

• INCLUDES STAGES 5.1 & 5.2

STAGE 5.3

MATHSCAPE

Working Mathematically

9

EXTENSION

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Preface

Mathscape 9 Extension is a comprehensive teaching and learning resource that has been written to address the new Stage 5.1/5.2/5.3 Mathematics syllabus in NSW. Our aim was to write a book that would allow more able students to grow in confidence, to improve their understanding of Mathematics and to develop a genuine appreciation of its inherent beauty. Teachers who wish to inspire their students will find this an exciting, yet very practical resource. The text encourages a deeper exploration of mathematical ideas through substantial, well-graded exercises that consolidate students' knowledge, understanding and skills. It also provides opportunities for students to explore the history of Mathematics and to address many practical applications in contexts that are both familiar and relevant.

From a teaching perspective, we sought to produce a book that would adhere as strictly as possible to both the content and spirit of the new syllabus. Together with *Mathscape 10 Extension*, this book allows teachers to confidently teach the Stage 5.1/5.2/5.3 courses knowing that they are covering all of the mandatory outcomes.

Content from Stage 4 has been included in each chapter, where appropriate. This will allow teachers to diagnose significant misconceptions and identify any content gaps. For those students who have achieved the relevant Stage 4 outcomes, this material could be used as a review to introduce the Stage 5.1/5.2/5.3 topics, or to revise important concepts when they occur. However, for those students who have not achieved these outcomes by the start of Year 9, this material will be new work. All content is clearly listed as either Stage 4 or Stage 5.1/5.2/5.3 in the contents section at the front of the book. A detailed syllabus correlation grid has been provided for teachers on the *Mathscape 9/9 Extension School CD-ROM*.

Mathscape 9 Extension has embedded cross-curriculum content, which will support students in achieving the broad learning outcomes defined by the Board of Studies. The content also addresses the important key competencies of the Curriculum Framework, which requires students to collect, analyse and organise information; to communicate mathematical ideas; to plan and organise activities; to work with others in groups; to use mathematical ideas and techniques; to solve problems; and to use technology.

A feature of each chapter which teachers will find both challenging and interesting for their students is the 'Focus on working mathematically' section. Although the processes of working mathematically are embedded throughout the book, these activities are specifically designed to provoke curiosity and deepen mathematical insight. Most begin with a motivating real-life context, such as television advertising, or the gradient of a ski run, but on occasion they begin with a purely mathematical question. (These activities can also be used for assessment purposes.)

In our view, there are many legitimate, time-proven ways to teach Mathematics successfully. However, if students are to develop a deep appreciation of the subject, they will need more than traditional methods. We believe that all students should be given the opportunity to appreciate Mathematics as an essential and relevant part of life. They need to be given the opportunity to begin a Mathematical exploration from a real-life context that is meaningful to them. To show interest and enjoyment in enquiry and the pursuit of mathematical knowledge, students need activities where they can work with others and listen to their arguments, as well as work individually. To demonstrate confidence in applying their mathematical knowledge and skills to the solution of everyday problems, they will need experience of this in the classroom. If they are to learn to persevere with difficult and challenging problems, they will need to experience these sorts of problems as well. Finally, to recognise that mathematics has been developed in many cultures in response to human needs, students will need experiences of what other cultures have achieved mathematically.

We have tried to address these values and attitudes in this series of books. Our best wishes to all teachers and students who are part of this great endeavour.

*Clive Meyers
Lloyd Dawe
Graham Barnsley
Lindsay Grimison*

How to use this book

Mathscape 9 Extension is a practical resource that can be used by teachers to supplement their teaching program. The exercises in this book and the companion text (*Mathscape 10 Extension*) provide a complete and thorough coverage of all content and skills in the Stage 5.1/5.2/5.3 course. The great number and variety of questions allow for the effective teaching of more able students. Each chapter contains:

- a set of chapter outcomes directed to the student
- all relevant theory and explanations, with important definitions and formulae boxed and coloured
- step-by-step instructions for standard questions
- a large number of fully worked examples preceding each exercise
- extensive, thorough and well-graded exercises that cover each concept in detail
- chapter-related, problem-solving activities called ‘Try this’ integrated throughout
- a language skills section linked to the *Macquarie Learners Dictionary*
- novel learning activities focusing on the process of working mathematically
- a thorough chapter review.

Explanations and examples

The content and skills required to complete each exercise have been introduced in a manner and at a level that is appropriate to the students in this course. Important definitions and formulae have been boxed and coloured for easy reference. For those techniques that require a number of steps, the steps have been listed in point form, boxed and coloured. Each exercise is preceded by several fully worked examples. This should enable the average student to independently complete the majority of relevant exercises if necessary.

The exercises

The exercises have been carefully graded into three distinct sections:

- **Introduction.** The questions in this section are designed to introduce students to the most basic concepts and skills associated with the outcome(s) being covered in the exercise. Students need to have mastered these ideas before attempting the questions in the next section.
- **Consolidation.** This is a major part of the exercise. It allows students to consolidate their understanding of the basic ideas and apply them in a variety of situations. Students may need to use content learned or skills acquired in previous exercises or topics to answer some of these questions. The average student should be able to complete most of the questions in this section, although the last few questions may be a little more difficult.
- **Further applications.** Some questions presented in this section will be accessible to the average student; however, the majority of questions are difficult. They might require a reverse procedure, the use of algebra, more sophisticated techniques, a proof, or simply time-consuming research. The questions can be open-ended, requiring an answer with a justification. They may also involve extension or off-syllabus material. In some questions, alternative techniques and methods of solution other than the standard method(s) may be introduced, which may confuse some students.

Teachers need to be selective in the questions they choose for their students. Some students may not need to complete all of the questions in the Introduction or Consolidations sections of each exercise, while only the most able students should usually be expected to attempt the questions in the Further applications section. Those questions not completed in class might be set as homework at the teacher’s discretion. It is not intended that any student would attempt to answer every possible question in each exercise.

Focus on working mathematically

The Working Mathematically strand of the syllabus requires a deeper understanding of Mathematics than do the other strands. As such, it will be the most challenging strand for students to engage with and for teachers to assess. The Working Mathematically outcomes listed in the syllabus have been carefully integrated into each chapter of the book; however, we also decided to include learning activities in each chapter that will

enable teachers to focus sharply on the processes of working mathematically. Each activity begins with a real-life context and the Mathematics emerges naturally. Teachers are advised to work through them before using them in class. Answers have not been provided, but notes for teachers have been included on the *Mathscape 9/9 Extension School CD-ROM*, with suggested weblinks. Teachers may wish to select and use the Learning activities in ‘Focus on working mathematically’ for purposes of assessment. This too is encouraged. The Extension activities will test the brightest students. Suggestions are also provided to assess the outcomes regarding Communication and Reflection.

Problem solving

Each chapter contains a number of small, chapter-related, problem-solving activities called ‘Try this’. They may be of some historical significance, or require an area outside the classroom, or require students to conduct research, or involve the use of algebra, while others relate the chapter content to real-life context. Teachers are advised to work through these exercises before using them in class.

Technology

The use of technology is a clear emphasis in the new syllabus. Innovative technology for supporting the growth of understanding of mathematical ideas is provided on the *Mathscape 9/9 Extension School CD-ROM*, which is fully networkable and comes free-of-charge to schools adopting *Mathscape 9 Extension* for student use.

Key features of the CD-ROM include:

- spreadsheet activities
- dynamic geometry
- animations
- executables
- student worksheets
- weblinks for ‘Focus on working mathematically’.

Language

The consistent use of correct mathematical terms, symbols and conventions is emphasised strongly in this book, while being mindful of the students’ average reading age. Students will only learn to use and spell correct mathematical terms if they are required to use them frequently in appropriate contexts. A language section has also been included at the end of each chapter titled ‘Language link with Macquarie’, where students can demonstrate their understanding of important mathematical terms. This might, for example, include explaining the difference between the mathematical meaning and the everyday meaning of a word. Most chapters include a large number of worded problems. Students are challenged to read and interpret the problem, translate it into mathematical language and symbols, solve the problem, then give the answer in an appropriate context.

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Rational numbers



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ evaluate numerical expressions using a calculator
- ⦿ estimate the result of a calculation
- ⦿ state the number of significant figures in a number
- ⦿ round off a number correct to a given number of significant figures
- ⦿ determine the effect of rounding during calculations on the accuracy of the results
- ⦿ convert fractions to recurring decimals
- ⦿ convert recurring decimals to fractions
- ⦿ express a rate in its simplest form
- ⦿ convert rates from one set of units to another
- ⦿ solve problems involving rates.

1.1 Significant figures

No quantity, such as length, mass or time, can be measured exactly. For a measurement to be of use, we need to know how accurate it is. That is, we must be confident that each digit in the measurement is significant.

A significant figure is a number that is correct within some stated degree of accuracy.

The rules for significant figures are:

- ⊙ All non-zero digits are significant.
- ⊙ Zeros between non-zero digits are significant.
- ⊙ Zeros at the end of a decimal are significant.
- ⊙ Zeros before the first non-zero digit in a decimal are not significant.
- ⊙ Zeros after the last non-zero digit in a whole number may or may not be significant.

When rounding off correct to a specified number of significant figures, choose the number that is closest in value to the given number and which also contains the required number of significant figures.

Example 1

State the number of significant figures in each number.

a 4.009

b 137.20

c 0.001 64

d 5000

Solutions

- a** In 4.009, the two non-zero digits (i.e. 4 and 9) are significant and the two zeros between these digits are significant. $\therefore$ The number has 4 significant figures.
- b** In 137.20, the four non-zero digits (i.e. 1, 3, 7 and 2) are significant and the zero at the end of the decimal is significant. $\therefore$ The number has 5 significant figures.
- c** In 0.001 64, the three non-zero digits (i.e. 1, 6 and 4) are significant; however, the zeros at the beginning of the decimal are not significant. $\therefore$ The number has 3 significant figures.
- d** In 5000, the non-zero digit (i.e. 5) is significant. Either some, all or none of the final zeros could possibly be significant. This would need to be determined from the context in which the number occurs. If we knew that the number had been rounded off correct to:
- i** 1 significant figure, then only the 5 would be significant
 - ii** 2 significant figures, then only the 5 and the first zero would be significant
 - iii** 3 significant figures, then only the 5 and the first two zeros would be significant
 - iv** 4 significant figures, then all of the digits would be significant.

**Example 2**

Round off 47.503 correct to:

- a** 4 significant figures **b** 3 significant figures
c 2 significant figures **d** 1 significant figure

Solutions

- a** $47.503 = 47.50$ (4 significant figures) **b** $47.503 = 47.5$ (3 significant figures)
c $47.503 = 48$ (2 significant figures) **d** $47.503 = 50$ (1 significant figure)

**Example 3**

Round off 39.99 correct to:

- a** 3 significant figures **b** 2 significant figures **c** 1 significant figure

Solutions

- a** $39.99 = 40.0$ (3 significant figures)
b $39.99 = 40$ (2 significant figures; both the 4 and the 0 are significant figures)
c $39.99 = 40$ (1 significant figure; only the 4 is significant)

Exercise**1.1****1** State the number of significant figures in each of the following.

- a** 45 **b** 7281 **c** 859 **d** 132 494
e 607 **f** 3012 **g** 4001 **h** 20 809

2 State the number of significant figures in each decimal.

- a** 5.28 **b** 7.152 **c** 38.5 **d** 254.883
e 0.4 **f** 0.005 **g** 0.0371 **h** 0.003 469
i 5.062 **j** 13.007 **k** 58.0208 **l** 0.001 09
m 9.30 **n** 0.10 **o** 1.4700 **p** 0.004 080
q 3.030 **r** 32.0040 **s** 409.010 00 **t** 0.010 203 00

Consolidation**3** Round off each of the following correct to 1 significant figure.

- a** 83 **b** 27 **c** 65 **d** 94
e 136 **f** 415 **g** 250 **h** 3810
i 9450 **j** 26 449 **k** 539 499 **l** 850 000

4 Round off each of these numbers correct to 2 significant figures.

- a** 128 **b** 171 **c** 234 **d** 675
e 1459 **f** 4026 **g** 8350 **h** 12 042
i 45 718 **j** 76 285 **k** 285 195 **l** 644 003

- 5 Round off each of the following decimals correct to the number of significant figures indicated in the brackets.

a	3.67	[1]	b	0.484	[1]	c	0.0731	[2]	d	6.2085	[4]
e	11.784	[2]	f	0.3	[2]	g	25.156	[3]	h	49.066 28	[5]
i	91.045	[3]	j	144.387	[2]	k	7.3855	[4]	l	10.9367	[2]
m	2018.68	[3]	n	3693.21	[2]	o	4002.142	[5]	p	9187.549	[6]

- 6 Round off the following correct to:

i	1 significant figure	ii	2 significant figures	iii	3 significant figures		
a	99.35	b	194.97	c	998.763	d	499.861

Further applications

- 7 Write down a possible number that is approximately equal to:

a	130, correct to 2 significant figures	b	2.47, correct to 3 significant figures

1.2 The calculator

As a wide variety of calculators is available, there are differences in the way they operate. The examples here have been worked with the use of a direct logic calculator. That is, the calculations are performed in the logical order in which they appear. For example, to evaluate $\sqrt{9}$ on a direct logic calculator, we press the square root key $\sqrt{\square}$ followed by the 9, then press $=$. For models that do not use direct logic, we enter the 9, then press the square root key. You will need to familiarise yourself with how your calculator works.

Example 1

Evaluate each of the following.

a	$\frac{6}{7} + 5\frac{2}{3}$	b	$-78 - 96$	c	15.98^2	d	$\sqrt{69.4}$
e	$\sqrt[3]{41}$	f	3.52^4	g	$\sqrt[5]{117.3}$	h	$\frac{1}{0.274}$

Solutions

Calculator steps:

a	$6 \left[\frac{a}{b} \right] 7 \left[+ \right] 5 \left[\frac{a}{b} \right] 2 \left[\frac{a}{b} \right] 3 \left[= \right]$
b	$\left[+/\!- \right] 78 \left[- \right] 96 \left[= \right]$
c	$15.98 \left[x^2 \right] \left[= \right]$
d	$\left[\sqrt{\square} \right] 69.4 \left[= \right]$
e	$\left[\sqrt[3]{\square} \right] 41 \left[= \right]$
f	$3.52 \left[x^y \right] 4 \left[= \right]$
g	$5 \left[\sqrt[x]{\square} \right] 117.3 \left[= \right]$
h	$0.274 \left[x^{-1} \right] \left[= \right]$

Answer:

$6\frac{11}{21}$
-174
255.3604
8.330 666 24
3.448 217 24
153.522 012 2
2.593 340 858
3.649 635 036

**Example 2**

Evaluate each of these, correct to 2 decimal places, using the grouping symbols keys $($ and $)$.

a $\frac{86.9 + 213.7}{5.6 \times 8.3}$

b $\sqrt{342.5 - 114.8}$

Solutions**Calculator steps:****Answer**

a $86.9 \boxed{+} 213.7 \boxed{=} \boxed{\div} \boxed{(} 5.6 \boxed{\times} 8.3 \boxed{)} \boxed{=}$

6.47

b $\boxed{\sqrt{}} \boxed{(} 342.5 \boxed{-} 114.8 \boxed{)} \boxed{=}$

15.09

**Example 3**

Use the memory function on the calculator to evaluate $\frac{72.6 + 153.9}{12.5^2 \times 0.98}$, correct to 3 decimal places.

Solution

- i Evaluate the denominator first and store the answer in the memory.

$12.5 \boxed{x^2} \boxed{\times} 0.98 \boxed{=} \boxed{\text{Min}}$

- ii Evaluate the numerator, then divide the answer by the number stored in the memory.

$72.6 \boxed{+} 153.9 \boxed{=} \boxed{\div} \boxed{\text{MR}} \boxed{=}$

Answer: 1.479 (3 decimal places)

Exercise**1.2**

- a Evaluate $\sqrt{72} \times 12.43$ correct to 3 decimal places, without rounding off during the calculation.

b Round off $\sqrt{72}$ to the nearest integer, then multiply by 12.43.

c Round off $\sqrt{72}$ to 1, 2 and 3 decimal places then multiply by 12.43. What do you notice?

d What effect does rounding off too early have on the accuracy of an answer?
- Evaluate each of these using the fraction key $\boxed{a \frac{b}{c}}$, then give the answers as decimals, correct to 2 decimal places.

a $\frac{3}{8} + \frac{1}{11}$ b $8\frac{7}{9} - 3\frac{4}{5}$ c $2\frac{5}{6} \times 4\frac{2}{7}$
- Evaluate each of the following using the sign change key $\boxed{+/-}$.

a $-98 - 156$ b $-49 + 32 - 77$ c $-156 \div -12$
- Evaluate each of these correct to 4 significant figures using the square key $\boxed{x^2}$.

a 7.8^2 b $(-12.7)^2$ c $(4\frac{7}{8})^2$

- 5 Evaluate each of these correct to the nearest tenth using the square root key $\sqrt{\square}$ and cube root key $\sqrt[3]{\square}$.

a $\sqrt{23}$

b $85 + \sqrt{72.6}$

c $\sqrt{90} \times 16.45$

d $\sqrt[3]{70}$

e $\sqrt[3]{110.4} \div 2.96$

f $\sqrt[3]{36.7} + \sqrt{152.6}$

- 6 Evaluate each of the following correct to 1 decimal place using the power key x^y .

a 6.5^3

b 3.72^4

c 4.08^5

d $(2\frac{3}{11})^6$

e $1.85^7 \times 4.3$

f $8.9^4 - 3.1^5$

- 7 Evaluate each of these correct to the nearest hundredth using the root key $\sqrt[x]{\square}$.

a $\sqrt[4]{11}$

b $\sqrt[5]{68.2}$

c $\sqrt[7]{212.9}$

d $\sqrt[5]{96} \times 12.5$

e $3 - \sqrt[6]{2.4}$

f $\sqrt[4]{7\frac{1}{5}}$

- 8 Evaluate each of the following correct to 3 significant figures using the reciprocal key x^{-1} or $\frac{1}{x}$.

a $\frac{1}{7}$

b $\frac{1}{0.245}$

c $\frac{1}{0.065^2}$

d $\frac{1}{\sqrt{3}}$

e $\frac{1}{\sqrt[3]{51.4}}$

f $\frac{1}{1.98^4}$

- 9 Evaluate each of these correct to 2 decimal places using the pi key π .

a $\pi + 16.82$

b 7π

c $\frac{9\pi}{2}$

d π^2

e $\frac{1}{\pi}$

f $5\sqrt{\pi}$

- 10 Evaluate each of the following correct to the nearest tenth using the grouping symbols keys $(\square)$ and $\square$ where necessary.

a $\frac{73 + 115}{14}$

b $\frac{172}{8.5 \times 3.1}$

c $\frac{19.3 \times 54.7}{6.4 + 9.8}$

d $\frac{12 \times 11 \times 10}{7 \times 8 \times 9}$

e $\frac{9.4^3}{5.1 \times 7.25}$

f $\frac{\sqrt{135} + 18.7}{11 - \pi}$

Consolidation

- 11 Find the value of each expression, correct to 2 decimal places.

a $10.65^2 \times 8.3$

b $\frac{83}{2.6^4}$

c $\sqrt{\frac{101}{7}}$

d $\sqrt{42 \times 7.5}$

e $\frac{28}{\sqrt{5}}$

f $3^4 - 4.1^3$

g $\sqrt[3]{74.9 + 87.2}$

h $\sqrt[5]{7.9^3}$

i $\frac{16.8^6}{13.9^4}$

j $\frac{\sqrt{10} + \sqrt{20}}{\sqrt{15}}$

k $\sqrt[3]{25 + 50.3 - 19.6}$

l $\frac{1}{\sqrt{0.06} \times 7}$

m $\frac{30}{\sqrt{2} + \sqrt{3}}$

n $\frac{-250}{5^2 - 8^2}$

o $\sqrt[4]{82.6 \times 16.1}$

p $\frac{24^2 + 23^2}{16^2 - 15^2}$

q $\frac{1}{2^5 - \sqrt[5]{2}}$

r $\frac{\sqrt[3]{116.7 + 99.8}}{2.1^2}$

s $(1.7 + 1.16)^6$

t $\sqrt{18^2 + 7^3}$

u $5^4 + \sqrt[3]{8\frac{1}{8}}$

v $\frac{10 + \sqrt{3}}{10 - \sqrt{3}}$

w $\frac{1}{\sqrt[3]{0.1} + 0.2^2}$

x $\frac{(-8.4)^3}{-6.3 + 11.4}$

12 Evaluate, correct to the nearest tenth.

a $\sqrt{\pi^3 \times 14}$

b $\frac{1}{3 - \sqrt{5}}$

c $(\sqrt{13.6})^7$

d $\frac{0.92}{2.3} + \frac{18.9}{5.14}$

e $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}}$

f $\frac{-40.6 + 15.35}{\sqrt{6.2 \times 7.7}}$

g $\frac{\sqrt{17} + \sqrt{18}}{\sqrt[3]{17} + \sqrt[3]{18}}$

h $4.6(19.83 - 7.12)^3$

i $\frac{1}{\sqrt{0.86 - 0.29^2}}$

j $\left(9\frac{2}{3}\right)^3 \div \left(1\frac{1}{2}\right)^7$

k $\frac{100}{\sqrt{10} + \sqrt[3]{10} + \sqrt[4]{10}}$

l $\sqrt{\frac{124.37 - 19.66}{9.7 + 11.75}}$

Further applications

13 Use the memory function on the calculator to evaluate each of these, correct to 1 decimal place.

a $\sqrt[3]{\frac{7.6^2 + 39}{1.4^3 \times 0.995}}$

b $\left(\frac{11.6}{2.3}\right)^5 \div \left(\frac{9.47}{1.02}\right)^3$

c $\frac{8.1}{1.9 + 2.64} + \frac{13.4}{7 \times 0.16^2}$

d $\frac{3.9 \times 15.6}{10.58 - 1.33^3} + \sqrt{\frac{21.4}{6.09}}$

e $\frac{57.5^3 - 13.6^4}{\sqrt{15 \times 98.2}} \times \frac{1}{12.4^2}$

f $\sqrt[4]{\frac{17.5 \times 5.3}{6.7}} - \sqrt[3]{\frac{1}{0.075^2}}$

1.3 Estimation

Calculators do not make arithmetic errors. But sometimes we get incorrect answers when we use a calculator. This is because we may have:

- left out a decimal point
- pressed the wrong key by mistake
- not pressed the equals key at the right time
- not understood the question
- set the calculator in the wrong mode
- not pressed the second function key.

By estimating the answer before using a calculator, we can work out whether the calculator answer is reasonable. An **estimate** is more than a guess. It is an approximate answer that is worked out logically. It does not have to be very close to the correct answer but it should be of the same **order of magnitude**. That is, if the estimate is in the tens, the correct answer should not be in the hundreds or the thousands.

For example, before evaluating 19.855×4.84 with a calculator, we could estimate that the answer would be close to 20×5 , that is, 100. If the calculator gives the answer as 9609.82, we might have made an error when entering the data. In fact, a decimal point was omitted, since the correct answer is 96.0982. It is also possible, of course, that our estimate is incorrect.

NOTE: Many different estimates can be given to calculations depending on the way that each individual number is rounded off.

Example

Estimate the answer to each of these calculations.

a 386×19

b $154.5 \div 11.2$

c 17.74×0.493

d $\frac{41.68 \times 21.19}{6.904}$

Solutions

a 386×19
 $\div 400 \times 20$
 $= 8000$

b $154.5 \div 11.2$
 $\div 150 \div 10$
 $= 15$

c 17.74×0.493
 $\div 18 \times \frac{1}{2}$
 $= 9$

d $\frac{41.68 \times 21.19}{6.904}$
 $\div \frac{42 \times 20}{7}$
 $= 6 \times 20$
 $= 120$

Exercise

1.3

1 Round off each number correct to 1 significant figure and hence estimate the value of:

a 48×33

b 385×11

c 69×114

d $19\,952 \times 9$

e $223 \div 52$

f $642 \div 22$

g $38\,840 \div 375$

h $8445 \div 23$

i 54×186

j $2751 \div 63$

k 297×42

l $96\,959 \div 4367$

2 Estimate the answer, as an integer, to each of the following calculations.

a $8.7 + 19.4 + 12.1$

b $96.5 - 27.3 + 15.046$

c $24.2 \times 3.75 \times 5.3$

d 24.8×3.88

e $32.42 \div 7.93$

f $126.7 \div 9.82$

g $5.34 \times 11.92 \times 8.15$

h $53.5 \div 6.12 \times 8.046$

i $189.4 - 47.75 - 283.19$

■ Consolidation

3 Estimate the answer to each of these.

a $(14.797 + 32.88) \div 8.1$

b $(348.5 - 102.7) \times 4.193$

c $495.13 \div (9.96 \times 10.02)$

4 Find the approximate value of:

a $18.8 + 6.84 \times 3.125$

b $183.4 - 31.2 \div 5.17$

c $20.4 \div 3.95 + 19.87 \times 5.02$

d $2117 - 12.13 \times 8.4 \times 4.96$

5 Estimate:

a 16.45×0.482

b 43.65×0.252

c 13.82×1.55

d 8.094×1.26

6 Estimate the answer for each of these, giving the answer as an integer.

a $\sqrt{23.67}$

b $\sqrt{84.77}$

c $\sqrt[3]{29.1}$

d $\sqrt[3]{119.8}$

7 Estimate the value of each calculation.

a $\frac{4.76 \times 9.27}{2.89}$

b $\frac{73.4 \times 15.2}{4.57}$

c $\frac{50.6 + 73.1}{15.8 - 4.593}$

d $\frac{\sqrt{106.2}}{7.046^2}$

8 The crowds at each day of a test cricket match played at the SCG between Australia and England were as follows:

• Day 1—34 356

• Day 2—29 875

• Day 3—26 234

• Day 4—18 558

• Day 5—9063

Round off each day's crowd to the nearest 5000 spectators and hence estimate the total match attendance.

9 A group of 4 people having dinner in a restaurant ordered the following meals from the menu:

• Tamara: spaghetti bolognese \$18.75

• Luke: steak Diane \$21.75

• Amanda: fettuccine boscaiola \$19.20

• Barry: veal parmigiana \$20.60

They also ordered 2 bottles of wine at \$11.45 each and 4 coffees at \$3.25 each.

a Estimate the total cost of the meal, allowing for a small tip.

b Approximately how much would each person expect to pay if they shared the bill equally?

10 Therese decided to re-carpet her lounge room using carpet squares of side length 50 cm. The lounge room is rectangular in shape and measures 5.2 m by 6.8 m.

a Estimate the area of the room in square metres.

b How many carpet squares are needed to cover an area of 1 m^2 ?

- c** Estimate the number of carpet squares that are needed to cover the entire lounge room floor.
- d** If the carpet squares are sold in packs of 40 at \$385 per pack, estimate the total cost of the re-carpeting.
- e** Should re-carpeting decisions be based on estimates or accurate measurements? Explain.

■ Further applications

- 11 a** Evaluate $\sqrt{4}$ and $\sqrt{9}$. Hence, find estimates for $\sqrt{5}$ and $\sqrt{7}$, correct to 1 decimal place.
- b** Evaluate $\sqrt{100}$ and $\sqrt{121}$. Hence, find estimates for $\sqrt{110}$, $\sqrt{105}$ and $\sqrt{115}$, correct to 1 decimal place.
- 12** Consider the statement $2^n = 12$.
- a** Show by substitution that:
- i** $3 < n < 4$ **ii** $3.5 < n < 3.6$
- iii** $3.58 < n < 3.59$ **iv** $3.584 < n < 3.585$
- b** Hence, estimate the value of n , correct to 3 decimal places.
- 13** By substituting and then refining estimates, find the approximate value of n in each of the following, correct to 3 decimal places.
- a** $2^n = 20$ **b** $3^n = 36$ **c** $5^n = 100$

TRY THIS Fermi problem

A Fermi problem is a problem solved by making a good estimation.
Try these problems:

- 1 How many telephone calls are made in one day in Australia?
- 2 What would be the total value of all the books in every library in Australia?

1.4 Recurring decimals

A **recurring decimal** has an infinite number of decimal places, with one or more of the digits repeating themselves indefinitely. Recurring decimals are written with a dot above the first and last digits in the repeating sequence.

For example: $0.444\ 444\ \dots = 0.\dot{4}$

$$0.329\,329\,\dots = 0.\dot{3}2\dot{9}$$
$$0.616\ 161\ \dots = 0.\dot{6}\dot{1}$$
$$1.288\ 888\ \dots = 1.2\dot{8}$$

A **rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are integers (whole numbers) and $b \neq 0$. Every recurring decimal can be expressed as a fraction, so recurring decimals are rational numbers.

To convert a fraction to a recurring decimal

- ⊙ divide the numerator by the denominator.

To convert a recurring decimal to a fraction:

- ⊙ let the decimal be x
- ⊙ multiply both sides by the smallest power of 10 so that the recurring part of the decimal becomes a whole number
- ⊙ subtract the first equation from the second
- ⊙ solve the resulting equation.

Example 1

Convert each of these fractions to a recurring decimal.

a $\frac{5}{9}$

b $\frac{7}{11}$

c $\frac{1}{12}$

Solutions

a
$$\begin{array}{r} 0.5\ 5\ 5\ldots \\ 9 \overline{)5.0^5 0^5 0^5} \\ \hline \end{array}$$

 $\therefore \frac{5}{9} = 0.\dot{5}$

b
$$\begin{array}{r} 0.6\ 3\ 6\ 3\ldots \\ 11 \overline{)7.0^4 0^7 0^4 0^7} \\ \hline \end{array}$$

 $\therefore \frac{7}{11} = 0.6\dot{3}$

c
$$\begin{array}{r} 0.08\ 3\ 3\ldots \\ 12 \overline{)1.00^4 0^4 0^4} \\ \hline \end{array}$$

 $\therefore \frac{1}{12} = 0.08\dot{3}$

Example 2

Convert each recurring decimal to a fraction in simplest form.

a $0.\dot{8}$

b $0.\dot{1}\dot{5}$

c $0.2\dot{4}$

Solutions

a Let $x = 0.\dot{8} \dots$ ①
 $\therefore 10x = 8.\dot{8} \dots$ ②
 Subtract ① from ②
 $\therefore 9x = 8$
 $\therefore x = \frac{8}{9}$

b Let $x = 0.\dot{1}\dot{5} \dots$ ①
 $\therefore 100x = 15.\dot{1}\dot{5} \dots$ ②
 Subtract ① from ②
 $\therefore 99x = 15$
 $\therefore x = \frac{15}{99}$
 $= \frac{5}{33}$

c Let $x = 0.2\dot{4} \dots$ ①
 $\therefore 10x = 2.\dot{4} \dots$ ②
 $\therefore 100x = 24.\dot{4} \dots$ ③
 Subtract ② from ③
 $\therefore 90x = 22$
 $\therefore x = \frac{22}{90}$
 $= \frac{11}{45}$

Exercise 1.4

This exercise should be completed without the use of a calculator, unless otherwise indicated.

1 Write each of these as a recurring decimal.

a $0.222 \dots$

b $0.777 \dots$

c $0.6444 \dots$

d $0.3555 \dots$

e 0.272 727 ...	f 0.919 191 ...	g 0.484 848 ...	h 0.030 303 ...
i 0.146 146 ...	j 0.029 029 ...	k 0.152 152 ...	l 0.698 698 ...
m 1.666 ...	n 3.818 181 ...	o 8.274 274 ...	p 13.955 555 ...

■ Consolidation

2 Use short division to convert each of these fractions to a recurring decimal.

a $\frac{1}{3}$	b $\frac{1}{9}$	c $\frac{2}{3}$	d $\frac{4}{9}$
e $\frac{1}{11}$	f $\frac{3}{11}$	g $\frac{1}{6}$	h $\frac{2}{15}$
i $\frac{5}{12}$	j $\frac{7}{22}$	k $\frac{5}{6}$	l $\frac{11}{12}$

3 **a** Convert $1\frac{2}{3}$ to a decimal using a calculator.

b Does the calculator round off the answer at the last digit?

4 Express each of the following as a recurring decimal.

a $\frac{1}{7}$	b $\frac{5}{7}$	c $\frac{1}{13}$	d $\frac{4}{13}$

5 **a** Write down the recurring decimal for $\frac{1}{9}$.

b Hence, write down recurring decimals for $\frac{2}{9}$, $\frac{5}{9}$, $\frac{7}{9}$ and $\frac{8}{9}$.

c What meaning should be given to $0.\dot{9}$? Why?

6 Convert each of these recurring decimals to a fraction or mixed numeral, in simplest form.

a $0.\dot{2}$	b $0.\dot{7}$	c $0.\dot{3}$	d $0.\dot{6}$
e $0.\dot{1}\dot{9}$	f $0.\dot{3}\dot{5}$	g $0.\dot{2}\dot{7}$	h $0.\dot{7}\dot{5}$
i $0.1\dot{5}$	j $0.4\dot{8}$	k $0.7\dot{3}$	l $0.9\dot{4}$
m $2.\dot{1}$	n $1.\dot{6}\dot{0}$	o $7.8\dot{3}$	p $3.41\dot{6}$

■ Further applications

7 **a** Write down the recurring decimal for $\frac{1}{3}$.

b Hence, express $\frac{1}{30}$ and $\frac{1}{300}$ as recurring decimals.

8 **a** Express $\frac{11}{30}$ and $\frac{1}{6}$ as recurring decimals.

b Show that $\frac{1}{6} + \frac{1}{5} = \frac{11}{30}$ by adding fractions.

c Show that $\frac{1}{6} + \frac{1}{5} = \frac{11}{30}$ by adding decimals.



9 a Express $\frac{2}{3}$ as a recurring decimal.

b Use the fact that $\frac{1}{15} = \frac{2}{30} = \frac{2}{3} \times \frac{1}{10}$ to express $\frac{1}{15}$ as a recurring decimal.

1.5 Rates

A **rate** is a comparison of two *unlike quantities*. This is different from a ratio, in that a ratio is a comparison of two or more *like quantities*. In particular, a rate is a measure of how one quantity is *changing* with respect to another. In a ratio, units are not written, whereas in a rate, the units must be written if the rate is to have any meaning.

A rate is a comparison of two unlike quantities.

Equivalent rates can be formed by changing the units in either or both quantities. For example, a rate of 5 cm/s is equivalent to 50 mm/s since, in both cases, the object moves the same distance (5 cm or 50 mm) in equal amounts of time (1 s).

To be in simplest form, a rate must be expressed as a quantity per 1 unit of another quantity. For example, a rate of 60 km/h is in simplest form because it represents a change in distance of 60 km for every 1 hour of time.

Example 1

Express each of the following statements as a rate in simplest form.

a \$150 in 3 hours

b 48 L in 12 min

Solutions

a \$150 in 3 hours
 $\div 3 \quad \div 3$
 = \$50 in 1 hour
 = \$50/h

b 48 L in 12 min
 $\div 12 \quad \div 12$
 = 4 in 1 min
 = 4 L/min

Example 2

Convert:

a 2.4 kg/day to g/day

b 3.5 cm³/s to cm³/min

c 18 m/s to km/h

Solutions

a 2.4 kg in 1 day
 = 2400 g in 1 day
 = 2400 g/day

b 3.5 cm³ in 1 s
 $\times 60 \quad \times 60$
 = 210 cm³ in 1 min
 = 210 cm³/min

c 18 m in 1 s
 $\times 60 \quad \times 60$
 = 1080 m in 1 min
 $\times 60 \quad \times 60$
 = 64 800 m in 1 h
 = 64.8 km/h

Exercise 1.5

1 Express each statement as a rate in simplest form.

a 30 m in 3 s	b 80 km in 2 h	c 45 L in 5 min
d 42 kg over 7 m ²	e 32 g in 4 s	f 200 trees in 8 h
g 108 km on 9 L	h \$180 in 4 h	i 90c for 5 min
j \$12 for 8 kg	k 119 runs in 34 overs	l 150 crates in 4 days
m 240 beats in $2\frac{1}{2}$ min	n 72 kL in 1.5 h	o 13 km on 1.25 L

2 Complete these equivalent rates.

a 3 cm/s = _____ cm/min	b 5 g/min = _____ g/h	c \$2.30/kg = \$ _____ /t
d 7.5 L/h = _____ L/day	e 0.9 km/min = _____ km/h	f 0.4 kg/m ² = _____ kg/ha

3 Complete these equivalent rates.

a 2 L/min = _____ mL/min	b 9 m/s = _____ cm/s
c 3.8 cm/s = _____ mm/s	d \$1.15/g = _____ c/g
e 14.6 t/day = _____ kg/day	f 2.35 ha/week = _____ m ² /week

4 Complete these equivalent rates.

a 70 mm/s = _____ cm/s	b 850 cm/min = _____ m/min
c 4900 mL/day = _____ L/day	d 24c/min = \$ _____ /min
e 25 g/m ³ = _____ kg/m ³	f 59 600 L/year = _____ kL/year

■ Consolidation

5 Complete the following equivalent rates.

a 75 cm/s = _____ m/min	b 8c/g = \$ _____ /kg
c 9 m/mL = _____ km/L	d 150 kg/h = _____ t/day
e 81.25 mL/h = _____ L/day	f 142 m/min = _____ km/h

6 Complete the following equivalent rates.

a 25 m/s = _____ km/h	b 40 mL/s = _____ L/h
c 27.5 g/s = _____ kg/h	d 5 mm/min = _____ m/day
e 0.8 m/min = _____ km/day	f 2.4c/mm = \$ _____ /m
g 72 km/h = _____ m/s	h 12.24 t/day = _____ kg/min

7 Convert these annual interest rates to monthly rates.

a 12% p.a.	b 6% p.a.	c 18% p.a.	d 4.2% p.a.

8 Convert these monthly interest rates to annual rates.

a 0.75% per month	b 0.9% per month	c 1.25% per month

9 Calculate the daily interest rate on a credit card if the annual rate is 15.33% p.a.

10 Convert:

- | |
|---|
| a \$240/week to an equivalent monthly rate |
| b \$1352/month to an equivalent fortnightly rate |

- c \$2.80/week to an equivalent quarterly rate
 d \$44.20/quarter to an equivalent fortnightly rate.

■ Further applications

11 Complete these equivalent rates.

a $5\text{c}/\text{cm}^2 = \$\text{____}/\text{m}^2$ b $60\text{ mL}/\text{m}^2 = \text{____ L}/\text{km}^2$ c $1.2\text{ g}/\text{cm}^3 = \text{____ t}/\text{m}^3$

12 Complete this equivalent rate: $\$25/\text{L} = \text{____ c}/\text{cm}^3$.

TRY THIS Desert walk

James is a cross-country walker. He comes to a 60 km stretch of desert where there is neither water nor food. He can walk 20 km per day and he can carry enough food and water for 2 days. How many days will it take him to cross the desert, and how many kilometres will he travel if he has to build up depots of food and water?

Difficult part

If he was considering a 100 km trip across the desert, how many days' supply of food would be necessary?



1.6 Solving problems with rates

We use many different types of rates every day, often without realising it. For example:

- driving speed
- petrol consumption rates
- electricity rates
- bank interest rates
- sporting strike rates
- pollution rates
- currency exchange rates
- rates of pay
- medical recovery rates

As most adults drive a car, the concept of **speed** plays a very important role in our daily lives. We need to know how fast to drive in order to reach a particular destination on time. It is also important to know at what speed we can safely drive under various conditions, such as on narrow roads, in wet weather, near pedestrian crossings and so on.

Informally, we think of speed as a measure of how fast an object is travelling. Formally, however, speed is defined as the rate of change of distance with respect to time. If we know the distance that an object has travelled from one point to another and the amount of time that it took to get there, then we can calculate how fast it was travelling. You should already be familiar with the following formulae relating speed, distance and time.

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$ $S = \frac{D}{T}$	$\text{Distance} = \text{Speed} \times \text{Time}$ $D = S \times T$	$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$ $T = \frac{D}{S}$

There is an important distinction that needs to be made between **average speed** and **instantaneous speed**. The formulae above are usually associated with average speed, since the speed of the object may vary at different times throughout its journey. It may start moving slowly, speed up at times and slow down or even stop at other times. If, however, a speed camera had been used to measure the speed of the object at a single moment in time, then it would have measured the instantaneous speed of the object. The instantaneous speed at a split second may therefore differ from the average speed over the entire journey.

The degrees and minutes key on the calculator can be used to simplify the working in some questions, particularly when the time is given in hours and minutes or minutes and seconds.

Example 1

EG
+S

- a The entry price to an amusement park is \$7.50 per child. Find the total entry cost for a group of 90 children.
- b A farmer used 145 kg of super phosphate to cover an area of 5 ha. How many kilograms were used per hectare?

Solutions

- a The entry cost for 1 child = \$7.50
 $\therefore$ cost for 90 children = $90 \times \$7.50$
 $= \$675$
- b 145 kg covers an area of 5 ha
 $\div 5$
 $\therefore$ 29 kg covers an area of 1 ha
- b 145 kg covers an area of 5 ha
 $\div 5$
 $\therefore$ 29 kg covers an area of 1 ha

Example 2

EG
+S

A car can travel 138 km on 15 L of petrol. How far can it travel on a full tank of 35 L?

Solution

Using the unitary method,

138 km on 15 L

$\div 15$ $\div 15$

= 9.2 km on 1 L

$\times 35$ $\times 35$

= 322 km on 35 L

$\therefore$ The car can travel 322 km on a full tank of 35 L of petrol.

**Example 3**

- a Jenny ran 600 metres in 80 seconds. What was her running speed?
 b A man drove at an average speed of 60 km/h for 7 hours. How far did he drive?
 c Shona's average walking speed is 5 km/h. How long would it take her to walk 20 km?

Solutions

$$\begin{aligned} \text{a } S &= \frac{D}{T} \\ &= \frac{600}{80} \\ &= 7.5 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \text{b } D &= S \times T \\ &= 60 \times 7 \\ &= 420 \text{ km} \end{aligned}$$

$$\begin{aligned} \text{c } T &= \frac{D}{S} \\ &= \frac{20}{5} \\ &= 4 \text{ h} \end{aligned}$$

Exercise 1.6

- 1 a An author writes at a rate of 3 pages per hour. How many pages would she write in 6 hours?
 b A shearer was able to shear 18 sheep per hour. How many sheep could he shear in $2\frac{1}{2}$ hours?
 c If petrol costs 97.4 cents/L, find how much it would cost to fill the tank in a car if the tank holds 42 L.
 d A tap is dripping at the rate of 3 mL per minute. How many litres of water will be lost in 2 days?
 e The crew on a fishing boat put out the nets every 2 hours and catch an average of 240 kg of fish. How many tonnes would the crew expect to catch if they fish for 10 hours?
- 2 a Trevor earns \$15.20 per hour as a sales assistant. How many hours would he need to work in order to earn \$562.40?
 b Janine has a typing speed of 54 words per minute. How long would it take her to type a 1350 word article?
 c A cricket side scored 243 runs in 50 overs during a limited overs cricket match. Calculate the average scoring rate in runs per over.
 d A plumber charged \$200 for $2\frac{1}{2}$ hours labour to repair a broken pipe. Find the plumber's hourly rate.
 e A machine prints 150 newspapers per minute. How long would it take to print 18 000 newspapers?

Consolidation

- 3 a Georgina drove 12 km in 10 minutes. At the same speed, how far would she drive in 30 minutes?
 b Gino's pulse rate was 100 beats per minute. How many times would his heart beat in 15 seconds?

- c** A fruit picker claimed that he could pick 1200 apples per hour. How many apples could he pick in 20 minutes?
- d** A bank teller can serve 20 customers per hour. How many customers can she serve in 45 minutes?
- e** A tap drips 12 times in 20 seconds. How many times would it drip in 30 seconds?
- 4** Use the unitary method to answer the following questions.
- a** Dianne paid \$3.75 for 3 kg of oranges. How much would she have paid for 7 kg?
- b** In a walking race, Paul took 40 minutes to walk 8 km. How long would it take him to walk 13 km?
- c** Susan's car uses petrol at the rate of 10.6 L/100 km. How much petrol would she use on a journey of 250 km?
- d** If it takes $1\frac{1}{2}$ hours to remove 36 t of sugar from a silo, how long it would take to remove 30 t?
- e** George delivered 400 pamphlets in 50 minutes. How many pamphlets would he deliver in $2\frac{1}{2}$ hours?
- f** If sausages are being sold for \$2.80 per kilogram, find the cost of purchasing 350 grams of sausages.
- 5** The following currency conversions show the value of 1 Australian dollar (A\$1) in US\$, euro and NZ\$.
- A\$1 = US\$0.6075 A\$1 = 0.5636 euro A\$1 = NZ\$1.0887
- Use these currency conversions to convert:
- a** A\$20 into US\$ **b** A\$50 into euro **c** A\$175 into NZ\$
- d** A\$250 into euro **e** A\$600 into NZ\$ **f** A\$4500 into US\$
- 6** Use the currency conversions in Q5 to convert the following amounts into Australian dollars. Give your answers correct to the nearest cent.
- a** NZ\$30 **b** US\$95 **c** 110 euro **d** NZ\$200
- e** US\$565 **f** 782 euro **g** NZ\$1400 **h** US\$2378
- 7 a** Dave drove 350 km in 5 hours. What was his average speed?
- b** A plane travelled 1960 km in 7 hours. What was the speed of the plane?
- c** Jennifer ran at a speed of 8 km/h for $1\frac{1}{2}$ hours. How far did she run?
- d** A ship sailed at 42 km/h for 25 hours. What distance did it sail?
- e** Morgan rode his motor bike a distance of 340 km at a speed of 85 km/h. How long was the trip?
- f** A satellite orbits the Earth at a speed of 22 500 km/h. How long will it take for the satellite to travel a distance of 78 750 km?

- 8 Use the degrees and minutes key on your calculator to answer the following questions.
- a How far will a bus travel in 4 h 25 min at an average speed of 90 km/h?
 - b Calculate the average speed of a battleship which sails 600 km in 11 h 45 minutes. Answer correct to the nearest km/h.
 - c How long will it take for a plane to fly 615 km at a speed of 180 km/h? Answer correct to the nearest minute.

■ Further applications

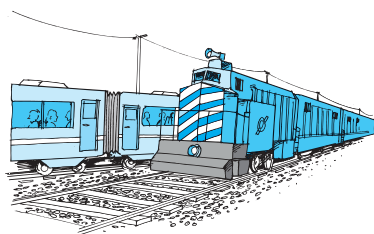
- 9 The speed of ships and sometimes of aircraft is usually measured in knots. A knot is a speed of 1 nautical mile per hour, where 1 nautical mile is equivalent to 1852 metres.
- a Express 1 knot in km/h.
 - b If an aircraft is travelling at 120 knots, how long would it take to travel 5000 km?
 - c If another aircraft is travelling at 760 knots, how many kilometres will it travel in 6 hours?
- 10 The petrol consumption (C) of a car is measured in litres of petrol (L) used per 100 km (K) travelled.
- a Write down a formula connecting C , L and K .
 - b Calculate the petrol consumption of a car that travels 1038 km in a month and uses 95 L of petrol.
 - c Meera is planning a tour of the Australian outback and expects to travel 10 000 km. Her vehicle's petrol consumption is expected to average 12 L/100 km. If the average price of petrol in the outback is \$1.12 per litre, calculate the expected cost of petrol for this trip.

TRY THIS Passing trains

A slow train leaves Canberra at 9:17 am and arrives at Goulburn at 12:02 pm. On the same day, the express leaves Canberra at 9:56 am and arrives in Goulburn at 11:36 am.

At what times does the express pass the slow train if each is travelling at a constant speed?

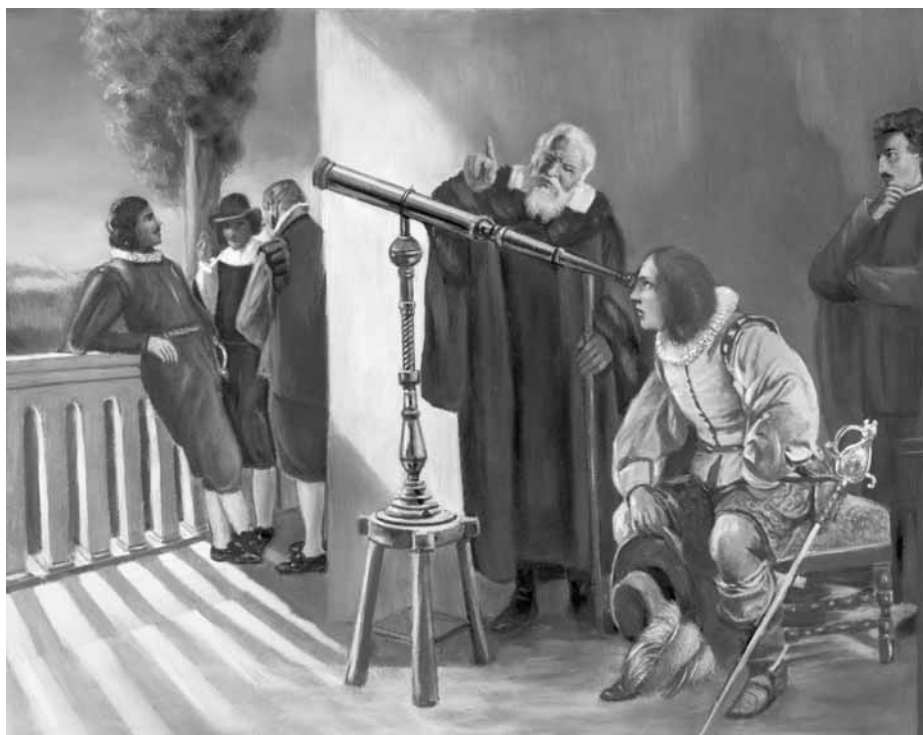
HINT: A travel graph would give an approximate time.





FOCUS ON WORKING MATHEMATICALLY

A NUMBER PATTERN FROM GALILEO 1615



Galileo looking through a telescope in his observatory

Introduction

Galileo Galilei (1564–1642), the famous Italian mathematician, is better known for his scientific achievements than his mathematical ones. For example, in 1610 he made a series of telescopes that enabled him to discover four of the moons of Jupiter, to see mountains on the Moon, and to prove that the Milky Way was made up of stars. The four moons of Jupiter he discovered centuries ago are today called the Galilean satellites in his honour. Their names are Io, Europa, Ganymede and Callisto. We now know, thanks to space probes, that Jupiter has, in fact, 16 moons, 13 of which have been discovered from Earth.

In this activity, however, you will investigate a number pattern for the fraction $\frac{1}{3}$. In 1615,

Galileo wrote one of the earliest manuscripts describing this pattern, so we can see how interested he was in pure mathematics. First, we search for a pattern among specific cases using *inductive* reasoning, and then we use algebra to generalise the pattern using *deductive* reasoning.





LEARNING ACTIVITIES

- 1 Check that the following statement is true: $\frac{1}{3} = \frac{1+3}{5+7}$
- 2 Notice that the numbers in the numerator and denominator form the pattern of odd numbers 1, 3, 5 and 7.
- 3 Continue the pattern to obtain $\frac{1+3+5}{7+9+11}$. Does it still equal $\frac{1}{3}$?
- 4 Write down the next term of the sequence and continue, checking that in each case the fraction is equivalent to $\frac{1}{3}$.
- 5 Why is this true? Don't try a formal proof, but see if you can draw a diagram to show that it must be. Use dots to represent the odd numbers and choose some specific cases. Ask for help as needed.



CHALLENGE

This is suggested as a group activity for extension stage 5 classes as an exercise in *collaborative learning*.

- 1 Investigate the pattern of odd numbers $1 + 3 + 5 + 7 + 9 + 11 + \dots$
- 2 Notice that the partial sums $1 + 3$, $1 + 3 + 5$, $1 + 3 + 5 + 7$, ... are perfect squares.
- 3 See if you can find the pattern for the sum of 2 terms, 3 terms, 4 terms, ...
- 4 Make a hypothesis about the sum of n terms.
- 5 Make a hypothesis about the sum of $2n$ terms.
- 6 If there were n terms in the numerator, how many would there be in the denominator? How many altogether?
- 7 Look carefully at the following patterns:

$$1 + 3 + 5 = 3^2$$

$$\text{and } 7 + 9 + 11 = (1 + 3 + 5 + 7 + 9 + 11) - (1 + 3 + 5) = 6^2 - 3^2$$

$$\text{So } \frac{1 + 3 + 5}{7 + 9 + 11} = \frac{3^2}{6^2 - 3^2} = \frac{9}{27} = \frac{1}{3}$$

- 8 See if you can show that the next term is also $\frac{1}{3}$ using this same pattern:

$$\frac{1 + 3 + 5 + 7}{9 + 11 + 13 + 15} = \dots$$

- 9 From the pattern of your results, see if you can write down an expression for the fraction you would get if there were n terms on top. Ask your teacher for help if you need it, and discuss the possibilities between yourselves. Check that the expression reduces to $\frac{1}{3}$.





LET'S COMMUNICATE

Discuss what you have learned from this activity with a classmate or, perhaps, if you have worked in a group for this activity, with the group members. Can you see the value of *inductive* thinking in mathematics, that is, finding a pattern to suggest a general rule?

If you worked in a group, write a short account of whether you enjoyed collaborating with others. Is it a good way to learn?



REFLECTING

Mathematical thinking can be *inductive*, searching for a pattern to suggest a general rule, or *deductive*, reasoning in a chain of argument that leads to a mathematical proof. Both are very important in learning mathematics and are often used together.

Think over how much of your learning in Year 9 is inductive and how much deductive. Discuss with your teacher as to how the two go together in mathematics lessons.



MACQUARIE

Language link with Macquarie

- 1 What is a small word for **magnitude**?
- 2 Explain the difference between a **guess** and an **estimate**.
- 3 What is a **rational number**?
- 4 When is a digit in a number **significant**?
- 5 Read the Macquarie Learners Dictionary entry for **rate**:

rate noun 1. speed: to work at a steady rate | The car was travelling at a rate of 100 kilometres an hour.
 2. a charge or payment: The interest rate on the loan is 10 per cent per year. 3. **rates**, the tax paid to the local council by people who own land
 —verb 4. to set a value on, or consider as: The council rated the land at \$20 000. | I rate him a very good friend.
 —phrase 5. **at any rate**, in any case: We enjoyed ourselves at any rate.
 6. **at this rate**, if things go on like this: At this rate we will soon run out of money.

How is the word 'rate' used in this chapter?

- 1** State the number of significant figures in:
- a** 406 **b** 7.2009
c 0.0031 **d** 12.0560
- 2** Round off each number correct to 1 significant figure.
- a** 76 **b** 150
c 4278 **d** 894 000
- 3** Round off each number correct to 2 significant figures.
- a** 341 **b** 725
c 15 049 **d** 369 412
- 4** Round off each number correct to the number of significant figures shown in the brackets.
- a** 198 [1] **b** 4316 [1]
c 18 209 [1] **d** 572 [2]
e 2154 [2] **f** 36 587 [2]
- 5** Round off each decimal correct to the number of significant figures shown in the brackets.
- a** 4.83 [1] **b** 0.0723 [2]
c 3.4661 [3] **d** 22.018 [3]
e 106.84 [2] **f** 8994.7 [1]
- 6** Evaluate each of these correct to 2 decimal places, using a calculator.
- a** $5\frac{2}{3} - 1\frac{7}{10}$ **b** $-6.3 - 1.29$
c 5.84^2 **d** $\sqrt{136.4}$
e $\sqrt[3]{91}$ **f** 2.71^5
g $\sqrt[6]{101.9}$ **h** $\frac{1}{0.107}$
i $\frac{8\pi}{3}$
- 7** Evaluate each of the following, correct to 2 decimal places, using a calculator.
- a** $\frac{15.7 \times 34.15}{12.31 - 5.6}$ **b** $\sqrt{75.3 \times 29.1}$
c $\frac{1}{\sqrt[3]{0.57 + 4.5^2}}$ **d** $3.4^5 - (2\frac{3}{5})^4$
- e** $\frac{\sqrt[5]{92.8}}{4 - \sqrt{2}}$ **f** $\frac{\sqrt[3]{15} + \sqrt[4]{13}}{\sqrt{15} - \sqrt{13}}$
- 8** Estimate the value of each calculation.
- a** $9.84 \times 15.2 + 18.77$
b $7.97 + 47.3 \div \sqrt{15.49}$
c $\frac{194.7 \times 259.2}{53.6}$
- 9** Write each of these as a recurring decimal.
- a** 0.333 333 ... **b** 0.252 525 ...
c 0.346 346 ... **d** 5.918 181 ...
- 10** Convert these fractions to recurring decimals.
- a** $\frac{7}{9}$ **b** $\frac{4}{11}$ **c** $1\frac{7}{12}$
- 11** Convert these recurring decimals to fractions.
- a** 0.2 **b** $0.\dot{7}\dot{2}$ **c** $0.1\dot{3}$
- 12** Given that $\frac{1}{6} = 0.1\dot{6}$, express each of the following fractions as a recurring decimal.
- a** $\frac{1}{60}$ **b** $\frac{1}{600}$
- 13** Express each statement as a rate in simplest form.
- a** 80 m in 10 s
b \$45 for 9 min
c 72 L in 3 h
d 215 runs for 5 wickets
- 14 a** A car uses 18 L of petrol to travel 150 km. How much petrol would be needed to travel 350 km?
b A farmer spreads 25 kg of fertiliser over an area of 4000 m². How much fertiliser would be needed to cover an area of 1.5 ha?

15 Convert:

- a** 7 mm/min to mm/h
- b** 75 km/h to km/day
- c** 1.35 L/m^2 to mL/m^2
- d** 8.2 m/s to cm/s

16 Convert:

- a** 40 m/min to km/h
- b** 250 mL/h to L/day
- c** 13.5 g/m^2 to kg/ha
- d** 5 m/s to km/h

17 a A plane flew 6000 km in $7\frac{1}{2}$ hours. At what speed was the plane travelling?

b Karen walked 24 km at 5 km/h, for how long did she walk?

c Jude drove at 80 km/h for 4 h 15 min. What distance did he drive?

18 Daryl drove 527 km in 6 h 23 min. Find his speed, correct to 1 decimal place.

Algebra

2



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ use algebra to find rules for simple number patterns
- ⦿ use the method of finite differences to find rules for simple number patterns
- ⦿ evaluate algebraic expressions by substituting numbers for pronumerals
- ⦿ add and subtract algebraic expressions
- ⦿ multiply and divide algebraic expressions
- ⦿ simplify algebraic expressions using the order of operations
- ⦿ expand algebraic expressions that contain grouping symbols using the distributive law
- ⦿ factorise algebraic expressions by removing the highest common factor
- ⦿ add and subtract algebraic fractions
- ⦿ multiply and divide algebraic fractions
- ⦿ link algebra with generalised arithmetic
- ⦿ use algebra to prove general properties of numbers
- ⦿ use algebra to generalise solutions to problems
- ⦿ expand binomial products
- ⦿ expand perfect squares using the special identities
- ⦿ determine whether a given expression is a perfect square
- ⦿ complete a perfect square
- ⦿ expand expressions using the difference of two squares identity
- ⦿ expand expressions that involve a combination of algebraic techniques.

2.1 Describing simple patterns

Many complex problems can often be solved more easily by using algebra. Algebra lets us replace complex statements with short, simple expressions. Algebra also lets us generalise results that are always true, or are true under certain conditions, so that we do not have to keep solving the same types of problems over and over again.

Finite differences

It is not always easy to find the algebraic rule that describes the relationship between variables. The method of **finite differences** is a simple technique that can be used to help us find this relationship. Finite differences are the differences between the numbers in the bottom row of a table of values.

For example, the numbers in the bottom row of this table are increasing by 3. Therefore, the finite differences in the table are all 3s.

x	1	2	3	4
y	13	16	19	22

$\underbrace{\hspace{1.5cm}}_{+3}$
 $\underbrace{\hspace{1.5cm}}_{+3}$
 $\underbrace{\hspace{1.5cm}}_{+3}$

To find the rule that links the variables x and y in a linear relationship:

- ① write the standard rule in the form $y = \Delta x + \square$
- ② find Δ , the finite differences between the bottom numbers in the table
- ③ find $\square$ by substituting into the rule a pair of values from the table.

NOTE: This method can only be used for linear relationships when the x -values are consecutive integers (e.g. $x = 1, 2, 3 \dots$).

Example

Find the rule that describes the relationship between x and y in this table of values.

x	0	1	2	3
y	7	12	17	22

Solution

Let the rule be in the form $y = \Delta x + \square$, where Δ is the difference between each pair of consecutive y -values.

Now, the y -values are increasing by 5, $\therefore \Delta = 5$.

If $y = 5x + \square$ and $x = 0$ when $y = 7$,

$$7 = (5 \times 0) + \square$$

$$7 = 0 + \square$$

$$\therefore \square = 7$$

$$\therefore \text{The rule is } y = 5x + 7.$$

x	0	1	2	3
y	7	12	17	22

$\underbrace{\hspace{1.5cm}}_{+5}$
 $\underbrace{\hspace{1.5cm}}_{+5}$
 $\underbrace{\hspace{1.5cm}}_{+5}$

Exercise 2.1

- 1 Complete each table of values using the given rules.

$$y = x + 3$$

a

x	1	2	3	4
y				

$$y = 2x + 5$$

b

x	0	1	2	3
y				

$$y = 3x - 4$$

c

x	5	6	7	8
y				

$$y = 5x - 7$$

d

x	2	3	4	5
y				

- 2 For each table of values in Q1, compare the differences between the y -values and the co-efficient of x in the rule. What do you notice?

- 3 Use the method of finite differences to find a rule for each table of values.

a

x	1	2	3	4
y	4	8	12	16

b

x	0	1	2	3
y	6	7	8	9

c

x	4	5	6	7
y	11	13	15	17

d

p	2	3	4	5
q	5	8	11	14

e

p	1	2	3	4
q	9	14	19	24

f

p	7	8	9	10
q	47	54	61	68

g

a	4	5	6	7
b	17	19	21	23

h

a	0	1	2	3
b	3	7	11	15

i

a	3	4	5	6
b	18	24	30	36

j

s	5	6	7	8
t	17	22	27	32

k

s	1	2	3	4
t	13	20	27	34

l

s	2	3	4	5
t	19	31	43	55

Consolidation

4

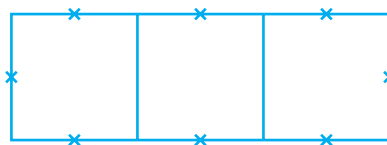
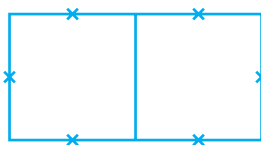
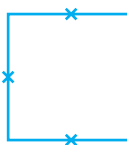


- a Copy and complete this table of values.

Number of pentagons (p)	1	2	3
Number of triangles (t)			

- b Write down an algebraic rule that links the number of triangles (t) to the number of pentagons (p).
- c How many triangles would there be in a figure with 9 pentagons?

5

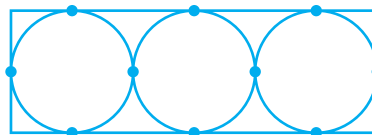
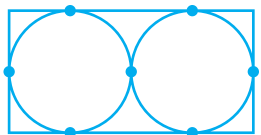
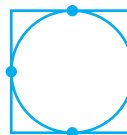


- a Copy and complete this table of values.

Number of squares (s)	1	2	3
Number of crosses (c)			

- b Write down an algebraic rule that links the number of crosses (c) to the number of squares (s).
- c How many crosses would there be in a figure with 20 squares?

6

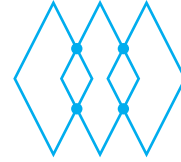
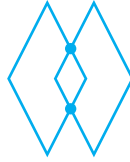


- a Copy and complete this table of values.

Number of circles (c)	1	2	3
Number of dots (d)			

- b Write down an algebraic rule that links the number of dots (d) to the number of circles (c).
- c How many dots would there be in a figure with 15 circles?

7

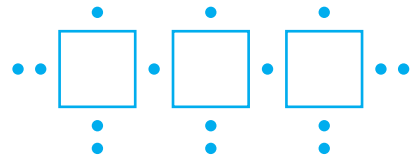
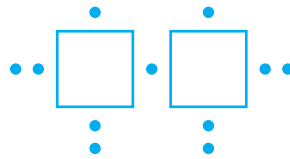
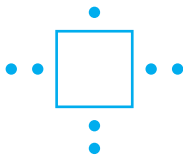


- a Copy and complete this table of values.

Number of large rhombuses (r)	1	2	3
Number of dots (d)			

- b Write down an algebraic rule that links the number of dots (d) to the number of large rhombuses (r).
 c How many dots would there be in a figure with 40 large rhombuses?

8

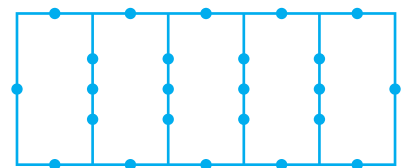
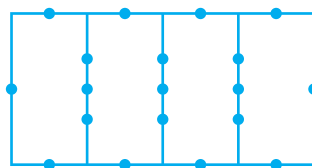
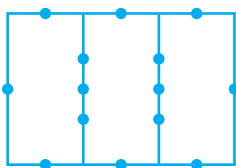


- a Copy and complete this table of values.

Number of squares (s)	1	2	3
Number of dots (d)			

- b Complete this rule that relates the number of dots to the number of squares: $d = \Delta s + \square$.

9

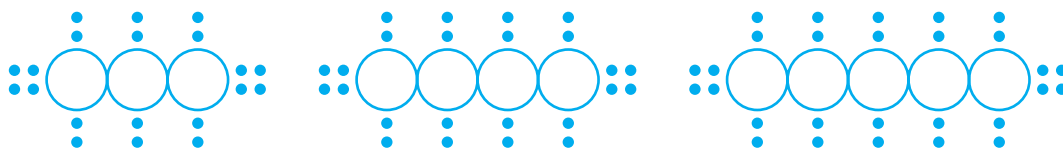


- a Copy and complete this table of values.

Number of rectangles (r)	3	4	5
Number of dots (d)			

- b Complete this rule that relates the number of dots to the number of rectangles:
 $d = \Delta r + \square$.

10

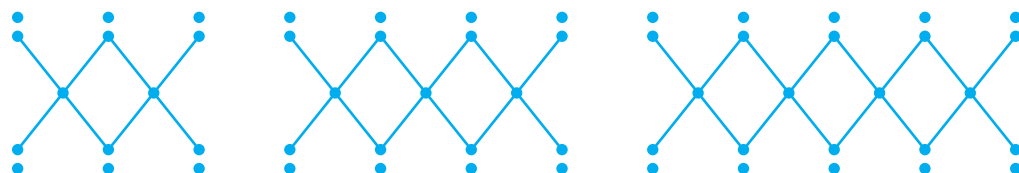


a Copy and complete this table of values.

Number of circles (c)	3	4	5
Number of dots (d)			

b Complete this rule that relates the number of dots to the number of circles: $d = \Delta c + \square$.

11



a Copy and complete this table of values.

Number of crosses (c)	2	3	4
Number of dots (d)			

b Complete this rule that relates the number of dots to the number of crosses: $d = \Delta c + \square$.

12 Use the method of finite differences to find a rule linking the x - and y -values in each table.

a

x	1	2	3	4
y	-7	-14	-21	-28

b

x	0	1	2	3
y	5	4	3	2

c

x	1	2	3	4
y	7	5	3	1

d

x	3	4	5	6
y	11	8	5	2

e

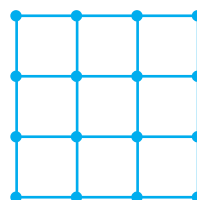
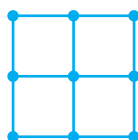
x	-4	-3	-2	-1
y	10	9	8	7

f

x	-2	-1	0	1
y	13	10	7	4

Further applications

13



- a Copy and complete this table of values.

Number of squares (s)	1	4	9
Number of dots (d)			

- b Write down an algebraic rule that links the number of dots (d) to the number of squares (s).
- c How many dots would there be in a figure with 64 squares?

14



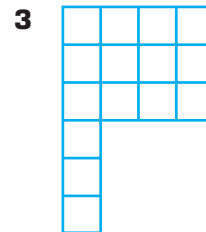
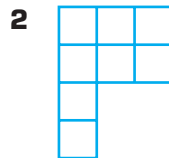
- a Copy and complete this table of values.

Number of cans in base (b)	1	2	3
Total number of cans (c)			

- b Write down an algebraic rule that links the total number of cans (c) to the number of cans in the base (b).
- c How many cans would there be in a pile with 10 cans in the base?

TRY THIS Flags

Consider the following diagrams, then complete the table.



Pole length	1	2	3	4	5	... n
Number of squares	3					

Find a rule relating the number of squares in the flag to the pole length.

HINT: The rule is not linear.

2.2 Substitution

When we **substitute** for a pronumeral, we give the pronumeral the value of a number. An algebraic expression can have a number of values, depending on the value(s) that are substituted for each pronumeral.

EG
+S

Example 1

Evaluate each of the following when $x = 3$ and $y = 7$.

a $8x - 2y$

b $2x^2$

c $\frac{x+y}{2}$

d $6(x+y)$

Solutions

$$\begin{aligned} \text{a} \quad 8x - 2y &= (8 \times 3) - (2 \times 7) \\ &= 24 - 14 \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{b} \quad 2x^2 &= 2 \times 3^2 \\ &= 2 \times 9 \\ &= 18 \end{aligned}$$

$$\begin{aligned} \text{c} \quad \frac{x+y}{2} &= \frac{3+7}{2} \\ &= \frac{10}{2} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{d} \quad 6(x+y) &= 6(3+7) \\ &= 6 \times 10 \\ &= 60 \end{aligned}$$

EG
+S

Example 2

Evaluate each of these when $m = 2$ and $n = 5$.

a $m - n + 9$

b $3m - 4n$

c $mn(m - n)$

Solutions

$$\begin{aligned} \text{a} \quad m - n + 9 &= 2 - 5 + 9 \\ &= -3 + 9 \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{b} \quad 3m - 4n &= (3 \times 2) - (4 \times 5) \\ &= 6 - 20 \\ &= -14 \end{aligned}$$

$$\begin{aligned} \text{c} \quad mn(m - n) &= 2 \times 5 \times (2 - 5) \\ &= 10 \times (-3) \\ &= -30 \end{aligned}$$

EG
+S

Example 3

Evaluate each of the following given that $p = 4$, $q = -3$ and $r = -6$.

a $p + q - r$

b pqr

c $q(p - r)$

Solutions

$$\begin{aligned} \text{a} \quad p + q - r &= 4 + (-3) - (-6) \\ &= 4 - 3 + 6 \\ &= 1 + 6 \\ &= 7 \end{aligned}$$

$$\begin{aligned} \text{b} \quad pqr &= 4 \times (-3) \times (-6) \\ &= -12 \times -6 \\ &= 72 \end{aligned}$$

$$\begin{aligned} \text{c} \quad q(p - r) &= -3(4 - (-6)) \\ &= -3 \times 10 \\ &= -30 \end{aligned}$$

Exercise 2.21 Evaluate each of the following when $k = 5$.

a $k + 7$	b $k - 2$	c $13 - k$	d $3k$
e $7k + 8$	f $12k - 23$	g $30 - 4k$	h k^2
i k^3	j $3k^2$	k $k^2 + 3k$	l $2k^2 - 9k$
m $\frac{40}{k}$	n $\frac{k}{15}$	o $\frac{k + 7}{4}$	p $\frac{5k + 11}{2k - 1}$

Consolidation2 Evaluate each of these when $m = 7$ and $n = 3$.

a $16 - m + n$	b $mn - 8$	c $6m - n$	d $2m + 5n$
e $13n - 4m$	f $50 - 2mn$	g $3m + 6n - 11$	h $100 - 5m - 3n$
i $n^2 + 10$	j $50 - m^2$	k $m^2 - n^2$	l $4n^2$
m $2m^2 + 13$	n $n^3 - 8m$	o mn^2	p $m^2n - mn^3$
q $5(m + n)$	r $12(m - n + 6)$	s $n(8m - 20)$	t $2n(5m + mn)$
u $\frac{24}{m - n}$	v $\frac{4m + 4n}{5}$	w $\frac{3m + 2n}{n^2}$	x $\sqrt{m^2 + 5n}$

3 Find the value of each expression using the substitutions $r = 6$, $s = 2$ and $t = 11$.

a $s - r$	b $r - t$	c $-s + t$	d $-t - r$
e $r - s - t$	f $s - t + r$	g $-r + s + t$	h $-t + s - r$
i $3s - t$	j $-5t + 4r$	k $-8r + st$	l $5s - rt$
m $100 - rst$	n $rs - st$	o $r^2 - 3rt$	p $s^2 - r^2 + t^2$
q $t - 5s^2$	r $r(s - t)$	s $5(2t - 4r - 9s)$	t $3s(r^2 - t^2)$

Further applications4 Evaluate each of the following given that $a = -3$, $b = 8$ and $c = -6$.

a $a + b$	b $b - c$	c $c + a$	d $a - b$
e $a - c + b$	f $c + b + a$	g $b - a - c$	h $-b + c + a$
i $4a - 2b - c$	j $b + 5a + 2c$	k $3b - 5a + 10c$	l $-4c + 3b - 7a$
m $b(a + c)$	n $c(b - a)$	o $2a(c + b)$	p $ac(b - 10)$
q $(b - a)(b + c)$	r a^2b	s $ab - c^3$	t $\sqrt{abc}$
u $\sqrt{b^2 + c^2}$	v $\frac{ab}{c}$	w $\frac{b - 2c}{a - 1}$	x $\frac{2(a^2 + c^2)}{ac}$

2.3**Adding and subtracting algebraic expressions**

Algebraic terms with identical pronumerals are called **like terms**. Only like terms can be added or subtracted.

Some examples of:

- like terms are $3m$ and $5m$, $7q$ and $-2q$, xy and yx , $4t^2$ and $9t^2$
- unlike terms are $4a$ and $4b$, ef and fg , $6u^2$ and $11u$.

To collect the like terms in an algebraic expression:

- add or subtract the co-efficients
- keep the same pronumeral(s).

EG
+
S

Example 1

Simplify each of these.

a $7s + 3s$

b $12w - 4w$

c $6y - y$

d $5r^2 + 2r^2$

e $14gh - 9gh$

f $7pq + 6qp$

Solutions

a $7s + 3s = 10s$

b $12w - 4w = 8w$

c $6y - y = 5y$

d $5r^2 + 2r^2 = 7r^2$

e $14gh - 9gh = 5gh$

f $7pq + 6qp = 13pq$

EG
+
S

Example 2

Simplify these expressions by collecting the like terms.

a $6e + 13 + 4e + 8$

b $9v^2 + 7v + v^2 - 3v$

c $8x + 7y - 5x - 12y$

Solutions

a $6e + 13 + 4e + 8$
 $= 6e + 4e + 13 + 8$
 $= 10e + 21$

b $9v^2 + 7v + v^2 - 3v$
 $= 9v^2 + v^2 + 7v - 3v$
 $= 10v^2 + 4v$

c $8x + 7y - 5x - 12y$
 $= 8x - 5x + 7y - 12y$
 $= 3x - 5y$

Exercise 2.3

1 a Simplify $7x + 3x$.

b Verify your answer by substituting several values for x .

2 a Simplify $5n + 2n$ and $2n + 5n$.

b Does $5n + 2n = 2n + 5n$?

c Does it matter in which order algebraic expressions are added?

3 a Simplify $5s - 3s$ and $3s - 5s$.

b Does $5s - 3s = 3s - 5s$?

c Does it matter in which order algebraic expressions are subtracted?

4 Simplify each of the following.

a $4y + 5y$

b $12n - 8n$

c $2c + c$

d $7k - k$

e $11z - 11z$

f $10b - 9b$

g $3a^2 + 4a^2$

h $13g^2 - 5g^2$

i $6pq + 5pq$

j $15xy - 8yx$

k $2abc + 6abc$

l $14m^2n + 5m^2n$

m $3t - 7t$	n $-2u + 12u$	o $-13p + 4p$	p $-8j - 7j$
q $5pq - 11pq$	r $-10yz + 9zy$	s $e^2 - 11e^2$	t $-9rs^2 + 7rs^2$

Consolidation

5 Simplify:

a $3a + 4a + 2a$	b $10b - 3b - b$	c $9k - 6k + 7k$	d $5m - 8m - 4m$
e $3p - 10p + 15p$	f $-6r + 4r + 9r$	g $-x - 7x - 5x$	h $-3c + 2c - 11c$
i $4e^2 - 7e^2 - 10e^2$	j $8a^2 - 12a^2 + 4a^2$	k $5ab + ab - 9ab$	l $-9pq + 6pq + 7pq$

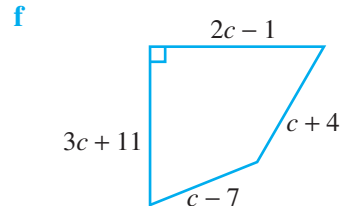
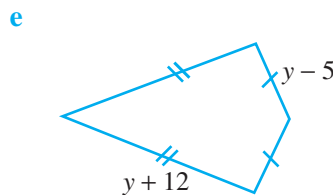
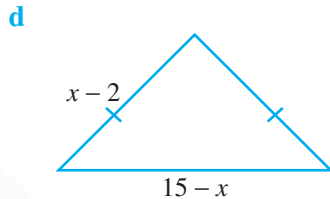
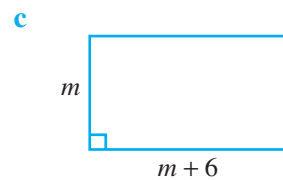
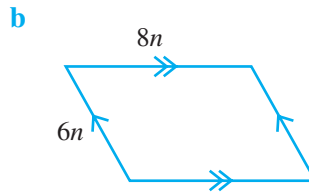
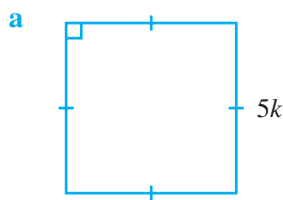
6 Collect the like terms in each expression.

a $4q + 3q + 2$	b $5g + 8 + 9$	c $15u - 7u - 3$	d $13 + 6t - 5t$
e $10c + 8c + d$	f $9j - 4k + 2j$	g $3a - 5a + 7$	h $12 - 2n - 4n$
i $x^2 + 4x + x$	j $8m + m^2 - 10m$	k $3w^2 + 2w^2 + w$	l $4a^2b + 6ab^2 - 3ab^2$

7 Simplify these expressions by collecting the like terms.

a $k + 2 + k + 3$	b $7c + 4 + 5c + 1$	c $8p + 3q + p + 7q$
d $8m + 5n + m - 4n$	e $5t + 12 - 2t + 4$	f $8u + 9v - 3u - v$
g $10g + 4g - 3h + 6h$	h $11p + 2q - 6q - 4p$	i $3b - 5c + 2c - 8b$
j $6s + 11 - 6s + 11$	k $5y - 9 + 5y + 9$	l $4m - 7n - 10m + 5n$
m $x + y - 4x - 7y$	n $-6a + 2b + 5a + 10b$	o $-5j - 12k + 15j - 4k$
p $x^2 + 6x + 2x^2 + 3x$	q $7a^2 + a^2 + a - 4a$	r $9u - 4u^2 - u^2 + 3u$
s $z^2 - 2z + 5z^2 - 3z$	t $d^2 + 7d + 5 - 4d$	u $4mn + 5m - 3mn - 9n$

8 Find, in simplest form, an algebraic expression for the perimeter of each figure.



Further applications

- 9 **a** Subtract $3x^2 - 4x + 10$ from $7x^2 + 2x - 4$.
b From $5a^2 + 9$, take $a^2 - 2a - 5$.
c Find the difference between $5p + 3$ and $2p^2 + 6p + 3$.
d By how much does $4k^2 + 7k + 11$ exceed $k^2 - 2k + 15$?
e Take the sum of $t^2 - t + 4$ and $2t^2 + 17t + 9$ from $4t^2 + 9t + 20$.

2.4

Multiplying and dividing algebraic expressions

Any algebraic terms can be multiplied or divided. They do not have to be like terms.

To multiply algebraic terms:

- ⊙ multiply the co-efficients
- ⊙ multiply the pronumerals.

To divide algebraic terms:

- ⊙ express the division as a fraction
- ⊙ divide the co-efficients
- ⊙ divide the pronumerals.

Example 1

Simplify each of the following:

a $b \times 3$

b $4r \times 5s$

c $\frac{1}{4} \times 24w$

d $8a \times 5a$

e $6xy \times 7yz$

f $-12u \times (-5v)$

Solutions

a $b \times 3 = 3b$

b $4r \times 5s = 20rs$

c $\frac{1}{4} \times 24w = 6w$

d $8a \times 5a = 40a^2$

e $6xy \times 7yz = 42xy^2z$

f $-12u \times (-5v) = 60uv$

Example 2

Simplify each of the following:

a $15p \div 5p$

b $21ab \div 3a$

c $45t^2 \div 9t$

d $64mn^2 \div (-8mn)$

Solutions

a $15p \div 5p$
 $= \frac{15p}{5p}$
 $= 3$

b $21ab \div 3a$
 $= \frac{21ab}{3a}$
 $= 7b$

c $45t^2 \div 9t$
 $= \frac{45t^2}{9t}$
 $= 5t$

d $64mn^2 \div (-8mn)$
 $= \frac{64mn^2}{-8mn}$
 $= -8n$

Exercise 2.4

- 1 **a** Simplify $2a \times 3b$.
b Verify your answer by substituting several pairs of values for a and b .
- 2 **a** Does $5n \times 4n$ equal $20n$ or $20n^2$?
b Substitute a value for n to verify your answer.

- 3 a** Does $12y \div 2y$ equal 6 or $6y$?
b Substitute a value for y to verify your answer.
- 4 a** Simplify $5x \times 3y$ and $3y \times 5x$.
b Does $5x \times 3y = 3y \times 5x$?
c Does it matter in which order algebraic expressions are multiplied?
- 5 a** Simplify $6p \div 12$ and $12 \div 6p$.
b Does $6p \div 12 = 12 \div 6p$?
c Does it matter in which order algebraic expressions are divided?
- 6** Simplify these products.
- | | | | |
|-----------------------------------|------------------------------------|-------------------------------------|-------------------------------------|
| a $5 \times 3n$ | b $6c \times 4$ | c $9w \times 7$ | d $11 \times 8g$ |
| e $u \times 5v$ | f $9m \times n$ | g $7a \times 2b$ | h $8x \times 5y$ |
| i $4c \times 9d$ | j $10r \times 7s$ | k $5p \times 12q$ | l $9v \times 9w$ |
| m $a \times a$ | n $2e \times e$ | o $4k \times 3k$ | p $5h \times 6h$ |
| q $mn \times mp$ | r $6cd \times 7c$ | s $5fg \times 4gh$ | t $4vw \times 8wx$ |
| u $\frac{1}{2}a \times 14$ | v $\frac{1}{3}m \times 12n$ | w $24pq \times \frac{3}{4}r$ | x $15c \times \frac{2}{3}cd$ |
- 7** Simplify these quotients.
- | | | | |
|--------------------------|--------------------------|--------------------------|-----------------------------|
| a $10b \div 2$ | b $21z \div 7$ | c $18k \div 3$ | d $40m \div 5$ |
| e $6w \div w$ | f $32n \div 4n$ | g $ab \div b$ | h $pqr \div pr$ |
| i $50gh \div 5h$ | j $42mn \div 6m$ | k $30xy \div 3y$ | l $54cde \div 9cd$ |
| m $t^2 \div t$ | n $13v^2 \div v$ | o $6u^2 \div 6u$ | p $15a^2 \div 5a$ |
| q $24m^2 \div 3m$ | r $72e^2 \div 8e$ | s $7a^2b \div 7a$ | t $60rs^2 \div 12rs$ |

Consolidation

- 8** Simplify:
- | | | | |
|---------------------------|----------------------------|-----------------------------|--------------------------------|
| a $-3 \times 7y$ | b $-8x \times (-5)$ | c $4g \times (-12h)$ | d $-10b \times (-c)$ |
| e $-j \times (-j)$ | f $-9v \times 3v$ | g $-7ab \times 5b$ | h $-8xy \times (-12yz)$ |
- 9** Simplify:
- | | | | |
|---------------------------------|------------------------------|-------------------------------|--------------------------------|
| a $\frac{-12c}{3}$ | b $\frac{-49n}{-7}$ | c $\frac{27k}{-9k}$ | d $\frac{-36ef}{4e}$ |
| e $\frac{-84mnp}{-12mp}$ | f $\frac{63k^2}{-7k}$ | g $\frac{-25t^2}{-5t}$ | h $\frac{-96u^2v}{8uv}$ |
- 10** Simplify each of the following expressions.
- | | | |
|--------------------------------------|-------------------------------------|---------------------------------------|
| a $3a \times 2b \times c$ | b $4m \times n \times 7p$ | c $5e \div 5 \times 2f$ |
| d $4g \times 3 \div 6g$ | e $24k \div 3k \div 2$ | f $30ab \div 3a \div 2b$ |
| g $9pq \div 3p \times 7q$ | h $6m \times 8n \div 12m$ | i $10a^2 \times 4b \div 5ab$ |
| j $27y \times 2yz \div 6y$ | k $5c \times 2d \times 6cd$ | l $72w^2 \div 9w \div 4w$ |
| m $-2x \times (-3y) \times 7$ | n $15p \times (-3q) \div 9p$ | o $-50rs \div 5r \times (-2s)$ |

Further applications

11 Find the missing term in each of these.

a $3m \times \square = 18m$

d $\square \div 5t = 6$

g $8e \times \square = 40ef$

j $\square \div w = 5w$

m $\square \times -4p = -32pq$

b $\square \times 4 = 28j$

e $\square \times 6x = 24xy$

h $\square \div 6k = 7m$

k $\square \times 9h = 72h^2$

n $-25gh \div \square = 5g$

c $12y \div \square = 3y$

f $36pq \div \square = 12p$

i $5a \times \square = 15a^2$

l $60c^2 \div \square = 5c$

o $\square \div 3x = -9x$

12 Simplify, giving your answers in simplest fraction form.

a $5c \div 10$

b $2 \div 2k$

c $9h \div 6$

d $4ab \div 12a$

e $12mn \div 20n$

f $14u \div 21uv$

g $25cd \div 35de$

h $42s^2 \div 49s$

i $18uv \div 27v^2$

j $35x^2 \div 60xy$

k $36abc \div 44bcd$

l $72e^2f \div 56ef^2$

TRY THIS

Overhanging the overhang

Place a ruler on the edge of a table. How far will it overhang the edge of a table before it topples?

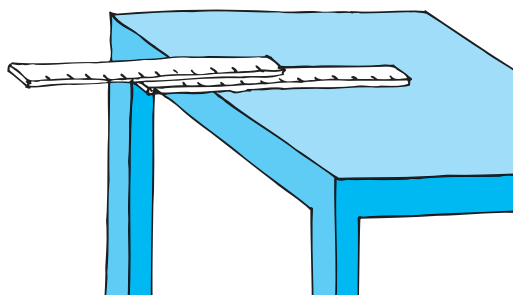
Now move the ruler so that it overhangs the table by 10 cm. Place another ruler on top of this first ruler. How far can this ruler overhang the first before it topples?

Now vary the bottom ruler each time. Continue to see how far you can overhang the top ruler.

Record your results.

Where should you place the two rulers so that you obtain the greatest possible overhang?

Now try three rulers and repeat the procedure. If possible, try four rulers. What conclusions can you make? Could you make a deduction if you had n rulers?



2.5

The order of operations

When simplifying expressions that contain several terms, follow the order of operations.

The order of operations is to:

- ⑥ simplify any expressions inside grouping symbols
- ⑥ simplify any multiplications and divisions, working from left to right
- ⑥ simplify any additions and subtractions, working from left to right.

**Example**

Simplify:

a $42cd \div 7c \times 5e$

b $40u - 9u \times 3 + 5u$

c $[25a - (3a + 12a)] \div 2a$

Solutions

$$\begin{aligned} \mathbf{a} \quad & 42cd \div 7c \times 5e \\ &= \frac{42cd}{7c} \times 5e \\ &= 6d \times 5e \\ &= 30de \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & 40u - (9u \times 3) + 5u \\ &= 40u - 27u + 5u \\ &= 13u + 5u \\ &= 18u \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & [25a - (3a + 12a)] \div 2a \\ &= [25a - 15a] \div 2a \\ &= \frac{10a}{2a} \\ &= 5 \end{aligned}$$

Exercise 2.5**1** Simplify:

a $3 \times (4n + 2n)$

d $12x - (5x + 3x)$

g $5p \times (3q + 9q)$

j $36y^2 \div (13y - 4y)$

m $2 \times (2f + 4f) \times 4$

b $(15q - 3q) \div 4$

e $3t \times (12t - 4t)$

h $(11c - c) \times 2d$

k $63gh \div (3g \times 3h)$

n $5 \times (17t - 9t) \div 4t$

c $(8j + 5j) \times 2$

f $(s + 7s) \times 4s$

i $21b \div (5b + 2b)$

l $50cd \div (8d + 2d)$

o $(17a^2 + 3a^2) \div (9a - 4a)$

Consolidation**2** Simplify these expressions by removing the innermost grouping symbols first.

a $[11t + (3 \times 4t)] + 2t$

d $6c + [9c - (10c - 5c)]$

g $32r - [12r + (45r \div 9)]$

b $[17y - (27y \div 3)] - y$

e $5 \times [(15n + 6n) \div 7]$

h $[(22f - 4f) \div 2] \times 5f$

c $[40g - (7g \times 5)] \times 4$

f $[8w + (4 \times 10w)] \div 12w$

i $-8k - [17k - (19k - 13k)]$

3 Simplify each expression using the order of operations.

a $5k + 3k \times 2$

d $25v^2 - 6v \times 4v$

g $18ef - 12ef \div 3$

j $24a \div 8 + 4a \times 2$

m $10g + 5g \times 3 + 2g$

b $20z - 14z \div 2$

e $22ab - 5a \times 3b$

h $7y + 20xy \div 4x$

k $8a \times 4b + a^2b \div a$

n $6x - 8 \times 2x + x$

c $4n \times 2n + 7n^2$

f $28pq \div 4p + 6q$

i $7 \times 2s - 5s \times 2$

l $100x^2 \div 2x - 8 \times 5x$

o $2k - 32k \div 4 - 3k$

4 Express each of these in simplest form.

a $\frac{10x \times 6}{4 \times 3x}$

b $\frac{19u + 9u}{13u - 6u}$

c $\frac{8p \times 3q}{12p - 6p}$

d $\frac{33rs - 15sr}{3r \times 2s}$

Further applications**5** Insert grouping symbols in each of these to make a true statement.

a $4 \times 2s + 3s = 20s$

d $24e^2 - 6e^2 \div 6e = 3e$

b $40pq \div 5p \times 2q = 4$

e $8 \times 4n - 5n \times 3 = 17n$

c $16a - 4a + 2a - 7a = 3a$

f $8w + 9w^2 \times 6 \div 3w = 26w$

2.6 The distributive law

Algebraic expressions can be expanded by the use of the **distributive law**.

To expand an expression by using the distributive law:

- ⊙ multiply the term outside the grouping symbols by each term inside.

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

Example 1

Expand:

a $4(k + 5)$

b $w(w - 1)$

c $6g(4g - 7h)$

Solutions

a $4(k + 5)$
 $= (4 \times k) + (4 \times 5)$
 $= 4k + 20$

b $w(w - 1)$
 $= (w \times w) - (w \times 1)$
 $= w^2 - w$

c $6g(4g - 7h)$
 $= (6g \times 4g) - (6g \times 7h)$
 $= 24g^2 - 42gh$

Example 2

Expand:

a $-5(n + 2)$

b $-7(e - 3)$

c $-8z(3x - 4y)$

Solutions

a $-5(n + 2)$
 $= -5n - 10$

b $-7(e - 3)$
 $= -7e + 21$

c $-8z(3x - 4y)$
 $= 24xz + 32yz$

Example 3

Expand and simplify:

a $3(b + 2) + 10$

b $12 + 4(a - 5)$

c $9(x + 5) - 4(x - 10)$

Solutions

a $3(b + 2) + 10$
 $= 3b + 6 + 10$
 $= 3b + 16$

b $12 + 4(a - 5)$
 $= 12 + 4a - 20$
 $= 4a - 8$

c $9(x + 5) - 4(x - 10)$
 $= 9x + 45 - 4x + 40$
 $= 5x + 85$

Exercise 2.6

1 Expand each of the following.

a $3(a + 4)$

b $5(p - 2)$

c $7(m + 1)$

d $8(5 - k)$

e $4(5h + 7)$

f $6(2y - 3)$

g $5(3m + 7n)$

h $2(9y - 10z)$

i $a(b + c)$	j $p(q - r)$	k $e(2f + g)$	l $k(4m - 11n)$
m $3t(u + v)$	n $6k(3m - 4)$	o $4f(5g - 7h)$	p $12r(3s + 5t)$
q $x(x + y)$	r $b(1 - b)$	s $7n(2n - 7)$	t $9vw(3v - 8w)$

2 Expand each of these.

a $-2(n + 7)$	b $-3(b - 6)$	c $-9(k - 1)$	d $-11(8 + u)$
e $-5(2j + 9)$	f $-6(7 - 10y)$	g $-x(y + z)$	h $-t(3u - v)$
i $-c(5d + 2e)$	j $-2n(p + q)$	k $-9r(5s - 3)$	l $-6h(4i - 11j)$
m $-s(s - t)$	n $-j(1 + j)$	o $-6y(5y - 12)$	p $-4mn(2m + 5n)$

3 Expand:

a $(x + 5)6$	b $(j - 2)7$	c $(k + 8)m$	d $(2p + 3)4$
e $(c - d)d$	f $(3a + 7b)5c$	g $(5s - 2t)4s$	h $(3m + 8n)2mn$

Consolidation

4 Expand and simplify each of these expressions.

a $5(n + 7) + 6$	b $4(c + 5) + 3c$	c $6(q + 4) - 11$
d $12(3 + t) - 5t$	e $-3(m + 2) + 10$	f $-7(2n - 3) - 5$
g $10a + 4(6 - a)$	h $7 + 3(4x - 1)$	i $2q - 6(5 + 2q)$
j $4m + 8(2m - 11)$	k $8 - (2x - 7)$	l $5c - 6(1 - 4c)$
m $5(2m + 9) + m + 15$	n $3k + 9 + 2(k - 4)$	o $12x + 17 - 2(3x - 5)$
p $7(5t + 3) - 10t - 15$	q $4y + 3(y + 7) + 8$	r $5w - 4(w - 3) - 9$

5 Expand each of these, then collect the like terms.

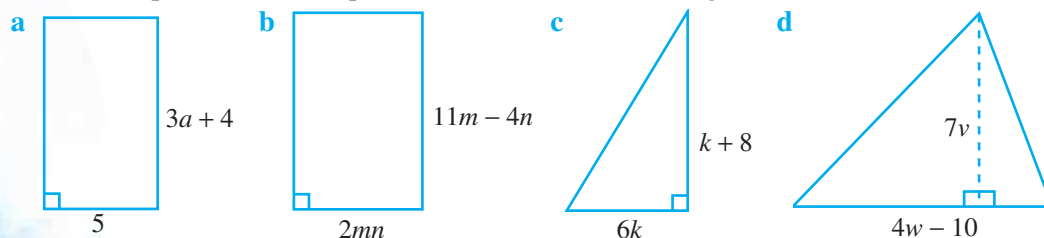
a $3(n + 4) + 5(n + 2)$	b $6(z + 5) + 4(z - 2)$	c $7(p - 2) + 8(p + 3)$
d $5(w + 2) + 2(w - 5)$	e $4(x + 3) - 3(x - 5)$	f $3(n - 1) - 7(n - 2)$
g $9(a + 6) - 7(3 - a)$	h $-4(s - 5) - 6(s - 1)$	i $8(2b + 3) + 3(3b - 2)$
j $6(3c - 4) - 5(4 - 3c)$	k $-3(7y + 2) + 5(2y + 3)$	l $-6(3k + 4) - 9(12 - 2k)$
m $x(x + 5) + 3(x + 9)$	n $y(y - 2) + 6(y - 7)$	o $3a(a + 6) + 2a(a + 4)$
p $4g(g + 3) - 6g(g - 2)$	q $8u(u - 2) - 5u(7 - u)$	r $10c(2d + e) + 5c(3d + 4e)$

6 Are the following statements true (T) or false (F)? Explain.

a $6(2p + 5) = (2p + 5)6$	b $7(3y + 2) = 21y + 2$	c $5 + 4(x - 1) = 9(x - 1)$
d $-2(5v - 3) = -10v - 6$	e $ab(a + b) = a^2b + ab^2$	f $-(w - 2) = 2 - w$

Further applications

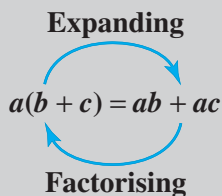
7 Find, in simplest form, an expression for the area of each figure.



2.7

The highest common factor

To **factorise** an expression means to write the expression as the product of its factors. This is the same as reversing or undoing the expansion process.



Many expressions can be factorised in several different ways. For example, we can factorise $8n + 16$ as $1(8n + 16)$ or $2(4n + 8)$ or $4(2n + 4)$ or $8(n + 2)$. However, by convention, we use the **highest common factor (HCF)**, that is, the *greatest possible factor* that is *common to every term* in the expression, when factorising. In this example, the HCF of $8n$ and 16 is 8 . Hence, the correct factorisation of $8n + 16$ is $8(n + 2)$.

To factorise an algebraic expression:

- ⊙ write the HCF of the terms outside the grouping symbols
- ⊙ divide each term in the expression by the HCF to find the terms inside the grouping symbols.

$$ab + ac = a(b + c) \quad \text{and} \quad ab - ac = a(b - c)$$

NOTE:

- If the first term of an expression is negative, then by convention, the HCF is also negative.
- Factorisations should be checked by expanding the answers.

Example 1

Factorise:

a $3x + 12$

d $a^2 + 8a$

b $2r - 14$

e $12t^2 - 16tu$

c $10p + 45$

f $m^2n + mn^2 - mnp$

Solutions

a $3x + 12$
 $= 3 \times x + 3 \times 4$
 $= 3(x + 4)$

d $a^2 + 8a$
 $= a \times a + a \times 8$
 $= a(a + 8)$

b $2r - 14$
 $= 2 \times r - 2 \times 7$
 $= 2(r - 7)$

e $12t^2 - 16tu$
 $= 4t \times 3t - 4t \times 4u$
 $= 4t(3t - 4u)$

c $10p + 45$
 $= 5 \times 2p + 5 \times 9$
 $= 5(2p + 9)$

f $m^2n + mn^2 - mnp$
 $= mn \times m + mn \times n - mn \times p$
 $= mn(m + n - p)$

**Example 2**

Factorise:

a $-7g - 28$

b $-ab + bc$

Solutions

$$\begin{aligned}\mathbf{a} \quad & -7g - 28 \\ & = -7 \times g - 7 \times (+4) \\ & = -7(g + 4)\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad & -ab + bc \\ & = -b \times a - b \times (-c) \\ & = -b(a - c)\end{aligned}$$

Exercise 2.7**1** Complete each of these factorisations.

a $2n + 6 = 2(\quad)$	b $3p - 15 = 3(\quad)$	c $7y + 7 = 7(\quad)$
d $4g + 10 = 2(\quad)$	e $12a - 9 = 3(\quad)$	f $15k - 25m = 5(\quad)$
g $ax + ay = a(\quad)$	h $pq - qr = q(\quad)$	i $st - t = t(\quad)$
j $m^2 + 3m = m(\quad)$	k $4r - r^2 = r(\quad)$	l $ab + b^2 = b(\quad)$
m $5d^2 + 10d = 5d(\quad)$	n $12p^2 - 21p = 3p(\quad)$	o $35yz + 28y^2 = 7y(\quad)$

2 Factorise each of these expressions by taking out the highest common factor.

a $2c + 8$	b $5y + 10$	c $18 + 3q$	d $35 + 7p$
e $2h - 14$	f $6t - 30$	g $33 - 3r$	h $48 - 4n$
i $5c + 5d$	j $3x - 6y$	k $21g + 7h$	l $8m - 40n$
m $ab + ac$	n $uv - uw$	o $ef - fg$	p $rs - r$
q $b^2 + bc$	r $k^2 - 8k$	s $11n + n^2$	t $a - a^2$

Consolidation**3** Factorise by removing the highest common factor.

a $6n + 9$	b $10b + 25$	c $10y + 12$	d $12k - 8$
e $21w - 35$	f $18s - 21$	g $16a + 24$	h $18t - 30$
i $30p + 27$	j $14c + 49$	k $30r - 80$	l $22e - 99$
m $35 - 55h$	n $90 + 63v$	o $39 + 26z$	p $24 - 60j$

4 Factorise each expression completely.

a $3ab + 9bc$	b $2xy + 8xz$	c $4pq - 20qr$	d $7gh - 14hi$
e $4uv + 6uw$	f $8ef + 20fg$	g $33rs - 77qr$	h $24mn - 20mp$
i $7c^2 + 21c$	j $24w^2 - 6w$	k $10g^2 - 22g$	l $15y + 40y^2$
m $mnp + mnq$	n $rst - rtu$	o $a^2b + ab^2$	p $def - de^2$
q $j^2k - jk^2m$	r $12tu + 15u^2v$	s $4ab^2 + 10a^2bc$	t $49x^2y^2 - 42xyz$

5 Factorise:

a $3a + 3b + 3c$	b $pq + pr - ps$	c $a^2 - ab - ac$
d $5r + 10s + 25$	e $4x^2 - 10x + 8xy$	f $6 + 24u - 18u^2$
g $42k^2 - 14k + 21$	h $3mn - m + mn^2$	i $2x^2 + 2xy - 6x$
j $30t - 15tu + 10t^2$	k $4cd + 28c^2 - 20ce$	l $21f - 70fg - 56f^2$
m $a^2b + ab^2 + ab$	n $8pq - p^2q + pq^2$	o $u^2vw - uv^2w - uvw^2$

6 Explain why each of these expressions has not been correctly or completely factorised.

a $8x + 12 = 2(4x + 6)$

b $p^2 + 7p = p(2 + 7)$

c $e^2 + e = e(e + 0)$

d $abc + abd = a(bc + bd)$

e $7uv + 14u = 7u(v + 14u)$

f $3p + 3q + 15 = 3(p + q) + 15$

Further applications

7 Factorise by taking out the greatest negative common factor.

a $-2p - 12$

b $-3x - 21$

c $-15g - 20$

d $-14u - 49$

e $-2t + 2$

f $-8w + 24$

g $-12k + 16$

h $-9r + 30$

i $-24 - 15m$

j $-18 + 45q$

k $-36 + 24y$

l $-63 - 77c$

m $-ab + bc$

n $-mn - km$

o $-x^2 - 2x$

p $-4e + e^2$

q $-9k^2 + 12k$

r $-20a - 28a^2$

s $-25b + 55bc$

t $-48x^2y - 60y^2$

8 Factorise by taking out the binomial common factor.

a $a(b + c) + 5(b + c)$

b $m(x - y) + n(x - y)$

c $p(p + 3) + 4(p + 3)$

d $x(a + 1) - 2(a + 1)$

e $3(m - 7) - n(m - 7)$

f $a^2(p + q) - 6(p + q)$

g $5c(c + 4) + 2(c + 4)$

h $8(1 - k) - 3m(1 - k)$

i $y(2s + 3) - z(2s + 3)$

j $4g(3w - 5) + 9h(3w - 5)$

k $x(x - 7) + (x - 7)$

l $(7b + 2c) - 3d(2c + 7b)$

2.8

Adding and subtracting algebraic fractions

To add or subtract algebraic fractions:

- ⊙ express the fractions with a common denominator
- ⊙ add or subtract the numerators
- ⊙ simplify if possible.

Example 1

Simplify:

a $\frac{11m}{12} + \frac{5m}{12}$

b $\frac{4}{3c} + \frac{5}{3c}$

c $\frac{11k}{10} - \frac{3k}{5}$

d $\frac{5h}{6} - \frac{3h}{4}$

Solutions

a $\frac{11m}{12} + \frac{5m}{12}$

$$= \frac{16m}{12}$$

$$= \frac{4m}{3}$$

b $\frac{4}{3c} + \frac{5}{3c}$

$$= \frac{9}{3c}$$

$$= \frac{3}{c}$$

c $\frac{11k}{10} - \frac{3k}{5}$

$$= \frac{11k}{10} - \frac{6k}{10}$$

$$= \frac{5k}{10}$$

$$= \frac{k}{2}$$

d $\frac{5h}{6} - \frac{3h}{4}$

$$= \frac{10h}{12} - \frac{9h}{12}$$

$$= \frac{h}{12}$$

EG
+S**Example 2**

Simplify:

a $\frac{1}{a} + \frac{5}{2a}$

Solutions

$$\begin{aligned} \text{a} \quad & \frac{1}{a} + \frac{5}{2a} \\ &= \frac{2}{2a} + \frac{5}{2a} \\ &= \frac{7}{2a} \end{aligned}$$

b $\frac{13}{20w} - \frac{2}{5w}$

$$\begin{aligned} \text{b} \quad & \frac{13}{20w} - \frac{2}{5w} \\ &= \frac{13}{20w} - \frac{8}{20w} \\ &= \frac{5}{20w} \\ &= \frac{1}{4w} \end{aligned}$$

c $\frac{7x}{12y} + \frac{5x}{8y}$

$$\begin{aligned} \text{c} \quad & \frac{7x}{12y} + \frac{5x}{8y} \\ &= \frac{14x}{24y} + \frac{15x}{24y} \\ &= \frac{29x}{24y} \end{aligned}$$

EG
+S**Example 3**

Simplify:

$\frac{k+4}{3} + \frac{k-2}{5}$

Solution

$$\begin{aligned} & \frac{k+4}{3} + \frac{k-2}{5} \\ &= \frac{5(k+4)}{15} + \frac{3(k-2)}{15} \\ &= \frac{5k+20+3k-6}{15} \\ &= \frac{8k+14}{15} \end{aligned}$$

Exercise 2.8**1** Simplify:

a $\frac{3a}{7} + \frac{2a}{7}$

e $\frac{3n}{8} + \frac{3n}{8}$

i $\frac{6b}{7} + \frac{8b}{7}$

b $\frac{5m}{9} - \frac{m}{9}$

f $\frac{11k}{12} - \frac{3k}{12}$

j $\frac{14w}{15} - \frac{4w}{15}$

c $\frac{9h}{13} - \frac{8h}{13}$

g $\frac{5c}{3} - \frac{2c}{3}$

k $\frac{19e}{24} - \frac{9e}{24}$

d $\frac{x}{4} + \frac{x}{4}$

h $\frac{9d}{10} + \frac{3d}{10}$

l $\frac{13s}{16} + \frac{9s}{16}$

2 Simplify:

a $\frac{5}{x} + \frac{2}{x}$

b $\frac{8}{p} - \frac{7}{p}$

c $\frac{10}{3y} + \frac{4}{3y}$

d $\frac{12}{7q} - \frac{3}{7q}$

e $\frac{1}{2n} + \frac{1}{2n}$

f $\frac{3}{4c} + \frac{5}{4c}$

g $\frac{11}{5g} + \frac{9}{5g}$

h $\frac{13}{12k} - \frac{4}{12k}$

i $\frac{17a}{10r} - \frac{9a}{10r}$

j $\frac{4m}{15b} + \frac{2m}{15b}$

k $\frac{7e}{20v} + \frac{8e}{20v}$

l $\frac{19t}{16z} - \frac{15t}{16z}$

Consolidation

3 Express these fractions with a common denominator, then simplify.

a $\frac{n}{2} + \frac{n}{4}$

b $\frac{a}{3} - \frac{a}{9}$

c $\frac{k}{3} + \frac{k}{12}$

d $\frac{d}{5} - \frac{d}{15}$

e $\frac{y}{5} + \frac{y}{2}$

f $\frac{t}{3} - \frac{t}{4}$

g $\frac{b}{4} - \frac{b}{7}$

h $\frac{h}{12} + \frac{h}{5}$

i $\frac{2c}{5} + \frac{3c}{10}$

j $\frac{5m}{12} - \frac{m}{3}$

k $\frac{3r}{5} + \frac{4r}{3}$

l $\frac{3u}{2} - \frac{6u}{7}$

m $\frac{w}{4} + \frac{5w}{6}$

n $\frac{7x}{6} - \frac{2x}{9}$

o $\frac{3f}{10} + \frac{5f}{8}$

p $\frac{11s}{12} - \frac{8s}{9}$

4 Explain why each of the following answers is not correct.

a $\frac{5m}{9} + \frac{2m}{9} = \frac{7m}{18}$

b $\frac{3w}{5} + \frac{2w}{3} = \frac{5w}{8}$

c $\frac{4}{5a} + \frac{3}{5a} = 1\frac{2}{5}a$

5 Simplify each of the following.

a $\frac{1}{x} + \frac{3}{2x}$

b $\frac{2}{3a} + \frac{1}{6a}$

c $\frac{17}{20e} - \frac{2}{5e}$

d $\frac{13}{12p} - \frac{2}{3p}$

e $\frac{3}{2u} + \frac{2}{3u}$

f $\frac{4}{5f} - \frac{3}{4f}$

g $\frac{2}{3t} - \frac{4}{7t}$

h $\frac{3}{5h} + \frac{5}{9h}$

i $\frac{5c}{4j} + \frac{11c}{6j}$

j $\frac{9m}{8z} - \frac{5m}{6z}$

k $\frac{9a}{10g} - \frac{3a}{4g}$

l $\frac{5k}{12n} + \frac{7k}{8n}$

Further applications

6 Simplify:

a $\frac{n+2}{2} + \frac{n+1}{6}$

b $\frac{b+3}{4} + \frac{b+4}{7}$

c $\frac{x+8}{5} + \frac{x-2}{3}$

d $\frac{m-3}{6} + \frac{m+6}{7}$

e $\frac{2w-5}{12} + \frac{w-1}{4}$

f $\frac{3s+2}{9} + \frac{2s-7}{5}$

g $\frac{x+7}{2} - \frac{x+3}{4}$

h $\frac{3c+10}{5} - \frac{c-3}{4}$

i $\frac{7e-1}{8} - \frac{2e-5}{3}$

2.9

Multiplying and dividing algebraic fractions

To multiply algebraic fractions:

- ⊙ cancel any common factors between the numerators and the denominators
- ⊙ multiply the numerators
- ⊙ multiply the denominators.

To divide algebraic fractions:

- ⊙ change the division sign to a multiplication sign and take the reciprocal of the second fraction
- ⊙ proceed as above for the multiplication of fractions.

NOTE: Any fractions can be multiplied or divided. They do not need to have a common denominator.

Example 1

Simplify:

a $\frac{m}{3} \times \frac{n}{4}$

Solutions

$$\begin{aligned} \text{a} \quad & \frac{m}{3} \times \frac{n}{4} \\ &= \frac{mn}{12} \end{aligned}$$

b $\frac{15x}{14y} \times \frac{7}{9x}$

$$\begin{aligned} \text{b} \quad & \frac{\overset{5}{\cancel{15}}x}{\underset{2}{\cancel{14}}y} \times \frac{\underset{3}{\cancel{7}}}{\underset{9}{\cancel{9}}x} \\ &= \frac{5}{6y} \end{aligned}$$

c $\frac{a^2}{bc^2} \times \frac{bc}{a}$

$$\begin{aligned} \text{c} \quad & \frac{\overset{2}{\cancel{a}}^2}{\cancel{b}c^2} \times \frac{\cancel{b}\overset{1}{\cancel{a}}}{\cancel{a}} \\ &= \frac{a}{c} \end{aligned}$$

Example 2

Simplify:

a $\frac{e}{4} \div \frac{7}{f}$

Solutions

$$\begin{aligned} \text{a} \quad & \frac{e}{4} \div \frac{7}{f} \\ &= \frac{e}{4} \times \frac{f}{7} \\ &= \frac{ef}{28} \end{aligned}$$

b $\frac{9c}{10d} \div \frac{12c^2}{25de}$

$$\begin{aligned} \text{b} \quad & \frac{9c}{10d} \div \frac{12c^2}{25de} \\ &= \frac{\overset{3}{\cancel{9}}c}{\underset{2}{\cancel{10}}d} \times \frac{\overset{5}{\cancel{25}}de}{\underset{4}{\cancel{12}}c^2} \\ &= \frac{15e}{8c} \end{aligned}$$

Exercise 2.9**1** Simplify:

a $\frac{a}{3} \times \frac{b}{2}$

b $\frac{u}{3} \times \frac{u}{4}$

c $\frac{a}{b} \times \frac{c}{d}$

d $\frac{1}{p} \times \frac{1}{q}$

e $\frac{1}{x} \times \frac{1}{4x}$

f $\frac{4c}{5} \times \frac{d}{3}$

g $\frac{9m}{7} \times \frac{3n}{4}$

h $\frac{5}{6x} \times \frac{7}{8x}$

2 Simplify:

a $\frac{x}{5} \div \frac{4}{y}$

b $\frac{v}{2} \div \frac{6}{v}$

c $\frac{t}{u} \div \frac{v}{w}$

d $\frac{1}{g} \div \frac{1}{h}$

e $\frac{1}{s} \div 2s$

f $\frac{3e}{7} \div \frac{5f}{6}$

g $\frac{10a}{11} \div \frac{3b}{4}$

h $\frac{4}{5h} \div \frac{3h}{13}$

Consolidation**3** Simplify each of the following by at first cancelling common factors.

a $\frac{n}{3} \times \frac{2}{n}$

b $\frac{a}{4} \times \frac{8}{b}$

c $\frac{3}{c} \times \frac{d}{15}$

d $\frac{3}{x} \times \frac{2x}{7}$

e $\frac{ab}{e} \times \frac{cd}{bc}$

f $\frac{5a}{3b} \times \frac{b}{10a}$

g $\frac{8d}{7c} \times \frac{21}{8e}$

h $\frac{4e}{10f} \times \frac{5f}{12e}$

i $\frac{9t}{14v} \times \frac{7u}{18tu}$

j $\frac{15w}{27y} \times \frac{18x}{25w}$

k $\frac{11i}{12h} \times \frac{21h}{22ij}$

l $\frac{44r}{35pq} \times \frac{10p}{99rs}$

4 Express each of these as a multiplication, then simplify.

a $\frac{x}{5} \div \frac{x}{3}$

b $\frac{m}{2} \div \frac{n}{6}$

c $\frac{5}{u} \div \frac{20}{v}$

d $\frac{6}{r} \div \frac{11s}{3r}$

e $\frac{ef}{g} \div \frac{eh}{hi}$

f $\frac{10k}{3} \div \frac{5m}{12}$

g $\frac{4s}{7t} \div \frac{16s}{t}$

h $\frac{9w}{28v} \div \frac{27w}{7v}$

i $\frac{4p}{33q} \div \frac{20pr}{11r}$

j $\frac{12c}{45b} \div \frac{16c}{25a}$

k $\frac{12e}{63d} \div \frac{20ef}{99d}$

l $\frac{42xy}{55x} \div \frac{49yz}{60w}$

5 Explain why each of the following solutions is incorrect.

a $\frac{5a}{3b} \times \frac{10}{11b} = \frac{2a}{33}$

b $\frac{4c}{7d} \div \frac{21}{c} = \frac{4}{3}$

6 Simplify each of the following.

a $\frac{p^2}{q} \times \frac{q^2}{p}$

b $\frac{5m}{4np} \times \frac{2n}{3m^2}$

c $\frac{2a^2}{3b^2} \times \frac{7b}{5a}$

d $\frac{ab^2}{p^2q} \times \frac{pq}{ab}$

e $\frac{r^2s}{tu^2} \div \frac{rs}{tuv}$

f $\frac{8e}{21f^2} \div \frac{24e^2}{35f}$

g $\frac{12x^2y}{25ab} \div \frac{28xy^2}{15bc}$

h $\frac{24tu^2}{33vw} \div \frac{36t^2u}{55wx}$

7 Simplify:

$$\text{a } \frac{ab}{bc} \times \frac{cd}{de} \times \frac{ef}{ag}$$

$$\text{b } \frac{5m}{7n} \times \frac{14p}{15m} \times \frac{9n}{16q}$$

$$\text{c } \frac{9r}{20s} \times \frac{15s}{22u} \div \frac{27r}{11t}$$

$$\text{d } \frac{15w}{7x} \div \frac{40y}{9x} \times \frac{16xy}{45w}$$

$$\text{e } \frac{21a^2}{32bc} \times \frac{55e^2}{63ab} \div \frac{45e}{24b^2c}$$

$$\text{f } \frac{14pq^2}{9ru} \div \frac{49qr}{18tu^2} \div \frac{30stu}{25r^2s}$$

Further applications

8 Factorise each expression where possible, then simplify.

$$\text{a } \frac{3x+12}{12} \times \frac{8}{x+4}$$

$$\text{b } \frac{5m+30}{3m-21} \times \frac{9m-63}{45}$$

$$\text{c } \frac{24m^2}{6k+42} \times \frac{5k+35}{18m}$$

$$\text{d } \frac{12t-12}{3u} \times \frac{2u+8}{8t-8}$$

$$\text{e } \frac{c^2+c}{3c+3} \times \frac{3c^2+6c}{6c^2}$$

$$\text{f } \frac{a^2+2a}{21x-21y} \times \frac{14x-14y}{5a+10}$$

$$\text{g } \frac{25a^2b}{18a-27b} \div \frac{35ab^2}{12a-18b}$$

$$\text{h } \frac{15u+20v}{24u-60v} \div \frac{30u+40v}{16u-40v}$$

$$\text{i } \frac{8bc-16c}{6ab-30a} \div \frac{4bc-8c}{3ab+15a}$$

2.10 Generalised arithmetic

We use generalised arithmetic to form a general expression to describe any value in a situation. For example, if Alicia is 10 years old, then:

- in 1 years time she will be $(10 + 1)$ years old
- in 2 years time she will be $(10 + 2)$ years old
- in k years time she will be $(10 + k)$ years old.

Her exact age in *any number of years time* can be worked out simply by adding that number to 10.

To form a general expression for a situation, choose a few numbers and look for a pattern in the answers. For example, to find the number of centimetres in y m, consider:

$$\begin{array}{llll} 1 \text{ m} = (1 \times 100) \text{ cm} & 2 \text{ m} = (2 \times 100) \text{ cm} & 3 \text{ m} = (3 \times 100) \text{ cm} & y \text{ m} = (y \times 100) \text{ cm} \\ = 100 \text{ cm} & = 200 \text{ cm} & = 300 \text{ cm} & = 100y \text{ cm} \end{array}$$

Being able to form a general expression is an essential skill in mathematics.

Listed below are some common key words and phrases and their meaning.

- Addition—sum, increase, add, plus, total, more than
- Subtraction—difference, decrease, subtract, take away, reduce, less than
- Multiplication—product, times, multiply, double, multiple
- Division—quotient, divide, halve, share

NOTE: In additions and subtractions where the second term is a pronumeral, the words ‘sum’ and ‘difference’ are usually preferred to phrases such as ‘more than’ and ‘less than’.

Odd and even numbers both have the same general expression because both odd numbers and even numbers increase by 2. So, if n is an odd number, then $n + 2$, $n + 4$, $n + 6$, ... are all odd.

However, if n is an even number, then $n + 2$, $n + 4$, $n + 6$, ... are all even. Whether such expressions are odd or even depends on whether n is odd or even.

EG
+
S**Example 1**

Write an algebraic expression for each of the following.

a five more than k	b two less than y
c the sum of m and n	d the difference between p and q
e the product of h and 3	f the quotient of d and e
g one-quarter of c	h two-thirds of u
i the square of w	j twice the cube of x

Solutions

a $k + 5$	b $y - 2$	c $m + n$	d $p - q$	e $3h$
f $\frac{d}{e}$	g $\frac{c}{4}$	h $\frac{2u}{3}$	i w^2	j $2x^3$

EG
+
S**Example 2**

Write the meaning of each expression in words.

a $3m - 5$	b $\frac{x}{y} + 7$	c $\frac{e - f}{10}$	d $4(g + 2)$

Solutions

- a** 5 less than the product of 3 and m
b 7 more than the quotient of x and y
c one-tenth of the difference between e and f
d 4 times the number which is 2 more than g

EG
+
S**Example 3**

Write down 3 consecutive numbers, the first of which is:

a n	b $n + 7$	c $n - 1$

Solutions

a $n, n + 1, n + 2$	b $n + 7, n + 8, n + 9$	c $n - 1, n, n + 1$

EG
+
S**Example 4**

Write down 3 consecutive:

- a** even numbers, the first of which is t
b even numbers, the first of which is $t + 5$
c odd numbers, the first of which is $3t$
d odd numbers, the first of which is $t - 1$

Solutions

a $t, t+2, t+4$ **b** $t+5, t+7, t+9$ **c** $3t, 3t+2, 3t+4$ **d** $t-1, t+1, t+3$

Exercise 2.10

1 Write an algebraic expression for each of the following.

a 3 more than x	b 5 less than t
c the sum of p and q	d the difference between m and n
e the sum of x, y and 7	f the product of m and 4
g 9 times the number n	h the product of $a, 2$ and b
i half of k	j one-quarter of z
k two-thirds of w	l the quotient of u and v
m the number of times that j divides into 4	n the square of k
o the cube of y	p the square root of g

2 Write each expression in words.

a $n+4$	b $q-6$	c $c+d$	d $x-y$
e $8u$	f $5ef$	g $\frac{h}{3}$	h $\frac{3v}{4}$
i $\frac{m}{n}$	j a^2	k g^3	l $\sqrt{d}$

Consolidation

3 Write an algebraic expression for each of these.

a 3 more than the product of 2 and x	b 1 less than the product of y and 5
c the sum of 7 and the product of p and q	d the difference between 4 and the square of u
e 6 more than half of c	f 9 less than one-fifth of w
g 2 more than the quotient of e and f	h 4 less than seven-tenths of r
i one-third of the sum of b and 1	j half the difference between g and h
k 3 times the number that is 12 more than a	l 9 times the number that is 3 less than p
m 4 times the sum of c and d	n 10 times the difference between r and s
o twice the square of y	p 8 times the cube of x
q the quotient of 5 and the square of j	r 1 more than half the cube of b

4 Write each of these algebraic expressions in words.

a $5x+7$	b $2n-3$	c $gh+4$	d $9-pq$
e $\frac{a+3}{4}$	f $\frac{b}{6}+8$	g $\frac{m-n}{7}$	h $u-\frac{v}{w}$
i $5(e+2)$	j $\frac{2(c-d)}{3}$	k $3r^2$	l $2s^3-9$

5 Write down an algebraic expression in simplest form for the number that is:

a 5 more than $t+2$	b 4 less than $p+13$
c 8 less than $3k-2$	d 6 more than $7y-4$

- 6 Write down three consecutive numbers, the first of which is:
- | | | | |
|-------------------|------------------|-------------------|------------------|
| a 3 | b a | c x^2 | d $t + 5$ |
| e $p - 11$ | f $k - 1$ | g $2n - 2$ | h $2 - u$ |
- 7 Write down three consecutive even numbers, the first of which is:
- | | | | | |
|------------|--------------|------------------|------------------|------------------|
| a 6 | b n | c $p + 8$ | d $x - 5$ | e $g - 2$ |
|------------|--------------|------------------|------------------|------------------|
- 8 Write down three consecutive odd numbers, the first of which is:
- | | | | | |
|------------|--------------|------------------|-------------------|------------------|
| a 3 | b k | c $y + 7$ | d $c - 12$ | e $s - 3$ |
|------------|--------------|------------------|-------------------|------------------|
- 9 Find three consecutive numbers such that:
- | | |
|-----------------------------------|------------------------------------|
| a the middle number is m | b the largest number is w |
|-----------------------------------|------------------------------------|
- 10 **a** Gary has k shirts hanging in his closet. How many shirts will he have after buying 3 more?
- b** Nerida has $\$d$ in her purse. How much will she have left after spending $\$4$?
- c** If there are c matches in a box, how many matches are there in 10 boxes?
- d** How much will each person receive if $\$x$ is shared equally among 6 people?
- 11 Brett had $\$p$ in his wallet and donated $q\%$ of this money to charity.
- | | |
|---|-------------------------------------|
| a How much money did Brett donate? | b How much did he have left? |
|---|-------------------------------------|
- 12 Write an expression for the average of u and v .
- 13 **a** Two of the angles in a triangle are p° and q° . What size is the third angle?
- b** Three of the angles in a quadrilateral are a° , b° and c° . What size is the fourth angle?
- 14 **a** Liesl is 9 years old. How old will she be in t years time?
- b** Vinoo is 16 years old. How old was he j years ago?
- 15 Matthew is x years older than Greg and Greg is y years older than Tim.
- | | |
|--|---------------|
| a How much older is Matthew than Tim? | |
| b If Matthew is w years old, find the age of: | |
| i Greg | ii Tim |
- 16 Find the perimeter of:
- | | |
|--|--------------------------------------|
| a an equilateral triangle with sides f cm | b a rhombus with sides n cm |
|--|--------------------------------------|
- 17 Convert:
- | | | |
|-------------------------|---------------------------|-----------------------|
| a x cm to mm | b y m to cm | c p km to m |
| d a cm to m | e q m to km | f t mm to cm |
| g $\$d$ to cents | h b c to dollars | i k h to min |
| j m s to min | k r L to mL | l e g to kg |
- 18 How many:
- | | |
|--|--|
| a millimetres are there in 5 cm v mm? | b metres are there in x km 150 m? |
| c minutes are there in a h b min? | d cents are there in $\$w$ and p c? |

- 19 From a 5-m roll of wallpaper, 12 strips each of length z cm are cut and then used to paper a wall. What length of the roll, in centimetres, was not used?
- 20 Farmer Frank wants to fence off a rectangular enclosure, using an existing fence as one side. If he has 50 m of fencing available and the width of the enclosure is to be y m, find:
a the length of the enclosure **b** the area of the enclosure
- 21 Max is a used-car dealer. He bought 10 cars from the manufacturer for $\$x$ each. He sold 7 cars at a profit of $\$c$ each and 3 cars at a profit of $\$d$ each. What was his total profit on the purchase and sale of the 10 cars?
- 22 Mrs Hadlee's fortnightly pension of $\$t$ is increased by 4%. How much will she receive each fortnight after the increase?
- 23 **a** A car travelled a distance of m km in n h. At what speed was the car travelling?
b A girl cycled at b km/h for h h. How far did she cycle?
c A man walked for p km at s km/h. For how long did he walk?

■ Further applications

- 24 Lydia and Jit share $\$c$ in the ratio $a : b$, where $a < b$.
a Who receives the greater amount? **b** How much money does Jit receive?
- 25 A 20-cm length of wire is cut into two pieces, the smaller piece having a length of x cm. The longer piece is bent into the shape of a rectangle with a width of 7 cm. How long is the rectangle?
- 26 **a** If 3 bags of seeds cost $\$c$, find the cost of k bags.
b If k boxes of cereal cost $\$m$, find the cost of 9 boxes.
c If v crates have a mass of 5 kg, find the mass of w crates.

TRY THIS Railway tickets

If a railway line had two stations, A and B , two types of tickets would be needed: A to B and B to A . If there were 3 stations, A , B and C , six tickets would be required: A to B , B to A , A to C , C to A , B to C , C to B .

Complete the following table and then find the general rule.

HINT: The rule is not linear.

Number of stations (s)	2	3	4	5	6	n
Number of tickets (t)	2					

2.11 Properties of numbers

There are a number of important properties of numbers that can be proven by the use of algebra. Many of these proofs involve the expansion of an expression or the removal of a common factor.

EG
+S

Example 1

If one number is divisible by 6 and another number is divisible by 4, show that the product of these numbers must be divisible by 8.

Solution

Let m, n be any two integers. Therefore, $6m$ is a multiple of 6 and $4n$ is a multiple of 4.

$$\begin{aligned} 6m \times 4n &= 24mn \\ &= 8 \times 3mn \\ &= 8y, \text{ say, where } y = 3mn \end{aligned}$$

As m and n are integers, $3mn$ is also an integer and thus y is an integer. Hence, 8 is a factor of the product.

Therefore, the product of the numbers must be divisible by 8.

EG
+S

Example 2

Show that the sum of five consecutive integers must be divisible by 5.

Solution

Let the integers be $x, x + 1, x + 2, x + 3, x + 4$.

$$\begin{aligned} x + (x + 1) + (x + 2) + (x + 3) + (x + 4) &= 5x + 10 \\ &= 5(x + 2) \\ &= 5y, \text{ say, where } y = x + 2. \end{aligned}$$

As $x + 2$ is an integer, y is an integer and 5 is a factor of the sum. Therefore, the sum of five consecutive integers must be divisible by 5.

Exercise 2.11

- 1 If a is an integer, what are the next three integers?
- 2
 - a If x is an even integer, what are the next three even integers?
 - b If x is an odd integer, what are the next three odd integers?
 - c If x is a multiple of 3, what are the next three multiples of 3?
 - d If x is a multiple of 7, what are the next three multiples of 7?
- 3 If m and n are positive integers, determine if each of the following will be odd or even.

a $2n$	b $2n + 1$	c $2m + 2n$
d $2(m + n + 1)$	e $2(m + n) + 1$	f $4(m - n) - 1$

Consolidation

- 4 Use algebra to prove each of these number properties.
- a The sum of two even numbers is an even number.
 - b The sum of two odd numbers is an even number.
 - c The sum of an odd number and an even number is an odd number.
 - d The product of two even numbers is an even number.
 - e The product of an even number and an odd number is an even number.
- 5 Prove that the sum of any three odd numbers is an odd number.
- 6 Janelle wanted to prove that the difference between two even numbers is always an even number. Here is her proof.
Let m and n be any two positive integers, then $2m$ and $2n$ must be even numbers.
Now, $2m - 2n = 2(m - n)$, which is divisible by 2 because 2 is a factor and m and n are integers.
Therefore, the difference between two positive integers must always be an even number.
Is Janelle's proof complete? Explain your answer.
- 7 Prove that the difference between:
- a two odd numbers must be an even number
 - b an odd number and an even number must be an odd number.
- 8 Morris claimed that the product of two even numbers is always an even number and, therefore, the quotient of two even numbers must also be an even number. Is Morris correct? If so, prove the result. If not, find an example where his claim is not true. (This is called a counter-example.)
- 9
- a Show that the sum of three consecutive integers is always divisible by 3.
 - b Show that the sum of seven consecutive integers is always divisible by 7.
 - c Is the sum of four consecutive integers always divisible by 4?
 - d Is the sum of six consecutive integers always divisible by 6?
- 10 Show that if a number is a multiple of 6, then it must also be a multiple of both 2 and 3.
- 11
- a If a number is divisible by 10, then by what other numbers must it also be divisible?
 - b If a number is divisible by 18, then by what other numbers must it also be divisible?

Further applications

- 12
- a A number is divisible by both 3 and 4. Must it also be divisible by 12?
 - b A number is divisible by both 2 and 7. Must it also be divisible by 14?
 - c A number is divisible by both 4 and 6. Must it also be divisible by 24?
 - d A number is divisible by both 6 and 9. Must it also be divisible by 54?

- 13 a** Look at your results in Q12. How do the numbers in **a** and **b** differ from those in **c** and **d**?
- b** Copy and complete this statement:
 'If a number is divisible by two positive integers p and q , then it will be divisible by their product pq if _____.'
- 14 a** Complete this expansion: $(a + b)(c + d) = a(c + d) + b(c + d)$
 $= \dots\dots$
- b** Hence, prove that the product of two odd numbers must be an odd number.

2.12

Generalising solutions to problems using patterns

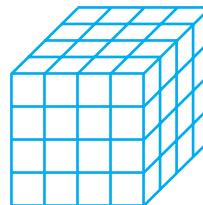
There are many problems in mathematics that can be solved by looking for patterns, then finding rules that describe them. In this section, we will extend our study of patterns in linear relationships to the solution of more general problems. This may involve the use of various problem-solving strategies as well as algebra.

**Example**

Cubes, similar to the one shown, but of any size, are constructed from small cubes.

The faces of the large cube are then painted blue.

Find an algebraic rule for the number of cubes that are painted, P , and the number that remain unpainted, U .

**Solution**

The large cube contains $4 \times 4 \times 4 = 4^3 = 64$ small cubes.

The cubes on the inside, which remain unpainted, form a smaller cube of side 2.

Thus, there are $2^3 = 8$ cubes which are not painted.

The remainder, $64 - 8 = 56$ will all be painted on at least one face.

If this case is generalised to a cube of side length s , then the inner cube of small cubes which are not painted will have side length $s - 2$.

There will be $(s - 2)^3$ unpainted cubes.

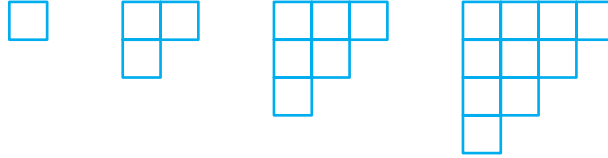
The remaining cubes $s^3 - (s - 2)^3$ will have been painted.

Painted cubes: $P = s^3 - (s - 2)^3$

Unpainted cubes: $U = (s - 2)^3$

Exercise 2.12

- 1 We are interested in finding the minimum number of straight lines of any length that are required to draw each figure.



- a Complete the table.

Number of rows of squares (n)	1	2	3	4
Minimum number of lines (l)		6		

- b Describe, in words, the relationship between the number of rows of squares and the minimum number of lines in each figure.
 c Write an algebraic statement linking l and n .
 d What is the minimum number of lines that are required to draw a figure with 30 rows of squares?

- 2 A sheet of writing paper is folded in half *horizontally*, then folded again and again. The number of creases is recorded at each stage.

- a Copy and complete the following table.

Number of folds (f)	1	2	3	4
Number of creases (c)				

- b Write down a formula to describe the relationship between the number of creases and the number of folds.
 c How many creases would there be if the paper had been folded 7 times?

Consolidation

- 3 Square rooms are tiled with white and black square tiles as shown. There are x tiles along each side of the room, and the top left tile is always white.

- a By considering square rooms of various sizes, find the number of tiles N that are needed to tile a square room of any size.
 b Find expressions for the number of white tiles that are needed to tile a square room of any size. (HINT: Consider separately squares with odd and even numbers of tiles on each side.)
 c How many black squares would be needed to tile a square room with a side length of 50 tiles?

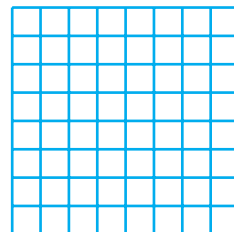


- 4 This triangular pattern is made up of black and white triangular tiles as shown. There are t black tiles along each side of the triangle. The top tile is always black.

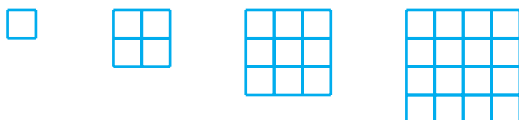
- Find an expression for the total number of tiles in a triangle of any size.
- Find an expression for the number of black tiles in a triangle of any size.
- How many white tiles would there be if there were 10 black tiles along each side of a triangle?



- 5 A chess board is in the shape of a square with a side length of 8 units. Consider the following problem.
How many squares of *any size* are there on a standard chess board?



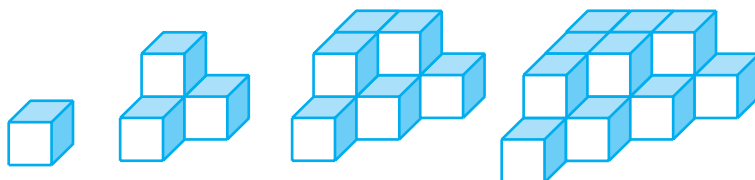
This problem can be made simpler by first drawing smaller diagrams such as those below.



- a Copy and complete this table of values from the diagrams above.

Side length (x)	1	2	3	4
Number of squares (N)				

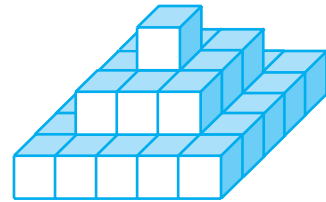
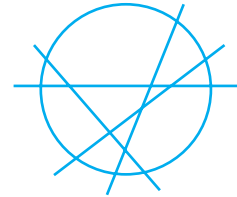
- Describe in words the relationship that exists between the side length and the total number of squares in the diagram.
 - Write this relationship as a formula linking N and x .
 - Use this formula to find the number of squares on a standard chess board.
- 6 Consider the following three-dimensional models.



- a Copy and complete the following table.

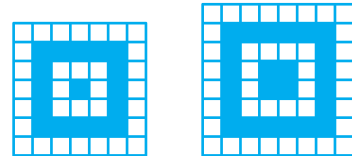
Number of rows	1	2	3	4
Number of cubes in bottom layer				
Number of cubes in second layer				
Total number of cubes				

- b** Show by substitution that the number of cubes in the bottom layer is $\frac{n(n+1)}{2}$, where n is the number of rows.
- c** Find an expression for the total number of cubes in each figure.
- d** Hence, find an expression for the number of cubes in the second layer.
- e** How many cubes will there be in the second layer of the 25th figure?
- 7** A circle has been divided into a number of regions by drawing several straight lines.
No more than two lines can intersect at any one point and the number of regions is to be a maximum.
- a** Form a table of values and use it to find a relationship between the number of lines (l) and the number of regions (r).
- b** How many regions would be formed by the intersection of 10 straight lines?
- 8** The pyramid shown has 3 storeys. Consider a similar pyramid with n storeys.
- a** Write down a number pattern that shows the number of cubes in each storey.
- b** Find an expression for the number of cubes on the bottom layer of a pyramid with n storeys.
- c** How many cubes would there be in the bottom layer of a pyramid that is 10 storeys high?
- d** Use the number pattern in **a** to find the total number of cubes in the pyramid.



Further applications

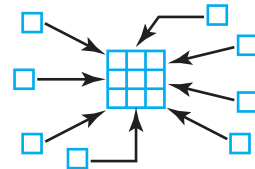
- 9** Square rooms are to be tiled using two colours as shown. Taking separate 'odd' and 'even' cases, establish a rule for the number of white tiles (T) needed, where there are n tiles along each side.



- 10** For this question refer to the diagrams in Q5.

A person wishes to produce a 10×10 square on a computer by repeating a small square. For example, the 3×3 square could be produced using 8 small squares, as the central square is outlined by its surrounding squares.

- a** Find the minimum number of small squares that can be used to produce the 10×10 square.
- b** Taking 'odd' and 'even' as separate cases, form equations to find the minimum number of small squares for an $n \times n$ square. Let S represent the total number of small squares required.



2.13 Binomial products

A **binomial** is an expression that contains two terms. Some examples of binomial expressions are $a + 7$, $3m - 4$ and $x^2 + 2x$. The product of two binomials is called a **binomial product**. Some examples of binomial products are $(x + 6)(x - 4)$, $(2t - 1)(t - 3)$ and $(3a - b)(2a + 7b)$. There are several methods that can be used to expand a binomial product.

Area diagrams

Consider a rectangle with length $(x + 3)$ units and width $(x + 2)$ units. This rectangle can be divided into four smaller rectangles as shown.

	x	3
x	x^2	$3x$
2	$2x$	6

To find the area of the large rectangle, we simply find the sum of the areas of the smaller rectangles.

Area of large rectangle = sum of areas of smaller rectangles.

$$\begin{aligned}
 (x + 3)(x + 2) &= (x \times x) + (x \times 3) + (2 \times x) + (2 \times 3) \\
 &= x^2 + 3x + 2x + 6 \\
 &= x^2 + 5x + 6
 \end{aligned}$$

The distributive law

The **distributive law** can be used to expand expressions such as $a(b + c)$. When using the distributive law, the term outside the grouping symbols is multiplied by each term inside the grouping symbols. That is, $a(b + c) = ab + ac$.

To expand a binomial product using the distributive law:

- ⊙ multiply the first term in the first factor by each term in the second factor
- ⊙ multiply the second term in the first factor by each term in the second factor.

$$\begin{aligned}
 (a + b)(c + d) &= a(c + d) + b(c + d) \\
 &= ac + ad + bc + bd
 \end{aligned}$$

NOTE: The distributive law can be used to expand expressions in which the factors contain more than two terms.

The FOIL method

The **FOIL method** is simply the use of the distributive law without writing the first line of working. The acronym FOIL stands for **F**irst, **O**utside, **I**nside and **L**ast, which is the order in which the terms in the grouping symbols should be multiplied together. This method is also referred to as expansion by **inspection**.

$$(a + b)(c + d) = ac + ad + bc + bd$$

To expand a binomial product using the FOIL method:

- ⊙ multiply the first terms
- ⊙ multiply the outside terms
- ⊙ multiply the inside terms
- ⊙ multiply the last terms
- ⊙ collect any like terms.

Example 1

Expand and simplify each expression by using the distributive law.

a $(a + 8)(a - 2)$

b $(2x + 7)(3x - 4)$

Solutions

a $(a + 8)(a - 2)$
 $= a(a - 2) + 8(a - 2)$
 $= a^2 - 2a + 8a - 16$
 $= a^2 + 6a - 16$

b $(2x + 7)(3x - 4)$
 $= 2x(3x - 4) + 7(3x - 4)$
 $= 6x^2 - 8x + 21x - 28$
 $= 6x^2 + 13x - 28$

Example 2

Expand and simplify $(x + 3)(x^2 - 5x + 1)$.

Solution

$$\begin{aligned}(x + 3)(x^2 - 5x + 1) &= x(x^2 - 5x + 1) + 3(x^2 - 5x + 1) \\ &= x^3 - 5x^2 + x + 3x^2 - 15x + 3 \\ &= x^3 - 2x^2 - 14x + 3\end{aligned}$$

Exercise 2.13

1 Expand each of the following binomial products by using an area diagram.

a $(x + 2)(x + 5)$

b $(a + 3)(a + 4)$

c $(n + 7)(n + 6)$

2 Expand $(x + 6)(x - 2)$ using an area diagram.

3 Expand these binomial products by using the distributive law.

a $(x + 3)(y + 2)$

b $(p + 7)(q + 4)$

c $(g + 1)(h + 6)$

d $(m + 5)(n - 2)$

e $(u - 10)(v + 6)$

f $(j - 3)(k - 4)$

g $(2x + 3)(y + 7)$

h $(3p + 1)(q - 4)$

i $(5a - 6)(b - 3)$

j $(3m + 2)(2n + 9)$

k $(6j + 5)(2k - 3)$

l $(8u - 9)(3v - 5)$

4 Expand and simplify each expression by using the distributive law.

a $(x+2)(x+3)$

b $(x+5)(x+4)$

c $(x+10)(x-6)$

d $(x+8)(x-5)$

e $(x-9)(x+1)$

f $(x-4)(x+3)$

g $(x-6)(x-4)$

h $(x-5)(x-2)$

i $(x+7)(x-7)$

■ Consolidation

5 Expand each of the following by using the FOIL method.

a $(a+2)(a+4)$

b $(y+3)(y+5)$

c $(m+7)(m+1)$

d $(p+7)(p-5)$

e $(t+9)(t-2)$

f $(w+6)(w-4)$

g $(k-4)(k+2)$

h $(u-7)(u+1)$

i $(j-10)(j+6)$

j $(z-2)(z-1)$

k $(n-4)(n-7)$

l $(q-8)(q-11)$

6 Expand each of the following products by using the FOIL method.

a $(2x+3)(x+1)$

b $(3a+4)(a+2)$

c $(p+7)(4p+1)$

d $(4m+5)(2m+3)$

e $(2k+7)(5k+4)$

f $(3w+2)(2w+9)$

g $(5t-2)(t+2)$

h $(y+8)(2y-3)$

i $(3h-4)(h-5)$

j $(6u+1)(u-3)$

k $(7b-2)(b-4)$

l $(n-3)(2n+11)$

m $(4s+7)(2s+1)$

n $(3j+8)(4j+5)$

o $(3q-5)(5q-3)$

p $(4+x)(5x+3)$

q $(8+5r)(6-r)$

r $(5-6e)(7-10e)$

s $(2c-3)(4+9c)$

t $(5-12k)(3k+2)$

u $(1+2g)(9-2g)$

7 Prove the expansion identity $(x+a)(x+b) = x^2 + (a+b)x + ab$.

8 Expand these binomial products by using the expansion identity derived in Q7.

a $(x+4)(x+1)$

b $(x+3)(x+5)$

c $(x+2)(x+4)$

d $(x+5)(x+2)$

e $(x+6)(x+3)$

f $(x+10)(x+2)$

g $(x-3)(x-1)$

h $(x-4)(x-5)$

i $(x-2)(x-7)$

j $(x-9)(x-3)$

k $(x-10)(x-4)$

l $(x-7)(x-6)$

m $(x+5)(x-2)$

n $(x-7)(x+3)$

o $(x-6)(x+2)$

p $(x+3)(x-5)$

q $(x-8)(x+4)$

r $(x+3)(x-10)$

s $(x-2)(x+11)$

t $(x+9)(x-5)$

u $(x-12)(x+9)$

9 Expand and simplify:

a $2(c+3)(c+2)$

b $3(z+4)(z-2)$

c $4(y-5)(y-1)$

d $-2(v-7)(v+3)$

e $a(b-2)(2b-5)$

f $x(2x+7)(3x-4)$

10 **a** Show by expanding both expressions that $(x-8)(x-3) = (8-x)(3-x)$.

b Write another binomial product that means the same as $(2z-5)(7-z)$. Verify your answer by expanding both expressions.

11 **a** Evaluate 13×12 by first expressing it as $(10+3)(10+2)$.

b Evaluate 19×17 by first expressing it as $(20-1)(20-3)$.

c Evaluate 14×18 by first expressing it as $(10+4)(20-2)$.

12 The smallest of three consecutive integers is x . Find their product.

Further applications

- 13** The perimeter of a square is $(4x + 20)$ cm. Write an expression in simplest form for the area of the square.
- 14** Expand and simplify:
- a** $(x + 1)(x^2 + 3x + 5)$ **b** $(a + 2)(a^2 - 2a + 6)$ **c** $(s - 5)(s^2 - 2s - 7)$
d $(g + 3)(2g^2 + g + 4)$ **e** $(2e - 3)(e^2 + 3e - 8)$ **f** $(6k - 5)(4k^2 - 2k - 1)$
- 15** Expand and simplify each of these.
- a** $(a + 1)(a + 2)(a + 3)$ **b** $(n - 3)(n + 7)(n - 2)$ **c** $(2t - 1)(t - 3)(4t - 5)$
- 16** By what expression should:
- a** $n + 3$ be multiplied to give $n^2 + 10n + 21$? **b** $x - 2$ be multiplied to give $x^2 - 9x + 14$?
c $d - 5$ be multiplied to give $d^2 - d - 20$? **d** $p + 4$ be multiplied to give $p^2 - 16$?

2.14 Perfect squares

A **perfect square** is the product of two identical expressions. Some examples of perfect squares are:

$$n^2, 4p^2, (x + y)^2 \text{ and } (2m - 3n)^2$$

To expand a perfect square of the form $(a + b)^2$ or $(a - b)^2$:

- ⊙ square the first term
- ⊙ add or subtract twice the product of the two terms, depending on the sign in the expression
- ⊙ add the square of the last term.

$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{and} \quad (a - b)^2 = a^2 - 2ab + b^2$$

NOTE: In the expanded form of the perfect square, the first sign is the same as the sign in the grouping symbols and the last sign is always +.

Proofs: $(a + b)^2 = (a + b)(a + b)$

$$= a(a + b) + b(a + b)$$

$$= a^2 + ab + ba + b^2$$

$$= a^2 + 2ab + b^2$$

$(a - b)^2 = (a - b)(a - b)$

$$= a(a - b) - b(a - b)$$

$$= a^2 - ab - ba + b^2$$

$$= a^2 - 2ab + b^2$$

Example 1

Expand each of these perfect squares.

a $(x + 4)^2$

b $(p - 7)^2$

c $(3m + 5)^2$

d $(ab - 6c)^2$

Solutions

$$\begin{aligned} \mathbf{a} \quad (x+4)^2 &= x^2 + (2 \times x \times 4) + 4^2 \\ &= x^2 + 8x + 16 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad (3m+5)^2 &= (3m)^2 + (2 \times 3m \times 5) + 5^2 \\ &= 9m^2 + 30m + 25 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad (p-7)^2 &= p^2 - (2 \times p \times 7) + 7^2 \\ &= p^2 - 14p + 49 \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad (ab-6c)^2 &= (ab)^2 - (2 \times ab \times 6c) + (6c)^2 \\ &= a^2b^2 - 12abc + 36c^2 \end{aligned}$$

Example 2

Complete each of these perfect squares.

$$\mathbf{a} \quad y^2 + \dots + 100 = (\dots)^2$$

$$\mathbf{b} \quad 9p^2 + 12p + \dots = (\dots)^2$$

Solutions

- a** The first and last terms in the grouping symbols are $\sqrt{y^2} = y$ and $\sqrt{100} = 10$, respectively.
The middle term in the trinomial = twice the product of the terms in the grouping symbols.

$$= 2 \times y \times 10$$

$$= 20y$$

$$\therefore y^2 + 20y + 100 = (y + 10)^2$$

- b** The first term in the grouping symbols is $\sqrt{9p^2} = 3p$.

The middle term in the trinomial = twice the product of the terms in the grouping symbols.

$$12p = 2 \times 3p \times \text{last term in the grouping symbols}$$

$$12p = 6p \times \text{last term}$$

$\therefore$ The last term in the grouping symbols is 2.

The last term in the trinomial = the square of the last term in the grouping symbols.

$$= 2^2$$

$$= 4$$

$$\therefore 9p^2 + 12p + 4 = (3p + 2)^2$$

Exercise 2.14

- 1 a** Express $(a+b)^2$ as $(a+b)(a+b)$. Hence, show that $(a+b)^2 = a^2 + 2ab + b^2$.

- b** Express $(a-b)^2$ as $(a-b)(a-b)$. Hence, show that $(a-b)^2 = a^2 - 2ab + b^2$.

- 2** Use the perfect square identities to expand:

$$\mathbf{a} \quad (p+q)^2$$

$$\mathbf{b} \quad (m+n)^2$$

$$\mathbf{c} \quad (x-y)^2$$

$$\mathbf{d} \quad (c-d)^2$$

- 3** Expand these perfect squares.

$$\mathbf{a} \quad (x+3)^2$$

$$\mathbf{b} \quad (m+5)^2$$

$$\mathbf{c} \quad (k-2)^2$$

$$\mathbf{d} \quad (y-7)^2$$

$$\mathbf{e} \quad (u+4)^2$$

$$\mathbf{f} \quad (t-1)^2$$

$$\mathbf{g} \quad (c-11)^2$$

$$\mathbf{h} \quad (b+9)^2$$

$$\mathbf{i} \quad (e-6)^2$$

$$\mathbf{j} \quad (p+10)^2$$

$$\mathbf{k} \quad (w+8)^2$$

$$\mathbf{l} \quad (n-12)^2$$

- 4** Expand:

$$\mathbf{a} \quad (5+a)^2$$

$$\mathbf{b} \quad (3-j)^2$$

$$\mathbf{c} \quad (11+y)^2$$

$$\mathbf{d} \quad (1-h)^2$$

Consolidation

5 Use a calculator to expand each of these.

a $(a + 13)^2$

b $(q - 17)^2$

c $(22 + r)^2$

d $(18 - v)^2$

6 Expand each of these perfect squares.

a $(y + 0.2)^2$

b $(f - 0.7)^2$

c $(s + 0.5)^2$

d $(p - 1.2)^2$

7 Expand each perfect square.

a $(2x + 3)^2$

b $(3a + 5)^2$

c $(4k - 1)^2$

d $(5h - 2)^2$

e $(3u + 4)^2$

f $(6d - 7)^2$

g $(2c - 11)^2$

h $(4w + 9)^2$

i $(8g - 1)^2$

j $(7p + 2)^2$

k $(10y + 3)^2$

l $(12f - 5)^2$

m $(6 + 5e)^2$

n $(9 - 2v)^2$

o $(5 - 8b)^2$

p $(7 + 12q)^2$

8 Expand:

a $(ab + c)^2$

b $(p - qr)^2$

c $(rs + st)^2$

d $(3ef - 4gh)^2$

9 Expand and simplify:

a $2(p + 5)^2$

b $-3(a - 4)^2$

c $x(2x + 7)^2$

d $5t(3t - 2u)^2$

10 Complete each of these perfect squares.

a $(x + 3)^2 = x^2 + 6x + \underline{\hspace{1cm}}$

c $(c + 4)^2 = c^2 + \underline{\hspace{1cm}} + 16$

e $(k + \underline{\hspace{1cm}})^2 = k^2 + \underline{\hspace{1cm}} + 36$

g $(\underline{\hspace{1cm}})^2 = u^2 - \underline{\hspace{1cm}} + 4$

i $(\underline{\hspace{1cm}})^2 = n^2 + 2n + \underline{\hspace{1cm}}$

k $(\underline{\hspace{1cm}})^2 = p^2 - 16p + \underline{\hspace{1cm}}$

b $(m - 5)^2 = m^2 - 10m + \underline{\hspace{1cm}}$

d $(w - 7)^2 = w^2 - \underline{\hspace{1cm}} + 49$

f $(\underline{\hspace{1cm}} - 10)^2 = y^2 - \underline{\hspace{1cm}} + 100$

h $(\underline{\hspace{1cm}})^2 = a^2 + \underline{\hspace{1cm}} + 81$

j $(\underline{\hspace{1cm}})^2 = t^2 + 24t + \underline{\hspace{1cm}}$

l $(\underline{\hspace{1cm}})^2 = z^2 - 22z + \underline{\hspace{1cm}}$

11 Complete each of these perfect squares.

a $(3m + 4)^2 = 9m^2 + 24m + \underline{\hspace{1cm}}$

c $(2q + 3)^2 = 4q^2 + \underline{\hspace{1cm}} + 9$

e $(\underline{\hspace{1cm}} + 1)^2 = 25s^2 + 10s + \underline{\hspace{1cm}}$

g $(\underline{\hspace{1cm}})^2 = 4g^2 + 44g + 121$

i $(\underline{\hspace{1cm}})^2 = 16j^2 - \underline{\hspace{1cm}} + 81$

k $(\underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}} + 36b + 4$

b $(2e - 7)^2 = 4e^2 - 28e + \underline{\hspace{1cm}}$

d $(3h - 8)^2 = 9h^2 - \underline{\hspace{1cm}} + 64$

f $(4k + \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}} + 40k + 25$

h $(\underline{\hspace{1cm}})^2 = 36a^2 - 60a + 25$

j $(\underline{\hspace{1cm}})^2 = 121r^2 + \underline{\hspace{1cm}} + 144$

l $(\underline{\hspace{1cm}})^2 = 49y^2 - 42y + \underline{\hspace{1cm}}$

12 State whether each expression is a perfect square.

a $a^2 + 9$

b $(b - 2)^2$

c $x^2 + 28x + 196$

d $c^2 + 10c - 25$

e $n^2 - 4n + 16$

f $m^2 + n^2$

g $k^4 + 12k^2 + 36$

h x^2y^2

i $z^2 - 25$

j $4x^2 + 6x + 9$

k $49 - 14u + u^2$

l $e^2 + f^2 + 2ef$

13 Expand:

a $\left(z + \frac{1}{3}\right)^2$

b $\left(c - \frac{4}{5}\right)^2$

c $\left(m + \frac{5}{2}\right)^2$

d $\left(w - 1\frac{1}{6}\right)^2$

e $\left(a + \frac{1}{a}\right)^2$

f $\left(t - \frac{1}{t}\right)^2$

g $\left(2h + \frac{3}{h}\right)^2$

h $\left(\frac{c}{5} - \frac{5}{c}\right)^2$

14 Simplify:

a $\sqrt{y^2 + 20y + 100}$

b $\sqrt{g^2 - 16g + 64}$

c $\sqrt{9j^2 - 42j + 49}$

15 Evaluate:

a 101^2 by first expressing it as $(100 + 1)^2$ b 99^2 by first expressing it as $(100 - 1)^2$

16 Use the expansions for perfect squares to evaluate each of the following.

a 35^2

b 107^2

c 49^2

d 28^2

17 Expand $\left(x + \frac{1}{x}\right)^2$. Hence, evaluate $\left(2\frac{1}{2}\right)^2$ and $\left(3\frac{1}{3}\right)^2$.

■ Further applications

18 a Show that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ using the distributive law.

b Hence, expand each of the following.

i $(p + q + 3)^2$

ii $(x - y - 4)^2$

iii $(2f + 3g + 4h)^2$

19 Complete each of these perfect squares.

a $(\quad)^2 = \quad + 30c + 25$

b $(\quad)^2 = \quad + 56w + 16$

c $(\quad)^2 = \quad - 110t + 121$

d $(\quad)^2 = \quad - 168r + 49$

TRY THIS

Proof

1 If we take a two-digit number, reverse the digits to form another number, then subtract the smaller number from the larger number, why is the final number a multiple of 9?

Now continue!

2 If we add the two digits in our answers to the 9 times table (up to 10×9) we always obtain a 9 (e.g. $7 \times 9 = 63$ but $6 + 3 = 9$). Using algebraic expressions, can you prove why this happens?

3 Complete this expansion:

$$\begin{aligned}(n + 1)^2 &= (n + 1)(n + 1) \\ &= (n + 1)n + (n + 1)1 \\ &= \_ \_ \_ \end{aligned}$$

4 Use algebraic expressions to prove that the difference between the squares of consecutive numbers is equal to twice the smaller number plus one (e.g. $6^2 - 5^2 = 11 = 2 \times 5 + 1$).

HINT: Let n and $n + 1$ be consecutive numbers.

2.15 Difference of two squares

When the sum of two terms is multiplied by their difference, the resulting expression is called a difference of two squares.

To expand an expression of the form $(a + b)(a - b)$:

- ⊙ square the first term
- ⊙ subtract the square of the second term.

$$(a + b)(a - b) = a^2 - b^2$$

Proof: $(a + b)(a - b) = a(a - b) + b(a - b)$
 $= a^2 - ab + ba - b^2$
 $= a^2 - b^2$

Example 1

Expand each of these products using the identity $(a + b)(a - b) = a^2 - b^2$.

a $(x - 5)(x + 5)$

b $(2a - 7)(2a + 7)$

c $(3m + 4n)(3m - 4n)$

Solutions

a $(x - 5)(x + 5)$
 $= x^2 - 5^2$
 $= x^2 - 25$

b $(2a - 7)(2a + 7)$
 $= (2a)^2 - 7^2$
 $= 4a^2 - 49$

c $(3m + 4n)(3m - 4n)$
 $= (3m)^2 - (4n)^2$
 $= 9m^2 - 16n^2$

Example 2

Expand and simplify $7(3 - 2t)(3 + 2t)$.

Solution

$$7(3 - 2t)(3 + 2t) = 7(9 - 4t^2)$$

$$= 63 - 28t^2$$

Exercise 2.15

1 Show that $(a + b)(a - b) = a^2 - b^2$.

2 Use the difference of two squares identity to expand:

a $(p - q)(p + q)$

b $(x + y)(x - y)$

c $(e - f)(e + f)$

d $(m + n)(m - n)$

3 Expand the following expressions.

a $(x+3)(x-3)$	b $(p+2)(p-2)$	c $(y-5)(y+5)$
d $(h+4)(h-4)$	e $(1-b)(1+b)$	f $(8+m)(8-m)$
g $(7+c)(7-c)$	h $(6-w)(6+w)$	i $(k+11)(k-11)$
j $(9-j)(9+j)$	k $(u-10)(u+10)$	l $(12+d)(12-d)$

4 Expand:

a $(t+13)(t-13)$	b $(z-17)(z+17)$
c $(21+q)(21-q)$	d $(16-s)(16+s)$

■ Consolidation

5 Expand each of the following.

a $(2a+3)(2a-3)$	b $(5t-2)(5t+2)$	c $(4k-1)(4k+1)$
d $(3p+7)(3p-7)$	e $(8y-5)(8y+5)$	f $(2n+9)(2n-9)$
g $(7-10x)(7+10x)$	h $(4+5e)(4-5e)$	i $(1-6g)(1+6g)$

6 Expand:

a $(4b+c)(4b-c)$	b $(j-2k)(j+2k)$
c $(8m-n)(8m+n)$	d $(3p+2q)(3p-2q)$
e $(4x-7y)(4x+7y)$	f $(9c+5d)(9c-5d)$
g $(6g-11h)(6g+11h)$	h $(10u+3v)(10u-3v)$
i $(5r-12s)(5r+12s)$	j $(7j-6k)(7j+6k)$
k $(4e+9f)(4e-9f)$	l $(11m+10n)(11m-10n)$

7 Expand and simplify:

a $5(a-2)(a+2)$	b $4(3+y)(3-y)$	c $-10(c-7)(c+7)$
d $a(b-c)(b+c)$	e $6(2p+3q)(2p-3q)$	f $2uv(u+v)(u-v)$

8 Expand each of these expressions.

a $(pq-r)(pq+r)$	b $(f+gh)(f-gh)$	c $(ab-cd)(ab+cd)$

9 Expand and simplify:

a $\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)$	b $\left(k+\frac{3}{5}\right)\left(k-\frac{3}{5}\right)$	c $\left(m-2\frac{3}{4}\right)\left(m+2\frac{3}{4}\right)$
d $\left(y+\frac{1}{y}\right)\left(y-\frac{1}{y}\right)$	e $\left(3s-\frac{2}{s}\right)\left(3s+\frac{2}{s}\right)$	f $\left(\frac{a}{b}+\frac{b}{a}\right)\left(\frac{a}{b}-\frac{b}{a}\right)$

- 10 **a** Evaluate 13×7 by first expressing it as $(10+3)(10-3)$.
b Evaluate 21×19 by first expressing it as $(20+1)(20-1)$.
c Evaluate 35×25 by first expressing it as $(30+5)(30-5)$.

Further applications

11 For each of the following multiply two expressions together such that the result is a difference of two squares, then complete the expansion using the FOIL method.

a $(m + 1)(m - 1)(m + 6)$

b $(n - 3)(n + 3)(n - 7)$

c $(y + 5)(y - 2)(y - 5)$

d $(6 - p)(p + 3)(6 + p)$

e $(4a - 7b)(4a + 7b)(a - b)$

f $(2u + 5v)(3u + 4v)(2u - 5v)$

12 **a** Expand and simplify $(a + b + c)(a + b - c)$ by first expressing it as $(a + b)^2 - c^2$.

b Similarly, expand and simplify $(a - b - c)(a - b + c)$.

2.16 Miscellaneous expansions

The questions in this exercise are a combination of monomial products, binomial products, perfect squares and the difference of two squares.

NOTE: Grouping symbols should be written around the expansion of a binomial product if it is preceded by a minus sign.

Example

Expand and simplify:

a $(x + 4)^2 + (2x + 5)(x - 3)$

b $(x + 8)(x - 8) - (x - 1)^2$

Solutions

a
$$\begin{aligned} & (x + 4)^2 + (2x + 5)(x - 3) \\ &= x^2 + 8x + 16 + 2x^2 - 6x + 5x - 15 \\ &= 3x^2 + 7x + 1 \end{aligned}$$

b
$$\begin{aligned} & (x + 8)(x - 8) - (x - 1)^2 \\ &= x^2 - 64 - (x^2 - 2x + 1) \\ &= x^2 - 64 - x^2 + 2x - 1 \\ &= 2x - 65 \end{aligned}$$

Exercise 2.16

1 Expand and simplify each expression.

a $(a + 3)(a + 5)$

b $(m + 6)^2$

c $(k + 4)(k - 4)$

d $(2x + 7)(x + 3)$

e $(u - 5)(u + 5)$

f $(n - 9)(n + 2)$

g $(b - 7)^2$

h $(c - 8)(3c + 2)$

i $(z - 10)(z - 8)$

j $(6j - 1)(2j - 3)$

k $(3e - f)^2$

l $(1 - l)(1 + l)$

m $(4p + 7q)^2$

n $(2g + 5h)(2g - 5h)$

o $(a + b)(c + d)$

p $(2v - 11w)^2$

q $(bc - 8)(bc + 8)$

r $(2m + 5)(3m - 4)$

2 Expand and simplify each expression.

a $3n + 5(n + 2)$

b $(x + 6)(x + 4) + 5$

c $(t - 4)(t + 4) - 3$

d $7g + (g - 1)(g + 3)$

e $2k^2 + (k + 5)^2$

f $a^2 + (6 - a)(6 + a) - 20$

■ Consolidation

3 Expand and simplify each of the following expressions.

a $(y + 4)^2 + 2(y + 3)$

b $(e + 5)(e + 6) + 4(e + 2)$

c $(x + 3)^2 + (x + 1)(x + 5)$

d $(c + 8)^2 + (c + 5)^2$

e $(d + 3)(d + 8) + (d + 2)(d + 5)$

f $(h - 2)^2 + 7h(h + 6)$

g $(b + 7)(b - 7) + 10(b + 4)$

h $3j(j + 2) + (j - 1)(j + 2)$

i $(z + 1)(z - 8) + (z - 1)(z + 8)$

j $(3r + 5)(r + 5) + (r + 2)^2$

k $(2f - 3g)(2f + 3g) + (2f + 3g)^2$

l $(5t + 2)(5t - 2) + (t - 4)(3t + 1)$

4 Expand and simplify each of the following expressions.

a $12x - 5(2x - 3)$

b $4p(p - 7) - p(p - 1)$

c $(k - 4)^2 - 3(4 - k)$

d $(n - 7)(n - 3) - 2(n + 10)$

e $(a + 6)^2 - (a + 6)(a - 6)$

f $(b + 3)(b - 3) - (b - 1)^2$

g $(u + 10)^2 - (u + 8)^2$

h $(p + q)^2 - (p - q)^2$

i $(a + b)(a - b) - (a + b)^2$

j $(2c + 3)(c + 4) - 5(c - 3)$

k $(v + 8)(v + 5) - (v - 2)(v - 9)$

l $(3k + 11)^2 - (3k - 7)(k + 10)$

m $(6q - 7)(6q + 7) - (6q - 5)^2$

n $(a + b)(c + d) - (a + c)(b + d)$

■ Further applications

5 Expand and simplify each of these.

a $(a + 1)^2 + (a + 2)^2 + (a + 3)^2$

b $(n + 1)(n - 1) + (n + 2)(n - 2) + (n + 3)(n - 3)$

c $(x + 3)(x + 4) + (x + 3)(x - 4) + (x - 3)(x + 4)$

d $(h + 1)^2 + (h - 1)(h + 1) - (h - 1)^2$

e $(k + 5)^2 - 2(k + 5)(k + 6) + (k + 6)^2$

f $(x + y)(x^2 + 3xy + 2y^2) - (x + y)^2$

g $(e + 5)(e^2 + 2e + 3) + (e - 3)(e^2 - 4e + 2)$

h $(a + b)(a^2 - ab + b^2) - (a - b)(a^2 + ab + b^2)$

6 **a** Expand and simplify $(a + b)^3$ by first expressing it as $(a + b)(a + b)^2$.

b Similarly, expand and simplify $(a - b)^3$.

c Hence, find $(a + b)^3 + (a - b)^3$.

7 Expand and simplify:

a $(x + 2)^3$

b $(t - 5)^3$

c $(2p + 3)^3$

d $(4x - 3y)^3$

8 Expand and simplify $(a + b)^4$.



FOCUS ON WORKING MATHEMATICALLY

A NUMBER PATTERN FROM BLAISE PASCAL 1654



Introduction

Blaise Pascal (1623–1662), the brilliant French mathematician, philosopher, writer and theologian, was a young contemporary of Pierre de Fermat and René Descartes. Although he made many important contributions to the study of geometry, his name is associated in school mathematics with a famous series of numbers written in the shape of a triangle known as *Pascal's triangle*

				1					
				1		1			
			1	2	1				
		1	3	3	1				
	1	4	6	4	1				
1	5	10	10	5	1				

in which each number is the sum of the two numbers immediately above it. The array was already well known by many predecessors, including the Ancient Chinese, but it has survived with his name attached. However, Pascal was the first to investigate its patterns systematically. In particular, he used it to obtain the coefficients of the terms in the expansion of $(a + b)^n$ where n is a positive whole number. For example, in the expansion of $(a + b)^2$ we have $1a^2 + 2ab + 1b^2$.

Taking the apex of the triangle 1 as row 0, you can see that the array of coefficients 1 2 1 is the second row of the triangle above. It is this work which is the focus of this activity.





LEARNING ACTIVITIES

- 1 Copy and complete the first 10 rows of Pascal's triangle.
- 2 Write down the expansions of $(a + b)^n$ for the cases $n = 0, 1, 2, 3, 4$ and 5 . Copy and complete the table. Use a different colour for the coefficients and compare with the rows of Pascal's triangle.

n	Expansion
0	$(a + b)^0 = 1$
1	$(a + b)^1 = 1a + 1b$
2	$(a + b)^2 =$
3	
4	
5	

- 3 Notice how the powers of a decrease as the powers of b increase. Use the pattern to predict:
 - a the first and last terms of $(a + b)^6$
 - b the second term of $(a + b)^6$
 - c the fourth term of $(a + b)^6$.
- 4 Write out the whole expansion of $(a + b)^6$ and $(a + b)^{10}$.
- 5 By taking $b = 3$, predict the expansion of $(a + 3)^5$.
- 6 Add the numbers in each row and record the number pattern. How can you explain this? Can you predict the sum of the numbers in the n th row? Take the top number 1 as row 0.
- 7 Consider each row as a single number, that is 1, 11, 121, 1331, 14641, ... What do you notice?
- 8 Look along the diagonals and see if you can find the triangle numbers 1, 3, 6, 10, 15, ... and the Fibonacci numbers 1, 1, 2, 3, 5, 8, 13, ... What other number patterns can you find?



CHALLENGE

- 1 At the local ice cream shop there are 19 different flavours to choose from. You decide to buy a cone with 3 scoops. Show that there are 1330 different ways to choose your cone. You may find it helpful to consider the separate cases: all 3 flavours the same, all 3 different, and 2 the same and 1 different.



NOTE: Two scoops of strawberry with a scoop of vanilla, for example, is counted as only one way, no matter what order the scoops are chosen.

- 2 Draw up a table showing the total number of triple scoops you can have from the number of flavours n . Verify the data in the table.

Number of flavours, n	Total number of choices
1	1
2	4
3	10
4	20

- 3 Continue the pattern in your table until you reach $n = 19$. Did you get 1330 choices?
- 4 See if you can find the number pattern 1, 4, 10, 20, ... in Pascal's triangle.
- 5 There is a general solution to this problem. If n is the number of flavours, there are $\frac{1}{6}n(n+1)(n+2)$ choices. Verify that this works for $n = 1, 2, 3, 4$ and check $n = 19$.



LET'S COMMUNICATE

Divide the class into groups to make two charts for display:

- 1 Make a chart to illustrate the link between the expansion of $(a + b)^n$ (called the binomial theorem) and Pascal's triangle showing how to write down the terms of the expansion without multiplying out. You could include the special case of what happens when $a = b = 1$.
- 2 Make a chart to illustrate the patterns and different types of numbers you found in Pascal's triangle.



REFLECTING

Pascal valued intuition, the ability to perceive truth independently of reasoning. In his mathematical work he loved to anticipate results. He often made superb guesses and had the ability to see shortcuts.

Do you like making guesses and taking shortcuts to a solution? One of Pascal's famous declarations regarding this was 'Reason is the slow and tortuous method by which those who do not know the truth discover it'. Would you agree? Reflect on the usefulness of intuition in the learning of mathematics and discuss it with your teacher.





MACQUARIE

Language link with Macquarie

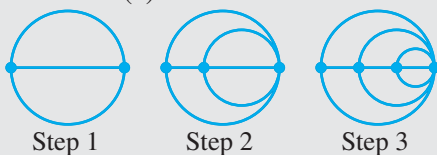
- 1 Explain each of the following and give an example of each:
 - a like terms
 - a binomial product
 - a perfect square
- 2 Compare the meanings of *expand* and *factorise*.
- 3 Read the Macquarie Learners Dictionary entry for *expand*.

expand verb **1.** to increase in size or to swell: The balloon expanded as the hot air flowed into it. **2.** to express in greater detail: to expand a short story into a novel **3.** to spread, stretch out or unfold: A bird expands its wings to fly.

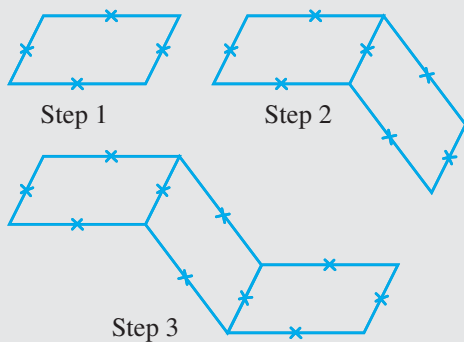
Word family: **expansion** noun

In what way is the mathematical meaning similar to the idea of stretching out?

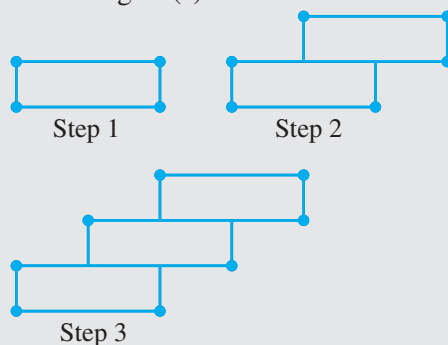
- 1 Write down an algebraic rule that links:
 - a the number of dots (d) to the number of circles (c).



- b the number of crosses (c) to the number of parallelograms (p).



- c the number of dots (d) to the number of rectangles (r).



- 2 Use the method of finite differences to find the rule that links x and y in each table of values.

(HINT: Start $y = \dots$).

a

x	0	1	2	3
y	9	11	13	15

b

x	1	2	3	4
y	3	8	13	18

c	x	4	5	6	7
	y	31	40	49	58

3 Write an algebraic expression that shows:

- a 5 more than the product of p and 7
- b 2 less than the product of c and d
- c the sum of 8 and the product of x , y and z
- d 3 more than one-quarter of n
- e the difference between r and half of s
- f 7 times the cube of c
- g one-sixth of the number that is 4 more than h
- h twice the number that is 9 less than e
- i the sum of one-third of a and two-fifths of b
- j the square of the difference between v and w .

4 Write down an algebraic expression in simplest form for the number that is:

- a 8 more than $y - 2$
- b 5 less than $k - 4$

5 Write down three consecutive numbers, the first of which is:

- a t b $e + 8$
- c $2c - 5$ d $d - 1$

6 Write down three consecutive even numbers, the first of which is:

- a x b $n + 7$
- c $a - 2$

7 Write down three consecutive odd numbers, the first of which is:

- a b b $v + 4$
- c $z - 3$

8 Convert:

- a p km to m b b mm to cm
- c $\$k$ to cents d f min to h

9 How many:

- a centimetres are there in 3 m q cm?
- b seconds are there in a min b s?

10 Find the value of each expression using the substitutions $p = 4$, $q = 7$ and $r = 8$.

- a $3p + r$ b $pq - 12$
- c $3p^2$ d $30 - 5q$
- e $2p + 10q - 3r$ f $p^2 - r^2$
- g $r(p + q)$ h $\frac{5p + qr - 3r}{q + 6}$

11 Evaluate each of the following expressions using the substitutions $u = 5$, $v = -2$ and $w = -6$.

- a $u - v$ b $w + u$
- c $v + w$ d $u - w + v$
- e $w - u - v$ f $4w^2$
- g $3u - 4w$ h $w(v - u)$
- i $u^2 + w^2$ j $\sqrt{uvw} + 4$

12 Simplify each of these.

- a $12ef + 5fe$ b $6u^2 + 4u^2$
- c $8a^2 - 3a^2$ d $7a^2b - 3a^2b$
- e $-8d + 5d$ f $-9v - 3v$
- g $2c - 10c$ h $-4r + 8r$

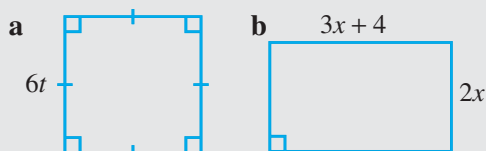
13 Simplify:

- a $-6h + 9h - 13h$
- b $4t^2 - t^2 - 3t^2$

14 Simplify each of the following by collecting the like terms.

- a $a + 7 + a + 4$
- b $5c + 8d + 3c + d$
- c $14p + 11q + 9p - 8q$
- d $20g + 10h - 13g - 8h$
- e $4m + 5 - m - 5$
- f $17a - 6b + 2a + 4b$
- g $-10x + 4y + 7x - 9y$
- h $a^2 + 3a + a^2 - 8a$

- 15 Find, in simplest form, an algebraic expression for the perimeter of each figure.



- 16 Simplify:

a $8k \times 3$	b $m \times 6n$
c $5c \times 7d$	d $n \times n$
e $4g \times 9g$	f $ab \times bc$
g $\frac{1}{2}p \times 12q$	h $\frac{2}{3}rs \times 9st$
i $-5 \times 3e$	j $-8u \times -4v$
k $-2a \times -3b \times -4c$	l $7pq \times -6qr$

- 17 Simplify:

a $12w \div 4$	b $2c \div c$
c $ef \div e$	d $30h \div 5h$
e $21ab \div 3a$	f $48mnp \div 8mp$
g $b^2 \div b$	h $18s^2 \div 6s$
i $40u \div (-4)$	j $(-20c) \div (-4c)$
k $(-60xy) \div 5x$	l $(-24d^2) \div (-6d)$

- 18 Simplify:

a $\frac{63ab}{7a}$ **b** $\frac{72x^2}{12x}$ **c** $\frac{36uv^2}{3uv}$

- 19 Simplify each of the following.

a $28a \div 4a \times 5b$
b $3c \times 12d \div 9c$
c $40xy \div 4x \div 2y$
d $60z^2 \div 12z \times 7w$

- 20 Simplify each of these, giving the answers in simplest fraction form.

a $3k \div 15$ **b** $16ab \div 20a$
c $35m \div 45m^2$ **d** $14u^2v \div 21uv^2$

- 21 Find the simplest answer for each of these.

a $(13h - 8h) \times 6$
b $100y \div (9y + y)$
c $4 \times (9t - 6t) \times 5t$

d $[20g - (7g + g)] \times 4$
e $40w - [19w + (42w \div 6)]$
f $5x \times [(-14x + 10x) \div 2]$

- 22 Simplify:

a $\frac{15c + 9c}{13c - 7c}$ **b** $\frac{9u \times 8v}{7u + 5u}$

- 23 Use the order of operations to simplify:

a $4n + 3n \times 5$ **b** $30j - 56j \div 7$
c $18e + 7e \times 4 - 12e$

- 24 Expand:

a $4(3n + 8)$ **b** $(4a - 5b)7$
c $2f(g + 11h)$ **d** $x(x - 6)$
e $5c(3c - 10)$ **f** $pq(p + q)$
g $-3a(b + 9c)$ **h** $-12r(3s - 4r)$

- 25 Expand and simplify each of these expressions.

a $5(u + 6) - 8$ **b** $3(t - 8) + 10$
c $9(p + 2) - 25$ **d** $-7(2x - 5) + 13$
e $6 + 4(3n - 4)$ **f** $15 - (8 - m)$
g $10k - 3(8 - 2k) + 11$

- 26 Factorise each of the following by removing the highest common factor.

a $5r - 20$ **b** $ab - bc$
c $xy + y$ **d** $e^2 + 11e$
e $tuv - uvw$ **f** $18p - 21$
g $20km + 15mn$ **h** $24rs - 30s^2$
i $cd^2 + c^2d$ **j** $-14a + 49$
k $-15ef - 24eg$ **l** $-77w^2 + 132w$

- 27 Factorise each of these by removing a binomial common factor.

a $a(b + 7) + 5(b + 7)$
b $m(m - n) - 4(m - n)$
c $x(y + 2z) + (y + 2z)$

- 28 Simplify:

a $\frac{7c}{9} - \frac{c}{9}$ **b** $\frac{5u}{12} + \frac{11u}{12}$
c $\frac{13k}{10} + \frac{7k}{10}$ **d** $\frac{17}{20s} - \frac{3}{20s}$

e $\frac{a}{5} + \frac{a}{10}$

f $\frac{m}{3} - \frac{m}{7}$

g $\frac{3w}{4} + \frac{w}{6}$

h $\frac{a}{4b} + \frac{a}{12b}$

29 Simplify:

a $\frac{b}{5} \times \frac{c}{8}$

b $\frac{p}{16} \times \frac{12}{p}$

c $\frac{14m}{45} \times \frac{20n}{21m}$

d $\frac{x^2}{42y} \times \frac{36yz}{7x}$

e $\frac{5a}{12} \div \frac{5}{3b}$

f $\frac{32}{h} \div \frac{40}{h}$

g $\frac{20v}{49u} \div \frac{25v}{28w}$

h $\frac{ab^2}{54c} \div \frac{a^2b}{45cd}$

30 Expand and simplify:

a $(y+4)(y+5)$ b $(m-7)(m-3)$

c $(t+8)(t-2)$ d $(a-11)(a+4)$

e $(n-3)(n-9)$ f $(k+12)(k-2)$

g $(e+6)(e+7)$ h $(s+1)(s-13)$

31 Expand and simplify:

a $(2b+5)(b+3)$ b $(2p-3)(3p-8)$

c $(5n+3)(7n-6)$ d $(4-3r)(2+r)$

32 Expand these perfect squares.

a $(p+3)^2$ b $(m-7)^2$

c $(2c+5)^2$ d $(4y-3)^2$

e $(3a+4b)^2$ f $(7j-6k)^2$

g $(x+\frac{3}{2})^2$ h $(3t-\frac{8}{t})^2$

33 Expand and simplify $2x(9x-2y)^2$.

34 Complete these perfect squares.

a $(a-6)^2 = a^2 - 12a + \underline{\hspace{1cm}}$

b $(d+4)^2 = d^2 + \underline{\hspace{1cm}} + 16$

c $(\underline{\hspace{1cm}})^2 = t^2 + 22t + \underline{\hspace{1cm}}$

d $(\underline{\hspace{1cm}})^2 = n^2 - \underline{\hspace{1cm}} + 81$

e $(\underline{\hspace{1cm}})^2 = 9x^2 + \underline{\hspace{1cm}} + 25$

f $(\underline{\hspace{1cm}})^2 = 16u^2 - 88u + \underline{\hspace{1cm}}$

35 State whether each expression is a perfect square.

a $n^2 + 4$

b $k^2 + 6k - 9$

c $a^2 + 2a + 1$

d $e^2 - 36$

e $q^2 - 10q + 100$

f $9c^2 + 24c + 16$

36 Expand:

a $(x+5)(x-5)$

b $(k-8)(k+8)$

c $(12-m)(12+m)$

d $(2t-7)(2t+7)$

e $(3a+11b)(3a-11b)$

f $(ab-6c)(ab+6c)$

37 Expand and simplify:

a $(a+3)(a^2+5a+4)$

b $(3n-2)(4n^2-n-7)$

c $(y-2)(y+2)(y+9)$

d $(2k-1)(k-4)(5k+3)$

38 Expand and simplify:

a $(x-5)^2 + (x+4)(x-4)$

b $(n+6)(n+2) + (3n-4)^2$

c $(a+10)(a-10) - (a-7)^2$

d $(2u+9)(u-4) - (u-3)(u+6)$

3

Consumer arithmetic



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ calculate earnings for employees in the form of salaries, wages, commission, piece work and casual work
- ⦿ calculate overtime pay at special pay rates
- ⦿ calculate holiday leave loading and bonuses
- ⦿ calculate weekly, fortnightly, monthly and yearly incomes
- ⦿ calculate net pay after deductions such as taxation and superannuation
- ⦿ calculate the taxable income after allowable tax deductions have been considered
- ⦿ calculate the tax payable and net income on a given gross income
- ⦿ calculate the Medicare levy payable on a given taxable income
- ⦿ prepare a budget for a given income and solve budget problems
- ⦿ calculate a best buy
- ⦿ solve problems involving discounts
- ⦿ solve problems involving profit and loss.

3.1 Salaries and wages

Self-employed people charge a fee to those for whom they provide their services. However, men and women who work for other people are paid in a variety of different ways. Most people are paid either a wage or a salary.

Salaries

A **salary** is a fixed amount paid for one year's work. It is usually divided into equal weekly, fortnightly or monthly payments. Employees who are paid a salary are also entitled to sick pay, holiday pay and the required superannuation employer contributions. Salary earners are not paid for extra hours worked; however, they may be able to negotiate some time off with their employer. This may, for example, allow for a late start on one day or an early departure.

Typical salary earners include teachers, police, public servants and scientists.

Wages

A wage is a fixed amount paid for each hour's work for a specified number of hours per week. Employees who are paid a wage are also entitled to sick pay, holiday pay and the required superannuation employer contributions. Wage earners are paid for extra hours worked, usually at a higher hourly rate of pay.

Typical wage earners include sales assistants, construction workers, mechanics and factory workers.

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Example 1

Alice is paid \$620 per week.
Find her:

- a annual salary
- b fortnightly pay
- c monthly pay

Solutions

- a Annual salary = weekly pay $\times$ 52
 $= \$620 \times 52$
 $= \$32\,240$
- b Fortnightly pay = weekly pay $\times$ 2
 $= \$620 \times 2$
 $= \$1240$
- c Monthly pay = annual salary $\div$ 12
 $= \$32\,240 \div 12$
 $\div \$2686.67$

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Example 2

- a Virgil works 40 hours per week as a mechanic and earns \$13.45 per hour. Calculate his regular weekly wage.

Solutions

- a Weekly wage = hourly rate of pay $\times$ number of hours worked
 $= \$13.45 \times 40$
 $= \$538$

- b** Paul works 36 hours per week as a glazier and earns a regular weekly wage of \$525.60. Find his hourly rate of pay.

$$\begin{aligned}\text{b Hourly rate of pay} &= \text{weekly wage} \div \text{number of hours worked} \\ &= \$525.60 \div 36 \\ &= \$14.60\end{aligned}$$

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S**Example 3**

John's annual salary is \$28 490. Find his fortnightly pay after receiving a 6% pay rise.

Solution

- $$\begin{aligned}\text{i New annual salary} &= 1.06 \times \text{old salary} \\ &= 1.06 \times \$28\,490 \\ &= \$30\,199.40 \\ \text{ii Fortnightly pay} &= \text{annual salary} \div 26 \\ &= \$30\,199.40 \div 26 \\ &\div \$1161.52\end{aligned}$$

Exercise 3.1

- Carrie earns an annual salary of \$28 080. How much would Carrie receive if she asked to be paid:
 - weekly?
 - fortnightly?
 - monthly?
- Find the annual salary of a worker whose:
 - weekly pay is \$498
 - fortnightly pay is \$1362.50
 - monthly pay is \$4423.92
- Calculate the weekly wage for a tradesperson who works:
 - a 38-hour week at \$12.20/h
 - a 33-hour week at \$16.15/h
- For how many hours must an employee work each week at the rate of:
 - \$16/h to earn \$592?
 - \$17.40/h to earn \$635.10?
- Increase an annual salary of \$31 400 by 5%.
 - Increase a monthly salary of \$2925 by 7.2%.
- Decrease an annual salary of \$48 990 by 12%.
 - Decrease a fortnightly salary of \$2738 by 8.7%.

Consolidation

- Gary the gardener works at a municipal golf course and is paid an annual salary of \$29 640. How much would Gary have been paid after 11 weeks?
- Moiria is paid \$891.60 each fortnight as a hairdresser. How much would she have been paid after working for 18 weeks?

- 9 Mitchell is considering applying for these two jobs which have been advertised in the newspaper.
- i Pharmacist's assistant—Salary: \$1040 per fortnight
 - ii Security guard—Wage: \$14.67/h, 35 hours per week
- Which position offers the better pay and by how much per week?
- 10 Sergeant Carlton is paid \$1856.40 per month as a soldier in the Australian Army. What is his equivalent weekly rate of pay?
- 11 A helicopter pilot is paid \$506.75 per week. Calculate his equivalent monthly pay.
- 12 A sales position is advertised in a weekend newspaper with an annual salary of \$32K. What does this mean?
- 13 Denise earns \$3740 per month as an executive producer at a television station. Find her equivalent fortnightly pay.
- 14 Malek's annual salary is \$45 820. Calculate his new weekly pay if Malek receives a pay rise of 5%.
- 15 By how much will Kevin's weekly pay increase if his annual salary of \$41 066 is increased by 4%?
- 16 Leila earns \$1072.90 per week as an architect. Find her new annual salary if she receives a pay rise of $7\frac{1}{2}\%$.
- 17 The employees at a fruit company are required to take a pay cut of 10% to ensure that the business does not go bankrupt. Calculate the new fortnightly pay of a fruit canner whose annual salary was \$25 400 before the pay cut.
- 18 Due to falling sales, the employees at a small shoe factory have their pay reduced by 8%. Calculate the new annual pay for a worker who previously earned \$385 per week.
- 19 Copy and complete the following wage table.

Employee	Hours worked	Hourly rate of pay	Total weekly pay
Bart	34	\$15.20	
Kristina		\$10.95	\$416.10
Marta	36		\$532.80

- 20 Dean works from 8:30 am to 5 pm each day, Monday to Friday, as a shop assistant. Calculate Dean's fortnightly pay if he earns \$12.75 per hour.
- 21 Rudy earns \$15.65/h for a 37-hour week, while George is paid an annual salary of \$33 189. By how much must Rudy's hourly rate of pay be increased in order to earn the same annual pay as George?

- 22** Anise has a casual job selling whitegoods. She is paid \$10.40 per hour during the week and \$15.60 per hour on weekends. Find her pay for a week in which she works from 5 pm–9 pm on Thursday night and 11 am–4:30 pm on Saturday.
- 23** A casual waitress is paid \$9.20/h for any hours worked before noon and \$11.50/h for hours worked after noon. Calculate her pay for a week where she worked for the hours shown.
- Tues.—10 am to 3 pm
 - Thurs.—12 noon to 6 pm
 - Fri.—8:30 am to 1 pm
- 24** William is an apprentice plumber. He is paid a trade allowance of \$8.80 per hour and works 40 hours each week.
- a** Calculate William's regular weekly pay.
 - b** If he receives a 5% increase, find William's new hourly rate of pay.
 - c** By how much has William's weekly pay increased?
- 25** Last year Yvette was paid an annual salary of \$36 500 as a chef in a French restaurant. This year, her annual salary rose to \$39 420 owing to an increase in the number of customers. By what percentage did Yvette's salary increase?
- 26** A teacher's salary increased from \$48 700 to \$50 891.50. Calculate her percentage pay rise.
- 27** Brett was paid \$585 per week last year as a car mechanic. He was laid off at the end of the year and this year, he receives \$2408.25 per month in his new job.
- a** Express each pay rate as an annual salary.
 - b** Did Brett's pay increase or decrease by changing jobs and by what percentage?

■ Further applications

- 28** Find the annual salary of each of these employees before they received the given pay rise/cut.

	Change in pay	New salary
Greg	Pay rise of 8%	\$29 160
Taleisha	Pay rise of 5%	\$35 700
Lyselle	Pay rise of 7.4%	\$45 967.20
Martin	Pay cut of 9%	\$51 870
Taylor	Pay cut of 11%	\$57 049
Peter	Pay cut of 6.3%	\$79 410.75

- 29** Allen's hourly rate of pay was increased by 4% and he now earns \$686.40 for a 40-hour week. How much would Allen have earned for a 35-hour week before the increase?
- 30** After receiving a pay rise of 7.5%, Laurie's rate of pay increased to \$19.35/h. How much extra will Laurie earn over a full year if he works for 76 hours each fortnight?

3.2 Other methods of payment

While most permanent workers are paid a wage or a salary, an increasing number of Australians are being employed in casual positions or in positions where they are paid according to what they produce or sell.

Commission

A **commission** is usually quoted as a percentage of the value of goods sold. A small retainer or basic weekly wage may also be paid. The greater the value of goods sold, the greater the income is for that pay period. Sick pay, holiday pay and superannuation may or may not be paid. Those who are paid on a commission basis must budget carefully for the weeks when their income is substantially less than usual.

Typical occupations include salespeople and real estate agents.

Piece work

With **piece work**, the employee is paid a fixed amount of money for each item produced. The more items a worker produces, the greater the pay for that pay period. Piece work positions are usually of a temporary or casual nature, rather than permanent positions. Sick pay, holiday pay and superannuation are not paid.

Typical occupations include textile workers, fruit pickers, decorators and printers.

Casual

Casual employees do not work a set number of hours per week but are employed when needed, sometimes at short notice. A fixed amount is paid for each hour's work. Higher hourly rates are paid because they are not entitled to receive sick pay or holiday pay. Casual employees are paid superannuation.

Typical occupations include tennis coaches, temporary secretaries and casual teachers.

Example 1

Caleb is a used car salesman. He is paid a weekly retainer of \$150 plus a commission of 2% on the value of his sales in excess of \$30 000. Find his total pay for a week when his sales total \$53 000.

Solution

- i Sales in excess of \$30 000 = $\$53\,000 - \$30\,000$
= \$23 000
- ii Commission = 2% of \$23 000
= $0.02 \times \$23\,000$
= \$460
- iii Total pay = retainer + commission
= $\$150 + \$460$
= \$610

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EG
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A printer charges \$0.60 for each colour pamphlet produced and \$0.15 for each black and white leaflet. How much would he receive for printing 250 colour pamphlets and 600 black and white leaflets?

Solution

$$\begin{aligned}\text{Total pay} &= (250 \times \$0.60) + (600 \times \$0.15) \\ &= \$150 + \$90 \\ &= \$240\end{aligned}$$

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Trudi is a swimming instructor. She charges \$15 per lesson for children aged 5 to 10 years and \$12 per lesson for children aged 11 to 15 years. She is paid \$219 altogether for her 8 am class on Saturday morning. If 9 of the children are aged between 5 and 10 years, how many children are aged between 11 and 15 years?

Solution

- i Pay for children 5–10 years = $\$15 \times 9$
= \$135
 - ii Pay for children 11–15 years = $\$219 - \$135$
= \$84
 - iii Number of children 11–15 years = $\$84 \div \$12$
= 7
- $\therefore$ There are 7 children in the 11–15 years age group.

Exercise 3.2

- Calculate each of these commissions.
 - a 5% on sales of \$12 000 b $2\frac{3}{4}\%$ on sales of \$9442 c 6.8% on sales of \$3127
- 2 Kerrie is paid a commission of 12% on all perfume sales that she makes. Find her commission for a week when she sells perfume products to the value of \$2340.
- 3 A telesales representative is paid a fortnightly commission of 13.5% on the total value of his sales. Find his commission for the previous fortnight when sales totalled \$4200.
- 4 A factory worker is paid 22c for each gadget she constructs on the assembly line. Calculate her pay for a week in which she constructs 2135 gadgets.
- 5 Karen is a textile worker who receives \$11.75 for each garment she knits. How many garments did Karen knit last week if she was paid \$411.25?

Consolidation

- 6 Bill and Ben are furniture salesmen. Bill is paid a commission of $2\frac{1}{2}\%$ on his weekly sales, while Ben is paid an annual salary of \$29 500. Last week Bill sold furniture to the value of \$23 600. Who received the greater pay last week, and by how much?
- 7 Mohammed's rate of commission was 7.5% in December and rose to 8% in January. Calculate Mohammed's total pay for these two months if his sales amounted to \$27 800 in December and \$29 250 in January.
- 8 Petra is a sales representative for a large pay TV company. She is paid a weekly retainer of \$165 plus a commission of \$25 for each new customer that she signs up. Find Petra's total pay for a week when she signs up 17 new pay TV customers.
- 9 Tonia's weekly pay consists of a retainer of \$240 plus a commission of $6\frac{1}{2}\%$ on her sales. Calculate Tonia's total pay for the week in which her daily sales were:
- | | | |
|----------------|---------------|--------------|
| • Mon.—\$480 | • Tues.—\$535 | • Wed.—\$513 |
| • Thurs.—\$675 | • Fri.—\$608 | • Sat.—\$320 |
- 10 Norito is paid a basic fortnightly salary of \$395 as well as a commission of 18% on the value of all sales in excess of \$16 500. Calculate his pay for a fortnight where his sales totalled \$21 475.
- 11 A real estate agent is paid a commission of 3% on the first \$180 000 of the value of a property and 2% on the remaining value. Find her total commission on the sale of each of the following properties.
- a A home unit sold for \$175 000 b A house sold for \$263 000
- 12 Miss Elle Itquik is a real estate agent. To sell a property, she charges a commission of $3\frac{1}{2}\%$ on the first \$200 000 of the value of the property, $2\frac{1}{2}\%$ on the next \$150 000 and 1% on the remaining value. If the house is sold at auction, there is an additional auction fee of \$2875. How much will she receive for selling a house at auction for \$865 000?
- 13 Wade sold goods last week to the value of \$7260. His pay for the week was \$535.40, which comprised a retainer plus a commission of 4% on his sales. How much is Wade's retainer?
- 14 Joshua is employed to sell earth-moving equipment to construction companies. He is paid an annual salary of \$22 400 plus a commission of 11% on all sales in excess of \$440 000. Calculate his average fortnightly pay for a year when he sells \$469 300 worth of equipment.
- 15 Roy is a junior Rugby League referee. He is paid \$65 for each game that he referees on weekdays and \$85 for each weekend game. Find his total pay for a week in which he referees games on Tuesday and Thursday evenings, two games on Saturday and another game on Sunday.

- 16** Joy takes in extra ironing to help pay the bills. She charges \$12 per basket for shirts and \$7 per basket for trousers. Altogether Joy was paid \$117 for 3 baskets of trousers and several baskets of shirts. How many baskets of shirts did she iron?
- 17** A doctor charges \$22.50 for consultations that last less than 10 minutes and \$28.75 for those that last longer than 10 minutes. How much will she be paid for a day in which she sees 19 patients each for less than 10 minutes and 14 patients for longer than 10 minutes?
- 18** To pick up and deliver furniture the TLC Removal Company charges customers \$50 per hour for 2 men and \$70 per hour for 3 men. The hourly rates apply from the time the truck leaves the depot until its estimated return. There is an extra fee of \$75 for customers moving to or from an above-ground floor unit or double storey house.
- Calculate the total fee payable on a job where the truck leaves the depot at 7:30 am and returns at 2:30 pm, 3 men are needed and the furniture is being delivered from a top-floor unit to a house.
 - If the company takes 34% of the money and the men split the remainder equally, find the amount that each man will be paid.
- 19** An advertisement is placed in the major newspapers for experienced fruit pickers to pick grapes in South Australia. The successful applicants are to be paid \$180 per day, as well as a dislocation allowance of \$35 per week if they are from interstate. How much would an experienced fruit picker from New South Wales earn if she was hired to pick grapes for 12 weeks?
- 20** Paul is a car windscreen fitter. He pays \$110 for each standard windscreen and charges a mark-up of 15% plus \$28 for labour. Calculate Paul's profit for a day when he installs 6 windscreens.
- 21** Adam runs a small printing company. He charges \$21.80 to print 100 personal business cards and \$43 to print 50 colour brochures. How much will Adam be paid by a customer who orders 350 business cards and 4000 colour brochures.
- 22** Deborah is contracted by a local toy store to make teddy bears. She is paid \$8.20 per bear for the first 40 bears, then \$9.35 for each bear thereafter. How much will Deborah be paid in a month where she makes 48 teddy bears?
- 23** As a travelling salesman, Nathan is paid a weekly retainer of \$140, a commission of 8% on his sales, plus a travel allowance of 48c per kilometre. The table below shows his daily sales and distance travelled for the first week of February.
- Find Nathan's total sales.
 - How far did he travel altogether?
 - Calculate Nathan's total pay for the week.

	Sales	Distance travelled (km)
Monday	\$942	215
Tuesday	\$564	72
Wednesday	\$1088	148
Thursday	\$740	95
Friday	\$916	266
Saturday	\$625	310

- 24 A stockbroker purchases 15 000 NRMA shares at \$2.72 per share on behalf of a client. Her brokerage fees are 2% on the first \$2000 and 1.5% on the remaining value of the shares.
- Calculate the purchase price of the shares.
 - How much will the broker receive in fees?
- 25 Tony the television repairman charges a call-out fee of \$35, \$42 per hour for labour, plus the cost of parts. Mrs Anania books Tony to come to her house and repair the family's television set. The job takes $2\frac{1}{2}$ hours and requires new parts to the value of \$55. How much should Tony charge for the repair job?

Further applications

- 26 Diana is paid a retainer of \$255 per week as well as a commission based on the value of her weekly sales. By the end of the week she had sold \$10 400 worth of automobile parts to several retailers. If her total pay for the week was \$463, find Diana's rate of commission.
- 27 Roberta is a real estate agent. To sell a property, Roberta charges 3% on the first \$200 000 of the value of a property, 2.5% on the next \$150 000 and 2% on the remaining value. If the property is sold at auction, she charges an extra \$2350. Roberta successfully auctioned a property and was paid \$20 000 altogether in commission and auction fees. What was the sale price of the property?

3.3 Overtime and other payments

Overtime is extra time worked beyond the regular number of hours. Overtime hours are usually paid at $1\frac{1}{2}$ times or 2 times the normal hourly rate of pay. Such overtime rates are commonly called 'time and a half' or 'double time' respectively.

A **bonus** is an extra payment or other benefit given to an employee as a reward for exceptional performance in their job.

Holiday leave loading is a bonus given to all permanent wage and salary earners and some other employees when they take their annual holidays. Employees are paid an *additional* $17\frac{1}{2}\%$ of 4 weeks normal pay, assuming that they take the maximum 4 weeks leave all at once.

Example 1

Ethan earns \$15.10 per hour for a 38-hour week as an electrician. Overtime is paid at the time and a half rate for the first 5 hours and double time thereafter. Calculate Ethan's pay for a week when he works for 46 hours.

Solution

The first 38 hours are paid at the normal hourly rate, the next 5 hours are paid at 1.5 times the normal hourly rate and the final 3 hours are paid at twice the normal hourly rate.

$$\begin{aligned}\text{Pay} &= (38 \times \$15.10) + (5 \times 1.5 \times \$15.10) + (3 \times 2 \times \$15.10) \\ &= \$573.80 + \$113.25 + \$90.60 \\ &= \$777.65\end{aligned}$$

EG
+S**Example 2**

Juanita works as a waitress and is normally paid \$324 for a 30-hour week. For how many hours would she have worked in a week where her total pay is \$405 if all overtime is paid at the time and a half rate?

Solution

- i Overtime pay = $\$405 - \$324$
= \$81
 - ii Normal hourly rate of pay = $\$324 \div 30$
= \$10.80
 - iii Hourly overtime rate of pay = $\$10.80 \times 1.5$
= \$16.20
 - iv Number of overtime hours = $\$81 \div \$16.20$
= 5
 - v Total hours worked = $30 + 5$
= 35
- $\therefore$ Juanita worked for 35 hours.

EG
+S**Example 3**

Keryn's annual salary is \$29 744. At Christmas, Keryn takes her annual holidays and is paid 4 weeks normal pay plus a holiday leave loading of $17\frac{1}{2}\%$ on this amount.

- a Calculate the holiday leave loading.
- b Find Keryn's total holiday pay.

Solutions

- a i 4 weeks normal pay = $\$29\,744 \div 52 \times 4$
= \$2288
- ii Holiday loading = $17\frac{1}{2}\%$ of \$2288
= $0.175 \times \$2288$
= \$400.40
- b Total pay = $\$2288 + \$400.40$
= \$2688.40

Exercise 3.3

- 1 An employee's normal rate of pay is \$12 per hour. Find her overtime rate of pay at the:
 - a time and a half rate
 - b double time rate
- 2 Merv is a motor car mechanic. His normal rate of pay is \$16 per hour. How much overtime pay would Merv earn by working for:
 - a 3 hours at the time and a half rate?
 - b 4 hours at the double time rate?
- 3 For how many hours would a person need to work at the normal hourly pay rate in order to earn the same amount of money as that earned for working:
 - a 5 hours at the double time rate?
 - b 4 hours at the time and a half rate?
- 4 Calculate the total pay for a week in which a tradesperson works:
 - a 37 hours at \$16/h and 4 hours at the time and a half rate
 - b 39 hours at \$20.85/h and 3 hours at the double time rate
 - c 32 hours at \$21.50/h and 3 hours at the time and a half rate
 - d 36 hours at \$17.40/h and $5\frac{1}{2}$ hours at the double time rate
 - e 38 hours at \$15.65/h, 2 hours at the time and a half rate and 4 hours at the double time rate

Consolidation

- 5 Harry is an electrician who this week worked for 40 hours at the normal rate of \$19.40 per hour and 6 hours overtime at the time and a half rate. Find Harry's total pay for the week.
- 6 A plumber worked for 36 hours at \$22.50 per hour and 3 hours overtime at the double time rate. Calculate his total pay for the week.
- 7 Elise is a textile worker. She is paid \$12.30/h for the first 40 hours, time and a half for the next 5 hours and double time thereafter. Calculate her pay for a week when she works:
- a 40 hours b 44 hours c 50 hours
- 8 A construction worker earns \$17.25/h for the first 38 hours worked and double time thereafter. Find his total pay for a week when he worked for 46 hours.
- 9 Antoinette is paid at the rate of \$14.50/h before 1 pm and at the time and a half rate after 1 pm. Find her total pay for a week when she worked 10 am–5 pm, Monday to Friday.
- 10 Hayden's normal rate of pay is \$24 per hour, Monday to Friday. He is paid at the time and a half rate after 6 pm during the week and at the double time rate on weekends. Calculate Hayden's total pay for a week when he worked the following hours.
- Mon.—10 am to 6 pm • Tues.—10 am to 6 pm • Wed.—10 am to 8 pm
 - Thurs.—1 pm to 9 pm • Sat.—10 am to 4 pm
- 11 Calculate the number of overtime hours worked by an employee who was paid:
- a \$20 per hour for the first 40 hours, earning \$920 altogether, with overtime being paid at the double time rate
- b \$18.80 per hour for the first 31 hours, earning \$780.20 altogether, with overtime being paid at the time and a half rate
- c \$17.45 per hour for the first 37 hours, earning \$924.85 altogether, with overtime being paid at the double time rate
- 12 A tradesperson normally earns \$636.50 for a 38-hour week. For how many hours would she have worked in a week where her total pay is \$804, if all overtime is paid at the double time rate?
- 13 Calculate the normal hourly rate of pay for an employee who was paid:
- a \$649.30 for working 34 hours at the normal rate and 6 hours at the time and a half rate
- b \$1029 for working 32 hours at the normal rate and 5 hours at the double time rate
- c \$684.45 for working 36 hours at the normal rate and 3 hours at the time and a half rate
- 14 A locksmith worked for 36 hours at \$14.70/h and 6 hours overtime at the double time rate.
- a Calculate his total pay for the week.
- b How much less would the locksmith be paid if the overtime was paid at the time and a half rate?
- 15 Last week, a groundsman worked for 34 hours at \$13.80 per hour and 8 hours overtime, which was paid at the time and a half rate.
- a How much was the groundsman paid for the week?

- b** For how many hours would he need to work at the normal rate to earn the same amount of money?
- 16** Madeleine's employer gave her a Christmas bonus equivalent to 3% of her annual salary. Find Madeleine's bonus if she normally earns \$1950 per fortnight.
- 17** An advertising salesman was given a bonus of 20% of one month's pay as a reward for creating a series of successful advertising campaigns. Find the amount of his bonus if he usually earns \$840 per week.
- 18** Leave loading is an annual bonus calculated on $17\frac{1}{2}\%$ of 4 weeks normal pay. Find the holiday leave loading that is due to an employee who earns:
- | | |
|---------------------------|-------------------------------|
| a \$640 per week | b \$1106 per fortnight |
| c \$3120 per month | d \$48 340 per year |
- 19** Pedro works for 38 hours each week and is paid \$16.25 per hour. At Christmas, Pedro takes his annual 4 weeks leave. He is paid 4 weeks pay plus $17\frac{1}{2}\%$ leave loading on this amount. Calculate Pedro's total holiday pay.
- 20** A stockbroker earns an annual salary of \$82 460. Calculate her total holiday pay if she takes her annual 4 weeks leave in July.

■ Further applications

- 21** Calculate the fortnightly salary of an employee whose 4 weeks annual leave loading amounts to \$980.
- 22** Surangi was paid \$3946 when she took her annual 4 weeks leave. This amount is composed of her normal 4 weeks pay, holiday leave loading plus a bonus of \$280. Find Surangi's normal fortnightly pay.

3.4 Wage deductions

A person's total income before any amounts are deducted is called their **gross income**. Typical deductions include income tax, superannuation, union fees and medical insurance. The amount of money remaining after the deductions have been made is called the **net income**.

$$\text{Net income} = \text{gross income} - \text{deductions}$$

■ Income tax

The Australian government requires employers to deduct a certain amount of money from the gross pay of their employees each pay period. This money is then paid to the government on a monthly basis. Because the tax is taken out of employees' pay packets on a regular basis, it is called **Pay As You Go** or **PAYG** tax.

■ Superannuation

Superannuation is money set aside by an employee for their retirement. Upon retirement, it can be taken either in a lump sum or in the form of regular payments. An employee can decide how much, if any, of their gross income to save in this manner. Employers, however, are required by law to contribute a percentage of each permanent employee's wage or salary into a superannuation fund on their behalf.

Example

A carpenter is paid a gross annual salary of \$34 630. Each week he contributes 5% of his gross pay to a voluntary superannuation fund. His other weekly deductions are \$166.50 in PAYG tax and \$6.90 for union fees.

a Calculate the total deductions.

b Find the net weekly income.

Solutions

$$\begin{aligned} \text{a i Weekly superannuation contributions} &= 5\% \text{ of } \$34\,630 \div 52 \\ &= 0.05 \times \$34\,630 \div 52 \\ &\div \$33.30 \end{aligned}$$

$$\begin{aligned} \text{ii Total weekly deductions} &= \$33.30 + \$166.50 + \$6.90 \\ &= \$206.70 \end{aligned}$$

$$\begin{aligned} \text{b Net weekly income} &= \text{gross weekly income} - \text{deductions} \\ &= (\$34\,630 \div 52) - \$206.70 \\ &\div \$459.26 \end{aligned}$$

Exercise 3.4

- 1 Jack earns a gross weekly salary of \$480. His employer deducts \$116 in PAYG tax instalments each week. What is Jack's net weekly pay?
- 2 Julianne has a net weekly salary of \$372 and deductions totalling \$143. What is her gross weekly pay?
- 3 Eve's gross fortnightly pay is \$1070. Each fortnight her employer deducts \$313.40 in PAYG tax instalments and she contributes \$51.70 to a voluntary superannuation fund for her retirement. Find Eve's net weekly pay.
- 4 Calculate the net pay for each of these employees.
 - a Mick has a gross fortnightly wage of \$964 and deductions of \$210 in tax and \$14.50 in union fees.
 - b Yumi has a gross weekly wage of \$755.40 and deductions of \$213.10 in tax, \$37.75 in superannuation and \$13.50 in health insurance.
 - c Nicholas has a gross monthly salary of \$2690.80 and deductions of \$794.65 in tax, \$21.30 in life insurance and \$32.82 in union fees.

■ Consolidation

- 5 Mrs Hadlee has a gross annual salary of \$32 568. Each week, the following amounts are deducted from her pay: tax, \$151.80; superannuation, \$32.95; health insurance, \$15.45. Find her:
- a gross weekly pay b total weekly deductions c net weekly pay
- 6 A printer is paid a gross weekly wage of \$13.25 per hour for a 40-hour week. He has weekly deductions totalling \$195.20. What is his net weekly wage?
- 7 A fork-lift driver is paid a gross wage of \$13.40 per hour for 75 hours per fortnight. He has deductions of \$303.15 in PAYG tax instalments and \$15.50 for home and contents insurance. Find his net fortnightly wage.
- 8 The gross hourly wage for a fitter and turner is \$16 per hour for a 34-hour week. He has \$152.32 deducted in weekly tax instalments. What percentage of his:
- a gross wage is paid in tax?
b net wage is paid in tax? (Answer correct to the nearest whole percentage.)
- 9 Maryanne receives a gross fortnightly wage of \$1366 and her fortnightly deductions total \$524. Find her net monthly wage.
- 10 A ferry master is paid \$13.80 per hour for a 40-hour week, with overtime paid at the time and a half rate.
- a Find his net pay for a week when he worked for 45 hours and had PAYG tax deductions of \$229.98.
b What percentage of his gross pay was paid in tax? (Answer correct to the nearest whole percentage.)
- 11 A construction worker earns \$19.60 per hour for a 38-hour week plus a weekly site allowance of \$18.90. Each week his employer deducts \$267.30 in tax and 5% of the gross wage is paid into a superannuation fund.
- a How much money is paid into the superannuation fund each week?
b Calculate the worker's net weekly pay.
- 12 Working as a crane operator, Barry earns \$14.50 per hour. His normal working hours are from 8 am to 4 pm, Monday to Friday, with overtime paid at the double time rate. Last week, Barry worked the following hours:
- | | | |
|-----------------------|------------------------|------------------------|
| • Mon.—8 am to 4 pm | • Tues.—8 am to 5 pm | • Wed.—8 am to 5:30 pm |
| • Thurs.—8 am to 4 pm | • Fri.—8 am to 4:30 pm | |
- Each week Barry pays 7.5% of his gross wage (not including overtime pay) into a superannuation fund. Last week his employer deducted \$195.65 in PAYG tax instalments.
- a What was Barry's gross pay for last week, including overtime pay?
b How much money is deducted in superannuation payments each week?
c Calculate Barry's net pay for last week.

Further applications

- 13 Dominique has a regular net weekly income of \$468. Her employer deducts 35% of her gross weekly income in tax. Find Dominique's gross annual salary.
- 14 Last week Zachary worked for 36 hours at the normal pay rate and 4 hours overtime at the double time rate. His employer deducted 43% of his gross income in tax, leaving a net income for that week of \$478.80. Calculate Zachary's normal gross hourly pay rate.

3.5 Taxation

Under the Pay As You Go or PAYG tax system, employers deduct money each pay period from the gross pay of their employees. Self-employed people need to organise this themselves. This money is then paid to the government on a monthly basis. The amount of money deducted varies with each employee according to their gross income and the tax scales in operation at the time.

The **financial year** begins on 1 July and ends on 30 June. At the end of each financial year, employers are required to give each employee a **payment summary form**. The payment summary form shows the annual income, deductions such as union fees and superannuation, and the amount of tax that has been withheld during the year. Employees, and those who are self-employed, must then fill in a tax return form and lodge it with the Australian Taxation Office (ATO) on or before 31 October. If a person does not lodge the form in time, they may be fined.

The amount of tax that each person must pay is based on their **taxable income**. The taxable income is calculated by finding the total gross income (e.g. salary, bank interest, rent from an investment property) and then subtracting any allowable tax deductions (e.g. work-related expenses, travel expenses, charitable donations). People can minimise the amount of tax that they have to pay by claiming as many legitimate tax deductions as possible.

Taxable income = total gross income – allowable tax deductions

The table below is the 2000/2001 tax table for Australian wage and salary earners. It can be used to work out how much tax a person has to pay. The taxable income is always rounded down to the nearest dollar.

Taxable income	Tax payable
\$1–\$6000	Nil
\$6001–\$20 000	17 cents for each \$1 over \$6000
\$20 001–\$50 000	\$2380 + 30 cents for each \$1 over \$20 000
\$50 001–\$60 000	\$11 380 + 42 cents for each \$1 over \$50 000
\$60 001 and over	\$15 580 + 47 cents for each \$1 over \$60 000

Sometimes the amount of tax taken out by employers is not enough because the employee has earned extra income from other sources. In this case, the person may have to pay more tax to the government. If, however, the employer has taken out too much tax because the employee is able to claim several tax deductions, then the person may receive a tax refund.

As well as paying income tax, Australians contribute to the cost of funding the public health-care system through the payment of an annual Medicare levy. The table below shows the levy payable by individual taxpayers. The amount payable may also vary according to an individual's circumstances.

Taxable income	Medicare levy
\$1–\$13 807	Nil
\$13 808–\$14 926	20 cents for each \$1 over \$13 807
\$14 927 and over	Flat 1.5% of taxable income

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Example 1

Imelda earns \$28 490 as a floor polisher. Last year her employer took out \$5290 in PAYG tax instalments.

- Calculate the tax payable.
- Is Imelda entitled to a tax refund or does she have to pay more tax? Explain.

Solutions

- A taxable income of \$28 490 falls into the \$20 001 to \$50 000 tax bracket.
 - We first need to calculate by how much this salary exceeds \$20 000.
Now, $\$28\,490 - \$20\,000 = \$8490$.
 - Tax payable = $\$2380 + (30\text{c for each } \$1 \text{ over } \$20\,000)$
 $= \$2380 + (0.3 \times \$8490)$
 $= \$2380 + \$2547$
 $= \$4927$
- $\$5290 - \$4927 = \$363$. The amount of tax taken out is \$5290, which is greater than the tax payable, \$4927.
Therefore, Imelda is entitled to a tax refund of \$363.

EG
+ S

Example 2

Arjuna earns \$56 315 as a construction engineer. In the last financial year he also earned \$436 in interest on his savings and he had allowable tax deductions totalling \$3720.

- Find the taxable income.
- How much tax should he have paid?
- Find the average net weekly income.

Solutions

- Taxable income = total gross income – allowable tax deductions
 $= (\$56\,315 + \$436) - \$3720$
 $= \$53\,031$

- b**
- i** $\$53\,031 - \$50\,000 = \$3031.$
 - ii** Tax payable = $\$11\,380 + (42c \text{ for each } \$1 \text{ over } \$50\,000)$
 $= \$11\,380 + (0.42 \times \$3031)$
 $= \$11\,380 + \$1273.02$
 $= \$12\,653.02$
- c**
- i** Net annual income = total gross income – tax
 $= (\$56\,315 + \$436) - \$12\,653.02$
 $= \$44\,097.98$
 - ii** Average net weekly income = net annual income $\div$ 52
 $= \$44\,097.98 \div 52$
 $\div \$848.04$

Exercise 3.5

- 1** Use the tax table on page 93 to calculate the tax payable on each annual salary.

a \$5200	b \$13 000	c \$34 750	d \$53 345
e \$71 260	f \$46 863	g \$58 758	h \$172 510
- 2** Mr Ford has an annual salary of \$56 300. His tax accountant told him that he could claim a total of \$2580 in tax deductions.

a Find Mr Ford's taxable income.	b Calculate the tax payable on this income.

Consolidation

- 3** Kurt has two jobs. During the day he works as a builder's labourer and in the evening he works in a café. As a labourer, Kurt is paid \$601.25 per week while the owner of the café pays him \$446 per fortnight.
 - a** Find Kurt's total annual income.
 - b** Calculate the tax payable on this income.
- 4** After leaving school, Denise got a part-time job working at a service station and was paid a weekly wage of \$285. Because she was studying at university in the evenings, Denise was able to claim tax deductions of \$744 at the end of the financial year for educational expenses.
 - a** What was Denise's gross annual income?
 - b** Find her taxable income.
 - c** How much tax must Denise pay on her annual income?
- 5** As a computer systems analyst, Tim earns an annual salary of \$74 300. Last year he earned an additional dividend of \$4065 on his share portfolio and \$710 interest on his savings. He had allowable tax deductions of \$4351.80.
 - a** Calculate Tim's gross annual income.
 - b** Find his taxable income.
 - c** Find the amount of tax that Tim must pay.

- 6 Stefan's gross fortnightly pay is \$1610.58. His employer took out \$384.60 each fortnight in PAYG tax instalments.
- a Find Stefan's gross annual income.
 - b Calculate the tax payable on this income.
 - c Is Stefan entitled to a tax refund? If so, how much?
- 7 Bettina's gross monthly pay is \$4447.08. In the last 12 months she earned interest of \$294.50 on her savings, \$2870 on a real estate investment and had allowable tax deductions totalling \$385.40. Her employer deducted \$985.34 each month in PAYG tax instalments.
- a Find Bettina's gross annual income.
 - b Find her taxable income.
 - c Calculate the tax payable.
 - d Will Bettina receive a tax refund or does she have to pay more tax? Explain your answer.
- 8 Alana's gross weekly income is \$475.48 and she has tax deductions of \$412.72. Find:
- a her gross annual income
 - b her taxable income
 - c the tax payable
 - d her net weekly income
- 9 Find the net fortnightly income on a gross annual salary of \$64 312, with allowable tax deductions totalling \$2140.
- 10 Ian's gross weekly pay last year was \$924. At the end of the year he received a 10% pay rise.
- a What was Ian's net weekly pay before the pay rise?
 - b What was Ian's net weekly pay after the pay rise?
 - c By how much has Ian's weekly take home pay increased?
- 11 The normal working week for a steelworker is 39 hours at \$18.75 per hour, with overtime being paid at the time and a half rate. Last year the steelworker worked 146 hours overtime on top of his regular hours and had allowable tax deductions of \$1178.40.
- a Calculate his gross income for the year.
 - b Find the tax payable on this income.
 - c Calculate the tax refund to which he is entitled if PAYG tax instalments of \$168.23 were deducted from his gross pay each week.
- 12 Doug's gross fortnightly salary is \$1448. At Christmas he receives 4 weeks pay plus a holiday leave loading of $17\frac{1}{2}\%$. During the year, he spent \$4200 on dental work and was able to claim 20% of this amount in excess of \$1250 as a tax deduction.
- a Calculate Doug's gross annual salary including the leave loading.
 - b Find the taxable income.
 - c How much tax should Doug pay on this taxable income?
- 13 Erlinda invested \$16 400 and was paid interest on this amount at 6% per annum for the financial year ending 30 June. She was required to declare this interest as part of her taxable income. Erlinda's gross fortnightly salary is \$2209.
- a How much interest did she receive on the investment?
 - b Calculate the amount of tax that Erlinda paid last financial year.
- 14 Dobey's monthly salary for the first 6 months of the financial year was \$3095.15. He then received a 12% pay rise. How much tax should Dobey pay for that financial year?

- 15** Paula's gross annual salary is \$45 000. Her tax advisor approved the following tax deductions:
- 40% of the cost of a new computer that was purchased for \$3570
 - \$396.20 for stationery
 - \$572.90 for work-related travel expenses
 - \$151.75 for dry-cleaning of uniforms
 - \$384.55 for union fees
- a** What is the total value of Paula's tax deductions?
b What is her taxable income?
c Calculate the tax payable on this income.
- 16** Prior to the reform of the Australian tax system in 2000, a person with an annual income of \$49 000 paid tax of \$8942 plus 43 cents for each \$1 earned over \$38 000.
- a** Calculate the tax payable on an income of \$49 000 under the old tax scale.
b Calculate the tax payable on this income under the new tax scale.
c How much better off per week is a person with this income under the new tax scale?
- 17** Giselle is paid a weekly retainer of \$145.65 plus a 5% commission on the value of her sales. Last year, Giselle sold books to the value of \$652 375. She claimed a total of \$2966.30 in tax deductions for the year.
- a** Find her gross annual income.
b Find the tax payable.
c What was Giselle's *average* net weekly pay last year?
- 18** Individual Australian taxpayers whose taxable income is greater than \$13 807 are required to pay an annual Medicare levy to contribute to the cost of public health care. The payment scale is shown on page 94. An additional 1% Medicare levy surcharge is payable by individual taxpayers with a taxable income greater than \$50 000 per annum who have not taken out private health insurance with a registered health fund.
Calculate the annual Medicare levy payable on a taxable income of:
- a** \$12 500 **b** \$21 000 **c** \$14 260
- 19** Harley is an engineer with a taxable income of \$95 400. He does not have private health insurance.
- a** Does Harley have to pay the extra Medicare levy surcharge of 1%?
b How much will he pay altogether for Medicare?
c Would Harley be better off by paying \$675 per year for private hospital cover? If so, by how much?
- 20** Glenda is an industrial chemist with a taxable income of \$48 460. Her employer has deducted \$185 per week in PAYG tax instalments.
- a** Calculate the tax payable on this income.
b Calculate the Medicare levy payable on this income.
c How much tax has been deducted from Glenda's annual salary?
d Is Glenda entitled to a refund or does she have to pay more tax? In either case, state the amount.

- 21 Dilini has a gross annual salary of \$75 484 and last year she earned \$4620 in dividends on her shares. She had allowable tax deductions of \$725 for work-related travel expenses and \$594 for other expenses. Dilini has top private hospital cover. Her employer deducted \$528.30 in PAYG tax instalments each week from her gross pay.
- Find her taxable income.
 - Calculate the amount of tax that she should pay on her income.
 - How much should Dilini pay for Medicare?
 - Is Dilini entitled to receive a tax refund? If so, how much?

■ Further applications

- 22 Find the taxable income of an employee who paid the amounts below in tax. (Ignore the Medicare levy.)
- a \$1598 b \$4870 c \$13 358.20 d \$31 847.64
- 23 Victor earns a regular hourly rate of pay for working 35 hours per week as an electrician and is not eligible to earn overtime pay. In the previous financial year he paid \$14 160.40 in tax and also paid a Medicare levy. He had total tax deductions of \$3790.35 and does not have private health insurance. Find:
- a his taxable income b the Medicare levy c his hourly rate of pay

3.6 Budgeting

A **budget** is an organised spending plan based upon a person's net income. It is important to work out your spending habits and make an accurate budget so that you have enough money at hand when bills come in. Some bills vary a lot at different times of the year and unexpected expenses may also occur. It is wise to save as much money as you can so that you are not caught out.

Example

Helen has just moved out of home. She has a net income of \$540 per week as a video technician and spends $\frac{1}{9}$ of her income on food. Helen has created this weekly budget to manage her finances.

- How much does Helen spend on food each week?
- How much money does she save each week?
- What percentage of her income does Helen spend on rent, correct to 1 decimal place?
- If Helen's budget was represented on a sector graph, what would be the central angle in the 'clothes' sector?

Item	Weekly expenses
Rent	\$160
Car loan repayment	\$125
Petrol	\$35
Food	
Clothes	\$30
Entertainment	\$45
Savings	

Solutions

- a $\frac{1}{9} \times \$540 = \$60$
- b Savings = $\$540 - (\$160 + \$125 + \$35 + \$60 + \$30 + \$45)$
 $= \$540 - \$455$
 $= \$85$
- c $\frac{160}{540} \times 100\% \div 29.6\%$
- d Central angle = $\frac{30}{540} \times 360^\circ$
 $= 20^\circ$

Exercise 3.6

- 1 Partima has a net weekly income of \$410. She has formulated a weekly budget plan in order to manage her money.
- How much does Partima save each week?
 - Partima pays the rent on a fortnightly basis. How much rent does she pay each fortnight?
 - What percentage of her net income does she spend on food? Answer correct to the nearest whole percentage.
- 2 Kendra earns \$572 per week after tax. She lives alone and has just bought a new car on terms. She sets aside equal amounts of money for bills and entertainment.
- How much does Kendra set aside for bills?
 - What are her annual expenses for rent?
 - What percentage of her income does Kendra save? Answer correct to the nearest whole percentage.
- 3 Reggie shares a house with 3 friends. They pay \$320 per week in rent and share equally the payment of the rent and all bills. This year they estimate that the bills will amount to approximately \$2080.
- How much does Reggie contribute each week for: **i** rent? **ii** bills?
 - What is Reggie's net weekly income?
 - What fraction of his income is spent on food?
 - How much more does Reggie spend on rent than on entertainment?

Partima's weekly budget

Rent	\$205
Transport	\$28
Food	\$75
Entertainment	\$50
Savings	

Kendra's weekly budget

Rent	\$185
Car loan repayment	\$76
Petrol	\$42
Bills	
Clothes	\$35
Entertainment	
Food	\$75
Savings	\$29

Reggie's weekly budget

Rent	
Bills	
Entertainment	\$65
Transport	\$22
Food	\$60
Clothes	\$45
Savings	\$38

Consolidation

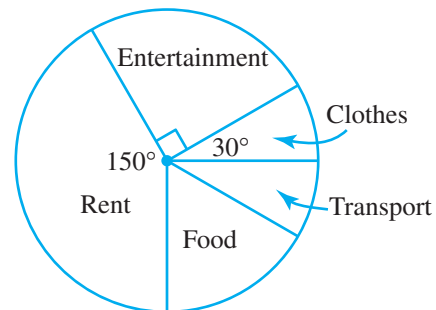
- 4 Stuart lives at home with his parents. He goes to university at night and works as a storeman and packer by day. He pays 30% of his net income to his parents for food and board.
- What is Stuart's net weekly income?
 - He budgets \$10 more per week for entertainment than for his university expenses. How much does he budget for each item?
 - If this budget was to be shown on a sector graph, what size would the central angle be for savings?
 - Stuart received a pay rise of 12% and he decided to split this extra amount between what he spends on clothes and what he pays for food and board. How much will he spend on clothes after the pay rise?

Stuart's weekly budget

Food and board	\$105
Entertainment	
Uni. expenses	
Petrol	\$40
Clothes	\$45
Savings	\$70

- 5 Virgil constructed this sector graph to illustrate his weekly budget.
- Virgil spent twice as much on food as he did on transport. What angle should be shown for each of these sectors?
 - What fraction of his weekly income does Virgil spend on clothes?
 - Virgil pays \$225 per week in rent. Find his net weekly income.
 - Is this a good budget? Explain.

Virgil's weekly budget



- 6 Anthea has a net annual salary of \$27 300. She has weekly expenses of \$17.50 for health insurance, \$37 for petrol and \$80 for food. She pays \$480 per fortnight for rent and \$78 per month for car insurance. She wants to allow \$75 per week for savings. Draw up a weekly budget for Anthea, with the remaining money being divided equally between savings and 'other expenses'.
- 7 Brett earns \$16.50 per hour after tax for a 36-hour week. He pays \$354 each fortnight for rent and has weekly expenses of \$110 for food and \$48 for petrol. His annual expenses are \$1550 for bills and \$940 for car insurance. Brett pays 5% of his weekly wage into a superannuation fund for his retirement. The rest of his money is deposited into a savings account. Draw up a weekly budget for Brett, rounding off all amounts to the nearest dollar.
- 8 Jillian wants to buy a new car. She took out a car loan and agreed to make regular monthly repayments for 5 years. She will also need to budget for the following costs:
- Petrol—\$35 per week
 - Loan repayment—\$355 per month
 - Insurance—\$75.40 per month
 - Driver's licence renewal—\$35 per annum
 - Servicing—\$280 every 6 months
 - Green slip—\$315 per annum
 - Registration—\$225 per annum

- a How much will Jillian eventually pay on the car loan?
- b What is the total annual running cost of this car?
- c How much will Jillian need to budget each week to cover the running costs?
- d Calculate the total cost of buying and running this car for 5 years.

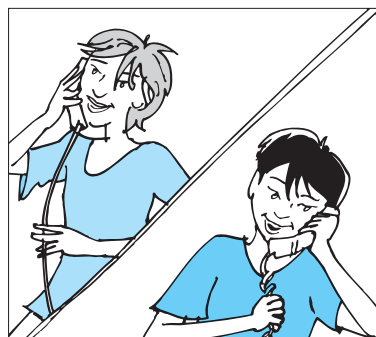
■ Further applications

- 9 a Select a job that appeals to you from the employment section of a newspaper and note the annual salary/pay conditions.
- b Read through the real estate section and select a house/home unit that you can afford to rent on this salary.
- c Choose a new/used car from the motoring section and note the purchase price and repayment costs.
- d Get copies of recent family bills for electricity, water, gas, telephone and so on and estimate the weekly costs for a single person.
- e Prepare a complete weekly budget, including rent, car payments, bills, petrol, clothes, entertainment, insurance, food and savings. How much money is left over?

TRY THIS Telephone charges

The following is a summary of a report that was published in a newspaper.

'Charges for Telco Blue long-distance telephone calls will be slashed by up to 31%. This will save consumers \$100m per year. The biggest reductions will be on the longest distances, for example Sydney-to-Melbourne calls are slashed from 67c to 46c for 3 minutes and Brisbane-to-Perth calls will fall from 90c to 68c for 3 minutes.'



- a Find the percentage decrease in the costs of the 2 calls quoted in the article.
- b A firm had a telephone bill in 2002 of \$12 580 for calls. An examination of the account showed that 35% of the sum was due to long-distance calls. In making a budget for 2003, the firm assumes that the number of local and long-distance telephone calls will stay in the same ratio and both will increase by 10%. It is also allowing for a 25% reduction in the cost of long-distance calls because of the Telco Blue announcement (since the phrase 'up to 31%' was used!). Calculate the firm's estimate of its 2003 telephone bill for calls.

3.7 Best buys

Many products are sold in containers or packets of varying sizes. Usually, buying the larger quantity provides better value for money as the manufacturer is able to cut down on packaging and delivery costs. However, on some occasions, the retailer may have a surplus of other-sized stock that needs to be sold to make room for new stock. This stock may be discounted considerably, making it the best buy.

EG
+S

Example 1

Which of these is the best buy?

- A** 10 kg for \$13 **B** 15 kg for \$19.95 **C** 20 kg for \$25 **D** 30 kg for \$38.40

Solution

The lowest common multiple of 10, 15, 20 and 30 is 60. We therefore compare the costs of purchasing 60 kg of the product in each case.

A 10 kg for \$13	B 15 kg for \$19.95	C 20 kg for \$25	D 30 kg for \$38.40
$\times 6$ $\times 6$	$\times 4$ $\times 4$	$\times 3$ $\times 3$	$\times 2$ $\times 2$
= 60 kg for \$78	= 60 kg for \$79.80	= 60 kg for \$75	= 60 kg for \$76.80

$\therefore$ **C** is the best buy.

EG
+S

Example 2

Which of these is the best buy?

- A** 5 L for \$42.25 **B** 6.8 L for \$57.12
C 10.2 L for \$86.70 **D** 12 L for \$100.20

Solution

The lowest common multiple of 5, 6.8, 10.2 and 12 is not obvious. We therefore find the cost of 1 L of the product in each case.

A 5 L for \$42.25	B 6.8 L for \$57.12
$\div 5$ $\div 5$	$\div 6.8$ $\div 6.8$
= 1 L for \$8.45	= 1 L for \$8.40
C 10.2 L for \$86.70	D 12 L for \$100.20
$\div 10.2$ $\div 10.2$	$\div 12$ $\div 12$
= 1 L for \$8.50	= 1 L for \$8.35

$\therefore$ **D** is the best buy.

Exercise 3.7

1 Determine the best buy in each of these by finding the cost of:

- a** 100 g of each item.

A 10 g for \$2.40

C 50 g for \$11.50

B 20 g for \$4.50

D 100 g for \$23.80



- b** 2 L of each item.
A 200 mL for \$8.52
C 500 mL for \$20.90
- c** 1500 mL of each item.
A 100 mL for \$1.27
C 300 mL for \$3.75
- 2** Find the best buy in each of these.
- a** **A** 5 cm for \$1.25
C 10 cm for \$2.70
- b** **A** 5 g for \$1.80
C 25 g for \$9.50
- c** **A** 6 kg for \$4.86
C 9 kg for \$7.02
- d** **A** $\frac{1}{4}$ L for \$23.13
C $\frac{1}{2}$ L for \$45.75
- B** 400 mL for \$16.84
D 1 L for \$41.90
- B** 8 cm for \$2.32
D 20 cm for \$5.56
- B** 15 g for \$4.95
D 75 g for \$29.25
- B** 8 kg for \$6.80
D 12 kg for \$9.90
- B** $\frac{2}{5}$ L for \$35.60
D 1 L for \$87.50

Consolidation

- 3** Find the unit cost of each item and hence determine the better buy in each of these.
- a** 30 mL for \$2.10 or 65 mL for \$4.88 **b** 85 cm for \$5.19 or 95 cm for \$5.89
c 7 kg for \$29.61 or 12 kg for \$50.28 **d** 75 g for \$10.16 or 125 g for \$17.03
- 4** A 1.25-L bottle of lemonade is priced at \$1.45 while a 2-L bottle is priced at \$2.40. Which size represents the better value?
- 5** Bird seed is sold in 3 sizes: 500 g for \$1.78, 1 kg for \$3.45 and 1.75 kg for \$5.90. Which size represents the best value for money?
- 6** Oz Tea is sold in 3 different-sized packets: 100 g for \$1.75, 200 g for \$3.40 and 250 g for \$4.35. Which sized packet of tea is the most expensive per gram?
- 7** Screen doors are advertised for sale at \$180 each or 3 for \$500. Daniel needs to replace the screen door at the front of his house after an attempted break-in. Which purchase option is better for Daniel? Why?
- 8** Gary needs 2 tins of paint to paint his son's bedroom. Paint is advertised for sale at \$9.50 per tin or 'buy 3 tins and get 1 free'. Which purchase option should Gary make? Why?

Further applications

- 9** A 1 kg bag of prawns costs \$7.60 while a $3\frac{1}{2}$ kg bag costs \$26.75.
- a** Which size represents the better buy?
- b** At what price should a $2\frac{1}{2}$ kg bag be sold if it is to represent equal value for money with the best buy?
- 10** Potatoes are sold in 2 kg bags for \$2.80 and 5 kg bags for \$7.10. Find the maximum price at which a 3 kg bag should be sold if it is to be the best buy by 10%.

3.8 Discounts

A **discount** is a reduction in price of an item that is for sale. It is usually expressed as a percentage of the marked price. Retailers offer discounts to encourage customers to shop in their store or to quickly sell any remaining stock before the new stock arrives.

Some stores offer **consecutive discounts** on the marked price of an item. That is, they offer a discount on the already discounted price. It is important to note that the two discounts must be worked separately. You cannot simply add the percentages and make one calculation.

Consecutive discounts are usually offered for customers who are paying cash rather than using a credit card.

Example 1

A Christmas tree with a marked price of \$35 is marked for sale at 40% off in early January.

a Calculate the discount.

b Find the new retail price.

Solutions

$$\begin{aligned}\text{a} \quad \text{Discount} &= 40\% \text{ of } \$35 \\ &= 0.4 \times \$35 \\ &= \$14\end{aligned}$$

$$\begin{aligned}\text{b} \quad \text{New retail price} &= \text{marked price} - \text{discount} \\ &= \$35 - \$14 \\ &= \$21\end{aligned}$$

Example 2

A customer purchased a lounge chair with a marked price of \$1100 at the mid-year sales, where everything in the store was advertised at 15% off. He was also given a further discount of 2% because he paid cash. How much did the customer pay for the chair?

Solution

- i** If the chair was discounted by 15%, then the customer would only have paid 85% of the marked price.
- $$\begin{aligned}85\% \text{ of } \$1100 &= 0.85 \times \$1100 \\ &= \$935\end{aligned}$$
- ii** After a further discount of 2% was allowed, the customer only paid 98% of the discounted price.
- $$\begin{aligned}98\% \text{ of } \$935 &= 0.98 \times \$935 \\ &= \$916.30\end{aligned}$$
- $\therefore$ The customer paid \$916.30 for the chair.

Example 3

Xiang paid \$42 for a pair of jeans at a 25% off sale. What was the marked price of the jeans before the discount was allowed?



Solution

The unitary method is used to find the original price of a discounted item. Since a discount of 25% was allowed, the purchase price must have been 75% of the marked price.

75% of the marked price represents \$42

$\div 75$ $\div 75$

$\therefore$ 1% of the marked price represents \$0.56

$\times 100$ $\times 100$

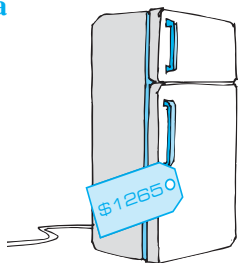
$\therefore$ 100% of the marked price represents \$56

$\therefore$ The jeans cost \$56 before the discount was allowed.

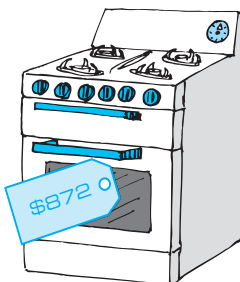
Exercise 3.8

- 1 Niles was given a discount of 25% when he purchased a new set of tyres for his car. What percentage of the retail price did he pay?
- 2 Find the following discounts:
 - a 25% off the retail price of a \$580 dishwasher
 - b 40% off the retail price of a \$130 heater
 - c $33\frac{1}{3}\%$ off the retail price of a \$216 bicycle
- 3 An electrical goods super store is holding a post-Christmas sale, with discounts of 15% on all television sets, 20% on refrigerators and $12\frac{1}{2}\%$ on ovens and ranges. Find the new sale price for each of these electrical goods.

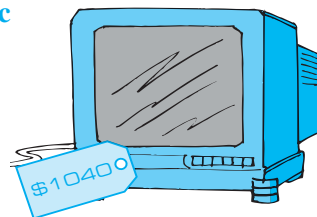
a



b



c

**Consolidation**

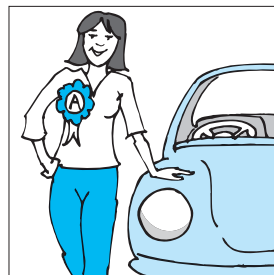
- 4 A book store offers teachers a 15% discount on all books. Mrs Hosogoe purchased 2 books priced at \$29.95 each and 3 books priced at \$24.75 each. Find:
 - a the total cost without the discount
 - b the total cost with the discount
- 5 A dishwasher was advertised at \$640 with a further \$175 for installation. Calculate the total purchase and installation costs if the cost of the dishwasher (excluding installation) is reduced by 18% due to minor scratching.

- 6 An outdoor television antenna is advertised with a marked price of \$115 with \$25 extra for installation. Find the total purchase and installation cost of an antenna during a sale in which all antennas are being discounted by 10% and installation costs are being discounted by 20%.
- 7 John purchased a 4-cylinder mower with a price tag of \$340. He received the advertised discount, plus a further 5% off the discounted price for paying cash. How much did John pay for the mower?



- 8 Mrs Lee purchased the following items for her son in preparation for the new school year:
- 4 pens at 95c each
 - an eraser costing 50c
 - a ruler costing 43c
 - a bottle of liquid paper costing \$1.85
 - 8 exercise books at \$1.14 each
 - a pencil sharpener costing 24c
- Find the change from \$20 if a discount of 10% was given on the cost of each item.
- 9 The owner of a local video rental store placed the following sign in the window: 'Rent any 4 movies and get one free'. What percentage discount is being offered on the hire of 5 movies?
- 10 Find the percentage discount that was allowed if a pair of shoes with a retail price of \$150 was sold for \$117.
- 11 The price of a book was reduced from \$42 to \$36.54. What percentage discount is this?
- 12 The retail price of a school bag was reduced by \$11 to \$28. Find, correct to 1 decimal place, the percentage discount.
- 13 A motorbike was discounted by 10% and sold for \$5220. What was the cost of the motorbike before the discount was applied?
- 14 A set of encyclopaedias was sold for \$410.40 after being discounted by 28%. Find the cost of the encyclopaedias before the discount was applied.
- 15 A pair of jeans was reduced by 35% and sold for \$61.75. What was the price of the jeans before the discount was applied?
- 16 Tamara bought a blouse at a sale, where items were listed as 30% off. What was the original retail price of the blouse if Tamara paid \$43.40?

- 17 Kerryanne bought a guitar at a 35% off sale at the markets and paid \$136.50. How much did she save?
- 18 Drivers who do not make a claim on their car insurance receive a 10% no-claim bonus each year.
- a Would a driver receive a no-claim bonus in their first year?
 - b Catherine has been driving for 4 years and has a perfect driving record. What percentage would her no-claim bonus be?
 - c How much will Catherine have to pay to insure her car if the full insurance premium is \$810?



■ Further applications

- 19 The children's entry price into a pantomime is set at 60% of the adult's price. Find the total entry price for a family of 2 adults and 3 children if 1 adult and 1 child can enter for \$24.
- 20 The cost of a camera was reduced by 20% in January with a further 5% discount being given for paying cash. A tourist paid \$516.80 cash for a camera. What was the original price?
- 21 A leather handbag was discounted by \$ x and then sold for \$ y . Find the percentage discount in terms of x and y .

TRY THIS Progressive discounting

A sales representative is eager to sell you a car. He offers you three successive discounts on the car (5%, 10%, 15%) in any order that you wish. Which order should you choose?



3.9 Profit and loss

The terms profit and loss refer to the difference between the cost price and selling price of an item. This difference is called a **profit** if the selling price is greater than the cost price and a **loss** if the selling price is less than the cost price.

⊙ Profit = selling price – cost price

⊙ Loss = cost price – selling price

In business, the manufacturer makes products, marks up the cost, then sells them to a wholesaler. The wholesaler marks up the cost and then distributes the products to a retailer. The retailer marks up the price and then sells the products to customers in their store. The mark-up is usually a percentage of the purchase price at each stage.

To express the profit or loss as a percentage of the cost price or selling price:

⊙ calculate the profit or loss

⊙ divide the profit or loss by the cost price or selling price

⊙ multiply by $\frac{100}{1} \%$.

Percentage profit or loss is calculated on the cost price unless otherwise stated.

EG
+S

Example 1

Quentin bought a bicycle for \$150 and sold it six months later, making a loss of 30% on the purchase price. Find the selling price.

Solution

$$\begin{aligned} \text{i} \quad \text{Loss} &= 30\% \text{ of } \$150 \\ &= 0.3 \times \$150 \\ &= \$45 \end{aligned}$$

$$\begin{aligned} \text{ii} \quad \text{Selling price} &= \text{cost price} - \text{loss} \\ &= \$150 - \$45 \\ &= \$105 \end{aligned}$$

EG
+S

Example 2

A retailer purchased an electric fan for \$30 and sold it for \$42. Calculate the percentage profit.

Solution

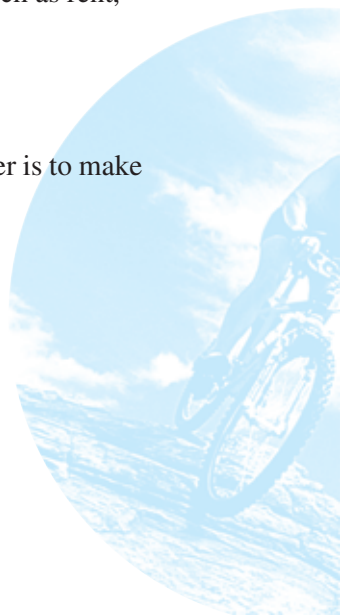
$$\begin{aligned} \text{i} \quad \text{Profit} &= \text{selling price} - \text{cost price} \\ &= \$42 - \$30 \\ &= \$12 \end{aligned}$$

$$\begin{aligned} \text{ii} \quad \text{Percentage profit} &= \frac{\text{profit}}{\text{cost price}} \times \frac{100}{1} \% \\ &= \frac{12}{30} \times \frac{100}{1} \% \\ &= 40\% \end{aligned}$$

- 13** Tori bought 7000 shares in a telecommunications company in 1996 and sold them 5 years later. If the share price was \$3.25 at the time of purchase and \$3.08 at the time of selling, find:
- a** the total loss made on these shares
 - b** the percentage loss, correct to 1 decimal place
- 14** Yvonne sold her home unit for \$224 000, making a profit of 28% on the purchase price. How much did she pay for the unit?
- 15** Daryl sold his car for \$8500, which represented a loss of 55% on the original purchase price due to depreciation. How much did Daryl pay for the car originally? Give your answer correct to the nearest dollar.
- 16** A shop owner sold a computer for \$2875, making a profit of 32% on the wholesale price. What was the wholesale price of the computer? Give your answer correct to the nearest dollar.
- 17** The owner of a sports store marks up the cost of cricket balls by 15% and cricket bats by 40%. One boy paid \$280 for a bat and three balls while a second boy paid \$248 for a bat and a ball.
- a** Find the retail price of each bat and ball.
 - b** Find the wholesale price of each.
 - c** Calculate the total profit made on these sales.

■ Further applications

- 18** A manufacturer sells plates to a wholesaler at cost plus 20%. The wholesaler then marks up the price by a further 25% and sells them to a retailer. The retailer then sells the plates for \$4.20 each, making a profit of 40%. How much would it cost to manufacture 200 plates?
- 19** A factory owner pays his 3 employees \$12.50 per hour for 36 hours per week to manufacture mouse traps. Each employee can make 40 mouse traps per hour. The owner pays 75c per trap for parts and \$490 per week in other business overheads, such as rent, electricity and telephone charges.
- a** Find the weekly wages bill.
 - b** How many traps are produced each week?
 - c** Calculate the total cost of running the business per week
 - d** For how much should each trap be sold to a wholesaler if the factory owner is to make a profit of at least \$800 per week?





FOCUS ON WORKING MATHEMATICALLY

SYDNEY MARKET PRICES IN 1831



Introduction

The first edition of the *Sydney Herald* was published on Monday 18 April 1831. It was not until 1840 that the *Herald* became a daily, the *Sydney Morning Herald*. One of the interesting columns was headed *Markets*. It included prices at the Sydney markets on a wide range of goods. In those days, in fact up to 1966, Australia's currency was in pounds, shillings and pence. An extract from a column is reprinted here.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
Ale, English . . doz.	0	12	6	Girard.	14s	0d—11s	0d	black.	0	3	0	Colon.leaf	0	0	3
Colonial . . gall.	0	6	0	Hall	14s	0d—11s	0d	Poultry, Turkeys, ea.	0	4	3	„ fig..	0	1	6
Arrow Root. . . . lb.	0	1	6	Fruit, Oranges, doz.	0	2	0	Geese.	0	3	3	„ stlks..	0	0	6
Beer, English, gall.	0	4	0	Lemons, „	0	1	0	Fowls, pair	0	1	9	Negrohd..	0	3	6
Colonial „	0	2	0	Apples, „	0	2	0	Ducks, ea.	0	1	6	Segars,Col.box	0	7	0
Biscuit. . . . per cwt.	0	16	0	Peaches, „	0	0	3	Rice. lb.	0	0	3	Hav. „	0	15	0
Blankets. Col. p.pr	0	10	0	Grain, Wheat, Bush.	0	4	0	Salt, colonial . . cwt	0	5	0	Chinsura	0	10	0
Blue. lb.	0	2	0	Maize, „	0	2	6	Soap, colonial . . lb	0	0	4 ¹ / ₂	Manilla .	1	10	0
Bread, 2 lb loaf. . .	0	0	3 ¹ / ₂	Barley	0	2	9	Spirits, Rum. . . gall.	0	9	0	Snuff, colon . . . lb	0	7	0
Butter, fresh	0	1	0	Oats	0	0	0	Brandy	0	13	0	Vegetables,			
salt.	0	0	8	Meat, Beef, p.qtr.lb.	0	0	1 ¹ / ₂	Gin	0	12	6	Potatoes, . . p.cwt	0	3	6
Candles, moulds. . lb	0	0	7	joint, „	0	0	2 ¹ / ₂	Colon.	0	7	6	do old do..	0	0	0
Dips.	0	0	5 ¹ / ₂	salted „	0	0	3	Starch lb.	0	2	6	Cabbages, . . p.doz.	0	1	6
Cheese, English . lb	0	1	9	Mutton, carcass „	0	0	2	Straw, Oat . . . load	0	12	6	Turnips, .p. bunch	0	0	2
Colonial	0	0	7	per joint „	0	0	3	Barley	0	12	6	Carrots, do.	0	0	3
Cloth, Parramat. . yd	0	1	3	Pork, carcass „	0	0	4	Sugar, Loaf, per lb.	0	0	8	Beans, broad, pk.	0	0	0
Coffee, ground . . lb	0	1	6	per joint „	0	0	5 ¹ / ₂	Moist	0	0	3 ¹ / ₂	French, „	0	0	0
Eggs, per doz. . . .	0	2	6	salted	0	0	4	Tallow, melted.cwt.	1	8	0	Onions lb	0	0	1 ¹ / ₂
Flax, New Zea . ton	22	0	0	Bacon. . . per lb.	0	0	7	Rough fat	1	3	4	Vinegar, colon. gall.	0	3	0
Flour, 100 lbs. fine	2nds			Hams . . per lb.	0	0	9	Tea, Hyson . . . lb	0	1	6	Wine, Port, p. doz.	35s—40s		
Barker. 14s 0d—11s 0d				Veal, p. quarter „	0	0	3	Young Hyson „	0	2	6	Madeira „	35s—40s		
Cooper 14s 0d—11s 0d				per joint „	0	0	4	Souchong	0	3	3	Sherry „	35s—40s		
Darlg.Mill. . . 14s 6d—11s 6d				Milk, per quart . . .	0	0	6	Pekoe.	0	3	3	Claret „	10s—60s		
Dixon 13s 0d—11s 0d				Oil, sperm gall.	0	5	0	Gunpowder . . .	0	4	6	Cape gall.	0	4	6
								Tobacco, Brazil. . lb	0	2	4	Elder.	0	5	6





LEARNING ACTIVITIES

Work in groups to do this activity. You will need to know the following about Australia's currency in 1831:

- *Coins*: the farthing ($\frac{1}{4}$ penny), the halfpenny ($\frac{1}{2}$ penny), the penny, the threepenny 'bit' (3 pence), the sixpenny coin (6 pence), the shilling (12 pence), the two shilling coin called 'two bob', and the crown (worth 2 shillings and 6 pence).
- *Notes*: the pound (called a 'quid'), a 5 pound note (called a 'fiver'), a 10 pound note (called a 'tenner') and a 20 pound note. One pound was worth 20 shillings.
- *Units*: the gallon (gall, 1 gall = 4.8 L), the quart (qt, $\frac{1}{4}$ gallon), the pound weight (lb, 1 kg = 2.2 lb) and the bushel (1 bush = 8 gall).

- Using the table on page 111 for prices at Sydney's markets, work out the cost of each of the following items and the total cost of the bill in £ s d (pounds, shillings and pence).

Item	Cost	Item	Cost
2 loaves of bread		2 bushels barley	
$\frac{1}{2}$ gallon of Colonial beer		12 lb salted beef	
$\frac{1}{2}$ lb English cheese		$\frac{1}{2}$ lb bacon	
$\frac{1}{4}$ lb ground coffee		1 quart of milk	
1 dozen eggs		3 ducks	
6 lemons		2 lb Colonial soap	
$\frac{1}{2}$ dozen oranges		1 $\frac{1}{2}$ lb loaf sugar	

- Estimate the price of each of the above items if you bought them today. A bushel of barley can be taken as 48 lb. Total the bill in dollars and cents.
- Compare the prices of items in 1831 with prices today. How can you account for the big change? Write down some factors that you would have to take into account (e.g. how much people earned). (NOTE: When Australia changed to decimal currency in 1966, 1 pound became \$2.)
- The toll on the Sydney Harbour Bridge was raised to \$3 in January 2002. It was sixpence when the bridge was opened in 1932. Is this price rise reasonable over 70 years? What things need to be taken into account (e.g. construction of the Sydney Harbour Tunnel)?
- Suppose you gained a pay rise of \$20 per week. If inflation is 5%, what would be your *real* increase in terms of how much more money you gained? If your new salary was \$280 per week after the pay rise, what is your total amount of available money after taking inflation into account?





CHALLENGE

Discuss the following in class or use as a project idea.

- 1 Why is the 'Cost of Living Index' also called the 'Consumer Price Index'? How is it calculated in Australia today?
- 2 What is inflation? How is it measured? Inflation was low in Australia in 2001. How can mathematics be used to help predict what might happen in 2004 and succeeding years?



LET'S COMMUNICATE

Using the price list on page 111 as a guide, write a letter to a student at school in 1831, explaining why the cost of living is so much more in Sydney today. Explain what Sydney is now like, what technological progress has been made, what people earn and what they have to spend compared to the early years of settlement. You might mention we went to decimal currency in 1966!



REFLECTING

Ask your grandparents, parents or guardian if they have experienced the effects of inflation on their income. Why do governments need to control inflation? Why do you need to know about it? How can inflation be measured mathematically?



MACQUARIE

Language link with Macquarie

- 1 Explain the difference between a *salary* and a *wage*.
- 2 What is an example of *casually* paid work?
- 3 Explain the difference between *gross* and *net* income.
- 4 Define *taxable income* for a new mathematics dictionary.
- 5 Read the Macquarie Learners Dictionary entry for *budget*:

budget *noun* **1.** a plan showing how much money a person, organisation or country will earn and how it will be spent

—verb **2.** to make such a plan: We *budgeted* on the basis that we would both have full-time jobs next year.

—adjective **3.** not costing much: *budget* clothes

□ **Word family:** *budgetary* adjective

Why is it important to be able to manage your income and spending?

- 1 Barry earns an annual salary of \$50 600. How much would Barry receive if he was paid:
 - a weekly?
 - b fortnightly?
 - c monthly?
- 2 A tradesman is paid \$14.65 per hour for a 36-hour week. Calculate his weekly wage.
- 3 A personal assistant receives a pay rise of 6% on his annual salary of \$38 850. Find his new monthly pay.
- 4 Catherine is a sales assistant at a petrol station. She works 40 hours each week and is paid \$12.25 per hour. Calculate her equivalent monthly pay.
- 5 A bus driver is paid \$13.95 per hour and his fortnightly wage is \$1060.20. For how many hours does he work each fortnight?
- 6 Ximena's annual salary increased from \$45 300 to \$48 244.50. Calculate the percentage increase in her salary.
- 7 Joseph is paid a retainer of \$160 per week plus a commission of 4.5% on his weekly sales. Calculate Joseph's pay for a week when he sells goods to the value of \$9560.
- 8 An encyclopaedia saleswoman is paid a weekly retainer plus a commission of \$22 for each set of encyclopaedias that she sells. How much is she paid as a retainer if in one week she sold 9 sets of encyclopaedias and was paid \$449?
- 9 To sell a property, the Second National Real Estate Agency charges a commission of 3.5% on the first \$150 000 of the sale price and 2% on the remaining value. The agent responsible for the sale receives 34% of the agency's commission.
 - a Calculate the commission paid to the agency on the sale of a house for \$410 000.
 - b How much does the agent responsible for the sale receive?
- 10 A factory worker is paid 15c for each mouse trap put together on an assembly line. How many mouse traps did the worker assemble in a week when they were paid \$348?
- 11 A doctor charges \$21.80 for consultations that last less than 10 minutes and \$27.50 for those that last longer than 10 minutes. Yesterday the doctor earned \$1077.90. If 28 consultations lasted less than 10 minutes, how many patients stayed for longer than 10 minutes?
- 12 Stan earns \$16.40 an hour for a 39-hour week as a crane operator. Overtime is paid at the time and a half rate for the first 5 hours and at the double time rate thereafter. Calculate Stan's total pay for a week when he works for 48 hours.
- 13 Ricky worked for 35 hours last week at \$11.35 per hour. He also worked 6 hours overtime, which was paid at the time and a half rate.
 - a Calculate Ricky's total pay for the week.
 - b For how many hours would he need to work at the normal rate in order to earn the same pay?
- 14 Nadia's annual salary is \$23 660. She took her annual 4 weeks holidays in January and was paid 4 weeks normal pay plus a holiday leave loading of $17\frac{1}{2}\%$ on this amount. How much did she receive altogether?

- 15** Mr Knight's gross fortnightly pay is \$1352. Each fortnight, his employer deducts \$266.37 in PAYG tax instalments and he has other deductions of \$23.50 for superannuation and \$12.65 for health insurance. Calculate Mr Knight's net fortnightly pay.
- 16** Find the gross weekly pay of an employee with a net annual salary of \$35 600 and weekly deductions totalling \$392.55.
- 17** A painter earns an annual salary of \$36 750 and is paid monthly. His annual deductions are \$7405 in PAYG tax and 5% of his gross salary is paid into a superannuation fund. Find the painter's net monthly income.
- 18** Use the tax table on page 93 to calculate the amount of tax payable on each of these annual salaries.
- a** \$5750 **b** \$18 430
c \$49 600 **d** \$72 195
- 19** Jelena has an annual salary of \$41 060. She has allowable tax deductions totalling \$2825.
- a** What is her taxable income?
b How much tax is she required to pay?
c Calculate Jelena's average net fortnightly pay.
- 20** Eugene has a gross annual salary of \$47 850. Each week, his employer takes out \$226.50 in PAYG tax instalments.
- a** Calculate the amount of tax that Eugene is required to pay on his salary.
b Is he entitled to a tax refund or does he have to pay more tax? Justify your answer.
- 21 a** A dental patient was billed for two fillings and an X-ray. She paid

\$107.80 for each of the fillings, \$74 for the X-ray and 10% GST on the total bill. How much was she charged altogether?

- b** Alex paid \$90.75 for her weekly groceries, which included 10% GST. How much would the groceries have cost without the GST?
- 22** Sarah earns \$450 per week after tax and rents a house with her sister. The women agree to contribute equally towards the payment of any gas, telephone or electricity bills, which they estimate will come to \$480 per quarter, and the rent, which is \$280 per week. So she can manage her money wisely, Sarah has prepared a weekly budget.

Budget item	Budget amount
Rent	
Food	\$75
Bills	
Clothes	\$30
Transport	\$35
Entertainment	\$60
Savings	

- a** How much should Sarah budget for rent each week?
b What should the weekly budget amount be for bills?
c How much is left over for savings?
d What percentage of her income, correct to 1 decimal place, is spent on clothes?
e If this information was shown on a sector graph, what would be the angle at the centre of the food sector?

- f** Adjust this budget to allow for an 8% pay rise with one-third of the extra money being spent on entertainment and the rest of the money going towards savings.
- 23** Determine the best buy by finding the cost of 200 g of the item.
A 20 g for \$3.60 **B** 25 g for \$4.25
C 40 g for \$8 **D** 50 g for \$9.50
- 24** Determine the best buy by finding the cost of 1 mL of the item.
A 55 mL for \$3.41
B 70 mL for \$4.13
C 90 mL for \$5.49
D 115 mL for \$6.67
- 25** A calculator wholesaler gives retail stores a discount of 5% on orders of more than 50 units. How much would a store owner save if he purchased 80 calculators, given that the normal wholesale price is \$16.40?
- 26** A music store offered 20% off the cost of all classical CDs. Johann purchased 5 CDs with a marked price of \$22. How much did he pay altogether?
- 27 a** A kitchen appliance was discounted from \$42 to \$36.75. What was the percentage discount?
b The retail price of a reclining chair was reduced by \$152 to \$798. What was the percentage discount?
- 28** The cost of a treadmill was reduced by 15% and sold for \$632.40. What was the original price?
- 29** Ian saved \$34.80 when he purchased 4 new tyres for his car at a tyre sale. If he paid \$109.30 per tyre, find the percentage discount. Answer correct to 1 decimal place.
- 30** Ian bought 4 new tyres for his car and received a $7\frac{1}{2}\%$ discount on the total cost. If Ian paid \$429.20 altogether, find how much he saved.
- 31** An antiques dealer purchased a rare book for \$3500 and sold it a week later, making a profit of 15%. Find the sale price of the book.
- 32** Aislinn bought a chest of drawers for \$380 and sold it 2 years later for \$247. Calculate the percentage loss.
- 33 a** Shane sold his car for \$10 200, which represented a loss of 40% on the original price. How much did Shane pay for the car?
b A retailer bought a chandelier from a wholesaler and sold it in her lighting store for \$899, making a profit of 45%. Calculate the wholesale price of the chandelier.

Equations, inequations and formulae



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ solve simple one- and two-step linear equations
- ⦿ solve simple quadratic equations of the form $ax^2 = c$
- ⦿ solve equations with pronumerals on both sides
- ⦿ solve equations with grouping symbols
- ⦿ solve equations that contain a single fraction
- ⦿ solve equations that contain two or more fractions
- ⦿ write down the integer solution set for a given inequality
- ⦿ graph inequalities on the number line
- ⦿ solve simple inequations
- ⦿ solve inequations that involve the reversal of the inequality sign
- ⦿ solve word problems, geometric problems and measurement problems by the use of equations
- ⦿ solve problems involving inequalities
- ⦿ evaluate the subject of a formula
- ⦿ solve equations that arise from substitution into a formula
- ⦿ change the subject of a formula
- ⦿ determine restrictions on variables after a formula has been re-arranged.

4.1 One- and two-step equations

Linear equations

A **linear equation** is a number sentence in which one of the numbers has been replaced by a pronumeral and the highest power of the pronumeral is 1. Some examples of linear equations are $x + 5 = 9$, $3x - 4 = 8$, $2x - 7 = 6x + 1$ and $\frac{x}{2} + 9 = 13$.

The solution to an equation is the value of the pronumeral which makes the statement true. For example, the solution to the equation $x + 4 = 7$ is $x = 3$, because when x is replaced by 3 we have $3 + 4 = 7$, which is a true statement. There is only one solution for any linear equation. Thus, we are able to check whether a solution is correct by direct substitution.

To solve an equation formally, we need to isolate the pronumeral on one side and take all of the numbers to the other side. This is done by performing inverse operations.

To solve a linear equation:

- ⊙ isolate the pronumeral by performing inverse operations to both sides of the equation.

Quadratic equations

A **quadratic equation** is an equation in which the highest power of the pronumeral is 2. In Year 9 you will only study quadratic equations of the form $ax^2 = c$, where a and c are constants.

Whereas linear equations have only one solution, quadratic equations have either two solutions, one solution or no solution.

Consider the quadratic equation $x^2 = 25$. Now, $5^2 = 25$ and $(-5)^2 = 25$, so x could be either 5 or -5 . These solutions may be written as $x = \pm 5$.

How many solutions does the equation $x^2 = -4$ have? Why?

Example 1

Solve each of the following equations.

a $x + 5 = 11$

b $y - 7 = 2$

c $3k = 18$

d $\frac{w}{2} = 9$

Solutions

a $x + 5 = 11$

$$\begin{array}{r} -5 \quad -5 \\ \hline \therefore x = 6 \end{array}$$

b $y - 7 = 2$

$$\begin{array}{r} +7 \quad +7 \\ \hline \therefore y = 9 \end{array}$$

c $3k = 18$

$$\begin{array}{r} \div 3 \quad \div 3 \\ \hline \therefore k = 6 \end{array}$$

d $\frac{w}{2} = 9$

$$\begin{array}{r} \times 2 \quad \times 2 \\ \hline \therefore w = 18 \end{array}$$

EG
+S**Example 2**

Solve these equations.

a $5 - t = -3$

b $-4 - p = 8$

Solutions

a $5 - t = -3$

$-5 \quad -5$

$-t = -8$

$\div (-1) \quad \div (-1)$

$\therefore t = 8$

b $-4 - p = 8$

$+4 \quad +4$

$-p = 12$

$\div (-1) \quad \div (-1)$

$\therefore p = -12$

EG
+S**Example 3**

Solve:

a $3x + 10 = 22$

b $5y - 2 = 38$

c $9 - 8a = 3$

Solutions

a $3x + 10 = 22$

$-10 \quad -10$

$3x = 12$

$\div 3 \quad \div 3$

$\therefore x = 4$

b $5y - 2 = 38$

$+2 \quad +2$

$5y = 40$

$\div 5 \quad \div 5$

$\therefore y = 8$

c $9 - 8a = 3$

$-9 \quad -9$

$-8a = -6$

$\div (-8) \quad \div (-8)$

$a = \frac{-6}{-8}$

$\therefore a = \frac{3}{4}$

EG
+S**Example 4**

Solve:

a $x^2 = 36$

b $9x^2 = 121$

c $x^2 = -9$

Solutions

a $x^2 = 36$

$\therefore x = \pm 6$

b $9x^2 = 121$

$\div 9 \quad \div 9$

$x^2 = \frac{121}{9}$

$\therefore x = \pm \frac{11}{3}$

c $x^2 = -9$

There are no solutions as there is no number that can be squared to give a negative number.

Exercise 4.1**1** Solve each of these equations using a guess, check and refine approach.

a $x + 4 = 10$

b $p - 3 = 5$

c $7t = 28$

d $\frac{m}{5} = 6$

e $2n + 1 = 9$

f $3k - 2 = 22$

g $7h + 21 = 0$

h $30 - 2y = 16$

2 Determine by substitution whether the solution given in brackets is correct.

a $b + 15 = 33$

$[b = 19]$

b $q - 19 = 8$

$[q = 27]$

c $31 - a = 14$

$[a = -17]$

d $4x + 9 = 5$

$[x = 12]$

e $7c + 3 = -11$

$[c = -2]$

f $8 - 5t = -12$

$[t = -4]$

3 Solve each of these equations.

a $a + 4 = 9$

b $p - 5 = 12$

c $y + 8 = 3$

d $q - 15 = -16$

e $7w = 42$

f $-11r = 88$

g $\frac{b}{5} = 6$

h $\frac{f}{-7} = -9$

■ Consolidation

4 Solve the following equations.

a $3u = 2$

b $2e = 7$

c $8a = 4$

d $15c = 12$

e $12d = -9$

f $-16t = -24$

g $-27k = 21$

h $-18w = -30$

i $x + \frac{1}{2} = 2$

j $y - \frac{1}{4} = 3$

k $m + \frac{1}{3} = \frac{1}{2}$

l $q - \frac{1}{2} = 1\frac{1}{4}$

m $3n = \frac{1}{4}$

n $2v = \frac{3}{5}$

o $4t = -1\frac{1}{3}$

p $-5h = -3\frac{2}{3}$

q $\frac{e}{2} = \frac{1}{7}$

r $\frac{r}{4} = \frac{3}{13}$

s $\frac{w}{3} = 1\frac{1}{2}$

t $\frac{k}{-2} = 4\frac{1}{4}$

u $c + 0.4 = 1.2$

v $g - 1.3 = 0.7$

w $0.9b = 2.7$

x $\frac{v}{0.8} = 0.7$

5 Solve:

a $4 - k = 1$

b $-2 + y = 4$

c $-t - 3 = -8$

d $-6 + z = -7$

e $5 - h = 8$

f $-9 - r = -2$

g $13 = -10 + c$

h $-9 = -a + 4$

i $11 - w = 30$

6 Solve these equations.

a $3x + 2 = 14$

b $2p + 5 = 19$

c $7e + 9 = 30$

d $6a - 1 = 5$

e $4g - 7 = 25$

f $9s - 4 = 59$

g $3c + 10 = 4$

h $11f - 1 = -23$

i $12d + 17 = 5$

j $11 + 2y = 11$

k $8m - 1 = -33$

l $3p - 10 = -1$

m $1 - 2z = 17$

n $4 - 5t = 34$

o $-7 - 3u = -4$

7 Solve each of these equations, giving the solutions as fractions or mixed numerals, in simplest form.

a $4p + 20 = 23$

b $7m - 3 = 2$

c $5h + 8 = 17$

d $12r - 5 = 4$

e $8k - 13 = 7$

f $7 + 12x = 3$

g $13 - 4n = 15$

h $10k - 8 = 17$

i $-6 - 21w = 9$

8 **a** Solve $5x - 8 = 19$ by:

i using a guess, check and refine approach

ii by performing inverse operations to both sides

b Which method was easier? Why?

9 Solve these quadratic equations, giving the answers as either integers or fractions.

a $x^2 = 4$

b $x^2 = 9$

c $x^2 = 49$

d $x^2 = 100$

e $4x^2 = 4$

f $9x^2 = 36$

g $3x^2 = 48$

h $5x^2 = 180$

i $9x^2 = 4$

j $49x^2 = 16$

k $9x^2 = 100$

l $81x^2 = 144$

10 Solve, correct to 1 decimal place:

a $x^2 = 5$

b $2x^2 = 24$

c $5x^2 = 100$

d $8x^2 = 56$

Further applications

11 Change one number in the equation:

a $4x - 1 = 6$ so that the solution is $x = 3$ **b** $5 - 2x = 8$ so that the solution is $x = -7$

12 Write down an equation whose solution is:

a $x = 5$

b $x = -3$

c $x = \frac{1}{2}$

4.2

Equations with pronumerals on both sides

To solve an equation with pronumerals on both sides:

- take all pronumerals to one side and all numerals to the other side by performing inverse operations.

Example 1

Solve:

a $8t = 5t + 21$

b $4n = 9n - 20$

Solutions

a $8t = 5t + 21$

$-5t \quad -5t$

$3t = 21$

$\div 3 \quad \div 3$

$\therefore t = 7$

b $4n = 9n - 20$

$-9n \quad -9n$

$-5n = -20$

$\div (-5) \quad \div (-5)$

$\therefore n = 4$

Example 2

Solve:

a $7k - 13 = 3k + 35$

b $14 - 9y = 3y + 26$

Solutions

a $7k - 13 = 3k + 35$

$-3k \quad -3k$

$4k - 13 = 35$

$+13 \quad +13$

$4k = 48$

$\div 4 \quad \div 4$

$\therefore k = 12$

b $14 - 9y = 3y + 26$

$+9y \quad +9y$

$14 = 12y + 26$

$-26 \quad -26$

$-12 = 12y$

$\div 12 \quad \div 12$

$-1 = y$

$\therefore y = -1$

Exercise 4.2

- Solve each of these equations using the guess, check and refine approach.

a $3m = 2m + 7$	b $4k + 15 = k + 21$	c $9y + 5 = 4y - 10$
- Determine by substitution whether the solution given in brackets is correct.

a $5p + 28 = 12p$ [$p = 4$]	b $7c - 24 = -5c$ [$c = 2$]
c $8w - 13 = 3w + 7$ [$w = 5$]	d $1 - 2h = 6h + 9$ [$h = -1$]
- Solve each equation by taking all of the algebraic terms to one side.

a $5k = 4k + 3$	b $9m = 8 + 7m$	c $3s = 28 - s$
d $45 - 2p = 7p$	e $x = 2x + 5$	f $3b = 6b + 21$
g $7r - 55 = 2r$	h $3m + 42 = -4m$	i $24 + 4u = -8u$
j $4g + 60 = 9g$	k $40 - 4c = c$	l $-88 + 7k = -4k$

Consolidation

- Solve $5y - 7 = 2y + 5$ by first:

i adding 7 to both sides	ii subtracting 5 from both sides
iii subtracting $2y$ from both sides	iv subtracting $5y$ from both sides
 - Discuss whether one approach was easier than the others.
- Solve:

a $3x + 8 = 2x + 11$	b $6p - 1 = 5p + 7$	c $9a + 2 = 7a + 12$
d $4m - 9 = 2m + 15$	e $10y - 11 = 6y + 5$	f $8t - 11 = 5t - 2$
g $7k + 8 = 2k - 7$	h $9w + 3 = 5w - 21$	i $12q - 13 = 3q - 13$
j $b + 20 = 2b + 15$	k $6s - 4 = 9s + 23$	l $8e - 21 = 10e + 5$
m $14u - 15 = 8u + 9$	n $5 - 2x = 3x + 25$	o $4n + 7 = 13 - 2n$
p $4 - 3t = 4t - 31$	q $4k + 16 = 52 - 5k$	r $7h - 26 = -2 + 19h$
- Solve each of these equations, giving the solutions as fractions or mixed numerals, in simplest form.

a $m + 9 = 3m$	b $17k - 8 = 5k$	c $1 - 3c = c$
d $15 - 7u = 11u$	e $16y = 27 - 5y$	f $9p - 33 = -13p$
g $8t + 10 = 5t + 18$	h $5z - 4 = 18 + z$	i $23 - 7a = 5a - 19$
j $15 + 3e = 35 + 10e$	k $-6 - 4g = 3 - 25g$	l $6 - 3k = 21 + k$

Further applications

- Change one number in the equation $5x + 6 = 2x + 21$ so that the solution is:

a $x = 7$	b $x = 0$	c $x = -4$
- If $p = 2q + 9$, find values for p and q given that p is 5 more than q .
 - If $y = 11 - 3x$, find values for x and y given that y is equal to 25 more than the product of 4 and x .

4.3 Equations with grouping symbols

To solve an equation with grouping symbols:

- ⊙ expand the expressions in grouping symbols
- ⊙ collect like terms if possible
- ⊙ solve the resulting equation by performing inverse operations.

EG
+S

Example 1

Solve each of these equations.

a $5(e - 4) = 65$

b $7(m + 6) = 4(m - 3)$

Solutions

a $5(e - 4) = 65$

$$5e - 20 = 65$$

$$+20 \quad +20$$

$$5e = 85$$

$$\div 5 \quad \div 5$$

$$\therefore e = 17$$

b $7(m + 6) = 4(m - 3)$

$$7m + 42 = 4m - 12$$

$$-4m \quad -4m$$

$$3m + 42 = -12$$

$$-42 \quad -42$$

$$3m = -54$$

$$\div 3 \quad \div 3$$

$$\therefore m = -18$$

EG
+S

Example 2

Solve:

$$3(2x + 5) + 12 = 7 - 5(5x + 6)$$

Solution

$$3(2x + 5) + 12 = 7 - 5(5x + 6)$$

$$6x + 15 + 12 = 7 - 25x - 30$$

$$6x + 27 = -25x - 23$$

$$+25x \quad +25x$$

$$31x + 27 = -23$$

$$-27 \quad -27$$

$$31x = -50$$

$$\div 31 \quad \div 31$$

$$\therefore x = -1\frac{19}{31}$$

Exercise 4.3

1 In each of the following, expand the expression that contains grouping symbols, then solve the resulting equation.

a $2(m + 3) = 16$

b $3(k - 4) = 12$

c $5(c + 2) = 25$

d $7(4 + n) = 84$

e $6(5 - t) = 48$

f $9(2 - x) = 9$

g $4(2y + 3) = 20$

h $2(3a - 1) = 28$

i $3(5w + 2) = 36$

j $6(1 - 2p) = 54$

k $5(3 - 4c) = 75$

l $11(3 - 2m) = 121$

2 Solve each of these equations.

a $-2(n - 7) = 24$

b $-5(p + 2) = 30$

c $-4(y - 3) = 28$

d $-3(m - 8) = 6$

e $-8(t + 4) = 24$

f $-6(10 - a) = 6$

g $-4(2k + 5) = 12$

h $-7(3r - 1) = 49$

i $-2(6 - 5d) = 38$

3 Solve each of these equations, giving the solutions as fractions or mixed numerals, in simplest form.

a $3(a + 2) = 11$

b $4(m - 2) = 15$

c $5(k + 4) = 23$

d $3(8y + 1) = 21$

e $11(3p - 2) = 5$

f $6(5 + 2z) = 8$

g $-2(h + 8) = 5$

h $-8(2v + 3) = 16$

i $-12(5c + 2) = 18$

4 **a** Solve $7(x + 3) = 35$ by first:

i expanding the expression on the left-hand side

ii dividing both sides by 7

b Discuss whether one approach was easier than the other.

Consolidation

5 Solve:

a $4(p - 5) = 3p + 8$

b $7(n - 2) = 6n + 4$

c $4e - 7 = 5(e - 3)$

d $5(s + 7) = 2s + 50$

e $9(b + 4) = 26 + 4b$

f $5y - 57 = 8(y - 3)$

g $7(2u + 1) = 6u + 71$

h $12(3z - 2) = 20z - 72$

i $5(4 - 3t) = 42 - 4t$

6 Solve:

a $3(x + 4) = 2(x + 9)$

b $6(n - 2) = 5(n + 1)$

c $4(y + 2) = 3(y - 6)$

d $5(g - 1) = 4(g + 4)$

e $7(a + 1) = 5(a + 3)$

f $5(r + 2) = 3(r - 2)$

g $3(z + 5) = 6(z - 1)$

h $12(p - 3) = 7(p - 3)$

i $2(2w + 9) = 3(w + 10)$

j $8(c + 5) = 4(3c - 1)$

k $9(2e - 3) = 3(e + 6)$

l $10(3m + 6) = 6(m + 2)$

Further applications

7 Simplify the expressions on each side of these equations, then solve for x .

a $3(x + 2) + 2(x + 1) = 33$

b $7(x - 3) + 4(x + 2) = 64$

c $2(8x - 1) + 5(2x - 3) = 35$

d $4(3x + 5) - 6(x + 6) = 26$

e $5(x + 8) = 120 - (x + 2)$

f $9 - 2(x - 8) = 2(x - 4) + 1$

g $23 - 4x = 16 - 3(2x + 7)$

h $11(2x - 3) + 4 = 35 + 2(x - 2)$

i $8(3x - 2) - 2(5 - 4x) + 58 = 0$

j $25x - 4(2x - 7) = 3(3x + 5) + 45$

4.4

Equations with one fraction

To solve an equation that contains one fraction:

- ⊙ take all constant terms to one side by performing inverse operations
- ⊙ multiply both sides of the equation by the denominator
- ⊙ solve the resulting equation.

Example

Solve:

a $\frac{3a}{5} = 12$

b $\frac{m}{6} + 5 = 8$

c $\frac{11 - 4x}{3} = 9$

Solutions

$$\begin{aligned} \text{a} \quad \frac{3a}{5} &= 12 \\ \times 5 \quad \times 5 & \\ 3a &= 60 \\ \div 3 \quad \div 3 & \\ \therefore a &= 20 \end{aligned}$$

$$\begin{aligned} \text{b} \quad \frac{m}{6} + 5 &= 8 \\ -5 \quad -5 & \\ \frac{m}{6} &= 3 \\ \times 6 \quad \times 6 & \\ \therefore m &= 18 \end{aligned}$$

$$\begin{aligned} \text{c} \quad \frac{11-4x}{3} &= 9 \\ \times 3 \quad \times 3 & \\ 11-4x &= 27 \\ -11 \quad -11 & \\ -4x &= 16 \\ \div (-4) \quad \div (-4) & \\ \therefore x &= -4 \end{aligned}$$

Exercise 4.4

1 Solve:

a $\frac{d}{4} = 3$

b $\frac{k}{7} = -5$

c $\frac{y}{-2} = 9$

d $\frac{w}{-3} = -8$

e $\frac{2x}{3} = 8$

f $\frac{3a}{5} = 6$

g $\frac{5m}{2} = 10$

h $\frac{4k}{3} = 12$

i $-\frac{7w}{2} = 21$

j $\frac{6c}{7} = -12$

k $-\frac{10s}{9} = 20$

l $-\frac{8u}{3} = -24$

2 a Solve $\frac{m}{4} - 3 = 5$ by first:

i adding 3 to both sides

ii multiplying both sides by 4

b Discuss whether one approach was easier than the other.

Consolidation

3 Solve each of the following equations.

a $\frac{n}{5} + 1 = 4$

b $\frac{k}{8} + 7 = 9$

c $\frac{c}{3} - 4 = 10$

d $\frac{a}{9} - 2 = 5$

e $\frac{z}{4} + 8 = 4$

f $\frac{h}{5} - 6 = -2$

g $\frac{p}{9} - 8 = -6$

h $-\frac{x}{12} + 4 = -1$

i $5 + \frac{j}{6} = 13$

j $-3 + \frac{u}{11} = -1$

k $12 - \frac{s}{9} = 8$

l $-10 - \frac{d}{7} = -4$

m $\frac{3a}{2} + 4 = 13$

n $\frac{5e}{6} - 3 = 12$

o $30 + \frac{7h}{6} = 2$

p $40 - \frac{9z}{2} = 4$

4 Solve these equations.

a $\frac{m+5}{3} = 2$

b $\frac{k-2}{5} = 4$

c $\frac{s+9}{6} = 4$

d $\frac{t+15}{4} = 2$

e $\frac{2x-3}{3} = 5$

f $\frac{3c-1}{5} = 7$

g $\frac{4-5d}{3} = 3$

h $\frac{65+10b}{9} = 5$

i $\frac{11-3z}{7} = 5$

j $\frac{7r+1}{6} = -8$

k $\frac{10+4f}{7} = 6$

l $\frac{12-5q}{8} = 9$

5 Solve:

a $\frac{m+3}{2} + 4 = 9$

b $\frac{x-2}{5} + 8 = 11$

c $\frac{7+w}{3} - 1 = 5$

d $\frac{2b+7}{3} - 2 = 7$

e $\frac{9+8u}{7} + 6 = 13$

f $\frac{t-12}{7} + 5 = 2$

g $\frac{2+a}{5} + 7 = 3$

h $\frac{k-19}{2} - 3 = -15$

i $\frac{18-4c}{3} + 9 = 11$

Further application

6 Solve:

a $\frac{3}{4}x = 2$

b $\frac{x+7}{2} = x$

c $2x = \frac{3x-2}{4}$

d $\frac{4x}{5} = 2x - 1$

e $3x + 11 = \frac{2}{3}x$

f $\frac{1}{2}(2x-5) = 3$

g $\frac{4x-3}{6} = 9 - 2x$

h $\frac{5}{3}(x-4) = 3 + x$

i $\frac{x}{3} + x = 8$

4.5

Equations with more than one fraction

To solve an equation that contains more than one fraction:

- ⊙ multiply the expression on each side by the lowest common multiple (LCM) of the denominators
- ⊙ solve the resulting equation by performing inverse operations.

EG
+S

Example 1

Solve:

$$\frac{a}{6} + \frac{a}{4} = 15$$

Solution

$$\frac{a}{6} + \frac{a}{4} = 15$$

$\times 12 \quad \times 12$ (The LCM of 6 and 4 is 12.)

$$\left(\frac{a}{6} \times 12\right) + \left(\frac{a}{4} \times 12\right) = 15 \times 12$$

$$2a + 3a = 180$$

$$5a = 180$$

$$\div 5 \quad \div 5$$

$$\therefore a = 36$$

EG
+S**Example 2**

Solve:

$$\frac{2x}{3} + 7 = \frac{x}{2} + 8$$

Solution

$$\frac{2x}{3} + 7 = \frac{x}{2} + 8 \quad \times 6 \quad \times 6 \quad (\text{The LCM of 3 and 2 is 6.})$$

$$\left(\frac{2x}{3} \times 6\right) + (7 \times 6) = \left(\frac{x}{2} \times 6\right) + (8 \times 6)$$

$$4x + 42 = 3x + 48$$

$$\begin{array}{r} -3x \end{array} \quad \begin{array}{r} -3x \end{array}$$

$$x + 42 = 48$$

$$\begin{array}{r} -42 \end{array} \quad \begin{array}{r} -42 \end{array}$$

$$\therefore x = 6$$

EG
+S**Example 3**

Solve:

$$\frac{x+8}{2} = \frac{x-4}{5}$$

Solution

$$\frac{x+8}{2} \times 10^5 = \frac{x-4}{5} \times 10^2 \quad (\text{The LCM of 2 and 5 is 10.})$$

$$5(x+8) = 2(x-4)$$

$$5x + 40 = 2x - 8$$

$$\begin{array}{r} -2x \end{array} \quad \begin{array}{r} -2x \end{array}$$

$$3x + 40 = -8$$

$$\begin{array}{r} -40 \end{array} \quad \begin{array}{r} -40 \end{array}$$

$$3x = -48$$

$$\begin{array}{r} \div 3 \end{array} \quad \begin{array}{r} \div 3 \end{array}$$

$$\therefore x = -16$$

Exercise 4.5

1 Multiply each term by the lowest common denominator, then solve the resulting equation.

a $\frac{x}{3} + \frac{x}{2} = 5$

b $\frac{a}{10} + \frac{a}{5} = 3$

c $\frac{t}{6} + \frac{t}{10} = 8$

d $\frac{k}{3} - \frac{k}{6} = 4$

e $\frac{n}{6} - \frac{n}{8} = 3$

f $\frac{y}{3} - \frac{y}{7} = 4$

g $\frac{3u}{5} + \frac{u}{2} = 22$

h $\frac{2m}{3} + \frac{3m}{4} = 17$

i $\frac{3c}{4} - \frac{5c}{8} = 2$

j $\frac{6h}{7} - \frac{2h}{3} = 8$

k $\frac{4w}{9} + \frac{5w}{12} = 31$

l $\frac{7e}{8} - \frac{4e}{5} = 6$

Consolidation2 **a** Solve $\frac{c+3}{14} = \frac{4}{7}$ by first:**i** multiplying both sides by 14**ii** expressing the fractions with a common denominator and equating the numerators**b** Discuss whether one approach was easier than the other.

3 Solve:

a $\frac{a+4}{10} = \frac{1}{2}$

b $\frac{n-5}{12} = \frac{1}{3}$

c $\frac{y+2}{8} = \frac{3}{4}$

d $\frac{c+7}{15} = \frac{3}{5}$

e $\frac{3b-1}{21} = \frac{2}{3}$

f $\frac{7t+4}{30} = \frac{5}{6}$

g $\frac{11-4x}{40} = \frac{7}{8}$

h $\frac{3-5m}{42} = 1\frac{1}{7}$

4 Solve:

a $\frac{2y}{5} = 11 - \frac{y}{3}$

b $\frac{a}{3} = \frac{3a}{10} + 2$

c $\frac{e}{4} = \frac{5e}{6} - 21$

d $\frac{x}{8} + \frac{x}{6} = 1 + \frac{x}{4}$

e $\frac{3t}{8} + \frac{2t}{5} = \frac{t}{4} + 21$

f $\frac{u}{4} - \frac{2u}{5} = \frac{u}{10} - 5$

5 Solve:

a $\frac{m+3}{2} = \frac{m+5}{3}$

b $\frac{x+6}{5} = \frac{x-3}{2}$

c $\frac{k-4}{7} = \frac{k+2}{4}$

d $\frac{2d-5}{9} = \frac{d-3}{6}$

e $\frac{3n-2}{4} = \frac{2n+5}{3}$

f $\frac{5t+2}{6} = \frac{7t-4}{5}$

g $\frac{4-7z}{5} = \frac{2-11z}{7}$

h $\frac{9h+4}{8} = \frac{11h+5}{10}$

i $\frac{3h+7}{2} = \frac{1+4h}{11}$

6 Solve each of these equations.

a $\frac{x+1}{6} + \frac{x+5}{4} = 6$

b $\frac{2f+3}{5} + \frac{7f-2}{8} = 8$

c $\frac{4b+2}{10} + \frac{3b-5}{4} = 7$

d $\frac{n-7}{4} - \frac{n-1}{9} = 1$

e $\frac{2q+1}{3} - \frac{q-8}{5} = 8$

f $\frac{10-4m}{3} - \frac{6-4m}{7} = 4$

g $\frac{2}{5}x = \frac{1}{2}(x-1)$

h $\frac{1}{3}(2x+3) = \frac{1}{2}x$

i $\frac{1}{4}(x+2) + \frac{1}{2}(x-1) = 3$

j $\frac{1}{2}(x-3) + \frac{3}{5}(x+1) = \frac{3}{4}$

k $\frac{1}{3}a - \frac{3}{4}(a-2) = \frac{5}{6}a$

l $\frac{5(t+3)}{6} - \frac{3(1+2t)}{8} = 2\frac{2}{3}$

Further applications

7 Solve the following.

a $\frac{2}{a} = 3$

b $\frac{5}{x} + \frac{2}{3} = 0$

c $\frac{4}{x} - \frac{2}{x} = 1$

d $\frac{2}{3a} + \frac{1}{2} = \frac{1}{6}$

e $\frac{5}{2x} + \frac{3}{x} = \frac{1}{3}$

f $\frac{7}{x} - \frac{1}{3x} = \frac{2}{3}$

g $\frac{5}{2x} + \frac{3}{4} = \frac{-2}{x}$

h $\frac{a+4}{3a} = 7 + \frac{1}{a}$

i $\frac{4}{5a} - \frac{3}{2a} = 7$

8 Solve:

a $\frac{1}{a} + \frac{1}{a-1} = \frac{5}{a}$

b $\frac{2}{c-2} - \frac{3}{c+2} = \frac{7}{c^2-4}$

c $\frac{5}{y+2} = \frac{3}{y-2}$

$$\text{d} \quad \frac{3}{1-a} - \frac{2}{1+a} = 0$$

$$\text{e} \quad \frac{x}{x-2} = \frac{x+3}{x+2}$$

$$\text{f} \quad \frac{y}{y+4} = \frac{y-3}{y+2}$$

$$\text{g} \quad \frac{6}{x} - \frac{3}{x} = \frac{2}{x+4}$$

$$\text{h} \quad \frac{y+5}{y-1} = \frac{y+3}{y-2}$$

$$\text{i} \quad \frac{u+8}{u-6} = \frac{u+4}{u-2}$$

4.6 Inequations

An **inequation** is a number sentence in which two quantities are not equal. An inequation is similar to an equation except that it has an inequality sign instead of an equals sign. The most commonly used inequality signs are:

Inequality sign	Meaning
$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to
$\neq$	not equal to

While the equations looked at so far have a unique solution, inequations can have many solutions. The set of all possible solutions to an inequation is called the **solution set**.

Consider the equation $x + 2 = 5$ and the inequation $x + 2 > 5$. The equation has only one solution, $x = 3$. The inequation, however, has an infinite number of solutions. Some of these solutions are $x = 4$, $x = 5$, $x = 8\frac{1}{2}$, $x = 12.3$, $x = 100$, because when 2 is added to each number, the result is always greater than 5.

The solutions can be integers, fractions or decimals. If we specify that x is an integer, the solutions for this inequality would be $\{4, 5, 6, 7, 8, \dots\}$ —all integers greater than 3. The number 3 is not a solution because, on substituting $x = 3$, we note that $3 + 2$ is not greater than 5. If the inequation were $x + 2 \geq 5$, then $x = 3$ would be one of the solutions.

Graphing solutions on the number line

The solution to an inequation can be graphed on the number line. This gives an instant picture of what the solution actually means.

To graph the solution set of an inequality on a number line:

- ① place the number that occurs in the solution at the centre of the number line segment
- ② draw a closed dot ● on this number if the inequality sign is $\geq$ or $\leq$
- ③ draw an open dot ○ on this number if the inequality sign is $>$ or $<$
- ④ from the dot, draw an arrow along the number line in the direction indicated by the inequality sign.

Solving inequations

Inequations are solved in the same way as equations—by performing inverse operations to both sides of the inequation. The pronumeral is written on the left-hand side in the solution so that the arrow is drawn correctly when the solution is graphed on the number line.

Consider the statement $6 > -2$, which is clearly true. Now, when both sides of this inequality are multiplied (or divided) by -1 , we have $-6 > 2$, which is not a true statement. To overcome this problem, we need to reverse the inequality sign in the answer. In general:

When multiplying or dividing both sides of an inequation by a negative number, reverse the inequality sign.

Example 1

Graph each of these inequations on a number line.

a $x > 5$

b $x \leq -2$

c $3 < x \leq 6$

d $x \leq 5 \text{ or } x > 7$

Solutions



Example 2

Solve each of the following inequations.

a $3x + 4 < 22$

b $6x - 7 \geq 2x + 13$

c $11 \leq \frac{x}{2} - 3$

Solutions

a $3x + 4 < 22$

$$\begin{array}{r} -4 \quad -4 \\ 3x < 18 \end{array}$$

$$3x < 18$$

$$\begin{array}{r} \div 3 \quad \div 3 \\ \therefore x < 6 \end{array}$$

$$\therefore x < 6$$

b $6x - 7 \geq 2x + 13$

$$\begin{array}{r} -2x \quad -2x \\ 4x - 7 \geq 13 \end{array}$$

$$4x - 7 \geq 13$$

$$\begin{array}{r} +7 \quad +7 \\ 4x \geq 20 \end{array}$$

$$4x \geq 20$$

$$\begin{array}{r} \div 4 \quad \div 4 \\ \therefore x \geq 5 \end{array}$$

$$\therefore x \geq 5$$

c $11 \leq \frac{x}{2} - 3$

$$\begin{array}{r} +3 \quad +3 \\ 14 \leq \frac{x}{2} \end{array}$$

$$14 \leq \frac{x}{2}$$

$$\begin{array}{r} \times 2 \quad \times 2 \\ 28 \leq x \end{array}$$

$$28 \leq x$$

$$\therefore x \geq 28$$

Example 3

Solve the inequation $3x - 1 < 14$, where:

a x is a real number

b x is an integer

c x is a positive integer

Solutions

$$\begin{array}{rcl} \text{a} & 3x - 1 < 14 \\ & +1 & +1 \\ & 3x < 15 \\ & \div 3 & \div 3 \\ \therefore & x < 5 \end{array}$$

b If x is an integer (and < 5), then the solutions are $x = 4, 3, 2, 1, 0, -1, -2, \dots$

c If x is a positive integer (and < 5), then the solutions are $x = 1, 2, 3, 4$.

Example 4

Solve:

$$\text{a} \quad -\frac{x}{9} \geq 3$$

$$\text{b} \quad 2 - 3x < 23$$

Solutions

$$\begin{array}{rcl} \text{a} & -\frac{x}{9} \geq 3 \\ \times (-9) & \times (-9) \\ \therefore & x \leq -27 \end{array}$$

$$\begin{array}{rcl} \text{b} & 2 - 3x < 23 \\ -2 & -2 \\ -3x & < 21 \\ \div (-3) & \div (-3) \\ \therefore & x > -7 \end{array}$$

Note the reversal of the inequality sign in these examples.

Exercise 4.6

1 Write down the integer solutions for each of these inequalities.

$$\text{a} \quad x \geq 5$$

$$\text{b} \quad x \leq 4$$

$$\text{c} \quad x > 0$$

$$\text{d} \quad x < 12$$

$$\text{e} \quad x \geq -7$$

$$\text{f} \quad x < -15$$

$$\text{g} \quad x \leq -7$$

$$\text{h} \quad x > -11$$

$$\text{i} \quad 2 \leq x \leq 7$$

$$\text{j} \quad 8 < x < 14$$

$$\text{k} \quad -4 \leq x < 2$$

$$\text{l} \quad -12 < x < -10$$

2 State the inequality that has been graphed on each of these number lines.



3 Graph each of these inequalities on a number line.

$$\text{a} \quad x > 4$$

$$\text{b} \quad x \leq 10$$

$$\text{c} \quad x \geq 1$$

$$\text{d} \quad x < 6$$

$$\text{e} \quad x < 0$$

$$\text{f} \quad x \geq -7$$

$$\text{g} \quad 2 \leq x < 5$$

$$\text{h} \quad 3 \leq x \leq 8$$

$$\text{i} \quad -4 < x < 0$$

$$\text{j} \quad x < 2 \text{ or } x > 4$$

$$\text{k} \quad x \leq -1 \text{ or } x \geq 2$$

$$\text{l} \quad 0 < x < 5 \text{ or } x \geq 7$$

4 Solve these inequations and graph the solutions on a number line.

a $x + 3 < 9$

b $a - 4 \geq 1$

c $y - 7 \leq 8$

d $9 + k > 12$

e $3w \leq 21$

f $5n > 25$

g $6b \geq 3$

h $10u < -15$

i $\frac{m}{4} \geq 7$

j $\frac{d}{6} < 2$

k $\frac{h}{8} > -3$

l $\frac{p}{5} \leq -2$

5 Solve the following inequations.

a $2 > y - 1$

b $7 \leq c + 3$

c $-6 < z - 1$

d $-10 \geq g + 8$

e $12 < 3p$

f $30 \geq 6q$

g $-26 > 13m$

h $-28 \leq 4t$

i $4 \geq \frac{v}{6}$

j $-8 < \frac{s}{2}$

k $9 \leq \frac{b}{3}$

l $-12 > \frac{r}{5}$

Consolidation

6 Solve:

a $3x + 1 \geq 7$

b $4k - 3 < 17$

c $2t - 9 > 5$

d $8 + 5g \leq 23$

e $11 + 2m < 19$

f $7 + 3w \geq 31$

g $5q + 14 > 4$

h $1 + 6y < -17$

i $4p - 36 \leq 0$

j $19 < 2z + 9$

k $13 \geq 3a - 14$

l $23 > 7e - 12$

m $3(u + 3) > 30$

n $2(f - 4) \leq 22$

o $-6(2 - a) \geq 9$

p $6 < 8(3d - 2)$

q $30 \leq 5(2c + 1)$

r $40 > 12(4 + 3n)$

7 Solve the following inequations.

a $2a + 12 < a + 20$

b $7m - 8 \geq 2m + 17$

c $3h + 11 > 9h - 7$

d $2 - 4x \leq 23 - x$

e $2(5p - 3) > 6p + 14$

f $3(2e - 7) < 5(e - 8)$

8 Solve these inequations.

a $\frac{x}{4} + 7 > 10$

b $\frac{2m}{3} - 2 < 8$

c $15 + \frac{k}{2} \geq 8$

d $\frac{t-2}{4} \leq 3$

e $\frac{3n+7}{5} > 5$

f $\frac{4u-11}{3} \geq 7$

9 **a** Solve $2x + 5 < 19$, where x is a positive integer

b Solve $5x + 8 \geq 33$, where x is an integer

c Solve $\frac{x}{3} - 1 \leq 6$, where x is an integer greater than 15

d Solve $12x + 7 > 9x - 5$, where x is a negative integer

10 Solve each of these equations, which involve the reversal of the inequality sign.

a $-2a < 10$

b $-3y \geq 6$

c $-5n \leq -20$

d $-4d > -36$

e $-m > 2$

f $-c \leq -4$

g $-\frac{k}{3} < 10$

h $-\frac{w}{6} \geq -5$

i $-\frac{1}{2}x \leq 9$

j $-\frac{1}{3}e > -4$

k $24 < -3d$

l $-50 > -10s$

m $32 \leq -4g$

n $7 \geq -\frac{t}{5}$

o $-6 < -\frac{z}{3}$

p $-9 \leq -\frac{f}{6}$

11 Solve:

a $2 - 3c < 8$

d $21 - 5h \geq 11$

g $20 - 3v \leq 2$

j $2(4 - 5n) < 28$

m $3(5 - 2t) \leq 4$

b $5 - 2s > 17$

e $7 - 4t < 23$

h $53 - 12d > 5$

k $6(8 - 3r) < -24$

n $2(7 - 10n) \geq 6$

c $14 - p \leq 9$

f $13 - 9g > 4$

i $10 - 7q \geq 66$

l $5(3 - 7j) > 20$

o $-9(3s - 1) > -27$

12 Solve:

a $\frac{x}{2} + \frac{x}{3} > 10$

d $\frac{x-1}{4} - \frac{x}{2} < -3$

g $\frac{3x-7}{6} - \frac{x-2}{4} < 1\frac{1}{2}$

b $\frac{2x}{3} - \frac{x}{5} \leq 21$

e $\frac{x}{5} - 4 > \frac{3x}{4} - 6$

h $\frac{2-x}{2} - \frac{x-3}{3} \leq 8$

c $12 - x \geq \frac{x}{3}$

f $\frac{x-1}{5} + \frac{2x-3}{6} \geq \frac{2}{3}$

i $\frac{x}{4} - \frac{x-2}{3} > \frac{5}{8} + \frac{1-x}{6}$

13 Solve these inequality problems.

- a** If a certain integer is increased by 3, the result is greater than 7 but less than 13. Find all possible values for the integer.
- b** If a certain number is halved then decreased by 1, the result would lie between 3 and 9. Between what possible values could the number lie?
- c** The sum of 3 consecutive integers is greater than 6 but no more than 15. What could the integers be?
- d** A rectangle is to be constructed with length x cm and width $(x - 7)$ cm. The perimeter of the rectangle is to be less than 34 cm. What are the possible values for x ?
- e** Two sides of a given triangle are 5 cm and 11 cm. What is the range of possible lengths for the third side of the triangle?

14 a and b are two positive integers where $a > b$. State whether the following are true (T) or false (F).

a $b < a$

b $-b < -a$

c $\frac{a}{2} > \frac{b}{2}$

d $\frac{1}{a} > \frac{1}{b}$

e $2a > 2b$

f $a + 5 > b + 5$

g $a^2 > b^2$

h $\sqrt{a} < \sqrt{b}$

Further applications

15 How would you graph $x \neq 3$ on a number line?

16 Solve:

a $4 < x - 3 < 9$

b $6 \leq 2x \leq 10$

c $-1 \leq \frac{x}{2} < 7$

d $7 < 3x + 1 \leq 16$

e $10 < \frac{3x}{4} - 2 < 22$

f $15 \leq \frac{x}{2} + \frac{x}{3} < 20$

17 Solve $-9 \leq 1 - 2x \leq -15$ and graph your solution on a number line.

4.7

Solving worded problems

To solve worded problems:

- ⊙ express each unknown in terms of x
- ⊙ form an equation and solve it
- ⊙ give the answer in the context of the question.

Example 1

Form an equation and solve it to find the number in each of these.

- a Nine less than 4 times a number is equal to 23.
- b Thirteen is added to one-quarter of a number. The result is 19.
- c Increase a number by 7, then double it. The result is 30.

Solutions

In each example, let the number be x .

a $4x - 9 = 23$

$$+9 \quad +9$$

$$4x = 32$$

$$\div 4 \quad \div 4$$

$$\therefore x = 8$$

$\therefore$ The number is 8.

b $\frac{x}{4} + 13 = 19$

$$-13 \quad -13$$

$$\frac{x}{4} = 6$$

$$\times 4 \quad \times 4$$

$$\therefore x = 24$$

$\therefore$ The number is 24.

c $2(x + 7) = 30$

$$2x + 14 = 30$$

$$-14 \quad -14$$

$$2x = 16$$

$$\div 2 \quad \div 2$$

$$\therefore x = 8$$

$\therefore$ The number is 8.

Example 2

The sum of 3 consecutive odd numbers is 129. Find the numbers.

Solution

Let the numbers be x , $x + 2$, $x + 4$.

$$x + (x + 2) + (x + 4) = 129$$

$$3x + 6 = 129$$

$$-6 \quad -6$$

$$3x = 123$$

$$\div 3 \quad \div 3$$

$$\therefore x = 41$$

$\therefore$ The numbers are 41, 43, 45.

EG
+S**Example 3**

The cost of a cricket ball is 80c more than the cost of a tennis ball. If 3 cricket balls and 4 tennis balls cost \$19.90, find the cost of each ball.

Solution

Let the cost of a tennis ball be x cents

$\therefore$ the cost of a cricket ball is $(x + 80)$ cents.

$$3(x + 80) + 4x = 1990 \quad (\text{NOTE: } \$19.90 = 1990\text{c})$$

$$3x + 240 + 4x = 1990$$

$$7x + 240 = 1990$$

$$\begin{array}{r} -240 \quad -240 \\ 7x + 240 = 1990 \\ \hline 7x = 1750 \end{array}$$

$$7x = 1750$$

$$\begin{array}{r} \div 7 \quad \div 7 \\ 7x = 1750 \\ \hline \therefore x = 250 \end{array}$$

$$\therefore x = 250$$

$\therefore$ Each tennis ball costs \$2.50 and each cricket ball costs \$3.30.

Exercise 4.7

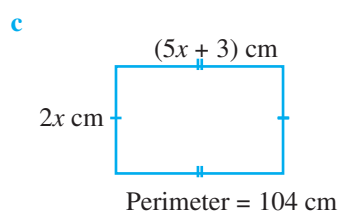
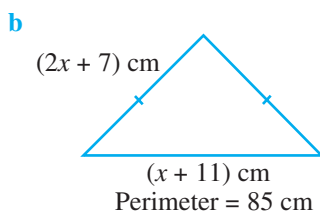
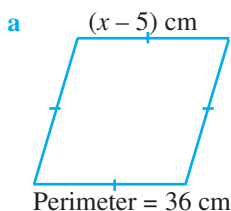
In Q1 to Q5, form an equation and solve it to find the number.

- 1
 - a Six more than 4 times a number is equal to 18.
 - b Five less than twice a number is equal to 9.
 - c When a number is multiplied by 3, then increased by 7, the result is 43.
 - d Double a number, then reduce it by 9. The result is 13.
 - e When a number is multiplied by 5 and this is then subtracted from 60, the result is 20.
- 2
 - a A number is increased by 4 and then multiplied by 6. The result is 30.
 - b When 3 is subtracted from a number and this is then multiplied by 8, the result is 64.
 - c The difference between a number and 9 is tripled. The result is 45.
- 3
 - a If 12 is added to half of a number, the result is 25.
 - b A number is divided by 7, then decreased by 3. The result is 4.
 - c Two-thirds of a number is 18.
 - d Eight less than three-quarters of a number is 31.
- 4
 - a A number is added to 17 and then divided by 4. The result is 7.
 - b Think of a number. Double it, add 5, then divide by 3. The result is 11.
 - c A certain number is decreased by 4, doubled, then divided by 5. The result is 6.
- 5
 - a A number is doubled, then decreased by 9. The result is equal to 13 more than the number.
 - b Eighteen less than the product of a number and 5 is equal to double the number.
 - c Think of a number. Double it, add 20, then divide by 4. The result is equal to 3 times the number.
 - d One-third of a number is equal to 5 less than twice the number.

■ Consolidation

- 6 Form an equation and solve it to answer each of the following.
- The sum of two consecutive numbers is 151. What are the numbers?
 - The sum of three consecutive numbers is 54. What are the numbers?
 - The sum of four consecutive numbers is 98. What are the numbers?
- 7 Form an equation and solve it to find the numbers in each of these.
- The sum of three consecutive even numbers is 102. Find the numbers.
 - The sum of four consecutive odd numbers is 48. Find the numbers.
 - The sum of two consecutive even numbers is equal to 27 more than the odd number that lies between them. Find the even numbers.
 - The sum of three consecutive odd numbers is equal to 39 more than the sum of the even numbers that lie between them. Find the odd numbers.

- 8 Form an equation and solve it to find the value of the pronumeral in each of these.



- 9 Form an equation, then solve it to answer each of the following problems.
- In a group of 29 men and women, there are 7 more women than men. How many people of each gender are there?
 - Annika has \$9 less than Kris. If together they have \$41, find the amount of money that each girl has.
 - The perimeter of a parallelogram is 56 cm and one side is 6 cm shorter than an adjacent side. Find the lengths of the sides.
 - The cost of a new tyre is \$35 more than the cost of a retread. If the cost of two new tyres and two retreads is \$370, find the unit cost of each tyre.
 - An isosceles trapezium has two equal sides of length 7 cm. One of the parallel sides is 5 cm longer than the other parallel side. Find the lengths of the parallel sides if the trapezium has a perimeter of 35 cm.
 - A 2.5 m length of timber is cut into 3 pieces. One piece is twice the length of the shortest piece and the other is 30 cm longer than the shortest piece. Find, in centimetres, the length of each piece of timber.
 - Raymond is half the age of his father. The sum of their ages is 78 years. How old is each person?
- 10 Form an equation, then solve it to answer each of these.
- An imported brand of sugar costs 60c more per kilogram than an Australian brand. If 2 kg of imported sugar plus 5 kg of Australian sugar costs \$13.80, find the cost per kilogram of the imported sugar.

- b Jonathan is twice as old as Darren and Darren is three times as old as Bettina. The sum of their ages is 120 years. Find the age of each person.
- c At a local fruit shop, tomatoes are sold at $24c$ each and pears are sold at $28c$ each. Keryn bought 8 more pears than tomatoes and paid the fruiterer \$3.80. How many pears and tomatoes did Keryn purchase?
- d If the numerator and denominator in the fraction $\frac{5}{11}$ are increased by a certain number, n , the value of the fraction would then be $\frac{2}{3}$. Find the number.
- e Penny has saved \$18 in 20c and 50c coins. There are 8 more 50c coins than 20c coins. What is the total value of the 20c coins?
- f An apprentice mechanic agrees to be paid \$90 for each day that he comes to work and to pay his employer \$40 for each day that he does not come to work. How many days did the apprentice work in April if his total pay for the month was \$1790?

■ Further applications

- 11 a A woman has a daughter who is half her age and a son who is two-thirds her age. The sum of the children's ages is 12 years more than the age of their mother. How old is each person?
 - b A man is 37 years old and his daughter is 5 years old. In how many years time will the man be 3 times the age of his daughter?
 - c Anita is 4 times as old as Frank. In 5 years time Anita will only be 3 times as old as Frank. Find their present ages.
 - d Six years ago, Wendy was twice the age of Thao. At present, Wendy is 30 years older than Thao. Find the present age of each woman.
- 12 Emma tries to guess the number of beads in a jar but guesses 75 too many. Laura guesses 63 too few. If the average of their guesses is 350, how many beads are in the jar?
- 13 A Boeing 729 airliner has a total mass at take-off 94 000 kg. The fuel and crew are $\frac{1}{4}$ the mass of the unloaded plane and the passengers and luggage are $\frac{1}{3}$ the mass of the fuel and crew. What is the mass of the unloaded plane?

TRY THIS A prince and a king

The following problems are from a translation of a collection of ancient Greek works.

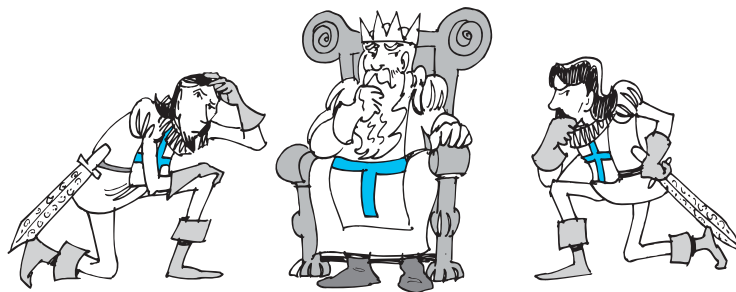
Problem 1

'I wish', said the Prince, 'for my two sons to receive the 9000 gold coins which I possess so that the fifth part of the elder one's share exceeds by 90 the fourth part of what goes to the younger. Please calculate what each son shall receive.'

Problem 2

Croesus the King
blessed five bowls
weighing 10 minae in
total. Each was
10 drachmae heavier
than the one before.
How much did each
bowl weigh?

(100 drachmae = 1 mina)



4.8 Evaluating the subject of a formula

A **formula** is an algebraic statement that shows the relationship between various quantities. Formulae are also known as **literal equations**. Most formulae are written with a single pronumeral on the left-hand side. This pronumeral is called the **subject of the formula**. The subject can be evaluated by substituting values for all of the other pronumerals in the formula.

A lot of our work in mathematics relies on memorising and using formulae. You do not need to memorise the formulae in this exercise, or the next, although you will certainly recognise many of them.

ExampleEG
+S

- a** If $v = u + at$, find the value of v when $u = 45$, $a = -2$ and $t = 10.4$.
b Given that $A = \frac{h}{2}(a + b)$, find the value of A when $h = 14.6$, $a = 9.7$ and $b = 8.5$.
c If $E = \frac{1}{2}mv^2$, find the value of E when $m = 17.25$ and $v = -8$.

Solutions

$$\begin{aligned}\text{a } v &= u + at \\ &= 45 - 2 \times 10.4 \\ &= 45 - 20.8 \\ &= 24.2\end{aligned}$$

$$\begin{aligned}\text{b } A &= \frac{h}{2}(a + b) \\ &= \frac{14.6}{2}(9.7 + 8.5) \\ &= 7.3 \times 18.2 \\ &= 132.86\end{aligned}$$

$$\begin{aligned}\text{c } E &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 17.25 \times (-8)^2 \\ &= \frac{1}{2} \times 17.25 \times 64 \\ &= 552\end{aligned}$$

Exercise 4.8

- 1 a If $A = lb$, find the value of A when $l = 7$, $b = 5$.
 b If $F = ma$, find the value of F when $m = 8.5$, $a = 2.3$.
 c If $A = \frac{1}{2}bh$, find the value of A when $b = 9$, $h = 6.45$.
- 2 a Given that $S = \frac{D}{T}$, find S when $D = 90$, $T = 5$.
 b If $M = \frac{5K}{18}$, find M when $K = 27$.
 c Given that $I = \frac{PRN}{100}$, find I if $P = 200$, $R = 7$, $N = 4$.
- 3 Find the value of the subject in each formula given that:
- | | | |
|-----------------|----------------------------------|--|
| a $P = 2L + 2B$ | i $L = 9$, $B = 4$ | ii $L = 5.8$, $B = 11.3$ |
| b $y = mx + b$ | i $m = 2$, $x = 7$, $b = 1$ | ii $m = -3$, $x = 6$, $b = -4$ |
| c $v = u + at$ | i $u = 8$, $a = 1.5$, $t = 10$ | ii $u = 32$, $a = -\frac{1}{2}$, $t = 8$ |
- 4 a If $A = s^2$, find A when $s = 8$.
 b If $V = x^3$, find V when $x = 12$.
 c If $y = ax^2 + c$, find y when $a = 2$, $x = -3$, $c = -10$.
- 5 a Find the value of V if $V = \sqrt{2gR}$, $g = 9.8$, $R = 2.5$.
 b Find the value of v if $v = \sqrt{\frac{g}{k}}$, $g = 9.8$, $k = 0.392$.
 c Find the value of T if $T = \sqrt{R^3}$, $R = 4$.
- 6 a Given that $A = \frac{h}{2}(a + b)$, find the value of A when $h = 16$, $a = 5$, $b = 7$.
 b Given that $S = \frac{n}{2}(a + l)$, find the value of S when $n = 17$, $a = 3$, $l = 11$.
 c Given that $C = \frac{5}{9}(F - 32)$, find the value of C when $F = 86$.

Consolidation

- 7 a If $E = \frac{1}{2}mv^2$, find E when $m = 21$, $v = 0.2$.
 b If $A = \frac{1}{2}r^2\theta$, find A when $r = 6$, $\theta = 2.4$.
 c If $F = \frac{mv^2}{r}$, find F when $m = 18$, $v = -5$, $r = 12$.
 d If $S = \frac{v^2 - u^2}{2}$, find S when $v = 13$, $u = 7$.
 e If $h = \frac{dv^2}{gr}$, find h correct to 2 decimal places when $d = 6$, $v = 2.5$, $g = 9.8$, $r = 15.4$.
 f If $y = tx - at^2$, find y when $t = 2$, $x = 5$, $a = -3$.
 g If $s = ut + \frac{1}{2}at^2$, find s when $u = 3.5$, $t = 8$, $a = -22$.

8 Find the value of:

- a D if $D = \frac{360}{n}$, when $n = 5$
- b m if $m = \frac{y_2 - y_1}{x_2 - x_1}$, when $y_2 = 23$, $y_1 = 5$, $x_2 = 4$, $x_1 = -2$
- c S if $S = \frac{a}{1-r}$, when $a = 6$, $r = \frac{1}{3}$
- d T if $T = \frac{m_1 - m_2}{1 + m_1 m_2}$, when $m_1 = -4$, $m_2 = \frac{1}{2}$

9 Evaluate, correct to 1 decimal place:

- a A when $A = 4\pi r^2$ and $r = 2.75$
- b V when $V = \pi r^2 h$ and $r = 1.2$, $h = 5$
- c V when $V = \frac{4}{3}\pi r^3$ and $r = 3\frac{5}{12}$
- d A when $A = \pi(R^2 - r^2)$ and $R = 10.5$, $r = 6.5$
- e S when $S = \pi r^2 + \pi r s$ and $r = 1.6$, $s = 2.75$

10 a If $S = 2(ab + bc + ca)$, find S when $a = 3$, $b = 4$, $c = 6$.

- b Given that $A = \frac{h}{3}(d_F + 4d_M + d_L)$, find A when $h = 4$, $d_F = 3.3$, $d_M = 6.7$, $d_L = 4.1$.
- c Find the value of T if $T = a + (n-1)d$ when $a = 7$, $n = 15$, $d = 1\frac{1}{4}$.
- d Evaluate S if $S = \frac{n}{2}[2a + (n-1)d]$ given that $n = 10$, $a = 4$, $d = -3$.

Further applications

11 Evaluate:

- a R if $R = \sqrt{a^2 + b^2}$ and $a = 2$, $b = \sqrt{5}$
- b v if $v = n\sqrt{a^2 - x^2}$ and $n = 3$, $a = 10$, $x = -6$
- c T if $T = 2\pi\sqrt{\frac{l}{g}}$ and $l = 32$, $g = 9.8$ (Answer correct to 1 decimal place.)
- d v if $v = \sqrt{u^2 + 2as}$ and $u = 11$, $a = -5$, $s = -7.5$
- e E if $E = \sqrt{1 - \frac{b^2}{a^2}}$ and $a = 5$, $b = 3$

12 a If $A = Pr^n$, find correct to 2 decimal places the value of A when $P = 250\,000$, $r = 0.14$, $n = 5$.

b If $T = ar^{n-1}$, find T when $a = 8$, $r = \frac{1}{2}$, $n = 7$.

c If $A = P\left(1 + \frac{r}{100}\right)^n$, find A correct to 2 decimal places when $P = 3250$, $r = 17$, $n = 4$.

d If $E = \frac{(1+r)^{n-1}}{n}$, find E correct to 2 decimal places when $r = 0.12$, $n = 15$.

4.9 Equations arising from substitution

To find the value of a pronumeral other than the subject of a formula, you will need to solve an equation.

Example

- a** If $P = 2L + 2B$, find the value of B when $P = 42.2$ and $L = 12.7$.
b Given that $S = \frac{n}{2}(a + l)$, find the value of a when $S = 630$, $n = 18$ and $l = 44$.
c If $E = \frac{1}{2}mv^2$, find the value of v when $E = 213.6$, $m = 26.7$ and $v > 0$.

Solutions

$$\begin{aligned} \mathbf{a} \quad P &= 2L + 2B \\ 42.2 &= 2 \times 12.7 + 2B \\ 42.2 &= 25.4 + 2B \\ -25.4 \quad -25.4 \\ 16.8 &= 2B \\ \div 2 \quad \div 2 \\ \therefore B &= 8.4 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad S &= \frac{n}{2}(a + l) \\ 630 &= \frac{18}{2}(a + 44) \\ 630 &= 9(a + 44) \\ 630 &= 9a + 396 \\ -396 \quad -396 \\ 234 &= 9a \\ \div 9 \quad \div 9 \\ \therefore a &= 26 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad E &= \frac{1}{2}mv^2 \\ 213.6 &= \frac{1}{2} \times 26.7 \times v^2 \\ 213.6 &= 13.35v^2 \\ +13.35 \quad +13.35 \\ 16 &= v^2 \\ \therefore v &= 4 \quad (v > 0) \end{aligned}$$

Exercise 4.9

- a** If $F = ma$, find m when $F = 10.5$, $a = 7$.
b If $V = lbh$, find b when $V = 480$, $l = 12$, $h = 8$.
- If $P = 2L + 2B$, find the value of:
a L when $P = 32$, $B = 6$ **b** B when $P = 46$, $L = 13.5$
- If $S = \frac{D}{T}$, find:
a D when $S = 15$, $T = 6$ **b** D when $S = 25.4$, $T = 3.5$
c T when $S = 45$, $D = 315$ **d** T when $D = 36.256$, $S = 4.4$
- If $M = \frac{5K}{18}$, find the value of K when $M = 30$.

Consolidation

- If $v = u + at$, find the value of:
a u when $v = 50$, $a = 5$, $t = 6$ **b** u when $v = 14$, $a = -12$, $t = 8$
c a when $v = 167.3$, $u = 10.5$, $t = 12.8$ **d** t when $v = 9.76$, $u = 15.2$, $a = -3.4$

- 6** If $S = \frac{n}{2}(a + l)$, find the value of:
- a** n when $S = 96$, $a = 5$, $l = 7$ **b** l when $S = -28$, $n = 7$, $a = 3$
- c** a when $S = 689$, $n = 26$, $l = 48$
- 7** If $v = \sqrt{2gR}$, find the value of:
- a** g when $v = 9$, $R = 4.05$ **b** R when $v = 12$, $g = 10$
- 8** If $v = \sqrt{\frac{g}{k}}$, find the value of:
- a** g when $v = 4$, $k = 0.5$ **b** k when $v = 6$, $g = 12$
- 9** If $E = \frac{1}{2}mv^2$, find the value of:
- a** m when $E = 24$, $v = 4$ **b** v when $E = 14.4$, $m = 12.8$ and $v > 0$
- 10** If $F = \frac{mv^2}{r}$, find the value of:
- a** m when $F = 20$, $v = 5$, $r = 20$ **b** v when $F = 49$, $m = 10$, $r = 2.5$ and $v > 0$
- c** r when $F = 56.32$, $v = -12.8$, $m = 22$
- 11** If $S = \frac{v^2 - u^2}{2}$, find the value of:
- a** v when $S = 36$, $u = 7$ and $v > 0$ **b** u when $S = 17.1$, $v = 10.4$ and $u > 0$
- 12** If $S = ut + \frac{1}{2}at^2$, find the value of:
- a** u when $S = 116$, $t = 4$, $a = 12$ **b** a when $S = 3.92$, $u = 7$, $t = 1.4$
- 13** If $m = \frac{y_2 - y_1}{x_2 - x_1}$, find the value of:
- a** y_2 when $m = 3$, $y_1 = 5$, $x_2 = 8$, $x_1 = 4$ **b** y_1 when $m = -3$, $y_2 = 13$, $x_2 = -5$, $x_1 = 2$
- c** x_2 when $m = \frac{3}{4}$, $y_2 = 21$, $y_1 = 6$, $x_1 = 4$ **d** x_1 when $m = -\frac{5}{7}$, $y_2 = -8$, $y_1 = 2$, $x_2 = 9$
- 14** If $V = \pi r^2 h$, find the value of:
- a** h when $V = 32\pi$, $r = 2$
- b** r correct to 1 decimal place when $V = 1170$, $h = 5.4$ and $r > 0$
- 15** If $V = \frac{4}{3}\pi r^3$, find the value of:
- a** r when $V = 288\pi$ **b** r correct to 1 decimal place when $V = 100$
- 16** If $S = \frac{a}{1-r}$, find the value of:
- a** a when $S = 30$, $r = \frac{1}{4}$ **b** a when $S = 50$, $r = -0.6$
- c** r when $S = 36$, $a = 12$ **d** r when $S = 28$, $a = 42$

■ Further applications

- 17** If $A = \pi(R^2 - r^2)$, find the value of:
- a** R when $A = 40\pi$, $r = 3$ and $R > 0$
- b** R correct to 1 decimal place when $A = 78.5$, $r = 6.5$ and $R > 0$

- c** r when $A = 85\pi$, $R = 11$ and $r > 0$
d r correct to 1 decimal place when $A = 480.3$, $R = 13.7$ and $r > 0$
- 18** If $S = \frac{n}{2}[2a + (n-1)d]$, find the value of:
a a when $S = 250$, $n = 10$, $d = 4$ **b** d when $S = -44$, $n = 8$, $a = -3$
- 19** If $T = ar^{n-1}$, find the value of:
a a when $T = 175$, $r = 5$, $n = 3$ **b** r when $T = 80$, $a = 5$, $n = 5$
c n when $T = 56$, $a = 7$, $r = 2$
- 20** If $T = \frac{m_1 - m_2}{1 + m_1 m_2}$, find the value of:
a m_1 when $T = \frac{1}{8}$, $m_2 = 5$ **b** m_2 when $T = -4\frac{1}{2}$, $m_1 = -\frac{1}{2}$

TRY THIS Floodlighting by formula

The formula below is used by lighting engineers to work out how many floodlights are needed to illuminate a given area. Many factors are taken into account, including the size of the area and the strength of illumination required (measured in lumens per square metre).

Different uses require different strengths of illumination. If a car park, a tennis court and a beach park are all of the same area, which one should have the strongest illumination?

$$\text{Number of floodlights} = \frac{A \times L^2}{B \times C}$$

where A = area

L = light intensity per square metre desired

B = light intensity emitted by one light

C = the constant factor (depends on height of poles, reflection from walls, etc.)

- Calculate the number of floodlights required to light a rectangular area 50 m $\times$ 30 m with an intensity of 2 lumens per square metre if the type of lights that have been chosen emit 2000 lumens each and the constant factor is 0.26.
- If the intensity required was dropped to 1 lumen per square metre, how many lights would be needed?
- If the original number of lights is to be kept (because the poles and fixtures have already been installed) but new lights can be purchased that are less than 2000 lumens, find to the nearest lumen the output required from each light to give 1 lumen per square metre.

4.10 Changing the subject of a formula

To find the value of one of the pronumerals in a formula, we could either:

- 1 substitute the given values into the formula then solve the resulting equation, or
- 2 change the subject of the formula to the desired variable then substitute the given values.

The phrase ‘solve this literal equation for x ’ means the same as ‘make x the subject of this formula’.

Literal equations are solved in the same way as other equations, that is, by performing inverse operations to both sides and isolating the desired pronumeral.

To solve a literal equation, or change the subject of a formula:

- ⊙ expand any grouping symbols
- ⊙ perform inverse operations to both sides until the left-hand side contains only the desired subject.

NOTE: After a formula has been re-arranged, the variable that is the new subject must only appear once. That is, the subject cannot appear on both sides of the formula, nor can it appear twice on the same side.

To change the subject of a formula, where the desired subject appears more than once:

- ⊙ take all terms that contain this variable to one side
- ⊙ take the remaining terms to the other side
- ⊙ factorise the expression that contains the desired subject
- ⊙ divide both sides by the expression in the grouping symbols.

NOTE: It may be necessary to place restrictions on the possible values of the variables once a formula has been re-arranged. In particular:

- 1 An expression that occurs in the denominator cannot be equal to zero.
- 2 An expression that occurs under a square root sign cannot be negative.

EG
+S**Example 1**Solve each literal equation for x .

a $px + q = r$

Solutions

a $px + q = r$

$-q \quad -q$

$px = r - q$

$\div p \quad \div p$

$\therefore x = \frac{r - q}{p}$

b $y = a(x - z)$

b $y = a(x - z)$

$y = ax - az$

$+az \quad +az$

$y + az = ax$

$\div a \quad \div a$

$\frac{y + az}{a} = x$

$\therefore x = \frac{y + az}{a}$

EG
+S**Example 2**Make c the subject of each formula.

a $b^2 = a^2 - c^2$

Solutions

a $b^2 = a^2 - c^2$

$+c^2 \quad +c^2$

$b^2 + c^2 = a^2$

$-b^2 \quad -b^2$

$c^2 = a^2 - b^2$

$\sqrt{\quad} \quad \sqrt{\quad}$

$\therefore c = \pm\sqrt{a^2 - b^2}$

b $p = \frac{q + \sqrt{c}}{r}$

b $p = \frac{q + \sqrt{c}}{r}$

$\times r \quad \times r$

$pr = q + \sqrt{c}$

$-q \quad -q$

$pr - q = \sqrt{c}$

$(\quad)^2 \quad (\quad)^2$

$(pr - q)^2 = c$

$\therefore c = (pr - q)^2$

EG
+S**Example 3**Make y the subject of the formula $\frac{x}{3} + \frac{y}{5} = z$.**Solution**

$\frac{x}{3} + \frac{y}{5} = z$

$\times 15 \quad \times 15$

$5x + 3y = 15z$

$-5x \quad -5x$

$3y = 15z - 5x$

$\div 3 \quad \div 3$

$\therefore y = \frac{15z - 5x}{3}$

EG
+S**Example 4**Make m the subject of the

formula $y = \frac{m}{m+5}$.

Solution

$$y = \frac{m}{m+5}$$

$$\times(m+5) \quad \times(m+5)$$

$$y(m+5) = m$$

$$my + 5y = m$$

$$-my \quad -my$$

$$5y = m - my$$

$$5y = m(1 - y)$$

$$\div(1 - y) \quad \div(1 - y)$$

$$\frac{5y}{1 - y} = m$$

$$\therefore m = \frac{5y}{1 - y}$$

Exercise 4.10**1** Make x the subject of each formula.

a $y = a + x$

b $x - q = p$

c $m - x = n$

d $2x = k$

e $c = dx + e$

f $y = \frac{x}{c}$

g $p = \frac{q}{x}$

h $\frac{ux}{v} = w$

i $\frac{x}{a} = y - z$

j $\frac{bx}{c} = \frac{d}{e}$

k $\frac{m}{x} = \frac{n}{p}$

l $\frac{k}{x} = g + h$

2 Expand the grouping symbols in each formula, then make a the subject.

a $c = 3(a + b)$

b $y = 2(a - 7)$

c $w = m(a + c)$

d $p = k(1 - a)$

e $z = 5y(2 - a)$

f $A = 2\pi(r - a)$

Consolidation**3** Multiply each term by the lowest common denominator, then solve for n .

a $m = \frac{kn}{3} - p$

b $\frac{n}{2} + \frac{k}{5} = 1$

c $\frac{a}{b} - \frac{n}{k} = 1$

d $\frac{n}{3} + \frac{n}{7} = k$

e $\frac{n-a}{2} = \frac{b+c}{3}$

f $h = \frac{m+n}{3} + \frac{h+m}{4}$

4 Transpose each formula so that t is the subject.

a $at^2 = b$

b $t^2 + m = n$

c $k = u - t^2$

d $b^2 = a^2 - t^2$

e $\frac{t^2}{s} = r$

f $h = \frac{ut^2}{v}$

g $\frac{t^2}{4} = \frac{9}{a^2}$

h $\frac{a}{2t} = \frac{8t}{b}$

5 Solve for c :

$$\begin{array}{llll} \text{a} & \sqrt{c} = j & \text{b} & \sqrt{cd} = a \\ \text{c} & y = \sqrt{c-4} & \text{d} & p = \frac{\sqrt{c}}{3} \\ \text{e} & M = \frac{a\sqrt{c}}{n} & \text{f} & L = K + \sqrt{c} \\ \text{g} & a = b + d\sqrt{c} & \text{h} & p - q\sqrt{c} = N \end{array}$$

6 Make x the subject of each formula.

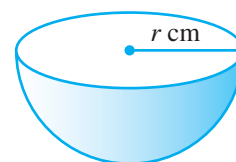
$$\begin{array}{lll} \text{a} & x - p = q - x & \text{b} & mx = p + nx \\ \text{c} & cx - d = x + d & \text{d} & a = \frac{1-x}{x} \\ \text{e} & y = \frac{x}{x+1} & \text{f} & t = \frac{x}{x-3} \\ \text{g} & g = \frac{x+1}{x-1} & \text{h} & v = \frac{a-x}{a+x} \\ \text{i} & c = \frac{a-bx}{x-b} \end{array}$$

7 Solve for x : $px - q = qx - p$.

8 Solve each of the following for the letter shown in the brackets.

$$\begin{array}{llll} \text{a} & D = \frac{M}{V} & [M] & \text{b} & v = u + at & [a] & \text{c} & P = 2(L + B) & [L] \\ \text{d} & T = \frac{D}{S} & [S] & \text{e} & A = \frac{x+y}{2} & [x] & \text{f} & I = \frac{PRN}{100} & [P] \\ \text{g} & V = \frac{1}{3}Ah & [A] & \text{h} & E = \frac{1}{2}mv^2 & [v] & \text{i} & R^3 = T^2 & [R] \\ \text{j} & v^2 = u^2 + 2as & [s] & \text{k} & V = \sqrt{2gR} & [R] & \text{l} & A = 2\pi r(r+h) & [h] \\ \text{m} & S = \frac{n}{2}(a+l) & [n] & \text{n} & A = \frac{h}{2}(a+b) & [a] & \text{o} & F = \frac{9C}{5} + 32 & [C] \\ \text{p} & C = \frac{5}{9}(F-32) & [F] & \text{q} & V = \frac{4}{3}\pi r^3 & [r] & \text{r} & s = \frac{v^2 - u^2}{2} & [u] \\ \text{s} & v = n\sqrt{a^2 - x^2} & [a] & \text{t} & S = \frac{a}{1-r} & [r] & \text{u} & s = ut + \frac{1}{2}at^2 & [u] \\ \text{v} & T = 2\pi\sqrt{\frac{l}{g}} & [l] & \text{w} & A = \pi(R-r)(R+r) & [R] & \text{x} & \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} & [R] \end{array}$$

9 The volume $V \text{ cm}^3$ of a hemisphere with radius $r \text{ cm}$ is given by the formula $V = \frac{2}{3}\pi r^3$. Find the length of the radius if the hemisphere has a volume of $18\pi \text{ cm}^3$.

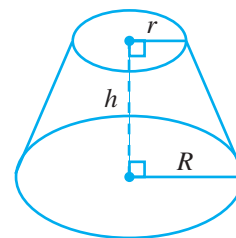


- 10** The area of an equilateral triangle of side x cm is given by the formula $A = \frac{\sqrt{3}}{4}x^2$.

Find, correct to 2 decimal places, the side length of an equilateral triangle with an area 30 cm^2 .

- 11** The distance d kilometres that a person can see to the visible horizon from a height h metres above sea level is given by the formula $d = 5\sqrt{\frac{h}{2}}$. Liesl can see 40 km out to sea from the top of a cliff which is h metres above sea level. How high is the cliff?

- 12** A frustum is a solid that results when a cone is cut by a plane parallel to its base. The volume of a frustum is given by the formula $V = \frac{\pi h}{3}(R^2 + Rr + r^2)$, where R and r are the radii of the circular faces and h is the height between these faces. Find the height of a frustum in which the radii are 12 cm and 8 cm, and the volume is $608\pi \text{ cm}^3$.



■ Further applications

- 13** For each of the formulae below:

- i state any restrictions that must exist on the variables
- ii re-arrange the formula so that the variable shown in the brackets is the subject
- iii state any restrictions that must exist on the variables after the formula has been re-arranged.

a $a = bx + c$	$[x]$	b $y = ax^2$	$[x]$	c $\frac{p}{q} = \frac{r}{s}$	$[r]$
d $c = a\sqrt{b^2 + 9}$	$[b]$	e $k = \frac{m}{nu^2 + 1}$	$[u]$	f $z = \frac{t}{t-2}$	$[t]$

- 14** Make m_1 the subject of the formula $T = \frac{m_1 - m_2}{1 + m_1 m_2}$.

- 15** Make r the subject of each equation:

a $A = Pr^n$	b $A = P\left(1 + \frac{r}{100}\right)^n$



FOCUS ON WORKING MATHEMATICALLY

SPLITTING THE ATOM

Introduction

$$E = mc^2$$

The famous equation derived by Albert Einstein in 1904 revealed the relationship between energy (E) and mass (m) to the scientific world. At that time it was only a theory. The means of testing it slowly increased with technological progress. As so often happens, theory was well ahead of practical reality. But, Einstein was right, energy and mass are interconnected. The one can be changed into the other. Einstein also predicted that the mass of a body depends on its speed. The greater the velocity of a body, the greater the increase in its mass. The effect is undetectably small at lower speeds, for example a jet plane, but as the speed of the object approaches the speed of light, its mass increases without limit.

Note that it is the speed of light c in the equation above which is the link between the mass and energy. The value of c is about $3 \times 10^8 \text{ m s}^{-1}$, a very large value. If even a small mass could be converted into pure energy, the output would be enormous. For example, a small piece of coal converted completely into its mass equivalent in electrical energy would keep a single light globe burning for over 1600 billion years.

In 1939, scientists had the technical means to successfully bombard the nucleus of a uranium atom, in effect splitting the atom, reducing its mass, and liberating the equivalent in enormous amounts of energy. Unfortunately, this technology, called nuclear fission, coincided with the outbreak of war in Europe. Einstein's equation could be used to produce an atomic bomb, more devastating than anything the world had ever known.

The impact of Einstein's discovery, born out of thinking mathematically about the nature of the physical world, brought about changes to the lives of human beings on planet Earth, which at the turn of the 20th century were unimaginable. It heralded the beginning of the nuclear age.





LEARNING ACTIVITIES

We will now see how the equation works. Remember that the total energy and mass always remain the same. There is simply a conversion from one to the other. It is simpler to read $E = mc^2$ to mean:

The change in energy of a body = the change in mass $\times c^2$.

If there is a loss in mass there will be a corresponding increase in energy (e.g. splitting the atom). Similarly, if there is a gain in mass there will be a corresponding decrease in energy.

Example

Suppose that 1 kg of the fuel in a nuclear reactor undergoes fission (i.e. is broken down into a smaller mass). In such a nuclear reaction the loss in mass is typically of the order of 1/1000 of the total mass, in this case 1 g. Calculate the liberated energy using Einstein's equation and $c = 3 \times 10^8 \text{ m s}^{-1}$.

- 1 Copy and complete: Increase in energy = loss in mass $\times c^2$
 $= 1/1000 \times 1 \times 3 \times 10^8 \times 3 \times 10^8$ (mass must be in kg)
 $= \dots\dots\dots$ joules (J)
- 2 Note how large this amount of energy is. To see, write the answer out in full.
- 3 In a nuclear reactor this energy is released in a controlled fashion over a period of time, not in a single explosion such as happens with nuclear weapons. Note again that only a small portion of the mass is lost.



CHALLENGE

According to Einstein, the mass of a body moving from rest to a speed v will increase. In the case of an aircraft, for example, some of the energy required to accelerate it to cruising speed goes into increasing its kinetic energy and hence its mass. The rest goes into overcoming external forces. The following example shows how small the mass increase is.

- 1 A jumbo jet of mass 350 tonnes is flying at 900 km h^{-1} . Calculate its kinetic energy (KE) using the formula $KE = \frac{1}{2}mv^2$
Units: Mass must be in kg, speed in m s^{-1} . Take $100 \text{ km h}^{-1} = 30 \text{ m s}^{-1}$
 The kinetic energy calculated will be in joules (J).



- 2 Use your answer for kinetic energy and the equation $E = mc^2$ to calculate the increase in the mass of the jumbo jet. Expect a small answer! The unit will be kg. You may like to change it to milligrams for a final answer.

NOTE: In everyday life, it is in applications in electronics when beams of electrons move at close to the speed of light that Einstein's equation becomes significant. Some examples of this are TV sets, scanning machines for use in medicine, and computers. Check this with your science teacher.



LET'S COMMUNICATE

- 1 Write a summary of the meaning of the equation $E = mc^2$ to you. You may first like to re-read the introduction above.
- 2 Do you feel that the continued burning of coal to produce electricity is better than the dangers of using nuclear reactors? Discuss the advantages and disadvantages. How do the two methods relate to Einstein's equation?



REFLECTING

Reflect on how important a role mathematics has played in the development of the nuclear age and in our understanding of the natural world and beyond to the furthest reaches of the universe. Reflect on how human beings can use such information for the common good but also for the destruction of life.



MACQUARIE

Language link with Macquarie

- 1 What is the difference between an **equation** and an **inequation**?
- 2 List 2 words that mean the same thing as **inverse**.
- 3 What is the difference between **solve** and **substitute**?
- 4 Define **formula** for a new mathematics dictionary.
- 5 Read the Macquarie Learners Dictionary entry for **equate**:

equate verb 1. to regard, treat, or represent as equal or connected: *He equates money with happiness.*

| *The decrease in the incidence of lung cancer equates with a decrease in the number of smokers.*

2. to be the same in number, value, etc.: *The cost of a home mortgage equates to about 25 per cent of an average income.*

Why is the skill of equating important to science and medicine?

1 Solve each of the following equations.

a $12 - j = 4$ b $7 = -3 + b$
 c $-x - 6 = 2$ d $-9 = 5 - c$

2 Solve:

a $2p + 7 = 23$ b $5k - 4 = 26$
 c $17 - 4y = 29$ d $8m + 3 = -45$

3 Solve:

a $8h = 3h + 40$ b $9d + 42 = 2d$
 c $3y = 7y - 44$ d $60 - 7s = 5s$

4 Solve these equations.

a $5r + 8 = 2r + 29$
 b $9b + 11 = 5b - 25$
 c $23 - 3u = 41 - 5u$

5 Solve each of the following.

a $9(2a + 5) = 99$
 b $-8(2 - 3f) = -112$

6 Solve:

a $6(x + 2) = 5x + 16$
 b $4p - 54 = 2(7p + 3)$
 c $5(2r - 3) = -3(1 - 3r)$

7 Expand and simplify the expressions on each side of these equations, then solve for x .

a $5(x + 3) + 2(x + 2) = 54$
 b $9(x - 4) + 3(5 - x) = 33$
 c $8 - 3(x - 4) = 5(x + 2) + 2$

8 Solve each of these equations, giving the solutions in simplest fraction form.

a $-15g = -9$ b $x + \frac{1}{3} = 5$
 c $u - 2\frac{1}{2} = 1\frac{2}{3}$ d $\frac{w}{5} = -\frac{3}{4}$
 e $4h + 13 = 27$ f $7 - 15s = -5$

9 Solve:

a $\frac{3x}{2} = 12$ b $-\frac{2z}{3} = -8$
 c $\frac{t}{11} - 7 = 2$ d $\frac{4a}{3} + 5 = 13$
 e $6 - \frac{2u}{5} = -2$ f $\frac{n+4}{7} = 3$

g $\frac{w+3}{12} = \frac{2}{3}$ h $\frac{5r-14}{20} = \frac{4}{5}$
 i $\frac{e+6}{4} = \frac{e-2}{5}$ j $\frac{7h-8}{4} = \frac{4h-5}{2}$

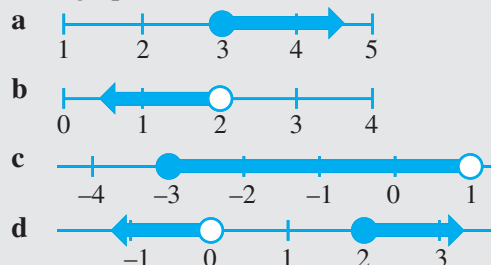
10 Solve:

a $\frac{x}{4} + \frac{x}{5} = 18$ b $\frac{3m}{5} - \frac{m}{6} = 13$
 c $\frac{2a}{7} = 11 - \frac{a}{2}$
 d $\frac{3u+2}{4} + \frac{u-5}{3} = 1$
 e $\frac{z+1}{4} + \frac{2z}{5} = 2 - \frac{7z}{10}$
 f $\frac{3}{4}w = \frac{5}{6}(w - 2)$

11 Write down the integer solution set for these inequalities.

a $x < 7$ b $x \geq 5$
 c $x > -10$ d $x \leq 0$
 e $3 \leq x \leq 8$ f $-2 < x < 5$
 g $x \leq 1$ or $x \geq 5$ h $x < -4$ or > 7

12 State, in terms of x , the inequality that has been graphed on each number line.



13 Solve each of these inequations.

a $y - 5 < 3$ b $k + 7 > 4$
 c $6m \geq 30$ d $\frac{t}{9} \leq 3$
 e $15 > u - 4$ f $3c + 11 \leq 23$
 g $5(z + 6) \geq 10$ h $7w + 27 < 4w - 12$
 i $a - \frac{a}{3} \geq 14$ j $\frac{2b}{3} - \frac{b}{4} \leq 10$

14 Solve:

- a $-3m < 21$ b $9 - 4c \geq 37$
 c $5(9 - 2r) \leq 85$ d $2 - \frac{n}{4} > -6$
 e $\frac{2e + 5}{-3} \geq -7$ f $9 < \frac{-3(7 - 2t)}{5}$

15 Form an equation and solve it to find the number in each of these.

- a Seven less than three times a number is equal to 38.
 b When a number is multiplied by 3 and this is then subtracted from 50, the result is 23.
 c If 8 is added to half of a number, the result is 13.
 d A number is increased by 6, then multiplied by 4. The result is 68.
 e Think of a number. Double it. Add 17. Divide by 5. The result is 7.
 f A number is multiplied by 5, then decreased by 30. The result is equal to 18 more than double the number.

16 Form an equation and solve it to answer each of the following.

- a The sum of four consecutive numbers is 70. What are the numbers?
 b The sum of three consecutive odd numbers is 87. What are the numbers?

- 17 a Samantha has \$47 less than James and together they have \$283. How much does each person have?
 b At a local cafeteria, the cost of a salad sandwich is 40c more than for a ham sandwich. A group of friends bought 4 salad sandwiches and 3 ham sandwiches and paid \$17 altogether. Find the cost of each sandwich.

18 Solve these inequality problems.

- a If a certain integer is increased by 3, the result is greater than 7 but less than 13. Find all possible values for the integer.
 b If a certain number is halved, then decreased by 1, the result would lie between 3 and 9. Between what possible values could the number lie?
 c Two sides of a given triangle are 8 cm and 13 cm. What is the range of possible lengths for the third side of the triangle?
- 19 a If $v = \sqrt{2gR}$, find the value of v when $g = 9.8$ and $R = 250$.
 b If $v = u + at$, find the value of v when $u = 60$, $a = -3$ and $t = 8.7$.
 c If $E = \frac{1}{2}mv^2$, find the value of E when $m = 12.8$ and $v = 4.5$.
 d If $S = \frac{n}{2}(a + l)$, find the value of S when $n = 16$, $a = 13$ and $l = 18$.
 e If $S = \frac{n}{2}[2a + (n - 1)d]$, find the value of S when $n = 42$, $a = 50$, $d = -2$.
- 20 a If $P = 2L + 2B$, find the value of L when $P = 40$ and $B = 12$.
 b If $y = mx + b$, find the value of m when $y = 8$, $x = 15$ and $b = -2$.
 c If $M = \frac{5K}{18}$, find the value of K when $M = 45$.
 d If $S = ut + \frac{1}{2}at^2$, find the value of a when $S = 245$, $u = 87.5$ and $t = 7$.
 e If $m = \frac{y_2 - y_1}{x_2 - x_1}$, find the value of x_1 , when $m = 3$, $y_2 = 9$, $y_1 = -6$, $x_2 = 7$.

21 Rewrite each formula so that x is the subject.

a $\frac{ax}{b} = \frac{c}{d}$

b $\frac{e}{x} = f - g$

c $m = 2a(3 + x)$

d $u = v - x^2$

e $p = q + r\sqrt{x}$

f $\frac{x}{2} + \frac{y}{5} = z$

g $x - b = a - x$

h $y = \frac{x+2}{x-1}$

22 Consider the formula $z = w\sqrt{x-y}$.

a State any restrictions that exist on the variables.

b Make x the subject of the formula.

c State any restrictions that must exist on the variables after the formula has been re-arranged.

Measurement



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ state the limits of accuracy for a given measurement
- ⦿ convert between units of length, mass, capacity and time
- ⦿ solve problems involving length, mass, capacity and time
- ⦿ perform time calculations with and without the use of a calculator
- ⦿ convert between 12-hour time and 24-hour time
- ⦿ find the length of a side in a right-angled triangle using Pythagoras' theorem
- ⦿ solve problems using Pythagoras' theorem
- ⦿ calculate the perimeter of a polygon
- ⦿ calculate the circumference of a circle
- ⦿ calculate the perimeters of sectors and composite figures
- ⦿ solve practical problems involving perimeter
- ⦿ convert between units of area
- ⦿ use area formulae to find the areas of triangles and the special quadrilaterals
- ⦿ calculate the area of a circle
- ⦿ calculate the areas of sectors and composite figures
- ⦿ solve practical problems involving area.

5.1 Length, mass, capacity and time

Length

The basic unit of **length** in the metric system is the **metre**. Originally, the metre was defined as one ten-millionth of the distance from the North Pole to the equator along the line of longitude that runs through Paris. This definition was revised in 1960, and the metre is now defined in terms of the wavelength of light.

Other units of length in common use are the millimetre (mm), centimetre (cm) and kilometre (km). Each unit of length can be compared to the metre by referring to its **prefix**. For example, the prefix *milli-* means ‘one-thousandth of’, so 1 millimetre literally means ‘one-thousandth of a metre’. Similarly, the prefixes *centi-* and *kilo-* mean ‘one-hundredth of’ and ‘one thousand times’ respectively. Hence, 1 centimetre means ‘one-hundredth of a metre’ and 1 kilometre means ‘one-thousand metres’.

Prefix	Meaning
milli-	$\frac{1}{1000}$
centi-	$\frac{1}{100}$
kilo-	1000

The common conversions for length are:

- ⊙ 10 mm = 1 cm (i.e. 10 millimetres = 1 centimetre)
- ⊙ 100 cm = 1 m (i.e. 100 centimetres = 1 metre)
- ⊙ 1000 m = 1 km (i.e. 1000 metres = 1 kilometre)

Mass

The **mass** of an object is the amount of matter that it contains. Mass is not the same as weight, although the two terms are commonly used interchangeably. Weight refers to the force with which an object is being attracted to the Earth due to gravity. In the metric system, the base unit for mass is the **kilogram**.

The common conversions for mass are:

- ⊙ 1000 mg = 1 g (i.e. 1000 milligrams = 1 gram)
- ⊙ 1000 g = 1 kg (i.e. 1000 grams = 1 kilogram)
- ⊙ 1000 kg = 1 t (i.e. 1000 kilograms = 1 tonne)

Capacity

Capacity refers to the amount of liquid that a container can hold. In the metric system, the base unit for capacity is the **litre**.

The common conversions for capacity are:

- ⊙ 1000 mL = 1 L (i.e. 1000 millilitres = 1 litre)
- ⊙ 1000 L = 1 kL (i.e. 1000 litres = 1 kilolitre)
- ⊙ 1000 kL = 1 ML (i.e. 1000 kilolitres = 1 megalitre)

- ⊙ To convert to a smaller unit, multiply by the conversion factor.
- ⊙ To convert to a larger unit, divide by the conversion factor.

Time

We use **time** to *order* the events that take place in our everyday lives. Without time, it would not be possible to say which event came before or after another event. We often measure the degree of change in a particular situation according to the amount of time that passes, for example when calculating the speed of a moving object. The speed is, in fact, a measure of the change in distance with respect to the elapsed time.

The common conversions for time are:

- ⊙ $60 \text{ s} = 1 \text{ min}$ (i.e. 60 seconds = 1 minute)
- ⊙ $60 \text{ min} = 1 \text{ h}$ (i.e. 60 minutes = 1 hour)
- ⊙ $24 \text{ h} = 1 \text{ day}$ (i.e. 24 hours = 1 day)

We use instruments such as watches and clocks to tell the time. These are either **analog** or **digital**. Time pieces with rotating hands are called analog, whereas those that display digits only are called digital. Many digital watches and clocks operate in 24-hour time, that is from 00:00 to 24:00 hours, rather than in am or pm time.

To convert from 12-hour time to 24-hour time:

- ⊙ add 12 hours to the time if it is 1 pm or greater
- ⊙ write the time using 4 digits.

Many time calculations can be more easily performed with the use of the degrees and minutes, or $\boxed{\text{DMS}}$ and $\boxed{^{\circ}''}$ keys on the calculator. It may first be necessary to express one of the given times in 24-hour time.

Example 1

Convert:

a 8 cm to mm

d 70 mm to cm

b 5.2 m to cm

e 129 cm to m

c 0.04 km to m

f 2300 m to km

Solutions

a $8 \text{ cm} = (8 \times 10) \text{ mm}$
 $= 80 \text{ mm}$

d $70 \text{ mm} = (70 \div 10) \text{ cm}$
 $= 7 \text{ cm}$

b $5.2 \text{ m} = (5.2 \times 100) \text{ cm}$
 $= 520 \text{ cm}$

e $129 \text{ cm} = (129 \div 100) \text{ m}$
 $= 1.29 \text{ m}$

c $0.04 \text{ km} = (0.04 \times 1000) \text{ m}$
 $= 40 \text{ m}$

f $2300 \text{ m} = (2300 \div 1000) \text{ km}$
 $= 2.3 \text{ km}$

EG
+S**Example 2**

Convert:

a 0.57 m to mm

b 98 000 cm to km

Solutions

$$\begin{aligned}\mathbf{a} \quad 0.57 \text{ m} &= (0.57 \times 100) \text{ cm} \\ &= 57 \text{ cm} \\ &= (57 \times 10) \text{ mm} \\ &= 570 \text{ mm}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 98\,000 \text{ cm} &= (98\,000 \div 100) \text{ m} \\ &= 980 \text{ m} \\ &= (980 \div 1000) \text{ km} \\ &= 0.98 \text{ km}\end{aligned}$$

EG
+S**Example 3**

Convert:

a 5 L to mL

b 6.8 kL to L

c 910 L to kL

Solutions

$$\begin{aligned}\mathbf{a} \quad 5 \text{ L} &= (5 \times 1000) \text{ mL} \\ &= 5000 \text{ mL}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 6.8 \text{ kL} &= (6.8 \times 1000) \text{ L} \\ &= 6800 \text{ L}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad 910 \text{ L} &= (910 \div 1000) \text{ kL} \\ &= 0.91 \text{ kL}\end{aligned}$$

EG
+S**Example 4**

Convert:

a 4 kg to g

b 3.72 g to mg

c 9100 g to kg

d 384 kg to t

Solutions

$$\begin{aligned}\mathbf{a} \quad 4 \text{ kg} &= (4 \times 1000) \text{ g} \\ &= 4000 \text{ g}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 3.72 \text{ g} &= (3.72 \times 1000) \text{ mg} \\ &= 3720 \text{ mg}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad 9100 \text{ g} &= (9100 \div 1000) \text{ kg} \\ &= 9.1 \text{ kg}\end{aligned}$$

$$\begin{aligned}\mathbf{d} \quad 384 \text{ kg} &= (384 \div 1000) \text{ t} \\ &= 0.384 \text{ t}\end{aligned}$$

EG
+S**Example 5**

Use the degrees and minutes key on the calculator to convert:

a 1.25 h to hours and minutes

b 3 h 21 min to hours

Solutions

$$\mathbf{a} \quad \text{Press } 1.25 \text{ [2nd F] [DMS]}. \text{ The display of } 1^{\circ}15' \text{ is then interpreted as } 1 \text{ h } 15 \text{ min.}$$

$$\mathbf{b} \quad \text{Press } 3 \text{ [DMS] } 21 \text{ [DMS] [2nd F] [DMS]}. \text{ Therefore, } 3 \text{ h } 21 \text{ min} = 3.35 \text{ h.}$$

Exercise 5.1**1** Choose the most appropriate unit (mm, cm, m, km) that could be used to measure:**a** the length of a fly**b** the height of a 4-year-old girl**c** the length of a caravan**d** the distance between two towns

- e** the height of a table
g the width of a postage stamp
- f** the distance run in a sprint race
h the distance between two bus depots
- 2** Choose the most appropriate unit (mg, g, kg, t) that could be used to measure the mass of:
- a** an orange **b** a bee's wing **c** a railway carriage
d a television set **e** a clump of hair **f** a calculator
g a baby **h** a truck **i** a box of pencils
- 3** Choose the most appropriate unit (mL, L, kL) that could be used to measure the capacity of:
- a** a glass of water **b** a backyard fountain **c** a swimming pool
d Sydney Harbour **e** a bird bath **f** a tea cup
g a teaspoon **h** the petrol tank of a bus **i** a small dam
- 4** Convert:
- a** 6 km to m **b** 300 cm to m **c** 9 cm to mm **d** 2500 m to km
e 0.46 m to cm **f** 4 mm to cm **g** 178 m to km **h** 2.3 cm to mm
i 0.8 km to m **j** 0.1 m to cm **k** 200 cm to mm **l** 16 m to km
m 30 m to cm **n** 0.07 mm to cm **o** 2 cm to m **p** 0.3 m to km
- 5** Complete each of the following conversions.
- a** 5 m = mm **b** 2 km = cm **c** 4000 mm = m
d 900 000 cm = km **e** 3.8 km = mm **f** 1 650 000 mm = km
- 6** Complete each of these conversions.
- a** 4 g = mg **b** 8000 kg = t **c** 1.5 kg = g
d 14 500 mg = g **e** 2790 g = kg **f** 70 000 kg = t
g 12.4 g = mg **h** 1.82 kg = g **i** 375 g = kg
j 140 mg = g **k** 0.87 t = kg **l** 0.046 kg = g
m 20 kg = t **n** 6 mg = g **o** 0.005 47 g = mg
- 7** Complete each of these conversions.
- a** 4 L = mL **b** 3000 mL = L **c** 8 kL = L
d 7500 L = kL **e** 2.4 L = mL **f** 1950 L = kL
g 3610 mL = L **h** 5.07 kL = L **i** 0.73 L = mL
j 195 L = kL **k** 11 mL = L **l** 0.0068 kL = L
- 8** Convert:
- a** 1 min = s **b** 1 h = min **c** 1 day = h
d 3 h = min **e** 2 days = h **f** 5 min = s
g $\frac{1}{2}$ h = min **h** $\frac{3}{4}$ min = s **i** $\frac{2}{3}$ day = h
j $1\frac{1}{2}$ min = s **k** $3\frac{1}{4}$ days = h **l** $2\frac{5}{6}$ h = min
m 180 s = min **n** 72 h = days **o** 420 s = min
p 90 s = min **q** 75 min = h **r** 32 h = days
- 9 a** Explain why 1.25 h does not mean 1 h 25 min.
b Express 1.25 h in hours and minutes.

■ Consolidation

- 10** Express each time in minutes and seconds, without the use of a calculator.
- a** 1.1 min **b** 2.4 min **c** 3.25 min **d** 4.75 min
- 11** Use the degrees and minutes key on your calculator to express each time in hours and minutes.
- a** 1.9 h **b** 0.35 h **c** 3.45 h **d** 2.8 h
- 12** Use the degrees and minutes key on your calculator to express each time in hours.
- a** 1 h 24 min **b** 2 h 42 min **c** 4 h 45 min **d** 36 min
- 13** Express each of these in 24-hour time.
- a** 2 am **b** 7 pm **c** 12 midnight **d** 12 noon
e 4:30 am **f** 1:45 pm **g** 11:59 pm **h** 12:24 am
- 14** Express each of these in standard 12-hour time.
- a** 04:00 **b** 07:30 **c** 13:00 **d** 15:20
e 08:15 **f** 16:35 **g** 20:00 **h** 23:47
- 15** Simplify, giving the answer in metres.
- a** 1 m + 37 cm + 9 mm **b** 3.6 m + 228 cm + 15 mm
c 12.7 km + 83 m + 54 cm **d** 1 km + 455 m + 38 cm
- 16**
- a** Which distance is greater, 15.8 m or 14 950 mm, and by how many metres?
 - b** How many toothpicks of length 65 mm can be cut from a 1.3 m strip of wood?
 - c** A snooker table is to have 6 legs made and each leg is to be 72 cm long. How many metres of wood are needed?
 - d** How many laps of a 400 m running track must an athlete complete in order to finish a 10 km run?
 - e** From a 3.6 m piece of timber, 5 pieces of equal length are cut, leaving 28 cm. What lengths of timber were cut?
 - f** The average length of Lucy's walking stride is 38 cm. How far, in kilometres, would Lucy walk if she took 9500 strides?
 - g** Fourteen cars each of width 1.65 m are parked side by side in a car park. The distance between each car is 85 cm. Find, in metres, the total distance taken up by the cars.
- 17**
- a** Find, in kilograms, the mass of 24 tins of soup, if each tin has a mass of 535 g.
 - b** The total mass of 8 small cars in a shipping container is 7.6 t. What is the mass, in kilograms, of each car?
 - c** Find the mass, in kilograms, of 3000 pumpkin seeds if each seed weighs 450 mg.
 - d** A bunch of 64 grapes has a mass of 430 g. Find the average mass of each grape, correct to 1 decimal place.
 - e** How many 225 g bags of lollies can be filled completely from a container that holds 12 kg of lollies?
 - f** A builder wants to construct a brick wall consisting of 18 layers, with 15 bricks in each layer. Calculate, in kilograms, the total mass of bricks needed for the job if each brick weighs 2150 g.

- g** A box of 15 fresh pineapples has a mass of 58.6 kg and each pineapple has a mass of 3680 g. Determine the mass of the box when empty. Give your answer in kilograms.
- h** If 7 containers of wheat have a total mass of 980 kg, find, in tonnes, the mass of 19 containers of wheat.
- 18 a** A 300 mL bottle of salad dressing contains enough dressing for 15 serves. What is the serving size?
- b** **i** How many 250 mL glasses can be filled from a juice container that holds 20 L?
ii How many 300 mL glasses can be filled completely?
- c** Cary purchased a 250 mL bottle of cough medicine. The adult dosage is 10 mL, three times a day. How many full days will the cough medicine last?
- d** The local council pool has a capacity of 1500 kL. During a hot spell, 15 000 L of water was lost due to evaporation. How many kilolitres of water remain?
- e** The dam on old Henry's farm has a capacity of 8.3 ML. How many litres is this? (1 ML = 1000 kL)
- f** A recipe requires $\frac{1}{2}$ cup of water for each person. Find, in litres, the amount of water that is needed for 11 people if 1 metric cup is equivalent to 250 mL.
- g** **i** Amber's garden tap is dripping at the rate of 16 drops per minute, with each drop of water having a volume of 0.5 mL. How many litres of water will be lost in one day?
ii If on a subsequent day the tap drips at the rate of 10 drops per minute and loses 36 L over the course of a day, find the volume of water in each drop.
- 19** How long is it, in hours and minutes, from:
- a** 8 am to 2:15 pm? **b** 4:45 am to 10:30 am? **c** 11:19 am to 10:08 pm?
- 20** Calculate the time difference between:
- a** 7:20 pm Saturday and 2:05 am Sunday **b** 9:12 am Thursday and 12 noon Friday
- 21** What will the time be:
- a** 9 h 26 min after 12:57 am? **b** 3 h 10 min before 8:05 pm?

■ Further applications

- 22** The carat is a unit of mass that is used to measure precious stones and some expensive metals such as gold. If a certain amount of gold is described as 24 carat, it means that the entire mass is composed of 100% pure gold with no impurities. The purity of the gold can be worked out by expressing the number of carats as a fraction of 24. For example, a 10 kg ingot of 12 carat gold is only 50% pure, because 12 is one-half or 50% of 24. Therefore, in this 10 kg ingot, 5 kg of the mass is pure gold and the other 5 kg is made up of impurities (such as other metals).
- How many grams of gold are there in:
- a** a 1 kg ingot of 12 carat gold? **b** a 2 kg ingot of 18 carat gold?
c a 1.5 kg ingot of 8 carat gold? **d** a 6 kg ingot of 14 carat gold?

5.2 Accuracy and precision

It is not possible to measure any length, mass, time, temperature or other quantity exactly. The value read off a measuring instrument is affected by physical factors, such as the thickness of the ink that is used in the markings on the instrument. However, we know that the exact value of the quantity being measured lies between two adjacent markings. The **accuracy** of a measurement refers to how close the reading is to the exact value of the quantity. The degree of accuracy in any measurement depends very much on the accuracy of the measuring instrument that is being used. Before measuring something, we need to consider the following questions:

- 1 Why are we conducting the measurement?
- 2 How precise does the measurement have to be?
- 3 What measuring instrument would be the most appropriate to use?

The **precision** of a measuring instrument refers to the smallest unit that is marked on it. For example, a metre ruler that is marked in 1 cm intervals has a precision of 1 cm. It is not possible to be more precise than the smallest unit that is marked on the instrument.

- ⊙ All measurements are accurate to within $\pm \frac{1}{2}$ of the smallest unit marked on the measuring instrument.
- ⊙ The smaller limit of accuracy is called the lower bound.
- ⊙ The greater limit of accuracy is called the upper bound.
- ⊙ The exact value is greater than or equal to the lower bound but less than the upper bound.

Example 1

State the limits of accuracy for each of the following measurements.

- a The temperature of a sick child is 39°C , correct to the nearest 1°C .
- b The height of a man is 180 cm, correct to the nearest 10 cm.

Solutions

- a The temperature is given correct to the nearest 1°C , so the possible error is $\pm(\frac{1}{2} \times 1^{\circ}\text{C})$, i.e. $\pm 0.5^{\circ}\text{C}$.

Therefore, the limits of accuracy are $39 \pm 0.5^{\circ}\text{C}$, i.e. 38.5°C and 39.5°C . The actual temperature of the child must lie between 38.5°C and 39.5°C .

- b The height is given correct to the nearest 10 cm, so the possible error is $\pm(\frac{1}{2} \times 10 \text{ cm})$, i.e. $\pm 5 \text{ cm}$.

Therefore, the limits of accuracy are $180 \pm 5 \text{ cm}$, i.e. 175 cm and 185 cm. The actual height of the man must lie between 175 cm and 185 cm.

**Example 2**

State the upper and lower bounds for each measurement.

a 6.2 kg

b 4.18 m

Solutions

- a** The mass is given correct to the nearest tenth of a kilogram, so the possible error is $\pm(\frac{1}{2} \times 0.1 \text{ kg})$, i.e. $\pm 0.05 \text{ kg}$.

Therefore, the limits of accuracy are $6.2 \pm 0.05 \text{ kg}$.

$$\begin{aligned} \text{Lower bound} &= 6.2 \text{ kg} - 0.05 \text{ kg} \\ &= 6.15 \text{ kg} \end{aligned}$$

$$\begin{aligned} \text{Upper bound} &= 6.2 \text{ kg} + 0.05 \text{ kg} \\ &= 6.25 \text{ kg} \end{aligned}$$

- b** The length is given correct to the nearest hundredth of a metre, so the possible error is $\pm(\frac{1}{2} \times 0.01 \text{ m})$, i.e. $\pm 0.005 \text{ m}$.

Therefore, the limits of accuracy are $4.18 \pm 0.005 \text{ m}$.

$$\begin{aligned} \text{Lower bound} &= 4.18 \text{ m} - 0.005 \text{ m} \\ &= 4.175 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Upper bound} &= 4.18 \text{ m} + 0.005 \text{ m} \\ &= 4.185 \text{ m} \end{aligned}$$

Exercise 5.2

- Within what limits are the following instruments accurate?
 - A dressmaker's tape marked in millimetres.
 - A speedometer with 20 km/h markings.
 - A metre ruler marked in centimetres.
 - A trundle wheel with a circumference of 1 m and no other markings.
 - A clock with only hours marked.
 - A flood sign marked in intervals of 50 cm.
 - A thermometer marked in intervals of 2°C .
 - A measuring tape with 1 cm markings.
 - A set of balance scales marked in intervals of 1 kg.
 - An altimeter marked in intervals of 1250 m.
- The length of the hem of a pair of trousers is measured as 4 cm, correct to the nearest centimetre.
 - What is the lower bound for this measurement?
 - What is the upper bound?
- The size of a television screen is quoted as being 64 cm.
 - Between what limits does the actual size lie?
 - Which one of these limits could not possibly be the size of the screen?

Consolidation

- The following measurements are given correct to the nearest kilogram. Between what limits does the actual mass lie?
 - 2 kg
 - 9 kg
 - 14 kg
 - 48 kg

- 5 The following measurements are given correct to the nearest 10 cm. Between what limits does the actual length lie?
- a 30 cm b 50 cm c 120 cm d 260 cm
- 6 Peter asked his English teacher, 'how many pages are there in our next novel?'. His teacher replied, 'there are about 300 pages'. Between what limits could the actual number of pages lie if the teacher's estimate was given correct to the nearest:
- a 100 pages? b 50 pages? c 10 pages?
- 7 In the year 2000 an archaeologist carbon dated a piece of pottery and found that it was approximately 1800 years old, correct to the nearest 100 years. Between what years was the pottery probably made?
- 8 The capacity of a rainwater tank is 8700 L, correct to the nearest 100 L. Within what limits does the actual capacity of the tank lie?
- 9 State the lower and upper bounds for each of the following.
- a The height of a child is 140 cm, correct to the nearest 10 cm.
 b The time required to fly from Sydney to Perth is 3 h 50 min, correct to the nearest 20 min.
 c The capacity of a swimming pool is 1500 kL, correct to the nearest 50 kL.
 d The mass of a woman is 84 kg, correct to the nearest kilogram.
 e The length of a beetle is 12 mm, correct to the nearest millimetre.
 f The mass of a cargo container is 46 t, correct to the nearest tonne.
 g The length of an envelope is 16 cm, correct to the nearest centimetre.
 h The height of a building is 135 m, correct to the nearest metre.
- 10 a The masses of a set of objects are given as 15.2 g, 13.7 g, 9.4 g and 20.1 g. What do you think were the smallest intervals on the measuring instrument used?
 b Would it be appropriate to give a measurement of 10.63 g using this instrument?
- 11 State the lower and upper bounds for each of these measurements.
- | | | | |
|----------|-----------|-----------|-----------|
| a 7 mm | b 2 cm | c 8 km | d 15 m |
| e 1.6 cm | f 4.3 km | g 6.8 m | h 12.1 mm |
| i 2.8 m | j 17.3 mm | k 23.6 cm | l 30.0 km |
- 12 State the limits between which each measurement lies.
- | | | | |
|----------|----------|------------|-----------|
| a 6 kg | b 6.5 L | c 22.36 t | d 25°C |
| e 5.1 mL | f 0.8 kL | g 160.4 cm | h 11.7 g |
| i 10.0 L | j 103 dB | k 4.9 kg | l 0.05 Hz |

■ Further applications

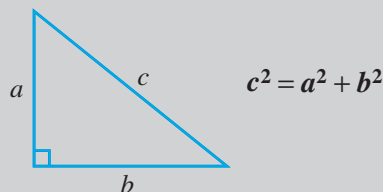
- 13 A rectangular piece of glass is to be cut with dimensions 72 cm × 85 cm.
- a Find the greatest possible length and width.
 b Find the least possible length and width.
 c Within what limits should the area of the glass lie?



5.3 Pythagoras' theorem

Pythagoras' theorem describes the relationship between the lengths of the sides in any right-angled triangle.

In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

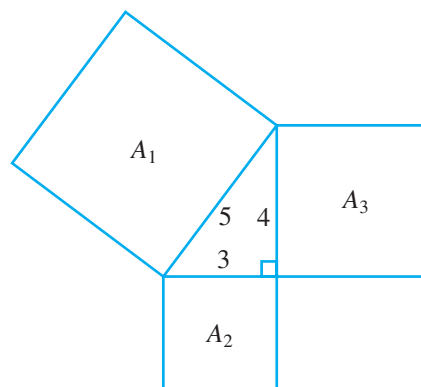


For example, in this diagram we can see that the areas are:

$$\begin{array}{lll} A_1 = 5^2 & A_2 = 3^2 & A_3 = 4^2 \\ = 25 & = 9 & = 16 \end{array}$$

Now, $25 = 9 + 16$

$$\therefore A_1 = A_2 + A_3$$



A **Pythagorean triad** is a set of three positive integers, a , b and c , for which $c^2 = a^2 + b^2$. That is, they are integral dimensions of a right-angled triangle. All multiples of Pythagorean triads are also triads. For example, we know that $\{3, 4, 5\}$ is a Pythagorean triad. Hence, $\{6, 8, 10\}$, $\{9, 12, 15\}$ and $\{12, 16, 20\}$ are also Pythagorean triads.

The converse of Pythagoras' theorem

The *converse* of Pythagoras' theorem can be used to show that a triangle is right angled.

If the square on one side of a triangle is equal to the sum of the squares on the other two sides, then the angle between the two shorter sides is a right angle.

Example 1

State Pythagoras' theorem for this triangle using:

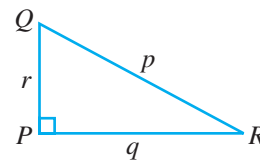
a side notation

b angle notation

Solutions

a $p^2 = q^2 + r^2$

b $QR^2 = PQ^2 + PR^2$



**Example 2**

Determine whether the following are Pythagorean triads.

a 5, 12, 13

b 7, 8, 9

Solutions

a $13^2 = 5^2 + 12^2$

$$169 = 25 + 144$$

$$169 = 169$$

This is a true statement,

$\therefore$ 5, 12, 13 is a Pythagorean triad.

b $9^2 = 7^2 + 8^2$

$$81 = 49 + 64$$

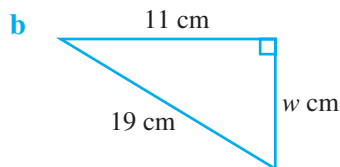
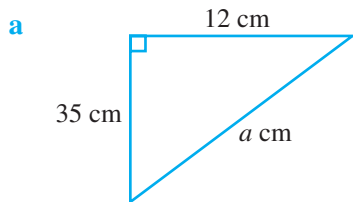
$$81 = 113$$

This is not a true statement,

$\therefore$ 7, 8, 9 is not a Pythagorean triad.

**Example 3**

Find the value of the pronumeral in each diagram. Give your answer correct to 1 decimal place where necessary.

**Solutions**

a $a^2 = 12^2 + 35^2$

$$= 144 + 1225$$

$$= 1369$$

$$\therefore a = \sqrt{1369}$$

$$= 37$$

b $w^2 + 11^2 = 19^2$

$$w^2 + 121 = 361$$

$$\begin{array}{r} -121 \\ -121 \end{array}$$

$$w^2 = 240$$

$$\therefore w = \sqrt{240}$$

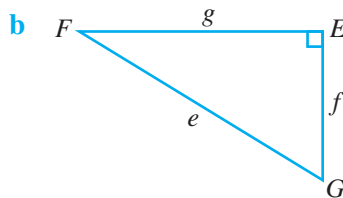
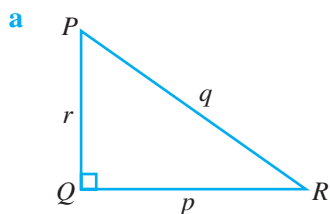
$$= 15.5 \text{ (1 decimal place)}$$

Exercise 5.3

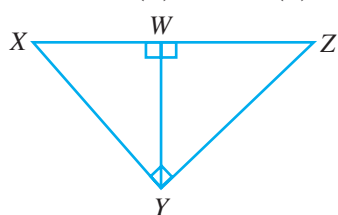
1 Write down Pythagoras' theorem for each triangle using:

i side notation

ii angle notation

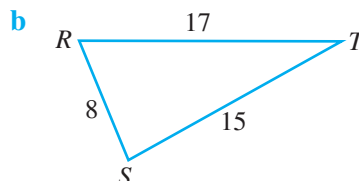
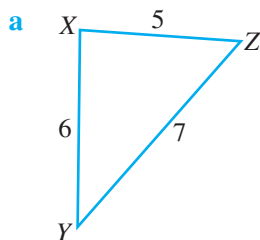


- 2 Write true (T) or false (F) for each of these.



- a** $XY^2 = XW^2 + WY^2$ **b** $XW^2 = WZ^2 + XZ^2$
c $YZ^2 = ZW^2 + WY^2$ **d** $WY^2 = XW^2 + XY^2$
e $YZ^2 = WY^2 + XZ^2$ **f** $XZ^2 = XY^2 + YZ^2$

- 3 Use Pythagoras' theorem to determine which of the following triangles, not drawn to scale, are right-angled. If the triangle is right-angled, name the hypotenuse.

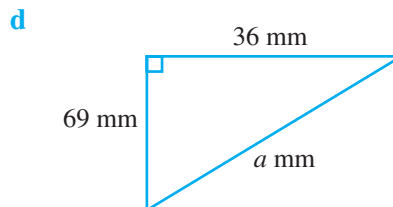
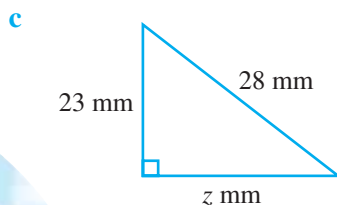
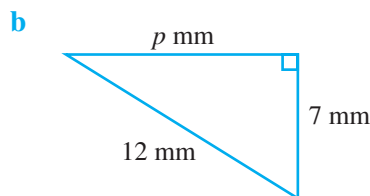
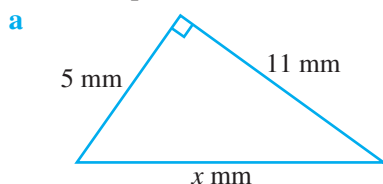


- 4 Which of the following are Pythagorean triads?

- a** 6, 8, 10 **b** 3, 5, 7 **c** 12, 35, 37 **d** 10, 15, 20

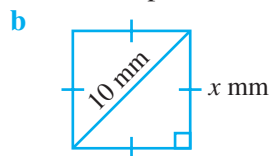
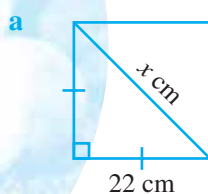
- 5 Explain, without calculation, why 2, 3, 6 could not be a Pythagorean triad.

- 6 Find the value of the pronumeral in each of the following. Give your answers correct to 1 decimal place.

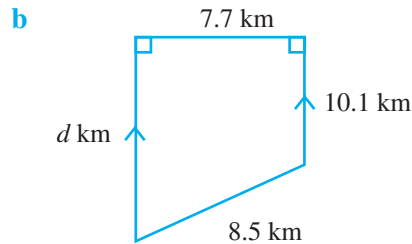
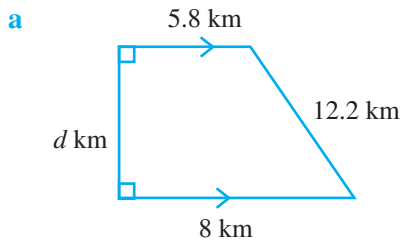


Consolidation

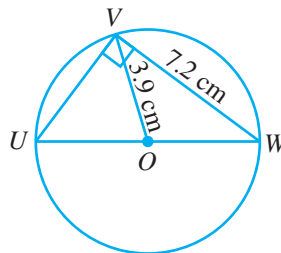
- 7 Find the value of x in each of these, correct to 1 decimal place.



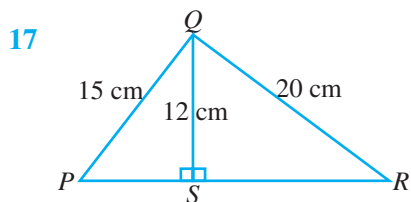
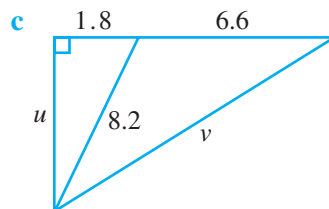
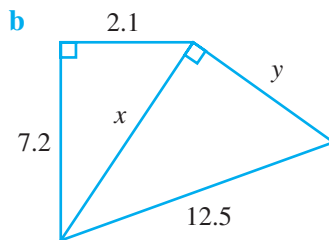
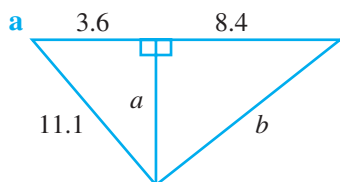
- 8 a In $\triangle QRS$, $\angle R = 90^\circ$, $QR = 25$ km and $RS = 38$ km. Find the length of QS , correct to 4 significant figures.
- b In $\triangle ABC$, $\angle A = 90^\circ$, $AC = 41$ cm and $BC = 75$ cm. Find the length of AB , correct to the nearest centimetre.
- 9 Find, correct to 1 decimal place, the possible lengths of the third side of a right-angled triangle in which two of the sides measure 6 m and 14.4 m.
- 10 a A ladder reaches 7.5 m up a wall and the foot of the ladder is 2.4 m away from the base of the wall. Find the length of the ladder correct to the nearest centimetre.
- b A ladder of length 4.5 m leans against a wall. The foot of the ladder is 1.2 m away from the base of the wall. How far up the wall does the ladder reach, correct to 1 decimal place?
- 11 a A ship sailed 6 km due north, then changed course and sailed 14 km due east. How far is the ship from its starting point?
- b Emilia drove due east from J to K , then turned and drove 15 km due south to L . If L is 48 km from J , find how far east Emilia drove.
- 12 While out orienteering, a group of students walked 350 m due west, 290 m due north then 560 m due east. How far, to the nearest metre, are the students from their starting point?
- 13 a A non-right-angled isosceles triangle has a base length of 66 cm and a height of 56 cm. How long are the equal sides?
- b A non-right-angled isosceles triangle has equal sides of length 75 mm and a height of 21 mm. How long is the base?
- 14 Find the value of d in each figure.



- 15 Find the length of the chord UV in this circle.

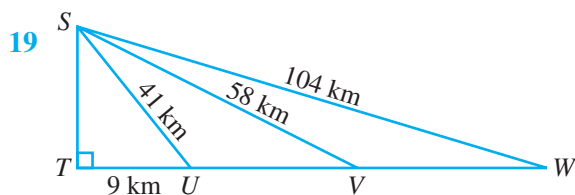
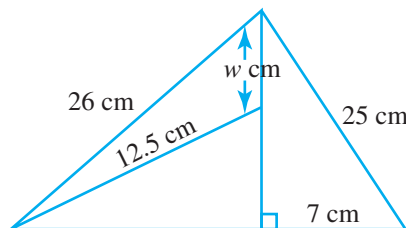


- 16 Find the values of the pronumerals in each of these. Answer correct to 1 decimal place where necessary.

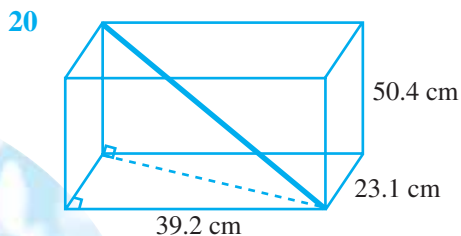


In $\triangle PQR$, S is a point on PR such that QS is perpendicular to PR .
If $QS = 12$ cm, $PQ = 15$ cm and $QR = 20$ cm, prove that $\angle PQR$ is a right angle.

- 18 Find the value of w .

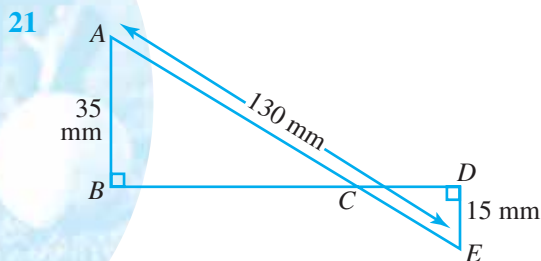


In the diagram, $TU = 9$ km, $SU = 41$ km, $SV = 58$ km and $SW = 104$ km.
Find the length of VW .



Find the length of the longest rod that will fit completely inside this rectangular prism.

Further applications



In the diagram, $AB = 35$ mm, $DE = 15$ mm and $AE = 130$ mm.

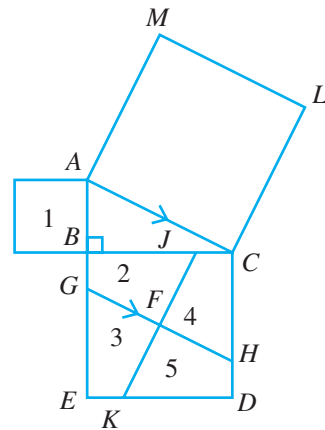
- a** Find the length of BD .
b Find the length of CD if $AC = 91$ mm.

- 22 If a, b, c are the sides of a right-angled triangle and c is the hypotenuse, prove that any multiple of a, b, c will also be the sides of a right-angled triangle.
- 23 The expressions $p^2 - q^2, 2pq, p^2 + q^2$, where p and q are positive integers and $p > q$ can be used to generate Pythagorean triads. By substituting values for p and q , find at least 5 Pythagorean triads.
- 24 Two air force jets took off from the same airport at 3 pm. One jet flew due south at 320 km/h while the other flew due west at 370 km/h. How far apart are the jets at 5:30 pm if each maintains the same course and speed? (Answer correct to the nearest whole kilometre.)
- 25 The Mountain Top Ski Resort is situated on top of a 3.6 km high mountain. A cable car from the resort travels along the cable at 5 m/s and takes 13 min to reach the ground station.
- How long is the cable?
 - How far is the ground station from the foot of the mountain?

TRY THIS Pythagorean proof by Perigal

The mathematician Perigal produced this proof for Pythagoras' theorem. Try it!

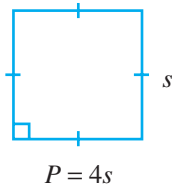
- Draw a right-angled triangle, ABC .
- Draw squares on all three sides.
- Find the centre of square $BCDE$. Call it F .
- Draw a line GH such that GH is parallel to AC and passes through F .
- Draw a line JK such that JK is perpendicular to GH .
- Cut pieces 1, 2, 3, 4 and 5 out and use them to cover totally square $ACLM$.
- Can you find your own way of proving Pythagoras' theorem?



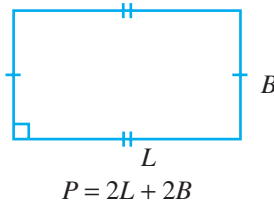
5.4 Perimeter

The **perimeter** of a two-dimensional figure is the total distance around its boundary. The formulae below can also be used to find the perimeter of some common figures.

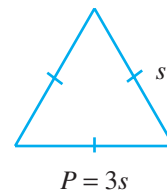
Square



Rectangle



Equilateral triangle

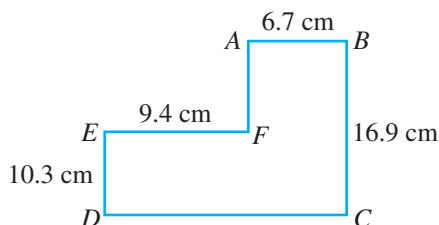


To find the perimeter of general figures:

- ⊙ find the lengths of any unknown sides
- ⊙ add the lengths of all sides that form part of the boundary.

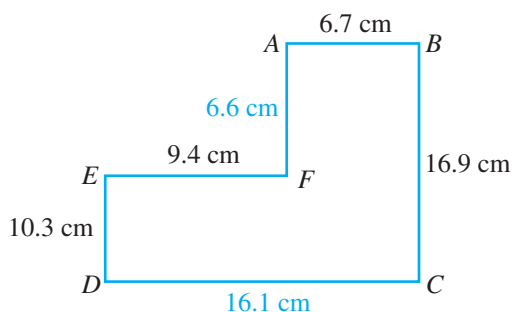
Example 1

Find the total perimeter of this figure.



Solution

- i $AF = BC - ED$
 $= 16.9 - 10.3$
 $= 6.6 \text{ cm}$
- ii $DC = EF + AB$
 $= 9.4 + 6.7$
 $= 16.1 \text{ cm}$
- iii $P = 6.7 + 16.9 + 16.1 + 10.3 + 9.4 + 6.6$
 $= 66 \text{ cm}$

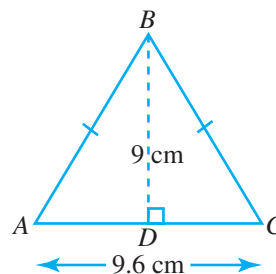


Example 2

- a Find the length of AB .
- b Hence, find the perimeter of $\triangle ABC$.

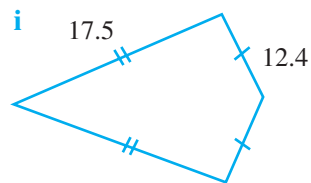
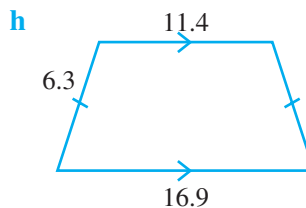
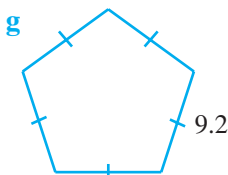
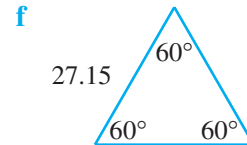
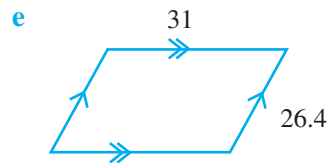
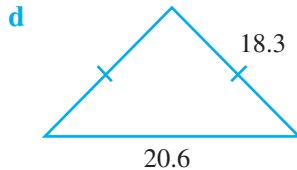
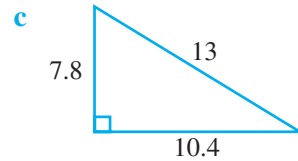
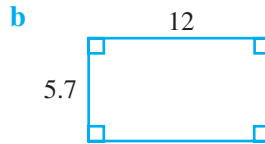
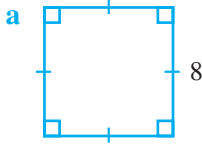
Solutions

- i $AD = 9.6 \div 2$
 $= 4.8 \text{ cm}$
- ii $AB^2 = AD^2 + BD^2$
 $= 4.8^2 + 9^2$
 $= 104.04$
 $\therefore AB = \sqrt{104.04}$
 $= 10.2 \text{ cm}$
- b Perimeter $= AB + BC + AC$
 $= 10.2 + 10.2 + 9.6$
 $= 30 \text{ cm}$



Exercise 5.4

1 Find the perimeter of each figure. (All dimensions are in cm.)



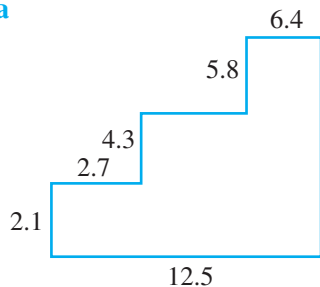
- 2 **a** Find the perimeter of a rhombus whose sides are 14.8 mm.
b A rectangle has a length of 10 cm and a width of 6.5 cm. Find the perimeter.
c A regular octagon has sides of length 7.1 m. What is its perimeter?
d An isosceles triangle has a base of length 19.4 cm and equal sides of length 23.55 cm. Find its perimeter.

Consolidation

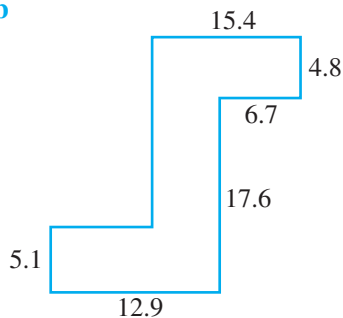
- 3 **a** A square has a perimeter of 52 mm. How long are the sides?
b A regular dodecagon has a perimeter of 105 mm. Find its side length.
c An isosceles triangle has a base length of 16.2 cm and a perimeter of 35.8 cm. What length are the equal sides?
d A parallelogram has a perimeter of 48 m and the longer parallel sides each have a length of 15 m. How long are the shorter sides?
- 4 **a** Find the side length of a square whose perimeter is equal to that of an equilateral triangle with sides 12 cm.
b Find the width of a rectangle whose length is 18.5 cm and perimeter is the same as that of a regular hexagon of side length 9 cm.
c Find the perimeter of a rhombus whose sides are the same length as those of a regular heptagon whose perimeter is 56 cm.

- 5 Calculate the total perimeter of each figure. All angles are right angles and all measurements are in millimetres.

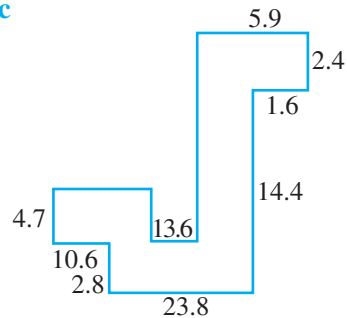
a



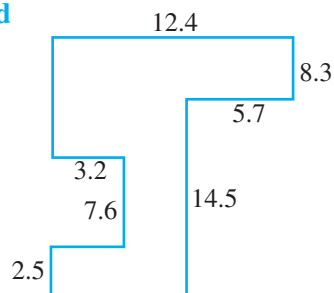
b



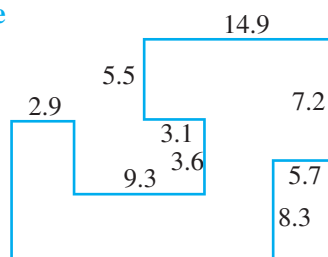
c



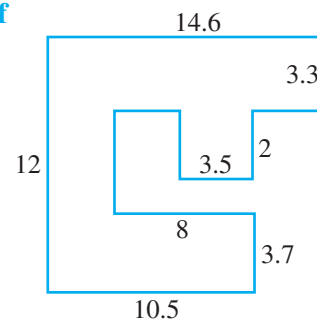
d



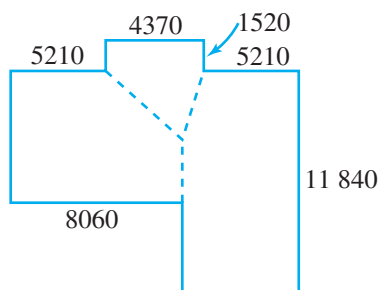
e



f



- 6 An athletics field is 130 m long and 45 m wide. How far, in kilometres, will an athlete run if she completes 15 laps of the field?
- 7 A frame 3.5 cm wide is made to surround a print measuring 45 cm by 18 cm. Find the perimeter of the framed print.
- 8 Find the cost to replace the guttering around the roof of this house at \$36.25 per metre. (All measurements are in millimetres)



9



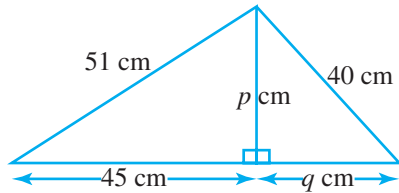
The frieze shown is to be placed around the top of the walls in Tamara's bedroom, which measures 4.2 m by 2.75 m. The frieze is sold in 4 m rolls at \$16.80 per roll.

- a What length of frieze is required?
- b How many rolls of frieze are needed?
- c Calculate the total cost.

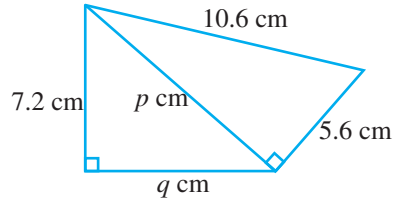
- 10 Rebecca walks once around the boundary of a square park every morning. Her average walking speed is 3.2 km/h and it takes her 45 min to complete one lap of the park. Find, in metres, the side length of the park.

- 11 Find values for p and q , then find the perimeter of each figure.

a



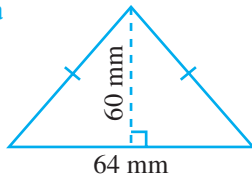
b



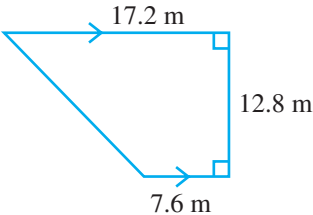
- 12 a A rectangle has a perimeter of 51 mm and a length of 18 mm. Find the length of its diagonals.
 b A square has a perimeter of 12 m. Find, correct to 1 decimal place, the length of its diagonals.

- 13 Find the perimeter of each figure.

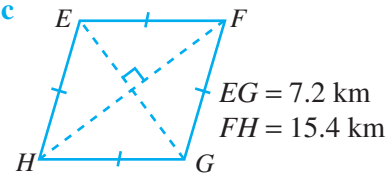
a



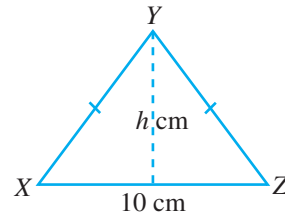
b



c



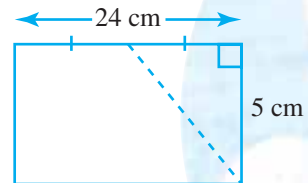
- 14 In the isosceles triangle XYZ , $XY = YZ$ and $XZ = 10$ cm. The perimeter of the triangle is 36 cm. Find the height h cm.



- 15 Find, correct to 3 significant figures, the altitude of an equilateral triangle whose perimeter is 300 mm.
 16 The rhombus $QRST$ has a perimeter of 388 mm and the diagonal $QS = 130$ mm. Find the length of the other diagonal RT .

Further applications

- 17 A rectangle with dimensions 5 cm by 24 cm is cut into a triangle and a trapezium as shown. The pieces are then rearranged to form a right-angled triangle. How much greater is the perimeter of the resulting triangle than that of the rectangle?



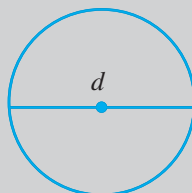
- 18 Three circles with centres O , P and Q are drawn so that each circle touches the other two. Two of these circles have diameters of length 12 cm and 18 cm. The perimeter of $\triangle OPQ$ is 46 cm. Find the length of the radius in the third circle.
- 19 The length and perimeter of a rectangle are in the ratio 5 : 16. Find the length and perimeter of the rectangle if the width is 72 cm.
- 20 The perpendicular sides of a right-angled triangle have lengths in the ratio 3 : 4. The perimeter of the triangle is 168 cm. Find the lengths of the sides. (HINT: Let the perpendicular sides have lengths $3x$ and $4x$, then show that the length of the hypotenuse is $5x$.)

5.5 Circumference

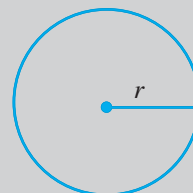
Circumference

Circumference is the perimeter of a closed curve. The irrational number π (pronounced pi) is defined as the length of the circumference (C) divided by the length of the diameter (d). That is, $\pi = \frac{C}{d}$. This ratio is the same for any circle, no matter the size, and its approximate value is 3.142 (3 decimal places).

The circumference of a circle with diameter d units or radius r units is given by:



$$C = \pi d$$

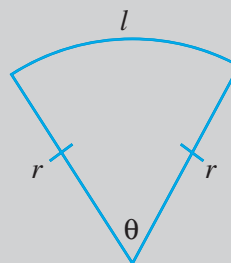


$$C = 2\pi r$$

Length of an arc

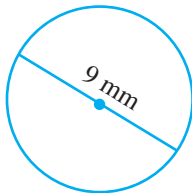
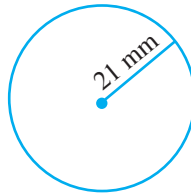
The length of an arc which subtends an angle θ at the centre of a sector is given by:

$$l = 2\pi r \times \frac{\theta}{360}$$



EG
+
S**Example 1**

Calculate the circumference of each circle, correct to 1 decimal place.

a**b****Solutions**

$$\begin{aligned}\mathbf{a} \quad C &= \pi d \\ &= \pi \times 9 \\ &= 28.3 \text{ mm (1 decimal place)}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad C &= 2\pi r \\ &= 2 \times \pi \times 21 \\ &= 131.9 \text{ mm (1 decimal place)}\end{aligned}$$

NOTE: In these examples, the exact circumferences would be 9π mm and 42π mm.EG
+
S**Example 2**

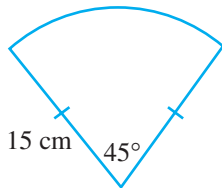
Find, correct to 2 decimal places, the length of the diameter in a circle whose circumference is 35 m.

Solution

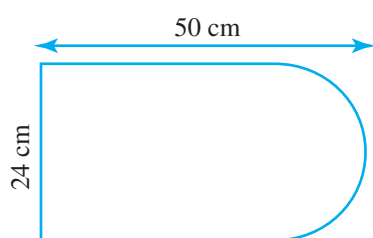
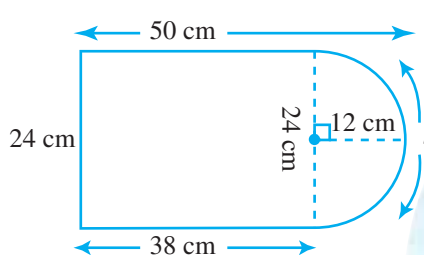
$$\begin{aligned}C &= \pi d \\ 35 &= \pi d \\ \therefore d &= \frac{35}{\pi} \\ &= 11.14 \text{ m (2 decimal places)}\end{aligned}$$

EG
+
S**Example 3**

Find the total perimeter of each figure, correct to 1 decimal place.

a**Solutions**

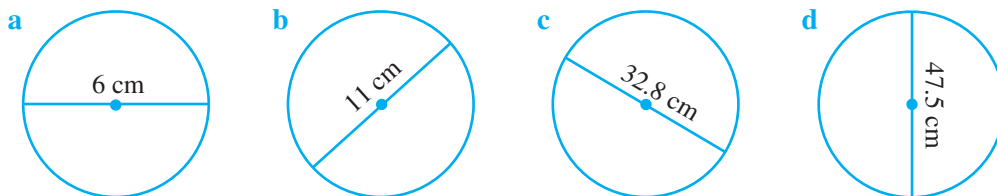
$$\begin{aligned}\mathbf{a} \quad \mathbf{i} \quad l &= 2\pi r \times \frac{\theta}{360} \\ &= 2 \times \pi \times 15 \times \frac{45}{360} \\ &= 11.8 \text{ cm (1 decimal place)} \\ \mathbf{ii} \quad P &= 11.8 + 15 + 15 \\ &= 41.8 \text{ cm}\end{aligned}$$

b**b**

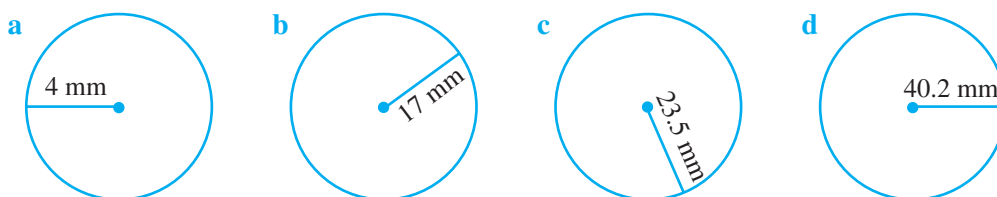
$$\begin{aligned}\mathbf{i} \quad l &= \pi d \times \frac{1}{2} \\ &= \pi \times 24 \times \frac{1}{2} \\ &= 37.7 \text{ cm (1 decimal place)} \\ \mathbf{ii} \quad P &= 37.7 + 24 + 38 + 38 \\ &= 137.7 \text{ cm}\end{aligned}$$

Exercise 5.5

- 1 Find, correct to 1 decimal place, the circumference of each circle using the formula $C = \pi d$.



- 2 Find, correct to 1 decimal place, the circumference of each circle using the formula $C = 2\pi r$.



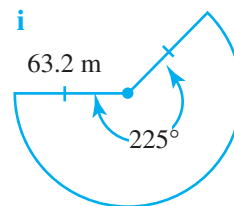
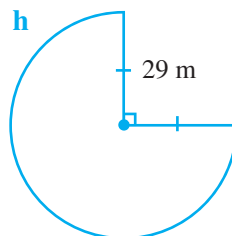
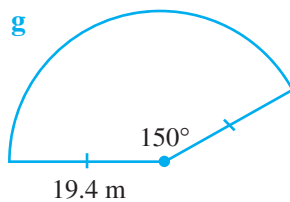
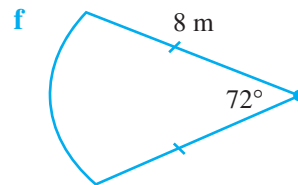
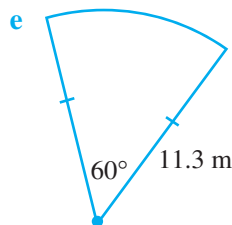
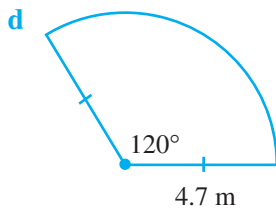
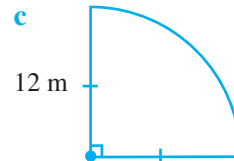
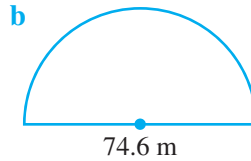
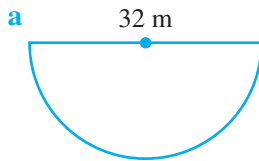
- 3 Find, correct to 3 significant figures, the circumference of a circle whose:

- a** diameter is 9.6 cm **b** diameter is 13.45 m
c radius is 22.7 mm **d** radius is 51.8 cm

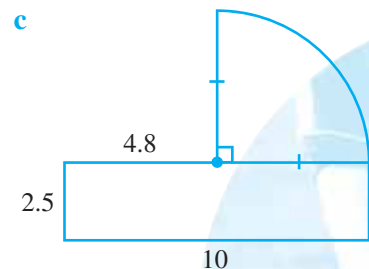
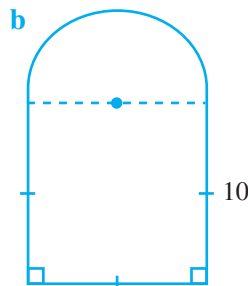
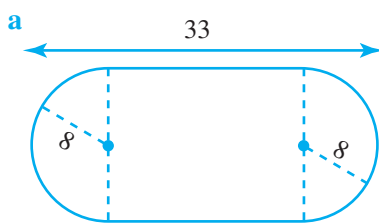
Consolidation

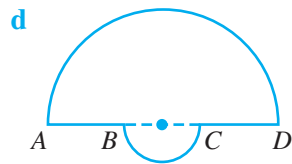
- 4 Find, correct to 1 decimal place, the diameter of a circle whose circumference is:
a 20 m **b** 35.5 m **c** 42.74 m
- 5 Find the length of the radius in a circle whose circumference is 24.1 cm. Give your answer correct to 3 significant figures.
- 6 Determine, in terms of π , the circumference of a circle with:
a diameter 7 cm **b** diameter 13 cm **c** radius 10 cm **d** radius 31 cm
- 7 Write down the length of the diameter and radius in a circle whose circumference is:
a 8π mm **b** 22π mm **c** 36π mm **d** 50π mm
- 8 The wheels in Goran's jeep have a diameter of 42 cm. How far will the jeep travel if the wheels rotate through 50 000 revolutions? Answer correct to the nearest kilometre.
- 9 The diameter of a bicycle wheel is 25 cm. How many complete revolutions are needed for the bicycle to travel a distance of 400 m?
- 10 A 37.7 km motor race is to be run around a circular race track with a radius of 60 m.
a Find the circumference of the track, correct to the nearest metre.
b How many laps of the track must be completed by each car?

- 11** The second hand in a clock is 5 cm long. How far would the tip of the hand travel in 2 h 25 min? Give your answer in metres, correct to the nearest centimetre.
- 12** A wire square of side 32 cm is re-formed into the shape of a circle. Find the length of the radius, correct to 1 decimal place.
- 13** Find, correct to the nearest centimetre, the perimeter of the smallest square into which a circle with circumference 75.4 cm can be inscribed.
- 14** Calculate the total perimeter of each figure, correct to 1 decimal place.

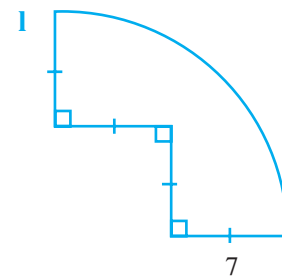
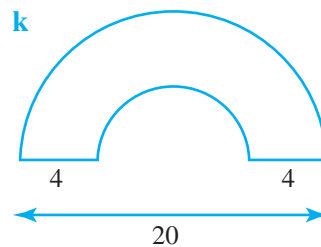
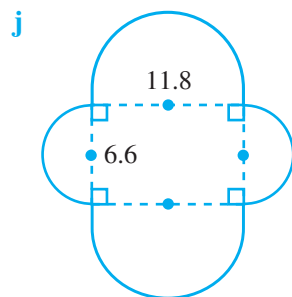
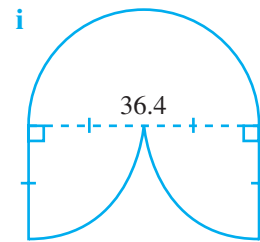
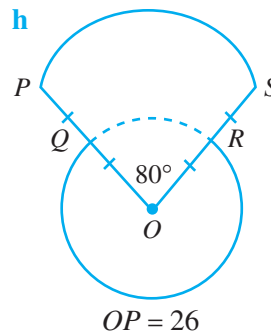
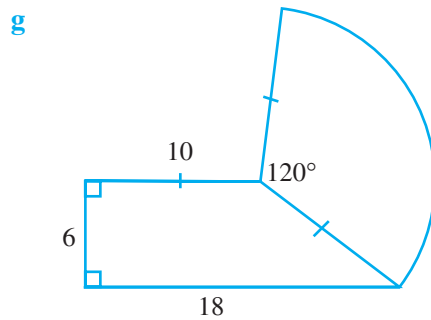
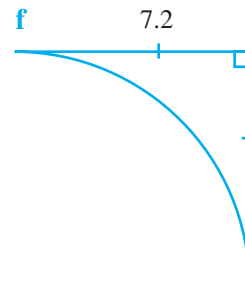
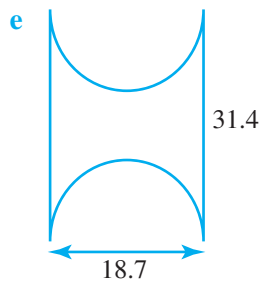


- 15** Find the total perimeter of each figure, correct to 1 decimal place. All measurements are in cm.

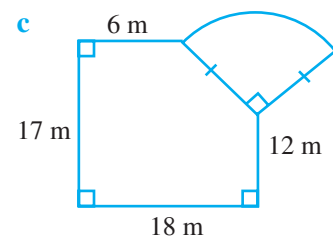
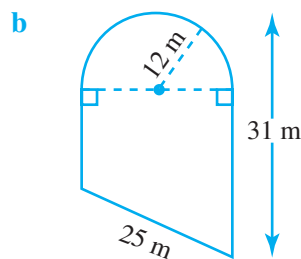
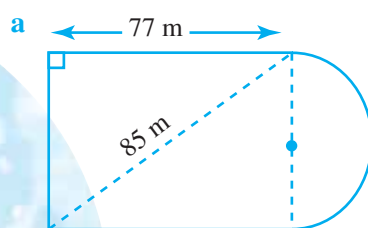




$AB = BC = CD$,
 $AD = 12$



16 Use Pythagoras' theorem to help you find the perimeter of each figure, correct to 1 decimal place.



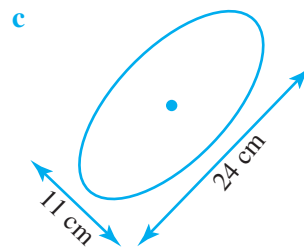
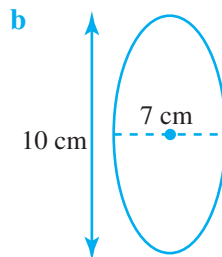
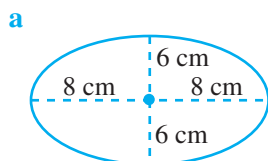
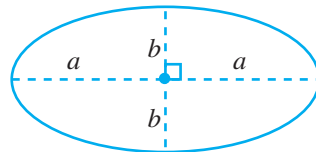
Further applications

17 A chord AB of length 30 cm is drawn inside a circle centre O with circumference 78π cm and parallel to a diameter CD . How far is the chord from the centre of the circle?

18 The circumference of an ellipse can be approximated using

the formula $C \div 2\pi \sqrt{\frac{a^2 + b^2}{2}}$, where a, b are the semi-major and semi-minor axes respectively.

Find, correct to 1 decimal place, the approximate circumference of each ellipse.



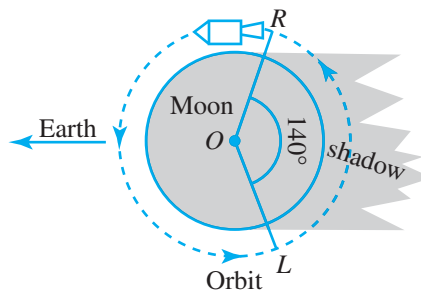
TRY THIS Command module

Humans first stepped on the Moon in 1969. While Neil Armstrong and Edwin Aldrin stepped out of the *Apollo 11* lunar landing craft, Michael Collins remained in the command module circling the Moon. When the command module passed behind the Moon as viewed from the Earth, it lost contact with mission control. This was quite a worrying time. The point L represents where the signal was lost and the point R represents where the signal was regained.

The crucial question is:

For how long was the lunar craft out of contact with the Earth?

Given that the speed of the command module was 5960 km/h, the height of orbit above the Moon's surface was 110 km, the diameter of the Moon is 3477 km and angle $LOR = 140^\circ$, find how long the command module was out of contact with Earth.



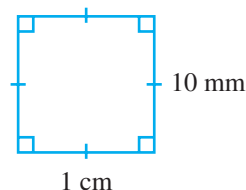
5.6 Converting units of area

Units of area

Consider a square with side length 1 cm, or 10 mm.

Taking the side length as 1 cm, $\text{area} = 1 \text{ cm} \times 1 \text{ cm}$
 $= 1 \text{ cm}^2$

Taking the side length as 10 mm, $\text{area} = 10 \text{ mm} \times 10 \text{ mm}$
 $= 100 \text{ mm}^2$



Equating these results, we see that $1 \text{ cm}^2 = 100 \text{ mm}^2$. Similarly, it can be shown that $1 \text{ m}^2 = 10\,000 \text{ cm}^2$ and $1 \text{ km}^2 = 1\,000\,000 \text{ m}^2$. In each case, we could have converted to the smaller unit of area by multiplying by the square of the linear conversion factor— 10^2 , 100^2 or 1000^2 .

To convert between units of area:

- ☉ multiply or divide by the square of the linear conversion factor.

Units such as square millimetres (mm^2), square centimetres (cm^2) and square metres (m^2) are used to describe small areas. However, large areas are more conveniently measured in square kilometres (km^2) or hectares (ha), where $1 \text{ ha} = 10\,000 \text{ m}^2$.

$$1 \text{ ha} = 10\,000 \text{ m}^2 \text{ (i.e. 1 hectare = 10\,000 square metres)}$$

Example 1

Convert:

a 3.7 m^2 to cm^2

b 45 mm^2 to cm^2

Solutions

a $3.7 \text{ m}^2 = (3.7 \times 100^2) \text{ cm}^2$
 $= (3.7 \times 10\,000) \text{ cm}^2$
 $= 37\,000 \text{ cm}^2$

b $45 \text{ mm}^2 = (45 \div 10^2) \text{ cm}^2$
 $= (45 \div 100) \text{ cm}^2$
 $= 0.45 \text{ cm}^2$

Example 2

Convert:

a 18 ha to m^2

b $53\,000 \text{ m}^2$ to ha

Solutions

a $18 \text{ ha} = (18 \times 10\,000) \text{ m}^2$
 $= 180\,000 \text{ m}^2$

b $53\,000 \text{ m}^2 = (53\,000 \div 10\,000) \text{ ha}$
 $= 5.3 \text{ ha}$

Exercise 5.6

1 Complete each of the following conversions.

a $1 \text{ cm}^2 = \square \text{ mm}^2$

b $1 \text{ m}^2 = \square \text{ cm}^2$

c $1 \text{ km}^2 = \square \text{ m}^2$

d $1 \text{ mm}^2 = \square \text{ cm}^2$

e $1 \text{ cm}^2 = \square \text{ m}^2$

f $1 \text{ m}^2 = \square \text{ km}^2$

2 Convert each of the following by multiplying by the square of the linear conversion factor.

a $3 \text{ m}^2 \text{ to cm}^2$

b $7 \text{ km}^2 \text{ to m}^2$

c $6 \text{ cm}^2 \text{ to mm}^2$

d $1.24 \text{ km}^2 \text{ to m}^2$

e $4.5 \text{ m}^2 \text{ to cm}^2$

f $9.7 \text{ cm}^2 \text{ to mm}^2$

g $2.56 \text{ m}^2 \text{ to cm}^2$

h $18.75 \text{ cm}^2 \text{ to mm}^2$

i $0.16 \text{ km}^2 \text{ to m}^2$

j $0.4 \text{ cm}^2 \text{ to mm}^2$

k $0.057 \text{ km}^2 \text{ to m}^2$

l $0.0013 \text{ m}^2 \text{ to cm}^2$

3 Convert each of the following by dividing by the square of the linear conversion factor.

a $400 \text{ mm}^2 \text{ to cm}^2$

b $90\,000 \text{ cm}^2 \text{ to m}^2$

c $5\,000\,000 \text{ m}^2 \text{ to km}^2$

d $28\,000 \text{ cm}^2 \text{ to m}^2$

e $650 \text{ mm}^2 \text{ to cm}^2$

f $7\,400\,000 \text{ m}^2 \text{ to km}^2$

g $198 \text{ mm}^2 \text{ to cm}^2$

h $3\,280\,000 \text{ m}^2 \text{ to km}^2$

i $43\,390 \text{ cm}^2 \text{ to m}^2$

j $7000 \text{ cm}^2 \text{ to m}^2$

k $15 \text{ mm}^2 \text{ to cm}^2$

l $1\,095\,600 \text{ m}^2 \text{ to km}^2$

Consolidation

4 Copy and complete each of these conversions.

a $5 \text{ cm}^2 = \square \text{ mm}^2$

b $700 \text{ mm}^2 = \square \text{ cm}^2$

c $8 \text{ m}^2 = \square \text{ cm}^2$

d $6\,400\,000 \text{ m}^2 = \square \text{ km}^2$

e $150\,000 \text{ cm}^2 = \square \text{ m}^2$

f $9 \text{ km}^2 = \square \text{ m}^2$

g $56.7 \text{ m}^2 = \square \text{ cm}^2$

h $37 \text{ mm}^2 = \square \text{ cm}^2$

i $0.9 \text{ cm}^2 = \square \text{ mm}^2$

j $816\,000 \text{ m}^2 = \square \text{ km}^2$

k $0.02 \text{ m}^2 = \square \text{ cm}^2$

l $0.045 \text{ km}^2 = \square \text{ m}^2$

m $0.3 \text{ mm}^2 = \square \text{ cm}^2$

n $6 \text{ cm}^2 = \square \text{ m}^2$

o $0.000\,78 \text{ cm}^2 = \square \text{ mm}^2$

5 Complete each of the following conversions.

a $1 \text{ ha} = \square \text{ m}^2$

b $4 \text{ ha} = \square \text{ m}^2$

c $9.5 \text{ ha} = \square \text{ m}^2$

d $20\,000 \text{ m}^2 = \square \text{ ha}$

e $75\,000 \text{ m}^2 = \square \text{ ha}$

f $360\,000 \text{ m}^2 = \square \text{ ha}$

g $0.5 \text{ ha} = \square \text{ m}^2$

h $6000 \text{ m}^2 = \square \text{ ha}$

i $900 \text{ m}^2 = \square \text{ ha}$

j $1.26 \text{ ha} = \square \text{ m}^2$

k $3 \text{ m}^2 = \square \text{ ha}$

l $0.02 \text{ ha} = \square \text{ m}^2$

Further applications

6 Convert:

a $3 \text{ m}^2 \text{ to mm}^2$

b $0.0005 \text{ m}^2 \text{ to mm}^2$

c $169\,000 \text{ mm}^2 \text{ to m}^2$

d $750\,000 \text{ mm}^2 \text{ to m}^2$

e $21\,700\,000 \text{ cm}^2 \text{ to km}^2$

f $0.000\,000\,744 \text{ km}^2 \text{ to mm}^2$

7 Convert:

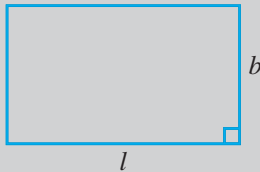
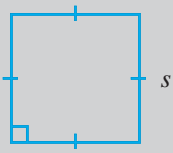
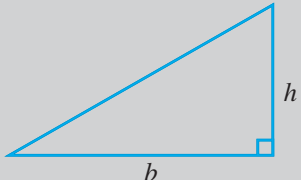
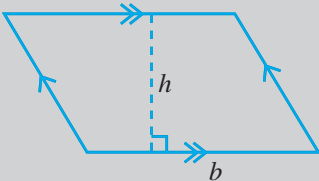
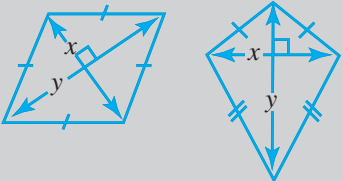
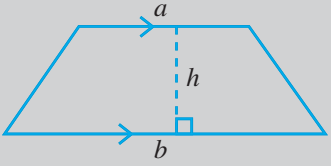
a $0.004\,75 \text{ ha to cm}^2$

b $120\,000 \text{ cm}^2 \text{ to ha}$



5.7 Calculating area

The area of a plane figure is the amount of space enclosed by its boundary. It can be calculated by finding the number of unit squares that will fit inside the figure. For common figures, however, the following formulae may be used.

Rectangle  $A = lb$	Square  $A = s^2$	Triangle  $A = \frac{1}{2}bh$
Parallelogram  $A = bh$	Rhombus and kite  $A = \frac{1}{2}xy$	Trapezium  $A = \frac{h}{2}(a + b)$

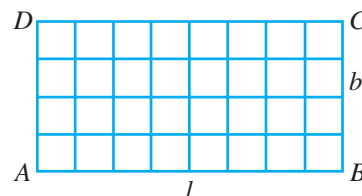
The last three formulae can be derived by dividing each figure into rectangles and triangles.

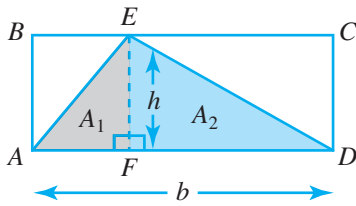
Proofs

1 Rectangle

Divide the rectangle into l columns and b rows, each of width 1 unit. The rectangle has now been divided into small squares, each of which has an area of 1 unit². As there are l squares in each of the b rows, the total number of unit squares inside the rectangle must be $l \times b$.

$\therefore$ The area of rectangle $ABCD = l \times b$.



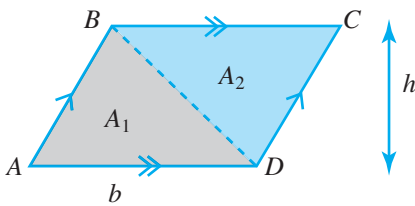
2 Triangle

$$A_1 = \frac{1}{2} \text{ area of } ABEF$$

$$A_2 = \frac{1}{2} \text{ area of } FECD$$

$$\begin{aligned} A_1 + A_2 &= \frac{1}{2} (\text{area of } ABEF + \text{area of } FECD) \\ &= \frac{1}{2} \times \text{area of } ABCD \\ &= \frac{1}{2} \times bh \end{aligned}$$

$$\therefore \text{area of } \triangle AED = \frac{1}{2}bh$$

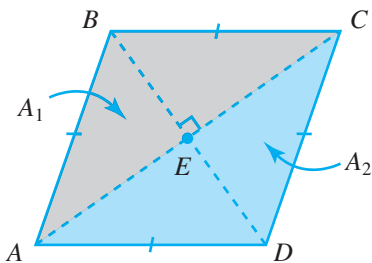
3 Parallelogram

$$\text{Area} = A_1 + A_2$$

$$= \frac{bh}{2} + \frac{bh}{2}$$

$$= \frac{2bh}{2}$$

$$\therefore \text{area of parallelogram } ABCD = bh$$

4 Rhombus

Let $AC = x$ and $BD = y$.

$BE = DE = \frac{y}{2}$ (diagonals bisect each other in a rhombus)

$$\text{Area} = A_1 + A_2$$

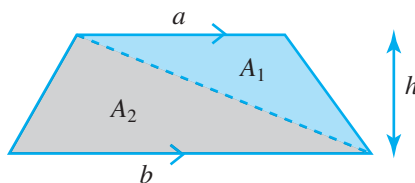
$$= \left(\frac{1}{2} \times AC \times BE\right) + \left(\frac{1}{2} \times AC \times DE\right)$$

$$= \left(\frac{1}{2} \times x \times \frac{y}{2}\right) + \left(\frac{1}{2} \times x \times \frac{y}{2}\right)$$

$$= \frac{xy}{4} + \frac{xy}{4}$$

$$= \frac{2xy}{4}$$

$$\therefore \text{area of rhombus } ABCD = \frac{1}{2}xy$$

5 Trapezium

$$\text{Area} = A_1 + A_2$$

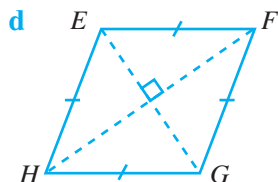
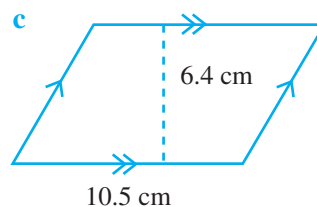
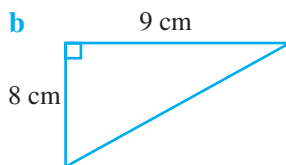
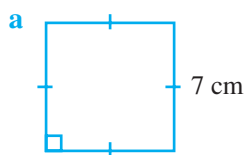
$$= \frac{ah}{2} + \frac{bh}{2}$$

$$\therefore \text{area of trapezium } ABCD = \frac{h}{2}(a + b)$$

NOTE: The proof for the area of a square obviously follows directly from that of the rectangle. The proof for the area of a kite is similar to that of the rhombus.

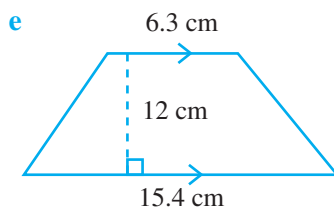
EG
+S**Example**

Find the area of each figure.

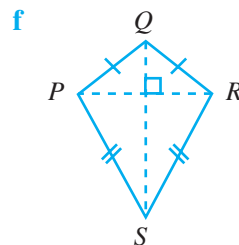


$$EG = 9.2 \text{ cm}$$

$$FH = 14.6 \text{ cm}$$



□



$$PR = 8.5 \text{ cm}$$

$$QS = 21 \text{ cm}$$

Solutions

a

$$A = s^2$$

$$= 7^2$$

$$\therefore A = 49 \text{ cm}^2$$

b

$$A = \frac{bh}{2}$$

$$= \frac{9 \times 8}{2}$$

$$\therefore A = 36 \text{ cm}^2$$

c

$$A = bh$$

$$= 10.5 \times 6.4$$

$$\therefore A = 67.2 \text{ cm}^2$$

d

$$A = \frac{1}{2}xy$$

$$= \frac{1}{2} \times 9.2 \times 14.6$$

$$\therefore A = 67.16 \text{ cm}^2$$

e

$$A = \frac{h}{2}(a + b)$$

$$= \frac{12}{2}(6.3 + 15.4)$$

$$= 6 \times 21.7$$

$$\therefore A = 130.2 \text{ cm}^2$$

f

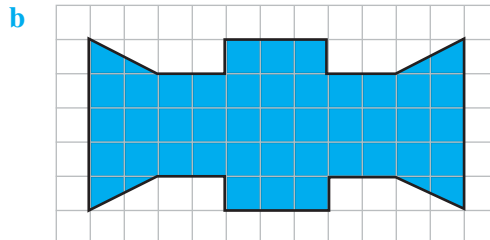
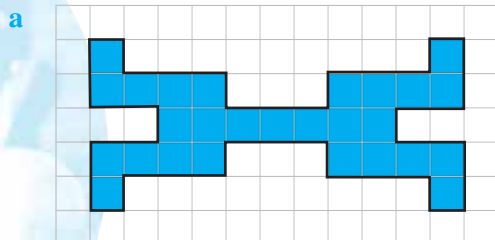
$$A = \frac{1}{2}xy$$

$$= \frac{1}{2} \times 8.5 \times 21$$

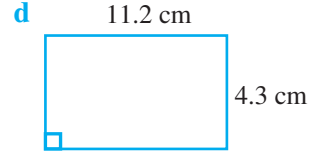
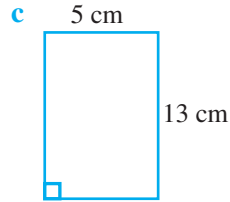
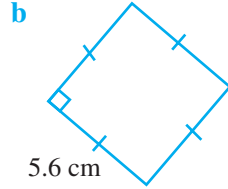
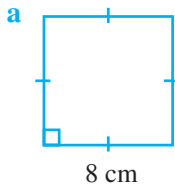
$$\therefore A = 89.25 \text{ cm}^2$$

Exercise 5.7

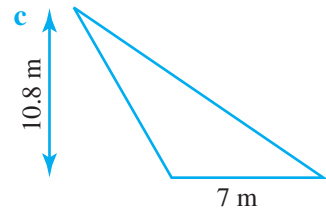
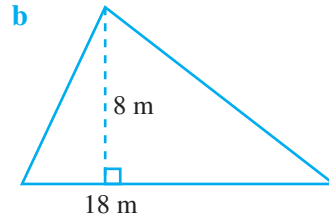
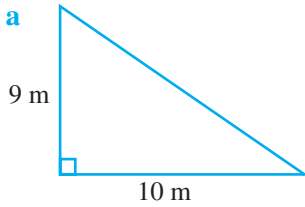
1 Write down the area of each figure, in square units.



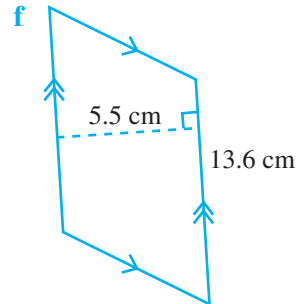
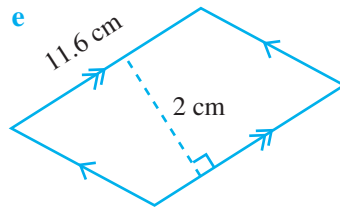
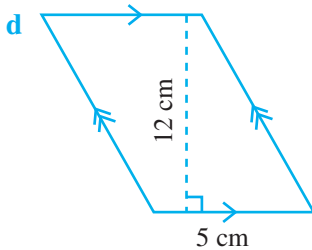
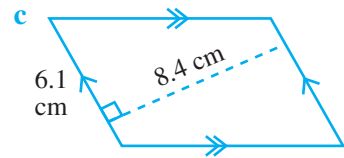
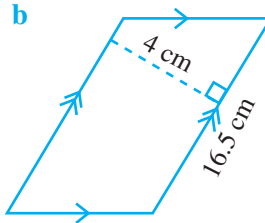
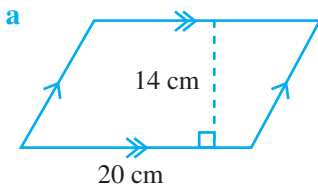
2 Find the area of each figure.



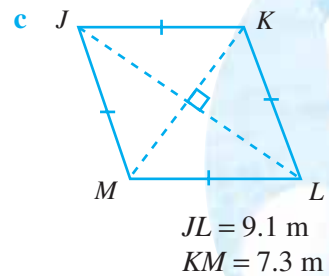
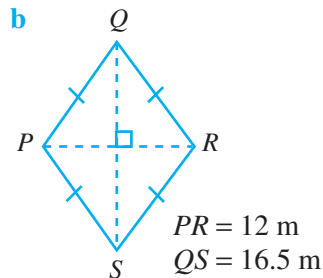
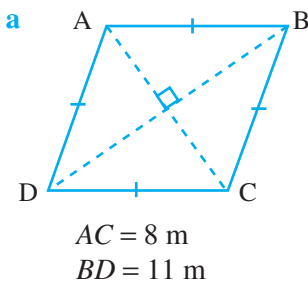
3 Find the area of each triangle.

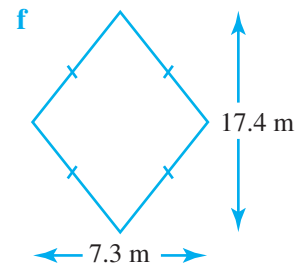
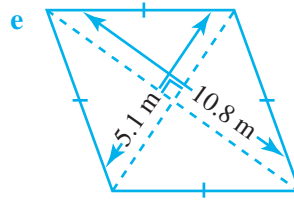
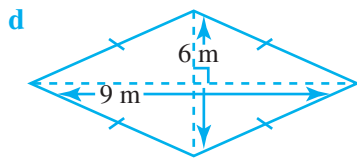


4 Find the area of each parallelogram.

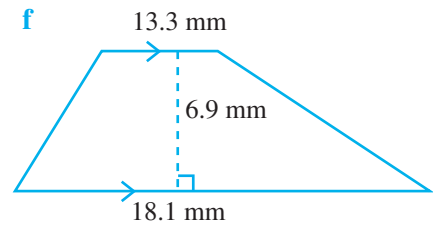
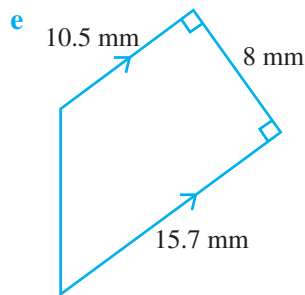
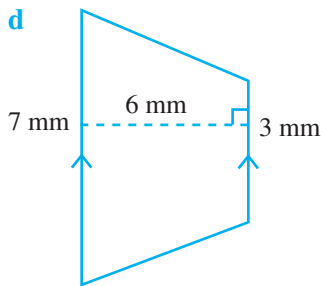
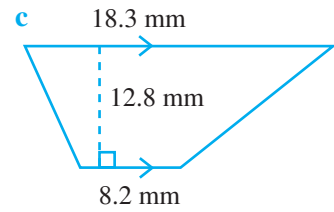
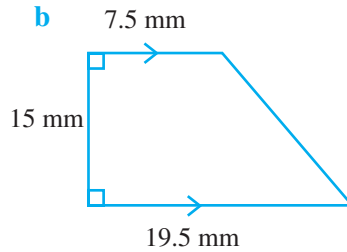
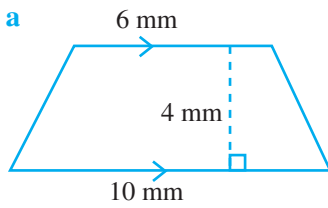


5 Find the area of each rhombus.

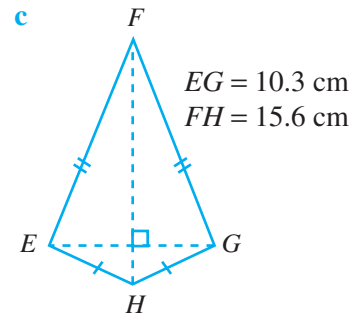
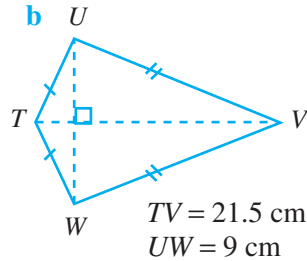
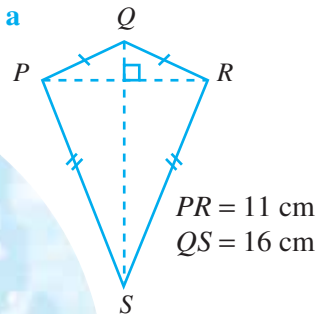


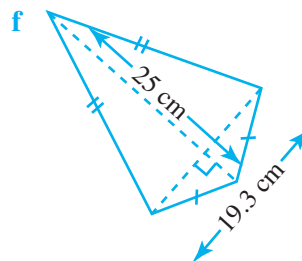
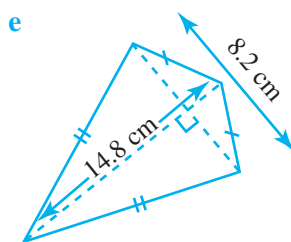
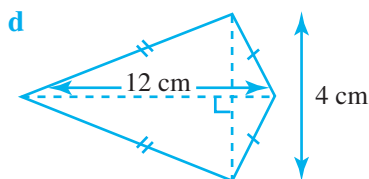


6 Find the area of each trapezium.



7 Find the area of each kite.

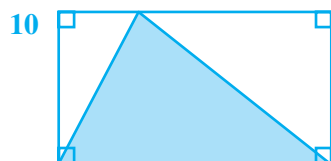




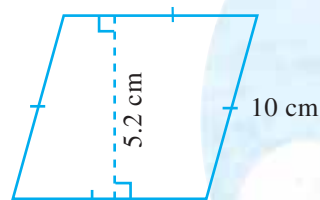
Consolidation

- 8 Calculate the area of:
- a square of side 8.5 m
 - a rectangle with length 17 cm and width 9.5 cm
 - a triangle whose base is 24 mm and height is 15.2 mm
 - a rhombus with diagonals measuring 16 m and 21 m
 - a parallelogram with a base of 11.3 m and a perpendicular height of 6.5 m
 - a trapezium with parallel sides of length 15 cm and 23 cm and a perpendicular height of 10.5 cm
 - a kite whose diagonals are of length 8.6 mm and 19.5 mm

- 9 A rectangle has a length of 1.2 m and a breadth of 40 cm. Calculate the area in square metres.

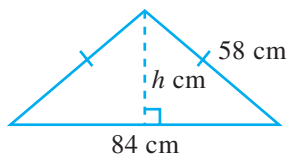


- What fraction of this rectangle has been shaded?
 - State the area of the shaded triangle if the area of the rectangle is 66 cm^2 .
 - State the area of the rectangle if the area of the shaded triangle is 24 cm^2 .
- 11
- Find the area of a square whose perimeter is 68 m.
 - Find the perimeter of a square whose area is 196 m^2 .
 - Find the side length of a square whose area is equal to that of a rectangle with dimensions 18 m by 8 m.
- 12
- The length of a rectangle is 14 m and its area is 112 m^2 . Find the width.
 - The width of a rectangle is 6 m and its perimeter is 44 m. Find the area.
 - The length of a rectangle is 22 m and its area is 198 m^2 . Find the perimeter.
- 13
- Explain why a rhombus is also a parallelogram.
 - Hence, find the area of this rhombus.
- 14 Find the area of $\triangle ABC$ in which $\angle B = 90^\circ$, $BC = 9 \text{ mm}$ and $AC = 41 \text{ mm}$.



15 Find the area of a rectangle with a width of 10 mm and diagonals of 26 mm.

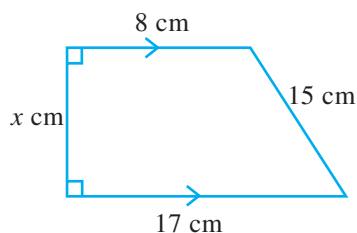
16



- a Find the perpendicular height, h cm, of this isosceles triangle.
b Hence, calculate the area.

17 a Find the value of x .

b Hence, calculate the area of the trapezium.



18 The shorter sides in a parallelogram have a length of 5 cm and the perpendicular distance between the longer sides is 4 cm. Find the area of the parallelogram given that its perimeter is 38 cm.

19 A rhombus $EFGH$ has a perimeter of 180 mm and the longer diagonal FH is 72 mm long.

a Find the length of the shorter diagonal EG .

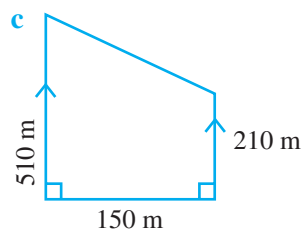
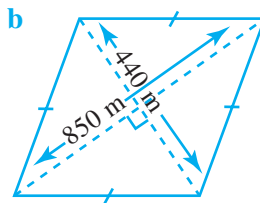
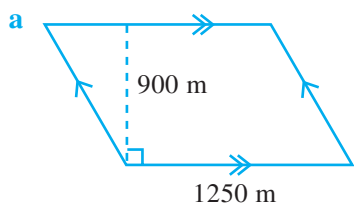
b Hence, calculate the area of the rhombus.

20 A rhombus $CDEF$ has an area of 384 mm^2 and the shorter diagonal DF is 24 mm long.

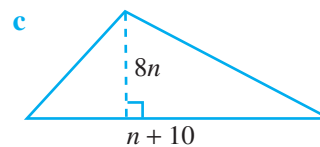
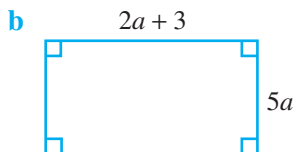
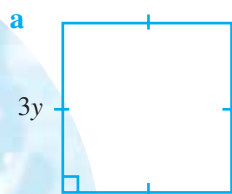
a Find the length of the longer diagonal CE .

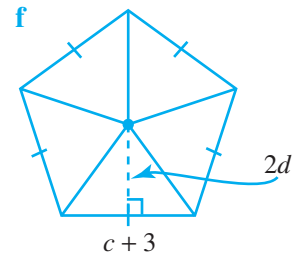
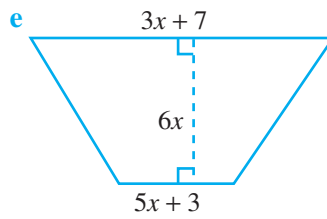
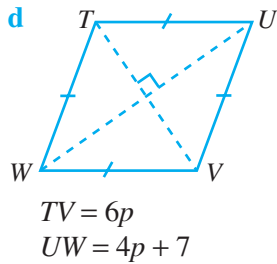
b Hence, calculate the perimeter of the rhombus.

21 Find, in hectares, the area of each of these fields.

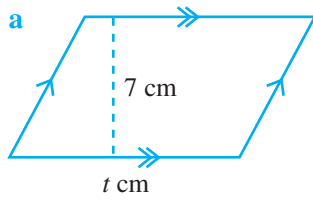


22 Find, in simplest form, an algebraic expression for the area of each figure.

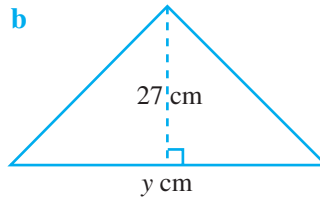




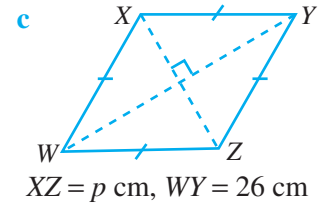
23 Find the value of the pronumeral in each of these.



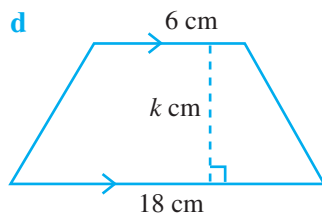
Area = 133 cm^2



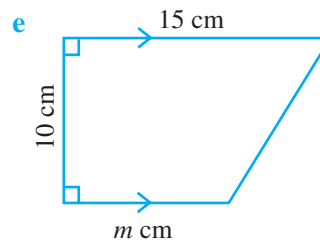
Area = 243 cm^2



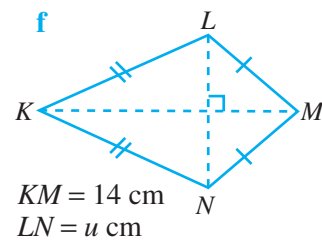
Area = 195 cm^2



Area = 84 cm^2



Area = 130 cm^2

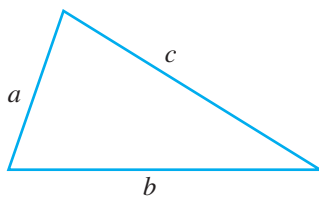


Area = 63 cm^2

Further applications

24 The sides of a right-angled triangle are in the ratio 8 : 15 : 17. If the perimeter of the triangle is 160 cm, find the area.

25

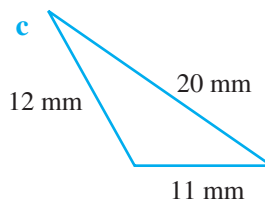
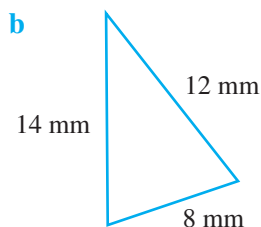
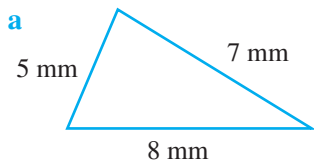


Heron's formula states that the area of any triangle with sides a , b , c is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}.$$

- a** Show that a triangle with sides of length 5 cm, 12 cm and 13 cm is right-angled.
- b** Find the area of the triangle using Heron's formula.
- c** Verify this answer using $A = \frac{bh}{2}$.

26 Use Heron's formula to find the area of each triangle. Answer correct to 1 decimal place.

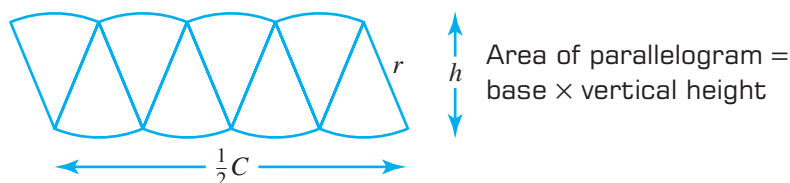


TRY THIS The area of a circle

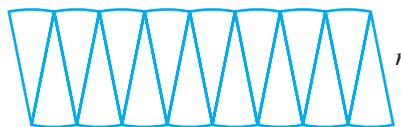
One of the most difficult problems in mathematics is to find the area of a circle whose radius is known. Archimedes (c. 287–212 BC), a famous Greek mathematician, discovered one method which is similar to that given below.

Step 1 Draw a circle of any radius and accurately draw 4 diameters at 45° to one another to form 8 equal sectors.

Step 2 Cut out the 8 sectors and arrange them as shown. The figure is roughly a parallelogram. What is its length and vertical height?



Step 3 Cut each sector exactly in half to form 16 sectors. Place the sectors side by side. The figure is now much closer to a parallelogram. What is its area?



- 1** Describe the problems with assuming the shape is a parallelogram.
- 2** How could you reduce these problems?
- 3** What do you think of the following argument?

If the sectors are made very very narrow, then the shape becomes very very close to a parallelogram. If we imagine the sectors to be so thinly sliced that their width could not be measured, then the shape becomes an exact parallelogram.

$$\begin{aligned}
 \text{Area of circle} &= \text{area of parallelogram} \\
 &= \text{base} \times \text{vertical height} \\
 &= \frac{1}{2} \text{ circumference} \times \text{radius}
 \end{aligned}$$

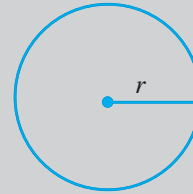
5.8 Area of a circle

Area of a circle

The previous activity should have convinced you that the area of a circle can be found using the formula below.

The area of a circle with radius r units is given by:

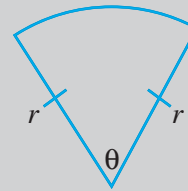
$$A = \pi r^2$$



Area of a sector

The area of a sector which subtends an angle θ at the centre is given by:

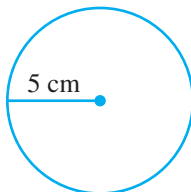
$$A = \pi r^2 \times \frac{\theta}{360}$$



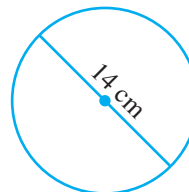
Example 1

Find the area of each circle, correct to 1 decimal place.

a



b



Solutions

$$\begin{aligned} \text{a } A &= \pi r^2 \\ &= \pi \times 5^2 \\ &= 78.5 \text{ cm}^2 \text{ (1 decimal place)} \end{aligned}$$

$$\begin{aligned} \text{b } A &= \pi r^2 \\ &= \pi \times 7^2 \\ &= 153.9 \text{ cm}^2 \text{ (1 decimal place)} \end{aligned}$$

NOTE: In these examples, the exact areas would be $25\pi \text{ cm}^2$ and $49\pi \text{ cm}^2$.

EG
+S**Example 2**

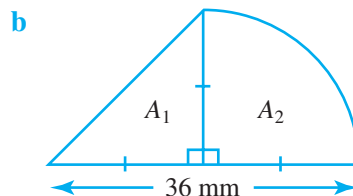
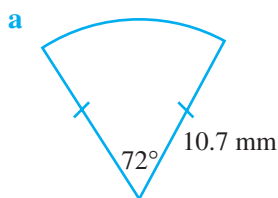
Find, correct to 2 decimal places, the length of the radius in a circle whose area is 150 m^2 .

Solution

$$\begin{aligned}
 A &= \pi r^2 \\
 150 &= \pi r^2 \\
 r^2 &= \frac{150}{\pi} \\
 \therefore r &= \sqrt{\frac{150}{\pi}} \\
 &= 6.91 \text{ m (2 decimal places)}
 \end{aligned}$$

EG
+S**Example 3**

Find the total area of each figure, correct to 4 significant figures.

**Solutions**

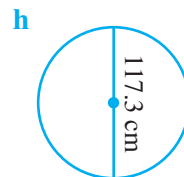
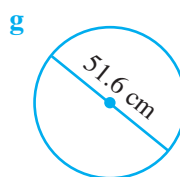
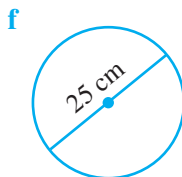
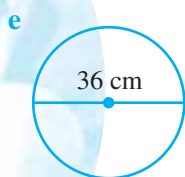
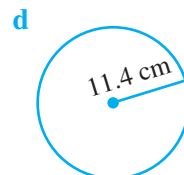
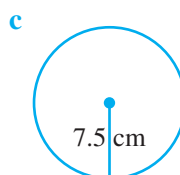
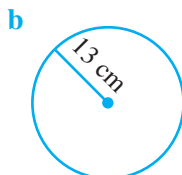
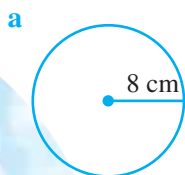
a $A = \pi r^2 \times \frac{\theta}{360}$
 $= \pi \times 10.7^2 \times \frac{72}{360}$
 $= 71.94 \text{ mm}^2 \text{ (4 significant figures)}$

b **i** $A_1 = \frac{1}{2} \times 18 \times 18$ **ii** $A_2 = \pi r^2 \times \frac{\theta}{360}$
 $= 162 \text{ mm}^2$ $= \pi \times 18^2 \times \frac{90}{360}$
 $= 254.5 \text{ mm}^2 \text{ (4 significant figures)}$

iii $A = A_1 + A_2$
 $= 162 + 254.5$
 $= 416.5 \text{ mm}^2$

Exercise 5.8

1 Find the area of each circle, correct to 1 decimal place.



2 Calculate the area of a circle with:

- a** radius 6 cm **b** radius 95.3 mm **c** diameter 42 cm **d** diameter 19.6 m

■ Consolidation

3 Find, correct to the nearest millimetre, the radius of a circle whose area is:

- a** 153.9 mm^2 **b** 452.4 mm^2 **c** 4300.8 mm^2

4 Determine, correct to the nearest centimetre, the diameter of a circle whose area is 1722 cm^2 .

5 Find the exact area of a circle with radius:

- a** 3 cm **b** 7 mm **c** 19 m

6 Find the exact area of a circle with diameter:

- a** 8 m **b** 18 cm **c** 32 mm

7 **a** Find the radius of a circle whose area is $9\pi \text{ cm}^2$.

b Find the diameter of a circle whose area is $64\pi \text{ cm}^2$.

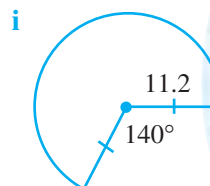
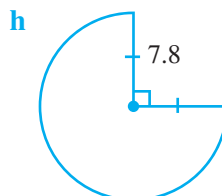
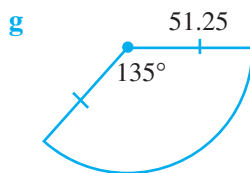
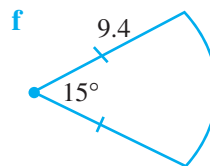
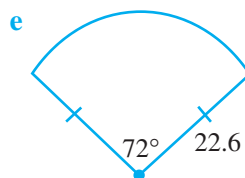
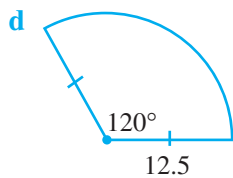
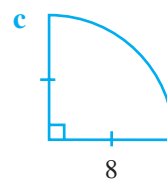
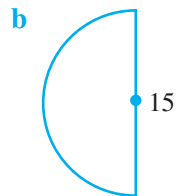
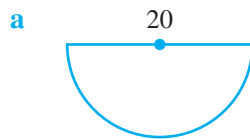
8 Find the exact area of a circle whose circumference is:

- a** $10\pi \text{ cm}$ **b** $16\pi \text{ cm}$ **c** $26\pi \text{ cm}$

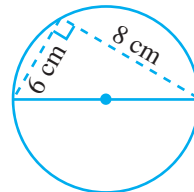
9 Find the exact circumference of a circle whose area is:

- a** $36\pi \text{ cm}^2$ **b** $121\pi \text{ cm}^2$ **c** $289\pi \text{ cm}^2$

10 Calculate the area of each figure, correct to 1 decimal place. All measurements are in metres.

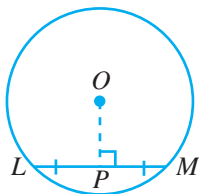


- 11 Find, correct to 1 decimal place, the radius of a circle whose area is equal to that of:
- a square of side 8 cm
 - a rhombus with diagonals 28 cm and 52 cm
- 12 Which has the greater area, a semicircle with diameter 12 cm or a parallelogram with base 8 cm and perpendicular height 7 cm?
- 13 a Find the length of the diameter.
b Hence, find the area of the circle, correct to 1 decimal place.

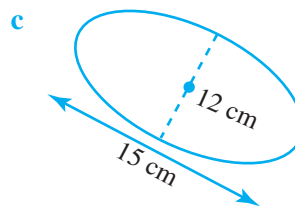
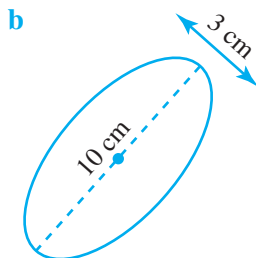
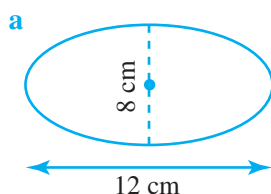
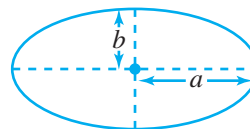


Further applications

- 14 In the diagram, O is the centre of the circle and OP is a perpendicular bisector of the chord LM . If $LM = 8$ cm and $OP = 9.6$ cm, find the area of the circle, correct to the nearest square centimetre.



- 15 The area of an ellipse is given by the formula $A = \pi ab$, where a , b are the semi-major and semi-minor axes respectively. Find, correct to 1 decimal place, the area of each ellipse.



- 16 Find the length of the diameter in a circle whose area is equal to that of an ellipse with axes of length 18 cm and 8 cm.

5.9

Composite areas

Not all figures have a single formula that can be used to calculate their area.

To calculate the area of a composite figure, use one of the following methods.

Method 1

When the figure can be subdivided into smaller figures:

- ⊙ divide the figure up into smaller standard figures
- ⊙ calculate the area of each part
- ⊙ add the areas.

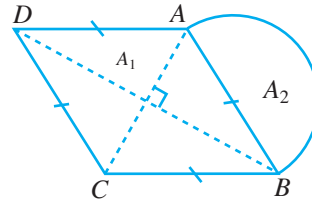
Method 2

When a figure(s) has been cut out from a larger figure:

- ⊙ calculate the area of the larger figure
- ⊙ calculate the areas of any smaller figures that have been cut out
- ⊙ subtract the smaller areas from the large area.

Example 1

Find the total area, correct to 1 decimal place, if $AC = 20$ mm, $BD = 48$ mm, $AD = 26$ mm.

**Solution**

i $A_1 = \frac{1}{2}xy$

$$= \frac{1}{2} \times 20 \times 48$$

$$= 480 \text{ cm}^2$$

ii $A_2 = \pi r^2 \times \frac{1}{2}$

$$= \pi \times 13^2 \times \frac{1}{2}$$

$$= 265.5 \text{ cm}^2$$

(1 decimal place)

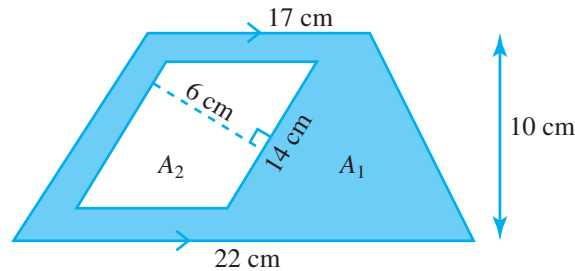
iii $A = A_1 + A_2$

$$= 480 + 265.5$$

$$= 745.5 \text{ cm}^2$$

Example 2

Find the shaded area.

**Solution**

i $A_1 = \frac{h}{2}(a + b)$

$$= \frac{10}{2}(17 + 22)$$

$$= 195 \text{ cm}^2$$

ii $A_2 = bh$

$$= 14 \times 6$$

$$= 84 \text{ cm}^2$$

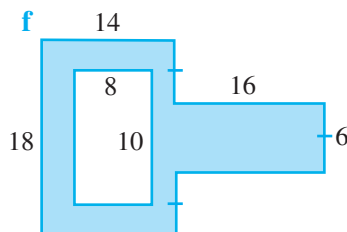
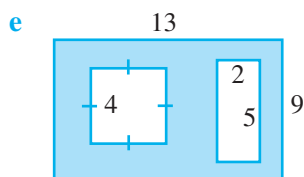
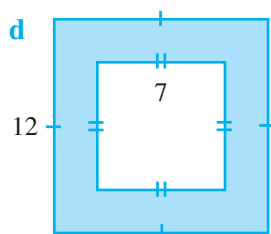
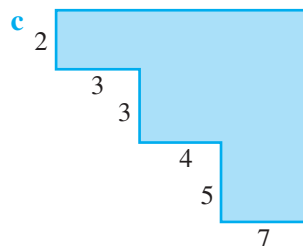
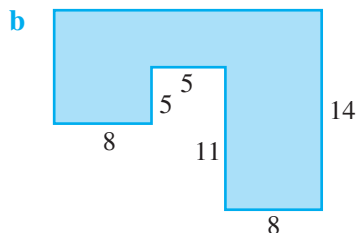
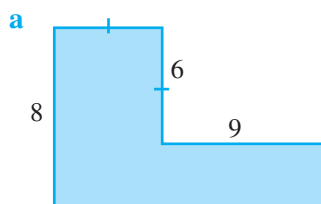
iii $A = A_1 - A_2$

$$= 195 - 84$$

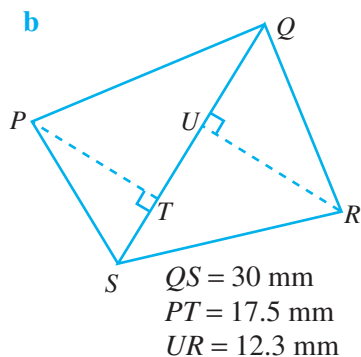
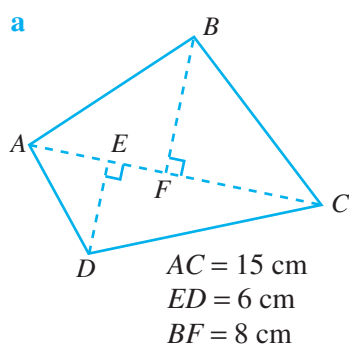
$$= 111 \text{ cm}^2$$

Exercise 5.9

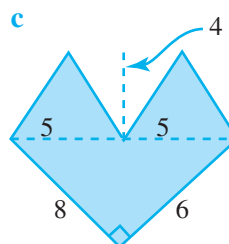
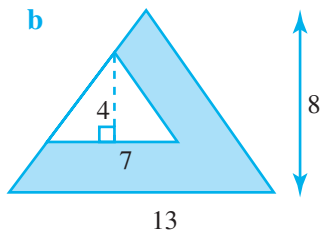
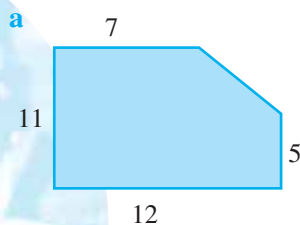
- 1 Find the shaded area in each of the following. All measurements are in centimetres and all angles are right angles.

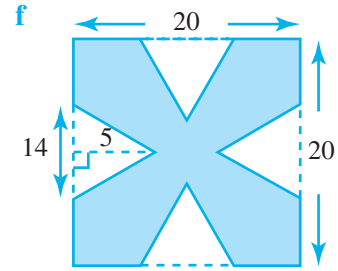
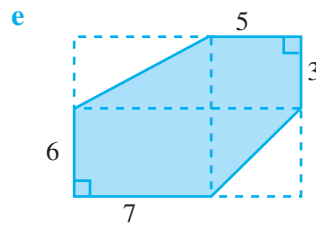
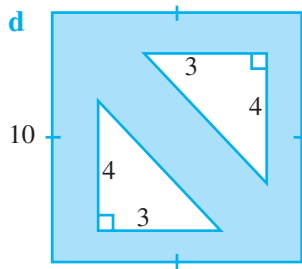


- 2 Calculate the total area of each quadrilateral by summing the areas of individual triangles.



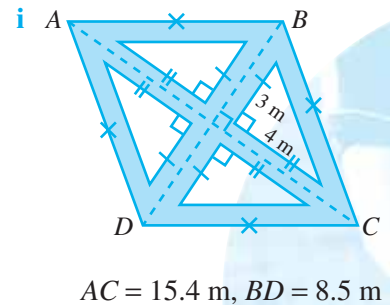
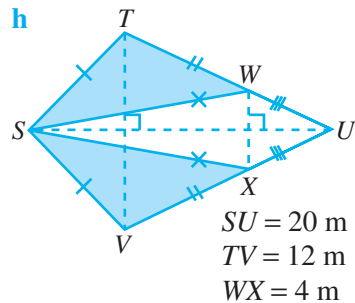
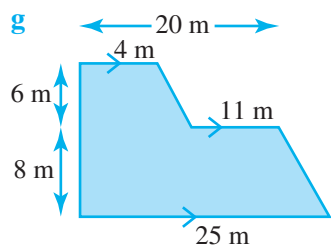
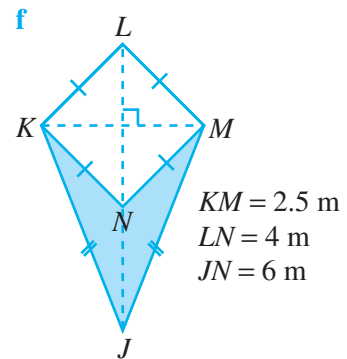
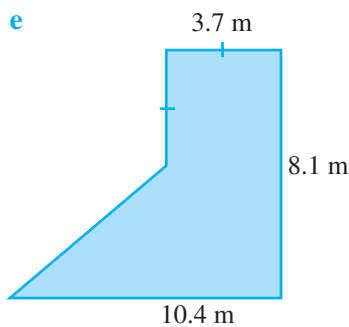
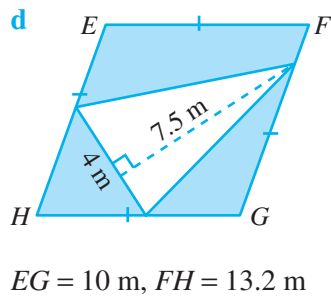
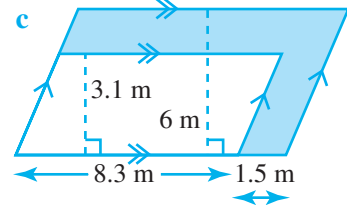
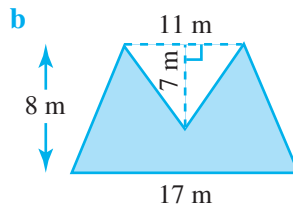
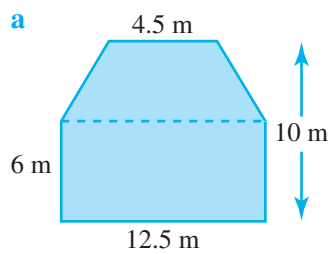
- 3 Find the shaded area in each of the following. All measurements are in cm.



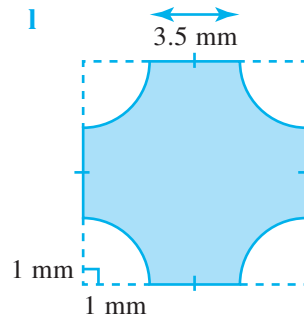
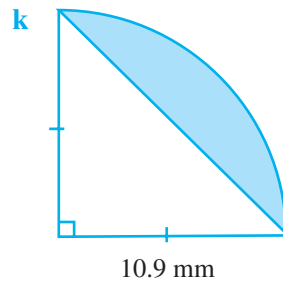
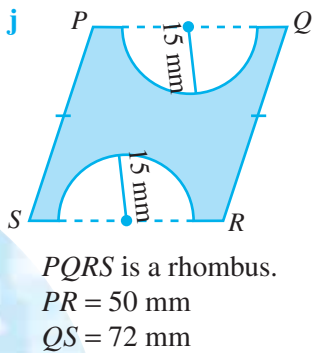
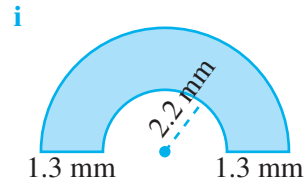
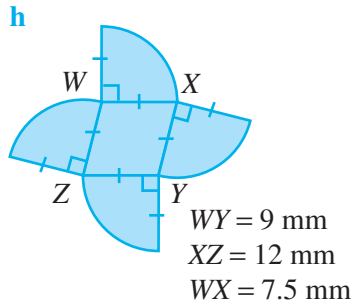
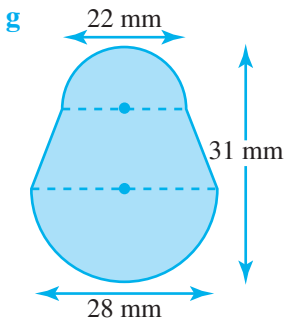
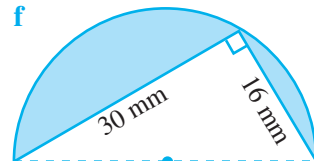
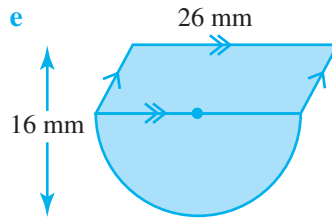
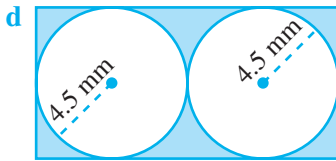
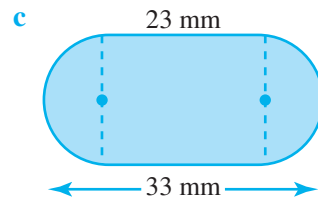
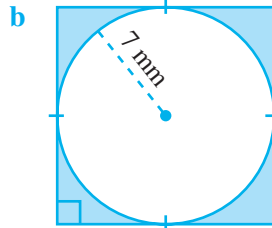
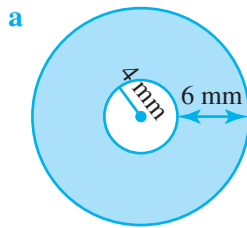


Consolidation

4 Find the shaded area in each figure.

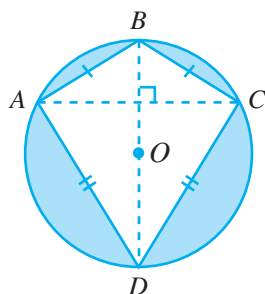


5 Determine the shaded area in each of these, correct to 1 decimal place.



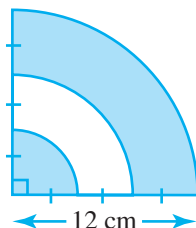
Further applications

6



Find the shaded area, correct to 1 decimal place, given that $AC = 24$ mm, $BC = 13$ mm and $CD = 37$ mm.

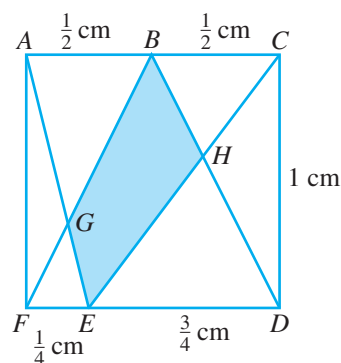
7 Find the shaded area in terms of π .



TRY THIS Area

Find the area of $BHEG$.

HINT: Subtract areas that are fractions of the square.



5.10 Problems involving area

Example 1

A rectangular patio measuring 7.3 m long and 3.4 m wide is to be concreted. Determine the cost of concreting this patio if the concrete costs \$22.50 per square metre.

Solution

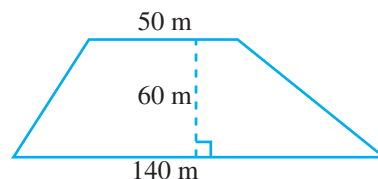
i Area = 7.3×3.4
 $= 24.82 \text{ m}^2$

ii Cost = area $\times$ cost per square metre
 $= 24.82 \times \$22.50$
 $= \$558.45$

**Example 2**

A sheep farmer's property is trapezoidal in shape. He receives \$125 for every sheep that he sells to the abattoirs. If each sheep on the property has 15 m^2 of land on which to graze, calculate:

- the area of the property
- the number of sheep on the property
- the total value of the sheep when they are eventually sold

**Solutions**

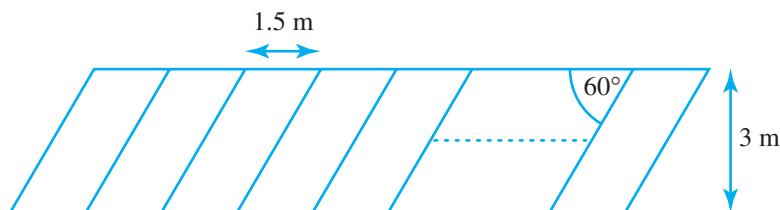
$$\begin{aligned} \text{a } A &= \frac{h}{2}(a + b) \\ &= \frac{60}{2}(50 + 140) \\ &= 5700 \text{ m}^2 \end{aligned} \quad \begin{array}{l} \text{b Number of sheep} = 5700 \div 15 \\ = 380 \end{array} \quad \begin{array}{l} \text{c Value of sheep} = \$125 \times 380 \\ = \$47\,500 \end{array}$$

Exercise 5.10

- Eliza has a poster of the pop group Savage Garden on her bedroom wall. The poster measures 1.2 m by 40 cm and the wall measures 3 m by 2.4 m. What fraction of this wall area is taken up by the poster?
- A sprinkler is situated in the middle of a square garden of side 9 m. The sprinkler waters a circular section of lawn with a radius of 1.5 m. What area of the lawn is not being watered? Answer correct to 1 decimal place.
- The pendulum of a grandfather clock is 90 cm long and swings through an angle of 60° . Find the exact area swept out by the pendulum in one complete swing from left to right.

Consolidation

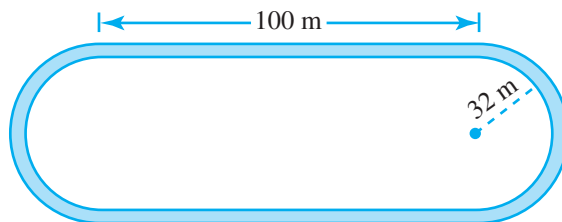
- Street parking outside a small shopping centre is in the form of 14 diagonal car spaces, each on an angle of 60° to the kerb. Each car space is 1.5 m wide and extends 3 m into the road from the kerb. Calculate the total parking area.



- A framed print measures 80 cm by 65 cm. The frame is 5 cm wide. Find the area of the unframed print.

- 6 A roundabout is to be constructed at a busy road intersection. It is to have a diameter of 4.6 m and the road around it is to be 5.2 m wide. Calculate the area of the road, not including the roundabout. Answer correct to the nearest square metre.
- 7 A stained-glass window consists of a semicircle above a rectangle. The total height of the window is 1.6 m and the width is 50 cm.
- Calculate the total area of glass required for 4 windows, correct to 1 decimal place.
 - Find the cost if stained glass costs \$312 per square metre.
- 8 The four walls and ceiling in Robyn's bedroom are to be painted. The dimensions of her room are: length 4500 mm, width 3100 mm, height 2700 mm. The total area of the door and windows is 6.3 m^2 .
- Calculate, in square metres, the total area that is to be painted.
 - How many litres of paint are needed if 1 L of paint covers an area of 5 m^2 ?
- 9 The council is to lay an 80 cm wide footpath down one side of a 320 m long street. There is a 5 m wide side street every 60 m.
- How many side streets are there?
 - Find the total area of the footpath.
 - Calculate the cost of laying the path if concrete and labour costs amount to \$12 per square metre.
- 10 Find, correct to 1 decimal place, the radius of a circle whose area is equal to that of a rhombus with diagonals 18 mm and 13 mm.

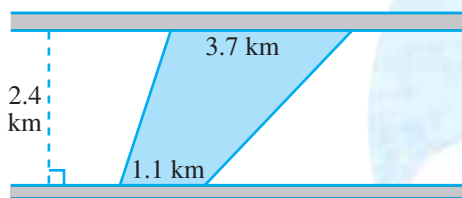
- 11 A 5 m wide running track consists of two straight sections each 100 m long and a semicircular section at each end with an inner radius of 32 m. Calculate the area of the running track, correct to the nearest square metre.



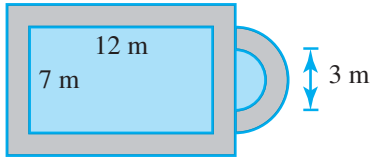
- 12
-

Carpet is to be laid in the lounge room, dining room and games room of this house. Find:

- the total area to be carpeted
 - the cost of carpeting these rooms at \$22.50 per square metre
- 13 A farmer intends to buy this area of land situated between two parallel highways 2.4 km apart with frontages of 1.1 km and 3.7 km.
- Calculate, in hectares, the area of the farm.
 - Find the cost of the farm at \$1400 per hectare.



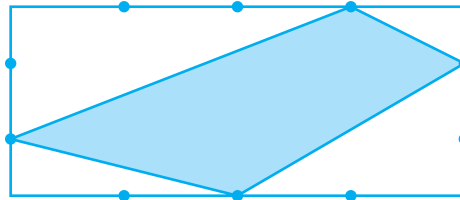
14



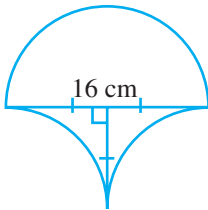
A swimming pool measures 12 m by 7 m and has a heated spa of diameter 3 m at one end. The pool and spa is surrounded by a 1 m wide tiled path. Calculate the total area of the path, correct to 1 decimal place.

Further applications

15 What fraction of this rectangle has been shaded?



16



- a Calculate the area of this figure and hence show that it is independent of π .
- b How could you explain this result without actually finding the area?



FOCUS ON WORKING MATHEMATICALLY

THE SOLAR SYSTEM



Introduction

In this activity we will work from tabulated data to compare the sizes of the planets in the solar system and their distances from the Sun. The nine planets revolve around the Sun in elliptical paths which approximate circles. It is interesting that the planets all lie in nearly the same plane. You might first like to check out an atlas or the solar system exploration home page of NASA to get a good look.



LEARNING ACTIVITIES

The table below shows the diameters (at the equator) and the average distance of each planet from the Sun in the solar system. Distances are given in *millions of kilometres*.

Planet	Diameter	Average distance from Sun
Sun	1.4	0
Mercury	0.0049	58
Venus	0.012	108
Earth	0.013	150 (1 AU)*
Mars	0.068	228
Jupiter	0.14	778
Saturn	0.12	1429
Uranus	0.047	2871
Neptune	0.045	4504
Pluto	0.0023	5914

* The mean distance from the centre of the Earth to the centre of the Sun is called an Astronomical Unit (symbol AU).

$$1 \text{ AU} = 150 \text{ million kilometres} = 1.5 \times 10^8 \text{ km}$$

This is a convenient yardstick for measuring distance between objects in the solar system, and is very useful in solving equations of planetary motion.

- Using a calculator, work out the ratio of the diameter of the Sun to the diameter of the Earth (nearest whole number).
If the diameter of Earth was 1 m, what would be the diameter of the Sun? Does this surprise you?
- Compare the diameters of the nine planets in the table. (You might like to multiply each diameter by 1000 to make it easier.)
- Which planets are close to each other in size? Which planet is the smallest? the largest? Draw a graph to illustrate your findings.



- 4 Which planet is nearest to Earth? How far away is it?
- 5 The planets are shown in the order of their distance in millions of km from the Sun. Draw up a table and convert these distances to AU. What do you notice?
- 6 On 27 August 2003, Mars was at its closest point to Earth in approximately 70 000 years. It was just 55.76 million km away. How much closer was it? Why such a long time gap?



CHALLENGE

In this activity take the speed of light to be 3×10^5 km/s, and use the data in the table as you need it.

- 1 The length of Jupiter's year is 4333 days. How many Earth years does this correspond to? Describe in words what the 'Jupiter year' means.
- 2 Calculate the average speed of the Earth in its orbit around the Sun in km/h, given that the length of the orbit is 942 million km. Does the answer surprise you?
- 3 How long would it take a ray of light from the Sun to reach Earth?
- 4 A 'light year' is defined as the distance light travels in 1 year. It is a unit of distance used in astronomy. Calculate the distance that light travels in 1 year in km and convert it to AU.
- 5 The Sun is relatively close to us when compared to the other stars of our galaxy. Discuss with your teacher or classmate how long it would take for light to reach us from Alpha Centauri, the nearest star (after our Sun), if it is about $4\frac{1}{3}$ light years away. What does this imply in terms of time when we study the stars?

NOTE: You can see Alpha Centauri in the south in the night sky. It is the lower of the two 'Pointers' to the constellation of the Southern Cross.



LET'S COMMUNICATE

- 1 What have you learned about the solar system in these activities? Write a summary of what has surprised you about our solar system and space to the stars beyond.
- 2 What do you think is the most important thing for humans to find out next about the solar system in which we live? Can mathematics help?



REFLECTING

Reflect on how important a role mathematics has played in sending spacecraft to the Moon (relatively close), to Mars and to the outer planets. You might like to check out the NASA website <www.nasa.gov/> to see the extent of mathematics in learning about our solar system, our galaxy and the universe beyond.





MACQUARIE

Language link with Macquarie

- 1 What do the following **prefixes** mean:
centi-, **milli-**, **kilo-**?
- 2 Write the following in words:
 $100 \text{ mg} \times 10 = 1 \text{ g}$.
- 3 Explain the difference between **accuracy** and **precision**.
- 4 What is the **mass** of an object?
- 5 Read the Macquarie Learners Dictionary entry for **estimate**:

estimate verb **1.** to roughly work out the value, size or other qualities: We *estimated* the cost to be \$20.
 –noun **2.** a rough judgment: I think it will take two hours to get there, but it's only an *estimate*.

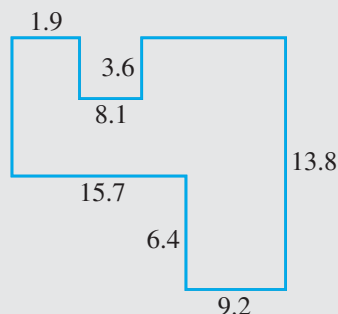
□ **Word family:** **estimation** noun

Why are good estimates important in business and in science?

- 1 Choose the most appropriate unit that could be used to measure:
 - a the time needed to boil an egg
 - b the distance between Australia and the United States
 - c the mass of a light bulb
 - d the length of an eyelash
 - e the capacity of an eyedropper
 - f the mass of a washing machine
 - g the time needed to drive from Sydney to Newcastle
 - h the capacity of a petrol tanker
- 2 Convert:
 - a 410 cm to m
 - b 2.3 km to m
 - c 106 mm to cm
 - d 52 cm to m
 - e 75 m to km
 - f 0.06 cm to mm
 - g 3480 cm to m
 - h 0.02 m to cm
- 3 Convert:
 - a 3.9 m to mm
 - b 0.56 km to cm
- 4 Convert:
 - a 3640 kg to t
 - b 1.8 g to mg
 - c 0.9 t to kg
 - d 310 g to kg
 - e 86 g to mg
 - f 0.003 t to kg

- 5 Convert:
 - a 1670 mL to L
 - b 1.4 kL to L
 - c 420 L to kL
 - d 5.671 L to mL
 - e 0.0087 kL to L
 - f 59 mL to L
- 6 Complete each of these area conversions.
 - a $6 \text{ cm}^2 = \square \text{ mm}^2$
 - b $3.9 \text{ m}^2 = \square \text{ cm}^2$
 - c $580 \text{ mm}^2 = \square \text{ cm}^2$
 - d $0.004 \text{ km}^2 = \square \text{ m}^2$
 - e $72\,000 \text{ cm}^2 = \square \text{ m}^2$
 - f $41\,000 \text{ m}^2 = \square \text{ km}^2$
- 7 Convert:
 - a 8 ha to m^2
 - b 45 000 m^2 to ha
 - c 0.64 ha to m^2
 - d 7310 m^2 to ha
- 8 Within what limits are the following instruments accurate?
 - a A speedometer with 10 km/h markings.
 - b A dressmaker's tape with 1 cm markings.
 - c An altimeter marked in intervals of 1000 m.

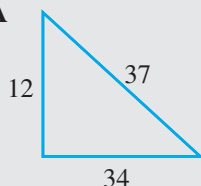
- 9** Within what limits will each of these measurements lie?
- The mass of a tomato is 80 g, correct to the nearest 10 g.
 - The age of an ancient vase is 2800 years, correct to the nearest 100 years.
 - The length of a dragonfly is 36 mm, correct to the nearest mm.
- 10** State the upper and lower bounds for each measurement.
- 9 mm
 - 206 L
 - 1.8 g
 - 17.1 m
- 11** Convert:
- 4 min to s
 - $1\frac{2}{3}$ days to h
 - 90 min to h
 - $\frac{3}{5}$ h to min
 - 75 s to min
 - 216 h to days
- 12**
- Express 2 h 23 min 42 s in hours
 - Express 7.205 h in hours, minutes and seconds.
- 13** Simplify, using the degrees and minutes key on your calculator:
- 4 h 23 min – 1 h 38 min
 - 45 min $\div$ 12
- 14** Express each of these in 24-hour time.
- 5 am
 - 9 pm
 - 1:50 am
 - 11:26 pm
- 15** Express each of these 24-hour times in standard 12-hour time.
- 0800 h
 - 1400 h
 - 0620 h
 - 2238 h
- 16** How long is it in hours and minutes between 9:25 pm Monday and 4:10 am Tuesday?
- 17** Nicole's average stride when jogging is 57 cm. How many strides will she take in running a distance of 2.5 km? Answer correct to the nearest 100 strides.
- 18** Find the mass in kilograms of 820 bananas if each banana has an average mass of 93 g.
- 19** How many 350 g bags of popcorn can be filled to the top from a machine that holds 25 kg of popcorn?
- 20** An empty industrial water tank is filled at the rate of 75 L/min. How long, in hours and minutes, will it take to fill the tank if it has a capacity of 12 kL?
- 21** Warwick needs to take 15 mL of his medicine four times a day for 3 weeks. Find, in litres, the amount of medicine that he must take in order to finish the course.
- 22** Calculate the perimeter of this figure. (All measurements are in millimetres.)



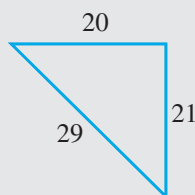
- 23** Find the side length of a rhombus whose perimeter is equal to that of an equilateral triangle of side 9.6 m.
- 24** A rug measuring 4.6 m by 5.4 m is placed in the centre of a lounge room floor. The rug is 180 cm away from the wall on each side. Find the perimeter of the lounge room.

25 Which triangle is right-angled? Why?

A

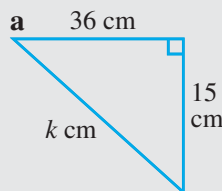


B

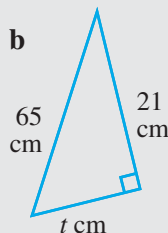


26 Find the value of the pronumeral in each of these. Answer correct to 1 decimal place where necessary.

a

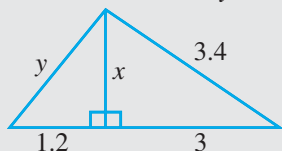


b



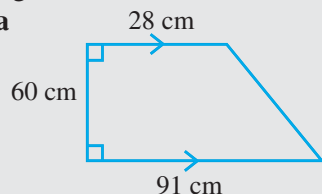
- 27 a In $\triangle FGH$, $\angle H = 90^\circ$, $FG = 38$ mm and $GH = 20$ mm. Find the length of FH , correct to 2 decimal places.
 b In $\triangle XYZ$, $\angle X = 90^\circ$, $XY = 72$ mm and $XZ = 41$ mm. Find the length of YZ , correct to 3 significant figures.

28 Find values for x and y .

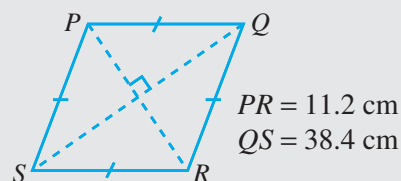


29 Calculate the total perimeter of each figure.

a



b

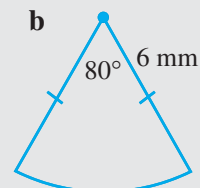


- 30 Find, correct to 1 decimal place, the circumference of a circle with:
 a diameter 16 cm b radius 51 cm
 31 a Find, correct to 2 decimal places, the diameter of a circle with circumference 54 cm.
 b Find, correct to 3 significant figures, the radius of a circle with circumference 61 cm.
 32 The diameter of a bicycle wheel is 46 cm. How many complete revolutions are needed for a girl to cycle 2 km to school?
 33 Calculate the total perimeter of each figure, correct to 1 decimal place.

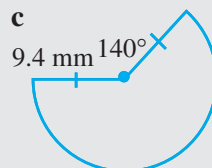
a



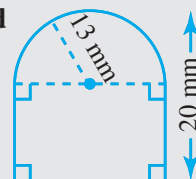
b



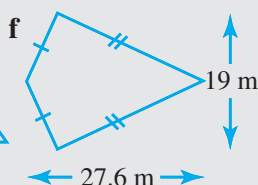
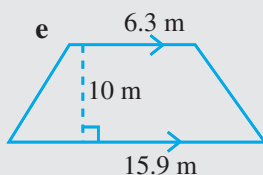
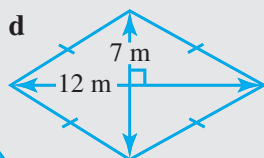
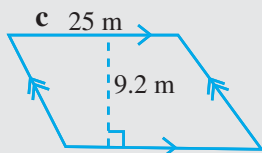
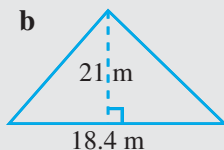
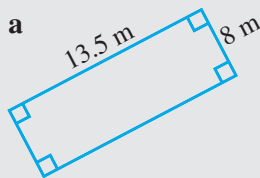
c



d

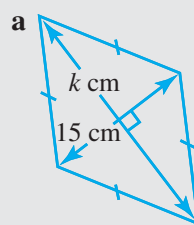


34 Calculate the area of each figure.

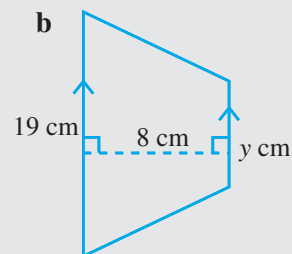


- 35 a Find the area of a square whose perimeter is 76 cm.
 b Find the perimeter of a square whose area is 169 cm^2 .
- 36 a Find the perimeter of a rectangle whose area is 84 cm^2 and width is 6 cm.
 b Find the area of a rectangle whose perimeter is 71 cm and length is 22.5 cm.
- 37 Determine the area of a rectangle whose length is 8.4 m and diagonals are 9.1 m.

38 Find the value of the pronumeral in each of these.

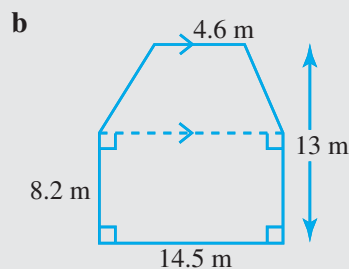
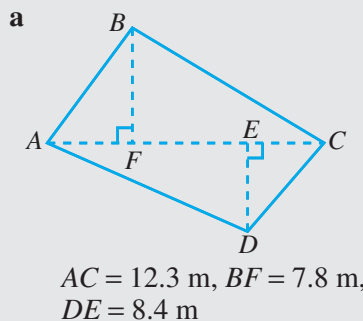


Area = 135 cm^2



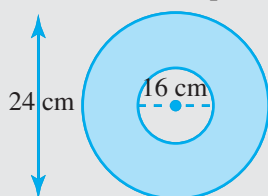
Area = 120 cm^2

- 39 Find, correct to 2 decimal places, the area of a circle whose:
 a radius is 11 cm
 b diameter is 13 cm
- 40 a Find the exact area of a circle whose circumference is $14\pi \text{ cm}$.
 b Find the exact circumference of a circle whose area is $25\pi \text{ cm}^2$.
- 41 Find the total area of each figure.

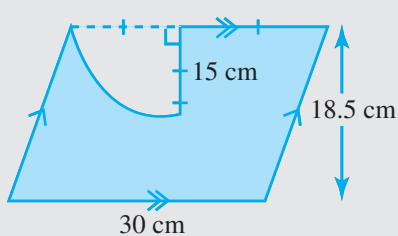


- 42** Find the shaded area in each of these, correct to 1 decimal place.

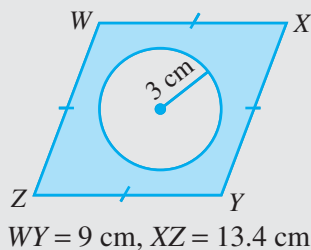
a



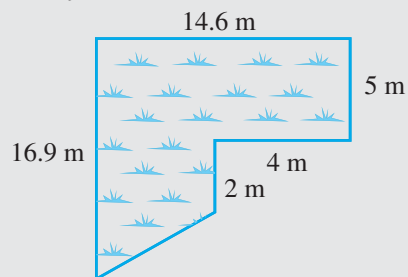
b



c



- 43** Steven wants to lay new turf in his backyard.



- Determine the area of Steve's backyard.
- Calculate the cost of returfing if the grass costs \$21.40 per square metre.

6

Data representation and analysis



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ read, interpret and draw a variety of graphs
- ⦿ construct and interpret frequency distribution tables, frequency histograms, frequency polygons, dot plots, and stem and leaf plots
- ⦿ calculate the mean, median, mode and range of a set of scores
- ⦿ calculate the mean, median, mode and range of the scores in a frequency table, frequency histogram, frequency polygon, dot plot and stem and leaf plot
- ⦿ solve problems involving the mean, median, mode and range
- ⦿ interpret the data in the cumulative frequency column of a frequency table
- ⦿ use the cumulative frequency column in a frequency table to find the median
- ⦿ construct and interpret a cumulative frequency histogram and a cumulative frequency polygon (ogive)
- ⦿ use a cumulative frequency polygon to find the median
- ⦿ construct and interpret frequency distribution tables, frequency histograms and frequency polygons for grouped data
- ⦿ find the approximate mean of a set of grouped data using the class centres
- ⦿ find the modal class of a set of grouped data.

6.1 Graphs

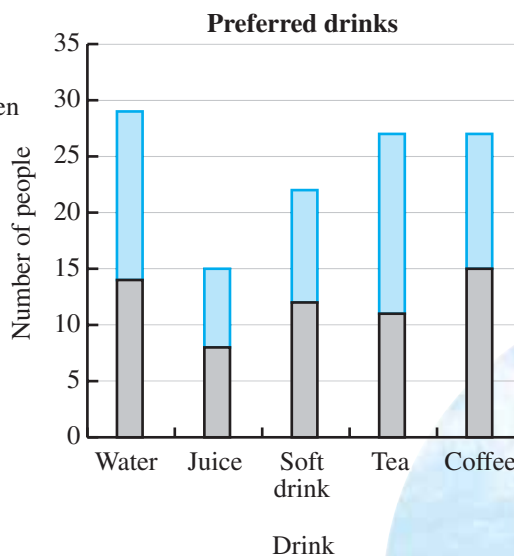
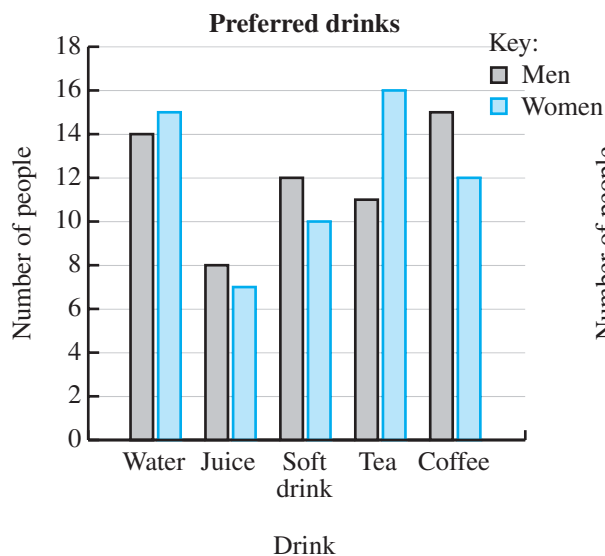
Graphs are used to represent both numerical and categorical data in a way that makes the data easier to understand and analyse. A graph must have a title, the axes must be clearly labelled and the scale chosen must be appropriate. If the scale is inconsistent or incomplete, the graph can be misleading. These techniques are often used deliberately to give a false impression about the data.

This table shows the survey results of a group of 60 men and 60 women who were asked to state their preferred non-alcoholic drink. The graphs below illustrate some or all of the data in the table.

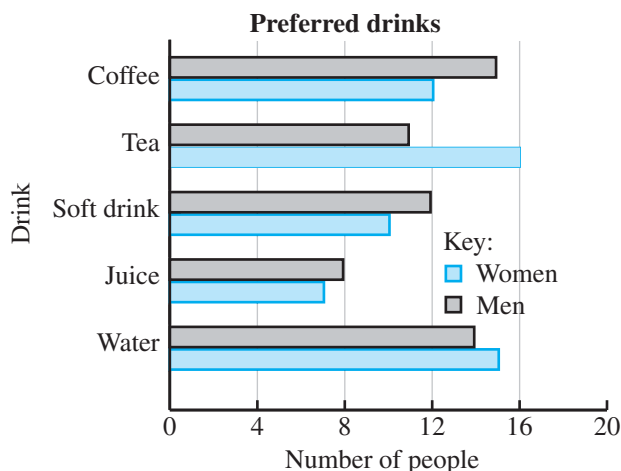
Preferred drink	Men	Women
Water	14	15
Juice	8	7
Soft drink	12	10
Tea	11	16
Coffee	15	12

Column graph

A **column graph** consists of a number of vertical columns, which may be either separate or joined. The data (either numerical or non-numerical) is marked on the horizontal axis and the frequency or number is marked on the vertical axis. Column graphs may also use multiple columns or stacked columns to compare two or more quantities.



Bar graph



A **bar graph** is a column graph that has been drawn on its side. The data is marked on the vertical axis and the frequency is marked on the horizontal axis. Like column graphs, bar graphs are often used to represent non-numerical data.

Divided bar graph

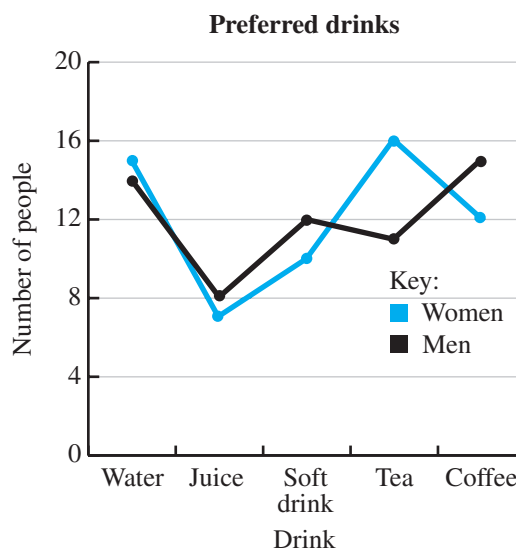
A **divided bar graph** consists of a large rectangle divided into smaller rectangles. The lengths of the small rectangles are in proportion to the sizes of the categories. The scale used is 1 drink = 2 mm.

Men's preferred drink



Line graph

A **line graph** is usually used to compare two sets of numerical data, such as temperature versus time, but it can also be used to represent non-numerical data. It consists of a series of line segments. Line graphs are often used to *extrapolate* information. This is done by extending the graph and making predictions based on a *trend*. This is particularly important in areas such as business and finance. Line graphs are also used to *interpolate* information. This is done by taking a limited number of readings, drawing a line graph, then reading off values between the original points. This technique is used in areas such as science and health care.



Picture graph

A **picture graph** is a simple graph in which a picture or symbol is used a number of times to represent the data. A key is used to explain the meaning of the symbol. Picture graphs are visually appealing, with the picture related in some way to the data.

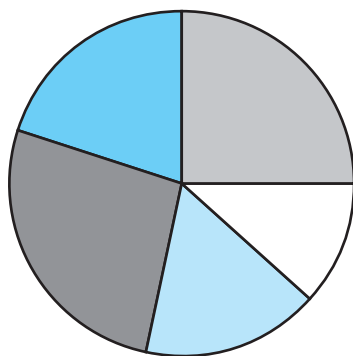
Women's preferred drink

Water	      
Juice	   
Soft drink	   
Tea	      
Coffee	    

Key:

 = 2 women

Women's preferred drink



Key:

-  Water
-  Juice
-  Soft drink
-  Tea
-  Coffee

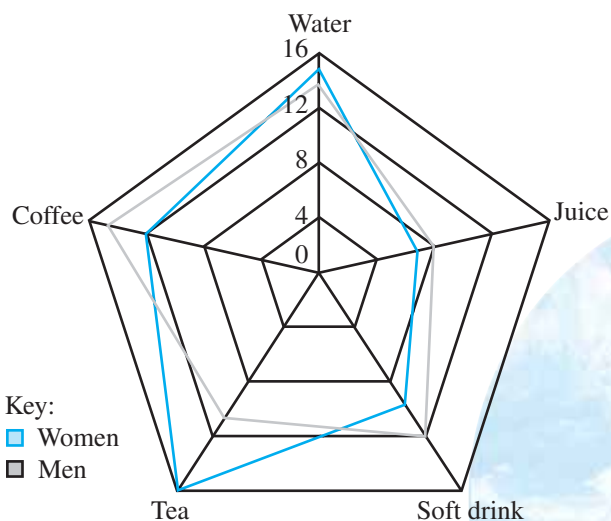
Sector graph

A **sector graph** or **pie chart** consists of a circle that has been divided into sectors. The sizes of the sectors are in proportion to the sizes of the categories. The size of the data in each sector can be worked out by measuring the central angles.

Radar chart

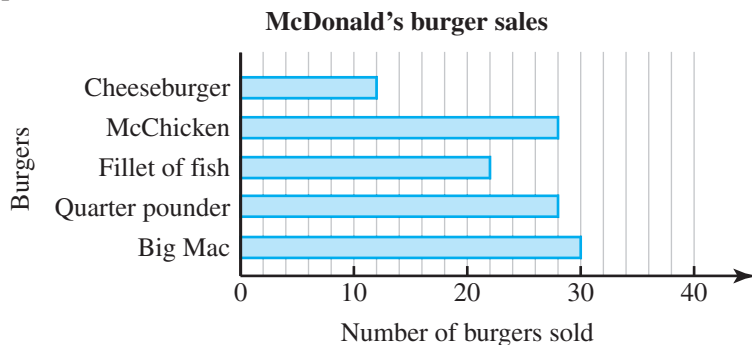
A **radar chart** is used to plot changes in quantities, such as temperature or water level over a period of time. It can also be used to compare two sets of data.

Preferred drinks

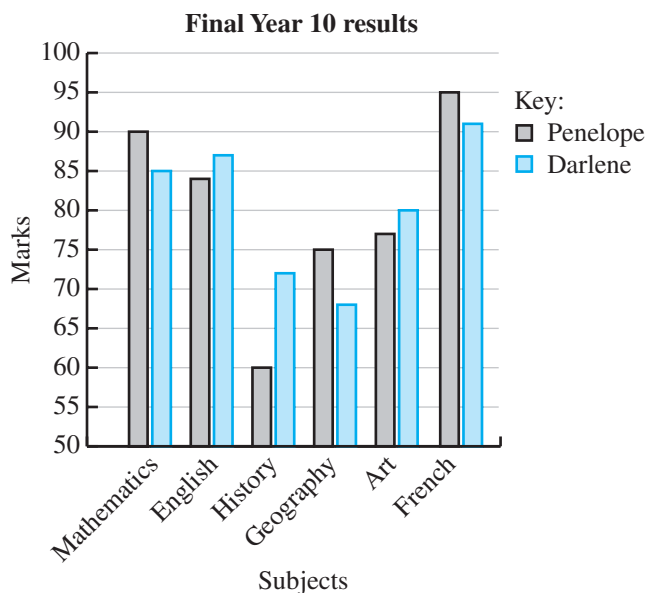


Exercise 6.1

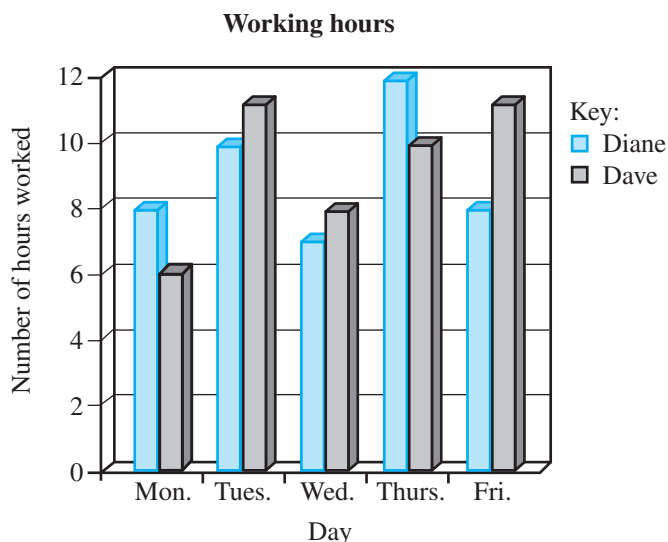
- 1 This bar graph shows the numbers of burgers sold at a McDonald's restaurant in a 30-minute period.



- a Which burger was the most popular?
 - b Which was the least popular burger?
 - c 22 of which burger were sold during this time?
 - d Calculate the total number of burgers sold.
 - e What percentage of sales were Fillet of fish?
 - f What was the average number of burgers sold per minute?
- 2 Penelope and Darlene are identical twins. Being very competitive, they decided to graph their final Year 10 results and compare their performance in each subject.
- a Which girl was better at Mathematics?
 - b Which was Darlene's best subject?
 - c In which subject was the difference between their marks most noticeable?
 - d In which subject did Penelope score 77%?
 - e By how many marks did Penelope 'beat' Darlene in geography?
 - f Does the graph give a false impression about the girls' performance in one particular subject? Explain.



3

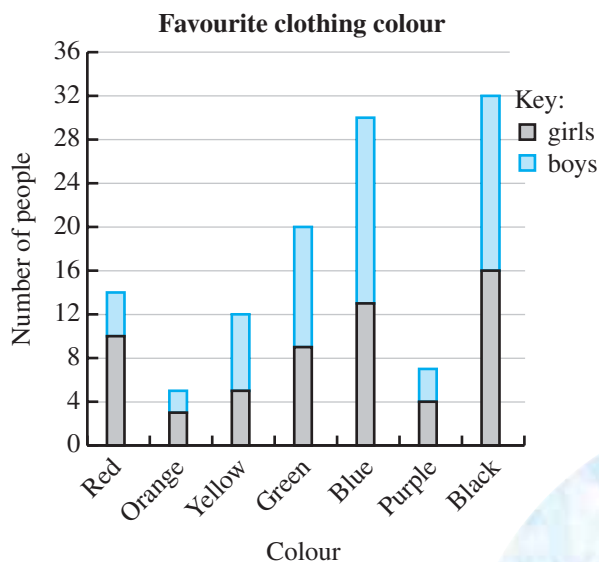


This three-dimensional column graph shows the number of hours worked last week by Dave and Diane.

- a Why does the graph look as though the tops of the columns do not reach the lines? Do they?
- b How many hours did Dianne work on Monday?
- c How many hours did Dave work on Tuesday?
- d Who worked the longest day during the week?
- e Who worked the most hours last week, and by how much?
- f Who worked for 7 hours on one day? Which day was that?

Consolidation

- 4 A group of 60 girls and 60 boys were asked to state their favourite clothing colour. This stacked column graph shows their responses.
 - a What was the most popular colour overall?
 - b What colour was most favoured by:
 - i girls?
 - ii boys?
 - c How many more boys than girls preferred green clothing?
 - d Which colour was favoured equally by both boys and girls?
 - e Which colour was preferred by exactly 7 boys?
 - f Of those who preferred red clothing, what fraction were girls?

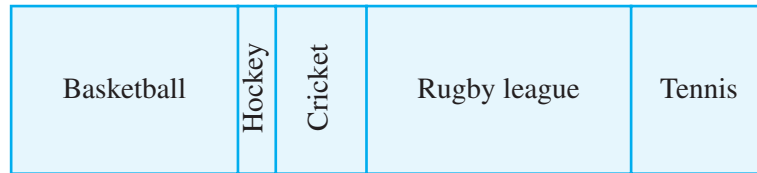


- 5 Draw a column graph to represent the following television viewing habits. Write the channels on the horizontal axis and the percentage of viewers on the vertical axis.

Type of service	Free to air					Pay TV
Channel	SBS	2	7	9	10	
% of view	8	12	18	21	15	26

- 6 A group of 300 people were surveyed to find their favourite sport. The results are recorded in this divided bar graph.

Favourite sports



- a Complete this scale for the graph: 1 mm = ___ people.
 b Which sport was the most popular?
 c How many people named basketball as their favourite sport?
 d What fraction of those surveyed said cricket was their favourite sport?
 e What percentage of respondents voted for hockey?
- 7 A group of 200 retirees listed the following activities as hobbies. Draw a divided bar graph to represent this information. Use a scale of 1 person = 2 mm.

Hobby	Gardening	TV/video	Reading	Bushwalking	Music
Number of people	33	45	42	52	28

- 8 This graph shows the number of children who played junior soccer in Melbart from 1998 to 2003.

Soccer registrations

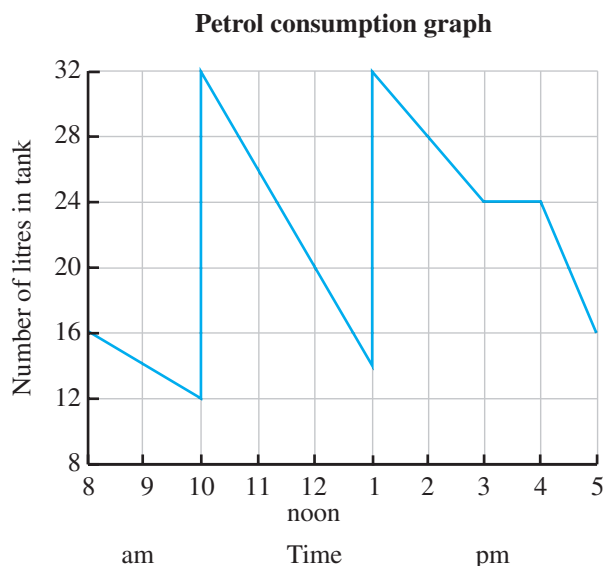
Year	Number of players
1998	40 000
1999	50 000
2000	60 000
2001	50 000
2002	40 000
2003	70 000

Key:

= 20 000 players

- a What type of graph is this?
 b How many children played soccer in:
 i 2000? ii 1999?
 c How many more children played soccer in 2003 than 2002?
 d In which years did the number of children playing soccer decline?
 e In which year(s) did the greatest increase in participation occur?
 f In which year did 70 000 children play soccer?

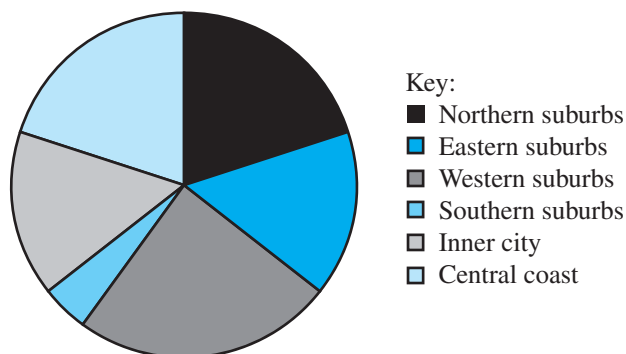
- 9 This line graph shows the amount of petrol in the tank of a car.



- a How much petrol was originally in the tank?
- b What happened at 10 am and 1 pm?
- c What is the capacity of the petrol tank? How do you know?
- d At what times did the tank contain 14 L of petrol?
- e How much petrol was in the tank at 12:30 pm?
- f How much petrol did the car use between 10 am and 1 pm?
- g How much petrol was used between 3 pm and 4 pm? What does this mean?
- h Between what times was the car probably travelling with the greatest speed? How can you tell?
- 10 The height of a 50-storey building was regularly recorded during construction as a measure of progress. Draw a line graph to represent the data with a horizontal scale of 1 cm = 4 months and a vertical scale of 1 cm = 4 storeys.

Time in months	4	8	12	16	20	24	28	32	36	40	44	48
Storeys completed	0	1	5	12	19	26	32	38	40	44	48	50

- 11 Houses built by Knockemdown Constructions



- d How many houses did the company build on the Central Coast?
- e What percentage of the houses were built in the northern suburbs?
- f What fraction of the total number of houses were built outside of Sydney?

This sector graph illustrates the number of houses constructed by Knockemdown Constructions in Sydney and the Central Coast.

- a What other name is given to a sector graph?
- b In which region does the company build the:
- i most houses?
 - ii least houses?
- c If 45 houses were built altogether, how many of these were in the southern suburbs?

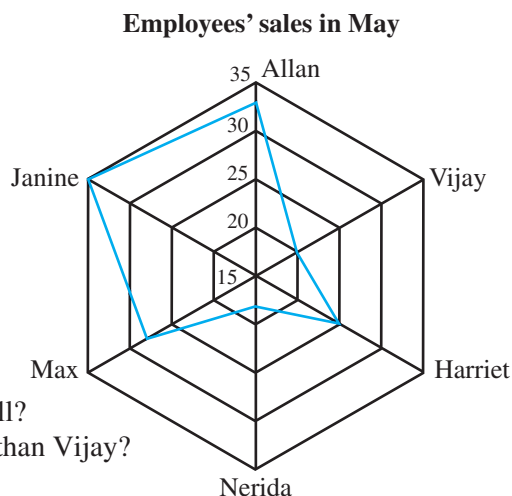
- 12 At Esperanto High School, every student in Year 8 must study a language. The table below shows the number of students who studied each of the four languages that were offered. Draw a sector graph with a radius of 3 cm to represent this data.

Language	French	Spanish	Cantonese	Japanese
Number of students	42	29	11	38

Further applications

- 13 A computer store manager drew this radar chart to compare the performance of his salespeople. The graph shows the number of computers sold during May by each employee.

- How many computers were sold last month by Harriet?
- Who sold 28 computers?
- Who sold the most computers? How many did they sell?
- Who sold the least? How many did they sell?
- How many more computers did Allan sell than Vijay?



6.2 Organising data

Once data has been *collected*, it must then be *organised* into a table or graph so that it can be *analysed*. In Year 8 you learned to organise statistical data into a frequency distribution table, histogram, polygon, dot plot and stem and leaf plot. The data in the diagrams below relate to the number of cars per household in a small street.

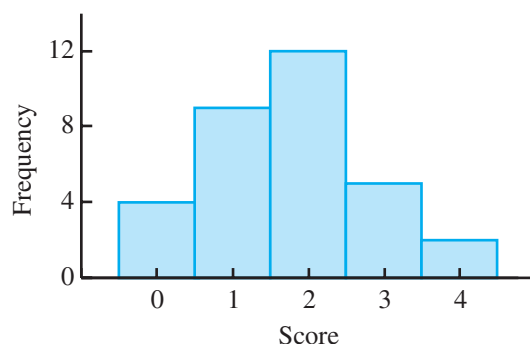
The frequency distribution table

In a frequency distribution table:

- the scores are placed in the left-hand column
- a tally column may be used to enter the scores one at a time
- the frequency indicates the number of times that each score occurs.

Score	Tally	Frequency
0		4
1		9
2		12
3		5
4		2
$\Sigma f =$		32

The frequency histogram



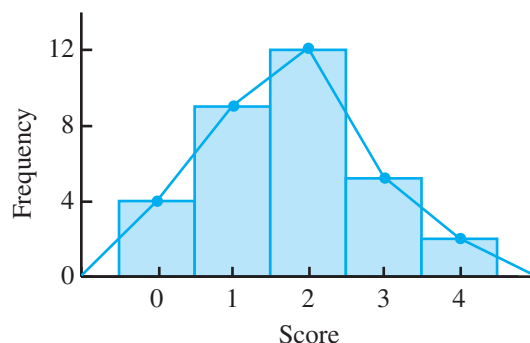
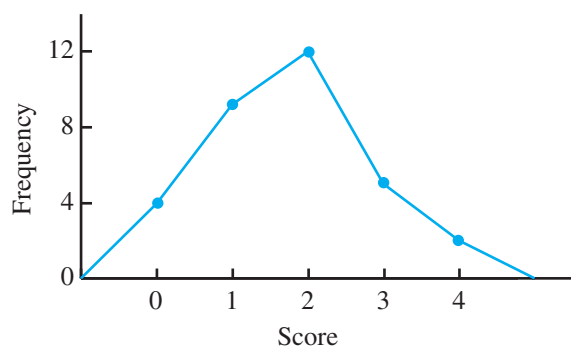
The frequency histogram is a type of column graph. In a histogram:

- the scores are placed on the horizontal axis
- the frequencies are shown on the vertical axis
- the columns straddle the scores and are drawn next to each other without a gap
- a space of half the width of one column is left on the horizontal axis before the first column is drawn.

The frequency polygon

The frequency polygon is a type of line graph. In a polygon:

- the scores are placed on the horizontal axis
- the frequencies are shown on the vertical axis
- the polygon begins and ends on the horizontal axis
- the first score is marked one full unit away from the vertical axis.

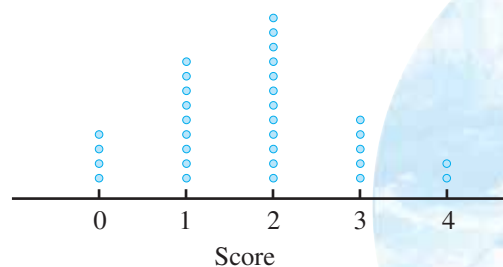


When a polygon and histogram are drawn on the same set of axes, the polygon joins the midpoints of the tops of the columns. The area under the histogram is equal to the area under the polygon.

The dot plot

The dot plot is a simplified version of the histogram. In the dot plot:

- the scores are placed along a horizontal line
- one dot is placed above the score in a vertical line each time that score occurs.



Clusters or bunches are easily seen, as well as any **outliers**, that is, scores that are a long way from the other scores.

The stem and leaf plot

The stem and leaf plot is similar to a histogram that has been drawn on its side, except that the rows are made up of digits. In the stem and leaf plot:

- the first part of each score is called the **stem**, and is written on the left-hand side of the plot
- the remaining part of the number is called the **leaf**, and is written on the right-hand side of the plot.

Stem	Leaf
2	8 9
3	2 5 6 8
4	4 7 8
5	1 3

For example, the ordered stem and leaf plot above shows the scores 28, 29, 32, 35, 36, 38, 44, 47, 48, 51, 53.

When a large number of scores begin with the same digit(s), the scores can be written in class intervals of 5. For example, this stem and leaf plot shows the scores 70, 71, 74, 75, 77, 77, 78, 80, 81, 82, 83, 84, 87, 89.

Stem	Leaf
7 ⁽⁰⁾	0 1 4
7 ⁽⁵⁾	5 7 7 8
8 ⁽⁰⁾	0 1 2 3 4
8 ⁽⁵⁾	7 9

EG
+S

Example 1

A fruit grower delivered his bananas to the market. The crates were then opened and the contents of each crate were counted. The number of bananas per crate is shown below. Show this data in a:

a frequency distribution table

b frequency histogram

c frequency polygon

d dot plot

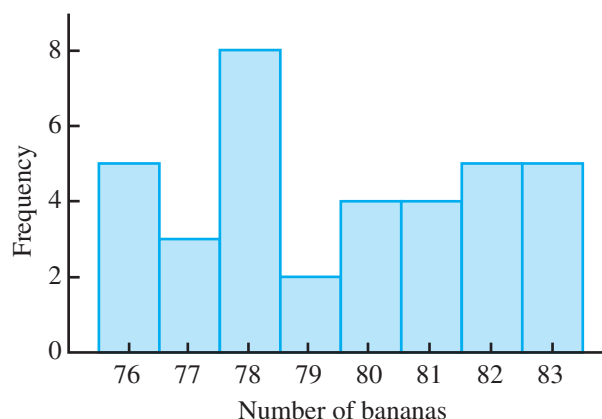
78 78 81 76 83 82 80 79 77 78 83 78
81 82 83 76 79 78 76 77 80 82 82 83
78 80 76 81 83 78 78 77 76 82 81 80

Solutions

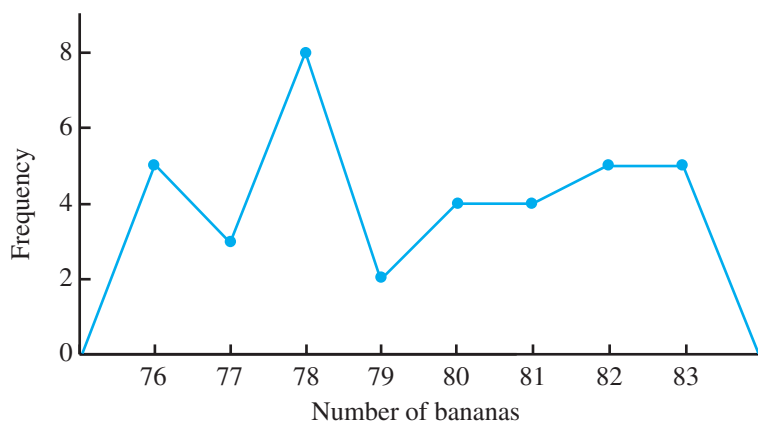
a

Number of bananas	Tally	Frequency
76		5
77		3
78		8
79		2
80		4
81		4
82		5
83		5
$\Sigma f =$		36

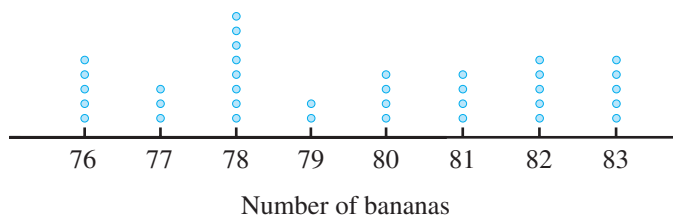
b



c



d

EG
+S**Example 2**

The stem and leaf plot shows the essay marks out of 25 for a group of Year 9 history students.

- How many students are in the class?
- Write down the highest and lowest marks.
- How many students scored more than 60%?

Stem	Leaf
0 ⁽⁰⁾	3 4
0 ⁽⁵⁾	8 9 9
1 ⁽⁰⁾	2 2 3 4 4 4
1 ⁽⁵⁾	5 6 6 7 7 7 8 8
2 ⁽⁰⁾	0 0 1 1 1 2 2 3 4

Solutions

- To find the number of students, count the leaves. There are 28 students in the class.
- The highest mark was 24 and the lowest mark was 3.
- $60\% \times 25 = 15$. There are 16 students with a mark greater than 15.

Exercise 6.2

1

Score	Tally	Frequency
12	III	
13	HHH	
14	IIII	
15	HHH III	
16	HHH II	
17	II	
$\Sigma f =$		

- Copy and complete this frequency distribution table.
- Draw a frequency histogram and polygon on the same set of axes to illustrate the data.

- 2 A teacher marked his Year 9 students' essays out of 10 and posted the following results.

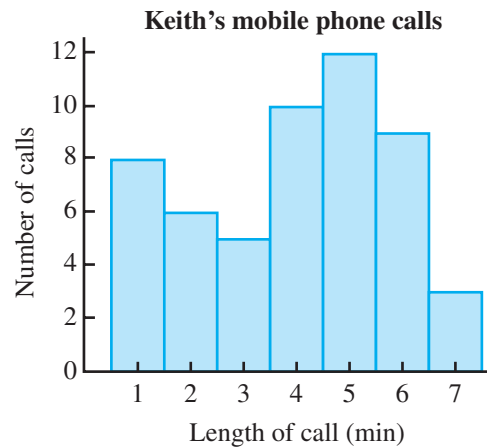
9	4	6	4	8	5	7	6
8	8	6	3	9	6	6	6
7	2	3	10	5	7	6	9

- Organise the data into a frequency distribution table with score, tally and frequency columns.
- How many students handed in an essay?
- What were the highest and lowest scores?
- What mark was scored by most students?
- What percentage of students scored 6 out of 10?

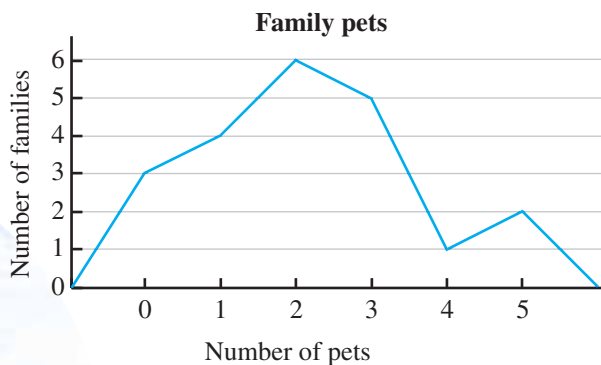
Consolidation

- 3 The histogram shows Keith's mobile phone call times last week, in minutes.

- How many calls lasted for 4 minutes?
- For how many calls did he talk for 3 minutes or less?
- How many telephone calls did he make?
- Draw a frequency polygon to illustrate the data.



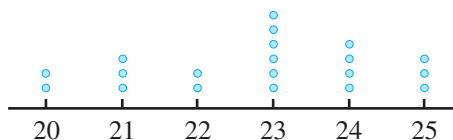
4



A community group was surveyed to find out the number of pets that each family owned. The results of the survey were recorded in this frequency polygon.

- What is the greatest number of pets in any family?
- What was the most typical number of pets per family?
- How many families had exactly 2 pets?
- What percentage of families had more than 2 pets?

5



- a Copy and complete this frequency table using the data in the dot plot.

Score	20	21	22	23	24	25
Frequency						

- b What fraction of the scores are either even or divisible by 3?

- c Show this data on a frequency histogram and polygon.

- 6 40 boxes of matches were opened and their contents counted to check the accuracy of an automatic packing machine in a factory. The number of matches in each box is given below.

54 49 52 50 51 50 48 49 49 51
 53 50 50 51 49 47 52 50 50 50
 51 52 48 49 47 50 52 53 54 48
 48 47 48 47 49 50 51 50 52 55

- a Draw a dot plot to illustrate this data.

- b How many matches do you think were supposed to be in each box?

- 7 Study each of the dot plots below.

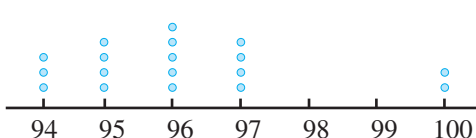
- i Are there any clusters? If so, where?

- ii Are there any outliers?

a



b



8

Stem	Leaf
10	6 4 9 3
11	7 8 4 2 7 1
12	8 5 2 0 8 6 1 9 3
13	7 6 7 2 4 1 0
14	5 6 0 9 1 4

Arrange the data in this stem and leaf plot into an ordered stem and leaf plot.

- 9 The stem and leaf plot shows the number of seeds per watermelon in a batch of watermelons.

- a How many watermelons are there?

- b What was the maximum number of seeds found in a single watermelon?

- c What was the most typical number of seeds in a watermelon?

- d In what percentage of the watermelons were there less than 50 seeds?

Stem	Leaf
3	1 2 5 6 7
4	0 3 3 4 7 8 8
5	2 2 2 2 5 7 7 9
6	4 4 5 6 8

10	Stem	Leaf
	0 ⁽⁵⁾	7 8 8 9
	1 ⁽⁰⁾	0 2 3 3 3 4
	1 ⁽⁵⁾	5 5 5 6 6 7 8 8 9
	2 ⁽⁰⁾	1 1 2 3 4 4
	2 ⁽⁵⁾	5 6 7 7 9

The stem and leaf plot shows the maximum daily temperatures in an outer Sydney suburb during one month.

- Find the difference between the highest and lowest temperatures in the table.
- On how many days was the temperature in the 20s?
- In which season do you think these temperatures were taken?

Further applications

- 11 50 students from Barnsley High School aged from 12 to 17 years were invited to take part in a swimming carnival. The number of 13 year olds was 2 more than the number of 12 year olds. The number of 14 year olds was 3 times the number of 12 year olds. The number of 15 year olds was twice the number of 14 year olds. There were equal numbers of 13 year olds and 17 year olds and there were twice as many 16 year olds as 17 year olds.

- Find the number of students that were invited from each age group and hence complete this frequency table.

Age	12	13	14	15	16	17
Number of students						

- Which age group had the highest representation?
- Find the average age of the swimming squad, correct to 1 decimal place.

6.3 Analysing data

Measures of central tendency

When data has been collected and organised into a table or graph, the next step is to analyse this information. We can try to find a value that is typical or representative of the data. In particular, we often speak of an **average**; however, the term 'average' is frequently used incorrectly. Depending on the context, the word average may refer to the middle score (i.e. the *median*), the score that occurs most frequently (i.e. the *mode*) or to the sum of the scores divided by the number of scores (i.e. the *mean*). Each of these measures is central to the data in some sense. Hence, they are called **measures of central tendency**.

The mean

The **mean** is the sum of the scores divided by the number of scores. The symbol for the mean is $\bar{x}$.

$$\text{Mean } (\bar{x}) = \frac{\text{sum of the scores}}{\text{number of scores}}$$

i.e. $\bar{x} = \frac{\sum x}{n}$ where

- $\bar{x}$ is the mean
- $\sum x$ is the sum of the scores
- n is the number of scores

The mean of the data in a frequency distribution table is given by:

$$\bar{x} = \frac{\sum fx}{\sum f} \quad \text{where}$$

- $\bar{x}$ is the mean
- $\sum fx$ is the sum of the scores
- $\sum f$ is the number of scores

To find the mean of a large number of multiple scores using a calculator:

- ⊙ set the calculator to statistics mode (SD)
- ⊙ clear the statistics memory by pressing **2ndF** **AC**
- ⊙ enter multiple scores by pressing score **×** frequency **M+** (or score **2ndF** **'** frequency **M+** depending on the calculator)
- ⊙ repeat this until all of the scores have been entered
- ⊙ press the mean key **$\bar{x}$** .

NOTE: These steps may vary for different calculators.

■ The median

When a set of scores has been arranged in ascending order, the **median** is the number in the middle; that is, the number of scores below it is equal to the number of scores above it. The median is equal to the middle score if the number of scores is odd, or to the average of the two middle scores if the number of scores is even.

When a set of n scores has been arranged in ascending order, the median is:

- ⊙ the $\left(\frac{n+1}{2}\right)$ th score if n is odd
- ⊙ the average of the $\frac{n}{2}$ th and $\left(\frac{n}{2} + 1\right)$ th scores if n is even.

■ The mode

The **mode** is the score with the highest frequency. That is, it is the score that occurs more times than any other. If a set of scores has two or more scores each with the highest frequency, then there would be two or more modes.

The mode is the score with the highest frequency.

■ Measure of spread

Measure of spread refers to the way in which data is spread out or clustered together. The simplest measure of spread is the **range**. This tells us the difference between the highest and lowest scores but only shows the extreme values in a distribution. It does not tell us whether the scores bunch up or how spread out they might be relative to each other. In Year 10 you will examine other measures of spread that give this information.

Range = highest score – lowest score.

Example 1

Find the mean of 3, 8, 2, 5, 7, 1, 5, 1.

Solution

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of the scores}}{\text{number of scores}} \\ &= \frac{3 + 8 + 2 + 5 + 7 + 1 + 5 + 1}{8} \\ &= \frac{32}{8} \\ &= 4\end{aligned}$$

Example 2

Copy and complete this frequency distribution table, then find the mean, correct to 2 decimal places.

Score (x)	Tally	Frequency (f)	fx
7	IIII		
8	IIII II		
9	IIII IIII		
10	IIII		
11	IIII I		
12	II		
Totals =			

Solution

Score (x)	Tally	Frequency (f)	fx
7		4	28
8		7	56
9		10	90
10		3	30
11		6	66
12		2	24
		$\sum f = 32$	$\sum fx = 294$

$$\begin{aligned}\bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{294}{32} \\ &= 9.19 \\ &\quad \text{(2 decimal places)}\end{aligned}$$

Example 3

Find the mean of these scores, correct to 2 decimal places.

Score	1	2	3	4
Frequency	8	7	12	10

Solution

Set the calculator to statistics mode and clear the memory. Enter the scores as either:

$$1 \times 8 \text{ [M+]}$$

$$2 \times 7 \text{ [M+]}$$

$$3 \times 12 \text{ [M+]}$$

$$4 \times 10 \text{ [M+]}$$

or $1 \text{ [2ndF] ['] 8 [M+]}$

$2 \text{ [2ndF] ['] 7 [M+]}$

$3 \text{ [2ndF] ['] 12 [M+]}$

$4 \text{ [2ndF] ['] 10 [M+]}$

Press the mean key $\bar{x}$.

Answer: $\bar{x} = 2.65$ (2 decimal places)

Example 4

Find the median of each set of scores.

a 12, 17, 13, 19, 8, 23, 10

b 42, 18, 36, 14, 29, 7

Solutions

a In ascending order, the scores are 8, 10, 12, 13, 17, 19, 23. There are an odd number of scores and the middle score is 13. Therefore, the median is 13.

b In ascending order, the scores are 7, 14, 18, 29, 36, 42. There are an even number of scores and the two middle scores are 18 and 29. The median is the number that lies halfway

$$\begin{aligned}\text{between 18 and 29. Median} &= \frac{18 + 29}{2} \\ &= 23.5\end{aligned}$$

EG
+
S**Example 5**

Find the mode of each set of scores.

a 3, 7, 9, 7, 8, 2, 4, 7**b** 15, 26, 17, 15, 20, 17, 11**Solutions****a** There are more 7s than any other score. Therefore, 7 is the mode.**b** There are two 15s and two 17s, which is more than any other score. Therefore, 15 and 17 are both modes.EG
+
S**Example 6**

Find the range of the scores 19, 52, 37, 66, 102, 36, 99.

Solution

$$\begin{aligned}\text{Range} &= \text{highest score} - \text{lowest score} \\ &= 102 - 19 \\ &= 83\end{aligned}$$

Exercise 6.3**1** Find the mean of each set of scores, correct to 1 decimal place where necessary.**a** 9, 7, 10, 3, 6, 5**b** 14, 19, 11, 14, 20, 16, 17**c** 37, 61, 72, 90, 83, 55, 46, 12**d** 21.6, 22.3, 20.9, 25.2, 29.4**e** 42.9, 50.1, 36.5, 62.7, 31.2, 53.8**f** 5, -2, 8, 4, -3, -7, 0, -9**2** Find the median of each set of scores.**a** 20, 34, 17, 15, 41, 38, 9**b** 7, 6, 1, 9, 4, 10, 3, 4, 12**c** 182, 101, 147, 118, 132**d** 12.3, 6.8, 11.4, 19.1, 17.5, 14.8, 20.2**e** 8, 5, 2, 10, 6, 6**f** 29, 46, 72, 51, 18, 40, 67, 33**g** 4, 9, 8, 3, 9, 7, 0, 4, 7, 3**h** 17.6, 13.5, 14.1, 8.2, 19.7, 10.4**3** Find the mode of each set of scores.**a** 3, 8, 6, 1, 8, 4, 9**b** 14, 16, 11, 10, 18, 11, 17, 5**c** 42, 37, 39, 42, 30, 39, 21, 45**d** 106, 110, 106, 103, 110, 110, 106**4** Find the range of each set of scores.**a** 5, 9, 4, 8, 2, 10**b** 21, 13, 17, 25, 19, 8, 23**c** 9.4, 7.2, 6.6, 8.7, 1.5**d** -3, 9, 4, -7, 0, -2, 11

■ Consolidation

- 5 Find the mean, median, mode and range for each set of scores, correct to 1 decimal place where necessary.

- a 36, 30, 37, 38, 30, 31, 34 b 213, 251, 240, 239, 251, 295, 201
c 5.9, 15.3, 20.6, 5.9, 18.7, 5.9 d -46, 17, 0, -3, 75, 31, -20, -64

- 6 Use your calculator to find the mean of each set of scores, correct to 2 decimal places.

a

Score	1	2	3	4	5
Frequency	4	7	9	12	6

b

Score	10	11	12	13	14	15
Frequency	3	16	4	10	8	11

c

Score	48	49	50	51	52
Frequency	23	15	20	18	21

d

Score	95	96	97	98	99	100
Frequency	18	10	3	7	3	14

- 7 Find the median, mode and range of each set of scores in Q6.

- 8 How many scores are in a set if:

- a the median is the 17th score and there are an odd number of scores?
b the median lies between the 23rd and 24th scores and there are an even number of scores?
c there are 12 scores below the median and there are an odd number of scores?
d there are 39 scores after the median and there are an even number of scores?

- 9 Copy and complete these frequency distribution tables, then find the mode and mean, correct to 1 decimal place where necessary.

a

x	Tally	f	fx
10			
11			
12			
13			
14			
15			

b

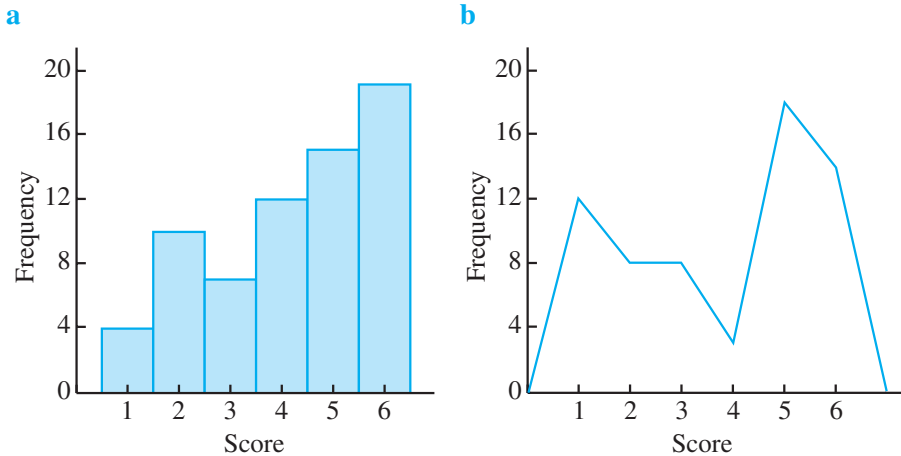
x	Tally	f	fx
54			
55			
56			
57			
58			
59			

- 10 The number of points scored in each game by a soccer team during the season is shown below.

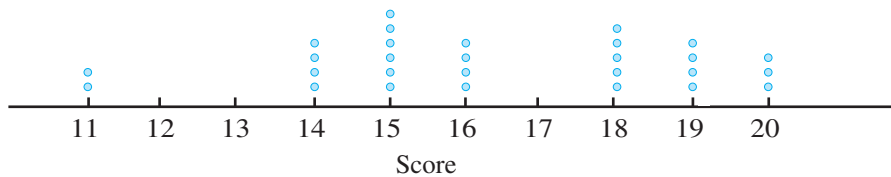
1	3	2	5	1	0	1	3	2	4	0	1
2	0	1	1	3	2	0	0	1	0	4	2
1	1	3	2	0	1	1	5	3	1	2	1

- a Organise the data into a frequency distribution table with score (x), tally, frequency (f) and fx columns.
- b Calculate the mean, correct to 1 decimal place.
- c Write down the mode.
- d Find the median.
- e In what percentage of games did the team fail to score? Answer correct to the nearest whole per cent.

11 Find the mean, mode, median and range for the data shown in each diagram.



12 The students in 9C were given a list of 20 countries and asked to name the capital city of each. The dot plot shows the number of correct responses by each student.



- a Find the mean and mode for the data.
- b Which scores could be classified as outliers?
- c Find the mean of the scores, excluding the outliers.

13 This stem and leaf plot shows the ages of patients who arrived after 9 pm at a certain medical centre on Saturday night.

- a How many patients visited the centre?
- b What was the most typical age of a patient?
- c What was the median age of the patients?
- d What was the average age of the patients?
- e What was the range of the patients' ages?

Stem	Leaf
0	8 9
1	1 2 4 6 7 7
2	0 1 3 4 5 5 6 8 9
3	3 4 8 8
4	2 2 2 4 5 9
5	0 1 3 3 8
6	2 5 5 6 7 8

14	Stem	Leaf
	10 ⁽⁰⁾	1 4
	10 ⁽⁵⁾	7 5 6 9 8 6
	11 ⁽⁰⁾	4 2 3 2 0 4 4
	11 ⁽⁵⁾	9 7 8 8 7 6
	12 ⁽⁰⁾	3 0 1 2 0
	12 ⁽⁵⁾	5 5 5

The stem and leaf plot shows the heights of children in a Year 6 class in centimetres.

- a** Arrange the data into an ordered stem and leaf plot.
- b** Find the modal height(s).
- c** What is the range of the students' heights?
- d** Find the median height.

- 15** The data below shows the number of grapes on each bunch of grapes on sale at a local fruit shop.

47	36	42	37	45	48	41	35	28
36	33	24	44	32	27	35	31	40
43	47	34	28	38	32	46	33	36

- a** Arrange the data into a stem and leaf plot with stems 2⁽⁰⁾, 2⁽⁵⁾, etc.
- b** What is the modal number of grapes?
- c** What is the median number of grapes?

Further applications

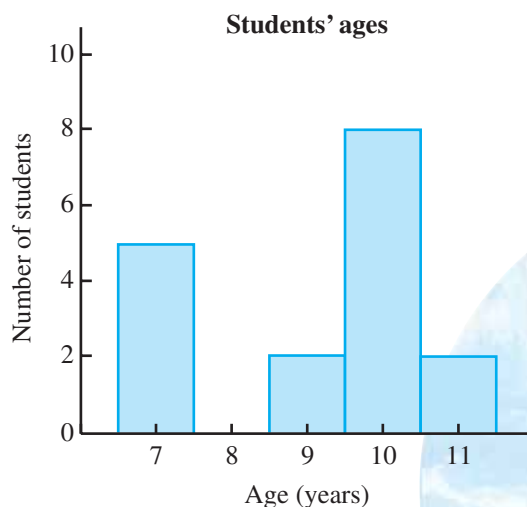
- 16** The maximum temperatures on the first 9 days of July in a small town were:

3°C, 4°C, 3°C, 7°C, 5°C, 3°C, 4°C, 5°C, 6°C

Explain carefully what the temperature could have been on 10 July if for the first 10 days:

- a** the modal temperature was 3°C
- b** the temperature range was 5°C
- c** the median temperature was 4°C
- d** the mean temperature was 4°C

- 17** The histogram shows the ages of a group of children. Each child is aged from 7 to 11 years. Copy and complete the histogram so that the average age of the children is exactly 9 years.



6.4 Problems involving the mean

The mean can be affected by the addition (or subtraction) of scores from a set. When the new score(s) are well above or well below the mean, the mean may increase or decrease significantly.

When a score x is added to a set of scores and x is:

- ⊙ greater than the mean, the mean will increase
- ⊙ less than the mean, the mean will decrease
- ⊙ equal to the mean, the mean will stay the same.

Example 1

EG
+
S

- a A set of 13 scores has a mean of 15. Find the sum of the scores.
- b A set of scores has a mean of 7 and a sum of 119. Find the number of scores.

Solutions

$$\text{a} \quad \bar{x} = \frac{\sum x}{n}$$

$$15 = \frac{\sum x}{13}$$

$$\times 13 \quad \times 13$$

$$\sum x = 15 \times 13$$

$$\therefore \sum x = 195$$

$\therefore$ The sum of the scores is 195.

$$\text{b} \quad \bar{x} = \frac{\sum x}{n}$$

$$7 = \frac{119}{n}$$

$$\times n \quad \times n$$

$$7n = 119$$

$$\div 7 \quad \div 7$$

$$\therefore n = 17$$

$\therefore$ There are 17 scores.

Example 2

EG
+
S

The mean of 3 scores is 14. If two of the scores are 17 and 9, find the third score.

Solution

$$\bar{x} = \frac{\sum x}{n}$$

$$14 = \frac{9 + 17 + x}{3}$$

$$14 = \frac{x + 26}{3}$$

$$\times 3 \quad \times 3$$

$$42 = x + 26$$

$$-26 \quad -26$$

$$\therefore x = 16$$

$\therefore$ The third score is 16.

EG
+ S**Example 3**

Without performing any calculations, state whether a mean of 50 will increase, decrease or stay the same when a score of:

a 20 is added**b** 90 is added**c** 50 is added**Solutions****a** The score being added is less than the mean, $\therefore$ the mean will decrease.**b** The score being added is greater than the mean, $\therefore$ the mean will increase.**c** The score being added is equal to the mean, $\therefore$ the mean will stay the same.EG
+ S**Example 4**

The mean of a set of 18 scores is 12. Find the mean of the scores, correct to 2 decimal places, after a score of 25 is added.

Solution**i** Find the sum of the scores before the extra score is added:

$$\bar{x} = \frac{\sum x}{n}$$

$$12 = \frac{\sum x}{18}$$

$$\times 18 \quad \times 18$$

$$\therefore \sum x = 216$$

ii After the extra score is added, the sum of the scores will increase by 25, while the number of scores will increase by 1.

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{216 + 25}{18 + 1}$$

$$= \frac{241}{19}$$

$$= 12.68 \text{ (2 decimal places)}$$

Exercise 6.4

- 1 a** A set of 5 scores has a sum of 30. Find the mean of the scores.
- b** The sum of a set of 16 scores is 624. What is the mean of the scores?
- c** Find the mean of a set of 27 scores whose sum is 788.4.
- 2 a** A set of 9 scores has a mean of 4. Find the sum of the scores.
- b** The mean of a set of 22 scores is 57. What is the sum of the scores?
- c** Find the sum of a set of 35 scores whose mean is 12.8.
- 3 a** A set of scores has a mean of 7 and a sum of 28. Find the number of scores.
- b** The mean of a set of scores is 15 and the sum of the scores is 540. What is the number of scores?
- c** Find the number of scores in a set whose mean is 43.6 and sum is 2616.

- 4** **a** The mean of a set of three scores is 20. If two of the scores are 35 and 15, find the 3rd score.
b The mean of a set of four scores is 17. If three of the scores are 11, 18 and 23, find the 4th score.
- 5** A set of 15 scores has a mean of 24. Find, correct to 1 decimal place, the new mean after each of these scores is added to the set.
a 48 **b** 17 **c** 24
- 6** State whether the mean will increase, decrease or stay the same when a score is added to a set and the score is:
a greater than the mean **b** less than the mean **c** equal to the mean
- 7** A set of 28 scores has a mean of 16. Find, correct to 1 decimal place, the new mean after each of these scores is taken out of the set.
a 40 **b** 10 **c** 16
- 8** State whether the mean will increase, decrease or stay the same when a score is taken out from a set and the score is:
a greater than the mean **b** less than the mean **c** equal to the mean
- 9** The average weight of a forward in a certain Rugby team is 94 kg. Find the approximate weight of the forward pack, given that there are 8 forwards in a Rugby team.
- 10** After 16 games this season, Meera's goal average in netball is 5.5 goals per game. How many goals has she scored so far this season?
- 11** The average weight of a baby in a maternity ward is 3.2 kg and the total weight of the babies is 80 kg. How many babies are there?
- 12** Makhaya opens the batting for his cricket team. His first four scores this season have been 17, 82, 43 and 35. How many runs must he score in his next innings to have a batting average of 50?
- 13** Janine was absent on the day her class sat for a mathematics test. Her teacher marked the tests that night and found that the class average was 72% for the 29 students who sat for the test. Janine returned to school the next day and scored 66% on the test.
a What effect will her mark have on the class average? Why?
b Calculate the new average, correct to 1 decimal place.
c The teacher decided that Joelle's mark of 18% was an outlier. She decided to work out the mean again, taking this score out of the calculation. What effect will the loss of this mark have on the class average? Why?
d Calculate the new average, including Janine's mark and excluding Joelle's mark.

Further applications

- 14 a** The mean of a set of 6 scores is 13. After a 7th score is added to the set, the new mean is then 15. Find the 7th score.
- b** The mean of a set of 11 scores is 27. After a 12th score is added to the set, the new mean is then 30. Find the 12th score.
- 15 a** The mean of a set of 19 scores is 31. When one of the scores is taken out of the set, the new mean is then 29. Find the score that was taken out.
- b** The mean of a set of 44 scores is 52. When one of the scores is taken out of the set, the new mean is then 53. Find the score that was taken out.
- 16** Herschel has an average of 76% on the first four tests of the semester. Find the highest possible mark he can score this semester if he has two more tests to go and each test contributes equally towards his report mark.
- 17** A real estate agent sold 15 houses last month at an average price of \$420 000. A second agent sold 25 houses at an average price of \$480 000. What was the average price overall for the houses sold by these two agents last month?

TRY THIS The English language

Use the skills learned in this chapter to investigate some or all of the following.

- a** How often is each of the letters of the alphabet used? A newspaper or a novel would be a great source to test.
- b** Using your dictionary estimate how many words start with a, b, . . . z.
- c** What is the average length of a word? Look at words in both adults' and children's books.
- d** What is the average number of words in a sentence? Compare words found in adults' and children's books.
- e** Can you formulate a rule based on word length that will decide whether a book is suitable to be read by, for example, a 6-year-old, an 8-year-old or a 12-year-old child?

6.5 Cumulative frequency

The **cumulative frequency** column (*cf*) in a frequency distribution table gives a progressive total of the frequencies. Each cumulative frequency represents the sum of the frequencies for that score and those scores that are less than it. The cumulative frequency column can be used to answer questions such as 'how many scores are less than or equal to 8?' It can also be used to calculate the median.

A frequency histogram and polygon can be drawn using the frequencies as the heights of the columns. Similarly, a cumulative frequency histogram and cumulative frequency polygon, or **ogive**, can be drawn using the cumulative frequencies.

To draw an ogive:

- ⊙ draw a cumulative frequency histogram with the columns having heights equal to the cumulative frequencies of the scores
- ⊙ join the top right-hand corners of the columns with a line graph, starting with the bottom left-hand corner of the first column.

NOTE: The ogive finishes at the top of the last column and is not drawn back down to the horizontal axis as for the frequency polygon.

The median can also be determined graphically from the ogive.

To find the median of a set of discrete, individual scores from an ogive:

- ⊙ draw a horizontal line from the halfway mark on the vertical axis to the ogive
- ⊙ draw a vertical line down to the horizontal axis
- ⊙ read off the median.

Example 1

For each of these frequency tables:

- i add a cumulative frequency column
- ii write down the number of scores that are less than 4
- iii find the median

a

x	f
1	3
2	5
3	9
4	7
5	12
6	13
$\sum f =$	49

b

x	f
1	4
2	2
3	9
4	10
5	3
6	2
$\sum f =$	30

Solutions

a i	x	f	Cumulative frequency
	1	3	3
	2	5	8
	3	9	17
	4	7	24
	5	12	36
	6	13	49
	$\Sigma f =$	49	

- ii The cumulative frequency for 3 is 17.
 $\therefore$ There are 17 scores less than 4.
- iii There are 49 scores, so the median is the $\left(\frac{49+1}{2}\right)$ th, or 25th score. Now, the 24th score is a 4, so the 25th score is a 5.
 $\therefore$ The median is 5.

b i

x	f	Cumulative frequency
1	4	4
2	2	6
3	9	15
4	10	25
5	3	28
6	2	30
$\Sigma f =$		30

- ii The cumulative frequency for 3 is 15.
 $\therefore$ There are 15 scores less than 4.
- iii There are 30 scores, so the median lies halfway between the 15th and 16th scores.
 Now, the 15th score is a 3 and the 16th score is a 4.
 $\therefore$ The median is 3.5.

EG
+S**Example 2**

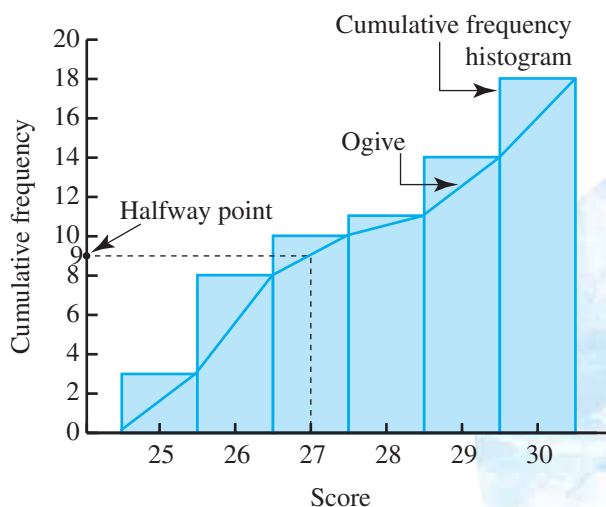
Draw a cumulative frequency histogram and ogive for the data in this frequency distribution table and hence find the median.

Score	Frequency	Cumulative frequency
25	3	3
26	5	8
27	2	10
28	1	11
29	3	14
30	4	18
$\Sigma f =$		18

Solution

As there are 18 scores, the halfway mark on the vertical axis is at 9.

The median of the scores in this distribution is 27.



Exercise 6.5

- 1 Roman programmed his computer to simulate tossing 10 coins 50 times. The results are shown in the frequency distribution table below.

a Copy and complete this frequency distribution table.

Number of tails	Tally	Frequency (f)	Cumulative frequency (cf)
2	III		
3	IIII		
4	IIII IIII		
5	IIII IIII IIII		
6	IIII IIII I		
7	IIII		
8	III		
		$\Sigma f =$	

b On how many occasions were there:

i exactly 4 tails?	ii 4 tails or less?	iii less than 4 tails?
iv exactly 6 tails?	v 6 tails or less?	vi less than 6 tails?
vii exactly 8 tails?	viii 8 tails or less?	ix less than 8 tails?

- 2 Copy and complete these frequency distribution tables, then find the median using the cumulative frequency column.

a

Score (x)	Tally	f	cf
1	IIII II		
2	III		
3	IIII II		
4	IIII I		
5	IIII IIII		
6	IIII IIII		
7	IIII I		
		$\Sigma f =$	

b

Score (x)	Tally	f	cf
100	IIII II		
101	IIII IIII		
102	IIII IIII IIII II		
103	IIII IIII IIII		
104	IIII III		
105	IIII I		
106	IIII		
		$\Sigma f =$	

■ Consolidation

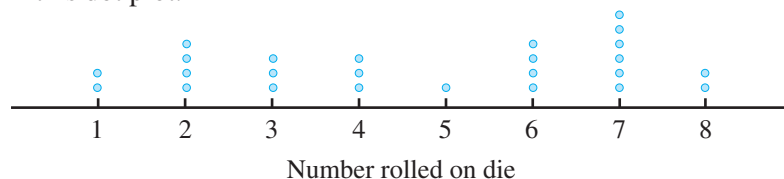
- 3 The number of take-away meals sold each day by a Chinese restaurant is shown below.

18	22	19	25	22	21	20	23	24	18
20	20	21	23	22	24	22	23	21	23
23	22	21	19	20	23	20	22	22	23
24	23	20	19	25	18	23	22	23	23

- Organise this data into a frequency distribution table with score, tally, frequency and cumulative frequency columns.
 - On how many days did the restaurant sell:
 - 20 meals?
 - less than 24 meals?
 - more than 22 meals?
 - at least 21 meals?
 - Use the cumulative frequency column to find the median number of meals sold per day.
- 4 A preschool teacher recorded the time taken by a group of children to individually complete a jigsaw puzzle. The completion times in minutes are:

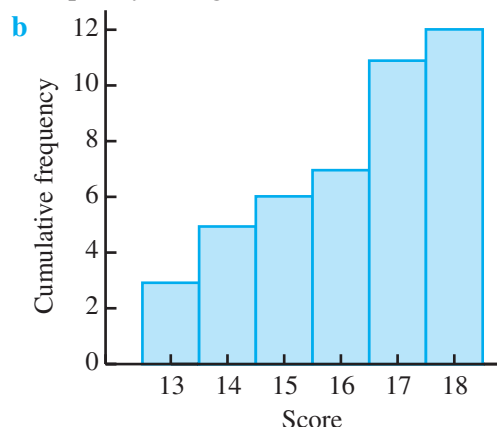
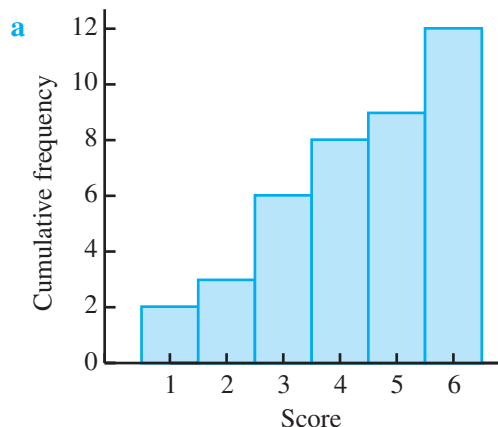
7	9	13	15	10	8	14	8	11	12
9	7	10	11	9	8	8	13	15	10
12	13	8	7	10	11	8	9	9	8

- Organise the data into a frequency distribution table and include a cumulative frequency column.
 - How many children took part?
 - What fraction of the children completed the puzzle in:
 - 10 minutes?
 - less than 9 minutes?
 - no more than 11 minutes?
 - What percentage of the children completed the puzzle in 12 minutes or less?
 - Find the median completion time.
- 5 An 8-sided die in the shape of a regular octahedron has its faces numbered from 1 to 8. The die was rolled 25 times and the number showing on the uppermost face each time was recorded in this dot plot.



- Draw a frequency distribution table with score, frequency and cumulative frequency columns.
- What is the median?
- How many scores are above the median but less than 8?

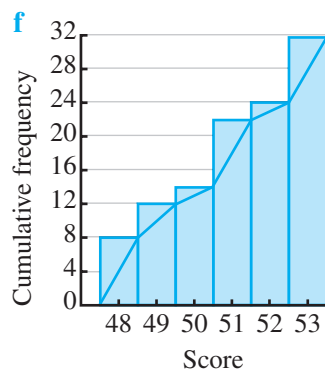
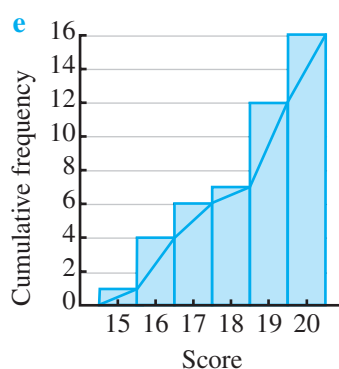
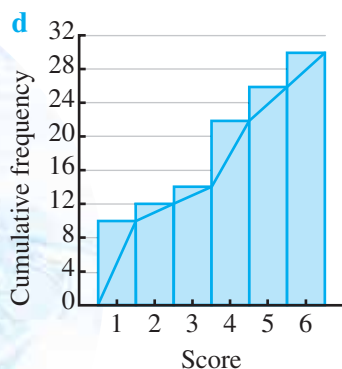
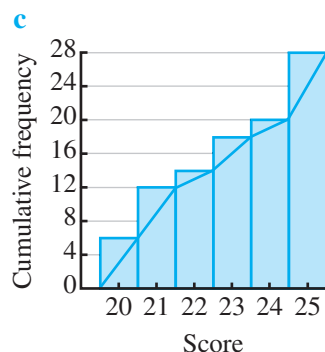
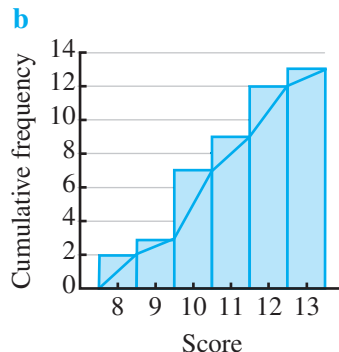
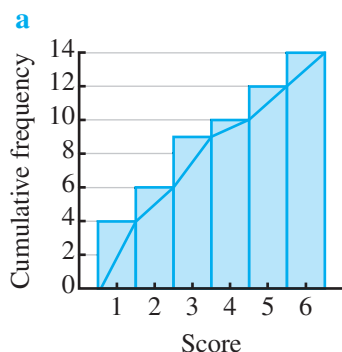
- 6 Draw a frequency distribution table with score, frequency and cumulative frequency columns for the data in each of these cumulative frequency histograms.



- 7 Draw on the same set of axes, a cumulative frequency histogram and ogive for the data in this table.

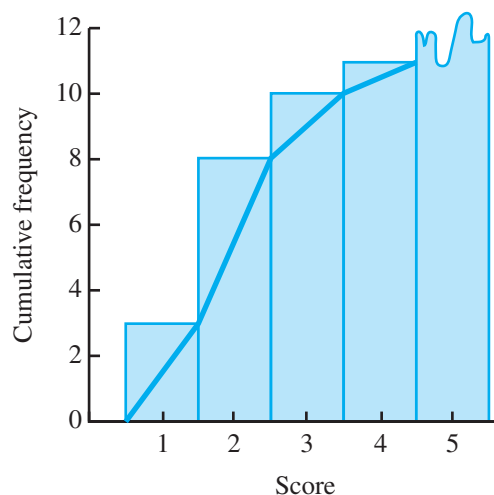
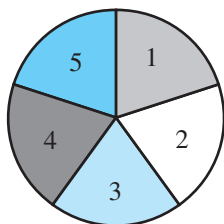
Score	80	81	82	83	84	85	86
Frequency	2	1	3	2	4	3	4

- 8 Use the ogive to find the median for each set of discrete data.



Further applications

- 9 Jocelyn threw a dart several times at the dart board shown and recorded her results in the form of a cumulative frequency histogram. However, she was interrupted while drawing the last column and did not complete it. If the median score was 3.5, find the number of times she hit the number 5.



- 10 Copy and complete this frequency distribution table and hence find the median of the scores.

Score (x)	Frequency (f)	Cumulative frequency	fx
1	3		
2		7	
3			12
4		20	
5	7		
6			60
7			
$\sum f =$		$\sum fx =$	189

6.6 Grouped data

Frequency graphs and tables can be constructed easily when the number of scores is small. However, this method is not very practical when there is a large number of scores. For example, how do we create a frequency distribution table for a set of 50 different scores that ranges from 1 to 100, and of what use would it be?

In these cases we group the data into class intervals, such as 1–10, 11–20 etc., to form a grouped frequency distribution. This allows us to organise the data into tables or graphs and hence make judgements about the data. We usually include the **class centres** in a grouped frequency distribution table. Class centres are the values that lie halfway between the lower and upper limits of each class interval. For example, in the class interval 11–15, the class centre is 13.

When grouping data:

- ⊙ the class intervals should not overlap
- ⊙ the intervals should be of equal width
- ⊙ all intervals within the range of the data should be included, even if they do not contain any scores in the distribution.

The main disadvantage with grouping data is that some information will be lost. For example, we no longer know what the individual scores are in the distribution so we cannot calculate the mean, median, mode or range exactly. We also can't determine whether the scores are distributed evenly within each class interval or are clustered together at one end. (In practice, we assume that the scores are evenly distributed.) Since individual scores are not known, we find the modal class rather than the mode. With single data we use an fx column, where fx is the frequency $\times$ the score. With grouped data we use an $(f \times cc)$ column, where $f \times cc$ is the frequency $\times$ the class centre.

NOTE: $\frac{\sum(f \times cc)}{\sum f}$ is then only an *estimate* for the mean, not the exact mean. Why?

Example 1

- a Organise the following scores into a grouped frequency distribution table with class, class centre (cc), tally, frequency (f) and $(f \times cc)$ columns. Use class intervals of 60–64, 65–69, etc.

65	72	69	73	84	79	66	77	81	75
66	68	78	83	74	89	84	75	71	68
70	81	64	62	67	72	75	88	81	86
65	63	72	79	80	67	87	66	73	71

- b Estimate the mean of the scores correct to 1 decimal place.
c What is the modal class?

Solutions

a

Class	Class centre (cc)	Tally	Frequency (f)	$f \times cc$
60–64	62		3	186
65–69	67		10	670
70–74	72		9	648
75–79	77		7	539
80–84	82		7	574
85–89	87		4	348
Totals:			40	2965

b
$$\bar{x} = \frac{\sum(f \times cc)}{\sum f}$$

$$= \frac{2965}{40}$$

$$= 74.1 \text{ (1 decimal place)}$$

- c The modal class is 65–69, since it contains 10 scores, which is more than any other class.

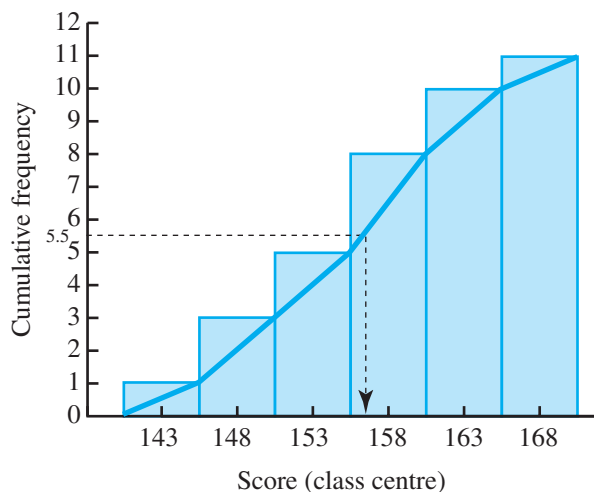
EG
+ S**Example 2**

- a** Draw a cumulative frequency histogram and ogive for the data in this grouped frequency distribution table.
- b** Use the ogive to estimate the median.

Class	Class centre	Frequency	Cumulative frequency
141–145	143	1	1
146–150	148	2	3
151–155	153	2	5
156–160	158	3	8
161–165	163	2	10
166–170	168	1	11
		$\Sigma f =$	11

Solutions

- a** With grouped data, the individual scores are not known. Therefore, we can only estimate the median.



- b** Median $\doteq 153 + (0.7 \times 5)$
 $= 156.5$

Exercise 6.6

- 1** Write down the class centre for each of these class intervals.

a 5–7**b** 10–14**c** 22–25**d** 30–35

- 2 a Copy and complete this grouped data frequency distribution table.

Class	Class centre (cc)	Tally	Frequency (f)	$f \times cc$
1–5				
6–10				
11–15				
16–20				
21–25				
26–30				
Totals:				

- b What do the totals $\sum f$ and $\sum (f \times cc)$ represent in a grouped data frequency table?
- 3 a Organise these scores into a frequency distribution table with class intervals of 20–24, 25–29, etc. Your table should include the following headings: class, class centre, tally, frequency and fx .

22	30	42	35	49	41	36	35	30	25
24	49	34	22	40	33	30	47	21	29
41	36	22	45	38	37	28	31	44	39

- b What is the modal class?
- c How many scores are there in the table?
- d Estimate the mean of these scores using the totals $\sum f$ and $\sum (f \times cc)$, correct to 1 decimal place.

Consolidation

- 4 A security firm has set a minimum height requirement of 160 cm for its security guards. The heights (in cm) of the guards are:

165	171	183	174	172	185	183	169	175
179	170	184	181	178	173	162	177	179
172	178	166	164	171	187	181	176	172
177	189	190	173	175	188	192	160	187
163	171	164	185	173	174	179	184	191

- a Organise the data into a frequency distribution table with class intervals of 160–164, 165–169, etc.
- b Draw a frequency histogram and polygon for the data, using the class centres on the horizontal axis.
- c What is the modal class?
- d Estimate the average height of the guards using the totals $\sum f$ and $\sum (f \times cc)$.

- 5 The table below gives an indication of the number of people that attended all first-grade Rugby league matches during a season. The figures have been rounded to the nearest 1000 people, then grouped.

Crowd (in '000s)	1–7	8–14	15–21	22–28	29–35	36–42
Frequency	15	52	45	39	27	4

- How many matches were played during the season?
- Write down the class centre for each class interval.
- Using the class centres, estimate:
 - the total number of people who attended matches during the season
 - the average match crowd size

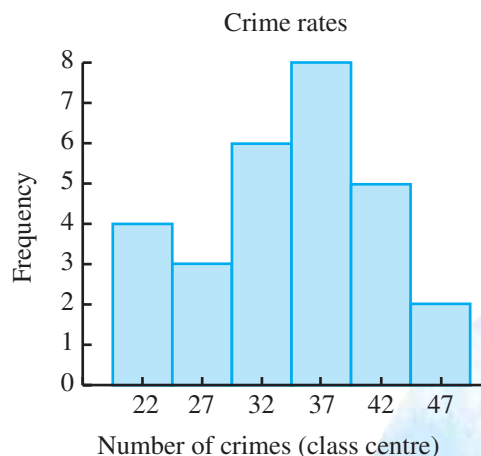
- 6 This stem and leaf plot shows the marks of a group of Year 11 students in a Business Studies examination marked out of 50.

Stem	Leaf
0	7 9 9
1	2 5 1 6 8 4 1 6
2	0 4 9 7 3 5 6 2 4 4
3	6 5 8 4 5 0 3 7 9 0 1 8
4	8 7 9 4 2 5 7

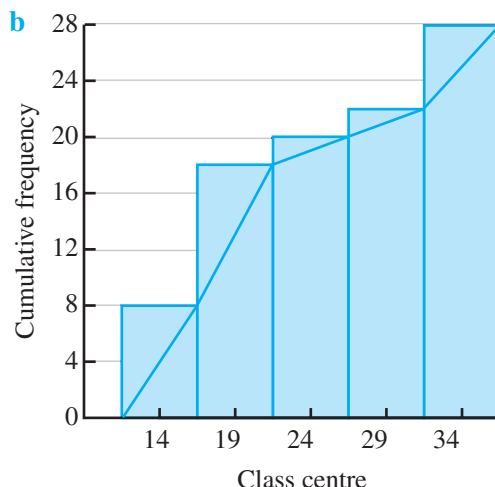
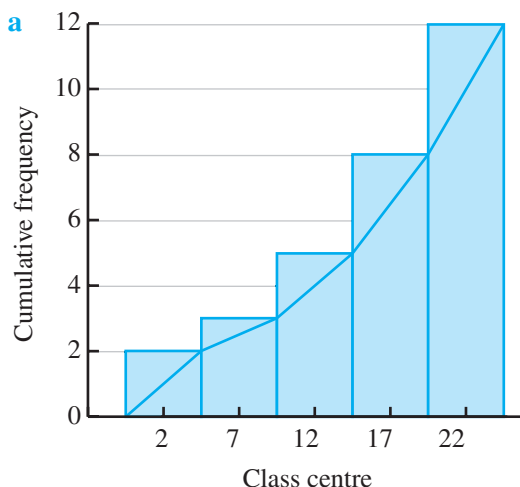
- Why would it be inappropriate to organise this data into a frequency distribution table with individual scores?
- Draw a grouped data frequency distribution table with class intervals of 0–9, 10–19, etc.
- Use the frequency table to estimate the mean of the scores.
- What is the modal class?
- What is the median class?

- 7 This histogram gives an indication of the number of crimes that were committed in a suburb over a six-month period.

- What are the class intervals?
- What is the greatest number of crimes that could have been committed in any one month?
- What is the modal class?
- Estimate the average number of crimes that occurred per month during this period.



- 8 Use these ogives to estimate the median score.



- 9 The daily number of calls made to the 000 telephone number over a period of 4 weeks is:

134	137	145	122	155	148	152
144	143	149	135	121	140	151
146	133	128	125	127	136	136
149	150	147	145	134	131	124

- a** Draw a grouped data frequency distribution table for this data, with class, class centre, tally, frequency and cumulative frequency columns. Use class intervals of 120–124, 125–129, etc.
- b** Draw a cumulative frequency histogram and ogive.
- c** Use the ogive to find the median number of 000 calls made per day.
- 10** This stem and leaf plot shows the average daily noise level in decibels (dB) at a construction site.
- a** Draw a cumulative frequency histogram and ogive for this data using class intervals 70–74, 75–79, etc. Write the class centres on the horizontal axis.
- b** Use the ogive to estimate the median daily noise level in decibels.

Stem	Leaf
7 ⁽⁰⁾	1 2 2 4
7 ⁽⁵⁾	5 5 7 8 9
8 ⁽⁰⁾	0 1 3 3 3 4 4
8 ⁽⁵⁾	6 6 7 7 8 8 9 9 9
9 ⁽⁰⁾	0 0 1 1 2 3
9 ⁽⁵⁾	5 6 7 8 8

Further applications

- 11** By convention, when calculating the mean of a grouped frequency distribution, we assume that the average score within each class is equal to the class centre. If, however, the scores are clustered about the lower end of each class interval and we calculate the mean in the usual manner, would our estimate for the mean be too large or too small? Why?

12 Consider the following set of scores.

15	16	18	19	26	26	27	29	30	31	31	33	40	41
43	44	46	46	47	48	50	50	51	53	55	57	57	58

- a Calculate the mean of the individual scores, correct to 2 decimal places.
- b Construct a frequency histogram with the following class intervals:
 - i 10–19, 20–29, ...
 - ii 15–24, 25–34, ...
 - iii 15–19, 20–24, ...
- c Write down the modal class of the data in each histogram.
- d Find the mean of the data in each histogram, correct to 2 decimal places where necessary. Compare these answers with the mean of the individual scores calculated in part a.
- e Which histogram best illustrates the distribution of the data? Why?



FOCUS ON WORKING MATHEMATICALLY

WORLD HEALTH



Introduction

The health of people from different nations around the world varies tremendously. In the so-called 'developed' countries, such as Japan, the United States and Australia, people can expect to live longer than others living in 'third world' countries, such as Malawi or Zambia. This statistic is called *life expectancy*. It is the average age at which people die in any given year. In 1999 in Australia, for example, this figure was 79 years. On the other hand in Zambia in Africa in 1999, the life expectancy was 38 years.

Another indicator of health is the number of infants who die in a country before the age of 1 year. In 1999 in Australia, five children died per 1000 live births. This is called the



infant mortality rate. In the early 1990s, young children were dying without apparent reason while asleep in their cots. This led to a huge increase in funding for research to find out why. Red Nose Day became the opportunity to give to this appeal. SIDS (Sudden Infant Death Syndrome), as it became known, has now been dramatically decreased. Not all countries are so lucky.

Statistical data on health is used by organisations like the World Health Organization and UNICEF to target support for countries in need. Documenting causes of death, such as malaria, typhoid, AIDS, heart disease and so on, is helpful information for immunisation programs, and educational programs to help people understand how to combat disease and malnutrition in their communities. It helps focus attention on the need for supporting clean water programs, water conservation, efficient farming, sewage disposal, hospital construction, training of health workers and so on. Today the concept of 'global health', in which attention is given to controlling disease across international boundaries, is widely accepted as a duty of care to the peoples of the world. The United Nations plays a major role in meeting this goal.



LEARNING ACTIVITIES

In the table below, two indicators of public health—life expectancy and infant mortality rates—are shown for 24 selected countries. The data is sorted by highest life expectancy in 1999 to lowest. The intention is to use the data to review the ideas you have learned in this chapter.

World health indicators for 24 selected countries

Country	Life expectancy at birth in years		Infant mortality rate per 1000 live births	
	1980	1999	1980	1999
Japan	76	81	8	4
Australia	74	79*	11	5†
France	74	79	10	5
Sweden	76	79	7	4
Finland	73	77	8	4
United Kingdom	74	77	12	6
United States	74	77	13	7
Cuba	74	76	20	7
Argentina	70	74	35	18
China	67	70	42	30
Brazil	63	67	70	32
Indonesia	55	66	90	42
Guatemala	57	65	84	40
India	54	63	115	71
Bangladesh	48	61	132	61
Iraq	62	59	80	101
Gambia	40	53	159	75



Country	Life expectancy at birth in years		Infant mortality rate per 1000 live births	
	1980	1999	1980	1999
Kenya	55	48	75	76
Mozambique	44	43	145	131
Ethiopia	42	42	155	104
Zimbabwe	55	40	80	70
Botswana	58	39	71	58
Malawi	44	39	169	132
Zambia	50	38	90	114

* As compared to 60 for Aboriginal and Torres Strait Islander Australians.

† As compared to 20 for Aboriginal and Torres Strait Islander Australians.

Source: 2001 World Development Indicators, World Bank at <www.devdata.worldbank.org/hnpstats/DCselection.asp>

- 1 Look closely at the data for life expectancy and infant mortality rates for the whole table. What do you immediately notice about countries with high life expectancies? About countries with low life expectancies?
- 2 Compare the life expectancy data for 1980 with 1999. What do you notice? What reasons could you give for life expectancy to actually fall rather than rise?
- 3 What was the average age at which people died in Indonesia in 1999?
- 4 What type of graph would best suit the life expectancy data for the countries in the table? Draw your graph.
- 5 What statistics might be most appropriate to describe the life expectancy data? The infant mortality data? Calculate these statistics.
- 6 If men lived on average to 75 years in Australia in 1999, can the life expectancy of women be calculated? What assumption would you have to make?
- 7 Does the data in the table imply that people in Kenya did not live beyond 48 in 1999? Justify your answer.
- 8 It might be claimed that Japan is the healthiest country in which to live. Would you agree? What additional statistical data would you need to make a more informed decision?
- 9 What was the chance that a child born in Australia in 1999 would not live beyond 1 year? Are there some sections of the Australian community where this is more likely to happen than others?



CHALLENGE

The *World Health Report* (1999), *Making a Difference*, charts the 20th century revolution in health which has led to a drop in birth rates and dramatic gains in life expectancy. But not everyone has benefited. The report predicted that more than a billion people will enter the 21st century without having participated in the health revolution.

- 1 Compare the data in the table with the general finding of the above *Report* that great progress has been made in world health. To what extent would you agree? How can mathematics be used to encourage governments of wealthy countries to take more action?
- 2 Is it reasonable to expect that all countries will eventually have the same life expectancy? Why? Why not?
- 3 What causes, other than disease, malnutrition and poverty, might contribute to the low life expectancy for adults and children in the developing world?



LET'S COMMUNICATE

Discuss in class the kind of data you would like to see in order to make a sensible conclusion about the health of the Australian people, children, adults, men and women. Are statistics misleading sometimes? Discuss how statistical evidence can inform:

- the success or otherwise of community campaigns against smoking and other addictive substances
- whether child immunisation is a good thing
- the health of Indigenous Australians.



REFLECTING

Reflect on the important role statistics has played in the case for the development of programs to support global health. Is it true that a healthy Australia depends on a healthy world? On this basis, what action could the Australian government take, for example, with respect to pollution of the Earth's air? How much do we pollute it? Should we try to reduce our production of carbon dioxide from motor cars, or the burning of coal to produce electricity?



MACQUARIE

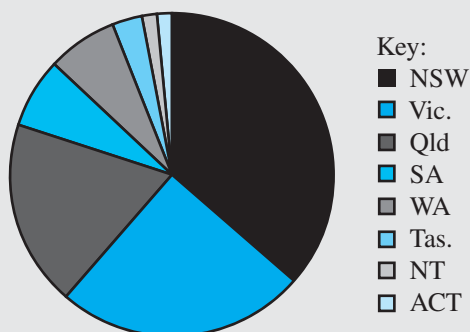
Language link with Macquarie

- 1 Sketch the difference between a **column** and a **line graph**.
- 2 Explain what is meant by a **sector graph**.
- 3 Write down what is meant by the **mean** of a set of scores.
- 4 Read the Macquarie Learners Dictionary entry for **graph**:

graph *noun* a diagram which shows the relationship between two or more things by dots, lines or bars

Why is the ability to read data from a graph so important for doctors and nurses in hospitals?

1 Population of Australian states September 2001



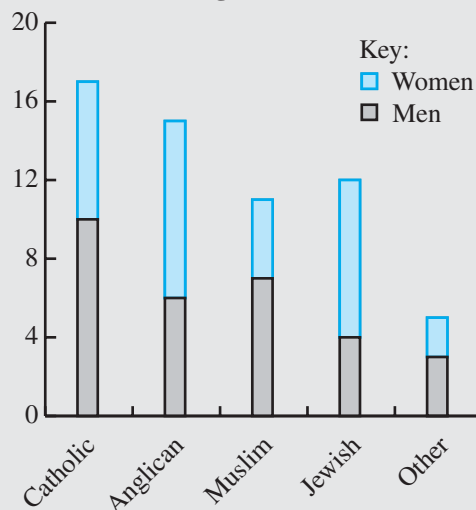
This sector graph is based on data collected by the Australian Bureau of Statistics. It shows the relative populations of each Australian state and territory in September 2001.

- a Which state or territory contains approximately one-third of the Australian population?
- b Which state or territory has the second largest population behind New South Wales?
- c Measure the angle at the centre of the Queensland sector. Hence, estimate the total Australian population, correct to the nearest 100 000 people, if the population of Queensland is 3 642 400.
- d Measures the angles for the Western Australia and South Australia sectors. Does the total population of these states exceed that of Queensland?
- e If the Australian population is 19 442 300 and the population of South Australia is 1 503 700, find the angle at the centre of the South Australian sector.

- f What percentage of Australians live in Tasmania? Answer correct to the nearest whole percentage.

2

Religious beliefs



A group of 30 men and 30 women were surveyed about their religious beliefs.

- a How many Catholics were surveyed?
 - b How many of the women surveyed were Muslim?
 - c Which was greater, the number of Jewish women or the number of Catholic women?
 - d What percentage of those surveyed were Jewish?
 - e How many more Catholic men were surveyed than Anglican women?
 - f For which religions were more women surveyed than men?
- 3 Find the mean, median, mode and range for each set of scores.
 - a 9, 13, 6, 15, 8
 - b 24, 21, 30, 23, 26, 26, 20, 22
 - c 26, 11, 26, 40, 11, 12

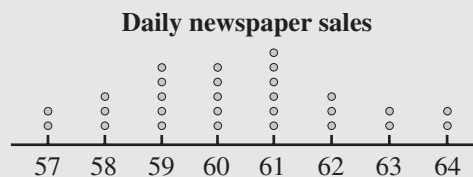
- 4 The number of sandwiches sold each day at a busy café is shown below.

53 52 48 50 47 51 51 53 48 50
49 48 49 47 52 51 53 53 51 48
51 52 51 49 47 53 51 51 48 50

- Organise the data into a frequency distribution table with score, tally, frequency, cumulative frequency and fx columns.
 - Draw a frequency histogram and polygon on the same set of axes.
 - What was the least number of sandwiches sold on any day?
 - What was the modal number of sandwiches sold?
 - On how many days were less than 50 sandwiches sold?
 - Calculate the average number of sandwiches sold per day.
- 5 Complete the data in this frequency distribution table, then find the mean.

Score (x)	Frequency (f)	Cumulative frequency	fx
1		3	
2		12	
3		19	
4		22	
5			40
6			24
7			
$\Sigma f =$	40	$\Sigma fx =$	

- 6 This dot plot shows the number of newspapers sold each day by a newsagent over a 28-day period.



- What was the greatest number of newspapers sold in one day?
 - What was the modal number of sales?
 - Find the range of the daily sales.
 - How many newspapers were sold altogether?
 - What was the average number of newspapers sold per day?
 - Are there any outliers?
 - Draw a frequency polygon for this data.
- 7 This stem and leaf plot shows the number of hours worked each week by a casual teacher over a school year.
- | Stem | Leaf |
|------|---------------------------|
| 0 | 7 7 8 9 |
| 1 | 3 4 4 5 6 6 6 8 8 8 9 |
| 2 | 0 0 1 2 2 3 3 4 5 7 7 8 9 |
| 3 | 1 1 2 3 4 4 6 9 |
- For how many weeks did the teacher work?
 - What was the range of hours worked per week?
 - What was the median number of hours worked per week?
 - Write down the modal number of hours worked.
 - Redraw this stem and leaf plot using stems of $0^{(5)}$, $1^{(0)}$, $1^{(5)}$ etc.

- 8 Find the mean, median, mode and range for the data in each of these frequency tables, correct to 1 decimal place where necessary.

a

x	8	9	10	11	12
f	7	5	2	6	3

b

x	1	2	3	4	5	6
f	5	9	3	12	15	14

- 9 **a** A set of 18 scores has a mean of 7.5. What is the sum of the scores?
b A set of scores has a mean of 13 and a sum of 286. How many scores are there?
- 10 The mean of a set of 5 scores is 11.8. If 4 of the scores are 17, 9, 14, and 6, find the 5th score.
- 11 A set of 16 scores has a mean of 41. Find the new mean, correct to 1 decimal place, after a score of:
a 25 is added **b** 17 is taken out
- 12 The table below shows the number of tickets written by a parking officer during the first four days of the week.

Day	Mon.	Tues.	Wed.	Thurs.
Number of tickets	16	10	13	21

- a** Why were the number of tickets highest on Thursday?
b How many tickets would the officer need to write on Friday to write an average of 15 tickets per day?
- 13 **a** State whether the mean would increase, decrease or stay the same if a score was added to a set and that score was:

- i** equal to the mean
ii less than the mean
iii greater than the mean

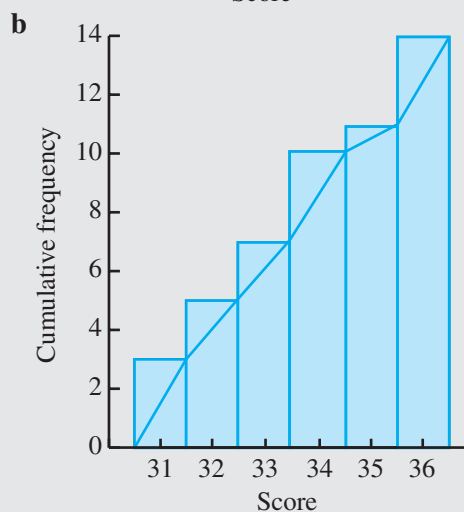
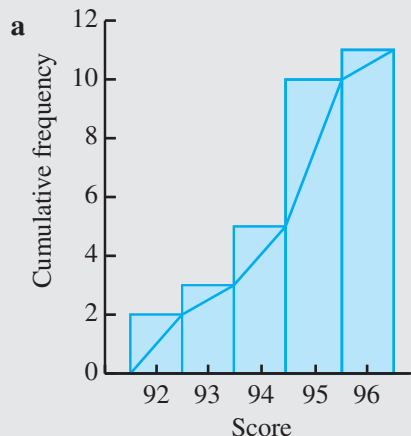
- b** Determine the corresponding effects on the mean if the score was taken out of the set.

- 14 **a** The mean of a set of 9 scores is 24. When a further score is added to the set, the new mean is 25.5. Find the score that was added.
b The mean of a set of 15 scores is 12. After a score is taken out of the set, the new mean is 11.5. Find the score that was taken out.
- 15 The data in the table shows the age groups of people who responded to a survey that was conducted in a music store.

Age (years)	Frequency
8–14	21
15–21	40
22–28	53
29–35	39
36–42	28
43–49	19

- a** How many of those surveyed were 21 years of age or younger?
b What was the modal class?
c What age could the oldest person have been in this survey?
d What percentage of those surveyed were older than 28 years of age?
e Estimate the average age of those who took part in the survey. (Answer correct to the nearest whole year.)

16 Find the median score in each of these.



17 Matt's best 30 times for the 100 m sprint are given below, in seconds.

11.6	11.1	10.9	10.5	11.4	12.3
11.5	10.6	10.5	11.0	12.1	11.9
11.6	11.7	10.4	10.5	10.7	12.0
11.2	11.7	12.4	10.2	11.0	11.8
11.3	10.9	11.5	12.3	10.1	10.2

- Organise the data into a grouped frequency distribution table with class, class centre, tally, frequency, cumulative frequency and $f \times cc$ columns. Use class intervals of 10.0–10.4, 10.5–10.9, etc.
- What is the modal class?
- On how many occasions did Matt run under 11.5 seconds?
- Use the totals in the table to estimate Matt's average time for the 100 m sprint.
- Draw a cumulative frequency histogram and ogive.
- Use the ogive to estimate Matt's median time.

Probability



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ use the language of probability to describe the likelihood of an event occurring
- ⦿ repeat an experiment a number of times to determine the relative frequency of an event
- ⦿ estimate the probability of an event from experimental data using relative frequencies
- ⦿ simulate a probability experiment by generating random numbers on a calculator or computer
- ⦿ design a device that could be used to produce a given relative frequency
- ⦿ calculate the theoretical probability of an event occurring.

7.1 Probability and its language

In many everyday situations, events take place that we cannot predict with any certainty. For example, when a person leaves home by car, they do not know whether a particular set of traffic lights 10 km away will be green when they reach it. A farmer plants canola hoping that it will succeed, but cannot confidently predict the weather. We would all love to know which questions will appear in the mathematics examination but we can only guess what the teacher will include. While we cannot be *certain* what will happen in any of these cases, it is important that we make the best possible prediction. The branch of mathematics that we use to do this is called **probability**.

People often discuss the likelihood of a particular event happening, to make judgements about what might happen, and to take action based on their decisions. The person driving from home may decide that the traffic lights will probably be red because they have driven that way many times and usually have to stop. A student in the mathematics class could decide that there will almost certainly be a question on probability in the examination. They might decide this because they have looked at past papers and found that most of them had such a question.

We often discuss the likelihood of events occurring using terms such as ‘very likely’, ‘good chance’, ‘almost certain’, ‘probably’, ‘unlikely’, and ‘extremely unlikely’. We use probability to make these statements more precise by giving a numerical value to the likelihoods.

Example 1

Ten balls are identical except that they have a number from 1 to 10 written on them. The balls are placed in a bag and a blindfolded person draws one ball from the bag. How likely is it that:

- a ball number 3 is drawn?
- b the ball drawn has an even number?
- c the ball is number 14?
- d the ball has a number greater than 2?
- e the ball has a number less than 20?

Solutions

- a Not very likely. Only one out of the 10 balls has 3.
- b Quite likely. Five of the balls have an even number.
- c Impossible. The highest number is 10!
- d Highly likely. Most of the numbers are greater than 2.
- e Certain. All the numbers are less than 20!

All the balls in example 1 were identical apart from the number, and the person drawing them was blindfolded. This means that each ball is equally likely to be drawn. We say that the drawing is **random**.

Ten different balls can be drawn. Each of the possible results is called an **outcome**, and the results—1, 2, 3, 4, 5, 6, 7, 8, 9, 10—form the **set of all possible outcomes**.

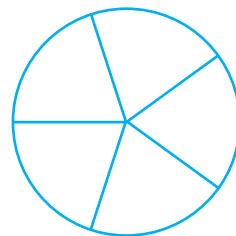
The types of occurrences described in example 1 are called **events**, for example, the ball is either 3 or 7.

The whole procedure described in example 1 is called an **experiment** and each drawing by the person is called a **trial**.

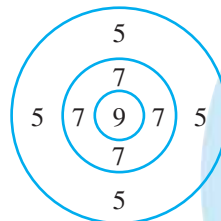
Exercise 7.1

- 1 Describe the probability of each of the following events occurring.
 - a You will become a pilot at some time during your life.
 - b You will captain an Australian cricket team.
 - c You will have a mathematics lesson tomorrow.
 - d Your best friend will marry and have children.
 - e You will have a job sometime before you are 30.
 - f Your brother will pass his driver's licence test at the first attempt.
 - g Someone in your mathematics class will become a mathematics teacher.
- 2 A die has 6 faces, with 4 faces coloured blue, 1 green and 1 yellow. An experiment is conducted in which this die is thrown and the colour on the uppermost face is noted.
 - a List all possible outcomes for this experiment.
 - b Are all the outcomes equally likely?
 - c How likely do you consider each of these events?
The uppermost face is:

i yellow	ii yellow or blue
iii red	iv blue or yellow or green
- 3 You are told that a spinner has 5 equal sectors coloured red, green and blue. You are also informed that the chance of spinning a red is much greater than that of spinning a green or a blue. How many sectors of each colour do you think the spinner has?



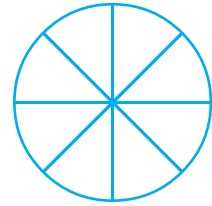
- 4 For each of the following experiments:
 - i list all possible outcomes
 - ii indicate, with reasons, if all the outcomes are equally likely.
 - a A normal die (with faces numbered 1 to 6) is rolled.
 - b Three cards with numbers 8, 9 and 10 are placed face down. Two are chosen at random and their numbers added.
 - c Three friends are interested in which of them will achieve the highest mark in a mathematics test.
 - d An archer shoots a single arrow at this target.
 - e An archer shoots two arrows and the scores are added.



- 5 A spinner has 8 equal sectors. Make three copies of the diagram and colour it using red, blue and yellow sectors so that if it is spun:

- a it is most likely to stop on blue
- b red and yellow are equally likely
- c red is most likely, blue next, and yellow least likely.

Can you colour this spinner so that all these colours have equal chances?



■ Consolidation

- 6 A perfect cube has blank faces. You can write any numbers you wish on the faces. Select numbers for the faces so that, in each case:
- a an even number is more likely than an odd number
 - b a number greater than 10 is very likely
 - c a number less than 20 is very likely
 - d parts a, b and c apply.
- 7 Choose coloured balls to be put in a bag so that the following conditions are all met. When one ball is drawn:
- a blue is the most likely colour *and* crimson is possible
 - b yellow is more likely than crimson *and* red is not possible
- 8 For each of the following situations suggest how you would perform a *random* selection.
- a A basketball coach wishes to appoint a team secretary from the squad of 10 players.
 - b You need to select one letter from the alphabet.
 - c You need to select 3 numbers less than 100.
 - d There are 3 movies you would like to see tonight.
 - e The school choir has 30 members but only 18 can perform in the combined high schools' choir.
- In each case, is random selection really appropriate?
- 9 As a visitor to a school you visit a class where the students are doing a test. The teacher says 'The person who tops the class will probably be a girl'. What can you deduce from this remark?
- 10 a You read in a newspaper that, to raise money for a charity, three prominent citizens (yet to be named) will race over 100 m during the lunch break at an international cricket match. Are the outcomes equally likely?
- b The next day the announcement is made that the three people will be the prime minister of Australia, the world 100 m champion and a famous film star. Are the outcomes equally likely?
 - c The 100 m champion runs backwards and the race is won by the film star. Are the outcomes equally likely?

- 11** For each of the following decide whether the choice has been random.
- a** A kindergarten class of 30 pupils sit in pairs at their desks. The teacher chooses two children to take a message to another teacher. He chooses one of the desks and sends the two children seated at that desk.
 - b** A cube, cylinder, cone, square pyramid and octahedron are placed in a bag and a blindfolded student draws one shape from the bag.
 - c** A researcher wishes to survey opinions of a group of people. He places their names in an alphabetical list and chooses every 7th person for interview.
 - d** A teacher often needs to choose individuals or groups of students from her class. She prepares a spinner with all the students' names on it and spins it to select students one at a time until she has the group size she requires.
 - e** A teacher chooses a child as class captain by numbering the children 1 to 20 and throwing a dart at a dartboard with 20 sectors numbered 1 to 20.

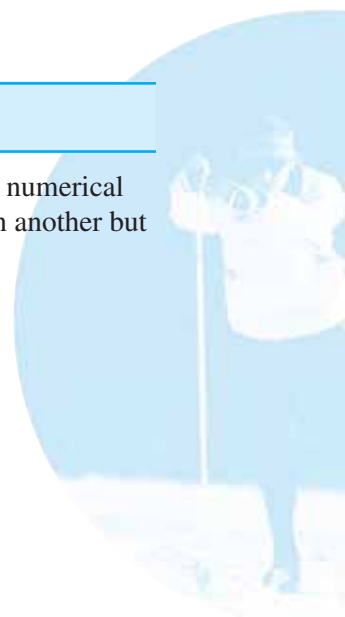
■ Further applications

- 12 a** One year there are 24 horses competing in the Melbourne Cup. We are interested only in the winning horse.
- i** How many outcomes are there?
 - ii** Are all outcomes equally likely?
- b** A girl runs a sweep in which she sells 31 tickets with a prize for only the winner. After all 31 tickets are sold, she conducts a draw to determine which ticket has which horse.
- i** Before the draw, do all tickets have an equal chance of winning?
 - ii** After the draw, do all tickets have an equal chance of winning?
- 13 a** Gamblers have often been heard to say 'There's no such thing as a certainty.' Precisely what do they mean? Are they correct?
- b** A scientist might claim that no event can be certain, while a mathematician may claim that certain events can easily be found. How are they differing in their views?
- 14** In a probability experiment, Jessica tosses a coin repeatedly until it comes up tails, and records the number of tosses required. Describe the sample space for this experiment. Are all outcomes equally likely?
- 15** Design a probability experiment for which the sample space is infinite.

7.2

Experimental probability

In exercise 7.1 we examined some aspects of probability but we did not give any numerical values to probabilities. We may have decided that one event was more likely than another but we did not say by how much it was more likely.



We represent probability numerically by finding the proportion of times a particular event occurs. Suppose that there are red and green balls in a bag and for each trial we draw one ball and note its colour. We know that for one trial the ball will be either red or green but that is all we know. We don't know how often it would be green (or red) if we repeated the trial many times. One way to answer our question is to carry out an experiment—repeat the trial over and over to see what proportion of the time we have selected a green ball.

The table shows the results from such an experiment.

Trial	1	2	3	4	5	6	7	8	9	10	6 green and 4 red
Colour	R	G	G	G	G	R	R	G	G	R	
Trial	11	12	13	14	15	16	17	18	19	20	14 green and 6 red
Colour	G	G	R	G	G	G	R	G	G	G	
Trial	21	22	23	24	25	26	27	28	29	30	21 green and 9 red
Colour	G	R	G	G	R	G	G	R	G	G	

After 2 trials we have 1 red and 1 green so we might guess that we will draw green $\frac{1}{2}$ the time (50%).

After 5 trials we have 1 red and 4 green, which suggests drawing green $\frac{4}{5}$ or 80% of the time.

After 10 trials, 4 red and 6 green— $\frac{6}{10}$ or 60%.

After 30 trials, 9 red and 21 green— $\frac{21}{30}$ or 70%.

We can see that the more trials we conduct the better estimate we have of correct theoretical probability.

For any particular event A, the **probability of A, P(A)**, is the proportion of trials favourable to A out of the total number of trials. This experimental probability is called a **relative frequency**.

$$P(A) = \frac{\text{number of trials favourable to A}}{\text{total number of trials}}$$

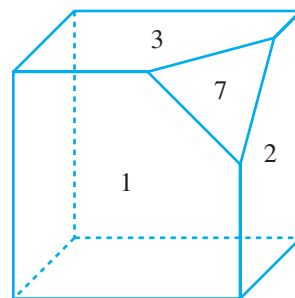
For the above experiment, after 30 trials we have:

$$\begin{aligned}
 P(A) &= \frac{\text{number of green balls}}{\text{number of trials}} \\
 &= \frac{21}{30} \\
 &= \frac{7}{10}
 \end{aligned}$$

where event A = drawing a green ball.

EG
+ S**Example**

An ordinary die with faces labelled 1 to 6 has had one corner sliced off, thus creating a seventh face labelled 7. We want to know the experimental probability for each number being face down when the die is thrown. The results of throwing the die 60 times are shown below:



4	2	5	7	1	4	6	3	5	3	4	6
6	5	5	6	2	3	5	7	5	2	2	4
3	2	7	4	6	3	1	6	4	1	3	1
4	3	5	6	2	3	7	4	2	5	7	6
5	3	6	2	7	6	5	1	5	4	1	4

What are the experimental probabilities for each of the possible outcomes?

Solution

The frequencies of each outcome are:

1—6 times

2—8 times

3—9 times

4—10 times

5—11 times

6—10 times

7—6 times

The probabilities are:

$$P(1) = \frac{6}{60}$$

$$P(2) = \frac{8}{60}$$

$$P(3) = \frac{9}{60}$$

$$P(4) = \frac{10}{60}$$

$$P(5) = \frac{11}{60}$$

$$P(6) = \frac{10}{60}$$

$$P(7) = \frac{6}{60}$$

Remember that these are estimates based on the 60 trials. The results would probably vary a little if we conducted more trials. The more trials we perform, the more accurate our results are likely to be.

If an outcome is impossible (e.g. getting 8 in example 1), then the probability will be 0:

$$P(8) = \frac{0}{60} = 0.$$

If an outcome is certain (e.g. getting a number less than 8 in example 1), then the probability will be 1: $P(1 \text{ to } 7) = \frac{60}{60} = 1.$

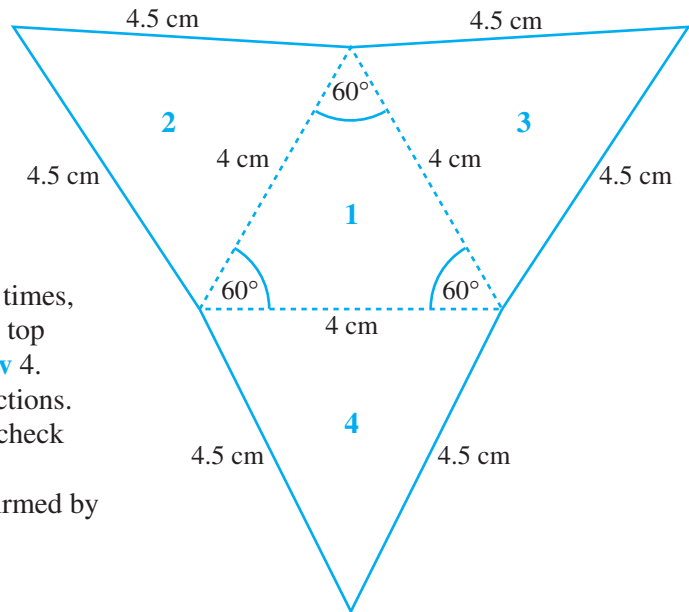
- ⊙ An impossible event has a probability of zero.
- ⊙ A certain event has a probability of 1.

All probabilities are fractions in the range $0 \leq P \leq 1.$

Exercise 7.2

For most of these questions, work in pairs or small groups.

- 1 a** Copy this diagram onto light cardboard, and then cut it out. Fold it along the dotted lines and use sticky tape to form the triangular pyramid. Label the sides 1 to 4 with the equilateral triangle as 1 (see diagram).
- b** If this shape was rolled 100 times, *predict* how many times the top face would be: **i** 1 **ii** 2 **iii** 3 **iv** 4. Give reasons for your predictions.
- c** Carry out an experiment to check your predictions.
- d** Were your predictions confirmed by the experiment?



- 2** Seven friends play handball in the school playground. They often argue about which of them is the best player, so they decide to record the results of 100 games. For each game they recorded in a table the player who spent most time at the top position, and the player who was eliminated most.

Name	Player with most time at top position		Player with most times eliminated	
Varia		14		10
Thanh		15		16
Seeza		16		20
Roger		18		23
Leigh		9		15
Lee-Ann		7		15
Greg		21		1
Total		100	Total	

- a** For each player find that player's probability in any particular game of:
 - i** spending most time in top position
 - ii** being eliminated most
- b** Which player do you consider to be the best at handball? Give reasons for your answer.
- c** Which player do you consider to be
 - i** worst?
 - ii** second best?
 Give reasons for your answer.

- 3 a** Three students were discussing the possible outcomes when 3 coins are tossed simultaneously. They decided that only 4 different outcomes were possible: **i** 3 heads, **ii** 2 heads and 1 tail, **iii** 1 head and 2 tails, and **iv** 3 tails. They decided that since there were four outcomes the probability for each one would be $\frac{1}{4}$. Do you agree?
- b** As a check they decided to perform an experiment tossing one coin each and recording their results. The results of 100 trials are shown in the table.

3 heads	12
2 heads and 1 tail	36
1 head and 2 tails	38
3 tails	14

Taking into account the results of their experiment, what are the probabilities of each event? Give your answers as decimals.

How would you explain this result?

- 4** The Australian netball selectors want to select a player for the goals shooter position. Each of 10 players' results over recent matches are collected and presented in this table.

Player	Shots	Baskets	Player	Shots	Baskets
A	80	60	F	50	41
B	50	34	G	90	72
C	60	48	H	100	68
D	60	50	I	80	68
E	70	49	J	75	50

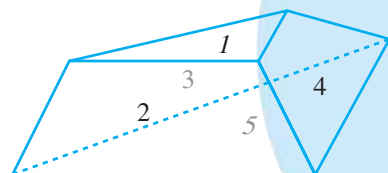
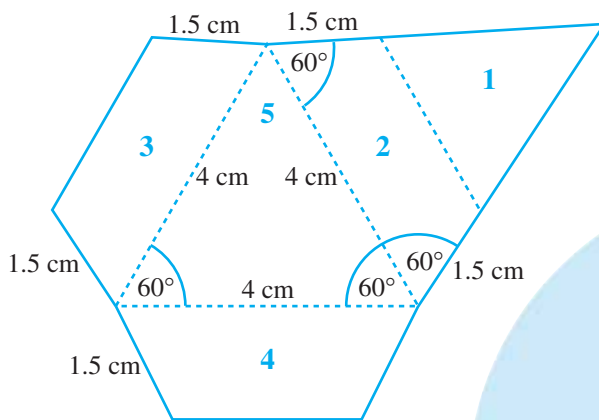
- a** State, as a fraction, the relative frequency of success for each player.
- b** Use your calculator to express each probability of shooting a basket as a decimal.
- c** Which player has the highest probability of success based on this data?

Consolidation

- 5 a** Copy this diagram onto light cardboard, and then cut it out. Fold it along the dotted lines and use sticky tape to form the solid shown. Label the sides 1 to 5 with 1 as the smallest triangle and 5 as the largest triangle.
- b** If this shape was rolled 100 times, *predict* how many times the top face would be:
- i** 1 **ii** 2 **iii** 3 **iv** 4 **v** 5.

Give reasons for your predictions.

- c** Carry out an experiment to check your predictions.
- d** Were your predictions confirmed by the data?



- 6 Take a drawing pin, drop it from a height of about 40 cm onto your desk and note whether it stops point up or point down. Continue to repeat the experiment and complete this table.

Number of trials	Number point up	Estimated probability of point up
5		
10		
15		
20		
25		
30		
35		
40		

- a Compare your different estimates for the probability of point up. What changes do you notice as the number of trials increases?
 - b Compare your probabilities with those of two or three of your classmates. What do you notice?
 - c What basic principle do your results suggest for finding experimental probabilities?
- 7 A card sharp wishes to know the probability of a hand of 5 cards having at least 1 heart.
- a Shuffle a deck of cards and deal 4 hands of 5 cards, dealing one at a time to each hand. Record the number of hands dealt and the number that had 3 or more cards of any particular suit.
Repeat this experiment, shuffling the cards well between deals, until you are confident of the probability. Record the probability for this event.
 - b Would the result be different if the cards had been dealt one full hand at a time? Give reasons for your answer.
 - c Would the result be different if 10 hands were dealt at a time? Give reasons for your answer.
You may wish to experiment further to check your answers.

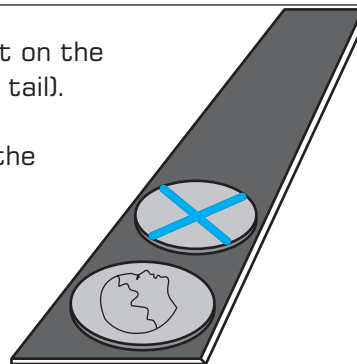
■ Further applications

- 8 a A bag contains 3 objects of identical size but not all the same colour. A blindfolded person draws one object from the bag, its colour is noted by an assistant, and the object is returned to the bag.
 - i How many trials would you require to be confident that you knew the colours of the 3 objects?
 - ii Ask a friend to place 3 objects in a bag without letting you see the colours. Perform the experiment to test the accuracy of your answer to part i.
- b i If the bag in part a contained 10 objects, how many trials would you need to be confident of the colours of the objects?

- ii With a friend, perform the experiment to check your answer.
- c From your results in parts a and b, what factors influence the number of trials needed to make good predictions?
- 9 A gold mining company is aware of a gold-bearing reef 200 m below the surface. To check the value of the reef and decide whether to mine it, 10 exploratory holes are drilled and the gold content, in grams of gold per tonne (gAu/t), for each sample is assayed. The results are:
- | | | | | |
|------------|------------|------------|------------|------------|
| 8.3 gAu/t | 14.8 gAu/t | 22.1 gAu/t | 15.7 gAu/t | 10.4 gAu/t |
| 16.7 gAu/t | 14.9 gAu/t | 16.8 gAu/t | 9.6 gAu/t | 17.5 gAu/t |
- a If another hole was drilled, what would be the probability of it assaying at 15 gAu/t or higher?
- b On the basis of these results a company employee suggests that when the reef is mined there is a probability of $\frac{1}{2}$ that gold production will exceed 15 gAu/t. Do you agree? Give reasons for your results.
- c What do you think is the best method of using the drill results?
- 10 Five friends are about to play a game but need to decide who will have first turn. The only equipment available is a 50c coin. Devise a procedure that gives each person the same chance of being chosen. Check your procedure experimentally to make sure it provides equal chances for all 5 friends.

TRY THIS**Two-up**

In two-up, two coins are tossed into the air. Players bet on the outcome—both heads, both tails, or odds (a head and a tail). What is the probability of obtaining five odds in a row? Perform a trial and then state your results. How does the total class result compare with the theoretical result (1 in 32)?

**7.3****Computer simulations**

Most computers, and some calculators, have a function that generates random numbers. The function may vary between computer and calculator brands, though.

A common form of random number generated is to provide a number between 0 and 1 (not including 1) with *all numbers in that range being equally likely*. The number of decimal places provided varies but for this exercise we will assume 6 decimal places. We will also use the word RANDOM to indicate that the generator is to produce a random number.

EG
+ S**Example 1**

Show how a random number generator could be used to simulate the throwing of a die where all 6 numbers are equally likely.

Solution

- 1 We use our computer to produce a random number that lies in the range $0 \leq \text{RANDOM} < 1$.
- 2 We need a random number from 1 to 6 so we multiply the random number by 6. Our number now lies in the range $0 \leq 6 \times \text{RANDOM} < 6$.
- 3 We now *truncate the number*, that is, we ignore all of the figures after the decimal point. (Note that truncating is not the same as rounding off because the number always goes down to the nearest whole number.)
At this stage our numbers will be one of 0, 1, 2, 3, 4, 5.
- 4 The last step is to add 1 to the number. Our number will now be one of the numbers 1, 2, 3, 4, 5, 6 with all numbers equally likely.

The following table summarises this procedure.

Step	Range of values	Example
1 Obtain RANDOM number.	$0 \leq \text{RANDOM} < 1$	$0 \leq 0.412869 < 1$
2 Multiply by 6.	$0 \leq 6 \times \text{RANDOM} < 6$	$0 \leq 2.477214 < 6$
3 Truncate to whole number.	RANDOM is 0, 1, 2, 3, 4 or 5	Number is 2
4 Add 1.	RANDOM is 1, 2, 3, 4, 5 or 6	Number is 3

The procedure gives us a number from 1 to 6, with all numbers equally likely in just the same way as throwing a normal 6-sided die.

The procedure for simulating the experiment in example 1 using a commonly available graphics calculator is provided here.

Casio (CFX 9850G plus) procedure

The keys required for producing a random number 1 to 6 are:

AC/ON, EXE, OPTN, F6 (), F4 (NUM), F2 (INT), (, 6, EXIT, F3 (PROB),
F4 (Ran #), +, 1,)

The command is now prepared and each time you press EXE you will get another random number from 1 to 6.

If you want random numbers from 1 to 50, say, just replace the 6 by 50. All other steps remain the same.

EG
+ S**Example 2**

Show how a random number generator could be used to simulate drawing 1 ball from a bag of 10 balls (4 black, 3 red, 2 blue and 1 yellow).

Solution

Step	Procedure	Example
1	Assign a colour to each of the numbers 1 to 10 as the question requires.	1, 2, 3 and 4 all black 5, 6 and 7 all red 8 and 9 both blue 10 yellow (It doesn't matter which numbers are which colours as long as we have the correct number of each colour.)
2	Obtain a random number.	0.821615
3	Multiply by 10.	8.21615
4	Truncate the number.	8
5	Add 1.	9
6	Check for the colour chosen.	blue

We can simulate an experiment where a number of balls are selected one at a time, the colour noted and the ball returned. All we need to do is repeat the process several times. For example:

	$\times 10$		truncate		add 1		convert to colour
0.712194	$\rightarrow$	7.12194	$\rightarrow$	7	$\rightarrow$	8	$\rightarrow$ blue
0.134728	$\rightarrow$	1.34728	$\rightarrow$	1	$\rightarrow$	2	$\rightarrow$ black
0.024555	$\rightarrow$	0.24555	$\rightarrow$	0	$\rightarrow$	1	$\rightarrow$ black

In this simulation the balls drawn were blue, black and black. Of course, many other sequences were possible.

EG
+ S**Example 3**

Show how a random number generator could be used to simulate dealing a hand of 5 cards from a normal playing deck of 52 cards.

Solution

Assign numbers to the cards:

Numbers 1 to 13 to spades	Ace, 2, 3, ... Jack, Queen, King
Numbers 14 to 26 to hearts	Ace, 2, 3, ... Jack, Queen, King
Numbers 27 to 39 to diamonds	Ace, 2, 3, ... Jack, Queen, King
Numbers 40 to 52 to clubs	Ace, 2, 3, ... Jack, Queen, King



Select 5 random numbers

	0.600032	0.001278	0.437151	0.912349	0.721638
Multiply by 52	↓	↓	↓	↓	↓
	31.201664	0.066456	22.731852	47.442148	37.525176
	↓	↓	↓	↓	↓
Truncate	31	0	22	47	37
Add 1	32	1	23	48	38
Cards represented	↓	↓	↓	↓	↓
	6 of diamonds	Ace of spades	10 of hearts	9 of diamonds	Queen of diamonds

Exercise 7.3

You will need access to a random number generator, such as a computer spreadsheet, graphics calculator or other computer or calculator facility.

1 Complete these as a class or group using a graphics calculator. Follow the directions for the calculator for each simulation.

- a** *Driving test simulation.* This assumes that the probability of passing a driving test at any particular attempt is $\frac{1}{4}$. Prepare your calculator to give a random number from 1 to 4. Take 1, 2 or 3 as 'fail' and 4 as 'pass'.

Each person is to take a turn to make one trial at the driving test until you pass.

- i** At which attempt did you pass?
- ii** How many attempts did most people take?
- iii** How many attempts did the last person take?

- b** *Die elimination.* This is best with a group of 8–10 people.

Prepare your calculator to yield a random number from 1 to 6 (for the faces of a die). All members of the group are to simulate one throw of the die. The lowest score is eliminated. If two or more people have the lowest score, they all stay in the game.

- i** How many trials were required to find a winner?
- ii** Early in the game, how likely was it that someone would be eliminated?
- iii** Late in the game, how likely was it that someone would be eliminated?
- iv** How do you explain your answers to parts **i** and **ii**?
- v** What would happen if 50 people played this game?

- c** *Pick a card elimination.* This simulates a game in which people take it in turns to draw a card from 5 cards (10, Jack, Queen, King, Ace of spades). If the 10 of spades is drawn, the person is eliminated (and sits down). If the Jack, Queen or King is drawn, the person is still in and remains standing. If the Ace is drawn, the person can choose any other person to be eliminated. Prepare your graphics calculator to give a random number from 1 to 5. The whole class is to stand and takes turns to try your luck.

- i** How likely were you to be eliminated at any one drawing?
- ii** How many times were people eliminated by a classmate?
- iii** How many trials were there altogether?

- iv** Using your answers to parts **ii** and **iii**, what is the experimental probability of one particular person having the opportunity to eliminate someone else?
- d** *Babies, babies, babies.* This is best done as a whole class activity so that results can be compared. The situation simulated is that you are married and each year you and your spouse have the following possible outcomes:

baby boy—72/300

baby girl—70/300

twin boys—2/300

twin girls—2/300

twins, one boy and one girl—4/300

no child—150/300

Prepare your graphics calculator to give a random number from 1 to 300.

Assign the code:

boy 1 to 72

twin boys 293 or 294

girl 73 to 142

twin girls 295 or 296

no child 143 to 292

twins, boy/girl 297 to 300

Take turns to simulate the results for each year for a period of 10 years, recording your results.

- i** How many sets of twins would you have expected in the class? How many were there?
- ii** How many couples had more than 6 children?
- iii** How many couples had fewer than 3 children?
- iv** What were the smallest and largest numbers of children?
- v** Did any couple have all boys or all girls?
- e** Cutting cards. Use teams of about 5 members. One person from each team is to cut the cards. The highest card wins a point for that team. Take turns cutting the cards until 50 points have been awarded.

You will need to prepare your calculator to give random numbers 1 to 52.

Numbers could be assigned to cards:

1—Ace of hearts

2—Ace of diamonds

3—Ace of clubs

4—Ace of spades

5—King of hearts

6—King of diamonds

7—King of clubs

8—King of spades

9—Queen of hearts

10—Queen of diamonds, etc.

■ Consolidation

- 2** Program your graphics calculator (or computer) to provide a random number:
- a** from 1 to 10 **b** from 1 to 100 **c** from 1 to 500
- d** from 10 to 20 **e** from 50 to 100 **f** from 100 to 120
- g** which is an even number less than 41
- h** which is an even number between 19 and 31
- 3** Use your random number generator to simulate each of the following experiments.
- a** One card is drawn from a set of 10 cards labelled 1 to 10.
- b** One person is selected from 100 people numbered 1 to 100.
- c** A number is selected from the even numbers 2, 4, 6, ... 40.
- d** A spinner with 40 sections is spun once.
- e** A die with faces 2, 4, 6, 8, 10, 12 is rolled.
- f** A 4-sided die with faces 3, 6, 9, 12 is rolled.

■ Further applications

- 4 Simulate each of these experiments:
 - a A cube with 3 red, 2 blue and 1 white face is rolled.
 - b A spinner has 12 sections with these pictures—1 television set, 1 CD player, 2 soccer balls, 3 tins of tennis balls and 5 bottles of soft drink. It costs \$50 to spin the wheel and the prize is the picture at which the wheel stops.
 - c A pack of cards has 13 spades, 13 clubs, 13 hearts and 13 diamonds. One card is selected at random and the suit noted.
 - d A class has 13 boys and 15 girls. One class member is chosen at random.
- 5 What are the advantages of simulating experiments? Suggest situations in which simulation would be better than real experimentation.

TRY THIS The game of craps

The game of craps was invented by African-Americans in about 1800. Two dice are thrown. The player rolling the dice wins if they obtain a total of 7 or 11 on the first throw. If they throw 2, 3 or 12 it is called a 'crap' and the player loses their money. Any other total is called a point. The player continues rolling the dice until they can gain another point by rolling this total again. They win except if rolling a total of 7, which is a loss.

Can you simulate this game using a spreadsheet?

First we need to generate a random number between 1 and 6 to act as a die. In Microsoft Excel we put in cell A1: = INT(RAND() * 6 + 1). Then we use the 'Edit' function to fill down to A100. Similarly, we put the formula into cell B1 and again fill down. In cell C1 we can put the total of the dice rolled in A1 and B1. Hence, put in cell C1: = A1 + B1.

Thus, you would have the following but not with the same numbers.

	A	B	C
1	2	6	8
2	4	3	7
3	5	1	6
.			
.			
.			
.			

Now continue and finish this game simulation.

7.4 Theoretical probability

It is not always necessary to experiment to discover the probabilities of certain events. Sometimes we may be able to construct a theoretical description of the situation.

For example, if a die is a perfect cube and is thrown in a game, there is no reason to assume that the six possible outcomes—1, 2, 3, 4, 5, 6—are not equally likely. Similarly, if we draw one card from a deck of 52 all the different cards should have the same chance of being drawn.

The probability of an event A is given by:

$$P(A) = \frac{\text{number of outcomes favourable to A}}{\text{total number of possible outcomes}}$$

With this formula all outcomes must be equally likely.

Example 1

Ten identical pieces of cardboard are numbered 1 to 10. One card is chosen at random. What is the probability that it is:

- a** 3? **b** even? **c** either 4 or 7? **d** less than 7? **e** greater than 12?

Solutions

- a** $\frac{1}{10}$ (number of outcomes favourable = 1)
b $\frac{5}{10} = \frac{1}{2}$ (2, 4, 6, 8, 10 are all favourable outcomes)
c $\frac{2}{10} = \frac{1}{5}$ (4 and 7 are favourable)
d $\frac{6}{10} = \frac{3}{5}$ (1, 2, 3, 4, 5, 6 are favourable)
e 0 (there are no cards greater than 12)

A theoretical probability gives a *measure* of the rate of success of an event if many trials were conducted. It does not mean that you will get exactly that score in a single experiment. That is, what happens in an experimental trial may be different from what we expected theoretically.

Example 2

A person tosses a coin 6 times, recording each result.

- a** What is the probability of a head at each toss?
b Theoretically, how many heads would you expect in 6 tosses?
c Perform this experiment. How many heads did you get?
d What was the experimental probability of a head in a single toss?



Solutions

- a** $\frac{1}{2}$
b 3 (You would expect half heads and half tails.)
c One possible result: H T T T H T 2 heads
d Experimental probability = $\frac{2}{6} = \frac{1}{3}$

The experimental result is not what we had expected theoretically. This is not surprising because we know that the theoretical result will not always occur. It is the *average* of all the results over a very large number of trials.

Exercise 7.4

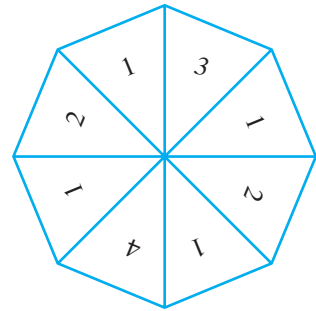
- 1 The spinner shown is spun. What is the probability of it stopping on:

a 1? **b** 2? **c** 3?
- 2 A card is drawn at random from a normal deck of 52 cards. What is the probability of it being:

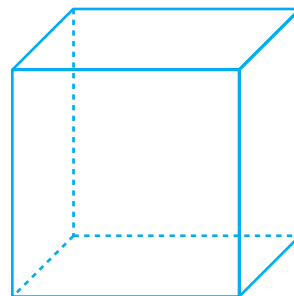
a the three of spades? **b** any of the four threes?
c a club? **d** a black card?
e a Jack, Queen or King? **f** a red Ace?
- 3 At a fête a raffle has 100 tickets at \$1 each. The prize is valued at \$50. The first five people to come along are very keen to win the prize. Mrs Alix buys 40 tickets, Mr Steptoe 30, Mr Naba 5, Mrs Naba 5 and Dr Georgiou 20.

a Find the probability of each of these people winning the prize.
b Which person has the greatest probability of winning the prize?
c Who will win the prize?
d If the raffle was run 1000 times, with each person buying the same number of tickets as they bought this time, how many times would you expect each person to win? How sure are you that this number would occur in practice?
- 4 The probability of any mathematics test being on a Wednesday is $\frac{1}{5}$. If there were 200 mathematics tests, how many of them would you expect to be on a Wednesday? Can you be certain that there will be exactly that number on a Wednesday?
- 5 A 4-digit number is formed using the digits 8, 7, 5, 2. What is the probability that the number:

a begins with 5? **b** is less than 4000? **c** is greater than 4000?
d is odd? **e** is at least 9000? **f** is less than 9000?

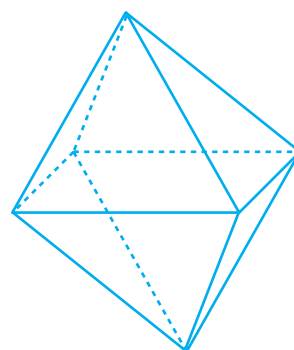


- 6 A cube has 6 faces.
- Show how you could colour its faces so that, when thrown, it has $P(\text{red}) = \frac{1}{2}$, $P(\text{yellow}) = \frac{1}{3}$, $P(\text{white}) = \frac{1}{6}$.
 - Suppose that you toss this cube 60 times.
 - How many of each colour would you expect?
 - You tried the experiment and the results were 28 red, 21 yellow, 11 white. Do these results surprise you?
 - Explain why this experimental result might be different from your theoretical prediction.



Consolidation

- 7 An 8-sided die has faces labelled 1 to 8. If the die is rolled, what is the probability of obtaining:
- a 5?
 - a number greater than 3?
 - an even number?
 - a number divisible by 3?



- 8 A 20-sided die (a regular icosahedron) is labelled 2, 4, 6, 8, ... 38, 40. If the die is rolled, what is the probability of:
- an even number?
 - a multiple of 5?
 - a number less than 20?
 - an odd number?
 - a number divisible by 12?
 - a number greater than 4?
- 9 A basketballer has a $\frac{7}{10}$ chance of scoring from the free-throw line. If the player has 60 free throws, how many would you expect to be successful? In a game would the player definitely score that number?

- 10 A bag contains 20 balls. When a ball is drawn at random we know that:

$$P(\text{silver}) = \frac{1}{2}$$

$$P(\text{gold}) = \frac{2}{5}$$

$$P(\text{white}) = \frac{1}{20}$$

$$P(\text{black}) = \frac{1}{20}$$

What are the contents of the bag?

- 11 A bag contains 24 balls. If a ball is drawn at random, we know that:

$$P(\text{red}) = \frac{1}{8}$$

$$P(\text{blue}) = \frac{1}{4}$$

$$P(\text{green}) = \frac{5}{8}$$

What are the contents of the bag?

- 12 Two friends each have a pack of 52 cards. They each draw a card at random from the pack. What is the probability that they have both drawn:

a the same suit?

b the same number?

c the same card?

Further applications

- 13** Four students are about to play cards and draw one card each to decide who deals first. The person with the highest card will deal first. The cards are, from highest to lowest, Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3, 2. If numbers are equal, then they compare suits with hearts highest, then diamonds, clubs and finally spades.

Peter draws a Jack of clubs then replaces his card in the pack. Ellie draws a 6 of hearts then replaces it. Sasha draws a 10 of diamonds then replaces it. Cecilia is the last to draw. What is her chance of winning?

- 14** Design a spinner, with coloured sectors with the following probabilities:

$$P(\text{blue}) = \frac{1}{3}$$

$$P(\text{red}) = \frac{1}{4}$$

$$P(\text{hyacinth}) = \frac{1}{4}$$

$$P(\text{tan}) = \frac{1}{6}$$

- 15** A boy holds his mathematics textbook open as shown. His friend tosses an ordinary die into the book, and it falls with an edge in the centre and two faces up. What is the probability that:

- a** the two faces up are 3 and 5?
- b** one of the two faces up is a 4?
- c** the sum of the two faces is 6?
- d** the sum of the two faces is 7?



- 16** A girl rolls two ordinary dice. What is the probability that both dice show the same number?
- 17** Archie rolls a red die with faces numbered 1, 2, 3, 4, 5, 6. Alicia rolls a blue die with faces numbered 2, 3, 4, 5, 6, 7. Archie wins if his red die shows the higher number *or* if the numbers are equal. Alicia wins if her blue die shows the higher number. Who has the better chance of winning? Why?

TRY THIS Winning chances

A slot machine has 3 dials, each with 10 positions using any of four different symbols: tennis, soccer, Rugby or golf.

For \$1 per spin the machine pays \$20 for 3 tennis, \$10 for 3 soccer, \$100 for 3 Rugby, and \$900 for 3 golf.

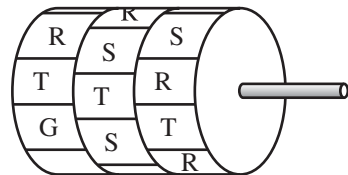
Secret investigators have discovered for you that the number of symbols on each dial are—

dial 1: 2T, 3S, 1R, 4G

dial 2: 1T, 2S, 2R, 5G

dial 3: 4T, 4S, 2R, 0G

Can you calculate what are the real chances of winning?





FOCUS ON WORKING MATHEMATICALLY

A PARTY GAME: ROLL A SIX AND EAT THE CHOCOLATE



Introduction

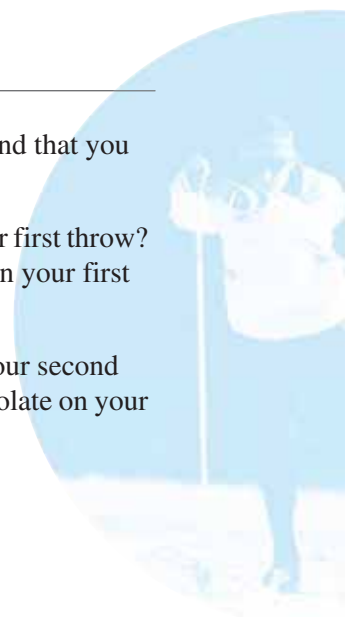
Have you ever played this party game? Players sit in a circle and take it in turn to roll a die. If you get a six you immediately put on a funny hat and a bib and begin to eat a bar of chocolate using a knife and fork. You cannot pick up the chocolate with your fingers! You continue eating until someone else throws a six. They then grab the knife and fork, put on the hat and bib and try to eat as much as they can before another six is thrown. The game ends when there is no more chocolate.



LEARNING ACTIVITIES

For these activities we will assume that there are 4 players (including yourself) and that you throw first.

- 1 What is the probability that you will get the chance to eat the chocolate on your first throw? What is the probability that you will *not* get the chance to eat the chocolate on your first throw?
- 2 What is the probability that you will get the chance to eat the chocolate on your second throw? What is the probability that you will *not* get the chance to eat the chocolate on your second throw?



- 3 The results of a hypothetical game are shown in the table below. The letter N indicates NOT A SIX and the letter Y indicates YES A SIX. The players are Marcia, Ali, Jo and yourself. In this game you ended up eating the rest of the chocolate just after Jo had her last turn and the game ended.

Look down the first column. You threw the die first and did not get a six. Then Marcia tried but did not get a six. Neither did Ali. Does Jo have a greater chance of getting a six? Why? Why not? Would the event 'throwing a six' in this game be a dependent or independent event? Why?

Player	1st throw	2nd throw	3rd throw	4th throw
You	N	Y	N	Y
Marcia	N	N	Y	N
Ali	N	N	N	N
Jo	Y	N	N	N

- 4 Draw a probability tree to show the possible outcomes for the game. Show the pathway N, N, N, Y on your tree diagram. What is the probability of observing this outcome for the first round?
- 5 What is the probability that Ali did not throw a six in the game?
- 6 Who threw a six but had the *least* time to dress up and eat the chocolate? Who had the *most* time?



CHALLENGE

This activity can be done in a group.

- Carry out the following experiment: Throw a single die 4 times and count the number of times you get a six. Repeat this 49 times. Draw up a table to show the results for the total of 50 trials. You may wish to do this on a spreadsheet using a random number generator. The formula =INT(RAND()*6 + 1) will generate numbers from 1 to 6 at random.
- There are four possible outcomes for each experiment, 0 sixes, 1 six, 2 sixes, 3 sixes and 4 sixes. Summarise the data, showing the observed frequencies and relative frequencies of the number of sixes. Graph the results. What is the shape of the distribution? How do the results compare with the theoretical results from your probability tree?
- Calculate the relative frequency of observing *at least* one six in 50 trials.
- Calculate the theoretical probability of observing *at least* one six in 4 throws of a single unbiased die. How does the result compare with the observed result of your experiment?
- Show that the theoretical probability of observing *at least* one six in n throws of a single unbiased die is $1 - \left(\frac{5}{6}\right)^n$.





LET'S COMMUNICATE

The mathematical analysis of probability was prompted by a French gentleman gambler Antoine Gombaud, known as the Chevalier de Mere, in the 17th century. He attempted to make money gambling with dice. Probability theory had not been developed, but de Mere made money by betting that he could roll at least one six in four throws of a die. Experience led him to believe that he would win more times than he would lose with this bet.

- 1 De Mere reasoned that since the chance of rolling a six with a die are 1 in 6, the chance of rolling a six in 4 tries would be $\frac{4}{6}$ or $\frac{2}{3}$. Discuss with your neighbour why his reasoning was wrong.
- 2 From the results of your work in this activity, discuss why de Mere did in fact make money from this bet.



REFLECTING

In 1654, de Mere approached the famous mathematician Blaise Pascal with a problem. He had lost money on a new game he had invented. He bet that he could roll a double six in 24 throws of two dice. Pascal explained to de Mere why in the long term he had won in the first game but had lost in the other. However, his ideas prompted Pascal to contact his mathematician friend Pierre Fermat. From their correspondence the theory of probability was born.

Reflect on how mathematics is often developed through a human need, in this case the analysis of risk. The insurance industry today was founded on this analysis.



MACQUARIE

Language link with Macquarie

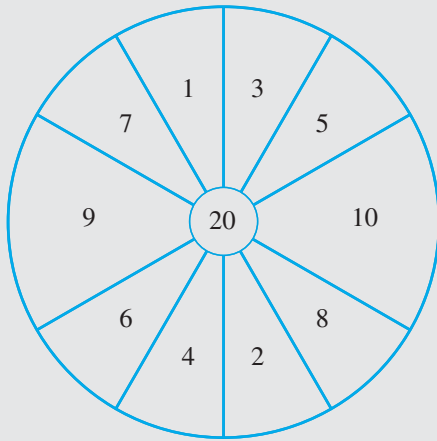
- 1 What is meant by the following terms:
 - a at random?
 - b equally likely?
 - c experimental probability?
- 2 Explain the difference between **relative frequency** and **theoretical probability** in a simple experiment.
- 3 Read the Macquarie Learners Dictionary entry for **simulate**:

simulate verb *Formal* **1.** to make a pretence of: *He simulated admiration to flatter her.* **2.** to imitate or make a copy of: *They simulated diamonds to make the cheap jewellery.*

□ **Word family:** **simulation** noun—**simulator** noun a device used in training or experiments that simulates movement or flight.

How does the common English meaning differ from the mathematical meaning?

- 1 A darts player throws one dart at this dart board and their score is recorded.



- a List all possible outcomes.
b Are all the outcomes equally likely? If not, indicate which ones are **i** most likely **ii** least likely.
- 2 Choose coloured balls to be placed in a bag so that *all* the following conditions are met. When one ball is drawn:
a green is the least likely colour
b red is the most likely colour
c brown and yellow are equally likely
- 3 a Explain what is meant by random selection.
b Outline a method for randomly choosing two students to represent your class at an interschool event.

- 4 A wheel is spun to decide which prize a quiz show contestant wins. Possible results are:
- | | | |
|----------------|------------------------|-----------------------------|
| car (\$52 000) | holiday (\$4650) | TV (\$1200) |
| spin again | dinner for two (\$180) | two concert tickets (\$150) |
| no prize | | |

The results of 60 spins are:

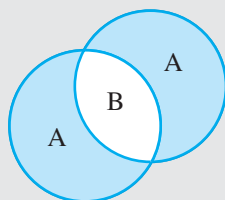
TV	spin	holiday	concert	concert	dinner
dinner	car	dinner	no prize	spin	concert
spin	dinner	concert	concert	dinner	TV
dinner	no prize	car	dinner	holiday	spin
spin	spin	concert	concert	concert	no prize
TV	concert	dinner	concert	concert	dinner
dinner	no prize	TV	dinner	dinner	spin
dinner	spin	dinner	concert	dinner	holiday
car	holiday	spin	no prize	TV	dinner
spin	dinner	TV	concert	dinner	concert

- a Complete the table below.

Prize	Tally	Total	Experimental probability
Car			
Holiday			
TV			
Dinner			
Concert			
Spin			
No prize			

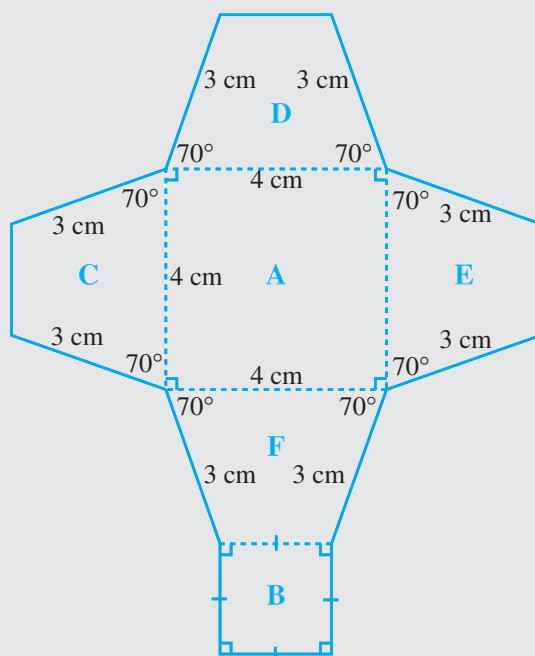
- b** If the wheel was a circle divided into 20 sectors, estimate the number of sectors for each prize.

- 5** Draw two small circles (radius approximately 5 cm) that intersect to give three regions of approximately equal size. Create an experiment in which you toss a small object, such as a coin, onto the circles drawn. Perform a sufficient number of trials of the experiment to enable you to estimate the probabilities of events A, B, C and D:



A = the object stops completely inside one of the shaded regions
 B = the object stops completely inside region B
 C = some part of the object is touching a circle
 D = the object stops completely outside the figure

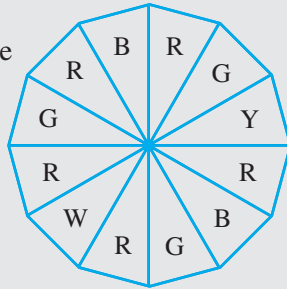
- 6 a** Copy this diagram onto light cardboard, and cut it out. Fold it along the dotted lines and use sticky tape to form a solid shape. Label the large square face A, the small square face B, and the other faces C, D, E and F.



- b** If this shape was rolled 100 times, *predict* how many times the top face would be
- | | | |
|-------------|-------------|--------------|
| i A | ii B | iii C |
| iv D | v E | vi F |
- Give reasons for your predictions.
- c** Carry out an experiment to check your predictions.
- d** Were your predictions confirmed by the experimental data?

- 7 Program your graphics calculator (or computer) to:
- provide a random number:
 - from 1 to 30
 - from 50 to 100
 - simulate a game where an 8-sided die (labelled 1, 2, 3, ..., 8) is rolled. You win if you roll a 1 or an 8. How often will you win? Use your simulation to experiment and check your findings.

- 8 This spinner is spun. What is the probability of it stopping on:



- green (G)?
- red (R)?
- blue (B)?
- yellow (Y)?
- white (W)?

- 9 A regular octahedron (8 identical faces) is labelled 10, 12, 14, ..., 24. If it is rolled, what is the probability of:

- an even number?
- a number greater than 20?
- a multiple of 3?
- a multiple of 5?

- 10 Choose coloured balls to put in a bag so that a random selection of one ball has the following probabilities:

- $P(\text{crimson}) = \frac{1}{4}$
- $P(\text{aqua}) = \frac{1}{3}$
- $P(\text{hyacinth}) = \frac{1}{12}$
- $P(\text{lemon}) = \frac{1}{6}$
- $P(\text{white}) = \frac{1}{6}$
- $P(\text{black}) = 0$

8

Surds



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ determine whether a number is rational or irrational
- ⦿ distinguish between surds and square roots
- ⦿ define a real number
- ⦿ arrange rational and irrational numbers in ascending order
- ⦿ locate the position of a surd on the real number line
- ⦿ express surds in simplest form
- ⦿ express surds of the form $a\sqrt{b}$ as entire surds
- ⦿ add and subtract surds
- ⦿ multiply and divide surds
- ⦿ expand binomial products that contain surds
- ⦿ rationalise the denominator of a fraction
- ⦿ add and subtract fractions that have surds in the denominator.

8.1

Rational and irrational numbers

Real numbers are numbers that can be represented by points on the number line. They can be divided into two sets of numbers: the *rational numbers* and the *irrational numbers*. These terms are defined below.

Rational and irrational numbers

- ⊙ A rational number is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.
- ⊙ An irrational number is a number that cannot be written in this form.

More simply, a number is rational if it can be expressed as a fraction, or ratio. Integers, mixed numerals, percentages, terminating and recurring decimals can all be expressed as fractions. Therefore, they are all rational numbers.

Some examples of rational numbers are 3, -2 , 0, $\frac{3}{4}$, $1\frac{7}{9}$, 0.87, $0.\dot{4}$ and 15%. There is an infinite number of rational numbers between any two numbers on a number line.

The set of irrational numbers includes π , decimals that neither terminate nor recur and surds (see below). When an irrational number is expressed as a decimal, the decimal continues indefinitely; however, there is no recurring pattern in the digits.

Surds

When a real number other than zero is squared, the result is always a positive number, while the square of zero is zero. Therefore, every positive number has two square roots. One square root is a positive number, the other square root is a negative number. For example, the two square roots of 9 are 3 and -3 , as $3^2 = 9$ and $(-3)^2 = 9$. The notation $\sqrt{\quad}$, however, only refers to the **positive square root** of a number; that is, $\sqrt{9} = 3$, and not -3 .

- ⊙ $\sqrt{x}$ is the positive square root of x for $x > 0$
- ⊙ $\sqrt{x} = 0$ if $x = 0$
- ⊙ $\sqrt{x}$ is undefined if $x < 0$

Square roots and other roots of numbers may be either rational or irrational. A **surd** is a root of a rational number that is itself irrational. For example:

- 1 Numbers such as $\sqrt{2}$ and $\sqrt[3]{7}$ are called surds because they are *irrational* roots. These numbers cannot be written as integers or fractions and do not have decimal equivalents that either terminate or recur. We can only find approximate decimal values for surds.

- 2 Numbers such as $\sqrt{9}$ and $\sqrt[3]{8}$ are not called surds because they are *rational* roots. These numbers can be written as integers ($\sqrt{9} = 3$ and $\sqrt[3]{8} = 2$).

A surd is an irrational number of the form $\sqrt[n]{x}$, where x is a rational number and $n \geq 2$ is an integer.

NOTE: All surds are irrational numbers; however, all irrational numbers are not surds.

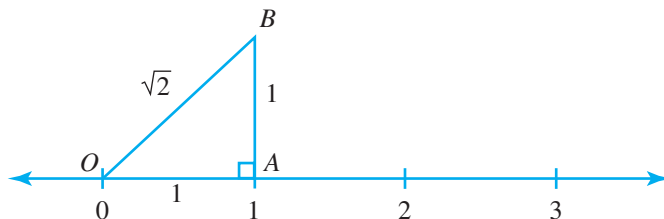
■ Locating surds on the number line

Although surds do not have exact decimal equivalents, we can still locate their exact position on the number line. For example, we could locate the exact position of $\sqrt{2}$ on a number line as follows.

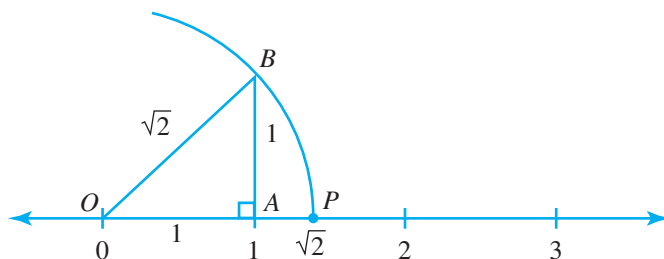
- Step 1 Draw a number line using a scale of 1 unit = 2 cm. Leave enough space above the line to perform the construction.



- Step 2 Construct $\triangle OAB$ with $OA = AB = 1$ unit. Show by Pythagoras' Theorem that the exact length of OB is $\sqrt{2}$ units.



- Step 3 Place the compass point at O and using a radius of length OB , make an arc to cut the number line at P as shown. The interval OP is the same length as the interval OB , as OP and OB are equal radii of the circle with centre O . Therefore, $OP = \sqrt{2}$ units and P represents the exact position of the irrational number $\sqrt{2}$ on the number line.



■ A proof that $\sqrt{2}$ is not rational

This proof is based on contradiction!

If $\sqrt{2}$ is rational then it must be able to be expressed in the form $\frac{a}{b}$ ($b \neq 0$) where a and b are integers and have no common factor.

$$\text{i.e.} \quad \sqrt{2} = \frac{a}{b} \quad (b \neq 0)$$

$$\text{so} \quad 2 = \frac{a^2}{b^2}$$

$$\therefore a^2 = 2b^2$$

$\therefore a^2$ is divisible by 2.

Hence, a is divisible by 2.

Let $a = 2k$.

$$\therefore (2k)^2 = 2b^2$$

$$\therefore b^2 = 2k^2$$

$\therefore b^2$ is divisible by 2.

Hence, b is also divisible by 2.

This means that a and b have a common factor of 2, which is a contradiction of our original assumption that $\sqrt{2}$ is rational and could be expressed in the form $\frac{a}{b}$, where a and b had no common factor.

Hence, the assumption is false and $\sqrt{2}$ is not rational.

Example 1

State whether each number is rational or irrational.

a 27%

b $0.\dot{3}$

c $\sqrt{6}$

d $\frac{\pi}{4}$

Solutions

a $27\% = \frac{27}{100}$, $\therefore 27\%$ is a rational number.

b $0.\dot{3} = \frac{1}{3}$, $\therefore 0.\dot{3}$ is a rational number.

c $\sqrt{6} = 2.449\,489\,743 \dots$, which neither terminates nor recurs, $\therefore \sqrt{6}$ is an irrational number.

d $\frac{\pi}{4} = 0.785\,398\,163 \dots$, which neither terminates nor recurs, $\therefore \frac{\pi}{4}$ is an irrational number.

EG
+ S**Example 2**Arrange $\sqrt{30}$, 4.6, 5, $3\sqrt{3}$ in ascending order.**Solution**

Write any surds as decimals, correct to 3 decimal places.

$$\sqrt{30}, 4.6, 5, 3\sqrt{3}$$

$$= 5.477, 4.6, 5, 5.196$$

In ascending order, the numbers are $4.6, 5, 5.196, 5.477$
 i.e. $4.6, 5, 3\sqrt{3}, \sqrt{30}$.

Exercise 8.1**1** State whether or not each of these numbers is a surd.

a $\sqrt{3}$	b $\sqrt{4}$	c $\sqrt{10}$	d $\sqrt{15}$	e $\sqrt{25}$
f $\sqrt{33}$	g $\sqrt{36}$	h $\sqrt{49}$	i $\sqrt{50}$	j $\sqrt{90}$
k $\sqrt[3]{6}$	l $\sqrt[3]{8}$	m $\sqrt[3]{30}$	n $\sqrt[3]{88}$	o $\sqrt[3]{100}$

2 State whether each of these numbers is rational or irrational.

a $\frac{1}{2}$	b $\sqrt{9}$	c $\sqrt{2}$	d $1\frac{2}{3}$	e 0.6
f π	g $0.\dot{3}$	h 1.7	i $2\sqrt{5}$	j $\sqrt{144}$
k 25%	l $3 + \sqrt{11}$	m 8^2	n $\sqrt{99}$	o 5.3%
p -5	q $\sqrt{1}$	r $\sqrt[3]{27}$	s $\frac{1}{\pi}$	t $\sqrt{16} + \sqrt{49}$
u $\sqrt{4} + \sqrt{5}$	v $\sqrt[3]{3}$	w $\sqrt{\sqrt{81}}$	x 2:3	y $\frac{\sqrt{5}}{2}$

3 State whether each number is rational or irrational.

a 0.7	b $5\frac{1}{2}\%$	c $\sqrt{12}$	d $\frac{\pi}{3}$
e 4:1	f 0	g $\sqrt[3]{343}$	h 2005
i $\frac{\sqrt{18}}{2}$	j $(\sqrt{3})^2$	k $2\sqrt{2}$	l $\frac{22}{7}$
m 3×10^2	n 4π	o $0.\dot{2}\dot{7}$	p 2^{2^2}

4 Between which two consecutive integers do each of these surds lie?

a $\sqrt{5}$	b $\sqrt{19}$	c $\sqrt{76}$	d $\sqrt{150}$

Consolidation

5 Express each of these irrational numbers as a decimal, correct to 1 decimal place.

a $\sqrt{7}$

b $\sqrt[3]{13}$

c $\sqrt[4]{47}$

d $\sqrt[5]{61}$

e $1 + \sqrt{5}$

f $11 - \sqrt{3}$

g $3\sqrt{29}$

h $3\sqrt{2} + 2\sqrt{7}$

i $\frac{1}{\sqrt{5}}$

j $\frac{15}{\sqrt{2}}$

k $\frac{1}{2\sqrt{3}}$

l $\frac{\sqrt{43}}{5\sqrt{7}}$

m $\frac{1 + \sqrt{10}}{2}$

n $\frac{\sqrt{3} + \sqrt{11}}{5}$

o $\frac{14 - 3\sqrt{2}}{2\sqrt{5}}$

p $\frac{2 + \sqrt{6}}{2 - \sqrt{6}}$

6 Arrange each group of numbers in ascending order.

a $\sqrt{13}, 3.4, \sqrt{15}, 3.7$

b $9.6, \sqrt{96}, \sqrt{83}, 9.15$

c $3\sqrt{2}, 3.6, \sqrt[3]{90}, 4$

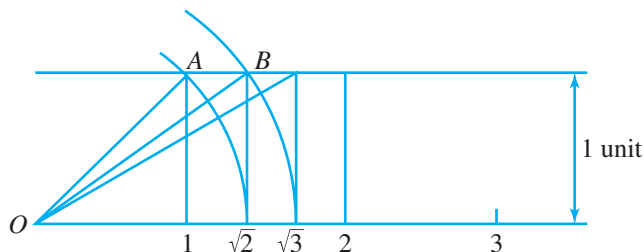
d $4\sqrt{3}, 7, \frac{41}{6}, 3\sqrt{5}$

e $\sqrt{26}, 5, 2\sqrt{6}, 3\sqrt{3}$

f $1 + \sqrt{67}, 4\sqrt{5}, 12 - \sqrt{11}, 10.1$

7 The method outlined here allows for the location of all possible surds on the real number line.

a Construct a number line with intervals of 1 unit marked on it, then construct another line parallel to this such that the lines are 1 unit apart.



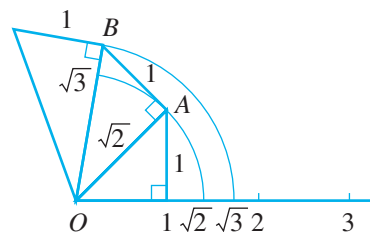
b Show that the interval OA has length $\sqrt{2}$ units. Use a compass to locate the position of $\sqrt{2}$ on the number line.

c Find the length of the interval OB and hence locate the position of $\sqrt{3}$ on the number line.

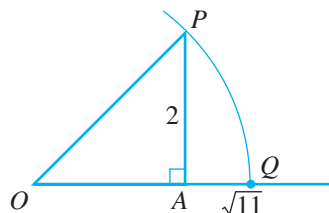
d Repeat this process two more times to locate the position of $\sqrt{5}$ on the number line.

8 Here is another method for locating surds on the number line.

a Construct a number line with intervals of 1 unit marked on it. On this number line, construct a right-angled triangle with perpendicular sides of length 1 unit and hypotenuse OA as shown.



- b Show that the interval OA has length $\sqrt{2}$ units. Use a compass to locate the position of $\sqrt{2}$ on the number line.
- c At A , construct an interval AB of length 1 unit, perpendicular to OA . Show that the interval OB has length $\sqrt{3}$ units. Use a compass to locate the position of $\sqrt{3}$ on the number line.
- d Repeat this process two more times to locate the position of $\sqrt{5}$ on the number line.
- 9 This diagram, not drawn to scale, shows the position of $\sqrt{11}$ marked at Q on a number line. If the interval AP has length 2 units, find:
- the irrational number that is represented by the point A on the number line
 - the exact length of the interval AQ .
- 10 Construct a number line with intervals of length 1 unit marked on it, where 1 unit is equal to the diameter of a 10 cent coin.



- Determine the exact circumference of the coin, in units.
- Explain how the position of π could now be located on this number line.

Further applications

- 11 Classify each of the following as always rational, sometimes rational or never rational, for positive integers a, b . If the expression is sometimes rational, explain under what conditions this is the case.
- | | | |
|------------------|------------------------------|-------------------------------|
| a $(\sqrt{a})^2$ | b $\sqrt{a^2}$ | c $\sqrt{a} + \sqrt{b}$ |
| d $\sqrt{a-b}$ | e $\sqrt{a} \times \sqrt{b}$ | f $\frac{\sqrt{a}}{\sqrt{b}}$ |

8.2

Simplifying surds

When a surd is expressed in the form $\sqrt{n}$ it is called an **entire surd**. Some examples of entire surds are $\sqrt{2}$, $\sqrt{5}$ and $\sqrt{14}$. A surd $\sqrt{n}$ is in its simplest form when n is not divisible by a square number (other than 1). That is, the number is not divisible by 4, 9, 16, 25, 36, ...

When we simplify surds we make use of the fact that $(\sqrt{x})^2 = x = \sqrt{x^2}$.

To simplify a surd of the form $\sqrt{n}$:

- ⊙ Express $\sqrt{n}$ in the form $\sqrt{p} \times \sqrt{q}$ where $n = pq$ and p is the largest square number which divides into n .
- ⊙ Write the answer in the form $a\sqrt{q}$, where $a = \sqrt{p}$.

Example 1

Simplify each of these surds.

a $\sqrt{12}$

b $\sqrt{200}$

c $5\sqrt{63}$

Solutions

$$\begin{aligned} \text{a } \sqrt{12} &= \sqrt{4} \times \sqrt{3} \\ &= 2 \times \sqrt{3} \\ &= 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b } \sqrt{200} &= \sqrt{100} \times \sqrt{2} \\ &= 10 \times \sqrt{2} \\ &= 10\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{c } 5\sqrt{63} &= 5 \times \sqrt{9} \times \sqrt{7} \\ &= 5 \times 3 \times \sqrt{7} \\ &= 15\sqrt{7} \end{aligned}$$

Example 2

Express $3\sqrt{7}$ as an entire surd.

Solution

$$\begin{aligned} 3\sqrt{7} &= \sqrt{9} \times \sqrt{7} \\ &= \sqrt{63} \end{aligned}$$

Example 3

Simplify:

a $\sqrt{16p}$

b $\sqrt{a^7}$

c $\sqrt{18x^2y^3}$

Solutions

$$\begin{aligned} \text{a } \sqrt{16p} &= \sqrt{16} \times \sqrt{p} \\ &= 4 \times \sqrt{p} \\ &= 4\sqrt{p} \end{aligned}$$

$$\begin{aligned} \text{b } \sqrt{a^7} &= \sqrt{a^6} \times \sqrt{a} \\ &= a^3 \times \sqrt{a} \\ &= a^3\sqrt{a} \end{aligned}$$

$$\begin{aligned} \text{c } \sqrt{18x^2y^3} &= \sqrt{18} \times \sqrt{x^2} \times \sqrt{y^3} \\ &= \sqrt{9} \times \sqrt{2} \times \sqrt{x^2} \times \sqrt{y^2} \times \sqrt{y} \\ &= 3 \times \sqrt{2} \times x \times y \times \sqrt{y} \\ &= 3xy\sqrt{2y} \end{aligned}$$

Exercise 8.2

1 Express each product in the form $\sqrt{n}$.

a $\sqrt{2} \times \sqrt{5}$

b $\sqrt{3} \times \sqrt{7}$

c $\sqrt{7} \times \sqrt{2}$

d $\sqrt{5} \times \sqrt{3}$

e $\sqrt{11} \times \sqrt{2}$

f $\sqrt{3} \times \sqrt{13}$

g $\sqrt{10} \times \sqrt{3}$

h $\sqrt{5} \times \sqrt{11}$

2 Evaluate:

a $\sqrt{2} \times \sqrt{2}$

b $\sqrt{3} \times \sqrt{3}$

c $\sqrt{7} \times \sqrt{7}$

d $\sqrt{11} \times \sqrt{11}$

e $2\sqrt{2} \times 2\sqrt{2}$

f $4\sqrt{3} \times 4\sqrt{3}$

g $5\sqrt{2} \times 5\sqrt{2}$

h $3\sqrt{7} \times 3\sqrt{7}$

i $(\sqrt{5})^2$

j $(\sqrt{13})^2$

k $(2\sqrt{5})^2$

l $(4\sqrt{2})^2$

3 Express each of these in simplest surd form.

a $\sqrt{8}$

b $\sqrt{12}$

c $\sqrt{18}$

d $\sqrt{20}$

e $\sqrt{24}$

f $\sqrt{27}$

g $\sqrt{28}$

h $\sqrt{32}$

i $\sqrt{40}$

j $\sqrt{45}$

k $\sqrt{48}$

l $\sqrt{50}$

m $\sqrt{54}$

n $\sqrt{60}$

o $\sqrt{63}$

p $\sqrt{72}$

q $\sqrt{75}$

r $\sqrt{80}$

s $\sqrt{84}$

t $\sqrt{90}$

u $\sqrt{96}$

v $\sqrt{99}$

w $\sqrt{150}$

x $\sqrt{200}$

Consolidation

4 Simplify:

a $5\sqrt{12}$

b $6\sqrt{20}$

c $2\sqrt{27}$

d $4\sqrt{28}$

e $8\sqrt{45}$

f $11\sqrt{48}$

g $10\sqrt{50}$

h $3\sqrt{63}$

i $4\sqrt{72}$

j $6\sqrt{75}$

k $3\sqrt{88}$

l $5\sqrt{300}$

5 Express in simplest surd form:

a $\sqrt{128}$

b $\sqrt{160}$

c $\sqrt{175}$

d $\sqrt{242}$

e $\sqrt{243}$

f $\sqrt{245}$

g $\sqrt{288}$

h $\sqrt{396}$

i $\sqrt{405}$

j $\sqrt{448}$

k $\sqrt{675}$

l $\sqrt{720}$

6 Express each of the following as an entire surd.

a $2\sqrt{2}$

b $3\sqrt{2}$

c $2\sqrt{5}$

d $3\sqrt{3}$

e $4\sqrt{2}$

f $2\sqrt{11}$

g $3\sqrt{5}$

h $4\sqrt{3}$

i $5\sqrt{2}$

j $2\sqrt{13}$

k $3\sqrt{6}$

l $2\sqrt{14}$

m $2\sqrt{15}$

n $3\sqrt{7}$

o $6\sqrt{2}$

p $5\sqrt{3}$

q $4\sqrt{5}$

r $2\sqrt{22}$

s $3\sqrt{10}$

t $4\sqrt{6}$

7 Find the value of each pronumeral.

a $2\sqrt{3} = \sqrt{k}$

b $\sqrt{a} = 7\sqrt{2}$

c $m\sqrt{7} = \sqrt{112}$

d $\sqrt{275} = 5\sqrt{y}$

Further applications

8 Simplify these algebraic surds.

a $\sqrt{9a}$

b $\sqrt{25a}$

c $\sqrt{a^3}$

d $\sqrt{a^5}$

e $\sqrt{a^2b}$

f $\sqrt{ab^2}$

g $\sqrt{a^4b}$

h $\sqrt{ab^6}$

i $\sqrt{a^3b^4}$

j $\sqrt{a^8b^5}$

k $\sqrt{a^5b^3}$

l $\sqrt{a^9b^7}$

m $\sqrt{4a^3}$

n $\sqrt{12a^4}$

o $\sqrt{18a^5}$

p $\sqrt{27a^2b}$

q $\sqrt{36ab^4}$

r $\sqrt{45a^3b^2}$

s $\sqrt{48a^4b^2}$

t $\sqrt{50a^7b^{11}}$

9 Express as entire surds:

a $7\sqrt{a}$

b $a\sqrt{a}$

c $a^4\sqrt{a}$

d $ab\sqrt{a}$

e $5\sqrt{ab}$

f $2\sqrt{2a}$

g $4a^2\sqrt{3ab}$

h $3a^2b^3\sqrt{5ab}$

10 Show that $\frac{x-y}{x+y}\sqrt{\frac{x+y}{x-y}} = \sqrt{\frac{x-y}{x+y}}$

TRY THIS Greater number

Without using a calculator, determine which number is greater:

$$1 + 2\sqrt{3} \text{ or } 8 - 2\sqrt{3}$$

8.3

Addition and subtraction of surds

Like surds are surds that have the same number or expression under the radical sign.

- Examples of like surds are $3\sqrt{2}$ and $5\sqrt{2}$, $4\sqrt{3}$ and $\sqrt{3}$, $-2\sqrt{a}$ and $6\sqrt{a}$.
- Examples of unlike surds are $\sqrt{2}$ and $\sqrt{3}$, $5\sqrt{7}$ and $5\sqrt{3}$, $\sqrt{3a}$ and $\sqrt{3b}$.

Surds can be treated in the same way as pronumerals when adding or subtracting. For example, to simplify $3a + 2a$, we add the co-efficients and keep the same pronumeral. That is, we write $3a + 2a = 5a$ because '3 lots of a number a ' plus '2 lots of the number a ' is equal to '5 lots of the number a '. However, pronumerals can stand for irrational numbers as well as for rational numbers. Therefore, by substituting $a = \sqrt{7}$, for example, we could write $3\sqrt{7} + 2\sqrt{7} = 5\sqrt{7}$. That is, '3 lots of $\sqrt{7}$ ' plus '2 lots of $\sqrt{7}$ ' is equal to '5 lots of $\sqrt{7}$ '.

When adding or subtracting surds, we add or subtract the rational parts and keep the same irrational part.

$$a\sqrt{c} + b\sqrt{c} = (a + b)\sqrt{c} \quad \text{and} \quad a\sqrt{c} - b\sqrt{c} = (a - b)\sqrt{c}$$

Expressions such as $3a + 2b$ cannot be simplified in algebra because $3a$ and $2b$ are not like terms. That is, we would be adding 3 lots of one number with 2 lots of a different number. So it is with surds. We cannot simplify $3\sqrt{7} + 2\sqrt{5}$ because '3 lots of $\sqrt{7}$ ' plus '2 lots of $\sqrt{5}$ ' does not equal '5 lots of $\sqrt{12}$ '.

Only like surds can be added or subtracted.

NOTE: In some questions, the surds that are to be added or subtracted may not initially be like surds. However, once simplified, they may become like surds, in which case they can then be added or subtracted.

To add or subtract surds:

- ⊙ express each surd in its simplest form
- ⊙ add or subtract the rational parts of like surds
- ⊙ keep the same irrational parts.

Example 1

Simplify:

a $3\sqrt{5} + 7\sqrt{5}$

b $7\sqrt{2} - \sqrt{2}$

c $8\sqrt{6} + 5\sqrt{6} - 4\sqrt{6}$

Solutions

$$\begin{aligned} \mathbf{a} \quad & 3\sqrt{5} + 7\sqrt{5} \\ &= (3 + 7)\sqrt{5} \\ &= 10\sqrt{5} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & 7\sqrt{2} - \sqrt{2} \\ &= (7 - 1)\sqrt{2} \\ &= 6\sqrt{2} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & 8\sqrt{6} + 5\sqrt{6} - 4\sqrt{6} \\ &= (8 + 5 - 4)\sqrt{6} \\ &= 9\sqrt{6} \end{aligned}$$

Example 2

Simplify $5\sqrt{3} + 6\sqrt{2} - 2\sqrt{3} + 4\sqrt{2}$.

Solution

$$\begin{aligned} 5\sqrt{3} + 6\sqrt{2} - 2\sqrt{3} + 4\sqrt{2} &= 5\sqrt{3} - 2\sqrt{3} + 6\sqrt{2} + 4\sqrt{2} \\ &= 3\sqrt{3} + 10\sqrt{2} \end{aligned}$$

Example 3

Simplify:

a $\sqrt{18} + \sqrt{32}$

b $5\sqrt{20} - 2\sqrt{45}$

Solutions

$$\begin{aligned}
 \text{a} \quad & \sqrt{18} + \sqrt{32} \\
 &= (\sqrt{9} \times \sqrt{2}) + (\sqrt{16} \times \sqrt{2}) \\
 &= 3\sqrt{2} + 4\sqrt{2} \\
 &= 7\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad & 5\sqrt{20} - 2\sqrt{45} \\
 &= (5 \times \sqrt{4} \times \sqrt{5}) - (2 \times \sqrt{9} \times \sqrt{5}) \\
 &= (5 \times 2 \times \sqrt{5}) - (2 \times 3 \times \sqrt{5}) \\
 &= 10\sqrt{5} - 6\sqrt{5} \\
 &= 4\sqrt{5}
 \end{aligned}$$

Exercise 8.3

1 a Does $9 + 16 = 25$?

b Evaluate $\sqrt{9}$, $\sqrt{16}$ and $\sqrt{25}$. Does $\sqrt{9} + \sqrt{16} = \sqrt{25}$?

c Evaluate $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ correct to 3 decimal places. Does $\sqrt{2} + \sqrt{3} = \sqrt{5}$?

d In general, does $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$?

2 Simplify:

a $5\sqrt{2} + 2\sqrt{2}$

b $8\sqrt{3} - 5\sqrt{3}$

c $2\sqrt{5} + 7\sqrt{5}$

d $10\sqrt{11} - 6\sqrt{11}$

e $2\sqrt{7} + \sqrt{7}$

f $6\sqrt{3} - \sqrt{3}$

g $\sqrt{5} + \sqrt{5}$

h $\sqrt{13} - \sqrt{13}$

i $8\sqrt{10} - 7\sqrt{10}$

j $12\sqrt{11} + 5\sqrt{11}$

k $9\sqrt{7} - 4\sqrt{7}$

l $6\sqrt{2} + 8\sqrt{2}$

m $2\sqrt{3} - 6\sqrt{3}$

n $-8\sqrt{5} + 5\sqrt{5}$

o $-4\sqrt{6} - 3\sqrt{6}$

3 Simplify:

a $5\sqrt{2} + 3\sqrt{2} + \sqrt{2}$

b $7\sqrt{3} - 5\sqrt{3} + 4\sqrt{3}$

c $12\sqrt{10} - 3\sqrt{10} - 2\sqrt{10}$

d $\sqrt{6} + \sqrt{6} + \sqrt{6}$

e $9\sqrt{7} - 2\sqrt{7} - 6\sqrt{7}$

f $4\sqrt{5} - 3\sqrt{5} + 8\sqrt{5}$

g $4\sqrt{3} - 6\sqrt{3} + 5\sqrt{3}$

h $-3\sqrt{11} + 6\sqrt{11} + 4\sqrt{11}$

i $2\sqrt{10} - 3\sqrt{10} - 4\sqrt{10}$

Consolidation

4 Simplify by collecting the like surds:

a $4\sqrt{2} + 2\sqrt{3} + 3\sqrt{2} + \sqrt{3}$

b $5\sqrt{5} + 4\sqrt{3} + 7\sqrt{5} + 6\sqrt{3}$

c $8\sqrt{7} + 2\sqrt{5} - 3\sqrt{7} + 2\sqrt{5}$

d $10\sqrt{6} + 9\sqrt{2} - 3\sqrt{2} - \sqrt{6}$

e $9\sqrt{3} + 3\sqrt{5} + 5\sqrt{3} - 3\sqrt{5}$

f $2\sqrt{10} + 4\sqrt{11} - \sqrt{10} + 5\sqrt{11}$

g $10\sqrt{7} + 2\sqrt{3} - 3\sqrt{7} - 6\sqrt{3}$

h $3\sqrt{5} - 2\sqrt{2} - 11\sqrt{5} + 7\sqrt{2}$

i $8\sqrt{2} + \sqrt{5} + 9\sqrt{2} - 6\sqrt{5}$

j $4\sqrt{6} - 3\sqrt{10} + 11\sqrt{6} + 12\sqrt{10}$

k $\sqrt{3} - 10\sqrt{7} + \sqrt{3} + 2\sqrt{7}$

l $9\sqrt{13} - \sqrt{2} - 10\sqrt{13} - \sqrt{2}$

5 Express each surd in simplest form, then collect the like surds.

a $\sqrt{8} + \sqrt{2}$

b $\sqrt{12} - \sqrt{3}$

c $\sqrt{5} + \sqrt{20}$

d $\sqrt{27} + \sqrt{12}$

e $\sqrt{45} - \sqrt{20}$

f $\sqrt{32} - \sqrt{8}$

g $\sqrt{63} + \sqrt{7}$

h $\sqrt{18} + \sqrt{50}$

i $\sqrt{40} - \sqrt{10}$

j $\sqrt{45} + \sqrt{80}$

k $\sqrt{98} - \sqrt{32}$

l $\sqrt{200} + \sqrt{50}$

m $8\sqrt{2} + \sqrt{72}$

n $4\sqrt{7} + \sqrt{28}$

o $11\sqrt{3} - \sqrt{48}$

p $7\sqrt{5} - \sqrt{20}$

q $\sqrt{96} + 6\sqrt{6}$

r $\sqrt{63} + 8\sqrt{7}$

s $13\sqrt{6} - 3\sqrt{150}$

t $15\sqrt{11} - 5\sqrt{44}$

u $6\sqrt{13} + 2\sqrt{52}$

v $8\sqrt{45} + 3\sqrt{20}$

w $4\sqrt{98} - 5\sqrt{50}$

x $7\sqrt{75} - 6\sqrt{27}$

6 Express in simplest surd form.

a $\sqrt{20} + \sqrt{27} + \sqrt{45} + \sqrt{12}$

b $\sqrt{63} + 7\sqrt{2} - \sqrt{28} - \sqrt{18}$

c $5\sqrt{8} + 2\sqrt{40} - \sqrt{32} + 4\sqrt{90}$

d $4\sqrt{75} - 2\sqrt{48} + 7\sqrt{6} - 3\sqrt{54}$

e $7\sqrt{18} + \sqrt{125} - 5\sqrt{32} - 2\sqrt{80}$

f $\sqrt{300} - 11\sqrt{10} + 8\sqrt{27} + 3\sqrt{90}$

Further applications

7 Simplify:

a $5\sqrt{p} + 3\sqrt{q} + 4\sqrt{p} - \sqrt{q}$

b $7\sqrt{u} + 2\sqrt{v} - 3\sqrt{u} - 4\sqrt{v}$

c $9\sqrt{x} - \sqrt{y} + 2\sqrt{x} - 6\sqrt{y}$

d $3\sqrt{m} - 4\sqrt{n} - 3\sqrt{m} + 5\sqrt{n}$

8 Simplify:

a $\sqrt{4a} + 5\sqrt{a}$

b $\sqrt{9p} + \sqrt{16p}$

c $\sqrt{27x} - \sqrt{12x}$

d $\sqrt{k^3} + 3k\sqrt{k}$

e $\sqrt{m^5} + 7m^2\sqrt{m}$

f $\sqrt{18t^2} - \sqrt{8t^2}$

g $\sqrt{75y^3} - \sqrt{48y^3}$

h $\sqrt{49u^2v^7} - 4uv^3\sqrt{v}$

i $\sqrt{75c^3d^4} + 10cd^2\sqrt{3c}$

8.4

Multiplication and division of surds

The following rules should be used to multiply or divide surds, where $a > 0$ and $b > 0$:

$$(\sqrt{a})^2 = \sqrt{a^2} = a$$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$$

$$\frac{a\sqrt{b}}{c\sqrt{d}} = \frac{a}{c}\sqrt{\frac{b}{d}}$$

To multiply or divide surds:

- ⊙ multiply or divide the rational parts
- ⊙ multiply or divide the irrational parts
- ⊙ simplify if possible.

NOTE: In questions where the product is large, it is often easier to simplify the surds first, then multiply.

EG
+ S

Example 1

Simplify:

a $\sqrt{5} \times \sqrt{5}$

b $3\sqrt{2} \times 3\sqrt{2}$

Solutions

a $\sqrt{5} \times \sqrt{5} = \sqrt{5^2}$
 $= 5$

b $3\sqrt{2} \times 3\sqrt{2}$
 $= (3 \times 3) \times (\sqrt{2} \times \sqrt{2})$
 $= 9 \times 2$
 $= 18$

EG
+ S

Example 2

Simplify:

a $\sqrt{7} \times \sqrt{2}$

b $4\sqrt{3} \times 2\sqrt{5}$

c $\sqrt{10} \times \sqrt{2}$

Solutions

a $\sqrt{7} \times \sqrt{2}$
 $= \sqrt{7 \times 2}$
 $= \sqrt{14}$

b $4\sqrt{3} \times 2\sqrt{5}$
 $= 4 \times 2 \times \sqrt{3 \times 5}$
 $= 8\sqrt{15}$

c $\sqrt{10} \times \sqrt{2}$
 $= \sqrt{20}$
 $= \sqrt{4} \times \sqrt{5}$
 $= 2\sqrt{5}$

EG
+ S

Example 3

Simplify:

a $\sqrt{30} \div \sqrt{6}$

b $24\sqrt{55} \div 8\sqrt{5}$

c $\sqrt{48} \div \sqrt{3}$

Solutions

a $\sqrt{30} \div \sqrt{6}$
 $= \frac{\sqrt{30}}{\sqrt{6}}$
 $= \sqrt{\frac{30}{6}}$
 $= \sqrt{5}$

b $24\sqrt{55} \div 8\sqrt{5}$
 $= \frac{24\sqrt{55}}{8\sqrt{5}}$
 $= \frac{24}{8} \sqrt{\frac{55}{5}}$
 $= 3\sqrt{11}$

c $\sqrt{48} \div \sqrt{3}$
 $= \frac{\sqrt{48}}{\sqrt{3}}$
 $= \sqrt{\frac{48}{3}}$
 $= \sqrt{16}$
 $= 4$

Exercise 8.4**1** Simplify:

a $\sqrt{5} \times \sqrt{3}$

b $\sqrt{2} \times \sqrt{11}$

c $\sqrt{7} \times \sqrt{5}$

d $\sqrt{10} \times \sqrt{3}$

e $7 \times 2\sqrt{2}$

f $3\sqrt{5} \times 4$

g $6\sqrt{2} \times 5$

h $2 \times 8\sqrt{3}$

i $5\sqrt{2} \times 4\sqrt{3}$

j $3\sqrt{7} \times 9\sqrt{3}$

k $4\sqrt{5} \times 7\sqrt{11}$

l $9\sqrt{3} \times 5\sqrt{13}$

m $\sqrt{5} \times \sqrt{5}$

n $\sqrt{17} \times \sqrt{17}$

o $3\sqrt{2} \times \sqrt{2}$

p $5\sqrt{3} \times 5\sqrt{3}$

q $\sqrt{2} \times \sqrt{3} \times \sqrt{5}$

r $\sqrt{5} \times \sqrt{7} \times \sqrt{3}$

s $2\sqrt{7} \times \sqrt{2} \times 4\sqrt{3}$

t $3\sqrt{5} \times 2\sqrt{7} \times 8\sqrt{2}$

2 Simplify:

a $\sqrt{10} \div \sqrt{2}$

b $\sqrt{30} \div \sqrt{5}$

c $\sqrt{77} \div \sqrt{11}$

d $\sqrt{42} \div \sqrt{7}$

e $10\sqrt{3} \div 2$

f $21\sqrt{2} \div 7$

g $20\sqrt{6} \div 4$

h $48\sqrt{11} \div 6$

i $2\sqrt{6} \div \sqrt{6}$

j $5\sqrt{3} \div \sqrt{3}$

k $9\sqrt{5} \div 3\sqrt{5}$

l $24\sqrt{7} \div 6\sqrt{7}$

m $8\sqrt{30} \div \sqrt{2}$

n $3\sqrt{26} \div \sqrt{2}$

o $4\sqrt{70} \div \sqrt{10}$

p $9\sqrt{35} \div \sqrt{5}$

q $25\sqrt{21} \div 5\sqrt{3}$

r $48\sqrt{22} \div 8\sqrt{2}$

s $54\sqrt{30} \div 6\sqrt{3}$

t $32\sqrt{65} \div 4\sqrt{5}$

Consolidation**3** Multiply each of the following and give the answer in simplest surd form.

a $\sqrt{6} \times \sqrt{2}$

b $\sqrt{2} \times \sqrt{10}$

c $\sqrt{3} \times \sqrt{6}$

d $\sqrt{12} \times \sqrt{2}$

e $\sqrt{2} \times \sqrt{14}$

f $\sqrt{8} \times \sqrt{5}$

g $\sqrt{3} \times \sqrt{15}$

h $\sqrt{6} \times \sqrt{8}$

i $\sqrt{5} \times \sqrt{10}$

j $\sqrt{18} \times \sqrt{3}$

k $\sqrt{12} \times \sqrt{6}$

l $\sqrt{15} \times \sqrt{5}$

m $4\sqrt{6} \times \sqrt{2}$

n $2\sqrt{8} \times \sqrt{3}$

o $\sqrt{20} \times 5\sqrt{2}$

p $3\sqrt{22} \times 6\sqrt{2}$

q $2\sqrt{7} \times 7\sqrt{8}$

r $3\sqrt{8} \times 4\sqrt{10}$

s $4\sqrt{3} \times 2\sqrt{30}$

t $5\sqrt{12} \times 2\sqrt{8}$

u $4\sqrt{33} \times 5\sqrt{3}$

4 Simplify each of the surds, then multiply. Give your answers in simplest form.

a $\sqrt{6} \times \sqrt{18}$

b $\sqrt{12} \times \sqrt{20}$

c $\sqrt{18} \times \sqrt{12}$

d $\sqrt{24} \times \sqrt{27}$

e $\sqrt{45} \times \sqrt{63}$

f $\sqrt{27} \times \sqrt{44}$

g $\sqrt{50} \times \sqrt{18}$

h $\sqrt{12} \times \sqrt{48}$

i $\sqrt{80} \times \sqrt{20}$

j $\sqrt{28} \times 2\sqrt{3}$

k $3\sqrt{20} \times \sqrt{8}$

l $\sqrt{54} \times 7\sqrt{32}$

m $8\sqrt{24} \times 3\sqrt{12}$

n $6\sqrt{45} \times 5\sqrt{72}$

o $3\sqrt{98} \times 5\sqrt{112}$

5 Simplify each of the following.

a $\sqrt{24} \div \sqrt{3}$

b $\sqrt{60} \div \sqrt{5}$

c $\sqrt{120} \div \sqrt{6}$

d $\sqrt{72} \div \sqrt{3}$

e $\sqrt{56} \div \sqrt{7}$

f $\sqrt{54} \div \sqrt{3}$

g $\sqrt{18} \div \sqrt{2}$

h $\sqrt{20} \div \sqrt{5}$

i $\sqrt{75} \div \sqrt{3}$

j $3\sqrt{24} \div \sqrt{2}$

k $4\sqrt{90} \div \sqrt{5}$

l $4\sqrt{72} \div 2\sqrt{8}$

m $8\sqrt{120} \div 2\sqrt{3}$

n $21\sqrt{98} \div 7\sqrt{2}$

o $10\sqrt{96} \div 2\sqrt{3}$

Further applications

6 Simplify:

a $\sqrt{a} \times \sqrt{b}$

b $3\sqrt{m} \times 5\sqrt{n}$

c $\sqrt{x} \times \sqrt{y} \times \sqrt{z}$

d $5\sqrt{p} \times 2\sqrt{q} \times 3\sqrt{r}$

e $3\sqrt{2a} \times 2\sqrt{7b} \times 4\sqrt{5c}$

f $k\sqrt{k} \times \sqrt{k}$

g $a\sqrt{b} \times b\sqrt{a}$

h $5\sqrt{c} \times 3\sqrt{cd}$

i $4\sqrt{3a} \times 2\sqrt{12a}$

j $\sqrt{2p} \times \sqrt{14q}$

k $2\sqrt{45m} \times 3\sqrt{18m}$

l $3\sqrt{24p^2q} \times 5\sqrt{32pq^5}$

7 Simplify each of these quotients.

a $\sqrt{xy} \div \sqrt{y}$

b $\sqrt{pqr} \div \sqrt{pr}$

c $10\sqrt{a} \div 2\sqrt{a}$

d $24\sqrt{uvw} \div 3\sqrt{vw}$

e $\sqrt{28gh} \div \sqrt{7g}$

f $\sqrt{a^7} \div \sqrt{a}$

g $\sqrt{p^2q^5} \div \sqrt{pq}$

h $14\sqrt{75a^5} \div 2\sqrt{3a^2}$

i $10\sqrt{40x^6} \div 2\sqrt{5x^3}$

TRY THIS Imaginary numbers

In the Extension 2 Mathematics Course in Year 12, students study imaginary numbers such as i , where $i = \sqrt{-1}$.

1 Is $\sqrt{-1}$ a surd? Explain your answer.

2 Is $\sqrt{-1}$ a real number?

3 Find the values of i^2 , i^3 , i^4 .

4 Find the values of i^5 , i^6 , i^7 , i^8 . What do you notice?

5 What would be the value of

a i^{100} ?

b i^{45} ?

c i^{79} ?

8.5

Binomial products with surds

The rules for the expansion of binomial products can be applied to expressions that involve surds.

- ⊙ $a(b + c) = ab + ac$
- ⊙ $(a + b)(c + d) = ac + ad + bc + bd$
- ⊙ $(a + b)(a - b) = a^2 - b^2$
- ⊙ $(a + b)^2 = a^2 + 2ab + b^2$
- ⊙ $(a - b)^2 = a^2 - 2ab + b^2$

Example 1

Expand and simplify:

a $2\sqrt{3}(\sqrt{6} + 5\sqrt{3})$

Solutions

$$\begin{aligned}\mathbf{a} \quad & 2\sqrt{3}(\sqrt{6} + 5\sqrt{3}) \\ &= 2\sqrt{18} + 10\sqrt{9} \\ &= (2 \times 3\sqrt{2}) + (10 \times 3) \\ &= 6\sqrt{2} + 30\end{aligned}$$

b $(\sqrt{2} - 3)(\sqrt{5} + 4)$

$$\begin{aligned}\mathbf{b} \quad & (\sqrt{2} - 3)(\sqrt{5} + 4) \\ &= \sqrt{2}(\sqrt{5} + 4) - 3(\sqrt{5} + 4) \\ &= \sqrt{10} + 4\sqrt{2} - 3\sqrt{5} - 12\end{aligned}$$

Example 2

Expand and simplify:

a $(3 + \sqrt{5})^2$

Solutions

$$\begin{aligned}\mathbf{a} \quad & (3 + \sqrt{5})^2 \\ &= 3^2 + (2 \times 3 \times \sqrt{5}) + (\sqrt{5})^2 \\ &= 9 + 6\sqrt{5} + 5 \\ &= 14 + 6\sqrt{5}\end{aligned}$$

b $(5\sqrt{2} - \sqrt{6})^2$

$$\begin{aligned}\mathbf{b} \quad & (5\sqrt{2} - \sqrt{6})^2 \\ &= (5\sqrt{2})^2 - (2 \times 5\sqrt{2} \times \sqrt{6}) + (\sqrt{6})^2 \\ &= 50 - 10\sqrt{12} + 6 \\ &= 56 - 10\sqrt{12} \\ &= 56 - (10 \times 2\sqrt{3}) \\ &= 56 - 20\sqrt{3}\end{aligned}$$

EG
+ S**Example 3**

Expand and simplify:

a $(4 + \sqrt{7})(4 - \sqrt{7})$

b $(3\sqrt{5} - 2\sqrt{10})(3\sqrt{5} + 2\sqrt{10})$

Solutions

$$\begin{aligned}
 \mathbf{a} \quad & (4 + \sqrt{7})(4 - \sqrt{7}) \\
 &= 4^2 - (\sqrt{7})^2 \\
 &= 16 - 7 \\
 &= 9
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & (3\sqrt{5} - 2\sqrt{10})(3\sqrt{5} + 2\sqrt{10}) \\
 &= (3\sqrt{5})^2 - (2\sqrt{10})^2 \\
 &= 45 - 40 \\
 &= 5
 \end{aligned}$$

Exercise 8.5**1** Expand and simplify each of the following.

a $\sqrt{2}(\sqrt{3} + 5)$

b $\sqrt{7}(\sqrt{5} - 2)$

c $\sqrt{11}(\sqrt{2} + 6)$

d $2\sqrt{3}(5\sqrt{2} + \sqrt{5})$

e $4\sqrt{2}(2\sqrt{5} - \sqrt{11})$

f $3\sqrt{5}(6\sqrt{7} - 9\sqrt{6})$

g $\sqrt{3}(\sqrt{3} + 2)$

h $\sqrt{5}(6 + \sqrt{5})$

i $\sqrt{7}(\sqrt{7} - \sqrt{2})$

j $3\sqrt{2}(\sqrt{2} + 5)$

k $4\sqrt{3}(\sqrt{5} - \sqrt{3})$

l $5\sqrt{6}(4\sqrt{6} - 2\sqrt{5})$

m $\sqrt{2}(\sqrt{6} + 8)$

n $\sqrt{3}(\sqrt{15} - \sqrt{6})$

o $\sqrt{6}(\sqrt{8} - \sqrt{12})$

p $5\sqrt{3}(7 + \sqrt{8})$

q $2\sqrt{10}(\sqrt{2} - \sqrt{5})$

r $3\sqrt{15}(2\sqrt{3} - 3\sqrt{6})$

Consolidation**2** Expand and simplify:

a $(\sqrt{2} + 3)(\sqrt{5} + 2)$

b $(\sqrt{3} + 1)(\sqrt{7} - 4)$

c $(\sqrt{5} - 2)(\sqrt{3} - 6)$

d $(2 + \sqrt{10})(7 - \sqrt{3})$

e $(\sqrt{6} + \sqrt{2})(\sqrt{5} + \sqrt{7})$

f $(\sqrt{5} + \sqrt{3})(\sqrt{2} - \sqrt{11})$

g $(5 + \sqrt{6})(2 + \sqrt{6})$

h $(4 - \sqrt{3})(7 - \sqrt{3})$

i $(\sqrt{5} + \sqrt{2})(\sqrt{2} - \sqrt{3})$

j $(\sqrt{7} + 1)(\sqrt{7} - \sqrt{2})$

k $(3\sqrt{2} + \sqrt{5})(\sqrt{2} + \sqrt{7})$

l $(5\sqrt{3} - \sqrt{2})(2\sqrt{7} + \sqrt{3})$

m $(2\sqrt{5} + \sqrt{11})(2\sqrt{5} - \sqrt{3})$

n $(4\sqrt{2} - 2\sqrt{5})(2\sqrt{2} - \sqrt{5})$

o $(\sqrt{6} + 5)(2 + \sqrt{2})$

p $(4 + \sqrt{3})(\sqrt{8} + 1)$

q $(\sqrt{2} - 3)(\sqrt{10} + 4)$

r $(\sqrt{12} + \sqrt{10})(\sqrt{3} - \sqrt{2})$

s $(3\sqrt{2} + 2\sqrt{3})(5\sqrt{6} - 3\sqrt{8})$

t $(2\sqrt{5} - 4\sqrt{2})(3\sqrt{8} - 5\sqrt{10})$

3 Expand and simplify these perfect squares.

a $(\sqrt{2} + 1)^2$

b $(\sqrt{3} - 2)^2$

c $(\sqrt{5} + 3)^2$

d $(4 + \sqrt{7})^2$

e $(3 - \sqrt{6})^2$

f $(5 - \sqrt{2})^2$

g $(\sqrt{2} + \sqrt{3})^2$

h $(\sqrt{7} + \sqrt{5})^2$

i $(\sqrt{5} - \sqrt{2})^2$

j $(2\sqrt{2} - 3)^2$

k $(3\sqrt{2} + 4)^2$

l $(7 - 2\sqrt{5})^2$

m $(2\sqrt{3} + 3\sqrt{2})^2$

n $(5\sqrt{2} - 4\sqrt{5})^2$

o $(3\sqrt{7} + 2\sqrt{2})^2$

p $(\sqrt{6} + \sqrt{2})^2$

q $(\sqrt{10} - \sqrt{5})^2$

r $(\sqrt{3} + \sqrt{12})^2$

s $(3\sqrt{2} - \sqrt{10})^2$

t $(\sqrt{14} - 4\sqrt{2})^2$

u $(2\sqrt{15} + 6\sqrt{3})^2$

4 Expand each expression into a difference of two squares, then simplify.

a $(\sqrt{2} + 1)(\sqrt{2} - 1)$

b $(\sqrt{7} - 2)(\sqrt{7} + 2)$

c $(5 - \sqrt{2})(5 + \sqrt{2})$

d $(4 + \sqrt{3})(4 - \sqrt{3})$

e $(\sqrt{10} + \sqrt{3})(\sqrt{10} - \sqrt{3})$

f $(\sqrt{5} - \sqrt{11})(\sqrt{5} + \sqrt{11})$

g $(\sqrt{13} - \sqrt{2})(\sqrt{13} + \sqrt{2})$

h $(\sqrt{7} + \sqrt{15})(\sqrt{7} - \sqrt{15})$

i $(2\sqrt{3} + 1)(2\sqrt{3} - 1)$

j $(3\sqrt{2} - 2)(3\sqrt{2} + 2)$

k $(7 - 3\sqrt{3})(7 + 3\sqrt{3})$

l $(10 + 2\sqrt{3})(10 - 2\sqrt{3})$

m $(2\sqrt{5} + 3)(2\sqrt{5} - 3)$

n $(7 - 4\sqrt{3})(7 + 4\sqrt{3})$

o $(4\sqrt{3} + \sqrt{5})(4\sqrt{3} - \sqrt{5})$

p $(\sqrt{19} + 2\sqrt{2})(\sqrt{19} - 2\sqrt{2})$

q $(5\sqrt{2} - 2\sqrt{5})(5\sqrt{2} + 2\sqrt{5})$

r $(3\sqrt{6} - 5\sqrt{2})(3\sqrt{6} + 5\sqrt{2})$

5 Find the value of all pronumerals in each of the following.

a $(5\sqrt{2} + 2\sqrt{5})^2 = a + b\sqrt{10}$

b $(2\sqrt{6} - 3\sqrt{2})^2 = x + y\sqrt{3}$

6 Find the values of m and n in each of these, where $m > 0$.

a $(m + \sqrt{n})^2 = 14 + 6\sqrt{5}$

b $(\sqrt{m} + n)^2 = 43 - 12\sqrt{7}$

Further applications

7 Expand and simplify:

a $(1 + \sqrt{2})^3$

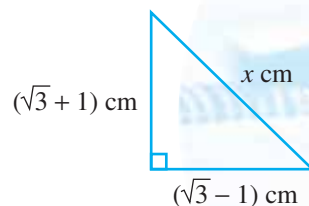
b $(2 - \sqrt{3})^4$

c $(\sqrt{3} + 2\sqrt{2})^4$

8 Show that $(\sqrt{2} + \sqrt{3} + \sqrt{6})^2 = 11 + 6\sqrt{2} + 4\sqrt{3} + 2\sqrt{6}$.

9 **a** Show that $(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2$ is rational.

b Using Pythagoras' theorem and your result from part **a**, find the length of the hypotenuse in this right-angled triangle.



10 Expand and simplify $(\sqrt{3} + \sqrt{2} + \sqrt{5})(\sqrt{3} + \sqrt{2} - \sqrt{5})$ using $(a + b)(a - b) = a^2 - b^2$.

8.6 Rationalising the denominator

Fractions that have a surd in the denominator are not easy to work with. To overcome this problem, we form an equivalent fraction with a rational denominator. That is, we **rationalise** the denominator.

Rationalising a monomial denominator

If the denominator of a fraction contains a single term, then the fraction is said to have a **monomial** denominator. To rationalise a monomial denominator, we use the fact that $\sqrt{a} \times \sqrt{a} = a$.

To rationalise a monomial denominator:

- ⊙ multiply the numerator and denominator by the surd in the denominator
- ⊙ simplify the surd in the numerator if possible
- ⊙ cancel any common factors.

Rationalising a binomial denominator (Extension)

If the denominator of a fraction contains two terms, then the fraction is said to have a binomial denominator.

The **conjugate** of the binomial $a + b$ is $a - b$ and vice-versa. To rationalise a binomial denominator, we make use of the identity $(a + b)(a - b) = a^2 - b^2$.

To rationalise a binomial denominator:

- ⊙ multiply the numerator and denominator by the conjugate of the denominator
- ⊙ simplify the surd in the numerator if possible
- ⊙ cancel any common factors.

NOTE: If a and/or b are surds, then both a^2 and b^2 must be integers. Hence $a^2 - b^2$ must be rational.

Example 1

Rationalise the denominator in each of these.

a $\frac{1}{\sqrt{5}}$

b $\frac{6}{5\sqrt{2}}$

c $\frac{\sqrt{3}}{\sqrt{8}}$

Solutions

$$\begin{aligned} \text{a} \quad & \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{5}}{5} \end{aligned}$$

$$\begin{aligned} \text{b} \quad & \frac{6}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{6\sqrt{2}}{10} \\ &= \frac{3\sqrt{2}}{5} \end{aligned}$$

$$\begin{aligned} \text{c} \quad & \frac{\sqrt{3}}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} \\ &= \frac{\sqrt{24}}{8} \\ &= \frac{2\sqrt{6}}{8} \\ &= \frac{\sqrt{6}}{4} \end{aligned}$$

EG
+S**Example 2**

Express $\frac{\sqrt{14+6}}{\sqrt{2}}$ in simplest surd form with a rational denominator.

Solution

$$\begin{aligned} \frac{\sqrt{14+6}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} &= \frac{\sqrt{28+6\sqrt{2}}}{2} \\ &= \frac{2\sqrt{7}+6\sqrt{2}}{2} \\ &= \frac{2(\sqrt{7}+3\sqrt{2})}{2} \\ &= \sqrt{7}+3\sqrt{2} \end{aligned}$$

EG
+S**Example 3 (Extension)**

Rationalise the denominator in each of these.

$$\text{a} \quad \frac{1}{\sqrt{7}-2}$$

$$\text{b} \quad \frac{3}{2\sqrt{3}+3}$$

$$\text{c} \quad \frac{\sqrt{5}+2}{\sqrt{5}-2}$$

Solutions

$$\begin{aligned} \text{a} \quad & \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} \\ &= \frac{\sqrt{7}+2}{(\sqrt{7})^2-2^2} \\ &= \frac{\sqrt{7}+2}{7-4} \\ &= \frac{\sqrt{7}+2}{3} \end{aligned}$$

$$\begin{aligned} \text{b} \quad & \frac{3}{2\sqrt{3}+3} \times \frac{2\sqrt{3}-3}{2\sqrt{3}-3} \\ &= \frac{3(2\sqrt{3}-3)}{(2\sqrt{3})^2-3^2} \\ &= \frac{3(2\sqrt{3}-3)}{12-9} \\ &= \frac{3(2\sqrt{3}-3)}{3} \\ &= 2\sqrt{3}-3 \end{aligned}$$

$$\begin{aligned} \text{c} \quad & \frac{\sqrt{5}+2}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} \\ &= \frac{(\sqrt{5}+2)^2}{(\sqrt{5})^2-2^2} \\ &= \frac{5+4\sqrt{5}+4}{5-4} \\ &= \frac{9+4\sqrt{5}}{1} \\ &= 9+4\sqrt{5} \end{aligned}$$

Exercise 8.6

1 Express each of these fractions with a rational denominator.

a $\frac{1}{\sqrt{2}}$

b $\frac{1}{\sqrt{3}}$

c $\frac{2}{\sqrt{5}}$

d $\frac{3}{\sqrt{7}}$

e $\frac{5}{\sqrt{6}}$

f $\frac{1}{3\sqrt{5}}$

g $\frac{1}{4\sqrt{3}}$

h $\frac{2}{3\sqrt{7}}$

i $\frac{3}{2\sqrt{5}}$

j $\frac{7}{8\sqrt{2}}$

k $\frac{\sqrt{2}}{\sqrt{3}}$

l $\frac{\sqrt{3}}{\sqrt{7}}$

m $\frac{3\sqrt{5}}{\sqrt{2}}$

n $\frac{\sqrt{6}}{3\sqrt{11}}$

o $\frac{5\sqrt{3}}{3\sqrt{2}}$

Consolidation

2 Express each fraction in simplest form with a rational denominator.

a $\frac{2}{\sqrt{2}}$

b $\frac{3}{\sqrt{3}}$

c $\frac{6}{\sqrt{2}}$

d $\frac{10}{\sqrt{5}}$

e $\frac{12}{\sqrt{3}}$

f $\frac{3}{\sqrt{6}}$

g $\frac{2}{\sqrt{10}}$

h $\frac{7}{\sqrt{21}}$

i $\frac{5}{\sqrt{30}}$

j $\frac{6}{\sqrt{42}}$

k $\frac{5}{2\sqrt{5}}$

l $\frac{8}{3\sqrt{2}}$

m $\frac{6}{5\sqrt{3}}$

n $\frac{9}{4\sqrt{6}}$

o $\frac{15}{2\sqrt{10}}$

p $\frac{3\sqrt{2}}{4\sqrt{3}}$

q $\frac{5\sqrt{3}}{2\sqrt{5}}$

r $\frac{9\sqrt{7}}{2\sqrt{6}}$

s $\frac{12\sqrt{5}}{5\sqrt{6}}$

t $\frac{6\sqrt{11}}{5\sqrt{10}}$

u $\frac{\sqrt{10}}{\sqrt{2}}$

v $\frac{\sqrt{3}}{\sqrt{21}}$

w $\frac{\sqrt{8}}{\sqrt{6}}$

x $\frac{3\sqrt{6}}{\sqrt{2}}$

y $\frac{4\sqrt{10}}{5\sqrt{5}}$

3 Express each fraction in simplest form with a rational denominator.

a $\frac{1 + \sqrt{2}}{\sqrt{2}}$

b $\frac{4 - \sqrt{3}}{\sqrt{3}}$

c $\frac{3 + \sqrt{7}}{\sqrt{5}}$

d $\frac{\sqrt{5} - 2}{\sqrt{6}}$

e $\frac{5 + 2\sqrt{2}}{\sqrt{3}}$

f $\frac{7 - 2\sqrt{3}}{\sqrt{10}}$

g $\frac{3\sqrt{2} + 4}{2\sqrt{5}}$

h $\frac{5\sqrt{3} - 3\sqrt{5}}{2\sqrt{7}}$

i $\frac{4 + \sqrt{2}}{\sqrt{2}}$

j $\frac{15 - 2\sqrt{3}}{\sqrt{3}}$

k $\frac{3\sqrt{5} - 20}{\sqrt{10}}$

l $\frac{\sqrt{3} - 9}{3\sqrt{6}}$

4 Rationalise the denominator in each fraction, then simplify.

a $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{5}}$

b $\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{7}}$

c $\frac{2}{\sqrt{11}} + \frac{3}{\sqrt{2}}$

d $\frac{3}{\sqrt{5}} + \frac{5}{\sqrt{3}}$

e $\frac{8}{\sqrt{2}} + \frac{2}{\sqrt{6}}$

f $\frac{6}{\sqrt{3}} - \frac{3}{\sqrt{6}}$

g $\frac{\sqrt{2}}{\sqrt{5}} + \frac{\sqrt{7}}{\sqrt{3}}$

h $\frac{5}{\sqrt{12}} - \frac{2}{\sqrt{3}}$

i $\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{3}}$

j $\frac{2}{\sqrt{5}} - \frac{3}{4\sqrt{2}}$

k $\frac{5}{2\sqrt{3}} - \frac{4}{3\sqrt{3}}$

l $\frac{3\sqrt{2}}{2\sqrt{3}} + \frac{2\sqrt{3}}{5\sqrt{2}}$

Further applications

5 Express each of these fractions in simplest surd form with a rational denominator.

a $\frac{1}{\sqrt{2} + 1}$

b $\frac{1}{\sqrt{3} - 1}$

c $\frac{1}{\sqrt{5} + \sqrt{2}}$

d $\frac{1}{\sqrt{7} - \sqrt{3}}$

e $\frac{2}{3 + \sqrt{3}}$

f $\frac{3}{4 - \sqrt{2}}$

g $\frac{7}{\sqrt{6} - \sqrt{5}}$

h $\frac{\sqrt{2}}{\sqrt{3} + 7}$

i $\frac{1}{2\sqrt{3} - 1}$

j $\frac{1}{3\sqrt{5} + 2}$

k $\frac{\sqrt{5}}{4\sqrt{3} - 2\sqrt{2}}$

l $\frac{11\sqrt{7}}{5\sqrt{2} + 2\sqrt{5}}$

m $\frac{\sqrt{15}}{2\sqrt{3} - \sqrt{5}}$

n $\frac{6}{3\sqrt{2} - 2\sqrt{3}}$

o $\frac{1}{\sqrt{63} + \sqrt{28}}$

p $\frac{\sqrt{10}}{\sqrt{20} - \sqrt{18}}$

q $\frac{\sqrt{2} + 1}{\sqrt{2} - 1}$

r $\frac{5 - \sqrt{3}}{5 + \sqrt{3}}$

s $\frac{\sqrt{10} + \sqrt{7}}{\sqrt{10} - \sqrt{7}}$

t $\frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}}$

6 Show that $\frac{1}{2\sqrt{2} - 1} - \frac{1}{2\sqrt{2} + 1}$ is rational, and find its value.

TRY THIS Exact values

Find the exact value of the following series.

$$\frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{99} + \sqrt{100}}$$

HINT: Try rationalising the denominators.



FOCUS ON WORKING MATHEMATICALLY

FIBONACCI NUMBERS AND THE GOLDEN MEAN

Introduction

There is a fascinating sequence of numbers called *Fibonacci* numbers which are closely related to patterns of growth in nature. Here it is:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Can you see the pattern? What would be the next three numbers in the sequence?

The sequence itself is named after an Italian mathematician Leonardo Pisano, better known by his nickname Fibonacci, who described the pattern when solving a problem about the breeding of rabbits. The date was 1202, well before the invention of the printing press, so his book was handwritten. You can find out more about him, and the problem he solved, by checking out the Internet. The properties of the sequence have been investigated for quite a long time. Today, there is even a learned journal called *Fibonacci Quarterly* where mathematicians publish their findings about Fibonacci numbers.

The learning activities below lead to a surprising finding about a special surd $\frac{1 + \sqrt{5}}{2}$, which figures prominently in art and architecture.



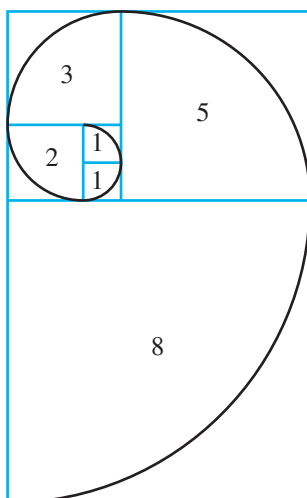


LEARNING ACTIVITIES

In this activity we are going to explore some of the patterns in the sequence visually. We will look at examples of many different types of real numbers: integers, rational numbers and irrational numbers. If you can, get access to the Internet for this activity.

1 The link of the numbers to patterns of growth

Let's begin with a visual representation of each number as a square. Start with two squares of length 1 cm, and build a square of side 2 as shown in the diagram. Continue the process and see how the growth spirals outwards. You can draw $\frac{1}{4}$ circles to show the complete effect. Allocate about $\frac{1}{2}$ page for the whole picture.



2 The occurrence of Fibonacci numbers in nature

The Internet has some excellent pictures of Fibonacci numbers occurring at growth points in plants, seed heads (a sunflower is a good example), pine cones, cauliflowers, petals on flowers and leaf arrangements. Try Ron Knott's web page at the University of Surrey, for example, www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html or discuss using a search engine with your teacher. You will also see Fibonacci numbers occurring in the spiral growth of shells. Compare the shell spiral with the Fibonacci spiral shown above. You can zoom in on these pictures for excellent viewing.

If examples could be brought to school for the class to see, it would be even better. Bring in cauliflower florets, pine cones and so on. Draw up a table to show your results. See if you can find more examples at the fruit market, in the garden or at the beach.



3 The link to the golden number

An interesting pattern in the sequence is found by forming a new sequence as follows.

- 1 Start with 1, 1, 2, 3, 5, 8, 13, 21, ... Now, divide each term by the one before it, i.e. term 2 by term 1, term 3 by term 2, term 4 by term 3, and so on, to get

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \dots$$

- 2 Record your results and plot them on a number line. You will see that these rational numbers (fractions) jump back and forth about a fixed number we call a limit, in this case the golden mean, or golden number. The further you go, the closer you get to the golden number. Write down its approximate value from your graph.
- 3 The golden number is actually an irrational number for which we use the Greek letter Φ (pronounced phi). Its exact value is $\frac{1 + \sqrt{5}}{2}$. Enter this into your calculator and see what you get as an approximation. Write down the value of Φ correct to 3 decimal places.



CHALLENGE

- 1 Starting from 1, 1, 2, 3, 5, 8, ... create another sequence, but this time dividing each term by the one after it, i.e. term 1 by term 2, term 2 by term 3, and so on. These terms are the reciprocals of the terms in the sequence above. You will get:

$$\frac{1}{1}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \dots$$

Choose a suitable scale and plot the numbers on a graph as before. What is the approximate value of the limit?

- 2 By forming the sequence of the reciprocals we end up with a limit which is close to 0.618. What do you notice?
- 3 Calculate the reciprocal of the golden number Φ (use 1.618 034) with your calculator. What do you notice? Make a hypothesis about Φ and its reciprocal $\frac{1}{\Phi}$.
- 4 If you can, work out the reciprocal of Φ as an irrational number, you may need help from your teacher. See if this confirms your hypothesis in Q3.





LET'S COMMUNICATE

Does it surprise you that growth in nature has a tendency to conform to numbers of the Fibonacci sequence? Check out the Internet first and then write a summary of what you have learned about the occurrence of mathematics in plant and animal life. You could collaborate with others in a group and make a poster to illustrate your group findings.

and/or

Design a poster for your classroom that illustrates what you have learned about the golden number Φ .



REFLECTING

In this activity you have seen a close link between the sequences of real numbers, the use of algebra to generalise a relationship, and the geometry of growth in plants and animals. Notice how important mathematics is as a whole discipline, its various branches being only part of a much bigger human activity. Reflect on the value of describing nature mathematically and tracing its beauty in the shapes of natural things.



MACQUARIE

Language link with Macquarie

- 1 Write down the meaning of each of the following and give an example of:
 - a a real number
 - b a rational number
 - c an irrational number
 - d an integer
- 2 Read the Macquarie Learners Dictionary entry for the words **rational** and **irrational**.

rational adjective 1. sensible or reasonable: a rational decision 2. sane or in possession of your reason: He was quite rational when he regained consciousness.

❑ **Word family: rationality** noun—**rationally** adverb
irrational adjective not rational; not based on logical judgement: She has an irrational fear of water.

❑ **Word family: irrationally** adverb

Now note the special meaning given to these words in mathematics. How would you explain the difference to a student whose first language is not English?

- 1 State whether each number is rational or irrational.

a $1\frac{3}{5}$ b $\sqrt{5}$ c 0.82

d $\sqrt[3]{20}$ e -6 f 71%

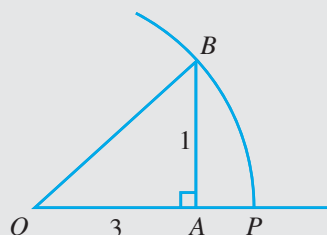
g $4 + \sqrt{10}$ h π i $\sqrt{81}$

- 2 Between which two consecutive integers does $3 + 5\sqrt{2}$ lie?

- 3 Arrange these numbers in ascending order.

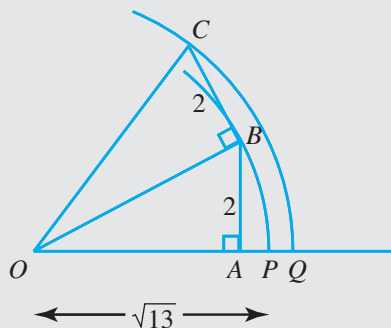
$\sqrt[3]{700}$, $\sqrt{83}$, 8.6, $2\sqrt{17}$

- 4 If $OA = 3$ units and $AB = 1$ unit:



- a find the irrational number represented by P
b write down the exact length of the interval AP .

- 5 If $OP = \sqrt{13}$ units, $AB = BC = 2$ units:



- a find the integer represented by A
b find the irrational number represented by Q
c write down the exact length of

i AP ii AQ iii PQ

- 6 Evaluate:

a $\sqrt{5} \times \sqrt{5}$ b $4\sqrt{7} \times \sqrt{7}$

c $3\sqrt{2} \times 3\sqrt{2}$ d $(5\sqrt{3})^2$

- 7 Simplify each of these surds.

a $\sqrt{12}$ b $\sqrt{32}$ c $2\sqrt{45}$

d $3\sqrt{112}$ e $\sqrt{a^3}$ f $\sqrt{y^5}$

g $\sqrt{27p}$ h $\sqrt{50x^3y^4}$

- 8 Express the following as entire surds.

a $2\sqrt{7}$ b $3\sqrt{5}$ c $4\sqrt{3}$

d $5\sqrt{6}$ e $3\sqrt{n}$ f $k\sqrt{k}$

g $c^3\sqrt{c}$ h $2e\sqrt{6ef}$

- 9 a If $\sqrt{m} = 3\sqrt{7}$, find m .

b If $\sqrt{12z} = 6\sqrt{3}$, find z .

- 10 Simplify:

a $\sqrt{3} + \sqrt{3}$ b $9\sqrt{2} + 5\sqrt{2}$

c $8\sqrt{11} - 3\sqrt{11}$ d $6\sqrt{7} - 5\sqrt{7}$

e $8\sqrt{3} - \sqrt{3}$ f $\sqrt{6} + \sqrt{6} + \sqrt{6}$

g $4\sqrt{10} + 7\sqrt{10} - 5\sqrt{10}$

h $-12\sqrt{5} + 7\sqrt{5} - 4\sqrt{5}$

- 11 Simplify:

a $7\sqrt{3} + 5\sqrt{2} + 4\sqrt{3} + \sqrt{2}$

b $8\sqrt{2} + 6\sqrt{5} - 3\sqrt{2} - \sqrt{5}$

c $10\sqrt{7} - 3\sqrt{3} - 2\sqrt{7} + 3\sqrt{3}$

d $5\sqrt{10} - 12\sqrt{11} + 4\sqrt{11} - 9\sqrt{10}$

- 12 Simplify:

a $\sqrt{20} + \sqrt{45}$ b $\sqrt{98} - \sqrt{8}$

c $4\sqrt{27} + \sqrt{48}$ d $7\sqrt{80} - 3\sqrt{125}$

e $5\sqrt{18} + 7\sqrt{48} - 2\sqrt{32} - 6\sqrt{12}$

13 Simplify:

- a** $\sqrt{3} \times \sqrt{7}$ **b** $5 \times 4\sqrt{3}$
c $2\sqrt{5} \times 6\sqrt{2}$ **d** $\sqrt{3} \times \sqrt{2} \times \sqrt{11}$
e $\sqrt{30} \div \sqrt{6}$ **f** $18\sqrt{5} \div 2\sqrt{5}$
g $20\sqrt{42} \div 4\sqrt{7}$ **h** $\sqrt{44} \div \sqrt{11}$

14 Simplify:

- a** $\sqrt{20} \times \sqrt{5}$ **b** $\sqrt{12} \times \sqrt{6}$
c $2\sqrt{8} \times 3\sqrt{10}$ **d** $\sqrt{27} \times \sqrt{32}$
e $\sqrt{54} \div \sqrt{3}$ **f** $\sqrt{48} \div 2$
g $14\sqrt{120} \div 2\sqrt{5}$ **h** $16\sqrt{63} \div 2\sqrt{7}$

15 Expand and simplify:

- a** $\sqrt{2}(\sqrt{7} + 4)$
b $2\sqrt{3}(\sqrt{3} - 5)$
c $7\sqrt{2}(\sqrt{10} - 2\sqrt{2})$
d $(\sqrt{7} + \sqrt{3})(\sqrt{5} - \sqrt{2})$
e $(2\sqrt{5} + \sqrt{6})(\sqrt{8} - 3\sqrt{3})$
f $(\sqrt{5} + 4)^2$
g $(2\sqrt{3} - 3)^2$
h $(2\sqrt{6} - 3\sqrt{2})^2$
i $(\sqrt{11} - 2)(\sqrt{11} + 2)$

j $(3\sqrt{10} + 4\sqrt{2})(3\sqrt{10} - 4\sqrt{2})$

16 a Find values for a and b given that $(2\sqrt{7} + 3)^2 = a + b\sqrt{7}$.**b** Find values for m and n given that $(3\sqrt{10} - 2\sqrt{6})^2 = m + n\sqrt{15}$.**17** Express each fraction in simplest form with a rational denominator.

- a** $\frac{1}{\sqrt{7}}$ **b** $\frac{2}{3\sqrt{5}}$
c $\frac{\sqrt{15}}{\sqrt{6}}$ **d** $\frac{\sqrt{2}}{3\sqrt{10}}$
e $\frac{1 + 2\sqrt{3}}{\sqrt{3}}$ **f** $\frac{4 + 5\sqrt{2}}{\sqrt{6}}$

Extension**18** Express each fraction in simplest form with a rational denominator.

- a** $\frac{1}{\sqrt{5} + 1}$ **b** $\frac{1}{\sqrt{11} - 3}$
c $\frac{1}{\sqrt{6} - \sqrt{2}}$ **d** $\frac{12}{6 + \sqrt{3}}$
e $\frac{5\sqrt{6}}{\sqrt{8} + \sqrt{2}}$ **f** $\frac{\sqrt{10} + 3}{\sqrt{10} - 3}$

Indices



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ state the base and the index in a number that has been written in index form
- ⦿ write in the expanded form a number that has been written in index form and vice versa
- ⦿ evaluate numbers that have been written in index form
- ⦿ express a given number in index form with a specified base or index
- ⦿ simplify numerical and algebraic expressions using the index law for multiplication
- ⦿ simplify numerical and algebraic expressions using the index law for division
- ⦿ simplify numerical and algebraic expressions using the index law for further powers
- ⦿ simplify and evaluate where possible expressions that contain a zero index
- ⦿ write as fractions expressions that contain negative indices
- ⦿ write fractions in index form using negative indices
- ⦿ write square roots, cube roots and other roots in index form
- ⦿ simplify expressions that contain fraction indices by first writing them with a radical sign
- ⦿ solve numerical problems involving indices
- ⦿ express very large numbers and very small numbers in scientific notation
- ⦿ write the basic numeral for a number in scientific notation
- ⦿ enter numbers in scientific notation on a calculator
- ⦿ write a calculator display in scientific notation
- ⦿ write in ascending order numbers that are in scientific notation
- ⦿ solve problems involving scientific notation
- ⦿ convert expressions from surd form to index form and vice versa.

9.1 Index notation

Index notation is used to shorten the way products of numbers or pronumerals are written. In the expression 3^4 , 3 is the **base** and 4 is the **index**, **power** or **exponent**. The index indicates the number of times the base has been multiplied by itself. That is, 3^4 means $3 \times 3 \times 3 \times 3$, or 4 factors of 3. By convention, the index is usually omitted when it is 1.

EG
+S

Example 1

Express each of these in index form.

a $5 \times 5 \times 5$

b $n \times n \times n \times n \times n$

c $2 \times 3 \times 2 \times 2 \times 3$

Solutions

a $5 \times 5 \times 5 = 5^3$

b $n \times n \times n \times n \times n = n^5$

c $2 \times 3 \times 2 \times 2 \times 3$
 $= 2 \times 2 \times 2 \times 3 \times 3$
 $= 2^3 \times 3^2$

EG
+S

Example 2

Find values for x and y , if $2^x \times 5^y = 400$.

Solution

To find the values for x and y , we first express 400 as the product of its prime factors. To do this we could use a factor tree; however, it is easier simply to divide 400 by 2s and 5s and count the number of each factor.

$$\begin{array}{ll} 400 \div 2 = 200 & \text{We have been able to divide 400 by 2 four times, so there must be} \\ 200 \div 2 = 100 & \text{4 factors of 2. The remaining term, 25, is clearly equal to } 5^2, \\ 100 \div 2 = 50 & \therefore 400 = 2^4 \times 5^2. \\ 50 \div 2 = 25 & \\ \text{and } 25 = 5^2 & \end{array}$$

Exercise 9.1

1 Express in index form:

a 5×5

d $a \times a \times a$

g $a \times b \times a \times b$

j $4 \times e \times e$

m $a \times a \times b$

p $6 \times y \times y - z \times z$

r $7 \times e \times e \times e - 8 \times f \times f + 4 \times g \times g$

b $2 \times 2 \times 2$

e $p \times p \times p \times p \times p$

h $m \times m \times n \times m \times n$

k $q \times q \times q \times 7 \times q$

n $m \times m \times m - n \times n$

q $2 \times r + 3 \times s \times s$

c $3 \times 3 \times 3 \times 3$

f $y \times y \times y \times y$

i $g \times h \times h \times g \times h \times h$

l $u \times 5 \times v \times u \times v \times v$

o $c \times c \times c \times c + d \times d \times d$

2 Write each of these in the expanded form.

a 3^2

e n^3

i pq^2

m $a^2 + b^2$

b 5^3

f y^5

j p^2q

n $m^3 + 3n$

c 2^5

g $3m^2$

k x^3y^4

o $p^3 - 2q^2$

d 7^4

h $11q^6$

l a^2bc^3

p $3a^3b^2 + 5a^2b$

Consolidation

- 3 Write each of the following in index form.
- | | | |
|----------------------|----------------------|------------------------|
| a three factors of 2 | b two factors of 3 | c four factors of k |
| d x factors of 5 | e m factors of n | f $2p$ factors of $3q$ |
- 4 Evaluate each of the following using a calculator.
- | | | | |
|------------------------|---------------|------------------|----------------|
| a 3^5 | b 8^3 | c 7^4 | d 4^6 |
| e $\frac{2^{10}}{4^2}$ | f $3^4 + 4^3$ | g 2×5^7 | h $12^5 - 8^6$ |
- 5 Find the value of n in each of these, where $n > 0$.
- | | | | |
|--------------|--------------|--------------|-----------------|
| a $7^n = 49$ | b $2^n = 8$ | c $3^n = 81$ | d $10^n = 1000$ |
| e $n^2 = 36$ | f $n^5 = 32$ | g $n^3 = 64$ | h $n^4 = 625$ |
- 6 Find values for all pronumerals in each of these.
- | | | |
|--------------------------|---------------------------|---|
| a $36 = 2^m \times 3^n$ | b $50 = 2^a \times 5^b$ | c $375 = 3^p \times 5^q$ |
| d $400 = 2^x \times 5^y$ | e $392 = 2^u \times 7^v$ | f $648 = a^4 \times b^3$ |
| g $1701 = 3^e \times f$ | h $3872 = 2^r \times s^2$ | i $12\,168 = 2^a \times 3^b \times c^2$ |
- 7 If $2^p \times 3^q = 72$, find the value of $2^q \times 3^p$.
- 8 Show, by substituting values for n , that:
- | | |
|------------------------------|---------------------------|
| a $3 \times n^2 \neq (3n)^2$ | b $3n \times 4n \neq 12n$ |
|------------------------------|---------------------------|

Further applications

- 9 Evaluate 2^{2^2} .
- 10 Find the value of m if $2^{(2^m)} = 256$.
- 11 Simplify $k^2 \div \underbrace{(1 + 1 + 1 + 1 + \dots + 1)}_{k \text{ times}}$.
- 12 If $2^x = 20$, find the value of x correct to 2 decimal places using a guess, check and refine method.
- 13 What is the units digit in 2^{63} ?

9.2

Simplifying numerical expressions using the index laws

Expressions that contain indices and which have the same base can be simplified using various **index laws**. Four of these laws will be developed in Exercise 9.2 for numerical expressions and then generalised to algebraic expressions in the exercises that follow.

Because of the discovery nature of this exercise, the questions are presented without worked examples.

Exercise 9.2

- Express each term in the expanded form and hence simplify the expressions, giving the answers in simplest index form.

a $2^2 \times 2^3$	b $3^3 \times 3^4$	c $5^3 \times 5^3$	d $7^4 \times 7$
- Verify each of the following statements using a calculator.

a $5^3 \times 5^4 = 5^7$	b $2^8 \times 2^5 = 2^{13}$	c $3^6 \times 3 = 3^7$
- Write down a rule that could be used to quickly multiply terms in index form.
- Express each term in the expanded form and hence simplify the expressions, giving the answers in simplest index form.

a $3^6 \div 3^2$	b $7^5 \div 7^3$	c $2^7 \div 2^4$	d $5^3 \div 5$
- Verify each of the following statements using a calculator.

a $2^{11} \div 2^6 = 2^5$	b $5^9 \div 5^2 = 5^7$	c $3^{14} \div 3^6 = 3^8$
- Write down a rule that could be used to quickly divide terms in index form.
- Express each term in the expanded form and hence simplify the expressions, giving the answers in simplest index form.

a $(2^3)^2$	b $(5^2)^3$	c $(7^3)^3$	d $(3^2)^5$
- Verify each of the following statements using a calculator.

a $(5^4)^2 = 5^8$	b $(3^2)^6 = 3^{12}$	c $(2^3)^3 = 2^9$
- Write down a rule that could be used to quickly raise terms in index form to a further power.

Consolidation

- Use the index laws developed above to write each expression in simplest index form.

a $2^5 \times 2^3$	b $3^4 \times 3^2$	c $7^6 \times 7^5$	d $5^3 \times 5^9$
e $3^{11} \div 3^7$	f $5^8 \div 5^6$	g $2^{13} \div 2^5$	h $7^{15} \div 7^9$
i $(5^3)^4$	j $(7^8)^3$	k $(3^{10})^2$	l $(2^4)^6$
- Explain the error in each of these statements, then correct it.

a $3^3 \times 3^2 = 9^5$	b $5^4 \times 5^2 = 5^8$	c $7^5 \div 7^5 = 1^5$
d $2^{12} \div 2^3 = 2^4$	e $(5^4)^2 = 25^8$	f $(6^5)^4 = 6^9$
- Explain the error in each of these statements, then correct it.

a $6^3 \times 6 = 6^3$	b $10^4 \div 10 = 10^4$
- Consider the following pattern with powers of 2.

$$2^4 = 2 \times 2 \times 2 \times 2$$

$$2^3 = 2 \times 2 \times 2$$

$$2^2 = 2 \times 2$$

$$2^1 = 2$$

- a How is each line obtained from the previous line?
- b Write the next line in the pattern.
- c Write a similar pattern using a base of 3.
- d What is the value of 3^0 ?
- e What do you think the value of 5^0 would be?

Further application

- 14 Use this table of powers of 2 and the index laws developed above to evaluate the expressions below.

2^1	2	2^5	32	2^9	512
2^2	4	2^6	64	2^{10}	1024
2^3	8	2^7	128	2^{11}	2048
2^4	16	2^8	256	2^{12}	4096

a 16×8

b 64×32

c 128×4

d 256×16

e $64 \div 4$

f $256 \div 32$

g $1024 \div 16$

h $4096 \div 128$

i 4^3

j 16^2

k 8^4

l 4^5

9.3 The index laws

The index law for multiplication

When multiplying expressions that contain indices:

- ⊙ multiply any co-efficients
- ⊙ keep the same base
- ⊙ add the indices.

$$a^m \times a^n = a^{m+n}$$

Proof: $a^m \times a^n = \underbrace{(a \times a \times \dots \times a)}_{m \text{ factors}} \times \underbrace{(a \times a \times \dots \times a)}_{n \text{ factors}}$

$$= \underbrace{(a \times a \times a \times \dots \times a)}_{(m+n) \text{ factors}}$$

$$= a^{m+n}$$

The index law for division

When dividing expressions that contain indices:

- ⊙ divide any co-efficients
- ⊙ keep the same base
- ⊙ subtract the indices.

$$a^m \div a^n = a^{m-n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

Proof: $a^m \div a^n = \frac{a^m}{a^n}$

$$= \frac{a \times a \times \dots \times a \text{ (} m \text{ factors)}}{a \times a \times \dots \times a \text{ (} n \text{ factors)}}$$

$$= \underbrace{(a \times a \times \dots \times a)}_{(m-n) \text{ factors}}$$

$$= a^{m-n}$$

The index law for further powers

When raising an expression that contains an index to a further power:

- ⊙ raise any co-efficient to the power outside the grouping symbols
- ⊙ keep the same base
- ⊙ multiply the indices.

$$(a^m)^n = a^{mn}$$

Proof: $(a^m)^n = (a^m \times a^m \times a^m \times \dots \times a^m)$

$$= a^{m+m+m+\dots+m} \text{ (} n \text{ times)}$$

$$= a^{mn}$$

Example 1

Simplify, giving answers in index form:

a $2^3 \times 2^4$

d $t^2 \times t^3 \times t^7$

b $a^4 \times a^5$

e $7n^3 \times 4n^6$

c $x^5 \times x$

f $8p^2q^4 \times 5p^6q^9$

Solutions

a $2^3 \times 2^4$
 $= 2^{3+4}$
 $= 2^7$

b $a^4 \times a^5$
 $= a^{4+5}$
 $= a^9$

c $x^5 \times x$
 $= x^{5+1}$
 $= x^6$

$$\begin{aligned} \mathbf{d} \quad t^2 \times t^3 \times t^7 \\ = t^{2+3+7} \\ = t^{12} \end{aligned}$$

$$\begin{aligned} \mathbf{e} \quad 7n^3 \times 4n^6 \\ = 28n^{3+6} \\ = 28n^9 \end{aligned}$$

$$\begin{aligned} \mathbf{f} \quad 8p^2q^4 \times 5p^6q^9 \\ = 40p^{2+6}q^{4+9} \\ = 40p^8q^{13} \end{aligned}$$

Example 2

Simplify, giving answers in index form:

$$\mathbf{a} \quad 5^9 \div 5^3$$

$$\mathbf{b} \quad m^6 \div m^4$$

$$\mathbf{c} \quad p^9 \div p$$

$$\mathbf{d} \quad \frac{x^{10}}{x^2}$$

$$\mathbf{e} \quad 24y^{12} \div 3y^5$$

$$\mathbf{f} \quad 28c^8d^5 \div 7cd^2$$

Solutions

$$\begin{aligned} \mathbf{a} \quad 5^9 \div 5^3 \\ = 5^{9-3} \\ = 5^6 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad m^6 \div m^4 \\ = m^{6-4} \\ = m^2 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad p^9 \div p \\ = p^{9-1} \\ = p^8 \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad \frac{x^{10}}{x^2} \\ = x^{10-2} \\ = x^8 \end{aligned}$$

$$\begin{aligned} \mathbf{e} \quad 24y^{12} \div 3y^5 \\ = 8y^{12-5} \\ = 8y^7 \end{aligned}$$

$$\begin{aligned} \mathbf{f} \quad 28c^8d^5 \div 7cd^2 \\ = 4c^{8-1}d^{5-2} \\ = 4c^7d^3 \end{aligned}$$

Example 3

Simplify, giving answers in index form:

$$\mathbf{a} \quad (2^5)^4$$

$$\mathbf{b} \quad (a^4)^3$$

$$\mathbf{c} \quad (5k)^2$$

$$\mathbf{d} \quad (7m^5)^2$$

$$\mathbf{e} \quad (2c^9)^3$$

$$\mathbf{f} \quad (x^6y^7)^4$$

Solutions

$$\begin{aligned} \mathbf{a} \quad (2^5)^4 \\ = 2^{5 \times 4} \\ = 2^{20} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad (a^4)^3 \\ = a^{4 \times 3} \\ = a^{12} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad (5k)^2 \\ = 5^2k^2 \\ = 25k^2 \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad (7m^5)^2 \\ = 7^2m^{5 \times 2} \\ = 49m^{10} \end{aligned}$$

$$\begin{aligned} \mathbf{e} \quad (2c^9)^3 \\ = 2^3c^{9 \times 3} \\ = 8c^{27} \end{aligned}$$

$$\begin{aligned} \mathbf{f} \quad (x^6y^7)^4 \\ = x^{6 \times 4}y^{7 \times 4} \\ = x^{24}y^{28} \end{aligned}$$

Exercise 9.3

1 Simplify each expression by adding the indices.

$$\mathbf{a} \quad n^2 \times n^3$$

$$\mathbf{b} \quad a^5 \times a^4$$

$$\mathbf{c} \quad y^6 \times y^2$$

$$\mathbf{d} \quad t^3 \times t$$

$$\mathbf{e} \quad e^4 \times e^6$$

$$\mathbf{f} \quad x^2 \times x^5$$

$$\mathbf{g} \quad m \times m^8$$

$$\mathbf{h} \quad d^5 \times d^5$$

$$\mathbf{i} \quad p^7 \times p^6$$

$$\mathbf{j} \quad r^3 \times r^5$$

$$\mathbf{k} \quad b^9 \times b$$

$$\mathbf{l} \quad z^8 \times z^4$$

$$\mathbf{m} \quad c^4 \times c^2 \times c^4$$

$$\mathbf{n} \quad k^5 \times k^7 \times k^3$$

$$\mathbf{o} \quad w^2 \times w \times w^8$$

2 Simplify each expression by subtracting the indices.

$$\mathbf{a} \quad p^5 \div p^2$$

$$\mathbf{b} \quad x^7 \div x^3$$

$$\mathbf{c} \quad q^4 \div q^2$$

$$\mathbf{d} \quad y^9 \div y$$

$$\mathbf{e} \quad t^{10} \div t^3$$

$$\mathbf{f} \quad b^6 \div b^4$$

$$\begin{array}{l} \mathbf{g} \quad n^8 \div n^5 \\ \mathbf{j} \quad r^9 \div r^3 \div r^2 \end{array}$$

$$\begin{array}{l} \mathbf{h} \quad m^{11} \div m^{10} \\ \mathbf{k} \quad d^{11} \div d^7 \div d \end{array}$$

$$\begin{array}{l} \mathbf{i} \quad f^{13} \div f^7 \\ \mathbf{l} \quad j^{13} \div j^6 \div j^5 \end{array}$$

3 Simplify each expression by multiplying the indices.

$$\mathbf{a} \quad (a^3)^2$$

$$\mathbf{b} \quad (p^4)^2$$

$$\mathbf{c} \quad (x^5)^3$$

$$\mathbf{d} \quad (b^3)^4$$

$$\mathbf{e} \quad (m^3)^3$$

$$\mathbf{f} \quad (y^5)^4$$

$$\mathbf{g} \quad (t^6)^3$$

$$\mathbf{h} \quad (n^8)^2$$

$$\mathbf{i} \quad (q^2)^{11}$$

$$\mathbf{j} \quad (c^5)^5$$

$$\mathbf{k} \quad (h^3)^9$$

$$\mathbf{l} \quad (w^5)^8$$

4 Simplify each of these.

$$\mathbf{a} \quad 3a^4 \times a^2$$

$$\mathbf{b} \quad n^3 \times 2n^5$$

$$\mathbf{c} \quad 2k^6 \times 9k$$

$$\mathbf{d} \quad 4y^3 \times 2y^4$$

$$\mathbf{e} \quad 3c^6 \times 5c^2$$

$$\mathbf{f} \quad 5t^7 \times 4t^2$$

$$\mathbf{g} \quad 8d \times 3d^9$$

$$\mathbf{h} \quad 6n^7 \times 5n^4$$

$$\mathbf{i} \quad 4u^8 \times 8u^4$$

$$\mathbf{j} \quad 2n^3 \times 3n^2 \times n^4$$

$$\mathbf{k} \quad 3p^4 \times 4p^5 \times 3p$$

$$\mathbf{l} \quad 7z^3 \times 3z^4 \times 2z^{10}$$

5 Simplify each of these.

$$\mathbf{a} \quad 2n^6 \div n^2$$

$$\mathbf{b} \quad 3b^5 \div b^3$$

$$\mathbf{c} \quad 6c^4 \div c$$

$$\mathbf{d} \quad 10m^4 \div 2m^2$$

$$\mathbf{e} \quad 8y^9 \div 4y^3$$

$$\mathbf{f} \quad 21k^{10} \div 7k^4$$

$$\mathbf{g} \quad 30z^6 \div 5z$$

$$\mathbf{h} \quad 24p^{12} \div 3p^4$$

$$\mathbf{i} \quad 22x^8 \div 2x^6$$

$$\mathbf{j} \quad 40m^{10} \div 5m^3 \div 2m^2$$

$$\mathbf{k} \quad 32s^{12} \div 2s^3 \div 4s$$

$$\mathbf{l} \quad 60e^{14} \div 5e^2 \div 3e^4$$

6 Simplify:

$$\mathbf{a} \quad \frac{a^7}{a^4}$$

$$\mathbf{b} \quad \frac{d^8}{d^2}$$

$$\mathbf{c} \quad \frac{n^6}{n^5}$$

$$\mathbf{d} \quad \frac{k^{10}}{k^3}$$

$$\mathbf{e} \quad \frac{2e^9}{e^2}$$

$$\mathbf{f} \quad \frac{10m^2}{2m}$$

$$\mathbf{g} \quad \frac{18u^5}{3u^2}$$

$$\mathbf{h} \quad \frac{15h^{10}}{5h^6}$$

$$\mathbf{i} \quad \frac{24s^7}{6s^5}$$

$$\mathbf{j} \quad \frac{90z^{12}}{9z^5}$$

$$\mathbf{k} \quad \frac{49r^9}{7r^8}$$

$$\mathbf{l} \quad \frac{54f^{16}}{6f^{10}}$$

7 Simplify each of these.

$$\mathbf{a} \quad (4a^3)^2$$

$$\mathbf{b} \quad (3m^5)^2$$

$$\mathbf{c} \quad (6g^4)^2$$

$$\mathbf{d} \quad (2k^9)^2$$

$$\mathbf{e} \quad (2n^5)^3$$

$$\mathbf{f} \quad (3e^4)^3$$

$$\mathbf{g} \quad (2q^7)^4$$

$$\mathbf{h} \quad (7y^6)^2$$

$$\mathbf{i} \quad (10b^4)^3$$

$$\mathbf{j} \quad (2w^8)^5$$

$$\mathbf{k} \quad (5g^6)^2$$

$$\mathbf{l} \quad (10x^5)^4$$

$$\mathbf{m} \quad (4c^{10})^3$$

$$\mathbf{n} \quad (3f)^4$$

$$\mathbf{o} \quad (2v)^6$$

$$\mathbf{p} \quad (6s^7)^3$$

Consolidation

8 Simplify:

$$\mathbf{a} \quad a^2b^3 \times a^2$$

$$\mathbf{b} \quad x^3 \times x^4y^2$$

$$\mathbf{c} \quad pq^3 \times p^5$$

$$\mathbf{d} \quad m^4n^2 \times m^8n$$

$$\mathbf{e} \quad j^2k^3 \times j^3k^4$$

$$\mathbf{f} \quad y^3z^5 \times y^5z^2$$

$$\mathbf{g} \quad 3ab^2 \times 4a^2b$$

$$\mathbf{h} \quad 5mn^3 \times 2m^2n^5$$

$$\mathbf{i} \quad 4u^3v^6 \times 6u^3v^3$$

$$\mathbf{j} \quad 7r^5s \times 3rs^5$$

$$\mathbf{k} \quad 4w^2x^2 \times 9w^4x^9$$

$$\mathbf{l} \quad 6c^5d^9 \times 5c^8d$$

$$\mathbf{m} \quad 8fh^8 \times 4f^5h^9$$

$$\mathbf{n} \quad 12s^7t^9 \times 5s^4t^7$$

$$\mathbf{o} \quad 7i^5j^4 \times 8i^4j^{12}$$

9 Simplify each of the following:

$$\mathbf{a} \quad a^5b^6 \div a^3b^2$$

$$\mathbf{b} \quad m^6n^9 \div m^4n^2$$

$$\mathbf{c} \quad g^7h^{10} \div g^3h$$

$$\mathbf{d} \quad x^4y^8 \div xy^3$$

$$\mathbf{e} \quad u^7v^3 \div u^4v$$

$$\mathbf{f} \quad c^8d^5 \div c^6d^3$$

$$\begin{aligned} \mathbf{g} \quad & 18y^6z^{11} \div 9yz^4 \\ \mathbf{j} \quad & 35m^8n^6 \div 5m^7n^2 \\ \mathbf{m} \quad & 45e^{11}f^{14} \div 9e^5f^8 \end{aligned}$$

$$\begin{aligned} \mathbf{h} \quad & 21a^5b^2 \div 7a^4b \\ \mathbf{k} \quad & 48c^6d^{13} \div 8cd^9 \\ \mathbf{n} \quad & 42x^4y^{10} \div 6xy^6 \end{aligned}$$

$$\begin{aligned} \mathbf{i} \quad & 32i^{10}j^5 \div 4i^3j^2 \\ \mathbf{l} \quad & 33p^{10}q^7 \div 3p^3q^4 \\ \mathbf{o} \quad & 72b^{12}c^{15} \div 6b^7c^{10} \end{aligned}$$

10 Simplify:

$$\begin{aligned} \mathbf{a} \quad & (x^2y^4)^2 \\ \mathbf{d} \quad & (u^2v^3)^4 \\ \mathbf{g} \quad & (5m^3n^4)^2 \\ \mathbf{j} \quad & (2yz^5)^4 \\ \mathbf{m} \quad & (5q^{12}r)^3 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & (p^5q^3)^2 \\ \mathbf{e} \quad & (r^4s^3)^5 \\ \mathbf{h} \quad & (9s^2t^6)^2 \\ \mathbf{k} \quad & (3p^4q^9)^3 \\ \mathbf{n} \quad & (2x^5y^2)^6 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & (cd^4)^3 \\ \mathbf{f} \quad & (a^8b)^4 \\ \mathbf{i} \quad & (2c^6d^3)^3 \\ \mathbf{l} \quad & (2g^{11}h^4)^5 \\ \mathbf{o} \quad & (10e^3d^8)^4 \end{aligned}$$

11 Copy and complete these products.

$$\begin{aligned} \mathbf{a} \quad & a^4 \times \square = a^6 \\ \mathbf{d} \quad & 3m^2 \times \square = 3m^8 \\ \mathbf{g} \quad & \square \times 6b^7 = 42b^{10} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & p^3 \times \square = p^8 \\ \mathbf{e} \quad & 2n^4 \times \square = 14n^8 \\ \mathbf{h} \quad & \square \times 3x^6 = 24x^{13} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & \square \times s = s^7 \\ \mathbf{f} \quad & \square \times 6t^8 = 30t^{11} \\ \mathbf{i} \quad & 7h^8 \times \square = 63h^9 \end{aligned}$$

12 Copy and complete these quotients.

$$\begin{aligned} \mathbf{a} \quad & e^8 \div \square = e^3 \\ \mathbf{d} \quad & 15d^7 \div \square = 5d^3 \\ \mathbf{g} \quad & \square \div 20x^6 = 5x^7 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & \square \div y^4 = y^2 \\ \mathbf{e} \quad & \square \div 2g^7 = 10g^5 \\ \mathbf{h} \quad & 72k^{17} \div \square = 8k^{13} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & \square \div q = q^3 \\ \mathbf{f} \quad & 40u^{11} \div \square = 8u^9 \\ \mathbf{i} \quad & \square \div 12j^8 = 5j^8 \end{aligned}$$

13 Copy and complete each of the following.

$$\begin{aligned} \mathbf{a} \quad & (\quad)^2 = 9p^4 \\ \mathbf{d} \quad & (\quad)^2 = 100s^8 \\ \mathbf{g} \quad & (\quad)^4 = 16n^{24} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & (\quad)^2 = 25z^6 \\ \mathbf{e} \quad & (\quad)^3 = 8d^{15} \\ \mathbf{h} \quad & (\quad)^4 = 81x^{32} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & (\quad)^2 = 64m^{10} \\ \mathbf{f} \quad & (\quad)^3 = 27w^9 \\ \mathbf{i} \quad & (\quad)^5 = 32b^{35} \end{aligned}$$

14 Copy and complete:

$$\begin{aligned} \mathbf{a} \quad & a^3b^5 \times \square = a^7b^8 \\ \mathbf{d} \quad & 3c^4d^6 \times \square = 15c^5d^9 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & \square \times p^3q^4 = p^8q^{11} \\ \mathbf{e} \quad & 12m^5n \times \square = 36m^7n^5 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & \square \times y^2z^4 = y^8z^{12} \\ \mathbf{f} \quad & \square \times 5u^5v^4 = 40u^{10}v^{13} \end{aligned}$$

15 Copy and complete:

$$\begin{aligned} \mathbf{a} \quad & p^{10}q^8 \div \square = p^6q^5 \\ \mathbf{d} \quad & 30x^7y^3 \div \square = 6xy^2 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & \square \div e^2f^2 = e^5f^8 \\ \mathbf{e} \quad & \square \div 5m^3n^4 = 11m^{10}n \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & u^{11}v^9 \div \square = u^7v^8 \\ \mathbf{f} \quad & \square \div 7c^4d^6 = 6c^3d^6 \end{aligned}$$

16 Copy and complete:

$$\begin{aligned} \mathbf{a} \quad & (\quad)^2 = 16x^4y^{10} \\ \mathbf{d} \quad & (\quad)^3 = 8u^3v^{18} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & (\quad)^2 = 49m^8n^{12} \\ \mathbf{e} \quad & (\quad)^3 = 1000e^6f^{21} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & (\quad)^2 = 144p^6q^{16} \\ \mathbf{f} \quad & (\quad)^4 = 16c^{16}d^{44} \end{aligned}$$

Further applications

17 If $2^x = 3$, evaluate:

$$\mathbf{a} \quad 2^{x+1}$$

$$\mathbf{b} \quad 2^{x+2}$$

$$\mathbf{c} \quad 2^{x+5}$$

18 If $3^x = 5$, evaluate:

$$\mathbf{a} \quad 3^{x+1}$$

$$\mathbf{b} \quad 3^{x+2}$$

$$\mathbf{c} \quad 3^{x+4}$$

19 If $2^x = 80$, evaluate:

$$\mathbf{a} \quad 2^{x-1}$$

$$\mathbf{b} \quad 2^{x-2}$$

$$\mathbf{c} \quad 2^{x-4}$$

$$\mathbf{d} \quad 2^{x-7}$$

20 If $5^x = 1000$, evaluate:

a 5^{x-1}

b 5^{x-2}

c 5^{x-3}

d 5^{x-5}

21 If $2^x = 3$, evaluate:

a 4^x

b 8^x

c 16^x

22 If $3^x = 10$, evaluate

a 9^x

b 27^x

c 81^x

9.4

Miscellaneous questions on the index laws

EG
+
SS

Example 1

Simplify:

a $k^8 \div k^2 \times k^5$

b $y^{14} \div (y^3)^2$

c $\left(\frac{e^{11}}{e^3}\right)^5$

Solutions

a $k^8 \div k^2 \times k^5$
 $= k^{8-2+5}$
 $= k^{11}$

b $y^{14} \div (y^3)^2$
 $= y^{14} \div y^6$
 $= y^8$

c $\left(\frac{e^{11}}{e^3}\right)^5$
 $= (e^8)^5$
 $= e^{40}$

EG
+
SS

Example 2

Simplify:

a $45s^{11} \div 5s^4 \times 2s^6$

b $(5m^6)^2 \times (2m^4)^3$

c $\frac{10n^4 \times 8n^{10}}{4n \times 5n^2}$

Solutions

a $45s^{11} \div 5s^4 \times 2s^6$
 $= 9s^7 \times 2s^6$
 $= 18s^{13}$

b $(5m^6)^2 \times (2m^4)^3$
 $= 25m^{12} \times 8m^{12}$
 $= 200m^{24}$

c $\frac{10n^4 \times 8n^{10}}{4n \times 5n^2}$
 $= \frac{80n^{14}}{20n^3}$
 $= 4n^{11}$

Exercise 9.4

1 Simplify:

a $y^3 \times y^5$

d $b^{10} \div b^4$

g $(z^9)^2$

j $(e^8)^4$

b $m^7 \div m^2$

e $(a^5)^3$

h $d^4 \times d^7$

k $v^{13} \div v$

c $(t^3)^4$

f $k^6 \times k^8$

i $p^{15} \div p^8$

l $c \times c^{12}$

2 Simplify:

a $12y^8 \div 3y^6$

d $9p^6 \times 4p$

g $12w^5 \times 6w^7$

j $(5r^{12})^3$

b $5t^4 \times 7t^5$

e $49s^{17} \div 7s$

h $(2k^8)^5$

k $11g^8 \times 10g^{11}$

c $(6c^4)^2$

f $(3f^7)^3$

i $45b^{16} \div 5b^6$

l $56m^4 \div 8m^3$

Consolidation

3 Simplify these expressions using the index laws.

a $n^4 \times n^5 \times n^2$

d $y^5 \times y^7 \div y^3$

g $(n^6)^5 \div n^{10}$

j $\frac{m^5 \times m^4}{m^2}$

b $p^8 \div p^3 \times p^5$

e $(a^4)^2 \times a^3$

h $e^7 \times (e^4)^3$

k $\frac{v^{15}}{v^3 \times v^4}$

c $y^{13} \div y^4 \div y^2$

f $x^{10} \div (x^2)^3$

i $(h^4)^7 \div (h^2)^9$

l $\frac{(b^7)^3}{b^5}$

4 Simplify:

a $(e^5 \times e^4)^2$

d $\left(\frac{d^8}{d^3}\right)^2$

g $2g^4 \times g^7 \times 5g^2$

j $(5m^4)^2 \times 4m^7$

m $\frac{5b^4 \times 9b^{10}}{3b^5}$

b $(v^9 \div v^2)^3$

e $\left(\frac{n^6}{n^2}\right)^3$

h $30t^{12} \div 2t^2 \div 3t^6$

k $40w^{16} \div (2w^3)^3$

n $\frac{12c^7 \times 5c^{11}}{10c^4 \times 3c^6}$

c $(m^4 \times m^3 \div m)^5$

f $\frac{(a^5)^3}{(a^6)^2}$

i $8r^5 \div 2r^3 \times 5r^9$

l $(3a^6)^2 \times (2a^4)^3$

o $\frac{(8u^{13})^2}{(2u^5)^4}$

5 Simplify:

a $3a^4 \times 5a^3 \times 2a$

d $(3u^5)^3 \times 2u^4$

g $\frac{9v^8 \times 8v^6}{12v^{11}}$

b $30g^{10} \div 6g^3 \times 4g^5$

e $56d^{19} \div (2d^4)^3$

h $\frac{8m^{11} \times 10m^2}{5m \times 4m^3}$

c $100x^{13} \div 2x \div 5x^4$

f $(4k^6)^5 \div (2k^2)^5$

i $\frac{(12b^5)^2}{4b^2 \times (3b^3)^2}$

6 Simplify:

a $\frac{(m^6)^3 \times m^4}{m^7}$

d $\frac{s^{15} \times (s^4)^4}{(s^4)^3 \times s^7}$

b $\frac{(t^8)^5 \times (t^2)^3}{t^{10}}$

e $(k^9)^5 \div \left(\frac{k^4}{k}\right)^7$

c $\frac{c^{30}}{(c^2)^4 \times c^6}$

f $\left(\frac{h^{11}}{h^3}\right)^3 \times \frac{h^5}{h^3}$

7 Simplify:

a $\frac{(3u^4)^2 \times 6u^{10}}{u^4 \times (3u^2)^3 \times u^2}$

b $\frac{10(pq)^2 \times 20p^{20}q^{14}}{(2p^6q^3)^2 \times 5p^5q}$

Further applications

8 Simplify each of the following expressions and evaluate where possible.

a $2^a \times 2^b$

e $2^{5t} \times 2^t$

b $k^m \times k^n$

f $p^q \times p^q \times p^q$

c $5^x \times 5$

g $5^{m+1} \times 5^{m+3}$

d $3^n \times 3^n$

h $a^{2b+5} \times a^{5-2b}$

i $3^u \div 3^v$

j $4^v \div 4^3$

k $5^{2k} \div 5^k$

l $11^w \div 11$

m $\frac{6^{e+3}}{6^3}$

n $\frac{5^{x+2}}{5^x}$

o $\frac{m^{p+q}}{m^{p-q}}$

p $\frac{x^{7y+6}}{x^{4+7y}}$

q $(2^a)^b$

r $(n^5)^k$

s $(e^f)^f$

t $(p^{2q})^6$

u $(5^3)^{7u}$

v $(6^{2a})^{3b}$

w $(2^x)^{y+z}$

x $(a^{m-n})^k$

9 Half of 2^{2n} would be:

A 2^n

B 1^{2n}

C 1^n

D 2^{2n-1}

10 Find one quarter of 2^n in index form.

9.5 The zero index

The index laws for multiplication, division and further powers have used examples in which the indices are positive integers. We now consider the meaning of the zero index.

$$a^n \div a^n = a^{n-n} \quad (\text{using the index law for division})$$

$$= a^0$$

But, $a^n \div a^n = 1$ (the quotient of any number and itself is 1)

Putting these two results together, we have $a^0 = 1$.

$$a^0 = 1, (a \neq 0)$$

Example

Simplify and evaluate where possible:

a 3^0

b $(3a)^0$

c $3a^0$

Solutions

a $3^0 = 1$

b $(3a)^0 = 1$

$$\begin{aligned} \text{c} \quad 3a^0 &= 3 \times a^0 \\ &= 3 \times 1 \\ &= 3 \end{aligned}$$

Exercise 9.5

1 Copy and complete the following tables.

a

2^5	$2 \times 2 \times 2 \times 2 \times 2$	32
2^4	$2 \times 2 \times 2 \times 2$	16
2^3		
2^2		
2^1		
2^0		

b

3^5	$3 \times 3 \times 3 \times 3 \times 3$	243
3^4	$3 \times 3 \times 3 \times 3$	81
3^3		
3^2		
3^1		
3^0		

2 Find the value of each expression using the index key on your calculator.

a 6^0

b 13^0

c 28^0

d 51^0

Consolidation

3 Simplify and evaluate where possible.

a 5^0

b $(-3)^0$

c a^0

d $(2k)^0$

e $2k^0$

f $3n^0$

g $7t^0$

h $-6p^0$

i ab^0

j p^0q

k $(xy)^0$

l m^2n^0p

4 Evaluate:

a $6^0 + 4$

b $5 - 12^0$

c 4×7^0

d $5^0 + 3^0$

e $8a^0 - 2$

f $9p^0 \times 5$

g $20 \div 4t^0$

h $3 - 10x^0$

i $4k^0 + 7m^0$

j $p^0 + q^0 + r^0$

k $x^0 - y^0 - z^0$

l $3 + 9u^0 - 4^0$

m $6a^0 - b^0 + 3c^0$

n $14m^0 + (14m)^0 - 8n^0$

o $5^0 + (5a)^0 + 5a^0$

p $3p^0 \times 7q^0 \times 2$

q $(11g^2)^0 + 11^0 - 11h^0$

r $8x^0 + 36 \div 4y^0$

Further applications

5 a What do you think the value of 0^0 would be?

b Try to find the value of 0^0 using a calculator.

c Can you explain the problem?

TRY THIS Smallest to largest

Put the following numbers in order from smallest to largest: 3^{100} , 5^{75} , 2^{125} .

HINT: Some lateral thinking is required—use one of the index laws.

9.6 The negative index

Until now our study of indices has only included examples where the index is a positive integer or zero. We will now look at cases where the index is a negative integer. The meaning of the negative index can best be determined by studying the pattern opposite. To find the next expression in each line, we divide the expression in the previous line by a . From this pattern, we can generalise the meaning of the negative index.

$$a^3 = a \times a \times a$$

$$a^2 = a \times a$$

$$a^1 = a$$

$$a^0 = 1$$

$$a^{-1} = \frac{1}{a}$$

$$a^{-2} = \frac{1}{a^2}$$

$$a^{-3} = \frac{1}{a^3}$$

$$a^{-1} = \frac{1}{a} \quad \text{and} \quad a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$$

NOTE: a^{-n} is the reciprocal of a^n .

EG
+S**Example 1**

Express each of these as a fraction in simplest form.

a 5^{-1} b 6^{-2}

Solutions

a $5^{-1} = \frac{1}{5}$

b $6^{-2} = \frac{1}{6^2}$
 $= \frac{1}{36}$

EG
+S**Example 2**

Write each of these with a negative index.

a $\frac{1}{x}$ b $\frac{1}{w^4}$

Solutions

a $\frac{1}{x} = x^{-1}$

b $\frac{1}{w^4} = w^{-4}$

EG
+S**Example 3**

Express each of these without grouping symbols or negative indices.

a $(3t)^{-1}$

b $(5q^4)^{-2}$

c $(j^5k^7)^{-3}$

Solutions

a $(3t)^{-1} = \frac{1}{3t}$

b $(5q^4)^{-2} = \frac{1}{(5q^4)^2}$
 $= \frac{1}{25q^8}$

c $(j^5k^7)^{-3} = \frac{1}{(j^5k^7)^3}$
 $= \frac{1}{j^{15}k^{21}}$

Exercise 9.6

1 Copy and complete each of the following tables.

a

2^4	$2 \times 2 \times 2 \times 2$	16
2^3	$2 \times 2 \times 2$	8
2^2	2×2	4
2^1	2	2
2^0	1	1
2^{-1}	$\frac{1}{2}$	$\frac{1}{2}$
2^{-2}	$\frac{1}{2 \times 2}$	$\frac{1}{4}$
2^{-3}		
2^{-4}		

b

3^4	$3 \times 3 \times 3 \times 3$	81
3^3	$3 \times 3 \times 3$	27
3^2		
3^1		
3^0		
3^{-1}		
3^{-2}		
3^{-3}		
3^{-4}		

2 Find the value of each expression by using the index key $[x^y]$ on your calculator, then convert the decimal to a fraction in simplest form.

a 2^{-1}

b 4^{-1}

c 5^{-1}

d 10^{-1}

3 Express each of these as a fraction in simplest form.

a 3^{-1}	b 7^{-1}	c 8^{-1}	d 12^{-1}	e 4^{-2}	f 5^{-2}
g 7^{-2}	h 11^{-2}	i 2^{-3}	j 3^{-3}	k 2^{-5}	l 3^{-4}
m 5^{-3}	n 2^{-4}	o 10^{-3}	p 3^{-5}	q 9^{-2}	r 2^{-6}

4 Write each expression with a positive index.

a m^{-1}	b p^{-1}	c h^{-2}	d n^{-3}	e e^{-6}	f y^{-4}

5 Write each of these with a negative index.

a $\frac{1}{a}$	b $\frac{1}{x}$	c $\frac{1}{c^2}$	d $\frac{1}{u^3}$	e $\frac{1}{p^5}$	f $\frac{1}{y^8}$

Consolidation

6 Find the value of n in each of these.

a $\frac{1}{5} = 5^n$	b $\frac{1}{7} = 7^n$	c $\frac{1}{4} = 2^n$	d $\frac{1}{9} = 3^n$
e $\frac{1}{8} = 2^n$	f $\frac{1}{32} = 2^n$	g $\frac{1}{36} = 6^n$	h $\frac{1}{27} = 3^n$
i $\frac{1}{49} = 7^n$	j $\frac{1}{64} = 2^n$	k $\frac{1}{125} = 5^n$	l $\frac{1}{81} = 3^n$

7 Express each of these without grouping symbols or negative indices.

a $(2m)^{-1}$	b $(7c)^{-1}$	c $(5r)^{-1}$	d $(4q)^{-1}$	e $(3u)^{-2}$	f $(6g)^{-2}$
g $(11t)^{-2}$	h $(9k)^{-2}$	i $(2p)^{-3}$	j $(5s)^{-3}$	k $(3b)^{-4}$	l $(2n)^{-5}$
m $(7x^3)^{-1}$	n $(10y^5)^{-2}$	o $(8w^4)^{-2}$	p $(2z^6)^{-3}$	q $(2c^6)^{-4}$	r $(3a^4)^{-3}$
s $(13v^7)^{-2}$	t $(5y^{12})^{-3}$	u $(a^2b^3)^{-2}$	v $(p^3q^5)^{-4}$	w $(9ef^6)^{-2}$	x $(3y^8z^7)^{-3}$

8 Evaluate without the use of a calculator.

a $2^3 + 2^{-1}$	b $3^2 - 3^{-2}$	c $2^{-1} + 2^{-2}$
d $5^{-1} + 5^0$	e $7^0 - 7^{-1}$	f $4^2 + 4^{-1} + 4^0$
g $2^{-1} \times 10$	h 20×2^{-2}	i $3^{-2} \times 54$
j $3^{-1} \times 2^{-1}$	k $3 \times 2^{-2} \times 12$	l $4^{-3} \times 4^2 \times 8$
m $2 \div 2^{-1}$	n $3^{-2} \div \frac{1}{36}$	o $5^0 \div 2^{-3}$
p $8 \div 5^{-1}$	q $6^{-1} \div 2^{-1}$	r $2^{-5} \div 4^{-2}$

Further applications

9 Express each of these in simplest form, without negative indices.

a $\left(\frac{1}{2}\right)^{-1}$	b $\left(\frac{1}{3}\right)^{-1}$	c $\left(\frac{5}{2}\right)^{-1}$	d $\left(\frac{3}{4}\right)^{-1}$
e $\left(\frac{3}{2}\right)^{-2}$	f $\left(\frac{7}{3}\right)^{-2}$	g $\left(\frac{4}{5}\right)^{-2}$	h $\left(\frac{7}{10}\right)^{-2}$
i $\left(\frac{2}{3}\right)^{-3}$	j $\left(\frac{5}{2}\right)^{-3}$	k $\left(\frac{3}{2}\right)^{-4}$	l $\left(\frac{10}{3}\right)^{-4}$

m $\left(3\frac{1}{2}\right)^{-1}$

n $\left(2\frac{1}{4}\right)^{-2}$

o $\left(1\frac{5}{6}\right)^{-2}$

p $\left(1\frac{2}{3}\right)^{-3}$

10 Express as fractions without grouping symbols or negative indices.

a $\left(\frac{n}{6}\right)^{-1}$

b $\left(\frac{5}{k}\right)^{-1}$

c $\left(\frac{a}{b}\right)^{-1}$

d $\left(\frac{2m}{3n}\right)^{-1}$

e $\left(\frac{x}{3}\right)^{-2}$

f $\left(\frac{4}{h}\right)^{-2}$

g $\left(\frac{y}{2}\right)^{-3}$

h $\left(\frac{2}{d}\right)^{-4}$

i $\left(\frac{3}{m}\right)^{-3}$

j $\left(\frac{e}{2}\right)^{-5}$

k $\left(\frac{c}{d}\right)^{-2}$

l $\left(\frac{e}{f}\right)^{-5}$

m $\left(\frac{2a}{3}\right)^{-2}$

n $\left(\frac{7}{8p}\right)^{-2}$

o $\left(\frac{3m}{4n}\right)^{-3}$

p $\left(\frac{2x}{5y}\right)^{-4}$

q $\left(\frac{u^2}{4}\right)^{-2}$

r $\left(\frac{ab}{2}\right)^{-3}$

s $\left(\frac{10m^4}{11p^{12}}\right)^{-2}$

t $\left(\frac{2a^2b^3}{3c^4}\right)^{-5}$

9.7

Products and quotients with negative indices

Example 1

Express each of these products as a fraction without any negative indices.

a ab^{-1}

b $m^{-1}n^{-1}$

c $5p^{-2}q^4r^{-3}$

d $\frac{3}{8}w^{-6}$

Solutions

$$\begin{aligned} \text{a} \quad ab^{-1} &= a \times \frac{1}{b} \\ &= \frac{a}{b} \end{aligned}$$

$$\begin{aligned} \text{b} \quad m^{-1}n^{-1} &= \frac{1}{m} \times \frac{1}{n} \\ &= \frac{1}{mn} \end{aligned}$$

$$\begin{aligned} \text{c} \quad 5p^{-2}q^4r^{-3} &= 5 \times \frac{1}{p^2} \times q^4 \times \frac{1}{r^3} \\ &= \frac{5q^4}{p^2r^3} \end{aligned}$$

$$\begin{aligned} \text{d} \quad \frac{3}{8}w^{-6} &= \frac{3}{8} \times \frac{1}{w^6} \\ &= \frac{3}{8w^6} \end{aligned}$$

Example 2

Express each fraction as a product containing negative indices.

a $\frac{e^2}{f^6}$

b $\frac{1}{a^3b^9}$

c $\frac{8k^2}{m^7n}$

d $\frac{2}{5r^4}$

Solutions

$$\begin{aligned} \text{a} \quad \frac{e^2}{f^6} &= e^2 \times \frac{1}{f^6} \\ &= e^2f^{-6} \end{aligned}$$

$$\begin{aligned} \text{b} \quad \frac{1}{a^3b^9} &= \frac{1}{a^3} \times \frac{1}{b^9} \\ &= a^{-3}b^{-9} \end{aligned}$$

$$\begin{aligned} \text{c} \quad \frac{8k^2}{m^7n} &= 8 \times k^2 \times \frac{1}{m^7} \times \frac{1}{n} \\ &= 8k^2m^{-7}n^{-1} \end{aligned}$$

$$\begin{aligned} \text{d} \quad \frac{2}{5r^4} &= \frac{2}{5} \times \frac{1}{r^4} \\ &= \frac{2}{5}r^{-4} \end{aligned}$$

**Example 3**

Simplify each expression by adding, subtracting or multiplying the indices, then give the answers without any negative indices.

a $n^{-6} \times n^2$

b $t^{-9} \div t^{-7}$

c $(2v^{-5})^{-4}$

Solutions

$$\begin{aligned}\mathbf{a} \quad n^{-6} \times n^2 &= n^{-6+2} \\ &= n^{-4} \\ &= \frac{1}{n^4}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad t^{-9} \div t^{-7} &= t^{-9-(-7)} \\ &= t^{-2} \\ &= \frac{1}{t^2}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad (2v^{-5})^{-4} &= 2^{-4}v^{-5 \times (-4)} \\ &= \frac{1}{2^4}v^{20} \\ &= \frac{v^{20}}{16}\end{aligned}$$

Exercise 9.7

1 Express each of these products as a fraction without any negative indices.

a xy^{-1}

b $p^{-1}q$

c ab^{-2}

d $u^{-4}v^3$

e $e^{-3}f^2$

f $g^{-1}h^5$

g w^3x^{-7}

h b^6c^{-1}

i $r^{-4}s^9$

j jk^{-3}

k $a^{-1}b^{-1}$

l $x^{-2}y^{-1}$

m $e^{-3}f^{-2}$

n $p^{-4}q^{-6}$

o $g^{-5}h^{-2}$

p $3a^{-1}$

q $5c^{-2}$

r $2e^{-4}$

s $7b^{-5}$

t $9k^{-3}$

u $\frac{1}{2}u^{-1}$

v $\frac{1}{4}p^{-2}$

w $\frac{2}{3}t^{-4}$

x $\frac{4}{7}w^{-9}$

y $\frac{3}{5}n^{-6}$

2 Express each fraction as a product containing negative indices.

a $\frac{a}{b}$

b $\frac{x^3}{y}$

c $\frac{p}{q^2}$

d $\frac{m^3}{n^4}$

e $\frac{1}{e^2f^4}$

f $\frac{c^4}{d^5}$

g $\frac{j^{10}}{k^6}$

h $\frac{1}{g^7h^9}$

i $\frac{y^6}{z}$

j $\frac{1}{u^3v^{10}}$

k $\frac{5}{n}$

l $\frac{3}{p^2}$

m $\frac{9}{a^7}$

n $\frac{4}{z^3}$

o $\frac{11}{t^5}$

p $\frac{10}{m^6}$

q $\frac{1}{3m}$

r $\frac{1}{5k}$

s $\frac{1}{4g^2}$

t $\frac{1}{12a^3}$

u $\frac{2}{7t}$

v $\frac{3}{4r^2}$

w $\frac{5}{8y^6}$

x $\frac{9}{11f^4}$

Consolidation

3 Express each product as a fraction without any negative indices.

a $a^2b^3c^{-1}$

b $x^3y^{-2}z^4$

c $e^{-3}f^2g^2$

d $i^{-1}j^{-2}k^3$

e $u^2v^{-2}w^{-5}$

f $m^{-1}n^{-3}p^{-2}$

g $r^{-4}s^2t^{-7}$

h c^5de^{-9}

i $p^{-1}q^{-1}r^{-1}$

j $w^2x^{-3}y^{-5}$

k $4a^{-2}b^3$

l $3mn^{-1}$

m $7e^{-3}f^{-2}$

n $5a^4b^{-6}$

o $12r^{-1}s^{-1}$

p $8x^{-4}y^7$

q $\frac{1}{3}c^{-2}d^5$

r $\frac{1}{8}y^3z^{-4}$

s $\frac{5}{6}a^{-1}b^{-1}$

t $\frac{3}{10}u^2v^{-8}$

4 Express each fraction as a product containing negative indices.

a $\frac{p^2q^4}{r}$

b $\frac{ab^2}{c^5}$

c $\frac{u^6}{v^2w^4}$

d $\frac{1}{e^2f^3g^7}$

e $\frac{m^4}{np^3}$

f $\frac{1}{a^4bc}$

g $\frac{i^3j^5}{k^3}$

h $\frac{c^6}{d^8e^{11}}$

i $\frac{1}{xyz}$

j $\frac{m}{np^8}$

k $\frac{5a^4}{b^2}$

l $\frac{3u^5}{v^4}$

m $\frac{9}{e^3f^4}$

n $\frac{6m^7}{n^5}$

o $\frac{10}{pq}$

p $\frac{2r^2}{3s^2}$

q $\frac{4a^5}{5b^3}$

r $\frac{9}{10gh^4}$

s $\frac{5v^4}{12w^9}$

t $\frac{8}{9y^5z^{10}}$

- 5 Simplify each expression by adding, subtracting or multiplying the indices, then give the answers as fractions if they contain a negative index.

a $m^5 \times m^{-3}$

b $k^{-7} \times k^4$

c $y^{-6} \times y^{-2}$

d $g^3 \times g^{-3}$

e $5a^{-2} \times a^7$

f $2z^9 \times 9z^{-2}$

g $4t^{-1} \times 6t^{-8}$

h $7p^{-5} \times 2p^5$

i $e^2 \div e^3$

j $j^{-2} \div j^9$

k $q^{-7} \div q^{-4}$

l $y^{-5} \div y^{-6}$

m $10u^{-1} \div u^4$

n $12c^3 \div 3c^{-7}$

o $30 \div 6w^{-2}$

p $40n^{-6} \div 5n^{-6}$

q $(d^{-3})^2$

r $(s^{-2})^{-4}$

s $(z^6)^{-3}$

t $(f^{-4})^{-7}$

u $(3a^{-5})^2$

v $(2h^{-8})^{-3}$

w $(4m^{-7})^3$

x $(7x^{-9})^{-2}$

- 6 Simplify, giving your answers as fractions.

a $a^3 \div a^8 \times a^2$

b $n \times n^4 \div n^9$

c $q^2 \div q^3 \div q^4$

d $15w^4 \div 3w^6 \div w^3$

e $40p^3 \div 2p^7 \times 4p$

f $6y \times 7y^2 \div 3y^7$

g $3m^2 \div 4m^5 \times 12m$

h $2c^4 \times 3c \div 7c^{11}$

i $4x^3 \times 6x^2 \div 36x^8$

Further applications

- 7 Simplify, giving answers without negative indices.

a $\frac{a}{b^{-1}}$

b $\frac{x^3}{x^{-2}}$

c $\frac{m^2}{m^{-2}n^3}$

d $\frac{x^3y^{-2}}{x^{-4}y^{-1}}$

- 8 Simplify, giving answers without negative indices.

a $\frac{a^5 \times a^{-2}}{(a^3)^2}$

b $\left(\frac{n^4}{n^6}\right)^{-3}$

c $\frac{w^{-3}}{w^7} \times \frac{w^{-2}}{w^5}$

d $\frac{b^3 \div b^{-5}}{(b^{-1})^{-2}}$

e $\left(\frac{x^3}{x^2}\right)^5 \times \left(\frac{x^4}{x}\right)^{-2}$

f $\frac{(u^{-2})^{-3} \times (u^5)^{-1}}{u^{-3} \times u^5}$

TRY THIS Digit patterns

Complete these and note the pattern of the last digits.

$2^1 = \square$

$2^2 = \square$

$2^3 = \square$

$2^4 = \square$

$2^5 = \square$

$2^6 = \square$

$2^7 = \square$

$2^8 = \square$

etc.

From such patterns, work out the last digit of the following.

$2^{13} \times 3^{10}$

9.8 The fraction index

The meaning of the fraction index can be seen from the following examples.

1 Using the index law for multiplication: $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^{\frac{1}{2} + \frac{1}{2}}$
 $= a^1$
 $= a$

But $\sqrt{a} \times \sqrt{a} = a$.

Putting these results together, we have $a^{\frac{1}{2}} = \sqrt{a}$.

2 Using the index law for multiplication: $a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} = a^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$
 $= a^1$
 $= a$

However, $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$.

Putting these results together, we have $a^{\frac{1}{3}} = \sqrt[3]{a}$.

$$a^{\frac{1}{2}} = \sqrt{a}$$

$$a^{\frac{1}{3}} = \sqrt[3]{a}$$

■ The general unit fraction index

The results above can be extended to expressions other than the square root and cube root of a number. For example, $\sqrt[4]{a} = a^{\frac{1}{4}}$, $\sqrt[5]{a} = a^{\frac{1}{5}}$ and $\sqrt[6]{a} = a^{\frac{1}{6}}$. In general:

$$\sqrt[n]{a} = a^{\frac{1}{n}}, (n > 0)$$

NOTE: The calculator keys $\sqrt[x]{}$ or $\left[\frac{1}{x^y} \right]$ can be used to find roots greater than the cube root.

■ The non-unit fraction index

Consider the expression $a^{\frac{p}{q}}$, where $q > 0$. This expression can be interpreted in two ways.

$$a^{\frac{p}{q}} = (a^p)^{\frac{1}{q}} \quad \text{or} \quad a^{\frac{p}{q}} = \left(a^{\frac{1}{q}} \right)^p$$

$$= \sqrt[q]{a^p} \quad \quad \quad = (\sqrt[q]{a})^p$$

$$a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p, (q > 0)$$

EG
+S**Example 1**

Evaluate:

a $16^{\frac{1}{2}}$

b $27^{\frac{1}{3}}$

c $\left(\frac{9}{49}\right)^{\frac{1}{2}}$

d $\left(\frac{8}{125}\right)^{\frac{1}{3}}$

Solutions

$$\begin{aligned} \text{a} \quad 16^{\frac{1}{2}} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{b} \quad 27^{\frac{1}{3}} \\ &= \sqrt[3]{27} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{c} \quad \left(\frac{9}{49}\right)^{\frac{1}{2}} \\ &= \sqrt{\frac{9}{49}} \\ &= \frac{\sqrt{9}}{\sqrt{49}} \\ &= \frac{3}{7} \end{aligned}$$

$$\begin{aligned} \text{d} \quad \left(\frac{8}{125}\right)^{\frac{1}{3}} \\ &= \sqrt[3]{\frac{8}{125}} \\ &= \frac{\sqrt[3]{8}}{\sqrt[3]{125}} \\ &= \frac{2}{5} \end{aligned}$$

EG
+S**Example 2**

Simplify:

a $\sqrt{a^{14}}$

b $\sqrt[3]{a^{15}}$

c $\sqrt{9a^8}$

d $\sqrt[3]{8a^{33}}$

Solutions

$$\begin{aligned} \text{a} \quad \sqrt{a^{14}} \\ &= (a^{14})^{\frac{1}{2}} \\ &= a^7 \end{aligned}$$

$$\begin{aligned} \text{b} \quad \sqrt[3]{a^{15}} \\ &= (a^{15})^{\frac{1}{3}} \\ &= a^5 \end{aligned}$$

$$\begin{aligned} \text{c} \quad \sqrt{9a^8} \\ &= (9a^8)^{\frac{1}{2}} \\ &= 3a^4 \end{aligned}$$

$$\begin{aligned} \text{d} \quad \sqrt[3]{8a^{33}} \\ &= (8a^{33})^{\frac{1}{3}} \\ &= 2a^{11} \end{aligned}$$

EG
+S**Example 3**

Evaluate:

a $9^{\frac{3}{2}}$

b $16^{-\frac{3}{4}}$

Solutions

$$\begin{aligned} \text{a} \quad 9^{\frac{3}{2}} &= (\sqrt{9})^3 \\ &= 3^3 \\ &= 27 \end{aligned}$$

$$\begin{aligned} \text{b} \quad 16^{-\frac{3}{4}} &= \frac{1}{16^{\frac{3}{4}}} \\ &= \frac{1}{(\sqrt[4]{16})^3} \\ &= \frac{1}{2^3} \\ &= \frac{1}{8} \end{aligned}$$

Exercise 9.8

1 Express each of the following with a radical sign ($\sqrt[n]{\quad}$), then evaluate.

a $4^{\frac{1}{2}}$ **b** $9^{\frac{1}{2}}$ **c** $49^{\frac{1}{2}}$ **d** $81^{\frac{1}{2}}$ **e** $8^{\frac{1}{3}}$ **f** $27^{\frac{1}{3}}$
g $125^{\frac{1}{3}}$ **h** $1000^{\frac{1}{3}}$ **i** $36^{\frac{1}{2}}$ **j** $64^{\frac{1}{3}}$ **k** $121^{\frac{1}{2}}$ **l** $100^{\frac{1}{2}}$

Consolidation

2 Express each of the following with a radical sign, then simplify.

a $\left(\frac{4}{9}\right)^{\frac{1}{2}}$ **b** $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ **c** $\left(\frac{25}{49}\right)^{\frac{1}{2}}$ **d** $\left(\frac{27}{125}\right)^{\frac{1}{3}}$ **e** $\left(\frac{36}{121}\right)^{\frac{1}{2}}$ **f** $\left(\frac{16}{81}\right)^{\frac{1}{2}}$
g $\left(\frac{27}{64}\right)^{\frac{1}{3}}$ **h** $\left(\frac{81}{100}\right)^{\frac{1}{2}}$ **i** $\left(1\frac{7}{9}\right)^{\frac{1}{2}}$ **j** $\left(3\frac{3}{8}\right)^{\frac{1}{3}}$ **k** $\left(1\frac{24}{25}\right)^{\frac{1}{2}}$ **l** $\left(13\frac{4}{9}\right)^{\frac{1}{2}}$

3 Express each of these as fractions in simplest form.

a $16^{-\frac{1}{2}}$ **b** $25^{-\frac{1}{2}}$ **c** $8^{-\frac{1}{3}}$ **d** $27^{-\frac{1}{3}}$ **e** $36^{-\frac{1}{2}}$ **f** $125^{-\frac{1}{3}}$
g $121^{-\frac{1}{2}}$ **h** $64^{-\frac{1}{2}}$ **i** $64^{-\frac{1}{3}}$ **j** $\left(\frac{4}{25}\right)^{-\frac{1}{2}}$ **k** $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$ **l** $\left(11\frac{1}{9}\right)^{-\frac{1}{2}}$

4 Express each of these in the form $(a^m)^n$, then simplify.

a $\sqrt{a^6}$ **b** $\sqrt{a^{10}}$ **c** $\sqrt{a^{22}}$ **d** $\sqrt[3]{a^6}$ **e** $\sqrt[3]{a^{12}}$ **f** $\sqrt[3]{a^{30}}$

5 Simplify:

a $(9t^8)^{\frac{1}{2}}$ **b** $(25u^{12})^{\frac{1}{2}}$ **c** $(8c^9)^{\frac{1}{3}}$ **d** $(125b^{21})^{\frac{1}{3}}$
e $(16e^{12})^{\frac{1}{2}}$ **f** $(169w^{18})^{\frac{1}{2}}$ **g** $(64n^{18})^{\frac{1}{3}}$ **h** $(27v^{15})^{\frac{1}{3}}$

6 Simplify:

a $16^{\frac{1}{4}}$ **b** $81^{\frac{1}{4}}$ **c** $32^{\frac{1}{5}}$ **d** $64^{\frac{1}{6}}$

7 Evaluate each of the following by using the $\boxed{\frac{1}{x^y}}$ or $\boxed{\sqrt[y]{x}}$ key on your calculator.

a $\sqrt[4]{625}$ **b** $\sqrt[6]{4096}$ **c** $\sqrt[5]{7776}$ **d** $\sqrt[8]{256}$
e $\sqrt[7]{2187}$ **f** $\sqrt[10]{1024}$ **g** $\sqrt[9]{19\,683}$ **h** $\sqrt[6]{1\,000\,000}$

8 Express each of these in the form $(a^m)^n$, then simplify.

a $\sqrt[4]{2^4}$

b $\sqrt[5]{p^{35}}$

c $\sqrt[7]{k^{28}}$

d $\sqrt[6]{y^{72}}$

9 Express each of these in index form.

a $\sqrt{a^3}$

b $\sqrt[3]{m^2}$

c $\sqrt{e^5}$

d $\sqrt[3]{p^4}$

e $\sqrt[4]{y^3}$

f $\sqrt[3]{k^5}$

g $\sqrt{t^7}$

h $\sqrt[5]{h^6}$

i $\sqrt[3]{z^7}$

j $\sqrt[8]{u^5}$

k $\sqrt[7]{r^2}$

l $\sqrt[9]{w^{10}}$

10 Express each of the following in the form $(\sqrt[q]{a})^p$, then evaluate.

a $4^{\frac{3}{2}}$

b $8^{\frac{2}{3}}$

c $16^{\frac{3}{2}}$

d $27^{\frac{2}{3}}$

e $16^{\frac{3}{4}}$

f $25^{\frac{3}{2}}$

g $8^{\frac{4}{3}}$

h $81^{\frac{3}{4}}$

i $100^{\frac{3}{2}}$

j $32^{\frac{2}{5}}$

k $8^{\frac{7}{3}}$

l $64^{\frac{5}{6}}$

m $\left(\frac{1}{9}\right)^{\frac{3}{2}}$

n $\left(\frac{1}{8}\right)^{\frac{4}{3}}$

o $\left(\frac{4}{25}\right)^{\frac{3}{2}}$

p $\left(\frac{27}{125}\right)^{\frac{2}{3}}$

q $9^{\frac{-3}{2}}$

r $125^{\frac{-2}{3}}$

s $32^{\frac{-4}{5}}$

t $49^{\frac{-3}{2}}$

11 Write each of these in surd form.

a $a^{\frac{3}{2}}$

b $m^{\frac{2}{3}}$

c $p^{\frac{4}{3}}$

d $n^{\frac{3}{5}}$

e $k^{\frac{5}{6}}$

f $y^{\frac{7}{2}}$

g $x^{\frac{-1}{2}}$

h $c^{\frac{-1}{3}}$

i $e^{\frac{-3}{2}}$

j $s^{\frac{-2}{3}}$

k $v^{\frac{-4}{5}}$

l $b^{\frac{-7}{6}}$

12 Write each of these in index form.

a $x\sqrt{x}$

b $x^2\sqrt{x}$

c $x \times \sqrt[3]{x}$

d $x^2 \times \sqrt[3]{x}$

e $x \times \sqrt[5]{x}$

f $x^2 \times \sqrt[4]{x}$

g $x^3\sqrt{x}$

h $x^4 \times \sqrt[7]{x}$

i $\frac{1}{\sqrt{x}}$

j $\frac{1}{\sqrt[3]{x}}$

k $\frac{1}{\sqrt[5]{x}}$

l $\frac{1}{\sqrt[7]{x}}$

m $\frac{1}{\sqrt[3]{x}}$

n $\frac{1}{x \times \sqrt[3]{x}}$

o $\frac{1}{x^2 \times \sqrt{x}}$

p $\frac{1}{x^2 \times \sqrt[3]{x}}$

Further applications

13 Write each of these in the form $a^m \times \sqrt[n]{a}$, where m and n are positive integers.

a $a^{\frac{3}{2}}$

b $a^{\frac{5}{2}}$

c $a^{\frac{4}{3}}$

d $a^{\frac{5}{3}}$

14 a Which is greater, $2^{\frac{1}{2}}$ or $3^{\frac{1}{3}}$? [HINT: Raise each number to the power of 6.]

b Which is greater, $5^{\frac{1}{3}} \times 3^{\frac{1}{2}}$ or $9^{\frac{1}{4}} \times 2^{\frac{5}{6}}$?

9.9

Scientific notation

A number is in *scientific notation* if it is written in the form $m \times 10^n$, where m is a number between 1 and 10 and n is an integer.

Scientific notation is also called **standard notation**. It is used to write very large numbers or very small numbers.

To express a number in scientific notation:

- ⊙ move the decimal point so that the number is between 1 and 10
- ⊙ multiply this number by a power of 10, where the index indicates the direction and the number of places that the decimal point would have to be moved to be in its original position.

To write the basic numeral for a number that has been written in scientific notation:

- ⊙ move the decimal point in the direction and the number of places indicated by the index in the power of 10.

If the number is greater than 1, then the index in the power of 10 will be positive. If the number is less than 1, then the index in the power of 10 will be negative.

Example 1

Express each of the following in scientific notation.

a 50 000

b 473 000

c 0.002

d 0.000 681

Solutions

a $50\,000$

$\therefore 50\,000 = 5 \times 10^4$

b $473\,000$

$\therefore 473\,000 = 4.73 \times 10^5$

c 0.002

$\therefore 0.002 = 2 \times 10^{-3}$

d $0.000\,681$

$\therefore 0.000\,681 = 6.81 \times 10^{-4}$

**Example 2**

Write the basic numeral for each of these.

a 8×10^2

b 3.052×10^5

c 6×10^{-4}

d 7.44×10^{-3}

Solutions

a $8.\overbrace{00}$

$\therefore 8 \times 10^2 = 800$

c $0\overbrace{000}\overbrace{6}.$

$\therefore 6 \times 10^{-4} = 0.0006$

b $3.\overbrace{052}\overbrace{00}$

$\therefore 3.052 \times 10^5 = 305\,200$

d $0\overbrace{00}\overbrace{7}.\overbrace{44}$

$\therefore 7.44 \times 10^{-3} = 0.007\,44$

Exercise 9.9**1** Express each number in scientific notation.

a 200

b 5000

c 90 000

d 700 000

e 40

f 60 000

g 3000

h 8 000 000

2 Express each number in scientific notation.

a 0.09

b 0.002

c 0.0004

d 0.000 06

e 0.3

f 0.0007

g 0.08

h 0.000 005

3 Write the basic numeral for each of these.

a 5×10^2

b 2×10^3

c 8×10^4

d 3×10^6

e 7×10^4

f 9×10^2

g 6×10^3

h 4×10^5

4 Write the basic numeral for each of these.

a 2×10^{-1}

b 5×10^{-2}

c 9×10^{-3}

d 3×10^{-5}

e 7×10^{-2}

f 6×10^{-4}

g 4×10^{-1}

h 8×10^{-6}

Consolidation**5** Express in scientific notation.

a 1400

b 37 000

c 120 000

d 9 600 000

e 120

f 2500

g 93 000

h 810 000

i 49

j 723

k 2466

l 1490

m 56 700

n 137 000

o 9990

p 6 520 000

q 58.9

r 190.2

s 30.3

t 154.26

6 Write the basic numeral for each of these.

a 9.1×10^2

b 3.2×10^3

c 8.4×10^4

d 2.6×10^6

e 4.3×10^3

f 5.9×10

g 6.5×10^5

h 7.1×10^4

i 2.9×10

j 4.163×10^3

k 5.24×10^2

l 1.1315×10^4

m 3.85×10^3

n 8.92×10^4

o 2.685×10^5

p 9.003×10^6

7 Express in scientific notation.

a 0.71	b 0.062	c 0.0094	d 0.000 035
e 0.54	f 0.0017	g 0.036	h 0.000 79
i 0.0051	j 0.09	k 0.000 067	l 0.088
m 0.125	n 0.0352	o 0.4	p 0.000 663
q 0.007 54	r 0.4761	s 0.003 924	t 0.000 011 62

8 Write the basic numeral for each of these.

a 1.9×10^{-1}	b 8.6×10^{-3}	c 6.4×10^{-2}	d 5.8×10^{-5}
e 3.1×10^{-2}	f 9.3×10^{-1}	g 4.8×10^{-3}	h 7.2×10^{-6}
i 8.42×10^{-1}	j 5.87×10^{-2}	k 1.06×10^{-4}	l 4.19×10^{-5}
m 2.132×10^{-2}	n 4.076×10^{-1}	o 7.119×10^{-3}	p 1.002×10^{-4}

9 Write each of the following in scientific notation, correct to 3 significant figures.

a 17 349	b 2069	c 199 610	d 15.287
e 903.148	f 3566.81	g 0.1662	h 0.022 593
i 0.007 384 9	j 0.000 106 6	k 0.000 009 172	l 0.05

Further applications

10 Evaluate each of the following using the index laws and without the use of a calculator. Give your answers in scientific notation.

a $(2 \times 10^3) \times (4 \times 10^6)$	b $(5 \times 10^2) \times (1.5 \times 10^4)$
c $(1.2 \times 10^4) \times (6 \times 10^{-1})$	d $(3 \times 10^{-5}) \times (2.5 \times 10^{-2})$
e $\frac{8.6 \times 10^{12}}{2 \times 10^4}$	f $\frac{6 \times 10^3}{1.5 \times 10^8}$
g $\frac{9.2 \times 10^5}{2.3 \times 10^{-1}}$	h $\frac{5.4 \times 10^{-7}}{6 \times 10^{-4}}$

9.10

Scientific notation on the calculator

Calculators are programmed to give very large and very small numbers in scientific notation. If you enter $940\,000 \times 730\,000$ into a calculator, it will show the answer as 6.862^{11} because the number of digits in the answer is more than is available on the display. The answer is actually in scientific notation and means 6.862×10^{11} . It does not mean '11 factors of 6.862'.

Numbers can be entered in scientific notation using the exponent key **[EXP]**. For example, to enter 3.4×10^{-7} , press 3.4 **[EXP]** **[$\pm$]** 7. The basic numeral can be found by pressing the equals key.

**Example 1**

Evaluate each of the following using a calculator and give the answers in scientific notation, correct to 4 significant figures.

a 0.0649^8

b $\sqrt{9.5 \times 10^{25}}$

Solutions

a Press 0.0649 x^y 8 $=$

Calculator readout: $3.14744083 \times 10^{-10}$

Answer = $3.147\ 440\ 83 \times 10^{-10}$

= 3.147×10^{-10} (4 significant figures)

b Press $\sqrt{}$ $($ 9.5 EXP 25 $)$ $=$

Calculator readout: $9.746794345 \times 10^{12}$

Answer = $9.746\ 794\ 345 \times 10^{12}$

= 9.747×10^{12} (4 significant figures)

**Example 2**

Evaluate $(6.5 \times 10^7) \times (9.2 \times 10^{11})$ using a calculator and give the answer in scientific notation.

Solution

Press 6.5 EXP 7 $\times$ 9.2 EXP 11 $=$

Calculator readout: 5.98×10^{19}

Answer = 5.98×10^{19}

Exercise 9.10

1 Use the exponent function EXP on the calculator to find the value of:

a 2.3×10^2

b 5.91×10^3

c 7.04×10^4

d 1.608×10^6

e 5.2×10^{-1}

f 6.42×10^{-3}

g 9.27×10^{-2}

h 3.899×10^{-4}

2 Write each calculator display in scientific notation.

a 4.7^{06}

b 3.14^{05}

c 9.05^{09}

d 1.782^{12}

e 5.7^{-04}

f 6.22^{-07}

g 8.013^{-11}

h 2.637^{-18}

3 a Which two of these expressions have the same meaning:

4×10^3 , 4^3 and the calculator display 4^{03} ?

b Why don't all three expressions have the same meaning?

Consolidation

4 Evaluate each of the following using a calculator and give your answers in scientific notation, correct to 4 significant figures.

a 526^3

b 9317^4

c 12.739^6

d 0.0037^7

e 0.0469^9

f $0.000\ 715^{12}$

g $(5.6 \times 10^4)^5$

h $(9.47 \times 10^{-7})^2$

i $(7.21 \times 10^{-6})^{-7}$

j $\sqrt{8.46 \times 10^5}$

k $\sqrt[3]{7.604 \times 10^{-8}}$

l $\sqrt[4]{(5.299 \times 10^{-2})^{10}}$

m $\frac{1}{1.74 \times 10^3}$

n $\frac{1}{2.3 \times 10^{-4}}$

o $\frac{1}{(5.83 \times 10^{-2})^3}$

- 5 Evaluate each of the following using a calculator and give your answers in scientific notation, correct to 4 significant figures.

a $(5.4 \times 10^8) \times (9.3 \times 10^6)$

b $(8.73 \times 10^5) \times (7.36 \times 10^7)$

c $(3.9 \times 10^{28}) \div (5.1 \times 10^{11})$

d $(2.13 \times 10^{15}) \div (1.6 \times 10^9)$

e $(7.2 \times 10^{19}) \times (2.5 \times 10^{-4})$

f $(5.98 \times 10^{-6}) \times (3.47 \times 10^{-12})$

g $\frac{8.64 \times 10^{-17}}{1.02 \times 10^{-3}}$

h $\frac{6.022 \times 10^{-4}}{5.97 \times 10^{20}}$

- 6 Arrange these numbers in ascending order:

5.2×10^8 , 6.7×10^5 , 3.94×10^{-12} , 8.41×10^{-9} , 1.5×10^6 , 3.959×10^{-12}

■ Further applications

- 7 The Earth has a mass of 6×10^{24} kg while the planet Jupiter has a mass of 1.2×10^{27} kg.
- Which planet has the greater mass and by how much?
 - How many times greater is the mass of the larger planet than that of the smaller planet?
- 8 The Earth orbits the Sun at an approximate speed of 8333 m/s. Express this speed in km/h, giving your answer in scientific notation, correct to 1 significant figure.
- 9 The average distance between the Earth and the Sun (approximately 150 million km) is defined as one astronomical unit (AU). If the average distance between Pluto and the Sun is 39.44 AU, express this distance in kilometres in scientific notation, correct to 4 significant figures.
- 10 The speed of light is approximately 300 000 km/s. If the Sun is about 1.495×10^8 km from Earth, find the time that it takes for sunlight to reach the Earth. Give your answer in minutes, correct to 1 decimal place.
- 11 a The diameter of the Earth is 1.275×10^7 metres. What is the radius of the Earth in kilometres? Give your answer in scientific notation.
- b What is the circumference of the Earth at the equator, in kilometres? Give your answer correct to the nearest kilometre.
- 12 a The Moon's radius is 1.738×10^6 metres and the Earth's radius is 6.38×10^6 metres. How many times larger is the radius of the Earth than that of the Moon? Give your answer correct to 2 decimal places.
- b The Sun's radius is 6.96×10^8 metres. How many times larger is the radius of the Sun than that of the Earth? Give your answer correct to the nearest whole number.
- 13 a The mass of an atom of hydrogen is 1.6×10^{-24} grams and the mass of an electron is 9×10^{-28} grams. Which is heavier, and how many times heavier is it? Give your answer correct to the nearest whole number.
- b How many millions of atoms of hydrogen are needed to weigh 1 gram? Give your answer in scientific notation.



FOCUS ON WORKING MATHEMATICALLY

MATHEMATICS IS AT THE HEART OF SCIENCE



Introduction

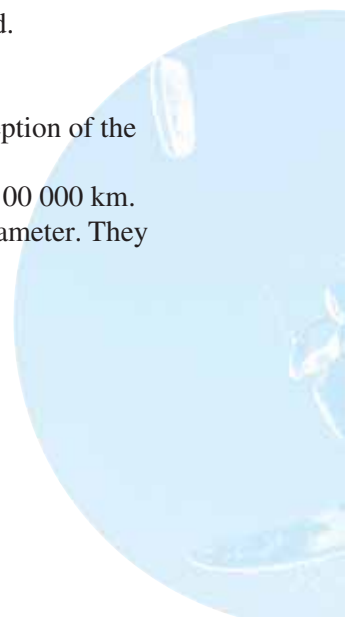
In this chapter, you have learned about the need for a standard way of writing large and small numbers. This is called 'scientific notation' and a calculator is a very useful device for working with them. We are now going to explore the usefulness of indices to record important scientific information.



LEARNING ACTIVITIES

Facts about our bodies

- 1 Write each of the following facts about the human body in scientific notation.
 - a Your bone marrow produces about 2 million red blood cells every second.
 - b On average, the human brain contains about 100 billion nerve cells.
 - c There are about 100 000 hairs on your head.
 - d Each of your eyes has about 120 million rods, which give you your perception of the world.
 - e Placed end to end, the blood vessels in your body would measure about 100 000 km.
 - f The tiny tubes (called capillaries) in your kidneys are about 0.0001 m in diameter. They are so small red blood cells pass through them in single file.



- 2 Investigate the accuracy of the following statements. A calculator will help.
- a 'Your heart will beat about 3 billion times in your lifetime.' Assume that your life expectancy will be 85 years.
 - b 'You will breathe about 10 million times in a year.'
 - c 'One light year, the distance that light travels through a vacuum in one year, is approximately 9.46×10^{15} m.' Take the speed of light to be 299 792 458 m/s.



CHALLENGE

- 1 The mass of a water molecule is 0.000 000 000 000 000 000 029 9 gram. Write this in scientific notation.
- 2 The thickness of the anti-reflection coating on a camera lens is 0.000 000 12 m. Write this in scientific notation.
- 3 The diameter of a human hair is 0.000 07 m. Write this in scientific notation. If 1 micron (μ) is 10^{-6} m, what is this diameter in microns?
- 4 The accuracy of a clock made by John Harrison to measure longitude (position on the Earth's surface east or west of Greenwich) is an amazing story. He managed this superb technological feat around 1770. Here is a quote from the book *Longitude* by Dava Sobel (Griffin Press, 1998), which tells how accurate his clock was:

'In some trials [the clock] lost only 7.287×10^{-7} seconds per second, an accuracy that was not to be achieved again for at least 100 years ... And for his own time finding—to see how accurate his results were—he used the passage of a particular star past a neighbor's chimney.'

Show that the time lost by Harrison's clock on a 30-day voyage was theoretically less than 2 seconds.



LET'S COMMUNICATE

Write a short account of the usefulness of indices in recording small and large scientific measurements. What new scientific facts did you learn?



REFLECTING

Reflect on the way in which mathematics is so basic to the study of medicine, astronomy, chemistry and physics. Think over this statement: 'Without mathematics there would be no science'.





MACQUARIE

Language link with Macquarie

- 1 What is another word for **index**?
- 2 In the expression $100 = 10^2$, which number is the **base**?
- 3 A number is in s _____ n _____ if it is written as the product of a number between 1 and 10 and a power of 10.
- 4 What is meant by a **negative index**? A **fraction index**?

- 5 Read the Macquarie Learners Dictionary entry for **notation**:

notation noun a way of writing down things like music or dance by using signs or symbols, such as notes or lines to stand for sounds or marks to stand for movement

What sorts of jobs require skills in the use of notation?

- 1 Express each of these in simplest index form.

a $3 \times 3 \times 3 \times 3$

b $a \times a \times a \times a \times a$

c $7 \times p \times 2 \times q \times q \times p$

- 2 Write each of these in simplest index form.

a 7^2

b k^6

c a^3b^4

d $5e^2 + 3f^4$

- 3 Write an expression in index form for 'three factors of five'.

- 4 Find values for all pronumerals in each of these.

a $72 = 2^x \times 3^y$

b $675 = 3^m \times 5^n$

- 5 Express in simplest index form:

a $5^3 \times 5^7$

b $2^{10} \div 2^6$

c $(3^5)^4$

- 6 Simplify:

a $n^3 \times n^4$

b $5p^2 \times 3p^4$

c $a^8b^3 \times a^4b^7$

d $\frac{1}{2}k^6 \times 8k$

e $ab \times bc \times cd$

f $-5x^2y^3 \times (-8xy^4)$

g $y^9 \div y^4$

h $21c^9 \div 3c^5$

i $u^{12}v^8 \div u^3v$

j $3t^{11} \div \frac{1}{2}t$

k $\frac{24m^{13}}{3m^8}$

m $(z^4)^7$

o $(2x^5)^4$

q $(a^6b^3)^8$

- 7 Simplify:

a $t^3 \times t^5 \times t^2$

c $y^9 \times y^5 \div y^3$

e $p^{19} \div (p^2)^3$

g $\frac{h^{11}}{h^3 \times h^2}$

i $\frac{(k^5)^7 \times k^4}{k^{12}}$

l $\frac{-42e^8f^{12}}{7e^2f^5}$

n $(3p)^3$

p $(rs)^5$

r $(-2j^4k^9)^3$

b $d^{13} \div d^4 \div d^2$

d $(p^6)^4 \times p^7$

f $(z^{10})^4 \div (z^3)^5$

h $\left(\frac{v^8}{v^4}\right)^6$

8 Simplify $\frac{9d^{13} \times (2d^3)^3}{12d^{14}}$.

- 9 Evaluate each of these if $2^n = 5$.

a 2^{n+1}

b 2^{n+2}

c 2^{n+4}

- 10 Evaluate each of these if $5^k = 500$.

a 5^{k-1}

b 5^{k-2}

c 5^{k-4}

11 Evaluate each of these if $3^y = 10$.

a 3^{2y} b 3^{3y}
c 3^{4y-1} d 3^{5y+2}

12 Express each of these as a fraction in simplest form.

a 4^{-1} b 6^{-2} c 3^{-4}
d 5^{-3} e 2^{-5}

13 Write each of these with a negative index.

a $\frac{1}{m}$ b $\frac{1}{k^2}$ c $\frac{1}{a^5}$
d $\frac{1}{r^9}$ e $\frac{1}{x^{11}}$

14 Find the value of n if:

a $\frac{1}{7} = 7^n$ b $\frac{1}{64} = 8^n$
c $\frac{1}{16} = 2^n$ d $\frac{1}{1000} = 10^n$

15 Express each of these without any negative indices.

a ab^{-1} b $p^{-1}q$
c $x^{-1}y^{-1}$ d c^3d^{-2}
e $m^{-4}n^{-5}$ f $u^2v^3w^{-7}$
g $r^4s^{-1}t^{-6}$ h $a^{-1}b^{-2}c^{-3}$
i $9t^{-2}$ j $7e^{-1}f^{-5}g$
k $\frac{3}{4}p^{-3}$ l $\frac{5}{9}k^{-1}m^2$

16 Express each of the following as a product without negative indices.

a $\frac{x}{y}$ b $\frac{m^3}{n^5}$ c $\frac{1}{p^2q^4}$
d $\frac{a^4b^5}{c^7}$ e $\frac{m^6}{kn^5}$ f $\frac{2}{x^3yz^8}$
g $\frac{5m^9}{n^2p^3}$ h $\frac{2e^4}{3f}$

17 Express the following in simplest form, without negative indices.

a $\left(\frac{1}{4}\right)^{-1}$ b $\left(\frac{3}{2}\right)^{-1}$ c $\left(\frac{5}{3}\right)^{-2}$
d $\left(2\frac{1}{2}\right)^{-3}$ e $\left(\frac{6}{t}\right)^{-1}$ f $\left(\frac{m}{7}\right)^{-2}$
g $\left(\frac{2p}{q}\right)^{-3}$ h $\left(\frac{8c^4}{5d^7}\right)^{-2}$

18 Simplify, then give the answers without any negative indices.

a $a^8 \times a^{-3}$ b $k^{-9} \times k^2$
c $c^{-4} \times c^{11}$ d $2p^{-7} \times 8p^{-4}$
e $s^4 \div s^{10}$ f $x^{-3} \div x^5$
g $r^{-12} \div r^{-7}$ h $63u^{-2} \div 9u^{-6}$
i $(v^6)^{-1}$ j $(n^{-5})^2$
k $(q^{-4})^{-8}$ l $(2y^{-7})^{-3}$

19 Express each of these with a radical sign, then evaluate.

a $36^{\frac{1}{2}}$ b $64^{\frac{1}{2}}$ c $121^{\frac{1}{2}}$
d $8^{\frac{1}{3}}$ e $27^{\frac{1}{3}}$ f $125^{\frac{1}{3}}$
g $81^{\frac{1}{4}}$ h $32^{\frac{1}{5}}$ i $1\,000\,000^{\frac{1}{6}}$

20 Simplify:

a $9^{-\frac{1}{2}}$ b $49^{-\frac{1}{2}}$
c $8^{-\frac{1}{3}}$ d $1000^{-\frac{1}{3}}$
e $16^{-\frac{1}{4}}$ f $100\,000^{-\frac{1}{5}}$

21 Evaluate each of these by using the $\left[\frac{1}{x^y}\right]$ or $\left[\sqrt[y]{x}\right]$ key on your calculator.

a $\sqrt[4]{1296}$ b $\sqrt[5]{16\,807}$
c $\sqrt[7]{16\,384}$

22 Simplify each of the following.

a $\sqrt{a^{10}}$ b $\sqrt[3]{w^{12}}$ c $\sqrt{4k^6}$
 d $\sqrt{49p^{22}}$ e $\sqrt[3]{8u^{18}}$ f $\sqrt[3]{27s^{21}}$

23 Evaluate:

a $9^{\frac{3}{2}}$ b $8^{\frac{4}{3}}$ c $32^{\frac{3}{5}}$
 d $27^{-\frac{4}{3}}$ e $\left(\frac{1}{16}\right)^{\frac{3}{2}}$ f $\left(\frac{8}{27}\right)^{\frac{2}{3}}$
 g $\left(\frac{100}{9}\right)^{\frac{-3}{2}}$ h $\left(\frac{64}{125}\right)^{\frac{-2}{3}}$

24 Write each of these in surd form.

a $m^{\frac{5}{2}}$ b $k^{\frac{2}{3}}$ c $e^{\frac{3}{4}}$
 d $q^{\frac{-2}{9}}$ e $a^{\frac{-5}{6}}$ f $z^{\frac{-7}{5}}$

25 Write each of these in index form.

a $a\sqrt{a}$ b $p^3\sqrt{p}$ c $n \times \sqrt[4]{n}$
 d $t^2 \times \sqrt[3]{t}$ e $\frac{1}{\sqrt{c}}$ f $\frac{1}{\sqrt[4]{y}}$

g $\frac{1}{\sqrt[3]{x^4}}$ h $\frac{1}{b^2\sqrt{b}}$

26 Express each of these numbers in scientific notation.

a 3000 b 190 000
 c 28 600 d 173.4
 e 0.0004 f 0.026
 g 0.001 98 h 0.000 583 1

27 Write the basic numeral for each of these.

a 9×10^2 b 6×10^4
 c 8.7×10^3 d 1.04×10^5
 e 3×10^{-1} f 7×10^{-3}
 g 1.9×10^{-2} h 4.61×10^{-6}

28 Use the exponent key $\boxed{\text{EXP}}$ to find the value of each calculator display:

a 9^{05} b 5.6^{04}
 c 2^{-03} d 3.27^{-05}

29 Evaluate each of the following, giving the answer in scientific notation, correct to 4 significant figures.

a 7265^3 b $10\,288^7$
 c 0.0847^{13} d $0.009\,29^{15}$

Geometry



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ determine whether two angles are adjacent
- ⦿ apply the properties of complementary, supplementary, vertically opposite angles and angles at a point to find unknown angles, giving reasons
- ⦿ state whether a pair of angles on parallel lines are alternate, corresponding or co-interior
- ⦿ find unknown angles on parallel lines, giving reasons
- ⦿ classify triangles according to their sides and angles
- ⦿ use the angle sum of a triangle to find unknown angles, giving reasons
- ⦿ use the exterior angle property of triangles to find unknown angles, giving reasons
- ⦿ use the properties of isosceles and equilateral triangles to find unknown sides and angles, giving reasons
- ⦿ use the angle sum of a quadrilateral to find unknown angles, giving reasons
- ⦿ classify the special quadrilaterals according to their side, angle and diagonal properties
- ⦿ use the properties of the special quadrilaterals to find unknown sides and angles, giving reasons
- ⦿ name a polygon according to the number of sides
- ⦿ determine whether a polygon is convex or non-convex
- ⦿ determine whether a polygon is regular or irregular
- ⦿ find the interior angle sum of a polygon by dissecting it into triangles and by the formula
- ⦿ find the size of the interior and exterior angles of a regular polygon
- ⦿ find the number of sides in a regular polygon given the size of its interior or exterior angles
- ⦿ state the minimum set of conditions for two triangles to be congruent
- ⦿ identify matching sides and angles in congruent triangles
- ⦿ apply the congruence tests to justify that two triangles are congruent
- ⦿ apply the congruence tests to establish properties of triangles and quadrilaterals
- ⦿ apply the congruence tests to prove unfamiliar results.

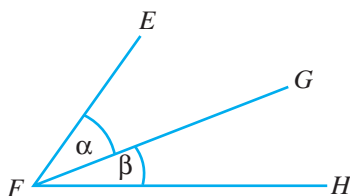
10.1 Angles

In this course, the emphasis will be on not only finding the correct size of an angle but, more importantly, on giving correct reasons for *each step* in your working. While geometric reasons can be given in many ways, it is important to spell important terms correctly, use only standard abbreviations and be consistent in your setting out. By convention, the degrees symbol is not written when Greek letters are used in angles. Relationships between two or more angles can be used to prove many properties of geometric figures.

Adjacent angles

Two angles are adjacent if they:

- ⊙ have a common vertex, and
- ⊙ have a common ray, and
- ⊙ lie on opposite sides of this common ray.



For example, $\angle EFG$ is adjacent to $\angle GFH$ because:

- 1 F is a common vertex, and
- 2 FG is a common ray, and
- 3 the angles lie on opposite sides of FG .

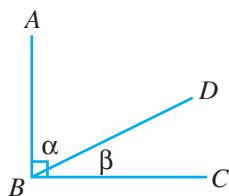
Complementary and supplementary angles

Complementary angles are two angles whose sum is 90° . Adjacent angles in a right angle are complementary.

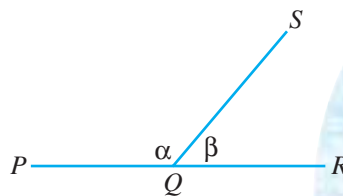
Supplementary angles are two angles whose sum is 180° . Adjacent angles on a straight line are supplementary.

Complementary angles have a sum of 90° .
Supplementary angles have a sum of 180° .

For example:



$\angle ABD$ and $\angle DBC$ are complementary angles: $\alpha + \beta = 90^\circ$



$\angle PQS$ and $\angle SQR$ are supplementary angles: $\alpha + \beta = 180^\circ$

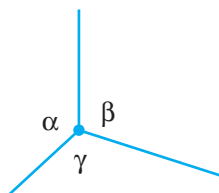
NOTE: When giving reasons, the terms complementary angles and supplementary angles must not be used. These terms simply mean that the angles have a sum of 90° or 180° . In geometric reasoning, you need to explain *why* the angles have that sum (e.g. angles in a right angle, or angles on a straight line).

Angles at a point

Angles at a point are two or more angles that have a common vertex and whose sum is 360° , or one complete revolution.

Angles at a point have a sum of 360° .

For example:



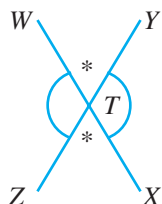
$$\alpha + \beta + \gamma = 360^\circ$$

Vertically opposite angles

Vertically opposite angles are formed by the intersection of two straight lines. Vertically opposite angles must be equal because they are adjacent and supplementary to a common angle.

Vertically opposite angles are equal.

For example:



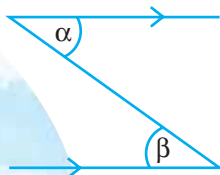
$\angle WTZ$ and $\angle YTX$ are vertically opposite angles.

$\angle WTY$ and $\angle ZTX$ are vertically opposite angles.

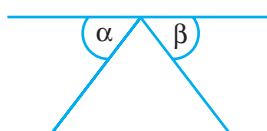
Example 1

Explain why α and β are not adjacent angles in each of the following.

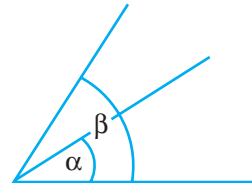
a



b



c

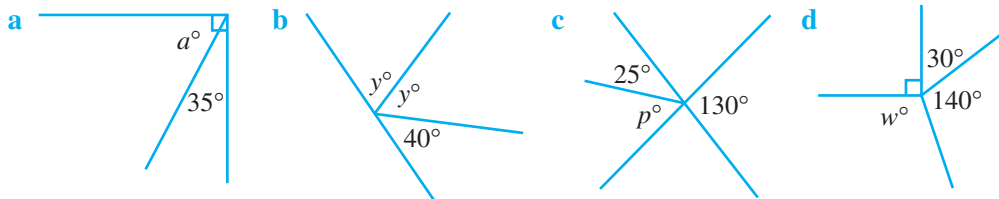


Solutions

- a The angles do not have a common vertex.
- b The angles do not have a common ray.
- c The angles do not lie on opposite sides of the common ray.

EG
+ S**Example 2**

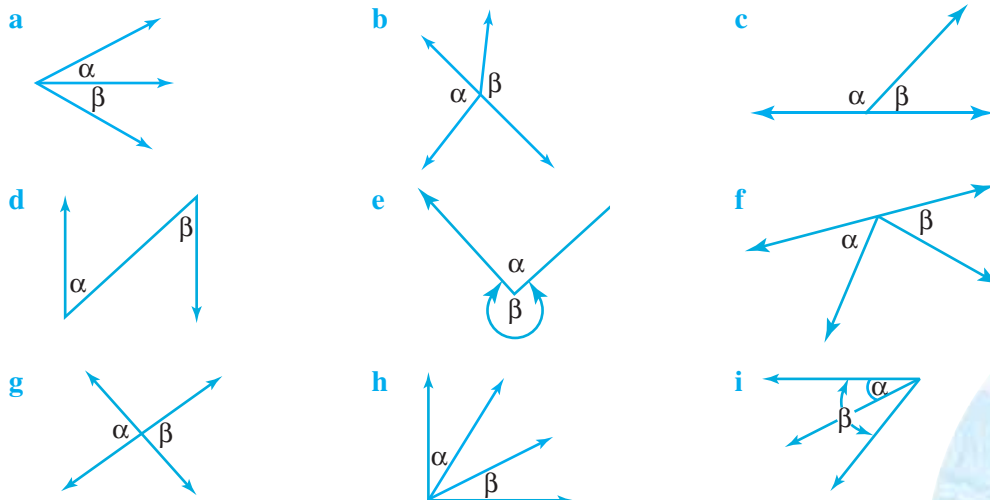
Find the value of the pronumeral in each of these, giving reasons.

**Solutions**

- a** $a + 35 = 90$ (angles in a right angle)
 $\therefore a = 55$
- b** $y + y + 40 = 180$ (angles on a straight line)
 $2y + 40 = 180$
 $2y = 140$
 $\therefore y = 70$
- c** $p + 25 = 130$ (vertically opposite angles are equal)
 $\therefore p = 105$
- d** $w + 30 + 90 + 140 = 360$ (angles at a point)
 $w + 260 = 360$
 $\therefore w = 100$

Exercise 10.1

- 1** For each of the following, state whether the angles α and β are adjacent. If they are not adjacent, give a reason why.



- 2** Write down the:

- a** complement of 50° **b** supplement of 60°
c complement of the supplement of 155° **d** supplement of the complement of 28°

3 What size is the angle that is vertically opposite to an angle measuring 42° ?

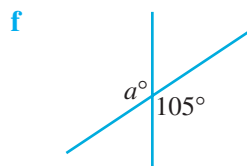
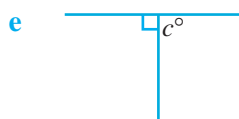
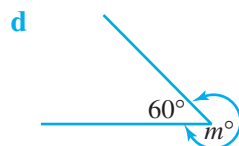
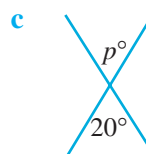
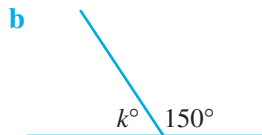
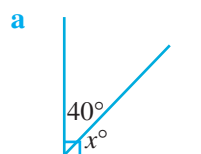
4 How many degrees are there in:

a $\frac{1}{2}$ revolution?

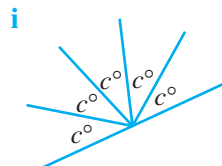
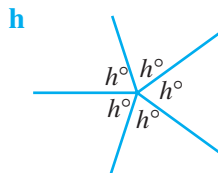
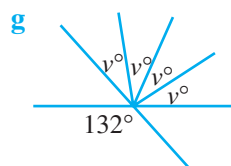
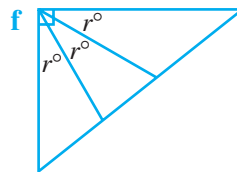
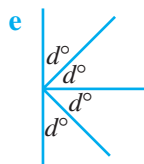
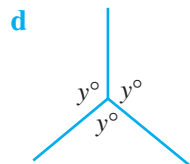
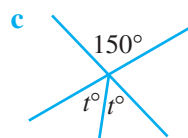
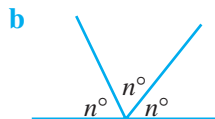
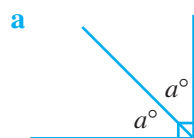
b $\frac{3}{4}$ revolution?

c $\frac{7}{12}$ revolution?

5 Find the value of the pronumeral in each of these, giving brief reasons.

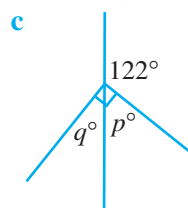
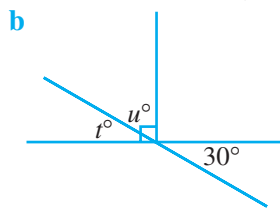
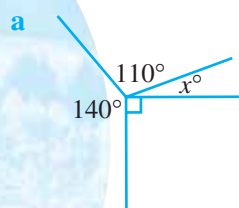


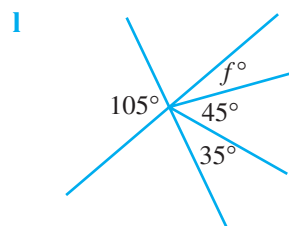
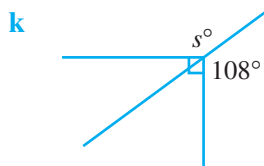
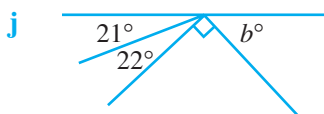
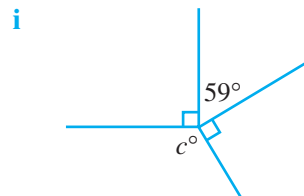
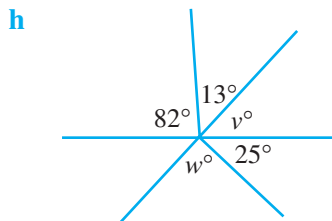
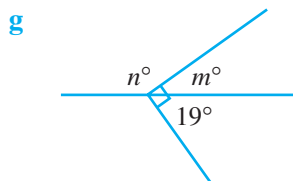
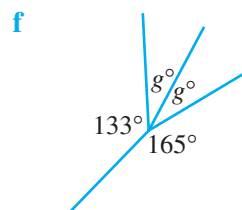
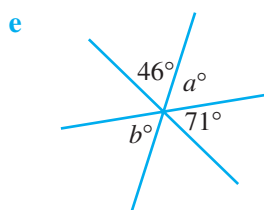
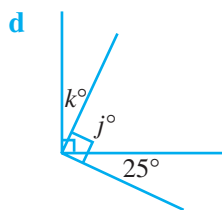
6 Find the value of the pronumeral in each of these, giving brief reasons.



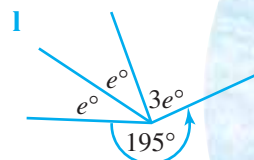
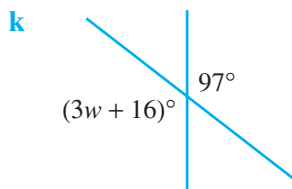
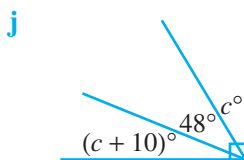
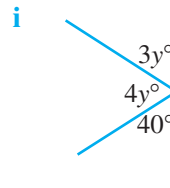
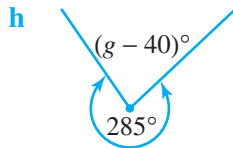
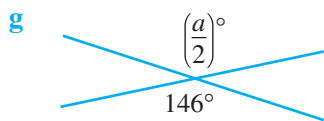
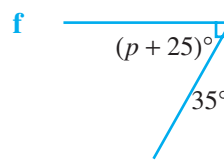
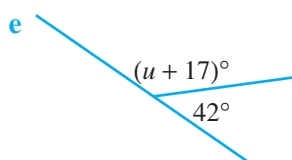
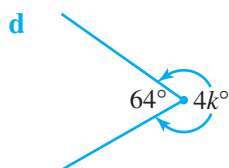
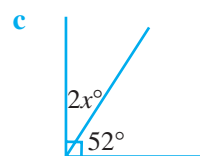
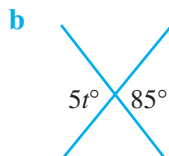
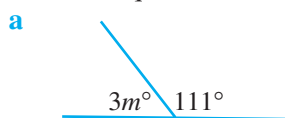
Consolidation

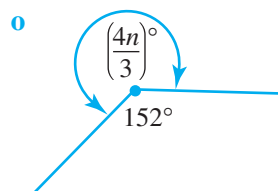
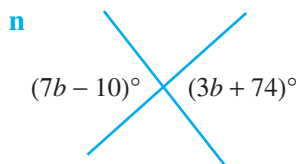
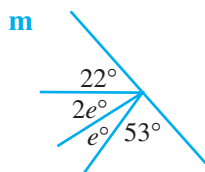
7 Find the value of the pronumeral in each of these. (Do not give reasons.)



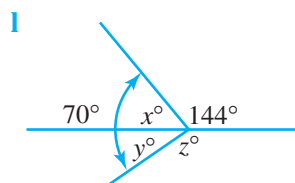
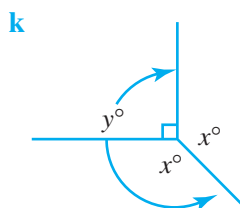
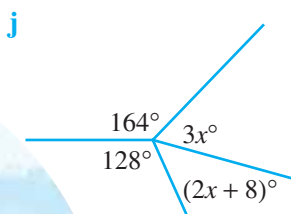
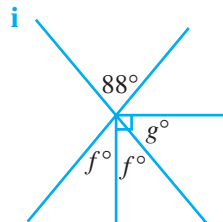
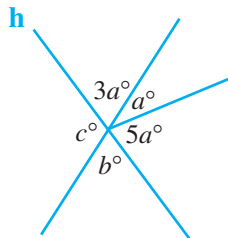
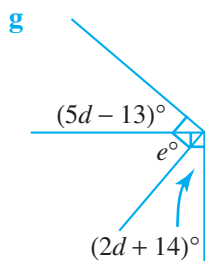
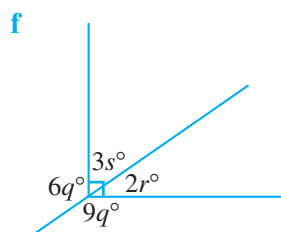
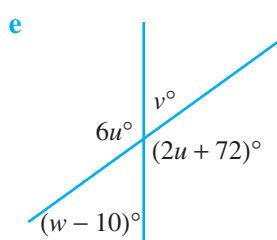
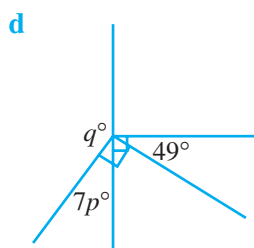
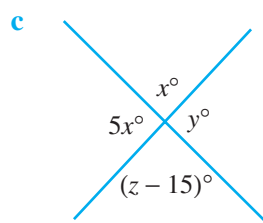
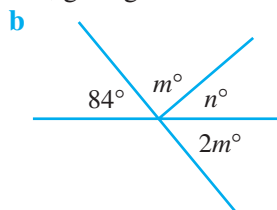
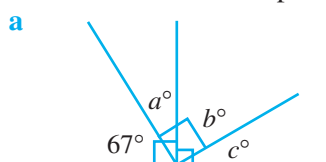


8 Form an equation and solve it to find the value of each pronumeral. Give brief reasons.





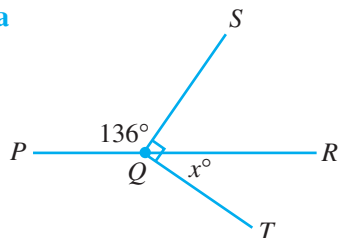
9 Find the value of each pronumeral, giving reasons.



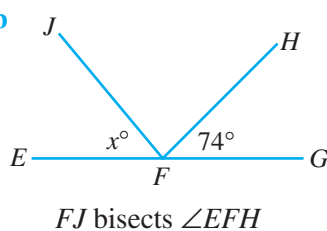
Further applications

10 Find the value of x in each of these, giving reasons.

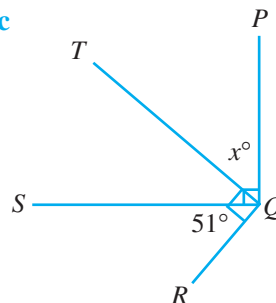
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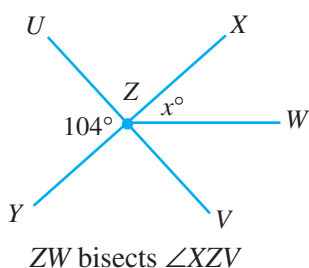
b



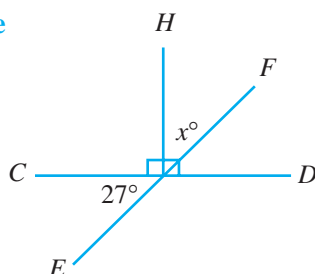
c



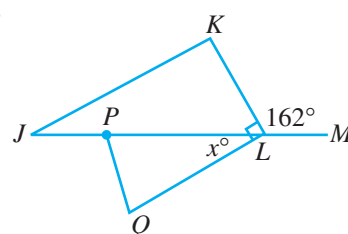
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e



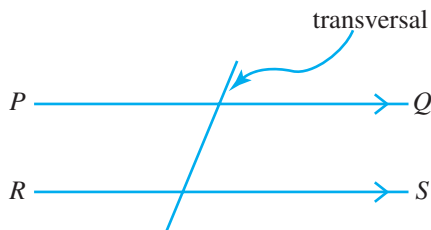
f



10.2 Parallel lines

Parallel lines are two or more lines that have been drawn in the same plane and never meet. We say that parallel lines are *equidistant*. That is, the distance between the lines remains constant. Arrowheads are used to show that two lines are parallel.

The notation $\parallel$ means 'is parallel to'. In this diagram we could say that $PQ \parallel RS$, meaning ' PQ is parallel to RS '.

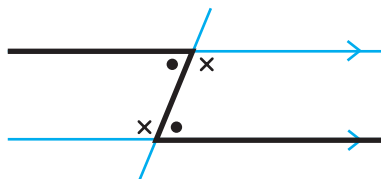


A line that cuts two or more parallel lines is called a **transversal**. When a pair of parallel lines is cut by a transversal, 8 angles are formed. These angles can be classified into 3 special pairs of angles: **alternate angles**, **corresponding angles** and **co-interior angles**.

Alternate angles

Alternate angles:

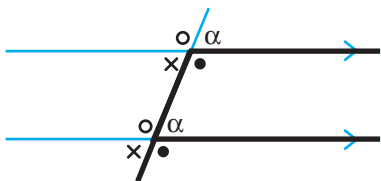
- ⊙ lie between the parallel lines and on opposite sides of the transversal
- ⊙ are equal in size
- ⊙ form a Z shape.



Corresponding angles

Corresponding angles:

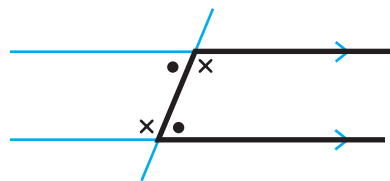
- ⊙ lie on the same side of the parallel lines and on the same side of the transversal
- ⊙ are equal in size
- ⊙ form a F shape.



Co-interior angles

Co-interior angles:

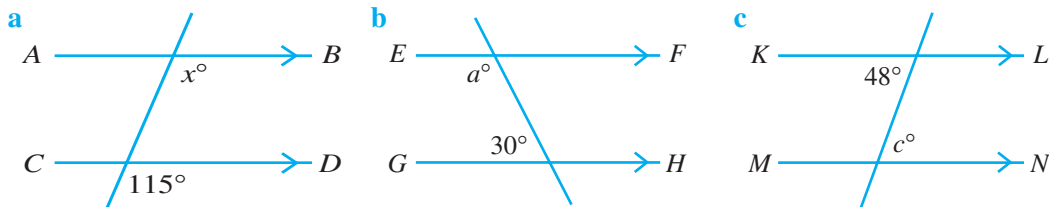
- ⊙ lie between the parallel lines and on the same side of the transversal
- ⊙ are supplementary
- ⊙ form a C shape.



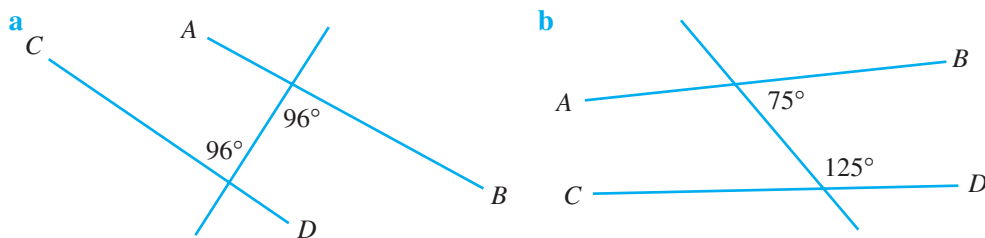
NOTE: In questions where reasons are required to be given, you *must* refer to the parallel lines and name them if they are labelled. It is not good enough to write only 'alternate angles', for example, as a reason.

EG
+S**Example 1**

Find the value of the pronumeral in each of the following, giving reasons.

**Solutions**

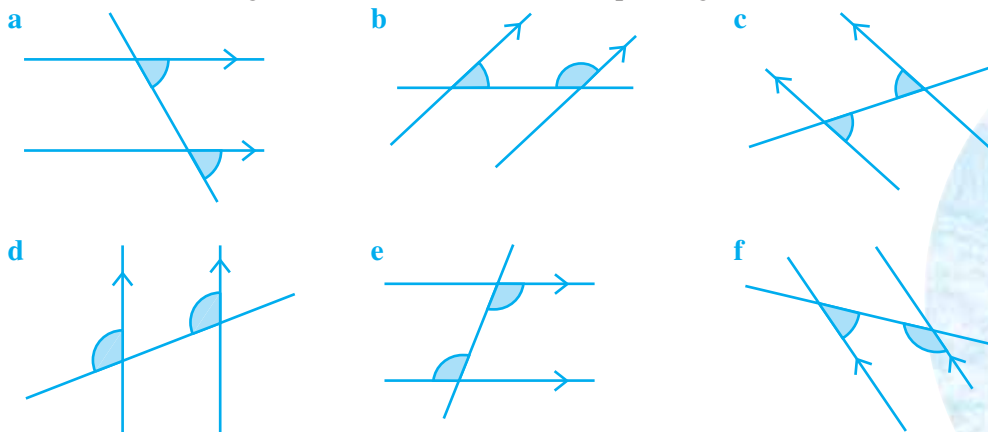
- a** $x = 115$ (corresponding angles, $AB \parallel CD$).
b $a = 150$ (co-interior angles, $EF \parallel GH$).
c $c = 48$ (alternate angles, $KL \parallel MN$).

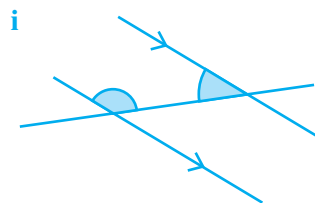
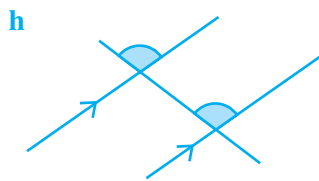
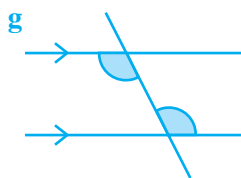
EG
+S**Example 2**Determine whether $AB \parallel CD$ in each of these. Diagrams are not drawn to scale.**Solutions**

- a** AB is parallel to CD (alternate angles are equal).
b AB is not parallel to CD (co-interior angles are *not* supplementary).

Exercise 10.2

1 State whether the angles shown are alternate, corresponding or co-interior.





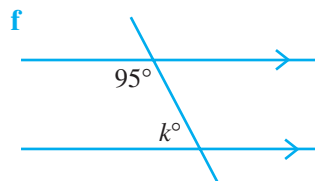
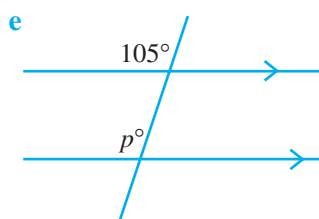
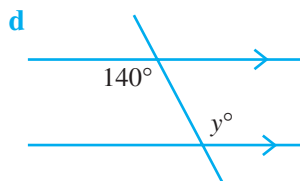
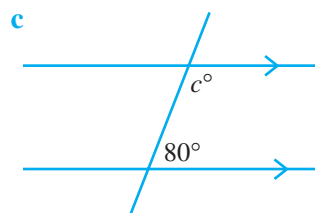
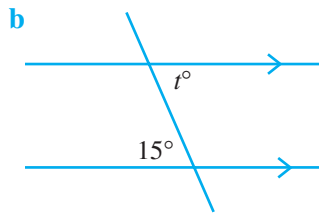
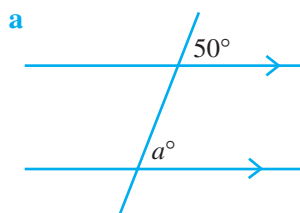
2 State whether these angles are equal or supplementary.

a corresponding angles

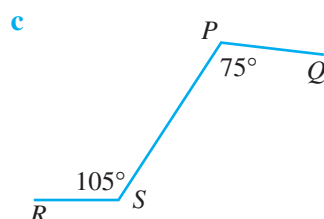
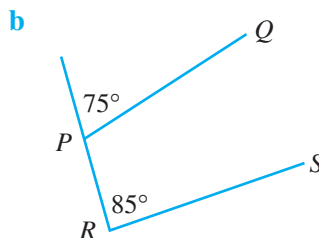
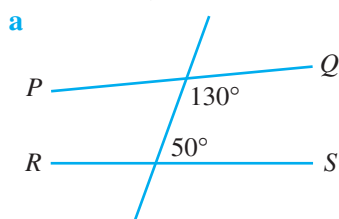
b alternate angles

c co-interior angles

3 Find the value of the pronumeral in each of the following, giving brief reasons.

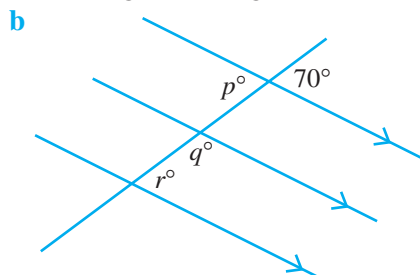
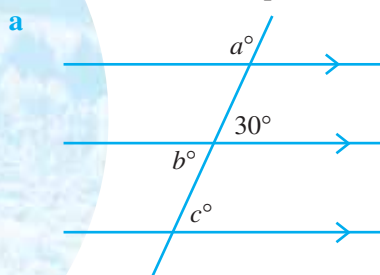


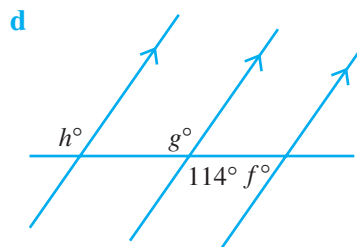
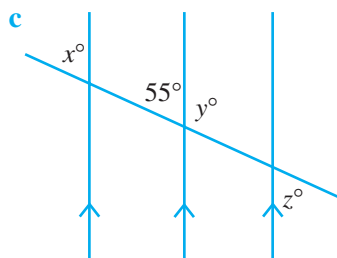
4 State whether $PQ \parallel RS$ in each of these. If they are parallel, give a reason. (Diagrams are not to scale.)



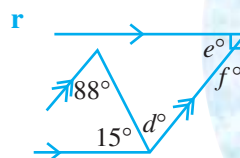
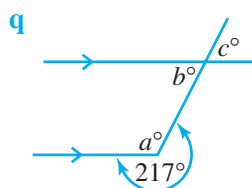
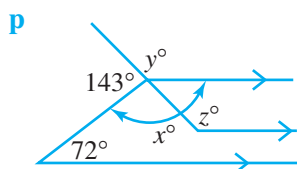
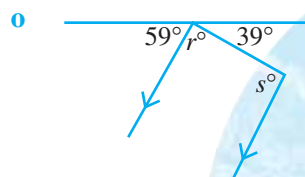
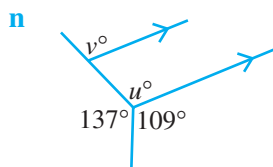
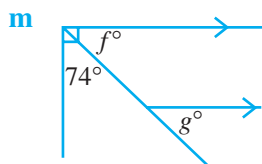
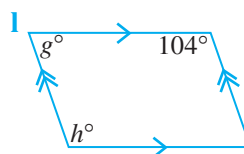
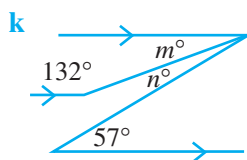
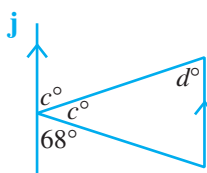
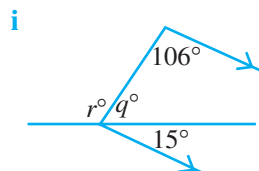
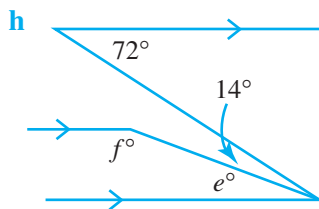
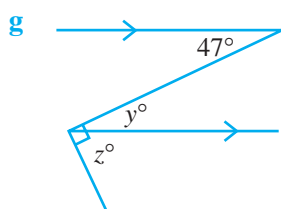
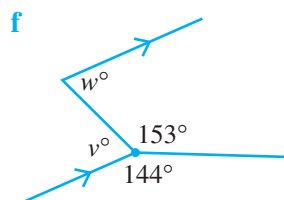
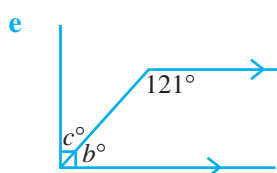
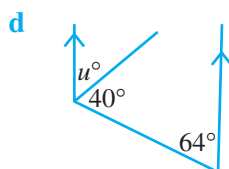
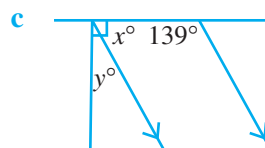
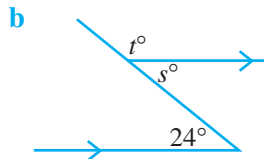
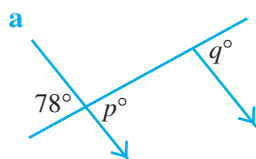
Consolidation

5 Find the values of all pronumerals in each the following. (Do not give reasons.)

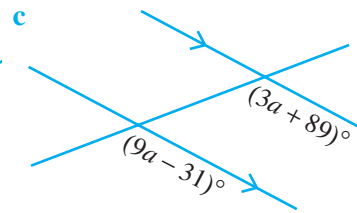
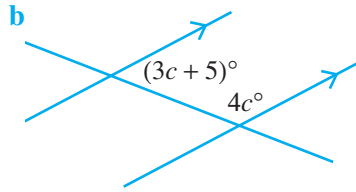
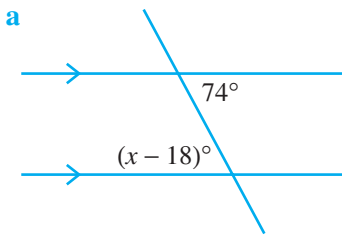




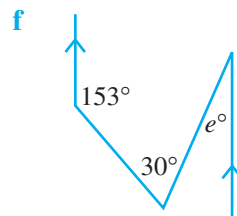
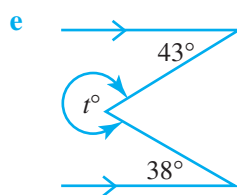
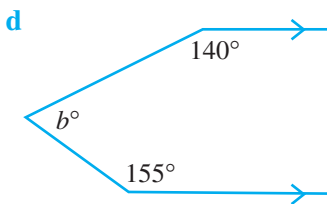
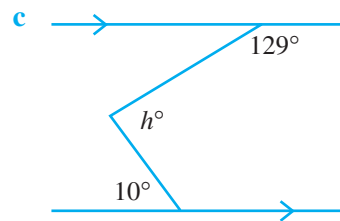
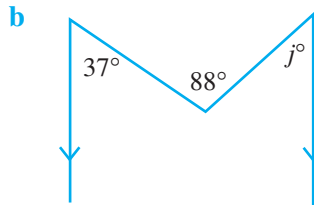
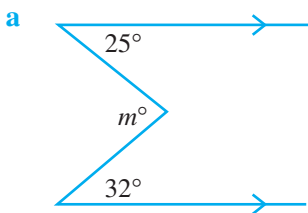
6 Find the value of the pronumeral in each of these, giving reasons.



7 Form an equation and solve it to find the value of each pronumeral. Give brief reasons.

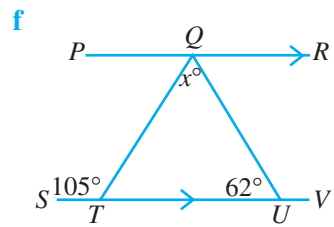
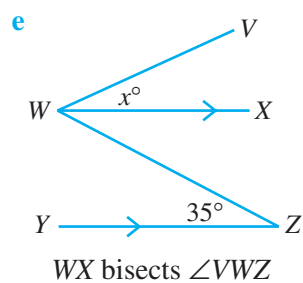
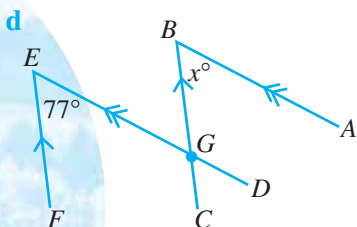
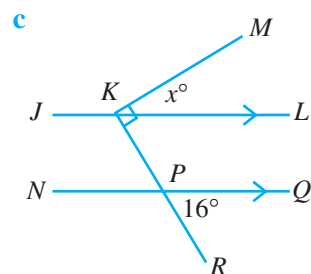
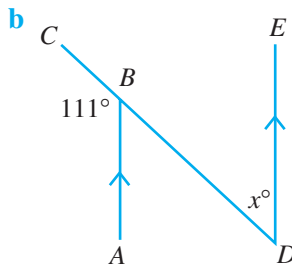
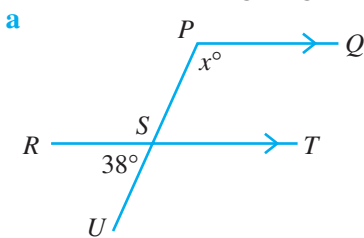


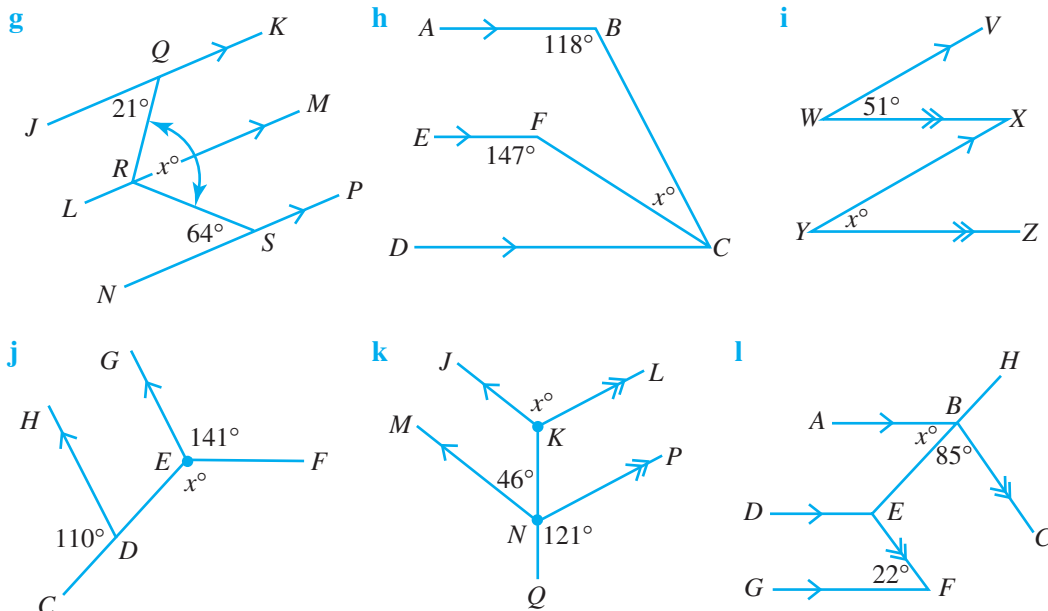
8 Find the value of the pronumeral in each of these. (HINT: You will need to draw a line parallel to the given parallel lines.)



Further applications

9 Find the value of x , giving reasons.





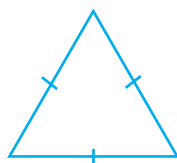
10.3 Triangles

Classification of triangles

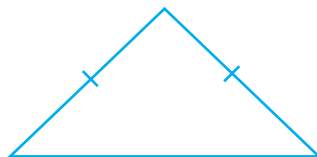
A triangle can be classified according to the length of its sides or the size of its angles.

Classification according to sides:

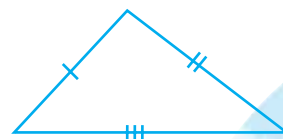
- ⊙ An equilateral triangle is a triangle in which all three sides are equal in length.
- ⊙ An isosceles triangle is a triangle in which two sides are equal in length.
- ⊙ A scalene triangle is a triangle in which no sides are equal in length.



Equilateral



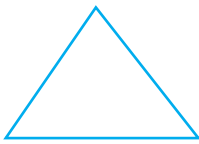
Isosceles



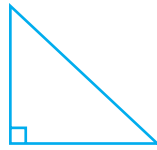
Scalene

Classification according to angles:

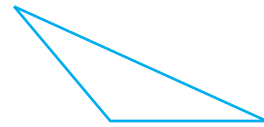
- ⊙ An acute-angled triangle is a triangle in which all three angles are acute.
- ⊙ A right-angled triangle is a triangle in which there is one right angle.
- ⊙ An obtuse-angled triangle is a triangle in which there is one obtuse angle.



Acute-angled



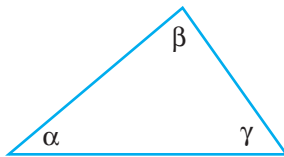
Right-angled



Obtuse-angled

The angle sum of a triangle

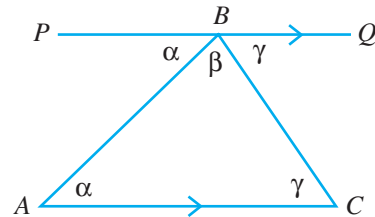
The angle sum of a triangle is 180° .



That is, $\alpha + \beta + \gamma = 180^\circ$.

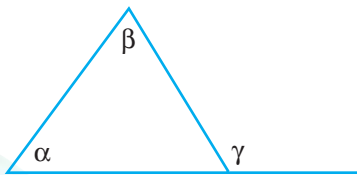
Proof:

- Construct PQ through B , parallel to AC .
- $\angle PBA = \angle BAC$ (alternate angles, $PQ \parallel AC$)
 $\therefore \angle PBA = \alpha$
- $\angle QBC = \angle ACB$ (alternate angles, $PQ \parallel AC$)
 $\therefore \angle QBC = \gamma$
- $\angle PBA + \angle ABC + \angle QBC = 180^\circ$ (angles on a straight line)
 $\therefore \alpha + \beta + \gamma = 180^\circ$
 $\therefore$ The angle sum of the triangle is 180° .



The exterior angle of a triangle

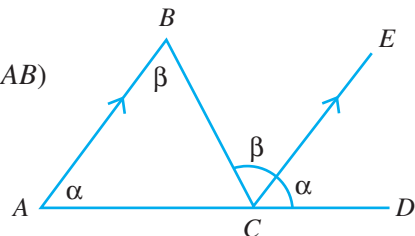
The exterior angle of a triangle is equal to the sum of the two interior opposite angles.



That is, $\gamma = \alpha + \beta$.

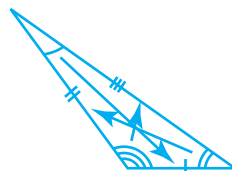
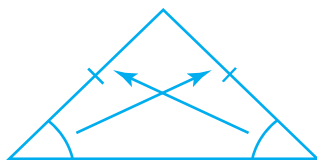
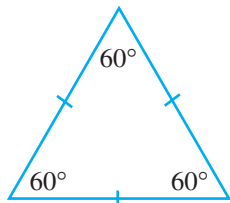
Proof:

- Produce AC to D .
- Construct CE parallel to AB .
- $\angle ECD = \angle BAC$ (corresponding angles, $CE \parallel AB$)
 $\therefore \angle ECD = \alpha$
- $\angle BCE = \angle ABC$ (alternate angles, $CE \parallel AB$)
 $\therefore \angle BCE = \beta$
- $\angle BCD = \angle ECD + \angle BCE$ (adjacent angles)
 $\therefore \gamma = \alpha + \beta$
 $\therefore$ The exterior angle of the triangle is equal to the sum of the two interior opposite angles.



Some other properties of triangles

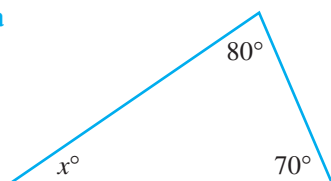
- In an equilateral triangle, all angles are 60° .
- In an isosceles triangle, the equal angles are opposite the equal sides.
- In any triangle, the longest side is opposite the largest angle and the shortest side is opposite the smallest angle.

EG
+S

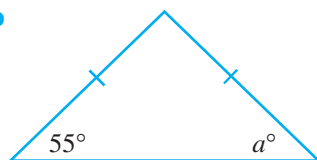
Example 1

Find the value of the pronumeral in each of the following, giving reasons.

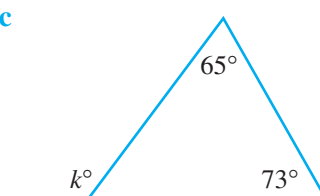
a



b



c



Solutions

a $x + 70 + 80 = 180$ (angle sum of a Δ)

$$x + 150 = 180$$

$$\therefore x = 30$$

b $a = 55$ (base angles of an isosceles Δ are equal)

c $k = 65 + 73$ (exterior angle of a Δ)

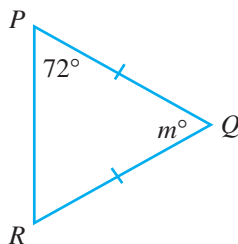
$$\therefore k = 138$$

EG
+S

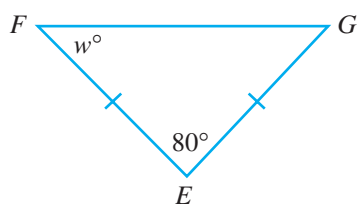
Example 2

Find the value of the pronumeral in each of these, giving reasons.

a



b



Solutions

a $\angle QRP = 72^\circ$ (base angles of an isosceles Δ , $PQ = QR$)

$$\therefore m + 72 + 72 = 180 \text{ (angle sum of a } \Delta)$$

$$m + 144 = 180$$

$$\therefore m = 36$$

b

$\angle EGF = w^\circ$ (base angles of an isosceles Δ , $EF = EG$)

$$\therefore w + w + 80 = 180 \text{ (angle sum of a } \Delta)$$

$$2w + 80 = 180$$

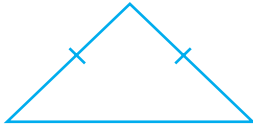
$$2w = 100$$

$$\therefore w = 50$$

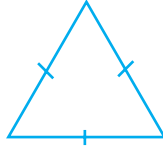
Exercise 10.3

1 Classify each triangle as scalene, isosceles or equilateral.

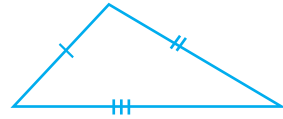
a



b

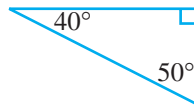


c

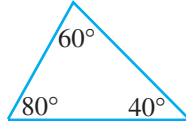


2 Classify each triangle as acute-angled, right-angled or obtuse-angled.

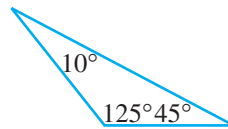
a



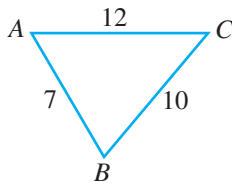
b



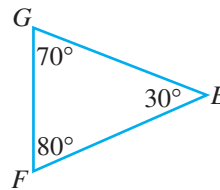
c



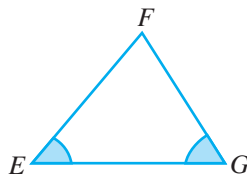
3 a State the **i** smallest angle and **ii** largest angle.



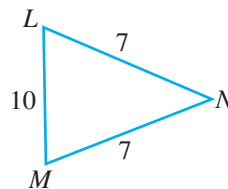
b State the **i** shortest side and **ii** longest side.



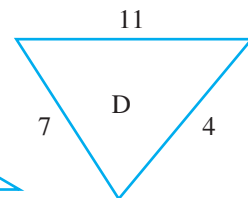
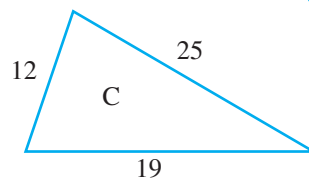
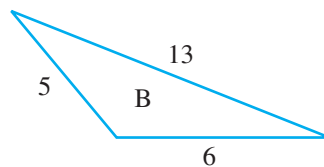
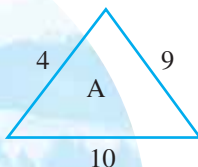
4 a Name the equal sides in this isosceles triangle.



b Name the equal angles in this isosceles triangle.



5 Which two of these triangles could not possibly exist? Why?

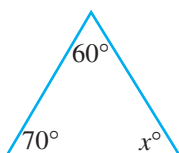


6 Is it possible for:

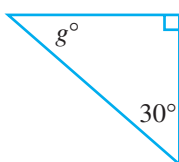
- a a triangle to have two right angles?
- b an equilateral triangle to have an obtuse angle?
- c an isosceles triangle to be right-angled?
- d an obtuse-angled triangle to be isosceles?
- e a scalene triangle to be right-angled?
- f a triangle to have two obtuse angles?

7 Find the value of the pronumeral in each of these, giving brief reasons.

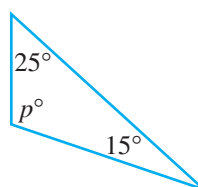
a



b

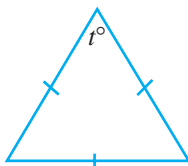


c

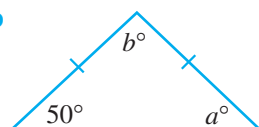


8 Find the value of the pronumerals in each of these, giving reasons.

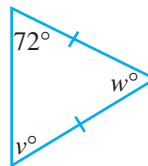
a



b

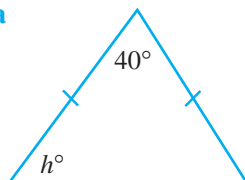


c

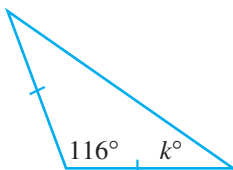


9 Find the value of each pronumeral. (Do not give reasons.)

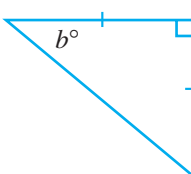
a



b



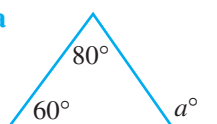
c



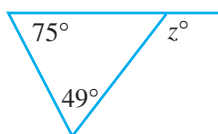
Consolidation

10 Use the exterior angle property to find the value of the pronumeral in each of these, giving brief reasons.

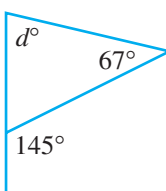
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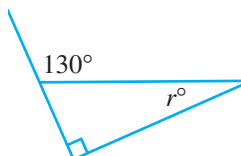
b



c

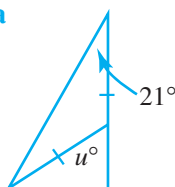


d

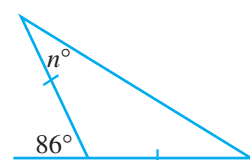


11 Use the exterior angle property to find the value of each pronumeral. (Do not give reasons.)

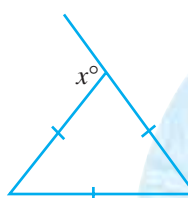
a



b

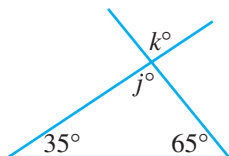


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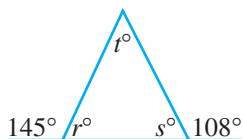


12 Find the value of the pronumeral in each of the following, giving reasons.

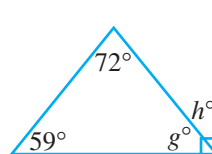
a



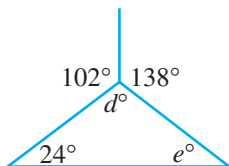
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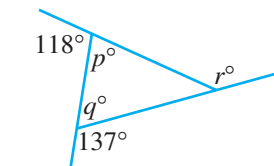
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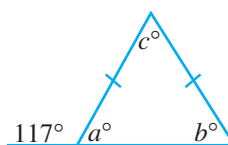
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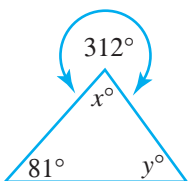
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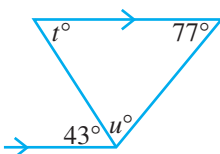
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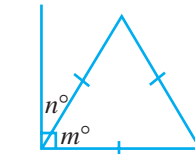
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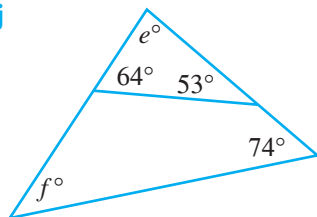
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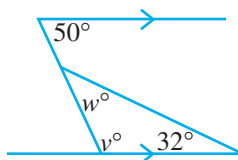
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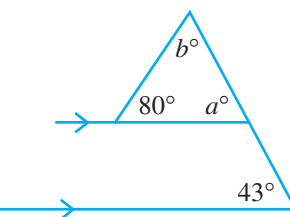
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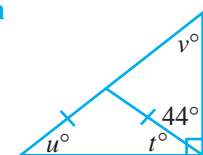
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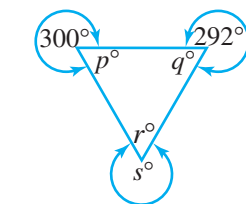
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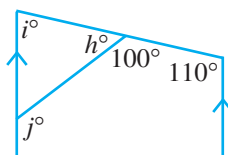
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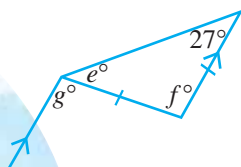
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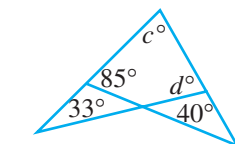
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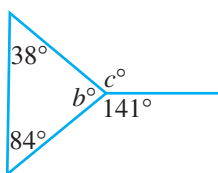
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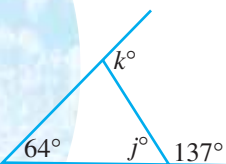
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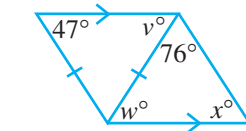
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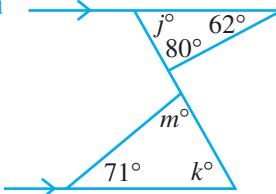
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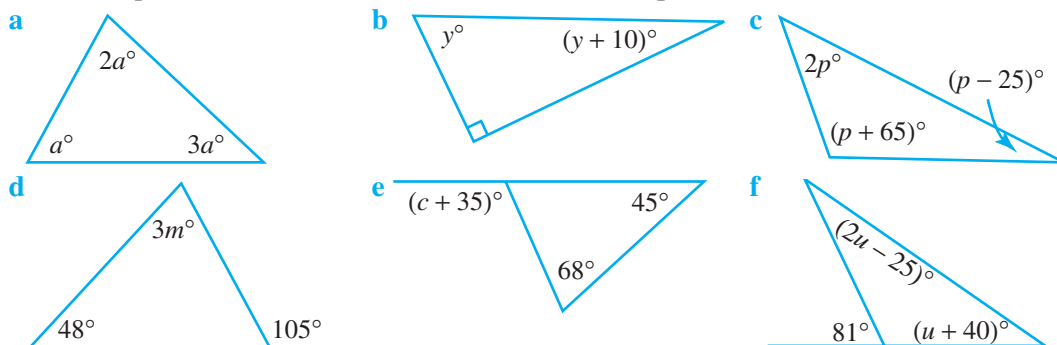
t



u

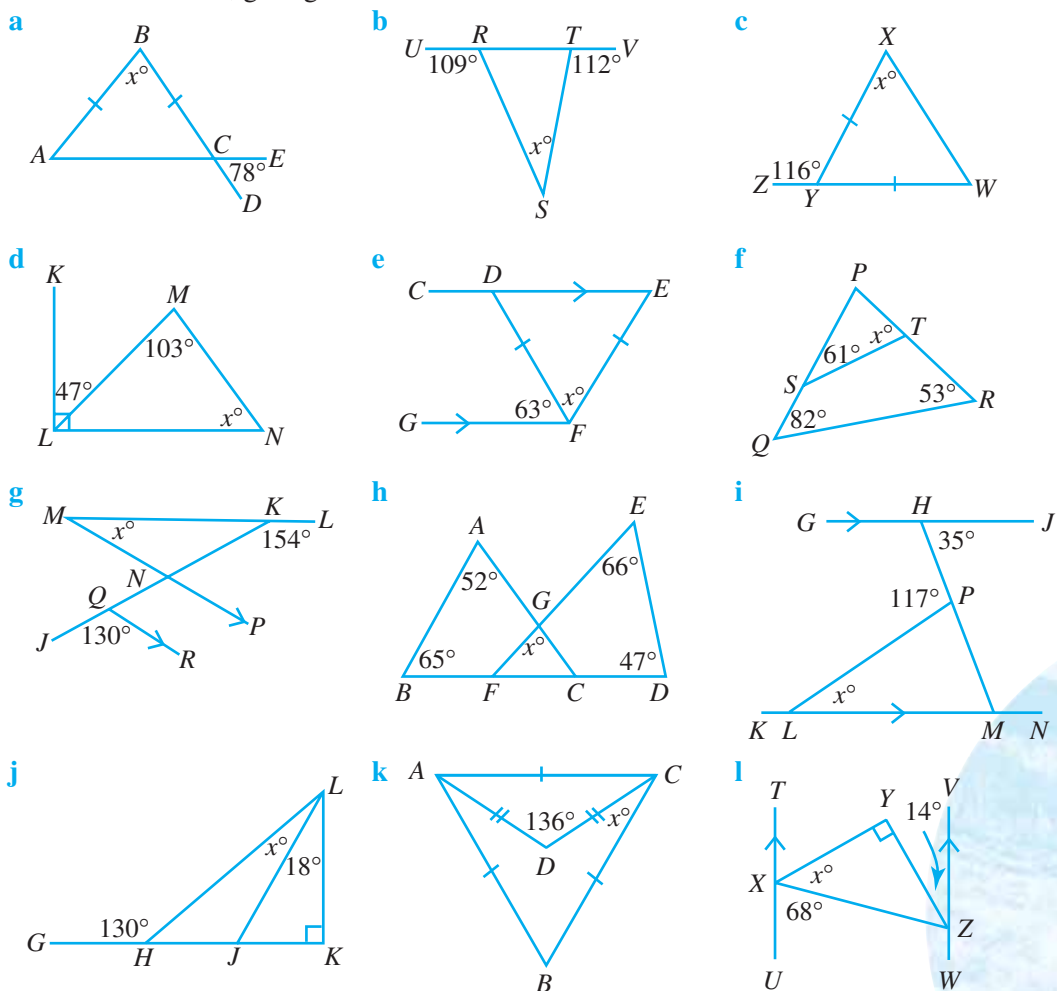


13 Form an equation and solve it to find the value of each pronumeral. Give brief reasons.

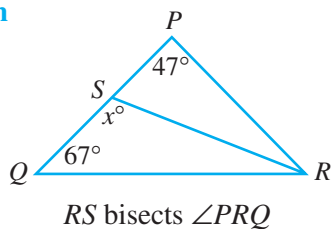


Further applications

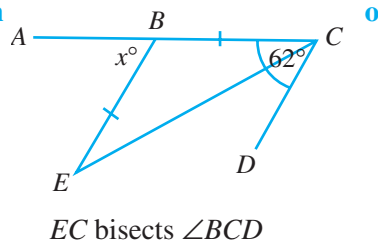
14 Find the value of x , giving reasons.



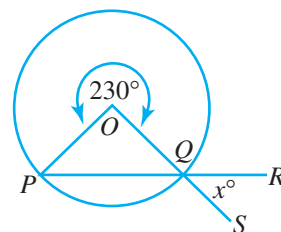
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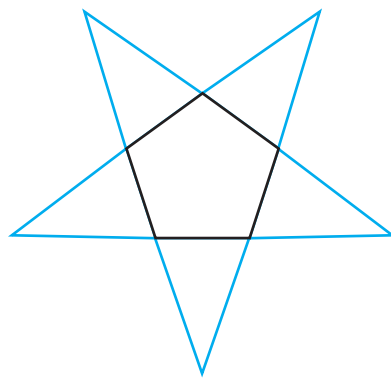


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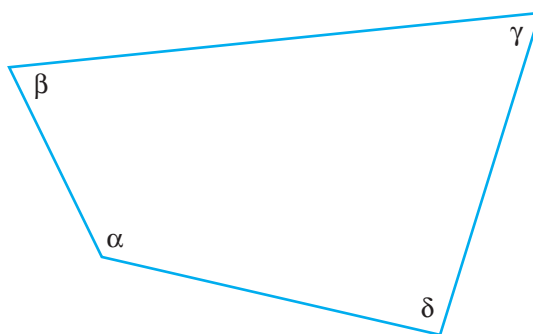
**TRY THIS****The badge of the Pythagoreans**

Pythagoras (c. 580–500 BC) formed a secret society among his followers for the study of mathematics. The penalty for revealing secrets of the society was death. The Pythagoreans had their own special sign—by extending the sides of a regular pentagon, a five-pointed star was formed. This was believed to have magical mathematical properties.

What is the size of the angles at the points of the star?

**10.4 Angle sum of a quadrilateral**

The angle sum of a quadrilateral is 360° .

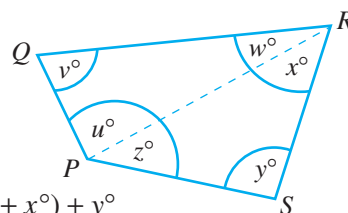


That is, $\alpha + \beta + \gamma + \delta = 360^\circ$.

- Proof:
- Construct the diagonal PR .
 - $u^\circ + v^\circ + w^\circ = 180^\circ$ (angle sum of $\triangle PQR$)
 - $x^\circ + y^\circ + z^\circ = 180^\circ$ (angle sum of $\triangle RSP$)

$$\begin{aligned}
 \text{Now, } \angle P + \angle Q + \angle R + \angle S &= (u^\circ + z^\circ) + v^\circ + (w^\circ + x^\circ) + y^\circ \\
 &= (u^\circ + v^\circ + w^\circ) + (x^\circ + y^\circ + z^\circ) \\
 &= 180^\circ + 180^\circ \\
 &= 360^\circ
 \end{aligned}$$

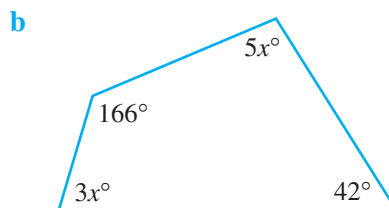
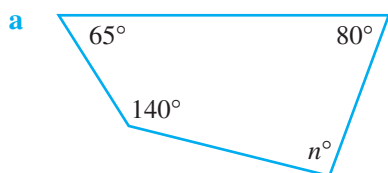
$\therefore$ The angle sum of the quadrilateral is 360° .



EG
+
S

Example

Find the value of the pronumeral in each of the following, giving reasons.



Solutions

- a** $n + 140 + 65 + 80 = 360$ (angle sum of a quadrilateral)

$$n + 285 = 360$$

$$\therefore n = 75$$

- b** $3x + 166 + 5x + 42 = 360$ (angle sum of a quadrilateral)

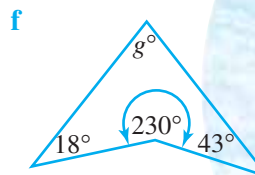
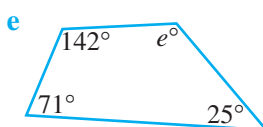
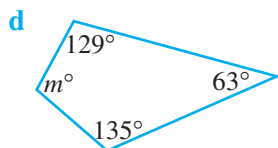
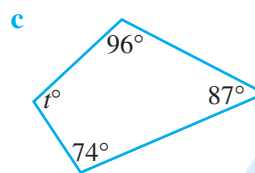
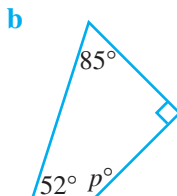
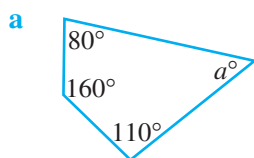
$$8x + 208 = 360$$

$$8x = 152$$

$$\therefore x = 19$$

Exercise 10.4

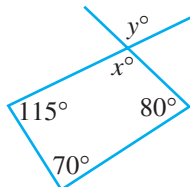
- 1 Find the value of the pronumeral in each of these, giving brief reasons.



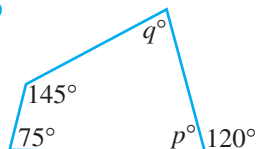
Consolidation

2 Find the values of all pronumerals in each of the following, giving brief reasons.

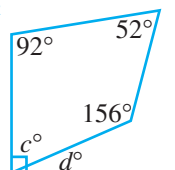
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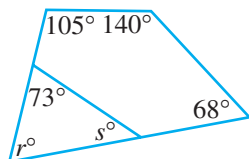
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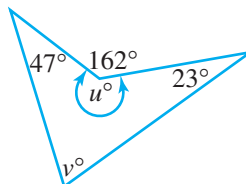
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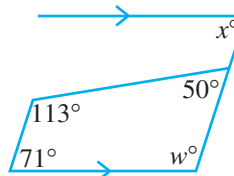
d



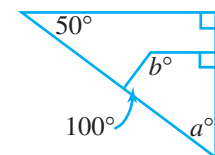
e



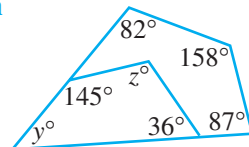
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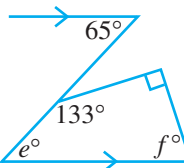
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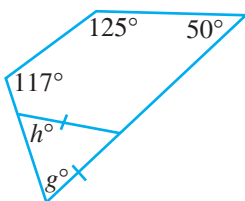
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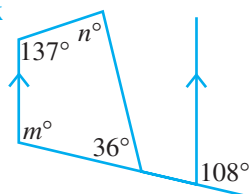
i



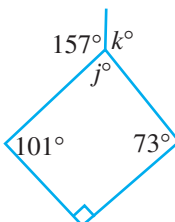
j



k

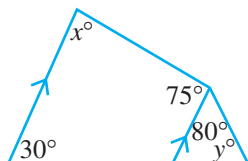


l

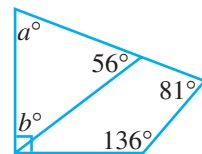


3 Find the values of all pronumerals in each of these.

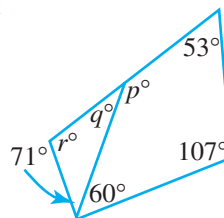
a



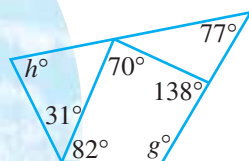
b



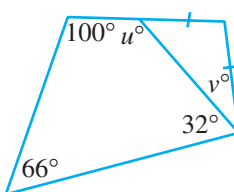
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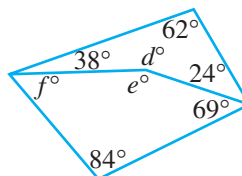
d



e

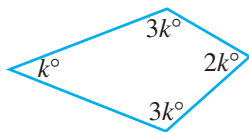


f

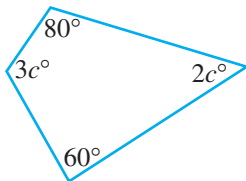


4 Form an equation and solve it to find the value of the pronumeral in each of these.

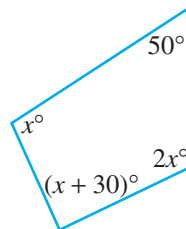
a



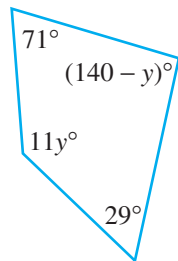
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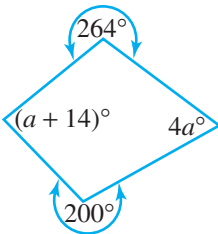
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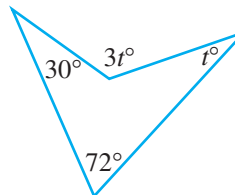
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e



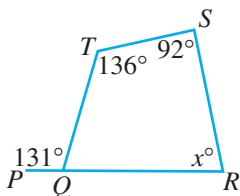
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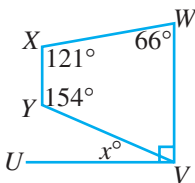
Further applications

5 Find the value of x , giving reasons.

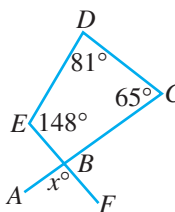
a



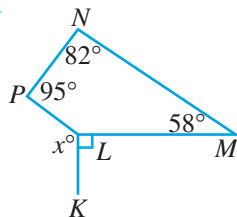
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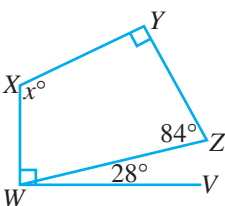
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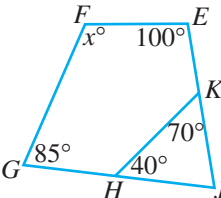
d



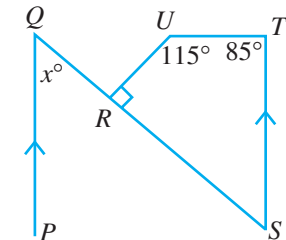
e



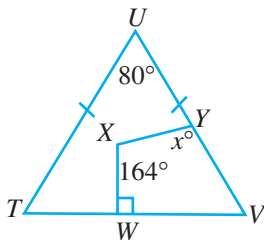
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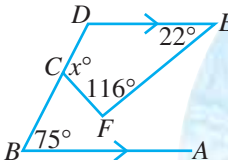
g



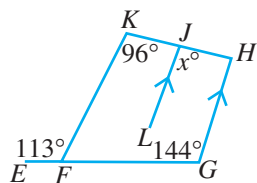
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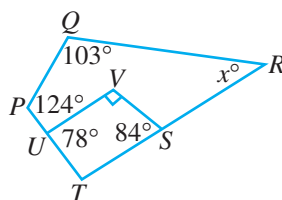
i



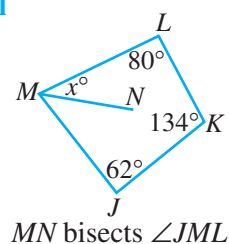
j



k



l



10.5 Special quadrilaterals

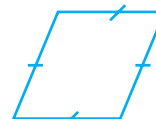
The diagrams below show the classic representations of the special quadrilaterals. However, we should not rely totally on these stereotypes. We will now define these quadrilaterals more clearly, look at their properties and consider questions such as ‘Is a square a rhombus?’.



Trapezium



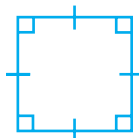
Parallelogram



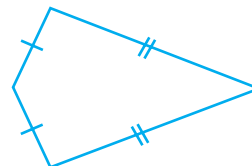
Rhombus



Rectangle



Square



Kite

Definitions

- ⊙ A trapezium is a quadrilateral with at least one pair of opposite sides parallel.
- ⊙ A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
- ⊙ A rhombus is a parallelogram with two adjacent sides equal in length.
- ⊙ A rectangle is a parallelogram in which one angle is a right angle.
- ⊙ A square is a rectangle with two adjacent sides equal in length.
- ⊙ A kite is a quadrilateral with two pairs of adjacent sides equal in length.

☉ The angle sum of a quadrilateral is 360° .

Properties of a parallelogram:

- ☉ opposite sides are parallel
- ☉ opposite sides are equal
- ☉ opposite angles are equal
- ☉ diagonals bisect each other

Properties of a rectangle:

- ☉ all properties of a parallelogram
- ☉ all angles are right angles
- ☉ diagonals are equal

Properties of a trapezium:

- ☉ one pair of opposite sides are parallel

Properties of a rhombus:

- ☉ all properties of a parallelogram
- ☉ all sides are equal
- ☉ diagonals are perpendicular
- ☉ diagonals bisect the angles at the vertices

Properties of a square:

- ☉ all properties of a rectangle
- ☉ all sides are equal
- ☉ diagonals bisect the angles at the vertices
- ☉ diagonals are perpendicular

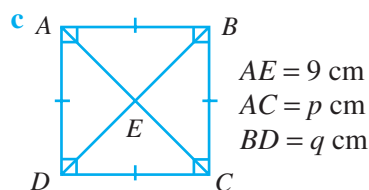
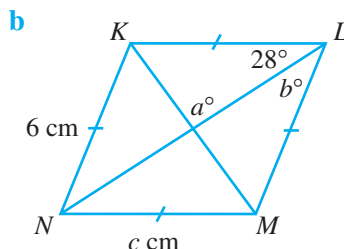
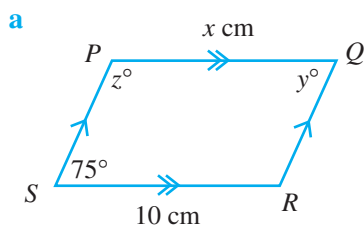
Properties of a kite:

- ☉ two pairs of adjacent sides are equal

Think about the question ‘Is a square a rhombus?’. One of the tests for a rhombus is that it must have all sides equal. We know that in a square all sides are equal. Therefore, a square is a rhombus because it meets one of the tests for a rhombus. This means that the square has all of the properties of a rhombus, and some other properties as well. If a square is a rhombus, must a rhombus be a square?

Example

Find the values of all pronumerals in each of the following, giving reasons.



Solutions

- a**
- i $x = 10$ (opposite sides of a parallelogram are equal)
 - ii $y = 75$ (opposite angles of a parallelogram are equal)
 - iii $z + 75 = 180$ (co-interior angles, $PQ \parallel SR$)
 $\therefore z = 105$
- b**
- i $a = 90$ (diagonals of a rhombus are perpendicular)
 - ii $b = 28$ (diagonals of a rhombus bisect the angles at the vertices)
 - iii $c = 6$ (sides of a rhombus are equal)
- c**
- i $p = 18$ (diagonals of a square bisect each other)
 - ii $q = 18$ (diagonals of a square are equal)

Exercise 10.5

1 Complete this table using ticks to show which of the properties apply to each quadrilateral.

	Square	Rectangle	Parallelogram	Rhombus	Trapezium	Kite
All sides are equal						
Opposite sides are equal						
All angles are right angles						
Opposite angles are equal						
Opposite sides are parallel						
Equal diagonals						
Diagonals bisect each other						
Diagonals are perpendicular						
Diagonals bisect the angles at the vertices						

2 How many axes of symmetry has a:

a square?

b rectangle?

c parallelogram?

d rhombus?

e trapezium?

f kite?

Consolidation

3 Write true (T) or false (F) for each of these.

a a square is a rectangle

b a rectangle is a square

c a rhombus is a parallelogram

d a square is a rhombus

e a trapezium is a parallelogram

f a parallelogram is a trapezium

g a rectangle is a parallelogram

h a rhombus is a square

i a parallelogram is a rhombus

j a rhombus is a trapezium

4 Classify each of the following quadrilaterals giving brief reasons. (Diagrams are not drawn to scale.)

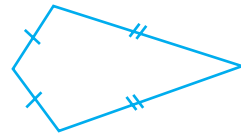
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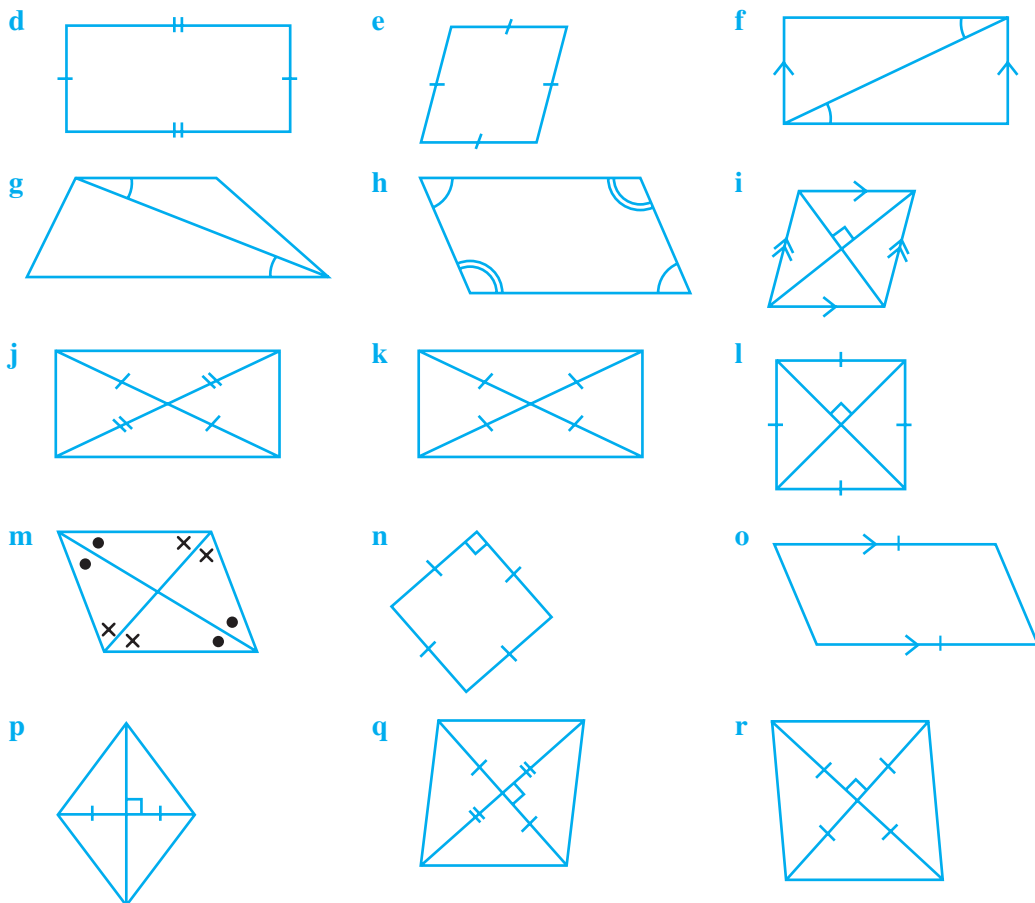


b

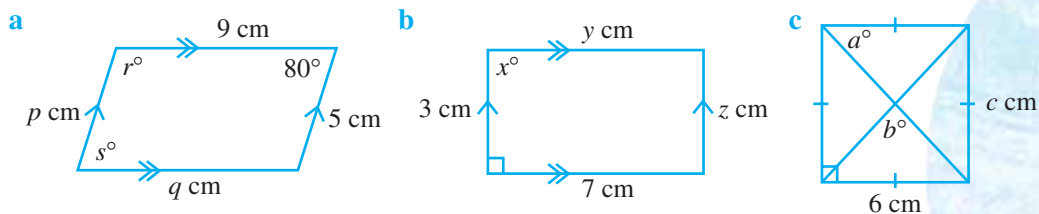


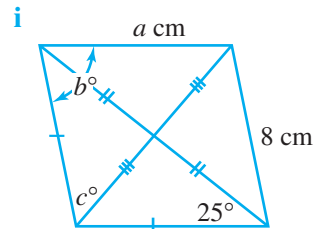
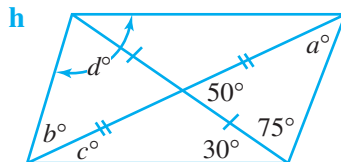
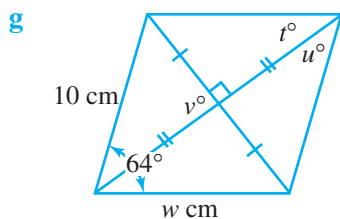
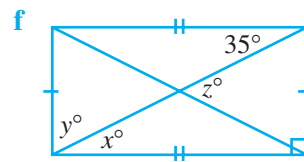
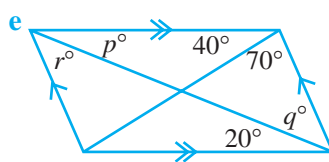
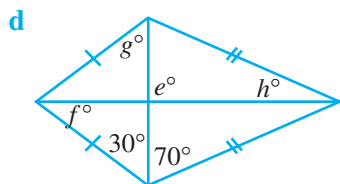
c



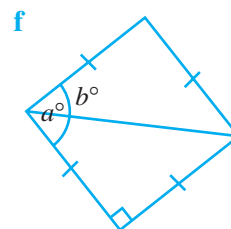
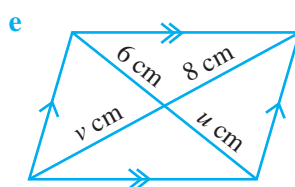
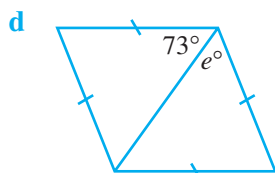
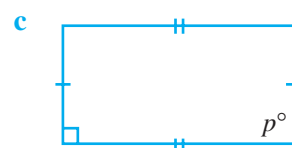
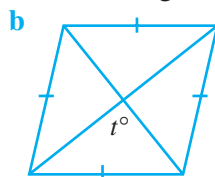
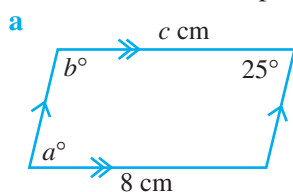


- 5** In each of the following, draw a diagram and use it to find the required lengths.
- The diagonals of the rectangle $ABCD$ intersect at E . If $AC = 12$ cm, find the length of BD and BE .
 - The diagonals of the parallelogram $PQRS$ intersect at T . If $PT = 10$ cm, find the length of TR and PR .
 - The diagonals of the rhombus $JKLM$ intersect at N . If $JL = 18$ cm and $KN = 11$ cm, find the length of JN and KM .
- 6** Classify each of these quadrilaterals. Hence, find the value of all pronumerals.

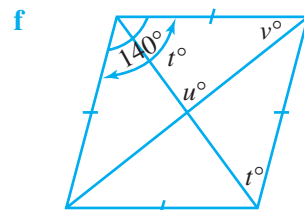
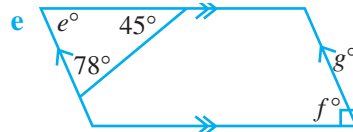
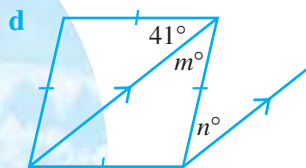
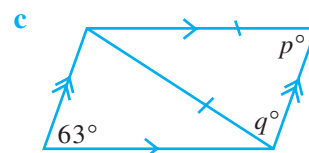
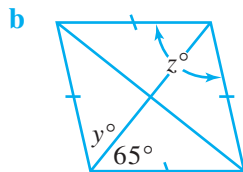
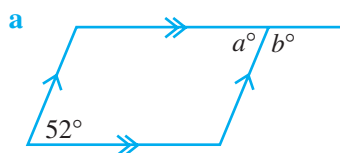


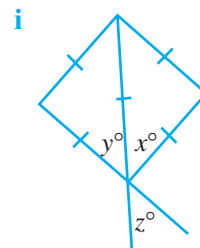
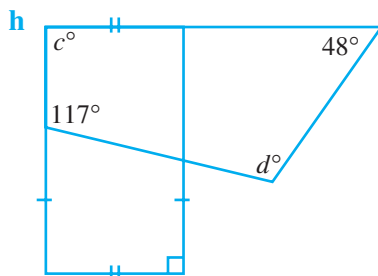
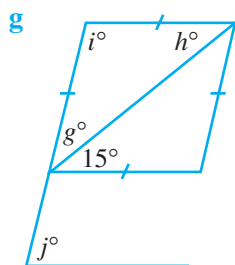


7 Find the value of the pronumerals in each figure, giving reasons.

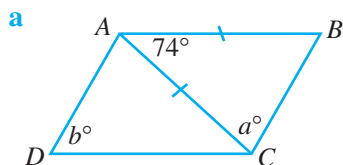


8 Find the value of the pronumerals in each figure, giving reasons.

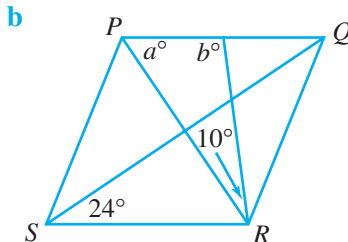




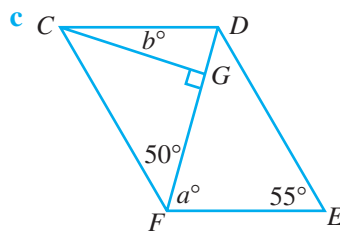
9 Find the values of a and b in each of the following. (Do not give reasons.)



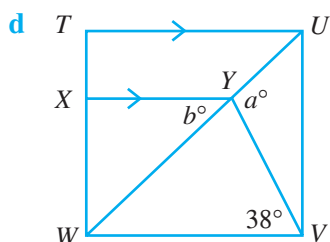
$ABCD$ is a parallelogram,
 $AB = AC$.



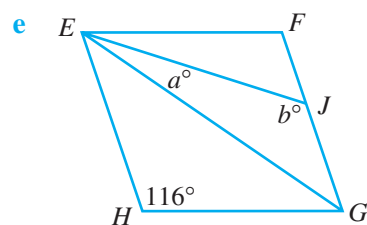
$PQRS$ is a rhombus.



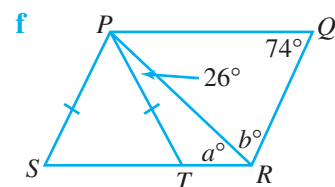
$CDEF$ is a parallelogram,
 $CG \perp FD$.



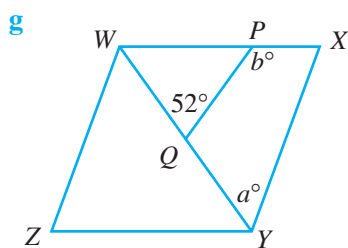
$TUVW$ is a square,
 $TU \parallel XY$.



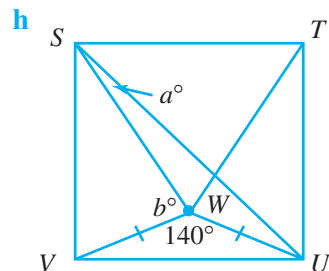
$EFGH$ is a rhombus,
 EJ bisects $\angle FEG$.



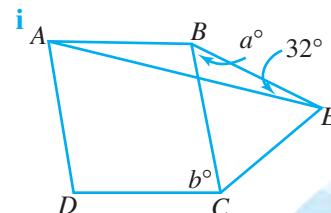
$PQRS$ is a parallelogram,
 $PS = PT$.



$WXYZ$ is a rhombus,
 $WX = WY$.



$STUV$ is a square, $VW = UW$,
 $\triangle SWT$ is equilateral.

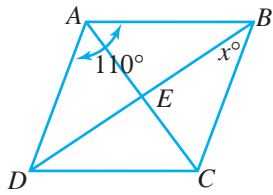


$ABCD$ is a rhombus,
 $\triangle BEC$ is equilateral.

Further applications

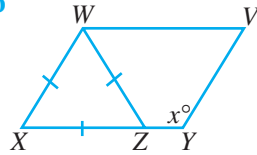
10 Find the value of x , giving reasons.

a



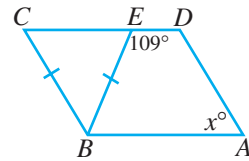
$ABCD$ is a rhombus.

b



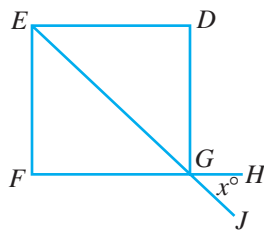
$VWXY$ is a parallelogram,
 ΔWXZ is equilateral.

c



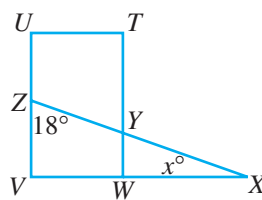
$ABCD$ is a parallelogram,
 $BC = BE$.

d



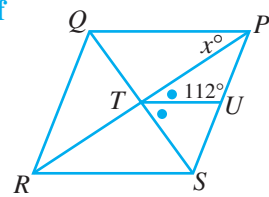
$DEFG$ is a square.

e



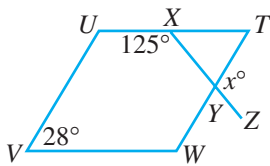
$TUVW$ is a rectangle.

f



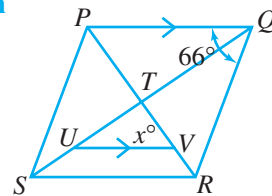
$PQRS$ is a rhombus,
 TU bisects $\angle PTS$.

g



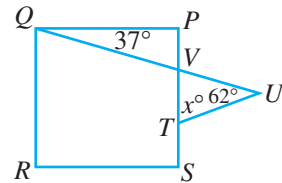
$TUVW$ is a parallelogram.

h



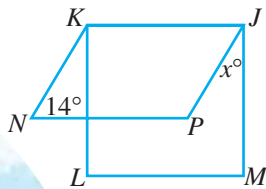
$PQRS$ is a rhombus,
 $UV \parallel PQ$.

i



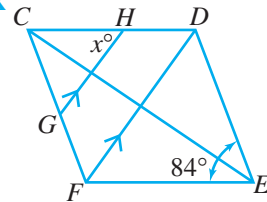
$PQRS$ is a square.

j



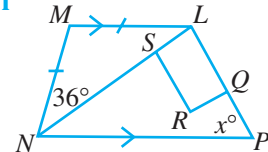
$JKLM$ is a square,
 $JKNP$ is a parallelogram.

k



$CDEF$ is a rhombus,
 $GH \parallel FD$.

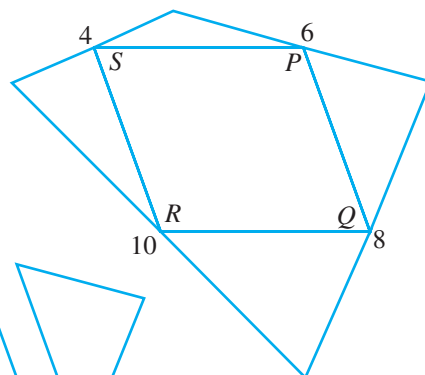
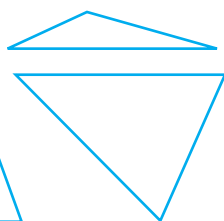
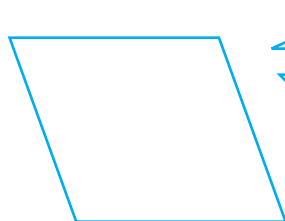
l



$QRSL$ is a rectangle,
 $ML = MN$, $ML \parallel NP$.

TRY THIS Five shapes

Draw an irregular quadrilateral with sides 4, 6, 8 and 10 cm long. Join the midpoints of the sides $PQRS$, and cut out the five shapes formed. Can you arrange the four triangles to cover the large central piece exactly? What shape is it?



10.6 Polygons

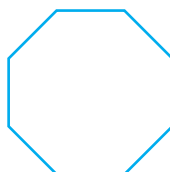
Common polygons

A **polygon** is a closed figure bounded by only straight sides. The name of a polygon is based on the number of sides that make up the boundary of the figure. The names of the first 10 polygons are shown below.

Sides	Polygon	Sides	Polygon
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	11	Undecagon
7	Heptagon	12	Dodecagon

Convex and non-convex polygons

A polygon can be convex or non-convex. A **convex polygon** is a polygon in which all of the diagonals lie within the figure. All interior angles are less than 180° . A **non-convex polygon** is a polygon in which at least one diagonal does not lie completely within the figure. One or more interior angles is greater than 180° .



Convex polygon



Non-convex polygon

The interior angle sum of a polygon

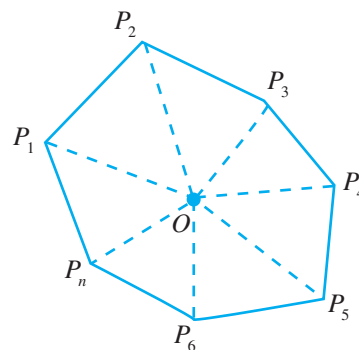
The sum S of the interior angles of any n -sided polygon is given by $S = (n - 2) \times 180^\circ$.

Proof: Let $P_1P_2P_3P_4\dots P_n$ be a convex polygon with n sides. Choose any point O inside the polygon and join it to each of the vertices, forming n triangles.

The angle sum of each triangle is 180° , therefore, the sum of the angles in n triangles is $180n^\circ$. However, this includes the angles around O whose sum is 360° . These angles must be subtracted from the angles around the boundary of the polygon to give the interior angle sum S .

$$\begin{aligned}\text{Hence, } S &= 180n^\circ - 360^\circ \\ &= 180^\circ(n - 2), \text{ on factorising}\end{aligned}$$

An alternative proof of this result is developed in Q8–10 of the following exercise.



The exterior angle sum of a polygon

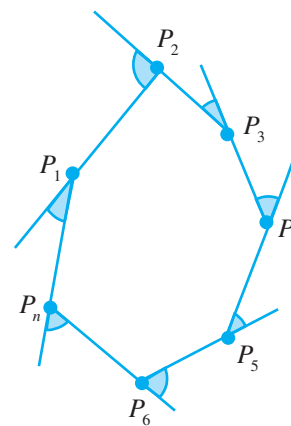
When one side of a polygon is produced, the angle between this produced side and an adjacent side of the polygon is called an exterior angle of the polygon.

The sum of the exterior angles of any convex polygon is 360° .

Proof: Let $P_1P_2P_3P_4\dots P_n$ be a convex polygon with n sides. If each side of the polygon is produced as shown, then the sum of the interior and exterior angles at each vertex is 180° . As there are n interior angles and n exterior angles, the total sum of these angles is $180n^\circ$.

We know that the sum of the interior angles is $180^\circ(n - 2)$. The interior angles must be subtracted from the total angle sum to find S , the sum of the exterior angles of the polygon.

$$\begin{aligned}\text{Hence, } S &= 180n^\circ - 180^\circ(n - 2) \\ &= 180n^\circ - 180n^\circ + 360^\circ \\ &= 360^\circ\end{aligned}$$



Regular polygons

A **regular polygon** is a polygon in which all of the sides are equal and all of the angles are equal. The size of the interior and exterior angles in a regular polygon can be found by dividing the sum of these angles by the number of angles.

In any regular n -sided convex polygon:

- ⊙ each interior angle measures $\frac{180^\circ(n-2)}{n}$
- ⊙ each exterior angle measures $\frac{360^\circ}{n}$.

EG
+S

Example 1

$ABCDE$ is a regular pentagon. Find:

- a the angle sum of the pentagon
- b the size of each interior angle
- c the size of each exterior angle

Solutions

$$\begin{aligned} \text{a } S &= 180^\circ(n-2) \\ &= 180^\circ(5-2) \\ &= 180^\circ \times 3 \\ &= 540^\circ \end{aligned}$$

$$\begin{aligned} \text{b Interior angles} &= \frac{540^\circ}{5} \\ &= 108^\circ \end{aligned}$$

$$\begin{aligned} \text{c Exterior angles} &= \frac{360^\circ}{5} \\ &= 72^\circ \end{aligned}$$

EG
+S

Example 2

A regular polygon has exterior angles of 40° . Calculate:

- a the size of the interior angles
- b the number of sides

Solutions

$$\begin{aligned} \text{a Interior angle} &= 180^\circ - \text{exterior angle} \\ &\quad (\text{angles on a straight line}) \\ &= 180^\circ - 40^\circ \\ &= 140^\circ \end{aligned}$$

$$\begin{aligned} \text{b Exterior angle} &= \frac{360^\circ}{n}, \text{ where } n \text{ is the number of sides} \\ 40 &= \frac{360}{n} \\ 40n &= 360 \\ \therefore n &= 9 \\ \therefore \text{The polygon has 9 sides.} \end{aligned}$$

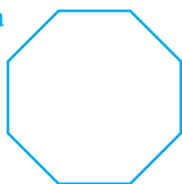
Exercise 10.6

1 Name the polygon that has the following number of sides.

a 3	b 4	c 5	d 6	e 7
f 8	g 9	h 10	i 11	j 12

2 State whether each polygon is convex or non-convex.

a



b



c



d

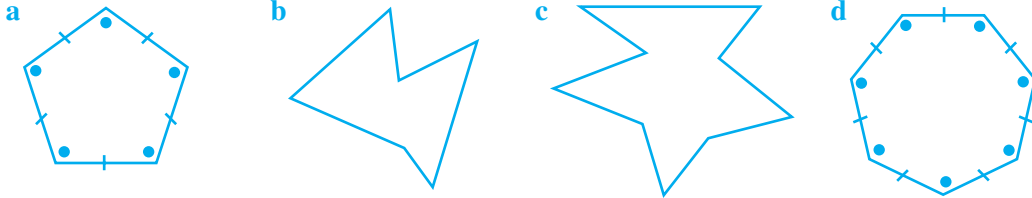


3 Draw each of the following:

- a** a convex hexagon **b** a convex octagon
c a non-convex quadrilateral **d** a non-convex pentagon

4 Can a triangle be non-convex? Explain.

5 Give a two-word name for each figure, including whether it is regular or irregular.



6 What name is given to a regular:

- a** 3-sided polygon? **b** 4-sided polygon?

7 **a** Name a quadrilateral whose:

- i** sides are equal but angles are not **ii** angles are equal but sides are not
b Are the figures in **a** regular?
c If the sides are equal in a polygon, does this mean that the angles must be equal?
d If the angles are equal in a polygon, does this mean that the sides must be equal?

Consolidation

8 **a** Draw a convex pentagon $ABCDE$ and divide it into triangles by drawing a line from one vertex to each of the other vertices.

- b** How many triangles are formed?
c Hence, find the angle sum of the pentagon.
d Would the angle sum be different if the figure was non-convex?

9 Use the method outlined in Q8 to find the angle sum of:

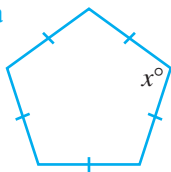
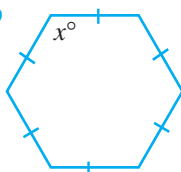
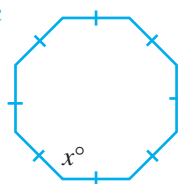
- a** a hexagon **b** a heptagon **c** an octagon

10 **a** Copy and complete this table, using your results from Q8 and Q9.

- b** How many triangles would be formed in a polygon that has n sides?
c By how much must the number of triangles be multiplied to give the angle sum?
d Write down a formula that could be used to find the angle sum of any n -sided polygon.

Number of sides	Number of triangles	Angle sum
3	1	180°
4	2	360°
5		
6		
7		
8		

- 11** Use your formula from Q10 to find the angle sum of each of these regular polygons. Hence, find the value of x .

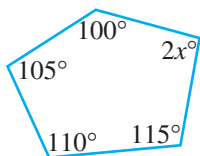
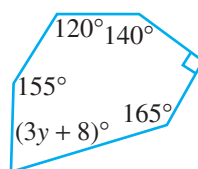
a**b****c**

- 12** Find the size of the interior angles in a regular:

a nonagon**b** decagon**c** dodecagon

- 13** Calculate the size of the interior angles in a regular polygon that has 15 sides.

- 14** Form an equation and solve it to find the value of the pronumeral in each polygon.

a**b**

- 15** Use the formula $\theta = \frac{360^\circ}{n}$ to find the size of each exterior angle θ in a regular:

a pentagon**b** octagon**c** decagon**d** hexagon**e** nonagon**f** dodecagon

- 16** Find the size of the exterior angles in a regular polygon that has 15 sides.

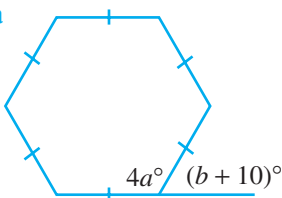
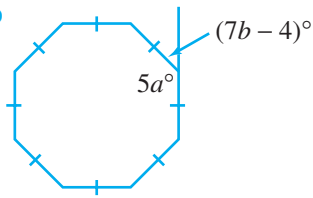
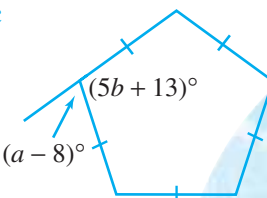
- 17** A regular polygon has 24 sides.

a Find the size of the exterior angles.**b** Hence, find the size of the interior angles.**c** Use your answer in part **b** to find the angle sum of the polygon.

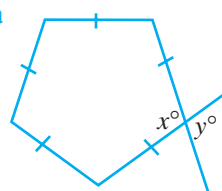
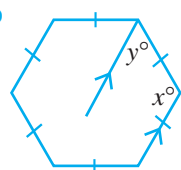
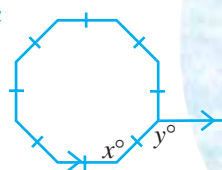
- 18** How many sides are there in a regular polygon whose exterior angles each measure:

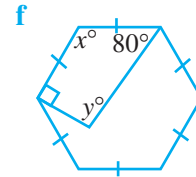
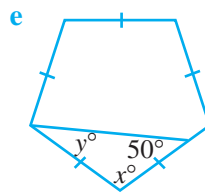
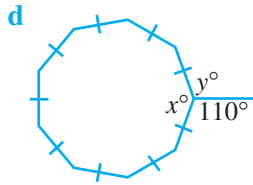
a 120° ?**b** 72° ?**c** 30° ?**d** 12° ?

- 19** Find values for a and b in each of these regular polygons.

a**b****c**

- 20** Find values for x and y , giving reasons. All polygons are regular.

a**b****c**



- 21** Find, correct to the nearest minute, the size of the interior angles in a regular:
- a** heptagon **b** undecagon

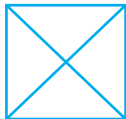
■ Further applications

- 22** Calculate the angle sum of a polygon whose exterior angles are 20° .
- 23** Explain why a regular polygon cannot have interior angles measuring 80° .
- 24** A *tessellation* is a pattern of shapes that fit together exactly so that they completely cover a given area. Which of the regular polygons with up to 12 sides will tessellate? Why?
- 25** Construct a regular pentagon in a circle using a pair of compasses, then measure the size of the interior angles.

TRY THIS

How many diagonals in a polygon?

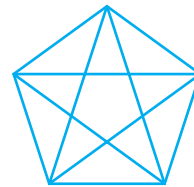
A diagonal is a line that goes from one corner of a figure to an opposite corner.



A square has 2 diagonals.

How many diagonals are there in a pentagon?

Clearly there are 5.



Copy and complete the following table.

Polygon	Sides (S)	Number of diagonals (D)
Square	4	2
Pentagon	5	5
Hexagon	6	
	7	
	8	
	9	

Can you find the pattern?
What is the rule linking S with D ?

TRY THIS An investigation of triangles

Triangles have only three sides and are the simplest of all polygons. As a result, they are frequently used in many aspects of our everyday lives. In addition, all of the other polygons can be subdivided into triangles, and therefore any knowledge of triangles is helpful in dealing with other polygons.

This raises an important question: 'What information do we need in order to completely determine a triangle?' That is, how many measurements do we need in order to be able to construct a particular triangle?

In this investigation, we will test all of the various possibilities in order to establish the *minimum set of conditions* needed to construct a given triangle, or to say that two triangles are identical.

A triangle has three sides and three angles, so at first there appear to be six possible measurements we might need. We remember, however, that the three angles of a triangle have a sum of 180° , so knowing the sizes of two angles is the same as knowing all three. This means that we need only consider the sizes of three sides and two angles.

A number of different possibilities need to be considered:

One measurement	• Just one angle • Just one side
Two measurements	• Two sides • One angle and one side
Three measurements	• Three sides • Two angles (effectively three angles) • Two sides and one angle, with the known angle <i>between</i> the two sides • Two sides and one angle, with the known angle not between the two sides
Four measurements	• Three sides and one angle • Two angles and one side (effectively three angles and one side)
Five measurements	• Three sides and two angles (effectively three sides and three angles)

It seems clear that the first two in the list above, 'just one angle' and 'just one side', do not provide enough information. There is, therefore, no need to test them.

Now let us test the other possibilities.

For this investigation you will need to work in pairs. One person in each pair should be *A*, and the other *B*. (You may decide to swap roles between cases.) Each pair should test two or three possibilities and report their findings to the class.

- Step 1* A draws any triangle without showing it to B.
- Step 2* A then measures the required number of sides and/or angles. Be as accurate as you can with these measurements.
- Step 3* A tells B the lengths of any required sides and sizes of any required angles.
- Step 4* B then tries to construct the triangle, using ruler, protractor, compasses and the measurements provided by A.
- Step 5* A and B then compare triangles and consult to determine whether the particular case provided sufficient information to ensure that an accurate copy of the original triangle was constructed.
- Step 6* After testing several cases, report your findings to the class.

10.7 Tests for congruent triangles

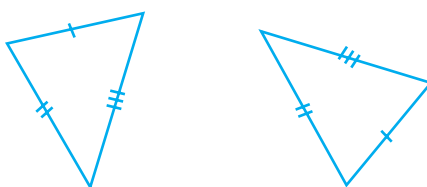
Two triangles are said to be **congruent** if they have exactly the same size and shape. Each triangle can be obtained from the other by performing one or more of the following transformations—translation, rotation or reflection. The sides and angles that are in the same positions relative to other sides and angles are called **matching sides** and **matching angles**. The symbols $\equiv$ and $\cong$ are used to mean ‘is congruent to’.

If two triangles are congruent, then:

- ⊙ the matching sides are equal in length
- ⊙ the matching angles are equal in size
- ⊙ the figures are equal in area.

It is not necessary to know the measurements of every side and every angle in a pair of triangles in order for us to be certain that the triangles are congruent. Having completed the previous investigation, you should have deduced the following four tests for congruent triangles. These tests represent the *minimum amount of information* that is needed to prove that two triangles are congruent.

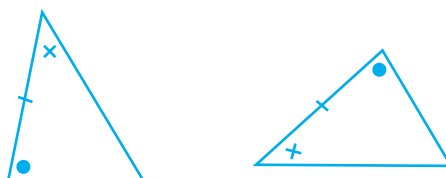
- 1 If the three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent (SSS).**



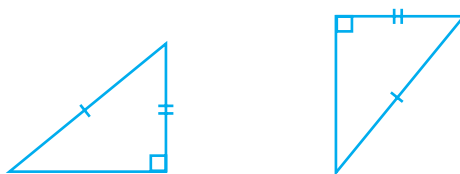
- 2 If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the two triangles are congruent (SAS).



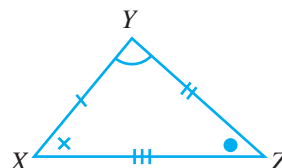
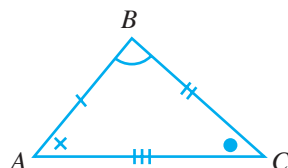
- 3 If two angles and one side of one triangle are equal to two angles and the matching side of another triangle, then the two triangles are congruent (AAS).



- 4 If the hypotenuse and a second side of one right-angled triangle are equal to the hypotenuse and a second side of another right-angled triangle, then the two triangles are congruent (RHS).

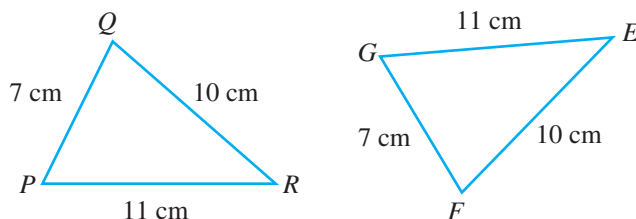


- NOTE: 1 If the three angles of one triangle are equal to the three angles of another triangle, then the triangles are not necessarily congruent. That is, **AAA** is *not* a test for congruent triangles.
- 2 When naming congruent figures, the vertices must be given in matching order. Thus, for the triangles below, we would write $\triangle ABC \equiv \triangle XYZ$.



EG
+
SS**Example 1**

The triangles shown are congruent. Name these congruent triangles, giving the vertices in matching order.

**Solution**

$\angle P$ and $\angle G$ are matching angles (both angles are opposite the 10 cm sides).

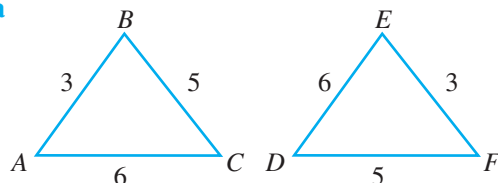
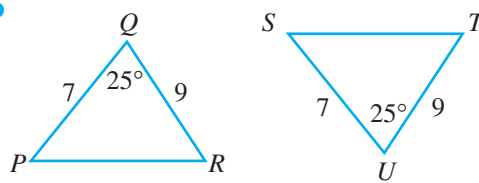
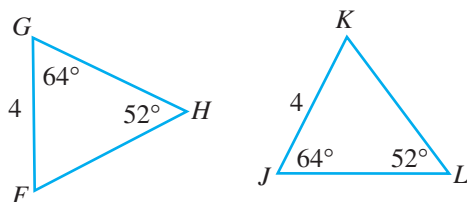
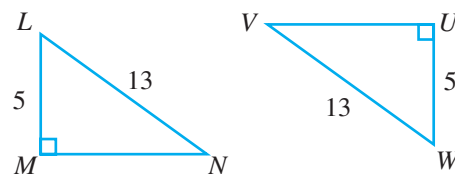
$\angle Q$ and $\angle F$ are matching angles (both angles are opposite the 11 cm sides).

$\angle R$ and $\angle E$ are matching angles (both angles are opposite the 7 cm sides).

$\therefore \triangle PQR \equiv \triangle GFE$.

EG
+
SS**Example 2**

State the test that could be used to prove that the following pairs of triangles are congruent.

a**b****c****d****Solutions**

a $AB = EF = 3$, $BC = DF = 5$, $AC = DE = 6$, $\therefore \triangle ABC \equiv \triangle EFD$ (SSS)

b $PQ = SU = 7$, $\angle PQR = \angle SUT = 25^\circ$, $QR = TU = 9$, $\therefore \triangle PQR \equiv \triangle SUT$ (SAS)

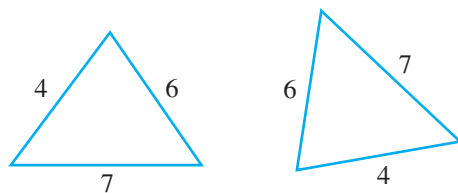
c $\angle FGH = \angle KJL = 64^\circ$, $\angle GHF = \angle JLK = 52^\circ$, $FG = JK = 4$, $\therefore \triangle FGH \equiv \triangle KJL$ (AAS)

d $\angle LMN = \angle VWU = 90^\circ$, $LN = VW = 13$, $LM = UW = 5$, $\therefore \triangle LMN \equiv \triangle VWU$ (RHS)

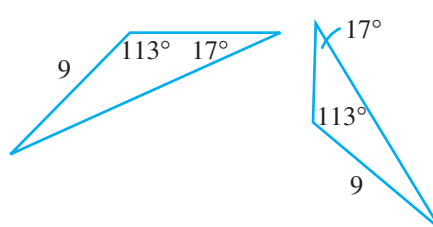
Exercise 10.7

1 State the test that could be used to prove that these pairs of triangles are congruent.

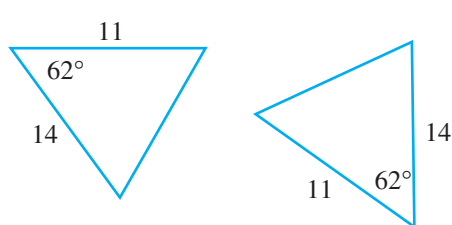
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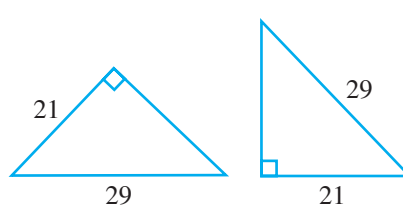
b



c

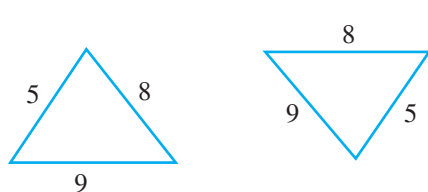


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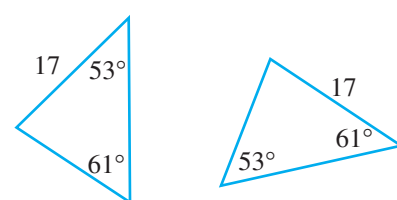


2 State whether each pair of triangles is congruent. If they are congruent, state the test used.

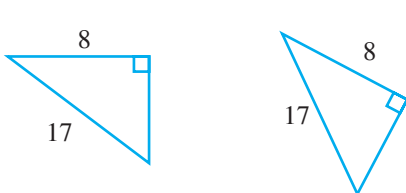
a



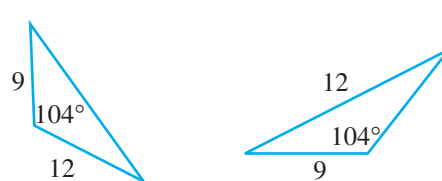
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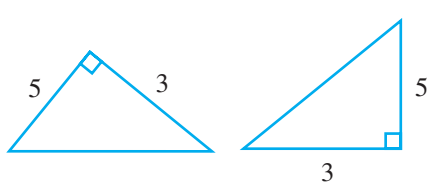
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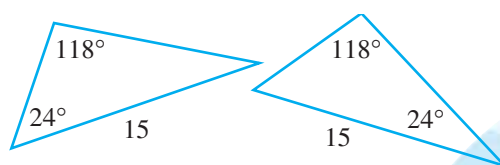
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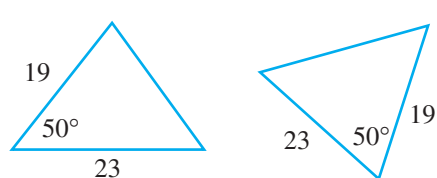
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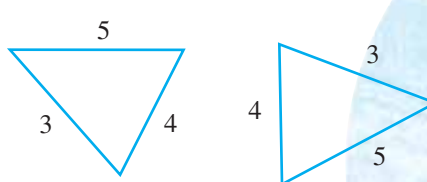
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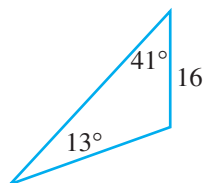
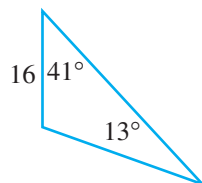
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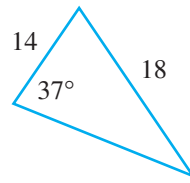
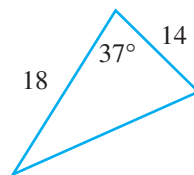
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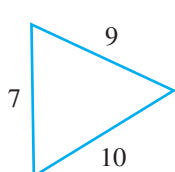
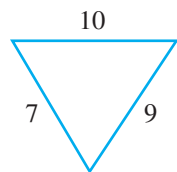
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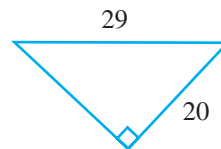
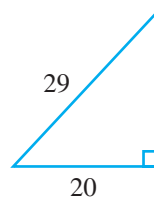
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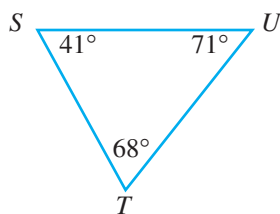
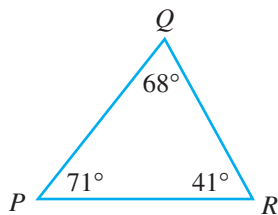


l



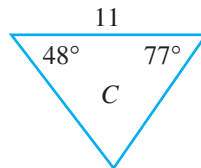
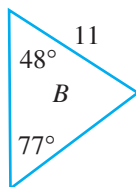
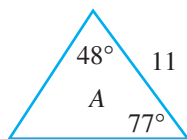
Consolidation

3 Is $\triangle PQR \equiv \triangle UTS$? Why/why not?

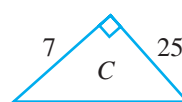
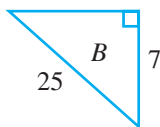
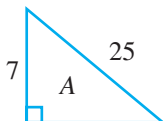


4 Find a pair of congruent triangles in each of these and state the test used.

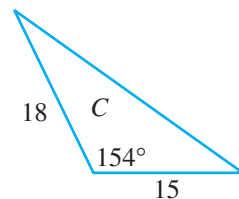
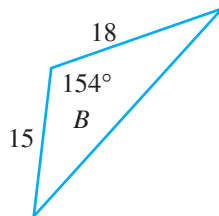
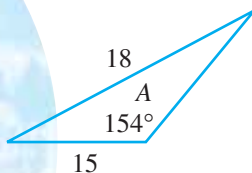
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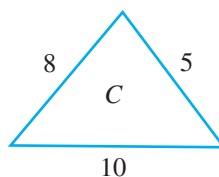
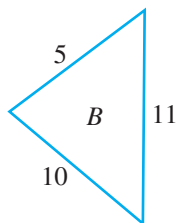
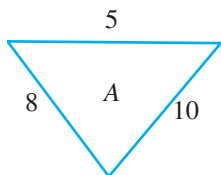
b



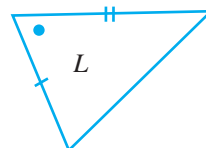
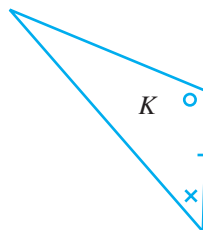
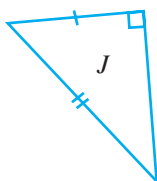
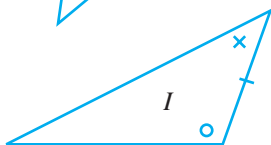
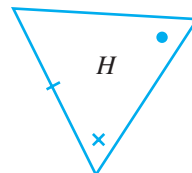
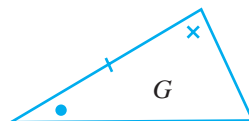
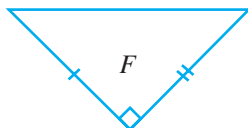
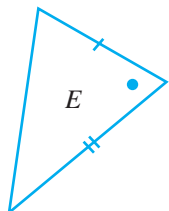
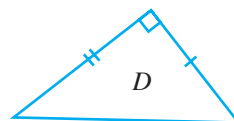
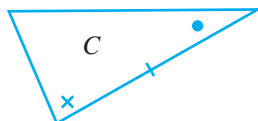
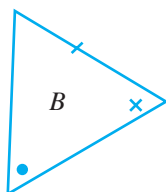
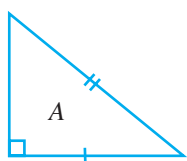
c



d



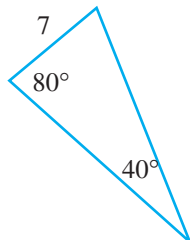
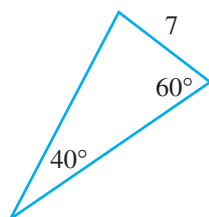
- 5 Find six pairs of congruent triangles from those below and state the test used to justify the congruence.



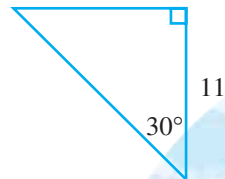
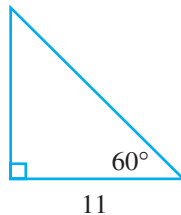
Further applications

- 6 Determine whether the triangles in each pair below are congruent. All lengths are in mm.

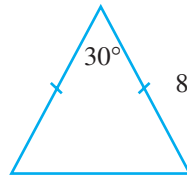
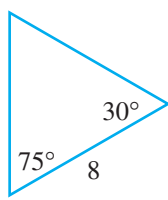
a



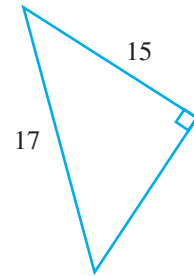
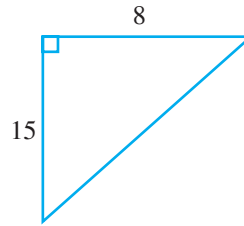
b



c



d



7 In $\triangle ABC$, $\angle A = 56^\circ$, $\angle C = 42^\circ$ and $BC = 8$ cm. Determine whether each of the following triangles is congruent to $\triangle ABC$.

- a In $\triangle PQR$, $\angle Q = 56^\circ$, $\angle R = 42^\circ$, $QR = 8$ cm.
- b In $\triangle XYZ$, $\angle Y = 82^\circ$, $\angle Z = 42^\circ$, $YZ = 8$ cm.
- c In $\triangle LMN$, $\angle L = 82^\circ$, $\angle N = 56^\circ$, $MN = 8$ cm.

10.8 Congruence proofs

The standard congruence proof for triangles has five steps.

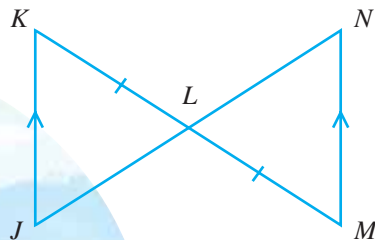
To prove that two triangles are congruent:

- ① identify the triangles that are being used in the proof
- ② name the three pairs of equal sides or angles
- ③ name the congruent triangles, giving the vertices of the triangles in matching order, and state the congruence test used.

NOTE: By convention, the sides or angles on the LHS of the proof should belong to one triangle and the sides or angles on the RHS should belong to the other triangle.

Example 1

EG
+
S



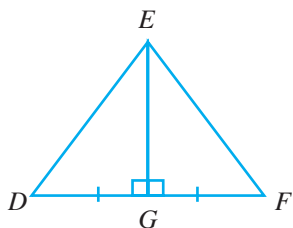
- $KL = LM$
- $JK \parallel MN$

Prove that $\triangle JKL \equiv \triangle NML$.

Solution

In $\triangle JKL$ and $\triangle NML$:

- $KL = LM$ (given)
 - $\angle JKL = \angle LMN$ (alternate angles, $JK \parallel MN$)
 - $\angle KLJ = \angle NLM$ (vertically opposite angles are equal)
- $\therefore \triangle JKL \equiv \triangle NML$ (AAS).

EG
+S**Example 2**

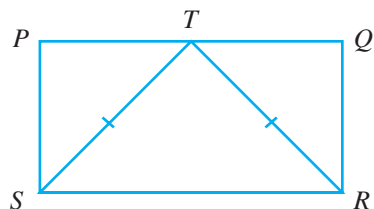
- EG bisects DF
- $EG \perp DF$

Prove that $\triangle EGD \equiv \triangle EGF$.

Solution

In $\triangle EGD$ and $\triangle EGF$:

- $DG = GF$ (EG bisects DF)
 - $\angle EGD = \angle EGF = 90^\circ$ ($EG \perp DF$)
 - EG is a common side
- $\therefore \triangle EGD \equiv \triangle EGF$ (SAS).

EG
+S**Example 3**

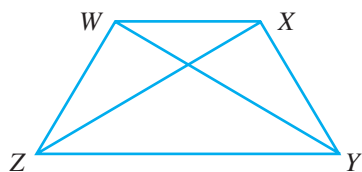
- $PQRS$ is a rectangle
- $TS = TR$

Prove that $\triangle TPS \equiv \triangle TQR$.

Solution

In $\triangle TPS$ and $\triangle TQR$:

- $TS = TR$ (given)
 - $\angle TPS = \angle TQR = 90^\circ$ ($\angle$ s in a rectangle are right angles)
 - $PS = QR$ (opposite sides of a rectangle are equal)
- $\therefore \triangle TPS \equiv \triangle TQR$ (RHS).

EG
+S**Example 4**

- $XZ = WY$
- $WZ = XY$

Prove that $\triangle XWZ \equiv \triangle WXY$.

Solution

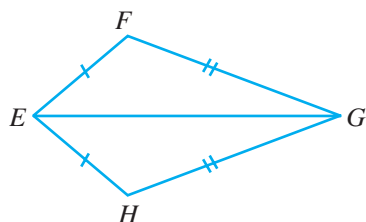
In $\triangle XWZ$ and $\triangle WXY$:

- $XZ = WY$ (given)
 - $WZ = XY$ (given)
 - WX is a common side
- $\therefore \triangle XWZ \equiv \triangle WXY$ (SSS).

Exercise 10.8

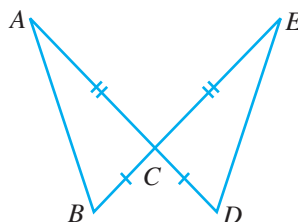
1 Copy and complete each of the following congruence proofs.

a



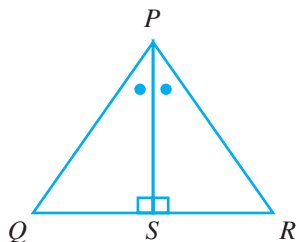
In $\triangle EFG$ and $\triangle EHG$:
 $EF = \dots$ (given)
 $FG = HG$ (.....)
 $\dots$ is a common side
 $\therefore \triangle EFG \equiv \triangle \dots$ (SSS)

b



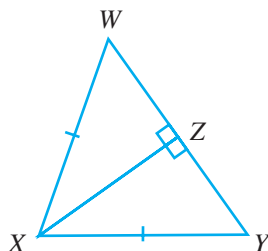
In $\triangle ABC$ and $\triangle EDC$:
 $BC = CD$ (.....)
 $AC = \dots$ (given)
 $\angle ACB = \dots$ (.....)
 $\therefore \triangle ABC \equiv \triangle \dots$ (...)

2 a



$\angle QPS = \angle RPS$ and $PS \perp QR$.
 Prove that $\triangle PQS \equiv \triangle PRS$.

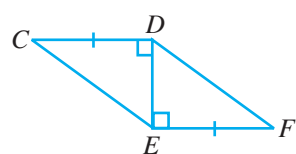
b



$XW = XY$ and $XZ \perp WY$.
 Prove that $\triangle XWZ \equiv \triangle XYZ$.

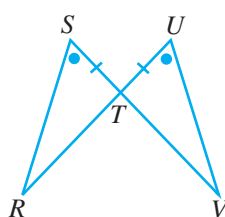
Consolidation

3 a



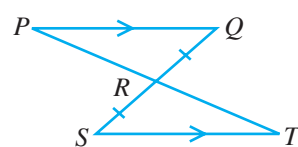
Prove that $\triangle CDE \equiv \triangle FED$.

b

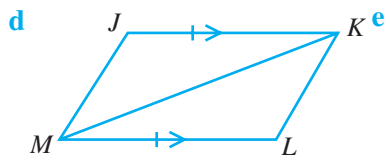


Prove that $\triangle RST \equiv \triangle VUT$.

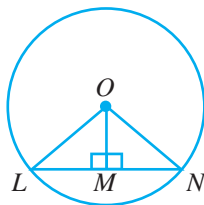
c



Prove that $\triangle PQR \equiv \triangle TSR$.

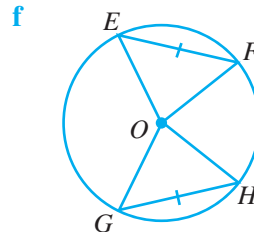


Prove that $\Delta MJK \equiv \Delta KLM$.



O is the centre of the circle.

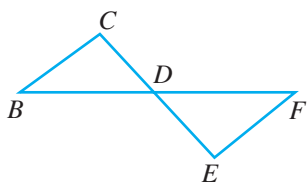
Prove that $\Delta OLM \equiv \Delta ONM$.



O is the centre of the circle.

Prove that $\Delta EOF \equiv \Delta GOH$.

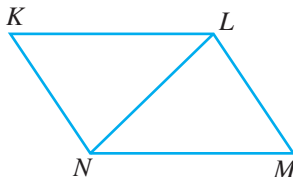
4 a



- CE and BF bisect each other.

Prove that $\Delta BCD \equiv \Delta FED$.

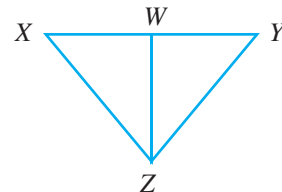
b



- $KLMN$ is a parallelogram.

Prove that $\Delta KLN \equiv \Delta MNL$.

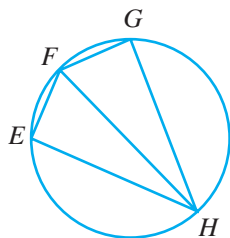
c



- ZW bisects XY
- $ZW \perp XY$

Prove that $\Delta XWZ \equiv \Delta YWZ$.

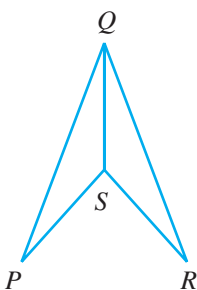
d



- $FE \perp EH$
- $FG \perp GH$
- $EH = GH$

Prove that $\Delta FEH \equiv \Delta FGH$.

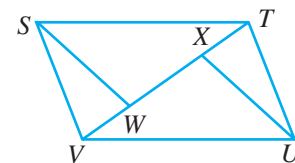
e



- $\angle QPS = \angle QRS$
- SQ bisects $\angle PQR$

Prove that $\Delta QPS \equiv \Delta QRS$.

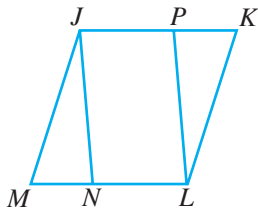
f



- $STUV$ is a parallelogram.
- $SW = XU$
- $VW = XT$

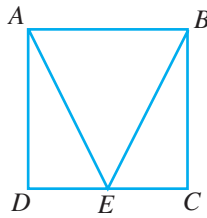
Prove that $\Delta SVW \equiv \Delta UTX$.

g



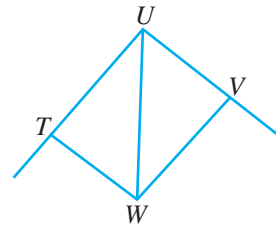
- $JKLM$ is a rhombus.
 - $MN = PK$
- Prove that $\triangle JMN \cong \triangle LKP$.

h



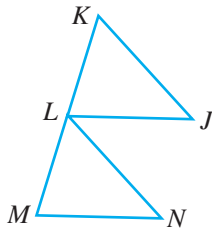
- $ABCD$ is a square.
 - $AE = BE$
- Prove that $\triangle ADE \cong \triangle BCE$.

i



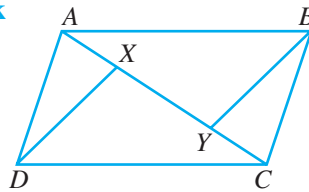
- $TU \parallel WV$
 - $TW \parallel UV$
- Prove that $\triangle UTW \cong \triangle WVU$.

j



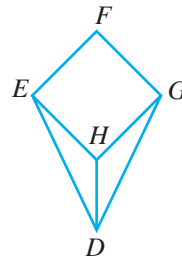
- $LJ \parallel MN$
 - $KJ \parallel LN$
 - JL bisects KM
- Prove that $\triangle K LJ \cong \triangle LMN$.

k



- $ABCD$ is a parallelogram.
 - $AY = XC$
- Prove that $\triangle AYB \cong \triangle CXD$.

l



- $EFGH$ is a rhombus.
 - $DE = DG$
- Prove that $\triangle DEH \cong \triangle DGH$.

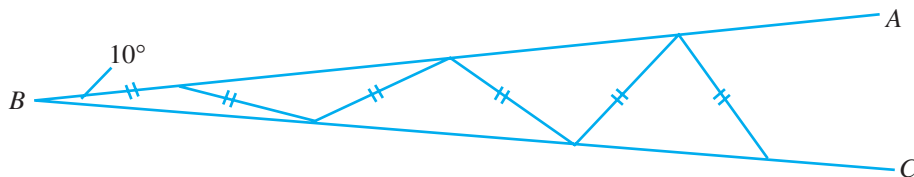
Further applications

5 For each of the following, draw a diagram and label it with all of the given information, then complete the proof.

- LMN is an isosceles triangle with $LM = LN$. PMN is another isosceles triangle with $PM = PN$, where P lies on the opposite side of MN to L . Prove that $\triangle LMP \cong \triangle LNP$.
- $JKLM$ is a rectangle. N is a point on KL such that $JN = MN$. Prove that $\triangle JKN \cong \triangle MLN$.
- ABC is an isosceles triangle with $AB = AC$. P and Q are the midpoints of AB and AC respectively. Prove that $\triangle PBC \cong \triangle QCB$.

TRY THIS Triangle angles

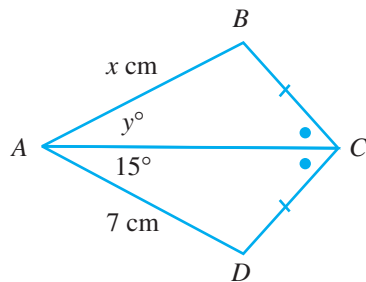
Suppose that we continue drawing triangles as shown in the diagram below. What is the size of the angles in the last triangle that you are able to draw? $\angle ABC = 10^\circ$.



10.9 Deductive reasoning and congruent triangles

The properties of many figures as well as other general geometric results can be deduced by first proving that two triangles are congruent. It follows that if one triangle is congruent to another triangle, then the matching sides and angles in those triangles must be equal. The equivalence of these matching sides and angles can be used, for example, to prove that:

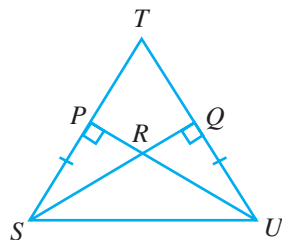
- two lines are parallel
- two lines are perpendicular
- a line bisects an interval
- a line bisects an angle
- a given triangle is isosceles
- a given quadrilateral is a parallelogram, a rectangle, a rhombus or a square.

Example 1

- AC bisects $\angle BCD$
 - $BC = CD$
- a** Prove that $\triangle ABC \equiv \triangle ADC$.
- b** Find values for x, y giving reasons.

Solution

- a** In $\triangle ABC$ and $\triangle ADC$:
- $BC = CD$ (given)
 - $\angle BCA = \angle DCA$ (AC bisects $\angle BCD$)
 - AC is a common side
- $\therefore \triangle ABC \equiv \triangle ADC$ (SAS)
- b**
- $AB = AD$ (matching sides of congruent $\triangle$ s)
- $\therefore x = 7$
- $\angle BAC = \angle DAC$ (matching angles of congruent $\triangle$ s)
- $\therefore y = 15$

EG
+S**Example 2**

- $PS = QU$
- $UP \perp ST$
- $SQ \perp TU$

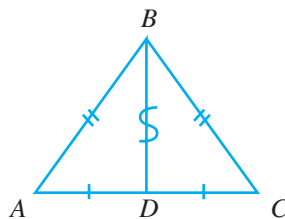
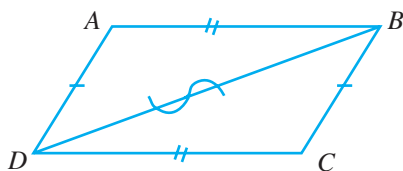
- a** Prove that $\triangle PRS \equiv \triangle QRU$.
b Hence, prove that $\triangle SRU$ is isosceles.

Solution

- a** In $\triangle PRS$ and $\triangle QRU$:
- $PS = QU$ (given)
 - $\angle RPS = \angle RQU = 90^\circ$ ($UP \perp ST$, $SQ \perp TU$)
 - $\angle PRS = \angle QRU$ (vertically opposite $\angle$ s are equal)
- $\therefore \triangle PRS \equiv \triangle QRU$ (AAS)
- b** • $RS = RU$ (matching sides of congruent Δ s)
 $\therefore \triangle SRU$ is isosceles.

EG
+S**Example 3**

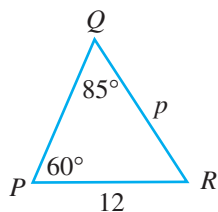
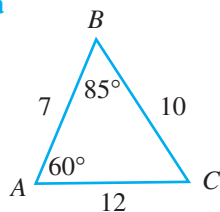
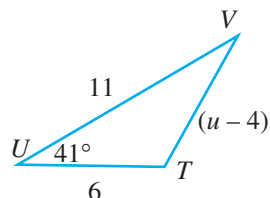
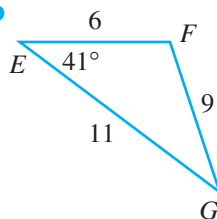
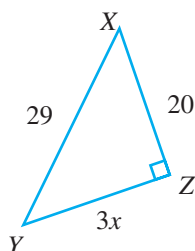
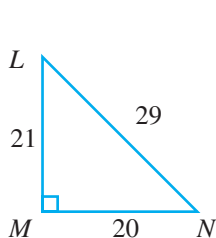
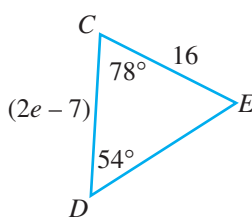
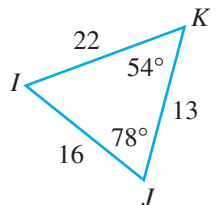
- a** If $\triangle ABD \equiv \triangle CDB$, prove that $AB \parallel DC$. **b** If $\triangle ABD \equiv \triangle CBD$, prove that $BD \perp AC$.

**Solutions**

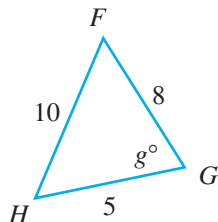
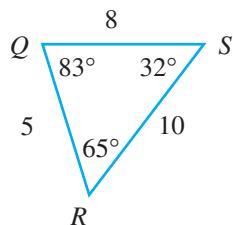
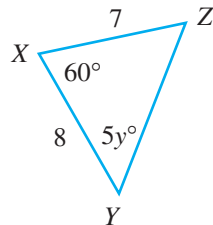
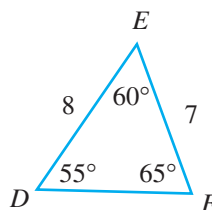
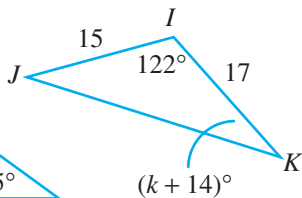
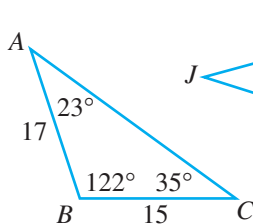
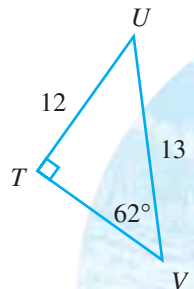
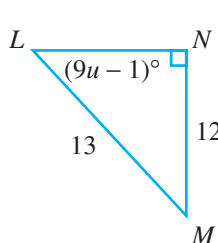
- a** $\triangle ABD \equiv \triangle CDB$ (SSS)
 $\therefore \angle ABD = \angle BDC$ (matching angles of congruent Δ s)
 $\therefore AB \parallel DC$ (alternate $\angle$ s are equal)
- b** $\triangle ABD \equiv \triangle CBD$ (SSS)
- $\angle BDA = \angle BDC$ (matching angles of congruent Δ s)
 - $\angle BDA + \angle BDC = 180^\circ$ ($\angle$ s on a straight line are supplementary)
- $\therefore \angle BDA = \angle BDC = 90^\circ$
 $\therefore BD \perp AC$

Exercise 10.9

- 1 Each pair of triangles below is congruent. Name the congruent triangles with the vertices in matching order and state the test that justifies their congruence. Hence, find the value of the pronumeral, giving a reason. All lengths are in cm.

a**b****c****d**

- 2 Each pair of triangles below is congruent. Name the congruent triangles with the vertices in matching order and state the test that justifies their congruence. Hence, find the value of the pronumeral, giving a reason. All lengths are in cm.

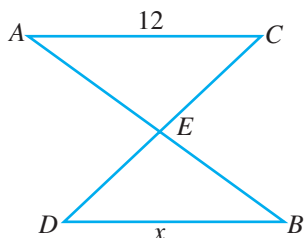
a**b****c****d**

Consolidation

3 For each of the following:

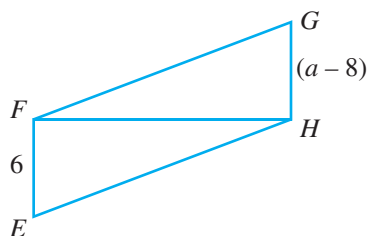
- copy the diagram and mark on it all of the given information
- prove that the two triangles are congruent
- hence, find the value of the pronumeral, giving a reason. (All lengths are in mm.)

a



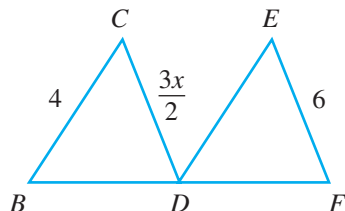
AB and CD bisect each other.

c



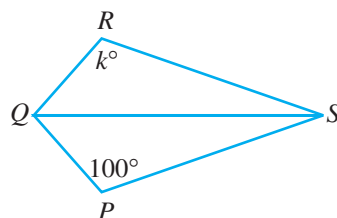
$EF \perp FH$, $GH \perp FH$, $FG = EH$.

e



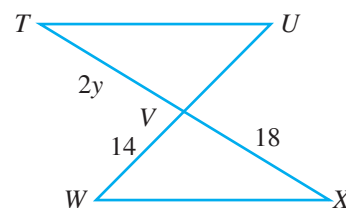
$BC \parallel DE$, $CD \parallel EF$, $BD = DF$.

b



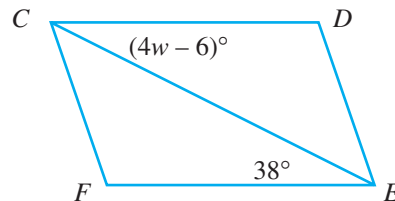
$PS = SR$, QS bisects $\angle PSR$.

d



$TU \parallel WX$, $TU = WX$.

f



$CF \parallel DE$, $CF = DE$.

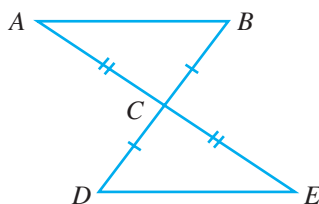
4 PQR is an isosceles triangle with $PQ = PR$. S is a point on the base QR .

- If PS is the altitude from the apex P to the base QR , prove that $\triangle PQS \equiv \triangle PRS$.
 - Hence, show that PS bisects the base QR .
- If PS is the median joining the apex P to the midpoint of the base QR , prove that $\triangle PQS \equiv \triangle PRS$.
 - Hence, show that PS bisects the apex angle P .
- If PS is the angle bisector of angle P , prove that $\triangle PQS \equiv \triangle PRS$.
 - Hence, show that PS is also a perpendicular bisector of the base.

5 In $\triangle PQR$, S is a point on QR such that $PS \perp QR$. Show that $\triangle PQR$ is isosceles if:

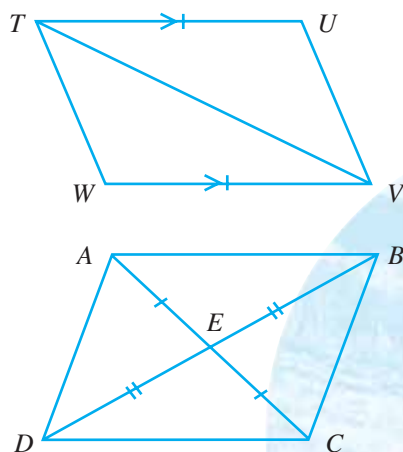
- PS also bisects $\angle P$.
- PS also bisects the side QR .

- 6 $EFGH$ is a kite with $EF = EH$ and $GF = GH$.
- Prove that $\triangle EFG \equiv \triangle EHG$.
 - Hence, show that the diagonal EG bisects $\angle FEH$.
- 7 Prove the following properties of parallelograms by first proving that two triangles are congruent.
- The opposite sides of a parallelogram are equal.
 - The opposite angles of a parallelogram are equal.
- 8 $ABCD$ is a parallelogram. The diagonals AC and BD intersect at X .
- Prove that $\triangle AXD \equiv \triangle CXB$.
 - Hence, show that the diagonals of a parallelogram bisect each other.
- 9 $WXYZ$ is a rectangle.
- Prove that $\triangle WZY \equiv \triangle XYZ$.
 - Hence, show that the diagonals of a rectangle are equal in length.
- 10 $QRST$ is a rhombus. The diagonals intersect at X .
- Prove that $\triangle QRX \equiv \triangle SRX$.
 - Hence, show that the diagonals of a rhombus are perpendicular.
- 11 AE and BD bisect each other.
- Prove that $\triangle ABC \equiv \triangle EDC$.
 - Hence, prove that $AB \parallel DE$.



■ Further applications

- 12 In the quadrilateral $TUVW$, $TU = WV$ and $TU \parallel WV$.
- Prove that $\triangle TUV \equiv \triangle VWT$.
 - Hence, show that $TW \parallel UV$.
 - What kind of quadrilateral is $TUVW$? Why?
- 13 $ABCD$ is a quadrilateral in which the diagonals AC and BD bisect each other at E .
- Prove that $\triangle AED \equiv \triangle CEB$. Hence, show that $AD \parallel BC$.
 - Prove that $\triangle AEB \equiv \triangle CED$. Hence, show that $AB \parallel DC$.
 - Hence, show that $ABCD$ is a parallelogram.





FOCUS ON WORKING MATHEMATICALLY

DOES A TRIANGLE HAVE A CENTRE?



Napoleon Bonaparte

Introduction

Establishing the centre of a circle, a square or a rectangle is easy enough. There is a unique point which qualifies as the centre of each of these figures. In this activity, we will investigate the question ‘Does a triangle have a centre?’ Various points that could stake a claim to being the centre of a triangle will be considered and a new point called a **Fermat point** will be introduced. You will also learn about a famous theorem that carries the name of Napoleon Bonaparte.



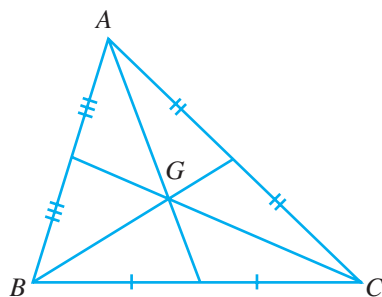
LEARNING ACTIVITIES

You will need a set square, ruler, a sharp pencil and a pair of compasses for this activity. Alternatively, the activity will work well using the software packages *Geometer's Sketchpad* or *Cabri Geometry*. This activity is very suitable for group work.



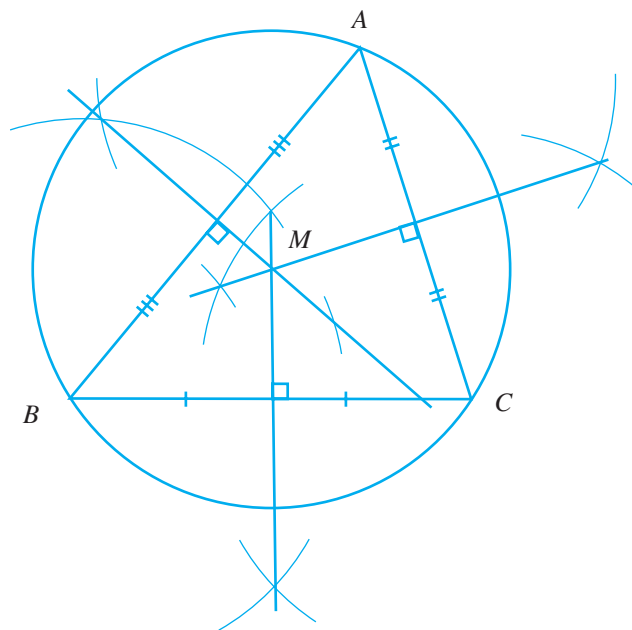
Draw a scalene acute-angled triangle ABC and construct each of the following:

- 1 the medians of the triangle



The medians are *concurrent* at the centroid G . Write down some important geometrical properties of G .

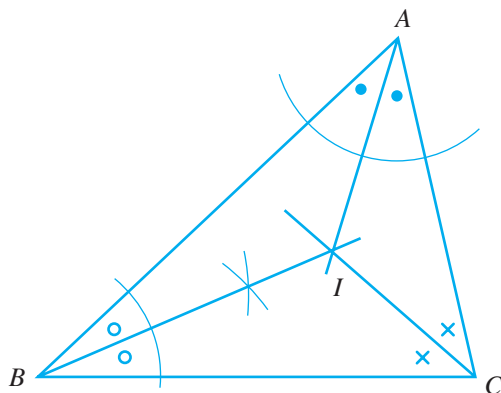
- 2 the right bisectors of the sides



The right bisectors are *concurrent* at the circumcentre M . Write down some important geometrical properties of M .

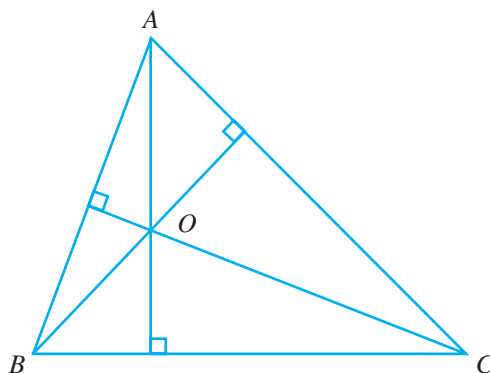


3 the angle bisectors



The angle bisectors are *concurrent* at the incentre I . Write down some important geometrical properties of I .

4 the orthocentre



The altitudes are *concurrent* at the orthocentre O . Write down *some* important geometrical properties of O .

- 5 Which of the above points best qualify as the centre of a triangle based on your results so far? Discuss this in groups and use the geometrical properties of G , M , I and O to make a case. You may disagree!
- 6 Finally, using about half a page, construct the points G , M , I and O for a single triangle ABC . What do you notice? If you were accurate, the points G , M and O will lie on a straight line. This line is called the *Euler line* of the triangle, after Leonhard Euler (1707–1783), the Swiss mathematician who discovered it. Why does the incentre I not lie on this line? Discuss with your teacher.
- 7 If you are using *Geometer's Sketchpad* or *Cabri Geometry* software, drag the vertex A and note that G , M and O are always collinear.

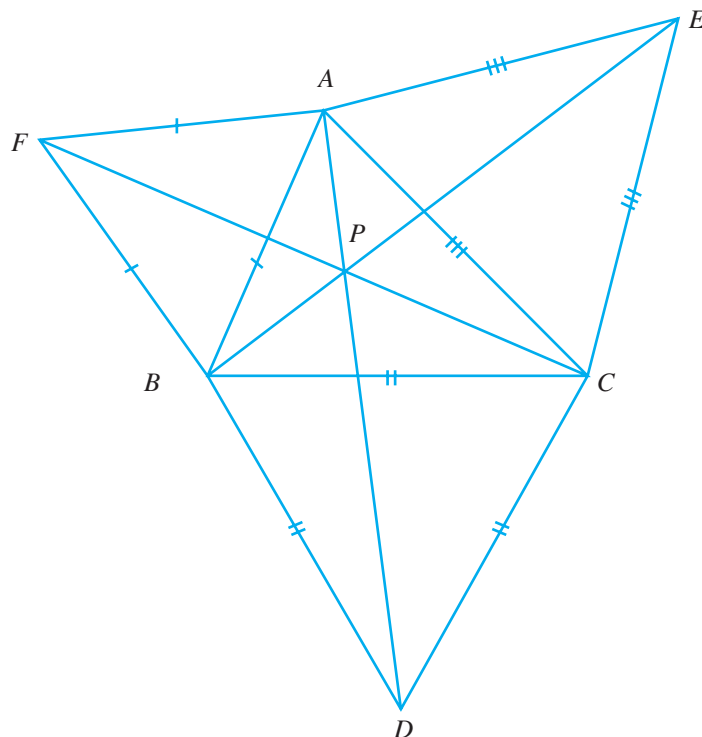




CHALLENGE

We will now construct one further point of concurrence called a **Fermat point**. It is named after the French mathematician, Pierre de Fermat (1601–1665) who discovered it.

Draw a scalene acute-angled triangle ABC . Construct an equilateral triangle on each side as shown, and join AD , BE and CF . These lines are concurrent at the Fermat point P .



- 1 Make a list of some of the geometrical properties of the figure. For example, angles APB , BPC and CPA are all 120° . By using congruent triangles show that $AD = BE = CF$.
- 2 The Italian mathematician Evangelista Torricelli (1608–1647), a contemporary of Fermat, showed that the Fermat point P is such that the sum $PA + PB + PC$ is a minimum. Would this make the Fermat point an ideal candidate for the centre of a triangle? Compare the Fermat point P with the *circumcentre* M . How do they differ?
- 3 There was a proviso to Torricelli's result. The angles of triangle ABC must be less than 120° . Why do you think this is so?



- 4 Show that $PA + PB + PC$ is equal in length to the lines AD , BE and CF .
- 5 Draw a new triangle ABC and construct the Fermat point P . Now construct the circumcircles of the equilateral triangles on each side of triangle ABC . What do you notice?
- 6 Verify Napoleon's theorem: The centres of the equilateral triangles drawn on each side of $\triangle ABC$, themselves form an equilateral triangle. Check it out on the Internet.



LET'S COMMUNICATE

Write a conclusion to your work to answer the original question 'What is the centre of a triangle?'



REFLECTING

Reflect on the practical uses of the 'centres' you have drawn in this activity. The website www.punahou.edu/acad/sanders/CenterTriangle.html has some fascinating applications.



MACQUARIE

Language link with Macquarie

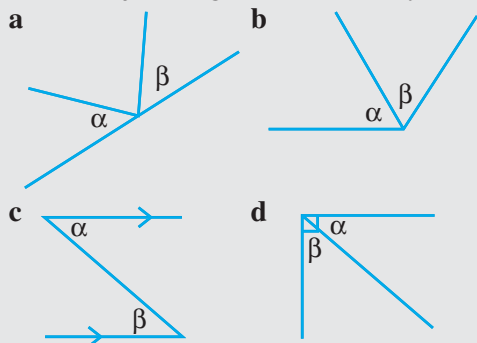
- 1 Two lines drawn in the same plane that never meet are called **parallel**.
- 2 Compare and contrast the **scalene** and **isosceles** triangles.
- 3 Define **polygon** for a new mathematics dictionary.
- 4 Match the following words with their meanings:
- 5 Read the Macquarie Learners Dictionary entry for **intersect**:

intersect verb 1. to cut or divide by passing through or across: This line intersects the circle. 2. to cross: There is a signpost where the streets intersect.

Why do architects and engineers have to understand geometrical principles?

Trapezium	A quadrilateral with two pairs of adjacent sides equal
Rhombus	A parallelogram in which one angle is a right angle
Parallelogram	A parallelogram with a pair of adjacent sides equal
Rectangle	A quadrilateral that is both a rhombus and a rectangle
Square	A quadrilateral with both pairs of opposite sides parallel
Kite	A quadrilateral with at least one pair of opposite sides parallel

- 1 For each of the following, state whether the angles α and β are adjacent. If they are not adjacent, give a reason why.



- 2 What kind of angle measures:

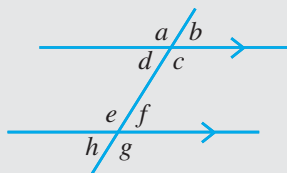
- a 180° ? b 41° ? c 360° ?
d 125° ? e 90° ? f 273° ?

- 3 Copy and complete these statements.

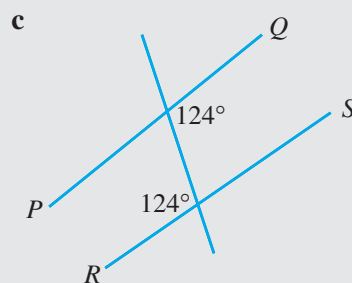
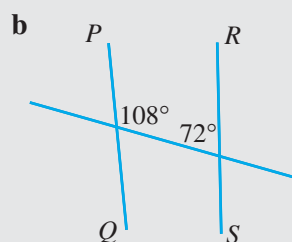
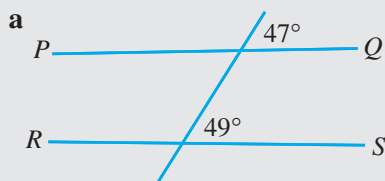
- a Vertically opposite angles are _____.
b Complementary angles add up to _____.
c Supplementary angles add up to _____.
d Angles at a point add up to _____.

- 4 List all pairs of:

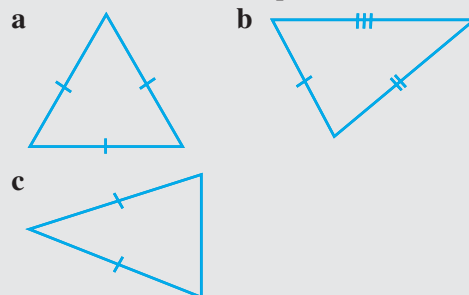
- a alternate angles
b corresponding angles
c co-interior angles



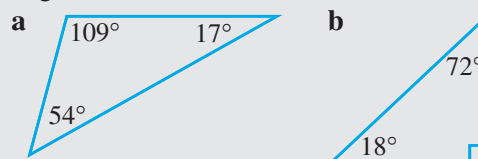
- 5 State whether or not $PQ \parallel RS$ in each of these, giving a reason.



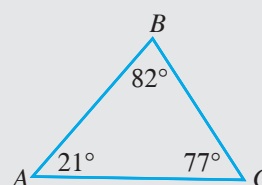
- 6 Classify each of these triangles as either scalene, isosceles or equilateral.



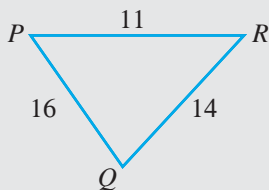
- 7 Classify each of these triangles as either right-angled, acute-angled or obtuse-angled.



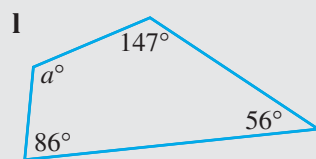
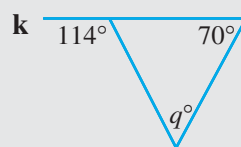
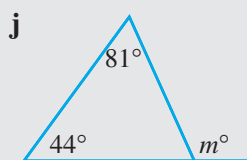
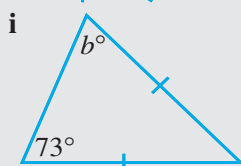
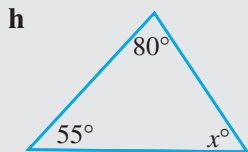
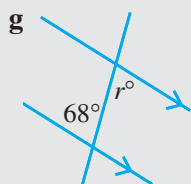
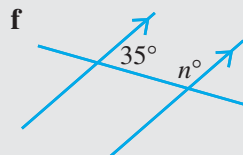
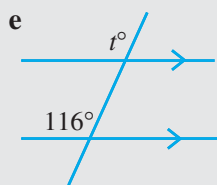
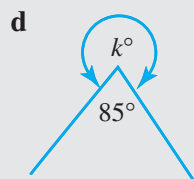
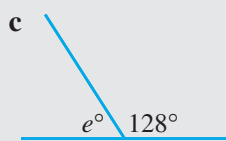
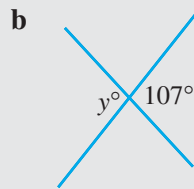
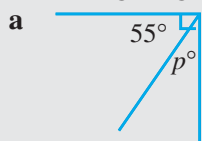
- 8 a Name the shortest and longest sides in this triangle.



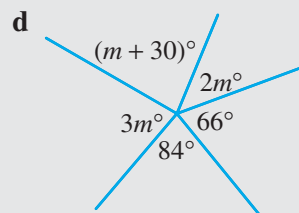
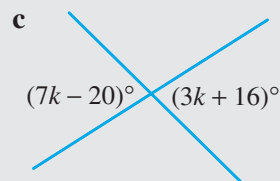
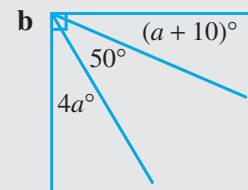
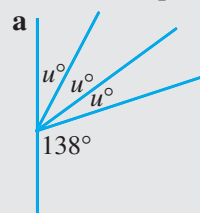
- b** Name the smallest and largest angles in this triangle.



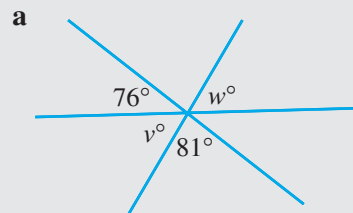
- 9** Can a triangle with sides 23 mm, 51 mm and 25 mm possibly exist? Explain.
- 10** Find the value of the pronumeral in each of these, giving brief reasons.

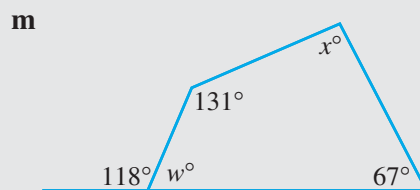
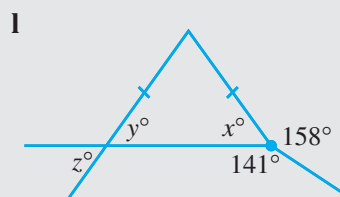
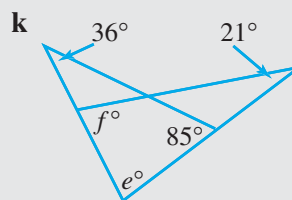
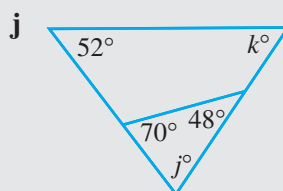
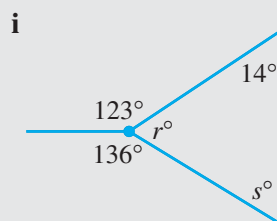
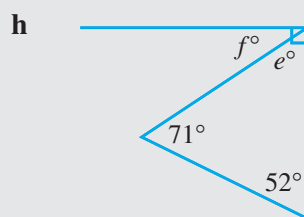
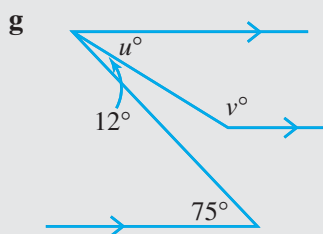
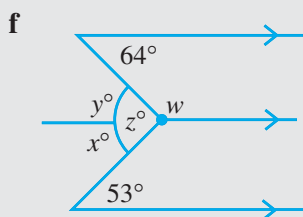
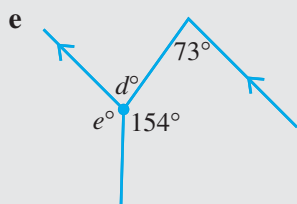
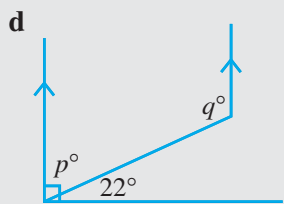
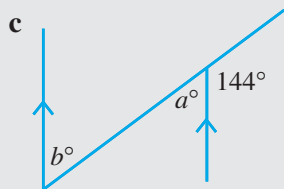
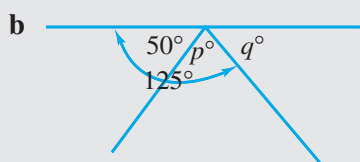


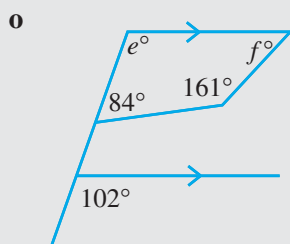
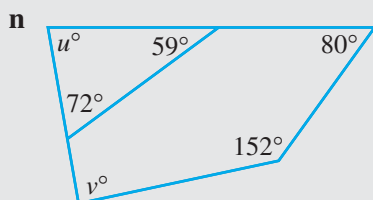
- 11** Form an equation and solve it to find the value of the pronumeral in each of these.



- 12** Find the values of the pronumerals in each of these, giving reasons.

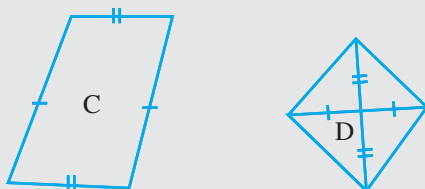




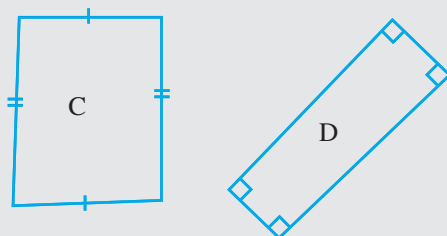
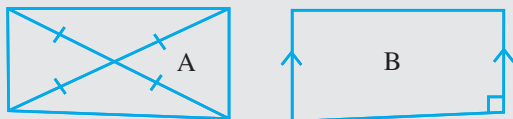


13 Determine which figures (not drawn to scale) are:

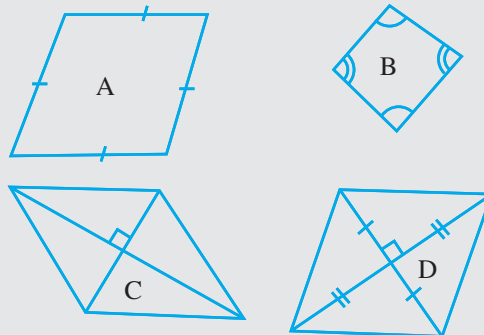
a parallelograms and give a reason.



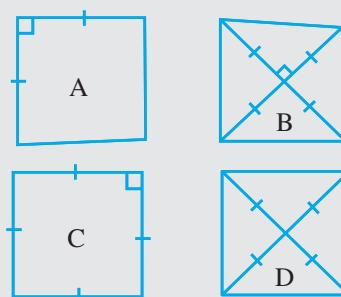
b rectangles and give a reason.



c rhombuses and give a reason.



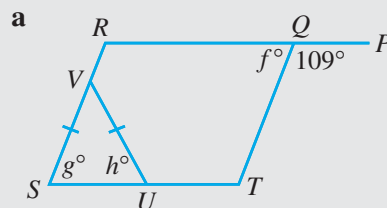
d squares and give a reason.



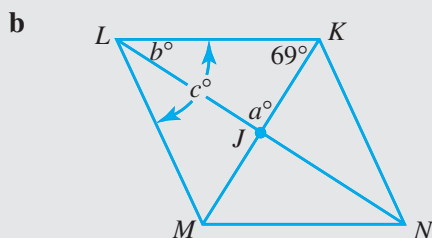
14 Write true (T) or false (F) for each of these.

- The opposite angles in a rhombus are equal.
- The diagonals in a rectangle are perpendicular.
- The diagonals bisect each other in a parallelogram.
- The diagonals bisect the angles at the vertices in a square.

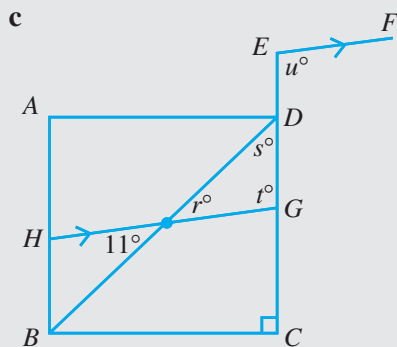
15 Find the values of the pronumerals in each of these, giving reasons.



- $QRST$ is a parallelogram
- $VS = VU$



- $KLMN$ is a rhombus



- $ABCD$ is a square
- $HG \parallel EF$

- 16** Name the polygon that has the following number of sides.

a 4	b 5	c 6
d 8	e 10	f 12

- 17** Define carefully each of the following terms.

- a** convex polygon
- b** regular polygon

- 18** Is a rhombus a regular polygon? Why/why not?

- 19** Calculate the interior angle sum of each of these regular polygons, then find the size of their interior angles.

- a** pentagon
- b** nonagon

- 20** Find the size of the exterior angles in a regular:

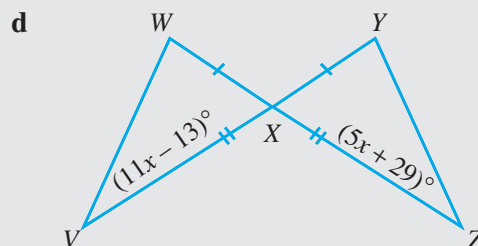
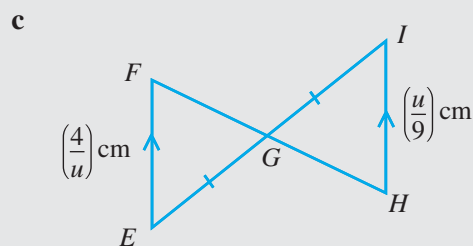
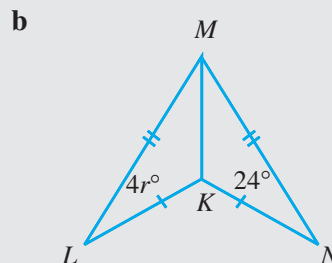
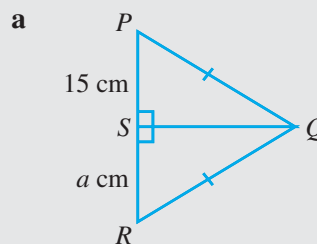
- a** decagon
- b** dodecagon

- 21** Find the size of the interior and exterior angles in a regular polygon with 20 sides.

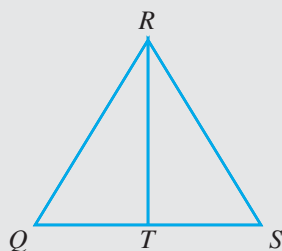
- 22** How many sides has a regular polygon with:

- a** exterior angles measuring 15°
- b** interior angles measuring 175°

- 23** The triangles in each pair are congruent. In each case, name the congruent triangles giving the vertices in matching order and state the test used. Hence, find the value of the pronumeral, giving reasons.

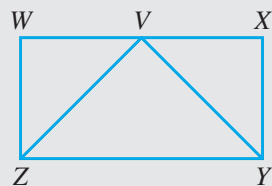


24



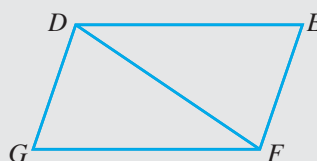
- $RT \perp QS$
- RT bisects $\angle QRS$
- a** Prove that $\triangle RQT \cong \triangle RST$.
- b** Hence, prove that $\triangle QRS$ is isosceles.

25



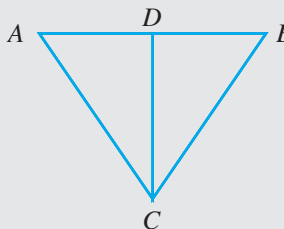
- WXYZ is a rectangle
- $VZ = VY$
- a** Prove that $\triangle VWZ \cong \triangle VXY$.
- b** Hence, prove that V is the midpoint of WX .

26



- $DG = EF$
- $DG \parallel EF$
- a** Prove that $\triangle DGF \cong \triangle FED$.
- b** Hence, prove that $DE \parallel GF$.

27



- CD bisects AB
- $AC = CB$
- a** Prove that $\triangle DCA \perp \triangle DCB$.
- b** Hence, prove that $CD \perp AB$.

The linear function



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

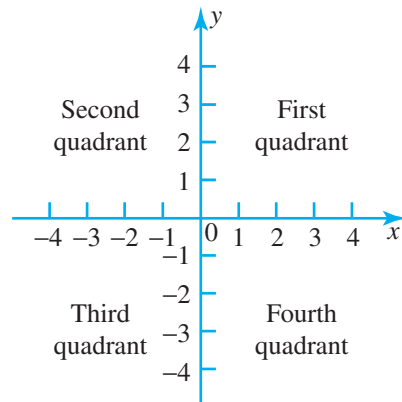
- ⊙ read, plot and name ordered pairs on the number plane
- ⊙ determine the quadrant in which a given point lies
- ⊙ find the length of a horizontal or vertical interval given the co-ordinates of its end points
- ⊙ solve simple co-ordinate problems involving the perimeter and area of triangles and quadrilaterals
- ⊙ derive an equation that describes a discrete linear relationship and graph it on the number plane
- ⊙ draw the graph of a line given its equation by plotting points
- ⊙ find the x - and y -intercepts of a line given its equation
- ⊙ determine by substitution whether a point lies on a line
- ⊙ graph vertical and horizontal lines with equations $x = a$ and $y = b$
- ⊙ find the gradient of a straight line using the ratio of the vertical rise and horizontal run
- ⊙ determine whether a line will increase or decrease from left to right by considering the sign of its gradient
- ⊙ state the gradient and y -intercept of a line given its equation
- ⊙ write the equation of a line given its gradient and y -intercept
- ⊙ explain the effect on the graph of a line by changing the gradient or y -intercept
- ⊙ determine whether two lines are parallel by considering gradients
- ⊙ find the co-ordinates of the fourth vertex of a parallelogram by considering gradients
- ⊙ sketch the graph of a line given its equation by considering the gradient and y -intercept
- ⊙ find by substitution into $y = mx + b$, the equation of the line that has a given gradient and passes through a given point
- ⊙ graph a straight line by plotting the y -intercept, then using the gradient to find another point that will lie on the line
- ⊙ compare similarities and differences of linear relationships
- ⊙ graph two lines on the same number plane and hence determine their point of intersection
- ⊙ find the equation of a line after translation or reflection.

11.1 The number plane

The **number plane** is made up of four quadrants separated by two perpendicular lines called **axes**. The horizontal number line is referred to as the x -axis and the vertical number line is referred to as the y -axis.

A point is located by giving the horizontal reading first (i.e. the x -value), followed by the vertical reading (i.e. the y -value). The readings are given in the form of an **ordered pair** or **pair of co-ordinates** (x, y) . The point where the two axes intersect is called the **origin** and its co-ordinates are $(0, 0)$.

The distance between two points in the number plane is measured in **units**.

EG
+
S

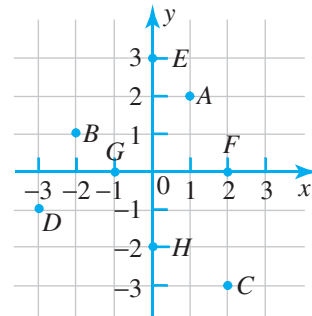
Example 1

State the co-ordinates of each point.

a A	b B	c C	d D
e E	f F	g G	h H

Solutions

a $A(1, 2)$	b $B(-2, 1)$	c $C(2, -3)$
d $D(-3, -1)$	e $E(0, 3)$	f $F(2, 0)$
g $G(-1, 0)$	h $H(0, -2)$	

EG
+
S

Example 2

In which quadrant would each point lie?

a $(-3, -4)$	b $(2, 5)$
c $(1, -2)$	d $(-6, 7)$

Solutions

- a** $(-3, -4)$ lies in the third quadrant
b $(2, 5)$ lies in the first quadrant
c $(1, -2)$ lies in the fourth quadrant
d $(-6, 7)$ lies in the second quadrant

EG
+
S

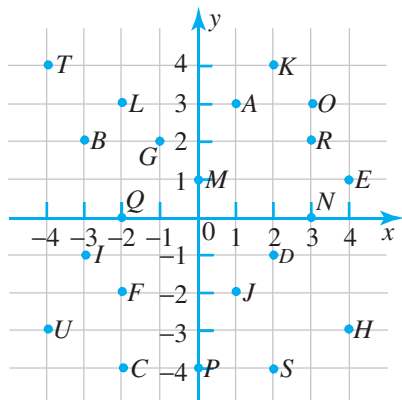
Example 3

Find the distance between the points:

- | |
|------------------------------------|
| a $A(3, 10)$ and $B(8, 10)$ |
| b $P(4, -2)$ and $Q(4, 1)$ |

Solutions

- a** The x -values (i.e. 8 and 3) differ by 5 and the y -values are equal, $\therefore$ the distance $AB = 5$ units.
b The y -values (i.e. 1 and -2) differ by 3 and the x -values are equal, $\therefore$ the distance $PQ = 3$ units.

Exercise 11.1**1**

Write down the co-ordinates of each point.

a A	b B	c C
d D	e E	f F
g G	h H	i I
j J	k K	l L
m M	n N	o O
p P	q Q	r R
s S	t T	u U

2 In which quadrant does each point lie?

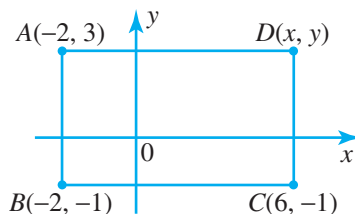
a $(-2, 4)$	b $(1, 7)$	c $(6, -2)$	d $(-3, -4)$
e $(4, 1)$	f $(-1, -1)$	g $(-3, 7)$	h $(8, -5)$
i $(-5, -7)$	j $(-6, 1)$	k $(9, -4)$	l $(6, 11)$

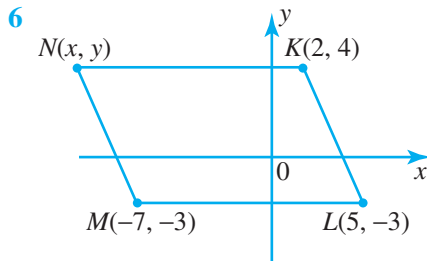
Consolidation**3** Find the distance between each pair of points.

a $(2, 3)$ and $(7, 3)$	b $(-1, 0)$ and $(3, 0)$	c $(-8, -4)$ and $(-1, -4)$
d $(5, 3)$ and $(5, 14)$	e $(0, -2)$ and $(0, 3)$	f $(-2, -11)$ and $(-2, -2)$

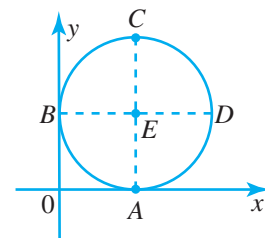
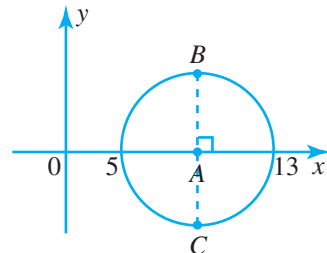
4 Write down the co-ordinates of the point that is:

a 4 units to the right of $(2, 5)$	b 7 units to the left of $(0, 6)$
c 8 units above $(4, -5)$	d 10 units below $(-7, 1)$

5 a Find the co-ordinates of D in the first quadrant such that $ABCD$ is a rectangle.**b** Find the perimeter of $ABCD$.**c** Find the area of $ABCD$.

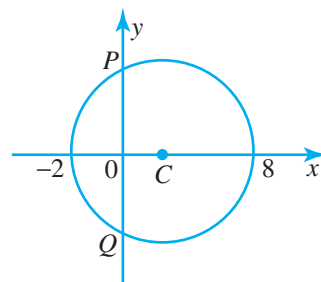


- a** Find the co-ordinates of N , a point in the second quadrant, such that $KLMN$ is a parallelogram.
- b** Find the area of $KLMN$.
- 7 The vertices of $\triangle ABC$ are $A(1, 2)$, $B(5, 4)$ and $C(9, 2)$. The perpendicular from B meets AC at D .
- a** Mark this information on a diagram.
- b** By inspection, what kind of triangle is ABC ?
- c** Find the area of $\triangle ABC$.
- 8 **a** Plot the points $T(-2, 2)$, $U(10, -3)$ and $V(-2, -8)$ on a number plane.
- b** Find the area of $\triangle TUV$.
- 9 A set of points is said to be *collinear* if they lie in a straight line. Plot each set of points on a number plane, then state whether they are collinear.
- a** $(-2, 1)$, $(1, 2)$, $(4, 3)$ **b** $(0, 3)$, $(2, 5)$, $(5, 6)$
- 10 Find the co-ordinates of the centre and the length of the radius of a circle that passes through the points $(5, 10)$, $(1, 6)$, $(5, 2)$ and $(9, 6)$.
- 11 The circle shown has centre A and the diameter BC is parallel to the y -axis.
- a** Find the co-ordinates of A , B and C .
- b** Find the exact circumference of the circle.
- c** Find the exact area of the circle.
- 12 The circle shown has centre $E(5, 5)$ and touches the x -axis and y -axis at A and B respectively. Find the co-ordinates of A , B , C and D .



Further applications

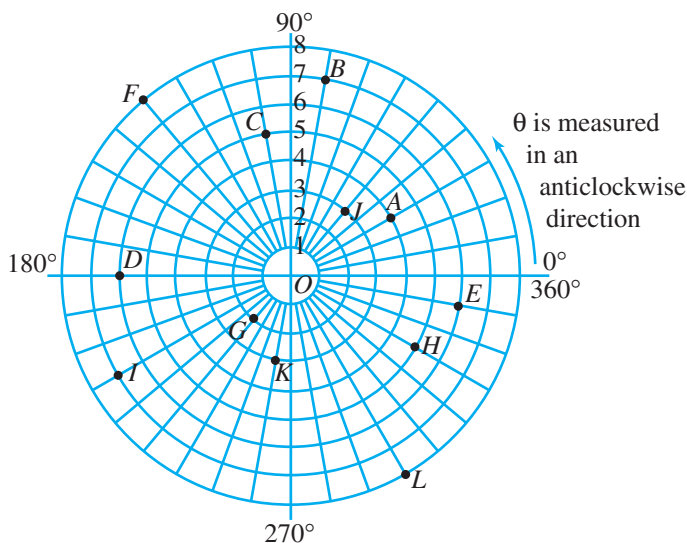
- 13** A circle centre C , where C lies on the x -axis, cuts the x -axis at $x = -2$ and $x = 8$ and the y -axis at P and Q .
- Find the co-ordinates of C .
 - Find the length of the radius.
 - Find the co-ordinates of P and Q , and hence calculate the length of the chord PQ .



On some occasions it is more useful to describe the position of a point in terms of its distance (r units) from another point, O , and the angular rotation, θ , about the point O .

This is usually written as (r, θ) .

Such ordered pairs are called polar co-ordinates. Use the diagram opposite to answer Q14.



- 14** Give the polar co-ordinates of each point.

a	A	b	B	c	C
d	D	e	E	f	F
g	G	h	H	i	I
j	J	k	K	l	L

11.2 Graphing straight lines (1)

If the terms in a number pattern increase or decrease by a common difference, then they are said to form a **linear relationship**. Linear relationships can be described by an algebraic expression or by drawing their graph. When graphed on a number grid or number plane, the points lie in a straight line.

In chapter 2, we described linear relationships with the equation $y = \Delta x + \square$, where Δ represented the common difference between the y -values, or bottom numbers, in a table of values. We will now write this equation as $y = mx + b$.

The equation of a line describes the relationship between the x - and y -co-ordinates of every point on the line. For example, the equation:

- $y = 2x$ describes a line in which the y -value of each point is twice the x -value.
- $y = x + 3$ describes a line in which the y -value of each point is 3 more than the x -value.
- $y = \frac{1}{2}x - 1$ describes a line in which the y -value of each point is 1 less than half the x -value.

To graph a linear relationship using a table of values:

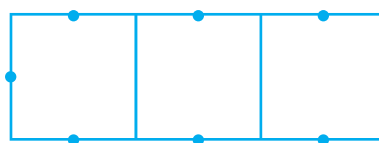
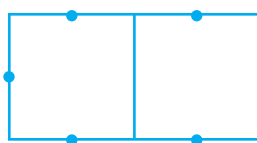
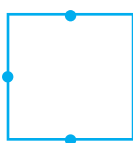
- ⊙ substitute each x -value into the equation to find the corresponding y -value
- ⊙ plot the points on a number plane
- ⊙ draw a straight line through the points if appropriate.

To graph a linear relationship by its intercepts:

- ⊙ substitute $x = 0$ into the equation to find the y -intercept
- ⊙ substitute $y = 0$ into the equation to find the x -intercept
- ⊙ draw a straight line through the intercepts.

EG
+
S

Example 1



- a Copy and complete this table of values.

Number of squares (x)	1	2	3	4	5
Number of dots (y)					

- b How many dots are being added in each step?
 c Write down an equation in the form $y = mx + b$ that shows the relationship between the number of dots and the number of squares in each step.
 d Plot the values from this table on a number grid, with the number of squares along the horizontal axis and the number of dots along the vertical axis. Is the relationship linear?

Solutions

a

Number of squares (x)	1	2	3	4	5
Number of dots (y)	3	5	7	9	11

- b 2 dots are being added in each step.
 c The equation is of the form $y = mx + b$, where m is the common difference between the y -values.
 $\therefore m = 2$.

We now use $m = 2$ and one of the points in the table (1, 3) to find the value of b .

$$y = mx + b$$

$$3 = 2(1) + b$$

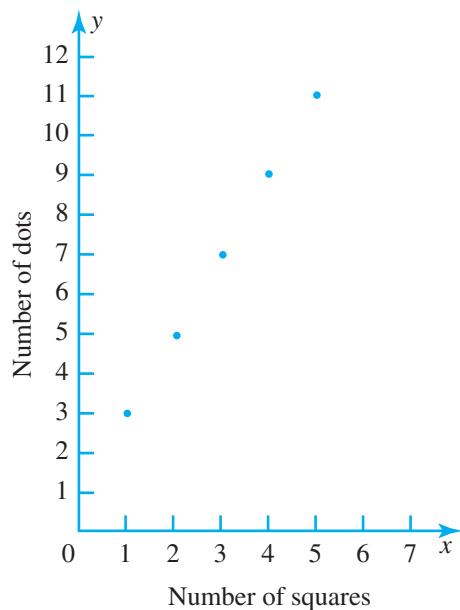
$$3 = 2 + b$$

$$\therefore b = 1$$

$$\therefore \text{The equation is } y = 2x + 1.$$

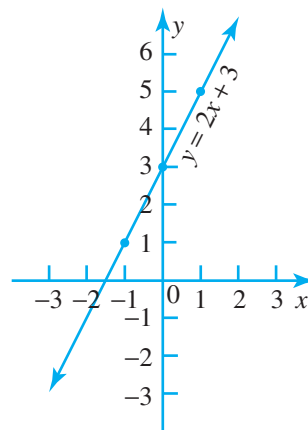
The graph shows a linear relationship. The dots are not joined because the number of squares must be a whole number.

d

**Example 2**Draw the graph of the line $y = 2x + 3$ using a table of values.**Solution**

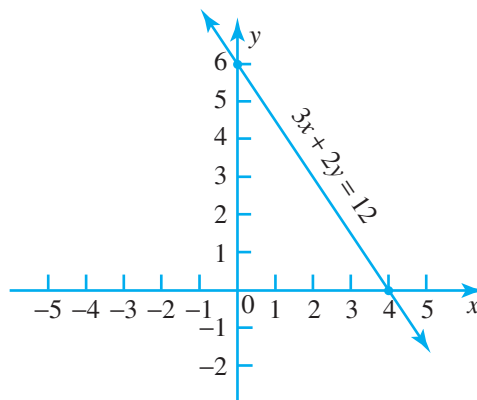
$y = 2x + 3$	$y = 2x + 3$	$y = 2x + 3$
$y = 2(-1) + 3$	$y = 2(0) + 3$	$y = 2(1) + 3$
$y = -2 + 3$	$y = 0 + 3$	$y = 2 + 3$
$\therefore y = 1$	$\therefore y = 3$	$\therefore y = 5$

x	-1	0	1
y	1	3	5

**Example 3**Find the x - and y -intercepts and hence sketch the line $3x + 2y = 12$.**Solution**

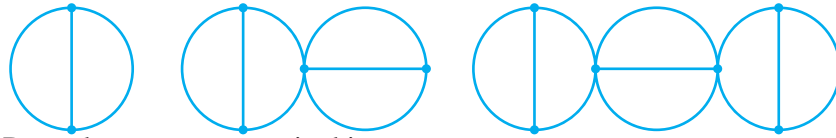
When $x = 0$: $3(0) + 2y = 12$
 $2y = 12$
 $\therefore y = 6$

When $y = 0$: $3x + 2(0) = 12$
 $3x = 12$
 $\therefore x = 4$



Exercise 11.2

1

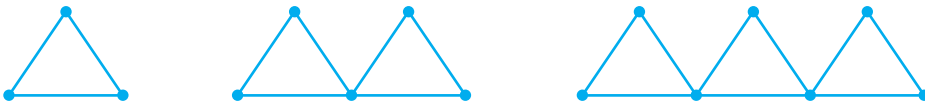


- a Draw the next two steps in this pattern.
b Copy and complete this table of values.

Number of circles (x)	1	2	3	4	5
Number of dots (y)					

- c How many dots are being added in each step?
d Write down an equation in the form $y = mx + b$ that shows the relationship between the number of dots and the number of circles in each step.
e Plot the values from this table on a number grid, with the number of circles along the horizontal axis and the number of dots along the vertical axis.
f Should the points be joined? Explain.
g Is the relationship linear?

2

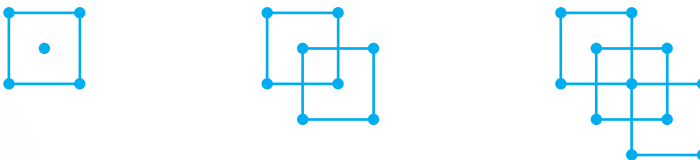


- a Draw the next two steps in this pattern.
b Copy and complete this table of values.

Number of triangles (x)	1	2	3	4	5
Number of dots (y)					

- c How many dots are being added in each step?
d Write down an equation in the form $y = mx + b$ that shows the relationship between the number of dots and the number of triangles in each step.
e Plot the values from this table on a number grid, with the number of triangles along the horizontal axis and the number of dots along the vertical axis.

3



- a Draw the next two steps in this pattern.
b Copy and complete this table of values.

Number of squares (x)	1	2	3	4	5
Number of dots (y)					

- c How many dots are being added in each step?

- d** Write down an equation in the form $y = mx + b$ that shows the relationship between the number of dots and the number of squares in each step.
- e** Plot the values from this table on a number grid, with the number of squares along the horizontal axis and the number of dots along the vertical axis.

■ Consolidation

- 4** Copy and complete these tables of values, then graph each line on a separate number plane.

$y = x + 3$

a	x	0	1	2
	y			

$y = 3x$

b	x	-1	0	1
	y			

$y = 5 - x$

c	x	1	2	3
	y			

$y = 2x - 3$

d	x	0	1	2
	y			

$y = 3x + 1$

e	x	-1	0	1
	y			

$x + y = 4$

f	x	0	1	2
	y			

- 5** Graph each of these lines on a separate number plane by plotting at least 3 points.

a $y = x$	b $y = 4x$	c $y = -x$	d $y = x + 4$
e $y = x - 3$	f $y = 1 - x$	g $y = 2x + 1$	h $y = 3x - 2$
i $y = 2 - 2x$	j $x + y = 2$	k $2x + y = 6$	l $3x - y = 3$

- 6** Find the x - and y -intercepts and hence sketch each of the following lines.

a $y = x + 3$	b $y = x - 2$	c $y = 4 - x$	d $y = 2x + 6$
e $y = 12 - 3x$	f $y = -2x - 5$	g $x - 2y = 8$	h $4x + y = 6$
i $3x + 4y = 36$	j $y = \frac{1}{2}x + 1$	k $y = 2 - \frac{1}{3}x$	l $y = \frac{2}{3}x - 6$

- 7 a** Graph the lines $y = x$, $y = 2x$ and $y = 3x$ on the same number plane.

- b** What is the effect of the co-efficient in each equation?

- 8** Which line would be steeper, $y = \frac{1}{2}x$ or $y = 2x$? Why?

- 9 a** Graph the lines $y = x$, $y = x + 2$ and $y = x - 2$ on the same number plane.

- b** Are the lines parallel?

- c** What is the effect of the constant term in each question?

- 10** What transformation must be performed on the line $y = x$ to obtain the graph of:

a $y = x + 5$?	b $y = x - 3$?	c $y = x + 2$?	d $y = x - 7$?

- 11** What transformation must be performed on the line $y = x$ to obtain the graph of $y = -x$?

- 12** Describe the transformation that would map:

a $y = x + 2$ onto $y = x + 5$	b $y = -x + 4$ onto $y = -x - 1$
c $y = 2x - 4$ onto $y = 2x$	d $y = -3x - 5$ onto $y = -3x - 6$



13 What would be the equation of each line after it was reflected in the y -axis?

- a** $y = x$ **b** $y = -2x$ **c** $y = \frac{1}{2}x$ **d** $y = -\frac{1}{3}x$
e $y = x + 1$ **f** $y = x - 4$ **g** $y = -3x + 2$ **h** $y = 5 - 2x$

Further applications

14 Describe the two transformations that would be necessary to map:

- a** $y = x$ onto $y = -x + 3$ **b** $y = 2x$ onto $y = -2x - 4$ **c** $y = -3x$ onto $y = 3x - 7$
d $y = x + 3$ onto $y = -x$ **e** $y = 2x - 1$ onto $y = -2x + 2$ **f** $y = 5 - 4x$ onto $y = 4x - 5$

TRY THIS Size 8

Find out the different sizes given for women's dresses. How does the size of the dress relate to the actual measurement of the dress? Draw a graph relating the dress size to this dress measurement. Is it linear?

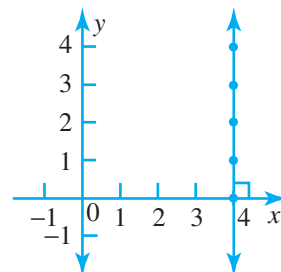
11.3 Graphing straight lines (2)

Horizontal and vertical lines

When an equation of the form $x = a$ is graphed on the number plane, the result is a vertical line that cuts the x -axis at a .

Consider the line that passes through the given points. The x - and y -values have no direct relationship but the x -values for all these points are 4s. Hence, the equation of the line is $x = 4$. By plotting the points, it is clear that the line must be vertical and cut the x -axis at 4.

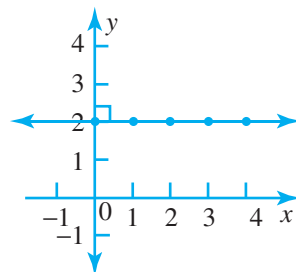
x	4	4	4	4	4
y	0	1	2	3	4



When an equation of the form $y = b$ is graphed on the number plane, the result is a horizontal line that cuts the y -axis at b .

Consider the line that passes through the given points. The x - and y -values have no direct relationship but the y -values for all these points are 2s. Hence, the equation of the line is $y = 2$. By plotting the points, it is clear that the line must be horizontal and cut the y -axis at 2.

x	0	1	2	3	4
y	2	2	2	2	2



- ⊙ $x = a$ is the equation of a vertical line, cutting the x -axis at a .
- ⊙ $y = b$ is the equation of a horizontal line, cutting the y -axis at b .

The condition for a point to lie on a line

To determine whether a point lies on a line, we substitute the co-ordinates of the point into the equation of the line. If the co-ordinates satisfy the equation, then the point lies on the line. If the co-ordinates do not satisfy the equation, then the point does not lie on the line. It is not necessary to draw the graph of the line.

A point lies on a line if its co-ordinates satisfy the equation of the line.

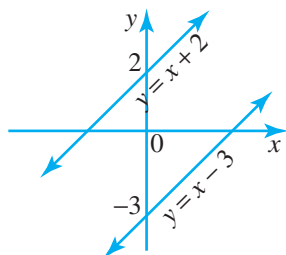
For example, the points $(0, 2)$, $(1, 3)$, $(5, 7)$, $(-2, 0)$ and $(-3, -1)$ would all lie on the line $y = x + 2$ because in each point, the y -value is 2 more than the x -value. Would the point $(7, 5)$ lie on this line? Why?

The intersection of straight lines

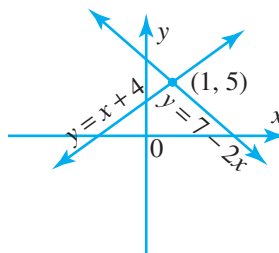
If two straight lines graphed on a number plane are:

- **parallel**, they will have no points of intersection
- **concurrent**, they will have exactly one point of intersection.

For example:



Parallel lines



Concurrent lines

If two straight lines intersect, then they do so at a unique point. As this point lies on both lines, its co-ordinates must satisfy both equations. This fact allows us to find the simultaneous solutions of two equations in x and y .

The co-ordinates of the point of intersection of two straight lines satisfy the equations of both lines.

EG
+S**Example 1**

Determine whether the point $(-3, 11)$ lies on the line $y = 5 - 2x$.

Solution

$$\begin{aligned}y &= 5 - 2x \\11 &= 5 - 2(-3) \\11 &= 5 + 6 \\11 &= 11\end{aligned}$$

The co-ordinates satisfy the equation,
 $\therefore$ the point lies on the line.

EG
+S**Example 2**

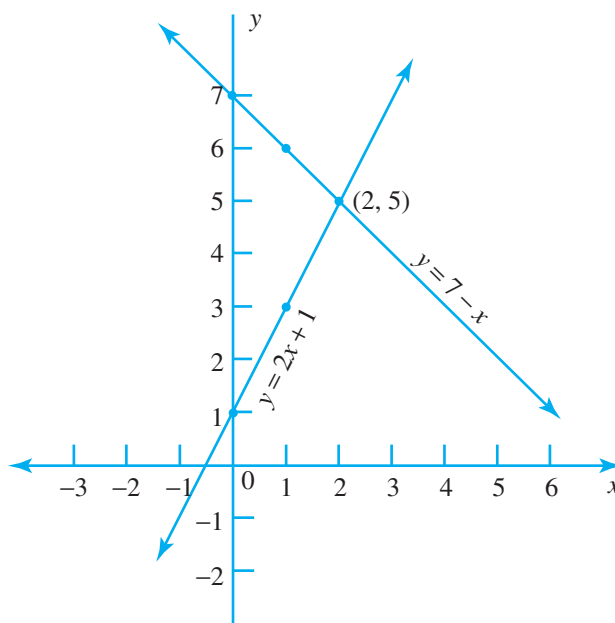
Find the point of intersection of the lines $y = 2x + 1$ and $y = 7 - x$.

Solution

$y = 2x + 1$				
x	0	1	2	
y	1	3	5	

$y = 7 - x$				
x	0	1	2	
y	7	6	5	

From the graph, the point of intersection is $(2, 5)$.

**Exercise 11.3**

- 1 Plot each set of points on a number plane. Draw the line that passes through these points and write down its equation.

a

x	3	3	3	3	3
y	0	1	2	3	4

b

x	-2	-1	0	1	2
y	3	3	3	3	3

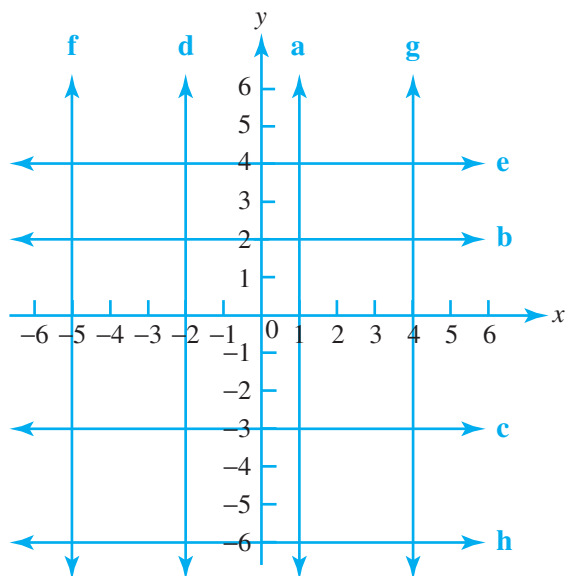
c

x	-1	-1	-1	-1	-1
y	-1	0	2	2	3

d

x	-2	-1	0	1	2
y	-2	-2	-2	-2	-2

2 Write down the equation of each line.



3 At what point do these lines intersect?

a $x = 3$ and $y = 2$

b $y = 1$ and $x = 4$

c $y = -3$ and $x = -6$

4 Find the co-ordinates of the point of intersection of:

a the line $x = 1$ and the x -axis

b the line $y = 7$ and the y -axis

5 a What is the equation of the x -axis? Why?

b What is the equation of the y -axis? Why?

6 Write down the equation of the line that is equidistant from the lines:

a $x = 2$ and $x = 8$

b $y = 1$ and $y = -7$

7 Find the equation of the line that is:

a parallel to the x -axis and passes through the point $(2, 6)$

b parallel to the y -axis and passes through the point $(-1, 4)$

c perpendicular to the x -axis and passes through the point $(5, -2)$

d perpendicular to the y -axis and passes through the point $(-8, -1)$

8 Which of the following points lie on the line $y = 2x - 7$?

A(4, 1)

B(0, 7)

C(12, 17)

D(-8, -23)

E($\frac{1}{2}$, -6)

F(-6, 5)

G($3\frac{1}{2}$, 0)

H(-10, -13)

9 Which of the following lines pass through the point $(-2, 3)$?

$y = x + 5$

$y = 5 - x$

$y = 2x + 7$

$y = -x + 1$

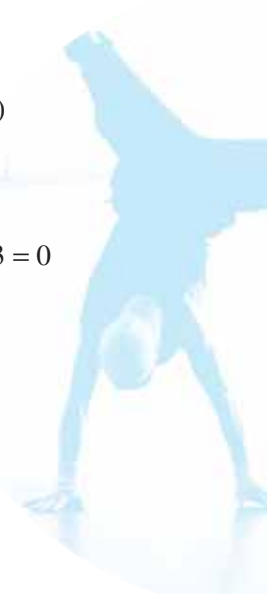
$y = 9 - 3x$

$x + 2y = 4$

$y = -\frac{3}{2}x$

$2x + 3y + 13 = 0$

10 Find the co-ordinates of three points that lie on the line $2x - y = 12$.



- 11** Find the value of the pronumeral in each ordered pair if:
- a** $(2, r)$ lies on the line $y = x + 7$ **b** $(k, 4)$ lies on the line $y = 11 - x$
c $(-5, t)$ lies on the line $2x + y = 8$ **d** $(p, -6)$ lies on the line $y = \frac{3}{4}x$
- 12** Find the value of the pronumeral in each ordered pair if the line:
- a** $y = 5x + 4$ passes through $(g, 4)$ **b** $y = 2 - 9x$ passes through $(-2, s)$
c $y = -\frac{2}{3}x$ passes through $(-15, u)$ **d** $3x - 4y + 5 = 0$ passes through $(a, 8)$
- 13 a** Find the value of c if $(3c - 1, -2)$ lies on the line $y = \frac{1}{5}x - 6$.
b Find the value of m if $(-1, 3)$ lies on the line $y = mx + 7$.
c Find two possible values for a if the line $ax - y + 8 = 0$ passes through the point $(a, 17)$.
- 14** Graph each pair of lines on the same number plane, then write down the co-ordinates of their point of intersection.
- a** $y = 3$ and $x = -2$ **b** $y = x$ and $y = 5$
c $y = -x$ and $y = x - 6$ **d** $y = x + 4$ and $y = 2 - x$
e $y = 3x$ and $y = x + 2$ **f** $y = x - 3$ and $y = 2x - 1$
g $y = 2x$ and $y = -3x - 5$ **h** $y = 1 - x$ and $y = 3 - 2x$
- 15 a** Does $(-2, 3)$ lie on the line $y = 2x + 7$?
b Does $(-2, 3)$ lie on the line $y = x + 5$?
c What is the geometric significance of this result?
- 16** Determine by substitution whether each pair of lines intersect at the given point.
- a** $y = 5x - 4$ and $y = 8 - x$, $(2, 6)$ **b** $y = x + 1$ and $y = 1 - x$, $(-1, 0)$
c $y = 2x - 10$ and $y = 7 + x$, $(-3, 4)$ **d** $y = 6x - 5$ and $y = 1 - 6x$, $(\frac{1}{2}, -2)$
- 17** Graph the lines $y = 2x + 1$ and $2x - y = 3$ on the same number plane and hence explain why they have no point of intersection.

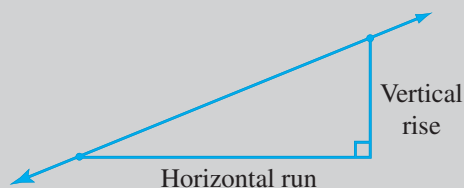
■ Further applications

- 18** In a stable there are x horses and y jockeys, where $x + y = 9$. Between them, the total number of legs is 26.
- a** Show that, in simplest form, the total number of legs is given by $2x + y = 13$.
b Graph the lines $x + y = 9$ and $2x + y = 13$ on the same number plane and find their point of intersection.
c How many horses and jockeys are there?
- 19** A child's money box contains x 5c coins and y 10c coins. There are 20 coins altogether with a total value of \$1.65.
- a** Show that $x + y = 20$ and $x + 2y = 33$ (in simplest form).
b Graph the lines on the same number plane and find their point of intersection.
c Find the number of 5c coins and 10c coins.

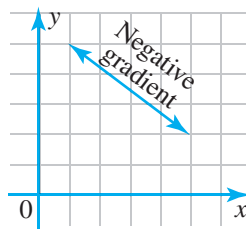
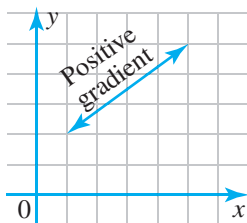
11.4 Gradient of a line

The **gradient** or **slope** of a line is a measure of how steep it is. The symbol for gradient is m .

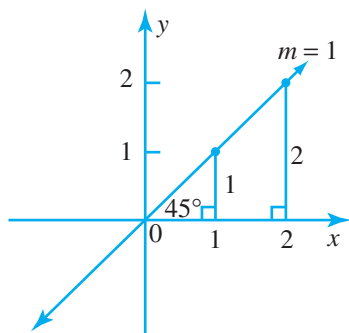
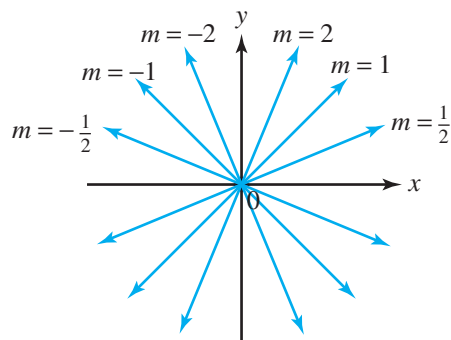
$$\text{Gradient } (m) = \frac{\text{vertical rise}}{\text{horizontal run}}$$



If the line is 'going up', or *increasing* from left to right, then its gradient is said to be *positive*. If the line is 'going down', or *decreasing* from left to right, then its gradient is said to be *negative*.



The greater the gradient (+ or -), the steeper is the line. The gradient of a horizontal line is 0, while the gradient of a vertical line is not defined. Why?

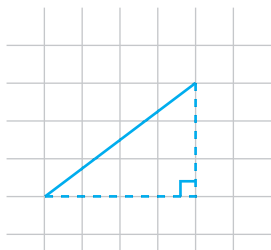
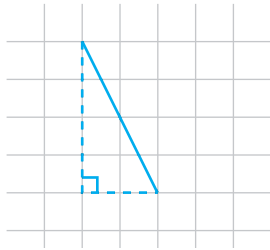
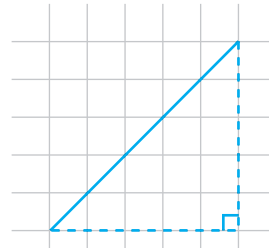


If a line is inclined to the x -axis at an angle of 45° , then for every point on that line, the vertical rise is equal to the horizontal run. Hence, the gradient of this line is equal to 1. It follows then that a line that is inclined to the x -axis at an angle of less than 45° has a gradient of less than 1, and a line that is inclined to the x -axis at an angle that is greater than 45° has a gradient greater than 1.

- ⊙ If a line is increasing from left to right, then it has a positive gradient.
- ⊙ If a line is decreasing from left to right, then it has a negative gradient.
- ⊙ The gradient of a horizontal line is 0.
- ⊙ The gradient of a vertical line is not defined.
- ⊙ The gradient of a line that is inclined to the x -axis at an angle of 45° is 1.

Example 1

Find the gradient of each interval.

a**b****c****Solutions**

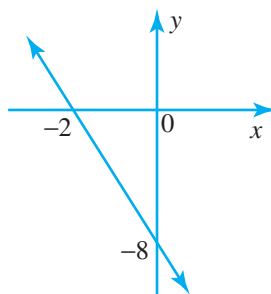
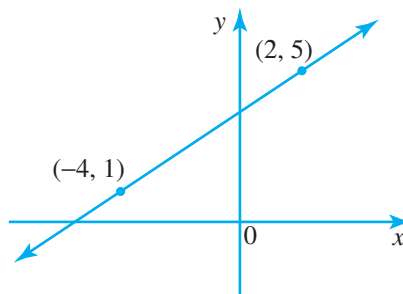
$$\begin{aligned} \mathbf{a} \quad m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-4}{2} \\ &= -2 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{5}{5} \\ &= 1 \end{aligned}$$

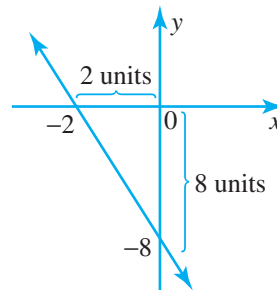
Example 2

Find the gradient of each line.

a**b****Solutions**

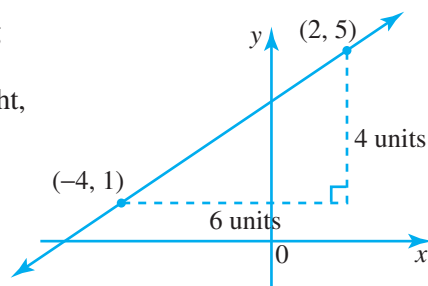
- a** The rise and run are distances, and are therefore positive. However, the line is decreasing from left to right, so the gradient is negative.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{8}{-2} \\ &= -4 \end{aligned}$$



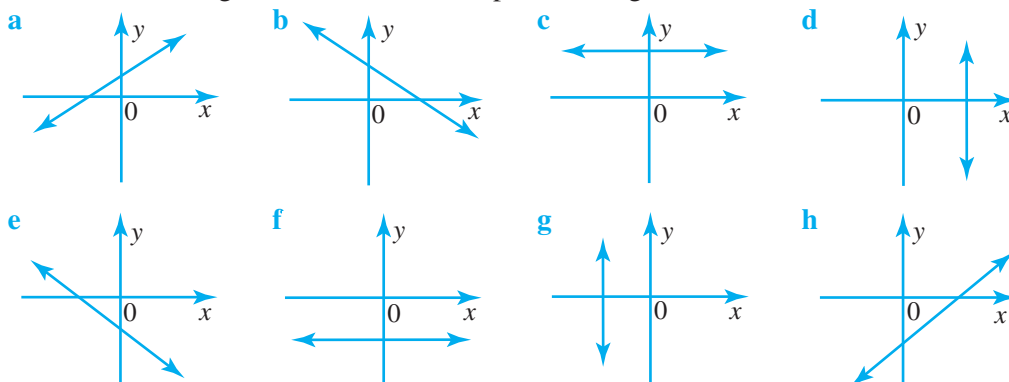
- b** Draw a right-angled triangle on the line using the given points and hence find the rise and the run. The line is increasing from left to right, so the gradient is positive.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

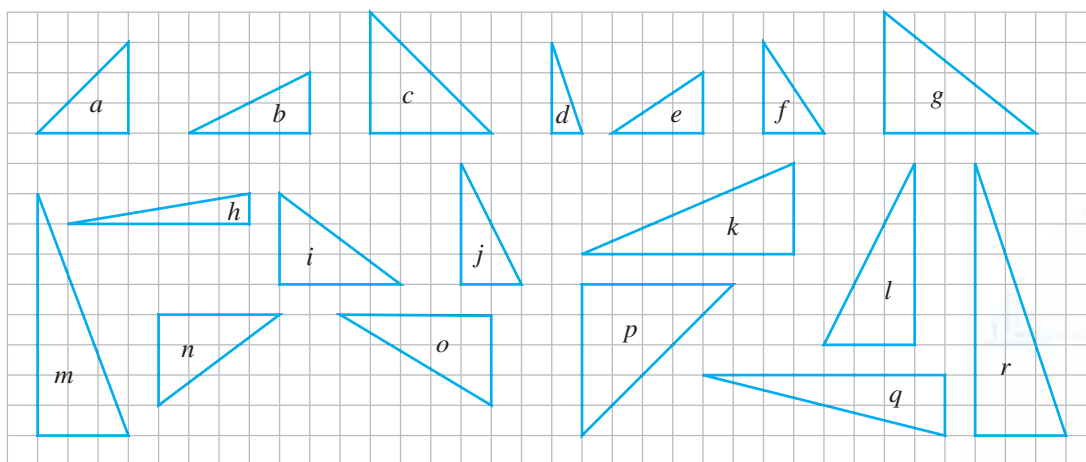


Exercise 11.4

- 1** State whether the gradient of each line is positive, negative, zero or undefined.

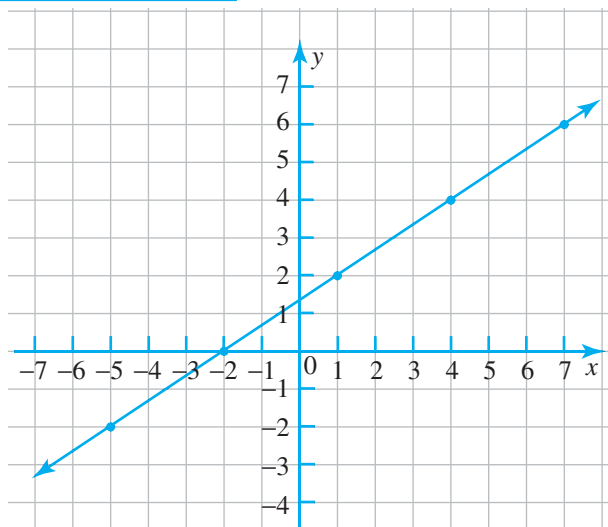


- 2** Find the gradient of each interval.



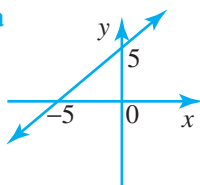
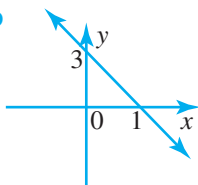
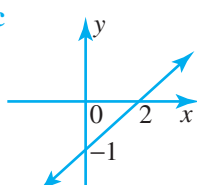
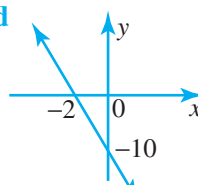
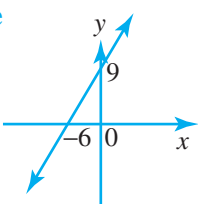
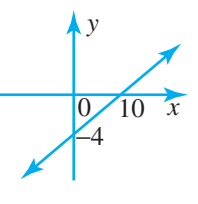
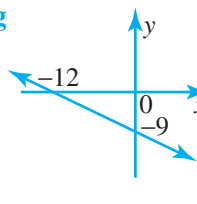
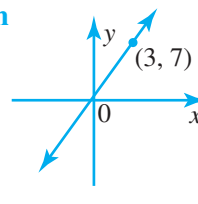
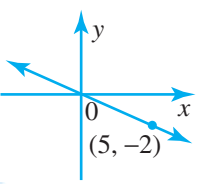
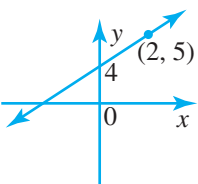
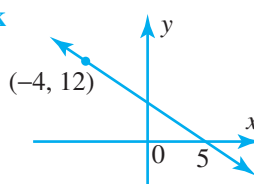
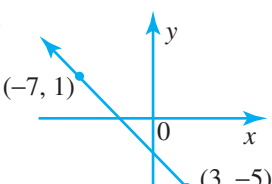
Consolidation

3



- a** Choose 3 different pairs of points that lie on the line and use them to calculate its gradient.
- b** Copy and complete this statement: 'Any two points that lie on a line can be used to calculate its _____.'

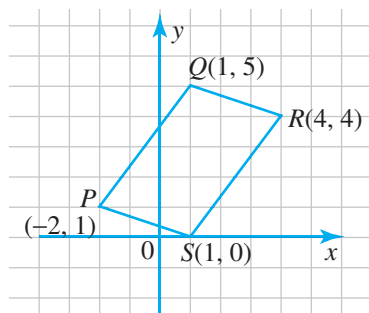
4 Find the gradient of each line.

a

b

c

d

e

f

g

h

i

j

k

l


5 Plot each pair of points on a number plane. Hence, find the gradient of the interval joining these points.

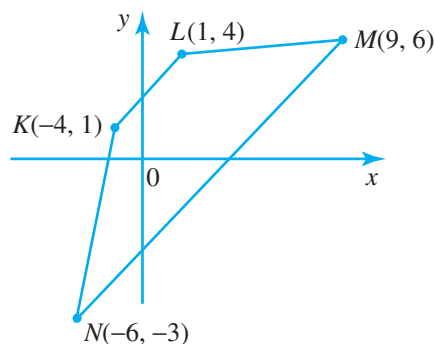
a $P(1, 1)$ and $Q(3, 7)$
b $G(-2, 5)$ and $H(2, 1)$
c $C(-4, -5)$ and $D(4, 5)$
d $J(11, -2)$ and $K(1, 10)$
e $E(-1, 0)$ and $F(7, 4)$
f $R(-8, -5)$ and $S(1, 1)$

6

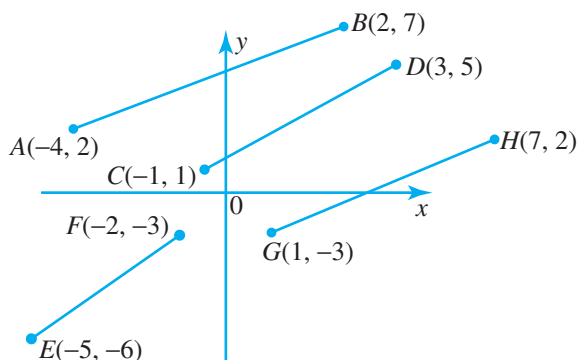


- a** Find the gradient of PQ and SR . Is $PQ \parallel SR$?
b Find the gradient of QR and PS . Is $QR \parallel PS$?
c What type of quadrilateral is $PQRS$? Why?
d Copy and complete this statement:
 'If two lines are parallel, then they have the same _____.'

- 7 a** Find the gradient of each side of the quadrilateral $KLMN$.
b What type of quadrilateral is $KLMN$? Why?



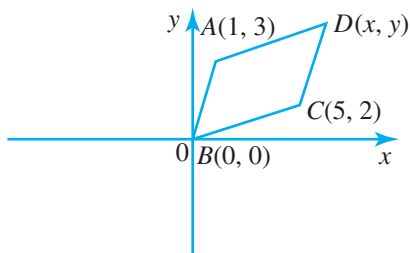
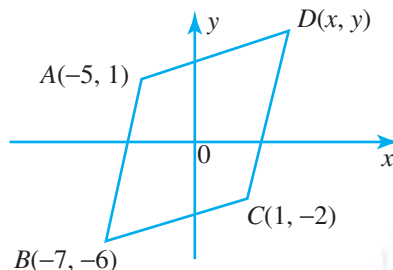
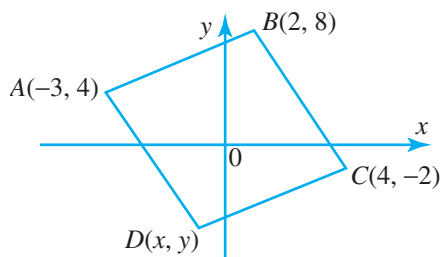
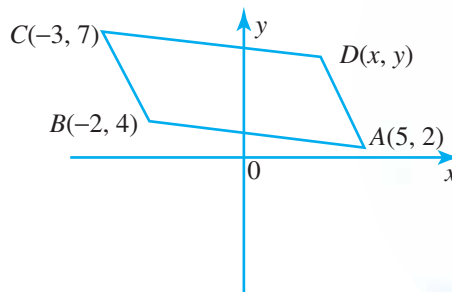
8



- a** Find the gradient of each interval.
b Which intervals are parallel?

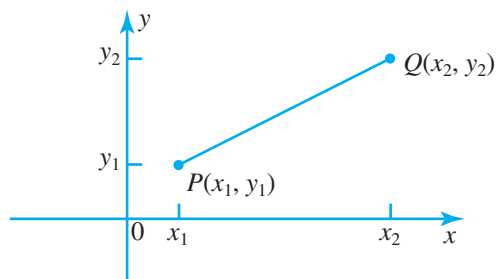
(Diagram not to scale.)

- 9** Use the fact that parallel lines have the same gradient to find the co-ordinates of D , the fourth vertex of the parallelogram $ABCD$, in each of these.

a**b****c****d**

Further applications

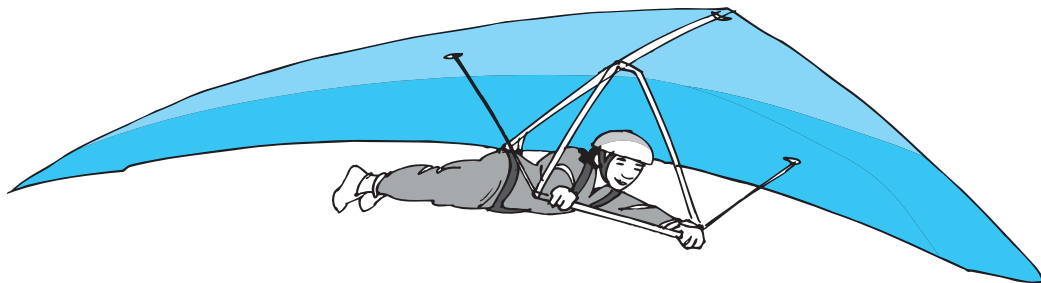
10



- a Derive a formula for the gradient of the interval PQ , using the co-ordinates of P and Q .
- b Hence, find the gradient of the line that passes through the points:
 - i $(0, 1)$ and $(2, 9)$
 - ii $(5, -1)$ and $(-1, 3)$
 - iii $(-2, -3)$ and $(-3, -8)$

TRY THIS Hanging around

A hang-glider is at the top of a cliff 400 m high. The cliff has a slope, or gradient, of 1 in 4. At the bottom of the cliff there is a lake 1500 m wide. The hang-glider jumps off the cliff with a rate of descent of 1 in 40. Will it clear the lake?



11.5 The linear equation $y = mx + b$

When the equation of a straight line is written in the form $y = mx + b$, it is said to be in **gradient-intercept form**, where m is the gradient of the line and b is the y -intercept. For example, if a line has a gradient of 3 and cuts the y -axis at 7, then its equation is $y = 3x + 7$.

Notice that when the gradient is 0 (i.e. $m = 0$), the equation $y = mx + b$ becomes $y = b$, which is the standard equation of a horizontal line.

The gradient-intercept form of the linear equation is $y = mx + b$, where:

- ⊙ m is the gradient, and
- ⊙ b is the y -intercept.

EG
+
S**Example 1**

State the gradient and y-intercept of each line.

a $y = 3x + 5$

b $y = 4x - 2$

c $y = -2x - 11$

d $y = 9 - 5x$

e $y = x$

f $y = \frac{2x}{3}$

Solutions

a $m = 3$ and $b = 5$

b $m = 4$ and $b = -2$

c $m = -2$ and $b = -11$

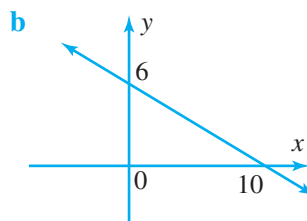
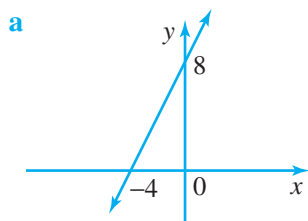
d $m = -5$ and $b = 9$

e $m = 1$ and $b = 0$

f $m = \frac{2}{3}$ and $b = 0$

EG
+
S**Example 2**

Find the equation of each of these lines.

**Solutions**

- a i** The line is increasing from left to right, so the gradient is positive.

$$\begin{aligned}
 m &= \frac{\text{rise}}{\text{run}} \\
 &= \frac{8}{4} \\
 &= 2
 \end{aligned}$$

- ii** The line cuts the y-axis at 8, $\therefore b = 8$.

- iii** Substituting into $y = mx + b$, the equation of the line is $y = 2x + 8$.

- b i** The line is decreasing from left to right, so the gradient is negative.

$$\begin{aligned}
 m &= \frac{\text{rise}}{\text{run}} \\
 &= -\frac{6}{10} \\
 &= -\frac{3}{5}
 \end{aligned}$$

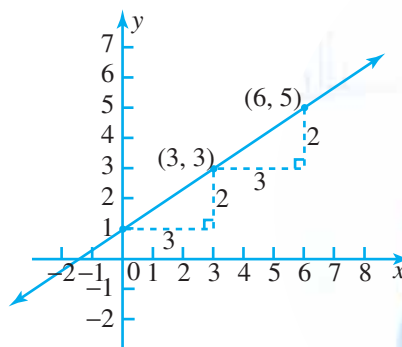
- ii** The line cuts the y-axis at 6, $\therefore b = 6$.

- iii** Substituting into $y = mx + b$, the equation of the line is $y = -\frac{3}{5}x + 6$.

EG
+
S**Example 3**Graph the line $y = \frac{2}{3}x + 1$ by plotting the y-intercept, then using the gradient to find two other points on the line.**Solution**The y-intercept is 1 and the gradient is $\frac{2}{3}$.

Beginning at the y-intercept (0, 1), count 3 units to the right and 2 units up, then mark the point (3, 3).

Beginning at this point, count 3 units to the right and 2 units up, then mark the point (6, 5). Draw the line that passes through these 3 points.



Exercise 11.5

1 For each of the following, state the gradient and y-intercept.

a $y = 2x + 3$

b $y = 3x - 1$

c $y = -2x + 5$

d $y = -4x - 3$

e $y = x + 4$

f $y = -x + 2$

g $y = 3 + 5x$

h $y = 6 - x$

i $y = 2x$

j $y = -7x$

k $y = \frac{1}{2}x + 4$

l $y = \frac{1}{3}x - 5$

m $y = \frac{2}{3}x + 1$

n $y = 8 - \frac{3}{4}x$

o $y = -2 - \frac{6}{5}x$

p $y = 3(x - 2)$

q $y = 4(2x + 5)$

r $y = -2(5 - 7x)$

2 Write down the equation of the line that has:

a a gradient of 4 and y-intercept of 2

b a gradient of -3 and y-intercept of 5

c a slope of 1 and y-intercept of -4

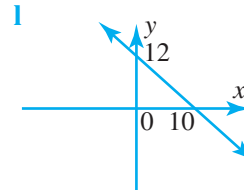
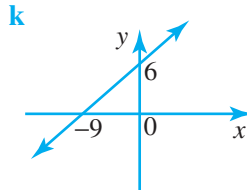
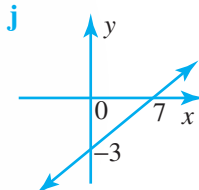
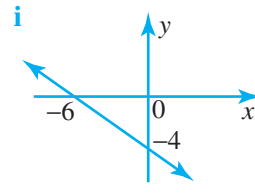
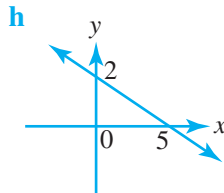
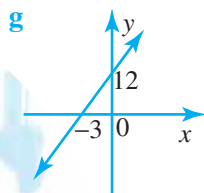
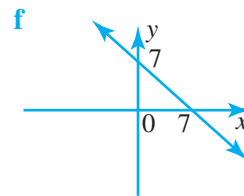
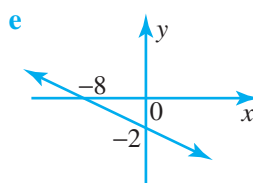
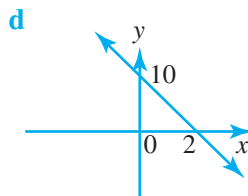
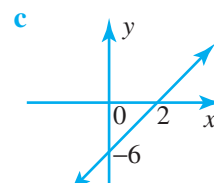
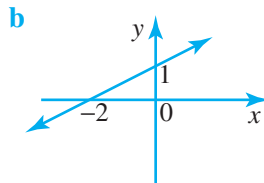
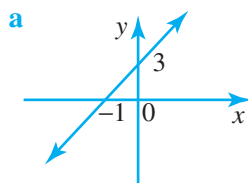
d a slope of $-\frac{1}{2}$ and cuts the y-axis at -7

e a gradient of 5 and passes through the origin

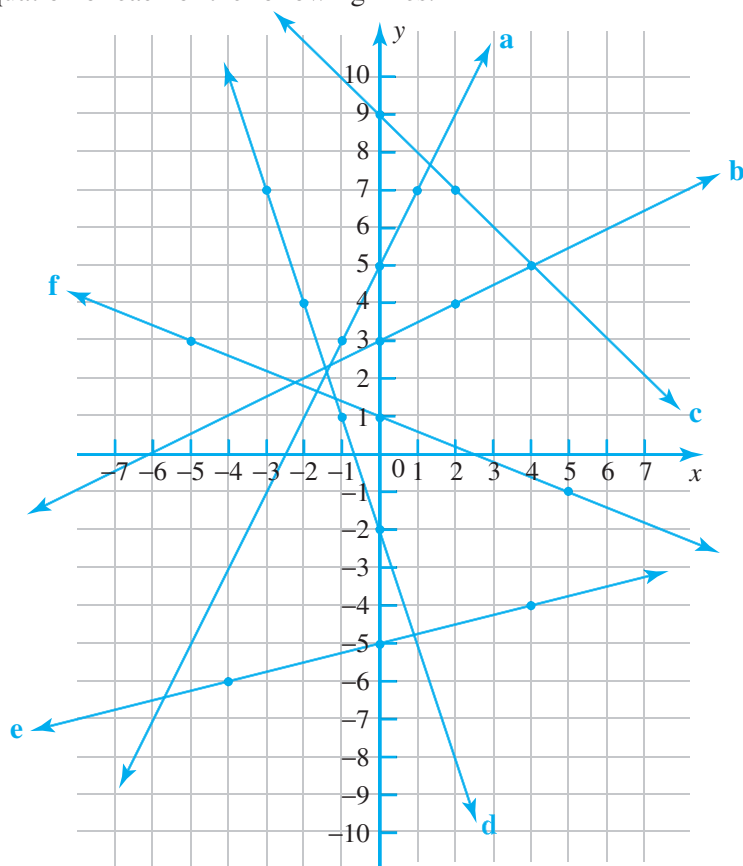
f a slope of -1 and passes through the origin

Consolidation

3 Find the gradient and y-intercept of each line, and hence write down its equation.



4 Find the equation of each of the following lines.



5 By substitution into the equation $y = mx + b$, find the equation of the line that passes through the given point with the given gradient.

a $(1, 1), m = 2$

b $(2, 3), m = 1$

c $(-1, 2), m = 3$

d $(-2, -4), m = -2$

e $(5, 0), m = -1$

f $(0, 4), m = -3$

g $(2, 4), m = \frac{1}{2}$

h $(-4, 5), m = -\frac{1}{2}$

i $(6, -1), m = \frac{1}{3}$

j $(-6, 2), m = -\frac{2}{3}$

k $(4, -3), m = -\frac{5}{4}$

l $(8, -3), m = \frac{3}{2}$

6 Graph each of these lines by plotting the y-intercept, then use the gradient to find two other points on the line.

a $y = x + 4$

b $y = 2x + 3$

c $y = \frac{1}{2}x - 1$

d $y = 2 - x$

e $y = -3x + 4$

f $y = -\frac{1}{3}x - 2$

g $y = \frac{1}{4}x + 1$

h $y = 2 - 4x$

i $y = 5 - \frac{1}{2}x$

j $y = \frac{2}{3}x + 2$

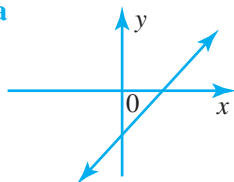
k $y = \frac{3}{2}x - 3$

l $y = -\frac{3}{4}x - 1$

7 Choose the equation that best describes each graph below.

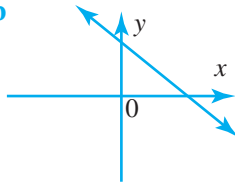
• $y = 2x + 5$

a



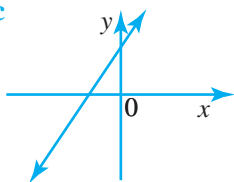
• $y = 2x - 5$

b



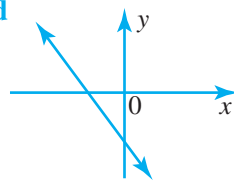
• $y = 5 - 2x$

c



• $y = -2x - 5$

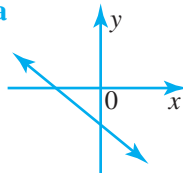
d



8 Choose the equation that best describes each graph below.

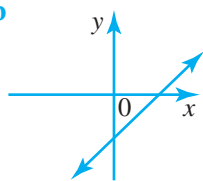
• $y = 7 - 3x$

a



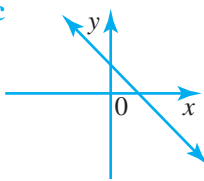
• $y = 3x + 7$

b



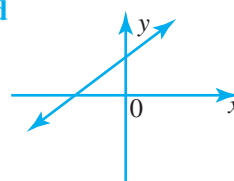
• $y = -3x - 7$

c



• $y = 3x - 7$

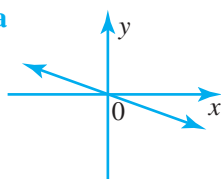
d



9 Choose the equation that best describes each graph below.

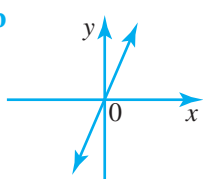
• $y = 2x$

a



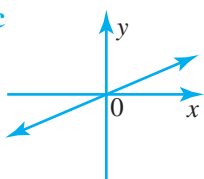
• $y = -2x$

b



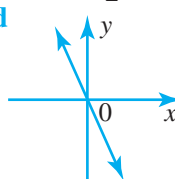
• $y = \frac{1}{2}x$

c



• $y = -\frac{1}{2}x$

d



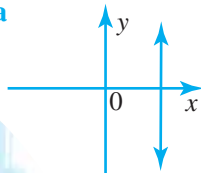
10 Choose the equation that best describes each graph below.

• $y = 5x$

• $x = -5$

• $y = 1 - 5x$

a

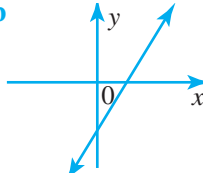


• $y = -5x$

• $y = -5$

• $y = -5x - 1$

b

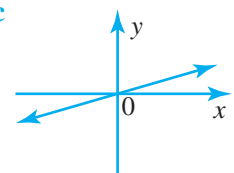


• $x = 5$

• $y = 5x + 1$

• $y = \frac{1}{5}x$

c

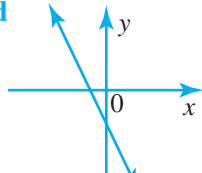


• $y = 5$

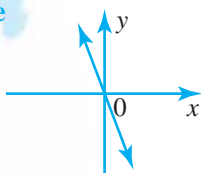
• $y = 5x - 1$

• $y = -\frac{1}{5}x$

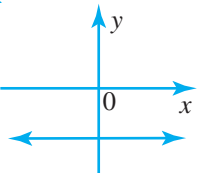
d



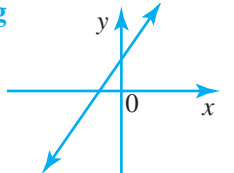
e



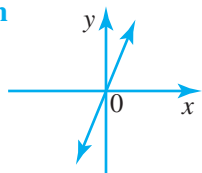
f

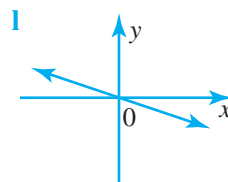
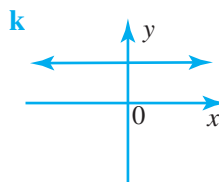
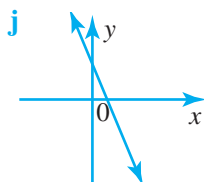
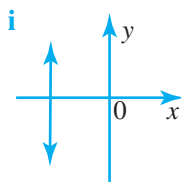


g



h





11 i What does each set of lines have in common?

ii How do they differ?

a $y = 2x$, $y = 2x + 7$, $y = 2x - 4$

b $y = 3x + 5$, $y = x + 5$, $y = 5 - \frac{1}{2}x$

c $y = x$, $y = 4x$, $y = \frac{1}{2}x$, $y = -\frac{2}{3}x$

d $y = 4$, $y = -3$, $y = 0$, $y = \frac{3}{5}$

12 Choose from the equations below, the lines that:

a slope to the left

b pass through the origin

c have a positive y-intercept

d have a gradient that is greater than 1

e are perpendicular to the x-axis

A $y = 5 - 2x$

B $y = x$

C $y = \frac{1}{2}x - 1$

D $x = 4$

E $y = 3x$

F $y = -x - 2$

G $y = 7 + 2x$

H $y = -\frac{5}{6}x$

I $x = 0$

J $y = \frac{5}{4}x - 3$

K $y = -x$

L $x = -2$

13 Consider the line with equation $y = \frac{1}{2}x + 3$. What would be the effect on the graph by changing:

a the gradient to 2?

b the gradient to $-\frac{1}{2}$

c the y-intercept to -1?

14 Consider the line with equation $y = \frac{1}{3}x + 2$.

a What would be the effect on the graph if the gradient was changed to 0?

b What would the equation become as a result of that change?

■ Further applications

15 Find the equation of the line that passes through the point:

a (2, 10) and cuts the y-axis at 4

b (-1, 11) and cuts the y-axis at 9

c (4, -6) and cuts the y-axis at -4

16 Find the equation of the line that passes through the points:

a (2, 11) and (5, 20)

b (6, 9) and (-10, 1)

c (12, 2) and (-6, 14)

17 Find the equation of the line that cuts the x-axis at p and the y-axis at q , where $p > 0$ and $q > 0$.



TRY THIS Latitude and temperature

Below is a list of coastal Australian cities with their latitude (in degrees and minutes) and average annual air temperature ($^{\circ}\text{C}$). Can you find a relationship of the form $T = kL + c$ where k and c are constants, T = temperature in degrees Celsius and L = latitude in degrees?

City	Latitude	$^{\circ}\text{C}$	City	Latitude	$^{\circ}\text{C}$
Hobart	$42^{\circ}53'$	12.4	Perth	$31^{\circ}57'$	18.2
Melbourne	$37^{\circ}49'$	14.9	Brisbane	$27^{\circ}28'$	20.6
Sydney	$33^{\circ}50'$	17.4	Darwin	$12^{\circ}25'$	27.5

Source: *Commonwealth Year Book*

When you have found a rule, use a Commonwealth or State Year Book to look up temperatures of other coastal cities and test the rule to see how well it works. Does your rule work for inland cities? (If not, can you suggest a reason?)

**FOCUS ON WORKING MATHEMATICALLY****TELEVISION ADVERTISING**

Introduction

Every month, companies spend thousands of dollars on advertising. It has been found by market research that television is generally an excellent medium in which to advertise a product, particularly if it is shown during a popular program. However, it is very expensive.

In this activity we will look at some hypothetical data to investigate a possible relationship between the amount of money spent on advertising and the volume of sales. A company wants to know whether television advertising is worthwhile, that is whether advertising is actually related to sales.



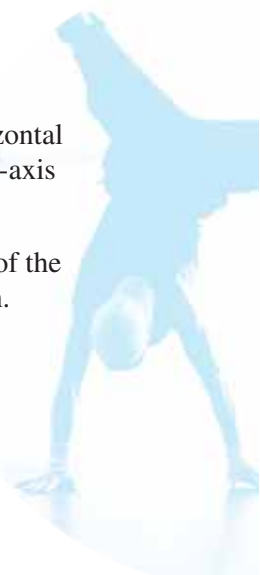
LEARNING ACTIVITIES

Materials: graph paper, ruler and pencil, a graphics calculator or Excel spreadsheet

The following table shows company data for monthly sales (y) against television advertising expenditure (x) for a random sample of 10 months taken from the two previous years. Monthly sales (y) is the dependent variable and monthly TV advertising expenditure (x) is the independent variable. The mathematical objective is to obtain a linear equation that will predict monthly sales as a function of advertising expenditure.

Month	TV advertising (x) $\times$ \$100 000	Sales (y) $\times$ \$100 000
1	1.1	110
2	0.7	70
3	0.8	82
4	0.9	90
5	1.0	94
6	1.2	105
7	0.7	86
8	1.2	100
9	0.9	75
10	1.0	98

- 1 On graph paper draw a scatter diagram of the data in the table showing x on the horizontal axis and y on the vertical axis. Choose a suitable scale, say 1 cm = 0.1 units on the x -axis and 1 cm = 10 units on the y -axis. Adjust the origin as convenient.
- 2 Examine the shape of the scatter diagram. What do you notice? Calculate the mean of the x -values $\bar{x}$ and the mean of the y -values $\bar{y}$ and plot this point $M(\bar{x}, \bar{y})$ on the graph.



- 3 Draw a line of best fit, using your eye to guide you. Assume that this line will pass through M . It should be as close to as many of the points on the scatter diagram as possible.
- 4 Now measure the gradient (or slope) of the graph. Let this number be m .
- 5 Where does the graph cross the y -axis? Let this number be b .
- 6 Write down the equation of the graph using your values for m and b .
- 7 Use your model to predict the sales volume for a month if the TV advertising expenditure is raised to \$140 000.
- 8 Do you feel that there is enough evidence for the company to conclude that TV advertising expenditure is indeed related to volumes of sales? Why? What assumptions would the company have to make in using the equation to predict sales volumes from TV advertising expenditure?
- 9 Do you think the figures for TV advertising expenditure are realistic? See if you can find out from the Internet what some companies spend on TV advertising per month as a fraction of their sales.



CHALLENGE

- 1 Use a graphics calculator or a spreadsheet such as Excel to plot the data in the table on a scatter diagram, and calculate the slope and the y -intercept of the line of best fit. What is the correlation coefficient for this data? Discuss the goodness of fit to a linear model with your teacher.
- 2 You may have found that the calculated line of best fit $y = 58x + 36$ (rounding to the nearest whole numbers) is quite different from what you obtained by eye. What methods do people use when they want greater accuracy, but a computer is not available? Discuss this in class with your teacher.



LET'S COMMUNICATE

Discuss what you have learnt from this activity with a classmate or perhaps if you have worked in groups for the activity, with the group members. Can the group pose another problem that arises from what you have found? For example, is a linear model the best one for predicting sales? What else could be used?



REFLECTING

Mathematics is a powerful tool for predicting relationships in the world of business to guide decision-making. Think over the use of mathematics for economic forecasting and the indices that are regularly reported on TV as evidence of the health of the economy.





MACQUARIE

Language link with Macquarie

- 1 Explain the difference between an **axis** and an **origin**.
- 2 If the terms in a number pattern increase or decrease by a common difference, they are said to form a **linear relationship**.
- 3 What is another word for **gradient**?
- 4 When is the word **concurrent** used to describe two lines on a graph?

- 5 Read the Macquarie Learners Dictionary entries for **linear** and **lineage**:

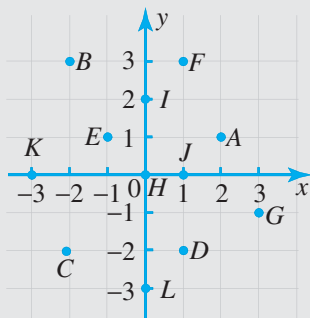
linear adjective 1. relating to lines or length: a linear measure 2. arranged in a line: a linear series

lineage noun 1. descent from a line of ancestors: My family's lineage can be traced back to the First Fleet.

Compare the uses of these related words.

- 1 Write down the co-ordinates of:

a	A	b	B	c	C	d	D
e	E	f	F	g	G	h	H
i	I	j	J	k	K	l	L



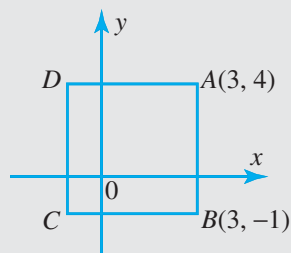
- 2 In which quadrant does each point lie?

a	(2, -4)	b	(-3, -1)
c	(1, 5)	d	(-7, 6)
- 3 How far apart are the points:

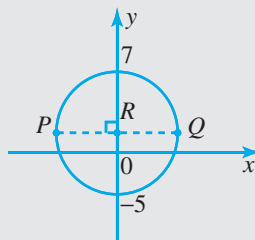
a	(3, 5) and (10, 5)
b	(1, -4) and (1, -2)?
- 4 Find the co-ordinates of the point that is:

a	6 units above (4, -1)
b	3 units to the left of (-2, -3)

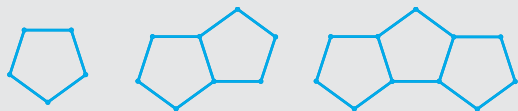
5



- Write down the co-ordinates of C and D such that ABCD is a square.
 - Calculate the perimeter.
- 6 a Plot the points R(4, 7), S(4, -3) and T(-5, -3) on a number plane.
 - Find the area of $\triangle RST$.
 - 7 The circle shown has centre R and diameter PQ perpendicular to the y-axis. Find the co-ordinates of P, Q and R.



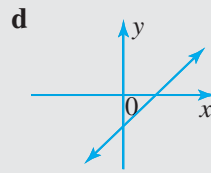
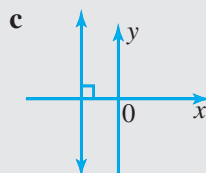
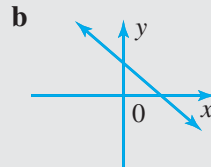
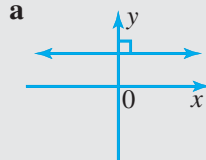
- 8 a Draw the next step in this pattern.



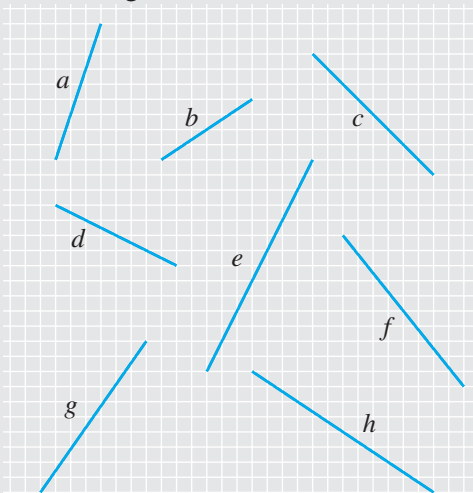
- b Copy and complete this table of values.

Number of pentagons (x)	1	2	3	4	5	6
Number of dots (y)						

- c How many dots are added in each step?
- d Write down an equation in the form $y = mx + b$ that shows the relationship between the number of dots and the number of pentagons.
- e Plot the values from this table on a number grid, with the number of pentagons along the horizontal axis and the number of dots along the vertical axis. Is the relationship linear?
- 9 Graph each of these lines on a number plane by plotting at least 3 points.
- a $y = 3x$ b $y = 2x - 3$
 c $y = 4 - x$ d $3x + y = 3$
- 10 Describe the transformations that would be necessary to map:
- a $y = -x$ to $y = x - 2$
 b $y = 3x$ to $y = 7 - 3x$
 c $y = 6x - 5$ to $y = -6x$
- 11 Find the x - and y -intercepts and hence sketch each of these lines.
- a $y = 6 - 2x$
 b $3x - 2y - 12 = 0$
 c $y = \frac{1}{3}x + 1$
- 12 Which two of these points lie on the line $y = 5 - 3x$?
- A $(2, -1)$ B $(0, 2)$
 C $(-4, 17)$ D $(-\frac{1}{3}, 4)$
- 13 Which two of these lines pass through the point $(3, -4)$?
- A $y = -x + 1$ B $y = -2x - 2$
 C $y = x - 7$ D $2x - y = 10$
- 14 a The point $(a, 5)$ lies on the line $y = 2x - 7$. Find the value of a .
 b The line $4x + 3y - 7 = 0$ passes through the point $(-2, k)$. Find the value of k .
- 15 Sketch each of the following lines.
- a $x = 4$ b $x = -2$
 c $y = 1$ d $y = -3$
- 16 At what point do the lines $y = 6$ and $x = 1$ intersect?
- 17 Write down the equation of each of the co-ordinate axes.
- 18 Find the equation of the line that is:
- a parallel to the y -axis and passes through the point $(4, -2)$
 b parallel to the x -axis and passes through the point $(-1, -3)$
- 19 State whether the gradient of each line is positive, negative, zero or undefined.



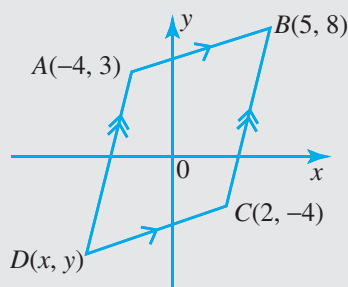
- 20 Find the gradient of each interval.



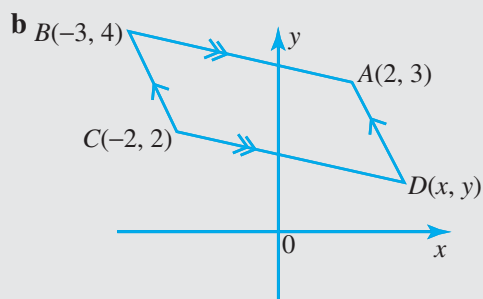
- 21 Complete this statement: 'If two lines are parallel, then they have the same _____'.

- 22 Find the co-ordinates of D , the fourth vertex of each parallelogram.

a



b



- 23 State the gradient and y-intercept of each line.

a $y = 3x + 8$ b $y = 4x - 4$

c $y = 7 - x$ d $y = -2x$

e $y = \frac{1}{2}x + 11$ f $y = \frac{4}{3}x - 1$

- 24 Write down the equation of the line that has:

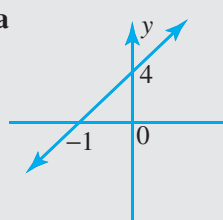
a a gradient of 2 and a y-intercept of -4

b a slope of -3 and cuts the y-axis at 5

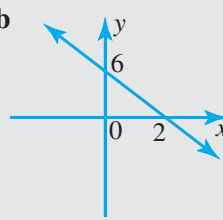
c a gradient of $\frac{1}{4}$ and passes through the origin

- 25 For each line find the gradient and the y-intercept and then write down its equation.

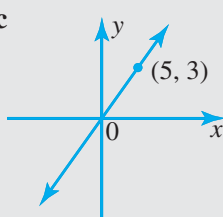
a



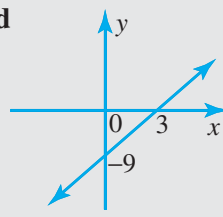
b



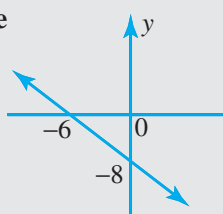
c



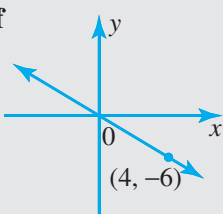
d



e



f



- 26** By substitution into the equation $y = mx + b$, find the equation of the line that passes through the given point and has the given gradient.

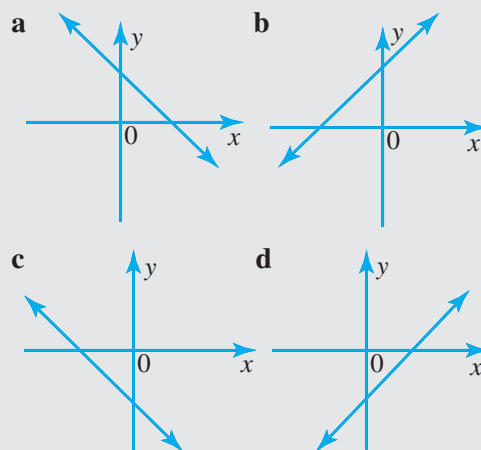
- a** $(2, 13)$, $m = 3$
b $(-9, 23)$, $m = -2$
c $(18, 8)$, $m = \frac{2}{3}$

- 27** Graph each of these lines by plotting the y -intercept, then using the gradient find two more points on the line.

- a** $y = x + 3$ **b** $y = \frac{1}{2}x - 1$
c $y = -3x + 2$ **d** $y = -\frac{2}{3}x$

- 28** Match each of these equations with one of the graphs provided.

- $y = 7x + 4$ • $y = 7x - 4$
 • $y = -7x + 4$ • $y = -7x - 4$



- 29** Graph the lines $y = x + 3$ and $y = -3x - 1$ on the same number plane. Hence, find their point of intersection.
- 30** Determine by substitution whether the lines $3x - 2y + 18 = 0$ and $y = 5 - 2x$ intersect at $(4, -3)$.

12 Trigonometry



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ recognise that the ratio of matching sides in similar right-angled triangles is constant for equal angles
- ⦿ name the hypotenuse, opposite and adjacent sides in a right-angled triangle in relation to an acute angle
- ⦿ state the values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ given the side lengths in a right-angled triangle
- ⦿ find decimal approximations of the trigonometric ratios of a given angle
- ⦿ find an angle correct to the nearest degree using a calculator, given one of the trigonometric ratios of the angle
- ⦿ find the length of a short side in a right-angled triangle using trigonometry
- ⦿ find the length of the hypotenuse in a right-angled triangle using trigonometry
- ⦿ find the size of an acute angle in a right-angled triangle using trigonometry
- ⦿ solve practical problems that involve finding the length of a side by trigonometry
- ⦿ solve practical problems that involve finding an angle by trigonometry
- ⦿ identify angles of elevation and depression on diagrams
- ⦿ solve practical problems that involve angles of elevation and depression
- ⦿ draw diagrams showing the compass bearing of one point from another point
- ⦿ draw diagrams showing the true bearing of one point from another point
- ⦿ find the bearing of A from B given the bearing of B from A
- ⦿ solve practical trigonometry problems involving bearings.

Trigonometry is a branch of geometry. The word trigonometry comes from the Greek words *trigonon*, meaning ‘a triangle’, and *metron*, meaning ‘a measure’. It is concerned with the measurement of triangles, specifically with finding the length of a side or the size of an angle. Trigonometry is used in many fields, such as surveying, navigation and astronomy. In surveying, for example, trigonometry allows us to find the height of a tall building or to calculate the distance to an inaccessible location, such as a tree that is on the other side of a river.

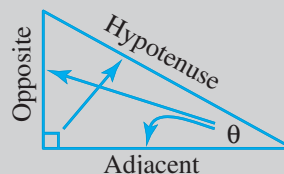
Trigonometry is based on the properties of similar triangles. You should remember from Year 8 that in two similar triangles, all angles are equal and the corresponding sides are in the same ratio.

12.1 Side ratios in right-angled triangles

In a right-angled triangle it is common to use the Greek letter θ (pronounced *theta*) to refer to an angle whose size is unknown. Other Greek letters such as α and β are also used often. The *hypotenuse* is the side that lies opposite the right angle. It is the longest side in the triangle. The two shorter sides are named according to their positions relative to the angle θ .

The sides of a right-angled triangle are named as follows:

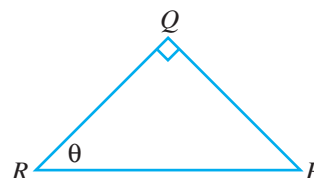
- ⊙ the side opposite the right angle is called the *hypotenuse*
- ⊙ the side opposite the angle θ is called the *opposite*
- ⊙ the side adjacent to the angle θ is called the *adjacent*.



Example 1

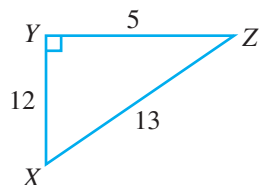
In $\triangle PQR$, name the:

- a hypotenuse
- b side opposite the angle θ
- c side adjacent to the angle θ



Solutions

- a PR is the hypotenuse. b PQ is the opposite side. c QR is the adjacent side.

EG
+ S**Example 2**Find the value of each ratio in $\triangle XYZ$.

a $\frac{\text{side opposite } \angle X}{\text{hypotenuse}}$

b $\frac{\text{side adjacent to } \angle X}{\text{hypotenuse}}$

c $\frac{\text{side opposite } \angle Z}{\text{side adjacent to } \angle Z}$

Solutions

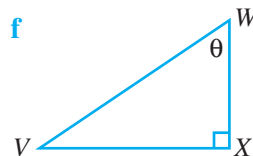
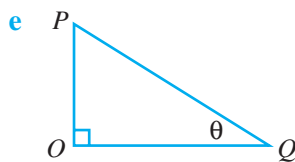
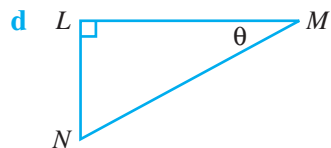
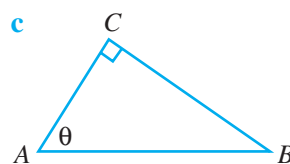
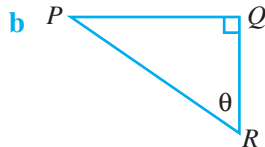
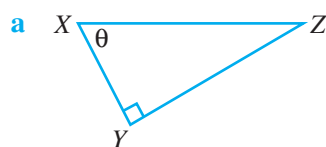
a $\frac{\text{side opposite } \angle X}{\text{hypotenuse}} = \frac{5}{13}$

b $\frac{\text{side adjacent to } \angle X}{\text{hypotenuse}} = \frac{12}{13}$

c $\frac{\text{side opposite } \angle Z}{\text{side adjacent to } \angle Z} = \frac{12}{5}$

Exercise 12.1

1 For each triangle below, name:

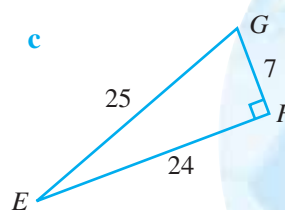
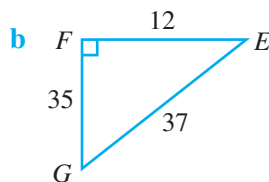
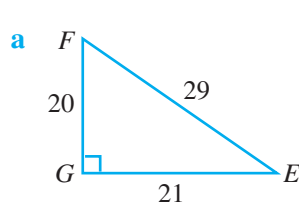
i the hypotenuse**ii** the opposite side**iii** the adjacent side2 Draw $\triangle TUV$ in which $\angle U = 90^\circ$. Which side is:**a** opposite $\angle T$?**b** adjacent to $\angle T$?**c** opposite $\angle V$?**d** adjacent to $\angle V$?

3 Find the value of each ratio for the triangles below.

i $\frac{\text{side opposite } \angle E}{\text{side adjacent to } \angle E}$

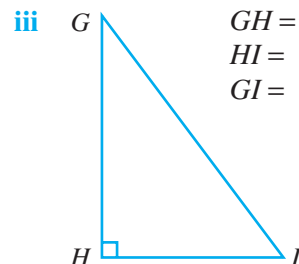
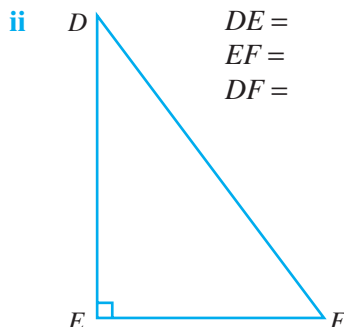
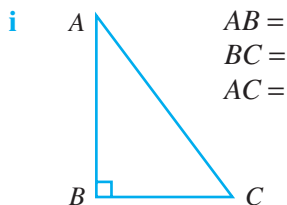
ii $\frac{\text{side opposite } \angle E}{\text{hypotenuse}}$

iii $\frac{\text{side adjacent to } \angle E}{\text{hypotenuse}}$



Consolidation

4 a Measure the lengths of the sides in each triangle, correct to the nearest millimetre.



b Measure the acute angles in the triangles. Which angles are equal to:

i $\angle A$?

ii $\angle C$?

c Are the triangles similar? Why?

d Find, as a fraction in simplest form, the following side ratios.

i $\frac{BC}{AB}$

ii $\frac{EF}{DE}$

iii $\frac{HI}{GH}$

iv $\frac{BC}{AC}$

v $\frac{EF}{DF}$

vi $\frac{HI}{GI}$

vii $\frac{AB}{AC}$

viii $\frac{DE}{DF}$

ix $\frac{GH}{GI}$

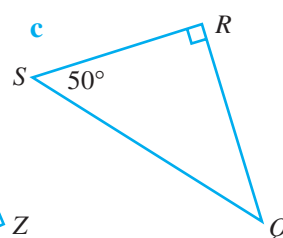
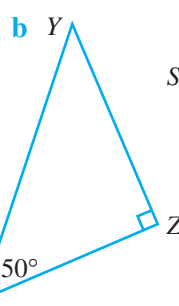
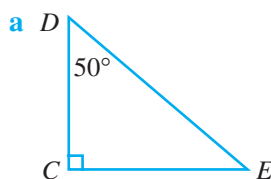
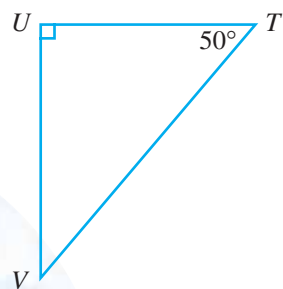
e What do you notice about the ratios of the matching sides?

5 Without measuring, write down the side ratio in each triangle that is equal to:

i $\frac{UV}{TU}$

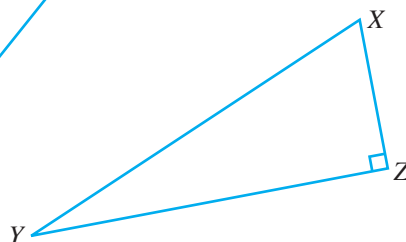
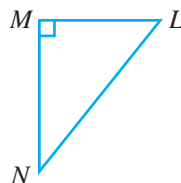
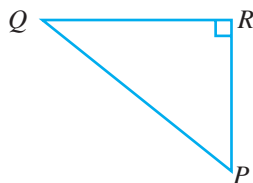
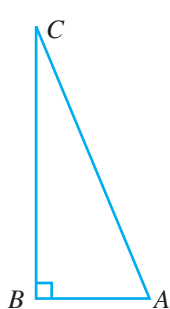
ii $\frac{TU}{TV}$

iii $\frac{UV}{TV}$



Further applications

6



a Find each of these ratios in simplest form.

i $\frac{AB}{BC}$

ii $\frac{PR}{QR}$

iii $\frac{LM}{MN}$

iv $\frac{XZ}{YZ}$

b Using your results from part a, write down the angle that is equal to:

i $\angle C$

ii $\angle A$

iii $\angle Q$

iv $\angle P$

12.2 The trigonometric ratios

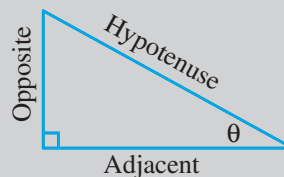
In the previous exercise we saw that the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$ and $\frac{\text{opposite}}{\text{adjacent}}$ have constant values in similar, right-angled triangles. Later, we will use this fact to find the length of a side or the size of an angle in a triangle. Since they are used so often, these ratios are given special names: the **sine** ratio ($\sin \theta$), the **cosine** ratio ($\cos \theta$) and the **tangent** ratio ($\tan \theta$). Together they are known as the **trigonometric ratios** (or trig ratios for short).

The definitions of the trigonometric ratios are:

⊙ $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

⊙ $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

⊙ $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$



NOTE: The abbreviations **SOH**, **CAH** and **TOA** can be used to help you memorise these definitions.

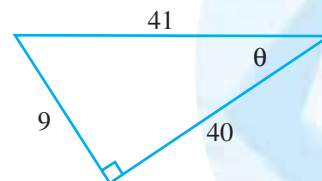
Example 1

Find each of the following trigonometric ratios for the given triangle.

a $\sin \theta$

b $\cos \theta$

c $\tan \theta$



Solutions

$$\begin{aligned} \text{a } \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{9}{41} \end{aligned}$$

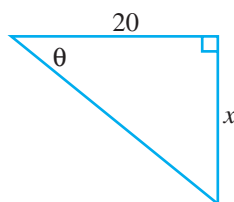
$$\begin{aligned} \text{b } \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{40}{41} \end{aligned}$$

$$\begin{aligned} \text{c } \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{9}{40} \end{aligned}$$

Example 2

Find the value of the pronumeral given that

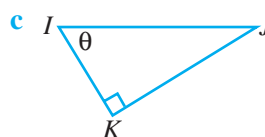
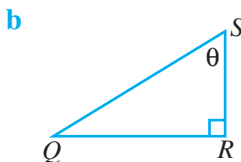
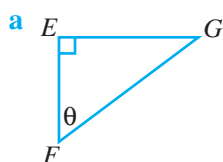
$$\tan \theta = \frac{3}{4}.$$

**Solution**

$$\begin{aligned} \tan \theta &= \frac{3}{4} \\ \frac{x}{20} &= \frac{3}{4} \\ \times 20 \quad \times 20 \\ \therefore x &= 15 \end{aligned}$$

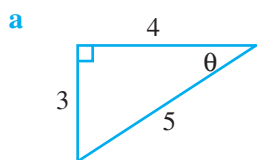
Exercise 12.2

1 Name the opposite side, adjacent side and hypotenuse in each triangle.

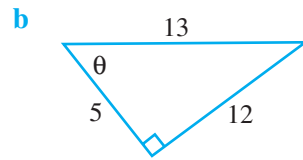


2 For each of the following triangles, state as a fraction, the value of:

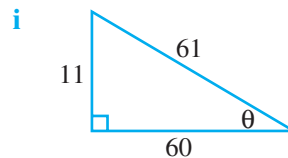
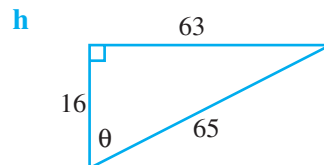
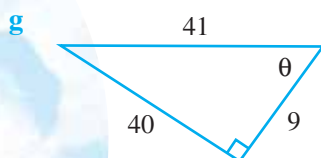
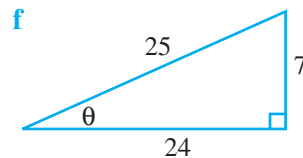
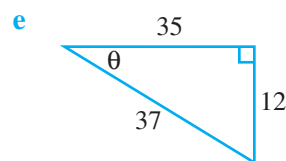
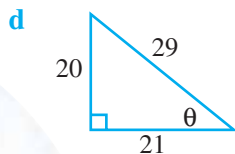
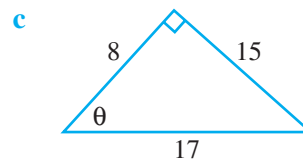
i $\sin \theta$



ii $\cos \theta$

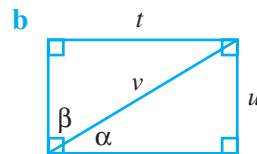
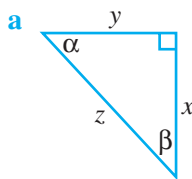


iii $\tan \theta$



3 For each figure, find:

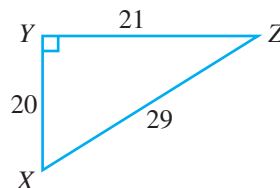
i	$\sin \alpha$	ii	$\cos \alpha$
iii	$\tan \alpha$	iv	$\sin \beta$
v	$\cos \beta$	vi	$\tan \beta$



Consolidation

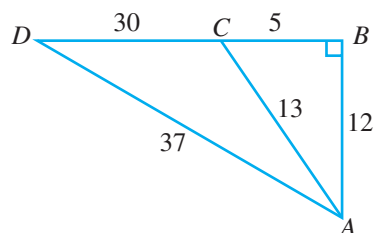
4 Which angle has a:

a	sine of $\frac{20}{29}$?	b	tangent of $\frac{21}{20}$?
c	cosine of $\frac{21}{29}$?	d	sine of $\frac{21}{29}$?
e	cosine of $\frac{20}{29}$?	f	tangent of $\frac{20}{21}$?



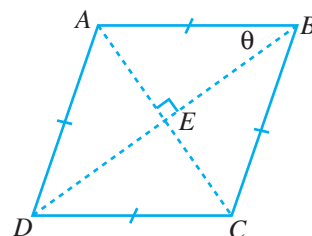
5 Find the value of:

a	$\sin \angle BAC$	b	$\tan \angle BDA$
c	$\cos \angle BAD$	d	$\sin \angle ACB$
e	$\cos \angle CAB$	f	$\tan \angle BAD$
g	$\sin \angle ADB$	h	$\tan \angle BAC$
i	$\cos \angle BCA$	j	$\tan \angle BCA$
k	$\cos \angle BDA$	l	$\sin \angle DAB$

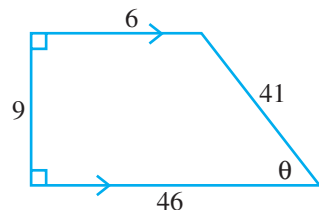


6 A rhombus $ABCD$ has diagonals AC and BD of length 16 cm and 30 cm respectively, intersecting at E .

- a Find the side length of the rhombus.
b Find values for $\sin \theta$, $\cos \theta$ and $\tan \theta$, where $\angle ABE = \theta$.



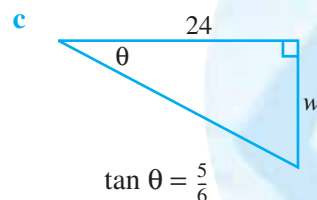
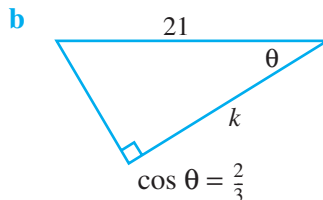
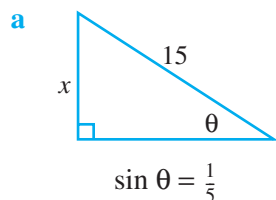
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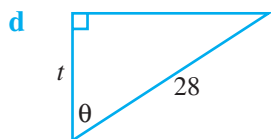


Find values for $\sin \theta$, $\cos \theta$ and $\tan \theta$ in this trapezium.
(HINT: Draw a right-angled triangle.)

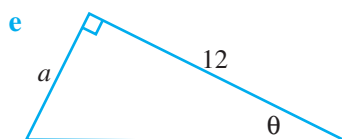
8 In $\triangle ABC$, AB is perpendicular to BC and $AB = BC$. Find the value of $\tan A$.

9 Find the value of the pronumeral in each of these:

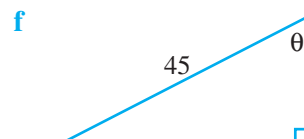




$$\cos \theta = \frac{2}{7}$$

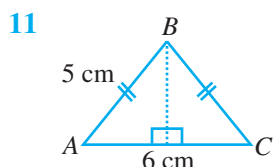


$$\tan \theta = \frac{3}{4}$$



$$\sin \theta = \frac{8}{9}$$

- 10** In $\triangle PQR$, $\angle Q = 90^\circ$, $PQ = 14$ mm and $QR = 48$ mm. Find values for $\tan P$ and $\tan R$ in simplest form.

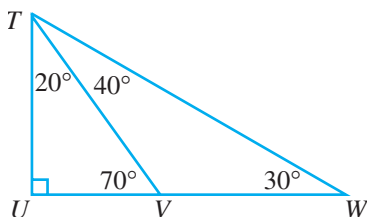


In the isosceles $\triangle ABC$, $AB = BC = 5$ cm and $AC = 6$ cm. Find values for:

- a** $\cos A$ **b** $\tan A$ **c** $\sin A$

- 12** In the isosceles $\triangle PQR$, $PQ = QR$ and $\cos R = \frac{7}{25}$. Find the length of PR if $QR = 50$ mm.

- 13** Write true (T) or false (F) for each of these.



a $\sin 20^\circ = \frac{UV}{TV}$

b $\tan 70^\circ = \frac{UV}{TV}$

c $\tan 30^\circ = \frac{TV}{TW}$

d $\sin 70^\circ = \frac{TV}{TW}$

e $\cos 30^\circ = \frac{VW}{TW}$

f $\tan 40^\circ = \frac{VW}{TV}$

g $\cos 20^\circ = \frac{TV}{TW}$

h $\sin 60^\circ = \frac{UV}{TW}$

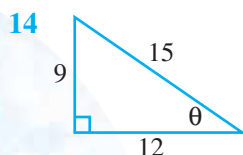
i $\cos 70^\circ = \frac{UV}{TV}$

j $\tan 20^\circ = \frac{UV}{TW}$

k $\sin 30^\circ = \frac{TV}{TW}$

l $\cos 60^\circ = \frac{TV}{TW}$

Further applications



- a** If $\sin^2 \theta$ means $(\sin \theta)^2$, write down values for $\sin^2 \theta$ and $\cos^2 \theta$.

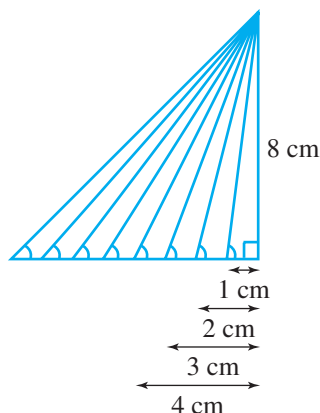
- b** Hence, find the value of $\sin^2 \theta + \cos^2 \theta$.

- c** Repeat this question using any two triangles from Q2. What do you notice?

- 15** For each of the following, draw a right-angled triangle and label one of the acute angles as θ . Use the given ratio and Pythagoras' theorem to find values for the other ratios.

- a** If $\sin \theta = \frac{7}{25}$, find values for $\cos \theta$ and $\tan \theta$.

- b** If $\cos \theta = \frac{60}{61}$, find values for $\sin \theta$ and $\tan \theta$.

TRY THIS**Height to base ratio**

Draw a line of height 8 cm and a base line of 1 cm. Calculate the ratio of height : base and use your protractor to measure the shaded base angle. Now extend the base line 1 cm at a time, and again calculate the ratio and find the angle. Complete a table:

Distance from base	1	2	3	4	5	6	7	8	9	10
Ratio, height : base										
Angle										

- 1 What can you conclude about the ratio and angle size relationship?
- 2 Is the decrease in the angle size constant each time?

12.3**Trigonometric ratios using a calculator****Degrees and minutes**

Angles are measured in degrees and minutes (1 degree = 60 minutes, i.e. $1^\circ = 60'$). When we round off angles correct to the nearest degree, angles with less than 30 minutes are rounded down and angles with 30 minutes or more are rounded up. The degrees, minutes and seconds key on your calculator should look like either **DMS** or **° ' "**.

Evaluating trigonometric expressions

A calculator can be used to find the value of a trigonometric expression. The order in which you press the keys will vary between calculator models.

To evaluate a trigonometric expression, either:

- ⊙ press the appropriate trigonometric ratio key, then enter the angle, or
- ⊙ enter the angle, then press the appropriate trigonometric ratio key.

Finding an angle

The inverse key $\boxed{\text{INV}}$, shift key $\boxed{\text{shift}}$, or second function key $\boxed{2\text{nd F}}$, can be used to undo the process of finding the sine, cosine or tangent of an angle, and hence find the size of the angle θ .

To find the size of an angle given either a fraction or decimal:

- ⊙ press either the $\boxed{\text{INV}}$, $\boxed{\text{shift}}$ or $\boxed{2\text{nd F}}$ key followed by the appropriate trigonometric function key
- ⊙ enter the fraction or decimal into the calculator, then press $\boxed{=}$
- ⊙ round off the angle as required.

Example 1

Round off each angle correct to the nearest degree.

a $17^\circ 28'$

b $32^\circ 49'$

c $71^\circ 30'$

Solutions

a $28'$ is less than $30'$, so we round down, $\therefore 17^\circ 28' \doteq 17^\circ$.

b $49'$ is greater than $30'$, so we round up, $\therefore 32^\circ 49' \doteq 33^\circ$.

c $30'$ is halfway, so we round up, $\therefore 71^\circ 30' \doteq 72^\circ$.

Example 2

Evaluate each expression, correct to 2 decimal places.

a $\sin 48^\circ$

b $12 \cos 39^\circ$

c $\frac{25.6}{\tan 55^\circ}$

d $\frac{\sin 73^\circ}{\cos 10^\circ + \tan 25^\circ}$

Solutions

	Calculator steps	Calculator readout	Answer
a	$\boxed{\sin} \boxed{48} \boxed{=}$	0.743144825	0.74
b	$\boxed{12} \boxed{\times} \boxed{\cos} \boxed{39} \boxed{=}$	9.325751537	9.33
c	$\boxed{25.6} \boxed{\div} \boxed{\tan} \boxed{55} \boxed{=}$	17.92531298	17.93
d	$\boxed{\sin} \boxed{73} \boxed{\div} \boxed{(} \boxed{\cos} \boxed{10} \boxed{+} \boxed{\tan} \boxed{25} \boxed{)} \boxed{=}$	0.659013575	0.66

Example 3

Find θ correct to the nearest degree.

a $\sin \theta = 0.4275$

b $\tan \theta = 1.1482$

Solutions

	Calculator steps	Calculator readout	Answer
a	$\boxed{2\text{nd F}} \boxed{\sin} \boxed{0.4275} \boxed{=}$	25.30900817	25°
b	$\boxed{2\text{nd F}} \boxed{\tan} \boxed{1.1482} \boxed{=}$	48.94646772	49°

Exercise 12.3

1 Round off each angle, correct to the nearest degree.

a $18^\circ 53'$

b $26^\circ 10'$

c $35^\circ 30'$

d $7^\circ 49'$

e $41^\circ 4'$

f $50^\circ 22'$

g $12^\circ 58'$

h $63^\circ 30'$

i $102^\circ 17'$

j $125^\circ 30'$

k $142^\circ 54'$

l $166^\circ 51'$

2 Evaluate these trigonometric expressions, correct to 2 decimal places.

a $\sin 35^\circ$

b $\tan 54^\circ$

c $\cos 12^\circ$

d $\tan 18^\circ$

e $38 \sin 65^\circ$

f $51 \cos 29^\circ$

g $16.4 \tan 83^\circ$

h $42.7 \sin 10^\circ$

i $\frac{23}{\cos 74^\circ}$

j $\frac{15}{\tan 18^\circ}$

k $\frac{57.2}{\sin 45^\circ}$

l $\frac{10.4}{\cos 67^\circ}$

m $\frac{1}{\sin 55^\circ}$

n $\frac{1}{\tan 21^\circ}$

o $\frac{1}{\cos 14.6^\circ}$

p $\frac{1}{\tan 87.5^\circ}$

Consolidation

3 Evaluate, correct to the nearest hundredth:

a $\frac{10 \sin 56^\circ}{7 \sin 23^\circ}$

b $\frac{18 \cos 12^\circ}{13 \tan 68^\circ}$

c $\frac{21 \tan 84^\circ}{15 \cos 71^\circ}$

d $\frac{\cos 70^\circ \sin 24^\circ}{\tan 55^\circ}$

e $\frac{\sin 57^\circ + \tan 9^\circ}{\cos 18^\circ}$

f $\frac{\cos 15^\circ - \tan 40^\circ}{\sin 60^\circ}$

g $\frac{\sin 82^\circ}{\tan 16^\circ \cos 53^\circ}$

h $\frac{\tan 65^\circ}{\cos 72^\circ - \sin 10^\circ}$

i $\frac{\cos 33^\circ}{\tan 14^\circ + \cos 15^\circ}$

j $\frac{\tan 69^\circ - \sin 24^\circ}{\sin 36^\circ + \cos 9^\circ}$

k $\frac{\sin 45^\circ + \cos 72^\circ}{\tan 29^\circ \tan 50^\circ}$

l $\frac{\cos 40^\circ \sin 16^\circ}{\sin 40^\circ \cos 16^\circ}$

4 Find the value of θ , correct to the nearest degree.

a $\tan \theta = 0.2867$

b $\cos \theta = 0.9063$

c $\sin \theta = 0.9744$

d $\cos \theta = 0.4179$

e $\sin \theta = 0.6111$

f $\tan \theta = 0.9041$

g $\sin \theta = 0.8845$

h $\tan \theta = 3.0624$

i $\cos \theta = 0.6088$

j $\tan \theta = 0.5276$

k $\sin \theta = 0.8301$

l $\cos \theta = 0.0615$

m $\sin \theta = 0.0247$

n $\cos \theta = 0.9589$

o $\tan \theta = 5.7594$

5 Find the acute angle θ , correct to the nearest degree.

a $\cos \theta = \frac{3}{7}$

b $\sin \theta = \frac{4}{11}$

c $\tan \theta = \frac{1}{10}$

d $\cos \theta = \frac{5}{6}$

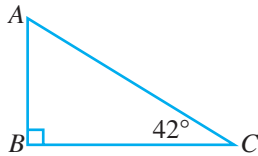
e $\sin \theta = \frac{8}{9}$

f $\tan \theta = \frac{6}{17}$

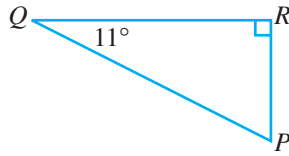
g $\cos \theta = \frac{2}{13}$

h $\tan \theta = \frac{12}{5}$

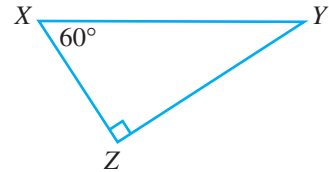
- $$\mathbf{a} \quad \frac{BC}{AC}$$



b $\frac{PR}{PQ}$



c $\frac{XZ}{YZ}$



a $\sin \alpha$

b $\sin 2\alpha$

c $2 \sin \alpha$

d $\sin\left(\frac{\alpha}{2}\right)$

e $\sin (\alpha + 30^\circ)$

f $3 \sin (2\alpha - 65^\circ)$

NOTE: In those questions involving the tangent ratio where the pronumeral would be in the denominator, it is often easier to find the other acute angle in the triangle and use it to find the required side length. By using the other angle, the pronumeral should then be in the numerator.

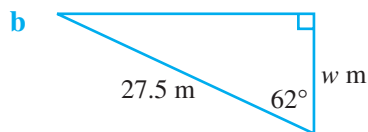
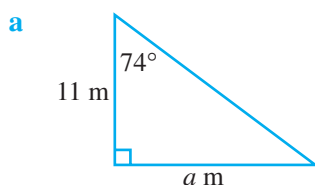
Finding the length of the hypotenuse

To find the length of the hypotenuse:

- ⊙ determine whether the sine or cosine ratio is to be used
- ⊙ write down a trigonometric equation
- ⊙ take the reciprocal of both sides
- ⊙ multiply both sides by the denominator under the pronumeral
- ⊙ evaluate using a calculator.

Example 1

Find the value of the pronumeral in each of the following, correct to 1 decimal place.



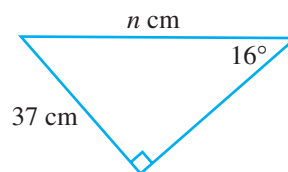
Solutions

$$\begin{aligned} \text{a } \tan 74^\circ &= \frac{a}{11} \\ \times 11 \quad \times 11 \\ a &= 11 \times \tan 74^\circ \\ &= 38.361\,558\,88 \\ \therefore a &= 38.4 \text{ (1 decimal place)} \end{aligned}$$

$$\begin{aligned} \text{b } \cos 62^\circ &= \frac{w}{27.5} \\ \times 27.5 \quad \times 27.5 \\ w &= 27.5 \times \cos 62^\circ \\ &= 12.910\,467\,98 \\ \therefore w &= 12.9 \text{ (1 decimal place)} \end{aligned}$$

Example 2

Find the length of the hypotenuse, correct to 1 decimal place.

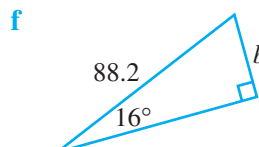
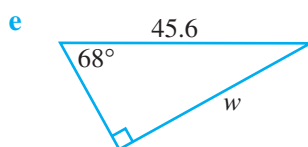
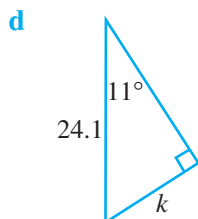
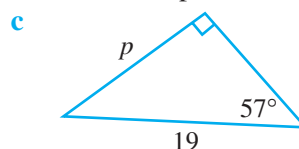
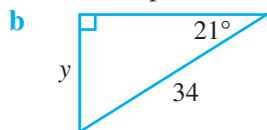
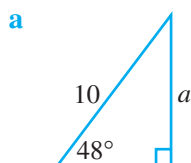


Solution

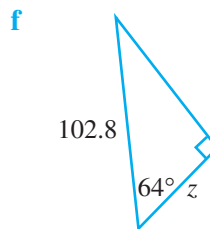
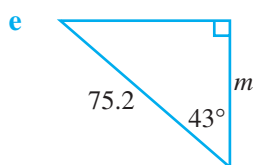
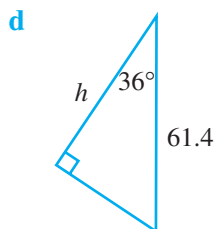
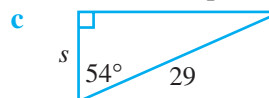
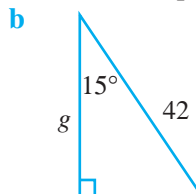
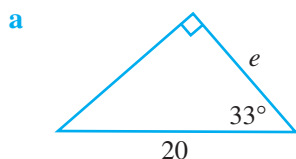
$$\begin{aligned} \sin 16^\circ &= \frac{37}{n} \\ \frac{1}{\sin 16^\circ} &= \frac{n}{37} \\ \times 37 \quad \times 37 \\ n &= \frac{37}{\sin 16^\circ} \\ &= 134.234\,345\,3 \\ \therefore \text{The length of the hypotenuse is } 134.2 \text{ cm (1 decimal place).} \end{aligned}$$

Exercise 12.4

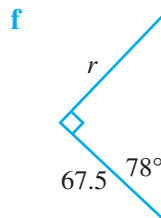
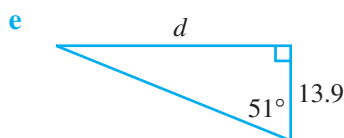
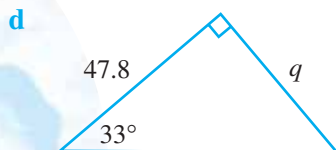
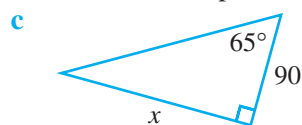
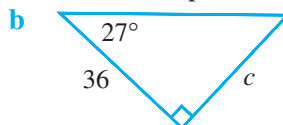
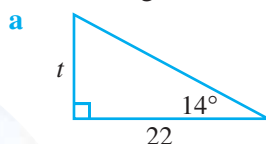
1 Use the sine ratio to find the value of each pronumeral, correct to 1 decimal place.



2 Use the cosine ratio to find the value of each pronumeral, correct to 1 decimal place.

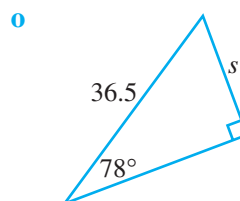
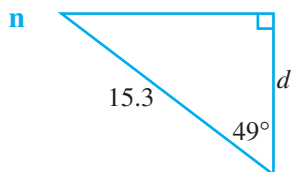
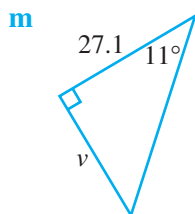
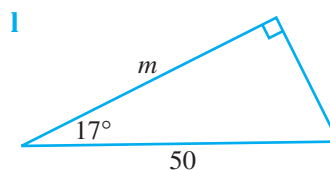
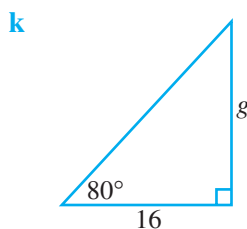
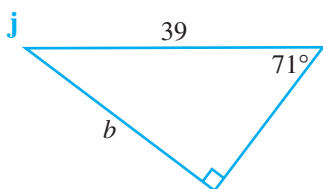
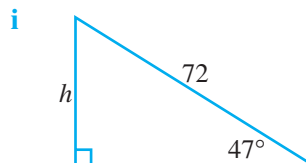
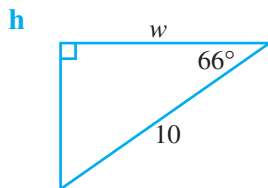
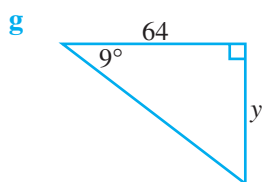
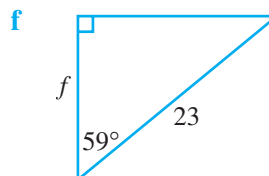
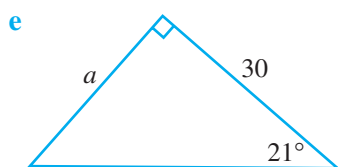
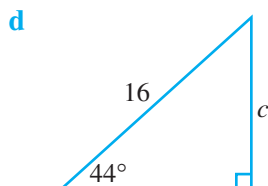
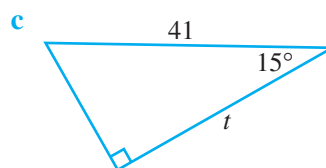
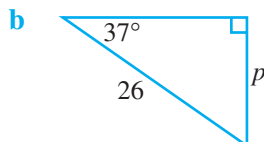
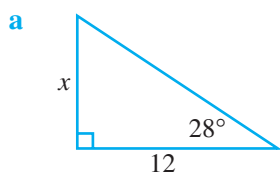


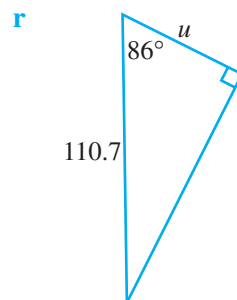
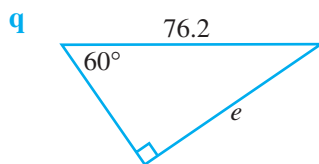
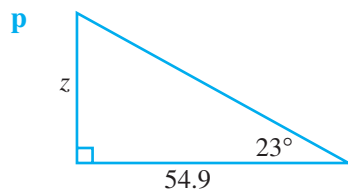
3 Use the tangent ratio to find the value of each pronumeral, correct to 1 decimal place.



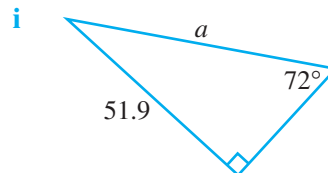
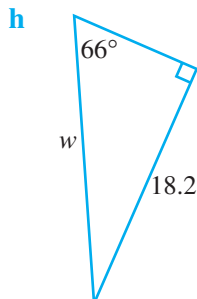
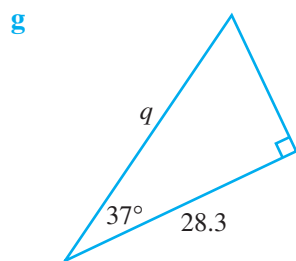
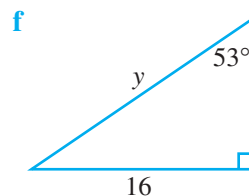
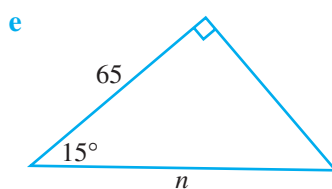
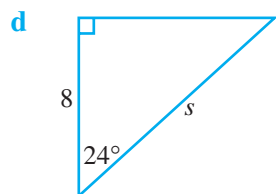
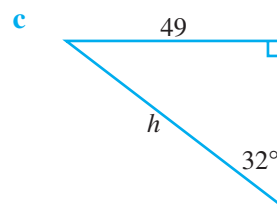
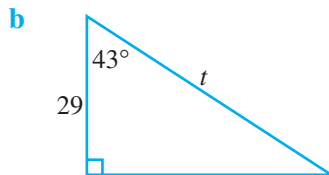
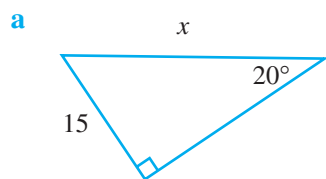
Consolidation

- 4 Find the value of the pronumeral in each triangle, correct to 2 decimal places.
All measurements are in centimetres.

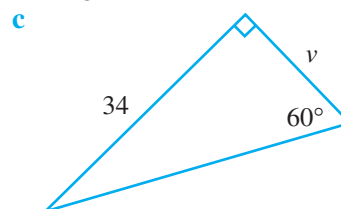
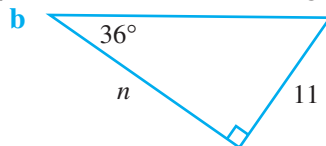
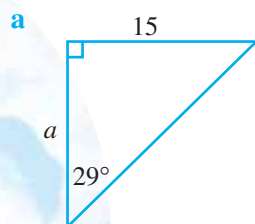




- 5** Find the length of the hypotenuse in each of the following, correct to 1 decimal place. All measurements are in millimetres.



- 6** Find the value of each pronumeral, correct to 3 significant figures.



Further applications

- 7 Draw a diagram and mark on it all of the given information to answer each of the following.
- In $\triangle PQR$, $\angle Q = 90^\circ$, $\angle R = 29^\circ$ and $PR = 18$ cm. Find the length of QR , correct to 1 decimal place.
 - In $\triangle XYZ$, $\angle Z = 90^\circ$, $\angle X = 42^\circ$ and $XY = 27$ mm. Find the length of YZ , correct to 4 significant figures.
 - In $\triangle TUV$, $\angle V = 90^\circ$, $\angle T = 75^\circ$ and $TV = 51$ m. Find the length of UV , correct to the nearest metre.
 - In $\triangle LMN$, $\angle M = 90^\circ$, $\angle L = 16^\circ$ and $LM = 34$ km. Find the length of LN , correct to the nearest metre.

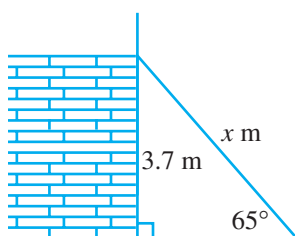
12.5 Problems involving finding sides

Many practical problems involving length or height can be solved by the use of trigonometry.

Example 1

A ladder leans against a wall and makes an angle of 65° with the ground. The ladder reaches 3.7 m up the wall. Find the length of the ladder, correct to the nearest centimetre.

Solution



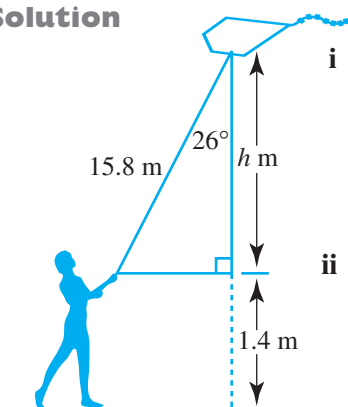
$$\begin{aligned}\sin 65^\circ &= \frac{3.7}{x} \\ \frac{1}{\sin 65^\circ} &= \frac{x}{3.7} \\ \times 3.7 &\quad \times 3.7 \\ x &= \frac{3.7}{\sin 65^\circ} \\ &= 4.082\,498\,3\end{aligned}$$

$\therefore$ The length of the ladder is 4.08 m (to nearest cm).

Example 2

A girl is flying a kite which is attached to the end of a 15.8 m length of string. The angle between the string and the vertical is 26° . Find the height of the kite above the ground if the girl is holding the string 1.4 m above the ground.

Solution

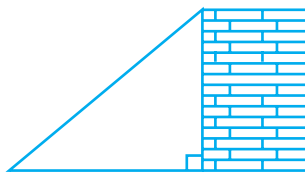


$$\begin{aligned}\text{i } \cos 26^\circ &= \frac{h}{15.8} \\ h &= 15.8 \times \cos 26^\circ \\ &= 14.200\,945\,93 \\ &= 14.2 \text{ m (to 1 decimal place)} \\ \text{ii } \text{height} &= 14.2 \text{ m} + 1.4 \text{ m} \\ &= 15.6 \text{ m}\end{aligned}$$

$\therefore$ The height of the kite above the ground is 15.6 m.

Exercise 12.5

1



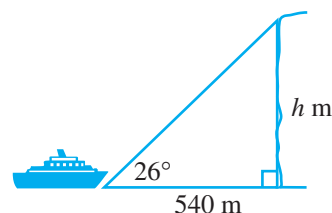
Answer the following questions, correct to 1 decimal place.

a A ladder leans against a wall, making an angle of 62° with the ground. The foot of the ladder is 3 m from the base of the wall. How far up the wall will the ladder reach?

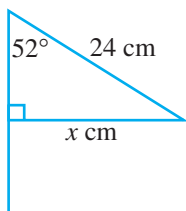
b A second ladder of length 6 m leans against a wall and makes an angle of 22° with the wall. How far is the foot of the ladder from the base of the wall?

c A third ladder leans against a wall and makes an angle of 71° with the ground. Find the length of the ladder if it reaches 8 m up the wall.

- 2** A ship at sea is anchored 540 m away from the base of a vertical cliff. The captain used a clinometer to measure the angle to the top of the cliff and found it to be 26° . Calculate the height of the cliff, correct to the nearest metre.



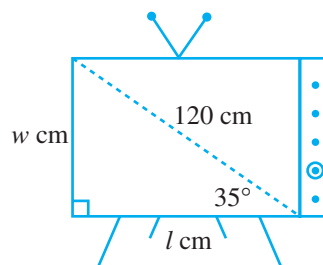
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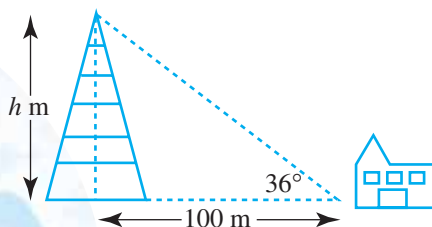
The diagonal edge of a sports club pennant makes an angle of 52° with the stick to which it is attached. Find the horizontal length of the pennant, given that the diagonal edge is 24 cm. Answer correct to 1 decimal place.

- 4** The size of a television set is determined by the diagonal length of the picture screen. In a 120 cm big screen television, the diagonal makes an angle of 35° with the length of the screen. Calculate, correct to the nearest millimetre:

- a** the length l cm, of the screen
b the width w cm, of the screen

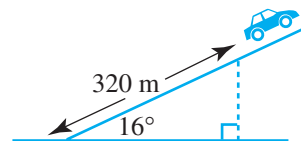


5



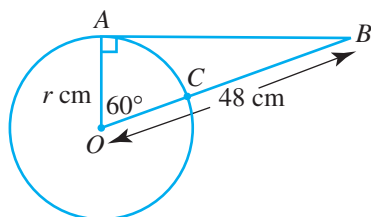
A mobile phone tower was built 100 m away from the front gate of a preschool. A child standing at the gate looked up to the top of the tower at an angle of 36° . Calculate the height of the tower, correct to the nearest metre.

- 6** A car travelled 320 m along a road that rises steadily at an angle of 16° to the horizontal. Through what horizontal distance did the car travel? Answer correct to 3 significant figures.



Consolidation

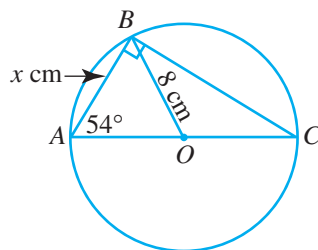
7



AB is a tangent to a circle with centre O . The interval OB is 48 cm long and cuts the circle at C . Find:

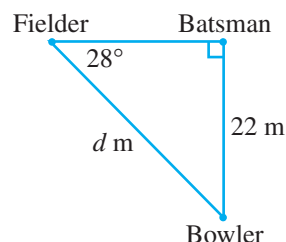
- a** the length of the radius, r cm
- b** the length of the interval BC

- 8** At a certain time of day when the altitude of the sun is 38° , a tree casts a shadow of 22.5 m on the ground. Find the height of the tree, correct to the nearest metre.
- 9** The diagonal strut of a rectangular gate makes an angle of 55° with the width. Find, correct to the nearest centimetre, the length of the diagonal strut given that the gate has a length of 1.2 m.
- 10** A square has a diagonal of length 12 cm.
- a** What size is the angle between the diagonal and the length? Why?
 - b** Use trigonometry to calculate the side length of the square, correct to 2 decimal places.
 - c** Use Pythagoras' theorem to calculate the side length of the square, correct to 2 decimal places.
- 11** In the diagram, AC is a diameter of the circle and O is the centre. If $OB = 8$ cm, $\angle A = 54^\circ$ and $\angle ABC = 90^\circ$, find:
- a** the length of the diameter AC
 - b** the length of the chord AB , correct to 1 decimal place



- 12** The angle between the vertical and the slant edge of a cone is 24° and the perpendicular height of the cone is 9 cm. Calculate the length of the diameter, correct to the nearest centimetre.
- 13** A small child is flying a balloon which is attached to the end of a 14.5 m long string. The angle between the string and the vertical is 33° . How high is the balloon above the ground if the hand holding the string is 1.2 m above the ground? Answer correct to the nearest metre.
- 14** A surveyor walked 35 m from A to B along a river bank, then measured the angle to a point C , which lies on the opposite side of the river and is directly in line with the point A . Calculate the width of the river, correct to the nearest metre, given that $\angle ABC = 37^\circ$.
- 15** At a local shopping centre, a wheelchair ramp is inclined to the horizontal at an angle of 14° . Find the length of the ramp if it leads to an entrance that is 2.4 m higher than the surrounding floor area. Answer correct to the nearest tenth of a metre.

- 16 During a cricket match, one of the batsmen played the ball square of the wicket to a fielder. To run the batsman out at the bowler's end, the fielder must throw the ball at an angle of 28° to the line in which he fielded the ball. If the length of the pitch is 22 m, find how far the fielder must throw the ball in order to run the batsman out. Give your answer correct to 1 decimal place.

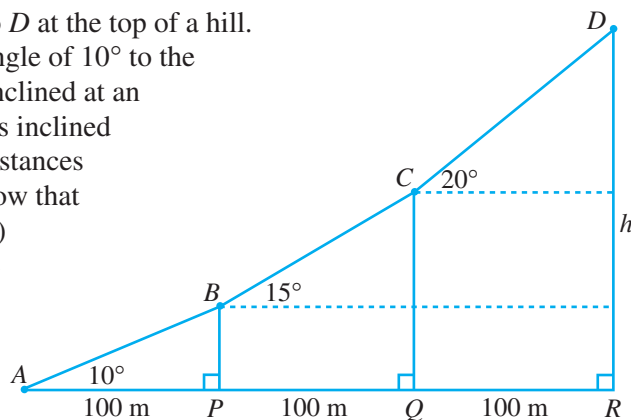


- 17 During recess, Jenny stood in the school playground and looked up at an angle of 27° to find the time on a clock which is positioned on a wall, 8.2 m above the playground. Find how far Jenny is standing from the wall if her eye level is 1.2 m above the ground. Answer correct to 3 significant figures.
- 18 The rhombus $ABCD$ has a perimeter of 52 cm. The diagonals AC and BD intersect at P and $\angle ABC = 46^\circ$.
- Explain why $\angle ABP = 23^\circ$.
 - Find the length of the diagonal AC , correct to the nearest millimetre.
- 19 For each of the following, draw an isosceles triangle and mark on it all of the given information.
- $\triangle ABC$ is isosceles with $AB = BC = 24$ mm and $\angle BAC = 56^\circ$. Find, correct to 1 decimal place, the length of AC .
 - $\triangle PQR$ is isosceles with $PQ = QR$ and $\angle PQR = 132^\circ$. S is a point on PR such that $QS \perp PR$. If $QS = 37$ mm, find, correct to 1 decimal place, the length of PQ .
 - $\triangle XYZ$ is isosceles with $XY = YZ$, $XZ = 82$ mm and $\angle XYZ = 118^\circ$. W is a point on XZ such that $YW \perp XZ$. Find, correct to 1 decimal place, the length of YW .

Further applications

- 20 A plane took off and left the tarmac at an angle of 25° . The plane continued to fly on this line for 5 minutes at a speed of 300 km/h before levelling off. Calculate, in metres, the height of the plane when it levels off, correct to the nearest metre.

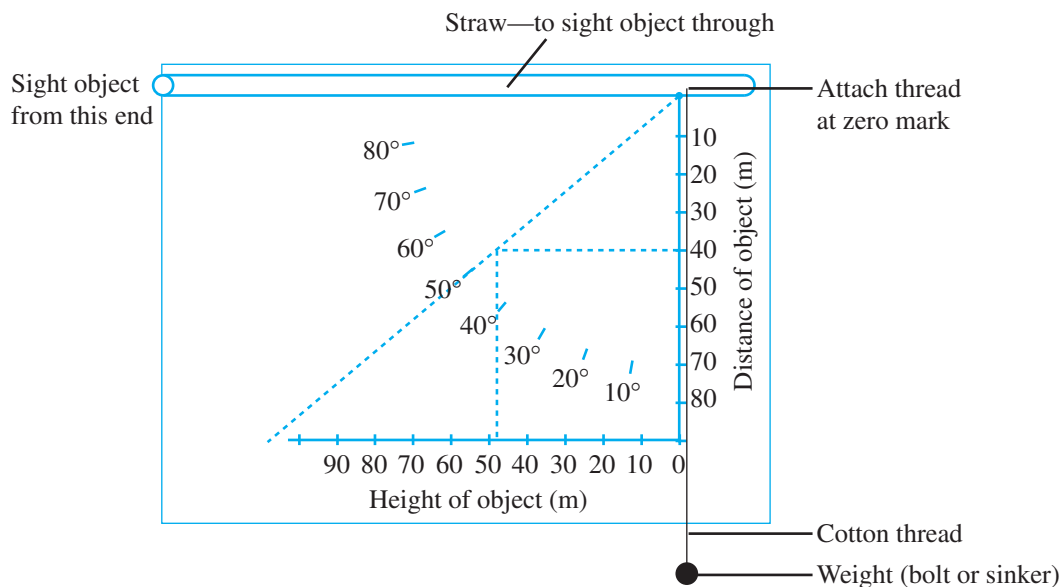
- 21 Three roads, AB , BC and CD lead to D at the top of a hill. The first road AB is inclined at an angle of 10° to the horizontal. The second road BC is inclined at an angle of 15° and the third road CD is inclined at an angle of 20° . The horizontal distances AP , PQ and QR are each 100 m. Show that $h = 100 (\tan 10^\circ + \tan 15^\circ + \tan 20^\circ)$ and hence find DR , the height of the hill, correct to the nearest metre.



TRY THIS

Make a hypsometer

A hypsometer allows you to find the height of an object without doing any calculations. On graph paper, draw the following, then stick it on thin cardboard. Make sure you use the same scale on the vertical and horizontal axes. (Ignore the dotted lines in the diagram for now. They relate to the steps listed overleaf.)



How to use it

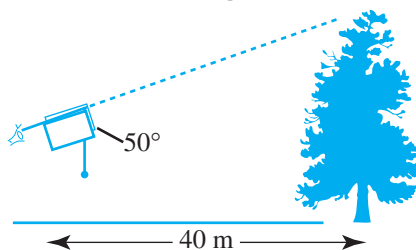
- 1 Walk a certain number of metres from the base of an object (e.g. 40 m).
- 2 Sight the top of the object through the straw.
- 3 Place your finger against the thread, then read off the angle (e.g. 50°).
- 4 Imagine a line (see dotted line in the diagram above) being drawn from 40 m across to the thread then down to the horizontal axis. The height is 47 m.

$$\text{Check: } \tan 50^\circ = \frac{h}{40}$$

$$40 \tan 50^\circ = h$$

$$47.67 = h$$

Is the tree 47.67 m tall? Does the height the hypsometer is held above the ground make any difference?



12.6 Finding the size of an angle

Trigonometry can also be used to find sizes of angles in triangles. In Exercise 12.3 we used the inverse key $\boxed{\text{INV}}$, shift key $\boxed{\text{shift}}$, or second function key $\boxed{2\text{nd F}}$, followed by the $\boxed{\sin}$, $\boxed{\cos}$ or $\boxed{\tan}$ keys to find the size of an angle given a fraction or decimal value. When we find an angle, we are actually using the **inverse trigonometric functions**— $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$. The notation $\sin^{-1}x$, read as the *inverse sine of x* , means ‘the angle whose sine is x ’. The expressions $\cos^{-1}x$ and $\tan^{-1}x$ have similar meanings. The inverse trigonometric functions ‘undo’ the basic trigonometric functions of the sine, cosine and tangent ratios, and hence give the size of the original angle.

The inverse trigonometric functions are used to find the size of an angle, where:

- ⊙ $\sin^{-1}x$ means ‘the angle whose sine is x ’
- ⊙ $\cos^{-1}x$ means ‘the angle whose cosine is x ’
- ⊙ $\tan^{-1}x$ means ‘the angle whose tangent is x ’.

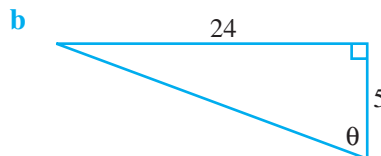
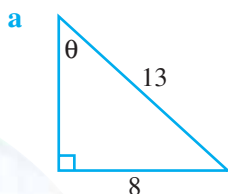
To find the size of an angle:

- ⊙ press either the $\boxed{\text{INV}}$, $\boxed{\text{shift}}$ or $\boxed{2\text{nd F}}$ key followed by the appropriate trigonometric function key
- ⊙ enter the given fraction or decimal, then press $\boxed{=}$
- ⊙ round off the angle as required.

NOTE: If the side lengths are decimals you will need to use the division and grouping symbols keys on the calculator rather than the fraction key.

Example

Find the acute angle θ , correct to the nearest degree.



Solutions

a $\sin \theta = \frac{8}{13}$

$$\theta = \sin^{-1}\left(\frac{8}{13}\right)$$

Press $\boxed{2\text{nd F}} \boxed{\sin} 8 \boxed{\frac{b}{c}} 13 \boxed{=}$

$$\theta = 37^\circ 58' 47.54''$$

$$= 38^\circ \text{ (to the nearest degree)}$$

b $\tan \theta = \frac{24}{5}$

$$\theta = \tan^{-1}\left(\frac{24}{5}\right)$$

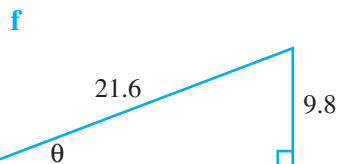
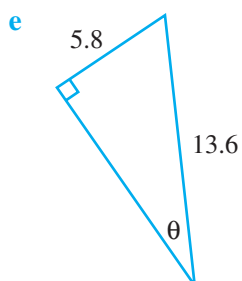
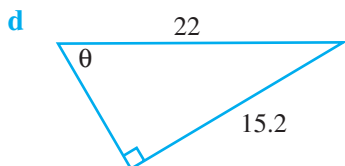
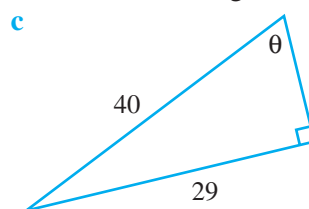
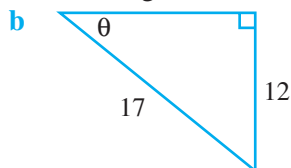
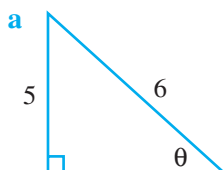
Press $\boxed{2\text{nd F}} \boxed{\tan} 24 \boxed{\frac{b}{c}} 5 \boxed{=}$

$$\theta = 78^\circ 13' 54.16''$$

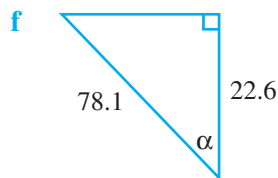
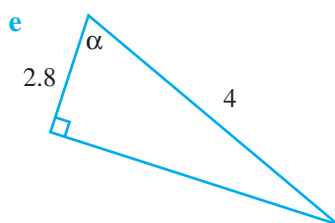
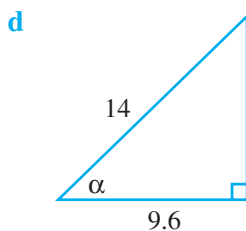
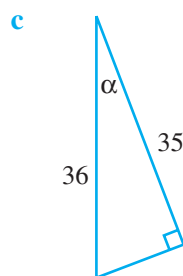
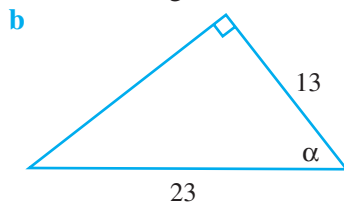
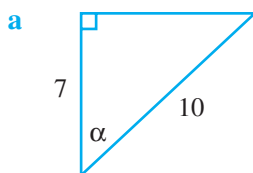
$$= 78^\circ \text{ (to the nearest degree)}$$

Exercise 12.6

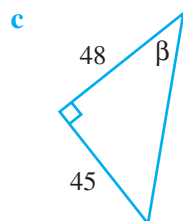
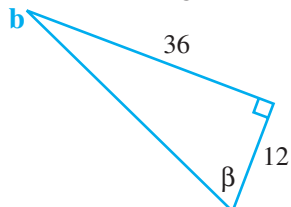
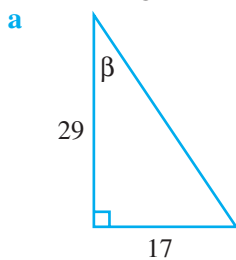
1 Use the sine ratio to find the size of the angle marked θ , correct to the nearest degree.

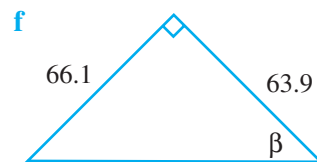
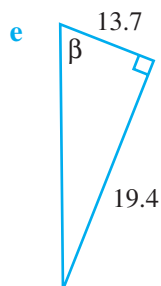
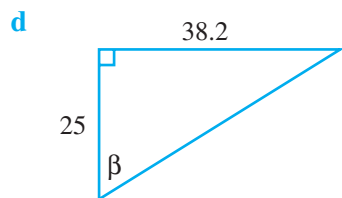


2 Use the cosine ratio to find the size of the angle marked α , correct to the nearest degree.



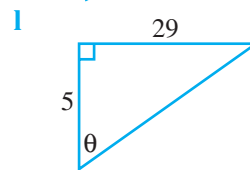
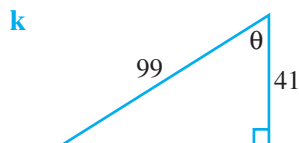
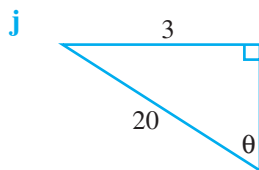
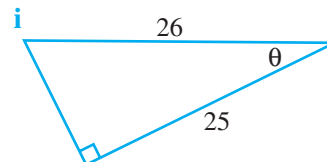
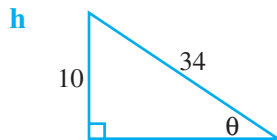
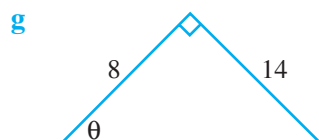
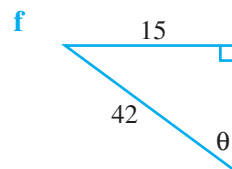
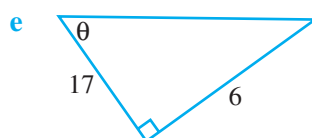
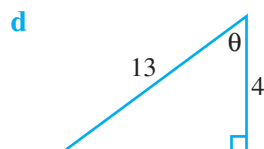
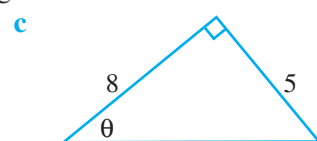
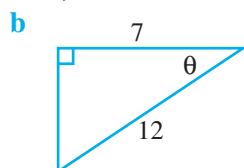
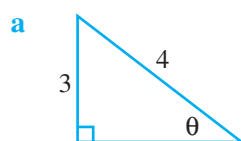
3 Use the tangent ratio to find the size of the angle marked β , correct to the nearest degree.



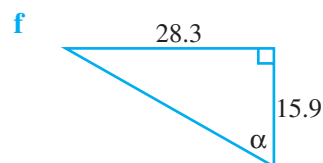
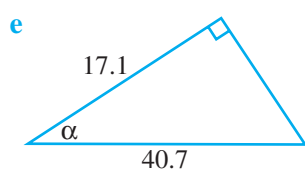
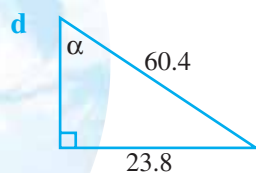
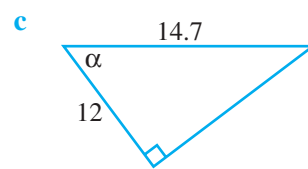
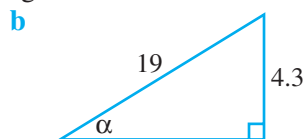
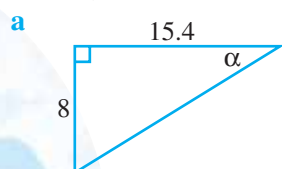


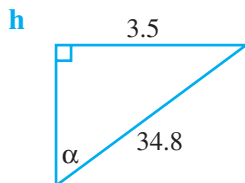
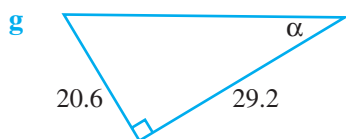
Consolidation

4 Find the size of the angle marked θ , correct to the nearest degree.



5 Find α , correct to the nearest degree.

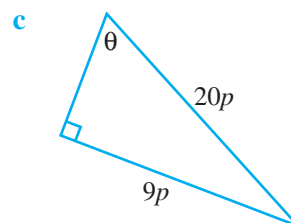
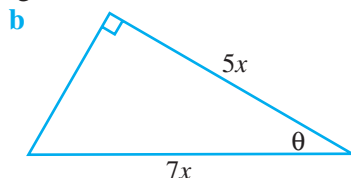
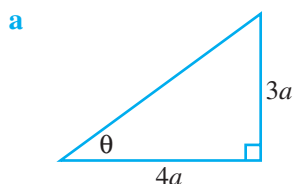




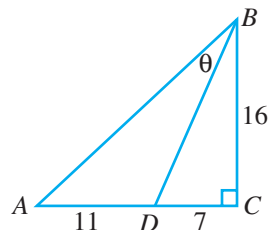
- 6** Draw a diagram and mark on it all of the given information to answer each of the following.
- In $\triangle UVW$, $\angle V = 90^\circ$, $UV = 10$ cm and $VW = 13$ cm. Find the size of $\angle W$, correct to the nearest degree.
 - In $\triangle PQR$, $\angle R = 90^\circ$, $PQ = 63$ mm and $QR = 24$ mm. Find the size of $\angle P$, correct to the nearest degree.
 - In $\triangle ABC$, $\angle A = 90^\circ$, $AB = 12.2$ m and $BC = 13.5$ m. Find the size of $\angle B$, correct to the nearest degree.

Further applications

- 7** Find θ , correct to the nearest degree.



- 8** Find θ , correct to the nearest degree.



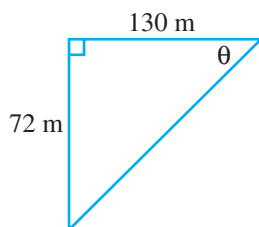
12.7 Problems involving finding angles

Practical problems that involve finding angles can be solved by the use of trigonometry.

Example 1

A ship dropped anchor off the coast of a resort. The anchor fell 72 m to the sea bed. During the next 2 hours, the ship drifted 130 m. Calculate the angle between the anchor line and the surface of the water, correct to the nearest degree.

Solution



$$\begin{aligned}\tan \theta &= \frac{72}{130} \\ \therefore \theta &= \tan^{-1} \frac{72}{130} \\ &= 28^\circ 58' 46.95'' \\ &= 29^\circ \text{ (to nearest degree)}\end{aligned}$$

$\therefore$ The anchor line makes an angle of 29° with the surface of the water.

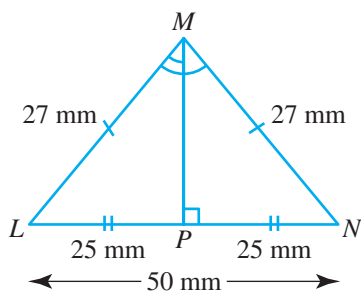
EG
+
S**Example 2**

$\triangle LMN$ is isosceles with $LM = MN = 27$ mm and $LN = 50$ mm. Find the size of $\angle LMN$, correct to the nearest degree.

Solution

Draw MP , the perpendicular bisector of $\triangle LMN$, as shown.

Now, $LP = \frac{1}{2}LN$, $\therefore LP = 25$ mm.



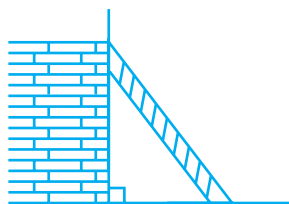
$$\begin{aligned} \text{i} \quad \sin \angle LMP &= \frac{LP}{LM} \\ &= \frac{25}{27} \\ \therefore \angle LMP &= \sin^{-1} \frac{25}{27} \end{aligned}$$

$$\begin{aligned} \text{ii} \quad \angle LMN &= 2 \times \angle LMP \\ &= 2 \times \sin^{-1} \frac{25}{27} \\ &= 136^\circ \\ &\quad (\text{to the nearest degree}) \end{aligned}$$

Exercise 12.7

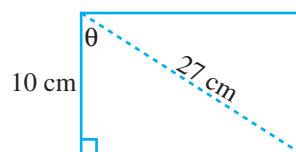
In each of the following questions, give all angles correct to the nearest degree.

1



- a A ladder of length 5 m leans against a wall. The foot of the ladder is 1.5 m from the base of the wall. Find the angle formed between the ladder and the ground.
- b The foot of a ladder is 2.3 m from the base of a wall and the ladder reaches 8.4 m up the wall. Find the angle made by the ladder with the wall.

- 2 A rectangle has a diagonal of length 27 cm and a width of 10 cm. Find the size of the angle formed between the diagonal and the width.

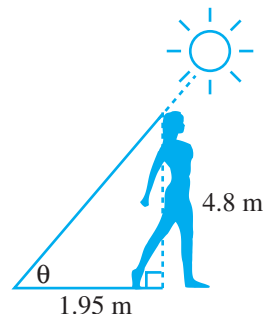


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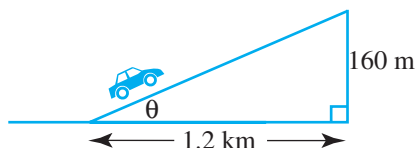


A ski slope of length 815 m has a vertical drop of 320 m. Calculate the angle between the ski slope and the horizontal.

- 4 Find the angle at which the Sun's rays strike the ground at a certain time of day if a statue of height 4.8 m casts a shadow of length 1.95 m.



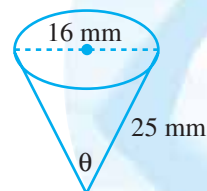
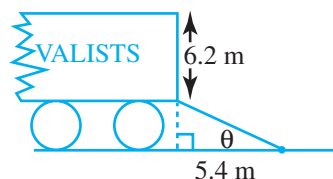
5



A straight section of road rises 160 m over a horizontal distance of 1.2 km. Calculate the angle at which the road rises.

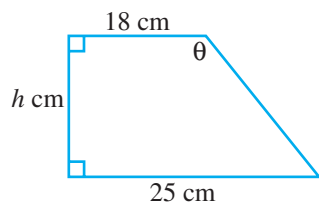
Consolidation

- 6 The rear door of a removalist's van is opened and lowered, touching the ground at a point 5.4 m from the back of the van. The height of the door is 6.2 m. What angle does the door make with the ground?
- 7 A wheelchair access ramp at a local school rises 1 m for each 4.5 m horizontally. Find the angle of inclination of the ramp.
- 8 A straight section of railway track climbs 5.2 km up a hill to a station situated 1.25 km above the surrounding area. At what angle does the track rise?
- 9 A rectangle has an area of 108 cm^2 and a length of 12 cm. Find the angle formed between the length and the diagonal.
- 10 Ben's kite is attached to a 30 m length of string and is flying at a height of 26 m. The hand holding the string is at eye level, 1.5 m above the ground.
 - a Find the height of the kite above eye level.
 - b At what angle must Ben look up in order to see the kite?
- 11 Maree's driveway is 11.3 m long and slopes downward from her house to the street. If the house is set back 9.1 m from the street, find the angle at which the driveway rises.
- 12 Colleen and Gary live on opposite sides of a 35 m wide river but directly in line with each other. Colleen's house is situated 15 m above the river and Gary's house is situated 6 m above the river. At what angle must Colleen look down to see Gary's house?
- 13 For each of the following, first draw an isosceles triangle and mark on it all of the given information.
 - a $\triangle ABC$ is isosceles with $AB = BC = 20 \text{ cm}$ and $AC = 24 \text{ cm}$. Find the size of the base angles.
 - b $\triangle LMN$ is isosceles with $LM = MN = 34 \text{ mm}$. K is a point on LN such that $MK \perp LN$ and $MK = 18 \text{ mm}$. Find the size of the apex angle LMN .
 - c $\triangle TUV$ is isosceles with $TU = UV$ and $TV = 32 \text{ m}$. W is a point on TV such that $UW \perp TV$ and $UW = 11 \text{ m}$. Find the size of the apex angle TUV .
- 14 A cone has a diameter of 16 mm and a slant height of 25 mm. Calculate the size of the vertical angle θ .
- 15 A step ladder has legs of length 2.2 m and the maximum distance between the legs is 0.9 m. Find the largest possible angle in which the ladder can be opened.



Further applications

16



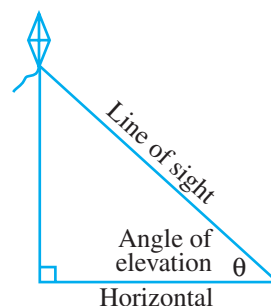
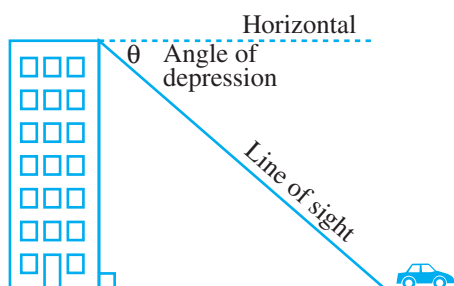
A trapezium has parallel sides of length 18 cm and 25 cm. The area of the trapezium is 516 cm^2 .

- Find the height, h , of the trapezium.
- Find the size of the angle marked θ .

- 17 A chord AB of length 16 cm is drawn in a circle with centre O and area $100\pi \text{ cm}^2$. Find the size of the angle that the chord AB subtends at the centre of the circle.

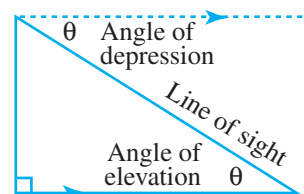
12.8 Angles of elevation and depression

The **angle of elevation** is the angle between the horizontal and the line of sight when the observer is looking upward.



The **angle of depression** is the angle between the horizontal and the line of sight when the observer is looking downward.

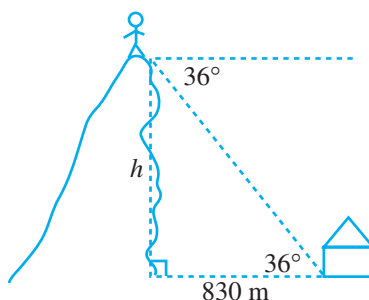
The angles of elevation and depression involving the same line of sight are equal in size, since the angles are alternate angles between parallel lines.



Example 1

The angle of depression from a farmer on top of a ridge to a farmhouse below is 36° . If the farmhouse is 830 m from the base of the ridge, find the height of the ridge, correct to the nearest metre.

Solution

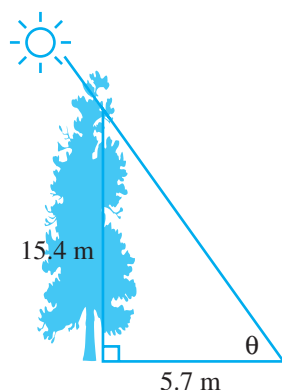


$$\begin{aligned}\tan 36^\circ &= \frac{h}{830} \\ h &= 830 \times \tan 36^\circ \\ &= 603.030\ 298\ 2\end{aligned}$$

$\therefore$ The height of the ridge is 603 m (to nearest metre).

EG
+S**Example 2**

A tree of height 15.4 m casts a shadow on the ground of length 5.7 m. Calculate the angle of elevation of the Sun, correct to the nearest degree.

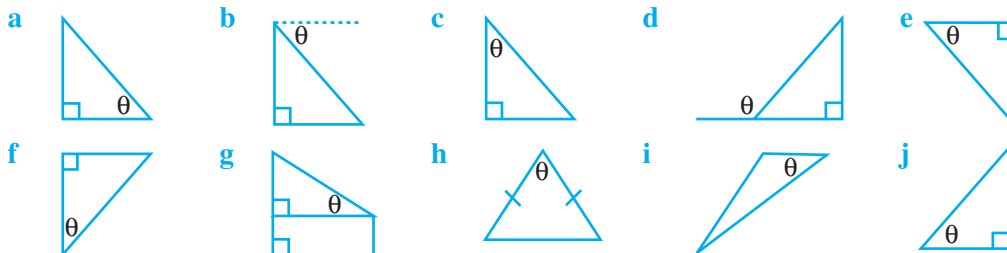
Solution

$$\begin{aligned}\tan \theta &= \frac{15.4}{5.7} \\ \theta &= \tan^{-1}\left(\frac{15.4}{5.7}\right) \\ &= 69.688\,981\,57\end{aligned}$$

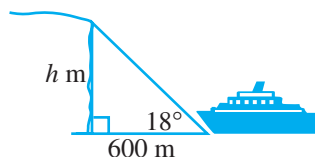
$\therefore$ The angle of elevation of the Sun is 70° (to nearest degree).

Exercise 12.8

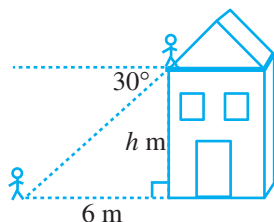
- 1 State whether the angle marked θ is an angle of elevation, angle of depression or neither.



- 2 From a ship 600 m out to sea, the angle of elevation of a lighthouse on top of a vertical cliff is 18° . Find the height of the cliff, correct to the nearest metre.

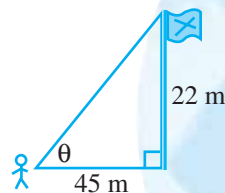


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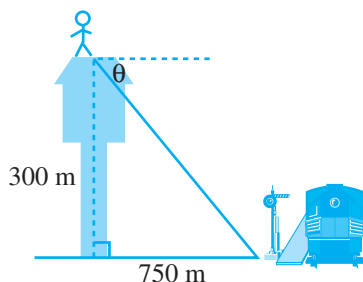


While cleaning leaves from the guttering around his roof, a man observes his son playing in the backyard, 6 m away. From the roof of the house, the angle of depression to the child is 30° . Find the height of the man above the ground, correct to the nearest tenth of a metre.

- 4 An observer stands on level ground, 45 m away from the base of a 22 m high flag pole. Find the angle of elevation of the top of the pole, correct to the nearest degree.

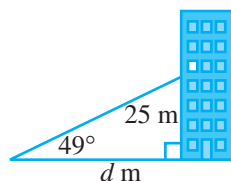


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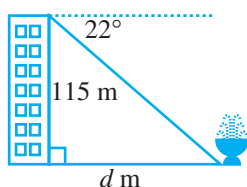


A repairer working at the top of a 300 m high tower sees a train station 750 m away from the foot of the tower. Find, to the nearest degree, the angle of depression of the train station from the top of the tower.

- 6 From a bus stop on level ground the angle of elevation of an open window in an office building is 49° . If the window is 25 m above the street, calculate the distance between the bus stop and the building, correct to 2 decimal places.

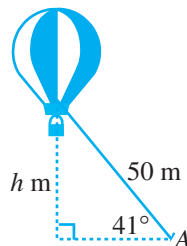


7



From the top of a 115 m high tower, the angle of depression of a fountain is 22° . Find, correct to 1 decimal place, the distance between the fountain and the base of the tower.

- 8 A hot-air balloon hovering above an airfield remains tethered to the ground at A by a 50 m long rope. A worker standing on the ground at A observes the angle of elevation of the balloon to be 41° . Calculate the height of the balloon, correct to the nearest metre.



Consolidation

- 9 The control tower at an airport casts a shadow of length 180 m on the tarmac when the angle of elevation of the Sun is 55° . Calculate the height of the tower, correct to the nearest metre.
- 10 From the edge of the footpath, the elevation of a 54 m high crane on a building site is 72° . How far is the crane from the footpath, to the nearest tenth of a metre?
- 11 A quarantine station situated on a headland is 475 m above sea level. Calculate, to the nearest degree, the angle of elevation of the quarantine station from a ferry that is 620 m from the base of the headland.
- 12 Two hours after a boat dropped anchor, the captain found that it had drifted 45 m towards the coast. The angle of depression of the anchor from the bow of the boat was then 76° . Calculate the length of the anchor line, correct to the nearest metre.

- 13** During a cricket test at the Sydney Cricket Ground, an airship is hovering above the ground at an altitude of 2 km. Find the angle of depression of the arena from the airship if the horizontal distance between the airship and the arena is 800 m. Answer correct to the nearest degree.
- 14** A spire of length 9 m stands on top of a cathedral. While preparing to carry out repairs, a surveyor standing on the ground 48 m from the building measures the angle of elevation of the top of the spire to be 39° . Find the height of the cathedral excluding the spire, correct to 1 decimal place.
- 15** Two buildings, each of height 30 m, stand 15 m apart on opposite sides of a street. From the top of one building the angle of depression of a balcony in the other building is 52° . Find the height of the balcony above the street, correct to the nearest tenth of a metre.

■ Further applications

- 16** From the deck of a ship at sea, the angle of elevation of the top of a vertical cliff is 45° . The angle of elevation of the top of a hotel on the edge of the cliff is 60° . If the ship is anchored 300 m from the base of the cliff, find correct to the nearest metre, the height of the hotel.
- 17** From the cockpit of an airplane flying at an altitude of 2500 m, the angle of depression of the airport is 50° . The airplane continues to fly in the same straight line and, after a few minutes, the angle of depression of the airport is then 66° . Find the distance travelled between the two sightings, correct to the nearest metre.
- 18** The angle of elevation of the top of a tree from a point P due west of the tree is 40° . From a second point Q due east of the tree, the angle of elevation is 32° . If the distance between P and Q is 200 m, find the height of the tree, correct to 4 significant figures. (HINT: Let the height of the tree be h m and the distances west and east of the tree be x m and y m.)

TRY THIS

Pilot instructions

Imagine you are a pilot and have been given the following instructions.

You must approach the airport at an angle of descent of 10° until you are a horizontal distance of 5 km from the airport. You are then to approach at an angle of 5° .

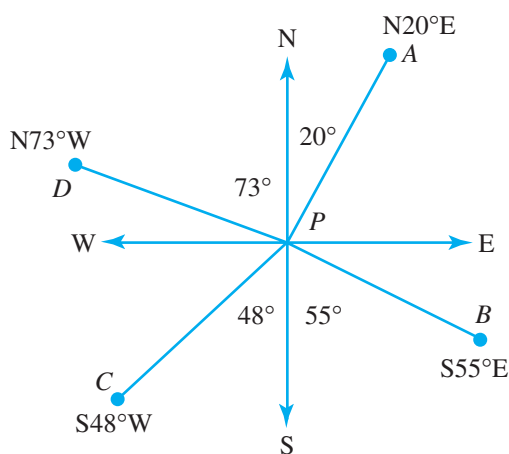
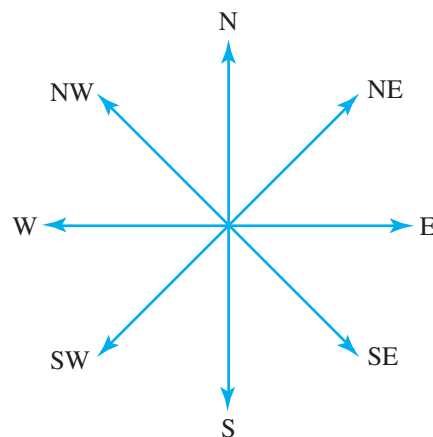
If you are currently flying at a height of 5000 metres, at what horizontal and vertical distance should you begin your descent?

12.9 Bearings

A **bearing** is a measure of the direction of one point from another point. There are two types of bearings: compass bearings and true bearings.

Compass bearings

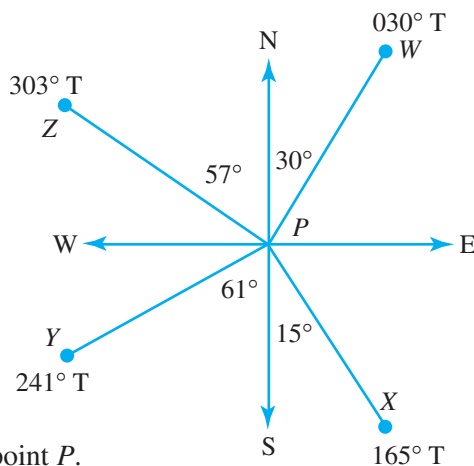
A **compass bearing** is a deviation involving the four cardinal directions north, south, east and west. For example, a bearing of $N20^\circ E$ means a deviation of 20° from the north towards the east. Compass bearings are always measured from the north or south and towards the east or west. Thus, it is not correct to write a bearing such as $W30^\circ S$, as this implies that we are deviating from the west towards the north by an angle of 30° .



A bearing such as NE means $N45^\circ E$. Similarly, NW means $N45^\circ W$, SE means $S45^\circ E$ and SW means $S45^\circ W$. These bearings are shown on the diagram above. The diagram to the left shows the bearings of the four points A, B, C and D from a point P.

True bearings

A **true bearing** is a deviation from north, measured in a clockwise direction. By convention, a true bearing is written using 3 digits. For bearings that are less than 100° it is customary to place 1 or 2 zeroes at the front as needed. For example, a clockwise rotation of 74° from north would be written as 074° . Similarly, a rotation of 8° would be written as 008° . The diagram to the right shows the bearings of the four points W, X, Y and Z from a point P.



Opposite bearings

The **opposite bearing** of B from A is the bearing of A from B . To find the opposite bearing (or any change in direction) it will be necessary to draw a new compass at the end of the ray.

To find the bearing of A from B given the bearing of B from A :

- ⊙ draw a compass at B
- ⊙ mark on this compass the angle from north around to the ray BA
- ⊙ on the compass with centre A , find the acute angle between BA and the north–south axis
- ⊙ use parallel line properties to find the required bearing on the compass with centre B .

NOTE: (1) Opposite bearings always differ by 180° . That is, the new bearing will be either 180° more than the original bearing, or 180° less.

(2) The reference given after the word 'FROM' should be at the centre of the *active compass* (i.e. the compass in which you are working).

Example 1

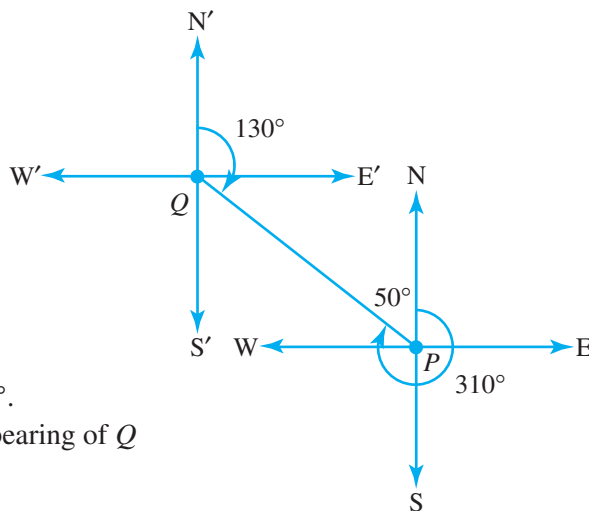
The bearing of Q from P is 310° .
Find the bearing of P from Q .

Solution

- i $\angle NPQ = 360^\circ - 310^\circ$
 $= 50^\circ$
- ii $\angle N'QP = 180^\circ - 50^\circ$
(Co-interior $\angle$ s, $N'Q \parallel NP$)
 $= 130^\circ$

$\therefore$ The bearing of P from Q is 130° .

NOTE: The bearing of P from Q and the bearing of Q from P differ by 180° .



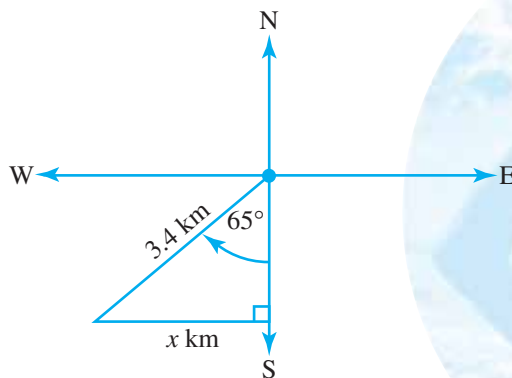
Example 2

Marta walked on a bearing of $S65^\circ W$ for 3.4 km. Find how far west she has walked, correct to 1 decimal place.

Solution

$$\sin 65^\circ = \frac{x}{3.4}$$

$$\begin{aligned}\therefore x &= 3.4 \times \sin 65^\circ \\ &= 3.08144676 \\ &= 3.1 \text{ km (correct to 1 decimal place)}\end{aligned}$$



EG
+ S**Example 3**

Kevin drove due north from A to B for 50 km. He then turned and drove due west to C , which is 72 km from B . Find the bearing of C from A .

Solution

$$\tan \theta = \frac{72}{50}$$

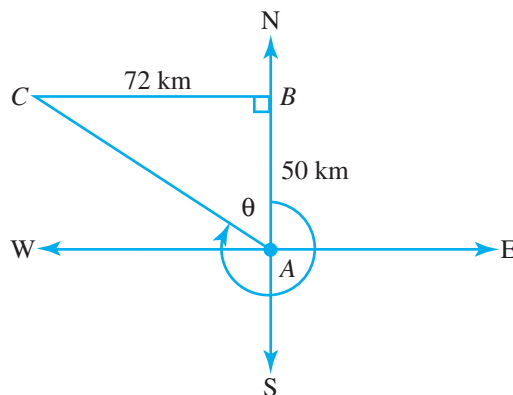
$$\therefore \theta = \tan^{-1}\left(\frac{72}{50}\right)$$

$$= 55^\circ 13' 19.81''$$

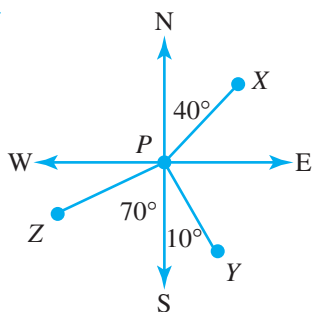
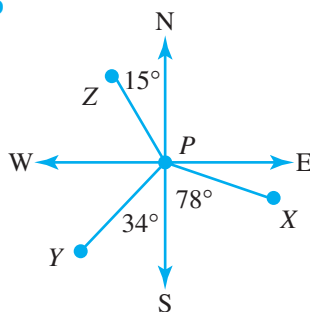
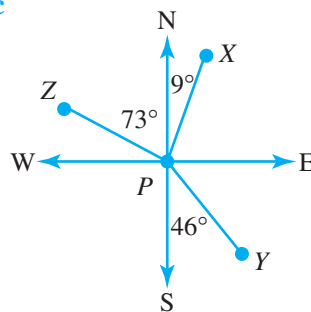
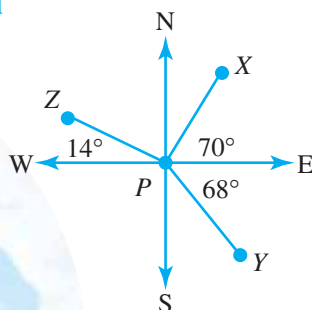
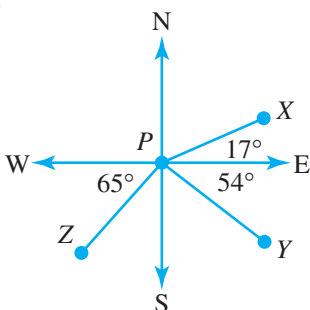
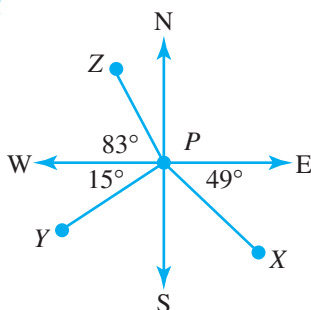
$$= 55^\circ \text{ (to the nearest degree)}$$

$$\text{Bearing} = 360^\circ - 55^\circ$$

$$= 305^\circ$$

**Exercise 12.9**

- 1
 - i Find the compass bearings from P of the points X , Y and Z .
 - ii Find the true bearings from P of the points X , Y and Z .

a**b****c****d****e****f**

- 2 Use a pair of alternate angles to find the true bearing of J from K , given that the bearing of K from J is:

a 126°

b 057°

c 339°

d 228°

e 023°

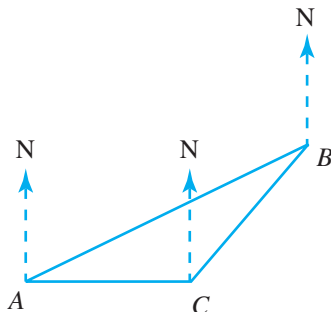
f 352°

g 205°

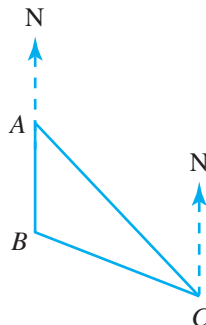
h 144°

3 In each of the following, find the size of $\angle ABC$ given that:

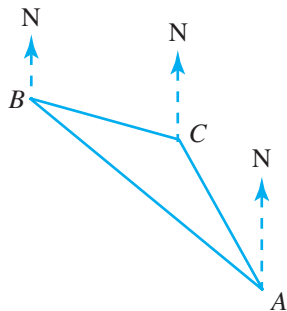
- a i the bearing of B from A is 028°
 ii the bearing of B from C is 016°



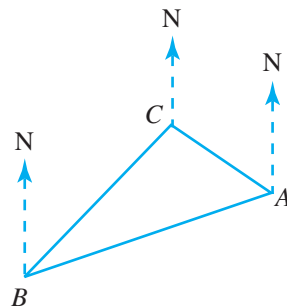
- b i the bearing of C from A is 150°
 ii the bearing of B from C is 305°



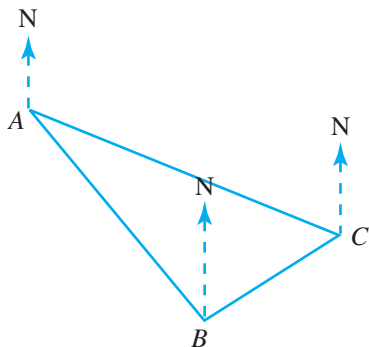
- c i the bearing of C from B is 105°
 ii the bearing of B from A is 327°



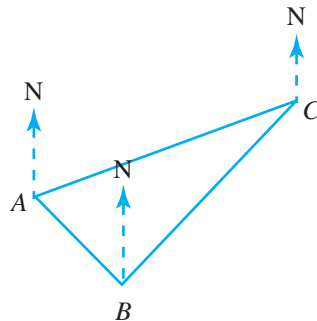
- d i the bearing of B from A is 250°
 ii the bearing of B from C is 232°



- e i the bearing of B from C is 241°
 ii the bearing of A from B is 316°



- f i the bearing of B from C is 210°
 ii the bearing of A from B is 320°



Consolidation

4 For each of the following, draw a neat diagram, then find the required distance, correct to 1 decimal place.

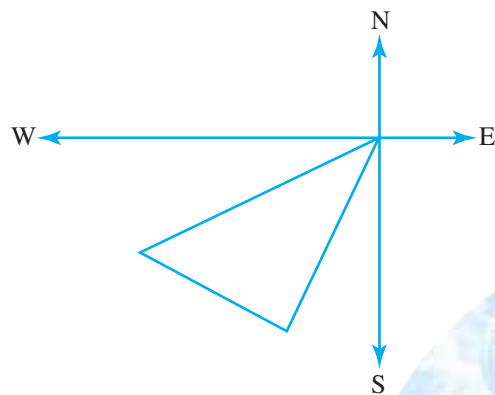
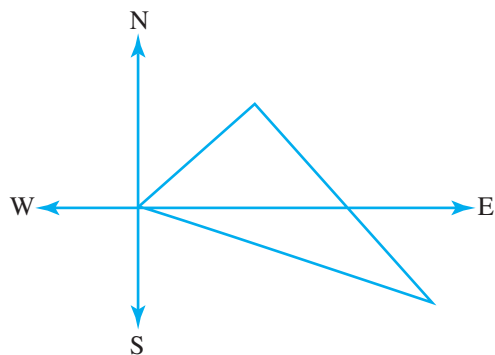
- a Lou drove for 72 km on a bearing of 051° . How far did she drive:
 i north?
 ii east?

- b** Annabel jogged for 13 km on a bearing of 122° . How far did she jog:
i east? **ii** south?
- c** Erin cycled for 37 km on a bearing of 254° . How far did she cycle:
i south? **ii** west?
- d** Michael sailed for 115 km on a bearing of 343° . How far did he sail:
i west? **ii** north?
- 5** Answer each of the following, correct to 4 significant figures.
- a** A ship leaves port and sails 25 km due east from A to B , then turns and sails due north to C . If the bearing of C from A is 036° , find the distance BC .
- b** A car travels 148 km due south from P to Q , then turns and travels due east to R . If the bearing of R from P is 143° , find the distance QR .
- c** A helicopter flew due west from X to Y , then changed course and flew 86 km due south to Z . If Z is on a bearing of 205° from X , find the distance between X and Y .
- d** A yacht sailed due north from F to G , then changed course and sailed 52 km due west to H . If H is on a bearing of 337° from F , find the distance between F and G .
- 6** Answer each of the following, correct to the nearest km.
- a** Moonee is 73 km due east of Lewisville. Newdale is due north of Moonee and on a bearing of $N43^\circ E$ from Lewisville. Find the distance between Newdale and Lewisville.
- b** Ulladatta is due south of Tarramoorra. Vineyard is 53 km due west of Ulladatta and on a bearing of $S15^\circ W$ from Tarramoorra. Find the distance between Tarramoorra and Vineyard.
- c** Benning is due west of Ascot. Cartwright is 126 km due north of Benning and on a bearing of $N28^\circ W$ from Ascot. Find the distance between Ascot and Cartwright.
- d** Frankston is 78 km due south of Eaglevale. Glenmore is due east of Frankston and on a bearing of $S17^\circ E$ from Eaglevale. Find the distance between Eaglevale and Glenmore.
- 7** Answer each of the following, correct to the nearest degree.
- a** A catamaran sailed 11 km due south from P to Q , then sailed 16 km due east to R . Find the true bearing of:
i R from P **ii** P from R
- b** Zachary rode due north from A to B , then rode 15 km due west to C , which is 33 km from A . Find the true bearing of:
i C from A **ii** A from C
- c** Ellen drove 107 km due south from V to W , then drove due west to X , which is 135 km from V . Find the true bearing of:
i X from V **ii** V from X
- d** A park ranger flew his helicopter 76 km due east from C to D , then flew 210 km due south to E . Find the true bearing of:
i E from C **ii** C from E

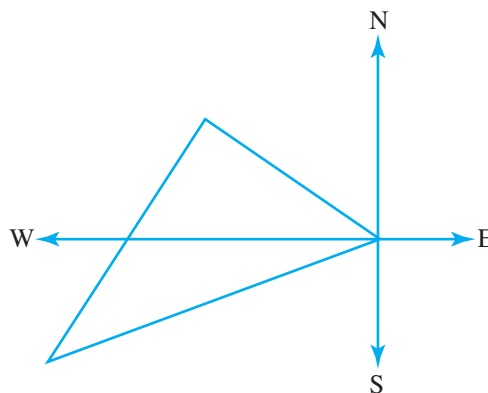
- 8 a** Martin walked from home (H) to the local store (S) on a bearing of 035° . He then walked on a bearing of 125° to the park (P), which is 450 m due east of his home.
- Show that $\angle HSP = 90^\circ$.
 - Find the distance between Martin's home and the store, correct to 1 decimal place.
- b** Walter sailed his new boat from the marina (M) to a buoy (B) on a bearing of 243° . He then sailed on a bearing of 333° to a small wharf (W), which is 14 km due west of the marina to pick up a friend.
- Show that $\angle MBW = 90^\circ$.
 - Find the distance between the buoy and the wharf, correct to 3 significant figures.
- c** Tamsin ran 7 km from home (H) to the beach (B) on a bearing of 104° . She then ran on a bearing of 194° to the gym (G), which is due south of her home.
- Show that $\angle HBG = 90^\circ$.
 - How far is Tamsin's home from the gym, correct to the nearest km?

9 In each of the following:

- Copy the diagram provided and mark on it all of the given information.
 - Show that the triangle is right-angled.
 - Find the required bearing, correct to the nearest degree.
- a** A homing pigeon flew 7 km from its home on a bearing of 037° . After a short rest, the pigeon flew 16 km to its destination on a bearing of 127° . Find the bearing of the pigeon from its home.
- b** While participating in an orienteering activity, a group of children walked from the starting position on a bearing of 212° until they reached a waterhole. They then walked on a bearing of 302° for 4 km to a waterfall, which is 5.5 km from the starting position. Find the bearing of the waterfall from the starting position.



- c The captain of a ship at sea sights a lighthouse 5 nautical miles away on a bearing of 341° . At the same time, the lighthouse supervisor sights a yacht on a bearing of 251° . The distance between the yacht and the ship is 30 nautical miles. Find the bearing of the ship from the yacht.



- 10 Jeff and Erica leave home at 7 pm. Jeff drove due north at 80 km/h while Erica drove due east at 60 km/h. Find the distance and bearing of Erica from Jeff at 8:15 pm.
- 11 Anita and Barbara set out at the same time. Anita walked NE at 4 km/h while Barbara walked SE at 6 km/h. Find the distance and bearing of Anita from Barbara after 2 hours.

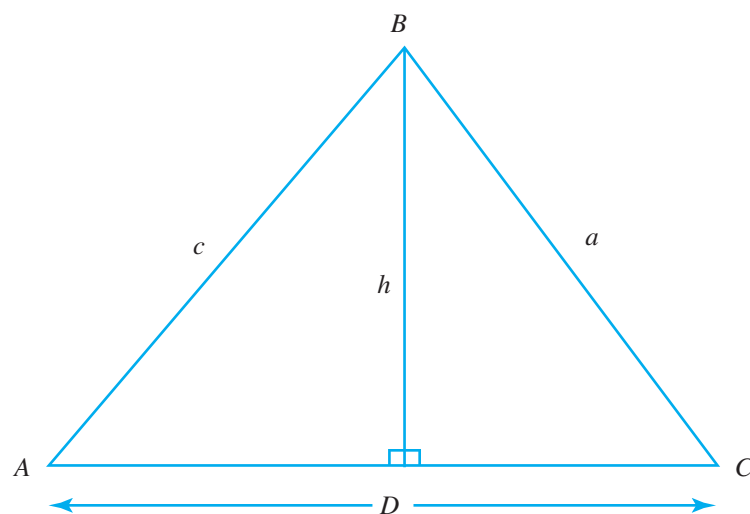
Further applications

- 12 A submarine sailed 30 nautical miles from X to Y on a course of 040° . It then sailed 55 nautical miles due east to Z . Find the distance and bearing of Z from X .
- 13 A fire observation tower T in a national park is 10 km from ranger headquarters R on a bearing of $N52^\circ W$. A second tower M is 24 km from headquarters on a bearing of $N38^\circ E$.
- Find the distance between the towers.
 - Find the bearing of:

i M from T	ii T from M
iii R from T	iv R from M
- 14 The bearing of a ship F from a lighthouse L is 320° , while the bearing of a second ship G from the same lighthouse is 285° . Both ships are due north of an oil rig H , which is 15.2 nautical miles due west of the lighthouse. Calculate the distance between the ships, correct to the nearest nautical mile.

TRY THIS

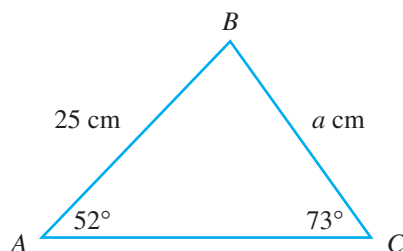
The sine rule



$\triangle ABC$ is a non-right-angled triangle with altitude BD .

- a** Show that $h = c \sin A$
- b** Show that $h = a \sin C$
- c** Hence, show that $\frac{a}{\sin A} = \frac{c}{\sin C}$

This is known as the sine rule and it can be used to find the length of a side in a non-right-angled triangle. Use this rule to find the value of a in this triangle, correct to 1 decimal place.





FOCUS ON WORKING MATHEMATICALLY

FINDING YOUR LATITUDE FROM THE SUN

**Introduction**

The latitude of a place on the Earth's surface is a measure of how far the place is north or south of the equator. It is measured in degrees. For example, Australia stretches between 10° South and 40° South of the equator. These days, the *global positioning system* (GPS) lets us read our position from a satellite very accurately. For example, GPS receivers are regularly installed on boats and aeroplanes as a safety device. Bushwalkers carry them for the same reason.

The early mariners were able to find their latitude from the Sun by day and from the stars by night. In this activity we will find out how to measure latitude by measuring the angle of the Sun's rays at its highest point in the sky. This angle is called the *zenith angle*.

At noon on about 21 March and 22 September (the time and date varies slightly each year), the Sun is overhead at the equator. On these days (called the equinox, meaning equal day and equal night), we can get a direct measure of our latitude. However, we can do the calculation on any day of the year if we make an appropriate adjustment.



LEARNING ACTIVITIES

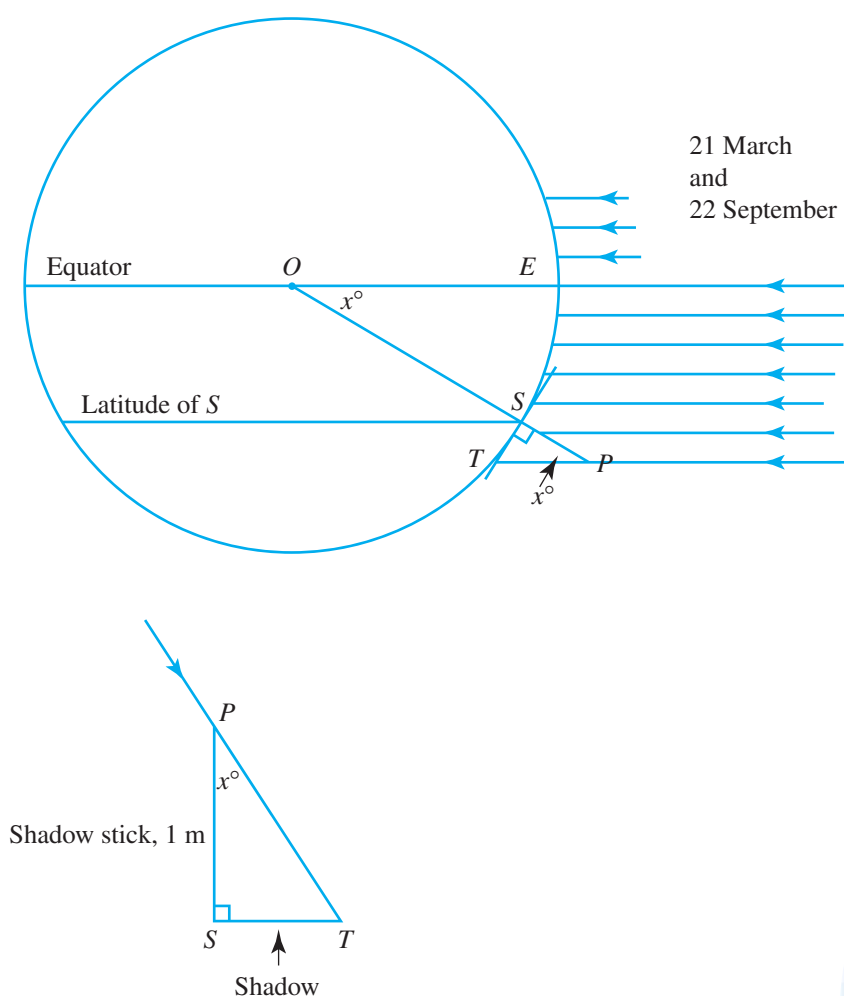
Materials needed: a 1 m rule, string, a measuring tape and a calculator and a sunny day.

- 1 Write down the date on which you are taking this measurement, at the top of your page. This is especially important if the date is different from 21 March or 22 September.



- 2 If you can, set up the shadow stick (1 m rule) so that the measurements can be taken just before and just after noon. Twenty minutes before and after at 10-minute intervals would be suitable. The idea is to measure the length of the shadow when the Sun is at its highest point in the sky, that is when the shadow of the stick is shortest. Make sure the stick is vertical.
- 3 Choose the shortest shadow. The angle x in the diagram is the latitude of your city or town (strictly speaking, wherever you took the measurement). Use the tangent ratio to calculate x . Check using an atlas.

The latitude of a point S measured by a shadow stick at noon on 21 March or 22 September (diagram not to scale).



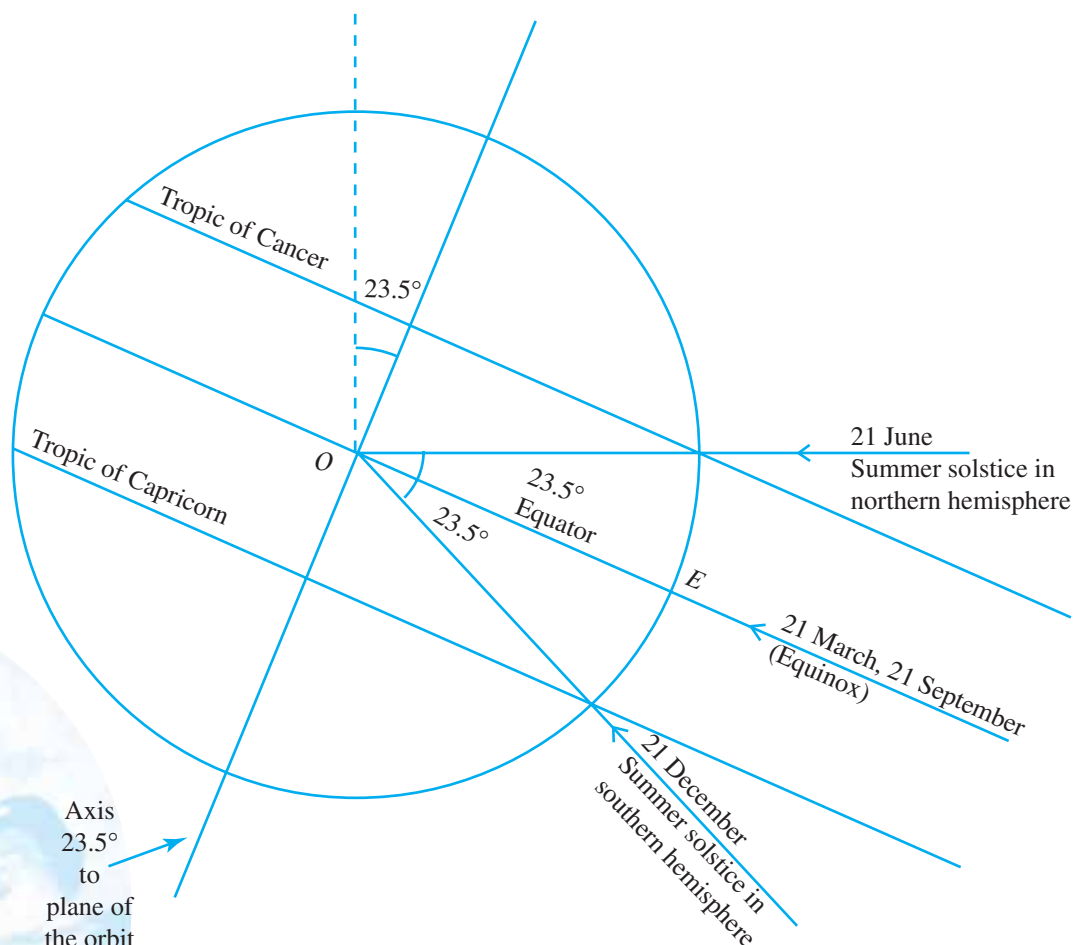


CHALLENGE

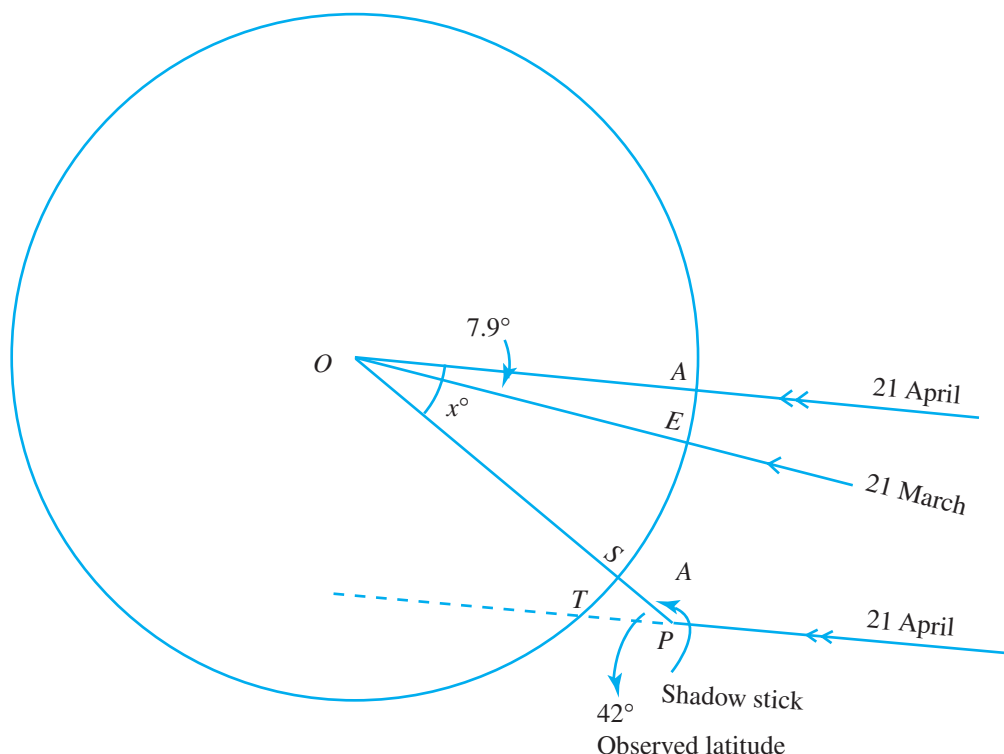
Because of the tilt of the Earth to the plane of its orbit (23.5°) the Sun appears to move across the sky so that it is overhead no further north than 23.5° (Tropic of Cancer), and no further south than 23.5° (Tropic of Capricorn). Globes (models of the Earth) are tilted at this angle to show this clearly.

The diagram below illustrates this idea.

- 1 It takes 3 months from 21 March to 21 June (92 days) for the Sun to move from the equator and be overhead at the Tropic of Cancer. During this time it moves through 23.5° . The Sun then begins the journey back, reaching the equator on 21 September. It is overhead at the Tropic of Capricorn on 21 December. Make a copy of the diagram in your book.



- 2 If a measurement of the latitude of S is taken on a day other than 21 March or 21 September, you can use the following model to make your own calculations. The calculation below is for 21 April at 12 noon.



The number of days from 21 March to 21 April is 31. So, by 21 April, the Sun has moved a fraction of $\frac{31}{92}$ of 23.5° north across the sky. By using a calculator, a value close to 7.9 degrees is obtained. The Sun is now overhead at A .

Suppose the observed 'latitude' of Sydney (S) at noon on 21 April was 42° .

By using alternate angles we see that x , the true latitude of Sydney, is given by $x = 42^\circ - 7.9^\circ = 34.1^\circ$

- 3 If a measurement is taken between 21 September and 21 December, the adjustment will have to be *added*. Draw a new diagram to show the position of the Sun on 11 October. Calculate the number of degrees it is south of the equator and then find your latitude.



LET'S COMMUNICATE

Make a poster to illustrate what you have learned about calculating latitude from the position of the Sun in the sky when it is overhead at the equator. Include information about the tilt of the Earth, the days of the equinox, and the days of the solstice and the movement of the Sun across the sky through the year.



REFLECTING

The early mariners of the Mediterranean, such as the Greeks and Phoenicians in the fourth century BC, experienced the sight of land rising as their ships approached land. Similarly, a ship's mast appeared to sink below the horizon when sailing out of port. It was also noticed that noon shadows always point north above the tropics, but within the tropics they will point north at some seasons and south in others.

In learning to navigate by latitude, the early mariners came to an inescapable conclusion. What was it?



MACQUARIE

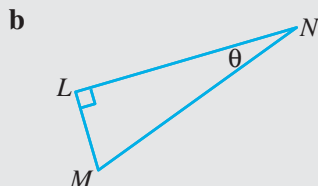
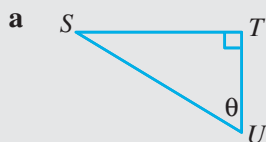
Language link with Macquarie

- 1 Use each of the following in a simple sentence:
 - a *angle of depression*
 - b *the tangent ratio*
 - c *the bearing of Q from P*
 - d *latitude of Sydney*
- 2 Read the Macquarie Learners Dictionary entry for **equinox**:

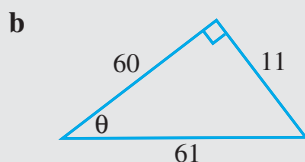
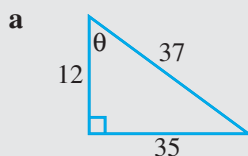
equinox *noun* either of two times of the year when the sun is directly over the earth's equator, making night and day all over the earth of equal length, occurring about 21 March and 22 September.

Many other words in mathematics contain the prefix **equi-**, for example, *equilateral* (equal sides). Can you think of others?

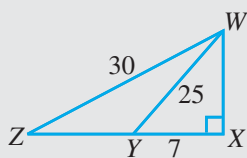
- 1 Name the hypotenuse, opposite side and adjacent side in each triangle.



- 2 Find values for $\sin \theta$, $\cos \theta$ and $\tan \theta$ for each triangle.



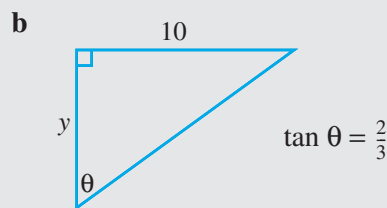
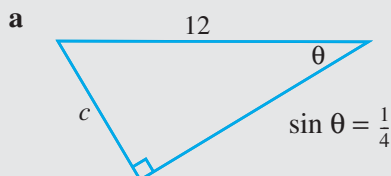
- 3 a Find the lengths WX and YZ .



- b Hence, find in simplest form, the value of:

- i $\sin \angle WYX$ ii $\cos \angle WZX$
 iii $\tan \angle XWZ$ iv $\sin \angle XWY$
 v $\cos \angle XYW$ vi $\tan \angle XZW$

- 4 Find the value of the pronumeral in each of these.



- 5 Evaluate each of the following, correct to 1 decimal place.

- a $24 \cos 37^\circ$ b $\frac{75}{\tan 52^\circ}$
 c $\frac{\sin 28^\circ + \tan 49^\circ}{5 \cos 32^\circ}$

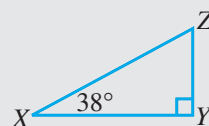
- 6 Find θ , correct to the nearest degree.

- a $\tan \theta = 4.0108$ b $\cos \theta = 0.0773$
 c $\sin \theta = 0.1246$

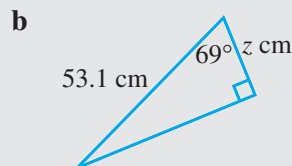
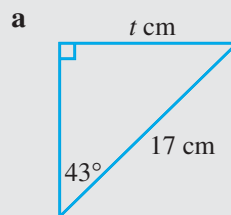
- 7 If $\tan \theta = 2.0503$, find, correct to 2 decimal places, the value of:

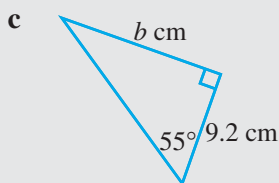
- a $\sin \theta$ b $\cos \theta$

- 8 Find the value of $\frac{YZ}{XY}$, correct to 3 decimal places.

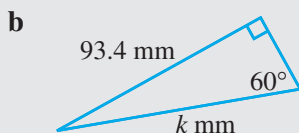
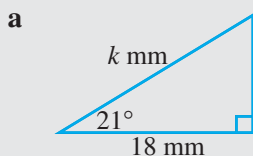


- 9 Find the value of each pronumeral, correct to 1 decimal place.



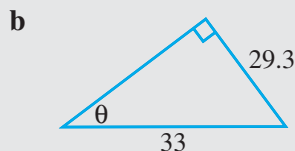
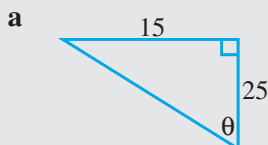


- 10 Find the value of k in each of these, correct to 2 decimal places.

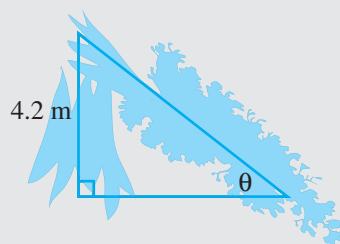


- 11 In $\triangle DEF$, $\angle F = 90^\circ$, $\angle E = 17^\circ$ and $DF = 42$ cm. Find, correct to 2 decimal places, the length of:
 a EF b DE
- 12 A ladder 5.2 m long leans against a wall and makes an angle of 61° with the ground. Find how far the ladder reaches up the wall, correct to 1 decimal place.
- 13 A tree of height 18.5 m casts a shadow on the ground. Find the length of the shadow when the sun has an altitude of 32° .
- 14 a In the isosceles $\triangle FGH$, $FG = GH = 28$ cm and $\angle FHG = 67^\circ$. Find the length of FH , correct to 1 decimal place.
 b In the isosceles $\triangle UVW$, $UW = VW$, $UV = 13$ cm and $\angle UWV = 102^\circ$. Find the length of UW , correct to 3 significant figures.

- 15 Find θ , correct to the nearest degree.



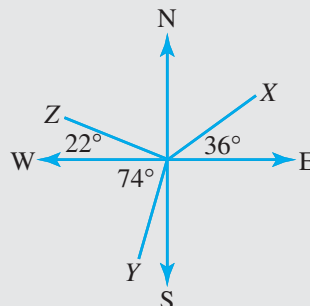
- 16 In $\triangle XYZ$, $\angle Z = 90^\circ$, $XZ = 25$ m and $XY = 83$ m. Find the size of $\angle ZXY$, correct to the nearest degree.
- 17 A rectangle has a length of 9.4 cm and a perimeter of 34.2 cm.
 a Find the width of the rectangle.
 b Calculate, to the nearest degree, the angle between the diagonal and the width.
- 18 After a strong storm, a tree of height 34 m broke in two, with the top section resting on the tree stump. If the stump has a height of 4.2 m, find to the nearest degree, the angle between the tree and the ground.



- 19 a In the isosceles $\triangle PQR$, $PQ = QR$. S is a point on PR such that QS is perpendicular to PR . If $QS = 54$ mm and $PR = 90$ mm, find the size of $\angle P$, correct to the nearest degree.

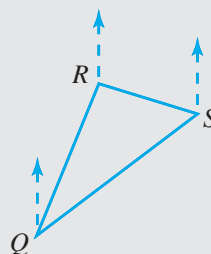
- b** In the isosceles triangle IKJ , $IJ = JK = 35$ mm and $IK = 44$ mm. Find the size of $\angle IJK$, correct to the nearest degree.
- 20 a** From a boat 452 m out to sea, the angle of elevation of the top of a cliff is 54° . Find the height of the cliff, correct to 1 decimal place.
- b** The angle of depression of a point E on the ground from the top of a 150 m high tower is 26° . Find the distance between E and the foot of the tower, correct to 4 significant figures.
- 21** Calculate the angle of depression of a small fountain from the top of a telegraph pole if the pole is 15 m tall and the fountain is 21 m from the base of the pole. Give your answer correct to the nearest degree.
- 22** Clem is standing at a point A , 38 m away from the front door of a building. From this point, the angle of elevation of the top of the building is 75° .
- a** Find the height of the building, correct to the nearest centimetre.
- b** Clem walks 52 m further away from the building to a point B such that B , A and the front door of the building are in a straight line. Find the angle of elevation of the top of the building then. Answer correct to the nearest degree.
- 23 a** Find the compass bearings from P of the points X , Y , Z .

- b** Find the true bearings from P of the points X , Y , Z .



- 24** The bearing of R from Q is 043° , the bearing of R from S is 288° , and the bearing of S from Q is 068° . Find the size of:

a $\angle RQS$ **b** $\angle QRS$ **c** $\angle RSQ$



- 25** A ship leaves port and sails due north for 115 km, then turns and sails 175 km due west. Find the distance and bearing of the ship from port.
- 26** Tatum drove 74 km on a bearing of 138° . How far east did she drive, correct to 1 decimal place?
- 27** Ochre Hill is due south of Eastern Valley. Jackville is 107 km due west of Eastern Valley and on a bearing of $N13^\circ W$ from Ochre Hill. Find, correct to 4 significant figures, the distance between Ochre Hill and Jackville.

- 28** A pilot flew his light plane 240 km due east from F to G , then flew 475 km due north to H . Find as a true bearing the bearing of:

a H from F **b** F from H

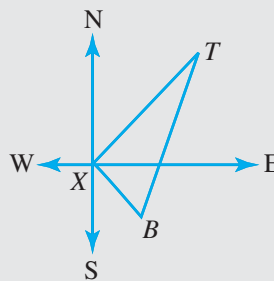
- 29** Soo Ji walked on a bearing of 295° from home (H) to the local park (P) to meet her sister. Together they walked on a bearing of 205° to the beach (B), which is 380 m due west of their home.

a Show that $\angle HPB = 90^\circ$.
b Find, correct to the nearest metre, the distance between Soo Ji's house and the park.

- 30** A surveyor standing at X sights a tower T , 840 m away on a bearing of 037° .

A second surveyor standing at B , 365 m from X , measures the bearing of X from his position as 307° .

a Show that $\angle TXB = 90^\circ$.
b Find the bearing of the tower from B .



13

Simultaneous equations



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

- ⦿ solve simultaneous equations by informal methods
- ⦿ solve a pair of simultaneous equations by graphing two lines and finding their point of intersection
- ⦿ recognise that equations of parallel lines cannot be solved simultaneously
- ⦿ solve a pair of simultaneous equations using the substitution method
- ⦿ solve a pair of simultaneous equations using the elimination method, by direct addition or subtraction
- ⦿ solve a pair of simultaneous equations using the elimination method, where one or both equations must first be multiplied by a constant
- ⦿ solve word problems, geometric problems and measurement problems using simultaneous equations.

13.1 Equations with two unknowns

A linear equation that contains only a single pronumeral must have a unique solution. For example, the only solution to the equation $2x + 1 = 7$ is $x = 3$, as 3 is the only number that can be doubled and then increased by 1 to give 7. However, equations that contain more than one pronumeral, such as $x + y = 6$, have an infinite number of solutions. Some possible solutions for this equation are:

$$x = 6, y = 0 \text{ or } x = 5, y = 1 \text{ or } x = 3, y = 3 \text{ or } x = 7, y = -1 \text{ or } x = 3\frac{1}{2}, y = 2\frac{1}{2} \text{ or } x = 4.8, y = 1.2.$$

A unique solution can be obtained only if the number of equations being solved is equal to the number of different pronumerals involved. In these situations, the given equations are solved *simultaneously* (i.e. at the same time). For example, it is possible to solve the equations $x + y = 6$ and $y = 2x$ simultaneously and obtain a unique solution because there are two unknowns (x and y) and there are two equations. By inspection, the solution is $x = 2, y = 4$ because $2 + 4 = 6$ and $4 = 2 \times 2$.

In this section, we will concentrate on solving simultaneous equations by informal methods. This may involve a guess and check approach, forming a table of values or using a spreadsheet.

EG
+S

Example 1

Form a table of values to find two positive integers x and y , which simultaneously satisfy the equations $x + y = 9$ and $2x + 3y = 23$.

Solution

x	y	$x + y$	$2x + 3y$
0	9	9	27
1	8	9	26
2	7	9	25
3	6	9	24
4	5	9	23

Therefore, the solution is $x = 4, y = 5$.

Exercise 13.1

- 1 State whether it is possible to find unique solutions for each of these problems.
 - a The sum of two numbers is 7. Find the numbers.
 - b The product of two numbers is 12. Find the numbers.
 - c The sum of two numbers is 7 and their product is 12. Find the numbers.
 - d The total cost of a drink and a sandwich is \$3.50. Find the cost of the drink.
 - e The product of two consecutive positive integers is 20. Find the integers.
 - f Jan is 8 years younger than her sister Harriet. How old is Jan?
 - g A straight line passes through the point (3, 1). Find its equation.
 - h Peter has saved twice as much money as Darren, and together they have saved \$36. How much money has each boy saved?

- 2 Why is it that some of the problems in Q1 have unique solutions while others do not?
- 3 Write down three pairs of integers a and b that satisfy each of the following equations.
a $a + b = 9$ **b** $3a - b = 11$ **c** $2a + 4b = 20$ **d** $3a - 3b = 12$
- 4 Is it possible to find values for x and y such that $x + y = 5$ and $x + y = 9$? Explain your answer.
- 5 Determine by substitution whether the given values satisfy each pair of simultaneous equations.
- | | | | |
|------------------------|-----------------|-----------------------|-----------------|
| a $x + y = 5$ | $x = 2, y = 3$ | b $x - y = 3$ | $x = 7, y = 4$ |
| $x + 3y = 11$ | | $x - 2y = 1$ | |
| c $2x + 3y = 4$ | $x = -1, y = 2$ | d $x - 2y = 9$ | $x = 5, y = -2$ |
| $4x + 5y = -14$ | | $2x + y = 8$ | |

■ Consolidation

- 6 Find the value of each pronumeral by using a guess and check approach.
- | | |
|------------------------|-------------------------|
| a $x + y = 13$ | b $p + q = 45$ |
| $x - y = 3$ | $p = 2q$ |
| c $a + 2b = 17$ | d $3m + 2n = 11$ |
| $2a + b = 13$ | $4m + 3n = 16$ |
- 7 Complete this table of values to find integers x and y which satisfy the equations $y = x + 8$ and $y = 2x + 5$.

x	$x + 8$	$2x + 5$
0		
1		
2		
3		
4		
5		

- 8 Find integers x and y , $x > 0$, which satisfy both equations, by completing a table of values similar to that in Q7.
- | | | |
|-------------------|-----------------------|------------------------|
| a $y = 2x$ | b $y = 22 - x$ | c $y = x + 6$ |
| $y = 3x - 5$ | $y = 4x + 7$ | $y = \frac{1}{2}x + 9$ |

- 9 Complete this table of values to find integers x and y which satisfy the equations $x + y = 7$ and $2x + 5y = 23$.

x	y	$x + y$	$2x + 5y$
0	7		
1	6		
2	5		
3	4		
4	3		

- 10 Find integers x and y , $x > 0$, which satisfy each pair of equations, by completing a table of values similar to that in Q9.

a $x + y = 14$
 $3x - y = 10$

b $x + y = 10$
 $2x + 3y = 27$

c $x - y = 8$
 $x - 5y = 12$

- 11 Solve the following problems by using a guess and check approach or by completing a table of values.

- a** Sharon is three times as old as Julian and the sum of their ages is 52 years. How old is each person?
- b** Dylan is 15 cm taller than Yuri and the sum of their heights is 345 cm. How tall is each boy?
- c** Tina paid \$1.60 for eight apples while Eloise paid the same amount for two apples and three pears. Find the cost of each piece of fruit.
- d** The length of a rectangle is 12 cm more than the width and the perimeter is 42 cm. Find the dimensions of the rectangle.

■ Further applications

Many pairs of simultaneous equations can be solved by using a spreadsheet. For example, to solve the equations $x + y = 8$ and $3x + 5y = 26$, set up the spreadsheet as follows.

	A	B	C	D
1	X	Y	$X + Y$	$3X + 5Y$
2	0	$= 8 - A2$	$= A2 + B2$	$= (3 * A2) + (5 * B2)$
3	$= A2 + 1$			

Now, use the **Fill Down** command to fill the cells down to line 10.

The spreadsheet should now look like this.

	A	B	C	D
1	X	Y	$X + Y$	$3X + 5Y$
2	0	8	8	40
3	1	7	8	38
4	2	6	8	36
5	3	5	8	34
⋮				
9	7	1	8	26
10	8	0	8	24

The solutions to the simultaneous equations can be found on line 9, i.e. $x = 7$, $y = 1$.

12 Use a spreadsheet to solve the following pairs of simultaneous equations. (Start the x values at -5).

a $x + y = 10$
 $3x + 4y = 33$

b $x - y = -6$
 $5x + 6y = 25$

c $x - y = 5$
 $4x - 9y = 30$

13.2 The graphical method

Equations such as $y = 4 - x$ and $y = 3x$ are called **linear equations** because they result in straight line graphs when graphed on a number plane. The solution to a pair of simultaneous linear equations is given by the co-ordinates of the point of intersection of their straight line graphs.

If the lines are parallel, then they have no point of intersection. In this case, there would be no solutions to the simultaneous equations.

To solve a pair of simultaneous linear equations graphically:

- ⊙ graph the straight lines on the same number plane
- ⊙ find the co-ordinates of the point of intersection
- ⊙ write the solution that corresponds to this point.

NOTE: The solutions to simultaneous equations are not always integers. In some questions, therefore, it is necessary to estimate the co-ordinates of the point of intersection of the graphs. The approximate solution to the simultaneous equations is then written using either fractions or decimals.

EG
+ S**Example**Solve graphically the simultaneous equations $y = 3x$ and $y = 4 - x$.**Solution**

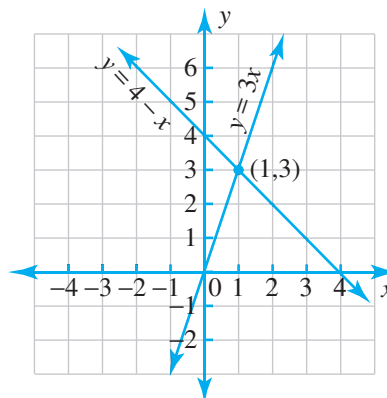
$$y = 3x$$

x	0	1	2
y	0	3	6

$$y = 4 - x$$

x	0	1	2
y	4	3	2

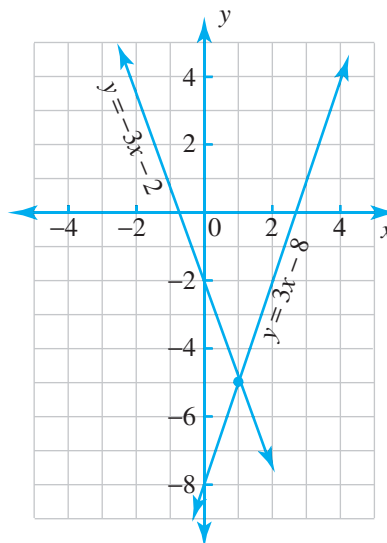
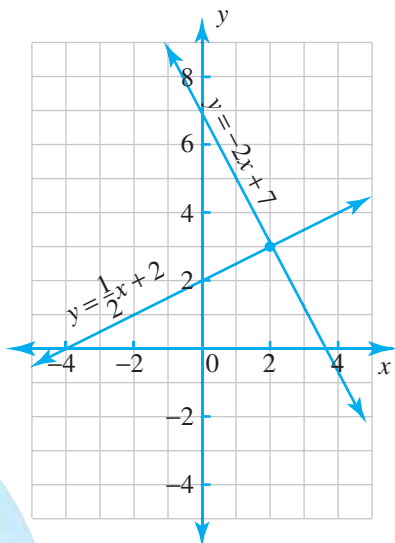
The lines intersect at $(1, 3)$. Therefore, the solution is $x = 1, y = 3$.

**Exercise 13.2**

1 Solve each pair of simultaneous equations by using the graphs below.

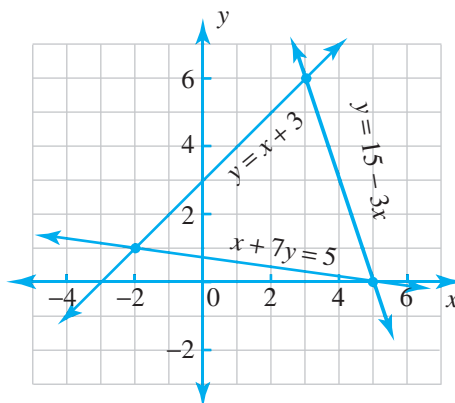
a $y = \frac{1}{2}x + 2, y = -2x + 7$

b $y = 3x - 8, y = -3x - 2$

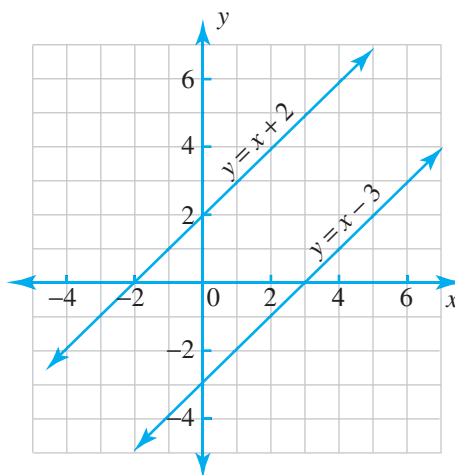


- 2 Use the given graphs to solve each pair of simultaneous equations.

- a** $y = x + 3$
 $y = 15 - 3x$
- b** $x + 7y = 5$
 $y = x + 3$
- c** $y = 15 - 3x$
 $x + 7y = 5$



- 3 The equations $y = x + 2$ and $y = x - 3$ have been graphed on the same number plane. Is it possible to find values for x and y that satisfy both equations? Explain your answer.



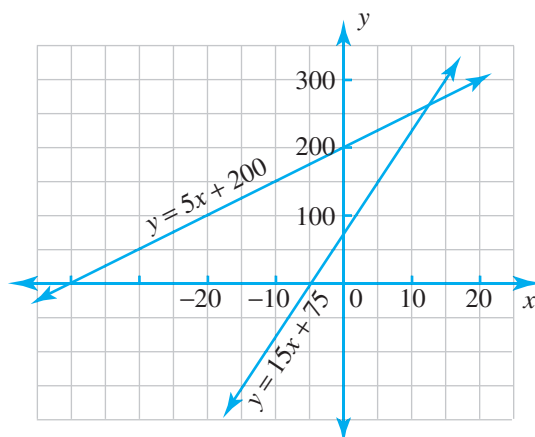
Consolidation

- 4 Graph each pair of equations on the same number plane and find their point of intersection. Hence, write down the solution to each pair of simultaneous equations.

a $y = x + 3$	b $y = -x$	c $y = 2x$
$y = 2x + 1$	$y = x + 6$	$y = 3 - x$
d $x + y = 5$	e $y = 3x$	f $x - y = 6$
$x - y = 1$	$2x + y = 5$	$y = -2x$
g $4p + q = 9$	h $a + b = 1$	i $m - 2n = 2$
$p - q = 1$	$2a + 3b = 6$	$n = m - 2$

- 5 **a** Graph the equations $y = \frac{1}{2}x + 1$ and $x - 2y = 2$ on the same number plane.
- b** Will solutions be obtained if the equations $y = \frac{1}{2}x + 1$ and $x - 2y = 2$ are solved simultaneously? Why?

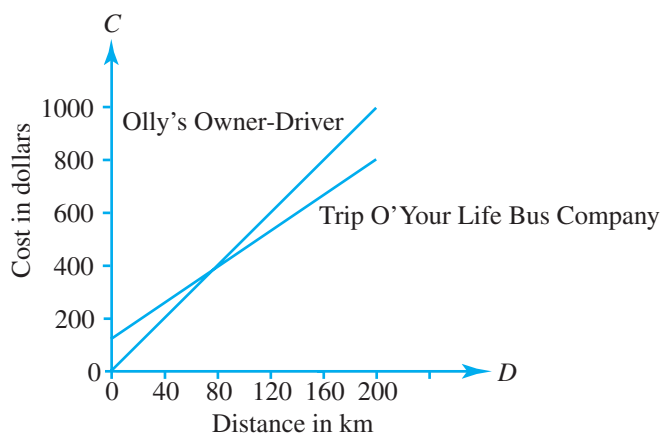
- 6 On some occasions, it may be necessary to estimate the co-ordinates of the point of intersection of two straight lines. This estimate can then be refined by substituting into both equations.



Use this method to solve the equations $y = 5x + 200$ and $y = 15x + 75$ simultaneously.

Further applications

- 7 The school wishes to hire buses to transport students on excursions. It is estimated that shorter excursions will be about a 40 km return trip while longer excursions may be up to 200 km return.

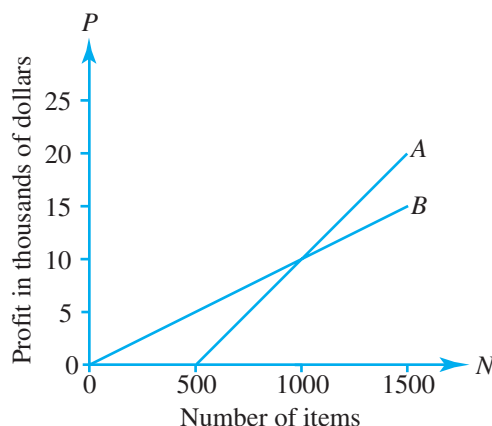


Olly's Owner-Driver offers buses at a charge of \$4.80 per km, and its rival Trip O' Your Life Bus Company charges \$80 up front to cover insurance plus \$4 per km.

- Write down an equation that represents each offer.
- On graph paper, make an accurate copy of the graph provided and label each line with the name of the company.
- Over what distance would the fee charged by both companies be the same?
- Over what distances will Olly's Owner-Driver be cheaper?

- 8 A certain factory produces products A and B. Product A makes a profit only after 500 items have been sold, after which a steady profit is made. Product B makes a profit of \$10 on the sale of each item.

- a What profit is made on the sale of 1500 items of:
 i product A? ii product B?
 b How many items of each product are sold if the profits from each are equal?
 c How many items of each product must be sold to make a profit of \$8000?



13.3 The substitution method

The graphical method of solution for simultaneous equations is quite time-consuming and it is not sufficiently accurate when the solutions are not integers. Simultaneous equations can be solved more quickly and with greater accuracy by using an algebraic approach. In this exercise, we look at one such method—the **substitution method**.

To solve a pair of simultaneous equations using the substitution method:

- ① **number the equations**
- ② **make one of the pronumerals the subject of one equation**
- ③ **substitute this expression into the other equation**
- ④ **solve the resulting equation**
- ⑤ **substitute this solution into either of the original equations to find the value of the other pronumeral.**

Example 1

Solve the equations $y = 2x - 11$ and $x + 4y = 19$ simultaneously, by using the substitution method.

Solution

$$y = 2x - 11 \dots\dots (1)$$

$$x + 4y = 19 \dots\dots (2)$$

- i Substitute $2x - 11$ for y in equation (2). ii Substitute $x = 7$ into equation (1).

$$x + 4(2x - 11) = 19$$

$$x + 8x - 44 = 19$$

$$9x - 44 = 19$$

$$9x = 63$$

$$\therefore x = 7$$

$$y = 2(7) - 11$$

$$= 14 - 11$$

$$= 3$$

$$\therefore \text{The solution is } x = 7, y = 3$$

**Example 2**

Solve the simultaneous equations $2x + 3y = 12$ and $5x - 2y = 15$ using the substitution method.

Solution

$$2x + 3y = 12 \dots\dots (1)$$

$$5x - 2y = 15 \dots\dots (2)$$

i Make x the subject of equation (1).

$$2x + 3y = 12$$

$$2x = 12 - 3y$$

$$\therefore x = \frac{12 - 3y}{2}$$

ii Substitute $\frac{12 - 3y}{2}$ for x in equation (2).

$$5\left(\frac{12 - 3y}{2}\right) - 2y = 15$$

$$\times 2 \qquad \qquad \times 2$$

$$5(12 - 3y) - 4y = 30$$

$$60 - 15y - 4y = 30$$

$$60 - 19y = 30$$

$$-19y = -30$$

$$\therefore y = \frac{30}{19}$$

iii Substitute $y = \frac{30}{19}$ into (2).

$$5x - 2\left(\frac{30}{19}\right) = 15$$

$$5x - \frac{60}{19} = 15$$

$$5x - \frac{60}{19} = \frac{285}{19}$$

$$5x = \frac{345}{19}$$

$$\therefore x = \frac{69}{19}$$

$\therefore$ The solution is

$$x = \frac{69}{19}, y = \frac{30}{19}.$$

Exercise 13.3

1 Use the fact that if $a = c$ and $b = c$, then $a = b$ to solve the following pairs of simultaneous equations.

a $y = 4x - 7$
 $y = 2x + 1$

b $y = x - 11$
 $y = 3x - 21$

c $y = 6 - 2x$
 $y = x - 15$

2 Solve each pair of simultaneous equations using the substitution method.

a $x + y = 5$
 $y = 3$

b $y = x + 3$
 $x = 4$

c $x - y = -8$
 $y = -5$

d $y = 3 - x$
 $x = -1$

e $x + y = 6$
 $y = x$

f $x + 3y = 14$
 $y = 2x$

g $2x - y = 12$
 $y = -x$

h $3x - 2y = -18$
 $y = -3x$

Consolidation

3 Solve each pair of equations using the substitution method.

a $x + y = 7$
 $y = x + 3$

b $x + y = 6$
 $x = y - 10$

c $4x + y = 13$
 $y = 2x - 11$

d $x + 2y = 8$
 $y = x + 1$

e $2x + 3y = 16$
 $x = y + 3$

f $3x + 11y = 12$
 $y = x - 4$

g $2x + 5y = 30$
 $x = 2y - 3$

h $3x + y = 19$
 $x = 3y + 13$

i $5x + 2y = 24$
 $y = 14 - 2x$

4 Solve the following simultaneous equations using the substitution method.

a $x - y = 1$

$y = 9 - x$

d $2x - y = 4$

$y = x - 3$

g $x - 2y = 5$

$y = x - 1$

j $2x - 3y = 19$

$y = 2x - 13$

b $x - y = 4$

$y = 6 - x$

e $3x - y = -11$

$y = x + 7$

h $3x - 5y = 2$

$y = x - 2$

k $4x - 7y = 26$

$y = 13 - 5x$

c $x - y = -3$

$y = -x - 11$

f $5x - y = 4$

$y = x - 16$

i $x - 4y = 17$

$y = -3 - x$

l $3x - 5y = 14$

$y = 6x + 8$

5 Make either x or y the subject of one equation, then substitute it into the other equation to solve these simultaneous equations.

a $x + y = 7$

$x + 7y = 19$

d $x - y = 14$

$2x + 3y = 3$

g $4x + y = 8$

$5x - 3y = 10$

b $3x + y = 2$

$2x + 3y = 20$

e $2x - y = 6$

$3x + 4y = -46$

h $x - 3y = 3$

$2x - 7y = 5$

c $2x + 7y = 1$

$x + 2y = 2$

f $x - 5y = 22$

$2x + 9y = 6$

i $2x - y = 4$

$3x - 8y = 32$

6 Solve the following pairs of simultaneous equations. [The solutions contain fractions].

a $a + 6b = 7$

$3a + 2b = 13$

d $u + 4v = 5$

$3u - 8v = 50$

b $9p + q = 5$

$15p - 2q = 12$

e $8c - 4d = 9$

$c + 2d = 3$

c $4m + 8n = 11$

$3m + n = 2$

f $6g - h = 8$

$4g + 4h = 3$

7 Solve these simultaneous equations using the substitution method.

a $2x + 3y = 8$

$3x + 2y = 7$

b $2x + 5y = 23$

$7x - 4y = 16$

c $3x - 4y = -1$

$4x - 7y = 2$

■ Further applications

8 Solve the equations $\frac{x}{2} + \frac{y}{4} = 5$ and $\frac{4x}{3} + \frac{5y}{8} = 13$.

9 Find values for x , y , z in each of these using the substitution method.

a $x + y + z = 9$

$y = 5x - 6$

$z = 2y - 5$

b $x - y + z = 13$

$y = 6 - x$

$z = 4y + 9$

c $x + y + z = 11$

$x - y + z = 15$

$x + y - z = -7$

TRY THIS Find the values

1 You are given that $x + y = 42$, $y + z = 67$ and $z + x = 55$.

a Find $x + y + z$.

b Hence, find values for x , y , z .

2 You are given that $xy = 117$, $yz = 286$ and $zx = 198$. Find values for x , y , z .

13.4 The elimination method

The second algebraic method of solution is the **elimination method**. It involves the addition or subtraction of corresponding terms in each equation to *eliminate* or remove one of the pronumerals. In some questions, it may be necessary to first multiply by a constant either one or both of the equations in order to eliminate a pronumeral.

To solve a pair of simultaneous equations using the elimination method:

- ⊙ number the equations
- ⊙ add the corresponding terms if two like pronumerals have co-efficients which are numerically equal but opposite in sign, or
- ⊙ subtract the corresponding terms if two like pronumerals have co-efficients which are numerically equal and have the same sign, or
- ⊙ multiply one or both equations by a constant such that two like pronumerals will have numerically equal co-efficients, then add or subtract the corresponding terms in each equation
- ⊙ solve the resulting equation
- ⊙ substitute this solution into either one of the original equations to find the value of the other pronumeral.

Example 1

Solve each pair of simultaneous equations using the elimination method.

a $7x + 2y = 24$
 $3x + 2y = 16$

b $6x + 5y = 19$
 $3x - 5y = 17$

Solutions

a **i** $7x + 2y = 24$ (1)
 $3x + 2y = 16$ (2)
 Subtract equation (2) from equation (1).
 $4x = 8$
 $\therefore x = 2$

ii Substitute $x = 2$ into equation (2).
 $3(2) + 2y = 16$
 $6 + 2y = 16$
 $2y = 10$
 $\therefore y = 5$
 $\therefore$ The solution is $x = 2, y = 5$.

b **i** $6x + 5y = 19$ (1)
 $3x - 5y = 17$ (2)
 Add equations (1) and (2).
 $9x = 36$
 $\therefore x = 4$

ii Substitute $x = 4$ into equation (2).
 $6(4) + 5y = 19$
 $24 + 5y = 19$
 $5y = -5$
 $\therefore$ The solution is $x = 4, y = -1$.

EG
+ S**Example 2**Solve the equations $4x - 3y = 9$ and $3x - 2y = 5$ simultaneously using the elimination method.**Solution**

i $4x - 3y = 9$ (1)

$3x - 2y = 5$ (2)

Multiply equation (1) by 2 and
equation (2) by 3.

$8x - 6y = 18$ (3)

$9x - 6y = 15$ (4)

Subtract (4) from (3).

$-x = 3$

$\therefore x = -3$

ii Substitute $x = -3$ into equation (2).

$3(-3) - 2y = 5$

$-9 - 2y = 5$

$-2y = 14$

$\therefore y = -7$

 $\therefore$ The solution is $x = -3, y = -7$.**Exercise 13.4****1** Add these expressions vertically.

a $2x + y$

$x + y$

e $5x - 3y$

$-x + 6y$

b $x - y$

$3x + y$

f $-10x - 9y$

$4x + y$

c $4x - 3y$

$9x + 3y$

g $3x - y$

$x - y$

d $-x + 2y$

$x + 7y$

h $x - 5y$

$2x - y$

2 Subtract these expressions vertically.

a $3x + 5y$

$3x + y$

e $3x + y$

$5x - 6y$

b $6x + 3y$

$2x + 3y$

f $5x - y$

$4x - y$

c $4x - y$

$x + 6y$

g $7x - 8y$

$3x - 5y$

d $3x - 4y$

$5x + y$

h $6x - 2y$

$-4x - 11y$

3 Solve each pair of simultaneous equations using the elimination method by adding the equations.

a $x + y = 5$

$x - y = 1$

d $-x + y = 9$

$x + y = 3$

g $x + 3y = 17$

$x - 3y = 5$

j $7x - 2y = 37$

$3x + 2y = 13$

b $x - y = 7$

$x + y = 5$

e $-5x + y = 3$

$5x + 3y = 29$

h $-x + 11y = 4$

$x + 2y = -4$

k $x + y = 5$

$-x + 2y = 16$

c $3x + y = 14$

$x - y = 2$

f $4x + 2y = 14$

$-4x + 5y = 35$

i $3x - 5y = 23$

$4x + 5y = 19$

l $7x + 4y = -29$

$-7x + 5y = 11$

- 4 Solve each pair of simultaneous equations using the elimination method by subtracting the equations.

a $2x + y = 13$

$x + y = 8$

d $x + 5y = 21$

$x + y = 1$

g $2x - y = 8$

$x - y = 3$

j $x - 5y = 22$

$x - y = 6$

b $4x + y = 2$

$x + y = 5$

e $x + 9y = 3$

$x + 12y = 0$

h $3x - y = 20$

$x - y = 8$

k $2x - y = 5$

$2x - 7y = 59$

c $5x + y = 22$

$3x + y = 16$

f $2x + 3y = 12$

$2x + 7y = 36$

i $11x - 2y = 30$

$3x - 2y = -2$

l $-3x - 9y = 48$

$-3x - 4y = 33$

Consolidation

- 5 Solve each pair of simultaneous equations using the elimination method.

a $x + y = 11$

$x - y = 5$

d $x + y = 4$

$x - 2y = 13$

g $7x - 2y = 27$

$3x - 2y = 7$

j $x + 3y = 33$

$5x + 3y = 45$

m $x - 3y = 4$

$-x - 2y = -14$

b $2x + y = 20$

$x + y = 8$

e $x + 4y = 3$

$x + y = 6$

h $2x + 7y = 73$

$2x + 3y = 29$

k $5x - 2y = 1$

$5x - 6y = 13$

n $x - 3y = 14$

$x - 9y = 2$

c $3x - y = 19$

$x - y = 5$

f $3x - 5y = 46$

$2x + 5y = 14$

i $3x + y = 7$

$8x - y = 37$

l $-6x - 7y = 31$

$3x + 7y = -19$

o $5x - 2y = 37$

$-5x + 8y = -13$

- 6 Multiply either one or both equations by a suitable constant, then add or subtract to find x, y .

a $x + y = 8$

$3x + 2y = 21$

d $x - 4y = 29$

$4x + 3y = 2$

g $x - y = 5$

$3x - 4y = 12$

j $2x + 5y = 24$

$3x + 2y = 14$

m $4x + 3y = 5$

$7x - 4y = -19$

p $3x - 7y = 13$

$4x + 5y = 3$

b $x + 3y = 10$

$3x + y = 14$

e $2x - y = -3$

$5x + 2y = 15$

h $4x - 3y = 10$

$8x - 4y = 16$

k $3x + 4y = 29$

$2x - 6y = 2$

n $6x - 13y = 1$

$5x + 2y = -12$

q $2x + 5y = 7$

$3x + 2y = -6$

c $3x + 4y = 22$

$2x + y = 3$

f $x + 2y = 1$

$6x - 5y = 23$

i $5x - 2y = 8$

$7x - 8y = -20$

l $2x - 3y = 6$

$3x - 2y = 19$

o $2x - 4y = 20$

$7x - 9y = 55$

r $2x - 9y = 8$

$-5x - 6y = 37$

- 7 Solve the following pairs of simultaneous equations. [The solutions contain fractions].

a $7p + q = 6$

$9p + 5q = 22$

b $5a + 3b = 1$

$13a - 9b = 4$

c $6m - 9n = 5$

$8m - 7n = 10$

Further applications

8 Solve each pair of equations simultaneously using the elimination method.

a $\frac{x}{4} + y = 9$

$\frac{x}{6} + 2y = 2$

b $\frac{x}{4} + \frac{y}{3} = 16$

$\frac{x}{8} - \frac{y}{12} = 2$

c $\frac{10-x}{3} - \frac{y}{4} = 6$

$\frac{3x}{4} + \frac{9-y}{5} = 8$

9 Solve for x, y, z using the elimination method.

a $x + y + z = 11$

$x + y - z = 7$

$x - y + z = 1$

d $2x + y - z = 0$

$3x - y + 4z = 10$

$x + 2y - z = -6$

b $x - y + z = 5$

$x + y + z = 9$

$x + y - z = 15$

e $2x + 3y - z = 18$

$3x + 2y + 2z = 6$

$5x + y - 4z = 1$

c $x + 2y + 3z = 10$

$2x - y - 3z = 9$

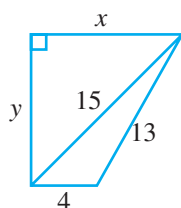
$5x + 4y + 3z = 32$

f $x + 2y + 3z = 4$

$2x + 3y - 5z = 65$

$4x - y - 4z = 31$

TRY THIS A Pythagorean problem



Form a pair of simultaneous equations, then solve them to find values for x, y .

13.5 Solving problems using simultaneous equations

We have already seen that a single equation that contains two pronumerals will not have a unique solution. We need to solve a pair of equations simultaneously in order to find unique values for the two unknowns. Similarly, in real-life or for other practical problems, a single statement relating two unknown quantities may not provide sufficient information to enable us to solve the problem. However, if two statements are given relating the quantities, then we can use simultaneous equations to find a unique solution.

To solve a problem using simultaneous equations:

- ⊙ use pronumerals (x and y) to represent the quantities that are to be found
- ⊙ form a pair of equations that show the relationship between the pronumerals
- ⊙ solve the equations simultaneously
- ⊙ use the algebraic solutions for x and y to answer the problem.

Example 1

EG
+S

The cost of 1 banana and 3 tomatoes is \$0.76, while 2 bananas and 5 tomatoes cost \$1.34. Find the cost of each.

Solution

Let the cost of 1 banana be x cents and the cost of 1 tomato be y cents.

i $x + 3y = 76 \dots\dots (1)$

$2x + 5y = 134 \dots\dots (2)$

Multiply equation (1) by 2.

$2x + 6y = 152$

$2x + 5y = 134$

Subtract equation (2) from equation (1).

$\therefore y = 18$

ii Substitute $y = 18$ into equation (1).

$x + 3(18) = 76$

$x + 54 = 76$

$\therefore x = 22$

$\therefore$ The cost of a banana is 22c and the cost of a tomato is 18c.

Example 2

EG
+S

A jar contains 25 coins made up of only 5c and 20c coins. The total amount of money in the jar is \$2.90. How many of each coin are there?

Solution

Let the number of 5-cent coins be x and the number of 20-cent coins be y .

The total number of coins is given by the equation $x + y = 25 \dots\dots (1)$.

If there are x 5-cent coins, then the amount of money in 5-cent coins is $5x$ cents.

If there are y 20-cent coins, then the amount of money in 20-cent coins is $20y$ cents.

The total amount of money, in cents, is given by the equation $5x + 20y = 290 \dots\dots (2)$.

i $x + y = 25 \dots\dots (1)$

$5x + 20y = 290 \dots\dots (2)$

Multiply equation (1) by 5.

$5x + 5y = 125 \dots\dots (3)$

$5x + 20y = 290 \dots\dots (2)$

Subtract equation (2) from equation (3).

$-15y = -165$

$\therefore y = 11$

ii Substitute $y = 11$ into equation (1).

$x + 11 = 25$

$\therefore x = 14$

$\therefore$ There are 14 5c coins and 11 20c coins.

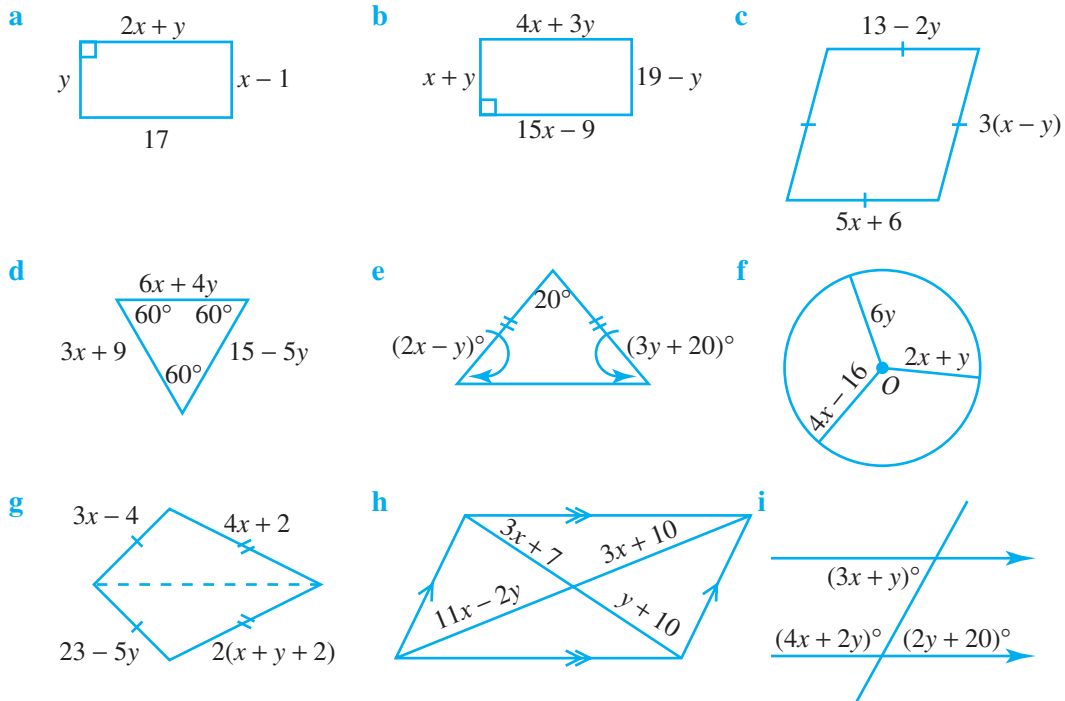
Exercise 13.5

- 1 For each of the following, form a pair of simultaneous equations, then solve them to find the numbers. [Let the numbers be x and y .]
 - a The sum of two numbers is 22 and their difference is 12.
 - b The difference between two numbers is 27 and one number is four times the other.
 - c The sum of two numbers is 38 and one number is 8 less than the other.
 - d The sum of two numbers is 14. The smaller number plus five times the larger number is 54.
 - e The first number plus twice the second number is equal to 27, while twice the first number plus the second number is equal to 24.
 - f Find two numbers which differ by 3 such that three times the larger number plus twice the smaller number is equal to 54.

Consolidation

- 2 Form a pair of simultaneous equations and solve them to answer each of the following problems.
 - a A man is five times the age of his son and the sum of their ages is 42 years. How old is each person?
 - b A drink and three ice-creams cost \$3.60, while three drinks and two ice-creams cost \$5.20. Find the cost of each.
 - c Two apples and three peaches cost \$1.35, while four apples and nine peaches cost \$3.75. Find the cost of each.
 - d The total weight of three tables and ten chairs is 1850 kg while two tables and twelve chairs weigh 1660 kg. Find the weight of each.
 - e In a game of legball, the Rhinos scored three tries and four goals for a total of 27 points, while the Dingoes scored two tries and five goals for a total of 25 points. How many points are scored for each try and goal?
 - f Tammy's piggy bank contains only five-cent and ten-cent coins. If it contains 48 coins with a total value of \$3.45, find the number of each type of coin.
 - g In the HSC, Craig's exam result for mathematics exceeded his school assessment result by 8 marks. The total of the two results was 166 marks. Find Craig's exam result and his school assessment result.
 - h A small community theatre charges admission prices of \$15 for adults and \$8 for children. If 85 tickets were sold on opening night and the total takings were \$1114, find the number of adults and children who attended.
 - i Last weekend a corner store sold 40 bottles of Coola and 55 bottles of Melonade, which contributed \$108 to their total takings for the week. One customer bought a bottle of each drink and paid \$2.25. Find the cost of each drink.
 - j A rectangle is three times as long as it is wide and the total perimeter is 112 cm. Find the dimensions of the rectangle.
 - k The adjacent sides in a parallelogram are in the ratio 3 : 2 and the total perimeter is 150 cm. Find the length of the sides.

- 1 At a children's birthday party, the children are aged either 12 years or 13 years. If there are 14 children at the party and the total of their ages is 176 years, find the number of children in each age group.
- 3 Form a pair of simultaneous equations by using a geometric property, then solve for x, y . (All measurements are in cm.)



- 4 **a** When the numerator and denominator of a certain fraction are each increased by 1, the value of the fraction is then $\frac{5}{6}$. However, when the numerator and denominator are each decreased by 5, the value of the fraction is then $\frac{3}{4}$. Find the original fraction.
(Hint: Let the fraction be $\frac{x}{y}$.)
- b** After a bill was debated in parliament, the members of the House of Representatives voted in favour of the bill by a majority of 39. If there were 97 politicians present, find the number who voted in favour of the bill.
- c** The straight line with equation $y = mx + b$ passes through the points (2, 10) and (-3, -25). Find values for m and b and hence find the equation of the line.
- d** Three times the sum of two numbers exceeds seven times their difference by 14, while half the difference of the numbers is equal to 12 less than their sum. Find the numbers.
- e** The diagonals of a rhombus are such that one diagonal is 14 cm longer than the other, while the sum of their lengths is 46 cm. Find the lengths of the diagonals and hence determine the perimeter of the rhombus.

- f** The linear equation $F = Ca + b$ links the temperature in degrees Celsius ($^{\circ}\text{C}$) and degrees Fahrenheit ($^{\circ}\text{F}$). If 10°C is equivalent to 50°F and 30°C is equivalent to 86°F , find values for a and b . Hence, express the boiling point of water in degrees Fahrenheit.

■ Further applications

- 5 a** Six years ago, Lydia was three times as old as Keiko. In three years time, Lydia will be twice as old as Keiko. What are their present ages?
- b** In a certain two-digit number, twice the tens digit is equal to one less than five times the units digit. If the digits are reversed and this number is subtracted from the original number, the result is equal to 14 less than five times the sum of the digits. Find the number.
- c** A man paddles his canoe 60 km downstream (i.e. with the current) for 5 hours. Later that day, he paddles 48 km back upstream (i.e. against the current) for 6 hours. The man's rowing speed is constant at x km/h and the water is flowing at a constant speed of y km/h.
- Explain why $x + y = 12$ and $x - y = 8$
 - Hence, find the man's rowing speed in still water and the speed of the river current.



FOCUS ON WORKING MATHEMATICALLY

EXPLORING FOR WATER, OIL AND GAS— THE DENSITY OF AIR-FILLED POROUS ROCK



Introduction

In 2002–2003, NSW was devastated by drought, which caused enormous hardship. Water is crucial for survival in the outback and many farmers, especially those in dry regions, sink bores on their properties to gain access to sufficient water. The geology of the rocks on the property informs the farmer of where water might be found.

The density of different types of rock is an important clue to where water may be found. Sedimentary rocks such as sandstone are much more porous than igneous rocks such as granite. Inside porous rocks are tiny holes (pores). These pores may trap water, oil or gas under great pressure deep underground, and so are very important in geological exploration. In this activity, we will see how simultaneous equations may turn up in the solution of such scientific problems.

The density D of a material is a measure of its mass M per unit volume V . We can express this mathematically using the equation:

$$D = \frac{M}{V} \quad (1)$$

As you would expect, the density of air-filled porous rock is less than the density of the grains making up the rock. The formula that links the total density D of air-filled rock to the density of the rock grains themselves D_r is

$$D = D_r \left(1 - \frac{V_p}{V} \right) \quad (2)$$

In this formula, V_p is the volume occupied by the pore space and V is the total volume of the rock.



LEARNING ACTIVITIES

- 1 The simultaneous equations (1) and (2) can be used to determine the density D and volume V of a porous air-filled rock. Write down the other variables which we must know in order to do this.
- 2 Examine equation (1). For a given M , what is the shape of the graph of D against V ?
- 3 Examine equation (2). Is V_p always less than V ? What happens in the special case when $V_p = V$? Is this possible?
- 4 Use an algebraic method to solve the simultaneous equations (1) and (2) for D and V , for an air-filled porous rock, given that the mass M is 12 kg, the volume of the pore space V_p is 3 m^3 and the density of the rock grains D_r is 1 kg/m^3 .



CHALLENGE

- 1 For this problem, why is the graph of equation (2) appropriate only in the first quadrant? What happens at $V = 3$? What happens to D as V tends to infinity?
- 2 Check your solution above by drawing the graphs and finding the point of intersection. Draw up a table of values for each graph and choose your scales carefully.



- 3 Eliminate D from equations (1) and (2) to obtain an expression for V . Substitute the given values for M , D_r and V_p and hence find V . Does it agree with your answer above?
- 4 Eliminate V from equations (1) and (2) to obtain an expression for D . Substitute the values for M , D_r and V_p and hence find D . Does it agree with your answer above?
- 5 Comment on the power of the different methods you have used to solve the simultaneous equations. What does each one reveal that the others do not?



LET'S COMMUNICATE

Write a summary of what you have learned in this chapter about techniques for solving simultaneous equations. What do you have to be careful about? What particular skills do you need? What are the advantages and disadvantages of different methods? Discuss this with your teacher.



REFLECTING

In most of the examples in this chapter, the equations to be solved simultaneously were straight lines. Straight lines can intersect in, at most, one point, so there was only ever the possibility of one solution. What might happen when one or more of the equations are not straight lines? What does this indicate about the possible strengths and weaknesses of solving two simultaneous equations by drawing their graphs?



MACQUARIE

Language link with Macquarie

- 1 Explain each of the following in a single sentence and give an example.
 - a **variable**
 - b **simultaneous equations**
 - c **linear equations**
 - d **coefficient of x .**
- 2 Read the Macquarie Learners Dictionary entry for **simultaneous**:

simultaneous adjective happening at the same time

□ **Word family:** **simultaneously** adverb

In what way is the mathematical meaning similar to the idea of happening at the same time?

1 State whether it is possible to find unique solutions to these problems.

- a The difference between two numbers is 5. Find the numbers.
- b Ryan is 11 years younger than his sister Mary-Jane. How old is each person?
- c Brittany spent three times as much time on her Commerce assignment as she did on her Maths assignment. Altogether, she spent 6 hours on the two assignments. How long did she spend on each?

2 Find three pairs of integers p, q that satisfy the equation $5p - 2q = 13$.

3 Determine by substitution whether the solution to each pair of simultaneous equations is $x = 3, y = -1$.

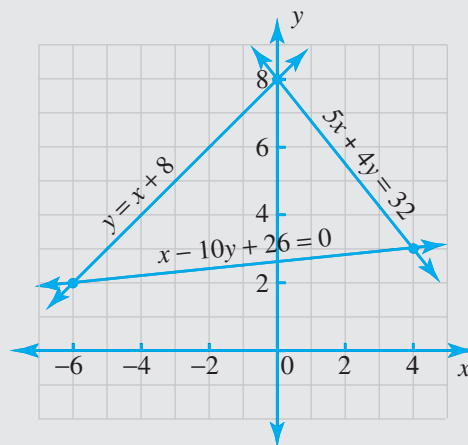
- a $2x + y = 5$ b $4x + 3y = 9$
 $x - y = 2$ $2x - 5y = 11$

4 Complete this table and hence solve the simultaneous equations $x + y = 8$ and $2x - y = 7$.

x	y	$x + y$	$2x - y$
2		8	
3		8	
4		8	
5		8	
6		8	

5 Use the graphs provided to solve each pair of simultaneous equations.

- a $y = x + 8$ b $x - 10y + 26 = 0$
 $5x + 4y = 32$ $y = x + 8$
- c $5x + 4y = 32$
 $x - 10y + 26 = 0$



6 Graph the equations $2x - y = 1$ and $y = 2x + 3$ on the same number plane. Is there a solution for these simultaneous equations? Explain.

7 Solve for x, y :
 $y = 5 - 3x$
 $y = x - 7$

8 Solve these simultaneous equations using the substitution method.

- a $x + y = 12$ b $2x + y = 5$
 $y = x - 2$ $x = y - 5$
- c $3x + 4y = 13$ d $x - y = 2$
 $x = 3 - 2y$ $y = 3x - 8$
- e $4x - y = 10$ f $x - 3y = 9$
 $y = 4 - 3x$ $y = 2x + 7$

9 Make either x or y the subject of one equation, then solve for x, y using the substitution method.

- a $2x - 3y = 4$ b $5x - y = 1$
 $x + 2y = 9$ $7x - 2y = 5$
- c $4x + y = 5$ d $6x - y = 18$
 $8x - 5y = -4$ $9x + 5y = 14$

10 Solve these simultaneous equations using the elimination method.

- a $3x + y = 13$ b $2x + 5y = 8$
 $x - y = 7$ $-2x + y = 4$

$$\begin{array}{ll} \text{c} & 7x + y = 25 \\ & 4x + y = 16 \\ \text{e} & 2x - y = 10 \\ & -x - y = -5 \end{array} \quad \begin{array}{ll} \text{d} & 3x + 11y = 5 \\ & 3x + 4y = -2 \\ \text{f} & 6x - 7y = 9 \\ & 8x - 7y = 5 \end{array}$$

- 11** Multiply one or both equations by a suitable constant, then solve for x, y using the elimination method.

$$\begin{array}{ll} \text{a} & x + 2y = 7 \\ & 3x + 5y = 19 \\ \text{c} & 3x + 2y = 4 \\ & 2x + 5y = 21 \\ \text{e} & 6x + 5y = 7 \\ & 4x - 3y = 11 \end{array} \quad \begin{array}{ll} \text{b} & 3x - 2y = 1 \\ & 5x - 8y = 11 \\ \text{d} & 2x - y = -12 \\ & 4x + 5y = 18 \\ \text{f} & 5x - 3y = -17 \\ & -2x - 7y = -26 \end{array}$$

- 12** Solve these simultaneous equations using the elimination method.

$$\begin{array}{ll} \text{a} & 2e + 6f = 5 \\ & 18e - 12f = 1 \\ \text{b} & 24u - 6v = 31 \\ & 36u - 24v = 19 \end{array}$$

- 13** Solve simultaneously to find for m, n :

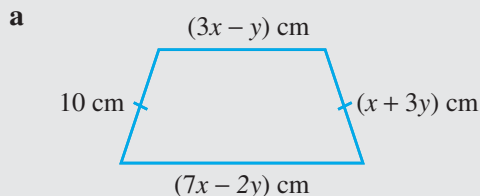
$$\frac{m}{3} - \frac{n}{2} = 4$$

$$\frac{5m}{2} + \frac{3n}{4} = 12$$

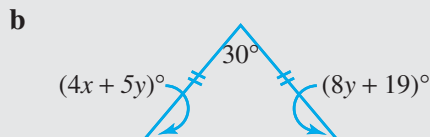
- 14** Form a pair of simultaneous equations and solve them to answer each of the following problems.

- a** The sum of two numbers is 39 and one number is 7 more than the other. Find the numbers.
- b** Three bananas and two rockmelons cost \$3.05 while four bananas and one rockmelon cost \$1.90. Find the cost of each.
- c** Bronya's piggy bank contains only 10-cent and 20-cent coins. If there are 60 coins with a total value of \$7.50, find the number of 10-cent and 20-cent coins.

- 15** Form a pair of simultaneous equations, then solve them to find x, y .



Perimeter = 54 cm



14

Co-ordinate geometry



This chapter at a glance

Stage 5.1/5.2/5.3

After completing this chapter, you should be able to:

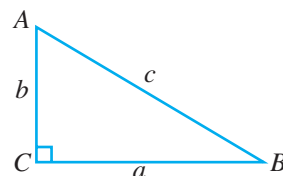
- find the distance between two points on a number plane using Pythagoras' theorem
- find the distance between two points using the distance formula
- find the midpoint of an interval
- find one of the end points of an interval given the other end point and the midpoint
- find the gradient of an interval using the ratio of the vertical rise and horizontal run
- find the gradient of an interval using the gradient formula
- rearrange the equation of a line from the general form to the gradient-intercept form and vice versa
- solve problems that involve distance, midpoint, gradient and the equation of a line
- find the equation of a line given its gradient and one point on the line
- find the equation of a line passing through two points
- determine whether two lines are parallel or perpendicular by considering gradients
- find the equation of a line that is parallel or perpendicular to a given line
- solve problems that involve parallel and perpendicular lines
- graph inequalities on the number plane
- write an inequality that describes a given region in the number plane
- shade the region that represents the intersection of two or more inequalities
- use co-ordinate geometry to classify triangles and quadrilaterals and establish their properties
- use co-ordinate geometry and deductive reasoning to solve general problems.

14.1 The distance between two points

Distance using Pythagoras' theorem

Pythagoras' theorem can be used to find the distance between two points, or length of an interval, on a number plane. Distances that are not an integral may be given either as a decimal or as a **surd** (i.e. in the form $\sqrt{n}$).

In $\triangle ABC$, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a, b are the lengths of the shorter sides.



To find the distance between two points:

- ⊙ form a right-angled triangle with the interval as the hypotenuse
- ⊙ find the lengths of the vertical and horizontal sides by counting units
- ⊙ use Pythagoras' theorem to find the required distance.

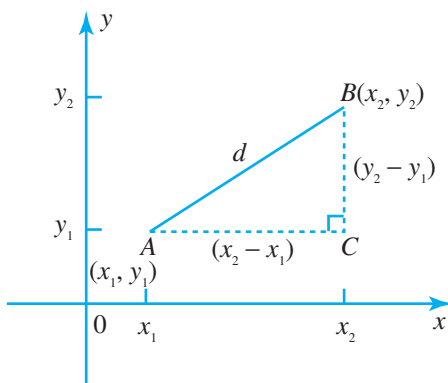
The distance formula

Pythagoras' theorem can be used to derive a formula for the distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$. The formula lets us calculate distance without the need to draw a diagram.

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Proof:



Consider the interval AB with $A(x_1, y_1)$ and $B(x_2, y_2)$ and the distance between these points d units.

Construct $\triangle ABC$ on AB such that AC is perpendicular to BC .

Now, the length of AC is $(x_2 - x_1)$ units, and the length of BC is $(y_2 - y_1)$ units.

By Pythagoras' theorem,

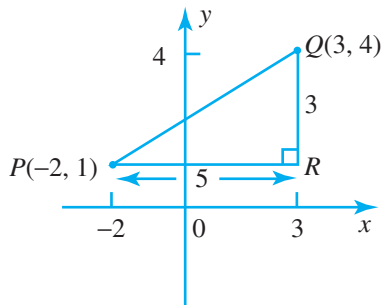
$$AB^2 = AC^2 + BC^2$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EG
+S**Example 1**

Use Pythagoras' theorem to find the distance between $P(-2, 1)$ and $Q(3, 4)$. Give your answer in surd form.

Solution

The points P and Q are joined and the point R is chosen so that the right-angled triangle PQR is formed with PQ as hypotenuse.

$$\begin{aligned} PR &= 3 - (-2) & PQ^2 &= PR^2 + QR^2 \\ &= 5 \text{ units} & &= 5^2 + 3^2 \\ & & &= 25 + 9 \\ & & &= 34 \\ QR &= 4 - 1 & & \\ &= 3 \text{ units} & \therefore PQ &= \sqrt{34} \text{ units} \end{aligned}$$

EG
+S**Example 2**

Use the distance formula to find the distance between the points $P(-2, 3)$ and $Q(6, 9)$.

Solution

Let $P(-2, 3)$ be (x_1, y_1) and $Q(6, 9)$ be (x_2, y_2) .

$$\therefore x_1 = -2, y_1 = 3, x_2 = 6 \text{ and } y_2 = 9$$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - (-2))^2 + (9 - 3)^2} \\ &= \sqrt{8^2 + 6^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10 \text{ units} \end{aligned}$$

EG
+S**Example 3**

$\triangle LMN$ has vertices $L(-2, 5)$, $M(4, 5)$ and $N(1, 8)$. Show that $\triangle LMN$ is isosceles.

Solution

$$\begin{aligned} \text{i} \quad LM &= 4 - (-2) \\ &= 6 \text{ units} \end{aligned}$$

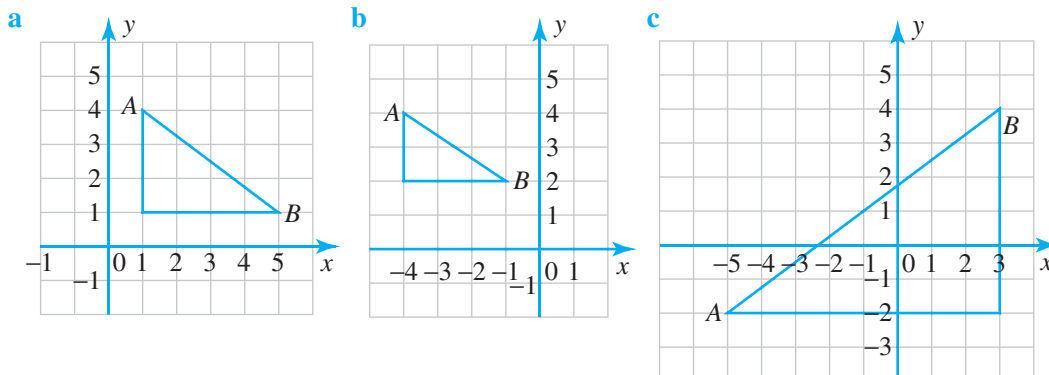
$$\begin{aligned} \text{iii} \quad NL &= \sqrt{(1 - (-2))^2 + (8 - 5)^2} \\ &= \sqrt{3^2 + 3^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

$$\begin{aligned} \text{ii} \quad MN &= \sqrt{(1 - 4)^2 + (8 - 5)^2} \\ &= \sqrt{(-3)^2 + 3^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

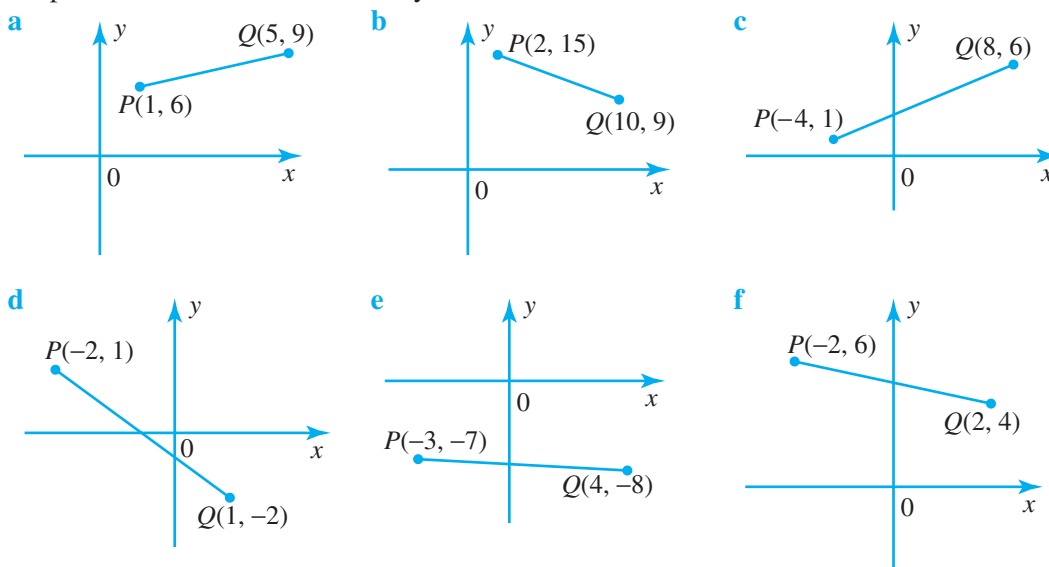
$$\begin{aligned} \text{iv} \quad MN &= NL = 3\sqrt{2} \text{ units,} \\ \therefore \triangle LMN &\text{ is isosceles.} \end{aligned}$$

Exercise 14.1

- 1 Use Pythagoras' theorem to find the length of the interval AB . Give your answers correct to 1 decimal place where necessary.

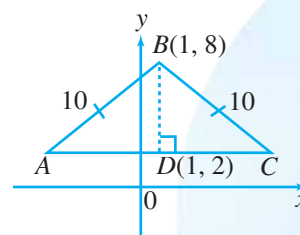


- 2 Use Pythagoras' theorem to find the length of each interval. Give your answers in simplest surd form where necessary.



- 3 $\triangle ABC$ is isosceles with $AB = BC = 10$ units and apex $B(1, 8)$. The base AC is parallel to the x -axis. $D(1, 2)$ is a point on AC such that BD is perpendicular to AC .

- a** Find the co-ordinates of A and C .
b Find the area of $\triangle ABC$.

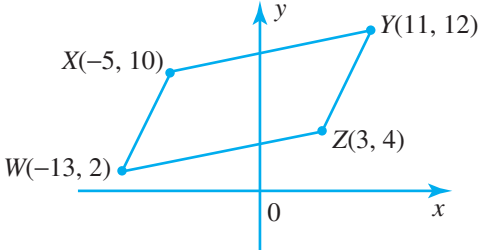


- 4 Use the distance formula to calculate the distance between each pair of points. Answer correct to 1 decimal place where necessary.

- a** $(5, 2)$ and $(8, 6)$ **b** $(0, 1)$ and $(12, 6)$ **c** $(2, 3)$ and $(10, 9)$

d (3, 4) and (5, 7)	e (4, 1) and (3, 3)	f (7, 5) and (5, 3)
g (2, 9) and (0, 3)	h (2, -1) and (-2, 1)	i (3, -5) and (-2, -3)
j (-4, -2) and (-1, 5)	k (6, 0) and (-2, 4)	l (-7, -6) and (-2, -11)
m (8, 9) and (2, 12)	n (-1, -3) and (3, 2)	o (-2, -3) and (2, -7)

Consolidation

- 5 **a** Which point is closer to the origin— $G(1, 6)$ or $H(3, 5)$?
b Which point is further from $N(1, 2)$ — $L(3, 5)$ or $M(5, 3)$?
- 6 Show that the points $Q(8, 2)$ and $R(-1, -7)$ are equidistant from $S(3, -2)$.
- 7 Show that $\triangle LMN$ with vertices $L(3, 4)$, $M(7, 1)$ and $N(10, 5)$ is isosceles.
- 8 $\triangle ABC$ has vertices $A(-3, 2)$, $B(1, 8)$ and $C(7, 4)$.
a Show that $\triangle ABC$ is isosceles.
b Use Pythagoras' theorem to determine whether the triangle is also right angled.
- 9 Find the perimeter of the trapezium with vertices $A(-7, -2)$, $B(-1, 6)$, $C(3, 6)$ and $D(18, -2)$.
- 10 The points $T(-4, -1)$, $U(8, 8)$ and $V(16, 14)$ are collinear.
a Find the distance between T and U .
b Find the distance between U and V .
c In what ratio does U divide TV ?
- 11 A circle with centre $(7, 6)$ passes through the point $(1, -2)$.
a Find the length of the radius.
b Find the exact circumference of the circle.
- 12  WXYZ is a parallelogram. Find $XZ:WY$, the ratio of the lengths of the diagonals.
- 13 Show that $\triangle PQR$ with vertices $P(-2, 5)$, $Q(4, 5)$ and $R(1, 5 - 3\sqrt{3})$ is equilateral.
- 14 A quadrilateral $EFGH$ has vertices $E(4, 6)$, $F(7, 1)$, $G(2, -2)$ and $H(-1, 3)$.
a Find the lengths of the sides.
b Find the lengths of the diagonals.
c What kind of quadrilateral is $EFGH$? Why?
- 15 The vertices of a quadrilateral $KLMN$ are $K(6, 4)$, $L(4, 8)$, $M(8, 6)$ and $N(9, 3)$.
a Find the lengths of the sides.
b What kind of quadrilateral is $KLMN$? Why?

16 $P(-1, 7)$, $Q(2, 3)$, $R(2, -2)$ and $S(-1, 2)$ are the vertices of a quadrilateral.

- Find the length of each side.
- Find the lengths of the diagonals.
- What kind of quadrilateral is $PQRS$? Why?

Further applications

17 The distance between the points $C(-5, 1)$ and $D(7, p)$ is 15 units. Show with the use of a diagram that there are two possible values for p . Hence, find the possible co-ordinates of D .

14.2 The midpoint of an interval

The point that lies halfway between the two end points of an interval is called the **midpoint** of the interval.

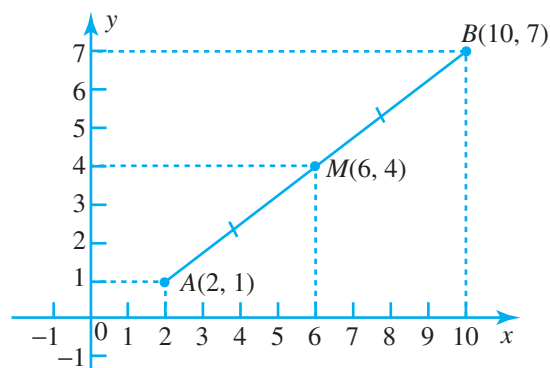
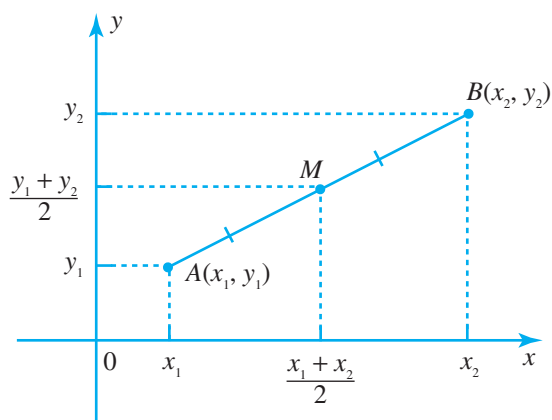
To find the number that lies halfway between two numbers, we add those numbers and divide by 2. That is, we take their average. In the midpoint of an interval:

- the x -value lies halfway between the x -values of the two end points,

$$\text{i.e. } x = \frac{x_1 + x_2}{2}.$$

- the y -value lies halfway between the y -values of the two end points,

$$\text{i.e. } y = \frac{y_1 + y_2}{2}.$$



For example, in this diagram, the x -value halfway between $x = 2$ and $x = 10$ is

$$x = \frac{2 + 10}{2}, \text{ i.e. } x = 6.$$

The y -value halfway between $y = 1$ and

$$y = 7 \text{ is } y = \frac{1 + 7}{2}, \text{ i.e. } y = 4.$$

$\therefore$ The midpoint of $A(2, 1)$ and $B(10, 7)$ is $M(6, 4)$.

The midpoint $M(x, y)$ of the interval AB where $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example 1

Find the co-ordinates of M , the midpoint of the interval joining $A(-3, 2)$ and $B(7, 6)$.

EG
+S**Solution**

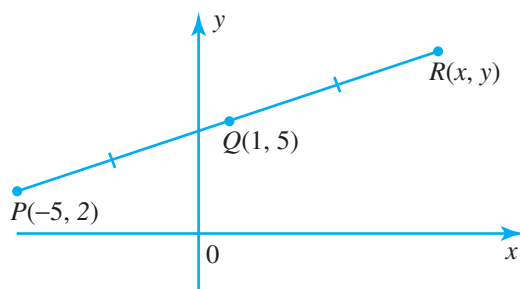
Let $A(-3, 2)$ be (x_1, y_1) and $B(7, 6)$ be (x_2, y_2) .

$$\therefore x_1 = -3, y_1 = 2, x_2 = 7 \text{ and } y_2 = 6$$

$$\begin{aligned} M(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-3 + 7}{2}, \frac{2 + 6}{2} \right) \\ &= (2, 4) \end{aligned}$$

Example 2

Find the co-ordinates of R if Q is the midpoint of PR and $P(-5, 2)$, $Q(1, 5)$.

EG
+S**Solution**

- i The x -value of Q is equal to the average of the x -values of P and R .

$$\frac{x + (-5)}{2} = 1$$

$$\frac{x - 5}{2} = 1$$

$$\times 2 \quad \times 2$$

$$x - 5 = 2$$

$$+5 \quad +5$$

$$\therefore x = 7$$

- ii The y -value of Q is equal to the average of the y -values of P and R .

$$\frac{y + 2}{2} = 5$$

$$\times 2 \quad \times 2$$

$$y + 2 = 10$$

$$-2 \quad -2$$

$$\therefore y = 8$$

$\therefore$ The co-ordinates of R are $(7, 8)$.

Exercise 14.2

- 1 Find the co-ordinates of the midpoint of the interval joining each pair of points.

a $(0, 5)$ and $(2, 9)$

d $(7, 3)$ and $(7, 13)$

g $(4, 1)$ and $(12, -7)$

j $(2, 0)$ and $(-8, -2)$

m $(1, -1)$ and $(-1, 13)$

p $(-15, -8)$ and $(-9, 4)$

b $(1, 6)$ and $(5, 8)$

e $(-2, 3)$ and $(2, 5)$

h $(-6, 3)$ and $(8, -11)$

k $(-5, 4)$ and $(5, -4)$

n $(-7, -4)$ and $(-9, -14)$

q $(0, -5)$ and $(14, -11)$

c $(4, 11)$ and $(10, 17)$

f $(-1, -2)$ and $(5, 6)$

i $(-2, -9)$ and $(-10, -1)$

l $(-12, -2)$ and $(-8, 4)$

o $(-2, 6)$ and $(6, -2)$

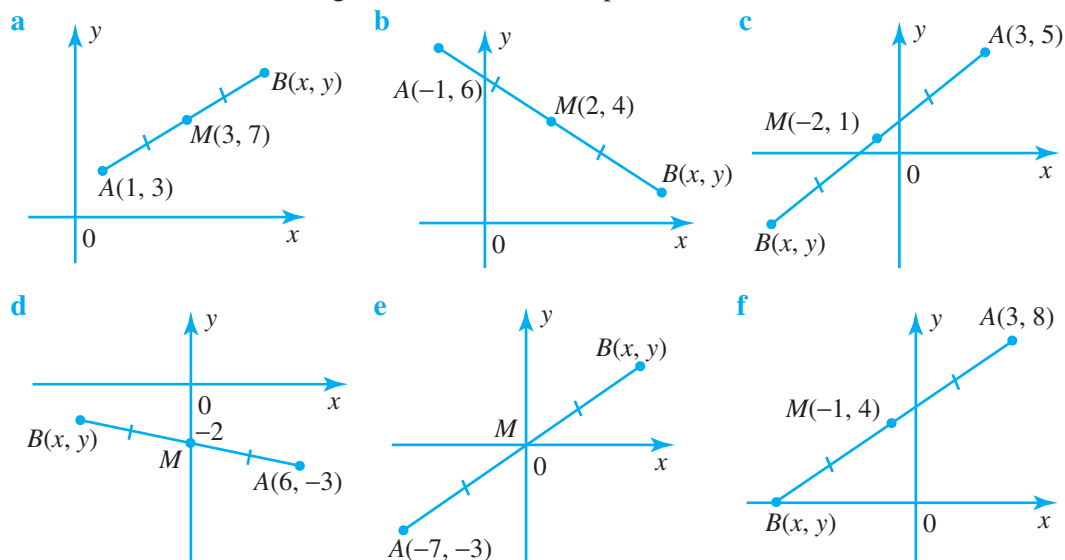
r $(-3, 13)$ and $(-19, 7)$

2 Find the co-ordinates of M , the midpoint of the interval LN with:

a $L(1, 3), N(2, 5)$	b $L(4, 0), N(7, 3)$	c $L(6, 7), N(1, 10)$
d $L(8, -3), N(3, -1)$	e $L(-2, 5), N(-3, -3)$	f $L(9, -8), N(-2, 13)$
g $L(0, -1), N(-9, -10)$	h $L(-4, 3), N(4, -2)$	i $L(-2, 11), N(-13, -6)$

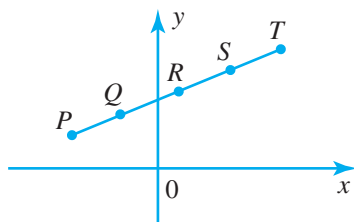
Consolidation

3 Find the co-ordinates of B , given that M is the midpoint of AB .



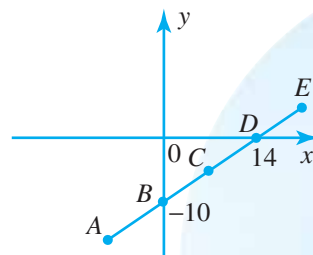
- 4 **a** Find the co-ordinates of D where $E(-3, 1), F(7, 5)$ and D the midpoint of EF .
b Find the co-ordinates of D where $E(1, 4), F(8, -1)$ and E the midpoint of DF .

5 **a**



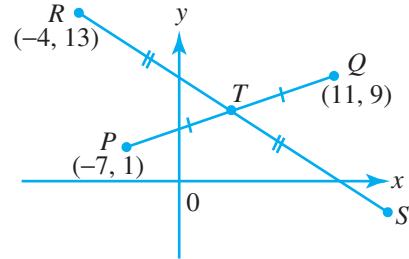
The points P, Q, R, S, T are equidistant. Find the co-ordinates of Q, R and S given $P(-4, 2)$ and $T(8, 10)$.

- b** The points A, B, C, D, E are equidistant. Find the co-ordinates of C, A and E .



- 6 The vertices of a quadrilateral $ABCD$ are $A(7, 0)$, $B(-3, 0)$, $C(-7, -2)$ and $D(3, -2)$.
- Find the midpoint of AC .
 - Find the midpoint of BD .
 - What kind of quadrilateral is $ABCD$? Why?

- 7 The intervals PQ and RS bisect each other at T .
- Find the co-ordinates of T .
 - Find the co-ordinates of S .
 - What type of quadrilateral is $PRQS$? Why?



- 8 The vertices of $\triangle TUV$ are $T(-9, 4)$, $U(-1, -2)$ and $V(7, 4)$. The midpoints of TU and UV are P and Q respectively.
- Find the co-ordinates of P and Q .
 - Show that the line joining these midpoints is half the length of the third side of the triangle.
- 9 The midpoint of an interval is $(-1, 4)$. What could the co-ordinates of the end points be?
- 10 A median is a line drawn from a vertex of a triangle to the midpoint of the opposite side. The vertices of $\triangle XYZ$ are $X(1, 5)$, $Y(5, -3)$ and $Z(7, 4)$.
- Find the co-ordinates of W , the midpoint of XY .
 - Find the length of the median ZW .
- 11 The end points of the diameter of a circle are $(-2, 2)$ and $(8, -10)$.
- Find the co-ordinates of the centre of the circle.
 - Find the length of the radius.
- 12 A circle with centre $C(4, 1)$ passes through the point $P(9, 13)$. Find the co-ordinates of Q , the other end point of the diameter PQ .

Further applications

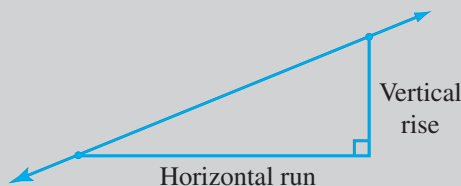
- 13 Find the co-ordinates of V , the midpoint of the interval joining the points:
- $U(2a, 3b)$ and $W(6a, 9b)$
 - $U(-a, 10b)$ and $W(7a, 0)$
 - $U(-3a, -11b)$ and $W(5a, -3b)$
- 14 In each of the following, form an equation and solve it to find the values of the pronumerals, given that M is the midpoint of the interval EF .
- $E(-1, 7)$, $M(p + 3, 2q)$, $F(11, 17)$
 - $E(m - 4, n + 2)$, $M(-1, 6)$, $F(3, 1)$
 - $E(-9, -7)$, $M(\frac{2a}{3}, -4)$, $F(5, 2b - 11)$

14.3 The gradient formula

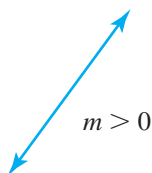
Gradient

The **gradient** or **slope** of a line is a measure of how steep it is. In exercise 11.4 we defined the gradient of a line as the *vertical rise* divided by the *horizontal run*. Gradient is represented by the pronumeral m .

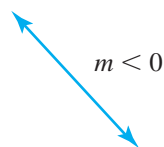
$$\text{Gradient } (m) = \frac{\text{vertical rise}}{\text{horizontal run}}$$



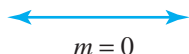
If a line slopes to the right, its gradient is positive.



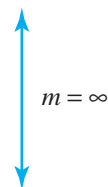
If a line slopes to the left, its gradient is negative.



The gradient of a horizontal line is 0.



The gradient of a vertical line is undefined.

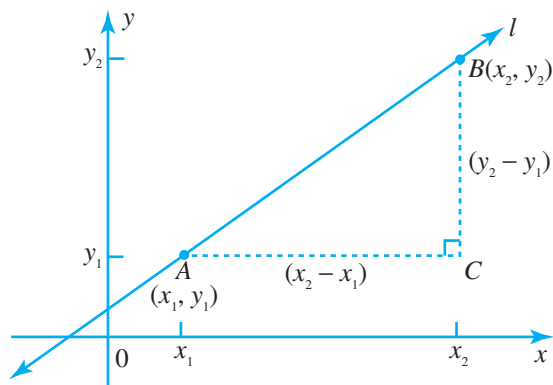


The definition above can be used to derive a formula for the gradient of a line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$. This formula can then be used to calculate the gradient of a line without the need to draw a diagram.

The gradient of the line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Proof:



Consider the line l , which passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ and has gradient m .

Construct $\triangle ABC$ on the interval AB such that AC is perpendicular to BC .

Now, the length of AC is $(x_2 - x_1)$ units, and the length of BC is $(y_2 - y_1)$ units.

By definition, $m = \frac{\text{vertical rise}}{\text{horizontal run}}$

$$m = \frac{BC}{AC}$$

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$

NOTE:

- When the definition $m = \frac{\text{rise}}{\text{run}}$ was used to find the gradient of a line, we had to consider whether the gradient was positive or negative according to whether the line sloped to the right or to the left. When the gradient formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ is used, this is not necessary as the correct sign is determined automatically by the algebra.
- Any two points on a line can be used to find the gradient.

Angle of inclination

The gradient of a line is related to the tangent ratio in trigonometry. In fact, the gradient of a line can be calculated by finding the tangent of the angle at which the line is inclined to the positive direction of the x -axis.

If a line l is inclined to the positive direction of the x -axis at an angle θ , then its gradient is given by:

$$m = \tan \theta$$

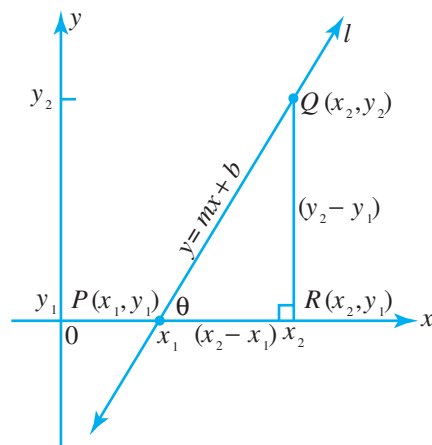
Proof:

Consider the line l with equation $y = mx + b$, which makes an angle θ with the x -axis as shown.

Choose $P(x_1, y_1)$ on the x -axis for convenience and $Q(x_2, y_2)$ as any other point on the line. The co-ordinates of R are then (x_2, y_1) .

Now, the length of PR is $(x_2 - x_1)$ units and the length of QR is $(y_2 - y_1)$ units.

$$\begin{aligned} \text{By trigonometry in } \triangle PQR, \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{QR}{PR} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ \therefore \tan \theta &= m_{PQ} \end{aligned}$$



That is, the gradient of the line is equal to the tangent of the angle of inclination to the x -axis.

EG
+S

Example 1

Find the gradient of the line that passes through the points:

a $C(3, 2)$ and $D(11, 8)$

b $U(4, -6)$ and $V(-1, 4)$

Solutions

- a** Let $C(3, 2)$ be (x_1, y_1)
and $D(11, 8)$ be (x_2, y_2) .
 $\therefore x_1 = 3, y_1 = 2, x_2 = 11$ and $y_2 = 8$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8 - 2}{11 - 3} \\ &= \frac{6}{8} \\ &= \frac{3}{4} \end{aligned}$$

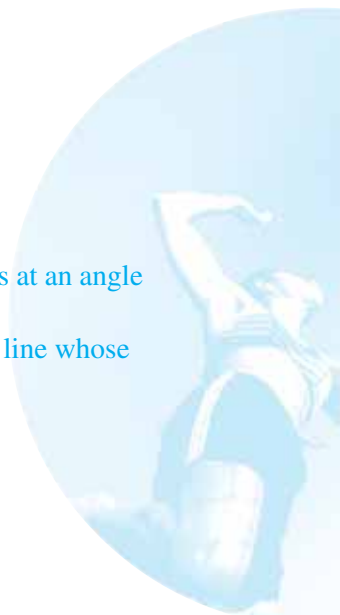
- b** Let $U(4, -6)$ be (x_1, y_1)
and $V(-1, 4)$ be (x_2, y_2) .
 $\therefore x_1 = 4, y_1 = -6, x_2 = -1$ and $y_2 = 4$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - (-6)}{-1 - 4} \\ &= \frac{10}{-5} \\ &= -2 \end{aligned}$$

EG
+S

Example 2

- a** Find, correct to 1 decimal place, the gradient of a line which meets the x -axis at an angle of 58° .
b Find, correct to the nearest degree, the angle of inclination to the x -axis of a line whose gradient is $\frac{3}{7}$.



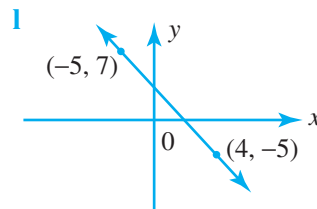
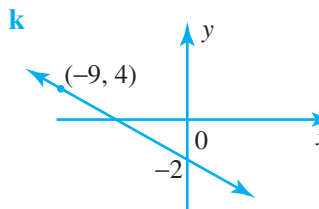
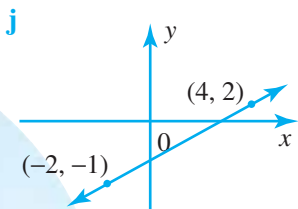
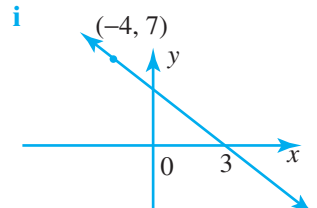
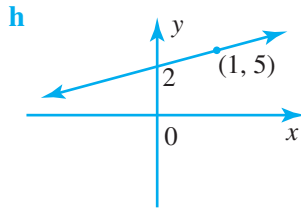
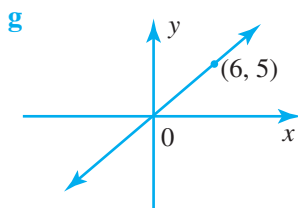
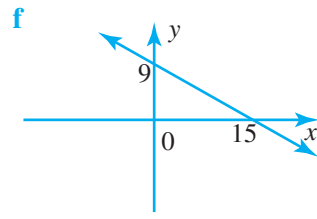
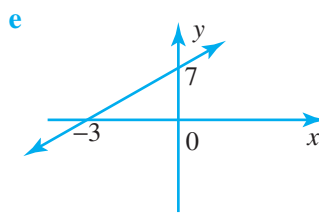
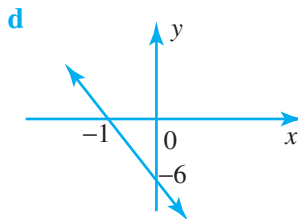
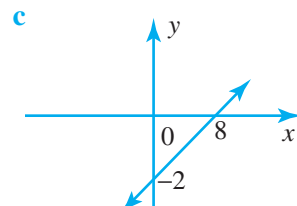
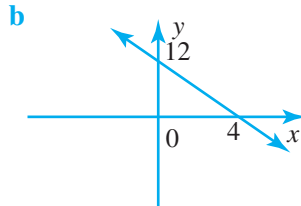
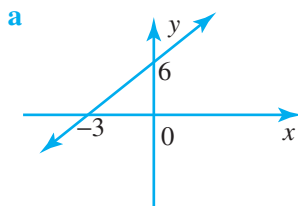
Solutions

$$\begin{aligned} \text{a} \quad m &= \tan \theta \\ \therefore m &= \tan 58^\circ \\ &\doteq 1.6 \end{aligned}$$

$$\begin{aligned} \text{b} \quad m &= \tan \theta \\ \frac{3}{7} &= \tan \theta \\ \therefore \theta &= \tan^{-1}\left(\frac{3}{7}\right) \\ &\doteq 23^\circ \end{aligned}$$

Exercise 14.3

1 Find the gradient of each line, using $m = \frac{\text{rise}}{\text{run}}$.



2 Find the gradient of the line which passes through the given points, using $m = \frac{y_2 - y_1}{x_2 - x_1}$.

a (1, 3) and (5, 7)

b (6, 2) and (12, 5)

c (-1, 1) and (2, 6)

d (-4, -3) and (0, 6)

e (1, -3) and (-1, 3)

f (-4, -2) and (4, 3)

g (3, 5) and (-7, 11)

h (4, 0) and (-8, -2)

i (-4, -6) and (-1, -2)

j (2, -4) and (6, -10)

k (2, 7) and (11, 13)

l (7, -5) and (1, 3)

m (-2, 3) and (5, -4)

n (-3, 7) and (-4, 2)

o (-1, -2) and (-7, -3)

p (5, 3) and (-2, 11)

q (-7, 6) and (-3, 14)

r (0, 2) and (5, -2)

Consolidation

- 3 A straight line passes through the points $A(-1, -6)$, $B(0, -4)$ and $C(1, -2)$. Find the gradient of the line using the points:

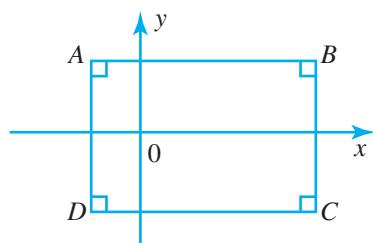
a A and B **b** B and C **c** C and A

- 4 The gradient of a line passing through the points:

a $(1, 2)$ and $(k, 10)$ is 2 . Find the value of k .
b $(14, -5)$ and $(11, t)$ is -4 . Find the value of t .
c $(-6, c)$ and $(24, -25)$ is $-\frac{3}{5}$. Find the value of c .

- 5 If P is the midpoint of $K(1, 2)$ and $L(5, 8)$ and Q is the midpoint of $M(4, 6)$ and $N(10, 20)$, find the gradient of the line passing through the points P and Q .

6



$ABCD$ is a rectangle with $A(-2, 3)$ and $C(4, -1)$. Find:

- a** the co-ordinates of B and D
b the slope of each diagonal

- 7 $V(3, 6)$ is the midpoint of $U(-2, 4)$ and $W(x, y)$. Find the co-ordinates of W and the gradient of the line that passes through W and $T(6, -3)$.

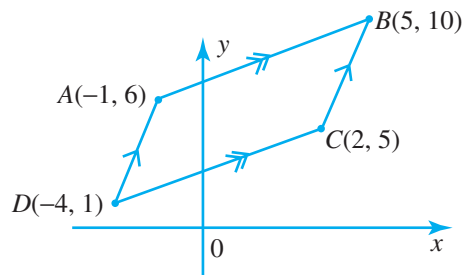
- 8 Consider the points $F(2, 3)$, $G(0, 7)$ and $H(-3, 13)$.

- a** Find the gradient of FG .
b Find the gradient of GH .
c What can you say about F , G and H ?

- 9 The lines $x = 2$ and $y = -9$ intersect at R while the lines $x = -1$ and $y = 3$ intersect at S . Find the gradient of the line RS .

- 10 **a** Find the gradient of the line that passes through the points $(-1, 2)$ and $(3, 2)$. What can you say about the line?
b Find the gradient of the line that passes through the points $(5, 1)$ and $(5, 7)$. What can you say about the line?

11



$ABCD$ is a parallelogram with vertices as shown.

- a** Find the gradient of AB and DC .
b Find the gradient of AD and BC .
c Copy and complete this statement: 'Parallel lines have the same _____'.

- 12 The vertices of $\triangle LMN$ are $L(-2, 1)$, $M(4, 9)$ and $N(6, 7)$. The midpoints of LM and LN are P and Q respectively. Show that the line passing through the points P and Q is parallel to MN .

Further applications

- 13 Find, correct to 1 decimal place where necessary, the gradient of a line that meets the x -axis at an angle of:

a 10°	b 20°	c 25°	d 30°	e 37°
f 45°	g 51°	h 60°	i 75°	j 89°

- 14 What happens to the gradient of a line as the angle of inclination increases?

- 15 Find, correct to the nearest degree, the angle of inclination to the x -axis of the line whose gradient is:

a $\frac{1}{10}$	b $\frac{1}{5}$	c $\frac{1}{4}$	d $\frac{1}{3}$	e $\frac{1}{2}$
f 1	g 2	h 3.4	i 7.8	j 25

TRY THIS

A line with no integer co-ordinates

The straight line with equation $2x + 6y = 7$ does not pass through any points on the number plane in which both the x - and y -values are integers. Why is this?

14.4 General form of the equation of a line

In exercise 11.5 we saw that the equation of a straight line can be written as $y = mx + b$, where m is the gradient and b is the y -intercept. When the equation is written in this way, it is said to be in the **gradient–intercept form**. In some questions, however, it is easier to write the equation as $ax + by + c = 0$, where a , b and c are integers and $a > 0$. When the equation is written in this way it is said to be in the **general form**.

The equation of a straight line in general form is:

$$ax + by + c = 0, \text{ where } a, b \text{ and } c \text{ are integers and } a > 0.$$

NOTE: If the equation of a line is written in the general form, we must rewrite it in the gradient–intercept form to find the gradient.

EG
+
S**Example 1**

- a Express the equation $3x - 2y + 10 = 0$ in the gradient–intercept form.
 b Find the gradient and y-intercept.

Solutions

$$\begin{aligned}
 \text{a } 3x - 2y + 10 &= 0 \\
 &\quad +2y \quad +2y \\
 3x + 10 &= 2y \\
 &\quad \div 2 \quad \div 2 \\
 \therefore y &= \frac{3}{2}x + 5
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \text{The equation } y &= \frac{3}{2}x + 5 \text{ is in the form} \\
 y &= mx + b, \text{ where } m \text{ is the gradient and} \\
 b &\text{ is the } y\text{-intercept.} \\
 \therefore m &= \frac{3}{2} \text{ and } b = 5.
 \end{aligned}$$

EG
+
S**Example 2**

Express each equation in the general form.

$$\text{a } y = 4x - 9$$

$$\text{b } y = \frac{1}{2}x - 7$$

Solutions

$$\begin{aligned}
 \text{a } y &= 4x - 9 \\
 -y &\quad -y \\
 0 &= 4x - y - 9 \\
 \text{i.e. } 4x - y - 9 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{b } y &= \frac{1}{2}x - 7 \\
 \times 2 \quad \times 2 \\
 2y &= x - 14 \\
 -2y \quad -2y \\
 0 &= x - 2y - 14 \\
 \text{i.e. } x - 2y - 14 &= 0
 \end{aligned}$$

EG
+
S**Example 3**Find the x- and y-intercepts of the line $2x + 3y - 12 = 0$, then sketch the line.**Solution**

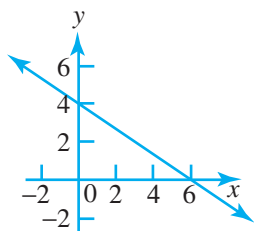
- i To find the x-intercept, let $y = 0$.

$$\begin{aligned}
 2x + 3(0) - 12 &= 0 \\
 2x - 12 &= 0 \\
 &\quad +12 \quad +12 \\
 2x &= 12 \\
 &\quad \div 2 \quad \div 2 \\
 \therefore x &= 6
 \end{aligned}$$

- ii To find the y-intercept, let $x = 0$.

$$\begin{aligned}
 2(0) + 3y - 12 &= 0 \\
 3y - 12 &= 0 \\
 &\quad +12 \quad +12 \\
 3y &= 12 \\
 &\quad \div 3 \quad \div 3 \\
 \therefore y &= 4
 \end{aligned}$$

iii



Exercise 14.4

1 Write down the gradient and y-intercept of each line.

a $y = 3x + 2$

b $y = 5x - 1$

c $y = -4x + 3$

d $y = 6 - 5x$

e $y = -2x - 4$

f $y = 7x$

g $y = -x$

h $y = 8 - x$

i $y = \frac{1}{2}x + 9$

j $y = \frac{3}{4}x - 2$

k $y = 6 - \frac{2}{3}x$

l $y = \frac{3}{5} - \frac{1}{7}x$

2 Write each of these equations in the general form.

a $y = x + 6$

b $y = 4x + 1$

c $y = 3x - 2$

d $y = 5 - x$

e $y = -x + 3$

f $y = -2x - 6$

g $y = x$

h $y = -5x$

i $x + y = 1$

j $x - y = 3$

k $4x + y = 7$

l $7x - y = 12$

m $x + 2y = 5$

n $3x + 4y = 8$

o $x = 3y + 10$

p $3x = 2y + 6$

q $y = \frac{1}{3}x$

r $y = \frac{1}{4}x$

s $y = \frac{1}{2}x + 4$

t $y = \frac{1}{5}x - 3$

u $y = \frac{2}{3}x - 1$

v $y = 7 - \frac{1}{2}x$

w $y = -\frac{3}{4}x - 5$

x $y = 2 - \frac{5}{6}x$

3 Write each equation in the gradient-intercept form.

a $x - y + 2 = 0$

b $x - y - 4 = 0$

c $2x - y + 1 = 0$

d $x + y + 5 = 0$

e $x + y - 1 = 0$

f $3x + y - 7 = 0$

g $2x - y - 9 = 0$

h $4x + y + 6 = 0$

i $5x - y + 2 = 0$

j $x - 2y + 6 = 0$

k $x + 3y - 12 = 0$

l $x + 2y = 0$

m $4x + 4y - 3 = 0$

n $6x - 3y + 1 = 0$

o $3x - 7y = 0$

p $2x - 3y + 3 = 0$

q $3x - 2y + 8 = 0$

r $5x + 6y - 12 = 0$

s $3x - 4y + 5 = 0$

t $2x + 7y - 1 = 0$

u $x - 8y - 10 = 0$

v $x - 9y - 6 = 0$

w $3x + 5y - 10 = 0$

x $4x + 6y + 6 = 0$

Consolidation

4 By substitution into the equation $y = mx + b$, find the value of b and hence write down the equation of the line that passes through the point A with gradient m . Give each equation in the general form.

a $A(2, 7) \quad m = 3$

b $A(-4, 1) \quad m = 2$

c $A(5, -3) \quad m = -1$

d $A(-1, -6) \quad m = -4$

e $A(8, 0) \quad m = \frac{1}{2}$

f $A(-12, 5) \quad m = -\frac{2}{3}$

5 A straight line passes through the point $(-1, 4)$ with a gradient of 2.

a Find the equation of the line in the form $y = mx + b$.

b Express this equation in the general form.

c Find the x - and y -intercepts of the line.

d Draw a neat sketch of the line, showing the intercepts with the axes.

6 Which line is steeper, $2x - 3y + 7 = 0$ or $3x - 4y - 2 = 0$? Why?

7 Match each of these equations with one of the graphs below.

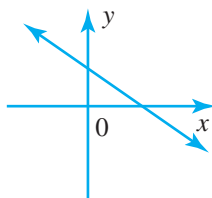
a $y = 3x - 7$

b $y = 7 - 3x$

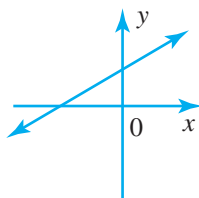
c $y = -3x - 7$

d $y = 3x + 7$

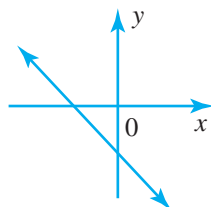
A



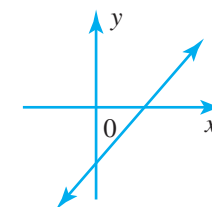
B



C



D



8 Match each of these equations with one of the graphs below.

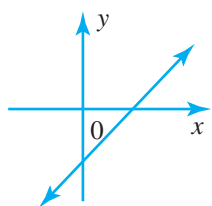
a $6x - y - 2 = 0$

b $6x - y + 2 = 0$

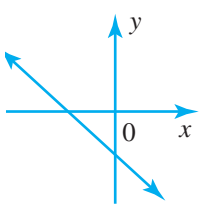
c $6x + y + 2 = 0$

d $6x + y - 2 = 0$

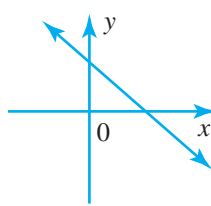
A



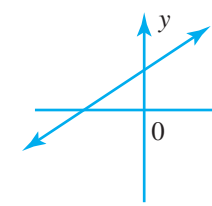
B



C

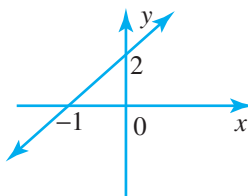


D

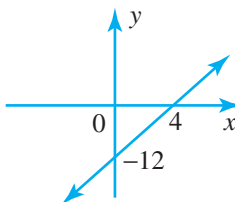


9 Find the equation of each line in the general form.

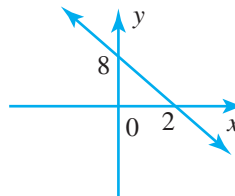
a



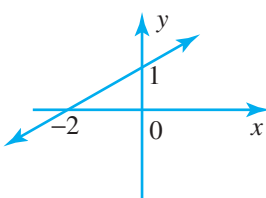
b



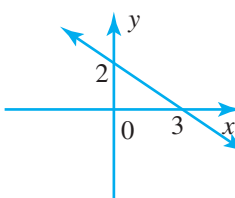
c



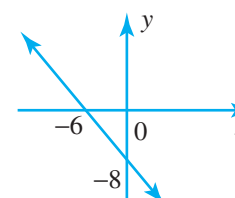
d



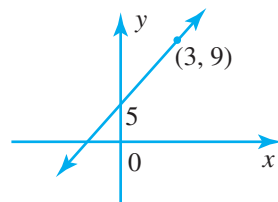
e



f



10



a Find the gradient of this line.

b Find the equation of the line, giving your answer in the general form.

11 A straight line cuts the y-axis at 3 and passes through the point $(-7, 2)$.

a Find the gradient of the line.

b Find the equation of the line in the general form.

- 12 a** Express the equation $2x + y - 4 = 0$ in the gradient–intercept form, then sketch the line.
b Determine whether the point $(15, -26)$ lies on the line.
- 13** The point $(2, -4)$ lies on the line $kx - 4y - 22 = 0$. Find the value of k and hence find the gradient of the line.
- 14** Find, by inspection, the equation of the line that passes through all of the points in each table. Give the equations in the general form.

a

x	0	1	2	3
y	5	6	7	8

b

x	0	1	2	3
y	-3	-2	-1	0

c

x	0	1	2	3
y	0	4	8	12

d

x	3	4	5	6
y	9	11	13	15

e

x	0	1	2	3
y	-1	2	5	8

f

x	0	1	2	3
y	8	7	6	5

g

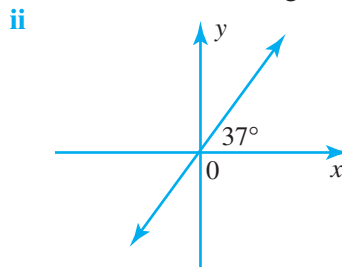
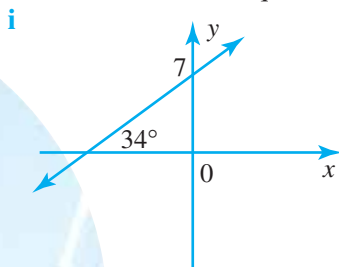
x	0	2	4	6
y	0	1	2	3

h

x	0	3	6	9
y	2	4	6	8

Further applications

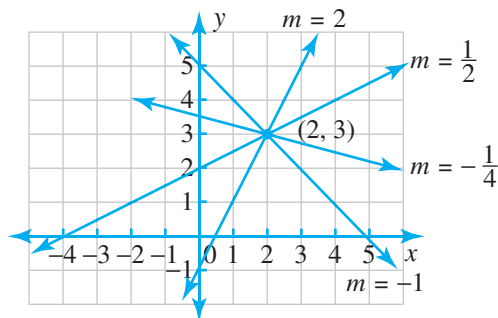
- 15** Find the gradient and y -intercept of the line that passes through each pair of points. Hence, find the equation of each line, giving your answers in the general form.
a $(1, 7)$ and $(3, 11)$ **b** $(-1, 4)$ and $(2, 1)$ **c** $(-6, 11)$ and $(-2, 9)$
- 16** Find, in the general form, the equation of the line that passes through the origin and the midpoint of $(1, 13)$ and $(11, 5)$.
- 17 a** Show that $\tan 34^\circ \doteq \frac{2}{3}$ and $\tan 37^\circ \doteq \frac{3}{4}$.
b Hence, find the equation of each line below in the general form.



- 18** Find the angle at which each line is inclined to the x -axis, correct to the nearest degree.
a $x - 2y + 6 = 0$ **b** $2x - 5y - 20 = 0$ **c** $4x - 3y + 21 = 0$

14.5 The equation of a line given the gradient and a point

There are an infinite number of lines that pass through any given point. What distinguishes between these lines, however, is that each has a different gradient. The equation of any straight line can therefore be specified exactly if we know the gradient of the line and the co-ordinates of one point through which it passes. There are two standard methods that can be used to find the equation.



The gradient-intercept form of the equation

The given point and gradient can be used to find the y-intercept of the line so that its equation can then be found.

To find the equation of the line that passes through the point (x_1, y_1) with gradient m :

- ⊙ substitute the point and gradient into the equation $y = mx + b$ to find the value of b
- ⊙ rewrite the equation $y = mx + b$ using the values for m and b .

The point-gradient formula

The point-gradient formula can be used to find the equation of a line without first having to find the y-intercept.

The equation of the straight line that passes through the point (x_1, y_1) with gradient m is given by:

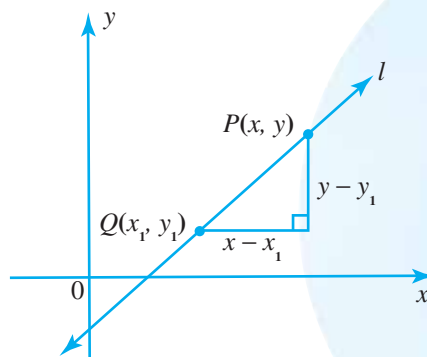
$$y - y_1 = m(x - x_1).$$

Proof:

Let $P(x, y)$ be any point on the line l which has gradient m and passes through the fixed point $Q(x_1, y_1)$.

Using the gradient formula, $m = \frac{y - y_1}{x - x_1}$

This can be re-written as $y - y_1 = m(x - x_1)$.



EG
+S**Example 1**

By substituting into the equation $y = mx + b$, find the equation of the line that has a gradient of 4 and passes through the point $(-2, 7)$.

Solution

Substituting $x = -2$, $y = 7$, $m = 4$ into $y = mx + b$ gives

$$7 = 4(-2) + b$$

$$7 = -8 + b$$

$$\therefore b = 15$$

$\therefore$ The equation of the line is $y = 4x + 15$.

EG
+S**Example 2**

Use the point–gradient formula to find the equation of the line that has a gradient of -3 and passes through the point $(2, -4)$.

Solution

$$y - y_1 = m(x - x_1)$$

$$y - -4 = -3(x - 2)$$

$$y + 4 = -3x + 6$$

$$\therefore y = -3x + 2$$

EG
+S**Example 3**

Use the point–gradient formula to find the equation of the line that has a gradient of $\frac{2}{3}$ and passes through the point $(5, 1)$. Give your answer in the general form.

Solution

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{2}{3}(x - 5)$$

$$\times 3 \quad \times 3$$

$$3y - 3 = 2(x - 5)$$

$$3y - 3 = 2x - 10$$

$$\therefore 2x - 3y - 7 = 0$$

Exercise 14.5

- 1 By substituting into $y = mx + b$, find the equation of the line that passes through the given point with the given gradient. Give the answers for **e–h** in the general form.

a $(1, 5)$, $m = 2$ **b** $(-2, 4)$, $m = 3$ **c** $(1, -7)$, $m = -1$ **d** $(-5, -4)$, $m = -2$

e $(6, 0)$, $m = \frac{1}{3}$ **f** $(4, -3)$, $m = -\frac{1}{2}$ **g** $(-12, 5)$, $m = \frac{2}{3}$ **h** $(-8, -9)$, $m = -\frac{3}{4}$

- 2 Use the point–gradient formula to find the equation of the line that passes through the given point with the given gradient. Give your answers in the gradient–intercept form.

a $(3, 5)$, $m = 1$ **b** $(1, 2)$, $m = 3$ **c** $(-3, 4)$, $m = 2$

d $(-2, 0)$, $m = 6$ **e** $(4, 2)$, $m = -1$ **f** $(0, 7)$, $m = -4$

g $(5, -6)$, $m = -3$ **h** $(-1, -4)$, $m = -2$ **i** $\left(\frac{1}{2}, 7\right)$, $m = 2$

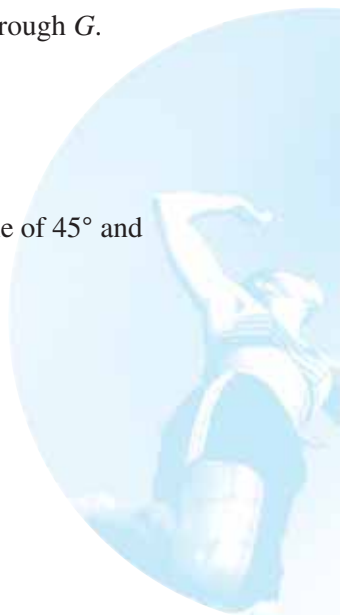
j $\left(\frac{3}{4}, 5\right)$, $m = -8$ **k** $\left(-\frac{2}{3}, -9\right)$, $m = -6$ **l** $\left(-\frac{4}{7}, -1\right)$, $m = 0$

Consolidation

- 3 Use the point–gradient formula to find, in the general form, the equation of the line which has a gradient of:
- a $\frac{1}{2}$ and passes through (3, 4) b $\frac{2}{3}$ and passes through (5, -2)
- c $-\frac{1}{3}$ and passes through (-1, 2) d $-\frac{3}{4}$ and passes through (-3, 7)
- e $\frac{1}{5}$ and cuts the x -axis at 4 f $-1\frac{5}{6}$ and cuts the y -axis at -2
- 4 The point (-7, 6) lies on a line whose gradient is -3. Find the equation of the line.
- 5 a Find the equation of the line which passes through (-4, 9) and has a gradient of -2.
b Does the point (8, -15) lie on this line ?
- 6 Find the equation of the line which has a gradient of $\frac{1}{4}$ and passes through the midpoint of (2, 6) and (-4, 10). Give your answer in the general form.
- 7 a A circle with centre $C(5, 10)$ has a diameter with end points $B(1, 14)$ and $D(x, y)$. Find the co-ordinates of D .
b Find, in the general form, the equation of the line passing through D that has a slope of $\frac{3}{5}$.
- 8 Find the x - and y -intercepts of the line that passes through the point (-6, 8) and has a gradient of 4.
- 9 a Find the co-ordinates of E , the x -intercept of the line with equation $2x + y + 10 = 0$.
b Hence, find the equation of the line passing through E which has a gradient of $-\frac{2}{7}$. Give your answer in the general form.
- 10 a Find the co-ordinates of G , the point of intersection of the lines $2x + 3y = 10$ and $5x + 2y = 3$.
b Hence, find the equation of the line with a gradient of -5 which passes through G . Give your answer in the gradient–intercept form.

Further applications

- 11 a Find the value of $\tan 45^\circ$.
b Hence, find the equation of the line that is inclined to the x -axis at an angle of 45° and passes through (-2, 11). Give your answer in the gradient–intercept form.



- 12 a** Show that $\tan 36^\circ 52'$ is approximately equal to $\frac{3}{4}$.
- b** Hence, find in the general form, the equation of the line that passes through $(-3, 1)$ and is inclined to the x -axis at an angle of $36^\circ 52'$.

14.6 The equation of a line given two points

There is only one straight line that can be drawn through any two points on a plane surface. If we know the co-ordinates of two points that lie on a line, then we can find its equation. In the previous chapter, we found the equation of a line given two points, using simultaneous equations. This involved substituting each pair of co-ordinates into the equation $y = mx + b$, giving two equations in terms of m and b . These equations were then solved simultaneously to find m and b and thus the equation of the line. In this exercise, we will look at the two standard methods for finding the equation of a line given two points.

The gradient and point-gradient formulae

The equation of the line can be determined using the gradient and point-gradient formulae that have already been covered.

To find the equation of the line that passes through the points (x_1, y_1) and (x_2, y_2) :

- ⊙ find the gradient of the line using the gradient formula $m = \frac{y_2 - y_1}{x_2 - x_1}$
- ⊙ substitute the gradient and one point into the point-gradient formula $y - y_1 = m(x - x_1)$
- ⊙ give the equation in the required form.

The two-point formula

The two-point formula is simply another version of the point-gradient formula. It combines the two steps above into a single formula.

The equation of the straight line that passes through the points (x_1, y_1) and (x_2, y_2) is given by:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

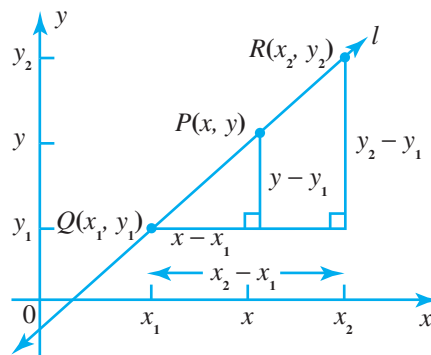
Proof:

Let $P(x, y)$ be any point on the line l which also passes through the points $Q(x_1, y_1)$ and $R(x_2, y_2)$.

$$\text{Now, } m_{PQ} = \frac{y - y_1}{x - x_1} \text{ and } m_{QR} = \frac{y_2 - y_1}{x_2 - x_1}$$

But, the points P , Q , and R are collinear, $\therefore m_{PQ} = m_{QR}$.

$$\therefore \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$



EG
+S

Example 1

Find the gradient of the line that passes through the points $(3, 1)$ and $(5, 9)$. Hence, find the equation of the line.

Solution

Let $(3, 1)$ be (x_1, y_1) and $(5, 9)$ be (x_2, y_2) .

$$\begin{aligned} \text{i } m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{9 - 1}{5 - 3} \\ &= \frac{8}{2} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{ii } y - y_1 &= m(x - x_1) \\ y - 1 &= 4(x - 3) \\ y - 1 &= 4x - 12 \\ \therefore y &= 4x - 11 \end{aligned}$$

EG
+S

Example 2

Use the two-point formula to find the equation of the line that passes through the points $(-2, 5)$ and $(4, -1)$.

Solution

Let $(-2, 5)$ be (x_1, y_1) and $(4, -1)$ be (x_2, y_2) .

$$\begin{aligned} \frac{y - y_1}{x - x_1} &= \frac{y_2 - y_1}{x_2 - x_1} \\ \frac{y - 5}{x - (-2)} &= \frac{-1 - 5}{4 - (-2)} \\ \frac{y - 5}{x + 2} &= \frac{-6}{6} \\ \frac{y - 5}{x + 2} &= -1 \\ y - 5 &= -1(x + 2) \\ y - 5 &= -x - 2 \\ \therefore y &= -x + 3 \end{aligned}$$



Exercise 14.6

- 1 For each of the following:
 - i find the gradient of the line that passes through the given points
 - ii find the equation of the line by substituting the gradient and one point into $y = mx + b$.

a (2, 5) and (6, 9)	b (1, 4) and (-2, 13)	c (10, 3) and (2, -1)
- 2 For each of the following:
 - i find the gradient of the line that passes through the given points
 - ii find the equation of the line by substituting the gradient and one point into $y - y_1 = m(x - x_1)$.

a (1, 7) and (2, 10)	b (-1, -6) and (3, 14)	c (-3, 11) and (-7, 19)
- 3 Use the two-point formula to find the equation of the line that passes through each pair of points. Give your answers in the general form for **g-l**.

a (3, 5) and (6, 8)	b (1, 3) and (3, 11)	c (1, 5) and (-2, -4)
d (-3, 4) and (1, 0)	e (0, 3) and (1, -2)	f (1, -9) and (-3, -1)
g (3, 1) and (5, 2)	h (-1, -5) and (2, -3)	i (8, 4) and (6, 5)
j (-5, 16) and (3, 10)	k (-8, -9) and (4, 0)	l (-3, 1) and (6, -14)

Consolidation

- 4 Find the equation of the line that passes through $P(7, 20)$ and Q , the midpoint of $R(-3, 5)$ and $S(5, 11)$.
- 5 **a** Determine, in the general form, the equation of the line that passes through the points $L(3, 12)$ and $M(9, 20)$.
b Find the x - and y -intercepts of the line.
- 6 **a** Find the equation of the line passing through the points $A(-3, 3)$ and $B(5, -13)$.
b Show that $C(-2, 1)$ lies on the line AB .
c What can you thus say about the points A , B and C ?
- 7 Show that the points $X(4, -2)$, $Y(10, 7)$ and $Z\left(3, -3\frac{1}{2}\right)$ are collinear.
- 8 Find the value of t given that the points $U(2, 7)$, $V(-1, -11)$ and $W\left(-\frac{2}{3}, t\right)$ are collinear.
- 9 **a** Find the co-ordinates of E , the point of intersection of the lines $y = 2x - 3$ and $4x - y = 5$.
b Hence, find the equation of the line that passes through E and the point $F(-1, -9)$.

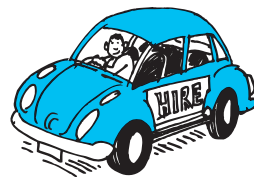
Further applications

- 10 Find, in the general form, the equation of the line that passes through the points $S\left(\frac{1}{4}, -3\right)$ and $T\left(1\frac{3}{4}, 1\frac{1}{2}\right)$
- 11 Consider the points $A(5, 20)$, $B(-3, -4)$, $C(3, -5)$ and $D(-5, 9)$. Find the co-ordinates of P , the point of intersection of the lines AB and CD .

TRY THIS

Car hire

The cost of renting cars at two hiring companies is \$70 per day or \$30 per day plus \$0.50 per kilometre. Write each cost as an equation and draw graphs to decide which option is the best.



14.7 Parallel lines

If the line l_1 has gradient m_1 and the line l_2 has gradient m_2 , then the lines l_1 and l_2 are parallel if:

$$m_1 = m_2$$

Proof:

Let l_1 and l_2 be parallel lines.

Now, $m_1 = \frac{AC}{BC}$

$$\therefore m_1 = \tan \angle ABC$$

and $m_2 = \frac{DF}{EF}$

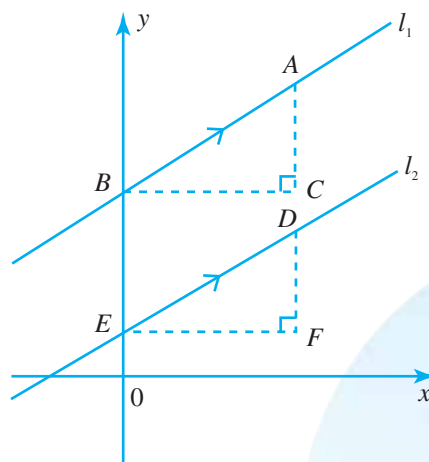
$$\therefore m_2 = \tan \angle DEF$$

But, $\angle ABC = \angle DEF$ (corresponding angles, $l_1 \parallel l_2$)

$$\therefore \tan \angle ABC = \tan \angle DEF$$

$$\therefore m_1 = m_2$$

That is, if two lines are parallel, then their gradients are equal.



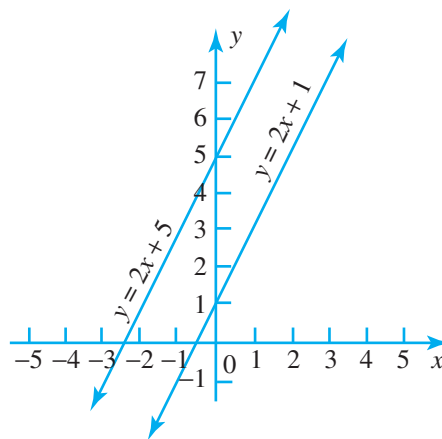
EG
+ SG**Example 1**Find the equation of the line that is parallel to $y = 2x + 5$ and cuts the y-axis at 1.**Solution**

Let the line $y = 2x + 5$ be of the form $y = m_1x + b_1$ and the line parallel to this be of the form $y = m_2x + b_2$.

The line $y = 2x + 5$ has gradient, $m_1 = 2$, so the parallel line has gradient $m_2 = 2$, as $m_1 = m_2$ for parallel lines.

If the parallel line cuts the y-axis at 1, then $b_2 = 1$.

$\therefore$ The equation of the parallel line is $y = 2x + 1$.

EG
+ SG**Example 2**

Find the equation of the line which is parallel to $5x - y + 7 = 0$ and passes through $(2, 4)$.

Solution

- i In order to find the gradient of the given line, we need to re-arrange the equation into the form $y = mx + b$.

$$5x - y + 7 = 0$$

$$\therefore y = 5x + 7$$

- ii The line $5x - y + 7 = 0$ has gradient $m_1 = 5$, so the line parallel to this has gradient $m_2 = 5$, as $m_1 = m_2$ for parallel lines. We now use the point-gradient formula to find the equation of the line through the point $(2, 4)$ with gradient 5.

$$y - 4 = 5(x - 2)$$

$$y - 4 = 5x - 10$$

$$\therefore y = 5x - 6$$

EG
+ SG**Example 3**

Show, by considering gradients, that the lines $y = 3x$, $y = 4 - 2x$, $y = 3x - 5$ and $y = -2x - 1$ enclose a parallelogram.

Solution

For the line $y = 3x$, $m_1 = 3$

For the line $y = 4 - 2x$, $m_2 = -2$

For the line $y = 3x - 5$, $m_3 = 3$

For the line $y = -2x - 1$, $m_4 = -2$

Now, $m_1 = m_3 \quad \therefore y = 3x$ is parallel to $y = 3x - 5$

Also, $m_2 = m_4 \quad \therefore y = 4 - 2x$ is parallel to $y = -2x - 1$

$\therefore$ The figure is a parallelogram (two pairs of opposite sides are parallel).

Exercise 14.7

- Write down the equation of the line that is parallel to:
 - the x -axis and passes through $P(4, 3)$
 - the y -axis and passes through $L(-1, 6)$
 - the x -axis and passes through $T(7, -9)$
 - the y -axis and passes through $C(-5, 2)$
 - the line $x = 3$ and passes through $J(-2, -8)$
 - the line $y = -4$ and passes through $R(10, 1)$
 - the line $y = 2$ and passes through $Z(6, -2)$
 - the line $x = -6$ and passes through $S(9, 4)$
- State whether or not these lines are parallel.
 - $y = 4x + 9$ and $y = 4x + 7$
 - $y = 3x + 5$ and $y = 5x + 3$
 - $y = 6 - 2x$ and $y = 2x + 1$
 - $y = 3x$ and $y = 3x + 7$
 - $y = x + 7$ and $y = x - 2$
 - $y = x - 4$ and $y = 5 - x$
 - $y = 5 - 7x$ and $y = 5 + 7x$
 - $y = 2x + 7$ and $y = 4 + 2x$
 - $y = \frac{2}{3}x - 4$ and $y = 6 + \frac{3}{2}x$
 - $y = \frac{1}{4}x + 5$ and $y = 2 + \frac{1}{4}x$
- Choose the parallel lines from each set.
 - $y = 3x - 2$, $y = 4 - 3x$, $y = 3x + 10$
 - $y = 2 - x$, $y = -x$, $y = x + 2$
- Find the equation of the line that is parallel to:
 - $y = 2x + 5$ and cuts the y -axis at 1
 - $y = 4x - 1$ and cuts the y -axis at -3
 - $y = 3x$ and has a y -intercept of 6
 - $y = -x$ and has a y -intercept of -4
 - $y = \frac{1}{2}x + 4$ and meets the y -axis at 2
 - $y = \frac{4}{3}x - 7$ and passes through the origin
- Consider the line with equation $y = x - 2$. What would be the equation of this line if it was shifted:
 - up by 5 units?
 - down by 1 unit?
 - up by 9 units?
 - down by 4 units?

Consolidation

- Express each equation in the gradient–intercept form, then determine whether the lines are parallel.
 - $2x + y - 8 = 0$ and $y = 2x + 1$
 - $3x + y + 7 = 0$ and $3x + y - 1 = 0$
 - $x + 4y - 4 = 0$ and $y = 5 - 4x$
 - $5x = 2y - 6$ and $5x - 2y + 9 = 0$
 - $y = 2x - 3$ and $6x - 3y + 7 = 0$
 - $y = -\frac{3}{4}x - 2$ and $3x - 4y = 10$
- Find, in the general form, the equation of the line that is parallel to:
 - $y = x - 2$ and passes through $P(2, 5)$
 - $y = -4x + 1$ and passes through $Q(-1, 3)$
 - $y = \frac{1}{4}x - 2$ and passes through $C(3, -4)$
 - $y = 6 + \frac{1}{5}x$ and passes through $S(2, 0)$
 - $2x + 3y - 15 = 10$ and passes through $T(4, 1)$
 - $3x - 8y = 32$ and passes through $L(-3, -2)$

- 8 $A(5, 8)$, $B(3, 1)$, $C(-4, -3)$ and $D(-2, 4)$ are four points.
- Find the gradient of:
 - AB
 - BC
 - CD
 - DA
 - Which lines are parallel?
 - What kind of quadrilateral is $ABCD$? Why?
- 9 Show by considering gradients that the lines $l_1: 3x - 2y + 8 = 0$, $l_2: x - 4y - 3 = 0$, $l_3: 6x - 4y - 5 = 0$ and $l_4: 3x - 12y + 1 = 0$ enclose a parallelogram.
- 10 $EFGH$ is a quadrilateral with vertices $E(1, 5)$, $F(6, 7)$, $G(8, 3)$, $H(-2, -1)$.
- Find the gradient of each side.
 - What kind of quadrilateral is $EFGH$? Why?
- 11
 - Write down the equation of the line l which is parallel to $y = 4 - 3x$ and cuts the y -axis at 6.
 - Show that l passes through the point $P(4, -6)$.
- 12 Find the equation of the line that:
 - cuts the y -axis at -2 and is parallel to the line joining the points $L(-1, 4)$ and $M(5, 7)$
 - passes through the origin and is parallel to the line that passes through the points $X(2, -3)$ and $Y(5, 6)$.
- 13
 - Find the co-ordinates of G , the point where the line $3x + 4y + 24 = 0$ cuts the y -axis.
 - Find the equation of the line that passes through G and is parallel to $y = -\frac{1}{2}x$.
- 14 A is the midpoint of PQ with $P(-3, 1)$ and $Q(7, 5)$. B is the midpoint of RS with $R(-10, -4)$ and $S(-2, 2)$. Find the equation of the line that is parallel to AB and cuts the y -axis at 13.
- 15 Find the gradient of each line and hence determine the value of k given that:
 - $y = 2kx - 9$ is parallel to $y = 1 - 10x$
 - $y = 5x + 7$ and $kx - 3y + 1 = 0$ are parallel lines
 - $4x - ky + 8 = 0$ is parallel to $2x - 3y - 12 = 0$

■ Further applications

- 16 This question outlines an alternate method for finding the equation of a line that passes through a given point and is parallel to a given line. It is often referred to as the k -method for parallel lines.
- Show that the lines $ax + by + c = 0$ and $ax + by + k = 0$ are parallel.
 - Write down the equation of the line l with constant term k that is parallel to $3x + 2y + 8 = 0$.
 - If l passes through $(1, -4)$, find the value of k .
 - Hence, find in the general form the equation of l , the line that is parallel to $3x + 2y + 8 = 0$ and passes through $(1, -4)$.

17 Use the k -method outlined in Q16 to find, in the general form, the equation of the line which passes through the point:

- a (2, 1) and is parallel to the line $x - 3y + 6 = 0$
- b (3, -4) and is parallel to the line $2x + y - 5 = 0$
- c (-5, 2) and is parallel to the line $3x + 7y + 9 = 0$
- d (-1, -7) and is parallel to the line $5x - 4y - 11 = 0$

TRY THIS

Temperature rising

Wendy has three thermometers with different linear scales (T_1 , T_2 , T_3). When T_1 reads 18° and 34° , T_2 reads 20° and 38° , respectively. When T_2 reads 24° and 32° , T_3 reads 46° and 64° . If the temperature on T_1 's scale rises 14° , how many degrees does it rise on T_3 's scale?

14.8 Perpendicular lines

If the line l_1 has gradient m_1 and the line l_2 has gradient m_2 , then the lines l_1 and l_2 are perpendicular if:

$$m_1 m_2 = -1$$

Proof:

Consider the perpendicular lines l_1 and l_2 which intersect at the origin. Construct $\triangle POQ$ as shown, with $OQ = a$ units and $PQ = b$ units. Rotate this triangle 90° anticlockwise about O to produce the image $\triangle P'OQ'$ in which P' lies on the line l_2 , $OQ' = a$ units and $P'Q' = b$ units.

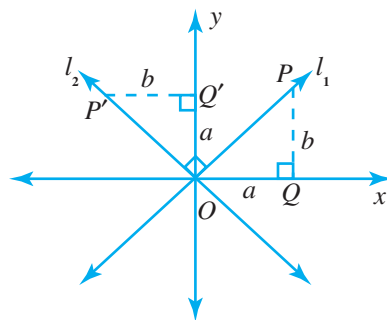
Let m_1 be the gradient of the line l_1 and m_2 be the gradient of the line l_2 .

$$\text{Now, } m_1 = \frac{b}{a} \text{ and } m_2 = -\frac{a}{b}.$$

(NOTE: m_2 is negative because the line l_2 leans to the left.)

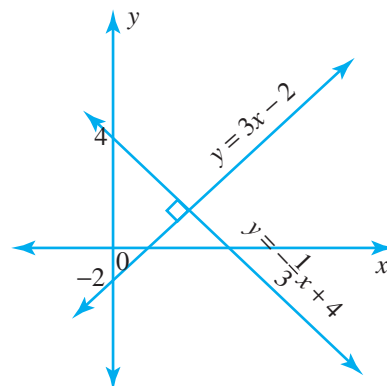
$$\begin{aligned} \text{Now, } m_1 m_2 &= \frac{b}{a} \times -\frac{a}{b} \\ &= -1 \end{aligned}$$

$\therefore$ If two lines are perpendicular, the product of their gradients is -1 .



EG
+S**Example 1**Find the equation of the line that is perpendicular to $y = 3x - 2$ and cuts the y -axis at 4.**Solution**Let the line $y = 3x - 2$ be of the form $y = m_1x + b_1$ and the line perpendicular to this be of the form $y = m_2x + b_2$.The line $y = 3x - 2$ has gradient $m_1 = 3$, so the perpendicular line has gradient $m_2 = -\frac{1}{3}$, since

$$3 \times -\frac{1}{3} = -1.$$

If the perpendicular line cuts the y -axis at 4, then $b_2 = 4$. $\therefore$ The equation of the perpendicular line is $y = -\frac{1}{3}x + 4$.EG
+S**Example 2**Find the equation of the line that is perpendicular to $y = \frac{1}{4}x + 2$ and passes through $(-1, 5)$.**Solution**

i The line $y = \frac{1}{4}x + 2$ has gradient $m_1 = \frac{1}{4}$, so any line perpendicular to it has gradient $m_2 = -4$, as $\frac{1}{4} \times -4 = -1$.

$$\begin{aligned} y - 5 &= -4(x + 1) \\ y - 5 &= -4x - 4 \\ \therefore y &= -4x + 1 \end{aligned}$$

ii We now use the point-gradient formula to find the equation of the line through the point $(-1, 5)$ with gradient -4 .

EG
+S**Example 3**Show by considering gradients that the lines $y = 2x + 1$, $y = 5 - \frac{1}{2}x$, $y = 2x - 4$ and $y = -\frac{1}{2}x - 3$ enclose a rectangle.**Solution**For the line $y = 2x + 1$, $m_1 = 2$ For the line $y = 5 - \frac{1}{2}x$, $m_2 = -\frac{1}{2}$ For the line $y = 2x - 4$, $m_3 = 2$ For the line $y = -\frac{1}{2}x - 3$, $m_4 = -\frac{1}{2}$ Now, $m_1m_2 = 2 \times -\frac{1}{2} = -1$, $\therefore y = 2x + 1$ is perpendicular to $y = 5 - \frac{1}{2}x$ Similarly, $m_2m_3 = -1$, $\therefore y = 5 - \frac{1}{2}x$ is perpendicular to $y = 2x - 4$ $m_3m_4 = -1$, $\therefore y = 2x - 4$ is perpendicular to $y = -\frac{1}{2}x - 3$ $m_4m_1 = -1$, $\therefore y = -\frac{1}{2}x - 3$ is perpendicular to $y = 2x + 1$ $\therefore$ The figure is a rectangle (all adjacent sides are perpendicular).

Exercise 14.8

- 1 Write down the equation of the line that is perpendicular to:
 - a the x -axis and passes through $A(1, 3)$
 - b the y -axis and passes through $V(2, -5)$
 - c the x -axis and passes through $G(-4, -1)$
 - d the y -axis and passes through $Z(-7, 3)$
- 2 State whether or not these lines are perpendicular.

a $y = x + 3$ and $y = 2 - x$	b $y = 2x + 5$ and $y = -\frac{1}{2}x + 1$
c $y = 3x + 2$ and $y = 4 - 3x$	d $y = 5x$ and $y = \frac{1}{5}x - 3$
e $y = -4x - 7$ and $y = \frac{1}{4}x + 6$	f $y = x + 2$ and $y = x - 2$
g $y = -\frac{1}{3}x + 8$ and $y = 3x + 1$	h $y = 5 - 7x$ and $y = \frac{1}{7}x - 2$
i $y = \frac{2}{3}x + 2$ and $y = -\frac{2}{3}x - 4$	j $y = -\frac{5}{4}x$ and $y = \frac{4}{5}x + 9$
- 3 Choose the perpendicular lines from each set.

a $y = 3x - 6$, $y = -3x - 1$, $y = -\frac{1}{3}x - 2$	b $y = \frac{2}{5}x - 4$, $y = -\frac{5}{2}x$, $y = -\frac{2}{5}x + 7$
- 4 Find the equation of the line that is perpendicular to:

a $y = x + 4$ and cuts the y -axis at 2	b $y = -2x - 1$ and cuts the y -axis at -3
c $y = \frac{1}{6}x + 2$ and has a y -intercept of -1	d $y = 4 - \frac{1}{5}x$ and has a y -intercept of 8
e $y = -\frac{2}{7}x + 3$ and meets the y -axis at 6	f $y = \frac{5}{12}x - 7$ and meets the y -axis at -4

Consolidation

- 5 Express each equation in the gradient–intercept form, then determine whether the lines are perpendicular.

a $y = \frac{1}{2}x + 3$ and $2x + y + 4 = 0$	b $3x - y + 1 = 0$ and $x + 3y - 12 = 0$
c $x + y + 4 = 0$ and $y = -x$	d $5x = 6y - 2$ and $6y = 5x + 3$
e $y = 7 - \frac{3}{4}x$ and $8x - 6y + 11 = 0$	f $4x - 6y = 3$ and $6x + 9y + 2 = 0$



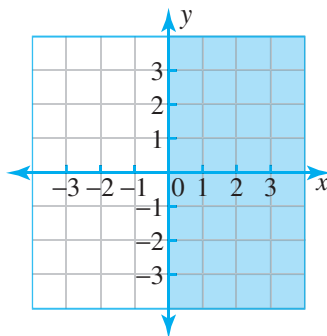
- 6 Find, in the general form, the equation of the line that is perpendicular to the line:
- a $y = x + 6$ and passes through $A(5, 2)$
 - b $y = 1 - 7x$ and passes through $N(-2, 3)$
 - c $y = \frac{1}{4}x - 5$ and passes through $H(-3, -4)$
 - d $y = -\frac{5}{8}x + 4$ and passes through $K(-1, 0)$
 - e $x - 3y - 1 = 0$ and passes through $C(-2, 0)$
 - f $4x + 2y - 5 = 0$ and passes through $J(8, -9)$
- 7 $P(5, 11)$, $Q(9, 5)$, $R(-3, -3)$ and $S(-7, 3)$ are four points.
- a Find the gradient of:
 - i PQ
 - ii QR
 - iii RS
 - iv SP
 - b Show that $PQRS$ is a parallelogram.
 - c Is the figure a rectangle? Why?
- 8 Show by considering gradients that the lines $5x + 3y - 2 = 0$, $3x - 5y + 1 = 0$, $10x + 6y + 7 = 0$ and $9x - 15y - 4 = 0$ enclose a rectangle.
- 9 $KLMN$ is a quadrilateral with vertices $K(1, 2)$, $L(4, 6)$, $M(8, 3)$ and $N(5, -1)$.
- a Find the gradient of each side.
 - b Find the gradients of the diagonals.
 - c What kind of quadrilateral is $KLMN$? Why?
- 10 a Write down the equation of the line that is perpendicular to $y = 5x - 6$ and cuts the y -axis at 3.
- b Show that this line passes through the point $A(10, 1)$.
- 11 Find the equation of the line which:
- a cuts the y -axis at -1 and is perpendicular to the line joining the points $U(-1, -13)$ and $V(4, 7)$
 - b passes through the origin and is perpendicular to the line that passes through the points $E(-2, 1)$ and $F(4, 9)$.
- 12 a Find the co-ordinates of S , the point where the line $3x - 2y + 14 = 0$ cuts the y -axis.
- b Hence, find the equation of the line that passes through S and is perpendicular to the line $5x - 4y - 1 = 0$.
- 13 Find the gradient of each line and, hence, determine the value(s) of k given that the line:
- a $y = (k - 7)x + 1$ is perpendicular to $y = \frac{1}{4}x + 6$
 - b $y = \frac{1}{2}x - 3$ is perpendicular to $6x - ky + 9 = 0$
 - c $kx + 4y - 2 = 0$ is perpendicular to $kx - 9y + 7 = 0$

Further applications

- 14** This question outlines an alternate method for finding the equation of a line that passes through a given point and is perpendicular to a given line. It is often referred to as the *k*-method for perpendicular lines.
- Show that the lines $ax + by + c = 0$ and $bx - ay + k = 0$ are perpendicular.
 - Write down the equation of the line l with constant term k that is perpendicular to $4x + 5y + 7 = 0$.
 - If l passes through $(2, 3)$, find the value of k .
 - Hence, find in the general form the equation of l , the line that is perpendicular to $4x + 5y + 7 = 0$ and passes through $(2, 3)$.
- 15** Use the *k*-method outlined in Q14 to find in the general form the equation of the line that passes through the point:
- $(4, 5)$ and is perpendicular to the line $3x + 2y + 1 = 0$
 - $(-7, 2)$ and is perpendicular to the line $4x - y + 2 = 0$
 - $(-1, -8)$ and is perpendicular to the line $2x + 5y - 8 = 0$
 - $(6, -3)$ and is perpendicular to the line $x - 3y - 12 = 0$

14.9 Regions in the number plane

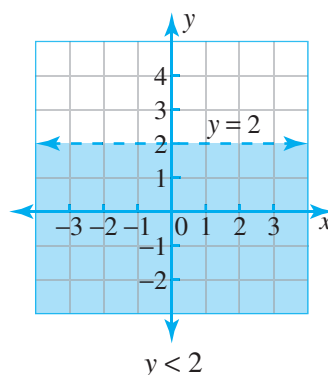
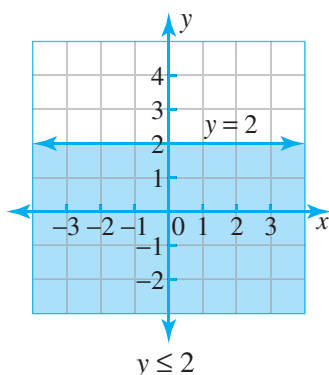
When a linear equation is graphed on the number plane, the result is a straight line. When a linear inequation is graphed, the result is a region or half-plane. A **region** is a set of points in the number plane that have something in common. For example, one region is the set of all points in which the x -values are greater than or equal to zero. It can be represented by shading the area to the right of the y -axis. The region here includes the points that lie on the y -axis, as well as those that lie in the shaded area.



The line which divides the number plane into two regions or half-planes is called the **boundary line**. In the example above, the boundary line is the y -axis.

When an inequation is graphed on a *number line*, a closed dot is used if we want to include a particular number in the solution set, while an open dot is used if we want to exclude it. Similarly, if we want to include the points that lie on the boundary line in our region, we draw the boundary line as a *continuous* line. If we do not wish to include these points, then we draw the boundary line as a *discontinuous* or *broken* line. That is, the boundary line may or may not be a part of a particular region.

For example, the regions represented by the inequalities $y \leq 2$ and $y < 2$ are shown below. The first region includes all of the points that lie on or below the line $y = 2$, while the second region does not include any points that lie on the line.



To shade the region represented by a linear inequation:

- ⊙ **determine whether the boundary line is continuous or discontinuous and sketch it**
- ⊙ **choose a point that does not lie on the boundary line and substitute its co-ordinates into the inequation**
- ⊙ **if the co-ordinates satisfy the inequation, shade the half-plane in which that point lies**
- ⊙ **if the co-ordinates do not satisfy the inequation, shade the half-plane in which the point does not lie.**

NOTE: A simpler method for determining the correct region is to solve the inequation for y . Then, if the inequality sign is either $>$ or $\geq$, shade the half-plane above the line. If the inequality sign is either $<$ or $\leq$, shade the half-plane below the line.

Example

Sketch each region on a separate number plane.

a $y \leq 2x + 4$

b $y > 3 - x$

Solutions

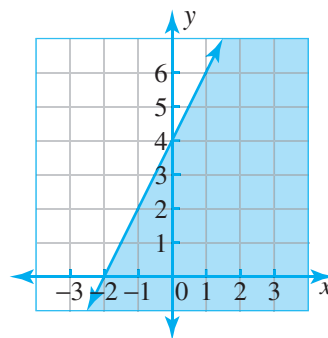
- a** The inequality sign is $\leq$, so the boundary line is continuous. Choose the origin as the test-point, as it does not lie on the boundary line.

$$y \leq 2x + 4$$

Substitute $(0, 0)$: $0 \leq 2(0) + 4$

$$0 \leq 4$$

Now, $0 \leq 4$ is a true statement, therefore we shade the half-plane that contains the test-point $(0, 0)$.



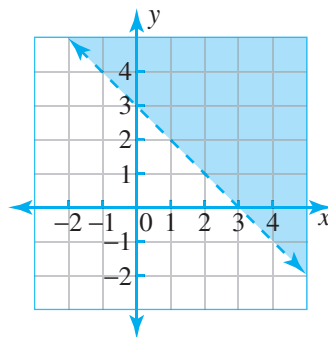
- b** The inequality sign is $>$, so the boundary line is discontinuous. Choose the origin as the test-point, as it does not lie on the boundary line.

$$y > 3 - x$$

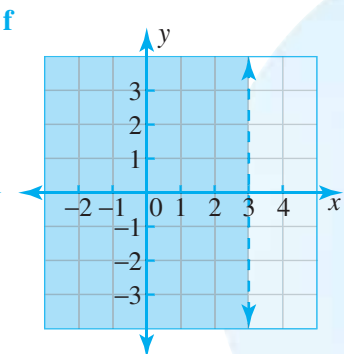
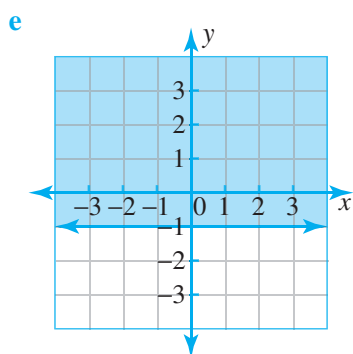
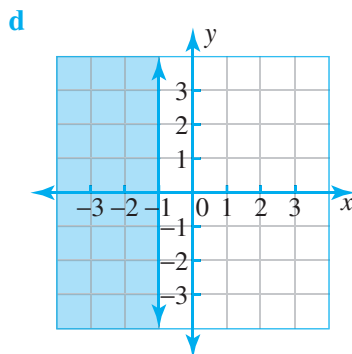
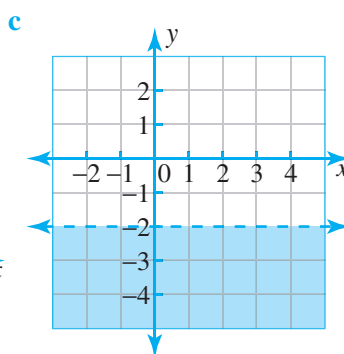
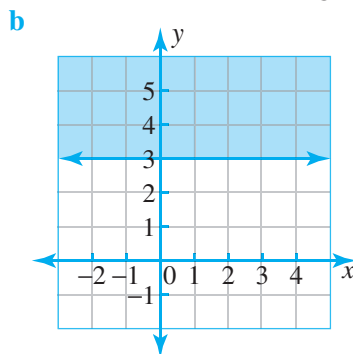
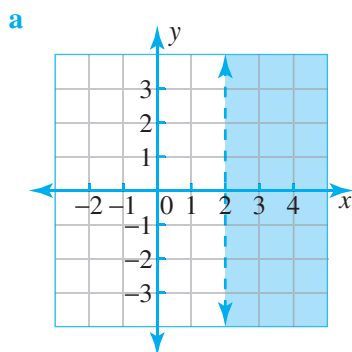
Substitute $(0, 0)$: $0 > 3 - 0$

$$0 > 3$$

Now, $0 > 3$ is not a true statement, therefore we shade the half-plane that does not contain the test-point $(0, 0)$.

**Exercise 14.9**

- 1** Write down the inequation that describes each of these regions.



2 Sketch each region on a separate number plane.

a $x \leq 1$

b $y < 4$

c $x > -2$

d $y \leq -1$

3 In each of the following, name the points that lie within the given region.

a $y > 2$; $A(3, 1)$, $B(0, 4)$, $C(1, 2)$, $D(-2, 3)$

b $x \leq -1$; $A(0, 0)$, $B(-2, 1)$, $C(-1, -3)$, $D(3, -5)$

c $y \geq x$; $A(2, 2)$, $B(4, 3)$, $C(-4, -5)$, $D(5, 7)$

d $y < x + 3$; $A(0, 7)$, $B(3, 4)$, $C(1, 1)$, $D(-2, 0)$

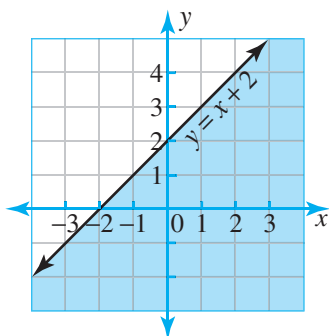
e $y > 2x - 1$; $A(5, 8)$, $B(3, 5)$, $C(-4, -6)$, $D(-2, -3)$

f $y \leq 2 - 3x$; $A(0, 2)$, $B(-2, 6)$, $C(1, 2)$, $D(-4, 15)$

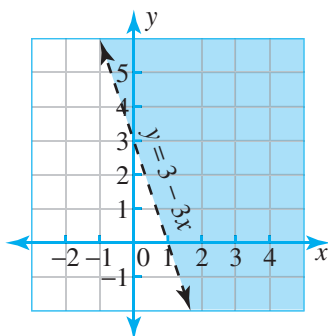
Consolidation

4 Write down the inequation that describes each of these regions.

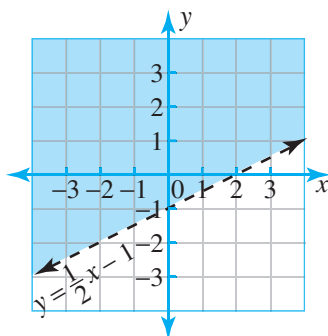
a



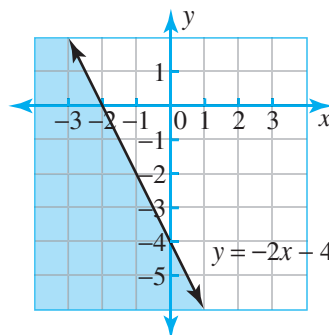
b



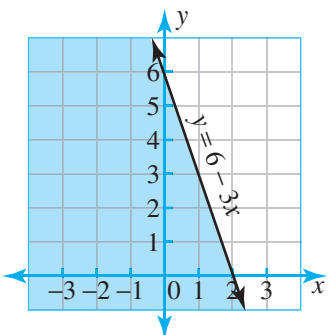
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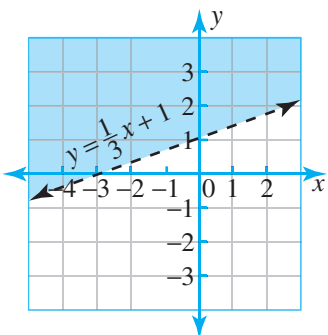
d



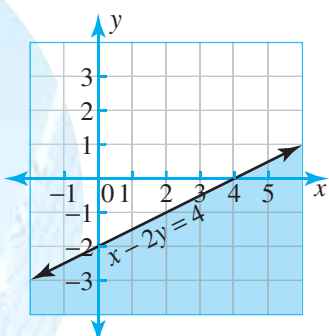
e



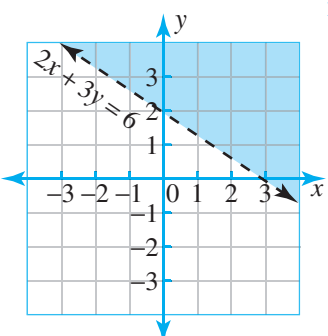
f



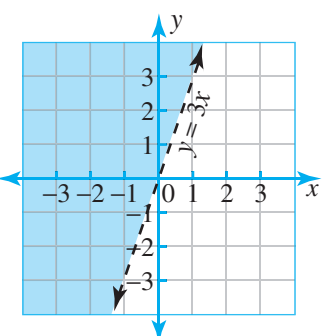
g



h



i



5 Sketch the following regions on separate number planes.

a $y \leq x$

b $y > x + 3$

c $y < 4 - x$

d $y \geq 2x + 1$

e $y \leq -2x$

f $y < 3x - 6$

g $y \geq -x - 5$

h $y > 2 - 2x$

6 Sketch each of the following regions on a separate number plane.

a $2x + y \leq 4$

b $x - 3y > 3$

c $3x - 2y \geq 12$

7 Shade the region on the number plane where both of these inequations hold.

a $x > 0$ and $y > 0$

b $x < 0$ and $y < 0$

c $x < 0$ and $y > 0$

d $x > 0$ and $y < 0$

8 Shade the region on the number plane enclosed by each set of lines, then find its area.

a $y = 0, y = 2x, x + y = 6$

b $x = 1, y = x - 5, y = 7 - x$

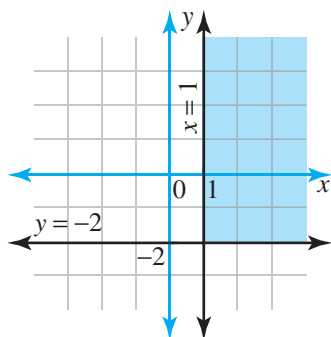
c $x = -2, x = 3, y = 0, y = 2x + 10$

d $x = 0, y = 0, y = \frac{1}{2}x + 3, y = 5 - \frac{1}{2}x$

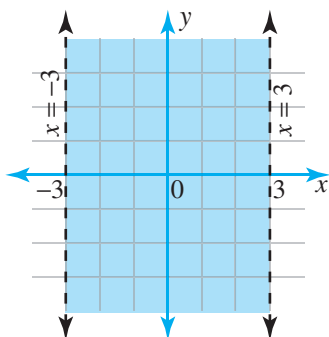
Further applications

9 Write down inequations which completely describe the shaded region.

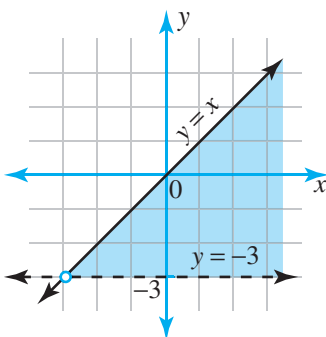
a



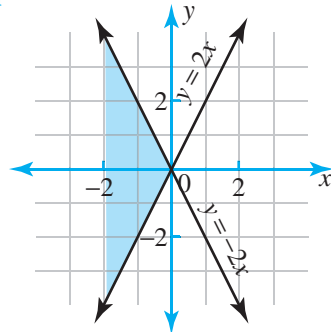
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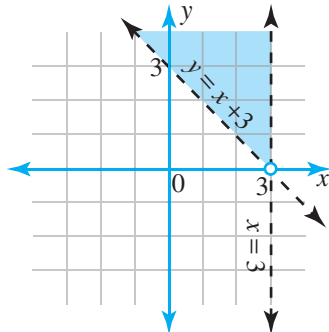
c



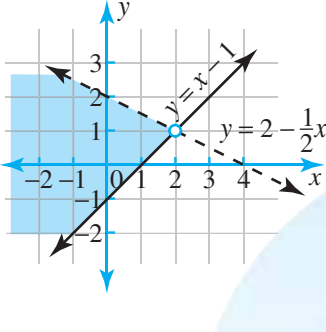
d



e



f



10 **a** Find the co-ordinates of the point of intersection of the lines $y = 2x - 3$ and $y = x + 1$.

b Sketch the region where the inequations $y \leq x + 1$ and $y \geq 2x - 3$ hold simultaneously.

11 Shade the region where both inequations hold simultaneously.

a $y \leq 2$ and $y > -1$

b $x < 0$ and $y > 1$

c $y < x$ and $x > 0$

d $y > 3$ and $y \geq 2x$

e $y \leq x + 1$ and $y \geq 2 - \frac{1}{2}x$

f $x + y < 3$ and $y > -4x$

g $x + y < 5$ and $x - 2y \geq 6$

h $2x - y < 3$ and $3x + 2y < 6$

14.10 Co-ordinate geometry problems

Co-ordinate geometry problems usually take several steps to solve and often require the use of a number of different formulae. The original purpose of co-ordinate geometry was to prove various geometric properties of triangles and quadrilaterals by using algebraic techniques. For example, to prove that the diagonals of a parallelogram bisect each other, we could show that the diagonals have the same midpoint. To show that a triangle is right-angled, we could show that the product of the gradients of two sides is -1 .

The following terms are used frequently in co-ordinate geometry problems.

- ⊙ **Median:** A median of a triangle is an interval drawn from one vertex to the midpoint of the opposite side.
- ⊙ **Altitude:** An altitude of a triangle is a perpendicular drawn from one vertex to the opposite side.
- ⊙ **Concurrent lines:** Concurrent lines are three or more lines that intersect at a single point.
- ⊙ **Collinear points:** Collinear points are three or more points that lie in the same straight line.
- ⊙ **Perpendicular bisector:** The perpendicular bisector of an interval is the line that passes through the midpoint of the interval and is at right angles to it.

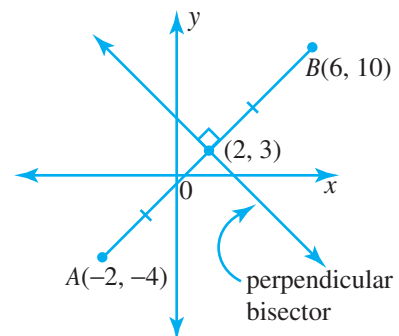
Example 1

Find the equation of the perpendicular bisector of the interval AB , where $A(-2, -4)$ and $B(6, 10)$. Give your answer in the general form.

Solution

- i Find the midpoint of AB .

$$\begin{aligned}\text{midpoint} &= \left(\frac{-2 + 6}{2}, \frac{-4 + 10}{2} \right) \\ &= (2, 3)\end{aligned}$$



- ii Find m_1 , the gradient of AB .

$$\begin{aligned} m_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{10 - -4}{6 - -2} \\ &= \frac{14}{8} \\ &= \frac{7}{4} \end{aligned}$$

- iii Find m_2 , the gradient of the perpendicular bisector.

$$\begin{aligned} m_1 m_2 &= -1 \text{ for } \perp \text{ lines} \\ \frac{7}{4} \times m_2 &= -1 \\ \therefore m_2 &= -\frac{4}{7} \end{aligned}$$

- iv Find the equation of the perpendicular bisector using $(2, 3)$ and $m = -\frac{4}{7}$.

$$\begin{aligned} y - 3 &= -\frac{4}{7}(x - 2) \\ 7y - 21 &= -4(x - 2) \\ 7y - 21 &= -4x + 8 \\ \therefore 4x + 7y - 29 &= 0 \end{aligned}$$

Example 2

How far is the point $(1, 8)$ from the point of intersection of the lines $y = 1 - x$ and $x + 2y = 5$?

Solution

- i To find the point of intersection of the lines, we solve the equations simultaneously.

$$y = 1 - x \dots\dots (1)$$

$$x + 2y = 5 \dots\dots (2)$$

Substitute (1) into (2).

$$x + 2(1 - x) = 5$$

$$x + 2 - 2x = 5$$

$$2 - x = 5$$

$$-x = 3$$

$$\therefore x = -3$$

Substitute $x = -3$ into (1).

$$y = 1 - -3$$

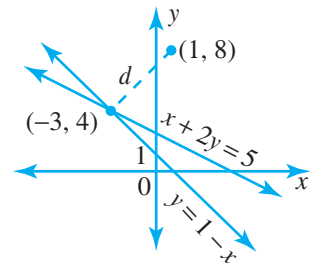
$$\therefore y = 4$$

$\therefore$ The point of intersection of the lines is $(-3, 4)$.

- ii Find the distance between the points $(-3, 4)$ and $(1, 8)$.

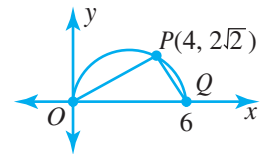
Let $(x_1, y_1) = (-3, 4)$ and $(x_2, y_2) = (1, 8)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1 - -3)^2 + (8 - 4)^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \\ &= 4\sqrt{2} \text{ units} \end{aligned}$$



Exercise 14.10

- 1 The line $2x + ky - 18 = 0$ passes through the point $(3, 4)$.
 - a Find the value of k and, hence, find the gradient of the line.
 - b Find the x - and y -intercepts of the line.
- 2 Find in simplest surd form the length of the interval cut off on the co-ordinate axes by the line $y = \frac{1}{3}x + 4$.
- 3 Find the equation of the line that passes through the point $(4, -6)$ and the x -intercept of $3x - 2y + 9 = 0$.
- 4 The interval RS has end points $R(-9, 14)$ and $S(11, 2)$.
 - a Find the midpoint of RS .
 - b Find the co-ordinates of P , the point which divides RS in the ratio $3:1$.
- 5 A circle with centre $(3, -5)$ passes through the point $(7, -2)$.
 - a Find the length of the radius.
 - b Does this circle also pass through the point $(0, -1)$? Why?
- 6 $\triangle DEF$ has vertices $D(3, -4)$, $E(0, -2)$, $F(-5, 12)$.
 - a Find the co-ordinates of G , the midpoint of DF .
 - b Hence, find the equation of the median EG .
- 7 OPQ is a semicircle with diameter OQ . Show that $OP \perp PQ$.

**Consolidation**

- 10 Prove that $\triangle ABC$ is both isosceles and right-angled if $A(-1, 2)$, $B(3, 4)$ and $C(5, 0)$.
- 11 A quadrilateral $PQRS$ has vertices $P(-2, 4)$, $Q(1, 6)$, $R(3, 3)$ and $S(-1, -4)$.
 - a Show that the diagonals are perpendicular.
 - b Show that QS bisects PR .
 - c Find the lengths of the sides.
 - d What kind of quadrilateral is $PQRS$? Why?
- 12 For what value of r will the line $y = \frac{1}{2}x + r$ bisect the interval which joins the points $(-3, 5)$ and $(1, -7)$?

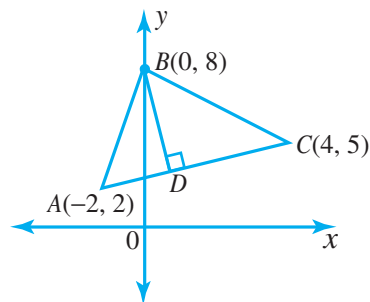
- 13** The vertices of $\triangle FGH$ are $F(-2, 3)$, $G(2, 5)$, $H(6, -3)$.
- Show that the triangle is right-angled and name the hypotenuse.
 - Find the co-ordinates of E , the midpoint of the hypotenuse.
 - Show that E is equidistant from each of the vertices.
- 14** The $\triangle XYZ$ has vertices $X(3, 6)$, $Y(3, -4)$ and $Z(3 - 5\sqrt{3}, 1)$. Show that $\triangle XYZ$ is equilateral.
- 15** The vertices of $\triangle LMN$ are $L(-7, -2)$, $M(1, 6)$ and $N(5, 2)$. The sides LM and LN have midpoints P and Q , respectively. Show that the line joining these midpoints is parallel to the third side MN and half its length.
- 16** The points $(-2, -1)$, $(1, 4)$, $(5, 2)$ are three vertices of a parallelogram. Find the three possible pairs of co-ordinates of the fourth vertex.
- 17**
 - Find the point of intersection of the lines $x + 2y = 13$ and $y = 3x - 11$.
 - Hence, show that the lines $x + 2y = 13$, $y = 3x - 11$ and $2x - 5y + 10 = 0$ are concurrent.
- 18** Show that the lines $y = 2x + 7$, $x + y = 1$ and $3x + 4y - 6 = 0$ are concurrent.
- 19** Consider the points $I(-3, -5)$, $J(1, 7)$, $K(2, 10)$.
- Find the gradient of the interval IJ .
 - Find the gradient of the interval JK .
 - What can you thus say about the points I, J, K ? Why?
- 20**
 - Find the equation of the line that passes through the points $(-1, 11)$ and $(6, -3)$.
 - Hence, show that the points $(-1, 11)$, $(6, -3)$ and $(2, 5)$ are collinear.
- 21** Show that the points $(2, -2)$, $(6, 0)$, $(-8, -7)$ are collinear.
- 22** Find the equation of the line that is perpendicular to $x - 2y + 7 = 0$ and passes through the point of intersection of the lines $y = x - 3$ and $4x + 3y + 2 = 0$.
- 23** Show that the lines $y = -3x - 4$, $x - 3y = 3$, $3x + y = 8$ and $x - 3y + 15 = 0$ enclose a rectangle.
- 24** A quadrilateral $WXYZ$ has vertices $W(2, 6)$, $X(7, 4)$, $Y(10, -3)$, $Z(5, -1)$.
- Show that $WX = YZ$ and $WX \parallel YZ$.
 - What kind of quadrilateral is $WXYZ$? Why?
- 25** The vertices of a parallelogram are $P(-9, 5)$, $Q(3, 3)$, $R(7, -3)$, $S(-5, -1)$. The midpoints of the sides PQ , QR , RS , SP are T , U , V , W , respectively.
- Find the co-ordinates of the midpoints.
 - Show that the intervals PR and QS bisect each other.
 - What kind of quadrilateral is $TUVW$? Why?
- 26** The quadrilateral $CDEF$ has vertices $C(3, 5)$, $D(5, 1)$, $E(1, -1)$, $F(-1, 3)$.
- Show that the diagonals bisect each other at right angles.
 - Find the lengths of the diagonals.
 - What kind of quadrilateral is $CDEF$? Why?
 - Find the length of one side.
 - Find the area of the quadrilateral.



- 27 The vertices of $\triangle ABC$ are $A(-2, 2)$, $B(0, 8)$, $C(4, 5)$.

D is a point on AC such that $BD \perp AC$.

- Find the length of AC .
- Find the gradient of AC .
- Find the equation of AC . Answer in the general form.
- What is the gradient of BD ?
- Find the equation of BD .
- Find the co-ordinates of D .
- Find the distance BD .
- Find the area of $\triangle ABC$.



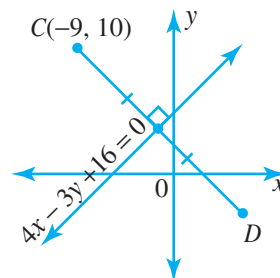
- 28
- Find the equation of the line that passes through the points $G(2, 4)$ and $H(7, -1)$.
 - Find the point of intersection of GH and the line $y = x - 2$.
 - In what ratio does the line $y = x - 2$ divide the interval GH ?

- 29 An interval PQ has end points $P(5, 8)$ and $Q(-1, 2)$.

- Find the co-ordinates of M , the midpoint of PQ .
- Find the gradient of PQ .
- Write down the gradient of a line that is perpendicular to PQ .
- Hence, find the equation of the perpendicular bisector of PQ .

- 30 Find, in the general form, the equation of the perpendicular bisector of the interval YZ , where $Y(-1, -2)$ and $Z(-3, 6)$.

- 31 The perpendicular bisector of the interval CD has equation $4x - 3y + 16 = 0$. If C has co-ordinates $(-9, 10)$, find the co-ordinates of D .



- 32 $R(e, 5)$ lies on the perpendicular bisector of the points $P(-4, -3)$ and $Q(2, 7)$. Find the value of e .

Further applications

- 33 Find the co-ordinates of the point on the line $y = x - 9$ which is equidistant from the points $(1, -4)$ and $(5, -2)$.

- 34 The points $A(-1, 2)$, $B(3, 6)$, $C(7, -5)$ are vertices of $\triangle ABC$. The midpoints of AB , BC , CA are L , M , N , respectively.

- Find the co-ordinates of L , M , N .
- Find the equations of the three medians AM , BN , CL .

- c Find the co-ordinates of P , the point of intersection of AM and BN . This point is called the **centroid** of the triangle.
- d Hence, show that the three medians of the triangle are concurrent.
- 35 The points $A(-3, 0)$, $B(1, 8)$, $C(7, 0)$ are vertices of $\triangle ABC$. The points L, M, N are points on the sides AB, BC, CA , respectively, such that $CL \perp AB$, $AM \perp BC$, $BN \perp CA$.
- a Find co-ordinates of P , the points of intersection of AM and BN . This point is called the **orthocentre** of the triangle.
- b Hence, show that the three altitudes of the triangle are concurrent.
- 36 The points $A(-2, 3)$, $B(2, 7)$, $C(8, 3)$ are vertices of $\triangle ABC$. The midpoints of AB, BC, CA are L, M, N , respectively.
- a Find the co-ordinates of L, M, N .
- b Find the gradient of each side of the triangle.
- c Find the equation of the perpendicular bisector of each side.
- d Find the co-ordinates of P , the point of intersection of the perpendicular bisectors of AB and CA . This point is called the **circumcentre** of the triangle.
- e Hence, show that the three perpendicular bisectors are concurrent.



FOCUS ON WORKING MATHEMATICALLY

FINDING THE GRADIENT OF A SKI RUN



Introduction

A *contour map* is used to show the shape of an area of land, its height above sea level and the steepness of its slopes. On the map, *contour lines* join places of equal height above sea level. The *contour interval* is the difference in height between two adjacent contour lines. On a given map, this difference is always kept the same.

With practice, you can easily learn to visualise the three-dimensional shape of features of the map by looking carefully at the patterns of the contour lines. For example, areas of land where contour lines are close together have steep slopes. Areas of land where there are only a few contour lines that are widely spaced are flat. Evenly spaced contours indicate a uniform slope. Mountains and hills are indicated by sets of contour loops that get smaller and smaller as you get nearer to the top.

In this activity, we study a contour map and use our knowledge of co-ordinate geometry to find the average slope (or gradient) of a ski run.

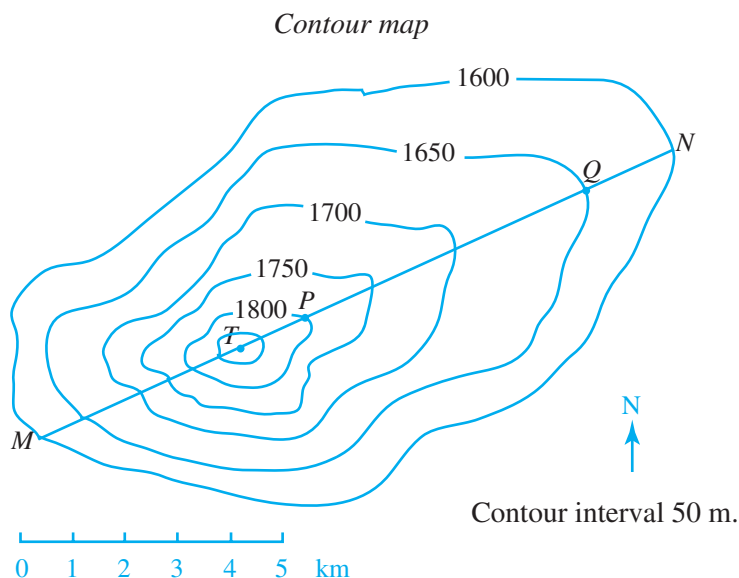


LEARNING ACTIVITIES

How do you find the gradient of a ski run from a contour map? In the following map, what is the average slope of the ski run from P to Q ?

Materials needed: A sheet of A4 paper creased to form a sharp edge, a ruler and a workbook.

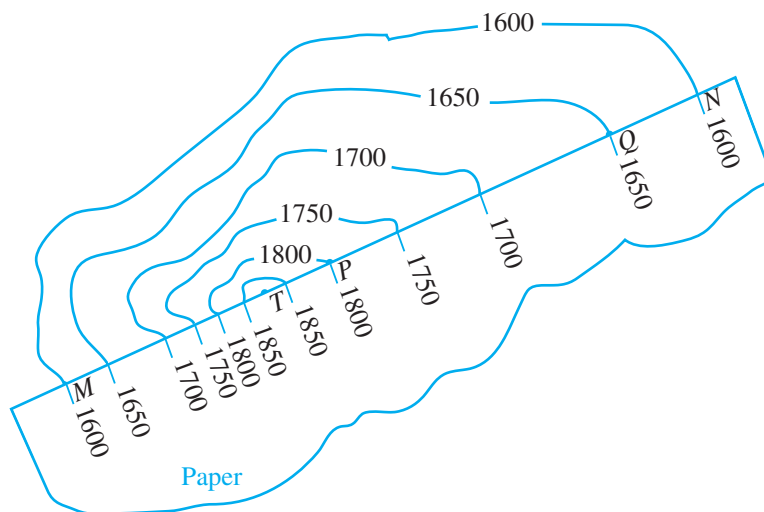
Study the contour map below.



- 1 Can you visualise the shape of the land? Draw it roughly in your workbook. Write down the scale of the map and the contour interval.

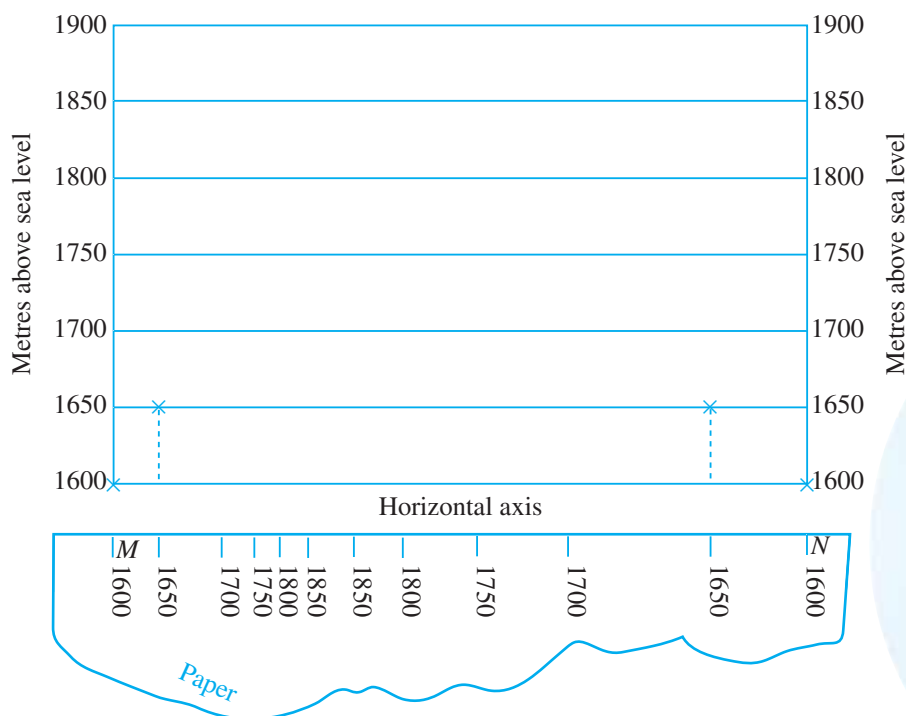
- 2 Now, look carefully at the figure below, which shows how to draw a cross-section of the map along the line MN . Place the straight edge of your piece of paper along the line MN . Mark M and N on the edge. Then, starting from M , make a mark where the edge of your paper cuts each contour line.

Marking the paper



- 3 Draw the axes for your cross-section in your workbook. The horizontal axis must be equal to the length of the line MN . For the vertical axis, which shows the height above sea level, choose a scale to suit your needs. It should look like the figure below.

Preparation of grid for the cross-sectional drawing



- 4 Place your sheet of paper along the horizontal axis and plot the contour points and heights to make a line graph. Several points have been plotted for you. Join the points with a single smooth, curved line and shade with a coloured pencil the area beneath the graph to highlight the shape.
- 5 Mark the points P and Q on your graph. What information do you now need to find the gradient of the ski run? The scale on the contour map shows the horizontal distance between points on the map.
- 6 What is the gradient of the ski run, PQ ? Make sure you use the same units for vertical rise and horizontal run. Finally, express your answer as a ratio in the form $1 : x$, where x is rounded off to the nearest whole number.

$$\text{Average slope of } PQ = \frac{\text{vertical rise}}{\text{horizontal run}}$$



CHALLENGE

- 1 Estimate the height above sea level of the mountain (point T) in the contour map. On a map, such a height is called a 'spot height'. Major spot heights on prominent features are often shown as 'trig stations'. You may have been at the top of a mountain and seen a trig station marked with a brass plate.
- 2 In the *Northern* Hemisphere, ski slopes are always preferred on the eastern or southern sides of a mountain. In the *Southern* Hemisphere, preferred ski slopes are on the eastern or northern sides of a mountain. Why is this?
- 3 Where are the ski slopes in Australia located? Can you say whether the mountain in this activity might be located in Australia? Look again at the maps for evidence. If you like, use the Internet to check the heights of Australian ski resorts.
- 4 List as many examples as you can of the use of contours in other contexts, for example, in weather maps. What are lines of equal pressure on a weather map called?



LET'S COMMUNICATE

Write a set of clear instructions for calculating the average gradient between two points on a contour map. Explain why the method can only give you the average gradient.





REFLECTING

Gradients are *very* important in many different situations. The following are all concerned with safety issues:

- A gradient sign on a steep descent (say 1 : 5) with a warning that trucks and buses must use low gears. Have you seen such a road sign?
- The need to know the gradient of a ski slope. Beginners and professionals need different slopes. Olympic competitions are especially important.
- When building a house, local council rules govern the slope of the driveway you wish to put in.
- Train lines in Europe built for high-speed trains with operating speeds of 300 km/h, such as the Train Grand Vitesse (TGV) in France, generally have a maximum gradient of 25 in 1000.

People may use different methods of calculation depending on the practical context, but the basic mathematical knowledge required is the same. That is what makes mathematics such a powerful tool in our daily living.



MACQUARIE

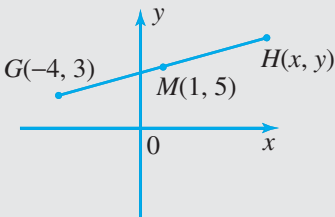
Language link with Macquarie

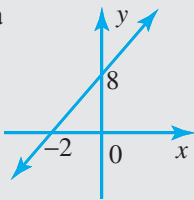
- 1 Explain the difference between a **number line** and a **number plane**.
- 2 What are **co-ordinates**?
- 3 Use the term **linear relationship** in a simple sentence.
- 4 What are **co-ordinate axes**?
- 5 Read the Macquarie Learners Dictionary entry for **gradient**:

gradient noun **1.** Specialised the amount of slope or steepness in a road, railway or path. **2.** a sloping surface

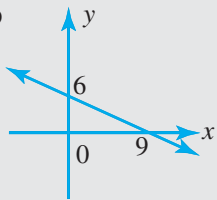
Write a precise mathematical definition for 'gradient' from what you have learned in this chapter.

- 1 Use the distance formula to find the distance between the points R and S .
Answer in surd form.
 - a $R(2, 4), S(6, 9)$
 - b $R(-2, 5), S(1, -3)$
- 2 Show that $\triangle IJK$ is isosceles with $I(3, 5)$, $J(-1, 3)$ and $K(1, 7)$.
- 3 The vertices of $\triangle UVW$ are $U(-2, 1)$, $V(8, 7)$ and $W(5, 12)$.
 - a Find the lengths of the sides in surd form.
 - b Use Pythagoras' theorem to determine whether the triangle is right angled.
- 4 The vertices of a quadrilateral $ABCD$ are $A(1, 1)$, $B(3, 2)$, $C(4, 4)$ and $D(-2, 7)$.
 - a Find the lengths of the sides.
 - b What kind of quadrilateral is $ABCD$? Why?
- 5 Find the co-ordinates of M , the midpoint of the interval CD with:
 - a $C(5, 4), D(11, 6)$
 - b $C(-2, 3), D(-6, -1)$
 - c $C(4, 0), D(7, -5)$.
- 6 Find the co-ordinates of H , given that M is the midpoint of the interval GH .


- 7 The points P, Q, R, S, T are equidistant with $P(1, 10)$ and $T(13, -18)$. Find the co-ordinates of Q, R and S .
- 8 The vertices of a quadrilateral $TUVW$ are $T(-4, 6)$, $U(3, 4)$, $V(2, -2)$, $W(-5, 0)$.
 - a Find the midpoint of each diagonal.
 - b What kind of quadrilateral is $TUVW$? Why?
- 9 Find values for e and f given that $M(e - 3, 2f - 9)$ is the midpoint of $L(-3, 2)$ and $N(5, 8)$.
- 10 Find the gradient of each line.



a



b
- 11 Use the gradient formula to find the slope of a line that passes through the points:
 - a $A(2, 7), B(4, 13)$
 - b $J(-1, 3), K(9, 5)$
 - c $Y(-4, -3), Z(0, -9)$
- 12 Find the value of the pronumeral if the gradient of the line passing through:
 - a $Q(g, 4)$ and $R(2, -5)$ is -3
 - b $D(-7, -5)$ and $E(t, 3)$ is $\frac{2}{3}$
- 13 $IJKL$ is a quadrilateral with vertices $I(6, 7)$, $J(4, 2)$, $K(-3, -2)$ and $L(-1, 3)$. Find the gradient of each side and hence show that $IJKL$ is a parallelogram.
- 14 For each of the following equations:
 - i write down the gradient and y-intercept
 - ii sketch the line on a number plane

- a $y = 2x + 5$
 - c $y = \frac{1}{2}x - 1$

- b $y = -x - 4$
 - d $y = -3x$

- 15** Write down the equation of the line:
- whose gradient is 1 and y-intercept is 6
 - that has a slope of $\frac{3}{4}$ and cuts the y-axis at -2
 - that passes through the origin and has a gradient of $-\frac{5}{2}$

- 16** Write each of these equations in the general form.

a $y = 5x - 2$ **b** $y = -3x + 4$

c $y = \frac{1}{4}x + 7$ **d** $y = 6 - \frac{2}{5}x$

- 17** Write each of these equations in the gradient-intercept form, then state the gradient.

a $x - y + 3 = 0$ **b** $3x + y - 7 = 0$

c $x - 2y - 10 = 0$ **d** $6x + 9y + 4 = 0$

- 18** By substitution into the equation $y = mx + b$, find in the general form, the equation of the line that passes through:

a $(-1, 5)$ and has a gradient of -2

b $(6, 11)$ and has a slope of $\frac{2}{3}$

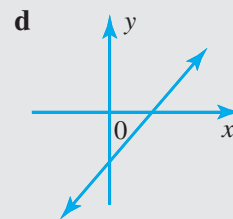
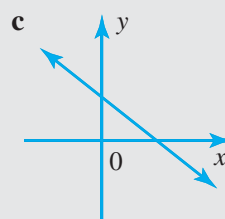
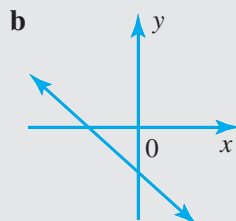
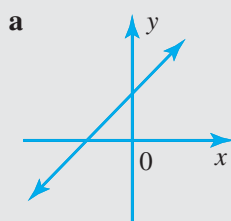
- 19 a** Find the x- and y-intercepts of the line $4x + 3y - 24 = 0$.

b Sketch the line.

c Use your sketch to find the gradient of the line.

- 20** Match each of these equations with one of the graphs below.

- $y = -5x + 4$
- $y = 5x + 4$
- $y = -5x - 4$
- $y = 5x - 4$



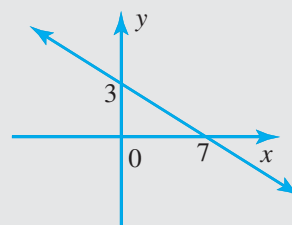
- 21** Determine, by substitution, whether each point lies on the line $9x - 2y + 5 = 0$.

a $(3, 16)$ **b** $(-1, -7)$

- 22 a** Find the value of k given that the point $(-4, -3)$ lies on the line $2x + ky - 7 = 0$.

b Hence, find the gradient of the line.

- 23** Find the equation of this line in the gradient-intercept form.



- 24** Find, by inspection, the equation of the line that passes through each set of points.

a

x	-2	-1	0	1	2
y	-5	-4	-3	-2	-1

b

x	-2	-1	0	1	2
y	1	3	5	7	9

c

x	-1	0	1	2	3
y	8	7	6	5	4

d

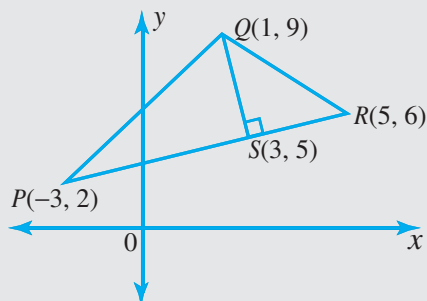
x	-3	-2	-1	0	1
y	3	1	-1	-3	-5

e

x	-2	0	2	4	6
y	0	1	2	3	4

- 25** Find, correct to 1 decimal place, the gradient of a line that is inclined to the x -axis at an angle of:
a 36° **b** 50° **c** 72°
- 26** Find, correct to the nearest degree, the angle at which a line is inclined to the x -axis if it has a gradient of:
a $\frac{1}{6}$ **b** $\frac{3}{2}$ **c** 5.4
- 27** Write down the equation of the line that is parallel to the line:
a $x = 2$ and passes through $(-3, 5)$
b $y = -1$ and passes through $(1, -4)$
- 28** Find, in the general form, the equation of the line that passes through the point:
a $(1, 5)$ and has a gradient of -3
b $(7, -4)$ and has a slope of $\frac{1}{2}$
c $(-3, 2)$ and has a gradient of $-\frac{5}{4}$
- 29** Find, in the general form, the equation of the line that passes through the points:
a $(1, 6)$ and $(5, 14)$
b $(11, 1)$ and $(3, -5)$
c $(-4, 2)$ and $(-2, -3)$
- 30** Show that the lines:
a $y = \frac{1}{3}x - 2$ and $x - 3y + 5 = 0$ are parallel.
b $6x - 4y - 3 = 0$ and $2x + 3y = 7$ are perpendicular.
- 31** Find the equation of the line that is:
a parallel to $y = \frac{1}{2}x + 3$ and cuts the y -axis at -2
b parallel to $y = 5 - 4x$ and passes through $(-2, 5)$
c perpendicular to $y = \frac{3}{4}x - 1$ and passes through the origin
- d** perpendicular to $y = \frac{1}{6}x + 2$ and passes through $(3, -4)$
- 32** Find the value of k if the lines:
a $y = \frac{k}{2}x - 7$ and $6x - 2y + 9 = 0$ are parallel
b $3x - 4y = 10$ and $5x + ky - 1 = 0$ are perpendicular.
- 33** Sketch the following regions on separate number planes.
a $x > 3$ **b** $y \leq 2$
c $y \geq x + 1$ **d** $y < 2x - 3$
e $y > 6 - 2x$ **f** $3x + 2y \leq 6$
- 34** On a number plane, shade the region where the inequalities $x + y < 8$ and $y \geq 2x - 1$ hold simultaneously.
- 35** Shade the region of the number plane enclosed by the lines $x = 0$, $y = x - 3$, $x + y = 5$, then find its area.
- 36** Find the co-ordinates of D , the fourth vertex of the parallelogram $ABCD$ given that $A(-6, -3)$, $B(1, -8)$, $C(3, -2)$ and D is a point in the second quadrant.
- 37** Find the equation of the perpendicular bisector of the interval AB , where $A(-3, 2)$ and $B(5, 8)$. Give your answer in the general form.
- 38** Show that the points $L(-2, -13)$, $M(3, 2)$, $N(7, 14)$ are collinear.
- 39** Show that the lines $y = 2x - 7$, $5x + 2y = 4$, $4x - 3y - 17 = 0$ are concurrent.
- 40** Show that $W(4, 8)$, $X(7, 2)$, $Y(1, -1)$, $Z(-2, 5)$ are the vertices of a square.

- 41 A triangle has vertices $P(-3, 2)$, $Q(1, 9)$ and $R(5, 6)$. S is a point on PR such that $QS \perp PR$.



- a Find the gradient of PR .
- b Find, in the general form, the equation of PR .
- c Write down the gradient of QS .
- d Find, in the gradient-intercept form, the equation of QS .
- e Find the co-ordinates of S .
- f Find the distance PR , in the simplest form.
- g Find the distance QS , in the simplest form.
- h Hence, find the area of $\triangle PQR$.

Answers

1 Rational numbers

Exercise 1.1

1 a 2, b 4, c 3, d 6, e 3, f 4, g 4, h 5 2 a 3, b 4, c 3, d 6, e 1, f 1, g 3, h 4, i 4, j 5, k 6, l 3, m 3, n 2, o 5, p 4, q 4, r 6, s 8, t 7 3 a 80, b 30, c 70, d 90, e 100, f 400, g 300, h 4000, i 9000, j 30 000, k 500 000, l 900 000 4 a 130, b 170, c 230, d 680, e 1500, f 4000, g 8400, h 12 000, i 46 000, j 76 000, k 290 000, l 640 000 5 a 4, b 0.5, c 0.073, d 6.209, e 12, f 0.30, g 25.2, h 49.066, i 91.0, j 140, k 7.386, l 11, m 2020, n 3700, o 4002.1, p 9187.55 6 a i 100, ii 99, iii 99.4, b i 200, ii 190, iii 195, c i 1000, ii 1000, iii 999, d i 500, ii 500, iii 500

Exercise 1.2

1 a 105.472, b 99.44, c 105.655, 105.5307, 105.46855, d The answer is less accurate. 2 a 0.47, b 4.98, c 12.14 3 a -254, b -94, c 13 4 a 60.84, b 161.3, c 23.77 5 a 4.8, b 93.5, c 156.1, d 4.1, e 1.6, f 15.7 6 a 274.6, b 191.5, c 1130.6, d 137.8, e 318.9, f 5987.9 7 a 1.82, b 2.33, c 2.15, d 31.14, e 1.84, f 1.64 8 a 0.143, b 4.08, c 237, d 0.577, e 0.269, f 0.0651 9 a 19.96, b 21.99, c 14.14, d 9.87, e 0.32, f 1.26 10 a 13.4, b 6.5, c 65.2, d 2.6, e 22.5, f 3.9 11 a 941.41, b 1.82, c 3.80, d 17.75, e 12.52, f 12.08, g 5.45, h 3.46, i 602.28, j 1.97, k 3.82, l 0.58, m 9.54, n 6.41, o 6.04, p 35.65, q 0.03, r 1.36, s 547.26, t 25.83, u 627.01, v 1.42, w 1.98, x -116.22 12 a 20.8, b 1.3, c 9276.5, d 4.1, e 1.7, f -3.7, g 1.6, h 9444.8, i 1.1, j 52.9, k 14.1, l 2.2 13 a 3.3, b 4.1, c 76.6, d 9.3, e 26.4, f -3.7

Exercise 1.3

1 a 1500, b 4000, c 7000, d 200 000, e 4, f 30, g 100, h 400, i 10 000, j 50, k 12 000, l 25
Many answers are possible for Q2–Q7 2 a 40, b 85, c 480, d 100, e 4, f 13, g 480, h 72, i -140 3 a 6, b 1000, c 5 4 a 40, b 177, c 105, d 1600 5 a 8, b 11, c 21, d 10 6 a 5, b 9, c 3, d 5 7 a 15, b 225, c 10, d 0.2 8 120 000 9 a \$120, b \$30 10 a 35 m², b 4, c 140, d \$1600 11 a 2, 3, 2.2, 2.6, b 10, 11, 10.5, 10.2, 10.7 12 b 3.585 13 a 4.322, b 3.262, c 2.861

Exercise 1.4

1 a 0.2̇, b 0.7̇, c 0.64̇, d 0.35̇, e 0.27̇, f 0.91̇, g 0.48̇, h 0.03̇, i 0.146̇, j 0.029̇, k 0.152̇, l 0.698̇, m 1.6̇, n 3.81̇, o 8.274̇, p 13.95̇ 2 a 0.3̇, b 0.1̇, c 0.6̇, d 0.4̇, e 0.09̇, f 0.27̇, g 0.16̇, h 0.13̇, i 0.416̇, j 0.318̇, k 0.83̇, l 0.916̇ 3 a 1.6̇, b yes 4 a 0.142857̇, b 0.714285̇, c 0.076923̇, d 0.307692̇ 5 a 0.1̇, b 0.2̇, 0.5̇, 0.7̇, 0.8̇, c 1 6 a $\frac{2}{9}$, b $\frac{7}{9}$, c $\frac{1}{3}$, d $\frac{2}{3}$, e $\frac{19}{99}$, f $\frac{35}{99}$, g $\frac{3}{11}$, h $\frac{25}{33}$, i $\frac{7}{45}$, j $\frac{22}{45}$, k $\frac{11}{15}$, l $\frac{17}{18}$, m $2\frac{1}{9}$, n $1\frac{20}{33}$, o $7\frac{5}{6}$, p $3\frac{5}{12}$ 7 a 0.3̇, b 0.03̇, 0.003̇ 8 a $\frac{11}{30} = 0.3\bar{6}$, $\frac{1}{6} = 0.1\bar{6}$
9 a 0.6̇, b 0.06̇

Exercise 1.5

1 a 10 m/s, **b** 40 km/h, **c** 9 L/min, **d** 6 kg/m², **e** 8 g/s, **f** 25 trees/h, **g** 12 km/L, **h** \$45/h, **i** 18c/min, **j** \$1.50/kg, **k** 3.5 runs/over, **l** 37.5 crates/day, **m** 96 beats/min, **n** 48 kL/h, **o** 10.4 km/L **2** a 180 cm/min, **b** 300 g/h, **c** \$2300/t, **d** 180 L/day, **e** 54 km/h, **f** 4000 kg/ha **3** a 2000 mL/min, **b** 900 cm/s, **c** 38 mm/s, **d** 115c/g, **e** 14 600 kg/day, **f** 23 500 m²/week **4** a 7 cm/s, **b** 8.5 m/min, **c** 4.9 L/day, **d** \$0.24/min, **e** 0.025 kg/m³, **f** 59.6 kL/year **5** a 45 m/min, **b** \$80/kg, **c** 9 km/L, **d** 3.6 t/day, **e** 1.95 L/day, **f** 8.52 km/h **6** a 90 km/h, **b** 144 L/h, **c** 99 kg/h, **d** 7.2 m/day, **e** 1.152 km/day, **f** \$24/m, **g** 20 m/s, **h** 8.5 kg/min **7** a 1%, **b** 0.5%, **c** 1.5%, **d** 0.35% **8** a 9% pa, **b** 10.8% pa, **c** 15% pa **9** 0.042% **10** a \$1040/month, **b** \$624/fortnight, **c** \$36.40/quarter, **d** \$6.80/fortnight **11** a \$500/m², **b** 60 000 L/km², **c** 1.2 t/m³ **12** 2.5 c/cm³

Exercise 1.6

1 a 18, **b** 45, **c** \$40.91, **d** 8.64 L, **e** 1.2 t **2** a 37, **b** 25 min, **c** 4.86 runs/over, **d** \$80/h, **e** 2 h **3** a 36 km, **b** 25, **c** 400, **d** 15, **e** 18 **4** a \$8.75, **b** 65 min, **c** 26.5 L, **d** $1\frac{1}{4}$ h, **e** 1200, **f** \$0.98 **5** a US\$12.15, **b** 28.18 euro, **c** NZ\$190.52, **d** 140.90 euro, **e** NZ\$653.22, **f** US\$2733.75 **6** a A\$27.56, **b** A\$156.38, **c** A\$195.17, **d** A\$183.71, **e** A\$930.04, **f** A\$1387.51, **g** A\$1285.94, **h** A\$3914.40 **7** a 70 km/h, **b** 280 km/h, **c** 12 km, **d** 1050 km, **e** 4 h, **f** $3\frac{1}{2}$ h **8** a 397.5 km, **b** 51 km/h, **c** 3 h 25 min **9** a 1.852 km/h, **b** 22.5 h, **c** 8445.12 km **10** a $C = \frac{L}{K}$, **b** 9.2 L/100 km, **c** \$1344

Chapter 1 Review

1 a 3, **b** 5, **c** 2, **d** 6 **2** a 80, **b** 200, **c** 4000, **d** 900 000 **3** a 340, **b** 730, **c** 15 000, **d** 370 000 **4** a 200, **b** 4000, **c** 20 000, **d** 570, **e** 2200, **f** 37 000 **5** a 5, **b** 0.072, **c** 3.47, **d** 22.0, **e** 110, **f** 9000 **6** a 3.97, **b** -7.59, **c** 34.11, **d** 11.68, **e** 4.50, **f** 146.17, **g** 2.16, **h** 9.35, **i** 8.38 **7** a 79.90, **b** 46.81, **c** 0.36, **d** 408.66, **e** 0.96, **f** 16.32 **8** a 170, **b** 20, **c** 1000 **9** a 0.3, **b** 0.25, **c** 0.346, **d** 5.918 **10** a 0.7, **b** 0.36, **c** 1.583 **11** a $\frac{2}{9}$, **b** $\frac{8}{11}$, **c** $\frac{2}{15}$ **12** a 0.016, **b** 0.0016 **13** a 8 m/s, **b** \$5/min, **c** 24 L/h, **d** 43 runs/wicket **14** a 42 L, **b** 93.75 kg **15** a 420 mm/h **b** 1800 km/day, **c** 1350 mL/m², **d** 820 cm/s **16** a 2.4 km/h **b** 6 L/day, **c** 135 kg/ha, **d** 18 km/h **17** a 800 km/h, **b** 4.8 h, **c** 340 km **18** a 82.6 km/h

2**Algebra****Exercise 2.1****1 a**

x	1	2	3	4
y	4	5	6	7

b

x	0	1	2	3
y	5	7	9	11

c

x	5	6	7	8
y	11	14	17	20

d

x	2	3	4	5
y	3	8	13	18

2 They are the same. **3** a $y = 4x$, **b** $y = x + 6$, **c** $y = 2x + 3$, **d** $q = 3p - 1$, **e** $q = 5p + 4$, **f** $q = 7p - 2$, **g** $b = 2a + 9$, **h** $b = 4a + 3$, **i** $b = 6a$, **j** $t = 5s - 8$, **k** $t = 7s + 6$, **l** $t = 12s - 5$

4 a

Number of pentagons (p)	1	2	3
Number of triangles (t)	5	10	15

b $t = 5p$, **c** 45**5 a**

Number of squares (s)	1	2	3
Number of crosses (c)	4	6	8

b $c = 2s + 2$, **c** 42**6 a**

Number of circles (c)	1	2	3
Number of dots (d)	4	7	10

b $d = 3c + 1$, **c** 46**7 a**

Number of rhombuses (r)	1	2	3
Number of dots (d)	0	2	4

b $d = 2r - 2$, **c** 78

8 a

Number of squares (s)	1	2	3
Number of dots (d)	7	11	15

$$b \ d = 4s + 3$$

10 a

Number of circles (c)	3	4	5
Number of dots (d)	20	24	28

$$b \ d = 4c + 8$$

12 a $y = -7x$, **b** $y = -x + 5$, **c** $y = -2x + 9$, **d** $y = -3x + 20$, **e** $y = -x + 6$, **f** $y = -3x + 7$

13 a

Number of squares (s)	1	4	9
Number of dots (d)	4	9	16

$$b \ d = (\sqrt{s} + 1)^2, \ c \ 81$$

9 a

Number of rectangles (r)	3	4	5
Number of dots (d)	14	19	24

$$b \ d = 5r - 1$$

11 a

Number of crosses (c)	2	3	4
Number of dots (d)	14	19	24

$$b \ d = 5c + 4$$

14 a

Number of cans in base (b)	1	2	3
Total number of cans (c)	1	3	6

$$b \ c = \frac{b(b+1)}{2}, \ c \ 55$$

Exercise 2.2

1 a 12, **b** 3, **c** 8, **d** 15, **e** 43, **f** 37, **g** 10, **h** 25, **i** 125, **j** 75, **k** 40, **l** 15, **m** 8, **n** $\frac{1}{3}$, **o** 3, **p** 4 **2 a** 12, **b** 13, **c** 39, **d** 29, **e** 11, **f** 8, **g** 28, **h** 56, **i** 19, **j** 1, **k** 40, **l** 36, **m** 111, **n** -29, **o** 63, **p** -42, **q** 50, **r** 120, **s** 108, **t** 336, **u** 6, **v** 8, **w** 3, **x** 8 **3 a** -4, **b** -5, **c** 9, **d** -17, **e** -7, **f** -3, **g** 7, **h** -15, **i** -5, **j** -31, **k** -26, **l** -56, **m** -32, **n** -10, **o** -162, **p** 89, **q** -9, **r** -54, **s** -100, **t** -510 **4 a** 5, **b** 14, **c** -9, **d** -11, **e** 11, **f** -1, **g** 17, **h** -17, **i** -22, **j** -19, **k** -21, **l** 69, **m** -72, **n** -66, **o** -12, **p** -36, **q** 22, **r** 72, **s** 192, **t** 12, **u** 10, **v** 4, **w** -5, **x** 5

Exercise 2.3

1 a $10x$, **2 a** $7n$, $7n$, **b** Yes, **c** No **3 a** $2s$, $-2s$, **b** No, **c** Yes **4 a** $9y$, **b** $4n$, **c** $3c$, **d** $6k$, **e** 0, **f** b , **g** $7a^2$, **h** $8g^2$, **i** $11pq$, **j** $7xy$, **k** $8abc$, **l** $19m^2n$, **m** $-4t$, **n** $10u$, **o** $-9p$, **p** $-15j$, **q** $-6pq$, **r** $-yz$, **s** $-10e^2$, **t** $-2rs^2$ **5 a** $9a$, **b** $6b$, **c** $10k$, **d** $-7m$, **e** $8p$, **f** $7r$, **g** $-13x$, **h** $-12c$, **i** $-13e^2$, **j** 0, **k** $-3ab$, **l** $4pq$ **6 a** $7q + 2$, **b** $5g + 17$, **c** $8u - 3$, **d** $13 + t$, **e** $18c + d$, **f** $11j - 4k$, **g** $7 - 2a$, **h** $12 - 6n$, **i** $x^2 + 5x$, **j** $m^2 - 2m$, **k** $5w^2 + w$, **l** $4a^2b + 3ab^2$ **7 a** $2k + 5$, **b** $12c + 5$, **c** $9p + 10q$, **d** $9m + n$, **e** $3t + 16$, **f** $5u + 8v$, **g** $14g + 3h$, **h** $7p - 4q$, **i** $-5b - 3c$, **j** 22, **k** $10y$, **l** $-6m - 2n$, **m** $-3x - 6y$, **n** $-a + 12b$, **o** $10j - 16k$, **p** $3x^2 + 9x$, **q** $8a^2 - 3a$, **r** $12u - 5v^2$, **s** $6z^2 - 5z$, **t** $d^2 + 3d + 5$, **u** $mn + 5m - 9n$ **8 a** $20k$, **b** $28n$, **c** $4m + 12$, **d** $x + 11$, **e** $4y + 14$, **f** $7c + 7$ **9 a** $4x^2 + 6x - 14$, **b** $4a^2 + 2a + 14$, **c** $-2p^2 - p$, **d** $3k^2 + 9k - 4$, **e** $t^2 - 7t + 7$

Exercise 2.4

1 a $6ab$, **2 a** $20n^2$, **3 a** 6, **4 a** $15xy$, $15xy$, **b** Yes, **c** No **5 a** $\frac{p}{2}$, $\frac{2}{p}$, **b** No, **c** Yes **6 a** $15n$, **b** $24c$, **c** $63w$, **d** $88g$, **e** $5uv$, **f** $9mn$, **g** $14ab$, **h** $40xy$, **i** $36cd$, **j** $70rs$, **k** $60pq$, **l** $81vw$, **m** a^2 , **n** $2e^2$, **o** $12k^2$, **p** $30h^2$, **q** m^2np , **r** $42c^2d$, **s** $20fg^2h$, **t** $32vw^2x$, **u** $7a$, **v** $4mn$, **w** $18pqr$, **x** $10c^2d$ **7 a** $5b$, **b** $3z$, **c** $6k$, **d** $8m$, **e** 6, **f** 8, **g** a , **h** q , **i** $10g$, **j** $7n$, **k** $10x$, **l** $6e$, **m** t , **n** $13v$, **o** u , **p** $3a$, **q** $8m$, **r** $9e$, **s** ab , **t** $5s$ **8 a** $-21y$, **b** $40x$, **c** $-48gh$, **d** $10bc$, **e** j^2 , **f** $-27v^2$, **g** $-35ab^2$, **h** $96xy^2z$ **9 a** $-4c$, **b** $7n$, **c** -3 , **d** $-9f$, **e** $7n$, **f** $-9k$, **g** $5t$, **h** $-12u$ **10 a** $6abc$, **b** $28mnp$, **c** $2ef$, **d** 2, **e** 4, **f** 5, **g** $21q^2$, **h** $4n$, **i** $8a$, **j** $9yz$, **k** $60c^2d^2$, **l** 2, **m** $42xy$, **n** $-5q$, **o** $20s^2$ **11 a** 6, **b** $7j$, **c** 4, **d** $30t$, **e** $4y$, **f** $3q$, **g** $5f$, **h** $42mk$, **i** $3a$, **j** $5w^2$, **k** $8h$, **l** $12c$, **m** $8q$, **n** $-5h$, **o** $-27x^2$ **12 a** $\frac{c}{2}$, **b** $\frac{1}{k}$, **c** $\frac{3h}{2}$, **d** $\frac{b}{3}$, **e** $\frac{3m}{5}$, **f** $\frac{2}{3v}$, **g** $\frac{5c}{7e}$, **h** $\frac{6s}{7}$, **i** $\frac{2u}{3v}$, **j** $\frac{7x}{12y}$, **k** $\frac{9a}{11d}$, **l** $\frac{9e}{7f}$

Exercise 2.5

1 a $18n$, **b** $3q$, **c** $26j$, **d** $4x$, **e** $24t^2$, **f** $32s^2$, **g** $60pq$, **h** $20cd$, **i** 3, **j** $4y$, **k** 7, **l** $5c$, **m** $48f$, **n** 10, **o** $4a$ **2 a** $25t$, **b** $7y$, **c** $20g$, **d** $10c$, **e** $15n$, **f** 4, **g** $15r$, **h** $45f^2$, **i** $-19k$ **3 a** $11k$, **b** $13z$, **c** $15n^2$, **d** v^2 , **e** $7ab$, **f** $13q$, **g** $14ef$, **h** $12y$, **i** $4s$, **j** $11a$, **k** $33ab$, **l** $10x$, **m** $27g$, **n** $-9x$, **o** $-9k$ **4 a** 5, **b** 4, **c** $4q$, **d** 3 **5 a** $4 \times (2s + 3s) = 20s$, **b** $40pq \div (5p \times 2q) = 4$, **c** $16a - (4a + 2a) - 7a = 3a$, **d** $(24e^2 - 6e^2) \div 6e = 3e$, **e** $(8 \times 4n) - (5n \times 3) = 17n$, **f** $8w + (9w^2 \times 6 \div 3w) = 26w$

Exercise 2.6

1 **a** $3a + 12$, **b** $5p - 10$, **c** $7m + 7$, **d** $40 - 8k$, **e** $20h + 28$, **f** $12y - 18$, **g** $15m + 35n$, **h** $18y - 20z$, **i** $ab + ac$, **j** $pq - pr$, **k** $2ef + eg$, **l** $4km - 11kn$, **m** $3tu + 3tv$, **n** $18km - 24k$, **o** $20fg - 28fh$, **p** $36rs + 60rt$, **q** $x^2 + xy$, **r** $b - b^2$, **s** $14n^2 - 49n$, **t** $27v^2w - 72vw^2$ 2 **a** $-2n - 14$, **b** $-3b + 18$, **c** $-9k + 9$, **d** $-88 - 11u$, **e** $-10j - 45$, **f** $-42 + 60y$, **g** $-xy - xz$, **h** $-3tu + tv$, **i** $-5cd - 2ce$, **j** $-2np - 2nq$, **k** $-45rs + 27r$, **l** $-24hi + 66hj$, **m** $-s^2 + st$, **n** $-j - j^2$, **o** $-30y^2 + 72y$, **p** $-8m^2n - 20mn^2$ 3 **a** $6x + 30$, **b** $7j - 14$, **c** $km + 8m$, **d** $8p + 12$, **e** $cd - d^2$, **f** $15ac + 35bc$, **g** $20s^2 - 8st$, **h** $6m^2n + 16mn^2$ 4 **a** $5n + 41$, **b** $7c + 20$, **c** $6q + 13$, **d** $36 + 7t$, **e** $-3m + 4$, **f** $-14n + 16$, **g** $6a + 24$, **h** $4 + 12x$, **i** $-10q - 30$, **j** $20m - 88$, **k** $15 - 2x$, **l** $29c - 6$, **m** $11m + 60$, **n** $5k + 1$, **o** $6x + 27$, **p** $25t + 6$, **q** $7y + 29$, **r** $w + 3$ 5 **a** $8n + 22$, **b** $10z + 22$, **c** $15p + 10$, **d** $7w$, **e** $x + 27$, **f** $-4n + 11$, **g** $16a + 33$, **h** $-10s + 26$, **i** $25b + 18$, **j** $33c - 44$, **k** $-11y + 9$, **l** -132 , **m** $x^2 + 8x + 27$, **n** $y^2 + 4y - 42$, **o** $5a^2 + 26a$, **p** $-2g^2 + 24g$, **q** $13u^2 - 51u$, **r** $35cd + 30ce$ 6 **a** True, **b** False, **c** False, **d** False, **e** True, **f** True 7 **a** $15a + 20$, **b** $22m^2n - 8mn^2$, **c** $3k^2 + 24k$, **d** $14vw - 35v$

Exercise 2.7

1 **a** $n + 3$, **b** $p - 5$, **c** $y + 1$, **d** $2g + 5$, **e** $4a - 3$, **f** $3k - 5m$, **g** $x + y$, **h** $p - r$, **i** $s - 1$, **j** $m + 3$, **k** $4 - r$, **l** $a + b$, **m** $d + 2$, **n** $4p - 7$, **o** $5z + 4y$ 2 **a** $2(c + 4)$, **b** $5(y + 2)$, **c** $3(6 + q)$, **d** $7(5 + p)$, **e** $2(h - 7)$, **f** $6(t - 5)$, **g** $3(11 - r)$, **h** $4(12 - n)$, **i** $5(c + d)$, **j** $3(x - 2y)$, **k** $7(3g + h)$, **l** $8(m - 5n)$, **m** $a(b + c)$, **n** $u(v - w)$, **o** $f(e - g)$, **p** $r(s - 1)$, **q** $b(b + c)$, **r** $k(k - 8)$, **s** $n(11 + n)$, **t** $a(1 - a)$ 3 **a** $3(2n + 3)$, **b** $5(2b + 5)$, **c** $2(5y + 6)$, **d** $4(3k - 2)$, **e** $7(3w - 5)$, **f** $3(6s - 7)$, **g** $8(2a + 3)$, **h** $6(3t - 5)$, **i** $3(10p + 9)$, **j** $7(2c + 7)$, **k** $10(3r - 8)$, **l** $11(2e - 9)$, **m** $5(7 - 11h)$, **n** $9(10 + 7v)$, **o** $13(3 + 2z)$, **p** $12(2 - 5j)$ 4 **a** $3b(a + 3c)$, **b** $2x(y + 4z)$, **c** $4q(p - 5r)$, **d** $7h(g - 2i)$, **e** $2u(2v + 3w)$, **f** $4f(2e + 5g)$, **g** $11r(3s - 7q)$, **h** $4m(6n - 5p)$, **i** $7c(c + 3)$, **j** $6w(4w - 1)$, **k** $2g(5g - 11)$, **l** $5y(3 + 8y)$, **m** $mn(p + q)$, **n** $rt(s - u)$, **o** $ab(a + b)$, **p** $de(f - e)$, **q** $jk(j - km)$, **r** $3u(4t + 5uv)$, **s** $2ab(2b + 5ac)$, **t** $7xy(7xy - 6z)$ 5 **a** $3(a + b + c)$, **b** $p(q + r - s)$, **c** $a(a - b - c)$, **d** $5(r + 2s + 5)$, **e** $2x(2x - 5 + 4y)$, **f** $6(1 + 4u - 3u^2)$, **g** $7(6k^2 - 2k + 3)$, **h** $m(3n - 1 + n^2)$, **i** $2x(x + y - 3)$, **j** $5t(6 - 3u + 2t)$, **k** $4c(d + 7c - 5e)$, **l** $7f(3 - 10g - 8f)$, **m** $ab(a + b + 1)$, **n** $pq(8 - p + q)$, **o** $uvw(u - v - w)$ 6 **a** HCF is 4, not 2, **b** $p^2 = p \times p$, not $p \times 2$, **c** $e = e \times 1$, not $e \times 0$, **d** b is a common factor also, **e** $14u$ has not been divided by $7u$, **f** 3 is a common factor of 15 also 7 **a** $-2(p + 6)$, **b** $-3(x + 7)$, **c** $-5(3g + 4)$, **d** $-7(2u + 7)$, **e** $-2(t - 1)$, **f** $-8(w - 3)$, **g** $-4(3k - 4)$, **h** $-3(3r - 10)$, **i** $-3(8 + 5m)$, **j** $-9(2 - 5q)$, **k** $-12(3 - 2y)$, **l** $-7(9 + 11c)$, **m** $-b(a - c)$, **n** $-m(n + k)$, **o** $-x(x + 2)$, **p** $-e(4 - e)$, **q** $-3k(3k - 4)$, **r** $-4a(5 + 7a)$, **s** $-5b(5 - 11c)$, **t** $-12y(4x^2 + 5y)$ 8 **a** $(b + c)(a + 5)$, **b** $(x - y)(m + n)$, **c** $(p + 3)(p + 4)$, **d** $(a + 1)(x - 2)$, **e** $(m - 7)(3 - n)$, **f** $(p + q)(a^2 - 6)$, **g** $(c + 4)(5c + 2)$, **h** $(1 - k)(8 - 3m)$, **i** $(2s + 3)(y - z)$, **j** $(3w - 5)(4g + 9h)$, **k** $(x - 7)(x + 1)$, **l** $(7b + 2c)(1 - 3d)$

Exercise 2.8

1 **a** $\frac{5a}{7}$, **b** $\frac{4m}{9}$, **c** $\frac{h}{13}$, **d** $\frac{x}{2}$, **e** $\frac{3n}{4}$, **f** $\frac{2k}{3}$, **g** c , **h** $\frac{6d}{5}$, **i** $2b$, **j** $\frac{2w}{3}$, **k** $\frac{5e}{12}$, **l** $\frac{11s}{8}$ 2 **a** $\frac{7}{x}$, **b** $\frac{1}{p}$, **c** $\frac{14}{3y}$, **d** $\frac{9}{7q}$, **e** $\frac{1}{n}$, **f** $\frac{2}{c}$, **g** $\frac{4}{g}$, **h** $\frac{3}{4k}$, **i** $\frac{4a}{5r}$, **j** $\frac{2m}{5b}$, **k** $\frac{3e}{4v}$, **l** $\frac{t}{4z}$ 3 **a** $\frac{3n}{4}$, **b** $\frac{2a}{9}$, **c** $\frac{5k}{12}$, **d** $\frac{2d}{15}$, **e** $\frac{7y}{10}$, **f** $\frac{t}{12}$, **g** $\frac{3b}{28}$, **h** $\frac{17h}{60}$, **i** $\frac{7c}{10}$, **j** $\frac{m}{12}$, **k** $\frac{29r}{15}$, **l** $\frac{9u}{14}$, **m** $\frac{13w}{12}$, **n** $\frac{17x}{18}$, **o** $\frac{37f}{40}$, **p** $\frac{s}{36}$ 5 **a** $\frac{5}{2x}$, **b** $\frac{5}{6a}$, **c** $\frac{9}{20e}$, **d** $\frac{5}{12p}$, **e** $\frac{13}{6u}$, **f** $\frac{1}{20f}$, **g** $\frac{2}{21t}$, **h** $\frac{52}{45h}$, **i** $\frac{37c}{12j}$, **j** $\frac{7m}{24z}$, **k** $\frac{3a}{20g}$, **l** $\frac{31k}{24n}$ 6 **a** $\frac{4n + 7}{6}$, **b** $\frac{11b + 37}{28}$, **c** $\frac{8x + 14}{15}$, **d** $\frac{13m + 15}{42}$, **e** $\frac{5w - 8}{12}$, **f** $\frac{33s - 53}{45}$, **g** $\frac{x + 11}{4}$, **h** $\frac{7c + 55}{20}$, **i** $\frac{5e + 37}{24}$

Exercise 2.9

1 **a** $\frac{ab}{6}$, **b** $\frac{u^2}{12}$, **c** $\frac{ac}{bd}$, **d** $\frac{1}{pq}$, **e** $\frac{1}{4x^2}$, **f** $\frac{4cd}{15}$, **g** $\frac{27mn}{28}$, **h** $\frac{35}{48x^2}$ 2 **a** $\frac{xy}{20}$, **b** $\frac{v^2}{12}$, **c** $\frac{tw}{uv}$, **d** $\frac{h}{g}$, **e** $\frac{1}{2s^2}$, **f** $\frac{18e}{35f}$, **g** $\frac{40a}{33b}$, **h** $\frac{52}{15h^2}$ 3 **a** $\frac{2}{3}$, **b** $\frac{2a}{b}$, **c** $\frac{d}{5c}$, **d** $\frac{6}{7}$, **e** $\frac{ad}{e}$, **f** $\frac{1}{6}$, **g** $\frac{3d}{ce}$, **h** $\frac{1}{6}$, **i** $\frac{1}{4v}$, **j** $\frac{2x}{5y}$, **k** $\frac{7}{8j}$, **l** $\frac{8}{63qs}$ 4 **a** $\frac{3}{5}$

$$\begin{aligned} & \mathbf{b} \frac{3m}{n}, \mathbf{c} \frac{v}{4u}, \mathbf{d} \frac{18}{11s}, \mathbf{e} \frac{fi}{g}, \mathbf{f} \frac{8k}{m}, \mathbf{g} \frac{1}{28}, \mathbf{h} \frac{1}{12}, \mathbf{i} \frac{1}{15q}, \mathbf{j} \frac{5a}{12b}, \mathbf{k} \frac{33}{35f}, \mathbf{l} \frac{72w}{77z} \quad \mathbf{6} \mathbf{a} pq, \mathbf{b} \frac{5}{6mp}, \mathbf{c} \frac{14a}{15b}, \\ & \mathbf{d} \frac{b}{p}, \mathbf{e} \frac{rv}{u}, \mathbf{f} \frac{5}{9ef}, \mathbf{g} \frac{9cx}{35ay}, \mathbf{h} \frac{10ux}{9vt} \quad \mathbf{7} \mathbf{a} \frac{f}{g}, \mathbf{b} \frac{3p}{8q}, \mathbf{c} \frac{t}{8u}, \mathbf{d} \frac{6x}{35}, \mathbf{e} \frac{11ae}{36}, \mathbf{f} \frac{10pq}{21} \quad \mathbf{8} \mathbf{a} 2, \mathbf{b} \frac{m+6}{3}, \\ & \mathbf{c} \frac{10m}{9}, \mathbf{d} \frac{u+4}{u}, \mathbf{e} \frac{c+2}{6}, \mathbf{f} \frac{2a}{15}, \mathbf{g} \frac{10a}{21b}, \mathbf{h} \frac{1}{3}, \mathbf{i} \frac{b+5}{b-5} \end{aligned}$$

Exercise 2.10

1 $\mathbf{a} x+3$, $\mathbf{b} t-5$, $\mathbf{c} p+q$, $\mathbf{d} m-n$, $\mathbf{e} x+y+7$, $\mathbf{f} 4m$, $\mathbf{g} 9n$, $\mathbf{h} 2ab$, $\mathbf{i} \frac{k}{2}$, $\mathbf{j} \frac{z}{4}$, $\mathbf{k} \frac{2w}{3}$, $\mathbf{l} \frac{u}{v}$, $\mathbf{m} \frac{4}{j}$, $\mathbf{n} k^2$, $\mathbf{o} y^3$,
 $\mathbf{p} \sqrt{g}$ **2** $\mathbf{a}$ 4 more than n , $\mathbf{b}$ 6 less than q , $\mathbf{c}$ the sum of c and d , $\mathbf{d}$ the difference between x and y ,
 $\mathbf{e}$ the product of 8 and u , $\mathbf{f}$ the product of 5, e and f , $\mathbf{g}$ one-third of h $\mathbf{h}$ three-quarters of v ,
 $\mathbf{i}$ the quotient of m and n , $\mathbf{j}$ the square of a , $\mathbf{k}$ the cube of g , $\mathbf{l}$ the square root of d **3** $\mathbf{a} 2x+3$, $\mathbf{b} 5y-1$,
 $\mathbf{c} pq+7$, $\mathbf{d} 4-u^2$, $\mathbf{e} \frac{c}{2}+6$, $\mathbf{f} \frac{w}{5}-9$, $\mathbf{g} \frac{e}{f}+2$, $\mathbf{h} \frac{7r}{10}-4$, $\mathbf{i} \frac{b+1}{3}$, $\mathbf{j} \frac{g-h}{2}$, $\mathbf{k} 3(a+12)$, $\mathbf{l} 9(p-3)$, $\mathbf{m} 4(c+d)$,
 $\mathbf{n} 10(r-s)$, $\mathbf{o} 2y^2$, $\mathbf{p} 8x^3$, $\mathbf{q} \frac{5}{j^2}$, $\mathbf{r} \frac{b^3}{2}+1$ **4** $\mathbf{a}$ 7 more than the product of 5 and x , $\mathbf{b}$ 3 less than the product of
2 and n , $\mathbf{c}$ 4 more than the product of g and h , $\mathbf{d}$ the difference between 9 and the product of p and q , $\mathbf{e}$ one-quarter
of 3 more than a , $\mathbf{f}$ 8 more than one-sixth of b , $\mathbf{g}$ one-seventh of the difference between m and n , $\mathbf{h}$ the difference
between u and the quotient of v and w , $\mathbf{i}$ the product of 5 and 2 more than e , $\mathbf{j}$ two-thirds of the difference between
 c and d , $\mathbf{k}$ 3 times the square of r , $\mathbf{l}$ 9 less than twice the cube of s **5** $\mathbf{a} t+7$, $\mathbf{b} p+9$, $\mathbf{c} 3k-10$, $\mathbf{d} 7y+2$
6 $\mathbf{a} 3, 4, 5$, $\mathbf{b} a, a+1, a+2$, $\mathbf{c} x^2, x^2+1, x^2+2$, $\mathbf{d} t+5, t+6, t+7$, $\mathbf{e} p-11, p-10, p-9$, $\mathbf{f} k-1, k$,
 $k+1$, $\mathbf{g} 2n-2, 2n-1, 2n$, $\mathbf{h} 2-u, 3-u, 4-u$ **7** $\mathbf{a} 6, 8, 10$, $\mathbf{b} n, n+2, n+4$, $\mathbf{c} p+8, p+10, p+12$, $\mathbf{d} x-5$,
 $x-3, x-1$, $\mathbf{e} g-2, g, g+2$ **8** $\mathbf{a} 3, 5, 7$, $\mathbf{b} k, k+2, k+4$, $\mathbf{c} y+7, y+9, y+11$, $\mathbf{d} c-12, c-10, c-8$, $\mathbf{e} s-3$,
 $s-1, s+1$ **9** $\mathbf{a} m-1, m, m+1$, $\mathbf{b} w-2, w-1, w$ **10** $\mathbf{a} k+3$, $\mathbf{b} \$(d-4)$, $\mathbf{c} 10c$, $\mathbf{d} \$\frac{x}{6}$ **11** $\mathbf{a} \$\left(\frac{pq}{100}\right)$,
 $\mathbf{b} \$\left(p-\frac{pq}{100}\right)$ **12** $\frac{u+v}{2}$ **13** $\mathbf{a} (180-p-q)^\circ$, $\mathbf{b} (360-a-b-c)^\circ$ **14** $\mathbf{a} (9+t)$ years old, $\mathbf{b} (16-j)$ years old
15 $\mathbf{a} (x+y)$ years, $\mathbf{b} i (w-x)$ years, $\mathbf{ii} (w-x-y)$ years **16** $\mathbf{a} 3f$ cm, $\mathbf{b} 4n$ cm **17** $\mathbf{a} 10x$ mm, $\mathbf{b} 100y$ cm,
 $\mathbf{c} 1000p$ m, $\mathbf{d} \frac{a}{100}$ m, $\mathbf{e} \frac{q}{1000}$ km, $\mathbf{f} \frac{t}{10}$ cm, $\mathbf{g} 100d$ cents, $\mathbf{h} \$\frac{b}{100}$, $\mathbf{i} 60k$ min, $\mathbf{j} \frac{m}{60}$ min, $\mathbf{k} 1000r$ mL,
 $\mathbf{l} \frac{e}{1000}$ kg **18** $\mathbf{a} 50+v$, $\mathbf{b} 1000x+150$, $\mathbf{c} 60a+b$, $\mathbf{d} 100w+p$ **19** $(500-12z)$ cm **20** $\mathbf{a} (50-2y)$ m,
 $\mathbf{b} (50y-2y^2)$ m² **21** $\$(7c+3d)$ **22** $\left(\$ \frac{26}{25} t\right)$ **23** $\mathbf{a} \frac{m}{n}$ km/h, $\mathbf{b} bh$ km, $\mathbf{c} \frac{p}{s}$ h **24** $\mathbf{a}$ Jit, $\mathbf{b} \$\frac{bc}{a+b}$
25 $\left(\frac{6-x}{2}\right)$ cm **26** $\mathbf{a} \$\frac{ck}{3}$, $\mathbf{b} \$\frac{9m}{k}$, $\mathbf{c} \frac{5w}{v}$ kg

Exercise 2.11

1 $\mathbf{a} +1, a+2, a+3$ **2** $\mathbf{a} x+2, x+4, x+6$, $\mathbf{b} x+2, x+4, x+6$, $\mathbf{c} x+3, x+6, x+9$, $\mathbf{d} x+7, x+14, x+21$
3 $\mathbf{a}$ even, $\mathbf{b}$ odd, $\mathbf{c}$ even, $\mathbf{d}$ even, $\mathbf{e}$ odd, $\mathbf{f}$ odd **6** no, she needs to state that $m > n$. **8** no, $12 \div 4$ is not even
9 $\mathbf{c}$ no, $\mathbf{d}$ no **11** $\mathbf{a} 1, 2, 5$, $\mathbf{b} 1, 2, 3, 6, 9$ **12** $\mathbf{a}$ yes, $\mathbf{b}$ yes, $\mathbf{c}$ no, $\mathbf{d}$ no **13** $\mathbf{a}$ the numbers are 'relatively prime',
that is they have no common divisor (apart from 1), $\mathbf{b}$ they are relatively prime

Exercise 2.12

1 a

n	1	2	3	4
l	4	6	8	10

$\mathbf{b}$ the minimum number of lines is equal
to double the number of rows of squares
plus two, $\mathbf{c} l=2n+2$, $\mathbf{d} 62$

2 a

f	1	2	3	4
c	1	3	7	15

$\mathbf{b} c=2^f-1$, $\mathbf{c} 127$

3 **a** x^2 , **b** $\frac{x^2}{2}$ if x is even, $\frac{x^2+1}{2}$ if x is odd, **c** 1250 **4 a** t^2 , **b** $\frac{t(t+1)}{2}$, **c** 45

5 **a**

x	1	2	3	4
N	1	5	14	30

b the total number of squares equals the sum of all square numbers up to and including x^2 , **c** $N = 1^2 + 2^2 + 3^2 + \dots + x^2$, **d** 204

6 **a**

Number of rows (n)	1	2	3	4
Number of cubes in bottom layer	1	3	6	10
Number of cubes in second layer	0	1	3	6
Total number of cubes	1	4	9	16

7 **a**

Number of lines (l)	1	2	3	4
Number of regions (r)	2	4	7	11

b 56

c n^2 , **d** $\frac{n(n-1)}{2}$, **e** 300

8 **a** 1, 9, 25, 49, ..., **b** $(2n-1)^2$, **c** 361, **d** 1330 **9** if n is even $T = \frac{n^2+2n}{2}$; if n is odd and $n+1$ is divisible by 4, $T = \frac{(n+1)^2}{2}$; if n is odd and $n-1$ is divisible by 4, $T = \frac{(n-1)^2}{2}$

10 **a** 68, **b** if n is even, $S = 4(n-1) + \frac{(n-2)^2}{2}$; if n is odd, $S = 4(n-1) + \frac{(n-2)^2-1}{2}$

Exercise 2.13

1 **a** $x^2+7x+10$, **b** $a^2+7a+12$, **c** $n^2+13n+42$ **2** $x^2+4x-12$ **3 a** $xy+2x+3y+6$, **b** $pq+4p+7q+28$,
c $gh+6g+h+6$, **d** $mn-2m+5n-10$, **e** $uv+6u-10v-60$, **f** $jk-4j-3k+12$, **g** $2xy+14x+3y+21$,
h $3pq-12p+q-4$, **i** $5ab-15a-6b+18$, **j** $6mn+27m+4n+18$, **k** $12jk-18j+10k-15$, **l** $24uv-40u-27v+45$
4 a x^2+5x+6 , **b** $x^2+9x+20$, **c** $x^2+4x-60$, **d** $x^2+3x-40$, **e** x^2-8x-9 , **f** x^2-x-12 , **g** $x^2-10x+24$,
h $x^2-7x+10$, **i** x^2-49 **5 a** a^2+6a+8 , **b** $y^2+8y+15$, **c** m^2+8m+7 , **d** $p^2+2p-35$, **e** $t^2+7t-18$,
f $w^2+2w-24$, **g** k^2-2k-8 , **h** u^2-6u-7 , **i** $j^2-4j-60$, **j** z^2-3z+2 , **k** $n^2-11n+28$, **l** $q^2-19q+88$
6 a $2x^2+5x+3$, **b** $3a^2+10a+8$, **c** $4p^2+29p+7$, **d** $8m^2+22m+15$, **e** $10k^2+43k+28$, **f** $6w^2+31w+18$,
g $5t^2+8t-4$, **h** $2y^2+13y-24$, **i** $3h^2-19h+20$, **j** $6u^2-17u-3$, **k** $7b^2-30b+8$, **l** $2n^2+5n-33$,
m $8s^2+18s+7$, **n** $12j^2+47j+40$, **o** $15q^2-34q+15$, **p** $5x^2+23x+12$, **q** $48+22r-5r^2$, **r** $60e^2-92e+35$,
s $18c^2-19c-12$, **t** $10-9k-36k^2$, **u** $9+16g-4g^2$ **8 a** x^2+5x+4 , **b** $x^2+8x+15$, **c** x^2+6x+8 ,
d $x^2+7x+10$, **e** $x^2+9x+18$, **f** $x^2+12x+20$, **g** x^2-4x+3 , **h** $x^2-9x+20$, **i** $x^2-9x+14$, **j** $x^2-12x+27$,
k $x^2-14x+40$, **l** $x^2-13x+42$, **m** $x^2+3x-10$, **n** $x^2-4x-21$, **o** $x^2-4x-12$, **p** $x^2-2x-15$, **q** $x^2-4x-32$,
r $x^2-7x-30$, **s** $x^2+9x-22$, **t** $x^2+4x-45$, **u** $x^2-3x-108$ **9 a** $2c^2+10c+12$, **b** $3z^2+6z-24$,
c $4y^2-24y+20$, **d** $42+8v-2v^2$, **e** $2ab^2-9ab+10a$, **f** $6x^3+13x^2-28x$ **10 b** $(5-2z)(z-7)$ **11 a** 156, **b** 323,
c 252 **12** x^3+3x^2+2x **13** $x^2+10x+25$ **14 a** x^3+4x^2+8x+5 , **b** $a^3+2a+12$, **c** $s^3-7s^2+3s+35$,
d $2g^3+7g^2+7g+12$, **e** $2e^3+3e^2-25e+24$, **f** $24k^3-32k^2+4k+5$ **15 a** $a^3+6a^2+11a+6$,
b $n^3+2n^2-29n+42$, **c** $8t^3-38t^2+47t-15$ **16 a** $n+7$, **b** $x-7$, **c** $d+4$, **d** $p-4$

Exercise 2.14

2 **a** $p^2+2pq+q^2$, **b** $m^2+2mn+n^2$, **c** $x^2-2xy+y^2$, **d** $c^2-2cd+d^2$ **3 a** x^2+6x+9 , **b** $m^2+10m+25$,
c k^2-4k+4 , **d** $y^2-14y+49$, **e** $u^2+8u+16$, **f** t^2-2t+1 , **g** $c^2-22c+121$, **h** $b^2+18b+81$, **i** $e^2-12e+36$,
j $p^2+20p+100$, **k** $w^2+16w+64$, **l** $n^2-24n+144$ **4 a** $25+10a+a^2$, **b** $9-6j+j^2$, **c** $121+22y+y^2$,
d $1-2h+h^2$ **5 a** $a^2+26a+169$, **b** $q^2-34q+289$, **c** $484+44r+r^2$, **d** $324-36v+v^2$ **6 a** $y^2+0.4y+0.04$,
b $f^2-1.4f+0.49$, **c** $s^2+s+0.25$, **d** $p^2-2.4p+1.44$ **7 a** $4x^2+12x+9$, **b** $9a^2+30a+25$, **c** $16k^2-8k+1$,
d $25h^2-20h+4$, **e** $9u^2+24u+16$, **f** $36d^2-84d+49$, **g** $4c^2-44c+121$, **h** $16w^2+72w+81$, **i** $64g^2-16g+1$,
j $49p^2+28p+4$, **k** $100y^2+60y+9$, **l** $144f^2-120f+25$, **m** $36+60e+25e^2$, **n** $81-36v+4v^2$, **o** $25-80b+64b^2$,
p $49+168q+144q^2$ **8 a** $a^2b^2+2abc+c^2$, **b** $p^2-2pqr+q^2r^2$, **c** $r^2s^2+2rs^2t+s^2t^2$, **d** $9e^2f^2-24efgh+16g^2h^2$
9 a $2p^2+20p+50$, **b** $-3a^2+24a-48$, **c** $4x^3+28x^2+49x$, **d** $45t^3-60t^2u+20tu^2$ **10 a** $(x+3)^2=x^2+6x+9$,
b $(m-5)^2=m^2-10m+25$, **c** $(c+4)^2=c^2+8c+16$, **d** $(w-7)^2=w^2-14w+49$, **e** $(k+6)^2=k^2+12k+36$,
f $(y-10)^2=y^2-20y+100$, **g** $(u-2)^2=u^2-4u+4$, **h** $(a+9)^2=a^2+18a+81$, **i** $(n+1)^2=n^2+2n+1$,
j $(t+12)^2=t^2+24t+144$, **k** $(p-8)^2=p^2-16p+64$, **l** $(z-11)^2=z^2-22z+121$

- 11 **a** $(3m+4)^2 = 9m^2 + 24m + 16$, **b** $(2e-7)^2 = 4e^2 - 28e + 49$, **c** $(2q+3)^2 = 4q^2 + 12q + 9$,
d $(3h-8)^2 = 9h^2 - 48h + 64$, **e** $(5s+1)^2 = 25s^2 + 10s + 1$, **f** $(4k+5)^2 = 16k^2 + 40k + 25$,
g $(2g+11)^2 = 4g^2 + 44g + 121$, **h** $(6a-5)^2 = 36a^2 - 60a + 25$, **i** $(4j-9)^2 = 16j^2 - 72j + 81$,
j $(11r+12)^2 = 121r^2 + 264r + 144$, **k** $(9b+2)^2 = 81b^2 + 36b + 4$, **l** $(7y-3)^2 = 49y^2 - 42y + 9$ 12 **a** no, **b** yes,
c yes, **d** no, **e** no, **f** no, **g** yes, **h** yes, **i** no, **j** no, **k** yes, **l** yes 13 **a** $z^2 + \frac{2z}{3} + \frac{1}{9}$, **b** $c^2 - \frac{8c}{5} + \frac{16}{25}$,
c $m^2 + 5m + \frac{25}{4}$, **d** $w^2 - \frac{7w}{3} + \frac{49}{36}$, **e** $a^2 + 2 + \frac{1}{a^2}$, **f** $t^2 - 2 + \frac{1}{t^2}$, **g** $4h^2 + 12 + \frac{9}{h^2}$, **h** $\frac{c^2}{25} - 2 + \frac{25}{c^2}$
14 **a** $y+10$, **b** $g-8$, **c** $3j-7$ 15 **a** 10 201, **b** 9801 16 **a** 1225, **b** 11 449, **c** 2401, **d** 784
17 $x^2 + 2 + \frac{1}{x^2}$, $6\frac{1}{4}$, $11\frac{1}{9}$ 18 **i** $p^2 + q^2 + 9 + 2pq + 6p + 6q$, **ii** $x^2 + y^2 + 16 - 2xy - 8x + 8y$,
iii $4f^2 + 9g^2 + 16h^2 + 12fg + 16fh + 24gh$ 19 **a** $(3c+5)^2 = 9c^2 + 30c + 25$, **b** $(7w+4)^2 = 49w^2 + 56w + 16$,
c $(5t-11)^2 = 25t^2 - 110t + 121$, **d** $(12r-7)^2 = 144r^2 - 168r + 49$

Exercise 2.15

- 2 **a** $p^2 - q^2$, **b** $x^2 - y^2$, **c** $e^2 - f^2$, **d** $m^2 - n^2$ 3 **a** $x^2 - 9$, **b** $p^2 - 4$, **c** $y^2 - 25$, **d** $h^2 - 16$, **e** $1 - b^2$, **f** $64 - m^2$,
g $49 - c^2$, **h** $36 - w^2$, **i** $k^2 - 121$, **j** $81 - j^2$, **k** $u^2 - 100$, **l** $144 - d^2$ 4 **a** $t^2 - 169$, **b** $z^2 - 289$, **c** $441 - q^2$,
d $256 - s^2$ 5 **a** $4a^2 - 9$, **b** $25t^2 - 4$, **c** $16k^2 - 1$, **d** $9p^2 - 49$, **e** $64y^2 - 25$, **f** $4n^2 - 81$, **g** $49 - 100x^2$, **h** $16 - 25e^2$,
i $1 - 36g^2$ 6 **a** $16b^2 - c^2$, **b** $j^2 - 4k^2$, **c** $64m^2 - n^2$, **d** $9p^2 - 4q^2$, **e** $16x^2 - 49y^2$, **f** $81c^2 - 25d^2$, **g** $36g^2 - 121h^2$,
h $100u^2 - 9v^2$, **i** $25r^2 - 144s^2$, **j** $49j^2 - 36k^2$, **k** $16e^2 - 81f^2$, **l** $121m^2 - 100n^2$ 7 **a** $5a^2 - 20$, **b** $36 - 4y^2$,
c $-10c^2 + 490$, **d** $ab^2 - ac^2$, **e** $24p^2 - 54q^2$, **f** $2u^3v - 2uv^3$ 8 **a** $p^2q^2 - r^2$, **b** $f^2 - g^2h^2$, **c** $a^2b^2 - c^2d^2$ 9 **a** $x^2 - \frac{1}{4}$,
b $k^2 - \frac{9}{25}$, **c** $m^2 - \frac{121}{16}$, **d** $y^2 - \frac{1}{y^2}$, **e** $9s^2 - \frac{4}{s^2}$, **f** $\frac{a^2}{b^2} - \frac{b^2}{a^2}$ 10 **a** 91, **b** 399, **c** 875 11 **a** $m^3 + 6m^2 - m - 6$,
b $n^3 - 7n^2 - 9n + 63$, **c** $y^3 - 2y^2 - 25y + 50$, **d** $108 + 36p - 3p^2 - p^3$, **e** $16a^3 - 16a^2b - 49ab^2 + 49b^3$,
f $12u^3 + 16u^2v - 75uv^2 - 100v^3$ 12 **a** $a^2 + 2ab + b^2 - c^2$, **b** $a^2 - 2ab + b^2 - c^2$

Exercise 2.16

- 1 **a** $a^2 + 8a + 15$, **b** $m^2 + 12m + 36$, **c** $k^2 - 16$, **d** $2x^2 + 13x + 21$, **e** $u^2 - 25$, **f** $n^2 - 7n - 18$, **g** $b^2 - 14b + 49$,
h $3c^2 - 22c - 16$, **i** $z^2 - 18z + 80$, **j** $12j^2 - 20j + 3$, **k** $9e^2 - 6ef + f^2$, **l** $1 - l^2$, **m** $16p^2 + 56pq + 49q^2$, **n** $4g^2 - 25h^2$,
o $ac + ad + bc + bd$, **p** $4v^2 - 44vw + 121w^2$, **q** $b^2c^2 - 64$, **r** $6m^2 + 7m - 20$ 2 **a** $8n + 10$, **b** $x^2 + 10x + 29$,
c $t^2 - 19$, **d** $g^2 + 9g - 3$, **e** $3k^2 + 10k + 25$, **f** 16 3 **a** $y^2 + 10y + 22$, **b** $e^2 + 15e + 38$, **c** $2x^2 + 12x + 14$,
d $2c^2 + 26c + 89$, **e** $2d^2 + 18d + 34$, **f** $8h^2 + 38h + 4$, **g** $b^2 + 10b - 9$, **h** $4j^2 + 7j - 2$, **i** $2z^2 - 16$, **j** $4r^2 + 24r + 29$,
k $8f^2 + 12fg$, **l** $28t^2 - 11t - 8$ 4 **a** $2x + 15$, **b** $3p^2 - 27p$, **c** $k^2 - 5k + 4$, **d** $n^2 - 12n + 1$, **e** $12a + 72$, **f** $2b - 10$,
g $4u + 36$, **h** $4pq$, **i** $-2b^2 - 2ab$, **j** $2c^2 + 6c + 27$, **k** $24v + 22$, **l** $6k^2 + 43k + 191$, **m** $60q - 74$, **n** $ac + bd - ab - cd$
5 **a** $3a^2 + 12a + 14$, **b** $3n^2 - 14$, **c** $3x^2 + 7x - 12$, **d** $h^2 + 4h - 1$, **e** 1, **f** $x^3 + 4x^2y + 5xy^2 + 2y^3 - x^2 - 2xy - y^2$,
g $2e^3 + 27e + 9$, **h** $2b^3$ 6 **a** $a^3 + 3a^2b + 3ab^2 + b^3$, **b** $a^3 - 3a^2b + 3ab^2 - b^3$, **c** $2a^3 + 6ab^2$ 7 **a** $x^3 + 6x^2 + 12x + 8$,
b $t^3 - 15t^2 + 75t - 125$, **c** $8p^3 + 36p^2 + 54p + 27$, **d** $64x^3 - 144x^2y + 108xy^2 - 27y^3$ 8 $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Chapter 2 Review

- 1 **a** $d = c + 1$, **b** $c = 3p + 1$, **c** $d = 2r + 2$ 2 **a** $y = 2x + 9$, **b** $y = 5x - 2$, **c** $y = 9x - 5$ 3 **a** $7p + 5$, **b** $cd - 2$, **c** $8 + xyz$,
d $\frac{n}{4} + 3$, **e** $r - \frac{s}{2}$, **f** $7c^3$, **g** $\frac{h+4}{6}$, **h** $2(e-9)$, **i** $\frac{a}{3} + \frac{2b}{5}$, **j** $(v-w)^2$ 4 **a** $y + 6$, **b** $k - 9$ 5 **a** $t, t + 1, t + 2$,
b $e + 8, e + 9, e + 10$, **c** $2c - 5, 2c - 4, 2c - 3$, **d** $d - 1, d, d + 1$ 6 **a** $x, x + 2, x + 4$, **b** $n + 7, n + 9, n + 11$,
c $a - 2, a, a + 2$ 7 **a** $b, b + 2, b + 4$, **b** $v + 4, v + 6, v + 8$, **c** $z - 3, z - 1, z + 1$ 8 **a** $1000p$ m, **b** $\frac{b}{10}$ cm,
c $100k$ c, **d** $\frac{f}{60}$ h 9 **a** $300 + q$, **b** $60a + b$ 10 **a** 20, **b** 16, **c** 48, **d** -5, **e** 54, **f** -48, **g** 88, **h** 4 11 **a** 7, **b** -1,
c -8, **d** 9, **e** -9, **f** 144, **g** 39, **h** 42, **i** 61, **j** 8 12 **a** $17ef$, **b** $10u^2$, **c** $5a^2$, **d** $4a^2b$, **e** $-3d$, **f** $-12v$, **g** $-8c$, **h** $4r$
13 **a** $-10h$, **b** 0 14 **a** $2a + 11$, **b** $8c + 9d$, **c** $23p + 3q$, **d** $7g + 2h$, **e** $3m$, **f** $19a - 2b$, **g** $-3x - 5y$, **h** $2a^2 - 5a$
15 **a** $24t$, **b** $10x + 8$ 16 **a** $24k$, **b** $6mn$, **c** $35cd$, **d** n^2 , **e** $36g^2$, **f** ab^2c , **g** $6pq$, **h** $6rs^2t$, **i** $-15e$, **j** $32uv$, **k** $-24abc$,

$l - 42pq^2r$ 17 **a** $3w$, **b** 2 , **c** f , **d** 6 , **e** $7b$, **f** $6n$, **g** b , **h** $3s$, **i** $-10u$, **j** 5 , **k** $-12y$, **l** $4d$ 18 **a** $9b$, **b** $6x$, **c** $12v$
 19 **a** $35b$, **b** $4d$, **c** 5 , **d** $35wz$ 20 **a** $\frac{k}{5}$, **b** $\frac{4b}{5}$, **c** $\frac{7}{9m}$, **d** $\frac{2u}{3v}$ 21 **a** $30h$, **b** 10 , **c** $60t^2$, **d** $48g$, **e** $14w$, **f** $-10x^2$
 22 **a** 4 , **b** $6v$ 23 **a** $19n$, **b** $22j$, **c** $34e$ 24 **a** $12n + 32$, **b** $28a - 35b$, **c** $2fg + 22fh$, **d** $x^2 - 6x$, **e** $15c^2 - 50c$,
 $f p^2q + pq^2$, **g** $-3ab - 27ac$, **h** $-36rs + 48r^2$ 25 **a** $5u + 22$, **b** $3t - 14$, **c** $9p - 7$, **d** $-14x + 48$, **e** $12n - 10$,
 $f 7 + m$, **g** $16k - 13$ 26 **a** $5(r - 4)$, **b** $b(a - c)$, **c** $y(x + 1)$, **d** $e(e + 11)$, **e** $uv(t - w)$, **f** $3(6p - 7)$, **g** $5m(4k + 3n)$,
 $h 6s(4r - 5s)$, **i** $cd(d + c)$, **j** $-7(2a - 7)$, **k** $-3e(5f + 8g)$, **l** $-11w(7w - 12)$ 27 **a** $(b + 7)(a + 5)$, **b** $(m - n)(m - 4)$,
 $c(y + 2z)(x + 1)$ 28 **a** $\frac{2c}{3}$, **b** $\frac{4u}{3}$, **c** $2k$, **d** $\frac{7}{10s}$, **e** $\frac{3a}{10}$, **f** $\frac{4m}{21}$, **g** $\frac{11w}{12}$, **h** $\frac{a}{3b}$ 29 **a** $\frac{bc}{40}$, **b** $\frac{3}{4}$, **c** $\frac{8n}{27}$, **d** $\frac{6xz}{49}$,
 $e \frac{ab}{4}$, **f** $\frac{4}{5}$, **g** $\frac{16w}{35u}$, **h** $\frac{5bd}{6a}$ 30 **a** $y^2 + 9y + 20$, **b** $m^2 - 10m + 21$, **c** $t^2 + 6t - 16$, **d** $a^2 - 7a - 44$, **e** $n^2 - 12n + 27$,
 $f k^2 + 10k - 24$, **g** $e^2 + 13e + 42$, **h** $s^2 - 12s - 13$ 31 **a** $2b^2 + 11b + 15$, **b** $6p^2 - 25p + 24$, **c** $35n^2 - 9n - 18$,
 $d 8 - 2r - 3r^2$ 32 **a** $p^2 + 6p + 9$, **b** $m^2 - 14m + 49$, **c** $4c^2 + 20c + 25$, **d** $16y^2 - 24y + 9$, **e** $9a^2 + 24ab + 16b^2$,
 $f 49j^2 - 84jk + 36k^2$, **g** $x^2 + 3x + \frac{9}{4}$, **h** $9t^2 - 48 + \frac{64}{t^2}$ 33 $162x^3 - 72x^2y + 8xy^2$ 34 **a** $(a - 6)^2 = a^2 - 12a + 36$,
 $b(d + 4)^2 = d^2 + 8d + 16$, **c** $(t + 11)^2 = t^2 + 22t + 121$, **d** $(n - 9)^2 = n^2 - 18n + 81$, **e** $(3x + 5)^2 = 9x^2 + 30x + 25$,
 $f(4u - 11)^2 = 16u^2 - 88u + 121$ 35 **a** no, **b** no, **c** yes, **d** no, **e** no, **f** yes 36 **a** $x^2 - 25$, **b** $k^2 - 64$, **c** $144 - m^2$,
 $d 4t^2 - 49$, **e** $9a^2 - 121b^2$, **f** $a^2b^2 - 36c^2$ 37 **a** $a^3 + 8a^2 + 19a + 12$, **b** $12n^3 - 11n^2 - 19n + 14$, **c** $y^3 + 9y^2 - 4y - 36$,
 $d 10k^3 - 39k^2 - 7k + 12$ 38 **a** $2x^2 - 10x + 9$, **b** $10n^2 - 16n + 28$, **c** $14a - 149$, **d** $u^2 - 2u - 18$

3

Consumer arithmetic

Exercise 3.1

1 **a** \$540, **b** \$1080, **c** \$2340 2 **a** \$25 896, **b** \$35 425, **c** \$53 087.04 3 **a** \$463.60, **b** \$532.95 4 **a** 37, **b** 36.5
 5 **a** \$32 970, **b** \$3135.60 6 **a** \$43 111.20, **b** \$2499.79 7 \$6270 8 \$8024.40 9 Pharmacist's assistant by
 \$6.55 10 \$428.40 11 \$2195.92 12 \$32 000 per annum 13 \$1726.15 14 \$925.21 15 \$31.59
 16 \$59 975.11 17 \$879.23 18 \$18 418.40 19 Bart, \$516.80 per week; Kristina, 38 hours worked;
 Marta, \$14.80 per hour 20 \$1083.75 21 \$1.60 22 \$127.40 23 \$165.60 24 **a** \$352, **b** \$9.24 per hour,
c \$17.60 25 8% 26 4.5% 27 **a** \$30 420, \$28 899, **b** Decrease by 5% 28 \$27 000, \$34 000, \$42 800,
 \$57 000, \$64 100, \$84 750 29 \$577.50 30 \$2667.60

Exercise 3.2

1 **a** \$600, **b** \$259.66, **c** \$212.64 2 \$280.80 3 \$567 4 \$469.70 5 35 6 Bill, by \$22.69 7 \$4425
 8 \$590 9 \$443.52 10 \$1290.50 11 **a** \$5250, **b** \$7060 12 \$18 775 13 \$245 14 \$985.50 15 \$385
 16 8 17 \$830 18 **a** \$565, **b** \$124.30 19 \$15 540 20 \$267 21 \$3516.30 22 \$402.80
 23 **a** \$4875, **b** 1106 km, **c** \$1060.88 24 **a** \$40 800, **b** \$622 25 \$195 26 2% 27 \$745 000

Exercise 3.3

1 **a** \$18, **b** \$24 2 **a** \$72, **b** \$128 3 **a** 10, **b** 6 4 **a** \$688, **b** \$938.25, **c** \$784.75, **d** \$817.80, **e** \$766.85
 5 \$950.60 6 \$945 7 **a** \$492, **b** \$565.80, **c** \$707.25 8 \$931.50 9 \$652.50 10 \$1164 11 **a** 3, **b** 7,
c 8 12 43 13 **a** \$15.10, **b** \$24.50, **c** \$16.90 14 **a** \$705.60, **b** \$44.10 15 **a** \$634.80, **b** 46 16 \$1521
 17 \$728 18 **a** \$448, **b** \$387.10, **c** \$504, **d** \$650.73 19 \$2902.25 20 \$7453.12 21 \$2800 22 \$1560

Exercise 3.4

1 \$364 2 \$515 3 \$352.45 4 **a** \$739.50, **b** \$491.05, **c** \$1842.03 5 **a** \$626.31, **b** \$200.20, **c** \$426.11
 6 \$334.80 7 \$686.35 8 **a** 28%, **b** 39% 9 \$1824.33 10 **a** \$425.52, **b** 35% 11 **a** \$38.19, **b** \$458.21
 12 **a** \$667, **b** \$43.50, **c** \$427.85 13 \$37 440 14 \$19.09

Exercise 3.5

1 a Nil, b \$1190, c \$6805, d \$12 784.90, e \$20 872.20, f \$10 438.90, g \$15 058.36, h \$68 459.70
 2 a \$53 720, b \$12 942.40 3 a \$42 861, b \$9238.30 4 a \$14 820, b \$14 076, c \$1372.92 5 a \$79 075, b \$74 723, c \$22 499.81 6 a \$41 875.08, b \$8942.50, c Yes, \$1057.10 7 a \$56 529.46, b \$56 144, c \$13 960.48, d Must pay \$2136.40. Tax payable is greater than tax paid. 8 a \$24 724.96, b \$24 312, c \$3673.60, d \$404.83 9 \$1835.04 10 a \$716.42, b \$774.52, c \$58.09 11 a \$42 131.25, b \$8665.60, c \$82.36 12 a \$38 154.80, b \$37 564, c \$7649.20 13 a \$984, b \$14 915.56 14 \$8191 15 a \$2933.40, b \$42 066, c \$8999.80 16 a \$13 672, b \$11 080, c \$49.85 17 a \$40 192.55, b \$7547.80, c \$627.78
 18 a Nil, b \$315, c \$90.60 19 a Yes, b \$2385, c Yes, \$279 20 a \$10 918, b \$726.90, c \$9620, d Needs to pay \$2024.90 21 a \$78 785, b \$24 408.95, c \$1181.78, d Yes, \$1880.88 22 a \$15 400, b \$28 300, c \$54 710, d \$94 612 23 a \$56 620, b \$1415.50, c \$33.19

Exercise 3.6

1 a \$52, b \$410, c 18% 2 a \$65, b \$9620, c 5% 3 a i \$80, ii \$10, b \$320, c $\frac{3}{16}$, d \$15 4 a \$350, b Entertainment = \$50; University expenses = \$40, c 72°, d \$66 5 a Food = 60°; Transport = 30°, b $\frac{1}{12}$, c \$540, d No, as no savings are made 6 Weekly budget: Health insurance = \$17.50, Petrol = \$37, Food = \$80, Rent = \$240, Car insurance = \$18, Savings = \$103.75, Other = \$28.75. Total \$525 7 Weekly budget: Rent = \$177, Food = \$110, Petrol = \$48, Bills = \$30, Car insurance = \$18, Superannuation = \$30, Savings = \$181. Total \$594 8 a \$21 300, b \$3859.80, c \$74.23, d \$40 599

Exercise 3.7

1 a B, b C, c B 2 a A, b B, c C, d D 3 a 30 mL for \$2.10, b 85 cm for \$5.19, c 12 kg for \$50.28, d 75 g for \$10.16 4 1.25 L bottle 5 1.75 kg for \$5.90 6 100 g for \$1.75 7 The single purchase is the better option as only one screen door is needed 8 Gary should purchase two tins at \$9.50 each, as only two tins are needed, not four 9 a 1 kg bag, b \$19 10 \$3.78

Exercise 3.8

1 75% 2 a \$145, b \$52, c \$72 3 a \$1012, b \$763, c \$884 4 a \$134.15, b \$114.03 5 \$699.80
 6 \$123.50 7 \$274.55 8 \$5.65 9 20% 10 22% 11 13% 12 28.2% 13 \$5800 14 \$570
 15 \$95 16 \$62 17 \$73.50 18 a No, b 40%, c \$486 19 \$57 20 \$680 21 $\frac{100x}{x+y}$

Exercise 3.9

1 a \$870, b \$2030 2 a \$1400, b \$4900 3 \$13 875 4 a 25%, b 20% 5 30% 6 31% 7 83% 8 55%
 9 Cost price is half the selling price 10 4 : 1 11 Profit = \$90.40 12 \$600.20 13 a \$1190, b 5.2%
 14 \$175 000 15 \$18 889 16 \$2178 17 a Ball = \$16; Bat = \$232, b Ball = \$13.91; Bat = \$165.71,
 c \$140.94 18 \$400 19 a \$1350, b 4320, c \$5080, d \$1.37

Chapter 3 Review

1 a \$973.08, b \$1946.15, c \$4216.67 2 \$527.40 3 \$3431.75 4 \$2123.33 5 76 6 6.5% 7 \$590.20
 8 \$251 9 a \$10 450, b \$3553 10 2320 11 17 12 \$893.80 13 a \$499.40, b 44 14 \$2138.50
 15 \$1049.48 16 \$1077.17 17 \$2292.29 18 a Nil, b \$2113.10, c \$11 260, d \$21 311.65 19 a \$38 235,
 b \$7850.50, c \$1277.29 20 a \$10 735, b PAYG tax = \$11 778 ∴ Refund due (\$1043) 21 a \$318.56, b \$82.50
 22 a \$140, b \$18.46, c \$91.54, d 6.7%, e 60°, f Entertainment = \$72 (extra \$12), Savings = \$115.54 (extra \$24)
 23 B 24 D 25 \$65.60 26 \$88 27 a 12.5%, b 16% 28 \$744 29 7.4% 30 \$34.80 31 \$4025
 32 35% 33 a \$17 000, b \$620

4

Equations, inequations and formulae

Exercise 4.1

1 $a x = 6$, $b p = 8$, $c t = 4$, $d m = 30$, $e n = 4$, $f k = 8$, $g h = -3$, $h y = 7$ 2 **a** Incorrect, **b** Correct, **c** Incorrect, **d** Incorrect, **e** Correct, **f** Incorrect 3 **a** $a = 5$, **b** $p = 17$, **c** $y = -5$, **d** $q = -1$, **e** $w = 6$, **f** $r = -8$, **g** $b = 30$, **h** $f = 63$
 4 **a** $u = \frac{2}{3}$, **b** $e = 3\frac{1}{2}$, **c** $a = \frac{1}{2}$, **d** $c = \frac{4}{5}$, **e** $d = -\frac{3}{4}$, **f** $t = 1\frac{1}{2}$, **g** $k = -\frac{7}{9}$, **h** $w = 1\frac{2}{3}$, **i** $x = 1\frac{1}{2}$, **j** $y = 3\frac{1}{4}$, **k** $m = \frac{1}{6}$,
l $q = 1\frac{3}{4}$, **m** $n = \frac{1}{12}$, **n** $v = \frac{3}{10}$, **o** $t = -\frac{1}{3}$, **p** $h = \frac{11}{15}$, **q** $e = \frac{2}{7}$, **r** $r = \frac{12}{13}$, **s** $w = 4\frac{1}{2}$, **t** $k = -8\frac{1}{2}$, **u** $c = 0.8$, **v** $g = 2$,
w $b = 3$, **x** $v = 0.56$ 5 **a** $k = 3$, **b** $y = 6$, **c** $t = 5$, **d** $z = -1$, **e** $h = -3$, **f** $r = -7$, **g** $c = 23$, **h** $a = 13$, **i** $w = -19$
 6 **a** $x = 4$, **b** $p = 7$, **c** $e = 3$, **d** $a = 1$, **e** $g = 8$, **f** $s = 7$, **g** $c = -2$, **h** $f = -2$, **i** $d = -1$, **j** $y = 0$, **k** $m = -4$, **l** $p = 3$,
m $z = -8$, **n** $t = -6$, **o** $u = -1$ 7 **a** $p = \frac{3}{4}$, **b** $m = \frac{5}{7}$, **c** $h = 1\frac{4}{5}$, **d** $r = \frac{3}{4}$, **e** $k = 2\frac{1}{2}$, **f** $x = -\frac{1}{3}$, **g** $n = -\frac{1}{2}$, **h** $k = 2\frac{1}{2}$,
i $w = -\frac{5}{7}$ 8 **a** $x = 5\frac{2}{5}$ **b** Inverse operations 9 **a** $x = \pm 2$, **b** $x = \pm 3$, **c** $x = \pm 7$, **d** $x = \pm 10$, **e** $x = \pm 1$, **f** $x = \pm 2$,
g $x = \pm 4$, **h** $x = \pm 6$, **i** $x = \pm \frac{4}{3}$, **j** $x = \pm \frac{4}{7}$, **k** $x = \pm 3\frac{1}{3}$, **l** $x = \pm 1\frac{1}{3}$ 10 **a** $x = \pm 2.2$, **b** $x = \pm 3.5$, **c** $x = \pm 4.5$, **d** $x = \pm 2.6$

Exercise 4.2

1 **a** $m = 7$, **b** $k = 2$, **c** $y = -3$ 2 **a** Correct, **b** Correct, **c** Incorrect, **d** Correct 3 **a** $k = 3$, **b** $m = 4$,
c $s = 7$, **d** $p = 5$, **e** $x = -5$, **f** $b = -7$, **g** $r = 11$, **h** $m = -6$, **i** $u = -2$, **j** $g = 12$, **k** $c = 8$, **l** $k = 8$ 4 **a** $y = 4$
 5 **a** $x = 3$, **b** $p = 8$, **c** $a = 5$, **d** $m = 12$, **e** $y = 4$, **f** $t = 3$, **g** $k = -3$, **h** $w = -6$, **i** $q = 0$, **j** $b = 5$, **k** $s = -9$, **l** $e = -13$,
m $u = 4$, **n** $x = -4$, **o** $n = 1$, **p** $t = 5$, **q** $k = 4$, **r** $h = -2$ 6 **a** $m = 4\frac{1}{2}$, **b** $k = \frac{2}{3}$, **c** $c = \frac{1}{4}$, **d** $u = \frac{5}{6}$, **e** $y = 1\frac{2}{7}$,
f $p = 1\frac{1}{2}$, **g** $t = 2\frac{2}{3}$, **h** $z = 5\frac{1}{2}$, **i** $a = 3\frac{1}{2}$, **j** $e = -2\frac{6}{7}$, **k** $g = \frac{3}{7}$, **l** $k = -3\frac{3}{4}$ 8 **a** $p = 1$, $q = -4$ **b** $x = -2$, $y = 17$

Exercise 4.3

1 **a** $m = 5$, **b** $k = 8$, **c** $c = 3$, **d** $n = 8$, **e** $t = -3$, **f** $x = 1$, **g** $y = 1$, **h** $a = 5$, **i** $w = 2$, **j** $p = -4$, **k** $c = -3$, **l** $m = -4$
 2 **a** $n = -5$, **b** $p = -8$, **c** $y = -4$, **d** $m = 6$, **e** $t = -7$, **f** $a = 11$, **g** $k = -4$, **h** $r = -2$, **i** $d = 5$ 3 **a** $a = 1\frac{2}{3}$, **b** $m = 5\frac{3}{4}$,
c $k = \frac{3}{5}$, **d** $y = \frac{3}{4}$, **e** $p = \frac{9}{11}$, **f** $z = -1\frac{5}{6}$, **g** $h = -10\frac{1}{2}$, **h** $v = -2\frac{1}{2}$, **i** $c = -\frac{7}{10}$ 4 **a** $x = 2$ 5 **a** $p = 28$, **b** $n = 18$,
c $e = 8$, **d** $s = 5$, **e** $b = -2$, **f** $y = -11$, **g** $u = 8$, **h** $z = -3$, **i** $t = -2$ 6 **a** $x = 6$, **b** $n = 17$, **c** $y = -26$, **d** $g = 21$,
e $a = 4$, **f** $r = -8$, **g** $z = 7$, **h** $p = 3$, **i** $w = 12$, **j** $c = 11$, **k** $e = 3$, **l** $m = -2$ 7 **a** $x = 5$, **b** $x = 7$, **c** $x = 2$, **d** $x = 7$,
e $x = 13$, **f** $x = 8$, **g** $x = -14$, **h** $x = 3$, **i** $x = -1$, **j** $x = 4$

Exercise 4.4

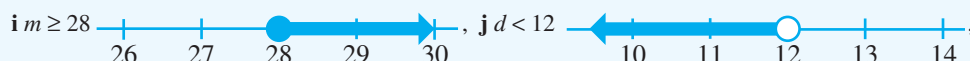
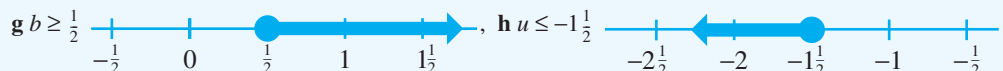
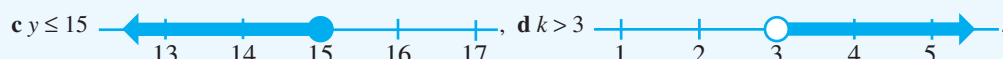
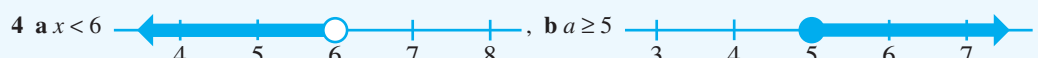
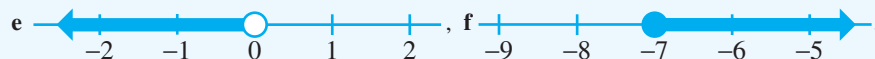
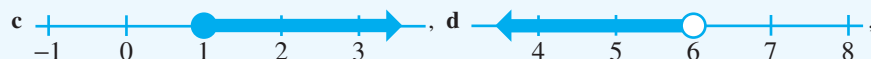
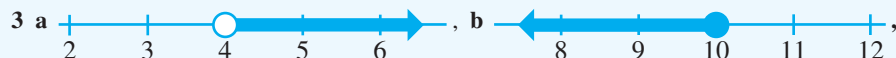
1 **a** $d = 12$, **b** $k = -35$, **c** $y = -18$, **d** $w = 24$, **e** $x = 12$, **f** $a = 10$, **g** $m = 4$, **h** $k = 9$, **i** $w = -6$, **j** $c = -14$, **k** $s = -18$,
l $u = 9$ 2 **a** $m = 32$ 3 **a** $n = 15$, **b** $k = 16$, **c** $c = 42$, **d** $a = 63$, **e** $z = -16$, **f** $h = 20$, **g** $p = 18$, **h** $x = 60$, **i** $j = 48$,
j $u = 22$, **k** $s = 36$, **l** $d = -42$, **m** $a = 6$, **n** $e = 18$, **o** $h = -24$, **p** $z = 8$ 4 **a** $m = 1$, **b** $k = 22$, **c** $s = 15$, **d** $t = -7$,
e $x = 9$, **f** $c = 12$, **g** $d = -1$, **h** $b = -2$, **i** $z = -8$, **j** $r = -7$, **k** $f = 8$, **l** $q = -12$ 5 **a** $m = 7$, **b** $x = 17$, **c** $w = 11$,
d $b = 10$, **e** $u = 5$, **f** $t = -9$, **g** $a = -22$, **h** $k = -5$, **i** $c = 3$ 6 **a** $x = 2\frac{2}{3}$, **b** $x = 7$, **c** $x = -\frac{2}{5}$, **d** $x = \frac{5}{6}$, **e** $x = -4\frac{5}{7}$,
f $x = 5\frac{1}{2}$, **g** $x = 3\frac{9}{16}$, **h** $x = 14\frac{1}{2}$, **i** $x = 6$

Exercise 4.5

1 **a** $x = 6$, **b** $a = 10$, **c** $t = 30$, **d** $k = 24$, **e** $n = 72$, **f** $y = 21$, **g** $u = 20$, **h** $m = 12$, **i** $c = 16$, **j** $h = 42$, **k** $w = 36$,
l $e = 80$ 2 **a** $c = 5$ 3 **a** $a = 1$, **b** $n = 9$, **c** $y = 4$, **d** $c = 2$, **e** $b = 5$, **f** $t = 3$, **g** $x = -6$, **h** $m = -9$ 4 **a** $y = 15$,
b $a = 60$, **c** $e = 36$, **d** $x = 24$, **e** $t = 40$, **f** $u = 20$ 5 **a** $m = 1$, **b** $x = 9$, **c** $k = -10$, **d** $d = 1$, **e** $n = 26$, **f** $t = 2$,
g $z = -3$, **h** $h = 0$, **i** $h = -3$ 6 **a** $x = 11$, **b** $f = 6$, **c** $b = 7$, **d** $n = 19$, **e** $q = 13$, **f** $m = -2$, **g** $x = 5$, **h** $x = -6$,
i $x = 4$, **j** $x = 1\frac{1}{2}$, **k** $a = 1\frac{1}{5}$, **l** $t = 6\frac{1}{2}$ 7 **a** $a = \frac{2}{3}$, **b** $x = -7\frac{1}{2}$, **c** $x = 2$, **d** $a = -2$, **e** $x = 16\frac{1}{2}$, **f** $x = 10$, **g** $x = -6$,
h $a = \frac{1}{20}$, **i** $a = -\frac{1}{10}$ 8 **a** $a = 1\frac{1}{3}$, **b** $c = 3$, **c** $y = 8$, **d** $a = -\frac{1}{3}$, **e** $x = -6$, **f** $y = -12$, **g** $x = -12$, **h** $y = 7$, **i** $u = -1$

Exercise 4.6

1 $a x = 5, 6, 7, \dots$, $b x = 4, 3, 2, \dots$, $c x = 1, 2, 3, \dots$, $d x = 11, 10, 9, \dots$, $e x = -7, -6, -5, \dots$,
 $f x = -16, -17, -18, \dots$, $g x = -7, -8, -9, \dots$, $h x = -10, -9, -8, \dots$, $i x = 2, 3, 4, 5, 6, 7$, $j x = 9, 10, 11, 12, 13$,
 $k x = -4, -3, -2, -1, 0, 1$, $l x = -11$ 2 $a x < 3$, $b x \geq 2$, $c x > 8$, $d x \leq -3$, $e x > 13$, $f x \leq -6$, $g 2 \leq x \leq 6$,
 $h -3 < x \leq 0$, $i x < 4$ or $x > 6$, $j x \leq -1$ or $x > 2$



5 $a y < 3$, $b c \geq 4$, $c z > -5$, $d g \leq -18$, $e p > 4$, $f q \leq 5$, $g m < -2$, $h t \geq -7$, $i v \leq 24$, $j s > -16$, $k b \geq 27$,
 $l r < -60$ 6 $a x \geq 2$, $b k < 5$, $c t > 7$, $d g \leq 3$, $e m < 4$, $f w \geq 8$, $g q > -2$, $h y < -3$, $i p \leq 9$, $j z > 5$, $k a \leq 9$,
 $l e < 5$, $m u > 7$, $n f \leq 15$, $o a \geq 3\frac{1}{2}$, $p d > \frac{11}{12}$, $q c \geq 2\frac{1}{2}$, $r n < -\frac{2}{9}$ 7 $a a < 8$, $b m \geq 5$, $c h < 3$, $d x \geq -7$,
 $e p > 5$, $f e < -19$ 8 $a x > 12$, $b m < 15$, $c k \geq -14$, $d t \leq 14$, $e n > 6$, $f u \geq 8$ 9 $a x = 1, 2, 3, 4, 5, 6$,
 $b x = 5, 6, 7, 8, \dots$, $c x = 16, 17, 18, 19, 20, 21$, $d x = -3, -2, -1$ 10 $a a > -5$, $b y \leq -2$, $c n \geq 4$, $d d < 9$,
 $e m < -2$, $f c \geq 4$, $g k > -30$, $h w \leq 30$, $i x \geq -18$, $j e < 12$, $k d < -8$, $l s > 5$, $m g \leq -8$, $n t \geq -35$, $o z < 18$,
 $p f \leq 54$ 11 $a c > -2$, $b s < -6$, $c p \geq 5$, $d h \leq 2$, $e t > -4$, $f g < 1$, $g v \geq 6$, $h d < 4$, $i q \leq -8$, $j n > -2$, $k r > 4$,
 $l j < -\frac{1}{7}$, $m t \geq 1\frac{5}{6}$, $n n \leq \frac{2}{5}$, $o s < 1\frac{1}{3}$ 12 $a x > 12$, $b x \leq 45$, $c x \leq 9$, $d x > 11$, $e x < 3\frac{7}{11}$, $f x \geq 2\frac{9}{16}$, $g x < 8\frac{2}{3}$,
 $h x \geq -7\frac{1}{3}$, $i x > 1\frac{1}{2}$ 13 $a x = 5, 6, 7, 8, 9$, $b 8 < x < 20$, $c 2, 3, 4; 3, 4, 5; 4, 5, 6$, $d 8 < x < 12$,

e Between 6 and 16 cm 14 a T, b F, c T, d F, e T, f T, g T, h F 15 

16 a $7 < x < 12$, b $3 \leq x \leq 5$, c $-2 \leq x < 14$, d $2 < x \leq 5$, e $16 < x < 32$, f $18 \leq x < 24$



Exercise 4.7

1 a 3, b 7, c 12, d 11, e 8 **2** a 1, b 11, c 24 **3** a 26, b 49, c 27, d 52 **4** a 11, b 14, c 19 **5** a 22, b 6, c 2, d 3 **6** a 75, 76, b 17, 18, 19, c 23, 24, 25, 26 **7** a 32, 34, 36, b 9, 11, 13, 15, c 26, 28, d 37, 39, 41
8 a $x = 14$, b $x = 12$, c $x = 7$ **9** a 11 men, 18 women, b Kris has \$25, Annika has \$16, c 17 cm, 11 cm, d Retread tyre = \$75, New tyre = \$110, e 8 cm, 13 cm, f 55 cm, 85 cm, 110 cm, g Raymond is 26 years, Father is 52 years **10** a \$2.40, b Bettina is 12 years, Darren is 36 years, Jonathan is 72 years, c 11 pears, 3 tomatoes, d $n = 7$, e \$4, f 23 days **11** a Son is 24 years, Daughter is 18 years, Woman is 36 years, b 11 years, c Frank is 10 years, Anita is 40 years, d Thao is 36 years, Wendy is 66 years **12** 344 **13** 70 500 kg

Exercise 4.8

1 a $A = 35$, b $F = 19.55$, c $A = 29.025$ **2** a $S = 18$, b $M = 7.5$, c $I = 56$ **3** a i $P = 26$, ii $P = 34.2$, b i $y = 15$, ii $y = -22$, c i $v = 23$, ii $v = 28$ **4** a $A = 64$, b $V = 1728$, c $y = 8$ **5** a $V = 7$, b $v = 5$, c $T = 8$ **6** a $A = 96$, b $S = 119$, c $C = 30$ **7** a $E = 0.42$, b $A = 43.2$, c $F = 37.5$, d $S = 60$, e $h = 0.25$, f $y = 22$, g $s = -676$
8 a $D = 72$, b $m = 3$, c $S = 9$, d $T = 4.5$ **9** a $A = 95.0$, b $V = 22.6$, c $V = 167.1$, d $A = 213.6$, e $S = 21.9$
10 a $S = 108$, b $A = 45.6$, c $T = 24.5$, d $S = -95$ **11** a $R = 3$, b $v = 24$, c $T = 11.4$, d $v = 14$, e $E = 0.8$
12 a $A = 13.45$, b $T = 0.125$, c $A = 6090.13$, d $E = 0.33$

Exercise 4.9

1 a $m = 1.5$, b $b = 5$ **2** a $L = 10$, b $B = 9.5$ **3** a $D = 90$, b $D = 88.9$, c $T = 7$, d $T = 8.24$ **4** $K = 108$
5 a $u = 20$, b $u = 110$, c $a = 12.25$, d $t = 1.6$ **6** a $n = 16$, b $l = -11$, c $a = 5$ **7** a $g = 10$, b $R = 7.2$
8 a $g = 8$, b $k = \frac{1}{3}$ **9** a $m = 3$, b $v = 1.5$ **10** a $m = 16$, b $v = 3.5$, c $r = 64$ **11** a $v = 11$, b $u = 8.6$
12 a $u = 5$, b $a = -6$ **13** a $y_2 = 17$, b $y_1 = -8$, c $x_2 = 24$, d $x_1 = -5$ **14** a $h = 8$, b $r = 8.3$ **15** a $r = 6$, b $r = 2.9$ **16** a $a = 22.5$, b $a = 80$, c $r = \frac{2}{3}$, d $r = -\frac{1}{2}$ **17** a $R = 7$, b $R = 8.2$, c $r = 6$, d $r = 5.9$ **18** a $a = 7$, b $d = \frac{5}{7}$ **19** a $a = 7$, b $r = \pm 2$, c $n = 4$ **20** a $m_1 = 13\frac{2}{3}$, b $m_2 = 1\frac{3}{13}$

Exercise 4.10

1 a $x = y - a$, b $x = p + q$, c $x = m - n$, d $x = \frac{k}{2}$, e $x = \frac{c - e}{d}$, f $x = cy$, g $x = \frac{q}{p}$, h $x = \frac{vw}{u}$, i $x = a(y - z)$,
j $x = \frac{cd}{be}$, k $x = \frac{mp}{n}$, l $x = \frac{k}{g + h}$ **2** a $a = \frac{c - 3b}{3}$, b $a = \frac{y + 14}{2}$, c $a = \frac{w - mc}{m}$, d $a = \frac{k - p}{k}$, e $a = \frac{10y - z}{5y}$,
f $a = \frac{2\pi r - A}{2\pi}$ **3** a $n = \frac{3(m + p)}{k}$, b $n = \frac{10 - 2k}{5}$, c $n = \frac{ka - kb}{b}$, d $n = \frac{21k}{10}$, e $n = \frac{3a + 2b + 2c}{3}$,
f $n = \frac{9h - 7m}{4}$ **4** a $t = \pm \sqrt{\frac{b}{a}}$, b $t = \pm \sqrt{n - m}$, c $t = \pm \sqrt{u - k}$, d $t = \pm \sqrt{a^2 - b^2}$, e $t = \pm \sqrt{rs}$, f $t = \pm \sqrt{\frac{vh}{u}}$,
g $t = \pm \frac{6}{a}$, h $t = \pm \frac{\sqrt{ab}}{4}$ **5** a $c = j^2$, b $c = \frac{a^2}{d}$, c $c = y^2 + 4$, d $c = 9p^2$, e $c = \frac{M^2 n^2}{a^2}$, f $c = (L - K)^2$, g $c = \left(\frac{a - b}{d}\right)^2$,
h $c = \left(\frac{p - N}{q}\right)^2$ **6** a $x = \frac{p + q}{2}$, b $x = \frac{p}{m - n}$, c $x = \frac{2d}{c - 1}$, d $x = \frac{1}{a + 1}$, e $x = \frac{y}{1 - y}$, f $x = \frac{3t}{t - 1}$, g $x = \frac{g + 1}{g - 1}$,
h $x = \frac{a - av}{1 + v}$, i $x = \frac{a + bc}{b + c}$ **7** $x = -1$ **8** a $M = DV$, b $a = \frac{v - u}{t}$, c $L = \frac{P - 2B}{2}$, d $S = \frac{D}{T}$, e $x = 2A - y$,
f $P = \frac{100I}{RN}$, g $A = \frac{3V}{h}$, h $v = \pm \sqrt{\frac{2E}{m}}$, i $R = \sqrt[3]{T^2}$, j $s = \frac{v^2 - u^2}{2a}$, k $R = \frac{V^2}{2g}$, l $h = \frac{A - 2\pi r^2}{2\pi r}$, m $n = \frac{2S}{a + l}$,
n $a = \frac{2A - bh}{h}$, o $C = \frac{5F - 160}{9}$, p $F = \frac{9C + 160}{5}$, q $r = \sqrt[3]{\frac{3V}{4\pi}}$, r $u = \pm \sqrt{v^2 - 2s}$, s $a = \pm \sqrt{\frac{v^2}{n^2} + x^2}$, t $r = \frac{S - a}{S}$,
u $u = \frac{2s - at^2}{2t}$, v $l = \frac{gT^2}{4\pi^2}$, w $R = \pm \sqrt{\frac{A + \pi r^2}{\pi}}$, x $R = \frac{R_1 R_2}{R_1 + R_2}$ **9** 3 cm **10** 8.32 cm **11** 128 m

12 6 cm 13 a i none, ii $x = \frac{a-c}{b}$, iii $b \neq 0$, b i none, ii $x = \pm \sqrt{\frac{y}{a}}$, iii $\frac{y}{a} \geq 0$ and $a \neq 0$, c i $q \neq 0, s \neq 0$,
 ii $r = \frac{ps}{q}$, iii $q \neq 0$, d i none, ii $b = \pm \sqrt{\frac{c^2}{a^2} - 9}$, iii $a \neq 0, \frac{c^2}{a^2} \geq 9$, e i $nu^2 \neq -1$, ii $u = \pm \sqrt{\frac{m-k}{kn}}$,
 iii $k \neq 0, n \neq 0, \frac{m-k}{kn} \geq 0$, f i $t \neq 2$, ii $t = \frac{2z}{z-1}$, iii $z \neq 1$ 14 $m_1 = \frac{T+m_2}{1-Tm_2}$ 15 a $r = \sqrt[n]{\frac{A}{P}}$, b $r = 100 \left(\sqrt[n]{\frac{A}{P}} - 1 \right)$

Chapter 4 Review

1 a $j=8$, b $b=10$, c $x=-8$, d $c=14$ 2 a $p=8$, b $k=6$, c $y=-3$, d $m=-6$ 3 a $h=8$, b $d=-6$, c $y=11$,
 d $s=5$ 4 a $r=7$, b $b=-9$, c $u=9$ 5 a $a=3$, b $f=-4$ 6 a $x=4$, b $p=-6$, c $r=12$ 7 a $x=5$, b $x=9$,
 c $x=1$ 8 a $g=\frac{3}{5}$, b $x=4\frac{2}{3}$, c $u=4\frac{1}{6}$, d $w=-3\frac{3}{4}$, e $h=3\frac{1}{2}$, f $s=\frac{4}{5}$ 9 a $x=8$, b $z=12$, c $t=99$, d $a=6$,
 e $u=20$, f $n=17$, g $w=5$, h $r=6$, i $e=-38$, j $h=2$ 10 a $x=40$, b $m=30$, c $a=14$, d $u=2$, e $z=1\frac{8}{27}$,
 f $w=20$ 11 a $x=6, 5, 4, 3, \dots$, b $x=5, 6, 7, 8, \dots$, c $x=-9, -8, -7, -6, \dots$, d $x=0, -1, -2, -3, \dots$,
 e $x=3, 4, 5, 6, 7, 8$, f $x=-1, 0, 1, 2, 3, 4$, g $x=1, 0, -1, -2, \dots$ or $x=5, 6, 7, 8, \dots$,
 h $x=-5, -6, -7, -8, \dots$ or $x=8, 9, 10, 11, \dots$ 12 a $x \geq 3$, b $x < 2$, c $-3 \leq x < 1$, d $x < 0$ or $x \geq 2$
 13 a $y < 8$, b $k > -3$, c $m \geq 5$, d $t \leq 27$, e $u < 19$, f $c \leq 4$, g $z \geq -4$, h $w < -13$, i $a \geq 21$, j $b \leq 24$
 14 a $m > -7$, b $c \leq -7$, c $r \geq -4$, d $n < 32$, e $s \leq 8$, f $t > 11$ 15 a 15, b 9, c 10, d 11, e 9, f 16
 16 a 16, 17, 18, 19, b 27, 29, 31 17 a James has \$165, Samantha has \$118, b Ham sandwich costs \$2.20,
 Salad sandwich costs \$2.60 18 a $x=7, 8, 9$, b 8 and 20, c Between 5 cm and 21 cm 19 a $v=70$, b $v=33.9$,
 c $E=129.6$, d $S=248$, e $S=378$ 20 a $L=8$, b $m=\frac{2}{3}$, c $K=162$, d $a=-15$, e $x_1=2$ 21 a $x=\frac{bc}{ad}$,
 b $x=\frac{e}{f-g}$, c $x=\frac{m-6a}{2a}$, d $x=\pm\sqrt{v-u}$, e $x=\left(\frac{p-q}{r}\right)^2$, f $x=\frac{10z-2y}{5}$, g $x=\frac{a+b}{2}$, h $x=\frac{y+2}{y-1}$
 22 a $x \geq y$, b $x=\frac{z^2}{w^2}+y$, c $w \neq 0$

5

Measurement

Exercise 5.1

1 a mm, b cm, c m, d km, e cm, f m, g mm, h km 2 a g, b mg, c t, d kg, e mg, f g, g kg, h t, i g
 3 a mL, b L, c kL, d kL, e L, f mL, g mL, h L, i kL 4 a 6000 m, b 3 m, c 90 mm, d 2.5 km, e 46 cm,
 f 0.4 cm, g 0.178 km, h 23 mm, i 800 m, j 10 cm, k 2000 mm, l 0.016 km, m 3000 cm, n 0.007 cm, o 0.02 m,
 p 0.0003 km 5 a 5000, b 200 000, c 4, d 9, e 3 800 000, f 1.65 6 a 4000, b 8, c 1500, d 14.5, e 2.79,
 f 70, g 12 400, h 1820, i 0.375, j 0.14, k 870, l 46, m 0.02, n 0.006, o 5.47 7 a 4000, b 3, c 8000, d 7.5,
 e 2400, f 1.95, g 3.61, h 5070, i 730, j 0.195, k 0.011, l 6.8 8 a 60, b 60, c 24, d 180, e 48, f 300, g 30,
 h 45, i 16, j 90, k 78, l 170, m 3, n 3, o 7, p $1\frac{1}{2}$, q $1\frac{1}{4}$, r $1\frac{1}{3}$ 9 b 1 h 15 min 10 a 1 min 6 s, b 2 min 24 s,
 c 3 min 15 s, d 4 min 45 s 11 a 1 h 54 min, b 21 min, c 3 h 27 min, d 2 h 48 min 12 a 1.4 h, b 2.7 h,
 c 4.75 h, d 0.6 h 13 a 02:00, b 19:00, c 0:00, d 12:00, e 04:30, f 13:45, g 23:59, h 00:24 14 a 4 am,
 b 7:30 am, c 1 pm, d 3:20 pm, e 8:15 am, f 4:35 pm, g 8 pm, h 11:47 pm 15 a 1.379 m, b 5.895 m,
 c 12 783.54 m, d 1455.38 m 16 a 15.8 m, 0.85 m, b 20, c 4.32 m, d 25, e 664 mm, f 3.61 km, g 34.15 m
 17 a 12.84 kg, b 950 kg, c 1.35 kg, d 6.7 g, e 53, f 580.5 kg, g 3.4 kg, h 2.66 t 18 a 20 mL, b i 80, ii 66,
 c 8, d 1485 kL, e 8 300 000 L, f 1.375 L, g i 11.52 L, ii 2.5 mL 19 a 6 h 15 min, b 5 h 45 min, c 10 h 49 min
 20 a 6 h 45 min, b 1 d 2 h 48 min, 21 a 10:23 am, b 4:55 pm 22 a 500 g, b 1500 g, c 500 g, d 3500 g

Exercise 5.2

1 **a** ± 0.5 mm, **b** ± 10 km/h, **c** ± 0.5 cm, **d** ± 0.5 m, **e** ± 0.5 h, **f** ± 25 cm, **g** $\pm 1^\circ\text{C}$, **h** ± 0.5 cm, **i** ± 0.5 kg, **j** ± 625 m
 2 **a** 3.5 cm, **b** 4.5 cm **3 a** 63.5 cm and 64.5 cm, **b** 64.5 cm **4 a** 1.5 kg and 2.5 kg, **b** 8.5 kg and 9.5 kg,
c 13.5 kg and 14.5 kg, **d** 47.5 kg and 48.5 kg **5 a** 25 cm and 35 cm, **b** 45 cm and 55 cm, **c** 115 cm and 125 cm,
d 255 cm and 265 cm **6 a** 250 and 350, **b** 275 and 325, **c** 295 and 305 **7** 150 AD and 250 AD
8 8650 L and 8750 L **9 a** 135 cm and 145 cm, **b** 3 h 40 min and 4 h, **c** 1475 kL and 1525 kL,
d 83.5 kg and 84.5 kg, **e** 11.5 mm and 12.5 mm, **f** 45.5 t and 46.5 t, **g** 15.5 cm and 16.5 cm,
h 134.5 m and 135.5 m **10 a** 0.1 g, **b** No **11 a** 6.5 mm and 7.5 mm, **b** 1.5 cm and 2.5 cm, **c** 7.5 km and 8.5 km,
d 14.5 m and 15.5 m, **e** 1.55 cm and 1.65 cm, **f** 4.25 km and 4.35 km, **g** 6.75 m and 6.85 m,
h 12.05 mm and 12.15 mm, **i** 2.75 m and 2.85 m, **j** 17.25 mm and 17.35 mm, **k** 23.55 cm and 23.65 cm,
l 29.95 km and 30.05 km **12 a** 5.5 kg and 6.5 kg, **b** 6.45 L and 6.55 L, **c** 22.355 t and 22.365 t,
d 24.5°C and 25.5°C , **e** 5.05 mL and 5.15 mL, **f** 0.75 kL and 0.85 kL, **g** 160.35 cm and 160.45 cm,
h 11.65 g and 11.75 g, **i** 9.95 L and 10.05 L, **j** 102.5 dB and 103.5 dB, **k** 4.85 kg and 4.95 kg,
l 0.045 Hz and 0.055 Hz **13 a** 72.5 cm, 85.5 cm, **b** 71.5 cm, 84.5 cm, **c** Between 6041.75 cm^2 and 6198.75 cm^2

Exercise 5.3

1 **a** i $q^2 = p^2 + r^2$, **ii** $PR^2 = PQ^2 + QR^2$, **b** i $e^2 = f^2 + g^2$, **ii** $FG^2 = EF^2 + EG^2$ **2 a** True, **b** False, **c** True,
d False, **e** False, **f** True **3 a** No, **b** Yes, RT , **4 a** Yes, **b** No, **c** Yes, **d** No **5** $6^2 \neq 2^2 + 3^2$ **6 a** $x = 12.1$,
b $p = 9.7$, **c** $z = 16.0$, **d** $a = 77.8$ **7 a** $x = 31.1$, **b** $x = 7.1$ **8 a** 45.49 km, **b** 63 cm **9** 13.1 m or 15.6 m
10 a 7.87 m, **b** 4.3 m **11 a** 15.2 km, **b** 45.6 km **12** 358 m **13 a** 65 cm, **b** 144 mm **14 a** $d = 12$,
b $d = 13.7$ **15** 3 cm **16 a** $a = 10.5$, $b = 13.4$, **b** $x = 7.5$, $y = 10$, **c** $u = 8$, $v = 11.6$ **17** $PR^2 = PQ^2 + QR^2$
18 $w = 16.5$ **19** 54 cm **20** 67.9 cm **21 a** 120 mm, **b** 36 mm **24** 1223 km **25 a** 3900 m, **b** 1500 m

Exercise 5.4

1 **a** 32 cm, **b** 35.4 cm, **c** 31.2 cm, **d** 57.2 cm, **e** 114.8 cm, **f** 81.45 cm, **g** 46 cm, **h** 40.9 cm, **i** 59.8 cm
 2 **a** 59.2 mm, **b** 33 cm, **c** 56.8 m, **d** 66.5 cm **3 a** 13 mm, **b** 8.75 mm, **c** 9.8 cm, **d** 9 m **4 a** 9 cm, **b** 8.5 cm,
c 32 cm **5 a** 49.4 mm, **b** 84 mm, **c** 115 mm, **d** 76.8 mm, **e** 92.4 mm, **f** 73.2 mm **6** 5.25 km **7** 154 cm
8 \$2040.88 **9 a** 13.9 m, **b** 4, **c** \$67.20 **10** 600 m **11 a** $p = 24$, $q = 32$, perimeter 168 cm,
b $p = 9$, $q = 5.4$, perimeter 28.8 cm **12 a** 19.5 mm, **b** 4.2 m **13 a** 200 mm, **b** 53.6 m, **c** 34 km **14** 12 cm
15 86.6 mm **16** 144 mm **17** 2 cm **18** 8 cm **19** length = 120 cm, perimeter = 384 cm
20 42 cm, 56 cm, 70 cm

Exercise 5.5

1 **a** 18.8 cm, **b** 34.6 cm, **c** 103.0 cm, **d** 149.2 cm **2 a** 25.1 mm, **b** 106.8 mm, **c** 147.7 mm, **d** 252.6 mm
3 a 30.2 cm, **b** 42.3 m, **c** 143 mm, **d** 325 cm **4 a** 6.4 m, **b** 11.3 m, **c** 13.6 m **5** 3.84 cm **6 a** 7π cm,
b 13π cm, **c** 20π cm, **d** 62π cm **7 a** $d = 8$ mm, $r = 4$ mm, **b** $d = 22$ mm, $r = 11$ mm, **c** $d = 36$ mm, $r = 18$ mm,
d $d = 50$ mm, $r = 25$ mm **8** 66 km **9** 510 **10 a** 377 m, **b** 100 **11** 45.55 m **12** 20.4 cm **13** 96 cm
14 a 82.3 m, **b** 191.8 m, **c** 42.8 m, **d** 19.2 m, **e** 34.4 m, **f** 26.1 m, **g** 89.6 m, **h** 194.7 m, **i** 374.6 m
15 a 84.3 cm, **b** 45.7 cm, **c** 33.2 cm, **d** 33.1 cm, **e** 121.5 cm, **f** 25.7 cm, **g** 64.9 cm, **h** 125.8 cm, **i** 150.8 cm,
j 57.8 cm, **k** 58.3 cm, **l** 50.0 cm **16 a** 246.5 m, **b** 93.7 m, **c** 86.4 m **17** 36 cm **18 a** 44.4 cm, **b** 27.1 cm,
c 58.6 cm

Exercise 5.6

1 **a** 100, **b** 10 000, **c** 1 000 000, **d** 0.01, **e** 0.0001, **f** 0.000 001 **2 a** 30 000 cm^2 , **b** 7 000 000 m^2 , **c** 600 mm^2 ,
d 1 240 000 m^2 , **e** 45 000 cm^2 , **f** 970 mm^2 , **g** 25 600 cm^2 , **h** 1875 mm^2 , **i** 160 000 m^2 , **j** 40 mm^2 , **k** 57 000 m^2 ,
l 13 cm^2 **3 a** 4 cm^2 , **b** 9 m^2 , **c** 5 km^2 , **d** 2.8 m^2 , **e** 6.5 cm^2 , **f** 7.4 km^2 , **g** 1.98 cm^2 , **h** 3.28 km^2 , **i** 4.339 m^2 ,
j 0.7 m^2 , **k** 0.15 cm^2 , **l** 1.0956 km^2 **4 a** 500, **b** 7, **c** 80 000, **d** 6.4, **e** 15, **f** 9 000 000, **g** 567 000, **h** 0.37, **i** 90,
j 0.816, **k** 200, **l** 45 000, **m** 0.003, **n** 0.0006, **o** 0.078 **5 a** 10 000, **b** 40 000, **c** 95 000, **d** 2, **e** 7.5, **f** 36,
g 5000, **h** 0.6, **i** 0.09, **j** 12 600, **k** 0.0003, **l** 200 **6 a** 3 000 000 mm^2 , **b** 500 mm^2 , **c** 0.169 m^2 , **d** 0.75 m^2 ,
e 0.002 17 km^2 , **f** 744 000 mm^2 **7 a** 475 000 cm^2 , **b** 0.0012 ha

Exercise 5.7

1 a 27, b 43 2 a 64 cm^2 , b 31.36 cm^2 , c 65 cm^2 , d 48.16 cm^2 3 a 45 m^2 , b 72 m^2 , c 37.8 m^2
 4 a 280 cm^2 , b 66 cm^2 , c 51.24 cm^2 , d 60 cm^2 , e 23.2 cm^2 , f 74.8 cm^2 5 a 44 m^2 , b 99 m^2 , c 33.215 m^2 ,
 d 27 m^2 , e 27.54 m^2 , f 63.51 m^2 6 a 32 mm^2 , b 202.5 mm^2 , c 169.6 mm^2 , d 30 mm^2 , e 104.8 mm^2 ,
 f 108.33 mm^2 7 a 88 cm^2 , b 96.75 cm^2 , c 80.34 cm^2 , d 24 cm^2 , e 60.68 cm^2 , f 241.25 cm^2 8 a 72.25 m^2 ,
 b 161.5 cm^2 , c 182.4 mm^2 , d 168 m^2 , e 73.45 m^2 , f 199.5 cm^2 , g 83.85 mm^2 9 0.48 m^2 10 a $\frac{1}{2}$, b 33 cm^2 ,
 c 48 cm^2 11 a 289 m^2 , b 56 m , c 12 m 12 a 8 m , b 96 m^2 , c 62 m 13 a It is a quadrilateral with opposite
 sides parallel, b 52 cm^2 14 180 mm^2 15 240 mm^2 16 a 40 cm , b 1680 cm^2 17 a $x = 12$, b 150 cm^2
 18 56 cm^2 19 a 54 mm , b 1944 mm^2 20 a 32 mm , b 80 mm 21 a 112.5 ha , b 18.7 ha , c 5.4 ha
 22 a $9y^2$, b $10a^2 + 15a$, c $4n^2 + 40n$, d $12p^2 + 21p$, e $24x^2 + 30x$, f $5cd + 15d$ 23 a $t = 19$, b $y = 18$, c $p = 15$,
 d $k = 7$, e $m = 11$, f $u = 9$ 24 960 cm^2 25 b 30 cm^2 26 a 17.3 mm^2 , b 47.9 mm^2 , c 56.7 mm^2

Exercise 5.8

1 a 201.1 cm^2 , b 530.9 cm^2 , c 176.7 cm^2 , d 408.3 cm^2 , e 1017.9 cm^2 , f 490.9 cm^2 , g 2091.2 cm^2 ,
 h 10806.5 cm^2 2 a 113.1 cm^2 , b 28532.2 mm^2 , c 1385.4 cm^2 , d 301.7 m^2 3 a 7 mm , b 12 mm , c 37 mm
 4 47 cm 5 a $9\pi \text{ cm}^2$, b $49\pi \text{ mm}^2$, c $361\pi \text{ m}^2$ 6 a $16\pi \text{ m}^2$, b $81\pi \text{ cm}^2$, c $256\pi \text{ mm}^2$ 7 a 3 cm , b 16 cm
 8 a $25\pi \text{ cm}^2$, b $64\pi \text{ cm}^2$, c $169\pi \text{ cm}^2$ 9 a $12\pi \text{ cm}$, b $22\pi \text{ cm}$, c $34\pi \text{ cm}$ 10 a 157.1 m^2 , b 88.4 m^2 ,
 c 50.3 m^2 , d 163.6 m^2 , e 320.9 m^2 , f 11.6 m^2 , g 3094.3 m^2 , h 143.4 m^2 , i 240.8 m^2 11 a 4.5 cm , b 15.2 cm
 12 semicircle 13 a 10 cm , b 78.5 cm^2 14 340 cm^2 15 a 75.4 cm^2 , b 23.6 cm^2 , c 141.4 cm^2 16 12 cm

Exercise 5.9

1 a 66 cm^2 , b 191 cm^2 , c 96 cm^2 , d 95 cm^2 , e 91 cm^2 , f 268 cm^2 2 a 105 cm^2 , b 447 mm^2 3 a 117 cm^2 ,
 b 38 cm^2 , c 44 cm^2 , d 88 cm^2 , e 82.5 cm^2 , f 260 cm^2 4 a 109 m^2 , b 73.5 m^2 , c 33.07 m^2 , d 51 m^2 , e 44.71 m^2 ,
 f 7.5 m^2 , g 219 m^2 , h 80 m^2 , i 41.45 m^2 5 a 263.9 mm^2 , b 42.1 mm^2 , c 308.5 mm^2 , d 34.8 mm^2 , e 343.5 mm^2 ,
 f 214.0 mm^2 , g 647.9 mm^2 , h 230.7 mm^2 , i 11.6 mm^2 , j 1093.1 mm^2 , k 33.9 mm^2 , l 27.1 mm^2 6 776.6 mm^2
 7 $24\pi \text{ cm}^2$

Exercise 5.10

1 $\frac{1}{15}$ 2 73.9 m^2 3 $1350\pi \text{ cm}^2$ 4 63 m^2 5 3850 cm^2 6 160 m^2 7 a 3.1 m^2 , b $\$967.20$ 8 a 48.69 m^2 ,
 b 10 L 9 a 4, b 240 m^2 , c $\$2880$ 10 6.1 mm 11 2084 m^2 12 a 78.42 m^2 , b $\$1764.45$ 13 a 576 ha ,
 b $\$806400$ 14 48.3 m^2 15 $\frac{11}{24}$ 16 128 cm^2

Chapter 5 Review

1 a min, b km, c g, d mm, e mL, f kg, g h, h L 2 a 4.1 m , b 2300 m , c 10.6 cm , d 0.52 m , e 0.075 km ,
 f 0.6 mm , g 34.8 m , h 2 cm 3 a 3900 mm , b 56000 cm 4 a 3.64 t , b 1800 mg , c 900 kg , d 0.31 kg ,
 e 86000 mg , f 3 kg 5 a 1.67 L , b 1400 L , c 0.42 kL , d 5671 mL , e 8.7 L , f 0.059 L 6 a 600 , b 39000 ,
 c 5.8 , d 4000 , e 7.2 , f 0.041 7 a 80000 m^2 , b 4.5 ha , c 6400 m^2 , d 0.731 ha 8 a $\pm 5 \text{ km/h}$, b $\pm 0.5 \text{ cm}$,
 c $\pm 500 \text{ m}$ 9 a 75 g and 85 g , b 2750 and 2850 years, c 35.5 mm and 36.5 mm 10 a 8.5 mm and 9.5 mm ,
 b 205.5 L and 206.5 L , c 1.75 g and 1.85 g , d 17.05 m and 17.15 m 11 a 240 s , b 40 h , c 1.5 h , d 36 min ,
 e 1.25 min , f 9 days 12 a 2.395 h , b $7 \text{ h } 12 \text{ min } 18 \text{ s}$ 13 a $2 \text{ h } 45 \text{ min}$, b $3 \text{ min } 45 \text{ s}$ 14 a $05:00 \text{ h}$,
 b $21:00 \text{ h}$, c $01:50 \text{ h}$, d $23:26 \text{ h}$ 15 a $8:00 \text{ am}$, b $2:00 \text{ pm}$, c $6:20 \text{ am}$, d $10:38 \text{ pm}$ 16 $6 \text{ h } 45 \text{ min}$ 17 4400
 18 76.26 kg 19 71 20 $2 \text{ h } 40 \text{ min}$ 21 1.26 L 22 84.6 mm 23 7.2 m 24 34.4 m 25 B 26 a $k = 39$,
 b $t = 61.5$ 27 a 32.31 mm , b 82.9 mm 28 $x = 1.6$, $y = 2$ 29 a 266 cm , b 80 cm 30 a 50.3 cm , b 320.4 cm
 31 a 17.19 cm , b 9.71 cm 32 1384 33 a 56.6 mm , b 20.4 mm , c 54.9 mm , d 80.8 mm 34 a 108 m^2 ,
 b 193.2 m^2 , c 230 m^2 , d 42 m^2 , e 111 m^2 , f 262.2 m^2 35 a 361 cm^2 , b 52 cm 36 a 40 cm , b 292.5 cm^2
 37 29.4 m^2 38 a $k = 18$, b $y = 11$ 39 a 380.13 cm^2 , b 132.73 cm^2 40 a $49\pi \text{ cm}^2$, b $10\pi \text{ cm}$
 41 a 99.63 m^2 , b 164.74 m^2 42 a 251.3 cm^2 , b 378.3 cm^2 , c 32.0 cm^2 43 a 146.67 m^2 , b $\$3138.74$

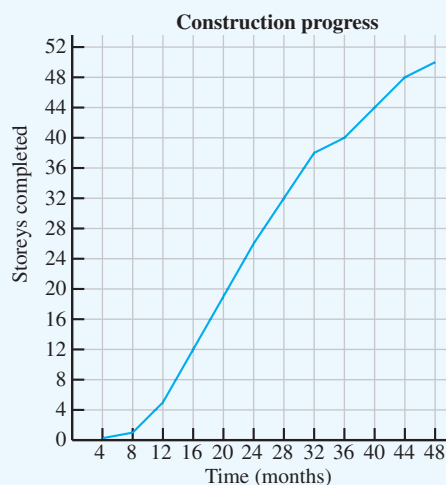
6

Data representation and analysis

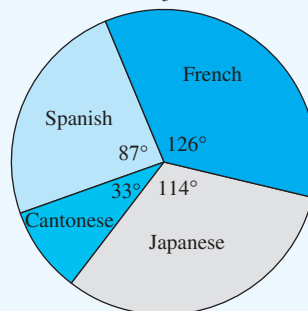
Exercise 6.1

- 1 a Big Mac, b Cheeseburger, c Fillet of fish, d 120, e $18\frac{1}{3}\%$, f 4 burgers per minute 2 a Penelope, b French, c History, d Art, e 7, f History 3 b 8, c 11, d Diane, e Dave by 1 hour, f Diane on Wednesday.
 4 a Black, b i Black, ii Blue, c 2, d Black, e Yellow, f $\frac{5}{7}$ 6 a 3, b Rugby league, c 90, d $\frac{3}{25}$, e 5%
 8 a Picture graph, b i 140 000, ii 100 000, c 40 000, d 2001, 2002, e 2000 and 2003, f 1998 9 a 16 L, b Tank was filled with petrol, c 32 L, d 9 am, 1 pm, e 17 L, f 18 L, g 0 L, Car is stationary, h 4 pm, 5 pm, Greatest use of petrol

10



- 11 a Pie chart, b i Western suburbs, ii Southern suburbs, c 2, d 9, e 20%, f $\frac{1}{5}$
 12



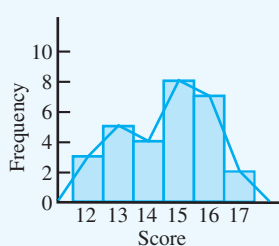
- 13 a 25, b Max, c Janine sold 35, d Nerida sold 18, e 13

Exercise 6.2

1 a

Score	Tally	Frequency
12	III	3
13	IIII	5
14	IIII	4
15	IIII III	8
16	IIII II	7
17	II	2
Total:		29

b



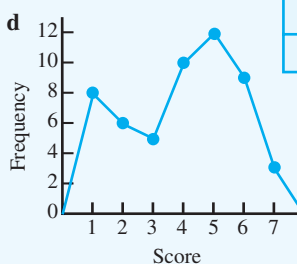
2 a

Score	Tally	Frequency
2	I	1
3	II	2
4	II	2
5	II	2
6	IIII II	7
7	III	3
8	III	3
9	III	3
10	I	1
Total:		24

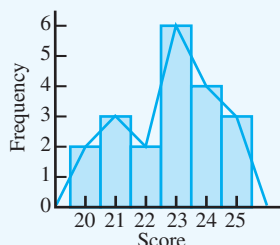
- b 24, c 10, 2, d 6, e 29% 3 a 10, b 19, c 53, d 4 a 5, b 2, c 6, d 38%

5 a

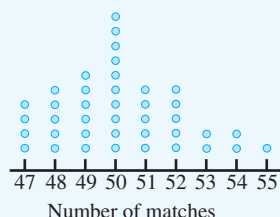
Score	20	21	22	23	24	25
Frequency	2	3	2	6	4	3



b $\frac{11}{20}$, c



6 a



b 50 7 a i Yes, around 5, ii Yes, 1, b i Yes, around 96, ii Yes, 100

8	Stem	Leaf
	10	3 4 6 9
	11	1 2 4 7 7 8
	12	0 1 2 3 5 6 8 8 9
	13	0 1 2 4 6 7 7
	14	0 1 4 5 6 9

9 a 25, b 68, c 52, d 48%

10 a 22°, b 11, c Winter

11

Age	12	13	14	15	16	17
Number of students	3	5	9	18	10	5

b 15 year olds, c 14.8 years

Exercise 6.3

1 a 6.7, b 15.9, c 57, d 23.9, e 46.2, f -0.5 2 a 20, b 6, c 132, d 14.8, e 6, f 43, g 5.5, h 13.8 3 a 8, b 11, c 39, 42, d 106, 110 4 a 8, b 17, c 7.9, d 18 5 a mean = 33.7, median = 34, mode = 30, range = 8, b mean = 241.4, median = 240, mode = 251, range = 94, c mean = 12.1, median = 10.6, mode = 5.9, range = 14.7, d mean = -1.3, median = -1.5, mode = none, range = 139 6 a 3.24, b 12.71, c 49.99, d 97.16 7 a median = 3, mode = 4, range = 4, b median = 13, mode = 16, range = 5, c median = 50, mode = 48, range = 4, d median = 96, mode = 95, range = 5 8 a 33, b 46, c 25, d 78

9 a

x	Tally	f	fx
10	III	3	30
11	IIII	5	55
12	IIII II	7	84
13	IIII	4	52
14	IIII IIII	10	140
15	IIII III	8	120
Totals:		37	481

mode = 14, mean = 13,

10 a

x	Tally	f	fx
0	IIII II	7	0
1	IIII IIII III	13	13
2	IIII II	7	14
3	IIII	5	15
4	II	2	8
5	II	2	10
Totals:		36	60

b

x	Tally	f	fx
54	IIII IIII	10	540
55	IIII	4	220
56	IIII II	7	392
57	IIII	3	171
58	IIII IIII III	14	812
59	IIII IIII	9	531
Totals:		47	2666

mode = 58, mean = 56.7

b 1.7, c 1, d 1, e 19% 11 a mean = 4.2, median = 5, mode = 6, range = 5, b mean = 3.8, median = 5, mode = 5, range = 5

12 a mean = 16.4, mode = 15, b The two scores of 11,

c 16.8 13 a 38, b 42, c 36, d 36.6, e 60

14 a Stem Leaf b 114 cm, 125 cm, c 24 cm, d 114 cm

Stem	Leaf
10 ⁽⁰⁾	1 4
10 ⁽⁵⁾	5 6 6 7 8 9
11 ⁽⁰⁾	0 2 2 3 4 4 4
11 ⁽⁵⁾	6 7 7 8 8 9
12 ⁽⁰⁾	0 0 1 2 3
12 ⁽⁵⁾	5 5 5

15 a

Stem	Leaf
2 ⁽⁰⁾	4
2 ⁽⁵⁾	7 8 8
3 ⁽⁰⁾	1 2 2 3 3 4
3 ⁽⁵⁾	5 5 6 6 6 7 8
4 ⁽⁰⁾	0 1 2 3 4
4 ⁽⁵⁾	5 6 7 7 8

b 36, c 36

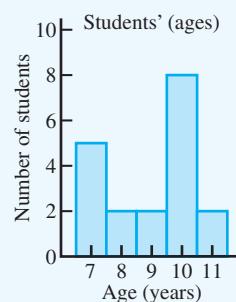
16 a Anything but 4°C or 5°C,

b 2°C or 8°C,

c 4°C or less,

d 0°C

17



Exercise 6.4

1 a 6, b 39, c 29.2 2 a 36, b 1254, c 448 3 a 4, b 36, c 60 4 a 10, b 16 5 a 25.5, b 23.6, c 24

6 a increase, **b** decrease, **c** stay the same **7 a** 15.1, **b** 16.2, **c** 16 **8 a** decrease, **b** increase, **c** stay the same

9 752 kg **10** 88 **11** 25 **12** 73 **13 a** It will lower the class average, **b** 71.8%, **c** It will increase the average,

d 73.7% **14 a** 27, **b** 63 **15 a** 67, **b** 9 **16** 84% **17** \$457 500

Exercise 6.5

1 a

Number of tails	Tally	f	cf
2	III	3	3
3	IIII	5	8
4	IIII IIII	9	17
5	IIII IIII IIII	15	32
6	IIII IIII I	11	43
7	IIII	4	47
8	III	3	50
$\Sigma f =$		50	

b i 9, ii 17, iii 8, iv 11, v 43, vi 32, vii 3, viii 50, ix 47

2 a

x	Tally	f	cf
1	IIII II	7	7
2	III	3	10
3	IIII II	7	17
4	IIII I	6	23
5	IIII IIII	10	33
6	IIII IIII	10	43
7	IIII I	6	49
$\Sigma f =$		49	

median = 5,

b

x	Tally	f	cf
100	IIII II	7	7
101	IIII IIII	9	16
102	IIII IIII IIII II	17	33
103	IIII IIII IIII	15	48
104	IIII IIII	8	56
105	IIII I	6	62
106	IIII	4	66
$\Sigma f =$		66	

median = 102.5

3 a

x	Tally	f	cf
18	III	3	3
19	III	3	6
20	IIII I	6	12
21	IIII	4	16
22	IIII III	8	24
23	IIII II I	11	35
24	III	3	38
25	II	2	40
$\Sigma f =$		40	

b i 6, ii 35, iii 16, iv 28, c median = 22

4 a

x	Tally	f	cf
7	III	3	3
8	IIII II	7	10
9	IIII	5	15
10	IIII	4	19
11	III	3	22
12	II	2	24
13	III	3	27
14	I	1	28
15	II	2	30
$\Sigma f =$		30	

b 30, c i $\frac{2}{15}$, ii $\frac{1}{3}$,
 iii $\frac{11}{15}$, d 80%,
 e 9.5 min

5 a

x	f	cf
1	2	2
2	4	6
3	3	9
4	3	12
5	1	13
6	4	17
7	6	23
8	2	25
$\Sigma f =$		25

b 5, c 10

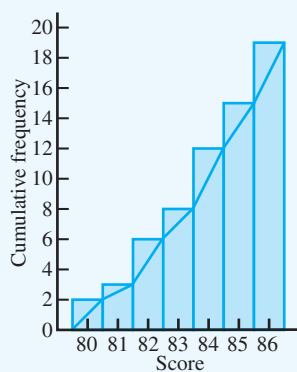
6 a

x	f	cf
1	2	2
2	1	3
3	3	6
4	2	8
5	1	9
6	3	12
$\Sigma f =$		12

b

x	f	cf
13	3	3
14	2	5
15	1	6
16	1	7
17	4	11
18	1	12
$\Sigma f =$		12

7



8 a 3, b 10, c 22.5, d 4, e 19, f 51

9 9

10

x	f	cf	fx
1	3	3	3
2	4	7	8
3	4	11	12
4	9	20	36
5	7	27	35
6	10	37	60
7	5	42	35
$\Sigma f =$		42	$\Sigma fx =$
			189

median = 5

Exercise 6.6**1** a 6, b 12, c 23.5, d 32.5

2 a

Class	cc	Tally	f	$f \times cc$
1–5	3	III	3	9
6–10	8	IIII	5	40
11–15	13	IIII IIII	9	117
16–20	18	IIII IIII IIII I	16	288
21–25	23	IIII IIII II	12	276
26–30	28	IIII II	7	196
Totals:			52	926

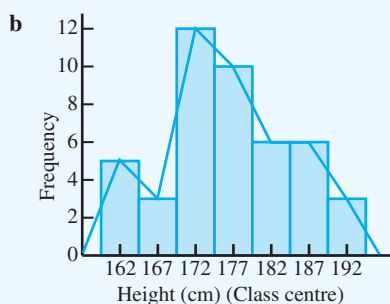
3 a

Class	cc	Tally	f	$f \times cc$
20–24	22	IIII	5	110
25–29	27	III	3	81
30–34	32	IIII I	6	192
35–39	37	IIII II	7	259
40–44	42	IIII	5	210
45–49	47	IIII	4	188
Totals:			30	1040

b Σf = no. of scores, $\Sigma(f \times cc)$ = approx. sum of scores b 35–39, c 30, d 34.7

4 a

Class	cc	Tally	f	$f \times cc$
160–164	162	IIII	5	810
165–169	167	III	3	501
170–174	172	IIII IIII II	12	2064
175–179	177	IIII IIII	10	1770
180–184	182	IIII I	6	1092
185–189	187	IIII I	6	1122
190–194	192	III	3	576
Totals:			45	7935

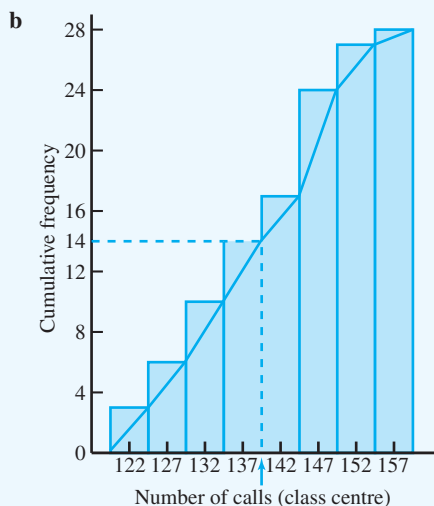
c 170–174, d 176.3 cm **5** a 182, b 4, 11, 18, 25, 32, 39, c i 3 437 000, ii 18 885 **6** a Too many different scores,b

Class	cc	f	$f \times cc$
0–9	4.5	3	13.5
10–19	14.5	8	116
20–29	24.5	10	245
30–39	34.5	12	414
40–49	44.5	7	311.5
Totals:		40	1100

, c 27.5, d 30–39, e 20–29 **7** a 20–24, 25–29, 30–34, 35–39, 40–44, 45–49, b 49, c 35–39, d 34 **8** a 16, b 19.5

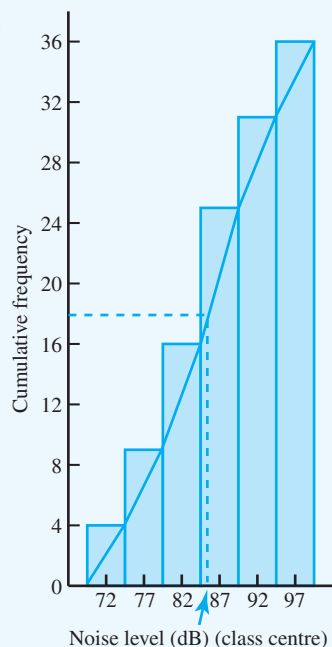
9 a

Class	cc	Tally	f	cf
120–124	122	III	3	3
125–129	127	III	3	6
130–134	132	IIII	4	10
135–139	137	IIII	4	14
140–144	142	III	3	17
145–149	147	IIII II	7	24
150–154	152	III	3	27
155–159	157	I	1	28
$\Sigma f =$			28	



c 139.5

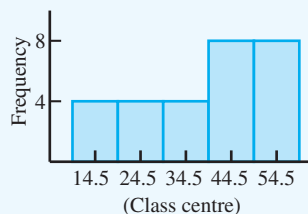
10 a



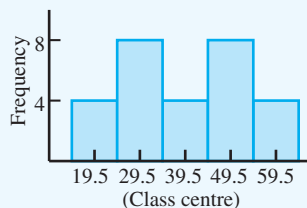
b 85.5 11 Too large.

12 a 38.82,

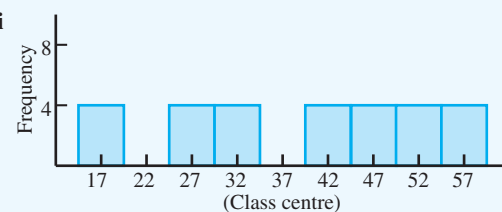
b i



ii



iii



c i 40–49, 50–59 ii 25–34, 45–54, iii 15–19, 25–29, 30–34, 40–44, 45–49, 50–54, 55–59,

d i 38.79, ii 39.5, iii 39.14, e iii, the classes are smaller

Chapter 6 Review

1 a NSW, b Vic., c 19 600 000, d No, total of angles is less than angle representing Qld, e 28° , f 3% 2 a 17,

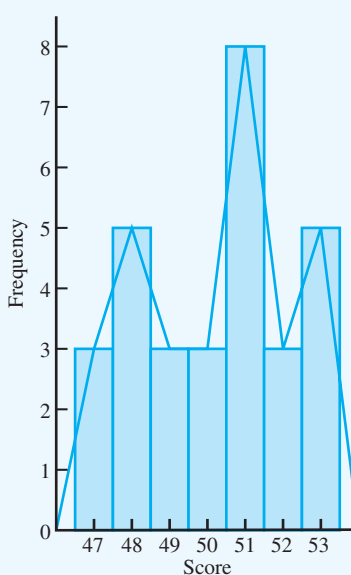
b 4, c Jewish women, d 20%, e 1, f Anglican, Jewish 3 a mean = 10.2, median = 9, no mode, range = 9,

b mean = 24, median = 23.5, mode = 26, range = 10, c mean = 21, median = 19, mode = 11, 26, range = 29

4 a

x	Tally	f	cf	fx
47	III	3	3	141
48	IIII	5	8	240
49	III	3	11	147
50	III	3	14	150
51	IIII III	8	22	408
52	III	3	25	156
53	IIII	5	30	265
		30		1507

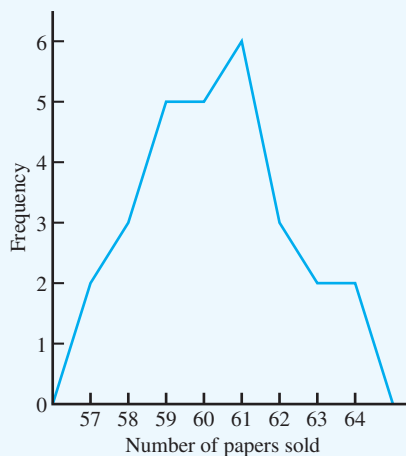
b



c 47, d 51, e 11, f 50.2

5 $\Sigma fx = 160$, mean = 4

6 a 64, b 61, c 7, d 1689, e 60.3, f No g



7 a 36, b 32, c 21.5, d 16 and 18, e

Stem	Leaf
0 ⁽⁵⁾	7 7 8 9
1 ⁽⁰⁾	3 4 4
1 ⁽⁵⁾	5 6 6 6 8 8 8 9
2 ⁽⁰⁾	0 0 1 2 2 3 3 4
2 ⁽⁵⁾	5 7 7 8 9
3 ⁽⁰⁾	1 1 2 3 4 4
3 ⁽⁵⁾	6 9

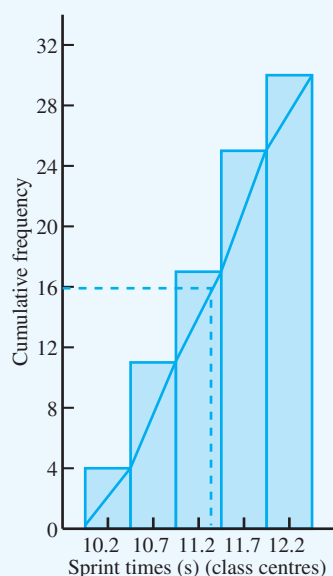
8 a mean = 9.7, median = 9, mode = 8, range = 4, b mean = 4.1, median = 4.5, mode = 5, range = 5 9 a 135, b 22 10 13 11 a 40.1, b 42.6 12 a Late night shopping, b 15 13 a i stay the same, ii decrease, iii increase, b stay the same, ii increase, iii decrease 14 a 39, b 19 15 a 61, b 22–28, c 49, d 43%, e 27 16 a 95, b 33.5

17 a

Class	cc	Tally	f	cf	f × cc
10.0–10.4	10.2	IIII	4	4	40.8
10.5–10.9	10.7	IIII II	7	11	74.9
11.0–11.4	11.2	IIII I	6	17	67.2
11.5–11.9	11.7	IIII III	8	25	93.6
12.0–12.4	12.2	IIII	5	30	61
Totals:			30		337.5

b 11.5–11.9, c 17, d 11.25 s,

e



f 11.3 s

7 Probability

Exercise 7.1

2 a blue, green, yellow, **b** No, **c** i not very likely, **ii** very likely, **iii** impossible, **iv** certain **3** 3 red, 1 green, 1 blue
4 a i 1, 2, 3, 4, 5, 6, **ii** Yes, **b** i 17, 18, 19, **ii** Yes, **c** i friend 1, friend 2, friend 3, **ii** No, **d** i 0, 5, 7, 9, **ii** No,
e i 0, 5, 7, 9, 10, 12, 14, 16, 18, **ii** No **10 a** Yes, **b** No, **c** No **11 a** No, **b** No, **c** No, **d** Yes, **e** Yes
12 a i 24, **ii** No, **b** i Yes, **ii** No

Exercise 7.2

2 a Varia $\frac{14}{100}$, $\frac{10}{100}$, Thanh $\frac{15}{100}$, $\frac{16}{100}$, Seeza $\frac{16}{100}$, $\frac{20}{100}$, Roger $\frac{18}{100}$, $\frac{23}{100}$, Leigh $\frac{9}{100}$, $\frac{15}{100}$, Lee-Ann $\frac{7}{100}$, $\frac{15}{100}$, Greg $\frac{21}{100}$, $\frac{1}{100}$
3 a No, **b** i 0.12, **ii** 0.36, **iii** 0.38, **iv** 0.14 **4 a** A $\frac{60}{80}$, B $\frac{34}{50}$, C $\frac{48}{60}$, D $\frac{50}{60}$, E $\frac{49}{70}$, F $\frac{41}{50}$, G $\frac{72}{90}$, H $\frac{68}{100}$, I $\frac{68}{80}$, J $\frac{50}{75}$,
b A 0.75, B 0.68, C 0.8, D 0.83, E 0.7, F 0.82, G 0.8, H 0.68, I 0.85, J 0.67, **c** Player I

Exercise 7.4

1 a $\frac{1}{2}$, **b** $\frac{1}{4}$, **c** $\frac{1}{8}$ **2 a** $\frac{1}{52}$, **b** $\frac{1}{13}$, **c** $\frac{1}{4}$, **d** $\frac{1}{2}$, **e** $\frac{3}{13}$, **f** $\frac{1}{26}$ **3 a** Mrs Alix: $\frac{2}{5}$, Mr Steptoe: $\frac{3}{10}$, Mr Naba: $\frac{1}{20}$,
Mrs Naba: $\frac{1}{20}$, Dr Georgiou: $\frac{1}{5}$, **b** Mrs Alix, **d** Mrs Alix: 400, Mr Steptoe: 300, Mr Naba: 50, Mrs Naba: 50,
Dr Georgiou: 200 **4** 40, No **5 a** $\frac{1}{4}$, **b** $\frac{1}{4}$, **c** $\frac{3}{4}$, **d** $\frac{1}{2}$, **e** 0, **f** 1 **6 a** Red = 3 faces, Yellow = 2 faces,
White = 1 face, **b** i Red = 30, Yellow = 20, White = 10 **7 a** $\frac{1}{8}$, **b** $\frac{1}{2}$, **c** $\frac{5}{8}$, **d** $\frac{1}{4}$ **8 a** 1, **b** 0, **c** $\frac{1}{5}$, **d** $\frac{3}{20}$, **e** $\frac{9}{20}$,
f $\frac{9}{10}$ **9** 42. Not necessarily. **10** 10 silver, 8 gold, 1 white, 1 black **11** 3 red, 6 blue, 15 green **12 a** $\frac{1}{4}$, **b** $\frac{1}{13}$,
c $\frac{1}{52}$ **13** $\frac{7}{26}$ **15 a** $\frac{1}{12}$, **b** $\frac{1}{3}$, **c** $\frac{1}{6}$, **d** 0 **16** $\frac{1}{6}$ **17** Alicia. Her probability of winning is $\frac{7}{12}$.

Chapter 7 Review

1 a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, **b** No, **i** 9 and 10, **ii** 20

4 a

Prize	Tally	Total	Experimental probability
Car	III	3	$\frac{1}{20}$
Holiday	IIII	4	$\frac{1}{15}$
TV	IIII I	6	$\frac{1}{10}$
Dinner	IIII III	18	$\frac{3}{10}$
Concert	IIII III	14	$\frac{7}{30}$
Spin	IIII	10	$\frac{1}{6}$
No prize	IIII	5	$\frac{1}{12}$

b Car 1, Holiday 1, TV 2, Dinner 6, Concert 4, Spin 3, No prize 2

8 a $P(G) = \frac{1}{4}$, **b** $P(R) = \frac{5}{12}$, **c** $P(B) = \frac{1}{6}$,

d $P(Y) = \frac{1}{12}$, **e** $P(W) = \frac{1}{12}$ **9 a** 1, **b** $\frac{1}{4}$,

c $\frac{3}{8}$, **d** $\frac{1}{4}$ **10** Crimson = 3, Aqua = 4, Hyacinth = 1, Lemon = 2, White = 2, Black = 0. Other answers are possible.

8

Surds

Exercise 8.1

1 a yes, b no, c yes, d yes, e no, f yes, g no, h no, i yes, j yes, k yes, l no, m yes, n yes, o yes
 2 a rational, b rational, c irrational, d rational, e rational, f irrational, g rational, h rational, i irrational, j rational, k rational, l irrational, m rational, n irrational, o rational, p rational, q rational, r rational, s irrational, t rational, u irrational, v irrational, w rational, x rational, y irrational 3 a rational, b rational, c irrational, d irrational, e rational, f rational, g rational, h rational, i irrational, j rational, k irrational, l rational, m rational, n irrational, o rational, p rational 4 a 2, 3, b 4, 5, c 8, 9, d 12, 13 5 a 2.6, b 2.4, c 2.6, d 2.3, e 3.2, f 9.3, g 16.2, h 9.5, i 0.4, j 10.6, k 0.3, l 0.5, m 2.1, n 1.0, o 2.2, p -9.9
 6 a 3.4, $\sqrt{13}$, 3.7, $\sqrt{15}$, b $\sqrt{83}$, 9.15, 9.6, $\sqrt{96}$, c 3.6, 4, $3\sqrt{2}$, $\sqrt[3]{90}$, d $3\sqrt{5}$, $\frac{41}{6}$, $4\sqrt{3}$, 7,
 e $2\sqrt{6}$, 5, $\sqrt{26}$, $3\sqrt{3}$, f $12 - \sqrt{11}$, $4\sqrt{5}$, $1 + \sqrt{67}$, 10.1 9 a $\sqrt{7}$ b $(\sqrt{11} - \sqrt{7})$ units 10 a π units,
 b Roll the coin along the line through one complete revolution 11 a always, b always, c sometimes,
 d sometimes, e sometimes, f sometimes

Exercise 8.2

1 a $\sqrt{10}$, b $\sqrt{21}$, c $\sqrt{14}$, d $\sqrt{15}$, e $\sqrt{22}$, f $\sqrt{39}$, g $\sqrt{30}$, h $\sqrt{55}$ 2 a 2, b 3, c 7, d 11, e 8, f 48, g 50,
 h 63, i 5, j 13, k 20, l 32 3 a $2\sqrt{2}$, b $2\sqrt{3}$, c $3\sqrt{2}$, d $2\sqrt{5}$, e $2\sqrt{6}$, f $3\sqrt{3}$, g $2\sqrt{7}$, h $4\sqrt{2}$, i $2\sqrt{10}$,
 j $3\sqrt{5}$, k $4\sqrt{3}$, l $5\sqrt{2}$, m $3\sqrt{6}$, n $2\sqrt{15}$, o $3\sqrt{7}$, p $6\sqrt{2}$, q $5\sqrt{3}$, r $4\sqrt{5}$, s $2\sqrt{21}$, t $3\sqrt{10}$, u $4\sqrt{6}$
 v $3\sqrt{11}$, w $5\sqrt{6}$, x $10\sqrt{2}$ 4 a $10\sqrt{3}$, b $12\sqrt{5}$, c $6\sqrt{3}$, d $8\sqrt{7}$, e $24\sqrt{5}$, f $44\sqrt{3}$, g $50\sqrt{2}$, h $9\sqrt{7}$,
 i $24\sqrt{2}$, j $30\sqrt{3}$, k $6\sqrt{22}$, l $50\sqrt{3}$ 5 a $8\sqrt{2}$, b $4\sqrt{10}$, c $5\sqrt{7}$, d $11\sqrt{2}$, e $9\sqrt{3}$, f $7\sqrt{5}$, g $12\sqrt{2}$,
 h $6\sqrt{11}$, i $9\sqrt{5}$, j $8\sqrt{7}$, k $15\sqrt{3}$, l $12\sqrt{5}$ 6 a $\sqrt{8}$, b $\sqrt{18}$, c $\sqrt{20}$, d $\sqrt{27}$, e $\sqrt{32}$, f $\sqrt{44}$, g $\sqrt{45}$,
 h $\sqrt{48}$, i $\sqrt{50}$, j $\sqrt{52}$, k $\sqrt{54}$, l $\sqrt{56}$, m $\sqrt{60}$, n $\sqrt{63}$, o $\sqrt{72}$, p $\sqrt{75}$, q $\sqrt{80}$, r $\sqrt{88}$, s $\sqrt{90}$, t $\sqrt{96}$
 7 a $k = 12$, b $a = 98$, c $m = 4$, d $y = 11$ 8 a $3\sqrt{a}$, b $5\sqrt{a}$, c $a\sqrt{a}$, d $a^2\sqrt{a}$, e $a\sqrt{b}$, f $b\sqrt{a}$, g $a^2\sqrt{b}$,
 h $b^3\sqrt{a}$, i $ab^2\sqrt{a}$, j $a^4b^2\sqrt{b}$, k $a^2b\sqrt{ab}$, l $a^4b^3\sqrt{ab}$, m $2a\sqrt{a}$, n $2a^2\sqrt{3}$, o $3a^2\sqrt{2a}$, p $3a\sqrt{3b}$,
 q $6b^2\sqrt{a}$, r $3ab\sqrt{5a}$, s $4a^2b\sqrt{3}$, t $5a^3b^5\sqrt{2ab}$ 9 a $\sqrt{49a}$, b $\sqrt{a^3}$, c $\sqrt{a^9}$, d $\sqrt{a^3b^2}$, e $\sqrt{25ab}$, f $\sqrt{8a}$,
 g $\sqrt{48a^5b}$, h $\sqrt{45a^5b^7}$

Exercise 8.3

1 a Yes, b No, c No, d No 2 a $7\sqrt{2}$, b $3\sqrt{3}$, c $9\sqrt{5}$, d $4\sqrt{11}$, e $3\sqrt{7}$, f $5\sqrt{3}$, g $2\sqrt{5}$, h 0, i $\sqrt{10}$,
 j $17\sqrt{11}$, k $5\sqrt{7}$, l $14\sqrt{2}$, m $-4\sqrt{3}$, n $-3\sqrt{5}$, o $-7\sqrt{6}$ 3 a $9\sqrt{2}$, b $6\sqrt{3}$, c $7\sqrt{10}$, d $3\sqrt{6}$, e $\sqrt{7}$,
 f $9\sqrt{5}$, g $3\sqrt{3}$, h $7\sqrt{11}$, i $-5\sqrt{10}$ 4 a $7\sqrt{2} + 3\sqrt{3}$, b $12\sqrt{5} + 10\sqrt{3}$, c $5\sqrt{7} + 4\sqrt{5}$, d $9\sqrt{6} + 6\sqrt{2}$,
 e $14\sqrt{3}$, f $\sqrt{10} + 9\sqrt{11}$, g $7\sqrt{7} - 4\sqrt{3}$, h $-8\sqrt{5} + 5\sqrt{2}$, i $17\sqrt{2} - 5\sqrt{5}$, j $15\sqrt{6} + 9\sqrt{10}$, k $2\sqrt{3} - 8\sqrt{7}$,
 l $-\sqrt{13} - 2\sqrt{2}$ 5 a $3\sqrt{2}$, b $\sqrt{3}$, c $3\sqrt{5}$, d $5\sqrt{3}$, e $\sqrt{5}$, f $2\sqrt{2}$, g $4\sqrt{7}$, h $8\sqrt{2}$, i $\sqrt{10}$, j $7\sqrt{5}$, k $3\sqrt{2}$,
 l $15\sqrt{2}$, m $14\sqrt{2}$, n $6\sqrt{7}$, o $7\sqrt{3}$, p $5\sqrt{5}$, q $10\sqrt{6}$, r $11\sqrt{7}$, s $-2\sqrt{6}$, t $5\sqrt{11}$, u $10\sqrt{13}$, v $30\sqrt{5}$,
 w $3\sqrt{2}$, x $17\sqrt{3}$ 6 a $5\sqrt{5} + 5\sqrt{3}$, b $\sqrt{7} + 4\sqrt{2}$, c $6\sqrt{2} + 16\sqrt{10}$, d $12\sqrt{3} - 2\sqrt{6}$, e $\sqrt{2} - 3\sqrt{5}$,
 f $34\sqrt{3} - 2\sqrt{10}$ 7 a $9\sqrt{p} + 2\sqrt{q}$, b $4\sqrt{u} - 2\sqrt{v}$, c $11\sqrt{x} - 7\sqrt{y}$, d $\sqrt{n}$ 8 a $7\sqrt{a}$, b $7\sqrt{p}$, c $\sqrt{3x}$,
 d $4k\sqrt{k}$, e $8m^2\sqrt{m}$, f $t\sqrt{2}$, g $y\sqrt{3y}$, h $3uv^3\sqrt{v}$, i $15cd^2\sqrt{3c}$

Exercise 8.4

1 a $\sqrt{15}$, b $\sqrt{22}$, c $\sqrt{35}$, d $\sqrt{30}$, e $14\sqrt{2}$, f $12\sqrt{5}$, g $30\sqrt{2}$, h $16\sqrt{3}$, i $20\sqrt{6}$, j $27\sqrt{21}$, k $28\sqrt{55}$,
 l $45\sqrt{39}$, m 5, n 17, o 6, p 75, q $\sqrt{30}$, r $\sqrt{105}$, s $8\sqrt{42}$, t $48\sqrt{70}$ 2 a $\sqrt{5}$, b $\sqrt{6}$, c $\sqrt{7}$, d $\sqrt{6}$, e $5\sqrt{3}$,

$f\ 3\sqrt{2}$, $g\ 5\sqrt{6}$, $h\ 8\sqrt{11}$, $i\ 2$, $j\ 5$, $k\ 3$, $l\ 4$, $m\ 8\sqrt{15}$, $n\ 3\sqrt{13}$, $o\ 4\sqrt{7}$, $p\ 9\sqrt{7}$, $q\ 5\sqrt{7}$, $r\ 6\sqrt{11}$, $s\ 9\sqrt{10}$,
 $t\ 8\sqrt{13}$ **3** $a\ 2\sqrt{3}$, $b\ 2\sqrt{5}$, $c\ 3\sqrt{2}$, $d\ 2\sqrt{6}$, $e\ 2\sqrt{7}$, $f\ 2\sqrt{10}$, $g\ 3\sqrt{5}$, $h\ 4\sqrt{3}$, $i\ 5\sqrt{2}$, $j\ 3\sqrt{6}$, $k\ 6\sqrt{2}$,
 $l\ 5\sqrt{3}$, $m\ 8\sqrt{3}$, $n\ 4\sqrt{6}$, $o\ 10\sqrt{10}$, $p\ 36\sqrt{11}$, $q\ 28\sqrt{14}$, $r\ 48\sqrt{5}$, $s\ 24\sqrt{10}$, $t\ 40\sqrt{6}$, $u\ 60\sqrt{11}$
4 $a\ 6\sqrt{3}$, $b\ 4\sqrt{15}$, $c\ 6\sqrt{6}$, $d\ 18\sqrt{2}$, $e\ 9\sqrt{35}$, $f\ 6\sqrt{33}$, $g\ 30$, $h\ 24$, $i\ 80$, $j\ 4\sqrt{21}$, $k\ 12\sqrt{10}$, $l\ 168\sqrt{3}$,
 $m\ 288\sqrt{2}$, $n\ 540\sqrt{10}$, $o\ 420\sqrt{14}$ **5** $a\ 2\sqrt{2}$, $b\ 2\sqrt{3}$, $c\ 2\sqrt{5}$, $d\ 2\sqrt{6}$, $e\ 2\sqrt{2}$, $f\ 3\sqrt{2}$, $g\ 3$, $h\ 2$, $i\ 5$, $j\ 6\sqrt{3}$,
 $k\ 12\sqrt{2}$, $l\ 6$, $m\ 8\sqrt{10}$, $n\ 21$, $o\ 20\sqrt{2}$ **6** $a\ \sqrt{ab}$, $b\ 15\sqrt{mn}$, $c\ \sqrt{xyz}$, $d\ 30\sqrt{pqr}$, $e\ 24\sqrt{70abc}$, $f\ k^2$,
 $g\ ab\sqrt{ab}$, $h\ 15c\sqrt{d}$, $i\ 48a$, $j\ 2\sqrt{7pq}$, $k\ 54m\sqrt{10}$, $l\ 240pq^3\sqrt{3p}$ **7** $a\ \sqrt{x}$, $b\ \sqrt{q}$, $c\ 5$, $d\ 8\sqrt{u}$, $e\ 2\sqrt{h}$,
 $f\ a^3$, $g\ q^2\sqrt{p}$, $h\ 35a\sqrt{a}$, $i\ 10x\sqrt{2x}$

Exercise 8.5

1 $a\ \sqrt{6}+5\sqrt{2}$, $b\ \sqrt{35}-2\sqrt{7}$, $c\ \sqrt{22}+6\sqrt{11}$, $d\ 10\sqrt{6}+2\sqrt{15}$, $e\ 8\sqrt{10}-4\sqrt{22}$, $f\ 18\sqrt{35}-27\sqrt{30}$,
 $g\ 3+2\sqrt{3}$, $h\ 6\sqrt{5}+5$, $i\ 7-\sqrt{14}$, $j\ 6+15\sqrt{2}$, $k\ 4\sqrt{15}-12$, $l\ 120-10\sqrt{30}$, $m\ 2\sqrt{3}+8\sqrt{2}$,
 $n\ 3\sqrt{5}-3\sqrt{2}$, $o\ 4\sqrt{3}-6\sqrt{2}$, $p\ 35\sqrt{3}+10\sqrt{6}$, $q\ 4\sqrt{5}-10\sqrt{2}$, $r\ 18\sqrt{5}-27\sqrt{10}$
2 $a\ \sqrt{10}+2\sqrt{2}+3\sqrt{5}+6$, $b\ \sqrt{21}-4\sqrt{3}+\sqrt{7}-4$, $c\ \sqrt{15}-6\sqrt{5}-2\sqrt{3}+12$, $d\ 14-2\sqrt{3}+7\sqrt{10}-\sqrt{30}$,
 $e\ \sqrt{30}+\sqrt{42}+\sqrt{10}+\sqrt{14}$, $f\ \sqrt{10}-\sqrt{55}+\sqrt{6}-\sqrt{33}$, $g\ 16+7\sqrt{6}$, $h\ 31-11\sqrt{3}$, $i\ \sqrt{10}-\sqrt{15}+2-\sqrt{6}$,
 $j\ 7-\sqrt{14}+\sqrt{7}-\sqrt{2}$, $k\ 6+3\sqrt{14}+\sqrt{10}+\sqrt{35}$, $l\ 10\sqrt{21}+15-2\sqrt{14}-\sqrt{6}$, $m\ 20-2\sqrt{15}+2\sqrt{55}-\sqrt{33}$,
 $n\ 26-8\sqrt{10}$, $o\ 2\sqrt{6}+2\sqrt{3}+10+5\sqrt{2}$, $p\ 8\sqrt{2}+4+2\sqrt{6}+\sqrt{3}$, $q\ 2\sqrt{5}+4\sqrt{2}-3\sqrt{10}-12$,
 $r\ 6-2\sqrt{6}+\sqrt{30}-2\sqrt{5}$, $s\ 30\sqrt{3}-36+30\sqrt{2}-12\sqrt{6}$, $t\ 12\sqrt{10}-50\sqrt{2}-48+40\sqrt{5}$ **3** $a\ 3+2\sqrt{2}$,
 $b\ 7-4\sqrt{3}$, $c\ 14+6\sqrt{5}$, $d\ 23+8\sqrt{7}$, $e\ 15-6\sqrt{6}$, $f\ 27-10\sqrt{2}$, $g\ 5+2\sqrt{6}$, $h\ 12+2\sqrt{35}$, $i\ 7-2\sqrt{10}$,
 $j\ 17-12\sqrt{2}$, $k\ 34+24\sqrt{2}$, $l\ 69-28\sqrt{5}$, $m\ 30+12\sqrt{6}$, $n\ 130-40\sqrt{10}$, $o\ 71+12\sqrt{14}$, $p\ 8+4\sqrt{3}$,
 $q\ 15-10\sqrt{2}$, $r\ 27$, $s\ 28-12\sqrt{5}$, $t\ 46-16\sqrt{7}$, $u\ 168+72\sqrt{5}$ **4** $a\ 1$, $b\ 3$, $c\ 23$, $d\ 13$, $e\ 7$, $f\ -6$, $g\ 11$,
 $h\ -8$, $i\ 11$, $j\ 14$, $k\ 22$, $l\ 88$, $m\ 11$, $n\ 1$, $o\ 43$, $p\ 11$, $q\ 30$, $r\ 4$ **5** $a\ a=70$, $b=20$, $b\ x=42$, $y=-24$
6 $a\ m=3$, $n=5$, $b\ m=7$, $n=-6$ **7** $a\ 7+5\sqrt{2}$, $b\ 97-56\sqrt{3}$, $c\ 217+88\sqrt{6}$ **9** $a\ 8$, $b\ 2\sqrt{2}\text{ cm}$ **10** $2\sqrt{6}$

Exercise 8.6

1 $a\ \frac{\sqrt{2}}{2}$, $b\ \frac{\sqrt{3}}{3}$, $c\ \frac{2\sqrt{5}}{5}$, $d\ \frac{3\sqrt{7}}{7}$, $e\ \frac{5\sqrt{6}}{6}$, $f\ \frac{\sqrt{5}}{15}$, $g\ \frac{\sqrt{3}}{12}$, $h\ \frac{2\sqrt{7}}{21}$, $i\ \frac{3\sqrt{5}}{10}$, $j\ \frac{7\sqrt{2}}{16}$, $k\ \frac{\sqrt{6}}{3}$, $l\ \frac{\sqrt{21}}{7}$, $m\ \frac{3\sqrt{10}}{2}$,
 $n\ \frac{\sqrt{66}}{33}$, $o\ \frac{5\sqrt{6}}{6}$ **2** $a\ \sqrt{2}$, $b\ \sqrt{3}$, $c\ 3\sqrt{2}$, $d\ 2\sqrt{5}$, $e\ 4\sqrt{3}$, $f\ \frac{\sqrt{6}}{2}$, $g\ \frac{\sqrt{10}}{5}$, $h\ \frac{\sqrt{21}}{3}$, $i\ \frac{\sqrt{30}}{6}$, $j\ \frac{\sqrt{42}}{7}$, $k\ \frac{\sqrt{5}}{2}$,
 $l\ \frac{4\sqrt{2}}{3}$, $m\ \frac{2\sqrt{3}}{5}$, $n\ \frac{3\sqrt{6}}{8}$, $o\ \frac{3\sqrt{10}}{4}$, $p\ \frac{\sqrt{6}}{4}$, $q\ \frac{\sqrt{15}}{2}$, $r\ \frac{3\sqrt{42}}{4}$, $s\ \frac{2\sqrt{30}}{5}$, $t\ \frac{3\sqrt{110}}{25}$, $u\ \sqrt{5}$, $v\ \frac{\sqrt{7}}{7}$, $w\ \frac{2\sqrt{3}}{3}$,
 $x\ 3\sqrt{3}$, $y\ \frac{4\sqrt{2}}{5}$ **3** $a\ \frac{\sqrt{2}+2}{2}$, $b\ \frac{4\sqrt{3}-3}{3}$, $c\ \frac{3\sqrt{5}+\sqrt{35}}{5}$, $d\ \frac{\sqrt{30}-2\sqrt{6}}{6}$, $e\ \frac{5\sqrt{3}+2\sqrt{6}}{3}$, $f\ \frac{7\sqrt{10}-2\sqrt{30}}{10}$,
 $g\ \frac{3\sqrt{10}+4\sqrt{5}}{10}$, $h\ \frac{5\sqrt{21}-3\sqrt{35}}{14}$, $i\ 2\sqrt{2}+1$, $j\ 5\sqrt{3}-2$, $k\ \frac{3\sqrt{2}-4\sqrt{10}}{2}$, $l\ \frac{\sqrt{2}-3\sqrt{6}}{6}$ **4** $a\ \frac{5\sqrt{2}+2\sqrt{5}}{10}$,
 $b\ \frac{7\sqrt{3}-3\sqrt{7}}{21}$, $c\ \frac{4\sqrt{11}+33\sqrt{2}}{22}$, $d\ \frac{9\sqrt{5}+25\sqrt{3}}{15}$, $e\ \frac{12\sqrt{2}+\sqrt{6}}{3}$, $f\ \frac{4\sqrt{3}-\sqrt{6}}{2}$, $g\ \frac{3\sqrt{10}+5\sqrt{21}}{15}$, $h\ \frac{\sqrt{3}}{6}$,
 $i\ \frac{3\sqrt{2}+\sqrt{3}}{6}$, $j\ \frac{16\sqrt{5}-15\sqrt{2}}{40}$, $k\ \frac{7\sqrt{3}}{18}$, $l\ \frac{7\sqrt{6}}{10}$ **5** $a\ \sqrt{2}-1$, $b\ \frac{\sqrt{3}+1}{2}$, $c\ \frac{\sqrt{5}-\sqrt{2}}{3}$, $d\ \frac{\sqrt{7}+\sqrt{3}}{4}$, $e\ \frac{3-\sqrt{3}}{3}$,
 $f\ \frac{12+3\sqrt{2}}{14}$, $g\ 7\sqrt{6}+7\sqrt{5}$, $h\ \frac{7\sqrt{2}-\sqrt{6}}{46}$, $i\ \frac{2\sqrt{3}+1}{11}$, $j\ \frac{3\sqrt{5}-2}{41}$, $k\ \frac{2\sqrt{15}+\sqrt{10}}{20}$, $l\ \frac{55\sqrt{14}-22\sqrt{35}}{30}$,
 $m\ \frac{6\sqrt{5}+5\sqrt{3}}{7}$, $n\ 3\sqrt{2}+2\sqrt{3}$, $o\ \frac{\sqrt{7}}{35}$, $p\ 5\sqrt{2}+3\sqrt{5}$, $q\ 3+2\sqrt{2}$, $r\ \frac{14-5\sqrt{3}}{11}$, $s\ \frac{17+2\sqrt{70}}{3}$, $t\ 5-2\sqrt{6}$
6 $\frac{2}{7}$

Chapter 8 Review

- 1 a rational, b irrational, c rational, d irrational, e rational, f rational, g irrational, h irrational, i rational
 2 10, 11 3 $2\sqrt{17}$, 8.6, $\sqrt[3]{700}$, $\sqrt{83}$ 4 a $\sqrt{10}$, b $(\sqrt{10}-3)$ units 5 a 3, b $\sqrt{17}$, c i $(\sqrt{13}-3)$ units,
 ii $(\sqrt{17}-3)$ units, iii $(\sqrt{17}-\sqrt{13})$ units 6 a 5, b 28, c 18, d 75 7 a $2\sqrt{3}$, b $4\sqrt{2}$, c $6\sqrt{5}$, d $12\sqrt{7}$,
 e $a\sqrt{a}$, f $y^2\sqrt{y}$, g $3\sqrt{3p}$, h $5xy^2\sqrt{2x}$ 8 a $\sqrt{28}$, b $\sqrt{45}$, c $\sqrt{48}$, d $\sqrt{150}$, e $\sqrt{9n}$, f $\sqrt{k^3}$, g $\sqrt{c^7}$,
 h $\sqrt{24e^3f}$ 9 a $m=63$, b $z=9$ 10 a $2\sqrt{3}$, b $14\sqrt{2}$, c $5\sqrt{11}$, d $\sqrt{7}$, e $7\sqrt{3}$, f $3\sqrt{6}$, g $6\sqrt{10}$, h $-9\sqrt{5}$
 11 a $11\sqrt{3}+6\sqrt{2}$, b $5\sqrt{2}+5\sqrt{5}$, c $8\sqrt{7}$, d $-4\sqrt{10}-8\sqrt{11}$ 12 a $5\sqrt{5}$, b $5\sqrt{2}$, c $16\sqrt{3}$, d $13\sqrt{5}$,
 e $7\sqrt{2}+16\sqrt{3}$ 13 a $\sqrt{21}$, b $20\sqrt{3}$, c $12\sqrt{10}$, d $\sqrt{66}$, e $\sqrt{5}$, f 9, g $5\sqrt{6}$, h 2 14 a 10, b $6\sqrt{2}$,
 c $24\sqrt{5}$, d $12\sqrt{6}$, e $3\sqrt{2}$, f $2\sqrt{3}$, g $14\sqrt{6}$, h 24 15 a $\sqrt{14}+4\sqrt{2}$, b $6-10\sqrt{3}$, c $14\sqrt{5}-28$,
 d $\sqrt{35}-\sqrt{14}+\sqrt{15}-\sqrt{6}$, e $4\sqrt{10}-6\sqrt{15}+4\sqrt{3}-9\sqrt{2}$, f $21+8\sqrt{5}$, g $21-12\sqrt{3}$, h $42-24\sqrt{3}$, i 7, j 58
 16 a $a=37$, $b=12$, b $m=114$, $n=-24$ 17 a $\frac{\sqrt{7}}{7}$, b $\frac{2\sqrt{5}}{15}$, c $\frac{\sqrt{10}}{2}$, d $\frac{\sqrt{5}}{15}$, e $\frac{\sqrt{3}+6}{3}$, f $\frac{2\sqrt{6}+5\sqrt{3}}{3}$
 18 a $\frac{\sqrt{5}-1}{4}$, b $\frac{\sqrt{11}+3}{2}$, c $\frac{\sqrt{6}+\sqrt{2}}{4}$, d $\frac{24-4\sqrt{3}}{11}$, e $\frac{5\sqrt{3}}{3}$, f $19+6\sqrt{10}$

9 Indices

Exercise 9.1

- 1 a 5^2 , b 2^3 , c 3^4 , d a^3 , e p^5 , f y^4 , g a^2b^2 , h m^3n^2 , i g^2h^4 , j $4e^2$, k $7q^4$, l $5u^2v^3$, m a^2+b , n m^3-n^2 , o c^4+d^3 ,
 p $6y^2-z^2$, q $2r+3s^2$, r $7e^3-8f^2+4g^2$ 2 a 3×3 , b $5 \times 5 \times 5$, c $2 \times 2 \times 2 \times 2 \times 2$, d $7 \times 7 \times 7 \times 7$, e $n \times n \times n$,
 f $y \times y \times y \times y \times y$, g $3 \times m \times m$, h $11 \times q \times q \times q \times q \times q$, i $p \times q \times q$, j $p \times p \times q$, k $x \times x \times x \times y \times y \times y \times y$,
 l $a \times a \times b \times c \times c \times c$, m $a \times a + b \times b$, n $m \times m \times m + 3 \times n$, o $p \times p \times p - 2 \times q \times q$,
 p $3 \times a \times a \times a \times b \times b + 5 \times a \times a \times b$ 3 a 2^3 , b 3^2 , c k^4 , d 5^x , e n^m , f $(3q)^{2p}$ 4 a 243, b 512, c 2401, d 4096,
 e 64, f 145, g 156 250, h -13 312 5 a 2, b 3, c 4, d 3, e 6, f 2, g 4, h 5 6 a $m=2$, $n=2$, b $a=1$, $b=2$,
 c $p=1$, $q=3$, d $x=4$, $y=2$, e $u=3$, $v=2$, f $a=3$, $b=2$, g $e=5$, $f=7$, h $r=5$, $s=11$, i $a=3$, $b=2$, $c=13$
 7 108 9 65 536 10 $m=3$ 11 k 12 $x=4.32$ 13 8

Exercise 9.2

- 1 a 2^5 , b 3^7 , c 5^6 , d 7^5 3 a $a^m \times a^n = a^{m+n}$ 4 a 3^4 , b 7^2 , c 2^3 , d 5^2 6 $a^m \div a^n = a^{m-n}$ 7 a 2^6 , b 5^6 ,
 c 7^9 , d 3^{10} 9 $(a^m)^n = a^{mn}$ 10 a 2^8 , b 3^6 , c 7^{11} , d 5^{12} , e 3^4 , f 5^2 , g 2^8 , h 7^6 , i 5^{12} , j 7^{24} , k 3^{20} , l 2^{24}
 11 a $3^3 \times 3^2 = 3^5$, b $5^4 \times 5^2 = 5^6$, c $7^5 \div 7^5 = 1$, d $2^{12} \div 2^3 = 2^9$, e $(5^4)^2 = 5^8$, f $(6^5)^4 = 6^{20}$ 12 a $6^3 \times 6 = 6^4$,
 b $10^4 \div 10 = 10^3$ 13 a By dividing by 2, b $2^0 = 1$, c $3^4 = 3 \times 3 \times 3 \times 3$, $3^3 = 3 \times 3 \times 3$, $3^2 = 3 \times 3$, $3^1 = 3$, $3^0 = 1$,
 d 1, e 1 14 a 128, b 2048, c 512, d 4096, e 16, f 8, g 64, h 32, i 64, j 256, k 4096, l 1024

Exercise 9.3

- 1 a n^5 , b a^9 , c y^8 , d t^4 , e e^{10} , f x^7 , g m^9 , h d^{10} , i p^{13} , j r^8 , k b^{10} , l z^{12} , m c^{10} , n k^{15} , o w^{11} 2 a p^3 , b x^4 ,
 c q^2 , d y^8 , e t^7 , f b^2 , g n^3 , h m , i f^6 , j r^4 , k d^3 , l j^2 3 a a^6 , b p^8 , c x^{15} , d b^{12} , e m^9 , f y^{20} , g t^{18} , h n^{16} , i q^{22} ,
 j c^{25} , k h^{27} , l w^{40} 4 a $3a^6$, b $2n^8$, c $18k^7$, d $8y^7$, e $15c^8$, f $20t^9$, g $24d^{10}$, h $30n^{11}$, i $32u^{12}$, j $6n^9$, k $36p^{10}$,
 l $42z^{17}$ 5 a $2n^4$, b $3b^2$, c $6c^3$, d $5m^2$, e $2y^6$, f $3k^6$, g $6z^5$, h $4p^8$, i $11x^2$, j $4m^5$, k $4s^8$, l $4e^8$ 6 a a^3 , b d^6 ,
 c n , d k^7 , e $2e^7$, f $5m$, g $6u^3$, h $3h^4$, i $4s^2$, j $10z^7$, k $7r$, l $9f^6$ 7 a $16a^6$, b $9m^{10}$, c $36g^8$, d $4k^{18}$, e $8n^{15}$,
 f $27e^{12}$, g $16q^{28}$, h $49y^{12}$, i $1000b^{12}$, j $32w^{40}$, k $25g^{12}$, l $10000x^{20}$, m $64c^{30}$, n $81f^4$, o $64v^6$, p $216s^{21}$
 8 a a^4b^3 , b x^7y^2 , c p^6q^3 , d $m^{12}n^3$, e j^5k^7 , f y^8z^7 , g $12a^3b^3$, h $10m^3n^8$, i $24u^6v^9$, j $21r^6s^6$, k $36w^6x^{11}$, l $30c^{13}d^{10}$,
 m $32f^6h^{17}$, n $60s^{11}t^{16}$, o $56i^{19}j^{16}$ 9 a a^2b^4 , b m^2n^7 , c g^4h^9 , d x^3y^5 , e u^3v^2 , f c^2d^2 , g $2y^5z^7$, h $3ab$, i $8i^7j^3$,
 j $7mn^4$, k $6c^5d^4$, l $11p^7q^3$, m $5e^6f^6$, n $7x^3y^4$, o $12b^5c^5$ 10 a x^4y^8 , b $p^{10}q^6$, c c^3d^{12} , d u^8v^{12} , e $r^{20}s^{15}$, f $a^{32}b^4$,
 g $25m^6n^8$, h $81s^4t^{12}$, i $8c^{18}d^9$, j $16y^4z^{20}$, k $27p^{12}q^{27}$, l $32g^{55}h^{20}$, m $125q^{36}r^3$, n $64x^{30}y^{12}$, o $10000e^{12}d^{32}$

11 a^2 , b^5 , c^6 , d^6 , e^7n^4 , $f5t^3$, $g7b^3$, $h8x^7$, $i9h$ 12 a^5 , b^6 , c^4 , $d3d^4$, $e20g^{12}$, $f5u^2$, $g100x^{13}$, $h9k^4$, $i60j^{16}$ 13 $a3p^2$, $b5z^3$, $c8m^5$, $d10s^4$, $e2d^5$, $f3w^3$, $g2n^6$, $h3x^8$, $i2b^7$ 14 a^4b^3 , b^5q^7 , c^6z^8 , $d5cd^3$, $e3m^2n^4$, $f8u^5v^9$ 15 a^4q^3 , b^7f^{10} , c^4u^v , $d5x^6y$, $e55m^{13}n^5$, $f42c^7d^{12}$ 16 $a4x^2y^5$, $b7m^4n^6$, $c12p^3q^8$, $d2uv^6$, $e10e^2f^7$, $f2c^4d^{11}$ 17 $a6$, $b12$, $c96$ 18 $a15$, $b45$, $c405$ 19 $a40$, $b20$, $c5$, $d\frac{5}{8}$
20 $a200$, $b40$, $c8$, $d\frac{8}{25}$ 21 $a9$, $b27$, $c81$ 22 $a100$, $b1000$, $c10000$

Exercise 9.4

1 a^8 , b^5 , c^{12} , d^6 , e^{15} , f^{14} , g^{18} , h^{11} , i^7 , j^{32} , k^{12} , l^{13} 2 $a4y^2$, $b35t^9$, $c36c^8$, $d36p^7$, $e7s^{16}$, $f27f^{21}$, $g72w^{12}$, $h32k^{40}$, $i9b^{10}$, $j125r^{36}$, $k110g^{19}$, $l7m$ 3 a^{11} , b^{10} , c^7 , d^9 , e^{11} , f^4 , g^{20} , h^{19} , i^{10} , j^{10} , k^8 , l^{16} 4 a^{18} , b^{21} , c^{30} , d^{10} , e^{12} , f^3 , g^{10} , h^{13} , i^{20} , j^{100} , k^{15} , l^{72} , m^{15} , n^{15} , o^{46} 5 $a30a^8$, $b20g^{12}$, $c10x^8$, $d54u^{19}$, $e7d^7$, $f32k^{20}$, $g6v^3$, $h4m^9$, $i4b^2$ 6 a^{15} , b^{16} , c^{16} , d^{12} , e^{24} , f^{26} 7 $a2u^6$, $b10p^5q^9$ 8 $a2a+b$, b^{k+m+n} , $c5^{x+1}$, $d3^{2n}$, $e2^6$, f^{3q} , $g5^{2m+4}$, h^{10} , $i3^{u-v}$, $j4^{y-3}$, $k5^k$, $l11^{w-1}$, $m6^e$, $n25$, o^{m^2q} , p^{x^2} , $q2^{ab}$, $r^{n^{5k}}$, $s^{e^{f^2}}$, $t^{p^{12q}}$, $u5^{21u}$, $v6^{6ab}$, $w2^{xy+xz}$, $x^{a^{mk-nk}}$ 9 a^D 10 2^{n-2}

Exercise 9.5

1 a	2^5	$2 \times 2 \times 2 \times 2 \times 2$	32
	2^4	$2 \times 2 \times 2 \times 2$	16
	2^3	$2 \times 2 \times 2$	8
	2^2	2×2	4
	2^1	2	2
	2^0	1	1
b	3^5	$3 \times 3 \times 3 \times 3 \times 3$	243
	3^4	$3 \times 3 \times 3 \times 3$	81
	3^3	$3 \times 3 \times 3$	27
	3^2	3×3	9
	3^1	3	3
	3^0	1	1

2 $a1$, $b1$, $c1$, $d1$ 3 $a1$, $b1$, $c1$, $d1$, $e2$, $f3$, $g7$, $h-6$, ia , jq , $k1$, lm^2p 4 $a5$, $b4$, $c4$, $d2$, $e6$, $f45$, $g5$, $h-7$, $i11$, $j3$, $k-1$, $l11$, $m8$, $n7$, $o7$, $p42$, $q-9$, $r17$ 5 0^0 is not defined

Exercise 9.6

1 a	2^4	$2 \times 2 \times 2 \times 2$	16
	2^3	$2 \times 2 \times 2$	8
	2^2	2×2	4
	2^1	2	2
	2^0	1	1
	2^{-1}	$\frac{1}{2}$	$\frac{1}{2}$
	2^{-2}	$\frac{1}{2 \times 2}$	$\frac{1}{4}$
	2^{-3}	$\frac{1}{2 \times 2 \times 2}$	$\frac{1}{8}$
	2^{-4}	$\frac{1}{2 \times 2 \times 2 \times 2}$	$\frac{1}{16}$
b	3^4	$3 \times 3 \times 3 \times 3$	81
	3^3	$3 \times 3 \times 3$	27
	3^2	3×3	9
	3^1	3	3
	3^0	1	1
	3^{-1}	$\frac{1}{3}$	$\frac{1}{3}$
	3^{-2}	$\frac{1}{3 \times 3}$	$\frac{1}{9}$
	3^{-3}	$\frac{1}{3 \times 3 \times 3}$	$\frac{1}{27}$
	3^{-4}	$\frac{1}{3 \times 3 \times 3 \times 3}$	$\frac{1}{81}$

2 $a\frac{1}{2}$, $b\frac{1}{4}$, $c\frac{1}{5}$, $d\frac{1}{10}$ 3 $a\frac{1}{3}$, $b\frac{1}{7}$, $c\frac{1}{8}$, $d\frac{1}{12}$, $e\frac{1}{16}$, $f\frac{1}{25}$, $g\frac{1}{49}$, $h\frac{1}{121}$, $i\frac{1}{8}$, $j\frac{1}{27}$, $k\frac{1}{32}$, $l\frac{1}{81}$, $m\frac{1}{125}$, $n\frac{1}{16}$, $o\frac{1}{1000}$, $p\frac{1}{243}$, $q\frac{1}{81}$, $r\frac{1}{64}$ 4 $a\frac{1}{m}$, $b\frac{1}{p}$, $c\frac{1}{h^2}$, $d\frac{1}{n^3}$, $e\frac{1}{e^6}$, $f\frac{1}{y^4}$ 5 a^{-1} , b^{-1} , c^{-2} , d^{-3} , e^{-5} , f^{-8} 6 $a-1$, $b-1$, $c-2$, $d-2$, $e-3$, $f-5$, $g-2$, $h-3$, $i-2$, $j-6$, $k-3$, $l-4$ 7 $a\frac{1}{2m}$, $b\frac{1}{7c}$, $c\frac{1}{5r}$, $d\frac{1}{4q}$

$$\begin{aligned}
 & e \frac{1}{9u^2}, f \frac{1}{36g^2}, g \frac{1}{121t^2}, h \frac{1}{81k^2}, i \frac{1}{8p^3}, j \frac{1}{125s^3}, k \frac{1}{81b^4}, l \frac{1}{32n^5}, m \frac{1}{7x^3}, n \frac{1}{100y^{10}}, o \frac{1}{64w^8}, p \frac{1}{8z^{18}}, \\
 & q \frac{1}{16c^{24}}, r \frac{1}{27a^{12}}, s \frac{1}{169v^{14}}, t \frac{1}{125y^{36}}, u \frac{1}{a^4b^6}, v \frac{1}{p^{12}q^{20}}, w \frac{1}{81e^2f^{12}}, x \frac{1}{27y^{24}z^{21}} \quad 8 \ a \ 8\frac{1}{2}, \ b \ 8\frac{8}{9}, \ c \ \frac{3}{4}, \\
 & d \ 1\frac{1}{5}, \ e \ \frac{6}{7}, \ f \ 17\frac{1}{4}, \ g \ 5, \ h \ 5, \ i \ 6, \ j \ \frac{1}{6}, \ k \ 9, \ l \ 2, \ m \ 4, \ n \ 4, \ o \ 8, \ p \ 40, \ q \ \frac{1}{3}, \ r \ \frac{1}{2} \quad 9 \ a \ 2, \ b \ 3, \ c \ \frac{2}{5}, \ d \ 1\frac{1}{3}, \ e \ \frac{4}{9}, \\
 & f \ \frac{9}{49}, \ g \ 1\frac{9}{16}, \ h \ 2\frac{2}{49}, \ i \ 3\frac{3}{8}, \ j \ \frac{8}{125}, \ k \ \frac{16}{81}, \ l \ \frac{81}{10000}, \ m \ \frac{2}{7}, \ n \ \frac{16}{81}, \ o \ \frac{36}{121}, \ p \ \frac{27}{125} \quad 10 \ a \ \frac{6}{n}, \ b \ \frac{k}{5}, \ c \ \frac{b}{a}, \ d \ \frac{3n}{2m}, \ e \ \frac{9}{x^2}, \\
 & f \ \frac{h^2}{16}, \ g \ \frac{8}{y^3}, \ h \ \frac{d^4}{16}, \ i \ \frac{m^3}{27}, \ j \ \frac{32}{e^5}, \ k \ \frac{d^2}{c^2}, \ l \ \frac{f^5}{e^5}, \ m \ \frac{9}{4a^2}, \ n \ \frac{64p^2}{49}, \ o \ \frac{64n^3}{27m^3}, \ p \ \frac{625y^4}{16x^4}, \ q \ \frac{16}{u^4}, \ r \ \frac{8}{a^3b^3}, \\
 & s \ \frac{121p^{24}}{100m^8}, \ t \ \frac{243c^{20}}{32a^{10}b^{15}}
 \end{aligned}$$

Exercise 9.7

$$\begin{aligned}
 & 1 \ a \ \frac{x}{y}, \ b \ \frac{q}{p}, \ c \ \frac{a}{b^2}, \ d \ \frac{v^3}{u^4}, \ e \ \frac{f^2}{e^3}, \ f \ \frac{h^5}{g}, \ g \ \frac{w^3}{x^7}, \ h \ \frac{b^6}{c}, \ i \ \frac{s^9}{r^4}, \ j \ \frac{j}{k^3}, \ k \ \frac{1}{ab}, \ l \ \frac{1}{x^2y}, \ m \ \frac{1}{e^3f^2}, \ n \ \frac{1}{p^4q^6}, \ o \ \frac{1}{g^5h^2}, \\
 & p \ \frac{3}{a}, \ q \ \frac{5}{c^2}, \ r \ \frac{2}{e^4}, \ s \ \frac{7}{b^5}, \ t \ \frac{9}{k^3}, \ u \ \frac{1}{2u}, \ v \ \frac{1}{4p^2}, \ w \ \frac{2}{3t^4}, \ x \ \frac{4}{7w^9}, \ y \ \frac{3}{5n^6} \quad 2 \ a \ ab^{-1}, \ b \ x^3y^{-1}, \ c \ pq^{-2}, \ d \ m^3n^{-4}, \\
 & e \ e^{-2}f^{-4}, \ f \ c^4d^{-5}, \ g \ j^{10}k^{-6}, \ h \ g^{-7}h^{-9}, \ i \ y^6z^{-1}, \ j \ u^{-3}v^{-10}, \ k \ 5n^{-1}, \ l \ 3p^{-2}, \ m \ 9a^{-7}, \ n \ 4z^{-3}, \ o \ 11t^{-5}, \ p \ 10m^{-6}, \\
 & q \ \frac{1}{3}m^{-1}, \ r \ \frac{1}{5}k^{-1}, \ s \ \frac{1}{4}g^{-2}, \ t \ \frac{1}{12}a^{-3}, \ u \ \frac{2}{7}t^{-1}, \ v \ \frac{3}{4}r^{-2}, \ w \ \frac{5}{8}y^{-6}, \ x \ \frac{9}{11}f^{-4} \quad 3 \ a \ \frac{a^2b^3}{c}, \ b \ \frac{x^3z^4}{y^2}, \ c \ \frac{f^2g^2}{e^3}, \ d \ \frac{k^3}{ij^2}, \\
 & e \ \frac{u^2}{v^2w^5}, \ f \ \frac{1}{mn^3p^2}, \ g \ \frac{s^2}{r^4t^7}, \ h \ \frac{c^5d}{e^9}, \ i \ \frac{1}{pqr}, \ j \ \frac{w^2}{x^3y^5}, \ k \ \frac{4b^3}{a^2}, \ l \ \frac{3m}{n}, \ m \ \frac{7}{e^3f^2}, \ n \ \frac{5a^4}{b^6}, \ o \ \frac{12}{rs}, \ p \ \frac{8y^7}{x^4}, \ q \ \frac{d^5}{3c^2}, \\
 & r \ \frac{y^3}{8z^4}, \ s \ \frac{5}{6ab}, \ t \ \frac{3u^2}{10v^8} \quad 4 \ a \ p^2q^4r^{-1}, \ b \ ab^2c^{-5}, \ c \ u^6v^{-2}w^{-4}, \ d \ e^{-2}f^{-3}g^{-7}, \ e \ m^4n^{-1}p^{-3}, \ f \ a^{-4}b^{-1}c^{-1}, \ g \ i^3j^5k^{-3}, \\
 & h \ c^6d^{-8}e^{-11}, \ i \ x^{-1}y^{-1}z^{-1}, \ j \ mn^{-1}p^{-8}, \ k \ 5a^4b^{-2}, \ l \ 3u^5v^{-4}, \ m \ 9e^{-3}f^{-4}, \ n \ 6m^7n^{-5}, \ o \ 10p^{-1}q^{-1}, \ p \ \frac{2}{3}r^2s^{-2}, \ q \ \frac{4}{5}a^5b^{-3}, \\
 & r \ \frac{9}{10}g^{-1}h^{-4}, \ s \ \frac{5}{12}v^4w^{-9}, \ t \ \frac{8}{9}y^{-5}z^{-10}, \quad 5 \ a \ m^2, \ b \ \frac{1}{k^3}, \ c \ \frac{1}{y^8}, \ d \ 1, \ e \ 5a^5, \ f \ 18z^7, \ g \ \frac{24}{t^9}, \ h \ 14, \ i \ \frac{1}{e}, \ j \ \frac{1}{j^{11}}, \ k \ \frac{1}{q^3}, \\
 & l \ y, \ m \ \frac{10}{u^5}, \ n \ 4c^{10}, \ o \ 5w^2, \ p \ 8, \ q \ \frac{1}{d^6}, \ r \ s^8, \ s \ \frac{1}{z^{18}}, \ t \ f^{28}, \ u \ \frac{9}{a^{10}}, \ v \ \frac{h^{24}}{8}, \ w \ \frac{64}{m^{21}}, \ x \ \frac{x^{18}}{49} \quad 6 \ a \ \frac{1}{a^3}, \ b \ \frac{1}{n^4}, \ c \ \frac{1}{q^5}, \\
 & d \ \frac{5}{w^5}, \ e \ \frac{80}{p^3}, \ f \ \frac{14}{y^4}, \ g \ \frac{9}{m^2}, \ h \ \frac{6}{7c^6}, \ i \ \frac{2}{3x^3} \quad 7 \ a \ ab, \ b \ x^5, \ c \ \frac{m^4}{n^3}, \ d \ \frac{x^7}{y} \quad 8 \ a \ \frac{1}{a^3}, \ b \ n^6, \ c \ \frac{1}{w^{17}}, \ d \ b^6, \ e \ \frac{1}{x}, \ f \ \frac{1}{u}
 \end{aligned}$$

Exercise 9.8

$$\begin{aligned}
 & 1 \ a \ 2, \ b \ 3, \ c \ 7, \ d \ 9, \ e \ 2, \ f \ 3, \ g \ 5, \ h \ 10, \ i \ 6, \ j \ 4, \ k \ 11, \ l \ 10 \quad 2 \ a \ \frac{2}{3}, \ b \ \frac{2}{3}, \ c \ \frac{5}{7}, \ d \ \frac{3}{5}, \ e \ \frac{6}{11}, \ f \ \frac{4}{9}, \ g \ \frac{3}{4}, \ h \ \frac{9}{10}, \\
 & i \ 1\frac{1}{3}, \ j \ 1\frac{1}{2}, \ k \ 1\frac{2}{5}, \ l \ 3\frac{2}{3} \quad 3 \ a \ \frac{1}{4}, \ b \ \frac{1}{5}, \ c \ \frac{1}{2}, \ d \ \frac{1}{3}, \ e \ \frac{1}{6}, \ f \ \frac{1}{5}, \ g \ \frac{1}{11}, \ h \ \frac{1}{8}, \ i \ \frac{1}{4}, \ j \ 2\frac{1}{2}, \ k \ 1\frac{1}{2}, \ l \ \frac{3}{10} \quad 4 \ a \ a^3, \ b \ a^5, \\
 & c \ a^{11}, \ d \ a^2, \ e \ a^4, \ f \ a^{10} \quad 5 \ a \ 3t^4, \ b \ 5u^6, \ c \ 2c^3, \ d \ 5b^7, \ e \ 4e^6, \ f \ 13w^9, \ g \ 4n^6, \ h \ 3v^5 \quad 6 \ a \ 2, \ b \ 3, \ c \ 2, \ d \ 2 \\
 & 7 \ a \ 5, \ b \ 4, \ c \ 6, \ d \ 2, \ e \ 3, \ f \ 2, \ g \ 3, \ h \ 10 \quad 8 \ a \ a^6, \ b \ p^7, \ c \ k^4, \ d \ y^{12} \quad 9 \ a \ a^{\frac{3}{2}}, \ b \ m^{\frac{2}{3}}, \ c \ e^{\frac{5}{2}}, \ d \ p^{\frac{4}{3}}, \ e \ y^{\frac{4}{3}}, \ f \ k^{\frac{5}{3}}, \\
 & g \ t^{\frac{7}{2}}, \ h \ h^{\frac{5}{2}}, \ i \ z^{\frac{7}{3}}, \ j \ u^{\frac{5}{8}}, \ k \ r^{\frac{2}{7}}, \ l \ w^{\frac{10}{9}} \quad 10 \ a \ 8, \ b \ 4, \ c \ 64, \ d \ 9, \ e \ 8, \ f \ 125, \ g \ 16, \ h \ 27, \ i \ 1000, \ j \ 4, \ k \ 128, \\
 & l \ 32, \ m \ \frac{1}{27}, \ n \ \frac{1}{16}, \ o \ \frac{8}{125}, \ p \ \frac{9}{25}, \ q \ \frac{1}{27}, \ r \ \frac{1}{25}, \ s \ \frac{1}{16}, \ t \ \frac{1}{343} \quad 11 \ a \ \sqrt{a^3}, \ b \ \sqrt[3]{m^2}, \ c \ \sqrt[3]{p^4}, \ d \ \sqrt[5]{n^3}, \ e \ \sqrt[6]{k^5}, \\
 & f \ \sqrt[7]{y^7}, \ g \ \frac{1}{\sqrt{x}}, \ h \ \frac{1}{\sqrt[3]{c}}, \ i \ \frac{1}{\sqrt[3]{e^3}}, \ j \ \frac{1}{\sqrt[3]{s^2}}, \ k \ \frac{1}{\sqrt[5]{v^4}}, \ l \ \frac{1}{\sqrt[6]{b^7}} \quad 12 \ a \ x^{\frac{3}{2}}, \ b \ x^{\frac{5}{2}}, \ c \ x^{\frac{4}{3}}, \ d \ x^{\frac{7}{3}}, \ e \ x^{\frac{6}{5}}, \ f \ x^{\frac{9}{4}}, \ g \ x^{\frac{7}{2}}, \\
 & h \ x^{\frac{29}{7}}, \ i \ x^{-\frac{1}{2}}, \ j \ x^{-\frac{1}{3}}, \ k \ x^{-\frac{1}{5}}, \ l \ x^{-\frac{1}{7}}, \ m \ x^{-\frac{1}{x}}, \ n \ x^{-\frac{4}{3}}, \ o \ x^{-\frac{5}{2}}, \ p \ x^{-\frac{7}{3}} \quad 13 \ a \ a\sqrt{a}, \ b \ a^2\sqrt{a}, \ c \ a \times \sqrt[3]{a}, \\
 & d \ a \times \sqrt[3]{a^2} \quad 14 \ a \ 3\frac{1}{3}, \ b \ 9\frac{1}{4} \times 2\frac{5}{6}
 \end{aligned}$$

Exercise 9.9

1 $a 2 \times 10^2$, $b 5 \times 10^3$, $c 9 \times 10^4$, $d 7 \times 10^5$, $e 4 \times 10$, $f 6 \times 10^4$, $g 3 \times 10^3$, $h 8 \times 10^6$ 2 $a 9 \times 10^{-2}$,
 $b 2 \times 10^{-3}$, $c 4 \times 10^{-4}$, $d 6 \times 10^{-5}$, $e 3 \times 10^{-1}$, $f 7 \times 10^{-4}$, $g 8 \times 10^{-2}$, $h 5 \times 10^{-6}$ 3 $a 500$, $b 2000$, $c 80\,000$,
 $d 3\,000\,000$, $e 70\,000$, $f 900$, $g 6000$, $h 400\,000$ 4 $a 0.2$, $b 0.05$, $c 0.009$, $d 0.000\,03$, $e 0.07$, $f 0.0006$, $g 0.4$,
 $h 0.000\,008$ 5 $a 1.4 \times 10^3$, $b 3.7 \times 10^4$, $c 1.2 \times 10^5$, $d 9.6 \times 10^6$, $e 1.2 \times 10^2$, $f 2.5 \times 10^3$, $g 9.3 \times 10^4$,
 $h 8.1 \times 10^5$, $i 4.9 \times 10$, $j 7.23 \times 10^2$, $k 2.466 \times 10^3$, $l 1.49 \times 10^3$, $m 5.67 \times 10^4$, $n 1.37 \times 10^5$, $o 9.99 \times 10^3$,
 $p 6.52 \times 10^6$, $q 5.89 \times 10$, $r 1.902 \times 10^2$, $s 3.03 \times 10$, $t 1.5426 \times 10^2$ 6 $a 910$, $b 3200$, $c 84\,000$, $d 2\,600\,000$,
 $e 4300$, $f 59$, $g 650\,000$, $h 71\,000$, $i 29$, $j 4163$, $k 524$, $l 11\,315$, $m 3850$, $n 89\,200$, $o 268\,500$ $p 9\,003\,000$
7 $a 7.1 \times 10^{-1}$, $b 6.2 \times 10^{-2}$, $c 9.4 \times 10^{-3}$, $d 3.5 \times 10^{-5}$, $e 5.4 \times 10^{-1}$, $f 1.7 \times 10^{-3}$, $g 3.6 \times 10^{-2}$, $h 7.9 \times 10^{-4}$,
 $i 5.1 \times 10^{-3}$, $j 9 \times 10^{-2}$, $k 6.7 \times 10^{-5}$, $l 8.8 \times 10^{-2}$, $m 1.25 \times 10^{-1}$, $n 3.52 \times 10^{-2}$, $o 4 \times 10^{-1}$, $p 6.63 \times 10^{-4}$,
 $q 7.54 \times 10^{-3}$, $r 4.761 \times 10^{-1}$, $s 3.924 \times 10^{-3}$, $t 1.162 \times 10^{-5}$ 8 $a 0.19$, $b 0.0086$, $c 0.064$, $d 0.000\,058$, $e 0.031$,
 $f 0.93$, $g 0.0048$, $h 0.000\,007\,2$, $i 0.842$, $j 0.0587$, $k 0.000\,106$, $l 0.000\,041\,9$, $m 0.021\,32$, $n 0.4076$,
 $o 0.007\,119$, $p 0.000\,100\,2$ 9 $a 1.73 \times 10^4$, $b 2.07 \times 10^3$, $c 2.00 \times 10^5$, $d 1.53 \times 10$, $e 9.03 \times 10^2$, $f 3.57 \times 10^3$,
 $g 1.66 \times 10^{-1}$, $h 2.26 \times 10^{-2}$, $i 7.38 \times 10^{-3}$, $j 1.07 \times 10^{-4}$, $k 9.17 \times 10^{-6}$, $l 5.56 \times 10^{-2}$ 10 $a 8 \times 10^9$, $b 7.5 \times 10^6$,
 $c 7.2 \times 10^3$, $d 7.5 \times 10^{-7}$, $e 4.3 \times 10^8$, $f 4 \times 10^{-5}$, $g 4 \times 10^6$, $h 9 \times 10^{-4}$

Exercise 9.10

1 $a 230$, $b 5910$, $c 70\,400$, $d 1\,608\,000$, $e 0.52$, $f 0.006\,42$, $g 0.0927$, $h 0.000\,389\,9$ 2 $a 4.7 \times 10^6$, $b 3.14 \times 10^5$,
 $c 9.05 \times 10^9$, $d 1.782 \times 10^{12}$, $e 5.7 \times 10^{-4}$, $f 6.22 \times 10^{-7}$, $g 8.013 \times 10^{-11}$, $h 2.637 \times 10^{-18}$ 3 $a 4 \times 10^3$, 40^3 ,
 $b 4^3 = 4 \times 4 \times 4$ 4 $a 1.455 \times 10^8$, $b 7.535 \times 10^{15}$, $c 4.274 \times 10^6$, $d 9.493 \times 10^{-18}$, $e 1.098 \times 10^{-12}$,
 $f 1.785 \times 10^{-38}$, $g 5.507 \times 10^{23}$, $h 8.968 \times 10^{-13}$, $i 9.873 \times 10^{35}$, $j 9.198 \times 10^2$, $k 4.237 \times 10^{-3}$, $l 6.464 \times 10^{-4}$,
 $m 5.747 \times 10^{-4}$, $n 4.348 \times 10^3$, $o 5.047 \times 10^3$ 5 $a 5.022 \times 10^{15}$, $b 6.425 \times 10^{13}$, $c 7.647 \times 10^{16}$, $d 1.331 \times 10^6$,
 $e 1.800 \times 10^{16}$, $f 2.075 \times 10^{-17}$, $g 8.471 \times 10^{-14}$, $h 1.009 \times 10^{-24}$ 6 3.94×10^{-12} , 3.959×10^{-12} , 8.41×10^{-9} ,
 6.7×10^5 , 1.5×10^6 , 5.2×10^8 7 a Jupiter by 1.194×10^{27} kg b 200 times 8 3×10^4 km/h 9 5.916×10^9 km
10 8.3 min 11 $a 6.375 \times 10^3$ km, $b 40\,055$ km 12 $a 3.67$ times, $b 109$ 13 a hydrogen, 1778 times,
 $b 6.25 \times 10^{17}$ million

Chapter 9 Review

1 $a 3^4$, $b a^5$, $c 14p^2q^2$ 2 $a 7 \times 7$, $b k \times k \times k \times k \times k \times k$, $c a \times a \times a \times b \times b \times b \times b$,
 $d 5 \times e \times e + 3 \times f \times f \times f \times f$ 3 5^3 4 $a x = 3$, $y = 2$, $b m = 3$, $n = 2$ 5 $a 5^{10}$, $b 2^4$, $c 3^{20}$ 6 $a n^7$, $b 15p^6$,
 $c a^{12}b^{10}$, $d 4k^7$, $e ab^2c^2d$, $f 40x^3y^7$, $g y^5$, $h 7c^4$, $i u^9v^7$, $j 6t^{10}$, $k 8m^5$, $l -6e^6f^7$, $m z^{28}$, $n 27p^3$, $o 16x^{20}$,
 $p r^{5s^5}$, $q a^{48}b^{24}$, $r -8j^{12}k^{27}$ 7 $a t^{10}$, $b d^7$, $c y^{11}$, $d p^{31}$, $e p^{13}$, $f z^{25}$, $g h^6$, $h v^{24}$, $i k^{27}$ 8 $6d^8$ 9 $a 10$, $b 20$,
 $c 80$ 10 $a 100$, $b 20$, $c \frac{4}{5}$ 11 $a 100$, $b 1000$, $c 3333\frac{1}{3}$, $d 900\,000$ 12 $a \frac{1}{4}$, $b \frac{1}{36}$, $c \frac{1}{81}$, $d \frac{1}{125}$, $e \frac{1}{32}$
13 $a m^{-1}$, $b k^{-2}$, $c a^{-5}$, $d r^{-9}$, $e x^{-11}$ 14 $a -1$, $b -2$, $c -4$, $d -3$ 15 $a \frac{a}{b}$, $b \frac{q}{p}$, $c \frac{1}{xy}$, $d \frac{c^3}{d^2}$, $e \frac{1}{m^4n^5}$, $f \frac{u^2v^3}{w^7}$,
 $g \frac{r^4}{st^6}$, $h \frac{1}{ab^2c^3}$, $i \frac{9}{t^2}$, $j \frac{7g}{ef^5}$, $k \frac{3}{4p^3}$, $l \frac{5m^2}{9k}$ 16 $a xy^{-1}$, $b m^3n^{-5}$, $c p^{-2}q^{-4}$, $d a^4b^5c^{-7}$, $e m^6k^{-1}n^{-5}$, $f 2x^{-3}y^{-1}z^{-8}$,
 $g 5m^9n^{-2}p^{-3}$, $h \frac{2}{3}e^4f^{-1}$ 17 $a 4$, $b \frac{2}{3}$, $c \frac{9}{25}$, $d \frac{8}{125}$, $e \frac{t}{6}$, $f \frac{49}{m^2}$, $g \frac{q^3}{8p^3}$, $h \frac{25d^{14}}{64c^8}$ 18 $a a^5$, $b \frac{1}{k^7}$, $c c^7$, $d \frac{16}{p^{11}}$,
 $e \frac{1}{v^6}$, $f \frac{1}{x^8}$, $g \frac{1}{r^5}$, $h 7u^4$, $i \frac{1}{v^6}$, $j \frac{1}{n^{10}}$, $k q^{32}$, $l \frac{y^{21}}{8}$ 19 $a 6$, $b 8$, $c 11$, $d 2$, $e 3$, $f 5$, $g 3$, $h 2$, $i 10$ 20 $a \frac{1}{3}$,
 $b \frac{1}{7}$, $c \frac{1}{2}$, $d \frac{1}{10}$, $e \frac{1}{2}$, $f \frac{1}{10}$ 21 $a 6$, $b 7$, $c 4$ 22 $a a^5$, $b w^4$, $c 2k^3$, $d 7p^{11}$, $e 2u^6$, $f 3s^7$ 23 $a 27$, $b 16$,
 $c 8$, $d \frac{1}{81}$, $e \frac{1}{64}$, $f \frac{4}{9}$, $g \frac{27}{1000}$, $h l \frac{9}{16}$ 24 $a \sqrt{m^5}$, $b \sqrt[3]{k^2}$, $c \sqrt[4]{e^3}$, $d \frac{1}{\sqrt[9]{q^2}}$, $e \frac{1}{\sqrt[6]{a^5}}$, $f \frac{1}{\sqrt[5]{z^7}}$ 25 $a a^{\frac{3}{2}}$, $b p^{\frac{7}{2}}$,
 $c n^{\frac{5}{4}}$, $d t^{\frac{7}{3}}$, $e c^{-\frac{1}{2}}$, $f y^{-\frac{1}{4}}$, $g x^{-\frac{4}{3}}$, $h b^{-\frac{5}{2}}$ 26 $a 3 \times 10^3$, $b 1.9 \times 10^5$, $c 2.86 \times 10^4$, $d 1.734 \times 10^2$, $e 4 \times 10^{-4}$,
 $f 2.6 \times 10^{-2}$, $g 1.98 \times 10^{-3}$, $h 5.831 \times 10^{-4}$ 27 $a 900$, $b 60\,000$ $c 8700$, $d 104\,000$, $e 0.3$, $f 0.007$, $g 0.019$,
 $h 0.000\,004\,61$ 28 $a 900\,000$, $b 56\,000$, $c 0.002$, $d 0.000\,032\,7$ 29 $a 3.834 \times 10^{11}$, $b 1.220 \times 10^{28}$,
 $c 1.155 \times 10^{-14}$, $d 3.313 \times 10^{-31}$

10

Geometry

Exercise 10.1

1 **a** Yes, **b** No; no common ray, **c** Yes, **d** No; no common vertex, **e** Yes, **f** No; no common ray, **g** No; no common ray, **h** No; no common ray, **i** No; the angles don't lie on opposite sides of the common ray 2 **a** 40° , **b** 120° , **c** 65° , **d** 118° 3 **a** 42° 4 **a** 180° , **b** 270° , **c** 210° 5 **a** $x=50$, **b** $k=30$, **c** $p=20$, **d** $m=300$, **e** $c=90$, **f** $a=105$ 6 **a** $a=45$, **b** $n=60$, **c** $t=75$, **d** $y=120$, **e** $d=45$, **f** $r=30$, **g** $v=33$, **h** $h=72$, **i** $c=36$ 7 **a** $x=20$, **b** $t=30$, **u** $=60$, **c** $p=58$, **q** $=32$, **d** $j=65$, **k** $=25$, **e** $a=63$, **b** $=63$, **f** $g=31$, **g** $m=71$, **n** $=109$, **h** $v=85$, **w** $=70$, **i** $c=121$, **j** $b=47$, **k** $s=162$, **l** $f=25$ 8 **a** $m=23$, **b** $t=17$, **c** $x=19$, **d** $k=74$, **e** $u=121$, **f** $p=30$, **g** $a=292$, **h** $g=115$, **i** $y=20$, **j** $c=16$, **k** $w=27$, **l** $e=33$, **m** $e=35$, **n** $b=21$, **o** $n=156$ 9 **a** $a=23$, **b** $=67$, **c** $=23$, **b** $m=42$, **n** $=54$, **c** $x=30$, **y** $=150$, **z** $=45$, **d** $p=7$, **q** $=131$, **e** $u=18$, **v** $=72$, **w** $=82$, **f** $q=18$, **s** $=24$, **r** $=9$, **g** $d=9$, **e** $=58$, **h** $a=20$, **b** $=60$, **c** $=120$, **i** $f=44$, **g** $=46$, **j** $x=12$, **k** $x=135$, **y** $=225$, **l** $x=36$, **y** $=34$, **z** $=146$ 10 **a** $x=46$, **b** $x=53$, **c** $x=51$, **d** $x=52$, **e** $x=63$, **f** $x=72$

Exercise 10.2

1 **a** corresponding, **b** co-interior, **c** alternate, **d** corresponding, **e** alternate, **f** co-interior, **g** alternate, **h** corresponding, **i** co-interior 2 **a** equal, **b** equal, **c** supplementary 3 **a** $a=50$, **b** $t=15$, **c** $c=100$, **d** $y=140$, **e** $p=105$, **f** $k=85$ 4 **a** Yes (co-interior angles are supplementary), **b** No, **c** No 5 **a** $a=150$, **b** $=30$, **c** $=30$, **b** $p=70$, **q** $=110$, **r** $=70$, **c** $x=55$, **y** $=125$, **z** $=55$, **d** $f=66$, **g** $=114$, **h** $=114$ 6 **a** $p=78$, **q** $=78$, **b** $s=24$, **t** $=156$, **c** $x=41$, **y** $=49$, **d** $u=76$, **e** $b=59$, **c** $=31$, **f** $v=63$, **w** $=63$, **g** $y=47$, **z** $=43$, **h** $e=58$, **f** $=122$, **i** $q=59$, **r** $=121$, **j** $c=56$, **d** $=56$, **k** $m=48$, **n** $=9$, **l** $g=76$, **h** $=104$, **m** $f=16$, **g** $=16$, **n** $u=114$, **v** $=114$, **o** $r=82$, **s** $=98$, **p** $x=108$, **y** $=109$, **z** $=109$, **q** $a=143$, **b** $=37$, **c** $=37$, **r** $d=88$, **e** $=77$, **f** $=13$ 7 **a** $x=92$, **b** $c=25$, **c** $a=20$ 8 **a** $m=57$, **b** $j=51$, **c** $h=61$, **d** $b=65$, **e** $t=279$, **f** $e=3$ 9 **a** $x=142$, **b** $x=69$, **c** $x=74$, **d** $x=77$, **e** $x=35$, **f** $x=43$, **g** $x=85$, **h** $x=29$, **i** $x=51$, **j** $x=109$, **k** $x=105$, **l** $x=73$

Exercise 10.3

1 **a** isosceles, **b** equilateral, **c** scalene 2 **a** right-angled, **b** acute-angled, **c** obtuse-angled 3 **a** $\angle ACB$, **ii** $\angle ABC$, **b** $\angle FG$, **ii** EG 4 **a** EF , FG , **b** LN , MN 5 **a** B and D ; The third side must be shorter than the sum of the other two sides. 6 **a** No, **b** No, **c** Yes, **d** Yes, **e** Yes, **f** No 7 **a** $x=50$, **b** $g=60$, **c** $p=140$ 8 **a** $t=60$, **b** $a=50$, **b** $=80$, **c** $v=72$, **w** $=36$ 9 **a** $h=70$, **b** $k=32$, **c** $b=45$ 10 **a** $a=140$, **b** $z=124$, **c** $d=78$, **d** $r=40$ 11 **a** $u=42$, **b** $n=43$, **c** $x=120$ 12 **a** $j=80$, **k** $=80$, **b** $r=35$, **s** $=72$, **t** $=73$, **c** $g=49$, **h** $=41$, **d** $d=120$, **e** $=36$, **e** $p=62$, **q** $=43$, **r** $=105$, **f** $a=63$, **b** $=63$, **c** $=54$, **g** $x=48$, **y** $=51$, **h** $t=43$, **u** $=60$, **i** $m=60$, **n** $=30$, **j** $e=63$, **f** $=43$, **k** $v=130$, **w** $=18$, **l** $a=43$, **b** $=57$, **m** $t=46$, **u** $=46$, **v** $=44$, **n** $p=60$, **q** $=68$, **r** $=52$, **s** $=308$, **o** $h=80$, **i** $=70$, **j** $=150$, **p** $e=27$, **f** $=126$, **g** $=126$, **q** $c=55$, **d** $=92$, **r** $b=58$, **c** $=161$, **s** $j=43$, **k** $=107$, **t** $v=47$, **w** $=47$, **x** $=57$, **u** $j=38$, **k** $=38$, **m** $=71$ 13 **a** $a=30$, **b** $y=40$, **c** $p=35$, **d** $m=19$, **e** $c=78$, **f** $u=22$ 14 **a** $x=24$, **b** $x=41$, **c** $x=58$, **d** $x=34$, **e** $x=54$, **f** $x=74$, **g** $x=24$, **h** $x=50$, **i** $x=82$, **j** $x=22$, **k** $x=38$, **l** $x=36$, **m** $x=80$, **n** $x=62$, **o** $x=25$

Exercise 10.4

1 **a** $a=10$, **b** $p=133$, **c** $t=103$, **d** $m=33$, **e** $e=122$, **f** $g=69$ 2 **a** $x=95$, **y** $=95$, **b** $p=60$, **q** $=80$, **c** $c=60$, **d** $=30$, **d** $r=47$, **s** $=60$, **e** $u=198$, **v** $=92$, **f** $w=126$, **x** $=54$, **g** $a=40$, **b** $=130$, **h** $y=33$, **z** $=146$, **i** $e=65$, **f** $=72$, **j** $g=68$, **h** $=68$, **k** $m=108$, **n** $=79$, **l** $j=96$, **k** $=107$ 3 **a** $x=105$, **y** $=70$, **b** $a=53$, **b** $=71$, **c** $p=140$, **q** $=40$, **r** $=69$, **d** $g=70$, **h** $=100$, **e** $u=162$, **v** $=18$, **f** $d=236$, **e** $=124$, **f** $=83$ 4 **a** $k=40$, **b** $c=44$, **c** $x=70$, **d** $y=12$, **e** $a=18$, **f** $t=51$ 5 **a** $x=83$, **b** $x=71$, **c** $x=66$, **d** $x=145$, **e** $x=124$, **f** $x=105$, **g** $x=70$, **h** $x=56$, **i** $x=117$, **j** $x=127$, **k** $x=25$, **l** $x=42$

Exercise 10.5

	Square	Rectangle	Parallelogram	Rhombus	Trapezium	Kite
All sides are equal	✓			✓		
Opposite sides are equal	✓	✓	✓	✓		
All angles are right angles	✓	✓				
Opposite angles are equal	✓	✓	✓	✓		
Opposite sides are parallel	✓	✓	✓	✓		
Equal diagonals	✓	✓				
Diagonals bisect each other	✓	✓	✓	✓		
Diagonals are perpendicular	✓			✓		✓
Diagonals bisect the angles at the vertices	✓			✓		

2 **a** 4, **b** 2, **c** 0, **d** 2, **e** 0, **f** 1 **3 a** True, **b** False, **c** True, **d** True, **e** False, **f** True, **g** True, **h** False, **i** False, **j** True **4 a** parallelogram, **b** rectangle, **c** kite, **d** parallelogram, **e** rhombus, **f** parallelogram, **g** trapezium, **h** parallelogram, **i** rhombus, **j** parallelogram, **k** rectangle, **l** rhombus, **m** rhombus, **n** square, **o** parallelogram, **p** kite, **q** rhombus, **r** square **5 a** $BD = 12$ cm, $BE = 6$ cm, $TR = 10$ cm, $PR = 20$ cm, $JN = 9$ cm, $KM = 22$ cm **6 a** parallelogram; $p = 5$, $q = 9$, $r = 100$, $s = 80$ **b** rectangle; $x = 90$, $y = 7$, $z = 3$, **c** square; $a = 45$, $b = 90$, $c = 6$, d kite; $e = 90$, $f = 60$, $g = 30$, $h = 20$, **e** parallelogram; $p = 20$, $q = 50$, $r = 50$, **f** rectangle; $x = 35$, $y = 55$, $z = 70$, **g** rhombus; $t = 32$, $u = 32$, $v = 90$, $w = 10$, **h** parallelogram; $a = 55$, $b = 55$, $c = 20$, $d = 105$, **i** rhombus; $a = 8$, $b = 50$, $c = 65$ **7 a** $a = 25$, $b = 155$, $c = 8$, $b t = 90$, $c p = 90$, $d e = 73$, $e u = 6$, $v = 8$, $f a = 90$, $b = 45$ **8 a** $a = 52$, $b = 128$, $b y = 65$, $z = 130$, $c p = 63$, $q = 63$, $d m = 41$, $n = 41$, $e e = 57$, $f = 57$, $g = 33$, $f t = 70$, $u = 90$, $v = 20$, $g g = 15$, $h = 15$, $i = 150$, $j = 30$, $h c = 90$, $d = 105$, $i x = 60$, $y = 60$, $z = 60$ **9 a** $a = 53$, $b = 53$, $b a = 66$, $b = 104$, $c a = 75$, $b = 15$, $d a = 83$, $b = 45$, $e a = 16$, $b = 132$, $f a = 48$, $b = 58$, $g a = 60$, $b = 112$, $h a = 15$, $b = 80$, $i a = 60$, $b = 124$ **10 a** $x = 35$, $b x = 120$, $c x = 71$, $d x = 45$, $e x = 72$, $f x = 23$, $g x = 83$, $h x = 57$, $i x = 65$, $j x = 76$, $k x = 48$, $l x = 54$

Exercise 10.6

1 **a** triangle, **b** quadrilateral, **c** pentagon, **d** hexagon, **e** heptagon, **f** octagon, **g** nonagon, **h** decagon, **i** undecagon, **j** dodecagon **2 a** convex, **b** non-convex, **c** non-convex, **d** convex **4** No; it would need at least four sides. **5 a** regular pentagon, **b** irregular hexagon, **c** irregular nonagon, **d** regular heptagon **6 a** equilateral triangle, **b** square **7 a** **i** rhombus, **ii** rectangle, **b** No, **c** No, **d** No **8 b** 3, **c** 540° , **d** No **9 a** 720° , **b** 900° , **c** 1080°

No. of sides	No. of triangles	angle sum
3	1	180°
4	2	360°
5	3	540°
6	4	720°
7	5	900°
8	6	1080°

b $n - 2$, **c** 180° , **d** $S = (n - 2) \times 180^\circ$
11 a 540° , $x = 108$, **b** 720° , $x = 120$, **c** 1080° , $x = 135$
12 a 140° , **b** 144° , **c** 150° **13** 156° **14 a** $x = 55$, **b** $y = 14$ **15 a** 72° , **b** 45° , **c** 36° , **d** 60° , **e** 40° , **f** 30°
16 24° **17 a** 15° , **b** 165° , **c** 3960° **18 a** 3, **b** 5, **c** 12, **d** 30 **19 a** $a = 30$, $b = 50$, $b a = 27$, $b = 7$, $c a = 80$, $b = 19$ **20 a** $x = 108$, $y = 108$, **b** $x = 120$, $y = 60$, $c x = 135$, $y = 135$, **d** $x = 140$, $y = 110$, **e** $x = 108$, $y = 22$, **f** $x = 120$, $y = 70$ **21 a** $128^\circ 34'$, **b** $147^\circ 16'$ **22** 2880°
24 Equilateral triangle, square, hexagon; their interior angles are factors of 360° **25** 108°

Exercise 10.7

1 a SSS, b AAS, c SAS, d RHS **2** a yes, SSS, b no, c yes, RHS, d no, e yes, SAS f yes, AAS, g yes, SAS, h no, i yes, AAS, j no, k yes, SSS, l yes, RHS **3** No, AAA is not a congruence test **4** a A, C, AAS, b A, B, RHS, c B, C, SAS, d A, C, SSS **5** A, J, RHS; D, F, SAS; C, G, AAS; I, K, AAS; E, L, SAS; B, H, AAS
6 a yes, b no, c yes, d yes **7** a no, b yes, c no

Exercise 10.8

- 1** a $EF = EH$ (given)
 $FG = HG$ (given)
 EG is a common side
 $\therefore \triangle EFG \equiv \triangle EHG$ (SSS)
- 2** a $\angle QPS = \angle RPS$ (given)
 $\angle PSQ = \angle PSR = 90^\circ$ ($PS \perp QR$)
 PS is a common side
 $\therefore \triangle PQS \equiv \triangle PRS$ (AAS)
- 3** a $CD = FE$ (given)
 $\angle CDE = \angle DEF = 90^\circ$ (given)
 DE is a common side
 $\therefore \triangle CDE \equiv \triangle FED$ (SAS)
- c $\angle QPR = \angle RTS$ (alternate $\angle$ s, $PQ \parallel ST$)
 $\angle PQR = \angle SRT$ (vert. opp. $\angle$ s)
 $SR = RQ$ (given)
 $\therefore \triangle PQR \equiv \triangle TSR$ (AAS)
- e $\angle OML = \angle OMN = 90^\circ$ (given)
 $OL = ON$ (equal radii)
 OM is a common side
 $\therefore \triangle OLM \equiv \triangle ONM$ (RHS)
- 4** a $BD = DF$ (CE bisects BF)
 $\angle CDB = \angle EDF$ (vert. opp. $\angle$ s)
 $CD = DE$ (CE bisects CE)
 $\therefore \triangle BCD \equiv \triangle FED$ (SAS)
- c $XW = WY$ (ZW bisects XY)
 $\angle XWZ = \angle YWZ = 90^\circ$ ($ZW \perp XY$)
 ZW is a common side
 $\therefore \triangle XWZ \equiv \triangle YWZ$ (SAS)
- e $\angle QPS = \angle QRS$ (given)
 $\angle PQS = \angle RQS$ (SQ bisects $\angle PQR$)
 SQ is a common side
 $\therefore \triangle QPS \equiv \triangle QRS$ (AAS)
- g $MN = PK$ (given)
 $\angle JMN = \angle PKL$ (opp. $\angle$ s of a rhombus)
 $JM = KL$ (opp. sides of a rhombus)
 $\therefore \triangle JMN \equiv \triangle LKP$ (SAS)
- i $\angle TUV = \angle UWV$ (alternate $\angle$ s, $TU \parallel WV$)
 $\angle TWU = \angle WUV$ (alternate $\angle$ s, $TW \parallel UV$)
 UW is a common side
 $\therefore \triangle UTW \equiv \triangle WVU$ (AAS)
- k $AY = XC$ (given)
 $\angle BAY = \angle XCD$ (alternate $\angle$ s, $AB \parallel DC$)
 $AB = DC$ (opp. sides of a parallelogram)
 $\therefore \triangle AYB \equiv \triangle CXD$ (SAS)
- b $BC = CD$ (given)
 $\angle ACB = \angle ECD$ (vert. opp. $\angle$ s)
 $AC = CE$ (given)
 $\therefore \triangle ABC \equiv \triangle EDC$ (SAS)
- b $\angle XZW = \angle XZY = 90^\circ$ ($XZ \perp WY$)
 $XW = XY$ (given)
 XZ is a common side
 $\therefore \triangle XWZ \equiv \triangle XYZ$ (RHS)
- b $\angle RST = \angle TUV$ (given)
 $\angle STR = \angle UTV$ (vert. opp. $\angle$ s)
 $ST = UT$ (given)
 $\therefore \triangle RST \equiv \triangle VUT$ (AAS)
- d $JK = ML$ (given)
 $\angle JKM = \angle KML$ (alternate $\angle$ s, $JK \parallel ML$)
 KM is a common side
 $\therefore \triangle MJK \equiv \triangle KLM$ (SAS)
- f $OE = OG$ (equal radii)
 $OF = OH$ (equal radii)
 $EF = GH$ (given)
 $\therefore \triangle EOF \equiv \triangle GOH$ (SSS)
- b $KL = NM$ (opp. sides of a parallelogram)
 $KN = LM$ (opp. sides of a parallelogram)
 LN is a common side
 $\therefore \triangle KLN \equiv \triangle MNL$ (SSS)
- d $\angle FEH = \angle FGH = 90^\circ$ ($FE \perp EH$, $FG \perp GH$)
 FH is a common side
 $EH = GH$ (given)
 $\therefore \triangle FEH \equiv \triangle FGH$ (RHS)
- f $SW = XU$ (given)
 $VW = XT$ (given)
 $SV = TU$ (opp. sides of a parallelogram)
 $\therefore \triangle SVW \equiv \triangle UTX$ (SSS)
- h $\angle ADE = \angle BCE = 90^\circ$ ($\angle$ s in a square)
 $AE = BE$ (given)
 $AD = BC$ (opp. sides of a square)
 $\therefore \triangle ADE \equiv \triangle BCE$ (RHS)
- j $\angle LKJ = \angle MLN$ (corresp. $\angle$ s, $KJ \parallel LN$)
 $\angle KLJ = \angle LMN$ (corresp. $\angle$ s, $LJ \parallel MN$)
 $KL = LM$ (JL bisects KM)
 $\therefore \triangle KLJ \equiv \triangle LMN$ (AAS)
- i $DE = DG$ (given)
 $EH = HG$ (adj. sides of a rhombus)
 DH is a common side
 $\therefore \triangle DEH \equiv \triangle DGH$ (SSS)

- 5 a $LM = LN$ (given)
 $PM = PN$ (given)
 LP is a common side
 $\therefore \triangle LMP \equiv \triangle LNP$ (SSS)
- c $AB = AC$ (given)
 $PB = QC$ (P, Q are midpoints of equal sides)
 $\angle PBC = \angle QCB$ (base $\angle$ s of isosceles $\triangle$, $AB = AC$)
 BC is a common side
 $\therefore \triangle PBC \equiv \triangle QCB$ (SAS)
- b $\angle JKN = \angle MLN = 90^\circ$ ($\angle$ s in a rectangle)
 $JN = MN$ (given)
 $JK = ML$ (opp. sides of a rectangle)
 $\therefore \triangle JKN \equiv \triangle MLN$ (RHS)

Exercise 10.9

- 1 a $\triangle ABC \equiv \triangle PQR$ (AAS), $p = 10$
 c $\triangle LMN \equiv \triangle LZX$ (RHS), $x = 7$
- 2 a $\triangle QRS \equiv \triangle GHF$ (SSS), $g = 83$
 c $\triangle ABC \equiv \triangle KIJ$ (SAS), $k = 9$
- 3 a ii $AE = EB$ (CD bisects AB)
 $\angle AEC = \angle DEB$ (vertically opp. $\angle$ s)
 $DE = EC$ (AB bisects CD)
 $\therefore \triangle AEC \equiv \triangle BED$ (SAS)
- iii $AC = DB$ (matching sides of congruent $\triangle$ s),
 $x = 12$
- c ii $\angle EFH = \angle FGH = 90^\circ$ ($EF \perp FH$, $GH \perp FH$)
 $EH = FG$ (given)
 FH is a common side
 $\therefore \triangle EFH \equiv \triangle GHF$ (RHS)
- iii $GH = EF$ (matching sides of congruent $\triangle$ s),
 $a = 14$
- e ii $\angle CBD = \angle EDF$ (corresp. $\angle$ s, $BC \parallel DE$)
 $\angle CDB = \angle EFD$ (corresp. $\angle$ s, $CD \parallel EF$)
 $BD = DF$ (given)
 $\therefore \triangle BCD \equiv \triangle DEF$ (AAS)
- iii $CD = EF$ (matching sides of congruent $\triangle$ s),
 $x = 4$
- 4 a i $\angle PSQ = \angle PSR = 90^\circ$ (PS is an altitude)
 $PQ = PR$ (given)
 PS is a common side
 $\therefore \triangle PQS \equiv \triangle PRS$ (RHS)
- ii $QS = SR$ (matching sides of congruent $\triangle$ s)
 $\therefore PS$ bisects QR
- c i $PQ = PR$ (given)
 $\angle QPS = \angle RPS$ (PS bisects $\angle QPR$)
 PS is a common side
 $\therefore \triangle PQS \equiv \triangle PRS$ (SAS)
- 5 a $\angle QPS = \angle RPS$ (PS bisects $\angle P$)
 $\angle PSQ = \angle PSR = 90^\circ$ ($PS \perp QR$)
 PS is a common side
 $\therefore \triangle PQS \equiv \triangle PRS$ (AAS)
 $PQ = PR$ (matching sides of congruent $\triangle$ s)
 $\therefore \triangle PQR$ is isosceles
- b $\triangle EFG \equiv \triangle UTV$ (SAS), $u = 13$
 d $\triangle IJK \equiv \triangle ECD$ (AAS), $e = 10$
 b $\triangle DEF \equiv \triangle YXZ$ (SAS), $y = 11$
 d $\triangle LMN \equiv \triangle VUT$ (RHS), $u = 7$
- b ii $PS = SR$ (given)
 $\angle PSQ = \angle RSQ$ (QS bisects $\angle PSR$)
 QS is a common side
 $\therefore \triangle PQS \equiv \triangle RQS$ (SAS)
- iii $\angle QRS = \angle QPS$ (matching $\angle$ s of congruent $\triangle$ s),
 $k = 100$
- d ii $\angle TUV = \angle VWX$ (alternate $\angle$ s, $TU \parallel WX$)
 $\angle TVU = \angle WVX$ (vert. opp. $\angle$ s)
 $TU = WX$ (given)
 $\therefore \triangle TVU \equiv \triangle XVW$ (AAS)
- iii $TV = VX$ (matching sides of congruent $\triangle$ s),
 $y = 9$
- f ii $CF = DE$ (given)
 $\angle FCE = \angle CED$ (alternate $\angle$ s, $CF \parallel DE$)
 CE is a common side
 $\therefore \triangle CFE \equiv \triangle EDC$ (SAS)
- iii $\angle DCE = \angle CEF$ (matching $\angle$ s of congruent $\triangle$ s),
 $w = 11$
- b i $PQ = PR$ (given)
 $QS = SR$ (PS is a median)
 PS is a common side
 $\therefore \triangle PQS \equiv \triangle PRS$ (SSS)
- ii $\angle QPS = \angle RPS$ (matching $\angle$ s of congruent $\triangle$ s)
 $\therefore PS$ bisects $\angle QPR$
- ii $QS = SR$ (matching sides of congruent $\triangle$ s)
 $\therefore PS$ bisects QR
 $\angle PSQ = \angle PSR$ (matching $\angle$ s of congruent $\triangle$ s)
 $\angle PSQ + \angle PSR = 180^\circ$ ($\angle$ s on a straight line)
 $\therefore \angle PSQ \equiv \angle PSR = 90^\circ$
 $\therefore PS$ is a perp. bisector of QR
- b PS is a common side
 $\angle PSQ = \angle PSR = 90^\circ$ ($PS \perp QR$)
 $QS = SR$ (PS bisects QR)
 $\therefore \triangle PQS \equiv \triangle PRS$ (SAS)
 $PQ = PR$ (matching sides of congruent $\triangle$ s)
 $\therefore \triangle PQR$ is isosceles

- 6 a** $EF = EH$ (given)
 $GF = GH$ (given)
 EG is a common side
 $\therefore \triangle EFG \equiv \triangle EHG$ (SSS)
- 7** Let the parallelogram be $STUV$.
a In $\triangle STV$ and $\triangle UVT$:
 $\angle SVT = \angle VTU$ (alternate $\angle$ s, $SV \parallel TU$)
 $\angle STV = \angle TVU$ (alternate $\angle$ s, $ST \parallel VU$)
 TV is a common side
 $\therefore \triangle STV \equiv \triangle UVT$ (AAS)
 $SV = TU$ (matching sides of congruent Δ s)
 $ST = VU$ (matching sides of congruent Δ s)
 $\therefore$ the opp. sides of a parallelogram are equal
- 8 a** $\angle AXD = \angle BXC$ (vert. opp. $\angle$ s)
 $\angle DAX = \angle XCB$ (alternate $\angle$ s, $AD \parallel BC$)
 $AD = BC$ (opp. sides of a parallelogram)
 $\therefore \triangle AXD \equiv \triangle CXB$ (AAS)
- 9 a** $WZ = XY$ (opp. sides of a rectangle)
 $\angle WZY = \angle XYZ = 90^\circ$ ($\angle$ s in a rectangle)
 ZY is a common side
 $\therefore \triangle WZY \equiv \triangle XYZ$ (SAS)
- 10 a** $QR = RS$ (adj. sides of a rhombus)
 $QX = XS$ (diagonals of a rhombus bisect each other)
 RX is a common side
 $\therefore \triangle QRX \equiv \triangle SRX$ (SSS)
- 11 a** $AC = CE$ (given)
 $\angle ACB = \angle DCE$ (vert. opp. $\angle$ s)
 $BC = CD$ (given)
 $\therefore \triangle ABC \equiv \triangle EDC$ (SAS)
- 12 a** $TU = WV$ (given)
 $\angle UTV = \angle TVW$ (alternate $\angle$ s, $TU \parallel WV$)
 TV is a common side
 $\therefore \triangle TUV \equiv \triangle VWT$ (SAS)
- 13 a** $AE = EC$ (BD bisects AC)
 $DE = EB$ (AC bisects DB)
 $\angle AED = \angle BEC$ (vert. opp. $\angle$ s)
 $\therefore \triangle AED \equiv \triangle CEB$ (SAS)
 $\angle DAE = \angle ECB$ (matching $\angle$ s of congruent Δ s)
 $\therefore AD \parallel BC$ (alternate $\angle$ s are equal)
- b** $\angle FEG = \angle HEG$ (matching $\angle$ s of congruent Δ s)
 $\therefore EG$ bisects $\angle FEH$
- b** In $\triangle STV$ and $\triangle UVT$:
 $SV = TU$ (opp. sides of a parallelogram)
 $ST = VU$ (opp. sides of a parallelogram)
 TV is a common side
 $\therefore \triangle STV \equiv \triangle UVT$ (SSS)
 $\angle VST = \angle VUT$ (matching $\angle$ s of congruent Δ s)
- In $\triangle SVU$ and $\triangle UTS$:
 $SV = TU$ (opp. sides of a parallelogram)
 $ST = VU$ (opp. sides of a parallelogram)
 SU is a common side
 $\therefore \triangle SVU \equiv \triangle UTS$ (SSS)
 $\angle SVU = \angle UTS$ (matching $\angle$ s of congruent Δ s)
 $\therefore$ the opp. $\angle$ s of a parallelogram are equal
- b** $AX = XC$ (matching sides of congruent Δ s)
 $DX = XB$ (matching sides of congruent Δ s)
 $\therefore$ the diagonals of a parallelogram bisect each other
- b** $WY = XZ$ (matching sides of congruent Δ s)
 $\therefore$ the diagonals of a rectangle are equal
- b** $\angle RXQ = \angle RXS$ (matching $\angle$ s of congruent Δ s)
 $\angle RXQ + \angle RXS = 180^\circ$ ($\angle$ s on a straight line)
 $\therefore \angle RXQ = \angle RXS = 90^\circ$
 $\therefore$ the diagonals of a rhombus are perpendicular
- b** $\angle ABC = \angle CDE$ (matching $\angle$ s of congruent Δ s)
 $\therefore AB \parallel DE$ (alternate $\angle$ s are equal)
- b** $\angle WTV = \angle TVU$ (matching $\angle$ s of congruent Δ s)
 $\therefore TW \parallel UV$ (alternate $\angle$ s are equal)
- c** $TUVW$ is a parallelogram (both pairs of opp. sides are parallel)
- b** $AE = EC$ (BD bisects AC)
 $DE = EB$ (AC bisects DB)
 $\angle AEB = \angle DEC$ (vert. opp. $\angle$ s)
 $\therefore \triangle AEB \equiv \triangle CED$ (SAS)
 $\angle ABE = \angle EDC$ (matching $\angle$ s of congruent Δ s)
 $\therefore AB \parallel DC$ (alternate $\angle$ s are equal)
- c** $AD \parallel BC$ and $AB \parallel DC$, $\therefore ABCD$ is a parallelogram

Chapter 10 Review

- 1 a** No, no common ray, **b** Yes, **c** No, no common vertex, **d** Yes **2 a** straight angle, **b** acute angle, **c** revolution, **d** obtuse angle, **e** right angle, **f** reflex angle **3 a** equal, **b** 90° , **c** 180° , **d** 360° **4 a** $d, f; c, e$, **b** $a, e; b, f; d, h; c, g, c, f; d, e$ **5 a** No, corresponding angles are not equal, **b** Yes, co-interior angles are supplementary,

- c Yes, alternate angles are equal **6** a equilateral, b scalene, c isosceles **7** a obtuse-angled, b right-angled
8 a shortest = BC ; longest = AC , b smallest = $\angle Q$, largest = $\angle R$ **9** No, the sum of any two sides must be greater than the third side **10** a $p = 35$, b $y = 107$, c $e = 52$, d $k = 275$, e $t = 116$, f $n = 145$, g $r = 68$, h $x = 45$,
i $b = 73$, j $m = 125$, k $q = 44$, l $a = 71$ **11** a $u = 14$, b $a = 6$, c $k = 9$, d $m = 30$ **12** a $v = 23$, w = 23,
b $p = 75$, q = 55, c $a = 36$, b = 36, d $p = 68$, q = 112, e $d = 73$, e = 133, f $x = 53$, y = 64, z = 117, g $u = 63$, v = 117,
h $e = 57$, f = 33, i $r = 101$, s = 65, j $j = 62$, k = 66, k $e = 59$, f = 100, l $x = 61$, y = 61, z = 61, m $w = 62$, x = 100,
n $u = 49$, v = 79, o $e = 102$, f = 13 **13** a A, opposite angles are equal, C, opposite sides are equal, D, diagonals
bisect each other, b A, diagonals are equal and bisect each other, D, all angles are right angles, c A, all sides are
equal, D, diagonals bisect each other at right angles, d B, diagonals are equal and bisect each other at right angles,
C, all sides are equal and one angle is a right angle **14** a True, b False, c True, d True
15 a $f = 71$, g = 71, h = 71, b $a = 90$, b = 21, c = 42, c $r = 11$, s = 45, t = 124, u = 124 **16** a quadrilateral,
b pentagon, c hexagon, d octagon, e decagon, f dodecagon **18** No, because angles are not equal.
19 a 540° , 108° , b 1260° , 140° **20** a 36° , b 30° **21** interior = 162° , exterior = 18° **22** a 24, b 72
23 a $\triangle PSQ \equiv \triangle RSQ$, RHS, a = 15, b $\triangle MKL \equiv \triangle MKN$, SSS, r = 6, c $\triangle FGE \equiv \triangle HGI$, AAS, u = 6,
d $\triangle WXV \equiv \triangle YXZ$, SAS, x = 7
24 a $\angle RTQ = \angle RTS = 90^\circ$ ($RT \perp QS$)
 $\angle QRT = \angle SRT$ (TR bisects $\angle QRS$)
 TR is a common side
 $\therefore \triangle RQT \equiv \triangle RST$ (AAS)
b $RQ = RS$ (matching sides of congruent $\triangle$ s)
 $\therefore \triangle QRS$ is isosceles
26 a $DG = EF$ (given)
 $\angle DGF = \angle DFE$ (alternate $\angle$ s, $DG \parallel EF$)
 DF is a common side
 $\therefore \triangle DGF \equiv \triangle FED$ (SAS)
b $\angle EDF = \angle DFG$ (matching $\angle$ s of congruent $\triangle$ s)
 $\therefore DE \parallel GF$ (alternate $\angle$ s are equal)
25 a $\angle VWZ = \angle VXY = 90^\circ$ ($\angle$ s in a rectangle)
 $VZ = VY$ (given)
 $WZ = XY$ (opp. sides of a rectangle)
 $\therefore \triangle VWZ \equiv \triangle VXY$ (RHS)
b $VW = VX$ (matching sides of congruent $\triangle$ s)
 $\therefore V$ is the midpoint of WX
27 a $AD = DB$ (CD bisects AB)
 $AC = CB$ (given)
 CD is a common side
 $\therefore \triangle DCA \equiv \triangle DCB$ (SSS)
b $\angle ADC = \angle BDC$ (matching $\angle$ s of congruent $\triangle$ s)
 $\angle ADC + \angle BDC = 180^\circ$ ($\angle$ s on a straight line)
 $\therefore \angle ADC = \angle BDC = 90^\circ$
 $\therefore CD \perp AB$

11

The linear function

Exercise 11.1

- 1** a A(1, 3), b B(-3, 2), c C(-2, -4), d D(2, -1), e E(4, 1), f F(-2, -2), g G(-1, 2), h H(4, -3), i I(-3, -1),
j J(1, -2), k K(2, 4), l L(-2, 3), m M(0, 1), n N(3, 0), o O(3, 3), p P(0, -4), q Q(-2, 0), r R(3, 2), s S(2, -4),
t T(-4, 4), u U(-4, -3) **2** a 2nd, b 1st, c 4th, d 3rd, e 1st, f 3rd, g 2nd, h 4th, i 3rd, j 2nd, k 4th, l 1st
3 a 5, b 4, c 7, d 11, e 5, f 9 **4** a (6, 5), b (-7, 6), c (4, 3), d (-7, -9) **5** a D(6, 3), b 24 units, c 32 units²
6 a N(-10, 4), b 84 units² **7** b isosceles triangle, c 8 units² **8** b 60 units² **9** a Yes, b No
10 centre = (5, 6), radius = 4 units **11** a A(9, 0), B(9, 4), C(9, -4), b 8π units, c 16π units²
12 A(5, 0), B(0, 5), C(5, 10), D(10, 5) **13** a C(3, 0), b 5 units, c P(0, 4), Q(0, -4), PQ = 8 units
14 a (4, 30°), b (7, 80°), c (5, 100°), d (6, 180°), e (6, 350°), f (8, 130°), g (2, 230°), h (5, 330°), i (7, 210°),
j (3, 50°), k (3, 260°), l (8, 300°)

Exercise 11.2

1 a

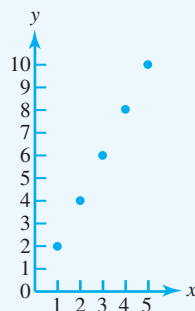


b

No. of circles (x)	1	2	3	4	5
No. of dots (y)	2	4	6	8	10

c 2, d $y = 2x$, e

f No, g Yes



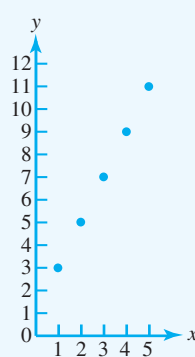
2 a



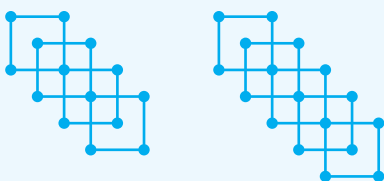
b

No. of triangles (x)	1	2	3	4	5
No. of dots (y)	3	5	7	9	11

c 2, d $y = 2x + 1$, e



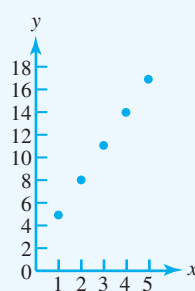
3 a



b

No. of squares (x)	1	2	3	4	5
No. of dots (y)	5	8	11	14	17

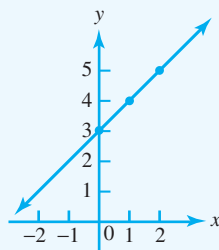
c 3, d $y = 3x + 2$, e



4 a

x	0	1	2
y	3	4	5

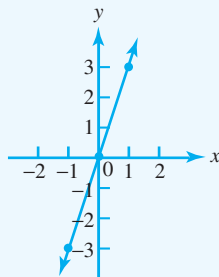
$y = x + 3$



b

x	-1	0	1
y	-3	0	3

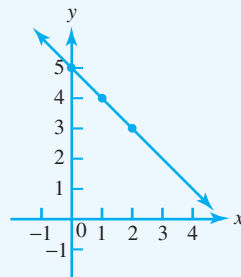
$y = 3x$



c

x	1	2	3
y	4	3	2

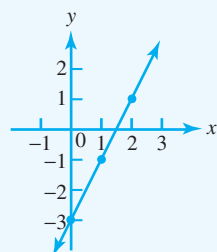
$y = 5 - x$



d

x	0	1	2
y	-3	-1	1

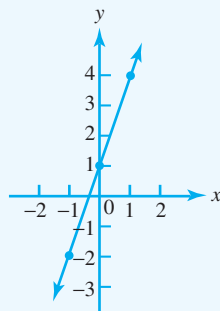
$$y = 2x - 3$$



e

x	-1	0	1
y	-2	1	4

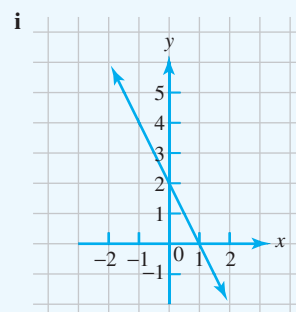
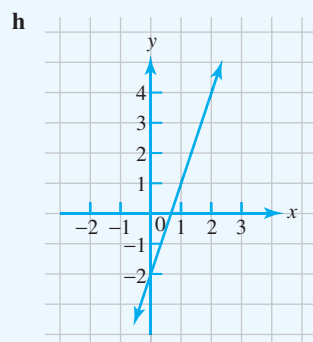
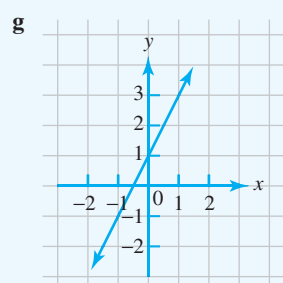
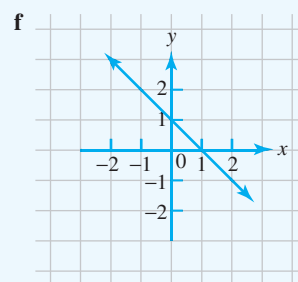
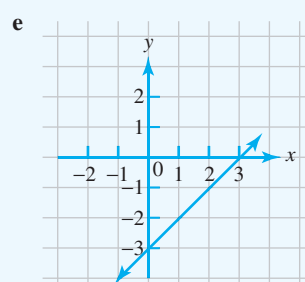
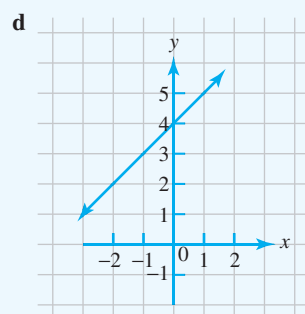
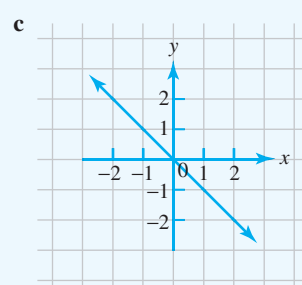
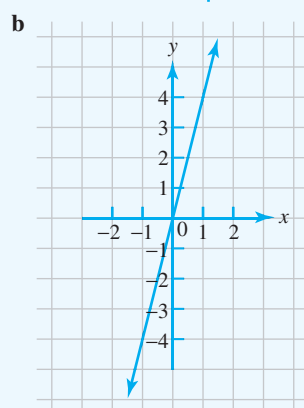
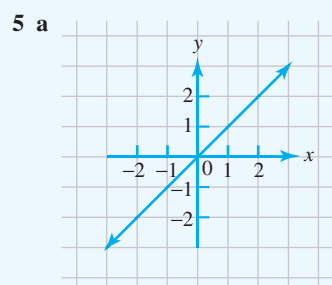
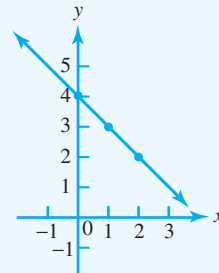
$$y = 3x + 1$$

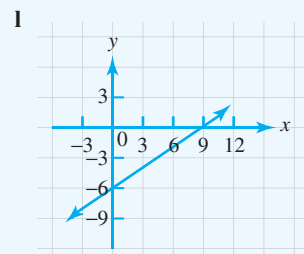
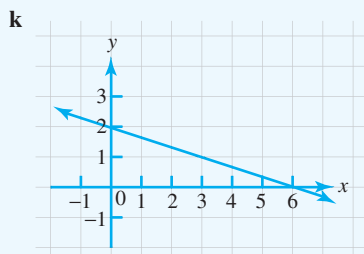
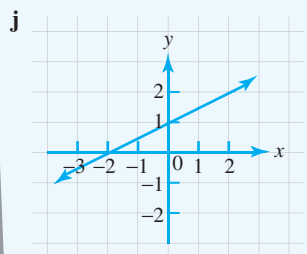
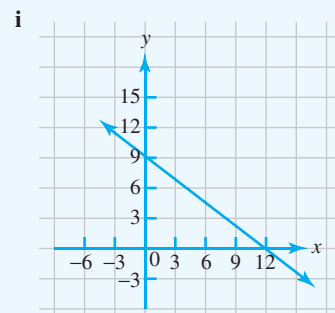
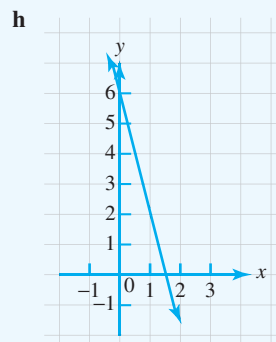
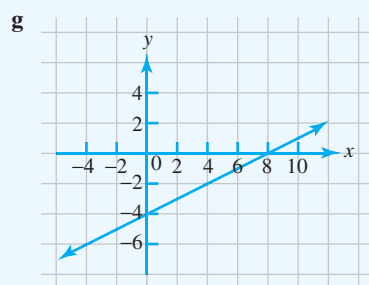
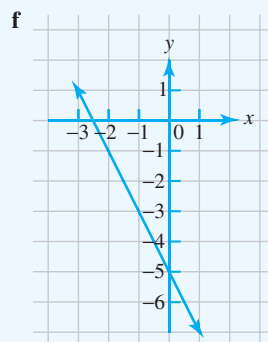
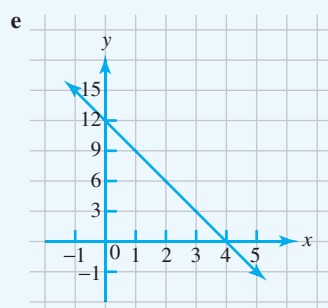
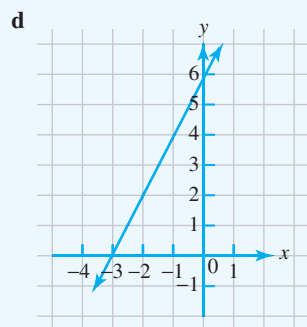
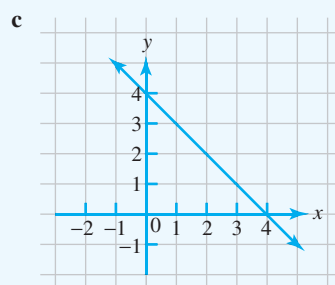
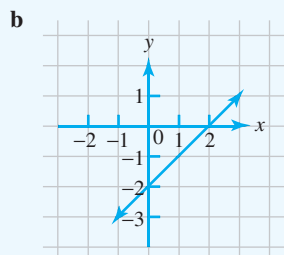
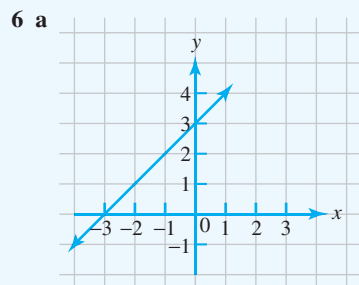
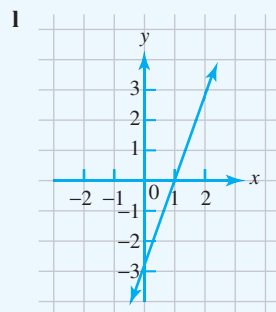
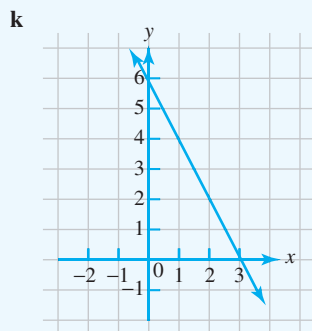
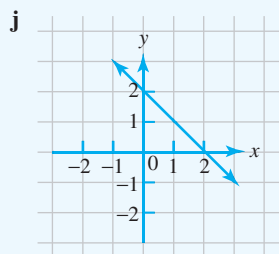


f

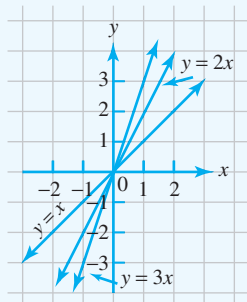
x	0	1	2
y	4	3	2

$$x + y = 4$$



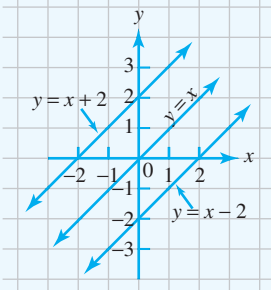


7 a



- b** The larger the co-efficient, the steeper the line.
8 $y = 2x$ is steeper because the co-efficient is larger.

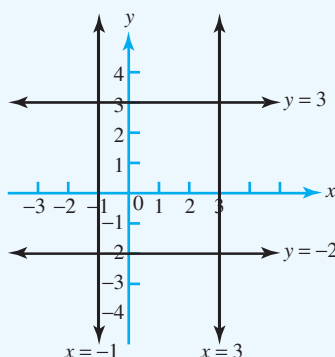
9 a



- b** Yes, **c** The constant term translates the line upward or downward.
10 **a** translate up 5 units, **b** translate down 3 units, **c** translate up 2 units,
d translate down 7 units **11** reflection in the y-axis
12 **a** translate up 3 units, **b** translate down 5 units, **c** translate up 4 units,
d translate down 1 unit **13** **a** $y = -x$, **b** $y = 2x$, **c** $y = -\frac{1}{2}x$, **d** $y = \frac{1}{3}x$,
e $y = -x + 1$, **f** $y = -x - 4$, **g** $y = 3x + 2$, **h** $y = 5 + 2x$
14 **a** reflect in the y-axis then translate up 3 units, **b** reflect in the y-axis then
translate down 4 units, **c** reflect in the y-axis then translate down 7 units,
d reflect in the y-axis then translate down 3 units, **e** reflect in the y-axis then
translate up 3 units, **f** reflect in the y-axis then translate down 10 units

Exercise 11.3

1



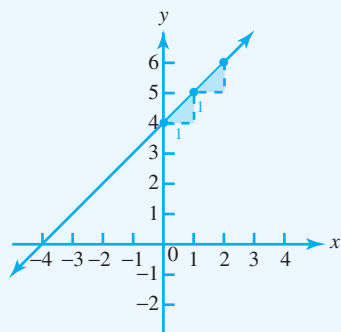
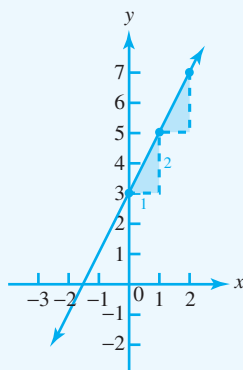
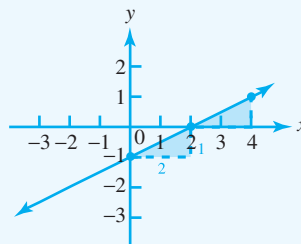
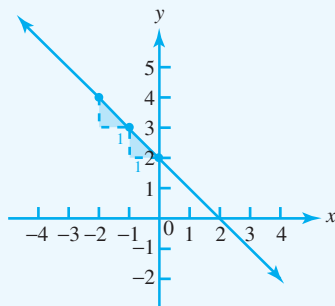
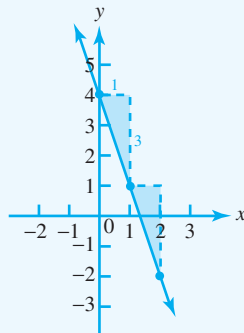
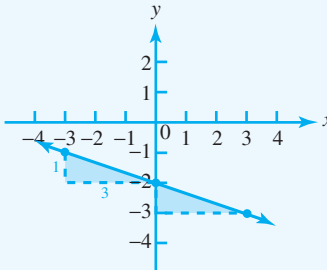
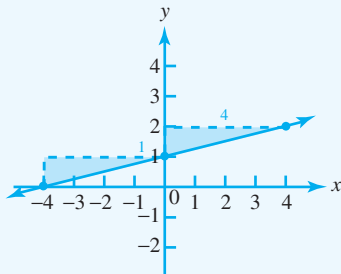
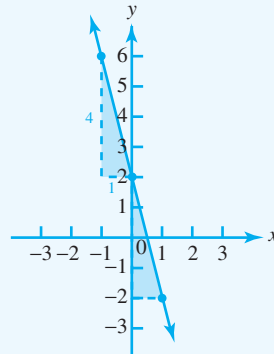
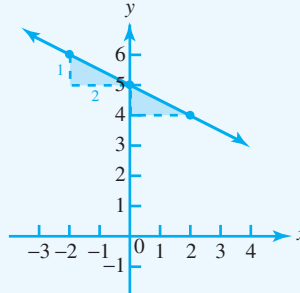
- a** $x = 3$, **b** $y = 3$, **c** $x = -1$, **d** $y = -2$ **2** **a** $x = 1$, **b** $y = 2$, **c** $y = -3$,
d $x = -2$, **e** $y = 4$, **f** $x = -5$, **g** $x = 4$, **h** $y = -6$ **3** **a** (3, 2), **b** (4, 1),
c (-6, -3) **4** **a** (1, 0), **b** (0, 7) **5** **a** $y = 0$, **b** $x = 0$ **6** **a** $x = 5$,
b $y = -3$ **7** **a** $y = 6$, **b** $x = -1$, **c** $x = 5$, **d** $y = -1$ **8** A, C, D, E, G
9 $y = x + 5$, $y = 2x + 7$, $y = -x + 1$, $x + 2y = 4$, $y = -\frac{3}{2}x$ **11** **a** $r = 9$,
b $k = 7$, **c** $t = 18$, **d** $p = -8$ **12** **a** $g = 0$, **b** $s = 20$, **c** $u = 10$, **d** $a = 9$
13 **a** $c = 7$, **b** $m = 4$, **c** $a = \pm 3$ **14** **a** (-2, 3), **b** (5, 5), **c** (3, -3),
d (-1, 3), **e** (1, 3), **f** (-2, -5), **g** (-1, -2), **h** (2, -1) **15** **a** Yes, **b** Yes,
c The lines intersect at (-2, 3) **16** **a** Yes, **b** No, **c** No, **d** Yes
17 The lines are parallel **18** **a** four legs for horses and two legs for
jockeys, $4x + 2y = 26 \Rightarrow 2x + y = 13$, **b** (4, 5), **c** 4 horses and 5 jockeys
19 **b** (7, 13), **c** 7 five-cent coins and 13 ten-cent coins.

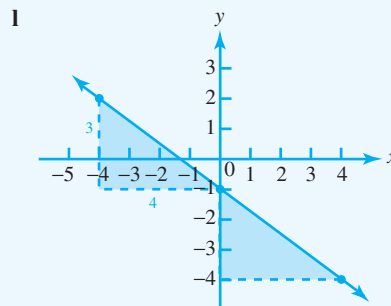
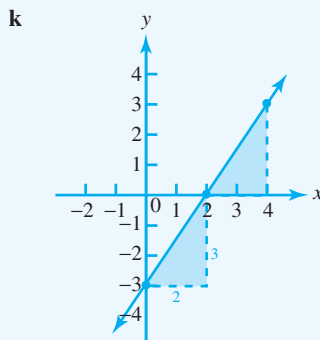
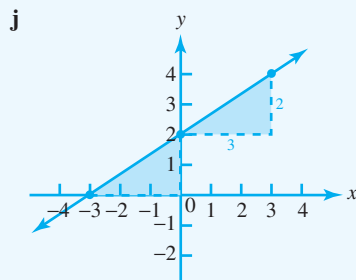
Exercise 11.4

- 1** **a** positive, **b** negative, **c** zero, **d** undefined, **e** negative, **f** zero, **g** undefined, **h** positive **2** **a** 1, **b** $\frac{1}{2}$, **c** -1,
d -3, **e** $\frac{2}{3}$, **f** $-\frac{3}{2}$, **g** $-\frac{4}{5}$, **h** $\frac{1}{6}$, **i** $-\frac{3}{4}$, **j** -2, **k** $\frac{3}{7}$, **l** 2, **m** $-\frac{8}{3}$, **n** $\frac{3}{4}$, **o** $-\frac{3}{5}$, **p** 1, **q** $-\frac{1}{4}$, **r** -3 **3** **a** $\frac{2}{3}$, **b** gradient
4 **a** 1, **b** -3, **c** $\frac{1}{2}$, **d** -5, **e** $\frac{3}{2}$, **f** $\frac{2}{5}$, **g** $-\frac{3}{4}$, **h** $\frac{7}{3}$, **i** $-\frac{2}{5}$, **j** $\frac{1}{2}$, **k** $-\frac{4}{3}$, **l** $-\frac{3}{5}$ **5** **a** 3, **b** -1, **c** $\frac{5}{4}$, **d** $-\frac{6}{5}$, **e** $\frac{1}{2}$, **f** $\frac{2}{3}$
6 **a** $m_{PQ} = \frac{4}{3}$, $m_{SR} = \frac{4}{3}$, Yes, $PQ \parallel SR$, **b** $m_{QR} = -\frac{1}{3}$, $m_{PS} = -\frac{1}{3}$, Yes, $QR \parallel PS$, **c** Parallelogram, because the opposite
sides are parallel, **d** gradient **7** **a** $m_{KL} = \frac{3}{5}$, $m_{LM} = \frac{1}{4}$, $m_{NM} = \frac{3}{5}$, $m_{KN} = 2$, **b** Trapezium, one pair of opposite sides
parallel **8** **a** $m_{AB} = \frac{5}{6}$, $m_{CD} = 1$, $m_{EF} = 1$, $m_{GH} = \frac{5}{6}$, **b** $AB \parallel GH$, $CD \parallel EF$ **9** **a** (6, 5), **b** (3, 5), **c** (-1, -6), **d** (4, 5),
10 **a** $m = \frac{y_2 - y_1}{x_2 - x_1}$, **b** **i** 4, **ii** $-\frac{2}{3}$, **iii** 5

Exercise 11.5

1 **a** $m=2, b=3$, **b** $m=3, b=-1$, **c** $m=-2, b=5$, **d** $m=-4, b=-3$, **e** $m=1, b=4$, **f** $m=-1, b=2$, **g** $m=5, b=3$,
h $m=-1, b=6$, **i** $m=2, b=0$, **j** $m=-7, b=0$, **k** $m=\frac{1}{2}, b=4$, **l** $m=\frac{1}{3}, b=-5$, **m** $m=\frac{2}{3}, b=1$, **n** $m=-\frac{3}{4}, b=8$,
o $m=-\frac{6}{5}, b=-2$, **p** $m=3, b=-6$, **q** $m=8, b=20$, **r** $m=14, b=-10$ 2 **a** $y=4x+2$, **b** $y=-3x+5$, **c** $y=x-4$,
d $y=-\frac{1}{2}x-7$, **e** $y=5x$, **f** $y=-x$ 3 **a** $y=3x+3$, **b** $y=\frac{1}{2}x+1$, **c** $y=3x-6$, **d** $y=-5x+10$, **e** $y=-\frac{1}{4}x-2$,
f $y=-x+7$, **g** $y=4x+12$, **h** $y=-\frac{2}{5}x+2$, **i** $y=-\frac{2}{3}x-4$, **j** $y=\frac{3}{7}x-3$, **k** $y=\frac{2}{3}x+6$, **l** $y=-\frac{6}{5}x+12$
4 **a** $y=2x+5$, **b** $y=\frac{1}{2}x+3$, **c** $y=-x+9$, **d** $y=-3x-2$, **e** $y=\frac{1}{4}x-5$, **f** $y=-\frac{2}{5}x+1$ 5 **a** $y=2x-1$, **b** $y=x+1$,
c $y=3x+5$, **d** $y=-2x-8$, **e** $y=-x+5$, **f** $y=-3x+4$, **g** $y=\frac{1}{2}x+3$, **h** $y=-\frac{1}{2}x+3$, **i** $y=\frac{1}{3}x-3$, **j** $y=-\frac{2}{3}x-2$,
k $y=-\frac{5}{4}x+2$, **l** $y=\frac{3}{2}x-15$

6 **a****b****c****d****e****f****g****h****i**

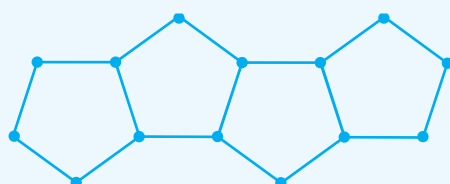


7 **a** $y = 2x - 5$, **b** $y = 5 - 2x$, **c** $y = 2x + 5$, **d** $y = -2x - 5$ **8** **a** $y = -3x - 7$, **b** $y = 3x - 7$, **c** $y = 7 - 3x$, **d** $y = 3x + 7$ **9** **a** $y = -\frac{1}{2}x$, **b** $y = 2x$, **c** $y = \frac{1}{2}x$, **d** $y = -2x$ **10** **a** $x = 5$, **b** $y = 5x - 1$, **c** $y = \frac{1}{5}x$, **d** $y = -5x - 1$, **e** $y = -5x$, **f** $y = -5$, **g** $y = 5x + 1$, **h** $y = 5x$, **i** $x = -5$, **j** $y = 1 - 5x$, **k** $y = 5$, **l** $y = -\frac{1}{5}x$ **11** **a** i Same gradient ($m = 2$), ii Different y-intercepts, **b** i Same y-intercepts ($b = 5$), ii Different gradients, **c** i All pass through origin, ii Different gradients, **d** i All horizontal lines, ii Different y-intercepts **12** **a** A, F, H, K, **b** B, E, H, I, K, **c** A, G, d E, G, J, **e** D, I, L **13** **a** The line would be steeper, **b** The line would be reflected in the y-axis, **c** The line would be translated down 4 units **14** **a** The line would be horizontal, **b** $y = 2$ **15** **a** $y = 3x + 4$, **b** $y = -2x + 9$, **c** $y = -\frac{1}{2}x - 4$ **16** **a** $y = 3x + 5$, **b** $y = \frac{1}{2}x + 6$, **c** $y = -\frac{2}{3}x + 10$ **17** $y = -\frac{q}{p}x + q$

Chapter 11 Review

1 **a** (2, 1), **b** (-2, 3), **c** (-2, -2), **d** (1, -2), **e** (-1, 1), **f** (1, 3), **g** (3, -1), **h** (0, 0), **i** (0, 2), **j** (1, 0), **k** (-3, 0), **l** (0, -3) **2** **a** 4th, **b** 3rd, **c** 1st, **d** 2nd **3** **a** 7 units, **b** 2 units **4** **a** (4, 5), **b** (-5, -3) **5** **a** C (-2, -1), D (-2, 4), **b** 20 units **6** **b** 45 units² **7** P (-6, 1), R (0, 1), Q (6, 1)

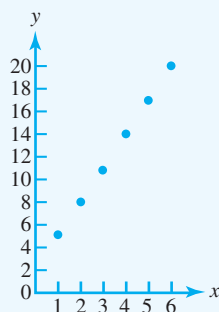
8 a



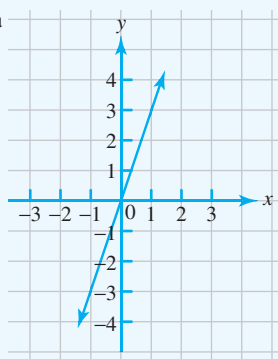
b

No. of pentagons (x)	1	2	3	4	5	6
No. of dots (y)	5	8	11	14	17	20

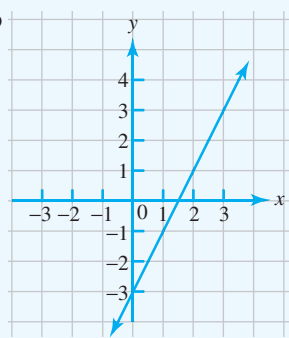
c 3, **d** $y = 3x + 2$, **e**

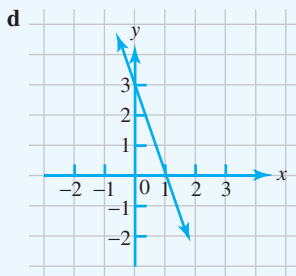
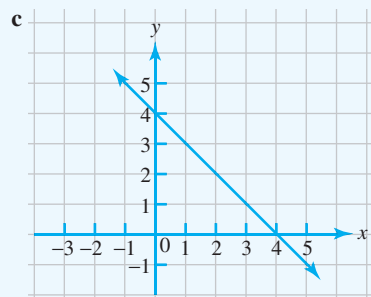


9 a



b





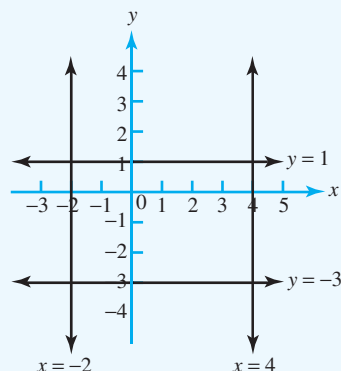
10 **a** reflect in the y-axis, then translate down 2 units, **b** reflect in the y-axis, then translate up 7 units, **c** reflect in the y-axis, then translate up 5 units

11 **a** $x = 3, y = 6$, **b** $x = 4, y = -6$,

c $x = -3, y = 1$ **12** A, C

13 C, D **14** **a** $a = 6$, **b** $k = 5$

15



16 (1, 6) **17** x-axis: $y = 0$, y-axis: $x = 0$ **18** **a** $x = 4$, **b** $y = -3$ **19** **a** 0,

b negative, **c** undefined, **d** positive **20** **a** 3, **b** $\frac{2}{3}$, **c** -1, **d** $-\frac{1}{2}$, **e** 2,

f $-\frac{5}{4}$, **g** $\frac{10}{7}$, **h** $-\frac{2}{3}$ **21** gradient **22** **a** D (-7, -9), **b** D (3, 1)

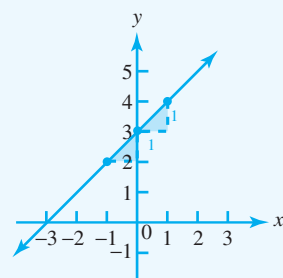
23 **a** $m = 3, b = 8$, **b** $m = 4, b = -4$, **c** $m = -1, b = 7$, **d** $m = -2, b = 0$,

e $m = \frac{1}{2}, b = 11$, **f** $m = \frac{4}{3}, b = -1$ **24** **a** $y = 2x - 4$, **b** $y = -3x + 5$,

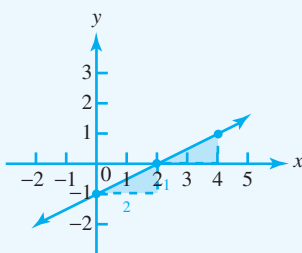
c $y = \frac{1}{4}x$ **25** **a** $y = 4x + 4$, **b** $y = -3x + 6$, **c** $y = \frac{3}{5}x$, **d** $y = 3x - 9$,

e $y = -\frac{4}{3}x - 8$, **f** $y = -\frac{3}{2}x$ **26** **a** $y = 3x + 7$, **b** $y = -2x + 5$, **c** $y = \frac{2}{3}x - 4$

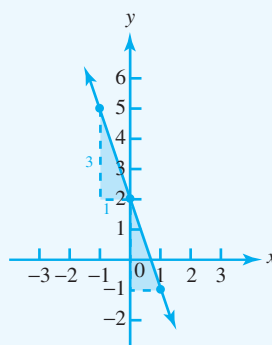
27



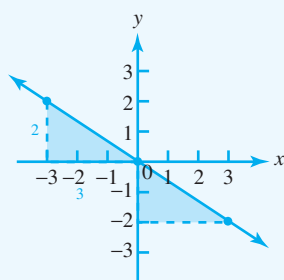
b



c



d



28 **a** $y = -7x + 4$, **b** $y = 7x + 4$, **c** $y = -7x - 4$, **d** $y = 7x - 4$

29 (-1, 2) **30** No

12 Trigonometry

Exercise 12.1

1 a i XZ, ii ZY, iii XY, b i PR, ii PQ, iii QR, c i AB, ii BC, iii AC, d i MN, ii LN, iii LM, e i PQ, ii PO, iii OQ, f i VW, ii VX, iii WX 2 a UV, b TU, c TU, d UV 3 a i $\frac{20}{21}$, ii $\frac{20}{29}$, iii $\frac{21}{29}$, b i $\frac{35}{12}$, ii $\frac{35}{37}$, iii $\frac{12}{37}$, c i $\frac{7}{24}$, ii $\frac{7}{25}$, iii $\frac{24}{25}$ 4 a i AB = 24 mm, BC = 18 mm, AC = 30 mm, ii DE = 40 mm, EF = 30 mm, DF = 50 mm, iii GH = 32 mm, HI = 24 mm, GI = 40 mm, b i $\angle A = \angle D = \angle G = 37^\circ$, ii $\angle C = \angle F = \angle I = 53^\circ$, c Yes, the matching angles are equal, d i $\frac{3}{4}$, ii $\frac{3}{4}$, iii $\frac{3}{4}$, iv $\frac{3}{5}$, v $\frac{3}{5}$, vi $\frac{3}{5}$, vii $\frac{4}{5}$, viii $\frac{4}{5}$, ix $\frac{4}{5}$, e They are equal 5 a i $\frac{CE}{CD}$, ii $\frac{CD}{DE}$, iii $\frac{CE}{DE}$, b i $\frac{YZ}{XZ}$, ii $\frac{XZ}{XY}$, iii $\frac{YZ}{XY}$, c i $\frac{QR}{RS}$, ii $\frac{RS}{QS}$, iii $\frac{QR}{QS}$ 6 a i $\frac{5}{12}$, ii $\frac{3}{4}$, iii $\frac{3}{4}$, iv $\frac{5}{12}$, b i Y, ii X, iii N, iv L

Exercise 12.2

1 a opposite = EG, adjacent = EF, hypotenuse = GF, b opposite = QR, adjacent = SR, hypotenuse = QS, c opposite = KJ, adjacent = IK, hypotenuse = IJ 2 a i $\frac{3}{5}$, ii $\frac{4}{5}$, iii $\frac{3}{4}$, b i $\frac{12}{13}$, ii $\frac{5}{13}$, iii $\frac{12}{5}$, c i $\frac{15}{17}$, ii $\frac{8}{17}$, iii $\frac{15}{8}$, d i $\frac{20}{29}$, ii $\frac{21}{29}$, iii $\frac{20}{21}$, e i $\frac{12}{37}$, ii $\frac{35}{37}$, iii $\frac{12}{35}$, f i $\frac{7}{25}$, ii $\frac{24}{25}$, iii $\frac{7}{24}$, g i $\frac{40}{41}$, ii $\frac{9}{41}$, iii $\frac{40}{9}$, h i $\frac{63}{65}$, ii $\frac{16}{65}$, iii $\frac{63}{16}$, i i $\frac{11}{61}$, ii $\frac{60}{61}$, iii $\frac{11}{60}$ 3 a i $\frac{x}{z}$, ii $\frac{y}{z}$, iii $\frac{x}{y}$, iv $\frac{y}{z}$, v $\frac{x}{z}$, vi $\frac{y}{x}$, b i $\frac{u}{v}$, ii $\frac{t}{v}$, iii $\frac{u}{t}$, iv $\frac{t}{v}$, v $\frac{u}{v}$, vi $\frac{t}{u}$ 4 a Z, b X, c Z, d X, e X, f Z 5 a $\frac{5}{13}$, b $\frac{12}{35}$, c $\frac{12}{37}$, d $\frac{12}{13}$, e $\frac{12}{13}$, f $\frac{35}{12}$, g $\frac{12}{37}$, h $\frac{5}{12}$, i $\frac{5}{13}$, j $\frac{12}{5}$, k $\frac{35}{37}$, l $\frac{35}{37}$ 6 a 17 cm, b $\sin \theta = \frac{8}{17}$, $\cos \theta = \frac{15}{17}$, $\tan \theta = \frac{8}{15}$ 7 $\sin \theta = \frac{9}{41}$, $\cos \theta = \frac{40}{41}$, $\tan \theta = \frac{9}{40}$ 8 1 9 a $x = 3$, b $k = 14$, c $w = 20$, d $t = 8$, e $a = 9$, f $c = 40$ 10 a $\tan P = \frac{24}{7}$, $\tan R = \frac{7}{24}$ 11 a $\frac{3}{5}$, b $\frac{4}{3}$, c $\frac{4}{5}$ 12 28 mm 13 a True, b False, c False, d True, e False, f False, g False, h True, i True, j False, k False, l True 14 a $\sin^2 \theta = \frac{9}{25}$, $\cos^2 \theta = \frac{16}{25}$, b 1, c $\sin^2 \theta + \cos^2 \theta = 1$ 15 a $\cos \theta = \frac{24}{25}$, $\tan \theta = \frac{7}{24}$, b $\sin \theta = \frac{11}{61}$, $\tan \theta = \frac{11}{60}$

Exercise 12.3

1 a 19° , b 26° , c 36° , d 8° , e 41° , f 50° , g 13° , h 64° , i 102° , j 126° , k 143° , l 167° 2 a 0.57, b 1.38, c 0.98, d 0.32, e 34.44, f 44.61, g 133.57, h 7.41, i 83.44, j 46.17, k 80.89, l 26.62, m 1.22, n 2.61, o 1.03, p 0.04 3 a 3.03, b 0.55, c 40.91, d 0.10, e 1.05, f 0.15, g 5.74, h 15.84, i 0.69, j 1.40, k 1.54, l 0.34 4 a 16° , b 25° , c 77° , d 65° , e 38° , f 42° , g 62° , h 72° , i 52° , j 28° , k 56° , l 86° , m 1° , n 16° , o 80° 5 a 65° , b 21° , c 6° , d 34° , e 63° , f 19° , g 81° , h 67° 6 a 0.743, b 0.191, c 0.577 7 a $\cos \theta = 0.6692$, $\tan \theta = 1.1105$, b $\sin \theta = 0.8387$, $\cos \theta = 0.5446$, c $\tan \theta = 8.1423$, $\sin \theta = 0.9925$ 8 a 0.77, b 0.98, c 1.53, d 0.42, e 0.98, f 1.72

Exercise 12.4

1 a $a = 7.4$, b $y = 12.2$, c $p = 15.9$, d $k = 4.6$, e $w = 42.3$, f $b = 24.3$ 2 a $e = 16.8$, b $g = 40.6$, c $s = 17.0$, d $h = 49.7$, e $m = 55.0$, f $z = 45.1$ 3 a $t = 5.5$, b $c = 18.3$, c $x = 193.0$, d $q = 31.0$, e $d = 17.2$, f $r = 317.6$ 4 a $x = 6.38$, b $p = 15.65$, c $t = 39.60$, d $c = 11.11$, e $a = 11.52$, f $f = 11.85$, g $y = 10.14$, h $w = 4.07$, i $h = 52.66$, j $b = 36.88$, k $g = 90.74$, l $m = 47.82$, m $v = 5.27$, n $d = 10.04$, o $s = 35.70$, p $z = 23.30$, q $e = 65.99$, r $u = 7.72$ 5 a 43.9 mm, b 39.7 mm, c 92.5 mm, d 8.8 mm, e 67.3 mm, f 20.0 mm, g 35.4 mm, h 19.9 mm, i 54.6 mm 6 a $a = 27.1$, b $n = 15.1$, c $v = 19.6$ 7 a 15.7 cm, b 18.07 mm, c 190 m, d 35.370 km

Exercise 12.5

1 a 5.6 m, b 2.2 m, c 8.5 m 2 263 m 3 18.9 cm 4 a 98.3 cm, b 68.8 cm 5 73 m 6 308 m 7 a 24 cm, b 24 cm 8 18 m 9 1.46 m 10 a 45°, b 8.49 cm, c 8.49 cm 11 a 16 cm, b 9.4 cm 12 8 cm 13 13 m 14 26 m 15 9.9 m 16 46.9 m 17 13.7 m 18 b 10.2 cm 19 a 26.8 mm, b 91.0 mm, c 24.6 mm 20 10565 m 21 81 m

Exercise 12.6

1 a 56°, b 45°, c 46°, d 44°, e 25°, f 27° 2 a 46°, b 56°, c 14°, d 47°, e 46°, f 73° 3 a 30°, b 72°, c 43°, d 57°, e 55°, f 46° 4 a 49°, b 54°, c 32°, d 72°, e 19°, f 21°, g 60°, h 17°, i 16°, j 9°, k 66°, l 80° 5 a 27°, b 13°, c 35°, d 23°, e 65°, f 61°, g 35°, h 6°, i 70° 6 a 38°, b 22°, c 25° 7 a 37°, b 44°, c 27° 8 25°

Exercise 12.7

1 a 73°, b 15° 2 68° 3 23° 4 68° 5 8° 6 29° 7 13° 8 14° 9 37° 10 a 24.5 m, b 55° 11 36° 12 14° 13 a 53°, b 116°, c 111° 14 37° 15 24° 16 a 24 cm, b 106° 17 106°

Exercise 12.8

1 a Elevation, b Depression, c Neither, d Neither, e Depression, f Neither, g Elevation, h Neither, i Depression, j Elevation 2 $h = 195$ m 3 $h = 3.5$ m 4 26° 5 22° 6 21.73 m 7 284.6 m 8 33 m 9 257 m 10 17.5 m 11 37° 12 186 m 13 68° 14 29.9 m 15 10.8 m 16 220 m 17 985 m 18 71.63 m

Exercise 12.9

1 a i $X = N40^\circ E$, $Y = S10^\circ E$, $Z = S70^\circ W$, ii $X = 040^\circ$, $Y = 170^\circ$, $Z = 250^\circ$, b i $X = S78^\circ E$, $Y = S34^\circ W$, $Z = N15^\circ W$, ii $X = 102^\circ$, $Y = 214^\circ$, $Z = 345^\circ$, c i $X = N9^\circ E$, $Y = S46^\circ E$, $Z = N73^\circ W$, ii $X = 009^\circ$, $Y = 134^\circ$, $Z = 287^\circ$; d i $X = N20^\circ E$, $Y = S22^\circ E$, $Z = N76^\circ W$, ii $X = 020^\circ$, $Y = 158^\circ$, $Z = 284^\circ$, e i $X = N73^\circ E$, $Y = S36^\circ E$, $Z = S25^\circ W$; ii $X = 073^\circ$, $Y = 144^\circ$, $Z = 205^\circ$, f i $X = S41^\circ E$, $Y = S75^\circ W$, $Z = N7^\circ W$, ii $X = 139^\circ$, $Y = 255^\circ$, $Z = 353^\circ$ 2 a 306°, b 237°, c 159°, d 048°, e 203°, f 172°, g 025°, h 324° 3 a 12°, b 125°, c 42°, d 18°, e 105°, f 70° 4 a i 45.3 km, ii 56.0 km, b i 11.0 km, ii 6.9 km, c i 10.2 km, ii 35.6 km, d i 33.6 km, ii 110.0 km 5 a 34.41 km, b 111.5 km, c 40.10 km, d 122.5 km 6 a 107 km, b 205 km, c 143 km, d 82 km 7 a i 125°, ii 305°, b i 333°, ii 153°, c i 218°, ii 038°, d i 160°, ii 340° 8 a ii 258.1 m, b ii 6.36 km, c ii 29 km 9 a iii 103°, b iii 259°, c iii 081° 10 125 km, 143° 11 14.4 km, 349° 12 77.8 nm, 073° 13 a 26 km, b i 061°, ii 241°, iii 128°, iv 218° 14 14 nautical miles

Chapter 12 Review

1 a hypotenuse = SU , opposite = ST , adjacent = TU , b hypotenuse = MN , opposite = LM , adjacent = LN 2 a $\sin \theta = \frac{35}{37}$, $\cos \theta = \frac{12}{37}$, $\tan \theta = \frac{35}{12}$, b $\sin \theta = \frac{11}{61}$, $\cos \theta = \frac{60}{61}$, $\tan \theta = \frac{11}{60}$ 3 a $WX = 24$, $YZ = 11$, b i $\frac{24}{25}$, ii $\frac{3}{5}$, iii $\frac{3}{4}$, iv $\frac{7}{25}$, v $\frac{7}{25}$, vi $\frac{4}{3}$ 4 a $c = 3$, $b y = 15$ 5 a 19.2, b 58.6, c 0.4 6 a 76°, b 86°, c 7° 7 a 0.90, b 0.44 8 0.781 9 a $t = 11.6$, b $z = 19.0$, c $b = 13.1$ 10 a $k = 19.28$, b $k = 107.85$ 11 a 137.38 cm, b 143.65 cm 12 4.5 m 13 29.6 m 14 a 21.9 cm, b 8.36 cm 15 a 31°, b 63°, c 80° 16 72° 17 a 7.7 cm, b 51° 18 8° 19 a 50°, b 78° 20 a 622.1 m, b 307.5 m 21 36° 22 a 141.82 m, b 58° 23 a $X = N54^\circ E$, $Y = S16^\circ W$, $Z = N68^\circ W$, b $X = 054^\circ$, $Y = 196^\circ$, $Z = 292^\circ$ 24 a 25°, b 115°, c 40° 25 209.4 km, 303° 26 49.5 km 27 475.7 km 28 a 027°, b 207° 29 b 344 m 30 b 014°

13 Simultaneous equations

Exercise 13.1

1 **a** No, **b** No, **c** Yes; 3, 4, **d** No, **e** Yes; 4, 5, **f** No, **g** No, **h** Yes; \$12, \$24 **4** No **5** **a** Yes, **b** No, **c** No, **d** Yes **6** **a** $x=8, y=5$, **b** $p=30, q=15$, **c** $a=3, b=7$, **d** $m=1, n=4$ **7** $x=3, y=11$ **8** $ax=5, y=10$, $bx=3, y=19$, $cx=6, y=12$ **9** $x=4, y=3$ **10** $ax=6, y=8$, $bx=3, y=7$, $cx=7, y=-1$
11 **a** Sharon is 39, Julian is 13, **b** Dylan is 180 cm, Yuri is 165 cm, **c** Apples are 20c each, pears are 40c each, **d** length = 16.5 cm, width = 4.5 cm **12** $ax=7, y=3$, $bx=-1, y=5$, $cx=3, y=-2$

Exercise 13.2

1 $ax=2, y=3$, $bx=1, y=-5$ **2** $ax=3, y=6$, $bx=-2, y=1$, $cx=5, y=0$ **3** No, the lines are parallel
4 $ax=2, y=5$, $bx=-3, y=3$, $cx=1, y=2$, $dx=3, y=2$, $ex=1, y=3$, $fx=2, y=-4$, $gp=2, q=1$,
h $a=-3, b=4$, **i** $m=2, n=0$ **5** **b** No, the lines are parallel **6** $x=12.5, y=262.5$ **7** **a** Olly's Owner-Driver:
 $C=4.8D$; Trip O' Your Life: $C=4D+80$, **c** 100 km, **d** less than 100 km **8** **a** i \$20 000, ii \$15 000, **b** 1000,
c Product A – 900 items; Product B – 800 items

Exercise 13.3

1 $ax=4, y=9$, $bx=5, y=-6$, $cx=7, y=-8$ **2** $ax=2, y=3$, $bx=4, y=7$, $cx=-13, y=-5$, $dx=-1, y=4$,
 $ex=3, y=3$, $fx=2, y=4$, $gx=4, y=-4$, $hx=-2, y=6$ **3** $ax=2, y=5$, $bx=-2, y=8$, $cx=4, y=-3$,
 $dx=2, y=3$, $ex=5, y=2$, $fx=4, y=0$, $gx=5, y=4$, $hx=7, y=-2$, $ix=-4, y=22$ **4** $ax=5, y=4$,
 $bx=5, y=1$, $cx=-7, y=-4$, $dx=1, y=-2$, $ex=-2, y=5$, $fx=-3, y=-19$, $gx=-3, y=-4$, $hx=4, y=2$,
 $ix=1, y=-4$, $jx=5, y=-3$, $kx=3, y=-2$, $lx=-2, y=-4$ **5** $ax=5, y=2$, $bx=-2, y=8$, $cx=4, y=-1$,
 $dx=9, y=-5$, $ex=-2, y=-10$, $fx=12, y=-2$, $gx=2, y=0$, $hx=6, y=1$, $ix=0, y=-4$ **6** $aa=4, b=\frac{1}{2}$,
 $bp=\frac{2}{3}, q=-1$, $cm=\frac{1}{4}, n=1\frac{1}{4}$, $du=12, v=-1\frac{3}{4}$, $ec=1\frac{1}{2}, d=\frac{3}{4}$, $fg=1\frac{1}{4}, h=-\frac{1}{2}$ **7** $ax=1, y=2$,
 $bx=4, y=3$, $cx=-3, y=-2$ **8** $x=6, y=8$ **9** $ax=2, y=4, z=3$, $bx=7, y=-1, z=5$, $cx=4, y=-2, z=9$

Exercise 13.4

1 $a3x+2y$, $b4x$, $c13x$, $d9y$, $e4x+3y$, $f-6x-8y$, $g4x-2y$, $h3x-6y$ **2** $a4y$, $b4x$, $c3x-7y$, $d-2x-5y$,
 $e-2x+7y$, fx , $g4x-3y$, $h10x+9y$ **3** $ax=3, y=2$, $bx=6, y=-1$, $cx=4, y=2$, $dx=-3, y=6$,
 $ex=1, y=8$, $fx=0, y=7$, $gx=11, y=2$, $hx=-4, y=0$, $ix=6, y=-1$, $jx=5, y=-1$, $kx=-2, y=7$,
 $lx=-3, y=-2$ **4** $ax=5, y=3$, $bx=-1, y=6$, $cx=3, y=7$, $dx=-4, y=5$, $ex=12, y=-1$, $fx=-3, y=6$,
 $gx=5, y=2$, $hx=6, y=-2$, $ix=4, y=7$, $jx=2, y=-4$, $kx=-2, y=-9$, $lx=-7, y=-3$ **5** $ax=8, y=3$,
 $bx=12, y=-4$, $cx=7, y=2$, $dx=7, y=-3$, $ex=7, y=-1$, $fx=12, y=-2$, $gx=5, y=4$, $hx=-2, y=11$,
 $ix=4, y=-5$, $jx=3, y=10$, $kx=-1, y=-3$, $lx=-4, y=-1$, $mx=10, y=2$, $nx=20, y=2$, $ox=9, y=4$
6 $ax=5, y=3$, $bx=4, y=2$, $cx=-2, y=7$, $dx=5, y=-6$, $ex=1, y=5$, $fx=3, y=-1$, $gx=8, y=3$,
 $hx=1, y=-2$, $ix=4, y=6$, $jx=2, y=4$, $kx=7, y=2$, $lx=9, y=4$, $mx=-1, y=3$, $nx=-2, y=-1$,
 $ox=4, y=-3$, $px=2, y=-1$, $qx=-4, y=3$, $rx=-5, y=-2$ **7** $ap=\frac{4}{13}, q=3\frac{11}{13}$, $ba=\frac{1}{4}, b=-\frac{1}{12}$,
 $cm=1\frac{5}{6}, n=\frac{2}{3}$ **8** $ax=48, y=-3$, $bx=32, y=24$, $cx=4, y=-16$ **9** $ax=4, y=5, z=2$,
 $bx=10, y=2, z=-3$, $cx=8, y=-5, z=4$, $dx=2, y=-4, z=0$, $ex=-2, y=7, z=-1$, $fx=4, y=9, z=-6$

Exercise 13.5

1 **a** 5, 17, **b** 36, 9, **c** 23, 15, **d** 4, 10, **e** 7, 10, **f** 12, 9 **2** **a** man = 35 years, son = 7 years,
b ice cream 80c, drink \$1.20, **c** apple 15 c, peach 35 c, **d** chair 80 kg, table 350 kg, **e** try 5, goal 3,
f 27 five-cent coins, 21 ten-cent coins, **g** exam 87%, assessment 79%, **h** 62 adults, 23 children,
i Coola \$1.05, Melonade \$1.20, **j** length = 42 cm, width = 14 cm, **k** 45 cm, 30 cm, **l** 6 aged 12 years, 8 aged 13 years
3 $ax=6, y=5$, $bx=3, y=8$, $cx=3, y=-4$, $dx=7, y=-3$, $ex=50, y=20$, $fx=10, y=4$, $gx=4, y=3$,

h $x = 2, y = 3$, **i** $x = 15, y = 25$ **4 a** $\frac{14}{17}$, **b** 68, **c** $y = 7x - 4$, **d** 5, 9, **e** diagonals = 16 cm, 30 cm, perimeter = 68 cm, **f** $a = \frac{9}{5}$, $b = 32$, 212°F **5 a** Keiko is 15 years, Lydia is 33 years, **b** 73, **c** man's rowing speed = 10 km/h, current speed = 2 km/h

Chapter 13 Review

1 a No, **b** No, **c** Yes **3 a** No, **b** Yes **4** $x = 5, y = 3$ **5 a** $x = 0, y = 8$, **b** $x = -6, y = 2$, **c** $x = 4, y = 3$
6 no solution, lines are parallel **7** $x = 3, y = -4$ **8 a** $x = 7, y = 5$, **b** $x = 0, y = 5$, **c** $x = 7, y = -2$, **d** $x = 3, y = 1$,
e $x = 2, y = -2$, **f** $x = -6, y = -5$ **9 a** $x = 5, y = 2$, **b** $x = -1, y = -6$, **c** $x = \frac{3}{4}, y = 2$, **d** $x = 2\frac{2}{3}, y = -2$
10 a $x = 5, y = -2$, **b** $x = -1, y = 2$, **c** $x = 3, y = 4$, **d** $x = -2, y = 1$, **e** $x = 5, y = 0$, **f** $x = -2, y = -3$ **11 a** $x = 3, y = 2$, **b** $x = -1, y = -2$, **c** $x = -2, y = 5$, **d** $x = -3, y = 6$, **e** $x = 2, y = -1$, **f** $x = -1, y = 4$ **12 a** $e = \frac{1}{2}, f = \frac{2}{3}$,
b $u = 1\frac{3}{4}, v = 1\frac{5}{6}$ **13** $m = 6, n = -4$ **14 a** 23, 16, **b** bananas 15c, rockmelon \$1.30,
c 15 twenty-cent coins, 45 ten-cent coins **15 a** $x = 4, y = 2$, **b** $x = 10, y = 7$

14

Co-ordinate geometry

Exercise 14.1

1 a 5, **b** 3.6, **c** 10 **2 a** 5, **b** 10, **c** 13, **d** $3\sqrt{2}$, **e** $5\sqrt{2}$, **f** $2\sqrt{5}$ **3 a** $A(-7, 2)$, $C(9, 2)$, **b** 48 units² **4 a** 5,
b 13, **c** 10, **d** 3.6, **e** 2.2, **f** 2.8, **g** 6.3, **h** 4.5, **i** 5.4, **j** 7.6, **k** 8.9, **l** 7.1, **m** 6.7, **n** 6.4, **o** 5.7 **5 a** H , **b** M
6 $QS = RS = \sqrt{41}$ **7** $LM = MN = 5 \therefore \triangle LMN$ is isosceles **8 a** $AB = BC = \sqrt{52}$, $\therefore \triangle ABC$ is isosceles,
b Yes **9** 56 **10 a** 15, **b** 10, **c** 3 : 2 **11 a** 10, **b** 20π **12** 5 : 13 **13** $PQ = QR = RP = 6$
14 a $EF = \sqrt{34}$, $FG = \sqrt{34}$, $GH = \sqrt{34}$, $HE = \sqrt{34}$, **b** $EG = \sqrt{68}$, $HF = \sqrt{68}$, **c** square, sides equal and diagonals
equal **15 a** $KL = \sqrt{20}$, $LM = \sqrt{20}$, $MN = \sqrt{10}$, $NK = \sqrt{10}$ **b** kite, two pairs of adjacent sides equal
16 a $PQ = 5$, $QR = 5$, $RS = 5$, $SP = 5$, **b** $PR = \sqrt{90}$, $SQ = \sqrt{10}$, **c** rhombus, 4 sides equal and diagonals unequal
17 (7, 10), (7, -8)

Exercise 14.2

1 a (1, 7), **b** (3, 7), **c** (7, 14), **d** (7, 8), **e** (0, 4), **f** (2, 2), **g** (8, -3), **h** (1, -4), **i** (-6, -5), **j** (-3, -1), **k** (0, 0),
l (-10, 1), **m** (0, 6), **n** (-8, -9), **o** (2, 2), **p** (-12, -2), **q** (7, -8), **r** (-11, 10) **2 a** $(1\frac{1}{2}, 4)$, **b** $(5\frac{1}{2}, 1\frac{1}{2})$,
c $(3\frac{1}{2}, 8\frac{1}{2})$, **d** $(5\frac{1}{2}, -2)$, **e** $(-2\frac{1}{2}, 1)$, **f** $(3\frac{1}{2}, 2\frac{1}{2})$, **g** $(-4\frac{1}{2}, -5\frac{1}{2})$, **h** $(0, \frac{1}{2})$, **i** $(-7\frac{1}{2}, 2\frac{1}{2})$ **3 a** (5, 11), **b** (5, 2),
c (-7, -3), **d** (-6, -1), **e** (7, 3), **f** (-5, 0) **4 a** (2, 3), **b** (-6, 9) **5 a** $Q(-1, 4)$, $R(2, 6)$, $S(5, 8)$,
b $A(-7, -15)$, $C(7, -5)$, $E(21, 5)$ **6 a** (0, -1), **b** (0, -1), **c** parallelogram, diagonals bisect each other.
7 a (2, 5), **b** (8, -3), **c** parallelogram, diagonals bisect each other **8 a** $P(-5, 1)$, $Q(3, 1)$,
b $PQ = 8$, $TV = 16 \therefore PQ = \frac{1}{2}TV$ **10 a** $W(3, 1)$, **b** 5 units **11 a** (3, -4), **b** $\sqrt{61}$ units **12** (-1, -11)
13 a (4a, 6b), **b** (3a, 5b), **c** (a, -7b) **14 a** $p = 2, q = 6$, **b** $m = -1, n = 9$, **c** $a = -3, b = 5$

Exercise 14.3

1 a 2, **b** -3, **c** $\frac{1}{4}$, **d** -6, **e** $\frac{7}{3}$, **f** $-\frac{3}{5}$, **g** $\frac{5}{6}$, **h** 3, **i** -1, **j** $\frac{1}{2}$, **k** $-\frac{2}{3}$, **l** $-\frac{4}{3}$ **2 a** 1, **b** $\frac{1}{2}$, **c** $\frac{5}{3}$, **d** $\frac{9}{4}$, **e** -3, **f** $\frac{5}{8}$, **g** $-\frac{3}{5}$,
h $\frac{1}{6}$, **i** $\frac{4}{3}$, **j** $-\frac{3}{2}$, **k** $\frac{2}{3}$, **l** $-\frac{4}{3}$, **m** -1, **n** 5, **o** $\frac{1}{6}$, **p** $-\frac{8}{7}$, **q** 2, **r** $-\frac{4}{5}$ **3 a** 2, **b** 2, **c** 2 **4 a** $k = 5$, **b** $t = 7$, **c** $c = -7$
5 2 **6 a** $B(4, 3)$, $D(-2, -1)$, **b** $m_{AC} = -\frac{2}{3}$, $m_{BD} = \frac{2}{3}$ **7** $W(8, 8)$, $\frac{11}{2}$ **8 a** -2, **b** -2, **c** collinear points **9** -4
10 a $m = 0$, horizontal, **b** m is undefined, vertical **11 a** $\frac{2}{3}, \frac{2}{3}$, **b** $\frac{5}{3}, \frac{5}{3}$, **c** gradient
12 $m_{PQ} = m_{MN} = -1 \therefore PQ \parallel MN$ **13 a** 0.2, **b** 0.4, **c** 0.5, **d** 0.6, **e** 0.8, **f** 1, **g** 1.2, **h** 1.7, **i** 3.7, **j** 57.3
14 the gradient increases **15 a** 6° , **b** 11° , **c** 14° , **d** 18° , **e** 27° , **f** 45° , **g** 63° , **h** 74° , **i** 83° , **j** 88°

Exercise 14.4

1 **a** $m=3, b=2$, **b** $m=5, b=-1$, **c** $m=-4, b=3$, **d** $m=-5, b=6$, **e** $m=-2, b=-4$, **f** $m=7, b=0$, **g** $m=-1, b=0$, **h** $m=-1, b=8$, **i** $m=\frac{1}{2}, b=9$, **j** $m=\frac{3}{4}, b=-2$, **k** $m=-\frac{2}{3}, b=6$, **l** $m=-\frac{1}{7}, b=\frac{3}{5}$ 2 **a** $x-y+6=0$, **b** $4x-y+1=0$, **c** $3x-y-2=0$, **d** $x+y-5=0$, **e** $x+y-3=0$, **f** $2x+y+6=0$, **g** $x-y=0$, **h** $5x+y=0$, **i** $x+y-1=0$, **j** $x-y-3=0$, **k** $4x+y-7=0$, **l** $7x-y-12=0$, **m** $x+2y-5=0$, **n** $3x+4y-8=0$, **o** $x-3y-10=0$, **p** $3x-2y-6=0$, **q** $x-3y=0$, **r** $x-4y=0$, **s** $x-2y+8=0$, **t** $x-5y-15=0$, **u** $2x-3y-3=0$, **v** $x+2y-14=0$, **w** $3x+4y+20=0$, **x** $5x+6y-12=0$ 3 **a** $y=x+2$, **b** $y=x-4$, **c** $y=2x+1$, **d** $y=-x-5$, **e** $y=-x+1$, **f** $y=-3x+7$, **g** $y=2x-9$, **h** $y=-4x-6$, **i** $y=5x+2$, **j** $y=\frac{1}{2}x+3$, **k** $y=-\frac{1}{3}x+4$, **l** $y=-\frac{1}{2}x$, **m** $y=-x+\frac{3}{4}$, **n** $y=2x+\frac{1}{3}$, **o** $y=\frac{3}{7}x$, **p** $y=\frac{2}{3}x+1$, **q** $y=\frac{3}{2}x+4$, **r** $y=-\frac{5}{6}x+2$, **s** $y=\frac{3}{4}x+\frac{5}{4}$, **t** $y=-\frac{2}{7}x+\frac{1}{7}$, **u** $y=\frac{1}{8}x-\frac{5}{4}$, **v** $y=\frac{1}{9}x-\frac{2}{3}$, **w** $y=-\frac{3}{5}x+2$, **x** $y=-\frac{2}{3}x-1$ 4 **a** $3x-y+1=0$, **b** $2x-y+9=0$, **c** $x+y-2=0$, **d** $4x+y+10=0$, **e** $x-2y-8=0$, **f** $2x+3y+9=0$ 5 **a** $y=2x+6$, **b** $2x-y+6=0$, **c** x -intercept $=-3$, y -intercept $=6$ 6 $3x-4y-2=0$, it has the larger gradient 7 **a** **D**, **b** **A**, **c** **C**, **d** **B** 8 **a** **A**, **b** **D**, **c** **B**, **d** **C** 9 **a** $2x-y+2=0$, **b** $3x-y-12=0$, **c** $4x+y-8=0$, **d** $x-2y+2=0$, **e** $2x+3y-6=0$, **f** $4x+3y+24=0$ 10 **a** $\frac{4}{3}$, **b** $4x-3y+15=0$ 11 **a** $\frac{1}{7}$, **b** $x-7y+21=0$ 12 **a** $y=-2x+4$, **b** **Yes** 13 **k** $=3, m=\frac{3}{4}$ 14 **a** $x-y+5=0$, **b** $x-y-3=0$, **c** $4x-y=0$, **d** $2x-y+3=0$, **e** $3x-y-1=0$, **f** $x+y-8=0$, **g** $x-2y=0$, **h** $2x-3y+6=0$ 15 **a** $2x-y+5=0$, **b** $x+y-3=0$, **c** $x+2y-16=0$ 16 $3x-2y=0$ 17 **a** $\tan^{-1}(\frac{2}{3}) \div 34^\circ$, $\tan^{-1}(\frac{3}{4}) \div 37^\circ$, **b** **i** $2x-3y+21=0$, **ii** $3x-4y=0$ 18 **a** 27° , **b** 22° , **c** 53°

Exercise 14.5

1 **a** $y=2x-3$, **b** $y=3x+10$, **c** $y=-x-6$, **d** $y=-2x-14$, **e** $x-3y-6=0$, **f** $x+2y+2=0$, **g** $2x-3y+39=0$, **h** $3x+4y+60=0$ 2 **a** $y=x+2$, **b** $y=3x-1$, **c** $y=2x+10$, **d** $y=6x+12$, **e** $y=-x+6$, **f** $y=-4x+7$, **g** $y=-3x+9$, **h** $y=-2x-6$, **i** $y=2x+6$, **j** $y=-8x+11$, **k** $y=-6x-13$, **l** $y=-1$ 3 **a** $x-2y+5=0$, **b** $2x-3y-16=0$, **c** $x+3y-5=0$, **d** $3x+4y-19=0$, **e** $x-5y-4=0$, **f** $11x+6y+12=0$ 4 $y=-3x-15$ 5 **a** $y=-2x+1$, **b** **yes** 6 $x-4y+33=0$ 7 **a** $D(9, 6)$, **b** $3x-5y+3=0$ 8 x -int. $=-8$, y -int. $=32$ 9 **a** $E(-5, 0)$, **b** $2x+7y+10=0$ 10 **a** $G(-1, 4)$, **b** $y=-5x-1$ 11 **a** **1**, **b** $y=x+13$ 12 **b** $3x-4y+13=0$

Exercise 14.6

1 **a** **i** **1**, **ii** $y=x+3$, **b** **i** -3 , **ii** $y=-3x+7$, **c** **i** $\frac{1}{2}$, **ii** $y=\frac{1}{2}x-2$ 2 **a** **i** **3**, **ii** $y=3x+4$, **b** **i** **5**, **ii** $y=5x-1$, **c** **i** -2 , **ii** $y=-2x+5$ 3 **a** $y=x+2$, **b** $y=4x-1$, **c** $y=3x+2$, **d** $y=-x+1$, **e** $y=-5x+3$, **f** $y=-2x-7$, **g** $x-2y-1=0$, **h** $2x-3y-13=0$, **i** $x+2y-16=0$, **j** $3x+4y-49=0$, **k** $3x-4y-12=0$, **l** $5x+3y+12=0$ 4 $y=2x+6$ 5 **a** $4x-3y+24=0$, **b** x -int. $=-6$, y -int. $=8$ 6 **a** $y=-2x-3$, **c** **A, B, C** are collinear points 7 $m_{XY}=m_{YZ}=\frac{3}{2}$, $\therefore X, Y, Z$ are collinear. 8 $t=-9$ 9 **a** $E(1, -1)$, **b** $y=4x-5$ 10 $12x-4y-15=0$ 11 $P(-1, 2)$

Exercise 14.7

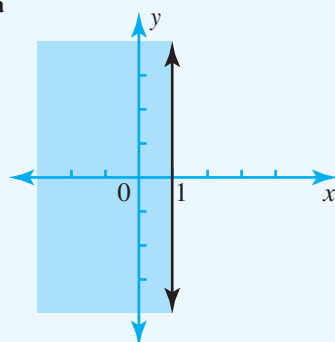
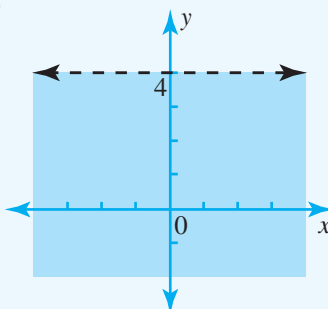
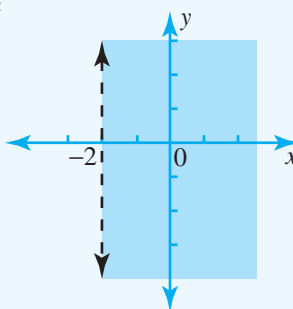
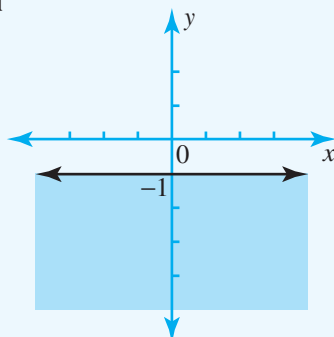
1 **a** $y=3$, **b** $x=-1$, **c** $y=-9$, **d** $x=-5$, **e** $x=-2$, **f** $y=1$, **g** $y=-2$, **h** $x=9$ 2 **a** **Yes**, **b** **No**, **c** **No**, **d** **Yes**, **e** **Yes**, **f** **No**, **g** **No**, **h** **Yes**, **i** **No**, **j** **Yes** 3 **a** $y=3x-2$ and $y=3x+10$, **b** $y=2-x$ and $y=-x$ 4 **a** $y=2x+1$, **b** $y=4x-3$, **c** $y=3x+6$, **d** $y=-x-4$, **e** $y=\frac{1}{2}x+2$, **f** $y=\frac{4}{3}x$ 5 **a** $y=x+3$, **b** $y=x-3$, **c** $y=x+7$, **d** $y=x-6$ 6 **a** $y=-2x+8$, $y=2x+1$, **no**, **b** $y=-3x-7$, $y=-3x+1$, **yes**, **c** $y=-\frac{1}{4}x+1$, $y=5-4x$, **no**, **d** $y=\frac{5}{2}x+3$, $y=\frac{5}{2}x+\frac{9}{2}$, **yes**, **e** $y=2x-3$, $y=2x+\frac{7}{3}$, **yes**, **f** $y=-\frac{3}{4}x-2$, $y=\frac{3}{4}x-\frac{5}{2}$, **no** 7 **a** $x-y+3=0$, **b** $4x+y+1=0$, **c** $x-4y-19=0$, **d** $x-5y-2=0$, **e** $2x+3y-11=0$, **f** $3x-8y-7=0$ 8 **a** **i** $\frac{7}{2}$, **ii** $\frac{4}{7}$, **iii** $\frac{7}{2}$, **iv** $\frac{4}{7}$, **b** $BC \parallel DA, AB \parallel CD$, **c** **parallelogram**, opposite sides are parallel 9 $m_1=m_3=\frac{3}{2}$, $m_2=m_4=\frac{1}{4}$, $l_1 \parallel l_3, l_2 \parallel l_4$, $\therefore$ the lines enclose a parallelogram 10 **a** $m_{EF}=\frac{2}{5}$, $m_{FG}=-2$, $m_{GH}=\frac{2}{5}$, $m_{HE}=2$, **b** $EF \parallel GH \therefore EFGH$ is a trapezium 11 **a** $y=-3x+6$ 12 **a** $y=\frac{1}{2}x-2$, **b** $y=3x$ 13 **a** $(0, -6)$, **b** $y=-\frac{1}{2}x-6$ 14 $y=\frac{1}{2}x+13$ 15 **a** $k=-5$, **b** $k=15$, **c** $k=6$ 16 **a** both have gradient $-\frac{a}{b}$, **b** $3x+2y+k=0$, **c** $k=5$, **d** $3x+2y+5=0$ 17 **a** $x-3y+1=0$, **b** $2x+y-2=0$, **c** $3x+7y+1=0$, **d** $5x-4y-23=0$

Exercise 14.8

- 1 **a** $x=1$, **b** $y=-5$, **c** $x=-4$, **d** $y=3$ 2 **a** yes, **b** yes, **c** no, **d** no, **e** yes, **f** no, **g** yes, **h** yes, **i** no, **j** yes
 3 **a** $y=3x-6$ and $y=-\frac{1}{3}x-2$, **b** $y=\frac{2}{3}x-4$ and $y=-\frac{5}{2}x$ 4 **a** $y=-x+2$, **b** $y=\frac{1}{2}x-3$, **c** $y=-6x-1$,
d $y=5x+8$, **e** $y=\frac{7}{2}x+6$, **f** $y=-\frac{12}{5}x-4$ 5 **a** yes, **b** yes, **c** no, **d** no, **e** yes, **f** no 6 **a** $x+y-7=0$,
b $x-7y+23=0$, **c** $4x+y+16=0$, **d** $8x-5y+8=0$, **e** $3x+y+6=0$, **f** $x-2y-26=0$ 7 **a** $i-\frac{3}{2}$, **ii** $\frac{2}{3}$,
iii $-\frac{3}{2}$, **iv** $\frac{2}{3}$ **b** opposite sides are parallel, **c** yes, adjacent sides are perpendicular 8 gradients are $-\frac{5}{3}, \frac{3}{5}, -\frac{5}{3}, \frac{3}{5}$
 adjacent sides are perpendicular $\therefore$ rectangle 9 **a** $m_{KL}=\frac{4}{3}, m_{LM}=-\frac{3}{4}, m_{MN}=\frac{4}{3}, m_{NL}=-\frac{3}{4}$, **b** $m_{KM}=\frac{1}{7}, m_{NL}=-7$,
c square, rectangle with diagonals perpendicular 10 **a** $y=-\frac{1}{5}x+3$ 11 **a** $y=-\frac{1}{4}x-1$, **b** $y=-\frac{3}{4}x$ 12 **a** $(0, 7)$,
b $y=-\frac{4}{5}x+7$ 13 **a** $k=3$, **b** $k=-3$, **c** $k=\pm 6$ 14 **a** gradients are $-\frac{a}{b}$ and $\frac{b}{a}$, $-\frac{a}{b} \cdot \frac{b}{a} = -1$, **b** $5x-4y+k=0$,
c $k=2$, **d** $5x-4y+2=0$ 15 **a** $2x-3y+7=0$, **b** $x+4y-1=0$, **c** $5x-2y-11=0$, **d** $3x+y-15=0$

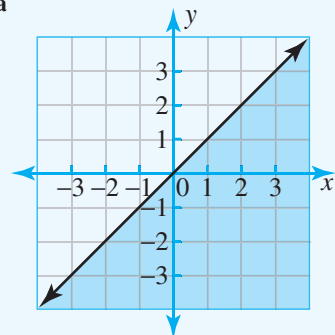
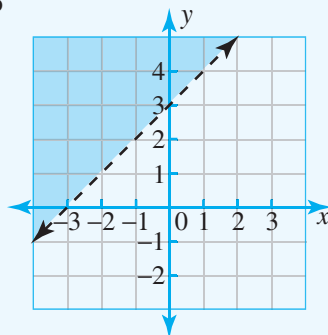
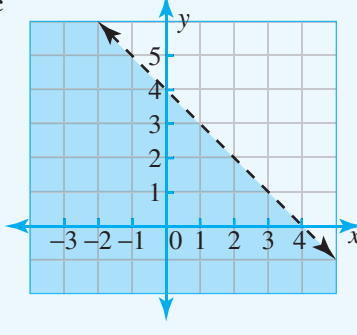
Exercise 14.9

- 1 **a** $x>2$, **b** $y\geq 3$, **c** $y<-2$, **d** $x\leq -1$, **e** $y\geq -1$, **f** $x<3$

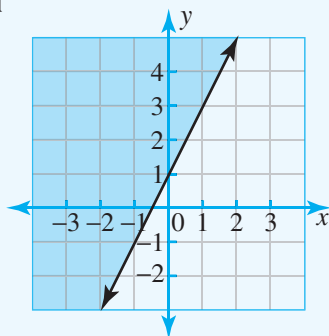
2 **a****b****c****d**

- 3 **a** B, D, **b** B, C, **c** A, D, **d** B, C, D, **e** C, D, **f** A, B

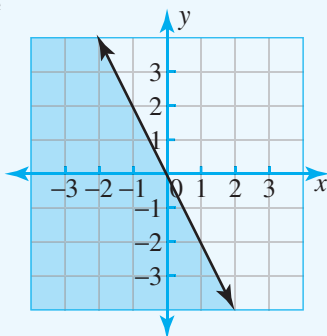
- 4 **a** $y\leq x+2$, **b** $y>3-3x$, **c** $y>\frac{1}{2}x-1$, **d** $y\leq -2x-4$,
e $y\leq 6-3x$, **f** $y>\frac{1}{3}x+1$, **g** $y\leq \frac{1}{2}x-2$, **h** $y>-\frac{2}{3}x+2$, **i** $y>3x$

5 **a****b****c**

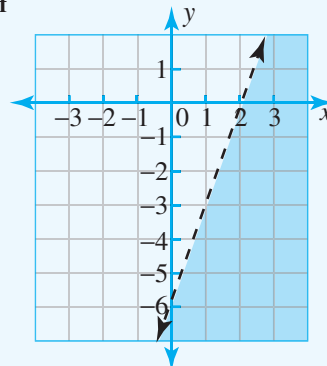
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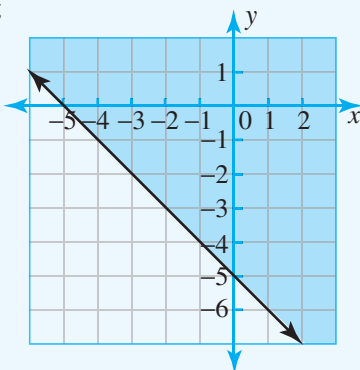
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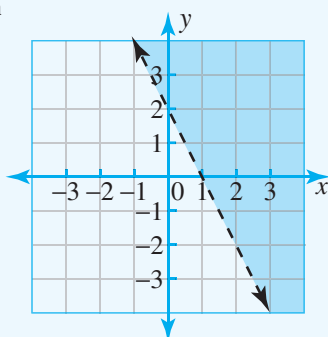
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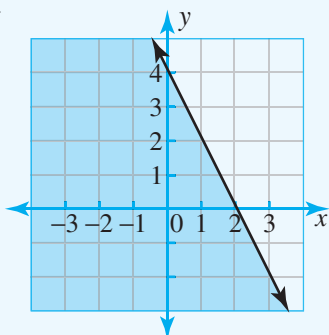
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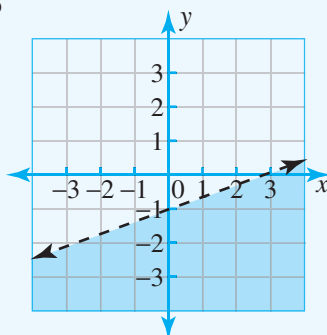
h



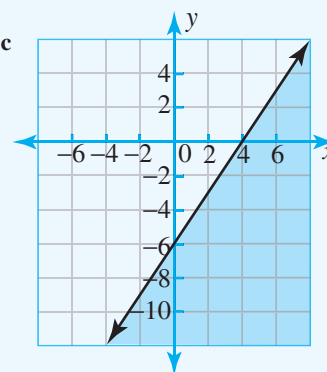
6 a



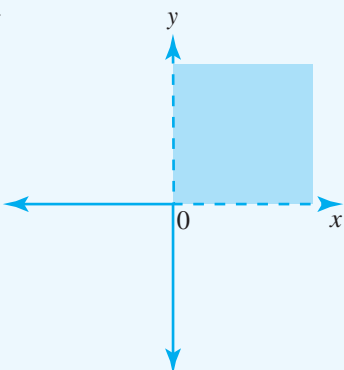
b



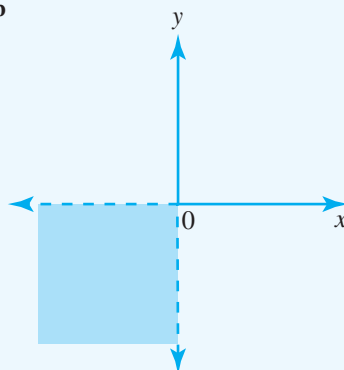
c

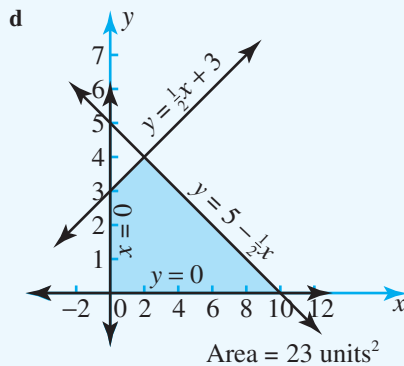
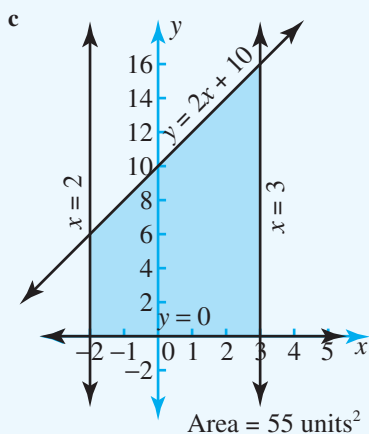
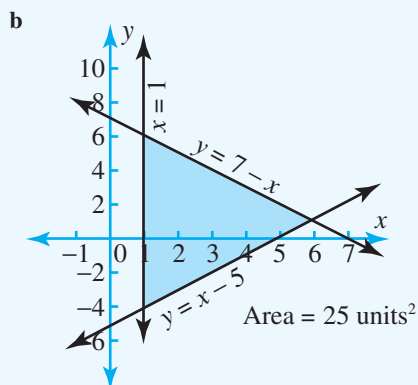
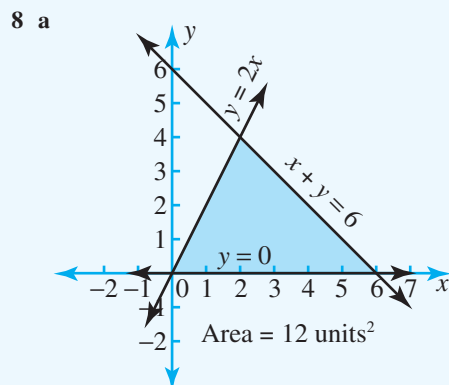
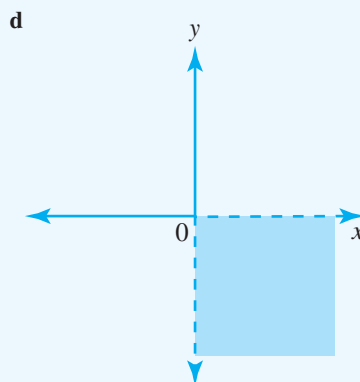
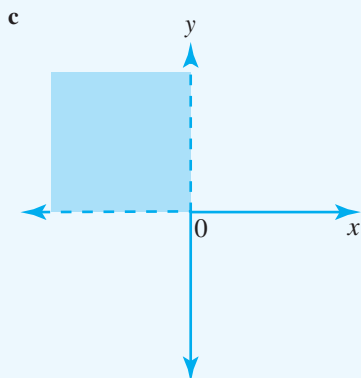


7 a



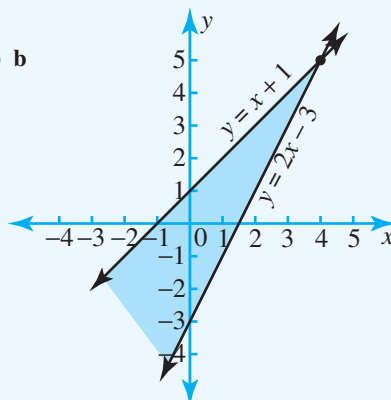
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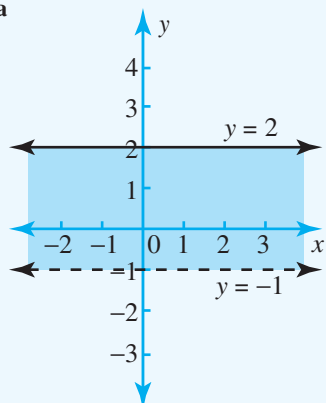


- 9 a $x \geq 1$, $y \geq -2$, b $-3 < x < 3$, c $y \leq x$, $y > -3$,
 d $y \geq 2x$, $y \leq -2x$, e $x < 3$, $y > x + 3$,
 f $y \geq x - 1$, $y < 2 - \frac{1}{2}x$

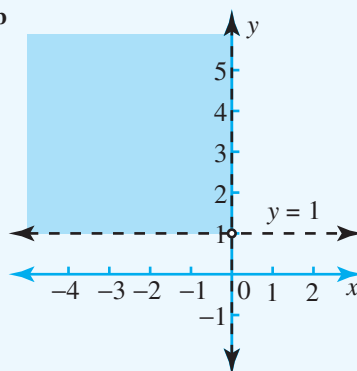
- 10 a (4, 5) b



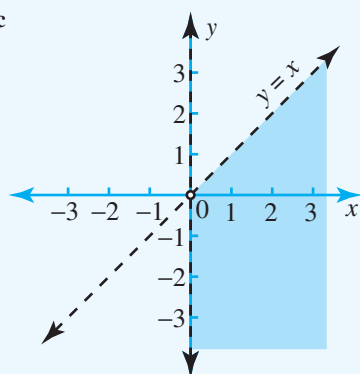
11 a



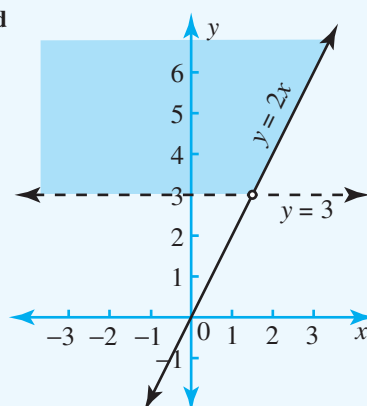
b



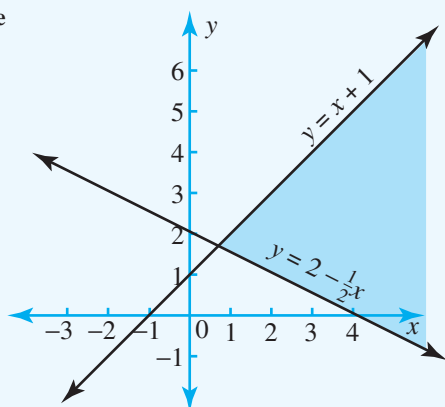
c



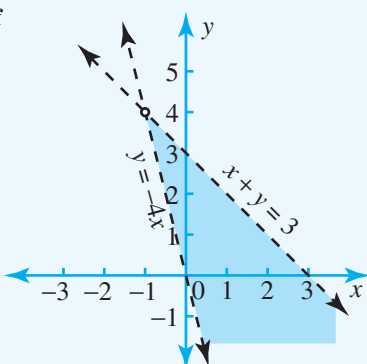
d

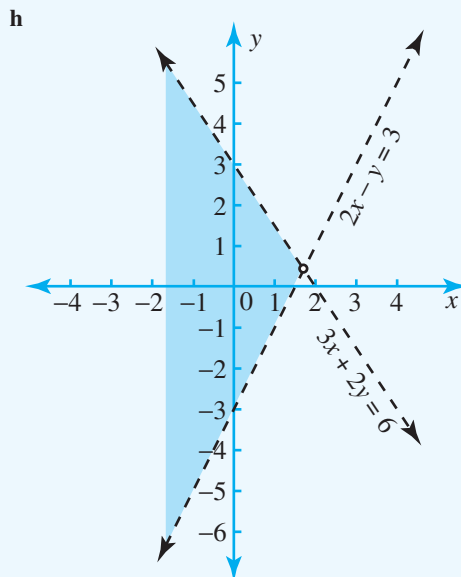
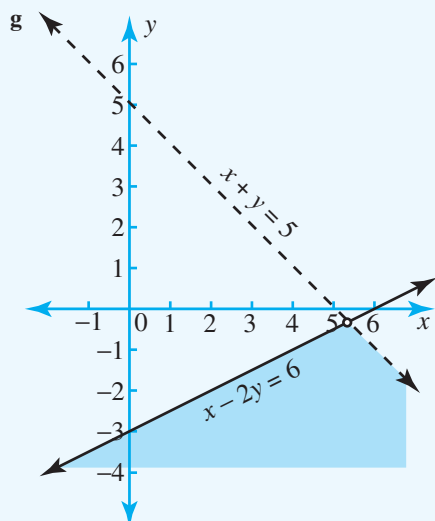


e



f



**Exercise 14.10**

1 **a** $k = 3$, $m = -\frac{2}{3}$, **b** x -int. = 9, y -int. = 6 **2** $4\sqrt{10}$ **3** $6x + 7y + 18 = 0$, **4** **a** (1, 8), **b** (6, 5) **5** **a** 5,

b yes, (0, -1) is 5 units from the centre **6** **a** $G(-1, 4)$, **b** $y = -6x - 2$ **7** $m_{OP} = \frac{\sqrt{2}}{2}$, $m_{PQ} = -\sqrt{2}$, $m_{OP} \times m_{PQ} = -1$

8 13 units **9** **a** $p = 6$, **b** $q = 6$ **10** $AB = BC = 2\sqrt{5}$, $m_{AB} = \frac{1}{2}$, $m_{BC} = -2$, $m_{AB} \times m_{BC} = -1$ **11** **a** $m_{PR} = -\frac{1}{5}$, $m_{QS} = 5$, $m_{PR} \times m_{QS} = -1$, **b** PR , QS have a common midpoint $(\frac{1}{2}, 3\frac{1}{2})$, **c** $PQ = \sqrt{13}$, $QR = \sqrt{13}$, $RS = \sqrt{65}$,

$SP = \sqrt{65}$, **d** kite, 2 pairs of adjacent sides equal **12** $r = -\frac{1}{2}$ **13** **a** $m_{FG} = \frac{1}{2}$, $m_{GH} = -2$, $m_{FG} \times m_{GH} = -1$,

FH is the hypotenuse, **b** (2, 0), **c** $EF = EG = EH = 5$ **14** **a** $XY = YZ = ZX = 10$ **15** $m_{PQ} = m_{MN} = -1$, $PQ = 2\sqrt{2}$, $MN = 4\sqrt{2}$ **16** (2, -3), (-6, 1), (8, 7) **17** **a** (5, 4), **b** (5, 4) also satisfies $2x - 5y + 10 = 0$

18 The lines all intersect at (-2, 3) **19** **a** 3, **b** 3, **c** I, J, K are collinear **20** **a** $y = -2x + 9$, **b** (2, 5) also satisfies $y = -2x + 9$ **21** $m_{AB} = m_{BC} = \frac{1}{2}$ **22** $y = -2x$ **23** gradients are $-\frac{1}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$, adj. sides are perpendicular

24 **a** $WX = YZ = \sqrt{29}$, $m_{WX} = m_{YZ} = -\frac{2}{5}$, **b** parallelogram, one pair of opp. sides equal and parallel

25 **a** $T(-3, 4)$, $U(5, 0)$, $V(1, -2)$, $W(-7, 2)$, **b** PR and QS have a common midpoint (-1, 1), **c** parallelogram, diagonals bisect each other **26** CE and DF have a common midpoint (2, 2), $m_{CE} = 3$, $m_{DF} = -\frac{1}{3}$, $m_{CE} \times m_{DF} = -1$, **b** $CE = 2\sqrt{10}$, $DF = 2\sqrt{10}$, **c** square, rhombus with equal diagonals **d** $2\sqrt{5}$, **e** 20 units² **27** **a** $3\sqrt{5}$, **b** $\frac{1}{2}$,

c $x - 2y + 6 = 0$, **d** -2, **e** $y = -2x + 8$, **f** (2, 4), **g** $2\sqrt{5}$, **h** 15 units² **28** **a** $y = -x + 6$, **b** (4, 2), **c** 2 : 3

29 **a** (2, 5), **b** 1, **c** -1, **d** $y = -x + 7$ **30** $x - 4y + 10 = 0$ **31** (7, -2) **32** $e = -6$ **33** (4, -5) **34** **a** $L(1, 4)$, $M(5, \frac{1}{2})$, $N(3, -1\frac{1}{2})$, **b** $AM: x + 4y - 7 = 0$, $BN: x = 3$, $CL: 3x + 2y - 11 = 0$, **c** (3, 1), **d** P also satisfies CL

35 **a** (1, 3), **b** P also satisfies BN **36** **a** $L(0, 5)$, $M(5, 5)$, $N(3, 3)$, **b** $m_{AB} = 1$, $m_{BC} = -\frac{2}{3}$, $m_{CA} = 0$,

c perp. bisector of $AB: y = -x + 5$, perp. bisector of $BC: 3x - 2y - 5 = 0$, perp. bisector of $CA: x = 3$, **d** (3, 2), **e** the perp. bisector of BC also passes through P

Chapter 14 Review

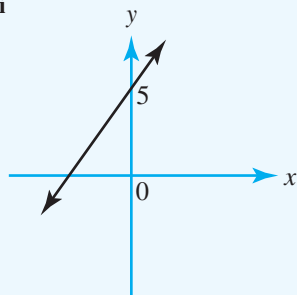
1 **a** $\sqrt{41}$, **b** $\sqrt{73}$ **2** $IJ = JK = 2\sqrt{5}$ **3** **a** $UV = \sqrt{136}$, $VW = \sqrt{34}$, $UW = \sqrt{170}$, **b** yes, $UW^2 = UV^2 + VW^2$

4 **a** $AB = \sqrt{5}$, $BC = \sqrt{5}$, $CD = 3\sqrt{5}$, $DA = 3\sqrt{5}$, **b** kite, two pairs of adjacent sides equal **5** **a** (8, 5), **b** (-4, 1), **c** $(5\frac{1}{2}, -2\frac{1}{2})$ **6** (6, 7) **7** $Q(4, 3)$, $R(7, -4)$, $S(10, -11)$ **8** **a** (-1, 2), **b** parallelogram, diagonals bisect each other

9 $e = 4, f = 7$ 10 **a** 4, **b** $-\frac{2}{3}$ 11 **a** 3, **b** $\frac{1}{5}$, **c** $-\frac{3}{2}$ 12 **a** $g = -1$, **b** $t = 5$ 13 $m_{IJ} = \frac{5}{2}, m_{JK} = \frac{4}{7}, m_{KL} = \frac{5}{2}$,
 $m_{LI} = \frac{4}{7}$, $IJ \parallel KL, JK \parallel LI$, two pairs of opposite sides parallel

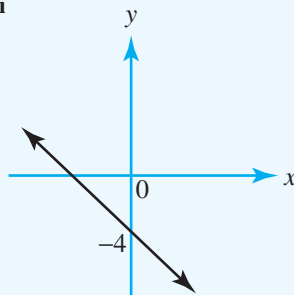
14 **a** **i** $m = 2, b = 5$,

ii



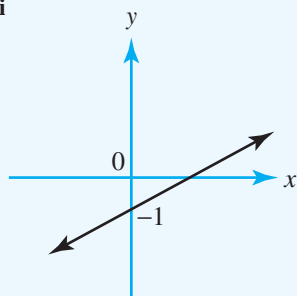
b **i** $m = -1, b = -4$,

ii



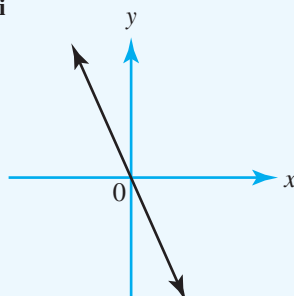
c **i** $m = \frac{1}{2}, b = -1$,

ii



d **i** $m = -3, b = 0$,

ii



15 **a** $y = x + 6$, **b** $y = \frac{3}{4}x - 2$, **c** $y = -\frac{5}{2}x$ 16 **a** $5x - y - 2 = 0$, **b** $3x + y - 4 = 0$, **c** $x - 4y + 28 = 0$,

d $2x + 5y - 30 = 0$ 17 **a** $y = x + 3, m = 1$, **b** $y = -3x + 7, m = -3$, **c** $y = \frac{1}{2}x - 5, m = \frac{1}{2}$, **d** $y = -\frac{2}{3}x - \frac{4}{9}, m = -\frac{2}{3}$

18 **a** $2x + y - 3 = 0$, **b** $2x - 3y + 21 = 0$ 19 **a** x -intercept = 6, y -intercept = 8, **c** $-\frac{4}{3}$ 20 **a** $y = 5x + 4$,

b $y = -5x - 4$, **c** $y = -5x + 4$, **d** $y = 5x - 4$ 21 **a** Yes, **b** No 22 **a** $k = -5$, **b** $\frac{2}{5}$ 23 $y = -\frac{3}{7}x + 3$

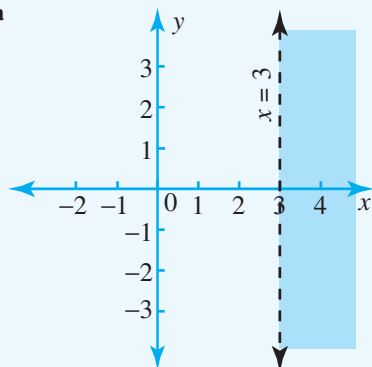
24 **a** $y = x - 3$, **b** $y = 2x + 5$, **c** $y = -x + 7$, **d** $y = -2x - 3$, **e** $y = \frac{1}{2}x + 1$ 25 **a** 0.7, **b** 1.2, **c** 3.1 26 **a** 9° , **b** 56° ,

c 80° 27 **a** $x = -3$, **b** $y = -4$, **c** $y = 3x + 1$, **d** $y = -2x - 4$ 28 **a** $3x + y - 8 = 0$, **b** $x - 2y - 15 = 0$,

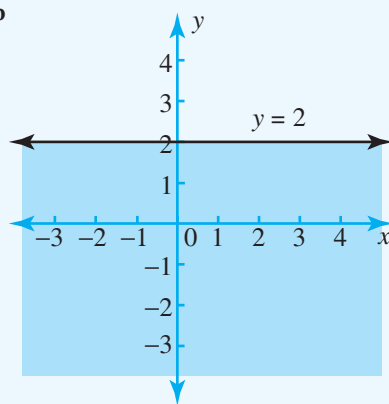
c $5x + 4y + 7 = 0$ 29 **a** $2x - y + 4 = 0$, **b** $3x - 4y - 29 = 0$, **c** $5x + 2y + 16 = 0$ 30 **a** $m_1 = m_2 = \frac{1}{3}$,

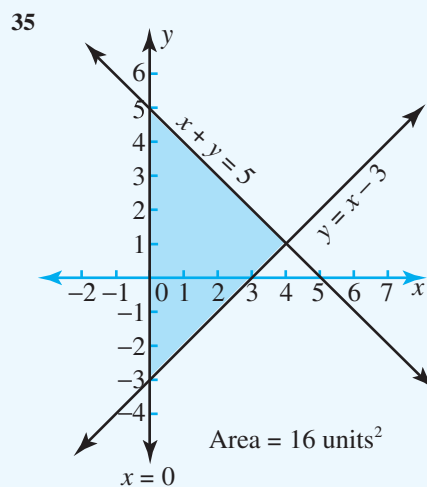
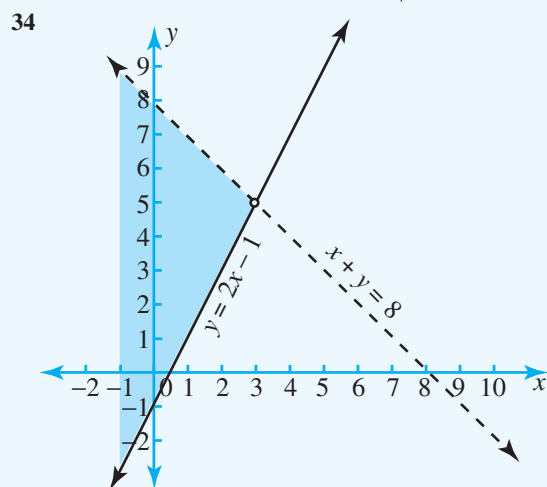
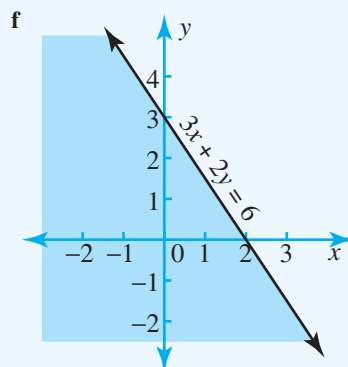
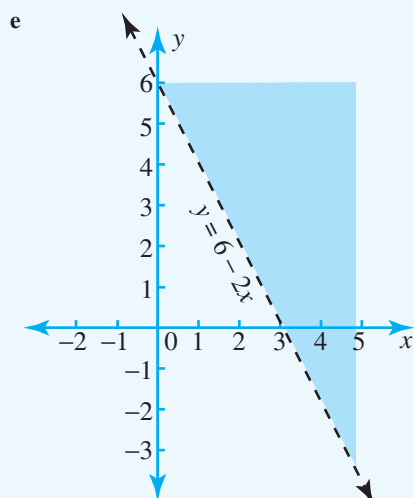
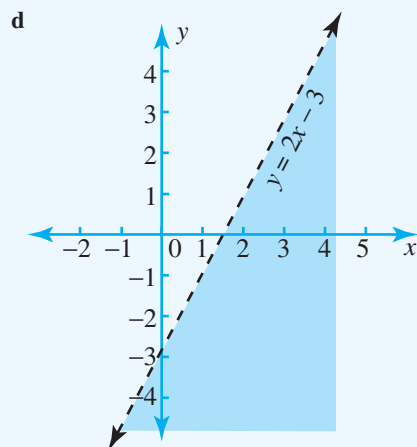
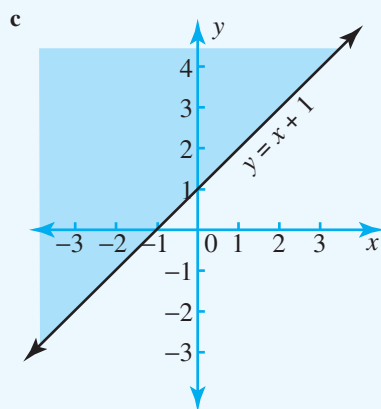
b $m_1 = \frac{3}{2}, m_2 = -\frac{2}{3}, \frac{3}{2} \times (-\frac{2}{3}) = -1$, 31 **a** $y = \frac{1}{2}x - 2$, **b** $y = -4x - 3$, **c** $y = -\frac{4}{3}x$, **d** $y = -6x + 14$ 32 **a** $k = 6$, **b** $k = \frac{15}{4}$

34 **a**



b





- 36 $D(-4, 3)$ 37 $4x + 3y - 19 = 0$ 38 $m_{LM} = m_{MN} = 3$ 39 lines intersect at $(2, -3)$ 40 $WX = XY = YZ = ZW = 3\sqrt{5}$ units, $m_{WX} = -2$, $m_{XY} = \frac{1}{2}$, $m_{WX} \times m_{XY} = -1$ 41 a $\frac{1}{2}$, b $x - 2y + 7 = 0$ c -2 d $y = 11 - 2x$ e $(3, 5)$ f $4\sqrt{5}$ units g $2\sqrt{5}$ units h 20 units²