^2 more than the surface area of the cube. What is the volume of the rectangular prism?

9 The height of a rectangular prism with a square base is increased by 2 cm to form a cube of side length x cm. The surface area of the new cube formed is 48 cm^2 more than the surface area of the rectangular prism.

a Write an expression to represent the surface area of the cube.

b Write an expression to represent the volume of the cube.

c What are the dimensions of the rectangular prism?

d What is the surface area of the rectangular prism?

e What is the value of x?

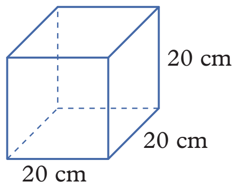
f What is the volume of the rectangular prism?

10 The height of a rectangular prism with a square base is increased by 1 cm to form a cube, and the surface area of the new cube formed is 12 cm^2 more than the surface area of the rectangular prism. What is the volume of the rectangular prism?

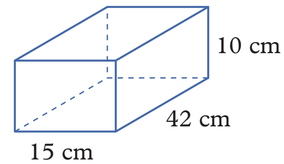
11 The length and width of a rectangular prism are the same, and the height of the prism is double the length. The sum of the length, the width and height is 12 cm. What is the volume of the prism?

12 By calculating the surface area and volume of both of these boxes, demonstrate that boxes with the same surface area do not always have the same volume.

a



b



13 A new kind of insulation is to be fitted in the roof of a house. It is measured in cubic metres rather than square metres because it fills the entire roof (normally insulation is just a thick layer). The roof is in the form of a triangular prism. If the roof is 2 m high, 9 m wide and 14 m long, calculate:

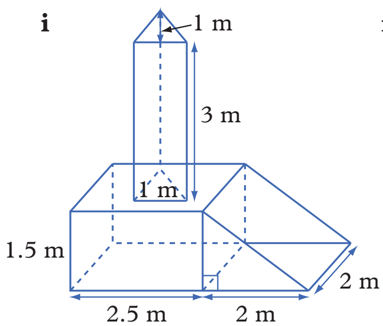
a how much insulation is needed (in cubic metres)

b how much it will cost if the insulation costs $30 per cubic metre.

14 Andrew likes to make geometric sculptures.

a If his materials cost $2 per cubic metre, calculate how much it costs him to produce each sculpture.

i



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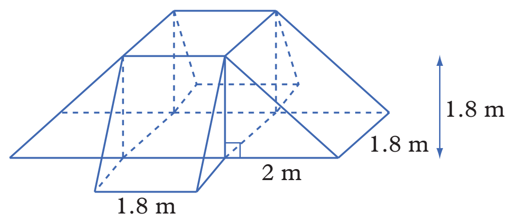

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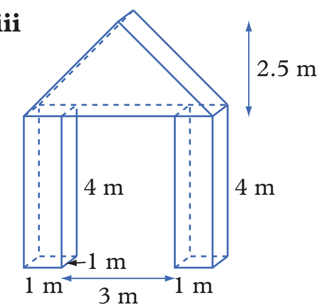


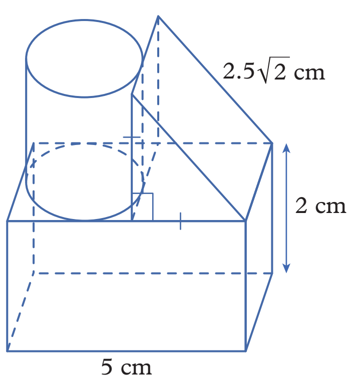
b Mel decides she wants to buy one of Andrew's pieces, and she wants to buy the lightest of the three sculptures. Use the following information to recommend which sculpture Mel should buy.

- sculpture **i** weighs 3 kg per cubic metre
- sculpture **ii** weighs 2 kg per cubic metre
- sculpture **iii** weighs 2.5 kg per cubic metre

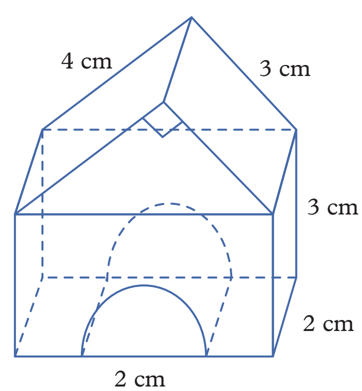
Exercise 5.5C: Challenge

15 Calculate the volume of the following sculptures.

a



b



Online resources:

Interactive skillsheet Volume of prisms	Interactive skillsheet Volume of cylinders	Interactive skillsheet Capacity	Investigation Prisms with the same volume	Quick quiz 5.5

Lesson 5.6

Errors in measurement

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate measurement errors, and find the lower and upper bounds of a measurement.

Inter-year links

Year 8	1.1 Rounding and estimating
Year 10	8.4 Compounding errors

Errors

Key content video
Errors in measurement

- An **error** is the difference between an estimated (or measured) value and the actual value.

  Error = estimated value − actual value
  - → When the estimated value is smaller than the actual value, the error is negative.
  - → When the estimated value is larger than the actual value, the error is positive.
- The **absolute value** of a number is the distance of that number from zero on a number line. Absolute values are never negative and are denoted by placing two vertical bars on either side of a number.
  - → The absolute value of a non-negative number is itself. For example, $|5| = 5$, $|0| = 0$.
  - → The absolute value of a negative number is equal to the number multiplied by -1. For example, $|-5| = 5$.
- An **absolute error** is the absolute value of an error.

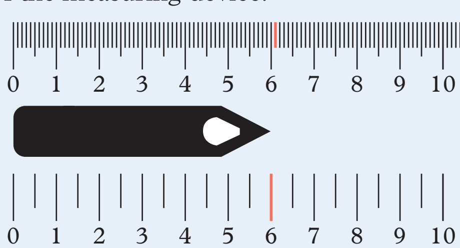
  Absolute error = |estimated value − actual value|
  - → An absolute error is never negative.
- A **relative error** is the ratio between the absolute error and the actual value of a quantity.

  $$\text{Relative error} = \frac{\text{absolute error}}{\text{actual value}}$$
- A **percentage error** is the relative error expressed as a percentage.

  $$\text{Percentage error} = \frac{\text{absolute error}}{\text{actual value}} \times 100$$

Measurements

- All measurements are estimates of actual values. Errors in measurement may arise due to a variety of factors, including human error (for example, incorrectly reading a device), systematic error (for example, if a device is incorrectly calibrated), and the accuracy of the measuring device.
- The accuracy of a measuring device depends on the smallest displayed units. For example, if the pencil on the right is measured using the top scale it would be estimated as 6.1 mm, whereas if it is measured using the bottom scale it would be estimated as 6 cm.



- The **lower bound** of a measurement is the smallest possible actual value of the quantity being measured, and the **upper bound** of a measurement is the highest possible actual value of the quantity being measured.
- When considering the accuracy of a measuring device, the lower and upper bounds can be found using these formulas:

  Lower bound = measurement − 0.5 × smallest unit of measurement

  Upper bound = measurement + 0.5 × smallest unit of measurement.

Worked example 5.6A Calculating absolute, relative and percentage errors

Using electronic scales, Alina measured the weight of a bunch of grapes as 355 g. If the actual weight of the grapes is 353.2 g, find the:

a absolute error

b relative error (correct to four decimal places)

c percentage error (correct to two decimal places).

THINK

a The absolute error is the absolute value of the actual value minus the estimated value. Substitute the values into the formula and evaluate.

b Divide the absolute error by the actual value to find the relative error. Round the answer to four decimal places.

c Convert the relative error into a percentage to find the percentage error. Round the answer to two decimal places.

WRITE

a $$\begin{aligned}\text{Absolute error} &= |\text{estimated value} - \text{actual value}| \\ &= |355 - 353.2| \\ &= 1.8 \text{ g}\end{aligned}$$

b $$\begin{aligned}\text{Relative error} &= \frac{1.8}{353.2} \\ &= 0.005\,09\ldots \\ &\approx 0.0051\end{aligned}$$

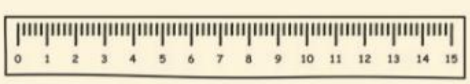
c $$\begin{aligned}\text{Percentage error} &= \frac{1.8}{353.2} \times 100\% \\ &= 0.509\ldots\% \\ &\approx 0.51\%\end{aligned}$$

Worked example 5.6B Finding the upper and lower bounds

Mitch measures the length of a pen as 9 cm. Find the lower and upper bounds for the actual length of the pen if the ruler has:

a 1 mm as the smallest unit of measurement

b 2 mm as the smallest unit of measurement.



THINK

a **1** The measurement of 9 cm is an estimation of the actual length of the pen. The error in the measurement depends on the accuracy of the ruler being used. Identify the smallest unit of measurement of this ruler as 1 mm and convert 1 mm to 0.1 cm.

WRITE

a $$\begin{aligned}\text{Measurement} &= 9 \text{ cm} \\ \text{Smallest unit} &= 1 \text{ mm} \\ &= 0.1 \text{ cm}\end{aligned}$$

2 Calculate the lower and upper bounds by substituting the measurement value and the smallest unit of measurement into the lower and upper bound formulas.	Lower bound = measurement − 0.5 × smallest unit = 9 − 0.5 × 0.1 = 8.95 cm Upper bound = measurement + 0.5 × smallest unit = 9 + 0.5 × 0.1 = 9.05 cm
b **1** Identify the smallest unit of measurement as 2 mm and convert 2 mm to 0.2 cm.	**b** Measurement = 9 cm Smallest unit = 2 mm = 0.2 cm
2 Calculate the lower and upper bounds by substituting the measurement value and the smallest unit of measurement into the lower and upper bound formulas.	Lower bound = measurement − 0.5 × smallest unit = 9 − 0.5 × 0.2 = 8.9 cm Upper bound = measurement + 0.5 × smallest unit = 9 + 0.5 × 0.2 = 9.1 cm

Helpful hints

- ✔ When calculating errors, be careful to correctly identify which value is the actual value and which value is the estimated value.
- ✔ Units of measurement must be converted into the same unit before substituting them into the given formulas.

Learning pathways

△ 1–6, 9–12 □ 2, 3, 6, 7, 9, 10, 12–14, 17 ⬠ 2, 3, 6, 8, 10, 13–17

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Exercise 5.6A: Understanding and fluency

1 State the absolute value of the following numbers.

a 3 **b** $\frac{2}{5}$ **c** −8 **d** −4.2 **e** 0 **f** $-\sqrt{2}$

2 Evaluate each of the following expressions.

a $|5 - 3|$ **b** $|3 - 5|$ **c** $|2.4 - 1|$

d $|-1 - 2.4|$ **e** $|0.24 - 0.172|$ **f** $|0.172 - 0.24|$

3 Many different numbers all round to the same value.

a What is the smallest decimal that could result in an answer of 1.7 when rounded to one decimal place?

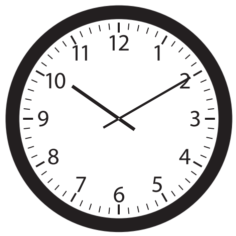
b What is the smallest decimal that could result in an answer of 2.33 when rounded to two decimal places?

4 Providing all these devices are accurately calibrated, which of the following will give the most accurate time?

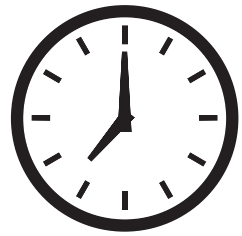
A

B

C



D



5 A cafe estimates 80 croissants will be sold in a day. Find the absolute error if the actual number of croissants sold is:

a 92 **b** 72.

WE 5.6A **6** Using a stopwatch, Rakesh measured the time it took him to make his bed as 3 minutes and 20 seconds. If the actual time taken was 3 minutes and 22 seconds, find the:

a absolute error

b relative error (correct to four decimal places)

c percentage error (correct to two decimal places).

7 A theatre estimates that 180 people will attend the matinee performance. In the end, only 160 people attended the performance. For the size of the actual audience and the estimated audience, determine:

a the absolute error **b** the relative error **c** the percentage error.

8 Art and Petal got married earlier this year. In their planning spreadsheet, they estimated that their wedding would cost a total of \$24 000. When they totalled their expenses, the actual cost was \$26 535. Find the:

a absolute error

b relative error (correct to four decimal places)

c percentage error (correct to two decimal places).

9 When 3.14 is used as an approximate value of π, determine the percentage error when estimating the circumference and the area of a circle with a radius of:

a 2 cm

b 10 cm.

WE 5.6B **10** A kitchen scale shows that a block of cheese weighs 430 g. Calculate the lower and upper bounds for this measurement value if the scale has:

a 1 g as the smallest unit of measurement **b** 5 g as the smallest unit of measurement.

Exercise 5.6B: Problem solving and reasoning

11 The length of an object is measured as 125 cm using a measuring tape that has markings every 5 cm. Between which two values would the actual length of the object lie?

12 The fraction $\frac{22}{7}$ can be used as an approximate value of π.

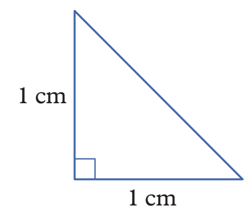
a If $\frac{22}{7}$ is used in place of π, find the percentage error when calculating the circumference of a circle with a radius of:

i 2 cm **ii** 10 cm **iii** r cm.

b Comment on your findings from the results in part **a**.

c Compare your answers to part **a** with your answers to question **9**. Which approximation of π is more accurate?

13 Katja wanted to estimate the value of $\sqrt{2}$, so she constructed a right-angled isosceles triangle with equal side lengths of 1 cm, as shown on the right.



a Explain why the length of the hypotenuse is equal to $\sqrt{2}$ cm.

b Katja measures the length of the hypotenuse as 1.4 cm. Find the:

i absolute error (correct to four decimal places)

ii relative error (correct to four decimal places)

iii percentage error (correct to two decimal places).

14 Jacob measures the side length of a square as 12 cm, correct to the nearest centimetre. Find:

a the lower and upper bounds for the side length of the square

b the lower and upper bounds for the perimeter of the square

c the lower and upper bounds for the area of the square.

15 The side length of a cube is measured to the nearest centimetre. The measurement is then used to calculate the volume of the cube as 64 cm^3.

a What is the largest possible volume of the cube, correct to two decimal places?

b What is the smallest possible volume of the cube, correct to two decimal places?

16 Arya wrapped a piece of string around the cylindrical pillar in front of her local art gallery to find the circumference of the circular cross-section of the pillar. The length of the string is then measured with a ruler that has 1 cm as the smallest division. The measurement is recorded as 113 cm.

a What are the lower and upper bounds of the measurement?

b What are the lower and upper bounds of the radius of the circular cross-section of the pillar, correct to the nearest centimetre?

c What are the lower and upper bounds of the area of the circular cross-section of the pillar, correct to the nearest square centimetre?

Exercise 5.6C: Challenge

17 Violet buys contact lenses to correct her eyesight. The contact lenses purchased have strengths of −2.5 and −2.4 for the left and right eye respectively, and they work well for her.

a Contact lenses are available with strength increments of 0.1. Find the lower and upper bounds for her left and right eye strengths.

b The optometrist measured her eyesight to be of strength −2.53 for the left eye and −2.37 for the right eye. Calculate the relative error for each contact lens.

Online resources:

Interactive skillsheet
Errors

Interactive skillsheet
Upper and lower bounds

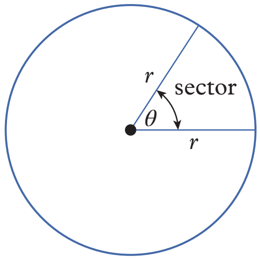
Quick quiz
5.6

Lesson 5.7

Review: Measurement

Module summary

Area of sectors

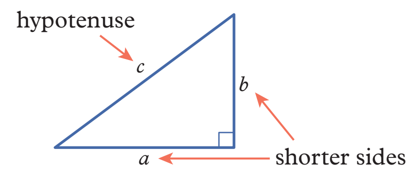


$$A = \frac{\theta}{360} \times \pi r^2$$

Pythagoras' theorem

$$c^2 = a^2 + b^2$$

hypotenuse   shorter sides



Any set of three whole numbers that satisfy Pythagoras' theorem is called a Pythagorean triple. For example, 3, 4, 5 is a Pythagorean triple as $3^2 + 4^2 = 5^2$.

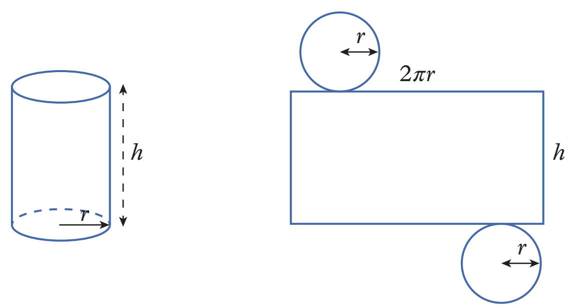
Area of composite shapes

1. Split the figure into simple shapes that have known area formulas.
2. Calculate any missing dimensions.
3. Calculate the areas of the individual shapes using the area formulas.
4. Add or subtract the areas to calculate the total area.

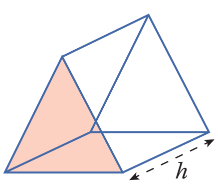
Surface area

1. Determine the shape of each face.
2. Calculate the area of each face.
3. Add the areas of the faces together.

$$\text{Cylinder SA} = 2\pi rh + 2r^2$$



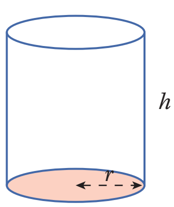
Volume of prisms



$$V = Ah$$

where A is the area of the base and h is the height

Volume of cylinders



$$V = \pi r^2 h$$

Capacity

The capacity of a 3D object is a measure of how much the object can hold. A container with an inside volume of:

- 1 cm^3 holds 1 mL of liquid
- 1000 cm^3 holds 1 L of liquid
- 1 m^3 holds 1 kL of liquid.

Errors

- error = estimated value − actual value
- absolute error = $|\text{estimated value} - \text{actual value}|$
- lower bound = measurement − 0.5 × smallest unit of measurement
- upper bound = measurement + 0.5 × smallest unit of measurement
- relative error = $\frac{\text{absolute error}}{\text{actual value}}$
- percentage error = $\frac{\text{absolute error}}{\text{actual value}} \times 100$

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Review questions 5.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- absolute error
- absolute value
- actual value
- arc
- area
- capacity
- circle
- circumference
- composite object
- composite shape
- cross-section
- cube
- cylinder
- edge
- error
- face
- kite
- lower bound
- net
- parallelogram
- percentage error
- prism
- Pythagoras' theorem
- Pythagorean triple
- rectangle
- rectangular prism
- relative error
- rhombus
- right prism
- sector
- square
- surface area
- trapezium
- triangle
- triangle inequality
- triangular prism
- upper bound
- volume

1 What is the collective name for three whole numbers that satisfy Pythagoras' theorem?

2 Explain the triangle inequality.

3 How can you calculate the percentage error when given the relative error?

4 Identify the key terms being referenced in each of these definitions.

a part of the circumference of a circle

b a 3D object with two circular bases and a constant circular cross-section parallel to the bases

5 Using an example, provide a definition in your own words for the following key terms.

a sector

b prism

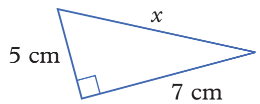
6 Complete the following sentences using words from the key terms list.

a A ________ of a 3D object is a 2D shape obtained when cutting through the object.

b A ________ has six faces that are all identical squares.

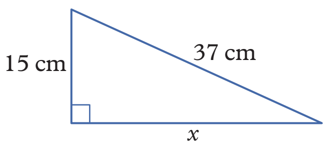
Review questions 5.7B: Multiple choice

5.1 **1** Which equation can be used to find the length of the hypotenuse in this triangle?



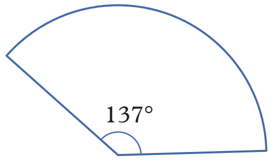
A $x = 7 + 5$ **B** $x^2 = 5^2 + 7^2$ **C** $7^2 = x^2 + 5^2$ **D** $5^2 = x^2 + 7^2$

5.1 **2** The unknown side length, x, in this triangle, correct to the nearest cm, is:



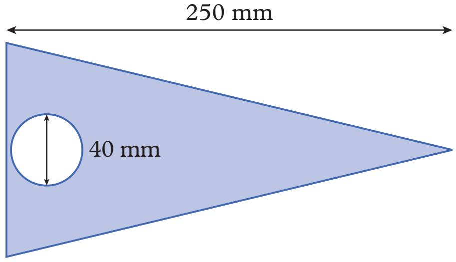
A 22 cm **B** 33 cm **C** 34 cm **D** 40 cm

5.2 **3** The area of this sector is closest to:



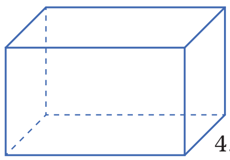
A 745 cm^2 **B** 432 cm^2 **C** 284 cm^2 **D** 108 cm^2

5.2 **4** The shaded area is closest to:



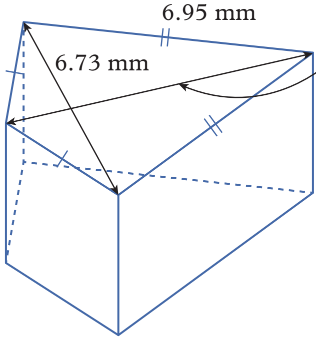
A 99.7 cm^2 **B** 137.43 cm^2 **C** 143.72 cm^2 **D** 162.57 cm^2

5.3 **5** The surface area of this rectangular prism is:



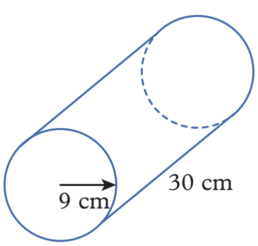
A 6708 cm^2 **B** 1080 cm^2 **C** 768 cm^2 **D** 678 cm^2

5.3 **6** The surface area of this prism is:



A 150.7444 mm^2 **B** 143.0216 mm^2 **C** 110.655 12 mm^2 **D** 221.217 792 mm^2

5.4 **7** The surface area of the following cylinder is closest to:

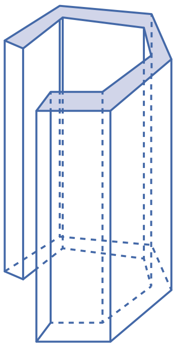


A 270 cm^2 **B** 7634.1 cm^2 **C** 2205.4 cm^2 **D** 7351.3 cm^2

5.4 **8** The surface area of a cylinder increased by 12π cm^2 after the height of the cylinder was increased by 3 cm^2. What is the radius of the base?

A 4π cm **B** 2π cm **C** 4 cm **D** 2 cm

5.5 **9** The volume of this prism is:



A 154.062 cm^3 **B** 80.2644 cm^3 **C** 114.4464 cm^3 **D** 34.182 cm^3

5.5 **10** Which of the following options is closest to the volume of the cylinder in question **7**?

A 7700 cm^3 **B** 7600 cm^3 **C** 700 cm^3 **D** 800 cm^3

5.6 **11** The lower and upper bounds of a measurement are 14.95 km and 15.05 km. What is the actual measurement?

A 14.9 km **B** 15.0 km **C** 15.00 km **D** 15 km

5.6 **12** Peter's height is measured as 163 cm. If his actual height is 164.4 cm, which of the following is closest to the percentage error of the measurement?

A 0.86% **B** 0.85% **C** 8.5% **D** 8.6%

Review questions 5.7C: Short answer

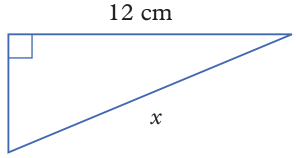
5.1 **1** A triangle has side lengths 30 mm, 40 mm and 50 mm.

a Show that the triangle is right-angled.

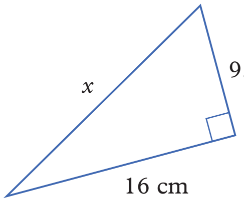
b Are the side lengths of the triangle an example of a Pythagorean triple? Briefly explain why or why not.

5.1 **2** Calculate the length of the hypotenuse in each triangle. Round your answers to two decimal places if necessary.

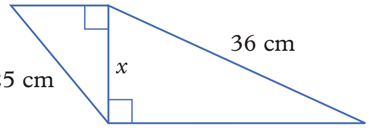
a



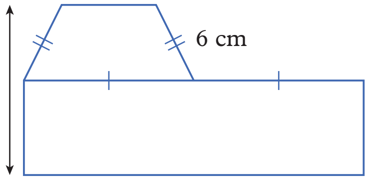
b



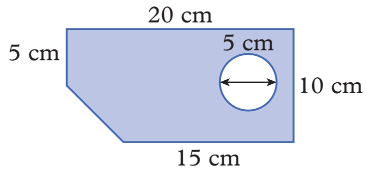
5.1 **3** Find the length of the sides represented by x and y in this diagram, correct to two decimal places.



5.2 **4** Calculate the area of this composite shape in square centimetres.

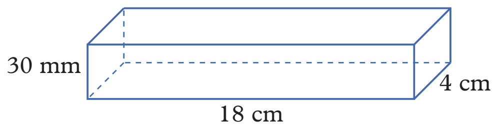


5.2 **5** Calculate the shaded area, correct to two decimal places.

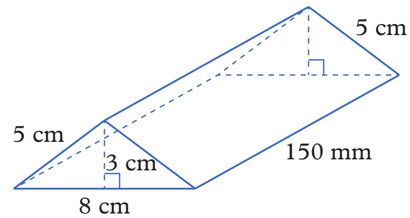


5.3 **6** Calculate the surface area of each prism.

a

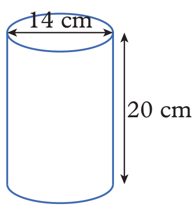


b



5.4 **7** Calculate the surface area of each cylinder, correct to two decimal places.

a



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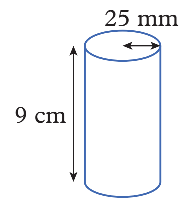


5.5 **8** Calculate the volume of each prism in question **6**.

5.5 **9** Calculate the volume of each cylinder in question **7**. Give your answers correct to two decimal places.

5.5 **10** The volume of a cylindrical water tank with radius 10 m is 4000 m^3.

a What is the capacity of the water tank?

b Calculate the height of the tank to the nearest metre.

5.6 **11** Calculate the upper and lower bounds for the following measurements.

a 55.0 cm using a ruler with 1 mm as the smallest unit of measurement

b 32.5 cm using a tape measure with 5 mm as the smallest unit of measurement

c 30 g using a scale with 1 g as the smallest unit of measurement

d 108 mL using a measuring cylinder with 1 mL as the smallest unit of measurement

Review questions 5.7D: Mathematical modelling

1 George is interested in finding out the thickness of a piece of A4 paper. As one sheet of paper is too thin to be measured alone, he decides to measure a stack of papers using a ruler and calculate the thickness for each piece from that. George records his measurement in a table below.

	Stack A	Stack B	Stack C	Stack D
Number of sheets	20	50	200	500
Thickness (mm)	2	4	17	42

a Stack C has 10 times as many papers as stack A, and stack D has 10 times as many papers as Stack B. Explain why the thicknesses recorded for stacks C and D are not exactly 10 times the thicknesses of stacks A and B.

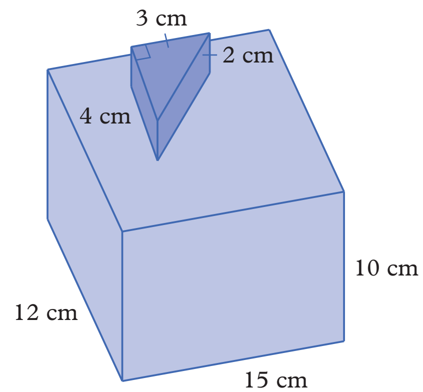
b Which measurement is likely to give the most accurate data for calculating the thickness of a piece of paper? Calculate the thickness of a piece of paper from this measurement.

c The actual thickness of this printing paper is 0.085 mm per sheet.

i Find the absolute error for each of the four measurements. Which measurement is most accurate?

ii Find the percentage error for each of the four measurements. Does smaller absolute error lead to smaller percentage error? Explain your reasoning in this context.

2 Angelique is making a sculpture. The rectangular prism will be painted purple and the triangular prism will be painted pink. The triangular prism has a height of 2 cm and the cross-section is a right-angled triangle with a base length of 3 cm and a perpendicular height of 4 cm.



a What is the length of the longest side of the triangle?

b Calculate the area to be painted in each colour (cm^2), assuming all sides, including the bottom, will be painted before assembly.

c Angelique has a tin of purple paint that will cover 1 m^2.
After painting two purple coats, what area would the remaining purple paint cover?

d Pink paint comes in smaller tins that contain enough paint to cover 90 cm^2. What is the maximum number of coats possible from one tin?

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Use Pythagoras' theorem to determine whether a triangle is right-angled ☐ Use Pythagoras' theorem to determine unknown side lengths	☐ Go back to Lesson 5.1 Pythagoras' theorem
☐ Calculate the area of composite shapes using addition or subtraction	☐ Go back to Lesson 5.2 Area of composite shapes
☐ Draw nets for right prisms ☐ Calculate the surface area of right prisms	☐ Go back to Lesson 5.3 Surface area of prisms
☐ Draw nets for cylinders ☐ Calculate the surface area of cylinders	☐ Go back to Lesson 5.4 Surface area of cylinders
☐ Calculate the volume and capacity of prisms and cylinders ☐ Calculate the volume of composite objects	☐ Go back to Lesson 5.5 Volume of prisms and cylinders
☐ Calculate absolute, relative and percentage errors ☐ Calculate the upper and lower bounds for measurements	☐ Go back to Lesson 5.6 Errors in measurement

6

Scale and trigonometry

Lessons

6.1 Ratios and rates
6.2 Similar figures
6.3 Area and volume scale factors
6.4 Trigonometric ratios
6.5 Using trigonometry to find side lengths
6.6 Using trigonometry to find angles

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Volume of prisms
- ✔ Substitution
- ✔ Solving equations using inverse operations
- ✔ Equivalent fractions

Curriculum links

- Solve spatial problems, applying angle properties, scale, similarity, ratio, Pythagoras' theorem and trigonometry in right-angled triangles (VC2M9M03)
- Use mathematical modelling to solve practical problems involving direct proportion, rates, ratio and scale, including financial contexts; formulate the problems and interpret solutions in terms of the situation; evaluate the model and report methods and findings (VC2M9M05)
- Recognise the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles using properties of similarity (VC2M9SP01)
- Apply the enlargement transformation to shapes and objects using dynamic geometry software as appropriate; identify and explain, using language of similarity, ratio and scale, aspects that remain the same and those that change (VC2M9SP02)

Lesson 6.1

Ratios and rates

Learning intentions

By the end of this lesson you will be able to ...

✔ solve problems involving ratios and rates.

Inter-year links

Year 7 3.8 Ratios

Year 8 3.6 Modelling with ratios

Year 8 3.7 Rates

Ratios

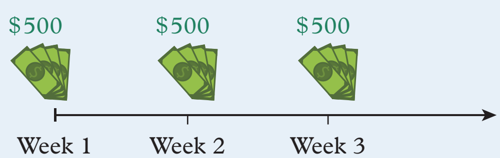
Key content video
Understanding ratios

- A **ratio** expresses a relationship between two or more different quantities with the same units. Ratios are written with a colon (:) between the quantities.
  For example, if there are 6 red balls and 2 pink balls in a bag, this could be expressed using the ratio 6 : 2.
- Ratios can be simplified by dividing each part of a ratio by a common factor. A ratio is in simplest form when the only common factor between the parts is 1.
  For example, the ratio 6 : 2 can be simplified to 3 : 1.

Rates

Key content video
Rates

- A **rate** is a comparison between two or more different quantities. It is a measure of how much one quantity changes for each unit of another quantity.
- A rate has a number and a unit, which indicates the two quantities being compared. The units of the two quantities are separated by the word 'per' (for each) or the symbol /.
  For example, \$500 per week or \$500/week represents a rate of \$500 each week.



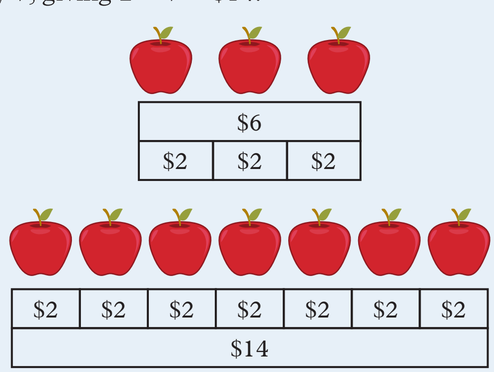
→ Order is important when writing a rate.

- For a rate to be in simplest form, the second of the two quantities being compared must have a value of 1.

The unitary method

Key content video
The unitary method

- The **unitary method** is a method where the value of a single unit of measure, the unitary rate, is determined. The unitary method can be used to determine the best buy for an item.
  For example, if 3 apples cost \$6, the cost of 1 apple (unitary rate) is \$2. To find the cost of 7 apples, multiply the unitary rate by 7, giving $2 \times 7 = \$14$.



Worked example 6.1A Solving problems involving ratios

In a packet of dried fruits, there are 5 raisins for every 3 sultanas. If there are 395 raisins in the packets:

a how many sultanas are in the packet

b how many raisins and sultanas are in the packet?

THINK	WRITE
a **1** Determine the number of pieces of fruit in one part of the ratio by dividing the given number of raisins (395) by the number of parts in this section (5).	**a** $395 \div 5 = 79$
2 Multiply the value from step 1 by the number of parts the sultanas represent.	$79 \times 3 = 237$ There are 237 sultanas in the packet.
b Add the number of sultanas to the number of raisins.	**b** $395 + 237 = 632$ There are 632 raisins and sultanas in the packet.

Worked example 6.1B Writing a rate in simplest form

Write this statement as a rate in simplest form: \$196.65 for 9 hours work.

THINK	WRITE
1 Write the two quantities as a rate. **2** For the rate to be in simplest form, the second quantity needs to be 1. Divide both quantities by 9.	Rate = \$196.65 per 9 hours $= \frac{\$196.65}{9}$ per $\frac{9 \text{ hours}}{9}$ = \$21.85 per 1 hour
3 Write your answer as a rate using the '/' to indicate 'per'.	The rate is \$21.85/hour.

Worked example 6.1C Solving problems using the unitary method

Determine which of these represents the best buy using the unitary method.

\$38.01 for 7 kg of oranges or \$28.50 for 4.5 kg of oranges

THINK

1. For each purchasing option, write the two quantities as a rate.
2. Divide each price by the number of kilograms to determine the cost of 1 kg. Where necessary, round each amount to the nearest cent.
3. Compare the prices for 1 kg to determine which option is better value.

WRITE

Option 1: \$38.01 for 7 kg $= \frac{\$38.01}{7}$ for $\frac{7 \text{ kg}}{7}$

$= \$5.43$ for 1 kg

Option 2: \$28.50 for 4.5 kg $= \frac{\$28.50}{4.5}$ for $\frac{4.5 \text{ kg}}{4.5}$

$\approx \$6.33$ for 1 kg

\$38.01 for 7 kg of oranges is better value than \$28.50 for 4.5 kg of oranges.

Helpful hints

✔ Ratios represent the relationship between quantities of the same unit, so therefore don't contain units. Rates represent the relationship between quantities of different units, so therefore contain units.

Learning pathways

△ 1, 2, 4–5(a–d), 7–9(a–d), 10, 11, 13, 15–17, 20

□ 2, 3, 4(e–h), 5(b–e), 6, 7–9(c–f), 12–14, 16–19

⬠ 3, 5(c–f), 6, 7–9(e–h), 12–14, 18–22

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Exercise 6.1A: Understanding and fluency

1 The ratio of germinated seeds to non-germinated seeds in Rita's garden is 9 : 4. There are 4500 germinated seeds.
- **a** How many germinated seeds are in each part of the ratio?
- **b** Use your answer to part **a** to determine how many non-germinated seeds are in Rita's garden.
- **c** How many seeds are in Rita's garden in total?

WE 6.1A 2 In a bag of mixed nuts, there are 4 cashews for every 7 hazelnuts.
If there are 119 hazelnuts in the bag:
- **a** how many cashews are in the bag
- **b** how many hazelnuts and cashews are in the bag?

3 Morning Sunshine fruit juice drink contains fruit juice and water in the ratio 1 : 3.
- **a** How much water (in mL) is in a 2 L container of Morning Sunshine? Hint: 1 L = 1000 mL.
- **b** How much fruit juice is in 600 mL of Morning Sunshine?

4 Write each statement as a rate with the appropriate unit.
- **a** $30 earned for each hour
- **b** $1.35 for 1 L of petrol
- **c** Hire cost of $55 for every hour
- **d** Cost of $2.45 for every jar
- **e** Call cost of 75 cents for every minute
- **f** Cost of $12.99 for every kilogram
- **g** Salary of $60 000 for every year
- **h** Charge of $6.85 for each parcel mailed

WE 6.1B 5 Use a rate in simplest form to model the following situations.
- **a** Mark drives 225 km in 3 hours.
- **b** Steph can run 10 km in 50 minutes.
- **c** Pablo can type 660 words in 12 minutes.
- **d** A box of 6 donuts costs $15.
- **e** It costs $78.12 to fill a 42-litre petrol engine.
- **f** The energy bill came to $160.50 for 30 days.

6 Write each statement as a rate in simplest form.
Where necessary, round each amount to the nearest cent.
- **a** $38.45 for 7 kg of oranges
- **b** $156.00 for 6.5 hours work
- **c** $22 collected in 60 minutes
- **d** 5.5 m length of timber costs $45.50

WE 6.1C 7 From each pair of options, determine which option represents the best buy using the unitary method.

	Option A	Option B
a	$28.40 for 4 kg	$34 for 5 kg
b	$411.55 for 5 L	$812.30 for 10 L
c	$31.92 for 3.5 m	$19.10 for 2.5 m
d	$312.50 for 25 m^2	$273.60 for 18 m^2

	Option A	Option B
e	\$8.22 for 1.2 kg	\$5.76 for 900 g
f	\$9.99 for 1.25 L	\$4.95 for 600 mL
g	\$11.73 for 1235 cm	\$8.65 for 9.4 m
h	\$78 for 8000 cm^2	\$765.60 for 8 m^2

8 The cost of a plumber's work is \$200 for 8 hours. Knowing this, we can determine the cost of the plumber for 1 hour of work by dividing the cost by 8. Then, we can multiply this amount to determine the cost of the plumber for any number of hours.

$$\begin{aligned} &\$200 \text{ per } 8 \text{ hours}\\ &= \frac{\$200}{8} \text{ per } \frac{8 \text{ hours}}{8}\\ &= \$25 \text{ per } 1 \text{ hour}\\ &= \$25 \times 5 \text{ per } 1 \times 5 \text{ hours}\\ &= \$125 \text{ per } 5 \text{ hours}\end{aligned}$$

- **a** It costs \$30 for 6 m. How much will it cost for 9 m?
- **b** It costs \$12 for 2 kg. How much will it cost for 93 kg?
- **c** It costs \$20 for 4 L. How much will it cost for 50 L?
- **d** It costs \$99 for 36 pieces. How much will it cost for 82 pieces?
- **e** It costs \$53 for 4 kg. How much will it cost for 85 kg?
- **f** It costs \$22 for 100 g. How much will it cost for 71 g?
- **g** It costs \$91 for 26 m^2. How much will it cost for 62 m^2?
- **h** It costs \$73 for 25 m^3. How much will it cost for 10 m^3?

9 Sometimes it is important to stick to a budget and determine what you can afford with a fixed amount. Using the plumber from the example in question **8**, you can determine the amount of time you can afford with \$120 by finding the time for \$1.

$$\begin{aligned} &\$200 \text{ per } 8 \text{ hours}\\ &= \frac{\$200}{200} \text{ per } \frac{8 \text{ hours}}{200}\\ &= \$1 \text{ per } 0.04 \text{ hours}\\ &= \$1 \times 120 \text{ per } 0.04 \times 120 \text{ hours}\\ &= \$120 \text{ per } 4.8 \text{ hours}\end{aligned}$$

- **a** It costs \$82 for 41 m. How much can be purchased for \$92?
- **b** It costs \$4 for 26 pieces. How much can be purchased for \$36?
- **c** It costs \$84 for 14 L. How much can be purchased for \$75?
- **d** It costs \$12 for 3 kg. How much can be purchased for \$3?
- **e** It costs \$5 for 3 kg. How much can be purchased for \$51?
- **f** It costs \$8 for 15 g. How much can be purchased for \$70?
- **g** It costs \$16 for 29 m^2. How much can be purchased for \$88?
- **h** It costs \$80 for 81 m^3. How much can be purchased for \$44?

10 An exchange rate between two currencies allows you to convert an amount of money in one currency to an amount of money in the other currency. On a given day, the exchange rate between Australian dollars (AUD) and United States dollars (USD) is 0.68, which means that each Australian dollar is worth 0.68 US dollars.

- **a** If you are given an amount in AUD, what calculation would you need to perform to convert it to USD?
- **b** If you are given an amount in USD, what calculation would you need to perform to convert it to AUD?
- **c** Convert the following amounts between AUD and USD.
  - **i** 3500 AUD to USD
  - **ii** 11 100 AUD to USD
  - **iii** 1224 USD to AUD
  - **iv** 5508 USD to AUD

Exercise 6.1B: Problem solving and reasoning

11 In 2022 there were 76 187 registered births in Victoria and the population of Victoria was approximately 6 593 000.

- **a** For Victoria, what was the birth rate per 1000 people in 2022? Round the rate to three decimal places.
- **b** If the birth rate for Australia in 2022 was 12.085 births per 1000 people, was the birth rate for Victoria higher or lower than the national average?

12 The resting heart rate of a professional athlete is 42 BPM (beats per minute).
- **a** How many heart beats would this athlete experience in 8 minutes of rest?
- **b** During exercise, the heart will beat faster to help pump blood around the body.
  If the same athlete's heart beats 774 times over a six-minute period of exercise, what was their average heart rate during that time?

13 A physiotherapist sees 54 patients every week.
- **a** If the physiotherapist works an average of 36 hours each week, how many patients does the physiotherapist see per hour?
- **b** On average, how many minutes does the physiotherapist spend with each patient?

14 The density of gold is 19.3 g/cm^3.
- **a** What is the weight of a piece of gold that is 1.2 cm^3?
- **b** What is the volume of a piece of gold that weighs 5.404 g?

15 A garden hose flows water at a rate of 12.2 litres/minute.
- **a** How much water will flow from the hose each hour?
- **b** How long will it take to fill a 9 L bucket? Give your answer to the nearest second.

16 The side lengths of a rectangle are in the ratio 4 : 7. If the perimeter of the rectangle is 115.5 cm, what are the side lengths?

17 The side lengths of a right-angled triangle are in the ratio 5 : 12 : 13. If the perimeter of the triangle is 270 cm^2, what is the area of the triangle? Hint: Draw a sketch of the triangle.

18 The price of petrol is given in cents per litre (c/L).
- **a** The price of petrol today is 175.2 c/L. How much will it cost for 20 L?
- **b** The price of petrol today is 199.9 c/L. How much will it cost for 15 L?
- **c** The price of petrol today is 162.8 c/L. How much will it cost for 1250 mL? Give your answer correct to the nearest cent.
- **d** I paid $51.55 by card for 25 L of petrol. What is the price of petrol today?
- **e** The price of petrol today is 183.5 c/L. What is the maximum number of litres I can buy with $50 in cash, correct to the nearest millilitre?

19 Jasmine lives 1200 m from school. She walks to school each day at a speed of 1 m/s. Rajesh lives 4.5 km from school. He rides in a car to school each day at a speed of 15 m/s.
- **a** If both Jasmine and Rajesh arrive at school at 8.30 am, what time did each student leave home?
- **b** Jasmine and Rajesh live 4.8 km apart. They plan to meet each other between their homes one morning and both leave their homes at 8 am. At what time will they meet?

20 Karl takes a photo on his phone with a resolution of 1200 × 675 pixels. He uploads the photo to a social media platform which converts the image to a size of 800 × 475 pixels.
- **a** Determine the aspect ratio (height : width) of both the original image and the resized image.
- **b** Was the ratio between the side lengths maintained in the image conversion?

Exercise 6.1C: Challenge

21 A plant grows 3.125% of its original height of 10 cm per day. After 45 days, the plant slows its growth to 1.5% of its original height per day. How tall will the plant be after 100 days?

22 In a bag of coloured marbles, there are six more blue marbles than red marbles and twice as many green marbles than red marbles.
If there are 54 blue, red and green marbles, what is the ratio of blue : red : green marbles in simplest form?

Online resources:

Interactive skillsheet Simplifying ratios	Interactive skillsheet Rates	Investigation Birth rates	Quick quiz 6.1

Lesson 6.2

Similar figures

Learning intentions

By the end of this lesson you will be able to ...

✔ understand and use the properties of similar figures.

Inter-year links

Year 7	3.2 Equivalent fractions
Year 8	9.4 Dilations and similar figures

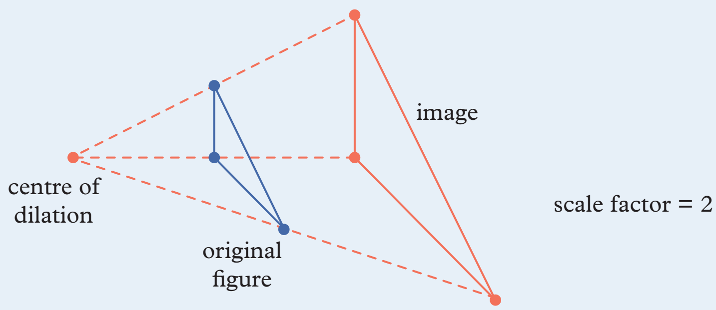
Similar figures

Key content video
Dilations and similar figures

- Two figures are **similar** if they have the same shape. Similar figures do not have to be the same size.
  - → The symbol ~ can be used to indicate similarity.
- If two figures are similar, the corresponding angles in the figures will be equal in size.
- **Dilations**, which transform the size of a shape, produce similar figures.
- The **scale factor** between similar figures is the ratio of corresponding side lengths between these figures.

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

  - → A scale factor between 0 and 1 indicates that the image is a **reduction** of (smaller than) the original shape.
  - → A scale factor greater than 1 indicates that the image is an **enlargement** of (larger than) the original shape.

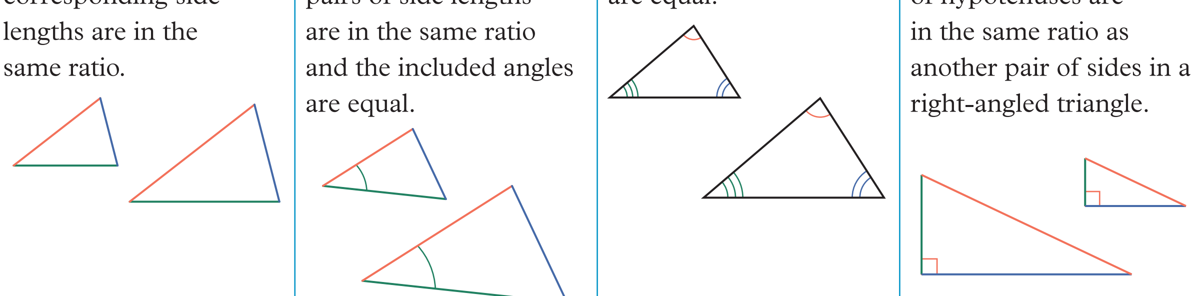


Similar triangles

Key content video
Similar triangles

- If a pair of triangles pass any of the following tests they are similar.

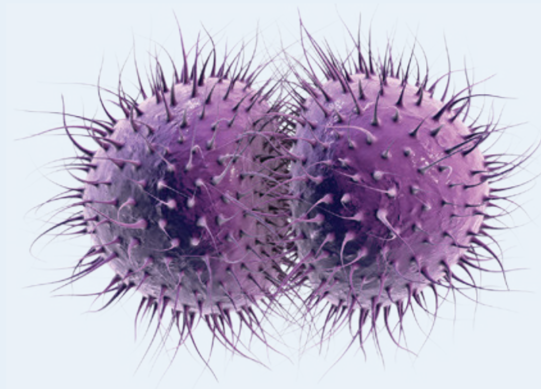
SSS	SAS	AAA	RHS
All three pairs of corresponding side lengths are in the same ratio.	Two corresponding pairs of side lengths are in the same ratio and the included angles are equal.	All three pairs of angles are equal.	The lengths of the pair of hypotenuses are in the same ratio as another pair of sides in a right-angled triangle.



Scale drawings

Key content video
Scale drawings

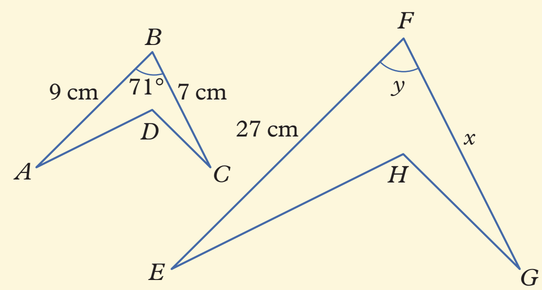
- A **scale drawing** is a drawing that represents a real-life object. Scale drawings can be used to accurately represent objects that are difficult to draw at their actual size, for example, house plans.
- In a scale drawing, the **scale** represents the ratio ‘drawing length : actual length’. For example, if the scale was 1 : 100 each cm in the drawing would represent 100 cm in the real-life object.
- Scale drawings can also be used to represent objects that are too small to see without using a microscope, for example, bacteria and viruses.
  → If the scale drawing was 10 000 : 1, each cm in the drawing would represent 0.0001 cm in real life.



Scale 10 000 : 1

Worked example 6.2A Finding unknown side lengths and angles in similar figures

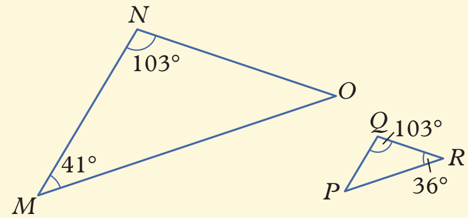
Find the values of x and y, given that this pair of shapes are similar.



THINK	WRITE
1 Identify two corresponding sides and calculate the scale factor. As A corresponds to E and B corresponds to F, then side AB corresponds to EF.	Sides AB and EF are corresponding. $\text{scale factor} = \dfrac{\text{image length}}{\text{original length}} = \dfrac{27}{9} = 3$
2 Identify a pair of corresponding sides that includes the unknown side length, x.	Sides BC and FG are corresponding.
3 Substitute in the known values into the scale factor formula and solve for x.	$\text{scale factor} = \dfrac{\text{image length}}{\text{original length}}$ $3 = \dfrac{x}{7}$ $x = 3 \times 7$ $x = 21$ cm
4 Identify a pair of corresponding angles that includes the unknown angle y.	Angles $\angle ABC$ and $\angle EFG$ are corresponding.
5 Corresponding angles in similar figures are equal in size.	$y = 71°$

Worked example 6.2B Determining if two triangles are similar

Determine if this pair of triangles are similar. State the similarity test used or explain why the triangles are not similar.



THINK

1 We are given two angles in each triangle, so we can use the angle sum of a triangle to calculate the third angle, and then use the AAA test to determine if the two triangles are similar.

2 Calculate the size of the third angle in $\triangle MNO$.

3 Calculate the size of the third angle in $\triangle PQR$.

4 Compare the size of the angles to determine whether the AAA test passes.

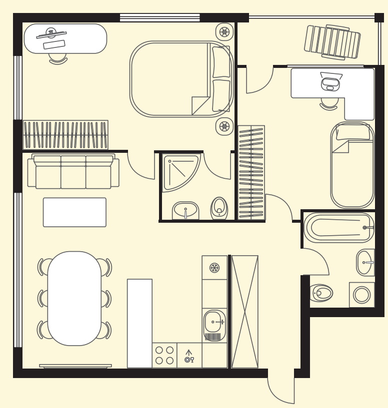
WRITE

2 $\angle NOM = 180° - 103° - 41°$
$= 36°$

3 $\angle QPR = 180° - 103° - 36°$
$= 41°$

4 The two triangles are similar as they pass the AAA similarity test.

Worked example 6.2C Interpreting scale drawings



The plan for the two-bedroom apartment on the right is drawn to a scale of 1 : 60. If, in the scale drawing, one of the bedrooms measures 8 cm by 5 cm, what is the size of the bedroom in real life? Give your answer in metres.

THINK

1 The scale is 1 : 60, so each cm in the scale drawing represents 60 cm in real life.

2 Multiply the scale drawing dimensions by 60 to determine the real-life dimensions.

3 Convert the units to metres by dividing by 100.

4 Write the answer.

WRITE

2 $8 \times 60 = 480$ cm
$5 \times 60 = 300$ cm

3 $\frac{480}{100} = 4.8$ m
$\frac{300}{100} = 3$ m

4 The real-life size of the bedroom is 4.8 m by 3 m.

Helpful hints

- ✔ If you are unsure which similarity test to use, start by writing down all the information that you know or that can be calculated.
- ✔ If two figures are similar, all corresponding side lengths on the two figures will be in the same ratio.

Learning pathways

△ 1–6, 8–10 ◻ 2–6, 8, 10–12 ⬠ 4, 5, 7, 8, 10–14

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Exercise 6.2A: Understanding and fluency

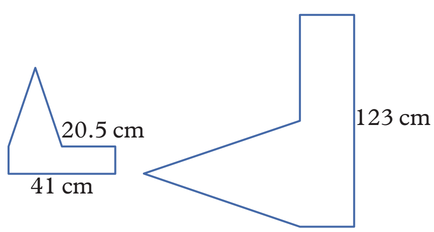
1 Decide whether each scale factor produces an enlargement or a reduction.

a 3 **b** $\frac{2}{5}$ **c** $\frac{1}{4}$

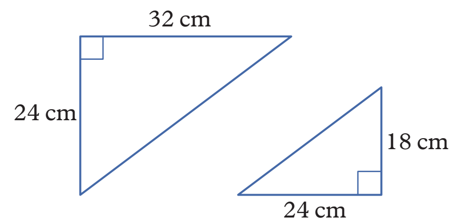
d $\frac{3}{2}$ **e** 10 **f** $\frac{1}{8}$

2 Find the scale factor for each pair of similar figures, given that the figure on the left is the original figure in each instance.

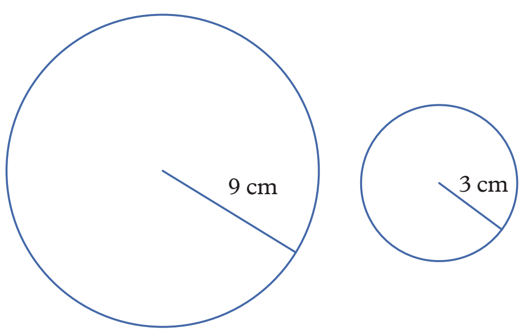
a



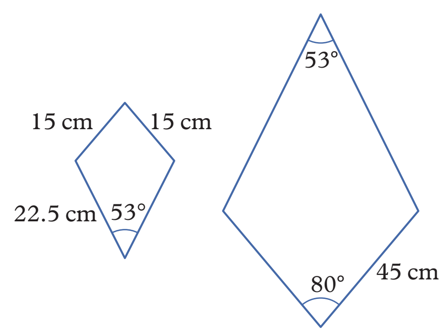
b



c

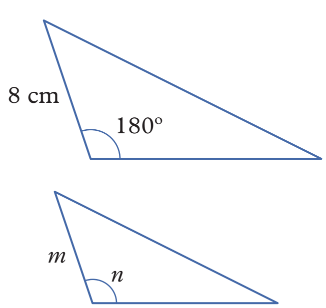


d



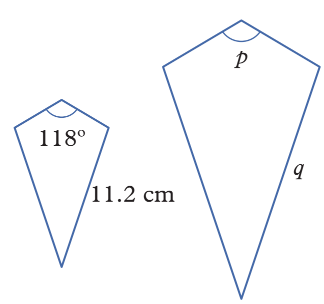
3 When a shape is dilated, the side lengths will be multiplied by the scale factor and the angles will stay the same. The shapes with labelled measurements have been dilated by the given scale factors to create the other shapes. Determine the value of the unknown pronumerals.

a



Scale factor = 0.75

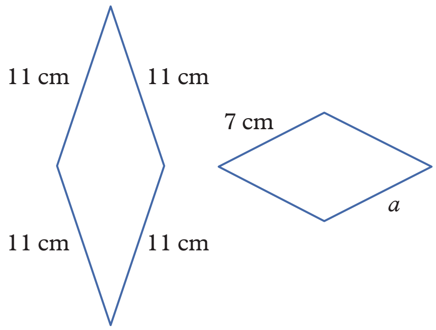
b



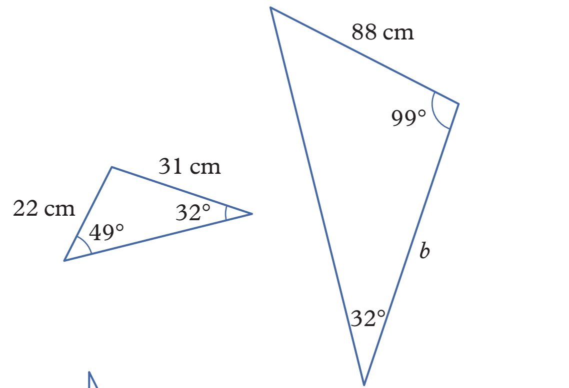
Scale factor = 2.5

WE 6.2A **4** Find the value of the pronumerals, given that each pair of shapes is similar.

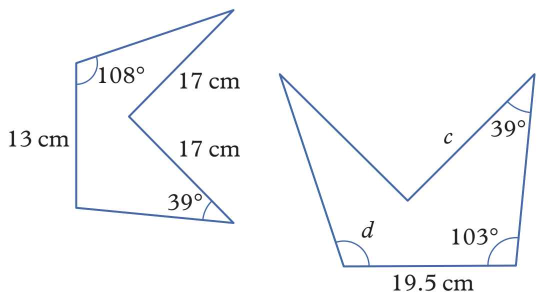
a



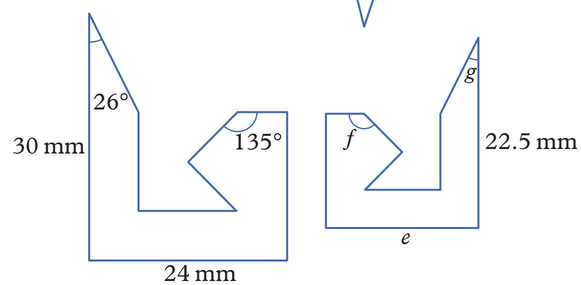
b



c

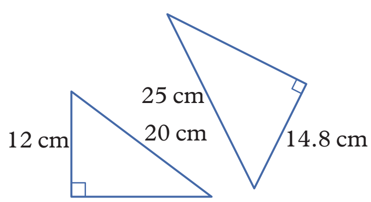


d

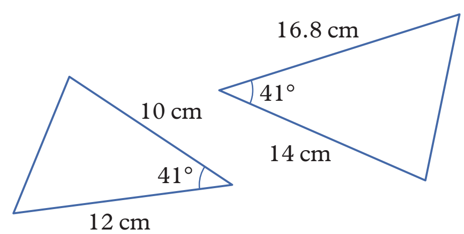


WE 6.2B **5** Determine if these pairs of triangles are similar, stating the similarity test used.

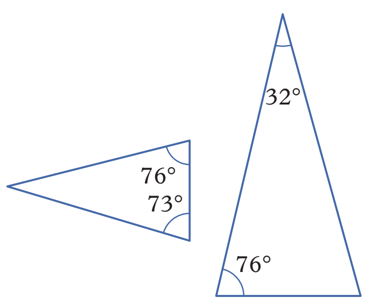
a



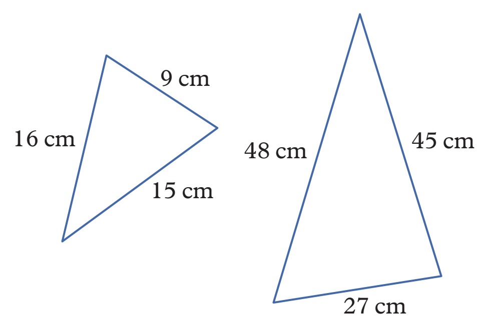
b



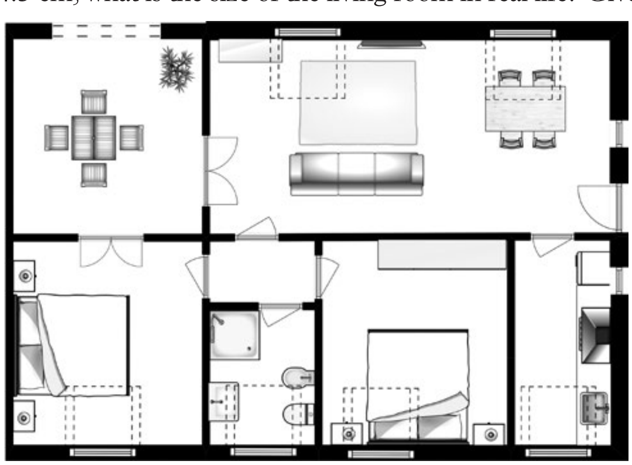
c



d



WE 6.2C **6** The plan for the two-bedroom apartment below is drawn to a scale of 1 : 80. If, in the scale drawing, the living room measures 9.8 cm by 4.5 cm, what is the size of the living room in real life? Give your answer in metres.



7 Ayesha creates a scale drawing of a house with a scale of 1 : 120. How big are each of the following rooms, given their sizes in the scale drawing?

a Bathroom (3 cm by 2.3 cm)

b Bedroom (3.5 cm by 2.8 cm)

c Kitchen (3.8 cm by 2.2 cm)

d Living room (4.2 cm by 4.05 cm)

Exercise 6.2B: Problem solving and reasoning

8 A photo of a red blood cell is printed at a scale of 5000 : 1. In the photo, the diameter of the red blood cell is 3.5 cm. What is the diameter of the red blood cell in micrometres (μm)?
Note: 1 cm = 10 000 μm

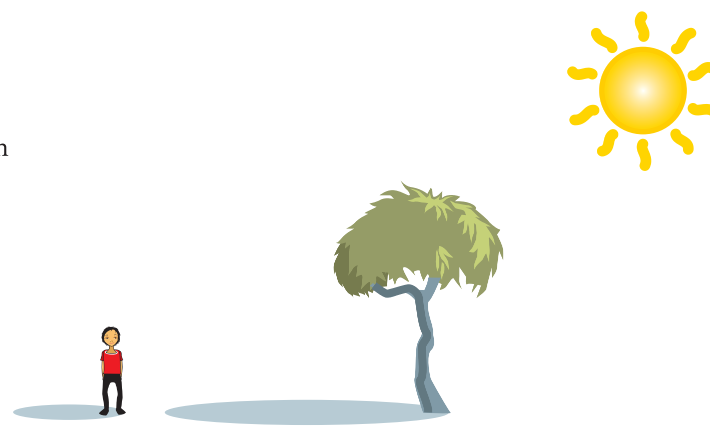
9 A model car is constructed using a ratio of 1 : 40.

a If the model car has a length of 10 cm, what is the length of the real car?

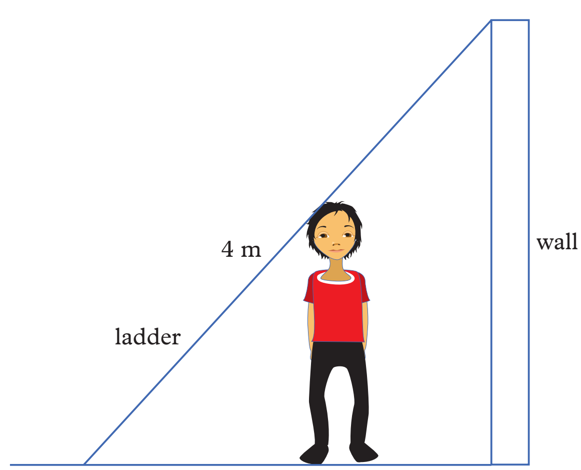
b If the real car has a height of 1.5 m, what is the height of the model car?

c Do you think that a model car can be a true similar figure to a real car? Explain your answer.

10 Similar triangles can be used to find the heights or lengths of objects that are difficult to measure. Consider this diagram showing the shadows cast by a person and a tree. The person is 1.76 m tall and casts a shadow of 2.1 m. If the tree casts a shadow of 7.8 m, how tall is the tree?



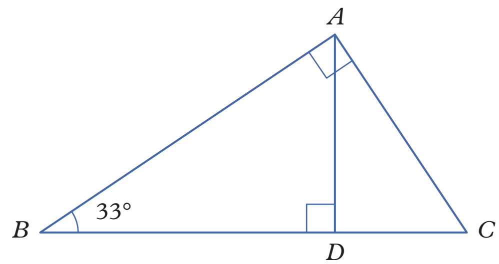
11 Luke wants to measure the heights of the walls in his house and then order some wall decorations. He places his 4 m ladder on an angle so that it hits the top of the wall, as shown in the diagram. The ladder has rungs every 50 cm: at 50 cm, 1 m, etc. If Luke, at 170 cm tall, stood directly under the ladder and his head came into contact with the fifth rung, find:



a the length of the ladder where it touched Luke's head

b the height of Luke's walls.

12 Consider the following diagram.

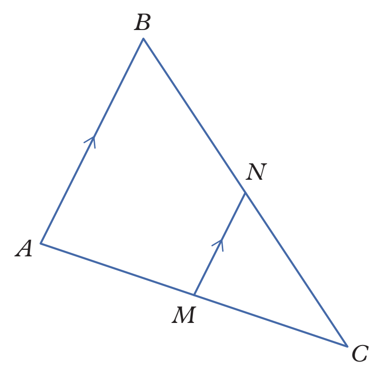


a What is the size of $\angle CAD$?

b Explain why $\triangle ABD$ is similar to $\triangle CAD$.

c There are two other pairs of similar triangles in this diagram. Use the similarity symbol, ~, to write similarity statements for both of these pairs of similar triangles.

13 Consider the following diagram.

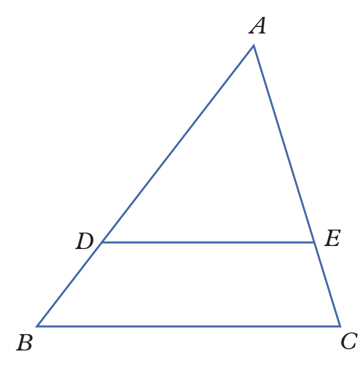


a Show that $\triangle ABC \sim \triangle MNC$.

b If $AB = 24$ cm, $MN = 12$ cm and $AC = 36$ cm, what is the length of AM?

Exercise 6.2C: Challenge

14 In $\triangle ABC$, points D and E lie on side AB and AC respectively, and DE is parallel to BC. Point D divides side AB in a ratio of 2 : 1.



a Find the ratio between the areas of $\triangle ABC$ and $\triangle ADE$ in the form $m : n$.

b Find the ratio between the areas of $\triangle ADE$ and the quadrilateral $BCED$ in the form $m : n$.

Online resources:

- **Interactive skillsheet** Similar figures
- **Interactive skillsheet** Similar triangles
- **Interactive skillsheet** Scale drawings
- **Worksheet** Scale factors
- **Quick quiz** 6.2

Lesson 6.3

Area and volume scale factors

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate and use area and volume scale factors.

Inter-year links

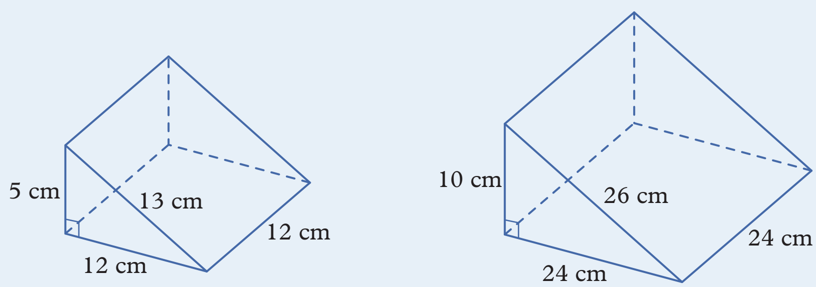
Year 7 9.5 Area of a triangle

Year 8 8.5 Volume and capacity

Similar objects

Key content video
Similar objects

- Two objects are similar if they are exactly the same shape. Similar objects do not need to be the same size.
- If two objects are similar, all side lengths will be in the same proportion.
  For example, these two prisms are similar, as every side length in the prism on the right is twice the corresponding side length in the prism on the left.



Scale factor

Key content video
Area scale factor

- A scale factor measures the enlargement or reduction factor between similar objects.
- The length scale factor is the ratio between an image side length and the original side length.

  $$\text{Length scale factor} = \frac{\text{image length}}{\text{original length}}$$

- The **area scale factor** is the ratio between an image area and the original area, and is equal to the square of the length scale factor between the objects.

  $$\text{Area scale factor} = \frac{\text{image area}}{\text{original area}} = \left(\frac{\text{image length}}{\text{original length}}\right)^2$$

- The **volume scale factor** is the ratio between an image volume and the original volume, and is equal to the cube of the length scale factor between the objects.

  $$\text{Volume scale factor} = \frac{\text{image volume}}{\text{original volume}} = \left(\frac{\text{image length}}{\text{original length}}\right)^3$$

- The length scale factor can be used to calculate the area and volume scale factors.
  For example, if the length scale factor between two similar objects is 4, the area scale factor is $4^2 = 16$ and the volume scale factor is $4^3 = 64$.
- The magnitude (size) of the scale factor determines if the image is a reduction or an enlargement of the original.
  → If the scale factor is *between 0 and 1*, the image is a *reduction* (smaller than the original object).
  → If the scale factor is *greater than 1*, the image is an *enlargement* (larger than the original object).

Worked example 6.3A Calculating area and volume scale factors

Two dolls from a set of Russian nesting dolls are identical in shape but differ in size. One of the dolls is twice as tall as the other.

a Find the area scale factor for the surface areas of the dolls.

b Find the volume scale factor for the volume of the dolls.

c If the smaller doll has a volume of 200 cm^3, find the volume of the larger doll.

THINK	WRITE
a The area scale factor is the square of the length scale factor.	**a** Area scale factor $= 2^2$ $= 4$
b The volume scale factor is the cube of the length scale factor.	**b** Volume scale factor $= 2^3$ $= 8$
c The volume of the smaller doll multiplied by the volume scale factor gives the volume of the larger doll.	**c** Volume $= 200 \times 8$ $= 1600\ \text{cm}^3$

Worked example 6.3B Using scale factors to find measurements (regular objects)

A rectangular prism has length 5 cm, width 4 cm and height 3 cm. The prism is dilated by a scale factor of 2.

a Find the surface area and volume of the original rectangular prism.

b Find the surface area and volume of the new prism by first dilating the length, width and height measurements.

c Find the surface area and volume of the new prism using the area and volume scale factors.

THINK	WRITE
a **1** The surface area of a prism is the sum of the areas of all six faces.	**a** Surface area $= 2 \times (3 \times 4 + 3 \times 5 + 4 \times 5)$ $= 94\ \text{cm}^2$
2 The volume of a rectangular prism is the product of the length, width and height of the prism.	Volume $= 3 \times 4 \times 5$ $= 60\ \text{cm}^3$

b **1** Find the dilated dimensions of the prism.	**b** Dilated length = 10 cm, dilated width = 8 cm, dilated height = 6 cm.
2 Find the image surface area using the dilated length, dilated width and dilated height measurements.	Surface area of image $= 2 \times (6 \times 8 + 6 \times 10 + 8 \times 10)$ $= 376\text{ cm}^2$
3 Find the image volume by multiplying the dilated length, width and height together.	Volume of image $= 6 \times 8 \times 10$ $= 480\text{ cm}^3$
c **1** Find the area scale factor as the square of the length scale factor, and then multiply the original surface area by the area scale factor.	**c** Area scale factor = (length scale factor)^2 $= 2^2$ $= 4$ Surface area of image $= 4 \times 94$ $= 376\text{ cm}^2$
2 Find the volume scale factor as the cube of the length scale factor, and then multiply the original volume by the volume scale factor.	Volume scale factor = (length scale factor)^3 $= 2^3$ $= 8$ Volume of image $= 8 \times 60$ $= 480\text{ cm}^3$

Helpful hints

✔ For similar objects with length scale factor k, the area scale factor is k^2 and the volume scale factor is k^3.
✔ The side length, area and volume of a dilated object can be found by multiplying the original measurement by the corresponding scale factor.

Learning pathways

△ 1–4, 6, 9, 10, 12 □ 3–6, 8, 10, 13, 15, 18 ⬠ 4, 7, 8, 11, 14–18

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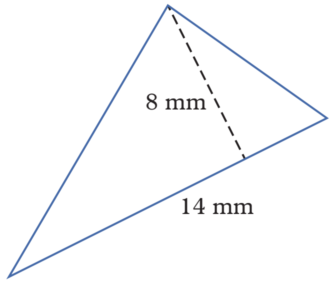
Exercise 6.3A: Understanding and fluency

1 Complete this table.

Length scale factor	Area scale factor	Volume scale factor
2		
$\frac{3}{4}$		
	16	
	$\frac{36}{25}$	
		$\frac{1}{27}$

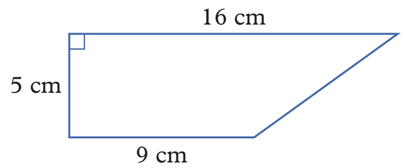
2 For each shape, state the area of the image if the original figure is dilated by the given length scale factor. Give your answers correct to two decimal places where necessary.

a



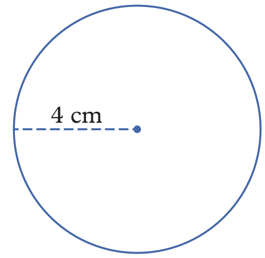
Length scale factor = 2

b



Length scale factor = $\frac{1}{5}$

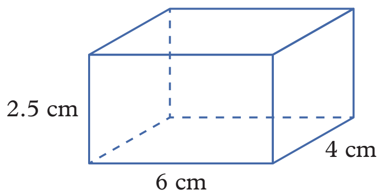
c



Length scale factor = $\frac{1}{2}$

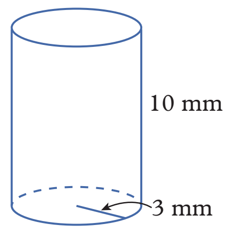
3 For each shape, state the volume of the image if the original figure is dilated by the given length scale factor.

a



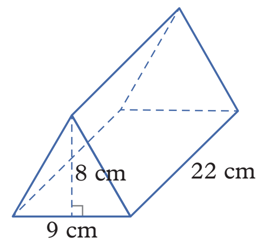
Length scale factor = $\frac{1}{2}$

b



Length scale factor = 5

c



Length scale factor = 1.5

WE 6.3A 4 A set of salt and pepper shakers are similar in shape. The salt shaker is 1.5 times taller than the pepper shaker.

- **a** Find the area scale factor for the surface areas of the shakers.
- **b** Find the volume scale factor for the volume of the shakers.
- **c** If the pepper shaker has a volume of 5 cm^3, find the volume of the salt shaker.

WE 6.3B 5 A cube of side length 5 mm is dilated by a scale factor of 3.

- **a** Find the surface area and volume of the original cube.
- **b** Find the surface area of the new cube.
- **c** Find the volume of the new cube.

6 Calculate the volume scale factor for these area scale factors.

a 49 **b** 25 **c** 100 **d** 64 **e** 4 **f** 400

Exercise 6.3B: Problem solving and reasoning

7 A cylinder with radius 12 cm and height 9 cm is dilated by a scale factor of $\frac{1}{3}$. Leave your answers as exact values for all parts of this question.

- **a** Find the surface area and volume of the original cylinder.
- **b** Find the surface area of the new cylinder.
- **c** Find the volume of the new cylinder.

8 An architect created a model of the Louvre Pyramid with a scale of 1 : 300.

- **a** The Louvre Pyramid is 21.6 m high. Find the height of the model in cm.
- **b** The base surface area of the Louvre Pyramid is 1000 m^2. Find the base surface area of the model to the nearest cm^2.

9 A floor plan with a scale of 1 : 50 illustrates a rectangular bedroom in a house. The dimensions of the room are measured to be 8 cm by 6 cm on the floor plan. Find the area of the bedroom in m^2.

10 Lake Eacham has an area of 640 000 m^2. On one map, the area of the lake is measured to be 4 cm^2. Find the scale of this map in the form 1 : n.

11 Andrea has a set of similar rectangular containers. She tries to store 400 mL of soup in a container with dimensions 15 cm × 5 cm × 18 cm and finds that the soup only fills part of the container. Find the length scale factor between this container and the smallest similar container that can store the soup.

12 Lillian grows herbs in a pot on her balcony. The herbs thrive under her care, and she needs to find a similar pot that has twice the volume for replanting. What is the scale factor comparing the dimensions of the two pots?

13 In low light, the pupils of our eyes dilate to let in more light. In normal light, Chantelle's pupils had a radius of 1.2 mm. When she walked into a dark room, they dilated to have a radius of 3.6 mm.

- **a** What is the length scale factor?
- **b** What is the area scale factor?
- **c** Using the formula $A = \pi r^2$ and your answer to part **b**, find correct to two decimal places:
  - **i** the original pupil area
  - **ii** the dilated pupil area
  - **iii** the difference in these two areas.

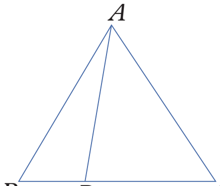
14 A radar has a scanning radius of roughly 5 km. If the owner wanted to cover an area of 100π km^2, by how much would the radius of the radar need to increase?

15 A community recreational centre is reconstructing their rectangular swimming pool of constant depth to accommodate the growing population. The new pool will be twice as long in width and length. Due to safety restrictions, the depth of the pool will remain the same as before. Find the scale factor for the volume of the new pool.

16 A student uses a 3D printer to create a solid sculpture with 25 g of filament. The student also spent 20 minutes painting its surface. This sculpture is later selected for an exhibition, and the student needs to print the same sculpture at three times the size for display.

- **a** How much filament does the student need to print the larger statue?
- **b** If this student can paint at a constant rate, approximately how long would it take to paint the surface of the larger sculpture?

17 In $\triangle ABC$, point D divides side BC into a ratio of 1 : 2. A student suggests that the area ratio between $\triangle ABD$ and $\triangle ADC$ is 1 : 4, calculated as the square of the length ratio. Do you agree with this statement? Explain your reasoning.



Exercise 6.3C: Challenge

18 A spherical balloon has a radius of 10 cm. It loses some air and deflates to half of its original radius over a week. The surface area of a sphere is $SA = 4\pi r^2$, and the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. Leave your answers as exact values for all parts of this question.

- **a** Find the surface area and the volume of the original balloon.
- **b** Find the radius of the balloon after a week.
- **c** Use the new radius to calculate the surface area of the balloon after a week.
- **d** Calculate the area scale factor. Comment on your result in relation to the radius scale factor.
- **e** Use the new radius to calculate the volume of the balloon after a week.
- **f** Calculate the volume scale factor. Comment on your result in relation to the radius scale factor.

Online resources:

Interactive skillsheet
Area and volume scale factors

Quick quiz
6.3

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

6.1 **1** Use a rate in simplest form to model the following situations.

a A cyclist rides 86 km in 4 hours. **b** A bag of 5 oranges costs \$4.80.

6.1 **2** Nicole walks to school at a speed of 1.6 m/s. If Nicole lives 1200 m from school, how long does it take her to walk to school?

6.1 **3** Use the unitary method to determine which of these represents the best buy.

a \$3.95 for 200 g or \$8.95 for 500 g **b** \$5.50 for 600 mL or \$9.99 for 1.25 L

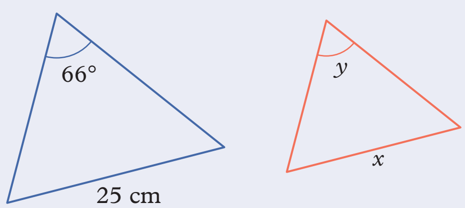
6.1 **4** The ratio of pink to red roses in a bunch of flowers is 2 : 3.

a If there are 6 pink roses, how many red roses are there?

b How many pink roses are there if there are 25 roses in total?

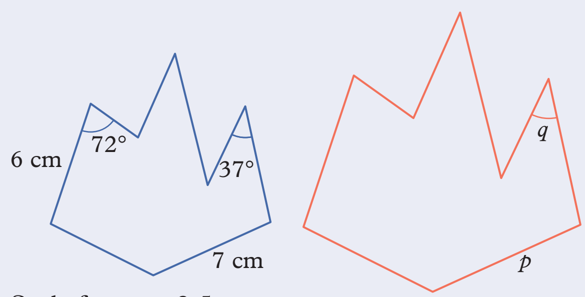
6.2 **5** The shapes on the left have been dilated by the given scale factor to create the image on the right. Determine the value of the unknown pronumerals in each image.

a



Scale factor = 0.8

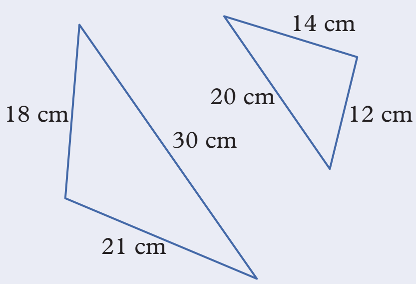
b



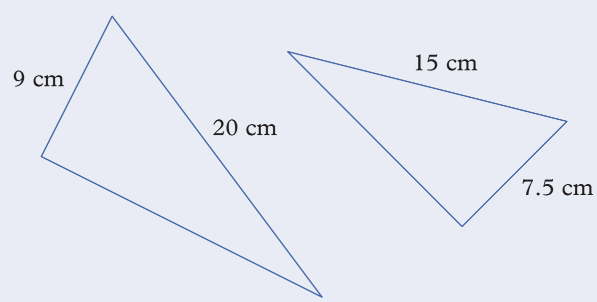
Scale factor = 2.5

6.2 **6** Determine if these pairs of triangles are similar. State the similarity test used.

a

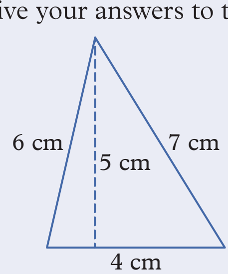


b



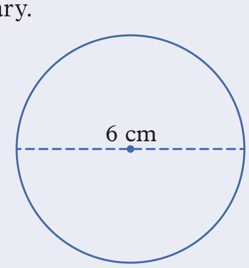
6.3 **7** For each shape, state the area of the image if the original figure is dilated by the given length scale factor. Give your answers to two decimal places where necessary.

a



Length scale factor = $\frac{1}{3}$

b



Length scale factor = 2

6.3 **8** A triangular flag has its dimensions increased by a factor of 5. If the original flag has a 30 cm base and a 45 cm height, find the difference in the area of material needed to make the two flags.

Lesson 6.4

Trigonometric ratios

Learning intentions

By the end of this lesson you will be able to …

✔ identify and use trigonometric ratios in right-angled triangles.

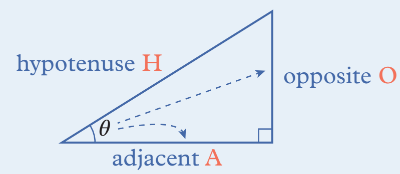
Inter-year links

Year 7	3.8 Ratios
Year 8	9.5 Similar triangles
Year 10	7.2 Trigonometry

Trigonometry

Key content video
Trigonometric ratios

- **Trigonometry** is the study of relationships between angles and side lengths in triangles.
- In a right-angled triangle with a reference angle:
  - → the **opposite side** is opposite the reference angle
  - → the **adjacent side** is next to or adjacent to the reference angle
  - → the hypotenuse is always the longest side and is opposite the right angle.
- The symbol θ (*theta*, a Greek letter) is often used to represent the reference angle.



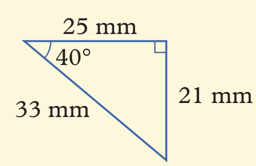
Trigonometric ratios

- For any right-angled triangle with a reference angle of θ:
  - → the **sine** of angle θ is the ratio of the lengths of the opposite side to the hypotenuse
  - → the **cosine** of angle θ is the ratio of the lengths of the adjacent side to the hypotenuse
  - → the **tangent** of angle θ is the ratio of the lengths of the opposite side to the adjacent side.
- The ratios can be written as fractions:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Worked example 6.4A Labelling the sides of a right-angled triangle with respect to a reference angle

Label the sides of this triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the angle of 40°.

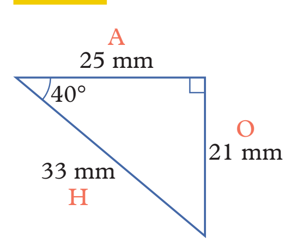


THINK

1 Label the hypotenuse with H. It is the longest side and is opposite the right angle.

2 Identify the opposite side and the adjacent side with respect to the reference angle of 40°. Label the side opposite 40° as O and the side next to 40° (adjacent) as A.

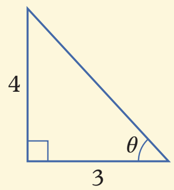
WRITE



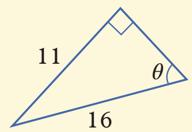
Worked example 6.4B Determining trigonometric ratios

For each triangle, identify which trigonometric ratio can be used and then determine its value in simplest form, based on the side lengths provided.

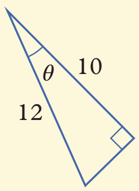
a

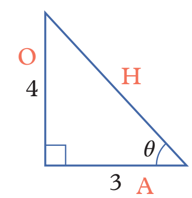
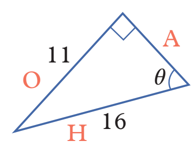
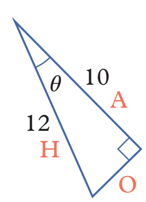


b



c



THINK	WRITE
a 1 Identify the given sides with respect to the reference angle. The side length opposite the angle is 4. The side length adjacent to the angle is 3. Label the sides accordingly.	**a**  O = 4, A = 3
2 Decide which trigonometric ratio to use. As O and A are involved, use tangent.	$\tan(\theta) = \frac{O}{A}$
3 Substitute the values into the equation to determine the ratio.	$= \frac{4}{3}$
b 1 Identify the given sides with respect to the angle. The side length opposite the angle is 11. The hypotenuse is 16. Label the sides accordingly.	**b**  O = 11, H = 16
2 Decide which trigonometric ratio to use. As O and H are involved, use sine.	$\sin(\theta) = \frac{O}{H}$
3 Substitute the values into the equation to determine the ratio.	$= \frac{11}{16}$
c 1 Identify the given sides with respect to the angle. The side length adjacent to the angle is 10. The hypotenuse is 12. Label the sides accordingly.	**c**  A = 10, H = 12
2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.	$\cos(\theta) = \frac{A}{H}$
3 Substitute the values into the equation to determine the ratio.	$= \frac{10}{12}$
4 Simplify the fraction.	$= \frac{5}{6}$

Helpful hints

- ✔ Trigonometry can involve a lot of calculator work. You will need to know how to enter the trigonometric function into your scientific calculator.
- ✔ Make sure your calculator is in degree mode, not radian mode. Otherwise, all your answers will be wrong! Check the DEG or D symbol is on screen, not RAD or R or GRAD or G.
- ✔ To remember the ratio of the two different sides according to the reference angle, you can use SOH CAH TOA. Like with Pythagoras' theorem, writing the equation from memory before substituting the values in will help you to memorise it quicker.

$$\sin(\theta) = \frac{O}{H} \quad \cos(\theta) = \frac{A}{H} \quad \tan(\theta) = \frac{O}{A} \longrightarrow \text{SOH CAH TOA}$$

- ✔ The sine, cosine or tangent values do not have units. The units cancel out in the ratio.
- ✔ Do not write sin =, cos = or tan = without including the angle. Sine, cosine and tangent do not have a value without an angle.
- ✔ The prefix 'co-' in cosine is short for complementary, as cosine is the sine of the complementary angle, which is the other non-right-angle in a right-angled triangle (opposite the side adjacent to the reference angle).

Learning pathways

△ 1–6, 7, 8(b, d, f), 9–11,14, 15, 18

□ 2–6, 8–10, 12, 16, 17–19

⬠ 3–6, 9, 10, 12, 13, 16, 18–20

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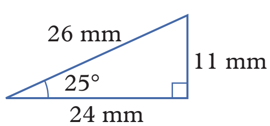
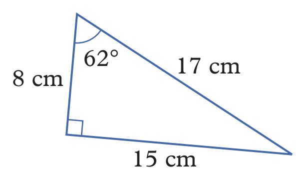
Exercise 6.4A: Understanding and fluency

WE 6.4A **1** Label the sides of each triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the given reference angle.

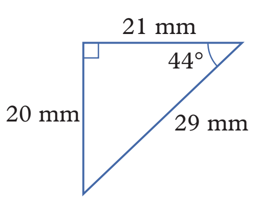
a



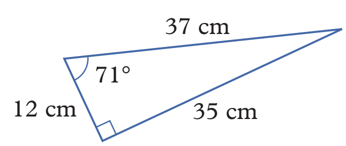
b



c

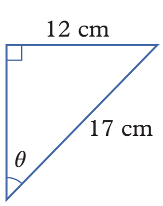


d

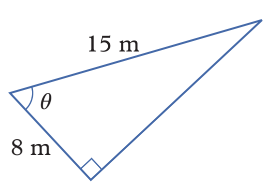


WE 6.4B **2** For each triangle, determine which trigonometric ratio can be used based on the side lengths provided.

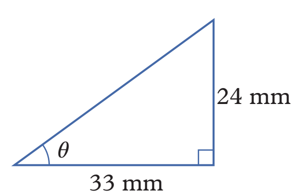
a



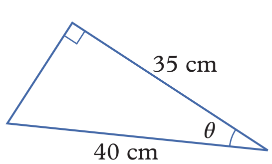
b



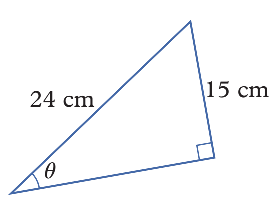
c



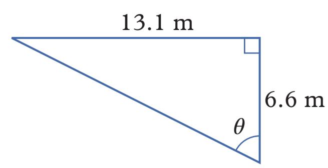
d



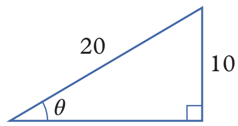
e

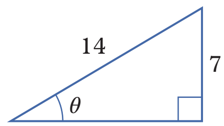


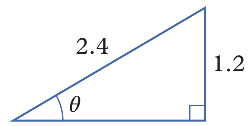
f

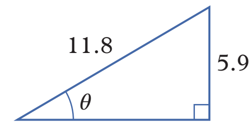


3 For each of the following triangles, determine the ratio equal to the sine of angle θ. Simplify your fractions where possible.

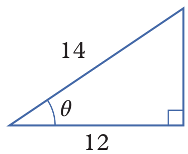
a


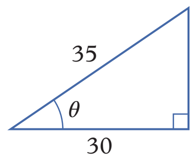
b


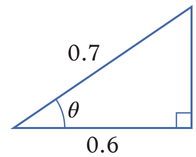
c


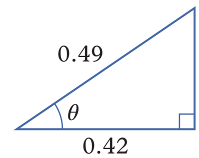
d


4 For each of the following triangles, determine the ratio equal to the cosine of angle θ. Simplify your fractions where possible.

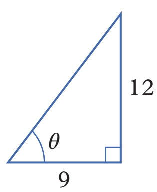
a


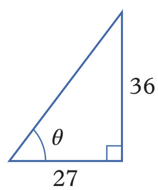
b


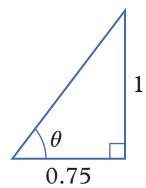
c


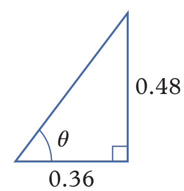
d


5 For each of the following triangles, determine the ratio equal to the tangent of angle θ. Simplify your fractions where possible.

a


b


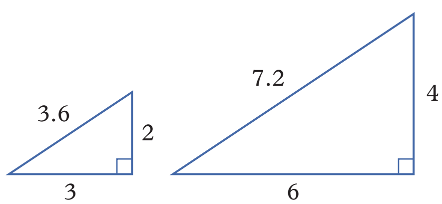
c


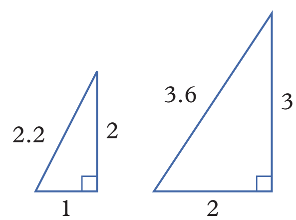
d


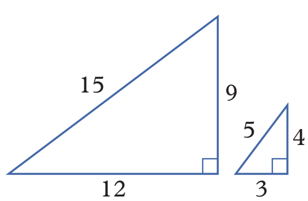
6 Consider the groups of triangles in questions **3**, **4** and **5**.

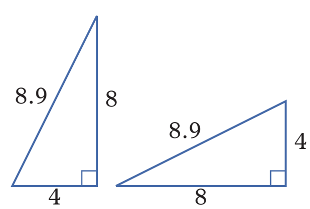
a Describe the relationship between the triangles in each group.

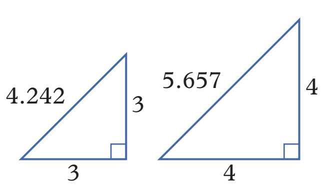
b Determine which of the following pairs of triangles have the same corresponding sine, cosine and tangent values for their non-right angles.

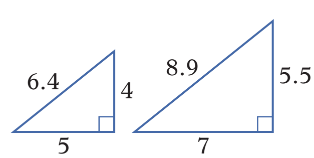
i


ii


iii


iv


v


vi


7 Evaluate the following using a calculator, correct to four decimal places. Ensure your calculator is in degree mode.

a $\sin(20°)$ **b** $\sin(36°)$ **c** $\cos(36°)$

d $\tan(36°)$ **e** $\tan(63°)$ **f** $\cos(63°)$

8 Evaluate the following using a calculator, correct to four significant figures. Ensure your calculator is in degree mode.

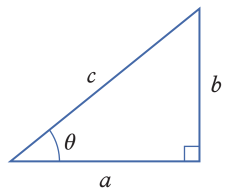
a $6\sin(20°)$ **b** $\dfrac{6}{\sin(20°)}$ **c** $\dfrac{\sin(20°)}{6}$

d $(\cos(20°))^2 + (\sin(20°))^2$ **e** $(\cos(20°))^2 - (\sin(20°))^2$ **f** $\dfrac{\sin(20°)}{\cos(20°)}$

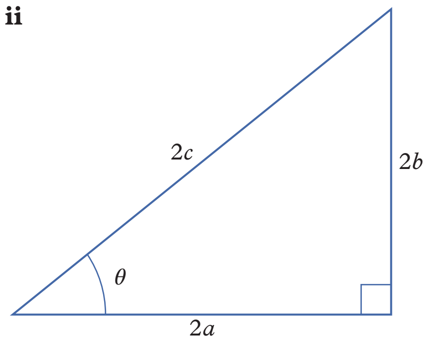
9 Consider the three similar triangles below.

a For each triangle, determine the sine, cosine and tangent of angle θ, simplifying where possible.

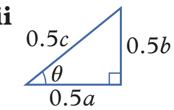
i



ii



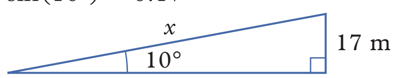
iii



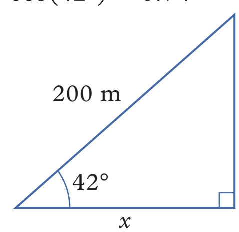
b Write a sentence about the values of the sine, cosine and tangent ratios of angles in similar triangles.

10 For each of the following, use the given approximate trigonometric ratio to determine the value of x.

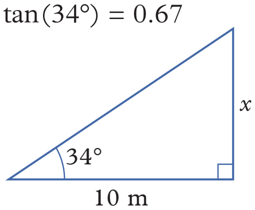
a $\sin(10°) = 0.17$



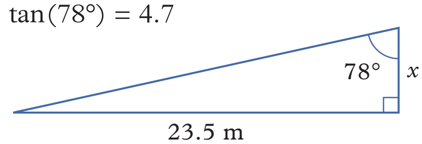
b $\cos(42°) = 0.74$



c $\tan(34°) = 0.67$



d $\tan(78°) = 4.7$



Exercise 6.4B: Problem solving and reasoning

11 The following table lists a number of trigonometric values.

a Using your calculator complete the table, giving the values correct to four decimal places.

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
1°			
5°			
15°			
30°			
45°			
60°			
75°			
85°			
89°			

b Which column(s) increase(s) as the angle θ increases? Which column(s) decrease(s) as the angle θ increases?

c Fill in the blanks of the following equations.

i $\sin(5°) = \cos(\_\_\_°)$

ii $\sin(\_\_\_°) = \cos(60°)$

iii $\sin(45°) = \cos(\_\_\_°)$

iv $\sin(\_\_\_°) = \cos(15°)$

d Describe the relationship between sine and cosine. Use the word *complementary* in your description.

12 Two special triangles can help us determine exact values of trigonometric ratios for common angles.

Diagram 1 shows a square with side lengths of 1 unit. If we cut the square in half along a diagonal, we obtain a right-angled isosceles triangle as shown in Diagram 2. This is the first special triangle.

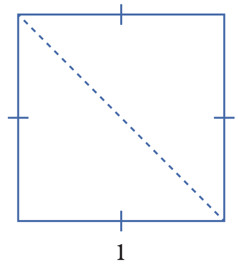


Diagram 1

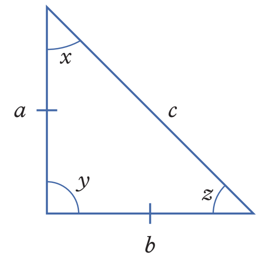


Diagram 2

a Determine the values of all side lengths and angles in Diagram 2 using geometry and Pythagoras' theorem. Write your answers exactly, as a square root, where required.

b Using the values determined in part **a**, write down the exact values of the sine, cosine and tangent ratios of 45°.

c The second special triangle is formed by cutting an equilateral triangle of side length 2 units in half down the middle, as shown in Diagrams 3 and 4.

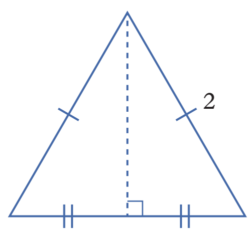


Diagram 3

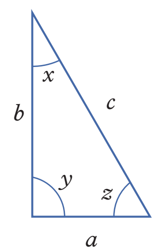


Diagram 4

Determine the values of all side lengths and angles in Diagram 4 using geometry and Pythagoras' theorem. Write your answers exactly, as a fraction or a square root, where required.

d Using the values determined in part **c**, write down the exact values of the sine, cosine and tangent ratios of 30° and 60°.

e Evaluate the following on a calculator.

i $(\sin(45°))^2$	**ii** $(\cos(30°))^2$	**iii** $(\tan(30°))^2$	**iv** $(\sin(60°))^2$
v $(\sin(30°))^2$	**vi** $(\cos(60°))^2$	**vii** $(\tan(45°))^2$	**viii** $(\cos(45°))^2$

f Describe how you could use the calculations in part **e** to obtain the exact values of the sine, cosine and tangent ratios of 30°, 45° and 60° without the special triangles.

13 The hypotenuse is always the longest side of a right-angled triangle.

a Use this piece of knowledge to explain why the values of sine and cosine of θ are never greater than 1 for $0° < \theta < 90°$.

b Explain why the values of tangent, unlike sine and cosine, can be greater than 1 for $0° < \theta < 90°$.

c Under what condition(s) is the value of tangent greater than 1?

14 Consider the table in question **11**.

a Add another column to the table with the heading $\frac{\sin(\theta)}{\cos(\theta)}$. Complete the column by performing the division $\sin(\theta) \div \cos(\theta)$ for each angle.

b Fill in the blanks of the following equations.

i $\frac{\sin(15°)}{\cos(15°)} = \tan(\_\_\_°)$

ii $\frac{\sin(\_\_\_°)}{\cos(45°)} = \tan(45°)$

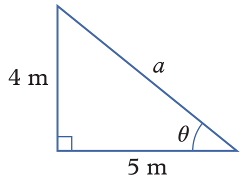
iii $\frac{\sin(60°)}{\cos(\_\_\_°)} = \tan(60°)$

iv $\frac{\sin(\_\_\_°)}{\cos(\_\_\_°)} = \tan(85°)$

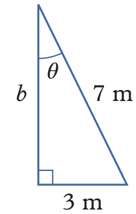
c Describe the relationship between $\frac{\sin(\theta)}{\cos(\theta)}$ and $\tan(\theta)$.

15 Consider each of the following triangles.

i



ii

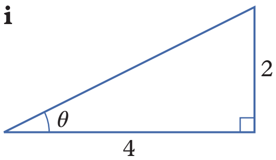


a Use Pythagoras' theorem to determine the missing side length, correct to two decimal places.

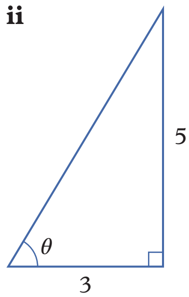
b Write the sine, cosine and tangent ratios of θ as a decimal, correct to four decimal places where required.

16 Consider each of the following right-angled triangles.

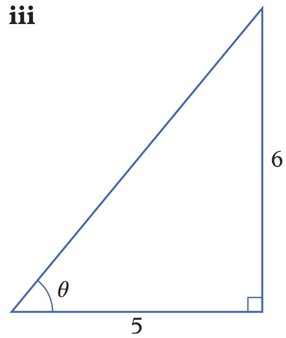
i



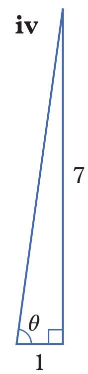
ii



iii



iv



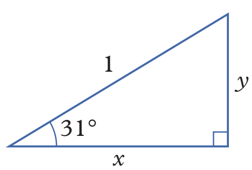
a Calculate the gradient of the hypotenuse in each triangle.

b Write the value of the tangent of θ for each right-angled triangle in part **a**.

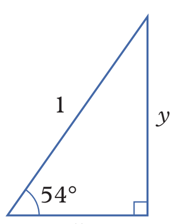
c Describe the relationship between the gradient of a line segment and the tangent of the angle from the horizontal.

17 Consider the following right-angled triangle, with a base of x and an altitude (height) of y.

a Write two equations using the sine ratio and cosine ratio for this triangle.



b Write two equations using the sine ratio and cosine ratio for this triangle.



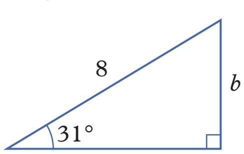
c Complete the following sentences by using the words 'base' and 'altitude'.

i The sine of an angle is the length of the _____ of a right-angled triangle with a hypotenuse of length 1.

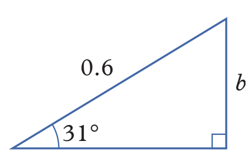
ii The cosine of an angle is the length of the _____ of a right-angled triangle with a hypotenuse of length 1.

d Use your answer to part **c** and the fact that trigonometric ratios are equal in similar triangles to determine the lengths of a and b in these similar triangles, correct to four significant figures.

i

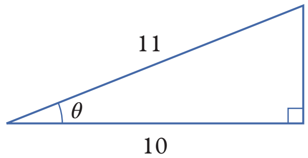


ii



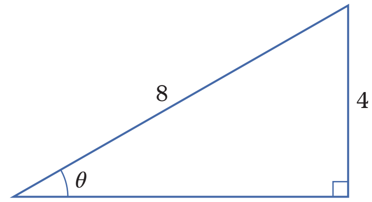
18 Explain and correct the error each person has made when writing the trigonometric ratio.

a



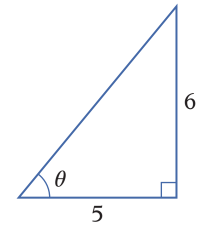
$\cos = \frac{10}{11}$

b



$\sin(\theta) = \frac{8}{4}$

c



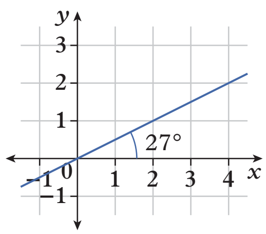
$\cos(\theta) = \frac{6}{5}$

Exercise 6.4C: Challenge

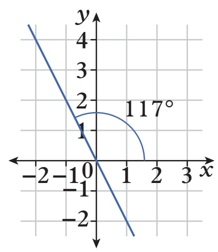
19 In question **16**, we saw that the tangent of the angle from the horizontal is equal to the gradient of the hypotenuse. This is also true for lines inclined at any angle, not just $0° < \theta < 90°$.

a State the tangent of the angle for each by determining the gradient of the line.

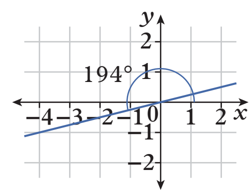
i



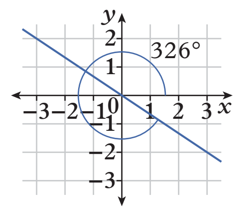
ii



iii



iv



b Evaluate the following with a calculator.

i $\tan(45°)$ **ii** $\tan(135°)$ **iii** $\tan(225°)$ **iv** $\tan(315°)$

c Explain the similarity of the values in part **b**. Hint: Draw a diagram to help.

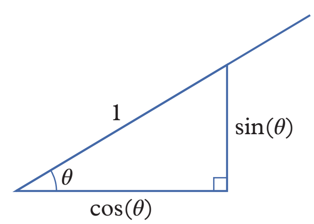
d Complete the following sentences.

i The gradient of a horizontal line is ______. Therefore, the tangent of 0° is ______.

ii The gradient of a vertical line is ______. Therefore, the tangent of 90° is ______.

20 We saw in question **14** that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$. The tangent of the angle from the horizontal is equal to the gradient, so $\frac{\sin(\theta)}{\cos(\theta)} = \frac{\text{rise}}{\text{run}}$.

Considering the ray on the right:



- the sine of the angle is the amount of rise when the hypotenuse is 1
- the cosine of the angle is the amount of run when the hypotenuse is 1.

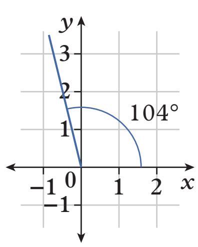
a Evaluate the following with a calculator.

i $\sin(30°)$ **ii** $\sin(150°)$ **iii** $\sin(210°)$ **iv** $\sin(330°)$

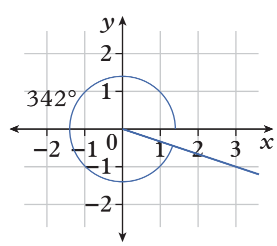
v $\cos(60°)$ **vi** $\cos(120°)$ **vii** $\cos(240°)$ **viii** $\cos(300°)$

b State whether the sine and cosine values are positive or negative for the angles shown.

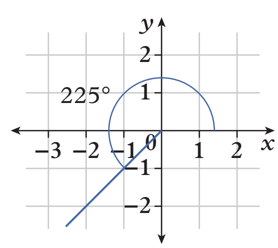
i



ii



iii



Online resources:

- **Interactive skillsheet** Trigonometry
- **Investigation** Nested similar triangles
- **CAS instructions** Trigonometry
- **Quick quiz** 6.4

Lesson 6.5

Using trigonometry to find side lengths

Learning intentions

By the end of this lesson you will be able to …

✔ find unknown side lengths in right-angled triangles.

Inter-year links

Year 7	6.6 Solving equations using inverse operations
Year 8	6.2 Solving linear equations
Year 10	7.3 Applications of trigonometry

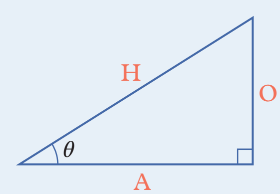
Finding side lengths

Key content video
Using trigonometry to find side lengths

- Trigonometric ratios can be used to find an unknown side length in a right-angled triangle if an angle (other than the right angle) and one side length are known.
- To find a side length:
  1. Identify the given sides with respect to the angle θ. Label the sides accordingly.
  2. Decide which trigonometric ratio to use.
  3. Substitute in the known side length and angle.
  4. Solve the equation for the unknown.
- Use a calculator to perform the final calculation, and where appropriate, round your answer.

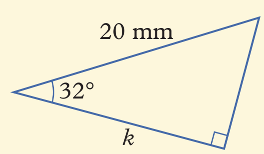
SOH CAH TOA

$$\sin(\theta) = \frac{\text{O}}{\text{H}} \qquad \cos(\theta) = \frac{\text{A}}{\text{H}} \qquad \tan(\theta) = \frac{\text{O}}{\text{A}}$$



Worked example 6.5A Identifying which trigonometric ratio to use to write an equation

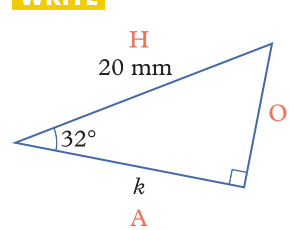
Determine which trigonometric ratio can be used, based on the side lengths provided, and hence write an equation that involves k.



THINK

1. Identify the given sides with respect to the angle 32°. The side length adjacent to the angle is k. The hypotenuse is 20 mm. Label the sides accordingly.
2. Decide which trigonometric ratio to use. As A and H are involved, use cosine.
3. Substitute for θ, A and H to write an equation involving k.

WRITE



A = k, H = 20 mm

$$\cos(\theta) = \frac{\text{A}}{\text{H}}$$

$$\cos(32°) = \frac{k}{20}$$

Worked example 6.5B Solving equations involving trigonometric ratios

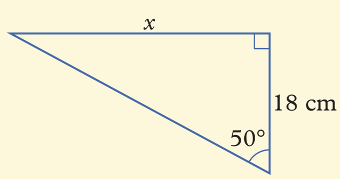
Solve the following equations, rounding your answers to two decimal places.

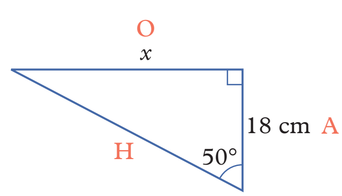
a $\sin(23°) = \dfrac{m}{4}$ **b** $\sin(55°) = \dfrac{12}{n}$

THINK	WRITE
a **1** Rearrange the equation to make m the subject of the formula.	**a** $\sin(23°) = \dfrac{m}{4}$ $m = 4 \times \sin(23°)$
2 Use a calculator to evaluate the trigonometric ratio $\sin(23°)$ and perform the multiplication operation $(\times 4)$.	$= 1.5629...$
3 Round the answer to two decimal places.	≈ 1.56
b **1** Rearrange the equation to make n the subject of the formula.	**b** $\sin(55°) = \dfrac{12}{n}$ $n = \dfrac{12}{\sin(55°)}$
2 Use a calculator to evaluate n.	$= 14.649...$
3 Round the answer to two decimal places.	≈ 14.65

Worked example 6.5C Using trigonometry to find an unknown side length in the numerator

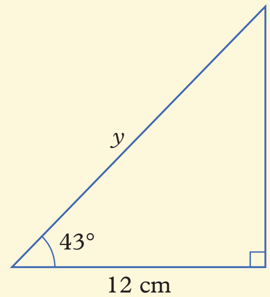
Use trigonometry to find the side length x in this triangle, correct to two decimal places.



THINK	WRITE
1 Identify the given sides with respect to the angle 50°. The side length adjacent to the angle is 18 cm. The side length opposite the angle is x. Label the sides accordingly.	 $O = x$, $A = 18$ cm
2 Decide which trigonometric ratio to use. As O and A are involved, use tangent.	$\tan(\theta) = \dfrac{O}{A}$
3 Substitute for θ, O and A.	$\tan(50°) = \dfrac{x}{18}$ $18 \times \tan(50°) = x$
4 Use a calculator to solve the equation for x.	$21.4515... = x$
5 Round the value of x to two decimal places.	$x \approx 21.45$ cm

Worked example 6.5D Using trigonometry to find an unknown side length in the denominator

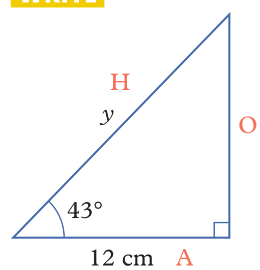
Use trigonometry to find the side length y in this triangle, correct to two decimal places.



THINK

1 Identify the given sides with respect to the angle 43°. The side length adjacent to the angle is 12 cm. The hypotenuse is y. Label the sides accordingly.

WRITE



A = 12 cm, H = y

THINK

2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.

WRITE

$$\cos(\theta) = \frac{A}{H}$$

THINK

3 Substitute for θ, A and H.

WRITE

$$\cos(43°) = \frac{12}{y}$$

THINK

4 Solve the equation for y. Multiply both sides of the equation by y, and then divide both sides of the equation by $\cos(43°)$.

WRITE

$$y \times \cos(43°) = 12$$

$$y = \frac{12}{\cos(43°)}$$

THINK

5 Use a calculator to divide 12 by $\cos(43°)$.

WRITE

$$\approx 16.4079...$$

THINK

6 Round the value of y to two decimal places.

WRITE

$$\approx 16.41 \text{ cm}$$

Helpful hints

✔ Remember that, when solving an equation for an unknown, you can check your answer by substituting it back into the original equation.

✔ SOH CAH TOA is still a useful mnemonic device for this section. You may find it helpful to write it at the top of your page before starting the exercise.

SOH CAH TOA

$$\sin(\theta) = \frac{O}{H} \qquad \cos(\theta) = \frac{A}{H} \qquad \tan(\theta) = \frac{O}{A}$$

Learning pathways

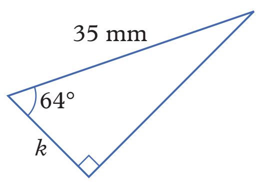
▲ 1–6, 8, 9, 13, 14(a–d) ■ 2, 3, 5–7, 10, 11, 13, 14, 18, 19 ⬟ 3, 5–7, 12, 13, 15–18, 20

ANS p505

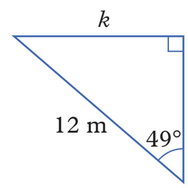
Exercise 6.5A: Understanding and fluency

WE 6.5A **1** Determine which trigonometric ratio can be used, based on the side lengths provided, and hence write an equation that involves k.

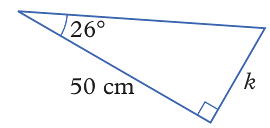
a



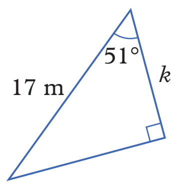
b



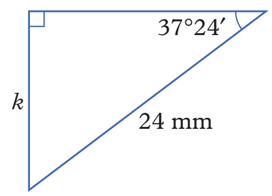
c



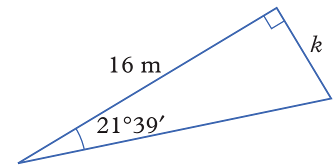
d



e



f



WE 6.5B **2** Solve the following equations for the unknown, correct to two decimal places.

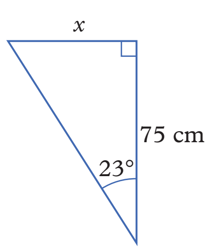
a $0.4226 = \frac{x}{5}$ **b** $0.4226 = \frac{5}{x}$ **c** $\sin(25°) = \frac{x}{5}$ **d** $\sin(25°) = \frac{5}{x}$

e $\cos(53°) = \frac{x}{11}$ **f** $\cos(53°) = \frac{11}{x}$ **g** $\tan(11°) = \frac{x}{20}$ **h** $\tan(79°) = \frac{20}{x}$

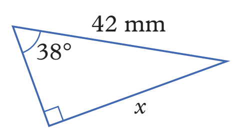
3 Solve each equation formed in question **1** to calculate the value of k, correct to two decimal places.

WE 6.5C **4** Use trigonometry to find the side length x in each triangle, correct to two decimal places.

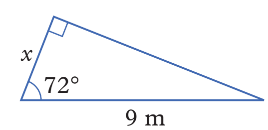
a



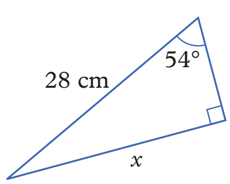
b



c



d



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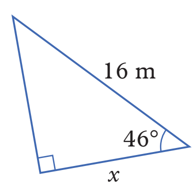
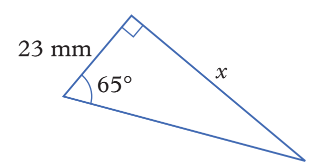
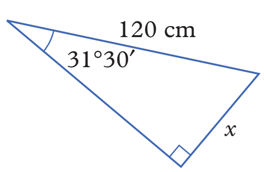


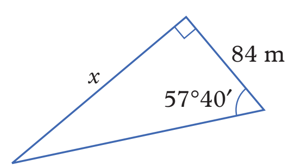
f



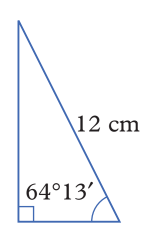
g



h

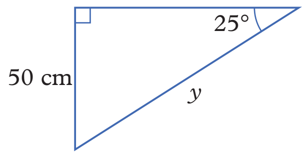


i

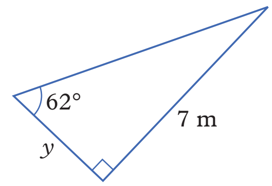


WE 6.5D **5** Find the side length y in each triangle, correct to two decimal places.

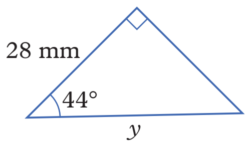
a



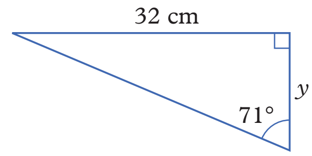
b



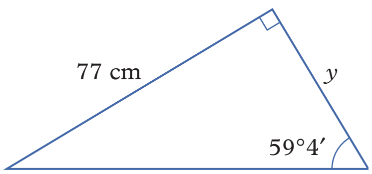
c



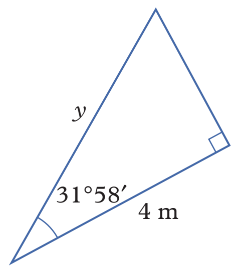
d



e

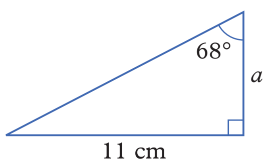


f

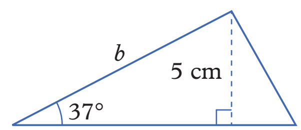


6 Find the value of the pronumeral in each triangle, correct to two decimal places.

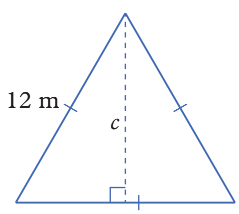
a



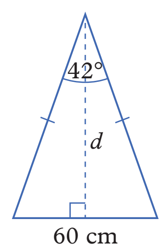
b



c

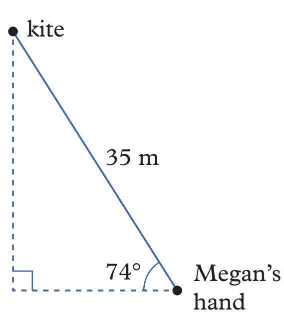


d



Exercise 6.5B: Problem solving and reasoning

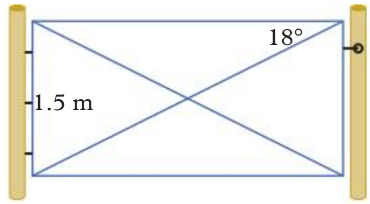
7 Megan holds the string attached to her kite at a height of 1 m above the ground. The 35 m long string makes an angle of 74° with the horizontal.



a Use trigonometry to calculate, correct to two decimal places, the vertical distance from one end of the string to the other.

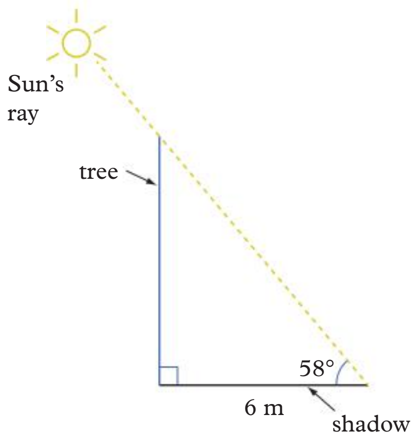
b What is the height of the kite above the ground, correct to two decimal places?

8 A farm gate has diagonal supporting braces that make an angle of 18° with the horizontal. If the height of the braces is 1.5 m, how wide is the gate, correct to two decimal places?



9 A shadow is formed when the Sun's rays are blocked by an object. The angle of the Sun's rays determine how long a shadow will be.

At a certain time of day, the Sun's rays make an angle of 58° with the ground, and a tree forms a shadow that is 6 m long. Find the following, correct to two decimal places.



a Use the diagram to calculate the height of the tree.

b At the same time, another tree forms a shadow that is 10 m long. How high is this tree?

10 The two identical sides of this ladder meet at an angle of 50° and are 1.8 m apart where they touch the ground. Find the following, correct to two decimal places.

a How high is the top of the ladder above the ground?

b How long is each side of the ladder?

11 The Great Hall of Parliament House in Canberra displays a tapestry based on a painting by the Australian artist Arthur Boyd. The tapestry measures 20 m wide and is said to be one of the largest tapestries in the world. The tapestry hangs from the ceiling and reaches all the way to the floor, with the angle between the diagonal and the floor being 24.2°.

a Draw a diagram to display this information.

b How high is the ceiling in the Great Hall, correct to the nearest metre?

c What is the perimeter of the tapestry, correct to the nearest metre?

12 A ramp for wheelchair access to a building is to be built at an angle of 3° to the horizontal. The front door of the building is 45 cm above ground level. How long should the ramp be? Give your answer in metres, correct to two decimal places.

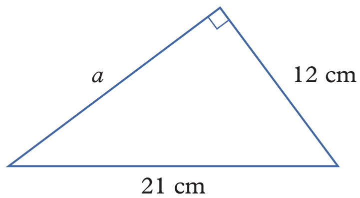
13 There are different ways to find an unknown side length in a right-angled triangle.

a What is the minimum amount of information needed to find an unknown side length in a right-angled triangle by using trigonometry?

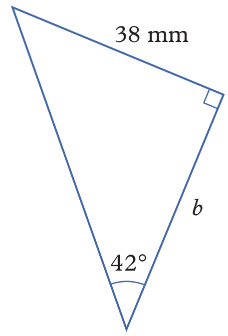
b What is the minimum amount of information needed to find an unknown side length in a right-angled triangle by using Pythagoras' theorem?

14 For each triangle, find the length of the unknown side, correct to one decimal place.

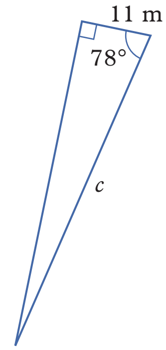
a



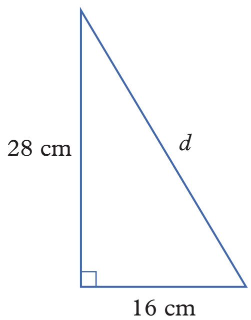
b



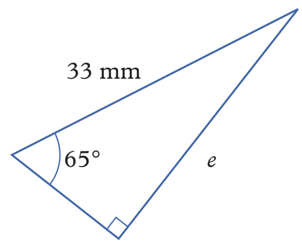
c



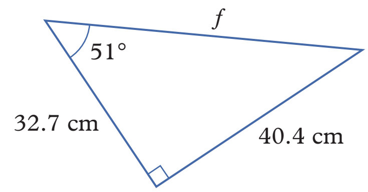
d



e

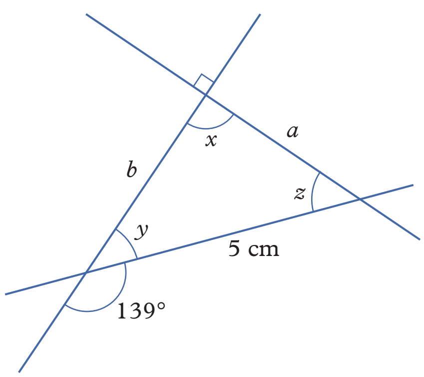


f

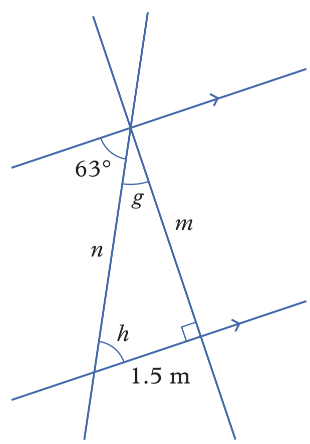


15 For each diagram, determine the values of the pronumerals, correct to two decimal places.

a

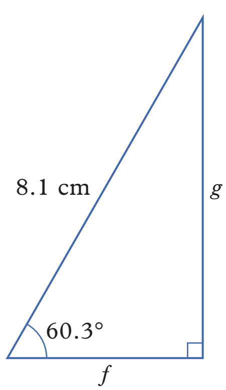


b

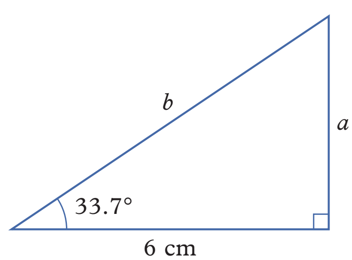


16 Use the specified ratio, geometry and Pythagoras' theorem to determine the value of the pronumerals, correct to two decimal places.

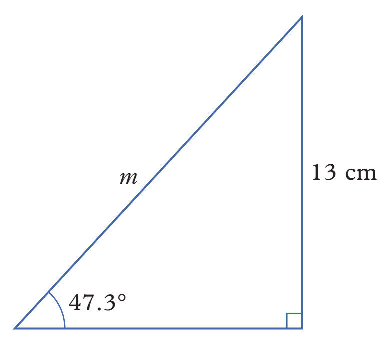
a sine



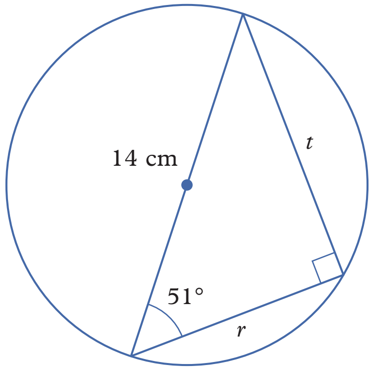
b tangent



c sine

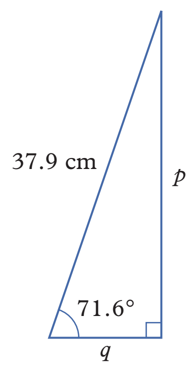


17 In the following diagram, determine the lengths r and t, correct to two decimal places.

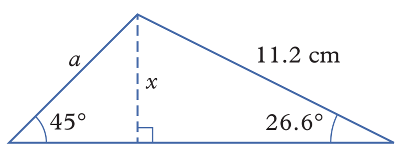


Exercise 6.5C: Challenge

18 Use only the tangent ratio and Pythagoras' theorem to determine the value of the pronumerals, correct to two decimal places. Hint: Start by writing p in terms of q.



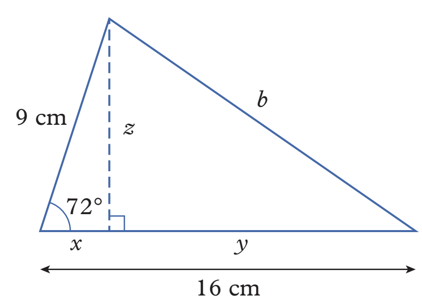
19 We can find lengths in non-right-angled triangles by constructing right-angled triangles made up of known lengths. Consider the non-right-angled triangle shown with a perpendicular height added as a dotted line.



- **a** Calculate the length of x, correct to two decimal places.
- **b** Using your answer to part **a**, calculate the length of a, correct to two decimal places.

20 Consider the non-right-angled triangle shown on the right.

- **a** Calculate the length of x, correct to two decimal places.
- **b** Calculate the length of y, correct to two decimal places.
- **c** Calculate the length of z, correct to two decimal places.
- **d** Hence, calculate the length of b.



Online resources:

Interactive skillsheet
Finding lengths by trigonometry

Investigation
Using trigonometry to find an approximate value for π

Quick quiz
6.5

Lesson 6.6

Using trigonometry to find angles

Learning intentions

By the end of this lesson you will be able to …

✔ find unknown angles in right-angled triangles.

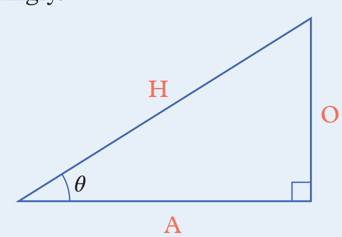
Inter-year links

Year 7	6.6 Solving equations using inverse operations
Year 8	6.2 Solving linear equations
Year 10	7.3 Applications of trigonometry

Finding angles

Key content video
Using trigonometry to find angles

- Trigonometric ratios can be used to find an unknown angle in a right-angled triangle if two side lengths are known.
- A calculator can be used to obtain the angle from its sine, cosine or tangent value. This is known as finding the **inverse sine** ($\sin^{-1}$), **inverse cosine** ($\cos^{-1}$) or **inverse tangent** ($\tan^{-1}$) of a value.
- To find an angle:
  1. Identify the given sides with respect to the angle. Label the sides accordingly.
  2. Decide which trigonometric ratio to use.
  3. Substitute in the known side lengths.
  4. Rearrange to make θ the subject of the equation using the inverse trigonometric ratio.
  5. Use the calculator to find the result.



Worked example 6.6A Solving trigonometric equations to find the value of an angle

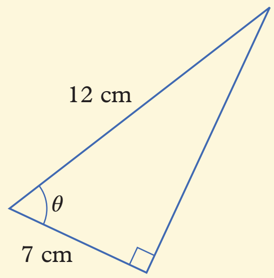
Solve each equation to find the value of θ, correct to the nearest degree.

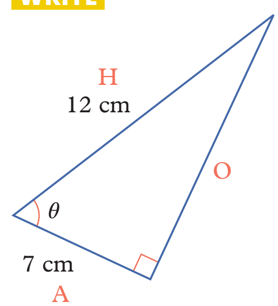
a $\tan(\theta) = 2.4$ **b** $\sin(\theta) = \frac{22}{35}$

THINK	WRITE
a 1 Rearrange to make θ the subject of the equation using the inverse tangent.	$\tan(\theta) = 2.4$ $\theta = \tan^{-1}(2.4)$
2 Use the $\tan^{-1}$ key on the calculator to find the inverse tangent of 2.4. Round the value of θ to the nearest degree.	$\approx 67.3801...$ $\approx 67°$
b 1 Rearrange to make θ the subject of the equation using the inverse sine.	$\sin(\theta) = \frac{22}{35}$ $\theta = \sin^{-1}\left(\frac{22}{35}\right)$
2 Use the $\sin^{-1}$ key on the calculator to find the inverse sine of $\frac{22}{35}$. Round the value of θ to the nearest degree.	$= 38.9448...$ $\approx 39°$

Worked example 6.6B Using trigonometry to find an unknown angle

Use trigonometry to find angle θ in this triangle, correct to the nearest degree.



THINK	WRITE
1 Identify the given sides with respect to the angle. The side length adjacent to the angle is 7 cm. The hypotenuse is 12 cm. Label the sides accordingly.	 A = 7 cm, H = 12 cm
2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.	$\cos(\theta) = \frac{A}{H}$
3 Substitute for A and H.	$\cos(\theta) = \frac{7}{12}$
4 Rearrange to make θ the subject of the equation using the inverse cosine.	$\theta = \cos^{-1}\left(\frac{7}{12}\right)$
5 Use the $\cos^{-1}$ key on the calculator to find the inverse cosine of $\frac{7}{12}$. Round the value of θ to the nearest degree.	$= 54.3146...$ $\approx 54°$

Helpful hints

- ✔ While a superscript of -1, for example, 2^{-1}, usually indicates 2 to the power of -1 (the reciprocal), $\sin^{-1}$ does *not* mean sine to the power of -1 (the reciprocal of the value of sine); instead it is the inverse of sine.
- ✔ It can be easy to choose the wrong trigonometric function when determining an angle. Labelling the sides of the triangle can help to minimise errors.
- ✔ If you get the ratio upside-down for sine and cosine when determining the angle, your calculator will display an error as no real angle has a sine or cosine ratio greater than 1. However, if you get the ratio upside-down for tangent, your calculator will not show an error.

Learning pathways

△ 1–3(a, c, e, f), 4–6, 8, 10, 12

□ 1–3(b, d, f, h), 4–7, 11–13, 17

⬠ 2–3(c, e, f), 4(b, d, f), 5, 7, 9, 13–17

ANS p505

Exercise 6.6A: Understanding and fluency

WE 6.6A **1** Calculate each of the following, correct to the nearest degree.

a $\sin^{-1}(0.23)$ **b** $\cos^{-1}(0.72)$ **c** $\tan^{-1}(1.46)$ **d** $\sin^{-1}(0.95)$

e $\sin^{-1}\left(\frac{2}{3}\right)$ **f** $\cos^{-1}\left(\frac{24}{25}\right)$ **g** $\tan^{-1}\left(\frac{39}{11}\right)$ **h** $\cos^{-1}\left(\frac{11}{17}\right)$

2 Find each unknown angle θ, correct to the nearest degree.

a $\sin(\theta) = 0.34$ **b** $\cos(\theta) = 0.81$ **c** $\tan(\theta) = 0.65$ **d** $\tan(\theta) = 0.47$

e $\cos(\theta) = 0.99$ **f** $\sin(\theta) = 0.01$ **g** $\tan(\theta) = 3.14$ **h** $\sin(\theta) = 0.71$

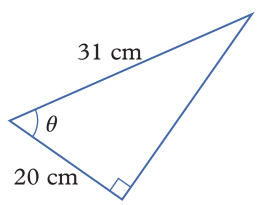
3 Solve each equation to find the value of θ, correct to the nearest degree.

a $\sin(\theta) = \frac{4}{5}$ **b** $\cos(\theta) = \frac{15}{19}$ **c** $\tan(\theta) = \frac{8}{21}$ **d** $\sin(\theta) = \frac{23}{26}$

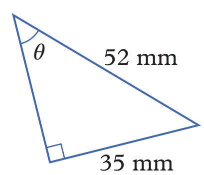
e $\cos(\theta) = \frac{11}{31}$ **f** $\sin(\theta) = \frac{9}{23}$ **g** $\cos(\theta) = \frac{17}{31}$ **h** $\tan(\theta) = \frac{26}{3}$

WE 6.6B **4** Use trigonometry to find angle θ in each triangle, correct to the nearest degree.

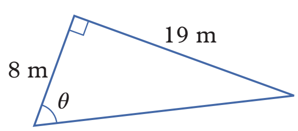
a



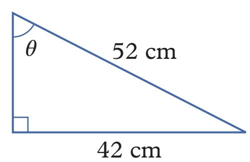
b



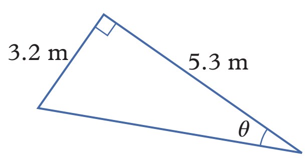
c



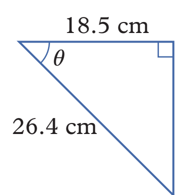
d



e

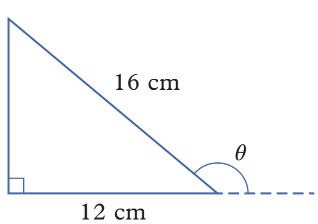


f

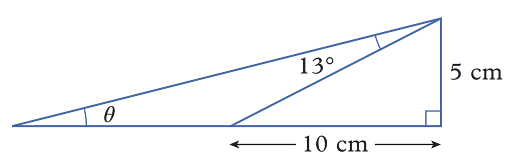


5 Determine the value of θ in each of the following diagrams, correct to one decimal place.

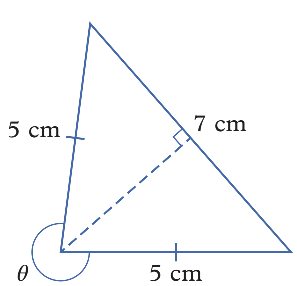
a



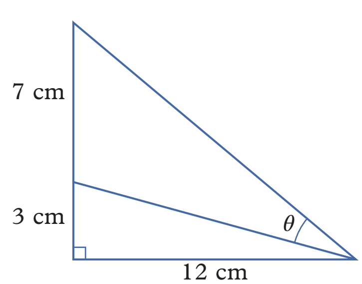
b



c

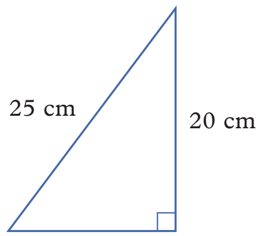


d

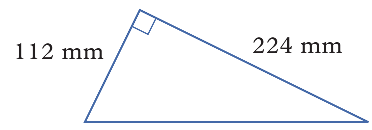


6 Calculate the size of the non-right angles in the following right-angled triangles. Give your answers in degrees, correct to one decimal place.

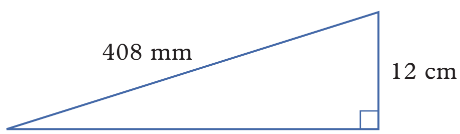
a



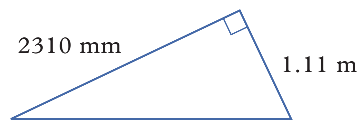
b



c

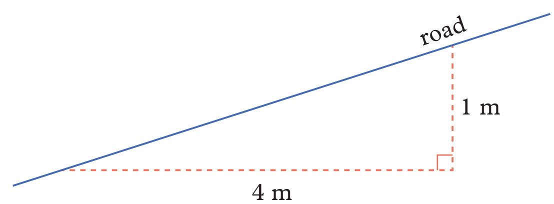


d



Exercise 6.6B: Problem solving and reasoning

7 A road has a gradient of 1 in 4; that is, it rises 1 m vertically for every 4 m horizontally.



a What angle does the road surface make with the horizontal? Give your answer correct to the nearest degree.

b How far have you travelled along the road if you are now 1.7 m higher than when you started? Give your answer correct to two decimal places.

8 The roof of a holiday house has the dimensions shown.

Calculate, correct to the nearest degree:

a the angle the roof makes with the horizontal

b the angle formed where the two sections of roof meet.

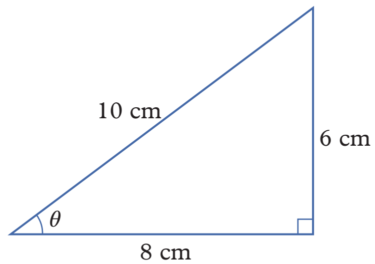
9 An anchor holding a boat in position lies on the seabed at a depth of 7.5 m. It is attached to the boat by a chain that is 8.4 m long.

a Draw a diagram of this scenario.

b What angle does the chain make with the vertical? Give your answer correct to the nearest degree.

c If the chain was longer, would this angle be larger or smaller? Explain your answer.

10 Consider the right-angled triangle shown below.



a Calculate the value of θ to the nearest degree using a calculator and:

i the sine ratio

ii the cosine ratio

iii the tangent ratio.

b Draw your own scale diagram of the triangle and measure angle θ with a protractor.

c Compare your answers to parts **a** and **b**. Comment on the advantages and disadvantages of using:

i trigonometry

ii measurements from a scale diagram.

11 Nadia and Alex are set a problem-solving task by their teacher. Their challenge is to work out the height of a tree in their school yard without climbing it or using a ladder. The only allowable equipment is a 1 m ruler, a tape measure and a calculator.

Nadia and Alex discuss some possible ideas, take some measurements simultaneously and draw these diagrams.

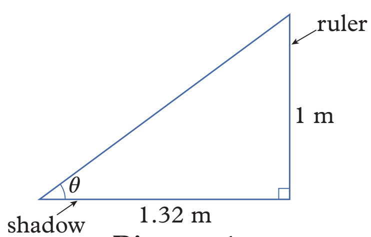


Diagram 1

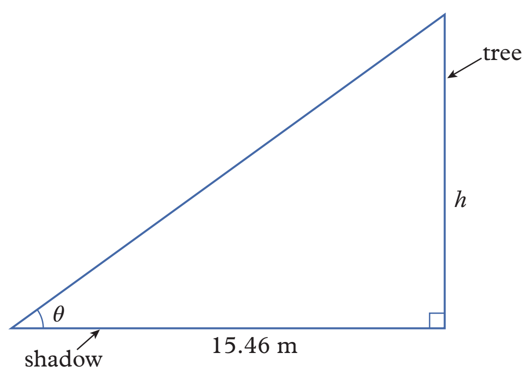


Diagram 2

a What do you think their strategy is?

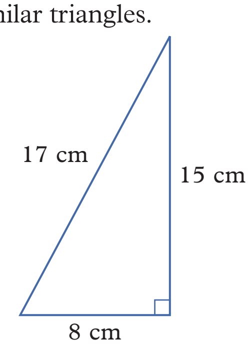
b Use trigonometry to calculate the value of θ to the nearest degree in Diagram 1.

c Explain why this value of θ can be used in Diagram 2.

d Use trigonometry to calculate the height of the tree, correct to two decimal places.

e Instead of using trigonometry, Nadia and Alex could have used their knowledge of similar triangles. Explain how this strategy could be used to calculate the height of the tree.

12 Consider the right-angled triangle shown on the right.



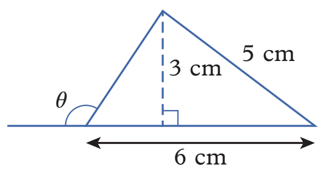
a Correct to the nearest degree, calculate the size of the non-right angles using a calculator and:

i the sine ratio **ii** the cosine ratio **iii** the tangent ratio.

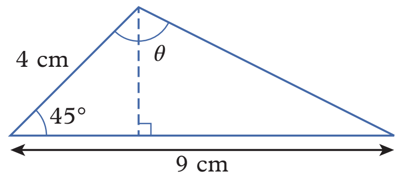
b After the size of one non-right angle has been determined, how else could the size of the other non-right angle be determined?

13 Determine the value of θ in each of the following diagrams, correct to one decimal place.

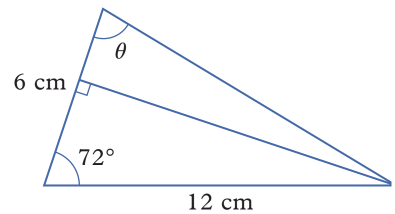
a



b

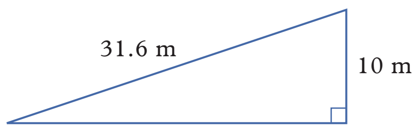


c

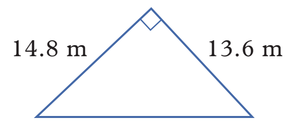


14 Calculate the size of the non-right angles and the missing side length in the following right-angled triangles using the sine ratio and Pythagoras' theorem, correct to one decimal place.

a

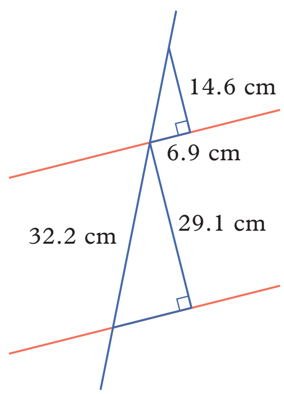


b

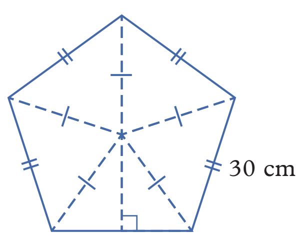


Exercise 6.6C: Challenge

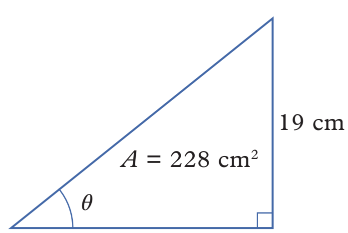
15 Determine if the two red lines below are parallel. Explain why or why not.



16 Calculate the area of the following regular pentagon, correct to one decimal place.



17 The area of the triangle shown below is 228 cm^2. Determine the size of the angle θ, correct to one decimal place.



Online resources:

Interactive skillsheet Finding angles by trigonometry

Worksheet Using trigonometry

Quick quiz 6.6

Lesson 6.7

Review: Scale and trigonometry

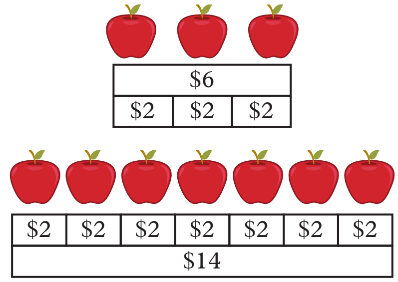
Module summary

Simplifying ratios and rates

120 : 72
÷ 24 ÷ 24
5 : 3

24 dishes/4 minutes
÷ 4 ÷ 4
6 dishes/minute

Unitary method



Similar figures

- Side lengths are in proportion
- Corresponding angles are equal

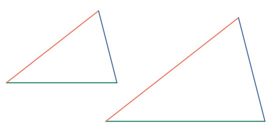
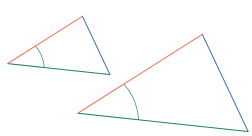
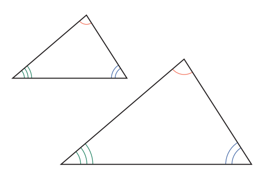
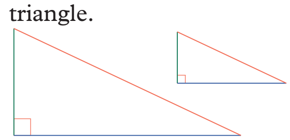
Scale factors

- Reduction: scale factor between 0 and 1
- Enlargement: scale factor greater than 1
- Length scale factor = $\dfrac{\text{image length}}{\text{original length}}$
- Area scale factor = (length scale factor)^2
- Volume scale factor = (length scale factor)^3

Scale drawings

- Scale is 'drawing length : actual length'
- 1 : 100 means 1 cm represents 100 cm

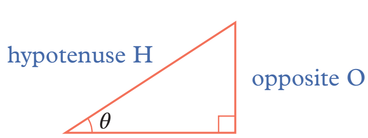
Similar triangles

SSS	SAS	AAA	RHS
All three pairs of corresponding side lengths are in the same ratio. 	Two corresponding pairs of side lengths are in the same ratio and the included angles are equal. 	All three pairs of angles are equal. 	The lengths of the pair of hypotenuses are in the same ratio as another pair of sides in a right-angled triangle. 

Trigonometry

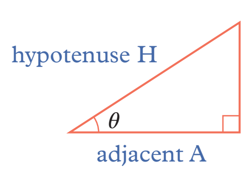
SOH

$\sin(\theta) = \dfrac{O}{H}$



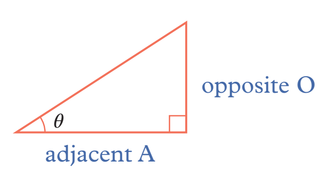
CAH

$\cos(\theta) = \dfrac{A}{H}$



TOA

$\tan(\theta) = \dfrac{O}{A}$



To find a side length:

1. Identify given sides with respect to the angle θ.
2. Decide which trigonometric ratio to use.
3. Substitute in known side length and angle.
4. Solve for the unknown.

To find an angle:

1. Identify given sides with respect to the angle.
2. Decide which trigonometric ratio to use.
3. Substitute in known side lengths.
4. Rearrange to make θ the subject of the equation using the inverse trigonometric ratio.
5. Solve for the unknown.

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Review questions 6.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- adjacent side
- area
- area scale factor
- cosine
- dilation
- enlargement
- hypotenuse
- inverse cosine
- inverse sine
- inverse tangent
- length scale factor
- opposite side
- rate
- ratio
- reduction
- scale
- scale drawing
- similar figures
- sine
- tangent
- trigonometry
- unitary method
- volume
- volume scale factor

1 What are the four tests for similar triangles?

2 How can the volume scale factor be calculated when given the length scale factor?

3 How can you identify the hypotenuse in a right-angled triangle?

4 Identify the key terms being referenced in each of these definitions.

a a drawing that represents a real-life object but which is drawn to a different scale

b the study of relationships between angles and side lengths in triangles

5 Using an example, provide a definition in your own words for the following key terms.

a ratio

b similar figures

6 Complete the following sentences using words from the key terms list.

a In a scale drawing, the ________ represents the ________ 'drawing length : actual length'.

b The ________ of angle θ is the ratio of the lengths of the opposite side to the adjacent side.

Review questions 6.7B: Multiple choice

6.1 **1** The exchange rate is 1 AUD = 0.66 EUR. How much is \$800 AUD worth?

A €528 **B** €800 **C** €1212 **D** €2424

6.1 **2** The side lengths of a parallelogram are in the ratio 5 : 8. If the perimeter of the parallelogram is 182 cm, what are the side lengths?

A 30 cm and 48 cm **B** 40 cm and 51 cm **C** 35 cm and 56 cm **D** 70 cm and 112 cm

6.2 **3** If a scale drawing is made with a scale of 1 : 500, how much does 1.5 cm in the drawing represent in real life in metres?

A 5 m **B** 7.5 m **C** 75 m **D** 750 m

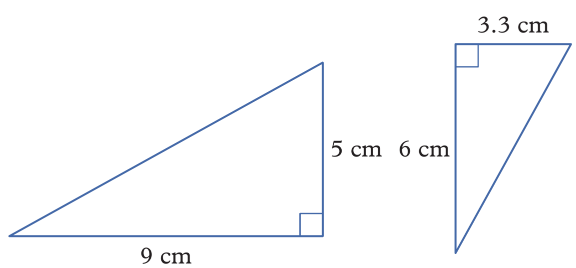
6.2 **4** State the similarity condition that can be used to determine whether this pair of triangles are similar.

A SSS

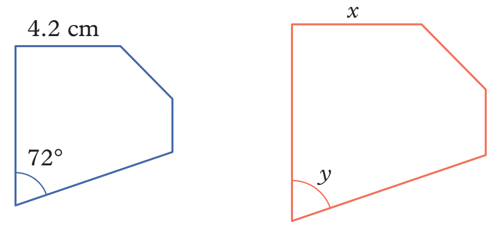
B AAA

C RHS

D SAS



6.2 **5** The shape on the left has been dilated by a scale factor of 2 to create the image on the right. What are the values of x and y?



A $x = 2.1$ cm, $y = 72°$ **B** $x = 8.4$ cm, $y = 144°$
C $x = 4.2$ cm, $y = 144°$ **D** $x = 8.4$ cm, $y = 72°$

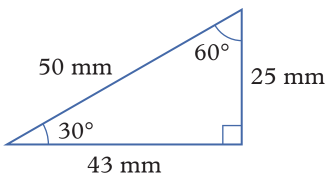
6.3 **6** A figure is dilated by a scale factor of $\frac{1}{2}$. What is its area scale factor?

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** 2 **D** 4

6.4 **7** Sine is the trigonometric ratio of which side to which side of a right-angled triangle?

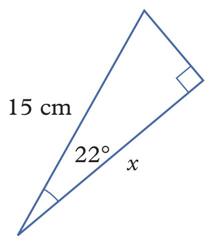
A Hypotenuse to opposite side **B** Opposite side to adjacent side
C Adjacent side to hypotenuse **D** Opposite side to hypotenuse

6.4 **8** Using the lengths on the triangle shown, the cosine of 30° is:



A $\frac{43}{50}$ **B** $\frac{25}{50}$ **C** $\frac{25}{43}$ **D** $\frac{50}{43}$

6.5 **9** Which equation can be used to find x in this triangle?

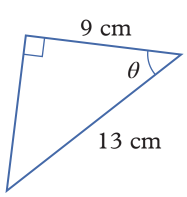


A $\cos(22°) = \frac{x}{15}$ **B** $\tan(22°) = \frac{x}{15}$
C $\sin(22°) = \frac{x}{15}$ **D** $\cos(22°) = \frac{15}{x}$

6.6 **10** If $\sin(\theta) = \frac{12}{15}$, then θ is equal to:

A $\sin\left(\frac{15}{12}\right)$ **B** $\sin^{-1}\left(\frac{12}{15}\right)$
C $\sin^{-1}\left(\frac{15}{12}\right)$ **D** $\sin\left(\frac{12}{15}\right)^{-1}$

6.6 **11** The angle θ in this triangle is closest to:



A 35° **B** 44°
C 46° **D** 55°

Review questions 6.7C: Short answer

6.1 **1** Use a rate in simplest form to model the following situations.

a It costs $7.20 to buy 4 avocados.

b A heart beats 510 times in 6 minutes.

c A train travels 546 km in 6.5 hours.

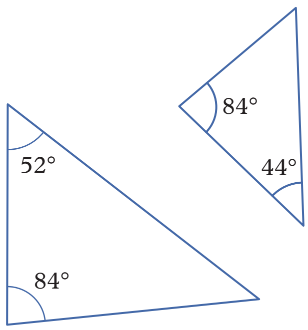
6.1 **2** In a box of milk and dark chocolates, there are 5 milk chocolates for every 4 dark chocolates. If there are 40 milk chocolates in the box:

a how many dark chocolates are in the box

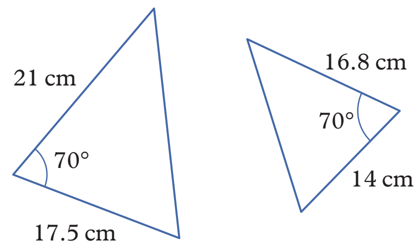
b how many chocolates are in the box?

6.2 **3** Determine if these pairs of triangles are similar. State the similarity test used.

a



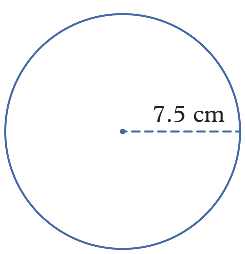
b



6.2 **4** Erica is standing next to a tree in the park. Erica casts a shadow that is 1 m in length, while the tree casts a shadow that is 4.5 m in length. If Erica is 1.6 m tall, what is the height of the tree?

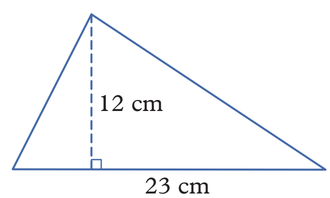
6.3 **5** For each shape, state the area of the image if the original figure is dilated by the given scale factor. If necessary, give your answers to two decimal places.

a



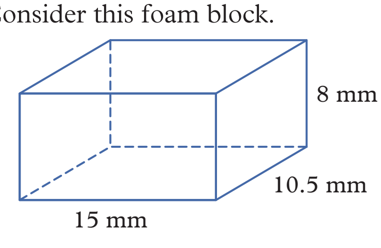
Scale factor = 3

b



Scale factor = $\frac{1}{3}$

6.3 **6** Consider this foam block.

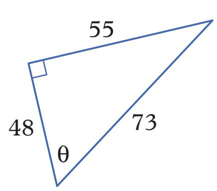


a What is the volume of the foam block?

b If the foam block was dilated by a scale factor of 3, what are the dimensions of the dilated foam block?

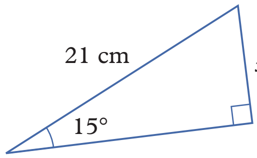
c What is the volume of the dilated foam block?

6.4 **7** Write equations for the sine, cosine and tangent of θ for this triangle.

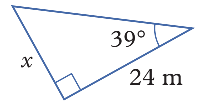


6.5 **8** Find the side length x in each triangle, correct to two decimal places.

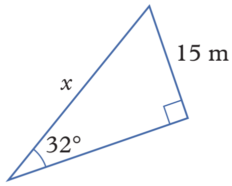
a



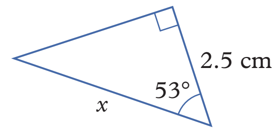
b



c

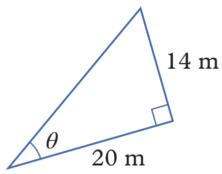


d

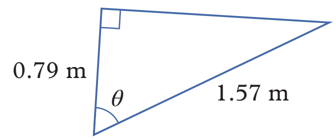


6.6 **9** Find the angle θ in each triangle, correct to the nearest degree.

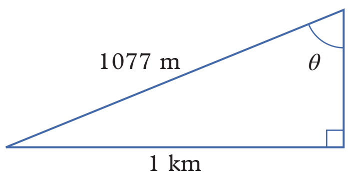
a



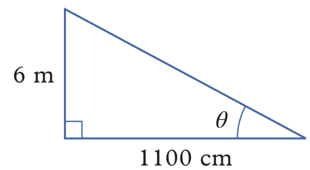
b



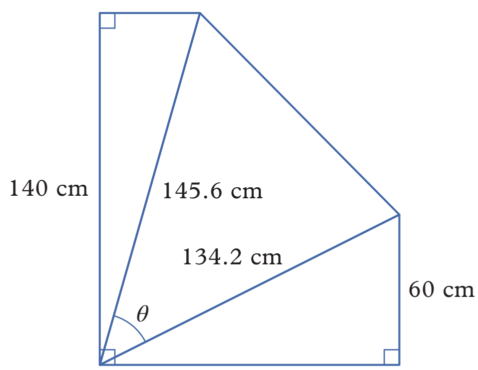
c



d



6.6 **10** Calculate the size of the angle θ, correct to one decimal place.



Review questions 6.7D: Mathematical modelling

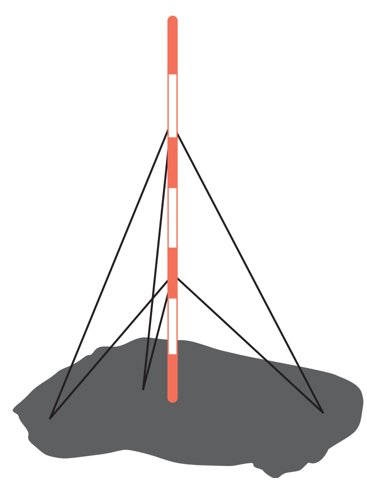
1 A 150 m tall AM radio tower is supported by six guy wires equally spaced around the tower, connecting to the ground at three points. Three of the wires are connected one-third of the way up the tower and the other three are connected one-third of the way down from the top of the tower.

a The longer wires are 125 metres in length. Which trigonometric ratio can be used to calculate the angle the longer wires make with the ground?

b What angle do the longer wires make with the ground, correct to one decimal place?

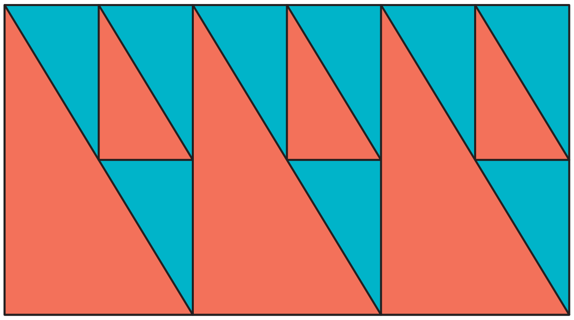
c The shorter wires make an angle of 33.7° with the ground. What is the length of the shorter wires, correct to the nearest metre?

d Calculate the total amount of wire used.



2 Julie is making a design for a blanket using right-angled triangles. The blanket design consists of small and large triangles in a repeating pattern, as shown below.

Each large triangle is 8 cm high and 4.8 cm wide and each small triangle is 4 cm high and 2.4 cm wide.



a Explain why the large and small triangles are similar triangles.

b If the large triangle was dilated to create the small triangle, what scale factor was used?

c What is the area scale factor between the large and small triangles?

d What is the ratio of red to blue material used in each blanket section?

e The blanket will be 168 cm long and 144 cm wide. How many large red triangles will be in the blanket?

f Calculate the areas of **i** red material and **ii** blue material in the blanket.

g The red material costs \$12 per square metre and the blue material costs \$10 per square metre. Calculate the total cost of material used in the blanket.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Solve problems involving ratios ☐ Solve problems involving rates	☐ Go back to Lesson 6.1 Ratios and rates
☐ Find unknown angles and side lengths in similar figures ☐ Determine if two triangles are similar ☐ Interpret scale drawings	☐ Go back to Lesson 6.2 Similar figures
☐ Calculate area and volume scale factors ☐ Use scale factors to find measurements of original and image objects	☐ Go back to Lesson 6.3 Area and volume scale factors
☐ Identify the hypotenuse, opposite and adjacent sides of a right-angled triangle containing a reference angle ☐ Calculate sine, cosine and tangent ratios of an angle in a right-angled triangle	☐ Go back to Lesson 6.4 Trigonometric ratios
☐ Use trigonometric ratios to find unknown lengths in a right-angled triangle	☐ Go back to Lesson 6.5 Using trigonometry to find side lengths
☐ Use trigonometric ratios to find an unknown angle in a right-angled triangle	☐ Go back to Lesson 6.6 Using trigonometry to find angles

Computational Thinking

Curriculum link

Design, test and refine algorithms involving a sequence of steps and decisions based on geometric constructions and theorems; discuss and evaluate refinements (VC2M9SP03)

Inter-year links

Year 7	Classifying polygons
Year 8	Binary numbers
Year 8	Geometric proofs
Year 10	Solving non-linear equations with digital tools
Year 10	Graph algorithms

Triangle inequality

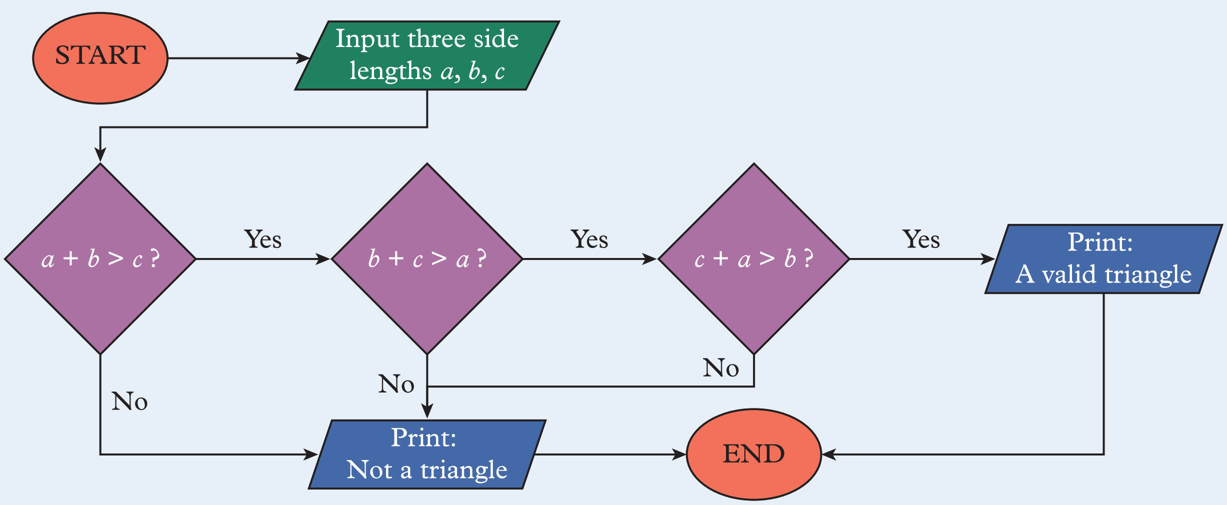
For any triangle, the sum of the lengths of any two sides is greater than the length of the third side. This is called the triangle inequality.

If a triangle has three sides with lengths a, b and c, then the triangle inequality states that the following inequalities hold:

$$a + b > c$$
$$b + c > a$$
$$c + a > b.$$

If the lengths of three line segments satisfy the triangle inequality, then they can form a triangle.

An algorithm for the triangle inequality can be written, checking whether three line segments can form a triangle. Here is the algorithm represented as a flow chart.



Check your understanding

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1 Answer the following questions.

a A triangle has three sides with integer lengths 5 cm, 8 cm and x cm. What is the maximum value of x?

b Use the flow chart to determine whether three line segments with lengths 2, 8 and 4 units can form a triangle. Write down all steps as you follow the flow chart.

c Do all three inequalities need to be checked in this algorithm? Explain.

d Show that if $a \le b \le c$ and $a < 0$, then $a + b < c$.
Hint: Start by substituting values for a, b and c and seeing what happens.

e Why does your answer to part **d** mean that for any negative input into the flow chart the result in the output will be 'Not a triangle'?

Pseudocode

Algorithms can also be represented using pseudocode. The steps of the algorithm are written in plain English, with key words in bold and indentations to indicate how the algorithm should be applied to an input. The numbers on the left refer to each line in the algorithm.

Here is the same algorithm as in the flow chart on the previous page, written in pseudocode.

```
1  input side lengths a, b, c
2      if (a + b > c) and (b + c > a) and (a + c > b) then
3          print This is a triangle.
4      else
5          print This is not a triangle.
6      end if
```

A function can be defined in pseudocode to complete a specific task for a given input. This is done to simplify pseudocode. The function, once defined, can be used in the future without rewriting all the steps in the function.

Check your understanding

2 Answer the following questions.

a What are the inputs to the algorithm?

b Run the algorithm with 4, 5, 8 as the inputs. Write down all steps as you follow the algorithm.

c Describe the role of the terms **input**, **if**, **and**, **print** and **else**. How are these represented in the flow chart?

d Line 1 in the algorithm can be replaced with the following to define a function that checks if three side lengths can form a triangle.

```
1  define CheckTriangleInequality(a, b, c)
```

Write the updated algorithm which defines the function. What is the output of CheckTriangleInequality(2, 3, 6)?

Task 1

An alternative way to express triangular inequality is checking these conditions:

$$a + b > c$$
$$a - b < c$$

where $a \geq b$.

1 A triangle has sides with lengths 5 units, 8 units and x units. What are the possible values of x?

2 Use a flow chart to represent this alternative version of the triangular inequality.

3 Use your flow chart to determine whether three line segments with lengths 10 units, 15 units and 6 units can form a triangle.

4 Represent this alternative version of the triangular inequality in pseudocode.

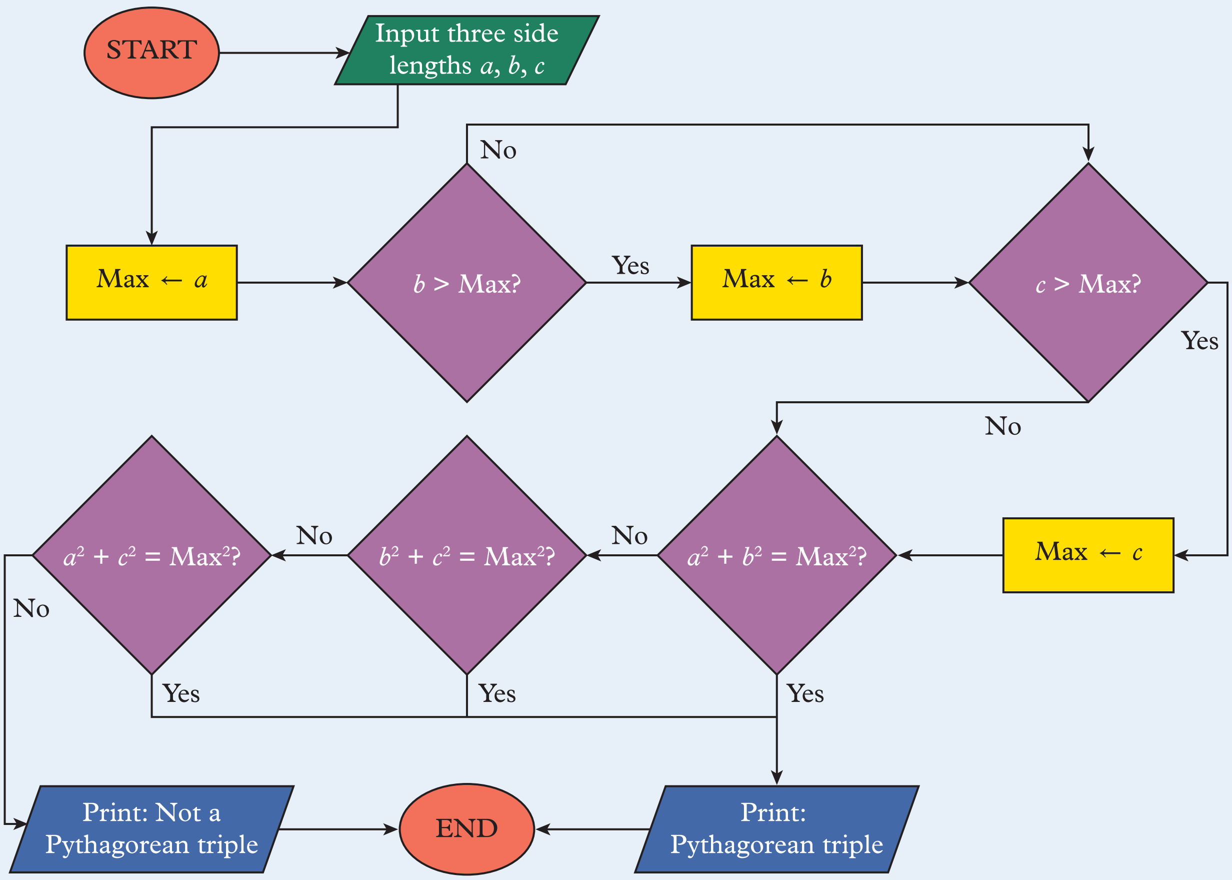
5 If $b \geq a$, what inequality should you check instead of $a - b < c$?

6 Update your flow chart and pseudocode algorithms based on your answer to question **4** so that you do not need to assume $a \geq b$.

Pythagorean triples

If three line segments with lengths a, b and c satisfy Pythagoras' theorem ($a^2 + b^2 = c^2$), then they form a right-angled triangle. The three positive integers a, b and c are called a Pythagorean triple, which is often written as (a, b, c).

Below is an algorithm, represented as a flow chart, that checks whether three inputs form a Pythagorean triple. Arrows pointing left are used to store and update the value of a valuable. The algorithm first determines which variable is the greatest in value, stores it as the variable 'Max', and then checks the possible variations of Pythagoras' theorem.



The following pseudocode algorithm defines the same algorithm as the flow chart above as a function so it can be run with different inputs more easily. Comments are added after # (hash) symbols to explain the purpose of specific lines of code.

```
define CheckPythagoreanTriple(a, b, c)
    Max ← a                # stores a as the variable Max (i.e. maximum value)
    if b > Max then
        Max ← b
    end if
    if c > Max then
        Max ← c
    end if
    if (a² + b² = Max²) or (b² + c² = Max²) or (a² + c² = Max²) then
        print Pythagorean triple.
    else
        print Not a Pythagorean triple.
    end if
```

Check your understanding

3 Answer the following questions.

a Determine the value of x given that (6, x, 10) is a Pythagorean triple.

b Write comments to explain the **if** control statements in lines 3–5 and 6–8 (or, equivalently, the first two decision points in the flow chart).

c Run CheckPythagoreanTriple(3, 5, 4) (or the flow chart algorithm with inputs 3, 5, 4). What happens at each step of the algorithm?

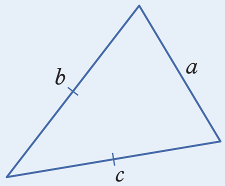
d Explain the difference between the terms '**or**' and '**and**' in the pseudocode algorithm.

e How could the algorithm above be simplified?
Hint: Can you write it without introducing the variable Max?

Classifying triangles

Triangles can be classified by their side lengths and by their internal angles. Suppose a triangle has three sides with lengths a, b and c, where c is its longest side.

Pythagoras' theorem and two related inequalities can be used to classify a triangle by its internal angles, using only its side lengths.



$a^2 + b^2 > c^2$
$a \neq b = c$

Classify by angles	Definition	Check
Right-angled	Has a right angle	$a^2 + b^2 = c^2$
Obtuse	Has an obtuse angle	$a^2 + b^2 < c^2$
Acute	All angles are acute	$a^2 + b^2 > c^2$

Triangles can also be classified by their side lengths.

Classify by side lengths	Definition	Check
Equilateral	Three equal sides	$a = b = c$
Isosceles	Exactly two equal sides	$a = b \neq c$ or $a \neq b = c$
Scalene	No equal sides	$a \neq b$, $a \neq c$, $b \neq c$

To fully classify a triangle, classify it by both its side lengths and angles. For example, a triangle may be an isosceles acute triangle.

Task 2

Represent the algorithms required in this task as a flow chart and then try to write them again using pseudocode.

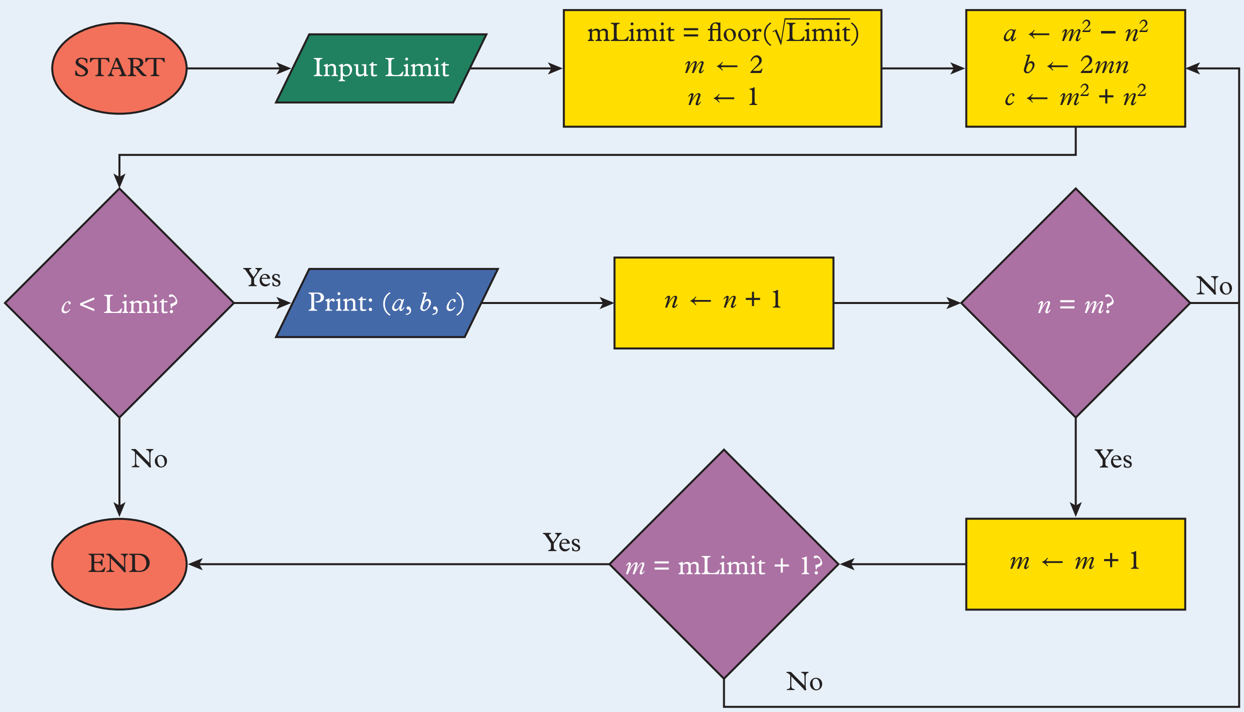
1. Write an algorithm to define the function ClassifyAngles(a, b, c), such that it takes three side lengths of a valid triangle as input, where c is its longest side, and prints the type of triangle according to its angles.
2. Write an algorithm to define the function ClassifySides(a, b, c), such that it takes three side lengths of a valid triangle as input, where c is its longest side, and prints whether the triangle is equilateral, isosceles or scalene.
3. Write a single algorithm to define a function ClassifyTriangles(a, b, c), such that it takes the lengths of three line segments as input and prints whether a triangle can be formed, and if yes, also prints the type of triangle by side lengths and internal angles. You may use any previously defined functions.

Generating Pythagorean triples

Euclid's formula is a common method to generate many Pythagorean triples. It states that for any two positive integers m and n where $m > n > 0$, the Pythagorean triple (a, b, c) can be generated as

$$a = m^2 - n^2$$
$$b = 2mn$$
$$c = m^2 + n^2.$$

Euclid's formula can be implemented using the flow chart below to generate many Pythagorean triples, where all values are less than a chosen limit. In this algorithm, all values need to be integers, so the 'floor' function is used to round the square root of Limit down to the closest integer.



The following pseudocode algorithm defines the same algorithm as the flow chart above as a function, so it can be run with different inputs more easily. In this version of the algorithm, two '**for**' loops are used to iterate over the possible values of m and n.

```
define GeneratePythagoreanTriple(Limit)
    mLimit ← floor(√limit)          # round (√limit) down to closest integer, store as mLimit
    for m from 2 to mLimit          # range over integer values 2 ≤ m ≤ mLimit
        for n from 1 to m − 1       # range over integer values 1 ≤ n ≤ m − 1
            a ← m² − n²
            b ← 2mn
            c ← m² + n²
            if c < Limit then
                print (a, b, c)
            else
                break
            end if
        end for
    end for
```

Check your understanding

4 Answer the following questions.

a Simplify the following values.

i floor(6.2) **ii** floor(5.9) **iii** floor($\sqrt{18}$) **iv** floor($\sqrt{30}$)

b Show that the values of a, b and c generated using Euclid's formula form Pythagorean triples.

c What are the smallest possible values of m and n? Which Pythagorean triple do these values produce?

d When the execution of the pseudocode reaches line 12, which line of code does it execute next? What happens to the variable n?

e Use the algorithm above to generate Pythagorean triples with values less than 30. Copy and complete the following table with the values of all variables, adding more rows as necessary.

m	n	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$

f Which Pythagorean triple is the last one to be printed by the algorithm and why?

Task 3

A primitive Pythagorean triple is a Pythagorean triple (a, b, c) where the highest common factor (HCF) of a, b and c is 1. Consider the values of a, b and c defined as before in terms of m and n. The Pythagorean triple (a, b, c) is primitive if:

- $(m - n)$ is odd, and
- the HCF of m and n is 1.

1 Amend the algorithm that defines GeneratePythagoreanTriple(Limit) to generate primitive Pythagorean triples with values less than a chosen limiting value.

2 Use your algorithm to generate all primitive Pythagorean triples with all values less than 40. Copy and complete the following table to keep track of the variables in the algorithm, adding more rows as necessary.

m	n	$m - n$	HCF(m, n)	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$

Extension

An alternative algorithm to generate primitive Pythagorean triples uses the following formulas.

$$a = uv \qquad b = \frac{u^2 - v^2}{2} \qquad c = \frac{u^2 + v^2}{2}.$$

Here u and v are positive, odd integers where $u > v$ and the HCF of u and v is 1.

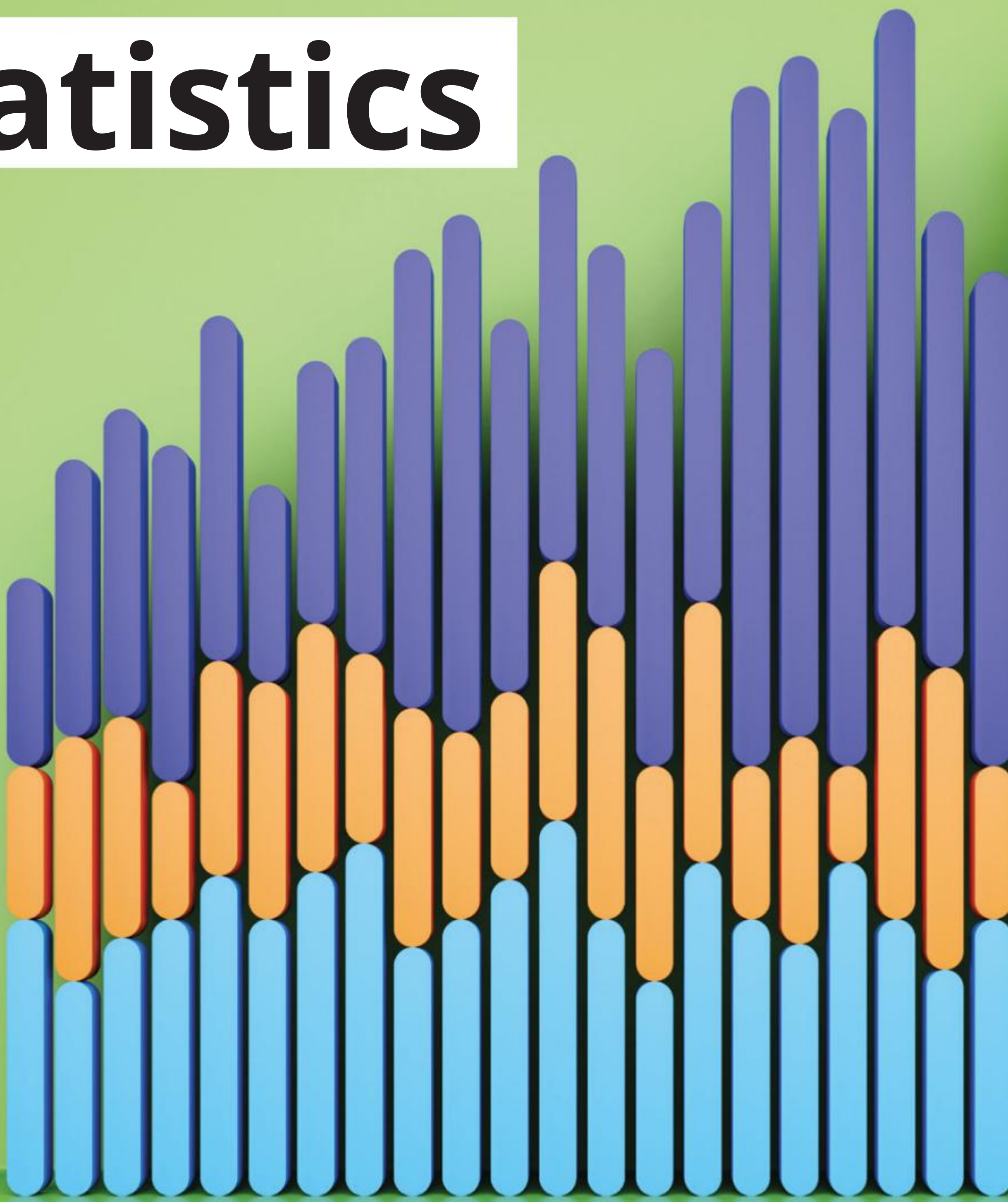
a Write a pseudocode algorithm to generate all primitive Pythagorean triples with values less than a limiting value.

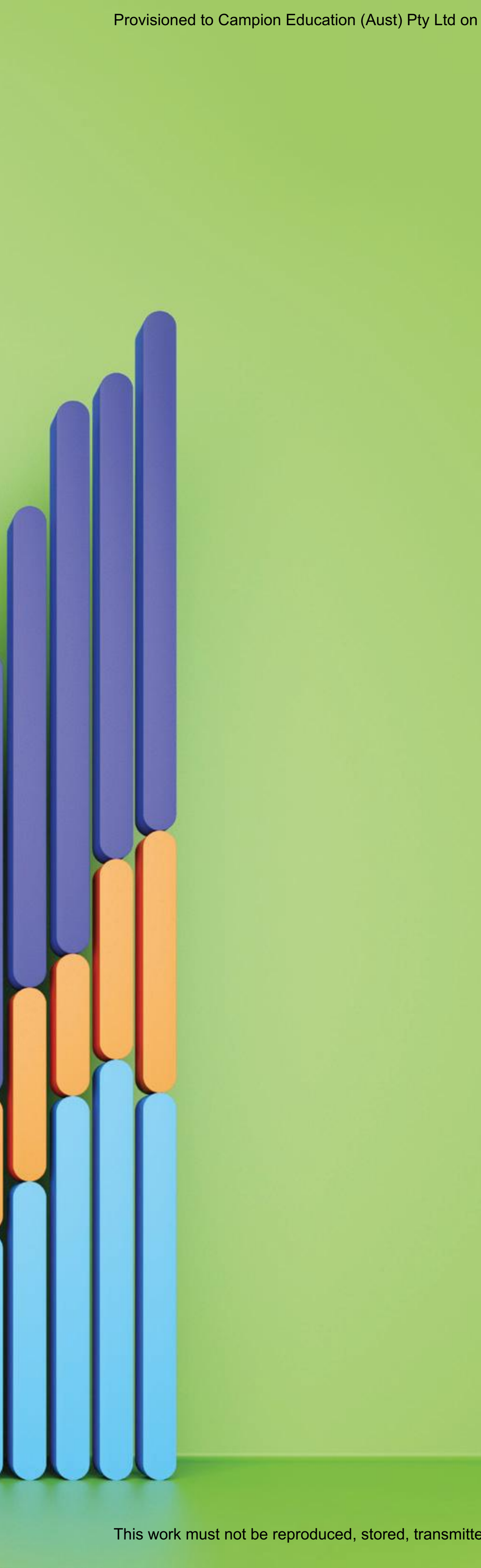
b Use the pseudocode above to generate all primitive Pythagorean triples with all values less than 40. It may help to use a table with the values of all variables, like the one below, to iterate through the algorithm.

i	u	j	v	HCF(u, v)	$a = uv$	$b = \frac{u^2 - v^2}{2}$	$c = \frac{u^2 + v^2}{2}$

7

Statistics





Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

✔ Interpreting graphs
✔ Stem-and-leaf plots

Curriculum links

- Analyse reports of surveys in digital media and elsewhere for information on how data was obtained ... to estimate population means and medians (VC2M9ST01)
- Analyse how different sampling methods ... can affect the results of surveys and how choice of representation can be used to support a particular point of view (VC2M9ST02)
- Represent the distribution of multiple data sets for numerical variables using comparative representations ... interpret numerical data sets with consideration of centre, spread and shape, and the effect of outliers on these measures (VC2M9ST03)
- Choose appropriate forms of display or visualisation for a given type of data; justify selections and interpret displays for a given context (VC2M9ST04)
- Plan and conduct statistical investigations involving the collection and analysis of different kinds of data; report findings and discuss the strength of evidence to support any conclusions (VC2M9ST05)

Surveys and sampling methods

Learning intentions

By the end of this lesson you will be able to ...

✔ identify sampling methods and evaluate surveys and statistical reports.

Inter-year links

Year 8 10.1 Collecting data and sampling methods

Surveys

- In statistics, a **population** is every potential piece of data under consideration.
- A **survey** is a list of questions designed to extract information about a population.
- Surveys usually take a **sample** of a population, and use the sample data to make **inferences** (conclusions drawn from evidence and reasoning) about the population.
  - → A **census** is a survey of an entire population and gives perfectly accurate data. However, taking a census is often impractical, expensive and time-consuming.
- Population means and medians can be estimated by using the information obtained in surveys.

Sampling methods

Key content video
Sampling methods

- The goal of selecting a sample is to get an accurate representation of the population, which means it is important to select the sample randomly to avoid possible **bias**.
  - → Bias is an inclination towards one person, group of people, thing or idea over another.
- When collecting a random sample from a population, natural variation means the sample statistics are unlikely to exactly match the population parameters.
  - → Large samples are more accurate and should better reflect the population parameters than small samples (assuming they are random).
- **Stratified sampling** divides the population into small groups known as strata (for example, based on age) and takes a random sample from each stratum. The size of each sample is proportional to how large the category is.
- **Systematic sampling** selects data at fixed intervals (for example, every fifth person). The starting point should be random.
- Non-random sampling methods include **convenience sampling** and **quota sampling**.
  - → Convenience sampling is when a sample that is easily accessible is selected.
  - → Quota sampling is similar to stratified sampling, but with the researcher selecting the size of each group.

Evaluating statistical reports

Key content video
Evaluating surveys and statistical reports

- When evaluating a statistical report or claim, you need to consider how the information was collected and how the information has been interpreted.
- Data should be fairly represented, so as not to skew the outcome. Presenting only some of the information or selecting inaccurate statistics to describe the data can be misleading.

Worked example 7.1A Identifying the sampling method

Jordan is conducting a survey to find out people's favourite foods. She asks people on the main street to complete her survey and finishes when she has 30 respondents. Determine the sampling method Jordan used.

THINK

This sample was taken from people who were easily accessible and is not randomly selected, so it is an example of convenience sampling.

WRITE

This is a convenience sample.

Worked example 7.1B Evaluating surveys

Decide whether each of these surveys will provide fair or biased results, giving a reason for your answer.

a Asking residents of every 20th household from a local community register if they enjoy running, to determine how many people in the local community exercise.

b Surveying 1000 random households in Melbourne in order to find the average number of people living in residential homes in Victoria.

THINK

a 1 Consider the surveying technique. Is the sample large enough and gathered in an unbiased way so that it theoretically represents the population (the local community)?

2 Consider the question asked. Does it answer what the questioner is trying to discover (how many people exercise)?

b Consider the surveying technique. Is the sample large enough and gathered in an unbiased way so that it theoretically represents the population (people in Victoria)?

WRITE

a The sample is randomly selected from the community and is of sufficient size. However, this survey is likely to provide biased results because the question does not ask if people exercise, just if they enjoy running.

b This survey is likely to provide biased results because, although the sample is of a reasonable size, it only samples from one city in Victoria and is therefore unlikely to be representative of the whole population.

Worked example 7.1C Identifying how a statement might misrepresent data

Explain how the statement 'Cricket is the world's most popular sport' might be a misrepresentation of the data that 67% of people surveyed at cricket matches in 15 countries answered that cricket was their favourite sport.

THINK

1 Look at the sample chosen. Does it reflect the population (the world)?

2 Look at the question or result given. Is the conclusion fair?

WRITE

This statement misrepresents the data by using a biased sample (people at cricket matches), who are much more likely to answer that cricket is their favourite sport than people not attending a cricket match. The question was also only asked in 15 countries, not every country in the world.

Worked example 7.1D Analysing surveys and results

Caleb wanted to find information about the age of people at a skate park. He surveyed every 10th person he saw there over one week, surveying a total of 20 people. Summary statistics gave a mode of 17, a median of 18, a mean of 22 and a range of 70. Caleb concluded that the mean age of people at the skate park of 22 was the most appropriate summary statistic to represent the data.

a Decide whether Caleb's data collection method is fair.

b Decide whether the interpretation of the data is fair.

c If appropriate, provide a suggestion to improve the survey.

THINK	WRITE
a Is the sampling method random, with an equal chance of selecting every person in the population?	**a** The method appears fair, although it could be biased depending on the times that Caleb visited the skate park. The sample could be larger.
b Does the conclusion fairly interpret the statistics?	**b** The interpretation could be improved. The sample size is small and the mean is distinctly larger than the median, which, along with a large range, implies the presence of an outlier. The median would be a better statistic to use.
c Are there any ways to improve this survey?	**c** Caleb could take a larger sample and ensure that he visits the skate park at different times of the day. His interpretation could also be improved by using the median as the measure of centre.

Helpful hints

✔ Make sure samples are taken from the whole population, not from a subset of the population. A sample will only be random if it is randomly selected from the whole population.

✔ Be sceptical when evaluating claims which use statistics. Data can be misrepresented, either intentionally or unintentionally, so consider the source of the data, and if possible, analyse it yourself.

Learning pathways

△ 1, 2–4(a, b), 6–8, 12

□ 1, 2–4(c, d), 5, 8–10, 13, 14

⬠ 1, 2, 4(c, d), 5, 9–12, 14–16

ANS p507

Exercise 7.1A: Understanding and fluency

WE 7.1A 1 Determine the sampling method used in the following situations.

a Xavier calls every 100th phone number from a register to complete a survey.

b Mia surveys 20 students from public and private schools in proportions that reflect the town's population.

c Ali surveys the first 50 people who will answer his questionnaire about school dinners.

d Tina surveys 10 adults from each age group (20–29, 30–39, etc.) about their voting intentions.

WE 7.1B **2** Decide whether each survey below will provide fair or biased results, giving reasons for your answers.

a Asking 500 random people at an AFL match their favourite sport, in order to determine the country's favourite sport

b Asking everybody at your school their opinion on a school issue

c Asking every 10th person on the electoral roll if they like dogs, to determine the most popular pet

d Asking a random sample of 100 people (stratified by age) from around the country who they think should be prime minister at the next election

WE 7.1C **3** Explain how each of the following headlines might be misrepresenting the data described in brackets.

a Hip-hop is the most popular music genre.' (70% of people surveyed said they enjoy listening to hip-hop.)

b 'New Apple tablet tops popularity stakes.' (48 people out of 50 surveyed in an Apple store said the Apple tablet was their favourite.)

c 'Women now earn more than men.' (10 people were randomly surveyed on a street.)

d 'New anti-ageing cream is unbeatable.' (100% of the anti-ageing cream company's shareholders were surveyed and agreed.)

WE 7.1D **4** For each scenario below:

i decide whether the data collection method is fair

ii decide whether the interpretation of the data collected is fair

iii if appropriate, provide a suggestion to improve the survey.

a Tess wanted to know which is the most popular beach in Victoria. She asked every 10th person on the electoral roll in her town, 'What is your favourite beach in Victoria?' Seventy-five per cent of 200 people said that Seaford was their favourite beach, so Tess concluded that Seaford was the most popular beach in Victoria.

b Carlos wanted to find information about the average height of basketball players in his town. He surveyed 10 players at random. Summary statistics gave a mode of 1.82 m, a median of 1.80 m, a mean of 1.79 m and a range of 25 cm. Carlos concluded the median height of 1.80 m of basketball players in his town was the most appropriate summary statistic to represent the data.

c Isaac wanted to know the most popular computer brands. He asked a random sample of 1000 people, 'Do you like Macs?' He found that 60% of people said yes. Isaac concluded that Mac computers were the most popular brand of computer.

d Asha wanted to know the most popular brand of soft drink in town. She asked all the local supermarkets which brand they sold the most of and found that Coke made up 37% of the sales, Pepsi 29%, Schweppes 17% and all other brands made up the remainder. She concluded that no single brand was the most popular.

Exercise 7.1B: Problem solving and reasoning

5 The marketing manager of a drug company claims that the company's new pain relief drug blocks pain for up to 18 hours. This is based on a sample of 15 people recording the number of hours they were pain-free. Here is that data:

2	18	3	2	2
1	2.5	2	1.5	2.5
2	1.5	2	1.5	1.5

a Why is the company's marketing manager making this claim? Explain why it is misleading.

b Calculate the three measures of centre (mean, median and mode) for the given data.

c The marketing manager changes tactics and decides to claim that the pain relief drug blocks pain for an average of 3 hours. Comment on this claim. Is the claim now fair? Explain.

6 What questions should you ask yourself when you are presented with a claim that uses statistics?

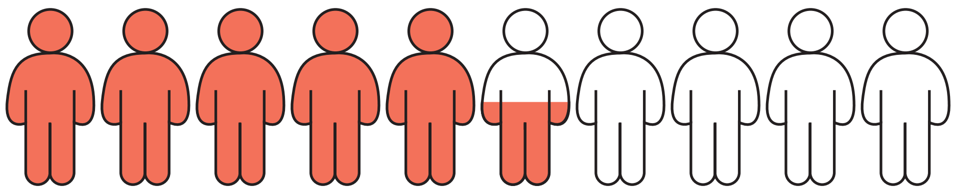
7 Each of these media statements is followed by a 'fine print' statement in brackets qualifying the original claim. Explain why the claims are not so 'amazing' when you read the fine print.

a An advertisement for a phone's battery life claims it 'lasts up to 7 days' (based on standby power consumption).

b An advertisement for toilet paper claims it is 'Voted Australia's favourite!' (a total of 42 people were surveyed).

c An advertisement for a moisturising lotion states '67% of women notice a difference after just one week!' (compared to the use of regular soap).

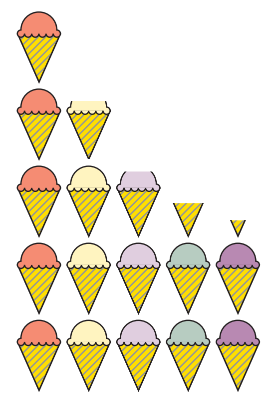
8 The following pictogram was created for an infographic about the popularity of the current government.



a Using the pictogram, what percentage of people do you think are satisfied with the current government?

b Discuss the strengths and weaknesses of using this pictogram to display the data.

9 A gelateria commissioned the pictogram on the right as part of their internal research into their most popular flavours.



a What is the purpose of the pictogram?

b What information does the pictogram provide?

c What information is missing in the pictogram?

d How well do you think the pictogram communicates the data?

10 Over the past 20 years, there has been a steady decrease in the use of landline phones and an increase in the screening of phone calls from unknown numbers. Discuss the impact that both of these factors may have had on collecting data from surveys.

11 Online surveys are very common. However, for an online survey to have any credibility, many issues need to be considered. For example:

- How can we ensure that a random sample has been surveyed that is representative of the population?
- How can we ensure that people completing the survey are who they claim to be?
- How do we deal with the non-response rate?

Write some guidelines you feel would be important to follow when choosing a sample for an online survey.

12 Use the statistics shown in the following table to comment on how the operating profit before tax of different Australian industries varied during the 2010s.

	Industry				
Year	**Mining**	**Manufacturing**	**Construction**	**Retail**	**Media**
2010–11	$85 156 m	$27 986 m	$30 282 m	$19 860 m	$10 430 m
2012–13	$54 374 m	$16 353 m	$26 340 m	$19 417 m	$8116 m
2014–15	$16 295 m	$20 652 m	$36 027 m	$18 724 m	$9400 m
2016–17	$33 258 m	$24 639 m	$36 043 m	$18 847 m	$7054 m
2018–19	$98 551 m	$33 822 m	$43 485 m	$17 582 m	$2935 m

(Source: ABS)

Note: figures are provided for every other year of the decade.

13 Consider the table and text below, which were published online.

'Australia has been ranked among the top 10 countries for its handling of the coronavirus pandemic, with New Zealand taking out the top spot in a poll conducted by a prominent Australian think tank.

The Lowy Institute assessed the response of 98 countries in how they managed the pandemic in the 36 weeks following their hundredth confirmed case of the virus.

It judged countries that had fewer reported cases and deaths (both in aggregate and per capita basis), as well as nations where testing rates were high.

China was excluded from the ranking because of a lack of publicly available data on testing.

The United States, Brazil, Iran and Mexico were ranked the worst handlers of the pandemic.

Countries that proved more successful in containing the virus were largely in the Asia-Pacific region.'

Rank	Country
1	New Zealand
2	Vietnam
3	Taiwan
4	Thailand
5	Cyprus
6	Rwanda
7	Iceland
8	Australia
9	Latvia
10	Sri Lanka

(Source: SBS news)

a From the table, where is Australia ranked in terms of their response to handling the coronavirus pandemic?

b Why would it be misleading to say that Australia ranked 8th out of 10 countries?

c Was the information collected a sample or a census? Justify your answer.

d Does this data support the claim that 'Australia has been ranked among the top 10 countries for its handling of the coronavirus pandemic'?

e The article does not directly provide the data to support its claims. What could you do to research this?

14 Find a media article that uses statistics to support its claims (including at least one visual display). Write a paragraph analysing these claims and commenting on how trustworthy you think the article is.

Exercise 7.1C: Challenge

15 Write an informative guide to teach somebody how and what to look for when reading statistics and the claims that they are supposedly supporting.

16 Conduct your own statistical investigation using data taken from the ABS (Australian Bureau of Statistics). Ask some questions in your investigation and provide answers after you have analysed the data. Provide a conclusion at the end of your report.

Online resources:

- **Interactive skillsheet** Sampling methods
- **Worksheet** Analysing statistics
- **Investigation** Comparing annual rainfall
- **Quick quiz** 7.1

Lesson 7.2

Classifying and displaying data

Learning intentions

By the end of this lesson you will be able to …

✔ classify, display and interpret data.

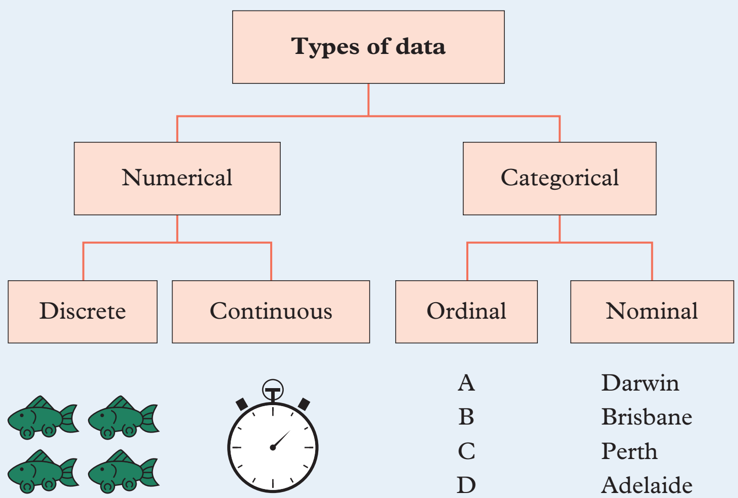
Inter-year links

Support	Interpreting data
Year 7	5.5 Line graphs
Year 10	10.6 Scatterplots

Classifying data

Key content video
Classifying data

- **Numerical data** can be classified as either **discrete** (countable) or **continuous** (measured).
- **Categorical data** can be classified as either **ordinal** (in a defined order) or **nominal** (unordered categories).



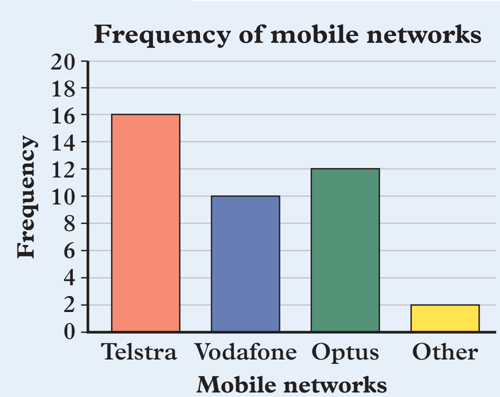
- **Univariate data** is data that only concerns a single variable.
- **Bivariate data** is data that shows the relationship between two variables. Bivariate data often falls within more than one classification. For example, comparing the amount of rainfall in different cities involves a numerical variable, the amount of rainfall, and a categorical variable, the city.
- **Multivariate data** is data that shows the relationship between three or more variables.

Displaying data

Key content video
Interpreting data displays

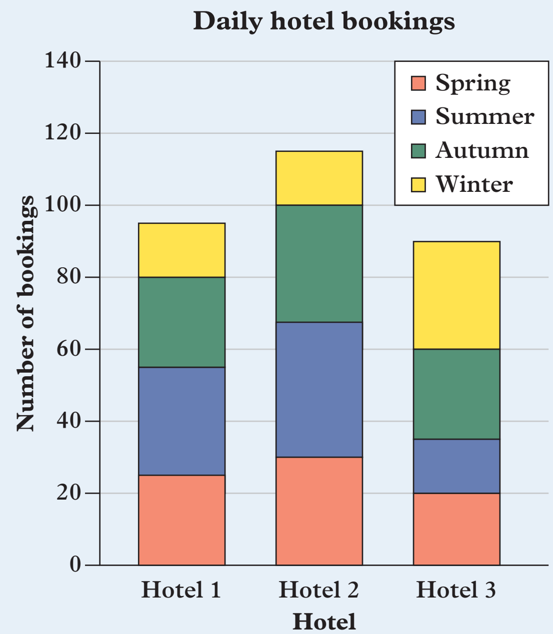
- **Bar charts** (or column graphs) can be used to display either univariate or bivariate data.
  → The horizontal axis will display either a discrete numerical variable or a categorical variable.
  → The vertical axis will display either the frequency (for univariate data) or a numerical variable (for bivariate data).

  For example, the bar chart on the right is displaying one categorical variable, with the frequency detailed in the vertical axis.

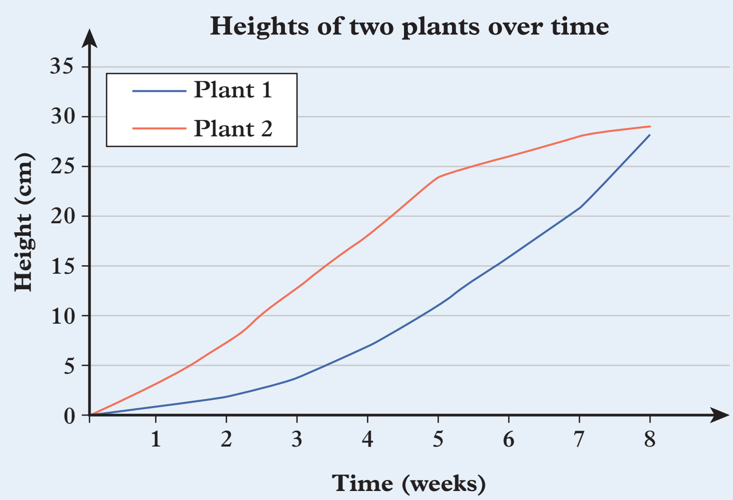


- A **stacked bar chart** displays an additional categorical variable within each bar.
  - → Each bar is divided into sections based on the additional categorical variable, which is detailed in a legend.

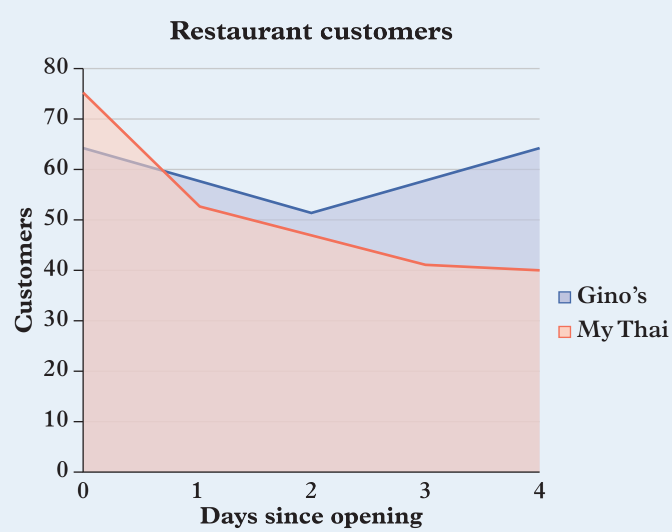
  For example, the stacked bar chart on the right is displaying two categorical variables (one on the horizontal axis and one in the legend), with a numerical variable on the vertical axis.



- **Line graphs** can be used to display data involving two numerical variables.
  - → The horizontal axis must be a continuous variable and usually displays the independent variable. The vertical axis usually displays the dependent variable. For example, the height of a plant depends on the length of time it has been growing.
  - → Travel graphs are an example of line graphs.
  - → If necessary, a break in the scale on the vertical axis, indicated by a zigzag, can be used to show the focus of the graph is a distance away from the origin.



- **Multiple line graphs** can be used to display data involving two numerical variables and one categorical variable. The categorical variable is detailed in the legend.
- **Area charts** shade the area between the lines and axes of a multiple line graph to create a sense of quantity over time.
  - → Area charts can be used to display data with two numerical variables (on the axes) and one categorical variable (in the legend).
  - → Area charts should only be used when it makes sense for the numerical variable in the vertical axis to accumulate over time. For example, in the multiple line graph above you cannot determine the total height of the plants by summing the values at each week, so an area chart should not be used for this data.



- All graphs should include clearly labelled axes with evenly spaced scales and a title and legend where necessary.

Worked example 7.2A Classifying bivariate data

A journalist is researching the time spent commuting to work from different suburbs. Identify each variable in this situation as either numerical or categorical.

THINK

1 Identify the two variables: time duration and name of suburb.

2 Classify each of these variables from the previous step. Time is a numerical variable and the suburb is a categorical variable.

WRITE

Time: numerical
Suburb: categorical

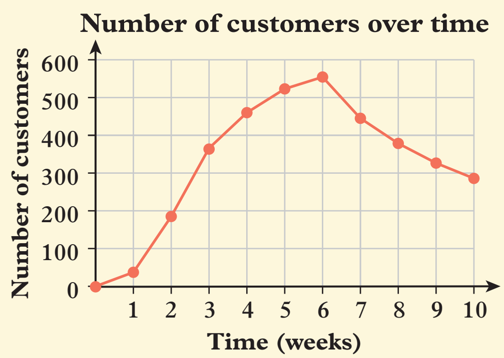
Worked example 7.2B Interpreting line graphs

Consider the line graph on the right.

a Classify the variables in the graph.

b What time period does it cover?

c Between which weeks does the number of customers increase the fastest?



THINK

a There are two variables in a line graph: a numerical variable on the vertical axis and a different numerical variable on the horizontal axis.

b Look at the range on the horizontal axis.

c Identify between which two points was the greatest increase on the vertical axis.

WRITE

a Time (weeks): continuous
Number of customers: discrete

b 10 weeks

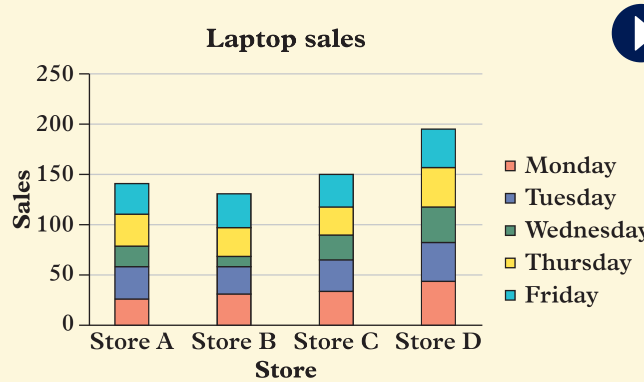
c Weeks 2 and 3

Worked example 7.2C Interpreting stacked bar charts

Consider the stacked bar chart on the right.

a Which variables are displayed in the graph?

b Which store sold the most laptops on the Monday?



THINK

a Look at the title of the axes and the legend.

b Identify the tallest section representing Monday, which is the bottom part of each bar. Use the horizontal axis to interpret which store this represents.

WRITE

a laptop sales, store and day

b Store D

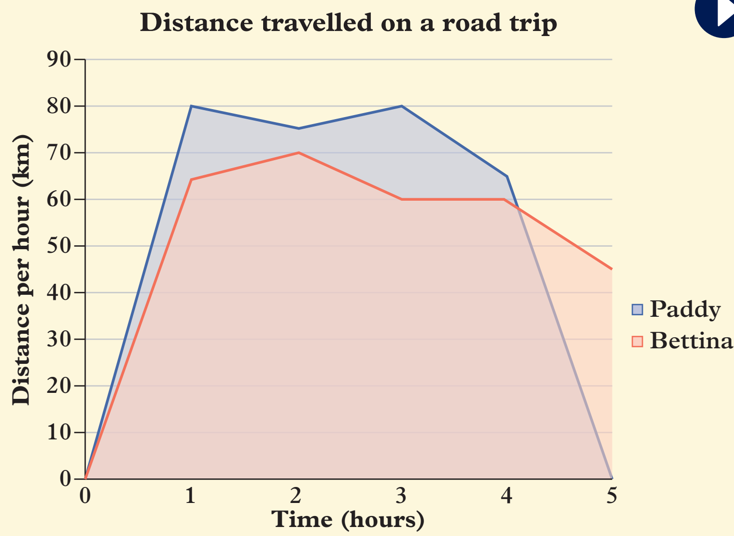
Worked example 7.2D
Interpreting area charts

Consider this area chart.

a Classify the variables in this chart.

b Who covered the most distance in an hour?

c Who completed their road trip first?



THINK

a There are three variables in an area chart: two numerical variables on the axes and a categorical variable in the legend.

b Identify the line with the highest point on the chart. Use the legend to identify who this represents.

c Identify which line reached 0 km first. Use the legend to identify who this represents.

WRITE

a Distance: continuous
Time: continuous
Person: categorical

b Paddy covered the most distance in an hour.

c Paddy completed his road trip first.

Worked example 7.2E Drawing a line graph

The data in the following table was recorded over 10 days.

Day	0	1	2	3	4	5	6	7	8	9
Weight of puppy (g)	92	100	110	125	123	131	142	150	156	164

Draw a line graph to represent this data.

THINK

1 The independent variable (days) should be placed on the horizontal axis and the dependent variable (weight) should be placed on the vertical axis.

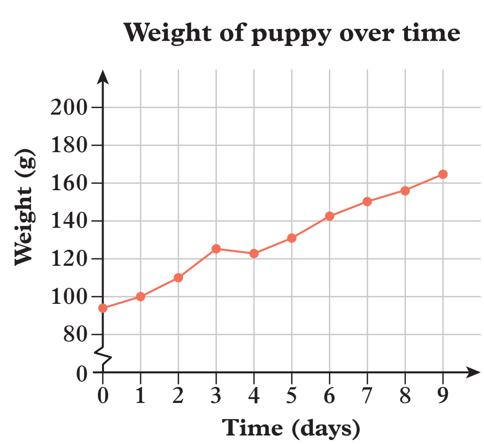
2 Draw the vertical axis. As the values start above 80, use a break in the axis to focus the graph above here.

3 Draw the horizontal scale, providing enough space to fit in all the independent values.

4 Plot the points from the table and join subsequent points with straight lines.

5 Include a title and axis labels.

WRITE



Helpful hints

- ✔ Univariate (single variable) data is usually displayed in a column graph with the frequency of the categories on the vertical axis. Bivariate data consists of two distinct variables.
- ✔ When classifying numerical data, consider whether the data needs to be counted (e.g. number of dogs) or measured (e.g. weight of dogs). Discrete data is counted, whereas continuous data is measured.
- ✔ When classifying categorical data, if the categories can be written in any order and still make sense (e.g. hair colour) the data is nominal, whereas if only one order makes sense (e.g. A, B, C, D) the data is ordinal.

Learning pathways

△ 1–5, 7, 8, 12, 14, 16 □ 2–4, 6, 7, 9–11, 15, 17, 18 ⬠ 3, 4, 6, 8, 11, 13, 15, 17–19

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Exercise 7.2A: Understanding and fluency

1 Match each of the following variables with the data classification.

- **a** Eye colour of students
- **b** Number of students in each class
- **c** Time (in minutes) spent travelling to school
- **d** Final grade on an end-of-year exam (A+, A, B+, B, etc.)

- **I** Numerical – discrete
- **II** Numerical – continuous
- **III** Categorical – ordinal
- **IV** Categorical – nominal

2 Consider the following four variables.

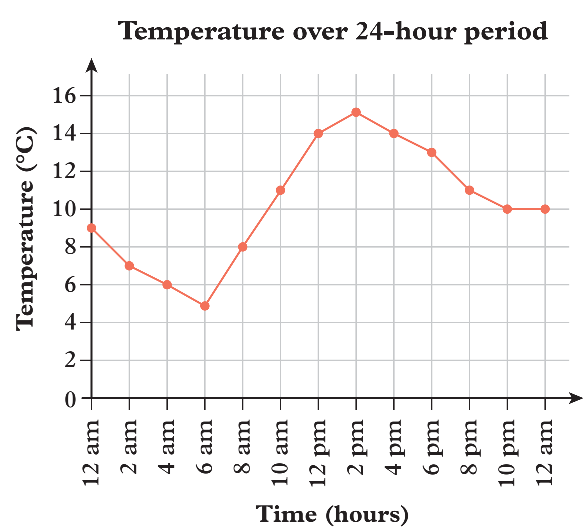
- **1** Height (in cm) of AFL players
- **2** Method of transport for commuting to school (car, bus, train, etc.)
- **3** Mark (out of 40) on a Mathematics test
- **4** Belt level in karate (white, orange, blue, etc.)
- **a** Classify each variable as numerical or categorical.
- **b** Classify each numerical variable as either discrete or continuous and each categorical variable as either ordinal or nominal.

WE 7.2A **3** In each of the following situations, identify the two variables and classify each as either numerical or categorical.

- **a** A car company hires an independent researcher to find the fuel consumption (kilometres per litre of petrol) for each of their car models.
- **b** A business owner investigates the relationship between the number of ice cream sales per day in her shop and the maximum daily temperature (in °C).
- **c** On each student's report, a school lists the student's exam score, as a percentage, for each subject.

WE 7.2B **4** Consider the line graph on the right.

- **a** What does the graph show?
- **b** Classify the variables in the graph.
- **c** What is the temperature at 10 am?
- **d** When is the temperature 15°C?
- **e** What is the coolest temperature recorded? At what time does it occur?

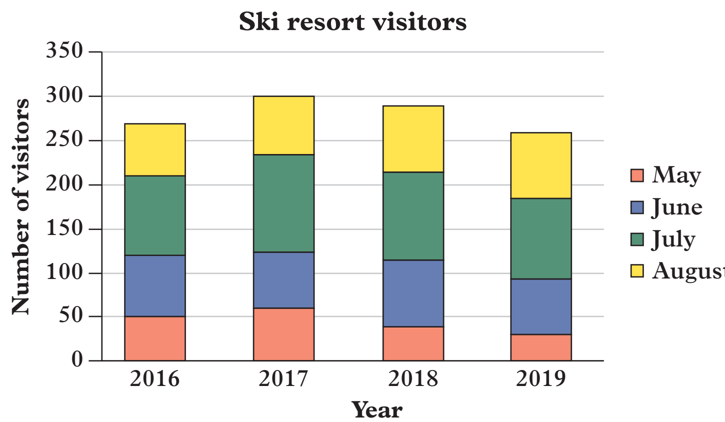


WE 7.2C **5** Consider the stacked bar chart on the right which shows the number of people who visited a ski resort during the months it was open from 2016 to 2019.

a What are the different variables in the graph?

b In which month of which year did the resort have the most visitors?

c In which month of which year did the resort have the least visitors?



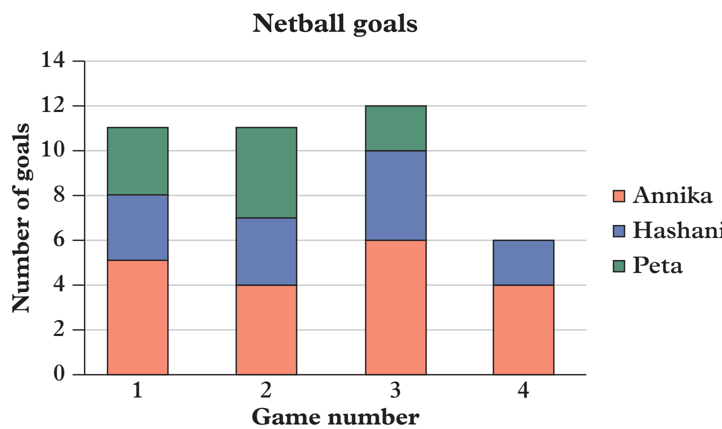
6 Consider the stacked bar chart on the right.

a What are the different variables in the graph?

b In which game did one of the three players not score a goal?

c In which game did the three players score the most goals combined?

d Which of the three players scored the least number of goals across the four games?



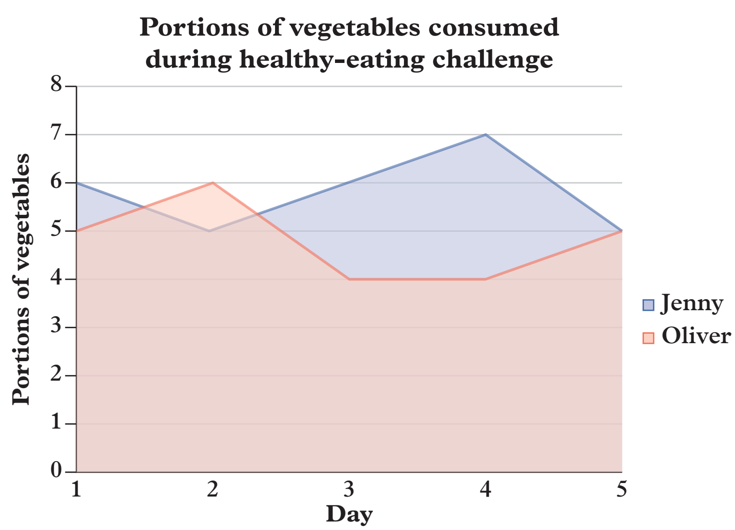
WE 7.2D **7** Consider the area chart on the right.

a Classify the variables in this chart.

b On which day did Oliver consume more portions of vegetables than Jenny?

c How many more portions of vegetables did Jenny consume than Oliver on day 4 of the challenge?

d How many portions of vegetables did Jenny and Oliver consume during the challenge?



WE 7.2E **8** The data in the following table shows the estimated market value of a gaming console for 10 months after its purchase. Draw a line graph to represent this data.

Time since purchase (months)	0	1	2	3	4	5	6	7	8	9	10
Market value ($)	600	400	350	320	300	270	250	250	220	220	220

9 The following data shows the number of kilometres driven by Antoine over a period of 9 days.

Day	1	2	3	4	5	6	7	8	9
Number of km driven	18	11	0	18	15	8	10	6	13

a Draw a line graph to represent this data.

b On which day(s) did Antoine drive the most kilometres?

c On how many days did Antoine drive more than 10 km?

Exercise 7.2B: Problem solving and reasoning

10 The following data is the monthly profit (rounded to the nearest $100) of a small business over the course of a year.

Month number	1	2	3	4	5	6	7	8	9	10	11	12
Profit ($)	4000	5300	3800	4100	4300	4000	3800	3400	5100	5400	4900	4600

a Choose an appropriate visualisation to display this data, justifying your decision.

b In which month was the greatest profit?

c In which month was the least profit?

d In which month was the greatest increase in profit over the previous month? How much was this increase?

e In which month was the greatest decrease in profit over the previous month? How much was this decrease?

11 Students in years 8, 9 and 10 were asked to choose their preferred ice-cream flavour from three choices given. The data is shown below.

	Year 8	**Year 9**	**Year 10**
Chocolate	140	135	130
Strawberry	100	90	70
Vanilla	60	75	60

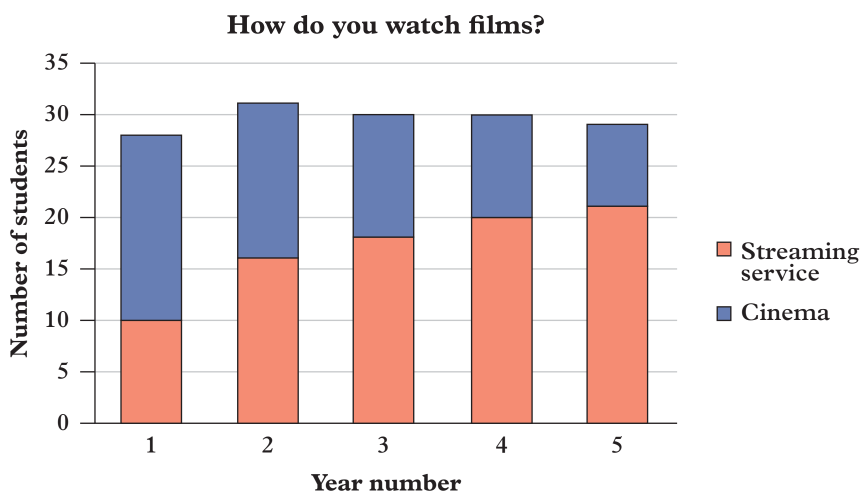
a Create a stacked bar chart for the set of data if the category on the horizontal axis is year level.

b Create a stacked bar chart for the set of data if the category on the horizontal axis is ice-cream flavour.

c Which of the two graphs is better for showing which ice-cream flavour is the most popular with Year 9 students?

d Is it accurate to say that, in general, Year 9 students prefer chocolate ice-cream more than Year 10 students? Why or why not?

12 Each year a Mathematics teacher has a Year 9 class with exactly 25 students. He asks each student, 'Do you regularly watch films on an online streaming service (like Netflix)?' and 'Do you go to the cinema regularly (once or more a month)?'. The following stacked bar chart shows the data collected over a 5-year period.



a How many students regularly watched films on streaming services in year 4?

b In which year was the number of students in each category closest to being equal?

c Explain why the values for each year can add to more than the number of students in the class.

d What would you expect the two columns to look like in 10 years, assuming the teacher continues collecting data? Explain your answer.

13 A specialty store sells Christmas items all year round. The table below shows the average number of customers per day, rounded to the nearest 10 customers, in each month over the course of 12 months.

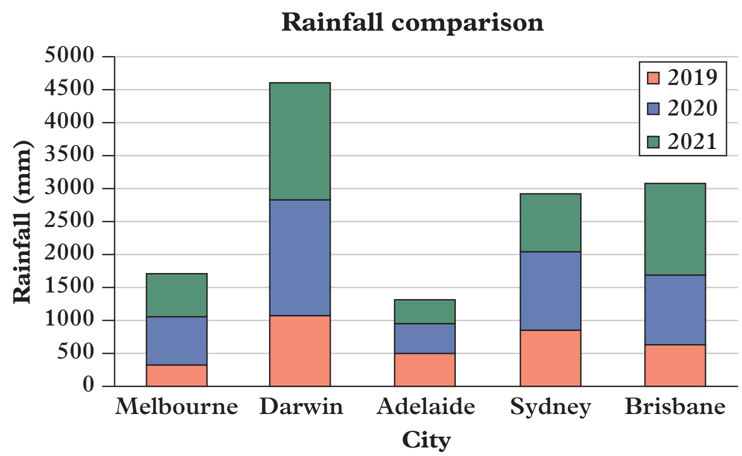
Month	1	2	3	4	5	6	7	8	9	10	11	12
Average daily customers	10	20	20	30	60	170	300	40	20	10	0	10

a Draw a line graph to represent this data.

b It is safe to assume that, in this data set, month 12 does not represent December? Which month number do you think represents December? Explain your answer.

c Is it accurate to say that the store had no customers at all in month 11? Explain your answer.

14 The following stacked bar chart shows the total rainfall (in mm) in different cities in Australia over the years 2019, 2020 and 2021.



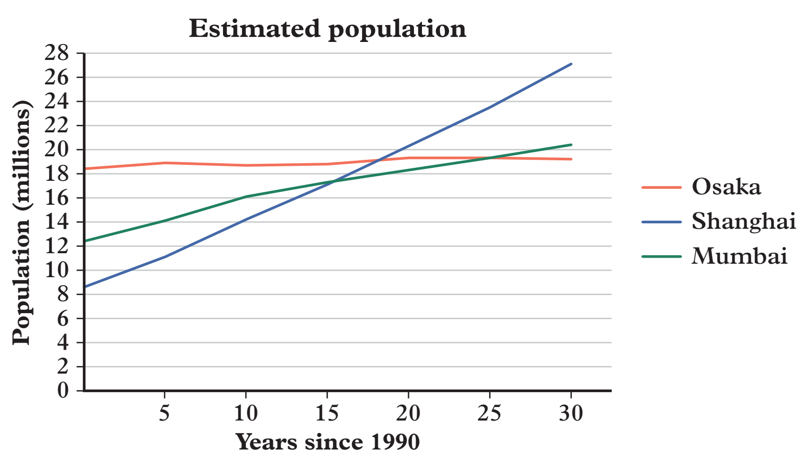
a Based on this data, which of the five cities is the wettest?

b Based on this data, which of the five cities is the driest?

c In which city did the rainfall decrease across the three years?

d Which city had the least rainfall in 2019?

15 The following multiple line graph shows the estimated metro population of three Asian cities since 1990.



a Which city had the highest population in 1990?

b Which city had the highest population in 2010?

c Approximately how many people were living in Mumbai in 2000?

d Which city had the highest population in 2020?

e In which year, approximately, did the population of Shanghai surpass the population of Osaka?

f Describe the change in each city's population between 1990 and 2020.

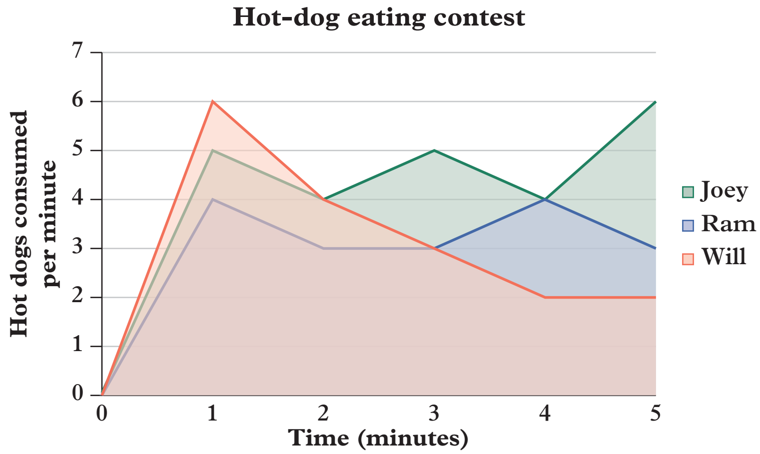
16 Consider the area chart on the right.

a Who won the hot-dog eating contest?

b Who came in second place?

c How many hot dogs were consumed by the three contestants?

d Who was most consistent over the 5 minutes? Justify your answer.



17 Otto is trying to decide which graph to use for several different research subjects. Decide which graph type (stacked bar chart, multiple line graph, area chart) Otto should choose in each instance, justifying your decisions.

a Number of steps taken by colleagues each day in a work stepathon challenge

b Favourite pizza toppings of children and adults

c Height of three children measured each year

Exercise 7.2C: Challenge

18 The table below shows the monthly profit, rounded to the nearest $1000, of a small ice-cream store in Geelong. In this case, January is represented by month 1.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Profit ($)	9000	8000	6000	3000	1000	−1000	−2000	0	3000	4000	7000	9000

a Draw a line graph to represent this data. Remember to make space below the horizontal axis for the negative values.

b Explain what the negative values in month 6 and month 7 mean.

c Explain what the 0 in month 8 means.

d What was the total profit of the ice-cream store over the course of the year?

e From month 8 to month 12 the profit is increasing each month. Do you expect this trend to continue forever? Explain your answer.

f How would you expect the monthly profit of an ice-cream store to be different for a similarly sized store in the northern hemisphere (for example, in a small town in Spain)? Explain your answer.

19 The following table shows the height (in cm) of a person over the first 15 years of their life.

Age (years)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Height (cm)	52	75	85	92	100	108	112	120	128	135	140	147	154	160	165	172

a Draw a line graph to represent this data.

b The person had a height of 52 cm when they were born and 172 cm 15 years later. Using these two points, find the average height they gained per year.

c Assuming the person continues growing at this average rate per year, how tall would they be at 25 years old?

d Instead, suggest a reasonable height that the person might be at 25 years old.

e Explain what you would expect the rest of the graph to look like if it continued to when the person was 25 years old.

Online resources:

Interactive skillsheet
Classifying data

Interactive skillsheet
Stacked bar charts

CAS instructions
Line graphs

Quick quiz
7.2

Grouped data and histograms

Learning intentions

By the end of this lesson you will be able to …

✔ understand and create histograms.

Inter-year links

Support Data displays

Year 7 10.3 Stem-and-leaf plots

Year 10 10.3 Distributions of data

Key content video
Grouped data and histograms

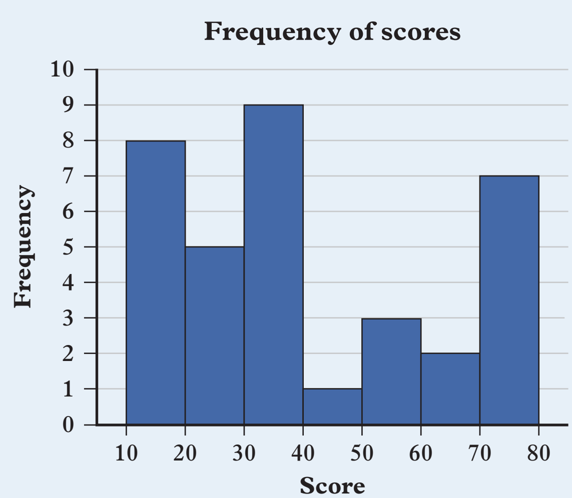
Grouping data into class intervals

- A **class interval** is a range of values that have been grouped together.
- **Grouped data** is numerical data that has been sorted into class intervals.
  Ungrouped data is raw data that has not been placed into class intervals.
- When grouping data, class intervals should be chosen so that a frequency table contains 5 to 10 classes.
  → Class intervals should start at a round number, for example, 0 or 10.
- Before constructing a frequency table with class intervals, identify whether the numerical data is discrete or continuous.
  → Discrete data uses class intervals such as 0–9, which means any value from 0 to 9 including both 0 and 9.
  → Continuous data uses class intervals such as 0–<10, which means any value from 0 to 10 including 0 but not including 10.

Discrete interval	Continuous interval
0–9	0–<10
10–19	10–<20

Histograms

- A **histogram** is a special kind of bar chart that can be used to represent grouped data.
  → Ungrouped discrete numerical data is usually represented by using a dot plot or bar chart.
- Histograms have no gap between columns, but there is a small gap between the vertical axis and the first column when the intervals do not start at 0.
- When displaying grouped data in a histogram, the **lower bound** of the class interval is shown on the left of each column and the **upper bound** of the class interval is shown on the right of each column.
  → For discrete data, where the upper and lower bounds of subsequent classes are not equal, the lower bound of the higher class interval should be shown. For example, for the discrete intervals 0–9, 10–19, 20–29, the intervals on the horizontal axis are labelled 0, 10, 20.



Worked example 7.3A Understanding histograms

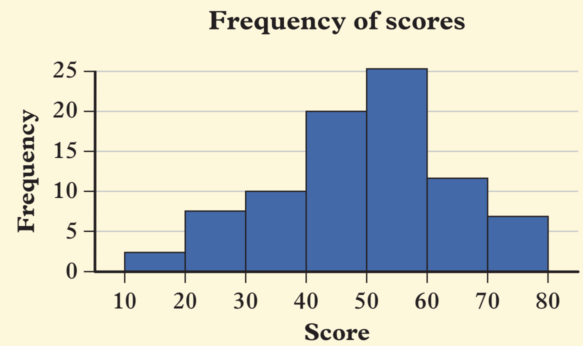
Consider the histogram on the right representing continuous data.

a What is the width of each class interval?

b What is the frequency of the class interval 30–<40?

c What is the most common class interval? What is its frequency?

d Which class has a frequency of 20?



THINK

a Look at the horizontal axis. The width of the class intervals is the width of each column.

b Look at the column covering the marks 30 to 40 and read off the frequency from the vertical axis.

c Identify the tallest column. Use the horizontal axis to identify the class interval and read off the frequency from the vertical axis.

d Identify the column with a frequency of 20 on the vertical axis, and then use the horizontal axis to identify the class interval.

WRITE

a The width of each class interval is 10.

b The class interval 30–<40 has a frequency of 10.

c The most common class interval is 50–<60 and it has a frequency of 25.

d The class interval with a frequency of 20 is 40–<50.

Worked example 7.3B Drawing a frequency table to represent grouped data

Use the frequency table below with class intervals of width 10 to represent this continuous data.

4.5, 11.6, 67.3, 33.7, 28.1, 36.4, 22.6, 54.8, 1.4, 66.8, 36.4, 29.3, 37.8, 42.3, 52.1, 38.3

Class							
Frequency							

THINK

1 Identify the minimum and maximum scores in the data set. The minimum score is 1.4 and the maximum score is 67.3.

2 The data is continuous so the class intervals must be in the form of 0–<10, etc.

3 Draw the frequency table and group the raw data into each class interval.

WRITE

Class	0–<10	10–<20	20–<30	30–<40	40–<50	50–<60	60–<70
Frequency	2	1	3	5	1	2	2

Worked example 7.3C Drawing a histogram

Draw a histogram to represent the data in this frequency table.

Class	10–<15	15–<20	20–<25	25–<30	30–<35	35–<40
Frequency	7	5	3	4	3	2

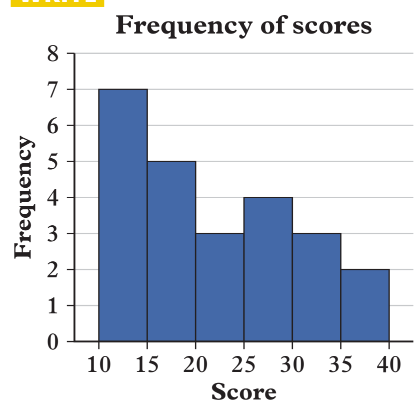
THINK

1 Draw the axes with an even scale that allows the minimum and maximum values to be shown. Ensure that there is a half space between the vertical axis and the first class interval.

2 Use the frequency table to draw the columns of the histogram.

3 Label both axes and provide a title.

WRITE



Helpful hints

- ✔ The discrete class interval 0–9 contains 10 values because it starts at 0, not 1.
- ✔ When grouping raw data, a tally column can help to ensure you don't miss any scores.
- ✔ You don't need to order the values in a data set before grouping data. Avoiding this unnecessary step can save you time when answering questions.

Learning pathways

△ 1–3, 4(a, c), 5(a, b), 6(a–c), 8, 10, 13, 16

□ 2, 4(b, c), 5(a, c), 6(d, e), 7, 11, 14, 16, 19

⬠ 4(d), 5(a, c), 6(e, f), 9, 12, 15, 17–20

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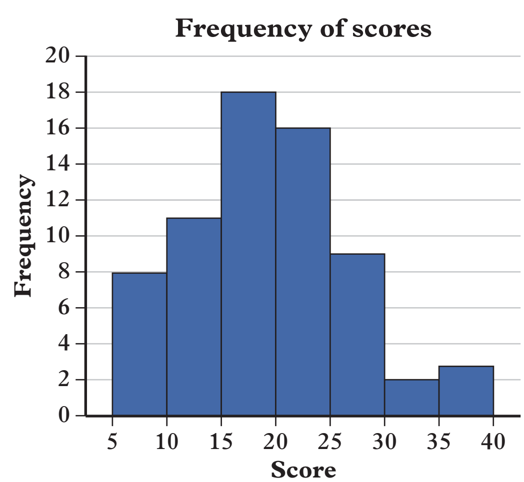
Exercise 7.3A: Understanding and fluency

WE 7.3A

1 Consider the histogram on the right representing continuous data.



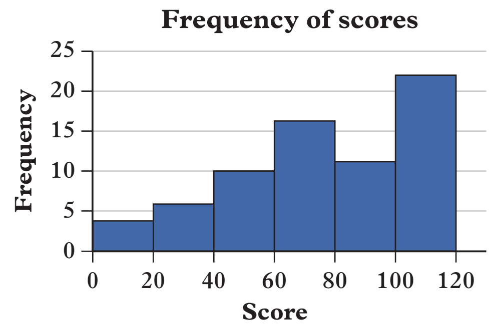
- **a** What is the width of each class interval?
- **b** What is the frequency of class interval 20–<25?
- **c** What is the most common class interval? What frequency does it have?
- **d** Which class interval has a frequency of 9?
- **e** What is the total frequency of scores that are larger than 25?
- **f** What is the total frequency of scores that are less than or equal to 15?
- **g** What percentage of the scores are more than 35?

2 Consider the histogram on the right representing continuous data.

a How many class intervals are there?

b What is the least common class interval? What frequency does it have?

c How many people were surveyed?



3 Joseph collected the following data on the heights of the people in his class.

145, 183, 167, 172, 161, 158, 153, 168, 165, 174, 157, 152, 173, 166, 158, 159, 160, 171, 171, 161, 165, 172, 165, 158, 154

a Identify the minimum value in the data set.

b Identify the maximum value in the data set.

c Calculate the range of the data.

d Group the data into class intervals of 5 (e.g. 150–<155) in a frequency table.

WE 7.3B 4 Draw a frequency table with the given class interval widths to represent each of these continuous data sets.

a 5, 16, 28, 24, 31, 39, 3, 18, 13, 11, 25, 33, 8, 12, 19, 21, 31, 28 [class interval width = 5]

b 14.5, 73.2, 22.1, 43.9, 42.0, 58.4, 19.8, 37.6, 62.1, 29.4, 34.5, 72.1, 59.1, 52.3, 63.1, 26.3, 34.0, 41.9, 48.5, 16.4, 31.2, 52.9 [class interval width = 10]

c 1.2, 5.4, 9.3, 11.4, 3.3, 4.7, 3.3, 3.9, 4.8, 6.6, 2.9, 1.9, 10.6, 9.7, 10.8, 3.6, 4.8, 2.7, 2.1, 1.7, 1.9, 11.9, 6.7, 5.4, 5.1, 1.6, 1.8 [class interval width = 2]

d 42, 79, 56, 49, 77, 50, 51, 46, 48, 72, 61, 78, 63, 45, 58, 53, 73, 58, 49, 61, 68, 67, 43, 49, 75, 77, 58, 54, 67, 72, 51, 56, 53, 48, 76, 78, 72, 42, 48, 53 [class interval width = 5]

WE 7.3C 5 Use each frequency table to draw a histogram.

a

Class	5–<10	10–<15	15–<20	20–<25	25–<30	30–<35	35–<40	40–<45	45–<50
Frequency	8	6	7	2	3	1	5	6	9

b

Class	0–<20	20–<40	40–<60	60–<80	80–<100	100–<120
Frequency	14	21	18	13	8	2

c

Class	0–<10	10–<20	20–<30	30–<40	40–<50	50–<60	60–<70
Frequency	5	8	12	3	11	9	6

6 Use each data set to draw an appropriate histogram with the given class interval widths.

a 13, 46, 13, 17, 35, 9, 22, 15, 8, 2, 35, 42, 42, 17, 16, 22, 29, 31, 47, 29, 13, 20, 36, 47, 28, 23, 30, 38 [class interval width = 10]

b 18.1, 24.5, 32.1, 15.6, 22.5, 29.1, 34.6, 16.7, 19.4, 17.5, 21.8, 27.5, 29.2, 30.1, 20.0, 33.1, 32.8, 31.9, 33.8, 14.3 [class interval width = 5]

c 64, 18, 120, 7, 29, 40, 145, 38, 72, 38, 18, 29, 2, 56, 49, 87, 99, 104, 59, 5, 29, 112, 118, 34, 59, 29, 19, 13 [class interval width = 20]

d 125, 726, 632, 465, 428, 257, 283, 399, 619, 402, 132, 196, 183, 743, 120, 703, 336, 652, 349, 402, 560, 144, 759, 717, 588, 185, 464, 685, 268, 352, 310, 408, 114, 782, 189 [class interval width = 100]

e 1.25, 1.89, 1.09, 1.76, 1.15, 1.36, 1.55, 1.67, 1.99, 1.32, 1.08, 1.14, 1.17, 1.62, 1.88, 4.9, 1.68, 1.49, 1.08, 1.16, 1.24, 1.19, 1.26, 1.83, 1.52, 1.18, 1.07, 1.42, 1.01, 1.19 [class interval width = 0.2]

f 25, 58, 48, 33, 26, 53, 42, 49, 58, 53, 46, 24, 58, 53, 46, 41, 38, 47, 44, 58, 53, 57, 39, 21, 48, 46, 42, 58, 52, 43, 42, 37, 36, 27, 46, 42, 49, 53, 57, 59 [class interval width = 5]

7 Data was collected on the time spent listening to music in hours per week, as shown below.

7	10	2	4	24	3	7	9	5	15
17	16	19	20	5	3.5	5	10	14	7
7	5	9	10	17	7	4	10	11	16
4	5.5	12	14	6	7	12	14	16	3

a Create a frequency table with class intervals of width 5 to collate the data.

b Draw a histogram to represent this data.

8 Data was collected on the weights (in kg) of various dogs, as shown below.

11.2	14.8	6.4	19.7	22.1	4.8	5.4	12.6	18.2	9.2
13.1	20.1	8.6	4.9	8.4	17	6.2	14.3	26.3	16.6
18.6	19.2	14.5	28.3	16.2	9	11.4	14.9	11.3	13.2

a Fill in the following frequency table.

Class	Frequency
0–<5	
5–<10	
10–<15	
15–<20	
20–<25	
25–<30	

b Based on your frequency table, draw a histogram to represent this data.

9 Data was collected on the ages of people in a train carriage one day.

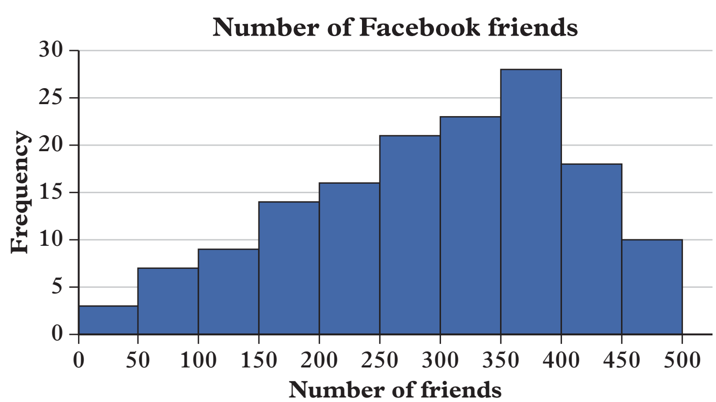
16	22	19	28	25	14	18	19	21
32	27	22	14	15	15	23	22	17
14	18	26	24	39	31	20	15	13
16	16	24	29	30	19	20	21	21

a Arrange this data in a frequency table with class intervals of width 5.

Note: Consider age as a continuous variable.

b Draw a histogram to represent this data.

10 Consider the following histogram.



a What does it show?

b What is the size of its class intervals?

c State the most common class interval and its frequency.

d If a data value of 140 was to be added to the histogram, to which class interval would it belong?

11 Data collected on the ages of customers in a clothes store in a day is shown in the table below.

a Why is this table difficult to read?

b Redraw the table with class intervals of width 10 so that it is easier to read.

Class	Frequency
<10	2
10–<12	6
12–<14	12
14–<16	16
16–<18	14
18–<20	13
20–<22	11
22–<24	9
24–<26	10
26–<28	8
28–<30	6

Class	Frequency
30–<32	4
32–<34	6
34–<36	5
36–<38	3
38–<40	2
40–<42	3
42–<44	1
44–<46	1
46–<48	0
48–<50	1
≥50	6

Exercise 7.3B: Problem solving and reasoning

12 Explain why you can't accurately decrease the size of the class intervals in this table.

Class	Frequency
0–<20	16
20–<40	48
40–<60	42

13 The following ordered data shows the number of kilometres travelled by different people to attend a conference.

4, 4, 11, 12, 12, 13, 19, 19, 20, 21, 21, 25, 28, 31, 33, 34, 42, 44, 52, 55, 57, 58

a Create a histogram with class intervals of width 20 to represent this data.

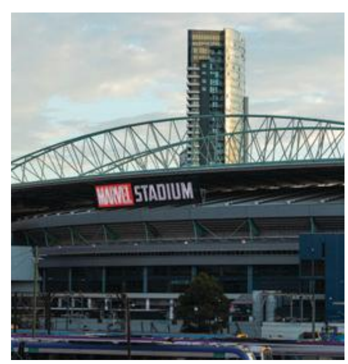
b Create a histogram with class intervals of width 10 to represent this data.

c Create a histogram with class intervals of width 5 to represent this data.

d Which of the three histograms do you think displays the data most appropriately? Explain your answer.

14 The following data was collected on how many thousands of people were present at AFL matches at Marvel Stadium, Melbourne.

10	51	35	22	25	46	34	21	8	28	11	19
22	31	38	42	22	22	48	30	25	26	15	21
18	18	41	39	33	33	38	25	28	10	17	13



a Create a histogram with class intervals of width 5 to represent this data.

b Create a histogram with class intervals of width 10 to represent this data.

c What is similar and what is different about your graphs for parts **a** and **b**?

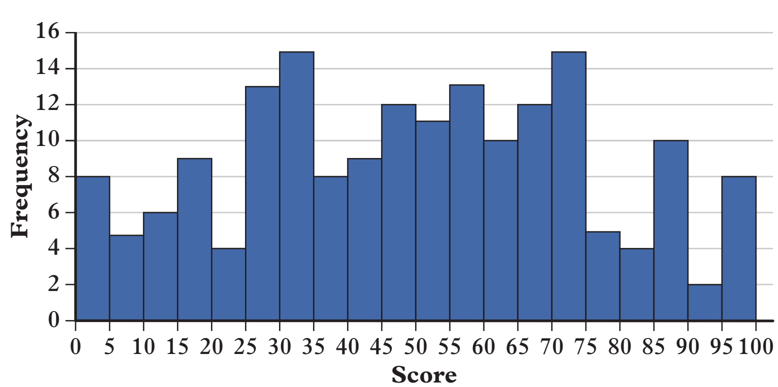
15 Consider the histogram on the right.

a What is the width of the class intervals?

b What can you say about the number of classes? Is it easy to read? Can you see a pattern?

c Redraw the histogram with class intervals of width 10.

d What patterns can you see now?



16 Stem-and-leaf plots can be used to display discrete data, where each piece of data is split into two parts (a stem and a leaf). The key indicates the value of the data. Consider this stem-and-leaf plot.

Stem	Leaf
1	2 6 7 8 8 9
2	0 1 5 6 7
3	3 6 8
4	0 2
5	1

Key: 1 | 2 = 12

a Which part of the plot represents the class intervals?

b What is the size of the class intervals?

c How many people were surveyed?

d What advantage does a stem-and-leaf plot have over a histogram?

17 Use the stem-and-leaf plot shown to draw a histogram.

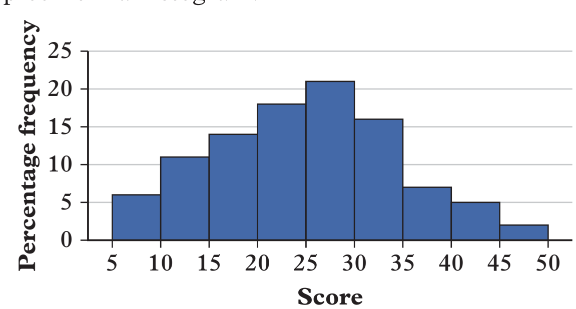
Stem	Leaf
1	0 0 1 1 3 4
1*	5 5 5 6 6 7 9
2	0 1 2 2 3 3 4 4 4
2*	5 6 6 7 8 8 9
3	0 1 1 2 2 3 4
3*	5 6 7 8

Key: 1 | 3 = 13

Exercise 7.3C: Challenge

18 Explain why you can't accurately construct a stem-and-leaf plot from a histogram.

19 Consider the percentage frequency histogram on the right.



a How is it different from a normal histogram?

b What percentage of scores are between 30 and 35?

c What percentage of scores are greater than 35?

d Without performing a calculation, state the sum of the percentage frequency columns. Explain how you know the answer.

e If there were 400 scores in total, calculate:

i the number of scores between 15 and 20

ii the number of scores less than 25

iii the number of scores between 20 and 40.

20 Create a percentage frequency histogram with class intervals of width 0.2 to represent this data set.

Weights of newborn babies at a particular hospital in one week (in kg):

3.25, 4.15, 2.75, 3.60, 3.95, 3.05, 2.85, 4.20, 1.95, 3.50, 3.65, 3.15, 3.70, 3.95, 4.10, 4.85, 2.90, 3.10, 3.30, 3.25, 3.50, 4.05, 3.45, 3.85, 3.75, 3.15, 3.45, 3.20, 3.25, 4.25, 2.55, 2.95, 3.40, 3.85, 3.80, 3.55, 3.20, 3.00, 3.20, 3.75, 4.00, 4.15, 3.80, 3.75, 3.40, 3.25, 3.15, 3.05, 3.85, 2.95

Online resources:

Interactive skillsheet — Histograms

Investigation — Pocket money

CAS instructions — Histograms

Quick quiz — 7.3

Statistical Investigations

The statistical investigation process

Statistics can be used to answer a wide range of real-life questions. By collecting and analysing data, we can draw evidence-based conclusions, allowing us to make informed decisions.

A statistical investigation involves the following steps:

1. Identifying a question to investigate
2. Collecting data
3. Analysing the data
4. Reporting on the findings.

Data used in a statistical investigation can either be collected directly by the researcher (primary data) or taken from a reputable source (secondary data).

Curriculum link

Plan and conduct statistical investigations involving the collection and analysis of different kinds of data; report findings and discuss the strength of evidence to support any conclusions (VC2M9ST05)

Climate change

In 2015, 195 countries from across the world signed the Paris Agreement, which is an international treaty on climate change. The agreement encourages countries to reduce their emissions and aim to reach net zero by 2050. In 2022, Australia updated its commitment to reduce greenhouse gas emissions to 43% below 2005 levels by 2030.

Australia is in the process of reducing our reliance on fossil fuels and moving towards renewable energy sources. The federal government's Department of Climate Change, Energy, the Environment and Water collects data each year on the generation of energy by source.

Investigation

The following table displays data taken from energy.gov.au about the total generation of energy in Australia from each source in the financial years 2001–02 and 2021–22. In Australia, the financial year runs from the start of July in one year to the end of June in the next year. Governments and corporations often collect data on the economy and industry using financial years rather than calendar years. Each value in this table is given in gigawatt hours (GWh).

	2001–02	2021–22
Coal	173 268	133 592
Gas	31 730	49 280
Oil	2446	4664
Renewable	17 426	83 996

Investigation questions

1 Explain why this is secondary data, not primary data.
2 Could you collect primary data on Australian energy generation? Why or why not?
3 Is the source of this data reliable? How do you know?
4 Only two financial years of data are shown. What do you think the data would look like for the years in between these two financial years?
5 How might the data be different for a financial year in the 1980s?
6 What do you think the data may look like 10 years from now? How about 50 years from now?

Analysis questions

1 In which of the financial years shown was the total energy generation larger?
2 What percentage of energy generation in Australia came from renewable energy sources in each of the financial years shown?
3 Copy the tables below into your workbook and fill in the missing values. Round each percentage to the nearest whole number.

	2001–02	%
Coal	173 268	
Gas	31 730	
Oil	2446	
Renewable	17 426	
Total		100%

	2021–22	%
Coal	133 592	
Gas	49 280	
Oil	4664	
Renewable	83 996	
Total		100%

4 Use the tables above to draw a stacked bar chart. The financial year will be on the horizontal axis and the percentage of energy generated by each source will be shown in the stacked columns. Remember to include a key.
5 Explain why the columns of your stacked bar chart are the same total height, despite the total energy generation being different between the two financial years.
6 Write a short report on your findings with respect to the change in Australia's energy generation between 2001–02 and 2021–22.

Investigate further

Go to energy.gov.au and find the data source used for this investigation. Extend your investigation by including data from more financial years, including a year before 2001–02 and a year between 2001–02 and 2021–22. Does the data from these additional years support the conclusion that Australia is reducing our reliance on fossil fuels and using more renewable energy?

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

7.1 **1** Determine the sampling method used in the following situations.

a Ms Walker surveys 10 students from each class about their opinions on school assembly.

b April surveys 20 people in her class about their opinions on the new school sports centre.

c Julio surveys 100 people from suburbs in his local government area in proportions that reflect their populations.

d Trent selects every 10th person from a list to complete a survey.

7.1 **2** For each scenario below:

i decide whether the data collection method is fair

ii decide whether the interpretation of the data collected is fair

iii if appropriate, provide a suggestion to improve the survey.

a Sienna wanted to know the average number of hours people in her town spent sitting down every day. She asked more than 100 people, chosen at random from people she saw walking through a mall, how many hours they sat down per day. Sienna came up with an average of 8.5 hours.

b Kane wanted to know the most popular party foods to serve at an upcoming school dance. He surveyed every second person in his neighbourhood about their favourite party foods. Kane decided to serve party pies, chocolates and sandwiches.

7.1 **3** Decide whether each of these surveys will provide fair or biased results, giving a reason for your answer.

a Asking 10 000 people randomly selected around the country their opinion on an issue in your community

b Asking everybody you see at a local shopping mall their opinion on a global issue

7.2 **4** Classify the following variables as numerical or categorical.

a Time spent watching television

b Most used smartphone app

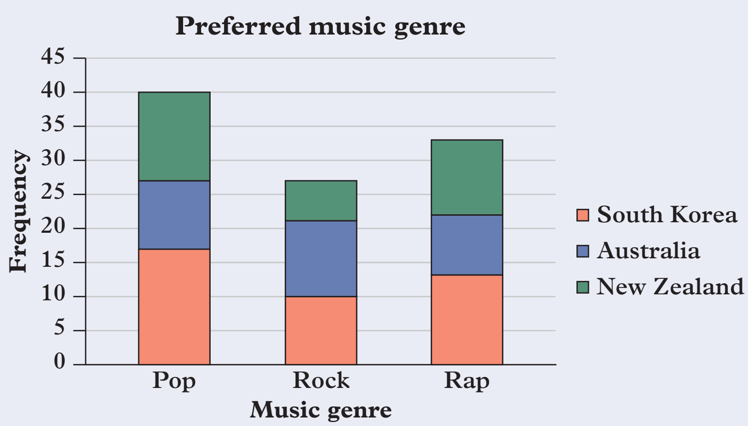
c Programming skill level (Beginner, Intermediate, Advanced)

d Number of members in a chess club

7.2 **5** Consider the following table of data. Draw a line graph to represent this data.

Hours since midnight	0	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (°C)	8	9	9	11	10	12	13	15	17	18	17	20	22

7.2 **6** Thirty teenagers from three different countries were asked to choose their preferred music genre. The results are shown in the following stacked bar chart graph.

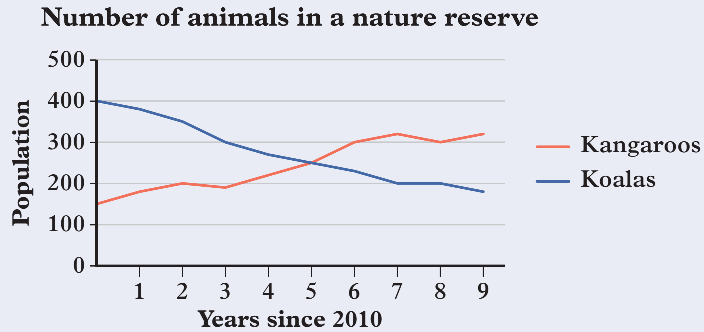


a Which music genre was most preferred by South Koreans?

b The teenagers of which country preferred rock to rap?

c What was the most popular music genre overall?

7.2 **7** Consider the following multiple line graph, which shows the number of two different animals in a nature reserve since 2010.

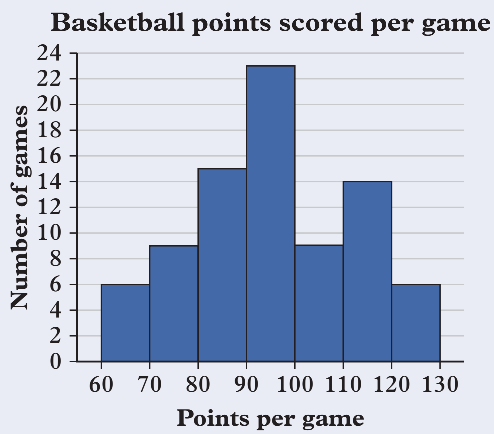


a The population of which animal in the reserve is increasing?

b During which year was the population of each animal the same?

c Between which two years did the population of koalas remain steady?

7.3 **8** Teams in the NBA play 82 games of basketball in a regular season. The number of points scored per game for a certain team is displayed in the following histogram.



a Which interval contains the most values?

b In how many games did the team score at least 120 points?

c In how many games did the team score less than 80 points?

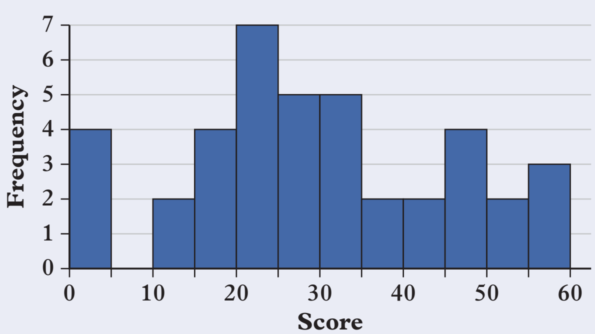
7.3 **9** A researcher asked 24 people how many minutes they spend in the shower on average. The results are shown below.

7	12	14	8	6	4	4	9	8	7	6	11
16	18	21	3	4	9	8	10	12	13	14	20

a Draw a frequency table with class intervals of 5 to represent this data.

b Use the frequency table to draw a histogram for this data.

7.3 **10** Redraw the following histogram with class intervals of 10, starting at 0.



Lesson 7.4

Summary statistics from tables and displays

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate summary statistics from tables and displays.

Inter-year links

Support	Understanding data
Year 7	10.1 Summary statistics
Year 8	10.2 Summary statistics
Year 10	10.1 Five-number summary and interquartile range

Measures of centre and spread

- The **mean**, **median** and **mode** are **measures of centre**, as they each represent a central point in a data set.

  $$\text{Mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$$

  Median = middle value when data is listed in order (divides the data set in half)

  Mode = most common value (there can be more than one mode)

- The **range** is a **measure of spread**, as it gives an indication of the spread of the data.

  Range = maximum value – minimum value

For example, consider the following data set.

2, 13, 16, 28, 35, 35, 46, 29, 16, 24

$$\begin{aligned}\text{Mean} &= \frac{2 + 13 + 16 + 28 + 35 + 35 + 46 + 29 + 16 + 24}{10}\\ &= \frac{244}{10}\\ &= 24.4\end{aligned}$$

Order the data set: 2, 13, 16, 16, 24, 28, 29, 35, 35, 46

The number of data values is even, so the median is the mean of the two middle values.

$$\text{Median} = \frac{24 + 28}{2} = 26$$

The highest frequency is 2 for both values 16 and 35.

Modes = 16 and 35

Range = 46 – 2 = 44

Calculating summary statistics from data displays

Key content video: Calculating summary statistics from tables and displays

- To calculate the mean from a frequency table, insert a column multiplying each individual score by its frequency, and divide the total of this column by the total of the frequencies.
- To calculate the median from a frequency table, insert a column for the **cumulative frequency** (a running total of the frequencies) to help identify where the middle value lies.
- To calculate summary statistics from a display that details individual scores (e.g. stem-and-leaf plots, dot plots or bar charts) the data can be treated as either a raw list or as a frequency table.

Worked example 7.4A Summary statistics from a frequency table

For the data shown in the following frequency table, find the:

a mean **b** median **c** mode **d** range.

Score (x)	Frequency (f)
1	4
2	7
3	8
4	8
5	2
6	1

THINK

a **1** Insert a 'Score × frequency' column and a 'Total' row and then complete the table. The total of the 'Frequency' column is the number of data values in the table.

2 To find the mean, divide the 'Score × frequency' total by the 'Frequency' total.

b **1** Insert a 'Cumulative frequency' column in the table. Check that the uppermost number matches the number of data values.

2 Divide the number of data values by 2 to find out how many values lie below the median. Then identify the row that contains the median score by using the cumulative frequency column.

c Identify the score with the highest frequency.

d Identify the minimum and maximum values in the score column of the data set. Subtract the minimum value from the maximum value.

WRITE

Score (x)	Frequency (f)	Score × frequency ($x \times f$)	Cumulative frequency
1	4	$1 \times 4 = 4$	4
2	7	$2 \times 7 = 14$	$4 + 7 = 11$
3	8	$3 \times 8 = 24$	$11 + 8 = 19$
4	8	$4 \times 8 = 32$	$19 + 8 = 27$
5	2	$5 \times 2 = 10$	$27 + 2 = 29$
6	1	$6 \times 1 = 6$	$29 + 1 = 30$
Total	**30**	**90**	

a $\text{Mean} = \frac{90}{30}$

$= 3$

b There are 30 values in the data set. $\frac{30}{2} = 15$ so 15 data values lie below the median.

The number of data values is even so the median is between the 15th and 16th value.

The 12th to 19th scores are 3, so both the 15th and 16th score are 3.

Median = 3

c The highest frequency is 8 for both scores 3 and 4.

Modes = 3, 4

d Range = 6 − 1

= 5

Worked example 7.4B Finding summary statistics from a dot plot

For the data shown in the following dot plot, find the:

a mean **b** median **c** mode **d** range.

THINK

a **1** Create a frequency table by counting the number of dots in each column.

2 Insert a 'Total' row, a 'Score × frequency' column and a 'Cumulative frequency' column in the table.

3 Find the mean by dividing the 'Score × frequency' total by the 'Frequency' total.

b Divide the number of values by 2 to find out how many values lie below the median.

c Find the mode by identifying the score with the highest frequency.

d Find the range by subtracting the minimum score from the maximum score.

WRITE

Score (x)	Frequency (f)	Score × frequency ($x \times f$)	Cumulative frequency
7	6	42	6
8	9	72	15
9	13	117	28
10	14	140	42
11	8	88	50
Total	**50**	**459**	

a Mean $= \frac{459}{50}$

$= 9.18$

b $\frac{50}{2} = 25$, so 25 data values lie below the mean.

The number of data values is even so the median is between the 25th and 26th score.

The 16th to 28th scores are 9, so both the 25th and 26th score are 9.

Median = 9

c The highest frequency is 14 for the score 10.

Mode = 10

d Range = 11 − 7

= 4

Worked example 7.4C Finding summary statistics from a stem-and-leaf plot

For the data shown in the following stem-and-leaf plot, find the:

a mean **b** median **c** mode **d** range.

Stem	Leaf
1	5 9
2	3 7 8 8 9
3	0 1 2 2 6 9 9
4	0 2 3 3 6 6 6 7 8
5	1 5

Key 2 | 3 = 23

THINK

a Find the mean by dividing the sum of scores by the number of scores.

b Divide the number of values by 2 to find out how many values lie below the median.

c Find the mode by identifying the most common scores.

d Find the range by calculating the difference between the minimum score and the maximum score.

WRITE

a Mean $= \dfrac{915}{25}$

$= 36.6$

b $\dfrac{25}{2} = 12.5$, so 12 data values lie below the mean.

The number of data values is odd so the median is the 13th score, which is 39.

Median = 39

c The highest frequency is 3 for the score 46.

Mode = 46

d Range = 55 − 15

= 40

Helpful hints

- ✔ When calculating summary statistics from a stem-and-leaf plot, make sure that you use the full data values by combining the stem and leaf and using the key.
- ✔ Remember that if the median falls between two different values, the median will be the average (mean) of these values.
- ✔ A data set can have more than one mode.
- ✔ Remember that a frequency table condenses raw data, so you do not have to read the same number many times. For example, if a score of 12 has a frequency of 7, then the raw data includes seven 12s.

Learning pathways

1(a, c, e), 2, 4, 5–6(a, b), 8, 9, 11(a, c, d, e), 13(a–c)

1(b, d, f), 3, 5, 6(b, c), 7, 8, 10, 13–15

1(d, f), 3, 5–6(b, c), 10, 12–14, 16

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Exercise 7.4A: Understanding and fluency

WE 7.4A **1** For the data shown in each frequency table, find the **i** mean, **ii** median, **iii** mode and **iv** range. Give your answers to two decimal places where necessary.

a

Score (x)	Frequency (f)
1	6
2	7
3	5
4	3
5	1

b

Score (x)	Frequency (f)
10	8
20	6
30	8
40	2

c

Score (x)	Frequency (f)
13	3
14	4
15	8
16	11
17	12
18	4

d

Score (x)	Frequency (f)
0	11
1	13
2	6
3	3
4	1

e

Score (x)	Frequency (f)
15	29
20	41
25	58
30	72

f

Score (x)	Frequency (f)
1	6
2	11
3	9
4	4
5	3
19	1

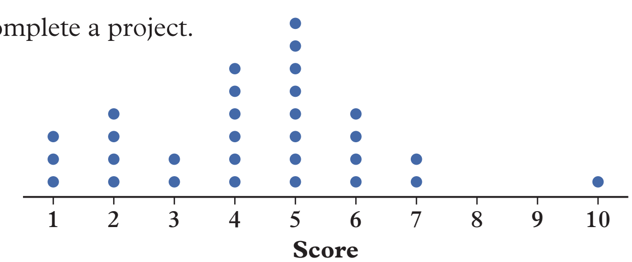
2 The dot plot on the right shows the time, in hours, to complete a project.

a How many people were surveyed?

b State the mode and range.

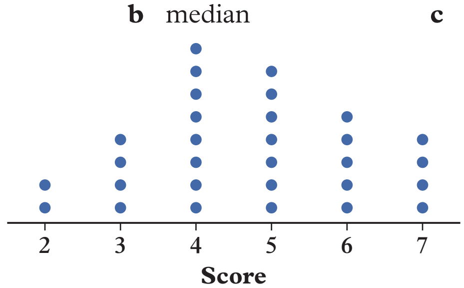
c Create a frequency table.

d Use the frequency table to calculate the median and mean of project completion time.

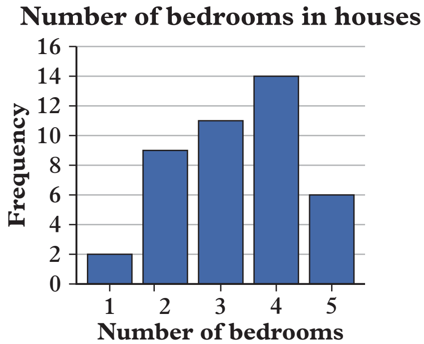


WE 7.4B **3** For the data shown in the following dot plot, find the:

a mean **b** median **c** mode **d** range.



4 The column graph below shows the number of bedrooms in a survey of houses.



a What is the most common number of bedrooms in the houses surveyed?

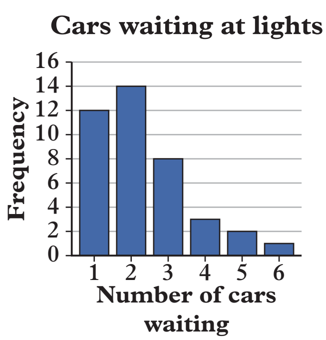
b What is the range of the number of bedrooms?

c Create a frequency table to represent the data shown in the column graph.

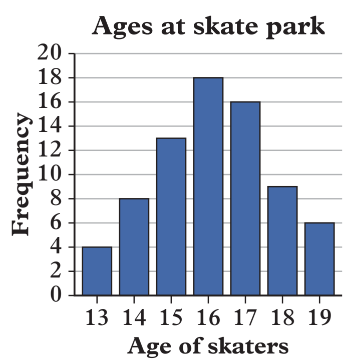
d Use the frequency table to calculate the median and the mean number of bedrooms. Give your answers to two decimal places where necessary.

5 Find the mean, median, mode and range for the data shown in each column graph. Give your answers to two decimal places where necessary.

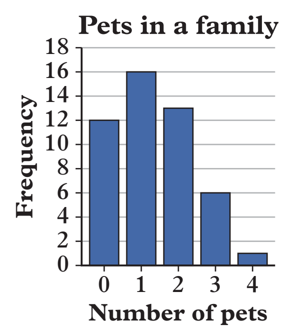
a



b



c



WE 7.4C **6** Find the mean, median, mode and range for the data displayed in each stem-and-leaf plot. Give your answers to two decimal places where necessary.

a

Stem	Leaf
3	2 4 7
4	2 2 2 6 9
5	3 6 8 9 9
6	1 1 4 6 7 7 8 8
7	0 4

Key 3 | 2 = 32

b

Stem	Leaf
1	2 3 8 8 8 9
2	0 0 1 2 3 6 7 8
3	2 3 7 9
4	4 6
5	2

Key 1 | 3 = 1.3

c

Stem	Leaf
1	0 1 2 4 5 6 7 7 9
2	2 3 4 4 6 8 8
3	0 1 1 1 3
4	3 7 9
5	1
6	
7	7

Key 1 | 2 = 120

Exercise 7.4B Problem solving and reasoning

7 A number of people were surveyed on how many pairs of shoes they bought in a year. Use the results shown in the table to find the mean, median, mode and range. Give your answers to two decimal places where necessary.

Score (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	9	13	21	18	13	11	7	4	1	3

8 Explain why the only summary statistic that you can find for categorical data is the mode.

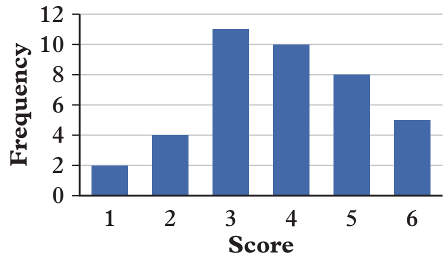
9 Match these summary statistics with the following graphs.

a mean = 3.3, median = 3, mode = 3, range = 5

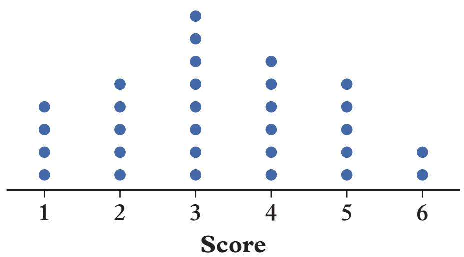
b mean = 3.825, median = 4, mode = 3, range = 5

c mean = 40.74, median = 36.5, mode = 74, range = 97

I



II



III

Stem	Leaf
0	1 2 6
1	2 6 7 7 8 9
2	0 1 1 2 4 6 7 7 8 9
3	0 3 4 4 5 6 7 9
4	0 1 2 2 5 9
5	0 1 2 3 5
6	0 1 4 8 9
7	4 4 4 4
8	1 9
9	8

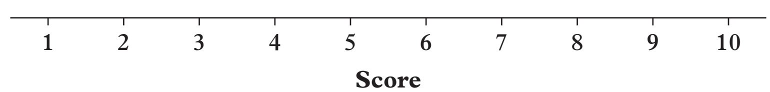
Key 2 | 1 = 21

10 Draw a dot plot on a number line as shown below using a different data set consisting of exactly 9 numbers for each of the following scenarios. In each case, write a sentence explaining your reasoning for placing the dots in the positions you chose.

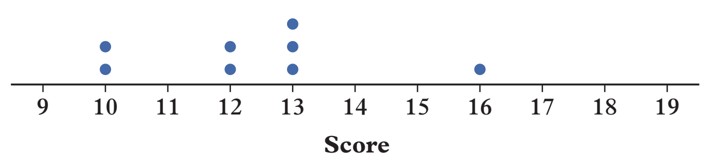
a The minimum value is 2 and the range is 7.

b The median and the maximum value are both 6.

c The mode includes four different numbers.



11 The dot plot on the right is missing one dot. Find a possible value of the missing dot for each of the following scenarios.
In each case, state whether your answer is unique or if it is one answer out of several that are possible.



a The maximum value is 17.

b The mean is 12.

c The median is 13.

d The modes are 10 and 13.

e The range is 7.

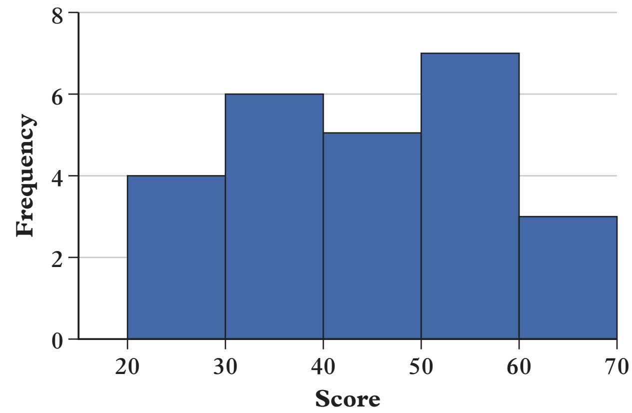
f The mean is a whole number.

12 A large data set contains 457 data points that are listed in a spreadsheet. The numbers are ordered from smallest to largest and are listed in rows of 30. This means the number in column 1 of row 1 is the smallest number in the set, the number in column 1 of row 2 is the 31st smallest number in the set, and so on.

a Find the row number and column number of the greatest number in the data set.

b Find the row number and column number of the median of the data set.

13 A researcher has collected discrete data and arranged it into the histogram on the right. The data is discrete, so the first interval is 20–29, the second interval is 30–39, etc. The raw data (i.e. the list of numbers that make up the data set) is not available.



a What is the maximum possible value of the range? Explain your reasoning.

b What is the minimum possible value of the range? Explain your reasoning.

c Explain why it is not possible to find the median.

d In which interval is the median contained?

e Explain why it is not possible to calculate the mean or even to solve for the interval in which it is contained.

14 For the histogram given in question **13**, it is not possible to calculate the exact value of the mean. However, it is possible to find the minimum and maximum possible values of the mean.

a Find the minimum possible value of the mean by assuming every data point occurs at the lowest possible value of its interval.

b Find the maximum possible value of the mean by assuming every data point occurs at the highest possible value of its interval.

c Find the average value of the minimum possible mean and the maximum possible mean. Is this a good way to estimate the mean for data presented in a histogram? Explain your answer.

Exercise 7.4C: Challenge

15 Elena has completed four Maths tests this year with an average score of 78%.

a What is the maximum value her average could increase to after completing the next test?

b What is the minimum value her average could drop to after completing the next test?

c What percentage score must she achieve on her next test to increase her average to 80%?

16 Another measure of spread is the standard deviation. It measures the average spread of each data value from the mean. A small standard deviation means that most values are close to the mean, and a large standard deviation means that the values are spread far from the mean. Standard deviation is best found using a calculator with the appropriate function. It can also be found (for a sample) using the formula:

$$s = \sqrt{\frac{\sum(\bar{x} - x)^2}{n - 1}}$$

where s represents standard deviation, $\sum$ means 'the sum of', $\bar{x}$ represents the mean, x represents an individual score and n represents the number of scores.

a Use your calculator or the standard deviation formula to find the standard deviation for each data set, correct to two decimal places.

i 4, 8, 2, 6, 4, 9, 7, 7, 4, 1, 2, 4, 6, 9, 7, 4, 6, 7, 8, 1, 2, 3, 1, 6, 2, 6

ii 22, 27, 35, 64, 12, 74, 37, 93, 27, 33, 11, 71, 64, 42, 81, 37, 13, 19, 88, 50

iii 103, 118, 109, 111, 117, 116, 117, 117, 105, 107, 116, 113, 108, 109, 112, 113

b Use your results to state whether each data set from part **a** has a large or small spread from the mean.

Online resources:

Interactive skillsheet	Interactive skillsheet	Investigation	CAS instructions	Quick quiz
Summary statistics from frequency tables	Summary statistics from displays	Estimating population means and medians	Measures of centre	7.4

Describing data

Learning intentions

By the end of this lesson you will be able to ...

✔ describe the distribution of a data set.

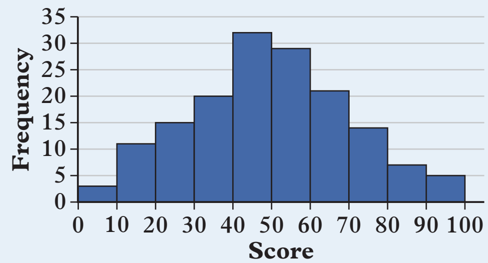
Inter-year links

Year 7	10.2 Describing data with dot plots
Year 10	10.3 Distributions of data

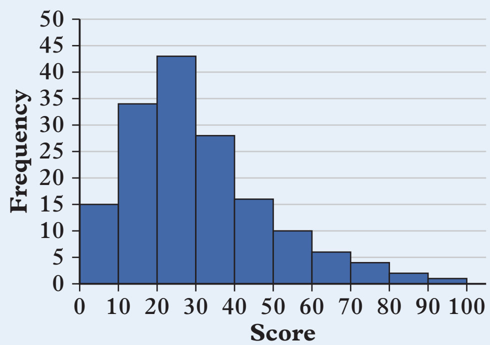
Describing data

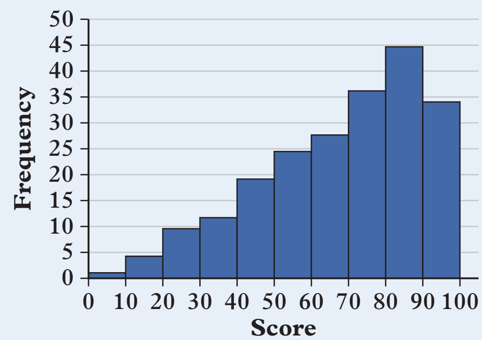
Key content video
Describing data

- By displaying numerical data, you can identify the shape of the distribution of the data and use this to describe data sets. **Symmetric distributions** have a peak in the middle and are roughly evenly spread on either side.

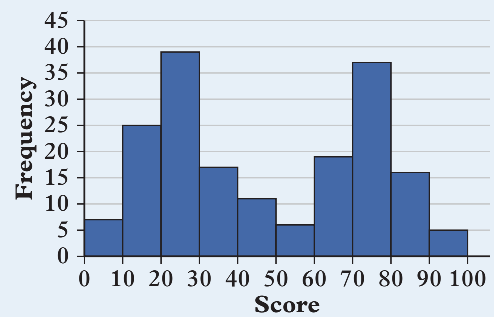


- A distribution is **skewed** when the data points are clustered on one side of the distribution. There is a tail on the other side of the distribution.
  - → **Positively skewed distributions** are skewed towards the vertical axis with a tail on the right of the distribution.
  - → **Negatively skewed distributions** are skewed away from the vertical axis with a tail on the left of the distribution.





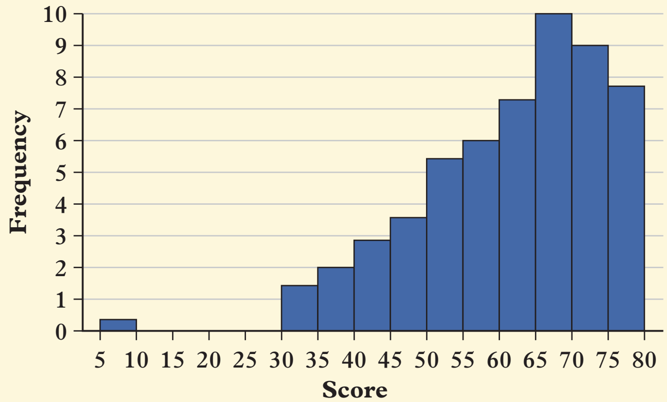
- A distribution is **bi-modal** if there are two clear peaks in the data.



- The mean, median and mode can all be used to represent a central point of a data set, but the shape of the distribution should be considered before deciding which measure of centre to use for this purpose.
- When describing symmetric distributions, either the mean or the median are appropriate representations of the centre. In perfect symmetric distributions, the mean, mode and median are equal.
- When describing skewed distributions, the median is a better representation of the centre than the mean, as the mean is skewed by the values in the tail.
- Bi-modal distributions are best described using the mode of each peak.
- An **outlier** is a data point that is significantly higher or lower than the other values in a data set.
  - → The presence of outliers can have a significant impact on the mean, so distributions with outliers are best described by using the median.
  - → Outliers can be determined using several different methods; in Year 10 you will be given a formula for determining whether a data point is an outlier. At this stage, only clear outliers will be considered.

Worked example 7.5A Describing distributions

Describe the distribution of this histogram.



THINK

1 Identify whether the distribution is symmetric, skewed or bi-modal.

2 Identify any outliers.

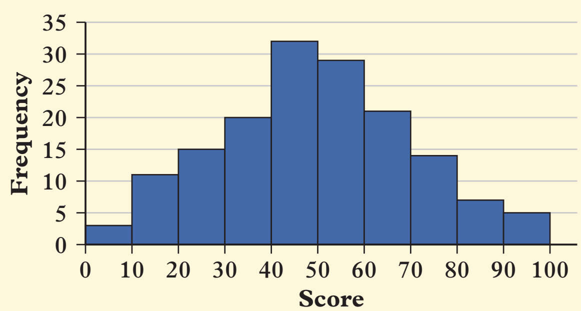
3 Write your answer.

WRITE

The distribution is negatively skewed with an outlier between 5 and 10.

Worked example 7.5B Deciding which measure of centre best describes a distribution

Decide which measure(s) of centre would best describe this distribution.



THINK

1 Identify whether the distribution is symmetric, skewed or bi-modal.

2 Identify how the shape of the distribution determines which measure of centre should be used to describe the distribution.

3 Write your answer.

WRITE

The distribution is symmetric so the centre of the distribution could be described by the mean or the median.

Worked example 7.5C Describing a histogram

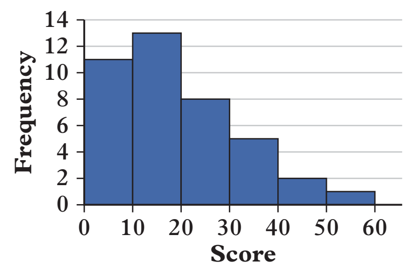
Use the data below to:

a draw a histogram with class intervals of width 10

b describe its distribution

c state the best measure of centre to describe the data, providing a reason.

49, 3, 16, 12, 20, 49, 22, 37, 32, 18, 34, 13, 4, 7, 17, 9, 13, 59, 25, 1,
15, 30, 23, 27, 4, 17, 26, 3, 10, 5, 8, 2, 31, 11, 8, 20, 27, 13, 16, 11

THINK	WRITE
a Use a frequency table to draw a histogram with an even scale and labels on both axes.	**a** 
b Decide whether the data is skewed.	**b** The distribution is positively skewed.
c Identify the best measure of centre to describe the data. The data is positively skewed, not symmetric or bi-modal, so the median should be used.	**c** The median should be used to describe the centre, because the distribution is skewed, which affects the value of the mean.

Helpful hints

✔ The peaks in a bi-modal distribution do not need to be of equal height.

✔ In geometry, we do not describe a shape as symmetric unless it has perfect symmetry. However, in statistics it is appropriate to describe a distribution as symmetric provided that it is approximately symmetric.

Learning pathways

△ 1–3, 5(a, b), 6, 8, 9

□ 1, 2, 4, 5(b, c), 6, 9–11, 12(a, b)

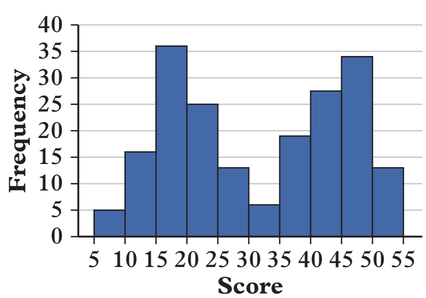
⬠ 2, 3, 5(b, c), 7, 9, 11–13

ANS p515

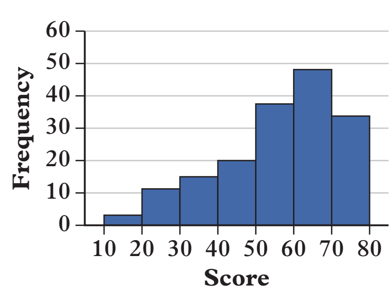
Exercise 7.5A: Understanding and fluency

WE 7.5A **1** Describe the distribution of each histogram.

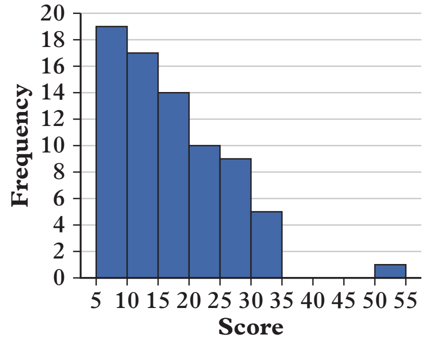
a



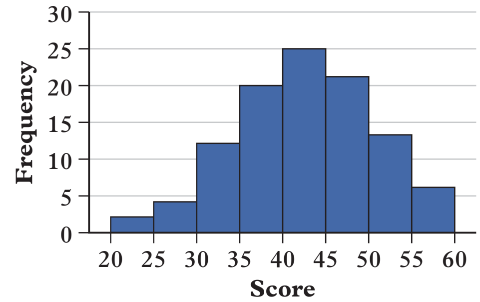
b



c



d



2 Describe the distribution of each stem-and-leaf plot.

a

Stem	Leaf
1	0 2
2	1 1 4
3	0 4 5 6
4	4 5
5	3

Key 1|2 = 12

b

Stem	Leaf
0	1 4 6 8
1	3 4 5 5 6
2	1 2 4 4
3	4 8 9
4	3
5	1

Key 1|3 = 13

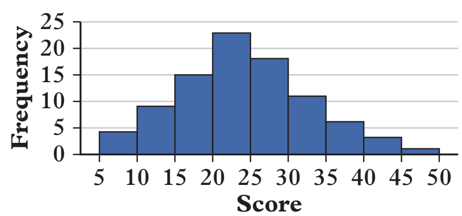
c

Stem	Leaf
0	9
1	
2	
3	1 9
4	4 5 6
5	4 5 5 6
6	0 1 1 2 4

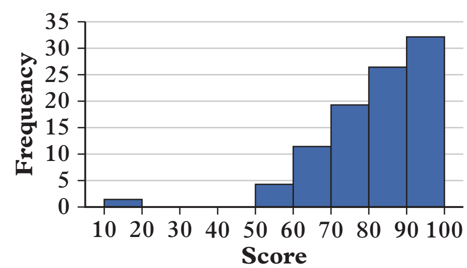
Key 3|9 = 39

WE 7.5B **3** Decide which measure(s) of centre would best describe each distribution.

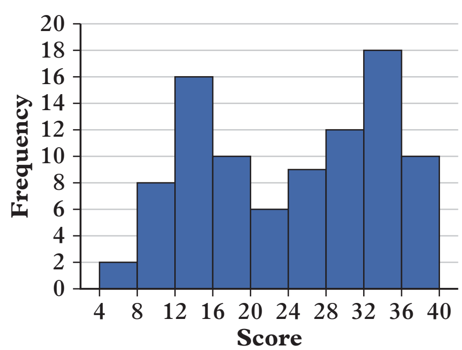
a



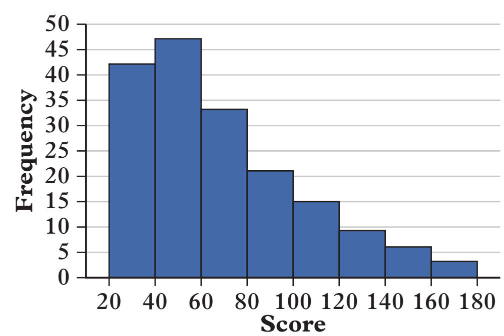
b



c



d



4 Decide which measure(s) of centre would best describe each distribution in question **1**.

WE 7.5C **5** For each data set:

i draw a histogram with the given class interval width

ii describe its distribution

iii state the best measure of centre to describe the data, providing a reason.

a 35, 33, 42, 99, 54, 68, 4, 91, 97, 55, 99, 86, 40, 58, 41, 95, 38, 62, 35, 88, 82, 98, 77, 69, 78, 78, 82, 81, 98, 57, 88, 41, 60, 85, 82, 85, 91, 90, 80, 49, 58, 66, 97, 95, 82, 84, 78, 91, 62, 42 [width = 10]

b 9, 2, 3, 9, 43, 8, 2, 15, 12, 17, 10, 10, 14, 9, 34, 7, 12, 18, 18, 47, 2, 12, 24, 34, 19, 1, 12, 18, 35, 47, 6, 14, 8, 35, 7, 9, 4, 17, 2, 20, 8, 12, 21, 24, 48, 6, 7, 8, 17, 41 [width = 5]

c 49, 23, 45, 23, 31, 77, 62, 21, 52, 51, 60, 54, 46, 69, 27, 60, 142, 41, 32, 80, 52, 21, 80, 65, 37, 33, 74, 45, 48, 78, 70, 21, 55, 64, 33, 42, 59, 67, 32, 79, 30 [width = 10]

Exercise 7.5B Problem solving and reasoning

6 This stem-and-leaf plot does not seem to have a noticeable pattern. However, it is possible to redraw this stem-and-leaf plot with more stems, just like histograms can be redrawn with different class sizes. For example, the number 1 can be the stem for two sets of leaves, with the numbers 10–<15 listed in the first stem labelled 1 and the numbers 15–<20 listed in the second stem labelled 1*.

Stem	Leaf
1	0 0 0 1 2 2 2 2 2 3 3 5 5 5 5 5 6 6 7 8 9
2	0 0 1 2 2 2 3 3 3 9

Key 1 | 3 = 13

a Redraw the plot above as a split stem-and-leaf plot with:

i four stems **ii** ten stems.

b Describe the distributions that you see from the split stem-and-leaf plots.

7 The following tables show the data that Ava collected on the weight of dogs (in kilograms) in her community.

Class	0–<2	2–<4	4–<6	6–<8	8–<10	10–<12	12–<14	14–<16	16–<18	18–<20
Frequency	1	6	3	5	6	5	8	6	7	3

Class	20–<22	22–<24	24–<26	26–<28	28–<30	30–<32	32–<34	34–<36	36–<38	38–<40
Frequency	7	2	5	3	4	5	4	2	1	4

a Draw a histogram to represent the data.

b What pattern can you see?

c Reorder the data into class intervals of width 4 and redraw your histogram.

d What pattern can you now see?

e Write a sentence describing the weight distribution of dogs as shown in your second histogram.

8 The histogram on the right shows the number of people at a skate park recorded at various times.

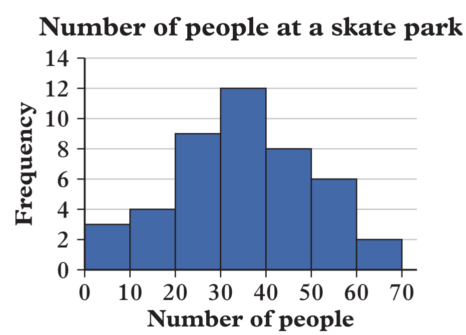
a Describe the shape of the distribution.

b What measure(s) of centre would be the most appropriate to use?

c Create a frequency table that represents the histogram.

d Use the frequency table to determine in which class interval the median lies.

e Write a sentence that describes the distribution in terms of the number of people at the skate park.



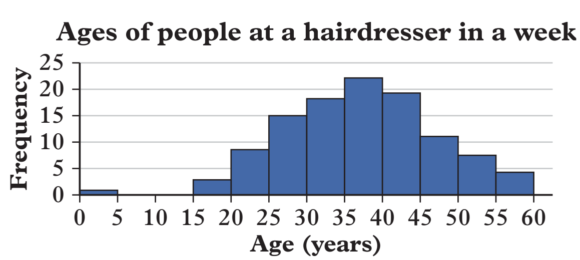
9 For each data distribution:

i describe its shape

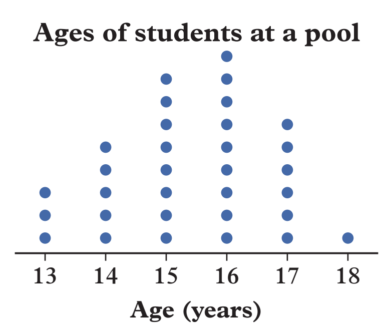
ii calculate the appropriate measure(s) of centre

iii write a sentence that summarises the graph in its context.

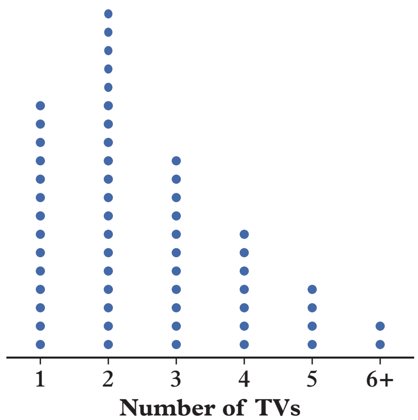
a



b



c **Number of TVs in a home**



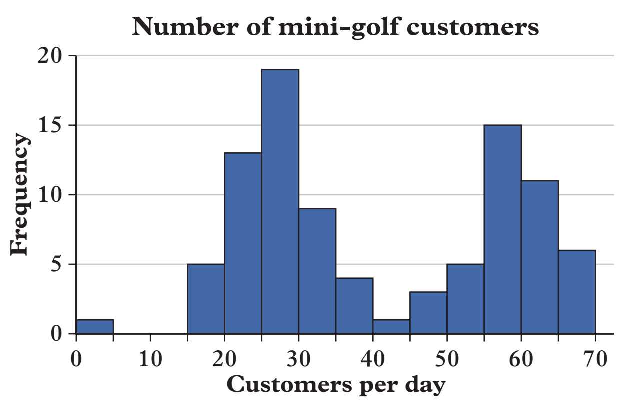
d **Length of hair in Year 9 students (cm)**

Stem	Leaf
0	1 2 2 2 3 3 3 4 4 6 6 6 6 7 8 8 8 9
1	0 2 4 6 8
2	0 1 5 8 8 9 9
3	0 1 2 2 2 2 2 4 5 5 5 5 6 7 8 9 9
4	1 2 2 4 4 5 6 6 8 9
5	0 4 5 6 8 8 8
6	2 9

Key 1 | 2 = 12

10 A mini-golf course is open every day of the year except Christmas day. The number of customers who visited the course each day in the months of October, November and December has been placed in the frequency table on the right. The histogram for this data is also shown.

Class	Frequency
0–<5	1
5–<10	0
10–<15	0
15–<20	5
20–<25	13
25–<30	19
30–<35	9
35–<40	4
40–<45	1
45–<50	3
50–<55	5
55–<60	15
60–<65	11
65–<70	6



a Suggest a reason why the class interval 0–<5 has a frequency of 1.

b Suggest a reason why the number of daily customers at the mini-golf course has a bi-modal distribution.

c Give examples of other businesses that will most likely have a bi-modal distribution for the number of daily customers.

11 A boat tour company offers ocean cruises where customers have a chance to see different wildlife, including orcas (killer whales). The following data set is the number of orcas that were spotted during tours for each day of February.

0	0	0	0	10	0	0	8	0	0	0	26	0	0
0	13	0	41	0	0	7	7	0	0	0	0	0	0

The whale watching company decides to run an advertisement during March on social media with the claim "Last month we saw an average of 4 killer whales per tour!"

a Explain why this claim, while technically true, is misleading.

b Why would the company not want to advertise the median number of whales they saw per tour?

c Create a histogram with class intervals of size 10 to display the data.

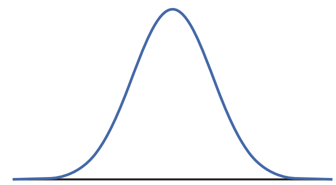
d Explain why the class interval 0–9 is misleading in the histogram.

e The company decides to change the advertisement to be more honest. The new advertisement will instead highlight the percentage of tours on which killer whales were spotted. Complete the sentence below to help the company write their new advertisement.

"Last month we saw killer whales on ___% of tours!"

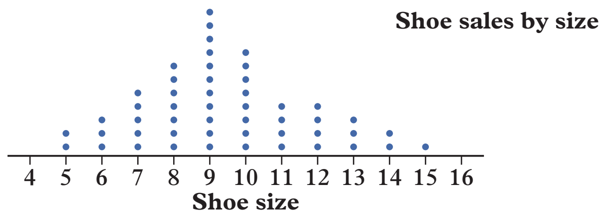
Exercise 7.5C: Challenge

12 A common distribution that arises from many sets of real-life data is called the normal distribution or bell curve. A perfect normal distribution has many nice features, including that it is symmetric and that its mean, median and mode are all equal. When a real-life data set follows this pattern closely, the data is described as 'approximately normally distributed'.



An example of this phenomenon is the number of sales of shoes by size, with the most common sales for adult males being size 9 and the number of sales decreasing for shoe sizes on either side of this centre.

The dot plot below shows one day's worth of sales by size in a shoe store that sells adult male shoes.



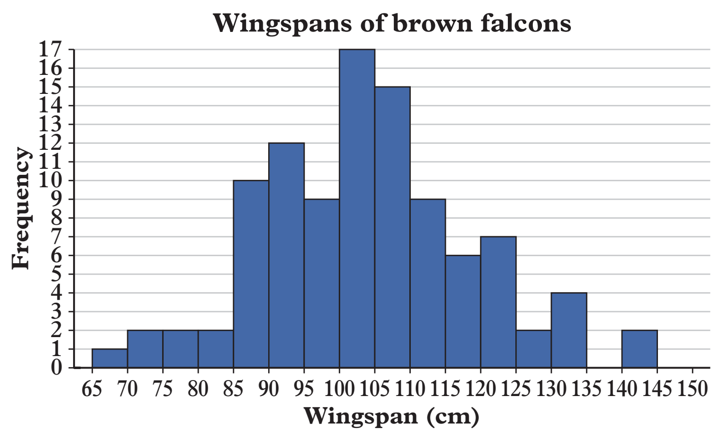
a Find the mean, median and mode of this data set. Round the value of the mean to one decimal place.

b Explain why, of the three values you found in part **a**, the mode is the most relevant statistic for the owner of the shoe store.

c What percentage of shoe sales were of sizes between 7 and 11, including 7 and 11?

d What percentage of shoe sales were of sizes between 5 and 13, including 5 and 13?

13 The bell-curve pattern can also be seen in histograms of continuous data. Approximately normally distributed data appears often in nature, from the heights and weights of most species of animals to the maximum daily temperatures in a city.



The histogram above was created from data collected on the wingspans of 100 adult brown falcons and placed in class intervals of size 5. The brown falcon is a bird species native to Australia and New Guinea, which is known to have a mean wingspan of approximately 105 cm.

a Redraw this histogram with class intervals of size 10, starting at 60.

b Redraw this histogram with class intervals of size 15, starting at 60.

c Which class interval size produces the histogram that most clearly shows the bell shape? Explain your answer.

d What percentage of the studied brown falcons had a wingspan between 90 cm and 120 cm?

e What percentage of the studied brown falcons had a wingspan between 75 cm and 135 cm?

Online resources:

Interactive skillsheet
Describing data

Quick quiz
7.5

Lesson 7.6

Comparing data

Learning intentions

By the end of this lesson you will be able to ...

✔ interpret and create graphs to compare multiple sets of data.

Inter-year links

Year 7	10.3 Stem-and-leaf plots
Year 8	10.3 Analysing data
Year 10	10.4 The mean and standard deviation (10A)

Comparing data sets

Key content video
Comparing data

- Data sets are easier to compare when displayed in the same format; for example, when two or more data sets are displayed in histograms.
- By placing the data in graphical displays, the range and shape of the distributions can be compared.
- To make a more thorough comparison of two or more data sets, summary statistics should be calculated so the centre and spread of each data set can be compared.

Back-to-back stem-and-leaf plots

- **Back-to-back stem-and-leaf plots** share the same stems with one data set on the right of the stems and the other data set on the left. For example, these two ordered stem-and-leaf plots show the ages of people swimming in two different pools.

Stem	Leaf Pool B
0	
1	7 7 7 7 8 9
2	2 3 4 4 6 7 8 9
3	2 2 4 5 5 7 9
4	2 2 6 7
5	1 9
6	2
7	

Key: 1 | 4 = 14

Stem	Leaf Pool A
0	2 7 9
1	1 4 5 6 7 9
2	0 1 1 5 6 8 8
3	1 2 2 4 9 9 9
4	2 4 5 7
5	2 3
6	
7	9

Key: 1 | 4 = 14

- The two stem-and-leaf plots can be combined into this back-to-back stem-and-leaf plot.

Leaf Pool B	Stem	Leaf Pool A
	0	2 7 9
9 8 7 7 7 7	1	1 4 5 6 7 9
9 8 7 6 4 4 3 2	2	0 1 1 5 6 8 8
9 7 5 5 4 2 2	3	1 2 2 4 9 9 9
7 6 2 2	4	2 4 5 7
9 1	5	2 3
2	6	
	7	9

Key: 1 | 4 = 14

→ On both sides of the back-to-back stem-and-leaf plot the values increase as they move away from the stem.

Worked example 7.6A Drawing a back-to-back stem-and-leaf plot to compare data sets

Draw an ordered back-to-back stem-and-leaf plot and make a brief comparison of the two data sets detailing the ages of people at a pool.

Winter: 45, 23, 15, 36, 57, 31, 9, 38, 44, 56, 52, 13, 36, 27, 48, 44, 48, 14, 27, 45

Summer: 31, 16, 14, 15, 23, 56, 24, 18, 17, 8, 11, 13, 16, 21, 17, 36, 20, 17, 14, 15

THINK

1 Identify the minimum and maximum numbers for both data sets.

2 The youngest is 8 and the oldest is 57, so use six stems: 0, 1, 2, 3, 4, 5.

3 Draw the back-to-back stem-and-leaf plot with these stems, and place the winter leaves on the left and the summer leaves on the right. Write the leaves in order and include a key.

4 Look at the centre and spread of the two data sets. Where does the centre appear to be for each set? What does this indicate about the ages at different times of the year?

WRITE

Leaf Winter	Stem	Leaf Summer
9	0	8
5 4 3	1	1 3 4 4 5 5 6 6 7 7 7 8
7 7 3	2	0 1 3 4
8 6 6 1	3	1 6
8 8 5 5 4 4	4	
7 6 2	5	6

Key: 1|4 = 14

The two data sets have a similar range of age brackets, but they are skewed in different directions and have different measures of centre. The data suggests that the pool attracts younger people in summer and older people in winter.

Worked example 7.6B Comparing data sets using summary statistics

Rachel investigated the average age of customers in two different cafes in her local area. Use her results to compare the ages of the customers in cafes.

Cafe	Mean	Median	Range
Gumtree	21	22	17
Jinx	27	21	45

THINK

1 Compare the given measure of spread (range).

2 Compare the given measures of centre (mean and median). The medians are similar, but Jinx has a higher mean than Gumtree. This difference combined with Jinx's large range suggests that Jinx's distribution may be skewed or affected by an outlier.

3 Summarise your findings.

WRITE

Jinx (45) covers a much wider range of ages than Gumtree (17).

The average ages are similar but Jinx (27) has a slightly higher mean than Gumtree (21). Jinx may have a positively skewed distribution or a data value for an older customer that is an outlier.

The mean customer age at the cafes is similar but Jinx has a larger range of ages.

Helpful hints

- ✔ When dealing with information from samples, the conclusions you draw will not be certain, so use words to show this, such as, 'The data indicates that …' or 'It appears that …'
- ✔ If you want to compare data sets but the data you're given is not in the same format, put the data sets in the same format before you begin the comparison.

Learning pathways

△ 1, 2(a, b), 3(a, b), 4(a–c), 5, 6, 8, 9, 13

□ 2(c, d), 3(b, c), 4(c–e), 6, 10, 11, 14, 16

⬠ 3(b, c), 4(c–e), 7, 11, 12, 14, 15, 17

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Exercise 7.6A: Understanding and fluency

1 Use the back-to-back stem-and-leaf plot shown below to answer these questions.

a What is the maximum score:

i in group A **ii** in group B **iii** overall?

Leaf Group A	Stem	Leaf Group B
9 7 4 3 1	0	7 9
9 8 8 6 4 3 2 1 0	1	3 4 6 8
7 5 3 0	2	0 1 1 4 4 4 5 8 9
8 3	3	2 2 3 4 7 9 9
1	4	0 3 5

Key: 1|3 = 13

b What is the most common score:

i in group A **ii** in group B **iii** overall?

c How would you describe the distribution of:

i group A **ii** group B?

2 Make a brief comparison of the two data sets in each back-to-back stem-and-leaf plot.

a

Leaf Group A	Stem	Leaf Group B
1	1	0 1 3 5 8 8
8 7 2	2	2 4 5 6 7 8 9 9
9 7 6 3 1	3	4 6 7 8
9 8 7 7 6 5 3 1 1	4	0 1 2
9 7 6 4 3 1 0	5	1 3

Key: 1|9 = 19

b

Leaf Group A	Stem	Leaf Group B
	3	0 0 1 2 4 5 6
9 8 7 6 5 4 4 4	4	1 2 5 5 6 7 8 9 9
8 7 6 6 5 4 3 1 1 0	5	0 3 4 5 5 6 8
8 7 6 5 4 3 3 0	6	
	7	
	8	1

Key: 3|2 = 3.2

c

Leaf Group A	Stem	Leaf Group B
	0	2 3 4
9 7 6 5 5 5	1	1 7 8 9 9
9 8 7 7 6 5 4 4 3	2	0 2 3 5 5 7 9
9 7 6 5 4 4 4 3	3	0 4 6 7 8 8
3 2 2 1	4	0 1 5 6
	5	0 6

Key: 1|7 = 170

d

Leaf Group A	Stem	Leaf Group B
	5	1
9 8 7 1	6	
7 6 5 4 3 3 3 2 1	7	8 9 9
9 8 7 5 4 2 2 1 0 0	8	1 2 3 4 6 7 8 8
	9	0 0 5 6 7 8 8 8 9

Key: 5|1 = 51

WE 7.6A **3** For each of the following draw a back-to-back stem-and-leaf plot and use it to make a brief comparison of the two data sets.

a Number of people at a cinema over 6 months of weekends:

Friday: 65, 48, 67, 55, 32, 92, 64, 51, 49, 57, 76, 61, 29, 46, 61, 59, 53, 67, 72, 88, 71, 58, 54, 57, 56, 42, 30

Saturday: 67, 78, 84, 26, 37, 42, 99, 84, 75, 68, 64, 55, 75, 85, 59, 66, 77, 78, 83, 81, 92, 94, 77, 76, 79, 89

b Daily maximum temperature in February:

Darwin: 32.3, 32.1, 32.9, 33.2, 33.5, 33.8, 33.4, 32.8, 32.5, 33.3, 32.5, 29.5, 32.8, 33.4, 33.1, 32.7, 33.4, 31.4, 33.0, 31.3, 29.9, 30.8, 30.0, 32.2, 32.1, 32.6, 31.8, 30.4

Adelaide: 20.6, 23.7, 25.4, 29.4, 34.2, 38.4, 27.0, 28.3, 28.0, 26.3, 28.3, 31.2, 33.8, 34.3, 36.0, 37.1, 39.2, 40.5, 26.6, 28.4, 32.1, 33.8, 35.6, 38.2, 28.7, 33.4, 21.5, 22.7

c Mass of dogs of two breeds (to the nearest hundred grams):

Boston Terriers: 5100, 6800, 7800, 10 500, 9200, 5500, 4900, 11 200, 8600, 9200, 10 500, 4900, 5500, 7600, 8400, 6800, 9200, 8600, 7500, 4600, 8300, 10 500, 11 000, 8000, 7600, 5100, 6700, 8300, 9200, 10 000, 4900, 7500, 7900, 6200, 8200, 4600, 7800, 8100, 6400, 9900

French Bulldogs: 9800, 12 900, 8600, 9900, 9800, 12 500, 11 200, 9100, 12 900, 11 200, 9500, 9200, 10 500, 11 900, 12 500, 11 800, 9400, 9800, 10 800, 10 600, 11 500, 9600, 12 300, 13 000, 11 000, 12 000, 10 600, 11 100, 12 700, 10 900, 9500, 11 500, 12 900, 10 100, 11 100, 9200, 11 500, 10 900, 9800, 11 300

WE 7.6B **4** For each table, use the given statistics to make a comparison of the two data sets.

a

Heights in class	Mean	Median	Range
9A	172 cm	168 cm	46 cm
9B	166 cm	167 cm	22 cm

b

Jeans prices	Mean	Median	Range
Store A	\$120	\$122	\$35
Store B	\$124	\$122	\$30

c

Internet usage per week	Mean	Median	Range
Primary school students	4.5 hours	4 hours	8 hours
Secondary school students	11.2 hours	12 hours	17 hours

d

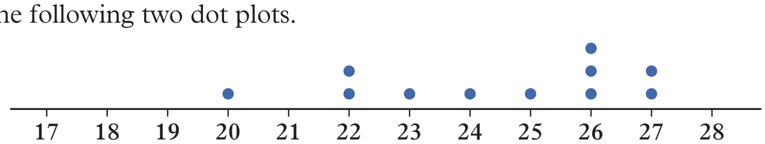
SD card capacity	Mean	Median	Range
Store A	20 GB	16 GB	60 GB
Store B	16 GB	16 GB	60 GB

e

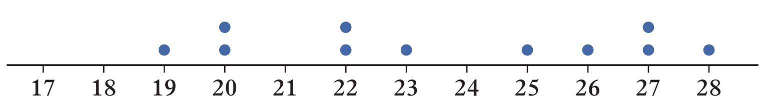
Goals per game	Mean	Median	Range
Player A	4.2	4	9
Player B	4.8	4	7

5 Consider the following two dot plots.

Group A



Group B

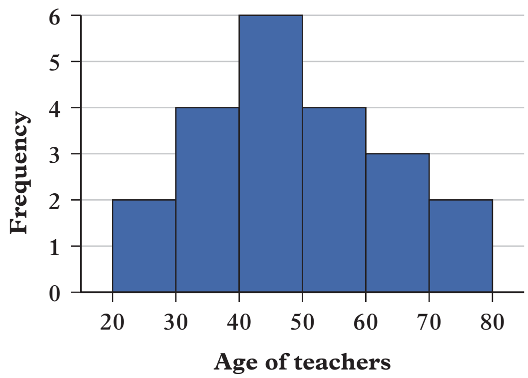


a Which group has the greater range? Justify your answer with calculations.

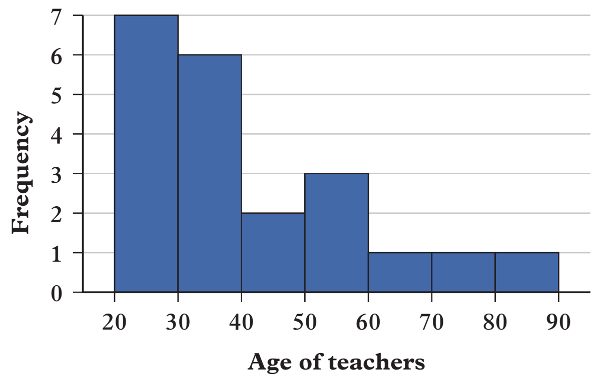
b Which group has the greater median? Justify your answer.

6 Consider the following two histograms that show the ages of teachers from two different schools.

School A



School B



- **a** Which school has the oldest teacher?
- **b** Which school has the most teachers below the age of 40?
- **c** Make a brief comparison of the two data sets.

Exercise 7.6B Problem solving and reasoning

7 Consider the back-to-back stem-and-leaf plot on the right, detailing the heights of Year 7 and Year 9 students (cm). Note the order of the data in the leaves of this stem-and-leaf plot.

Leaf Year 7	Stem	Leaf Year 9
8	14	
6 7 9 9	15	9
0 0 1 1 2 2 2 3 4 6 7 8	16	9 9 8 7 5 4 3 1
0 1 2 3 4	17	9 8 8 7 6 5 5 4 3 3 3 1
2	18	7 3 2 1
	19	2

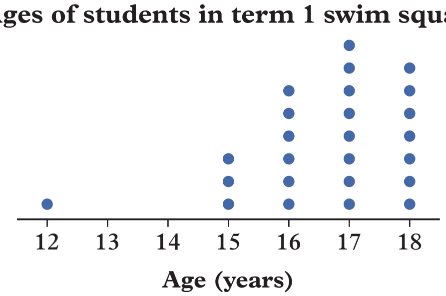
Key: 14|8 = 148

- **a** Make a brief comparison of the two data sets, writing your answer in context.
- **b** Calculate the mean (to two decimal places), median and range for both data sets displayed in the stem-and-leaf plot.
- **c** Use these statistics to make a more detailed comparison of the two data sets.
- **d** Does your answer to part **c** support your answer to part **a**? Explain.

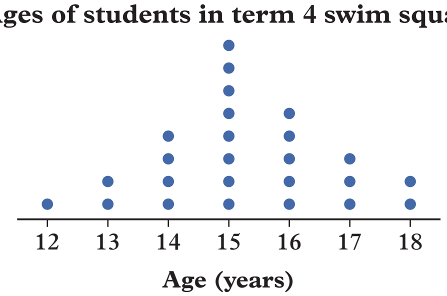
8 The ages of two swim squads were collected and displayed in the two dot plots below.

- **a** Compare the shape of the two dot plots.
- **b** Find the mean, median and range of each dot plot.
- **c** Write a couple of sentences comparing the two data sets with reference to their summary statistics. Be sure to explain the meaning of the range and centre in terms of the context of the data sets.

Ages of students in term 1 swim squad



Ages of students in term 4 swim squad



9 A class of Year 8 students and a class of Year 9 students were each asked to calculate the average number of minutes they spend studying per school night. The data is shown in the stem-and-leaf plots on the right.

a Redraw these separate stem-and-leaf plots as one back-to-back stem-and-leaf plot with the Year 8 group on the left and the Year 9 group on the right.

b Calculate the range of each group.

c Calculate the median of each group.

d Is it accurate to say that, in general, the Year 9 group studies more than the Year 8 group? Justify your answer.

Year 8

Stem	Leaf
0	0 5
1	4
2	3 5 7 7
3	0 0 2 3 8
4	1 4 4 9
5	5 8 9
6	0 5
7	1 4
8	3 6

Key: 4 | 1 = 41

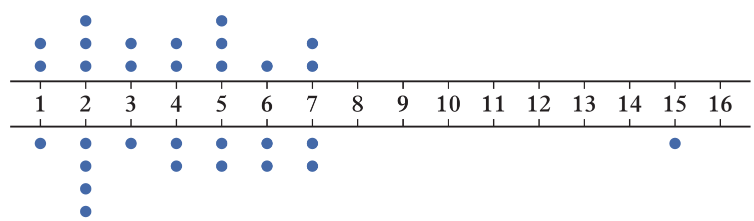
Year 9

Stem	Leaf
1	8 9
2	0 6
3	5 7
4	5 8 9
5	1 2 4 5
6	0 5
7	1 4 6 7 7
8	3 6
9	0 2 7

Key: 5 | 2 = 52

10 Two groups of 15 people were asked how many times they go to the hairdressers each year. Their responses are recorded in the back-to-back dot plot below. The data for Group A is above the number line and the data for Group B is below the number line.

Group A



Group B

a Find the median of each group.

b Complete the table below. Round the mean of each data set to one decimal place.

	Mean	Range
A		
B		

c Explain why the summary statistics in part **b** may be misleading to a person who has not seen the back-to-back dot plot.

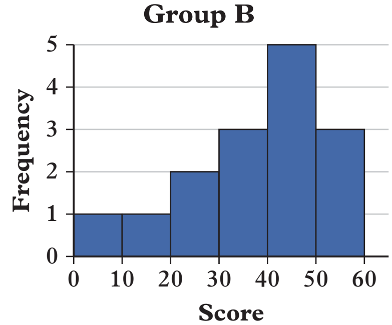
11 Consider the following stem-and-leaf plot and histogram.

Group A

Stem	Leaf
0	3 8 9
1	2 4 6 7
2	3 7 7
3	1 5
4	6 7
5	1

Key: 3 | 5 = 35

Group B



a Explain why you cannot redraw the histogram as a stem-and-leaf plot.

b Redraw the Group A data in the stem-and-leaf plot as a histogram with class intervals of 10 starting at 0.

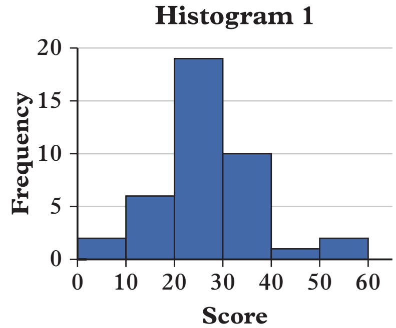
c Use the two histograms to make a brief comparison of the two data sets.

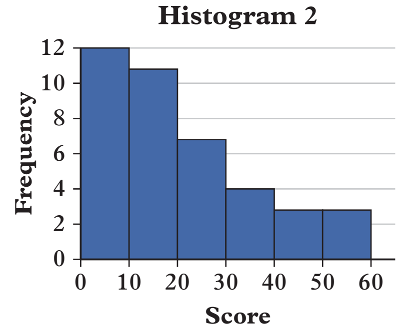
12 Consider the following summary statistics for two different data sets.

These data sets have been drawn as histograms, but they were mistakenly drawn without labels.

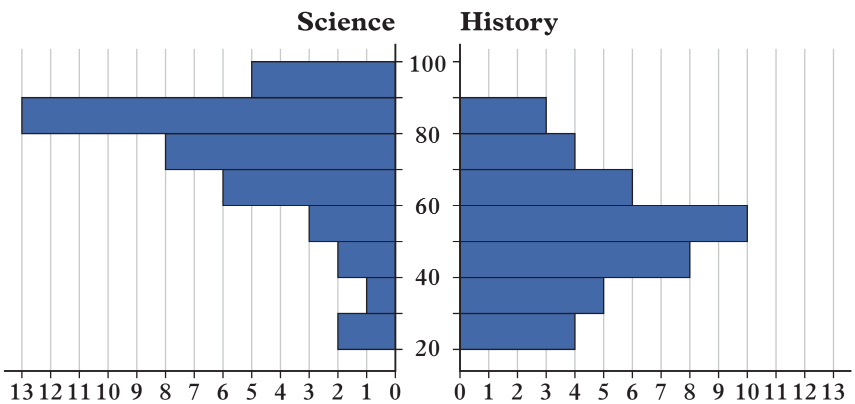
Which data set matches which histogram? Explain your answer.

	Mean	Median	Range
A	21	17	57
B	27	25	56





13 Two sets of grouped data can be displayed in one back-to-back histogram. The following back-to-back histogram shows the exam scores (out of 100) for the same 40 students in two different exams: Science and History.

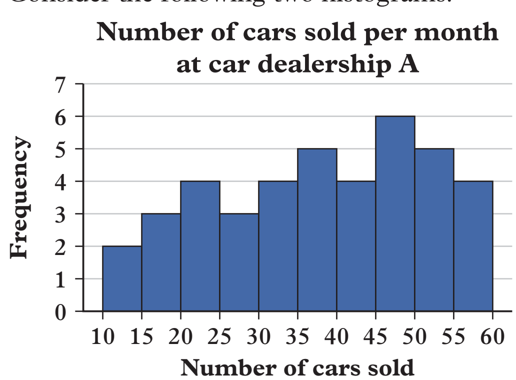


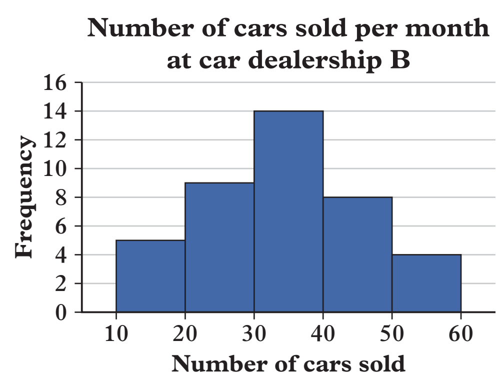
a Describe the distribution of the Science exam scores.

b Describe the distribution of the History exam scores.

c How many exam scores overall were in the interval 80–<90?

14 Consider the following two histograms.





a Why is it difficult to make a quick comparison of the two histograms as presented?

b Redraw the first histogram so that its class intervals match those of the second.

c Use your answer to part **b** to make a comparison of the two data sets.

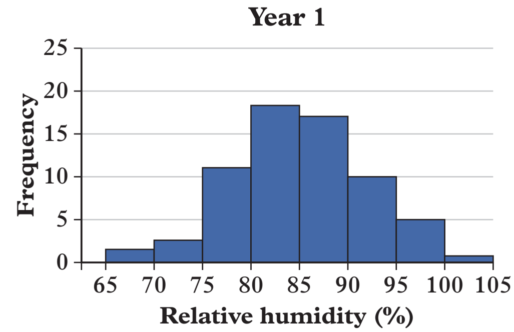
d Why can't you redraw the second histogram to match the first histogram?

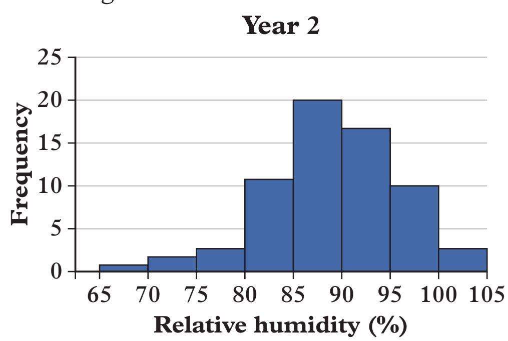
15 Relative humidity is an indicator of how much water vapor is in the air. As part of his research on climate change, Patrick recorded the relative humidity of this hometown every day for three consecutive years. The data he collected has been drawn as histograms shown on the right and below.

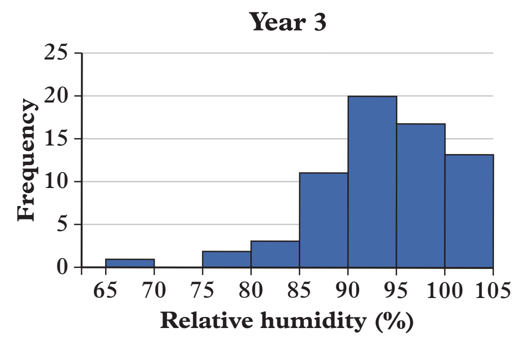
a What is the most common class interval in Year 1, Year 2 and Year 3?

b Which year has the greater median?

c What would you expect the histogram to look like in Year 4, assuming Patrick continues his research?







Exercise 7.6C: Challenge

16 A study on the gaming habits of Year 9 students focused on three separate classes, each with exactly 25 students. Each student was asked how many hours they spent per week playing video games. The study found the following results.

	Mean	Median	Range
Class A	5.5	5	12.5
Class B	7	6	28
Class C	6.5	8.5	14.5

a One class had an outlier. Which class had the outlier and how can you tell?

b The median for class C is much higher than the mean. Does this mean the data for class C is positively skewed or negatively skewed? Explain your answer.

c In general, which class plays video games the most? Justify your answer.

17 In a school there are 250 students in Year 8 and 200 students in Year 9. The following statistics have been calculated about the students' heights (in cm).

	Mean	Median	Minimum	Maximum
Year 8	160	161	142	173
Year 9	166	164	144	179

a Find the range for the heights of each year level.

b Find the range for the heights of all students in the two year levels combined.

c Find the average height of all students in the two year levels combined, rounded to one decimal place. Hint: The answer is not 163 cm.

d Explain why it is not possible to find the median height of all students in the two year levels combined.

Online resources:

Interactive skillsheet
Comparing data

Investigation
Closing the Gap

Quick quiz
7.6

Lesson 7.7

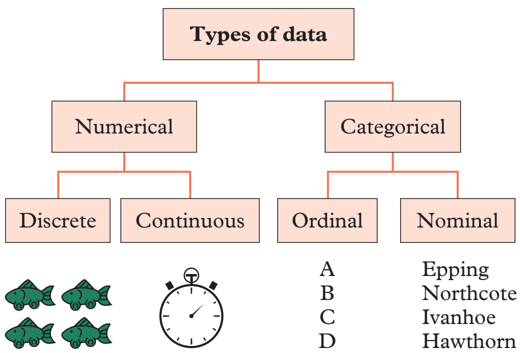
Review: Statistics

Module summary

Sampling

- *Stratified sampling* divides the population into strata and takes a random sample from each strata.
- *Systematic sampling* selects data at fixed intervals.
- *Convenience sampling* is when a sample that is easily accessible is selected.
- *Quota sampling* is when the researcher selects the size of each group to sample.

Classifying data



Bivariate data contains two variables.

Grouping data

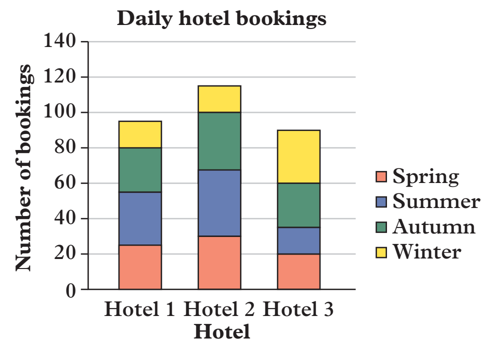
- Frequency tables: 5–10 class intervals
- Discrete data: 0–9
- Continuous data: 0–<10

Histograms

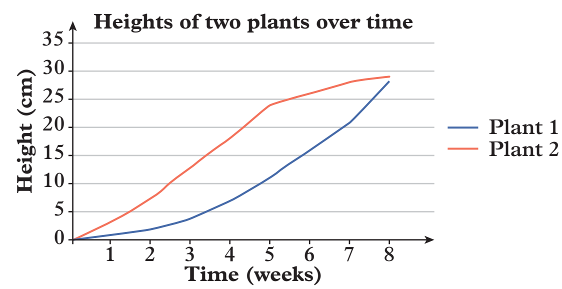
- Lower bound on left of each column
- For continuous data: upper bound on right of each column
- For discrete data: lower bound of next class interval on right of each column

Displaying data

- Bar charts: numerical variable or frequency (vertical axis), categorical data (horizontal axis)
- Stacked bar charts: second categorical variable in legend



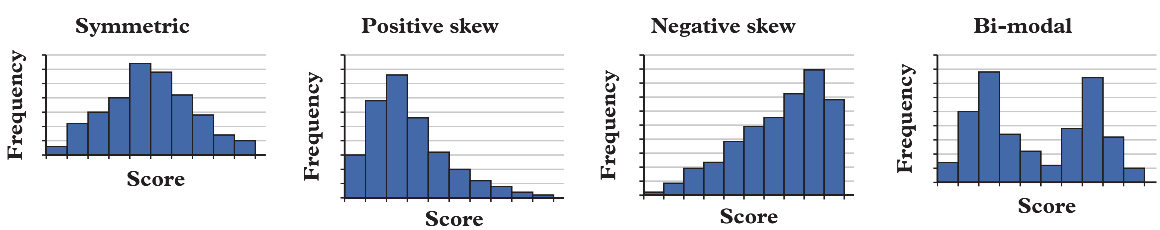
- Line graphs: two numerical variables
- Multiple line graphs/area charts: catagorical variable in legend



Measures of centre and spread

- Mean = $\dfrac{\text{sum of all data points}}{\text{total number of data points}}$
- Median = middle value when data is listed in order
- Mode = most common value
- Range = maximum value – minimum value
- To calculate the mean and median from a frequency table, add 'score × frequency' and 'cumulative frequency' columns to the table.

Data distributions



Describing distributions

- Symmetric: use mean or median
- Skewed: use median
- Bi-modal distribution: use mode of peaks
- Outliers: use median

Comparing data sets

- Put data sets in same format
- Compare centre and spread

Leaf	Stem	Leaf
9 8 7 7 7 7	1	1 4 5 6 7 9
9 8 7 6 4 4 3 2	2	0 1 1 5 6 8 8
9 7 5 5 4 2 2	3	1 2 2 4 9 9 9
7 6 2 2	4	2 4 5 7

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Review questions 7.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- area chart
- back-to-back stem-and-leaf plot
- bar chart
- bias
- bi-modal distribution
- bivariate data
- categorical data
- census
- class interval
- continuous data
- convenience sampling
- cumulative frequency
- discrete data
- grouped data
- histogram
- inference
- line graph
- lower bound
- mean
- measure of centre
- measure of spread
- median
- mode
- multiple line graphs
- multivariate data
- negatively skewed distribution
- nominal data
- numerical data
- ordinal data
- outlier
- population
- positively skewed distribution
- quota sampling
- range
- sample
- skewed
- stacked bar chart
- stratified sampling
- survey
- symmetric distribution
- systematic sampling
- ungrouped data
- univariate data
- upper bound

1 Explain the difference between a census and a sample.

2 Explain the difference between ordinal and nominal categorical data.

3 On which side of the distribution is the tail in a positively skewed distribution?

4 Identify the key terms being referenced in each of these definitions.

a a range of values that have been grouped together

b a distribution with two clear peaks

5 Using an example, provide a definition in your own words for the following key terms.

a cumulative frequency

b convenience sampling

6 Complete the following sentences using words from the key terms list.

a ________ is data that only concerns a single variable.

b A distribution is ________ when the data points are clustered on one side of the distribution.

Review questions 7.7B: Multiple choice

7.1 **1** Which of the following situations represents stratified sampling?

A Surveying the first 30 people you stop in the street

B Selecting 10 people from each of 5 age groups (20–29, 30–39, etc.) to complete a survey

C Asking your friends and family to complete a survey

D Surveying students from schools in a city in proportion to the number of students at each school

7.2 **2** Which of the following variables is categorical?

A Number of pets owned

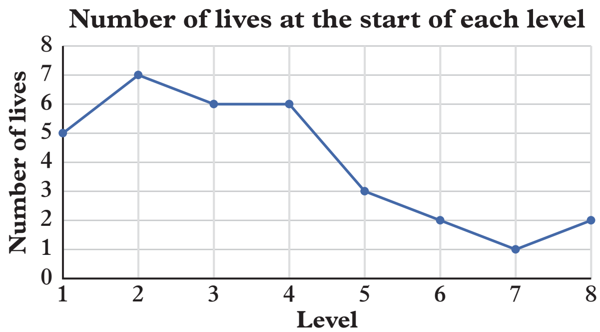
B Percentage score on a test

C Preferred music genre

D Time spent waiting in line at the canteen

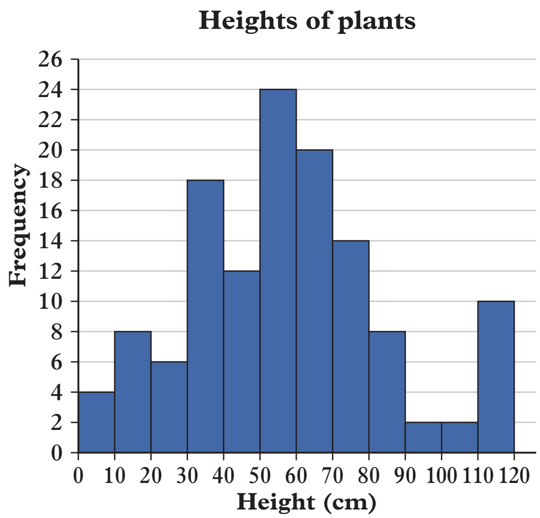
7.2 **3** The line graph below tracks the number of lives a gamer has when they begin each level of a video game. The gamer gains two lives during each level. During which level did they lose the most lives?

A 2
B 3
C 4
D 5



7.3 **4** The histogram below displays the results of research where the heights of plants (in cm) were measured. How many plants are less than 70 cm tall?

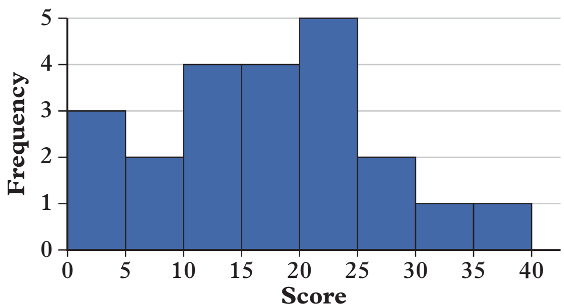
A 92
B 44
C 36
D 20



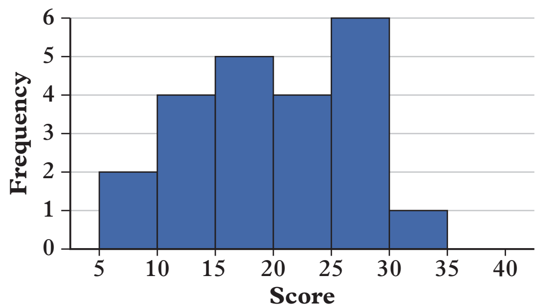
7.3 **5** The table of data below is represented in which histogram?

Class	0–<5	5–<10	10–<15	15–<20	20–<25	25–<30	30–<35	35–<40
Frequency	3	2	4	4	5	2	1	1

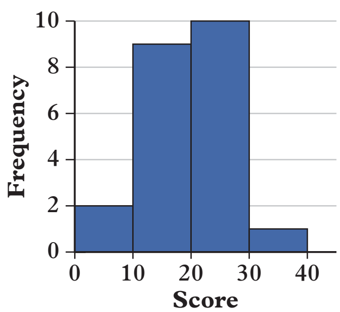
A



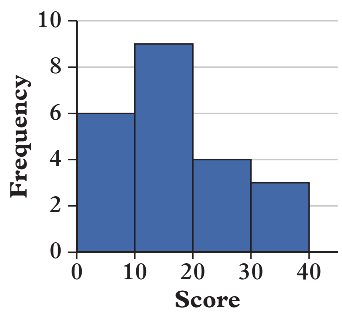
B



C



D



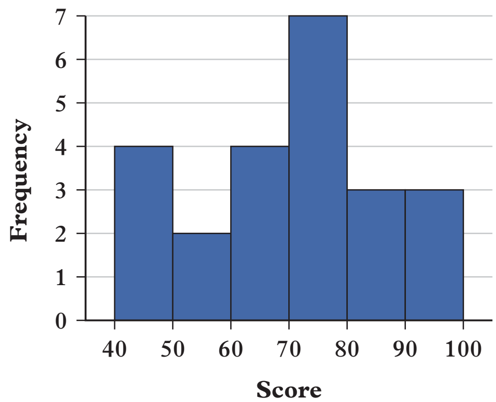
7.4 **6** In the histogram below, which class interval contains the median?

A 40–<50

B 50–<60

C 60–<70

D 70–<80



7.4 **7** What is the range of the data shown in the stem-and-leaf plot below?

A 75

B 11.6

C 7.5

D 116

Stem	Leaf
4	1
5	6 7
6	0 2 4 5
7	6 6
8	9
9	
10	1 4
11	2 6

Key: 5|6 = 5.6

7.5 **8** Which statement is true if describing distribution of data?

A Symmetric distributions have a centre closer to the right of the distribution, away from the *y*-axis.

B Any distribution that is skewed or has an outlier should have its centre described using the median rather than the mean.

C Positively skewed distributions have a middle peak and a roughly even spread on either side of this peak.

D Negatively skewed distributions have a centre closer to the left of the distribution, towards the vertical axis.

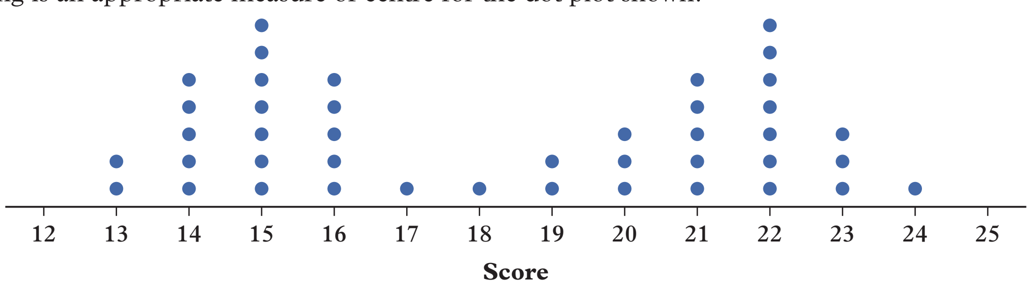
7.5 **9** Which of the following is an appropriate measure of centre for the dot plot shown?

A Mean

B Range

C Median

D Mode(s)



7.6 **10** How is each data set distributed in the back-to-back stem-and-leaf plot on the right?

A Both groups are positively skewed.

B Group A is negatively skewed and Group B is positively skewed.

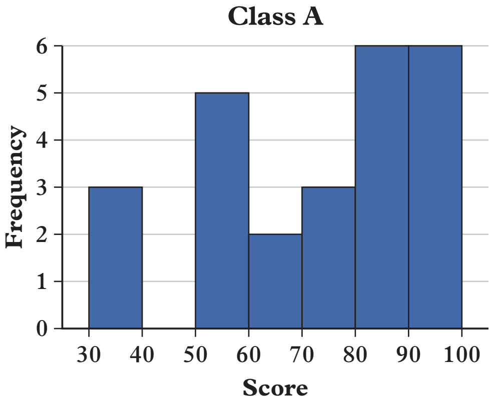
C Group A is positively skewed and Group B is negatively skewed.

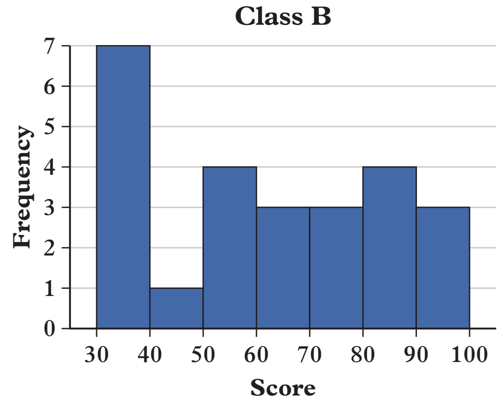
D Both groups are negatively skewed.

Leaf Group A	Stem	Leaf Group B
9 8 6 6 4 4 3 2	0	
8 7 5 4 3 1	1	9
7 7 1	2	1
5	3	3 4
4	4	2 4 7 9
1	5	2 3 3 4 6 8 9 9
	6	1 5 6 6 8

Key: 1|9 = 19

7.6 **11** The two histograms below show the Geography exam scores of two different Year 9 classes. Which of the following statements is definitely true?





A The range of each class is at least 60.

B There are more students in Class A than in Class B.

C The range of Class A is greater than the range of Class B.

D The median of Class A is greater than the median of Class B.

Review questions 7.7C: Short answer

7.1 **1** Decide whether each survey below will provide fair or biased results, giving reasons for your answers.

a Asking 20 of your friends their favourite film, in order to determine the favourite films of students at your school.

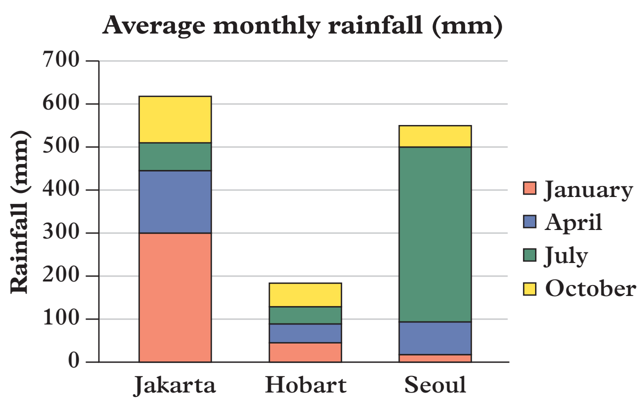
b Randomly asking 50 people (stratified by age) from your local council area who they want to win most seats at the next election to find the most popular Australian political party.

7.1 **2** Explain how each of the following headlines might be misrepresenting the data described in brackets.

a 'Swimming is the best form of exercise.' (18 of 20 swimming coaches were surveyed and agreed)

b 'Veganism increasing in popularity.' (75% of people liked a new vegan burger)

7.2 **3** The stacked bar chart on the right shows the average monthly rainfall in three cities across four different months.



a Identify the three variables shown in the graph and classify each as either numerical or categorical.

b Which city has the most consistent rainfall over the year?

c Of the months shown, when does Seoul have more rain than Jakarta?

d Suggest a reason that the wettest months in Seoul and Jakarta do not occur at the same time.

7.2 **4** Marina has started a new job selling cars at a car dealership. The number of cars she sold during each of her first 9 months is shown in the table below.

Month	1	2	3	4	5	6	7	8	9
Number of cars sold	2	4	8	14	12	15	19	21	22

a Draw a line graph to represent this data.

b In which month(s) did Marina sell fewer cars than in the previous month?

c At the car dealership, Marina is paid a fixed salary of \$4000 per month plus an extra \$200 for each car she sells. What was her total income from this job during month 6?

7.3 **5** Data was collected on the number of hours Year 9 students spent using various forms of social media over the course of a week.

45 21 37 21 20 17 31 32 11 15 17 18 20 31 48 32 21 5 11 7 19 18 27 42

40 21 32 23 24 19 38 37 14 15 19 28 22 35 41 33 27 2 10 5 18 18 25 43

a Construct a frequency table with class intervals of width 10.

b How many people were surveyed?

c What is the most common class interval?

d Draw a histogram to represent this data.

e Which interval is the most common?

7.4 **6** Find the mean, median, mode and range for this data set, correct to two decimal places where appropriate.

Score	3	4	5	6
Frequency	10	12	23	5

7.4 **7** Calculate the mean, median, mode and range for the data represented in this stem-and-leaf plot, correct to two decimal places where appropriate.

Stem	Leaf
0	2 4 5 7 7 7 7
1	5 8 8 8 9
2	1 1 3 4 7 9
3	7 8 8
4	1 2 5
5	8

Key 2|4 = 24

7.5 **8** The ages of 53 people are shown in the stem-and-leaf plot on the right.

a Find the median age of the group.

b Find the range of the data set.

c Draw a histogram to represent this data. Use class intervals of 5 with the first interval starting at 0.

d Describe the distribution of ages in the data set.

Stem	Leaf
0	4 4 5 5 7 8
1	0 1 1 1 3 4 4 5 6 6 6 6 7 9 9 9
2	0 0 1 3 4 4 4 4 5 6 6 6 7 7 7 8 9 9
3	0 0 0 0 1 1 1 2 2 3 3 3 3

Key: 1|3 = 13

7.6 **9** The results achieved in Maths tests by two different classes are recorded in a back-to-back stem-and-leaf plot.

Leaf Class 9A	Stem	Leaf Class 9B
8 5 5 3	4	6 7 8 8 9
9 7 3	5	1 3 6 7
7 6 5 4 3	6	3 4 7
0	7	2 4 8
9 7 5 3 1	8	2 5 6 7
0	9	8 9

Key: 4|6 = 46

a Find the mean, median, mode and range for each class.

b Compare the Maths results of the two classes.

7.6 **10** The following data comparing the number of hours spent at football training per fortnight were recorded for Year 9 boys from two different secondary schools.

School 1: 35 11 27 11 10 7 21 22 1 5 7 8 10 21 38 22 11 5 1 7 9 8 17 32

School 2: 12 14 16 17 20 11 22 11 11 16 19 14 2 4 10 19 17 6 4 6 2 7 16 31

a Collate the data in a back-to-back stem-and-leaf plot using class intervals of width 5.

b Calculate the mean, median and range for each school, correct to one decimal place.

c Which measure of centre would you use to discuss the results for each school?

Review questions 7.7D: Mathematical modelling

The stem-and-leaf plot on the right shows the measurements of shrubs taken at different nurseries.

Leaf Brisbane	Stem	Leaf Sydney
7 4 4 4	11	4 4 4 5 7 8 9
7 6 4 3 2 2	12	1 3 6 7
8 7 6 2 1	13	2 3 9
1 0	14	1 4 6 7 8 9
9 8 5 3 2	15	1 2 5 5

Key: 11 | 4 = 11.4 cm

a Compare the number of shrubs measured at each location.

b Calculate the mean, median, mode and range of shrub heights at each location, correct to two decimal places where necessary.

c Write a short comparison of the height of shrubs at the two locations.

d The manager of the nursery company wanted to collate the data for all of his businesses.

- **i** Generate lists of raw data for Brisbane and Sydney. Using this data and the lists below, create a frequency table using suitable class intervals and collate all of the data about shrub height.

  Melbourne: 12.1, 11.7, 18.3, 11.4, 11.4, 14.5, 15.6, 17.1, 16.5, 18.6, 13.0, 12.6, 12.4, 10.9, 11.4, 14.0, 16.9, 17.1, 16.5, 18.6

  Perth: 10.2, 10.6, 19.3, 11.4, 11.4, 15.9, 14.7, 18.3, 17.7, 13.4, 10.0, 12.5

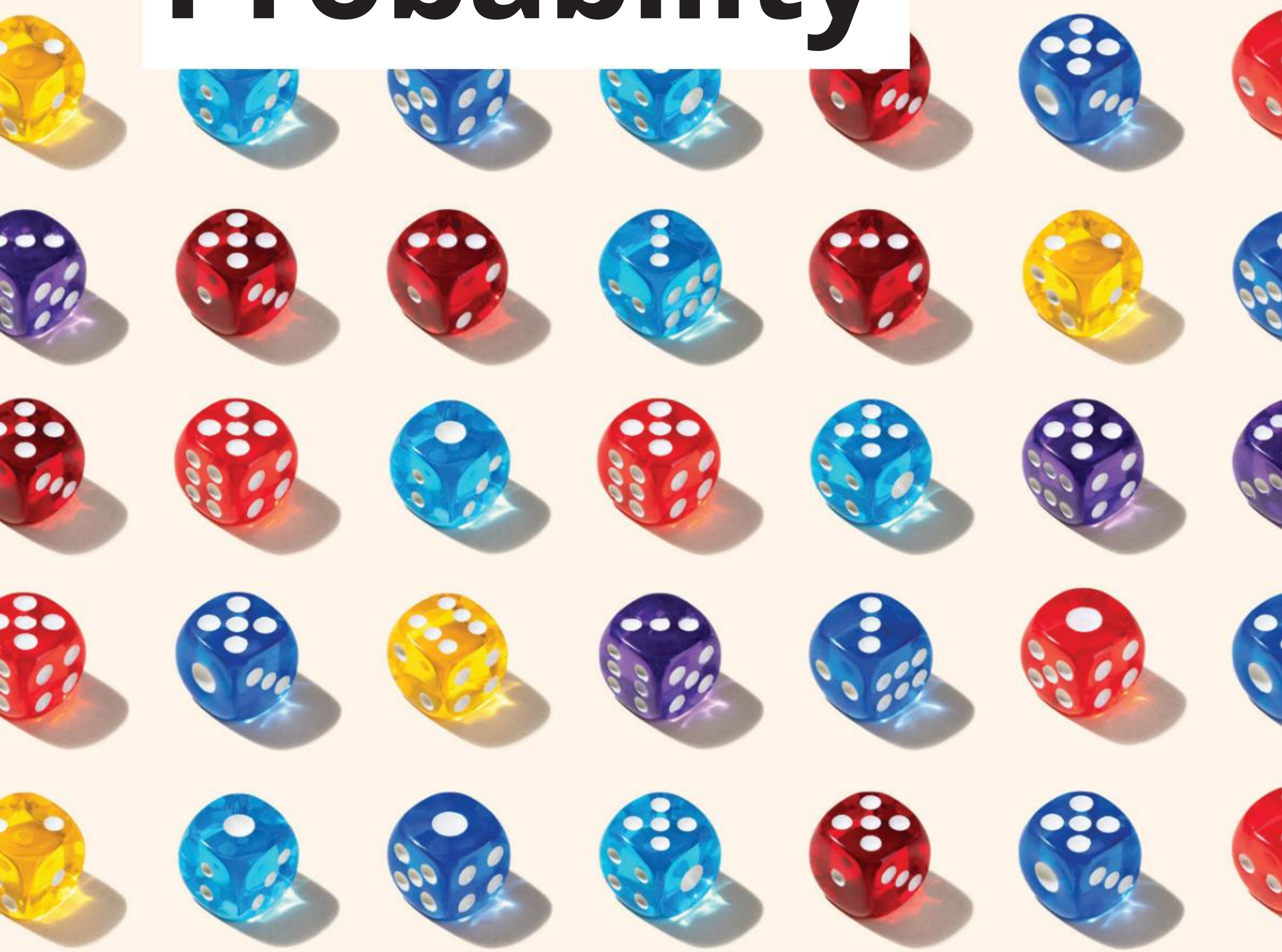
- **ii** How many plants were measured in total?
- **iii** What is the modal class?
- **iv** Represent this data as a histogram and comment on the shape of the distribution.
- **v** Calculate the mean shrub height for Melbourne and Perth, correct to two decimal places.
- **vi** Write a statement that could be used in a marketing campaign, which includes information about the smallest and largest shrubs and the average shrub height.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Evaluate the appropriateness of sampling methods ☐ Evaluate surveys and statistical reports	☐ Go back to Lesson 7.1 Surveys and sampling methods
☐ Classify data involving more than one variable ☐ Display data involving more than one variable ☐ Interpret data displayed in charts and graphs	☐ Go back to Lesson 7.2 Classifying and displaying data
☐ Group numerical data into class intervals ☐ Represent grouped numerical data in a histogram ☐ Interpret and analyse data displayed in a histogram	☐ Go back to Lesson 7.3 Grouped data and histograms
☐ Calculate summary statistics from frequency tables ☐ Calculate summary statistics from displays, such as stem-and-leaf plots, dot plots, column graphs and histograms	☐ Go back to Lesson 7.4 Summary statistics from tables and displays
☐ Identify whether a data set is symmetric, skewed or bi-modal ☐ Determine which measure of centre best describes a data set	☐ Go back to Lesson 7.5 Describing data
☐ Construct back-to-back stem-and-leaf plots ☐ Compare data sets in different graphical displays, such as dot plots, histograms and back-to-back stem-and-leaf plots ☐ Compare data sets using summary statistics	☐ Go back to Lesson 7.6 Comparing data

8 Probability



Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

✔ Calculating theoretical probability
✔ Equivalent fractions
✔ Adding and subtracting fractions
✔ Multiplying and dividing fractions

Curriculum links

- List all outcomes for two-step chance experiments both with and without replacement, using lists, tree diagrams, tables or arrays; assign probabilities to outcomes and events (VC2M9P01)
- Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and', inclusive 'or' and exclusive 'or' (VC2M9P02)
- Design and conduct repeated chance experiments and simulations using digital tools to estimate probabilities that cannot be determined exactly (VC2M9P03)

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Materials

✔ Calculator

Two-step chance experiments

Learning intentions

By the end of this lesson you will be able to …

✔ use tree diagrams and arrays to calculate theoretical probabilities.

Inter-year links

Support	Understanding probability
Year 7	10.5 Theoretical probability
Year 8	10.4 Theoretical probability
Year 10	11.1 Theoretical probability

Calculating theoretical probability

Key content video
Theoretical probability

Key content video
Complementary events

- The **sample space** of an **experiment** is a list of all possible outcomes in the experiment.
- An **event** is a collection of one or more outcomes from the sample space of an experiment.
- The probability of an event occurring can be written as Pr(A), where A represents the event.
- If all the outcomes in an experiment are equally likely to occur, the **theoretical probability** of an event occurring is the ratio of the number of outcomes in the event to the number of outcomes in the sample space. It can be expressed as a fraction, decimal or percentage and calculated using the formula:

$$\Pr(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$$

- The complement of an event A is the event that A does not occur. Events A and 'not A' are **complementary events**.
  - → Complementary events are denoted using the symbol prime (′). A and A′ are complementary events.
  - → All outcomes in the sample space that are not in A must be in A′. $\Pr(A) + \Pr(A') = 1$
- When calculating the probability of an event involving multiple outcomes, the probabilities of each outcome are added together to give the probability of the event.

Two-step chance experiments

Key content video
Arrays and tree diagrams

- Two-step chance experiments involve two steps, which may be completed at the same time or one after the other. For example, rolling a standard six-sided die and tossing a coin is a two-step chance experiment.

Arrays

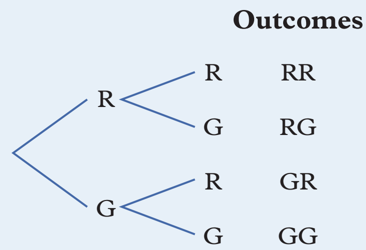
- An **array** is a list of items arranged in rows and columns.
- The sample space of a two-step chance experiment shown in an array has the outcomes of one event in columns and the outcomes of the other event in rows. For example, this array shows the outcomes from rolling a standard six-sided die and tossing a coin.
  - → There are two outcomes when a coin is tossed: heads (H) or tails (T).
  - → There are six outcomes when a die is rolled: 1, 2, 3, 4, 5, 6.

		Rolling a standard six-sided die					
		1	**2**	**3**	**4**	**5**	**6**
Tossing a coin	**H**	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
	T	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)

- When calculating probability using an array, all the outcomes are equally likely, and the theoretical probability formula can be used.
- Arrays are best used to display the sample space when there are a large number of outcomes in either step of a two-step experiment.

Tree diagrams

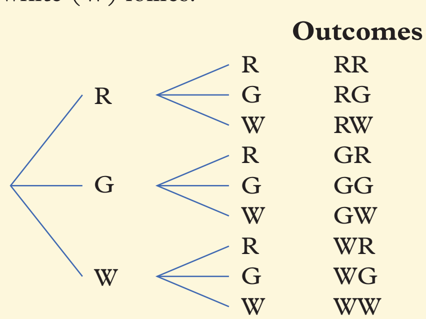
- In a **tree diagram**, the outcomes of each step of a chance experiment are listed vertically, with the events joined with branches. For example, this tree diagram shows the outcomes for selecting two marbles at random from a bag of red and green marbles.



- If the outcomes in a tree diagram are equally likely, the theoretical probability formula can be used.
- The probability of each final outcome can be calculated by multiplying together the individual probabilities along the branches that lead to that outcome. For example, Pr(G and R) = Pr(G) × Pr(R).

Worked example 8.1A Understanding tree diagrams

This tree diagram displays the outcomes of selecting two lollies at random from a jar containing red (R), green (G) and white (W) lollies.



Use this tree diagram to find:

a the total number of outcomes

b the number of outcomes containing at least 1 white lolly.

THINK	WRITE
a Count the number of final outcomes at the right end of the tree diagram.	**a** There are 9 possible outcomes: RR, RG, RW, GR, GG, GW, WR, WG, WW.
b Trace the branches carefully and count the number that contain at least one white lolly.	**b** 5 outcomes contain a white lolly: RW, GW, WR, WG, WW.

Worked example 8.1B Calculating probability using an array

Use an array to calculate the probability of a total sum of 10 or above when rolling two standard six-sided dice.

THINK

1 Draw an outline for the array, with enough columns to list all the outcomes for the first step and enough rows to list all the outcomes for the second step.

2 Complete the array by showing the result for the first step before the result for the second step in brackets, separated by a comma.

3 Count the outcomes that have a total sum of 10 or more (these cells have been highlighted in the answer).

4 Count the total number of outcomes in the array.

5 Use the theoretical probability formula and simplify if possible.

WRITE

		1st dice roll					
		1	**2**	**3**	**4**	**5**	**6**
2nd dice roll	**1**	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
	6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

$$\begin{aligned}\text{Pr(10 or above)} &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}\\ &= \frac{6}{36}\\ &= \frac{1}{6}\end{aligned}$$

Worked example 8.1C Calculating probability using a tree diagram

Use a tree diagram to calculate the probability of landing on exactly one tail when tossing a coin twice.

THINK

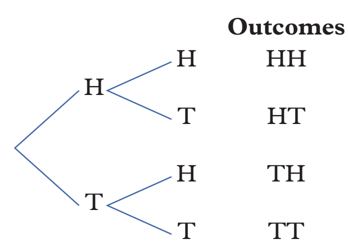
1 Draw the first two branches to represent the first coin toss. Label the ends of these branches with H and T to represent the two different outcomes, heads and tails.

2 From each branch, draw another two branches to represent the next coin toss and label them appropriately.

3 Write the final outcome on each of the four branches to complete the tree diagram.

4 There are four possible outcomes and two outcomes contain exactly one tail (HT, TH). As each of the outcomes are equally likely, the probability formula can be used.

WRITE



$$\begin{aligned}\text{Pr(exactly one tail)} &= \frac{2}{4}\\ &= \frac{1}{2}\end{aligned}$$

Helpful hints

✔ In other contexts, an event means an occurrence or an occasion, but in probability it means a collection of outcomes from the sample space. Don't get confused!

✔ When creating a tree diagram, make sure you leave plenty of space to draw in all the branches!

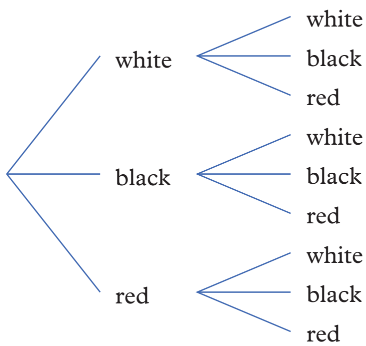
Learning pathways

△ 1–8, 10, 11 □ 2–4, 6, 8–12 ⬠ 2–4, 6, 9–13

ANS p521

Exercise 8.1A: Understanding and fluency

WE 8.1A **1** Consider this tree diagram that shows the results of choosing two coloured balloons at random.



a How many possible outcomes are there?

b How many of these outcomes contain at least one red?

WE 8.1B **2** Consider this array which shows the results of two people randomly picking a number between 1 and 3.

a How many possible outcomes are there?

b State the probability of obtaining a sum of:

i 3 **ii** an odd number

iii a prime number **iv** a square number

v a number greater than 6.

		1st number		
		1	**2**	**3**
2nd number	**1**	(1, 1)	(2, 1)	(3, 1)
	2	(1, 2)	(2, 2)	(3, 2)
	3	(1, 3)	(2, 3)	(3, 3)

3 Two four-sided dice (numbered 1–4) are rolled and the numbers showing are added together to give the final outcome.

a Create an array that lists all the outcomes.

b How many equally likely outcomes are detailed in the array?

c What is the most likely final outcome? What is the probability of this occurring?

d What are the least likely final outcomes? What is the probability of one of these outcomes occurring?

e State the probability of rolling two four-sided dice (numbered 1–4) and obtaining a sum:

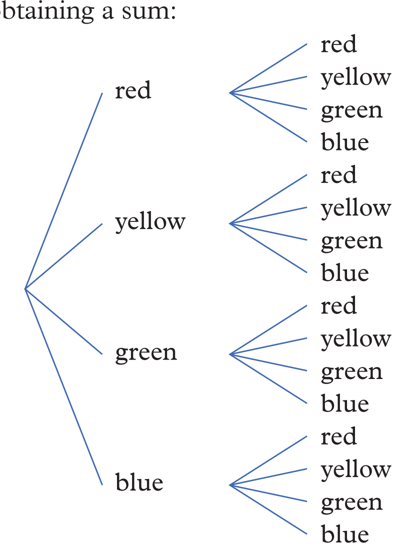
i of 3

ii greater than 6

iii which is an odd number

iv less than 5.

4 A spinner with four equally-sized coloured segments – red, yellow, green, blue – is spun twice. Assuming all outcomes are equally likely, use the tree diagram on the right to find the probability of landing on:



a the yellow segment (at least once)

b the green and the red segment (in any order)

c the blue segment twice

d two segments which are not red

e the yellow or blue segment

f the blue segment then the green segment.

WE 8.1C **5** A coin is tossed twice. Use a tree diagram to find the probability of landing on:

a two heads

b one head and one tail

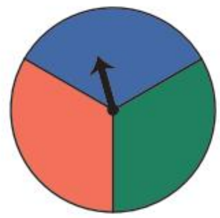
c no heads.

6 The spinner on the right is spun twice.
Use a tree diagram to find the probability of:

a two different results

b spinning blue at least once

c spinning the same colour both times.



Exercise 8.1B: Problem solving and reasoning

7 This array represents a single round of a game of rock (r), paper (p), scissors (s).

		Your choice		
		r	**p**	**s**
Opponent's choice	**r**	(r, r)	(p, r)	(s, r)
	p	(r, p)	(p, p)	(s, p)
	s	(r, s)	(p, s)	(s, s)

rock beats scissors

paper beats rock

scissors beats paper

a List the outcomes that result in you:

i winning

ii losing

iii drawing the game.

b Hence calculate the probability that you win a game of rock, paper, scissors.

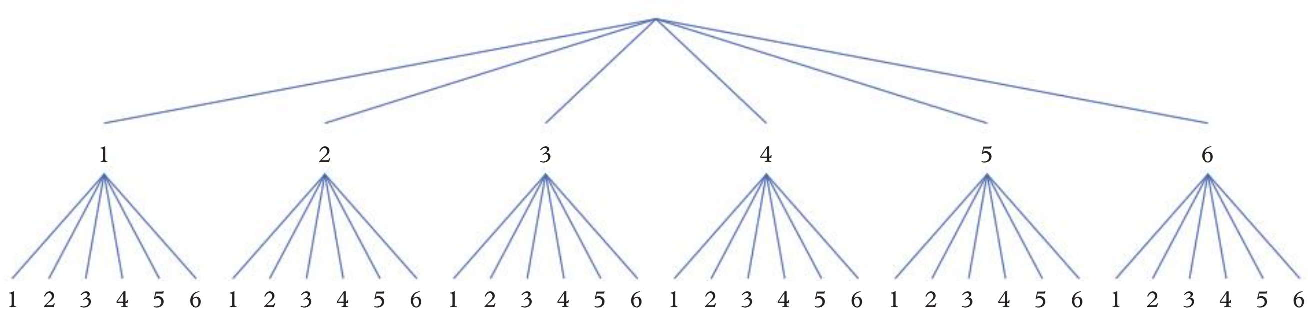
8 Consider sitting a quiz consisting of multiple-choice questions with answers A–D.

a Draw a tree diagram to represent the possible different options for the first two questions.

b Draw an array to represent the possible different options for the first two questions.

c Explain why you could use a tree diagram but not an array to represent guessing the answers to the first three questions.

9 The tree diagram below shows the result of rolling two standard six-sided dice.



a How many possible outcomes are there?

b Use the tree diagram to find the probability of rolling:

i two odd numbers

ii one even number

iii a double

iv a total of 6.

c Why would it be more appropriate to use an array to display the sample space?

10 Tree diagrams aren't limited to repeated trials of the same experiment. They can also be used to display unrelated trials. Imagine that you toss a coin and then roll a standard six-sided die.

a Draw a tree diagram to represent this two-step experiment.

b How many outcomes are there?

c What is the probability that you toss a tail:

i and roll a 6 **ii** and roll a number less than 4

iii or roll a 6 **iv** or roll a number less than 4?

d Explain the mistake made by someone who answers $\frac{8}{12}$ to part **c iii**.

11 Similar to tree diagrams, arrays are not limited to repeated trials of the same experiment. They can also be used to display unrelated trials. Imagine that you select a vowel (A, E, I, O, U) at random and roll a standard four-sided die.

a Draw an array to represent this two-step experiment.

b How many outcomes are there?

c What is the probability that:

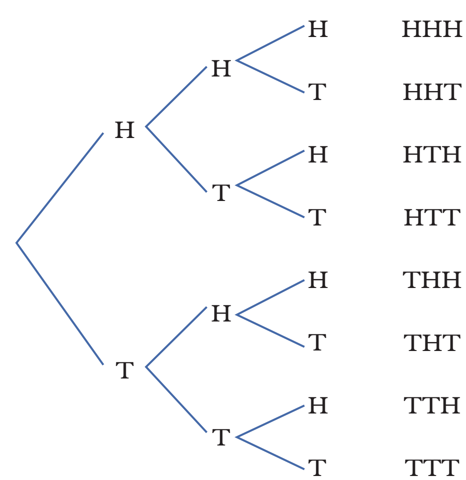
i you select E and roll a 3

ii you select E or roll a 3

iii you select a vowel with only straight lines and roll a number less than 4

iv you select a vowel with only straight lines or roll a number less than 4?

12 Experiments can have more than two steps. When using a tree diagram to represent this, we add more sets of branches to the end of each branch. For example, if a coin is tossed three times (or three coins are tossed), then the tree diagram that represents the sample space would look like this.



a Use the tree diagram shown to determine the probability of tossing a fair coin three times and getting:

i three heads

ii two heads and one tail in any order

iii two tails and one head in any order

iv an alternating sequence of heads and tails in any order.

b Explain why you couldn't use an array to show the sample space for an experiment with three or more steps where each set of outcomes at each step is shown. That is, without using the outcomes of a group of steps against the outcomes of another group.

Exercise 8.1C: Challenge

13 Anna is investigating the possibilities when rolling four fair six-sided dice.

a Find the probability of rolling four of the same number.

b Find the probability of rolling exactly three of the same number.

c Find the probability of rolling two of the same number and two different numbers.

d Find the probability of rolling two copies of each of two different numbers (e.g. two 2s and two 3s).

e Find the probability of rolling four different numbers.

Online resources:

Interactive skillsheet
Arrays

Interactive skillsheet
Tree diagrams

Investigation
Dominoes and tree diagrams

Quick quiz
8.1

Lesson 8.2

Experiments with replacement

Learning intentions

By the end of this lesson you will be able to …

✔ calculate probabilities for two-step chance experiments with replacement.

Inter-year links

Year 7	10.5 Theoretical probability
Year 8	10.4 Theoretical probability
Year 10	11.2 Experiments with and without replacement

Calculating probabilities for two-step experiments

Key content video
Experiments with replacement

- The probabilities for each step of a two-step experiment can be written on the branches of a tree diagram.
- The probabilities for each outcome can be found by multiplying together the probabilities of each branch leading to that outcome. Pr(A and B) = Pr(A) × Pr(B)
  → The probabilities of all the outcomes of an experiment always sum to 1.
- If the event you are looking for consists of more than one outcome, the probabilities of each outcome are added together to find the probability of the event.

Experiments with replacement

- Experiments with replacement involve selecting an item at random, recording the result, and then replacing the item before making another selection.
- When replacing the item before making another selection, the probabilities for each step of the experiment remain the same.

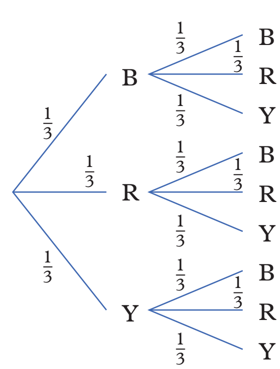
Worked example 8.2A Representing experiments with equally likely outcomes

A box contains equal numbers of blue, red and yellow activity cards. A card is selected at random, its colour is recorded and then it is replaced. Another card is then selected at random. Draw a tree diagram to represent this situation, complete with probabilities on each branch and for each final outcome.

THINK

1. Draw a tree diagram to represent this two-step experiment, listing all the final outcomes.
2. Write the probability of each branch on your tree diagram. Each colour is equally likely so there is a $\frac{1}{3}$ chance of selecting a blue card, a $\frac{1}{3}$ chance of selecting a red card and a $\frac{1}{3}$ chance of selecting a yellow card.
3. Multiply the probabilities along each branch to determine the probability of each final outcome.

WRITE



Outcomes	Probability
BB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
BR	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
BY	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
RB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
RR	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
RY	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
YB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
YR	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
YY	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$

Worked example 8.2B Calculating probabilities for experiments with equally likely outcomes

Use the tree diagram from **Worked example 8.2A** to find the probability that:

a two blue cards are selected

b a yellow card is selected first.

THINK	WRITE
a Identify the outcome in which both selections are blue cards.	**a** $\Pr(\text{two blue}) = \Pr(\text{BB})$ $= \frac{1}{9}$
b **1** Identify the outcomes in which a yellow activity card is selected first. **2** Find the sum of the probabilities and simplify.	**b** $\Pr(\text{yellow first}) = \Pr(\text{YB, YR, YY})$ $= \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$ $= \frac{3}{9}$ $= \frac{1}{3}$

Worked example 8.2C Representing experiments with outcomes that are not equally likely

A bag contains 15 red balls and 5 green balls. A ball is selected at random, its colour is recorded and then it is replaced. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment and find the probability of each outcome.

THINK

1 Draw a tree diagram to represent this experiment, listing all the final outcomes.

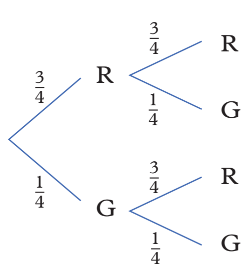
2 Find the probability of a green ball being selected: $\Pr(G) = \frac{5}{20} = \frac{1}{4}$

Find the probability of a red ball being selected: $\Pr(R) = \frac{15}{20} = \frac{3}{4}$

3 Write the probability on each branch.

4 Multiply the probabilities along each branch to determine the probability of each final outcome.

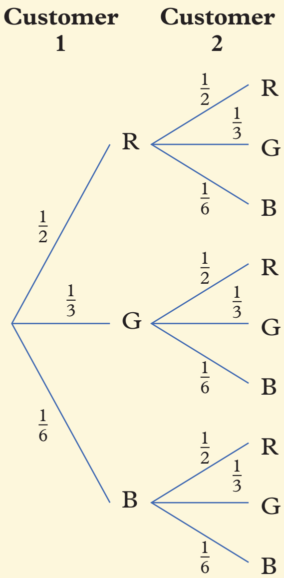
WRITE



Outcomes	Probability
RR	$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$
RG	$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$
GR	$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$
GG	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Worked example 8.2D Calculating probabilities for experiments with outcomes that are not equally likely

This tree diagram shows the possible outcomes when two balls are selected at random from a bag containing 3 red balls, 2 green balls and 1 blue ball. The first ball is returned after the first selection.



Outcomes	Probability
RR	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
RG	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
RB	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
GR	$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$
GG	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
GB	$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
BR	$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
BG	$\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$
BB	$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Calculate the probability of selecting:

a exactly one blue ball

b a blue ball and a green ball.

THINK	WRITE
a Identify the outcomes in which exactly one blue ball is selected. Find the sum of the probabilities of the outcomes and simplify.	**a** $\Pr(\text{exactly one blue}) = \Pr(RB) + \Pr(GB) + \Pr(BR) + \Pr(BG)$ $= \frac{1}{12} + \frac{1}{18} + \frac{1}{12} + \frac{1}{18}$ $= \frac{10}{36}$ $= \frac{5}{18}$
b Identify the outcomes in which a blue ball and a green ball are selected (in any order). Find the sum of the probabilities of the outcomes and simplify.	**b** $\Pr(\text{blue and green}) = \Pr(GB) + \Pr(BG)$ $= \frac{1}{18} + \frac{1}{18}$ $= \frac{2}{18}$ $= \frac{1}{9}$

Helpful hints

- ✔ The probabilities at each step of a two-step experiment always sum to 1.
- ✔ After calculating the probabilities of the final outcomes, check that the sum of these probabilities is equal to 1.

Learning pathways

△ 1–9, 13 □ 2, 4–7, 10, 11, 13–15 ⬠ 2, 4, 6, 7, 11, 12, 15, 16

ANS p522

Exercise 8.2A: Understanding and fluency

1 For each chance experiment, state the theoretical probability of the outcome in brackets in any given trial.

a Selecting a marble and recording its colour from a bag with 10 blue, 5 red, 10 yellow and 5 green marbles (selecting a green marble)

b Rolling a standard six-sided die and recording the number on top (rolling a 5)

c Drawing a card from a standard deck of 52 playing cards (drawing a red picture card – Jack, Queen or King)

d Selecting a gift voucher from a lucky dip containing twelve \$5 vouchers, eight \$10 vouchers and four \$20 vouchers (selecting a \$20 voucher)

WE 8.2A 2 Draw a tree diagram to represent each of the following situations, complete with probabilities on each branch and for each final outcome.

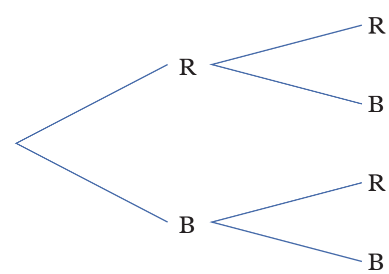
a A pencil case contains an equal number of red and blue pens. A pen is selected, its colour is recorded and then it is replaced. This is repeated one more time.

b A box contains an equal number of \$5, \$20 and \$75 vouchers. A voucher is selected, its value is recorded and then it is replaced. This is repeated one more time.

c A box contains 11 cards, numbered 1 to 11. A card is selected, it is recorded whether the number is even or odd, and then the card is replaced. This is repeated one more time.

d A ball pit contains an equal number of blue, red, green and yellow balls. A ball is selected, its colour is recorded and then it is replaced. This is repeated one more time.

WE 8.2B 3 A bag contains six red counters and six black counters. A counter is drawn twice, being replaced after the first draw. The tree diagram on the right shows the outcomes of the two selections.



a In any given trial, what is the probability of selecting a black counter?

b How many final outcomes are there?

c What is the probability of selecting:

i two black counters

ii exactly one black counter

iii two red counters?

4 A box contains milk, dark and white chocolates in equal numbers. A chocolate is selected at random from the box, its flavour recorded and then it is replaced. This is repeated once more. What is the probability that:

a both chocolates are white

b at least one chocolate is dark

c the first chocolate is white and the second chocolate is milk

d one chocolate is white and one chocolate is milk?

5 There are 52 cards in a standard pack of playing cards with 13 cards of each suit (clubs, diamonds, hearts and spades). A card is chosen at random from a pack, its suit is recorded and then it is replaced. This is repeated once more. What is the probability of selecting:

a two hearts

b at least one diamond

c exactly one club

d no spades

e at least one diamond or spade

f one heart and one club?

WE 8.2C **6** Draw a tree diagram with probabilities on the branches for each of these chance experiments.

a A pencil case contains five blue pens and two red pens. A pen is chosen, its ink colour is recorded and then it is replaced. This is repeated one more time.

b A box contains fifteen \$5 vouchers, ten \$20 vouchers and five \$75 vouchers. A voucher is chosen, its value is recorded and then it is replaced. This is repeated one more time.

c A box contains 15 cards, numbered 1 to 15. A card is chosen, it is recorded whether the number is even or odd, and then the card is replaced. This is repeated one more time.

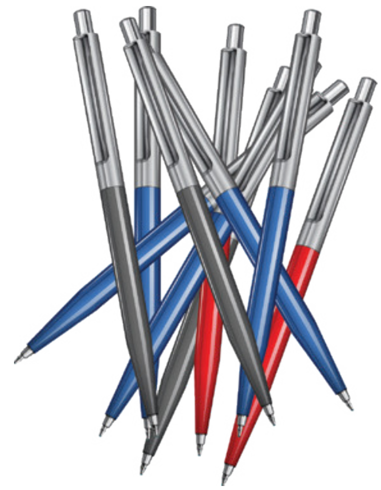
d A ball pit contains five blue, four red, three green and two yellow balls. A ball is chosen, its colour recorded and then it is replaced. This is repeated one more time.

WE 8.2D **7** A lucky dip contains 10 pink gift vouchers for \$100 and 40 green gift vouchers for \$10. A voucher is chosen, its value is recorded and then it is replaced. If this is repeated, find the probability that:

a a \$100 voucher is selected twice

b a \$100 voucher is not selected at all

c a \$100 voucher is selected first, and a \$10 voucher selected second.



8 A pencil case contains 5 blue pens, 3 black pens and 2 red pens. The owner of the pencil case takes out a pen at random for each lesson.

a Draw a tree diagram to represent the colours of the pens chosen for the first two lessons of the day. Remember to include the probabilities along each branch and for the final outcomes.

b Find the probability that the owner chooses:

i a blue pen each time

ii a red pen each time

iii a black pen each time

iv a blue pen, then a black pen

v a blue pen, then a red pen

vi a red pen, then a black pen.

Exercise 8.2B: Problem solving and reasoning

9 The probability of selecting a picture card from a standard 52-card pack is $\frac{3}{13}$.

A card is chosen from a standard pack of 52 cards and it is recorded whether it is a picture card or not before the card is replaced. This is repeated once more.

a Draw a tree diagram to represent this situation. Remember to include probabilities on the branches and calculate the final probability of each outcome as a decimal number rounded to four decimal places.

b Find the probability of selecting:

i exactly one picture card

ii at least one picture card

iii two picture cards

iv no picture cards.

10 In **Worked example 8.2D** part **a** there are four favourable outcomes out of nine possible outcomes. Explain why the theoretical probability of selecting exactly one blue ball is $\frac{5}{18}$ instead of $\frac{4}{9}$.

11 A store has a 'lucky dip' sale where you get a discount based upon the colour of a ball you choose from a box at random. If you take a red ball you get 10% off, if you take a green ball you get 25% off and if you take a blue ball you get 50% off. There are 10 balls of each colour in the box.

You can have a second chance if you don't take a blue ball out of the box first up, as long as you put the first ball back into the box.

Use a tree diagram or other means to show that you have a $\frac{5}{9}$ chance of selecting a blue ball from the box.

12 An array can be used to list the outcomes of a two-step experiment with replacement.

a Explain why an array may be a better choice to show the outcomes of a two-step experiment with replacement than a tree diagram when there is a large number of equally likely outcomes for each step (even when like outcomes are combined).

b Explain how you could represent outcomes that are not equally likely in an array.

13 A sock drawer contains 10 socks; some are black and some are white. You need to find out how many of each colour are in the drawer, but you can only select one sock at a time and place it back.

a If you selected with replacement 10 times and selected 3 black socks and 7 white socks, does this mean that there are 3 black and 7 white socks in the drawer? Explain.

b If you selected with replacement 50 times, selecting 21 black socks and 29 white socks, how many socks of each colour would you estimate are in the drawer?

c If you selected with replacement 80 times, selecting 34 black socks and 46 white socks, does this support your previous estimate?

14 A bag contains 12 aqua marbles and some green marbles.

a If the probability of selecting an aqua marble is 0.3, find the number of green marbles.

Three marbles are drawn from the bag and each marble is replaced in the bag before the next is drawn.

b Draw a tree diagram representing this situation. Include the probability of each step on the branches and calculate the probability of each outcome.

c Use your tree diagram to find the probability of selecting:

i three marbles of the same colour

ii exactly two green marbles

iii exactly two marbles of the same colour

iv a green marble first and an aqua marble third.

15 A bag contains some white, black and red tiles. Three tiles are drawn from the bag and their colours noted. Each tile is replaced in the bag before the next tile is drawn.

a If you were to represent this situation with a tree diagram, how many different outcomes would there be?

b Write out all possible outcomes where three different coloured tiles are drawn.

c If Pr (drawing a white tile) = w and Pr (drawing a black tile) = b, find the probability that three different coloured tiles are drawn.

Exercise 8.2C: Challenge

16 A bag contains different numbers of tiles labelled A, B, and C. A tile is taken out of the bag at random, the letter is observed, then the tile is replaced into the bag before a second tile is taken out of the bag at random. Let the probability of selecting a tile labelled A on any given selection be $\Pr(\text{A}) = a$, and the probability of selecting a tile labelled B on any given selection be $\Pr(\text{B}) = b$.

Write an expression for each of the following in terms of a and b.

a The probability of selecting two tiles labelled A

b The probability of selecting a tile labelled A and a tile labelled B in any order

c The probability of selecting a tile labelled C on any given selection

d The probability of selecting a tile labelled A and a tile labelled C in any order in:

i factorised form **ii** expanded form.

e The probability of selecting two tiles labelled C in:

i factorised form **ii** expanded form.

Online resources:

Interactive skillsheet Experiments with replacement

Investigation Three-step experiments with replacement

Quick quiz 8.2

Experiments without replacement

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate probabilities for two-step chance experiments without replacement.

Inter-year links

Year 7	10.5 Theoretical probability
Year 8	10.4 Theoretical probability
Year 10	11.2 Experiments with and without replacement

Experiments without replacement

Key content video
Experiments without replacement

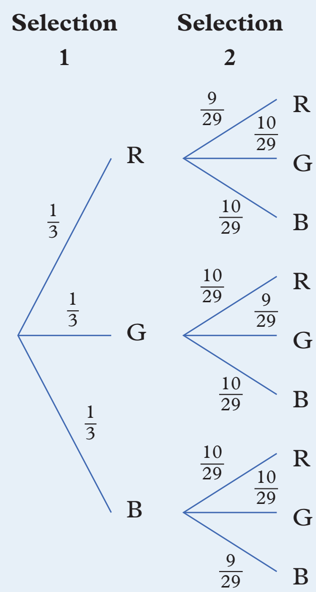
- Experiments without replacement involve selecting an item at random, recording the result and then not replacing the item before making another selection.
- The selected items are not replaced, so the probability of the remaining items being selected changes.
- A tree diagram or list of outcomes can help to find the probabilities of individual outcomes or events involving more than one outcome.
- For example, the following tree diagram shows the results of a person choosing two socks at random from 10 red, 10 green and 10 blue socks. The first sock is not replaced so the probabilities change for subsequent selections.
  - → If a pair of red socks is selected, the first selection is from 10 red socks out of 30, and the second selection is from 9 red socks out of 29.

    Selection 1: $\Pr(R) = \frac{10}{30} = \frac{1}{3}$

    Selection 2: $\Pr(R) = \frac{9}{29}$
  - → If a red then a green sock is selected, the first selection is from 10 red socks out of 30 and the second selection is from 10 green socks out of 29.

    Selection 1: $\Pr(R) = \frac{10}{30} = \frac{1}{3}$

    Selection 2: $\Pr(G) = \frac{10}{29}$
  - → The probabilities of the other outcomes are calculated in a similar manner.



Outcomes	Probability
RR	$\frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$
RG	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
RB	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
GR	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
GG	$\frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$
GB	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
BR	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
BG	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
BB	$\frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$

  - → The probability of selecting a pair of socks that are the same colour can be calculated:

$$\begin{aligned}\Pr(RR) + \Pr(GG) + \Pr(BB) &= \frac{9}{87} + \frac{9}{87} + \frac{9}{87}\\ &= \frac{27}{87}\\ &= \frac{9}{29}\end{aligned}$$

  - → The probability of selecting a pair of socks that are different colours can be calculated:

$$\begin{aligned}\Pr(RG) + \Pr(RB) + \Pr(GR) + \Pr(GB) + \Pr(BR) + \Pr(BG) &= \frac{10}{87} + \frac{10}{87} + \frac{10}{87} + \frac{10}{87} + \frac{10}{87} + \frac{10}{87}\\ &= \frac{60}{87}\\ &= \frac{20}{29}\end{aligned}$$

Selecting a pair of socks that are different colours is the complement of selecting a pair of socks that are the same colour, so the previous probability could also be calculated by subtracting $\frac{9}{29}$ from 1.

$1 - \frac{9}{29} = \frac{20}{29}$

Worked example 8.3A Representing experiments without replacement

A lucky dip contains five red gift vouchers for \$50 and five green gift vouchers for \$5. A gift voucher is selected and the customer keeps it. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment and find the probability of each possible outcome.

THINK

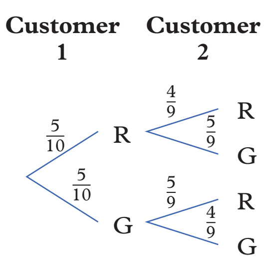
1 Draw a tree diagram to represent this two-step experiment, listing all the final outcomes.

2 Write the probability at the first branches of your tree diagram. Initially there is a $\frac{5}{10}$ chance of selecting a red gift voucher, and a $\frac{5}{10}$ chance of selecting a green gift voucher.

3 Determine the probabilities at the second branches and write them on your tree diagram.
- If you select a red gift voucher, 4 red and 5 green vouchers remain.
- If you select a green gift voucher, 5 red and 4 green gift vouchers remain.

4 Multiply the probabilities along each branch to determine the probability of each final outcome.

WRITE



Outcomes	Probability
RR	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} = \frac{2}{9}$
RG	$\frac{5}{10} \times \frac{5}{9} = \frac{25}{90} = \frac{5}{18}$
GR	$\frac{5}{10} \times \frac{5}{9} = \frac{25}{90} = \frac{5}{18}$
GG	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} = \frac{2}{9}$

Worked example 8.3B Calculating probabilities for experiments without replacement

Use the tree diagram from **Worked example 8.3A** to find the probability that:

a both customers select a \$50 voucher

b the first customer selects a \$5 voucher and the second customer selects a \$50 voucher.

THINK	WRITE
a Identify the outcome where both customers select a \$50 voucher.	**a** $\Pr(\text{both } \$50) = \Pr(RR) = \frac{20}{90} = \frac{2}{9}$
b Identify the outcome where the first customer selects a \$5 voucher and the second customer selects a \$50 voucher.	**b** $\Pr(\text{first } \$5 \text{ and second } \$50) = \Pr(GR) = \frac{25}{90} = \frac{5}{18}$

Helpful hints

- ✔ When objects are not replaced, the probabilities change after each step. Therefore you will need to re-evaluate how many total objects remain and how many of each type remain.
- ✔ When calculating probabilities involving multiple final outcomes, it is often easier not to simplify fractions until the end of your working, as it is easier to add fractions when they have a common denominator.

Learning pathways

△ 1–4, 6, 7, 9, 10, 14 ◻ 1(b, c), 2, 4–6, 8, 11, 12, 13(a), 15, 16 ⬠ 1(c), 4–6, 11–13, 15–18

ANS p524

Exercise 8.3A: Understanding and fluency

WE 8.3A **1** Draw a tree diagram with probabilities on the branches for each of these chance experiments without replacement.

a A drawer contains five black socks and five white socks. A sock is selected at random and its colour recorded. This is repeated one more time.

b An ice box contains six cans of Coke and six cans of Pepsi. A can is selected at random and its type recorded. This is repeated one more time.

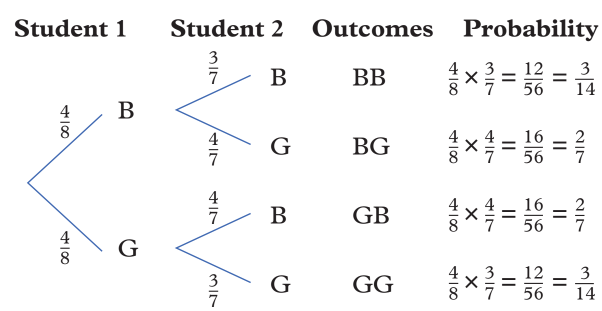
c A box contains five 16 GB SD cards, five 32 GB SD cards and five 64 GB SD cards. A card is selected at random and its capacity recorded. This is repeated once more.

WE 8.3B **2** The tree diagram on the right shows how two students can be selected at random from a group of four boys and four girls. Find the probability of selecting:

a two boys

b no boys

c a boy and a girl.



3 A lucky dip contains four purple gift vouchers for \$100 and four yellow gift vouchers for \$10. A gift voucher is chosen at random and the customer keeps it. If this was repeated for a second customer, find the probability that:

a both customers select a \$100 voucher

b the first customer selects a \$100 voucher and the second customer selects a \$10 voucher.

4 Each bonbon in a pack of 12 contains one toy, and there are three different kinds of toys: a whistle, a yo-yo and a bouncy ball. In total, the pack contains four of each kind of toy. You and a friend each select a bonbon at random from the pack.

a Draw a tree diagram to represent this situation. Be sure to include probabilities on each branch and the final probabilities for each outcome.

b Find the probability that:

i you both select a bonbon with a whistle

ii the first bonbon has a yo-yo and the second has a bouncy ball.

5 Consider the bonbons from question **4**. Find the probability that:

a you both select a bonbon with a different toy

b at least one of you selects a bonbon with a bouncy ball

c a bonbon with a whistle is not chosen.

6 Experiments without replacement can also start with unequally likely outcomes. A lucky dip contains 15 red, 10 green and 5 blue balls. Customers select one ball at random and they do not replace the ball before the next customer makes a selection.

a Complete the tree diagram on the right.

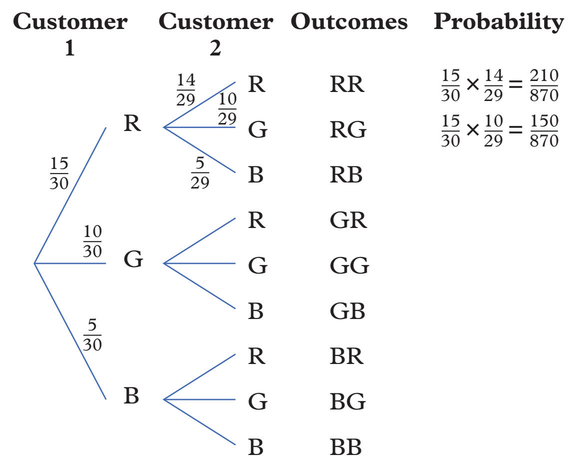
b Use the tree diagram to calculate the probability that:

i both customers select a blue ball

ii both customers select a green ball

iii both customers select a red ball

iv the first customer selects a red ball and the second selects a green ball.



7 Use the tree diagram from question **6** to calculate the probability of selecting:

a at least one green ball

b at least one red ball

c at least one blue ball

d exactly one red ball

e exactly one green ball

f a blue ball and a green ball.

Exercise 8.3B: Problem solving and reasoning

8 There are 52 cards in a standard pack of playing cards, with 13 cards of each suit (clubs, diamonds, hearts and spades). A card is selected and its suit is recorded. The card is not replaced and another card is selected. What is the probability of selecting:

a two hearts

b at least one diamond

c no spades

d one heart and one club?

9 A drawer contains two pink socks, two purple socks and two green socks. Use a tree diagram or other means to calculate the probability that you take out a pair when you select two socks from the drawer at random.

10 Imagine instead that the drawer from question **9** contains six socks of each colour. How does this change the probability of selecting a pair when you select two socks from the drawer at random?

11 If a drawer contained five red socks, four black socks and three white socks, find the probability that the first two socks selected at random from the drawer form a pair.

12 A sports team needs to select a captain and a vice-captain. Five people have put their names forward: Adrian, Chantelle, Katie, Guillermo and Sam.

a Draw a tree diagram to represent the selection (start with 'captain' branches).

b How does this tree diagram differ from the other ones you have created? Hint: Does the second set of branches contain the same number as the first set of branches?

c How many different combinations of captains and vice-captains are there? Remember that order is important!

d If each person has an equal chance, then find the probability that:

i Katie is selected captain

ii Sam is selected either captain or vice-captain

iii Adrian is captain and Chantelle is vice-captain

iv Guillermo does not get a position

v Katie and Sam both get a position.

13 An array can be used to show the outcomes for a two-step experiment without replacement. It is important to show the outcomes that cannot occur as they are the repeated outcomes that have been removed after the first step. We show this by placing a cross in that cell. For example, when selecting two marbles from a bag that contains five red marbles and four blue marbles without replacement, the array can be drawn as shown.

a Determine the probability of selecting the following combinations of marbles from the bag with five red marbles and four blue marbles.

- **i** Two red marbles
- **ii** Two blue marbles
- **iii** A red marble then a blue marble
- **iv** A red and a blue marble in either order.

		First								
		R	**R**	**R**	**R**	**R**	**B**	**B**	**B**	**B**
	R	×	RR	RR	RR	RR	BR	BR	BR	BR
	R	RR	×	RR	RR	RR	BR	BR	BR	BR
	R	RR	RR	×	RR	RR	BR	BR	BR	BR
Second	**R**	RR	RR	RR	×	RR	BR	BR	BR	BR
	R	RR	RR	RR	RR	×	BR	BR	BR	BR
	B	RB	RB	RB	RB	RB	×	BB	BB	BB
	B	RB	RB	RB	RB	RB	BB	×	BB	BB
	B	RB	RB	RB	RB	RB	BB	BB	×	BB
	B	RB	RB	RB	RB	RB	BB	BB	BB	×

b Another bag contains three green marbles and one purple marble. Two marbles are selected from the bag at random without replacement.

- **i** Draw an array to show the outcomes for this experiment.
- **ii** Determine the probability of selecting one purple marble out of the two marbles.
- **iii** Determine the probability of selecting two purple marbles.

14 A box of chocolates contains four milk chocolates, three white chocolates and two dark chocolates. Two chocolates are selected from the box, without looking. Find the probability of selecting:

- **a** two white chocolates
- **b** no dark chocolates
- **c** two different types of chocolate
- **d** at least one white chocolate.

15 Johan receives a chocolate box like the one in question **14**. He decides to eat three randomly chosen chocolates and share the rest with his friends.

a Draw a tree diagram to represent this situation. Include the probability of each step on the branches and calculate the probability of each outcome.

b Find the probability that Johan eats:

- **i** three milk chocolates
- **ii** one of each chocolate
- **iii** no milk chocolates.

16 At Oxford High there are 162 students in Year 7, 155 students in Year 8, 160 students in Year 9 and 148 students in Year 10. If two students from Years 7 to 10 are selected at random, what is the probability that both students are in Year 9?

Exercise 8.3C: Challenge

17 A lottery consists of 45 numbered balls of which six winning balls are chosen at random, with none of the six balls drawn replaced. To win the first division prize, you must pick all six winning numbers. What is the probability of winning the first division prize in this lottery? Express your answer as both a fraction and a decimal, correct to three significant figures.

18 An album contains nine songs and the lengths of the songs are shown in the table in minutes and seconds. Find the probability that the lengths of the first two different songs played on shuffle are above the mean length of a track on the album.

4:17	1:08	4:22	1:38	4:16	0:53	4:20	1:44	4:46

Online resources:

Interactive skillsheet
Experiments without replacement

Investigation
Draws for tennis tournaments

Quick quiz
8.3

ANS
p526

Checkpoint

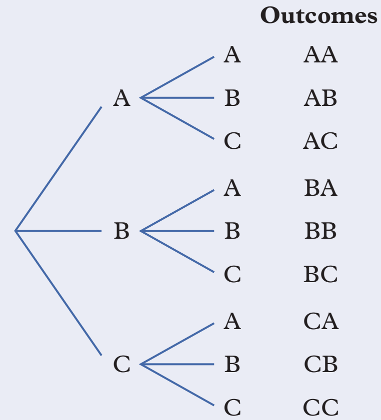
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

8.1 **1** A fair coin is tossed twice in a row. Draw a tree diagram that shows all outcomes for this experiment.

8.1 **2** A fair four-sided die numbered 1 to 4 is rolled twice in a row. Draw an array that shows all outcomes for this experiment.

8.1 **3** In the tree diagram shown on the right, all of the outcomes are equally likely. Determine the probability of the outcome consisting of:

a two consonants
b a vowel and a consonant
c at least one consonant.



8.1 **4** Use the array shown below to determine the probability of:

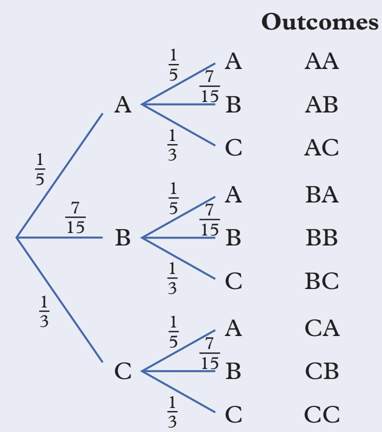
a B and Y
b B or Y or both
c B or Y but not both.

	A	B	C	D
X	(A, X)	(B, X)	(C, X)	(D, X)
Y	(A, Y)	(B, Y)	(C, Y)	(D, Y)
Z	(A, Z)	(B, Z)	(C, Z)	(D, Z)

8.2 **5** A bag contains three pieces of marble and four pieces of quartz. A piece is selected from the bag at random, returned to the bag, and then another piece is selected. Draw a tree diagram showing the probability of all outcomes, including probabilities on the branches for this experiment.

8.2 **6** Use the tree diagram shown on the right to determine the probability of:

a the same letter twice in a row
b at least one A
c less than two Cs.



8.2 **7** Use your tree diagram from question **5** to determine the probability of:

a selecting a piece of marble and a piece of quartz
b selecting a piece of marble second
c selecting at least one piece of quartz.

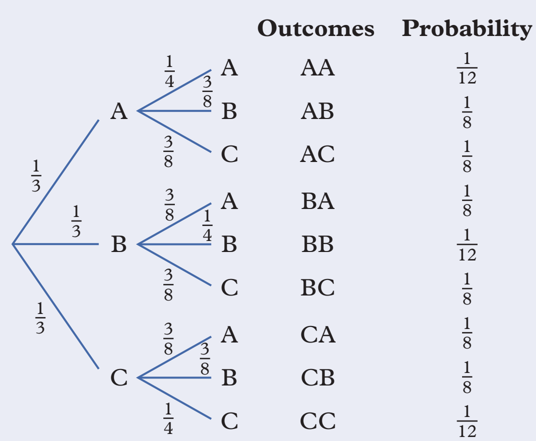
8.3 **8** A bag contains nine tiles; three tiles labelled A and six labelled B. A tile is drawn at random, the letter observed, then another tile is drawn without replacing the first. Draw a tree diagram with probabilities on the branches for this experiment and calculate the probability of each outcome.

8.3 **9** Use your tree diagram from question **8** to determine the probability of:

a the same letter twice in a row
b a tile labelled A second
c at least one B.

8.3 **10** Use the tree diagram shown on the right to determine the probability of:

a the same letter twice in a row
b at least one A
c anything but two Cs.



Simulations

Curriculum link

Design and conduct repeated chance experiments and simulations using digital tools to estimate probabilities that cannot be determined exactly (VC2M9P03)

Multi-step chance experiments

Simulations can be used to confirm theoretical probabilities from multi-step chance experiments. In **Lesson 8.1**, question **10**, you investigated a scenario which involved rolling a standard six-sided die and flipping a coin.

An array for this scenario is shown below.

	1	2	3	4	5	6
T	{T,1}	{T,2}	{T,3}	{T,4}	{T,5}	{T,6}
H	{H,1}	{H,2}	{H,3}	{H,4}	{H,5}	{H,6}

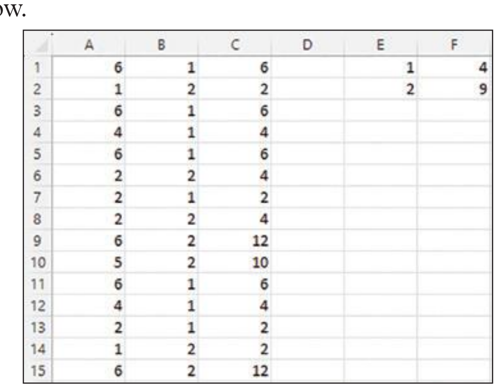
Simulation: Roll a die, toss a coin

Let's imagine that you play a game where you roll a die and then flip a coin. If the coin comes up heads, you double the score from your die roll. If the coin comes up tails, your score on the die remains the same.

The probability of each possible score occurring can be found theoretically, and these results can be confirmed by using a simulation. Due to the Law of Large Numbers, the relative frequency of a score will be very close to the theoretical probability of the score after a large number of trials.

Instructions

Open a new Microsoft Excel document or a similar spreadsheet program.

Code	Explanation
1 In cell A1, type in '=RANDBETWEEN(1,6)' and press Enter. In cell B1, type in '=RANDBETWEEN(1,2)' and press Enter.	This will randomly generate a whole number from 1 to 6 with equal chance in A1, and a 1 or 2 with equal chance in B1.
2 In cell C1, type in '=A1*B1'.	This will multiply the numbers in A1 and B1. This means that a 1 in B1 indicates that the simulated coin toss was tails and therefore the die score is multiplied by 1 (i.e. remains the same), whereas a 2 in B1 indicates that the simulated coin toss was heads and therefore the die score is multiplied by 2.
3 Highlight cells A1, B1 and C1 together and drag the bottom right corner down to cell C60.	This will simulate 60 trials of the game, storing each score in column C.
4 Enter the number 1 in cell E1. Then, in cell F1, type in '=COUNTIF(C:C,E1)'. Enter the number 2 in cell E2. Then, in cell F2, type in '=COUNTIF(C:C,E2)'.	The entries in cells F1 and F2 will count the number of times the overall score was 1 and 2, respectively. Your spreadsheet should look similar to the image below. 

	A	B	C	D	E	F
1	6	1	6		1	4
2	1	2	2		2	9
3	6	1	6			
4	4	1	4			
5	6	1	6			
6	2	2	4			
7	2	1	2			
8	2	2	4			
9	6	2	12			
10	5	2	10			
11	6	1	6			
12	4	1	4			
13	2	1	2			
14	1	2	2			
15	6	2	12			

Simulation questions

1 Copy the following table into your workbook. Fill in the empty spaces with the game score given by that scenario. For example, you will write 12 in the bottom right space, since H and 6 gives a score of 12.

	1	2	3	4	5	6
T						
H						

2 Use the table above to list the possible scores of this game.

A score of 2 is more likely than a score of 1 because there are two ways that a score of 2 can occur (1 and heads, 2 and tails), whereas there is only one way that a score of 1 can occur (1 and tails).

3 Which three scores are more likely than the other scores?

You may notice that every time you execute a cell, the whole spreadsheet refreshes. This can also be done manually by pressing F9. Refreshing the spreadsheet means all the formulas are executed again, and so all the randomly generated numbers are re-generated.

4 Press F9 ten times. How many times was the value in F1 greater than the value in F2?

Due to the Law of Large Numbers, if we increase the number of simulations, the relative frequencies of the scores of 1 and 2 will be closer to their theoretical probabilities. This means that if we increase the number of simulations enough, then it should always be the case that a score of 2 happens more often than a score of 1. Highlight cells A60, B60 and C60 together and drag the bottom right corner down to C600. Scroll back to the top of the spreadsheet.

5 Press F9 many times. Now that the number of trials has increased drastically, how often is the value in F2 greater than the value in F1?

Extension

There is a concept in probability theory called Expected Value (or Average Value). When an experiment has a numerical result, the expected value of the experiment is the score you ‘expect’ to achieve. Another way of interpreting the expected value is to think of it as the long-term average score. That is, the average score achieved if the experiment is repeated a large number of times will closely match the expected value.

The expected value can be calculated theoretically by multiplying each score by its probability, and then summing all these results. For example, the average value of a single die roll is 3.5 since. $1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = 3.5$. This means that if you rolled a die a large number of times and recorded all your scores, the average score would be close to 3.5.

Calculate the expected value for the die–coin game. Then, use ‘=AVERAGE(C:C)’ to calculate the average score across the 600 trials simulated in your spreadsheet. Does the simulated average score closely match the theoretical expected value?

Lesson 8.4

Relative frequency

Learning intentions

By the end of this lesson you will be able to …

✔ calculate expected numbers and relative frequencies.

Inter-year links

Support Chance experiments

Year 7 10.6 Experimental probability

Relative frequency

Key content video Relative frequency

- **Relative frequency** is calculated using the results of experiments or surveys rather than using theoretical probability.
- The relative frequency of a random event is the number of times a favourable outcome occurs divided by the total number of trials.
  - → Relative frequency = $\dfrac{\text{number of occurrences}}{\text{total number of trials}}$
- The **expected number** of each outcome in a chance experiment can be used to investigate whether there is any bias in an experiment. To calculate an expected number, multiply the theoretical probability by the number of trials in the experiment.
  - → Expected number = theoretical probability × number of trials
- If there is bias in a chance experiment, the experiment will favour one outcome over another outcome even when they are supposed to have an equal likelihood of occurring.
  - → When considering if a chance experiment is biased, consider both the expected number of occurrences and the actual number of occurrences. If there is a sizable difference, consider the total number of trials.
  - → With a small number of trials, the results often vary significantly from the expected numbers, but in the long term you can expect the relative frequency of a random event to approach the theoretical probability, due to a powerful concept in probability called the Law of Large Numbers.
  - → Start from the assumption that bias does not exist, and if you suspect bias may exist, carry out more trials to confirm your assumption.
- The theoretical probabilities of the outcomes from surveys, long-run experiments and statistical investigations with large data sets are not usually known. The relative frequency can be used to estimate the theoretical probability of outcomes in real-world events.

Worked example 8.4A Calculating relative frequencies

In Melbourne, it rained for 11 days in March. Find the relative frequency of rainy days in March, correct to two decimal places.

THINK	WRITE
1 Write the relative frequency formula.	Relative frequency = $\dfrac{\text{number of occurrences}}{\text{total number of outcomes}}$
2 Identify the number of occurrences (11) and the total number of outcomes (31). Substitute into the formula and solve, simplifying if possible.	$= \dfrac{11}{31}$
3 Convert to a decimal, correct to two decimal places, and write your answer.	≈ 0.35 The relative frequency of rainy days in Melbourne in March is 0.35.

Worked example 8.4B Calculating an expected number

Find the expected number of 5s or 6s if a standard six-sided die is rolled 90 times.

THINK

1 Write the formula for an expected number.

2 Identify the theoretical probability of rolling a 5 or a 6 $\left(\frac{2}{6}\right)$ and the number of trials (90) and substitute into the formula.

WRITE

$$\begin{aligned}\text{Expected number} &= \text{theoretical probability} \times \text{number of trials}\\ &= \frac{2}{6} \times 90\\ &= 30\end{aligned}$$

Worked example 8.4C Comparing the relative frequency and expected number

A standard six-sided die is rolled 90 times and a 5 is obtained 17 times.

a Find the expected number of 5s if a standard six-sided die is rolled 90 times.

b Comment on whether you believe this die may be biased.

THINK

a 1 Write the expected number formula.

2 Substitute the theoretical probability $\left(\frac{1}{6}\right)$ and the number of trials (90) into the formula and evaluate.

b Look to see if there is a significant difference between the expected number and the actual number. Consider the number of trials and make a reasonable assessment with the given information.

WRITE

a
$$\begin{aligned}\text{Expected number} &= \text{theoretical probability} \times \text{number of trials}\\ &= \frac{1}{6} \times 90\\ &= 15\end{aligned}$$

b Although there is a difference between the expected number (15) and the actual number (17), the difference is not significant. To determine whether the die is biased, more trials should be conducted.

Helpful hints

- ✔ The minimum value of a relative frequency is 0. For example, the number of rainy days cannot be −2 days out of 31 days. The minimum number of rainy days is 0 days out of 31 days.
- ✔ The maximum value of a relative frequency is 1. For example, the number of rainy days cannot be 32 days out of 31 days. The maximum number of rainy days is 31 days out of 31 days.
- ✔ The results from an experiment or survey are more reliable when the sample size is larger. This should be considered when drawing conclusions from data.

Learning pathways

△ 1–3, 5, 6, 7(a, b), 8, 10 □ 1, 2, 4–6, 8, 10, 11 ⬠ 1–2(d–f), 5, 6, 9–12

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Exercise 8.4A: Understanding and fluency

WE 8.4A **1** Find the relative frequency of the outcomes in the following situations. Give your answers as fractions and decimals, rounded to two decimal places where necessary.

- **a** Of 50 families surveyed about pets, 18 had a pet dog.
- **b** Of 300 people surveyed about being right-handed, left-handed or ambidextrous, 3 people were ambidextrous.
- **c** Of 56 people surveyed about playing sport, 21 people did not play a team sport.
- **d** Of 450 people surveyed, 300 people had less than two video game consoles.
- **e** Of 348 people surveyed about pizza or pasta, 87 people preferred pasta.
- **f** Of 1200 cats that had their tails measured, 655 had tails longer than 30 cm.

WE 8.4B **2** Find the expected number of:

- **a** heads if a coin is tossed 250 times
- **b** 1s or 2s if a standard six-sided die is rolled 120 times
- **c** consonants if a letter is selected randomly from the alphabet 130 times
- **d** 6s if a standard six-sided die is rolled 30 times
- **e** two-digit integers if an integer from 1 to 999 is selected 444 times
- **f** double heads if a pair of coins is tossed 856 times.

3 A standard six-sided die is rolled 150 times and the results are shown in this table.

Outcome	1	2	3	4	5	6
Frequency	38	32	19	24	21	16

- **a** Find the relative frequency of rolling a number greater than 2, based on this data. Give your answer as a fraction and a decimal, rounded to two decimal places.
- **b** Assuming the die is not biased, describe how you would expect the relative frequency of rolling a number greater than 2 to change in the long term.

4 A standard pack of 52 cards has four suits (diamonds, clubs, hearts and spades), with each suit consisting of 10 number cards (Ace to 10) and three picture cards (Jack, Queen and King).
A card is chosen at random from a pack and replaced. This is repeated 129 times for a total of 130 selections.

- **a** What is the expected number of picture cards?
- **b** If seven picture cards are obtained, find the relative frequency of selecting a picture card. Give your answer as a fraction and a decimal, correct to three decimal places.
- **c** Assuming the deck is not biased, describe how you would expect the relative frequency of selecting a picture card to change in the long term.

WE 8.4C **5** For each situation below:

- **i** Find the expected number of outcomes. Where necessary, give your answer to the nearest whole number
- **ii** Comment on whether you believe the experiment may be biased.
  - **a** Rolling a 4 if a standard six-sided die is rolled 180 times and a 4 is obtained 27 times
  - **b** Tossing a tail if a coin is tossed 36 times and a head is obtained 16 times
  - **c** Rolling an odd number if a standard six-sided die is rolled 200 times and an odd number is obtained 53 times

6 A number of experiments were performed and their results recorded below. For each experiment, find the number of times each outcome occurred.

a Total number of trials = 40

Outcome	Heads	Tails
Relative frequency	0.625	0.375

b Total number of trials = 120

Outcome	1	2	3	4	5	6
Relative frequency	0.15	0.2	0.175	0.1	0.125	0.25

c Total number of trials = 60

Outcome	Hearts	Diamonds	Clubs	Spades
Relative frequency	0.2	0.3	0.35	0.15

Exercise 8.4B: Problem solving and reasoning

7 A magician uses a number of props in his show to demonstrate magic tricks. He tosses a coin, rolls a six-sided die and selects a card from a pack. To test whether the props are fair (standard) or biased, the results over several shows were recorded and are shown in the tables below.

For each experiment:

i find the total number of trials

ii state the theoretical probability of each outcome

iii calculate the expected number of each outcome

iv calculate the relative frequency of each outcome based on the data

v state whether you think the prop used is fair, biased or if there are not enough trials to make a firm decision

vi give a reason to support your answer to part **v**.

a

Outcome	Heads	Tails
Frequency	8	2

b

Outcome	1	2	3	4	5	6
Frequency	966	971	1036	994	1031	1002

c

Outcome	Hearts	Diamonds	Clubs	Spades
Frequency	38	31	19	24

8 Three coins are tossed one at a time.

a There are 8 possible outcomes; for example, HTH represents a head, tail and head outcome. List all the possible outcomes.

b What is the theoretical probability of tossing a 'triple'?

c How many 'triples' would you expect to get if you performed 40 trials?

d The experiment is performed 40 times and 5 triples are recorded. Does the actual occurrence of a 'triple' match the expected number?

e Describe how you would expect the relative frequency to change if you performed 4000 trials.

9 A four-sided die (numbered 1–4) is rolled 20 times and the number 2 appears uppermost on 13 occasions. Explain why, considering the Law of Large Numbers, this does not mean that the die is biased, and what should be done to check for any bias.

10 Miguel's bag contains 60 coloured marbles. He wants to know if he can determine how many of each colour of marble are in his bag if he only picks out one marble at a time and then puts the selected marble back into the bag. Miguel records the colours of the first 10 marbles he picks out in the following table.

Colour	Red	Green	Blue	Purple
Frequency	4	2	2	2

a Based on the results in the above table, how many marbles of each colour do you expect to be in Miguel's bag?

b Miguel continues to pick out marbles from the bag, with the following table detailing the colours he has picked out after 50 marbles.

Colour	Red	Green	Blue	Purple
Frequency	21	13	7	9

Based on this new data, how many marbles of each colour do you now expect to be in Miguel's bag?

c Are you more confident in your prediction from part **a** or part **b**? Explain your answer.

11 Tulio read a headline that said, '51% of adults worldwide don't get enough sleep'. Concerned for his parents' health, he asks if they both get the recommended 7–8 hours of sleep, which they do. Tulio is now sceptical of the headline.

a Explain why Tulio's sample would not verify or refute the statistics in the headline regardless of his parents' responses.

b Later, Tulio finds another article that says, 'one in three Australians not getting enough sleep'. He believes this is more accurate because he only asked two people, so the third person he asks won't be getting enough sleep.

i Assuming the statistic is accurate, explain why Tulio would not be guaranteed to have a person that is not getting enough sleep next.

ii Give at least two reasons why the statistics from the two headlines are different.

Exercise 8.4C: Challenge

12 Sometimes people fake data because it can be either time-consuming or costly to collect primary data. Consider the instance of faking data on coin tosses, where people who create fake lists of results often avoid putting in long streaks of heads or tails because they think it would not happen in reality.

a If you toss a coin 7 times, what is the probability you get 7 heads?

b Explain why you would expect to get at least one streak of 7 or more heads if you toss a coin 1000 times.

c If you toss a coin 1000 times, how many streaks of exactly 5 heads in a row do you expect?

Online resources:

Interactive skillsheet
Relative frequency

Quick quiz
8.4

Two-way tables

Learning intentions

By the end of this lesson you will be able to ...

✔ create and use two-way tables to solve problems.

Inter-year links

Year 8	10.5 Two-way tables
Year 10	11.3 Two-way tables and Venn diagrams

Two-way tables

- **Two-way tables** display the results of a survey or experiment with two different categories or attributes of data. The results can be displayed as raw numbers or as relative frequencies. For example, both of these two-way tables display the same data.

Key content video
Understanding two-way tables

Key content video
Completing two-way tables

Key content video
Two-way relative frequency tables

	Dark hair	Light hair	Total
Dark eyes	16	4	20
Pale eyes	8	12	20
Total	24	16	40

	Dark hair	Light hair	Total
Dark eyes	0.4	0.1	0.5
Pale eyes	0.2	0.3	0.5
Total	0.6	0.4	1

Identifying the required outcome

- The language used to describe an outcome can help you to identify which cells of the two-way table are being referred to.
  - → 'and' means both outcomes must be met. For example, 16 people have dark eyes and dark hair.
  - → 'or' can be inclusive, meaning if both outcomes are met, the outcome is favourable. For example, 4 + 8 + 12 = 24 have pale eyes or light hair.
  - → 'or' can also be exclusive, meaning if both outcomes are met, the outcome is not favourable. For example, 4 + 8 = 12 people have pale eyes or light hair but not both.

Worked example 8.5A Understanding a two-way table

Consider this two-way table which shows whether Year 8 and Year 9 students prefer jam or Vegemite.

a How many students were surveyed?

b How many students are in Year 9 and prefer Vegemite?

	Year 8	Year 9	Total
Jam	28	38	66
Vegemite	25	34	59
Total	53	72	125

THINK

a The total number of people surveyed is displayed in the bottom right-hand corner.

b Identify the cell that is in the 'Year 9' column and the 'Vegemite' row.

WRITE

a 125 students were surveyed.

b 34 students are in Year 9 and prefer Vegemite.

Worked example 8.5B Creating a two-way relative frequency table

This two-way table shows whether or not a group of 50 people from Melbourne and Perth own a pet. Convert it into a two-way relative frequency table.

	Melbourne	Perth	Total
Owns pet	13	19	32
Does not own pet	11	7	18
Total	24	26	50

THINK

Divide the values in each cell by the value in the total cell in the bottom right-hand corner.
The totals should add to 1 in the bottom right-hand corner.

WRITE

	Melbourne	Perth	Total
Owns pet	$\frac{13}{50} = 0.26$	$\frac{19}{50} = 0.38$	$\frac{32}{50} = 0.64$
Does not own pet	$\frac{11}{50} = 0.22$	$\frac{7}{50} = 0.14$	$\frac{18}{50} = 0.36$
Total	$\frac{24}{50} = 0.48$	$\frac{26}{50} = 0.52$	1

Worked example 8.5C Creating and completing a relative frequency two-way table

A survey collected information on whether motorists from the country and city drive automatic or manual cars. Of the respondents, 45% are from the city and drive automatic cars, 43% are from the country and 34% drive manual cars.
Use this information to complete a relative frequency two-way table.

THINK

1 Draw the table with one of the categories (car type) placed in the rows and the other category (location) placed in the columns.

2 Convert the percentages to relative frequencies by dividing by 100, e.g. 43% = 0.43.

3 Insert the given information in the relevant cells. Remember that the total in the bottom right-hand corner is always 1.

4 The numbers in each row and column add to give the value in the total cells, so wherever there are two out of three cells in a row or column, the unknown value can be found by using addition or subtraction.

WRITE

	City	Country	Total
Automatic	0.45	$0.66 - 0.45 = 0.21$	$1 - 0.34 = 0.66$
Manual	$0.57 - 0.45 = 0.12$	$0.43 - 0.21 = 0.22$	0.34
Total	$1 - 0.43 = 0.57$	0.43	1

Worked example 8.5D Calculating probability using a two-way table

Consider the two-way table in **Worked example 8.5A**, showing whether Year 8 and 9 students prefer jam or Vegemite.

What is the probability of a student chosen randomly from the group being in Year 8 or preferring jam (or both)?

	Year 8	Year 9	Total
Jam	28	38	66
Vegemite	25	34	59
Total	53	72	125

THINK	WRITE
1 Identify the relevant cells in the table that represent this outcome (any cell in the Year 8 column or Jam row).	$\frac{28+38+25}{125}=\frac{91}{125}$
2 Divide the total of these cells by the total in the table. Simplify if possible.	

Helpful hints

- ✔ When adding the totals in rows and columns, be careful not to double the cells where the rows and columns intersect.
- ✔ In some problems it may appear that there is insufficient information to answer the question. Start by drawing a two-way table and filling in the given values. Then, use these values to find the missing values.

Learning pathways

△ 1, 3–10, 12

□ 2, 4–9, 11, 14, 15(a, b), 16(a–d)

⬠ 4, 6–9, 11, 13, 15(b, c), 16, 17

Exercise 8.5A: Understanding and fluency

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WE 8.5A WE 8.5D

1 Consider this two-way table that shows whether students in primary school (PS) and high school (HS) prefer to watch or play sport.

	PS	HS	Total
Watch sport	8	7	15
Play sport	22	18	40
Total	30	25	55

a How many students were surveyed in total?

b How many students in high school prefer to watch sport?

c What is the probability of a student chosen randomly from the group being in high school and preferring to watch sport?

2 Consider this two-way table that shows the hair colour and eye colour of a group of people.

	Fair	Dark	Total
Blue/green	23	11	34
Brown	6	35	41
Total	29	46	75

a How many people were surveyed in total?

b How many people with dark hair have blue/green eyes?

c What is the probability of a person chosen randomly from the group having dark hair or blue/green eyes but not both?

3 This two-way table shows whether Year 8 and Year 9 students prefer alternative or mainstream music. Use this two-way table to find the probability, as a simplified fraction, of a person chosen randomly from the group:

	Year 8	Year 9	Total
Alternative	14	23	37
Mainstream	29	19	48
Total	43	42	85

a being a Year 8 student who prefers mainstream music

b preferring alternative music

c being a Year 9 student

d being a Year 9 student who prefers alternative music.

4 This two-way table shows whether Year 7, 8 and 9 students prefer tennis, basketball or hockey. Use this table to find the probability, as a simplified fraction, that a person chosen randomly from this group:

	Year 7	Year 8	Year 9	Total
Tennis	23	26	37	86
Basketball	19	42	34	95
Hockey	31	13	25	69
Total	73	81	96	250

a is in Year 9

b plays basketball

c is in Year 8 and plays tennis

d plays hockey or tennis

e does not play hockey

f is in Year 7 and doesn't play basketball.

WE 8.5B 5 This two-way table shows whether students from two classes prefer sweet or savoury food. Convert the two-way table into a two-way relative frequency table.

	Class A	Class B	Total
Prefer sweet food	23	32	55
Prefer savoury food	29	16	45
Total	52	48	100

6 This two-way table shows whether students prefer to read books, play games or play sports, depending on their age. Convert the two-way table into a two-way relative frequency table. Write your answers as fractions in their simplest form.

	Reads books	Plays games	Plays sports	Total
Under 12	56	44	32	132
Over 12	12	44	68	124
Total	68	88	100	256

WE 8.5C 7 A survey collected information on whether people with light or dark hair preferred pop music or rock music. Of the respondents, 22% had light hair and preferred pop music, 65% had dark hair and 40% preferred rock music.

Use this information to complete a relative frequency two-way table.

8 A survey collected information on whether people who subscribe to Spotify also subscribe to Netflix. Of the respondents, $\frac{1}{6}$ had a subscription to both Spotify and Netflix, $\frac{1}{4}$ had a subscription to Spotify, and $\frac{1}{3}$ did not have a subscription to Netflix. Use this information to complete a relative frequency two-way table. Write your answers as fractions in their simplest form.

Exercise 8.5B: Problem solving and reasoning

9 The two-way table on the right shows whether people prefer to watch films at the cinema or at home and whether they prefer action or comedy films.

	Cinema	Home	Total
Action	22		
Comedy		14	33
Total			85

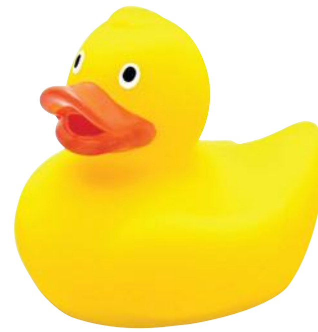
a Complete this two-way table.

b Calculate the probability, as a simplified fraction, that a person chosen randomly from the group prefers to:

i watch films at the cinema **ii** watch comedy films at home **iii** watch action films.

10 A group of 200 people, each with a single pet, were surveyed on their pets. Of the 113 who owned cats, 29 had a purebred. This gives a total of 104 pet owners who owned a purebred. Create a two-way table showing this information and use it to calculate the probability, as a simplified fraction, that a person chosen at random is an owner of a purebred that is not a cat.

11 In a group of people who were surveyed on their bathroom habits, 60% of those aged 50 or over preferred a bath to a shower, whereas 80% of under 50s said they preferred a shower rather than a bath; 55% of the group was aged 50 or over.



a Create a relative frequency two-way table to represent this information using decimals. Hint: The statement '60% of those aged 50 or over prefer a bath' refers to 60% of the proportion of those aged 50 or over, not the total.

b If a person was randomly selected from a group of 500 people, find the probability that they are:

i a person under 50 who prefers to shower

ii someone who prefers a bath

iii a person aged 50 or over who prefers to shower.

c Of the group of 500 people, find the number of people who:

i prefer a bath to a shower　　**ii** are aged 50 or over

iii are under 50 and prefer a bath.

12 Elsa recorded the make and colour of cars that went past her house over a week and recorded her results as shown. Add totals to her results and then find the probability, as a simplified fraction, that a car going past her house is:

a white　　**b** a Holden　　**c** a white Holden

d blue or red　　**e** not a Toyota　　**f** not black

g neither red nor a Ford　　**h** blue but not a Hyundai　　**i** a Mitsubishi but not silver.

	Ford	Hyundai	Holden	Mitsubishi	Toyota	Other
Silver	11	9	17	8	6	12
White	15	12	16	11	8	10
Red	12	10	8	6	13	14
Blue	8	15	12	7	9	11
Black	13	8	10	10	7	8
Other	4	3	4	2	1	2

13 $n(\text{A})$ is used to denote the number of elements in category A. For the relative frequency two-way table shown, determine the population size if:

a $n(\text{A}) = 30$　　**b** $n(\text{B}) = 72$　　**c** $n(\text{A and B}) = 98$　　**d** $n(\text{A or B}) = 102$

	A	A'	Total
B	0.1	0.35	0.45
B'	0.4	0.15	0.55
Total	0.5	0.5	1

14 Consider the relative frequency two-way table below. If the total frequency is 120, then draw the frequency two-way table.

	A	A'	Total
B	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{7}{12}$
B'	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{5}{12}$
Total	$\frac{1}{2}$	$\frac{1}{2}$	1

15 Complete each pair of frequency and relative frequency two-way tables.

a

	A	A'	Total
B	40		75
B'			
Total			

	A	A'	Total
B	0.2		
B'			
Total	0.4		1

b

	C	C'	Total
D			9
D'		9	
Total			

	C	C'	Total
D		0.25	
D'			
Total	0.3		1

c

	E	E'	Total
F	18		
F'		12	
Total			

	E	E'	Total
F		$\frac{1}{9}$	$\frac{11}{18}$
F'			
Total			1

Exercise 8.5C: Challenge

16 Consider this two-way table which shows whether a group of Year 8 and Year 9 students prefer Maths or English.

	Year 8	Year 9	Total
Maths		11	
English	10		
Total	22		50

a Complete the table.

b Use it to find the probability that a student selected randomly from the group:

i prefers Maths **ii** is in Year 9 **iii** is in Year 9 and prefers Maths.

c A student is selected randomly from the group. You know that they are in Year 9.

i How many students are in Year 9? **ii** How many students in Year 9 prefer Maths?

iii What is the probability that a student in Year 9 prefers Maths?

d What is the difference between parts **b iii** and **c iii**?

The problem represented in part **c iii** is an example of conditional probability. It looks at the probability of an outcome given certain conditions. In part **c iii**, you are looking for the probability that a student likes Maths, given that they are in Year 9. This means that you consider only the limited group of the condition rather than the entire population.

e Use these steps to calculate the probability of a student being in Year 8, given that they prefer English.

i What is the condition? How many students are in this group?

ii What is the specific group you are after? Hint: It is not just somebody in Year 8.

iii How many students are in this specific group?

iv Use your answers to parts **i** and **iii** to calculate the probability of selecting somebody in Year 8, given that they prefer English.

17 Use the two-way table in question **4** to calculate the probability of randomly selecting a person from the group that:

a plays hockey, given that they are in Year 8 **b** plays tennis, given that they are in Year 9

c is in Year 7, given that they play basketball **d** is in Year 8, given that they play tennis

e is in Year 9, given that they *don't* play hockey **f** is not in Year 7, given that they play basketball.

Online resources:

Interactive skillsheet
Two-way tables

Quick quiz
8.5

Lesson 8.6

Venn diagrams

Learning intentions

By the end of this lesson you will be able to ...

✔ use Venn diagrams to solve problems.

Inter-year links

Year 8 10.6 Venn diagrams

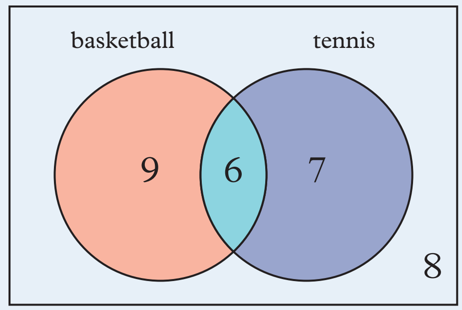
Year 10 11.3 Two-way tables and Venn diagrams

Venn diagrams

Key content video
Understanding Venn diagrams

- In mathematics, a **set** is a collection of distinct objects.
- A **Venn diagram** is used to display the relationship between different sets of data.
- In a Venn diagram, each circle represents a different set, and the rectangle containing the circles represents the entire data set.
- Frequencies are placed within Venn diagrams to represent the number of data points for each region of the diagram.

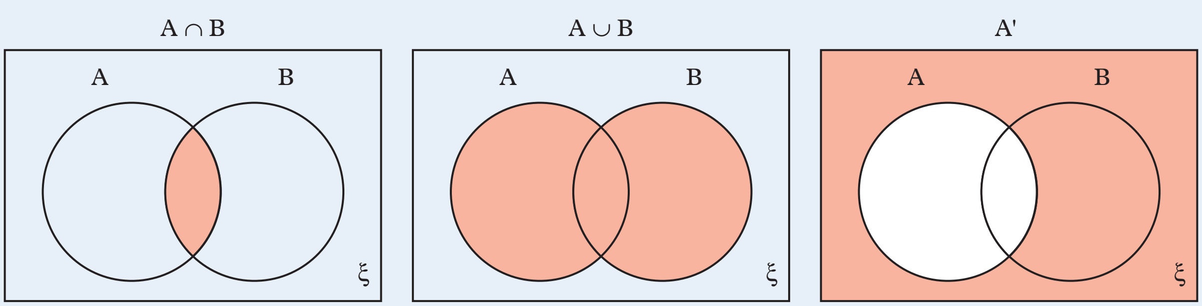
  For example, the Venn diagram on the right shows whether Year 9 students play either basketball or tennis. 6 students play both basketball and tennis (the region where these sets overlap).



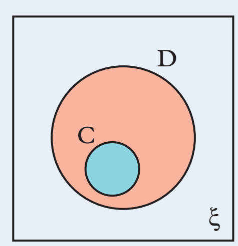
Set notation

Key content video
Set notation

- All elements contained in the rectangle of a Venn diagram belong to the **universal set**, which is represented by the Greek letter xi (ξ).
- $A \cap B$ means the **intersection** of sets A and B, which includes only the elements that are in A and are also in B. This can be read as 'A and B'.
- $A \cup B$ means the **union** of sets A and B, which includes all elements in either A or B or both. This can be read as 'A or B'.
  → The intersection ($A \cap B$) and union ($A \cup B$) of sets A and B are also sets themselves.
- A' means the **complement** of A, which includes all elements not in A.



- $C \subset D$ means C is a **subset** of D. This means that set C is contained within set D and every element in set C is also in set D.



Worked example 8.6A Understanding Venn diagrams

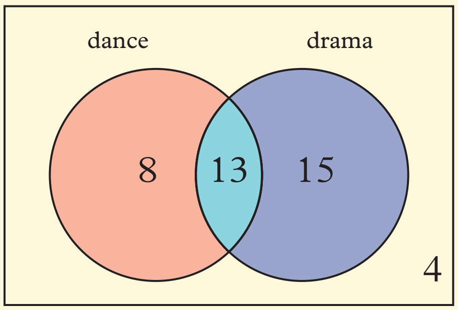
This Venn diagram shows the results of a survey where people were asked if they attend optional dance and drama classes.

a How many people attend both the dance and drama classes?

b How many people attend the drama class?

c How many people do not attend the dance class?

d How many people were surveyed in total?



THINK	WRITE
a Identify the required region, which is the intersection of both sets. This is the middle section where the two circles overlap.	**a** 13 people attend both classes.
b Identify the required region, which is the 'drama' circle. Add the numbers in this region.	**b** 13 + 15 = 28 attend the drama class.
c Identify the required region, which is everything outside the 'dance' circle. Add the numbers in this region.	**c** 15 + 4 = 19 people do not attend the dance class.
d Find the total of all numbers in the Venn diagram.	**d** 8 + 13 + 15 + 4 = 40 people were surveyed in total.

Worked example 8.6B Using Venn diagrams to calculate relative frequencies

Use the Venn diagram from **Worked example 8.6A** to find the relative frequency that a person chosen randomly from the survey group attends the dance class.

THINK	WRITE
1 Identify the required region, which is the 'dance' circle. Add the numbers in this region.	8 + 13 = 21 people attended the dance class.
2 Find the total number of people who were surveyed by adding all the numbers in the Venn diagram (don't forget the number outside the circles).	8 + 13 + 15 + 4 = 40 were surveyed.
3 To determine the relative frequency, divide the number in the required region by the total number.	Relative frequency $= \frac{21}{40}$ $= 0.525$

Worked example 8.6C Creating a Venn diagram

In a group of 40 people, 15 have a cat and 22 have a dog. There are 11 people who do not have either a cat or a dog. Draw a Venn diagram to represent this situation.

THINK

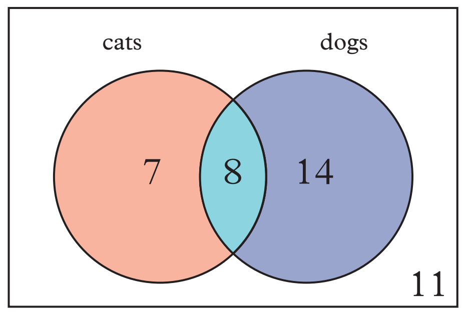
1. Draw two circles, one representing cats and the other representing dogs. Draw a rectangle around the two circles.
2. Place the number of people who do not have either a cat or a dog (11) in the rectangle outside the circles.
3. Determine the number of people who have a cat only by subtracting the number of people who have a dog (22) and the number of people who do not have either a cat or a dog (11) from the total (40).
4. Repeat step 3 to determine the number of people who have a dog only.
5. Determine the number of people who have a cat and a dog by subtracting the number of people who have a dog only (14) from the number of people who have a dog (22).

WRITE

Number of people who have a cat only
$= 40 - 22 - 11$
$= 7$

Number of people with a dog only
$= 40 - 15 - 11$
$= 14$

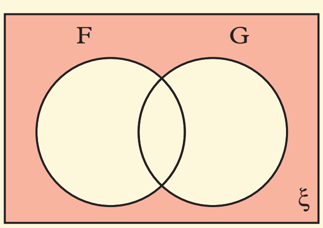
Number of people with a cat and a dog
$= 22 - 14$
$= 8$



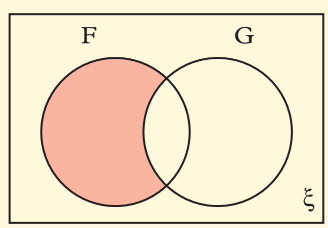
Worked example 8.6D Using set notation

Use set notation to describe the following regions in this Venn diagram.

a



b



THINK

a 1 Look at the shaded region. This represents the elements not in F and not in G.

2 Use set notation to represent the region. For example, usese the complement symbol G′ for 'not in G' and the symbol ∩ for 'and'.

b 1 Look at the shaded region. This represents the elements in F and not in G.

2 Use set notation to represent the region.

WRITE

a $F' \cap G'$

b $F \cap G'$

Note: There is sometimes more than one way to describe a region. For example, the region in part **a** could also be described as $(F \cup G)'$.

Helpful hints

- ✔ When creating Venn diagrams, check that the total in all the individual regions equals the number in the entire data set.
- ✔ Remember to include the value in the intersection when finding the total number of elements in a set. For example, in **Worked example 8.6A** the number of people attending drama classes is 28, not 15.

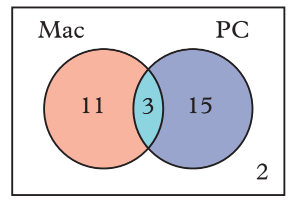
Learning pathways

△ 1, 3, 4, 6–8, 11, 12	□ 2, 4–9, 12, 13, 16	⬠ 5, 7–10, 12–15, 17

ANS p528

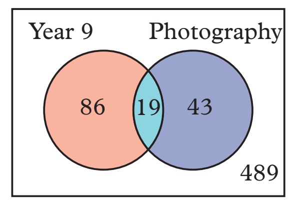
Exercise 8.6A: Understanding and fluency

WE 8.6A **1** Consider the Venn diagram on the right that shows whether people own a Mac or a PC.



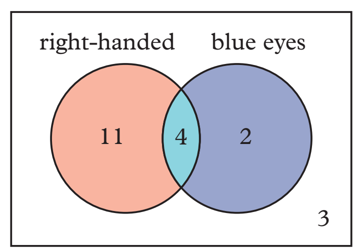
- **a** How many people own a Mac?
- **b** How many people own a PC but not a Mac?
- **c** How many people don't own either a Mac or a PC?
- **d** How many people were surveyed in total?

2 Consider the Venn diagram on the right that shows whether people are in Year 9 and whether they take a Photography class.



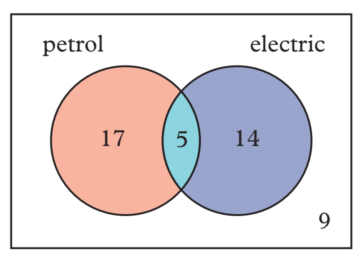
- **a** How many people take Photography?
- **b** How many people are in Year 9 and take Photography?
- **c** How many people are not in Year 9?
- **d** How many people were surveyed in total?

WE 8.6B **3** This Venn diagram shows whether people are right-handed and whether they have blue eyes. Use this diagram to find the relative frequency of a person:



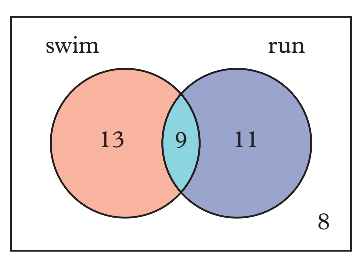
- **a** being right-handed with blue eyes
- **b** having blue eyes
- **c** being right-handed with eyes not blue
- **d** not having blue eyes
- **e** being left-handed with blue eyes
- **f** being left-handed.

4 This Venn diagram shows whether people drive petrol or electric cars. Use this diagram to find the relative frequency of a person who:



- **a** drives petrol cars only
- **b** drives both petrol and electric cars
- **c** drives electric cars
- **d** drives neither
- **e** does not drive petrol cars
- **f** drives petrol or electric cars (but not both).

5 This Venn diagram shows whether people like to swim or run for exercise. Use this diagram to find the relative frequency of a person who:



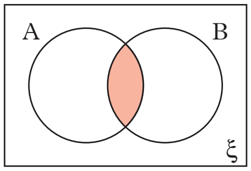
- **a** swims and runs
- **b** does not run
- **c** runs but does not swim
- **d** does not swim or run
- **e** swims
- **f** swims or runs.

WE 8.6C **6** In a group of 50 students, 24 are in Year 9, 19 walk to school and 16 are not in Year 9 and do not walk to school. Draw a Venn diagram to represent this situation.

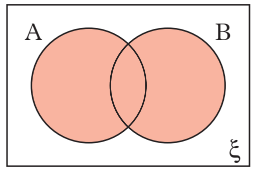
7 In a group of 160 students, 78 take music, 95 take drama and 46 take music and drama. Draw a Venn diagram to represent this situation.

WE 8.6D **8** Use set notation to describe the following shaded regions in the Venn diagrams below.

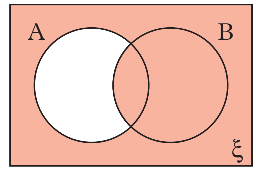
a



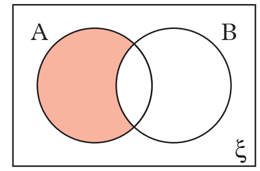
b



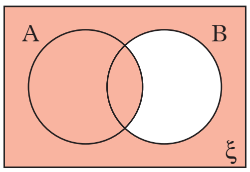
c



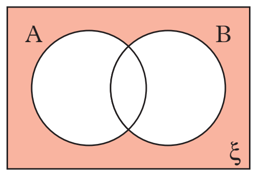
d



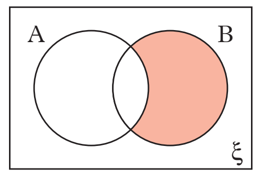
e



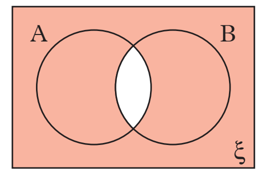
f



g

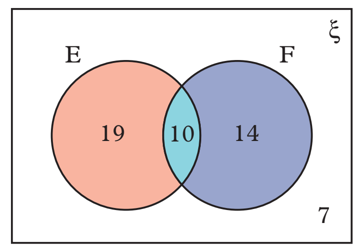


h



Exercise 8.6B: Problem solving and reasoning

9 Consider the Venn diagram on the right.



a Given that $n(\text{F})$ means the number of elements in set F, find:

i $n(\text{E})$ **ii** $n(\text{E} \cup \text{F})$ **iii** $n(\text{F}')$

iv $n(\text{E} \cap \text{F})$ **v** $n(\xi)$ **vi** $n(\text{E}' \cap \text{F})$.

b Given that Pr(E) means the probability of selecting an element from set E, find:

i $\Pr(\text{F})$ **ii** $\Pr(\text{E}')$ **iii** $\Pr(\text{E} \cap \text{F})$

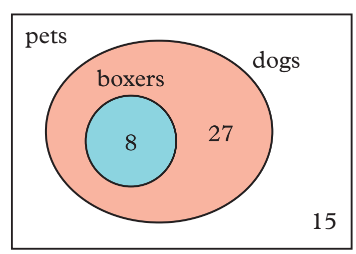
iv $\Pr(\text{E} \cup \text{F})'$ **v** $\Pr(\text{E}' \cap \text{F}')$ **vi** $\Pr(\text{F} \cup \text{E}')$.

10 Consider the Venn diagrams in question **8**.

a State which pairs are complementary by writing both set equalities using the symbol for the complementary event, ′. For example, $(\text{A} \cap \text{B})' = \text{A}' \cup \text{B}'$ and $(\text{A} \cup \text{B})' = \text{A}' \cap \text{B}'$.

b Explain the relationship between the complements of intersections and unions. (Note: These relationships are known as De Morgan's Laws.)

11 Venn diagrams can be used to show the relationship between three or more sets and also include subsets. Consider these two Venn diagrams, the first of which details the pets of pet owners and the second of which shows whether students take any of three elective subjects at school.

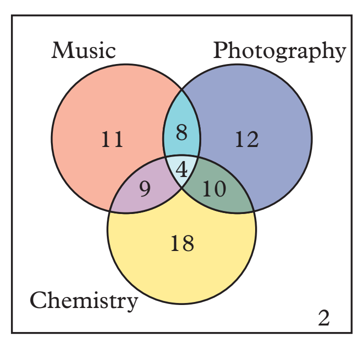


a Which Venn diagram shows subsets of data? Explain the relationship shown in this Venn diagram.

b Copy the first Venn diagram three times, and on separate copies, shade the section that represents:

i pets that are not dogs **ii** dogs that are not boxers

iii all dogs.



c The other Venn diagram shows the relationship between three different sets. Copy this Venn diagram five times and shade the section that represents students:

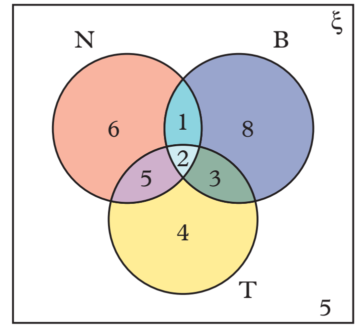
i taking all three elective subjects **ii** taking only Photography

iii taking Music or Chemistry **iv** taking Music

v not taking any of these three electives.

12 This Venn diagram shows the number of students who play netball, basketball and tennis. Find the probability that a student chosen randomly:

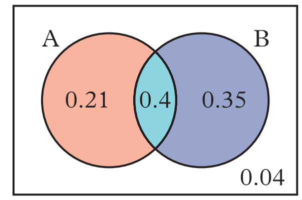
a plays netball but not tennis
b plays basketball only
c plays all three sports
d doesn't play any of these sports
e plays basketball or tennis
f plays tennis and netball.



13 Use the following information to draw a Venn diagram that shows the relationship between three different sets of data. In a group of 100 people surveyed:

- 35 liked action films
- 46 liked horror films
- 5 people liked all three genres
- 16 only liked horror
- 45 liked romance
- 55 did not like romance films
- 15 people liked both action and romance
- 18 liked both horror and romance
- 19 people did not like films in any of these three genres.

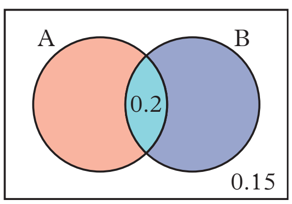
14 The Venn diagram on the right shows the relative frequency of each intersection between the events A, B and their complements. If the sample size was 400, redraw the Venn diagram using frequencies.

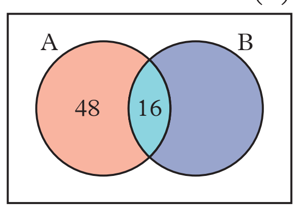


15 The following Venn diagrams represent the same sample of people for the events A and B.

a Complete the Venn diagrams.

b Calculate: **i** Pr(A) **ii** Pr(B)



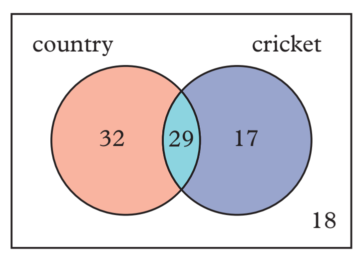


Exercise 8.6C: Challenge

16 The probability of an event can change when new information is obtained. For example, the probability of selecting a diamond from a deck of cards is $\frac{1}{4}$. However, if you are told the card is red, then the probability it is a diamond is now $\frac{1}{2}$, based on this new information. This would usually be asked with wording such as: "What is the probability that a randomly selected card from a deck is a diamond, given that it is red?"

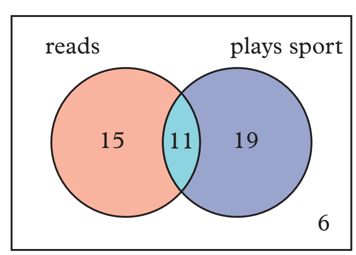
This Venn diagram shows whether people are from the country and if they like cricket or not. Use this diagram to calculate the probability that a person chosen randomly:

a likes cricket, given that they are from the country
b is from the country, given that they like cricket
c does not like cricket, given that they are from the country
d is not from the country, given that they do not like cricket.



17 This Venn diagram shows whether people like to read and/or play sport. Use this diagram to calculate the probability that a person chosen randomly:

a reads, given that they play sport
b plays sport, given that they read
c does not play sport, given that they read
d does not read, given that they do not play sport.



Online resources:

- Interactive skillsheet — Venn diagrams
- Interactive skillsheet — Using 'and' and 'or' in probability
- Interactive skillsheet — Set notation
- Investigation — Venn diagram applications
- Quick quiz — 8.6

Lesson 8.7

Review: Probability

Module summary

Theoretical probability

- Sample space: all possible outcomes
- Event: collection of one or more outcomes from the sample space
- If outcomes are equally likely:

$$\Pr(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$$

- A' means not A. A and A' are complementary events. $\Pr(A) + \Pr(A') = 1$

Experiments with/without replacement

- With replacement: probabilities for each step remain the same.
- Without replacement: probabilities for all remaining steps are changed.
- Tree diagrams and lists of outcomes can help to find the probabilities of multi-step experiments.

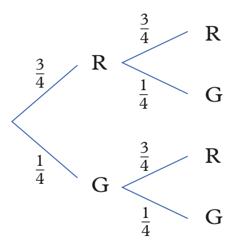
Arrays

- List of items arranged in rows and columns.
- Use when there is a large number of outcomes in either step of a two-step experiment.

	1	2	3	4	5	6
H	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
T	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)

Combining like outcomes

- When grouping like outcomes, the probability of the grouped outcome is the sum of the probabilities in the group.
- The probabilities of final outcomes can be found by multiplying the probabilities leading to that outcome.



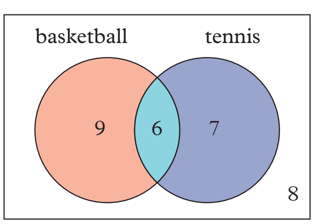
Outcomes	Probability
RR	$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$
RG	$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$
GR	$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$
GG	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Relative frequency

- Relative frequency $= \dfrac{\text{number of occurrences}}{\text{total number of trials}}$
- Expected number = theoretical probability × number of trials
- With a small number of trials, results may vary significantly from expected.
- In the long term, the relative frequency will approach the theoretical probability.

Venn diagrams

- Each region represents a different set.
- The rectangle represents the entire data set.
- Frequencies represent the number of data points for each region of the diagram.



Two-way tables

- Raw numbers or relative frequencies.

	Dark hair	Light hair	Total
Dark eyes	0.4	0.1	0.5
Pale eyes	0.2	0.3	0.5
Total	0.6	0.4	1

- 'and': both outcomes must be met
- Inclusive 'or': if both outcomes are met, the outcome is favourable.
- Exclusive 'or': if both outcomes are met, the outcome is not favourable.

Set notation

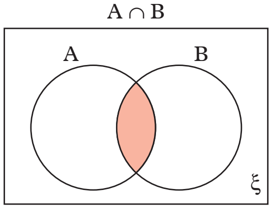
- Everything within the rectangle of a Venn diagram belongs to the universal set, ξ.

A ∩ B is the intersection of sets A and B (A and B)

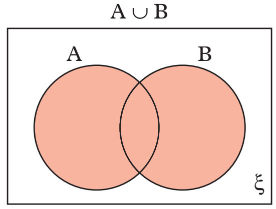
A ∪ B is the union of sets A and B (A or B)

A' is the complement of A (not A)

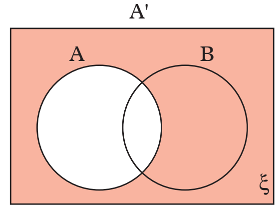
A ∩ B



A ∪ B



A'



ANS p529

Review questions 8.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- array
- bias
- complement
- complementary events
- expected number
- event
- experiment
- favourable outcomes
- intersection
- outcome
- relative frequency
- replacement
- sample space
- set
- subset
- theoretical probability
- tree diagram
- trial
- two-way table
- union
- universal set
- Venn diagram

1 Explain what the complement of an event is.

2 Explain why the probabilities at each step of an experiment without replacement change.

3 How can you calculate the relative frequency of a random event?

4 Identify the key terms being referenced in each of these definitions.

a a list of all the possible outcomes in an experiment

b the set of all elements

5 Using an example, provide a definition in your own words for the following key terms.

a expected number

b event

6 Complete the following sentences using words from the key terms list.

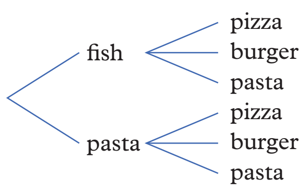
a C ⊂ D means C is a ________ of D.

b A ∩ B means the ________ of sets A and B.

Review questions 8.7B: Multiple choice

8.1 **1** If there is an equal probability of choosing every option on the following menu, the probability that a person orders pasta for both entree and main is:

Entree and main meal choices at a local cafe



A $\frac{1}{3}$ **B** $\frac{2}{3}$ **C** $\frac{1}{6}$ **D** $\frac{1}{2}$

8.1 **2** Two numbers are chosen randomly and their product is recorded. One number is chosen from the set $\{1, 2, 3, 4, 5\}$ and the other is chosen from the set $\{2, 3, 4\}$. The probability of which pair of events is not equal?

A Pr(4), Pr(8)

B Pr(odd), Pr(multiple of 5)

C Pr(multiple of 3), Pr(multiple of 2)

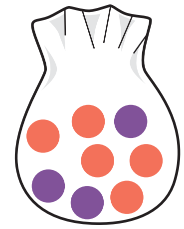
D Pr(square number), Pr(multiple of 6)

8.2 **3** There is a $\frac{1}{13}$ chance of selecting an Ace from a standard pack of playing cards. A card is selected at random and an Ace is chosen. The card is replaced into the pack. What is the probability of selecting an Ace on the second selection?

A $\frac{1}{13}$ **B** $\frac{1}{169}$ **C** $\frac{12}{13}$ **D** $\frac{144}{169}$

Questions **4** and **5** refer to the diagram on the right.

A marble is selected and not replaced and then a second selection is made.



8.3 **4** The first marble selected is orange. What is the probability the second marble selected is also orange?

A $\frac{4}{7}$ **B** $\frac{3}{7}$ **C** $\frac{1}{7}$ **D** $\frac{5}{7}$

8.3 **5** The probability of selecting two purple marbles is:

A $\frac{3}{8}$ **B** $\frac{2}{7}$ **C** $\frac{5}{14}$ **D** $\frac{3}{28}$

8.4 **6** A six-sided die is rolled many times. Consider this table showing the frequencies of the outcomes of this experiment.

Number on die	Frequency
1	45
2	35
3	27
4	30
5	32
6	31

Which one of these statements is true?

A The number of trials performed was 200 rolls.

B The relative frequency of rolling an even number is $\frac{13}{25}$.

C The number rolled the least frequently was 1.

D The relative frequency of rolling a 6 is $\frac{1}{6}$.

8.5 **7** Consider the following frequency table.

	Under 30s	30 and over	Total
Prefer coffee	176	98	
Prefer tea	101	25	126
Total	277		400

The relative frequency of a person preferring coffee is:

A $\frac{137}{200}$ **B** $\frac{49}{200}$ **C** $\frac{11}{25}$ **D** $\frac{101}{400}$

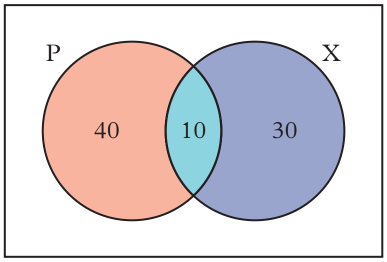
8.5 **8** This two-way table is to be converted to a relative frequency two-way table. Which relative frequency will not appear?

	A	A'	Total
B	5	5	10
B'	7	8	15
Total	12	13	25

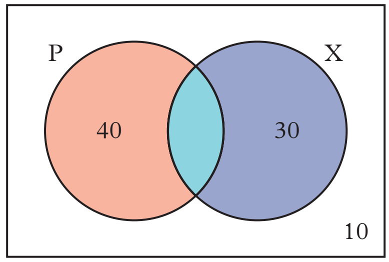
A 0.2 **B** 0.28 **C** 0.52 **D** 0.5

8.6 **9** A group of 80 students were asked if they owned a PlayStation (P) or an Xbox (X). 40 students owned a PlayStation, 30 students owned an Xbox and 10 said they owned both. Which of the following Venn diagrams correctly displays this information?

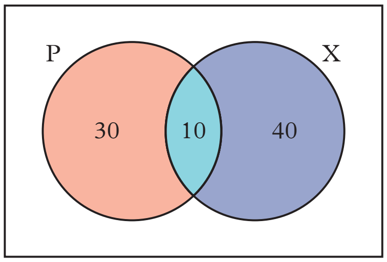
A



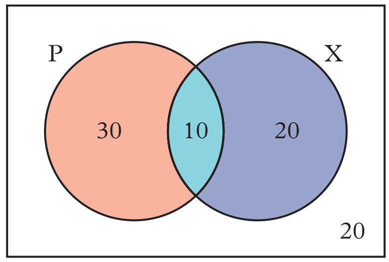
B



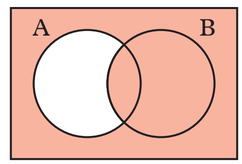
C



D



8.6 **10** Which of the following is represented by the shaded section in the Venn diagram?



A $A \cap B$ **B** $A \cup B$

C $A' \cup B$ **D** $A' \cap B$

Review questions 8.7C: Short answer

8.1 **1** A company runs a competition where one out of every four purchases contains the winning bar code for an ebook.

- **a** What is the probability of not winning an ebook?
- **b** Complete a tree diagram showing all probabilities on the branches for two trials.
- **c** Calculate the probability of:
  - **i** winning twice in a row
  - **ii** the complement of winning twice in a row
  - **iii** winning on the first try and then not winning on the second try.

8.1 **2** A coin is tossed and a fair standard six-sided die is rolled.

- **a** Show all possible outcomes on an array.
- **b** Calculate the probability that:
  - **i** the coin lands on heads and the die lands on 2
  - **ii** the coin lands on heads or the die lands on 2 or both
  - **iii** the coin lands on heads or the die lands on an even number, but not both.
- **c** Calculate the probability that the coin lands on heads and the die lands on a multiple of 3.
- **d** Calculate the probability that the coin lands on heads or the die lands on a factor of 6 or both.

8.2 **3** A bag contains six chocolates. Two have orange wrappers, one has a green wrapper and three have pink wrappers. A chocolate is chosen at random and the colour is recorded. The chocolate is replaced and then another selected.

- **a** Show all possible outcomes and probabilities on a tree diagram.
- **b** Calculate the probability that:
  - **i** both wrappers are orange **ii** both wrappers are green **iii** both wrappers are pink.
- **c** Calculate the probability that the first wrapper is green.
- **d** Calculate the probability that the second wrapper is pink.

8.3 **4** There are four suits (clubs, diamonds, hearts and spades) in a standard pack of 52 cards. Two cards are selected from a pack and the suits are noted. Assuming selection without replacement, calculate:

a Pr(two spades) **b** Pr(heart, then spade).

8.4 **5** Calculate the relative frequency of:

a a 3 being rolled, given that 45 out of 120 rolls of the die landed on 3

b the number 387 being generated when a random integer was generated 1000 times and 387 appeared 5 times

c selecting someone in a shoe store at random wearing boots when 300 people visited the shoe store in a day and 33 were wearing boots

d selecting a blue puzzle piece at random when a 1000-piece puzzle has 650 pieces with similar shades of blue on them.

8.4 **6** Calculate the expected number of the following.

a Rolling a 1 when a 12-sided die is rolled 420 times

b Generating 676 when a random integer from 1 to 1000 is generated 10 000 times

c *Hungry Like the Wolf* playing from a workout playlist with 73 tracks, when 365 tracks are played on random shuffle

d Rolling a double if a pair of dice is thrown 150 times while playing a board game.

8.5 **7** For the following questions, refer to this two-way table for Years 8, 9 and 10 students and their preferred brands of make-up.

	Year 8	**Year 9**	**Year 10**	**Total**
Napoleon Perdis	32	25	45	
Rimmel	22		22	54
Covergirl	17	31	13	
Maybelline		29		
Total	120		95	310

a Complete the table.

b Calculate the probability that a student chosen at random from the group:

i prefers Maybelline **ii** is in Year 8 and prefers Rimmel

iii is in Year 9 and prefers Napoleon Perdis.

c Calculate the probability that a student chosen from the Year 9 students:

i prefers Napoleon Perdis **ii** prefers Covergirl or Maybelline.

8.6 **8** A survey of 120 secondary school students was conducted about whether they like to play video games and board games. Of the 20 Year 9 students surveyed, 8 play video games, 10 play board games, and this includes 3 who play both. Of the students in other year levels who were surveyed, 55 play video games, 60 play board games, and this includes 35 who play both.

Let A be the set of Year 9 students, B be the set of students who play board games, and V be the set of students who play video games.

a Use a Venn diagram with circles for the sets A and V to display this information.

b Calculate:

i $\Pr(A \cap V)$ **ii** $\Pr(A \cup V)$

iii $\Pr(A' \cap V')$ **iv** $\Pr(V)$

c Use a Venn diagram with circles for the sets B and V to display the information.

d Calculate:

i $\Pr(B \cap V)$ **ii** $\Pr(B' \cup V)$

iii $\Pr(B \cap V')$ **iv** $\Pr(B)$

Review questions 8.7D: Mathematical modelling

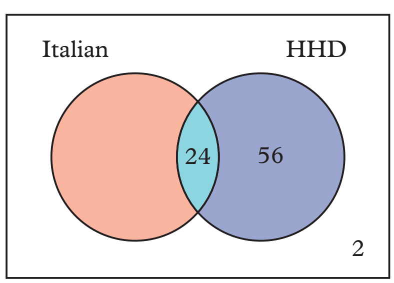
1 When moving from Year 9 into Year 10, students undertake core studies such as English and Mathematics and can select from a range of electives to complete their study program.

A student must select two from these four electives: Health and Human Development (HHD), Art, Italian and Food Technology. The student writes each subject on a card and then chooses two at random from a hat.

- **a** Would this selection be with or without replacement? Explain your reasoning.
- **b** Using your answer to part **a**:
  - **i** Draw a tree diagram to represent the possible subject combinations.
  - **ii** How many combinations are possible?
- **c** Find the probability of selecting:
  - **i** Art and Food Technology
  - **ii** Italian and HHD.

2 The senior students at the school were surveyed about the Food Technology and Art electives offered. 12% of the respondents were Year 12 students who preferred Art to Food Technology; 28% of the respondents were Year 11 students who preferred Food Technology to Art. Overall, 52% of the respondents were in Year 11.

- **a** Construct a two-way table showing these percentages as relative frequencies.
- **b** Based on these survey results, if a person was randomly selected from the 200 students who participated in the survey, find the probability that they are:
  - **i** a Year 12 student who prefers Food Technology
  - **ii** a Year 11 student who prefers Art
  - **iii** a student who prefers Art.
- **c** Of these 200 students, find the number of people who:
  - **i** are in Year 12 and prefer Food Technology
  - **ii** prefer Food Technology.
- **d** An analysis of numbers of students studying Italian and/or HHD was also undertaken and the results collated into this Venn diagram.



  - **i** How many of the 150 students study both Italian and HHD?
  - **ii** How many students study neither subject?
  - **iii** Use the information given to work out how many students study Italian only.
  - **iv** If selecting a student at random from the cohort, what is the probability that they study Italian only?
  - **v** If selecting a student at random from the cohort, what is the probability that they study HHD only?

3 Eric is researching the popularity of Ford and Holden cars amongst Year 8 boys and girls. He thinks it will be reasonably evenly distributed.

a Draw a tree diagram to represent the situation.
Hint: Start the first branches with gender.

b List the sample space (the possible outcomes) and, assuming all are equally likely, provide the theoretical probability for each outcome.

c Eric completes a survey and records these results.

	Ford	Holden
Boys	7	23
Girls	19	9

Copy and extend this table to make it a two-way table and find how many people were surveyed in total.

d Using your answer from part **c**, the probability that a person chosen randomly from the group:

i prefers Holdens to Fords

ii is a boy.

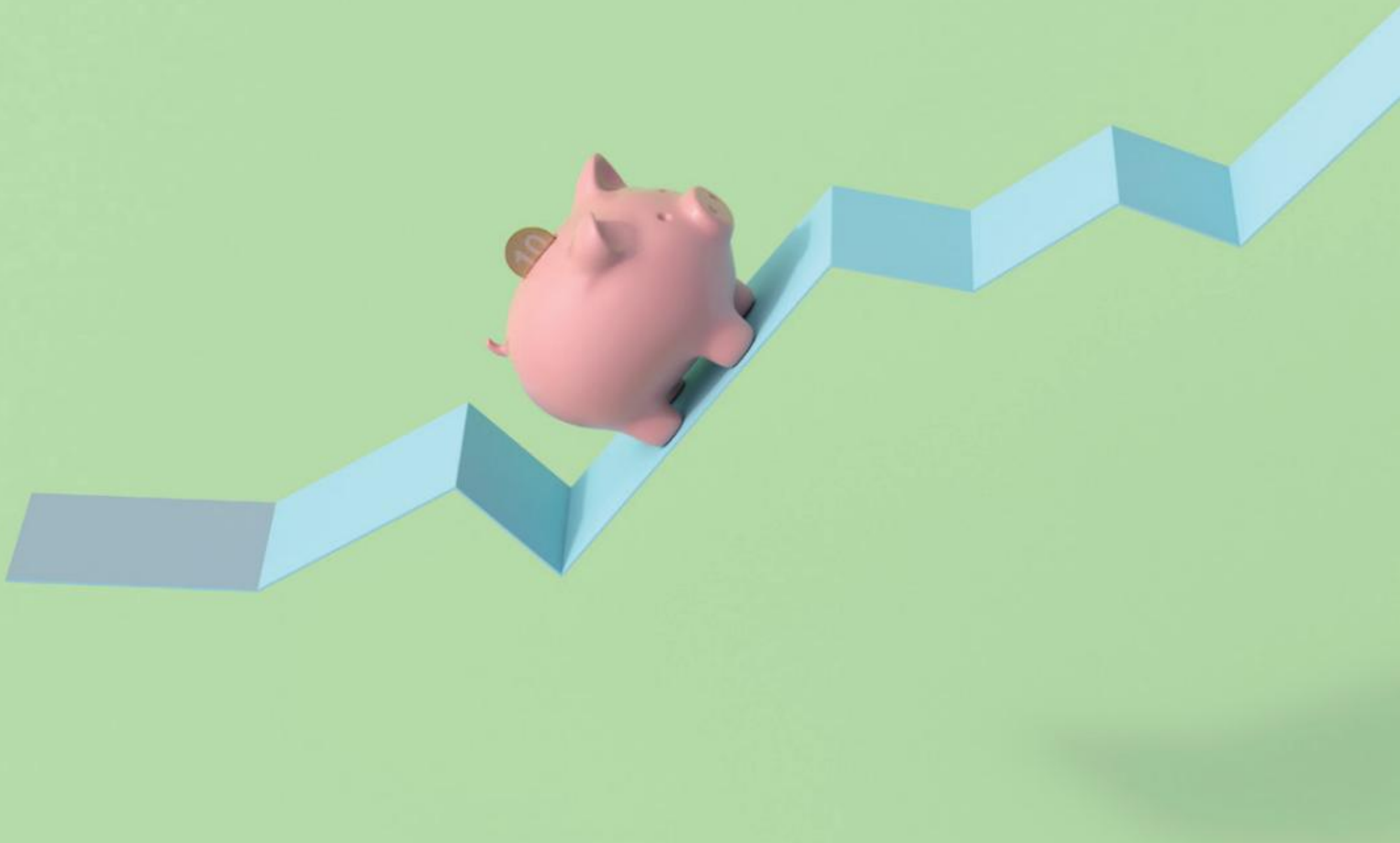
Checklist

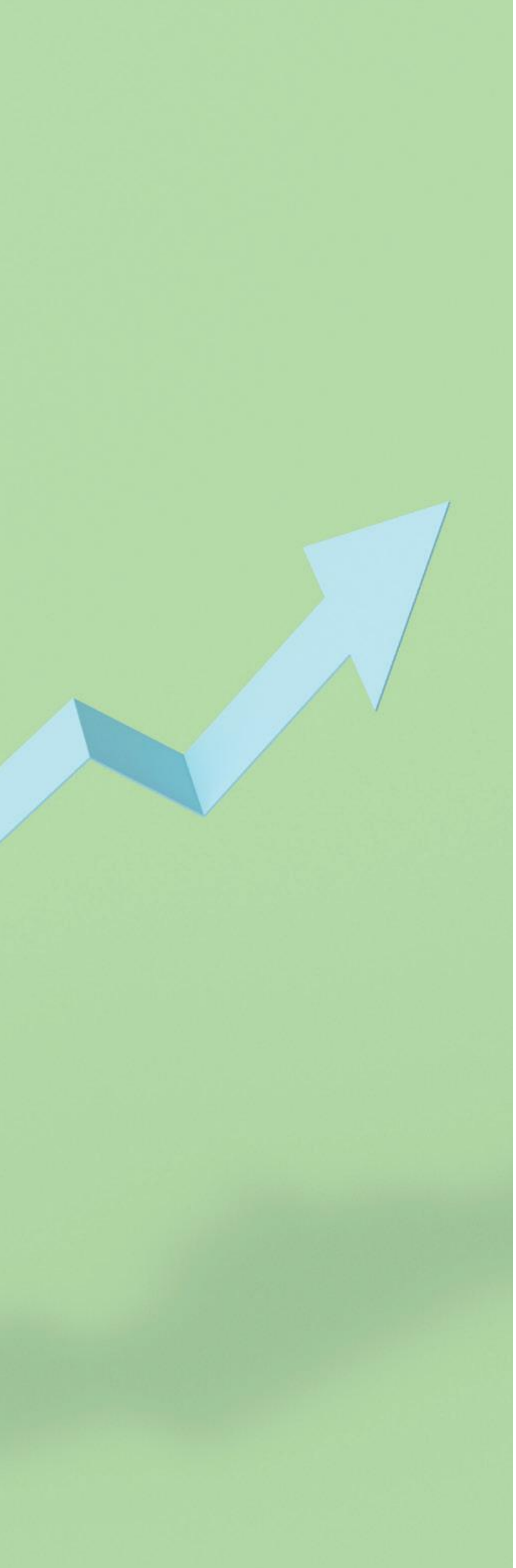
Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Create tree diagrams and arrays to represent sample spaces ☐ Use tree diagrams and arrays to calculate probabilities	☐ Go back to Lesson 8.1 Two-step chance experiments
☐ Draw tree diagrams with probabilities detailed on each branch ☐ Calculate probabilities for two-step chance experiments with replacement	☐ Go back to Lesson 8.2 Experiments with replacement
☐ Draw tree diagrams with probabilities changing at each step ☐ Calculate probabilities for two-step chance experiments without replacement	☐ Go back to Lesson 8.3 Experiments without replacement
☐ Calculate relative frequencies ☐ Calculate the expected number of times an outcome will occur in an experiment	☐ Go back to Lesson 8.4 Relative frequency
☐ Create two-way relative frequency tables ☐ Use two-way tables to calculate probabilities	☐ Go back to Lesson 8.5 Two-way tables
☐ Calculate the probabilities of outcomes using data displayed in Venn diagrams ☐ Create Venn diagrams to represent two or more sets of data ☐ Use set notation to represent sets of data	☐ Go back to Lesson 8.6 Venn diagrams

9

Financial mathematics





Lessons

9.1 Wages and salaries
9.2 Other forms of income
9.3 Taxation
9.4 Simple interest
9.5 Simple interest calculations

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

✔ Substitution
✔ Multiplying and dividing decimals
✔ Rounding decimals
✔ Calculating percentages
✔ Rates

Curriculum link

- Use mathematical modelling to solve applied problems involving change, including financial contexts involving simple interest; formulate problems, choosing to use either linear or quadratic functions or other simple variations; interpret solutions in terms of the context; evaluate the model and report methods and findings (VC2M9A06)

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Materials

✔ Calculator

Wages and salaries

Learning intentions

By the end of this lesson you will be able to …

✔ understand and calculate different types of income such as wages and salaries.

Inter-year links

Year 8 3.3 Mark-ups and discounts

Year 10 12.1 Earning money

Wages, penalty rates and leave loading

Key content video
Wages and salaries

- **Income** is money received in exchange for work or investment.
- A **wage** is a fixed rate of income an employee receives for working a specified time period, usually per hour. For example, an employee may receive a wage of \$25 per hour, or \$25/hour.
  - → An hourly wage can be calculated by dividing the amount someone earns by the number of hours they worked.
  - → When given a wage, you can work out how much an employee will earn for a given number of hours.
- **Penalty rates** are higher rates of pay that are given to employees for working particular hours or days, such as weekends, public holidays or overtime.
  - → Time-and-a-half means an employee is paid 1.5 times their standard wage.
  - → Double time means an employee is paid 2 times their standard wage.
- All full-time employees are entitled to a minimum of four weeks of paid annual leave per year. **Leave loading** is an extra payment on top of annual leave pay and is usually calculated as 17.5% of normal pay for this period.

Salaries and bonuses

- A **salary** is a fixed income which is paid to an employee on a regular basis. Salaries are usually expressed as an annual figure, with regular payments being made each week, fortnight or month.
  - → When calculating weekly or fortnightly pay, it is assumed there are 52 weeks and 26 fortnights in a year.
  - → Monthly pay can be calculated by dividing an annual salary by 12.
  - → Salaried employees do not typically receive penalty rates or leave loading.
- Some employees receive a **bonus** on top of their salary as an incentive or to reward good performance. Bonuses are often given as a percentage of an employee's salary.
  - → Bonuses may also be given to employees who receive a wage instead of a salary.

Worked example 9.1A Calculating wages

In a week, Petra works for 26 hours at her local cafe and earns \$650 without penalty rates. What is her hourly wage?

THINK	WRITE
Divide the amount earned by the number of hours to get the hourly wage.	$\frac{\$650}{26 \text{ hours}} = \$25/\text{hour}$

Worked example 9.1B Calculating income for a given period of time

Jordan works for 35 hours per week and is paid \$32.50 per hour. How much does he earn in a fortnight?

THINK	WRITE
1 Multiply the hourly wage by the number of hours worked each week to calculate the weekly wage.	Weekly wage = \$32.50 × 35 = \$1137.50
2 Multiply the weekly wage by 2 to calculate the fortnightly wage.	Fortnightly wage = \$1137.50 × 2 = \$2275

Worked example 9.1C Calculating income with penalty rates

Miriam's standard wage is \$38.30 per hour and on weekends she is paid time-and-a-half.

a What is Miriam's weekend rate of pay?

b How much does Miriam earn in a week when she works 6 hours per weekday and 5 hours on Saturday?

THINK	WRITE
a Multiply the weekday rate by the penalty rate (1.5) to calculate the weekend rate.	**a** Weekend rate = \$38.30 × 1.5 = \$57.45
b **1** Calculate the weekday income by multiplying the weekday rate of pay by the number of hours worked on weekdays.	**b** Weekday income = \$38.30 × (6 × 5) = \$38.30 × 30 = \$1149
2 Calculate the weekend income by multiplying the weekend rate of pay by the number of hours worked on the weekend.	Weekend income = \$57.45 × 5 = \$287.25
3 Add the weekday income to the weekend income to get the total income.	Total income = \$1149 + \$287.25 = \$1436.25

Worked example 9.1D Calculating leave loading

Will works 38 hours per week and is paid \$26 per hour. He takes one week of annual leave for which he receives an additional leave loading of 17.5%. How much does Will get paid for his week of annual leave?

THINK	WRITE
1 Calculate the weekly wage by multiplying the hourly wage by the number of hours worked in a week.	Weekly wage = \$26 × 38 = \$988
2 Calculate the leave loading by multiplying the weekly wage by 17.5%.	Leave loading = \$988 × 17.5% = \$988 × 0.175 = \$172.90
3 Add the leave loading to the weekly wage to calculate the annual leave weekly wage.	Annual leave weekly wage = \$988 + \$172.90 = \$1160.90

Helpful hints

✔ When calculating a monthly salary, you do not need to consider how many days that month has. All months should be considered as $\frac{1}{12}$ of a year. To calculate a monthly salary, divide the annual salary by 12. It should not be calculated by multiplying the weekly salary by 4.

Learning pathways

△ 1–3, 6–8, 11, 12, 15, 16

□ 2–4, 7–9, 12, 13, 16, 17

⬠ 2, 5, 8–10, 12, 14, 17–19

ANS p531

Exercise 9.1A: Understanding and fluency

WE 9.1A **1** Last week Iona worked for 32 hours and earned \$1318.40. What is her hourly wage?

2 Calculate the hourly wages for the given amount of time worked and money earned. Assume that no penalty rates were applied.

	Time worked	Amount earned
a	20 hours	\$560
b	35 hours	\$1575
c	12 hours	\$280.80
d	18.5 hours	\$444
e	27.25 hours	\$915.60

WE 9.1B **3** Hiro works for 26 hours per week and is paid \$39.20 per hour. How much does he earn in a fortnight?

4 Rafael works as a courier delivering parcels around the city. He earns \$28.50 per hour.

a Calculate Rafael's income for a week in which he works 20 hours.

b In one particular week, Rafael's wage totalled \$684. How many hours did Rafael work in this week?

5 Complete the following table.

Hourly wage	Time worked	Amount earned
\$32.20	25 hours	
	30 hours	\$735
\$27		\$756
\$30.30	38 hours	
	32 hours	\$1638.40

6 For each of these hourly wages, calculate:

i the time-and-a-half hourly wage

ii the double time rate.

a \$18 **b** \$24 **c** \$18.80 **d** \$25.90 **e** \$32.60 **f** \$29.90

WE 9.1C **7** Jo's standard wage is \$25.80 and on weekends she is paid time-and-a-half.

a What is Jo's weekend rate of pay?

b How much does Jo earn in a week when she works 5 hours per weekday and 7 hours on Saturday?

WE 9.1D **8** Raj works 34 hours per week and is paid \$31 per hour. He takes one week of annual leave for which he receives an additional leave loading of 17.5%. How much does Raj get paid for his week of annual leave?

9 Calculate the annual salary for the following given values.

a \$6450 per month **b** \$4480 per month **c** \$1240 per week

d \$950 per week **e** \$5100 per fortnight **f** \$3330 per fortnight

10 Lina works in research and earns a salary of \$66 612.

a If Lina is paid monthly, what is her salary each month?

b If Lina is paid fortnightly, what is her salary each fortnight?

Exercise 9.1B: Problem solving and reasoning

11 Dina earns a wage of \$34.50 per hour and works for 28 hours each week.

a What is Dina's weekly income?

b Calculate Dina's yearly income.

c Calculate Dina's monthly income.

12 As a bricklayer, Luka receives a standard rate of \$27.40 per hour. He receives a penalty rate of one-and-a-quarter for any overtime (above 30 hours per week).

a How much does Luka earn in a week in which he works 35 hours?

b How many hours does Luka work in a week in which he earned \$890.50?

13 Rahim earns \$3200 per month and Sal earns \$780 per week. Rahim argues that he earns more money than Sal because if you multiply Sal's wage by 4, the result is less than his wage, and there are four weeks in a month. Is Rahim correct? Justify your answer by showing appropriate calculations.

14 The following table shows the hours a group of workers worked for over the past week, as well as their hourly wage. All workers receive time-and-a-half for working on weekends. Calculate each worker's weekly income.

Worker	Hourly wage	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Nathan	\$31.50	9–5	9–5	–	9–3	9–1	9–3	–
Kristina	\$33.20	10–4	10–4	10–4	10–4	10–6	–	–
Jack	\$26.50	9–3	–	9–3	–	9–3	9–3	9–3
Mina	\$29.40	–	8–5	8–5	9–5	–	10–5	9–3

15 For two weeks of annual leave, Lola receives \$3172.50 including leave loading of 17.5%. Lola works a 36-hour week. What is Lola's hourly wage?

16 In 2024, the minimum wage in Australia for employees aged 21 and above is \$24.10 per hour.

a How much does someone receiving the minimum wage earn if they work a 38-hour week?

b A junior is an employee under 21 years old. Juniors usually get a percentage of the adult pay rate. If a junior is entitled to 80% of the adult pay rate, what is their minimum wage per hour to the nearest cent?

17 Marty works in a cafe on weekdays and at a bar on weekends. His cafe job pays \$24.80 an hour and his bar job pays \$37 an hour. One week Marty works for 22 hours at the cafe. How many hours did Marty need to work at the bar for his income for the week to total \$1000 or more?

18 Shivani works at an internet security company and receives a fortnightly salary of \$4246. At her end of year performance review, she's informed that she will receive a 6.5% bonus on her annual salary. How much is her bonus worth, correct to the nearest dollar?

Exercise 9.1C: Challenge

19 Exactly halfway through the financial year, Henrik received a promotion at work which gave him a 15% increase in his salary. In this financial year Henrik also received a 3% bonus on his new salary. If Henrik's total income from work for the year was \$75 446, what was his salary at the end of the year?

Online resources:

- **Interactive skillsheet** Wages and salaries
- **Worksheet** Wages and penalty rates
- **Worksheet** Salaries
- **Worksheet** Leave loading
- **Quick quiz** 9.1

Other forms of income

Learning intentions

By the end of this lesson you will be able to ...

✔ understand and calculate other forms of income.

Inter-year links

Year 7	4.8 Calculating percentages
Year 8	3.2 Calculating percentages
Year 10	12.1 Earning money

Commissions, piece work and royalties

Key content video
Commissions, piece work and royalties

- When an employee earns a **commission**, they are paid a proportion of the value of their sales. For example, a real estate agent may receive a 2% commission on their house sales.
  - → Employees who receive a commission usually also receive a base salary, which is a guaranteed income. This base salary may be lower than the national minimum wage.
- **Piece work** is income which is paid for each item (of group of items) produced or action performed. For example, an employee might earn a rate of \$20 for each haircut they perform.
  - → Piece rates are paid in place of a wage. The piece rate has to allow for the average worker to earn the equivalent of 15% above the minimum wage, and piece rate workers are guaranteed a minimum wage for each day that they work.
- A **royalty** is a payment made to a person or company for the use of their assets. For example, a royalty may be paid to an author for the publication of their content. Royalties are usually paid as a percentage of net sales.

Worked example 9.2A Calculating income including a commission

Oscar earns a base salary of \$450 per week plus a commission of 12% on all sales. For a week in which Oscar has sales of \$6500, calculate his:

a commission **b** total income.

THINK	WRITE
a Multiply the value of the sales by the commission percentage.	**a** Commission = \$6500 × 12% = \$6500 × 0.12 = \$780
b Add the base salary to the commission to calculate the total income.	**b** Total income = \$450 + \$780 = \$1230

Worked example 9.2B Calculating piece work income

Mishra is paid \$8.65 for each bouquet of flowers he arranges. In one week he arranges 56 bouquets of flowers. How much is he paid in that week?

THINK	WRITE
Multiply the amount earned per item by the number of items sold.	Income = \$8.65 × 56 = \$484.40

Worked example 9.2C Calculating royalty payments

Mikayla receives a 12.5% royalty on all sales of her book. If the book sales over the last financial year were \$72 610, how much will Mikayla receive in royalties?

THINK	WRITE
Multiply the value of the sales by the royalty percentage to calculate the royalty payment.	Royalty payment = \$72 610 × 12.5% = \$72 610 × 0.125 = \$9076.25

Helpful hints

✔ To multiply a value by a percentage, first convert the percentage into a decimal by dividing by 100.

Learning pathways

△ 2–5, 7, 9, 10, 12, 14

□ 1, 3, 5, 6(a–c), 8, 11, 13, 15

⬠ 1, 3, 6, 8, 11, 13, 14, 15, 16

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Exercise 9.2A: Understanding and fluency

1 Calculate the commission payable in each of the following situations.

	Commission rate	Sales
a	4%	\$22 500
b	9%	\$15 300
c	1.5%	\$250 000
d	3%	\$42 150
e	2.1%	\$56 050

WE 9.2A 2 Justine earns a base salary of \$520 per week plus a commission of 2.6% on all sales. For a week in which Justine has sales of \$48 000, calculate her:

a commission **b** total income.

3 Damo earns a base salary of \$310/week plus a commission of 5% on all sales. How much does he earn in a week if the weekly sales totals are as follows.

a \$22 000 **b** \$9000 **c** \$36 500

d \$17 800 **e** \$31 000 **f** \$42 200

WE 9.2B 4 Jermaine is paid \$24 for each haircut he completes. In one week, he cuts 42 people's hair. How much is he paid in that week?

5 Kylie is paid \$2.50 for each cup of coffee she makes.

a How much does Kylie earn if she makes:

i 40 cups **ii** 53 cups **iii** 64 cups?

b How many cups of coffee did Kylie make if she earned:

i \$90 **ii** \$177.50 **iii** \$212.50?

WE 9.2C **6** Calculate the amount authors will receive in royalties given the following rates and sales figures.

	Royalty rate	Sales
a	11%	\$135 000
b	1.2%	\$240 000
c	4.5%	\$65 000
d	15%	\$72 000
e	6%	\$110 500

Exercise 9.2B: Problem solving and reasoning

7 Last year, Juan received a royalty payment of \$557.20 on a 3.5% royalty. What were the sales for that year?

8 Hanna receives a royalty payment of \$1108.80 on sales of \$46 200. What is her royalty rate?

9 Denis received a base wage of \$400/week and a commission of 3.2% on sales. Denis' income for a week is \$1459.20. How much product did Denis sell that week?

10 Luna receives \$4.50 for every candle and \$3.50 for every bath bomb she sells. In one afternoon, Luna earned \$161.50. If Luna sold 18 candles, how many bath bombs did she sell?

11 Three friends, Abi, Beth and Craig, all start new jobs at the same time. Abi earns a wage of \$24.50 plus penalty rates, Beth earns a base salary of \$200 plus a commission of 4% on all sales, and Craig is paid \$25 for each item he produces. In one week:

- Abi works 20 hours at her standard rate, plus 6 hours at time-and-a-half
- Beth sells \$18 000 worth of product
- Craig produces 58 items.

Put the amount the three friends earn in this week in ascending order.

12 Jerome receives \$32 for each phone screen he replaces. If Jerome replaces 25 phone screens in 8 hours, what is his equivalent hourly rate of pay?

13 Greta earns \$2.80 for each small cup of coffee she makes and \$3.50 for each large cup of coffee she makes. If Greta makes 35 small cups of coffee and 14 large cups of coffee in 3 hours, what is her equivalent hourly rate of pay?

14 Owen receives \$3.50 for the first 20 trees he plants and \$4.40 for each additional tree planted.

a If Owen plants 34 trees, how much does he earn?

b If Owen earns \$188.80, how many trees does he plant?

15 Patrick earns \$3280 on \$80 000 worth of sales and Erica earns \$3010 on \$70 000 worth of sales.

a Who receives the higher rate of commission?

b If their rates of commission were swapped, how much would they each earn?

Exercise 9.2C: Challenge

16 When starting her new job, Nina is given the following options for her pay:

- Option A: base salary \$600, 2.4% commission
- Option B: base salary \$400, 2.8% commission
- Option C: base salary \$250, 3.0% commission

a Which option would give the highest income for Nina if she expects to sell \$55 000 of product per week?

b For what amount of expected sales should Nina choose each option?

Online resources:

Interactive skillsheet Commission, piece work and royalties

Worksheet Commission and royalties

Worksheet Piece work

Quick quiz 9.2

Taxation

Learning intentions

By the end of this lesson you will be able to …

✔ calculate net income after tax and other deductions.

Inter-year links

Year 7 4.8 Calculating percentages

Year 8 3.7 Rates

Year 10 12.1 Earning money

Key content video
Taxation

Income tax and deductions

- A person's total income includes wages, salaries, bonuses, royalties and interest earned.
- **Tax deductions** are expenses that can be subtracted from an individual's total income to determine their **taxable income**. Tax deductions include work-related expenses, gifts and donations, the cost of managing tax affairs and self-education expenses.

  Taxable income = Total income – Tax deductions
- **Income tax** is tax which is paid on an individual's taxable income.
  - → Income tax is a progressive tax, which means that the tax rate increases as the taxable income increases. This is based on the concept of the ability to pay, meaning that those who can afford to pay more, do pay more.
  - → Personal income tax is the largest source of government revenue in Australia, allowing the government to fund community services such as education and emergency services.
- The following table details the income tax rates for the 2024–2025 financial year:

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$45 000	16 cents for each $1 over $18 200
$45 001–$135 000	$4288 plus 30 cents for each $1 over $45 000
$135 001–$190 000	$31 288 plus 37 cents for each $1 over $135 000
$190 001 and over	$51 638 plus 45 cents for each $1 over $190 000

  - → For example, if someone has an income of $70 000 for the 2024–2025 financial year, their income tax payable would be:

$$\begin{aligned}\text{Income tax} &= \$4288 + 0.3 \times (\$70\,000 - \$45\,000) = \$4288 + 0.3 \times \$25\,000\\ &= \$4288 + \$7500\\ &= \$11\,788\end{aligned}$$

Pay-As-You-Go (PAYG) tax

- **Pay-As-You-Go (PAYG)** is a system whereby the approximate income tax owed by an employee is withheld from their pay, meaning they are paying their income tax in instalments instead of having to pay a lump sum at the end of the year.
  - → The PAYG taxation system helps to ensure people pay their income tax.
  - → Any discrepancies between the amount of income tax paid under the PAYG system and the amount of income tax owed in a financial year are resolved by completing a tax return. After reviewing the tax return, the Australian Taxation Office (ATO) will either provide a refund or let you know how much is still owed for each financial year.

Medicare levy and net earnings

- The **Medicare levy** is a 2% tax on taxable income which helps to fund the health care system.
  → Some individuals may be either exempt from the Medicare levy or eligible to pay a reduced levy.
- A person's **net income** can be calculated by subtracting their income tax and Medicare levy from their taxable income.

  Net income = Taxable income − Income tax − Medicare levy

Key content video
Budgeting

Budgets

- A **budget** is a financial plan. Budgets can be used to make sure you have enough money for future spending and to help you avoid going into debt.
- To prepare a budget you need to estimate your expected income and expenses. Expenses can be split up into essential items, such as food and housing, and discretionary items, such as entertainment and gifts.
  → To avoid going into debt, the total expenses in a budget should be less than the net income.

Worked example 9.3A Calculating taxable income

In the last financial year, Reina:

- earned a salary of \$77 800
- earned \$232 in interest on her investments
- donated \$300 to charity
- received a \$2500 bonus from work
- had \$950 of work-related expenses
- paid an accountant \$120 to manage her tax affairs.

Calculate Reina's taxable income.

THINK	WRITE
1 Calculate the total income by adding the salary, bonus and interest together.	Total income = \$77 800 + \$2500 + \$232 = \$80 532
2 Calculate the total deductions by adding the work-related expenses, charity donations and cost of managing tax affairs together.	Total deductions = \$950 + \$300 + \$120 = \$1370
3 Subtract the tax deductions from the total income to determine the taxable income.	Taxable income = \$80 532 − \$1370 = \$79 162

Worked example 9.3B Income tax

Use the 2024–2025 financial year tax rates table to calculate the income tax payable on income of \$144 900.

THINK	WRITE
1 Identify the row in the table into which the income falls. \$144 900 lies between \$135 001 and \$190 000.	\$144 900 − \$135 000 = \$9900
2 Follow the instructions in the second column of the table. To calculate '\$31 288 plus 37 cents for each \$1 over \$135 000', start by calculating the income over \$135 000. Then, multiply this figure by 0.37. Finally, add the result to \$31 288.	Income tax = \$31 288 + \$9900 × 0.37 = \$31 288 + \$3663 = \$34 951

Worked example 9.3C Creating an annual budget

Shai has just started a new job and wants to create a budget to keep track of his finances. He will earn \$1500 per week, and his estimated weekly expenses are \$450 on rent, \$150 on food, \$50 on transport and \$200 on other expenses. Each month, he also spends \$20 on his mobile phone, \$250 on entertainment and \$100 on clothes. Each month, he pays approximately \$300 for utilities like electricity and water. Shai can claim \$500 in tax deductions as work-related expenses and must pay the Medicare levy.

a Determine Shai's annual net income for the 2024–2025 financial year.

b Prepare an annual budget for Shai based on his estimated expenses. How much should he expect to spend each year?

c Is Shai's annual net income enough to cover his budget?

THINK	WRITE
a **1** Calculate the annual income by multiplying the weekly income by 52.	**a** Annual income = \$1500 × 52 = \$78 000
2 Calculate the taxable income by subtracting any tax deductions from the annual income. There are \$500 worth of tax deductions.	Taxable income = \$78 000 − \$500 = \$77 500
3 Calculate the income tax payable on the taxable income.	Income income = \$4288 + 0.3 × (\$77 500 − 45 000) = \$4288 + 0.3 × \$32 500 = \$4288 + \$9750 = \$14 038
4 Calculate the Medicare levy as 2% of the taxable income.	Medicare levy = 0.02 × \$77 500 = \$1550
5 Calculate the annual net income.	Net income = \$77 500 − \$14 038 − \$1550 = \$61 912
b **1** Determine the annual expense for each item by multiplying the weekly expenses by 52 and the monthly expenses by 12.	**b** Annual expenses Rent: \$450 × 52 = \$23 400 Food: \$150 × 52 = 7800 Transport: \$50 × 52 = \$2600 Other: \$200 × 52 = \$10 400 Mobile phone: \$20 × 12 = \$240 Entertainment: \$250 × 12 = \$3000 Clothes: \$100 × 12 = \$1200 Utilities: \$300 × 12 = \$3600
2 Calculate the sum of the annual expenses.	Total expenses = \$23 400 + \$7800 + \$2600 + \$10 400 + \$240 + \$3000 + \$1200 + \$3600 = \$52 240
c Subtract the total expenses from the annual net income and consider whether the budget is affordable.	**c** Net income − Total expenses = \$61 912 − \$52 240 = \$9672 Yes, Shai should be able to afford his budget.

Helpful hints

✔ A budget is only an estimate of future income and expenses, which is unlikely to be 100% accurate.
✔ '37 cents for each $1 over $135 000' means $\frac{37}{100}$ or 0.37 of the income over $135 000.

Learning pathways

△ 3, 4(a–c), 5, 6(a–c), 7, 9, 10, 11, 15, 17	□ 1, 2, 4(d–f), 6(d–f), 8, 10(d–f), 11, 13(a–c), 16, 18, 19	⬠ 1, 4(d–f), 7, 9, 11, 12, 13(d–f), 14, 16, 17, 20, 21

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Exercise 9.3A: Understanding and fluency

WE 9.3A **1** In the last financial year, Lisa:
- earned a salary of $57 500
- received a $1500 bonus from work
- earned $88 in interest on her bank account
- had $550 of work-related expenses
- donated $100 to charity
- paid an accountant $80 to manage her tax affairs.

Calculate Lisa's taxable income.

2 In the last financial year, Kosta's income from his two jobs was $55 000 and $34 800 respectively. Kosta also earned $395 in interest on his savings account and made a monthly donation of $30 to his favourite charity. Calculate Kosta's taxable income.

3 In each of the following situations, calculate the taxable income.
a income $4981.95; work-related expenses $56.80; donations $9.45
b 36 hours of work at $25.70 per hour; additional income $187.50; work-related expenses $38.90
c annual salary $91 200; work-related expenses $61.25 per month; donations $27 per month

WE 9.3B **4** Use the 2024–2025 financial year tax rates table to calculate the income tax payable on the following incomes.
a $59 000 **b** $111 000 **c** $80 300
d $16 000 **e** $231 500 **f** $146 700

5 Last year, Julie had a salary of $72 000 and received a 5% bonus at work. She earned $1045 in interest on her investments and had tax deductions that totalled $2420.
a Calculate Julie's taxable income.
b How much income tax does Julie need to pay?

6 Calculate the Medicare levy payable on the following incomes.
a $82 000 **b** $46 000
c $104 000 **d** $66 500
e $133 000 **f** $59 200

7 In the 2024–2025 financial year, Mika earned $122 400 and had tax deductions of $4200.
a What is Mika's taxable income?
b How much income tax must Mika pay?
c How much does Mika have to pay for the Medicare levy?
d What is Mika's net income?

8 Cody has a taxable income of $91 500 after deductions. What is Cody's net income after deducting income tax and the Medicare levy?

9 Sheldon comes up with a list of expected expenses for the year, as shown below. What are his expected total annual expenses?

- Rent: $350/week
- Food: $150/week
- Entertainment: $200/month
- Gifts: $300/year
- Clothes: $600/year
- Mobile phone: $45/month
- Transport: $240/month
- Other items: $100/month

WE 9.3C 10 Andrea has just started a new job and wants to create a budget to keep track of her finances. She will earn $3302.50 per fortnight, and her weekly expenses are $420 on rent, $140 on food, $80 on entertainment and $100 on personal items. Each month she also spends $75 on her mobile phone, $150 on clothes and $100 on transport. She also pays approximately $280 per month for utilities like electricity and water. She can claim $1500 in self-education expenses and $350 in work-related expenses as tax deductions. She must also pay the Medicare levy.

a Determine Andrea's annual net income for the 2024–2025 financial year, rounded to the nearest dollar.

b Prepare an annual budget for Andrea based on her estimated expenses. How much should she expect to spend each year?

c Is Andrea's annual net income enough to cover her budget?

Exercise 9.3B: Problem solving and reasoning

11 Last year, Portia had a salary of $86 520 and each month her employer withheld $1499 in tax for her. Portia earned an additional $872 in interest on her investments, had work-related expenses of $610 and donated $480 to charity.

a Calculate Portia's income tax for the year.

b Is Portia due a tax refund, or does she owe more money? By how much?

12 The tax brackets are named after the amount of cents paid for each dollar ($) over the given amount in that bracket. Explain why someone whose income falls into the 30% tax bracket does not pay 30% of their income in income tax.

13 Calculate the percentage, correct to one decimal place, of these taxable incomes that needs to be paid in income tax.

a $94 000 **b** $48 500 **c** $70 500

d $114 000 **e** $37 040 **f** $142 100

14 Explain how the fixed values in the 2024–2025 tax table ($4288, $31 288 and $51 638) are calculated.

15 Gil earns $6420 per month. He has no tax deductions and must pay the Medicare levy. He estimates the following expenses:

- Rent: $550/week
- Mobile phone: $60/month
- Food: $265/week
- Car: $500/month
- Entertainment: $350/month
- Health insurance: $110/month
- Clothes: $200/month
- Other items: $100/week.

a Determine Gil's annual net income for the 2024–2025 financial year. Round to the nearest dollar at each step in your calculations.

b Show that Gil cannot afford his estimated expenses.

16 The ATO produces weekly, fortnightly and monthly tax tables, which detail the amount of tax that should be withheld given an exact amount of earnings for that specified period. Use the ATO website to look up the weekly, fortnightly and monthly tax tables for regular payments and state the amount of income tax that should be withheld for the following earnings. State the amount with the tax-free threshold.

a $774 per week **b** $1950 per week **c** $3122 per fortnight

d $2150 per fortnight **e** $4797 per month **f** $7644 per month

17 This is a section from the ATO fortnightly tax table for the 2024–2025 financial year.

Fortnightly earnings 1 $	Amount to be withheld: With tax-free threshold 2 $	No tax-free threshold 3 $
2,462.00	436.00	658.00
2,464.00	436.00	658.00
2,466.00	436.00	658.00
2,468.00	436.00	658.00
2,470.00	438.00	660.00

Fortnightly earnings 1 $	Amount to be withheld: With tax-free threshold 2 $	No tax-free threshold 3 $
2,612.00	484.00	706.00
2,614.00	484.00	706.00
2,616.00	484.00	706.00
2,618.00	486.00	706.00
2,620.00	486.00	708.00

a In the 2024–2025 financial year, Ben earned $36.30 per hour and worked a 36-hour week. What was his fortnightly income?

b How much did Ben's employer withhold in tax for him each fortnight? Assume Ben is claiming the tax-free threshold.

18 Complete the information in this table.

Income	Deductions	Taxable income	Income tax	Medicare levy	Net income
$83 100	$300	$82 800	$15 628	$1656	
$55 400	$550	$54 850		$1097	
$102 310	$1100				
$68 440	$810				

Exercise 9.3C: Challenge

19 An alternative tax system is proposed, where all individuals pay 25% of their taxable income in income tax.

a If someone pays exactly the same income tax under the 2024–2025 system and this alternative system, what is their taxable income to the nearest dollar?

b Who would be worse off and who would be better off under this proposed tax system?

c Calculate the difference in income tax payable between the 2024–2025 system and this alternative system for people with the following taxable incomes.

i $38 600 **ii** $82 100 **iii** $163 000

d Use your answers to parts **a–c** to comment on the merits of using a progressive income tax system.

20 The 2023–2024 tax rates table is given below.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$45 000	19 cents for each $1 over $18 200
$45 001–$120 000	$5092 plus 32.5 cents for each $1 over $45 000
$120 001–$180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

a Calculate the amount of tax payable on the following incomes in the 2023–2024 financial year.

i $35 000 **ii** $75 000 **iii** $142 000 **iv** $210 000

b Are there any incomes, other than below $18 201, for which you would pay the same amount of income tax for both the 2023–2024 and 2024–2025 financial years?

21 Determine the taxable income of someone with a net income of the following amounts. Assume each person must pay the 2% Medicare levy. Write your answers to the nearest dollar.

a $95 000 **b** $125 000 **c** $210 000

Online resources:

Interactive skillsheet — Income tax

Worksheet — Calculating income tax

Quick quiz — 9.3

ANS
p532

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

9.1 **1** Complete the following table.

Hourly wage	Time worked	Amount earned
$32.20	12 hours	
$23.90		$478.00
$40.05	15 hours	
$36.25		$1051.25
	33 hours	$1402.50

9.1 **2** For each of these hourly wages, calculate:

i the time-and-a-half hourly wage **ii** the double time rate.

a $23 **b** $29 **c** $38 **d** $26.50 **e** $34.20 **f** $44.40

9.1 **3** Calculate the annual salary for the following given values.

a $5740 per month **b** $4510 per month **c** $1205 per week

d $2070 per week **e** $1840 per fortnight **f** $3430 per fortnight

9.1 **4** Grant works 30 hours per week and is paid $26.60 per hour. He takes one week of annual leave, for which he receives an additional leave loading of 17.5%. How much does Grant get paid for his week of annual leave?

9.2 **5** Calculate the piece work rate in each of these situations.

	Items produced	Income
a	59	$442.50
b	112	$403.20
c	46	$791.20
d	2450	$294.00
e	440	$171.60

9.2 **6** Last year, Angel received a payment of $3380 on a 5.2% royalty. What were the sales for that year?

9.3 **7** In the last financial year, Isaac earned a salary of $74 000 plus a 3.5% bonus. Isaac also earned $247 in interest on his bank account. He had $350 of work-related expenses, donated $240 to charity and paid an accountant $100 to manage his tax affairs. Calculate Isaac's taxable income.

9.3 **8** This is the income tax table for the 2024–2025 financial year.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$45 000	16 cents for each $1 over $18 200
$45 001–$135 000	$4288 plus 30 cents for each $1 over $45 000
$135 001–$190 000	$31 288 plus 37 cents for each $1 over $135 000
$190 001 and over	$51 638 plus 45 cents for each $1 over $190 000

Use the tax table to calculate the income tax payable on the following incomes.

a $75 300 **b** $94 200 **c** $47 000 **d** $53 990 **e** $115 040 **f** $66 090

9.3 **9** Calculate the Medicare levy payable on the following incomes.

a $50 500 **b** $63 420 **c** $76 100 **d** $82 090 **e** $104 820 **f** $137 950

9.3 **10** In the 2024–2025 financial year, Dina earns $81 300 and has tax deductions of $1800.

a What is Dina's taxable income? **b** How much income tax must Dina pay?

c How much does Dina have to pay for the Medicare levy? **d** What is Dina's net income?

Lesson 9.4

Simple interest

Learning intentions

By the end of this lesson you will be able to ...

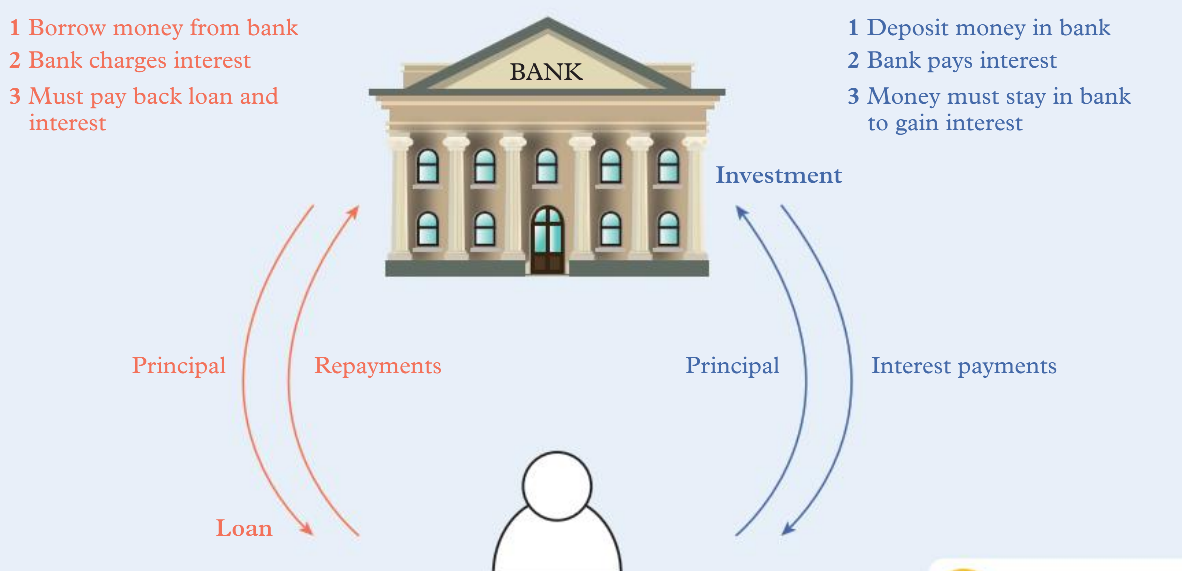
✔ calculate simple interest from loans and investments.

Inter-year links

Year 7	4.8 Calculating percentages
Year 8	5.2 Substitution
Year 10	12.2 Simple interest

Loans and investments

- A **loan** is an agreement to borrow money and pay it back later with interest. If you take out a loan from a bank, the total of your repayments will be more than the amount borrowed. Banks charge interest for providing access to their money.
- An **investment** is when money is deposited and interest is earned on the deposited money. If you invest money with a bank, the bank pays you interest for giving them access to your money.



Simple interest

Key content video
Simple interest

- The amount of interest you pay on a loan (or earn on an investment) depends on the original amount you borrow (or invest), the interest rate charged per time period and the number of time periods.
- **Simple interest** can be calculated using the formula:
  → I = interest
  → P = **principal**, the original amount of money borrowed or invested
  → r = **interest rate** per time period, usually a percentage converted to a fraction or decimal
  For example, 5% would be substituted as $\frac{5}{100}$ or 0.05. The interest rate 5% p.a. means 5% per annum or 5% per year.
  → n = number of time periods that form the duration of the loan or investment

$$I = Prn$$

simple interest
principal
interest rate
number of time periods

- The total you repay (or earn) includes the original amount plus the interest.
  Total amount = $P + I$

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Worked example 9.4A Calculating interest rates

Write an 8% p.a. interest rate as:

a a fraction in simplest form

b a decimal.

THINK	WRITE
a To convert a percentage to a fraction, the percentage becomes the numerator and the denominator is 100. Simplify the fraction if possible.	**a** $8\% \text{ p.a.} = \frac{\cancel{8}^{2}}{\cancel{100}_{25}}$ $= \frac{2}{25}$ per year
b To convert a percentage to a decimal, divide the percentage by 100.	**b** $8\% \text{ p.a.} = 8 \div 100$ $= 0.08$ per year

Worked example 9.4B Calculating simple interest

For an investment of $5200 at a simple interest rate of 6% p.a. for 4 years, calculate:

a the amount of simple interest earned

b the value of the investment after 4 years.

THINK	WRITE
a **1** Write the simple interest formula and identify the key terms: principal, interest rate and number of time periods. The rate must be written as a fraction or a decimal.	**a** $I = Prn$ $P = \$5200$ $r = 6\% = 0.06$ per year $n = 4$ years
2 Substitute the values into the formula and calculate the result.	$I = \$5200 \times 0.06 \times 4$ $= \$1248$
3 Write the answer.	The simple interest earned in 4 years is $1248.
b **1** The value of the investment after 4 years is calculated by adding the interest to the principal.	**b** Total amount $= P + I$ $= \$5200 + \$1248$ $= \$6448$
2 Write your final answer.	The value of the investment after 4 years is $6448.

Helpful hints

- ✔ Be careful when converting your interest rate to a decimal or fraction.
- ✔ When using cash, round your answers to the nearest 5 cents. For all other transactions, round to the nearest 1 cent.
- ✔ If you are finding the simple interest formula difficult, write it out in words.

  Simple interest = principal × interest rate × number of time periods
- ✔ Remember that the principal is the initial amount invested or borrowed.
- ✔ Make sure that the interest rate (r) and number of time periods (n) are in the same unit.
  - → If the interest rate is *per annum*, then the number of time periods must also be in *years*.
  - → If the interest rate is *per month*, then the number of time periods must be in *months*.

Learning pathways

△ 1–5, 7, 8(a–f), 9, 10, 11, 13, 15

□ 1–3, 4(b, d, f), 6, 7, 8(d–i), 10(a, c, e, g, i), 12, 13, 16, 18, 19(a)

⬠ 2, 3, 7, 8(g–l), 10(f–i), 13, 14, 16–20

ANS p532

Exercise 9.4A: Understanding and fluency

You can use your calculator for all questions in this exercise.

WE 9.4A **1** Write these interest rates as:

i a fraction in simplest form **ii** a decimal.

a 7% p.a. **b** 11% p.a. **c** 24% p.a. **d** 6% p.a. **e** 10% p.a. **f** 12% p.a.

WE 9.4B **2** For each investment, calculate:

i the amount of simple interest **ii** the value of the investment after the time period listed.

a a simple interest investment of \$5000 at an interest rate of 5% p.a. for 2 years

b a simple interest investment of \$4800 at an interest rate of 4% p.a. for 3 years

c a simple interest investment of \$12 500 at an interest rate of 8% p.a. for 5 years

3 For each loan, calculate:

i the amount of simple interest **ii** the total amount to be repaid.

a a loan of \$7500 at a simple interest rate of 5% p.a. over 3 years

b a loan of \$10 800 at a simple interest rate of 12% p.a. over 5 years

c a loan of \$25 000 at a simple interest rate of 7% p.a. over 8 years

4 Calculate the simple interest given each of these. All interest rates are per annum.

a $P = \$4000$, $r = 6\%$, $n = 5$ years **b** $P = \$8650$, $r = 7\%$, $n = 4$ years

c $P = \$15\,000$, $r = 8\%$, $n = 10$ years **d** $P = \$9200$, $r = 4\%$, $n = 3$ years

e $P = \$19\,999$, $r = 15\%$, $n = 6$ years **f** $P = \$20\,000$, $r = 20\%$, $n = 5$ years

5 Christian invests \$3500 in a bank that offers the simple interest rate of 4.8% per annum. He plans to leave the money invested for 2 years.

a Identify the values of P, r and n.

b How much simple interest does Christian earn?

c What is the total value of Christian's investment after 2 years?

6 Jenna plans to start her business in massage therapy and needs to borrow \$44 000 to assist with her set-up costs. She obtains an agreement with her lender to repay the money over 5 years with simple interest charged at 9.5% p.a.

a Identify the values of P, r and n.

b How much simple interest is Jenna charged?

c What is the total amount that Jenna repays?

7 The simple interest formula can be used to calculate interest for both loans and investments.

a Calculate the amount of simple interest in each of the following situations.

i \$5000 is invested at a simple interest rate of 0.4% per month for 30 months

ii \$5000 is borrowed at a simple interest rate of 0.4% per month for 30 months.

b Compare each of the answers in part **a**. Briefly explain how the simple interest formula is used for investment and loan situations.

c Given that the simple interest calculations involving loans and investments are identical, how are the calculations different when they are interpreted?

8 Convert each time period to years. Where appropriate, write the fraction in simplest form.

a 11 months **b** 7 weeks **c** 26 weeks

d 3 months **e** 271 days **f** 155 days

g 15 months **h** 48 months **i** 84 weeks

j 1241 days **k** 30 months **l** 286 weeks

9 A simple interest investment is made for 4 years and 3 months. Matthew thinks this is equivalent to 4.3 years but Lizzy is certain that Matthew is wrong. How is 4 years and 3 months written as a decimal in years?

10 For the investment values given in the table below, calculate:

i the amount of simple interest

ii the total amount at the end of the term.

	Principal $	Simple interest rate p.a.	Time
a	9000	6%	3 years
b	10 500	4.5%	6 months
c	7500	9.8%	130 weeks
d	29 000	3.2%	90 days
e	8600	6.4%	35 days
f	155 570	12.5%	11 months
g	19 999	19.9%	25 weeks
h	45 950	14.05%	2 years and 5 months
i	208 654	8.75%	5 weeks and 4 days

Exercise 9.4B: Problem solving and reasoning

11 Sade is investigating which is the best way to calculate her simple interest for a short-term investment. She invests $2400 for the month of June at a simple interest rate of 4.6% p.a.

a Calculate the simple interest amount after writing the time as a fraction of the total number of months in the year.

b Now calculate the simple interest amount after writing the number of days in June as a fraction of the total number of days in the year.

c Which method of calculation would Sade hope would be used? Briefly explain why.

d If the values given represented a short-term loan instead of an investment, which method of calculation would Sade prefer? Briefly explain why.

12 A bank is offering the simple interest rates shown in the table below for its customers to invest in a term deposit. The interest is calculated at the end of the investment. Jasmine has $20 000 to invest and plans to invest it for 12 months.

a What simple interest rate will Jasmine receive for her investment?

b How much interest does she earn?

c Jasmine's brother informed her that she would have earned more interest if she invested the money for one day less than 12 months. Investigate whether this statement is true and show working to support your finding.

Term (months)	Interest on investment amount (% p.a.)		
	$5000–<$10 000	$10 000–<$50 000	$50 000–<$100 000
1–2	2.5	2.5	2.8
3–6	3.25	3.25	3.25
7–11	5.5	5.55	5.5
12–24	5.3	5.25	5.2

13 Simple interest assumes the same amount of interest is added every period (year, month, day, etc.). Therefore, if we know the interest amount per period, we can multiply that by the number of periods to determine the total interest using the unitary method.

For example: $P = \$4000$, $r = 4\%$ p.a., $n = 5$ years, $I = \$?$
4% p.a. $\times \$4000 = \$160$ per 1 year
So for 5 years:
$\$160 \times 5$ per 1×5 years $= \$800$ per 5 years

For each of the following, calculate:

i the amount of interest per period

ii the total amount of interest

iii the total amount owing for a loan or in an account for an investment.

a $P = \$200$, $r = 5\%$ p.a., $n = 10$ years, $I = \$?$

b $P = \$1400$, $r = 6.2\%$ p.a., $n = 8$ years, $I = \$?$

c $P = \$500\,000$, $r = 8\%$ p.a., $n = 12.5$ years, $I = \$?$

d $P = \$750$, $r = 8\%$ p.a., $n = 20$ months, $I = \$?$

e $P = \$26\,000$, $r = 5\%$ p.a., $n = 72$ weeks, $I = \$?$

f $P = \$4380$, $r = 3.5\%$ p.a., $n = 153$ days, $I = \$?$

14 The total amount owing for a loan or in an account for an investment can be determined by adding the interest to the principal. Alternatively, we can find the percentage increase for the given time period, then apply the percentage increase to the principal.

For example: $P = \$4000$, $r = 4\%$ p.a., $n = 5$ years, $I = \$?$
4% per 1 year
So for 5 years:
$4\% \times 5$ per 1×5 years $= 20\%$ per 5 years
$(100\% + 20\%) \times \$4000 = \$4800$

For each part in question **13**:

i find the multiplier to determine the total amount using a percentage increase. Give your answers correct to four decimal places where appropriate

ii recalculate the total amount using the percentage increase in part **i**.

15 Banks vary in the ways in which they calculate simple interest on savings and transaction accounts. Some accounts earn no interest while others attract bonus interest rates if certain conditions are met. If an account provides interest, it is most likely to be calculated on the daily account balance. Consider the account balances for the month of February shown.

a The opening balance of \$640.90 applies for the first 7 days of the month and each new balance applies from the date the transaction is made. How many days does each balance on this account apply for?

Date	Transaction	Credits \$	Debits \$	Balance \$
01/02	Opening balance			640.90
08/02	Withdrawal at Handybank		−100.00	540.90
15/02	Deposit	+240.00		780.90
24/02	EFTPOS purchase		−125.40	655.50
28/02	Interest			

b The account attracts simple interest at a rate of 2.1% p.a. For each new balance in the account, calculate the simple interest based on the number of days each balance applies.

c Add all the amounts from part **b** to calculate the total interest for the month.

d What is the account balance at the end of February, if the total interest is added at the end of each month?

16 This bank statement shows the transactions made during the month of August. Interest is calculated daily at a rate of 1.8% p.a.

Date	Transaction	Credits \$	Debits \$	Balance \$
01/08	Opening balance			345.50
09/08	ATM Withdrawal		−50.00	295.50
14/08	Deposit – Pay	+370.00		665.50
16/08	ATM Withdrawal		−120.00	545.50
19/08	EFTPOS Purchase		−85.95	459.55
28/08	Deposit – Pay	+370.00		829.55
31/08	Interest			

a How much simple interest is earned during the month?

b What is the final account balance?

17 A bank offers a simple interest rate of 1.5% p.a. on its savings accounts, calculated daily. An extra 3.2% p.a. bonus rate is offered if no more than one withdrawal is made in the month and the account balance has increased by at least $200 for the month. Consider each of the account statements shown.

A

Date	Transaction	Credits $	Debits $	Balance $
01/09	Opening balance			1200.85
15/09	Deposit – Pay	+450.75		
24/09	Deposit – at branch	+820.00		
29/09	EFTPOS Purchase		−245.85	
30/09	Interest			

B

Date	Transaction	Credits $	Debits $	Balance $
01/01	Opening balance			1548.90
08/01	EFTPOS Purchase		−246.20	
15/01	Deposit – Pay	+1920.00		
29/01	EFTPOS Purchase		−85.94	
31/01	Interest			

a Will either of these accounts receive the bonus simple interest rate? Provide a reason to support your answer.

b Calculate the total interest earned on each account. You will need to determine the account balances following each transaction first.

c State the final account balance for each statement at the end of the month.

18 Joel plans to buy a second-hand car for $12 500. He has saved $2500 and plans to borrow the remaining money from his bank at a simple interest rate of 8.5% p.a. for 3 years.

a The car-seller asks for a deposit of 15% of the selling price. Are Joel's savings enough to cover the deposit? Note: A deposit is the first part of a payment often used as a promise to pay.

b How much does Joel borrow to buy the car?

c Calculate the total amount, including simple interest, that Joel pays for the car.

Exercise 9.4C: Challenge

19 Suppose you have $2000 and wish to double this amount over 3 years. You explore some different options to earn the most amount of simple interest possible.

a What is the annual simple interest rate that will enable your investment to double in 3 years?

b Explore how this rate changes if the time of the investment increases to:

i 4 years **ii** 5 years **iii** 6 years.

c Explore how this rate changes if the time of the investment decreases to:

i 2 years **ii** 1 year.

d Describe the effect of increasing and decreasing the number of years of the investment on the annual simple interest rate.

20 Provide three different annual interest rates and their corresponding time periods that would result in an investment of $5000 earning $1250 in simple interest.

Online resources:

Interactive skillsheet Calculating simple interest

Worksheet Calculating simple interest

Quick quiz 9.4

Lesson 9.5

Simple interest calculations

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate unknown values using the simple interest formula.

Inter-year links

Year 7	6.6 Solving equations using inverse operations
Year 8	6.2 Solving linear equations
Year 10	12.2 Simple interest

Calculating the principal, interest rate and number of time periods

Key content video
Simple interest calculations

- The simple interest formula is $I = Prn$.

$$I = Prn \quad \text{Solve for principal}$$

$$I = Prn \quad \text{Solve for interest rate}$$

$$I = Prn \quad \text{Solve for number of time periods}$$

- To solve the simple interest equation for P, r or n:
  1. Write the simple interest formula and identify the known variables.
  2. Substitute the values into the formula and simplify the calculation.
  3. Solve the equation for the unknown value using inverse operations.
  4. Write the answer.

Worked example 9.5A Calculating the number of time periods for an investment

How long will it take for an investment of \$4000 at an interest rate of 4% p.a. to earn \$800 in simple interest?

THINK	WRITE
1 Write the simple interest formula and identify the variables. Write r as a fraction or a decimal.	$I = Prn$ $P = \$4000$ $r = 4\% = 0.04$ p.a. $I = \$800$
2 Substitute the values into the formula and simplify the calculation.	$\$800 = \$4000 \times 0.04 \times n$ $\$800 = \$160 \times n$
3 Solve the equation for n.	$\dfrac{\$800}{\$160} = \dfrac{\$160 \times n}{\$160}$ $n = 5$
4 Write the answer and include the unit 'years' because r is 'per annum'.	It will take 5 years for an investment of \$4000 to earn \$800 in simple interest.

Worked example 9.5B Calculating the principal value

How much needs to be invested at an interest rate of 6% p.a. for 3 years to earn \$1440 in simple interest?

THINK	WRITE
1 Write the simple interest formula and identify the variables. r should be written as a fraction or a decimal.	$I = Prn$ $r = 6\% = 0.06$ p.a. $n = 3$ years $I = \$1440$
2 Substitute the values into the formula and simplify the calculation.	$\$1440 = P \times 0.06 \times 3$ $\$1440 = P \times 0.18$
3 Solve the equation for P.	$\frac{\$1440}{0.18} = \frac{P \times 0.18}{0.18}$ $P = \$8000$
4 Write the answer.	\$8000 needs to be invested to earn \$1440 in simple interest over 3 years.

Worked example 9.5C Calculating the interest rate

At what rate is \$6000 invested at over 6 years if \$864 is earned in simple interest?

THINK	WRITE
1 Write the simple interest formula and identify the variables. r should be written as a fraction or a decimal.	$I = Prn$ $P = \$6000$ $n = 6$ years $I = \$864$
2 Substitute the values into the formula and simplify the calculation.	$\$864 = \$6000 \times r \times 6$ $\$864 = \$36\,000 \times r$ $\frac{\$864}{\$36\,000} = \frac{\$36\,000 \times r}{\$36\,000}$
3 Solve the equation for r using inverse operations.	$r = 0.024$
4 Convert r to a percentage by multiplying the decimal by 100.	$= 2.4\%$
5 Write the answer and include the unit p.a. because n is in years.	\$6000 is invested at 2.4% p.a. to earn \$864 in simple interest over 6 years.

Helpful hints

- ✔ Remember the order of operations when solving equations.
- ✔ Remember to round your answers to the nearest cent.
- ✔ To find the solution, the pronumeral does not have to appear on the left-hand side of the equation – if the pronumeral is by itself on either side of the equation, you have found the solution!

Learning pathways

△ 1–7, 9–12	□ 1(d–f), 2–4, 7, 8, 10, 12–14	⬠ 4, 7(b, d, f), 8, 11–15

ANS p533

Exercise 9.5A: Understanding and fluency

You can use your calculator for all questions in this exercise.

1 Calculate the simple interest in each case. All interest rates are given per annum.

- **a** $P = \$2000$, $r = 7\%$, $n = 3$ years
- **b** $P = \$250$, $r = 11\%$, $n = 1$ year
- **c** $P = \$8500$, $r = 5\%$, $n = 4$ years
- **d** $P = \$25\,000$, $r = 4\%$, $n = 5$ years
- **e** $P = \$100\,000$, $r = 9.5\%$, $n = 4$ years
- **f** $P = \$16\,000$, $r = 6\%$, $n = 2.5$ years

WE 9.5A **2** Find the value for n in each of these situations.

- **a** How long will it take for an investment of \$8000 at a simple interest rate of 3% p.a. to earn \$1200 in simple interest?
- **b** How long will it take for an investment of \$1250 at a simple interest rate of 4% p.a. to earn \$350 in simple interest?
- **c** How long does an investment of \$15 000 at a simple interest rate of 9% p.a. take to earn \$5400 in simple interest?
- **d** How long will it take for an investment of \$5600 at a simple interest rate of 5% p.a. to earn \$1120 in simple interest?

WE 9.5B **3** Find the value for P in each of these situations.

- **a** How much needs to be invested at a simple interest rate of 8% p.a. for 5 years to earn \$2000 in simple interest?
- **b** How much is borrowed at a simple interest rate of 10% p.a. over 4 years if \$6000 is paid in simple interest?
- **c** How much is borrowed at a simple interest rate of 9% p.a. over 5 years if \$1800 is paid in simple interest?
- **d** How much needs to be invested at a simple interest rate of 6% p.a. for 2 years to earn \$576 in simple interest?

4 Find the unknown value in each of these situations. All interest rates are given per annum.

- **a** $I = \$600$, $P = \$3000$, $r = 4\%$, $n = ?$
- **b** $I = \$1200$, $P = ?$, $r = 5\%$, $n = 4$ years
- **c** $I = \$450$, $P = ?$, $r = 9\%$, $n = 2$ years
- **d** $I = \$850$, $P = \$8500$, $r = 5\%$, $n = ?$
- **e** $I = \$1000$, $P = ?$, $r = 8\%$, $n = 4$ years
- **f** $I = \$5060$, $P = \$9200$, $r = 11\%$, $n = ?$

5 Jessica has invested \$4500 in a bank that offers simple interest of 5.0% p.a. She plans to earn \$675 in interest.

- **a** From the simple interest formula, which variable do you not know the value of?
- **b** What variables do each of the given values represent?
- **c** How long does the money need to be invested to earn \$675 in simple interest?
- **d** At a higher interest rate of 7.5% p.a., how much sooner can Jessica earn \$675 in simple interest?

6 Throughout the course of a simple interest investment, Stefan's money increased in value from \$8400 to \$8946. The interest was earned at a simple interest rate of 0.25% per month.

- **a** What is the total amount of interest earned on this investment?
- **b** How many months was the initial amount of money invested for?

WE 9.5C 7 For the values given in the table, calculate the simple interest rate per annum that applies.

	Simple interest $	Principal $	Time
a	1640	8200	4 years
b	420	3500	2 years
c	985	9850	48 months
d	1680	12 000	30 months
e	3936	18 000	3 years and 5 months
f	680	6400	4 years and 3 months

8 Use the simple interest formula to determine the values for the missing amounts in the table.

	Simple interest $	Principal $	Rate % p.a.	Time
a		3700	5.6	4.5 years
b	234		4.8	13 months
c	42 532	70 000		6.2 years
d	3549	19 500	5.2	
e	1711		14.5	48 months
f	2631.60	15 480		130 weeks
g	56.88	948		1.2 years
h	1534.40	13 700	6.4	

Exercise 9.5B: Problem solving and reasoning

9 Priyansha borrowed a sum of money from her parents to help her buy her first laptop. They agreed to charge interest at a rate of 4% p.a. over a period of 3 years. The total interest charge for the term of the loan is $144.

a From the simple interest formula, which variable is unknown?

b Which variable does each of the given values represent?

c How much money does Priyansha borrow from her parents?

d Priyansha plans to pay her parents $35 each month for 3 years and believes this will cover the agreed terms of their loan. Determine whether Priyansha's plans are correct and show working to support your finding.

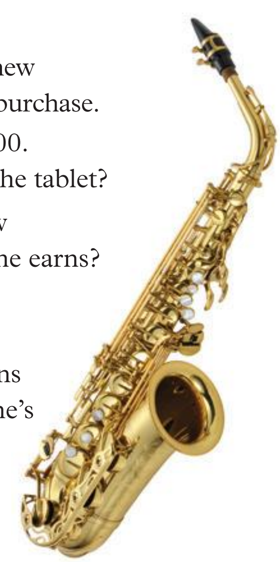
e What are the exact monthly payments Priyansha needs to make to repay her loan?

10 The cost of the latest tablet is $873. Although Gabriella has the savings to purchase the new tablet, she would rather let the interest earned from her investment cover the cost of the purchase.

a One bank offers her a simple interest rate of 7.2% p.a. for her investment of $10 000. How long does this money need to be invested to earn enough interest to pay for the tablet?

b Gabriella decides on 12 months to reach her goal. At the same rate of interest, how much does she need to invest in order to fully pay for the tablet with the interest she earns?

11 Daniel has decided to learn the alto saxophone through his school music program. To encourage his development, his parents bought the saxophone shown through a purchase program arranged by his school valued at $1200. The repayment conditions involve quarterly payments over 3 years. The simple interest charged on the saxophone's cost is $162.



a What is the quarterly interest rate charged?

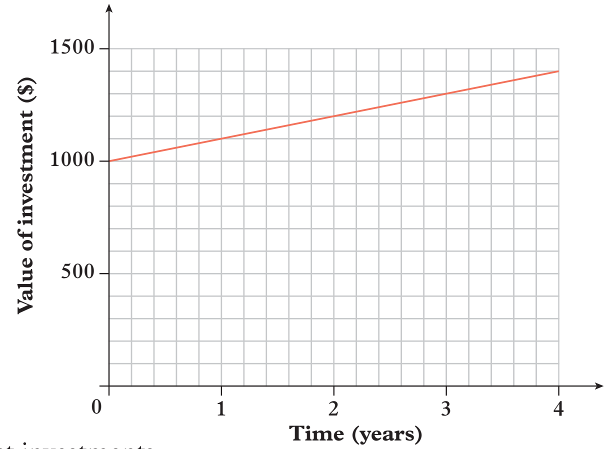
b What is the amount of each quarterly payment required?

12 The graph on the right details the value of a simple interest investment.

a What is the value of the principal?

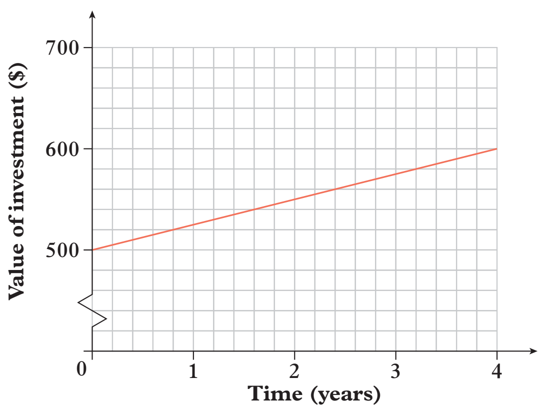
b What is the value of the investment after 4 years?

c What is the interest rate of the investment?

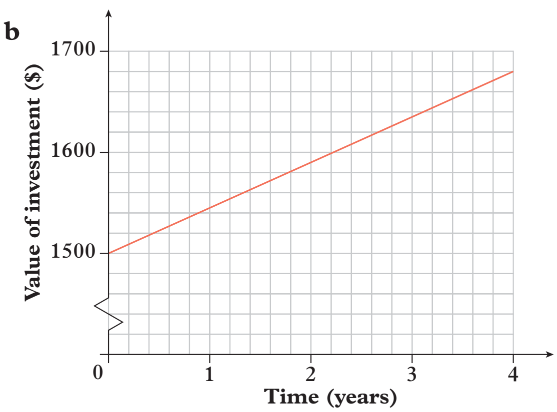


13 Determine the interest rate for each of these simple interest investments.

a



b



14 Consider the following situations.

a After 92 weeks, Alex is charged $63.25 on a simple interest loan with an annual interest rate of 6.5%. How much interest will he be charged after 100 weeks?

b Charlotte borrows $1000 at a simple interest rate of 3.65% p.a. and is charged $500 interest after a number of days. After how many days will Charlotte be charged $1000 interest?

c Helen has a simple interest investment of 20 cents. After 1000 months, she earns $1.05 in interest. Correct to the nearest year, how long will it take Helen to earn $1000 in interest?

Exercise 9.5C: Challenge

15 This bank statement is linked to a savings account and shows the transactions made during the month of April.

Date	Transaction	Amount $	Balance $
01/04	Opening balance		2905.60
03/04	Deposit – Pay	1230.75	4136.35
08/04	ATM Withdrawal	250.00	3886.35
15/04	EFTPOS Purchase	499.95	3386.40
17/04	Deposit – Pay	1230.75	4617.15
	Monthly interest	7.54	4624.69

What is the annual interest rate (% p.a.) that applies to this account? Remember that each new balance applies from the day of the transaction. Round your answer to one decimal place.

Online resources:

Interactive skillsheet Calculating simple interest rate

Interactive skillsheet Calculating the number of time periods for an investment

Interactive skillsheet Calculating the principal by simple interest formula

Investigation What percentage interest are you really paying?

Quick quiz 9.5

Lesson 9.6

Review: Financial mathematics

Module summary

Wages and salary

- Income: money received in exchange for work or investment
- Wage: a fixed rate of income for working a given period, e.g., $25/hour
- Salary: fixed income paid on a regular basis

$$\frac{\$78\,000}{12} = \$6500 \text{ per month}$$

$$\frac{\$78\,000}{52} = \$1500 \text{ per week}$$

Penalty rates and leave loading

- Time-and-a-half: 1.5 × standard wage
- Double time: 2 × standard wage
- Leave loading: 17.5% additional pay for annual leave

Commissions, royalties and piece work

- Commission: being paid a proportion of sales
- Royalties: payments made for the use of assets
  $$\begin{aligned} 2\% \text{ of } \$44\,000 &= 0.02 \times \$44\,000 \\ &= \$880 \end{aligned}$$
- Piece work: income paid for each item produced or action performed

Income tax

- Total income: wages, salaries, bonuses, royalties and interest earned
- Tax deductions: work-related expenses, gifts and donations and the cost of managing tax affairs
- Taxable income = Total income – Tax deductions
- PAYG tax is withheld to ensure employees meet their tax obligations
- Discrepancies are resolved by completing a tax return

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$45 000	16 cents for each $1 over $18 200
$45 001–$135 000	$4288 plus 30 cents for each $1 over $45 000
$135 001–$190 000	$31 288 plus 37 cents for each $1 over $135 000
$190 001 and over	$51 638 plus 45 cents for each $1 over $190 000

Medicare levy and net earnings

- 2% tax on taxable income to fund the health care system
- Net income = Taxable income – Income tax – Medicare levy

Simple interest

$$I = Prn$$

simple interest

principal

interest rate

number of time periods

- I = interest
- P = principal, the original amount of money borrowed or invested
- r = interest rate, usually a percentage converted to a fraction or decimal
- n = number of time periods

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Review questions 9.6A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- bonus
- budget
- commission
- double time
- expenses
- income
- income tax
- interest
- interest rate
- investment
- leave loading
- loan
- Medicare levy
- net income
- PAYG tax
- penalty rate
- piece work
- principal
- royalty
- salary
- simple interest
- tax deductions
- taxable income
- time-and-a-half
- wage

1 What percentage tax (on taxable income) is the Medicare levy?

2 How can you calculate a fortnightly salary if given an annual salary?

3 What does the acronym PAYG stand for?

4 Identify the key terms being referenced in each of these definitions.

a when money is borrowed and interest is paid on the amount borrowed

b a fixed rate of income an employee receives for working a given period

5 Using an example, provide a definition in your own words for the following key terms.

a leave loading

b simple interest

6 Complete the following sentences using words from the key terms list.

a ________ is income which is paid for each item produced or action performed.

b ________ are expenses that can be subtracted from a person's total income to determine their ________.

Review questions 9.6B: Multiple choice

9.1 **1** Emma worked for 30 hours and earned \$855. What is her hourly wage?

A \$28/hour **B** \$28.50/hour **C** \$30/hour **D** \$825/hour

9.1 **2** Mo receives a standard rate of \$30.20 per hour. He receives a penalty rate of one-and-a-quarter for any overtime (above 25 hours per week). How much does Mo earn in a week in which he works 32 hours?

A \$264.25 **B** \$755 **C** \$966.40 **D** \$1019.25

9.2 **3** Vita receives a base income of \$350/week plus a 3.5% commission on sales. How much does Vita earn in a week in which her sales total \$25 000?

A \$875 **B** \$1100 **C** \$1145 **D** \$1225

9.2 **4** Arthur receives a piece work rate for every haircut he performs. If Arthur cuts 17 people's hair in one day, he will earn \$399.50. What is his piece work rate of pay?

A \$23 **B** \$23.50 **C** \$24 **D** \$24.50

9.3 **5** Last year, Saheed had a salary of \$8420 per month and received a 4% bonus at work. He earned \$2304 in interest on his investments, spent \$1105 on work-related expenses and paid his accountant \$100 to manage his tax affairs. What was Saheed's taxable income?

A \$106 181 **B** \$101 573 **C** \$107 386 **D** \$105 082

9.3 **6** Jasmine comes up with the following list of expected expenses for the year. What are her expected total annual expenses?

- Rent: \$475/week
- Utility bills: \$450/quarter
- Food: \$160/week
- Home insurance: \$50/year
- Health insurance: \$108/month
- Entertainment: \$100/month
- Gifts: \$250/year
- Clothes: \$500/year
- Mobile phone: \$55/month
- Transport: \$200/month
- Other items: \$100/month

A \$2448
B \$29 376
C \$40 689
D \$42 376

9.4 **7** \$12 000 is invested at 4.2% p.a. simple interest for 18 months. Which values should be substituted into the simple interest formula?

A $P = 12\,000$, $r = 4.2$, $n = 18$
B $P = 12\,000$, $r = 0.42$, $n = 1.5$
C $P = 12\,000$, $r = 0.042$, $n = 1.5$
D $P = 12\,000$, $r = 0.042$, $n = 18$

9.4 **8** Ginger invests \$6400 in a bank that offers a simple interest rate of 7.2% per annum. What is the value of her investment after 3 years?

A \$921.60
B \$8243.20
C \$7782.40
D \$7321.60

9.5 **9** A loan of \$4500 with simple interest 8.5% p.a. is charged \$1530 in interest so that \$6030 is now owed. Which simple interest variable do you not know the value of?

A number of time periods
B principal
C interest rate
D interest amount

Review questions 9.6C: Short answer

9.1 **1** Complete the following table.

Hourly wage	Time worked	Amount earned
\$26.40	24 hours	
\$33.20	28.5 hours	
	30 hours	\$1131
\$41.80		\$877.80
\$23.90		\$872.35

9.1 **2** Katie works 36 hours per week and is paid \$32.50 per hour. She takes two weeks of annual leave, for which she receives an additional leave loading of 17.5%. How much does Katie earn for her two weeks of annual leave?

9.2 **3** Calculate the amount authors will receive in royalties, given the following rates and sales.

	Royalty rate	Sales
a	5.5%	\$45 000
b	3.2%	\$98 000
c	1.8%	\$30 000
d	9%	\$143 000
e	13%	\$57 200

9.3 **4** This is the income tax rates table for the 2024–2025 financial year.

Taxable income	Tax paid on income
0–\$18 200	Nil
\$18 201–\$45 000	16 cents for each \$1 over \$18 200
\$45 001–\$135 000	\$4288 plus 30 cents for each \$1 over \$45 000
\$135 001–\$190 000	\$31 288 plus 37 cents for each \$1 over \$135 000
\$190 001 and over	\$51 638 plus 45 cents for each \$1 over \$190 000

Use this table to calculate the income tax payable on the following incomes.

a \$113 000 **b** \$81 000 **c** \$59 500 **d** \$77 000 **e** \$35 000 **f** \$199 900

9.3 **5** Morgan has a taxable income of \$58 100 after deductions. Use the tax table in question **4** to calculate Morgan's net income after deducting income tax and the Medicare levy.

9.3 **6** Ellie has just started a new job. She will earn \$5275.50 each month. She has no tax deductions and must pay the Medicare levy. Her estimated expenses are:

- \$390/week on rent
- \$170/week on food
- \$100/week on entertainment
- \$100/week on personal items
- \$35/month on mobile phone
- \$200/month on clothes
- \$240/month on transport
- \$420/quarter on utilities.

a Calculate Ellie's annual net income.

b Prepare an annual budget for Ellie based on her estimated expenses. How much should she expect to spend each year?

c Is Ellie's annual net income enough to cover her budget?

9.4 **7** Complete the following interest rate conversions.

a Convert the given simple interest rates into a rate per annum.

i 0.1% per week **ii** 0.55% per month **iii** 1.8% per quarter

b Convert each given simple interest rate into the rate for the time period specified in brackets.

i 4.8% p.a. (per month) **ii** 6.24% p.a. (per week) **iii** 3.8% p.a. (per quarter)

9.4 **8** Calculate the simple interest in each case.

a $P = \$3000$, $r = 5\%$ p.a., $n = 4$ years **b** $P = \$6400$, $r = 2.5\%$ p.a., $n = 3$ years

c $P = \$35\,000$, $r = 4.8\%$ p.a., $n = 5$ months

9.5 **9** Find the unknown value P, r or n when:

a $I = \$240$, $P = \$2000$, $r = 4\%$ p.a. **b** $I = \$854.40$, $r = 8.9\%$ p.a., $n = 2$ years

c $I = \$1400$, $P = \$16\,000$, $r = 3.5\%$ p.a. **d** $I = \$630$, $P = \$3500$, $n = 2$ years

e $I = \$1011.50$, $P = \$8500$, $n = 3.5$ years

Review questions 9.6D: Mathematical modelling

1 Julia manages a bookstore and earns an annual salary of \$80 000. The normal hourly rate of \$24.50 applies to her casual staff, although the opportunity for overtime is available. The store's rent is \$1800 per week and Julia allows an extra \$200 per week to cover other costs.

a Julia's annual tax deductions total \$1240. What is Julia's monthly net income after subtracting her income tax and the Medicare levy? Use the income tax rates table for the 2024–2025 financial year.

b One week, Julia has three staff working. Simon works 24 hours at the normal hourly rate. Melanie works 15 hours at the normal rate, 3 hours at time-and-a-half and 5 hours at double time. Tahlia works 30 hours at the normal rate and 5 hours at time-and-a-half. Calculate the weekly income for each employee.

c What amount of money, to the nearest dollar, must Julia's store make that week to cover the cost of staff pay and store costs?

d The owners receive a quote for $48 000 to re-fit the store. They have half of this amount in savings and plan to borrow the remaining amount.

   i The bank lends the money at a simple interest rate of 8.2% p.a. over 3 years. What is the total amount of money that must be repaid?

   ii If the money is repaid in equal monthly instalments, what is the amount per month?

   iii In total, how much will the store makeover cost?

2 Jiwoo plans to put $4000 into a savings account for five years. His local bank offers him two account options.

Option 1: Simple interest at 4% p.a.

Option 2: Simple interest at 3.5% p.a. for the first two years and then 4.5% p.a. for the remaining 3 years.

a How much money will be in Jiwoo's account in 5 years if he chooses:

   i Option 1 ii Option 2?

b Which option should Jiwoo choose?

c The bank gives Jiwoo Option 3, which allows him to invest at a rate of 4% p.a. but to have the amount of interest earned be based on the amount of money in the account at the start of the year, instead of the original principal.

   i Complete the following table using this method of calculating the interest. Write all values correct to the nearest cent.

Year	Value at start of year	Interest	Value at end of year
1	$4000.00	$4000 × 0.04 = $160.00	$4160.00
2	$4160.00	$4160 × 0.04 = $166.40	$4326.40
3	$4326.40		
4			
5			

   ii Should Jiwoo choose Option 3 or his choice from part **b**?

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Understand the difference between a wage and a salary ☐ Calculate income for working a given period of time ☐ Apply penalty rates and leave loading	☐ Go back to Lesson 9.1 Wages and salaries
☐ Calculate income including a commission ☐ Calculate piece work income and royalty payments	☐ Go back to Lesson 9.2 Other forms of income
☐ Calculate an individual's taxable income ☐ Calculate income tax ☐ Calculate net income	☐ Go back to Lesson 9.3 Taxation
☐ Calculate interest using the simple interest formula ☐ Calculate the simple interest on investments and loans	☐ Go back to Lesson 9.4 Simple interest
☐ Solve for the number of time periods for a loan or investment using the simple interest formula ☐ Solve for the principal using the simple interest formula ☐ Solve for the interest rate using the simple interest formula	☐ Go back to Lesson 9.5 Simple interest calculations

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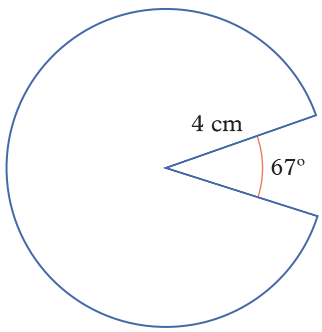
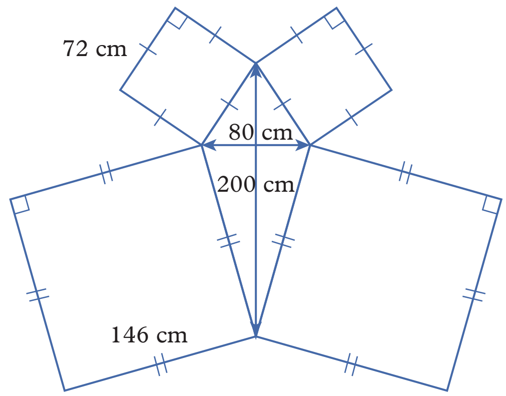
Semester 2 review

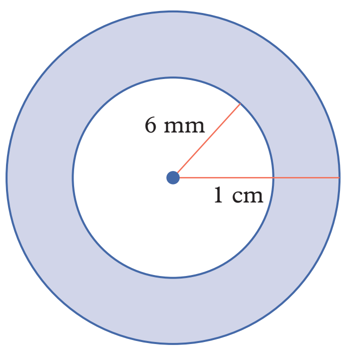
Short answer

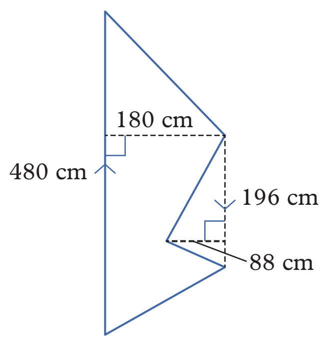
1 Ben works 32 hours per week and is paid $24 per hour. He takes two weeks of annual leave for which he receives an additional leave loading of 17.5%. How much does Ben get paid for his two weeks of annual leave?

2 Calculate the areas of these composite shapes, correct to two decimal places.

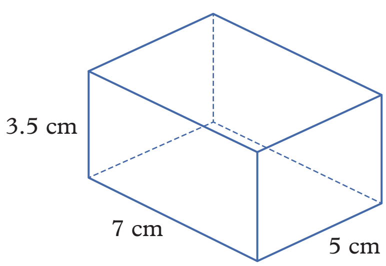
a


b


c


d


3 Calculate the surface area of the following objects, correct to one decimal place where necessary.

a


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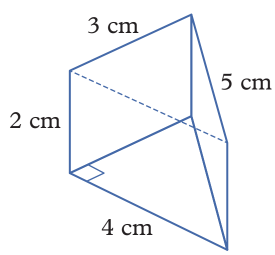

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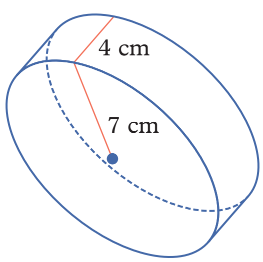
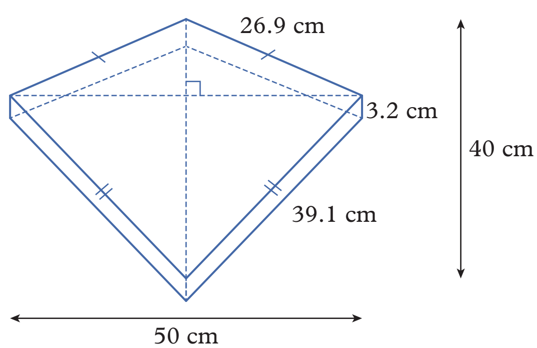


d


4 Calculate the volume of the objects in question **3**, correct to one decimal place where necessary.

5 Find the absolute value of the following:

a $|1 - 3|$ **b** $|2.4 - 1|$ **c** $|-1 - 2.4|$

6 Find the measurements, given the following lower and upper bounds:

a 24.5 mL and 25.5 mL **b** 54.95 g and 55.05 g **c** 4.55 m and 4.65 m

7 A scale drawing is drawn to a scale of 1 : 200. What is the size of each of these rooms given the size in the scale drawing?

a Bathroom (1.6 cm by 1.3 cm)

b Bedroom (2.4 cm by 1.8 cm)

c Kitchen (2.8 cm by 1.6 cm)

8 The exchange rate between Australian dollars (AUD) and United States dollars (USD) is 0.75. How many US dollars are equivalent to the following amounts of Australian dollars?

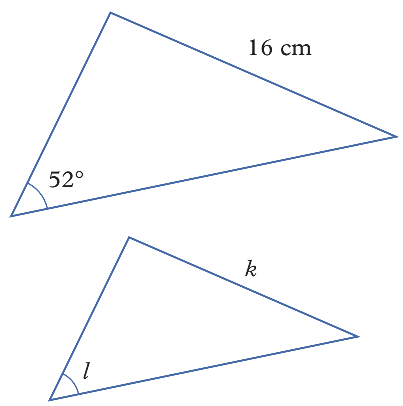
a \$8000 AUD

b \$420 AUD

c \$2250 AUD

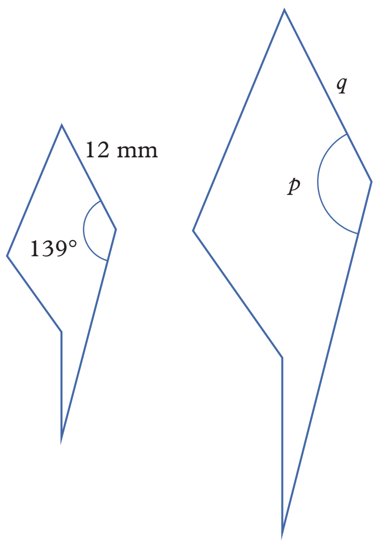
9 The shapes with labelled measurements have been dilated by the given scale factors to create the other shapes. Determine the value of the unknown pronumerals.

a



Scale factor = $\frac{3}{4}$

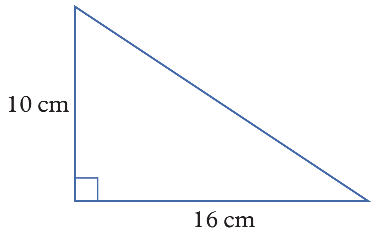
b



Scale factor = 2.5

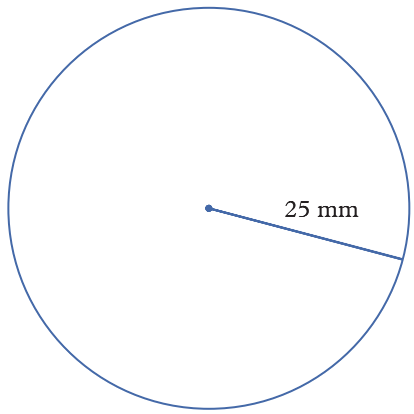
10 For each shape, state the area of the image if the original figure is dilated by the given length scale factor. Give your answers correct to two decimal places where necessary.

a



Length scale factor = 2

b

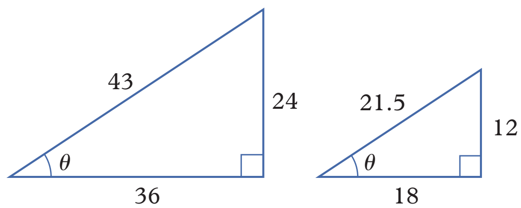


Length scale factor = $\frac{1}{4}$

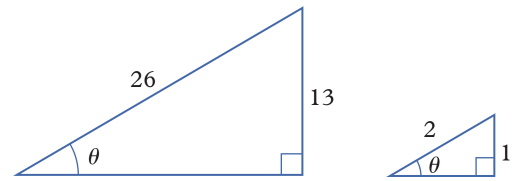
11 For each of the following pairs of similar triangles state the following values. Where necessary, round to four decimal places.

i the scale factor (original on left) **ii** $\sin(\theta)$ **iii** $\cos(\theta)$ **iv** $\tan(\theta)$

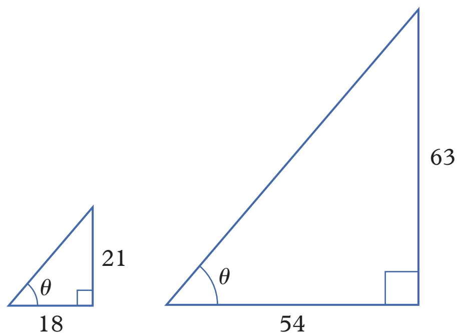
a



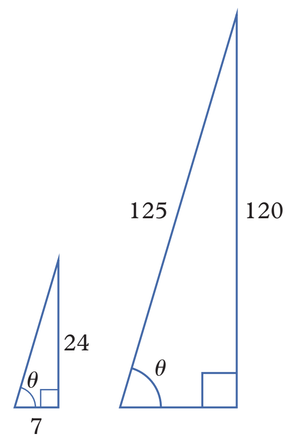
b



c

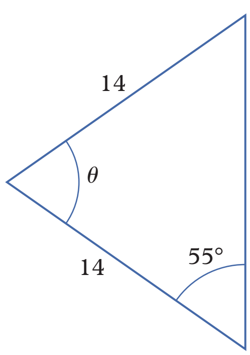


d

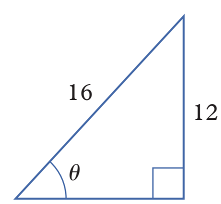


12 Determine the value of θ in each of the following diagrams, correct to the nearest degree.

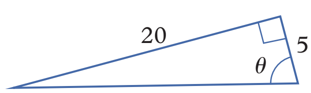
a



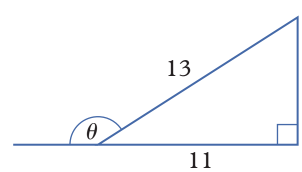
b



c

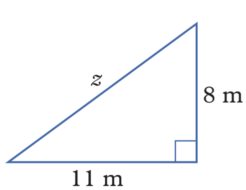


d

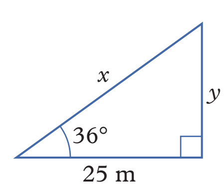


13 Determine the length of the unknown side lengths, correct to three significant figures.

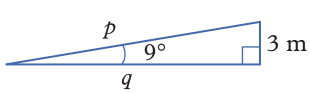
a



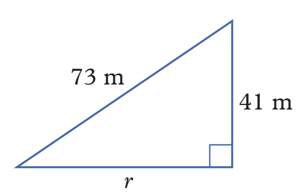
b



c



d



14 Determine whether these side lengths can form right-angled triangles.

a 7.7 cm, 41.7 cm, 42.7 cm

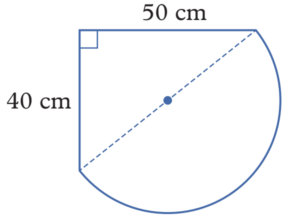
b 29 mm, 128 mm, 131 mm

c 20 cm, 48 cm, 52 cm

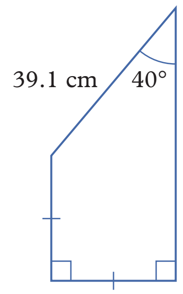
d 4 m, 4.2 m, 5.8 m

15 Calculate the area of these shapes, correct to one decimal place.

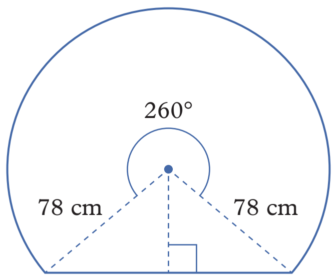
a



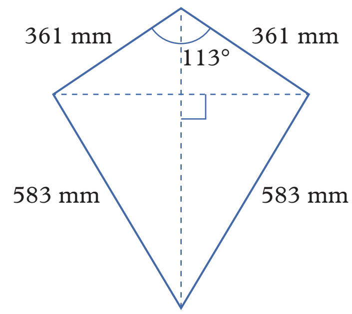
b



c



d



16 Simple interest is interest that is calculated on the principal (the original amount invested or borrowed).

a $4500 is invested at a simple interest rate of 8% p.a. How much interest is earned after nine years?

b $150 is borrowed and $35 interest is charged after two years. At what annual simple interest rate was the $150 borrowed, correct to two decimal places?

c $5 interest is earned on an investment after two weeks with a simple interest rate of 6% p.a. How much was invested, correct to the nearest cent?

d $100 000 is borrowed at a simple interest rate of 0.6% per month. After how many months is $1200 of interest charged?

17 Classify each of the following variables as:

i categorical or numerical

ii ordinal, nominal, discrete or continuous.

a Name of train line

b Distance to Southern Cross Station

c Number of passengers on the train

18 Draw an appropriate frequency table to represent each of these numerical data sets.

a

24.9,	31.1,	26.8,	19.2,	19.1,	29.8,	26,
25.5,	16,	23.7,	18.1,	25.8,	23,	18.3,
7.7,	15.3,	26.7,	27.6,	6.6,	23.8	

b

45,	94,	94,	75,	113,	75,	98,
87,	73,	51,	4,	90,	94,	48,
99,	62,	63,	64,	32,	101,	81,
119,	85,	98,	103,	92,	90,	86,
84,	56,	99,	81,	86,	82,	75,
38,	89,	89,	71,	46		

19 Construct the specified graph for each of the following data sets.

a Bar chart

Favourite game	Frequency
Board games	10
Card games	45
Video games	50
Physical games	25

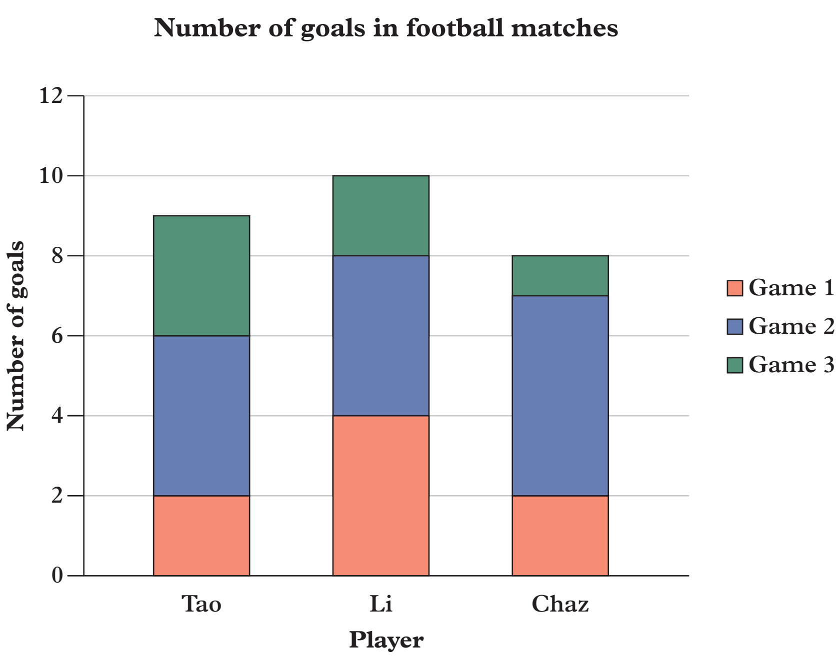
b Line graph

Day	1	2	3	4	5	6	7	8	9
Number	12	20	16	18	24	22	24	24	26

c Histogram

Class	Frequency
0–<10	30
10–<20	60
20–<30	10
30–<40	90
40–<50	30

20 Consider the stacked bar chart below.



a Who scored the most goals in total?

b Who scored the most goals in the second game?

c In which game did Tao score more goals than Li?

21 Determine the following statistics for each of the data sets. Round to two decimal places if required.

i mean **ii** median **iii** mode **iv** range

a 21, 19, 19, 19, 17, 16, 18, 26, 24, 19

b

Score (x)	10	11	12	13	14
Frequency (f)	6	12	18	20	14

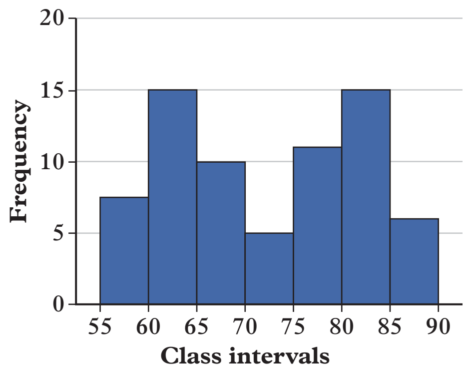
c

Stem	Leaf
2	0 3 4 6 7
3	1 2 3 3 4 4 4 5 7
4	1 2 5
5	1 3 3 6 8
6	0 1 1 2 4 6 7

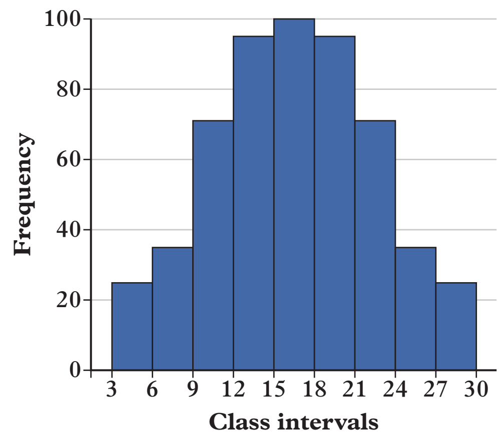
Key 2|1 = 2.1

22 Describe the distribution of the data in each of the following data displays.

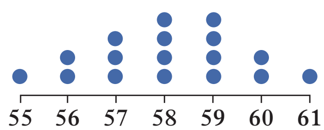
a



b



c



d

23 State which measures of centre are most appropriate to use to describe each of the distributions in question **22**.

24 Brea plans to play video games on two different consoles this afternoon. She has an Xbox, a PlayStation and a Nintendo Switch to choose from.

a Construct a tree diagram to show all outcomes in the sample space.

b Assuming each outcome is equally likely, calculate the probability that Brea plays:

i Xbox then PlayStation

ii Xbox second

iii Xbox either first or second

iv Xbox and PlayStation in either order.

25 A fair six-sided die is rolled and the result is noted. Following that a card is drawn from a standard 52-card deck and the suit (clubs, diamonds, hearts, spades) is noted.

a Construct an array to show all outcomes in the sample space.

b Calculate the probability of:

i rolling a 6 and drawing a spade

ii rolling an even number and drawing a red card (heart or diamond)

iii rolling an even number or drawing a red card or both

iv rolling an even number or drawing a red card but not both.

26 A bag contains 21 coloured buttons: 4 red, 8 blue and 9 yellow. Two buttons are selected at random from the bag. Calculate the probability that:

a a yellow and a blue button are selected, if the first button is replaced

b a yellow and a blue button are selected, if the first button is not replaced

c two red buttons are selected, if the first button is not replaced

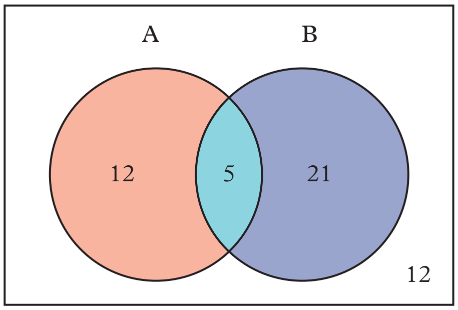
d two of the same-coloured buttons are selected, if the first button is not replaced.

27 Two-way tables and Venn diagrams can both be used to display the same sets of data.

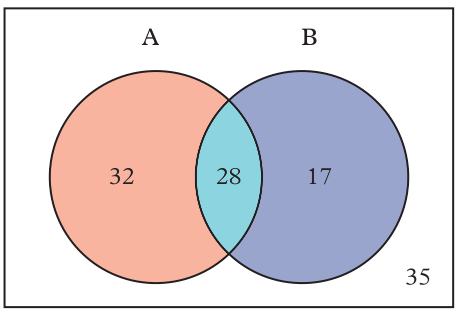
a Display the information in this two-way table in a Venn diagram.

	A	**A'**	**Total**
B	11	57	68
B'	47	25	72
Total	58	82	140

b Display the information in this Venn diagram in a two-way table.



28 Consider this Venn diagram.



a Given that $n(\text{A})$ means the number of elements in set A, find:

i $n(\text{A})$ **ii** $n(\text{A} \cup \text{B})$ **iii** $n(\text{A}')$

iv $n(\text{A} \cap \text{B})$ **v** $n(\xi)$ **vi** $n(\text{A}' \cap \text{B})$

b Given that Pr(A) means the probability of selecting an element from set A, find:

i $\Pr(\text{A})$ **ii** $\Pr(\text{A}')$ **iii** $\Pr(\text{A} \cap \text{B})$

iv $\Pr((\text{A} \cup \text{B})')$ **v** $\Pr(\text{A}' \cap \text{B}')$ **vi** $\Pr(\text{A} \cup \text{B}')$

29 A two-way relative frequency table displays the relative frequency of each outcome.

a Convert this two-way table to a relative frequency two-way table.

	A	**A'**	**Total**
B	75	5	80
B'	35	10	45
Total	110	15	125

b Convert this relative frequency two-way table to a two-way table given that the sample size is 120.

	A	**A'**	**Total**
B	$\frac{17}{40}$	$\frac{29}{120}$	$\frac{2}{3}$
B'	$\frac{1}{8}$	$\frac{5}{24}$	$\frac{1}{3}$
Total	$\frac{11}{20}$	$\frac{9}{20}$	1

30 A large box contains fluorescent and LED light globes from two companies: Phillips and Osram. A sample of 90 light globes is taken and 46 are Phillips light globes, 39 are fluorescent and 21 are Osram LED light globes.

a Show this information on a Venn diagram.

b From the sample of 90 light globes, calculate the relative frequency of:

i Phillips fluorescent light globes

ii LED light globes

iii Osram light globes

iv fluorescent or not Phillips or both.

31 A school has 212 Year 9 students. Of the 98 students with a smartphone, 56 have a gaming console. 80 students do not have a gaming console.

a Show this information in a two-way table.

b Calculate the relative frequency of students that:

i do not have a smartphone

ii have a gaming console

iii have neither a smartphone nor a gaming console

iv have a smartphone or a gaming console but not both.

Mathematical modelling

1 Lisa works at an aged care centre and receives a wage of \$24 per hour. For hours worked outside of the core working hours (8 am to 6 pm), Lisa receives time-and-a-quarter pay.

Lisa's regular working week is shown in the table below:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Hours	8–4	8–4	12–8	12–8	8–2	–	–

a How much does Lisa earn in a regular working week?

b Lisa is provided with 20 days of annual leave each year, for which she receives leave loading of 17.5%. How much is Lisa paid for her four weeks of annual leave?

c Calculate Lisa's annual income from her job if she works for 48 weeks and takes four weeks of annual leave.

d Lisa is also a co-author of a book on health care, for which she receives a 4.5% royalty. Over the last financial year, the book recorded sales of \$65 320.

Calculate the amount of income Lisa receives from her book royalty.

e Lisa also started the year with \$12 600 of savings, which she put into a simple interest account which paid 7.4% per annum.

i How much interest did Lisa earn on her savings?

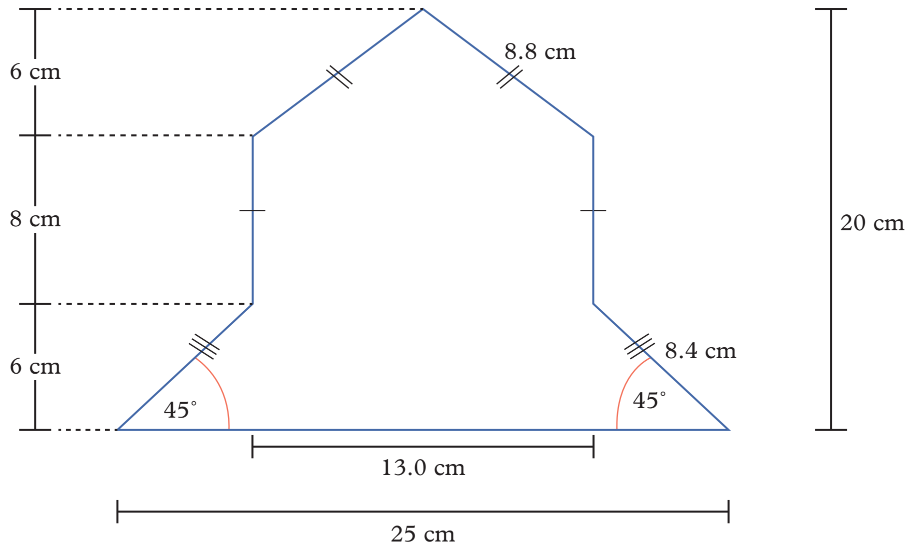
ii Given that Lisa had tax deductions which totalled \$484, what is Lisa's taxable income for the year?

iii Use the following tax rate table to determine Lisa's income tax for the year.

Taxable income	**Tax on this income**
0–\$18 200	Nil
\$18 201–\$45 000	16 cents for each \$1 over \$18 200
\$45 001–\$135 000	\$4288 plus 30 cents for each \$1 over \$45 000
\$135 001–\$190 000	\$31 288 plus 37 cents for each \$1 over \$135 000
\$190 001 and over	\$51 638 plus 45 cents for each \$1 over \$190 000

iv What is Lisa's net income for the year, after her income tax and the Medicare levy have been deducted from her taxable income?

2 Xav is planning to create a flat plywood spaceship to mount on his wall. He draws a rough diagram of the spaceship using a scale factor of 1 : 5. The desired lengths and angles in the spaceship are shown.



a Most of the measurements on the diagram are accurate, but Xav is unsure about the 8.4 cm and 13.0 cm lengths. Use trigonometry to check whether they are correct, and if they are not, provide new lengths correct to one decimal place.

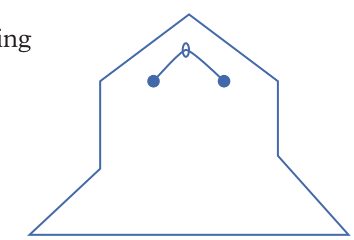
b Draw a labelled diagram of the spaceship showing the correct lengths and angles.

c Calculate the area of:

i the diagram of the spaceship

ii the actual plywood spaceship.

d To mount the plywood spaceship on the wall, Xav wants to attach an 80 cm string to two points on the back to hang on a hook. The hook will pull the string 20 cm up a vertical distance from where the string is connected, as shown.



i Determine the angle the string will make with the line between the two attachment points.

ii Determine the horizontal distance between the two points where Xav will attach the string, correct to one decimal place.

iii Determine whether the string is parallel to the edge of the plywood. Explain why or why not.

3 Arielle and Brenna have started to track their sleeping habits. They recorded how many minutes each slept in February, correct to the nearest 10 minutes.

Arielle:	550,	530,	480,	480,	470,	410,	480,
	510,	520,	490,	430,	480,	480,	370,
	460,	440,	490,	480,	500,	520,	470,
	470,	410,	480,	490,	500,	430,	470
Brenna:	570,	420,	560,	460,	530,	480,	510,
	410,	450,	510,	430,	540,	600,	570,
	600,	440,	530,	460,	630,	370,	580,
	500,	500,	550,	510,	400,	510,	460

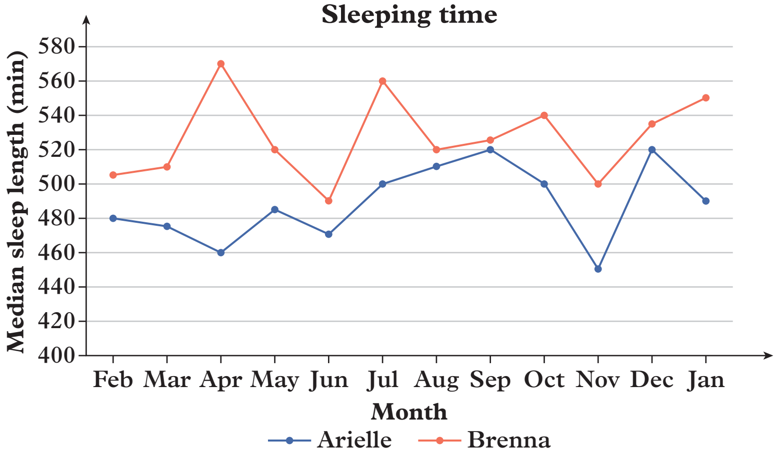
a Construct a back-to-back stem-and-leaf plot using the data above. Use intervals of 50 and the key 1 | 2 = 120 minutes.

b Compare the shape of the two data sets.

c Determine the median and range for Arielle's and Brenna's minutes of sleeping.

d Write two sentences comparing the two data sets with reference to their summary statistics from part **c**.

e The median values of Arielle's and Brenna's sleep times are calculated each month for a year and are plotted on the multiple line graph below.



i State the independent and dependent variables.

ii Compare the trends in Arielle's and Brenna's median sleep length over the year.

4 An avid puzzle collector has puzzles with 1000, 5000 and 10 000 pieces. Some of the puzzles have irregular-shaped designs, as shown in the table below.

	1000 pieces	5000 pieces	10 000 pieces
Number of puzzles in collection	20	9	1
Number with irregular-shaped designs	5	2	0

a Determine the probability of selecting a:

i 1000-piece puzzle out of all puzzles

ii puzzle with an irregular design out of all puzzles

iii 1000-piece puzzle with an irregular design out of all 1000-piece puzzles

iv 1000-piece puzzle with a regular design out of all puzzles with regular designs.

b The puzzle collector selects two puzzles at random with replacement.

Determine the probability of choosing:

i an irregular-shaped 1000-piece puzzle and then a regular-shaped puzzle that is not 1000 pieces

ii a 1000-piece puzzle and then an irregular shaped design.

c The puzzle collector starts with a rectangular 1000-piece puzzle. The puzzle actually consists of 1026 pieces in a 38-piece by 27-piece grid.

A piece is chosen at random from the puzzle. Calculate the probability of selecting:

i an edge piece connecting to any corner piece

ii a centre piece connecting to any edge piece.

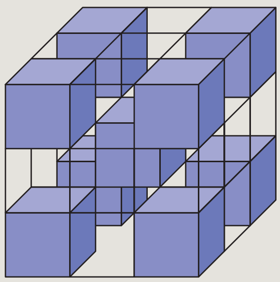
ANS p537

EXPLORATIONS 2

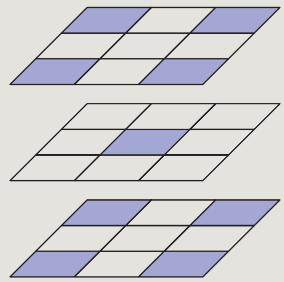
1 Cube stacks

Jali has a large supply of blue and clear cubes of identical size. She arranges 9 blue cubes and 18 clear cubes into a $3 \times 3 \times 3$ stack as shown below. The simpler diagram in the middle shows the position of the blue cubes in each 3×3 horizontal layer. Jali notices that this arrangement results in the same pattern, shown on the right, when she views the stack from all six directions: top, bottom, left, right, front and back. In fact, she realises that if two cubes are removed from the top and bottom layers, it is still possible to see the same final pattern from all directions.

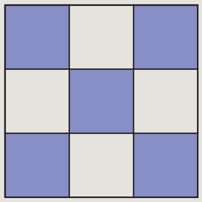
Jali wonders what other symmetrical patterns can be seen from all directions and how many blue cubes are needed to make them.



3D view



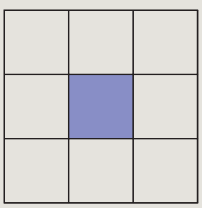
layer view



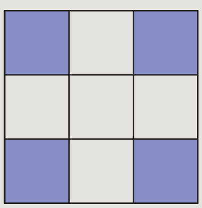
directional view

a For each of the following, determine both the maximum and minimum number of blue cubes that Jali can use in a $3 \times 3 \times 3$ stack so that the resulting pattern can be seen from all six directions. Draw a layer view for each.

i



ii



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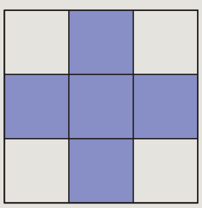

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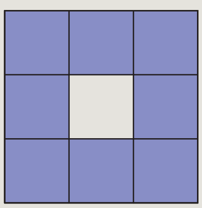
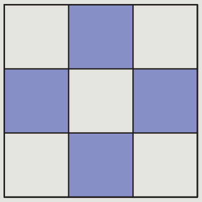


b Jali wants to make the symmetrical pattern shown on the right.



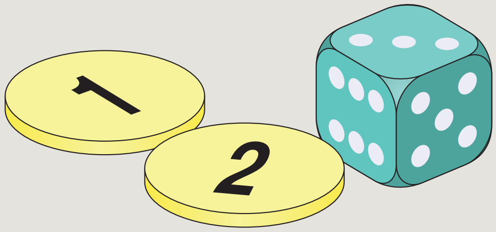
i Describe a stack in which this pattern can be seen from exactly two directions.

ii Describe a stack in which this pattern can be seen from exactly four directions.

iii Explain why it is not possible to see this pattern from all six directions.

2 Counter attack

Connie has a number of counters, each labelled 1 on one side and 2 on the other. She flips the counters and adds up the numbers showing on top. At the same time, Dieter selects a die and rolls it. The game ends in a draw if Connie and Dieter get the same score, otherwise the winner is the player with the higher score.



a Connie flips two counters and Dieter rolls a standard 6-sided die.

i Calculate the probability that both players get a score of 3.

ii Explain why Dieter is more likely to win than Connie.

b Connie flips three counters and Dieter rolls a 6-sided die. Is this version of the game fair? That is, do Connie and Dieter have the same chance of winning?

c Dieter selects an octahedral die numbered 1 to 8. How many counters must Connie flip for the game to be fair?

d Connie flips five counters. How many sides must there be on Dieter's die, assuming the sides are numbered 1, 2, 3 …, for the game to be fair?

e Generalise part **d** as follows: Connie flips n counters and Dieter spins a spinner with k wedges of equal size labelled 1, 2, 3 …, k. Find the relationship between n and k if the game is fair.

f Is there a fair version of the game if Dieter uses a dodecahedral die numbered 1 to 12? What about an icosahedral die numbered 1 to 20?

3 Differagons

A differagon is a polygon in which each side length is a different positive integer.

a For each of the following types of differagon, find the least possible perimeter:

i 3-sided
ii 3-sided with a right angle
iii 4-sided
iv 4-sided with a right angle
v 4-sided with two adjacent right angles
vi 4-sided with two opposite right angles
vii 5-sided with three consecutive right angles
viii 5-sided with three right angles, not all consecutive.

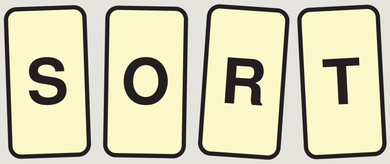
b Which types of differagon listed in part **a** can have an odd perimeter? Explain.

c Diago claims he has found a 6-sided differagon ABCDEF with the following properties:

- angles ABC, ACD, ADE and AFE are all right angles
- BC = 2, CD = 6, DE = 3 and EF = 7.

Show that Diago must have made a mistake.

4 All sorts



Imagine you have a row of cards in front of you, each with a single letter written on it. The goal is to sort them into alphabetical order. Here are three different approaches to this problem. As you work through them, see what shortcuts you can come up with to answer the questions as efficiently as possible.

a The SelectSort algorithm works as follows:

- Find the letter that appears earliest in the alphabet (in the case of repeats, choose the first occurrence).
- Swap it with the letter in the first position.
- Repeat the above steps with the remainder of the list.

For example, consider the cards S O R T in that order. The first step swaps O (the earliest letter) with S (the first letter), producing O S R T. Now repeat this procedure with the remaining list, S R T, swapping R and S. The final step applied to S T has no effect, since S is earliest and in the first position. So only two steps are required to arrive at the final order, O R S T.

For each of the following, find the number of steps that SelectSort will take to sort the cards into alphabetical order.

i O R D E R S **ii** S O R T E D **iii** A R R A N G I N G **iv** D I S O R G A N I S E D

b In the SwapThree algorithm, a move consists of reversing the order of three consecutive cards. For example, S O R T can be converted to R O S T by reversing the first three cards, or to S T R O by reversing the last three. For each of the following, decide whether it is possible to sort the cards into alphabetical order using SwapThree, and if so, determine the minimum number of moves required.

i O R D E R S **ii** S O R T E D **iii** A R R A N G I N G **iv** D I S O R G A N I S E D

c Each pass of the iSort algorithm works as follows:

- Compare the first two letters in the list. Whichever is earlier in the alphabet is moved to a new list. The other stays in the original list.
- Repeat the previous step with the remaining letters, moving one letter to the end of the new list at each stage.
- When there is one letter remaining, move it to the end of the new list.

For example, comparing the first two letters of S O R T, the letter O starts the new list and S R T remain in the original list. Comparing the first two letters again, R is moved to the new list, leaving S T. Finally, S and then T are moved to the new list, giving O R S T. This is already in alphabetical order, so the original list was sorted in one pass of the iSort algorithm. But in general, more passes may be needed.

For each of the following, find the number of passes of iSort needed to sort the cards into alphabetical order.

i O R D E R S **ii** S O R T E D **iii** A R R A N G I N G **iv** D I S O R G A N I S E D

Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

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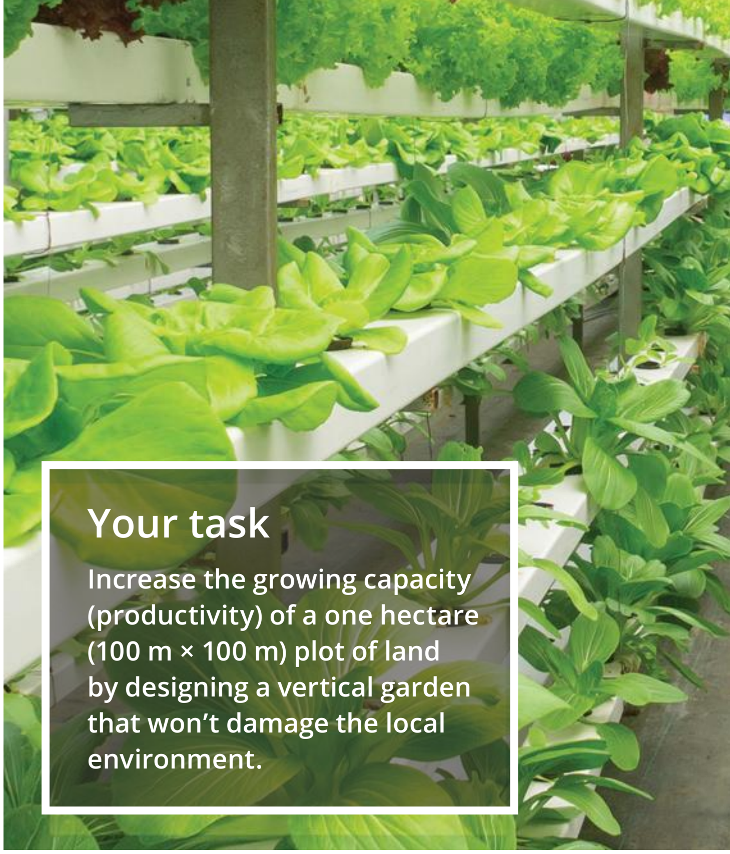
[STEAM project 1]

How can we use sustainable farming practices so that no one goes hungry in the future?



Your task

Increase the growing capacity (productivity) of a one hectare (100 m × 100 m) plot of land by designing a vertical garden that won't damage the local environment.

Figure 1 Vertical farming allows people to grow more food in a smaller space.

The United Nations ranks food shortages and hunger among the most serious issues affecting humankind. It predicts that more than 840 million people will be hungry by 2030. Even in a high-income country such as Australia, 5 per cent of the population are unable to access enough nutritious food. The experience of having inadequate access to food or having an inadequate supply of food is known as food insecurity. Food insecurity is linked to poor general health, higher rates of some cancers and higher mortality.

Rapid climate change is increasing threats to Australia's and the world's food security. Changes in the amount of rainfall, longer droughts and an increase in the number of extreme weather events are expected to disrupt the amount and quality of food that Australia can produce. A hotter climate is expected to cause stress in livestock animals, such as chickens, sheep and cattle, and to increase the amount of water needed for crop irrigation.

Figure 2 Drought impacts Australia's production of important crops, such as wheat.

Sustainable farming

Sustainable farming practices use methods that balance the needs of all members of the community. This means that new and old technologies are used to make sure that food production is:

- economically viable – if farmers cannot make enough money to survive, then the farming practice is not sustainable
- socially supportive – if the lifestyle of the farming community is not supported, then people will not want to live in the area
- ecologically sound – if the local environment is not supported, then the land will be unable to support food production.

Sustainable farming also works to maintain the diversity of the local wildlife.

Sustainable farming uses technology to increase the production of fresh, nutritious food while minimising the impact on the local environment.

HUMANITIES

In Geography this year, you will learn about food security around the world and food production in Australia. You will investigate the factors that influence crop yield (such as soil moisture) and how food production can alter a biome. In Economics and Business, you will study the agricultural resources (such as wheat) that form a large part of Australia's trade economy.

To complete this task successfully, you will need to investigate the environmental constraints on agricultural production in Australia, such as climate and distribution of water resources. You will also need to understand the extent to which agricultural innovations have overcome these constraints.

MATHS

In Maths this year, you will build on your knowledge of measurement and geometry to determine areas and volumes of more complicated shapes. You will study right-angled triangles using Pythagoras' theorem and trigonometry. You will also extend your skills in collecting, representing and investigating data.

To complete this task successfully, you will need to perform calculations involving angles, lengths and areas of two-dimensional and three-dimensional shapes. You will need to apply your understanding of scale factors to build a prototype of your designed product. To consider the situation at local, national and international scales, you will need skills in dealing with ratios and proportions. You may also find it helpful to use scientific notation for very large or very small numbers.

You will find help for applying these maths skills in Module 5 'Measurement', Module 6 'Scale and trigonometry', and Lesson 1.5 'Scientific notation' of *Oxford Maths 9 Victorian Curriculum (Second edition)*.

SCIENCE

In Science this year, you will learn about the carbon cycle and the ways human activity can disrupt it. You will also consider the consequences of disruption, including the enhanced greenhouse effect. You will also learn about asexual reproduction and investigate vegetative propagation.

To complete this task successfully, you will need to understand the factors required to keep a system, such as a vertical garden, alive. You may need to consider how these factors can be monitored and controlled automatically. You will also need to be familiar with the scientific method and understand how to conduct a fair test.

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, those who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems?

To answer these questions, you may need to investigate using different resources or conduct interviews or surveys.

Define

Before you start to design your vertical garden, you need to define the criteria that you will use to test the success of your vertical garden in achieving your goal.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide:

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1 What is the total area of the 100 m × 100 m plot of land? (Remember to use the correct units.)

2 If the plants are planted 25 cm apart in a 100 m row, and the rows are placed 1 m apart, how many plants could be planted in the plot of land?

Hint: Draw the plot of land to make sure you reach maximum capacity.

3 What criteria will you use to measure the success of your solution or design? How will you measure how much the end-users have been helped?

Ideate

Once you know who you're designing for, and you know what the criteria are, it's time to get creative!

As a group, brainstorm ways the problem can be solved. Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Once you have many possible solutions, it is time to sort them by possibility. Select three to five ideas that are possible. Research whether these ideas have already been produced by someone else. If they are already on the market, can you make a better version?

Build

Draw your top two vertical garden designs. Label each part of the designs. Include the materials that will be used for their construction.

Include in the designs:

a the total surface area available for plant growth

b a description of how food production will be increased

c a description of how the design (inputs and waste) will impact the local ecosystem

d a description of how the workers will access all areas of the design to tend the plants

e at least one advantage and disadvantage of each design.

Select one of the designs to take to the building and testing stage.

Build the prototype

You will need to build at least three versions of your vertical garden design prototype. The first prototype garden will be tested for effectiveness. The second prototype will be used to survey the group you are helping. The third prototype will be used for the presentation. The prototype may be full size, or it may be a scale model (10 cm represents 1 m). Use the following questions as a guideline for your prototype:

- What materials will you use?
- What material will you use to represent the plants?
- How will you represent the height, width and angle of the finished prototype?

Test

Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first vertical garden prototype. You will test the prototype more than once to compare results, so you will need to control your variables between tests. What criteria will you use to determine the success of your prototype? Conduct your tests and record your results.

Prototype 2

If your prototype will be used to help market gardeners, then you will need to generate a survey to test whether the prototype is appropriate for their use. (How would they use it? Would they consider buying it?)

If your prototype will be used to help another group, or native plants and animals, you will need to consider how you could test the impact it will have. (Will the prototype affect normal behaviours? How will the prototype affect the soil or waterways?)

Prototype 3

Use the information you have obtained from testing the first two versions to adapt your last prototype to be more effective and usable for the group you are helping. You may want to use the first two prototypes to demonstrate how the design has been improved.

Communicate

Present your vertical garden design to the class as though you are trying to get your peers to invest in it. Describe the criteria and testing used to measure the effectiveness of your vertical garden design.

In your presentation, you will need to:

- explain why we need to be more efficient with food production
- describe the key features of your design and how they improve or solve the problem of food shortages
- show a labelled, to-scale diagram of your prototype
- describe how the ecosystem will be affected by the installation of the prototype
- explain the relevant scientific principles that support your designed solution (e.g. water cycle, photosynthesis, nitrogen/carbon cycle)
- quantify the increase in food production that your design allows; present calculations to justify your claim
- present a calculation for the estimated cost of producing a full-size model of your design
- explain the implications of your design, at a state or national level, by comparing the benefits and costs.

Online resources:

Student booklet
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[STEAM project 2]

How can we harness technology so that we can live healthier lives?

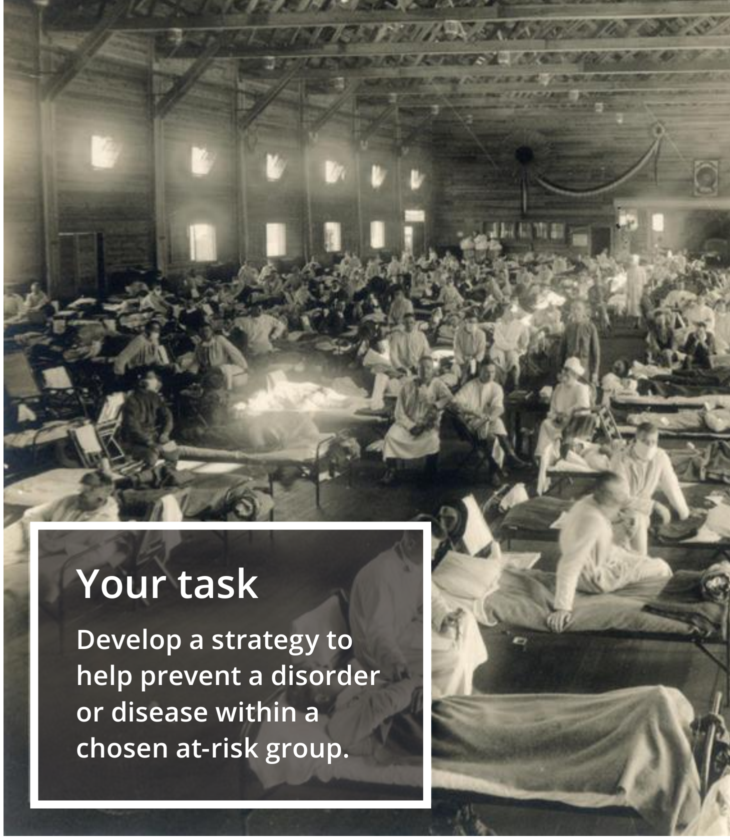

Figure 1 During the 1918 flu pandemic (sometimes called the Spanish flu), an estimated 500 million people, a third of the world's population, were infected with the virus.

A disorder or disease is a condition that affects the normal functioning of the body. Different disorders and diseases can affect many parts of the body. They can be caused by infectious agents such as bacteria or viruses that spread from person to person. Some disorders or diseases are inherited. Environmental factors (such as pollution or diet) can also have an impact on the development of disorders or diseases.

Heart disease, a non-infectious disease, is the leading cause of death globally. Mental health disorders, such as depression, bipolar disorder and dementia, also affect many people around the world.

Disorders and disease affect both high-income and low-income countries, but there are large differences in the ability of different healthcare systems to provide adequate care for people. The need for low-cost health care has led many researchers to investigate how technology can be used to help people live healthier lives.

Prevention of disorders and disease

There are many disorders and diseases that can be prevented through simple, low-cost interventions. Below are a few examples.

- Wearing a helmet or a seat belt has been shown to decrease the risk of brain injury from a road accident. In Vietnam, when wearing a helmet was made mandatory for motorcycle riders, it resulted in a 16 per cent decrease in head injuries.
- The use of mosquito nets can help to prevent malaria, a disease that can lead to life-long neurological impairment, such as epilepsy in children if they have a severe infection.

Figure 2 Healthcare workers wear personal protective equipment (PPE) to prevent the spread of infectious disease.

- Providing vaccinations for viruses such as polio and meningitis can also prevent neurological conditions.
- Promoting a healthy lifestyle and educating the population about the importance of diet can reduce the prevalence of stroke. In Japan, campaigns and treatment for high blood

pressure have reduced the rate of strokes by 70 per cent.

- Personal protective equipment (PPE) is used to protect people from catching infectious diseases, such as Covid-19.

HUMANITIES

In Economics and Business this year, you will learn about how health services are provided in Australia. In Geography, you will study how people are interconnected through travel, technology and trade. These connections affect where and how people access the services they need. In History, you will examine the experiences of different groups during the Industrial Revolution and the reforms made to improve living standards.

To complete this task successfully, you will need to research the demographics of your local area and the location and accessibility of health services. You should also consider the economic performance of your area to determine what type of preventative strategy would be most successful for your at-risk group.

MATHS

In Maths this year, you will extend your skills in representing and interpreting data. You will consider media reports that use statistics and collect secondary data to investigate social issues. You will relate real-world data to probabilities of events, and compare data sets using summary statistics and different graphical displays. You will evaluate and represent data, both with and without digital technology.

To complete this task successfully, you will need to find data to quantify the problem, work out how much your strategy will cost and calculate a quantitative, evidence-based estimate of the possible benefits of your strategy. You will need skills in dealing with ratios, proportions and percentages to consider the situation at local, national and international scales.

You will find help for applying these maths skills in Module 7 'Statistics' of *Oxford Maths 9 Victorian Curriculum (Second edition)*.

SCIENCE

In Science this year, you will learn about how the body coordinates and regulates its internal systems so that it can respond to changes. When things change in the environment (such as the emergence of a disease-causing agent) or a part of the body fails, the normal functioning of the body is interrupted. The body needs to respond and attempt to return to a normal homeostatic state before permanent damage is caused.

To complete this task successfully, you will need to identify how the body's systems work together to maintain a functioning body. You should consider the type of disorder or disease that you will be fighting and how it may cause changes in the body's normal function and response mechanisms.

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for? Is it the people directly affected by the disorder or disease, or do their families and carers need support too?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems? What words would you use to describe these feelings?

To answer these questions, you may need to investigate using different resources or conduct interviews or surveys.

Define

Before you start to design your solution to the problem, you need to define the parameters you are working towards.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1 Describe the type of life that the people you are helping lived before their lives were affected by the disorder or disease.

2 Describe how the people affected by the disease have needed to change their lives to cope with the effects of the disorder or disease.

3 Describe how you will know that you have made their lives better as a result of your prototype strategy.

Ideate

Once you know who you're designing for, and you know what the criteria are, it's time to get creative!

Outline the criteria or requirements your design must fulfil (i.e. usability, accessibility, cost).

Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Build

Each group member should draw an individually designed solution. Label each part of the design. Include the materials that will be used for its construction.

Include in the individual designs:

a a detailed diagram of the design

b a description of why it is needed by the individual or group

c a description of any similar designs that are already available to buy

d an outline of why your idea or design is better than others that can be purchased.

Present your design to your group.

Build the prototype

Choose one design and build two or three prototypes.

Use the following questions as a guideline for your prototype.

- What materials or technology will you need to build or represent your prototype design?
- What skills will you need to construct your prototype design?
- How will you make sure your prototype design is able to be used by the people who need it?
- How will you describe the way the prototype design will work?

Test

Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first prototype. You will test the prototype more than once to compare results, so you will need to control your variables between tests.

What criteria will you use to determine the success of your prototype?

Conduct your tests and record your results in an appropriate table.

Prototype 2

If your prototype will be used to help individuals with the disorder or disease, then you will need to generate a survey to test whether the prototype is appropriate for their use. (How would they use it? Would it make their life easier or harder? Would they consider buying it? How much would they be willing to pay to access the design?)

Prototype 3

Your last prototype should be adapted using the information gathered from testing the first two versions to make it more effective and usable for the group you are helping. You may want to use the first two prototypes as a demonstration of how the design has been improved over time.

Communicate

Present your design to the class as though you are trying to get your peers to invest in your design.

In your presentation, you will need to:

- outline the relevant disorder or disease and how it affects individuals, as well as society as a whole
- create a working model or a detailed series of diagrams, with a description of how it will be used by an individual or group
- explain how you changed your design as a result of testing or feedback
- describe how the design will improve the life of an individual or group
- describe how many people need or will use the design
- describe how individuals will be able to access the design (will they need to purchase it or will it be publicly funded?)
- describe how the design will improve an individual's ability to contribute to society as a whole.

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Answers

MODULE 1 Exponents

EX p7 1.1 Exponents

1 a $6 \times 6 \times 6 \times 6 = 1296$
b $8 \times 8 \times 8 = 512$
c $(-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$
d $(-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) = 729$
e $\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{125}{64}$
f $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{128}$
g $\left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) = \frac{16}{81}$
h $\left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) = -\frac{243}{3125}$

2 a $b \times b \times b \times b \times b \times b$
b $(-n) \times (-n) \times (-n) \times (-n) \times (-n)$
c $(-cd) \times (-cd)$
d $2pq \times 2pq \times 2pq \times 2pq$
e $2 \times p \times q \times q \times q \times q$
f $-4 \times a \times a \times b \times b \times b \times c$
g $3m^2 \times 3m^2 \times 3m^2 \times 3m^2 \times 3m^2$
h $3 \times m^2 \times m^2 \times m^2 \times m^2 \times m^2$

3 a 5^4 b a^4 c $7k^3v^4$
d $(qu)^5$ or q^5u^5 e $(-h)^3$ f $-h^3$
g $(n^3)^6$ h $(5b^3d^4)^2$

4 a 2×5^2 b $2^3 \times 3^2$ c $3^3 \times 5$
d $2 \times 3^3 \times 7$ e $2^3 \times 19$ f $2^2 \times 7 \times 29$
g $2 \times 5^2 \times 11$ h $3^2 \times 5 \times 37$

5 a 0.04 b 0.04 c 0.0004
d 0.008 e −0.008 f 0.000 008
g 0.0016 h 0.0016 i 0.000 000 16

6 a 5^4 b -5^3 c a^4b^4
d $5^8x^8y^8$ e $5x^8y^8$ f $\frac{11^6}{2^6}$
g $-3^5a^5b^5c^5$ h $3^8a^8b^8c^8$

7 a 343 b 6 c $-\frac{81}{320}$ d 2817

8 a $2^3 \times 3^2$ b $5^6 \times 6$ c $13^4 \times 17^5$
d $101^2 \times 103^5$ e $4^3 \times x^4$ f $7^2(xy)^7$ or $7^2x^7y^7$

9 a $2^3 \times 3 \times 5^2$ b $2 \times 3 \times 5^2$
c $2^2 \times 3^2 \times 5^2$ d $2^2 \times 3 \times 5^3$
e $2^3 \times 3 \times 5^3$ f $2^3 \times 3^2 \times 5^2$
g $2^2 \times 3 \times 5^2 \times 7$ h $2^5 \times 3 \times 5^2$

10 a x^7 b a^3b^2 c $3rs^2t^3$ d $4e^7f^2$

11 a Only k has the exponent 5, so the t should not be repeatedly multiplied. $tk^5 = t \times k \times k \times k \times k \times k$
b All of $2rw$ is raised to the power of 4, so $2rw$ should be repeatedly multiplied.
$(2rw)^4 = 2rw \times 2rw \times 2rw \times 2rw$
c The product was evaluated before the power.
$-3 \times (-2)^4 = -3 \times 16 = -48$

12 a 1 b 39 c −60
d $\frac{63}{4} = 15\frac{3}{4}$

13 a i 1 ii −1 iii 1 iv −1 v 1
vi −1 vii 1 viii −1 ix 1 x −1
b i When the exponent n, is odd, the basic numeral of $(-1)^n$ is −1.
ii When the exponent n, is even, the basic numeral of $(-1)^n$ is 1.
c i Negative ii Negative
iii Positive iv Positive
v Positive vi Negative
vii Positive viii Positive

14 a i 0.343 and 0.000 343
ii In exponent form, both are the cube of a decimal with the digit 7 in different place values: one is 7 tenths, the other is 7 hundredths. The basic numerals use the same digits 343 also in different place values; 0.000 343 is $10^3 = 1000$ times smaller than 0.343.
b i −0.064 and −0.064
ii In exponent form, both are the cube of 0.4 but one has a negative inside the brackets, the other outside. The basic numerals for both are exactly the same.
c i −1.728 and 2.0736
ii In exponent form, both raise −1.2 to a power: one is to the power of 3; the other to the power of 4. The basic numeral for the power of 3 is negative while the power of 4 is positive. The two numerals have different values.
d i 9.261 and 8.120 601
ii In exponent form, both are cubes of a number that uses the digits 2 and 1. The basic numerals share similar digits where 8.120 601 has its digits spaced out one place more than 9.261 does.

15 a 1.2 b 2024: 24, 2025: 29, 2028: 50

16 a 3 days: 2^3, 8 days: 2^8, 12 days: 2^{12}
b 3 days: 2^6, 8 days: 2^{16}, 12 days: 2^{24}
c 3 days: 2, 8 days: 2^4, 12 days: 2^6
d A: 24, B: 192, C: 6

17 a LCM: $2^8 \times 3^{15} \times 5^2 \times 7^4$ HCF: $2^4 \times 3^5 \times 5^2 \times 7$
b LCM: $a^8b^{15}c^2d^4$ HCF: $a^4b^5c^2d$
c LCM: $pq^5r^{10}s^4$ HCF: $pq^3r^7s^2$
d LCM: $24x^3y^9z^4$ HCF: $4xy^3z^4$

18 a 3 b 1 c 2 d 3 e 15

19 a 2^{10} b 5^{10} c $2^{12} \times 5^8$

20 $64^1 = 64$, $8^2 = 64$, $(-8)^2 = 64$, $4^3 = 64$, $2^6 = 64$, $(-2)^6 = 64$

21 a $-\frac{1}{12}$ b $\frac{243}{32}$ c $-\frac{2401}{10} = -240.1$

EX p12 1.2 Products and quotients of powers

1 **a** 3^9 **b** 7^6 **c** $(-2)^{12}$ **d** 6^3
e $(-8)^7$ **f** 10^{11} **g** 3^1 or 3 **h** 5^2
i 4^2 **j** $(-9)^9$ **k** 13^3 **l** 2^{10}

2 **a** 19 683 **b** 117 649 **c** 4096 **d** 216

3 **a** $3y^9$ **b** $7g^7$ **c** $6b^{11}$
d $-12k^{13}$ **e** $6b^{11}$ **f** $-80g^{11}$
g $27c^{14}$ **h** $15p^{10}$

4 **a** a^2 **b** d **c** g^{10}
d p^3 **e** a^5 **f** n^3
g r^8 **h** $8x^{11}$

5 **a** $12x^{11}$ **b** $10x^7$ **c** $-24x^9$ **d** $54x^{11}$
e $3x^4$ **f** $4x^4$ **g** $\frac{2x}{5}$ **h** $\frac{5x^8}{3}$ or $\frac{5}{3}x^8$
i $-8t^{12}$ **j** $\frac{5r^6}{8}$ or $\frac{5}{8}r^6$ **k** $5c^4$ **l** $\frac{5y^7}{2}$ or $\frac{5}{2}y^7$

6 **a** x^9 **b** $\frac{b}{a}$ **c** n **d** b^8
e x^5 **f** m^4 **g** $6a^6$ **h** n^5
i 5 **j** $-4t^2$ **k** 2 **l** 1

7 **a** a^9b^6 **b** $-18m^{11}n^3$ **c** c^2d^2
d k^2m^3 **e** x^8y^7 **f** $30g^{10}h^3$
g a^8b^6 **h** $15x^8y^{10}$ **i** $54w^4x^{13}y^4$
j $\frac{u^4}{5}$ or $\frac{1}{5}u^4$ **k** $-\frac{3}{4}ef^{10}$ **l** $-108v^{17}y^{10}$

8 **a** x^6 **b** $2k^3$ **c** $6a$
d $\frac{m^2}{x^2}$ **e** b^3 **f** $n^{13}p^3$
g $-2j^5q^6$ **h** $2w^2x^4$

9 **a** False: When multiplying you add, not multiply exponents. $x^3 \times x^4 = x^7$
b False: There is no exponent law for adding terms, but you can add these as they are like terms. $k^3 + k^3 = 2k^3$
c True
d False: There is a power of 1 in the middle term, which must be added. $a^5 \times a \times a^5 = a^{11}$
e False: Add the powers of m and n separately. $m^3n^5 \times m^2n^4 = m^5n^9$
f False: When dividing you subtract, not divide exponents. $100^8 \div 100^2 = 100^6$
g True
h False: The first b^6 has been missed. $\frac{a^5b^6}{a^2b^4} \times \frac{a^3b^5}{a^4b} = a^2b^6$

10 **a** **i** $\frac{2^3}{2^5} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2} = \frac{1}{2^2} = \frac{1}{2^{5-3}}$

ii $\frac{2^4}{2^8} = \frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$
$= \frac{1}{2 \times 2 \times 2 \times 2} = \frac{1}{2^4} = \frac{1}{2^{8-4}}$

iii $\frac{5^6}{5^7} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5} = \frac{1}{5^1} = \frac{1}{5^{7-6}}$

b **i** $\frac{1}{3^6}$ **ii** $\frac{1}{5^6}$ **iii** $\frac{1}{2^4 \times 3^5}$
iv $\frac{3^5}{2^4}$ **v** $\frac{2^4}{3^5}$

11 **a** 2^{15} **b** 3^{15} **c** $2^6 \times 3^6$ **d** 2^{30}

12 **a** $w = 5, x = 3, y = 2, z = 8$
b $x = 9, y = 6, z = 4$
c $a = 16, b = 9, c = 4, d = 6$

13 No. An exponent represents the number of times the base is repeatedly multiplied. If you write $2^4 \times 3^2$ as $(2 \times 3)^{4+2}$, you are saying that $2 \times 2 \times 2 \times 2 \times 3 \times 3$ is equal to $(2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$, which is not true. Similarly, if you write $y^8 \div x^5$ as $\left(\frac{y}{x}\right)^{8-5}$, you are saying that $\frac{y \times y \times y \times y \times y \times y \times y \times y}{x \times x \times x \times x \times x}$ is equal to $\left(\frac{y}{x}\right) \times \left(\frac{y}{x}\right) \times \left(\frac{y}{x}\right)$, which is not true.

14 **a** $2^3 = 8$ **b** $3^3 = 27$
c $5^2 = 25$ **d** $10^4 = 10\,000$

15 **a** $2^{12} = 8^4$ **b** $3^{15} = 27^5$ **c** $5^{18} = 25^9$
d $10^{12} = 10\,000^3$ **e** $4^{14} = 16^7$ **f** $2^{30} = 32^6$
g $6^6 = 216^2$ **h** $3^{30} = 243^6$

16 **a** $a^{m+n}b^{x+y}$ **b** $a^{m-n}b^{x-y}$

EX p17 1.3 Raising exponents and the zero exponent

1 **a** 6^{12} **b** 3^4 **c** $3^6 \times 4^2$ **d** 2^{24}
e $5^4 \times 2^{28}$ **f** $\frac{3^2}{4^2}$ **g** $\frac{5^{12}}{2^8}$ **h** $\frac{1}{8^{25}}$
i -3^{28} **j** 3^{20} **k** $7^{28} \times 11^{21}$ **l** $\frac{13^{48}}{17^{24}}$

2 **a** b^{10} **b** m^8 **c** j^{10} **d** j^{10} **e** n^{80} **f** p^{99}

3 **a** x^6y^6 **b** 2^3d^3 **c** $(-5)^7k^7 = -5^7k^7$
d $9^{10}p^{10}$ **e** 3^4m^4 **f** $\frac{8^2}{p^2}$
g $\frac{x^6}{y^6}$ **h** g^2h^2 **i** a^5b^5
j $\frac{k^3}{m^3}$ **k** 2^8x^8 **l** $-\frac{d^5}{3^5}$

4 **a** 3^4x^{24} **b** $5a^{28}b^7$
c $\frac{2^3m^3}{n^3}$ **d** $\frac{a^8}{b^{20}}$
e $-2u^{12}$ **f** $\frac{4}{3}v^{70}w^{30} = \frac{4v^{70}w^{30}}{3}$
g $\frac{7p^7}{9q^{42}}$ **h** $\frac{3^{10}}{2^3}r^{45} = \frac{3^{10}r^{45}}{2^3}$
i $\frac{8}{5^{24}t^{44}}$ **j** $3^2 \times 7 \times i^{34} = 63i^{34}$
k $2^{35}c^{40}$ **l** $\frac{5^4x^{75}}{7^4y^{90}}$

5 **a** 1 **b** 1 **c** 1 **d** 1

6 **a** 2 **b** 1 **c** −7 **d** 1
e −1 **f** 2 **g** 1 **h** 2
i 2 **j** 3 **k** 1 **l** 1
m 1 **n** 1 **o** −1 **p** −1

7 **a** x^{13} **b** x^{22} **c** x^{27} **d** x^{27}
e x^{10} **f** w^6 **g** $\frac{b}{3}$ **h** e^6
i 1 **j** $4a^9$ **k** t^{25} **l** f^{109}

8 **a** 1 **b** −7 **c** 1 **d** 3
e 1 **f** k^2 **g** $10g^4$ **h** 3
i x^{15} **j** 2 **k** $-8b$ **l** $\frac{2}{5}$

9 **a** −2 **b** k^{10} **c** $x^{11}y^9$
d $\frac{m^{12}}{2n^5}$ **e** 1 **f** $\frac{b^4}{a^2c}$

10 **a** x^9y^7 **b** $1568k^7$ **c** $81x^{24}$ **d** $-5a^{28}b^7$
e $\frac{x^{10}}{y^{11}}$ **f** $\frac{8m^3}{n^3}$ **g** $\frac{a^8}{b^{20}}$ **h** $\frac{w^{10}x^6}{y^8}$
i k^7mn^2 **j** $\frac{16t^{13}}{p^{21}}$ **k** ab^{30} **l** $72h^6$

11 **a** True **b** True
c False: Not every power in the bracket has been raised to the sixth power. $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y^6}$
d False: The bracket has been expanded incorrectly. $\frac{(k^3)^2 \times k^4}{k^2} = \frac{k^6 \times k^4}{k^2} = k^8$
e True **f** False, $100^9 \div 100^9 = 1$
g True **h** False, $a^0 \times b^0 = 1$

12 **a** 7 **b** 4 **c** 0 **d** 8
e 3 **f** 5 **g** 7 **h** 5

13 Eden added the 4 and 5 exponents before multiplying the 5 by the 3 exponent. $3^4 \times (3^5)^3 = 3^4 \times 3^{15} = 3^{19}$

14 **a** $(2^3)^5 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{3+3+3+3+3} = 2^{3\times 5}$
b $(x^7)^4 = x^7 \times x^7 \times x^7 \times x^7 = x^{7+7+7+7} = x^{7\times 4}$
c $(2\times 3)^4 = (2\times 3)\times(2\times 3)\times(2\times 3)\times(2\times 3) = 2\times 2\times 2\times 2\times 3\times 3\times 3\times 3 = 2^4\times 3^4$
d $\left(\frac{2}{3}\right)^6 = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{2^6}{3^6}$

15 **a** 3^{64} **b** 5^{81} **c** 7^{1024} **d** 10^{3125}

16 8^{15} cm^3

17 $\frac{4^5}{3^5}$

18 $(a^m)^n = a^{m\times n} = a^{n\times m} = (a^n)^m$

19 $x = \frac{3}{20}$

EX p19
Checkpoint

1 **a** $2\times 2\times 2\times 2\times 2\times 2 = 64$
b $(-3)\times(-3)\times(-3)\times(-3) = 81$
c $(-4)\times(-4)\times(-4) = -64$
d $\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6} = \frac{125}{216}$

2 **a** $a\times a\times a\times a\times a\times a$
b $(-b)\times(-b)\times(-b)\times(-b)$
c $3y\times 3y\times 3y\times 3y\times 3y$
d $3\times xy\times xy\times xy\times xy\times xy$

3 **a** 8^7 **b** u^4 **c** $(4b)^5$ **d** $-7k^3h^5$

4 **a** $2^2\times 7$ **b** $2^3\times 3^2$
c $2^2\times 11^2$ **d** $2\times 3^3\times 5$

5 **a** 8^{11} **b** $5^{10}\times 7^{12}$ **c** 6^5 **d** $3^8\times 10^7$

6 **a** a^{12} **b** $-12b^{18}c^{21}$ **c** u^5 **d** $\frac{5}{7}p^{14}q^{17}$

7 **a** 3^{10} **b** k^8
c $\frac{t}{c^{11}}$ **d** $-\frac{15}{4}d^{11}w^7$

8 **a** 1 **b** 1 **c** −1 **d** 8

9 **a** 3^{24} **b** j^{45}
c $(-5)^6a^{18}b^{42} = 5^6a^{18}b^{42}$ **d** $-\frac{3^8p^{40}}{2^8q^{56}}$

10 **a** 1 **b** $\frac{g^{20}}{3^7}$ **c** $5^7m^{66}n^{87}$ **d** $\frac{6}{j^{42}}$

EX p23
1.4 Negative exponents

1 **a** $\frac{7}{8}$ **b** 2 **c** $-\frac{1}{3}$ **d** $\frac{1}{3w}$
e d **f** $\frac{m}{r}$ **g** $5y$ **h** $\frac{1}{u^2}$

2 **a** $\frac{1}{5}$ **b** $\frac{1}{8}$ **c** $-\frac{1}{2}$
d $\frac{1}{x}$ **e** $\frac{1}{p}$ **f** $\frac{1}{3w}$

3 **a** 5^{-1} **b** $(-13)^{-1}$ **c** $\frac{1}{5^{-1}}$
d $-\frac{1}{8^{-1}}$ **e** m^{-1} **f** $\frac{1}{x^{-1}}$
g $(5y)^{-1}$ **h** $\frac{1}{(3w)^{-1}}$ **i** $\frac{3^{-1}}{4^{-1}} = \left(\frac{3}{4}\right)^{-1}$
j $\frac{6^{-1}}{5^{-1}} = \left(\frac{6}{5}\right)^{-1}$ **k** $\frac{a^{-1}}{w^{-1}} = \left(\frac{a}{w}\right)^{-1}$ **l** $\frac{1}{v^{-3}}$

4 **a** $\frac{1}{4^2}$ **b** $\frac{1}{2^6}$ **c** $\frac{1}{(-9)^3} = -\frac{1}{9^3}$
d $\frac{1}{(-5)^4} = \frac{1}{5^4}$ **e** $-\frac{1}{7^8}$ **f** $\frac{1}{10^5}$
g $\frac{1}{a^4}$ **h** $\frac{1}{x^7}$ **i** $\frac{1}{k^{10}}$
j $\frac{1}{m^2}$ **k** $\frac{1}{u^9}$ **l** $\frac{1}{g^{11}}$

5 **a** 3^{-4} **b** 4^{-7} **c** 6^{-5}
d $(-5)^{-3} = -5^{-3}$ **e** $(-9)^{-2} = 9^{-2}$ **f** -11^{-6}
g n^{-2} **h** g^{-11} **i** x^{-8}
j a^{-9} **k** p^{-4} **l** w^{-7}

6 **a** 2^3 **b** 5^6 **c** $(-8)^4 = 8^4$
d 3^9 **e** $(-7)^5 = -7^5$ **f** -4^2
g x^7 **h** y^3 **i** c^4
j z^{35} **k** $(5t)^4 = 5^4t^4 = 625t^4$
l $(uv)^{15} = u^{15}v^{15}$

7 **a** $\frac{5^2}{4^2}$ **b** $\frac{3}{7}$ **c** $-\frac{4^3}{3^3}$
d $\frac{2^4}{a^4}$ **e** $\frac{m}{6}$ **f** $\frac{u}{144}$
g $\frac{y^6}{x^6}$ **h** $-\frac{7^{11}}{9^{11}}$ **i** $\frac{17^{14}}{13^{14}}$
j $\frac{43^{11}}{500^{11}}$ **k** $\frac{97^{198}}{u^{198}}$ **l** $\frac{g^{654}}{123^{654}}$

8 **a** $\frac{y^3}{x^2}$ **b** $\frac{m^6}{n^4}$ **c** $\frac{c^7}{a}$

d $\frac{2k^5}{p^3}$ **e** $\frac{5b^2}{a^8}$ **f** $-\frac{4}{w^2x^6}$

g $\frac{a^4c^7}{b^5}$ **h** $\frac{m^5}{k^3n^8}$ **i** $\frac{7b^9d}{c^6}$

j $\frac{3}{x^2y^7z^4}$ **k** $\frac{-12p^{17}}{q^{18}u^{21}}$ **l** $\frac{-34}{j^{65}b^{78}}$

9 **a** $\frac{b^3}{a^5}$ **b** $\frac{p^2}{k^6}$ **c** $\frac{m^7}{h^4}$

d d^5e^8 **e** u^3w^8 **f** $3x^2y^4$

g $8g^6h^5$ **h** $-\frac{2c^9}{d^5}$ **i** $\frac{2n^3}{5m^7}$

j $\frac{1}{6e^5y^8}$ **k** $-\frac{1}{2g^{99}h^{11}}$ **l** $\frac{2c^{101}}{5r^{72}}$

10 **a** $\frac{n^4p^6}{m^3}$ **b** $\frac{4c^2e^6}{d^5}$ **c** $\frac{2p^4}{3kn^3}$

d $\frac{a^2b^3}{3c^5d^4}$ **e** $\frac{7k^2n}{m^6}$ **f** $\frac{u^5x^3}{10k^2w^8}$

g $-\frac{5pq^4}{r^4s^3}$ **h** $63b^{14}g^{15}k^{17}u^{10}$ **i** $\frac{1}{5t^4u^3v^7w^{12}}$

j $-\frac{49m^{59}n^{41}}{64l^{88}v}$ **k** $\frac{a^{97}c^{153}d^{167}f^{89}}{b^{105}e^{111}}$ **l** $k^{402}l^{812}m^{999}n^{571}$

11 **a** $\frac{1}{4^3}$ **b** $\frac{1}{7}$ **c** 2^2

d $\frac{1}{(-3)^6}=\frac{1}{3^6}$ **e** 5^4 **f** $\frac{1}{(-2)^7}=-\frac{1}{2^7}$

g $\frac{1}{9^2}$ **h** 3^8 **i** $\frac{1}{4^9}$

j $\frac{1}{10^3}$ **k** $\frac{1}{211^{14}}$ **l** $\frac{1}{13^{100}}$

12 **a** $\frac{1}{5^6}$ **b** $\frac{1}{3^8}$ **c** -2^4

d $\frac{1}{3^2}$ **e** $\frac{1}{6^4}$ **f** $\frac{1}{4}$

g $\frac{1}{9}$ **h** 5^8 **i** 7^3

j $\frac{1}{2^3}$ **k** 99^{127} **l** $\frac{1}{15^{162}}$

13 **a** $\frac{1}{64}$ **b** $\frac{1}{7}$ **c** 4

d $\frac{1}{729}$ **e** 625 **f** $-\frac{1}{128}$

14 **a** $\frac{1}{a^7b^7}$ **b** $\frac{1}{3^5x^5}=\frac{1}{243x^5}$ **c** $\frac{1}{4y}$

d $\frac{1}{7^2k^2}=\frac{1}{49k^2}$ **e** $\frac{1}{2^3p^3}=\frac{1}{8p^3}$ **f** $\frac{1}{m^{11}k^{11}}$

15 **a** −3 **b** −7 **c** −1 **d** 2

e −2 **f** −3 **g** −2 **h** −4

16 **a** $\frac{1}{64}$ mm **b** 0.015 625 mm

17 **a** $\frac{1}{100\,000\,000}$ **b** 0.000 000 01

18 **a** $\frac{1}{15\,625}$ **b** 0.000 064

19 **a**

Exponent form	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Basic numeral	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

b Each term is half of the term before it. The basic numeral of a term with base 2 and a negative exponent is the reciprocal of the value of the term with the matching positive exponent.

c $\frac{1}{64}$ **d** $\frac{1}{1024}$ **e** 128

20 **a**

Exponent form	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}	3^{-5}
Basic numeral	243	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

b Each term is one third of the term before it. The basic numeral of a term with base 3 and a negative exponent is the reciprocal of the value of the term with the matching positive exponent.

c $\frac{1}{729}$ **d** $\frac{1}{6561}$ **e** 2187

21 **a**

Exponent form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Basic numeral	10 000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$

b Each term is one tenth of the term before it. The basic numeral of a term with base 10 and a negative exponent is the reciprocal of the value of the term with the matching positive exponent.

c **i** 100 000 **ii** 1 000 000

iii 10 000 000 **iv** 100 000 000

v 1 000 000 000

d **i** $\frac{1}{100\,000}$ **ii** $\frac{1}{1\,000\,000}$

iii $\frac{1}{10\,000\,000}$ **iv** $\frac{1}{100\,000\,000}$

v $\frac{1}{1\,000\,000\,000}$

e **i** $\frac{1}{10}$ **ii** 0.1

f **i** $\frac{1}{100}=0.01$ **ii** $\frac{1}{1000}=0.001$

iii $\frac{1}{10\,000}=0.0001$

g **i** $\frac{1}{100\,000}=0.000\,01$

ii $\frac{1}{1\,000\,000}=0.000\,001$

iii $\frac{1}{10\,000\,000}=0.000\,000\,1$

iv $\frac{1}{100\,000\,000}=0.000\,000\,01$

v $\frac{1}{1\,000\,000\,000}=0.000\,000\,001$

h The number of decimal places in the decimal matches the number of zeros in the denominator of the fraction, which matches the power of 10.

22 a **i** 20 000 **ii** 7000 **iii** 300 000 **iv** 400 000 000 000 **v** 90 000 000

b **i** $\frac{5}{100}$ **ii** $\frac{8}{100\,000}$ **iii** $\frac{2}{1000}$ **iv** $\frac{7}{10\,000}$ **v** $\frac{6}{1000\,000\,000}$

c **i** 0.05 **ii** 0.000 08 **iii** 0.002 **iv** 0.0007 **v** 0.000 000 006

d When multiplying a whole number by a negative power of ten, move the decimal point one place to the left for each zero you see.

23 a **i** $2 \times 3^{-1} = 2 \times \frac{1}{3} = \frac{2}{3} = 2 \div 3$

ii $2 \div 3^{-1} = \frac{2}{3^{-1}} = 2 \times \frac{1}{3^{-1}} = 2 \times 3$

b Multiplication by the reciprocal of a number is equivalent to division by that number. Division by the reciprocal of a number is equivalent to multiplication by that number.

24 a **i** 3 **ii** 5 **iii** $\frac{5}{4}$

b Using the power of a power law, $(a^{-1})^{-1} = a^{(-1)\times(-1)} = a^1 = a$.

c The reciprocal of the reciprocal of a number is the original number.

25 a $2a^{-2}b^{-1}$ **b** $-3t^2v^{-3}$ **c** $5^{-1}x^4y^{-4}$ **d** $p^{-1}q^{-5}r^2$

26 a $\frac{1}{x^2}$ **b** $\frac{1}{x^4}$ **c** $8x^3$ **d** $\frac{30}{x^5}$ **e** 3 **f** x^9 **g** $\frac{1}{x^3}$ **h** $2x^5$ **i** $\frac{1}{3x^{10}}$ **j** $\frac{4}{7x^4}$ **k** $\frac{1}{x^2}$ **l** x^{11} **m** $\frac{4}{x^{13}}$ **n** $\frac{2}{x^8}$ **o** $\frac{1}{x^5}$ **p** $\frac{1}{x^7y^7}$

27 c $\frac{8}{x^{-3}} = \frac{1}{8^{-1}x^{-3}}$ **e** $\frac{1}{3^{-1}}$ **f** $\frac{1}{x^{-9}}$

h $\frac{2}{x^{-5}} = \frac{1}{2^{-1}x^{-5}}$ **l** $\frac{1}{x^{-11}}$

28 a $a^m \times a^{-n} = a^{m+(-n)}$

b $\frac{1}{a^{-m} \times a^n} = \frac{1}{a^{-m+n}}$

c $a^m \div a^n = a^{m-n} = a^{-n} \div a^{-m} = a^{-n-(-m)}$

EX p30

1.5 Scientific notation

1 a 540 **b** 73 600 **c** −1800 **d** 405 000 **e** 2 753 000 **f** 0.61 **g** 0.000 008 22 **h** −0.000 976 **i** 0.000 070 03

2 a 10^2 **b** 10^3 **c** 10^4 **d** 10^5 **e** 10^6 **f** 10^{-1} **g** 10^{-2} **h** 10^{-3} **i** 10^{-4} **j** 10^{-5}

3 a 320 000 **b** 8 140 000 000 **c** −500 **d** −23 450 000 **e** 11 000 **f** 0.0064 **g** 0.000 007 28 **h** 0.000 000 9 **i** −0.000 030 2 **j** −0.0541 **k** 450 000 000 000 **l** 0.000 000 006 12 **m** 0.57 **n** 1306.8 **o** 0.000 273 16

4 a Check the answers on your calculator with the answers in question **3**.

b Some of the very large and very small numbers are difficult to see on the calculator due to limited screen size.

5 a $4500 = 4 \times 10^3 + 5 \times 10^2$

b $7\,320\,000 = 7 \times 10^6 + 3 \times 10^5 + 2 \times 10^4$

c $200\,000 = 2 \times 10^5$

d $-190 = -1 \times 10^2 - 9 \times 10^1$

e $3216 = 3 \times 10^3 + 2 \times 10^2 + 1 \times 10^1 + 6 \times 10^0$

f $0.0063 = 6 \times 10^{-3} + 3 \times 10^{-4}$

g $0.000\,000\,18 = 1 \times 10^{-7} + 8 \times 10^{-8}$

h $0.05 = 5 \times 10^{-2}$

i $-0.000\,070\,2 = -7 \times 10^{-5} - 2 \times 10^{-7}$

j $0.427 = 4 \times 10^{-1} + 2 \times 10^{-2} + 7 \times 10^{-3}$

k $11\,220 = 1 \times 10^4 + 1 \times 10^3 + 2 \times 10^2 + 2 \times 10^1$

l $0.000\,004 = 4 \times 10^{-6}$

m $-568.2 = -5 \times 10^2 - 6 \times 10^1 - 8 \times 10^0 - 2 \times 10^{-1}$

n $0.000\,249 = 2 \times 10^{-4} + 4 \times 10^{-5} + 9 \times 10^{-6}$

o $679\,300 = 6 \times 10^5 + 7 \times 10^4 + 9 \times 10^3 + 3 \times 10^2$

p $-0.0102 = -1 \times 10^{-2} - 2 \times 10^{-4}$

6 a 4.5×10^3 **b** 7.32×10^6 **c** 2×10^5 **d** -1.9×10^2 **e** 3.216×10^3 **f** 6.3×10^{-3} **g** 1.8×10^{-7} **h** 5×10^{-2} **i** -7.02×10^{-5} **j** 4.27×10^{-1} **k** 1.122×10^4 **l** 4×10^{-6} **m** -5.682×10^2 **n** 2.49×10^{-4} **o** 6.793×10^5 **p** -1.02×10^{-2}

7 a 3 **b** 2 **c** 4 **d** 1 **e** 3 **f** 2 **g** 1 **h** 4 **i** 5 **j** 4 **k** 4 **l** 3

8 a 2 **b** 3 **c** 4 **d** 2 **e** 4 **f** 3

9 a 2.6×10^5 **b** -5.04×10^4 **c** 9.104×10^6 **d** -6.0×10^3 **e** 460 **f** 73 050 **g** 1000 **h** 40 000 **i** −5.14 **j** 0.035 **k** −42.06 **l** 0.9

10 a 3.3×10^2 **b** 4.87×10^4
c -1.908×10^5 **d** 3×10^3
e 4.03×10^{-1} **f** -5.4×10^{-2}
g 2.072×10^{-4} **h** -8×10^{-3}
i 7.6×10^2 **j** -2.070×10^4
k 4.02×10^1 **l** 5.4008×10^4

11 a A, B, F, H **b** B, F
c A, D, G **d** B, C, E, F, H

12 a 5×10^{-4} m **b** 2.3×10^4 ML
c 4.8×10^{-2} mm **d** 9.3×10^9 people

13 a 6400 times in a minute
b 0.000 08 mm
c 149 600 000 km
d 0.000 000 000 000 28 cm

14

	1234.56	4.0191
$\times 10^3$	$\underline{1.23456 \times 10^3}$	0.0040191×10^3
$\times 10^2$	12.3456×10^2	0.040191×10^2
$\times 10^1$	123.456×10^1	0.40191×10^1
$\times 10^0$	1234.56×10^0	$\underline{4.0191 \times 10^0}$
$\times 10^{-1}$	12345.6×10^{-1}	40.191×10^{-1}
$\times 10^{-2}$	123456×10^{-2}	401.91×10^{-2}
$\times 10^{-3}$	1234560×10^{-3}	4019.1×10^{-3}

	0.0492	0.007 40
$\times 10^3$	0.0000492×10^3	0.00000740×10^3
$\times 10^2$	0.000492×10^2	0.0000740×10^2
$\times 10^1$	0.00492×10^1	0.000740×10^1
$\times 10^0$	0.0492×10^0	0.00740×10^0
$\times 10^{-1}$	0.492×10^{-1}	0.0740×10^{-1}
$\times 10^{-2}$	$\underline{4.92 \times 10^{-2}}$	0.740×10^{-2}
$\times 10^{-3}$	49.2×10^{-3}	$\underline{7.40 \times 10^{-3}}$

15 a 6.8×10^7 **b** 2.0×10^2 **c** 4.5×10^4
d -4.0×10^4 **e** -1.23×10^{-1} **f** 3.0×10^5

16 a 7.3034×10^5 **b** 8.36×10^4 **c** 6.3×10^{-3}
d -2.68×10^{-2} **e** -1.057×10^4 **f** 9.35×10^4

17 1.08×10^9 km/h

18 a 2.4×10^6 km **b** 400 days

19 a i 86 coins **ii** 774 g
b 0.025 m = 2.5 cm

20 Approximately 507 seconds or about 8 minutes

21 a i 2 (1 s.f.), 1.9 (2 s.f.), 1.90 (3 s.f.)
ii 2 (1 s.f.), 2.0 (2 s.f.), 1.99 (3 s.f.)
iii 2 (1 s.f.), 2.0 (2 s.f.), 2.00 (3 s.f.)
iv 2 (1 s.f.), 2.0 (2 s.f.), 2.00 (3 s.f.)
v 2 (1 s.f.), 2.0 (2 s.f.), 2.01 (3 s.f.)
vi 2 (1 s.f.), 2.1 (2 s.f.), 2.10 (3 s.f.)
b If the trailing zeros were not written as significant, then many of these numbers would round to the same number for different significant figures, which means we would be unable to consistently determine the number of significant figures the number was rounded to.

22 a 0.4 (1 s.f.), 0.000 000 000 001 (1 s.f.), −0.000 03 (1 s.f.)
b 0, 0, and 0
c If leading zeros are significant, then all values just more than −0.5 and just less than 0.5 would round to 0 for 1 significant figure, which does not provide useful information about very small numbers and is no different to rounding to the nearest integer (or decimal place for more significant figures).

23 a Jane rounded to 3 decimal places, not 3 significant figures.
b Kaleb included the leading zeros as significant.
c Lisa did not include the 0 between the 1 and 2 as significant.
d Marius did not keep the place value of the significant figures.

24 a i 2.82×10^3 seconds
ii 1.2096×10^6 seconds
iii 2.4192×10^7 seconds
iv 3.1536×10^7 seconds
b i 2.78 hours **ii** 11.6 days
iii 31.7 years **iv** 31.7 millennia

25 a i 7.3×10^3 seconds
ii 9.1×10^{-6} seconds
iii 54×10^{-9} seconds
iv 82×10^{12} seconds
v 129×10^6 seconds
vi 974×10^{-12} seconds
b i 56.01×10^6 seconds, 56.01 megaseconds (56.01 Ms)
ii 920×10^3 seconds, 920 kiloseconds (920 ks)
iii 43.1×10^{-6} seconds, 43.1 microseconds (43.1 μs)
iv 788×10^{-9} seconds, 788 nanoseconds (788 ns)
v 80×10^{-3} seconds, 80 milliseconds (80 ms)
vi 103.56×10^{12} seconds, 103.56 teraseconds (103.56 Ts)

26 a i 3.3×10^{-7} seconds
ii about 0.30 seconds
b Watch for the smoke rather than listen for the shot (or listen for the shot and add 0.3 s onto the time).

27 Diameter of one atom = 2.54×10^{-9} cm
The line will be 2.54×10^{-3} cm long.

28 1.8×10^{356}

1.6 Rational and irrational numbers

EX p37

1 a IV **b** VI **c** II **d** V **e** I **f** III

2 a Rational **b** Rational **c** Irrational
d Rational **e** Irrational **f** Rational
g Rational **h** Irrational **i** Rational
j Rational **k** Rational **l** Irrational
m Rational **n** Irrational **o** Irrational

3 **a, c, d, f, g, i, j, l**

4 **a** 4.5826 **c** 2.8284 **d** –2.4495
f 11.1803 **g** –15.5885 **i** 1.4422
j 5.1962 **l** 0.4899

5 $\sqrt{3}, -3\sqrt{5}, \sqrt{8}, \sqrt[4]{32}$

6 **a** $2\sqrt{5}$ **b** $3\sqrt{3}$ **c** $2\sqrt{7}$ **d** $5\sqrt{2}$
e $6\sqrt{2}$ **f** $20\sqrt{2}$ **g** $7\sqrt{3}$ **h** $8\sqrt{6}$
i $12\sqrt{3}$ **j** $11\sqrt{7}$ **k** $9\sqrt{6}$ **l** $9\sqrt{11}$

7 **a** $\sqrt{45}$ **b** $\sqrt{96}$ **c** $\sqrt{12}$ **d** $\sqrt{44}$
e $\sqrt{250}$ **f** $\sqrt{448}$ **g** $\sqrt{507}$ **h** $\sqrt{720}$

8 **a** **i** $4\sqrt{3}$ **ii** $3\sqrt{5}$
iii $6\sqrt{6}$ **iv** $2\sqrt{10}$
b **i** $\sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$
ii $\sqrt{6} + \sqrt{6}$
iii $\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2}$
iv $\sqrt{11} + \sqrt{11} + \sqrt{11}$

9 **a** Rational **b** Irrational **c** Irrational **d** Rational
e Irrational **f** Irrational **g** Irrational **h** Irrational

10 Irrational, since it cannot be written exactly as a terminating or recurring decimal.

11 **a** $6\sqrt{10}$ **b** $99\sqrt{35}$ **c** $12\sqrt{105}$ **d** $50\sqrt{30}$

12 **a** $\sqrt{\frac{5}{8}} \approx 0.790\,569, \frac{\sqrt{5}}{\sqrt{8}} \approx 0.790\,569$
b Yes
c $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
d Irrational, because it cannot be written exactly as a terminating or recurring decimal.
e **i** $\sqrt{\frac{32}{2}} = \sqrt{16} = 4$, rational
ii $\sqrt{\frac{15}{3}} = \sqrt{5} = 2.236\ldots$, irrational
iii $\sqrt{\frac{7}{28}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$, rational

13 **a** **i** 2 **ii** 3
b 2, 3
c By choosing between which two perfect squares the number lies.
d **i** 3, 4 **ii** –4, –5 **iii** 11, 12
iv –6, –7 **v** 13, 14 **vi** 17, 18

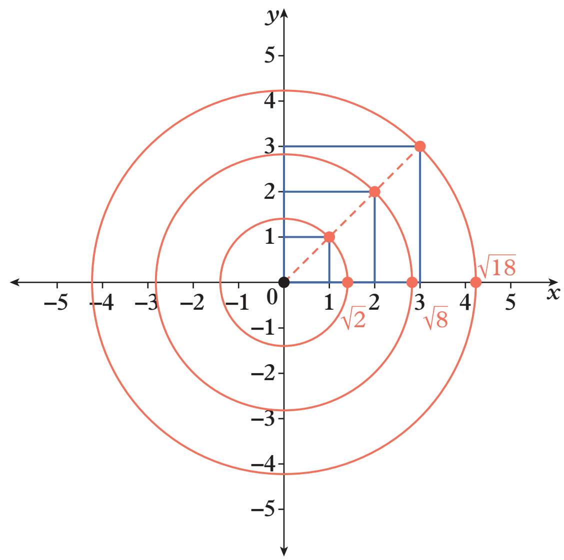
14 **a** **i** $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
ii $2 \times 3 \times 3 \times 3 \times 5$
iii $2 \times 2 \times 5 \times 7 \times 7$
iv $3 \times 3 \times 5 \times 5 \times 5$
b **i** $12\sqrt{2}$ **ii** $3\sqrt{30}$
iii $14\sqrt{5}$ **iv** $15\sqrt{5}$

15 **a** $\sqrt{3 \times 7} \times \sqrt{2 \times 7} = 7\sqrt{6}$
b $\sqrt{3^3} \times \sqrt{3 \times 11} = 9\sqrt{11}$
c $\sqrt{2 \times 3 \times 5} \times \sqrt{2 \times 3^2} = 6\sqrt{15}$
d $\sqrt{2 \times 3 \times 7} \times \sqrt{3 \times 7 \times 11} = 21\sqrt{22}$

16 **a** 8 cm
b **i** $8\sqrt{2}$ cm **ii** 11.31 cm

17 **a** **i** $\sqrt{35}$ cm **ii** 5.92 cm
b **i** $\sqrt{51}$ mm **ii** 7.14 mm
c **i** $\sqrt{95}$ cm **ii** 9.75 cm

18 **a, b, c**



d $\sqrt{8} = 2\sqrt{2}, \sqrt{18} = 3\sqrt{2}$
e They are all multiples of $\sqrt{2}$ so they are evenly spaced along the number line.

19 There are no positive real numbers that when raised to an even power will result in a negative solution; however, a negative number raised to an odd power will result in a negative solution.

20 **a**
$$\begin{aligned} \text{Let } x &= 0.363\,636\ldots \\ 100x &= 36.363\,636\ldots \\ 100x - x &= 36.363\,636\ldots - 0.363\,636\ldots \\ 99x &= 36 \\ x &= \frac{36}{99} \\ &= \frac{4}{11} \end{aligned}$$
So $0.\overline{36} = \frac{4}{11}$
b Because the recurring part of the decimal contained two digits, it was necessary to shift the decimal point two places to obtain the same recurring decimal again.

21 **a** $\frac{14}{99}$ **b** $\frac{7}{33}$ **c** $\frac{16}{33}$ **d** $\frac{172}{99}$

22 **a** $\frac{1}{3}$ **b** $\frac{8}{33}$ **c** $\frac{785}{999}$ **d** $\frac{41}{99}$
e $\frac{17}{30}$ **f** $\frac{35}{11}$ **g** $\frac{383}{90}$ **h** $\frac{3899}{1110}$

EX p42

Review: Exponents

Review questions 1.7A: Mathematical literacy review

1 factor tree

2 C

3 Sample answer using 12: Factors are numbers that multiply to make 12, such as 2 and 6 ($2 \times 6 = 12$), or 3 and 4 ($3 \times 4 = 12$). These factors are not necessarily prime numbers. Prime factors are factors that are prime, such as 2 and 3 ($2 \times 2 \times 3 = 12$).

4 Expanded form: $24 = 2 \times 2 \times 2 \times 3$
Exponent form: $24 = 2^3 \times 3$

5 **a** base, exponent form
b reciprocal, positive exponent

6 **a** True
b False. In 101, the zero is significant (for example).
c True
d False. In 100, the trailing zeros are not significant (for example).

Review questions 1.7B: Multiple choice

1 B **2** C **3** C **4** B **5** B
6 D **7** A **8** B **9** B **10** A

Review questions 1.7C: Short answer

1 **a** 81 **b** −125 **c** −64
d $\frac{243}{32}$ **e** 0.216 **f** 2.0736

2 **a** 17^6 **b** $(-5b^2)^5$
c $-10f^3v^7$ **d** $\left(\frac{b^4d^5}{6n^3}\right)^4$

3 **a** a^{16} **b** b **c** c^{16}
d $\frac{d^3}{3}$ **e** e^{47} **f** 9

4 **a** m^5n^8 **b** $54k^{18}l^{12}$

5 **a** $\frac{1}{5^5}$ **b** $-\frac{1}{11^3}$ **c** 4^4
d $-\frac{5^3}{4^3}$

6 **a** $\frac{1}{65\,536}$ **b** 343

7 **a** 58 760 **b** 0.000 009 02

8 **a** 5.4×10^5 **b** 7.6×10^{-4}

9 **a** 38 600
b 3.86×10^4 or 3.9×10^4 (rounded)

10 **a** 4 **b** 3

11 **a** $8\sqrt{2}$ **b** 33

Review questions 1.7D: Mathematical modelling

1 **a**

	Thy	Asha
Round 1	$\frac{2^{10}}{5^3}$	$\frac{3^6}{5^5}$
Round 2	$\frac{5^6}{2^3 \times 3}$	2×3^5
Round 3	$\frac{3^4}{2^3}$	$\frac{2^4 \times 5^3}{3^5}$

b Thy: $2^4 \times 3^3 \times 5^3$, Asha: $\frac{2^5 \times 3^6}{5^2}$
c Thy
d No. Asha had more factors, but the factors of 5 are divided (not multiplied) by the other factors, reducing the value. Thy's value was 54 000, whereas Asha's value was 933.12.
e Thy: $2^{-4} \times 3^{-3} \times 5^{-3} = \frac{1}{2^4 \times 3^3 \times 5^3}$,
Asha: $2^{-5} \times 3^{-6} \times 5^2 = \frac{5^2}{2^5 \times 3^6}$
f Thy: $2^2 \times 3 \times 5^4$, Asha: $2^9 \times 3$
g **i** Thy: $2^{-6} \times 4^1 \times 6^{-4}$, Asha: $1^6 \times 5^3 \times 5^{-4}$
ii Asha by two factors

2 **a**

Base (n)	n^2	n^3	n^4	n^5	n^6	n^7
3	9	27	81	243	729	2187
5	25	125	625	3125	15 625	78 125
7	49	343	2401	16 807	117 649	823 543
11	121	1331	14 641	161 051	1 771 561	19 487 171
13	169	2197	28 561	371 293	4 826 809	62 748 517
17	289	4913	83 521	1 419 857	24 137 569	410 338 673
19	361	6859	130 321	2 476 099	47 045 881	893 871 739

b For base numbers ending in 3, the final digit follows the pattern: 3, 9, 7, 1, 3, …
For base numbers ending in 5, the final digit is always 5.
For base numbers ending in 7, the final digit follows the pattern: 7, 9, 3, 1, 7, …
For base numbers ending in 9, the final digit follows the pattern: 9, 1, 9, …
For base numbers ending in 1, the final digit is always 1.

c

Base (n)	Last digit of n^{100}
3	1
5	5
7	1
11	1
13	1
17	1
19	1

d **i** 5 **ii** 1 **iii** 1
iv 1 **v** 1 **vi** 1
vii 1

3 **a** **i** Tas, NT, ACT
ii NSW, Vic, QLD, SA, WA

b–e

State	Population	Population (rounded to leading digit)	Population (scientific notation)
NSW	8 153 600	8 000 000	8×10^6
Vic	6 613 700	7 000 000	7×10^6
QLD	5 322 100	5 000 000	5×10^6
SA	1 820 500	2 000 000	2×10^6
WA	2 785 300	3 000 000	3×10^6
Tas	571 500	600 000	6×10^5
NT	250 600	300 000	3×10^5
ACT	456 700	500 000	5×10^5

f **i** NSW **ii** NT **iii** 7.7×10^6
iv 3.4×10^6 **v** 2.64×10^7

g 421 065; the difference is due to rounding.

MODULE 2 Algebra

EX p51

2.1 Simplifying

1 **a** $3x, -x, 20x$
b Like terms have exactly the same pronumerals raised to exactly the same powers. The coefficient can vary.
c 3, 7, −1, 2, 1, 20

2 $\{2ba^2, 3aab, 6a^2b\}$, $\{3a, 6a\}$, $\{2b^2a, 6ab^2, 3abb\}$, $\{6a^3, aaa, 6a^2a\}$

3 **a** $10a$ **b** $-8k$ **c** $6x^2$
d $4cd - 9cde$ **e** $12x + 6y$ **f** $4a + 6b$
g $9m + 2p$ **h** $1 - k$ **i** $3xy + 5x^2$
j $2d - 4de^2$ **k** $4m^3 + 2$ **l** $abc + 4ab + ac$

4 **a** $10x + y$ **b** $5ab - 5b + b^2 + a$
c $2km + 4$ **d** $-3x^2 + 3x - 4$
e $2a + a^2 + a^3 - 3$ **f** $6m^2n + 4mn^2 - 2n^2$

5 **a** $68a^2$ **b** $-27b^3$
c $-3c^2d^2$ **d** $3p^2q^2r + 2pq^2r$
e $8e + 8f + 4$ **f** $7x^2y + \frac{3xy}{2}$

6 **a** $6abcd$ **b** $-20mpxy$ **c** $9g^2h$
d $-24k^2mn$ **e** $56bjp^2t$ **f** ax^2y^2
g $18a^3bcd$ **h** $-20h^2k^2p$ **i** $-6b^3$
j $4km^2n^3$ **k** $15x^4y^2$ **l** $56a^4b^3c^2$

7 **a** $75a^{18}z^{36}$ **b** $-432b^{31}y^{16}$
c $-160c^8m^9x^9$ **d** $\frac{44d^7n^{10}}{w^7}$
e $\frac{1}{45e^{11}p^{24}t^{19}}$ **f** $\frac{168q^{33}}{g^{14}u^{28}}$

8 **a** b **b** $\frac{mn}{p}$ **c** $4c$ **d** $-\frac{2}{x}$
e $-\frac{1}{2}$ **f** $\frac{y}{4}$ **g** $\frac{6b}{5c}$ **h** $\frac{2}{11}$
i $6a$ **j** $-3xy$ **k** $\frac{5n^2}{3}$ **l** $-\frac{ac}{4}$

9 **a** $\frac{4}{d^5}$ **b** $\frac{1}{c^{24}d^{15}}$ **c** $-\frac{125u^3r^4}{49}$
d $\frac{16p^{38}}{9k^{24}}$ **e** $\frac{2x^9y^6}{z^3}$ **f** $20f$

10 **a** p^3q **b** $-2d$ **c** $-\frac{3x}{c}$
d $\frac{k}{2p}$ **e** $\frac{7c}{bd^2}$ **f** $\frac{m}{27n^4}$
g $-\frac{3}{2xy^2z^3}$ **h** $-16a^5b^7c^9d$ **i** $-\frac{2k^5m^4n^3}{3}$

11 **a** True **b** False **c** False **d** True

12 **a** Set B is correct. Set A: all signs changed to + when expression rearranged. Set C: The sign in front of the $2a$ term has changed from + to − and 2 − 7 should be −5, not 9.
b Set A is correct. Set B: −3 × 4 should be −12, not 12, and b should be squared in the last step. Set C: $b \times b$ (or b^2) has been written as $b \times 2$.
c Set C is correct. Set A: The terms are not in their appropriate locations in the fraction in the last step. Set B: error in cancelling the 'a' terms.

13 **a** $8x + y = 22$ **b** $-6xy = 36$ **c** $15xy = -90$
d $\frac{10}{y} = -5$ **e** $x^2y^3 = -72$ **f** $3x = 9$
g $6y - 2xy = 0$ **h** $x^2 + 4x - y = 23$
i $\frac{y^4}{x^3} = \frac{16}{27}$

14 **a** $2a + 3b + 2c = 11$ **b** $8ab - a = -18$
c $-2a^2b + ab^2 + 3ac = 40$ **d** $10a^2b^2c^2 = 1000$
e $3ab = -6$ **f** $\frac{4ac}{3} = \frac{40}{3} = 13\frac{1}{3}$

15 **a** $6x + 5y + 2$ **b** $4x + 2y - 2$ **c** $12x + 20$

16 **a** 51 cm **b** 24 cm **c** 68 cm

17 **a** $20xy$ **b** $49x^2$
c $9xy$ **d** $2y^2$

18 **a** 120 m^2 **b** 441 m^2
c 54 m^2 **d** 8 m^2

19 **a** $2y$ **b** 10 m, 160 m^2

20 **a** $3x$ **b** 96 cm^2, 16 cm

21 **a** **i** $6k$ **ii** $2k^2$
b 30 cm and 50 cm^2

22 **a** **i** $4x$ m **ii** x^2 m^2
iii $(4x + 8)$ m **iv** $(4x + 4)$ m^2
b **i** 36 m^2 **ii** 32 m
iii 40 m **iv** 64 m^2

23 **a** $10xy$ **b** 200 cm^2 **c** $12x + 8y$
d 88 cm **e** 158 cm

24 $2^{-48}x^{24}$

EX p57

2.2 Expanding

1 **a** $4a + 12$ **b** $7b + 35$ **c** $3c - 6$
d $5d - 5$ **e** $24 + 6e$ **f** $-2f - 16$
g $-3g - 12$ **h** $-8h + 40$ **i** $-4x + 36$
j $-10 + 5j$ **k** $kp + 6k$ **l** $ab - 4a$
m $18m + 6k$ **n** $2np + 4nq$ **o** $x^2 - 7xy$

2 **a** $4ab + 8bc + 12ac$ **b** $-3xy - 3x - 3y$
c $-m^2 + n^2 - 2m^2n^2$ **d** $a^4 - a^2b^2 + 2a^3b$
e $-pq + 4p - 8q$ **f** $2ab^2c + a^2bc + 4abc^2$

3 **a** $11x + 6$ **b** $5p + 6$ **c** $ab + 2a$
d $11y - 4$ **e** $4h + k + 2$ **f** $-6m$

4 **a** $5x - 8$ **b** $13k - 4$ **c** $7p + 1$
d $x^2 + 4x + 12$ **e** $m^2 + 5m + 6$ **f** $y^2 - 7y + 10$

5 **a** $12az + 15a$ **b** $56b + 40by$ **c** $14c - 6cx$
d $-20dw - 25d$ **e** $-48ev + 54e$ **f** $-40et - 30eu$
g $8g^2 - 28gt$ **h** $-30hr + 42h^2$ **i** $12i^2jk + 15ij^2k$

6 **a** $ab + 4a + 3b + 12$ **b** $cd + 7c + 2d + 14$
c $mn + m + 5n + 5$ **d** $jk - 5j - 9k + 45$
e $2ab + 6a + 7b + 21$ **f** $15cd - 20c + 6d - 8$

7 **a** $a^2 + 5a + 6$ **b** $x^2 + 15x + 50$
c $d^2 - 2d - 24$ **d** $y^2 - 5y - 24$
e $k^2 + 2k - 63$ **f** $m^2 - 3m - 18$
g $5e^2 - 22e + 8$ **h** $21a^2 - 31a + 8$
i $6y^2 - 13y + 5$

8 **a** $5b + 20 - ab - 4a$ **b** $-x^2 - 3x + 18$
c $-8c^2 - 2c + 15$ **d** $44d^2 - 102de + 54e^2$
e $-42p^2 + 13pq + 40p^2$ **f** $4x^2 - 9x - 1$
g $x^8 + x^7 + x^4 + x^3$ **h** $-20x^4 + 13x^2 + 84$
i $2x^2 + 11x + 11$

9 **a** $a^2 - 9$ **b** $b^2 - 4$ **c** $c^2 - 25$
d $9p^2 - 64$ **e** $h^2 - 1$ **f** $k^2 - m^2$

10 The middle two terms always combine to give zero, so you are left with the first term squared minus the second term squared. The two factors are almost the same with one set of brackets containing a sum of two terms and the other containing a difference of the same two terms.

11 **a** The result is the first term squared minus the second term squared.
b No, as you can multiply two factors in any order.
c **i** $x^2 - 4$ **ii** $y^2 - 81$ **iii** $m^2 - 36$
iv $9d^2 - 100$ **v** $m^2 - n^2$ **vi** $9 - x^2$

12 **a** $(100 + 3)(100 - 3) = 100^2 - 3^2$
b $103 \times 97 = (100 + 3)(100 - 3) = 10\,000 - 9 = 9991$
c **i** 9996 **ii** 9975 **iii** 999 999
iv 999 964

13 **a** $a^2 + 4a + 4$ **b** $b^2 + 14b + 49$
c $c^2 + 8c + 16$ **d** $9p^2 - 30p + 25$
e $h^2 - 2h + 1$ **f** $m^2 + 2mn + n^2$

14 The result is always the first term squared + 2 times the product of both terms + second term squared. The two factors are exactly the same.

15 **a** $(a + b)^2$ is a perfect square.
b **i** $x^2 + 6x + 9$ **ii** $y^2 + 12y + 36$
iii $m^2 + 4m + 4$ **iv** $16b^2 + 88b + 121$
v $m^2 + 2mn + n^2$ **vi** $25 + 10x + x^2$
c $(a - b)^2$ is also a perfect square. The rule or pattern holds true. The negative sign in the factor $(a - b)$ means that the middle term is $-2ab$ rather than $2ab$.
d **i** $a^2 - 4a + 4$ **ii** $b^2 - 8b + 16$
iii $c^2 - 14c + 49$ **iv** $25w^2 - 60w + 36$
v $k^2 - 2kp + p^2$ **vi** $9 - 6x + x^2$

16 **a** $6xw - 10x^2 + 14xy - 8xz$
b $6ax + 10bx - 12x + 9ay + 15by - 18y$
c $2x^2 + 7xy + 15x + 3y^2 + 15y + 18$
d $x^3 + 8x^2 + 19x + 14$

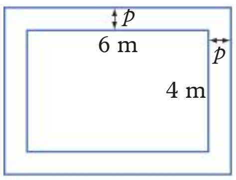
17 **a** **i** For Diagram 1: Area of large rectangle $= 7(3 + 2)$
$= 7 \times 5$
$= 35$
For Diagram 2: Area of large rectangle $= 5(x + 2)$
$= 5x + 10$
ii For Diagram 1:
Area of large rectangle $= 7 \times 3 + 7 \times 2$
$= 21 + 14$
$= 35$
For Diagram 2: Area of large rectangle $= 5 \times x + 5 \times 2$
$= 5x + 10$
b The area of the large rectangle is length × width or $a \times (b + c)$.
The total area of the two smaller rectangles is $a \times b + a \times c$.
As the two areas are equal, $a(b + c) = ab + ac$.

18 **a** **i** For Diagram 1:
Area of large rectangle $= (6 + 4)(3 + 5)$
$= 10 \times 8$
$= 80$
For Diagram 2:
Area of large rectangle $= (k + 2)(m + 3)$
$= km + 3k + 2m + 6$
ii For Diagram 1: Area of large rectangle
$= 6 \times 3 + 6 \times 5 + 4 \times 3 + 4 \times 5$
$= 18 + 30 + 12 + 20$
$= 80$
For Diagram 2: Area of large rectangle
$= k \times m + k \times 3 + 2 \times m + 2 \times 3$
$= km + 3k + 2m + 6$
b The area of the large rectangle is length × width or $(a + b) \times (c + d)$.
The total area of the four smaller rectangles is $a \times c + a \times d + b \times c + b \times d$.
As the two areas are equal,
$(a + b)(c + d) = ac + ad + bc + bd$.

19 D

20 **a** $8(2x + 5)$ **b** $16x + 40$
c **i** 88 cm² **ii** 88 cm²
d The answers should be the same if you have expanded correctly.

21 **a** **i** $(x + 7)(x + 3)$ **ii** $x^2 + 10x + 21$
iii 96 m²
b **i** $(x + 9)(x + 9)$ **ii** $x^2 + 18x + 81$
iii 196 cm²
c **i** $(2x - 1)(x + 2)$ **ii** $2x^2 + 3x - 2$
iii 63 cm²

22 **a**


b **i** $(2p + 6)$ m **ii** $(2p + 4)$ m
c $(2p + 6)(2p + 4) = 4p^2 + 20p + 24$
d $(4p^2 + 20p + 24) - (6 \times 4) = 4p^2 + 20p$
e **i** 24 m² **ii** 80 m² **iii** 56 m²

23 **a** $100y^2 - 140y + 49$ **b** $x^7 - x^6$
c $x^4 - 25$ **d** $y^{12} + 6y^7 + 9y^2$
e $-5a^3$ **f** $x^6y^6 + 2wx^5y^2$

24 $(a - b)^2 + (c - d)^2$
$= (a^2 - 2ab + b^2) + (c^2 - 2cd + d^2)$
$= (b^2 - 2ab + a^2) + (d^2 - 2cd + c^2)$
$= (b - a)^2 + (d - c)^2$

25 a Square: length = y, width = y
Rectangle: length = y, width = $x - y$

b Area of the original rectangle = xy
Total area of the two shapes $= (y)(x - y) + y^2$
$= xy - y^2 + y^2$
$= xy$

EX p64

2.3 Factorising using the HCF

1 a 4 **b** 2 **c** 5 **d** d **e** 2 **f** 3 **g** 4 **h** $3h$ **i** $12x$

2 a c **b** $2y$ **c** $9m$ **d** bd **e** $4y$ **f** $2k$ **g** p **h** 5 **i** $3p$

3 a $c(b + d)$ **b** $2y(x + 1)$ **c** $9m(2n - 1)$ **d** $bd(ac + f)$ **e** $4y(2x + 7)$ **f** $2k(3k - 5)$ **g** $p(1 + 11p)$ **h** $5(9ab - 8cd)$ **i** $3p(q + 2)$

4 a a^4 **b** b^2 **c** $2c^3$ **d** $6d^3z^5$ **e** $7e^6y^5$ **f** $2fx^9$ **g** $-2g$ **h** $6k^5$ **i** $x^9y^7z^{13}$

5 a $5(2x + 1)$ **b** $3(y - 7)$ **c** $4(2k + 3)$ **d** $3(5 - 2d)$ **e** $7(4x + 3y)$ **f** $10(2n - 5)$ **g** $xy(x + 3y^3)$ **h** $mn^2(mn - 9)$

6 a $5b(4a - 1)$ **b** $8d(1 + ce)$ **c** $5x(3x + 2)$ **d** $2k(2k - 11)$ **e** $6n(5 - 3n)$ **f** $a(16a + 1)$ **g** $2h(h - 7)$ **h** $6p(1 + p)$

7 a $8m^4(3m^2 + 2)$ **b** $7q^3r^2(3q^2 + 5r^4)$ **c** $5t^4u^2(3 - t^4u^5)$ **d** $6c^4d^4(2c^5d + 1)$ **e** $7e^5f^3(e^6 - 1)$ **f** $3ij^2k(i^4k^5 + 9j^5)$ **g** $b^3c^7(a^7b^9 + c^2d^4)$ **h** $pq(5m - 3np^9q^4)$ **i** $w^5z^3(u^4y^2 + v^4x^2)$

8 a $-5n(m + 2)$ **b** $-7x(2y + 1)$ **c** $-6c(1 - d)$ **d** $-a(a + 3)$ **e** $-2k(2k + 1)$ **f** $-8x(x - 1)$ **g** $-3(4 + xy)$ **h** $-2m(8 + 5m)$ **i** $-9xy(x - 2)$

9 a $(w + 4)(x + 2)$ **b** $(x - 1)(y + 7)$ **c** $(a + 6)(a - 3)$ **d** $(5 - n)(p + 8)$ **e** $(4 - k)(3k - 5)$ **f** $(3x - 4)(2x + 9)$ **g** $(2g + 1)(4g + 1)$ **h** $(8n - d)(2h - 1)$ **i** $(7x + 6y)(3a + 2b)$

10 a $(a + 5)(b + 4)$ **b** $(y - 6)(x + 7)$ **c** $(n + 4)(m - 2)$ **d** $(y + 3)(y + 5)$ **e** $(k - 7)(k + 2)$ **f** $(x + 3)(6 + x)$ **g** $(a - 7)(a - 2)$ **h** $(p + 5)(p - 2)$ **i** $(3c - 1)(2c + 3)$

11 Expanding means to write an expression without brackets. Factorising means to write an expression in factor form and generally involves using brackets. Factorising and expanding are opposite processes.

12 a i $6(x + 1)$ cm **ii** 36 cm
b i $2x(x + 4)$ cm **ii** 90 cm

13 a $(2x + 5)$ m **b** $(m + 15)$ cm

14 a $2(2x + 9)$ m **b** $2(2m + 15)$ cm

15 a $3(9x - 3y + 5z)$ **b** $5(9p - 10q - 1)$ **c** $4(1 - 5i + 10j - 15k)$ **d** $a^2(a^3 + a + 1)$ **e** $6b^3c^4(3c - 6b + 4b^5c)$ **f** $7r^5t^7(12r^7t + 1 + 7r^3t^7 + 2r^4t)$

16 a $\frac{1}{2}(5x + 3y)$ **b** $\frac{3}{5}(3g + h)$ **c** $\frac{5}{4}(2p + 3q)$ **d** $\frac{1}{12}(28m - 15n)$

17 a $x(10 - x)$ mm^2
b Length and width are x mm and $(10 - x)$ mm.
c Sample answers: 6 mm by 4 mm, 7 mm by 3 mm, 9 mm by 1 mm.

18 a Some possible answers are:
Height = $(7 - x)$ cm, base length = $6x$ cm
Height = $2(7 - x)$ cm, base length = $3x$ cm
Height = 6 cm, base length = $x(7 - x)$ cm
b Some possible answers, using the first sample answer in **a**:
Height = 3 cm, base length = 24 cm ($x = 4$)
Height = 6 cm, base length = 6 cm ($x = 1$)
Height = 2 cm, base length = 30 cm ($x = 5$)

19 $3x$ cm

20 a $(x + 3)(x - 4)$ **b** $(x - 4)(x + 3)$
c They are equivalent.

21 a $n + 1, n + 2$ **b** $3n + 3$
c $3(n + 1)$; three times middle number
d $n + (n + 2) + (n + 4) = 3n + 6 = 3(n + 2)$; also three times middle number

22 For four consecutive numbers, the sum is four times the average of the two middle numbers.
For six consecutive numbers, the sum is six times the average of the two middle numbers.
For 10 consecutive numbers, the sum is 10 times the average of the two middle numbers.

$$10 \times \left[\frac{(n + 4) + (n + 5)}{2}\right] = 10 \times \left(\frac{2n + 9}{2}\right) = 5(2n + 9)$$

23 a $(y + 5)(x - 3)$ **b** $(2q - 3)(p - 1)$
c $(a - 3)(2a - 3)$

24 Area of square $= l \times w = 2r \times 2r = 4r^2$
Area of circle $= \pi r^2$
Area of shaded section $= 4r^2 - \pi r^2 = r^2(4 - \pi)$

25 a $5\left(\frac{9x}{5} + \frac{17}{5}\right)$
b $12\left(x^2 + \frac{x}{4} + \frac{1}{2}\right)$
c $x\left(3x + 5y + \frac{y^2}{x}\right)$
d $(x + 2)\left(7(x + 2) + 5 + \frac{9}{x + 2}\right)$

EX p67

Checkpoint

1 a $4a - 2b - 5$ **b** $4t^2 + 12t - 4$ **c** $2x^3 - 7x^2 + 5x + 2$ **d** $4cd^2 - 10d^2 + 2c$

2 **a** $-35ab$ **b** $48a^2bc^2$
c $\frac{9h}{4}$ **d** $\frac{t}{3u}$

3 **a** $-800a^{16}b^{32}$ **b** $-\frac{729p^6}{8k^3}$
c $\frac{9t^4}{8r^7}$ **d** $\frac{2x^{28}}{3y^4}$

4 **a** $8a + 24$ **b** $-8b + 20$
c $21c - 3cy$ **d** $18d^2 - 9d$

5 **a** $3w + 41$ **b** $-x + 35$
c $15y - 9$ **d** $z^2 + 5z - 15$

6 **a** $pv + 2v + 5p + 10$ **b** $-qu - 3q + 4u + 12$
c $r^2 + 4r - 12$ **d** $6t^2 - 29t + 35$

7 **a** $a^7 + 2a^5 + 5a^4 + 10a^2$ **b** $b^3 + 7b^2 - 28b + 1$
c $c^2 - 11c - 10$ **d** $7d^2 - 8d + 19$

8 **a** $3(2a + 3)$ **b** $12(b - 3)$
c $-7(4c + 1)$ **d** $4d(2d + 9)$

9 **a** $(3g + 2)(2m + 5)$ **b** $(h - 3)(8 - n)$
c $(q + 7r)(p - 1)$ **d** $(x + 3)(2x - 5)$

10 **a** $(b + 5)(a + 4)$ **b** $(3d - 2)(4c - 7)$
c $(3f - 1)(5e + 1)$ **d** $(x + 6)(x + 5)$

EX p70

2.4 Factorising the difference of two squares

1 **a** $9x^2 - 49$
b Show each term as a square ($(3x)^2$ and 7^2) and then write the base of each term in two pairs of brackets. One pair of brackets contains the sum of the two bases and the other the difference.

2 **a** $(x + 6)(x - 6)$ **b** $(a + 10)(a - 10)$
c $(3 + y)(3 - y)$ **d** $(7 + x)(7 - x)$
e $(9 + a)(9 - a)$ **f** $(4 + y)(4 - y)$

3 **a** $(5m + 2)(5m - 2)$ **b** $(7 + 8c)(7 - 8c)$
c $(ab + 3)(ab - 3)$ **d** $(xy + w)(xy - w)$
e $(p + qr)(p - qr)$ **f** $(bn + dm)(bn - dm)$
g $(5jk + 6mn)(5jk - 6mn)$
h $(9ab + 7cd)(9ab - 7cd)$
i $(12pq + 11rt)(12pq - 11rt)$

4 **a** $3(x + 2)(x - 2)$ **b** $2(3p + 5)(3p - 5)$
c $5(10 + 7g)(10 - 7g)$ **d** $7(1 + 2u)(1 - 2u)$
e $4(3r + 5)(3r - 5)$ **f** $100(8 + 11z)(8 - 11z)$
g $6(x + y)(x - y)$ **h** $p(q + r)(q - r)$
i $u^2(t + d)(t - d)$ **j** $4d(3k + 2j)(3k - 2j)$
k $2a^2(9b + 11d)(9b - 11d)$ **l** $9x(1 + 3x)(1 - 3x)$

5 **a** $(k + 1)(k - 7)$ **b** $(8 - m)(6 - m)$
c $(y + 5)(1 - y)$ **d** $-a(a + 4)$
e $(y + 5 + x)(y + 5 - x)$ **f** $(u + t - 9)(u - t + 9)$
g $(3p + 4)(p + 4)$ **h** $(9 - 3m)(9 - 5m)$
i $3(2x + 3)$ **j** $(q + r + 1)(q - r + 11)$
k $(10x - 9)(4x + 1)$ **l** $4z$

6 **a** $(6 + x)(6 - x)$ **b** $(b + 4)(b - 4)$
c $(3h + 5)(3h - 5)$ **d** $(8n + 7d)(8n - 7d)$
e $(1 + rp)(1 - rp)$ **f** $(wx + tu)(wx - tu)$

7 **a** $\left(t + \frac{2}{3}\right)\left(t - \frac{2}{3}\right)$ **b** $\left(r + \frac{5}{7}\right)\left(r - \frac{5}{7}\right)$
c $\left(\frac{9}{4} + u\right)\left(\frac{9}{4} - u\right)$ **d** $\left(\frac{3}{10}d + \frac{11}{12}\right)\left(\frac{3}{10}d - \frac{11}{12}\right)$
e $\left(\frac{1}{8}k + \frac{1}{5}\right)\left(\frac{1}{8}k - \frac{1}{5}\right)$ **f** $\left(\frac{w}{13} + \frac{x}{14}\right)\left(\frac{w}{13} - \frac{x}{14}\right)$

8 **a** $(z + 0.4)(z - 0.4)$ **b** $(y + 1.1)(y - 1.1)$
c $(0.2 + x)(0.2 - x)$ **d** $(c + 0.05)(c - 0.05)$
e $(0.7g + 0.1h)(0.7g - 0.1h)$
f $(0.12 + 0.11k)(0.12 - 0.11k)$

9 **a** $(x^3 + 3)(x^3 - 3)$ **b** $(6 + y^5)(6 - y^5)$
c $(z^{11} + 7)(z^{11} - 7)$ **d** $(2p^7 + 5)(2p^7 - 5)$
e $\left(\frac{q^9}{4} + \frac{9}{11}\right)\left(\frac{q^9}{4} - \frac{9}{11}\right)$ **f** $(1 + r^{15}t^{25})(1 - r^{15}t^{25})$

10 **a** **i** 18×10 **ii** 25×-5 **iii** 197×1
iv 12×-4 **v** 16×10 **vi** 29×1
b **i** 180 **ii** –125 **iii** 197
iv –48 **v** 160 **vi** 29
c **i** 64 – 25 **ii** 144 – 324 **iii** 900 – 1
iv 81 – 9 **v** 100 – 900 **vi** 196 – 1
d **i** 39 **ii** –180 **iii** 899
iv 72 **v** –800 **vi** 195

11 **a** 2600 **b** 11.4 **c** $\frac{1}{25}$

12 **a** $2n + 1$ **b** Odd number

13 $4(x + 1)$

14 **a** $(x^2 + 4)(x + 2)(x - 2)$
b $(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

15 **a** $\pi R^2 - \pi r^2$ **b** $\pi(R + r)(R - r)$
c **i** 50.27 cm^2 **ii** 100.53 cm^2 **iii** 59.69 cm^2

16 5 cm and 7 cm

17 **a** $(5^2\pi - 4.75^2\pi + 3.75^2\pi - 3.5^2\pi + 2.5^2\pi - 2.25^2\pi + 1.25^2\pi - 1^2\pi)\text{ m}^2$
b $(5^2\pi - 4.75^2\pi + 3.75^2\pi - 3.5^2\pi + 2.5^2\pi - 2.25^2\pi + 1.25^2\pi - 1^2\pi)$

$$\begin{aligned}
&= \pi((5^2 - 4.75^2) + (3.75^2 - 3.5^2) + (2.5^2 - 2.25^2) + (1.25^2 - 1^2)) \\
&= \pi((5 + 4.75)(5 - 4.75) + (3.75 + 3.5)(3.75 - 3.5) + (2.5 + 2.25)(2.5 - 2.25) + (1.25 + 1)(1.25 - 1)) \\
&= \pi(0.25(5 + 4.75) + 0.25(3.75 + 3.5) + 0.25(2.5 + 2.25) + 0.25(1.25 + 1)) \\
&= 0.25\pi(5 + 4.75 + 3.75 + 3.5 + 2.5 + 2.25 + 1.25 + 1)\text{ m}^2
\end{aligned}$$

18
$$\begin{aligned}
&(x + 1)(x + y)^2 - (x + 1)(x - y)^2 \\
&= (x + 1)[(x + y)^2 - (x - y)^2] \\
&= (x + 1)[(x + y) + (x - y)][(x + y) - (x - y)] \\
&= (x + 1)(x + y + x - y)(x + y - x + y) \\
&= (x + 1)(2x)(2y) \\
&= 4xy(x + 1)
\end{aligned}$$

19 **a** $(\sqrt{a}+\sqrt{5})(\sqrt{a}-\sqrt{5})$ **b** $(b+\sqrt{7})(b-\sqrt{7})$
c $(\sqrt{c}+2\sqrt{2})(\sqrt{c}-2\sqrt{2})$ **d** $(\sqrt{3d}+4)(\sqrt{3d}-4)$
e $(2\sqrt{e}+3\sqrt{5})(2\sqrt{e}-3\sqrt{5})$
f $(f\sqrt{2}+5\sqrt{3})(f\sqrt{2}-5\sqrt{3})$
g $(x\sqrt{x}+y^2\sqrt{y})(x\sqrt{x}-y^2\sqrt{y})$
h $4(x\sqrt{2}+y\sqrt{3})(x\sqrt{2}-y\sqrt{3})$

20 **a** 9 m and 1 m **b** 7 m and 3 m
c 10 m and 3 m **d** 10.5 m and 4.5 m

21 9 cm^2

22 **a** x^3+y^3 **b** $(x+2)(x^2-2x+4)$
c $(x-2)(x^2+2x+4)$
d $(x-2)(x^2+2x+4)=x^3+8$
You can use the previous result, replacing 2 with −2.

23 31

2.5 Factorising quadratic expressions

EX p77

1 **a** 1 and 4 **b** 2 and 4 **c** 2 and 11
d 4 and 5 **e** 4 and 6 **f** 3 and 4
g 6 and 7 **h** 5 and 7 **i** 4 and 4

2 **a** −2 and 4 **b** −3 and 2 **c** −6 and −2
d −2 and 5 **e** −9 and 1 **f** −3 and −2
g −1 and 6 **h** −9 and 3 **i** −11 and −1

3 **a** $(x+1)(x+4)$ **b** $(x+2)(x+4)$
c $(x+2)(x+11)$ **d** $(x+4)(x+5)$
e $(x+4)(x+6)$ **f** $(x+3)(x+4)$
g $(x+6)(x+7)$ **h** $(x+5)(x+7)$
i $(x+4)^2$

4 **a** $(a+3)(a+1)$ **b** $(b+2)(b+7)$
c $(c+1)(c+6)$ **d** $(d+3)(d+7)$
e $(e+1)(e+7)$ **f** $(f+3)(f+5)$
g $(g+4)(g+7)$ **h** $(h+4)(h+9)$
i $(x+3)(x+6)$ **j** $(j+5)(j+9)$
k $(k+5)(k+6)$ **l** $(y+5)(y+8)$

5 **a** $(x-2)(x+4)$ **b** $(x-3)(x+2)$
c $(x-6)(x-2)$ **d** $(x-2)(x+5)$
e $(x-9)(x+1)$ **f** $(x-3)(x-2)$
g $(x-1)(x+6)$ **h** $(x-9)(x+3)$
i $(x-11)(x-1)$

6 **a** $(a-1)(a+3)$ **b** $(b-5)(b+3)$
c $(c-4)(c-1)$ **d** $(d-2)(d+7)$
e $(e-6)(e-4)$ **f** $(f-5)(f+2)$
g $(g-3)(g+4)$ **h** $(h-5)(h-3)$
i $(x-8)(x+3)$ **j** $(j-8)(j-2)$
k $(k-3)(k+6)$ **l** $(y-2)(y+1)$

7 **a** $3(x+1)(x+2)$ **b** $2(x+3)(x+5)$
c $5(x-1)(x+4)$ **d** $-4(x+2)(x+3)$
e $-6(x-5)(x-1)$ **f** $-(x-5)(x+7)$

8 **a** $a(x+2)(x+6)$ **b** $b(x-7)(x-4)$
c $10c(x-6)(x+5)$ **d** $-2d(x-5)(x-12)$
e $p^2(x+3)(x+10)$ **f** $x^2(q+11)(q+4)$

9 **a** $(x-3)^2$ **b** $(x-3)(x+3)$
c $(x+0)(x-6)=x(x-6)$

10 **a** $(x-5)(x+5)$ **b** $(x-5)^2$
c $x(x-25)$ **d** $(x+7)^2$
e $x(x-49)$ **f** $(x-7)(x+7)$

11 **a** $(a+3)(a+7)$ **b** $(b-5)(b+7)$
c $(c-3)(c+6)$ **d** $-(d+6)(d-8)$
e $-(e+4)(e+6)$ **f** $-(f-2)(f-8)$
g $(g+8)(g+9)$ **h** $(h-7)(h+2)$
i $-(x+4)(x+9)$

12 **a** $c=0;\ x^2+6x+0=(x+0)(x+6)$
$c=5;\ x^2+6x+5=(x+1)(x+5)$
$c=8;\ x^2+6x+8=(x+2)(x+4)$
$c=9;\ x^2+6x+9=(x+3)(x+3)$
b $c=-7;\ x^2+6x-7=(x-1)(x+7)$
$c=-16;\ x^2+6x-16=(x-2)(x+8)$
$c=-27;\ x^2+6x-27=(x-3)(x+9)$
$c=-40;\ x^2+6x-40=(x-4)(x+10)$
c When the coefficient of x is positive, look at the constant term. If the constant is positive, then both factors are positive. If the constant is negative, then one factor is positive and the other is negative.

13 **a** $c=0;\ x^2-6x+0=(x-0)(x-6)$
$c=5;\ x^2-6x+5=(x-1)(x-5)$
$c=8;\ x^2-6x+8=(x-2)(x-4)$
$c=9;\ x^2-6x+9=(x-3)(x-3)$
b $c=-7;\ x^2-6x-7=(x+1)(x-7)$
$c=-16;\ x^2-6x-16=(x+2)(x-8)$
$c=-27;\ x^2-6x-27=(x+3)(x-9)$
$c=-40;\ x^2-6x-40=(x+4)(x-10)$
c When the coefficient of x is negative, look at the constant term. If the constant is positive, then both factors are negative. If the constant is negative, then one factor is positive and the other is negative.

14 **a** $b=65;\ x^2+65x+64=(x+1)(x+64)$
$b=34;\ x^2+34x+64=(x+2)(x+32)$
$b=20;\ x^2+20x+64=(x+4)(x+16)$
$b=16;\ x^2+16x+64=(x+8)(x+8)$
b $b=-65;\ x^2-65x+64=(x-1)(x-64)$
$b=-34;\ x^2-34x+64=(x-2)(x-32)$
$b=-20;\ x^2-20x+64=(x-4)(x-16)$
$b=-16;\ x^2-16x+64=(x-8)(x-8)$
c **i** If the coefficient of x is positive, use positive factors when the constant is positive. If the coefficient of x is negative, use negative factors when the constant is positive.
ii Sum

15 **a** $b=63;\ x^2-63x-64=(x+1)(x-64)$
$b=30;\ x^2-30x-64=(x+2)(x-32)$
$b=12;\ x^2-12x-64=(x+4)(x-16)$
$b=0;\ x^2-0x-64=(x+8)(x-8)$

b $b = -63; x^2 + 63x - 64 = (x - 1)(x + 64)$
$b = -30; x^2 + 30x - 64 = (x - 2)(x + 32)$
$b = -12; x^2 + 12x - 64 = (x - 4)(x + 16)$
$b = 0x^2 - 0x - 64 = (x - 8)(x + 8)$

c **i** If the coefficient of x is positive, the larger factor is positive and the smaller factor is negative when the constant is negative. If the coefficient of x is negative, the larger factor is negative and the smaller factor is positive when the constant is negative.
ii Difference

16 **a** $(x + 4)(x + 4) = (x + 4)^2$
b $(y - 5)(y - 5) = (y - 5)^2$
c $(v + 1)(v + 1) = (v + 1)^2$
d $(2z - 3)(2z - 3) = (2z - 3)^2$
e $(3w + 7)(3w + 7) = (3w + 7)^2$
f $3(h - 2)(h - 2) = 3(h - 2)^2$
g $-(k + 9)(k + 9) = -(k + 9)^2$
h $(f + g)(f + g) = (f + g)^2$
i $(5p - 4q)(5p - 4q) = (5p - 4q)^2$

17 **a** Check for and factorise out any common factors. Halve the coefficient of the linear term and check if it is equal to the product of the square root of the squared terms (or squared term and constant). Both squared terms must be positive.
b **i** Both squared terms are not positive.
ii Both squared terms are not positive.
iii Half of the linear term is not the product of the square root of the squared terms.
iv Half of the linear term is not the product of the square root of the squared terms.

18 **a** $(x + 6)$ **b** $(x + 3)(x + 6)$
c **i** Length = 8 m, width = 5 m, area = 8 × 5 = 40 m²
ii Area = $2^2 + 9 \times 2 + 18 = 40$ m²

19 **a** **i** $(x + 9)$ cm **ii** $(3y + 4)$ m
b 10
c 7
d $x = 2, y = \frac{4}{3}$
e While one of the lengths is positive, the other length and area are both negative, which does not make sense for a physical rectangle. That is, the values for the length, width and area for both items must all be positive.

20 **a** Option 1: $(x - 2)(x - 3)$
Option 2: $(x - 2)(x - 5)$
Option 3: $(x - 2)(x - 2)$
Option 4: $(x - 2)(x - 4)$
b 2 m
c $x^2 - x - 6 = (x - 3)(x + 2)$ and $x^2 + x - 6 = (x - 2)(x + 3)$
Both are wider than their backyards' width, x m.
d $x^2 - 6x + 8$
e 8 m²
f 19.25 m²
g Melissa: 16 m², Lena: 22 m²

21 **a** $(x + 4)(x + 1)$ **b** $(x + 4)(x + 2)$
c $(x + 11)(x + 2)$ **d** $(x + 5)(x - 2)$
e $(x - 9)(x + 1)$ **f** $(x - 3)(x - 2)$

22 **a** $(3x - 2)(3x + 7)$ **b** $(11x + 10)(11x - 8)$
c $(5x - 4)(5x - 8)$ **d** $(x + 6)(x + 1)$
e $(x - 9)(x - 10)$ **f** $x(x - 4)$
g $3(x + 2)(3x - 7)$ **h** $2(x + 5)(2x - 3)$
i $16(2x - 1)(2x + 3)$

23 **a** $(x - 2)(x + 2)(x - 3)(x + 3)$
b $(x^2 + 4)(x - 4)(x + 4)$
c $(x^2 + 25)(x - 1)(x + 1)$

EX Review: Algebra

p82

Review questions 2.6A: Mathematical literacy review

1 **a** terms, binomial, trinomial
b monic quadratic trinomial
c linear term, constant term
d Factorising, highest common factor

2 **a** II **b** VI **c** VIII
d VII **e** I **f** IV
g III **h** V

Review questions 2.6B: Multiple choice

1 D	2 B	3 A	4 B
5 B	6 C	7 A	8 B
9 A	10 A	11 C	12 D
13 D	14 C	15 A	16 C

Review questions 2.6C: Short answer

1 **a** $16t$ **b** $13a - 18p$ **c** $-2k + km - 15$
d $13m^2n - 5m^2 + 11n^2 - 4mn^2$

2 **a** $44x^2y^2z$ **b** $72m^4n^3p^2$ **c** $\frac{5e}{6f}$ **d** $-\frac{n}{2m}$

3 **a** $4z - 28$
b $-40 + 24y$
c $35x - 30wx$
d $ut - 4u + 3t - 12$
e $40r - 15rv + 48 - 18v$
f $63mp - 27np + 77mq - 33nq$

4 **a** $8a - 11$ **b** $b^2 - 9b - 22$
c $12c^2 - 23c + 10$ **d** $d^2 - w^2$
e $36 + 12e + e^2$ **f** $f^2 - 18f + 81$

5 **a** $x^7 - x^6$ **b** $y^{12}z^4 - y^3z^6$
c $x^{11} + x^5y^4 - x^6y^3 - y^7$

6 **a** $4(a - 6)$ **b** $36pq(q + 4)$
c $(8 - d)(7d - 4)$ **d** $(1 + 3f)(5e + 2)$

7 **a** $z^4(6z - 5)$ **b** $3d(3dy + 5x^2)$
c $-12r^7t^8u^3(3r^3u^3 + 8)$
d $a^4b^2c^7d^5(ad^3 + a^2b^3c^2 + cd^2)$

8 **a** $8x(x^2+2)(3x-1)$ **b** $(r+q)(t+p+2)$
c $2(x+4)(2x-3)$

9 **a** $(a+8)(a-8)$ **b** $(11+b)(11-b)$
c $(6m+7n)(6m-7n)$ **d** $(p+3)(p-1)$
e $2(e+4)(e-4)$ **f** $9(2f-1)$

10 **a** $(y^5-z^5)(y^5+z^5)$ **b** $-8(q+2)$
c $(10+r)(10-r)$ **d** $(9j-8)(9j+8)$

11 **a** $\left(\frac{14}{5}x+\frac{13}{2}y\right)\left(\frac{14}{5}x-\frac{13}{2}y\right)$ **b** $\left(\frac{12}{c}+\frac{15}{d}\right)\left(\frac{12}{c}-\frac{15}{d}\right)$
c $\left(\frac{6}{7}+\frac{u}{4}\right)\left(\frac{6}{7}-\frac{u}{4}\right)$
d $(0.2v-1.1)(0.2v+1.1)$

12 **a** $(a+5)(a+1)$ **b** $(b-4)(b-3)$
c $(c-3)(c+7)$ **d** $(d-18)(d+2)$

13 **a** $(x+3)(x+5)$ **b** $-5(x-6)(x+2)$
c $-2(x-10)(x-6)$ **d** $-3(x+10)(x-8)$

14 **a** $2(7-x)(7+x)$ **b** $16(y^2+4x^2)$
c $(x-5)^2$ **d** $9x(x-4)$

Review questions 2.6D: Mathematical modelling

1 **a** **i** $(x+5)(x+11)$ **ii** $(x-6)(x-14)$
iii $(x+6)(x+16)$
b The numbers in the brackets in factor form are $a+b$ and $a-b$.
c **i** $x^2+16x+55$ **ii** $x^2-20x+84$
iii $x^2+22x+96$
d The coefficient of x in the expanded form is $2a$.
e The constant in the expanded form is a^2-b^2.
f **i** $(x+3)^2-4=x^2+6x+5=(x+1)(x+5)$
ii $(x-5)^2-16=x^2-10x+9=(x-9)(x-1)$
iii $(x+7)^2-100=x^2+14x-51=(x-3)(x+17)$

2 **a** $\frac{1}{2}n(n+1)=\frac{1}{2}n^2+\frac{1}{2}n, \frac{(n+1)^2-(n+1)}{2}=\frac{1}{2}n^2+\frac{1}{2}n$
b **i** 15 **ii** 15
c 5050
d **i** $n(n+1)$ **ii** 10 100
iii The sum of the first n even numbers is double the sum of the first n counting numbers.
e **i** $\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n=\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}$
ii Method 1: 55; Method 2: 55
iii 338 350

MODULE 3 Linear relationships

EX p90

3.1 Solving linear equations

1 **a** $x=6$ **b** $x=5$ **c** $x=36$ **d** $x=10$
e $x=-20$ **f** $x=4$ **g** $x=15$ **h** $x=-1$
i $x=-3$ **j** $x=-3$ **k** $x=12$ **l** $x=-6$

2 **a** $x=2$ **b** $x=-8$ **c** $x=9$ **d** $x=5$
e $x=20$ **f** $x=-8$ **g** $x=0$ **h** $x=-4$
i $x=-5$ **j** $x=10$ **k** $x=-20$ **l** $x=4$

3 **a** $x=6.9$ **b** $x=-2.6$ **c** $x=10.8$ **d** $x=-3.16$

4 **a** $x=6$ **b** Yes, $x=6$ **c** $x=5.6$ **d** $x=5.6$
e Expanding brackets first means you do not have to work with fractions or decimals until the last step.

5 **a** $x=3$ **b** $x=-9$ **c** $x=\frac{11}{2}$
d $x=-\frac{12}{5}$ **e** $x=4$ **f** $x=\frac{11}{6}$

6 **a** $x=4$ **b** Multiply both sides by x
c $x=4$

7 **a** $x=5$ **b** $x=-3$ **c** $x=-6$ **d** $x=4$
e $x=\frac{11}{2}$ **f** $x=-\frac{8}{5}$ **g** $x=4.8$ **h** $x=-\frac{1}{3}$

8 **a** $x=2$ **b** $x=7$ **c** $x=-1$ **d** $x=-3$
e $x=-2$ **f** $x=7$ **g** $x=4$ **h** $x=-4$

9 **a** $x=-1$ **b** $x=3$ **c** $x=7$ **d** $x=-3$
e $x=7$ **f** $x=-3$ **g** $x=-6$ **h** $x=9$

10 **a** $2x+5=x-4$ **b** $x=-9$

11 **a** $x=5$ **b** $x=-3$ **c** $x=2$ **d** $x=-3$

12 **a** $\frac{3(5x-1)}{6}=\frac{2(x+5)}{6}$ **b** $x=1$

13 **a** $x=6$ **b** $x=-4$ **c** $x=7$ **d** $x=-27$

14 **a** Let n = number of jellybeans
b $4n+2=34$
c $n=8$
d Each person received 8 jellybeans.

15 **a** Let x = number of months
b $70x+115=395$
c $x=4$ **d** 4 months

16 **a** \$185 **b** 14 goals
c 31 m × 19 m **d** 84 people

17 **a** B **b** \$3.50

18 **a** $P=3l$
b $w=10$ m

19 56 sausages

20 100°

21 Numbers are 5, 6 and 7.

22 **a** **i** Always **ii** Never **iii** Sometimes
iv Never **v** Always **vi** Never
b After expanding and simplifying, if the coefficient of the pronumeral is the same on both sides and the constants on both sides are equal. Alternatively, the equation reduces to $0=0$.
c After expanding and simplifying, if the coefficient of the pronumeral is the same on both sides and the constants on both sides are not equal. Alternatively, the equation reduces to an equation that is not true, such as $0=2$.

23 **a** $x=\frac{11}{5}=2.2$ **b** $x=\frac{19}{13}$ **c** $x=\frac{13}{7}$

EX p97

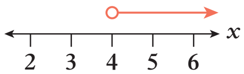
3.2 Solving linear inequalities

1 **a** $2<3$ **b** $3>2$
c $3\geq 2$ **d** $-3\leq -2$
e $3\neq -3$ **f** $-3\neq 3$

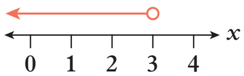
2 **a** $x\leq 5$ **b** $x>1$

c $x \geq -4$ **d** $x < -2$
e $x < 0$ **f** $x \neq 3$
g $x \geq 0$ **h** $x \neq -1$

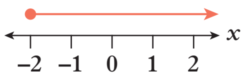
3 **a** $x > 4$



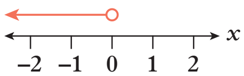
b $x \leq 3$



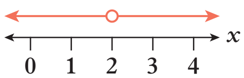
c $x \geq -2$



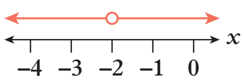
d $x < 0$



e $x \neq 2$



f $x \neq -2$

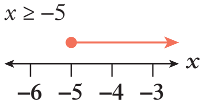


4 **a** **i** $1 < 14$ **ii** $-13 < 0$
iii $-12 < 27$ **iv** $-2 < \frac{9}{2}$
v $12 > -27$ **vi** $2 > -\frac{9}{2}$
vii $-\frac{4}{3} < 3$ **viii** $\frac{4}{3} > -3$

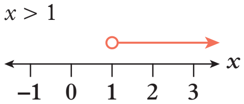
b **i** $x + 5 \geq 11$ **ii** $x - 9 \geq -3$
iii $3x \geq 18$ **iv** $\frac{x}{2} \geq 3$
v $-3x \leq -18$ **vi** $-\frac{x}{2} \leq -3$
vii $\frac{x}{3} \geq 2$ **viii** $-\frac{x}{3} \leq -2$

5 **a** $x > 14$ **b** $x \leq 300$ **c** $x < -5$
d $x \leq 5$ **e** $x < -70$ **f** $x \leq 5$
g $x \geq 10$ **h** $x > -3$ **i** $x \leq -9$
j $x \leq 7$ **k** $x > -37$ **l** $x > -\frac{13}{10}$
m $x < -\frac{1}{7}$ **n** $x \leq \frac{29}{6}$ **o** $x > \frac{11}{6}$
p $x \leq -\frac{95}{33}$

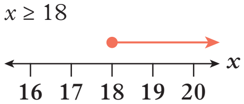
6 **a** $x \geq -5$



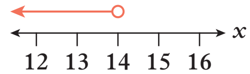
b $x > 1$



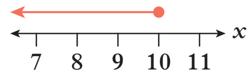
c $x \geq 18$



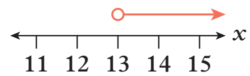
d $x < 14$



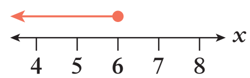
e $x \leq 10$



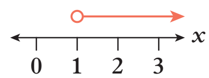
f $x > 13$



g $x \leq 6$



h $x > 1$



7 **a** $x > 4$ **b** $x \leq -2$ **c** $x \geq 3$
d $x > 4$ **e** $x < 1$ **f** $x \geq -1$
g $x \leq 5$ **h** $x < 2$

8 **a** $x > 11$ **b** $x \leq 13$ **c** $x \geq -2$
d $x > -19$ **e** $x > -15$ **f** $x \leq 1$
g $x \leq 3$ **h** $x > -\frac{1}{4}$ **i** $x \geq -\frac{13}{2}$

9 **a** $x \leq -7$ **b** $x > 1$ **c** $x \leq -3$
d $x < 1$ **e** $x > 4$ **f** $x > -5$

10 **a** $x > 13.5$ **b** $x > 2$ **c** $x \geq 1.5$
d $x < 3$ **e** $x \geq 0.5$ **f** $x < -3.5$

11 **a** $p \geq 750\,000$, where p is the selling price in dollars
b $h < 200$, where h is the height of a person in centimetres
c $s \leq 55$, where s is the speed in km/h
d $h < 196$, where h is the height of a person in centimetres

12 **a** $m = \frac{1}{2}(20 - x)$, where m is the number of watermelons they will each take home
b $\frac{1}{2}(20 - x) \leq 3$; $x \geq 14$
c 14 or more watermelons

13 **a** 4 **b** No **c** 5

14 **a** $3n \leq 25$, where n is the number of packs of playing cards Hassan could buy; $n \leq \frac{25}{3}$
b Hassan could buy 1, 2, 3, 4, 5, 6, 7 or 8 packs of playing cards.

15 **a** **i** $4 < 9$ **ii** $4 < 9$
iii $4 < 9$ **iv** $4 < 9$
v $\frac{1}{4} > \frac{1}{9}$ **vi** $\frac{1}{4} > \frac{1}{9}$
vii $4 = 4$

b The square of a negative and a positive number are both positive, so when two values in an inequality have different signs, then the order of the inequality may change.

c **i** $\frac{1}{3} < 2$ **ii** $-\frac{1}{3} < 2$

iii $-\frac{1}{3} > -2$ **iv** $\frac{1}{3} > -2$

v $3 > 2$ **vi** $-3 < 2$

d The greater the positive number, the smaller its reciprocal will be. The smaller the negative number, the greater its reciprocal will be. Therefore, when the signs of the values are the same, the order will reverse. However, when the signs of the values are different, the order does not change as the negative value will still be smaller than the positive value.

16 a $4x + 5y \le 100$ **b** $x \ge 0$ and $y \ge 0$

c $x = 25$ **d** $y = 20$

e $x = 15$

17 a $x \ge -11$ **b** $x < 4$ **c** $x \ge 2$

d $x < -8$ **e** $x \le 2$ **f** $x \le -3$

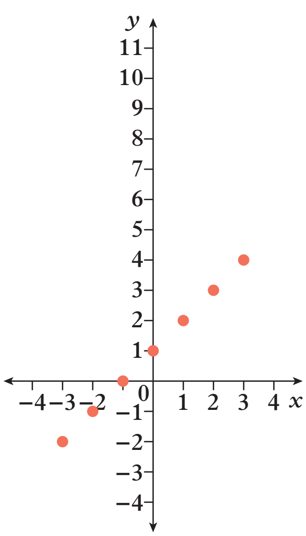
18 a $-1 \le x \le 4$ **b** $-\frac{9}{2} < x < -\frac{5}{2}$

c $-8 < x < 12$ **d** $-\frac{9}{2} < x \le \frac{9}{10}$

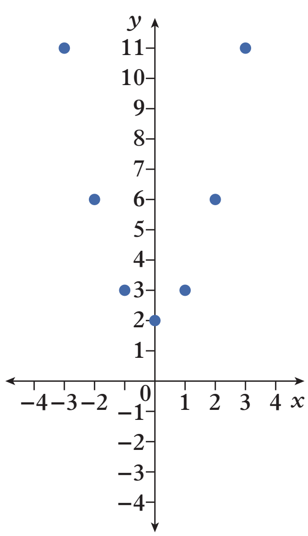
EX
p103

3.3 Plotting linear relationships

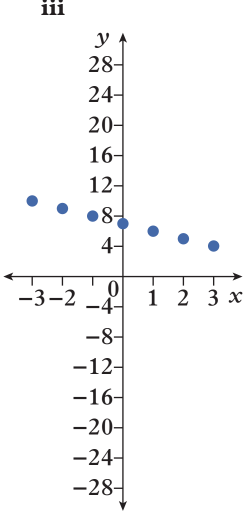
1 a i



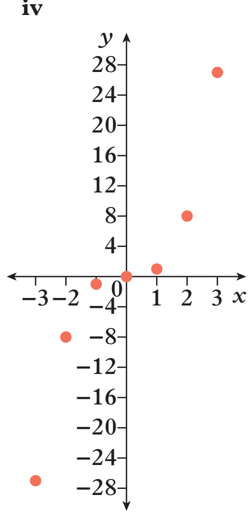
ii



iii



iv



b i Linear **ii** Non-linear

iii Linear **iv** Non-linear

2 a $y = x + 2$

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

(−3, −1), (−2, 0), (−1, −1), (0, 2), (1, 3), (2, 4), (3, 5)

b $y = x - 4$

x	−3	−2	−1	0	1	2	3
y	−7	−6	−5	−4	−3	−2	−1

(−3, −7), (−2, −6), (−1, −5), (0, −4), (1, −3), (2, −2), (3, −1)

c $y = 3 - x$

x	−3	−2	−1	0	1	2	3
y	6	5	4	3	2	1	0

(−3, 6), (−2, 5), (−1, 4), (0, 3), (1, 2), (2, 1), (3, 0)

d $y = 2 - x$

x	−3	−2	−1	0	1	2	3
y	5	4	3	2	1	0	−1

(−3, 5), (−2, 4), (−1, 3), (0, 2), (1, 1), (2, 0), (3, −1)

e $y = -x - 3$

x	−3	−2	−1	0	1	2	3
y	0	−1	−2	−3	−4	−5	−6

(−3, 0), (−2, −1), (−1, −2), (0, −3), (1, −4), (2, −5), (3, −6)

f $y = 4x$

x	−3	−2	−1	0	1	2	3
y	−12	−8	−4	0	4	8	12

(−3, -12), (−2, −8), (−1, −4), (0, 0), (1, 4), (2, 8), (3, 12)

g $y = 2x + 1$

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7

(−3, −5), (−2, −3), (−1, −1), (0, 1), (1, 3), (2, 5), (3, 7)

h $y = 3x - 2$

x	−3	−2	−1	0	1	2	3
y	−11	−8	−5	−2	1	4	7

(−3, −11), (−2, −8), (−1, −5), (0, −2), (1, 1), (2, 4), (3, 7)

i $y = 4 - 2x$

x	−3	−2	−1	0	1	2	3
y	10	8	6	4	2	0	−2

(−3, 10), (−2, 8), (−1, 6), (0, 4), (1, 2), (2, 0), (3, −2)

3 a IV: Radius
DV: Area because the formula for the area of a circle depends on the radius.

b IV: Number sold
DV: Revenue because it depends on the number of items sold.

c IV: Distance
DV: Light intensity because it depends on the distance from the source.

d IV: Number of words
DV: Number of pages because it depends on the number of words.

e IV: Amount of breath
DV: Volume because the volume of the note depends on the amount of breath used to play a note.

f IV: Number of cards
DV: Chance because the chance of drawing an ace of spades depends on the number of cards remaining.

4 **a** **i**

x	0	3	6	9	12	15	18
y	24	21	18	15	12	9	6

ii

x	−12	−6	−2	2	6	12
y	−2	−4	−12	12	4	2

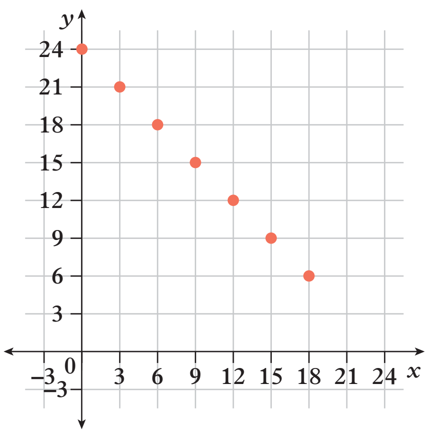
iii

x	−3	−2	−1	0	1	2	3
y	3	0	−1	0	3	8	15

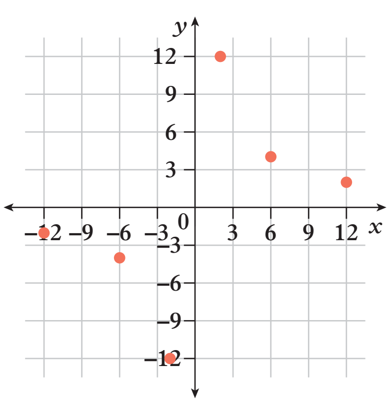
iv

x	−14	−10	−6	−2	2	6	10
y	−3	−2	−1	0	1	2	3

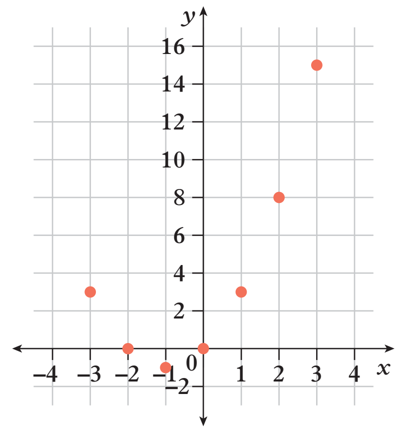
b **i** Linear



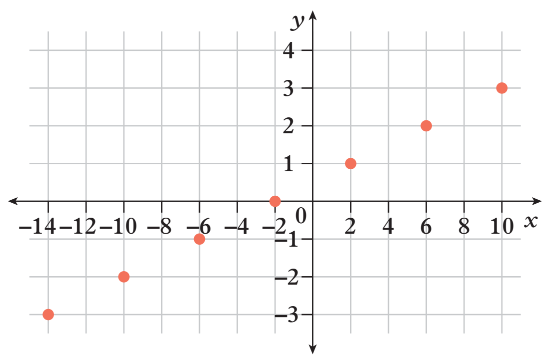
ii Non-linear



iii Non-linear



iv Linear



5 **a** **i** $y = x$

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

ii $y = 2x$

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

iii $y = 3x$

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9

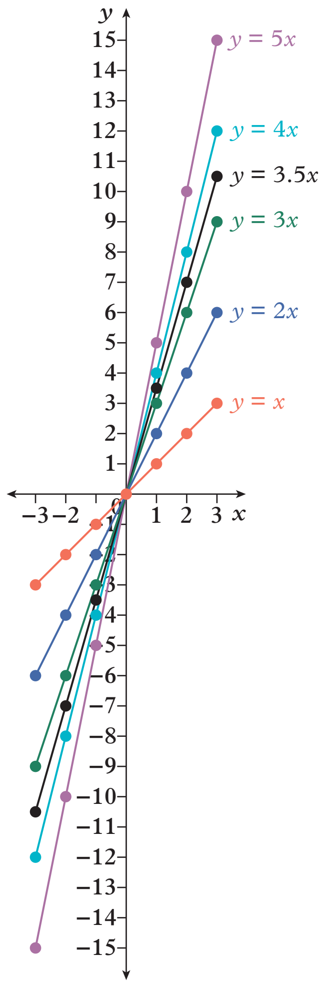
iv $y = 4x$

x	−3	−2	−1	0	1	2	3
y	−12	−8	−4	0	4	8	12

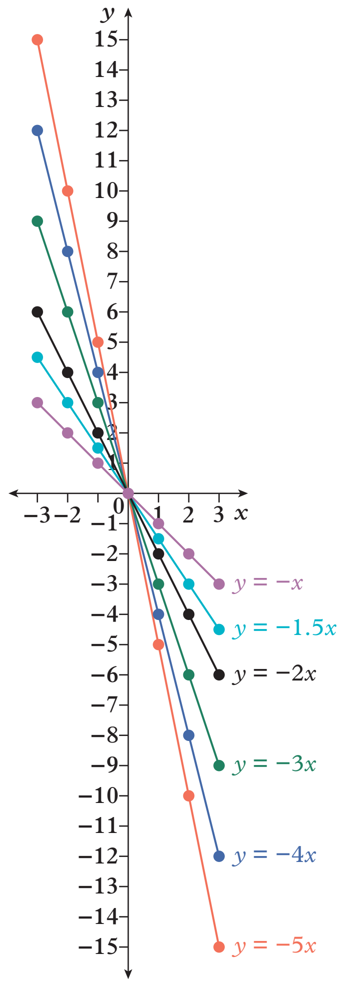
b All four graphs pass through the origin. However, the steepness of each line is different.

c As the coefficient of x increases, the steepness of the corresponding graph increases.

a, d



a, d



e The graph of $y = 0.5x$ is a straight line passing through the origin with an increase of 0.5 in the y-direction for each increase of 1 in the x-direction. The steepness of the line is half the steepness of the line of $y = x$.

6 a i $y = -x$

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3

ii $y = -2x$

x	−3	−2	−1	0	1	2	3
y	6	4	2	0	−2	−4	−6

iii $y = -3x$

x	−3	−2	−1	0	1	2	3
y	9	6	3	0	−3	−6	−9

iv $y = -4x$

x	−3	−2	−1	0	1	2	3
y	12	8	4	0	−4	−8	−12

b The values of y decrease from left to right.

c As the coefficient of x decreases, the steepness of the corresponding graph increases.

7 a i $y = x$

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

ii $y = x + 1$

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4

iii $y = x + 2$

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

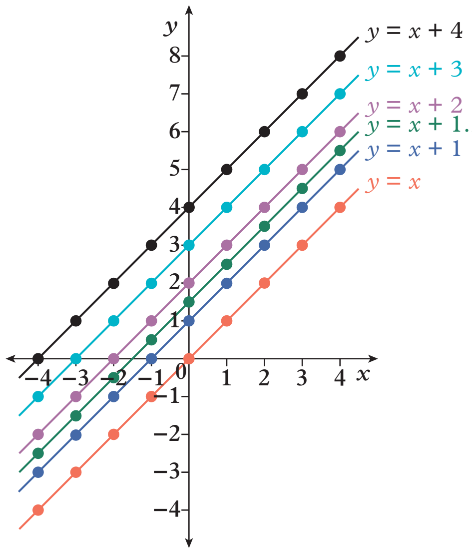
iv $y = x + 3$

x	−3	−2	−1	0	1	2	3
y	0	1	2	3	4	5	6

b All four graphs have the same steepness, but pass through different points on the x- and y-axes.

c As the value added to x increases, the graph moves upwards.

a, **d**



8 a i

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

ii

x	−3	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1	2

iii

x	−3	−2	−1	0	1	2	3
y	−5	−4	−3	−2	−1	0	1

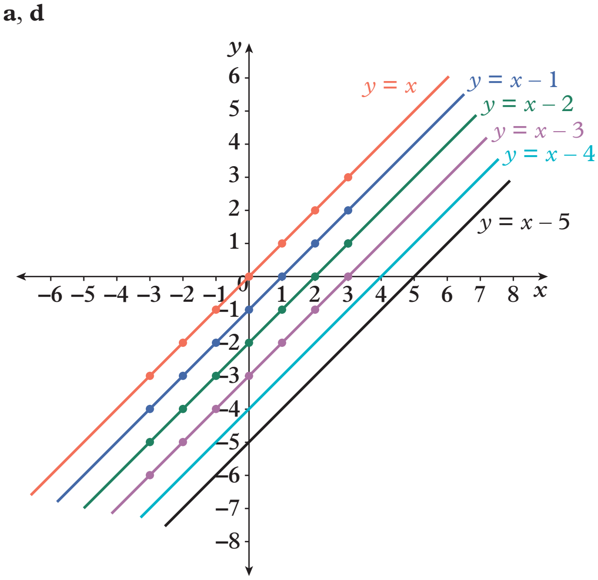
iv

x	−3	−2	−1	0	1	2	3
y	−6	−5	−4	−3	−2	−1	0

b All four graphs have the same steepness, but pass through different points on the x- and y-axes.

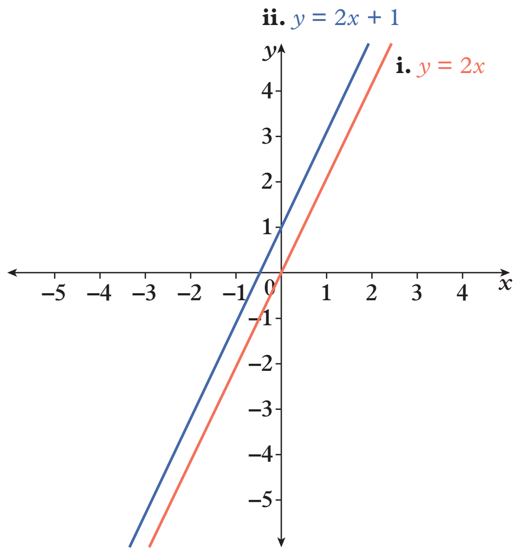
c As the value added to x decreases, the graph moves downwards.

a, **d**

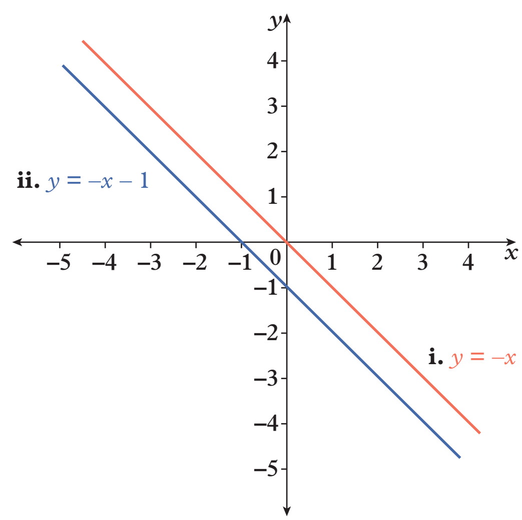


9 a A **b** D **c** B **d** C

10 a



b



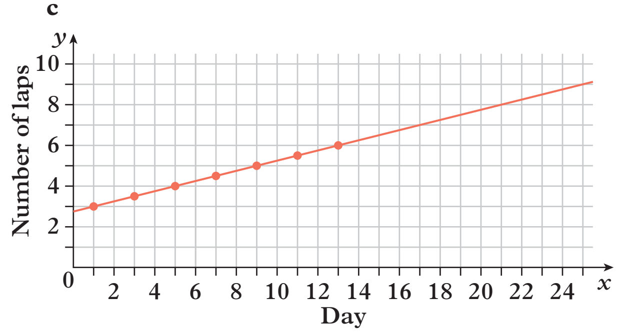
c Both graphs have the same steepness in each case, but the second graph is moved up in **a** and down in **b** in comparison to the first graph.

11 a IV: Day number, DV: Number of laps

b i The number of laps increases by the same amount every 2 days.

ii 2 points

c



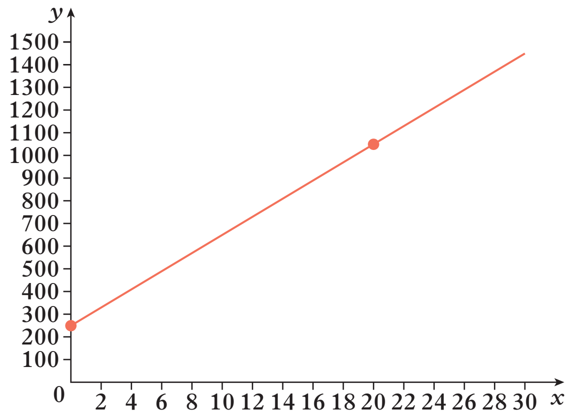
d i 7 laps **ii** Day 21

12 a $m = 40n + 250$

b Independent variable: number of students, n
Dependent variable: total amount of money collected for bus hire, m

c i \$250 **ii** \$1050

d



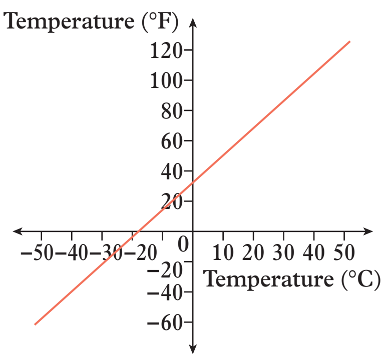
e Relationship is linear because it forms a straight line.

f \$1450

g 15 students

h 27 students

13 a



b 86°F

c −30°C

14 −40°F = −40°C

15 a (1, 2) **b** (0, 0) **c** (2, 0) **d** (4, 4)

EX p109

3.4 Gradient and intercepts

1 a $\frac{1}{3}$ **b** $-\frac{3}{2}$ **c** 2

2 a $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** 2 **d** $-\frac{1}{3}$

e $\frac{2}{5}$ **f** −3 **g** Undefined **h** −5

i $\frac{7}{3}$ **j** Undefined **k** −2 **l** 0

3 a Zero **b** Positive

c Negative **d** Undefined

4 a i 2 **ii** 1 **iii** −2

b i Undefined **ii** −3 **iii** None

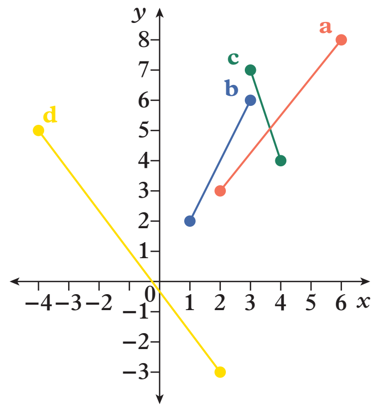
c i −2 **ii** −1 **iii** −2

d i 0 **ii** None **iii** 1

e i −6 **ii** 0 **iii** 0

f i $-\frac{2}{3}$ **ii** −3 **iii** −2

5 i



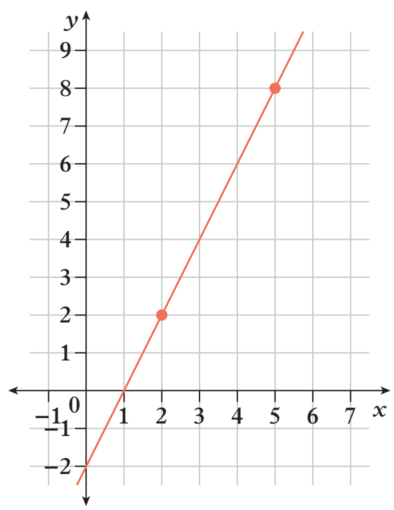
a ii $\frac{5}{4}$

b ii 2

c ii −3

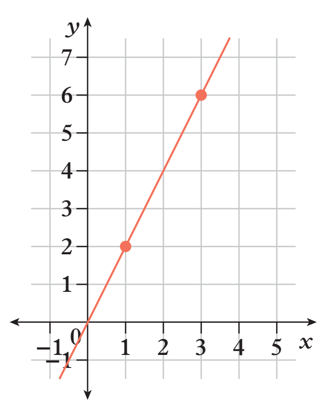
d ii $-\frac{4}{3}$

6 a i



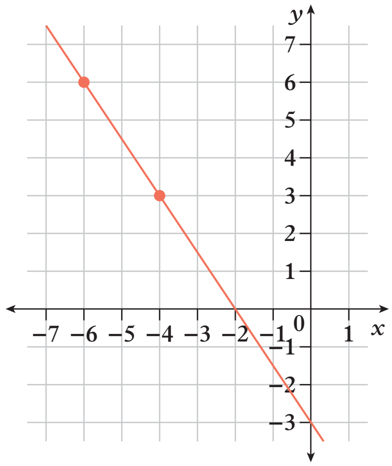
ii (1, 0), (0, −2)

b i



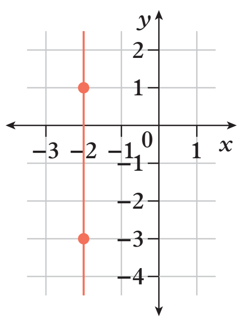
ii (0, 0)

c **i**



ii $(-2, 0)$, $(0, -3)$

d **i**



ii $(-2, 0)$

7 **a** **i** $\frac{3}{2}$ **ii** $\frac{3}{2}$

b The answers are the same. The gradient of a line is the same as the gradient of any line segment on that line.

8 **a** **i** Undefined

ii The value of $x_2 - x_1$ is 0. This is the denominator of the fraction in the gradient formula, but dividing by zero is undefined.

b **i** 0

ii The value of $y_2 - y_1$ is 0. This is the numerator of the fraction in the gradient formula, and dividing zero by a non-zero number always gives zero.

9 **a** **i** 0 **ii** 1 **iii** $\frac{1}{2}$ **iv** -1

b **i** $\frac{1}{2}$ **ii** 1 **iii** 0 **iv** -1

10 Kane subtracted the second y-coordinate from the first, and the first x-coordinate from the second; $m = -\frac{3}{4}$.

11 The order in which the coordinates are substituted into the gradient formula does not affect the value of the gradient.

12

$x_2 - x_1$	8	-20	-28	40	4	-6
$y_2 - y_1$	12	16	-4	-16	28	30
Gradient	$\frac{3}{2}$	$-\frac{4}{5}$	$\frac{1}{7}$	$-\frac{2}{5}$	7	-5

13 **a**

	Equation	Gradient	y-intercept
i	$y = 2x - 3$	2	$(0, -3)$
ii	$y = -\frac{3}{4}x + 5$	$-\frac{3}{4}$	$(0, 5)$
iii	$y = x + 4$	1	$(0, 4)$
iv	$y = -2x - 1$	-2	$(0, -1)$
v	$y = 4x$	4	$(0, 0)$
vi	$y = 2$	0	$(0, 2)$

b Gradient is coefficient of x; y-intercept is the constant

c **i** Gradient = 6, y-intercept: $(0, 4)$

ii Gradient = 1, y-intercept: $(0, -5)$

iii Gradient = -3, y-intercept: $(0, 0)$

14 **a** $x = 3$ **b** $y = 0$ **c** $y = 8$

d $y = 10$ **e** $y = 3$ **f** $x = 7$

15 **a** $\frac{y-4}{x} = 2$ **b** $y = 2x + 4$

c **i** $\frac{y-4}{x-1} = 2$ **ii** $y = 2x + 2$

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Checkpoint

1 **a** $x = -27$ **b** $x = 5$ **c** $x = 22$ **d** $x = 7$

2 **a** $x = 6$ **b** $x = \frac{1}{6}$ **c** $x = 6.25$ **d** $x = 40$

3 **a** $x = -4$ **b** $x = 2$ **c** $x = -5$ **d** $x = -3$

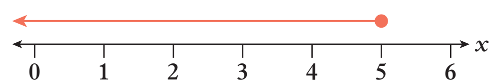
4 **a** Let n be the cost of one video game.

b $5n + 9.95 = 409.7$

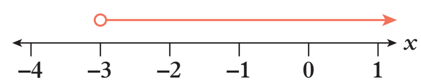
c $n = 79.95$

d $79.95

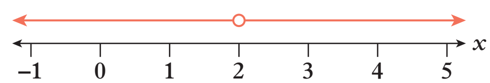
5 **a**



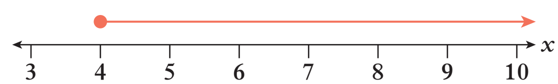
b



c



d



6 **a** $x < 5$ **b** $x \neq 6$

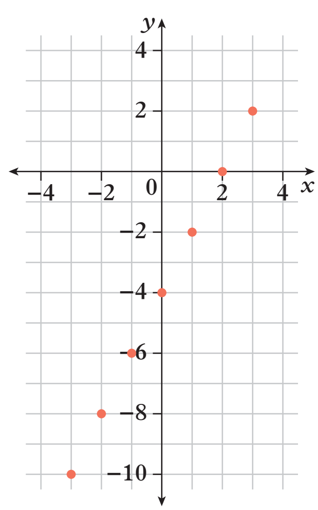
c $x \geq -5$ **d** $x \leq -2$

7 **a** $x > 3$ **b** $x \leq -5$

c $x > -25$ **d** $x > 1$

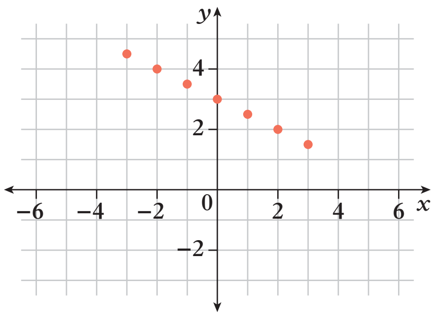
e $x \neq -3$ **f** $x \leq \frac{22}{3}$

8 **a** **i**



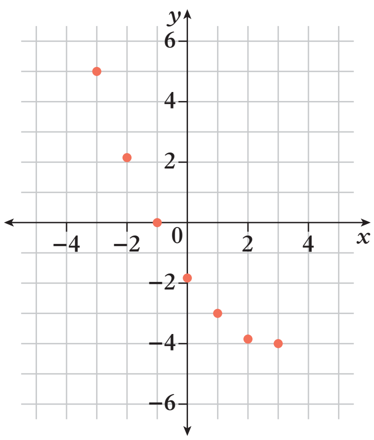
ii Linear

b **i**



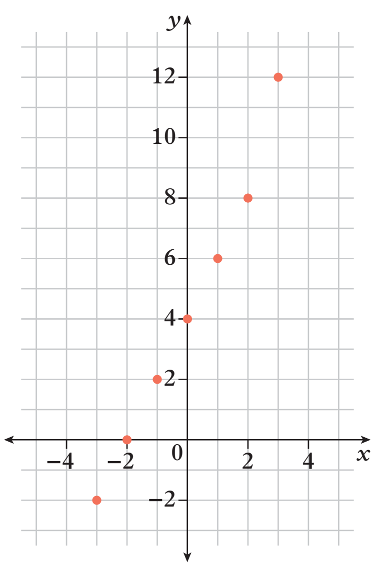
ii Linear

c **i**



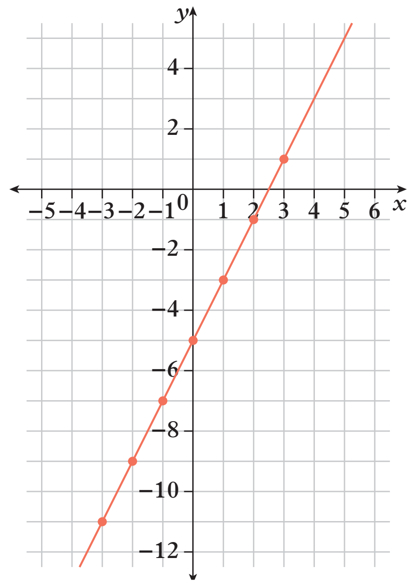
ii Non-linear

d **i**

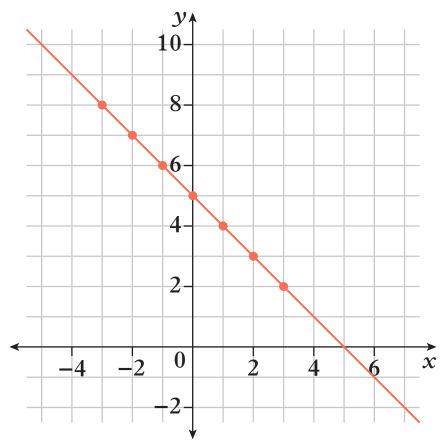


ii Non-linear

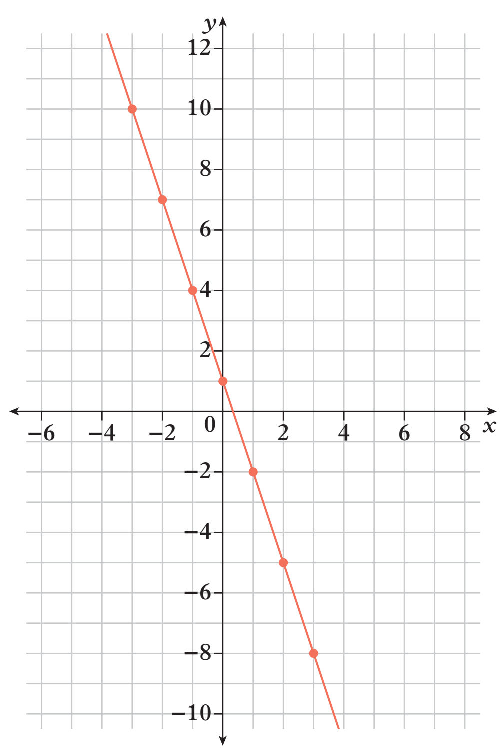
9 **a**



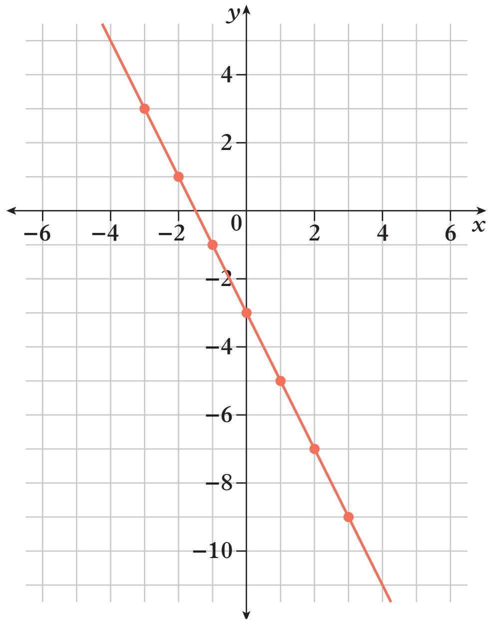
b



c



d



10 a IV: length, DV: volume

b IV: number of burgers, DV: calories

c IV: time, DV: number of achievements

11 a Zero **b** Negative **c** Undefined **d** Positive

12 a 3 **b** −3 **c** $-\frac{5}{3}$ **d** $-\frac{6}{5}$

13 a (3, 0), (0, 4)

b (−2, 0), (0, 3)

c (−6, 0), (0, −1)

EX
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3.5 Sketching linear graphs

1 a (12, 0); (0, 3) **b** (−4, 0); (0, 4)

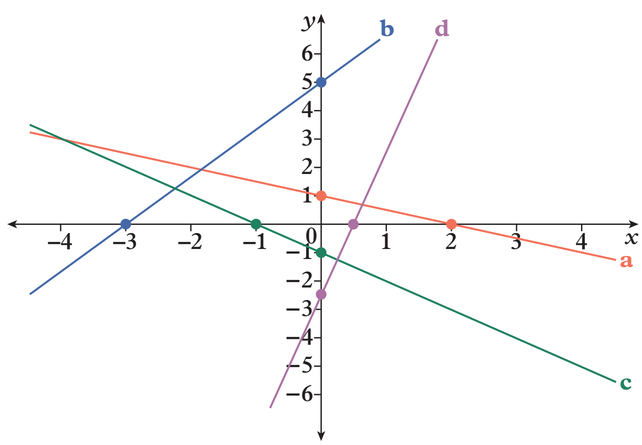
c (3, 0); (0, 6) **d** (5, 0); (0, −5)

e (4, 0); (0, 8) **f** (2, 0); (0, −6)

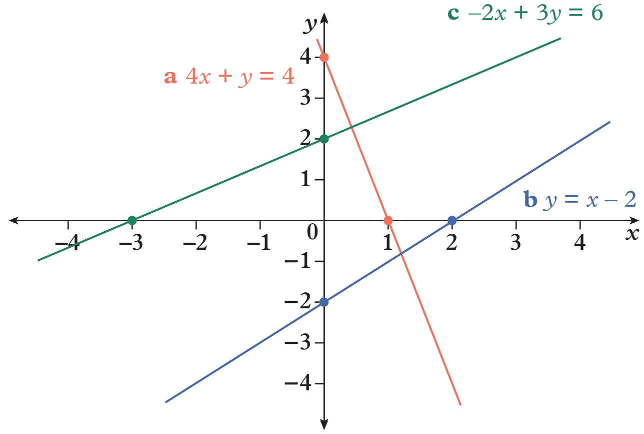
g (−2, 0); (0, −10) **h** (7, 0); (0, 7)

i (4, 0); (0, 4)

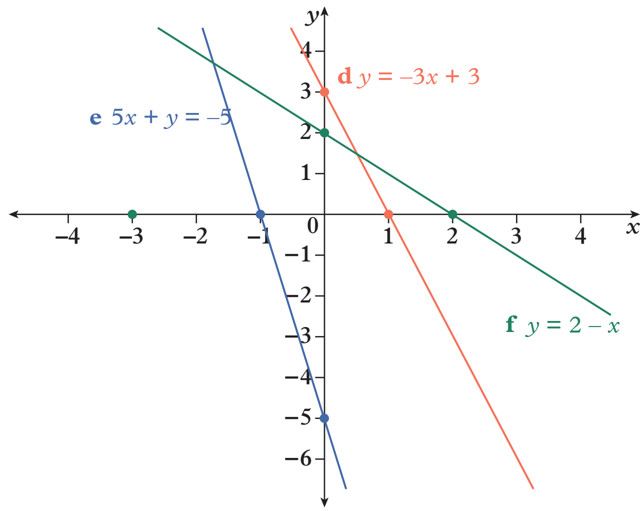
2 a–d



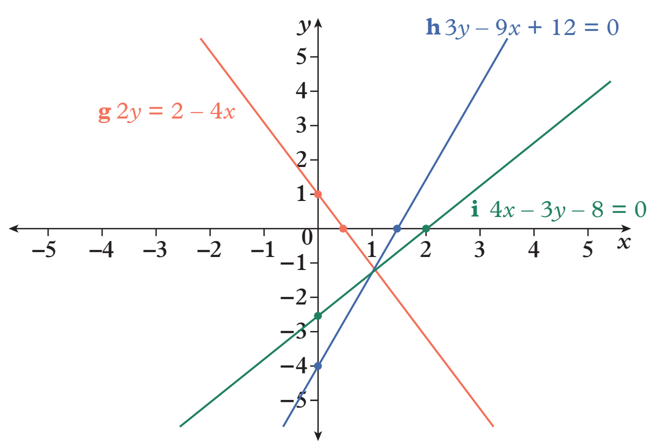
3 a–c



d–f

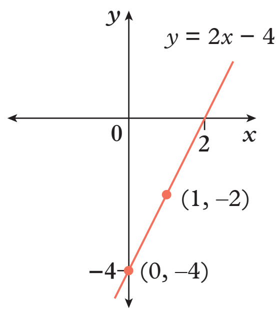


g–i

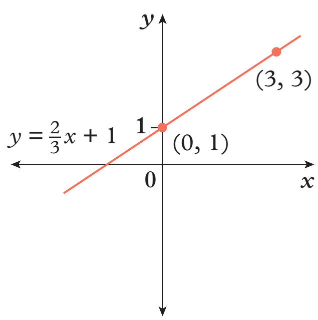


4 **a** $m = 2, c = 5$ **b** $m = 4, c = 1$
c $m = -3, c = 7$ **d** $m = -5, c = -3$
e $m = 1, c = -6$ **f** $m = -1, c = 1$
g $m = \frac{4}{3}, c = 2$ **h** $m = \frac{1}{2}, c = -8$
i $m = -\frac{4}{3}, c = \frac{1}{4}$ **j** $m = 0, c = 9$
k $m = -7, c = 0$ **l** $m = -\frac{2}{5}, c = 5$

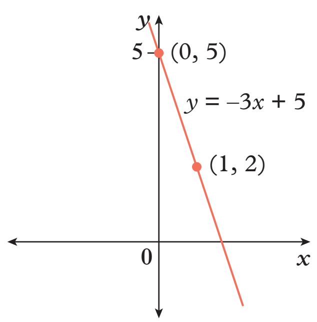
5 **a**



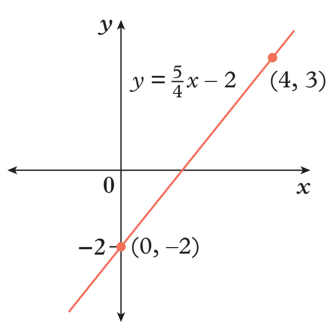
b



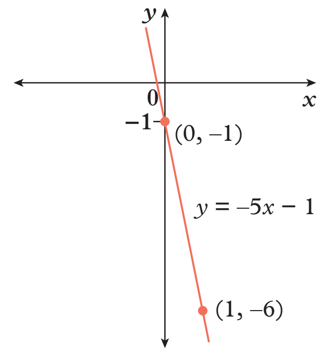
c



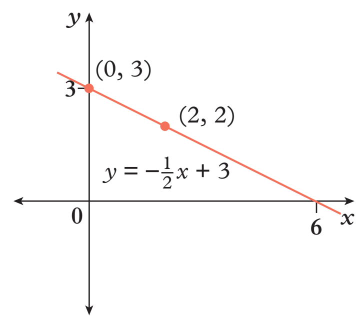
d



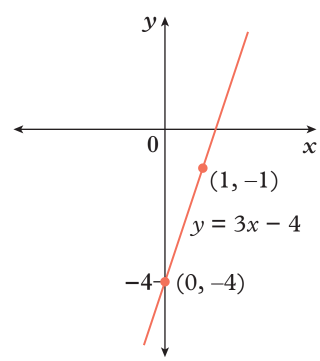
e



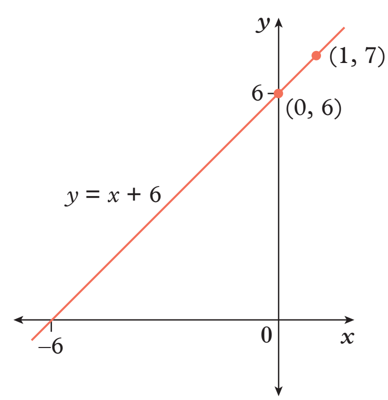
f



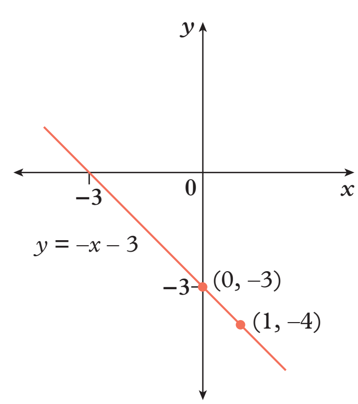
g



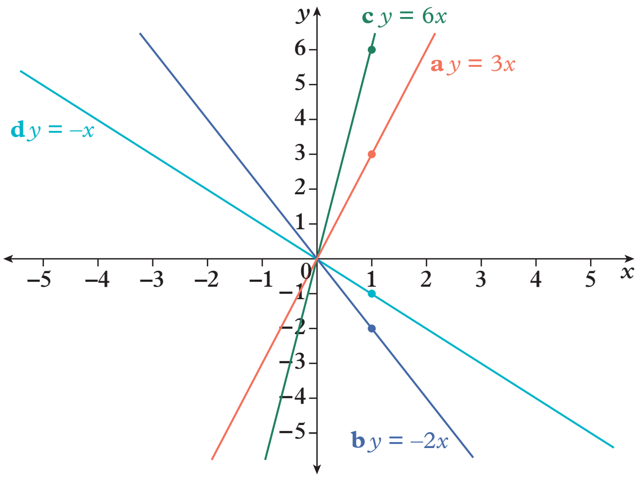
h



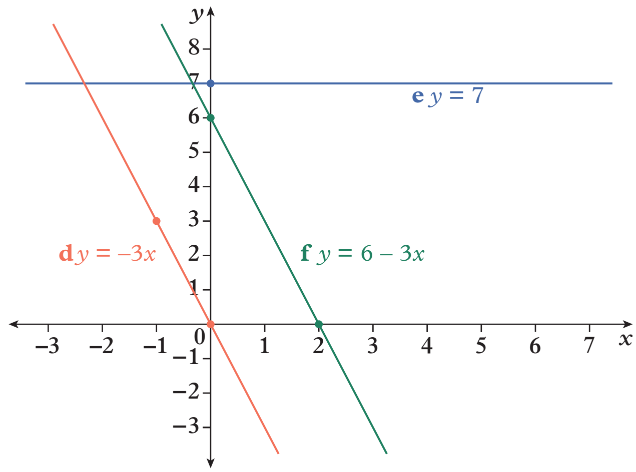
i



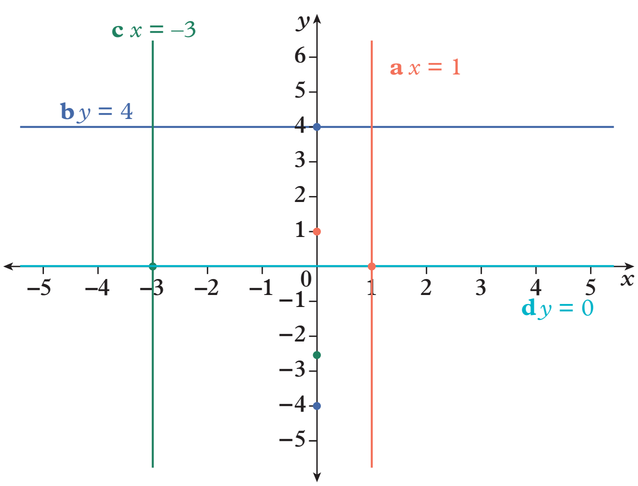
6



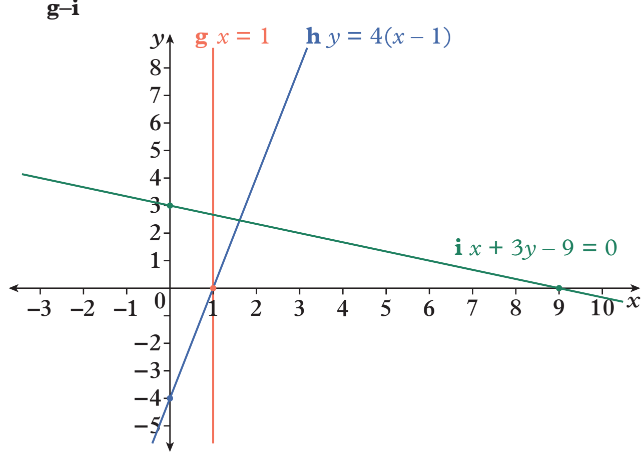
d–f



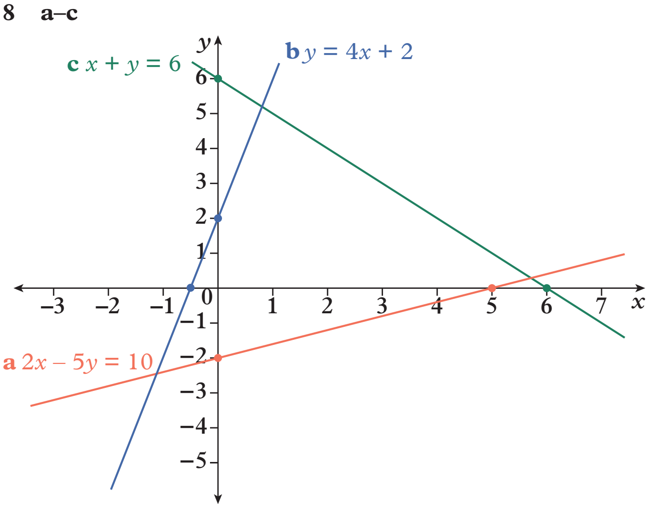
7



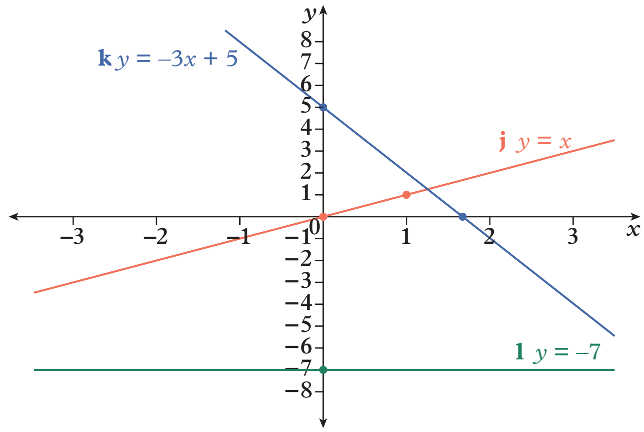
g–i



8 **a–c**



j–l



9 **a** 1 **b** 2 **c** 1
d 1 **e** 2 **f** 2
g 1 **h** 1 **i** 1

10 Graph B is correct. In Graph A, the coefficients of variables have been used as intercepts. Graph B is a correct sketch of $3x + 2y = 6$. Graph C has an x-intercept at -2 instead of $+2$.

11 a True

b False; the x-intercept is -2

c True

d True

e False; the line passes through $(-2, 0)$ and $(0, 6)$

f True

12 a False; the graph passes through the origin

b True

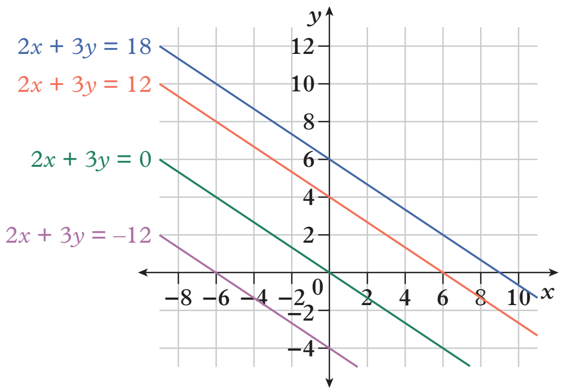
c True

d False; the point $(1, -4)$ lies on the line

e True

f False; the gradient of the line is -4

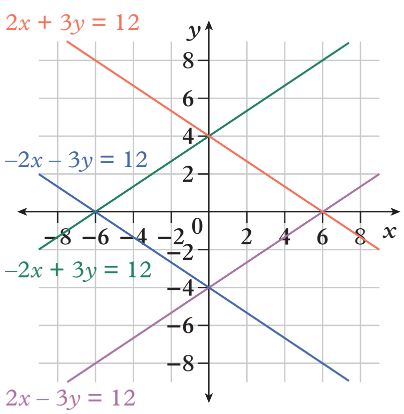
13 a



b The equations have the same variable sum, $2x + 3y$, and are parallel (have the same gradient).

c Increasing the constant translates the line up and to the right. Decreasing the constant translates the line down and to the left. If the constant is 0, the line passes through the origin. If the constant is positive, the line has positive intercepts. If the constant is negative, the line has negative intercepts.

14 a



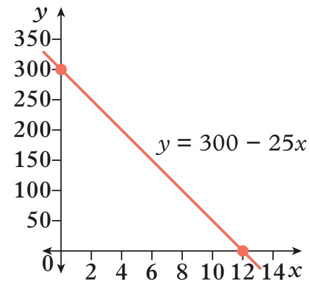
b All equations are equal to the same positive constant and have coefficients with the same unsigned values (but not necessarily the same sign).

c The term(s) with a positive coefficient have positive intercepts. The term(s) with a negative coefficient have negative intercepts. Compared to $2x + 3y = 12$, the line is reflected in the x-axis when the y-term is negative and reflected in the y-axis when the x-term is negative.

15 a $\frac{3}{5}$ **b** $-\frac{7}{6}$ **c** $-\frac{1}{2}$ **d** 2

16 a The independent variable is x (number of weeks) so Tony should calculate y (amount still owed in dollars), which depends on the number of payments over x weeks.

b



c Original cost of skateboard

d Number of weeks needed to pay off skateboard

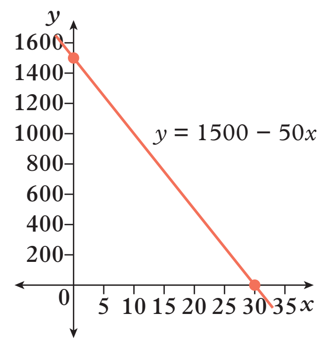
e Tony is paying $25 a week towards the skateboard. He will pay it off in 12 weeks.

17 a Yes, the constant rate of water use and no additions to the tank suggest a constant gradient and hence a linear relationship between time, x, and amount of water, y.

b $(30, 0)$

c $(0, 1500)$

d



e 1000 L

f End of day 18

18 a When $x = 0$, the equation becomes $by = 1$, which has the solution $y = \frac{1}{b}$ (the y-intercept), the reciprocal of b. Likewise, when $y = 0$, the equation becomes $ax = 1$, which has the solution $x = \frac{1}{a}$ (the x-intercept), the reciprocal of a.

b Divide both sides of the equation by d, then the reciprocals of the coefficients of x and y will be the x- and y-intercepts, respectively.

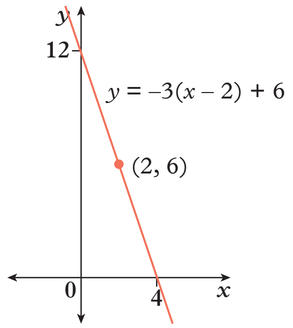
c $d = ab$

19 a $y = m(x - h) + k$

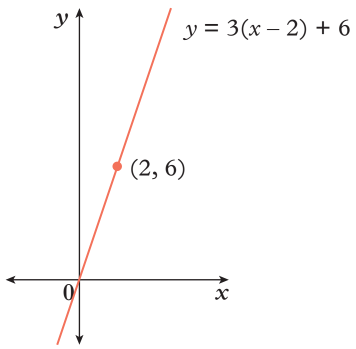
$y = m(x - 0) + c$

$y = mx + c$

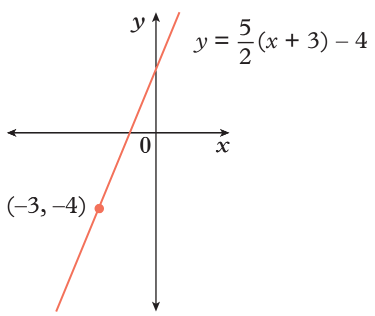
b **i**



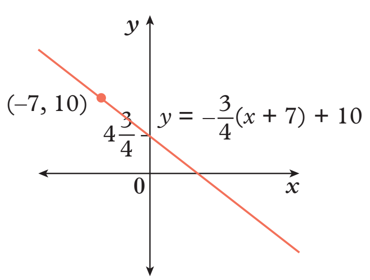
ii



iii



iv



c $(0, k - mh)$ and $\left(h - \frac{k}{m}, 0\right)$

d **i** $y = m(x - b)$; $x = \frac{y}{m} + b$
This is the x-intercept version of $y = mx + c$.

ii Take the gradient, m, out as if it were a factor of the expression $mx + c$ by dividing each term by m to write the equation as $y = m\left(x + \frac{c}{m}\right)$.

iii The x-intercept is the negative of the value added to x, $\frac{c}{m}$.

e **i** $y = 3(x - 4)$, $(4, 0)$

ii $y = 2\left(x - \frac{5}{2}\right)$, $\left(\frac{5}{2}, 0\right)$

iii $y = -3(x + 4)$, $(-4, 0)$

iv $y = 2\left(x + \frac{5}{2}\right)$, $\left(-\frac{5}{2}, 0\right)$

EX 3.6 Determining linear equations

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1 **a** $m = 2, c = 5$ **b** $m = 4, c = 1$
c $m = -3, c = 7$ **d** $m = -5, c = -3$
e $m = 1, c = -6$ **f** $m = -1, c = 1$
g $m = \frac{4}{3}, c = 2$ **h** $m = \frac{1}{2}, c = -8$
i $m = -\frac{4}{3}, c = \frac{1}{4}$ **j** $m = 0, c = 9$
k $m = -7, c = 0$ **l** $m = -\frac{2}{5}, c = 5$

2 **a** $y = 3x + 4$ **b** $y = -2x + 10$
c $y = x - 7$ **d** $y = -12x - 1$
e $y = -x + 20$ **f** $y = 5x$
g $y = \frac{1}{3}x + \frac{4}{5}$ **h** $y = -\frac{3}{4}x$

3 **a** $y = 3x + 6$ **b** $y = -\frac{2}{3}x + 4$
c $y = \frac{1}{2}x - 5$ **d** $y = -\frac{1}{5}x - 2$
e $y = 2x - 3$ **f** $y = -3x + 5$
g $y = -\frac{1}{2}x$ **h** $y = \frac{5}{2}x$

4 **a** $y = 2x - 22$ **b** $y = -2x + 8$
c $y = -x + 5$ **d** $y = 3x$
e $y = 2x + 1$ **f** $y = 5x - 49$
g $y = -\frac{1}{2}x - \frac{5}{2}$ **h** $y = \frac{4}{3}x - 11$

5 **a** $x = 5$ **b** $y = -2$
c $y = 60$ **d** $x = -40$
e $y = -\frac{3}{7}$ **f** $x = \frac{2}{3}$

6 **a** $y = -3x + 12$ **b** $y = 2x + 10$
c $y = 7x - 21$ **d** $y = -4x - 24$
e $y = 10x + 10$ **f** $y = -\frac{x}{3} + 6$
g $y = \frac{2}{5}x - 2$ **h** $y = \frac{3}{2}x + 9$

7 **a** $y = \frac{1}{2}x + \frac{9}{2}$ **b** $y = \frac{2}{3}x + \frac{14}{3}$
c $y = -\frac{1}{5}x + \frac{18}{5}$ **d** $y = \frac{3}{4}x + \frac{1}{2}$
e $y = -\frac{7}{6}x - \frac{1}{3}$ **f** $y = -\frac{11}{9}x - \frac{5}{3}$

8 **a** IV: number of years, DV: amount in Azami's account
b $A = 300n + 1900$

9 **a** $P = 60n + 30$ **b** $21 930

10 **a** IV: number of years from 2010, DV: length of stalactite in centimetres
b $g = \frac{1}{5}n + 85$
c The stalactite is 425 years old in 2010, which means it began to form in 1585.

11 **a** $y = 9x - 5$
b $y = \frac{3}{2}x + \frac{5}{2}$
c $y = \frac{3}{10}x + 45$
d $y = -2x + 30$

12 **a** $C = 45n + 125$
b $R = 99n + 275$
c $P = 54n + 150$

13 a i $y = 3(x + 4)$
ii $y = -5(x + 3)$
iii $y = 2(x - 5)$
iv $y = 2(3x - 4)$
v $y = 7(3 - x)$ or $y = -7(x - 3)$
vi $y = 8(2x - 5)$
b i $x = -4$ **ii** $x = -3$ **iii** $x = 5$
iv $x = \frac{4}{3}$ **v** $x = 3$ **vi** $x = \frac{5}{2}$
c The x-intercept is the value of x that makes the bracketed factor equal to zero. It is the opposite sign of the constant divided by the coefficient of x.

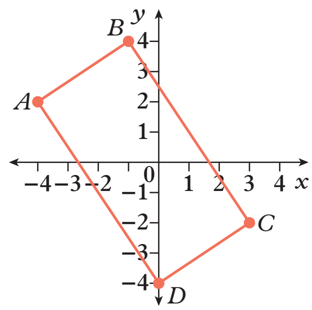
14 a $y = -\frac{21}{10}x + \frac{7}{5}$
b $y = -\frac{5}{3}x + \frac{1}{12}$
c $y = \frac{22}{81}x - \frac{19}{36}$

15 a $y = 9x + 11$ and $y = 11x + 9$ **b** $(1, 20)$

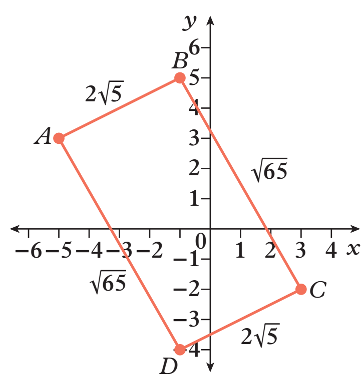
EX 3.7 Midpoint and length of a line segment

p129

1 a (2, 1) **b** (4, −2) **c** (−3, 3) **d** (3, 2)
2 a (2, 7) **b** (5, 4) **c** (3, 1) **d** (2, 8)
e (4, 7) **f** (5, 1) **g** (4, 3) **h** (1, −4)
i (−3, 0) **j** (3.5, 8.5) **k** (6.5, 5.5) **l** (0,0)
3 a 2.2 units **b** 4.5 units **c** 3.2 units
d 4.0 units **e** 6.3 units **f** 5.0 units
g 1.4 units **h** 14.0 units **i** 3.6 units
j 4.1 units **k** 4.5 units **l** 15.6 units
4 a i (3, 3) **ii** 4.5 units **iii** $-\frac{1}{2}$
b i (0.5, 0) **ii** 7.8 units **iii** $\frac{6}{5}$
c i (−2.5, 3) **ii** 8.5 units **iii** $\frac{8}{3}$
d i (−6, −5) **ii** 8.5 units **iii** −1
5 a 1.1 units **b** 2.2 units **c** 1.6 units **d** 2.0 units
e 3.2 units **f** 2.5 units **g** 0.7 units **h** 7.0 units
i 1.8 units **j** 2.1 units **k** 2.2 units **l** 7.8 units
6 (10, 5)
7 (−14, 9)
8 a 5 units **b** (5, 7)
9 20.3 units
10 a 20.5 units **b** 17.0 units
c 22.6 units **d** 19.6 units
11 a Leg 1: 21.5 km; leg 2: 16.5 km; leg 3: 28.8 km
b 66.9 km **c** 24.1 km
12 a (−5, −1), (−2, 3), (6, 3) and (3, −1)
b i (0.5, 1) **ii** (0.5, 1)
c Midpoints of both diagonals have same coordinates
d 26 cm
e 5.3 cm
13 a i 21.6 cm **ii** 10.8 cm
iii The perimeter of the orange shape is half the perimeter of the blue shape.
b i 20.7 cm
ii 16.7 cm
iii Perimeter of orange shape ≈ $\frac{4}{5}$ perimeter of blue shape
14 a AB: 3.6 m, BC: 7.2 m, CD: 3.6 m and AD: 7.2 m
b Midpoint for both diagonals (−0.5, 0)
c A: 4.0 m, B: 4.0 m, C: 4.0 m, D: 4.0 m
d The parallelogram is a rectangle because it has two opposite pairs of sides of equal length, diagonals equal in length and bisect each other, with each vertex the same distance from the midpoint.
e 26 m²



15 a Two sides are equal in length, $AB = AC = 4\sqrt{5}$ units and $BC = 4\sqrt{2}$ units.
b 24 units²
16 Two pairs of equal-length sides, $AB = CD = 2\sqrt{5}$ units and $BC = AD = \sqrt{65}$ units; two pairs of parallel sides with gradients $\frac{1}{2}$ and $-\frac{7}{4}$.



17 (0, 1)
18 $(\sqrt{5}, 2\sqrt{5})$ and $(-\sqrt{5}, -2\sqrt{5})$

EX 3.8 Direct proportion

p136

1 a $y \propto x$
b y is not directly proportional to x because the rate of change is not constant.
c $y \propto x$
d y is not directly proportional to x because when $x = 0$, $y = 1$.
e $y \propto x$
f y is not directly proportional to x as the graph does not pass through (0, 0).

g y is not directly proportional to x as the graph does not pass through (0, 0) and y does not increase as x increases.

h $y \propto x$

i y is not directly proportional to x as the gradient is not constant.

j y is not directly proportional to x as the graph does not pass through (0, 0).

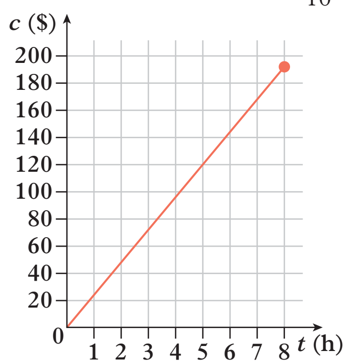
2 **a** $k = 4$ **b** $k = -10$ **c** $k = 3.5$ **d** $k = \frac{1}{3}$

3 **a** $y = 10x$ **b** $y = -9x$ **c** $y = \frac{1}{4}x$ **d** $y = -\frac{2}{3}x$

e $y = -6x$ **f** $y = 5x$ **g** $y = -\frac{1}{2}x$ **h** $y = 5x$

i $y = 2x$ **j** $y = 20x$ **k** $y = \frac{3}{10}x$ **l** $y = \frac{3}{4}x$

4 **a** **i**



ii 24

b \$24/hour

c $c = 24t$

d \$96

5 **a** The car travelled at a speed of 60 km/h for 1 hour, then at a speed of 120 km/h for second hour.

b $d = 60t$ **c** $d = 120t$

d No. The rate of change of d with respect to t is not constant over the duration of the journey.

6 **a** $k = 600$ **b** $C = 600A$ **c** 210 m^2

7 **a** 4 patients per hour

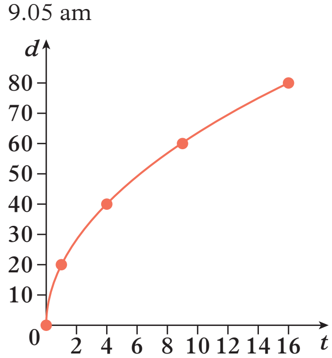
b 15 min

c \$62.50 per patient

8 **a** Jasmine 8.10 am, Nelson 8.25 am

b 9.05 am

9 **a**

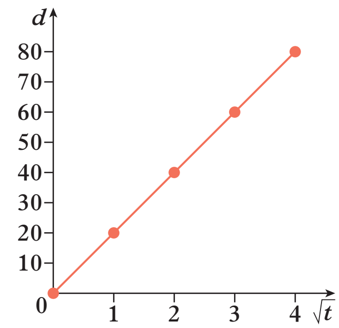


b No; as the graph is not a straight line passing through (0, 0)

c

$\sqrt{t}$	0	1	2	3	4
d	0	20	40	60	80

d



e Yes; it forms a straight-line graph that passes through (0, 0).

f $k = 20$, $d = 20\sqrt{t}$ **g** 120 km

10 Helena is 40 km from town A and Julia is 110 km from town A.

EX p140

Review: Linear relationships

Review questions 3.9A: Mathematical literacy review

1 **a** inequality, number line

b linear relationship, Cartesian plane

c linear equation, variable

2 **a** False **b** True **c** False

d False **e** True

3 **a** The *constant* in the gradient–intercept form of a linear equation is the y-value of the y-intercept; for example, the + 1 in Equation 1. The *constant of proportionality* is the coefficient of the dependent variable and is the same as the gradient or rate of change, for example, the 6 in Equation 2.

b A *linear relationship*, $y = mx + c$, is any relationship between two variables that produces a straight-line graph when plotted and a y-intercept at (0, c). For example, $y = 3x + 1$ has a y-intercept at (0, 1). A *directly proportional relationship* is a specific case of a linear relationship. The straight line passes through the origin and the y-intercept is at (0, 0), so the constant has a value of 0 in the equation, $C = 6t$.

Review questions 3.9B: Multiple choice

1 B	**2** D	**3** B	**4** A	**5** D
6 C	**7** B	**8** B	**9** C	**10** B
11 A	**12** C	**13** C	**14** A	**15** D

Review questions 3.9C: Short answer

1 **a** $x = 2$ **b** $x = \frac{2}{3}$ **c** $x = 3\frac{3}{4}$ **d** $x = 3$

2 **a** $x = 4$ **b** $x = -1$

3 **a** Let n be the number of people in the class.

b $\frac{3}{5}n + 1 = 16$ **c** $n = 25$ **d** 25 people

4 **a** $x \geq -3$ **b** $x < -10$ **c** $-1 < x \leq 2$

5 **a** $x > -1$ **b** $x < -2\frac{2}{3}$

6 **a**

x	−2	−1	0	1	2
y	3	4	5	6	7

b

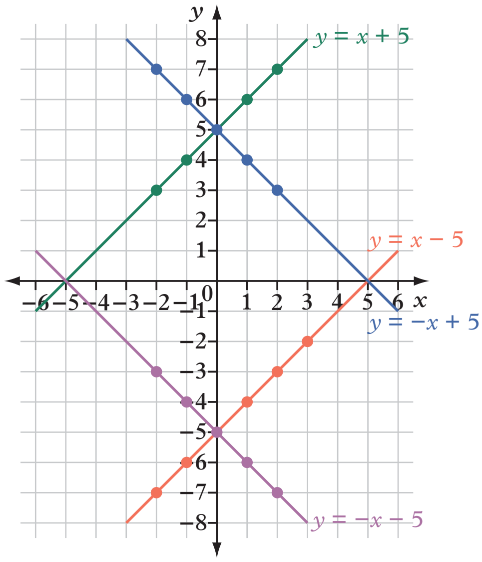
x	−2	−1	0	1	2
y	−7	−6	−5	−4	−3

c

x	−2	−1	0	1	2
y	7	6	5	4	3

d

x	−2	−1	0	1	2
y	−3	−4	−5	−6	−7



7 **a** **i** $\frac{1}{3}$ **ii** $(-6, 0)$ **iii** $(0, 2)$

b **i** Undefined **ii** $(3, 0)$

iii No y-intercept

c **i** 0 **ii** No x-intercept **iii** $(0, -2)$

d **i** -2 **ii** $(0, 0)$ **iii** $(0, 0)$

8 **a** $\frac{3}{2}$ **b** -1

9 **a** **i** $(9, 0)$ **ii** $(0, 6)$

b **i** $(2, 0)$ **ii** $(0, -6)$

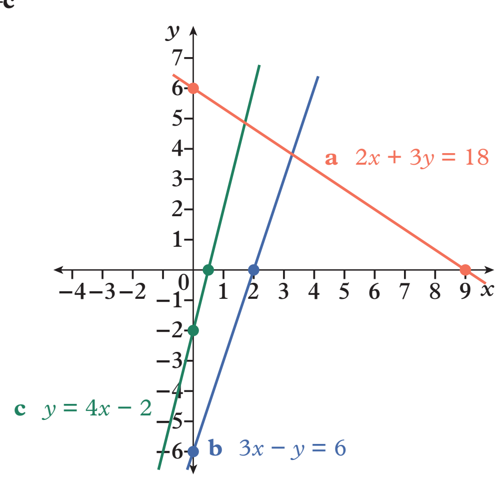
c **i** $\left(\frac{1}{2}, 0\right)$ **ii** $(0, -2)$

d **i** $\left(\frac{3}{5}, 0\right)$ **ii** $\left(0, -\frac{3}{2}\right)$

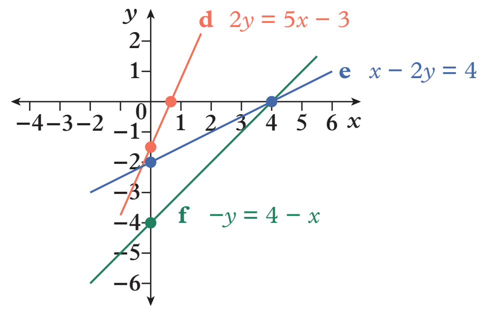
e **i** $(4, 0)$ **ii** $(0, -2)$

f **i** $(4, 0)$ **ii** $(0, -4)$

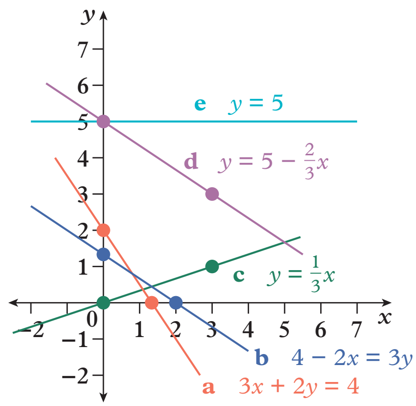
10 **a–c**



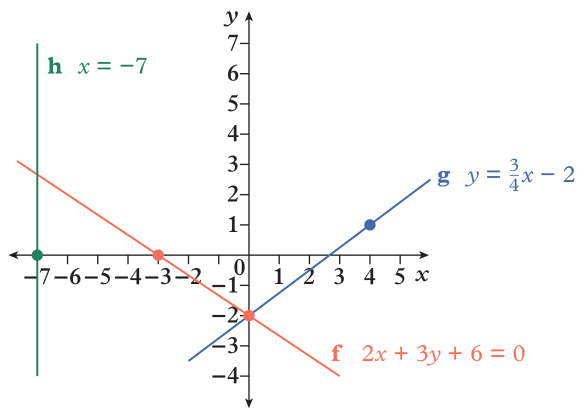
d–f



11 **a–e**



f–h



12 **a** $y = 4x - 2$ **b** $y = \frac{1}{4}x$ **c** $y = -\frac{1}{2}$

13 **a** $y = -2x - 4$ **b** $y = \frac{3}{2}x$

c $y = 1.25x + 1.25$ **d** $y = -\frac{1}{3}x + 1$

14 **a** $y = -x - 3$ **b** $y = 39$

c $y = -\frac{4}{3}x$ **d** $y = \frac{4}{5}x + \frac{32}{5}$

15 **a** **i** $(5, 5)$ **ii** 7.2 units

b **i** $(-3.5, 5.5)$ **ii** 1.4 units

c **i** $(0, 0)$ **ii** 8.9 units

d **i** $(-0.5, -5)$ **ii** 9.4 units

16 **a** y is not directly proportional to x; the rate of change is not constant.

b $y \propto x$ **c** $y \propto x$

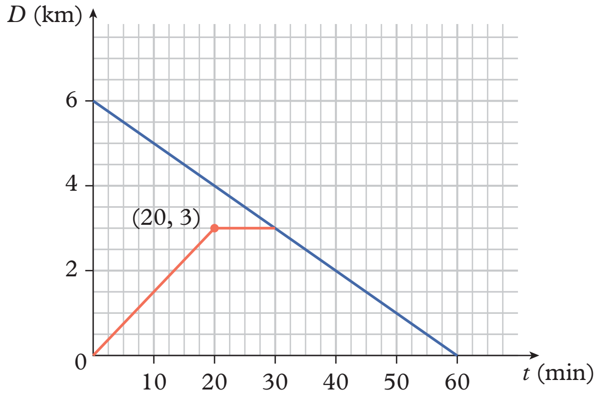
d y is not directly proportional to x as the gradient is not constant.

Review questions 3.9D: Mathematical modelling

1 **a** $-\frac{1}{10}$ **b** 0.1 km/min

c $D = -\frac{1}{10}t + 6$ **d** 3 km

e



f 10 minutes **g** 0.15 km/min

h $D = \frac{3}{20}t$

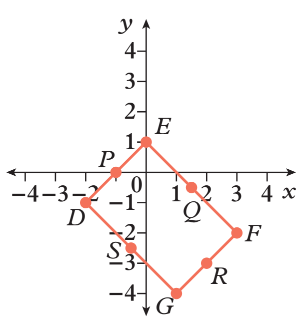
2 **a**

Line segment	Linear equation	x- or y-values involved
A	$y = 1$	$2 \le x \le 3$
B	$x = 3$	$1 \le y \le 2$
C	$y = 2$	$3 \le x \le 4$
D	$y = -\frac{4}{3}x + 7\frac{1}{3}$	$2.5 \le x \le 4$
E	$y = \frac{4}{3}x + \frac{2}{3}$	$1 \le x \le 2.5$
F	$y = 2$	$1 \le x \le 2$
G	$x = 2$	$1 \le y \le 2$

b

Line segment	Linear equation	x- or y-values involved
A	$y = ax + a$	$-1 \le x \le 0$
B	$y = -ax - a$	$-1 \le x \le 0$
C	$y = ax - a$	$0 \le x \le 1$
D	$y = -ax + a$	$0.5 \le x \le 1$
E	$y = ax$	$0 \le x \le 0.5$
F	$x = 0$	$0 \le y \le a$

3 **a**



b DE and FG are 2.8 units long, EF and DG 4.2 units long.

c **i** (0, 1) **ii** (0, 1) **iii** (0, −3) **iv** none

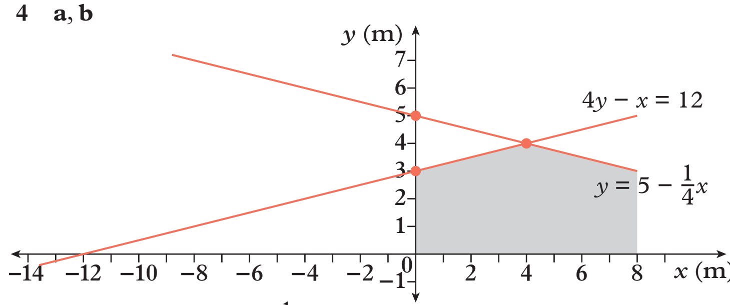
d **i** 1 **ii** −1 **iii** −1 **iv** 1

e **i** (−1, 0) **ii** (1.5, −0.5)

iii (2, −3) **iv** (−0.5, −2.5)

f $PQRS$ is a rhombus with all sides equal in length (PQ, RS, SP and QR).

4 **a, b**



c **i** 4.1 m **ii** $\frac{1}{4}$

iii Chimney will be at (2, 3.5).

MODULE 4 Non-linear relationships

4.1 Solving quadratic equations

EX p151

1 **a, c, e** and **h**

2 **a** Yes **b** No **c** No **d** No

e No **f** Yes **g** Yes **h** No

3 **a** $x = -3$ **b** $x = 6$ **c** $x = 2$ **d** $x = -\frac{1}{3}$

4 **a** $x = -2$ or $x = 3$ **b** $x = 1$ or $x = 7$

c $x = -4$ or $x = 4$ **d** $x = 0$ or $x = 6$

e $x = -5$ or $x = -1$ **f** $x = -2$ or $x = 0$

g $x = -8$ or $x = 8$ **h** $x = -1$ or $x = 7$

i $x = 0$ or $x = 11$ **j** $x = -\frac{3}{2}$ or $x = 5$

k $x = \frac{1}{2}$ **l** $x = -\frac{5}{3}$ or $-\frac{3}{2}$

5 **a** $x = -2$ or $x = -5$ **b** $x = 1$ or $x = 2$

c $x = 0$ or $x = -5$ **d** $x = 0$ or $x = 3$

e $x = -6$ or $x = 6$ **f** $x = -7$ or $x = -3$

6 **a** $x = -2$ or $x = 4$ **b** $x = -1$ or $x = 1$

c $x = 0$ or $x = -8$ **d** $x = 1$ or $x = 3$

e $x = -3$ **f** $x = 1$

7 See answers in questions **5** and **6**.

8 **a** $x = -8$ or $x = 2$ **b** $x = 1$ or $x = 4$

c $x = -6$ or $x = 6$ **d** $x = -9$ or $x = 0$

9 **a** $x = -2$ or $x = -1$ **b** $x = 0$ or $x = -4$

c $x = -2$ or $x = 4$ **d** $x = 2$

e $x = -2$ or $x = 1$ **f** $x = 0$ or $x = 2$

g $x = -7$ or $x = -3$ **h** $x = -4$

i $x = -4$ or $x = 4$

10 **a** $x = -8$ or $x = -6$ **b** $x = 0$ or $x = 4$

c $x = 0$ or $x = 1$ **d** $x = -8$ or $x = 8$

e $x = -3$ or $x = 1$ **f** $x = 4$

g $x = 2$ or $x = 3$ **h** $x = -5$

i $x = -2$ or $x = 2$

11 a i 2 **ii** 1 **iii** 0

b i The equation is factorised into two different linear expressions.

ii The equation is factorised into a perfect square.

iii The equation cannot be factorised.

12 a 2 **c** 1 **e** 2 **g** 2

b 2 **d** 0 **f** 2 **h** 0

13 a $w(w + 8)$ cm^2 **b** $w^2 + 8w$ cm^2

c $w^2 + 8w = 560$

d $w = 20$ or $w = -28$. $w = 20$ is the feasible solution and $w = -28$ is not, as it's not possible to have a negative width.

e Width = 20 cm and length = 28 cm

14 a $w^2 + 2w = 35$ **b** $w = -7$ or $w = 5$

c Length = 7 m, width = 5 m

15 $x(x - 12) = 640$, length = 32 cm, width = 20 cm

16 a i 8 m **ii** 0 m

b 8 m **c** 2 s

d Alex is above ground level when he throws the ball so it is only at ground level when the ball falls down and time cannot be -1.

17 a 2 seconds and 6 seconds

b The first time is when the rocket is ascending, the second time is when the rocket is descending.

18 a $x = 1, x = 3$ or $x = 5$ **b** $x = 0, x = -6$ or $x = 6$

c $x = -11$ or $x = 4$ **d** $x = 2, x = -\frac{1}{3}$ or $x = \frac{3}{2}$

EX
p156

4.2 Plotting quadratic relationships

1 a i $(-3, 0), (1, 0)$ **ii** $(0, -3)$

iii $(-1, -4)$ **iv** Minimum

v $x = -1$

b i $(0, 0), (4, 0)$ **ii** $(0, 0)$

iii $(2, 4)$ **iv** Maximum

v $x = 2$

c i $(-2, 0), (2, 0)$ **ii** $(0, 4)$

iii $(0, 4)$ **iv** Maximum

v $x = 0$

d i $(-1, 0), (5, 0)$ **ii** $(-5, 0)$

iii $(2, -9)$ **iv** Minimum

v $x = 2$

e i $(-5, 0), (1, 0)$ **ii** $(0, 5)$

iii $(-2, 9)$ **iv** Maximum

v $x = -2$

f i $(1, 0)$ **ii** $(0, 1)$

iii $(1, 0)$ **iv** Minimum

v $x = 1$

g i $(0, 0)$ **ii** $(0, 0)$

iii $(0, 0)$ **iv** Minimum

v $x = 0$

h i None **ii** $(0, -7)$

iii $(-2, -3)$ **iv** Maximum

v $x = -2$

i i $(3, 0)$ **ii** $(0, -6)$

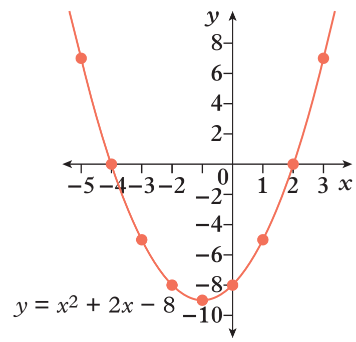
iii $(3, 0)$ **iv** Maximum

v $x = 3$

2 a i

x	−5	−4	−3	−2	−1	0	1	2	3
y	7	0	−5	−8	−9	−8	−5	0	7

ii



iii x-intercepts: $(-4, 0)$ and $(2, 0)$

y-intercept: $(0, -8)$

Turning point: $(-1, -9)$

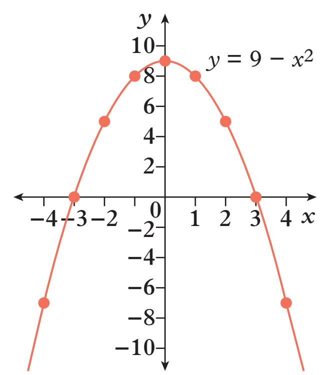
Axis of symmetry: $x = -1$

iv Minimum

b i

x	−4	−3	−2	−1	0	1	2	3	4
y	−7	0	5	8	9	8	5	0	−7

ii



iii x-intercepts: $(-3, 0)$ and $(3, 0)$

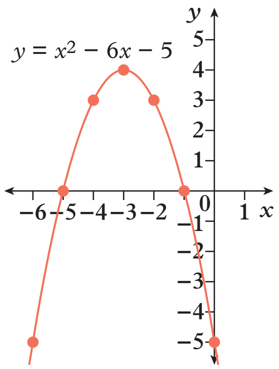
y-intercept: $(0, 9)$

Turning point: $(0, 9)$

Axis of symmetry: $x = 0$

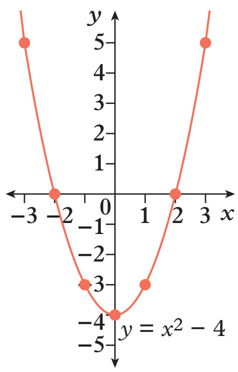
iv Maximum

3 **a** **i**



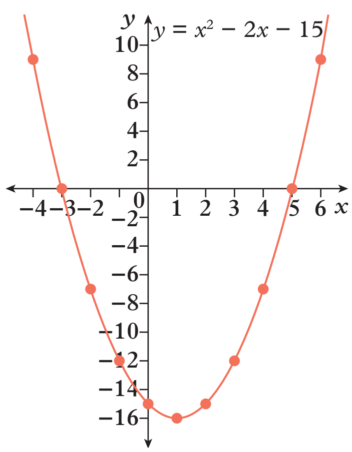
ii x-intercepts: $(-5, 0)$ and $(-1, 0)$
y-intercept: $(0, -5)$
Turning point: $(-3, 4)$
Axis of symmetry: $x = -3$

b **i**



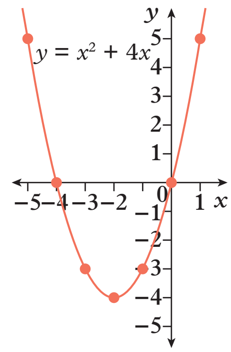
ii x-intercepts: $(-2, 0)$ and $(2, 0)$
y-intercept: $(0, -4)$
Turning point: $(0, -4)$
Axis of symmetry: $x = 0$

c **i**



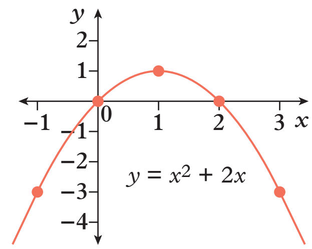
ii x-intercepts: $(-3, 0)$ and $(5, 0)$
y-intercept: $(0, -15)$
Turning point: $(1, -16)$
Axis of symmetry: $x = 1$

d **i**



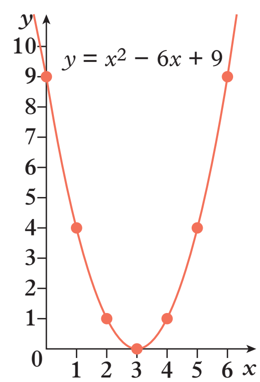
ii x-intercepts: $(-4, 0)$ and $(0, 0)$
y-intercept: $(0, 0)$
Turning point: $(-2, -4)$
Axis of symmetry: $x = -2$

e **i**



ii x-intercepts: $(0, 0)$ and $(2, 0)$
y-intercept: $(0, 0)$
Turning point: $(1, 1)$
Axis of symmetry: $x = 1$

f **i**



ii x-intercept: $(3, 0)$
y-intercept: $(0, 9)$
Turning point: $(3, 0)$
Axis of symmetry: $x = 3$

4 **a** **i** 2 **ii** 1 **iii** 0

b A parabola changes direction only once, so there can only be a maximum of two x-intercepts.

c One

d The y-coordinate of the y-intercept is given by the constant term, c, in the equation.

5 **a** **i** No x-intercepts **ii** 1 y-intercept

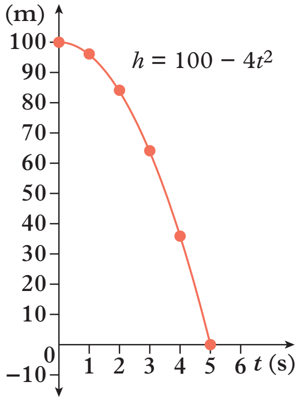
b **i** 1 x-intercept **ii** 1 y-intercept

6 **a** $x = 1$ **b** $x = 5.5$

c $x = -0.5$ **d** $x = -1.5$

7 **a** 20 m **b** 26 m **c** 5 s

8 a



b It is not possible to have negative time values and it is not possible for the hairclip to fall below ground level.

c **i** 84 m **ii** 64 m

d 100 m **e** 5 s

9 a (−2, 0) and (5, 0) **b** $x = -2$ or $x = 5$

c They are the same.

d The x-intercepts of a parabola represent the solutions to a quadratic equation.

10 a $x = 5$ **b** $x = 0$ **c** $x = -4$

11 a $x = 4$ and $x = 5$

b $x = 3$ and $x = 8$

c $x = -3$ and $x = -0.5$

d $x = 3$

12 (0, 0)

13 $-2 \le x \le 5$

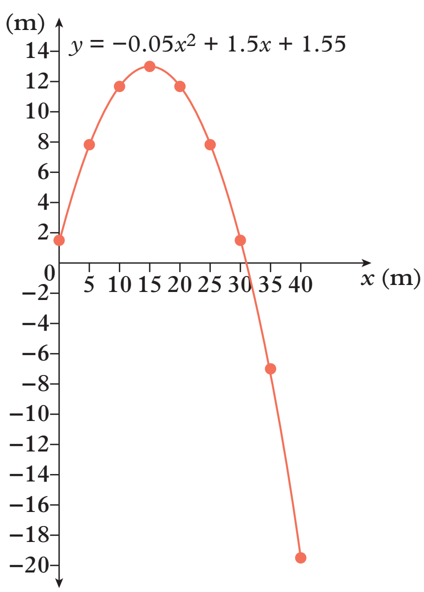
14 a x-intercepts: $(p, 0)$ and $(q, 0)$

b $y = (x + 7)(x - 5)$

c $y = -36$

d $y = -(x + 7)(x - 5)$

15 a



b 1.55 m **c** $x = -1$ and $x = 31$

d 31 m **e** 12.8 m

EX p163 **4.3 Sketching parabolas using intercepts**

1 a i (−7, 0) and (5, 0) **ii** (0, −35)

b i (2, 0) and (6, 0) **ii** (0, 36)

c i (−4, 0) and (−2, 0) **ii** (0, 96)

2 a i (0, 0) and (2, 0) **ii** (0, 0)

b i (−8, 0) and (0, 0) **ii** (0, 0)

c i (−4, 0) and (−2, 0) **ii** (0, 8)

d i (2, 0) and (6, 0) **ii** (0, 12)

e i (−1, 0) and (5, 0) **ii** (0, −5)

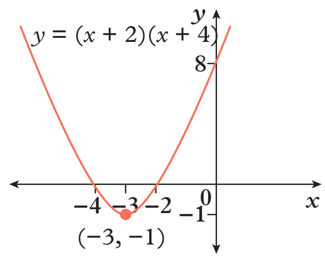
f i (−3, 0) and (3, 0) **ii** (0, −9)

3 a (2, −4) **b** (4, −1) **c** (1, −25) **d** (−2, 1)

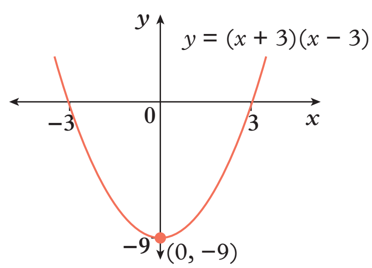
e (0, −4) **f** (1, 9) **g** $\left(-\frac{3}{2}, -\frac{9}{4}\right)$ **h** $\left(-\frac{7}{2}, \frac{49}{4}\right)$

i (5, −18)

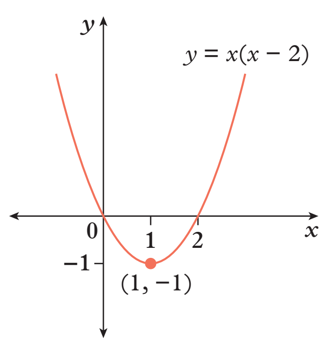
4 a



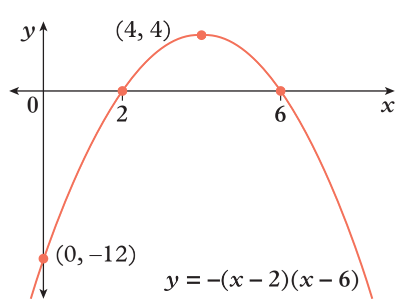
b



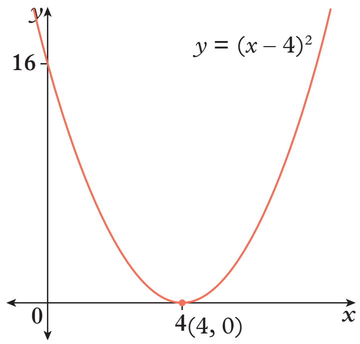
c



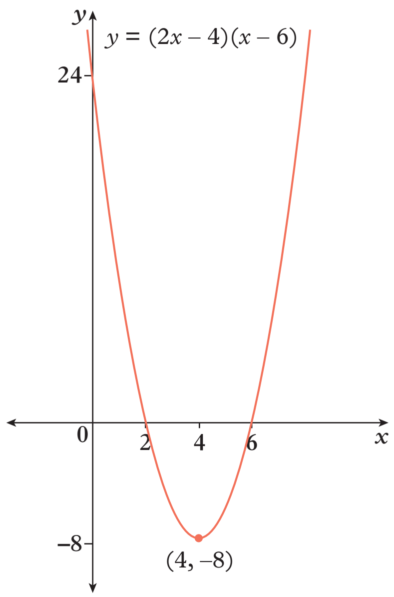
d



e



f

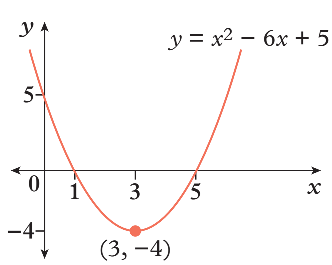


5 **a** Both graphs have x-intercepts at $(-3, 0)$ and $(1, 0)$.

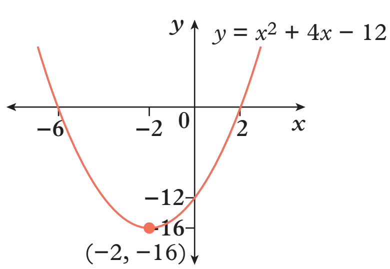
b $y = (x + 3)(x - 1)$ has turning point at $(-4, 0)$ and $y = -(x + 3)(x - 1)$ has turning point at $(4, 0)$.

c Both graphs have the same x-intercepts and the same shape, but they are reflections of each other in the x-axis. So $y = (x + 3)(x - 1)$ has a minimum turning point and $y = -(x + 3)(x - 1)$ has a maximum turning point.

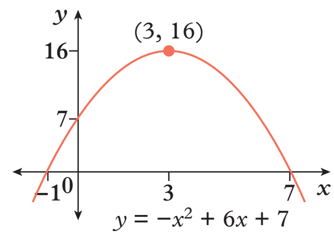
6 **a**



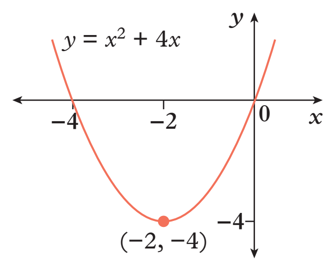
b



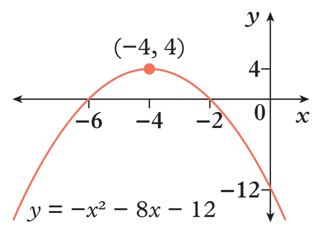
c



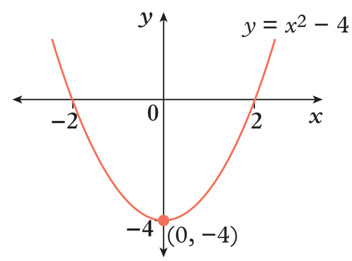
d



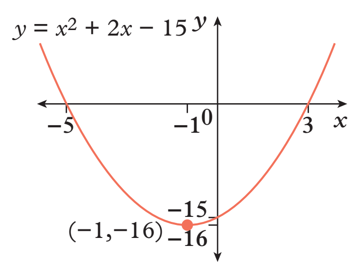
e



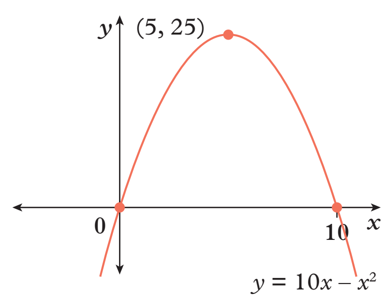
f



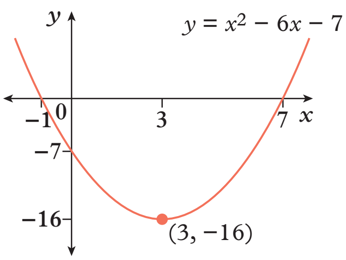
g



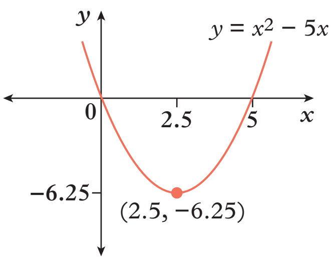
h



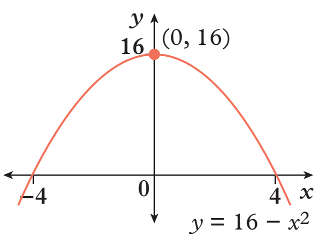
i



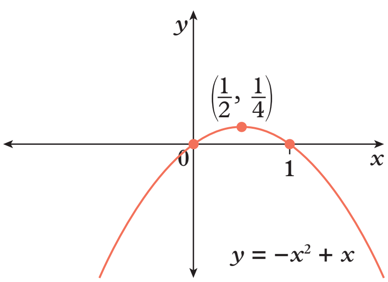
j



k



l



7 **a** C **b** A **c** B

8 Look at coefficient of x^2 term (a). If $a > 0$, parabola is upright, e.g. $y = x^2 + x + 6$ produces an upright parabola. If $a < 0$, parabola is inverted, e.g. $y = -x^2 - x + 6$ produces an inverted parabola.

9 **a** Upright **b** Upright **c** Inverted

d Inverted **e** Upright **f** Inverted

10 **a** Upright

b One (0, 9)

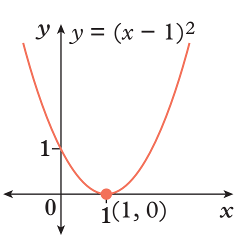
c One (3, 0)

d (3, 0)

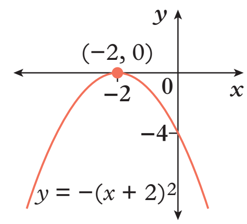
e The turning point is also the x-intercept.

f The negative value of p is the x-coordinate of the turning point. The turning point is also the x-intercept, so the y-coordinate of the turning point will be 0.

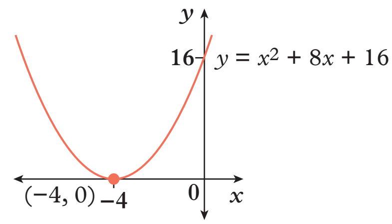
11 **a**



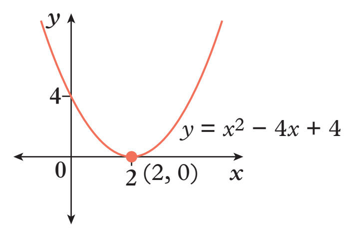
b



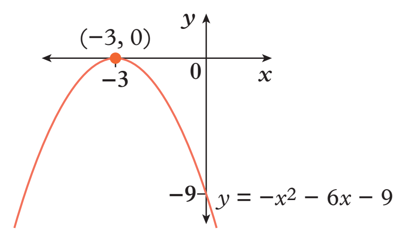
c



d



e

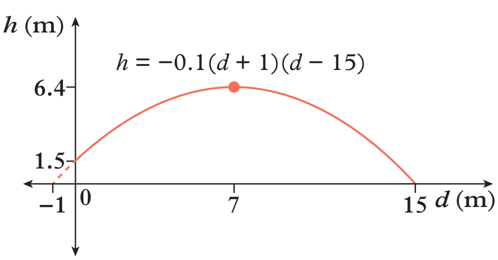


12 **a** Inverted **b** One (0, −1)

c None **d** (0, −1)

e The value of c is the y-coordinate of the turning point. The turning point is also the y-intercept, so the x-coordinate of the turning point will be 0.

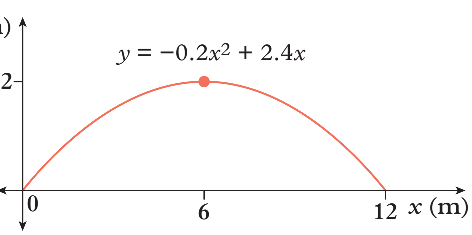
13 **a**



b The graph represents the path of the arrow from $d = 0$ m to $d = 15$ m.

c 6.4 m **d** 1.5 m **e** 15 m

14 **a**



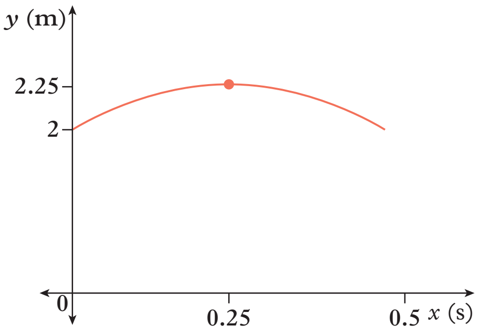
b 7.2 m **c** 6 m **d** 12 m

15 a 2 m

b (0, 2) and (0.5, 2), The ball is thrown and caught at the same height of 2 m.

c (0.25, 2.25)

d



e 2.25 m

16 a $y = \frac{1}{2}(x - 6)(x + 1)$

b $y = -2(x - 6)(x + 2)$

17 $y = \frac{3}{8}(x - 3)^2$

EX p166

Checkpoint

1 a $x = 3$ or $x = 4$ **b** $x = -2$ or $x = 2$

c $x = -9$ **d** $x = 0$ or $x = -4$

2 a $x = 5$ or $x = 7$ **b** $x = -7$ or $x = 4$

c $x = 6$ **d** $x = -9$ or $x = 9$

3 a $x = -4$ or $x = 4$ **b** $x = -2$ or $x = 4$

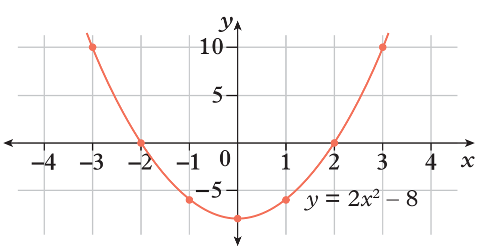
c $x = -5$ or $x = -1$ **d** $x = 0$ or $x = 15$

4 a (2, 0) and (–4, 0) **b** (0, 8)

c (–1, 9) **d** Maximum

e $x = -1$

5



6 a $x = 5$ **b** (5, 0)

7 a i (0, 0) and (−3, 0) **ii** (0, 0)

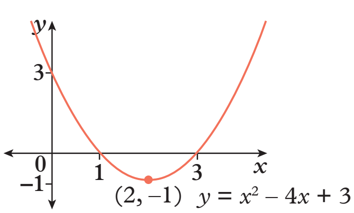
b i (−5, 0) and (5, 0) **ii** (0, −50)

c i (−4, 0) and (2, 0) **ii** (0, −8)

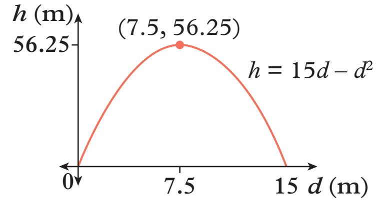
d i (8, 0) **ii** (0, 64)

8 a (−1, −9) **b** (2, −4) **c** (0, −36) **d** (10, 4)

9



10 a



b 15 m **c** 56.25 m

EX p170

4.4 Sketching parabolas using transformations

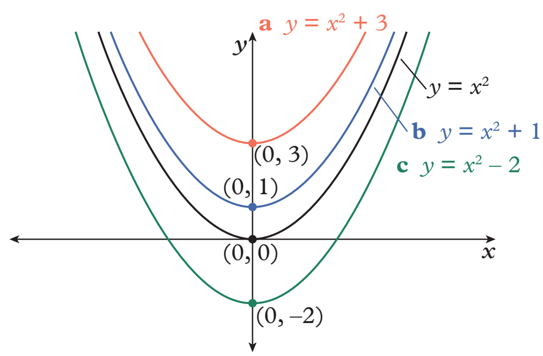
1 a 10, down

b 5, right

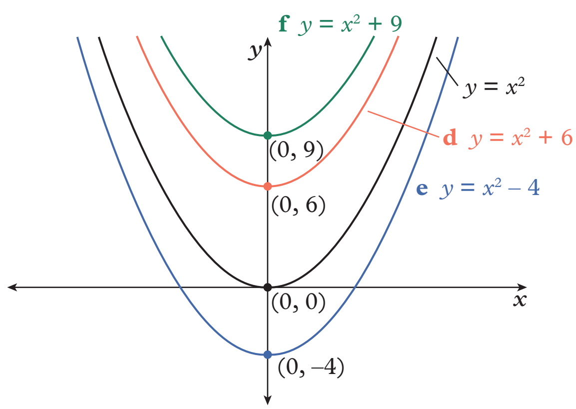
c 12, left

d 8, up

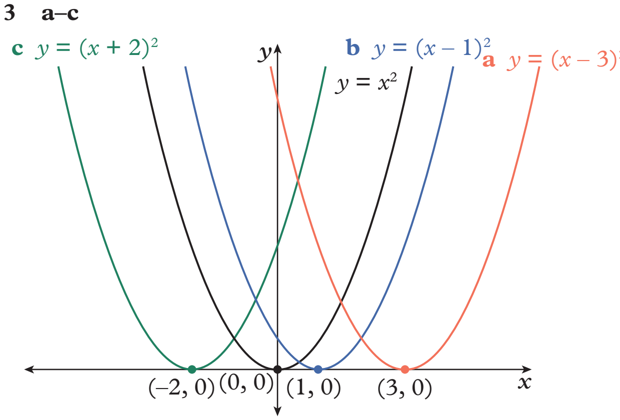
2 a–c



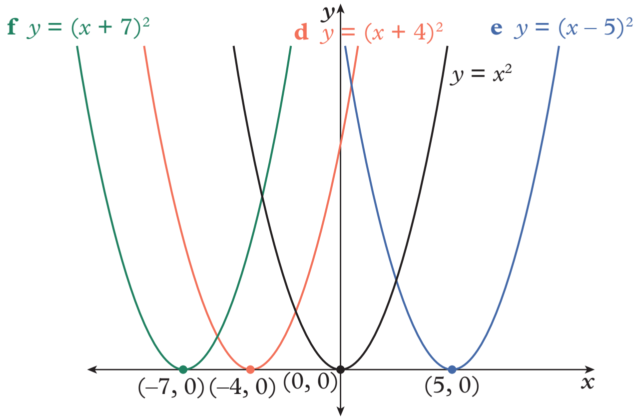
d–f



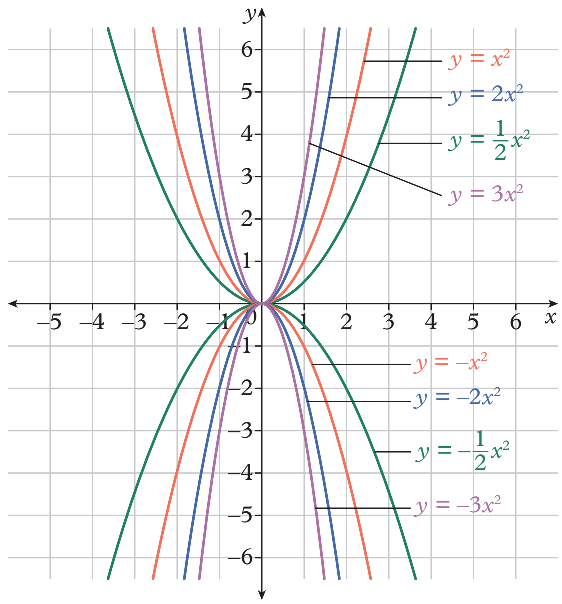
3 a–c



d–f

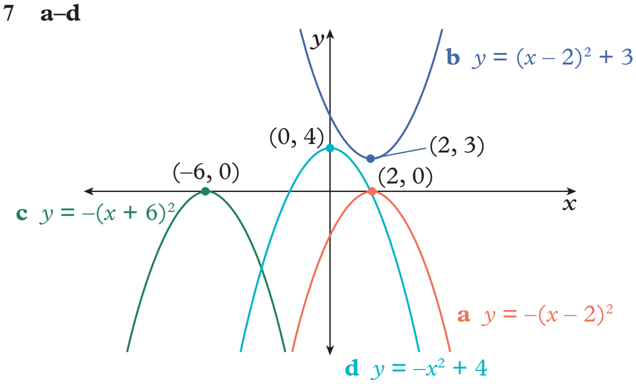


4, 5

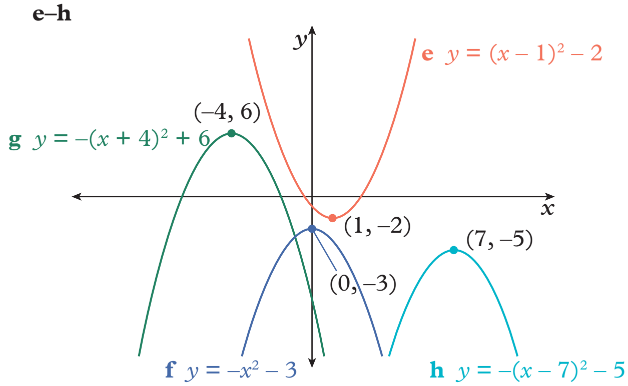


6 **a** 2, left, 3, down **b** Reflected, 4, up

7 **a–d**



e–h

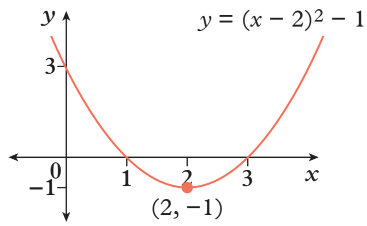


8 **a** **i** Upright

ii x-intercepts: (1, 0) and (3, 0); y-intercept: (0, 3)

iii (2, −1)

iv

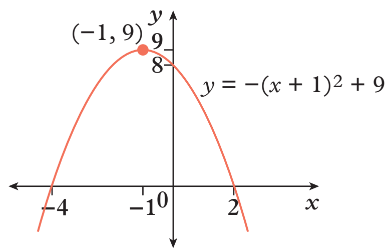


b **i** Inverted

ii x-intercepts: (−4, 0) and (2, 0); y-intercept: (0, 8)

iii (−1, 9)

iv

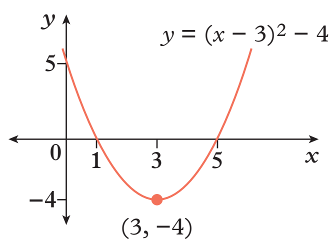


c **i** Upright

ii x-intercepts: (1, 0) and (5, 0); y-intercept: (0, 5)

iii (3, −4)

iv

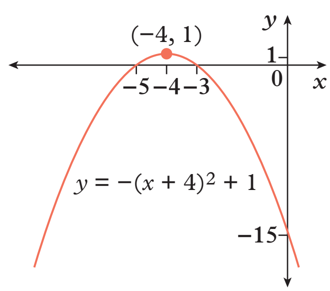


d **i** Inverted

ii x-intercepts: (−5, 0) and (−3, 0); y-intercept: (0, −15)

iii (−4, 1)

iv



9 **a** **i** Upright **ii** (1, −4)

iii a is positive **iv** $h = 1, k = -4$

b **i** Inverted **ii** (−3, 1)

iii a is negative **iv** $h = -3, k = 1$

c **i** Upright **ii** (3, −9)

iii a is positive **iv** $h = 3, k = -9$

10 **a** E **b** D **c** A **d** F **e** C

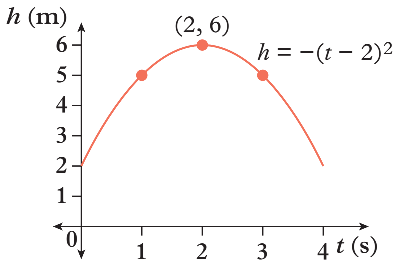
f B **g** H **h** I **i** G

11 **a** $y = (x - 3)^2 + 7$ **b** $y = (x + 2)^2 + 5$
c $y = -(x - 2)^2 + 4$ **d** $y = -(x - 6)^2 - 1$
e $y = (x - 9)^2$ **f** $y = -x^2 + 4$
g $y = -(x + 1)^2 - 2$ **h** $y = x^2 - 5$

12 **a** (4, 5) **b** 5
c There is no maximum value for y because the parabola is upright (it has a minimum turning point).

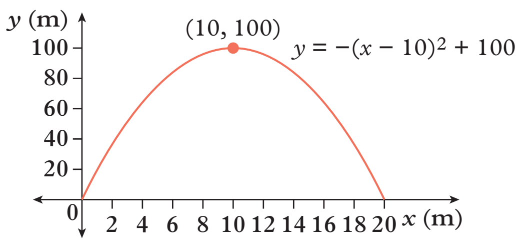
13 Graph is inverted so it has a maximum turning point. The y-coordinate of the turning point is 2, so the maximum y-value is 2.

14 **a** (2, 6)
b **i** 2 m
ii 2 m
c



d **i** 2 m
ii 6 m

15 **a**



b 100 m **c** 20 m

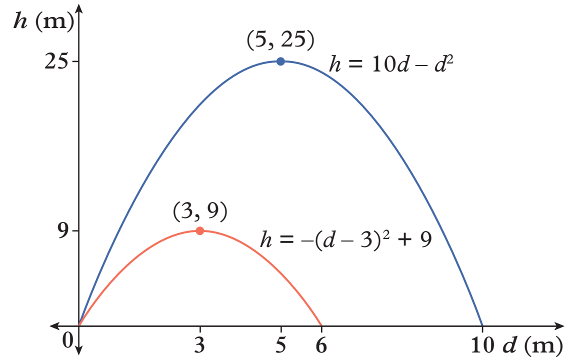
16 **a** $a = 1$
b $y = 2x^2, a = 2$
c $y = \frac{1}{2}x^2, a = \frac{1}{2}$
d If $a > 1$, the graph will be vertically enlarged.
If $0 < a < 1$, the graph will be vertically compressed.

17 **a** E **b** A **c** D **d** C **e** F **f** B

18 **a** **i** Narrower **ii** Upright **iii** (4, −3)
b **i** Wider **ii** Upright **iii** (−2, 6)
c **i** Narrower **ii** Upright **iii** (−2, 0)
d **i** Narrower **ii** Inverted **iii** (0, −4)
e **i** Wider **ii** Inverted **iii** (7, 0)
f **i** Narrower **ii** Inverted **iii** (−1, 5)
g **i** Wider **ii** Inverted **iii** (3, −4)
h **i** Wider **ii** Upright **iii** (0, 3)
i **i** The same **ii** Inverted **iii** (5, 4)
j **i** Narrower **ii** Upright **iii** (−3, −7)

19 **a** $y = 3(x - 2)^2$ **b** $y = -(x + 5)^2 - 4$
c $y = -\frac{1}{2}x^2$ **d** $y = -4(x + 2)^2$

20 **a, d ii**



b 9 m
c 6 m
d **i** $h = -(d - 5)^2 + 25$
iii 16 m

EX p178

4.5 Other non-linear relationships

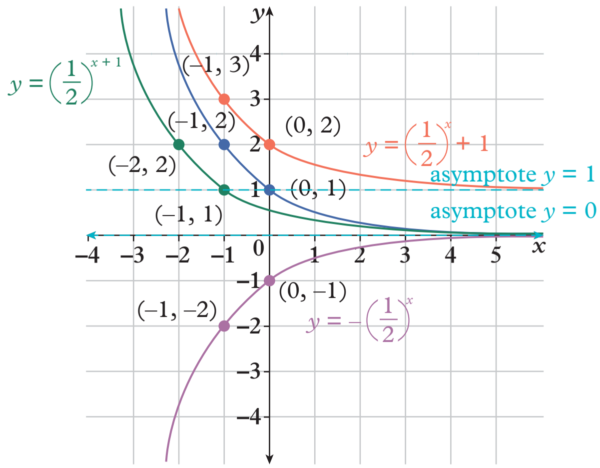
1 **a** B
b D
c A
d C

2 **a** x-intercepts (0, 0), y-intercepts (0, 0)
b x-intercepts (0, 0), y-intercepts (0, 0)
c x-intercepts (0, 0), y-intercepts (0, 0)
d x-intercepts undefined, y-intercepts undefined

3 **a** **i** (0, 0) **ii** (0, 0)
b **i** (−5, 0) **ii** $(0, \sqrt{5})$
c **i** (0, 0) **ii** (0, 0)
d **i** (0, 0) **ii** (0, 0)

4 **a** **i** (0, 0), (0, 0)
ii (2, 0), (2, 0)
iii (0, −5), (0, −5)
iv (3, 3), (3, 3)
b **i** $(-5, 0), (0, -\sqrt{5})$
ii $(-3, 0), (2, \sqrt{5})$
iii $(-5, -5), (0, \sqrt{5} - 5)$
iv $(-2, 3), (3, 3 + \sqrt{5})$
c **i** (0, 0), (0, 0)
ii (2, 0), (2, 0)
iii (0, −5), (0, −5)
iv (3, 3), (3, 3)
d **i** (0, 0), (0, 0)
ii (2, 0), (2, 0)
iii (0, −5), (0, −5)
iv (3, 3), (3, 3)

5 a–c



6 a To sketch the graph of $y = \frac{1}{x} + 20$, the graph of $y = \frac{1}{x}$ is translated 20 units up.

b To sketch the graph of $y = \frac{1}{x - 15}$, the graph of $y = \frac{1}{x}$ is translated 15 units right.

c To sketch the graph of $y = -1 + \sqrt[3]{x}$, the graph of $y = \sqrt[3]{x}$ is translated 1 units down.

d To sketch the graph of $y = \sqrt[3]{2 + x}$, the graph of $y = \sqrt[3]{x}$ is translated 2 units left.

7 a $y = 3 \times 5^x$

b $y = -5^x$

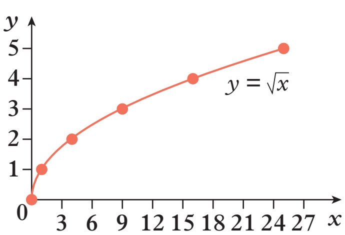
c $y = 5^{x-3}$

d $y = 5^x - 2$

8 a

x	0	1	4	9	16	25
y	0	1	2	3	4	5

b, d



No; not possible to take the square root of a negative number.

c As x increases, y increases, starting off steep and becoming less steep. The graph starts at the origin and extends into the first quadrant.

e i (3, 0)

ii Translate 3 units right

f i Translate 2 units up

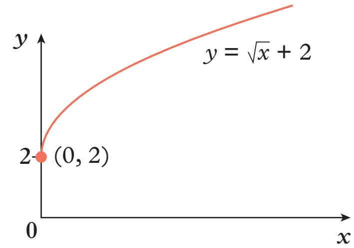
ii Translate 1 unit down

iii Translate 1 unit right

iv Translate 4 units left

v Translate 2 units right and 3 units up

g i



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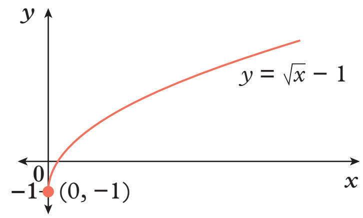
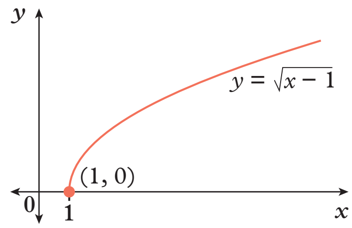
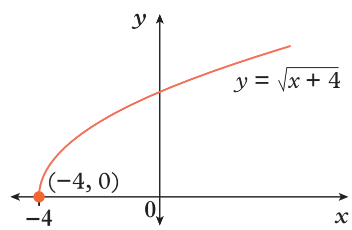


iii



iv



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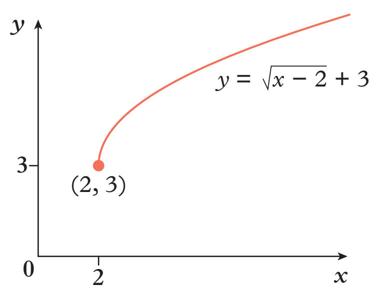
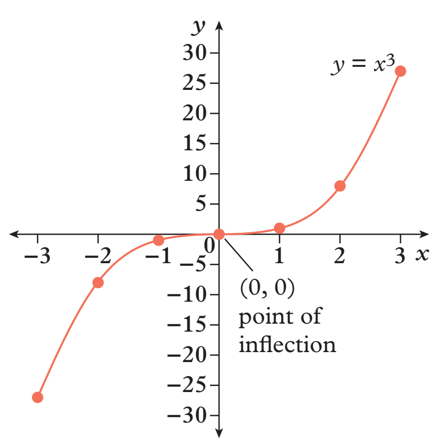


9 a

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

b



c As x increases, y increases. The graph appears to level out at $x = 0$ before rising more steeply again.

d **i** (0, 2) **ii** Translate graph 2 units up

e **i** Translate 1 unit up

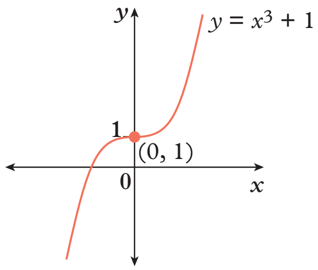
ii Translate 3 units down

iii Translate 2 units right

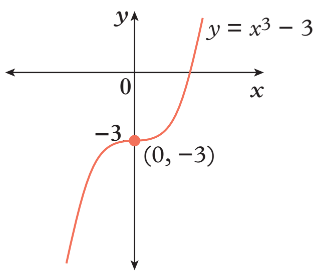
iv Translate 4 units left

v Translate 1 unit right and 2 units up

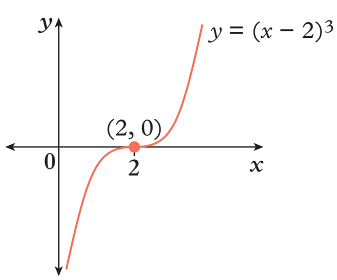
f **i**



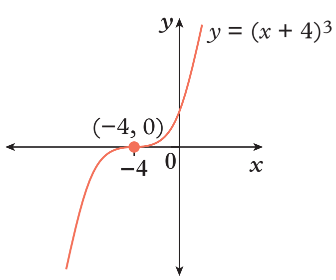
ii



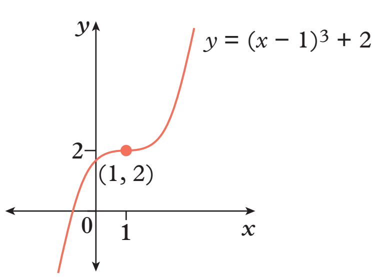
iii



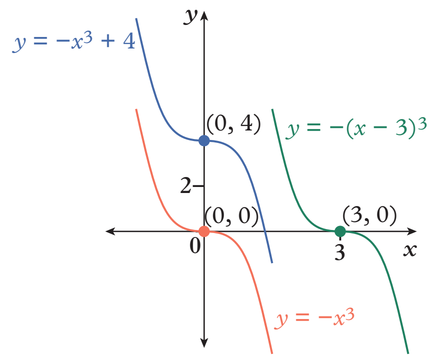
iv



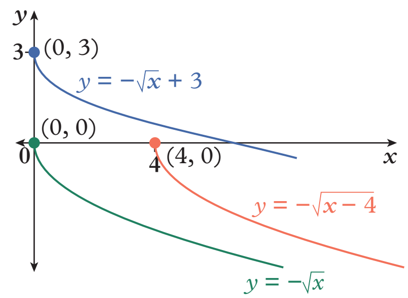
v



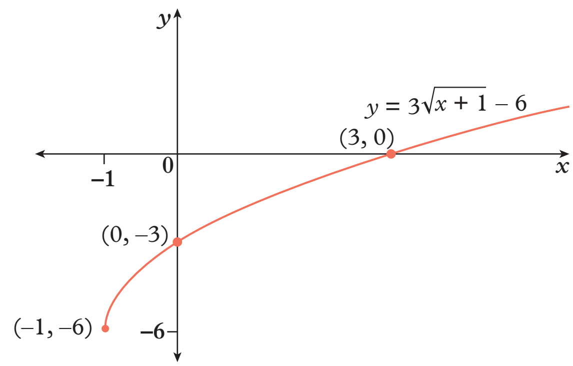
10 **a**



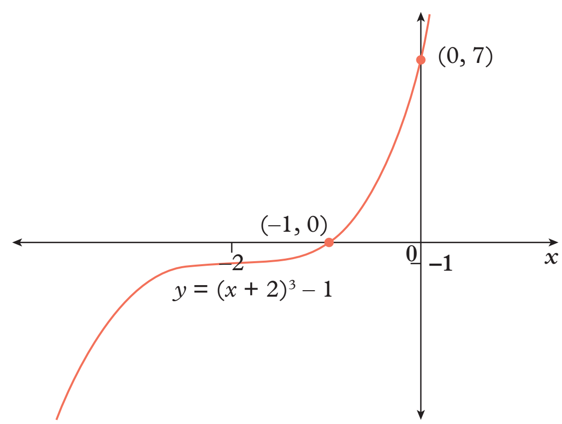
b



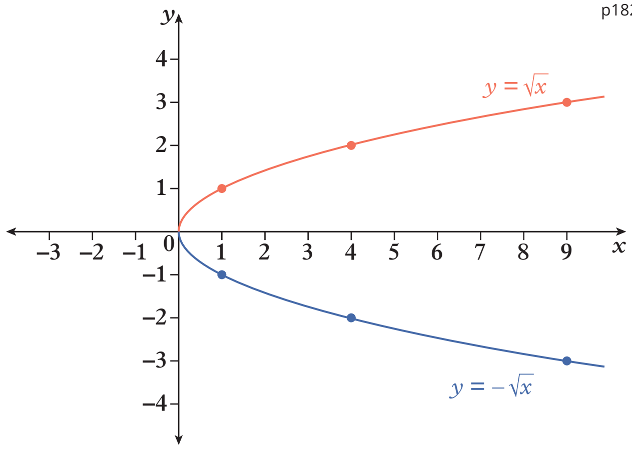
11 **a**



b

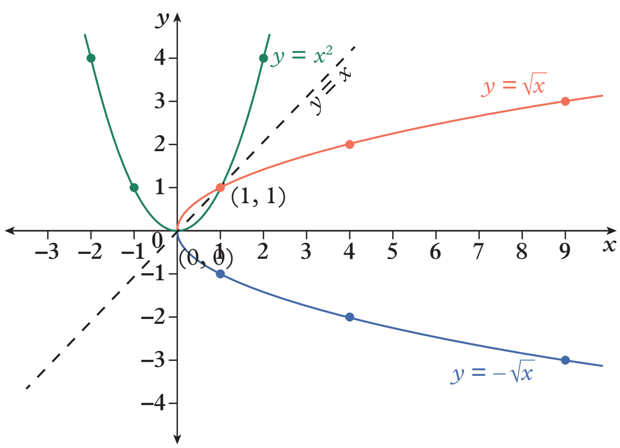


12 a



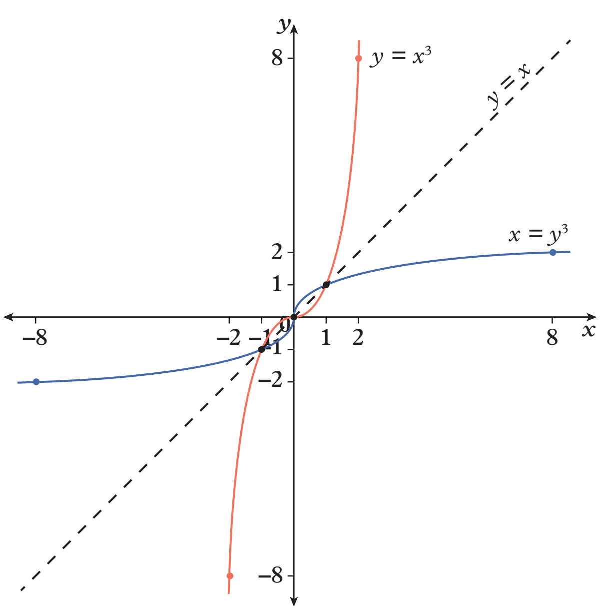
b $y = \pm\sqrt{x}$ or $y^2 = x$

c, d



e The graph of $y^2 = x$ and $y = x^2$ are reflected in the line $y = x$, or the graph of $y = x^2$ is rotated 90 degrees clockwise to make $y^2 = x^2$.

13



EX p182

Review: Non-linear relationships

Review questions 4.6A: Mathematical literacy review

1 highest common factor, factorise, Null Factor Law, factor

2 a $y = ax^2 + bx + c$, for example $y = 2x^2 + 3x + 1$

b $y = a(x - b)(x - c)$, for example $y = (x - 1)(x - 2)$

c $y = a(x - h)^2 + k$, for example $y = 2(x - 6)^2 + 3$

3 a IV **b** VII **c** I **d** III

e VIII **f** V **g** II **h** VI

4 a True

b False. If $a < -1$ then the graph has a vertical dilation and is enlarged.

c False. If $a = -1$ then the graph has a vertical reflection across the x-axis.

d True

e i True

ii False. The graph does not pass through the origin.

iii False. The coordinates of the vertex are $(0, k)$.

Review questions 4.6B: Multiple choice

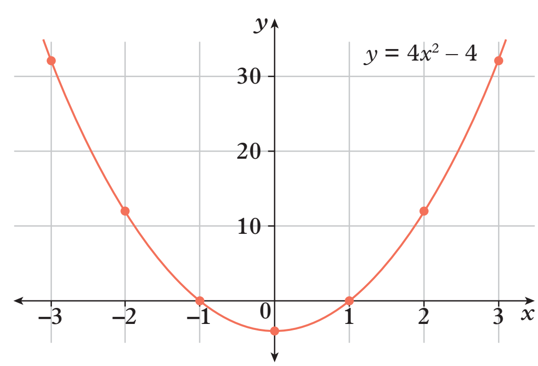
1 D	**2** A	**3** A	**4** D	**5** C
6 C	**7** D	**8** B	**9** C	**10** D

Review questions 4.6C: Short answer

1 a $x = 2$ or $x = 3$ **b** $x = -6$ or $x = 5$

c No solutions **d** $x = 0$ or $x = 12$

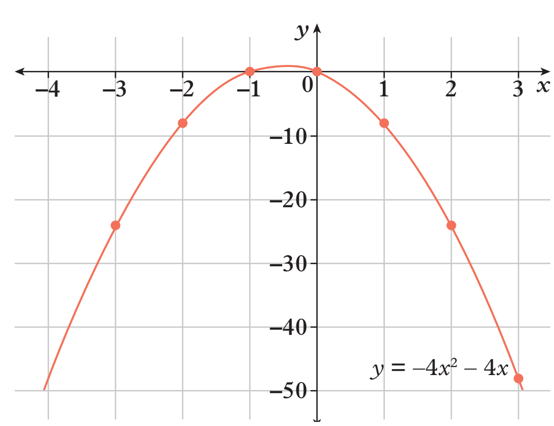
2 a



i x-intercepts: $(-1, 0)$ and $(1, 0)$, y-intercept: $(0, -4)$

ii $(0, -4)$ **iii** Minimum **iv** $x = 0$

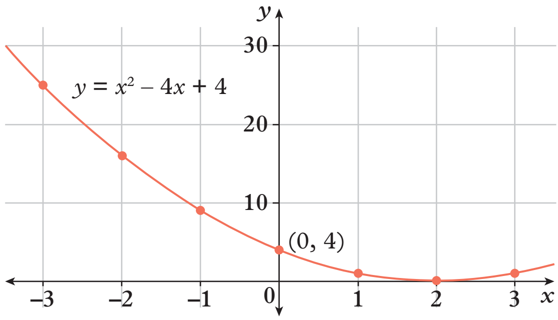
b



i x-intercepts: $(-1, 0)$ and $(0, 0)$, y-intercept: $(0, 0)$

ii $(-0.5, 1)$ **iii** Maximum **iv** $x = -0.5$

c



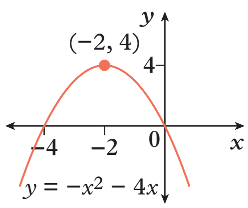
i x-intercept: $(2, 0)$, y-intercept: $(0, 4)$

ii $(2, 0)$ **iii** Minimum **iv** $x = 2$

3 a i x-intercepts: $(-4, 0)$ and $(0, 0)$, y-intercept: $(0, 0)$

ii $(-2, 4)$

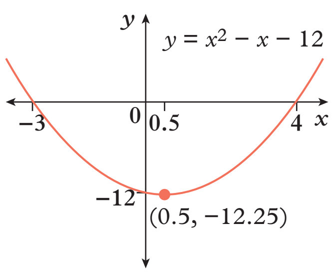
iii



b i x-intercepts: $(-3, 0)$ and $(4, 0)$, y-intercept: $(0, -12)$

ii $(0.5, -12.25)$

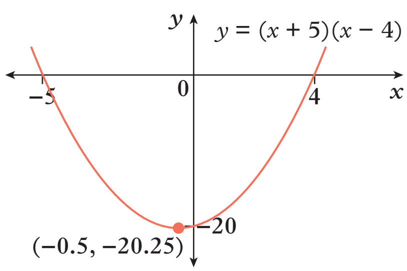
iii



c i x-intercepts: $(-5, 0)$ and $(4, 0)$, y-intercept: $(0, -20)$

ii $(-0.5, -20.25)$

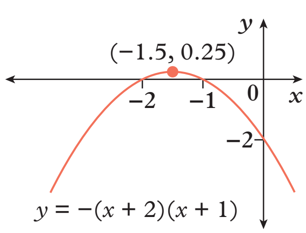
iii



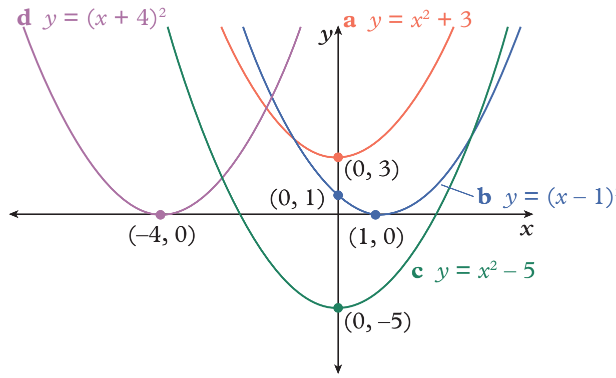
d i x-intercepts: $(-2, 0)$ and $(-1, 0)$, y-intercept: $(0, -2)$

ii $(-1.5, 0.25)$

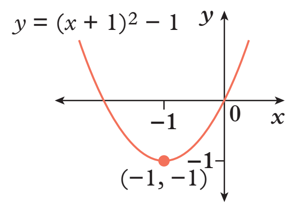
iii



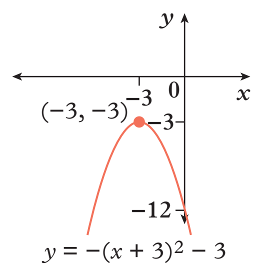
4 a–d



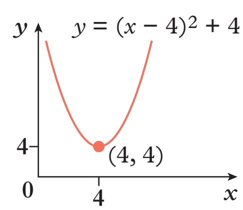
5 a



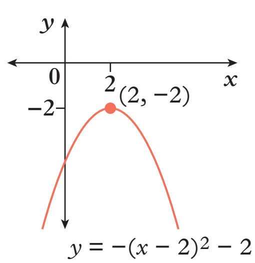
b



c



d



6 a Translate 3 units up

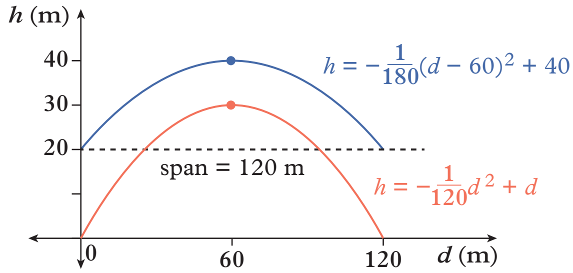
b Dilate by a factor of 2 in the y-direction and reflect in the x-axis

7 a Reflect across the x-axis

b Translate 2 units up

Review questions 4.6D: Mathematical modelling

1 a, d(i)



b **i** Upper arch: (0, 20), lower arch: (0, 0)

ii Upper arch: (60, 40), lower arch: (60, 30)

iii Both have the same axis of symmetry, $d = 60$.

c 20 m

d **i** 120 m

ii Sample answer:

Let $h = 0$: $-\frac{1}{120}d^2 + d = 0$

$-d\left(\frac{1}{120}d - 1\right) = 0$

$d = 0$ or $d = 120$

So, span = 120 m.

e **i** 30 m **ii** 40 m

f 10 m

g 12.5 m

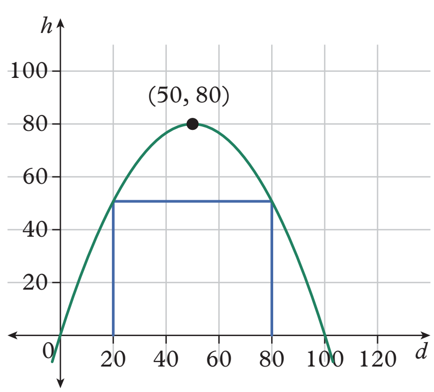
h The two arches are 20 m apart at the start. Approaching the highest points on both arches, the distance between the two arches decreases until it is a minimum of 10 m apart at the top of the arch. Approaching the far right side of the bridge, the distance between the two arches increases until they are a distance of 20 m apart at the end of the bridge.

2 a $h = ad(d - 100)$

b (0, 0) and (100, 0); $d = 0$ is where the ball starts, $d = 100$ is where the ball lands.

c (50, 80)

d



e $a = -0.032$

f $h = 51.2$ m

g 80 m

3 a, c

Equation	x	-2	-1	0	1	2
A	$y = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
B	$y = 2 \times 2^x$	$\frac{1}{2}$	1	2	4	8
C	$y = 2^{x+1}$	$\frac{1}{2}$	1	2	4	8

b Dilate by a factor of 2 in the y-direction

d $y = 4 \times 2^x$, $y = 2^{x+2}$

e Powers of 2 $(a = \ldots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots)$

f If $a = 2^n$ where n is an integer, then applying the product of powers law gives:

$$\begin{aligned} y &= a \times 2^x \\ &= 2^n \times 2^x \\ &= 2^{x+n} \end{aligned}$$

EX p186

SEMESTER 1 Review

Short answer

1 a 20.040 **b** 20.0

c 0.0008 **d** 7.827×10^{-4}

2 a 6 **b** 2 **c** 6 **d** 5

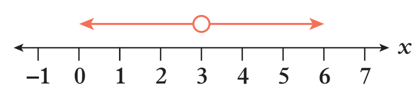
3 a 7.514×10^8 **b** 2.5360×10 or 2.5360×10^1

c 8.6×10^{-5} **d** 7.50×10^4

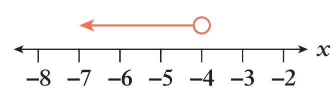
4 a $4\sqrt{2}$ **b** $6\sqrt{5}$ **c** $2\sqrt{21}$ **d** $12\sqrt{7}$

5 a $x < 4$ **b** $x \neq 2$ **c** $x \geq 7$ **d** $-5 < x \leq 2$

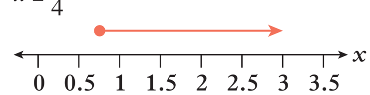
6 a $x \neq 3$



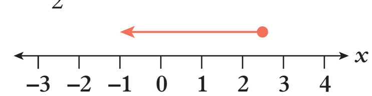
b $x < -4$



c $x \geq \frac{3}{4}$



d $x \leq \frac{5}{2}$



7 a 3^4 **b** $2^2 \times 5^3$

c $2^3 \times 3^3$ **d** $2^2 \times 3^3 \times 5^2$

8 a 343 **b** 375

c 82 130 **d** 0.010 530

e $\frac{27}{8} = 3.375$ **f** $\frac{49}{25} = 1.96$

9 a $2^{30} \times 7^{18}$ **b** $\frac{11^8}{17^9}$

c $\frac{1}{97^4 \times 101^{12}}$ **d** $41^{66} \times 57^{16}$

10 a $a^{16}b^{14}$ **b** $\frac{e}{c^{10}}$ **c** $\frac{g^4}{f}$ **d** $\frac{q^{51}}{p^4}$

11 a $56 - 16x$ **b** $-6x^5 + 16x^3$
c $x^2 - 5x - 14$ **d** $10xy + 15x - 4y - 6$
e $-12x - 23$ **f** $5x^2 + 6x + 6$

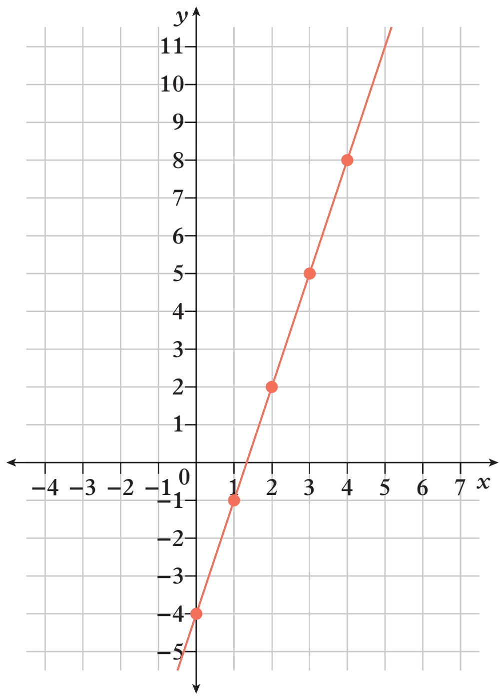
12 a $6(2a + 3b^2 - cd)$ **b** $7g(3g - 1)$
c $(a - 5)(a + 5)$ **d** $2(2m - 3n)(2m + 3n)$
e $(t - 5)(t + 9)$ **f** $(r - 4)(r + 9)$

13 a $x = \frac{11}{12}$ **b** $x = -3$
c $x = 8$ **d** $x = 7$
e $x = \frac{4}{5}$ **f** $x = -3, x = 7$
g $x = \frac{1}{9}, x = -\frac{3}{5}$ **h** $x = -5, x = -7$

14 a $\frac{1}{2}$ **b** 0 **c** $-\frac{3}{2}$
d Undefined

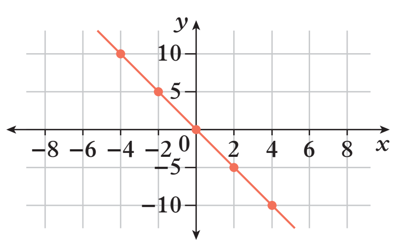
15 a

x	0	1	2	3	4
y	−4	−1	2	5	8



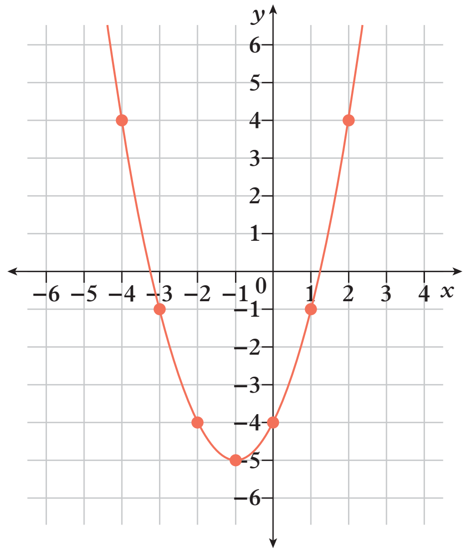
b

x	−4	−2	0	2	4
y	10	5	0	−5	−10



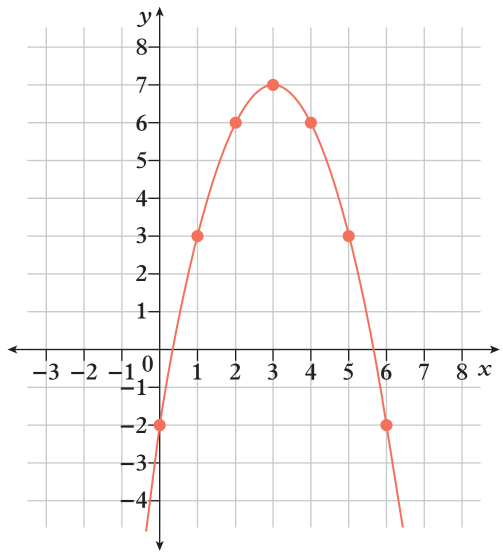
c

x	−4	−3	−2	−1	0	1	2
y	4	−1	−4	−5	−4	−1	4



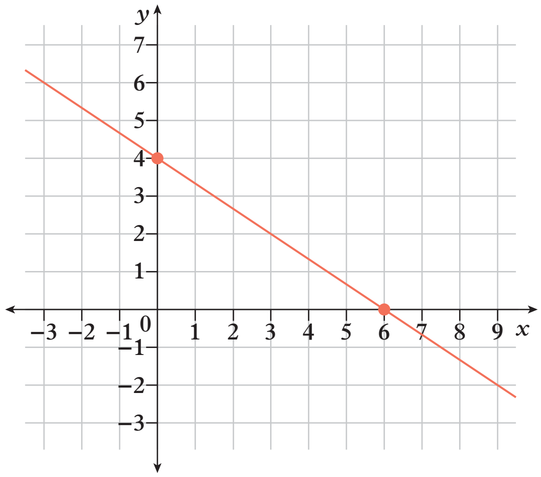
d

x	0	1	2	3	4	5	6
y	−2	3	6	7	6	3	−2

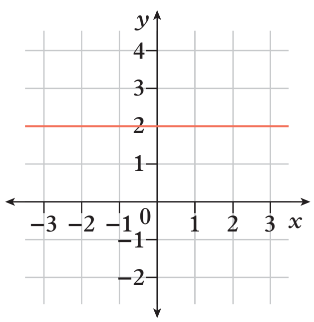


16 a $(2, 0), (0, -6)$
b $(-12, 0), \left(0, -\frac{15}{2}\right)$
c $(5, 0), (-9, 0), (0, -45)$
d $(10, 0), (-4, 0), (0, -40)$

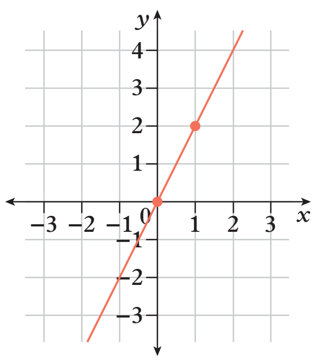
17 a



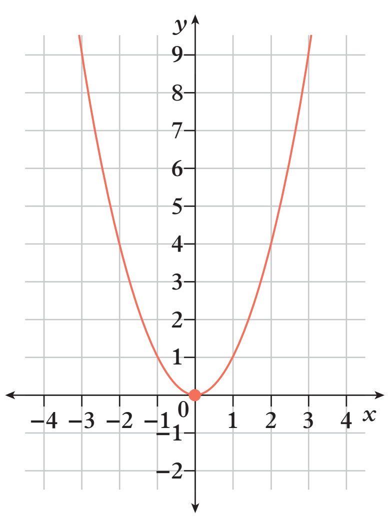
b



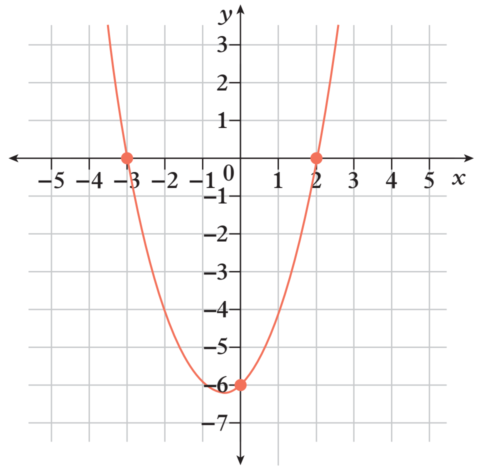
c



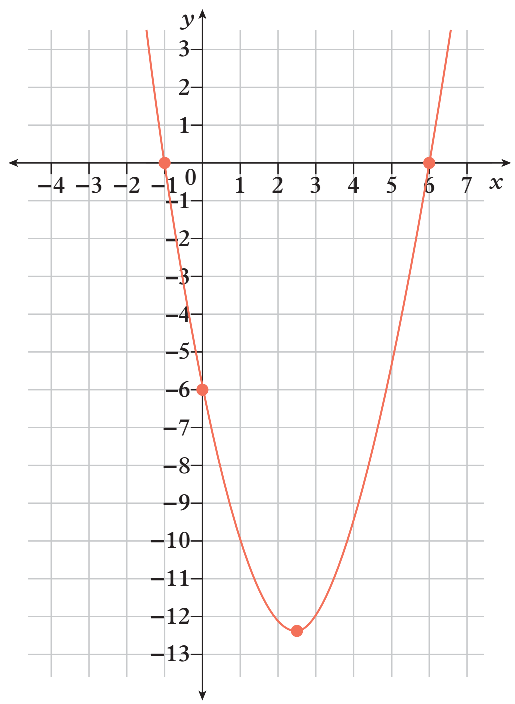
d



e



f



18 a $\frac{1}{4}$ **b** -3 **c** -5 **c** $-\frac{1}{4}$

19 a $y = -3x + 5$ **b** $x = -3$

20 a $(-1, 2)$ **b** $(2, 4.5)$

21 a 8.54 units **b** 6.32 units

22 a $x^2 - 14x + 52$ **b** $x^2 + 24x + 149$

c $-x^2 + 8x - 13$ **d** $-x^2 + 6x - 11$

23 a Horizontal translations of 2 units to the right

b Horizontal translations of 5 units to the left

c Horizontal translations of 3 units to the left, then vertical translation of 1 unit up

Mathematical modelling

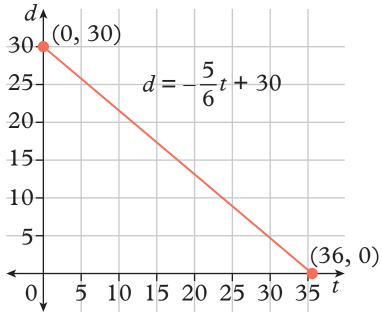
1 a d-intercept: $(0, 30)$, t-intercept: $(36, 0)$

b $\frac{5}{6}$ km/min

c From the axis intercepts, the graph must have a negative gradient, so the gradient, m, is the negative of the speed, $m = -\frac{5}{6}$.

d $d = -\frac{5}{6}t + 30$

e

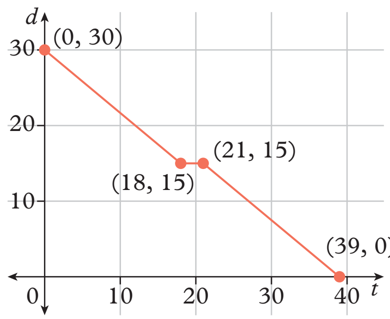


f **i** (18, 15)

ii Add a horizontal line segment at height $d = 15$ for $18 \leq t < 21$ since Jane's car isn't moving for those three minutes. The second half of the original graph is translated horizontally right by three units.

The t-intercept moves to (39, 0).

iii



iv $d = -\frac{5}{6}t + 30 \quad (0 \leq t < 18)$

$d = 15 \quad (18 \leq t < 21)$

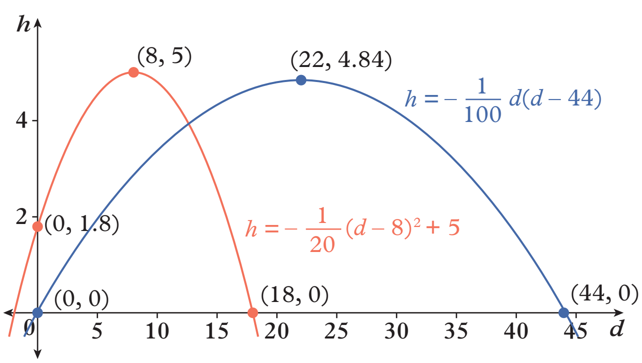
$d = -\frac{5}{6}t + \frac{65}{2} \quad (21 \leq t \leq 39)$

2 **a** **i** 0 m **ii** 1.8 m

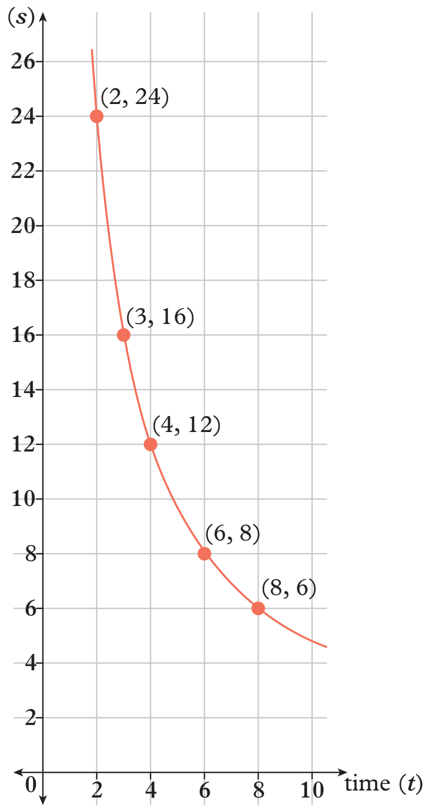
b **i** 4.84 m **ii** 5 m

c **i** 44 m **ii** 18 m

d



e



The relationship is non-linear.

EX
p190

EXPLORATIONS 1

1 **a** **i** The van's speed is 90 km/h and the bicycle's is 40 km/h. The van overtakes the bicycle at 4:24 pm.

ii 104 km/h **iii** 6:40 pm

b Speed of van $= \frac{1}{4}(c + 3t)$ km/h

Speed of bicycle $= (2t - c)$ km/h

The van overtakes the bicycle at 4:24 pm, which is independent of c and t.

2 **a** **i** The number of border squares is $2x + 2y - 4$ and the number of interior squares is $xy - 2x - 2y$.

ii $xy - 4x - 4y + 4 = 0$

iii 12, so the equation becomes $(x - 4)(y - 4) = 12$

iv $(x, y) = (5, 16), (6, 10), (7, 8), (8, 7), (10, 6)$ or $(16, 5)$

v Switching x and y rotates the design, so we can ignore the last three solutions.

b **i** Three: $(x, y) = (7, 34), (8, 20)$ or $(10, 13)$

ii One: $(x, y) = (4, 10)$

3 **a** To four decimal places: 2.4495, 2.9068, 2.9844, 2.9974, 2.9996, …, which appears to approach 3.

b Solve $x^2 - x - 6 = 0$ to give $x = -2$ or 3 but ignore $x = -2$ as x must be positive.

c **i** 4 and 1

ii $\sqrt{20 + \sqrt{20 + \ldots}}$ and $\sqrt{30 - \sqrt{30 - \ldots}}$

4 **a** $12 \longrightarrow 6 \longrightarrow 3 \longrightarrow 8 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$

b $13 \longrightarrow \cdots \longrightarrow 14 \longrightarrow 7 \longrightarrow 20 \longrightarrow 10 \longrightarrow 5$

c This cycle has length 18: There are other solutions.

$17 \longrightarrow 50 \longrightarrow 25 \longrightarrow \cdots \longrightarrow 136 \longrightarrow 68 \longrightarrow 34$

d Use the fact that a multiple of 3 can only be the result of halving.

e There is only one: the cycle in part **b**.

f One solution is $2^{500} + 1$. There are many others.

5 **a** Clockwise from the top left, the lengths and areas are as follows:

Length (cm)	14	14	12	12	6	6	8	8
Area (cm²)	80	96	72	54	18	17	30	32

b Dark length $= 14 + 12 + 6 + 8 = 40$ cm

$= \frac{1}{2}$ of total

Dark area $= 80 + 72 + 18 + 30 = 200$ cm²

$= \frac{1}{2}$ of total

c Show that, for any a and b, the total dark length is always $2c$ and the total dark area is always $\frac{1}{2}c^2$.

d Let P be at the centre and choose any three points on the boundary that divide the perimeter into three equal lengths. Show that cutting from P to the three boundary points creates three fair slices.

e It is possible for all positive integers by adapting the strategy in part **d**.

MODULE 5 Measurement

5.1 Pythagoras' theorem

EX p196

1 **a** Right-angled **b** Right-angled **c** Not right-angled

d Right-angled **e** Not right-angled

f Not right-angled

2 **c** Obtuse triangle

e Acute triangle

f Acute triangle

3 **a** 3.6 cm **b** 7.1 mm **c** 33.8 cm

d 7.3 cm **e** 2.0 cm **f** 75.0 mm

g 8.7 cm **h** 4.2 cm **i** 8.1 cm

4 **a** $10\sqrt{5}$ mm **b** $4\sqrt{13}$ cm **c** $5\sqrt{10}$ cm

d $3\sqrt{2}$ cm **e** $2\sqrt{10}$ mm **f** $4\sqrt{17}$ cm

5 **a** 22.4 mm **b** 14.4 cm **c** 15.8 cm

d 4.2 cm **e** 6.3 mm **f** 16.5 cm

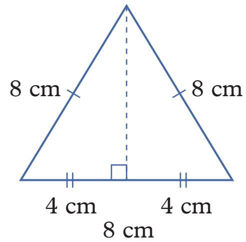
6 **a** $5\sqrt{2}$ cm **b** $20\sqrt{2}$ mm **c** $8\sqrt{2}$ cm

d $25\sqrt{2}$ cm

7 **a** 7.1 cm **b** 28.3 mm **c** 11.3 cm

d 35.4 cm

8 **a**



b **i** $4\sqrt{3}$ cm **ii** 6.9 cm

9 **a** **i** $10\sqrt{17}$ cm **ii** 41.2 cm

b **i** $(3\sqrt{11} + \sqrt{31})$ cm **ii** 15.5 cm

c **i** $(4\sqrt{14} - \sqrt{21})$ cm **ii** 10.4 cm

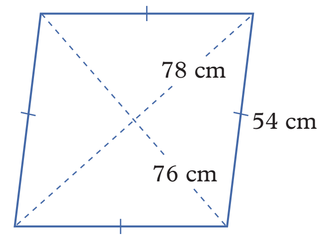
d **i** $(4\sqrt{21} + \sqrt{17})$ cm **ii** 22.5 cm

e **i** $8\sqrt{21}$ cm **ii** 36.7 cm

f **i** $5\sqrt{13}$ mm **ii** 18.0 mm

10 **a** $78^2 > 54^2 + 54^2$, so the frame is not truly square. The two sides meet at an obtuse angle.

b



11 **a** 11 cm **b** 2 folds **c** 2 folds

12 **a** (1) Afra used Pythagoras' theorem on the left right-angled triangle. (2) Afra used Pythagoras' theorem on the right right-angled triangle. Afra showed the 6 cm edge is the sum of the lengths of a and b.

b **i** $h^2 = 13 - a^2$ **ii** $b = 6 - a$

c
$$\begin{aligned} b^2 + h^2 &= 25 \\ (6 - a)^2 + (13 - a^2) &= 25 \\ 36 - 12a + a^2 + 13 - a^2 &= 25 \\ 49 - 12a &= 25 \end{aligned}$$

d $a = 2$ cm

e $b = 4$ cm, $h = 3$ cm

f $h = 1$ cm ($a = 2$, $b = 5$)

13 **a** All three sets of numbers (**i–iii**) satisfy Pythagoras' theorem.

b All three sets of numbers (**i–iii**) satisfy Pythagoras' theorem so all three triangles described are right-angled triangles.

c If the three numbers of a Pythagorean triad are multiplied or divided by the same number, the resulting numbers are also a Pythagorean triad.

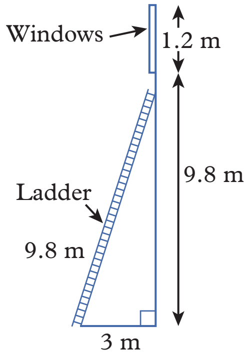
d Each side is multiplied by the same scale factor, so the corresponding sides are in the same ratio.

14 **a** $5\sqrt{5}$ cm, $10\sqrt{5}$ cm

b 11.2 cm, 22.4 cm

15 The only way the length of one side of a triangle could be equal to the sum of the lengths of the other two sides would be if they were all on the same straight line, but in that case the shape would not be a triangle, it would just be a straight line.

16 a



b 9.3 m

c The top of the window is 9.8 + 1.2 = 11 m above the ground. If Romesh stands on the very top of the ladder he is the same height above the ground as the top of the window (9.3 m + 1.7 m = 11 m). He will be able to clean the windows.

17 16.83 m

18 $x = 4\sqrt{2}$ cm, $y = 9$ cm, $z = 3\sqrt{91}$ cm

EX 5.2 Area of composite shapes

p203

1 a 20.79 cm² **b** 1800 mm² **c** 452.39 cm²
d 14.16 mm² **e** 44.2 cm² **f** 768 mm²

2 a 7.07 cm² **b** 127.41 cm² **c** 184.31 mm²

3 a Triangle, semicircle
b Three rectangles
c Triangle, rectangle, semicircle
d Triangle, rectangle
e Rectangles, semicircles
f Parallelogram, triangle, semicircle

4 a 122.27 cm² **b** 56 cm² **c** 31.57 cm²
d 69 cm² **e** 160 cm² **f** 174.63 cm²

5 a 70.87 cm² **b** 47.43 mm² **c** 424 cm²
d 69.73 cm² **e** 70.5 cm² **f** 60.5 cm²

6 a 141.02 cm² **b** 330 cm² **c** 557.08 cm²
d 34 cm² **e** 34.28 cm² **f** 63.26 cm²

7 a An annulus is the shape formed by two different-sized circles with a common centre. It is the area between the two circles.
b Area of outer circle – Area of inner circle
c 137.44 cm²

8 a 12.57 cm² **b** 65.97 cm² **c** 28.27 cm²
d 103.67 cm² **e** 226.19 cm² **f** 47.12 cm²

9 12 166.55 m²

10 a 35.5 cm²
b 19 525 cm²
Sample answers: To cater for wastage between tags when they are cut out; in case any mistakes are made in production of tags.

11 126.70 cm²

12 a 105.35 m² **b** $4740.75 **c** $1580.25

13 a 354.01 cm² **b** 336 cm² **c** 97.425 cm²
d 68.57 cm² **e** 35 cm² **f** 78.72 cm²

14 a i $4\sqrt{2}$ cm **ii** 44.93 cm²
b i $\sqrt{21}$ cm **ii** 71.56 cm²
c i $10\sqrt{2}$ cm **ii** 257.08 cm²

15 a A, B: 25 cm², C, E, G: 12.5 cm², D, F: 6.25 cm²
b i 100 cm² **ii** 100 cm² **iii** 100 cm²

16 47.75 cm²

17 15 509.73 cm²

18 71.78 cm²

19 a 44 cm² **b** 17.80 cm² **c** 15 m²

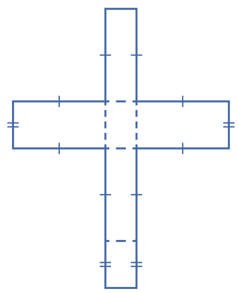
EX 5.3 Surface area of prisms

p211

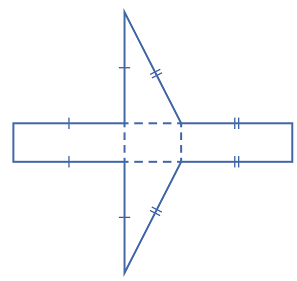
1 **b, c**

2 a i Triangular prism **ii** 36 cm²
b i Rectangular prism **ii** 70 cm²

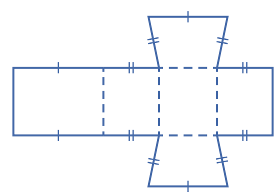
3 a



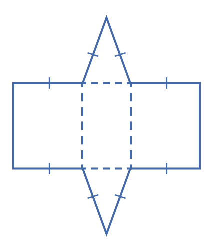
b



c

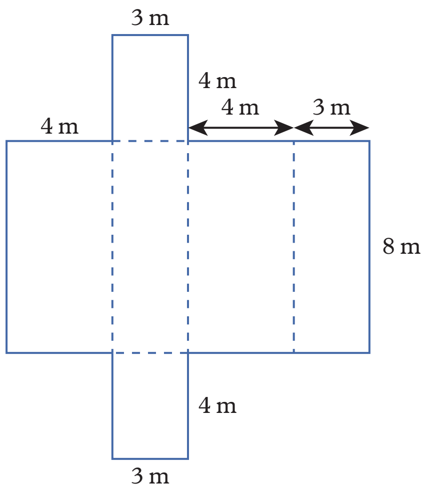


d



4 a 24 m² **b** 294 mm² **c** 600 cm²
d 384 mm² **e** 13.5 m² **f** 5400 cm²

5 **a**



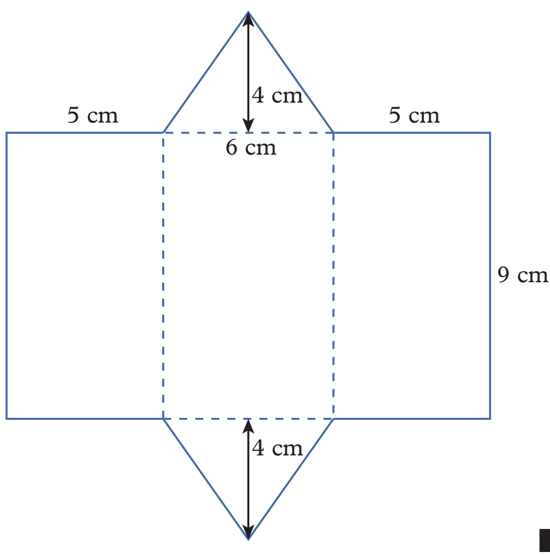
b 32 m², 24 m² and 12 m²

c 136 m²

6 **a** 344 mm² **b** 166 cm² **c** 378 mm²

d 268 m² **e** 76.96 m² **f** 738 cm²

7 **a**



b 12 cm², 12 cm², 45 cm², 45 cm², 54 cm²

c 168 cm²

8 **a** 2916 cm² **b** 510 mm² **c** 142 m²

d 768 mm² **e** 2408 cm² **f** 523 m²

9 **a** **i** 664 cm² **ii** 1328 cm²

b **i** 396 cm² **ii** 792 cm²

10 **a** 3820 cm² **b** 2961.2844 cm²

11 **a** 195.52 mm² **b** 162.174 m²

12 **a** 500 cm²

b **i** 292 cm²

ii 584 cm²

c Answer to **b ii** is 84 cm² greater because two more faces (7 cm by 6 cm) are created when the block of butter is cut in half.

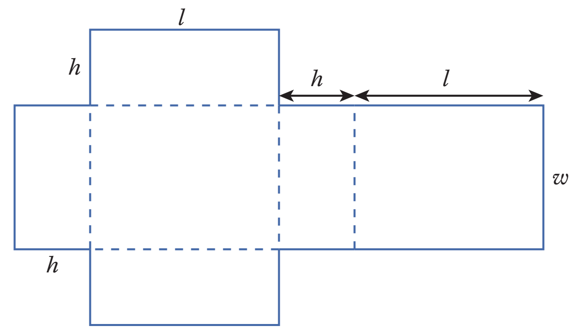
d 4032 cm²

e Sample answer: Cutting butter into smaller pieces increases the amount of surface exposed to the heat, so butter will melt more quickly.

13 **a** Cream: 30 m², blue: 53.575 m²

b Cream: 4 L, blue: 7.14 L

14 **a**



b $2(lw + hw + hl)$

c There are three dimensions for every rectangular prism and these can be represented by l, w and h, so this expression can be used to calculate the surface area of any rectangular prism.

d **i** 46 cm² **ii** 413.98 mm² **iii** $\frac{17}{18}$ cm²

15 **a** 25 cm² **b** 5 cm **c** $l = \sqrt{\frac{\text{TSA}}{6}}$

16 Sample answer: A cube has six identical square faces; hence all sides are equal in length. Therefore, the length of one face will determine the length of all sides. A rectangular prism could have a different length, breadth and height.

17 24.92 m²

18 **a** 1510 cm²

b 226.24 m²

19 **a** 50 cm²

b 30 cm²

EX p216

Checkpoint

1 **a** Yes **b** Yes **c** No **d** Yes

2 **a** **i** $\sqrt{41}$ cm **ii** 6.40 cm

b **i** $6\sqrt{14}$ cm **ii** 22.45 cm

3 **a** $(\sqrt{351} - \sqrt{15})$ cm

b $6\sqrt{7}$ cm

4 **a** 4.71 cm²

b 41.89 cm²

5 **a** 37.44 m²

b 284 m²

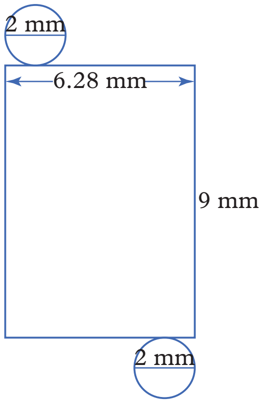
6 **a** 195 mm²

b 57.52 mm²

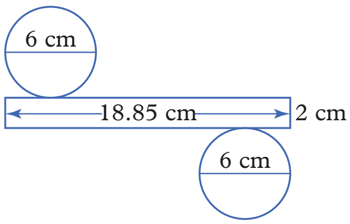
EX p218

5.4 Surface area of cylinders

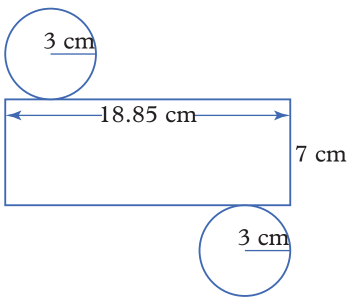
1 a



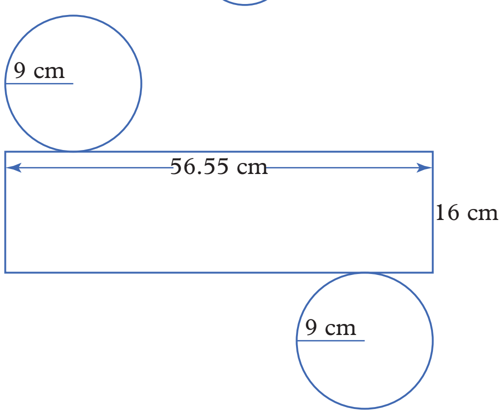
b



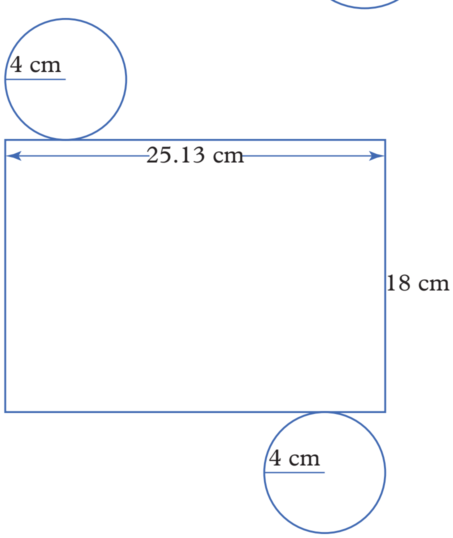
c



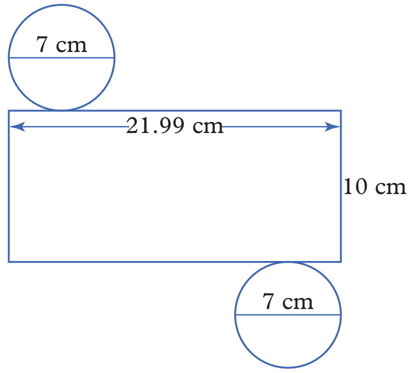
d



e

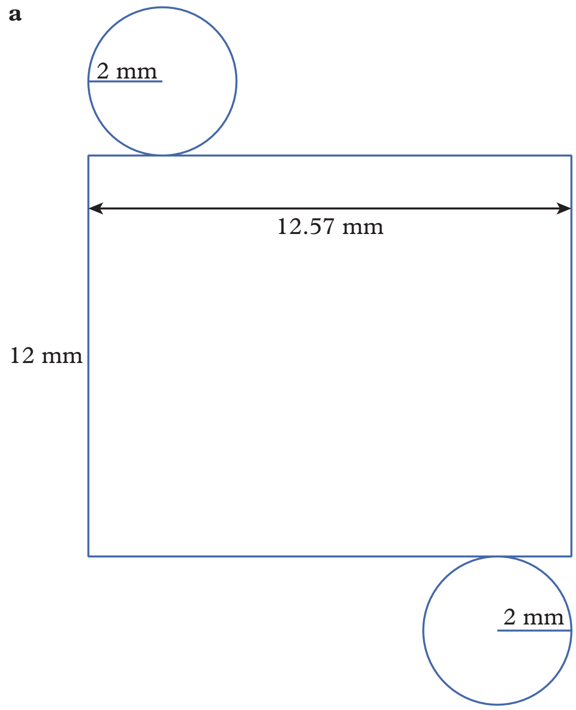


f

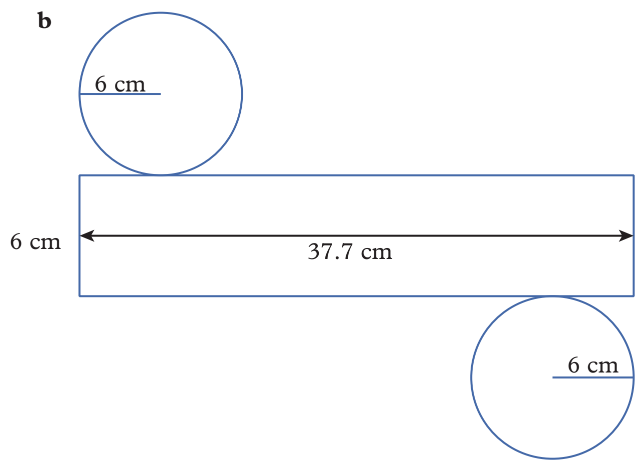


2 **a** 62.83 mm² **b** 94.25 cm² **c** 188.50 cm²
d 1413.72 cm² **e** 552.92 cm² **f** 296.88 cm²

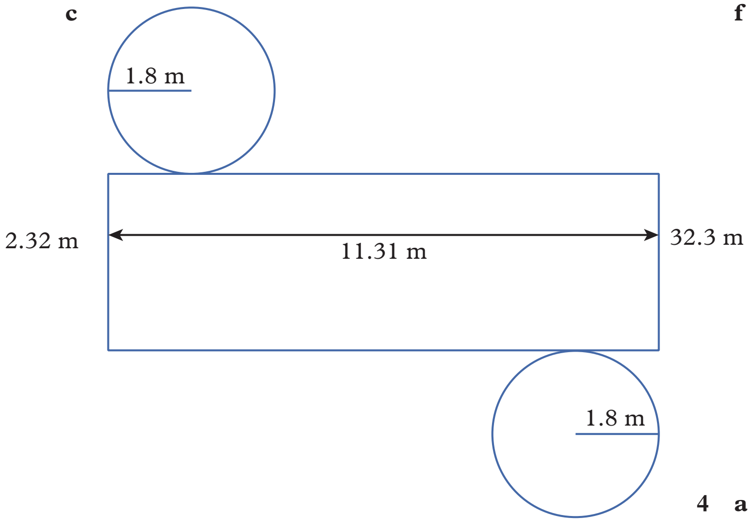
3 a



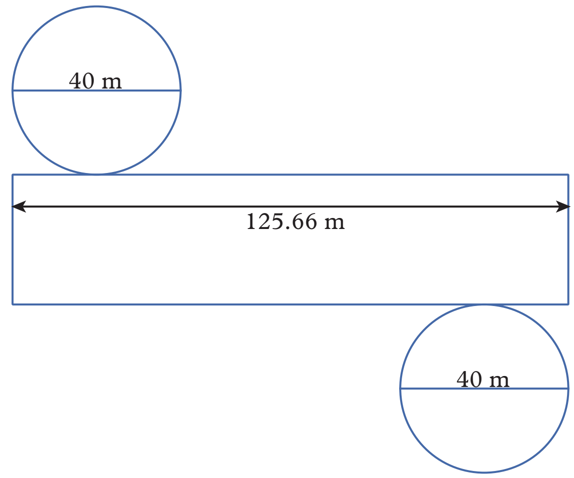
b



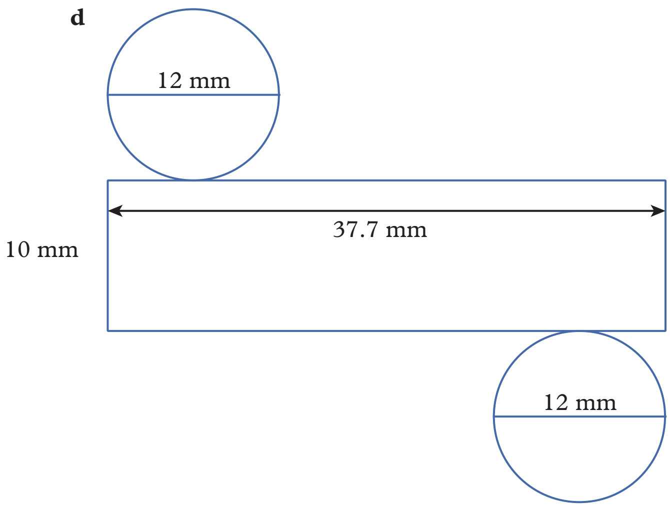
c



f



d



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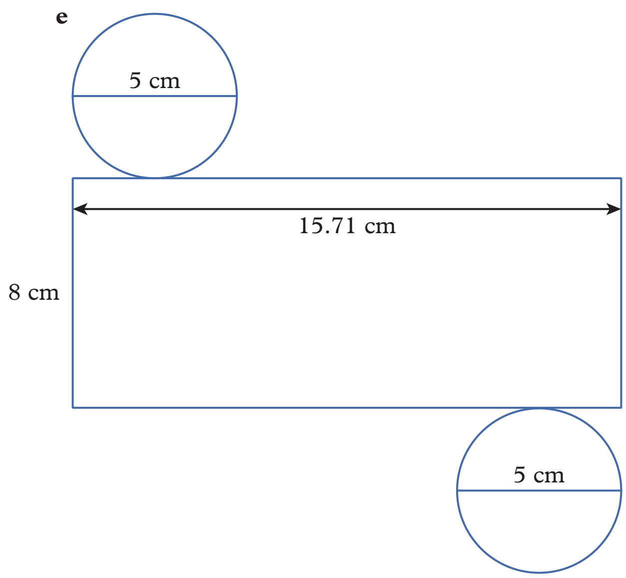


4 **a** 175.93 mm^2 **b** 452.39 cm^2 **c** 46.60 m^2
d 603.19 mm^2 **e** 164.93 cm^2 **f** 6572.21 m^2

5 **a** 100.53 cm^2 **b** 541.92 cm^2 **c** 603.19 cm^2
d 18.85 m^2 **e** 7.85 m^2 **f** 3.39 m^2

6 **a** 1130.97 cm^2 **b** 34.57 cm^2 **c** 96.13 m^2
d 157.10 m^2 **e** 15.36 mm^2 **f** 9.08 mm^2

7 3924.63 cm^2

8 \$1429 (\$1450 if rounded to needing 29 L of paint)

9 First roller with radius 4 cm and length 25 cm

10 **a** 376.99 cm^2
b Lucian has calculated the surface area of the whole cylinder (ignoring the hollowed-out section) and Curtis forgot to include the area of the inside surface.

11 8 cm

12 Sample answer: 2 × area of base calculates the surface area of the matching ends of a prism. The surface area of each of the rectangular faces joining the matching ends can be calculated by multiplying the section of the perimeter of the base that the face occupies by the height, so together the area of all of the rectangular faces joining the matching ends is perimeter of base × height.

13 No, surface area will not double if you double the height of the cylinder. The surface area of the curved surface will double, but the area of the ends will not change.

14 **a** 984.46 cm^2 **b** 70.85 m^2

15 $h = \dfrac{\text{SA} - 2\pi r^2}{2\pi r}$ or $h = \dfrac{\text{SA}}{2\pi r} - r$

16 48.42 cm^2

17 **a** 201.06 cm^2 **b** 615.75 mm^2
c 314.16 cm^2 **d** 127.32 cm^2

EX 5.5 Volume of prisms and cylinders

p224

1 **a** 48 cm^3 **b** 1080 cm^3 **c** 288 cm^3
d 210 cm^3 **e** 36 cm^3 **f** 82.5 cm^3
g 540 cm^3 **h** 2457 cm^3 **i** 140 mm^3

2 **a** 150.80 mm^3 **b** 678.58 cm^3 **c** 23.61 m^3
d 1.130 97 mm^3 **e** 157.08 cm^3 **f** 40 589.38 m^3

3 **a** 0.1508 mL **b** 678.58 mL **c** 23 610 L
d 1.130 97 mL **e** 157.08 mL **f** 40 589 380 L

4 **a** 249.31 cm^3 **b** 18.28 cm^3 **c** 608.58 cm^3
5 **a** 30 cm^2 **b** 4 cm
6 **a** 60 cm^2 **b** 40 mm
7 **a** $6x^2$ **b** x^3 **c** $x, x, x + 4$
d $6x^2 + 16x$ **e** 5 cm **f** 225 cm^3
8 160 cm^3
9 **a** $6x^2$ **b** x^3 **c** $x, x, x - 2$
d $6x^2 - 8x$ **e** 6 cm **f** 144 cm^3
10 18 cm^3
11 54 cm^3
12 **a** Surface area = 2400 cm^2, volume = 8000 cm^3
b Surface area = 2400 cm^2, volume = 6300 cm^3
13 **a** 126 m^3 **b** \$3780
14 **a** **i** \$24 **ii** \$37.58 **iii** \$28.50
b Sculpture **i** weighs 36 kg, sculpture **ii** weighs 37.584 kg, and sculpture **iii** weighs 35.625 kg. She should buy sculpture **iii.**
15 **a** 43.52 cm^3 **b** 38.86 cm^3

5.6 Errors in measurement

EX p229

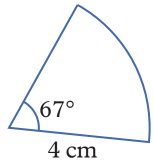
1 **a** 3 **b** $\frac{2}{5}$ **c** 8
d 4.2 **e** 0 **f** $\sqrt{2}$
2 **a** 2 **b** 2 **c** 1.4
d 3.4 **e** 0.068 **f** 0.068
3 **a** 1.65 **b** 2.325
4 B
5 **a** 12 **b** 8
6 **a** 2 seconds **b** 0.0099 **c** 0.99%
7 **a** 20 **b** 0.125 **c** 12.5%
8 **a** \$2535 **b** 0.0955 **c** 9.55%
9 **a** 0.05% **b** 0.05%
10 **a** Lower bound: 429.5 g, upper bound: 430.5 g
b Lower bound: 427.5 g, upper bound: 432.5 g
11 122.5 cm and 127.5 cm
12 **a** **i** 0.0402% **ii** 0.0402% **iii** 0.0402%
b The percentage error is the same regardless of the value of the radius.
c The approximation of $\frac{22}{7}$ is more accurate, because it has a smaller percentage error.
13 **a** The length of the hypotenuse is equal to the square root of the sum of the other two sides squared of a right-angled triangle.
b **i** 0.0142 cm **ii** 0.0101 **iii** 1.01%
14 **a** Lower bound: 11.5 cm, upper bound: 12.5 cm
b Lower bound: 46 cm, upper bound: 50 cm
c Lower bound 132.25 cm^2, upper bound: 156.25 cm^2
15 **a** 91.13 cm^3 **b** 42.88 cm^3
16 **a** Lower bound: 112.5 cm, upper bound: 113.5 cm
b Lower bound: 18 cm, upper bound: 18 cm
c Lower bound: 1007 cm^2, upper bound: 1025 cm^2
17 **a** Left eye: lower bound: −2.55, upper bound: −2.45
Right eye: lower bound: −2.45, upper bound: −2.35
b Left eye: 0.0119; right eye: 0.0127

Review: Measurement

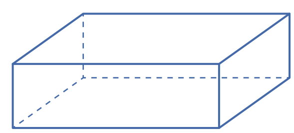
EX p233

Review questions 5.7A: Mathematical literacy review

1 A Pythagorean triple
2 The triangle inequality states that in every triangle the length of any one side must be less than the sum of the lengths of the other two sides.
3 Multiply the relative error by 100 to calculate the percentage error.
4 **a** arc **b** cylinder
5 **a** A sector is a portion of a circle formed by two radii and part of the circumference. For example:



b A prism is a 3D object with straight edges, two identical bases and a constant cross-section parallel to the bases. For example:



6 **a** cross-section
b cube

Review questions 5.7B: Multiple choice

1 B	2 C	3 D	4 B
5 C	6 B	7 C	8 D
9 D	10 B	11 B	12 B

Review questions 5.7C: Short answer

1 **a** $a^2 + b^2 = 30^2 + 40^2$
$= 2500$
$c^2 = 50^2$
$= 2500$
b Yes, as they are three whole numbers that satisfy Pythagoras' theorem.
2 **a** 13 cm **b** 18.41 cm
3 x = 19.90 cm, y = 15.13 cm
4 118 cm^2
5 167.87 cm^2
6 **a** 276 cm^2 **b** 294 cm^2
7 **a** 1187.52 cm^2 **b** 180.64 cm^2
8 **a** 216 cm^3 **b** 180 cm^3
9 **a** 3078.76 cm^3 **b** 176.71 cm^3
10 **a** 4000 kL or 4 ML **b** 13 m
11 **a** 55.05 cm and 54.95 cm
b 32.75 cm and 32.25 cm
c 30.5 g and 29.5 g
d 108.5 mL and 107.5 mL

Review questions 5.7D: Mathematical modelling

1 a Because all measurements are estimates. Errors exist in all measurements, especially when the instrument used is not accurate enough.
b Stack D. 0.084 mm.
c i A: 0.015 mm, B: 0.005 mm, C: 0 mm, D: 0.001 mm; Stack C.
ii 17.6%; 5.9%; 0%; 1.2%. Yes it does

2 a 5 cm
b Pink: 36 cm^2, purple: 900 cm^2
c 0.82 m^2
d 2

MODULE 6 Scale and trigonometry

6.1 Ratios and rates

EX p242

1 a 500 **b** 2000 **c** 6500
2 a 68 **b** 187
3 a 1500 mL **b** 150 mL
4 a \$30/h **b** \$1.35/L **c** \$55/h
d \$2.45/jar **e** \$0.75/min **f** \$12.99/kg
g \$60 000/year **h** \$6.85/parcel mailed
5 a 75 km/hour **b** 0.2 km/minute
c 55 words/minute **d** \$2.50/donut
e \$1.86/litre **f** \$5.35/day
6 a \$5.49/kg **b** \$24/h
c \$0.37/min **d** \$8.27/m
7 a B **b** B **c** B **d** A
e B **f** A **g** B **h** B
8 a \$45 **b** \$558 **c** \$250
d \$225.50 **e** \$1126.25 **f** \$15.62
g \$217 **h** \$29.20
9 a 46 m **b** 234 pieces **c** 12.5 L
d 0.75 kg **e** 30.6 kg **f** 131.25 g
g 159.5 m^2 **h** 44.55 m^3
10 a Multiply by 0.68
b Divide by 0.68
c i 2380 USD **ii** 7548 USD
iii 1800 AUD **iv** 8100 AUD
11 a 11.556 births per 1000 people
b Lower
12 a 336 beats
b 129 BPM
13 a 1.5 **b** 40 minutes
14 a 23.16 g **b** 0.28 cm^3
15 a 732 L **b** 44 seconds
16 21 cm and 36.75 cm
17 2430 cm^2
18 a \$35.04 **b** \$29.99 **c** \$2.04
d 206.2 c/L **e** 27.248 L
19 a Jasmine: 8:10 am, Nelson: 8:25 am
b 8:05 am
20 a Original image: 16 : 9, resized image: 32 : 19
b no
21 32.3125 cm
22 3 : 2 : 4

6.2 Similar figures

EX p248

1 a Enlargement **b** Reduction
c Reduction **d** Enlargement
e Enlargement **f** Reduction
2 a 3 **b** $\frac{3}{4}$ **c** $\frac{1}{3}$ **d** 3
3 a $m = 6$ cm, $n = 108°$
b $p = 118°$, $q = 28$ cm
4 a $a = 7$ cm
b $b = 124$ cm
c $c = 25.5$ cm, $d = 108°$
d $e = 18$ mm, $f = 135°$, $g = 26°$
5 a Not similar, RHS **b** Similar, SAS
c Not similar, AAA **d** Similar, SSS
6 7.84 m by 3.6 m
7 a 3.6 by 2.76 m **b** 4.2 m by 3.36 m
c 4.56 m by 2.64 m **d** 5.04 m by 4.86 m
8 7 µm
9 a 4 m **b** 3.75 cm
c If all proportions are in the same ratio, yes; however, it would be very difficult to manufacture a model car with every aspect in the same ratio.
10 $x = 8.30$ m
11 a 2.5 m
b 2.72 m tall
12 a 33°
b $\triangle ABD$ is similar to $\triangle CAD$ by the similarity test AAA
c $\triangle ABD \sim \triangle CBA$; $\triangle CAD \sim \triangle CBA$
13 a $\angle ABC = \angle MNC$ (corresponding angles on parallel lines)
$\angle CAB = \angle CMN$ (corresponding angles on parallel lines)
$\angle ACB = \angle MCN$ (same angle)
Therefore $\angle ABC \sim \angle MNC$ by the similarity test AAA.
b 18 cm
14 a 9 : 4 **b** 4 : 5

EX p254

6.3 Area and volume scale factors

1

Length scale factor	Area scale factor	Volume scale factor
2	4	8
$\frac{3}{4}$	$\frac{9}{16}$	$\frac{27}{64}$
4	16	64
$\frac{6}{5}$	$\frac{36}{25}$	$\frac{216}{125}$
$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

2 **a** 224 mm^2 **b** 2.5 cm^2 **c** 12.57 cm^2

3 **a** 7.5 cm^3 **b** 35 342 mm^3 **c** 2673 cm^3

4 **a** 2.25 **b** 3.375 **c** 16.875 cm^3

5 **a** SA = 150 mm^2, Volume = 125 mm^3
b 1350 mm^2
c 3375 mm^3

6 **a** 243 **b** 125 **c** 1000
d 512 **e** 8 **f** 8000

7 **a** SA = 504π cm^2, Volume = 1296π cm^3
b 56π cm^2
c 48π cm^3

8 **a** 7.2 cm **b** 111 cm^2

9 12 m^2

10 1 : 40 000

11 1.5

12 $\sqrt[3]{2}$

13 **a** 3
b 9
c **i** 4.52 mm^2
ii 40.72 mm^2
iii 36.19 mm^2

14 It would need to double to 10 km.

15 4

16 **a** 675 g **b** 180 min

17 The area scale factor is the square of the length scale factor only for similar objects. $\triangle ABD$ and $\triangle ADC$ are not similar. The bases have ratio 1 : 2, but the heights of the triangles are the same. So their area ratio is 1 : 2.

18 **a** SA = 400π cm^2, Volume = $\frac{4000}{3}\pi$ cm^3
b 5 cm
c 100π cm^2
d $\frac{1}{4}$. The area scale factor is the square of the radius scale factor.
e $\frac{500}{3}\pi$ cm^3
f $\frac{1}{8}$. The area scale factor is the cube of the radius scale factor.

EX p257

Checkpoint

1 **a** 21.5 km/h **b** \$0.96/orange

2 12.5 minutes

3 **a** \$8.95 for 500 g **b** \$9.99 for 1.25 L

4 **a** 9 **b** 10

5 **a** $x = 20$ cm, $y = 66°$ **b** $p = 17.5$ cm, $q = 37°$

6 **a** Similar, SSS **b** Not similar, RHS

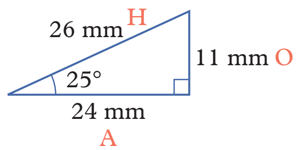
7 **a** 1.11 cm^2 **b** 113.10 cm^2

8 16 200 cm^2

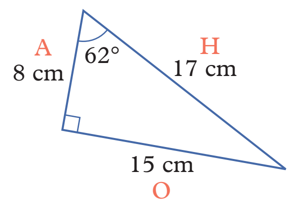
EX p260

6.4 Trigonometric ratios

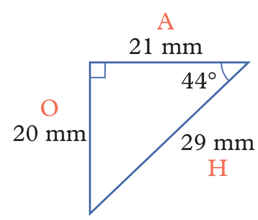
1 **a**



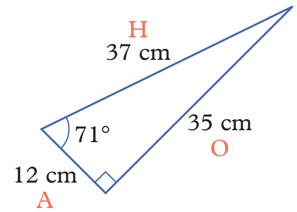
b



c



d



2 **a** $\sin(\theta) = \frac{12}{17}$ **b** $\cos(\theta) = \frac{8}{15}$
c $\tan(\theta) = \frac{24}{33} = \frac{8}{11}$ **d** $\cos(\theta) = \frac{35}{40} = \frac{7}{8}$
e $\sin(\theta) = \frac{15}{24} = \frac{5}{8}$ **f** $\tan(\theta) = \frac{13.1}{6.6} = \frac{131}{66}$

3 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

4 **a** $\frac{6}{7}$ **b** $\frac{6}{7}$ **c** $\frac{6}{7}$ **d** $\frac{6}{7}$

5 **a** $\frac{4}{3}$ **b** $\frac{4}{3}$ **c** $\frac{4}{3}$ **d** $\frac{4}{3}$

6 **a** They are similar triangles.
b **i, iii, iv, v**

7 **a** 0.3420 **b** 0.5878 **c** 0.8090
d 0.7265 **e** 1.9626 **f** 0.4540

8 **a** 2.052 **b** 17.54 **c** 0.057 00
d 1 **e** 0.7660 **f** 0.3640

9 a i $\sin(\theta) = \frac{b}{c}, \cos(\theta) = \frac{a}{c}, \tan(\theta) = \frac{b}{a}$

ii $\sin(\theta) = \frac{b}{c}, \cos(\theta) = \frac{b}{c}, \tan(\theta) = \frac{b}{a}$

iii $\sin(\theta) = \frac{b}{c}, \cos(\theta) = \frac{b}{c}, \tan(\theta) = \frac{b}{a}$

b The values of the sine, cosine and tangent ratios in similar triangles are equal.

10 a 100 m **b** 148 m **c** 6.7 m **d** 5 m

11 a

θ	sin (θ)	cos (θ)	tan (θ)
1	0.0175	0.9998	0.0175
5	0.0872	0.9962	0.0875
15	0.2588	0.9659	0.2679
30	0.5000	0.8660	0.5774
45	0.7071	0.7071	1.0000
60	0.8660	0.5000	1.7321
75	0.9659	0.2588	3.7321
85	0.9962	0.0872	11.4301
89	0.9998	0.0175	57.2900

b Sine and tangent increase, and cosine decreases.

c i $\sin(5°) = \cos(85°)$

ii $\sin(30°) = \cos(60°)$

iii $\sin(45°) = \cos(45°)$

iv $\sin(75°) = \cos(15°)$

d The cosine of an angle is the sine of the complementary angle. The sine of an angle is the cosine of the complementary angle.

12 a $a = 1, b = 1, c = \sqrt{2}, x = 45°, y = 90°, z = 45°$

b $\sin(45°) = \frac{1}{\sqrt{2}}, \cos(45°) = \frac{1}{\sqrt{2}}, \tan(45°) = 1$

c $a = 1, b = \sqrt{3}, c = 2, x = 30°, y = 90°, z = 60°$

d $\sin(30°) = \frac{1}{2}, \cos(30°) = \frac{\sqrt{3}}{2}, \tan(30°) = \frac{1}{\sqrt{3}}$

$\sin(60°) = \frac{\sqrt{3}}{2}, \cos(60°) = \frac{1}{2}, \tan(60°) = \sqrt{3}$

e i $\frac{1}{2}$ **ii** $\frac{3}{4}$ **iii** $\frac{1}{3}$ **iv** $\frac{3}{4}$

v $\frac{1}{4}$ **vi** $\frac{1}{4}$ **vii** 1 **viii** $\frac{1}{2}$

f Determine the square of the value of sine/cosine/tangent of 30°/45°/60°, then write the square root of the squared value.

13 a The values of sine and cosine of θ for $0° < \theta < 90°$ are the ratio of a shorter side to the longer hypotenuse. This means all values of sine and cosine can be considered as a smaller number divided by a larger number, which results in a value less than one.

b Tangent is the ratio of the two shorter sides. The two shorter sides have no restriction on which is longer or shorter.

c The tangent ratio is greater than one when the opposite side is longer than the adjacent side, which occurs when $45° < \theta < 90°$.

14 a

θ	tan (θ)	$\frac{\sin(\theta)}{\cos(\theta)}$
1	0.0175	0.0175
5	0.0875	0.0875
15	0.2679	0.2679
30	0.5774	0.5774
45	1	1
60	1.7321	1.7321
75	3.7321	3.7321
85	11.4301	11.4301
89	57.2900	57.2900

b i $\frac{\sin(15°)}{\cos(15°)} = \tan(15°)$

ii $\frac{\sin(45°)}{\cos(45°)} = \tan(45°)$

iii $\frac{\sin(60°)}{\cos(60°)} = \tan(60°)$

iv $\frac{\sin(85°)}{\cos(85°)} = \tan(85°)$

c $\frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$

15 a i $a = 6.40$ m

ii $b = 6.32$ m

b i $\sin(\theta) = 0.6247, \cos(\theta) = 0.7809, \tan(\theta) = 0.8$

ii $\sin(\theta) = 0.4286, \cos(\theta) = 0.9035, \tan(\theta) = 0.4743$

16 a i $\frac{1}{2}$ **ii** $\frac{5}{3}$ **iii** $\frac{6}{5}$ **iv** 7

b i $\frac{1}{2}$ **ii** $\frac{5}{3}$ **iii** $\frac{6}{5}$ **iv** 7

c The tangent of the angle from the horizontal is equal to the gradient of the hypotenuse.

17 a $x = \cos(31°), y = \sin(31°)$

b $x = \cos(54°), y = \sin(54°)$

c i The sine of an angle is the length of the altitude of a right-angled triangle with a hypotenuse of length 1.

ii The cosine of an angle is the length of the base of a right-angled triangle with a hypotenuse of length 1.

d i $a = 6.857, b = 4.120$

ii $a = 0.5143, b = 0.3090$

18 a The person has not written the angle, θ. $\cos(\theta) = \frac{10}{11}$

b The person has written the reciprocal of the sine ratio. So, the fraction is upside down.

$\sin(\theta) = \frac{4}{8}$ or $\frac{1}{\sin(\theta)} = \frac{8}{4}$

$\sin(\theta) = \frac{1}{2}$ $\frac{1}{\sin(\theta)} = 2$

c The ratio between the two given sides is the tangent ratio; however, the person has written a ratio using 'cos' instead of 'tan'. $\tan(\theta) = \frac{6}{5}$

19 a i $\tan(27°) = \frac{1}{2}$ **ii** $\tan(117°) = -2$

iii $\tan(194°) = \frac{1}{4}$ **iv** $\tan(326°) = -\frac{2}{3}$

b i 1 **ii** −1 **iii** 1 **iv** −1

c All four angles (45°, 135°, 225°, and 315°) are 45° rotations from the horizontal, so the gradient of the corresponding line is either 1 or −1 for a line inclined at a 45° angle depending on which way it is rotated.

d **i** The gradient of a horizontal line is zero. Therefore, the tangent of 0° is zero.

ii The gradient of a vertical line is undefined. Therefore, the tangent of 90° is undefined.

20 **a** **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** $-\frac{1}{2}$ **iv** $-\frac{1}{2}$

v $\frac{1}{2}$ **vi** $-\frac{1}{2}$ **vii** $-\frac{1}{2}$ **viii** $\frac{1}{2}$

b **i** sine: positive, cosine: negative

ii sine: negative, cosine: positive

iii sine: negative, cosine: negative

EX p269 6.5 Using trigonometry to find side lengths

1 **a** $\cos(\theta) = \frac{\text{A}}{\text{H}}, \cos(64°) = \frac{k}{35}$

b $\sin(\theta) = \frac{\text{O}}{\text{H}}, \sin(49°) = \frac{k}{12}$

c $\tan(\theta) = \frac{\text{O}}{\text{A}}, \tan(26°) = \frac{k}{50}$

d $\cos(\theta) = \frac{\text{A}}{\text{H}}, \cos(51°) = \frac{k}{17}$

e $\sin(\theta) = \frac{\text{O}}{\text{H}}, \sin(37°) = \frac{k}{24}$

f $\tan(\theta) = \frac{\text{O}}{\text{A}}, \tan(21°) = \frac{k}{16}$

2 **a** $x = 2.11$ **b** $x = 11.83$

c $x = 2.11$ **d** $x = 11.83$

e $x = 6.62$ **f** $x = 18.28$

g $x = 3.89$ **h** $x = 3.89$

3 **a** 15.34 mm **b** 9.06 m **c** 24.39 cm

d 10.70 m **e** 14.44 mm **f** 6.14 m

4 **a** 31.84 cm **b** 25.86 mm **c** 2.78 m

d 22.65 cm **e** 11.11 m **f** 49.32 mm

g 61.80 cm **h** 129.35 m **i** 5.26 cm

5 **a** 118.31 cm **b** 3.72 m **c** 38.92 mm

d 11.02 cm **e** 46.27 cm **f** 4.72 m

6 **a** 4.44 cm **b** 8.31 cm **c** 10.39 m

d 78.15 cm

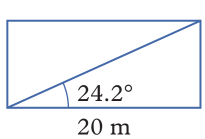
7 **a** 33.64 m **b** 34.64 m

8 4.62 m

9 **a** 9.60 m **b** 16.00 m

10 **a** 1.93 m **b** 2.13 m

11 **a**



b 9 m

c 58 m

12 8.60 m

13 **a** One angle size (other than the right angle) and one side length

b Two side lengths

14 **a** $a = 17.2$ cm **b** $b = 42.2$ mm

c $c = 52.9$ m **d** $d = 32.2$ cm

e $e = 29.9$ mm **f** $f = 52.0$ cm

15 **a** $a = 3.28$ cm, $b = 3.77$ cm, $x = 90°$, $y = 41°$, $z = 49°$

b $g = 27°$, $h = 63°$, $m = 2.94$ m, $n = 3.30$ m

16 **a** $f = 4.01$ cm, $g = 7.04$ cm

b $a = 4.00$ cm, $b = 7.21$ cm

c $m = 17.69$ cm, $n = 12.00$ cm

17 $r = 8.81$ cm and $t = 10.88$ cm

18 $p = 35.96$ cm, $q = 11.96$ cm

19 **a** $x = 5.01$ cm **b** $a = 7.09$ cm

20 **a** $x = 2.78$ cm **b** $y = 13.22$ cm

c $z = 8.56$ cm **d** $b = 15.75$ cm

EX p276 6.6 Using trigonometry to find angles

1 **a** 13° **b** 44° **c** 56° **d** 72°

e 42° **f** 16° **g** 74° **h** 50°

2 **a** 20° **b** 36° **c** 33° **d** 25°

e 8° **f** 1° **g** 72° **h** 45°

3 **a** 53° **b** 38° **c** 21° **d** 62°

e 69° **f** 23° **g** 57° **h** 83°

4 **a** 50° **b** 42° **c** 67° **d** 54°

e 31° **f** 46°

5 **a** $\theta = 138.6°$ **b** $\theta = 13.6°$

c $\theta = 271.1°$ **d** $\theta = 25.8°$

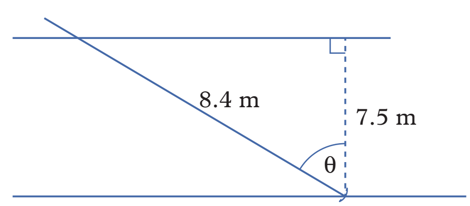
6 **a** 36.9°, 53.1° **b** 26.4°, 63.6°

c 72.9°, 17.1° **d** 25.7°, 64.3°

7 **a** 14° **b** 7.01 m

8 **a** 47° **b** 86°

9 **a**



b 27°

c Larger. If the chain is longer, the angle with the vertical would be greater. If the chain is shorter, the angle is closer to the vertical, hence making a smaller angle size with the vertical.

10 **a** **i** 37° **ii** 37° **iii** 37°

b $\theta = 37°$

c **i** Using trigonometry can be accurate to a required degree of accuracy but is difficult to perform without a calculator.

ii It is an advantage to use a scale diagram to check the results from trigonometric calculations, but it is difficult to record measurements with sufficient accuracy.

11 **a** First, calculate the angle of the Sun's rays at the time of measurement by using the length of a metre ruler's shadow. Then use the length of the tree's shadow as a measurement to calculate the height of the tree.

b 37°

c It refers to the angle of the Sun's rays at the time of measurement, both measurements being taken at the same time.

d 11.71 m

e By forming an equivalent ratio statement using the corresponding sides.

$x : 1 \text{ m} = 15.46 \text{ m} : 1.32 \text{ m}$

$$x = \frac{1 \times 15.46}{1.32} = 11.71 \text{ m}$$

12 **a** **i** 28°, 62° **ii** 28°, 62° **iii** 28°, 62°

b The two non-right angles are complementary, so $90° - \theta$.

13 **a** $\theta = 123.7°$ **b** $\theta = 110.4°$ **c** $\theta = 78.6°$

14 **a** 30.0 m, 18.4°, 71.6°

b 20.1 m, 42.6°, 47.4°

15 The corresponding angles in the bottom left of each triangle are approximately 64.707° and 64.652°, so the lines are very nearly parallel.

16 1548.4 cm^2

17 $\theta = 38.4°$

EX
p281

Review: Scale and trigonometry

Review questions 6.7A: Mathematical literacy review

1 AAA, SAS, RHS, SSS

2 The volume scale factor can be calculated by taking the cube of the length scale factor.

3 The hypotenuse is the side opposite the right angle in a right-angled triangle. It is also the longest side.

4 **a** scale drawing

b trigonometry

5 **a** a relationship between two or more quantities with the same units

b two or more figures with the same shape

6 **a** scale, ratio

b tangent

Review questions 6.7B: Multiple choice

1 A **2** C **3** B **4** C **5** D **6** A
7 D **8** A **9** A **10** B **11** C

Review questions 6.7C: Short answer

1 **a** \$1.80/avocado **b** 85 beats/minute

c 84 km/hour

2 **a** 32 **b** 72

3 **a** Similar, AAA **b** Similar, SAS

4 7.2 m

5 **a** 1590.43 cm^2 **b** 15.33 cm^2

6 **a** 1260 mm^3

b 45 mm by 31.5 mm by 24 mm

c 34 020 mm^3

7 $\sin(\theta) = \frac{55}{73}, \cos(\theta) = \frac{48}{73}, \tan(\theta) = \frac{55}{48}$

8 **a** 5.44 cm **b** 19.43 m

c 28.31 m **d** 4.15 cm

9 **a** 35° **b** 60°

c $\theta = 68°$ **d** $\theta = 29°$

10 $\theta = 47.5°$

Review questions 6.7D: Mathematical modelling

1 **a** sine **b** 53.1°

c 90 m **d** 645 m

2 **a** The ratio of the side lengths is directly proportional.

b $\frac{1}{2}$

c $\frac{1}{4}$

d 5 : 3

e 630

f **i** 15 120 cm^2 **ii** 9072 cm^2

g \$27.22

EX
p286

Computational thinking

1 **a** 12

b **1** Start

2 $a = 2, b = 8, c = 4$

3 Is $2 + 8 > 4$? Yes.

4 Is $8 + 4 > 2$? Yes.

5 Is $4 + 2 > 8$? No.

6 Report 'Not a triangle'.

7 End

c All inequalities need to be checked to conclude the three sides can form a triangle. However, if an inequality is evaluated to be false, the algorithm terminates and subsequent inequalities are not checked. The assignment of the values of a, b and c is random, and any random sum of two sides must be greater (as seen in part **b**). The first two inequalities are true, but the condition is rejected at the third inequality.

d If $a < 0$, then $a + b < b$. If $b \leq c$, then $a + b < b \leq c$. This means $a + b < c$.

e If the value assigned as a is negative, then the first condition fails and the flowchart reports 'Not a triangle'. If b or c are assigned as negative, the same working shows that the second and third conditions fail as they are the same with the variables relabelled. In all three cases, the flowchart reports 'Not a triangle'.

2 **a** Three side lengths a, b and c

b **1** input side lengths $a = 4, b = 5, c = 8$.

2 Is $4 + 5 > 8$? Yes.

3 Is $5 + 8 > 4$? Yes.

4 Is $4 + 8 > 5$? Yes.

5 Print 'This is a triangle.'

c **input** – add inputs for any starting variables (here a, b, c); corresponds to green input parallelogram

if – decision with possible answers Yes/No (or True/False); corresponds to purple decision point rhombuses

and – means that statements on either side must be Yes (True); corresponds to following a sequence of 'Yes' arrows

print – what the algorithm should report; corresponds to blue output parallelograms

else – what the algorithm should do if not all the if statements are Yes (True); corresponds to 'No' arrows

d This is not a triangle.

3 **a** $x = 8$

b **Lines 3–5:** #Replaces the variable *Max* with b if $b > a$, and leaves *Max* unchanged if $a > b$

Lines 6–8: #Replaces the variable *Max* with c if $c > Max$ (where max is either a or b depending on the result from the previous if control)

c **1** Input $a = 3$, $b = 5$, $c = 4$.

2 The value of the variable Max is assigned to be 3.

3–5 The inequality $b >$ Max is checked. The inequality is true, since 5>3. The value of Max is updated to be 5.

6–8 The inequality $c >$ Max is checked. The inequality is false, since 4<5. The value of Max is not updated.

9–13 The conditions ($a^2 + b^2 = \text{Max}^2$), ($b^2 + c^2 = \text{Max}^2$) and ($a^2 + c^2 = \text{Max}^2$) are checked. The last condition is true, since $3^2 + 4^2 = 5^2$. The statement 'Form a Pythagorean triple' is printed.

d The conjunction '**and**' means all conditions need to be satisfied simultaneously to evaluate this control statement as true, while '**or**' means just one condition needs to be satisfied for the statement to be evaluated as true.

e It could be simplified by not defining the variable Max and updating line **9** to check all three possible variations of the equation.

4 **a** **i** 6 **ii** 5 **iii** 4 **iv** 5

b

$$\begin{aligned} a^2 &= (m^2 - n^2)^2 = m^4 - 2m^2n^2 + n^4 \\ b^2 &= (2mn)^2 = 4m^2n^2 \\ c^2 &= (m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4 \\ a^2 + b^2 &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 + n^2 \\ &= c^2 \end{aligned}$$

c Line 4. The value of n is increased by 1.

d The values $m = 2$ and $n = 1$ produce the triple (3, 4, 5).

e In GeneratePythagoreanTriple(30), the '**for**' loop for m iterates between 2 and floor($\sqrt{30}$) =5.

m	n	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$
2	1	3	4	5
3	1	8	6	10
3	2	5	12	13
4	1	15	8	17
4	2	12	16	20
4	3	7	24	25
5	1	24	10	26
5	2	21	20	29
5	3	16	30	34 (break)

f The inner '**for**' loop terminates as the break condition is reached. The values generated on the last row will not be printed, so (21, 20, 29) = (20, 21, 29) is the last triple to be printed.

MODULE 7 Statistics

7.1 Surveys and sampling methods

EX p296

1 **a** Systematic sampling

b Stratified sampling

c Convenience sampling

d Quota sampling

2 **a** Biased; people at an AFL football match are not a representative sample of the whole country.

b Fair; this should provide an answer representative of the views of all the people in school.

c Biased; the question is not fair and leads to only responses regarding people's attitude to dogs.

d Biased; a sample of 100 is too small to represent the population.

3 **a** There is no indication of how people were chosen for survey or exactly what they were asked.

b People shopping in an Apple store are probably Apple customers/supporters, so this is a biased sample.

c The sample is too small, and there is no indication of the gender or employment status of people making up the sample or whether it was their opinions or facts about their personal earnings that they were questioned about.

d Shareholders in a company have a vested interest in seeing that company's product succeed.

4 **a** **i** The data collection method is not fair. Tess only surveyed people from her town, when the population needing to be considered is the entire poulation of Victoria.

ii The interpretation is biased because the sample is biased.

iii Tess should have surveyed people across all of Victoria rather than just in her town.

b **i** The sample is too small for any results to be reliable.
ii Because the sample is so small, the median would be the most reliable result to take.
iii Survey a larger sample, and watch out for outliers.

c **i** The question only asked if the people surveyed like Mac computers, but not if they prefer them to other types of computers. This is not a fair data collection method.
ii Because the question is biased, no assumption can be made.
iii The question could have been, 'Which brand of computer do you prefer?'

d **i** The data collection method is fair.
ii The interpretation of the data is poor. The figure does not need to be above 50% for the soft drink to be the most popular. It just needs to be a greater percentage than any other soft drink.
iii She should conclude Coke is the most popular.

5 **a** Only one person claimed 18 hours pain free. Most of the claims are far less than 18 hours but, because one person said 18 hours, the claim of 'up to 18 hours' has been made.
b mean = 3.1, median = 2, mode = 2
c This claim is not too far from the truth. However, the median or the mode would be a better representation of the data than the mean.

6 How was the data collected? How big was the sample? Does the data represent a sample of the population? How was the questionnaire designed? Is the interpretation fair?

7 **a** If a phone is in stand-by mode, its battery could last up to 7 days. But, in normal use, a phone would not be in stand-by mode for 7 days!
b The 42 people surveyed could have been chosen for the survey because they were purchasing that brand of toilet paper at a supermarket. It is also a very small sample size that is not representative of all of Australia.
c This moisturising lotion was not compared with another moisturising lotion, but with soap.

8 **a** ≈54%
b Sample answer: The infographic is visual and highlights the data, but it can be difficult to interpret the exact percentage portrayed in the infographic.

9 **a** To determine the most popular gelato flavour
b The popularity of five different gelato flavours
c The exact number of people who liked each gelato flavour
d Sample answer: The infographic is a good visual aid to get a quick snapshot of the results, however it lacks depth.

10 Sample answer: Both of these factors have made it more difficult to collect data from surveys over the phone and will have had an impact on the random sampling of phone surveys, as some demographics will be less likely to use landline phones and/or more likely to screen phone calls. Other surveying methods, such as online surveys, have become more prevalent during this time.

11 Teacher to check.

12 Sample answer: The operating profits of the mining industry were the most variable, with a big drop in the middle of the decade, followed by a big rise at the end of the decade. Manufacturing followed a similar pattern to mining, with a sharp initial drop in operating profit, followed by a steady rise to a new height at the end of the decade. After an initial drop in 2012–13, construction steadily increased in operating profit through the rest of the decade, while the operating of the retail industry gradually fell, and the profit of the media industry fell significantly after a slight boost in the middle of the decade.

13 **a** 8th
b Because 98 countries were assessed
c This is a sample, because data was not collected from all countries in the world, with China specifically being excluded.
d No, because other countries that were not included may have handled the coronavirus pandemic better than Australia did.
e Look up the Lowy Institute and find the data they used to rank the countries.

14 Teacher to check.

15 Teacher to check.

16 Teacher to check.

EX 7.2 Classifying and displaying data

p304

1 **a** IV **b** I **c** II **d** III

2 **a** 1 and 3 are numerical; 2 and 4 are categorical
b height: continuous; mark: discrete; method of transport: nominal; karate belt: ordinal

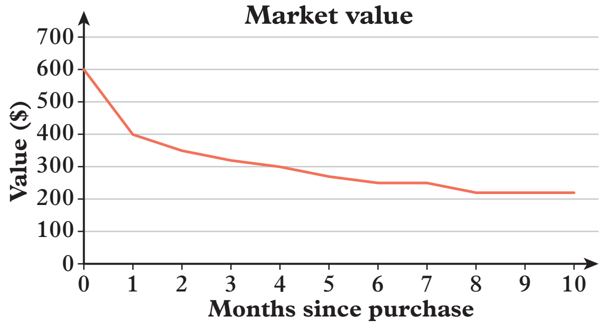
3 **a** fuel consumption: numerical; car model: categorical
b number of ice cream sales: numerical; maximum daily temperature: numerical
c exam score: numerical; subject: categorical

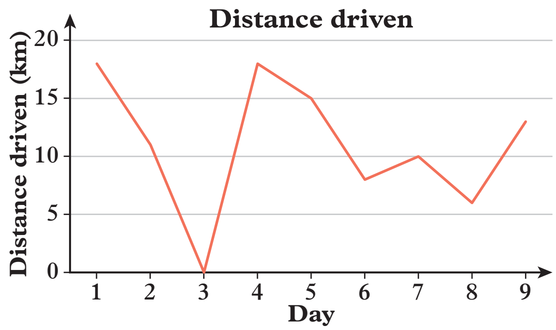
4 **a** Temperature over a 24-hour period, measured every 2 hours
b time: continuous; temperature: continuous
c 11°C **d** 2 pm **e** 5°C, 6 am

5 **a** Number of people, month, year
b July 2017
c May 2019

6 **a** Number of goals, player, game number
b Game 4
c Game 3
d Hashani

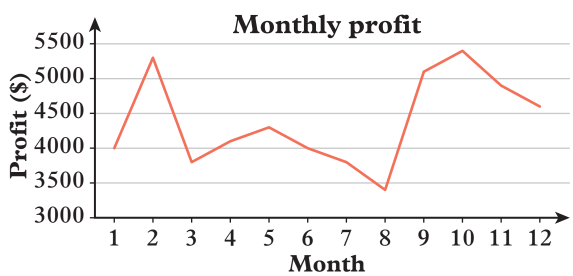
7 a Portions of vegetables consumed, day, person
b Day 2
c 3
d 53

8


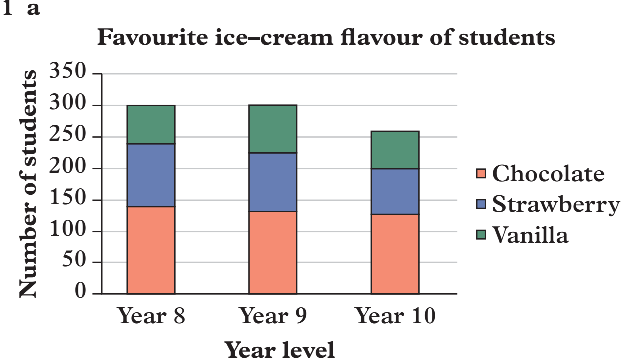
9 a


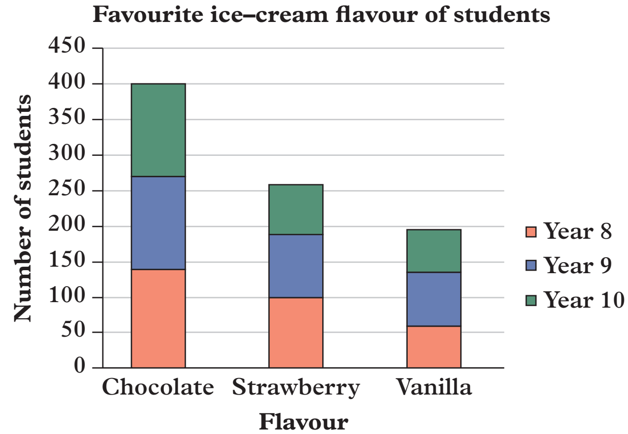
b Day 1 and day 4
c 5

10 a The data involves two numerical variables with one variable being continuous, so a line graph would be appropriate to visualise this data.



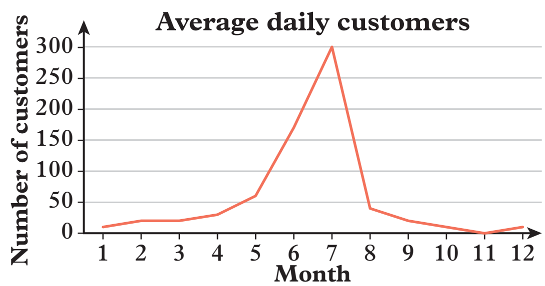
b Month 10
c Month 8
d Month 9, $1700
e Month 3, $1500

11 a


b


c The chart in part **a** with the year level on the horizontal axis
d No. Although slightly more Year 9 students preferred chocolate ice-cream than Year 10 students, there were more Year 9 students in the data.

12 a 20 **b** Year 2
c Students can regularly do both: watch films on a streaming service and go to the cinema.
d Sample answer: The columns will stay similar to how they are in Year 5, with a majority of students watching films regularly on streaming services, and some students regularly going to the cinema.

13 a


b Month 7. It is the most popular month, and it is followed by a large drop off, which makes sense because January would not be busy.
c No. The data is rounded to the nearest 10 customers and it is the average number of daily customers for the month, not the total number of customers in the month. The average could have been just below 5 customers per day, which would be around 150 customers total for the month.

14 a Darwin **b** Adelaide
c Adelaide **d** Melbourne

15 a Osaka **b** Shanghai
c 16 million **d** Shanghai
e 2008
f Sample answer: The population of Osaka stayed relatively level throughout the 30-year period, whereas the population of Shanghai increased quickly, and the population of Mumbai increased slowly.

16 a Joey **b** Ram

c 58

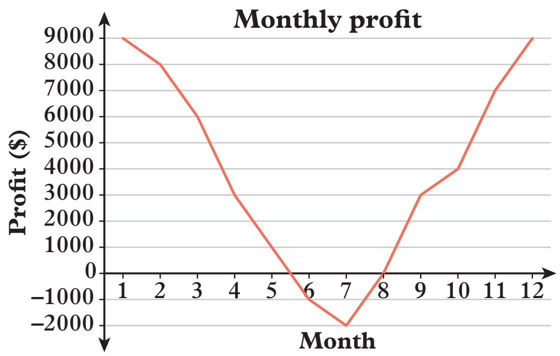
d Ram. He consistently ate either 3 or 4 hotdogs per minute.

17 a Area chart; it is useful to visualise the total number of steps taken in the challenge.

b Stacked bar chart; there are two categorical variables, and the favourite topping overall can be easily seen.

c Multiple line graph; the heights can be viewed as progressing over time.

18 a



b Negative profit means the store lost money.

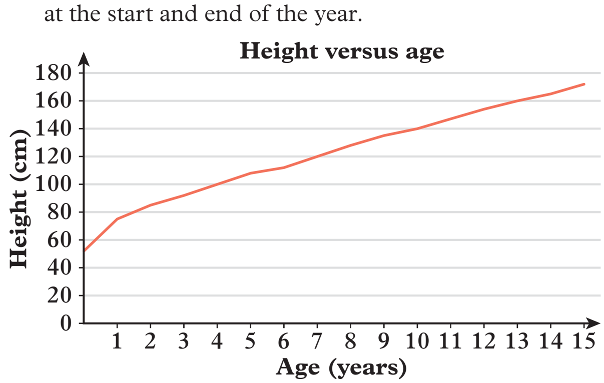
c $0 profit means the store broke even (did not make money or lose money).

d $47 000

e No. The profit is most likely associated with the time of year so it will be back down again in the middle of next year.

f Summer in the northern hemisphere occurs in the middle of the year so the months with the highest profit would be the months in the middle of the year, unlike the Geelong store, which has the highest profits at the start and end of the year.

19



b 8 cm

c 252 cm

d 185 cm (answers will vary)

e Continue increasing until the age of 18 or 19, then remain constant.

2 a 6 **b** 0–<20, 4 **c** 69

3 a 145 **b** 183 **c** 38

d

Class	Frequency
145–<150	1
150–<155	3
155–<160	5
160–<165	3
165–<170	6
170–<175	6
175–<180	0
180–<185	1
Total	25

4 a

Class	Frequency
0–<5	1
5–<10	2
10–<15	3
15–<20	3
20–<25	2
25–<30	3
30–<35	3
35–<40	1

b

Class	Frequency
10–<20	3
20–<30	3
30–<40	4
40–<50	4
50–<60	4
60–<70	2
70–<80	2

c

Class	Frequency
0–<2	6
2–<4	7
4–<6	6
6–<8	2
8–<10	2
10–<12	4

d

Class	Frequency
40–<45	3
45–<50	8
50–<55	7
55–<60	5
60–<65	3
65–<70	3
70–<75	4
75–<80	7

EX
p311

7.3 Grouped data and histograms

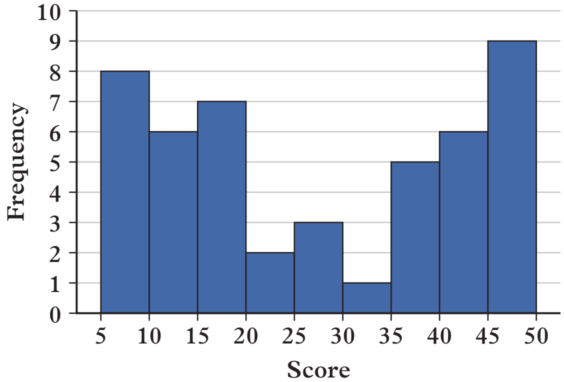
1 a 5 **b** 16

c 15–<20, 18 **d** 25–<30

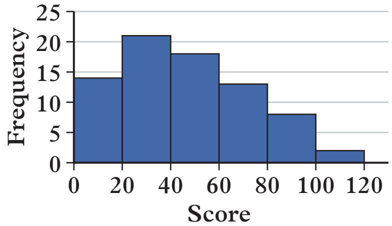
e 14 **f** 19

g 4.5%

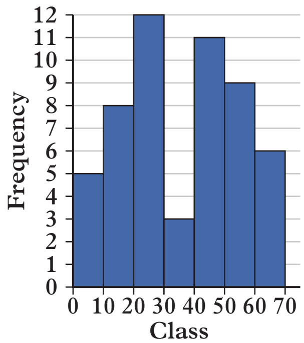
5 a



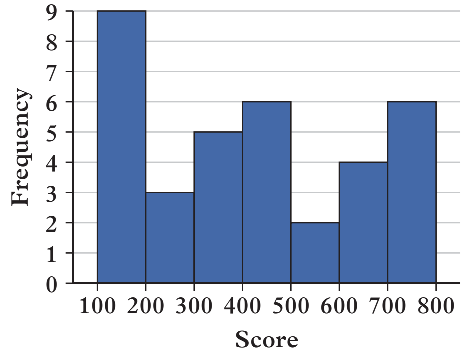
b



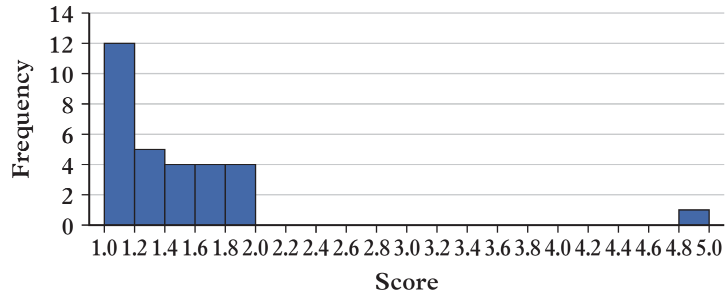
c



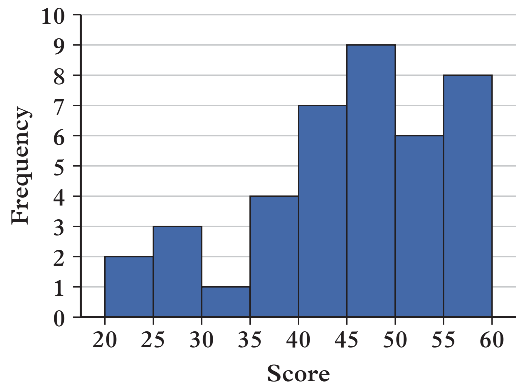
d



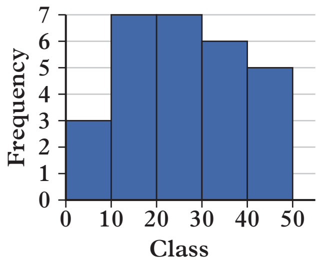
e



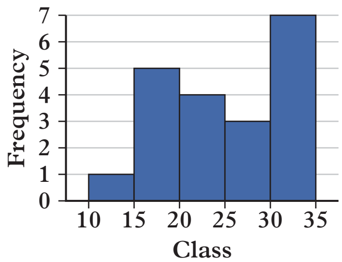
f



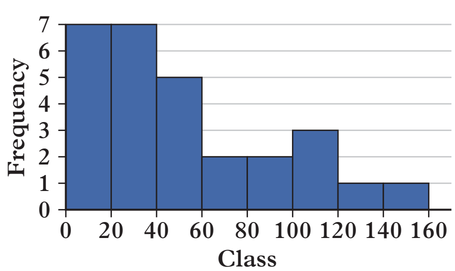
6 a



b



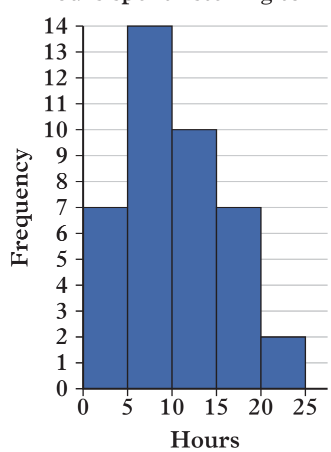
c



7 a

Hours	Frequency
0–<5	7
5–<10	14
10–<15	10
15–<20	7
20–<25	2

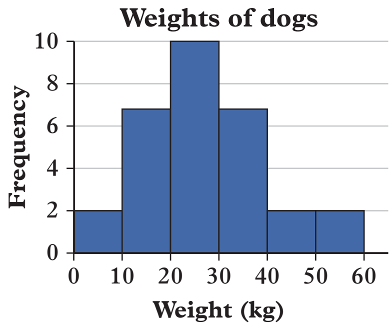
b Hours spent listening to music



8 a

Class	Frequency
0–<5	2
5–<10	7
10–<15	10
15–<20	7
20–<25	2
25–<30	2

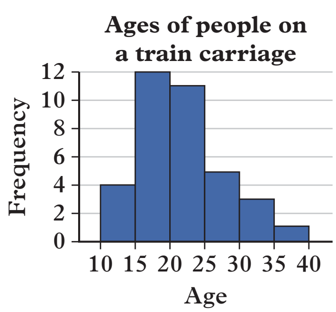
b



9 a

Ages of people on a train	Frequency
10–<15	4
15–<20	12
20–<25	11
25–<30	5
30–<35	3
35–<40	1

b



10 a Number of Facebook friends

b 50 **c** 350–399; 28

d 100–149

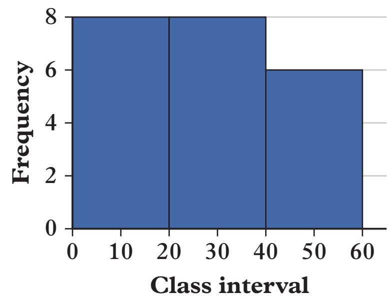
11 a There are too many class intervals, which makes the table long and difficult to read.

b

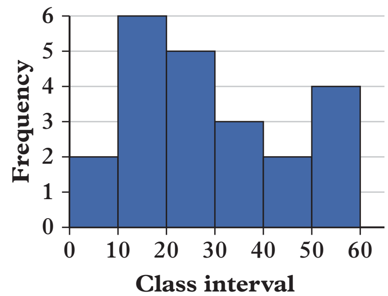
Ages of customers	Frequency
0–<10	2
10–<20	61
20–<30	44
30–<40	20
40–<50	6
≥50	6

12 Without the raw data it is not possible to determine the number in each of the smaller class intervals.

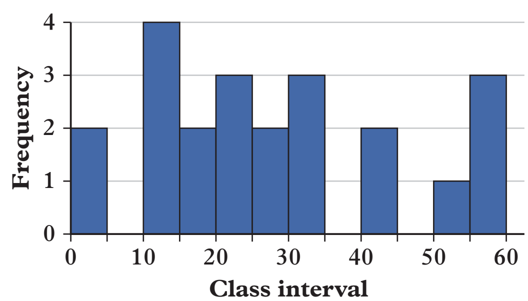
13 a



b

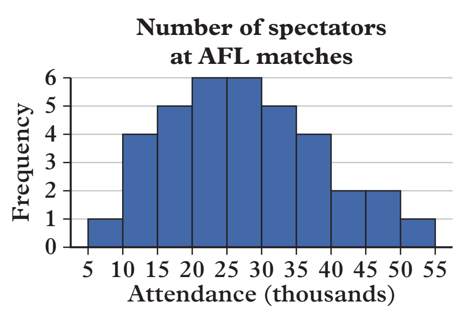


c

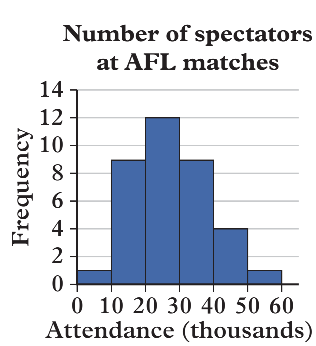


d Sample answer: The histogram in part **b** is most appropriate. The intervals in part **a** are too wide to give an accurate representation of the spread of the data. Both parts **b** and **c** display the data effectively, but the histogram in part **b** looks the neatest as it does not have intervals with frequency of 0 like part **c** does.

14 a



b

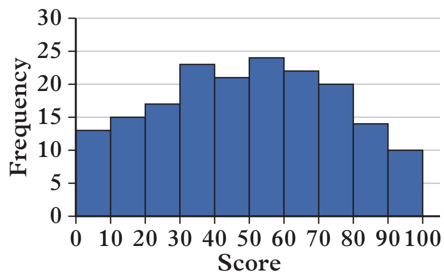


c Similar shape for each histogram (roughly symmetrical). Using smaller class intervals has resulted in smaller frequency values. That is, largest frequency is 6 compared with 12.

15 **a** 5

b There appears to be no pattern. Too many intervals make it harder to read.

c



d The distribution is symmetrical, with lower frequencies at either end and higher frequencies in the middle.

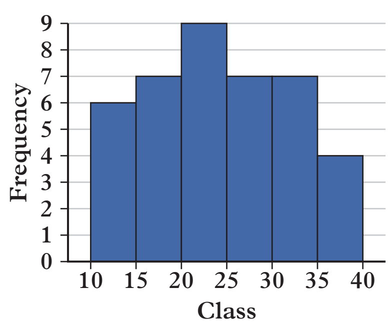
16 **a** The stems represent the class intervals.

b 10; e.g. 10–19, 20–29 etc.

c 17

d In a stem-and-leaf plot, each individual piece of data can be seen.

17



18 Histograms do not provide specific data within each class interval; they only provide the frequency of data for each class interval.

19 **a** The vertical axis represents percentage frequency rather than frequency.

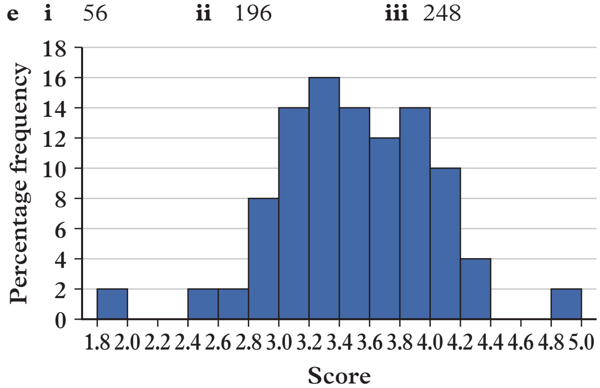
b 16%

c 14%

d 100%; because percentages of a whole add to 100%

e **i** 56 **ii** 196 **iii** 248

20



Checkpoint

EX p318

1 **a** Quota sampling **b** Convenience sampling

c Stratified sampling **d** Systematic sampling

2 **a** **i** The sample only consisted of people walking through the mall, so this could lead to biased results (especially when the survey is about time spent sitting down). It's also not clear who her population is.

ii A poor method of collection means the interpretation could be biased.

iii Sienna should have specified her population and sampled people in various locations for her survey to be representative of the population. It might also be best to use the median as the measure of centre for this data.

b **i** Kane's survey method is biased because his neighbourhood contains only a small section of the target population and also includes people who are not part of the target population.

ii The interpretation is not fair because the sample is biased.

iii Kane should have surveyed a sample of people who will be attending the school dance.

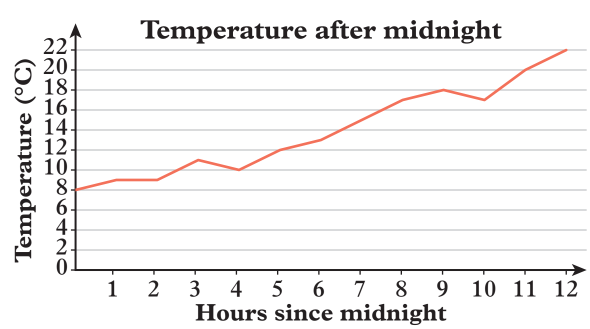
3 **a** Biased; people outside your community would probably not have any appreciation of particular community issues, so their views could skew the results.

b Biased; a small sample of one section of the population is not likely to be representative of the entire population.

4 **a** Numerical **b** Categorical

c Categorical **d** Numerical

5



6 **a** Pop **b** Australia **c** Pop

7 **a** Kangaroos **b** 2015

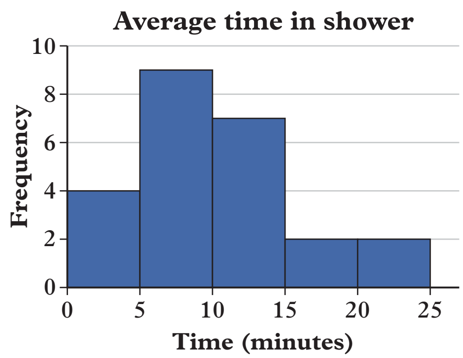
c Between 2017 and 2018

8 **a** 90–99 **b** 6 **c** 15

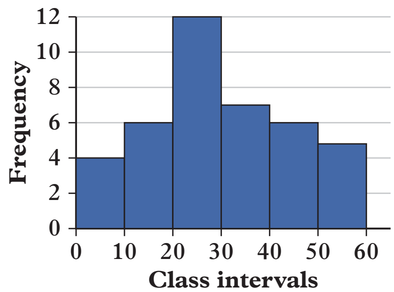
9 **a**

Class	Frequency
0–<5	4
5–<10	9
10–<15	7
15–<20	2
20–<25	2

b



10



EX 7.4 Summary statistics from tables and displays

p324

1 **a** **i** 2.36 **ii** 2 **iii** 2 **iv** 4
b **i** 21.67 **ii** 20 **iii** 10, 30 **iv** 30
c **i** 15.88 **ii** 16 **iii** 17 **iv** 5
d **i** 1.12 **ii** 1 **iii** 1 **iv** 4
e **i** 24.33 **ii** 25 **iii** 30 **iv** 15
f **i** 3.09 **ii** 2.5 **iii** 2 **iv** 18

2 **a** 30 people **b** Mode = 5, range = 9

c

Hours	Frequency
1	3
2	4
3	2
4	6
5	8
6	4
7	2
8	0
9	0
10	1
Total	30

d Mean = 4.3, median = 4.5

3 **a** Mean = 4.7 **b** Median = 5
c Mode = 4 **d** Range = 5

4 **a** Four bedrooms **b** 4

c

Number of bedrooms	Frequency
1	2
2	9
3	11
4	14
5	6
Total	42

d Mean = 3.31, median = 3

5 **a** Mean = 2.3, mode = 2, median = 2, range = 5
b Mean = 16.15, mode = 16, median = 16, range = 6
c Mean = 1.33, mode = 1, median = 1, range = 4

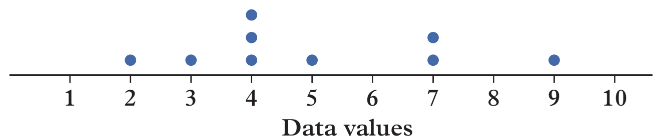
6 **a** Mean = 55.43, median = 59, mode = 42, range = 42
b Mean = 2.70, median = 2.3, mode = 1.8, range = 4
c Mean = 280.38, median = 250, mode = 310, range = 670

7 Mean = 4.21, median = 4, mode = 3, range = 9

8 Categorical data does not contain any numerical scores. You can only find the most popular/common category that is the mode.

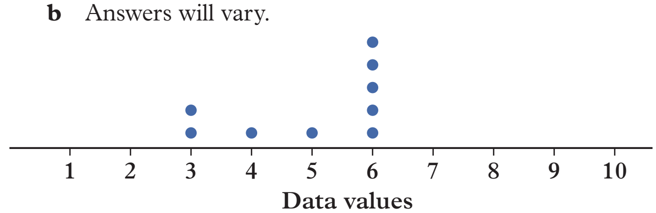
9 **a** II **b** I **c** III

10 **a** Answers will vary.



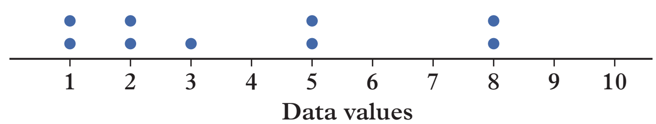
The lowest value must be 2, the highest value must be 9, and the other dots can be anywhere from 2 to 9.

b Answers will vary.



There must be at least five dots on 6 and all other dots must be below 6.

c Answers will vary.



There must be four values with exactly two dots each and one value with exactly one dot.

11 **a** 17 (unique)
b 9 (unique)
c Any number greater than or equal to 13 (not unique)
d 10 (unique)
e 9 or 17 (not unique)
f Any multiple of 9 (not unique)

12 **a** 16th row, 7th column **b** 8th row, 19th column

13 **a** 49, because the highest possible value is 69 and the lowest possible value is 20
b 31, because the minimum value of the highest number is 60 and the maximum value of the lowest number is 29.
c The raw data is not available.
d 40–<49

e It is not possible to calculate the mean because the raw data is not available. It is not possible to know which interval contains the mean since, unlike the median, the mean is not found based on position.

14 a 39.6 **b** 48.6

c 44.1. Yes, it is reasonable to assume that the values in each interval are evenly spread out rather than all clustered at the top or all clustered at the bottom.

15 a 82.4% **b** 62.4% **c** 88%

16 a i 2.56 **ii** 26.37 **iii** 4.73

b The second set of data has a large spread of data from the mean (standard deviation = 26.37), but the other two data sets have a small spread of data from the mean (standard deviations = 2.56 and 4.73 respectively).

EX
p330

7.5 Describing data

1 a Bi-modal

b Negatively skewed

c Positively skewed with an outlier

d Symmetric

2 a Symmetric

b Positively skewed

c Negatively skewed with an outlier

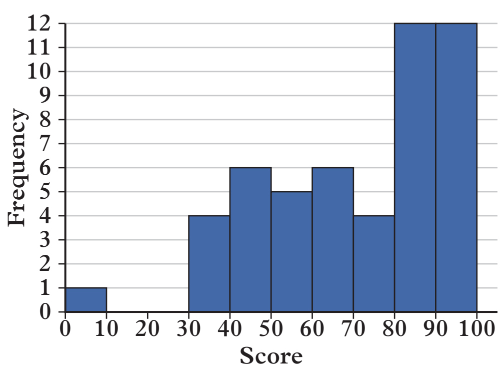
3 a Mean or median **b** Median

c Mode of each peak **d** Median

4 a Mode of each peak **b** Median

c Median **d** Mean or median

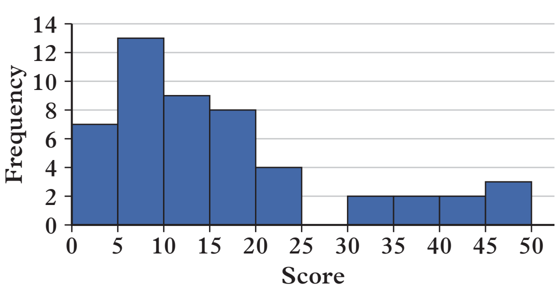
5 a i



ii Negatively skewed

iii Median. The graph is skewed and there is an outlier present, therefore the mean may not be an accurate measure of centre.

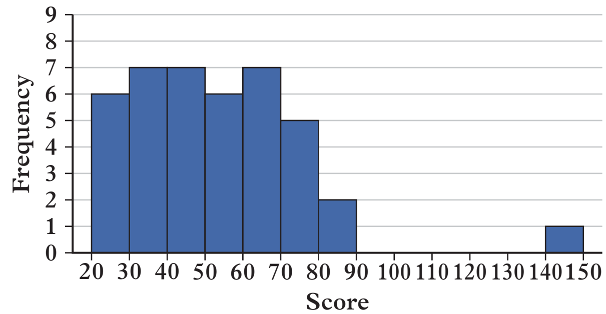
b i



ii Positively skewed

iii Median. The graph is skewed, therefore the mean may not be an accurate measure of centre.

c i



ii Symmetric or positively skewed with an outlier

iii Median. An outlier is present, therefore the mean may not be an accurate measure of centre.

6 a i

Stem	Leaf
1	0 0 0 1 2 2 2 2 2 2 3 3
1*	5 5 5 5 5 6 6 7 8 9
2	0 0 1 2 2 2 3 3 3
2*	9

Key: 1|3 = 13

ii

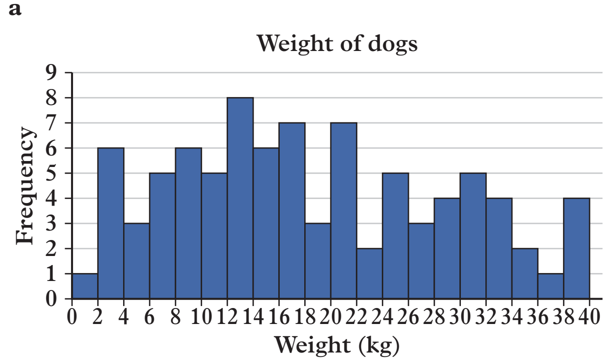
Stem	Leaf
1	0 0 0 1
1*	2 2 2 2 2 3 3
1^	5 5 5 5 5
1#	6 6 7
1+	8 9
2	0 0 1
2*	2 2 2 3 3 3
2^	
2#	
2+	9

Key: 1|3 = 13

b i Positively skewed

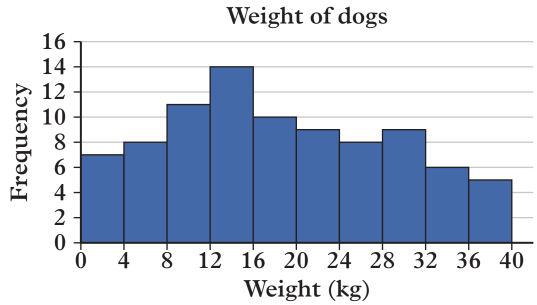
ii Bi-modal with an outlier

7 a



b No pattern visible

c



d Very slightly positively skewed

e Weight of dogs ranges from below 2 kg up to 40 kg. The distribution is fairly even and roughly symmetric with a peak (modal class) around 12–16 kg.

8 a Symmetric

b Mean or median

c

Number of people	Frequency
0–<10	3
10–<20	4
20–<30	9
30–<40	12
40–<50	8
50–<60	6
60–<70	2
Total	44

d 30–<40

e Sample answer: The average number of people in the skate park is between 30 and 40.

9 a i Symmetrical with an outlier

ii Median is 35–<40.

iii Ages of people at a hairdresser's in a week usually ranges from 15 to 60 years, with the median age being between 35 and 40 years.

b i Negatively skewed

ii Mean = 15.41, median = 15.5

iii Ages of students at a pool ranges from 13 to 18 years, with median age 15.5 years.

c i Positively skewed **ii** Median = 2

iii The majority of homes surveyed have one or two TVs, with the most common number of TVs in each home being two.

d i Bi-modal

ii Mode of each peak = 0–<10 cm and 30–<40 cm

iii Of the Year 9 students surveyed, most have either short hair ranging from 1 to 10 cm in length or longer hair ranging from 30 to 40 cm in length.

10 a There were 0 customers on Christmas Day because the course was closed.

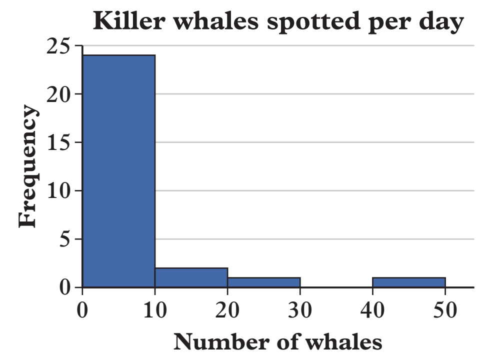
b A mini golf course is likely to have few customers during the week and do the majority of its business on Friday, Saturday, and Sunday.

c Any business that is much busier on weekends: cinemas, golf clubs, paintball, amusement parks, bars, hotel, etc.

11 a It could lead people to believe that customers usually see about 4 whales per tour, which is not true.

b The median number of whales spotted is 0, which would not be a statistic worth advertising.

c



d The 24 numbers in this interval are not evenly spread out. 21 of them are 0.

e 25

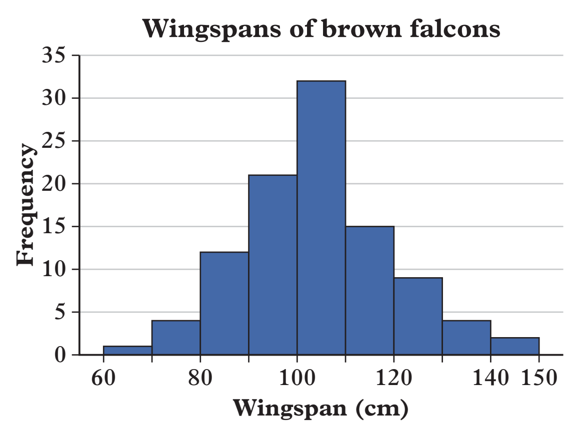
12 a Mean: 9.4, median: 9, mode: 9

b The mode tells the owner which shoe size is most popular. The median and mean do not tell the owner relevant data for stocktaking.

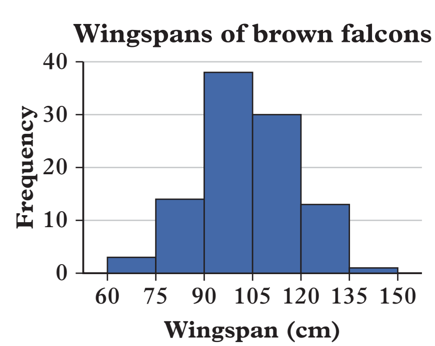
c 70%

d 94%

13 a



b



c The histogram with class intervals of size 10, because the smaller size creates intervals that do not follow the pattern properly and the larger size intervals make the pattern less obvious.

d 68%

e 95%

EX p337

7.6 Comparing data

1 **a** **i** 41 **ii** 45 **iii** 45

b **i** 18 **ii** 24 **iii** 18 and 24

c **i** Positively skewed **ii** Symmetrical

2 **a** Group A and Group B have a similar spread of data but are skewed in different directions. Group A is negatively skewed, whereas Group B is positively skewed. Group A has a larger centre than group B.

b Both groups A and B are symmetric, but group B has a larger range due to the presence of an outlier. The median should be used as a measure of centre for group B, which has a smaller centre than group A.

c Both groups have a symmetric distribution and have a similar centre, but Group B is spread over a larger range.

d Both groups are negatively skewed, but group A has a smaller centre than Group B. Group B has an outlier present and a larger range.

3 **a**

Leaf Friday	Stem	Leaf Saturday
9	2	6
2 0	3	7
9 8 6 2	4	2
9 8 7 7 6 5 4 3 1	5	5 9
7 7 5 4 1 1	6	4 6 7 8
6 2 1	7	5 5 6 7 7 8 8 9
8	8	1 3 4 4 5 9
2	9	2 4 9

Key: 2|9 = 29

Data sets are spread roughly over same range but data set for Friday night is symmetrical, whereas the data set for Saturday night is negatively skewed. More people seem to attend cinema on Saturday night than on Friday night.

b

Leaf Adelaide	Stem	Leaf Darwin
6	20	
5	21	
7	22	
7	23	
	24	
4	25	
6 3	26	
0	27	
7 4 3 3 0	28	
4	29	5 9
	30	0 4 8
2	31	3 4 8
1	32	1 1 2 3 5 5 6 7 8 8 9
8 8 4	33	0 1 2 3 4 4 4 5 8
3 2	34	
6	35	
0	36	
1	37	
4 2	38	
2	39	
5	40	

Key: 32|9 = 32.9

Adelaide has a much larger range of temperatures than Darwin. Adelaide seems to have a roughly symmetric distribution with a lower average temperature than Darwin. Darwin's temperatures cover a small range and appear to be negatively skewed, with a higher average temperature than Adelaide.

c

Leaf Boston terriers	Stem	Leaf French bulldogs
9 9 9 6 6	4	
5 5 1 1	5	
8 8 7 4 2	6	
9 8 8 6 6 5 5	7	
6 6 4 3 3 2 1 0	8	6
9 2 2 2 2	9	1 2 2 4 5 5 6 8 8 8 8 9
5 5 5 0	10	1 5 6 6 8 9 9
2 0	11	0 1 1 2 2 3 5 5 5 8 9
	12	0 3 5 5 7 9 9 9
	13	0

Key: 10|9 = 10.9 kg

Both dog breeds appear to have roughly symmetric distributions, with Boston terriers covering a larger range of masses. French bulldogs appear to be heavier on average than Boston terriers.

4 **a** Median heights of students in both classes are similar but 9A has a larger range of heights. The larger range and higher mean values suggests that 9A heights may be affected by presence of an outlier. Median should be used as measure of centre.

b The median and mean price of jeans and range of prices in all stores are similar. Distributions seem to be symmetrical. Mean should be used as measure of centre.

c Both the mean and median indicate that the use of internet in secondary school is greater than its use in primary school. Range of hours is also greater in secondary school. Median should be used as measure of centre.

d Both stores have the same range and median of storage capacity in their cards, however store A has a larger mean than store B (and larger than its median), which suggests that store A has a skewed distribution and the median is a better representation of centre.

e Players A and B have the same median but player B has a larger mean. Combined with the smaller range, it appears that player B is a more consistent goal kicker with a positively skewed distribution, which would make them a better goal kicker.

5 **a** B. The range of B is 9, whereas the range of A is 7.

b A. The median of A is 25, whereas the median of B is 23.

6 **a** School B

b School B (13 at school B and 6 at school A)

c The two data sets are spread out over roughly the same age brackets, but distributed differently. School A has a roughly symmetric distribution, whereas school B is positively skewed.

7 **a** The range of heights is similar for Year 7 and Year 9 but Year 9 heights are slightly skewed in the negative direction. Year 9 students seem to be taller overall.

b

Class heights	Mean (cm)	Median (cm)	Range (cm)
Year 7	164.22	162	34
Year 9	173.54	173.5	33

c Range of heights for Year 7 and Year 9 students is almost identical. Measures of centre are close for each group, but the mean and median for Year 9 is higher, indicating the Year 9 students are taller than the Year 7 students.

d Yes; mean and median indicate that the Year 9 students are slightly taller than the Year 7 students and ranges of heights for Year 7 students and Year 9 students are almost identical.

8 **a** Dot plot 1: negatively skewed;
Dot plot 2: symmetrical

b Dot plot 1: mean = 16.6, median = 17, range = 6
Dot plot 2: mean = 15.24, median = 15, range = 6

c Scores in both data sets are spread over same range, 6. The data set in first dot plot has a higher mean, median and mode than the data set in the second dot plot. Therefore, students participating in the swim squad in term 1 tended to be older than students who participated in the school play in term 4.

9 **a**

Leaf Year 8	Stem	Leaf Year 9
5 0	0	
4	1	8 9
7 7 5 3	2	0 6
8 3 2 0 0	3	5 7
9 4 4 1	4	5 8 9
9 8 5	5	1 2 4 5
5 0	6	0 5
4 1	7	1 4 6 7 7
6 3	8	3 6
	9	0 2 7

Key: 5|2 = 52

b Year 8: 86, Year 9: 79

c Year 8: 41, Year 9: 55

d Yes, it is true in general because the median for Year 9 students is much higher.

10 **a** Group A: 4, Group B: 4

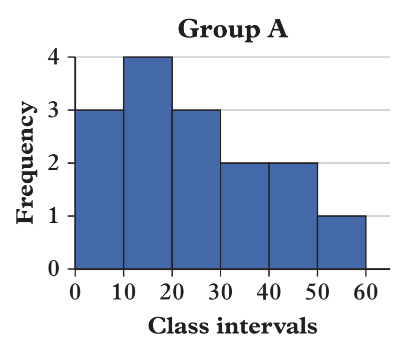
b

	Mean	Range
A	3.8	6
B	4.7	14

c The two data sets are very similar apart from the outlier in Group B.

11 **a** We do not have access to the raw data for the histogram. All that is known is how many numbers are in each interval, but not the actual values of those numbers.

b



c The two data sets are spread out over roughly the same values but distributed differently. Group A is positively skewed, whereas Group B is negatively skewed. Group B would have a higher centre than group A.

12 A matches with histogram 2 and B matches with histogram 1.

Reasons may vary. Any of the following are acceptable:

The median for histogram 2 is in the interval 10–<20 which matches with data set A.

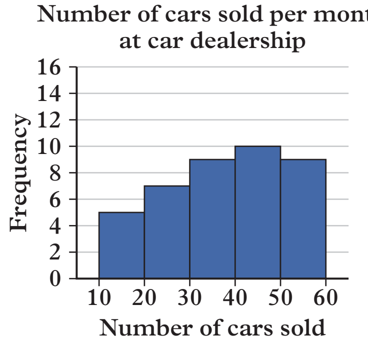
The median for histogram 1 is in the interval 20–<30, which matches with data set B. Histogram 2 will have a lower mean than histogram 1 because so many more values are stacked in the first two bins.

13 a Negatively skewed **b** Roughly symmetric

c 16

14 a Class intervals used are not same size.

b



c Both histograms display data spread over same range. The first histogram displays data that is negatively skewed. The median class for the number of cars sold each month by car dealership A was 30–40, with the most frequent number sold in any month being 40–50 cars. The second histogram displays data that is symmetrical. Car dealership B had the same median class, which is also the modal class for this data.

d We do not know how the data is spread over the class intervals to enable us to draw the histogram with smaller class intervals.

15 a Year: 1 80–85; Year: 2 85–90; Year: 3 90–95

b Year 3

c If the trend continues, in Year 4 the graph will be negatively skewed with a modal class of 95–100.

16 a Class B; it has a range of 28.

b Negatively skewed. A higher median than mean indicates that the centre of the distribution is a high value but there is a long tail of lower values to bring the mean down.

c Class C, because its median is the highest by far. Class B has the highest mean but was affected by one outlier, which does not indicate that class B plays video games more in general.

17 a Year 8: 31; Year 9: 35

b 37

c 162.7 cm

d Summary statistics cannot help us to find the middle value of all 450 student heights; the raw data is needed.

EX p344

Review: Statistics

Review questions 7.7A: Mathematical literacy review

1 A census is a survey of an entire population, whereas a sample is a survey of a selection of the population.

2 Ordinal data can be put into categories that have a defined order, whereas the categories that nominal data can be put into have no defined order.

3 The right side of the distribution

4 a class interval

b bi-modal distribution

5 a Cumulative frequency is a running total of frequencies in a frequency table; for example, if the frequency at a score was 5, and the total of the frequencies before that score was 12, then the cumulative frequency of that score would be 17.

b Convenience sampling is when a sample that is easily accessible is selected; for example, a researcher may choose to select students in their class to complete a survey

6 a univariate data

b skewed

Review questions 7.7B: Multiple choice

1 D	**2** C	**3** C	**4** A
5 A	**6** D	**7** C	**8** B
9 D	**10** C	**11** D	

Review questions 7.7C: Short answer

1 a Biased; your friends are not a representative sample of the whole school.

b Biased; people from your local council area are not representative of the population.

2 a Swimming coaches will be biased towards swimming as a form of exercise, so this is a biased sample.

b The data is about the popularity of a vegan burger, not about veganism.

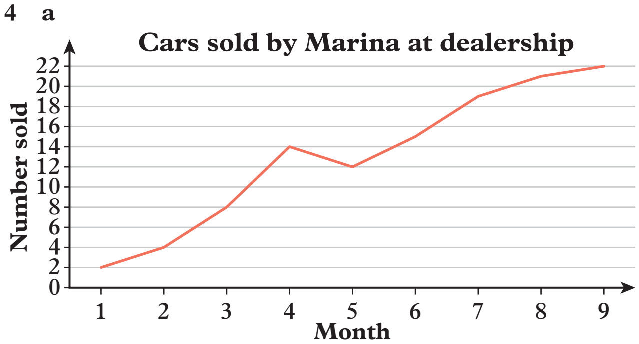
3 a Average monthly rainfall: numerical; city: categorical; month: categorical

b Hobart

c July

d Jakarta is in the southern hemisphere and Seoul is in the northern hemisphere, so they do not experience summer and winter at the same times.

4 a



b Month 5

c \$7000

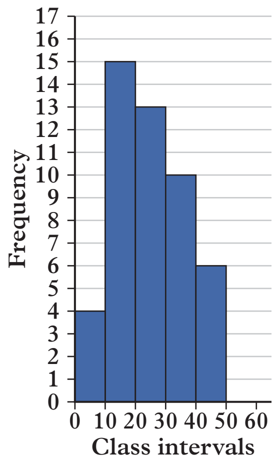
5 a

Class interval	Frequency
0–<10	4
10–<20	15
20–<30	13
30–<40	10
40–<50	6
Total	48

b 48

c 10–<20

d

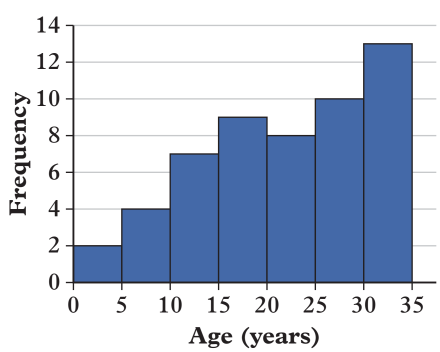


e 10–<20

6 Mean = 4.46, median = 5, mode = 5, range = 3

7 Mean = 22.84, median = 21, mode = 7, range = 56

8 a 24 **b** 29 **c** Age



d Negatively skewed

9 a Class 9A: mean = 66.32, median = 65, mode = 45, range = 47

Class 9B: mean = 67.14, median = 64, mode = 48, range = 53

b The means and medians are very similar despite the stem-and-leaf plots looking quite different, but Class 9B has a slightly larger range.

10 a

Leaf School 1	Stem	Leaf School 2
1 1	0	2 2 4 4
9 8 8 7 7 7 5 5	0*	6 6 7
1 1 1 0 0	1	0 1 1 1 2 4 4
7	1*	6 6 6 7 7 9 9
2 2 1 1	2	0 2
7	2*	
2	3	1
8 5	3*	

Key: 1|1 = 11

b School 1: mean = 14.4, median = 10.5, range = 37

School 2: mean = 12.8, median = 13, range = 29

c In discussing the results, the median will give the best measure of centre, since in school 1 the mean is affected by the positive skew.

Review questions 7.7D: Mathematical modelling

a Brisbane: 22 shrubs were measured, Sydney: 24 shrubs were measured

b Brisbane: mean = 13.34, median = 13.15, mode = 11.4, range = 4.5

Sydney: mean = 13.33, median = 13.25, mode = 11.4, range = 4.1

c The data for the two locations is very similar: 22 shrubs were measured in Brisbane and 24 were shrubs measured in Sydney, and the mean and mode for both locations is almost identical. The median is slightly higher for Sydney. It could be said that the average shrub height was 13.3 cm.

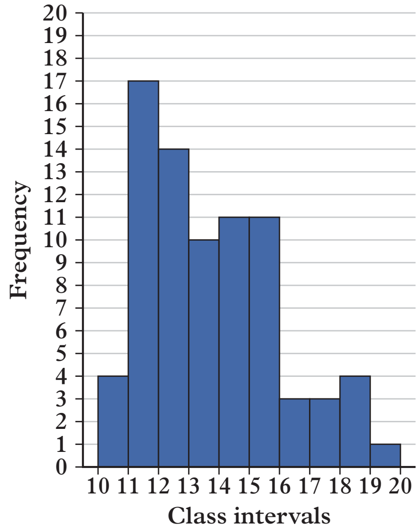
d i

Class interval	Frequency
10–<11	4
11–<12	17
12–<13	14
13–<14	10
14–<15	11
15–<16	11
16–<17	3
17–<18	3
18–<19	4
19–<20	1
Total	78

ii 78 plants

iii 11–<12

iv



The data displayed in the histogram is positively skewed.

v Melbourne = 14.53, Perth = 13.78

vi If purchasing a shrub from this nursery chain, it will most likely be in height range of 10–20 cm, with the average shrub height being about 14 cm.

MODULE 8 Probability

8.1 Two-step chance experiments

EX p355

1 **a** 9 **b** 5

2 **a** 9

b **i** $\frac{2}{9}$ **ii** $\frac{4}{9}$ **iii** $\frac{5}{9}$

iv $\frac{1}{3}$ **v** 0

3 **a**

		1st dice roll			
		1	**2**	**3**	**4**
2nd dice roll	**1**	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

b 16 **c** $5, \frac{1}{4}$ **d** $2 \text{ or } 8, \frac{1}{16}$

e **i** $\frac{1}{8}$ **ii** $\frac{3}{16}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{8}$

4 **a** $\frac{7}{16}$ **b** $\frac{1}{8}$ **c** $\frac{1}{16}$

d $\frac{9}{16}$ **e** $\frac{3}{4}$ **f** $\frac{1}{16}$

5 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

6 **a** $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{1}{3}$

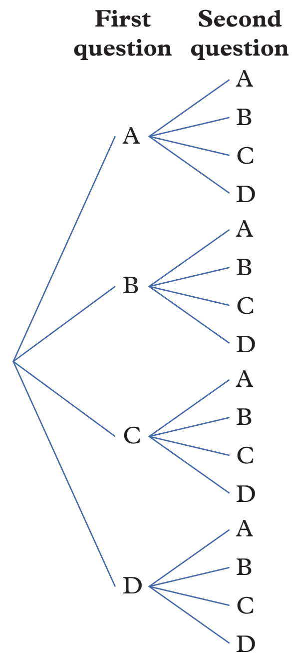
7 **a** **i** (r, s), (p, r), (s, p)

ii (r, p), (p, s), (s, r)

iii (r, r), (p, p), (s, s)

b $\frac{1}{3}$

8 **a**



b

		Second question			
		A	**B**	**C**	**D**
First question	**A**	A, A	A, B	A, C	A, D
	B	B, A	B, B	B, C	B, D
	C	C, A	C, B	C, C	C, D
	D	D, A	D, B	D, C	D, D

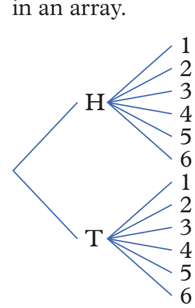
c With a tree diagram you can add a third step to the right of the second step, whereas it is not possible to add another dimension to an array to represent the third step.

9 **a** 36

b **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{6}$ **iv** $\frac{5}{36}$

c When there are a large number of outcomes in each step of a two-step chance experiment, the information is easier to interpret in an array than a tree diagram, and it can be easier to locate the favourable outcomes in an array.

10 **a**



b 12

c **i** $\frac{1}{12}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{12}$ **iv** $\frac{3}{4}$

d Tails on the coin toss and a 6 on the die is counted as one outcome, not two.

11 a

		Vowel				
		A	**E**	**I**	**O**	**U**
Die roll	**1**	(A, 1)	(E, 1)	(I, 1)	(O, 1)	(U, 1)
	2	(A, 2)	(E, 2)	(I, 2)	(O, 2)	(U, 2)
	3	(A, 3)	(E, 3)	(I, 3)	(O, 3)	(U, 3)
	4	(A, 4)	(E, 4)	(I, 4)	(O, 4)	(U, 4)

b 20

c **i** $\frac{1}{20}$ **ii** $\frac{2}{5}$ **iii** $\frac{9}{20}$ **iv** $\frac{9}{10}$

12 a **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{1}{4}$

b It is not possible to add another dimension to an array to represent the third or more steps.

13 a $\frac{1}{216}$ **b** $\frac{5}{54}$ **c** $\frac{5}{9}$

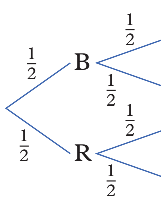
d $\frac{5}{72}$ **e** $\frac{5}{18}$

EX 8.2 Experiments with replacement

p361

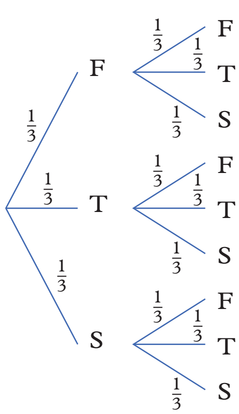
1 a $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{3}{26}$ **d** $\frac{1}{6}$

2 a



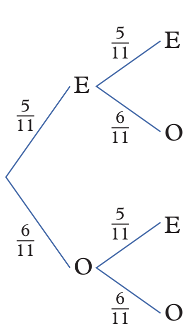
Outcomes	Probability
BB	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
BR	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
RB	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
RR	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

b



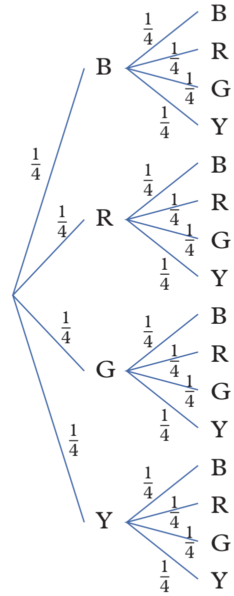
Outcomes	Probability
FF	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
FT	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
FS	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
TF	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
TT	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
TS	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
SF	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
ST	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
SS	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$

c



Outcomes	Probability
EE	$\frac{5}{11}\times\frac{5}{11}=\frac{25}{121}$
EO	$\frac{5}{11}\times\frac{6}{11}=\frac{30}{121}$
OE	$\frac{6}{11}\times\frac{5}{11}=\frac{30}{121}$
OO	$\frac{6}{11}\times\frac{6}{11}=\frac{36}{121}$

d



Outcomes	Probability
BB	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
BR	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
BG	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
BY	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
RB	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
RR	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
RG	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
RY	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
GB	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
GR	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
GG	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
GY	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
YB	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
YR	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
YG	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
YY	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$

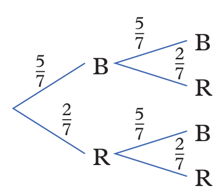
3 a $\frac{1}{2}$ **b** 4

c **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{4}$

4 a $\frac{1}{9}$ **b** $\frac{5}{9}$ **c** $\frac{1}{9}$ **d** $\frac{2}{9}$

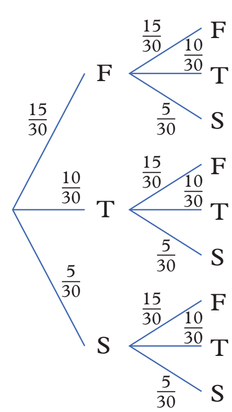
5 a $\frac{1}{16}$ **b** $\frac{7}{16}$ **c** $\frac{3}{8}$ **d** $\frac{9}{16}$ **e** $\frac{3}{4}$ **f** $\frac{1}{8}$

6 a



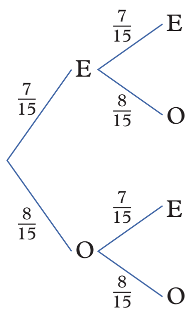
Outcomes	Probability
BB	$\frac{5}{7}\times\frac{5}{7}=\frac{25}{49}$
BR	$\frac{5}{7}\times\frac{2}{7}=\frac{10}{49}$
RB	$\frac{2}{7}\times\frac{5}{7}=\frac{10}{49}$
RR	$\frac{2}{7}\times\frac{2}{7}=\frac{4}{49}$

b



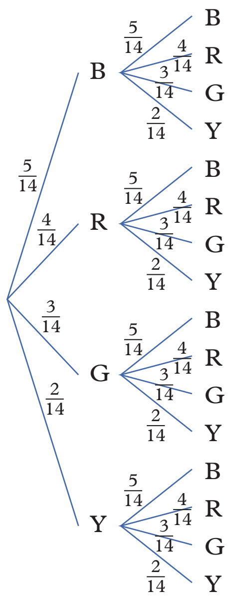
Outcomes	Probability
FF	$\frac{15}{30}\times\frac{15}{30}=\frac{225}{900}$
FT	$\frac{15}{30}\times\frac{10}{30}=\frac{150}{900}$
FS	$\frac{15}{30}\times\frac{5}{30}=\frac{75}{900}$
TF	$\frac{10}{30}\times\frac{15}{30}=\frac{150}{900}$
TT	$\frac{10}{30}\times\frac{10}{30}=\frac{100}{900}$
TS	$\frac{10}{30}\times\frac{5}{30}=\frac{50}{900}$
SF	$\frac{5}{30}\times\frac{15}{30}=\frac{75}{900}$
ST	$\frac{5}{30}\times\frac{10}{30}=\frac{50}{900}$
SS	$\frac{5}{30}\times\frac{5}{30}=\frac{25}{900}$

c



Outcomes	Probability
EE	$\frac{7}{15} \times \frac{7}{15} = \frac{49}{225}$
EO	$\frac{7}{15} \times \frac{8}{15} = \frac{56}{225}$
OE	$\frac{8}{15} \times \frac{7}{15} = \frac{56}{225}$
OO	$\frac{8}{15} \times \frac{8}{15} = \frac{64}{225}$

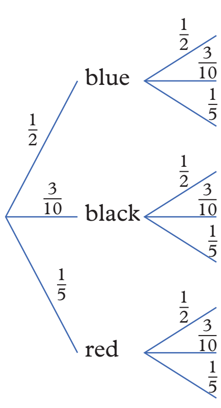
d



Outcomes	Probability
BB	$\frac{5}{14} \times \frac{5}{14} = \frac{25}{196}$
BR	$\frac{5}{14} \times \frac{4}{14} = \frac{20}{196}$
BG	$\frac{5}{14} \times \frac{3}{14} = \frac{15}{196}$
BY	$\frac{5}{14} \times \frac{2}{14} = \frac{10}{196}$
RB	$\frac{4}{14} \times \frac{5}{14} = \frac{20}{196}$
RR	$\frac{4}{14} \times \frac{4}{14} = \frac{16}{196}$
RG	$\frac{4}{14} \times \frac{3}{14} = \frac{12}{196}$
RY	$\frac{4}{14} \times \frac{2}{14} = \frac{8}{196}$
GB	$\frac{3}{14} \times \frac{5}{14} = \frac{15}{196}$
GR	$\frac{3}{14} \times \frac{4}{14} = \frac{12}{196}$
GG	$\frac{3}{14} \times \frac{3}{14} = \frac{9}{196}$
GY	$\frac{3}{14} \times \frac{2}{14} = \frac{6}{196}$
YB	$\frac{2}{14} \times \frac{5}{14} = \frac{10}{196}$
YR	$\frac{2}{14} \times \frac{4}{14} = \frac{8}{196}$
YG	$\frac{2}{14} \times \frac{3}{14} = \frac{6}{196}$
YY	$\frac{2}{14} \times \frac{2}{14} = \frac{4}{196}$

7 **a** $\frac{1}{25}$ **b** $\frac{16}{25}$ **c** $\frac{4}{25}$

8 **a**

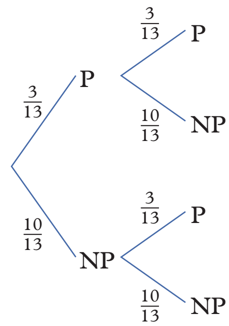


Outcomes	Probability
blue, blue	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
blue, black	$\frac{1}{2} \times \frac{3}{10} = \frac{3}{20}$
blue, red	$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$
black, blue	$\frac{3}{10} \times \frac{1}{2} = \frac{3}{20}$
black, black	$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$
black, red	$\frac{3}{10} \times \frac{1}{5} = \frac{3}{50}$
red, blue	$\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$
red, black	$\frac{1}{5} \times \frac{3}{10} = \frac{3}{50}$
red, red	$\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$

b **i** $\frac{1}{4}$ **ii** $\frac{1}{25}$ **iii** $\frac{9}{100}$

iv $\frac{3}{20}$ **v** $\frac{1}{10}$ **vi** $\frac{3}{50}$

9 **a**



Outcomes	Probability
P, P	$\frac{3}{13} \times \frac{3}{13} = \frac{9}{169} \approx 0.0533$
P, NP	$\frac{3}{13} \times \frac{10}{13} = \frac{30}{169} \approx 0.1775$
NP, P	$\frac{10}{13} \times \frac{3}{13} = \frac{30}{169} \approx 0.1775$
NP, NP	$\frac{10}{13} \times \frac{10}{13} = \frac{100}{169} \approx 0.5917$

b **i** $\frac{60}{169} \approx 0.3550$ **ii** $\frac{69}{169} \approx 0.4083$

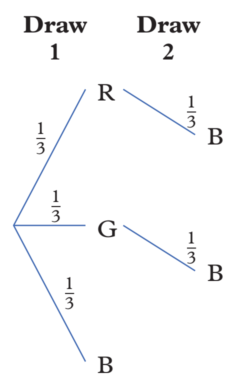
iii $\frac{9}{169} \approx 0.0533$ **iv** $\frac{100}{169} \approx 0.5917$

10 The grouped outcomes of this chance experiment are not equally likely, so the theoretical probability is calculated by adding the probabilities of the successful outcomes instead of using the theoretical probability formula.

11 Pr(blue ball) = Pr(blue ball on first draw) or Pr(not blue, then blue)

$$= \frac{1}{3} + \left(2 \times \frac{1}{3} \times \frac{1}{3}\right)$$

$$= \frac{5}{9}$$



Outcomes	Probability
RB	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
GB	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
	$\frac{1}{3}$

12 **a** If the two-step chance experiment with replacement has lots of outcomes, they may be easier to see in an array than a tree diagram, and it can be easier to locate the favourable outcomes.

b If outcomes are not equally likely, then include the outcomes in the array in the same proportion as the likelihood. For example, if the first event is a ball is drawn at random from 1 blue and 2 red, then the array would have three columns: blue, red, red.

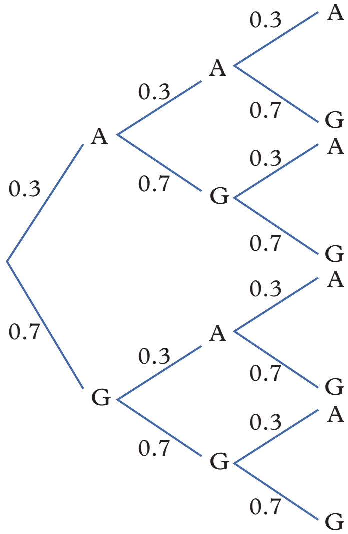
13 **a** Sample answer: No not necessarily. The socks are replaced each time, so you cannot say that this is the exact number of white and black socks. Some of the same socks may have been chosen multiple times.

b 4 black socks and 6 white socks

c Yes, this supports the previous estimate.

14 a 28 green marbles

b



c i 0.37

ii 0.441

iii 0.63

iv 0.21

15 a $3^3 = 27$

b BRW, BWR, RBW, RWB, WBR, WRB

c $6bw(1 - b - w)$

16 a a^2 **b** $2ab$ **c** $1 - a - b$

d i $2a(1 - a - b)$ **ii** $2a - 2a^2 - 2ab$

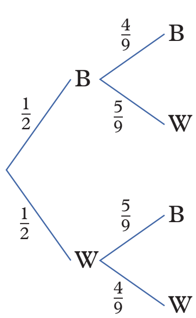
e i $(1 - a - b)^2$

ii $1 - 2a - 2b + 2ab + a^2 + b^2$

EX 8.3 Experiments without replacement

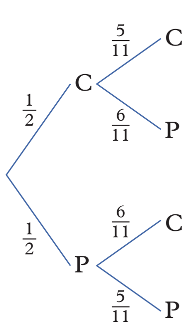
p366

1 a



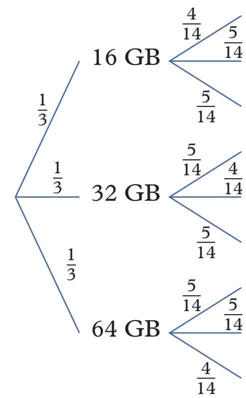
Outcomes	Probability
BB	$\frac{1}{2} \times \frac{4}{9} = \frac{4}{18}$
BW	$\frac{1}{2} \times \frac{5}{9} = \frac{5}{18}$
WB	$\frac{1}{2} \times \frac{5}{9} = \frac{5}{18}$
WW	$\frac{1}{2} \times \frac{4}{9} = \frac{4}{18}$

b



Outcomes	Probability
CC	$\frac{1}{2} \times \frac{5}{11} = \frac{5}{22}$
CP	$\frac{1}{2} \times \frac{6}{11} = \frac{6}{22}$
PC	$\frac{1}{2} \times \frac{6}{11} = \frac{6}{22}$
PP	$\frac{1}{2} \times \frac{5}{11} = \frac{5}{22}$

c

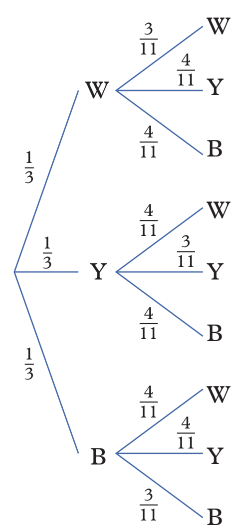


	Outcomes	Probability
16 GB	16, 16	$\frac{1}{3} \times \frac{4}{14} = \frac{4}{42}$
32 GB	16, 32	$\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$
64 GB	16, 64	$\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$
16 GB	32, 16	$\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$
32 GB	32, 32	$\frac{1}{3} \times \frac{4}{14} = \frac{4}{42}$
64 GB	32, 64	$\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$
16 GB	64, 16	$\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$
32 GB	64, 32	$\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$
64 GB	64, 64	$\frac{1}{3} \times \frac{4}{14} = \frac{4}{42}$

2 a 0.21 **b** 0.21 **c** 0.58

3 a $\frac{3}{14}$ **b** $\frac{2}{7}$

4 a

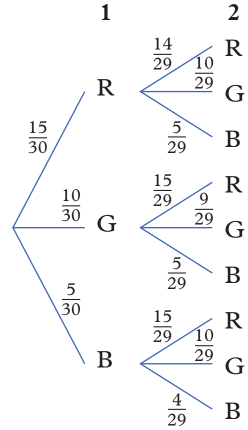


Outcomes	Probability
WW	$\frac{1}{3} \times \frac{3}{11} = \frac{3}{33}$
WY	$\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$
WB	$\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$
YW	$\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$
YY	$\frac{1}{3} \times \frac{3}{11} = \frac{3}{33}$
YB	$\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$
BW	$\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$
BY	$\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$
BB	$\frac{1}{3} \times \frac{3}{11} = \frac{3}{33}$

b i $\frac{1}{11}$ **ii** $\frac{4}{33}$

5 a $\frac{8}{11}$ **b** $\frac{19}{33}$ **c** $\frac{14}{33}$

6 a



Outcomes	Probability
RR	$\frac{15}{30} \times \frac{14}{29} = \frac{210}{870}$
RG	$\frac{15}{30} \times \frac{10}{29} = \frac{150}{870}$
RB	$\frac{15}{30} \times \frac{5}{29} = \frac{75}{870}$
GR	$\frac{10}{30} \times \frac{15}{29} = \frac{150}{870}$
GG	$\frac{10}{30} \times \frac{9}{29} = \frac{90}{870}$
GB	$\frac{10}{30} \times \frac{5}{29} = \frac{50}{870}$
BR	$\frac{5}{30} \times \frac{15}{29} = \frac{75}{870}$
BG	$\frac{5}{30} \times \frac{10}{29} = \frac{50}{870}$
BB	$\frac{5}{30} \times \frac{4}{29} = \frac{20}{870}$

b **i** $\frac{20}{870} = \frac{2}{87}$

ii $\frac{90}{870} = \frac{3}{29}$

iii $\frac{210}{870} = \frac{7}{29}$

iv $\frac{150}{870} = \frac{5}{29}$

7 **a** $\frac{490}{870} = \frac{49}{87}$ **b** $\frac{660}{870} = \frac{22}{29}$

c $\frac{270}{870} = \frac{9}{29}$ **d** $\frac{450}{870} = \frac{15}{29}$

e $\frac{400}{870} = \frac{40}{87}$ **f** $\frac{100}{870} = \frac{10}{87}$

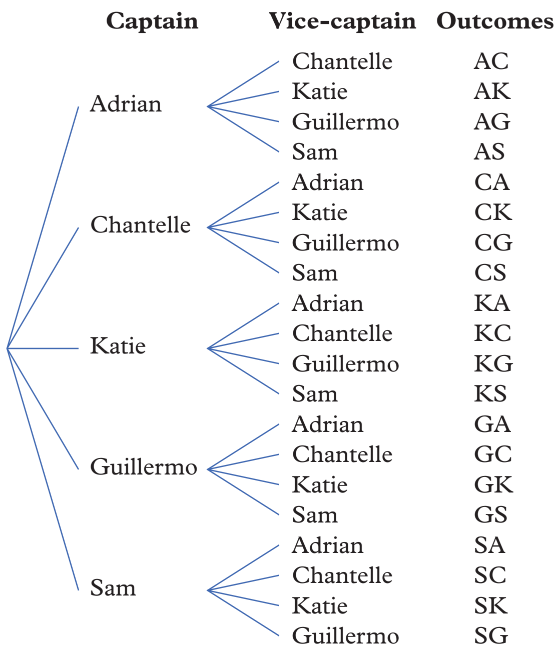
8 **a** $\frac{1}{17}$ **b** $\frac{15}{34}$ **c** $\frac{19}{34}$ **d** $\frac{13}{102}$

9 $\frac{1}{5}$

10 Pr(pair) $= \frac{5}{17}$; probability of a pair increases slightly.

11 $\frac{38}{132} = \frac{19}{66}$

12 **a**



b The second set of branches has one fewer branch. Once the captain has been selected there is one fewer person to choose from for vice-captain's position.

c 20

d **i** $\frac{1}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{1}{20}$

iv $\frac{3}{5}$ **v** $\frac{1}{10}$

13 **a** **i** $\frac{5}{18}$ **ii** $\frac{1}{6}$ **iii** $\frac{5}{18}$ **iv** $\frac{5}{9}$

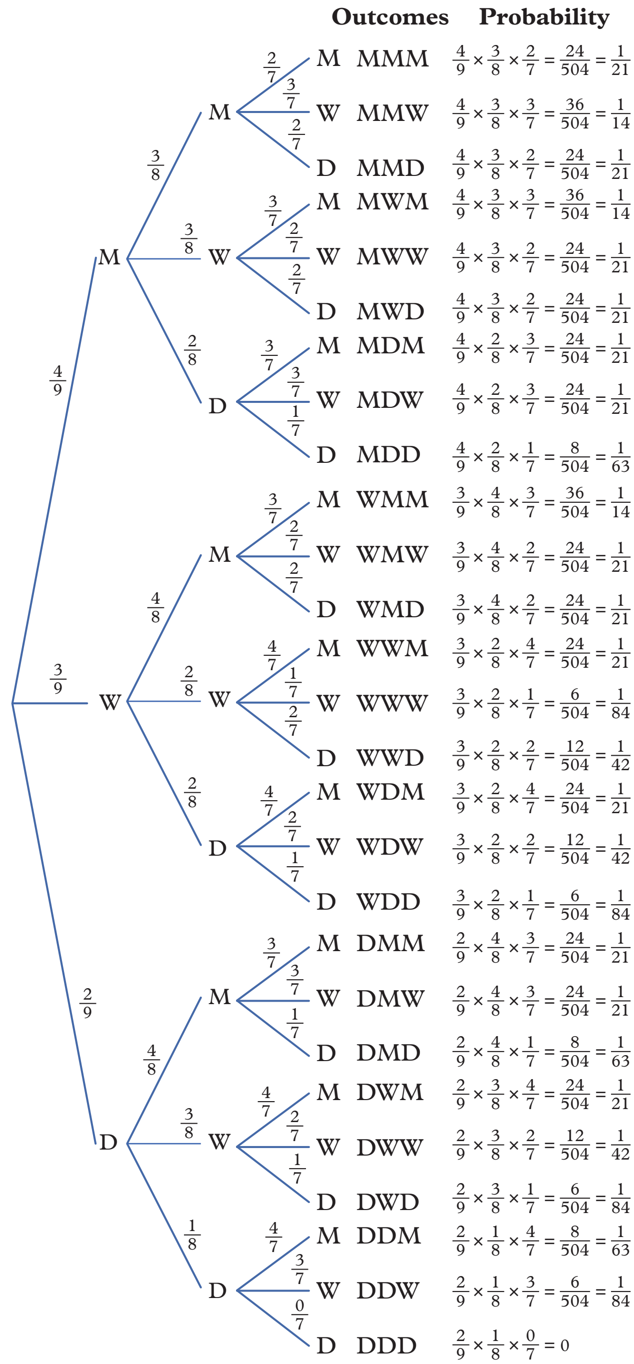
b **i**

	G	**G**	**G**	**P**
G	×	GG	GG	GP
G	GG	×	GG	GP
G	GG	GG	×	GP
P	PG	PG	PG	×

ii $\frac{1}{2}$ **iii** 0

14 **a** $\frac{1}{12}$ **b** $\frac{7}{12}$ **c** $\frac{13}{18}$ **d** $\frac{7}{12}$

15 **a**



b **i** $\frac{1}{21}$

ii $\frac{2}{7}$

iii $\frac{5}{42}$

16 $\frac{106}{1625}$

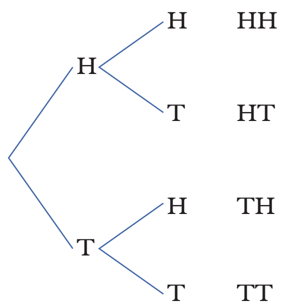
17 $\frac{1}{8\,145\,060} \approx 0.000\,000\,123$

18 $\frac{5}{18}$

Checkpoint

EX p369

1 **Outcomes**



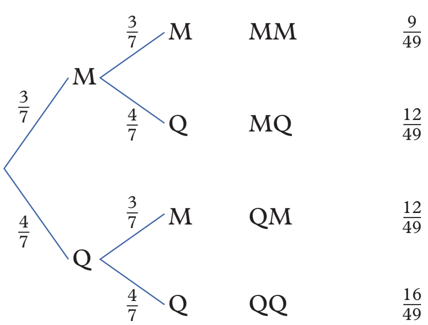
2

	1	2	3	4
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
3	(1, 3)	(2, 2)	(3, 3)	(3, 4)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

3 **a** $\frac{4}{9}$ **b** $\frac{4}{9}$ **c** $\frac{8}{9}$

4 **a** $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{5}{12}$

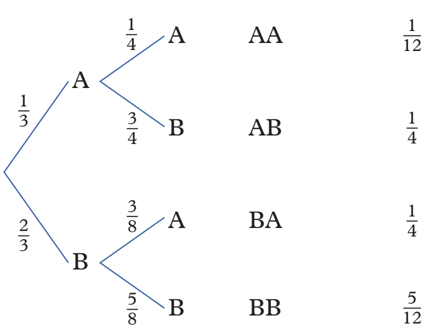
5 **Outcomes** **Probability**



6 **a** $\frac{83}{225}$ **b** $\frac{9}{25}$ **c** $\frac{8}{9}$

7 **a** $\frac{24}{49}$ **b** $\frac{3}{7}$ **c** $\frac{40}{49}$

8 **Outcomes** **Probability**



9 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{11}{12}$

10 **a** $\frac{1}{4}$ **b** $\frac{7}{12}$ **c** $\frac{11}{12}$

8.4 Relative frequency

EX p374

1 **a** $\frac{9}{25} \approx 0.36$ **b** $\frac{1}{100} = 0.01$
c $\frac{3}{8} \approx 0.38$ **d** $\frac{2}{3} \approx 0.67$
e $\frac{87}{348} \approx 0.25$ **f** $\frac{131}{240} \approx 0.55$

2 **a** 125 **b** 40 **c** 105
d 5 **e** 40 **f** 214

3 **a** $\frac{8}{15} \approx 0.53$
b In the long term, you would expect the experimental probability of rolling a die and obtaining a number greater than 2 to increase as it approaches the theoretical probability of $\frac{2}{3} \approx 0.67$.

4 **a** 30
b $\frac{7}{130} \approx 0.054$
c In the long term, you would expect the experimental probability of drawing a picture card to increase as it approaches the theoretical probability of $\frac{3}{13} \approx 0.231$.

5 **a** **i** 30
ii There is only a slight difference between the expected number (30) and the actual number (27), so there is no evidence that the experiment is biased.
b **i** 18
ii There is only a slight difference between the expected number (18) and the actual number (16), so there is no evidence that the experiment is biased.
c **i** 100
ii There is a significant difference between the expected number (100) and the actual number (53) and the number of trials is large. It appears that this experiment is biased.

6 **a** Heads: 25; tails: 15
b One: 18; two: 24; three: 21; four: 12; five: 15; six: 30
c Hearts: 12; diamonds: 18; clubs: 21; spades: 9

7 **a** **i** 10
ii Pr(heads) $= \frac{1}{2}$, Pr(tails) $= \frac{1}{2}$
iii Heads = 5, tails = 5
iv Pr(heads) $= \frac{4}{5} = 0.8$, Pr(tails) $= \frac{1}{5} = 0.2$
v There are not enough trials to make a firm decision.
vi There are only 10 trials; this is too few to tell if it is biased.
b **i** 6000
ii Pr(1) = Pr(2) = Pr(3) = Pr(4) = Pr(5) = Pr(6) $= \frac{1}{6} \approx 0.17$
iii Expected number = 1000 for all outcomes
iv Pr(1) ≈ 0.16, Pr(2) ≈ 0.16, Pr(3) ≈ 0.17, Pr(4) ≈ 0.17, Pr(5) ≈ 0.17, Pr(6) ≈ 0.17

v The prop is fair.

vi The frequency is very close to the expected number for each outcome. The relative frequency for each outcome is approximately equal to its theoretical probability after a large number of trials.

c **i** 112

ii Pr(hearts) = Pr(diamonds) = Pr(clubs) = Pr(spades) = $\frac{1}{4}$ = 0.25

iii Expected number = 28 for all outcomes

iv Pr(hearts) ≈ 0.34, Pr(diamonds) ≈ 0.28, Pr(clubs) ≈ 0.17, Pr(spades) ≈ 0.21

v The prop is biased.

vi Not all frequencies are close to expected numbers of each outcome (0.25). Hearts and diamonds are greater than expected, and clubs and spades are less than expected. Relative frequencies for all outcomes are not approximately equal to their theoretical probabilities after a reasonable number of trials.

8 **a** HHH, HHT, HTH, HTT, THH, TTH, THT, TTT

b $\frac{1}{4}$ **c** 10

d No, the actual occurrences are only half of the expected number.

e The relative frequency should get closer to the theoretical probability after 4000 trials.

9 With a small number of rolls, the actual number (13) can be significantly different from the expected number (5) due to random chance. As the number of rolls increases, the relative frequency will approach the theoretical probability if no bias exists. Therefore, to check for any bias, a higher number of trials should take place.

10 **a** 24 blue, 12 green, 12 blue, 12 purple

b 25 blue, 16 green, 8 blue, 11 purple

c The predictions from part **b** should be more accurate as the number of trials is greater.

11 **a** A sample size of two is not a broad enough sample to verify a claim about the worldwide adult population.

b **i** While the expected number would be 1 for a group of 3, 'one in three' does not mean one in any group of three. It means that any given Australian has a one in three chance of being an Australian who does not get enough sleep.

ii The first headline is worldwide while the second is Australian only. The samples may be biased. The sample sizes may be too small to draw wide-scale conclusions from.

12 **a** $\frac{1}{128}$

b A streak of 7 of more heads has a 1 in 128 chance of occurring, so it is very likely to occur if you toss a coin 1000 times.

c 31

EX p379

8.5 Two-way tables

1 **a** 55 **b** 7 **c** $\frac{7}{55}$

2 **a** 75 **b** 11 **c** $\frac{58}{75}$

3 **a** $\frac{29}{85}$ **b** $\frac{37}{85}$ **c** $\frac{42}{85}$ **d** $\frac{23}{85}$

4 **a** $\frac{48}{125}$ **b** $\frac{19}{50}$ **c** $\frac{13}{125}$ **d** $\frac{31}{50}$ **e** $\frac{181}{250}$ **f** $\frac{27}{125}$

5

	Class A	Class B	Total
Prefer sweet food	0.23	0.32	0.55
Prefer savoury food	0.29	0.16	0.45
Total	0.52	0.48	1

6

	Reads books	Plays games	Plays sports	Total
Under 12	$\frac{7}{32}$	$\frac{11}{64}$	$\frac{1}{8}$	$\frac{33}{64}$
Over 12	$\frac{3}{64}$	$\frac{11}{64}$	$\frac{17}{64}$	$\frac{31}{64}$
Total	$\frac{17}{64}$	$\frac{11}{32}$	$\frac{25}{64}$	1

7

	Light hair	Dark hair	Total
Prefer pop music	0.22	0.38	0.6
Prefer rock music	0.13	0.27	0.4
Total	0.35	0.65	1

8

	Spotify	No Spotify	Total
Netflix	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{2}{3}$
No Netflix	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{3}$
Total	$\frac{1}{4}$	$\frac{3}{4}$	1

9 **a**

	Cinema	Home	Total
Action	22	30	52
Comedy	19	14	33
Total	41	44	85

b **i** $\frac{41}{85}$ **ii** $\frac{14}{85}$ **iii** $\frac{52}{85}$

10

	Cat	No cat	Total
Purebred	29	75	104
Not purebred	84	12	96
Total	113	87	200

$$\text{Pr(non-cat purebred)} = \frac{75}{200} = \frac{3}{8}$$

11 **a**

	50 and over	Under 50	Total
Bath	0.33	0.09	0.42
Shower	0.22	0.36	0.58
Total	0.55	0.45	1

b **i** 0.36 **ii** 0.42 **iii** 0.22

c **i** 210 **ii** 275 **iii** 45

12 a $\frac{18}{83}$ **b** $\frac{67}{332}$ **c** $\frac{4}{83}$

d $\frac{125}{332}$ **e** $\frac{72}{83}$ **f** $\frac{69}{83}$

g $\frac{109}{166}$ **h** $\frac{47}{332}$ **i** $\frac{9}{83}$

13 a 60 **b** 160 **c** 980 **d** 120

14

	A	A'	Total
B	40	30	70
B'	20	30	50
Total	60	60	120

15 a

	A	A'	Total
B	40	35	75
B'	40	85	125
Total	80	120	200

	A	A'	Total
B	0.2	0.175	0.375
B'	0.2	0.425	0.625
Total	0.4	0.6	1

b

	C	C'	Total
D	4	5	9
D'	2	9	11
Total	6	14	20

	C	C'	Total
D	0.2	0.25	0.45
D'	0.1	0.45	0.55
Total	0.3	0.7	1

c

	E	E'	Total
F	18	4	22
F'	2	12	14
Total	20	16	36

	E	E'	Total
F	$\frac{1}{2}$	$\frac{1}{9}$	$\frac{11}{18}$
F'	$\frac{1}{18}$	$\frac{1}{3}$	$\frac{7}{18}$
Total	$\frac{5}{9}$	$\frac{4}{9}$	1

16 a

	Year 8	Year 9	Total
Maths	12	11	23
English	10	17	27
Total	22	28	50

b i $\frac{23}{50}$ **ii** $\frac{14}{25}$ **iii** $\frac{11}{50}$

c i 28 **ii** 11 **iii** $\frac{11}{28}$

d Part **c iii** says the student is in Year 9, hence the total number of possible outcomes is restricted to the total number of Year 9 students (28). In part **b iii** you are selecting from the total number of students (50), and of those you need to select a student in Year 9 who likes Maths.

e i Likes English; 27

ii Year 8 students who like English

iii 10

iv $\frac{10}{27}$

17 a $\frac{13}{81}$ **b** $\frac{37}{96}$ **c** $\frac{1}{5}$

d $\frac{13}{43}$ **e** $\frac{71}{181}$ **f** $\frac{4}{5}$

EX p386

8.6 Venn diagrams

1 a 14 **b** 15 **c** 2 **d** 31

2 a 62 **b** 19 **c** 532 **d** 637

3 a $\frac{1}{5}$ **b** $\frac{3}{10}$ **c** $\frac{11}{20}$

d $\frac{7}{10}$ **e** $\frac{1}{10}$ **f** $\frac{1}{4}$

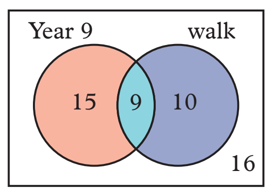
4 a $\frac{17}{45}$ **b** $\frac{1}{9}$ **c** $\frac{19}{45}$

d $\frac{1}{5}$ **e** $\frac{23}{45}$ **f** $\frac{31}{45}$

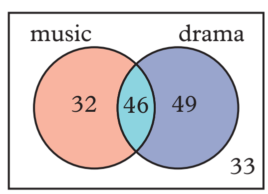
5 a $\frac{9}{41}$ **b** $\frac{21}{41}$ **c** $\frac{11}{41}$

d $\frac{8}{41}$ **e** $\frac{22}{41}$ **f** $\frac{33}{41}$

6



7



8 a $A \cap B$ **b** $A \cup B$

c $A' \cup B$ **d** $A \cap B'$

e $A \cup B'$ **f** $A' \cap B'$ $(A \cup B)'$

g $A' \cap B$ **h** $A' \cup B'$ or $(A \cap B)'$

9 a i 29 **ii** 43 **iii** 26

iv 10 **v** 50 **vi** 14

b i $\frac{12}{25}$ **ii** $\frac{21}{50}$ **iii** $\frac{1}{5}$

iv $\frac{7}{50}$ **v** $\frac{7}{50}$ **vi** $\frac{31}{50}$

10 a $(A \cap B)' = A' \cup B'$ and $(A' \cup B')' = A \cap B$

$(A' \cap B)' = A \cup B'$ and $(A \cup B')' = A' \cap B$

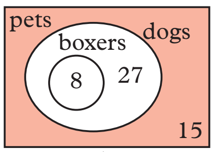
$(A \cap B')' = A' \cup B$ and $(A' \cup B)' = A \cap B'$

$(A' \cap B')' = A \cup B$ and $(A \cup B)' = A' \cap B'$

b The complement of an intersection is the union of the complements of the two events. The complement of a union is the intersection of the complements of the two events.

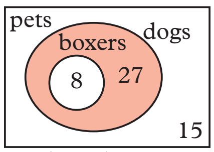
11 a The first Venn diagram shows a subset of data. All the set of 'Boxers' belongs to the set of 'Dogs', hence 'Boxers' is a subset of 'Dogs'.

b i



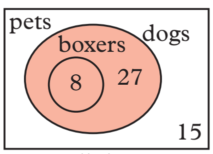
pets that are not dogs

ii



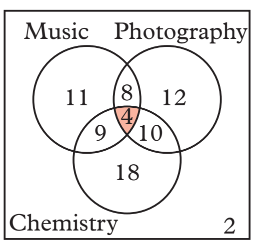
dogs that are not boxers

iii

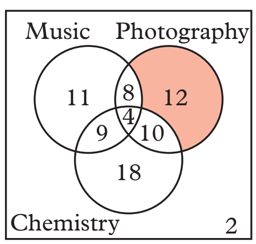


all dogs

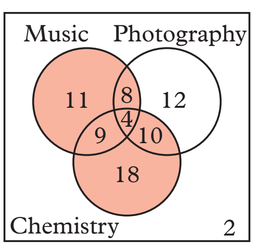
c i



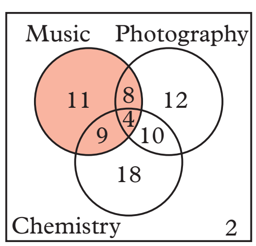
ii



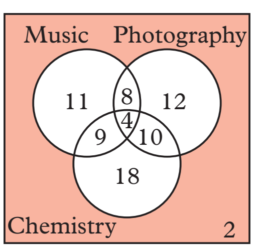
iii



iv



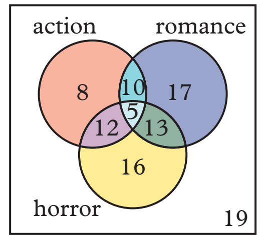
v



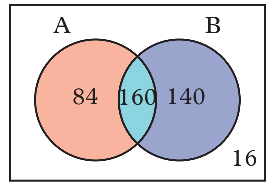
12 a $\frac{7}{34}$ **b** $\frac{4}{17}$ **c** $\frac{1}{17}$

d $\frac{5}{34}$ **e** $\frac{23}{34}$ **f** $\frac{7}{34}$

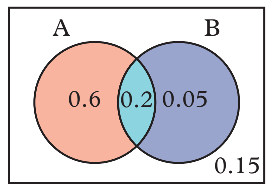
13 Sample answer:

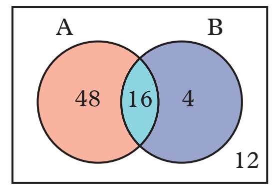


14



15 a





b i 0.8 **ii** 0.25

16 a $\frac{29}{61}$ **b** $\frac{29}{46}$ **c** $\frac{32}{61}$ **d** $\frac{9}{25}$

17 a $\frac{11}{30}$ **b** $\frac{11}{26}$ **c** $\frac{15}{26}$ **d** $\frac{2}{7}$

EX p390

Review: Probability

Review questions 8.7A: Mathematical literacy review

1 The complement of an event A is the event that A does not occur.

2 In an experiment without replacement, the sample space reduces after each step, so the probabilities change at each step.

3 The relative frequency of an event can be calculated by dividing the number of times a favourable outcome occurs by the total number of trials.

4 a Sample space

b Universal set

5 a The number of favourable outcomes expected in a probability outcome

b A collection of one or more outcomes from the sample space of an experiment

6 a Subset

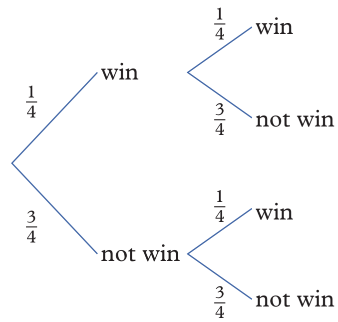
b Intersection

Review questions 8.7B: Multiple choice

1 C	**2** C	**3** A	**4** A	**5** D
6 A	**7** A	**8** D	**9** D	**10** C

Review questions 8.7C: Short answer

1 **a** $\frac{3}{4}$ **b** **Chance of winning an eBook**



c **i** $\frac{1}{16}$ **ii** $\frac{15}{16}$ **iii** $\frac{3}{16}$

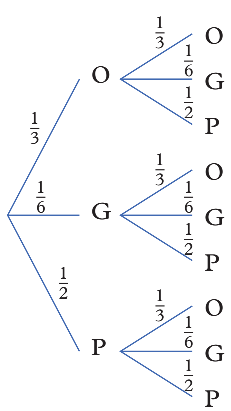
2 **a**

	1	2	3	4	5	6
H	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

b **i** $\frac{1}{12}$ **ii** $\frac{7}{12}$ **iii** $\frac{1}{2}$

c $\frac{1}{6}$ **d** $\frac{5}{6}$

3 **a**



Outcomes	Probability
OO	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
OG	$\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}$
OP	$\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$
GO	$\frac{1}{6}\times\frac{1}{3}=\frac{1}{18}$
GG	$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$
GP	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
PO	$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$
PG	$\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$
PP	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

b **i** $\frac{1}{9}$ **ii** $\frac{1}{36}$ **iii** $\frac{1}{4}$

c $\frac{1}{6}$ **d** $\frac{1}{2}$

4 **a** $\frac{1}{17}$ **b** $\frac{13}{204}$

5 **a** $\frac{3}{8}$ **b** $\frac{1}{200}$ **c** $\frac{11}{100}$ **d** $\frac{13}{20}$

6 **a** 35 **b** 10 **c** 5 **d** 25

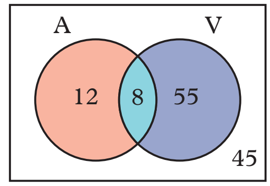
7 **a**

	Year 8	Year 9	Year 10	Total
Napoleon Perdis	32	25	45	102
Rimmel	22	10	22	54
Covergirl	17	31	13	61
Maybelline	49	29	15	93
Total	120	95	95	310

b **i** $\frac{3}{10}$ **ii** $\frac{11}{155}$ **iii** $\frac{5}{62}$

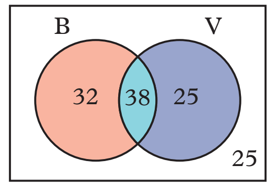
c **i** $\frac{5}{19}$ **ii** $\frac{12}{19}$

8 **a**



b **i** $\frac{1}{15}$ **ii** $\frac{5}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{21}{40}$

c

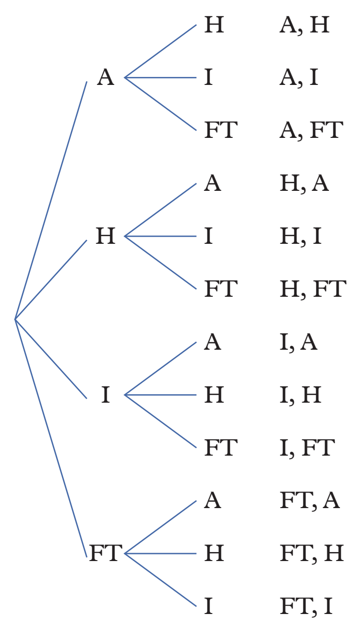


d **i** $\frac{19}{60}$ **ii** $\frac{11}{15}$ **iii** $\frac{4}{15}$ **iv** $\frac{7}{12}$

Review questions 8.7D: Mathematical modelling

1 **a** Without replacement, as the same subject cannot be selected twice

b **i**



Key:
A = Art
H = Health and Human Development
I = Italian
FT = Food Technology

ii 6

c **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$

2 **a**

	Year 11	Year 12	Total
Art	0.24	0.12	0.36
Food technology	0.28	0.36	0.64
Total	0.52	0.48	1.00

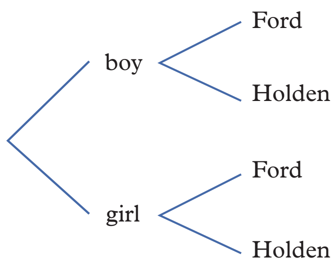
b **i** $\frac{9}{25}$ **ii** $\frac{6}{25}$ **iii** $\frac{9}{25}$

c **i** 72 **ii** 128

d **i** 24 **ii** 2 **iii** 68

iv $\frac{34}{75}$ **v** $\frac{28}{75}$

3 **a**



b {BF, BH, GF, GH};
B = boy, G = girl, F = Ford, H = Holden
Each outcome has a theoretical probability of 0.25.

c

	Ford	Holden	Total
Boys	7	23	30
Girls	19	9	28
Total	26	32	58

58 people were surveyed.

d **i** $\frac{32}{58}$ **ii** $\frac{15}{29}$

MODULE 9 Financial mathematics

9.1 Wages and salaries

EX p400

1 \$41.20
2 **a** \$28 **b** \$45 **c** \$23.40
d \$24 **e** \$33.60
3 \$2038.40
4 **a** \$570 **b** 24
5

Hourly wage	Time worked	Amount earned
\$32.20	25 hours	\$805
\$24.50	30 hours	\$735
\$27	28 hours	\$756
\$30.30	38 hours	\$1151.40
\$51.20	32 hours	\$1638.40

6 **a** **i** \$27 **ii** \$36
b **i** \$36 **ii** \$48
c **i** \$28.20 **ii** \$37.60
d **i** \$38.85 **ii** \$51.80
e **i** \$48.90 **ii** \$65.20
f **i** \$44.85 **ii** \$59.80
7 **a** \$38.70 **b** \$915.90
8 \$1238.45
9 **a** \$77 400 **b** \$53 760 **c** \$64 480
d \$49 400 **e** \$132 600 **f** \$86 580
10 **a** \$5551/month **b** \$2562/fortnight
11 **a** \$966 **b** \$50 232 **c** \$4186
12 **a** \$993.25 **b** 32 hours
13 Rahim is not correct. To calculate Sal's monthly wage, you first need to calculate his annual wage by multiplying by 52, and then you need to divide by 12.
$\$780 \times 52 = \$40\,560$
$\$40\,560 \div 12 = \$3380$
14 Nathan: \$1102.50, Kristina: \$1062.40, Jack: \$954, Mina: \$1337.70
15 \$37.62
16 **a** \$915.80 **b** \$19.28
17 13 hours
18 \$7176
19 \$78 200

9.2 Other forms of income

EX p403

1 **a** \$900 **b** \$1377 **c** \$3750
d \$1264.50 **e** \$1177.05
2 **a** \$1248 **b** \$1768
3 **a** \$1410 **b** \$760 **c** \$2135
d \$1200 **e** \$1860 **f** \$2420
4 \$1008
5 **a** **i** \$100 **ii** \$132.50 **iii** \$160
b **i** 36 cups **ii** 71 cups **iii** 85 cups
6 **a** \$14 850 **b** \$2880 **c** \$2925
d \$10 800 **e** \$6630
7 \$15 920
8 2.4%
9 \$33 100
10 23
11 Abi (\$710.50), Beth (\$920), Craig (\$1450)
12 \$100/hour
13 \$49/hour
14 **a** \$131.60 **b** 47
15 **a** Erica
b Patrick: \$3440, Erica: \$2870
16 **a** Option B
b Option A: Below \$50 000
Option B: Between \$50 000 and \$75 000
Option C: Above \$75 000

9.3 Taxation

EX p408

1 \$58 358
2 \$89 835
3 **a** \$4915.70 **b** \$1073.80 **c** \$90 141
4 **a** \$8488 **b** \$24 088 **c** \$14 878
d \$0 **e** \$70 313 **f** \$35 617
5 **a** \$74 225 **b** \$13 055.50
6 **a** \$1640 **b** \$920 **c** \$2080
d \$1330 **e** \$2660 **f** \$1184
7 **a** \$118 200 **b** \$26 248 **c** \$2364
d \$89 588
8 \$71 432
9 \$33 920

10 a $66 342

b Annual expenses

Rent: $420 × 52 = $21 840

Food: $140 × 52 = $7280

Entertainment: $80 × 52 = $4160

Personal items: $100 × 52 = $5200

Mobile phone: $75 × 12 = $900

Clothes: $150 × 12 = $1800

Transport: $100 × 12 = $1200

Utilities: $280 × 12 = $3360

Total: $45 740

c Yes

11 a $16 678.60

b Portia is due a tax refund of $1309.40.

12 People in the 30% tax bracket only pay 30% of their incomes over $45 000. The rest of their income is taxed at a lower percentage.

13 a 20.2% **b** 11.0% **c** 16.9%
d 21.9% **e** 8.1% **f** 23.9%

14 These are the values of the income tax payable for the amount of income at the top of the previous tax bracket (e.g. $4288 is payable for an income of $45 000).

15 a $61 599

b Total expenses = $62 220, which is greater than his net income of $61 599

16 Answers correct for the 2023–2024 financial year.

a $95 **b** $490 **c** $712
d $376 **e** $862 **f** $1846

17 a $2613.60 **b** $484

18

Income	Deductions	Taxable income	Income tax
$83 100	$300	$82 800	$15 628
$55 400	$550	$54 850	$7243
$102 310	$1100	$101 210	$21 151
$68 440	$810	$67 630	$11 077

Income	Medicare levy	Net income
$83 100	$1656	$65 516
$55 400	$1097	$46 510
$102 310	$2024.20	$78 034.80
$68 440	$1352.60	$55 200.40

19 a $155 517

b People earning less than $155 517 would be worse off and people earning more than $155 517 would be better off.

c i $6386 worse off

ii $5107 worse off

iii $898 better off

d Sample answer: A progressive income tax system reduces the tax burden on the people who can least afford to pay and reduces inequality.

20 a i $3192 **ii** $14 842
iii $37 607 **iv** $65 167

b No. This can be shown through calculation or by graphing the tax rates on the same Cartesian plane.

21 a $126 159 **b** $174 325 **c** $332 336

EX p411

Checkpoint

1

Hourly wage	Time worked	Amount earned
$32.20	12 hours	$386.40
$23.90	20 hours	$478.00
$40.05	15 hours	$600.75
$36.25	29 hours	$1051.25
$42.50	33 hours	$1402.50

2 a i $34.50 **ii** $46
b i $43.50 **ii** $58
c i $57 **ii** $76
d i $39.75 **ii** $53
e i $51.30 **ii** $68.40
f i $66.60 **ii** $88.80

3 a $68 880 **b** $54 120 **c** $62 660
d $107 640 **e** $47 840 **f** $89 180

4 $937.65

5 a $7.50 per item **b** $3.60 per item
c $17.20 per item **d** $0.12 per item
e $0.39 per item

6 $65 000

7 $76 147

8 a $13 378 **b** $19 048 **c** $4888
d $6985 **e** $25 300 **f** $10 615

9 a $1010 **b** $1268.40 **c** $1522
d $1641.80 **e** $2096.40 **f** $2759

10 a $79 500 **b** $14 638
c $1590 **d** $63 272

EX p414

9.4 Simple interest

1 a i $\frac{7}{100}$ **ii** 0.07
b i $\frac{11}{100}$ **ii** 0.11
c i $\frac{6}{25}$ **ii** 0.24
d i $\frac{3}{50}$ **ii** 0.06
e i $\frac{1}{10}$ **ii** 0.1
f i $\frac{3}{25}$ **ii** 0.12

2 a i $500 **ii** $5500
b i $576 **ii** $5376
c i $5000 **ii** $17 500

3 a i $1125 **ii** $8625
b i $6480 **ii** $17 280
c i $14 000 **ii** $39 000

4 a \$1200 **b** \$2422 **c** \$12 000
d \$1104 **e** \$17 999.10 **f** \$20 000

5 a P = \$3500, r = 4.8%, n = 2 years
b \$336
c \$3836

6 a P = \$44 000, r = 9.5%, n = 5 years
b \$20 900 **c** \$64 900

7 a i \$600 **ii** \$600
b Answers to parts **a i** and **ii** are identical. Simple interest is calculated in the same manner for investments and loans. In each case, simple interest is found by multiplying principal, rate (as a fraction or decimal) and number of time periods.
c Interest on an investment is a payment to the investor and on a loan is a charge.

8 a $\frac{11}{12}$ of a year **b** $\frac{7}{52}$ of a year **c** $\frac{1}{2}$ of a year
d $\frac{1}{4}$ of a year **e** $\frac{271}{365}$ of a year **f** $\frac{31}{73}$ of a year
g $1\frac{1}{4}$ years **h** 4 years **i** $1\frac{8}{13}$ years
j $3\frac{2}{5}$ years **k** $2\frac{1}{2}$ years **l** $5\frac{1}{2}$ years

9 4.25 years

10 a i \$1620 **ii** \$10 620
b i \$236.25 **ii** \$10 736.25
c i \$1837.50 **ii** \$9337.50
d i \$228.82 **ii** \$29 228.82
e i \$52.78 **ii** \$8652.78
f i \$17 825.73 **ii** \$173 395.73
g i \$1913.37 **ii** \$21 912.37
h i \$15 601.94 **ii** \$61 551.94
i i \$1950.77 **ii** \$210 604.77

11 a \$9.20 **b** \$9.07
c Sade would hope the method that writes time as a fraction of total number of months in year was used. She earns more interest with this method.
d Sade would prefer the method that uses the number of days in June as a fraction of total number of days in year. She would pay less interest on her loan using this method.

12 a 5.25% **b** \$1050
c This statement is true. If Jasmine invests her money for 364 days, the calculations are:

$$I = \frac{\$20\,000 \times 0.0555 \times 364}{365} = \$1106.96$$

She receives slightly more interest.

13 a i \$10 **ii** \$100 **iii** \$300
b i \$86.80 **ii** \$694.40 **iii** \$2094.40
c i \$40 000 **ii** \$500 000 **iii** \$1 000 000
d i \$5 per month or \$60 per year
ii \$100
iii \$850
e i \$25 per week or \$1300 per year
ii \$1800
iii \$27 800
f i \$0.42 per day or \$153.3 per year
ii \$64.26
iii \$4444.26

14 a i 1.5 **ii** \$300
b i 1.496 **ii** \$2094.40
c i 2 **ii** \$1 000 000
d i 1.1333 **ii** \$849.98
e i 1.0692 **ii** \$27 799.20
f i 1.0147 **ii** \$4444.39

15 a, b

Balance \$	Days applied for	Interest earned \$
640.90	7 days	0.26
540.90	7 days	0.22
780.90	9 days	0.40
655.50	5 days	0.19

c \$1.07
d \$656.57

16 a \$0.72 **b** \$830.27

17 a Yes, account A will receive the bonus interest rate. They have made only one withdrawal for the month and the account balance has increased by more than \$200. Account B will not receive the bonus because there is more than one withdrawal.
b Account A: \$6.24, Account B: \$3.06
c Account A: \$2231.99, Account B: \$3139.82

18 a The minimum deposit is \$1875, so Joel's savings are enough to cover it.
b \$10 000 **c** \$15 050

19 a $33\frac{1}{3}$% p.a.
b i 25% p.a. **ii** 20% p.a. **iii** $16\frac{2}{3}$% p.a.
c i 50% p.a. **ii** 100% p.a.
d As the number of years increases, the interest rate decreases. As the number of years decreases, the interest rate increases.

20 Sample answer: 5% p.a. for 5 years, 2.5% for 10 years, 10% for 2.5 years

EX p420 9.5 Simple interest calculations

1 a \$420 **b** \$27.50 **c** \$1700
d \$5000 **e** \$38 000 **f** \$2400

2 a n = 5 years **b** n = 7 years **c** n = 4 years
d n = 4 years

3 a P = \$5000 **b** P = \$15 000 **c** P = \$4000
d P = \$4800

4 a n = 5 years **b** P = \$6000 **c** P = \$2500
d n = 2 years **e** P = \$3125 **f** n = 5 years

5 a n, number of time periods
b $P = \$4500$, $r = 5.0\%$ p.a., $I = \$675$
c 3 years
d One year sooner ($n = 2$ years compared to $n = 3$ years)
6 a \$546 b 26 months
7 a 5.0% p.a. b 6.0% p.a. c 2.5% p.a.
d 5.6% p.a. e 6.4% p.a. f 2.5% p.a.
8 a \$932.40 b \$4500 c 9.8%
d 3.5 years or 3 years and 6 months
e \$2950 f 6.8% g 5.0%
h 1.75 years or 1 year and 9 months
9 a P, amount borrowed
b $r = 4.0\%$ p.a., $n = 3$ years, $I = \$144$
c \$1200
d Total amount to be repaid = \$1200 + \$144 = \$1344
Repayments = \$35 × 36 = \$1260
(3 years = 36 months)
Repayments of \$35 per month will not be enough to repay the loan over three years.
e \$37.33 (will be 12 cents short at the end, but can make that adjustment later)
10 a 1.2125 years = a year and 78 days or about a year and two and a half months
b \$12 125
11 a 1.125% per quarter b \$13.50
12 a \$1000 b \$1400 c 10% p.a.
13 a 5% p.a. b 3% p.a.
14 a \$68.75 b 10 000 days c 79 365 years
15 2.2%

EX
p424

Review: Financial mathematics

Review questions 9.6A: Mathematical literacy review

1 2%
2 Divide by 26
3 Pay-As-You-Go
4 a loan b wage
5 a Leave loading is an extra payment on top of annual leave that is usually calculated as 17.5% of normal pay for this period. For example, if someone usually earned \$3000 per week and they received leave loading, they would earn \$3000 × 1.175 = \$3525 for a week of annual leave.
b Simple interest is interest which is calculated on the principal which has been borrowed or invested. For example, the simple interest on a \$3000 investment at 5% p.a. for two years would be 3000 × 0.05 × 2 = \$300.
6 a Piece work
b Tax deductions, taxable income

Review questions 9.6B: Multiple choice

1 B 2 D 3 D 4 B 5 A
6 D 7 C 8 C 9 A

Review questions 9.6C: Short answer

1

Hourly wage	Time worked	Amount earned
\$26.40	24 hours	\$633.60
\$33.20	28.5 hours	\$946.20
\$37.70	30 hours	\$1131
\$41.80	21 hours	\$877.80
\$23.90	36.5 hours	\$872.35

2 \$2749.50
3 a \$2475 b \$3136 c \$540
d \$12 870 e \$7436
4 a \$24 688 b \$15 088 c \$8638
d \$13 888 e \$2688 f \$56 093
5 \$48 720
6 a \$52 260
b Annual expenses
Rent: \$390 × 52 = \$20 280
Food: \$170 × 52 = \$8840
Entertainment: \$100 × 52 = \$5200
Personal items: \$100 × 52 = \$5200
Mobile phone: \$35 × 12 = \$420
Clothes: \$200 × 12 = \$2400
Transport: \$240 × 12 = \$2880
Utility bills: \$420 × 4 = \$1680
Total: \$46 900
c Yes
7 a i 5.2% p.a.
ii 6.6% p.a.
iii 7.2% p.a.
b i 0.4% per month
ii 0.12% per week
iii 0.95% per quarter
8 a \$600 b \$480 c \$700
9 a $n = 3$ years b $P = \$4800$ c $n = 2.5$ years
d $r = 9\%$ p.a. e $r = 3.4\%$ p.a.

Review questions 9.6D: Mathematical modelling

1 a \$5230.73
b Simon: \$588; Melanie: \$722.75; Tahlia: \$918.75
c \$5437
d i \$29 904
ii \$830.67
iii \$53 904
2 a i \$4800 ii \$4820
b Option 2 as it yields the greatest return.

c **i**

Year	Value at start of year	Interest	Value at end of year
1	\$4000.00	\$4000 × 0.04 = \$160.00	\$4160.00
2	\$4160.00	\$4160 × 0.04 = \$166.40	\$4326.40
3	\$4326.40	\$4326.40 × 0.04 = \$173.06	\$4499.46
4	\$4499.46	\$4499.46 × 0.04 = \$179.98	\$4679.44
5	\$4679.44	\$4679.44 × 0.04 = \$187.18	\$4866.62

ii Jiwoo should choose Option 3 as it yields the greatest return.

EX
p428

SEMESTER 2 Review

Short answer

1 \$1804.80

2 **a** 40.91 cm² **b** 61 000.00 cm² **c** 201.06 mm² or 2.01 cm² **d** 52 216.00 cm²

3 **a** 154 cm² or 15 400 mm² **b** 36 cm² **c** 483.8 cm² **d** 2422.4 cm²

4 **a** 122.5 cm³ **b** 12 cm³ **c** 615.8 cm³ **d** 3200 cm³

5 **a** 2 **b** 1.4 **c** 3.4

6 **a** 25 mL **b** 55.0 g **c** 4.6 m

7 **a** 3.2 m by 2.6 m **b** 4.8 m by 3.6 m **c** 5.6 m by 3.2 m

8 **a** \$6000 USD **b** \$315 USD **c** \$1687.50 USD

9 **a** $k = 12$ cm, $l = 52°$ **b** $p = 139°$, $q = 30$ mm

10 **a** 320 cm² **b** 122.72 mm²

11 **a** **i** $\frac{1}{2}$ **ii** 0.5581 **iii** 0.8372 **iv** 0.6667
b **i** $\frac{1}{13}$ **ii** 0.5 **iii** 0.8660 **iv** 0.5774
c **i** 3 **ii** 0.7593 **iii** 0.6508 **iv** 1.1667
d **i** 5 **ii** 0.96 **iii** 0.28 **iv** 3.4286

12 **a** 70° **b** 49° **c** 76° **d** 148°

13 **a** $z = 13.6$ m
b $x = 30.9$ m, $y = 18.2$ m
c $p = 19.2$ m, $q = 18.9$ m
d $r = 60.4$ m

14 **a** No **b** No **c** Yes **d** Yes

15 **a** 2610.1 cm² **b** 1008.1 cm² **c** 16 799.9 cm² **d** 210 276.5 mm²

16 **a** \$3240 **b** 11.67% p.a. **c** \$2166.67 **d** 2 months

17 **a** **i** Categorical **ii** Nominal
b **i** Numerical **ii** Continuous
c **i** Numerical **ii** Discrete

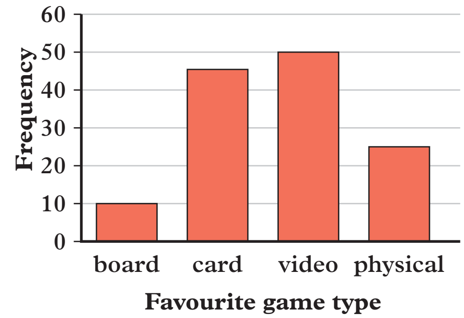
18 **a** Sample answer:

Class	Frequency
5–<10	2
10–<15	0
15–<20	6
20–<25	4
25–<30	7
30–<35	1

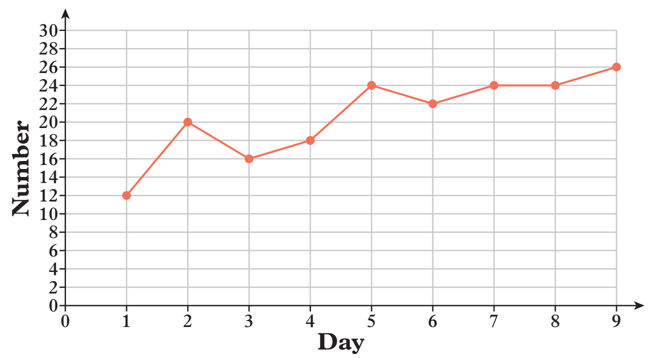
b

Class	Frequency
20–<40	2
40–<60	5
60–<80	8
80–<100	21
100–<120	4

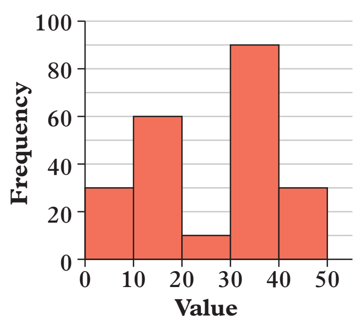
19 **a**



b



c



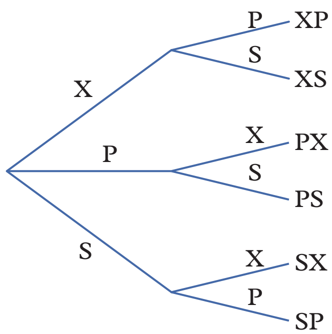
20 **a** Li **b** Chaz **c** Game 3

21 **a** **i** 19.8 **ii** 19 **iii** 19 **iv** 10
b **i** 12.34 **ii** 12 **iii** 13 **iv** 4
c **i** 4.36 **ii** 4.1 **iii** 3.4 **iv** 4.7

22 **a** Bi-modal symmetric **b** Perfectly symmetric **c** Symmetric **d** Positively skewed

23 **a** Mode of each peak **b** Mean, median or mode **c** Mean, median or mode **d** Median **e** Median or mode **f** Mode

24 a



b **i** $\frac{1}{6}$ **ii** $\frac{1}{3}$ **iii** $\frac{2}{3}$ **iv** $\frac{1}{3}$

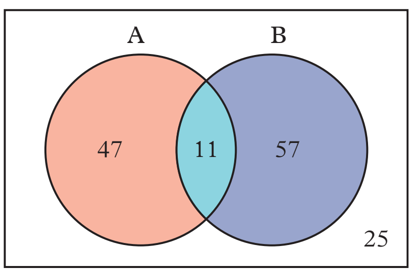
25 a

	1	2	3	4	5	6
Spade	(1, ♠)	(2, ♠)	(3, ♠)	(4, ♠)	(5, ♠)	(6, ♠)
Club	(1, ♣)	(2, ♣)	(3, ♣)	(4, ♣)	(5, ♣)	(6, ♣)
Diamond	(1, ♦)	(2, ♦)	(3, ♦)	(4, ♦)	(5, ♦)	(6, ♦)
Heart	(1, ♥)	(2, ♥)	(3, ♥)	(4, ♥)	(5, ♥)	(6, ♥)

b **i** $\frac{1}{24}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{4}$ **iv** $\frac{1}{2}$

26 a $\frac{16}{49}$ **b** $\frac{12}{35}$ **c** $\frac{1}{35}$ **d** $\frac{1}{3}$

27 a



b

	A	A′	Total
B	5	21	26
B′	12	12	24
Total	17	33	50

28 a **i** 60 **ii** 77 **iii** 52
iv 28 **v** 112 **vi** 17

b **i** $\frac{15}{28}$ **ii** $\frac{13}{28}$ **iii** $\frac{1}{4}$
iv $\frac{5}{16}$ **v** $\frac{5}{16}$ **vi** $\frac{95}{112}$

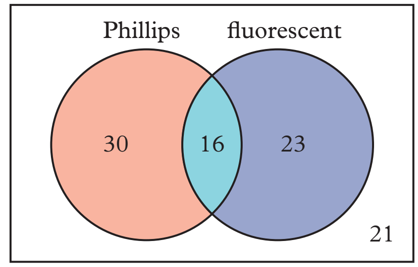
29 a

	A	A′	Total
B	$\frac{3}{5}$	$\frac{1}{25}$	$\frac{16}{25}$
B′	$\frac{7}{25}$	$\frac{2}{25}$	$\frac{9}{25}$
Total	$\frac{22}{25}$	$\frac{3}{25}$	1

b

	A	A′	Total
B	51	29	80
B′	15	25	40
Total	66	54	120

30 a



b **i** $\frac{8}{45}$ **ii** $\frac{17}{30}$ **iii** $\frac{22}{45}$ **iv** $\frac{2}{3}$

31 a

	S	S′	Total
G	56	76	132
G′	42	38	80
Total	98	114	212

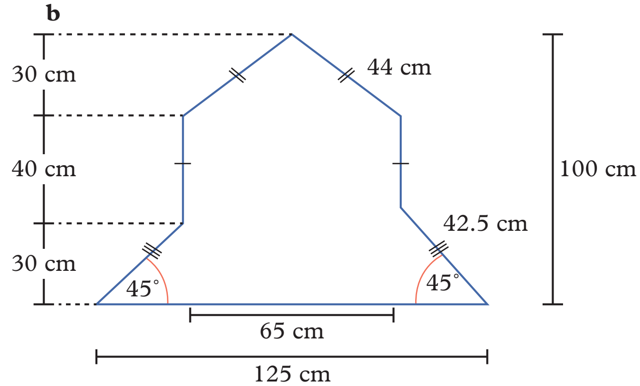
b **i** $\frac{57}{106}$ **ii** $\frac{33}{53}$ **iii** $\frac{19}{106}$ **iv** $\frac{59}{106}$

Mathematical modelling

1 **a** \$936 **b** \$4399.20 **c** \$49 327.20
d \$2939.40
e **i** \$932.40 **ii** \$52 715
iii \$6602.50 **iv** \$45 058.20

2 **a** 8.4 cm should be 8.5 cm, correct to one decimal place. 13.0 cm is correct.

b



c **i** 257 cm² **ii** 6425 cm²
d **i** 30° **ii** 69.3 cm
iii They are not parallel as the angle the edge of the plywood makes with the horizontal (≈ 42.7°) is not the same as the angle the string makes with the horizontal (30°), and these are corresponding angles.

3 **a**

Leaf Arielle	Stem	Leaf Brenna
7	3*	7
4 3 3 1 1	4	0 1 2 3 4
9 9 9 8 8 8 8 8 8 8 7 7 7 7 6	4*	5 6 6 6 8
3 2 2 1 0 0	5	0 0 1 1 1 1 3 3 4
5	5*	5 6 7 7 8
	6	0 0 3

Key: 1 | 2 = 120 minutes

b Arielle's minutes of sleeping is negatively skewed whereas Brenna's minutes of sleeping is approximately symmetric.

c Arielle: median = 480 minutes, range = 180 minutes
Brenna: median = 510 minutes, range = 260 minutes

d Arielle sleeps on average less than Brenna with a median of 480 minutes of sleep compared to 510 minutes of sleep for Brenna. Arielle has a more consistent length of sleep with a range of 180 minutes as compared to 260 minutes for Brenna.

e **i** Independent variable: Month
Dependent variable: Median sleep length

ii Arielle and Brenna have valleys in June and November, but their peaks don't coincide. Both Arielle and Brenna's sleep length is, on average, increasing.

4 **a** **i** $\frac{2}{3}$ **ii** $\frac{7}{30}$ **iii** $\frac{1}{4}$ **iv** $\frac{15}{23}$

b **i** $\frac{2}{45}$ **ii** $\frac{7}{45}$

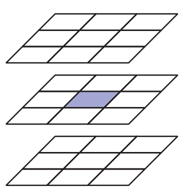
c **i** $\frac{4}{513}$ **ii** $\frac{61}{513}$

EX p438

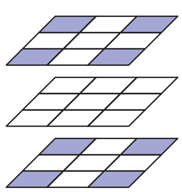
EXPLORATIONS 2

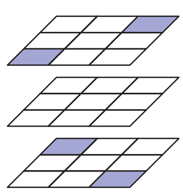
1 **a** Rotations and reflections of the minimum diagrams in **ii** and **iv** are also possible.

i maximum = minimum = 1

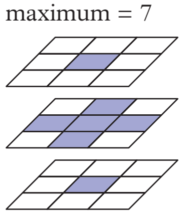


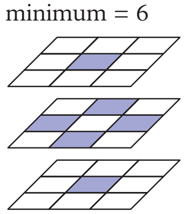
ii maximum = 8 minimum = 4



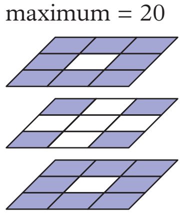


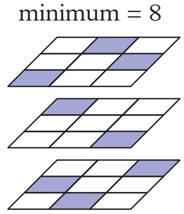
iii maximum = 7 minimum = 6





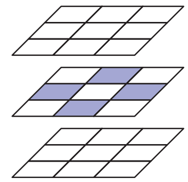
iv maximum = 20 minimum = 8

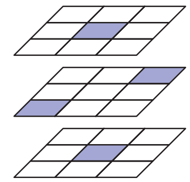




b One possible arrangement is given for **i** and **ii**. There are others.

i Top/bottom views: **ii** Four side views:





iii If the pattern can be viewed from all four sides, deduce that the top and bottom layers must have a cube in the centre, as in **ii** above, so the pattern cannot be seen from above or below.

2 **a** **i** $\frac{1}{12}$

ii Dieter has probability $\frac{1}{2}$ of winning but draws are also possible.

b No, Connie has probability $\frac{7}{12}$ of winning.

c 3 coins

d 14 sides

e $k = 3n - 1$

f No for dodecahedral (cannot have $n = 13$ counters). Yes for icosahedral (with $n = 7$ counters).

3 **a** **i** 9 **ii** 12 **iii** 10 **iv** 10
v 16 **vi** 20 **vii** 18 **viii** 22

b Types **i**, **iii** and **iv** only.

c $AB = AF$, so not all side lengths are different.

4 **a** **i** 2 steps **ii** 5 steps
iii 6 steps **iv** 9 steps

b **i** 2 moves **ii** impossible
iii 9 moves **iv** impossible

c **i** 2 passes **ii** 5 passes
iii 5 passes **iv** 9 passes

Glossary

absolute error
the magnitude of an error

absolute value
the distance of a number from zero on the number line

accuracy
the closeness of a measurement to its exact value

adjacent side
(trigonometry) the side, in a right-angled triangle, next to the reference angle that is not the hypotenuse

arc
a portion of the circumference of a circle. Angles are indicated using an arc

area chart
a graph created by shading the area between the lines and axes of a multiple line graph to create a sense of quantity over time

area scale factor
the scale factor between the areas of two similar shapes

array
items arranged in rows and columns. Used in probability to display the sample space of a two-step chance experiment

axis of symmetry
imaginary line that divides a symmetrical shape or graph so that one side is a reflection of the other

back-to-back stem-and-leaf plot
stem-and-leaf plot that displays two sets of data using a single set of stems

bar chart
graph used to display either univariate or bivariate data, where the horizontal axis displays a discrete numerical or categorical variable, and the vertical axis displays either the frequency or a numerical variable

base
in a value expressed in exponent form, the base is the number that is repeatedly multiplied; for example, 2^4 has a base of 2

bias
an inclination towards one person, group of people, thing or idea over another

BIDMAS
order of operations: brackets, indices (exponents), division and multiplication, addition and subtraction

bi-modal
a distribution with two clear peaks

binomial
an expression containing two terms

binomial product
a factorised expression that is the product of two binomial expressions such as $(a + b)(c + d)$

bivariate data
data that shows the relationship between two variables

bonus
an incentive or reward for good performance, which is given on top of a salary or wage

budget
a financial plan which details expected income and expenses for a given period of time

capacity
amount of a substance that a 3D object (container) can hold (mL, L)

Cartesian plane
number plane or region formed by a pair of horizontal and vertical axes that allows any point to be described by a pair of coordinates

categorical data
non-numerical data that can be organised into categories

census
a survey of an entire population

circumference
perimeter of a circle

class intervals
groupings of data into equal-sized classes

coefficient
the number multiplying any pronumerals in an algebraic term

commission
a payment which is a proportion of the total value of sales

complement
everything in the universal set that is not in the given set; for example, if $\xi = \{1, 2, 3, 4, 5\}$ and set A contains even numbers, $A = \{2, 4\}$, then the complement of A contains odd numbers, $A' = \{1, 3, 5\}$

complementary events
a pair of events, A and A′ (not A), where only one of the two events can take place and their probabilities sum to 1

composite object
an object which can be split into two or more simple objects

composite shape
a shape which is made up of more than one simple shape

conjugates
binomial products with the same terms but one different sign, such as $a + b$ and $a - b$

constant
a term without any pronumerals

constant of proportionality
if two quantities, x, and y are related so that $y = kx$, k is the constant of proportionality. k is equal to the rate of change of y with respect to x and the gradient of the graph of y vs. x

continuous data
numerical data that can be measured; for example, height of a plant

convenience sampling
when a sample that is easily accessible is selected; for example, selecting students from your class to complete a survey

coordinate points (coordinates)
pair of numbers that describe the position of a point on the Cartesian plane

cosine (cos)
in trigonometry, the cosine of an angle in a right-angled triangle equals the ratio of the length of the adjacent side to the length of the hypotenuse
$\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

cross-section
a 2D shape obtained when cutting through a 3D object

cube
a 3D object with six faces that are all identical squares

cumulative frequency
a running total of frequencies in a frequency table

cylinder
a 3D object with a uniform circular cross-section

dependent variable
a variable that depends on another variable (*see* independent variable) in a relationship of two variables. It is listed second in a table of values and in pairs of coordinates and shown on the vertical axis of a Cartesian plane

difference of two squares
rule for factorising a difference of two squares: $a^2 - b^2 = (a - b)(a + b)$

dilation
1 transformation that can enlarge or compress the graph of a function in the x- or y-direction
2 transformation that produces an enlargement or a reduction of the original figure

direct proportion
two quantities are in direct proportion if, as they increase or decrease, the ratio between the quantities stays the same

discrete data
numerical data that can be counted; for example, number of siblings

distributive law
states that the same result is obtained if you add the numbers inside a pair of brackets first, before multiplying by the common factor, or multiply each number inside the brackets by the number outside the brackets before adding: $a(b + c) = ab + ac$

edge
a line segment which joins two faces in a 3D object

enlargement
larger image produced after a figure has been dilated

equation
two expressions shown to be equivalent by an equals sign

error
the difference between the actual value and an estimated value of a quantity:
error = exact value – estimated value

event
outcome or group of outcomes in a sample space; for example, a single outcome is rolling a 2; a group of outcomes is rolling a 2 or a 4

expand
to convert an expression from factorised to expanded form

expanded form
algebraic expression written without brackets, as opposed to factor form; for example, $3(2x + 1)$ expands to $6x + 3$

expected number
number of favourable outcomes expected in a probability experiment:
expected number = theoretical probability × number of trials

experiment
repeatable procedure that has a clearly defined set of possible results

exponent
for a value expressed in exponent form, the exponent indicates the number of times the base is written as a repeated multiplication

exponent form
shorter form of writing a repeated multiplication, where a number is written with a base and an exponent; for example, 2^4 is written in exponent form

expression
quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations

faces
flat surfaces of a polyhedron, where each face is a polygon

factor tree
a tree diagram where composite numbers are broken down into factor pairs until prime numbers are found

factorise
to write an algebraic expression as the product of its factors; for example, $6x + 3$ factorises to $3(2x + 1)$, where the two factors are 3 and $2x + 1$

factorised form
algebraic expression written as the product of its factors, as opposed to expanded form

general form
the general form of a quadratic relationship is $y = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$

gradient
numerical measure of the slope of a graph
gradient $= m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

gradient–intercept form
general form of a linear equation, $y = mx + c$, where m is the gradient and c is the y-coordinate of the y-intercept of the corresponding linear graph

grouped data
numerical data that has been sorted into groups or class intervals

highest common factor (HCF)
greatest factor that is common to two or more given numbers or expressions

histogram
type of bar chart that can be used to display grouped continuous or discrete numerical data where the bounds of each class interval are located at the edges of the columns

horizontal translation
transformation where a shape or object is translated (moved) left or right in a straight line without turning or changing size

income
money received in exchange for work or investment

income tax
the tax which is paid on an individual's taxable income

independent variable
the quantity that changes independently in a relationship between two variables. It is listed first in a table of values and in pairs of coordinates and shown along the horizontal axis on the Cartesian plane

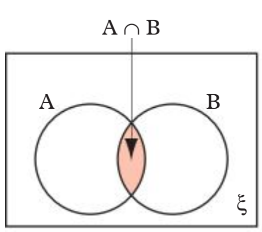
index
see exponent

inequality
a mathematical statement that compares the values of two expressions

inference
a conclusion based on facts and reasoning

interest rate
the proportion of a loan that is charged as interest to the borrower, or the proportion of a deposit that is given as interest to the investor

intersection
the intersection of two sets (A ∩ B) is the set of elements that appear in both A and B



inverse cosine ($\cos^{-1}$)
used to obtain an angle from its cosine value

inverse operation
operation that reverses the effect of a previous operation; for example, addition and subtraction are inverse operations

inverse sine ($\sin^{-1}$)
used to obtain an angle from its sine value

inverse tangent ($\tan^{-1}$)
used to obtain an angle from its tangent value

investment
a deposit of money, where interest is earned on the amount of money deposited

irrational number
a number that cannot be written as a fraction with both the denominator and numerator being integers; irrational numbers are non-terminating decimals that are not recurring

leading coefficient
the number multiplied by the highest power of a variable in an expression; for example, 3 is the leading coefficient in $3x^2 + 4x - 1$

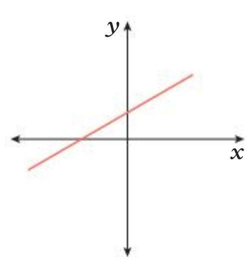
leave loading
an extra payment on top of annual leave that is usually calculated as 17.5% of normal pay for this period

like terms
terms that contain the same pronumerals in expanded form

line graph
graph that displays the relationship between two variables where points are plotted and joined by a straight line. A continuous variable must be displayed on the horizontal axis; for example, time

linear equation
equation that contains a pronumeral term or terms where the highest power is 1 with no two pronumerals multiplied together; for example, $2x + 3 = 6$

linear graph
straight line graph of a linear relationship



linear inequality
an inequality containing pronumerals raised only to the power of 1 with no two pronumerals multiplied together; for example, $2x + 3 > 1$

linear relationship
relationship between two variables that produces a linear graph

linear term
the term with a variable of power 1 in an expression; for example, $4x$ is the linear term in $3x^2 + 4x - 1$

loan
an agreement to borrow money, with interest paid on the amount of money borrowed

lower bound
1 abbreviation for lower boundary; the lowest value in a class interval
2 the smallest possible actual value of a quantity that has been measured

maximum turning point
a point at which a parabola changes direction and has its maximum y-value

mean
a measure of the centre of a data set (numerical average) calculated by adding all of the data values and then dividing by the number of values

measure of centre
number used to represent the central point in a data set. Measures of centre include the mean, median and mode

measure of spread
number used to indicate how spread out a data set is. The range is a measure of spread

median
a measure of the centre of a data set. For an ordered set of data, it is the middle number if there is an odd number of values and the average of the two middle numbers if there is an even number of values

Medicare levy
a 2% tax on taxable income which helps to fund the health care system

midpoint
halfway point or middle point of an interval or line segment

minimum turning point
a point at which a parabola changes direction and has its minimum y-value

mode
a measure of the centre of a data set determined by finding the value or values that occur most frequently in the data set

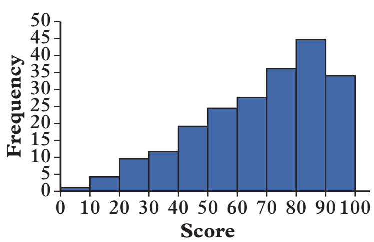
monic quadratic equation
a quadratic equation in which the leading coefficient is equal to 1

monic quadratic trinomial
a quadratic trinomial in which the leading coefficient is equal to 1

multiple line graph
graph used to display two or more sets of bivariate numerical data. The categorical variable is detailed in the legend

multivariate data
data that shows the relationship between three or more variables

negatively skewed
a distribution that is skewed away from the vertical axis with a tail to the left



net
2D plan that can be folded to form a three-dimensional object

net income
calculated by subtracting income tax and the Medicare levy from taxable income

nominal data
categorical data with categories that cannot be ordered; for example, eye colour

non-linear equation
an equation that contains at least one variable with a power other than 1 or a product of variables

non-linear relationship
relationship between two variables that produces a non-linear graph

Null Factor Law
states that if the product of two factors is 0, then one or both of the factors must equal 0

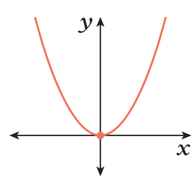
numerical data
data that can be counted or measured

opposite side
(trigonometry) the side opposite the reference angle in a right-angled triangle

ordinal data
categorical data that can be placed in categories in a specific order; for example, a rating system of 1–5

outlier
extreme piece of data that is much higher or lower than the rest of the data set

parabola
the graph of a quadratic relationship



Pay-As-You-Go (PAYG)
a tax system whereby the approximate income tax owed by an individual is withheld from their pay, to help ensure they fulfil their tax obligations

penalty rates
higher rates of pay that are given to employees for working particular hours or days, such as weekends, public holidays or overtime

percentage error
the relative error expressed as a percentage

perfect square
the square of an integer; for example, 1, 4, 9 and 25 are perfect squares

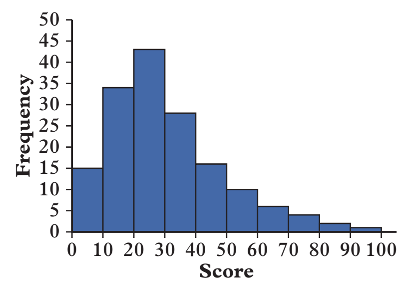
piece work
income which is paid for each item (or group of items) produced or action performed

plot
graphical representation of a relationship consisting of individual coordinate points

plus–minus symbol
± indicates both positive and negative values or both addition and subtraction

population
all potential pieces of data under consideration

positively skewed
a distribution that is skewed towards the vertical axis with a tail to the right



prime factorisation
the product of all the prime factors of an integer

principal
the original amount of money borrowed (for a loan) or deposited (for an investment)

prism
3D object with two ends that are identical polygons and joined by straight edges

pronumeral
letter or symbol that takes the place of a number

Pythagoras' theorem
states that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares on the two other sides, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the other side lengths

Pythagorean triple
any set of three positive whole numbers that satisfy Pythagoras' theorem

quadratic equation
an equation that contains a pronumeral term or terms with the highest power of 2; for example, $x^2 + 3x - 2 = 10$. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are constants

quadratic expression
an algebraic expression that contains a squared pronumeral, with no exponents greater than 2 in the expression

quadratic trinomial
expression of the form $ax^2 + bx + c$, where a, b and c are constants

quota sampling
dividing the population into separate categories (such as gender or age) and then taking a specified sample from each category that is chosen by the researcher

range
a measure of spread determined by finding the difference between the highest and lowest values in a data set

rate
comparison between two or more different quantities. It is a measure of how a quantity changes for each unit of another quantity

rate of change
a measure of how a quantity changes for each unit of change in another quantity

ratio
an expression of a relationship between two or more quantities, written with a colon between the quantities

rational number
a number that can be written as a fraction with both the numerator and denominator being integers. The set of rational numbers includes whole numbers, fractions, terminating decimals and recurring decimals

real number
a number that can be represented by a point on a number line. Real numbers consist of all rational and irrational numbers

reciprocal
the reciprocal of a number is the value which, when multiplied by that number, results in 1; for example, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

rectangular prism
prism with three pairs of identical faces (total of six faces)

reduction
smaller image produced after a figure has been dilated

reflection
transformation where a shape, object or graph is reflected (flipped) across a mirror line to produce its exact mirror image

relative error
the ratio between the absolute error and the actual value of a quantity

relative frequency
the number of times a favourable outcome occurs divided by the total number of outcomes

$$\text{relative frequency} = \frac{\text{number of occurrences}}{\text{total number of outcomes}}$$

right prism
prism with sides that make right angles with its base

royalty
a payment made to a person or company for the use of their assets

salary
a fixed income which is paid to an employee on a regular basis

sample
a selection of data that is part of a population

sample space
list of all the different outcomes possible for a probability experiment; for example, the sample space when rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$

scale
the ratio between the drawing length and the actual length in a scale drawing

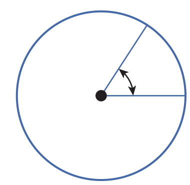
scale drawing
a drawing that accurately represents a real-life object but which is drawn to a different scale

scale factor
indicates how many times larger or smaller an image is after a figure has been dilated

$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

scientific notation
a value written in the form $a \times 10^m$, where $1 \le a < 10$ or $-10 < a \le -1$ and m is an integer

sector
portion of a circle formed by two radii and part of the circumference



set
a collection of distinct objects or numbers

significant figures
the number of digits in a value that contribute to its accuracy

similar
figures with the same shape but which do not have to be the same size

simple interest
interest which is calculated on the principal which has been borrowed or invested

sine (sin)
the sine of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the hypotenuse

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

sketch
a simple graph in which main features such as intercepts and turning points are labelled

skewed
a data set where data points are clustered to the left or right of the distribution

solution
value of the pronumeral that makes an equation a true statement

stacked bar chart
graph used to display frequency of two sets of categorical data. The categorical variable is detailed in the legend

standard form
see scientific notation

stratified sampling
dividing the population into separate categories (such as gender or age) and then taking a random sample from each category that is proportional to its size

subset
a set contained within another set

surd
irrational root of an integer that cannot be simplified to remove the root; for example, $\sqrt{2}$

surface area
area of the outer surface of an object, found by calculating the sum of the areas of each face of the object

survey
a list of questions designed to extract information about a population

symmetrical (symmetric)
a data set with a peak in the middle and data points are roughly evenly spread on either side of the peak

systematic sampling
selecting a sample at fixed intervals (such as every third person). The starting point should be random

tangent (tan)
in trigonometry, the tangent of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the adjacent side
$\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}}$

tax deductions
expenses that can be subtracted from an individual's total income to determine their taxable income, such as work-related expenses, gifts and donations

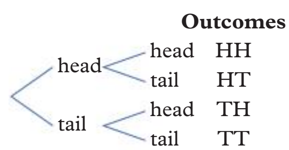
taxable income
an individual's total income minus any tax deductions, which is used to calculate the amount of income tax payable

term
a single pronumeral or number, or the product or quotient of numbers and pronumerals

theoretical probability
probability determined by mathematical reasoning, not experiments

transformations
general name for translations, reflections, rotations and dilations

tree diagram
a branched display of outcomes for a multi-step experiment; for example, the outcomes when two coins are tossed



triangular prism
3D object with five faces, two identical triangular faces whose respective vertices are joined by parallel lines

trigonometry
the study of relationships between angles and side lengths of a triangle

trinomial
an expression containing three terms

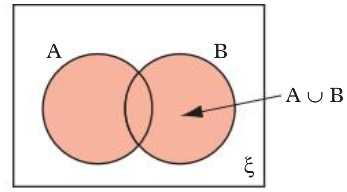
turning point
the point where a parabola changes direction

turning point form
general form of a quadratic equation $y = a(x - h)^2 + k$, where a is the dilation factor and the coordinates of the turning point are (h, k)

two-way table
display of outcomes of surveys or experiments with two different categories

ungrouped data
raw data that has not been placed into class intervals

union
the union of two sets $(A \cup B)$ is the set of elements in A or B or both



unitary method
a method where the value of a single unit of measure is calculated, which can then be used to determine the value of multiple units of measure

univariate data
a data set for one variable

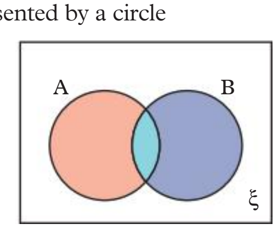
universal set
all possible outcomes in a chance experiment or the set of all elements, ξ, inside the rectangle of a Venn diagram

unknown
a quantity or measurement required in a problem that has one possible value only and is usually represented by a pronumeral

upper bound
1 abbreviation for upper boundary; the largest value in a class interval
2 the highest possible actual value of a quantity that has been measured

variable
quantity that can have different values and can be represented with a word, symbol or pronumeral

Venn diagram
a display of the relationship between different sets of data. The universal set, ξ, is represented by a rectangle and each set of data is represented by a circle



vertex
see turning point

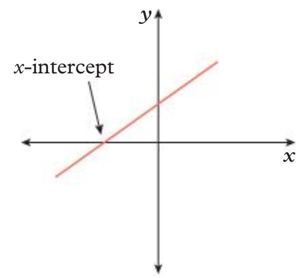
vertical translation
transformation in which a shape or object is translated (moved) up or down in a straight line without turning or changing size

volume
amount of space that a three-dimensional (3D) object occupies (cm^3, m^3)

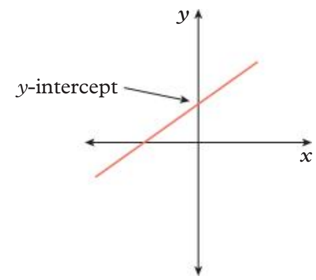
volume scale factor
the scale factor between the volumes of two similar objects

wage
a fixed rate of income an employee receives for working a specified time period; for example, $25/hour

***x*-intercept**
point where a line crosses the x-axis of a Cartesian plane



***y*-intercept**
point where a line crosses the y-axis of a Cartesian plane



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