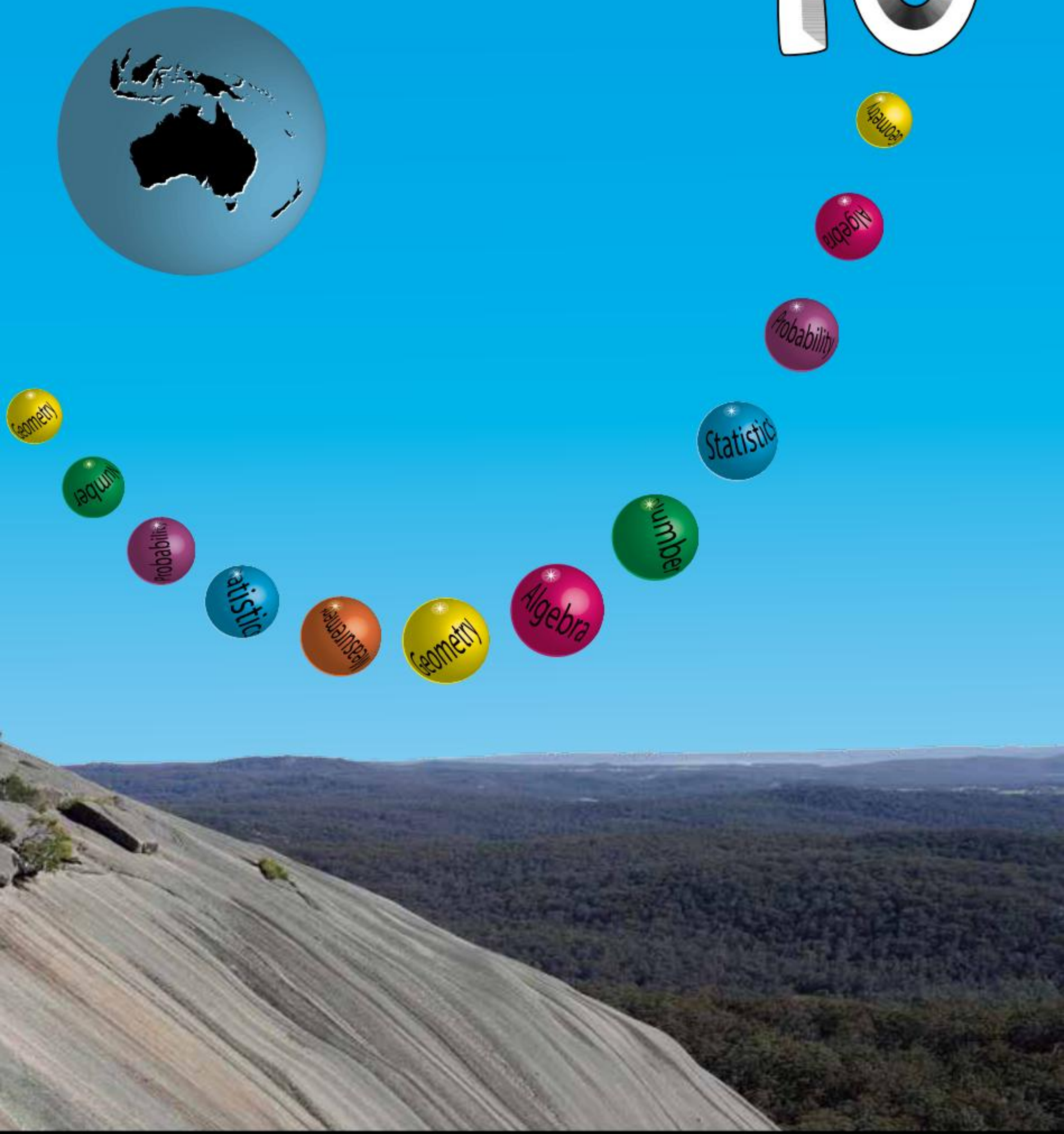
Mathematics 10



National Curriculum 10 &10A

Dr Terry Dwyer

Mathematics 10

National Curriculum 10 & 10A

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The cover: Maths is the pathway to success in our technological economy and society.

"In today's world, economic access and full citizenship depend crucially on math and science literacy." - Robert P. Moses.

"Mathematics makes a significant contribution to our modern society; its basic skills are vital for the life opportunities of the youth. Let us all embrace and encourage the study of this indispensable subject."
- Alfred L. Teye.

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Contents - Overview

10A *units indicated overleaf*

Preface

This text has been written for Year 10 students. The aim of the text is to assist students in investigating and understanding the exciting and very important world of Mathematics and to implement the intent of the Australian Mathematics Curriculum.

A literature review of learning from school textbooks was used to enhance the format of this textbook.

Each chapter, apart from Review, contains:

- Numerous worked examples
- Numerous sets of graded exercises
- An open-ended rich task
- Mental computation
- Technology in mathematics
- Investigations
- Puzzles
- Maths competition preparation
- A mathematics game
- A mathematics trick
- A bit of mathematics history
- Careers using mathematics
- Chapter review

Acknowledgments

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Resources

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- **Workprogram**
- **Study guides** for each term.
- Detailed **lesson plans** for each term.
- Sample **assessment items.**
- **PDF** of this textbook.

Algebra 1

1

Number and Algebra → Patterns and Algebra

★ Factorise algebraic expressions by taking out a common algebraic factor.
- use the distributive law and the index laws to factorise algebraic expressions.
- understand the relationship between factorisation and expansion.

★ Simplify algebraic products and quotients using index laws.
- apply knowledge of index laws to algebraic terms, and simplify algebraic expressions using both positive and negative integral indices.

★ Apply the four operations to simple algebraic fractions with numerical denominators.
- express the sum and difference of algebraic fractions with a common denominator.
- use the index laws to simplify products and quotients of algebraic fractions.

The square root of a perfect square is a whole number.

$\sqrt{64} = 8$

$\sqrt{9} = 3$

$\sqrt{36} = 6$

A TASK

$$x_1=2^2 + 3^2 + 6^2$$
$$x_2=3^2 + 4^2 + 12^2$$
$$x_3=4^2 + 5^2 + 20^2$$

Show that x_n is a perfect square.

A LITTLE BIT OF HISTORY

An ancient document, called the Moscow papyrus, consists of twenty-five mathematical problems and their solutions.

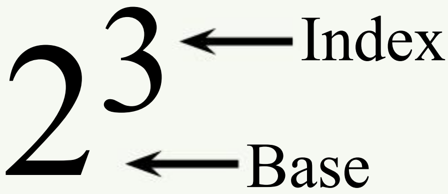
The document was discovered in the Necropolis of Dra Abu'l Neggra in Egypt and is estimated to have been written around 1850 BC.

If you are told: An enclosure of a set and 2 arurae, the breadth having ¾ of the length.

Can you solve the problem?
A rectangle has an area of 12 and the breadth is three-quarters the length.
What is the length?

Warmup

A convenient way of writing 2×2×2 is



Exercise 1.1
Write the following in index form:

2a×2×a×2a×a×a
$= \underline{2^3a^5}$

cddcdcccd
$= \underline{c^5d^4}$

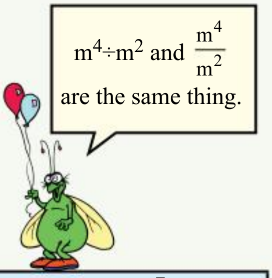
1 2b×2×2b
2 abbaaabb
3 3×3x×3x×3×3x×3
4 xyyyxxxyy
5 10d×10d×10d×10d
6 5pp55p5ppp5

Index Law 1

$a^m \times a^n = a^{m+n}$

Index Law 2

$a^m \div a^n = a^{m-n}$



Simplify and write the following in index form:

$10^3 \times 10^2 = 10\times10\times10 \times 10\times10 = \underline{10^5}$

or $10^3 \times 10^2 = 10^{3+2} = \underline{10^5}$

$a^2 \times a^5 = a\times a \times a\times a\times a\times a\times a = \underline{a^7}$

or $a^2 \times a^5 = a^{2+5} = \underline{a^7}$

7 $10^2 \times 10^4$
8 $3^3 \times 3^2$
9 $2^4 \times 2^3$
10 $10^5 \times 10^3$
11 $x^2 \times x^3$
12 $x^4 \times x^2$
13 $4.1^3 \times 4.1^3$
14 $d^3 \times d^5$
15 $x \times x^4$
16 $y^3 \times y$
17 $0.2^3 \times 0.2^4$
18 $a^3 \times a^2$
19 2.3×2.3^5
20 $10^2 \times 10^3$
21 $10^3 \times 10^5 \times 10^2$
22 $x^4 \times x^2 \times x$

$\boxed{y = y^1}$

Simplify and write the following in index form:

$10^3 \div 10^2 = \frac{2\times2\times2}{2\times2} = \underline{10}$ $\boxed{10 = 10^1}$

or $10^3 \div 10^2 = 10^{3-2} = \underline{10}$

$a^6 \div a^2 = \frac{a\times a\times a\times a\times a\times a}{a\times a} = a\times a\times a\times a = \underline{a^4}$

or $a^6 \div a^2 = a^{6-2} = \underline{a^4}$

23 $10^4 \div 10^2$
24 $10^4 \div 10^3$
25 $4^6 \div 4^2$
26 $2^2 \div 2^2$
27 $x^6 \div x^3$
28 $y^4 \div y^2$
29 $10^6 \div 10^3$
30 $a^4 \div a^3$
31 $10^5 \div 10$
32 $b^5 \div b^3$
33 $3^5 \div 3^4$
34 $10^4 \div 10$
35 $x^8 \div x^3$
36 $4.3^5 \div 4.3^2$
37 $10^7 \times 10^3 \div 10^5$
38 $y^5 \div y^5$
39 $\frac{10^5}{10^3}$
40 $\frac{x^7}{x^4}$
41 $\frac{a^7 \times a^2}{a^4}$
42 $\frac{10^7 \times 10^3}{10^4 \times 10^6}$

Index Law 3	**Zero Index**	**Negative Index**
$(a^m)^n = a^{m\times n}$	$a^0 = 1$	$a^{-m} = \frac{1}{a^m}$

Exercise 1.2

Simplify and write the following in index form:

$(b^4)^2 = (b\times b\times b\times b)^2$ $= (b\times b\times b\times b)\times(b\times b\times b\times b)$ $= \underline{b^8}$ **or** $(b^4)^2 = b^{4\times 2} = \underline{b^8}$	$10^4\times(10^2)^3 = 10^4\times 10^6 = \underline{10^{10}}$ $(b^4)^2b^3 = b^8\times b^3 = \underline{b^{11}}$

1 $(b^2)^4$

2 $(b^2)^3$

3 $(b^3)^2$

4 $(10^3)^2$

5 $(x^2)^2$

6 $(x^2)^5$

7 $(y^3)^4$

8 $(y^5)^2$

9 $10^3(10^2)^2$

10 $x^5(x^3)^2$

11 $(2^3)^2 2^5$

12 $b^3(b^3)^5$

Simplify each of the following:

$10^0 = \underline{1}$	$h^0 = \underline{1}$	$3\times 5^0 = 3\times 1 = \underline{3}$	$5b^0 = 5\times 1 = \underline{5}$

13 10^0

14 h^0

15 x^0

16 a^0

17 5×10^0

18 $5a^0$

19 4×3^0

20 2×1^0

21 $(x^0)^2\times x$

22 $b^2(b^0)^3$

23 $10(10^5)^0$

24 $10\times(10^0)^2$

Write each of the following using a negative index:

$\frac{1}{10^3} = \underline{10^{-3}}$	$\frac{1}{b^5} = \underline{b^{-5}}$	$\frac{1}{10} = \underline{10^{-1}}$	$\frac{1}{10000} = \frac{1}{10^4} = \underline{10^{-4}}$

25 $\frac{1}{10^5}$

26 $\frac{1}{b^4}$

27 $\frac{1}{10}$

28 $\frac{1}{100}$

Simplify and write the following in index form:

$10^2\times 10^{-3} = 10^{2-3} = \underline{10^{-1}}$	$10^{-3}\div 10^{-4} = 10^{-3-\,-4} = 10^{-3+4} = \underline{10}$

29 $10^{-3}\times 10^2$

30 $10^{-2}\times 10^4$

31 $10^5\div 10^{-3}$

32 $10^{-4}\div 10^{-2}$

33 $5^{-2}\times 5^3$

34 $10^{-2}\times 10^6$

35 $x^4\div x^{-2}$

36 $10^{-2}\div 10^4$

37 $x^{-5}\times x^4\times x^3$

38 $y^4\times y^{-7}\times y^2$

39 $10^{-5}\div 10^3$

40 $y^{-4}\div y^{-5}$

$(10^{-2})^4 = 10^{-2\times 4} = \underline{10^{-8}}$	$9(10^0)^{-3} = 9\times 10^{0\times -3} = 9\times 1 = \underline{9}$

41 $(10^{-2})^4$

42 $(2^{-3})^5$

43 $(a^2)^{-3}$

44 $(10^{-5})^{-2}$

45 $(y^2)^4$

46 $2(x^{-3})^0$

47 $(y^4)^{-4}$

48 $(y^{-1})^{-7}$

Distributive Law

Distribute - to spread out, to cover everything.

Multiply each inside term by the outside term.

$$a(b + c) = ab + ac$$

Exercise 1.3

Expand each of the following:

$4(a + 3) = \underline{4a + 12}$

$^{-}a(a + 3) = \underline{{}^{-}a^2 - 3a}$

$5x^2(3x - 2y)$
$= \underline{15x^3 - 10x^2y}$

$3(2b - 5) = \underline{6b - 15}$

$^{-}b^2(2b - 5) = \underline{{}^{-}2b^3 + 5b^2}$

$^{-}5x^3(3x^2 - 2y^3)$
$= \underline{{}^{-}15x^5 + 10x^3y^3}$

+ times + = +
+ times − = −
− times + = −
− times − = +

1 $5(x + 2)$
2 $4(a + 3)$
3 $y(y + 2)$
4 $^{-}x(x + 2)$
5 $x^2(3x - 3)$
6 $4x(2x - 5)$
7 $^{-}5x^2(3x - 2)$
8 $^{-}2y(2y^2 - 1)$
9 $4a^2(3a^3 + 2b)$

Simplify each of the following by expanding and then collecting like terms:

$4(x + 3) - 3(x + 4)$
$= 4x + 12 - 3x - 12$
$= \underline{x}$

$^{-}x(x - 1) - x(x - 4)$
$= {}^{-}x^2 + x - x^2 + 4x$
$= \underline{{}^{-}2x^2 + 5x}$

10 $3(x + 5) - 2(x + 3)$
11 $^{-}x(x - 1) - x(x - 2)$
12 $2(x + 5) - 3(x + 1)$
13 $^{-}a(a + 2) - a(a + 6)$
14 $2(x + 5) - 3(x + 2)$
15 $^{-}a^2(a + 2) - a^2(a + 6)$

$(x + 5)(x + 4)$
$= x(x + 4) + 5(x + 4)$
$= x^2 + 4x + 5x + 20$
$= \underline{x^2 + 9x + 20}$

$(x^3 + 5)(x^2 - 3)$
$= x^3(x^2 - 3) + 5(x^2 - 3)$
$= \underline{x^5 - 3x^3 + 5x^2 - 15}$

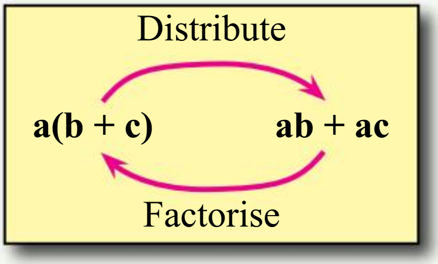
16 $(x + 2)(x + 1)$
17 $(x^3 + 2)(x^2 - 1)$
18 $(x + 3)(x + 1)$
19 $(x^2 + 3)(x^3 - 1)$
20 $(x + 3)(x + 1)$
21 $(x^3 + 3)(x^3 - 1)$

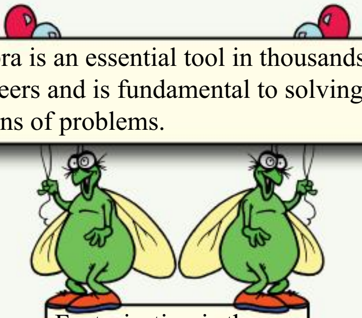
$(x + 3)^2 = (x + 3)(x + 3)$
$= x(x + 3) + 3(x + 3)$
$= x^2 + 3x + 3x + 9$
$= \underline{x^2 + 6x + 9}$

$(x^2 - 4)^2 = (x^2 - 4)(x^2 - 4)$
$= x^2(x^2 - 4) - 4(x^2 - 4)$
$= x^4 - 4x^2 - 4x^2 + 16$
$= \underline{x^4 - 8x^2 + 16}$

22 $(x + 1)^2$
23 $(x - 1)^2$
24 $(x^2 + 2)^2$
25 $(x - 2)^2$
26 $(x + 3)^2$
27 $(x^2 - 3)^2$
28 $(x + 3)^2$
29 $(x^3 - 3)^2$

Factorisation





Algebra is an essential tool in thousands of careers and is fundamental to solving millions of problems.

Factorisation is the inverse of distribution.

Exercise 1.4

Factorise each of the following:

$6x + 9$	$4xy - 6x$	$10x^2 - 8x$
$= 3\times2x + 3\times3$	$= 2x\times2y - 2x\times3$	$= 2x\times5x - 2x\times4$
$= \underline{3(2x + 3)}$	$= \underline{2x(2y - 3)}$	$= \underline{2x(5x - 4)}$

1 $6a + 9$
2 $4ab - 6a$
3 $10c^2 - 8c$
4 $14x + 10$
5 $4ab - 6b$
6 $8d^2 - 6d$
7 $9c + 12$
8 $8xy + 10x$
9 $16x^2 - 12x$
10 $6x + 10$
11 $12st - 15t$
12 $15p^5 - 36p^3$

$18 - 20a^4$	$4x^5 - 10x$	$8x^5 - 12x^3$
$= 2\times9 - 2\times10a^4$	$= 2x\times2x^4 - 2x\times5$	$= 4x^3\times2x^2 - 4x^3\times3$
$= \underline{2(9 - 10a^4)}$	$= \underline{2x(2x^4 - 5)}$	$= \underline{4x^3(2x^2 - 3)}$

13 $6 - 9b^3$
14 $4x^4 - 6x$
15 $10c^4 - 8c^5$
16 $4 + 10a^4$
17 $4x^5 + 8x$
18 $9d^5 + 6d^3$
19 $9 - 12x^5$
20 $8y^3 - 10y$
21 $9x^2 - 12x^3$
22 $6 + 10y^2$
23 $12x^2 + 15x$
24 $12y^5 - 36y^3$
25 $10a^3 + 15a^2$
26 $21x^3 + 18x^2$
27 $24a^3 - 27a^2$

The common term, a+b, is taken out and put at the front.

$$\mathbf{c\underline{(a + b)} + d\underline{(a + b)} = \underline{(a + b)}(c + d)}$$

$x(x - 5) + 4(x - 5)$	$x(x - 2) - 3(x - 2)$
$= \underline{(x - 5)(x + 4)}$	$= \underline{(x - 2)(x - 3)}$

28 $x(x + 5) + 3(x + 5)$
29 $x(x + 5) - 4(x + 5)$
30 $x(x - 1) + 4(x - 1)$
31 $x(x - 1) - 2(x - 1)$
32 $x(x - 6) + 3(x - 6)$
33 $x(x - 5) - 4(x - 5)$
34 $x(x - 2) + 5(x - 2)$
35 $x(x - 3) - 7(x - 3)$

Simplify Algebraic Products

5a means 5 **multiplied** by a
- 5a is a **product**
- 5 is a factor
- a is a factor

$6p \times {}^{-}2p^3$ means 6p **multiplied** by ${}^{-}2p^3$
- $6p \times {}^{-}2p^3$ is a **product**
- 6p is a factor
- ${}^{-}2p^3$ is a factor

To simplify is to reduce to a simpler form.

$6p \times {}^{-}2p^3 = {}^{-}12p^4$

Exercise 1.5

Simplify the following algebraic expressions:

$$7x \times 2x = 7 \times 2 \times x \times x = \underline{14x^2}$$

$$5ab^2 \times {}^{-}3a^2b^3 = 5 \times {}^{-}3 \times a \times a^2 \times b^2 \times b^3 = \underline{{}^{-}15a^3b^5}$$

1 $5x \times 2x$
2 $3y \times 4y$
3 $3a \times 2a$
4 $8b \times 3b$
5 $e^7 \times 2e^3$
6 $8x^3 \times 2x^5$
7 $5x^2 \times {}^{-}2x$
8 $3x \times {}^{-}4x$
9 ${}^{-}5x \times 4x^3$
10 ${}^{-}4x^2 \times {}^{-}2x$
11 $6m \times {}^{-}2m^3$
12 ${}^{-}5w \times 3w^4$
13 ${}^{-}4h^2 \times {}^{-}4h$
14 ${}^{-}3x^5 \times 5x$
15 ${}^{-}3p^2d \times {}^{-}2pd$
16 $4ab \times {}^{-}2a^2b^3$
17 $7de \times {}^{-}4d^2e$
18 $5mn \times {}^{-}3m^2n$
19 ${}^{-}4a^2b^2c \times {}^{-}5a^2bc$
20 $5x^4y^3z^5 \times {}^{-}3x^2yz^3$

Index Law 1

$a^m \times a^n = a^{m+n}$

Multiply the numbers.
Multiply the letters.

+ times − = −
− times + = −
− times − = +

$${}^{-}8x^3 \times 3x^{-2} = {}^{-}8 \times 3 \times x^3 \times x^{-2} = \underline{{}^{-}24x}$$

$$5a^3b^{-2} \times {}^{-}3ab^3 = 5 \times {}^{-}3 \times a^3 \times a \times b^{-2} \times b^3 = \underline{{}^{-}15a^4b}$$

$a = a^1$

21 $10x^4 \times 2x^{-2}$
22 $5a^{-2} \times 4a^4$
23 $3y^5 \times 3y^{-2}$
24 $2p^{-6} \times 7p^3$
25 $4x^6 \times {}^{-}2x^{-4}$
26 ${}^{-}2x^{-7} \times 3x^4$
27 $4x^4y^6 \times 7x^{-2}y^{-2}$
28 $2a^3b^5 \times 4a^{-2}b^{-3}$
29 ${}^{-}6x^{-3}y^2 \times 2x^6y^{-3}$
30 ${}^{-}2e^5f^3 \times {}^{-}2e^{-2}f^{-1}$
31 ${}^{-}3a^{-2}b^6 \times 7a^4b^{-3}$
32 ${}^{-}2a^{-2}b^{-2} \times {}^{-}2a^4b^3$

Negative Index

$a^{-m} = \dfrac{1}{a^m}$

$$4x^{-5} \times 2x^3 = 4 \times 2 \times x^{-5} \times x^3 = \underline{8x^{-2}}$$

$${}^{-}3a^{-3}b^{-2} \times 2ab^{-3} = {}^{-}3 \times 2 \times a^{-3} \times a \times b^{-2} \times b^{-3} = \underline{{}^{-}6a^{-2}b^{-5}}$$

$x = x^1$

33 $3x^{-4} \times 4x^2$
34 ${}^{-}8x \times 4x^{-6}$
35 $6y^{-2} \times {}^{-}3y^{-1}$
36 ${}^{-}10d^{-3} \times {}^{-}2d^5$
37 ${}^{-}3g^3 \times 2g^{-4}$
38 ${}^{-}2a^{-6} \times {}^{-}4a^2$
39 $4x^{-4}y^{-6} \times 3x^{-2}y^{-2}$
40 $2a^3b^{-5} \times 4a^{-2}b^{-3}$
41 ${}^{-}5a^{-3}b^2 \times 3a^{-6}b^{-3}$
42 ${}^{-}2x^{-5}y^3 \times {}^{-}2x^{-2}y^{-4}$
43 ${}^{-}5e^{-2}f^{-6} \times 2e^4f^{-3}$
44 ${}^{-}4a^{-5}b^{-3} \times {}^{-}3a^4b^{-3}$

Zero Index

$a^0 = 1$

Simplify Algebraic Quotients

$8a \div 2$ means $8a$ **divided** by 2
- $8a$ is the dividend
- 2 is the divisor
- $4a$ is the **quotient**

$6p \div {}^{-}2p^3$ means $6p$ **divided** by ${}^{-}2p^3$
- $6p$ is the dividend
- ${}^{-}2p^3$ is the divisor
- ${}^{-}3p^{-2}$ is the **quotient**

To simplify is to reduce to a simpler form.

$6p \div {}^{-}2p^3 = {}^{-}3p^{-2}$

Exercise 1.6

Simplify the following algebraic expressions:

A quotient is the result of division.

$8x \div 4 = \underline{2x}$

$12x^5 \div 4x^2 = 3x^{5-2}$

$= \underline{3x^3}$

$${}^{-}6x^5 \div 4x^2y = \frac{-6x^5}{4x^2y}$$

$$= \underline{\frac{-3x^3}{2y}}$$

1 $10x \div 5$

2 $16a \div 4$

3 $12x \div 3$

4 $14d \div 7$

5 ${}^{-}8x \div 4$

6 $6x \div {}^{-}3$

7 ${}^{-}10y \div {}^{-}2$

8 ${}^{-}10a \div {}^{-}2$

9 $14x^6 \div 2x^3$

10 $21x^7 \div 3x^4$

11 $8x^4 \div 4x^2$

12 ${}^{-}4g^3 \div 2g^2$

13 ${}^{-}12a^6 \div {}^{-}4a^2$

14 $8x^6 \div {}^{-}4x^4$

15 $16x^5y \div {}^{-}4x^3$

16 ${}^{-}14x^4y \div {}^{-}7x^2$

17 ${}^{-}20ab \div 4b$

18 ${}^{-}16a^9c^2 \div 12a^6$

19 ${}^{-}24e^5f^3 \div {}^{-}12e^2$

20 ${}^{-}21a^5b^6c \div 28a^4b^3$

Index Law 2

$a^m \div a^n = a^{m-n}$

Divide the numbers.
Divide the letters

$+$ divided by $- = -$
$-$ divided by $+ = -$
$-$ divided by $- = +$

$6x^{-3} \div 2x^2 = 3x^{-3-2}$

$= \underline{3x^{-5}}$

${}^{-}4x^2y^{-4} \div 2xy^{-3} = {}^{-}2x^{2-1}y^{-4--3}$ $\{x = x^1\}$

$= {}^{-}2x^1y^{-4+3}$ $\{- -3 = 3\}$

$= \underline{{}^{-}2xy^{-1}}$ $\boxed{x^1 = x}$

21 $6a^{-3} \div 3a^2$

22 $6b^3 \div 2b^{-1}$

23 $\dfrac{4x^{-3}}{2x^2}$

24 $\dfrac{12a^5}{3a^{-2}}$

25 ${}^{-}9w \div 3w^{-2}$

26 $8s^{-3} \div 2s^{-2}$

27 $15x^2 \div 5x^{-4}$

28 $3y^{-2} \div 2y$

29 $\dfrac{18x^{-2}}{6x^{-3}}$

30 $\dfrac{14n^{-5}}{7n^2}$

31 $12m^{-2}n \div {}^{-}3m^2n$

32 $8ab^{-3} \div {}^{-}4a^{-2}b$

33 $4ab^{-1} \div {}^{-}2a^{-2}b^4$

34 ${}^{-}8x^2y^{-1} \div {}^{-}2xy^{-1}$

35 $\dfrac{10m^3n^{-2}}{{}^{-}2m^{-1}n^2}$

36 $\dfrac{{}^{-}15ab^2}{{}^{-}3a^{-2}b^{-3}}$

37 ${}^{-}4c^{-2}d^2 \div 4c^2d^2$

38 ${}^{-}4a^2b^2c \div {}^{-}a^{-2}bc^{-2}$

Negative Index

$a^{-m} = \dfrac{1}{a^m}$

Zero Index

$a^0 = 1$

Operations with Algebraic Fractions

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$$

To add fractions, the denominator needs to be the same.

Exercise 1.7

Simplify the following algebraic expressions:

$$\frac{2x}{7}+\frac{3x}{7}$$
$$=\frac{2x+3x}{7}$$
$$=\frac{5x}{7}$$

$$\frac{3x}{4}+\frac{7x}{4}$$
$$=\frac{3x+7x}{4}$$
$$=\frac{10x}{4}$$
$$=\frac{5x}{2}$$

1 $\frac{x}{5}+\frac{2x}{5}$ **2** $\frac{3a}{4}+\frac{6a}{4}$

3 $\frac{5b}{3}+\frac{2b}{3}$ **4** $\frac{c}{6}+\frac{4c}{6}$

5 $\frac{4x}{3}+\frac{x}{3}$ **6** $\frac{4x^2}{5}+\frac{2x^2}{5}$

7 $\frac{3e}{4}+\frac{5e}{4}$ **8** $\frac{3a^3}{4}+\frac{5a^3}{4}$

9 $\frac{5a}{3}+\frac{4a}{3}$ **10** $\frac{3x^3}{7}+\frac{x^2}{7}$

11 $\frac{4x}{3}+\frac{x}{3}$ **12** $\frac{2y^5}{6}+\frac{7y^2}{6}$

$$\frac{3x^5}{8}+\frac{2x^5}{8}$$
$$=\frac{3x^5+2x^5}{8}$$
$$=\frac{5x^5}{8}$$

$3x^3+x^2 = 3x^3+x^2$
The terms are not the same - they can't be added.

$$\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

To subtract fractions, the denominator needs to be the same.

$$\frac{5x}{9}-\frac{3x}{9}$$
$$=\frac{5x-3x}{9}$$
$$=\frac{2x}{9}$$

$$\frac{9x}{4}-\frac{3x}{4}$$
$$=\frac{9x-3x}{4}$$
$$=\frac{6x}{4}$$
$$=\frac{3x}{2}$$

13 $\frac{3x}{5}-\frac{x}{5}$ **14** $\frac{3a}{2}-\frac{a}{2}$

15 $\frac{7y}{3}-\frac{2y}{3}$ **16** $\frac{4a}{6}-\frac{3a}{6}$

17 $\frac{5c}{3}-\frac{2c}{3}$ **18** $\frac{2e}{3}-\frac{e}{3}$

19 $\frac{7x}{4}-\frac{3x}{4}$ **20** $\frac{5x^3}{4}-\frac{3x^3}{4}$

21 $\frac{9y}{2}-\frac{5y}{2}$ **22** $\frac{7x^2}{5}-\frac{y^2}{5}$

23 $\frac{7z}{8}-\frac{z}{8}$ **24** $\frac{3x^5}{3}-\frac{2x^2}{3}$

$$\frac{4x^2}{5}-\frac{2x^2}{5}$$
$$=\frac{4x^2-2x^2}{5}$$
$$=\frac{2x^2}{5}$$

$3x^5-2x^2 = 3x^5-2x^2$
The terms are not the same - they can't be subtracted.

Operations with Algebraic Fractions

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

Multiply the numerators.
Multiply the denominators.

Exercise 1.8
Simplify the following algebraic expressions:

$$\frac{2}{5a}\times\frac{3}{4a}$$
$$=\frac{2\times3}{5a\times4a}$$
$$=\frac{6}{20a^2}=\underline{\frac{3}{10a^2}}$$

$$\frac{6x^5}{7}\times\frac{5}{8x^3}$$
$$=\frac{6x^5\times5}{7\times8x^3}$$
$$=\frac{30x^5}{56x^3}$$
$$=\underline{\frac{15x^2}{28}}$$

1 $\frac{3}{x}\times\frac{4}{x}$ **2** $\frac{5}{a}\times\frac{2}{a^2}$

3 $\frac{y}{2}\times\frac{y}{3}$ **4** $\frac{y^3}{4}\times\frac{y}{3}$

5 $x\times\frac{x}{5}$ **6** $\frac{4x}{5}\times3x$

7 $\frac{9x^3}{7}\times\frac{2x}{3}$ **8** $\frac{4x^4}{5}\times\frac{3}{2}$

9 $\frac{7a^2}{9}\times6a^3$ **10** $\frac{4x^3}{5}\times\frac{x}{6}$

11 $\frac{2x^6}{3}\times\frac{1}{4x^2}$ **12** $\frac{5x^3}{6}\times\frac{3}{15x^5}$

Index Law 1

$a^m\times a^n=a^{m+n}$

$3x=\frac{3x}{1}$

Multiply the numbers.
Multiply the letters.

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$$

In division, the second fraction is turned upside.

$$\frac{2x}{3}\div\frac{4x}{5}$$
$$=\frac{2x}{3}\times\frac{5}{4x}$$
$$=\frac{10x}{12x}=\underline{\frac{5}{6}}$$

$$\frac{6x^5}{5}\div\frac{3x^3}{2}$$
$$=\frac{6x^5}{5}\times\frac{2}{3x^3}$$
$$=\frac{12x^5}{15x^3}=\underline{\frac{4x^2}{5}}$$

13 $\frac{x}{2}\div\frac{x}{3}$ **14** $\frac{a}{3}\div\frac{a}{2}$

15 $\frac{m}{4}\div\frac{m}{3}$ **16** $\frac{2x}{5}\div\frac{3x}{4}$

17 $\frac{4e}{3}\div\frac{2}{3}$ **18** $\frac{6x}{7}\div\frac{4}{5}$

19 $\frac{3x^4}{2}\div\frac{2x}{5}$ **20** $\frac{3y^3}{5}\div\frac{3y^2}{4}$

21 $\frac{4t^5}{2}\div\frac{1}{2}$ **22** $\frac{3a^2}{2b}\div\frac{2a}{b^3}$

23 $\frac{6x^2}{y}\div\frac{4x}{y^2}$ **24** $\frac{12a^2b^3}{7c}\div\frac{8ab}{c}$

Index Law 2

$a^m\div a^n=a^{m-n}$

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 1.9

1. Spell Quotient
2. $5 - 7$
3. $3 - {}^{-}4$
4. $10^2 \times 10^3$
5. $x^3 \div x^2$
6. $(2^{-3})^2$
7. Simplify: $\frac{x}{2} + \frac{x}{3}$
8. Simplify: $\frac{x}{2} - \frac{x}{3}$
9. Increase \$6 by 10%
10. If I paid \$50 deposit and 10 payments of \$10. How much did I pay?

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

10% of \$6 is \$0.60

Exercise 1.10

1. Spell Distribution
2. ${}^{-}2 - 3$
3. $3 \times {}^{-}4$
4. $10^5 \times 10^{-3}$
5. $x^5 \div x^2$
6. $(2^3)^{-2}$
7. Simplify: $\frac{x}{2} + \frac{x}{5}$
8. Simplify: $\frac{x}{2} - \frac{x}{5}$
9. Increase \$8 by 10%
10. If I paid \$50 deposit and 10 payments of \$15. How much did I pay?

Algebra is the greatest labour saving device ever invented by humans.

Exercise 1.11

1. Spell Factorisation
2. ${}^{-}5 - 1$
3. $1 \div {}^{-}2$
4. $10^{-2} \times 10^3$
5. $x^{-3} \div x^7$
6. $(2^{-3})^{-2}$
7. Simplify: $\frac{x}{3} + \frac{x}{5}$
8. Simplify: $\frac{x}{3} - \frac{x}{5}$
9. Increase \$9 by 10%
10. If I paid \$100 deposit and 10 payments of \$25. How much did I pay?

'If you think dogs can't count, try putting three dog biscuits in your pocket and then giving Fido only two of them' - Phil Pastoret.

Stockbrokers buy and sell shares and bonds for clients.

- Relevant school subjects are Mathematics and English.
- Courses usually involve a Universtity Bachelor degree with a major in commerce/finance.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 1.12

1 Put the following fractions in order of increasing size:

$\frac{3}{4}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{2}$

2 Evaluate each of the following:

a) $1 + 2 \times 3 - 4$

b) $12 \div 3 \times 4 - 6 + 7$

c) $9 \times 8 \div 6 - 5$

d) $(10 + 2) \times 5 - 5$

e) $((((1 - 2) - 3) - 4) - 5)$

f) $6 - (5 - (4 - (3 - (2 - 1))))$

Order of Operations:
1 ( ) brackets first.
2 × and ÷ from left to right.
3 + and − from left to right.

3 Simplify each of the following:

a) $10^5 \times 10^2$

b) $10^5 \div 10^3$

c) $10^2 \div 10^3 \times 10^4$

d) $10^7 \div 10^9 \times 10^2$

$$10^7 \div 10^5$$
$$= 10^{7-5}$$
$$= \underline{10^2 \text{ or } 100}$$

4 Simplify each of the following:

a) $7x + 2y - 3x + 4y$

b) $4x - 2y - 8x + y$

c) $(2a + b) - (a + 5b)$

d) $3(x - 2) - 3(x - 5)$

e) $(x - 2) - (1 - x)$

f) $2x(x - 1) + 5x^2$

$$2(x - 1) - 3(x - 4)$$
$$= 2x - 2 - 3x + 12$$
$$= \underline{{}^{-}x + 10}$$

5 If $2^{(x+3)} = 32$, what is the value of x?

$32 = 2^5$

6 If $4^{(2x-1)} = 16$, what is the value of x?

7 Simplify: $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

8 If $\frac{1}{x} = \frac{1}{2} + \frac{2}{3}$, what is the value of x?

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$$
$$= \frac{3 \times 5}{2 \times 3 \times 5} + \frac{2 \times 5}{3 \times 2 \times 5} + \frac{2 \times 3}{5 \times 2 \times 3}$$
$$= \frac{15}{30} + \frac{10}{30} + \frac{6}{30}$$
$$= \underline{\frac{31}{30}}$$

Investigations

Investigation 1.1 Shortcut for squaring numbers less than 10

Complete the pattern:

$2^2 = 3\times1 + 1$ $\quad 6^2 =$
$3^2 = 4\times2 + 1$ $\quad 7^2 =$
$4^2 = 5\times3 + 1$ $\quad 8^2 =$
$5^2 = 6\times4 + 1$ $\quad 9^2 =$

Investigation 1.2 Shortcut for squaring numbers near 10

Example: 13^2 $\{13 = 10 + 3\}$

$$\begin{aligned} 13^2 &= 10^2 + 20\times3 + 3^2 \\ &= 100 + 60 + 9 \\ &= \underline{169} \end{aligned}$$

$$\begin{aligned} (10 + 3)^2 &= (10 + 3)(10 + 3) \\ &= 10(10 + 3) + 3(10 + 3) \\ &= 10^2 + 10\times3 + 10\times3 + 3^2 \\ &= 100 + 60 + 9 \\ &= \underline{169} \end{aligned}$$

Example: 16^2 $\{16 = 10 + 6\}$

$$\begin{aligned} 16^2 &= 10^2 + 20\times6 + 6^2 \\ &= 100 + 120 + 36 \\ &= \underline{256} \end{aligned}$$

$$\begin{aligned} (10 + a)^2 &= (10 + a)(10 + a) \\ &= 10(10 + a) + a(10 + a) \\ &= 10^2 + 10a + 10a + a^2 \\ &= \underline{100 + 20a + a^2} \end{aligned}$$

What is 15^2?
What is 14^2?
What is 17^2?

What is 9^2? $\{9 = 10 - 1\}$

$$\begin{aligned} 9^2 &= 10^2 + 20\times{}^-1 + {}^-1^2 \\ &= 100 + {}^-20 + 1 \\ &= \underline{81} \end{aligned}$$

Investigation 1.3 Shortcut for squaring numbers near 50

Example: 53^2 $\{53 = 50 + 3\}$

$$\begin{aligned} 53^2 &= 50^2 + 100\times3 + 3^2 \\ &= 2500 + 300 + 9 \\ &= \underline{2809} \end{aligned}$$

$$\begin{aligned} (50 + 3)^2 &= (50 + 3)(50 + 3) \\ &= 50(50 + 3) + 3(50 + 3) \\ &= 50^2 + 50\times3 + 50\times3 + 3^2 \\ &= 2500 + 300 + 9 \\ &= \underline{2809} \end{aligned}$$

Example: 46^2 $\{46 = 50 - 4\}$

$$\begin{aligned} 46^2 &= 50^2 + 100\times{}^-4 + {}^-4^2 \\ &= 2500 + {}^-400 + 16 \\ &= \underline{2116} \end{aligned}$$

$$\begin{aligned} (50 + a)^2 &= (50 + a)(50 + a) \\ &= 50(50 + a) + a(50 + a) \\ &= 50^2 + 50a + 50a + a^2 \\ &= \underline{2500 + 100a + a^2} \end{aligned}$$

What is 55^2?
What is 54^2?
What is 47^2?

Investigation 1.4

Investigate

squaring numbers near 100?

A Couple of Puzzles

Exercise 1.13

1 A cup and saucer together weigh 360 g. If the cup weighs twice as much as the saucer, what is the weight of the saucer?

2 A cup and saucer together costs $25. If the cup cost $9 more than the saucer, what is the cost of the saucer?

3 Complete the following multiplication problems:

a)

43
18 ×
□44
430
□74

b)

3 □
□ 8 ×
2□□
6 4 0
8 9 □

c)

□ 2
3 □ ×
□□8
1 2 □ 0
□4 2 8

d)

□ 5
□ 6 ×
2 7 0
9 □ 0
11□0

A Game

Target is played by two people, or two teams, using four dice.

1 Take turns to choose a number from the board. This is then the target number.

2 Roll the four dice. The first person, or team, to arrange the four numbers on the dice to equal the target number scores a hit.

11	**12**	**13**
14	**15**	**16**
17	**18**	**19**

Example:
Target = 14
Dice numbers = 2, 5, 3, 1

$14 = 5\times2+3+1$
$14 = 3\times5+1-2$
$14 = 2^3+5+1$

Is the game too easy?
Change the numbers to 21 to 29.
or 31 to 39

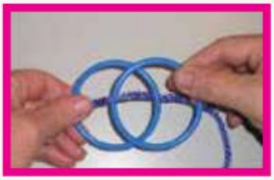
A Sweet Trick

Pass a ring through a string.

1 Thread a light rope/string through two rings.
2 Grip the two rings as shown in the photo.
3 With flair, very quickly pull the ring in the right hand down the rope.
4 Hey presto. The ring in the left hand is through the rope.

First practice the trick.

Then add exaggerated gestures.



Technology

Technology 1.1 Simplifying Fractions

Scientific calculators are excellent in working with fractions:

1 Simplify $\frac{15}{35}$ $\boxed{15}\ \boxed{a\frac{b}{c}}\ \boxed{35}\ \boxed{=}\ \boxed{3r7}$ meaning $\frac{3}{7}$

2 Simplify $\frac{18}{4}$ $\boxed{18}\ \boxed{a\frac{b}{c}}\ \boxed{4}\ \boxed{=}\ \boxed{4r1r2}$ meaning $4\frac{1}{2}$

To change to a vulgar fraction: $\boxed{\text{2ndF}}\ \boxed{a\frac{b}{c}}$ to give $\boxed{9r2}$ ie $\frac{9}{2}$

3 Use a scientific calculator to simplify the following ratios:

a) 3 : 9 **b)** 9 : 12 **c)** 16 : 24
d) 2.1 : 3.5 **e)** 14.4 : 12.6 **f)** 256 : 1024

Technology 1.2 Expanding and Factorising

Graphics calculators are capable of expanding and factorising:

1 Choose **expand** from the algebra menu.
2 Enter the algebraic expression: 3(4x − 5) to produce 12x − 15

1 Choose **factor** from the algebra menu.
2 Enter the algebraic expression: 12x − 15 to produce 3(4x − 5)

Technology 1.3 The Distributive Law and Factorising

There are a considerable number of resources about the Distributive Law and factorising on the Internet.

Try some of them.

The human mind has never invented a labor-saving machine equal to algebra.

Technology 1.4 Algebraic Fractions

Algebraic fractions

Watch videos on adding, subtracting, multiplying, and dividing fractions'.

'The essence of mathematics is not to make simple things complicated, but to make complicated things simple.' - S. Gudder.

Chapter Review 1

Exercise 1.14

Expand each of the following:

$4(a+3) = \underline{4a+12}$ ${}^{-}a(a+3) = \underline{{}^{-}a^2 - 3a}$ ${}^{-}b^2(2b-5) = \underline{{}^{-}2b^3 + 5b^2}$	$(x+5)(x+4)$ $= x(x+4) + 5(x+4)$ $= x^2 + 4x + 5x + 20$ $= \underline{x^2 + 9x + 20}$	$(x+3)^2 = (x+3)(x+3)$ $= x(x+3) + 3(x+3)$ $= x^2 + 3x + 3x + 9$ $= \underline{x^2 + 6x + 9}$

1 $5(x+2)$

2 ${}^{-}x(x+2)$

3 $(x^3+2)(x^2-1)$

4 $(x+3)(x+1)$

5 $(x+3)^2$

6 $(x^3+3)(x^3-1)$

Factorise each of the following:

$6x + 9$ $= 3{\times}2x + 3{\times}3$ $= \underline{3(2x+3)}$	$4xy - 6x$ $= 2x{\times}2y - 2x{\times}3$ $= \underline{2x(2y-3)}$	$10x^2 - 8x$ $= 2x{\times}5x - 2x{\times}4$ $= \underline{2x(5x-4)}$

7 $6x + 10$

8 $12st - 15t$

9 $15p^5 - 36p^3$

10 $6 + 10y^2$

11 $12x^2 + 15x$

12 $12y^5 - 36y^3$

Simplify the following algebraic expressions:

$7x \times {}^{-}2x = 7 \times {}^{-}2 \times x \times x$ $= \underline{{}^{-}14x^2}$	$5a^3b^{-2} \times {}^{-}3ab^3 = 5 \times {}^{-}3 \times a^3 \times a \times b^{-2} \times b^3$ $= \underline{{}^{-}15a^4b}$

13 $6m \times {}^{-}2m^3$

14 ${}^{-}5w \times 3w^4$

15 ${}^{-}4h^2 \times {}^{-}4h$

16 $10x^4 \times 2x^{-2}$

17 $5a^{-2} \times 4a^4$

18 $3y^5 \times 3y^{-2}$

19 $4x^4y^6 \times 7x^{-2}y^{-2}$

20 $2a^3b^5 \times 4a^{-2}b^{-3}$

21 ${}^{-}6x^{-3}y^2 \times 2x^6y^{-3}$

$6x^{-3} \div 2x^2 = 3x^{-3-2}$ $= \underline{3x^{-5}}$	${}^{-}4x^2y^{-4} \div 2xy^{-3} = {}^{-}2x^{2-1}y^{-4--3}$ $\{x = x^1\}$ $= {}^{-}2x^1y^{-4+3}$ $\{- -3 = 3\}$ $= \underline{{}^{-}2xy^{-1}}$

22 $14x^6 \div 2x^3$

23 ${}^{-}12a^6 \div {}^{-}4a^2$

24 $8x^6 \div {}^{-}4x^4$

25 ${}^{-}9w \div 3w^{-2}$

26 $8s^{-3} \div 2s^{-2}$

27 $4ab^{-1} \div {}^{-}2a^{-2}b^4$

28 $\dfrac{18x^{-2}}{6x^{-3}}$

29 $\dfrac{10m^3n^{-2}}{{}^{-}2m^{-1}n^2}$

30 $\dfrac{{}^{-}15ab^2}{{}^{-}3a^{-2}b^{-3}}$

$\dfrac{3x^5}{8} + \dfrac{2x^5}{8}$ $= \dfrac{3x^5 + 2x^5}{8}$ $= \dfrac{5x^5}{\underline{8}}$

31 $\dfrac{4x}{3} + \dfrac{x}{3}$

32 $\dfrac{4x^2}{5} + \dfrac{2x^2}{5}$

33 $\dfrac{3e}{4} + \dfrac{5e}{4}$

34 $\dfrac{3a^3}{4} + \dfrac{5a^3}{4}$

35 $\dfrac{7x}{4} - \dfrac{3x}{4}$

36 $\dfrac{5x^3}{4} - \dfrac{3x^3}{4}$

37 $\dfrac{9y}{2} - \dfrac{5y}{2}$

38 $\dfrac{7x^2}{5} - \dfrac{y^2}{5}$

$\dfrac{4x^2}{5} - \dfrac{2x^2}{5}$ $= \dfrac{4x^2 - 2x^2}{5}$ $= \dfrac{2x^2}{\underline{5}}$

Chapter Review 2

Exercise 1.15

Expand each of the following:

$4(a+3) = \underline{4a+12}$ $^{-}a(a+3) = \underline{^{-}a^2 - 3a}$ $^{-}b^2(2b-5) = \underline{^{-}2b^3 + 5b^2}$	$(x+5)(x+4)$ $= x(x+4) + 5(x+4)$ $= x^2 + 4x + 5x + 20$ $= \underline{x^2 + 9x + 20}$	$(x+3)^2 = (x+3)(x+3)$ $= x(x+3) + 3(x+3)$ $= x^2 + 3x + 3x + 9$ $= \underline{x^2 + 6x + 9}$

1 $3(x+4)$ **2** $^{-}a(a+5)$ **3** $(x^2+4)(x^3-1)$

4 $(x+2)(x+1)$ **5** $(x+2)^2$ **6** $(x^2+3)(x^2-1)$

Factorise each of the following:

$6x + 9$ $= 3\times2x + 3\times3$ $= \underline{3(2x+3)}$	$4xy - 6x$ $= 2x\times2y - 2x\times3$ $= \underline{2x(2y-3)}$	$10x^2 - 8x$ $= 2x\times5x - 2x\times4$ $= \underline{2x(5x-4)}$

7 $6x + 8$ **8** $12ab - 9a$ **9** $9x^5 - 15x^2$

10 $8 + 12y^2$ **11** $6x^3 + 15x$ **12** $18y^4 - 24y^3$

Simplify the following algebraic expressions:

$7x \times {}^{-}2x = 7 \times {}^{-}2 \times x \times x$ $= \underline{^{-}14x^2}$	$5a^3b^{-2} \times {}^{-}3ab^3 \quad = 5 \times {}^{-}3 \times a^3 \times a \times b^{-2} \times b^3$ $= \underline{^{-}15a^4b}$

13 $5x \times {}^{-}2x^4$ **14** $^{-}5x \times 4x^3$ **15** $^{-}2y^5 \times {}^{-}3y$

16 $7x^5 \times 2x^{-2}$ **17** $3a^{-2} \times 4a^5$ **18** $2y^6 \times 3y^{-2}$

19 $3x^5y^3 \times 6x^{-3}y^{-3}$ **20** $5x^3y^4 \times 4x^{-2}y^{-1}$ **21** $^{-}4a^{-2}b^4 \times 2a^5b^{-3}$

$6x^{-3} \div 2x^2 = 3x^{-3-2}$ $= \underline{3x^{-5}}$	$^{-}4x^2y^{-4} \div 2xy^{-3} = {}^{-}2x^{2-1}y^{-4--3} \quad \{x = x^1\}$ $= {}^{-}2x^1y^{-4+3} \quad \{- -3 = 3\}$ $= \underline{^{-}2xy^{-1}}$

22 $10x^5 \div 2x^3$ **23** $^{-}10y^7 \div {}^{-}4y^3$ **24** $6x^5 \div {}^{-}4x^4$

25 $^{-}15b^2 \div 3b^{-3}$ **26** $8c^{-4} \div 6c^{-2}$ **27** $10ab^{-3} \div {}^{-}2a^{-2}b^3$

28 $\dfrac{15x^{-4}}{18x^2}$ **29** $\dfrac{14a^5b^{-3}}{^{-}10a^2b^2}$ **30** $\dfrac{^{-}14xy^{-2}}{^{-}6x^2y^{-3}}$

$$\frac{3x^5}{8} + \frac{2x^5}{8}$$
$$= \frac{3x^5 + 2x^5}{8}$$
$$= \underline{\frac{5x^5}{8}}$$

31 $\dfrac{x}{5} + \dfrac{2x}{5}$ **32** $\dfrac{3a}{4} + \dfrac{6a}{4}$

33 $\dfrac{4x}{3} + \dfrac{x}{3}$ **34** $\dfrac{4x^2}{5} + \dfrac{2x^2}{5}$

35 $\dfrac{3x}{5} - \dfrac{x}{5}$ **36** $\dfrac{3a}{2} - \dfrac{a}{2}$

37 $\dfrac{7x}{4} - \dfrac{3x}{4}$ **38** $\dfrac{5x^3}{4} - \dfrac{3x^3}{4}$

$$\frac{4x^2}{5} - \frac{2x^2}{5}$$
$$= \frac{4x^2 - 2x^2}{5}$$
$$= \underline{\frac{2x^2}{5}}$$

Linear Equations

2

Number and Algebra ➞ **Linear and non-linear relationships**

★ Solve problems involving linear equations, including those derived from formulas.
  - represent word problems with simple linear equations and solve them to answer questions.

★ Solve linear inequalities and graph their solutions on a number line.
  - represent word problems with simple linear inequalities and solve them to answer questions.

★ Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology.
  - associate the solution of simultaneous equations with the coordinates of the intersection of their corresponding graphs.

A matrix is a rectangular array of numbers or symbols.

$$\begin{pmatrix}3 & 1\\2 & 5\end{pmatrix}$$

A TASK

Matrices are used to solve simultaneous equations. An example is shown below.

Learn how to use matrices to solve simultaneous equations such as:

$3x + y = {}^-7$ ...**(1)**

$2x + 5y = 4$ ...**(2)**

A LITTLE BIT OF HISTORY

Solving simultaneous equations dates as far back as 300 BC in the Chinese textbook *Nine Chapters of the Mathematical Art*.

Gauss (1777-1855) describes a method of solving simultaneous equations using matrices.

Todd (1906-1995) used matrices to analyse airplane vibrations during World War II.

Simultaneous equations and matrices are now a vital part of engineering, physics, chemistry, IT, and economics.

$3x - 2y = 8$ ...**(1)**

$x + 4y = {}^-2$ ...**(2)**

$$\begin{pmatrix}3 & {}^-2\\1 & 4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}8\\{}^-2\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{14}\begin{pmatrix}4 & 2\\{}^-1 & 3\end{pmatrix}\begin{pmatrix}8\\{}^-2\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{14}\begin{pmatrix}28\\{}^-14\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2\\{}^-1\end{pmatrix}$$

Solution: $x = 2$, $y = {}^-1$

Linear Equations

Each day, in Australia, millions of real-world problems are solved by the use of linear equations.

The basic idea is to:
1 Write the formula.
2 Substitute.
3 Solve for the unknown.

Exercise 2.1

The small car hire rate, in \$H, is given by the function: $H = 86d + 65$, where d is the number of days the car is hired.

If the hire charge was \$667, for how many days was the car hired?

$$\begin{aligned} H &= 86d + 65 && \{\text{write the formula}\} \\ 667 &= 86d + 65 && \{\text{substitute hire} = \$667\} \\ 667 - 65 &= 86d && \{\text{inverse of + is −}\} \\ 602 &= 86d && \\ 602 \div 86 &= d && \{\text{inverse of × is ÷}\} \\ 7 &= d && \end{aligned}$$

The car was hired for 7 days.

Expressions such as $H = 86d + 65$ are linear because the highest power of d is 1 ($d = d^1$).

1 The car hire rate, in \$H, is given by the function: $H = 73d + 60$, where d is the number of days the car is hired. If the hire charge was \$425, for how many days was the car hired?

2 The car hire rate, in \$H, is given by the function $H = 92d + 59$, where d is the number of days the car is hired. If the hire charge was \$1 071, for how many days was the car hired?

3 The mobile phone call rate, in \$C, is given by the function: $C = 0.39 + 0.88t$, where t is the time of the call in minutes. Find the length of a call for which the cost was \$4.13.

4 The mobile phone call rate, in \$C, is given by the function: $C = 0.38 + 0.92t$, where t is the time of the call in minutes. Find the length of a call for which the cost was \$3.83.

5 The labour cost, \$C, for a plumber's visit is given by $C = 85.50t + 45$, where t is the time in hours spent by the plumber on the job. If the cost for the plumber's visit was \$258.75, for how long was the plumber on the job?

6 The labour cost, \$C, for an electrician's visit is given by $C = 74.50t + 55$, where t is the time in hours spent by the electrician on the job. If the cost for the electrician's visit was \$539.25, for how long was the electrician on the job?

7 Speed, v, is given by the formula: $v = s \div t$ ($v = \frac{s}{t}$), where s is the distance and t is the time. What distance will a car, travelling at 100 km/h, cover in 2.5 h?

8 Speed, v, is given by the formula: $v = s \div t$ ($v = \frac{s}{t}$), where s is the distance and t is the time. If thunder is heard eight seconds after the lightning is seen, how far away was the lightning (Assume sound travels at 330 m/s)?

Exercise 2.2

1 The circumference, C, of a circle is given by the formula: $C = 2\pi r$, where r is the radius of the circle. A circular horse yard is to have a circumference of 56 m. What should be the radius of the horse yard?

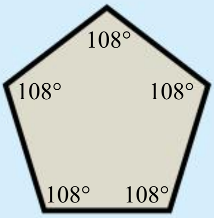
2 The circumference, C, of a circle is given by the formula: $C = 2\pi r$, where r is the radius of the circle. A circular horse yard is to have a circumference of 75 m. What should be the radius of the horse yard?

3 The perimeter, P, of a rectangle is given by $P = 2(l + b)$, where l is the length and b is the breadth. If the length of a house block is 43 m and the perimeter is 138 m, what is the breadth of the house block?

4 The perimeter, P, of a rectangle is given by $P = 2(l + b)$, where l is the length and b is the breadth. If a 80 m length of fencing is to fence a rectangular area with a breadth of 12 m, what must be the length?

The sum of the interior angles of a polygon is given by the formula:
$S = 90(2n - 4)$, where n is the number of sides on the polygon.
How many sides in a polygon with an interior angle sum of 540°?

$$
\begin{aligned}
S &= 90(2n - 4) && \{\text{write the formula}\}\\
540 &= 90(2n - 4) && \{\text{substitute angle} = 540\}\\
540 \div 90 &= (2n - 4) && \{\text{inverse of} \times \text{is} \div\}\\
6 &= 2n - 4 &&\\
6 + 4 &= 2n && \{\text{inverse of} - \text{is} +\}\\
10 &= 2n &&\\
10 \div 2 &= n && \{\text{inverse of} \times \text{is} \div\}\\
5 &= n &&
\end{aligned}
$$



The polygon has 5 sides.

5 The sum of the interior angles of a polygon is given by the formula:
$S = 90(2n - 4)$, where n is the number of sides on the polygon.
How many sides in a polygon with an interior angle sum of 540°?

6 The sum of the interior angles of a polygon is given by the formula:
$S = 90(2n - 4)$, where n is the number of sides on the polygon.
How many sides in a polygon with an interior angle sum of 1080°?

7 The volume, V, of a square based prism is given by the formula: $V = w^2h$, where w is the width of the base and h is the height of the prism. If the width of the prism is 4 cm and the volume is 96 cm^3, what is the height of the prism?

8 The volume of a cylinder, V, is given by the formula: $V = \pi r^2 h$, where r is the radius of the base of the cylinder and h is the height of the cylinder. If a cylinder with a base radius of 6.5 cm has a volume of 1 487 cm^3, what is the height of the cylinder?

9 The volume of a cone, a circular based pyramid, is given by the formula:
$V = \frac{\pi r^2 h}{3}$ where r is the radius of the base of the cone and h is the height of the cone. If a cone has a radius of 1.1 m and a volume of 6.8 m^3, what is the height of the cone?

Linear Inequations

is less than $<$

$\leqslant$ is less than or equal to

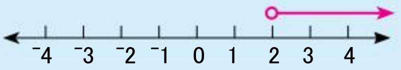
is greater than $>$

$\geqslant$ is greater than or equal to

Exercise 2.3

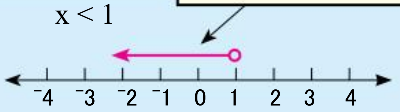
Graph the following inequations on the number line:

$x > 2$



$x < 1$

This shows that all numbers to the left of 1 are less than 1.



1 $x > 1$

2 $x > 0$

3 $x < 3$

4 $x < 2$

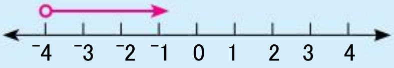
5 $x > {}^{-}3$

6 $x > {}^{-}1$

7 $x < 0$

8 $x < {}^{-}2$

$$\begin{aligned} x + 3 &> {}^{-}1 \\ x &> {}^{-}1 - 3 \qquad \{\text{inverse of + is −}\} \\ x &> {}^{-}4 \end{aligned}$$



Can you subtract the same thing from both sides of an inequation?

$7 > 3$ is true

Is $7-2 > 3-2$ true?

$5 < 9$ is true

Is $5-4 < 9-4$ true?

9 $x + 3 > 4$

10 $x + 2 > 2$

11 $x + 3 < {}^{-}1$

12 $x + 3 < 2$

13 $x + 2 > 1$

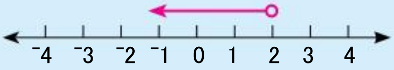
14 $x + 1 > {}^{-}1$

15 $x + 3 < {}^{-}1$

16 $x + 1 < {}^{-}2$

17 Even though an accounting fee of \$3 was added to the price, it was still less than \$10. What was the price?

$$\begin{aligned} x - 3 &< {}^{-}1 \\ x &< {}^{-}1 + 3 \qquad \{\text{inverse of + is −}\} \\ x &< 2 \end{aligned}$$



Can you add the same thing to both sides of an inequation?

$7 > 3$ is true

Is $7+2 > 3+2$ true?

$5 < 9$ is true

Is $5+4 < 9+4$ true?

18 $x + 3 > 4$

19 $x + 2 > 2$

20 $x + 3 < {}^{-}1$

21 $x + 3 < 2$

22 $x - 2 > 1$

23 $x - 1> {}^{-}1$

24 $x - 3< {}^{-}3$

25 $x - 4 < {}^{-}2$

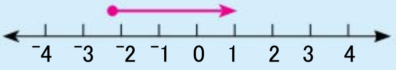
Exercise 2.4

Graph the solutions to the following inequations on the number line:

$4x \geq {}^{-}9$

$x \geq {}^{-}9 \div 4$ {inverse of × is ÷}

$x \geq {}^{-}2.25$



Can you multiply or divide both sides of an inequation by the same thing?

$7 > 3$ is true

Is $7 \times 2 > 3 \times 2$ true?

$5 < 9$ is true

Is $5 \div 2 < 9 \div 2$ true?

1 $5x > 6$

2 $x/2 > {}^{-}1$

3 $2x \leq {}^{-}3$

4 $x/3 < 1$

5 $6x \geq {}^{-}24$

6 $x/4 > {}^{-}1$

7 $3x \geq 9$

8 $x/2 < {}^{-}2$

9 Twice a number is less than 5.

10 When the land was divided into two equal parts, each part was larger than 2 hectares.

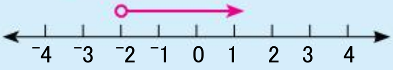
$2a - 3 > {}^{-}7$

$2a > {}^{-}7 + 3$ {inverse of − is +}

$2a > {}^{-}4$

$a > {}^{-}4 \div 2$ {inverse of × is ÷}

$\underline{a > {}^{-}2}$



Did you notice?

$>$ ○→

$\geq$ ●→

11 $2a + 1 > 6$

12 $4x + 2 > {}^{-}2$

13 $5b - 3 < 1$

14 $2x + 3 < 2$

15 $5x + 2 \leq {}^{-}7$

16 $6x - 7 > 18$

17 $4x + 3 \geq {}^{-}1$

18 $2x + 5 \leq 7$

19 How many apps does Amelia need to sell in order to nett more than \$500 if each app is sold for \$1.20 and the cost of development and hosting was \$189?

BUT

If you multiply or divide both sides of an inequation by a negative?

$7 > 3$

$7 \times {}^{-}2 \quad 3 \times {}^{-}2$

${}^{-}14 < {}^{-}6$

< becomes >

> becomes <

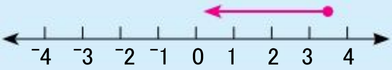
$5 - 2x \geq {}^{-}2$

${}^{-}2x \geq {}^{-}2 - 5$ {inverse of +5 is −5}

${}^{-}2x \geq {}^{-}7$

$x \leq {}^{-}7 \div {}^{-}2$ {inverse of ×¯2 is ÷¯2}

$\underline{x \leq 3.5}$



Did you also notice?

≥ became ≤

when ÷ by **¯2**

20 ${}^{-}2x > 6$

21 ${}^{-}3b \leq {}^{-}12$

22 $b \div {}^{-}3 < 1$

23 $x \div {}^{-}2 \geq 1$

24 $2 - 5x \leq {}^{-}8$

25 ${}^{-}6x + 3 > 9$

26 ${}^{-}4x + 3 \geq {}^{-}1$

27 ${}^{-}2x + 5 \leq 7$

Simultaneous Equations

Below is a pair of linear simultaneous equations:

$x + 2y = 12$

$4x - y = 3$

To solve means finding values of x and y that satisfy **both** equations.

A solution is: x=2, y=5

$2 + 2\times5 = 12$ ✓

$4\times2 - 5 = 3$ ✓

Graphing Simultaneous Equations

Exercise 2.5

Use a graphical **method** to solve the pairs of simultaneous equations:

$y = 3x + 4$

$y = 2x + 5$

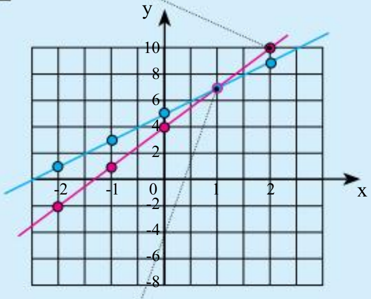
1 Complete a table of values

x=2

$y=3\times2+4 = 10$

x	$^{-}2$	$^{-}1$	0	1	2
y=3x+4	$^{-}2$	1	4	7	10

x	$^{-}2$	$^{-}1$	0	1	2
y=2x+5	1	3	5	7	9



2 Plot the points to graph each line.

3 The solution is where the lines intersect: x = 1, y = 7

1 $y = 2x + 2$
$y = 3x + 1$

x	$^{-}2$	$^{-}1$	0	1	2
y=2x+2					

x	$^{-}2$	$^{-}1$	0	1	2
y=3x+1					

2 $y = x + 3$
$y = 2x + 1$

x	$^{-}2$	$^{-}1$	0	1	2
y=x+3					

x	$^{-}2$	$^{-}1$	0	1	2
y=2x+1					

3 $y = 3x + 2$
$y = x + 3$

x	$^{-}2$	$^{-}1$	0	1	2
y=3x+2					

x	$^{-}2$	$^{-}1$	0	1	2
y=x+3					

4 $y = 2x + 3$
$y = 4x + 1$

x	$^{-}2$	$^{-}1$	0	1	2
y=2x+3					

x	$^{-}2$	$^{-}1$	0	1	2
y=4x+1					

Exercise 2.6

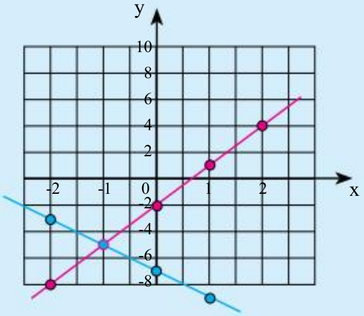
Use a graphical **method** to solve the pairs of simultaneous equations:

$y = 3x - 2$
$y = {}^{-}2x - 7$

1 Complete a table of values

x	$^{-}2$	$^{-}1$	0	1	2
y=3x− 2	$^{-}8$	$^{-}5$	$^{-}2$	1	4

x	$^{-}2$	$^{-}1$	0	1	2
y=$^{-}$2x−7	$^{-}3$	$^{-}5$	$^{-}7$	$^{-}9$	$^{-}11$



2 Plot the points to graph each line.

3 The solution is where the lines intersect: $x = {}^{-}1, y = {}^{-}5$

1 $y = 3$
$y = {}^{-}x + 1$

x	$^{-}2$	$^{-}1$	0	1	2
y=3	3	3	3	3	3

x	$^{-}2$	$^{-}1$	0	1	2
y=$^{-}$x+1					

2 $y = 4x$
$y = {}^{-}x + 5$

x	$^{-}2$	$^{-}1$	0	1	2
y=4x					

x	$^{-}2$	$^{-}1$	0	1	2
y=$^{-}$x+5					

3 $y = x - 3$
$y = {}^{-}3x + 1$

x	$^{-}2$	$^{-}1$	0	1	2
y=x−3					

x	$^{-}2$	$^{-}1$	0	1	2
y=$^{-}$3x+1					

First get y on its own:
$y + x + 1 = 0$
$y = {}^{-}x - 1$

Can you use technology to solve simultaneous equations?
You sure can - see Technology 2.1

4 $y = x - 3$
$y + x + 1 = 0$

x	$^{-}2$	$^{-}1$	0	1	2
y=x−3	3	3	3	3	3

x	$^{-}2$	$^{-}1$	0	1	2
y=$^{-}$x−1					

5 $y = x$
$x + y = 2$

x	$^{-}2$	$^{-}1$	0	1	2
y=x					

x	$^{-}2$	$^{-}1$	0	1	2
y=$^{-}$x+2					

First get y on its own:
$x + y = 2$
$y = {}^{-}x + 2$

6 $x + y = 1$
$2x + y = 3$

x	$^{-}2$	$^{-}1$	0	1	2
y=					

x	$^{-}2$	$^{-}1$	0	1	2
y=					

Substitution Method

Instead of x in (1) substitute 2y + 5

The x is on its own.

Substitute either x or y from one equation into the other equation

$x + y = 53$ .....**(1)**

$x = 2y + 5$ .....**(2)**

Exercise 2.7

Use the **substitution method** to solve the pair of simultaneous equations:

$x + y = 53$ ... **(1)**

$x = 2y + 5$ ... **(2)**

Substitute for x, from (2) in (1)

$2y + 5 + y = 53$ {x=2y+5}

$3y + 5 = 53$ {2y+y=3y}

$3y = 53 - 5$ {inverse of +is−}

$3y = 48$

$y = 48 \div 3$ {inverse of ×is÷}

$\underline{y = 16}$

From (2) $x = 2 \times 16 + 5$

$\underline{x = 37}$

Solution: $\underline{x=37, y=16}$

Check: substitute for x and y in (1)

$x+y=53$

$37+16=53$ ✓

$y = 2x - 1.9$ ... **(1)**

$^{-}4x + 3y = 7.5$ ... **(2)**

Substitute for y, from (1) in (2)

$^{-}4x+3(2x-1.9)=7.5$ {y=2x−1.9}

$^{-}4x + 6x - 5.7 = 7.5$ {distribute 3}

$2x - 5.7 = 7.5$

$2x = 7.5+5.7$ {inverse}

$2x = 13.2$

$x = 13.2 \div 2$ {inverse}

$\underline{x = 6.6}$

From (1) $y = 2 \times 6.6 - 1.9$

$\underline{y = 11.3}$

Solution: $\underline{x=11.3, y=6.6}$

Check: substitute for x and y in (2)

$^{-}4x + 3y = 7.5$

$^{-}4 \times 6.6 + 3 \times 11.3 = 7.5$ ✓

1 $x + y = 5$
$x = y - 1$

2 $x + y = 36$
$x = y + 10$

3 $x + y = 43$
$x = y - 11$

4 $x + y = 93$
$y = x + 55$

5 $x + 3y = 768$
$x = 2y - 12$

6 $x + 2y = 902$
$x = y + 569$

7 $2x + 5y = 43.2$
$x = 2y - 21.6$

8 $2x + 4y = 3.9$
$x = y + 5.7$

9 The sum of two numbers is ninety-eight ($x + y = 98$), and one number is twenty more than the other number ($x = y + 20$).

10 The sum of two numbers is seventy-four, and one number is sixty more than the other number.

11 The sum of two numbers is three hundred and sixty-one ($x + y = 361$), and one number is forty-three less than the other number ($x = y - 43$).

12 The sum of two numbers is five hundred and three, and one number is two hundred and fifteen less than the other number.

13 Entry to an entertainment park costs \$144.55 for one adult and three children. The price of an adult ticket is \$31.95 more than a child's ticket. Find the price of an adult ticket and a child's ticket.

14 The perimeter of a block of land is 2145 m. The length is 29 m longer than the breadth. Find the length and the breadth.

Elimination Method

Adding these two equations eliminates y

Because $3y + {}^{-}3y = 0$

Eliminate either x or y by adding or subtracting the equations.

$3x + 3y = 51$ .....**(1)**

$x - 3y = 5$ .....**(2)**

Exercise 2.8

Use the **elimination method** to solve the pair of simultaneous equations:

$3x + 3y = 51$ ... **(1)**

$x - 3y = 5$ ... **(2)**

Eliminate y by adding (1) to (2)

$4x = 56$ {3x+x=4x, 51+5=56}

$x = 56 \div 4$ {inverse}

$\underline{x = 14}$

Check: substitute for x and y in (2)

$x - 3y = 5$

$14 - 3x3 = 5$ ✓

From (1)

$3x14 + 3y = 51$ {x=14}

$42 + 3y = 51$

$3y = 51 - 42$ {inverse}

$3y = 9$

$\underline{y = 3}$ {inverse}

Solution: $\underline{x=14, y=3}$

1 $x + y = 15$
$x - y = 1$

2 $x + y = 63$
$x - y = 29$

3 $x + y = 121$
$x - y = 97$

4 $3x + y = 68$
$2x - y = 7$

5 $7x + y = 128$
$4x - y = 15$

6 $3x + y = 12.1$
$4x - y = 9.6$

7 $2x + 3y = 24$
$x - 3y = 3$

8 $5x + 2y = 41$
$2x - 2y = 8$

9 The sum of two numbers is twenty-three ($x + y = 23$). The difference between the two numbers is fifteen ($x - y = 15$). What are the numbers?

$5x - y = 31$ ... **(1)**

$2x - 3y = 2$ ... **(2)**

Prepare elimination of y by $(1)x^{-}3$

${}^{-}15x + 3y = {}^{-}93$ ... **(3)**

$2x - 3y = 2$ ... **(2)**

Eliminate y by adding (3) to (2)

${}^{-}13x = {}^{-}91$

$x = {}^{-}91 \div 13$ {inverse}

$\underline{x = 7}$

Substitute for x in (1)

$5 \times 7 - y = 31$ {x=7}

${}^{-}y = 31 - 35$ {inverse}

${}^{-}y = {}^{-}4$

$y = {}^{-}4 \div {}^{-}1$ {inverse}

$y = 4$

Solution: $\underline{x=7, y=4}$

Check: substitute for x and y in (2)

$2x - 3y = 2$

$2 \times 7 - 3 \times 4 = 2$ ✓

10 $2x + y = 7$
$x + 2y = 11$

11 $5x + 2y = 10$
$4x + 3y = 15$

12 $7x - y = 51$
$2x - 3y = {}^{-}18$

13 A pen has thirty-two heads (x+y=32) and one hundred and two legs (2x+4y=102). Assuming normal animals, how many hens and how many sheep?

14 A pen has sixty-five heads and one hundred and sixty-four legs. Assuming normal animals, how many emus and how many wombats?

Mental Computation

Mental computation gives you practice in thinking.

Exercise 2.9

1. Spell simultaneous.
2. Solve: $2x + 5 = 8$
3. Solve: $5x - 3 < 7$
4. Solve: $x + y = 8,\ x - y = 2$
5. Solve: $x + y = 10,\ 2x + y = 15$
6. $3 - {}^{-}5$
7. $x^3 \div x^2$
8. $(2^{-3})^2$
9. Simplify: $\frac{x}{2} + \frac{x}{3}$
10. Increase \$6 by 30%

Guess and check can be a quick way of solving simple simultaneous equations.

$x + y = 8, \qquad x - y = 2$
$7 + 1 = 8$
$6 + 2 = 8$ {This difference is 2}
$5 + 3 = 8$
$4 + 4 = 8$ Thus x=6, y=2

$$\begin{aligned} & 6 + 6 \times 30/100 \\ &= 6 + 180/100 \\ &= 6 + 1.8 \\ &= \$7.80 \end{aligned}$$

Exercise 2.10

1. Spell linear.
2. Solve: $4x + 2 = 10$
3. Solve: $2x - 5 > 3$
4. Solve: $x + y = 7,\ x - y = 3$
5. Solve: $x + y = 10,\ 2x + y = 14$
6. $3 - {}^{-}2$
7. $x^2 \div x^5$
8. $(3^{-3})^3$
9. Simplify: $\frac{x}{2} - \frac{x}{3}$
10. Increase \$7 by 30%

Two wrongs don't make a right, but three lefts do.

So does seven lefts.

'Half this game is ninety percent mental' - Philadelphia baseball manager, Danny Ozark.

Exercise 2.11

1. Spell equation.
2. Solve: $2x + 5 = 8$
3. Solve: $5x - 3 < 7$
4. Solve: $x + y = 7,\ x - y = 5$
5. Solve: $x + y = 10,\ 2x + y = 18$
6. $3 - 7$
7. $x^5 \div x^3$
8. $(2^{-2})^4$
9. Simplify: $\frac{x}{2} - \frac{x}{4}$
10. Increase \$8 by 30%

If you're short of everything but the enemy, you are in the combat zone. - Murphy's laws of combat.

Sports Scientists improve sporting performances using knowledge from medicine, biomechanics, nutrition, and psychology.

- Relevant school subjects are English and Mathematics.
- Courses usually involve a sports/exercise/movement related degree.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 2.12

If $a = 4 - 3b$ and $b = 2c - 1$, express a in terms of c.

$a = 4 - 3b$
$a = 4 - 3(2c - 1)$ {$b = 2c - 1$}
$a = 4 - 6c + 3$ {distribute $^-3$}
$\underline{a = 7 - 6c}$

$P = 2(b+w)$
$A = bw$
$2(b+w) = 10$
$b+w = 5$
$bw = 6$

Find two numbers whose sum is 5 and whose product is 6

1 If $a = 4 - 2b$ and $b = 2c - 3$, express a in terms of c.

2 If $a = 1 - 3b$ and $b = 2c - 1$, express a in terms of c.

3 Find the dimensions of a rectangle whose perimeter is 10 cm and area is 6 cm^2.

4 Find the dimensions of a rectangle whose perimeter is 14 cm and area is 10 cm^2.

Find two numbers whose sum is 7 and whose product is 10

5 Find the dimensions of a rectangle whose perimeter is 26 cm and area is 42 cm^2.

6 Solve the following sets of simultaneous equations:

$a + b = 18$ ...**(1)**
$b + c = 9$ ...**(2)**
$a + c = 13$ ...**(3)**

(1) − **(2)** gives: $a - c = 9$ ...**(4)**
$a + c = 13$ ...**(3)**

(4) + **(3)** gives: $2a = 22$
$\underline{a = 11}$

Sub a in **(1)** $11 + b = 18$
$\underline{b = 7}$

Sub a in **(3)** $11 + c = 13$
$\underline{c = 2}$

Solution: $\underline{a=11, b=7, c=2}$

a)
$a + b = 24$
$b + c = 17$
$a + c = 23$

b)
$a + b = 10$
$b + c = 1$
$a + c = 5$

c)
$a + b + c = 21$
$a + b + d = 15$
$c + d = 12$
$a + c = 13$

7 The fraction, $\frac{1}{2}$, is tripled by subtracting the same number from the numerator and the denominator. What is the number?

8 If a, b, and c are different integers from one to nine inclusive, what is the smallest possible value of:

$$\frac{abc}{a+b+c}$$

9 Find all positive real solutions of the simultaneous equations:

$x + y^2 + z^3 = 3$
$y + z^2 + x^3 = 3$
$z + x^2 + y^3 = 3$

A Couple of Puzzles

Exercise 2.13

1 Fill each empty square with either 3 or 5 so that the total of each row is 20.

6				
	7		2	
		8		
				2

2 9 15 23 33 ?

A Game

Odds and Evens is a simple game that is often used as an introduction to Game Theory. Game Theory originated in 1944 through the work of the economist Oskar Morgenstern. Game Theory is now applied to business, politics, economics, and even warfare.

1 Each of two players chooses either odds or evens.

2 On cue, each player holds up one of their hands with one, two, three, four, or five digits showing.

3 If the **total** number of digits is even, then evens wins. If the **total** number of digits is odd, then odds wins.

The first step in Game Theory is to make a matrix of outcomes.

	1	**2**	**3**	**4**	**5**
1	1	$^{-}1$	1	$^{-}1$	1
2	$^{-}1$	1	$^{-}1$	1	$^{-}1$
3	1	$^{-}1$	1	$^{-}1$	1
4	$^{-}1$	1	$^{-}1$	1	$^{-}1$
5	1	$^{-}1$	1	$^{-}1$	1

1 means that evens wins.
$^{-}1$ means that odds wins.
The game is fair.

How does this matrix prove that the game is fair?

A Sweet Trick

1 Ask your audience to add the following numbers in their head (no calculators allowed).

2 Ask for their answer.

1000
40
1000
30
1000
20
1000
10

Most of your audience will think that the answer is 5000.

The answer is 4100

Your audience may need some convincing.

Investigations

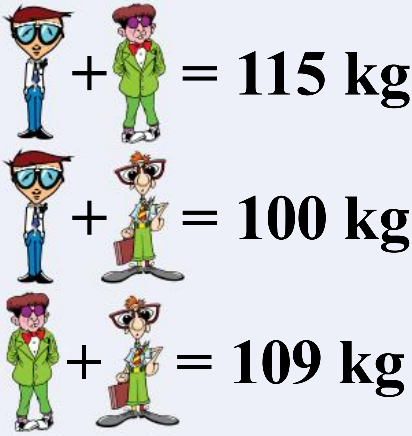
Investigation 2.1 Simultaneous Equations

1 Form a group of three.

2 Two at a time, stand on a set of scales and record the total weight. (Eg., a+b=115, a+c=100, b+c=109).

3 Can you solve the equations to find out how much each person weighs?

How accurate are your answers? Weigh each person.

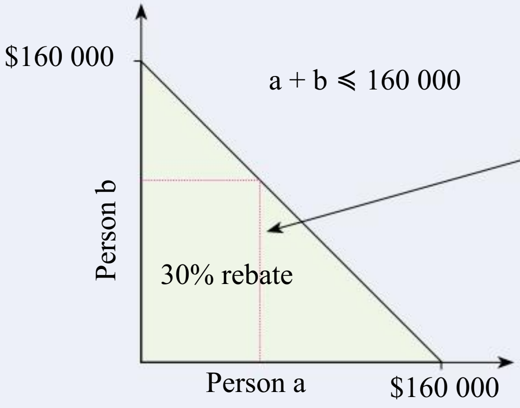


Investigation 2.1 Linear Inequations

Private health insurance rebate

	Singles Annual Income	Couples Annual Income	Rebate
1	$80,000 or less	$160,000 or less	30%
2	$80,001 to $93,000	$160,001 to $186,000	20%
3	$93,001 to $124,000	$160,001 to $248,000	10%
4	More than $124,001	More than $248,001	0%

For example, a couple earning $160 000 or less per year will have the cost of their private health insurance reduced by 30%.



Person a can earn $60 000 and person b can earn $100 000 per year and keep the 30% rebate.

Indicators of inequations

more	less
smaller	larger
faster	slower
lighter	heavier

Investigate

Examples of inequations in the newspaper

Technology

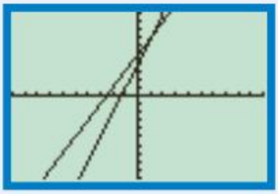
Technology 2.1 Simultaneous equations

Use a Graphics Calculator to solve the simultaneous equations in this chapter.

$y = 3x + 4$
$y = 2x + 5$

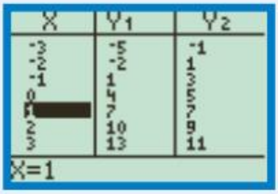
Press Y = and enter the two equations(3x + 4, 2x + 5).

Press Graph to see a graph.



Press Table to see a table of the values.

The intersection is where the y values are the same.



X	Y1	Y2
-3	-5	-1
-2	-2	1
-1	1	3
0	4	5
1	7	7
2	10	9
3	13	11

X=1

Follow your graphics calculator's procedure for finding the intersection points.

Technology 2.2 Internet Solvers

There are a very large number of 'graph, function, plotter" applets/applications on the Internet.

Simultaneous Equations

Can you find a good 'simultaneous equations', or 'system of equations' solver for linear simultaneous equations?

What great thing would you attempt if you knew you could not fail? - Robert H. Schuller.

In seeking wisdom thou art wise; in imagining that thou hast attained it thou art a fool - Lord Chesterfield.

Technology 2.3 Videos

Simultaneous Equations

Watch videos on methods of solving simultaneous linear equations.

Wisdom is the quality that keeps you from getting into situations where you need it - Doug Larson.

Chapter Review 1

Question 2.14

1 The mobile phone call rate, in \$C, is given by the function: $C = 0.42 + 0.88t$, where t is the time of the call in minutes. Find the length of a call for which the cost was \$4.16.

2 The sum of the interior angles of a polygon is given by the formula:
$S = 90(2n - 4)$, where n is the number of sides on the polygon.
How many sides in a polygon with an interior angle sum of 1440°?

3 Graph the solutions to the following inequations on the number line:

a) $x + 3 > 4$ **b)** $x/3 > {}^{-}1$ **c)** $3x + 2 \leq {}^{-}7$

4 Use a graphical **method** to solve the pairs of simultaneous equations:

a) $y = 2x + 3$
$y = 3x + 1$

x	$^{-}2$	$^{-}1$	0	1	2
y=2x+3					

x	$^{-}2$	$^{-}1$	0	1	2
y=3x+1					

b) $y = x - 2$
$y = {}^{-}2x + 1$

x	$^{-}2$	$^{-}1$	0	1	2
y=x−2					

x	$^{-}2$	$^{-}1$	0	1	2
y=$^{-}$2x+1					

$x + y = 53$ ... **(1)**
$x = 2y + 5$ ... **(2)**

Substitute for x, from (2) in (1)

$2y + 5 + y = 53$ {x=2y+5}
$3y + 5 = 53$ {2y+y=3y}
$3y = 53 - 5$ {inverse of +is−}
$3y = 48$
$y = 48 \div 3$ {inverse of ×is÷}
$\underline{y = 16}$

From (2) $x = 2 \times 16 + 5$
$\underline{x = 37}$

Solution: $\underline{x=37, y=16}$

Check: substitute for x and y in (1)
x+y=53
37+16=53 ✓

5 Use the **substitution method** to solve the pair of simultaneous equations:

a) $x + y = 5$, $x = y - 1$ **b)** $x + y = 3$, $x = y + 10$ **c)** $x + y = 93$, $y = x + 55$

6 The sum of two numbers is ninety-six, and one number is eighty more than the other number. Use the **substitution method** to find a solution.

$3x + 3y = 51$ ... **(1)**
$x - 3y = 5$ ... **(2)**

Eliminate y by adding (1) to (2)

$4x = 56$ {3x+x=4x, 51+5=56}
$x = 56 \div 4$ {inverse}
$\underline{x = 14}$

From (1)

$3x14 + 3y = 51$ {x=14}
$42 + 3y = 51$
$3y = 51 - 42$ {inverse}
$3y = 9$
$\underline{y = 3}$ {inverse}

Solution: $\underline{x=14, y=3}$

7 Use the **elimination method** to solve the pair of simultaneous equations:

a) $x + y = 15$, $x - y = 1$ **b)** $x + y = 63$, $x - y = 29$ **c)** $2x + y = 7$, $x + 2y = 11$

8 The sum of two numbers is forty. The difference between the two numbers is six. Use the elimination method to find the numbers?

Chapter Review 2

Question 2.15

1 The labour cost, \$C, for a plumber's visit is given by $C = 85.50t + 45$, where t is the time in hours spent by the plumber on the job. If the cost for the plumber's visit was \$258.75, for how long was the plumber on the job?

2 The perimeter, P, of a rectangle is given by $P = 2(l + b)$, where l is the length and b is the breadth. If the length of a house block is 43 m and the perimeter is 138 m, what is the breadth of the house block?

3 Graph the solutions to the following inequations on the number line:

a) $x + 3 < {}^{-}1$ **b)** $6x \geq {}^{-}24$ **c)** $6x - 7 > 18$

4 Use a graphical **method** to solve the pairs of simultaneous equations:

a) $y = 3x + 2$
$y = x + 3$

x	$^{-}2$	$^{-}1$	0	1	2
y=3x+2					

x	$^{-}2$	$^{-}1$	0	1	2
y=x+3					

b) $y = x - 3$
$y = {}^{-}3x + 1$

x	$^{-}2$	$^{-}1$	0	1	2
y=x−3					

x	$^{-}2$	$^{-}1$	0	1	2
y=$^{-}$3x+1					

$x + y = 53$ ... **(1)**
$x = 2y + 5$ ... **(2)**

Substitute for x, from (2) in (1)

$2y + 5 + y = 53$ {x=2y+5}
$3y + 5 = 53$ {2y+y=3y}
$3y = 53 - 5$ {inverse of + is −}
$3y = 48$
$y = 48 \div 3$ {inverse of × is ÷}
$\underline{y = 16}$

From (2) $x = 2 \times 16 + 5$
$x = 37$

Solution: $\underline{x=37, y=16}$

Check: substitute for x and y in (1)
$x+y=53$
$37+16=53$ ✓

5 Use the **substitution method** to solve the pair of simultaneous equations:

a) $x + y = 7$
$x = y - 3$

b) $x + y = 1$
$x = y - 5$

c) $x + y = 37$
$y = x - 1$

6 The sum of two numbers is one hundred and forty, and one number is twenty more than the other number. Use the **substitution method** to find a solution.

$3x + 3y = 51$ ... **(1)**
$x - 3y = 5$ ... **(2)**

Eliminate y by adding (1) to (2)

$4x = 56$ {3x+x=4x, 51+5=56}
$x = 56 \div 4$ {inverse}
$\underline{x = 14}$

From (1)

$3x14 + 3y = 51$ {x=14}
$42 + 3y = 51$
$3y = 51 - 42$ {inverse}
$3y = 9$
$\underline{y = 3}$ {inverse}

Solution: $\underline{x=14, y=3}$

7 Use the **elimination method** to solve the pair of simultaneous equations:

a) $x + y = 15$
$x - y = 11$

b) $x + y = 37$
$x - y = 11$

c) $2x + y = 1$
$x + 3y = 13$

8 The sum of two numbers is eighty-three. The difference between the two numbers is eleven. Use the elimination method to find the numbers?

Area Volume

3

Measurement and Geometry ➞ Using units of measurement

★ Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

- Investigate and determine the volumes and surface areas of composite solids by considering the individual solids from which they are constructed.

★ 10A Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

- use formulas to solve problems
- use authentic situations to apply knowledge and understanding of surface area and volume.

Q Volume of pizza
radius z, depth a?
A Pizza = $\text{pi}\times z^2\times a$

The Great Pyramid of Giza in Egypt originally had a square base of 230.4 m and a height of 146.5 m when it was faced with casing stones. The casing stones gave the pyramid a flat, white, polished limestone surface.

Can you calculate the total area of polished limestone that was originally on the pyramid?

A LITTLE BIT OF HISTORY

The Great Pyramid of Giza is thought to have been built for the Pharaoh Khufu, also known as Cheops, around 2560 BC after 20 years construction.

Taylor, 1859, proposed that it is not coincidence that the perimeter of the Great Pyramid divided by its height gives 2π.

Petrie, 1883, published detailed measurements of the Great Pyramid:

Some casing stones weigh 15 tons each.

2.5 million blocks varying in size from 2 to 70 tons each

Northwest corner: 89° 59’58”
Northeast corner: 90° 3’02”
Southeast corner: 89° 56’02”
Southwest corner: 90° 3’02”

Area

Area formulas

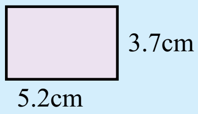
Area rectangle = length×breadth
Area triangle = ½base×height
Area circle = π×radius^2

A hectare is the area of a square 100 m by 100 m.

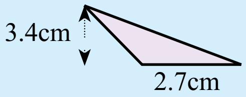
The diameter is twice the radius.

Exercise 3.1

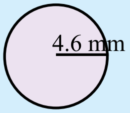
Calculate the area of each of the following shapes:



Area = l×b
= 5.2cm × 3.7cm
= 19.24 cm^2

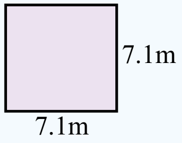


Area = ½bh
= 0.5×2.7cm×3.4cm
= 4.59 cm^2



Area = πr^2
= π×4.6mm×4.6mm
= 66.48 mm^2

1



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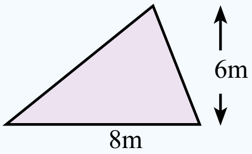

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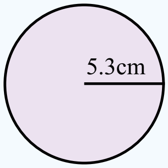
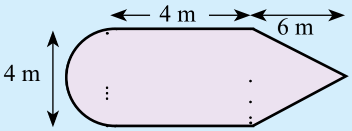


4 A rectangular paddock is 120 m by 140 m. What is the area of the paddock in square metres and hectares (1 hectare = 10 000m^2)?

5 A paddock, in the shape of a triangle, has a base of 230 m and a perpendicular height of 370 m. What is the area of the paddock in square metres and hectares?

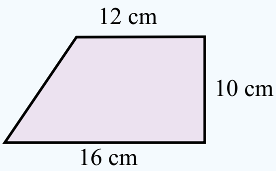
6 The sprinkler sprays water in a circle with a radius of 4.5 m. Calculate the area covered by the sprinkler.



Area = Area of circle + Area of rectangle + Area of triangle
= πr^2 + l×b + ½bh
= π×2^2 + 4×4 + ½×6×4
= 12.57 + 16 + 12
Area = 40.57 m^2

Composite shapes can be rectangles, triangles, and circles composed together.

7



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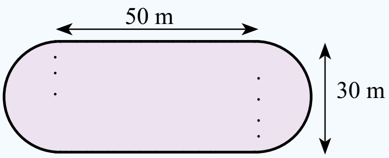

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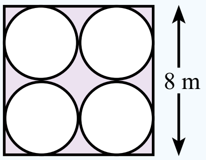
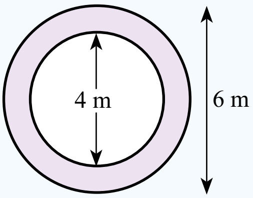


Find the shaded area.

10



Volume

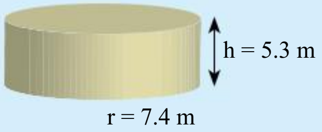
Prisms are three-dimensional shapes that have a constant cross-section.

V_{prism} = Area of base × height

The surface area of a solid is the total area of each face of the solid.

Exercise 3.2

Find the volume and the surface area of each of the following prisms:



V = Area of base × height

$= \pi r^2 \times h$

$= \pi\times7.4^2\times5.3$

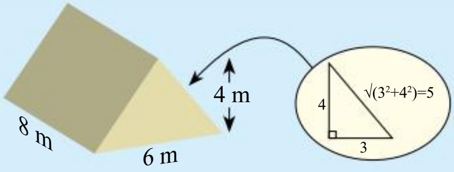
$V = 911.78 \text{ m}^3$

SA = area 2 circles + area rectangle

$= 2\times\pi r^2 + 2\pi rh$

$= 2\times\pi\times7.4^2 + 2\pi\times7.4\times5.3$

$SA = 590.49 \text{ m}^2$



V = Area of base × height

$= \frac{1}{2}b\times\perp h \times h$

$= \frac{1}{2}\times6\times4\times8$

$V = 96 \text{ m}^3$

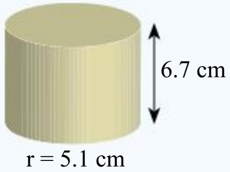
SA = area 2 triangles + area 3 rectangles

$= 2 \times \frac{1}{2}bh + 8\times6 + 8\times5 + 8\times5$

$= 2\times\frac{1}{2}\times6\times4 + 128$

$SA = 152 \text{ m}^2$

1



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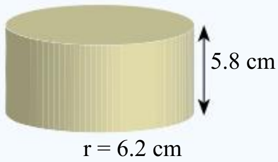

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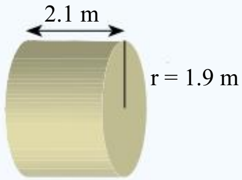

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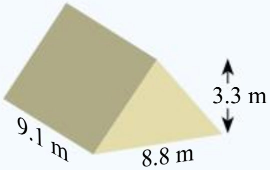

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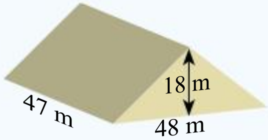

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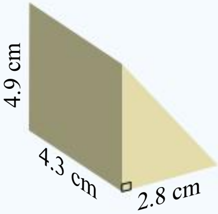

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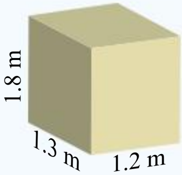

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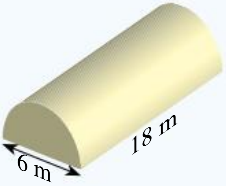

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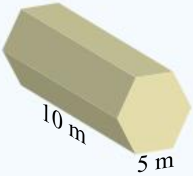
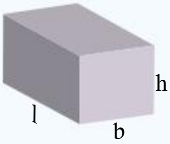


Volume of Composite Solids

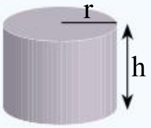
V_{box} = Area of base × height

$= l \times b \times h$



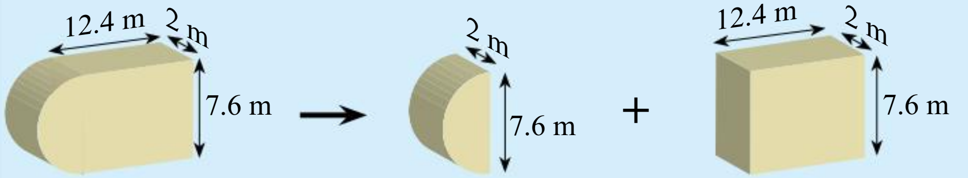
$V_{cylinder}$ = Area of base × height

$= \pi r^2 \times h$



Exercise 3.3

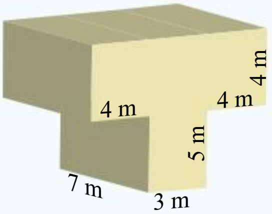
Calculate the volume of each of the following solids:



$$\begin{aligned}\text{Volume} &= \text{Volume half cylinder} + \text{Volume rectangular based prism}\\ &= \tfrac{1}{2}\times\pi\times r^2\times h + l\times b\times h\\ &= \tfrac{1}{2}\times\pi\times 3.8^2\times 2 + 12.4\times 7.6\times 2\\ &= 45.36 + 188.48\end{aligned}$$

$$\underline{\text{Volume} = 233.84\ \text{m}^3}$$

1



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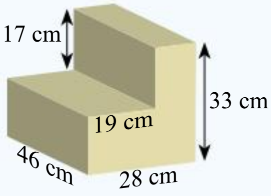

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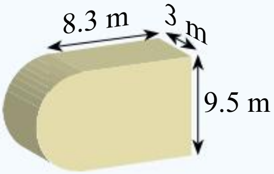

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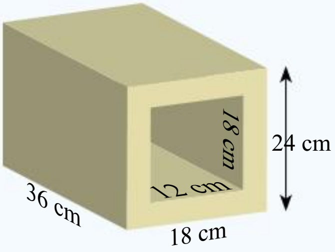

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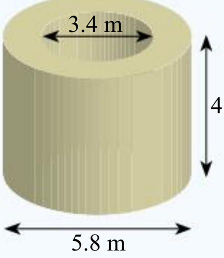
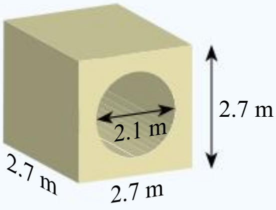


Volume, n. 1. amount of space contained by a three-dimensional object.
SI unit is cubic metre (m^3).

6



Surface Area of Composite Solids

The surface area of a solid is the total area of each face of the solid.

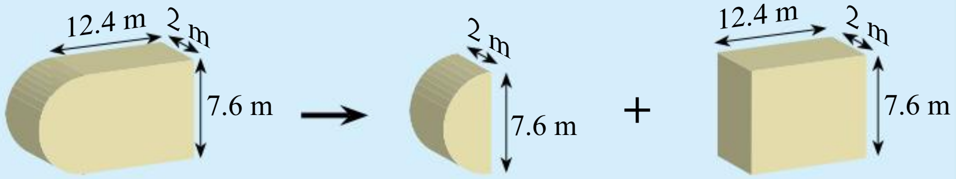
Area formulas

Area rectangle = length×breadth
Area triangle = ½base×height
Area circle = π×radius^2

The surface area of a composite solid is the sum of all areas of all exposed shapes.
SI unit is square metre (m^2).

Exercise 3.4

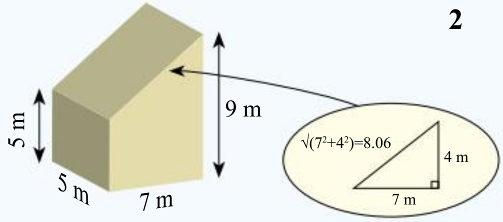
Calculate the surface area of each of the following solids:



Surface area = 2 half circles + half curved + 5 sides of rectangular prism
= $2\times\frac{1}{2}\times\pi\times r^2$ + $\frac{1}{2}\times 2\times\pi\times r\times h$ + $2\times l\times b$ + $2\times l\times d$ + $b\times d$
= $2\times\frac{1}{2}\times\pi\times 3.8^2$ + $\frac{1}{2}\times 2\times\pi\times 3.8\times 2$ + $2\times 12.4\times 7.6$ + $2\times 12.4\times 2$ + 7.6×2
= 45.36 + 23.88 + 188.48 + 49.6 + 15.2

Volume = 322.52 m^2

1



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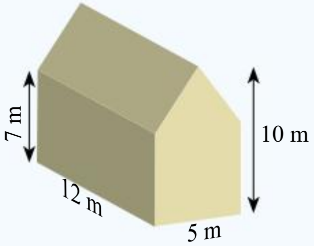

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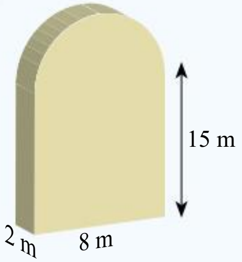

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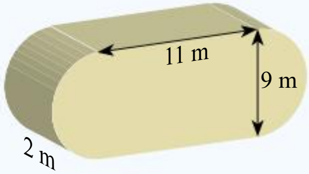

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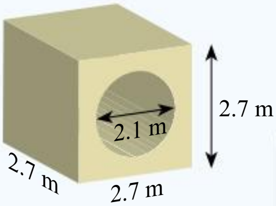
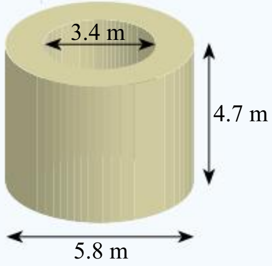


The surface area of Australia is 7 741 000 km^2.

6



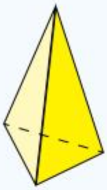
10A Right Pyramids

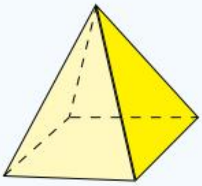
A right pyramid is a pyramid for which the apex lies directly above the centre of the base.

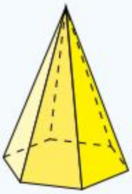
Volume Right Pyramid

$V_{pyramid} = ⅓ \times \text{Area base} \times \text{height}$

The name of a pyramid depends on the shape of the base.


Triangular pyramid

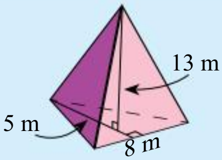

Rectangular pyramid


Hexagonal pyramid

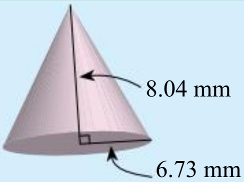

Circular pyramid

Exercise 3.5

Calculate the volume of each of the following pyramids:

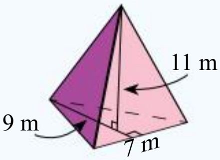


$V_{pyramid}$ = ⅓ × Areabase × height
= ⅓ × ½base×height × height
= ⅓ × ½×8×5× 13 m^3
= 86.67 m^3



$V_{pyramid}$ = ⅓ × Areabase × height
= ⅓ × πr^2 × height
= ⅓ × $\pi \times 6.73^2$ × 8.04 mm^3
= 381.34 m^3

1



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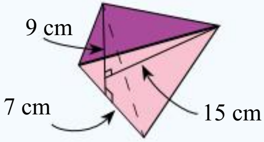

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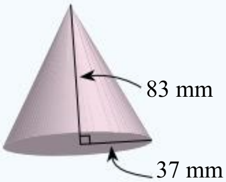

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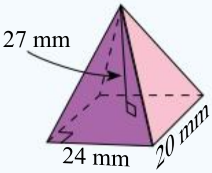

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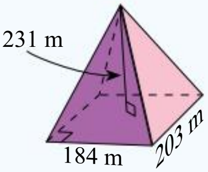

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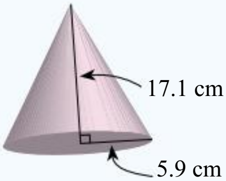


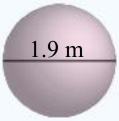
Volume Sphere

$V_{sphere} = \frac{4}{3}\pi r^3$

V_{sphere} $= \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi\times(6.3\div2)^3$
= 130.92 cm^3

7

8



9

10

10A Composite Solids

Volume Right Pyramid

$V_{pyramid} = \frac{1}{3} \times \text{Area base} \times \text{height}$

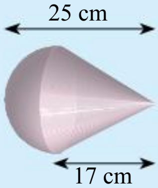
Volume Sphere

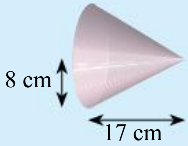
$V_{sphere} = \frac{4}{3}\pi r^3$

The volume of a composite solid is the sum of the volumes of individual solids. SI unit is cubic metre (m^3).

Exercise 3.6

Calculate the volume of each of the following composite solids:





$$V = V_{hemisphere} + V_{circular\ pyramid}$$

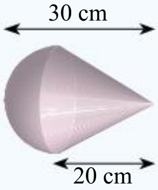
$$V = \frac{1}{2} \times \frac{4}{3}\pi r^3 + \frac{1}{3} \times \pi r^2 \times \text{height}$$

$$V = \frac{1}{2} \times \frac{4}{3}\pi \times 8^3\ cm^3 + \frac{1}{3} \times \pi \times 8^2 \times 17\ cm^3$$

$$V = 1072.33\ cm^3 + 1139.35\ cm^3$$

$$V = 2211.68\ cm^3$$

1



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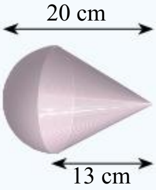

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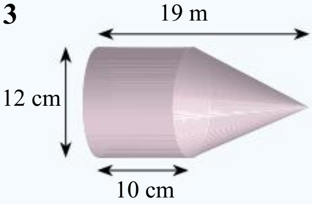

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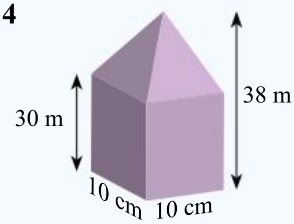

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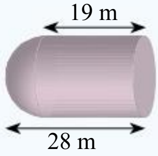

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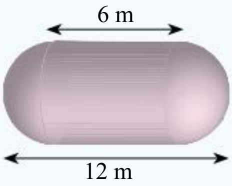

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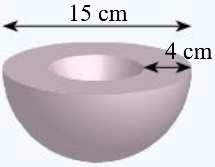

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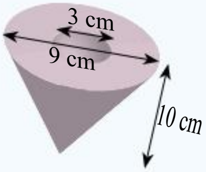


Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 3.7

1 Spell Right Pyramid.
2 What is the formula for the area of a circle?
3 What is the formula for the volume of a cylinder?
4 What is the formula for the volume of a pyramid?
5 Solve: $2x - 3 < 7$
6 Solve: $x + y = 9,\ x - y = 1$
7 $^{-}2 - {}^{-}5$
8 $(x^{-2})^3$
9 Simplify: $\frac{x}{2} + \frac{x}{3}$
10 What is the cube of 3?

The sum of two numbers is 9
Their difference is 1.
What are the two numbers?

3^3
$= 3 \times 3 \times 3$
$= 27$

Exercise 3.8

1 Spell sphere.
2 What is the formula for the area of a rectangle?
3 What is the formula for the volume of a rectangular prism?
4 What is the formula for the volume of a circular based pyramid?
5 Solve: $3x - 2 > 4$
6 Solve: $x + y = 10,\ x - y = 2$
7 $^{-}7 - 3$
8 $x^{-3} \div x^2$
9 Simplify: $\frac{x}{2} - \frac{x}{3}$
10 What is the cube of 4?

'The beginning is the most important part of the work' - Plato.

Exercise 3.9

1 Spell composite.
2 What is the formula for the area of a triangle?
3 What is the formula for the volume of a triangular prism?
4 What is the formula for the volume of a triangular based pyramid?
5 Solve: $4x + 1 > 9$
6 Solve: $x + y = 15,\ x - y = 3$
7 $4 - 5$
8 $x^5 \times x^{-2}$
9 Simplify: $\frac{x}{2} \div \frac{x}{3}$
10 What is the cube of 5?

'Putting off a hard thing makes it impossible' - Charles E Wilson.

Competition Questions

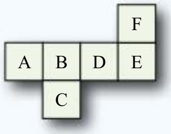
Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 3.10

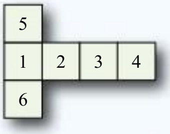
1 60 centicubes are glued together to form a rectangular based prism. If the area of the base is 20 cm^2, what is the height of the prism?

2 60 centicubes are glued together to form a square based prism. If the perimeter of the base is 8 cm, what is the height of the prism?

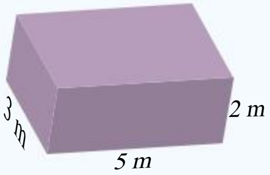
3 If the net is folded to form a cube, which letter is opposite C?



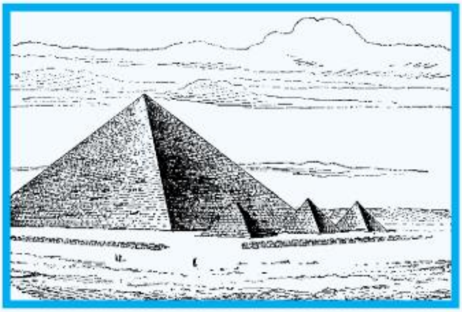
4 The net is folded to form a cube, If the three numbers at each corner are added, what is the largest sum?



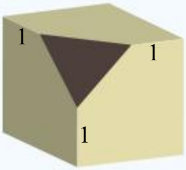
5 What would be the dimensions of the rectangular prism if its surface area was doubled.



6 The Great Pyramid of Giza originally had a square base of sides 230.4 m and a height of 146.5 m. If the Pyramid was built with an estimated 2.5 million blocks, what is an estimate of the size of each block (assuming the Pyramid was made with identical cubes).

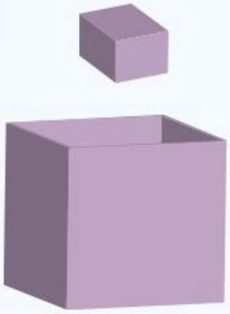


7 A 2 m cube has a corner cut off. What is then the surface area of the cube?



A cubic metre of water weights 1 tonne (1000L).

8 A 4 metre square-based tank has water to a depth of 4 metres. If a cube of side 2 metres is placed in the tank, what is then the level of water in the tank?



Investigations

Investigation 3.1 Prisms, Pyramids, and Spheres

To become familiar with the various shapes make a collection of everyday prisms, pyramids, and spheres. Name them.

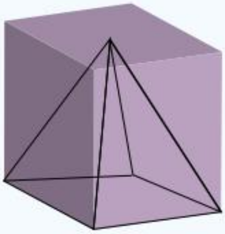
Investigation 3.2 Volume of a Square Pyramid

Build a Model

By making models, show that the volume of a square pyramid is a third the volume of the cube that it fits in.

Is the amount of sand needed to fill the pyramid one-third the amount of sand needed to fill the cube?

Volume Right Pyramid
$V_{pyramid} = ⅓ \times$ Area base $\times$ height



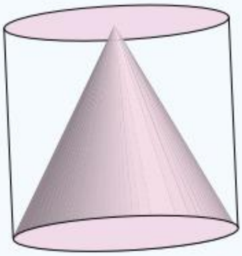
Investigation 3.3 Volume of a Cone

Build a Model

By making models, show that the volume of a cone is a third the volume of the cylinder that it fits in.

Is the amount of sand needed to fill the cone one-third the amount of sand needed to fill the cube?

Volume Right Pyramid
$V_{pyramid} = ⅓ \times$ Area base $\times$ height

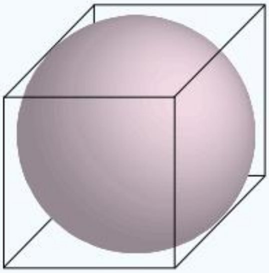


Investigation 3.4 Volume of a Sphere

Build a Model

By making models, show that the volume of a sphere is four-thirds the volume of the cube that it fits in.

Volume Sphere
$V_{sphere} = \frac{4}{3}\pi r^3$



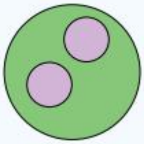
A Couple of Puzzles

Exercise 3.11

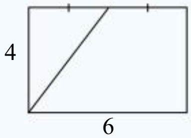
1 Tim exercises for 30 min every afternoon.
How many hours of exercise does Tim get in January?

2 The string is fixed to the two points.
What shape is made by the pencil
if the string is kept taut?

3 How many circles of radius 2 cm can
be cut from a circle of radius 6 cm?



4 The 6 x 4 rectangle is cut into two pieces
as shown and rearranged to make one
right-angled triangle.
What is the length of the hypotenuse?

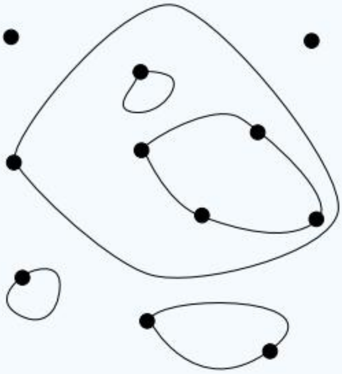


A Game

Loopy is a game for two players.

1 Place 8 to 12 dots on a piece of paper.
2 Take turns to make a loop.
3 The loser is the person who can't
make a loop.

A loop starts and finishes at the same dot.
A loop may go through a maximum of four dots.
A loop can't cross another loop.



A Sweet Trick

1 Show your audience a $50 note.

2 Tell them they can have the $50 if
they can fold it in half eight times.

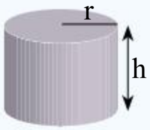
1 fold = 2 layers
2 folds = 4 layers
3 fold = 8 layers
4 folds = 16 layers
5 fold = 32 layers
6 folds = 64 layers
7 fold = 128 layers
8 folds = 256 layers

Try it. Eight folds is impossible.

What is the smallest number of folds that would be impossible?

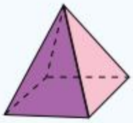
Technology

Technology 3.1 Spreadsheets



Enter the formula:
=2*pi()*c2*b2+2*pi()*c2^2

Prism	Height	Radius	Surface Area
Cylinder	4.92	1.2	46.14371



Volume Right Pyramid

$V_{pyramid} = \frac{1}{3} \times \text{Area base} \times \text{height}$

Enter the formula:
=1/3*c2^2*b2

Pyramid	Height	Base	Volume
Square pyramid	12	3.5	46.14371

Volume Right Pyramid

$V_{pyramid} = \frac{1}{3} \times \text{Area base} \times \text{height}$

Enter the formula:
=1/3*pi()*c2^2*b2

Pyramid	Height	Radius	Volume
Cone	8	1.3	46.14371

Volume Sphere

$V_{sphere} = \frac{4}{3}\pi r^3$

Enter the formula:
=4/3*pi()*b2^3

Sphere	Radius	Volume
Sphere	1.3	46.14371

Technology 3.2 Pyramid Activities

Search the Internet for some of the many prism activities.

Use search phrases such as:

a) 'Interactive pyramids'
b) 'Pyramid models'
c) 'Egyptian pyramids'
d) 'Pyramid Applets'

Q What do you get if you cross an Egyptian mummy and a car mechanic?

A Toot and Car Man.

Q: Why did the mummy call the doctor?

A: Because she was coffin.

Archaeologists analyse and interpret the history of human cultures.

- Relevant school subjects are English, Mathematics, History, Science.
- Courses generally involves a University degree and a postgraduate qualification in archaeology.

Chapter Review 1

Prisms are three-dimensional shapes that have a constant cross-section.

V_{prism} = Area of base × height

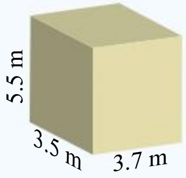
Area formulas

Area rectangle = length×breadth
Area triangle = ½base×height
Area circle = π×radius²

Exercise 3.12

Find the volume and the surface area of each of the following prisms:

1



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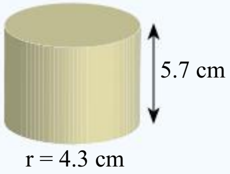

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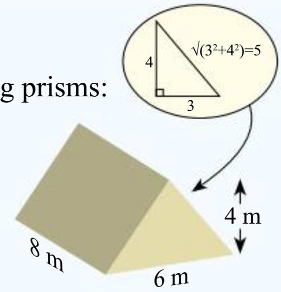

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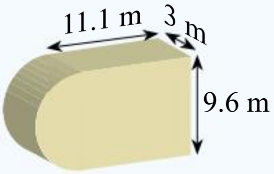

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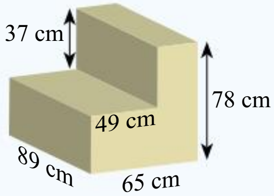

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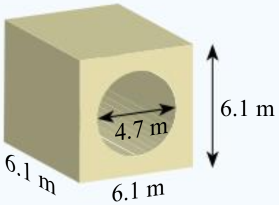
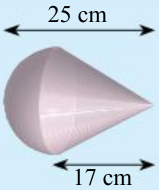


10A Calculate the volume of each of the following:



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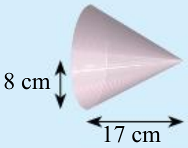


$V = V_{hemisphere} + V_{circular\ pyramid}$

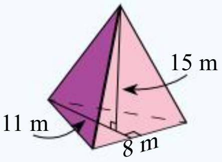
$V = \frac{1}{2} \times \frac{4}{3}\pi r^3 + \frac{1}{3} \times \pi r^2 \times \text{height}$

$V = \frac{1}{2} \times \frac{4}{3}\pi \times 8^3 \text{ cm}^3 + \frac{1}{3} \times \pi \times 8^2 \times 17 \text{ cm}^3$

$V = 1072.33 \text{ cm}^3 + 1139.35 \text{ cm}^3$

$V = \underline{2211.68 \text{ cm}^3}$

7



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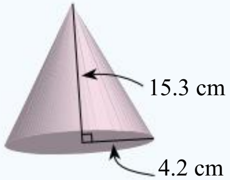

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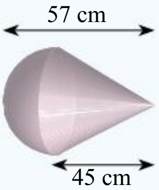

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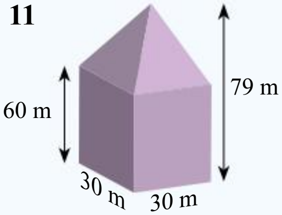

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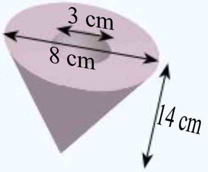


Chapter Review 2

Prisms are three-dimensional shapes that have a constant cross-section.

V_{prism} = Area of base × height

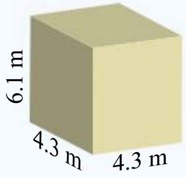
Area formulas

Area rectangle = length×breadth
Area triangle = ½base×height
Area circle = π×radius^2

Exercise 3.13

Find the volume and the surface area of each of the following prisms:

1



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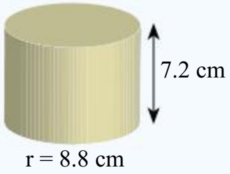

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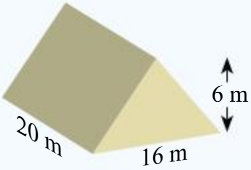

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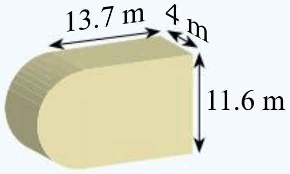

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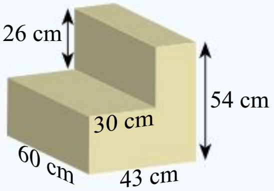

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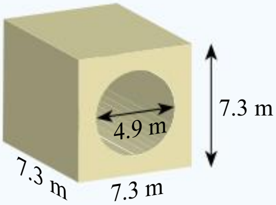
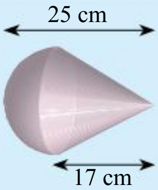


10A Calculate the volume of each of the following



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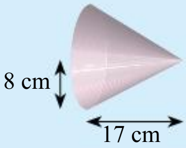


$V = V_{hemisphere} + V_{circular\ pyramid}$

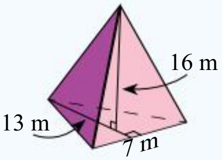
$V = \frac{1}{2} \times \frac{4}{3}\pi r^3 + \frac{1}{3} \times \pi r^2 \times \text{height}$

$V = \frac{1}{2} \times \frac{4}{3}\pi \times 8^3\ \text{cm}^3 + \frac{1}{3} \times \pi \times 8^2 \times 17\ \text{cm}^3$

$V = 1072.33\ \text{cm}^3 + 1139.35\ \text{cm}^3$

$\underline{V = 2211.68\ \text{cm}^3}$

7



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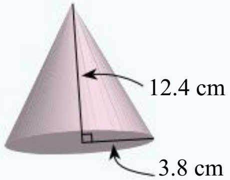

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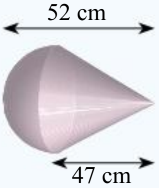

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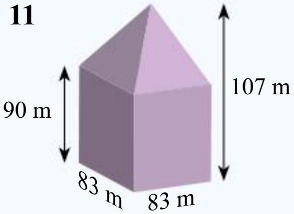

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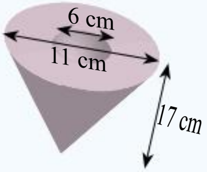


Indices & Surds & Logs

4

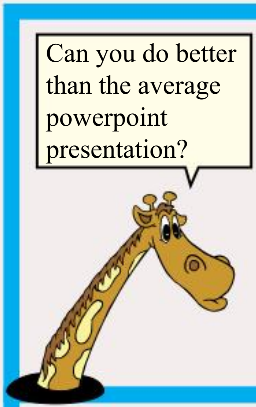
Number and Algebra ➞ Real Numbers

★ 10A Define rational and irrational numbers and perform operations with surds and fractional indices.

- understand that the real number system includes irrational numbers.
- extend the index laws to rational number indices.
- perform the four operations with surds.

★ 10A Use the definition of a logarithm to establish and apply the laws of logarithms.

- investigate the relationship between exponential and logarithmic expressions.
- simplify expressions using the logarithm laws.



A TASK

Choose one of the index laws and present a lesson in Powerpoint.

Some ideas:

- Write the law, examples, problems by hand, scan, paste the image in the slide.
- Insert hyperlinks and thus allow jumping around the slides.
- Record audio of voice. Find sound effects on the Internet. Insert the audio.
- Experiment with animation.

A LITTLE BIT OF HISTORY

Hypatia (370 AD-415 AD) has been described as the "pre-eminent mathematician of her time". Hypatia became head of the famous Platonist school at Alexandria and she lectured on mathematics and philosophy.

Little evidence of Hypatia's work in mathematics remains. Some letters exist that ask her advice on the construction of an astrolabe and a hydroscope (What is an astrolabe? What is a hydroscope?).

Hypatia's name is today used for mathematical achievement:

- *Hypatia scholarship for mathematically gifted women.*
- *Hypatia award for mathematical excellence.*
- *Hypatia award for excellence in philosophy.*

Our Number System

Our Hindu-Arabic number system is simpler and more efficient than other number systems.

A number system is a set of numbers used to count, compare, calculate, etc.

Real Numbers

Rational Numbers

Can be expressed as a ratio a/b where a and b are integers.

Integers.
¯3, 5, ¯18, 0, 3, 9

Fractions.
1/2, 3/4, 5/3, 6/7

Decimals.
(Either terminate or recurr.)

Irrational Numbers

Cannot be expressed as a ratio a/b
(a and b are integers).

Surds.
$\sqrt{2} = 1.4142135...$
$\sqrt{3} = 1.7320508...$

Special numbers.
$\pi = 3.1415926...$
$e = 2.7182818...$

Complex Numbers

a + bi
a and b are real numbers.
i is the imaginary part.

Integers are:		
	Positive whole numbers:	1, 2, 3, 4, 5, 6, ...
and	Zero:	0
and	Negative whole numbers:	¯1, ¯2, ¯3, ¯4, ¯5, ¯6, ...

Surds

Surds are numbers left in square root form or cube root form etc.

Surds are irrational numbers - we don't know their exact value.

Surds are left as square roots because it is less accurate to change them to decimals.

Exercise 4.1
Simplify the following surds:

$\sqrt{20}$ $= \sqrt{4} \times \sqrt{5}$ $= \underline{2\sqrt{5}}$	$\sqrt{54}$ $= \sqrt{9} \times \sqrt{6}$ $= \underline{3\sqrt{6}}$	$3\sqrt{75}$ $= 3\sqrt{25} \times \sqrt{3}$ $= \underline{15\sqrt{3}}$

Surd Rule
$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

1 $\sqrt{8}$
2 $\sqrt{12}$
3 $\sqrt{18}$
4 $\sqrt{20}$
5 $\sqrt{50}$
6 $\sqrt{24}$
7 $\sqrt{27}$
8 $\sqrt{200}$
9 $\sqrt{28}$
10 $\sqrt{40}$
11 $\sqrt{45}$
12 $\sqrt{44}$

Surds

Exercise 4.2

Simplify the following surds:

$$\begin{aligned}&\sqrt{5}\times\sqrt{3}\\=\ &\sqrt{5\times 3}\\=\ &\underline{\sqrt{15}}\end{aligned}$$

$$\begin{aligned}&2\sqrt{2}\times 5\sqrt{6}\\=\ &2\times 5\times\sqrt{2\times 6}\\=\ &10\times\sqrt{12}\\=\ &10\times 2\sqrt{3}\\=\ &\underline{20\sqrt{3}}\end{aligned}$$

$$\begin{aligned}&\sqrt{5}\,(\sqrt{3}+\sqrt{5})\\=\ &\sqrt{15}+\sqrt{25}\\=\ &\underline{\sqrt{15}+5}\end{aligned}$$

Surd Rule

$$\sqrt{x}\times\sqrt{y}=\sqrt{xy}$$

Surds happen when accurate answers are more important than rounded off answers.

1 $\sqrt{2}\times\sqrt{3}$

2 $\sqrt{3}\times\sqrt{2}$

3 $\sqrt{3}\times\sqrt{5}$

4 $\sqrt{5}\times\sqrt{2}$

5 $\sqrt{2}\times\sqrt{6}$

6 $\sqrt{3}\times\sqrt{6}$

7 $\sqrt{5}\times\sqrt{10}$

8 $\sqrt{6}\times\sqrt{4}$

9 $\sqrt{2}\times\sqrt{8}$

10 $\sqrt{3}\times\sqrt{12}$

11 $\sqrt{2}\times\sqrt{14}$

12 $\sqrt{3}\times\sqrt{15}$

13 $\sqrt{5}\times\sqrt{10}$

14 $\sqrt{3}\times\sqrt{21}$

15 $\sqrt{4}\times\sqrt{12}$

16 $\sqrt{7}\times\sqrt{14}$

17 $2\sqrt{5}\times 3\sqrt{10}$

18 $3\sqrt{2}\times 4\sqrt{6}$

19 $5\sqrt{6}\times 2\sqrt{3}$

20 $2\sqrt{6}\times 7\sqrt{10}$

21 $\sqrt{2}\,(\sqrt{2}+\sqrt{3})$

22 $\sqrt{3}\,(\sqrt{2}+\sqrt{3})$

23 $\sqrt{5}\,(\sqrt{2}+\sqrt{10})$

24 $\sqrt{3}\,(\sqrt{2}-\sqrt{6})$

25 $\sqrt{2}\,(3\sqrt{2}+\sqrt{3})$

26 $\sqrt{7}\,(2\sqrt{7}+2\sqrt{3})$

27 $\sqrt{2}\,(4\sqrt{2}-\sqrt{10})$

28 $\sqrt{5}\,(\sqrt{15}+3\sqrt{10})$

29 $2\sqrt{2}\,(3\sqrt{2}-5\sqrt{8})$

Exercise 4.3

Simplify the following surds:

$$\begin{aligned}&5\sqrt{21}-8\sqrt{21}\\=\ &\underline{{}^{-}3\sqrt{21}}\end{aligned}$$

$$\begin{aligned}&\sqrt{5}-9+3\sqrt{5}\\=\ &\underline{4\sqrt{5}-9}\end{aligned}$$

$$\begin{aligned}&7\sqrt{3}-3\sqrt{12}\\=\ &7\sqrt{3}-3\sqrt{3\times 4}\\=\ &7\sqrt{3}-6\sqrt{3}\\=\ &\underline{\sqrt{3}}\end{aligned}$$

1 $3\sqrt{2}+4\sqrt{2}$

2 $2\sqrt{7}+3\sqrt{7}$

3 $6\sqrt{5}-2\sqrt{5}$

4 $3\sqrt{10}-7\sqrt{10}$

5 $6\sqrt{3}-3\sqrt{3}+5$

6 $\sqrt{5}-9+3\sqrt{5}$

7 $5\sqrt{2}-4+2\sqrt{2}$

8 $\sqrt{3}+6-2\sqrt{3}+4$

9 $9\sqrt{3}-2\sqrt{12}$

10 $5\sqrt{3}+\sqrt{27}$

11 $4\sqrt{2}+7\sqrt{8}$

12 $3\sqrt{2}-\sqrt{18}$

13 $3\sqrt{12}-2\sqrt{48}$

14 $2\sqrt{12}-3\sqrt{3}+2\sqrt{27}$

Expand and simplify:

$$\begin{aligned}&(\sqrt{2}-2\sqrt{3})(3-\sqrt{6})\\=\ &\sqrt{2}\,(3-\sqrt{6})-2\sqrt{3}\,(3-\sqrt{6})\\=\ &3\sqrt{2}-\sqrt{12}-6\sqrt{3}+2\sqrt{18}\\=\ &3\sqrt{2}-2\sqrt{3}-6\sqrt{3}+6\sqrt{2}\\=\ &\underline{9\sqrt{2}-8\sqrt{3}}\end{aligned}$$

15 $(\sqrt{2}-3\sqrt{3})(2-\sqrt{6})$

16 $(2\sqrt{3}+\sqrt{2})(3\sqrt{6}-5)$

17 $(2\sqrt{5}-\sqrt{2})(\sqrt{10}-1)$

18 $(4-\sqrt{10})(3\sqrt{5}+\sqrt{2})$

Fractional Indices

$1^2 = 1$	$1^3 = 1$	$1^4 = 1$	$1^5 = 1$
$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$
$4^2 = 16$	$4^3 = 64$	$4^4 = 256$	$4^5 = 1024$
$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	$5^5 = 3125$

$x^{\frac{1}{2}} = \sqrt{x}$ the square root

$x^{\frac{1}{3}} = \sqrt[3]{x}$ the cube root

$x^{\frac{1}{4}} = \sqrt[4]{x}$ the fourth root

Exercise 4.4

Copy and complete the following table:

$1^{\frac{1}{2}} = 1$	$1^{\frac{1}{3}} = 1$	$1^{\frac{1}{4}} = 1$	$1^{\frac{1}{5}} = 1$
$4^{\frac{1}{2}} = 2$	$8^{\frac{1}{3}} = 2$	$16^{\frac{1}{4}} =$	$32^{\frac{1}{5}} =$
$9^{\frac{1}{2}} =$	$27^{\frac{1}{3}} =$	$81^{\frac{1}{4}} =$	$243^{\frac{1}{5}} =$
$16^{\frac{1}{2}} =$	$64^{\frac{1}{3}} =$	$256^{\frac{1}{4}} =$	$1024^{\frac{1}{5}} =$
$25^{\frac{1}{2}} =$	$125^{\frac{1}{3}} =$	$625^{\frac{1}{4}} =$	$3125^{\frac{1}{5}} =$
$36^{\frac{1}{2}} =$	$216^{\frac{1}{3}} =$	$1296^{\frac{1}{4}} =$	$7776^{\frac{1}{5}} =$

The connection between indices and surds. →

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Exercise 4.5

Evaluate the following:

$$8^{\frac{1}{3}} = \sqrt[3]{8}$$
$$= \underline{2} \qquad \{2^3 = 8\}$$

$$100000^{\frac{1}{5}} = \sqrt[5]{100000}$$
$$= \underline{10}$$

1 $4^{\frac{1}{2}}$

2 $8^{\frac{1}{3}}$

3 $16^{\frac{1}{4}}$

4 $32^{\frac{1}{5}}$

5 $100^{\frac{1}{2}}$

6 $27^{\frac{1}{3}}$

7 $9^{\frac{1}{2}}$

8 $1000^{\frac{1}{3}}$

9 $64^{\frac{1}{6}}$

10 $32^{\frac{1}{5}}$

11 $625^{\frac{1}{4}}$

12 $81^{\frac{1}{2}}$

What happens when the index is negative? →

$$a^{\frac{-1}{n}} = \frac{1}{\sqrt[n]{a}}$$

Negative index means 'put under 1'.

Technology 4.1 may be useful.

$$49^{\frac{-1}{2}} = \frac{1}{\sqrt{49}}$$
$$= \underline{\frac{1}{7}}$$

13 $4^{\frac{-1}{2}}$

14 $9^{\frac{-1}{2}}$

15 $36^{\frac{-1}{2}}$

16 $25^{\frac{-1}{2}}$

17 $100^{\frac{-1}{2}}$

18 $27^{\frac{-1}{3}}$

19 $8^{\frac{-1}{3}}$

20 $343^{\frac{-1}{3}}$

21 $243^{\frac{-1}{5}}$

Fractional Index Law

$$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = \underline{4}$$

or

$$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = \underline{4}$$

$$\rightarrow \quad \boxed{x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m}$$

In this case $(\sqrt[2]{9})^3$ is easier than $\sqrt[2]{9^3}$

$$= 3^3 \qquad = \sqrt[2]{729}$$
$$= \underline{27} \qquad = \underline{27}$$

Find the square root of 9, then cube.

Cube 9, then find the square root

Exercise 4.6

Evaluate the following:

$$9^{\frac{3}{2}} = (\sqrt[2]{9})^3 = 3^3 = \underline{27}$$

1	$4^{\frac{3}{2}}$	**2**	$8^{\frac{2}{3}}$	**3**	$16^{\frac{3}{4}}$	**4**	$100^{\frac{3}{2}}$
5	$27^{\frac{2}{3}}$	**6**	$32^{\frac{2}{5}}$	**7**	$81^{\frac{1}{4}}$	**8**	$1000^{\frac{1}{3}}$
9	$64^{\frac{1}{6}}$	**10**	$32^{\frac{1}{5}}$	**11**	$64^{\frac{3}{2}}$	**12**	$125^{\frac{2}{3}}$

$$(x^2)^{\frac{3}{2}} = \left(\sqrt[2]{x^2}\right)^3 = (x)^3 = \underline{x^3}$$

13	$(x^2)^{\frac{3}{2}}$	**14**	$(x^3)^{\frac{2}{3}}$	**15**	$(x^4)^{\frac{3}{4}}$
16	$(x^5)^{\frac{2}{5}}$	**17**	$(a^2)^{\frac{5}{2}}$	**18**	$27^{\frac{-1}{3}}$
19	$(p^2)^{\frac{3}{2}}$	**20**	$(b^4)^{\frac{3}{2}}$	**21**	$(e^6)^{\frac{2}{3}}$

$$(4a^2)^{\frac{3}{2}} = \left(\sqrt[2]{4a^2}\right)^3 = (2a)^3 = \underline{8a^3}$$

22	$(4a^2)^{\frac{3}{2}}$	**23**	$(9b^2)^{\frac{3}{2}}$	**24**	$(8x^3)^{\frac{2}{3}}$
25	$(4x^6)^{\frac{3}{2}}$	**26**	$(25x^6)^{\frac{3}{2}}$	**27**	$(100x^4)^{\frac{3}{2}}$
28	$(8c^3)^{\frac{4}{3}}$	**29**	$(4x^8)^{\frac{5}{2}}$	**30**	$(25a^6)^{\frac{3}{2}}$

$$(8x^3)^{-\frac{2}{3}} = \frac{1}{\left(\sqrt[3]{8x^3}\right)^2} = \frac{1}{(2x)^2} = \underline{\frac{1}{(2x)^2}}$$

31	$(8x^3)^{-\frac{2}{3}}$	**32**	$(9b^2)^{-\frac{3}{2}}$
33	$(4a^2)^{-\frac{3}{2}}$	**34**	$(4x^6)^{-\frac{3}{2}}$
35	$(8x^6)^{-\frac{4}{3}}$	**36**	$(27a^9)^{-\frac{2}{3}}$
37	$(32x^5)^{-\frac{2}{5}}$	**38**	$(64x^6)^{-\frac{3}{6}}$

Index Law 1

Multiplying Indices:

$$\begin{aligned} 2^4\times2^2 &= 2\times2\times2\times2\times2\times2 \\ &= 2^6 \end{aligned}$$

or

Multiplying Indices:

$$\begin{aligned} 2^4\times2^2 &= 2^{4+2} \\ &= 2^6 \end{aligned}$$

→

Index Law 1

$$a^m\times a^n = a^{m+n}$$

Exercise 4.7

Simplify each of the following:

$$\begin{aligned} 2x^3\times5x^2 &= 10x^{3+2} \\ &= \underline{10x^5} \\ 7^{1/4}\times7^{3/4} &= 7^{1/4+3/4} \\ &= 7^1 = \underline{7} \\ 8^{5/3}\times8^{-1} &= 8^{5/3-1} \\ &= 8^{2/3} = \underline{4} \\ 2x^{1/2}\times4x^{1/3} &= 8x^{1/2+1/3} \\ &= \underline{8x^{5/6}} \text{ or } \underline{8\sqrt[6]{x^5}} \\ a^{1/4}\times a^{3/4}\times a^{-1/2} &= a^{1/4+3/4-1/2} \\ &= \underline{a^{1/2}} \text{ or } \underline{\sqrt{a}} \end{aligned}$$

1 $a^3\times a^2$

2 $3a^2\times4a^3$

3 $x^4\times x^3$

4 $2x^2\times3x^2$

5 $5b^3\times2b^5$

6 $4p^5\times2p^{-4}$

7 $5^{2/3}\times5^{1/3}$

8 $4^{1/2}\times4^{1/2}$

9 $3^{1/4}\times3^{3/4}$

10 $10^{1/2}\times10^{1/2}$

11 $10^{-3/2}\times10^{1/2}$

12 $7^{5/4}\times7^{-1/4}$

13 $4^{3/2}\times4^{-1}$

14 $4^{5/2}\times4^{-1}$

15 $8^{2/3}\times8^{-1/3}$

16 $16^{3/4}\times16^{-1/2}$

17 $27^{2/3}\times27^{-1/3}$

18 $25^{3/2}\times25^{-1}$

19 $x^{2/3}\times x^{1/3}$

20 $x^{1/4}\times x^{3/4}$

21 $x^{2/3}\times x^{-1/3}$

22 $3a^{3/2}\times2a^{-1/2}$

23 $5e^{5/4}\times2e^{-1/4}$

24 $6y^{3/5}\times2y^{7/5}$

25 $x^{2/3}\times x^{2/3}\times x^{-1/3}$

26 $x^{5/4}\times x^{-3/4}\times x^{1/4}$

27 $h^{3/5}\times h^{1/5}\times2h^{-2/5}$

28 $2y\times3y^{5/4}\times2y^{-1/4}$

$$15x^{-3/4} = 15x^{-\frac{3}{4}}$$

They are the same thing.

Technology 4.1 may be useful.

Index Law 2

Dividing Indices:

$$a^3\div a^2 = \frac{a\times a\times a}{a\times a} = a$$

or

Dividing Indices:

$$a^3\div a^2 = a^{3-2} = a$$

→

Index Law 2

$$a^m\div a^n = a^{m-n}$$

Exercise 4.8

Simplify each of the following:

$$\begin{aligned} 6x^3\div2x^5 &= 3x^{3-5} \\ &= \underline{3x^{-2}} \\ 4^{3/4}\div4^{1/4} &= 4^{3/4-1/4} \\ &= 4^{1/2} = \underline{2} \\ 8^{-5/3}\div8^{-1} &= 8^{-5/3--1} \\ &= 8^{-2/3} = \frac{1}{(\sqrt[3]{8})^2} = \underline{\frac{1}{4}} \\ 6x^{1/4}\div3x^{-1/4} &= 2x^{1/4--1/4} \\ &= \underline{2x^{1/2}} \text{ or } \underline{2\sqrt{x}} \\ 54^{2/3}\div2^{1/3} &= 9\times2^{2/3}\div2^{1/3} \\ &= \underline{9\times2^{1/3}} \end{aligned}$$

$$\begin{aligned} &54^{2/3} \\ &= (27\times2)^{2/3} \\ &= 27^{2/3}\times2^{2/3} \\ &= 9\times2^{2/3} \end{aligned}$$

1 $x^3\div x^2$

2 $b^7\div b^3$

3 $x^4\div x^6$

4 $4x^5\div2x^2$

5 $8d^5\div2d^2$

6 $6p^5\div3p^4$

7 $4^{3/2}\div4^{1/2}$

8 $4^{3/4}\div4^{1/4}$

9 $4^{9/4}\div4^{3/4}$

10 $8^{2/3}\div8^{1/3}$

11 $16^{5/4}\div16^{1/2}$

12 $27^{5/3}\div27^{2/3}$

13 $4^{1/2}\div4^{-1}$

14 $4^{3/2}\div4^{-1}$

15 $8^{1/3}\div8^{-1/3}$

16 $2^{1/2}\div2^{-1/2}$

17 $9^{1/2}\div9^{-3/2}$

18 $25^1\div25^{1/2}$

19 $6x^{1/4}\div2x^{-1/4}$

20 $8x^{3/4}\div2x^{1/4}$

21 $x^{2/3}\div x^{-1/3}$

22 $10s^{1/2}\div2s^{-1/2}$

23 $12k^{-1/4}\div3k^{-5/4}$

24 $6y^{7/5}\div2y^{4/5}$

25 $54^{2/3}\div2^{2/3}$

26 $48^{1/2}\div3^{-1/2}$

27 $32^{3/4}\div2^{-1/4}$

28 $24^{5/2}\div6^{1/2}$

Index Law 3

Power Indices:

$$\begin{aligned}(2^3)^2 &= (2\times2\times2)^2\\ &= (2\times2\times2)\times(2\times2\times2)\\ &= 2^6\end{aligned}$$

or

Power Indices:

$$\begin{aligned}(2^3)^2 &= 2^{3\times2}\\ &= 2^6\end{aligned}$$

→

Index Law 3

$$(a^m)^n = a^{m\times n}$$

Exercise 4.9

Simplify each of the following:

$$\left(\frac{4}{9}\right)^{-\frac{3}{2}} = \frac{(\sqrt[2]{9})^3}{(\sqrt[2]{4})^3} = \underline{\frac{27}{8}}$$

$$(3a^2)^{-2} = 3^{-2}a^{-4} \text{ or } \underline{\frac{1}{9a^4}}$$

$$\begin{aligned}(2^3)^{\frac{2}{3}} &= 2^{\frac{3\times2}{3}}\\ &= 2^2 = \underline{4}\end{aligned}$$

$$(x^2y^4)^{-1/2} = \underline{x^{-1}y^{-2}} \text{ or } \underline{\frac{1}{xy^2}}$$

1 $\left(\frac{9}{25}\right)^{\frac{1}{2}}$ **2** $\left(\frac{9}{25}\right)^{\frac{3}{2}}$ **3** $\left(\frac{4}{9}\right)^{-\frac{3}{2}}$

4 $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ **5** $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **6** $\left(\frac{8}{64}\right)^{-\frac{2}{3}}$

7 $(2^3)^2$ **8** $(2^2)^2$ **9** $(x^{-3})^3$

10 $(x^3y)^2$ **11** $(2x^2)^{-3}$ **12** $(3a^4)^3$

13 $(2^3)^{\frac{2}{3}}$ **14** $(2^2)^{\frac{3}{2}}$ **15** $(3^2)^{\frac{3}{2}}$

16 $(3^3)^{\frac{2}{3}}$ **17** $(2^4)^{\frac{1}{2}}$ **18** $(4^3)^{\frac{1}{6}}$

19 $(x^3y^{-3})^{1/3}$ **20** $(x^2y^4)^{1/2}$ **21** $(ab^{1/3})^{-3}$

22 $(x^2y)^{-1/2}$ **23** $(27a^3)^{2/3}$ **24** $(x^{1/2}y^2)^{2/3}$

The more practice you get the easier it becomes.

Index Law 4

Try 8^0 on your calculator.
Is your answer 1?

Zero Index:

$$p^3 \div p^3 = 1$$

or $p^3 \div p^3 = p^{3-3} = p^0$

Which must be = 1

or

Zero Index:

$$p^0 = 1$$

→

Zero Index

$$a^0 = 1$$

Exercise 4.10

Simplify each of the following:

$$\begin{aligned}4e^6\times2e^{-6} &= 8e^{6+-6}\\ &= 8e^0 = \underline{8}\end{aligned}$$

$$\begin{aligned}3^{3/2}\times3^{-3/2} &= 3^{3/2+-3/2}\\ &= 3^0 = \underline{1}\end{aligned}$$

$$\begin{aligned}15x^{-3/4}\div3x^{-3/4} &= 5x^{-3/4--3/4}\\ &= 5x^0 = \underline{5}\end{aligned}$$

1 $2x^2\times3x^{-2}$ **2** $5b^{-3}\times2b^3$ **3** $4p^5\times2p^{-5}$

4 $4^{3/2}\times4^{-3/2}$ **5** $5^{-5/2}\times5^{5/2}$ **6** $8^{-1/3}\times8^{1/3}$

7 $4x^5\div2x^5$ **8** $8d^{-3}\div2d^{-3}$ **9** $6p^4\div3p^4$

10 $6x^{1/4}\div2x^{1/4}$ **11** $8x^{-3/4}\div2x^{-3/4}$ **12** $x^{2/3}\div x^{2/3}$

Logarithms

Logarithms are a useful tool for handling wide ranging quantities.

Logarithms have many uses such as in sound (decibels), earthquakes (Richter scale), soil (pH), share markets (index scales), and in many fields such as accounting, chemistry, physics, etc.

Index

$100 = 10^2$

Log

$\text{Log}_{10}100 = 2$

The log of 100 to base 10 is 2

Exercise 4.11

Rewrite each following index as a log:

$100 = 10^2$

Log 100 to base 10 is 2.
or $\underline{\log_{10}100 = 2}$

$81 = 3^4$

Log 81 to base 3 is 4.
or $\underline{\log_3 81 = 4}$

1 $100 = 10^2$
2 $125 = 5^3$
3 $25 = 5^2$
4 $1000 = 10^3$
5 $27 = 3^3$
6 $64 = 4^3$
7 $36 = 6^2$
8 $100\ 000 = 10^5$
9 $16 = 2^4$
10 $32 = 2^5$
11 $625 = 5^4$
12 $216 = 6^3$
13 $0.1 = 10^{-1}$
14 $0.01 = 10^{-2}$

Rewrite each following log as an index:

$\log_{10}1000 = 3$

$\underline{1000 = 10^3}$

$\log_2 512 = 9$

$\underline{512 = 2^9}$

15 $\log_2 8 = 3$
16 $\log_{10}1000 = 3$
17 $\log_{10}100 = 2$
18 $\log_5 125 = 3$
19 $\log_2 64 = 6$
20 $\log_{10}10 = 1$
21 $\log_2 32= 5$
22 $\log_3 27 = 3$
23 $\log_4 1024 = 5$
24 $\log_7 2401 = 4$
25 $\log_{10}0.1 = {}^{-}1$
26 $\log_{10}0.001 = {}^{-}3$

What is the value of each of the following logs:

$\log_{10}100$
$\log_{10}100 = x$
$100 = 10^x$ thus $x = 2$
$\underline{\log_{10}100 = 2}$

$\log_2 8$
$\log_2 8 = x$
$8 = 2^x$ thus $x = 3$
$\underline{\log_2 8 = x}$

27 $\log_2 16$
28 $\log_{10}10$
29 $\log_{10}100$
30 $\log_3 27$
31 $\log_4 16$
32 $\log_5 625$
33 $\log_7 49$
34 $\log_{10}0.1$
35 $\log_{10}1000$
36 $\log_3 81$
37 $\log_2 32$
38 $\log_5 125$
39 $\log_7 343$
40 $\log_6 216$
41 $\log_4 64$
42 $\log_{10}0.01$

Logarithm Laws

$$\log_b x + \log_b y = \log_b xy$$

Adding logs

$\log_{10}1000 + \log_{10}100$
$= \log_{10}10^3 + \log_{10}10^2$
$= \quad 3 \quad + \quad 2$
$= 5$

$\log_{10}1000\times100$
$= \log_{10}10^3\times10^2$
$= \log_{10}10^5$
$= 5$

Exercise 4.12
Simplify the following:

$\log_{10}2 + \log_{10}5$
$= \log_{10}2\times5$
$= \log_{10}10$
$= \underline{1}$

1 $\log_{10}2 + \log_{10}5$
2 $\log_{10}25 + \log_{10}4$
3 $\log_3 9 + \log_3 3$
4 $\log_6 9 + \log_6 4$
5 $\log_8 32 + \log_8 2$
6 $\log_{10}50 + \log_{10}20$

$$\log_b x - \log_b y = \log_b \frac{x}{y}$$

Subtracting logs

$\log_{10}1000 - \log_{10}100$
$= \log_{10}10^3 - \log_{10}10^2$
$= \quad 3 \quad - \quad 2$
$= 1$

$\log_{10}1000\div100$
$= \log_{10}10^3\div10^2$
$= \log_{10}10^1$
$= 1$

$\log_{10}150 - \log_{10}1.5$
$= \log_{10}150\div1.5$
$= \log_{10}100$
$= \underline{2}$

7 $\log_4 20 - \log_4 5$
8 $\log_{10}200 - \log_{10}2$
9 $\log_2 32 - \log_2 2$
10 $\log_2 40 - \log_2 2.5$
11 $\log_3 162 - \log_3 6$
12 $\log_6 1080 - \log_6 5$

$$\log_b x^d = d\log_b x$$

Powers

$\log_{10}10^4$
$= \log_{10}10000$
$= 4$

$4\log_{10}10$
$= 4\times1$
$= 4$

$\log_5 5^3 = 3\log_5 5$
$= \underline{3}$

13 $\log_4 4^6$
14 $\log_{10}10^8$
15 $\log_3 9^5$
16 $\log_2 4^3$
17 $\log_x x^3$
18 $\log_b b^6$

$\log_4 80 + \log_4 3 - \log_{10}15$
$= \log_4 80\times3\div15$
$= \log_4 16$
$= \underline{2}$

19 $\log_{10}4 + \log_{10}5 - \log_{10}2$
20 $\log_{10}5 + \log_{10}40 - \log_{10}2$
21 $\log_2 80 + \log_2 45 - \log_2 225$
22 $\log_3 54 - \log_3 18 + \log_3 12 - \log_3 4$
23 $2\log_{10}2 + \log_{10}50 - \log_{10}2$

Mental Computation

The majority of everyday problems are solved mentally by adults.

Exercise 4.13

1 Spell Fractional Indices.
2 Simplify: $\sqrt{12}$
3 Simplify: $4^{\frac{3}{2}}$
4 What is the value of: $\text{Log}_{10}100$
5 What is the formula for the volume of a cylinder?
6 Solve: $2x - 5 < 7$
7 Solve: $x + y = 7$, $xy = 12$
8 $^{-}2 - {}^{-}6$
9 $(x^{-2})^3$
10 Simplify: $\frac{x}{5} + \frac{2x}{5}$

It is no exaggeration to say that the undecided could go one way or another.

One-Half was talking to the Square Root of 2. But the Square Root of 2 was speaking very quietly, and One-Half asked the Square Root of 2 to 'Speak up'.

The pun, of course, is that fractions speak louder than surds.

Exercise 4.14

1 Spell Logarithms.
2 Simplify: $\sqrt{18}$
3 Simplify: $27^{\frac{2}{3}}$
4 What is the value of: $\text{Log}_{2}4$
5 What is the formula for the volume of a pyramid?
6 Solve: $3x + 2 > 5$
7 Solve: $x + y = 8$, $xy = 15$
8 $^{-}2 - 5$
9 $x^{-2}x^5$
10 Simplify: $\frac{2x}{3} - \frac{x}{3}$

'The road to success is always under construction' - Lily Tomlin.

Exercise 4.15

1 Spell Surd.
2 Simplify: $\sqrt{27}$
3 Simplify: $16^{\frac{3}{2}}$
4 What is the value of: $\text{Log}_{10}1000$
5 What is the formula for the area of a circle?
6 Solve: $5x - 1 < 7$
7 Solve: $x + y = 7$, $xy = 10$
8 $^{-}2 - {}^{-}6$
9 $x^{-2} \div x^3$
10 Simplify: $\frac{x}{2} - \frac{x}{3}$

Student: Pi r squared.
Baker: No! Pies are round, cakes are square!

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 4.16

1 Evaluate each of the following:

a) $10 \div 0.2$ **b)** $10 \div 0.02$ **c)** $10 \div 0.002$

d) $1+\frac{2}{10}+\frac{9}{100}$ **e)** $2+\frac{3}{100}+\frac{7}{1000}$ **f)** $5+\frac{4}{10}+\frac{6}{1000}$

g) $\frac{1}{2}(1+(-1)^2)$ **h)** $\frac{1}{2}(1+(-1)^7)$ **i)** $\frac{1}{2}(1+(-1)^8)$

j) $2^6 \div 2^4$ **k)** $3^7 \div 3^5$ **l)** $7^9 \div 7^7$

m) $(1-\sqrt{2})(1+\sqrt{2})$ **n)** $(1-\sqrt{3})(1+\sqrt{3})$ **o)** $(1+\sqrt{5})(1-\sqrt{5})$

2 Write each of the following in index form.

a) 100 **b)** 10 000

c) 2000 **d)** 50 000

e) 0.01 **f)** 0.0003

$100 = \underline{10^2}$ $\quad 3000 = 3\times1000$

$\quad = \underline{3\times10^3}$

$0.0003 = \underline{3\times10^{-4}}$

3 Simplify each of the following:

$$\begin{aligned}&= \frac{\sqrt{2}-\sqrt{8}}{\sqrt{2}}\\ &= \frac{(\sqrt{2}-\sqrt{8})\sqrt{2}}{\sqrt{2}\times\sqrt{2}}\\ &= \frac{\sqrt{2}\times\sqrt{2}-\sqrt{8}\times\sqrt{2}}{2}\\ &= \frac{2-4}{2}\\ &= \underline{{}^-1}\end{aligned}$$

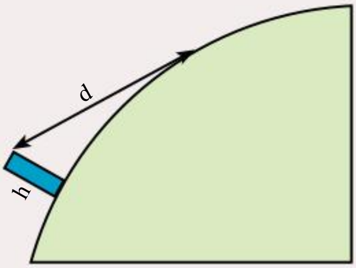
a) $\frac{\sqrt{8}+\sqrt{2}}{\sqrt{2}}$ **b)** $\frac{\sqrt{8}-\sqrt{2}}{\sqrt{2}}$

c) $\frac{\sqrt{27}+\sqrt{3}}{\sqrt{3}}$ **d)** $\frac{\sqrt{27}-\sqrt{3}}{\sqrt{3}}$

e) $\frac{\sqrt{18}+\sqrt{8}}{\sqrt{2}}$ **f)** $\frac{\sqrt{18}+\sqrt{8}}{\sqrt{50}}$

Don't do too much in your head. Pen and paper work will get better results.

4 A climber is very near the top of Mt Bartle Frere, Queensland's highest mountain, at 1620 m. How far out to sea can the climber see? The distance, d kilometers, to the horizon seen from a height of h metres is given by the formula: $d = 8\sqrt{\frac{h}{5}}$



5 If $a = 3$ and $b = {}^-2$, find the value of $a^2 - b^2$

6 If $a = {}^-3$ and $b = {}^-2$, find the value of $a^3 - b^3$

7 Solve each of the following:

a) $3^x = 81$ **b)** $2^x = 16$ **c)** $2^{x+1} = 16$

d) $3^x = 9^2$ **e)** $2^x = 4^3$ **f)** $10^x = 100^2\times10^3$

8 If $a^b = 2$, find the value of $a^{2b} + 10$

9 If $a^b = 2$, find the value of $(a^{2b})^3 - 60$

$a^{2b} = (a^b)^2$

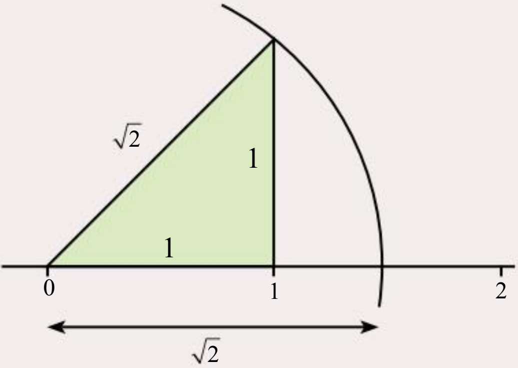
Investigations

Investigation 4.1 Surds

Surds such as $\sqrt{5}$ and $\sqrt{21}$ are irrational numbers because they don't have an exact decimal value.

Where is $\sqrt{2}$ on the number line?

1. Draw a triangle with a base of 1 unit and a height of 1 unit.
2. Find the hypotenuse.
   $h^2 = 1^2 + 1^2$
   $h^2 = 2$
   $h = \sqrt{2}$
3. Use a compass to measure $\sqrt{2}$ along the number line.
4. Compare the position of $\sqrt{2}$ with the calculator value of $\sqrt{2} = 1.41$



Use a similar method to place $\sqrt{3}$, $\sqrt{5}$ etc., on the number line.

Investigation 4.2 Surds

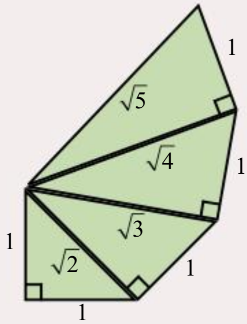
Investigate

Surds have caused considerable consternation throughout history.

Surds are radical.

Investigation 4.3 A Surd Spiral Wall Chart

Make and complete a large wall chart, for the classroom wall, of the surd spiral:



Mining Engineers prepare, plan, and oversee the development of mines and the procedures for mineral extraction.

- Relevant school subjects are English, Mathematics, and Science.
- Courses usually involve a Universtity engineering degree.

Technology

Technology 4.1 Calculators and Indices

Use a calculator with a power button such as y^x, x^y, or ^.

1 $2^3 \times 3^2 = \boxed{2}\ \boxed{y^x}\ \boxed{3}\ \boxed{\times}\ \boxed{3}\ \boxed{y^x}\ \boxed{2}\ \boxed{=}$ to give the answer 72.

2 $32^{\frac{3}{5}} = \boxed{32}\ \boxed{y^x}\ \boxed{(}\ \boxed{3}\ \boxed{\div}\ \boxed{5}\ \boxed{)}\ \boxed{=}$ to give the answer 8.

Note the need for brackets around 3/5.

3 $\sqrt{625} = 625^{\frac{1}{2}} = \boxed{625}\ \boxed{y^x}\ \boxed{0.5}\ \boxed{=}$ to give the answer 25.

4 $\sqrt[3]{8^2} = 8^{\frac{2}{3}} = \boxed{8}\ \boxed{(}\ \boxed{2}\ \boxed{\div}\ \boxed{3}\ \boxed{)}\ \boxed{=}$ to give the answer 4.

Technology 4.2 Surds and Indices

Videos

Watch online videos on surds and the index laws.

Surds are irrational numbers - we don't know their exact value.

Technology 4.3 Surd Calculator

Surds

Experiment with an online surd calculator.

$\sqrt{2} = 1.414214$
$\sqrt{3} = 1.732051$
$\sqrt{4} = 2$
$\sqrt{5} = 2.236068$

Technology 4.4 Surd Spiral

Use a spreadsheet to model the surd spiral on the previous page.

=c1

=sqrt(a1^2+b1^2)

1	1	1.414214
1	1.414214	1.732051
1	1.732051	2
1	2	2.236068

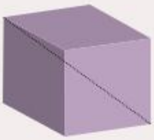
A Couple of Puzzles

Exercise 4.17

1 Arrange the following fractions in ascending order: $\boxed{\frac{1}{2}}$ $\boxed{\frac{3}{4}}$ $\boxed{\frac{1}{3}}$ $\boxed{\frac{2}{3}}$ $\boxed{\frac{1}{4}}$

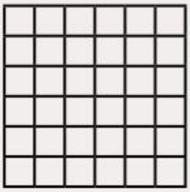
2 An approximate formula for conversion between kilometres k and miles m is: 5k = 8m. Approximately how many miles per hour is a speed limit of 100 km/h?

3 if $2^{x+y} = 16$ and $2^{x-y} = 256$, find x and y.



4 A cube has a side length of one metre What is the length of the diagonal?

A Game



Six and Out is a game for two players.

The game is played on a six by six square of squares as shown.

Players take turns and must mark either one, two, or three squares with an X.

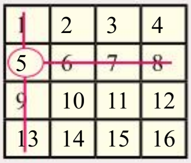
The loser is the player who marks the last square.

A Sweet Trick

1 Have the number 34 written on a piece of paper and hidden in the room.

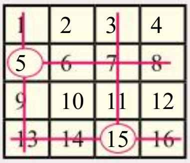
2 Write the following grid on a sheet of paper.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16



3 Your audience chooses any number and you cross off the other numbers in the same row and column.

4 Your audience chooses an uncrossed number and you cross off the other numbers in the same row and column.



5 This continues until the audience has chosen four numbers.

The total of the four chosen numbers equals the hidden number.

34 equals the sum of each diagonal.

Chapter Review 1

Exercise 4.18

1 Simplify the following surds:

a) $\sqrt{2} \times \sqrt{3}$ **b)** $\sqrt{4} \times \sqrt{12}$ **c)** $\sqrt{5}\,(\sqrt{2} + \sqrt{10})$

2 Expand and simplify:

$(\sqrt{2} - 2\sqrt{3})(3 - \sqrt{6})$
$= \sqrt{2}(3 - \sqrt{6}) - 2\sqrt{3}(3 - \sqrt{6})$
$= 3\sqrt{2} - \sqrt{12} - 6\sqrt{3} + 2\sqrt{18}$
$= 3\sqrt{2} - 2\sqrt{3} - 6\sqrt{3} + 6\sqrt{2}$
$= \underline{9\sqrt{2} - 8\sqrt{3}}$

a) $(\sqrt{2} - 3\sqrt{3})(2 - \sqrt{6})$
b) $(2\sqrt{3} + \sqrt{2})(3\sqrt{6} - 5)$
c) $(2\sqrt{5} - \sqrt{2})(\sqrt{10} - 1)$
d) $(4 - \sqrt{10})(3\sqrt{5} + \sqrt{2})$

3 Evaluate each of the following:

$9^{\frac{3}{2}} = (\sqrt[2]{9})^3$
$= 3^3$
$= \underline{27}$

a) $4^{\frac{1}{2}}$ **b)** $1000^{\frac{1}{3}}$ **c)** $4^{\frac{3}{2}}$
d) $16^{\frac{3}{4}}$ **e)** $(4a^2)^{\frac{3}{2}}$ **f)** $(8x^3)^{-\frac{2}{3}}$

4 Simplify each of the following:

a) $10^{1/2}\times10^{1/2}$ **b)** $10^{-3/2}\times10^{1/2}$ **c)** $3a^{3/2}\times2a^{-1/2}$ **d)** $x^{5/4}\times x^{-3/4}\times x^{1/4}$
e) $4^{3/2}\div4^{1/2}$ **f)** $4^{3/4}\div4^{1/4}$ **g)** $6x^{1/4}\div2x^{-1/4}$ **h)** $8x^{3/4}\div2x^{1/4}$
i) $\left(\frac{9}{25}\right)^{\frac{1}{2}}$ **j)** $\left(\frac{9}{25}\right)^{\frac{3}{2}}$ **k)** $\left(\frac{8}{64}\right)^{-\frac{2}{3}}$ **l)** $(x^{1/2}y^2)^{2/3}$

5 Rewrite each following index as a log:

a) $100 = 10^2$ **b)** $125 = 5^3$ **c)** $32 = 2^5$ **d)** $0.01 = 10^{-2}$

6 Rewrite each following log as an index:

a) $\log_2 8 = 3$ **b)** $\log_{10} 1000 = 3$ **c)** $\log_7 2401 = 4$ **d)** $\log_{10} 0.001 = {}^{-}3$

7 What is the value of each of the following logs:

a) $\log_2 16$ **b)** $\log_5 625$ **c)** $\log_7 49$ **d)** $\log_{10} 0.1$

8 Simplify the following:

$\log_{10}2 + \log_{10}5$
$= \log_{10}2\times5$
$= \log_{10}10$
$= \underline{1}$

$\log_{10}150 - \log_{10}1.5$
$= \log_{10}150\div1.5$
$= \log_{10}100$
$= \underline{2}$

$\log_5 5^3 = 3\log_5 5$
$= \underline{3}$

a) $\log_{10}2 + \log_{10}5$ **b)** $\log_{10}25 + \log_{10}4$
c) $\log_3 9 + \log_3 3$ **d)** $\log_6 9 + \log_6 4$
e) $\log_8 32 + \log_8 2 + \log_8 4 + \log_8 16$
f) $\log_4 20 - \log_4 5$ **g)** $\log_{10}200 - \log_{10}2$
h) $\log_2 32 - \log_2 2$ **i)** $\log_2 40 - \log_2 2.5$
j) $\log_3 162 - \log_3 6$ **k)** $\log_6 1080 - \log_6 5$
l) $\log_2 80 + \log_2 45 - \log_2 225$
m) $\log_4 4^6$ **n)** $\log_{10} 10^8$
o) $\log_3 9^5$ **p)** $\log_2 4^3$

Chapter Review 2

Exercise 4.19

1 Simplify the following surds:

a) $\sqrt{3} \times \sqrt{2}$ **b)** $\sqrt{5} \times \sqrt{10}$ **c)** $\sqrt{7}\,(2\sqrt{7} + 2\sqrt{3}\,)$

2 Expand and simplify:

$$\begin{aligned}&(\sqrt{2} - 2\sqrt{3}\,)(3 - \sqrt{6}\,)\\ &= \sqrt{2}\,(3 - \sqrt{6}\,) - 2\sqrt{3}\,(3 - \sqrt{6}\,)\\ &= 3\sqrt{2} - \sqrt{12} - 6\sqrt{3} + 2\sqrt{18}\\ &= 3\sqrt{2} - 2\sqrt{3} - 6\sqrt{3} + 6\sqrt{2}\\ &= \underline{9\sqrt{2} - 8\sqrt{3}}\end{aligned}$$

a) $(3 - 2\sqrt{3}\,)(\sqrt{2} - \sqrt{6}\,)$

b) $(2\sqrt{6} + \sqrt{2}\,)(\sqrt{3} - 4)$

c) $(\sqrt{5} - \sqrt{2}\,)(\sqrt{10} - \sqrt{2}\,)$

d) $(1 - \sqrt{10}\,)(2\sqrt{5} + \sqrt{2}\,)$

3 Evaluate each of the following:

$$\begin{aligned}9^{\frac{3}{2}} &= (\sqrt[2]{9})^3\\ &= 3^3\\ &= \underline{27}\end{aligned}$$

a) $4^{\frac{1}{2}}$ **b)** $8^{\frac{1}{3}}$ **c)** $8^{\frac{2}{3}}$

d) $27^{\frac{2}{3}}$ **e)** $(9b^2)^{\frac{3}{2}}$ **f)** $(9b^2)^{-\frac{3}{2}}$

4 Simplify each of the following:

a) $10^{1/3} \times 10^{1/3}$ **b)** $10^{5/2} \times 10^{-3/2}$ **c)** $5x^{3/2} \times 2x^{-1/2}$ **d)** $a^{3/4} \times a^{-1} \times a^{1/4}$

e) $9^{3/2} \div 9^{1/2}$ **f)** $16^{3/4} \div 16^{1/4}$ **g)** $8x^{1/2} \div 2x^{-1/2}$ **h)** $6b^{3/4} \div 3b^{1/4}$

i) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ **j)** $\left(\frac{9}{25}\right)^{\frac{3}{2}}$ **k)** $\left(\frac{4}{9}\right)^{-\frac{3}{2}}$ **l)** $(x^{1/3}y^4)^{3/2}$

5 Rewrite each following index as a log:

a) $9 = 3^2$ **b)** $64 = 4^3$ **c)** $64 = 2^6$ **d)** $0.001 = 10^{-3}$

6 Rewrite each following log as an index:

a) $\log_2 32 = 5$ **b)** $\log_{10} 100 = 2$ **c)** $\log_4 1024 = 5$ **d)** $\log_{10} 0.01 = {}^{-}2$

7 What is the value of each of the following logs:

a) $\log_2 8$ **b)** $\log_5 125$ **c)** $\log_7 343$ **d)** $\log_{10} 0.001$

8 Simplify the following:

$$\begin{aligned}&\log_{10} 2 + \log_{10} 5\\ &= \log_{10} 2 \times 5\\ &= \log_{10} 10\\ &= \underline{1}\end{aligned}$$

$$\begin{aligned}&\log_{10} 150 - \log_{10} 1.5\\ &= \log_{10} 150 \div 1.5\\ &= \log_{10} 100\\ &= \underline{2}\end{aligned}$$

$$\begin{aligned}\log_5 5^3 &= 3\log_5 5\\ &= \underline{3}\end{aligned}$$

a) $\log_2 4 + \log_2 8$ **b)** $\log_3 27 + \log_3 3$

c) $\log_4 8 + \log_4 2$ **d)** $\log_{10} 8 + \log_{10} 125$

e) $\log_6 54 + \log_6 18 + \log_6 12 + \log_6 4$

f) $\log_5 20 - \log_5 4$ **g)** $\log_2 80 - \log_2 5$

h) $\log_3 216 - \log_3 8$ **i)** $\log_6 54 - \log_6 1.5$

j) $\log_2 720 - \log_2 45$ **k)** $\log_4 560 - \log_4 35$

l) $\log_{10} 5 + \log_{10} 40 - \log_{10} 2$

m) $\log_2 2^5$ **n)** $\log_{10} 10^3$

o) $\log_3 9^4$ **p)** $\log_2 4^5$

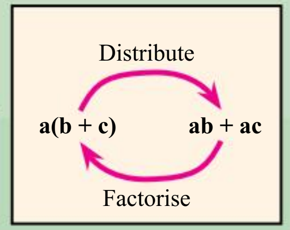
Review 1

5

Chapter 1 Algebra 1

$2\times2\times2 = 2^3$ ← Index ← Base

$(^-2)^2 = {}^-2\times{}^-2 = 4$
$(^-2)^3 = {}^-2\times{}^-2\times{}^-2 = {}^-8$
$(^-2)^4 = {}^-2\times{}^-2\times{}^-2\times{}^-2 = 16$



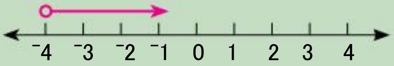
$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$
$\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$
$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$
$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$

Index Laws

$a^m\times a^n = a^{m+n}$
$a^m\div a^n = a^{m-n}$
$(a^m)^n = a^{m\times n}$
$a^0 = 1$
$a^{-m} = \frac{1}{a^m}$

Chapter 2 Linear Equations

$x + 3 > {}^-1$
$x > {}^-1 - 3$
$x > {}^-4$



Expressions such as H = 86d + 65 are linear because the highest power of d is 1 ($d = d^1$).

If you multiply or divide both sides of an inequation by a negative?

$7 > 3$
$7\times{}^-2 \quad 3\times{}^-2$
${}^-14 < {}^-6$

< becomes >
> becomes <

Substitute either x or y from one equation into the other equation

$x + y = 53$ **.....(1)**
$x = 2y + 5$ **.....(2)**

Eliminate either x or y by adding or subtracting the equations.

$3x \boxed{+ 3y} = 51$ **.....(1)**
$x \boxed{- 3y} = 5$ **.....(2)**

Chapter 3 Area & Volume

Area formulas
Area rectangle = length×breadth Area triangle = ½base×height Area circle = π×radius²

Prisms are three-dimensional shapes that have a constant cross-section.

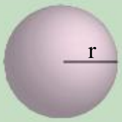
V_{prism} = Area of base × height

Right Pyramid 10A

$V_{pyramid}$ = ⅓ × Area base × height

Sphere 10A

$V_{sphere} = \frac{4}{3}\pi r^3$



Chapter 4 Indices & Logs 10A

Surd Rule

$\sqrt{x}\times\sqrt{y}=\sqrt{xy}$

Surd Rule

$\sqrt{xy}=\sqrt{x}\times\sqrt{y}$

$$(8x^3)^{-\frac{2}{3}} = \frac{1}{\left(\sqrt[3]{8x^3}\right)^2} = \frac{1}{(2x)^2} = \underline{\frac{1}{4x^2}}$$

$\log_b x + \log_b y = \log_b xy$

$\text{Log}_{10}100 = 2$

$\updownarrow \updownarrow \updownarrow \updownarrow$

$100 = 10^2$

$\log_b x^d = d\log_b x$

$\log_b x - \log_b y = \log_b \frac{x}{y}$

Review 1

Exercise 5.1 Mental computation

1 Spell Factorisation.
2 Simplify: $\sqrt{12}$
3 Simplify: $4^{\frac{3}{2}}$
4 What is the value of: $\text{Log}_{10}1000$
5 What is the formula for the volume of a cylinder?
6 Solve: $2x - 5 < 3$
7 Solve: $x + y = 7,\ xy = 10$
8 ${}^{-}2 - {}^{-}8$
9 $(x^{-3})^2$
10 Simplify: $\frac{x}{5} + \frac{2x}{5}$

$$\begin{aligned} 9^{\frac{3}{2}} &= (\sqrt[2]{9})^3 \\ &= 3^3 \\ &= 27 \end{aligned}$$

Paraprosdokian - a figure of speech in which the latter part of a sentence or phrase is surprising or unexpected.

If I agreed with you, we'd both be wrong.

Where there's a will, I want to be in it.

Exercise 5.2

1 Expand each of the following:

$4(a + 3) = \underline{4a + 12}$ ${}^{-}a(a + 3) = \underline{{}^{-}a^2 - 3a}$ ${}^{-}b^2(2b - 5) = \underline{{}^{-}2b^3 + 5b^2}$	$(x + 5)(x + 4)$ $= x(x + 4) + 5(x + 4)$ $= x^2 + 4x + 5x + 20$ $= \underline{x^2 + 9x + 20}$	$(x + 3)^2 = (x + 3)(x + 3)$ $= x(x + 3) + 3(x + 3)$ $= x^2 + 3x + 3x + 9$ $= \underline{x^2 + 6x + 9}$

a) $3(x + 4)$ **b)** ${}^{-}x(x + 3)$ **c)** $(x^3 + 1)(x^2 - 2)$
d) $(x + 2)(x + 3)$ **e)** $(x + 2)^2$ **f)** $(x^3 + 3)(x^3 - 1)$

2 Factorise each of the following:

$6x + 9$ $= 3{\times}2x + 3{\times}3$ $= \underline{3(2x + 3)}$	$4xy - 6x$ $= 2x{\times}2y - 2x{\times}3$ $= \underline{2x(2y - 3)}$	$10x^2 - 8x$ $= 2x{\times}5x - 2x{\times}4$ $= 2x(5x - 4)$

a) $4x + 10$ **b)** $8ab - 12a$ **c)** $15n^5 - 36n^2$
d) $8 + 10y^3$ **e)** $12x^2 + 20x$ **f)** $14y^4 - 28y^3$

3 Simplify the following algebraic expressions:

$5a^3b^{-2} \times {}^{-}3ab^3$ $= 5 \times {}^{-}3 \times a^3 \times a \times b^{-2} \times b^3$ $= \underline{{}^{-}15a^4b}$	${}^{-}4x^2y^{-4} \div 2xy^{-3}$ $= {}^{-}2x^{2-1}y^{-4--3} \quad \{x = x^1\}$ $= {}^{-}2x^1y^{-4+3} \quad \{- -3 = 3\}$ $= \underline{{}^{-}2xy^{-1}}$

a) $5a \times {}^{-}3a^3$ **b)** ${}^{-}5y \times 2y^3$ **c)** ${}^{-}3z^2 \times {}^{-}4z$
d) $8x^4 \times 2x^{-3}$ **e)** $3y^4 \times 5y^{-2}$ **f)** $6a^2b^5 \times 4a^{-2}b^{-3}$
g) $16x^5 \div 2x^3$ **h)** ${}^{-}12a^6 \div {}^{-}3a^2$ **i)** $12x^7 \div {}^{-}4x^4$
j) $\frac{18x^{-2}}{6x^{-3}}$ **k)** $\frac{10m^3n^{-2}}{{}^{-}2m^{-1}n^2}$ **l)** $\frac{{}^{-}15ab^2}{{}^{-}3a^{-2}b^{-3}}$

$$\frac{3x^5}{8}+\frac{2x^5}{8}$$
$$=\frac{3x^5+2x^5}{8}$$
$$=\underline{\frac{5x^5}{8}}$$

m) $\frac{4x}{3}+\frac{x}{3}$

n) $\frac{4x^2}{5}+\frac{2x^2}{5}$

o) $\frac{3e}{4}+\frac{5e}{4}$

p) $\frac{3a^3}{4}+\frac{5a^3}{4}$

q) $\frac{7x}{4}-\frac{3x}{4}$

r) $\frac{5x^3}{4}-\frac{3x^3}{4}$

$$\frac{4x^2}{5}-\frac{2x^2}{5}$$
$$=\frac{4x^2-2x^2}{5}$$
$$=\underline{\frac{2x^2}{5}}$$

4 Graph the solutions to the following inequations on the number line:

a) $x + 2 > 5$ **b)** $x/2 > {}^-1$ **c)** $3x + 1 \leq {}^-6$

5 Use a graphical **method** to solve the pairs of simultaneous equations:

a) $y = 6x + 1$
$y = 2x + 5$

x	¯2	¯1	0	1	2
y=6x+1					

x	¯2	¯1	0	1	2
y=2x+5					

b) $y = x + 3$
$y = 3x - 1$

x	¯2	¯1	0	1	2
y=x+3					

x	¯2	¯1	0	1	2
y=3x−1					

$x + y = 53$ ... **(1)**
$x = 2y + 5$ ... **(2)**

Substitute for x, from (2) in (1)

$2y + 5 + y = 53$ {x=2y+5}
$3y + 5 = 53$ {2y+y=3y}
$3y = 53 - 5$ {inverse of +is−}
$3y = 48$
$y = 48 \div 3$ {inverse of ×is÷}
$\underline{y = 16}$

From (2) $x = 2\times16 + 5$
$x = 37$

Solution: $\underline{x=37, y=16}$

Check: substitute for x and y in (1)
$x+y=53$
$37+16=53$ ✓

6 Use the **substitution method** to solve the pair of simultaneous equations:

a) $x + y = 6$
$x = y - 2$

b) $x + y = 2$
$x = y + 10$

c) $x + y = 87$
$y = x + 41$

7 The sum of two numbers is ninety-six, and one number is eighty more than the other number. Use the **substitution method** to find a solution.

$3x + 3y = 51$ ... **(1)**
$x - 3y = 5$ ... **(2)**

Eliminate y by adding (1) to (2)

$4x = 56$ {3x+x=4x, 51+5=56}
$x = 56 \div 4$ {inverse}
$\underline{x = 14}$

From (1)

$3x14 + 3y = 51$ {x=14}
$42 + 3y = 51$
$3y = 51 - 42$ {inverse}
$3y = 9$
$\underline{y = 3}$ {inverse}

Solution: $\underline{x=14, y=3}$

8 Use the **elimination method** to solve the pair of simultaneous equations:

a) $x + y = 15$
$x - y = 3$

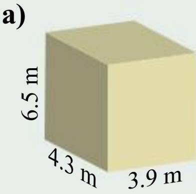
b) $x + y = 65$
$x - y = 39$

c) $2x + y = 10$
$x + 2y = 14$

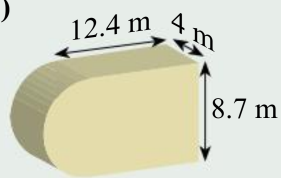
9 The sum of two numbers is one hundred and five. The difference between the two numbers is thirty-one. Use the elimination method to find the numbers?

10 Find the volume and the surface area of each of the following prisms:

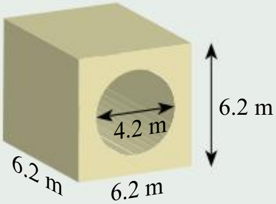
a)



b)



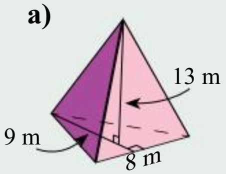
c)



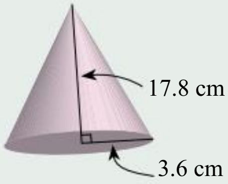
11 Calculate the volume of each of the following:

$$V = V_{\text{hemisphere}} + V_{\text{circular pyramid}}$$
$$V = \tfrac{1}{2} \times \tfrac{4}{3}\pi r^3 + \tfrac{1}{3} \times \pi r^2 \times \text{height}$$
$$V = \tfrac{1}{2} \times \tfrac{4}{3}\pi \times 8^3\ \text{cm}^3 + \tfrac{1}{3} \times \pi \times 8^2 \times 17\ \text{cm}^3$$
$$V = 1072.33\ \text{cm}^3 + 1139.35\ \text{cm}^3$$
$$\underline{V = 2211.68\ \text{cm}^3}$$

a)

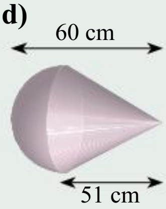


b)

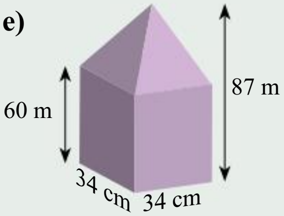


c)

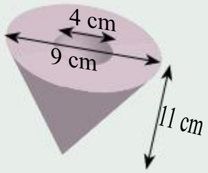
d)



e)



f)



12 Simplify the following surds:

a) $\sqrt{3} \times \sqrt{2}$ **b)** $\sqrt{3} \times \sqrt{12}$ **c)** $\sqrt{10}\,(\sqrt{2} + \sqrt{5})$

13 Expand and simplify:

$$(\sqrt{2} - 2\sqrt{3})(3 - \sqrt{6})$$
$$= \sqrt{2}(3 - \sqrt{6}) - 2\sqrt{3}(3 - \sqrt{6})$$
$$= 3\sqrt{2} - \sqrt{12} - 6\sqrt{3} + 2\sqrt{18}$$
$$= 3\sqrt{2} - 2\sqrt{3} - 6\sqrt{3} + 6\sqrt{2}$$
$$= \underline{9\sqrt{2} - 8\sqrt{3}}$$

a) $(\sqrt{3} - 1)(3\sqrt{2} + \sqrt{6})$

b) $(2\sqrt{6} + \sqrt{2})(3 - 5\sqrt{3})$

c) $(3\sqrt{5} - \sqrt{2})(\sqrt{10} - 1)$

d) $(3\sqrt{5} + \sqrt{2})(5 - \sqrt{10})$

14 10A Evaluate each of the following:

$$9^{\frac{3}{2}} = (\sqrt[2]{9})^3$$
$$= 3^3$$
$$= \underline{27}$$

a) $4^{\frac{1}{2}}$ **b)** $1000^{\frac{1}{3}}$ **c)** $4^{\frac{3}{2}}$

d) $16^{\frac{3}{4}}$ **e)** $(4a^2)^{\frac{3}{2}}$ **f)** $(8x^3)^{-\frac{2}{3}}$

15 10A Simplify each of the following:

a) $5^{1/2}\times5^{1/2}$ **b)** $10^{3/2}\times10^{{}^-1/2}$ **c)** $5x^{3/2}\times2x^{{}^-1/2}$ **d)** $a^{5/2}\times a^{{}^-3/2}\times a^{1/2}$

e) $9^{3/2}\div9^{1/2}$ **f)** $9^{3/4}\div9^{1/4}$ **g)** $6x^{1/4}\div3x^{{}^-1/4}$ **h)** $6n^{3/4}\div2n^{1/4}$

i) $\left(\frac{9}{25}\right)^{\frac{1}{2}}$ **j)** $\left(\frac{9}{25}\right)^{\frac{3}{2}}$ **k)** $\left(\frac{8}{64}\right)^{-\frac{2}{3}}$ **l)** $(x^{1/2}y^3)^{2/3}$

16 10A Rewrite each following index as a log:

a) $1000 = 10^3$ **b)** $64 = 2^6$ **c)** $81 = 3^4$ **d)** $0.01 = 10^{{}^-2}$

17 10A Rewrite each following log as an index:

a) $\log_2 16 = 4$ **b)** $\log_{10} 100 = 2$ **c)** $\log_7 343 = 3$ **d)** $\log_{10} 0.001 = {}^-3$

18 10A What is the value of each of the following logs:

a) $\log_2 8$ **b)** $\log_5 125$ **c)** $\log_6 216$ **d)** $\log_{10} 0.1$

19 10A Simplify the following:

$$\log_{10}2 + \log_{10}5$$
$$= \log_{10}2\times5$$
$$= \log_{10}10$$
$$= \underline{1}$$

$$\log_{10}150 - \log_{10}1.5$$
$$= \log_{10}150\div1.5$$
$$= \log_{10}100$$
$$= \underline{2}$$

a) $\log_{10}20 + \log_{10}5$ **b)** $\log_{10}125 + \log_{10}8$

c) $\log_3 9 + \log_3 9$ **d)** $\log_6 18 + \log_6 2$

e) $\log_8 16 + \log_8 2 + \log_8 4 + \log_8 32$

f) $\log_4 80 - \log_4 5$ **g)** $\log_{10}300 - \log_{10}3$

h) $\log_3 45 - \log_3 5$ **i)** $\log_6 1512 - \log_6 7$

j) $\log_2 48 + \log_2 25 - \log_2 75$

k) $\log_5 5^3$ **l)** $\log_{10} 10^4$

Review 2

Exercise 5.3 Mental computation

1 Spell Simultaneous.

2 Simplify: $\sqrt{18}$

3 Simplify: $27^{\frac{2}{3}}$

4 What is the value of: $\text{Log}_{10}100$

5 What is the formula for the volume of a prism?

6 Solve: $2x - 3 > 5$

7 Solve: $x + y = 10,\ xy = 21$

8 ${}^-2 - {}^-3$

9 $(x^{-2})^3$

10 Simplify: $\frac{2x}{3} - \frac{x}{3}$

Exercise 5.4

1 Expand each of the following:

$4(a+3) = \underline{4a+12}$ $^{-}a(a+3) = \underline{^{-}a^2 - 3a}$ $^{-}b^2(2b-5) = \underline{^{-}2b^3 + 5b^2}$	$(x+5)(x+4)$ $= x(x+4) + 5(x+4)$ $= x^2 + 4x + 5x + 20$ $= \underline{x^2 + 9x + 20}$	$(x+3)^2 = (x+3)(x+3)$ $= x(x+3) + 3(x+3)$ $= x^2 + 3x + 3x + 9$ $= \underline{x^2 + 6x + 9}$

a) $2(x+3)$ **b)** $^{-}a(a+5)$ **c)** $(x^2+1)(x^2-3)$

d) $(x+1)(x+2)$ **e)** $(x+4)^2$ **f)** $(x^3+1)(x^3-1)$

2 Factorise each of the following:

$6x + 9$ $= 3{\times}2x + 3{\times}3$ $= \underline{3(2x+3)}$	$4xy - 6x$ $= 2x{\times}2y - 2x{\times}3$ $= \underline{2x(2y-3)}$	$10x^2 - 8x$ $= 2x{\times}5x - 2x{\times}4$ $= \underline{2x(5x-4)}$

a) $6x + 10$ **b)** $9xy - 12x$ **c)** $15b^3 - 20b^2$

d) $8 + 12p^3$ **e)** $12x^3 + 18x$ **f)** $14y^3 - 21y$

3 Simplify the following algebraic expressions:

$5a^3b^{-2} \times {}^{-}3ab^3$ $= 5 \times {}^{-}3 \times a^3 \times a \times b^{-2} \times b^3$ $= \underline{^{-}15a^4b}$	$^{-}4x^2y^{-4} \div 2xy^{-3}$ $= {}^{-}2x^{2-1}y^{-4--3} \quad \{x = x^1\}$ $= {}^{-}2x^1y^{-4+3} \quad \{- -3 = 3\}$ $= \underline{^{-}2xy^{-1}}$

a) $^{-}2x^3 \times 5x$ **b)** $5y \times {}^{-}3y^2$ **c)** $^{-}3z^2 \times {}^{-}2z^2$

d) $6x^5 \times 2x^{-3}$ **e)** $3c^3 \times 2c^{-2}$ **f)** $2a^3b^4 \times 4a^{-2}b^{-3}$

g) $16x^5 \div 2x^3$ **h)** $^{-}15x^5 \div {}^{-}3x^2$ **i)** $10x^5 \div {}^{-}4x^4$

j) $\dfrac{15x^{-4}}{18x^2}$ **k)** $\dfrac{14a^5b^{-3}}{^{-}10a^2b^2}$ **l)** $\dfrac{^{-}14xy^{-2}}{^{-}6x^2y^{-3}}$

$\dfrac{3x^5}{8} + \dfrac{2x^5}{8}$ $= \dfrac{3x^5 + 2x^5}{8}$ $= \underline{\dfrac{5x^5}{8}}$

m) $\dfrac{4x}{3} + \dfrac{x}{3}$ **n)** $\dfrac{4x^2}{5} + \dfrac{2x^2}{5}$

o) $\dfrac{3x}{5} - \dfrac{x}{5}$ **p)** $\dfrac{3a}{2} - \dfrac{a}{2}$

q) $\dfrac{7x}{4} - \dfrac{3x}{4}$ **r)** $\dfrac{5x^3}{4} - \dfrac{3x^3}{4}$

$\dfrac{4x^2}{5} - \dfrac{2x^2}{5}$ $= \dfrac{4x^2 - 2x^2}{5}$ $= \underline{\dfrac{2x^2}{5}}$

4 Graph the solutions to the following inequations on the number line:

a) $x - 5 < {}^{-}2$ **b)** $x/3 > {}^{-}1$ **c)** $2x + 4 \leq {}^{-}1$

5 Use a graphical **method** to solve the pairs of simultaneous equations:

a) $y = 3x + 2$
$y = 7x - 2$

x	$^{-}2$	$^{-}1$	0	1	2
y=3x+2					

x	$^{-}2$	$^{-}1$	0	1	2
y=7x−2					

b) $y = 3x + 7$
$y = {}^-2x + 2$

x	$^-2$	$^-1$	0	1	2
y=3x+7					

x	$^-2$	$^-1$	0	1	2
y=$^-$2x+2					

6 Use the **substitution method** to solve the pair of simultaneous equations:

$x + y = 53$ ... **(1)**
$x = 2y + 5$ ... **(2)**

Substitute for x, from (2) in (1)

$2y + 5 + y = 53$ {x=2y+5}
$3y + 5 = 53$ {2y+y=3y}
$3y = 53 - 5$ {inverse of +is−}
$3y = 48$
$y = 48 \div 3$ {inverse of ×is÷}
$\underline{y = 16}$

From (2) $x = 2 \times 16 + 5$
$x = 37$

Solution: $\underline{x=37,\ y=16}$

Check: substitute for x and y in (1)
$x+y=53$
$37+16=53$ ✓

a) $x + y = 10$
$x = y - 4$

b) $x + y = 5$
$x = y - 11$

c) $x + y = 92$
$y = x - 14$

7 The sum of two numbers is one hundred and ten, and one number is thirty-two more than the other number. Use the **substitution method** to find a solution.

8 Use the **elimination method** to solve the pair of simultaneous equations:

$3x + 3y = 51$ ... **(1)**
$x - 3y = 5$ ... **(2)**

Eliminate y by adding (1) to (2)

$4x = 56$ {3x+x=4x, 51+5=56}
$x = 56 \div 4$ {inverse}
$\underline{x = 14}$

From (1)
$3x14 + 3y = 51$ {x=14}
$42 + 3y = 51$
$3y = 51 - 42$ {inverse}
$3y = 9$
$\underline{y = 3}$ {inverse}

Solution: $\underline{x=14,\ y=3}$

a) $x + y = 11$
$x - y = 7$

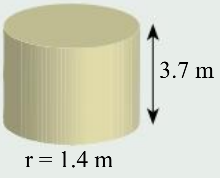
b) $x + y = 51$
$x - y = 15$

c) $2x + y = {}^-2$
$x + 2y = 5$

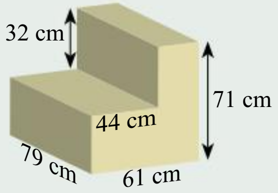
9 The sum of two numbers is one hundred and six. The difference between the two numbers is thirty-two. Use the elimination method to find the numbers?

10 Find the volume and the surface area of each of the following prisms:

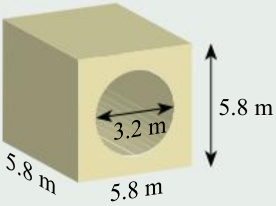
a)



b)

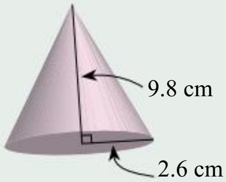


c)

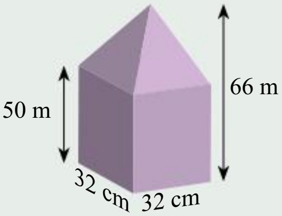


11 10A Calculate the volume of each of the following:

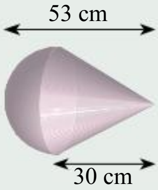
a)



b)



c)



12 10A Simplify the following surds:

a) $\sqrt{2} \times \sqrt{6}$ **b)** $\sqrt{3} \times \sqrt{6}$ **c)** $\sqrt{2}(\sqrt{2} + \sqrt{3})$

13 10A Expand and simplify:

$(\sqrt{2} - 2\sqrt{3})(3 - \sqrt{6})$
$= \sqrt{2}(3 - \sqrt{6}) - 2\sqrt{3}(3 - \sqrt{6})$
$= 3\sqrt{2} - \sqrt{12} - 6\sqrt{3} + 2\sqrt{18}$
$= 3\sqrt{2} - 2\sqrt{3} - 6\sqrt{3} + 6\sqrt{2}$
$= \underline{9\sqrt{2} - 8\sqrt{3}}$

a) $(\sqrt{3} + \sqrt{2})(1 + \sqrt{6})$
b) $(\sqrt{6} + \sqrt{2})(\sqrt{2} + \sqrt{6})$
c) $(3\sqrt{5} - \sqrt{2})(2\sqrt{2} - 1)$
d) $(2\sqrt{2} + \sqrt{5})(3\sqrt{2} - \sqrt{10})$

14 10A Evaluate each of the following:

$9^{\frac{3}{2}} = (\sqrt[2]{9})^3$
$= 3^3$
$= \underline{27}$

a) $4^{\frac{1}{2}}$ **b)** $32^{\frac{1}{5}}$ **c)** $125^{\frac{2}{3}}$
d) $64^{\frac{3}{2}}$ **e)** $(9b^2)^{\frac{3}{2}}$ **f)** $(4x^6)^{\frac{3}{2}}$

15 10A Simplify each of the following:

a) $3^{1/2}\times3^{1/2}$ **b)** $10^{5/2}\times10^{-3/2}$ **c)** $2x^{3/2}\times3x^{-1/2}$ **d)** $x^{5/2}\times x^{-3/2}\times x^{1/2}$
e) $9 \div 9^{1/2}$ **f)** $4^{3/4}\div4^{1/4}$ **g)** $9x^{1/2}\div3x^{-1/2}$ **h)** $4a^{3/4}\div2a^{1/4}$
i) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ **j)** $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **k)** $\left(\frac{8}{64}\right)^{-\frac{2}{3}}$ **l)** $(x^{1/3}y^4)^{3/2}$

16 10A Rewrite each following index as a log:

a) $100 = 10^2$ **b)** $32 = 2^5$ **c)** $243 = 3^5$ **d)** $0.1 = 10^{-1}$

17 10A Rewrite each following log as an index:

a) $\log_2 8 = 3$ **b)** $\log_{10} 10 = 1$ **c)** $\log_5 125 = 3$ **d)** $\log_{10} 0.01 = -2$

18 10A What is the value of each of the following logs:

a) $\log_2 16$ **b)** $\log_5 25$ **c)** $\log_4 64$ **d)** $\log_{10} 0.001$

19 10A Simplify the following:

$\log_{10}2 + \log_{10}5$
$= \log_{10}2\times5$
$= \log_{10}10$
$= \underline{1}$

$\log_{10}150 - \log_{10}1.5$
$= \log_{10}150\div1.5$
$= \log_{10}100$
$= \underline{2}$

a) $\log_{10}25 + \log_{10}4$ **b)** $\log_{10}125 + \log_{10}8$
c) $\log_8 16 + \log_8 4$ **d)** $\log_6 9 + \log_6 4$
e) $\log_6 12 + \log_6 4 + \log_6 9 + \log_6 3$
f) $\log_4 112 - \log_4 7$ **g)** $\log_{10}600 - \log_{10}6$
h) $\log_3 54 - \log_3 6$ **i)** $\log_5 500 - \log_5 4$
j) $\log_3 15 + \log_3 54 - \log_3 10$
k) $\log_5 5^7$ **l)** $\log_{10} 100^3$

Quadratics

6

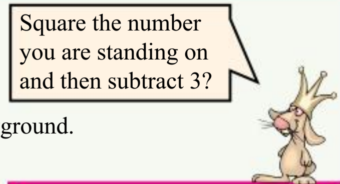
Number and Algebra → Patterns and Algebra

★ Expand binomial products and factorise monic quadratic expressions using a variety of strategies.

- explore the method of completing the square to factorise quadratic expressions and solve quadratic equations.
- identify and use common factors, including binomial expressions, to factorise algebraic expressions using the technique of grouping in pairs.
- use the identities for perfect squares and the difference of squares to factorise quadratic expressions.

A TASK

Physically demonstrate quadratic functions:



Example:

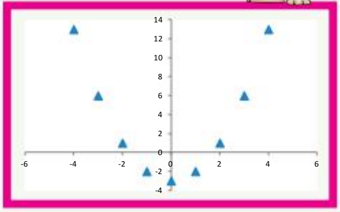
* Mark out −4,−3,−2,−1, 0, 1, 2, 3, 4 on the x-axis on the playground.

* Mark out −14 to 14 on the y-axis.

* Have members of your group stand on the points on the x-axis.

* For the function $y=x^2-3$, give the instruction "Square the number you are standing on and subtract three".

* Ask your group to step out the answers along the y-axis".



Make a powerpoint of your quadratic demonstrations by inserting digital photos in each slide.

A LITTLE BIT OF HISTORY

Rene Descartes (1596-1650) made a major contribution to analytical geometry. Descartes was quite clever at using geometry to solve problems.

Example: To find √3

1 Draw a line of length 3, AB

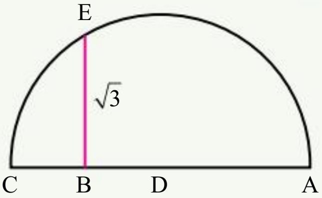
2 Add length 1 to AB, ABC

3 Find the midpoint, D

4 Draw a semi-circle centre D, radius AD

5 Draw a vertical line from B, BE

The length of BE = √3



Distributive Law

Distribute - to spread out, to cover everything.

Multiply each inside term by the outside term.

$$a(b + c) = ab + ac$$

Exercise 6.1

Expand each of the following:

$3(x + 1) = \underline{3x + 3}$

${}^{-}2(x - 4) = \underline{{}^{-}2x + 8}$

$a(a - 1) = \underline{a^2 - a}$

${}^{-}x(x + 5) = \underline{{}^{-}x^2 - 5x}$

1 $6(x + 2)$
2 $4(a - 3)$
3 ${}^{-}2(y + 5)$
4 ${}^{-}4(x - 3)$
5 $x(x + 2)$
6 $b(b - 2)$
7 ${}^{-}p(p + 3)$
8 ${}^{-}x(x - 3)$

$3 \times 2 = 6$
$3 \times {}^{-}2 = {}^{-}6$
${}^{-}3 \times 2 = {}^{-}6$
${}^{-}3 \times {}^{-}2 = 6$

+ times + = +
+ times − = −
− times + = −
− times − = +

Simplify each of the following by expanding and then collecting like terms:

$4(x + 3) - 3(x + 2)$
$= 4x + 12 - 3x - 6$
$= \underline{x + 6}$

${}^{-}x(x - 1) - 2(x - 4)$
$= {}^{-}x^2 + x - 2x + 8$
$= \underline{{}^{-}x^2 - x + 8}$

9 $3(x + 5) + 2(x + 3)$
10 $4(x + 1) + 3(x + 1)$
11 $2(x + 3) - 3(x + 2)$
12 $2(x + 5) - 3(x + 2)$
13 $a(a + 2) - a(a - 6)$
14 $x(x - 1) + x(x - 2)$
15 ${}^{-}d(d - 2) - d(d + 3)$
16 ${}^{-}y(y - 5) - y(y - 6)$

$ax^2 + bx + x$ is a quadratic because the highest power is two.

$(x + 4)(x + 3)$
$= x(x + 3) + 4(x + 3)$
$= x^2 + 3x + 4x + 12$
$= \underline{x^2 + 7x + 12}$

$(2x + 5)(x + 4)$
$= 2x(x + 4) + 5(x + 4)$
$= 2x^2 + 8x + 5x + 20$
$= \underline{2x^2 + 13x + 20}$

$(x - 2)(x + 3)$
$= x(x + 3) - 2(x + 3)$
$= x^2 + 3x - 2x - 6$
$= \underline{x^2 + x - 6}$

$(2x - 5)(x - 4)$
$= 2x(x - 4) - 5(x - 4)$
$= 2x^2 - 8x - 5x + 20$
$= \underline{2x^2 - 13x + 20}$

17 $(x + 2)(x + 1)$
18 $(x + 3)(x + 1)$
19 $(x + 1)(x + 4)$
20 $(x + 3)(x + 2)$

$$(x + 2)(x + 1) = x(x + 1) + 2(x + 1)$$

21 $(2x + 1)(x + 3)$
22 $(3x + 2)(x + 1)$
23 $(5x + 3)(x + 2)$
24 $(3x + 1)(2x + 4)$

$$(2x + 1)(x + 3) = 2x(x + 3) + 1(x + 3)$$

25 $(x - 1)(x + 3)$
26 $(x - 2)(x + 1)$
27 $(x + 5)(x - 1)$
28 $(x + 3)(x - 4)$

$$(x - 1)(x + 3) = x(x + 3) - 1(x + 3)$$

29 $(2x - 1)(x - 3)$
30 $(3x - 2)(x - 1)$
31 $(5x - 3)(x - 2)$
32 $(3x - 1)(2x - 4)$

$$(2x - 1)(x - 3) = 2x(x - 3) - 1(x - 3)$$

Factorisation

$$\begin{tikzcd}a(b + c) \arrow[r, bend left, "\text{Distribute}"] & ab + ac \arrow[l, bend left, "\text{Factorise}"]\end{tikzcd}$$

Factorisation turns a sum into a product.

Algebra is an essential tool in thousands of careers and is fundamental to solving millions of problems.

The more practice you get the easier it becomes.

Exercise 6.2

Factorise each of the following:

$6x + 9$ $= 3\times2x + 3\times3$ $= \underline{3(2x + 3)}$	$4xy - 6x$ $= 2x\times2y - 2x\times3$ $= \underline{2x(2y - 3)}$	$10x^2 - 8x$ $= 2x\times5x - 2x\times4$ $= \underline{2x(5x - 4)}$

1 $6a + 9$ **2** $4ab - 6a$ **3** $10c^2 - 8c$

4 $14x + 10$ **5** $4ab - 6b$ **6** $8d^2 - 6d$

7 $9c + 12$ **8** $8xy + 10x$ **9** $16x^2 - 12x$

10 $6x + 10$ **11** $12st - 15t$ **12** $15p^5 - 36p^3$

$18 - 20a^4$ $= 2\times9 - 2\times10a^4$ $= \underline{2(9 - 10a^4)}$	$4x^5 - 10x$ $= 2x\times2x^4 - 2x\times5$ $= \underline{2x(2x^4 - 5)}$	$8x^5 - 12x^3$ $= 4x^3\times2x^2 - 4x^3\times3$ $= \underline{4x^3(2x^2 - 3)}$

13 $6 - 9b^3$ **14** $4x^4 - 6x$ **15** $10c^4 - 8c^5$

16 $4 + 10a^4$ **17** $4x^5 + 8x$ **18** $9d^5 + 6d^3$

19 $9 - 12x^5$ **20** $8y^3 - 10x$ **21** $9x^2 - 12x^3$

22 $6 + 10y^2$ **23** $12x^2 + 15x$ **24** $12y^5 - 36y^3$

25 $18x^2 + 24x^3$ **26** $21x^4 - 15x$ **27** $9b^7 + 24b^3$

The common term, a+b, is taken out and put at the front.

$$\mathbf{c\underline{(a + b)} + d\underline{(a + b)} = \underline{(a + b)}(c + d)}$$

$x(x - 5) + 4(x - 5)$ $= \underline{(x - 5)(x + 4)}$	$x(x - 5) - 3(x - 5)$ $= \underline{(x - 5)(x - 3)}$

28 $x(x + 5) + 3(x + 5)$ **29** $x(x + 5) - 4(x + 5)$

30 $x(x - 1) + 4(x - 1)$ **31** $x(x - 1) - 2(x - 1)$

32 $x(x - 6) + 3(x - 6)$ **33** $x(x - 5) - 4(x - 5)$

34 $x(x - 2) + 5(x - 2)$ **35** $x(x - 3) - 7(x - 3)$

Factorising by Grouping Pairs

$$\underline{c(a + b)} + \underline{d(a + b)} = \underline{(a + b)}(c + d)$$

The common term, a+b, is taken out and put at the front.

Factorisation turns a sum into a product.

Exercise 6.3

Factorise each of the following:

$a(x - 1) + b(x - 1)$
$= \underline{(x - 1)(a + b)}$

$r(t - 5) - 3(t - 5)$
$= \underline{(t - 5)(r - 3)}$

1 $a(b + c) + d(b + c)$
2 $x(y + 1) + 4(y + 1)$
3 $b(c - 6) + 3(c - 6)$
4 $z(p - 2) + 5(p - 2)$

5 $s(s + 5) - 2(s + 5)$
6 $x(x + 1) - 3(x + 1)$
7 $t(x - 3) - 4(x - 3)$
8 $u(v - 2) - 7(v - 2)$

$$\mathbf{a\underline{(b + c)} + d\underline{(b + c)} = (b + c)(a + d)}$$

These are already grouped in pairs.

$ax - a + bx - b$
$= (ax - a) + (bx - b)$
$= a(x - 1) + b(x - 1)$
$= \underline{(x - 1)(a + b)}$

$ab - ac + b - c$
$= a(b - c) + 1(b - c)$
$= \underline{(b - c)(a + 1)}$

9 $ab + ac + db + dc$
10 $2x + 2y + bx + by$
11 $ab + ad + 3b + 3d$
12 $3ax + 2ay + 9x + 6y$

13 $ab - ac + b - c$
14 $ax - ay + x - y$
15 $ef - e + af - a$
16 $x^2 - 2x + xy - 2y$

$ab + ac + db + dc = (ab + ac) + (db + dc)$

Group the first two terms and the second two terms.

Would you get the same answer if you swapped the two middle terms?

$x^2 + 2x + 3x + 6$
$= x(x + 2) + 3(x + 2)$
$= \underline{(x + 2)(x + 3)}$

$x^2 + 3x - 2x - 6$
$= x(x + 3) + {}^{-}2(x + 3)$
$= \underline{(x + 3)(x - 2)}$

$5x^2 + 10x + 3x + 6$
$= 5x(x + 2) + 3(x + 2)$
$= \underline{(x + 2)(5x + 3)}$

$2x^2 + 6x - 3x - 9$
$= 2x(x + 3) + {}^{-}3(x + 3)$
$= \underline{(x + 3)(2x - 3)}$

17 $x^2 + x + 4x + 4$
18 $x^2 + x + 3x + 3$
19 $x^2 + 2x + 6x + 12$
20 $x^2 - 6x + 3x - 18$

21 $x^2 + 5x - 2x - 10$
22 $x^2 + 1x - 3x - 3$
23 $x^2 + 3x - 4x - 12$
24 $x^2 + 7x - 2x - 14$

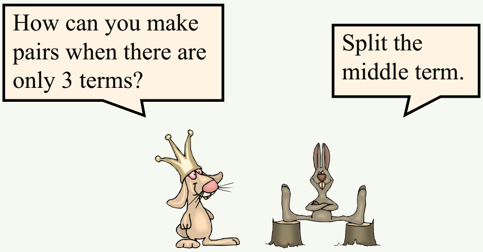
25 $3x^2 + x + 12x + 4$
26 $5x^2 + x + 15x + 3$
27 $2x^2 + 4x + 6x + 12$
28 $4x^2 - 24x + 3x - 18$

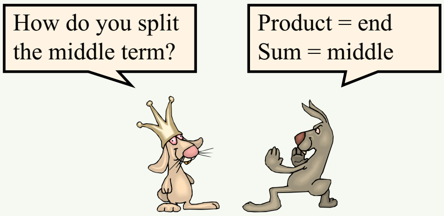
29 $3x^2 + 15x - 2x - 10$
30 $2x^2 + 2x - 3x - 3$
31 $4x^2 + 4x - 3x - 3$
32 $2x^2 + 14x - 3x - 21$

$x^2 - 6x + 3x - 18 = x(x - 6) + 3(x - 6)$

$3x^2 + 15x - 3x - 10$
$= 3x(x + 5) - 1(3x + 10)$

$-1\times-10 = 10$





$$x^2 + 5x + 6$$
$$= x^2 + 3x + 2x + 6$$
$$= x(x + 3) + 2(x + 3)$$
$$= (x + 3)(x + 2)$$

$3+2 = 5$ $\quad$ $3\times2 = 6$

$$x^2 + 5x + 6$$
$$= x^2 + 3x + 2x + 6$$

Exercise 6.4

Factorise each of the following:

$$x^2 + 6x + 8$$
$$= x^2 + 4x + 2x + 8$$
$$= x(x + 4) + 2(x + 4)$$
$$= \underline{(x + 4)(x + 2)}$$

$$x^2 + 8x + 12$$
$$= x^2 + 6x + 2x + 12$$
$$= x(x + 6) + 2(x + 6)$$
$$= \underline{(x + 6)(x + 2)}$$

$$x^2 + x - 12$$
$$= x^2 + 4x - 3x - 12$$
$$= x(x + 4) - 3(x + 4)$$
$$= \underline{(x + 4)(x - 3)}$$

$$x^2 - 14x - 15$$
$$= x^2 - 15x + 1x - 15$$
$$= x(x - 15) + 1(x - 15)$$
$$= \underline{(x - 15)(x + 1)}$$

$$x^2 - 9x + 8$$
$$= x^2 - 8x - 1x + 8$$
$$= x(x - 8) - 1(x - 8)$$
$$= \underline{(x - 8)(x - 1)}$$

$$t^2 - 13t - 30$$
$$= t^2 - 15t + 2t - 30$$
$$= t(t - 15) + 2(t - 15)$$
$$= \underline{(t - 15)(t + 2)}$$

1 $x^2 + 7x + 10$
2 $x^2 + 7x + 12$
3 $x^2 + 5x + 4$
4 $x^2 + 7x + 6$
5 $x^2 + 8x + 12$
6 $x^2 + 8x + 15$
7 $x^2 + 6x + 8$
8 $x^2 + 9x + 14$
9 $x^2 + 8x + 16$
10 $x^2 + 9x + 18$
11 $x^2 + 9x + 20$
12 $x^2 + 11x + 18$

13 $x^2 + x - 6$
14 $x^2 + 2x - 8$
15 $x^2 + 5x - 6$
16 $x^2 + 3x - 10$

17 $x^2 - 2x - 8$
18 $x^2 - 3x - 10$
19 $x^2 - 5x - 6$
20 $x^2 - 4x - 12$

21 $x^2 - 6x + 8$
22 $x^2 - 6x + 9$
23 $x^2 - 10x + 9$
24 $x^2 - 8x + 12$

25 $a^2 + 13a + 22$
26 $y^2 + 10y + 16$
27 $x^2 + 2x - 15$
28 $p^2 + 8p - 20$
29 $k^2 - 4k - 21$
30 $x^2 - 13x + 30$
31 $c^2 - 20c + 36$
32 $b^2 - 5b - 24$

$x^2 + 7x + 10$
Need Product=10 and Sum= 7
$10\times1=10$ ✓ $\quad$ $10+1=11$ ×
$5\times2=10$ ✓ $\quad$ $5+2=7$ ✓
$x^2 + 5x + 2x + 10$

$x^2 + x - 6$
Need Product=$^-6$ and Sum= 1
$2\times{}^-3={}^-6$ ✓ $\quad$ $2+{}^-3={}^-1$ ×
$6\times{}^-1={}^-6$ ✓ $\quad$ $6+{}^-1=5$ ×
$3\times{}^-2={}^-6$ ✓ $\quad$ $3+{}^-2=1$ ✓
$x^2 + 3x - 2x - 6$

$x^2 - 2x - 8$
Need Product=$^-8$ and Sum= $^-2$
$4\times{}^-2={}^-8$ ✓ $\quad$ $4+{}^-2=2$ ×
${}^-4\times2={}^-8$ ✓ $\quad$ ${}^-4+2={}^-2$ ✓

$x^2 - 6x + 8$
Need Product=8 and Sum= $^-6$
${}^-8\times{}^-1=8$ ✓ $\quad$ ${}^-8+{}^-1={}^-9$ ×
${}^-4\times{}^-2=8$ ✓ $\quad$ ${}^-4+{}^-2={}^-6$ ✓

Difference of Two Squares

$(a + b)(a - b)$
$= a(a - b) + b(a - b)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$

or

$(a - b)(a + b)$
$= a(a + b) - b(a + b)$
$= a^2 + ab - ab + b^2$
$= a^2 - b^2$

$$\begin{tikzcd} a^2 \arrow[d, leftarrow] & - & b^2 \arrow[d, leftarrow] \\ \text{square} & - & \text{square} \end{tikzcd}$$

Difference of two squares

Square the first term.
Square the second term.
Stick a − in the middle.

This looks easy.

Exercise 6.5

Expand and simplify the following products:

$(x - 5)(x + 5)$
$= x(x + 5) - 5(x + 5)$
$= x^2 + 5x - 5x - 25$
$= \underline{x^2 - 25}$

$(3 + 2b)(3 - 2b)$
$= 3(3 - 2b) + 2b(3 - 2b)$
$= 9 - 6b + 6b - 4b^2$
$= \underline{9 - 4b^2}$

1 $(x - 2)(x + 2)$
2 $(x - 6)(x + 6)$
3 $(x + 4)(x - 4)$
4 $(x + 3)(x - 3)$
5 $(x + 1)(x - 1)$
6 $(x + 5)(x - 5)$
7 $(x - 4)(x + 4)$
8 $(x - 3)(x + 3)$
9 $(3 - 2b)(3 + 2b)$
10 $(2x - 1)(2x + 1)$
11 $(5 + 3d)(5 - 3d)$
12 $(x + y)(x - y)$
13 $(a + b)(a - b)$
14 $(3e - 2f)(3e + 2f)$
15 $(t - 4g)(t + 4g)$
16 $(5w - x)(5w + x)$

Exercise 6.6

Factorise each of the following:

$x^2 - 25$
$= x^2 - 5^2$
$= \underline{(x - 5)(x + 5)}$

$4x^2 - y^2$
$= (2x)^2 - y^2$
$= \underline{(2x + y)(2x - y)}$

$x^2 - 5$
$= (x)^2 - (\sqrt{5})^2$
$= \underline{(x - \sqrt{5})(x + \sqrt{5})}$

$x^6 - 3$
$= (x^3)^2 - (\sqrt{3})^2$
$= \underline{(x^3 + \sqrt{3})(x^3 - \sqrt{3})}$

1 $x^2 - 9$
2 $a^2 - 16$
3 $1 - y^2$
4 $x^2 - 81$
5 $4a^2 - 1$
6 $9x^2 - 49$
7 $x^2 - 36a^2$
8 $100 - 9d^2$
9 $16a^2 - 25$
10 $4 - 9p^2$
11 $x^2 - 2$
12 $5 - d^2$
13 $3 - a^2$
14 $10 - 4m^2$
15 $x^2 - 20$
16 $7 - 2y^2$
17 $x^6 - 2$
18 $5 - d^4$
19 $y^6 - a^2$
20 $3 - 4m^6$
21 $a^2 - b^4$
22 $1 - 2y^8$

$$\begin{tikzcd} \mathbf{(a - b)(a + b)} \arrow[rr, bend left, "\text{Distribute}"] & & \mathbf{a^2 - b^2} \arrow[ll, bend left, "\text{Factorise}"] \end{tikzcd}$$

Exercise 6.7

Copy and complete the following:

$a^2 - b^2 = (a + b)(a - b)$
or
$a^2 - b^2 = (a - b)(a + b)$

Same thing

1 $x^2 - 4 = (x + 2)(? - ?)$
2 $16 - b^2 = (16 + ?)(16 - ?)$
3 $a^2 - 9 = (? - ?)(a + 3)$
4 $4 - 9b^2 = (2 - ?)(2 + ?)$
5 $?^2 - 9 = (x + 3)(? - ?)$
6 $? - ?^2 = (1 - 5d)(? + ?)$
7 $x^2 - ? = (? + \sqrt{7})(? - ?)$
8 $? - ? = (a^3 + b)(a^3 - b)$
9 $2x^2 - 9 = (? + ?)(? - ?)$
10 $x^5 - 3y^6 = (? - ?)(? + ?)$

Perfect Squares

$$\begin{aligned} &(a + b)^2 \\ &= (a + b)(a + b) \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2 \end{aligned}$$

and

$$\begin{aligned} &(a - b)^2 \\ &= (a - b)(a - b) \\ &= a(a - b) - b(a - b) \\ &= a^2 - ab - ab + b^2 \\ &= a^2 - 2ab + b^2 \end{aligned}$$

$$\underset{\text{square}}{(a + b)^{\underset{\uparrow}{2}}} \qquad \underset{\text{square}}{(a - b)^{\underset{\uparrow}{2}}}$$

Exercise 6.8

Expand and simplify the following perfect squares:

$(x + 3)^2 = (x + 3)(x +3)$
$= x(x + 3) + 3(x + 3)$
$= x^2 + 3x + 3x + 9$
$= \underline{x^2 + 6x + 9}$

$(x - 3)^2 = (x - 3)(x - 3)$
$= x(x - 3) - 3(x - 3)$
$= x^2 - 3x - 3x + 9$
$= \underline{x^2 - 6x + 9}$

$(2x + 3)^2$
$= (2x + 3)(2x +3)$
$= 2x(2x + 3) + 3(2x + 3)$
$= 4x^2 + 6x + 6x + 9$
$= \underline{4x^2 + 12x + 9}$

1 $(x + 2)^2$ **2** $(x + 6)^2$
3 $(y + 4)^2$ **4** $(b + 3)^2$
5 $(1 + d)^2$ **6** $(2x + 5)^2$
7 $(x + y)^2$ **8** $(a + b)^2$

9 $(x - 2)^2$ **10** $(x - 5)^2$
11 $(y - 1)^2$ **12** $(3 - n)^2$
13 $(2 - 3d)^2$ **14** $(3x - 2)^2$
15 $(x - y)^2$ **16** $(a - b)^2$

17 $(2x + 1)^2$ **18** $(2x + 5)^2$
19 $(3y + 1)^2$ **20** $(3y + 2)^2$
21 $(1 + 3x)^2$ **22** $(2 + 5y)^2$
23 $(2x - 1)^2$ **24** $(1 - 2b)^2$
25 $(3 - 5y)^2$ **26** $(2x - 3y)^2$

Exercise 6.9

factorise each of the following:

$a^2 + 2ab + b^2$
$= \underline{(a + b)^2}$

$x^2 - 2xy + y^2$
$= \underline{(x - y)^2}$

$x^2 + 6x + 9$
$= x^2 + 2\times3x + 9$
$= \underline{(x + 3)^2}$

$x^2 - 6x + 9$
$= x^2 - 2\times3x + 9$
$= \underline{(x - 3)^2}$

1 $x^2 + 2xy + y^2$ **2** $m^2 + 2mn + n^2$
3 $u^2 + 2uv + v^2$ **4** $y^2 + 2yz + z^2$

5 $a^2 - 2ab + b^2$ **6** $c^2 - 2cd + d^2$
7 $p^2 - 2pt + t^2$ **8** $k^2 + 2kz + z^2$

9 $x^2 + 4x + 4$ **10** $x^2 + 2x + 1$
11 $x^2 + 8x + 16$ **12** $x^2 + 10x + 25$
13 $x^2 + 12x + 36$ **14** $x^2 + 16x + 64$

15 $x^2 - 2x + 1$ **16** $x^2 - 4x + 4$
17 $x^2 - 6x + 9$ **18** $x^2 - 8x + 16$
19 $x^2 - 10x + 25$ **20** $x^2 - 12x + 36$

How do you know if $9x^2 - 12x + 4$ will factorise to a perfect square? →

$(3x)^2 - 12x + (-2)^2$
Does $2\times(3x)\times(-2)$ = middle term?
If so then $(3x - 2)^2$

$4x^2 + 20x + 25$
$= (2x)^2 + 2\times2x\times5 + (5)^2$
$= \underline{(2x + 5)^2}$

21 $4x^2 + 12x + 9$
22 $4x^2 - 12x + 9$ **23** $9x^2 - 12x + 4$
24 $9y^2 + 24y + 16$ **25** $9a^2 + 6x + 1$
26 $4x^2 - 4x + 1$ **27** $25x^2 - 30x + 9$

Solving Quadratics

This is a quadratic because the highest power of the unknown, x,is two.

Solving means find the values of x that makes this true.

$$ax^2 + bx + c = 0$$

Thousands of real life problems can be written as quadratics.

x^2 suggests that there could be none, one, or two solutions.

Exercise 6.10

Solve the following quadratics:

$x^2 + 6x + 9 = 0$
$(x + 3)^2 = 0$ {perfect square}
$x + 3 = 0$ {square root both sides}
$\underline{x = {}^-3}$ {inverse of +3}
Check: $({}^-3)^2 + 6\times{}^-3 + 9 = 0$ ✓

$x^2 - 8x + 16 = 0$
$(x - 4)^2 = 0$ {perfect square}
$x - 4 = 0$ {square root both sides}
$\underline{x = 4}$ {inverse of −4}
Check: $(4)^2 - 8\times4 + 16 = 0$ ✓

$4x^2 + 12x + 9 = 0$
$(2x)^2 + 2\times2x\times3 + (3)^2 = 0$
$(2x + 3)^2 = 0$ {perfect square}
$2x + 3 = 0$ {square root both sides}
$2x = {}^-3$ {inverse of +3}
$\underline{x = {}^-1.5}$ {inverse of ×2}
Check: $4\times({}^-1.5)^2 + 12\times{}^-1.5 + 9 = 0$ ✓

$x^2 + 6x + 8 = 0$
$x^2 + 4x + 2x + 8 = 0$ {grouping pairs}
$x(x + 4) + 2(x + 4) = 0$ {factorising }
$(x + 4)(x + 2) = 0$ {factorising}
Either $x + 4 = 0$ or $x + 2 = 0$
$\underline{x = {}^-4}$ or $\underline{x = {}^-2}$
Check: $({}^-4)^2 + 6\times{}^-4 + 8 = 0$ ✓
Check: $({}^-2)^2 + 6\times{}^-2 + 8 = 0$ ✓

$x^2 - 3x - 10 = 0$
$x^2 - 5x + 2x - 10 = 0$ {grouping pairs}
$x(x - 5) + 2(x - 5) = 0$ {factorising }
$(x - 5)(x + 2) = 0$ {factorising}
Either $x - 5 = 0$ or $x + 2 = 0$
$\underline{x = 5}$ or $\underline{x = {}^-2}$
Check: $(5)^2 - 4\times5 - 10 = 0$ ✓
Check: $({}^-2)^2 - 4\times{}^-2 - 10 = 0$ ✓

1 $x^2 + 2x + 1 = 0$
2 $x^2 + 4x + 4 = 0$
3 $x^2 + 8x + 16 = 0$
4 $x^2 + 10x + 25 = 0$
5 $x^2 + 14x + 49 = 0$

6 $x^2 - 2x + 1 = 0$
7 $x^2 - 6x + 9 = 0$
8 $x^2 - 8x + 16 = 0$
9 $x^2 - 10x + 25 = 0$
10 $x^2 - 16x + 64 = 0$

11 $4x^2 + 12x + 9 = 0$
12 $9a^2 + 6x + 1 = 0$
13 $4x^2 + 8x + 4 = 0$
14 $4x^2 - 4x + 1 = 0$
15 $9y^2 + 24y + 16 = 0$
16 $9x^2 - 12x + 4 = 0$
17 $25x^2 - 30x + 9 = 0$

18 $x^2 + 5x + 4 = 0$
19 $x^2 + 3x + 2 = 0$
20 $x^2 + 5x + 6 = 0$
21 $x^2 + 7x + 6 = 0$
22 $x^2 + 8x + 12 = 0$
23 $x^2 + 8x + 15 = 0$
24 $x^2 + 6x + 8 = 0$
25 $x^2 + 9x + 14 = 0$

26 $x^2 + x - 6 = 0$
27 $x^2 + 5x - 6 = 0$
28 $x^2 - 2x - 8 = 0$
29 $x^2 - 5x - 6 = 0$
30 $x^2 - 4x - 12 = 0$
31 $x^2 - 6x + 9 = 0$
32 $x^2 - 10x + 9 = 0$
33 $x^2 - 8x + 12 = 0$

Completing the Square

Use $a^2 + 2ab + b^2 = (a + b)^2$
to solve quadratics.

$x^2 + 6x + 2 = 0$

$x^2 + 6x = {}^{-}2$

$x^2 + 6x + 9 = {}^{-}2 + 9$

$(x + 3)^2 = 7$

$x + 3 = \pm\sqrt{7}$

$x = {}^{-}3 \pm \sqrt{7}$

$x = {}^{-}0.35$ or $x = {}^{-}5.65$

$x^2 + 6x + 9 = (x + 3)^2$

Add 9 to both sides to complete the square.

$(\sqrt{7})^2 = 7$ and $(-\sqrt{7})^2 = 7$
Which is why there are two solutions:
$+\sqrt{7}$ or $-\sqrt{7}$ thus $\pm\sqrt{7}$

Exercise 6.11

Solve the following quadratics by completing the square:

$x^2 + 8x - 3 = 0$

$x^2 + 8x = 3$ {move constant term}

$x^2 + 8x + 16 = (x + 4)^2$

$x^2 + 8x + 16 = 3 + 16$ {+16 to complete square}

$(x + 4)^2 = 19$ {perfect square}

$x + 4 = \pm\sqrt{19}$ {square root both sides}

$x = {}^{-}4 \pm \sqrt{19}$ {inverse of +4}

$\underline{x = 0.36 \text{ or } x = {}^{-}8.36}$ {use calculator}

Check: $(0.36)^2 + 8\times0.36 - 3 = 0$ ✓

Check: $({}^{-}8.36)^2 + 8\times{}^{-}8.36 - 3 = 0$ ✓

$x^2 - 10x + 1 = 0$

$x^2 - 10x = {}^{-}1$ {move constant term}

$x^2 - 10x + 25 = (x - 5)^2$

$x^2 - 10x + 25 = {}^{-}1 + 25$ {+25 to complete square}

$(x - 5)^2 = 24$ {perfect square}

$x - 5 = \pm\sqrt{24}$ {square root both sides}

$x = 5 \pm \sqrt{24}$ {inverse of +4}

$\underline{x = 9.90 \text{ or } x = 0.10}$ {use calculator}

Check: $(9.90)^2 - 10\times9.90 + 1 = 0$ ✓

Check: $(0.10)^2 - 10\times0.10 + 1 = 0$ ✓

$x^2 - 3x - 4 = 0$

$x^2 - 3x = 4$ {move constant term}

$x^2 - 3x + (3/2)^2 = (x - 3/2)^2$ {3/2 = half middle number}

$x^2 - 3x + (1.5)^2 = 4 + (1.5)^2$ {complete square, 3/2=1.5}

$(x - 1.5)^2 = 6.25$ {perfect square}

$x - 1.5 = \pm\sqrt{6.25}$ {square root both sides}

$x = 1.5 \pm \sqrt{6.25}$ {inverse of +4}

$\underline{x = 4 \text{ or } x = {}^{-}1}$ {use calculator}

Check: $(9.90)^2 - 10\times9.90 + 1 = 0$ ✓

Check: $(0.10)^2 - 10\times0.10 + 1 = 0$ ✓

There is more about solving quadratics in Chapter 7

1 $x^2 + 6x - 1 = 0$
2 $x^2 + 2x - 3 = 0$
3 $x^2 + 4x - 4 = 0$
4 $x^2 + 4x - 6 = 0$
5 $x^2 + 6x - 2 = 0$
6 $x^2 + 10x - 5 = 0$
7 $x^2 + 2x + 1 = 0$
8 $x^2 + 6x + 4 = 0$
9 $x^2 + 8x + 3 = 0$
10 $x^2 + 12x + 2 = 0$

11 $x^2 - 2x - 3 = 0$
12 $x^2 - 4x - 1 = 0$
13 $x^2 - 6x - 5 = 0$
14 $x^2 - 8x - 7 = 0$
15 $x^2 - 10x - 2 = 0$
16 $x^2 - 12x - 4 = 0$
17 $x^2 - 14x + 4 = 0$
18 $x^2 - 6x + 2 = 0$
19 $x^2 - 4x + 1 = 0$
20 $x^2 - 8x + 3 = 0$

21 $x^2 - 3x - 1 = 0$
22 $x^2 + 3x - 3 = 0$
23 $x^2 + 5x - 5 = 0$
24 $x^2 - 5x - 2 = 0$
25 $x^2 + 7x - 4 = 0$
26 $x^2 - 7x - 3 = 0$
27 $x^2 + 9x + 1 = 0$
28 $x^2 - 9x + 2 = 0$
29 $x^2 + 11x + 5 = 0$
30 $x^2 - 11x + 2 = 0$

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 6.12

1 Spell Factorisation

2 Expand $(x - 2)^2$

3 Factorise $a^2 + 2ab + b^2$

4 Simplify: $\sqrt{12}$

5 Simplify: $4^{\frac{3}{2}}$

6 What is the value of: $\text{Log}_{10}1000$

7 Solve: $x + y = 9,\ xy = 14$

8 $^{-}2 - {}^{-}6$

9 $(x^{-3})^2$

10 Calculate 21^2

$(a + b)^2$
$= a^2 + 2ab + b^2$

$(a - b)^2$
$= a^2 - 2ab + b^2$

$$\begin{aligned} 21^2 &= (20 + 1)^2 \\ &= 20^2 + 2\times20\times1 + 1^2 \\ &= 400 + 40 + 1 \\ &= \underline{441} \end{aligned}$$

Exercise 6.13

1 Spell Quadratic

2 Expand $(x - 3)^2$

3 Factorise $x^2 + 2xy + y^2$

4 Simplify: $\sqrt{18}$

5 Simplify: $9^{\frac{3}{2}}$

6 What is the value of: $\text{Log}_{10}10000$

7 Solve: $x + y = 8,\ xy = 15$

8 $6 - {}^{-}6$

9 $(y^{-2})^2$

10 Calculate 19^2

'Act as if what you do makes a difference. It does' - William James.

$$\begin{aligned} 19^2 &= (20 - 1)^2 \\ &= 20^2 - 2\times20\times1 + 1^2 \\ &= 400 - 40 + 1 \\ &= \underline{361} \end{aligned}$$

Exercise 6.14

1 Spell Distributive

2 Expand $(x + 2)^2$

3 Factorise $x^2 - 2xy + y^2$

4 Simplify: $\sqrt{20}$

5 Simplify: $8^{\frac{2}{3}}$

6 What is the value of: $\text{Log}_{10}1\ 000\ 000$

7 Solve: $x + y = 10,\ xy = 9$

8 $^{-}3 - 5$

9 $(a^{-5})^2$

10 Calculate 22^2

'When written in Chinese, the word "crisis" is composed of two characters. One represents danger and the other represents opportunity' - John F. Kennedy.

Microbiologists study microscopic life and develop products to either combat or utilise microscopic life.

- Relevant school subjects are English, Mathematics and Science.
- Courses usually involve a University science degree with a major in microbiology.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 6.15

1 Without using a calculator, calculate each of the following:

a) $59^2 = (60 - 1)^2$
b) $61^2 = (60 + 1)^2$
c) $95^2 = (100 - 5)^2$
d) $59^2 = (100 + 5)^2$

$19^2 = (20 - 1)^2$
$= 20^2 - 2\times20\times1 + 1^2$
$= 400 - 40 + 1$
$= \underline{361}$

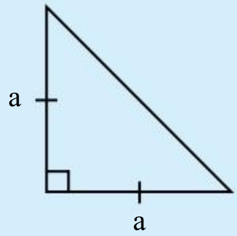
2 Simplify each of the following:

a) $5x - (2x - 5)$
b) $8x - 3(6 - 2x)$
c) $3x + 1 - (x - 2)$
d) $6a + 4b - (3a - 4b)$
e) $a(a - b) - b(a - b)$
f) $a(a - b) - a(b - a)$

Order of Operations:
1 ( ) brackets first.
2 × and ÷ from left to right.
3 + and − from left to right.

A right angled isosceles triangle has an area of 32.
What is the perimeter of the triangle?

Area $= \frac{1}{2}bh$
$32 = \frac{1}{2}a^2$
$64 = a^2$
$8 = a$



$hyp^2 = a^2 + a^2$
$hyp^2 = 64 + 64$
$hyp = \sqrt{128}$
$hyp = \sqrt{64\times2}$
$hyp = 8\sqrt{2}$

Perimeter $= 8 + 8 + 8\sqrt{2}$
$\underline{\text{Perimeter} = 16 + 8\sqrt{2}}$ or 27.31

Isosceles: having two sides equal.

Greek:
isos = equal
skelos = leg

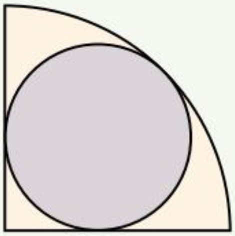
A forest is two kilometres across. How far can you go into it?
One kilometre.
After that, you are going out.

3 A right angled isosceles triangle has an area of 8.
What is the perimeter of the triangle?

4 A right angled isosceles triangle has an area of 18.
What is the perimeter of the triangle?

5 A right angled isosceles triangle has an area of 72.
What is the perimeter of the triangle?

6 What is the radius of the circle that fits in the quarter circle of radius 1?

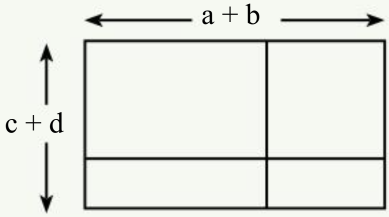


7 **Ŏ** is an operator between any two numbers such that:
a **Ŏ** b = $a^2 + ab + b^2$. Given that x **Ŏ** 2 = 0, what is the value of x?

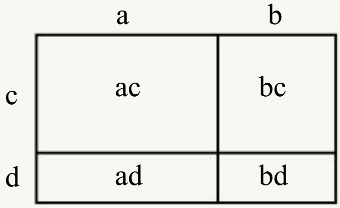
Investigations

Investigation 6.1 $(a + b)(c + d) = ac + ad + bc + bd$

1 An algebraic expression for the area of the rectangle on the right is $(a + b)(c + d)$



2 An algebraic expression for the sum of each of the four rectangles on the right is $ac + ad + bc + bd$



3 Thus: $(a + b)(c + d) = ac + ad + bc + bd$

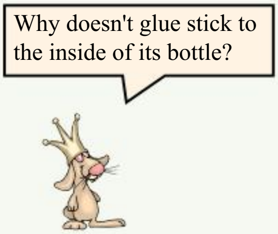
Similarly, show that
$(x + y)^2 = x^2 + 2xy + y^2$

Similarly, can you show that
$(x - y)^2 = x^2 - 2xy + y^2$

Investigation 6.2 Shortcut for squaring numbers?

Example: 53^2 $\{53 = 50 + 3\}$

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2\\ (50 + 3)^2 &= 50^2 + 2\times50\times3 + 3^2\\ &= 2500 + 300 + 9\\ &= \underline{2809}\end{aligned}$$



Example: 98^2 $\{98 = 100 - 2\}$

$$\begin{aligned}(a - b)^2 &= a^2 - 2ab + b^2\\ (100 - 2)^2 &= 100^2 - 2\times100\times2 + 2^2\\ &= 10000 - 400 + 4\\ &= \underline{9604}\end{aligned}$$

What is 203^2?
What is 198^2?
What is 896^2?

A Couple of Puzzles

Exercise 6.16

1 Which is larger 3^4 (ie 3×3×3×3) or 4^3 (ie 4×4×4)?

2 If $a^b = b^a$ and a and b are integers, what are the values of a and b?

3 A 4 m chain hangs between the tops of two 3 m high posts. How close must the posts be to each other so that the bottom of the chain hangs exactly 1 m above the ground?

A Game

Six Cut is played played on a calculator. You win if you can reduce the six-digit number to zero within six moves.

1 Enter any six-digit number on your calculator (No 0 and no repeated numbers). 839251

2 Either add, subtract, multiply, or divide the six-digit number by two-digit numbers until the result is zero.

1	Add 29	= 839280
2	Divide by 80	= 10491
3	Subtract 11	= 10480
4	Divide by 80	= 131
5	Subtract 90	= 41
6	Subtract 41	= 0

A Sweet Trick

1 Ask your audience to secretly write the result of a recent Rugby League, Union, or football game. 16 to 4

2 Your audience then enters the winning score on their calculator without letting you look. 16

3 Multiply by 20 16 × 20 = 320

4 Add 5 320 + 5 = 325

5 Multiply by 50 325 × 50 = 16250

6 Add the losing score 16250 + 4 = 16254

7 Multiply by 4 16254 × 4 = 65016

8 Subtract 1000 65016 − 1000 = 64016

9 Ask for their calculator

Make sure your audience presses '=' after each operation.

Divide their answer by 4000.
64016 ÷ 4000 = 16.004
Tell your audience the secret result.

Would this work for an AFL game?

Technology

Technology 6.1 Expanding and Factorising

Graphics calculators are capable of expanding and factorising:

1 Choose **expand** from the algebra menu.
2 Enter the algebraic expression: 3(4x − 5) to produce 12x − 15

1 Choose **factor** from the algebra menu.
2 Enter the algebraic expression: 12x − 15 to produce 3(4x − 5)

Technology 6.2 The Distributive Law and Factorising

There are a considerable number of resources about the Distributive Law and factorising on the Internet.

Try some of them.

Technology 6.3 Solving Quadratics by Factoring

Solving Quadratics

Watch videos on 'solving quadratics by factoring'.

In Alaska, where it gets very cold, π = 3.00. As you know, everything shrinks in the cold. They call it Eskimo pi.

Technology 6.4 Completing the Square

Completing the Square

Watch videos on 'completing the square'.

Joy of Mathematics

Can you find a reasonable video on the 'joy of mathematics'?

A biologist, a physicist and a mathematician were sitting in a street cafe watching the crowd. Across the street they saw a man and a woman entering a building. Ten minutes they reappeared together with a third person.

'They have multiplied', said the biologist.

'Oh no, an error in measurement', the physicist sighed.

'If exactly one person enters the building now, it will be empty again', the mathematician concluded.

Chapter Review 1

Exercise 6.17

1 Simplify each of the following by expanding and then collecting like terms:

$$\begin{aligned}&(2x-5)(x-4)\\&=2x(x+4)+5(x+4)\\&=2x^2+8x+5x+20\\&=\underline{2x^2+13x+20}\end{aligned}$$

a) $(x+2)(x+1)$

b) $(2x+1)(x+3)$

c) $(x-1)(x+3)$

d) $(2x-1)(x-3)$

$$(x+2)(x+1) = x(x+1) + 2(x+1)$$

2 Factorise each of the following:

$$\begin{aligned}&x^2+2x+3x+6\\&=x(x+2)+3(x+2)\\&=\underline{(x+2)(x+3)}\end{aligned}$$

a) $x^2+x+4x+4$

b) $3x^2+x+12x+4$

c) $3x^2+15x-2x-10$

3 Factorise each of the following:

$$\begin{aligned}&x^2+x-12\\&=x^2+4x-3x-12\\&=x(x+4)-3(x+4)\\&=\underline{(x+4)(x-3)}\end{aligned}$$

a) $x^2+8x+15$

b) x^2+x-6

c) $x^2-4x-12$

d) x^2-6x+8

$4+{}^{-}3 = 1$ **$4\times{}^{-}3 = {}^{-}12$**

$$\begin{aligned}&\mathbf{x^2 + x + {}^{-}12}\\&\mathbf{= x^2 + 4x + {}^{-}3x + 6}\end{aligned}$$

4 Solve each of the following:

$$\begin{aligned}x^2-8x+16&=0\\(x-4)^2&=0 &&\{\text{perfect square}\}\\x-4&=0 &&\{\text{square root both sides}\}\\\underline{x}&\underline{=4} &&\{\text{inverse of }-4\}\end{aligned}$$

Check: $(4)^2 - 8\times4 + 16 = 0$ ✓

$$\begin{aligned}4x^2+12x+9&=0\\(2x)^2+2\times2x\times3+(3)^2&=0\\(2x+3)^2&=0 &&\{\text{perfect square}\}\\2x+3&=0 &&\{\text{square root both sides}\}\\2x&={}^{-}3 &&\{\text{inverse of }+3\}\\\underline{x}&\underline{={}^{-}1.5} &&\{\text{inverse of }\times2\}\end{aligned}$$

Check: $4\times({}^{-}1.5)^2 + 12\times{}^{-}1.5 + 9 = 0$ ✓

a) $x^2+2x+1=0$

b) $x^2+4x+4=0$

c) $x^2-8x+16=0$

d) $x^2-10x+25=0$

e) $x^2-16x+64=0$

f) $4x^2+12x+9=0$

g) $x^2+5x+4=0$

h) $x^2+3x+2=0$

i) $x^2+5x+6=0$

j) $x^2+8x+15=0$

k) $x^2+6x+8=0$

l) $x^2+9x+14=0$

5 Solve the following quadratics by completing the square:

$$\begin{aligned}x^2-10x+1&=0\\x^2-10x&={}^{-}1 &&\{\text{move constant term}\}\\&x^2-10x+25=(x-5)^2\\x^2-10x+25&={}^{-}1+25 &&\{+25\text{ to complete square}\}\\(x-5)^2&=24 &&\{\text{perfect square}\}\\x-5&=\pm\sqrt{24} &&\{\text{square root both sides}\}\\x&=5\pm\sqrt{24} &&\{\text{inverse of }+4\}\\\underline{x=9.90}\text{ or }&\underline{x=0.10} &&\{\text{use calculator}\}\end{aligned}$$

Check: $(9.90)^2 - 10\times9.90 + 1 = 0$ ✓

Check: $(0.10)^2 - 10\times0.10 + 1 = 0$ ✓

a) $x^2+6x-1=0$

b) $x^2+2x-3=0$

c) $x^2+4x-4=0$

d) $x^2-2x-3=0$

e) $x^2-4x-1=0$

f) $x^2-6x-5=0$

g) $x^2-3x-1=0$

h) $x^2+3x-3=0$

i) $x^2-5x-2=0$

j) $x^2+7x-4=0$

Chapter Review 2

Exercise 6.18

1 Simplify each of the following by expanding and then collecting like terms:

$$\begin{aligned}&(2x-5)(x-4)\\&=2x(x+4)+5(x+4)\\&=2x^2+8x+5x+20\\&=\underline{2x^2+13x+20}\end{aligned}$$

a) $(x+3)(x+1)$
b) $(2x+2)(x+3)$
c) $(x-3)(x+1)$
d) $(2x-3)(x-2)$

$$(x+2)(x+1) = x(x+1) + 2(x+1)$$

2 Factorise each of the following:

$$\begin{aligned}&x^2+2x+3x+6\\&=x(x+2)+3(x+2)\\&=\underline{(x+2)(x+3)}\end{aligned}$$

a) $x^2+x+3x+3$
b) $3x^2+x+15x+5$
c) $2x^2+10x-3x-15$

3 Factorise each of the following:

$$\begin{aligned}&x^2+x-12\\&=x^2+4x-3x-12\\&=x(x+4)-3(x+4)\\&=\underline{(x+4)(x-3)}\end{aligned}$$

a) x^2+5x+6
b) x^2+2x-8
c) x^2-x-12
d) x^2-5x+6

$$4+{}^{-}3=1 \qquad 4\times{}^{-}3={}^{-}12$$

$$\begin{aligned}&x^2 + x + {}^{-}12\\&= x^2 + 4x + {}^{-}3x + 6\end{aligned}$$

4 Solve each of the following:

$$\begin{aligned}x^2-8x+16&=0\\(x-4)^2&=0 &&\{\text{perfect square}\}\\x-4&=0 &&\{\text{square root both sides}\}\\\underline{x}&\underline{=4} &&\{\text{inverse of }-4\}\end{aligned}$$

Check: $(4)^2-8\times4+16=0$ ✓

$$\begin{aligned}4x^2+12x+9&=0\\(2x)^2+2\times2x\times3+(3)^2&=0\\(2x+3)^2&=0 &&\{\text{perfect square}\}\\2x+3&=0 &&\{\text{square root both sides}\}\\2x&={}^{-}3 &&\{\text{inverse of }+3\}\\\underline{x}&\underline{={}^{-}1.5} &&\{\text{inverse of }\times2\}\end{aligned}$$

Check: $4\times({}^{-}1.5)^2+12\times{}^{-}1.5+9=0$ ✓

a) $x^2+2x+1=0$
b) $x^2+6x+9=0$
c) $x^2-6x+9=0$
d) $x^2-8x+16=0$
e) $x^2-12x+36=0$
f) $9x^2+6x+1=0$

g) $x^2+4x+3=0$
h) $x^2+6x+5=0$
i) $x^2-5x+6=0$
j) $x^2-2x-15=0$
k) $x^2-x-12=0$
l) $x^2-9x+14=0$

5 Solve the following quadratics by completing the square:

$$\begin{aligned}x^2-10x+1&=0\\x^2-10x&={}^{-}1 &&\{\text{move constant term}\}\end{aligned}$$

$x^2-10x+25=(x-5)^2$

$$\begin{aligned}x^2-10x+25&={}^{-}1+25 &&\{+25\text{ to complete square}\}\\(x-5)^2&=24 &&\{\text{perfect square}\}\\x-5&=\pm\sqrt{24} &&\{\text{square root both sides}\}\\x&=5\pm\sqrt{24} &&\{\text{inverse of }+4\}\\\underline{x=9.90}\text{ or }&\underline{x=0.10} &&\{\text{use calculator}\}\end{aligned}$$

Check: $(9.90)^2-10\times9.90+1=0$ ✓
Check: $(0.10)^2-10\times0.10+1=0$ ✓

a) $x^2+4x-3=0$
b) $x^2+2x-2=0$
c) $x^2+6x-2=0$
d) $x^2-2x-1=0$
e) $x^2-4x-1=0$
f) $x^2-8x-2=0$
g) $x^2-5x-1=0$
h) $x^2+3x-3=0$
i) $x^2-x-2=0$
j) $x^2+7x-5=0$

Solving Equations

7

Number and Algebra ➡ Linear and non-linear relationships

- ★ Solve linear equations involving simple algebraic fractions.
  - solve a wide range of linear equations, including those involving one or two simple algebraic fractions, and check solutions by substitution.
  - represent word problems, including those involving fractions, as equations and solve them to answer the question.
- ★ Solve simple quadratic equations using a range of strategies.
  - use a variety of techniques to solve quadratic equations, including grouping, completing the square, the quadratic formula and choosing two integers with the required product and sum.

Quadratic - the highest power of the variable is two.

$u^2 + 2P = 2h$
Air pressure and the Bernouilli effect.

$s = ut + ½at^2$
Distance travelled by accelerating object.

$$i\frac{\partial u}{\partial t} + \nabla^2 u + v(x)u = 0$$

Design of integrated circuits in mobile phones and computers.

A TASK

A graphics calculator can be used to solve quadratic equations.

Learn how to use a graphics calculator to solve quadratic equations such as:

$$\mathbf{6x^2 + 7x - 5 = 0}$$

The solutions are $x=0.5$, and $x = {}^{-}1.67$

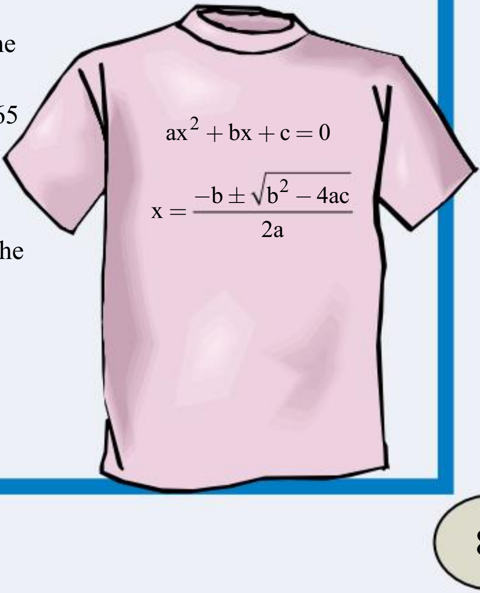
A LITTLE BIT OF HISTORY

The Babylonians (about 400 BC) appear to be the first to solve quadratic equations.

The Hindu mathematician Brahmagupta (598-665 AD) essentially used the quadratic formula to solve quadratics. Brahmagupta used letters for variables and considered negative numbers.

al-Khwarizmi (c 820 AD) also essentially used the quadratic formula to solve quadratics but didn't consider negative solutions.

Abraham bar Hiyya Ha-Nasi was the first to publish a book in 1145 which used the complete quadratic formula.



Solving Linear Equations

To solve an equation is to find the value of the unknown number (the variable) in the equation.

The formal way to solve an equation is to choose to do the same thing to each side of the equation until only the unknown number remains on one side of the equation.

Exercise 7.1

Solve each of the following equations:

$x + 7 = 5$ $x + 7 - 7 = 5 - 7$ $\underline{x = {}^{-}2}$	$5a = {}^{-}15$ $5a \div 5 = {}^{-}15 \div 5$ $\underline{a = {}^{-}3}$
$c - 4 = 7$ $c - 4 + 4 = 7 + 4$ $\underline{c = 11}$	$d \div 3 = 5$ $d \div 3 \times 3 = 5 \times 3$ $\underline{d = 15}$

The inverse of + is −
The inverse of − is +
The inverse of × is ÷
The inverse of ÷ is ×

1 $x + 8 = 3$
2 $b - 7 = {}^{-}11$
3 $5y = 25$
4 $t \div 4 = 3$
5 $a + 11 = 43$
6 $21m = 105$
7 $z \div 7 = 11$
8 ${}^{-}2w = 48$
9 $x - 29 = 41$

$4(d - 3) = {}^{-}20$
$4d - 12 = {}^{-}20$
$4d = {}^{-}20 + 12$
$4d = {}^{-}8$
$d = {}^{-}8 \div 4$
$\underline{d = {}^{-}2}$
Check: $4({}^{-}2 - 3) = 4 \times {}^{-}5 = {}^{-}20$ ✓

10 $3(x + 4) = 21$
11 ${}^{-}6(m - 2) = 18$
12 $2(a - 3) = 4$
13 $4(x + 2) = 16$
14 $2(x + 3) = {}^{-}16$
15 ${}^{-}2(f + 5) = 10$
16 $3(t - 5) = 12$
17 $4(p - 3) = {}^{-}20$
18 $5(b + 4) = 15$
19 $5(a + 6) = 25$
20 ${}^{-}4(n - 7) = 12$
21 $7(x - 3) = {}^{-}28$
22 $4(n - 7) = 12$
23 ${}^{-}3(x + 5) = 12$

${}^{-}5(2x - 3) = 25$
${}^{-}10x + 15 = 25$
${}^{-}10x = 25 - 15$
${}^{-}10x = 10$
$x = 10 \div {}^{-}10$
$\underline{x = {}^{-}1}$
Check: ${}^{-}5(2 \times {}^{-}1 - 3) = {}^{-}5 \times {}^{-}5 = 25$ ✓

24 $3(x - 4) = 12$
25 $2(2x - 5) = 10$
26 ${}^{-}2(x - 3) = 14$
27 $6(2x + 4) = {}^{-}36$
28 $3(x + 5) = {}^{-}18$
29 ${}^{-}4(5x - 4) = 24$
30 $4(2x - 2) = 16$
31 $2(2x - 5) = {}^{-}10$
32 ${}^{-}5(2d + 7) = 65$
33 $7({}^{-}3x + 2) = 35$
34 $5(2x - 3) = {}^{-}15$
35 $3(5x - 4) = {}^{-}15$
36 $7(2x - 1) = 21$
37 $5({}^{-}5x - 4) = 20$

$3({}^{-}2x - 3) - 4x = {}^{-}17$
${}^{-}6x - 9 - 4x = {}^{-}17$
${}^{-}10x - 9 = {}^{-}17$
${}^{-}10x = {}^{-}17 + 9$
${}^{-}10x = {}^{-}8$
$x = {}^{-}8 \div {}^{-}10$
$\underline{x = 0.8}$
Check: $3({}^{-}2 \times 0.8 - 3) - 4 \times 0.8$
$3 \times {}^{-}4.6 - 3.2 = {}^{-}17$ ✓

38 $2(x - 3) + 3x = 9$
39 $5(x - 3) - 2x = 9$
40 $3(x + 2) + 2x = 11$
41 $3(2x - 2) + 4x = 14$
42 $3x + 2(x + 2) = {}^{-}11$
43 $x + {}^{-}3(x - 1) = 5$
44 $2x + 3(x - 1) + 3x = 13$
45 $5x + 2(2x + 3) - x + 5 = 35$
46 ${}^{-}9x + 5x + 3(1 - 2x) + 5 = 29$

If the unknown occurs more than once, put the unknowns together ($5x - 3x = 2x$).

Solving Linear Equations

$$\begin{aligned}
5x - 3 &= 2x + 9 \\
5x - 3 - 2x &= 9 \\
3x - 3 &= 9 \\
3x &= 9 + 3 \\
3x &= 12 \\
x &= 12 \div 3 \\
\underline{x} &\underline{= 4}
\end{aligned}$$

Check: $(0.36)^2 + 8\times0.36 - 3 = 0$ ✓

If the unknown is on both sides of the =, put the unknowns together .

47 $5x - 8 = 2x + 4$

48 $2x + 2 = 9 - 5x$

49 $5h + 4 = 4h + 7$

50 $6y + 3 = y + 28$

51 $5w - 4 = 5 - 4w$

52 $4(t - 2) = 2t + 4$

53 $3(m + 3) + 2 = 15 - m$

54 $4x + 5 - x = 2(3 - x) + 4$

$$\begin{aligned}
\frac{x}{3} + 2 &= {}^-5 \\
x + 6 &= {}^-15 \\
x &= {}^-15 - 6 \\
\underline{x} &\underline{= {}^-21}
\end{aligned}$$

Check: $^-21/3 + 2 = {}^-7 + 2 = {}^-5$ ✓

Multiply by the denominator of the fraction.

55 $\frac{x}{4} + 3 = {}^-2$

56 $\frac{a}{5} - 2 = 4$

57 $7 + \frac{a}{2} = 3$

58 $\frac{y}{3} - 3 = {}^-1$

59 $6 = \frac{x}{4} - 1$

60 $\frac{x}{5} - 15 = {}^-3$

$$\begin{aligned}
\frac{2x - 3}{5} &= {}^-2 \\
2x - 3 &= {}^-2\times5 \\
2x - 3 &= {}^-10 \\
2x &= {}^-10 + 3 \\
2x &= {}^-7 \\
x &= {}^-7\div2 \\
\underline{x} &\underline{= {}^-3.5}
\end{aligned}$$

Check: $(2\times{}^-3.5-3)/5 = {}^-10/5 = {}^-2$ ✓

$$\begin{aligned}
\frac{x}{2} - \frac{2x}{3} &= {}^-2 \\
3x - 4x &= {}^-12 \qquad \{\times6\} \\
{}^-x &= {}^-12 \\
\underline{x} &\underline{= 12}
\end{aligned}$$

Check: $12/2-2\times12/3=6-8 = {}^-2$ ✓

$$\begin{aligned}
\frac{2x+1}{3} &= \frac{x}{2} - 4 \\
2(2x+1) &= 3x - 24 \qquad \{\times6\} \\
4x + 2 &= 3x - 24 \\
4x - 3x &= -24 - 2 \\
\underline{x} &\underline{= {}^-26}
\end{aligned}$$

Check: $(2\times{}^-26+1)/3 = {}^-26/2-4$

$({}^-52+1)/3 = {}^-13-4$

$^-17 = {}^-17$ ✓

61 $\frac{x-1}{2} = 1$

62 $\frac{x+2}{3} = 1$

63 $\frac{x-2}{2} = {}^-1$

64 $\frac{x-5}{3} = {}^-2$

65 $\frac{x+6}{2} = {}^-3$

66 $\frac{x+6}{3} = 2$

67 $\frac{2x}{3} + 5 = 2$

68 $\frac{3a}{4} - 2 = 3$

69 $\frac{x-5}{2} = 1$

70 $\frac{b-2}{3} = {}^-2$

71 $\frac{3x-2}{4} = 3$

72 $\frac{2b+4}{3} = 2$

73 $\frac{x}{2} + \frac{3}{5} = 1$

74 $\frac{x}{3} - \frac{1}{4} = 2$

75 $\frac{2x}{3} - \frac{x}{2} = 3$

76 $\frac{3x}{2} - \frac{x}{3} = 2$

77 $\frac{x}{2} = \frac{x}{3} + 4$

78 $\frac{x}{3} - 1 = \frac{x}{4}$

79 $\frac{2x}{3} + 3 = \frac{x}{2}$

80 $\frac{3x}{2} + 1 = \frac{{}^-x}{3}$

81 $\frac{2x+1}{3} + \frac{x}{2} = 5$

82 $\frac{3x-2}{2} = \frac{x}{3} + 6$

83 $\frac{3x-4}{2} = \frac{x}{5} - 1$

84 $\frac{3x-1}{5} = \frac{x}{2} + 4$

85 $\frac{2x}{3} + \frac{5}{2} = {}^-3x$

86 $\frac{3x}{2} - \frac{x}{3} - \frac{x}{4} = 2$

Solving Linear Equations

Each day, in Australia, millions of real-world problems are solved by the use of linear equations.

The basic idea is to:
1 Write the formula.
2 Substitute.
3 Solve for the unknown.

Exercise 7.2

The circumference, C, of a circle is given by the formula: $C = 2\pi r$, where r is the radius of the circle. A circular horse yard is to have a circumference of 56 m. What should be the radius of the horse yard?

$$\begin{aligned} C &= 2\pi r & &\{\text{write the formula}\} \\ 56 &= 2\pi r & &\{\text{substitute } C = 56\} \\ 56 \div (2\pi) &= r & &\{\text{inverse of } \times \text{ is } \div\} \\ 8.91 &= r \end{aligned}$$

Expressions such as $C = 2\pi r$ are linear because the highest power of r is 1 ($r = r^1$).

If you don't use the '(' and ')' on your calculator you will get $56 \div (2\pi)$ wrong.

Radius of horse yard = 8.91 m.

Check: $2\pi r = 2\pi \times 8.91 = 55.98$ m ✓

1 The circumference, C, of a circle is given by the formula: $C = 2\pi r$, where r is the radius of the circle. A circular horse yard is to have a circumference of 75 m. What should be the radius of the horse yard?

2 The perimeter, P, of a rectangle is given by $P = 2(l + b)$, where l is the length and b is the breadth. If the length of a house block is 43 m and the perimeter is 138 m, what is the breadth of the house block?

3 Speed, v, is given by the formula: $v = s \div t$ ($v = \frac{s}{t}$), where s is the distance and t is the time. If thunder is heard eight seconds after the lightning is seen, how far away was the lightning (Assume sound travels at 330 m/s)?

4 The sum of the interior angles of a polygon is given by the formula:
$S = 90(2n - 4)$, where n is the number of sides on the polygon.
How many sides in a polygon with an interior angle sum of 540°?

5 The volume, V, of a square based prism is given by the formula: $V = w^2h$, where w is the width of the base and h is the height of the prism. If the width of the prism is 4 cm and the volume is 96 cm^3, what is the height of the prism?

6 The volume of a cylinder, V, is given by the formula: $V = \pi r^2 h$, where r is the radius of the base of the cylinder and h is the height of the cylinder. If a cylinder with a base radius of 6.5 cm has a volume of 1487 cm^3, what is the height of the cylinder?

7 The volume of a cone, a circular based pyramid, is given by the formula:
$V = \frac{\pi r^2 h}{3}$ where r is the radius of the base of the cone and h is the height of the cone. If a cone has a radius of 1.1 m and a volume of 6.8 m^3, what is the height of the cone?

Exercise 7.3

Five times a number increased by four is divided by three to obtain twenty-two. What is the number?

Let the number be x,

$$\frac{5x + 4}{3} = 22$$

$$\begin{aligned} 5x + 4 &= 22 \times 3 \\ 5x + 4 &= 66 \\ 5x &= 66 - 4 \\ 5x &= 62 \\ x &= 62 \div 5 \\ \underline{x} &\underline{= 12.4} \end{aligned}$$

Check: $(5 \times 12.4 + 4)/3 = 66/3 = 22$ ✓

A rectangle has a length which is 5 m less than twice its width. What is the width of the rectangle, if the perimeter is 80 m?

Let the width be w,

$$\begin{aligned} 2 \times w + 2 \times (w - 5) &= 80 \\ 2w + 2w - 10 &= 80 \\ 4w - 10 &= 80 \\ 4w &= 80 + 10 \\ 4w &= 90 \\ w &= 90 \div 4 \\ \underline{w} &\underline{= 22.5 \text{ m}} \text{ (the width)} \end{aligned}$$

Check: $2 \times 22.5 + 2(22.5 - 5) = 80$ ✓

The crocodile was described as being one-third tail, one-quarter head and with a 300 cm body. What was the length of the crocodile?

Let the length of the crocodile be x,

$$\frac{1}{3}x + \frac{1}{4}x + 300 = x$$

$$\begin{aligned} 4x + 3x + 3600 &= 12x \qquad \{\times 12\} \\ 7x + 3600 &= 12x \\ 3600 &= 12x - 7x \\ 3600 &= 5x \\ 3600 \div 5 &= x \\ \underline{x} &\underline{= 720} \text{ (crocodile was 7.2 m)} \end{aligned}$$

Check: $720/3 + 720/4 + 300 = 720$ ✓

1 Three times a number increased by five is divided by two to obtain thirteen. What is the number?

2 Four times a number decreased by six is divided by seven to obtain twenty-two. What is the number?

3 Three consecutive numbers have a sum of forty-five. What are the numbers (4,5, and 6 are consecutive numbers)?

4 Find three consecutive numbers whose sum is six times the first number.

5 A rectangle has a length which is 3 m less than twice its width. What is the width of the rectangle, if the perimeter is 42 m?

6 A rectangle has a length twice the size of the width. What is the width of the rectangle if the perimeter is 180 cm?

7 An isosceles triangle has two angles each four times the size of the third angle. What is the size of each angle?

8 The crocodile was described as being one-third tail, one-quarter head and with a 200 cm body. What was the length of the crocodile?

9 The first side of a triangle is 20 cm longer that the second side. The third side is half the length of the second side. What is the length of the second side if the perimeter is 300 cm?

10 The difference between two numbers is 29. One number is 5 less than three times the other. What are the two numbers?

Solving Quadratics

This is a quadratic because the highest power of the unknown, x,is two.

Thousands of real life problems can be written as quadratics.

$ax^2 + bx + c = 0$

Solving means find the values of x that makes this true.

x^2 suggests that there could be none, one, or two solutions.

Exercise 7.4

Solve the following quadratics:

$$\begin{aligned} x^2 + 6x + 9 &= 0 \\ (x+3)^2 &= 0 && \{\text{perfect square}\} \\ x + 3 &= 0 && \{\text{square root both sides}\} \\ \underline{x} &\underline{= {}^{-}3} && \{\text{inverse of } +3\} \end{aligned}$$

Check: $({}^{-}3)^2 + 6\times{}^{-}3 + 9 = 0$ ✓

$$\begin{aligned} x^2 - 8x + 16 &= 0 \\ (x-4)^2 &= 0 && \{\text{perfect square}\} \\ x - 4 &= 0 && \{\text{square root both sides}\} \\ \underline{x} &\underline{= 4} && \{\text{inverse of } -4\} \end{aligned}$$

Check: $(4)^2 - 8\times4 + 16 = 0$ ✓

$$\begin{aligned} 4x^2 + 12x + 9 &= 0 \\ (2x)^2 + 2\times2x\times3 + (3)^2 &= 0 \\ (2x+3)^2 &= 0 && \{\text{perfect square}\} \\ 2x + 3 &= 0 && \{\text{square root both sides}\} \\ 2x &= {}^{-}3 && \{\text{inverse of } +3\} \\ \underline{x} &\underline{= {}^{-}1.5} && \{\text{inverse of } \times2\} \end{aligned}$$

Check: $4\times({}^{-}1.5)^2 + 12\times{}^{-}1.5 + 9 = 0$ ✓

$$\begin{aligned} x^2 + 6x + 8 &= 0 \\ x^2 + 4x + 2x + 8 &= 0 && \{\text{grouping pairs}\} \\ x(x+4) + 2(x+4) &= 0 && \{\text{factorising }\} \\ (x+4)(x+2) &= 0 && \{\text{factorising}\} \end{aligned}$$

Either $x + 4 = 0$ or $x + 2 = 0$

$\underline{x = {}^{-}4}$ or $\underline{x = {}^{-}2}$

Check: $({}^{-}4)^2 + 6\times{}^{-}4 + 8 = 0$ ✓

Check: $({}^{-}2)^2 + 6\times{}^{-}2 + 8 = 0$ ✓

$$\begin{aligned} x^2 - 3x - 10 &= 0 \\ x^2 - 5x + 2x - 10 &= 0 && \{\text{grouping pairs}\} \\ x(x-5) + 2(x-5) &= 0 && \{\text{factorising }\} \\ (x-5)(x+2) &= 0 && \{\text{factorising}\} \end{aligned}$$

Either $x - 5 = 0$ or $x + 2 = 0$

$\underline{x = 5}$ or $\underline{x = {}^{-}2}$

Check: $(5)^2 - 4\times5 - 10 = 0$ ✓

Check: $({}^{-}2)^2 - 4\times{}^{-}2 - 10 = 0$ ✓

1 $x^2 + 2x + 1 = 0$
2 $x^2 + 4x + 4 = 0$
3 $x^2 + 8x + 16 = 0$
4 $x^2 + 10x + 25 = 0$
5 $x^2 + 14x + 49 = 0$

6 $x^2 - 2x + 1 = 0$
7 $x^2 - 6x + 9 = 0$
8 $x^2 - 8x + 16 = 0$
9 $x^2 - 10x + 25 = 0$
10 $x^2 - 18x + 81 = 0$

11 $4x^2 + 12x + 9 = 0$
12 $9a^2 + 6x + 1 = 0$
13 $4x^2 + 8x + 4 = 0$
14 $4x^2 - 4x + 1 = 0$
15 $9y^2 + 24y + 16 = 0$
16 $9x^2 - 12x + 4 = 0$
17 $25x^2 - 30x + 9 = 0$

18 $x^2 + 5x + 4 = 0$
19 $x^2 + 3x + 2 = 0$
20 $x^2 + 5x + 6 = 0$
21 $x^2 + 7x + 6 = 0$
22 $x^2 + 8x + 12 = 0$
23 $x^2 + 8x + 15 = 0$
24 $x^2 + 6x + 8 = 0$
25 $x^2 + 9x + 14 = 0$

26 $x^2 + x - 6 = 0$
27 $x^2 + 5x - 6 = 0$
28 $x^2 - 2x - 8 = 0$
29 $x^2 - 5x - 6 = 0$
30 $x^2 - 4x - 12 = 0$
31 $x^2 - 6x + 9 = 0$
32 $x^2 - 10x + 9 = 0$
33 $x^2 - 8x + 12 = 0$

Completing the Square

Use $a^2 + 2ab + b^2 = (a + b)^2$
to solve quadratics.

$x^2 + 6x + 2 = 0$
$x^2 + 6x = {}^-2$
$x^2 + 6x + 9 = {}^-2 + 9$
$(x + 3)^2 = 7$
$x + 3 = \pm\sqrt{7}$
$x = {}^-3\pm\sqrt{7}$
$x = {}^-0.35$ or $x = {}^-5.65$

$x^2 + 6x + 9 = (x + 3)^2$

Add 9 to both sides to complete the square.

$(\sqrt{7})^2 = 7$ and $(-\sqrt{7})^2 = 7$
Which is why there are two solutions:
$+\sqrt{7}$ or $-\sqrt{7}$ thus $\pm\sqrt{7}$

Exercise 7.5

Solve the following quadratics by completing the square:

$x^2 + 8x - 3 = 0$
$x^2 + 8x = 3$ {move constant term}
$x^2 + 8x + 16 = (x + 4)^2$
$x^2 + 8x + 16 = 3 + 16$ {+16 to complete square}
$(x + 4)^2 = 19$ {perfect square}
$x + 4 = \pm\sqrt{19}$ {square root both sides}
$x = {}^-4\pm\sqrt{19}$ {inverse of +4}
$\underline{x = 0.36 \text{ or } x = {}^-8.36}$ {use calculator}
Check: $(0.36)^2 + 8\times0.36 - 3 = 0$ ✓
Check: $({}^-8.36)^2 + 8\times{}^-8.36 - 3 = 0$ ✓

$x^2 - 10x + 1 = 0$
$x^2 - 10x = {}^-1$ {move constant term}
$x^2 - 10x + 25 = (x - 5)^2$
$x^2 - 10x + 25 = {}^-1 + 25$ {+25 to complete square}
$(x - 5)^2 = 24$ {perfect square}
$x - 5 = \pm\sqrt{24}$ {square root both sides}
$x = 5\pm\sqrt{24}$ {inverse of +4}
$\underline{x = 9.90 \text{ or } x = 0.10}$ {use calculator}
Check: $(9.90)^2 - 10\times9.90 + 1 = 0$ ✓
Check: $(0.10)^2 - 10\times0.10 + 1 = 0$ ✓

$x^2 - 3x - 4 = 0$
$x^2 - 3x = 4$ {move constant term}
$x^2 - 3x + (3/2)^2 = (x - 3/2)^2$ {3/2 = half middle number}
$x^2 - 3x + (1.5)^2 = 4 + (1.5)^2$ {complete square, 3/2=1.5}
$(x - 1.5)^2 = 6.25$ {perfect square}
$x - 1.5 = \pm2.5$ {square root both sides}
$x = 1.5\pm2.5$ {inverse of +4}
$\underline{x = 4 \text{ or } x = {}^-1}$
Check: $(9.90)^2 - 10\times9.90 + 1 = 0$ ✓
Check: $(0.10)^2 - 10\times0.10 + 1 = 0$ ✓

1 $x^2 + 6x - 1 = 0$
2 $x^2 + 2x - 3 = 0$
3 $x^2 + 4x - 4 = 0$
4 $x^2 + 4x - 6 = 0$
5 $x^2 + 6x - 2 = 0$
6 $x^2 + 10x - 5 = 0$
7 $x^2 + 2x + 1 = 0$
8 $x^2 + 6x + 4 = 0$
9 $x^2 + 8x + 3 = 0$
10 $x^2 + 12x + 2 = 0$

11 $x^2 - 2x - 3 = 0$
12 $x^2 - 4x - 1 = 0$
13 $x^2 - 6x - 5 = 0$
14 $x^2 - 8x - 7 = 0$
15 $x^2 - 10x - 2 = 0$
16 $x^2 - 12x - 4 = 0$
17 $x^2 - 14x + 4 = 0$
18 $x^2 - 6x + 2 = 0$
19 $x^2 - 4x + 1 = 0$
20 $x^2 - 8x + 3 = 0$

21 $x^2 - 3x - 1 = 0$
22 $x^2 + 3x - 3 = 0$
23 $x^2 + 5x - 5 = 0$
24 $x^2 - 5x - 2 = 0$
25 $x^2 + 7x - 4 = 0$
26 $x^2 - 7x - 3 = 0$
27 $x^2 + 9x + 1 = 0$
28 $x^2 - 9x + 2 = 0$
29 $x^2 + 11x + 5 = 0$
30 $x^2 - 11x + 2 = 0$

The Quadratic Formula

Thousands of problems can be expressed as quadratics. Quadratics can be solved by guess and check, sketching graphs, factorising, and using the **quadratic formula**.

Given the quadratic: $$ax^2 + bx + c = 0$$

The solution is: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is the world famous quadratic formula.

Exercise 7.6

Use the quadratic formula to solve the following quadratics:

$x^2 + x - 2 = 0$

1 Write the quadratic → $\mathbf{x^2 + x - 2 = 0}$

2 Write the general quadratic → $ax^2 + bx + c = 0$

3 Decide values of a, b, c → $a=1 \quad b=1 \quad c={}^-2$

4 Write the quadratic formula → $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

5 Substitute for a, b, and c → $x = \frac{{}^-1 \pm \sqrt{1^2 - 4 \times 1 \times {}^-2}}{2 \times 1}$

6 Calculate

$$x = \frac{{}^-1 \pm \sqrt{1+8}}{2}$$

$$x = \frac{{}^-1 \pm \sqrt{9}}{2}$$

$$x = \frac{{}^-1 \pm 3}{2}$$

± means + or –

$$x = \frac{{}^-1 + 3}{2} \quad \text{or} \quad x = \frac{{}^-1 - 3}{2}$$

$$\underline{x = 1 \quad \text{or} \quad x = {}^-2}$$

Check: $x=1$, $1^2 + 1 - 2 = 0$ ✓

Check: $x={}^-2$, $({}^-2)^2 + {}^-2 - 2 = 0 = 4 - 4 = 0$ ✓

The numerator needs brackets if using a calculator.

1 $x^2 - x - 2 = 0$ which is the same as $(x - 2)(x + 1) = 0$

2 $x^2 + x - 2 = 0$ which is the same as $(x - 1)(x + 2) = 0$

3 $x^2 - 2x - 3 = 0$ which is the same as $(x + 1)(x - 3) = 0$

4 $x^2 + 2x + 1 = 0$ which is the same as $(x + 1)(x + 1) = 0$

5 $x^2 + 3x + 2 = 0$ which is the same as $(x + 2)(x + 1) = 0$

6 $x^2 + 5x + 6 = 0$ which is the same as $(x + 2)(x + 3) = 0$

7 $x^2 + 4x + 3 = 0$ which is the same as $(x + 1)(x + 3) = 0$

8 $x^2 - 2x + 1 = 0$ which is the same as $(x - 1)(x - 1) = 0$

9 $x^2 - 3x + 2 = 0$ which is the same as $(x - 1)(x - 2) = 0$

10 $x^2 - 5x + 6 = 0$ which is the same as $(x - 2)(x - 3) = 0$

The Quadratic Formula

$y = 2x^2 + 5x - 7$ is called a quadratic equation because the highest power of x is 2.

Exercise 7.7

Use the quadratic formula to solve the following quadratics:

$5x^2 - 2x - 4 = 0$

1 Write the quadratic ⟶ $\mathbf{5x^2 - 2x - 4 = 0}$

2 Write the general quadratic ⟶ $ax^2 + bx + c = 0$

3 Decide values of a, b, c ⟶ $a=5 \quad b={}^{-}2 \quad c={}^{-}4$

4 Write the quadratic formula ⟶ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

5 Substitute for a, b, and c ⟶ $x = \dfrac{-{}^{-}2 \pm \sqrt{({}^{-}2)^2 - 4 \times 5 \times {}^{-}4}}{2 \times 5}$

6 Calculate $x = \dfrac{2 \pm 9.17}{10}$ ⟵ See Technology 7.1

$x = \dfrac{(2+9.17)}{10}$ or $x = \dfrac{(2+9.17)}{10}$

$x = 1.12$ or $x = {}^{-}0.72$

Check: $x=1.12,\ 5\times1.12^2 - 2\times1.12 - 4 = 0$ ✓

Check: $x={}^{-}0.72,\ 5\times({}^{-}0.72)^2 - 2\times{}^{-}0.72 - 4 = 0$ ✓

1 $2x^2 + 3x - 2 = 0$

2 $x^2 + 2x - 5 = 0$

3 $5x^2 - 2x - 1 = 0$

4 $4x^2 - 3x - 3 = 0$

5 $2x^2 + x - 2 = 0$

6 $x^2 - 3x - 5 = 0$

7 $x^2 + 5x + 1 = 0$

8 $7x^2 - 2x - 5 = 0$

9 $4x^2 + 8x + 3 = 0$

10 $x^2 - 11x - 1 = 0$

11 $6x^2 + 8x - 4 = 0$

12 $4x^2 - 5x = 0$

13 $x^2 + 3x + 2 = 0$

14 $4x^2 - 8 = 0$

15 $8x^2 + 3x= 0$

16 $5x^2 - 4x = 0$

17 $16x^2 - 4= 0$

18 $14x^2 - 5 = 0$

19 ${}^{-}x^2 + 5x - 2 = 0$

20 ${}^{-}4x^2 - 3x + 1 = 0$

21 ${}^{-}2x^2 + 3x + 5 = 0$

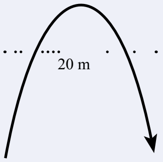
22 ${}^{-}3x^2 + 3x + 1 = 0$

23 ${}^{-}x^2 - 4x - 1 = 0$

24 ${}^{-}3x^2 - 6x - 2 = 0$

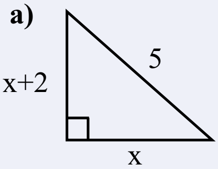
Nothing is all wrong. Even a busted clock is right twice a day.

25 A stone is thrown vertically with a velocity of 30 m/s. The motion of the stone is described by the relationship: $5t^2 - 30t + h = 0$, where h is the height in metres and t is the time in seconds. How long will it take the stone to reach a height of 20 m?

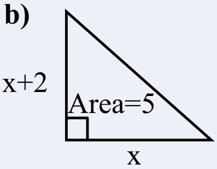


26 Calculate x in each of the following figures:

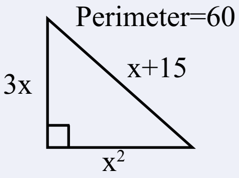
a)



b)



c)



Mental Computation

Mental computation gives you practice in thinking.

Exercise 7.8

1 Spell Quadratic.
2 What is the quadratic formula?
3 $3x^2 + 2x - 2 = 0$. What are the values of a, b, and c?
4 Expand $(x - 2)^2$
5 Factorise $x^2 + 4x + 3$
6 What is the value of: $\text{Log}_{10}100$
7 Solve: $x + y = 10$, $xy = 21$
8 $^-2 - {}^-5$
9 $(x^{-3})^4$
10 Increase \$6 by 30%

$x^2 + 4x + 3$
3+1=4
3×1=3
$= (x+3)(x+1)$

$6 + 6×30/100$
$= 6 + 180/100$
$= 6 + 1.8$
= \$7.80

Exercise 7.9

1 Spell 'Solving linear equations'.
2 What is the quadratic formula?
3 $5x^2 - x - 3 = 0$. What are the values of a, b, and c?
4 Expand $(x + 2)^2$
5 Factorise $x^2 + 7x + 10$
6 What is the value of: $\text{Log}_{10}1000$
7 Solve: $x + y = 11$, $xy = 30$
8 $^-2 - 3$
9 $(x^{-2})^4$
10 Increase \$7 by 30%

Learning without thought is labor lost; thought without learning is perilous - Confucius.

War does not determine who is right, only who is left - a paraprosdokian.

Exercise 7.10

1 Spell 'Completing the square'.
2 What is the quadratic formula?
3 $x^2 - 2x + 4 = 0$. What are the values of a, b, and c?
4 Expand $(x - 3)^2$
5 Factorise $x^2 + 5x + 4$
6 What is the value of: $\text{Log}_{10}10\,000$
7 Solve: $x + y = 6$, $xy = 8$
8 $^-7 + {}^-7$
9 $(x^{-2})^5$
10 Increase \$8 by 30%

You do not need a parachute to skydive. You only need a parachute to skydive twice - another paraprosdokian.

Mechanical engineers design and supervise the construction and operation of machinery.

- Relevant school subjects are English, Mathematics, and Physics.
- Courses usually involve an engineering degree.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 7.11

If $48a = b^2$ and a and b are positive integers, find the smallest value of a.

$48a = b^2$
$16\times3a = b^2$
$4^2\times3a = b^2$ thus $\underline{a = 3}$

If a and b are positive integers, find the smallest value of a:

1 $28a = b^2$

2 $24a = b^2$

3 $63a = b^2$

$y = \frac{3x-5}{2}$ What is the value of x when $y = {}^{-}3$?

${}^{-}3\times2 = 3x - 5$
${}^{-}6 + 5 = 3x$
${}^{-}1 \div 3 = x$
$x = \underline{{}^{-}⅓ \text{ or } {}^{-}0.33}$

What is the value of x when $y = {}^{-}2$?

4 $y = 4x + 3$

5 $y = \frac{2x}{5} + \frac{1}{3}$

6 $y = \frac{4x+3}{5}$

$(2x - 3)(x + 5) = 0$

Either $2x - 3 = 0$
$2x = 3$
$\underline{x = 1.5}$

Check: $(2\times1.5-3)(1.5+5)$
$= 0\times6.5$
$= 0$ ✓

Or $x + 5 = 0$
$\underline{x = {}^{-}5}$

Check: $(2\times{}^{-}5-3)({}^{-}5+5)$
$= {}^{-}13\times0$
$= 0$ ✓

Solve each of the following quadratic equations:

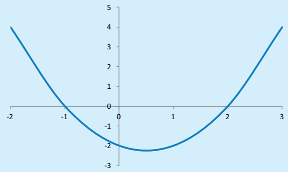
7 $(x + 1)(x - 1) = 0$

8 $(x - 2)(2x - 1) = 0$

9 $(3x + 1)(x - 2) = 0$

10 $(2x - 2)(x + 4) = 0$

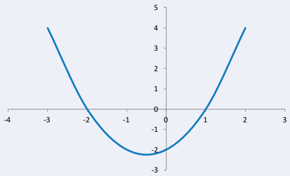
11 What is the equation of each of the following quadratics:



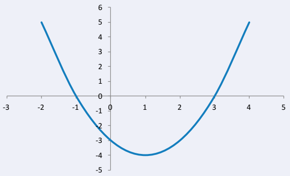
x-intercepts
x = ¯1 thus x + 1 = 0
x = 2 thus x − 2 = 0

$y = (x + 1)(x - 2)$
or $\underline{y = x^2 - x - 2}$

a)



b)

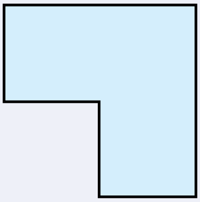


A Couple of Puzzles

Exercise 7.12

1 On Monday a plant is 2 cm high. Each day the plant doubles its height from the day before. How high will the plant be on Friday?

2 Can you cut this shape into
 a) Two conguent shapes?
 b) Three conguent shapes?
 c) Four congruent shapes?
 d) Six congruent shapes?



A Game

Calculator Hi Lo is played on a calculator. You try to guess the secret number in as few guesses as possible.

1 Someone else gives the limits of the secret number eg., 1 to 10, 1 to 100, 1 to 1000 and enters the secret number in the calculator's memory and clears the display.

74 **STO**

2 You enter a guess on the calculator.

50

3 Someone else divides your number by the number in memory.

÷ **RCL** **M+** =

4 If the display is:
greater than 1 then the guess was **too high**.
less than 1 then the guess was **too low**.
equal to 1 then the guess was **correct**.

13.867321
0.67567567

Try to guess the secret number in as few guesses as possible.

A Sweet Trick

Impress your audience by showing that you know some tricky patterns.

37×3 =
37×6 =
37×9 =
37×12 =
37×15 =

1×8 + 1 =
12×8 + 2 =
123×8 + 3 =
1234×8 + 4 =
12345×8 + 5 =

Think about your presentation:
- Think up some story about these numbers?
- Audience has calculators, you don't have a calculator?

$9^2 - 2^2 =$
$89^2 - 12^2 =$
$889^2 - 112^2 =$
$8889^2 - 1112^2 =$
$88889^2 - 11112^2 =$

$6^2 - 5^2 =$
$56^2 - 45^2 =$
$556^2 - 445^2 =$
$5556^2 - 4445^2 =$
$55556^2 - 44445^2 =$

Investigations

Investigation 7.1 A quadratic function

The distance an object falls in a certain time, when dropped, can be modelled by the quadratic function: $s = 4.9t^2$, where s is the distance in metres an object falls in t seconds.

Example: A stone takes 2.75 seconds to reach the bottom of a well.

$$s = 4.9t^2$$

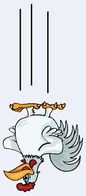
$$\text{Depth of well} = 4.9 \text{x} 2.75^2$$

$$= 37 \text{ m}$$

Investigate

Ways of testing the function

$$\mathbf{s = 4.9t^2}$$



A GPS unit and a stone jump off a cliff at the same time. Which will hit the ground first?

The GPS because it doesn't have to ask for directions.

Investigation 7.2 A linear function?

1 Turn on a tap and let the water run at a constant rate.

2 Ready the bucket by putting a ruler inside the bucket.

3 Put the bucket under the tap and record the height of water every 10 seconds (You may wish to record every 30 s or so dependent upon how much water is flowing from the tap).

4 Draw a graph.

You need:
- a bucket
- a ruler
- a stopwatch

Two statisticians were flying from Perth to Sydney. About an hour into the flight, the pilot announced, 'Unfortunately, we have lost an engine, but don't worry: There are three engines left. However, instead of five hours, it will take seven hours to get to Sydney.'
A little later, the pilot told the passengers that a second engine had failed. 'But we still have two engines left. We're still fine, except now it will take ten hours to get to Sydney.'
Somewhat later, the pilot again came on the intercom and announced that a third engine had died. 'But never fear, because this plane can fly on a single engine. Of course, it will now take 18 hours to get to Sydney.'
At this point, one statistician turned to another and said, 'Gee, I hope we don't lose that last engine, or we'll be up here forever!'

Technology

Technology 7.1 The Quadratic Formula and the Calculator

You can be more efficient and accurate in the use of your calculator.

Example: Solve: $5x^2 - 2x - 4 = 0$

1 Write the quadratic $\mathbf{5x^2 - 2x - 4 = 0}$

2 Write the general quadratic $ax^2 + bx + c = 0$

3 Decide values of a, b, c $a=5 \quad b={}^{-}2 \quad c={}^{-}4$

4 Write the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

5 Substitute for a, b, and c $x = \dfrac{-{}^{-}2 \pm \sqrt{({}^{-}2)^2 - 4 \times 5 \times {}^{-}4}}{2 \times 5}$

6 Calculate $x = \dfrac{2 \pm 9.17}{10}$

$$x = \frac{(2+9.17)}{10} \quad \text{or} \quad x = \frac{(2+9.17)}{10}$$

Use these brackets on your calculator.

$x = 1.12$ or $x = {}^{-}0.72$

To calculate this in one go. $\longrightarrow$ $\sqrt{({}^{-}2)^2 - 4 \times 5 \times {}^{-}4}$

you need an **extra set of brackets.** $\sqrt{(({}^{-}2)^2 - 4 \times 5 \times {}^{-}4)}$

You may need to practice this on a calculator.

$\boxed{\sqrt{\ }}\ \boxed{(}\ \boxed{(}\ \boxed{\pm}\ \boxed{2}\ \boxed{)}\ \boxed{x^2}\ \boxed{-}\ \boxed{4}\ \boxed{\times}\ \boxed{5}\ \boxed{\times}\ \boxed{\pm}\ \boxed{4}\ \boxed{)}\ \boxed{=}$

to give the answer 9.17

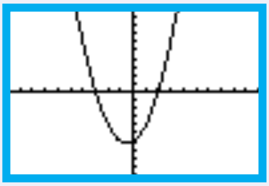
Technology 7.2 Quadratics and the Graphics Calculator

Use a graphics calculator to check your answers to Exercises 7.4, 7.5, 7.6, and 7.7.

Example: Graph: $y = x^2 + x - 6$ and solve: $x^2 + x - 6 = 0$

Press Y = and enter the function eg., $x^2 + x - 6$

Press Graph to see a graph of the function.



To solve: $x^2 + x - 6 = 0$ is to find the x-intercepts.

a) Use CALC to find the intercepts (some calculators use zero and value).

b) Use TRACE and move the cursor to the x-intercepts.

c) Use TABLE to find the x-intercept (where y = 0).

Can you get the answers: $x = {}^{-}3$ and $x = 2$

Chapter Review 1

Exercise 7.13

$$\frac{x}{2} - \frac{2x}{3} = {}^{-}2$$

$$3x - 4x = {}^{-}12 \qquad \{\times 6\}$$

$${}^{-}x = {}^{-}12$$

$$\underline{x = 12}$$

Check: $({}^{-}13+3)/5 = {}^{-}10/5 = {}^{-}2$ ✓

1 $3(x - 4) = 15$

2 ${}^{-}2(x - 3) = 8$

3 $7(2x - 1) = 14$

4 $2(x - 3) + 3x = 11$

5 $5(x - 3) - 2x = 12$

6 $6x - 8 = 2x + 4$

7 $2x + 2 = 16 - 5x$

8 $\frac{x}{3} + 5 = {}^{-}1$

9 $\frac{2x - 1}{3} = 2$

10 $\frac{3x}{2} - \frac{x}{3} = 7$

11 The sum of the interior angles of a polygon is given by the formula: $S = 90(2n - 4)$, where n is the number of sides on the polygon. How many sides in a polygon with an interior angle sum of 540°?

12 The perimeter of a block of land is 110 m. The length is 21 m longer than the breadth. Find the length and the breadth.

$$x^2 - 8x + 16 = 0$$

$$(x - 4)^2 = 0 \qquad \{\text{perfect square}\}$$

$$x - 4 = 0 \qquad \{\text{square root both sides}\}$$

$$\underline{x = 4} \qquad \{\text{inverse of } -4\}$$

Check: $(4)^2 - 8\times 4 + 16 = 0$ ✓

$$x^2 - 3x - 10 = 0$$

$$x^2 - 5x + 2x - 10 = 0 \qquad \{\text{grouping pairs}\}$$

$$x(x - 5) + 2(x - 5) = 0 \qquad \{\text{factorising }\}$$

$$(x - 5)(x + 2) = 0 \qquad \{\text{factorising}\}$$

Either $x - 5 = 0$ or $x + 2 = 0$

$\underline{x = 5}$ or $\underline{x = {}^{-}2}$

Check: $(5)^2 - 4\times 5 - 10 = 0$ ✓

Check: $({}^{-}2)^2 - 4\times{}^{-}2 - 10 = 0$ ✓

13 $x^2 + 4x + 4 = 0$

14 $x^2 + 10x + 25 = 0$

15 $x^2 - 2x + 1 = 0$

16 $x^2 - 8x + 16 = 0$

17 $x^2 + 5x + 4 = 0$

18 $x^2 + 8x + 12 = 0$

19 $x^2 + 5x - 6 = 0$

20 $x^2 - 2x - 8 = 0$

21 $x^2 - 5x - 6 = 0$

22 $x^2 - 4x - 12 = 0$

23 $x^2 - 10x + 9 = 0$

24 $x^2 - 8x + 12 = 0$

25 $x^2 + 5x + 2 = 0$

26 $3x^2 + 7x - 3 = 0$

27 $x^2 - 11x + 3 = 0$

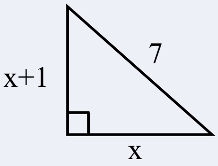
28 ${}^{-}4x^2 - 3x + 1 = 0$

Given the quadratic: $ax^2 + bx + c = 0$

The solution is: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

29 A stone is thrown vertically with a velocity of 40 m/s. The motion of the stone is described by the relationship: $5t^2 - 40t + h = 0$, where h is the height in metres and t is the time in seconds. How long will it take the stone to reach a height of 10 m?

30 Calculate x in the following figure:



Chapter Review 2

Exercise 7.14

$$\frac{x}{2} - \frac{2x}{3} = {}^{-}2$$

$3x - 4x = {}^{-}12$ $\{\times 6\}$

${}^{-}x = {}^{-}12$

$\underline{x = 12}$

Check: $({}^{-}13+3)/5 = {}^{-}10/5 = {}^{-}2$ ✓

1 $2(x - 3) = 10$

2 ${}^{-}5(x - 4) = 10$

3 $4(3x - 1) = 20$

4 $3(x - 1) + 2x = 12$

5 $7(x - 2) - 3x = 4$

6 $7x - 7 = 2x + 3$

7 $2x + 2 = 13 - 2x$

8 $\frac{x}{5} - 2 = {}^{-}5$

9 $\frac{3x - 2}{2} = 5$

10 $\frac{2x}{3} - \frac{x}{4} = 5$

11 The volume, V, of a square based prism is given by the formula: $V = w^2h$, where w is the width of the base and h is the height of the prism. If the width of the prism is 6 cm and the volume is 162 cm^3, what is the height of the prism?

12 An isosceles triangle has two angles each twice the size of the third angle. What is the size of each angle?

$x^2 - 8x + 16 = 0$

$(x - 4)^2 = 0$ {perfect square}

$x - 4 = 0$ {square root both sides}

$\underline{x = 4}$ {inverse of −4}

Check: $(4)^2 - 8\times4 + 16 = 0$ ✓

$x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$ {grouping pairs}

$x(x - 5) + 2(x - 5) = 0$ {factorising }

$(x - 5)(x + 2) = 0$ {factorising}

Either $x - 5 = 0$ or $x + 2 = 0$

$\underline{x = 5}$ or $\underline{x = {}^{-}2}$

Check: $(5)^2 - 4\times5 - 10 = 0$ ✓

Check: $({}^{-}2)^2 - 4\times{}^{-}2 - 10 = 0$ ✓

13 $x^2 + 6x + 9 = 0$

14 $x^2 + 8x + 16 = 0$

15 $x^2 - 4x + 4 = 0$

16 $x^2 - 10x + 25 = 0$

17 $x^2 + 4x + 3 = 0$

18 $x^2 + 6x + 8 = 0$

19 $x^2 + x - 6 = 0$

20 $x^2 - 3x - 10 = 0$

21 $x^2 - 5x - 6 = 0$

22 $x^2 - x - 12 = 0$

23 $x^2 - 9x + 8 = 0$

24 $x^2 - 13x + 12 = 0$

25 $x^2 + 3x + 1 = 0$

26 $3x^2 + 5x - 2 = 0$

27 $x^2 - 7x + 5 = 0$

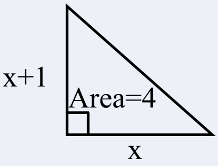
28 ${}^{-}2x^2 - x + 1 = 0$

Given the quadratic: $ax^2 + bx + c = 0$

The solution is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

29 A stone is thrown vertically with a velocity of 50 m/s. The motion of the stone is described by the relationship: $5t^2 - 50t + h = 0$, where h is the height in metres and t is the time in seconds. How long will it take the stone to reach a height of 40 m?

30 Calculate x in the following figure:



Chance

8

Statistics & Probability ➡ Chance

★ Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence.
  - recognise that an event can be dependent on another event and that this will affect the way its probability is calculated.

★ Use the language of 'if ....then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.
  - use two-way tables and Venn diagrams to understand conditional statements.
  - use arrays and tree diagrams to determine probabilities.

10A Investigate reports of studies in digital media and elsewhere for information on their planning and implementation.
  - evaluate the appropriateness of sampling methods in reports where statements about a population are based on a sample.
  - evaluate whether graphs in a report could mislead, and whether graphs and numerical information support the claims.

A TASK

Organise a class debate about the following topic:

Is it OK for a family to spend $55 every week playing Lotto?

A LITTLE BIT OF HISTORY

Sir Francis Bacon (1561-1626), the father of deductive reasoning, was the first to use inductive thinking as a basis for scientific procedure.

Thomas Bayes (1702-1761) provided the first mathematical basis for inductive reasoning and developed Bayesian probability theory.

Sir Thomas Bacon

Inductive reasoning begins with the facts and uses reasoned judgments to make general conclusions.

A box contains an unknown number and proportion of white and black balls. It is assumed that there are equal numbers.

A sample provides 3 white balls. Would you now begin to induce that there are more white than black balls in the box?

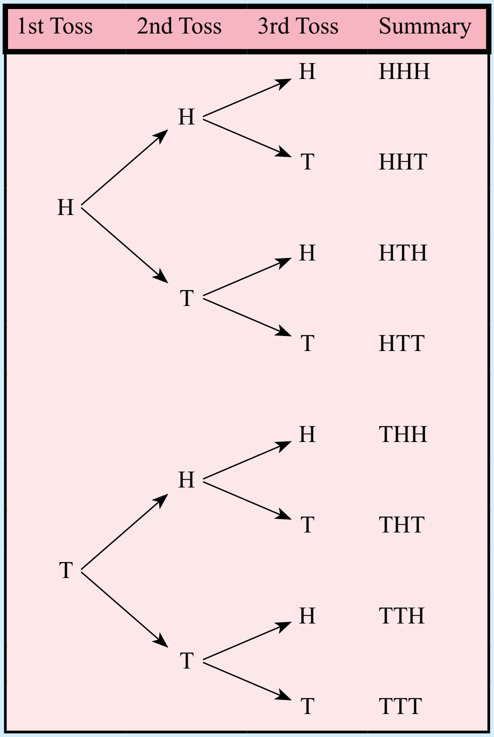
Theoretical Probability

$$\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}$$

Exercise 8.1

Assuming that the chance of a head or tail is equal, use a tree diagram to determine the following theoretical probabilities for tossing three coins:

a) P(3 heads) **b)** P(2 heads & 1 tail) **c)** P(3 tails)



2 Heads & 1 Tail
HHT, HTH, THH

a) P(HHH) $= \frac{1}{8} = 0.125$

b) P(2H & T) $= \frac{3}{8} = 0.375$

c) P(TTT) $= \frac{1}{8} = 0.125$

Tree diagrams and two-way tables are useful when calculating the theoretical probability of multi-outcome and compound events.

1 Assuming that the chances of a head or tail is equal, use a tree diagram to determine the following theoretical probabilities for the tossing of two coins:
a) P(2 heads) **b)** P(1 head & 1 tail) **c)** P(2 tails).

2 Assuming that the chances of a head or tail is equal, use a tree diagram to determine the following theoretical probabilities for the tossing of four coins:
a) P(4 heads) **b)** P(3 heads & 1 tail) **c)** P(2heads & 2tails)
d) P(1 head & 3 tails) **e)** P(4 tails).

3 Assuming that the chances of a girl or boy being born is equal, use a tree diagram to determine the following theoretical probabilities for a family of three children:
a) P(3 girls) **b)** P(2 girls & 1 boy) **c)** P(3 boys).

4 Assuming that the chances of a girl or boy being born is equal, use a tree diagram to determine the following theoretical probabilities for a family of four children:
a) P(4 girls) **b)** P(3 girls & 1 boy)
c) P(2 girls & 2 boys) **d)** P(4 boys).

Theoretical Probability

OR means put together.
AND means the intersection.

So many things involve probability.
Insurance is about probability.
Weather prediction is about probability.
Even atomic theory is about probability.

Exercise 8.2

Use a table to determine the following theoretical probabilities of the totals when tossing two dice:

Listing outcomes with a two-way table or tree diagram makes problem-solving easier.

a) P(5 or odd)

Mark the 5s **together** with the odds

		Dice 1					
		1	2	3	4	5	6
Dice 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$$P(5\,or\,odd) = \frac{no\,of\,5s\,or\,odds}{total} = \frac{18}{36} = \underline{0.5}$$

b) P(<7 and even)

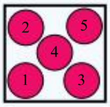
Mark the <7 **and** even

		Dice 1					
		1	2	3	4	5	6
Dice 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$$P(<7\,and\,even) = \frac{no\,of < 7\,and\,even}{total} = \frac{9}{36} = \underline{0.25}$$

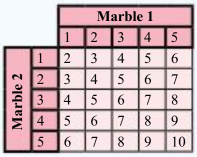
1 Determine the following theoretical probabilities of the **totals** when tossing two dice.

a) P(7 or odd) **b)** P(7 and odd) **c)** P(8 or even)
d) P(8 and even) **e)** P(<4 or odd) **f)** P(<4 and even)
g) P(>8 or even) **h)** P(>8 and odd) **i)** P(odd or divisible by 3)
j) P(odd or even) **k)** P(odd and even).



2 A box has 5 marbles numbered from 1 to 5.
Determine the following theoretical probabilities of the **totals** of two marbles (A marble is taken from the box, the number noted, put back in the box, and then a second marble taken from the box and the number noted).

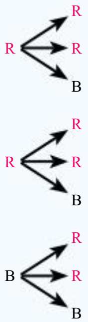
a) P(a total of 4) **b)** P(a total of 5)
c) P(an odd total) **d)** P(an even total)
e) P(3 and odd) **f)** P(3 or odd)
g) P(<3 and even) **h)** P(<3 or even).



		Marble 1				
		1	2	3	4	5
Marble 2	1	2	3	4	5	6
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10

3 A bag contains 2 red balls and a black ball. A ball is withdrawn, the colour noted, and replaced back in the bag. A second ball is then drawn. Find the probability of drawing:

a) 2 black balls one after the other
b) 2 red balls one after the other
c) A red and then a black
d) 2 red balls or 2 black balls.



Independence

Two events are independent if the probability of one event doesn't depend on the outcome of another event.

Two events are dependent if the probability of one event depends on the outcome of another event.

Exercise 8.3

Is event B dependent on event A?

Event A: Mia eats a fruit from a bowl containing 1 apple, 1 pear, and 1 banana.

Event B: Mia chooses another fruit from the bowl.

Event B **depends** on event A.
If Mia first eats an apple then secondly, Mia is then restricted to a pear or apple.
If Mia first eats a pear then secondly, Mia is then restricted to an apple or banana.
If Mia first eats a banana then secondly, Mia is then restricted to an apple or pear.

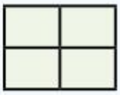
1 Event A: Simone chooses a fruit from a bowl containing 1 apple, 1 pear, and 1 banana and replaces the fruit in the bowl.

Event B: Simone then chooses another fruit from the bowl.

2 A game board of 2×2 blank squares is presented to Karen and Ethan.

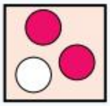
Event A: Karen chooses a blank square and puts a mark on it.

Event B: Ethan chooses a blank square and puts a mark on it.



3 Event A: Chloe takes a ball from a bag containing 2 red balls and 1 white ball leaving 2 balls in the bag.

Event B: Chloe then takes another ball from the bag.



4 Event A: Rex takes a ball from a bag containing 3 yellow balls and 2 white balls and then replaces the ball back in the bag.

Event B: Rex then takes another ball from the bag.

5 The numbers 2, 4, and 5 are each written on a card.

Event A: Ella takes one of the cards.

Event B: Noah takes one of the two cards left.

6 A circular spinner is shade with the colours red, blue, and green.

Event A: Emily spins and notes a colour.

Event B: Sophie spins and notes a colour.

7 Event A: Jack tosses a die and notes the top number.

Event B: Olivia tosses a die and notes the top number.

Exercise 8.4

The numbers 1, 4, and 5 are written on cards and put in a bag. A card is withdrawn, the number noted, and **not replaced**. A second card is then drawn.

a) Use a table or tree diagram to show all possible outcomes.

b) What is the probability that the order of numbers is:

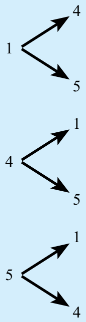
i) 41

ii) even

iii) divisible by 3

Because the two events are dependent there are less outcomes than if the first card had been replaced.

a) P(41) = $\frac{1}{6}$



b) P(even) = {14,54}

$= \frac{2}{6} \text{ or } \frac{1}{3}$

a) P(div 3) = {15,45,51,54}

$= \frac{4}{6} \text{ or } \frac{2}{3}$

1 The numbers 2, 3, and 4 are written on cards and put in a bag. A card is withdrawn, the number noted, and **not replaced**. A second card is then drawn.

a) Use a table or tree diagram to show all the possible outcomes.

b) What is the probability that the order of numbers is:

i) 32 **ii)** even **iii)** divisible by 3?

2 The numbers 2, 4, 5, and 7 are written on cards and put in a bag. A card is withdrawn, the number noted, and **not replaced**. A second card is then drawn.

a) Use a table or tree diagram to show all the possible outcomes.

b) What is the probability that the order of numbers is:

i) 54 **ii)** odd **iii)** divisible by 5?

3 Drawer A has three socks - one brown, one black, and one blue.
Drawer B has four socks - two brown and two black. Use a table or tree diagram to determine the following theoretical probabilities when selecting one sock from each drawer.

a) P(two brown) **b)** P(two black) **c)** P(1 black, 1 brown)

4 A bag contains 2 red and 3 blue balls. One ball is withdrawn, its colour noted, and not replaced. A second ball is then withdrawn.
What is the probability that the two balls are:

a) both red **b)** both blue **c)** a red and a blue?

5 A box of chocolates has 4 hard-centred chocolates and 6 soft-centred chocolates. A chocolate is chosen and eaten and then a second chocolate eaten.
What is the probability that the two chocolates are:

a) soft-centred **b)** hard-centred **c)** one of each?

Conditional Probability

Exercise 8.5

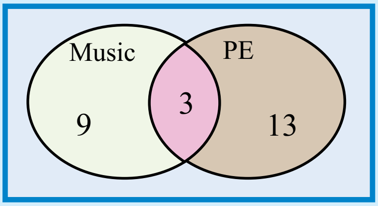
In a class of 25 students, 12 students study music, and 3 of the 16 PE students also study music. Find the probability that:

a) a student studies music and PE

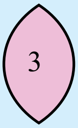
b) a student studies music or PE

c) a student does not study music

d) **if** a student studies music **then** the student also studies PE.

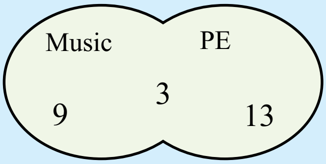


a) music **and** PE



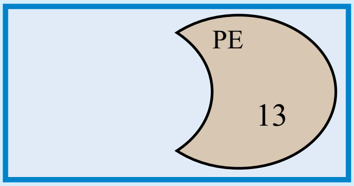
$= \frac{3}{25} = \underline{0.12}$

b) music **or** PE



$= \frac{25}{25} = \underline{1}$

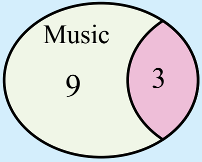
c) **not** music



$= \frac{13}{25} = \underline{0.52}$

d) **If** music **then** PE

$= \frac{3}{12} = \underline{0.25}$



If music means look only at all **music students.**

1 In a class of 25 students, 12 students study music, and 4 of the 17 PE students also study music. Draw a Venn Diagram and find the probability that a student:

a) studies music and PE.

b) studies music or PE.

c) does not study music.

d) studies PE **given that** the student studies music.

PE and music = 12+17−25

2 In a class of 23 students, 12 students play cricket, and 4 of the 15 students who play netball also play cricket.
Draw a Venn Diagram and find the probability that a student:

a) plays cricket and netball.

b) plays cricket or netball.

c) does not play cricket.

d) plays netball **knowing that** the student plays cricket.

Venn diagrams are a good way of representing probability.

3 In a class of 29 students, 27 students passed Maths, and 23 students passed English. Draw a Venn Diagram and find the probability that a student:

a) passed maths and English.

b) passed maths or English.

c) did not pass Maths.

d) passed English given that the student also passed maths.

Conditional Probability

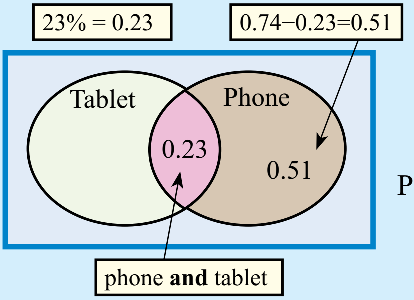
Conditional probability - probability conditional on a previous outcome or event.

Example: A bag has a red, a blue, and a green marble. What is the probability of drawing a blue marble after having drawn a green marble without replacement?

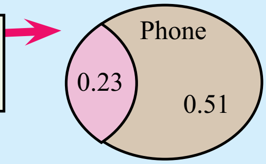
Language suggesting that probability is conditional on a previous event:
if .... then ....
given that ....
knowing that ....

Exercise 8.6

74% of all teenagers own a mobile phone and 23% of all teenagers own a mobile phone and a tablet. What is the probability that a teenager owns a tablet given that the teenager owns a mobile phone?



Given a phone means look only at phone owners.



$$\text{P(tablet given phone)} = \frac{\textit{Favourable outcomes}}{\textit{Total outcomes}}$$

$$= \frac{0.23}{0.74} = 0.31 \text{ or } \underline{31\%}$$

1 68% of all teenagers own a mobile phone and 17% of all teenagers own a mobile phone and a tablet. What is the probability that if a teenager owns a mobile phone then the teenager also owns a tablet?

2 32% of houses have a tiled roof and 13% of houses have a tiled roof and double brick. What is the probability that a house is double brick given that the house has a tiled roof?

3 A bag contains blue and red marbles. Two marbles are chosen without replacement. The probability of selecting a blue marble and then a red marble is 0.53, and the probability of selecting a blue marble on the first draw is 0.81. What is the probability of selecting a red marble on the second draw, given that the first marble drawn was blue?

4 In a survey of 50 coffee drinkers, 33 have milk. 11 have sugar with their coffee. 8 have both milk and sugar. Find the probability that a coffee drinker has sugar given that the coffee drinker uses milk.

5 The probability that a school day is Monday is 0.2. The probability that a student is absent on Monday is 0.05. What is the probability that a student is absent given that it is a Monday?

Collecting Data

The objective is to collect quality data with a minimum of effort.

Sample
A collection of data from **part** of the population.

Sample Bias:
The sample doesn't represent the population.

Sample Size:
Must be small enough to be economical but large enough to represent the population.

Example
What percentage of students have part-time jobs?
A census would collect data from **every** student.
A sample would collect data from **some** students.

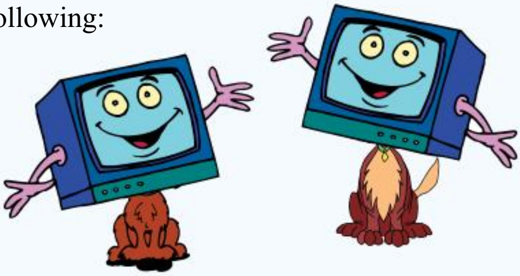
A stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample.

Exercise 8.7

A **Census** reflects the opinions of the whole population	
Advantages of samples ☺ A sample is more economical. ☺ Information can be gathered more quickly. ☺ A sample uses less resources.	**Disadvantages of samples** ☻ Tricky to get a good sample. ☻ The sample must be small but large enough to reflect the opinion of the population.

1 What is the meaning of each of the following:

- **a)** Census
- **b)** Sample
- **c)** Sample bias
- **d)** Sample size
- **e)** Random sample
- **f)** Stratified sample?



2 The Nielsen TV ratings in America attempts to measure the number of households watching a certain TV program. For example, the highest rating TV program in 2010-2011 was American Idol with a rating of 14.5 (14.5% of the television households).

a) If there is an estimated 120 million television households in America, how many households watched American Idol, on average, in 2010-2011?

b) Comment on the following criticisms of the Nielsen TV ratings:

- **i)** Only households that agree to be sampled are sampled
- **ii)** In 2009, 25 000 households participated in the ratings
- **iii)** Only households with a TV are sampled. Other TV viewers are not sampled (Cable TV, public places, Internet TV, etc).

c) Comment on the following statement:
'You can't believe TV ratings and you never will'.

Collecting Data

Using a random sample is the single most-important aspect of estimating population data.

A random sample helps avoid bias because each member of the population has an equal chance of being selected in the sample.

500-1000 is a typical size for political opinion polls.

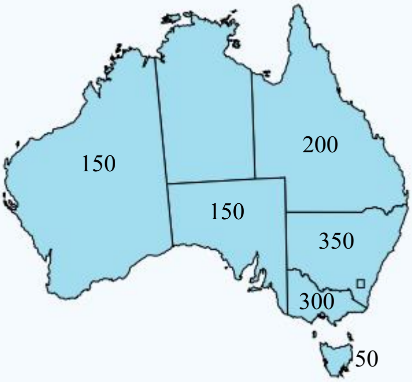
A random sample of size 2000 will be more accurate than a self-selected sample of size 200 000.

Exercise 8.8

1 It is often suggested that a random sample of 1000 people has a sampling error of ±3%, and that a random sample of 10 000 people would reduce the sampling error to ±1%.

An opinion poll of 1000 people reports that more people prefer product A (49%) to product B (47%). Add the sampling error to show that the conclusion that product A is preferred to product B may be incorrect.

2 An opinion poll survey uses the following stratified sample.
Comment on the appropriateness of the sample.



Population of Australian States	
NSW	7 200 000
Vic	5 600 000
Qld	4 500 000
WA	2 300 000
SA	1 600 000
Tas	500 000
ACT	400 000
NT	200 000

3 An opinion poll reports that public support for the National Broadband Network is 56% and opposition/don't know is 44%. The opinion poll used the above stratified sample.

a) Add the sampling error.

b) The opinion poll was conducted via landline telephones (ie no mobile phones). Comment on the following possible sample bias:

i) Non-response bias: Some people can't or won't answer. Are those who don't answer likely to have different views to those who do answer?

ii) Coverage bias: Are people with mobile phones likely to have different views to people with landline telephones?

iii) Response bias: Some people don't express their true beliefs.

c) Comment on the following statement:
'The opinion poll difference between A and B has to be at least 10% before we can start to think that there is a real difference'.

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 8.9

1 Spell Probability.
2 List all possible outcomes for tossing 3 coins
3 P(2 heads and a tail)
4 What is the quadratic formula?
5 $5x^2 - 3x - 1 = 0$. What are the values of a, b, and c?
6 Factorise $x^2 + 5x + 6$
7 What is the value of: $\text{Log}_2 8$
8 Solve: $x + y = 11$, $xy = 18$
9 ${}^{-}2 - {}^{-}7$
10 Evaluate 2^5?

2^5
$= 2\times2\times2\times2\times2$
$= 32$

Exercise 8.10

1 Spell Independence
2 List all possible outcomes for a family of three children
3 P(2 girls and a boy)
4 What is the quadratic formula?
5 $x^2 - 3x + 4 = 0$. What are the values of a, b, and c?
6 Factorise $x^2 - 6x + 5$
7 What is the value of: $\text{Log}_3 27$
8 Solve: $x + y = 11$, $xy = 28$
9 ${}^{-}5 - {}^{-}3$
10 Evaluate 3^5?

'I'd rather regret the things I've done than regret the things I haven't done' - Lucille Ball.

Exercise 8.11

1 Spell Conditional
2 List all possible outcomes for tossing three coins
3 P(3 heads)
4 What is the quadratic formula?
5 $2x^2 - x + 3 = 0$. What are the values of a, b, and c?
6 Factorise $x^2 + 3x - 10$
7 What is the value of: $\text{Log}_4 16$
8 Solve: $x + y = 9$, $xy = 14$
9 ${}^{-}2 - 4$
10 Evaluate 4^4?

'The probability of being watched is directly proportional to the stupidity of your act' - Law of Probability.

Agricultural Scientists improve the productivity of farms and agriculture.

- Relevant school subjects are English, Mathematics, Chemistry.
- Courses generally involves an agricultural science University degree.

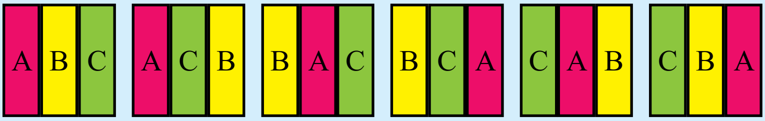
Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 8.12

Three books, A, B, and C, are randomly placed on a shelf.
What is the probability that A and B will be next to each other?

List all possible arrangements:



$$\text{P(AB next to each other)} = \frac{4}{6} = \underline{\frac{2}{3}}$$

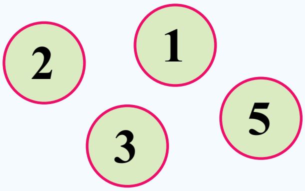
1 Three books, A, B, and C are randomly placed on a shelf.
What is the probability that B and C will be next to each other?

2 Three books, A, B, and C are randomly placed on a shelf.
What is the probability that A and C will be next to each other?

3 Four books, A, B, C, and D are randomly placed on a shelf.
What is the probability that A and B will be next to each other?

4 Three digits, 1, 2, and 3, are randomly arranged to form a three-digit number.
What is the probability that number is greater than 250?

5 Four digits, 1, 2, 3, and 5 are randomly arranged to form a four-digit number.
Find the probability that:
- a) the number is even.
- **b)** the number is odd.
- **c)** the first and the last digit is odd.
- **d)** the number is greater than 5 000.
- **e)** the number is even or greater than 5 000.
- **f)** The digits 3 and 5 are next to each other.



6 Three boys and two girls are randomly arranged in a row.
Find the probability that:
- **a)** the two girls are together.
- **b)** a boy is on each end.
- **c)** a girl is on one end and a boy is on the other.

7 Given that n! =n(n-1)(n-2) … 3x2x1.
- **a)** Find 4!
- **b)** Find 5!
- **c)** Form a problem to which the answer is 4!
- **d)** Form a different problem to which the answer is 5!
- **e)** Form a problem to which the answer is: $\frac{2\times 3!}{5!}$

Q In how many ways can three books A, B, and C be arranged on a shelf?

A $3! = 3\times2\times1 = 6$

Investigations

Investigation 8.1 A Blue Moon?

Use the Internet to research the following questions:

- **a)** Once in a blue moon. What is a blue moon?
- **b)** Buckley's chance. Who was Buckley?

Investigation 8.2 Research a gambling game.

There have been thousands and thousands of gambling games throughout the centuries.

- **a)** Select a gambling game.
- **b)** Investigate the theoretical probabilities of a gambler winning.
- **c)** Simulate the game a large number of times.
- **d)** Report your findings.

The one certainty about gambling is that the odds are against the gambler.

Investigation 8.3 Media Bias

- ☺ Internet search "media bias examples".
- ☺ Find media articles that involve sampling.
- ☺ Is the sample size sufficient?
- ☺ Is the sample biased?

The greater majority of people have more than the average number of legs?

Probably 2000 people out of 22 000 000 people have 1 leg.

$$\text{Average no legs} = \frac{2000 + 21998000 \times 2}{22000000}$$

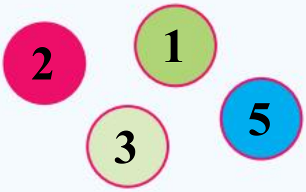
$$= 1.9999 \text{ legs}$$

21998000 people have more than the average.

Investigation 8.4 Dependent Events

Investigate

Is it true that if the first ball is not replaced then the choice of the second ball depends on which ball was first chosen?



Investigation 8.5 Probability Simulators

Investigate

Online probability simulators.

Probability means never having to say you're certain.

A Couple of Puzzles

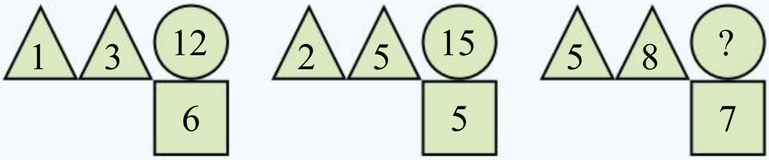
Straight from p168 Exercise 13.11 Year 10 2005

Exercise 8.13

1 What is the probability that a two-digit number contains a 5?

2 A drawer has five brown socks and three black socks. What is the smallest number of socks that need to be taken from the drawer, without looking, to be sure of having a matching pair?

3 What is the missing number?



A Game

Dingo is played with three dice by two players.
The playing board has the numbers 3 to 18 as shown.

3	4	5	6
7	8	9	10
11	12	13	14
15	16	17	18

1 Decide who goes first.
Highest total on throwing the three dice?

2 Each player, in turn, throws the three dice.

3 The player then has a limited time, say 1 minute, to make one of the numbers on the board using each of the three numbers showing.

4 The player marks the square as their own.

5 The player who owns the largest number of squares is the winner.

Example: 4, 3, 6
(6−4)×3 = 6 or 3×4+6 = 18
Which squares are the easiest to get?

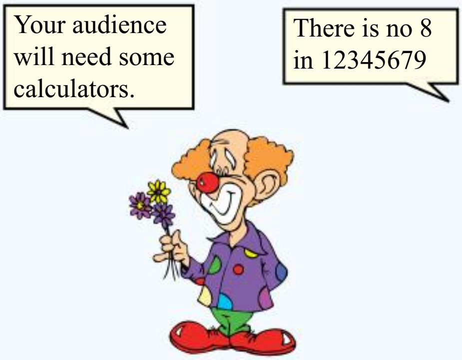
A Sweet Trick

Impress your audience by showing that you know some more tricky patterns.

1^2 =
11^2 =
111^2 =
1111^2 =
11111^2 =

1×9+2 =
12×9+3 =
123×9+4 =
1234×9+5 =
12345×9+6 =

9×6 =
99×66 =
999×666 =
9999×6666 =
99999×66666 =



15873×7 =
15873×14 =
15873×21 =
15873×28 =
15873×35 =

12345679×9 =
12345679×18 =
12345679×27 =
12345679×36 =
12345679×45 =

12345679×3 =
12345679×6 =
12345679×9 =
12345679×12 =
12345679×15 =

Technology

Technology 8.1

Use the Rand (random) on a calculator to simulate the throwing of a die.

A throw = 2ndF Rand x 6 + 0.5 = {Round the answer}

The random function gives a number between 0 and 1. This is scaled to give a number from 1 to 6.

Repeatedly pressing = may give a sequence of random numbers.

Technology 8.2

Use a spreadsheet to simulate the throwing of a die. This is a great way to estimate the experimental probability of the numbers 1, 2, 3, 4, 5, 6.

Enter the formula =Randbetween(1,6)

5		
4		
5	No 1s	86
6	No 2s	89
1	No 3s	73
3	No 4s	77
3	No 5s	91
5	No 6s	84
2		
1		

If using 500 rows enter: =CountIF(a1:a500,1)

If using 500 rows enter: =CountIF(a1:a500,6)

Press F9 to get a new set of random numbers.

Technology 8.3

Use a spreadsheet to simulate the throwing of two dice.

If using 500 rows enter: =CountIF(a2:a500,2)

Enter the formula =Randbetween(1,6)

Die 1	Die 2	Total			
3	3	6			
4	3	7			Probability
2	2	4	Total of 2 =	9	0.02
4	2	6	Total of 3 =	28	0.06
1	4	5	Total of 4 =	48	0.10
5	1	6	Total of 5 =	62	0.12
2	5	7	Total of 6 =	71	0.14
5	5	10	Total of 7 =	79	0.16
4	4	8	Total of 8 =	66	0.13
5	4	9	Total of 9 =	53	0.11
6	4	10	Total of 10 =	47	0.09
4	4	8	Total of 11 =	28	0.06
1	2	3	Total of 12 =	9	0.02
5	2	7	TOTAL	500	
2	6	8			
6	1	7			

Chapter Review 1

Exercise 8.14

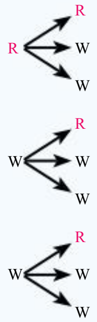
1 A bag contains 2 white balls and a red ball. A ball is withdrawn, the colour noted, and replaced back in the bag. A second ball is then drawn. Find the probability of drawing:

a) 2 white balls one after the other

b) 2 red balls one after the other

c) A red and then a white

d) at least 1 red ball.



2 The numbers 1, 3, and 5 are written on cards and put in a bag. A card is withdrawn, the number noted, and **not replaced**. A second card is then drawn.

a) Use a table or tree diagram to show all the possible outcomes.

b) What is the probability that the order of numbers is:

i) 51 **ii)** odd **iii)** divisible by 5?

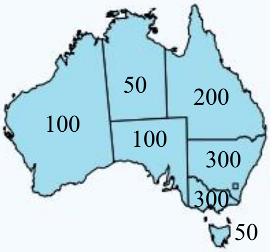
3 The hospital data showed that of the 90 patients, 34 patients had the A antigen, 23 had the B antigen. 9 patients had both the A and B antigens.
Find the probability that:

a) a patient had the A antigen only

b) a patient had no antigen (ie., neither the A nor B antigen)

c) a patient had no B antigen given that the patient had A Antigen.

4 35% of houses have a tiled roof and 17% of houses have a tiled roof and double brick. What is the probability that a house is double brick given that the house has a tiled roof?



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5 10A An opinion poll reports that public support for the National Broadband Network is 56% and opposition/don't know is 44%. The opinion poll used the above stratified sample.

a) Comment on the appropriateness of the stratified sample.

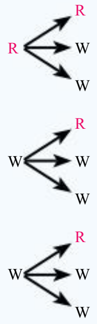
b) The opinion poll was conducted via landline telephones (ie no mobile phones). Comment on the following possible sample bias:

i) Non-response bias: Some people can't or won't answer. Are those who don't answer likely to have different views to those who do answer?

ii) Coverage bias: Are people with mobile phones likely to have different views to people with landline telephones?

Chapter Review 1

Exercise 8.15



1 A bag contains 2 white balls and a red ball. A ball is withdrawn, the colour noted, and replaced back in the bag. A second ball is then drawn. Find the probability of drawing:

- **a)** 2 white balls one after the other
- **b)** A white and then a red
- **c)** A red and then a white
- **d)** 2 whites or 2 reds.

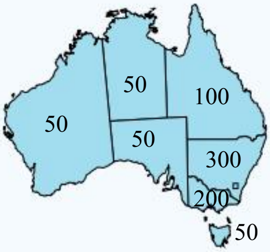
2 The numbers 3, 4, and 5 are written on cards and put in a bag. A card is withdrawn, the number noted, and **not replaced**. A second card is then drawn.

- **a)** Use a table or tree diagram to show all the possible outcomes.
- **b)** What is the probability that the order of numbers is:
  - **i)** 45 **ii)** even **iii)** divisible by 3?

3 The hospital data showed that of the 75 patients, 21 patients had the A antigen, 27 had the B antigen. 8 patients had both the A and B antigens.
Find the probability that:

- **a)** a patient had the B antigen only
- **b)** a patient had no antigen (ie., neither the A nor B antigen)
- **c)** a patient had no A antigen given that the patient had B Antigen.

4 24% of houses have a tiled roof and 13% of houses have a tiled roof and double brick. What is the probability that a house has a tiled roof given that the house is double brick?



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  - **ii)** Coverage bias: Are people with mobile phones likely to have different views to people with landline telephones?

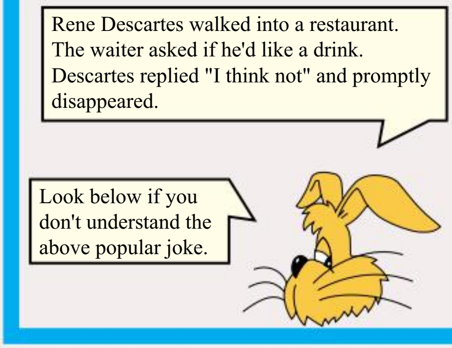
Polynomials

9

Number and Algebra ➞ Patterns and Algebra

10A Investigate the concept of a polynomial and apply the factor and remainder theorems to solve problems:

- investigate the relationship between algebraic long division and the factor and remainder theorems.



A TASK

While solving polynomials, Descartes arrived at 'The Rule of Signs'.

Research 'The rule of Signs'.

Does it work?
Can you give supporting evidence?

Make a powerpoint of your findings and then show it to the rest of the class.

A LITTLE BIT OF HISTORY

Rene Descartes (1596-1650) made a major contribution to analytical geometry.

Descartes:

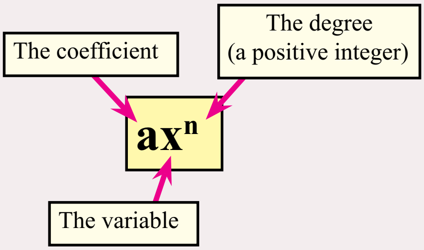
- initiated the use of superscripts for powers as in $aaaaa = a^5$.
- shifted the study of geometry from shapes to equations.
- stated the Factor Theorem: $x - a$ is a factor of a polynomial if and only if a is a solution (root).

Rene Descartes is also regarded as the father of modern philosophy. He is known for the saying:

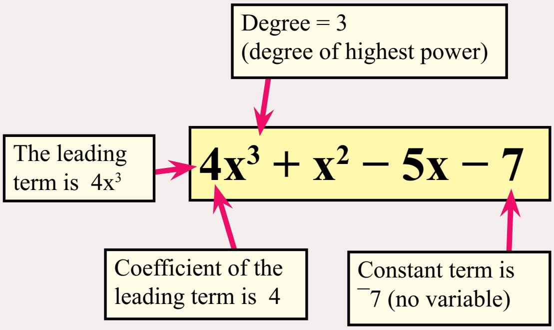
"cogito ergo sum" meaning "I think, therefore I am".

Polynomials

Monomial - 1 term



Polynomial - from 2 terms to many terms



Exercise 9.1

1 For each polynomial, state the variable, the degree, the leading coefficient, and the constant term:

$2m^3 + 4m - 5$

The variable is m

The degree is 3

The leading coefficient is 2

The constant term is $^{-}5$

a) $3x^3 - 5x^2 + 5x + 3$
b) $5a^4 - 4a^3 - a^2 + 2a - 1$
c) $^{-}3.5x^2 - \sqrt{2}x + 2.7$
d) $y^5 - 3y^2 - 8$
e) $^{-}2x^2 - 3x + 4$
f) $7x^5 + 5x^4 - 4x^3 - x^2 + 2x - 5$

2 Find the value of each polynomial for the given value of the variable:

$P(x) = 2x^2 + 4x - 5,\ P(^{-}2)$

$$\begin{aligned} P(^{-}2) &= 2(^{-}2)^2 + 4(^{-}2) - 5 \\ &= 2\times4 + 4\times{}^{-}2 - 5 \\ &= 8 + {}^{-}8 - 5 \\ &= \underline{^{-}5} \end{aligned}$$

$P(x) = 5x^4 + 4x^3 - 2,\ P(0)$

$$\begin{aligned} P(0) &= 5(0)^4 + 4(0)^3 - 2 \\ &= 5\times0 + 4\times0 - 2 \\ &= \underline{^{-}2} \end{aligned}$$

a) $P(x) = 3x^2 + 2x + 3,\ P(1)$
b) $P(c) = 2c^3 - c^2 + 3c - 1,\ P(2)$
c) $P(x) = {}^{-}x^2 - 2x + 1,\ P(^{-}2)$
d) $P(y) = y^5 - 2y^2 - 4,\ P(^{-}1)$
e) $P(x) = {}^{-}2x^2 + 5x + 7,\ P(3)$
f) $P(x) = x^5 - 2x^3 + 5x + 2,\ P(1)$
g) $P(a) = 5a^4 - 4a^3 - a^2 + 2a - 1,\ P(0)$
h) $P(x) = {}^{-}x^7 - 2x^4 + 7,\ P(0)$
i) $P(y) = y^3 - 3y^2 - 23,\ P(0)$
j) $P(x) = {}^{-}3x^2 - 3x + 4,\ P(b)$

3 Why are each of the following not polynomials:

$y^{3.1} + \sqrt{^{-}2}y^2 - 7.1$

The index 3.1 is not a positive whole number.

The coefficient of y^2 is not a real number.

a) $3x^4 - 7x^{0.5} + 2$
b) $b^{-2} - 4b^3 - 1$
c) $^{-}3.5x^{2/3} - \sqrt{3}x$
d) $\sqrt{^{-}2}y^3 + y^2 - 5.3$
e) $^{-}2.3x^2 - 3x^{-1} + 6$
f) $9x^5 - 2x^4 - 5x^3 - x^{2.3} + 2x - 5$?

The index must be a positive whole number.

The coefficient must be a real number.

Adding Polynomials

If P(x) and Q(x) are polynomials then P(x) + Q(x) is also a polynomial.

$$\begin{array}{r} 3x^3 - 2x^2 + 2x - 5 \\ 2x^3 + 3x^2 - 4x + 1 \; + \\ \hline \underline{5x^3 + 1x^2 - 2x - 4} \end{array}$$

Polynomials can be added vertically.

Exercise 9.2

1 Simplify each of the following polynomials:

$(5x^2 - 2x + 4) + (3x^2 + x - 5)$

$= 5x^2 - 2x + 4 + 3x^2 + x - 5$
$= 5x^2 + 3x^2 - 2x + x + 4 - 5$
$= \underline{8x^2 - x - 1}$

$(5x^2 - 2x + 4) + (3x^2 + x - 5)$

$$\begin{array}{r} 5x^2 - 2x + 4 \\ 3x^2 + x \;\; - 5 \; + \\ \hline \underline{8x^2 - x \;\; - 1} \end{array}$$

a) $(2x^2 - 3x + 5) + (5x^2 + 4x - 2)$
b) $(3x^2 + 2x + 2) + (3x^2 - 3x + 5)$
c) $(6x^2 - 5x + 2) + (3x^2 + 2x - 2)$
d) $(x^2 + 3x + 7) + (3x^2 - 6x - 3)$
e) $(^{-}4x^3 + 5x^2 - 3x + 5) + (5x^3 + 5x^2 + 4x - 2)$
f) $(7x^3 - 2x^2 + x + 2) + (4x^3 + 2x^2 - 5x + 3)$
g) $(4x^4 + 4x^3 - 6x^2 - 5x + 2) + (^{-}3x^4 - 5x^2 + 4x + 1)$
h) $(^{-}3x^5 - x^4 + 3x - 7) + (2x^4 - 4x^3 + 3x^2 - 6x + 3)$
i) $(3x^2 - 2x - 4) + (^{-}5x^2 + x - 5) + (6x^2 - 3x + 6)$
j) $(2x^2 + 3x + 6) + (x^2 - 5x - 2) + (^{-}x^2 + x + 2)$

Add polynomials by adding like terms.

Be careful with negative coefficients

2 Find P(x) + Q(x)

a) $P(x) = 2x^2 + 3x + 2$
$Q(x) = 3x^2 + 5x + 1$

b) $P(x) = 3x^2 + 2x + 1$
$Q(x) = x^2 + 3x + 5$

c) $P(x) = {}^{-}x^2 - 2x + 3$
$Q(x) = 4x^2 - 5x + 1$

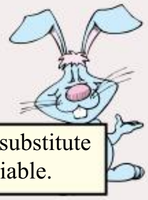
d) $P(x) = x^2 - 5x + 5$
$Q(x) = {}^{-}2x^2 + 4x - 3$

3 The sides of a triangle are $4x - 2$, $2x + 4$, and $5x - 6$.
What is the perimeter?

3P(x) = P(x)+P(x)+P(x)

4 Find 3P(x) given that $P(x) = x^4 - 2x^3 + 3x^2 + 5x - 3$

5 If R(x) − P(x) = Q(x), find R(x)
given that $P(x) = 2x^2 + 3x + 1$ and $Q(x) = x^2 + 4x + 2$



6 If R(x) = 2P(x) + Q(x), Find R(1)
$P(x) = {}^{-}x^3 - 2x^2 + 7$, $Q(x) = 3x^3 + x^2 + 3x - 2$

R(1) means substitute 1 for the variable.

7 The length of a rectangle is $2x^2 - 5$ and the breadth is $4x^2 + 5x$.
What is the perimeter of the rectangle?

Subtracting Polynomials

If P(x) and Q(x) are polynomials then P(x) − Q(x) is also a polynomial.

$^{-}3x^2 - {}^{+}2x^2 = {}^{-}5x^2$

$+2x - {}^{-}4x = {}^{+}6x$

$^{-}2 - {}^{-}1 = {}^{-}1$

$$\begin{array}{r} 5x^3 - 3x^2 + 2x - 2 \\ 2x^3 + 2x^2 - 4x - 1 \quad - \\ \hline \underline{3x^3 - 5x^2 + 6x - 1} \end{array}$$

As a check, the sum of these two rows should give the top row.

Polynomials can be subtracted vertically.

Exercise 9.3

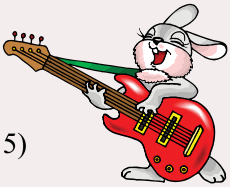
1 Simplify each of the following polynomials:

Distribute the − $(5x^2 - 2x + 4) - (3x^2 + x - 5)$ $= 5x^2 - 2x + 4 - 3x^2 - x + 5$ $= 5x^2 - 3x^2 - 2x - x + 4 + 5$ $= \underline{2x^2 - 3x + 9}$	$(5x^2 - 2x + 4) - (3x^2 + x - 5)$ $\begin{array}{r} 5x^2 - 2x + 4 \\ 3x^2 + \quad x - 5 \quad - \\ \hline \underline{2x^2 - 3x + 9} \end{array}$

a) $(2x^2 - 3x + 5) - (5x^2 + 4x - 2)$
b) $(3x^2 + 2x + 2) - (3x^2 - 3x + 5)$
c) $(6x^2 - 5x + 2) - (3x^2 + 2x - 2)$
d) $(x^2 + 3x + 7) - (3x^2 - 6x - 3)$
e) $({}^{-}4x^3 + 5x^2 - 3x + 5) - (5x^3 + 5x^2 + 4x - 2)$
f) $(7x^3 - 2x^2 + x + 2) - (4x^3 + 2x^2 - 5x + 3)$
g) $(4x^4 + 4x^3 - 6x^2 - 5x + 2) - ({}^{-}3x^4 - 5x^2 + 4x + 1)$
h) $({}^{-}3x^5 - x^4 + 3x - 7) - (2x^4 - 4x^3 + 3x^2 - 6x + 3)$
i) $(5x^4 - 2x^3 + 3x^2 - 2x - 4) - (6x^4 + 3x^3 - 5x^2 + x - 5)$
j) $(2x^2 + 3x + 6) + (x^2 - 5x - 2) - ({}^{-}x^2 + x + 2)$

Subtract like terms.

Be careful with negative coefficients



2 Find P(x) − Q(x)

a) $P(x) = 2x^2 + 3x + 2$
$Q(x) = 3x^2 + 5x + 1$

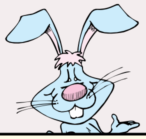
b) $P(x) = 3x^2 + 2x + 1$
$Q(x) = x^2 + 3x + 5$

c) $P(x) = {}^{-}x^2 - 2x + 3$
$Q(x) = 4x^2 - 5x + 1$

d) $P(x) = x^2 - 5x + 5$
$Q(x) = {}^{-}2x^2 + 4x - 3$

3 Two sides of a triangle are $2x - 2$ and $x + 4$. The perimeter is $5x - 6$. What is the length of the third side?

4 If $R(x) = 3P(x) - 2Q(x)$, Find $R({}^{-}1)$
$P(x) = {}^{-}x^3 - 2x^2 + 7$, $Q(x) = 3x^3 + x^2 + 3x - 2$



R(⁻1) means substitute ⁻1 for the variable.

5 The perimeter of a rectangle is $7x^2 - 5$.
The length is $3x^2 + 5x$.
What is the breadth of the rectangle?

Multiplying Polynomials

If P(x) and Q(x) are polynomials then P(x) × Q(x) is also a polynomial.

$$\begin{array}{rr} 3x^2 + 2x - 1 & \\ 4x - 1 & \times \\ \hline 12x^3 + 8x^2 - 4x & \\ -3x^2 - 2x + 1 & + \\ \hline 12x^3 + 5x^2 - 6x + 1 & \\ \hline \end{array}$$

$4x\times(3x^2 + 2x - 1)$ in this row

$^-1\times(3x^2 + 2x - 1)$ in this row

This row is the sum of the above 2 rows

Polynomials can be multiplied vertically.

Exercise 9.4

1 Find the following products:

$(5x^2 - 2x)(3x^2 - x + 5)$ $= 5x^2(3x^2 - x + 5) \quad - 2x(3x^2 - x + 5)$ $= 15x^4 - 5x^3 + 25x^2 - 6x^3 + 2x^2 - 10x$ $= \underline{15x^4 - 11x^3 + 27x^2 - 10x}$ $a(b + c) = ab + ac$	$(5x^2 - 2x)(3x^2 - x + 5)$ $\begin{array}{rr} 3x^2 - x + 5 & \\ 5x^2 - 2x & \times \\ \hline 15x^4 - 5x^3 + 25x^2 & \\ - 6x^3 + 2x^2 - 10x & + \\ \hline 15x^4 - 11x^3 + 27x^2 - 10x & \\ \hline \end{array}$

a) $(2x^2 - 3x)(5x^2 + 4x - 2)$
b) $(3x^2 + 2x)(3x^2 - 3x + 5)$
c) $(5x + 2)(3x^2 + 2x - 2)$
d) $(3x + 7)(3x^2 - 6x - 3)$
e) $(^-4x^3 + 1)(5x^3 + 5x^2 + 4x - 2)$
f) $(7x^3 - 2x^2)(4x^3 - 5x)$
g) $(4x^4 - 5x)(^-3x^4 + 4x + 1)$
h) $(^-3x^5 - 1)(2x^4 - 4x^3 + 3)$
i) $(3x^2 - 2x - 4)(x - 5)$
j) $(2x^2 - 6)(^-x^2 + x + 2)$

Commutative Law of Multiplication
$P(x)\times Q(x) = Q(x)\times P(x)$

Sometimes a different order makes it easier.

2 Find $P(x)\times Q(x)$

a) $P(x) = 3x + 2$
$Q(x) = 3x^2 + 5x + 1$

b) $P(x) = 2x + 1$
$Q(x) = x^2 + 3x + 5$

c) $P(x) = {}^-x^2 - 2x + 3$
$Q(x) = 4x^2 - 5x$

d) $P(x) = x^2 - 5x$
$Q(x) = {}^-2x^2 + 4x - 3$

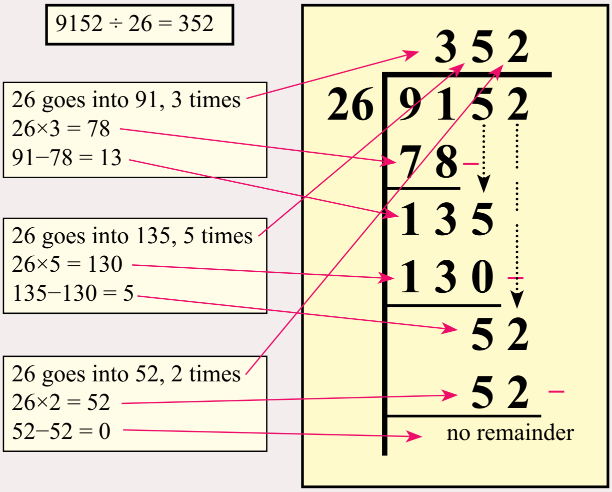
3 A rectangle has a length of $4x^2 - 2$ and a breadth of $2x^3 + 4$. What is the area of the rectangle?

4 If $R(x) = P(x)Q(x) - 2Q(x)$, Find R(x)
$P(x) = 2x^2 + 7, \; Q(x) = 3x^3 + 3x - 2$

Dividing Polynomials

We need 'long division' for P(x)÷Q(x)

It is tricky because there are 'lots' of steps.



9152 ÷ 26 = 352

26 goes into 91, 3 times
26×3 = 78
91−78 = 13

26 goes into 135, 5 times
26×5 = 130
135−130 = 5

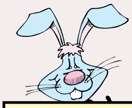
26 goes into 52, 2 times
26×2 = 52
52−52 = 0

Exercise 9.5

1 Use 'long division' to calculate the following divisions:

a)	690 ÷ 15	{15 is a factor of 690 thus no remainder}
b)	690 ÷ 46	{46 is a factor of 690 thus no remainder}
c)	713 ÷ 23	{23 is a factor of 713 thus no remainder}
d)	713 ÷ 31	{31 is a factor of 713 thus no remainder}
e)	1768 ÷ 34	{34 is a factor of 1768 thus no remainder}
f)	1768 ÷ 52	{52 is a factor of 1768 thus no remainder}

15 × 46 = 690
15 and 46 are factors of 690



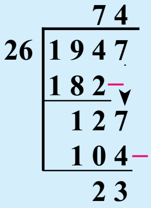
Lots of practice is the key to mastering this tricky process.

2 Use 'long division' to calculate the following divisions:
(The following have remainders - the divisor is not a factor).

1947 ÷ 26

= 74 remainder 23

(26 divides into 1947
74 times with 23 left over)



a) 463 ÷ 15
b) 643 ÷ 22
c) 782 ÷ 31
d) 2416 ÷ 45
e) 5482 ÷ 52
f) 9271 ÷ 73

3 Is A a factor of C?

a) A = 23
C = 7251

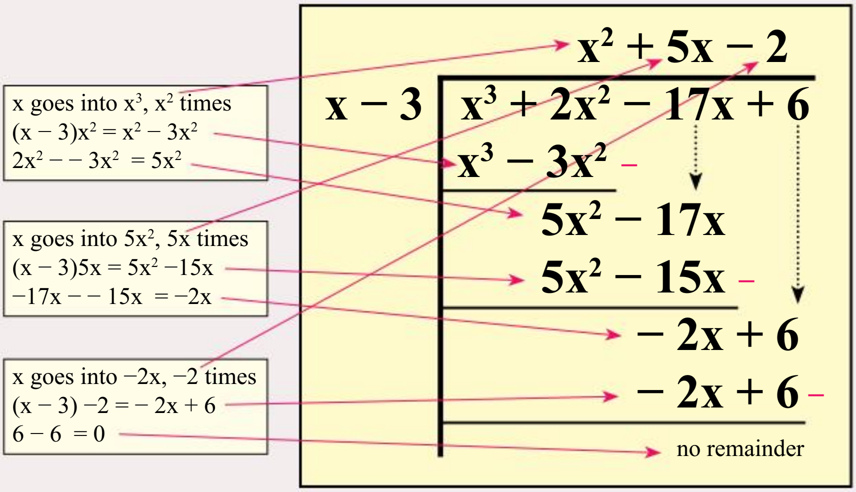
b) A = 36
C = 82908

If A×B = C then A and B are factors of C.

If C÷A has no remainder then A is a factor.

Dividing Polynomials

$(x^3 + 2x^2 - 17x + 6) \div (x - 3) = x^2 + 5x - 2$



x goes into x^3, x^2 times
$(x - 3)x^2 = x^2 - 3x^2$
$2x^2 - - 3x^2 = 5x^2$

x goes into $5x^2$, 5x times
$(x - 3)5x = 5x^2 - 15x$
$-17x - - 15x = -2x$

x goes into −2x, −2 times
$(x - 3) -2 = - 2x + 6$
$6 - 6 = 0$

Exercise 9.6

1 Complete the following divisions:

a) $(x^2 + 5x + 6) \div (x + 2)$ {x + 2 is a factor of $x^2 + 5x + 6$ thus no remainder}
b) $(x^2 + 5x + 6) \div (x + 3)$ {x + 3 is a factor of $x^2 + 5x + 6$ thus no remainder}
c) $(x^2 - 4x + 3) \div (x - 1)$ {x − 1 is a factor of $x^2 - 4x + 3$ thus no remainder}
d) $(x^2 - 4x + 3) \div (x - 3)$ {x − 3 is a factor of $x^2 - 4x + 3$ thus no remainder}
e) $(2x^2 - 3x - 2) \div (x - 2)$ {x − 2 is a factor of $2x^2 - 3x - 2$}
f) $(2x^2 - 3x - 2) \div (2x + 1)$ {2x + 1 is a factor of $2x^2 - 3x - 2$}
g) $(x^3 + 6x^2 + 11x + 6) \div (x + 2)$ {x + 2 is a factor of $x^3 + 6x^2 + 11x + 6$}
h) $(x^3 - 4x^2 + x + 6) \div (x - 3)$ {x − 3 is a factor of $x^3 - 4x^2 + x + 6$}

2 Find P(x) ÷ Q(x)

a) $P(x) = x^2 + 2x - 15$
$Q(x) = x + 5$

b) $P(x) = x^2 + 2x - 15$
$Q(x) = x - 3$

c) $P(x) = x^3 - 5x^2 + 2x + 32$
$Q(x) = x + 2$

d) $P(x) = x^2 - 5x$
$Q(x) = x - 5$

3 If x + 5 is a factor of $x^2 + 3x - 10$, find another factor.

4 If x − 3 is a factor of $x^2 + x - 12$, find another factor.

If P(x)×Q(X)=R(x) then P(x) and Q(x) are factors of R(x)

5 A rectangle has a an area of $x^3 - 5x^2 + 11x - 15$. Find the length given that the breadth is x − 3.

6 Is Q(x) a factor of P(x)?

a) $P(x) = x^2 + 2x - 15$
$Q(x) = x + 3$

b) $P(x) = x^2 + 2x - 15$
$Q(x) = x + 5$

c) $P(x) = x^3 + 5x^2 + 5x - 3$
$Q(x) = x + 3$

d) $P(x) = x^2 - 5x + 3$
$Q(x) = x - 2$

What is the clue that x−2 is not a factor?

The Factor Theorem

The Factor Theorem:

If $P(a) = 0$, then $x - a$ is a factor of $P(x)$

$P(x) = x^2 - 3x - 10$

$P(^-2) = (^-2)^2 - 3 \times {}^-2 - 10$

$P(^-2) = 4 + 6 - 10$

$P(^-2) = 0$

Thus $\underline{x + 2 \text{ is a factor.}}$

Exercise 9.7

1 Complete the following statements:

- **a)** If $P(2) = 0$ then .....?.... is a factor of $P(x)$
- **b)** If $P(1) = 0$ then .....?.... is a factor of $P(x)$
- **c)** If $P(^-2) = 0$ then ....?.... is a factor of $P(x)$
- **d)** If $x - 3$ is a factor of $P(x)$ then $P(.?.) = 0$
- **e)** If $x + 1$ is a factor of $P(x)$ then $P(.?.) = 0$
- **f)** If $x - 4$ is a factor of $P(x)$ then $P(.?.) = 0$

$P(1) = 0$ ➔ $x-1$ a factor
$P(^-1) = 0$ ➔ $x+1$ a factor
$P(2) = 0$ ➔ $x-2$ a factor
$P(^-3) = 0$ ➔ $x+3$ a factor
Do you get it?

Use the factor theorem to show that $x - 2$ is a factor of $x^2 + 2x - 8$.

If $x - 2$ is a factor then $P(2) = 0$

$P(x) = x^2 + 2x - 8$

$P(2) = 2^2 + 2 \times 2 - 8$

$P(2) = 4 + 4 - 8$

$P(2) = 0$ thus $\underline{x - 2 \text{ is a factor}}$

2 Is $x - 1$ a factor of $x^2 + 2x - 3$

3 Is $x - 2$ a factor of $x^2 - x - 2$

4 Is $x - 1$ a factor of $x^2 + 6x - 7$

5 Is $x - 3$ a factor of $x^2 - x - 12$

6 Is $x - 3$ a factor of $x^2 + 2x - 15$

7 Is $x - 2$ a factor of $x^2 + 2x - 8$

8 Is $x - 5$ a factor of $x^2 - 7x + 10$

Use the factor theorem to show that $x + 3$ is a factor of $x^2 + 2x - 3$.

If $x + 3$ is a factor then $P(^-3) = 0$

$P(x) = x^2 + 2x - 3$

$P(^-3) = (^-3)^2 + 2 \times {}^-3 - 3$

$P(^-3) = 9 - 6 - 3$

$P(^-3) = 0$ thus $\underline{x + 3 \text{ is a factor}}$

9 Is $x + 6$ a factor of $x^2 + 5x + 6$

10 Is $x + 1$ a factor of $x^2 + 3x + 2$

11 Is $x + 3$ a factor of $x^2 + 2x - 3$

12 Is $x + 4$ a factor of $x^2 + x - 20$

13 Is $x + 4$ a factor of $x^2 + 2x - 8$

14 Is $x + 1$ a factor of $x^2 + 4x + 3$

15 Is $x + 2$ a factor of $x^2 - 3x - 10$

Is $x + 3$ is a factor of $3x^3 + 10x^2 - x - 8$?

If $x + 3$ is a factor then $P(^-3) = 0$

$P(x) = 3x^3 + 10x^2 - x - 8$

$P(^-3) = 3 \times (^-3)^3 + 10 \times (^-3)^2 - {}^-3 - 8$

$P(^-3) = {}^-81 + 90 + 3 - 8$

$P(^-3) = 4$

$P(^-3) \neq 0$ thus $\underline{x + 3}$ **is not** a factor

16 Is $x + 1$ a factor of $5x^3 + 6x^2 + 2x + 1$

17 Is $x - 3$ a factor of $2x^3 - 8x^2 + 6x$

18 Is $x + 2$ a factor of $3x^3 + 2x^2 - 2x + 8$

19 Is $x - 1$ a factor of $3x^4 + 2x^3 - 5$

20 Is $x + 7$ a factor of $x^3 + 5x - 10$

21 Is $x + 2$ a factor of $x^3 - 4x^2 - 7x + 10$

22 Is $x - 5$ a factor of $x^3 - 4x^2 - 7x + 10$

23 Is $x - 1$ a factor of $x^3 - 4x^2 - 7x + 10t$

Solving Polynomial Equations

Use the Factor Theorem to find solutions to polynomials

If $P(a) = 0$, then $x - a$ is a factor of $P(x)$

Exercise 9.8

Solve the following polynomial equations:

$x^3 - 2x^2 - x + 2 = 0$

Try $P(1)$ then $P(^-1)$ then $P(2)$ then $P(^-2)$

$P(1) = 1^3 - 2\times1^2 - 1 + 2$
$P(1) = 1 - 2 - 1 + 2$
$P(1) = 0$ thus $\underline{x - 1}$ is a factor

$$\begin{array}{rl} & x^2 - x - 2 \\ x - 1 & \overline{)\, x^3 - 2x^2 - x + 2} \\ & x^3 - x^2 \; - \\ \hline & -x^2 - x \\ & -x^2 + x \; - \\ \hline & -2x + 2 \\ & -2x + 2 \; - \\ \hline & \cdot \quad \cdot \end{array}$$

Now factorise: $x^2 - x - 2$
$(x - 2)(x + 1)$
Either $x - 2 = 0$ or $x + 1 = 0$
$\underline{x = 2 \text{ or } x = {}^-1}$

Three solutions: $\underline{x = 1, x = 2, x = {}^-1}$

Check solutions to $x^3 - 2x^2 - x + 2 = 0$
$x = 1,\ 1^3 - 2\times1^2 - 1 + 2 = 0$ ✓
$x = 2,\ 2^3 - 2\times2^2 - 2 + 2 = 0$ ✓
$x = {}^-1,\ ({}^-1)^3 - 2\times({}^-1)^2 - {}^-1 + 2 = 0$ ✓

1 $x^3 - 2x^2 - x + 2 = 0$
2 $x^3 + 2x^2 - 14x + 5 = 0$
3 $x^3 - x^2 - 9x + 9 = 0$
4 $x^3 - 4x^2 + x + 6 = 0$
5 $x^3 + x^2 - 4x - 2 = 0$
6 $x^3 - 4x^2 - 7x + 10 = 0$

Basically 'Guess and Check' to find the first factor.

Factors of the constant term can give clues.

The Remainder Theorem

The Remainder Theorem:
If $P(x)$ is divided by $x - a$, then $P(a)$ is the remainder.

$P(x) = x^2 - 3x - 10$
$P(^-3) = (^-3)^2 - 3\times{}^-3 - 10$
$P(^-3) = 9 + 9 - 10$
$P(^-3) = 8$
The remainder is 8 when $P(x)$ is divided by $x + 3$

Exercise 9.9

1 Find the remainder when $3x^3 + 2x^2 - 2x + 4$ is divided by $x - 1$
2 Find the remainder when $x^3 - 5x^2 + x + 4$ is divided by $x + 1$
3 Find the remainder when $2x^3 + x^2 - x - 3$ is divided by $x - 2$
4 Find the remainder when $x^3 - 7x^2 - 5x + 8$ is divided by $x + 2$
5 Find the remainder when $4x^3 + 3x^2 - 2x - 15$ is divided by $x - 1$

Why not do a long division to check your answer?

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 9.10

1 Spell Polynomials
2 Is x − 1 a factor of $2x^3 + 3x^2 - 2x - 1$?
3 Simplify: $(2x^2 - 3x + 2) + (x^2 + 2x - 1)$
4 Expand: $3x^2(2x^2 - 2x - 1)$
5 Draw a tree diagram for tossing 3 coins
6 What is the quadratic formula?
7 Factorise $x^2 + 5x + 6$
8 What is the value of: $\text{Log}_2 16$
9 ${}^{-}2 - {}^{-}5$
10 What is 3^4?

3^4
$= 3\times3\times3\times3$
$= 81$

'I have no special talent. I am only passionately curious' - Albert Einstein.

Exercise 9.11

1 Spell Factor Theorem
2 Is x − 1 a factor of $x^3 - 3x^2 - 2x + 4$?
3 Simplify: $(2x^2 - 3x + 2) - (x^2 + 2x - 1)$
4 Expand: $2x(3x^2 + 4x - 2)$
5 Draw a tree diagram for a family of 3 children
6 What is the quadratic formula?
7 Factorise $x^2 + 6x + 5$
8 What is the value of: $\text{Log}_2 32$
9 ${}^{-}3 - 7$
10 What is 4^4?

'It's not that I'm so smart, it's just that I stay with problems longer' - Albert Einstein.

Exercise 9.12

1 Spell Remainder Theorem
2 Is x − 1 a factor of $x^3 - 2x^2 - 2x - 1$?
3 Simplify: $(x^3 - 2x + 1) - (3x^3 - 2x - 3)$
4 Expand: ${}^{-}2x(3x^3 - 2x - 3)$
5 Draw a tree diagram for tossing 3 coins
6 What is the quadratic formula?
7 Factorise $x^2 + 5x + 4$
8 What is the value of: $\text{Log}_2 64$
9 ${}^{-}5 + {}^{-}3$
10 What is 5^4?

'Do not worry about your difficulties in Mathematics. I can assure you mine are still greater' - Albert Einstein.

Multimedia Developers create multimedia by developing graphics, video, sound, text, and animations.

- Relevant school subjects are English and Mathematics.
- Courses generally involves a University degree in multimedia.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 9.13

1 a ÷ 4 = 5 remainder 1

a) what is the value of a?

b ÷ 4 also gives a remainder of 1

b) What is the smallest difference between a and b?

2 a ÷ b = 4 remainder 1

a) Write an expression for a

b) If (a+8) ÷ b = 5 remainder 1, what is the value of b?

3 Assuming that a, b, c are positive integers, solve:

Assuming that a, b, c are positive integers, solve: ab = 6, bc = 3, ac = 2.

ab = 6 (1)
bc = 3 (2)
ac = 2 (3)

(1)÷(3) b/c = 3
b = 3c
Sub for b in (2)
$3c^2 = 3$
$c^2 = 1$
$c = 1$

Sub for c in (3) gives a = 2
Sub for c in (2) gives b = 3

Thus: a=2, b=3, c = 1

a) ab = 12, bc = 3, ac = 4

b) ab = 10, bc = 12, ac = 30

The difference between the square of a positive integer and the square of the number four less is 104. Find the larger number.

Let the larger number be x
$x^2 - (x - 4)^2 = 104$
$x^2 - (x^2 - 8x + 16) = 104$
$x^2 - x^2 + 8x - 16 = 104$
$8x = 120$
$x = 15$ The larger number is 15

Law of Biomechanics - The severity of the itch is inversely proportional to the reach.

4 The difference between the square of a positive integer and the square of the number five less is 205. Find the larger number.

5 The difference between the squares of consecutive positive even numbers is 132. Find the larger number.

6 What is the remainder when $1 + x^3 + x^9$ is divided by $x - 1$

7 What is the remainder when $1 + x^3 + x^9$ is divided by $x + 1$

8 If $P(x) = ax^2 + bx + 5$, $P(-1) = 10$, and $P(1) = 4$, find a and b

Investigations

Investigation 9.1 Graphing Polynomials

Investigate

How do you graph a polynomial?

Graph by hand?

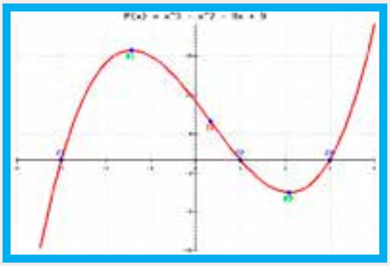
Graphics calculators?

Online Applets?

Geometry sofware?

Investigation 9.2 The Polynomial Graph

Before Rene Descartes most polynomial work was based on the graph of the polynomial.



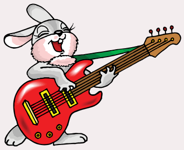
Investigate

What story does the graph of a polynomial tell?

Investigation 9.3 Practical Polynomials

Investigate

Some of the thousands of applications of polynomials in real life?



Ballistics

Electronics

You are a financial adviser and you are asked the following common type of question:

Which is the better investment?

\$100 000 in an online saver at 5.2% pa compounded monthly for 4 months (thus i=0.0173)? =100000(1+i)(1+i)(1+i)(1+i)	\$100 000 in a 4 month fixed deposit at 5.8% pa (thus i = 0.174)? = 100000(1 + i)

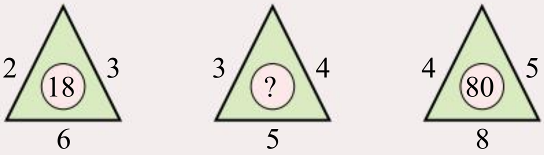
A Couple of Puzzles

Exercise 9.14

$9(9 + 9 \div 9) = 90$

1 Using four nines, 9, 9, 9, 9, can you write them in such a way that they are equal to 20?

2 Find the missing number:



A Game

Estimator is a calculator game. The winner is the person or group who scores three squares in any straight line (row, column, or diagonal).

1 Each person takes it in turns to select a square.

2 Each person estimates the answer to the square and writes down the estimate without the other person looking.

3 A calculator is used to find the correct answer.

4 The person with the closest estimate wins the square.

73^2	88×8	$9090\div91$	12% of 54	14^2+15^2
35% of 68	$\sqrt{5050}$	148^2	623×15	$7129\div68$
$7172\div15$	$7^2\times8^2$	46×54	$5^2\times6^2\times7^2$	95^2
77×91	$2051\div34$	14^3	48^2	$3^2\times4^2\times5^2$
23^2+42^2	15^3	15% of 92	$6234\div22$	39×85

A Sweet Trick

Your audience will need a six-sided die and a calculator. Ask your audience to do the following without them letting you see what they are doing.

1 Toss the die and have your audience enter the number on their calculator. 5

2 Multiply the number by 999 999. $5\times999999 = 4999995$

3 Divide the answer by 7. $4999995\div7 = 714285$

4 Rearrange the digits in ascending order. 124578

Tell them that the answer is 124578.

Brainstorm ways of embellishing the trick such as taking the answer from someone's pocket.

Technology

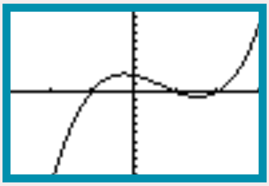
Technology 9.1 Solving Polynomials with the Graphics Calculator

Use a graphics calculator to check your answers to Exercise 9.8.

Example: Graph: $y = x^3 - 2x^2 - x + 2$ and solve: $x^3 - 2x^2 - x + 2 = 0$

Press Y = and enter the function eg., $x^2 + x - 6$

Press Graph to see a graph of the function.



To solve: $x^3 - 2x^2 - x + 2 = 0$ is to find the x-intercepts.

Use any of the following methods to find the x-intercepts:

a) Use CALC to find the intercepts (some calculators use zero and value).

b) Use TRACE and move the cursor to the x-intercepts.

c) Use TABLE to find the x-intercept (where y = 0).

Can you get the answers:
$x = 1, x = 2, x = {}^-1$?

Technology 9.2 Polynomial Long Division

Long Division

Watch videos on 'Long Division'.

'How many seconds are there in a year'?

'Twelve.
January second,
February second,
March second, ...'

Technology 9.3 The Factor Theorem

The Factor Theorem

Can you find a good video on 'The Factor Theorem'?

Technology 9.4 The Remainder Theorem

The Remainder Theorem

Can you find a good video on 'The Remainder Theorem'?

A statistician would never travel by air, because it had been estimated that the probability of there being a bomb on any given flight was one in a million, and these odds were too high.

One day, the statistician travelled by air. When asked why:

'Well, if the odds of one bomb are 1:million, then the odds of two bombs are (1/1,000,000) × (1/1,000,000). This is a very, very small probability, which I can accept. So now I bring my own bomb along!'

Chapter Review 1

Exercise 9.15

1 For each polynomial, state the variable, the degree, the leading coefficient, and the constant term:

a) $2x^3 - 7x^2 + 5x + 1$

b) $3a^4 - 3a^3 - a^2 + 2a - 5$

2 Find the value of each polynomial for the given value of the variable:

a) $P(x) = 2x^2 + 3x + 5,\ P(1)$

b) $P(x) = {}^-x^2 - 2x + 3,\ P({}^-2)$

3 Simplify each of the following polynomials:

a) $(2x^2 - 7x + 2) + (5x^2 + 2x - 2)$

b) $(5x^4 + 3x^3 - x^2 - 5x + 2) - ({}^-3x^4 - 2x^2 + 4x + 1)$

c) $({}^-2x + 1)(+x^2 + 4x - 2)$

d) $(x^2 - 4x + 3) \div (x - 1)$ {$x - 1$ is a factor of $x^2 - 4x + 3$ thus no remainder}

The Factor Theorem:
If $P(a) = 0$, then $x - a$ is a factor of $P(x)$

$P(x) = x^2 - 3x - 10$
$P({}^-2) = ({}^-2)^2 - 3\times{}^-2 - 10$
$P({}^-2) = 4 + 6 - 10$
$P({}^-2) = 0$
Thus $\underline{x + 2 \text{ is a factor.}}$

4 Use the factor theorem to show that $x - 2$ is a factor of $x^2 - 7x + 10$.

$x^3 - 2x^2 - x + 2 = 0$

Try $P(1)$ then $P({}^-1)$ then $P(2)$ then $P({}^-2)$

$P(1) = 1^3 - 2\times1^2 - 1 + 2$
$P(1) = 1 - 2 - 1 + 2$
$P(1) = 0$ thus $\underline{x - 1 \text{ is a factor}}$

Now factorise: $x^2 - x - 2$
$(x - 2)(x + 1)$
Either $x - 2 = 0$ or $x + 1 = 0$
$\underline{x = 2 \text{ or } x = {}^-1}$

$\underline{\text{Three solutions: } x = 1, x = 2, x = {}^-1}$

$$
\begin{array}{r|l}
 & \phantom{x^3 - {}}x^2 - x - 2 \\
\hline
x - 1 & x^3 - 2x^2 - x + 2 \\
 & x^3 - x^2 \ - \\
\cline{2-2}
 & -x^2 - x \\
 & -x^2 + x \ - \\
\cline{2-2}
 & -2x + 2 \\
 & -2x + 2 \ - \\
\cline{2-2}
 & \bullet \quad \bullet
\end{array}
$$

Check solutions to $x^3 - 2x^2 - x + 2 = 0$
$x = 1,\ 1^3 - 2\times1^2 - 1 + 2 = 0$ ✓
$x = 2,\ 2^3 - 2\times2^2 - 2 + 2 = 0$ ✓
$x = {}^-1,\ ({}^-1)^3 - 2\times({}^-1)^2 - {}^-1 + 2 = 0$ ✓

5 Solve each of the following polynomial equations:

a) $x^3 - 2x^2 - 5x + 6 = 0$

b) $x^3 - 6x^2 + 11x - 6 = 0$

The Remainder Theorem:
If $P(x)$ is divided by $x - a$, then $P(a)$ is the remainder.

6 Find the remainder when $x^3 - 5x^2 + x + 4$ is divided by $x + 1$

Chapter Review 2

Exercise 9.16

1 For each polynomial, state the variable, the degree, the leading coefficient, and the constant term:

a) $5x^3 + 7x^2 + 5x - 1$

b) $2b^5 - 4b^3 - 4b - 7$

2 Find the value of each polynomial for the given value of the variable:

a) $P(x) = 2x^2 + 3x + 5, \; P(2)$

b) $P(x) = 3x^2 + x - 3, \; P(^-1)$

3 Simplify each of the following polynomials:

a) $(x^2 + 2x + 3) + (3x^2 - 5x - 2)$

b) $(^-2x^2 - x + 3) - (^-3x^2 + 4x + 2)$

c) $(x + 2)(2x^2 + 3x - 2)$

d) $(x^2 + 2x - 8) \div (x - 2)$ {$x - 2$ is a factor of $x^2 + 2x - 8$ thus no remainder}

The Factor Theorem:
If $P(a) = 0$, then $x - a$ is a factor of $P(x)$

$P(x) = x^2 - 3x - 10$
$P(^-2) = (^-2)^2 - 3 \times {}^-2 - 10$
$P(^-2) = 4 + 6 - 10$
$P(^-2) = 0$
Thus $\underline{x + 2 \text{ is a factor.}}$

4 Use the factor theorem to show that $x - 3$ is a factor of $x^2 + 2x - 15$.

$x^3 - 2x^2 - x + 2 = 0$

Try $P(1)$ then $P(^-1)$ then $P(2)$ then $P(^-2)$

$P(1) = 1^3 - 2\times1^2 - 1 + 2$
$P(1) = 1 - 2 - 1 + 2$
$P(1) = 0$ thus $\underline{x - 1 \text{ is a factor}}$

$$\begin{array}{r|l} & \quad x^2 - x - 2 \\ \hline x - 1 & x^3 - 2x^2 - x + 2 \\ & x^3 - x^2 \; - \\ \hline & \quad -x^2 - x \\ & \quad -x^2 + x \; - \\ \hline & \qquad -2x + 2 \\ & \qquad -2x + 2 \; - \\ \hline & \qquad \bullet \quad \bullet \end{array}$$

Now factorise: $x^2 - x - 2$
$(x - 2)(x + 1)$
Either $x - 2 = 0$ or $x + 1 = 0$
$\underline{x = 2 \text{ or } x = {}^-1}$

$\underline{\text{Three solutions: } x = 1, x = 2, x = {}^-1}$

Check solutions to $x^3 - 2x^2 - x + 2 = 0$
$x = 1, \; 1^3 - 2\times1^2 - 1 + 2 = 0$ ✓
$x = 2, \; 2^3 - 2\times2^2 - 2 + 2 = 0$ ✓
$x = {}^-1, \; (^-1)^3 - 2\times(^-1)^2 - {}^-1 + 2 = 0$ ✓

5 Solve each of the following polynomial equations:

a) $x^3 + 2x^2 - x - 2 = 0$

b) $x^3 - 2x^2 - 9x + 18 = 0$

The Remainder Theorem:
If $P(x)$ is divided by $x - a$, then $P(a)$ is the remainder.

6 Find the remainder when $2x^3 - 3x^2 + x - 1$ is divided by $x + 2$

Review 2

10

Chapter 6 Quadratics

$ax^2 + bx + c = 0$

This is a quadratic because the highest power of the unknown, x,is two.

$$\begin{aligned}&(a + b)(a - b)\\ &= a(a - b) + b(a - b)\\ &= a^2 - ab + ab - b^2\\ &= a^2 - b^2\end{aligned}$$

$$\begin{aligned}x^2 + 6x + 2 &= 0\\ x^2 + 6x &= {}^{-}2\\ x^2 + 6x + 9 &= {}^{-}2 + 9\\ (x + 3)^2 &= 7\\ x + 3 &= \pm\sqrt{7}\\ x &= {}^{-}3\pm\sqrt{7}\\ x = {}^{-}0.35 \text{ or } x &= {}^{-}5.65\end{aligned}$$

$x^2 + 6x + 9 = (x + 3)^2$

Add 9 to both sides to complete the square.

$$\begin{aligned}&(a + b)^2\\ &= (a + b)(a + b)\\ &= a(a + b) + b(a + b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

$$\begin{aligned}&(a - b)^2\\ &= (a - b)(a - b)\\ &= a(a - b) - b(a - b)\\ &= a^2 - ab - ab + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

$(\sqrt{7})^2 = 7$ and $(-\sqrt{7})^2 = 7$

Which is why there are two solutions:

$+\sqrt{7}$ or $-\sqrt{7}$ thus $\pm\sqrt{7}$

Chapter 7 Solving Equations

To solve an equation is to find the value of the unknown number (the variable) in the equation.

$$\begin{aligned}5x - 3 &= 2x + 9\\ 5x - 3 - 2x &= 9\\ 3x - 3 &= 9\\ 3x &= 9 + 3\\ 3x &= 12\\ x &= 12 \div 3\\ x &= 4\end{aligned}$$

Check: $(0.36)^2 + 8\times0.36 - 3 = 0$ ✓

Given the quadratic: $ax^2 + bx + c = 0$

The solution is: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This is the world famous Quadratic Formula

$\mathbf{5x^2 - 2x - 4 = 0}$

$ax^2 + bx + c = 0$

$a=5 \quad b={}^{-}2 \quad c={}^{-}4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-{}^{-}2 \pm \sqrt{({}^{-}2)^2 - 4\times5\times{}^{-}4}}{2\times5}$$

$$x = \frac{2\pm9.17}{10}$$

$$x = \frac{(2+9.17)}{10} \quad \text{or} \quad x = \frac{(2+9.17)}{10}$$

$x = 1.12$ or $x = {}^{-}0.72$

Chapter 8 Chance

$$\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}$$

Listing outcomes with a two-way table or tree diagram makes problem-solving easier.

		Dice 1					
		1	2	3	4	5	6
Dice 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

OR means put together.
AND means the intersection.

Conditional probability - probability conditional on a previous outcome or event.

Two events are independent if the probability of one event doesn't depend on the outcome of another event.

Language suggesting that probability is conditional on a previous event:

if .... then ....
given that ....
knowing that ....

Chapter 9 10A Polynomials

$$\begin{array}{l l}3x^3 - 2x^2 + 2x - 5 & \\ 2x^3 + 3x^2 - 4x + 1 & + \\ \hline \underline{5x^3 + 1x^2 - 2x - 4} & \end{array}$$

$$\begin{array}{l l}5x^3 - 3x^2 + 2x - 2 & \\ 2x^3 + 2x^2 - 4x - 1 & - \\ \hline \underline{3x^3 - 5x^2 + 6x - 1} & \end{array}$$

$$\begin{array}{r l}3x^2 \quad + 2x \quad - 1 & \\ 4x \quad - 1 & \times \\ \hline 12x^3 + 8x^2 - 4x & \\ - 3x^2 - 2x + 1 & + \\ \hline \underline{12x^3 + 5x^2 - 6x + 1} & \end{array}$$

$$\begin{array}{r | r l} & x^2 + 5x - 2 & \\ \hline x - 3 & x^3 + 2x^2 - 17x + 6 & \\ & x^3 - 3x^2 & - \\ \hline & 5x^2 - 17x & \\ & 5x^2 - 15x & - \\ \hline & - 2x + 6 & \\ & - 2x + 6 & - \\ \hline & \text{no remainder} & \end{array}$$

The Factor Theorem:

If $P(a) = 0$, then $x - a$ is a factor of $P(x)$

$P(1) = 0$ ➔ $x-1$ a factor
$P({}^{-}1) = 0$ ➔ $x+1$ a factor
$P(2) = 0$ ➔ $x-2$ a factor
$P({}^{-}3) = 0$ ➔ $x+3$ a factor

The Remainder Theorem:

If $P(x)$ is divided by $x - a$, then $P(a)$ is the remainder.

Review 1

Exercise 10.1 Mental computation

1. Spell Polynomials
2. Is $x - 1$ a factor of $x^3 + 2x^2 - x - 2$?
3. Simplify: $(2x^2 - 3x + 2) + (x^2 + 4x - 4)$
4. Expand: $5x^2(2x^2 - x - 3)$
5. Draw a tree diagram for tossing 3 coins
6. What is the quadratic formula?
7. Factorise $x^2 + 5x + 4$
8. What is the value of: $\text{Log}_{10}100$
9. ${}^{-}2 - {}^{-}7$
10. What is 2^4?

$$\begin{aligned} &2^4 \\ &= 2\times2\times2\times2 \\ &= 16 \end{aligned}$$

Exercise 10.2

1 Simplify each of the following by expanding and then collecting like terms:

$$\begin{aligned} &(2x - 5)(x - 4) \\ &= 2x(x + 4) + 5(x + 4) \\ &= 2x^2 + 8x + 5x + 20 \\ &= \underline{2x^2 + 13x + 20} \end{aligned}$$

a) $(x + 2)(x + 1)$
b) $(x + 4)(x + 2)$
c) $(x - 5)(x + 1)$
d) $(2x - 1)(x - 3)$

$$(x + 2)(x + 1) = x(x + 1) + 2(x + 1)$$

2 Factorise each of the following:

$$\begin{aligned} &x^2 + x - 12 \\ &= x^2 + 4x - 3x - 12 \\ &= x(x + 4) - 3(x + 4) \\ &= \underline{(x + 4)(x - 3)} \end{aligned}$$

a) $x^2 + 5x + 6$
b) $x^2 + 3x - 10$
c) $x^2 - 3x - 10$
d) $x^2 - 5x + 6$

$4 + {}^{-}3 = 1$ **$4 \times {}^{-}3 = {}^{-}12$**

$$\mathbf{x^2 + x + {}^{-}12}$$
$$\mathbf{= x^2 + 4x + {}^{-}3x + 6}$$

3 Solve each of the following:

$$\begin{aligned} x^2 - 8x + 16 &= 0 \\ (x - 4)^2 &= 0 && \{\text{perfect square}\} \\ x - 4 &= 0 && \{\text{square root both sides}\} \\ \underline{x} &\underline{= 4} && \{\text{inverse of} -4\} \end{aligned}$$

Check: $(4)^2 - 8\times4 + 16 = 0$ ✓

$$\begin{aligned} 4x^2 + 12x + 9 &= 0 \\ (2x)^2 + 2\times2x\times3 + (3)^2 &= 0 \\ (2x + 3)^2 &= 0 && \{\text{perfect square}\} \\ 2x + 3 &= 0 && \{\text{square root both sides}\} \\ 2x &= {}^{-}3 && \{\text{inverse of} +3\} \\ \underline{x} &\underline{= {}^{-}1.5} && \{\text{inverse of} \times2\} \end{aligned}$$

Check: $4\times({}^{-}1.5)^2 + 12\times{}^{-}1.5 + 9 = 0$ ✓

a) $x^2 + 2x + 1 = 0$
b) $x^2 + 4x + 4 = 0$
c) $x^2 - 6x + 9 = 0$
d) $x^2 - 8x + 16 = 0$
e) $x^2 - 10x + 25 = 0$
f) $9x^2 + 6x + 1 = 0$

g) $x^2 + 4x + 3 = 0$
h) $x^2 + 5x + 6 = 0$
i) $x^2 - 5x + 4 = 0$
j) $x^2 - 2x - 15 = 0$
k) $x^2 + x - 12 = 0$
l) $x^2 - 9x + 14 = 0$

4 Solve each of the following quadratics by completing the square:

$x^2 - 10x + 1 = 0$

$x^2 - 10x = {}^{-}1$ {move constant term}

$x^2 - 10x + 25 = (x - 5)^2$

$x^2 - 10x + 25 = {}^{-}1 + 25$ {+25 to complete square}

$(x - 5)^2 = 24$ {perfect square}

$x - 5 = \pm\sqrt{24}$ {square root both sides}

$x = 5\pm\sqrt{24}$ {inverse of +4}

$\underline{x = 9.90 \text{ or } x = 0.10}$ {use calculator}

Check: $(9.90)^2 - 10\times9.90 + 1 = 0$ ✓

Check: $(0.10)^2 - 10\times0.10 + 1 = 0$ ✓

a) $x^2 + 4x + 2 = 0$
b) $x^2 + 6x - 2 = 0$
c) $x^2 + 10x - 1 = 0$
d) $x^2 - 2x - 2 = 0$
e) $x^2 - 2x - 3 = 0$
f) $x^2 - 5x - 1 = 0$
g) $x^2 - 6x - 1 = 0$
h) $x^2 + 7x - 3 = 0$
i) $x^2 - 2x - 4 = 0$
j) $x^2 + 7x - 5 = 0$

5 Solve each of the following:

$\frac{x}{2} - \frac{2x}{3} = {}^{-}2$

$3x - 4x = {}^{-}12$ {×6}

${}^{-}x = {}^{-}12$

$\underline{x = 12}$

Check: $({}^{-}13+3)/5 = {}^{-}10/5 = {}^{-}2$ ✓

a) $2(x - 1) = 8$
b) ${}^{-}5(x - 2) = 15$
c) $3(2x + 1) = 21$
d) $5(x - 1) + 2x = 9$
e) $4(x - 2) - 2x = 2$
f) $5x - 6 = 2x + 3$
g) $x + 2 = 11 - 2x$
h) $\frac{x}{2} - 3 = 1$
i) $\frac{2x - 1}{3} = 1$
j) $\frac{x}{3} + \frac{x}{2} = 1$

6 Solve each of the following:

$x^2 - 8x + 16 = 0$

$(x - 4)^2 = 0$ {perfect square}

$x - 4 = 0$ {square root both sides}

$\underline{x = 4}$ {inverse of −4}

Check: $(4)^2 - 8\times4 + 16 = 0$ ✓

$x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$ {grouping pairs}

$x(x - 5) + 2(x - 5) = 0$ {factorising }

$(x - 5)(x + 2) = 0$ {factorising}

Either $x - 5 = 0$ or $x + 2 = 0$

$\underline{x = 5}$ or $\underline{x = {}^{-}2}$

Check: $(5)^2 - 4\times5 - 10 = 0$ ✓

Check: $({}^{-}2)^2 - 4\times{}^{-}2 - 10 = 0$ ✓

a) $x^2 + 2x + 1 = 0$
b) $x^2 + 4x + 4 = 0$
c) $x^2 - 4x + 4 = 0$
d) $x^2 - 10x + 25 = 0$
e) $x^2 + 4x + 3 = 0$
f) $x^2 + 9x + 8 = 0$
g) $x^2 + 4x - 5 = 0$
h) $x^2 - 3x - 10 = 0$
i) $x^2 - 5x - 6 = 0$
j) $x^2 + x - 12 = 0$
k) $x^2 - 10x + 9 = 0$
l) $x^2 - 13x + 12 = 0$

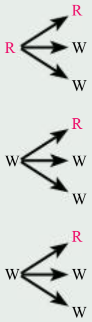
7 Use the quadratic formula to solve each of the following:

a) $x^2 + 3x + 1 = 0$
b) $x^2 + 5x - 2 = 0$
c) $2x^2 - 7x - 3 = 0$
d) ${}^{-}3x^2 - x + 1 = 0$

Given the quadratic: $ax^2 + bx + c = 0$

The solution is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

8 A bag contains 2 white balls and a red ball. A ball is withdrawn, the colour noted, and replaced back in the bag. A second ball is then drawn. Find the probability of drawing:



a) 2 white balls one after the other

b) A white and then a red

c) A red and then a white

d) 2 whites or 2 reds.

9 The numbers 2, 3, and 7 are written on cards and put in a bag. A card is withdrawn, the number noted, and **not replaced**. A second card is then drawn.

a) Use a table or tree diagram to show all the possible outcomes.

b) What is the probability that the order of numbers is:

i) 73 **ii)** odd **iii)** divisible by 3?

10 The hospital data showed that of the 70 patients, 29 patients had the A antigen, 18 had the B antigen. 9 patients had both the A and B antigens.
Find the probability that:

a) a patient had the B antigen only
b) a patient had no antigen (ie., neither the A nor B antigen)
c) a patient had no A antigen given that the patient had B Antigen.

Population of Australian States	
NSW	7 200 000
Vic	5 600 000
Qld	4 500 000
WA	2 300 000
SA	1 600 000
Tas	500 000
ACT	400 000
NT	200 000

11 10A An opinion poll reports that public support for the National Broadband Network is 53% and opposition/don't know is 47%. The opinion poll used the above stratified sample.

a) Comment on the appropriateness of the stratified sample.

b) The opinion poll was conducted via landline telephones (ie no mobile phones). Comment on the following possible sample bias:

i) Non-response bias: Some people can't or won't answer. Are those who don't answer likely to have different views to those who do answer?

ii) Coverage bias: Are people with mobile phones likely to have different views to people with landline telephones?

12 10A For each polynomial, state the variable, the degree, the leading coefficient, and the constant term:

a) $2x^3 + 5x^2 + 2x - 1$
b) $4a^5 - a^3 - 3a - 5$

13 Find the value of each polynomial for the given value of the variable:

a) $P(x) = 4x^2 + x + 1$, $P(1)$

b) $P(x) = x^2 + 2x - 3$, $P(^{-}2)$

The Factor Theorem:

If $P(a) = 0$, then $x - a$ is a factor of $P(x)$

14 Simplify each of the following polynomials:

a) $(x^2 + 3x + 7) + (3x^2 - 2x - 1)$

b) $(^{-}2x^2 - x + 3) - (^{-}5x^2 + 2x + 2)$

c) $(x + 3)(5x^2 + 2x - 1)$

d) $(x^2 - 5x + 6) \div (x - 3)$ {$x - 3$ is a factor of $x^2 - 5x + 6$ thus no remainder}

15 Use the factor theorem to show that $x - 2$ is a factor of $x^2 + 7x - 18$.

$x^3 - 2x^2 - x + 2 = 0$

Try $P(1)$ then $P(^{-}1)$ then $P(2)$ then $P(^{-}2)$

$P(1) = 1^3 - 2\times1^2 - 1 + 2$

$P(1) = 1 - 2 - 1 + 2$

$P(1) = 0$ thus $\underline{x - 1 \text{ is a factor}}$

$$\begin{array}{r|l}
 & \quad\;\; x^2 - x - 2 \\ \hline
x - 1 & x^3 - 2x^2 - x + 2 \\
 & x^3 - x^2 \; - \\ \hline
 & \quad\; -x^2 - x \\
 & \quad\; -x^2 + x \; - \\ \hline
 & \qquad\; -2x + 2 \\
 & \qquad\; -2x + 2 \; - \\ \hline
 & \qquad\quad \bullet \quad \bullet
\end{array}$$

Now factorise: $x^2 - x - 2$

$(x - 2)(x + 1)$

Either $x - 2 = 0$ or $x + 1 = 0$

$\underline{x = 2 \text{ or } x = ^{-}1}$

$\underline{\text{Three solutions: } x = 1, x = 2, x = ^{-}1}$

Check solutions to $x^3 - 2x^2 - x + 2 = 0$

$x = 1$, $1^3 - 2\times1^2 - 1 + 2 = 0$ ✓

$x = 2$, $2^3 - 2\times2^2 - 2 + 2 = 0$ ✓

$x = ^{-}1$, $(^{-}1)^3 - 2\times(^{-}1)^2 - ^{-}1 + 2 = 0$ ✓

16 Solve each of the following polynomial equations:

a) $x^3 + 7x^2 + 7x - 15 = 0$

b) $x^3 + 5x^2 + 2x - 8 = 0$

17 Find the remainder when $5x^3 - 2x^2 + 3x - 1$ is divided by $x + 1$

The Remainder Theorem:

If $P(x)$ is divided by $x - a$, then $P(a)$ is the remainder.

Review 2

Exercise 10.3 Mental computation

1 Spell Factor Theorem
2 Is $x - 1$ a factor of $2x^3 + x^2 - 2x + 4$?
3 Simplify: $(5x^2 - 2x + 2) - (x^2 + 2x - 1)$
4 Expand: $3x(2x^2 + 5x - 1)$
5 Draw a tree diagram for a family of 3 children
6 What is the quadratic formula?
7 Factorise $x^2 + 8x + 7$
8 What is the value of: $\text{Log}_{10}1000$
9 $^{-}3 - 3$
10 What is 2^6?

1 Simplify each of the following by expanding and then collecting like terms:

$$\begin{aligned}&(2x-5)(x-4)\\&=2x(x+4)+5(x+4)\\&=2x^2+8x+5x+20\\&=\underline{2x^2+13x+20}\end{aligned}$$

a) $(x+2)(x+1)$
b) $(x+5)(x+3)$
c) $(x-4)(x+1)$
d) $(2x-1)(x-2)$

$$\mathbf{(x+2)(x+1)=x(x+1)+2(x+1)}$$

2 Factorise each of the following:

$$\begin{aligned}&x^2+x-12\\&=x^2+4x-3x-12\\&=x(x+4)-3(x+4)\\&=\underline{(x+4)(x-3)}\end{aligned}$$

a) x^2+5x+6
b) x^2+2x-8
c) $x^2-4x-12$
d) x^2-5x+6

$$\mathbf{4+{}^{-}3=1} \qquad \mathbf{4\times{}^{-}3={}^{-}12}$$

$$\mathbf{x^2+x+{}^{-}12}$$
$$\mathbf{=x^2+4x+{}^{-}3x+6}$$

3 Solve each of the following:

$$\begin{aligned}x^2-8x+16&=0\\(x-4)^2&=0 &&\{\text{perfect square}\}\\x-4&=0 &&\{\text{square root both sides}\}\\\underline{x}&\underline{=4} &&\{\text{inverse of }-4\}\end{aligned}$$

Check: $(4)^2-8\times4+16=0$ ✓

$$\begin{aligned}4x^2+12x+9&=0\\(2x)^2+2\times2x\times3+(3)^2&=0\\(2x+3)^2&=0 &&\{\text{perfect square}\}\\2x+3&=0 &&\{\text{square root both sides}\}\\2x&={}^{-}3 &&\{\text{inverse of }+3\}\\\underline{x}&\underline{={}^{-}1.5} &&\{\text{inverse of }\times2\}\end{aligned}$$

Check: $4\times({}^{-}1.5)^2+12\times{}^{-}1.5+9=0$ ✓

a) $x^2+4x+4=0$
b) $x^2+2x+1=0$
c) $x^2-4x+4=0$
d) $x^2-8x+16=0$
e) $x^2-12x+36=0$
f) $4x^2+4x+1=0$

g) $x^2+4x+3=0$
h) $x^2+6x+5=0$
i) $x^2-8x+9=0$
j) $x^2-2x-15=0$
k) $x^2-x-12=0$
l) $x^2-8x+15=0$

4 Solve each of the following quadratics by completing the square:

$$\begin{aligned}x^2-10x+1&=0\\x^2-10x&={}^{-}1 &&\{\text{move constant term}\}\end{aligned}$$

$x^2-10x+25=(x-5)^2$

$$\begin{aligned}x^2-10x+25&={}^{-}1+25 &&\{+25\text{ to complete square}\}\\(x-5)^2&=24 &&\{\text{perfect square}\}\\x-5&=\pm\sqrt{24} &&\{\text{square root both sides}\}\\x&=5\pm\sqrt{24} &&\{\text{inverse of }+4\}\\x=9.90\text{ or }x&=0.10 &&\{\text{use calculator}\}\end{aligned}$$

Check: $(9.90)^2-10\times9.90+1=0$ ✓
Check: $(0.10)^2-10\times0.10+1=0$ ✓

a) $x^2+4x-3=0$
b) $x^2+2x-2=0$
c) $x^2+6x-2=0$
d) $x^2-2x-1=0$
e) $x^2-4x+2=0$
f) $x^2-8x+3=0$
g) $x^2-5x+1=0$
h) $x^2+3x-3=0$
i) $x^2-x-3=0$
j) $x^2+7x+5=0$

5 Solve each of the following:

a) $2(x-1)=8$ **b)** ${}^{-}2(x-4)=6$

c) $3(x+2)+2x=12$ **d)** $7x-7=2x+3$

e) $\frac{3x-2}{2}=5$ **f)** $\frac{2x}{3}-\frac{x}{4}=5$

6 Solve each of the following:

$$\begin{aligned} x^2 - 8x + 16 &= 0 \\ (x-4)^2 &= 0 && \{\text{perfect square}\} \\ x - 4 &= 0 && \{\text{square root both sides}\} \\ \underline{x} &\underline{= 4} && \{\text{inverse of } -4\} \end{aligned}$$

Check: $(4)^2 - 8\times4 + 16 = 0$ ✓

$$\begin{aligned} x^2 - 3x - 10 &= 0 \\ x^2 - 5x + 2x - 10 &= 0 && \{\text{grouping pairs}\} \\ x(x-5) + 2(x-5) &= 0 && \{\text{factorising }\} \\ (x-5)(x+2) &= 0 && \{\text{factorising}\} \end{aligned}$$

Either $x - 5 = 0$ or $x + 2 = 0$

$\underline{x = 5}$ or $\underline{x = {}^-2}$

Check: $(5)^2 - 4\times5 - 10 = 0$ ✓

Check: $({}^-2)^2 - 4\times{}^-2 - 10 = 0$ ✓

a) $x^2 + 6x + 9 = 0$

b) $x^2 + 8x + 16 = 0$

c) $x^2 - 4x + 4 = 0$

d) $x^2 - 10x + 25 = 0$

e) $x^2 + 6x + 5 = 0$

f) $x^2 + 6x + 8 = 0$

g) $x^2 + x - 6 = 0$

h) $x^2 - 3x - 10 = 0$

i) $x^2 + 5x - 6 = 0$

j) $x^2 - x - 12 = 0$

k) $x^2 - 9x + 8 = 0$

l) $x^2 - 13x + 12 = 0$

7 Use the quadratic formula to solve each of the following:

a) $x^2 + 5x + 1 = 0$

b) $x^2 + 3x - 5 = 0$

c) $3x^2 - 7x + 2 = 0$

d) ${}^-2x^2 - x + 1 = 0$

Given the quadratic: $ax^2 + bx + c = 0$

The solution is: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

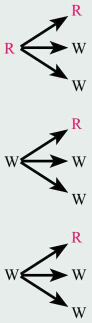
8 A bag contains 2 white balls and a red ball. A ball is withdrawn, the colour noted, and replaced back in the bag. A second ball is then drawn. Find the probability of drawing:

a) 2 white balls one after the other

b) 2 red balls one after the other

c) a red and then a white

d) at least 1 red ball.



9 The numbers 1, 3, 5, and 7 are written on cards and put in a bag. A card is withdrawn, the number noted, and **not replaced**. A second card is then drawn.

a) Use a table or tree diagram to show all the possible outcomes.

b) What is the probability that the order of numbers is:

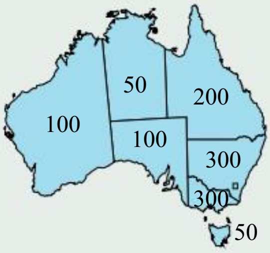
i) 75 **ii)** odd **iii)** divisible by 3?

10 The hospital data showed that of the 48 patients, 25 patients had the A antigen, 19 had the B antigen. 7 patients had both the A and B antigens.
Find the probability that:

a) a patient had the A antigen only

b) a patient had no antigen (ie., neither the A nor B antigen)

c) a patient had no B antigen given that the patient had A Antigen.



Population of Australian States	
NSW	7 200 000
Vic	5 600 000
Qld	4 500 000
WA	2 300 000
SA	1 600 000
Tas	500 000
ACT	400 000
NT	200 000

11 10A An opinion poll reports that public support for the National Broadband Network is 53% and opposition/don't know is 47%. The opinion poll used the above stratified sample.

a) Comment on the appropriateness of the stratified sample.

b) The opinion poll was conducted via landline telephones (ie no mobile phones). Comment on the following possible sample bias:

i) Non-response bias: Some people can't or won't answer. Are those who don't answer likely to have different views to those who do answer?

ii) Coverage bias: Are people with mobile phones likely to have different views to people with landline telephones?

12 10A For each polynomial, state the variable, the degree, the leading coefficient, and the constant term:

a) $5x^3 + 7x^2 + 5x - 1$

b) $2b^5 - 4b^3 - 4b - 7$

13 10A Find the value of each polynomial for the given value of the variable:

a) $P(x) = 2x^2 + 3x + 5$, $P(2)$

b) $P(x) = 3x^2 + x - 3$, $P(^-1)$

14 10A Simplify each of the following polynomials:

a) $(x^2 + 2x + 3) + (3x^2 - 5x - 2)$

b) $(^-2x^2 - x + 3) - (^-3x^2 + 4x + 2)$

c) $(x + 2)(2x^2 + 3x - 2)$

d) $(x^2 + 2x - 8) \div (x - 2)$ {$x - 2$ is a factor of $x^2 + 2x - 8$ thus no remainder}

15 10A Use the factor theorem to show that $x - 3$ is a factor of $x^2 + 2x - 15$.

16 10A Solve each of the following polynomial equations:

a) $x^3 + 2x^2 - x - 2 = 0$

b) $x^3 - 6x^2 + 11x - 6 = 0$

The Factor Theorem:
If $P(a) = 0$, then $x - a$ is a factor of $P(x)$

17 10A Find the remainder when $2x^3 - 3x^2 + x - 1$ is divided by $x + 2$

The Remainder Theorem:
If $P(x)$ is divided by $x - a$, then $P(a)$ is the remainder.

Finance

11

Number and Algebra → **Money and Financial Mathematics**

★ Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.

  - work with authentic information, data and interest rates to calculate compound interest and solve related problems.

A TASK

Play in the ASX sharemarket game (www.asx.com.au).

- Form a group of 2-4 students and decide your syndicate name.
- Ask a teacher to register your group, and get your syndicate number and password.
- Get a list of the game's companies and decide which ones to invest in.
- When the game starts, invest your $50 000.

A LITTLE BIT OF HISTORY

Simon Stevin (1548-1620), a mathematician and military engineer, was probably more responsible than anyone else for the widespread use of the decimal system.

Stevin, a Dutchman, wrote a number of mathematical books. The most important was De Thiende. De Thiende contained a complete decimal system applied to integers and fractions.

In De Thiende, Stevin pleaded for the application of the decimal system to all coinage, weights, and measures.

Australia adopted the decimal system in 1966.

Instructions, from De Thiende, on how to add decimal numbers.

Simple Interest

Interest is the charge for borrowing or lending money.

The charge, **simple interest**, is a percentage, the **interest rate**, of amount borrowed or lent, the **principal**.

Maths Bank

We have wonderful term deposit rates

Term	**<$5,000**	**<$20,000**	**<$100,000**
1 month	1.00%	4.00%	4.00%
2 months	1.25%	4.25%	4.25%
3 months	2.25%	5.30%*	5.30%
4 months	1.25%	4.50%	4.50%
6 months	2.25%	5.20%*	5.20%
9 months	2.00%	5.10%	5.10%
12 months	2.50%	5.00%*	5.00%
24 months	3.00%	5.00%	5.00%

The **interest rate** is normally per annum (pa, per year).

4.25% pa would be 4.25÷12% per month.

Exercise 11.1

Find the interest paid by Maths Bank for each of the following term deposits:

Principal = $8500
Term = 2 months
Interest rate = 4.25% {above table}

$$\text{Simple interest} = \$8500\times4.25\%\times2\text{ months}$$

$$= 8500\times\frac{4.25}{100}\times\frac{2}{12}$$

$$\underline{\text{Simple interest} = \$60.21}$$

1 $17, 200 for 2 months
2 $50, 000 for 4 months
3 $50, 000 for 12 months
4 $50, 000 for 24 months
5 $10, 000 for 9 months
6 $72, 500 for 3 months
7 $97, 000 for 6 months

The simple interest rule

$$I=Prt$$

I = simple interest
P = principal
r = the interest rate
t = the time

Exercise 11.2

Find the simple interest charged on each of the following:

$12 000 borrowed for 3 years at 13.5% pa

I = ?
P = $12 000
r = 13.5% pa
t = 3 years

$$I = Prt$$

$$I = 12000\times\frac{13.5}{100}\times3$$

$$\underline{I = \$4860}$$

1 $15 000 borrowed for 3 years at 13.5% pa
2 $20 000 borrowed for 5 years at 9.7% pa
3 $18 000 borrowed for 2 years at 11.25% pa
4 $7000 borrowed for 4 years at 16.75% pa
5 $59 000 borrowed for 2 years at 10.1% pa
6 $82 000 borrowed for 6 months at 5.7% pa {r in years thus t = 0.5 years}
7 $9000 borrowed for 1 year at 0.6% per month {r in months thus t = 12 months}

Exercise 11.3

If the charge for lending \$5600 at 0.7% per month was \$117.60, for how long was the \$5600 lent?

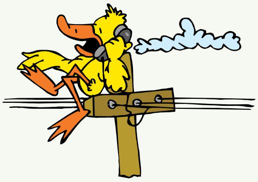
I = \$117.60
P = \$5600
r = 0.7% /month
t = ? months

$$\begin{aligned} I &= Prt \\ 117.60 &= 5600 \times \frac{0.7}{100} \times t \\ \frac{117.60}{5600} \times \frac{100}{0.7} &= t \\ \underline{t} &\underline{= 3 \text{ months}} \end{aligned}$$

If rate in years then time in years.

If rate in months then time in months.

If rate in days then time in days.



1 If the charge for lending \$2850 at 3.8% pa is \$433.20, for how long was the \$2850 lent?

2 If the charge for lending \$948 at 5.1% pa is \$96.70, for how long was the \$948 lent?

3 If the charge for borrowing \$40 000 at 1.3% per month is \$8320 for how long was the \$40 000 borrowed?

4 If the charge for borrowing \$105 500 at 0.02% per day is \$1899, for how long was the \$105 500 borrowed?

How much would need to be invested at 6.2% pa for 18 months in order to earn \$2500 interest?

I = \$2500
P = ?
r = 6.2% pa
t = 1.5 years {r in years, t in years}

$$\begin{aligned} I &= Prt \\ 2500 &= P \times \frac{6.2}{100} \times 1.5 \\ \frac{2500}{1.5} \times \frac{100}{6.2} &= P \\ \underline{P} &\underline{= \$26\ 882} \end{aligned}$$

5 How much would need to be invested at 3.8% pa for 18 months in order to earn \$750 interest (18 months = 1.5 years)?

6 How much would need to be invested at 5.3% pa for 6 months in order to earn \$250 interest (6 months = 0.5 years)?

Joe is charged \$315 for borrowing \$7500 for 6 months. What simple interest rate per month is Joe being charged?

I = \$315
P = \$7500
r = ? /month
t = 6 months {r = months, t = months}

$$\begin{aligned} I &= Prt \\ 315 &= \frac{7500 \times r \times 6}{100} \\ \frac{315 \times 100}{7500 \times 6} &= r \\ \underline{r} &\underline{= 0.7\% \text{ per month}} \end{aligned}$$

Can you get this answer on your calculator?

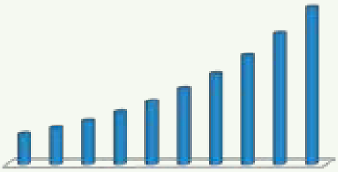
7 Andrea is charged \$3840 for borrowing \$45 000 for 8 months. What simple interest rate per month is Andrea being charged?

8 Tom is charged \$982 for borrowing \$68 000 for 90 days. What simple interest rate per day is Tom being charged?

Compound Interest

Compound interest is known as money-making magic.

Compounding is interest paid on interest



Exercise 11.4

$10 000 is invested at 12% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

Interest after 1st year = Prt = 10000×12%×1 = $1200
Principal after 1st year = 10000 + 1200
= $11 200

Interest after 2nd year = Prt = 11200×12%×1 = $1344
Principal after 2nd year = 11200 + 1344
= $12 544

Interest after 3rd year = Prt = 12544×12%×1 = $1505
Principal after 3rd year = 12544 + 1505
= $14 049

1 $10 000 is invested at 6% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

2 $10 000 is invested at 8% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

3 $10 000 is invested at 10% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

4 A house valued at $500 000 is expected to increase in value by 5% each year.
What is the expected value of the house after 5 years?

5 A house valued at $500 000 is expected to increase in value by 6% each year.
What is the expected value of the house after 5 years?

6 A house valued at $500 000 is expected to increase in value by 7% each year.
What is the expected value of the house after 5 years?

7 An investor places $60 000 with a management fund and expects a yearly increase in value of 9%.
What is the expected value of the investment after 4 years?

8 $5 000 is invested at 4.8% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

9 $5 000 is invested at 4.8% pa compounded each 6 months
(This means interest is added each 6 months @ 4.8%÷2 = 2.4%).
What will be the principal after 2 years (round to the nearest dollar)?

10 $5 000 is invested at 4.8% pa compounded each quarter
(This means interest is added each 3 months @ 4.8%÷4 = 1.2%).
What will be the principal after 2 years (round to the nearest dollar)?

Compound Interest

Growth Factor

$$(1+\frac{r}{100})$$

$P is invested at r% pa compounded yearly.

$$\text{Principal after 1st year} = P+P\times\frac{r}{100}$$
$$= \underline{P(1+\frac{r}{100})}$$

Exercise 11.5

$8000 is invested at 7.5% pa with interest added yearly.
What will be the principal after 3 years (round to the nearest dollar)?

$$\text{Growth Factor} = (1+\frac{r}{100})$$
$$= (1+\frac{7.5}{100})$$
$$= 1.075$$

$$\text{Principal after 1st year} = 8000\times\text{Growth Factor}$$
$$= 8000\times1.075$$
$$= \underline{\$8600}$$
$$\text{Principal after 2nd year} = 8600\times\text{Growth Factor}$$
$$= 8600\times1.075$$
$$= \underline{\$9245}$$
$$\text{Principal after 3rd year} = 9245\times\text{Growth Factor}$$
$$= 8000\times1.075$$
$$= \underline{\$9938}$$

1 $100 000 is invested at 10% pa compounded yearly.
What will be the principal after 5 years (round to the nearest dollar)?

2 $100 000 is invested at 12% pa compounded yearly.
What will be the principal after 5 years (round to the nearest dollar)?

3 A country town with a population of 10 000 people is assuming a population growth of 2% each year. What is the expected population after 3 years?

An investor is planning to increase $50 000 by 20% each year.
Roughly how long will it take to double the investment to $100 000?

$$\text{Growth Factor} = (1+\frac{r}{100})$$
$$= (1+\frac{20}{100})$$
$$= 1.20$$

$$\text{Value after each year} = \text{Value}\times\text{Growth Factor}$$
$$\text{Value after 1st year} = 50000\times1.2 = 60000$$
$$\text{Value after 2nd year} = 60000\times1.2 = 72000$$
$$\text{Value after 3rd year} = 72000\times1.2 = 86400$$
$$\text{Value after 4th year} = 86400\times1.2 = 103680$$

It will take about 4 years to double

4 An investor is planning to increase $50 000 by 10% each year.
Roughly how long will it take to double the investment to $100 000?

5 A farmer is planning to increase the herd of 100 by 10% each year.
Roughly how long will it take to double the herd to 200?

6 Assuming house prices match the inflation rate of 3%, roughly how long will it take for a house valued at $500 000 to be worth $600 000?

Compound Interest

$P is invested at r% pa compounded yearly.

Amount = Principal×Growth Factor

Amount after 1st year $= P(1+\frac{r}{100})$

Amount after 2nd year $= P(1+\frac{r}{100})\,(1+\frac{r}{100})$

Amount after 3rd year $= P(1+\frac{r}{100})\,(1+\frac{r}{100})\,(1+\frac{r}{100})$

Amount after n periods $= P(1+\frac{r}{100})^n$

Compound Interest Formula

$$A = P(1+\frac{r}{100})^n$$

P = principal
r = the interest rate
n = number of time periods
A = final amount

Exercise 11.6

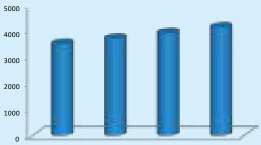
$3500 is invested at 5.8% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

A = ?
P = $3500
r = 5.8% pa
n = 3 times

$$A = P(1+\frac{r}{100})^n$$

$$A = 3500(1+\frac{5.8}{100})^3$$

$$\underline{A = \$4145}$$



1 $3500 is invested at 6.5% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

2 $70 000 is invested at 5.2% pa compounded yearly.
What will be the principal after 5 years (round to the nearest dollar)?

3 A house valued at $600 000 is expected to increase in value by 5% each year.
What is the expected value of the house after 10 years?

4 A house valued at $600 000 is expected to increase in value by 7% each year.
What is the expected value of the house after 10 years?

5 An investor places $57 800 with a management fund and expects a yearly increase in value of 7%. What is the expected value of the investment after 4 years (Round to the nearest dollar)?

6 An investor places $57 800 with a management fund and expects a yearly increase in value of 9%. What is the expected value of the investment after 4 years (Round to the nearest dollar)?

7 The Australian population of 23 million is expected to increase by 2% each year. What is the expected population after the next 10 years?

8 The Indonesian population of 150 million is expected to increase by 2.5% each year. What is the expected population after the next 10 years?

Which produces the better outcome over 1 year?

\$100 invested at 8% pa compounded yearly.

\$100 invested at 8% pa compounded monthly.

\$100 invested at 8% pa compounded daily.

A = ?

P = \$100

r = 8% pa

n = 1 time

$A = 100(1+\frac{8}{100})^1$

A = \$108.00

A = ?

P = \$100

r = 8/12% pd

n = 12 times

$A = 100(1+\frac{8}{12\times100})^{12}$

A = \$108.30

A = ?

P = \$100

r = 8/365% pd

n = 365 times

$A = 100(1+\frac{8}{365\times100})^{365}$

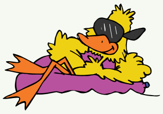
A = \$108.33

Exercise 11.7

1 Which produces the better outcome over 1 year?

- **a)** \$1000 invested at 8% pa compounded yearly?
- **b)** \$1000 invested at 8% pa compounded monthly?
- **c)** \$1000 invested at 8% pa compounded daily?

Maybe the difference magnifies over a longer time period and/or a larger principal?



2 Which produces the better outcome over 5 years?

- **a)** \$100 000 invested at 8% pa compounded yearly?
- **b)** \$100 000 invested at 8% pa compounded monthly?
- **c)** \$100 000 invested at 8% pa compounded daily?

Depreciation Formula

$$A = P(1-\frac{r}{100})^n$$

Depreciation is the decrease in value of assets.

Depreciation: the growth factor is < 1

Growth: the growth factor is > 1

A car is purchased for \$28 000. What is the value of the car after 5 years if it depreciates in value by 15% each year (round to nearest \$1000)?

A = ?

P = \$28 000

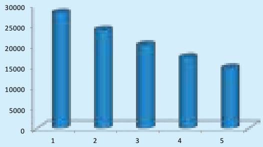
r = 15% pa

n = 5 times

$$A = P(1-\frac{r}{100})^n$$

$$A = 28000(1-\frac{15}{100})^5$$

A = \$12 000



3 A car is purchased for \$28 000. What is the value of the car after 5 years if it depreciates in value by 20% each year (round to nearest \$1000)?

4 A car is purchased for \$50 000. What is the value of the car after 5 years if it depreciates in value by 20% each year (round to nearest \$1000)?

5 A truck is purchased for \$123 000. What is the value of the truck after 3 years if it depreciates in value by 18% each year (round to nearest \$1000)?

Mental Computation

Use mental computation to check that charges and change are correct.

Exercise 11.8

1 Spell Compound
2 What is the Simple Interest Formula?
3 Compounded each month for 6 months - n?
4 Calculate (1 + 8/100)
5 Is x − 1 a factor of $x^3 + 3x^2 - 3x - 1$?
6 Simplify: $(x^2 - 5x + 3) + (x^2 + 2x - 1)$
7 What is the quadratic formula?
8 Factorise $x^2 - 5x + 4$
9 What is the value of: $\text{Log}_{10}100$
10 What is 60% of 30?

Each month for 6 months.
ie compounded 6 times
n = 6

60% of 30 = 0.6×30
= 18

Exercise 11.9

1 Spell Growth Factor
2 What is the Compound Interest Formula?
3 Compounted each month for 1 year - n?
4 Calculate (1 + 9/100)
5 Is x − 1 a factor of $2x^3 + x^2 - 3x - 1$?
6 Simplify: $(x + 2)(x + 3)$
7 What is the quadratic formula?
8 Factorise $x^2 + 7x + 6$
9 What is the value of: $\text{Log}_2 8$
10 What is 80% of 40?

'Go down deep enough into anything and you will find mathematics' - Dean Schlicter.

'If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is' - John Louis von Neumann.

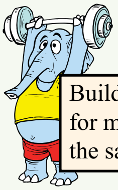
Exercise 11.10

1 Spell Depreciation
2 What is the Simple Interest Formula?
3 Compounted each month for 1½ years - n?
4 Calculate (1 + 6.5/100)
5 Is x − 1 a factor of $2x^3 + 3x^2 - 2x - 2$?
6 Simplify: $(x + 2)(x - 1)$
7 What is the quadratic formula?
8 Factorise $x^2 - 5x + 6$
9 What is the value of: $\text{Log}_3 27$
10 What is 40% of 70?

As a mathematician you could be engaged in research, computing, modelling, banking, insurance, engineering, forecasting, etc, etc.

Games Developers design, create, and produce games.
- Relevant school subjects are Mathematics and English.
- Courses usually involve a diploma or degree in information technology.

Competition Questions



Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 11.11

1 An item is discounted from \$600 to \$450. What is the discounted percentage?

2 An item is discounted from \$800 to \$750. What is the discounted percentage?

An item priced at \$200 is discounted 10% and then 10% GST is added.
What is the sale price?

$$\begin{aligned} \$200 \text{ discounted } 10\% &= 200 - 10\% \text{ of } 200 \\ &= 200 - 20 \\ &= \$180 \\ \$180 \text{ increased by } 10\% &= 180 + 10\% \text{ of } 180 \\ &= 180 - 18 \\ &= \underline{\$198} \end{aligned}$$

A shortcut

Increase by 10% then multiply by 1.1

Decrease by 10% then multiply by 0.9

Can you prove this is correct?

3 An item priced at \$300 is discounted 10% and then 10% GST is added. What is the sale price?

4 An item priced at \$500 is discounted 10% and then 10% GST is added. What is the sale price?

A litre of fruit juice has 20% orange juice. How much orange juice needs to be added so that the fruit juice has 50% orange juice?

$$\frac{200+x}{1000+x} = \frac{1}{2}$$

$$\begin{aligned} (200 + x)2 &= (1000 + x)1 \\ 400 + 2x &= 1000 + x \\ 2x - x &= 1000-400 \\ x &= 600 \end{aligned}$$

Thus add 600 mL

1 litre of 20% juice = 200 mm juice
then add x mm juice
= 200 + x of orange juice

1 litre and add x mm juice
= 1000 + x is total volume

Ratio of orange juice to total $= \frac{200+x}{1000+x}$ which is to be 50% $= \frac{1}{2}$

5 A litre of fruit juice has 40% orange juice. How much orange juice needs to be added so that the fruit juice has 50% orange juice?

6 A litre of fruit juice has 30% orange juice. How much orange juice needs to be added so that the fruit juice has 50% orange juice?

7 Entry costs \$15 for an adult and \$10 for a child. If a total of 50 adults and children enter and the total entry charge was \$640, how many children entered?

8 Entry costs \$25 for an adult and \$15 for a child. If a total of 80 adults and children enter and the total entry charge was \$1630, how many adults entered?

Investigations

Investigation 11.1 Which is more?

Investigate

Which is more:

10% every year

5% every 6 months?

This is important stuff. My lifestyle depends on it.

Investigation 11.2 Which is more?

Investigate

Which is more:

12% every year

6% every 6 months?

1% every month

The power of compound interest is that the amount of interest increases over time.

Investigation 11.3 Double your investment?

It is sometimes said that a very good investment will double in value in seven years.

Investigate

What compounding growth rate will increase the value of a property **from $300 000 to $600 000 in seven years?**

Investigate

What compounding growth rate will increase the value of a property **from $500 000 to $1 000 000 in seven years?**

Investigation 11.4 The Rule of 72

Investigate

The rule of 72

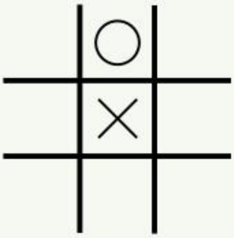
The rule of 114?
The rule of 144?

A Couple of Puzzles

Exercise 11.12

1 Play X to win.

2 Does a cube have more faces,
or more edges,
or more vertices?



A Game

Bingo is played by the whole class.
Beforehand, make up 25 finance problems with answers from 1 to 25.

1 Each person makes a 3×3 square and writes a number from 1 to 25 in each of the nine squares to make their bingo card.

2 Randomly select one of the 25 problems and tell the class the problem. If the answer matches a number on the bingo card then it is marked.

3 The winner calls BINGO when any three in a row, column, or diagonal is marked on their bingo card.

1 = 100% as a decimal
2 = 10% of 20
3 = \$1.75 + \$1.25
4 = 8 lots of 50c
5 = \$8.50 - \$3.50

17	**4**	**23**
6	**11**	**21**
2	**9**	**10**

A Sweet Trick

Amaze your relatives by being able to quickly multiply any two numbers each from 11 to 19 in your head.

Example 1 18 × 13

(18+3) × 10 = 210
8 × 3 = 24
234 is the answer

Example 2 17 × 15

(17+5) × 10 = 220
7 × 5 = 35
255 is the answer

With a little practice you will be able to do these fairly quickly in your head.

Technology

Technology 11.1 Simple Interest

Find the simple interest charged on $870 at 7.2% pa for 2 years.

The simple interest rule

$$I=Prt$$

	a	b	c	d
1	Principal	Rate	Time	Simple Interest
2	870	7.2	2	125.28

=a2*b2/100*c2

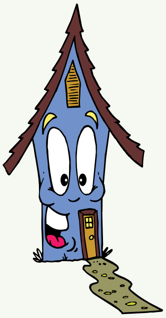
Use the spreadsheet to check the your answers to Exercises 11.1 and 11.2

Technology 11.2 Compound Interest

A $600 000 house increases in value by 5% each year.

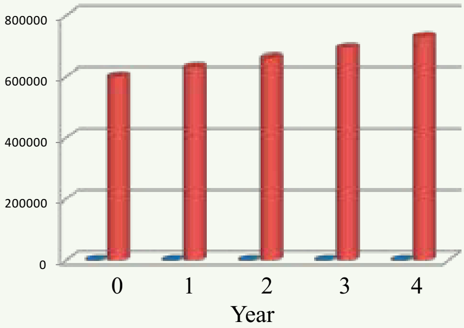
What is the value of the house after 10 years?

Use a spreadsheet to answer this question and to produce a graph of the yearly increase of the value of the house



	a	b	c
1	Year	Value	Interest
2	0	600000	5
3	1	630000	
4	2	661500	
5	3	694575	
6	4	729304	

=b5*(1+c2/100)



Technology 11.3 Compound Interest

Devise your own spreadsheet for compound interest.

Use your spreadsheet to check the answers to Exercises 11.4, 11.5. and 11.6

Compound Interest Formula

$$A = P(1+\frac{r}{100})^n$$

Chapter Review 1

Exercise 11.13

1 Find the simple interest charged on each of the following:

a) \$20 000 borrowed for 3 years at 11.5% pa

b) \$350 000 borrowed for 6 months at 7.2% pa

If the charge for lending \$5600 at 0.7% per month is \$789.60, for how long was the \$5600 lent?

I = \$117.60
P = \$5600
r = 0.7% /month
t = ? months

$$I = Prt$$
$$117.60 = 5600 \times \frac{0.7}{100} \times t$$
$$\frac{117.60}{5600} \times \frac{100}{0.7} = t$$
$$\underline{t = 3 \text{ months}}$$

The simple interest rule

$$I=Prt$$

I = simple interest
P = principal
r = the interest rate
t = the time

2 If the charge for lending \$5150 at 5.8% pa is \$448.05, for how long was the \$5150 lent?

3 How much would need to be invested at 6.3% pa for 6 months in order to earn \$500 interest (6 months = 0.5 years)?

4 Amelia is charged \$6121 for borrowing \$90 000 for 7 months. What simple interest rate per month is Amelia being charged?

\$3500 is invested at 5.8% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

A = ?
P = \$3500
r = 5.8% pa
n = 3 times

$$A = P(1+\frac{r}{100})^n$$
$$A = 3500(1+\frac{5.8}{100})^3$$
$$\underline{A = \$4145}$$

Compound Interest Formula

$$A = P(1+\frac{r}{100})^n$$

5 \$7500 is invested at 5.7% pa compounded yearly. What will be the principal after 4 years (round to the nearest dollar)?

6 \$125 500 is invested at 6.1% pa compounded monthly. What will be the principal after 3 years (round to the nearest dollar)?

7 Which produces the better outcome over 5 years?

a) \$100 000 invested at 7% pa compounded yearly?

b) \$100 000 invested at 7% pa compounded monthly?

c) \$100 000 invested at 7% pa compounded daily?

Depreciation Formula

$$A = P(1-\frac{r}{100})^n$$

8 A car is purchased for \$32 000. What is the value of the car after 5 years if it depreciates in value by 15% each year (round to nearest \$1000)?

Chapter Review 2

Exercise 11.14

1 Find the simple interest charged on each of the following:

a) \$15 000 borrowed for 3 years at 10.5% pa

b) \$250 000 borrowed for 4 months at 5.4% pa

If the charge for lending \$5600 at 0.7% per month is \$789.60, for how long was the \$5600 lent?

I = \$117.60
P = \$5600
r = 0.7% /month
t = ? years

$$I = Prt$$

$$117.60 = 5600 \times \frac{0.7}{100} \times t$$

$$\frac{117.60}{5600} \times \frac{100}{0.7} = t$$

$$\underline{t = 3 \text{ months}}$$

The simple interest rule

$$I=Prt$$

I = simple interest
P = principal
r = the interest rate
t = the time

2 If the charge for lending \$12 100 at 7.9% pa is \$2867.70, for how long was the \$12 100 lent?

3 How much would need to be invested at 7.1% pa for 8 months in order to earn \$500 interest (9 months = 0.75 years)?

4 Noah is charged \$6825 for borrowing \$50 000 for 18 months. What simple interest rate is Noah being charged?

\$3500 is invested at 5.8% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

A = ?
P = \$3500
r = 5.8% pa
n = 3 times

$$A = P(1+\frac{r}{100})^n$$

$$A = 3500(1+\frac{5.8}{100})^3$$

$$\underline{A = \$4145}$$

Compound Interest Formula

$$A = P(1+\frac{r}{100})^n$$

5 \$25 000 is invested at 5.6% pa compounded yearly.
What will be the principal after 5 years (round to the nearest dollar)?

6 \$200 000 is invested at 6.0% pa compounded monthly.
What will be the principal after 2 years (round to the nearest dollar)?

7 Which produces the better outcome over 5 years?

a) \$100 000 invested at 11% pa compounded yearly?

b) \$100 000 invested at 11% pa compounded monthly?

c) \$100 000 invested at 11% pa compounded daily?

Depreciation Formula

$$A = P(1-\frac{r}{100})^n$$

8 A car is purchased for \$38 000. What is the value of the car after 5 years if it depreciates in value by 16% each year (round to nearest \$1000)?

Trigonometry 1

12

Measurement and Geometry ➡ Pythagoras and Trigonometry

★ Solve right-angled triangle problems including those involving direction and angles of elevation and depression.

- apply Pythagoras' Theorem and trigonometry to problems in surveying and design.

What was the name of the first satellite to orbit the Earth?

The moon.

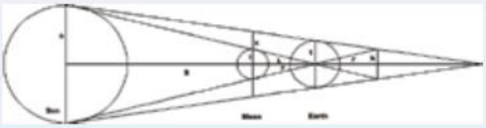
A TASK

Can you use trigonometry to estimate the distance to the Moon?

Diameter of the moon = 3500 km
Diameter of the Sun = 1.4×10^6 km
Distance to the Sun = 1.5×10^8 km

Demonstrate your method of estimating the distance to the moon to the rest of your class.

Drawings used by Hipparchus to calculate distances to the moon and the Sun.



A LITTLE BIT OF HISTORY

Trigonometry is a branch of mathematics that studies the relationships between the sides and angles of triangles (Greek: trigon - triangle, metron - measure).

The ancient Babylonians used trigonometry to record and predict astronomical events. The Egyptians used trigonometry to build pyramids.

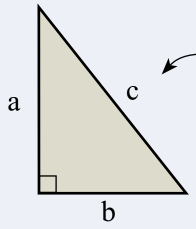
Hipparchus (180 - 125 BC) is considered the 'father of trigonometry.' Hipparchus used trigonometry to calculate the distance to the Sun and the moon, and to mathematically describe the non-uniform orbit of the moon and the apparent motion of the Sun.

Hipparchus

Pythagoras' Theorem

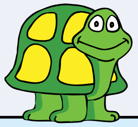
$$\boxed{c^2 = a^2 + b^2}$$

The square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.



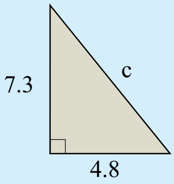
The hypotenuse is the longest side. It is opposite the right-angle (90°).

Given two sides of a right-angled triangle, Pythagoras will find the third.

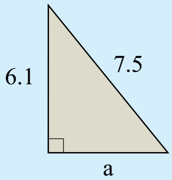


Exercise 12.1

1 Use Pythagoras' theorem to find the length of the unknown side (round to two decimal places):

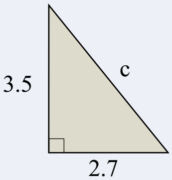


$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 7.3^2 + 4.8^2 \\ c^2 &= 76.33 \\ c &= \sqrt{76.33} \\ \underline{c} &\underline{= 8.74} \end{aligned}$$



$$\begin{aligned} c^2 &= a^2 + b^2 \\ 7.5^2 &= a^2 + 6.1^2 \\ 56.25 &= a^2 + 37.21 \\ 56.25 - 37.21 &= a^2 \\ \sqrt{19.04} &= a \\ \underline{4.36} &\underline{= a} \end{aligned}$$

a)



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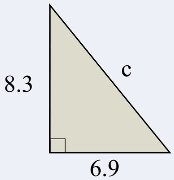

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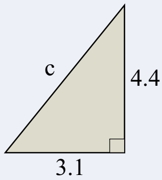

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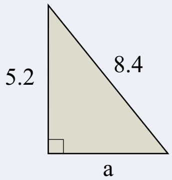

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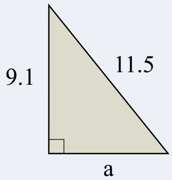

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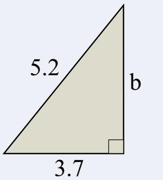

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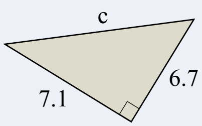

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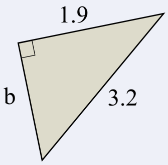
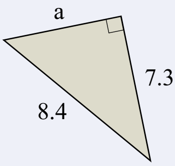
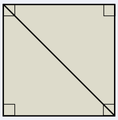


i)

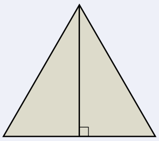


2 What is the length of the diagonal of a 10 m square?



3 A square has a diagonal of length 10 m, what is its side length?

4 A plane flies 60 km due East and then 110 km due South.
How far is the plane from its starting point?

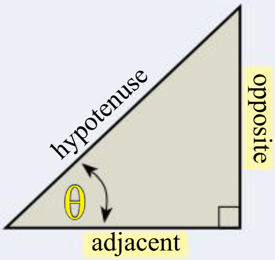


5 An equilateral triangle has sides of length 10 cm.
What is its perpendicular height?

The Tan Ratio

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

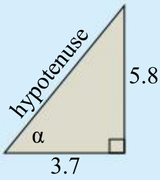
The Tan of an angle is the ratio of the side opposite the angle to the side adjacent to the angle.



Greek letters:
α is alpha
β is beta
θ is theta

Exercise 12.2

1 Use the Tan ratio to find the unknown (round to two decimal places):



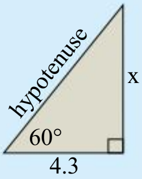
$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\alpha = \frac{5.8}{3.7}$$

$$\tan\alpha = 1.57$$

$$\alpha = \tan^{-1}(1.57)$$

$$\underline{\alpha = 57.51°}$$



$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 60 = \frac{x}{4.3}$$

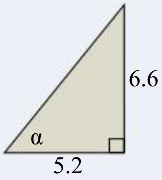
$$4.3\times\tan 60 = \text{x}$$

$$\underline{7.45 = \text{x}}$$

Use your calculator:
2ndF tan^{-1} 1.57 =

Make sure your calculator is on **degrees**.

a)



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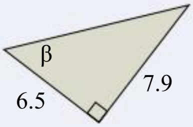

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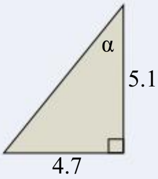

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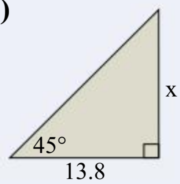

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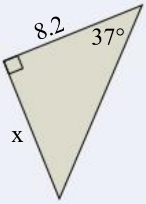
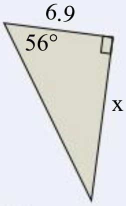
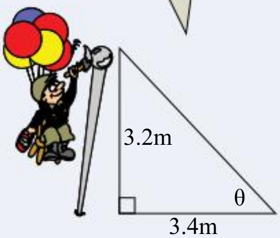


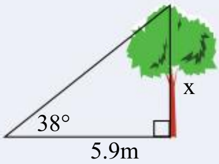
f)



2 A student with a clinometer, is lying on the ground 3.4 m out from the base of a 3.2 m flagpole. What angle should be showing on the clinometer?



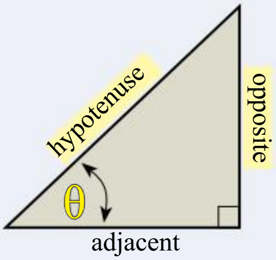
3 5.9 m out from the base of a tree, a clinometer measures the angle of elevation to the top of the tree as 38°. Find the height of the tree.



The Sine Ratio

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

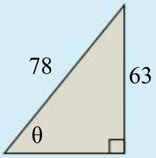
The Sine of an angle is the ratio of the side opposite the angle to the hypotenuse.



It is a kind of a tradition to use Greek letters for angles

Exercise 12.3

1 Use the Sine ratio to find the unknown (round to two decimal places):

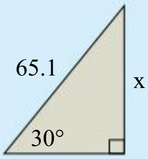


$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{63}{78}$$

$$\theta = \sin^{-1}\left(\frac{63}{78}\right)$$

$$\underline{\theta = 53.87°}$$



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 30 = \frac{x}{65.1}$$

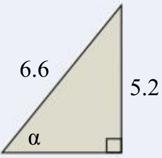
$$65.1 \times \sin 30 = x$$

$$\underline{32.55 = x}$$

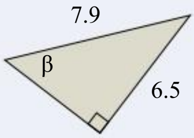
Use your calculator: $\boxed{\text{2ndF}}\ \boxed{\sin^{-1}}\ \boxed{0.81}\ \boxed{=}$

Make sure your calculator is on **degrees**.

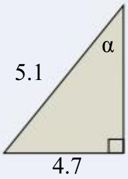
a)



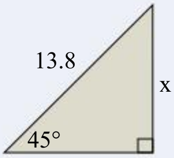
b)



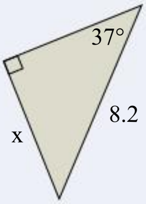
c)



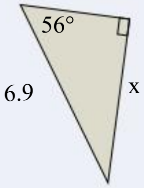
d)



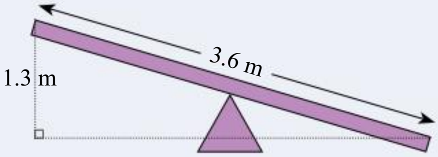
e)



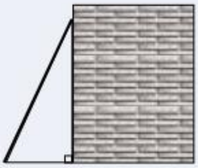
f)



2 The beam of a see-saw is 3.6 m long.
When one end is resting on the ground,
the other end is 1.3 m above the ground.
What angle does the beam make with the ground?



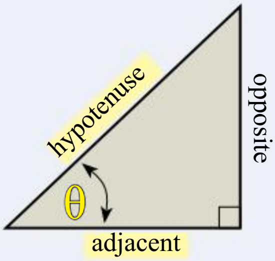
3 A 4.5 m ladder leaning against a wall makes an angle of 70° with the ground.
How far up the wall does the ladder reach?



The Cos Ratio

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

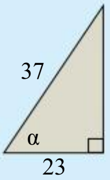
The Cosine of an angle is the ratio of the side adjacent to the angle to the hypotenuse.



Greek letters:
λ is lamda
μ is mu
σ is sigma

Exercise 12.4

1 Use the Cos ratio to find the unknown (round to two decimal places):

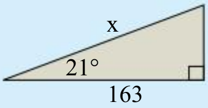


$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\alpha = \frac{23}{37}$$

$$\alpha = \cos^{-1}(\frac{23}{37})$$

$$\underline{\alpha = 51.57°}$$



$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 21 = \frac{163}{x}$$

$$x \times \cos 21 = 163$$

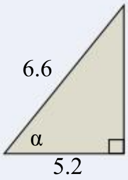
$$x = \frac{163}{\cos 21}$$

$$\underline{x = 174.60}$$

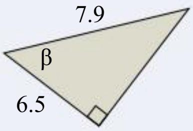
Use your calculator:
2ndF $\cos^{-1}$ 0.62 =

Make sure your calculator is on **degrees**.

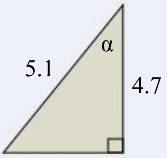
a)



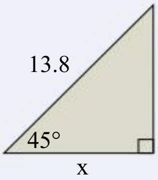
b)



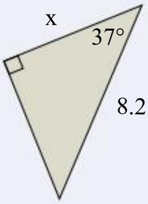
c)



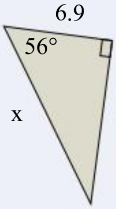
d)



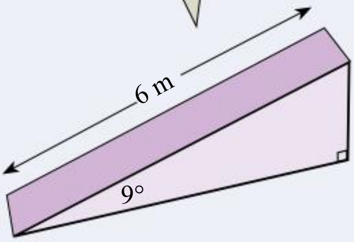
e)



f)

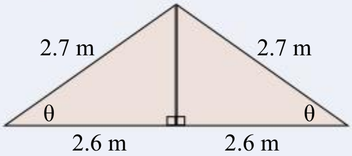


2 A ramp has an incline of 9° and a length of 6 m. What is the length of the horizontal base?



3 A ramp of length 12 m has a horizontal base of length 8 m.
What is the angle of incline of the ramp?

4 What is the pitch, θ, of the roof?



Trigonometry

Trigonometry can be used to solve right-angled triangle problems.

How to know which one to use.

1 Draw a diagram.
2 Write known values on the diagram.
3 Write the Pythagoras, sin, cos, tan formulae.
4 Tick known values in each formula.
5 Put a ? on the unknown wanted.
6 If only one unknown then use that formula.

The trick is which one?

$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$

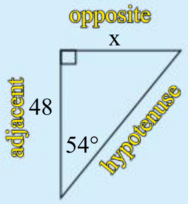
$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$

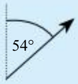
$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$

$c^2 = a^2 + b^2$

Exercise 12.5

A sailing boat sails south for 48 km, then on a bearing of 54°T until it is due east of its starting point. How far is the boat from its starting point?





54°T is 54° clockwise from North.

$\sin\overset{\checkmark}{\alpha} = \frac{\overset{?}{\text{opposite}}}{\text{hypotenuse}}$

$\cos\overset{\checkmark}{\alpha} = \frac{\text{adjacent}\checkmark}{\text{hypotenuse}}$

$\tan\overset{\checkmark}{\alpha} = \frac{\overset{?}{\text{opposite}}}{\text{adjacent}\checkmark}$ ← Use this one

$c^2 = \overset{\checkmark}{a^2} + \overset{?}{b^2}$

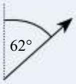
$$\tan 54 = \frac{x}{48}$$

$$48\times\tan 54 = x$$

$$66.06\text{ km} = x$$

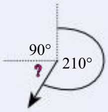
The boat is 66 km from its starting point.

1 A sailing boat sails south for 85 km, then on a bearing of 62°T until it is due east of its starting point. How far is the boat from its starting point?



62°T is 62° clockwise from North.

2 A plane flies due east for 93 km, then on a bearing of 210°T until the plane is due south of its starting point.
How far is the plane from its starting point?



210°T is 210° clockwise from North.

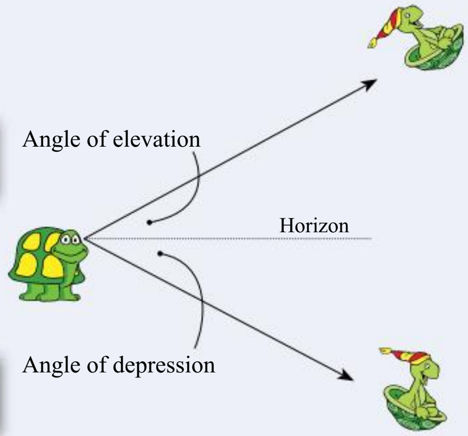
3 A plane flies due north for 138 km, then on a bearing of 120°T until the plane is due east of its starting point. The plane then flies due north back to its starting point. How far has the plane travelled?

120°T is 120° clockwise from North.

Elevation and Depression

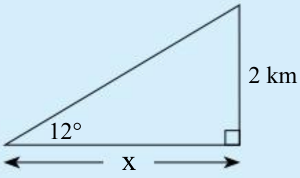
Angle of elevation - the object is *above the observer*.

Angle of depression - the object is *below the observer*.



Exercise 12.6

A radar station detects a plane at an angle of elevation of 12° and an altitude of 2 kilometres. What is the horizontal distance of the plane from the radar station?



$$\sin\alpha \checkmark = \frac{\text{opposite}\checkmark}{\text{hypotenuse}}$$

$$\cos\alpha \checkmark = \frac{\text{adjacent}?}{\text{hypotenuse}}$$

$$\tan\alpha \checkmark = \frac{\text{opposite}\checkmark}{\text{adjacent}?} \longleftarrow \boxed{\text{Use this one}}$$

$$c^2 = a^2 \checkmark + b^2 ?$$

$$\tan 12 = \frac{2}{x}$$

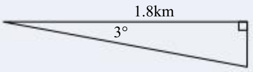
$$x\tan 12 = 2$$

$$x = \frac{2}{\tan 12}$$

The plane is 9.41 km from the radar station.

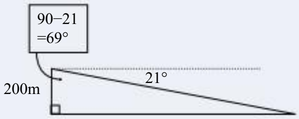
1 A radar station detects a plane at an angle of elevation of 15° and an altitude of 2.3 kilometres.
What is the horizontal distance of the plane from the radar station?

2 The radar on a fishing boat detects a school of fish at an angle of depression of 3° and a surface distance of 1.8 kilometres. At what depth is the school of fish?



3 An observer 350 m from the base of a cliff measures the angle of elevation to the top of the cliff to be 17°. What is the height of the cliff?

4 From the top of a 200 m tower, the angle of depression to a fire is 21°. How far away is the fire?



5 When the angle of elevation of the sun is 40°, a tree casts a shadow of length 18 m. What is the height of the tree?

Multi-Step Problems

Exercise 12.7

A person, from their window, measures the angle of elevation (34°) to the top and the angle of depression to the bottom (23°) of a building. The building is 28 m away. Find the height of the building.

In ΔABC,

$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 34 = \frac{BC}{28}$$

$$28\times\tan 34 = \text{BC}$$

$$\underline{18.89 = \text{BC}}$$

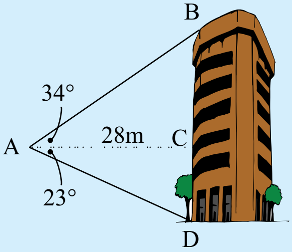
In ΔACD,

$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 23 = \frac{CD}{28}$$

$$28\times\tan 23 = \text{CD}$$

$$\underline{11.89 = \text{CD}}$$



$$\begin{aligned}\text{Height} &= \text{BC} + \text{CD}\\ &= 18.89 + 11.89\\ &= \underline{30.78\text{ m}}\end{aligned}$$

1 A person, from their window, measures the angle of elevation (20°) to the top and the angle of depression to the bottom (38°) of a building. The building is 35 m away. Find the height of the building.

2 A person, from their window, measures the angle of elevation (28°) to the top and the angle of depression to the bottom (39°) of a building. The building is 42 m away. Find the height of the building.

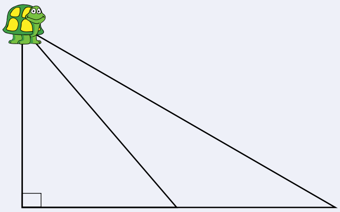
3 A person, from their window, measures the angle of elevation (28°) to the top of a building 36 m away. What is the angle of depression to the bottom of the building. The height of the building is 45 m.

4 Isabella stands 24 m from the foot of a vertical rock wall. She measures the angle of elevation to a rock climber to be 43°. She next measures the angle of elevation to the rock climber to be 52°.
How far up the rock wall has the climber ascended?

5 Tom stands 28 m from the foot of a vertical rock wall. He measures the angle of elevation to a rock climber to be 33°. He next measures the angle of elevation to the rock climber to be 46°. How far up the rock wall has the climber ascended?

6 An observer 35 m out from the foot of a vertical rock wall measures the angle of elevation to a rock climber to be 33°. If the climber ascends a further 15 m, what is the angle of elevation to the rock climber?

7 From the top of a 185 m tower,
the angle of depression to a fire at 1 pm was 9°.
At 3 pm, the angle of depression is 13°.
What is the speed of the fire?



8 From the top of a building 45 m high, the angle of depression to the bottom of a building across the street is 27°. The angle of elevation to the top of the building across the street is 46°. Find:

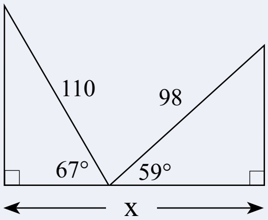
a) The height of the second building.

b) The width of the street.

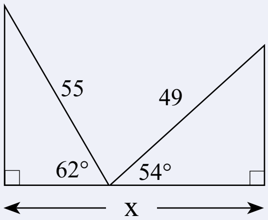
9 A person is standing between two rock walls. The person measures the angle of elevation to the top of one wall, 11 m high, to be 58°. The person measures the angle of elevation to the top of the second wall, 8 m high, to be 37°. How far apart are the two rock walls?

10 Find the unknowns in each of the following diagrams:

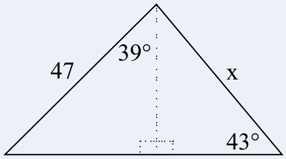
a)



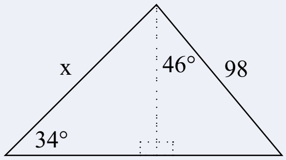
b)



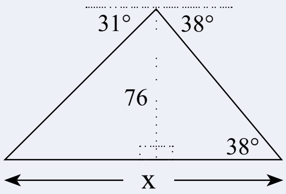
c)



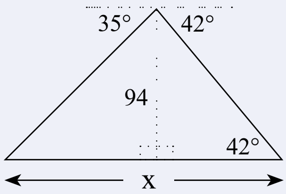
d)



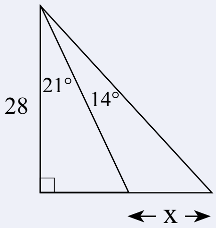
e)



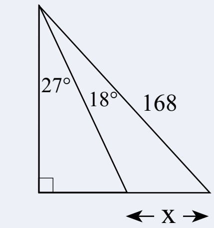
f)



g)



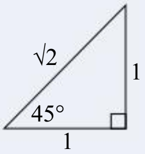
h)



Mental Computation

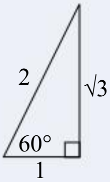
Mental computation gives you practice in thinking.

Exercise 12.8



1 Spell Trigonometry.
2 In the triangle, what is sin 45°?
3 In the triangle, what is cos 45°?
4 In the triangle, what is tan 45°?
5 Two sides of a right-angled triangle are 3 and 4, what is the hypotenuse?
6 What is the Simple Interest Formula?
7 Is x − 1 a factor of $x^3 + 3x^2 - 2x - 2$?
8 Simplify: $(x^2 - 2x + 1) + (x^2 + 3x - 4)$
9 Factorise $x^2 + 5x + 6$
10 What is the value of: $\text{Log}_{10}1000$

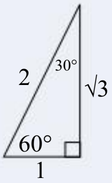
Exercise 12.9



'Thinking will not overcome fear but action will' - W Clement Stone.

1 Spell Pythagoras.
2 In the triangle, what is sin 60°?
3 In the triangle, what is cos 60°?
4 In the triangle, what is tan 60°?
5 Two sides of a right-angled triangle are 1 and 1, what is the hypotenuse?
6 What is the Compound Interest Formula?
7 Is x + 1 a factor of $x^3 + x^2 + x + 1$?
8 Simplify: $(x^2 + 2x + 3) + (x^2 - 3x - 1)$
9 Factorise $x^2 + 6x + 9$
10 What is the value of: $\text{Log}_2 8$

Exercise 12.10



'Out of intense complexities, intense simplicities emerge' - Winston Churchill.

1 Spell Depression.
2 In the triangle, what is sin 30°?
3 In the triangle, what is cos 30°?
4 In the triangle, what is tan 30°?
5 Two sides of a right-angled triangle are 1 and 2, what is the hypotenuse?
6 What is the Simple Interest Formula?
7 Is x − 1 a factor of $x^3 + x^2 + x - 1$?
8 Simplify: $(x^2 - 2x + 5) + (x^2 + 3x - 2)$
9 Factorise $x^2 + 6x + 8$
10 What is the value of: $\text{Log}_3 81$

Geophysicists use magnetic, seismic, and electrical data to study the geology below the surface of the earth.

- Relevant school subjects are English and Mathematics.
- Courses usually involve a geophysics or geoscience degree.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 12.11

1 What is the square root of 900?

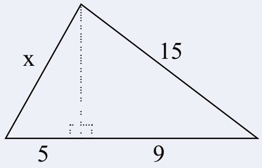
2 What is the square root of 9?

3 What is the square root of 0.09?

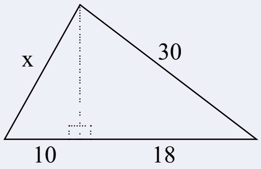
4 What is the square root of 0.0009?

5 Find the value of x in each of the following diagrams:

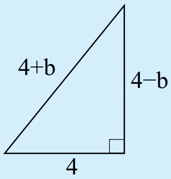
a)



b)

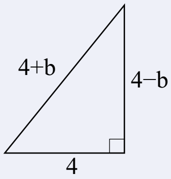


6 What is the value of b in each of the following diagrams:

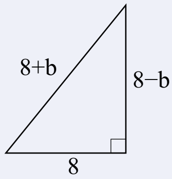


$$\begin{aligned}
(4+b)^2 &= 4^2 + (4-b)^2 && \{\text{Pythagoras}\}\\
(4+b)(4+b) &= 16 + (4-b)(4-b) &&\\
4(4+b) + b(4+b) &= 16 + 4(4-b) - b(4-b) && \{\text{Distribution}\}\\
16 + 4b + 4b + b^2 &= 16 + 16 - 4b - 4b + b^2 &&\\
16 + 8b + b^2 &= 32 - 8b + b^2 &&\\
8b + 8b &= 32 + b^2 - b^2 - 16 && \{\text{Inverse}\}\\
16b &= 16 &&\\
\underline{b} &\underline{= 1} &&
\end{aligned}$$

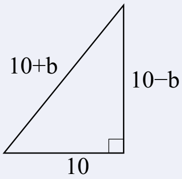
a)



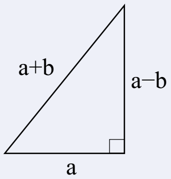
b)



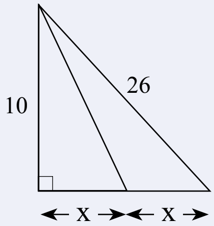
c)



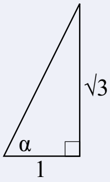
7 Find b in terms of a:



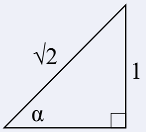
8 Find x



9 If tan α = √3, find sin α and cos α



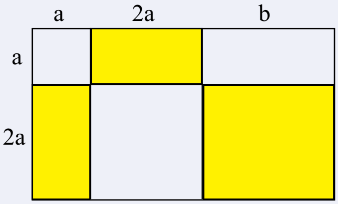
10 If $\sin\alpha = \dfrac{1}{\sqrt{2}}$ find cos α and tan α



A Couple of Puzzles

Exercise 12.12

1 Write an expression for the total shaded area?



2 Brett travels from A to B, 100 km, at an average speed of 80 km/h. How fast must Brett travel on the return journey so that the average speed for the complete trip, from A to B and back to A, is 100 km/h?

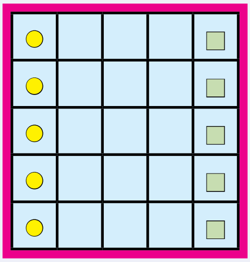
A Game

Kangaroos and Wallabies is a game for two players. The loser is the player who is unable to make a move.

1 Draw up a 5×5 board and place the kangaroos on the left and the wallabies on the right.

2 When it is your turn, move your macropod one square. Kangaroos move to the right and wallabies move to the left.

3 If your piece is immediately in front of an opponent, you may jump over your opponent to the square on the other side of your opponent.



For variety, change the number of rows and columns.

A Sweet Trick

This excellent trick has been known for centuries.

1 Ask your audience for any three digit number and write it twice on the board.

2 Ask for another three-digit number and write it under the number on the left.

3 You write the 9 complement on the right (The 9 complement of 734 is 265).

4 Tell your audience that you will, in your head, do the two multiplications and add the two products (ie., 264×734 + 264×265 = 263 736).

264	264
734	265

To get the answer:

1 Subtract 1 from 264 = 263

2 Append the 9 complement of 263 (736)

3 The answer is 263 736

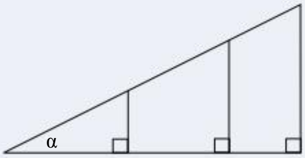
As always, this trick becomes very impressive with practice and drama.

Investigations

Investigation 12.1 The sine ratio

Calculate three sine ratios for the same angle.
Are they the same?

1 Draw right-angled triangles similar to the example.
2 Use a ruler to enter data in the spreadsheet.



	a	b	c	d	
1		Opposite	Hypotenuse	Sine ratio	Angle
2	Triangle 1				
3	Triangle 2				
4	Triangle 3				

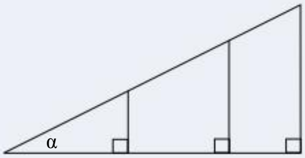
=b4/c4

=degrees(asin(d4))

Investigation 12.2 The cosine ratio

Calculate three cos ratios for the same angle.
Are they the same?

1 Draw right-angled triangles similar to the example.
2 Use a ruler to enter data in the spreadsheet.



	a	b	c	d	
1		Adjacent	Hypotenuse	Cos ratio	Angle
2	Triangle 1				
3	Triangle 2				
4	Triangle 3				

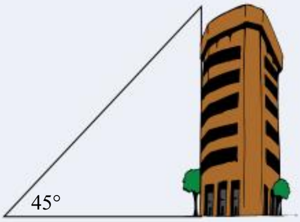
=b4/c4

=degrees(acos(d4))

Investigation 12.3 Height of a Building?

Measure out from the base of a building until the angle of the elevation to the top of the building is 45°.

The height of the building = distance from base. Why?



Investigate

Other ways of finding the height of a building

Investigate

Ways of finding the distance to the moon

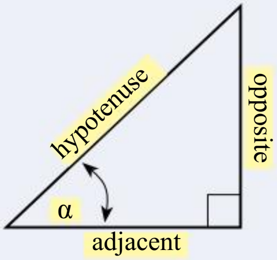
Technology

Use a spreadsheet to solve the previous exercises.

$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$

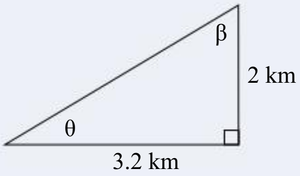
$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$



Technology 12.1 The Spreadsheet

Use spreadsheets to solve trigonometry problems.

a) Given the adjacent and opposite sides:



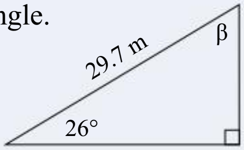
	a	b
1	Adjacent	3.2
2	Opposite	2
3	Hypotenuse	3.77
4	Sin θ	0.53
5	Cos θ	0.85
6	Tan θ	0.63
7	θ	32.01
	β	57.99

=sqrt(b1^2+b2^2)

=b1/b3

=degrees(atan(b6))

b) Given the angle and the hypotenuse, solve the triangle.



	a	b
1	θ	26
2	Hypotenuse	29.7
3	Adjacent	26.69
4	Opposite	13.02
5	Sin θ	0.44
6	Cos θ	0.90
7	Tan θ	0.49
	β	64

=b2*cos(radians(b1))

=b2*sin(radians(b1))

Spreadsheets normally use angles in radians thus the need to convert degrees to radians.

Technology 12.2 The Internet

Use the Internet to find videos and lessons about trigonometry.

Trigonometry is a branch of mathematics that studies the relationships between the angles and sides of triangles.

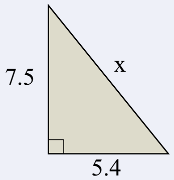
What about triangles that are not right-angled?

Chapter Review 1

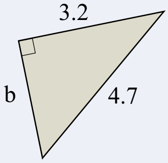
Exercise 12.13

1 Use Pythagoras' theorem to find the length of the unknown side (round to two decimal places):

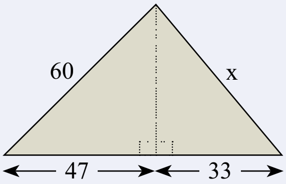
a)



b)

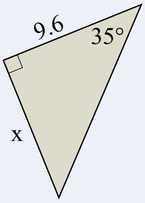


c)

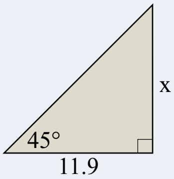


2 Use the sin, cos, or tan ratio to find the unknown (round to two decimal places):

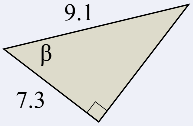
a)



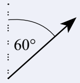
b)



c)

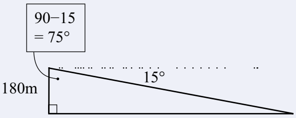


3 A plane flies due west for 127 km, then on a bearing of 60°T until the plane is due north of its starting point.
How far is the plane from its starting point?



60°T is 60° clockwise from North.

4 From the top of a 180 m tower, the angle of depression to a fire is 15°. How far away is the fire?



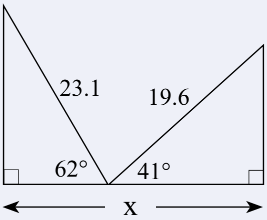
5 From the top of a building 55 m high, the angle of depression to the bottom of a building across the street is 42°. The angle of elevation to the top of the building across the street is 26°. Find:

a) The height of the building across the street.

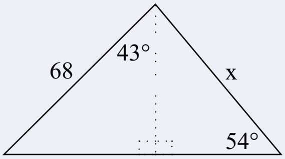
b) The width of the street.

6 Find the unknowns in each of the following diagrams:

a)



b)

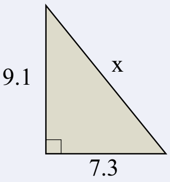


Chapter Review 2

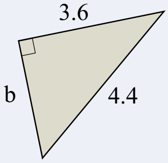
Exercise 12.14

1 Use Pythagoras' theorem to find the length of the unknown side (round to two decimal places):

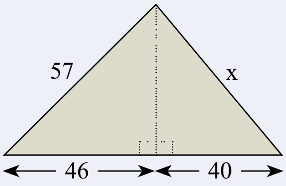
a)



b)

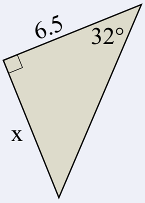


c)

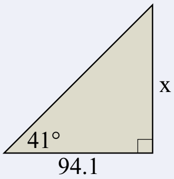


2 Use the sin, cos, or tan ratio to find the unknown (round to two decimal places):

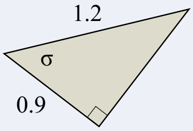
a)



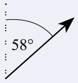
b)



c)

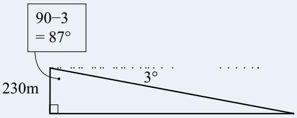


3 A plane flies due west for 201 km, then on a bearing of 58°T until the plane is due north of its starting point.
How far is the plane from its starting point?



58°T is 58° clockwise from North.

4 From the top of a 230 m tower, the angle of depression to a fire is 3°. How far away is the fire?



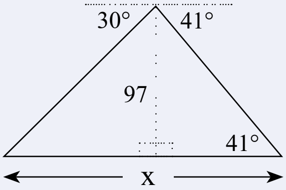
5 From the top of a building 62 m high, the angle of depression to the bottom of a building across the street is 59°. The angle of elevation to the top of the building across the street is 24°. Find:

a) The height of the building across the street.

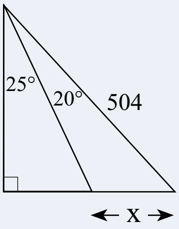
b) The width of the street.

6 Find the unknowns in each of the following diagrams:

a)



b)



Statistics 1

13

Statistics & Probability ➞ Data Representation & Interpretation

★ Determine quartiles and interquartile range.
  - find the five-number summary (minimum and maximum values, median and upper and lower quartiles) and use its graphical representation, the box plot, as tools for both numerically and visually comparing the centre and spread of data sets.

★ Construct and interpret box plots and use them to compare data sets.
  - understand that box plots are an efficient and common way of representing and summarising data and can facilitate comparisons between data sets.
  - use parallel box plots to compare data about the age distribution of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole.

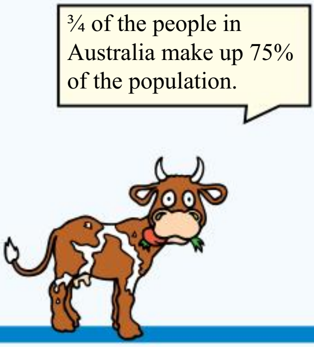
★ Compare shapes of box plots to corresponding histograms and dot plots.
  - Investigate data in different ways to make comparisons and draw conclusions.

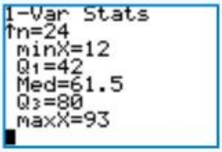
★ 10A Calculate and interpret the mean and standard deviation of data and use these to compare data sets.
  - use the standard deviation to describe the spread of a set of data.
  - use the mean and standard deviation to compare numerical data sets.

A TASK

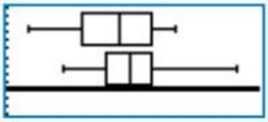
Show your class how to use a graphics calculator to produce box and whisker plots.

- Research a graphics calculator manual.
- Practice with the problems in Exercises 5, 6, 7
- Plan your lesson.
- Show your class.





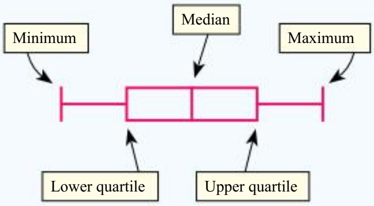
```
1-Var Stats
↑n=24
 minX=12
 Q1=42
 Med=61.5
 Q3=80
 maxX=93
```



A LITTLE BIT OF HISTORY

1900s Bowley used stem-and-leaf plots for initial data analysis.

1977 Tukey published the box plot as an efficient method for visualising five-number data summary. The five-numbers are (minimum, lower quartile, median, upper quartile, and the maximum).



Sometimes the whiskers are not the minimum and maximum. The whiskers might be the 2nd and 98th percentile so that non-included data are outliers.

Box Plots

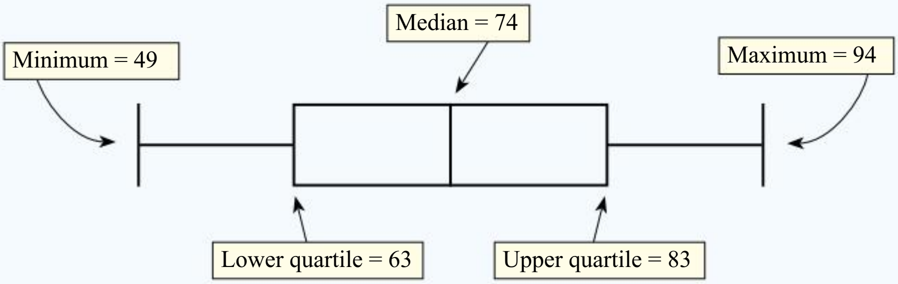
The **Box Plot**
is also known as the
box-and-whisker plot.

The box plot is a quick way of examining sets of data graphically.

The box plot is based on five numbers: Minimum, lower quartile, median, upper quartile, and the maximum.

Example data set: 73, 67, 49, 54, 76, 84, 73, 82, 94, 80, 74, 93, 61

In ascending order: $\boxed{49}$, 54, $\boxed{61, 65}$, 73, 73, $\boxed{74}$, 76, 80, $\boxed{82, 84}$, 93, $\boxed{94}$



The **Median**
is the **middle** of a set of scores.

The median ignores extreme high scores and extreme low scores.

Exercise 13.1
Find the median of each of the following set of scores:

2, 7, 5, 1, 3, 5, 7 Put the scores in ascending order 1, 2, 3, $\boxed{5}$, 5, 7, 7 Median = 5 {5 is in the middle }	2, 7, 5, 1, 3, 5, 7, 4 Put the scores in ascending order 1, 2, 3, $\boxed{4, 5}$, 5, 7, 7 Median = 4.5 {Average of 4 & 5}

1 1, 3, 5, 2, 7, 7, 5
2 1, 3, 5, 2, 7, 7, 500
3 51, 53, 55, 52, 57, 57, 55
4 1, 2, 3, 4, 1, 2, 4, 3
5 6.2, 9.8, 3.6, 3.2, 3.1, 3.3
6 21, 24, 23, 23, 24, 56
7 $^{-}5$, 2, 3, $^{-}2$, $^{-}4$, 3
8 1, $^{-}3$, 4, $^{-}2$, $^{-}1$, 2, 1

The **Lower Quartile**
is the **middle** of the bottom half
of a set of scores.

The lower quartile is sometimes given the symbol **Q1**.

The lower quartile is also known as the 25th percentile.

Exercise 13.2

Find the lower quartile of each of the following set of scores:

2, 7, 5, 1, 3, 5, 7 Put the scores in ascending order 1, 2, 3, 5, 5, 7, 7 Lower quartile = 2 {2 is in the middle of the three numbers 1,2,3 in the bottom half}	2, 7, 5, 1, 3, 5, 7, 4, 3, 7 Put the scores in ascending order 1, 2, 3, 3, 4, 5, 5, 7, 7, 7 Lower quartile = 3 {3 is in the middle of the five numbers 1,2,3,3,4 in the bottom half}

1 5, 4, 4, 3, 6, 4, 1

2 7, 5, 3, 7, 7, 7, 6

3 45, 44, 44, 43, 46, 44, 41

4 6, 2, 8, 4, 8, 7, 8, 5, 4, 5

5 6, 2, 9, 8, 3, 6, 3, 2, 3, 3

6 2, 2, 4, 2, 3, 2, 3, 2, 4, 3, 2, 3, 2, 3, 3

7 4, $^{-}2$, 1, $^{-}2$, 4, 3, $^{-}3$

8 9, 8, $^{-}4$, $^{-}9$, 6, $^{-}7$, 7, $^{-}6$, 9, 9

The **Upper Quartile**
is the **middle** of the top half
of a set of scores.

The upper quartile is sometimes given the symbol **Q3**.

The upper quartile is also known as the 75th percentile.

Exercise 13.3

Find the upper quartile of each of the following set of scores:

5, 3, 6, 2, 4, 4, 6, 3 Put the scores in ascending order 2, 3, 3, 4, 4, 5, 6, 6 Upper quartile = 5.5 {5.5 is in the middle of the four numbers 4,5,6,6 in the top half}	9, 4, 6, 6, 7, 6, 8, 8, 7 Put the scores in ascending order 4, 6, 6, 6, 7, 7, 8, 8, 9 Upper quartile = 8 {8 is in the middle of the four numbers 7,8,8,9 in the top half}

1 8, 7, 8, 9, 8, 7, 9, 7

2 3, 1, 3, 2, 2, 2, 3, 3, 1

3 78, 77, 78, 79, 78, 77, 79, 77

4 5, 1, 5, 4, 5, 4, 3, 5, 1, 5, 3

5 4, 4, 2, 5, 2, 5, 1, 3, 5, 6, 3, 2

6 4, 7, 4, 9, 5, 9, 5, 6, 5, 4, 7, 7, 6, 8, 6

7 8, 7, 8, $^{-}9$, $^{-}8$, 7, $^{-}9$, 7

8 3.2, 2.8, 4.1, 3.3, 4.5, 4.1, 5.7, 2.6

Box Plots

Exercise 13.4

Find the lower quartile, the median, and the upper quartile of each of the following sets of scores:

12, 15, 13, 11, 16, 13, 12, 15

Put the scores in ascending order

11, 12, 12, 13, 13, 15, 15, 16

Median = 13

Lower quartile = 12

Upper quartile = 15

2, 9, 4, 2, 5, 8, 9, 6, 6, 2, 3, 7, 5

Put the scores in ascending order

2, 2, 2, 3, 4, 5, 5, 6, 6, 7, 8, 9, 9

Median = 5

Lower quartile = 2.5

Upper quartile = 7.5

1 21, 22, 23, 24, 24, 24, 28, 28

2 11, 12, 13, 14, 14, 14, 18, 18

3 56, 52, 61, 55, 66, 53, 58

4 31, 46, 42, 51, 45, 43

5 82, 70, 53, 90, 95, 75, 57, 56, 89, 20, 81, 38, 79, 98, 43, 88, 73

6 151,176,136,142,152,163,141,156,166,152,142,153,162,148

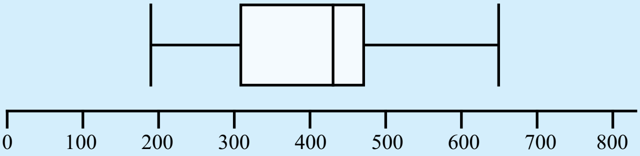
Exercise 13.5

Use a box and whisker plot to represent the following data.

Weekly rents from 13 properties:
435, 275, 195, 460, 565, 650, 335, 330, 470, 465, 365, 270, 460

First put the rents in order
195, 270, 275, 330, 335, 365, 435, 460, 460, 465, 470, 565, 650

Five number summary = (195, 302.5, 435, 467.5, 650)



1 Weekly wages of 11 employees rounded to the nearest $10:
$340, $220, $640, $990, $420, $270, $310, $340, $290, $560, $430

2 The result of a survey of Year 10 students and the amount of money in their pocket rounded to nearest $1:
$1, $3, $2, $22, $5, $0, $2, $0, $15, $3, $0, $7,
$1, $17, $33, $3, $0, $9, $2, $17, $20, $5, $8

3 A CD from a famous singer has songs with the following times in decimal minutes:
3.02, 3.57, 3.62, 2.23, 2.75, 2.22, 2.45, 2.30,
1.63, 2.07, 2.58, 2.78, 2.02, 1.98, 2.10, 2.93

4 10.4 were asked to rate the importance of mathematics on a scale from 1 to 100:
75, 80, 95, 80, 75, 65, 85, 85, 90, 85, 90, 95,
80, 85, 70, 95, 90, 95, 85,75, 90, 85, 95

Comparative Analysis

Exercise 13.6

Curious about whether there was a difference in performance on the end semester test, the following results were obtained from 10A and 10B. Analyse the data and make a comment.

10A 53,95,54,72,64,33,79,81,37,43,58,76,49,51,70,54,87,57,85,61,48,64,55,68

10B 92,68,76,47,52,77,57,84,63,42,68,97,72,62,44,79,81,77,60,75,85,52,94

First, put the data in order:

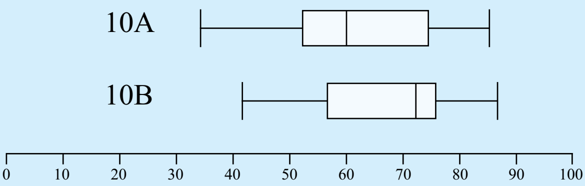
10A 33,37,43,48,49,51,53,54,54,55,57,58,61,64,64,68,70,72,76,79,81,85,87,95

10B 42,44,47,52,52,57,60,62,63,68,68,72,75,76,77,77,79,81,84,85,92,94,97

Second, calculate the five numbers for each data set:

	10A	10B
Minimum	33	42
Lower Q	52	57
Median	59.5	72
Upper Q	74	81
Maximum	95	97

Third, draw two box and whisker plots



Comment. Visually, 10B appears to be getting better results. The spread, standard deviation, of both forms are very similar, 15.7 and 15.6, with 10B having a 5% higher average. Essentially, both forms have similar distributions with 10B placed higher.

1 Curious about whether there was a difference in performance on the end semester test, the following results were obtained from 10A and 10B. Analyse the following data and make a comment.

End semester test (10A)	End semester test (10B)
60 ,42 ,70 ,47 ,85 ,80 ,74 ,92, 84 ,70 ,79 ,89 ,74 ,31 ,70 ,66, 46 ,81 ,86 ,71 ,44 ,82 ,62 ,97	67 ,75 ,59 ,83 ,45 ,55 ,59 ,35, 75, 65 ,69 ,43 ,77 ,68 ,44 ,49 45, 31, 62 ,71 ,81 ,57 ,51 ,75, 60 ,52

2 The following data has been obtained in order to compare the age distribution of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole.
Analyse the following data and make a comment.

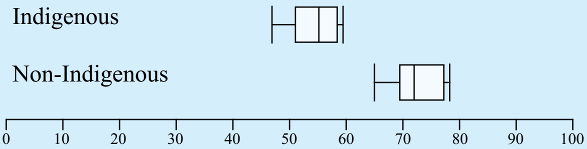
Indigenous	non-Indigenous
13, 52, 2, 9, 47, 6, 5, 22, 16, 3, 26, 64, 3, 10, 27, 6, 30, 11, 17, 35, 67, 44, 34, 38, 7	52, 3, 48, 23, 11, 27, 7, 36, 39, 16, 45, 82, 53, 61, 68, 19, 2, 38, 64, 14, 29, 57, 37, 76, 72

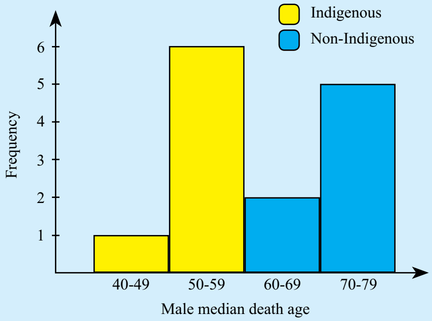
Comparative Analysis

Exercise 13.7
Compare the following data sets using a box plot, a histogram, a stem-and-leaf plot, and a dot plot. Compare the four different ways of displaying the data.

Mortality: Median age at death in various states of Australia	
Indigenous males	Non-Indigenous males
58.2, 54.6, 46.7, 50.3, 52.3, 56.8, 57.1	75.6, 77.2, 65.2, 69.7, 72.5, 78.1, 70.2

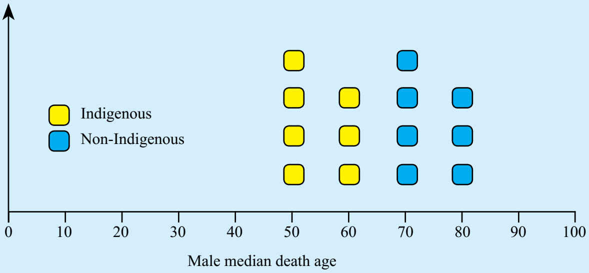
	Ind	Non
Minimum	46.7	65.2
Lower Q	50.3	69.7
Median	54.6	72.5
Upper Q	57.1	77.2
Maximum	58.2	78.1





Death age (6|5 is 65)

Indigenous		Non-Indigenous
7	4	
8 7 7 5 2 0	5	
	6	5
	7	0 0 3 6 7 8
	8	



Based on the data provided there is a clear difference in median ages at death between indigenous and non-indigenous groups.

The box plot, histogram, and the dot plot are related in shape. There are, however, differences in the presentation of information. The box plot is a quick way of presenting data and provides a clear and obvious comparison of the data. While the histogram and the dot plot provide useful representations of the data, they both tend to have different shapes dependent on the scale/grouping chosen.

The stem-and-leaf plot is very effective in providing a comparison between the two data sets but, again, the shape is dependent on the groupings chosen.

1 Median age at death in various states of Australia.

Mortality: Median age at death in various states of Australia	
Indigenous females	Non-Indigenous females
58, 62, 54, 55, 51, 56, 57	83, 84, 74, 81, 79, 83, 82

2 Some of the students who regularly bought lunch at the school tuckshop volunteered to have their cholesterol levels measured. Two years later, following a "Healthy Tuckshop" program, the same students again had their cholesterol levels measured. Analyse the data and make a comment.

Cholesterol levels (before)	Cholesterol levels (after)
5.2, 5.1, 4.4, 6.4, 4.8, 4.1, 5.1 5.4, 4.2, 5.1, 4.8, 4.7, 4.9, 4.7 4.4, 5.3, 5.1, 4.9, 4.6, 3.8, 6.2, 4.4, 5.2, 5.0	4.4, 5.8, 4.3, 4.8, 3.6, 5.1, 4.8 3.7, 4.8, 5.5, 4.1, 3.9, 3.9, 4.2 5.7, 4.9, 3.8, 4.9, 3.5, 5.3, 4.7 3.7, 5.5, 4.6,

3 A Year 10 class has been involved in a program of aerobic exercises during their physical education classes. The following data shows the heart rates one minute after a 1500 m run both before and after the aerobic program. Analyse the data and make a comment.

Heart rate (before)	Heart rate (after)
84, 78, 73, 98, 83, 91, 88, 67 98, 75, 61, 99, 89, 112, 97, 79 58, 79, 85, 93, 107, 67, 95, 86 70	72, 81, 54, 68, 76, 91, 51, 64, 72 61, 58, 47, 59, 67, 84, 73, 51, 69 56, 73, 66, 84, 52, 50, 64

4 The skin elasticity of 48 people at a workplace is measured. 24 of the people work mainly outdoors, while other 24 people work mainly indoors. Analyse the data and make a comment.

Skin elasticity (sun-exposed)	Skin elasticity (sun-protected)
76, 56, 74, 58, 82, 67, 79, 55 81, 39, 65, 81, 24, 45, 54, 42 38, 23, 93, 42, 81, 5, 86, 12	48, 88, 78, 55, 44, 81, 127, 67 67, 48, 31, 76, 92, 105, 74, 64 80, 77, 48, 59, 99, 57, 70, 54

5 Patients with multiple rib fractures were asked to provide a pain score one hour after receiving one of two analgesic drugs. A high score indicates a high level of pain. Analyse the data and make a comment.

Pain score (Analgesic A)	Pain score (Analgesic B)
11, 17, 8, 5, 14, 10, 7, 7, 10 10, 8, 4, 7, 5, 11, 6, 12, 10 5, 9, 13, 3, 10, 15, 5, 7, 9, 4	9, 5, 10, 10, 14, 13, 5, 15, 10 5, 7, 12, 9, 10, 12, 15, 13, 10 8, 5, 15, 10, 12, 8, 5, 9

The Mean

The **Mean**
describes the **middle** of the data.

The mean is also called the **average**.
The mean is heavily affected by extreme scores.

Exercise 13.8

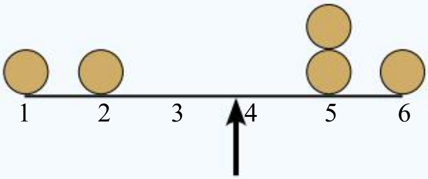
Find the mean of each of the following set of scores:

1, 2, 5, 5, 6

$$\text{mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$\text{mean} = \frac{1+2+5+5+6}{5}$$

mean = 3.8



Balance point is 3.8

1 1, 2, 3, 4, 4, 4
2 1, 2, 3, 4, 4, 80
3 1, 1, 2, 2, 3, 4, 4, 4, 4, 4
4 1, 6, 6, 2, 2, 3, 7, 9
5 31, 36, 36, 32, 32, 33
6 81, 86, 86, 82, 82, 83
7 5.1, 5.6, 5.6, 5.2, 5.2, 5.3
8 7.1, 7.6, 7.6, 7.2, 7.2, 7.3
9 −1, −6, −6, −2, −2, −3
10 −11,−16,−16,−12,−12,−13

The Standard Deviation

The **Standard Deviation**
describes the **spread** of the data.

The larger the standard deviation, the larger the spread.

The smaller the standard deviation, the smaller the spread.

Exercise 13.9

Use your calculator to find the mean and standard deviation of each of the following set of scores.

1, 2, 3, 4, 4

1 Change the calculator mode to Stat or SD
2 Enter a number then press M+
3 Repeat entering a number and then pressing M+
4 Find the $\bar{x}$ button, this is the mean.
5 Find the σ_x or σ_n button, this is the standard deviation of the population.

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

The standard deviation is an average of how far each score is from the mean.

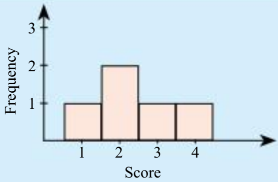
Mean = 2.8
Standard deviation = 1.17

1 1, 2, 3, 4, 4, 4
2 1, 2, 3, 4, 4, 80
3 1, 1, 2, 2, 3, 4, 4, 4, 4, 4
4 1, 6, 6, 2, 2, 3, 7, 9
5 31, 36, 36, 32, 32, 33
6 81, 86, 86, 82, 82, 83
7 101,106,106,102,102,103
8 721,726,726,722,722, 723
9 5.1, 5.6, 5.6, 5.2, 5.2, 5.3
10 7.1, 7.6, 7.6, 7.2, 7.2, 7.3
11 −1, −6, −6, −2, −2, −3
12 −11,−16,−16,−12,−12,−13

Comparative Analysis

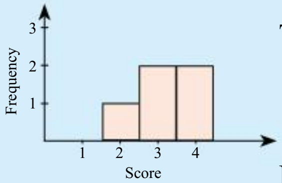
Exercise 13.10

Find the mean and standard deviation of each data set and use these descriptive statistics to compare the sets of data.



The scores are:
1, 2, 2, 3, 4

Mean = 2.4
SD = 1.02

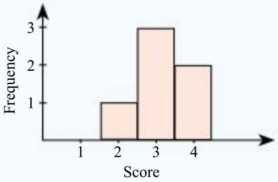


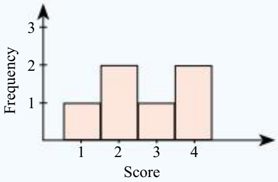
The scores are:
2, 3, 3, 4, 4

Mean = 3.2
SD = 0.75

The first data set has a lower mean - its centre is more to the left. The first data set is more spread out than the second data set (sd = 1.02 compared to 0.75).

1





2 Curious about whether there was a difference in performance on the end semester test, the following results were obtained from 10A and 10B.

End semester test (10A)	End semester test (10B)
60 ,42 ,70 ,47 ,85 ,80 ,74 ,92, 84 ,70 ,79 ,89 ,74 ,31 ,70 ,66, 46 ,81 ,86 ,71 ,44 ,82 ,62 ,97	67 ,75 ,59 ,83 ,45 ,55 ,59 ,35, 75, 65 ,69 ,43 ,77 ,68 ,44 ,49 45, 31, 62 ,71 ,81 ,57 ,51 ,75, 60 ,52

3 The following data has been obtained in order to compare the age distribution of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole.

Indigenous	Non-Indigenous
13, 52, 2, 9, 47, 6, 5, 22, 16, 3, 26, 64, 3, 10, 27, 6, 30, 11, 17, 35, 67, 44, 34, 38, 7	52, 3, 48, 23, 11, 27, 7, 36, 39, 16, 45, 82, 53, 61, 68, 19, 2, 38, 64, 14, 29, 57, 37, 76, 72

4 Water samples were taken from a river above and below an industrial plant and the dissolved oxygen measured (The greater the pollution, the lower the dissolved oxygen).

Dissolved oxygen (above)	Dissolved oxygen (below)
6.8, 6.3, 5.9, 5.7, 6.9, 6.5, 5.8, 6.4, 6.7, 6.9, 5.6, 6.2, 6.9, 5.7, 6.9, 5.8, 6.1, 6.8, 5.9, 6.8, 5.6	6.6, 5.9, 5.7, 4.3, 4.8, 5.9, 5.0 4.3, 4.8, 5.5, 4.9, 6.4, 4.7, 5.6 5.7, 6.3, 4.6, 4.7, 6.4, 5.1, 4.5

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 13.11

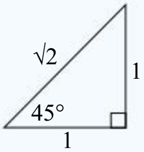
1 Spell Standard Deviation

Given the numbers: 2, 3, 4, 5, 6, 6, 6

2 What is the median?

3 What is the lower quartile?

4 What is the upper quartile?



5 In the triangle, what is sin 45°?

6 In the triangle, what is cos 45°?

7 In the triangle, what is tan 45°?

8 Two sides of a right-angled triangle are 3 and 4, what is the hypotenuse?

9 What is the Simple Interest Formula?

10 Is $x - 1$ a factor of $x^3 + 3x^2 - 2x - 1$?

'Wise people learn by other's mistakes, fools by their own' - Unknown.

Exercise 12.12

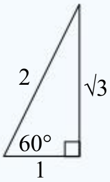
1 Spell Comparative

Given the numbers: 1, 1, 2, 3, 3, 4, 5, 7

2 What is the median?

3 What is the lower quartile?

4 What is the upper quartile?



5 In the triangle, what is sin 60°?

6 In the triangle, what is cos 60°?

7 In the triangle, what is tan 60°?

8 The hypotenuse in a right-angled triangle is 2 and a side is 1, what is the third side?

9 What is the Compound Interest Formula?

10 Is $x - 1$ a factor of $x^3 + 4x^2 - 3x - 2$?

'Learning is not a spectator sport' - D. Blocher.

Exercise 13.13

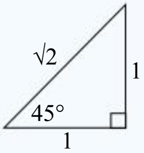
1 Spell Median

Given the numbers: 3, 4, 4, 5, 5, 6, 6, 7, 8

2 What is the median?

3 What is the lower quartile?

4 What is the upper quartile?



5 In the triangle, what is sin 45°?

6 In the triangle, what is cos 45°?

7 In the triangle, what is tan 45°?

8 Two sides of a right-angled triangle are 1 and 2, what is the hypotenuse?

9 What is the Compound Interest Formula?

10 Is $x + 1$ a factor of $x^3 + x^2 - 2x - 2$?

Orthoptists diagnose and treat vision and eye disorders.

- Relevant school subjects are English, Mathematics, Chemistry, Biology.
- Courses generally involves an orthoptic science University degree.

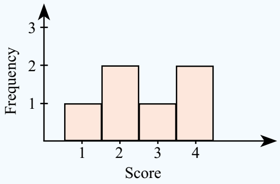
Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 13.14

1 Find the mode, median, and the mean of each of the following:

a)



b)

Score	Frequency
3	1
4	2
5	3
6	1

2 Find the average of:

a) 1, 0.1, 0.01 **b)** 11.1, 1.11, 0.111 **c)** 0.1, 0.11, 0.111

d) $\frac{1}{9}$ and $\frac{2}{9}$ **e)** $\frac{1}{2}$ and $\frac{1}{4}$ **f)** $\frac{1}{2}$ and $\frac{1}{5}$

3 The average of four numbers is 10. If 5 is added to each number, what is then the average of the four numbers?

4 The average of five numbers is 50. If 5 is subtracted from each number, what is then the average of the five numbers?

In a set of five numbers, the average of the first two numbers is 8 and the average of the last three numbers is 6. What is the average of the five numbers?

$8 = \frac{total}{2}$

Total of first two numbers = 16

$6 = \frac{total}{3}$

Total of last three numbers = 18

Total of the five numbers = 16 + 18 = 34

$mean = \frac{34}{5}$

Average = 6.8

5 In a set of five numbers, the average of the first two numbers is 6 and the average of the last three numbers is 9. What is the average of the five numbers?

6 In a set of ten numbers, the average of the first seven numbers is 8 and the average of the last three numbers is 9. What is the average of the ten numbers?

7 Three-quarters of the course is uphill and one-quarter of the course is downhill. If Tan runs uphill at an average speed of 8 km/h and runs downhill at an average speed of 12 km/h. What is Tan's average speed over the whole course?

8 Three-fifths of the course is uphill and two-fifths of the course is downhill. If Naqisa runs uphill at an average speed of 9 km/h and runs downhill at an average speed of 11 km/h.
What is Naqisa's average speed over the whole course?

9 The table shows Amelia's basketball scoring statistics for her last five games.
Give a possible set of five scores that would have the same mode, median, and mean.

Mode	Median	Mean
12	12	13

Investigations

Investigation 13.1 The Mean

Is the mean (or average) affected by large or small numbers?

Compare

The mean of: 31, 4, 85, 73, 72, 88, 38, 19, 36, **8**
with the mean of: 31, 4, 85, 73, 72, 88, 38, 19, 36, **800**
with the mean of: 31, 4, 85, 73, 72, 88, 38, 19, 36, **0.008**

Descriptive Stats
$\mu = 45.4$
$\Sigma x = 454$
$\Sigma x^2 = 29624$
$S_x = 31.64$
$\sigma_x = 30.02$
$n = 10$
min = 4
Q1 = 19
Med = 37
Q3 = 73
max = 88

Investigation 13.2 Percentiles

Investigate

Percentiles
What is the link between percentiles and quartiles?

Is this true?
lower quartile = ¼(n+1)
upper quartile = ¾(n+1)

Investigation 13.3 The Median

Is the median affected by large or small numbers?

Compare

The median of: 31, 4, 85, 73, 72, 88, 38, 19, 36, **8**
with the median of: 31, 4, 85, 73, 72, 88, 38, 19, 36, **800**
with the median of: 31, 4, 85, 73, 72, 88, 38, 19, 36, **0.008**

Investigation 13.4 Standard Deviation?

Investigate

10A

Standard Deviation
Where did it come from? What is it used for?

The standard deviation tells you how spread out the data is

Investigation 13.5 10A Standard Deviation?

Is the standard deviation affected by large or small numbers?

Compare

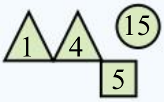
The standard deviation of: 31, 4, 85, 73, 72, 88, 38, 19, 36, **8**
with the standard deviation of: 31, 4, 85, 73, 72, 88, 38, 19, 36, **800**
with the standard deviation of: 31, 4, 85, 73, 72, 88, 38, 19, 36, **0.008**

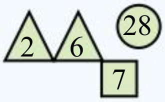
A Couple of Puzzles

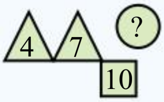
Exercise 13.15

1 What number needs to be included with 4 and 5 so that the average of the three numbers is 5?

2 Find the missing number:



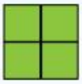




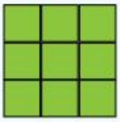
3 How many squares in each of the following squares?

a)

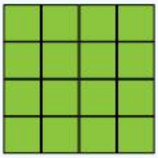
b)



c)



d)



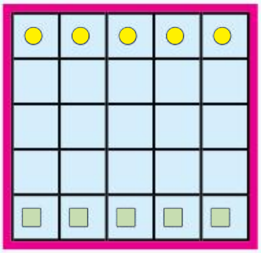
A Game

Line-up is a game played by two players.

1 Each player arranges each of their five pieces as shown.

2 Each player takes turns moving one square at a time. A move may be in any direction. Jumping is not allowed.

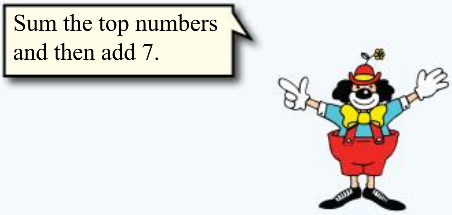
3 The winner is the first person to get their five pieces in a diagonal, horizontal, or vertical line.



A Sweet Trick

While your back is turned, ask your audience to:

1	Throw three dice	2, 4, 5
2	Sum the top three numbers	2+4+5 = 11
3	Select one of the dice and add its bottom number to the total	11+2 = 13
4	Ask your audience to throw the selected die again and add the top number to the total	13+3 = 16
5	Turn around, look at the three dice, and tell your audience their total.	



The secret to this trick is that the sum of the top number and the bottom number on a die is always 7.

Technology

Technology 13.1

A graphics calculator will automatically calculate a large number of descriptive statistics.

1 Select the Stat menu, EDIT, and enter data into one of the lists.

2 Return to the main screen.

3 Select the Stat menu, Calc, 1-Stats and enter L1.

L1	L2	L3
74		
74		
75		
76		
78		
78		
78		
80		

```
1_Stats
        x̄  = 76.63
        σx = 2.06
Lower Q    = 74.5
Median     = 77
Upper Q    = 78
```

The data can be entered in any order.

A graphics calculator will also produce a box plot. How?

How would you characterise the average statistician?

Probably mean.

Technology 13.2

A spreadsheet will calculate a very large number of descriptive statistics.

	a	b	c	d
1	74			
2	74		Mean	76.63
3	75		Standard D	2.06
4	76		Minimum	74
	78		Lower Q	74.75
	78		Median	77
	78		Upper Q	78
	80		Maximum	80

=average(a1:a8)

=stdevpa(a1:a8)

=min(a1:a8)

=quartile(a1:a8,1)

=quartile(a1:a8,2)

=quartile(a1:a8,3)

=max(a1:a8)

Technology 13.3

If you know how, a spreadsheet will draw box plots and other ways of visually displaying the shape of data sets.

Spreadsheet Box Plots

How can spreadsheets draw box plots?

Statistically speaking, in China, if you are a one in a million kind of person, there are one and half thousand others just like you.

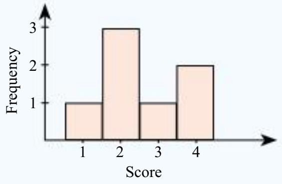
Chapter Review 1

Exercise 13.16

Mode = most often
Median = middle
Mean = average

1 Find the mode, median, and the mean of each of the following:

a)



b)

Score	Frequency
3	1
4	2
5	3
6	1

2 Find the mean, lower quartile, median, upper quartile, range, and 10A standard deviation for the following set of data:

8.1, 8.0, 8.3, 8.2, 8.6, 8.9, 8.2, 8.3, 8.8, 8.5.

3 Draw a box plot to represent the following data:

69, 34, 61, 60, 63, 52, 78, 57, 81, 65, 43, 58, 66, 41, 56, 76, 56

4 Depression affected patients were asked to self-rate their state of depression before and after treatment.
Use box plots to represent and then compare each data set.

Before treatment	After treatment
20, 23, 22, 19, 17, 15, 26, 15, 22, 26, 17, 24, 30, 22, 20, 23, 22, 26, 26, 27, 29, 20, 25, 30	19, 17, 14, 23, 19, 19, 20, 20, 21, 23, 25, 14, 13, 15, 19, 21, 23, 23, 18, 16, 17, 13, 14, 20

5 10A To test the effectiveness of two different product displays, shoppers were asked how much they spent on impulse buying after passing each shopping display.
Find the mean and standard deviation of each data set and use these descriptive statistics to compare the sets of data.

Display A	Display B
15, 9, 10, 11, 12, 6, 9, 7, 10, 6, 10, 13, 7, 11, 6, 9, 9, 15, 8, 15, 7, 11, 10	11, 12, 10, 11, 11, 16, 11, 12, 11, 16, 10, 18, 11, 16, 10, 18, 13, 13, 10, 16, 12, 10, 19, 10, 15

6 Five-sevenths of the course is uphill and two-sevenths of the course is downhill.
If Miri cycles uphill at an average speed of 20 km/h and cycles downhill at an average speed of 38 km/h.
What is Miri's average speed over the whole course?

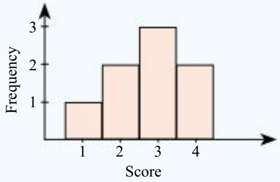
Chapter Review 2

Exercise 13.17

Mode = most often
Median = middle
Mean = average

1 Find the mode, median, and the mean of each of the following:

a)



b)

Score	Frequency
1	1
2	3
3	4
4	1

2 Find the mean, lower quartile, median, upper quartile, range, and 10A standard deviation for the following set of data:

3.4, 3.9, 3.7, 3.2, 3.7, 3.7, 3.6, 3.8, 3.5, 3.5, 3.8.

3 Draw a stem and leaf plot and a box and whisker plot to represent the following data:

61, 99, 71, 87, 53, 71, 84, 78, 84, 73, 85, 79, 61, 63, 53, 80, 91

4 Year 10 students were tested on their box plot knowledge before and after a week of inclass learning about box plots.
Use box plots to represent and then compare each data set.

Before learning	After learning
44, 70, 70, 43, 42, 56, 45, 56, 58, 40, 40, 64, 43, 57, 55, 56, 51, 47, 70, 58, 44	80, 90, 88, 81, 86, 67, 75, 70, 85, 77, 76, 70, 82, 88, 90, 69, 79, 85, 90, 65, 81

5 10A The following data has been obtained in order to compare the education of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole (5- Year 10, 7 - Year 12, 10 - Bachelor degree).
Find the mean and standard deviation of each data set and use these descriptive statistics to compare the sets of data.

Indigenous	Non-Indigenous
10, 5, 5, 7, 5, 5, 7, 5, 5, 5, 7, 7, 5, 10, 5, 5, 5, 7, 5, 7, 5,	5, 5, 7, 10, 5, 7, 7, 5, 10, 5, 7, 10, 5, 5, 7, 7, 7, 5, 5, 10, 7, 5

6 Three-eights of the course is uphill and five-eights of the course is downhill.
If Carla cycles uphill at an average speed of 24 km/h and cycles downhill at an average speed of 42 km/h.
What is Carla's average speed over the whole course?

Graphs

14

Number and Algebra ➞ Linear and Non-linear Relationships

★ 10A Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation.

- investigate the features of graphs of polynomials including axes intercepts and the effect of repeated factors.

★ 10A Solve simple exponential equations.

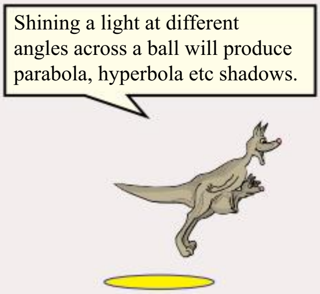
- investigate exponential equations derived from authentic mathematical models based on population growth.

★ 10A Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations.

- apply transformations, including translations, reflections in the axes and stretches to help graph parabolas, rectangular hyperbolas, circles and exponential functions.

★ 10A Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts.

- write quadratic equations that represent practical problems.



A TASK

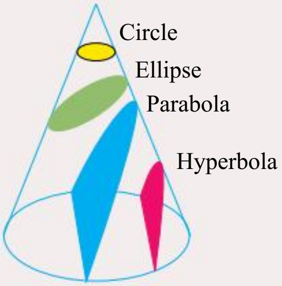
A function for cooling?

- Heat water and regularly record the temperature as it cools.
- Plot your results.
- Which fits best: parabola, hyperbola, exponential, polynomial?
- Can you create an equation that fits?

A LITTLE BIT OF HISTORY

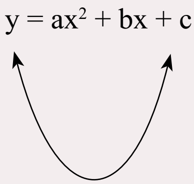
Menaechmus (c. 350 BC) appears to be the first to study conic sections as a means of trying to 'trisect the angle', double the cube', and 'square the circle'.

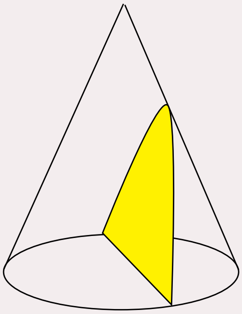
Appollonius (c. 220 BC) wrote eight books about the conic sections and named the ellipse, parabola, and hyperbola.



Parabolas

The graph of a quadratic function is a parabola.





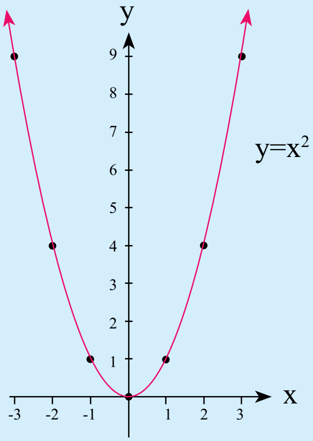
Exercise 14.1

Plot: $y = x^2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = x^2}$	9	4	1	0	1	4	9

$(-3)^2 = 9$

See Technology 14.1 for other ways of drawing graphs.



Graph the following quadratics by completing the table of values, plotting the points, and drawing a smooth curve through the points.

1 $y = 2x^2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 2x^2}$	18				2		

2 $y = 3x^2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 3x^2}$		12				12	

3 $y = 5x^2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 5x^2}$			5	0			

4 $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = x^2 + 2}$		6			3		

5 $y = x^2 + 5$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = x^2 + 5}$	14					9	

6 $y = x^2 - 3$

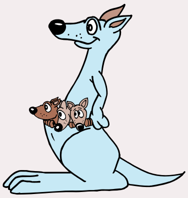
x	-3	-2	-1	0	1	2	3
$\mathbf{y = x^2 - 3}$		1		-3			

7 $y = x^2 - 6$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = x^2 - 6}$			-5			-2	

$\mathbf{y = ax^2}$

How does different values of **a** affect the parabola?

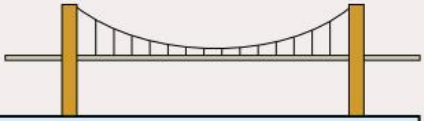


$\mathbf{y = ax^2 + c}$

How does different values of **c** affect the parabola?

A thrown stone follows a parabolic path.

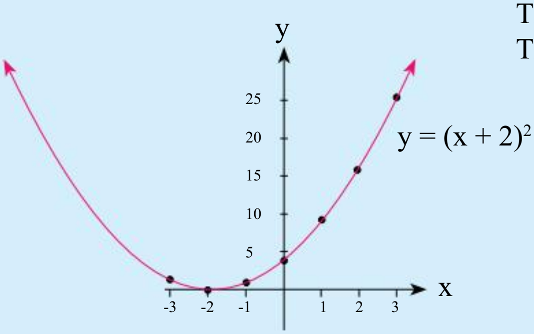
Parabolas can be used in the design of suspension bridges.



Exercise 14.2

Plot: $y = (x + 2)^2$

x	-3	-2	-1	0	1	2	3
$y = (x + 2)^2$	1	0	1	4	9	16	25



The parabola cuts the x-axis at -2
The parabola cuts the y-axis at 4

$$\begin{aligned}(x + \boxed{2})^2 &= (x + 2)(x + 2)\\ &= x(x + 2) + 2(x + 2)\\ &= x^2 + 2x + 2x + 4\\ &= x^2 + 4x + \boxed{4}\end{aligned}$$

x-intercept = -2
y-intercept = 4

Graph the following quadratics by completing the table of values.
Find the x-intercept and the y-intercept from your graph.

1 $y = (x + 1)^2$

x	-3	-2	-1	0	1	2	3
$y = (x + 2)^2$			1				

2 $y = (x - 1)^2$

x	-3	-2	-1	0	1	2	3
$y = (x - 1)^2$			4				

3 $y = (x + 3)^2$

x	-3	-2	-1	0	1	2	3
$y = (x + 3)^2$		1					

4 $y = (x - 3)^2$

x	-3	-2	-1	0	1	2	3
$y = (x - 3)^2$			16				

5 $y = -x^2$

x	-3	-2	-1	0	1	2	3
$y = -x^2$		-4					

6 $y = -x^2 + 4$

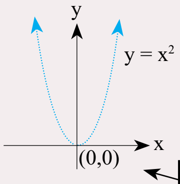
x	-3	-2	-1	0	1	2	3
$y = -x^2 + 4$	-5						

7 $y = -(x + 2)^2$

x	-3	-2	-1	0	1	2	3
$y = -(x + 2)^2$		0					

Parabolas

A parabola can be sketched by transformations of $\mathbf{y = x^2}$.



Sketch - no units shown.

Sketch - label the turning point.

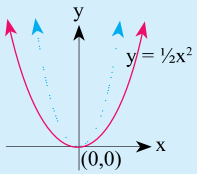
Vertical translation	$ax^2 + d$ the parabola is shifted **d units up**. $ax^2 - d$ the parabola is shifted **d units down**.
Horizontal translation	$a(x + d)^2$ the parabola is shifted **d units left.** $a(x^2 - d)^2$ the parabola is shifted **d units right**.
Vertical reflection	ax^2 the parabola has a ∪ shape. $-ax^2$ the parabola has a ∩ shape.
Dilation	ax^2 the parabola is ∪ for $a>1$. ax^2 the parabola is ⌒ for $a<1$.

Exercise 14.3

Sketch each of the following quadratics by first sketching $y = x^2$ and then applying the appropriate transformations.

$y = \frac{1}{2}x^2$

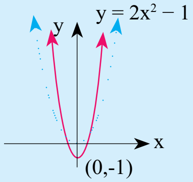
The coefficient, a, is less than 1, thus the shape is wider.



$y = 2x^2 - 1$

The coefficient = 2 thus the shape is narrower.

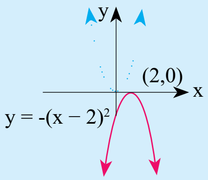
The parabola is also translated 1 unit down



$y = -(x - 2)^2$

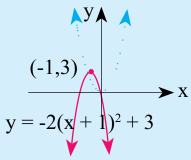
The parabola is translated 2 units to the right.

The parabola is also reflected vertically



$y = -2(x + 1)^2 + 3$

The parabola is reflected vertically, is narrower, is translated horizontally 1 unit to the left and is translated vertically 3 units.



1 $y = 3x^2$

2 $y = \frac{1}{4}x^2$

3 $y = 10x^2$

4 $y = 0.75x^2$

5 $y = 2x^2 + 3$

6 $y = 3x^2 - 1$

7 $y = 0.5x^2 + 1$

8 $y = \frac{3}{4}x^2 + 4$

9 $y = (x + 3)^2$

10 $y = (x - 1)^2$

11 $y = -(x + 1)^2$

12 $y = -3(x - 4)^2$

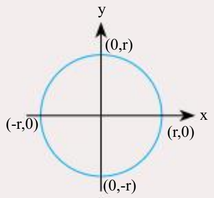
13 $y = 2x^2 + 3$

14 $y = 3x^2 - 1$

15 $y = 0.5x^2 + 1$

16 $y = \frac{3}{4}x^2 + 4$

Circles



Circle: $x^2 + y^2 = r^2$
Centre (0,0) Radius = r

Horizontal translation — $(x - p)^2 + y^2 = r^2$ the centre is shifted **p units right**.

Vertical translation — $x^2 + (y - q)^2 = r^2$ the centre is shifted **q units up**.

Exercise 14.4

1 Find the centre and radius of each of the following circles:

$x^2 + y^2 = 9$
centre = (0,0)
radius = 3

$(x - 2)^2 + (x + 3)^2 = 25$
centre = (2,-3)
radius = 5

a) $x^2 + y^2 = 9$
b) $x^2 + y^2 = 16$
c) $(x - 3)^2 + (y + 1)^2 = 4$
d) $(x + 1)^2 + (y - 5)^2 = 36$
e) $(x - 4)^2 + y^2 = 3$
f) $x^2 + (y + 2)^2 = 5$

2 Write the equation of each of the following circles:

centre (0,0), radius = 2
$x^2 + y^2 = 4$

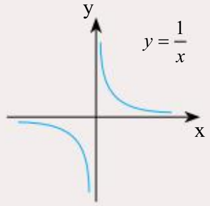
centre (-2,3), radius = $\sqrt{3}$
$(x + 2)^2 + (x - 3)^2 = 3$

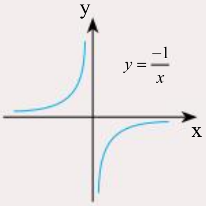
centre (0,5), radius = $\sqrt{7}$
$x^2 + (x - 5)^2 = 7$

a) centre (0,0), radius 2 units
b) centre (0,0), radius $\sqrt{5}$ units
c) centre (1,3), radius 3 units
d) centre (-4,1), radius 7 units
e) centre (6,-4), radius $\sqrt{3}$ units
f) centre (-2,-3), radius $\sqrt{2}$ units

Hyperbolas

Hyperbola: $y = \frac{a}{x}$
Hyperbolas have two distinct curves called arms or branches.





Exercise 14.5

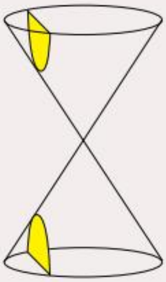
Graph the following hyperbolas by completing the table of values, plotting the points, and drawing a smooth curve through the points.

1 $y = \frac{9}{x}$

x	-3	-2	-1	0	1	2	3
$y = \frac{9}{x}$		-4.5		∞	9		

2 $y = \frac{-4}{x}$

x	-3	-2	-1	0	1	2	3
$y = \frac{-4}{x}$		2		∞			-1.3



Polynomials

Polynomial - from 2 terms to many terms

Degree = 3
(degree of highest power)

The leading term is $4x^3$

$$4x^3 + x^2 - 5x - 7$$

Coefficient of the leading term is 4

Constant term is $^{-}7$ (no variable)

The exponents must be non-negative integers.

$$y = x(x + 3)(x - 2)$$

A polynomial in factor form.

The same polynomial expanded.

$$y = x^3 + x^2 - 6x$$

The sketch of the same polynomial.

Exercise 14.6

Sketch each of the following polynomials:

$y = x(x + 3)(x - 2)$

x-intercepts when $y = 0$,

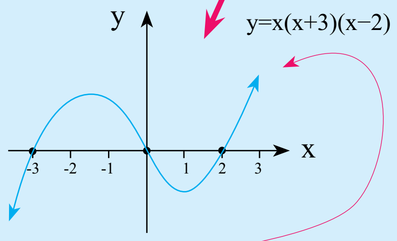
$x(x + 3)(x - 2) = 0$

either $x = 0$, $x = -3$, $x = 2$

Cuts x-axis at 3 places (0,0), (-3,0), (2,0)

If expanded, the leading term would be x^3

thus if x is large (eg x=+100), y is large (eg +1 000 000)



1 $y = (x + 2)(x - 1)$

2 $y = (x + 1)(x - 1)(x - 2)$

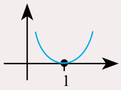
3 $y = (x + 1)(x + 2)(x - 1)(x - 3)$

4 $y = x(x + 2)(x - 3)$

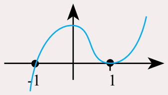
5 $y = 3x(x + 3)(x - 1)$

6 $y = (x + 1)(x - 4)(x - 3)$

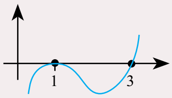
Two repeated factors eg $y=(x-1)^2$



Two repeated factors + single factor eg $y=(x+1)(x-1)^2$



Two repeated factors + single factor eg $y=(x-3)(x-1)^2$



$y = (x + 1)(x - 2)^2$

x-intercepts when $y = 0$,

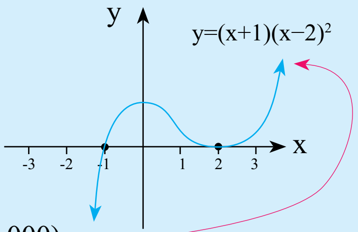
$(x + 1)(x - 2)^2 = 0$

either $x = -1$, $x = 2$

Cuts x-axis at 2 places (-1,0), (2,0)

If expanded, the leading term would be x^3

thus if x is large (eg x=+100), y is large (eg +1 000 000)



7 $y = (x + 1)(x - 3)^2$

8 $y = (x + 3)(x - 1)^2$

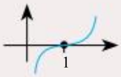
9 $y = (x - 1)^2$

10 $y = (x + 1)^2(x - 3)$

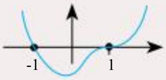
11 $y = x(x + 1)^2(x - 4)$

12 $y = x(x + 1)^2(x - 3)$

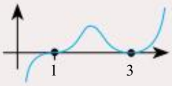
Three repeated factors
eg $y=(x-1)^3$



Three repeated factors + single factor eg $y=(x+1)(x-1)^3$



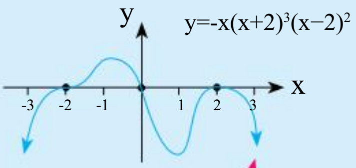
Three repeated factors + two repeated factors eg $y=(x-3)^2(x-1)^3$



$y = -x(x + 2)^3(x - 2)^2$

x-intercepts when $y = 0$,

$x(x + 2)^3(x - 2)^2 = 0$
either $x = 0$, $x = -2$, $x = 2$
Cuts x-axis at 3 places (0,0), (-2,0), (2,0)



If expanded, the leading term would be $-x^6$
thus if x is large (eg x=+100), y is large (eg -1 000 000 000 000)

13 $y = (x - 3)^3$
14 $y = (x + 1)^3$
15 $y = (x + 1)(x - 1)^3$
16 $y = (x - 1)^3(x + 2)$
17 $y = x(x + 1)^3(x - 4)$
18 $y = -x(x - 1)^3$
19 $y = -x(x + 1)(x - 4)^3$
20 $y = -x(x - 2)^2(x - 3)$

Exercise 14.7

Sketch each of the following polynomials after using the factor theorem to factorise each polynomial.

$y = x^3 - 2x^2 - x + 2$

Try P(1) then P($^-1$) then P(2) then P($^-2$)

$P(1) = 1^3 - 2\times1^2 - 1 + 2$
$P(1) = 1 - 2 - 1 + 2$
$P(1) = 0$ thus $\underline{x - 1 \text{ is a factor}}$

Now factorise: $x^2 - x - 2$
$(x - 2)(x + 1)$
Either $x - 2 = 0$ or $x + 1 = 0$
$\underline{x = 2 \text{ or } x = {^-1}}$

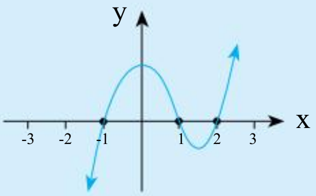
Thus: $y = (x - 1)(x - 2)(x + 1)$

x-intercepts when $y = 0$,
$(x - 1)(x - 2)(x + 1) = 0$
either $x = 1$, $x = 2$, $x = -1$
Cuts x-axis at 3 places (1,0), (2,0), (-1,0)

If expanded, the leading term would be x^3
thus if x is large (eg x=+100),
y is large (eg +1 000 000)

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline x - 1 & x^3 - 2x^2 - x + 2 \\ & x^3 - x^2 \; - \\ \hline & \quad -x^2 - x \\ & \quad -x^2 + x \; - \\ \hline & \qquad -2x + 2 \\ & \qquad -2x + 2 \; - \\ \hline & \qquad \bullet \quad \bullet \end{array}$$

$y = x^3 - 2x^2 - x + 2$

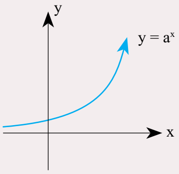


1 $x^3 - 2x^2 - x + 2 = 0$
2 $x^3 + 4x^2 + x - 6 = 0$
3 $x^3 + 7x^2 + 14x + 8 = 0$
4 $x^3 - 4x^2 + x + 6 = 0$
5 $x^3 - x^2 - 14x + 24 = 0$
6 $x^3 - 4x^2 - 11x + 30 = 0$

Exponential Functions

$y = a^x$

is an exponential function



$y = a^x$

The variable, x, is the exponent thus the name exponential functions.

Exercise 14.8

Plot: $y = 2^x$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 2^x}$	0.125	0.25	0.5	1	2	4	8

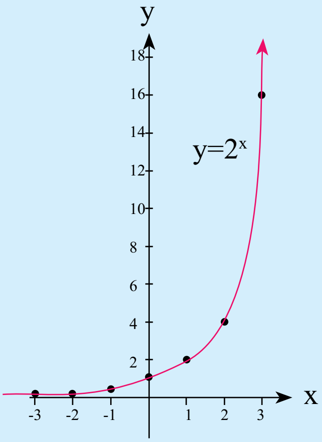
$2^{-3} = 0.125$

$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$

Calculator:

(2) (y^x) (+/–) (3) (=)

See Technology 14.2 for other ways of sketching exponential functions.



Sketch the following exponential functions by completing the table of values, plotting the points, and drawing a smooth curve through the points.

1 $y = 3^x$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 3^x}$		$\frac{1}{9}$					27

2 $y = 2 \times 2^x$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 2 \times 2^x}$			1			8	

3 $y = 5 \times 2^x$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 5 \times 2^x}$	$\frac{5}{8}$				10		

4 $y = 2^x + 3$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 2^x + 3}$		3.25					11

5 $y = 3^x - 2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 3^x - 2}$			-1.67			7	

6 $y = 2^{-x}$

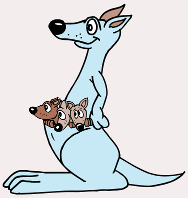
x	-3	-2	-1	0	1	2	3
$\mathbf{y = 2^{-x}}$	8					$\frac{1}{4}$	

7 $y = 3^{-x} - 4$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 3^{-x} - 4}$		5		-3			

$\mathbf{y = b \times a^x}$

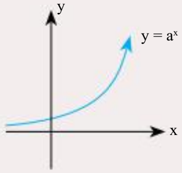
How does different values of **b** affect the function?



$\mathbf{y = a^x + c}$

How does different values of **c** affect the function?

Exponential GROWTH

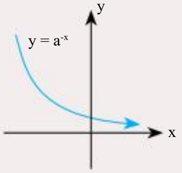


Exponential growth happens when a quantity repeatedly increases by a constant percentage of the quantity.

Examples

Compound interest growth
Asset depreciation
Microorganism population growth
Virus population growth
Animal population growth
Nuclear chain reaction

Exponential DECAY



Exponential decay happens when a quantity repeatedly decreases by a constant percentage of the quantity.

Exercise 14.9

A town's population of 10 000 increases each year by 5%.

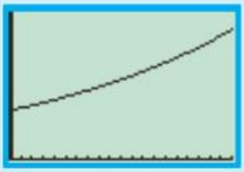
a) Use an exponential function to model the growth.
b) Estimate the town's population after 10 years.
c) When will the town's population double to 20 000?

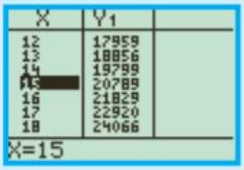
a) $P = 10000(1 + 0.05)^t$ {compound interest}
$P = 10000\times1.05^t$

b) $t = 10$
$P = 10000\times1.05^{10}$
$P = 16\,300$

c) Population doubles in 14 years

See Technology 14.2





```
X     Y1
12    17959
13    18856
14    19799
15    20789
16    21829
17    22920
18    24066
X=15
```

1 A town's population of 20 000 increases each year by 5%.
a) Use an exponential function to model the growth.
b) Estimate the town's population after 10 years.
c) When will the town's population double to 40 000?

2 A car purchased for $35 000 decreases in value each year by 12%.
a) Use an exponential function to model the depreciation.
b) Estimate the car's value after 5 years.

The amount of a drug in the bloodstream is modelled by the exponential decay function $A = 250\times1.2^{-t}$, where A is amount in milligrams and t is time in hours.

a) How much of the drug is in the bloodstream at time t = 0 hours?
b) How much of the drug is in the bloodstream after 3 hours?
c) Plot the function and use it to estimate when there is 100 mg.

a) $t = 0$
$A = 250\times1.2^{-0}$
$A = 250$ mg

b) $t = 3$
$A = 250\times1.2^{-3}$
$A = 145$ mg

c) using graphics calculator
100 mg after 5 hours

3 The amount of a drug in the bloodstream is modelled by the exponential decay function $A = 250\times1.2^{-t}$, where A is amount in milligrams and t is time in hours.
a) How much of the drug is in the bloodstream at time t = 0 hours?
b) How much of the drug is in the bloodstream after 2 hours?
c) Plot the function and use it to estimate when there is 50 milligrams.

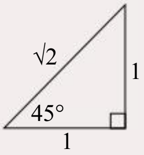
4 Assume a population of 1000 micro-organisms are able to triple every hour and be modelled by $P = 1000\times3^t$.
a) What is the population after 3 hours?
b) Plot the function and use it to estimate when the population is 10 000.

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 14.10

1 Spell Parabola
2 What is the general equation of a parabola?
3 What is the general equation of a hyperbola?
4 What is the general equation of a circle?
Given the numbers: 1, 2, 2, 3, 4, 5, 6
5 What is the median?
6 What is the lower quartile?
7 In the triangle, what is sin 45°?

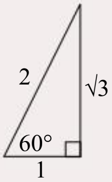


8 Two sides of a right-angled triangle are 3 and 4, what is the hypotenuse?
9 What is the Simple Interest Formula?
10 Is $x - 1$ a factor of $x^3 + x^2 - 2x - 1$?

'Success is stumbling from failure to failure with no loss of enthusiasm' - Winston S Churchill.

Exercise 14.11

1 Spell Exponential
2 What is the general equation of a parabola?
3 What is the general equation of a hyperbola?
4 What is the general equation of a circle?
Given the numbers: 1, 1, 3, 3, 4, 5, 6, 6
5 What is the median?
6 What is the upper quartile?
7 In the triangle, what is cos 60°?

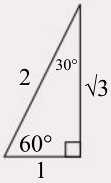


8 Two sides of a right-angled triangle are 1 and 2, what is the hypotenuse?
9 What is the Compound Interest Formula?
10 Is $x + 1$ a factor of $x^3 + x^2 - x - 1$?

'We are made to persist, that's how we find out who we are' - Tobias Wolff.

Exercise 14.12

1 Spell Hyperbola
2 What is the general equation of a parabola?
3 What is the general equation of a hyperbola?
4 What is the general equation of a circle?
Given the numbers: 1, 2, 3, 3, 3, 5, 5, 6, 6
5 What is the median?
6 What is the lower quartile?
7 In the triangle, what is sin 30°?



8 Two sides of a right-angled triangle are 2 and 2, what is the hypotenuse?
9 What is the Compound Interest Formula?
10 Is $x - 1$ a factor of $x^3 + 2x^2 - 2x + 1$?

Quantity Surveyors supervise construction sontracts and construction costs for all kinds of constructions.

- Relevant school subjects are English and Mathematics.
- Courses generally involves a relevant University degree.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 14.13

1 The point (2, -1) lies on which quadratic?

a) $y = x^2 + 2x - 3$

b) $y = x^2 + 2x - 6$

c) $y = x^2 + 2x - 9$

2 Numbered pegs are placed in uniform sequence around a circle.

a) If peg 1 is directly opposite peg 3, how many pegs in total?

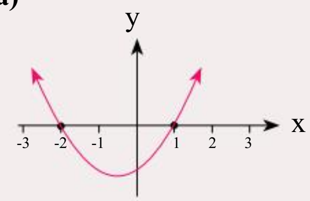
b) If peg 1 is directly opposite peg 5, how many pegs in total?

c) If peg 1 is directly opposite peg 7, how many pegs in total?

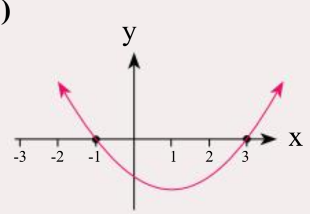
d) If peg 1 is directly opposite peg 41, how many pegs in total?

3 What is the equation of each of the following parabolas?

a)



b)

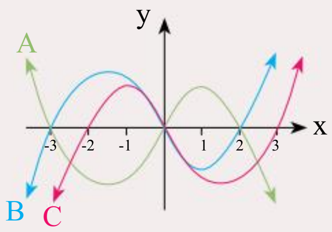


4 Match each polynomial equation with their graph

$y = x(x - 3)(x + 2)$

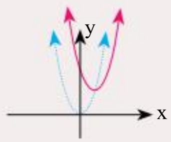
$y = x(x + 3)(x - 2)$

$y = -x(x + 3)(x - 2)$

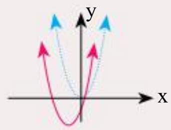


5 $y = x^2$ is shown. Which graph represents $y = (x - 1)^2 + 3$?

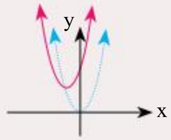
a)



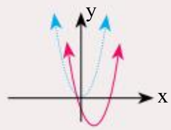
b)



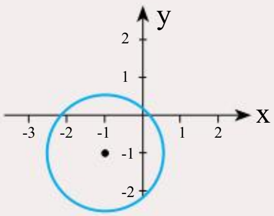
c)



d)

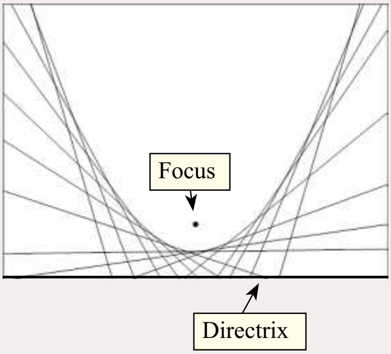


6 The circle with centre (-1, 1) is rotated 90° anticlockwise about the origin and then translated 2 units (to the left and 3 units up. What are the new coordinates of the centre of the circle?



Investigations

Investigation 14.1 Paper fold a Parabola



1 Make a dot above the center of the lower edge of the paper.

2 Fold the lower edge up so that it touches the dot.
Draw a line in the crease.

3 Fold again in a different position with the bottom edge touching the dot.

4 Repeat until the parabola shape is obvious.

A parabola is the set of all points so that the distance to a fixed point (called the focus) and a fixed line (called the directrix) is the same.

What affect does moving the focus, relative to the directrix, have on the shape of the parabola?

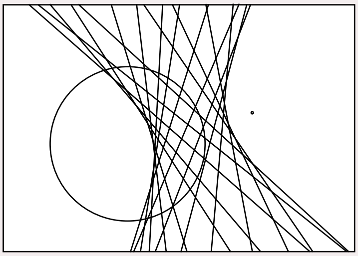
Investigation 14.2 Paper fold a Hyperbola

1 Draw a circle on a sheet of paper.
Put a dot outside the circle.

2 Fold the dot over until it touches the circle.
Draw a line in the crease.

3 Fold again so that the dot touches another part of the circle.

4 Repeat until the hyperbola shape is obvious.



A hyperbola is the set of all points so that the ratio of the distance to the focus and the directrix is the same.

What affect does moving the focus, relative to the circle, have on the shape of the hyperbola?

Investigate

Where is the directrix on a hyperbola?
What is the eccentricity of a hyperbola?

Investigation 14.3 Parabolas, hyperbolas, circles, exponential functions, etc

While there are thousands of examples of the use of parabolas, circles, exponential functions, etc in our modern and commercial world, can you find examples of these shapes in everday life?

Investigate

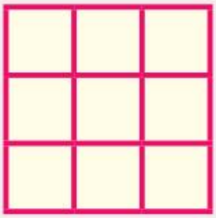
Can you find these shapes in your home?

A Couple of Puzzles

Exercise 14.14

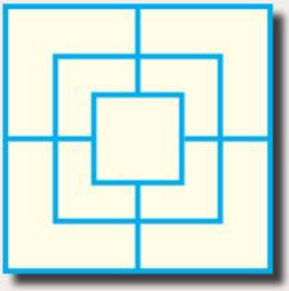
1 Jon has about 3.9 L of blood. If a vampire bat can drink about 30 mL of blood, how many vampire bats would suck Jon dry?

2 Place each of the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10 in each of the nine cells of the square so that the sum of each row, column, and diagonal is 18.



A Game

Nine Men Morris is an ancient game played by two players.



1 Each player starts with nine counters and takes turns to place each of the nine counters on a vertex on the board.

2 When a player forms a "mill", a line of 3 counters, the player may take one of the other player's counters off the board and out of play. A counter in a "mill" cannot be taken off the board.

3 After the nine counters have been placed on the board, play involves moving a counter to an adjacent vertex.

A player wins when the other player either can't move or has just two counters left.

The game is mentioned in Shakespeare's 'Midsummer Night's Dream'.

A dance is associated with the game where nine people dance the Morris dance.

A Sweet Trick

Three friends go to an outdoor café for coffee and cake.

The bill is exactly $30 so each of the three friends put in exactly $10.

The waiter takes the money to the cash register where the manager indicates that the three friends have been overcharged by $5.

The waiter takes five $1 coins but being dishonest keeps 2 for himself and gives one to each of the three friends.

Each of the friends paid $10 and got $1 back thus paying a total of 3×9 = $27.

The waiter has $2, thus total = $29.

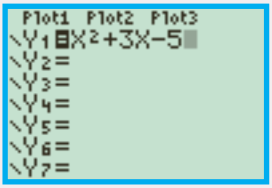
Where is the other $1?

Where is the other $1?

Technology

Technology 14.1

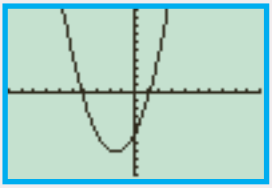
A graphics calculator will graph parabolas such as: $y = x^2 + 3x - 5$



```
Plot1 Plot2 Plot3
\Y1■X²+3X−5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

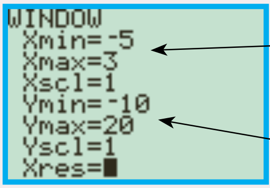
1 Press Y=

2 Enter: X,T,θ,n x² + 3 X,T,θ,n − 5



3 Press Graph

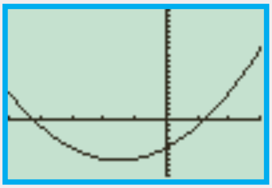
4 Change the scale by pressing Window



```
WINDOW
 Xmin=-5
 Xmax=3
 Xscl=1
 Ymin=-10
 Ymax=20
 Yscl=1
 Xres=■
```

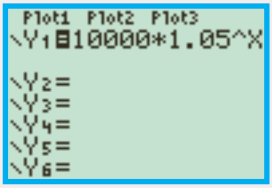
This sets the x-axis scale from -5 to 3

This sets the y-axis scale from -10 to 20



Technology 14.2

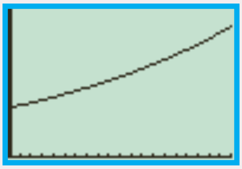
A graphics calculator will graph exponential functions such as: $P = 10000 \times 1.05^t$



```
Plot1 Plot2 Plot3
\Y1■10000*1.05^X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

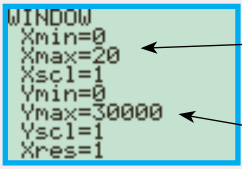
1 Press Y=

2 Enter: 10000 × 1.05 ^ X,T,θ,n



3 Press Graph

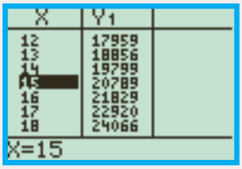
4 Change the scale by pressing Window



```
WINDOW
 Xmin=0
 Xmax=20
 Xscl=1
 Ymin=0
 Ymax=30000
 Yscl=1
 Xres=1
```

This sets the x-axis scale from 0 to 20 years

This sets the y-axis scale from 0 to $30000



X	Y1
12	17959
13	18856
14	19799
15	20789
16	21829
17	22920
18	24066

X=15

5 Press Table to find a value of $20 800 after 15 years

Technology 14.3

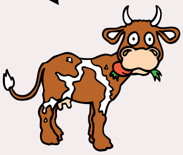
Graphs

Watch videos on 'Parabolas', 'Hyperbolas', 'Circles', 'Exponential Functions'.

What is: (x-a)(x-b)(x-c)...(x-z)?

0

[Hint: what is the 24th factor].



Chapter Review 1

Exercise 14.15

1 Graph the following quadratics by completing the table of values. Find the x-intercept and the y-intercept from your graph.

a) $y = 3x^2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 3x^2}$		12				12	

b) $y = x^2 - 5$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = x^2 - 5}$			-4			-1	

c) $y = (x - 2)^2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = (x - 2)^2}$							

2 Sketch each of the following quadratics by first sketching $y = x^2$ and then applying the appropriate transformations.

a) $y = 2x^2 + 3$ **b)** $y = -(x + 1)^2$

3 Find the centre and radius of the circle: $(x - 3)^2 + (y + 1)^2 = 4$

4 Write the equation of the circle with centre (3,-2) and radius $\sqrt{3}$ units.

5 Graph the following hyperbola by completing the table of values, plotting the points, and drawing a smooth curve through the points.

$$y = \frac{9}{x}$$

x	-3	-2	-1	0	1	2	3
$y = \frac{9}{x}$		-4.5		∞	9		

6 Sketch each of the following polynomials:

a) $y = (x + 1)(x - 1)(x - 2)$ **b)** $y = x(x + 1)^2(x - 4)$

c) $y = -x(x - 2)^2(x - 3)$ **d)** $y = x^3 - 2x^2 - x + 2$

7 Sketch the following exponential function by completing the table of values, plotting the points, and drawing a smooth curve through the points.

$y = 2^x + 3$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 2^x + 3}$		3.25					

8 A town's population of 10 000 increases each year by 5%.

a) Use an exponential function to model the growth.

b) Estimate the town's population after 10 years.

c) When will the town's population double to 20 000?

9 The amount of a drug in the bloodstream is modelled by the exponential decay function $A = 250 \times 1.3^{-t}$, where A is amount in milligrams and t is time in hours.

a) How much of the drug is in the bloodstream at time t = 0 hours?

b) How much of the drug is in the bloodstream after 3 hours?

c) Plot the function and use it to estimate when there is 100 mg.

Chapter Review 2

Exercise 14.16

1 Graph the following quadratics by completing the table of values. Find the x-intercept and the y-intercept from your graph.

a) $y = 2x^2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 2x^2}$		8					18

b) $y = x^2 - 3$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = x^2 - 3}$			-2				

c) $y = (x - 1)^2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = (x - 1)^2}$							

2 Sketch each of the following quadratics by first sketching $y = x^2$ and then applying the appropriate transformations.

a) $y = 2x^2 - 1$ **b)** $y = -(x + 2)^2$

3 Find the centre and radius of the circle: $(x + 1)^2 + (y - 2)^2 = 9$

4 Write the equation of the circle with centre (-2,3) and radius $\sqrt{2}$ units.

5 Graph the following hyperbola by completing the table of values, plotting the points, and drawing a smooth curve through the points.

$$y = \frac{6}{x}$$

x	-3	-2	-1	0	1	2	3
$y = \frac{6}{x}$		-3		∞			

6 Sketch each of the following polynomials:

a) $y = (x + 2)(x - 1)(x - 3)$ **b)** $y = x(x - 2)^2(x - 1)$

c) $y = -x(x + 1)^2(x - 3)$ **d)** $y = x^3 - 2x^2 - x + 2$

7 Sketch the following exponential function by completing the table of values, plotting the points, and drawing a smooth curve through the points.

$y = 2^x - 2$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 2^x - 2}$		-1.75					

8 A town's population of 5 000 increases each year by 2%.

a) Use an exponential function to model the growth.

b) Estimate the town's population after 10 years.

c) When will the town's population double to 10 000?

9 The amount of a drug in the bloodstream is modelled by the exponential decay function $A = 500 \times 1.4^{-t}$, where A is amount in milligrams and t is time in hours.

a) How much of the drug is in the bloodstream at time t = 0 hours?

b) How much of the drug is in the bloodstream after 2 hours?

c) Plot the function and use it to estimate when there is 250 mg.

Review 3

15

Chapter 11 Finance

The simple interest rule

$I=Prt$

I = simple interest
P = principal
r = the interest rate
t = the time

Compound Interest Formula

$$A = P(1+\frac{r}{100})^n$$

P = principal
r = the interest rate
n = number of time periods
A = final amount

Depreciation Formula

$$A = P(1-\frac{r}{100})^n$$

4.25% pa would be 4.25÷12% per month.

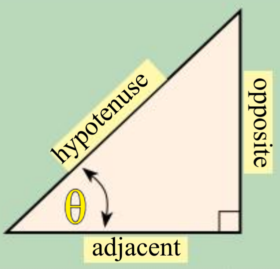
Chapter 12 Trigonometry 1

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$c^2 = a^2 + b^2$



Angle of elevation - the object is *above the observer*.

Angle of depression - the object is *below the observer*.

The square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.

Chapter 13 Statistics 1

The **Box Plot** is also known as the *box-and-whisker plot*.

The **Median** is the **middle** of a set of scores.

The **Lower Quartile** is the **middle** of the bottom half of a set of scores.

The **Upper Quartile** is the **middle** of the top half of a set of scores.

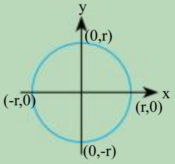
The **Mean** 10A describes the **middle** of the data.

The **Standard Deviation** 10A describes the **spread** of the data.

Chapter 14 Graphs

The graph of a quadratic function is a parabola.

Circle: $\mathbf{x^2 + y^2 = r^2}$
Centre (0,0) Radius = r

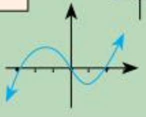


Hyperbola: $y = \frac{a}{x}$

Hyperbolas have two distinct curves called arms or branches.

$\mathbf{y = a^x}$
is an exponential function

y=x(x+3)(x−2)
is a polynomial



Review 1

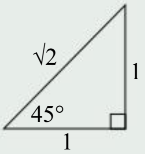
Exercise 15.1 Mental computation

1 Spell Parabola

2 What is the general equation of a parabola?

3 What is the general equation of a hyperbola?

4 What is the general equation of a circle?



Given the numbers: 1, 2, 2, 3, 4, 5, 6

5 What is the median?

6 What is the lower quartile?

7 In the triangle, what is cos 45°?

8 Two sides of a right-angled triangle are 1 and 2, what is the hypotenuse?

9 What is the Simple Interest Formula?

10 Is x − 1 a factor of $x^3 + x^2 - x - 1$?

Exercise 15.2

1 Find the simple interest charged on $50 000 borrowed for 3 years at 8.5% pa.

If the charge for lending $5600 at 0.7% per month is $789.60, for how long was the $5600 lent?

I = $117.60
P = $5600
r = 0.7% /month
t = ? months

$$I = Prt$$

$$117.60 = 5600 \times \frac{0.7}{100} \times t$$

$$\frac{117.60}{5600} \times \frac{100}{0.7} = t$$

$$\underline{t = 3 \text{ months}}$$

The simple interest rule

I=Prt

I = simple interest
P = principal
r = the interest rate
t = the time

2 If the charge for lending $6450 at 6.8% pa is $500, for how long was the $6450 lent?

3 How much would need to be invested at 6.5% pa for 6 months in order to earn $500 interest (6 months = 0.5 years)?

4 Cooper is charged $8000 for borrowing $70 000 for 9 months. What simple interest rate per month is Cooper being charged?

$3500 is invested at 5.8% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

A = ?
P = $3500
r = 5.8% pa
n = 3 times

$$A = P(1 + \frac{r}{100})^n$$

$$A = 3500(1 + \frac{5.8}{100})^3$$

$$\underline{A = \$4145}$$

Compound Interest Formula

$$A = P(1 + \frac{r}{100})^n$$

5 $25 500 is invested at 5.3% pa compounded yearly.
What will be the principal after 4 years (round to the nearest dollar)?

6 $250 000 is invested at 6.7% pa compounded monthly.
What will be the principal after 3 years (round to the nearest dollar)?

Depreciation Formula

$$A = P(1-\frac{r}{100})^n$$

7 Which produces the better outcome over 5 years?

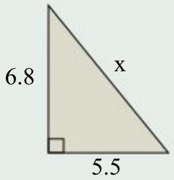
a) \$100 000 invested at 9% pa compounded yearly?

b) \$100 000 invested at 9% pa compounded monthly?

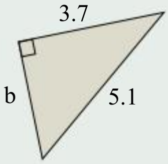
8 A car is purchased for \$35 000. What is the value of the car after 5 years if it depreciates in value by 10% each year (round to nearest \$1000)?

9 Use Pythagoras' theorem to find the length of the unknown side (round to two decimal places):

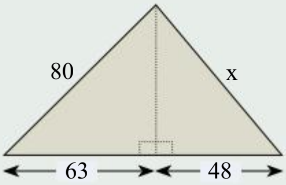
a)



b)

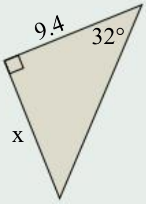


c)

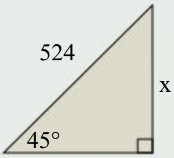


10 Use the sin, cos, or tan ratio to find the unknown (round to two decimal places):

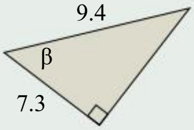
a)



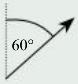
b)



c)

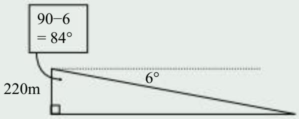


11 A plane flies due west for 147 km, then on a bearing of 60°T until the plane is due north of its starting point.
How far is the plane from its starting point?



60°T is 60° clockwise from North.

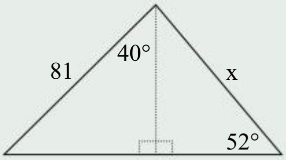
12 From the top of a 220 m tower, the angle of depression to a fire is 6°. How far away is the fire?



13 From the top of a building 47 m high, the angle of depression to the bottom of a building across the street is 40°. The angle of elevation to the top of the building across the street is 33°. Find:

a) The height of the building across the street.

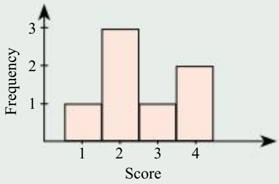
b) The width of the street.



14 Find the unknown in the diagram.

15 Find the mode, median, and the mean of each of the following:

a)



b)

Score	Frequency
3	1
4	2
5	3
6	1

16 Find the mean, lower quartile, median, upper quartile, range, and 10A standard deviation for the following set of data:

8.1, 8.0, 8.3, 8.2, 8.6, 8.9, 8.2, 8.3, 8.8, 8.5.

17 Draw a box plot to represent the following data:

69, 34, 61, 60, 63, 52, 78, 57, 81, 65, 43, 58, 66, 41, 56, 76, 56

18 Depression affected patients were asked to self-rate their state of depression before and after treatment.
Use box plots to represent and then compare each data set.

Before treatment	After treatment
20, 23, 22, 19, 17, 15, 26, 15, 22, 26, 17, 24, 30, 22, 20, 23, 22, 26, 26, 27, 29, 20, 25, 30	19, 17, 14, 23, 19, 19, 20, 20, 21, 23, 25, 14, 13, 15, 19, 21, 23, 23, 18, 16, 17, 13, 14, 20

19 10A To test the effectiveness of two different product displays, shoppers were asked how much they spent on impulse buying after passing each shopping display.
Find the mean and standard deviation of each data set and use these descriptive statistics to compare the sets of data.

Display A	Display B
15, 9, 10, 11, 12, 6, 9, 7, 10, 6, 10, 13, 7, 11, 6, 9, 9, 15, 8, 15, 7, 11, 10	11, 12, 10, 11, 11, 16, 11, 12, 11, 16, 10, 18, 11, 16, 10, 18, 13, 13, 10, 16, 12, 10, 19, 10, 15

20 Four-sevenths of the course is uphill and three-sevenths of the course is downhill. If Ella cycles uphill at an average speed of 15 km/h and cycles downhill at an average speed of 30 km/h.
What is Ella average speed over the whole course?

21 10A Graph the following quadratics by completing the table of values.
Find the x-intercept and the y-intercept from your graph.

a) $y = 2x^2$

x	-3	-2	-1	0	1	2	3
$y = 2x^2$		8					

b) $y = x^2 - 3$

x	-3	-2	-1	0	1	2	3
$y = x^2 - 3$			-2				

22 10A Sketch each of the following quadratics by first sketching $y = x^2$ and then applying the appropriate transformations.

a) $y = 2x^2 + 4$

b) $y = -(x - 2)^2$

23 10A Find the centre and radius of the circle: $(x-3)^2 + (y+1)^2 = 4$

24 10A Write the equation of the circle with centre (3,-2) and radius $\sqrt{3}$ units.

25 10A Graph the following hyperbola by completing the table of values, plotting the points, and drawing a smooth curve through the points.

$$y = \frac{9}{x}$$

x	-3	-2	-1	0	1	2	3
$y = \frac{9}{x}$		-4.5		∞	9		

26 10A Sketch each of the following polynomials:

a) $y = (x+1)(x-2)(x+2)$ **b)** $y = x(x+1)^2(x-2)$

c) $y = -x(x-1)^2(x+2)$ **d)** $x^3 - 2x^2 - x + 2 = 0$

27 10A Sketch the following exponential functions by completing the table of values, plotting the points, and drawing a smooth curve through the points.

$y = 2^x + 1$

x	-3	-2	-1	0	1	2	3
$\mathbf{y = 2^x + 1}$		1.25					

28 10A A town's population of 10 000 increases each year by 3%.

a) Use an exponential function to model the growth.

b) Estimate the town's population after 10 years.

c) When will the town's population double to 20 000?

Review 2

Exercise 15.3 Mental computation

1 Spell Exponential

2 What is the general equation of a parabola?

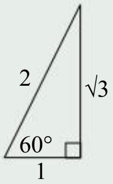
3 What is the general equation of a hyperbola?

4 What is the general equation of a circle?

Given the numbers: 1, 3, 3, 3, 4, 5, 6, 7

5 What is the median?

6 What is the upper quartile?



7 In the triangle, what is cos 60°?

8 Two sides of a right-angled triangle are 1 and 3, what is the hypotenuse?

9 What is the Compound Interest Formula?

10 Is $x + 1$ a factor of $2x^3 + x^2 - 2x - 1$?

Exercise 15.4

1 Find the simple interest charged on $50 000 borrowed for 3 years at 11.5% pa

The simple interest rule

$$I=Prt$$

I = simple interest
P = principal
r = the interest rate
t = the time

If the charge for lending \$5600 at 0.7% per month is \$789.60, for how long was the \$5600 lent?

I = \$117.60
P = \$5600
r = 0.7% /month
t = ? months

$$I = Prt$$
$$117.60 = 5600\times\frac{0.7}{100}\times t$$
$$\frac{117.60}{5600}\times\frac{100}{0.7} = t$$
$$\underline{t = 3 \text{ months}}$$

2 If the charge for lending \$25 000 at 6.8% pa is \$6800, for how long was the \$25 000 lent?

3 How much would need to be invested at 6.1% pa for 18 months in order to earn \$1000 interest (18 months = 1.5 years)?

4 Joe is charged \$2000 for borrowing \$80 000 for 6 months. What simple interest rate per month is Joe being charged?

\$3500 is invested at 5.8% pa compounded yearly.
What will be the principal after 3 years (round to the nearest dollar)?

A = ?
P = \$3500
r = 5.8% pa
n = 3 times

$$A = P(1+\frac{r}{100})^n$$
$$A = 3500(1+\frac{5.8}{100})^3$$
$$\underline{A = \$4145}$$

Compound Interest Formula

$$A = P(1+\frac{r}{100})^n$$

5 \$9500 is invested at 7.9% pa compounded yearly.
What will be the principal after 4 years (round to the nearest dollar)?

6 \$225 000 is invested at 6.2% pa compounded monthly.
What will be the principal after 4 years (round to the nearest dollar)?

7 Which produces the better outcome over 5 years?
a) \$100 000 invested at 9% pa compounded yearly?
b) \$100 000 invested at 9% pa compounded monthly?

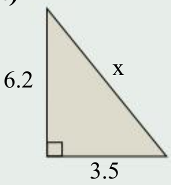
Depreciation Formula

$$A = P(1-\frac{r}{100})^n$$

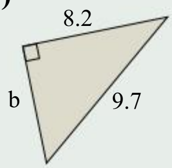
8 A car is purchased for \$25 000. What is the value of the car after 5 years if it depreciates in value by 12% each year (round to nearest \$1000)?

9 Use Pythagoras' theorem to find the length of the unknown side (round to two decimal places):

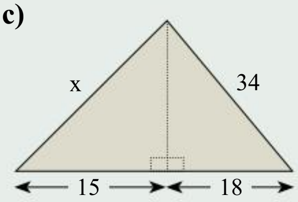
a)



b)

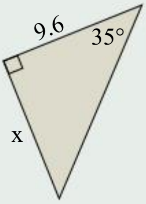


c)

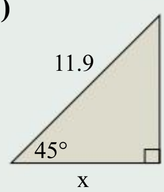


10 Use the sin, cos, or tan ratio to find the unknown (round to two decimal places):

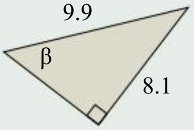
a)



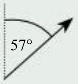
b)



c)

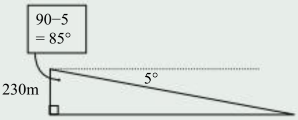


11 A plane flies due west for 96 km, then on a bearing of 57°T until the plane is due north of its starting point.
How far is the plane from its starting point?



57°T is 57° clockwise from North.

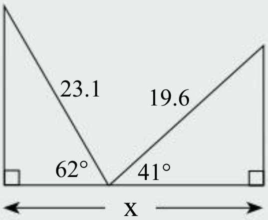
12 From the top of a 230 m tower, the angle of depression to a fire is 5°. How far away is the fire?



13 From the top of a building 52 m high, the angle of depression to the bottom of a building across the street is 46°. The angle of elevation to the top of the building across the street is 27°. Find:

a) The height of the building across the street.

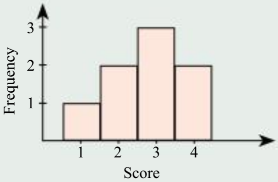
b) The width of the street.



14 Find the unknown in the diagram.

15 Find the mode, median, and the mean of each of the following:

a)



b)

Score	Frequency
1	1
2	3
3	4
4	1

16 Find the mean, lower quartile, median, upper quartile, range, and 10A standard deviation for the following set of data:

6.1, 6.9, 6.7, 6.5, 6.7, 6.7, 6.6, 6.8, 6.5, 6.5, 6.8.

17 Draw a stem and leaf plot and a box and whisker plot to represent the following data:

54, 97, 81, 82, 73, 66, 94, 58, 94, 83, 75, 79, 61, 63, 53, 80

18 Year 10 students were tested on their box plot knowledge before and after a week of inclass learning about box plots.
Use box plots to represent and then compare each data set.

Before learning	After learning
44, 70, 70, 43, 42, 56, 45, 56, 58, 40, 40, 64, 43, 57, 55, 56, 51, 47, 70, 58, 44	80, 90, 88, 81, 86, 67, 75, 70, 85, 77, 76, 70, 82, 88, 90, 69, 79, 85, 90, 65, 81

19 10A Investigating the results of a three week ball dexterity program, a PE teacher collected the following dexterity indexes before and after the program. Find the mean and standard deviation of each data set and use these descriptive statistics to compare the sets of data.

Dexterity (before)	Dexterity (after)
17, 22, 14, 20, 16, 18, 15, 18, 18, 19, 16, 17, 23, 18, 17, 15, 18, 22, 23, 17, 17, 17, 21, 22, 19, 22, 16	22, 24, 21, 26, 15, 18, 19, 21, 28, 23, 18, 20, 18, 17, 24, 16, 19, 23, 18, 22, 21, 25, 21, 26, 18, 19, 18

20 Four-ninths of the course is uphill and five-ninths of the course is downhill. If William cycles uphill at an average speed of 25 km/h and cycles downhill at an average speed of 37 km/h, what is William's average speed over the whole course?

21 10A Graph the following quadratics by completing the table of values. Find the x-intercept and the y-intercept from your graph.

$y = x^2 - 2$

x	-3	-2	-1	0	1	2	3
$y = x^2 - 2$			-1				

22 10A Sketch each of the following quadratics by first sketching $y = x^2$ and then applying the appropriate transformations.

a) $y = 3x^2 - 1$

b) $y = -(x + 2)^2$

23 10A Write the equation of the circle with centre (2,3) and radius $\sqrt{5}$ units.

24 10A Graph the following hyperbola by completing the table of values, plotting the points, and drawing a smooth curve through the points.

$y = \frac{2}{x}$

x	-3	-2	-1	0	1	2	3
$y = \frac{2}{x}$		-1		∞			

25 10A Sketch each of the following polynomials:

a) $y = (x + 2)(x - 2)(x - 3)$

b) $y = x(x - 1)^2(x + 1)$

c) $y = -x(x + 1)^2(x - 3)$

d) $x^3 - 2x^2 - x + 2 = 0$

26 10A Sketch the following exponential function by completing the table of values, plotting the points, and drawing a smooth curve through the points.

$y = 3^x - 1$

x	-3	-2	-1	0	1	2	3
$y = 3^x - 1$		-0.89					

27 10A The amount of a drug in the bloodstream is modelled by the exponential decay function $A = 400 \times 1.3^{-t}$, where A is amount in milligrams and t is time in hours.

a) How much of the drug is in the bloodstream at time t = 0 hours?

b) How much of the drug is in the bloodstream after 2 hours?

c) Plot the function and use it to estimate when there is 100 mg.

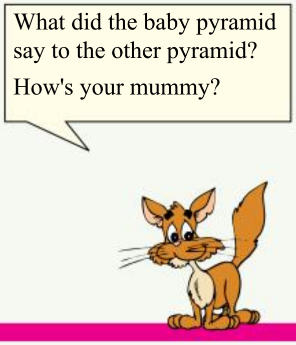
Coordinate Geometry

16

Number and Algebra → Linear and Non-linear Relationships

★ Solve problems involving parallel and perpendicular lines.
  - solve problems using the fact that parallel lines have the same gradient and conversely that if two lines have the same gradient then they are parallel.
  - solve problems using the fact that the product of the gradients of perpendicular lines is –1 and conversely that if the product of the gradients of two lines is –1 then they are perpendicular.

★ Explore the connection between algebraic and graphical representations of relations such as simple quadratics and circles using digital technology as appropriate.
  - sketch graphs of parabolas, and circles.
  - apply translations, reflections and stretches to parabolas and circles.



A TASK

What is the slope, gradient, of the sides of the Great Pyramid at Giza?

What is the slope, gradient, of other pyramids in Egypt?

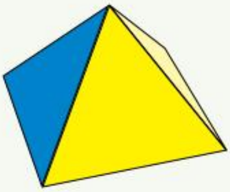
Are the slopes similar? Why did the Egyptians use these slopes?

A LITTLE BIT OF HISTORY

In 1858, Henry Rhind purchased a large papyrus - a 13 inch high roll 18 feet in length at Luxor in Egypt. The papyrus was then acquired by the British Museum.

The scroll became known as the Rhind Papyrus. It was a practical handbook of Egyptian mathematics written around 1700 BC. The Rhind is essentially a maths textbook containing practical examples and problems.

The second part of the Rhind papyrus involved geometry with problems about the slopes (gradient) of pyramids.



If a pyramid is 250 cubits high and the side of its base 360 cubits long, what is its seked?

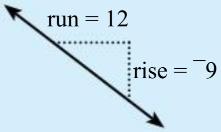
Gradient

The gradient, or slope, or m, is used to describe the steepness of lines.

$$m = \text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

Exercise 16.1

Find the gradient of each of the following line segments:

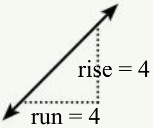


$$\text{Gradient} = m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{^-9}{12}$$

$$\underline{m = {}^-0.75}$$

1



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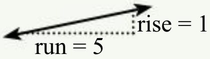

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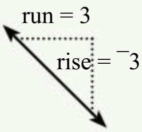

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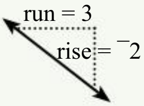
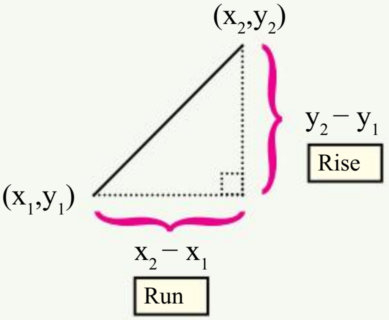
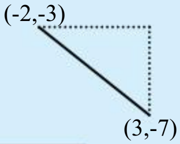




$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slopes up to the right ▶ positive gradient
Slopes down to the right ▶ negative gradient



First complete:

$x_1 = -2$, $y_1 = -3$
$x_2 = 3$, $y_2 = -7$

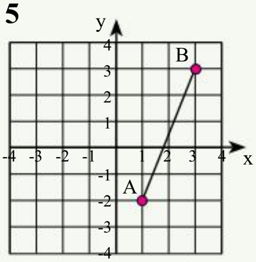
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{^-7 - {}^-3}{3 - {}^-2}$$

$$m = \frac{^-4}{5}$$

$$\underline{m = {}^-0.8}$$

5



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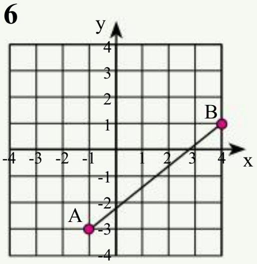

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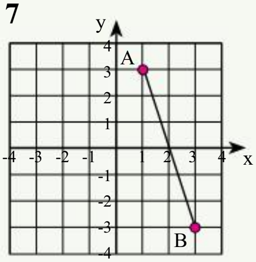

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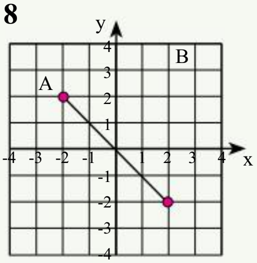


A(-1,4) and B(2,-3)

First complete: $x_1 = -1, y_1 = 4$
$x_2 = 2, \quad y_2 = -3$

$$\text{Gradient} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

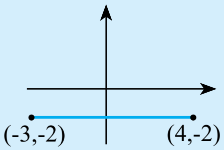
$$m = \frac{(-3-4)}{(2--1)}$$

$$m = -2.33$$

9 A(1,1), B(3,4)
10 A(2,5), B(6,1)
11 A(5,2), B(7,5)
12 A(2,3), B(4,1)
13 A(4,1), B(-2,3)
14 A(2,3), B(-5,-1)
15 A(3,0), B(-7,-8)
16 A(-3,1), B(-2,4)
17 A(-1,-2), B(3,1)

Exercise 16.2

Find the gradient of each of the following line segments and decide whether the line segment is parallel to the x-axis or the y-axis:



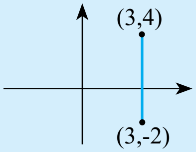
$x_1 = 4, y_1 = -2$
$x_2 = -3, y_2 = -2$

$$\text{Gradient} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(-2--2)}{(-3-4)}$$

$$m = \frac{0}{-7} = 0$$

The line is parallel to the x-axis.



$x_1 = 3, y_1 = 4$
$x_2 = 3, y_2 = -2$

$$\text{Gradient} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(4--2)}{(3-3)}$$

$$m = \frac{6}{0} = \infty$$

The line is parallel to the y-axis.

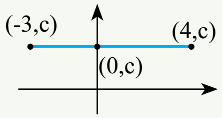
1 A(2,1), B(5,1)
2 A(5,3), B(6,3)
3 A(1,-2), B(7,-2)
4 A(2,-3), B(2,1)
5 A(4,1), B(4,-5)
6 A(-2,3), B(-2,-1)
7 A(3,0), B(-7,0)
8 A(-3,1), B(-3,4)
9 A(-1,-2), B(3,-2)

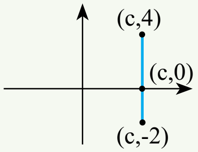
Gradient of lines parallel to the x-axis:

$$\mathbf{m = 0}$$

Gradient of lines parallel to the y-axis:

$$\mathbf{m = \infty}$$





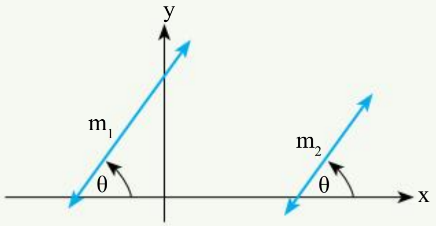
Equation of a line parallel to the x-axis:

$$\mathbf{y = c}$$

Equation of a line parallel to the y-axis:

$$\mathbf{x = c}$$

Parallel Lines



If two lines are parallel then they have the same gradient.

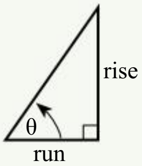
$m1 = m2$

Parallel lines make the same angle with the x-axis.

$m_1 = \frac{rise}{run}$ and $\tan\theta = \frac{rise}{run}$ thus $m_1 = \tan\theta$

$m_2 = \frac{rise}{run}$ and $\tan\theta = \frac{rise}{run}$ thus $m_2 = \tan\theta$

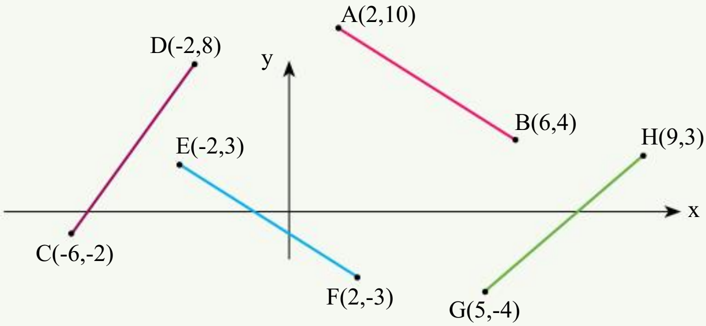
Therefore $m_1 = m_2$



Conversely, if two lines have the same gradient then they are parallel (AB//CD).

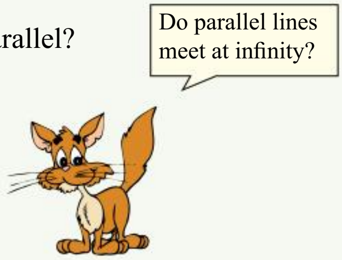
Exercise 16.3

1 Find the gradient of each of the following lines (not to scale) and thus show which lines are parallel:



2 Which of the following line segments are parallel?

- **a)** A(3,3), B(5,9)
- **b)** C(-2,5), D(-1,-6)
- **c)** E(4,-3), F(5,8)
- **d)** G(-2,2), H(-1,5)
- **e)** I(-3,3), J(5,3)



3 A quadrilateral is a parallelogram if the opposite sides are parallel. Is ABCD a parallelogram?

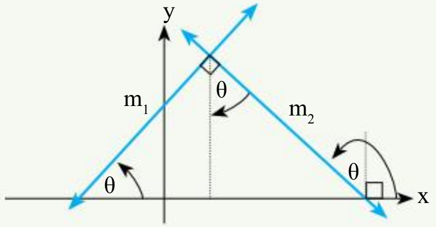
- **a)** A(8,8), B(5,4), C(2,3), D(5,7)
- **b)** A(-4,-5), B(-3,-3), C(-1,-6), D(-2,-8)
- **c)** A(-5,-4), B(2,-5), C(-7,4), D(0,3)

4 Given A(2,6), B(-3,5), and C(-4,-4), what should be the coordinates of D so that ABCD is a parallelogram?

Perpendicular Lines

If two lines are perpendicular then the product of their gradients is -1.

$$m_1 \times m_2 = {}^{-}1$$



$m_1 = \tan\theta$
$m_2 = \tan(90+\theta)$
$m_1 \times m_2 = \tan\theta \times \tan(90+\theta)$
Therefore $m_1 \times m_2 = {}^{-}1$

Examples:
$\tan 30 \times \tan(90+30) = {}^{-}1$
$\tan 50 \times \tan(90+50) = {}^{-}1$

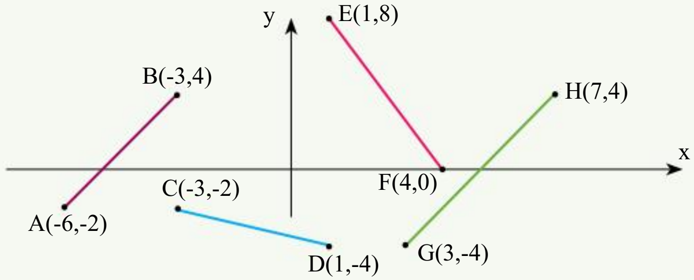
Conversely, if the product of the gradient of two lines is $^{-}1$ then the lines are perpendicular ($AB \perp CD$).

Exercise 16.4

1 Determine whether AB // CD or $AB \perp CD$ or neither:

a) $m_{AB} = 2$, $m_{CD} = 2$
b) $m_{AB} = -2$, $m_{CD} = 0.5$
c) $m_{AB} = 4$, $m_{CD} = -0.25$
d) $m_{AB} = -0.25$, $m_{CD} = 2$
e) $m_{AB} = \frac{1}{2}$, $m_{CD} = 2$
f) $m_{AB} = -3$, $m_{CD} = \frac{1}{3}$
g) $m_{AB} = \frac{5}{8}$, $m_{CD} = -1.6$
h) $m_{AB} = -\frac{2}{3}$, $m_{CD} = -\frac{2}{3}$

2 Find the gradient of each of the following lines (not to scale) and thus show which lines are parallel or perpendicular:



3 The triangle ABC is a right-angled triangle if one angle is 90°. Is ΔABC a right-angled triangle?

a) A(3,8), B(5,4), C(2,3)
b) A(2,-1), B(1,2), C(5,0)
c) A(-4,-3), B(2,-4), C(3,2)

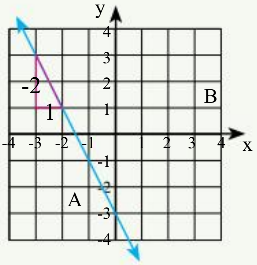
4 Given A(1,-3), B(-3,5), what should be the coordinates of C so that ABC is a right-angled triangle?

5 Is ABCD a rectangle given A(-7,1), B(-1,13), C(5,10), D(-1,-2)?

Gradient

Equation of a line:
$y = mx + c$
m = gradient
c = intercept on y-axis

$m = \frac{rise}{run} = \frac{-2}{1} = {}^{-}2$



$y = {}^{-}2x - 3$
$m = {}^{-}2$
y-intercept = $^{-}3$ {cuts y-axis at $(0,{}^{-}3)$ }

Exercise 16.5

1 Find the gradient and the y-intercept of each of the following linear equations:

a) $y = x + 2$
b) $y = x - 3$
c) $y = 2x + 1$
d) $y = 2x - 1$
e) $y = {}^{-}2x - 5$
f) $y = {}^{-}x + 7.5$
g) $y = 1.2x - 2$
h) $y = {}^{-}0.5x - 3.6$

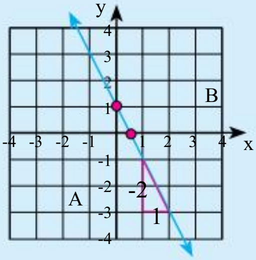
2 Rearrange each of the following linear equations into the form $y = mx + c$ and then find the gradient and the y-intercept:

a) $y - x = 2$
b) $y + x = 3$
c) $y - 2x = {}^{-}3$
d) $y + 3x = {}^{-}4$
e) $2y + 3x = 1$
f) $2y - 5x = {}^{-}1$
g) $5y + 4x - 1 = 0$
h) $3y - 2x - 6 = 0$

$2y + 3x = 9$
$2y = -3x + 9$
$y = 1.5x + 4.5$
$m = 1.5$
y-intercept = 4.5

3 For each of the following linear equations, find the gradient, the y-intercept, and the x-intercept. Use this information to sketch each linear equation.

$y = {}^{-}2x + 1$
$y = mx + c$
$m = {}^{-}2,\ c = 1$
gradient = $^{-}2$
y-intercept is at (0,1)
cuts the x-axis when y=0,
$0 = {}^{-}2x + 1$
$^{-}1 = {}^{-}2x$
$^{-}1/{}^{-}2 = x$
$0.5 = x$ thus x-intercept is at (0.5,0)



a) $y = x + 2$
b) $y = x - 3$
c) $y = 2x - 1$
d) $y = 2x + 4$
e) $y = {}^{-}x - 2$
f) $y = {}^{-}x + 5$
g) $y = 1.5x - 3$
h) $y = {}^{-}1.2x - 3$
i) $y + 2x = 1$
j) $y - 3x = {}^{-}3$
k) $3y + 1.5x - 4 = 0$
l) $2y - 4.2x + 3 = 0$

$m = 1$	$\theta = 45°$	↗
$m = 0$	$\theta = 0°$	→
$m = {}^{-}1$	$\theta = {}^{-}45°$	↘

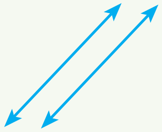
Exercise 16.6

1 Show that the following pairs of lines are parallel:

a) $y = 3x + 1$ and $y = 3x - 2$

b) $y = 2x - 3$ and $3y - 6x + 1 = 0$

PARALLEL
lines that never cross
$m_1 = m_2$



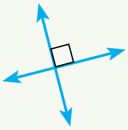
2 Which of the following pairs of lines are perpendicular:

a) $y = {}^{-}2x - 1$ and $y = 0.5x + 3$

b) $y = x - 3$ and $2y - x + 3 = 0$

c) $3y + 6x - 1 = 0$ and $2y - x + 3 = 0$

PERPENDICULAR
90°
$m_1 \times m_2 = {}^{-}1$



3 What is the value of m if the line $y = mx + 1$ is:

a) parallel to $3y + 2x - 4 = 0$?

b) perpendicular to $3y + 2x - 4 = 0$?

4 Are the following three points collinear:

a) A(2,3), B(5,4), and C(8,5)

b) A(5,4), B(-1,2), and C(-4,1)

c) A(-1,-1), B(-3,-3), and C(6,6)

Points are collinear if they are on the same line.

The points A, B, C are collinear if $m_{AB} = m_{BC}$

5 Find the angle at which each of the following lines cut the x-axis:

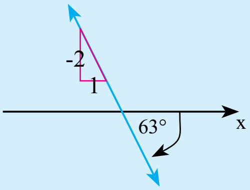
$y = -2x + 1$

$\tan \theta = m$

$\tan \theta = -2$

$\theta = \tan^{-1}(-2)$

$\underline{\theta = -63.43°}$



a) $y = x + 3$

b) $y = 2x - 5$

c) $y = {}^{-}2x + 1$

d) $y = 5x - 2$

e) $2y + 3x - 1 = 0$

6 If the gradient of the line segment A(3,-2), B(x,5) is 1, what is the value of x?

7 If the gradient of the line segment A(1,4), B(x,-2) is 0.5, what is the value of x?

8 What is the value of a if A(-1,3), B(2,0), and C(a, 4) are collinear?

9 What is the value of b if A(3,4), B(-3,0), and C(-2, b) are collinear?

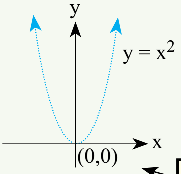
10 What is the equation of the line that cuts the y-axis at (0, 2) and is parallel to the line segment A(-1,3), B(2,-3)?

11 What is the equation of the line that cuts the y-axis at (0, -3) and is parallel to the line segment A(2,-1), B(4,-2)?

12 What is the equation of the line that cuts the y-axis at (0, 1) and is perpendicular to the line segment A(1,-4), B(-2,2)?

Quadratics

A quadratic can be sketched by transformations of $\mathbf{y = x^2}$.



Sketch - no units shown.

Sketch - label the turning point.

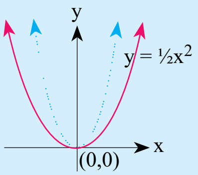
Exercise 16.7

Sketch each of the following quadratics by first sketching $y = x^2$ and then applying the appropriate transformations:

ax^2 the parabola is ∪ for **a>1.**
ax^2 the parabola is ∩ for a<1.

$y = \frac{1}{2}x^2$

The coefficient, a, is less than 1, thus the shape is wider.



1 $y = 3x^2$

2 $y = \frac{1}{4}x^2$

3 $y = 10x^2$

4 $y = 0.75x^2$

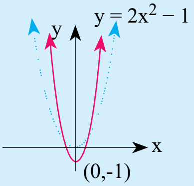
$ax^2 + d$ the parabola is shifted **d units up**.
$ax^2 - d$ the parabola is shifted **d units down**.

Use Technology to check your answers.

$y = 2x^2 - 1$

The coefficient = 2 thus the shape is narrower.

The parabola is also translated 1 unit down



5 $y = 2x^2 + 3$

6 $y = 3x^2 - 1$

7 $y = 0.5x^2 + 1$

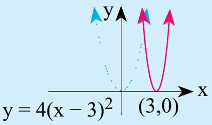
8 $y = \frac{3}{4}x^2 + 4$

$a(x + d)^2$ the parabola is shifted **d units left.**
$a(x^2 - d)^2$ the parabola is shifted **d units right**.

$y = 3(x - 3)^2$

The parabola is translated 3 units to the right.

The coefficient = 3 thus the shape is narrower.



9 $y = (x + 3)^2$

10 $y = (x - 1)^2$

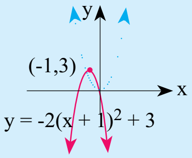
11 $y = (x + 1)^2$

12 $y = 3(x - 4)^2$

ax^2 the parabola has a ∪ shape.
$-ax^2$ the parabola has a ∩ shape.

$y = -2(x + 1)^2 + 3$

The parabola is reflected vertically, is narrower, is translated horizontally 1 unit to the left and is translated vertically 3 units.



13 $y = -2(x + 1)^2 + 3$

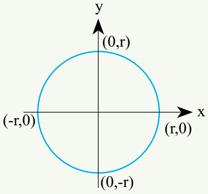
14 $y = -3(x + 2)^2 - 1$

15 $y = -0.5(x - 1)^2 + 1$

16 $y = -\frac{3}{4}(x - 2)^2 + 4$

Circles

Circle: $\mathbf{x^2 + y^2 = r^2}$
Centre (0,0) Radius = r



Exercise 16.8

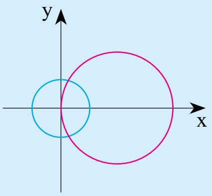
1 Find the centre and radius of each of the following circles:

$x^2 + y^2 = 9$ centre = (0,0) radius = 3	**a)** $x^2 + y^2 = 9$ **b)** $x^2 + y^2 = 16$ **c)** $x^2 + y^2 = 4$

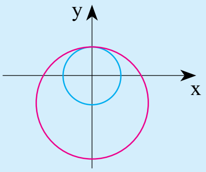
2 Sketch each of the following circles by first sketching $x^2 + y^2 = r^2$ and then applying the appropriate transformations:

Use Technology to check your answers.

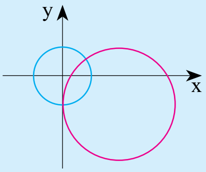
$(x - p)^2 + y^2 = r^2$ the circle is shifted **p units to the right**.

$(x - 2)^2 + y^2 = 4$ centre = (2,0) radius = 2 	**a)** $(x - 3)^2 + y^2 = 4$ **b)** $(x - 1)^2 + y^2 = 9$ **c)** $(x - 4)^2 + y^2 = 1$ **d)** $(x + 1)^2 + y^2 = 4$ **e)** $(x + 2)^2 + y^2 = 16$

$x^2 + (y - q)^2 = r^2$ the circle is shifted **q units up**.

$x^2 + (y + 1)^2 = 4$ centre = (0,-1) radius = 2 	**f)** $x^2 + (y + 1)^2 = 4$ **g)** $x^2 + (y + 3)^2 = 9$ **h)** $x^2 + (y + 2)^2 = 16$ **i)** $x^2 + (y - 5)^2 = 25$ **j)** $x^2 + (y - 2)^2 = 1$

$(x - p)^2 + (y - q)^2 = r^2$ the circle is shifted **p units right and q units up**.

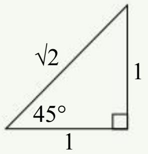
$(x - 2)^2 + (y + 1)^2 = 4$ centre = (2,-1) radius = 2 	**k)** $(x - 1)^2 + (y - 1)^2 = 1$ **l)** $(x - 3)^2 + (y - 1)^2 = 4$ **m)** $(x - 2)^2 + (y + 3)^2 = 9$ **n)** $(x + 4)^2 + (y - 2)^2 = 16$ **o)** $(x + 1)^2 + (y + 5)^2 = 25$

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 16.9

1 Spell Gradient
2 What is the gradient of the line $y = 3x - 5$?
3 What is the gradient of the line perpendicular to $y = 3x - 5$?
4 What angle does the line $y = x + 3$ make with the x-axis?
5 What is the general equation of a parabola?
6 What is the median of: 1, 2, 3, 3, 4, 5, 5?
7 Is $x - 1$ a factor of $x^3 + x^2 - 2x - 1$?
8 In the triangle, what is sin 45°?
9 Two sides of a right-angled triangle are 3 and 4, what is the hypotenuse?
10 Find 5% of \$60



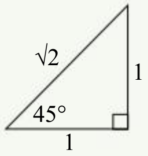
5% of \$60
= ½ of 10% of 60
= ½ of 6
= \$3

If two lines are perpendicular then the product of their gradients is -1.

$$m_1 \times m_2 = {}^{-}1$$

Exercise 16.10

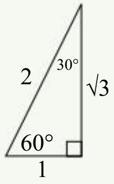
1 Spell Perpendicular
2 What is the gradient of the line $y = 2x - 5$?
3 What is the gradient of the line perpendicular to $y = 2x - 5$?
4 What angle does the line $y = {}^{-}x + 2$ make with the x-axis?
5 What is the general equation of a hyperbola?
6 What is the median of: 1, 1, 3, 3, 4, 5?
7 Is $x - 1$ a factor of $x^3 + x^2 - x - 1$?
8 In the triangle, what is cos 45°?
9 Two sides of a right-angled triangle are 1 and 2, what is the hypotenuse?
10 Find 5% of \$80



'The key of persistence opens all door closed by resistence' - John Di Lemme.

Exercise 16.11

1 Spell y-intercept
2 What is the gradient of the line $y = 5x - 2$?
3 What is the gradient of the line perpendicular to $y = -2x - 1$?
4 What angle does the line $y = x + 7$ make with the x-axis?
5 What is the general equation of a circle?
6 What is the median of: 2, 4, 5, 6, 6, 6, 7?
7 Is $x + 1$ a factor of $x^3 + x^2 - x - 1$?
8 In the triangle, what is tan 60°?
9 Two sides of a right-angled triangle are 1 and 3, what is the hypotenuse?
10 Find 5% of \$90



Astronomers observe, record, calculate, and develop theories about objects in the universe.

- Relevant school subjects are English, Mathematics and Physics.
- Courses usually involve a Universtity science degree with a major in astronomy.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

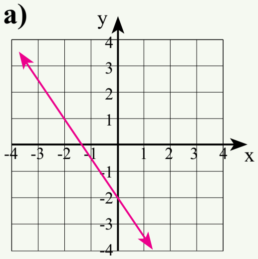
Exercise 16.12

1 Does the point (-2,3) lie on the line: $y = 2x + 1$?

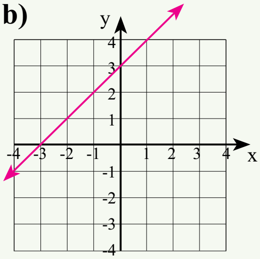
2 Does the point (-3,-1) lie on the line: $2x - 3y + 3 = 0$?

3 Which of the following represents the line: $2y + 3x + 2 = 0$?

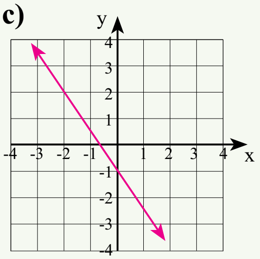
a)



b)

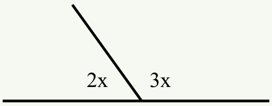


c)

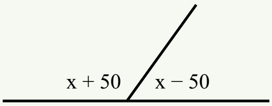


4 For each of the following, write an equation and then solve the equation:

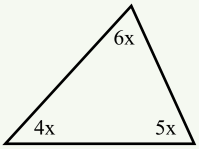
a)



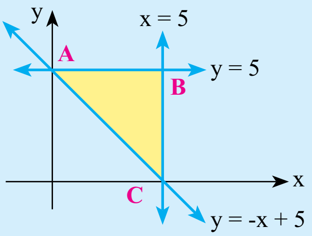
b)



c)

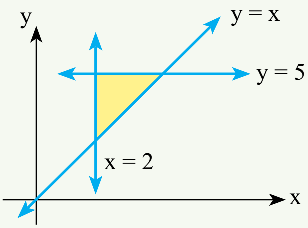


5 What is the area of each of the following triangles:

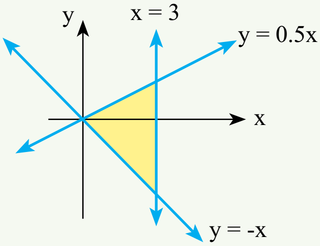


Coordinates of A,
simultaneously solve: $y = -x + 5$
$y = 5$
thus A(0,5)
Coordinates of B: B(5,5)
Coordinates of C: C(5,0)
Area ΔABC = ½base×height
= ½×5×5
= 12.5 units^2

a)



b)



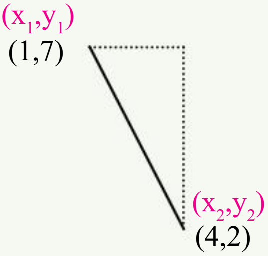
Investigations

Investigation 16.1 (x_1,y_1) and (x_2,y_2)?

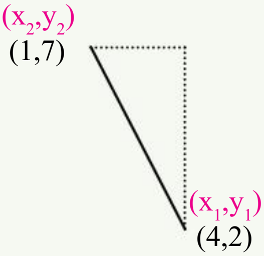
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Does it matter which point is (x_1,y_1) and which point is (x_2,y_2)?

Get the same answer anyway?



OR

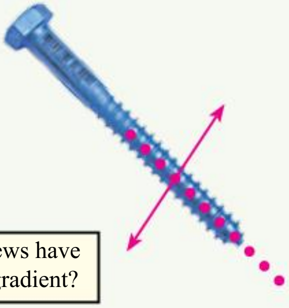


Investigation 16.2 Gradient

Investigate

The everyday use of gradient

Riding a bicycle up a 3% hill isn't too bad, a 11% hill is pure pain.



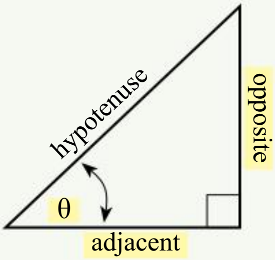
Do all screws have the same gradient?

Investigation 16.3 Gradient and Tan?

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Is gradient and tan θ the same thing?



Investigation 16.4 Ramp Gradient?

Building regulations suggest that disability access ramps should have a maximum ramp gradient of 1 : 15

What does a ramp gradient of 1 : 15 mean?

A roof is considered pitched if its pitch is greater than 3.2 in 12.

What angle is 3.2 in 12?

A Couple of Puzzles

Exercise 16.13

1 Does a cube have more faces,
or more edges,
or more vertices?

2 The square of a number plus four times the number is 60.
What is the number?

$$\boxed{x^2 + 4x = 60}$$

A Game

Bingo is played by the whole class. Beforehand, make up 25 finance problems with answers from 1 to 25.

1 Each person makes a 3x3 square and writes a number from 1 to 25 in each of the nine squares to make their bingo card.

2 Randomly select one of the 25 problems and tell the class the problem. If the answer matches a number on the bingo card then it is marked.

3 The winner calls BINGO when any three in a row, column, or diagonal is marked on their bingo card.

A BINGO card

17	**4**	**23**
6	**11**	**21**
2	**9**	**10**

Sample Problems

1 = 100% as a decimal
2 = 10% of 20
3 = $1.75 + $1.25
4 = 8 lots of 50c
5 = $8.50 - $3.50
6 = ...

A Sweet Trick

Amaze your relatives by being able to quickly multiply any two numbers each from 11 to 19 in your head.

Practice the following method:

Example 1: 18 × 13

$(18 + 3) \times 10 = 210$
$8 \times 3 = 24$
234 is the answer

Example 2: 17 × 15

$(17 + 5) \times 10 = 220$
$7 \times 5 = 35$
255 is the answer

With a little practice you will be able to do these fairly quickly in your head.

Technology

Technology 16.1 Calculating gradient

1 Calculate $\frac{(2-5)}{(4-1)}$ (2 − 5) ÷ (4 − 1) = to give $^-1$

2 Simplify $m = \frac{^-7 - ^-3}{3 - ^-2}$

The brackets are important in separating the numerator from the denominator.

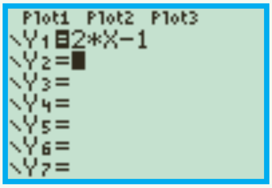
(+/- 7 − +/- 3) ÷ (3 − +/- 2) = to give $^-0.8$

Rounding to two decimal places, first look at the third decimal place:

56.231694	27.01769	1.07276	4.79634216
less than 5 thus **56.23**	5 or more thus **27.02**	less than 5 thus **1.07**	5 or more thus **4.80**

Technology 16.2 Graphics Calculators

A graphics calculator will graph linear equations such as: y = 2x − 1



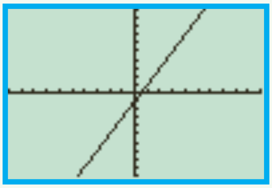
```
Plot1  Plot2  Plot3
\Y1■2*X−1
\Y2=■
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

1 Press Y=

2 Enter: 2 X,T,θ,n − 1

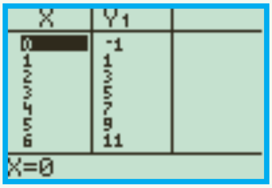
3 Press Graph

4 Change the scale by pressing Window



5 Find the intercepts by Table

or Calc then Zero or Intersect



X	Y1	
0	-1	
1	1	
2	3	
3	5	
4	7	
5	9	
6	11	

X=0

Technology 16.3 Applets

There are a very large number of interactive linear equation applets on the Internet.

Linear Equation Applets

Experiment with these applets.

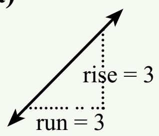
'The person ignorant of mathematics will be increasingly limited in their grasp of the main forces of civilization' - John Kemeny.

Chapter Review 1

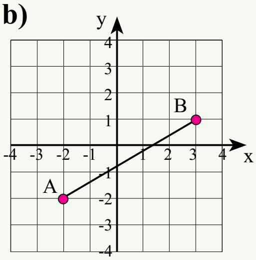
Exercise 16.14

1 Find the gradient of each of the following line segments:

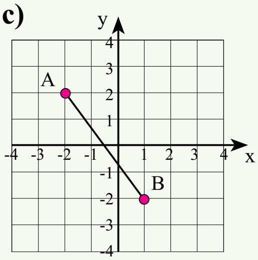
a)



b)



c)



2 Find the gradient of each of the following line segments and decide whether the line segment is parallel to the x-axis or the y-axis:

a) A(1,-2), B(1,1)

b) A(-2,3), B(1,3)

c) A(-2,-1), B(-5,-1)

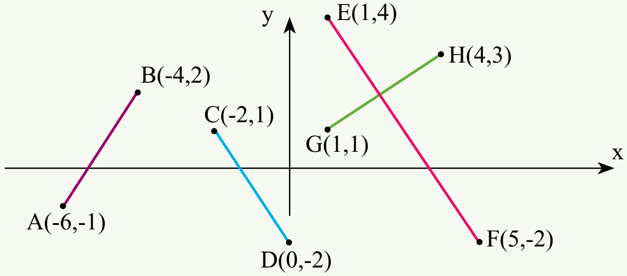
Gradient of lines parallel to the x-axis:

$$\mathbf{m = 0}$$

Gradient of lines parallel to the y-axis:

$$\mathbf{m = \infty}$$

3 Find the gradient of each of the following lines (not to scale) and thus show which lines are parallel or perpendicular:



4 Show that the following pairs of lines are parallel:

a) $y = 2x + 1$ and $y = 2x - 2$

b) $y = 3x - 3$ and $2y - 6x + 5 = 0$

If two lines are parallel then they have the same gradient.

$$\mathbf{m1 = m2}$$

5 Show that the following pairs of lines are perpendicular:

a) $y = {}^{-}2x + 3$ and $y = 0.5x + 5$

b) $y = {}^{-}x - 3$ and $3y - 3x + 4 = 0$

c) $4y + 3x - 2 = 0$ and $3y - 4x + 1 = 0$

If two lines are perpendicular then the product of their gradients is -1.

$$\mathbf{m_1 \times m_2 = {}^{-}1}$$

6 If the gradient of the line segment A(-2,5), B(x,-5) is 0.5, what is the value of x?

7 What is the value of b if A(1,-3), B(-2,0), and C(5, b) are collinear?

8 What is the equation of the line that cuts the y-axis at (0, 4) and is parallel to the line segment A(-2,5), B(1,-1)?

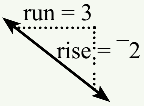
9 Sketch $y = 2(x - 1)^2$ by first sketching $y = x^2$ and then applying the appropriate transformations.

Chapter Review 2

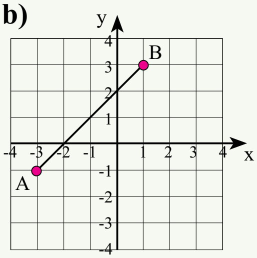
Exercise 16.15

1 Find the gradient of each of the following line segments:

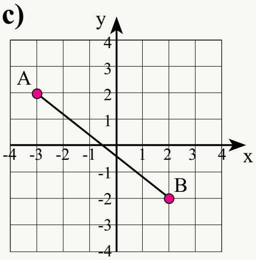
a)



b)



c)



2 Find the gradient of each of the following line segments and decide whether the line segment is parallel to the x-axis or the y-axis:

a) A(3,-2), B(1,-2)

b) A(1,4), B(1,-1)

c) A(-3,2), B(-3,5)

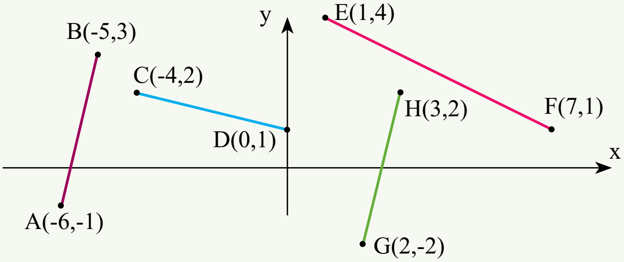
Gradient of lines parallel to the x-axis:

$$m = 0$$

Gradient of lines parallel to the y-axis:

$$m = \infty$$

3 Find the gradient of each of the following lines (not to scale) and thus show which lines are parallel or perpendicular:



4 Show that the following pairs of lines are parallel:

a) $y = -x + 3$ and $y = -x - 2$

b) $y = 2x - 3$ and $3y - 6x + 1 = 0$

If two lines are parallel then they have the same gradient.

$$m1 = m2$$

5 Show that the following pairs of lines are perpendicular:

a) $y = {}^{-}4x + 1$ and $y = 0.25x + 3$

b) $y = x - 3$ and $5y + 5x + 4 = 0$

c) $4y + x - 2 = 0$ and $2y - 8x + 7 = 0$

If two lines are perpendicular then the product of their gradients is -1.

$$m_1 \times m_2 = {}^{-}1$$

6 If the gradient of the line segment A(-1,4), B(x,-3) is 2, what is the value of x?

7 What is the value of b if A(-1,-2), B(5,-1), and C(3, b) are collinear?

8 What is the equation of the line that cuts the y-axis at (0, 3) and is parallel to the line segment A(-3,2), B(1,-4)?

9 Sketch $y = 3x^2 + 1$ by first sketching $y = x^2$ and then applying the appropriate transformations.

Geometric Reasoning

17

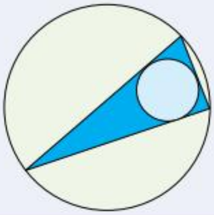
Measurement and Geometry ➡ Geometric Reasoning

★ Formulate proofs involving congruent triangles and angle properties.
  - apply an understanding of relationships to deduce properties of geometric figures (for example the base angles of an isosceles triangle are equal).

★ Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes.
  - distinguish between a practical demonstration and a proof (for example demonstrating triangles are congruent by placing them on top of each other, as compared to using congruence tests to establish that triangles are congruent).
  - perform a sequence of steps to determine an unknown angle giving a justification in moving from one step to the next.
  - communicate a proof using a sequence of logically connected statements.

★ 10A Prove and apply angle and chord properties of circles.
  - perform a sequence of steps to determine an unknown angle or length in a diagram involving a circle, or circles, giving a justification in moving from one step to the next.
  - communicating a proof using a logical sequence of statements.
  - proving results involving chords of circles.

How do you draw a circle through the three vertices of a triangle?



How do you draw a circle to fit inside a triangle?

A TASK

Find the centre of a triangle:

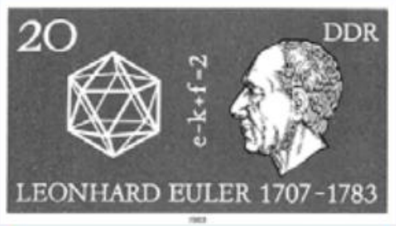
- Brainstorm different ways of finding the centre of a triangle.
- Use the Internet to research different triangle centres.
- Make large models of various centres of the same triangle.
- Post the models in your classroom.
- Brainstorm practical uses of the various triangle centres.

A LITTLE BIT OF HISTORY

Leonard Euler (1707-1783) made a massive contribution to mathematics.

For example, Euler laid the foundation of analytical mechanics, introduced the notations:

- f(x) for a function,
- e for the base of natural logs,
- i for the square root of -1,
- π for pi,
- ∑ for summation.

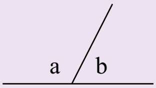
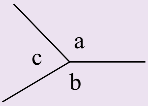
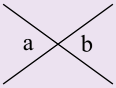


Euler's Formula:
For a simple polyhedron: **F − E + V = 2**
F is the number of faces, E is the number of edges, V is the number of vertices.

Lines

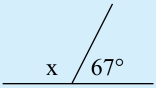
Axioms
An axiom is a statement that is simply accepted as being true.

Deductive Reasoning
Deductive reasoning involves using given true premises to reach a conclusion that is also true.

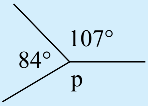
Figure	Axioms	
	The sum of the angles on a straight line is 180°	$a + b = 180°$
	The sum of the angles at a point is 360°	$a + b + c = 360°$
	Vertically opposite angles are equal	$a = b$

Exercise 17.1

Find the value of the unknowns. Show all working:

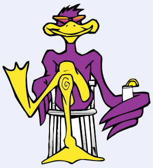


$$x + 67 = 180$$
$$x = 180 - 67$$
$$\underline{x = 113°}$$



$$p + 84 + 107 = 360$$
$$p + 191 = 360$$
$$p = 360 - 191$$
$$\underline{p = 169°}$$

Start with an axiom and use deductive reasoning to reach a conclusion.



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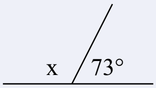

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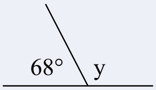

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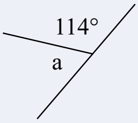

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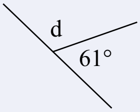

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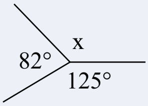

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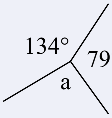

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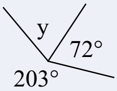

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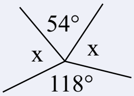

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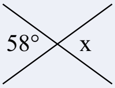

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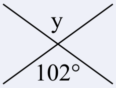

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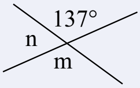

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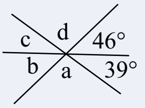
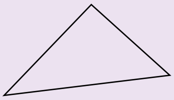
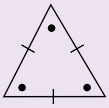
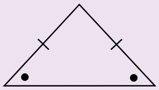
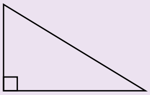
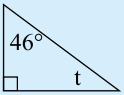


Triangles

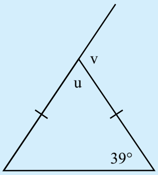
Triangle	Axioms	
	Scalene triangle	No sides equal Sum angles = 180°
	Equilateral triangle	Each side equal Each angle = 60°
	Isosceles triangle	Two sides equal Two angles equal
	Right-angled triangle	one angle is 90°

Exercise 17.2

Find the value of the unknowns. Show all working:



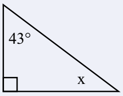
$$\begin{aligned} t + 46 + 90 &= 180 \\ t + 136 &= 180 \\ t &= 180 - 136 \\ \underline{t} &\underline{= 44°} \end{aligned}$$



$$\begin{aligned} u + 39 + 39 &= 180 \\ u + 78 &= 180 \\ u &= 180 - 78 \\ \underline{u} &\underline{= 102°} \end{aligned}$$

$$\begin{aligned} v + 102 &= 180 \\ v &= 180 - 102 \\ \underline{v} &\underline{= 78°} \end{aligned}$$

1



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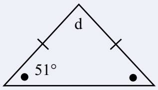

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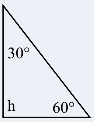

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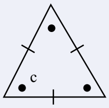

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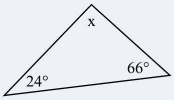

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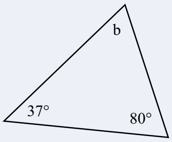

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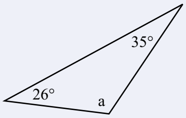

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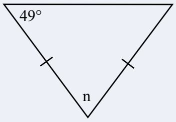

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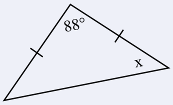

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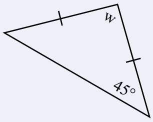

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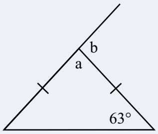

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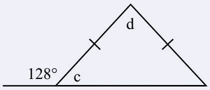
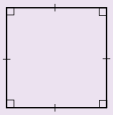
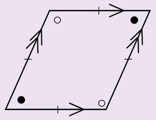
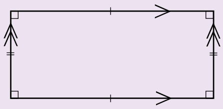
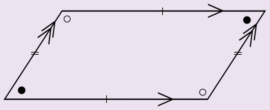
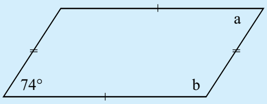


Quadrilaterals

Quadrilateral	Name	Axioms
	Square	Each side equal Each angle = 90°
	Rhombus	Each side equal Opposite angles equal Opposite sides parallel
	Rectangle	Opposite sides equal Each angle = 90° Opposite sides parallel
	Parallelogram	Opposite angles equal Opposite sides equal Opposite sides parallel

Exercise 17.3

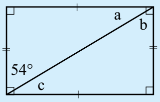
Find the value of the unknowns. Show all working:



$$
\begin{aligned}
a &= 74° && \{\text{opposite angles}\}\\
2b + 2\times74 &= 360 && \{360° \text{ in quadrilateral}\}\\
2b + 148 &= 360\\
2b &= 360 - 148\\
2b &= 212\\
b &= 212\div2\\
\underline{b} &\underline{= 106°}
\end{aligned}
$$

Check: $2a + 2b = 360°$

$2\times74 + 2\times106 = 148 + 212 = 360°$ ✓

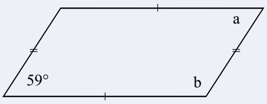


$$
\begin{aligned}
c + 54 &= 90\\
c &= 90 - 54\\
\underline{c} &\underline{= 36°}
\end{aligned}
$$

$$
\begin{aligned}
a + 54 + 90 &= 180 && \{180° \text{ in } \Delta\ \}\\
a + 144 &= 180\\
a &= 180 - 144\\
\underline{a} &\underline{= 36°}
\end{aligned}
$$

$$
\begin{aligned}
a + b &= 90\\
36 + b &= 90\\
b &= 90 - 36\\
\underline{b} &\underline{= 54°}
\end{aligned}
$$

1



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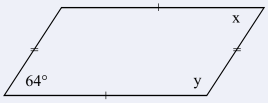

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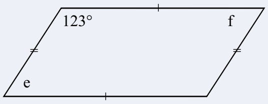

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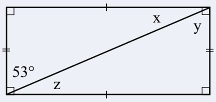

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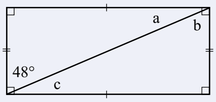

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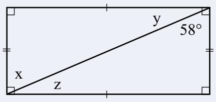
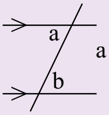
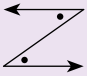
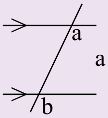
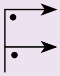
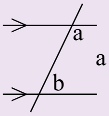
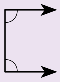
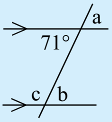


Parallel Lines

Angles in parallel lines	Name	Axioms
 a = b	Alternate angles 	Alternate angles are equal
 a = b	Corresponding angles 	Corresponding angles are equal
 a + b = 180°	Cointerior angles 	Cointerior angles sum to 180°

Exercise 17.4

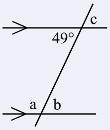
Find the value of the unknowns. Show all working:



b = 71° {Z ie alternate}
a = b {F ie corresponding}
a = 71°
c + 71 = 180 {U ie cointerior}
c = 180 − 71
c = 109°

There is often more than one way to solve these problems eg.,
a = 71 {vertically opposite}
b + c = 180° {angles straight line}

1



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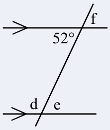

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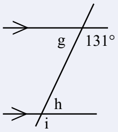

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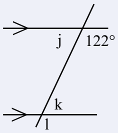

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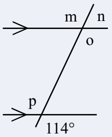

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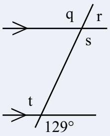

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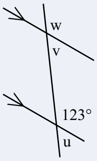

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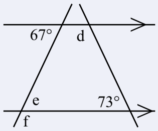

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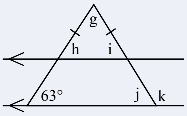

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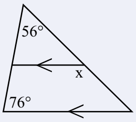

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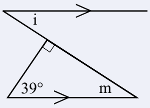

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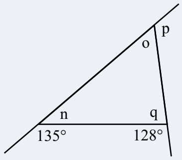
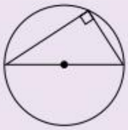
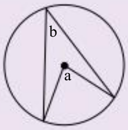
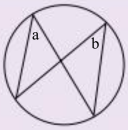
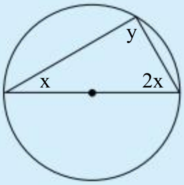


Circles

Angles in a circle	Description	Axioms
	Angles in a semicircle	The angle in a semicircle is 90°
	Central angles and angles on the circle	The angle at the centre is twice the angle on the circle $a = 2b$
	Angles on the same arc	Angles on a circle subtended by the same arc are equal $a = b$

Exercise 17.5

Find the value of the unknowns. Show all working:

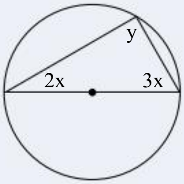


$\underline{y} = \underline{90}$ {semicircle angle}

$$\begin{aligned} x + 2x + 90 &= 180 \quad \{180° \text{ in a triangle}\} \\ 3x + 90 &= 180 \\ 3x &= 180-90 \\ 3x &= 90 \\ x &= 90 \div 3 \\ \underline{x} &= \underline{30°} \end{aligned}$$

There is an isosceles triangle here. Can you find it?

1



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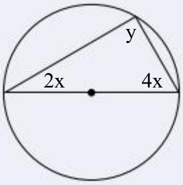

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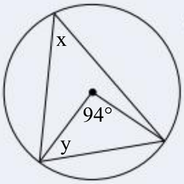

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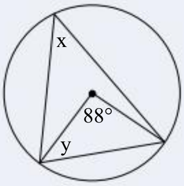

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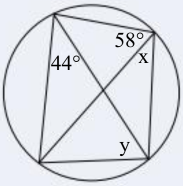

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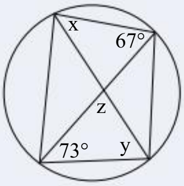

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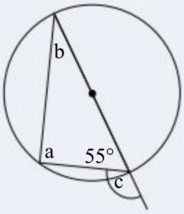

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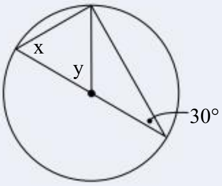

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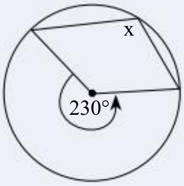
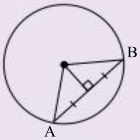
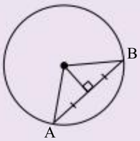
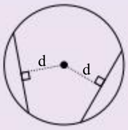
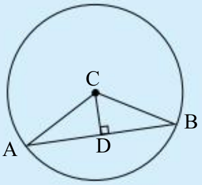


Circles

Chords in a circle	Description	Axioms
	Chord bisector (Chord AB)	The perpendicular from the centre to a chord bisects the chord
	Chord perpendicular	The line from the centre to the midpoint of a chord is perpendicular to the chord
	Equal chords	Congruent chords of a circle are the same distance from the centre and subtend equal angles at the centre

Exercise 17.6

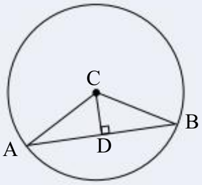
Find AB given that CD ⊥ AB, radius = 35 cm, and CD = 15 cm



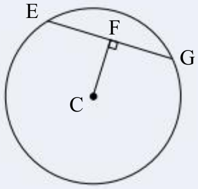
$$
\begin{aligned}
CD^2 + DB^2 &= CB^2 \quad \{\text{Pythagoras}\}\\
15^2 + DB^2 &= 35^2\\
DB^2 &= 35^2 - 15^2\\
DB &= \sqrt{1000}\\
DB &= 31.62\\
\underline{AB} &\underline{= 63.24 \text{ cm}} \quad \{\text{chord bisector}\}
\end{aligned}
$$

'I never knew what the word round meant until I saw Earth from space' - Aleksei Leonov.

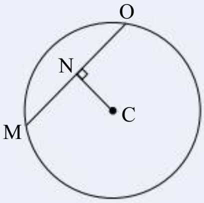
1 Find AD given that CD ⊥ AB, radius= 59 cm, AB = 45 cm



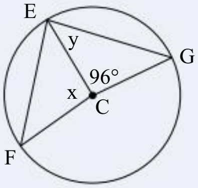
2 Find EG given that CF ⊥ EG, radius = 11.4 m, FG = 7.3 m



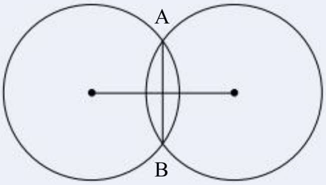
3 Find MN given that CN ⊥ OM, radius = 30 cm, CN = 20 cm.



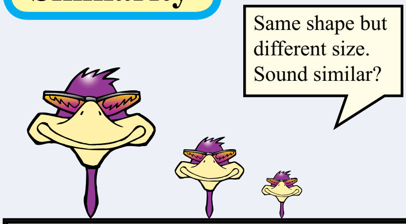
4 Find x and y given that EF = EG,



5 Prove that when two circles intersect, that the line line joining their centres bisects the common chord, AB, at right angles.



Similarity

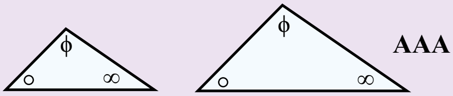


Similar triangles have **exactly the same shape** but not necessarily the same size.

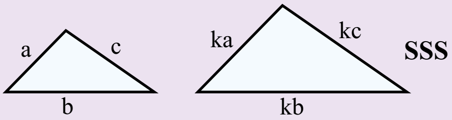
- The corresponding **angles are equal**.
- The corresponding sides have the same scale factor.
- The symbol for similarity is ~ .

Two triangles are similar if:

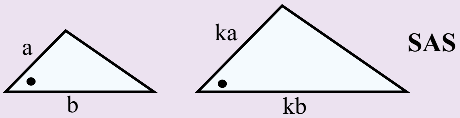
▶ The three matching angles are equal.



▶ The three matching sides are in the same ratio

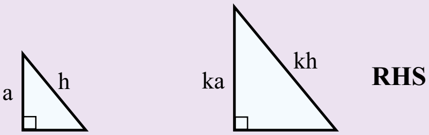


▶ Two matching sides are in the same ratio and the *included* angles are equal



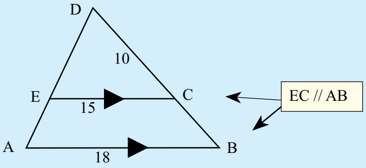
Two right-angled triangles are similar if:

▶ The hypotenuse and a matching side are in the same ratio



Exercise 17.7

Prove that △ABD ~ △ECD and find BD



∠D = ∠D {common angle}

∠DAB = ∠DEC {corresponding angles}

∠DCE = ∠DBA {corresponding angles}

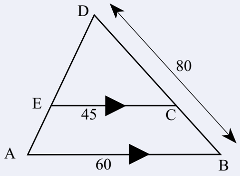
∴ △ABD ~ △ECD {**AAA**}

$\frac{BD}{CD} = \frac{AB}{EC}$ {same scale factor}

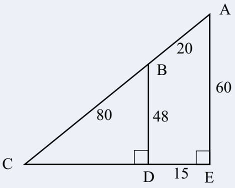
$\frac{BD}{10} = \frac{18}{15}$

$BD = \frac{18}{15} \times 10$ ∴ BD = 12

1 Prove that △ABD ~ △ECD and find DC

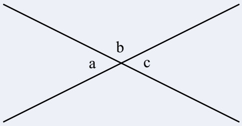


2 Prove that △ACE ~ △BCD and find CD



Proofs

A theorem is a statement that can be proved using deductive reasoning.



$a + b = 180°$ {180° on a straight line}
$c + b = 180°$ {180° on a straight line}
$\therefore a = c$

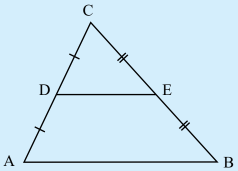
Deductive reasoning

Theorem → Vertically opposite angles are equal.

Exercise 17.8

Prove each of the following theorems:

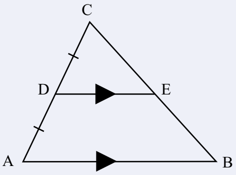
The line joining the midpoints of two sides of a triangle is half the length of the third side.



$\angle C = \angle C$ {common angle}
$\frac{CD}{CA} = \frac{1}{2}$ {midpoint}
$\frac{CE}{CB} = \frac{1}{2}$ {midpoint}
$\therefore \underline{\triangle CDE \sim \triangle CAB}$ {**SAS**}

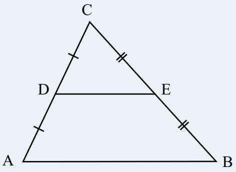
$\therefore \frac{DE}{AB} = \frac{1}{2}$ {same scale factor}
$\underline{DE = 0.5AB}$

1 A line from the midpoint of a side of a triangle and parallel to another side, bisects the third side.



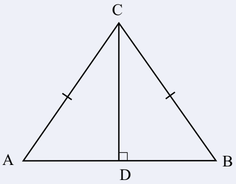
AD = DC
DE // AB
Show that CE = EB
or that $\frac{CE}{CB} = \frac{1}{2}$

2 The line joining the midpoints of two sides of a triangle is parallel to the third side.



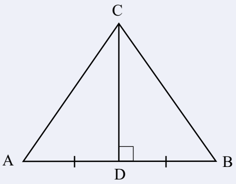
CD = DA
CE = EB
Show that DE // AB
or that ∠CDE = ∠A

3 The line from the vertex of an isosceles perpendicular to the base bisects the base.



CA = CB
∠CDA = ∠CDB
Show that AD = DB

4 If the line from one angle in a triangle is a perpendicular bisector of the opposite side, then the triangle is an isosceles triangle.



AD = BD
∠ADC = ∠BDC
Show that AC = BC

Mental Computation

Most everyday problems are solved mentally by adults.

Exercise 17.9

1 Spell Equilateral

2 Name two properties of an equilateral triangle.

3 Name two properties of a square

4 If the angle sum of a polygon = (n−2)×180°, what is the angle sum of a quadrilateral?

$(n-2)\times180$
$= (4-2)\times180$
$= 360$
Angle sum of a quadrilateral is 360°

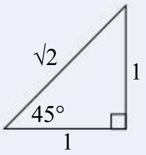
5 What is the gradient of the line $y = 2x + 1$?

6 What is the gradient of the line perpendicular to $y = 2x - 3$?

7 What angle does the line $y = x + 2$ make with the x-axis?

8 Is $x - 1$ a factor of $x^3 - x^2 + x - 1$?

9 In the triangle, what is cos 45°?



10 Two sides of a right-angled triangle are 1 and 1, what is the hypotenuse?

If two lines are perpendicular then the product of their gradients is -1.

$$m_1 \times m_2 = {}^-1$$

It's zero degrees now and it is predicted to be twice as cold later. How cold will it be?

Exercise 17.10

1 Spell Isosceles

2 Name two properties of an isosceles triangle.

3 Name two properties of a rectangle

4 If the angle sum of a polygon = (n−2)×180°, what is the angle sum of a pentagon?

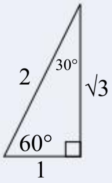
5 What is the gradient of the line $y = 3x + 2$?

6 What is the gradient of the line parallel to $y = x - 1$?

7 What angle does the line $y = {}^-x + 2$ make with the x-axis?

8 Is $x - 2$ a factor of $x^2 - x - 1$?

9 In the triangle, what is sin 60°?



10 Two sides of a right-angled triangle are 1 and 2, what is the hypotenuse?

Exercise 17.11

What did the little acorn say when it grew up? Geometry.

1 Spell Parallelogram

2 Name two properties of a rhombus.

3 Name two properties of a parallelogram

4 If the angle sum of a polygon = (n−2)×180°, what is the angle sum of a hexagon?

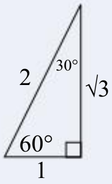
5 What is the gradient of the line $y = 5x + 3$?

6 What is the gradient of the line perpendicular to $y = 5x - 1$?

7 What angle does the line $y = 5$ make with the x-axis?

8 Is $x - 3$ a factor of $x^2 - 2x - 3$?

9 In the triangle, what is cos 30°?



10 Two sides of a right-angled triangle are 1 and 3, what is the hypotenuse?

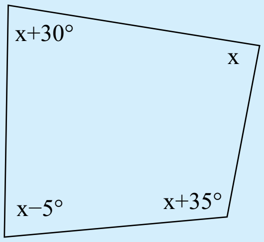
Metallurgists control and develop methods of extracting minerals.
- Relevant school subjects are English, Mathematics, Chemistry, Physics.
- Courses usually involve an engineering degree.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

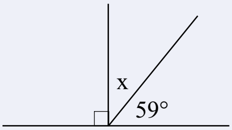
Exercise 17.12

Find the value of x in each of the following:



$$\begin{aligned} \text{Angle sum} &= 360° \\ x + 30 + x + x - 5 + x + 35 &= 360 \\ 4x + 60 &= 360 \\ 4x &= 360 - 60 \\ 4x &= 300 \\ x &= 300 \div 4 \\ \underline{x} &\underline{= 75°} \end{aligned}$$

1



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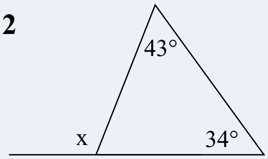
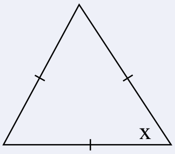


3



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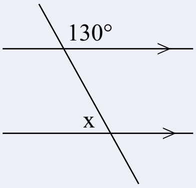

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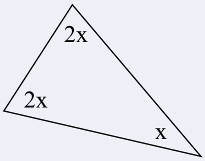

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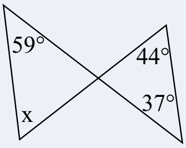

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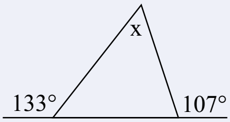

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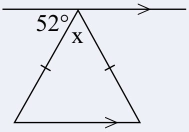

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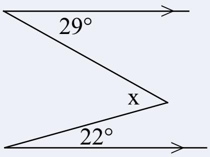

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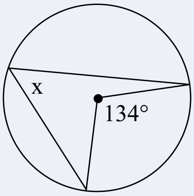

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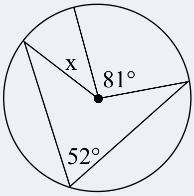

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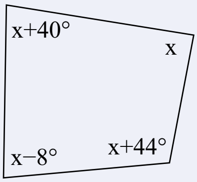

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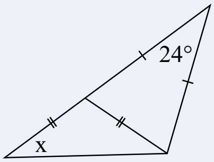

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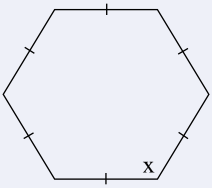

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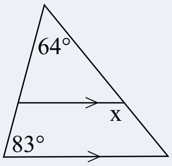
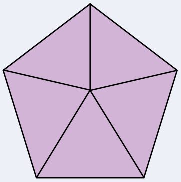


A Couple of Puzzles

Exercise 17.13

1 Fran earns \$650 per week and Fred earns \$x less per week. Together they earn \$1250. Find x.

2 An isosceles triangle has angles of a°, a°, and 30°. Find a.



3 A regular pentagon can be constructed with five isosceles triangles. What is the size of each angle of the isosceles triangle?

4 Find the numbers that correspond to each letter in the Alphanumerics:

a)
$$\begin{array}{r} \text{WE} \\ \text{ATE} \\ +\ \text{WE} \\ \hline \text{GREW} \end{array}$$

b)
$$\begin{array}{r} \text{HOCUS} \\ +\ \text{POCUS} \\ \hline \text{PRESTO} \end{array}$$

c)
$$\begin{array}{r} \text{LIES} \\ \text{LIES} \\ +\ \text{ARE} \\ \hline \text{SILLY} \end{array}$$

d)
$$\begin{array}{r} \text{HOME} \\ \text{HOME} \\ +\ \text{TO} \\ \hline \text{MOMMA} \end{array}$$

A Game

Fours is a calculator game in which the first person to have a calculator display a selected number is the winner.

1 Randomly select a number from 1 to 100.

Use the random button on your calculator?
100 × Rand =

2 Use only the 4, +, −, x, = keys and the y^x or ^ key to produce the selected number on the display of your calculator.

A Sweet Trick

1	Think of any number from 1 to 100.	92
2	Write down the name of the number.	Ninety-two
3	Count the number of letters in the number.	9
4	Write the name of the number.	nine
5	Count the number of letters in the number.	4
6	Write the name of the number.	four
7	Count the number of letters in the number.	4
8	Continue until a number repeats.	four, 4.

What is the number?

The number is always 4.

Investigations

Investigation 17.1 Isosceles Triangles

A practical demonstration.
Not a proof.

1 Draw the following triangles:
  - **a)** AB=10 cm, AC=10 cm, BC=5 cm
  - **b)** AB=10 cm, AC=10 cm, BC=10 cm
  - **c)** AB=10 cm, AC=10 cm, BC=15 cm

2 Measure the angles in each triangle and complete a table similar to the following:

3 What relationships do you notice?

	∠*A*	∠*B*	∠*C*
a)			
b)			
c)			

Investigation 17.2 Angles in a semicircle

A practical demonstration.
Not a proof.

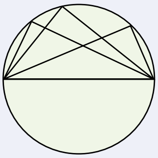
1 Draw a circle of radius 10 cm.

2 Draw a diameter.

3 Draw a number of angles in the semicircle. similar to the example shown.

4 Measure the angles on the semicircle. What do you notice?

5 Use scissors to cut out an interesting arrangement and post in your classroom.



Investigation 17.3 Central angles and angles on the circle

1 Draw a circle of radius 10 cm.

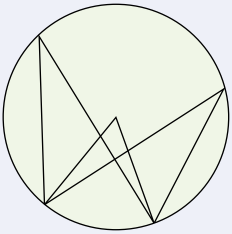
2 From an arc draw the central angle (to the centre of the circle).

A practical demonstration.
Not a proof.

3 From the same arc draw a number of angles on the circle similar to the example shown.

4 Measure the central angle and the angles on the circle. What do you notice?

5 Use scissors to cut out an interesting arrangement and post in your classroom.



Investigation 17.4

1 Construct a triangle ABC in which AB is 10 cm long.

2 Mark a point D on AB such that AD = 5 cm.

A practical demonstration.
Not a proof.

3 Draw DE parallel to BC, E being a point on AC.

4 Measure the lengths of AE and EC.
  - **a)** What is the ratio of AD to DB?
  - **b)** What is the ratio of AE to EC?

5 Repeat with the following:

AB	12	12	12	16
AD	6	4	9	12

Technology

Technology 17.1

Start computer programming. The LOGO computer language is a great starting point.

```
Forward 10
Right 90
Forward 10
Right 90
Forward 10
Right 90
Forward 10
```

1. Find a LOGO site on the Internet (Microworlds, LOGO, turtle).
2. Program the turtle to draw a square
3. Program the turtle to draw a regular hexagon (eg. Forward 10, Right 120, etc).
4. Try to produce some of the shapes described earlier in this chapter.

Technology 17.2

Produce a Powerpoint slide show of your LOGO programming

1. Use LOGO to produce a shape.
2. Press the Print Screen key on your keyboard.
3. Open Powerpoint, and paste the shape on a slide.

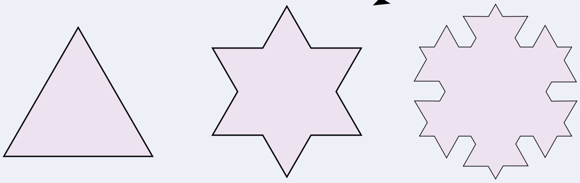
Fractals are found in nature. They have applications in soil mechanics, seismology, medicine and artwork.

Technology 17.3 Fractals

A fractal is a geometric shape that can be split into parts. Each part being similar to the original shape.

a) Draw the first four iterations of the Koch snowflake'

Start with an equilateral triangle
Repeat three times:
Add triangles a third the size to each side.



Each iteration produces smaller similar shapes.

b) Use Internet software to draw iterations of the Koch Snowflake.

c) Use search phrases such as 'Koch Snowflake' with 'applet', 'interactive' etc.

Technology 17.4 Fabulous Fern Fractals

Fern Fractals

Watch a Barnsley Fern fractal video.

Fern Fractals

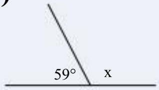
Form fabulous fern fractals.

Chapter Review 1

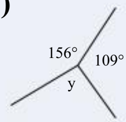
Exercise 17.14

1 Find the value of the unknowns. Show all working:

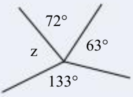
a)



b)



c)



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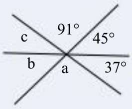

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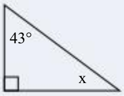

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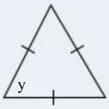
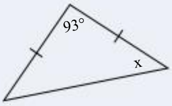
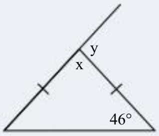


g)

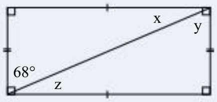


h)



i)

j)



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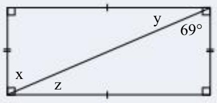

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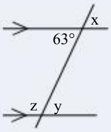

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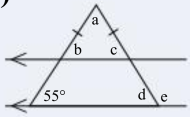

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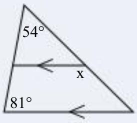
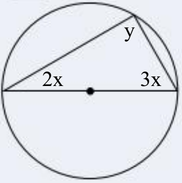
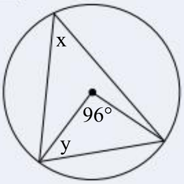


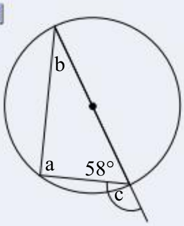
o) 10A



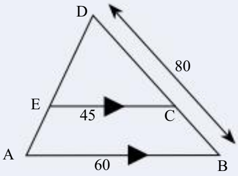
p) 10A



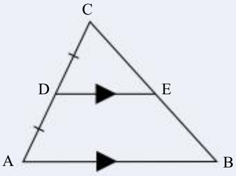
q) 10A



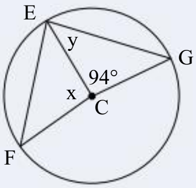
2 Prove that △ABD ~ △ECD and find DC



3 Prove that the line from the midpoint of a side of a triangle and parallel to another side, bisects the third side.



4 10A Find x and y given that EF = EG,



Deductive Reasoning

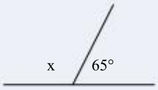
Deductive reasoning involves using given true premises to reach a conclusion that is also true.

Chapter Review 2

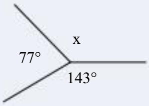
Exercise 17.15

1 Find the value of the unknowns. Show all working:

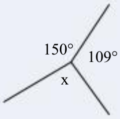
a)



b)



c)



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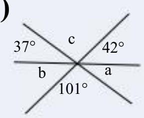

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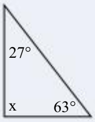

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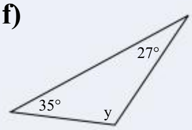
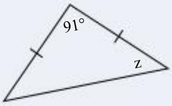


g)



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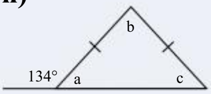

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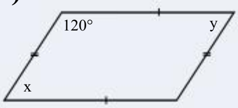

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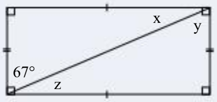

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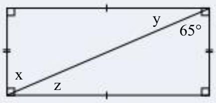

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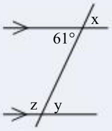

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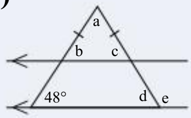

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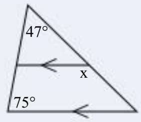
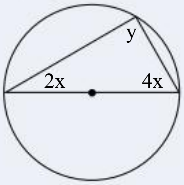
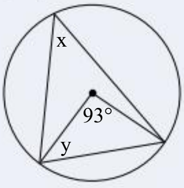


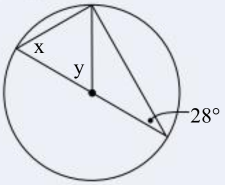
o) 10A



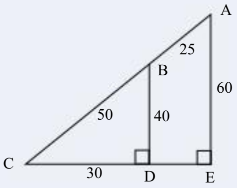
p) 10A



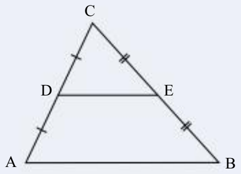
q) 10A



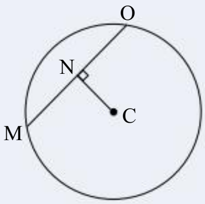
2 Prove that △ACE ~ △BCD and find DE



3 Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side.



4 10A Find MO given that CN ⊥ OM, radius = 33 cm, CN = 21 cm.



A theorem is a statement that can be proved using deductive reasoning.

Statistics 2

18

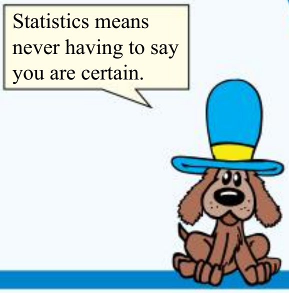
Statistics & Probability ➞ Data Representation & Interpretation

★ Use scatter plots to investigate and comment on relationships between two numerical variables.
  - use authentic data to construct scatter plots, make comparisons and draw conclusions.

★ Investigate and describe bivariate numerical data where the independent variable is time.
  - investigate biodiversity changes in Australia since European occupation.
  - construct and interpret data displays representing bivariate data over time.

★ Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data.
  - investigate the use of statistics in reports regarding the growth of Australia's trade with other countries of the Asia region.
  - evaluate statistical reports comparing the life expectancy of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole.

★ 10A Use information technologies to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation.
  - investigate different techniques for finding a 'line of best fit'.



A TASK

The longer a person's legs, the faster they can run?

- Collect data to answer the research question.
- Use a scatterplot to display the data.
- Analyse the data.
- Match the analysis with the research question.
- Publish your findings.

A LITTLE BIT OF HISTORY

1986 Space Shuttle Challenger disintegrated after the failure of o-ring seals. It is claimed that improper analysis of o-ring temperature failure led to the disaster. It has been suggested that proper analysis of a scatterplot and a line of best fit may have avoided the disaster.

Space Shuttle Challenger 1986

Bivariate Data

Bivariate data is represented by two variables.

What is the relationship between the two variables?

Are the values of one variable related to the values of another variable?

Can the values of one variable be used to predict the values of another variable?

Value of Australian exports to China	
Year	$Abillion
2006	20
2007	25
2008	30
2009	45
2010	60
2011	75

This is an example of bivariate data.
The two variables are **Year** and **$Abillion**.

Is there a relationship between Year and the value of exports to China?

Can a prediction be made about the value of exports to China in 2012?

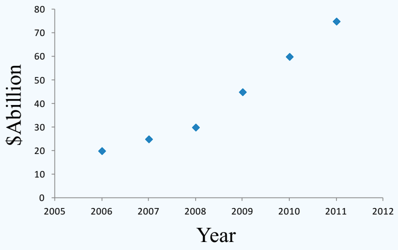
Scatterplots

Scatterplots give a visual representation of how the values of one variable are related to the values of another variable.

Value of Australian exports to China	
Year	$Abillion
2006	20
2007	25
2008	30
2009	45
2010	60
2011	75

See Technology for drawing scatterplots.

Value of Australian exports to China

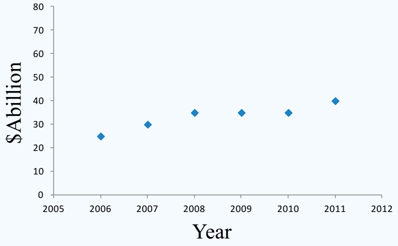


The dependent variable, Year, is placed on the x-axis.

$Abillion depends on the Year, thus $Abillion is the dependent variable. Year is the independent variable.

Value of Australian imports from China	
Year	$Abillion
2006	25
2007	30
2008	35
2009	35
2010	35
2011	40

Value of Australian imports from China



Exercise 18.1

1 For each of the following tables of bivariate data:

- **a)** indicate which variable is dependent and which variable is independent.
- **b)** draw a scatterplot.
- **c)** attempt to predict the value of exports or imports in 2012.
- **d)** What confidence might you have in your prediction?

i)

Value of Australian exports to Indonesia	
Year	$Abillion
2006	4.5
2007	4
2008	4.2
2009	4
2010	4.5
2011	5

ii)

Value of Australian imports from Indonesia	
Year	$Abillion
2006	4.5
2007	5
2008	5.2
2009	4.5
2010	5.3
2011	5.8

2 **a)** Indicate which variable is dependent and which variable is independent.

b) Draw a scatterplot.

c) Would you advise that the number of ads per day be increased?

Number of TV adverts per day and daily sales	
$Sales	Ads
150 000	2
200 000	3
220 000	6
240 000	5
170 000	1
220 000	8
180 000	5
200 000	4
250 000	6
200 000	3
200 000	2
220 000	5
200 000	7

3 **a)** Indicate which variable is dependent and which variable is independent.

b) Draw a scatterplot.

c) Can you give advice about the most economical speed?

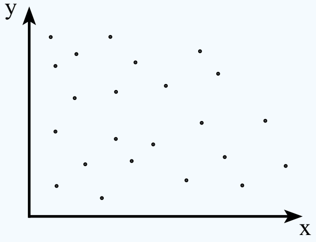
Speed and diesel use, litres per 100km, of a large truck	
L/100km	km/h
25	70
20	70
35	110
25	90
15	50
25	80
30	90
35	105
20	64
30	100
20	50
20	60

Correlation

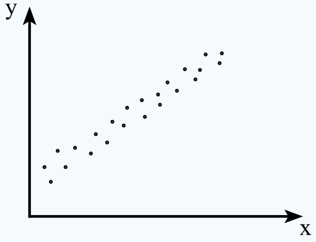
A correlation indicates a relationship between two variables.

Types of relationships

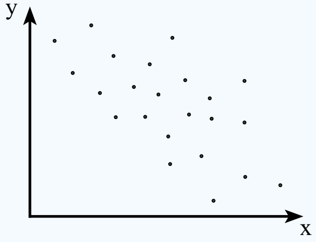
- linear or non-linear
- positive or negative
- strong or weak



The above scatterplot indicates no relationship between the two variables.



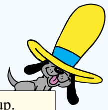
The above scatterplot indicates a positive linear relationship with a strong correlation between the two variables.



The above scatterplot indicates a negative possibly linear relationship with a weak correlation between the two variables.

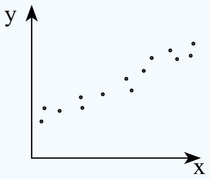
Exercise 18.2

Describe the relationship between the two variables as suggested by each of the following scatterplots:



Positive slopes up.
Negative slopes down.

1



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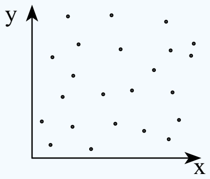

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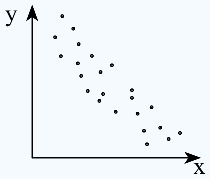

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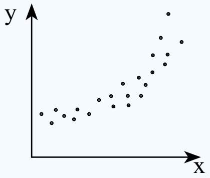

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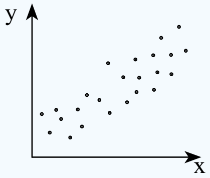

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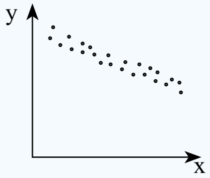

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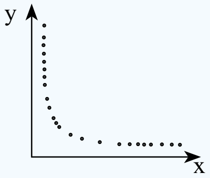

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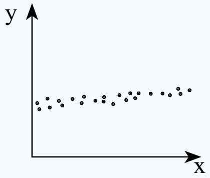

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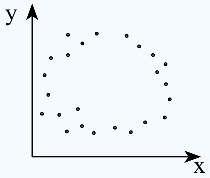


Exercise 18.3

1 Horse feeding, worming, and medication requires a reasonable estimate of the horse's weight. The following table shows bivariate data for 20 horses.

- **a)** Draw a scatterplot of the data.
- **b)** Describe the relationship as suggested by the scatterplot.
- **c)** If a worm paste dose is 5g per 100 kg weight, what dose needs to be given to a horse with a girth of 161 cm?
- **d)** How confident are you that you are giving the horse a reasonable amount of worm paste?

Measurement of girth (cm) and weight (kg)

Girth	Weight	Girth	Weight
173	430	144	250
165	380	156	310
138	225	193	560
147	260	179	460
133	190	164	360
176	440	185	520
163	360	155	300
191	550	158	340
174	430	198	610
168	390	177	450

2 The following table shows bivariate data of shoe size and spelling test scores for primary school students.

- **a)** Draw a scatterplot of the data.
- **b)** Describe the relationship as suggested by the scatterplot.
- **c)** It seems weird to think that spelling ability depends on shoe size. Can you explain the relationship?
- **d)** What relationship would you expect between shoe size and spelling ability for high school students?
- **e)** Test your hypothesis.

Shoe size and spelling test scores for primary school students

Size	Test	Size	Test
7	35	8.5	65
9	85	7	25
6.5	8	8	55
8.5	75	9.5	90
7.5	33	8.5	70
9	83	6.5	10
10.5	96	8	50
7	30	8	65
9	85	10	95
10	90	8	60

A strong correlation between A and B doesn't mean that A causes B.

It can be quite possible that another factor C causes both A and B.

Is it possible that increasing age causes an increase in shoe size?

Is it also possible that increasing age can cause increasing spelling test scores?

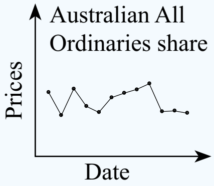
Time Series

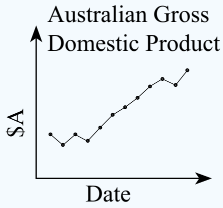
A time series is a sequence of data measured at regular time intervals.

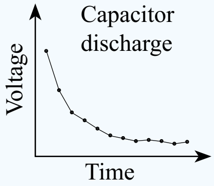
Time series are usually visually represented by data points joined by a line.

Types of time series trends

- linear or non-linear
- upward or downward
- no trend







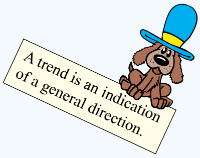
The above time series plot indicates no trend.

The above time series plot indicates a linear upward trend.

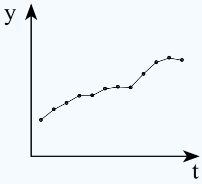
The above time series plot indicates a non-linear downward trend.

Exercise 18.4

Describe each of the following time series plots:


A trend is an indication of a general direction.

1



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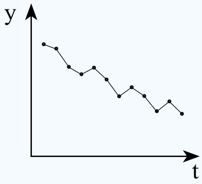

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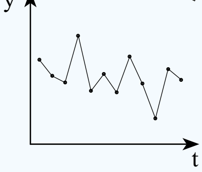

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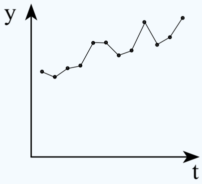

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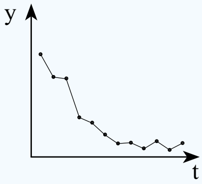

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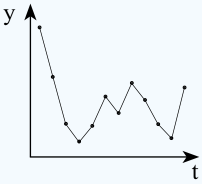

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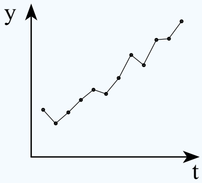

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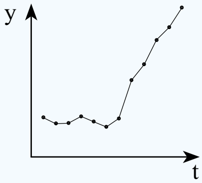

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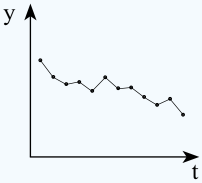


Exercise 18.5

1 Rainfall has a significant impact on may aspects of Australian life and there is a keen interest in weather forecasts.

a) Draw a time series plot of the data.

b) Describe the time series plot.

c) Attempt to make a forecast of the mean annual rainfall for the next five years.

d) How confident are you of your forecast?

Australian five-year mean annual rainfall

Year	mm	Year	mm
1905	420	1960	450
1910	460	1965	420
1915	420	1970	420
1920	430	1975	650
1925	410	1980	470
1930	470	1985	450
1935	400	1990	420
1940	480	1995	400
1945	430	2000	700
1950	560	2005	430
1955	530	2010	700

2 Temperature also has a significant impact on may aspects of Australian life and there is a keen interest in weather forecasts.

a) Draw a time series plot of the data.

b) Describe the time series plot.

c) Attempt to make a forecast of the mean annual temperature for the next five years.

d) How confident are you of your forecast?

Australian five-year mean annual temperature

Year	°C	Year	°C
1905	21.6	1960	21.6
1910	21.6	1965	21.6
1915	21.5	1970	21.7
1920	21.5	1975	21.7
1925	21.4	1980	21.8
1930	21.5	1985	22.0
1935	21.6	1990	22.1
1940	21.5	1995	22.1
1945	21.4	2000	22.2
1950	21.5	2005	22.3
1955	21.6	2010	22.3

3 It is thought that sea surface temperatures affect the Australian climate. The following table refers to sea surface temperatures in the Australian region.

a) Draw a time series plot of the data.

b) Describe the time series plot.

c) Attempt to make a forecast of the mean annual sea surface temperatures for the next five years.

d) How confident are you of your forecast?

Australian five-year mean annual sea surface temperatures

Year	°C	Year	°C
1905	21.4	1960	21.8
1910	21.5	1965	21.7
1915	21.5	1970	21.8
1920	21.5	1975	21.9
1925	21.5	1980	21.9
1930	21.5	1985	21.8
1935	21.6	1990	22.1
1940	21.7	1995	22.0
1945	21.7	2000	22.3
1950	21.6	2005	22.2
1955	21.6	2010	22.3

Line of Best Fit

A 'line of best fit' is a straight line that best represents the data on a scatterplot.

The line may pass through some of the points or none of the points.

Exercise 18.6

1 A student adjusts the power supply until a required current is reached and then reads the voltage drop across the resistor. The following table is produced:

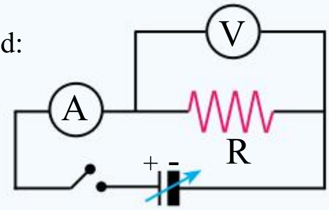
a) Produce a scatterplot.

b) Use technology to find a line of best fit

c) Use the line of best fit to predict the voltage drop across the resistor when the current is 20 microamps). Comment.

d) Use the line of best fit to predict the voltage drop across the resistor when the current is 120 microamps). Comment.

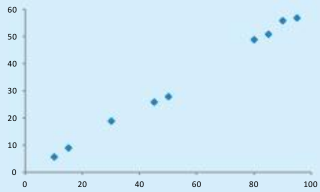
e) Use the points from c) and d) to plot the line of best fit.



The scatterplot and the line of best fit can be obtained from a spreadsheet, a graphics calculator, or online (for example, 'line of best fit applet').

See Technology 18.

Ohms Law experiment	
Current (microamps)	Voltage (millivolts)
10	5.8
15	9.1
30	19
45	26
50	28
80	49
85	51
90	56
95	57



b) Line of best fit: $\text{Voltage} = 0.61\times\text{Current} - 0.50$

(gradient(m) = 0.61, y-intercept = −0.50)

c) current = 20 microamps, $\text{Voltage} = 0.61\times\text{Current} - 0.50$

$\text{Voltage} = 0.61\times 20 - 0.50$

$\text{Voltage} = 12 \text{ milliamps}$

The scatterplot suggests a strong linear correlation between the two variables. The small number of points suggests that some care be taken in relying on using the line of best fit to make predictions.

d) current = 120 microamps, $\text{Voltage} = 0.61\times\text{Current} - 0.50$

$\text{Voltage} = 0.61\times 120 - 0.50$

$\text{Voltage} = 73 \text{ milliamps}$

The current of 120 microamps is outside the range of the small number of points taken (max current = 95 microamps). Care would need to be taken with this prediction even though the few points suggest a strong linear correlation.

e) Plot the two points (20,12) and (120,73) and draw the line of best fit through the two points.

Only male crickets chirp. The chirping sound is made by rubbing a wing against comb-like teeth on the bottom of the other wing.

2 Is there a linear relationship between the temperature and the frequency of cricket chirps?
Some data is shown:

a) Produce a scatterplot.

b) Use technology to find a line of best fit.

c) Use the line of best fit to predict the chirp frequency at a temperature of 23°C. Comment.

d) Use the line of best fit to predict the chirp frequency at a temperature of 18°C. Comment.

e) Use the points from c) and d) to plot the line of best fit.

Cricket chirps	
Temp (°C)	Frequency (chirps/sec)
34	20
24	16
21	15
28	17
29	18
26	15
25	14
27	16
22	16
31	20

3 The Gross Domestic product (GDP) of a country is the value of all goods and services produced by a country. The GDP is generally accepted as a measure of a country's economy. Australian GDP is shown:

a) Produce a scatterplot.

b) Use technology to find a line of best fit.

c) Use the line of best fit to predict the Australian GDP in 2002. Comment.

d) Use the line of best fit to predict the Australian GDP in 2015. Comment.

e) Use the points from c) and d) to plot the line of best fit.

Australian GDP	
Year	$billion
1999	390
2000	417
2001	380
2002	397
2003	468
2004	616
2005	696
2006	750
2007	857
2008	1062
2009	924
2010	1132

Population of Australia = 23 million
Population of China = 1400 million

4 The Gross Domestic product (GDP) of a country is the value of all goods and services produced by a country. The GDP is generally accepted as a measure of a country's economy. China's GDP is shown:

a) Produce a scatterplot.

b) Use technology to find a line of best fit.

c) Use the line of best fit to predict China's GDP in 2002. Comment.

d) Use the line of best fit to predict China's GDP in 2015. Comment.

e) Use the points from c) and d) to plot the line of best fit.

GDP of China	
Year	$billion
1999	1083
2000	1200
2001	1325
2002	1454
2003	1641
2004	1932
2005	2257
2006	2713
2007	3494
2008	4522
2009	4991
2010	5931

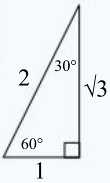
Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 18.7

1 Spell Scatterplot
2 Name two properties of an isosceles triangle
3 Name two properties of a square
4 If the angle sum of a polygon = (n−2)×180°, what is the angle sum of a quadrilateral?
5 What is the gradient of the line $y = x + 3$?
6 What is the gradient of the line perpendicular to $y = 2x - 1$?
7 What angle does the line $y = x + 2$ make with the x-axis?
8 In the triangle, what is cos 60°?
9 Factorise: $x^2 + 4x + 3$
10 Two sides of a right-angled triangle are 1 and 2, what is the hypotenuse?

$(n-2)\times180$
$= (4-2)\times180$
$= 360$
Angle sum of a quadrilateral is 360°



$x^2 + 4x + 3$
$= (x + 3)(x + 1)$

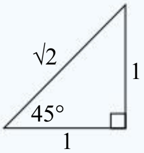
If two lines are perpendicular then the product of their gradients is -1.

$$m_1 \times m_2 = {}^{-}1$$

'All things are difficult before they are easy' - Thomas Fuller.

Exercise 18.8

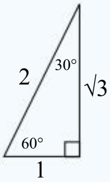
1 Spell Correlation
2 Name two properties of an equilateral triangle
3 Name two properties of a rectangle
4 If the angle sum of a polygon = (n−2)×180°, what is the angle sum of a hexagon?
5 What is the gradient of the line $y = 3x + 2$?
6 What is the gradient of the line parallel to $y = 3x - 1$?
7 What angle does the line $y = {}^{-}x + 2$ make with the x-axis?
8 In the triangle, what is sin 45°?
9 Factorise: $x^2 + 6x + 5$
10 Two sides of a right-angled triangle are 1 and 3, what is the hypotenuse?



'Opportunity is missed by most people because it is dressed in overalls and looks like work' - Thomas Edison.

Exercise 18.9

1 Spell Non-linear
2 Name two properties of a rhombus
3 Name two properties of a parallelogram
4 If the angle sum of a polygon = (n−2)×180°, what is the angle sum of a octagon?
5 What is the gradient of the line $y = 2x + 5$?
6 What is the gradient of the line perpendicular to $y = 2x - 3$?
7 What angle does the line $y = 2$ make with the x-axis?
8 In the triangle, what is cos 30°?
9 Factorise: $x^2 + 3x + 2$
10 Two sides of a right-angled triangle are 2 and 3, what is the hypotenuse?



Nuclear Physicists work on understanding nuclear science.

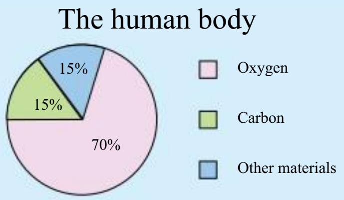
- Relevant school subjects are English, Mathematics, Physics.
- Courses generally involve a University degree with a major in nuclear physics.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

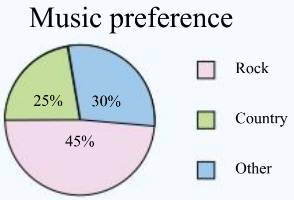
Exercise 18.10

1 What angle does carbon make at the centre of the pie chart?

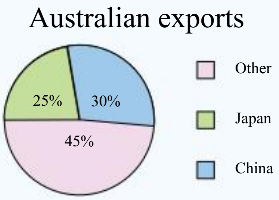


$$\text{Carbon angle} = \frac{15}{100}\times 360$$
$$= \underline{54^\circ}$$

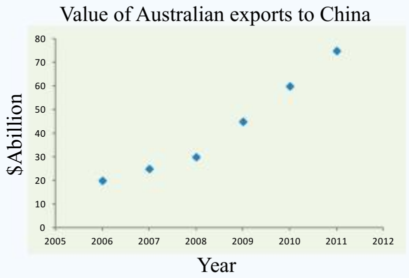
2 What angle does country music make at the centre of the pie chart?



3 What angle does China make at the centre of the pie chart?

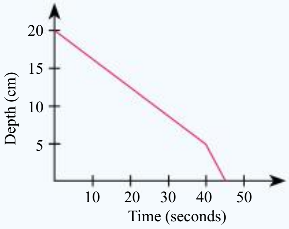


4 The graph shows the value of Australian exports to China. When was the biggest increase in exports to China?



Hint:
Where is the largest gradient?
Use a ruler to find the steepest slope.

5 A 20 cm container, initially full of water, is leaking through a hole in the bottom. Find the fastest change in the depth of water in cm/s.



Hint:

$$gradient = \frac{change\ in\ y}{change\ in\ x} = rate\ of\ change$$

Technology

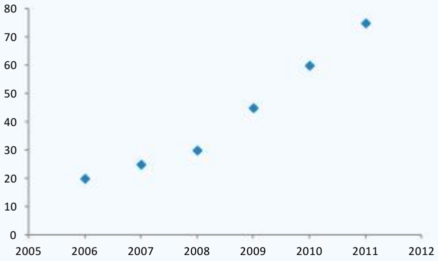
Technology 18.1 Spreadsheets

Spreadsheets will draw **Scatterplots**.

Year	$Abillion
2006	20
2007	25
2008	30
2009	45
2010	60
2011	75

Highlight data and insert a scatter chart.

Right-Click various parts of the plot to format parts to your liking.



Line of best fit 10A

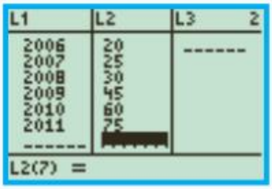
= LINEST(B1:B6,A1:A6,1,1)

y = 11.3x − 22600 or
$Abillion = 11.3Year − 22600

Technology 18.2 Graphics Calculators

A graphics calculator will draw **Scatterplots**.

1 Press STAT then Edit and enter the independent variable in L1 and the dependent variable in L2

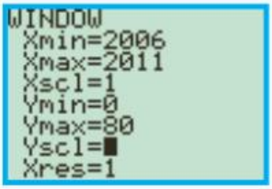


```
L1      L2      L3     2
2006    20      ------
2007    25
2008    30
2009    45
2010    60
2011    75
------
L2(7) =
```

2 Press STATPLOT then select Plot 1 and select the shown options.

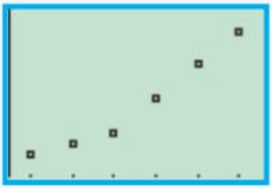
```
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L1
Ylist:L2
Mark: ■ + ·
```

3 Set the Window



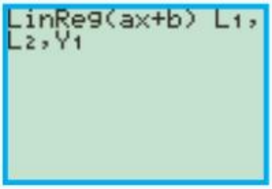
```
WINDOW
 Xmin=2006
 Xmax=2011
 Xscl=1
 Ymin=0
 Ymax=80
 Yscl=■
 Xres=1
```

4 Press ZOOM then choose Zoomstat

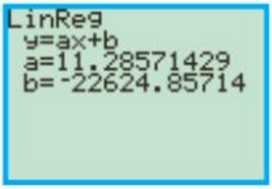


10A A graphics calculator will also draw and calculate the '**Line of Best Fit**'.

4 Press STAT CALC LinReg(ax+b) L1 , L2 , VARS YVARS 1 Y1



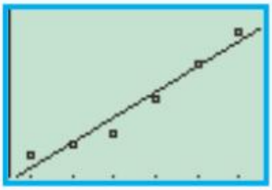
```
LinReg(ax+b) L1,
L2,Y1
```



```
LinReg
 y=ax+b
 a=11.28571429
 b=-22624.85714
```

y = 11.3x − 22600 or
$Abillion = 11.3Year − 22600

GRAPH



A Couple of Puzzles

Exercise 18.11

1 Each birthday I have had a birthday with the appropriate number of candles. If I have blown out 120 candles, how old am I?

2

9 7 23 5

8 15 ?

3

7 9 13 ? 37

4

10 18 34 66 ?

A Game

Play a couple of games and try to determine a winning strategy.

Twenty is played by two people with the numbers 1 to 20. The winner is the person who crosses out the number 20.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Starting at 1, take it in turns to cross out 1, 2, 3 or 4 numbers. The numbers must be in order.

A Sweet Trick

1 This trick is a variation of a trick in a previous chapter in which this grid of numbers was used.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

2 Change the grid by adding, or subtracting, the same number to each number in any row or column.

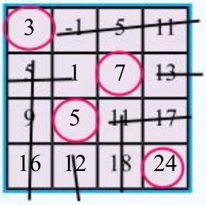
3 This grid happened by adding 2 to the 1st row, 3 to the 4th row, 5 to the 4th column, and subtracting 5 from the 2nd column.

3	-1	5	11
5	1	7	13
9	5	11	17
16	12	18	24

4 Your audience chooses an uncrossed number and you cross off the other numbers in the same row and column.

5 Repeat until all numbers are either chosen or crossed.

6 The sum of the four numbers equals 39.



39 equals the sum of each diagonal.
Try a 5x5 square, a 3x3 square, etc.
Embellish the trick with creative ideas.

Investigations

Investigation 18.1 Undertake real-life research.

1 Form a team and brainstorm an appropriate problem or issue.

2 Plan the research
- Define the overall research question.
- Define subset research questions.
- Decide how to obtain data to answer the research questions.
- Consider ways of ensuring that the data collection is unbiased.
- Consider the equipment needed for the research.

3 Conduct the research
- Rehearse the data collection method.
- Collect the data.

4 Analyse the data
- Look for errors/outliers and decide what to do with the errors/outliers.
- Calculate the appropriate descriptive statistics.
- Choose an appropriate method of presentation (Scattergrams, Histograms etc).

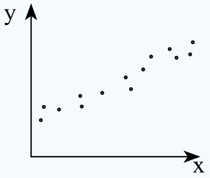
5 Report the conclusions
- Match the analysis with the research questions.
- Do the answers to the research questions indicate further research questions?

Investigation 18.2 Line of best fit and the spreadsheet

Investigate

A spreadsheet can be used to calculate and draw 'lines of best fit'.

How?

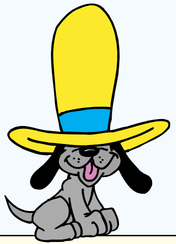


A line of best fit is a straight line that best represents the data on a scatter plot.

Investigation 18.3 Line of best fit

Investigate

Different 'lines of best fit'.



A line of best fit is also known as the trend line.

Chapter Review 1

Exercise 18.12

1 Bivariate data relating driver's age and road casualties (death or serious injury) was collected in Great Britian in 2010.

Driver's age and casualities	
Age	Casualties
1 (<17)	478
2 (17)	171
3 (18)	200
4 (19)	153
5 (20-24)	515
6 (25-29)	262
7 (30-39)	318
8 (40-59)	452

a) Draw a scatterplot of the data.

b) Describe the relationship as suggested by the scatterplot.

c) Which driver is more likely to be a road casuality, a 19 year-old or a 23 year-old?

d) Predict the number of casualities for the 60-69 year-old group.
How confident are you in making your prediction?

2 Estimates have been made of Australia's metric tonnes of carbon dioxide emissions (Mt CO$_2$ - e).

Emissions	
Year	MT CO$_2$ - e
2001	128
2002	128
2003	129
2004	132
2005	132
2006	134
2007	136
2008	138
2009	135
2010	138
2011	137

a) Draw a time series plot of the data.

b) Describe the time series plot.

c) Attempt to make a forecast of the mean annual rainfall for the next five years.

d) How confident are you of your forecast?

3 10A In an experiment measuring the growth of micro-organisms in a liquid culture, it was noted that as the number of micro-organisms increased, the culture became darker. Bivariate data was collected on the optical density (OD) of the culture and the concentration (C) of the organisms in the culture.

Optical Density	
OD	C
1	2
4	3
6	4
8	5
12	7
15	8
18	9
20	10

a) Produce a scatterplot.

b) Use technology to find a line of best fit.

c) Use the line of best fit to predict the concentration of micro-organisms with an optical density of 5. Comment.

d) Use the line of best fit to predict the concentration of micro-organisms with an optical density of 30. Comment.

e) Use the points from c) and d) to plot the line of best fit.

Chapter Review 2

Exercise 18.13

1 Global mean sea level set to base level of 0 mm in 1990 is shown in the table.

a) Draw a scatterplot of the data.

b) Describe the relationship as suggested by the scatterplot.

c) Use the scatterplot to estimate the sea level in 2015.

d) What confidence might you have in your estimation?

Global mean sea levels

Year	Sea level (mm)
1990	0
1993	10
1996	18
1999	25
2002	38
2005	49
2008	54
2011	65

2 Estimates have been made of Global mean sea level over the last 120 thousand years set to a base of 0 m twenty thousand years ago.

a) Draw a time series plot of the data.

b) Describe the time series plot.

c) Attempt to make a forecast of the global mean sea level in 10 thousand years time.

d) How confident are you of your forecast?

Global mean sea levels

Years	Sea level (m)
1 (100 000 years ago)	100
2 (90 000 years ago)	100
3 (80 000 years ago)	83
4 (70 000 years ago)	80
5 (60 000 years ago)	75
6 (50 000 years ago)	75
7 (40 000 years ago)	55
8 (30 000 years ago)	30
9 (20 000 years ago)	0
10 (10 000 years ago)	80
11 (now)	140

3 10A The relationship between the number of bedrooms and the price of a house is being examined by detailing advertisements of houses for sale (restricted to one suburb only).

a) Produce a scatterplot.

b) Use technology to find a line of best fit.

c) Use the line of best fit to initially set the price of a 3 bedroom house. Comment.

c) Use the line of best fit to initially set the price of a 5 bedroom house. Comment.

e) Use the points from c) and d) to plot the line of best fit.

Bedrooms and house prices

Bedrooms	$Price
1	350 000
1	370 000
2	440 000
2	450 000
3	520 000
3	530 000
4	610 000
4	605 000

Trigonometry 2

19

Measurement and Geometry ➞ Pythagoras and Trigonometry

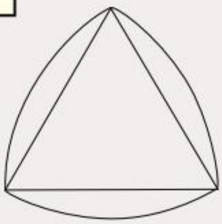
10A Establish the sine, cosine and area rules for any triangle and solve related problems.
- apply knowledge of sine, cosine and area rules to authentic problems such as those involving surveying and design.

10A Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.
- establish the symmetrical properties of trigonometric functions.
- investigate angles of any magnitude.
- understand that trigonometric functions are periodic and that this can be used to describe motion.

10A Solve simple trigonometric equations.
- use periodicity and symmetry to solve equations.

10A Apply Pythagoras' theorem and trigonometry to solving three-dimensional problems in right-angled triangles.
- investigate the applications of Pythagoras's theorem in authentic problems.

Trigonometry is a sine of the times.



A TASK

Demonstrate to your class that it is possible to drill square holes.

- Research the Reuleaux triangle.
- Construct a Reuleaux triangle.
- Prepare a powerpoint/poster to show how the Reuleaux triangle can drill a square hole.
- Demonstrate your work to your class.

A LITTLE BIT OF HISTORY

One of the more amazing archaeological finds has been the many mathematical tablets unearthed in the ancient cities of Babylonia.

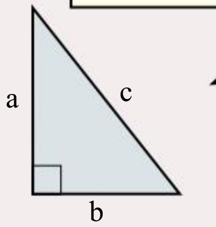
Incredibly, the oldest mathematical document (c. 1900 to 1600 BC) gives the answers to a problem involving Pythagorean triplets.

Most of the mathematical tablets appear to have been school texts showing how to solve problems.
This tablet shows a table of Pythagorean triplets as complicated as: 120, 119, 169

Use your calculator to show that:
$120^2 + 119^2 = 169^2$

Pythagoras's Theorem Review

In any right-angled triangle:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.



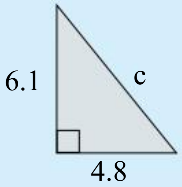
The hypotenuse is the longest side.
It is opposite the right-angle (90°).

$$c^2 = a^2 + b^2$$

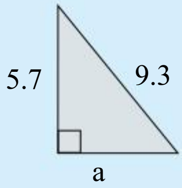
$$a^2 + b^2 = c^2$$

Exercise 19.1

Find the length of the unknown in each of the following
(round to two decimal places or leave in surd form as appropriate):

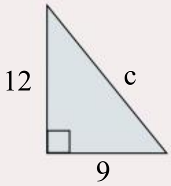


$c^2 = a^2 + b^2$
$c^2 = 6.1^2 + 4.8^2$
$c^2 = 60.25$
$c = \sqrt{60.25}$
$\underline{c = 7.76}$

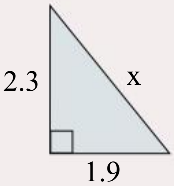


$a^2 + b^2 = c^2$
$a^2 + 5.7^2 = 9.3^2$
$a^2 = 9.3^2 - 5.7^2$
$a^2 = 54$
$a = \sqrt{54}$
$\underline{a = 3\sqrt{6}}$

1



2



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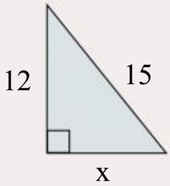

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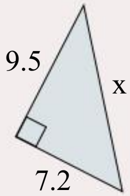

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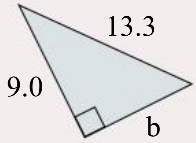

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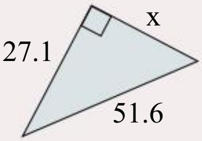
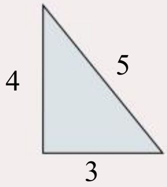


Converse:
If $c^2 = a^2 + b^2$ then the triangle is right-angled.

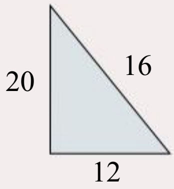
Exercise 19.2

Decide whether the following are right-angled triangles.

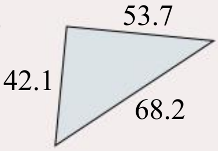
1



2



3



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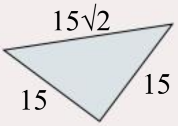
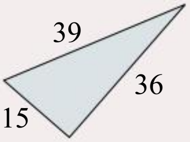


5



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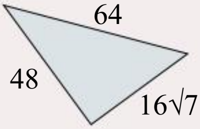
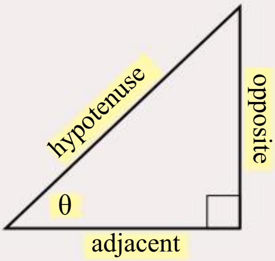


Trigonometry Review



$\sin\alpha = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\cos\alpha = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\alpha = \dfrac{\text{opposite}}{\text{adjacent}}$

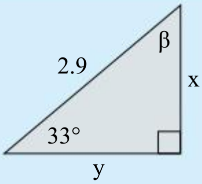
Can you learn these ratios off by heart?
SOHCAHTOA

These ratios only work in right-angled triangles.

Trigonometry is used to solve the thousands and thousands of triangle problems in engineering, surveying, architecture, astronomy, etc, etc, etc.

Exercise 19.3

Use a combination of the sin, cos, and tan ratios to find the unknowns in each of the following triangles (round to two decimal places).



$\sin\alpha\checkmark = \dfrac{\overset{?}{\text{opposite}}}{\text{hypotenuse}\checkmark}$

$\sin 33 = \dfrac{x}{2.9}$

$2.9\times\sin 33 = x$

$\underline{1.58 = x}$

$\cos\alpha\checkmark = \dfrac{\overset{?}{\text{adjacent}}}{\text{hypotenuse}\checkmark}$

$\cos 33 = \dfrac{y}{2.9}$

$2.9\times\cos 33 = y$

$\underline{2.43 = y}$

$\beta + 33 + 90 = 180$

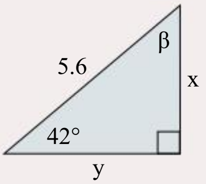
$\beta + 123 = 180$

$\beta = 180 - 123$

$\underline{\beta = 57°}$

Make sure your calculator is on degrees (deg).

1



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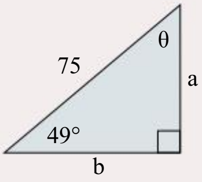

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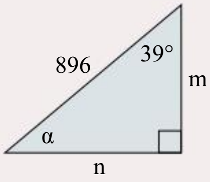

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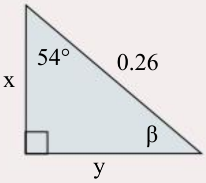

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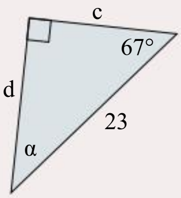

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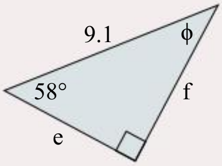

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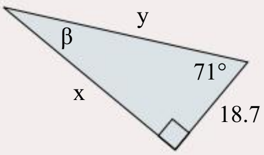

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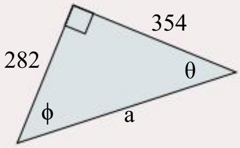
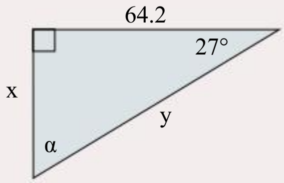


9



Rounding to two decimal places, first look at the third decimal place:

56.231694 ↑ less than 5 thus **56.23**	27.01769 ↑ 5 or more thus **27.02**	1.07276 ↑ less than 5 thus **1.07**	4.79634216 ↑ 5 or more thus **4.80**

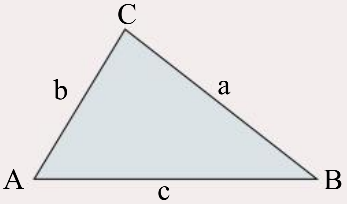
The Sine Rule

What if the triangle doesn't have a right angle? → Then use the sine rule and/or the cos rule.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

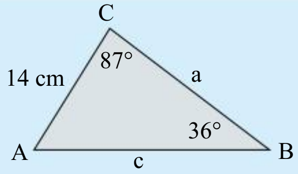


Side a is opposite angle A
Side b is opposite angle B
Side c is opposite angle C

Exercise 19.4

'solve' = find all unknowns.

Use the sine rule to help solve each of the following triangles.



$A + B + C = 180$
$A = 180 - 87 - 36$
$\underline{A = 57°}$

$$\frac{a?}{\sin A\checkmark} = \frac{b\checkmark}{\sin B\checkmark} = \frac{c\checkmark}{\sin ?C}$$

$$\frac{a}{\sin 57} = \frac{14}{\sin 36}$$

$$a = \frac{14 \times \sin 57}{\sin 36}$$

$\underline{a = 19.98 \text{ cm}}$

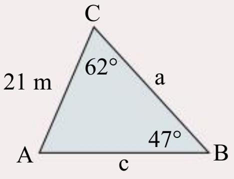
$$\frac{a\checkmark}{\sin A\checkmark} = \frac{b\checkmark}{\sin B\checkmark} = \frac{c?}{\sin C\checkmark}$$

$$\frac{14}{\sin 36} = \frac{c}{\sin 87}$$

$$\frac{14 \times \sin 87}{\sin 36} = c$$

$\underline{23.79 \text{ cm} = c}$

1



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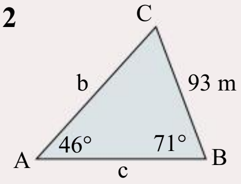

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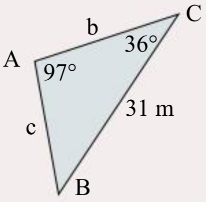
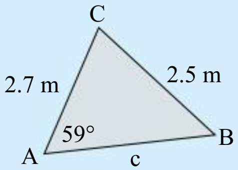




$$\frac{\sin A\checkmark}{a\checkmark} = \frac{\sin B?}{b\checkmark} = \frac{\sin C?}{c?}$$

$$\frac{\sin 59}{2.5} = \frac{\sin B}{2.7}$$

$$\frac{\sin 59 \times 2.7}{2.5} = \sin B$$

$0.93 = \sin B$
$\sin^{-1}(0.93) = B$
$\underline{68.4° = B}$

$A + B + C = 180$
$C = 180 - 59 - 68.4$
$\underline{C = 52.6°}$

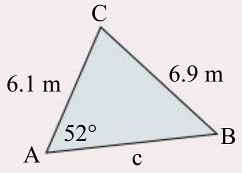
$$\frac{a\checkmark}{\sin A\checkmark} = \frac{b\checkmark}{\sin B\checkmark} = \frac{c?}{\sin C\checkmark}$$

$$\frac{2.5}{\sin 59} = \frac{c}{\sin 52.6}$$

$$\frac{2.5 \times \sin 52.6}{\sin 59} = c$$

$\underline{2.32 \text{ m} = c}$

4



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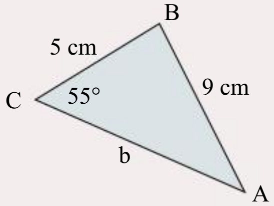

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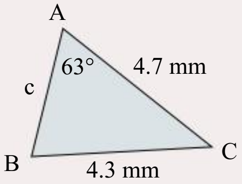


The Cos Rule

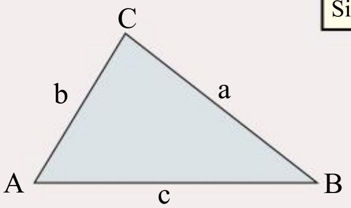
$$a^2 = b^2 + c^2 - 2bc\cos A$$

or

$$b^2 = a^2 + c^2 - 2ac\cos B$$

or

$$c^2 = a^2 + b^2 - 2ab\cos C$$

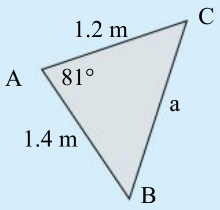


Side a is opposite angle A
Side b is opposite angle B
Side c is opposite angle C

The largest side is opposite the largest angle.

Exercise 19.5

Use the cos rule to help solve each of the following triangles.



$$\overset{?}{a}^2 = \overset{\checkmark}{b}^2 + \overset{\checkmark}{c}^2 - 2\overset{\checkmark}{b}\overset{\checkmark}{c}\cos \overset{\checkmark}{A}$$

$a^2 = 1.2^2 + 1.4^2 - 2 \times 1.2 \times 1.4 \times \cos 81$

$a^2 = 2.87$

$a = \sqrt{2.87}$

$\underline{a = 1.69\text{ m}}$

$$\frac{\sin \overset{\checkmark}{A}}{\overset{\checkmark}{a}} = \frac{\sin \overset{?}{B}}{\overset{\checkmark}{b}} = \frac{\sin \overset{?}{C}}{\overset{\checkmark}{c}}$$

$\frac{\sin 81}{1.69} = \frac{\sin B}{1.2}$

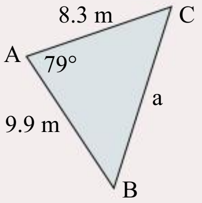
$\frac{\sin 81 \times 1.2}{1.69} = \sin B$

$\sin^{-1}(0.70) = B$

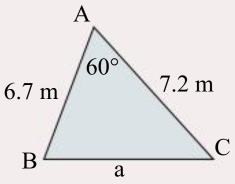
$\underline{44.4° = B}$

$C = 180 - 81 - 44.4 = \underline{54.6°}$

1



2



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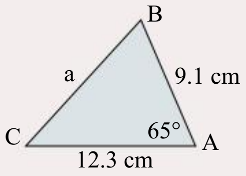
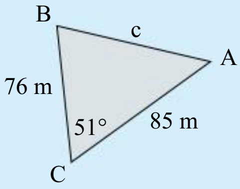




$$\overset{?}{c}^2 = \overset{\checkmark}{a}^2 + \overset{\checkmark}{b}^2 - 2\overset{\checkmark}{a}\overset{\checkmark}{b}\cos \overset{\checkmark}{C}$$

$c^2 = 76^2 + 85^2 - 2 \times 76 \times 85 \times \cos 51$

$c^2 = 4870.18$

$c = \sqrt{4870.18}$

$\underline{c = 67.8\text{ m}}$

$$\frac{\sin \overset{?}{A}}{\overset{\checkmark}{a}} = \frac{\sin \overset{?}{B}}{\overset{\checkmark}{b}} = \frac{\sin \overset{\checkmark}{C}}{\overset{\checkmark}{c}}$$

$\frac{\sin A}{76} = \frac{\sin 51}{67.8}$

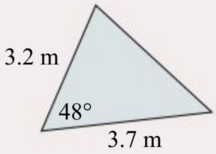
$\sin A = \frac{\sin 51 \times 76}{67.8}$

$A = \sin^{-1}(0.87)$

$\underline{A = 60.5°}$

$B = 180 - 51 - 60.5 = \underline{68.5°}$

4



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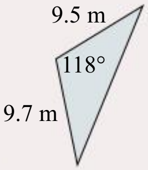

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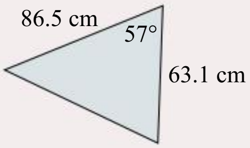


Give letters to the sides and angles.

Area

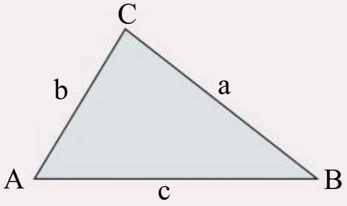
$Area = 0.5ab\sin C$

or

$Area = 0.5ac\sin B$

or

$Area = 0.5bc\sin A$

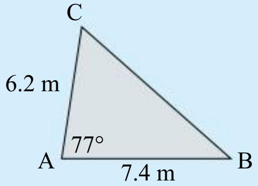


If right-angled:
Area Δ = 0.5base×⊥'ar height

Side a is opposite angle A
Side b is opposite angle B
Side c is opposite angle C

Exercise 19.6

1 Find the area of each of the following triangles.



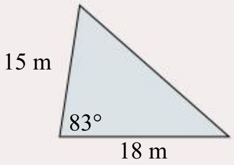
$Area = 0.5bc\sin A$

Area = ½×6.2×7.4×sin 77

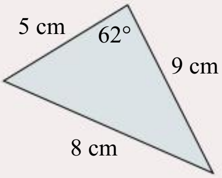
Area = 22.35 cm^2

To find area, look for two sides and the angle inbetween.

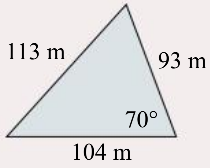
a)



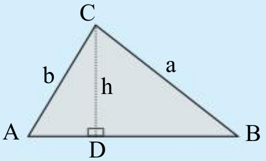
b)



c)



Show that: $Area = 0.5ac\sin B$



In ΔBCD, $\sin B = \frac{h}{a}$

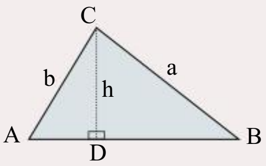
$h = a\sin B$

Area ΔABC = ½base×height

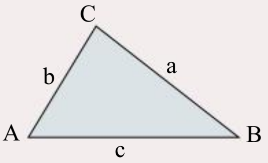
= ½c×a sin B

Thus $Area = 0.5ac\sin B$

2 Show that $Area = 0.5bc\sin A$



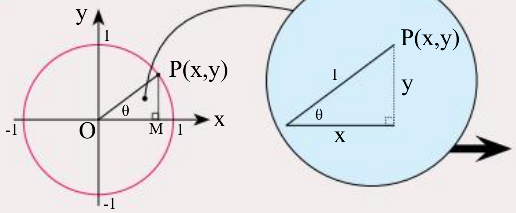
3 Show that: $Area = 0.5ab\sin C$



You need a right-angled triangle.
Where?

The Unit Circle

The unit circle has:
radius = 1 unit
centre = (0,0)



$\sin\theta = \frac{opp}{hyp} = \frac{y}{1} = y$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{1} = x$

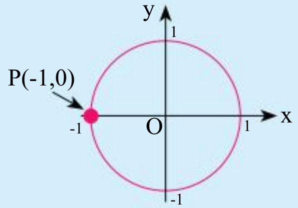
$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$

Exercise 19.7

Use the unit circle to calculate the following trigonometric ratios:

+ direction is anticlockwise.

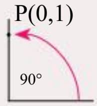
sin 180°, cos 180°, tan 180°

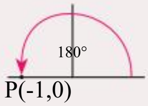


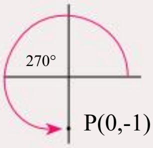
a) Plot 180° on the unit circle.
b) Write coordinates.

c) sin 180 = y = $\underline{0}$
cos 180 = x = $\underline{-1}$
tan 180 = y/x = 0/-1 = $\underline{0}$

Use calculator to check answers?

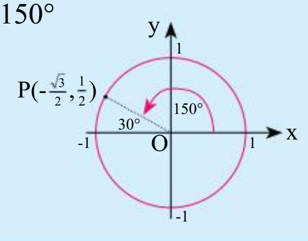


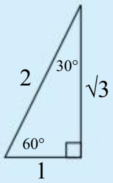




1 sin 90°
2 cos 90°
3 tan 90°
4 sin 180°
5 cos 180°
6 tan 180°
7 sin 270°
8 cos 270°
9 tan 270°
10 sin 0°
11 cos 0°
12 tan 0°

sin 150°, cos 150°, tan 150°

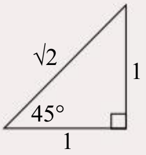




cos 30° = $\frac{\sqrt{3}}{2}$
sin 30° = $\frac{1}{2}$

a) Plot 150° on the unit circle.
b) Write coordinates.
x coordinate = cos θ = $-\frac{\sqrt{3}}{2}$
y coordinate = sin θ = $\frac{1}{2}$
c) sin 150 = y = $\underline{\frac{1}{2}}$
cos 150 = x = $\underline{-\frac{\sqrt{3}}{2}}$
tan 150 = x/y = $\frac{1}{2} \div -\frac{\sqrt{3}}{2} = \underline{-\frac{1}{\sqrt{3}}}$

13 sin 150°
14 cos 120°
15 tan 120°
16 sin 210°
17 cos 240°
18 tan 300°
19 sin 330°
20 cos 390°
21 tan 420°
22 sin 135°
23 cos 225°
24 tan 315°
25 sin -30°
26 cos -120°
27 tan -225°

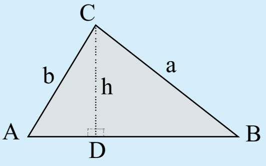


− direction is clockwise.

Miscellaneous Problems

Exercise 19.8

Show that: $\frac{a}{\sin A} = \frac{b}{\sin B}$



In ΔDBC, $\sin B = \frac{h}{a}$

$h = a \sin B$

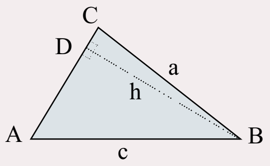
In ΔADC, $\sin A = \frac{h}{b}$

$h = b \sin A$

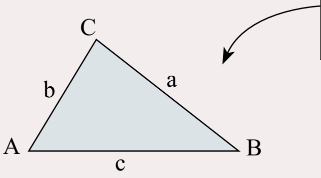
thus: $a \sin B = b \sin A$

or $\underline{\frac{a}{\sin A} = \frac{b}{\sin B}}$

1 Show that $\frac{a}{\sin A} = \frac{c}{\sin C}$



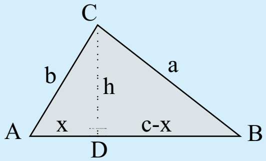
2 Show that: $\frac{b}{\sin B} = \frac{c}{\sin C}$



You need a right-angled triangle.

Where?

Show that: $a^2 = b^2 + c^2 - 2bc\cos A$



In ΔDBC, $a^2 = (c - x)^2 + h^2$

$a^2 = c^2 - 2cx + x^2 + h^2$

In ΔADC, $b^2 = x^2 + h^2$

sub for $x^2 + h^2$, $a^2 = c^2 - 2cx + b^2$ (1)

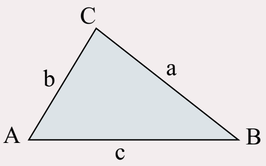
In ΔADC, $\cos A = \frac{x}{b}$

thus: $x = b \cos A$

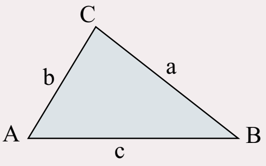
sub for x in (1), $a^2 = c^2 - 2cb\cos A + b^2$

or $\underline{a^2 = b^2 + c^2 - 2bc\cos A}$

3 Show that $b^2 = a^2 + c^2 - 2ac\cos B$

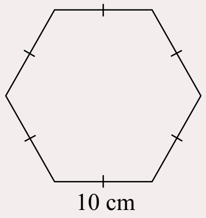


4 Show that: $c^2 = a^2 + b^2 - 2ab\cos C$

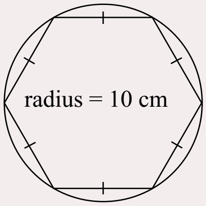


5 Find the area of each of the following figures:

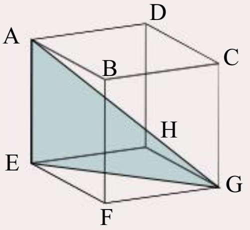
a)



b)



6 The cube has a side length of 10 cm. EG is a diagonal of the square EFGH. AG is a body diagonal of the cube.



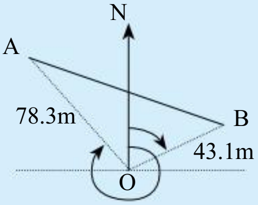
- **a)** Find the length of EG
- **b)** Find the length of AG
- **c)** Find angle EGA
- **d)** Find the area of ΔAEG

7 The Great Pyramid of Ghiza, a square based pyramid has a height of 146.5 m and a base length of 230.4 m

- **a)** Find the base diagonal
- **b)** Find the angle of the faces
- **c)** Find the gradient of the faces
- **d)** Find the total area of the four faces

A surveyor takes distances and bearings to boundary corners A (78.3 m and 321° 17') and B (43.1 m and 65° 34').
Find the distance along the boundary from A to B?

Bearings are clockwise from North



$\angle BOA = 360 - (321° 17' - 65° 34')$

$= 104° 17'$ or $104.28°$

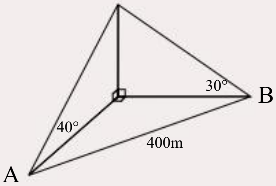
Try your calculator:
321 ° ' " 17 ° ' "
− 65 ° ' " 34 ° ' " =

$$o^2 = a^2 + b^2 - 2ab\cos O$$
$$o^2 = 43.1^2 + 78.3^2 - 2\times43.1\times78.3\times\cos104.28$$
$$o^2 = 9653.33$$
$$o = \sqrt{9653.33}$$
$$\underline{AB = 98.25\text{ m}}$$

8 A surveyor takes distances and bearings to boundary corners A (23.06 m and 54° 33') and B (37.92 m and 164° 15').
Find the distance along the boundary from A to B?

9 A surveyor takes distances and bearings to boundary corners A (653.96 m and 116° 05') and B (321.98 m and 294° 43').
The surveyor then measures the distance from A to B as 975.87 m.
It is vital that the surveyor's work is accurate.
Has the surveyor made accurate measurements?

10 The angle of elevation to the top of a hill from points A and B are 40° and 30° respectively.
Find the height of the hill to the nearest metre (points A and B are 400 m apart).



Mental Computation

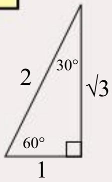
You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 19.9

If two lines are perpendicular then the product of their gradients is -1.

$$m_1 \times m_2 = {}^-1$$

1 Spell Trigonometry
2 What is the sine rule?
3 What is the cos rule?
4 What is the area of any triangle?
5 Name two properties of an isosceles triangle
6 What is the gradient of the line $y = 3x + 1$?
7 What is the gradient of the line perpendicular to $y = 3x + 1$?
8 In the triangle, what is cos 60°?
9 Factorise: $x^2 - 5x + 6$
10 Two sides of a right-angled triangle are 1 and 1, what is the hypotenuse?

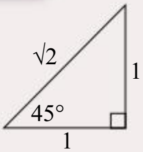


$x^2 - 5x + 6$
$= (x - 3)(x - 2)$

'You cannot plough a field by turning it over in your mind'.

Exercise 19.10

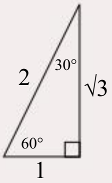
1 Spell Pythagoras
2 What is the sine rule?
3 What is the cos rule?
4 What is the area of any triangle?
5 Name two properties of an equilateral triangle
6 What is the gradient of the line $y = 2x + 3$?
7 What is the gradient of the line perpendicular to $y = 2x + 3$?
8 In the triangle, what is cos 45°?
9 Factorise: $x^2 - 4x + 3$
10 Two sides of a right-angled triangle are 1 and 2, what is the hypotenuse?



'Seek the lofty by reading, hearing and seeing great work at some moment every day' - Thornton Wilder.

Exercise 19.11

1 Spell Hypotenuse
2 What is the sine rule?
3 What is the cos rule?
4 What is the area of any triangle?
5 Name two properties of a parallelogram
6 What is the gradient of the line $y = -5x + 2$?
7 What is the gradient of the line perpendicular to $y = -5x + 2$?
8 In the triangle, what is sin 60°?
9 Factorise: $x^2 - 7x + 6$
10 Two sides of a right-angled triangle are 1 and 3, what is the hypotenuse?



Surveyors measure and assess property to produce maps and reports.

- Relevant school subjects are English and Mathematics.
- Courses generally involve a University degree in surveying.

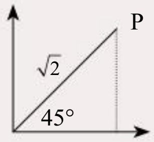
Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

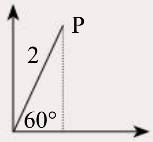
Exercise 19.12

1 Find the coordinates of the point P:

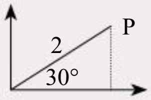
a)



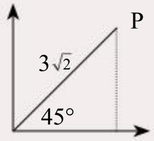
b)



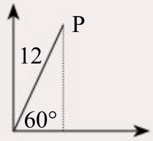
c)



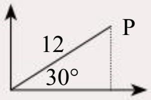
d)



e)

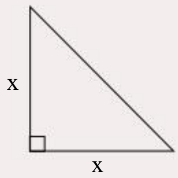


f)

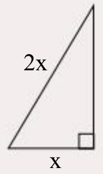


2 Write an expression for the area and perimeter of each of the following triangles:

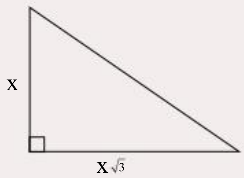
a)



b)



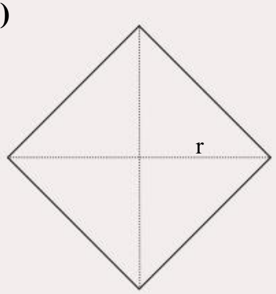
c)



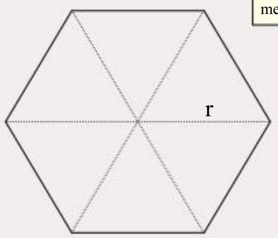
3 Find the area of each of the following regular polygons (r = √2).

A regular polygon means all sides equal.

a)

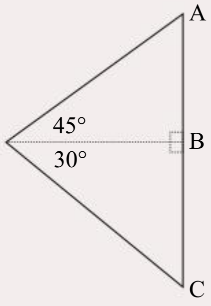


b)

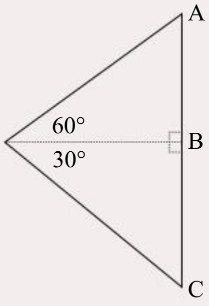


4 If AB = 21 m, find BC.

a)



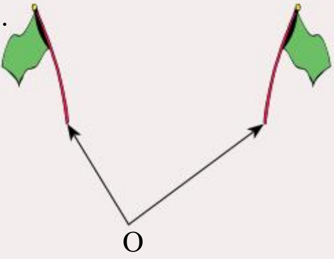
b)



Investigations

Investigation 19.1 Radial Method of Surveying

The radial method of surveying involves the taking of bearings and distances from a central point (The measurements radiate from the centre).



1. Brainstorm some ideas on how to measure bearings.
2. Make your bearing instrument.
3. Test your radial surveying by doing something similar to the problems in Exercise 19.8.

Investigation 19.2 The Sine Rule

1. Draw a right-angled triangle.
   Measure the three sides.
   Measure the three angles.
   Complete the table.

Side	sin(angle)	$\frac{\textit{side}}{\sin(\textit{angle})}$

Did you notice that in a right-angled triangle, the ratio of side to sin(angle) is equal to the hypotenuse.

2. Draw a non-right-angled triangle.
   Measure the three sides.
   Measure the three angles.
   Complete the table.

Side	sin(angle)	$\frac{\textit{side}}{\sin(\textit{angle})}$

Investigation 19.3 The Cos Rule (or The Law of Cosines)

Investigate

Triangulation and the Cos Rule

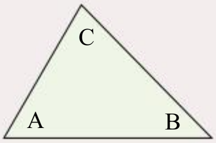
Investigate

GPS and the Cos Rule

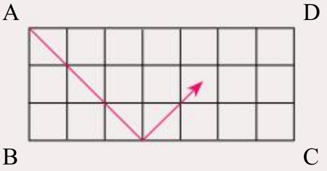
A Couple of Puzzles

Exercise 19.13

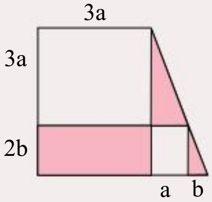
1 C is 50° less than B. B is 10° more than A. Find A, B, and C.



2 Which corner will the billiard ball hit?

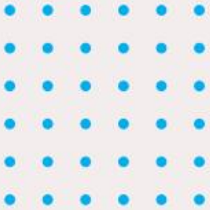


3 Write an expression for the total shaded area.



A Game

Dottie is a game for two players.
The loser is the player who is unable to make a move ie., join two dots.



For variety, change the number of dots.

1 Draw up a 6x6 dot board and decide who moves vertically and who moves horizontally.

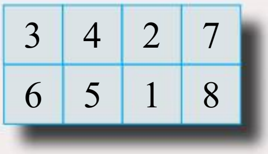
2 When it is your turn join two dots as in the example.

3 No dot can have two lines.

A Sweet Trick

This map folding trick has puzzled many people for centuries.

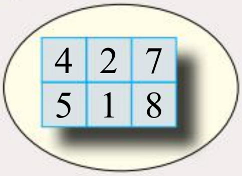
1 Divide and number an A4 sheet of paper as shown.



2 Do the same to the back of the sheet. Each number should be behind the same number on the front.

3 Fold the map so that only consecutive numbers touch. 1 touches only 2, 4 touches only 3 and 5.

a) Fold the 3 and 6 behind the 4 and 5.
b) Fold the 5, 1, and 8 up over the 4, 2, and 7.
c) Can you complete the last step?



Technology

Can you brainstorm twenty periodic things in our world?

Technology 19.1 Periodic Functions

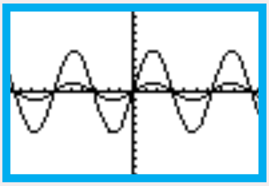
A very large number of things in our world are periodic, i.e., repetitive or cyclical. Some examples are our breathing (repeatedly breathing in and out, tides (repeatedly rising and falling), our heart (repeatedly pumping).

The sine and cosine functions help in forming mathematical models of these periodic events. The models allow deeper understanding of the periodic events and help in making predictions.

1 Draw a graph of $y = \sin x$

- **a)** Press `Y=` and enter sin x
- **b)** Press `GRAPH` and note the periodic nature of $y = \sin x$

2 Add a graph of $y = 5\sin x$



3 Add a graph of $y = \sin(5x)$

4 Add a graph of $y = 5 + \sin x$

5 What will $y = 5 + 5\sin(5x)$ look like?

The Sine Function

Experiment with an online 'Sine Applet'.

The Cos Function

Experiment with an online 'Cos Applet'.

The Tan Function

Experiment with an online 'Tan Applet'.

The Sine Rule

Watch some online videos about the 'sine rule'.

The Cos Rule

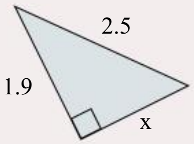
Watch some online videos about the 'Cos rule'.

Chapter Review 1

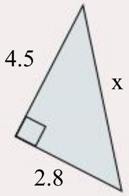
Exercise 19.14

1 Find the unknowns in each of the following right-angled triangles:

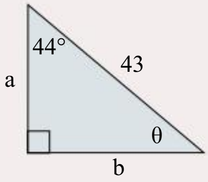
a)



b)

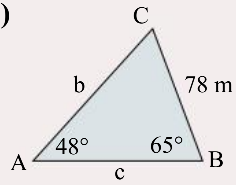


c)

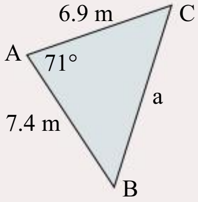


2 Use the sine rule and/or the cos rule to help solve each of the following triangles:

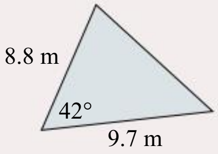
a)



b)



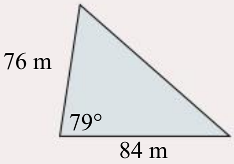
c)



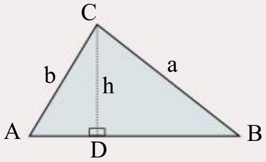
3 Use the unit circle to calculate the following trigonometric ratios:

a) cos 180° **b)** sin 210° **c)** tan 300°

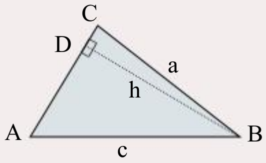
4 Find the area of the following triangle



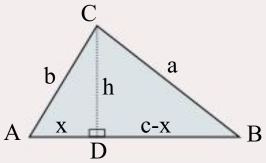
5 Show that $Area = 0.5bc\sin A$



6 Show that $\frac{a}{\sin A} = \frac{c}{\sin C}$



7 Show that: $a^2 = b^2 + c^2 - 2bc\cos A$



8 The Great Pyramid of Ghiza, a square based pyramid has a height of 146.5 m and a base length of 230.4 m

a) Find the base diagonal

b) Find the angle of the faces

c) Find the gradient of the faces

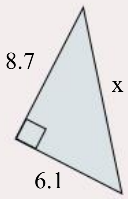
d) Find the total area of the four faces

Chapter Review 2

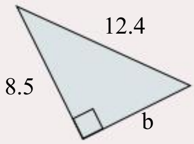
Exercise 19.15

1 Find the unknowns in each of the following right-angled triangles:

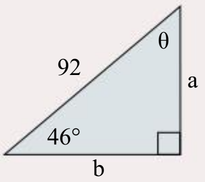
a)



b)

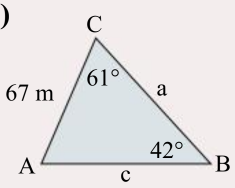


c)

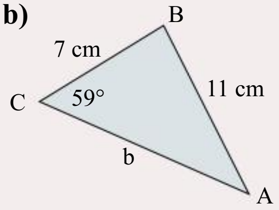


2 Use the sine rule and/or the cos rule to help solve each of the following triangles.

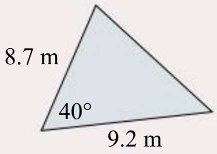
a)



b)



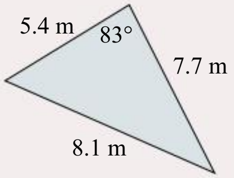
c)



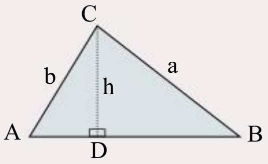
3 Use the unit circle to calculate the following trigonometric ratios:

a) cos 270°   **b)** sin 150°   **c)** tan 330°

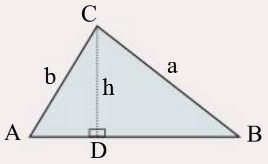
4 Find the area of the following triangle



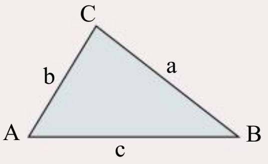
5 Show that: $Area = 0.5ac\sin B$



6 Show that: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$



7 Show that: $c^2 = a^2 + b^2 - 2ab\cos C$



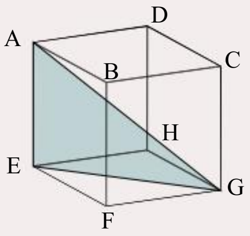
8 The cube has a side length of 10 cm. EG is a diagonal of the square EFGH. AG is a body diagonal of the cube.

a) Find the length of EG

b) Find the length of AG

c) Find angle EGA

d) Find the area of ΔAEG



Review 4

20

Chapter 16 Coordinate Geometry

$m = \text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

If two lines are parallel then they have the same gradient.
m1 = m2

Gradient of lines parallel to the x-axis:
m = 0

Gradient of lines parallel to the y-axis:
m = ∞

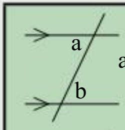
Equation of a line parallel to the x-axis:
y = c

Equation of a line parallel to the y-axis:
x = c

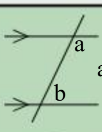
If two lines are perpendicular then the product of their gradients is -1.
$\mathbf{m_1 \times m_2 = {}^-1}$

Equation of a line:
y = mx + c
m = gradient
c = intercept on y-axis

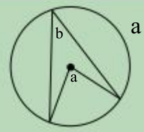
Chapter 17 Geometric Reasoning



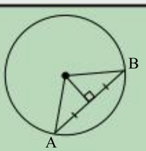
a = b

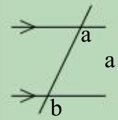


a + b = 180°

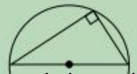


a = 2b

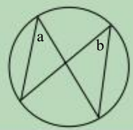




a = b



angle in semi-circle=90°



a = b

The perpendicular from the centre to a chord bisects the chord

Chapter 18 Statistics 2

Scatterplots give a visual representation of how the values of one variable are related to the values of another variable.

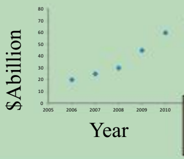
A correlation indicates a relationship between two variables.

linear or non-linear

positive or negative

strong or weak

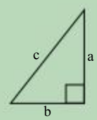
Value of Australian exports to China



The dependent variable, Year, is placed on the x-axis.

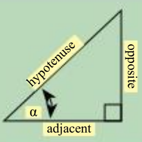
A 'line of best fit' is a straight line that best represents the data on a scatterplot.

Chapter 19 Trigonometry 2



$a^2 + b^2 = c^2$

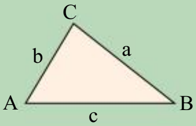
$c^2 = a^2 + b^2$



$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$



Side a is opposite angle A
Side b is opposite angle B
Side c is opposite angle C

The sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

The cos rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area $Area = 0.5ab\sin C$

Review 1

Exercise 20.1

1 Spell Trigonometry

2 What is the sine rule?

3 What is the cos rule?

4 What is the area of any triangle?

5 Name two properties of an isosceles triangle

6 What is the gradient of the line $y = 2x + 3$?

7 What is the gradient of the line perpendicular to $y = 2x + 5$?

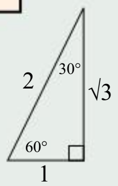
8 In the triangle, what is cos 60°?

9 Factorise: $x^2 - 5x + 6$

10 Two sides of a right-angled triangle are 1 and 2, what is the hypotenuse?

If two lines are perpendicular then the product of their gradients is -1.

$$\mathbf{m_1 \times m_2 = {}^-1}$$

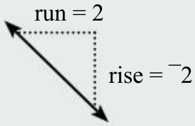


$x^2 - 5x + 6$
$= (x - 3)(x - 2)$

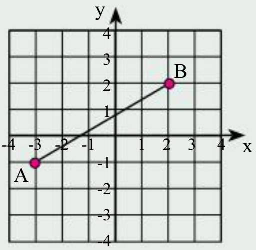
Exercise 20.2

1 Find the gradient of each of the following line segments:

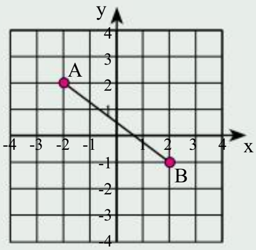
a)



b)



c)



2 Find the gradient of each of the following line segments and decide whether the line segment is parallel to the x-axis or the y-axis:

a) A(3,-3), B(1,-3)

b) A(2,5), B(2,-1)

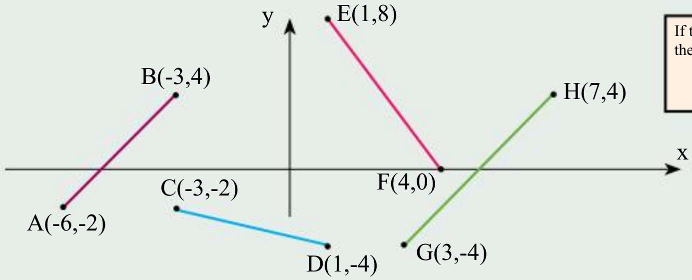
Gradient of lines parallel to the x-axis:

$$\mathbf{m = 0}$$

Gradient of lines parallel to the y-axis:

$$\mathbf{m = \infty}$$

3 Find the gradient of each of the following lines (not to scale) and thus show which lines are parallel or perpendicular:



If two lines are parallel then they have the same gradient.

$$\mathbf{m1 = m2}$$

4 Which pairs of lines are parallel and which are perpendicular:

a) $y = -2x + 3$ and $y = -2x - 2$

b) $y = 2x - 3$ and $3y - 6x + 1 = 0$

c) $y = {}^-4x + 1$ and $y = 0.25x + 3$

d) $4y + x - 2 = 0$ and $2y - 8x + 7 = 0$

If two lines are perpendicular then the product of their gradients is -1.

$$\mathbf{m_1 \times m_2 = {}^-1}$$

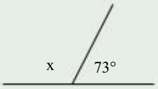
5 If the gradient of the line segment A(3,4), B(a,-2) is 1, what is the value of a?

6 What is the value of b if A(1,2), B(-3,-2), and C(2, b) are collinear?

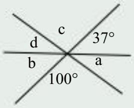
7 What is the equation of the line that cuts the y-axis at (0, 2) and is parallel to the line segment A(3,-1), B(1,2)?

8 Find the value of the unknowns. Show all working:

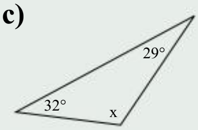
a)



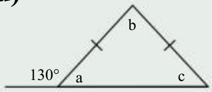
b)



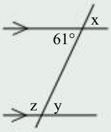
c)



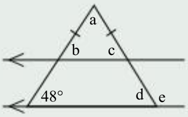
d)



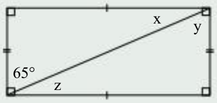
e)



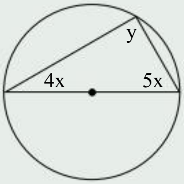
f)



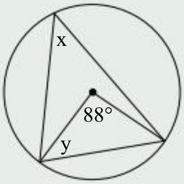
g)



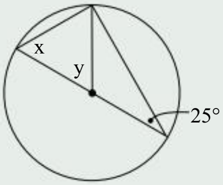
h) 10A



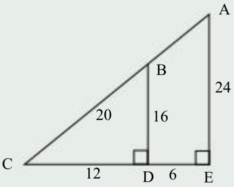
i) 10A



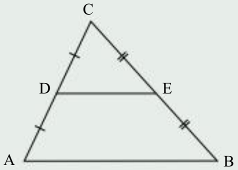
j) 10A



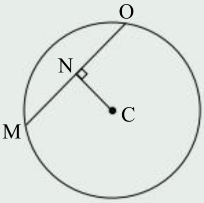
9 Prove that △ACE ~ △BCD and find AC



10 Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side.



11 10A Find MO given that CN ⊥ OM, radius = 25 cm, CN = 18 cm.



A theorem is a statement that can be proved using deductive reasoning.

12 The value of Australian exports to China is shown in the table.

a) Draw a scatterplot of the data.

b) Describe the relationship as suggested by the scatterplot.

c) Use the scatterplot to estimate the value of exports in 2015.

d) What confidence might you have in your estimation?

Value of Australian exports to China	
Year	$Abillion
2006	20
2007	25
2008	30
2009	45
2010	60
2011	75

13 10A The relationship between the current and the resulting voltage drop is shown in the table

a) Produce a scatterplot.

b) Use technology to find a line of best fit.

c) Use the line of best fit to predict the voltage drop when the current is 20 microamps. Comment.

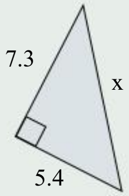
d) Use the line of best fit to predict the voltage drop when the current is 120 microamps. Comment.

e) Use the points from c) and d) to plot the line of best fit.

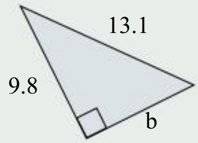
Ohms Law experiment	
Current (microamps)	Voltage (millivolts)
10	5.8
15	9.1
30	19
45	26
50	28
80	49
85	51
90	56
95	57

14 10A Find the unknowns in each of the following right-angled triangles:

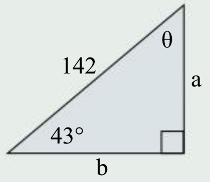
a)



b)

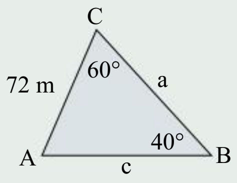


c)

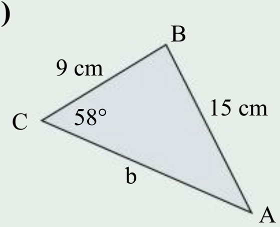


15 10A Use the sine rule and/or the cos rule to help solve each of the following triangles.

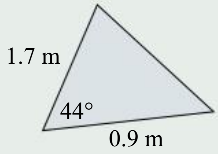
a)



b)



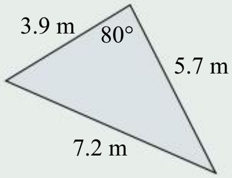
c)



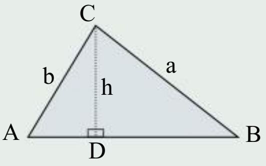
16 10A Use the unit circle to calculate the following trigonometric ratios:

a) cos 180° **b)** sin 150° **c)** tan 330°

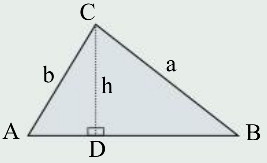
17 Find the area of the following triangle



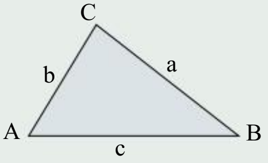
18 Show that: $Area = 0.5ac\sin B$



19 Show that: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

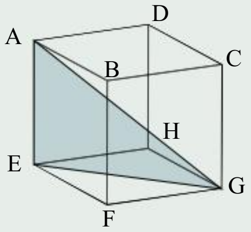


20 Show that: $c^2 = a^2 + b^2 - 2ab\cos C$



21 The cube has a side length of 20 cm. EG is a diagonal of the square EFGH. AG is a body diagonal of the cube.

a) Find the length of EG
b) Find the length of AG
c) Find angle EGA
d) Find the area of ΔAEG



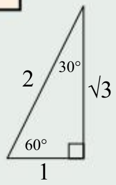
Review 2

Exercise 20.3

If two lines are perpendicular then the product of their gradients is -1.

$$\mathbf{m_1 \times m_2 = {}^-1}$$

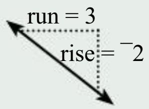
1 Spell Coordinate Geometry
2 What is the sine rule?
3 What is the cos rule?
4 What is the area of any triangle?
5 Name two properties of an equilateral triangle
6 What is the gradient of the line $y = 5x - 1$?
7 What is the gradient of the line perpendicular to $y = 3x + 4$?
8 In the triangle, what is cos 30°?
9 Factorise: $x^2 - 4x + 3$
10 Two sides of a right-angled triangle are 1 and 4, what is the hypotenuse?



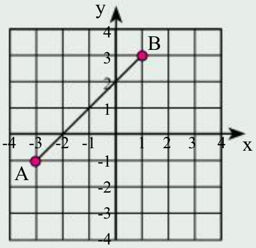
Exercise 20.4

1 Find the gradient of each of the following line segments:

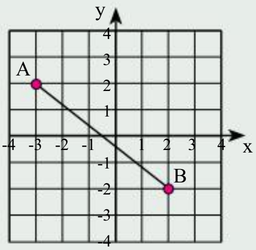
a)



b)



c)



2 Find the gradient of each of the following line segments and decide whether the line segment is parallel to the x-axis or the y-axis:

a) A(5,-2), B(1,-2)

b) A(1,3), B(1,-1)

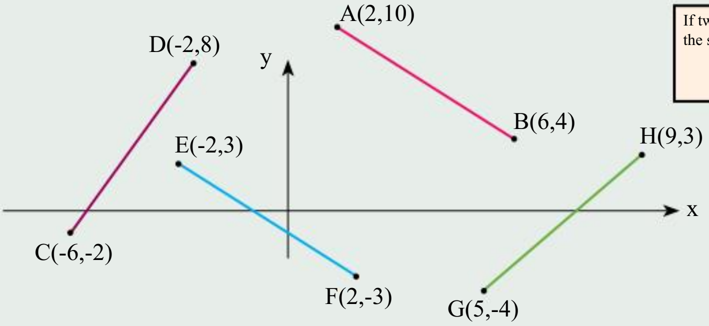
Gradient of lines parallel to the x-axis:

$$m = 0$$

Gradient of lines parallel to the y-axis:

$$m = \infty$$

3 Find the gradient of each of the following lines (not to scale) and thus show which lines are parallel or perpendicular:



If two lines are parallel then they have the same gradient.

$$m1 = m2$$

4 Which pairs of lines are parallel and which are perpendicular:

a) $y = x + 3$ and $y = x - 2$

b) $y = 2x - 3$ and $2y - 4x + 1 = 0$

a) $y = {}^{-}2x + 1$ and $y = 0.5x + 6$

c) $4y + x - 2 = 0$ and $2y - 8x + 7 = 0$

If two lines are perpendicular then the product of their gradients is -1.

$$m_1 \times m_2 = {}^{-}1$$

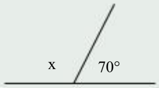
5 If the gradient of the line segment A(-1,4), B(x,-3) is 2, what is the value of x?

6 What is the value of b if A(-1,-2), B(5,-1), and C(3, b) are collinear?

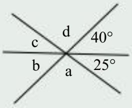
7 What is the equation of the line that cuts the y-axis at (0, 3) and is parallel to the line segment A(-3,2), B(1,-4)?

8 Find the value of the unknowns. Show all working:

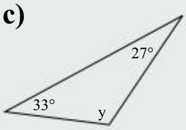
a)



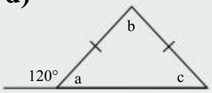
b)



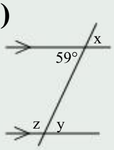
c)



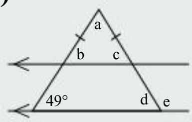
d)



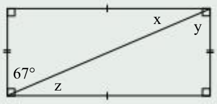
e)



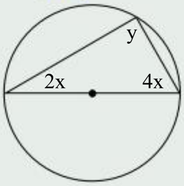
f)



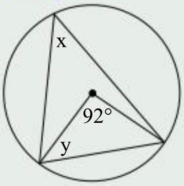
g)



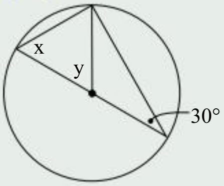
h) 10A



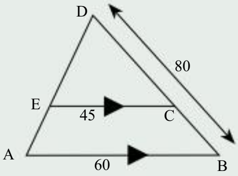
i) 10A



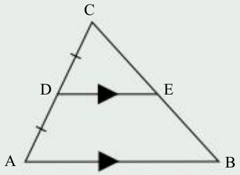
j) 10A



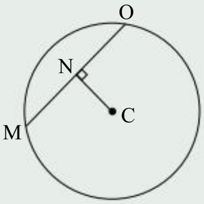
9 Prove that △ABD ~ △ECD and find DC



10 Prove that the line from the midpoint of a side of a triangle and parallel to another side, bisects the third side.



11 10A Find MO given that CN ⊥ OM, radius = 30 cm, CN = 22 cm.



A theorem is a statement that can be proved using deductive reasoning.

12 Global mean sea level set to base level of 0 mm in 1990 is shown in the table.

a) Draw a scatterplot of the data.

b) Describe the relationship as suggested by the scatterplot.

c) Use the scatterplot to estimate the sea level in 2015.

d) What confidence might you have in your estimation?

Global mean sea levels

Year	Sea level (mm)
1990	0
1993	10
1996	18
1999	25
2002	38
2005	49
2008	54
2011	65

13 10A The relationship between the number of bedrooms and the price of a house is being examined by detailing advertisements of houses for sale (restricted to one suburb only).

a) Produce a scatterplot.

b) Use technology to find a line of best fit.

c) Use the line of best fit to initially set the price of a 3 bedroom house. Comment.

d) Use the line of best fit to initially set the price of a 5 bedroom house. Comment.

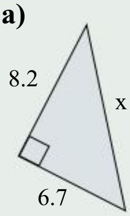
e) Use the points from c) and d) to plot the line of best fit.

Bedrooms and house prices

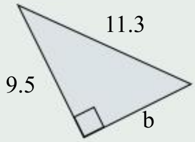
Bedrooms	$Price
1	350 000
1	370 000
2	440 000
2	450 000
3	520 000
3	530 000
4	610 000
4	605 000

14 10A Find the unknowns in each of the following right-angled triangles:

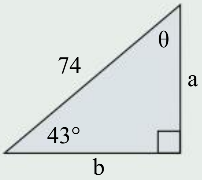
a)



b)

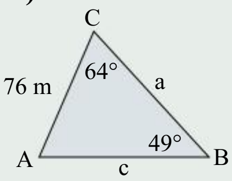


c)

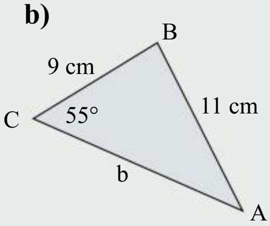


15 10A Use the sine rule and/or the cos rule to help solve each of the following triangles.

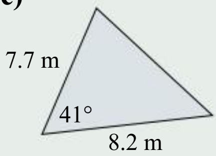
a)



b)



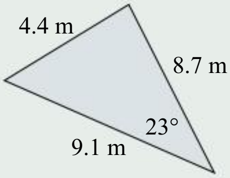
c)



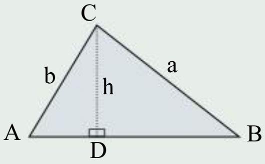
16 10A Use the unit circle to calculate the following trigonometric ratios:

a) cos 270° **b)** sin 120° **c)** tan 225°

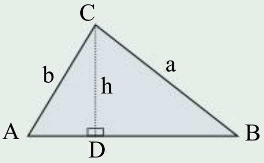
17 10A Find the area of the following triangle



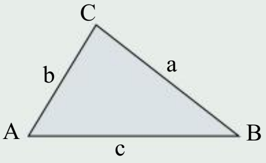
18 10A Show that: $Area = 0.5ac\sin B$



19 10A Show that: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$



20 10A Show that: $c^2 = a^2 + b^2 - 2ab\cos C$



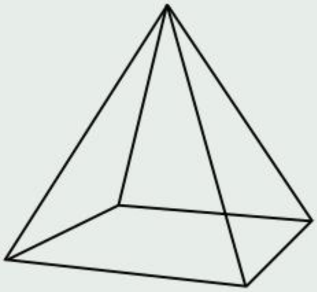
21 10A The Pyramid of Khafre, a square based pyramid, has a height of 143 m and a base length of 215 m

a) Find the base diagonal

b) Find the angle of the faces

c) Find the gradient of the faces

d) Find the total area of the four faces



Glossary

Acute − An acute angle is a sharp angle between 0° and 90°.

Angle − An angle is the measure of turn between two lines. Angles are measured in degrees from 0° to 360°, eg. 147°. In later studies other measures such as radians will be introduced.

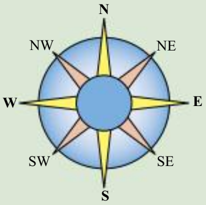
Angle sum of a polygon −
The sum of the interior angles of a triangle is 180°.
The sum of the interior angles of a quadrilateral is 360°.
The sum of the interior angles of a pentagon is 540°.
The general rule: Sum interior angles = (no sides − 2)x 180°

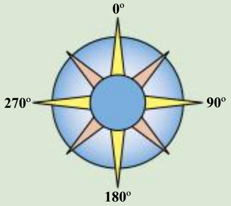
Ascending order of numbers is an order from smallest to largest.
Example: 2, 3, 5, 10 is in ascending order.

Average − An average is a central measure. Average and mean are the same.
(The mode, and median, although different, are also central measures).

Area − The area is the amount of surface.
Area of rectangle = length x breadth
Area of triangle = 1/2 x base x height
Area of circle = π x radius^2 = πr^2
Area of parallelogram = base x height

Bearing − The bearing is the angle measured clockwise from North.
The bearing of North is 0°.
The bearing of East is 90°.
The bearing of South is 180°.
The bearing of South West is 225°.
The bearing of West is 270°.





Bias − Unfair sampling. The sample does not represent the population.

Census − Collection of data from the entire population.

Centimetre − A centimetre is one-hundredth of a metre. 100 cm = 1 m.

Circumference − The circumference is the distance around the outside of a circle.
C = 2πr or C = πd

Complementary angles are angles that sum to 90°.
Example: 40° and 50° are complementary angles.

Complementary events - The complement of any event (A) is the event (not A). The probabilities of complementary events add to 1.

Composite number has more than two factors.
Example: 8 has factors of 1, 2, 4, 8. 8 is a composite number.

Compound stem-and-leaf plot − A compound stem-and-leaf plot has two stem-and-leaf plots joined together.

Congruent objects have the same shape and the same size.

The symbol for congruence is ≡ or ≅
The tests of congruent triangles are:
SSS (side, side, side).
SAS (side, angle, side).
ASA (angle, side, angle).
RHS (right-angle, hypotenuse, side).

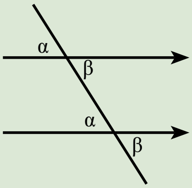
Consecutive numbers are numbers that follow one another.
Example: 3, 4, 5 are consecutive numbers.

Continuous numbers are numbers that can have any value. Weight is continuous because the weight of an object can be any number on the number line. Discrete numbers can have only certain values - the number of people in the class must be a whole number. There can't be 4.62 people in the class.

Compound Interest - Compound interest arises when interest is added to the principal. The interest that has been added also earns interest. This addition of interest to the principal is called compounding. Eg A bank account may have its interest compounded every year: in this case, an account with $1000 initial principal and 20% interest per year would have a balance of $1200 at the end of the first year, $1440 at the end of the second year, and so on.

Coordinates − An ordered pair on numbers that fix a point in the cartesian plane.
Example: P(2,5). The point P is 2 units to the right and 5 units up from the origin (0,0).

Corresponding angles − matching angles when a line cuts a pair of lines.
If the lines are parallel, the corresponding angles are equal.



α is one pair of corresponding angles.
β is another pair of corresponding angles.

Cube − A cube is a three-dimensional object with all six faces congruent and each face having the shape of a square. A cube is one of the five platonic solids.

Cubed − A cubed number is the number multiplied by itself three times.
Example: Two cubed = $2^3 = 8$.

Cubic centimetre is the amount of space occupied by a cube with each side of length 1 cm. The unit is 1 cm^3 (1 L = 1 000 cm^3).

Cubic metre is the amount of space occupied by a cube with each side of length 1 m. The unit is 1 m^3 (1 m^3 = 1 000 L).

Data - Information collected for analysis or reference.

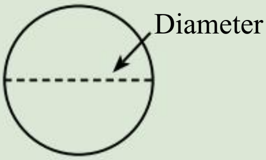
Decagon is a polygon with 10 sides and 10 angles.

Decimal place − The number of places after the decimal point.
Example: 5.281 has three decimal places.

Denominator in a fraction is the number at the bottom.

Descending order of numbers is an order from largest to smallest.
Example: 10, 5, 3, 2 is in descending order.

Diameter − The diameter of a circle is the length of the line joining two points on the circle and that passes through the centre of the circle.



Die − A die is a cube with each of the numbers 1, 2, 3, 4, 5, 6 on each of the six faces.
The opposite sides of a die sum to seven.
Die is singular, dice is plural.

Digit − A digit is a single number.
Example: The number 435 has the digits 4, 3, and 5.

Discrete numbers are numbers that can only have certain values, normally whole numbers.
Example: The number of people in the class is discrete (Can't be 4.62 people).

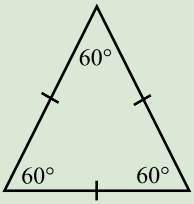
Distributive law − Each term in the brackets is multiplied by the term outside the brackets. Example: $3(a + 5) = 3a + 15$

Dividend − The dividend is the number being divided. In 45 ÷7, 45 is the dividend.

Divisor − The divisor is the number dividing. In 45 ÷7, 7 is the divisor.

Dodecahedron − A dodecahedron is a three-dimensional object with all twelve faces congruent and each face having the shape of a regular pentagon (5 sides of equal length). A dodecahedron is one of the five platonic solids.

Equation − An equation is a mathematical sentence with an equals sign.
Example: $2x + 5 = 9$ is an equation.



An **equilateral triangle** is a triangle with three equal sides.
Each of the three angles in an equilateral triangle are 60°.

Estimate − To make an approximate guess of the answer.
Example: An estimate of 43 x 26 is 40 x 30 = 1200

Evaluate − To evaluate an expression is to find the value of the expression.
Example: Evaluate $2x(3-1)$
$2x(3-1) = 2x2 = 4$

Even numbers are numbers that are exactly divisible by 2.
Example: 2, 4, 6, 8, 10 are even numbers.

Expand − Each term in the brackets is multiplied by the term outside the brackets.
Example: $3(a + 5) = 3a + 15$

Factors − The factors of a number are the numbers which divide exactly into the number.
Example: The factors of 6 are 1, 2, 3, 6.

Factorise − To make into a product.
Example: $3a + 15 = 3(a + 3)$

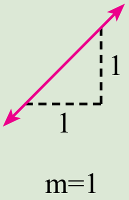
Finite − A definite number.
Example: {2, 3, 1, 6 } has a finite number of elements. It has 4 elements.
The opposite of finite is infinite.

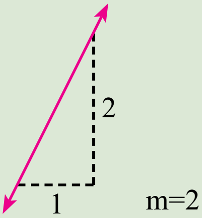
Formula − A formula is an equation.
The formula for the perimeter of a circle is: $C = 2\pi r$

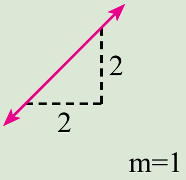
Frequency − The number of times a number occurs.
Example: 2, 3, 2, 3, 2, 2 The frequency of 2 is 4 {2 occurs 4 times}

Gram − A gram is a measure of mass and is one thousandth of a kilogram.
1 000 g = 1 kg

Gradient − A measure of the slope.
Example: The gradient of the line $y = 2x - 1$ is 2. $m = 2$







Greater than − The symbol for greater than is >
Example: 6 is greater than 4, $6 > 4$

Heptagon − A polygon with seven sides and seven angles.

Hexagon − A hexagon is a polygon with six sides and six angles.

Highest common factor − The largest factor that is common.
Example: The highest common factor of 12 {1,2,3,4,6,12} and 8 {1,2,4,8} is 4.

Hypotenuse - The longest side in a right-angled triangle.
The hypotenuse is opposite the right-angle.

Icosahedron − An icosahedron is a three-dimensional object with all twenty faces congruent and each face having the shape of an equilateral triangle.
A icosahedron is one of the five platonic solids.

Improper fraction − An improper fraction or vulgar fraction is a fraction with the numerator larger than the denominator.
Example: $\frac{5}{3}$ is an improper fraction.

Index − The power when a number is written in index form.
Example: $9 = 3^2$. The index is 2.

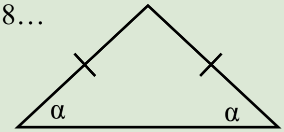
Infinite − Too large/many to be counted. Not finite.

Integers are whole positive numbers and whole negative numbers.
Example: −2, 3, 4, −7 are integers. 3.4 is not an integer.

Intersection − The point where two lines cross each other. The common numbers in a set of two numbers.

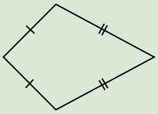
Irrational number − An irrational number is a number that cannot be written as a common fraction or as a decimal fraction that terminates or recurs.

Example: π is irrational because it cannot be written as a decimal that terminates or recurs. π = 3.14159265358…
$\sqrt{2}$ is irrational = 1.41421356….



Isosceles − A triangle with two sides of equal lengths.

The angles opposite the equal sides are equal.



Kite − A quadrilateral with two pairs of adjacent sides equal.

Kilogram − A kilogram is a measure of mass. 1 kg = 1 000 g. 1 tonne = 1 000 kg.

Kilometre − A kilometre is a measure of length. 1 km = 1 000 m.

Latitude − The latitude of a position on Earth is the angle North or South of the Equator.

Adelaide's latitude is 34.55°S, Adelaide's longitude is 138.35°E.
Brisbane's latitude is 27.28°S, Brisbane's longitude is 153.01°E.
Canberra's latitude is 35.27°S, Canberra's longitude is 149.12°E.
Darwin's latitude is 12.28°S, Darwin's longitude is 130.50°E.
Hobart's latitude is 42.53°S, Hobart's longitude is 147.19°E.
Melbourne's latitude is 37.82°S, Melbourne's longitude is 144.95°E.
Perth's latitude is 31.95°S, Brisbane's longitude is 115.83°E.
Sydney's latitude is 33.52°S, Sydney's longitude is 151.13°E.

Litre − A litre is a measure of volume. 1 kg = 1 000 mL.

Longitude − The longitude of a position on Earth is the angle East or West of the line of meridian through Greenwich. See Latitude for examples.

Mean − The mean of a set of numbers is the sum of the numbers divided by the number of numbers. The mean of 2, 4, 5, 7 =

Median − The median of a set of numbers is the middle number when the numbers have been put in order.

Example: Find the median of: 4, 5, 2, 3, 6, 7, 2
In order: 2, 2, 3, 4, 5, 6, 7
The median is 4
Find the median of: 1, 3, 1, 0, 4, 3
In order: 0, 1, 1, 3, 3, 4
The median is the mean of 1 and 3 = 2.

Metre − The metre, m, is the standard measure of length.

Millimetre − A millimetre is one thousandth of a metre. 1 m = 1 000 mm.

Mixed number − A mixed number consists of a whole number and a fraction.

Example: $2\frac{3}{5}$

Mode − The mode of a set of numbers is the number that occurs the most.
Example: 2, 4, 3, 3, 5, 3. The mode is 3 (3 occurs three times).
1, 5, 4, 1, 5, 3. The mode is 1 and 5 (bimodal).

Net − The net of a solid is the shape that can be folded to make the solid.

Numerator − The numerator is the top number in a fraction.

Obtuse angle − An obtuse angle is an angle between 90° and 180°.

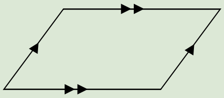
Octagon − An octagon is a polygon with eight sides and eight angles.

An **octahedron** is a three-dimensional object with all eight faces congruent and each face having the shape of an equilateral triangle. A octahedron is one of the five platonic solids.

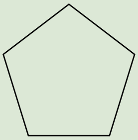
Odd numbers are numbers that are not exactly divisible by 2.
Example: 1, 3, 5, 7, 9 are odd numbers.

Obtuse angle − An angle greater than 90° and less than 180°.

Parallelogram − A parallelogram is a quadrilateral, four sided figure, in which the opposite sides are parallel.



Pentagon − A pentagon is a polygon with five sides and five angles.



Per annum − Per year.

Percentage − A percentage is a fraction of 100. $43\% = \frac{43}{100}$

Perimeter − The perimeter is the distance around the outside edge of a figure.

Perpendicular lines are lines that are at 90° to each other.

Pi, π, is the ratio of the circumference of a circle to the diameter.
$\pi = 3.14159265358\ldots$

Polygons are shapes made up of straight lines. Triangles (3 sides), quadrilaterals (4 sides), pentagons (5 sides), hexagons (6 sides) etc are polygons.

Polyhedron − A solid shape with flat sides. Cube, dodecahedron, icosahedron, etc

Power − The power of a number is the number of times the number is multiplied by itself.
Example: $3x3x3x3x3 = 3^5$. {3 to the fifth power

Probability − The chance of an event happening.
Probability ranges from a low of 0 (no chance) to a high of 1 (certain).

$$P(event) = \frac{No\ of\ \ favourable\ outcomes}{Total\ no\ of\ \ outcomes}$$

Prime number − A prime number is a number with just two factors, 1 and itself. 2, 3, 5, 7, 11, 13 are prime numbers. 1 is not a prime number.

Prism − A prism is a three-dimensional shape in which the base shape is repeated from the bottom to the top.

Cylinder.
A circular based prism.

Probability is the chance of an event happening.
If a die is thrown, the chance of a 3 showing is

Quadratic − An equation in which the highest power of x is 2
Example: $y = 2x^2 - 5x + 3$

Quadrilateral − A quadrilateral is a figure with four straight lines.

Quartile − A value that divides the data in quarters.
Upper quartile, median, lower quartile.

Quotient − The quotient is the result of a division.
Example: The quotient of 10÷5 is 2.

Radius − The radius of a circle is the distance from the centre of a circle to a point on the circle.

Range − The difference between the highest and the lowest value.

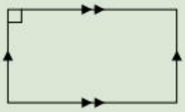
Ratio − A ratio is a comparison of two quantities. A certain two stroke petrol is made by mixing one part of two stroke oil to 32 parts of unleaded petrol (1: 32).

Rational number − A rational number is a number that can be written as a common fraction or as a decimal fraction that either terminates or recurs.

Example: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{-2}{3}$

Example: π is irrational because it cannot be written as a decimal fraction that terminates or recurs. π = 3.14159265358…

Rectangle − A rectangle is a four sided figure in which the opposite sides are parallel and the internal angles are 90° (right-angles).



Rectangular prism − A rectangular prism is a prism in which the base is a rectangle.

Right-angle − A right-angle is 90°.

Rounding - Giving an approximation of a number using the nearest more convenient number is called rounding. When rounding to the nearest ten, 11, 12, 13 and 14, round to 10, whereas 15, 16, 17, 18 and 19 round to 20.

Similar figures have the same shape. Congruent figures have a the same shape and the same size.

Square − A square is figure with four equal sides and each internal angle of size 90°.

Square centimetre, cm^2, is the area occupied by a square with each side of length 1 cm.

Square metre, m^2, is the area occupied by a square with each side of length 1 m ($1\ m^2 = 10\ 000\ cm^2$).

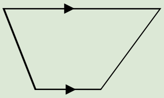
Symmetry - Property of regularity in shape by, for example, reflection or rotation. The letter T is symmetrical by reflection, the letter Z is symmetrical by rotation, the letter H is symmetrical by both reflection and rotation, the letter R is not symmetrical.

Surface area - The surface area of an object is the sum of the area of the various faces that make up the object.

Tonne − A tonne, t, is a measure of mass (1 t = 1 000 kg).

Transformation - A movement of figures and objects. The transformations translation (slide), rotation (turn) and reflection (flip) do not change the size or shape of the figure or object.

Trapezium − A trapezium is a four-sided figure with one pair of opposite sides parallel.



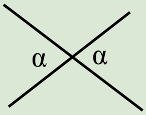
Triangle − A triangle is a figure with three sides.

Two-way table − A table that shows the sample space of a two-stage experiment.

Variables are letters used in equations, formulas, and expressions.
Example: x is a variable in the equation: $3x + 4 = 12$.

Vertex − The corner point of an angle.

Vertically opposite angles − A pair of non-adjacent angles formed when two lines intersect. Vertically opposite angles are equal.



Vinculum − The horizontal line separating the numerator from the denominator.

Volume of a figure is a measure of the amount of space occupied by the figure.
Example: The volume of a prism = area of base x height

Whole numbers are the positive counting numbers.
Example: 0, 1, 2, 3, 4, 5, etc.

x-intercept − The point where the graph cuts the x-axis.

y-intercept − The point where the graph cuts the y-axis.

Answers

Exercise 1.1 **1** 2^3b^2 **2** a^4b^4 **3** 3^6x^3 **4** x^4y^5 **5** 10^4d^4 **6** 5^5p^6 **7** 10^6 **8** 3^5 **9** 2^7 **10** 10^8 **11** x^5 **12** x^6 **13** 4.1^6 **14** d^8 **15** x^5 **16** y^4 **17** 0.2^7 **18** a^5 **19** 2.3^6 **20** y^4 **21** 10^{10} **22** x^7 **23** 10^2 **24** 10^3 **25** 4^2 **26** 1 **27** x^3 **28** y^2 **29** 10^3 **30** a **31** 10^4 **32** b^2 **33** 3 **34** 10^3 **35** x^5 **36** 4.3^3 **37** 10^5 **38** 1 **39** 10^2 **40** x^3 **41** a^5 **42** 1

Exercise 1.2 **1** b^8 **2** b^6 **3** b^6 **4** 10^6 **5** x^4 **6** x^{10} **7** y^{12} **8** y^{10} **9** 10^7 **10** x^{11} **11** 2^{11} **12** b^{18} **13** 1 **14** 1 **15** 1 **16** 1 **17** 5 **18** 5 **19** 4 **20** 2 **21** x **22** b^2 **23** 10 **24** 10 **25** 10^{-5} **26** b^{-4} **27** 10^{-1} **28** 10^{-2} **29** 10^{-1} **30** 10^2 **31** 10^8 **32** 10^{-2} **33** 5 **34** 10^4 **35** x^6 **36** 10^{-6} **37** x^2 **38** y^{-1} **39** 10^{-8} **40** y **41** 10^{-8} **42** 10^{-8} **43** a^{-6} **44** 10^{10} **45** y^8 **46** 2 **47** y^{-16} **48** y^7

Exercise 1.3 **1** 5x+10 **2** 4a+12 **3** y^2+2y **4** $-x^2-2x$ **5** $3x^3-3x^2$ **6** $8x^2-20x$ **7** $-15x^3+10x^2$ **8** $-4y^3+2y$ **9** $12a^5+8a^2b$ **10** x+9 **11** $-2x^2+3x$ **12** -x+7 **13** $-2a^2-8a$ **14** -x+4 **15** $-2a^3-8a^2$ **16** x^2+3x+2 **17** $x^5-x^3+2x^2-2$ **18** x^2+4x+3 **19** $x^5+3x^3-x^2-3$ **20** x^2+4x+3 **21** x^6+2x^3-3 **22** x^2+2x+1 **23** x^2-2x+1 **24** x^4+4x^2+4 **25** x^2-4x+4 **26** x^2+6x+9 **27** x^4-6x^2+9 **28** x^2+6x+9 **29** x^6-6x^3+9

Exercise 1.4 **1** 3(2a+3) **2** 2a(2b-3) **3** 2c(5c-4) **4** 2(7x+5) **5** 2b(2a-3) **6** 2d(4d-3) **7** 3(3c+4) **8** 2x(4y+5) **9** 4x(4x-3) **10** 2(3x+5) **11** 3t(4s-5) **12** $3p^3(5p^2-12)$ **13** $3(2-3b^3)$ **14** $2x(2x^3-3)$ **15** $2c^4(5-4c)$ **16** $2(2+5a^4)$ **17** $4x(x^4+2)$ **18** $3d^3(3d^2+2)$ **19** $3(3-4x^5)$ **20** $2y(4y^2-5)$ **21** $3x^2(3-4x)$ **22** $2(3+5y^2)$ **23** 3x(4x+5) **24** $12y^3(y^2-3)$ **25** $5a^2(2a+3)$ **26** $3x^2(7x+6)$ **27** $3a^2(8a-9)$ **28** (x+5)(x+3) **29** (x+5)(x-4) **30** (x-1)(x+4) **31** (x-1)(x-2) **32** (x-6)(x+3) **33** (x-5)(x-4) **34** (x-2)(x+5) **35** (x-3)(x-7)

Exercise 1.5 **1** $10x^2$ **2** $12y^2$ **3** $6a^2$ **4** $24b^2$ **5** $2e^{10}$ **6** $16x^8$ **7** $-10x^3$ **8** $-12x^2$ **9** $-20x^4$ **10** $8x^3$ **11** $-12m^4$ **12** $-15w^5$ **13** $16h^3$ **14** $-15x^6$ **15** $6p^3d^2$ **16** $-8a^3b^4$ **17** $-28d^3e^2$ **18** $-15m^3n^2$ **19** $20a^4b^3c^2$ **20** $-15x^6y^4z^8$ **21** $20x^2$ **22** $20a^2$ **23** $9y^3$ **24** $14p^{-3}$ **25** $-8x^2$ **26** $-6x^{-3}$ **27** $28x^2y^4$ **28** $8ab^2$ **29** $-12x^3y^{-1}$ **30** $4e^3f^2$ **31** $-21a^2b^3$ **32** $4a^2b$ **33** $12x^{-2}$ **34** $-32x^{-5}$ **35** $-18y^{-3}$ **36** $20d^2$ **37** $-6g^{-1}$ **38** $8a^{-4}$ **39** $12x^{-6}y^{-8}$ **40** $8ab^{-8}$ **41** $-15a^{-9}b^{-1}$ **42** $4x^{-7}y^{-1}$ **43** $-10e^2f^{-9}$ **44** $12a^{-1}b^{-6}$

Exercise 1.6 **1** 2x **2** 4a **3** 4x **4** 2d **5** -2x **6** -2x **7** 5y **8** 5a **9** $7x^3$ **10** $7x^3$ **11** $2x^2$ **12** -2g **13** $3a^4$ **14** $-2x^2$ **15** $-4x^2y$ **16** $2x^2y$ **17** -5a **18** $-4a^3c^2/3$ **19** $2e^3f^3$ **20** $-3ab^3c/4$ **21** $2a^{-5}$ **22** $3b^4$ **23** $2x^{-5}$ **24** $4a^7$ **25** $-3w^3$ **26** $4s^{-1}$ **27** $3x^6$ **28** $3y^{-3}/2$ **29** 3x **30** $2n^{-7}$ **31** $-4m^{-4}$ **32** $-2a^3b^{-4}$ **33** $-2a^3b^{-5}$ **34** 4x **35** $-5m^4n^{-4}$ **36** $5a^3b^5$ **37** $-c^{-4}$ **38** $4a^4bc^3$

Exercise 1.7 **1** 3x/5 **2** 9a/4 **3** 7b/3 **4** 5c/6 **5** 5x/3 **6** $6x^2/5$ **7** 2e **8** $8a^3/4$ **9** 3a **10** $(3x^3+x^2)/7$ **11** 5x/3 **12** $y^5+7y^2)/6$ **13** 2x/5 **14** a **15** 5y/3 **16** a/6 **17** c **18** e/3 **19** x **20** $x^3/2$ **21** 2y **22** $6y^2/5$ **23** 3z/4 **24** $(3x^5-2x^2)/3$

Exercise 1.8 **1** $12/x^2$ **2** $10/a^3$ **3** $y^2/6$ **4** $y^4/12$ **5** $x^2/5$ **6** $12x^2/5$ **7** $6x^4/7$ **8** $6x^4/5$ **9** $14a^5/3$ **10** $2x^4/15$ **11** $x^4/6$ **12** $x^{-2}/6$ **13** 3/2 **14** 2/3 **15** 3/4 **16** 8/15 **17** 2e **18** 15x/14 **19** $15x^3/4$ **20** 4y/5 **21** $4t^5$ **22** $3ab^2/4$ **23** 3xy/2 **24** $3ab^2/14$

Exercise 1.9 **2** -2 **3** 7 **4** 10^5 **5** x **6** 2^{-6} **7** 5x/6 **8** x/6 **9** \$6.60 **10** \$150

Exercise 1.10 **2** -5 **3** -12 **4** 10^2 **5** x^3 **6** 2^{-6} **7** 7x/10 **8** 3x/10 **9** \$8.80 **10** \$200

Exercise 1.11 **2** -6 **3** -0.5 **4** 10 **5** x^{-10} **6** 2^6 **7** 8x/15 **8** 2x/15 **9** \$9.90 **10** \$350

Exercise 1.12 **1** 1/4,1/3,1/2,2/3,3/4 **2a** 3 **2b** 17 **2c** 7 **2d** 55 **2e** -13 **2f** 3 **3a** 10^7 **3b** 10^2 **3c** 10^3 **3d** 1 **4a** 4x+6y **4b** -4x-y **4c** a-4b **4d** 9 **4e** 2x-3 **4f** $7x^2-2x$ **5** x=2 **6** x=1.5 **7** (ba+ac+ab)/abc **8** x=6/7

Exercise 1.13 **1** 120g **2** \$8 **3a** 43×18=(344+430)=774 **3b** 32×28=(256+640)=896 **3c** 42×34=(168+1260)=1428 **3d** 45×26=(270+900)=1170

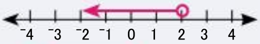
Exercise 1.14 **1** 5x+10 **2** $-x^2-2x$ **3** $x^5-x^3+2x^2-2$ **4** x^2+4x+3 **5** x^2+6x+9 **6** x^6+2x^3-3 **7** 2(3x+5) **8** 3t(4s-5) **9** $3p^3(5p^2-12)$ **10** $2(3+5y^2)$ **11** 3x(4x+5) **12** $12y^3(y^2-3)$ **13** $-12m^4$ **14** $-15w^5$ **15** $16h^3$ **16** $20x^2$ **17** $20a^2$ **18** $9y^3$ **19** $28x^2y^4$ **20** $8ab^2$ **21** $-12x^3y^{-1}$ **22** $7x^3$ **23** $3a^4$ **24** $-2x^2$ **25** $-3w^3$ **26** $4s^{-1}$ **27** $-2a^3b^{-5}$ **28** 3x **29** $-5m^4n^{-4}$ **30** $5a^3b^5$ **31** 5x/3 **32** $6x^2/5$ **33** 2e **34** $2a^3$ **35** x **36** $x^3/2$ **37** 2y **38** $(7x^2-y^2)/5$

Exercise 1.15 **1** 3x+12 **2** $-a^2-5a$ **3** $x^5+4x^3-x^2-4$ **4** x^2+3x+2 **5** x^2+4x+4 **6** x^4+2x^2-3 **7** 2(3x+4) **8** 3a(4b-3) **9** $3x^2(3x^3-5)$ **10** $4(2+3y^2)$ **11** $3x(2x^2+5)$ **12** $6y^3(3y-4)$ **13** $-10x^5$ **14** $-20x^4$ **15** $6y^6$ **16** $14x^3$ **17** $12a^3$ **18** $6y^4$ **19** $18x^2$ **20** $20xy^3$ **21** $-8a^3b$ **22** $5x^2$ **23** $5y^4/2$ **24** -3x/2 **25** $-5b^5$ **26** $4c^{-2}/3$ **27** $-5a^3b^{-6}$ **28** $5x^{-6}/6$ **29** $-7a^3b^{-5}/5$ **30** $7x^{-1}y/3$ **31** 3x/5 **32** 9x/4 **33** 5x/3 **34** $6x^3/5$ **35** 2x/5 **36** a **37** x **38** $2x^3/4$

Exercise 2.1 **1** 5days **2** 11days **3** 4.25mins **4** 3.75mins **5** 2.5hrs **6** 6.5hrs **7** 250km **8** 2640m

Exercise 2.2 **1** 8.91m **2** 11.94m **3** 26m **4** 28m **5** 5sides **6** 8sides **7** 6cm **8** 11.20cm **9** 5.37cm

Exercise 2.3

1 
2 
3 
4 
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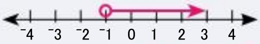
6 
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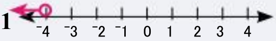
11 
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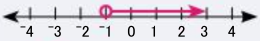
13 
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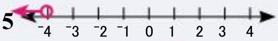
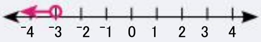
15 
16 
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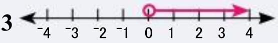
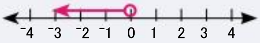
23 
24 
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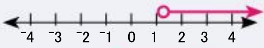
Exercise 2.4

1 
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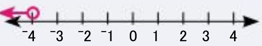
8 
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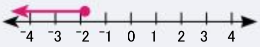
15 
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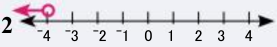
22 
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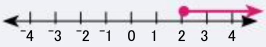
24 
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Exercise 2.5 **1** x=1,y=4 **2** x=2,y=5 **3** x=0.5,y=3.5 **4** x=1,y=5

Exercise 2.6 **1** x=-2,y=3 **2** x=1,y=4 **3** x=1,y=-2 **4** x=1,y=-2 **4** x=2,y=-1

Exercise 2.7 **1** x=2,y=3 **2** x=23,y=13 **3** x=16,y=27 **4** x=19,y=74 **5** x=300,y=156 **6** x=680,y=111 **7** x=-2.4,y=9.6 **8** x=4.45,y=-1.25 **9** two numbers are 59 and 39 **10** two numbers are 7 and 67 **11** two numbers are 159 and 202 **12** two numbers are 359 and 144 **13** adult=$60.10, child=$28.15 **14** l=550.75,b=521.75

Exercise 2.8 **1** x=8,y=7 **2** x=46,y=17 **3** x=109,y=12 **4** x=15,y=23 **5** x=13,y=37 **6** x=3.1,y=2.8 **7** x=9,y=2 **8** x=7,y=3 **9** the two numbers are 19 and 4 **10** x=1,y=5 **11** x=0,y=5 **12** x=9,y=12 **13** 13 hens, 19 sheep **14** 48 emus, 17 wombats

Exercise 2.9 **2** x=1.5 **3** x<2 **4** x=5,y=3 **5** x=5,y=5 **6** 8 **7** x **8** 2^{-6} **9** 5x/6 **10** $7.80

Exercise 2.10 **2** x=2 **3** x>4 **4** x=5,y=2 **5** x=4,y=6 **6** 5 **7** x^{-3} **8** 3^{-9} **9** x/6 **10** $9.10

Exercise 2.11 **2** x=1.5 **3** x<2 **4** x=6,y=4 **5** x=8,y=2 **6** -4 **7** x^2 **8** 2^{-8} **9** x/4 **10** $10.40

Exercise 2.12 **1** a=-4c+10 **2** a=-6c+4 **3** 3cm by 2cm **4** 5cm by 2cm **5** 7cm by 6cm **6a** a=15,b=9,c=8 **6b** a=7,b=3,c=-2 **6c** a=4,b=8,c=9,d=3 **7** 4 **8** 1 **9** x=y=z=1

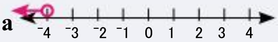
Exercise 2.13 **1** 6,3,3,3,5 : 3,7,3,2,5 : 3,3,8,3,3 : 3,5,5,5,2 **2** 45

Exercise 2.14 **1** 4.25mins **2** 10sides **3a**  **3b** 

3c  **4a** x=2,y=7 **4b** x=1,y=-1 **5a** x=2,y=3 **5b** x=6.5,y=-3.5 **5c** x=19,y=74

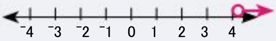
6 the two numbers are 8 and 88 **7a** x=8,y=7 **7b** x=46,y=17 **7c** x=1,y=5

Exercise 2.15 **1** 2.5hrs **2** b=26m **3a**



3b

3c



4a x=0.5,y=3.5 **4b** x=1,y=-2 **5a** x=2,y=5 **5b** x=-2,y=3 **5c** x=19,y=18 **6** the two numbers are 60 and 80 **7a** x=13,y=2 **7b** x=24,y=13 **7c** x=-2,y=5

Exercise 3.1 **1** $50.41m^2$ **2** $24m^2$ **3** $88.25cm^2$ **4** 16 $800m^2$, 1.68ha **5** 42 $550m^2$, 4.26ha **6** $63.62m^2$ **7** $140cm^2$ **8** $2206.86m^2$ **9** $13.73m^2$ **10** $15.71m^2$

Exercise 3.2 **1** $V=547.48m^3$, $SA=378.12m^2$ **2** $V=700.42cm^3$, $SA=467.47cm^2$ **3** $V=23.82m^3$, $SA=47.75m^2$ **4** $V=132.13m^3$, $SA=209.22m^2$ **5** $V=20304m^3$, $SA=5940m^2$ **6** $V=29.50cm^3$, $SA=71.08cm^2$ **7** $V=2.81m^3$, $SA=12.12m^2$ **8** $V=254.47m^3$, $SA=197.92m^2$ **9** $V=649.52m^3$, $SA=429.90m^2$

Exercise 3.3 **1** $V=413m^3$ **2** $V=27\ 646cm^3$ **3** $V=342.87m^3$ **4** $V=7776cm^3$ **5** $V=81.51m^3$ **6** $V=10.33m^3$

Exercise 3.4 **1** $243.31m^2$ **2** $367.74m^2$ **3** $391.40m^2$ **4** $425.78m^2$ **5** $54.63m^2$ **6** $170.53m^2$

Exercise 3.5 **1** $115.5m^3$ **2** $157.5cm^3$ **3** $118990mm^3$ **4** $4320mm^3$ **5** $2\ 876\ 104m^3$ **6** $623.35cm^3$ **7** $14137.17cm^3$ **8** $3.59m^3$ **9** $3156.55m^3$ **10** $217.45m^3$

Exercise 3.6 **1** $4188.79cm^3$ **2** $1385.44cm^3$ **3** $1470.27cm^3$ **4** $3266.67cm^3$ **5** $6361.73m^3$ **6** $282.74m^3$ **7** $535.12cm^3$ **8** $204.99cm^3$

Exercise 3.7 **2** πr^2 **3** πr^2h **4** ⅓AreaBase×height **5** $x<5$ **6** x=5,y=4 **7** 3 **8** x^{-6} **9** 5x/6 **10** 27

Exercise 3.8 **2** lb **3** lbh **4** ⅓ πr^2h **5** $x>2$ **6** x=6,y=4 **7** -10 **8** x^{-5} **9** x/6 **10** 64

Exercise 3.9 **2** ½base×⊥height **3** ½base×⊥height×h **4** ⅓½base×⊥height×h **5** $x>2$ **6** x=9,y=6 **7** -1 **8** x^3 **9** 3/2 **10** 125

Exercise 3.10 **1** 3cm **2** 15cm **3** F **4** 6+3+4=13 **5** 1cm×6cm×8cm is one answer **6** $1m^3$ **7** $23.37m^2$ **8** 4.5m

Exercise 3.11 **1** 15.5h **2** ellipse **3** 7 **4** 10

Exercise 3.12 **1** $V=71.23m^3$,$SA=105.1m^2$ **2** $V=331.10cm^3$,$SA=270.18cm^2$ **3** $V=96m^3$,$SA=152m^2$ **4** $V=428.25m^3$,$SA=426.14m^2$ **5** $V=289873cm^3$,$SA=31968cm^2$ **6** $V=121.15m^3$,$SA=278.63m^2$ **7** $V=220m^3$ **8** $V=282.63cm^3$ **9** $V=150533cm^3$ **10** $V=10405cm^3$ **11** $V=59700m^3$ **12** $V=227.50cm^3$

Exercise 3.13 **1** $V=112.79m^3$,$SA=141.9m^2$ **2** $V=1751.65cm^3$,$SA=884.67cm^2$ **3** $V=960m^3$,$SA=616m^2$ **4** $V=847.05m^3$,$SA=652.41m^2$ **5** $V=92520cm^3$,$SA=14724cm^2$ **6** $V=251.36m^3$,$SA=394.40m^2$ **7** $V=242.67m^3$ **8** $V=187.51cm^3$ **9** $V=91952cm^3$ **10** $V=1754.06cm^3$ **11** $V=659048m^3$ **12** $V=425.42cm^3$

Exercise 4.1 **1** $2\sqrt{2}$ **2** $2\sqrt{3}$ **3** $3\sqrt{2}$ **4** $2\sqrt{5}$ **5** $5\sqrt{2}$ **6** $2\sqrt{6}$ **7** $3\sqrt{3}$ **8** $10\sqrt{2}$ **9** $2\sqrt{7}$ **10** $2\sqrt{10}$ **11** $3\sqrt{5}$ **12** $2\sqrt{11}$

Exercise 4.2 **1** $\sqrt{6}$ **2** $\sqrt{6}$ **3** $\sqrt{15}$ **4** $\sqrt{10}$ **5** $2\sqrt{3}$ **6** $3\sqrt{2}$ **7** $5\sqrt{2}$ **8** $2\sqrt{6}$ **9** 4 **10** 6 **11** $2\sqrt{7}$ **12** $3\sqrt{5}$ **13** $5\sqrt{2}$ **14** $3\sqrt{7}$ **15** $4\sqrt{3}$ **16** $7\sqrt{2}$ **17** $30\sqrt{2}$ **18** $24\sqrt{3}$ **19** $30\sqrt{2}$ **20** $28\sqrt{15}$ **21** $2+\sqrt{6}$ **22** $3+\sqrt{6}$ **23** $\sqrt{10}+5\sqrt{2}$ **24** $\sqrt{6}-3\sqrt{2}$ **25** $6+\sqrt{6}$ **26** $14+2\sqrt{21}$ **27** $8-2\sqrt{5}$ **28** $5\sqrt{3}+15\sqrt{2}$ **29** 12-40

Exercise 4.3 **1** $7\sqrt{2}$ **2** $5\sqrt{7}$ **3** $4\sqrt{5}$ **4** $-4\sqrt{10}$ **5** $3\sqrt{3}+5$ **6** $4\sqrt{5}-9$ **7** $7\sqrt{2}-4$ **8** $-\sqrt{3}+10$ **9** $5\sqrt{3}$ **10** $8\sqrt{3}$ **11** $18\sqrt{2}$ **12** 0 **13** $-\sqrt{12}$ **14** $7\sqrt{3}$ **15** $11\sqrt{2}-8\sqrt{3}$ **16** $13\sqrt{2}+4\sqrt{3}$ **17** $11\sqrt{2}-4\sqrt{5}$ **18** $10\sqrt{5}-11\sqrt{2}$

Exercise 4.4 1,1,1,1: 2,2,2,2: 3,3,3,3: 4,4,4,4: 5,5,5,5: 6,6,6,6

Exercise 4.5 **1** 2 **2** 3 **3** 2 **4** 2 **5** 10 **6** 3 **7** 3 **8** 10 **9** 2 **10** 2 **11** 5 **12** 9 **13** 1/2 **14** 1/3 **15** 1/6 **16** 1/5 **17** 1/10 **18** 1/3 **19** 1/2 **20** 1/7 **21** 1/3

Exercise 4.6 **1** 8 **2** 4 **3** 8 **4** 1000 **5** 9 **6** 4 **7** 3 **8** 10 **9** 2 **10** 2 **11** 512 **12** 25 **13** x^3 **14** x^2 **15** x^3 **16** x^2 **17** a^5 **18** 1/3 **19** p^3 **20** b^6 **21** e^4 **22** $8a^3$ **23** $27b^3$ **24** $4x^2$ **25** $8x^9$ **26** $125x^9$ **27** $1000x^6$ **28** $16c^4$ **29** $32x^{20}$ **30** $125x^9$ **31** $1/(4x^2)$ **32** $1/(27b^3)$ **33** $1/(8a^3)$ **34** $1/(8x^9)$ **35** $1/(16x^8)$ **36** $1/(9a^6)$ **37** $1/(4x^2)$ **38** $1/()8x^3$

Exercise 4.7 **1** a^5 **2** $12a^5$ **3** x^7 **4** $6x^4$ **5** $10b^8$ **6** 8p **7** 5 **8** 4 **9** 3 **10** 10 **11** 1/10 **12** 7 **13** 2 **14** 8 **15** 2 **16** 2 **17** 3 **18** 5 **19** x **20** x **21** $x^{1/3}$ **22** 6a **23** 10e **24** $12y^2$ **25** x **26** $x^{3/4}$ **27** $2h^{2/5}$ **28** $12y^2$

Exercise 4.8 **1** x **2** b^4 **3** x^2 **4** $2x^3$ **5** $4d^3$ **6** 2p **7** 4 **8** 2 **9** 8 **10** 2 **11** 8 **12** 27 **13** 8 **14** 32 **15** 4 **16** 2 **17** 81 **18** 5 **19** $3x^{1/2}$ **20** $4x^{1/2}$ **21** x **22** 5s **23** 4k **24** $3y^{3/5}$ **25** 9 **26** 12 **27** 16 **28** 1152

Exercise 4.9 **1** 3/5 **2** 27/125 **3** 27/8 **4** 2/3 **5** 4/9 **6** 4 **7** 2^6 **8** 2^4 **9** x^{-9} **10** x^6y^2 **11** $1/(8x^6)$ **12** $27a^{12}$ **13** 4 **14** 8 **15** 27 **16** 9 **17** 4 **18** 2 **19** xy^{-1} **20** xy^2 **21** $a^{-3}b^{-1}$ **22** $x^{-1}y^{-1/2}$ **23** $9a^2$ **24** $x^{1/3}y^{4/3}$

Exercise 4.10 **1** 6 **2** 10 **3** 8 **4** 1 **5** 1 **6** 1 **7** 2 **8** 4 **9** 2 **10** 3 **11** 4 **12** 1

Exercise 4.11 **1** $\log_{10}100=2$ **2** $\log_5125=3$ **3** $\log_525=2$ **4** $\log_{10}1000=3$ **5** $\log_327=3$ **6** $\log_464=3$ **7** $\log_636=2$ **8** $\log_{10}100000=5$ **9** $\log_216=4$ **10** $\log_232=5$ **11** $\log_5625=5$ **12** $\log_6216=3$ **13** $\log_{10}0.1=-1$ **14** $\log_{10}0.01=-2$ **15** $8=2^3$ **16** $1000=10^3$ **17** $100=10^2$ **18** $125=5^3$ **19** $64=2^6$ **20** $10=10^1$ **21** $32=2^5$ **22** $27=3^3$ **23** $1024=4^5$ **24** $2401=7^4$ **25** $0.1=10^{-1}$ **26** $0.001=10^{-3}$ **27** 4 **28** 1 **29** 2 **30** 3 **31** 2 **32** 4 **33** 2 **34** -1 **35** 3 **36** 4 **37** 5 **38** 3 **39** 3 **40** 3 **41** 3 **42** -2 **33** **34** **35** **36** **37** **38** **39** **40** **41** **42**

Exercise 4.12 **1** 1 **2** 2 **3** 3 **4** 2 **5** 2 **6** 3 **7** 1 **8** 2 **9** 4 **10** 4 **11** 3 **12** 3 **13** 6 **14** 8 **15** 10 **16** 6 **17** 3 **18** 6 **19** 1 **20** 2 **21** 4 **22** 2 **23** 2

Exercise 4.13 **2** $2\sqrt{3}$ **3** 8 **4** 2 **5** πr^2h **6** $x<6$ **7** x=4,y=3 or vice versa **8** 4 **9** x^{-6} **10** 3x/5

Exercise 4.14 **2** $3\sqrt{2}$ **3** 9 **4** 2 **5** ⅓areabase×height **6** $x>1$ **7** x=5,y=3 or vice versa **8** -7 **9** x^3 **10** x/3

Exercise 4.15 **2** $3\sqrt{3}$ **3** 64 **4** 3 **5** πr^2 **6** $x<1.6$ **7** x=5,y=2 or vice versa **8** 4 **9** x^{-5} **10** x/6

Exercise 4.16 **1a** 50 **1b** 500 **1c** 5000 **1d** 1.29 **1e** 2.037 **1f** 5.406 **1g** 1 **1h** 0 **1i** 1 **1j** 4 **1k** 9 **1l** 49 **1m** -1 **1n** -2 **1o** -4 **2a** 10^2 **2b** 10^3 **2c** 2×10^3 **2d** 5×10^4 **2e** 10^{-2} **2f** 3×10^{-4} **3a** 3 **3b** 1 **3c** 4 **3d** 2 **3e** 5 **3f** 1 **4** 144km **5** 5 **6** -19 **7a** x=4 **7b** x=4 **7c** x=3 **7d** x=4 **7e** x=6 **7f** x=7 **8** 14 **9** 4

Exercise 4.17 **1** 1/4,1/3,1/2,2/3,3/4 **2** 62.5 km/h **3** x=6,y=-2 **4** $\sqrt{3}$

Exercise 4.18 **1a** $\sqrt{6}$ **1b** $4\sqrt{3}$ **1c** $\sqrt{10}+5\sqrt{2}$ **2a** $11\sqrt{2}-8\sqrt{3}$ **2b** $13\sqrt{2}-4\sqrt{3}$ **2c** $11\sqrt{2}-4\sqrt{5}$ **2d** $10\sqrt{5}-11\sqrt{2}$ **3a** 2 **3b** 10 **3c** 8 **3d** 8 **3e** $8a^3$ **3f** $1/(4x^2)$ **4a** 10 **4b** 10 **4c** 6a **4d** $x^{3/4}$ **4e** 4 **4f** 2 **4g** $3x^{1/2}$ **4h** $4x^{1/2}$ **4i** 3/5 **4j** 27/125 **4k** 4 **4l** $x^{1/3}y^{4/3}$ **5a** $\log_{10}100=2$ **5b** $\log_{5}125=3$ **5c** $\log_{2}32=5$ **5d** $\log_{10}0.01=-2$ **6a** $8=2^3$ **6b** $1000=10^3$ **6c** $2401=7^4$ **6d** $0.001=10^{-3}$ **7a** 4 **7b** 4 **7c** 2 **7d** -1 **8a** 1 **8b** 2 **8c** 3 **8d** 2 **8e** 4 **8f** 1 **8g** 2 **8h** 4 **8i** 4 **8j** 3 **8k** 3 **8l** 4 **8m** 6 **8n** 8 **8o** 10 **8p** 6

Exercise 4.19 **1a** $\sqrt{6}$ **1b** $5\sqrt{2}$ **1c** $14+2\sqrt{21}$ **2a** $9\sqrt{2}-5\sqrt{6}$ **2b** $2\sqrt{2}-7\sqrt{6}$ **2c** $5\sqrt{2}-\sqrt{10}-2\sqrt{5}+2$ **2d** $-9\sqrt{2}$ **3a** 2 **3b** 2 **3c** 4 **3d** 9 **3e** $27b^3$ **3f** $1/(27b^3)$ **4a** 10 **4b** 10 **4c** 10x **4d** 1 **4e** 9 **4f** 4 **4g** $4x^2$ **4h** $2b^{1/2}$ **4i** 2/3 **4j** 27/125 **4k** 27/8 **4l** x^2y^6 **5a** $\log_{3}9=2$ **5b** $\log_{4}64=3$ **5c** $\log_{2}64=6$ **5d** $\log_{10}0.001=-3$ **6a** $32=2^5$ **6b** $100=10^2$ **6c** $1024=4^5$ **6d** $0.01=10^{-2}$ **7a** 3 **7b** 3 **7c** 3 **7d** -3 **8a** 5 **8b** 4 **8c** 2 **8d** 3 **8e** 6 **8f** 1 **8g** 4 **8h** 3 **8i** 2 **8j** 4 **8k** 2 **8l** 2 **8m** 5 **8n** 3 **8o** 8 **8p** 10

Exercise 5.1 **2** $2\sqrt{3}$ **3** 8 **4** 3 **5** πr^2h **6** $x<4$ **7** x=5,y=2 or vice versa **8** 6 **9** x^{-6} **10** 3x/5

Exercise 5.2 **1a** 3x+12 **1d** $-x^2-3x$ **1c** $x^5-2x^3+x^2-2$ **1d** x^2+5x+6 **1e** x^2+2x+4 **1f** x^6+2x^3-3 **2a** 2(2x+5) **2b** 4a(2b-3) **2c** $3n^2(5n^3-12)$ **2d** $2(4+5y^3)$ **2e** 4x(3x+5) **2f** $7y^3(2y-4)$ **3a** $-15a^4$ **3b** $-10y^4$ **3c** $12z^3$ **3d** 16x **3e** $15y^2$ **3f** $24b^2$ **3g** $8x^2$ **3h** $4a^4$ **3i** $-3x^3$ **3j** 3x **3k** $-5m^4n^{-4}$ **3l** $5a^3b^5$ **3m** 5x/3 **3n** $6x^2/5$ **3o** 2e **3p** $2a^3$ **3q** x **3r** $x^3/2$

4a 

4b 

4c 

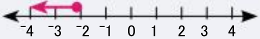
5a x=1,y=7 **5b** x=2,y=5 **6a** x=2,y=4 **6b** x=6,y=-4 **6c** x=23,y=64 **7** the two numbers are 8 and 88 **8a** x=9,y=6 **8b** x=52,y=13 **8c** x=2,y=6 **9** x=68,y=37 **10a** $V=109.01m^3,SA=140.14m^2$ **10b** $V=461.24m^3,SA=463.87m^2$ **10c** $V=152.43m^3,SA=284.74m^2$ **11a** $216m^3$ **11b** $241.58cm^3$ **11c** $179594cm^3$ **11d** $5852.79cm^3$ **11e** $79764cm^3$ **11f** $216.51cm^3$ **12a** $\sqrt{6}$ **12b** 6 **12c** $2\sqrt{5}+5\sqrt{2}$ **13a** $2\sqrt{6}$ **13b** $\sqrt{6}-27\sqrt{2}$ **13c** $16\sqrt{2}-5\sqrt{5}$ **13d** $13\sqrt{5}-10\sqrt{2}$ **14a** 2 **14b** 10 **14c** 8 **14d** 8 **14e** $8a^3$ **14f** $1/(4x^2)$ **15a** 5 **15b** 10 **15c** 10x **15d** $a^{3/2}$ **15e** 9 **15f** 3 **15g** $2x^{1/2}$ **15h** $3n^{1/2}$ **15i** 3/5 **15j** 27/125 **15k** 4 **15l** $x^{1/3}y^2$ **16a** $\log_{10}1000=3$ **16b** $\log_{2}64=6$ **16c** $\log_{3}81=4$ **16d** $\log_{10}0.01=-2$ **17a** $16=2^4$ **17b** $100=10^2$ **17c** $343=7^3$ **17d** $0.001=10^{-3}$ **18a** 3 **18b** 3 **18c** 3 **18d** -1 **19a** 2 **19b** 3 **19c** 4 **19d** 2 **19e** 4 **19f** 2 **19g** 2 **19h** 2 **19i** 3 **19j** 4 **19k** 3 **19l** 4

Exercise 5.3 **2** $3\sqrt{2}$ **3** 9 **4** 2 **5** ⅓areabase×height **6** $x>4$ **7** x=3,y=7 or vice versa **8** 1 **9** x^{-6} **10** x/3

Exercise 5.4 **1a** 2x+6 **1d** $-a^2-5a$ **1c** x^4-2x^2-3 **1d** x^2+3x+2 **1e** $x^2+8x+16$ **1f** x^6-1 **2a** 2(3x+5) **2b** 3x(3y-4) **2c** $5b^2(3b-4)$ **2d** $4(2+3p^3)$ **2e** $6x(2x^2+3)$ **2f** $7y(2y^2-3)$ **3a** $-10x^4$ **3b** $-15y^3$ **3c** $6z^4$ **3d** $12x^2$ **3e** 6c **3f** 8ab **3g** $8x^2$ **3h** $5x^3$ **3i** -5x/2 **3j** $5x^{-6}/6$ **3k** $-7a^3b^{-5}/5$ **3l** $7x^{-1}y/3$ **3m** 5x/3 **3n** $6x^2/5$ **3o** 2x/5 **3p** a **3q** x **3r** $x^3/2$

4a 

4b 

4c 

5a x=1,y=5 **5b** x=-1,y=4 **6a** x=3,y=7 **6b** x=-3,y=8 **6c** x=53,y=39 **7** the two numbers are 39 and 71 **8a** x=9,y=2 **8b** x=33,y=18 **8c** x=-3,y=4 **9** x=69,y=37 **10a** $V=22.78m^3,SA=44.86m^2$ **10b** $V=230917cm^3,SA=26702cm^2$ **10c** $V=148.47m^3,SA=244.06m^2$ **11a** $69.37cm^3$ **11b** $56661cm^3$ **11c** $42102cm^3$ **12a** $2\sqrt{3}$ **12b** $3\sqrt{2}$ **12c** $2+\sqrt{6}$ **13a** $3\sqrt{3}+4\sqrt{2}$ **13b** $4\sqrt{3}+8$ **13c** $6\sqrt{10}-3\sqrt{5}-4+\sqrt{2}$ **13d** $12-4\sqrt{5}+3\sqrt{10}-5\sqrt{2}$ **14a** 2 **14b** 2 **14c** 25 **14d** 512 **14e** $27b^3$ **14f** $8x^9$ **15a** 3 **15b** 10 **15c** 6x **15d** $x^{3/2}$ **15e** 3 **15f** 2 **15g** 3x **15h** $2a^{1/2}$ **15i** 2/3 **15j** 4/9 **15k** 4 **15l** $x^{1/2}y^6$ **16a** $\log_{10}100=2$ **16b** $\log_{2}32=5$ **16c** $\log_{3}243=5$ **16d** $\log_{10}0.1=-1$ **17a** $8=2^3$ **17b** $10=10^1$ **17c** $125=5^3$ **17d** $0.01=10^{-2}$ **18a** 4 **18b** 2 **18c** 3 **18d** -3 **19a** 2 **19b** 3 **19c** 2 **19d** 2 **19e** 4 **19f** 2 **19g** 2 **19h** 2 **19i** 3 **19j** 4 **19k** 7 **19l** 6

Exercise 6.1 **1** 6x+12 **2** 4a-12 **3** -2y-10 **4** -4x+12 **5** x^2+2x **6** b^2-2b **7** $-p^2-3p$ **8** $-x^2+3x$ **9** 5x+21 **10** 7x+7 **11** -x **12** -x+4 **13** 8a **14** $2x^2-3x$ **15** $-2d^2-d$ **16** $-2y^2+11y$ **17** x^2+3x+2 **18** x^2+4x+3 **19** x^2+5x+4 **20** x^2+5x+6 **21** $2x^2+7x+3$ **22** $3x^2+5x+2$ **23** $5x^2+13x+6$ **24** $6x^2+14x+4$ **25** x^2+2x-3 **26** x^2-x-2 **27** x^2+4x-5 **28** x^2-x-12 **29** $2x^2-7x+3$ **30** $3x^2-5x+2$ **31** $5x^2-13x+6$ **32** $6x^2-14x+4$

Exercise 6.2 **1** 3(2a+3) **2** 2a(2b-3) **3** 2c(5c-4) **4** 2(7x+5) **5** 2b(2a-3) **6** 2d(4d-3) **7** 3(3c+4) **8** 2x(4y+5) **9** 4x(4x-3) **10** 2(3x+5) **11** 3t(4s-5) **12** $3p^3(5p^2-12)$ **13** $3(2-3b^3)$ **14** $2x(2x^3-3)$ **15** $2c^4(5-4c)$ **16** $2(2+5a^4)$ **17** $4x(x^4+2)$ **18** $3d^3(3d^2+2)$ **19** $3(3-4x^5)$ **20** $2(4y^3-5x)$ **21** $3x^2(3-4x)$ **22** $2(3+5y^2)$ **23** 3x(4x+5) **24** $12y^3(y^2-3)$ **25** $6x^2(3+4x))$ **26** $3x(7x^3-5)$ **27** $3b^3(3b^4+8)$ **28** (x+5)(x+3) **29** (x+5)(x-4) **30** (x-1)(x+4) **31** (x-1)(x-2) **32** (x-6)(x+3) **33** (x-5)(x-4) **34** (x-2)(x+5) **35** (x-3)(x-7)

Exercise 6.3 **1** (b+c)(a+d) **2** (y+1)(x+4) **3** (c-6)(b+3) **4** (p-2)(z+5) **5** (s+5)(s-2) **6** (x+1)(x-3) **7** (x-3)(t-4) **8** (v-2)(u-7) **9** (b+c)(a+d) **10** (x+y)(2+b) **11** (b+d)(a+3) **12** (3x+2y)(a+3) **13** (b-c)(a+1) **14** (x-y)(a+1) **15** (f-1)(e+a) **16** (x-2)(x+y) **17** (x+1)(x+4) **18** (x+1)(x+3) **19** (x+2)(x+6) **20** (x-6)(x+3) **21** (x+5)(x-2) **22** (x+1)(x-3) **23** (x+3)(x-4) **24** (x+7)(x-2) **25** (3x+1)(x+4) **26** (5x+1)(x+3) **27** (x+2)(2x+6) **28** (x-6)(4x+3) **29** (x+5)(3x-2) **30** (x+1)(2x-3) **31** (x+1)(4x-3) **32** (x+7)(2x-3)

Exercise 6.4 **1** (x+5)(x+2) **2** (x+4)(x+3) **3** (x+4)(x+1) **4** (x+6)(x+1) **5** (x+2)(x+6) **6** (x+5)(x+3) **7** (x+4)(x+2) **8** (x+7)(x+2) **9** (x+4)(x+4) **10** (x+6)(x+3) **11** (x+5)(x+4) **12** (x+9)(x+2) **13** (x+3)(x-2) **14** (x+4)(x-2) **15** (x+6)(x-1) **16** (x+5)(x-2) **17** (x-4)(x+2) **18** (x-5)(x+2) **19** (x-6)(x+1) **20** (x-6)(x+2) **21** (x-4)(x-2) **22** (x-3)(x-3) **23** (x-9)(x-1) **24** (x-6)(x-2) **25** (a+11)(a+2) **26** (y+8)(y+2) **27** (x+5)(x-3) **28** (p+10)(p-2) **29** (k-7)(k+3) **30** (x-10)(x-3) **31** (c-18)(c-2) **32** (b-8)(b+3)

Exercise 6.5 **1** x^2-4 **2** x^2-36 **3** x^2-16 **4** x^2-9 **5** x^2-1 **6** x^2-25 **7** x^2-16 **8** x^2-9 **9** $9-4b^2$ **10** $4x^2-1$ **11** $25-9d^2$ **12** x^2-y^2 **13** a^2-b^2 **14** $9e^2-4f^2$ **15** t^2-16g^2 **16** $25w^2-x^2$

Exercise 6.6 **1** (x-3)(x+3) **2** (a+4)(a-4) **3** (1-y)(1+y) **4** (x+9)(x-9) **5** (2a-1)(2a+1) **6** (3x+7)(3x-7)

7 (x+6a)(x-6a) **8** (10-3d)(10+3d) **9** (4a+5)(4a-5) **10** (2-3p)(2+3p) **11** $(x+\sqrt{2})(x-\sqrt{2})$ **12** $(\sqrt{5}-d)(\sqrt{5}+d)$ **13** $(\sqrt{3}+a)(\sqrt{3}-a)$ **14** $(\sqrt{10}-2m)(\sqrt{10}+2m)$ **15** $(x+2\sqrt{5})(x-2\sqrt{5})$ **16** $(\sqrt{7}-\sqrt{2}y)(\sqrt{7}+\sqrt{2}y)$ **17** $(x^3+\sqrt{2})(x^3-\sqrt{2})$ **18** $(\sqrt{5}-d^2)(\sqrt{5}+d^2)$ **19** $(y^3+a)(y^3-a)$ **20** $(\sqrt{3}-2m^3)(\sqrt{3}+2m^3)$ **21** $(a+b^2)(a-b^2)$ **22** $(1-\sqrt{2}y^4)(1+\sqrt{2}y^4)$

Exercise 6.7 **1** (x+2)(x-2) **2** (16+b)(16-6) **3** (a-3)(a+3) **4** (2-3b)(2+3b) **5** $x^2-9=(x+3)(x-3)$ **6** $1-25d^2=(1-5d)(1+5d)$ **7** $x^2-7=(x+\sqrt{7})(x-\sqrt{7})$ **8** a^6-b^2 **9** $(\sqrt{2}x+3)(\sqrt{2}x-3)$ **10** $(x^{5/2}-\sqrt{3}y^3)(x^{5/2}+\sqrt{3}y^3)$

Exercise 6.8 **1** x^2+4x+4 **2** $x^2+12x+36$ **3** $y^2+8x+16$ **4** b^2+6x+9 **5** $1+2d+d^2$ **6** $4x^2+20x+25$ **7** $x^2+2xy+y^2$ **8** $a^2+2ab+b^2$ **9** x^2-4x+4 **10** $x^2-10x+25$ **11** y^2-2x+1 **12** $9-6n+n^2$ **13** $4-12d+9d^2$ **14** $9x^2-12x+4$ **15** $x^2-2xy+y^2$ **16** $a^2-2ab+b^2$ **17** $4x^2+4x+1$ **18** $4x^2+20x+25$ **19** $9y^2+6y+1$ **20** $9y^2+12y+4$ **21** $1+6x+9x^2$ **22** $4+20y+25y^2$ **23** $4x^2-4x+1$ **24** $1-4b+4b^2$ **25** $9-30y+25y^2$ **26** $4x^2-12xy+9y^2$

Exercise 6.9 **1** $(x+y)^2$ **2** $(m+n)^2$ **3** $(u+v)^2$ **4** $(y+z)^2$ **5** $(a-b)^2$ **6** $(c-d)^2$ **7** $(p-t)^2$ **8** $(k+z)^2$ **9** $(x+2)^2$ **10** $(x+1)^2$ **11** $(x+4)^2$ **12** $(x+5)^2$ **13** $(x+6)^2$ **14** $(x+8)^2$ **15** $(x-1)^2$ **16** $(x-2)^2$ **17** $(x-3)^2$ **18** $(x-4)^2$ **19** $(x-5)^2$ **20** $(x-6)^2$ **21** $(2x+3)^2$ **22** $(2x-3)^2$ **23** $(3x-2)^2$ **24** $(3y+4)^2$ **25** $(3a+1)^2$ **26** $(2x+1)^2$ **27** $(5x-3)^2$

Exercise 6.10 **1** x=-1 **2** x=-2 **3** x=-4 **4** x=-5 **5** x=-7 **6** x=1 **7** x=3 **8** x=4 **9** x=5 **10** x=8 **11** x=-1.5 **12** a=-1/3 **13** x=-1 **14** x=1/2 **15** y=-4/3 **16** x=2/3 **17** x=3/5 **18** x=-4 or x=-1 **19** x=-2 or x=-2 **20** x=-3 or x=-2 **21** x=-1 or x=-6 **22** x=-6 or x=-2 **23** x=-3 or x=-5 **24** x=-2 or x=-4 **25** x=-7 or x=-2 **26** x=-3 or x=2 **27** x=-6 or x=1 **28** x=4 or x=-2 **29** x=6 or x=-1 **30** x=6 or x=-2 **31** x=3 **32** x=1 or x=9 **33** x=2 or x=6

Exercise 6.11 **1** x=0.16 or x=-6.16 **2** x=1 or x=-3 **3** x=0.83 or x=-4.83 **4** x=1.16 or x=-5.16 **5** x=0.32 or x=-6.32 **6** x=0.48 or x=-10.48 **7** x=-1 **8** x=-0.76 or x=-5.24 **9** x=-0.39 or x=-7.61 **10** x=-0.17 or x=-11.83 **11** x=3 or x=-1 **12** x=4.24 or x=-0.24 **13** x=6.74 or x=-0.74 **14** x=8.80 or x=-0.80 **15** x=10.20 or x=-0.20 **16** x=12.32 or x=-0.32 **17** x=13.71 or x=0.29 **18** x=5.65 or x=0.35 **19** x=3.73 or x=0.27 **20** x=7.61 or x=0.39 **21** x=3.30 or x=-0.30 **22** x=0.79 or x=-3.79 **23** x=0.85 or x=-5.85 **24** x=5.37 or x=-0.37 **25** x=0.53 or x=-7.53 **26** x=7.41 or x=-0.41 **27** x=-0.11 or x=-8.89 **28** x=8.77 or x=0.23 **29** x=-0.48 or x=-10.52 **30** x=10.82 or x=0.18

Exercise 6.12 **2** x^2-4x+4 **3** $(a+b)^2$ **4** $2\sqrt{3}$ **5** 8 **6** 3 **7** 7 and 2 **8** 4 **9** x^{-6} **10** 441

Exercise 6.13 **2** x^2-6x+9 **3** $(x+y)^2$ **4** $3\sqrt{2}$ **5** 27 **6** 4 **7** 3 and 5 **8** 12 **9** y^{-4} **10** 361

Exercise 6.14 **2** x^2+4x+4 **3** $(x-y)^2$ **4** $2\sqrt{5}$ **5** 4 **6** 6 **7** 1 and 9 **8** -8 **9** a^{-10} **10** 484

Exercise 6.15 **1a** 3481 **1b** 3721 **1c** 9025 **1d** 3481 **2a** 3x+5 **2b** 14x-18 **2c** 2x+3 **2d** 3a+8b **2e** $a^2-2ab+b^2$ or $(a-b)^2$**2f** $2a^2-2ab$ or 2a(a-b) **3** $8+4\sqrt{2}$ **4** $12+6\sqrt{2}$ **5** $24+12\sqrt{2}$ **6** $1/\sqrt{8}$ **7** -2

Exercise 6.16 **1** 3^4 **2** a=2,b=4. If a=b then there are many answers **3** beside each other

Exercise 6.17 **1a** x^2+3x+2 **1b** $2x^2+7x+3$ **1c** x^2+2x-3 **1d** $2x^2-7x+3$ **2a** (x+1)(x+4) **2b** (3x+1)(x+4) **2c** (x+5)(3x-2) **3a** (x+3)(x+5) **3b** (x+3)(x-2) **3c** (x-6)(x+2) **3d** (x-4)(x-2) **4a** x=-1 **4b** x=-2 **4c** x=4 **4d** x=5 **4e** x=8 **4f** x=-3/2 **4g** x=-4 or x=-1 **4h** x=-1 or x=-2 **4i** x=-3 or x=-2 **4j** x=-5 or x=-3 **4k** x=-4 or x=-2 **4l** x=-2 or x=-7 **5a** x=0.16 or x=-6.16 **5b** x=1 or x=-3 **5c** x=0.83 or x=-4.83 **5d** x=3 or x=-1 **5e** x=4.24 or x=-0.24 **5f** x=6.74 or x=-0.74 **5g** x=3.30 or x=-0.30 **5h** x=0.79 or x=-3.79 **5i** x=5.37 or x=-0.37 **5j** x=0.53 or x=-7.53

Exercise 6.18 **1a** x^2+4x+3 **1b** $2x^2+8x+6$ **1c** x^2-2x-3 **1d** $2x^2-7x+6$ **2a** (x+1)(x+3) **2b** (3x+1)(x+5) **2c** (x+5)(2x-3) **3a** (x+3)(x+2) **3b** (x+4)(x-2) **3c** (x-4)(x+3) **3d** (x-3)(x-2) **4a** x=-1 **4b** x=-3 **4c** x=3 **4d** x=4 **4e** x=6 **4f** x=-1/3 **4g** x=-3 or x=-1 **4h** x=-1 or x=-5 **4i** x=3 or x=2 **4j** x=5 or x=-3 **4k** x=4 or x=-3 **4l** x=2 or x=7 **5a** x=0.65 or x=-4.65 **5b** x=0.73 or x=-2.73 **5c** x=0.32 or x=-6.32 **5d** x=2.41 or x=-0.41 **5e** x=4.24 or x=-0.24 **5f** x=8.24 or x=-0.24 **5g** x=5.19 or x=-0.19 **5h** x=0.79 or x=-3.79 **5i** x=2 or x=-1 **5j** x=0.65 or x=-7.65

Exercise 7.1 **1** x=-5 **2** b=-4 **3** y=5 **4** t=12 **5** a=32 **6** m=5 **7** z=77 **8** w=-24 **9** x=70 **10** x=3 **11** m=-1 **12** a=5 **13** x=2 **14** x=-11 **15** f=-10 **16** t=9 **17** p=-2 **18** b=-1 **19** a=-1 **20** n=4 **21** x=-1 **22** n=10 **23** x=-9 **24** x=8 **25** x=5 **26** x=-4 **27** x=-5 **28** x=-11 **29** x=-0.4 **30** x=3 **31** x=0 **32** d=-10 **33** x=-1 **34** x=0 **35** x=-0.2 **36** x=2 **37** x=-1.6 **38** x=3 **39** x=8 **40** x=1 **41** x=2 **42** x=-3 **43** x=-1 **44** x=2 **45** x=3 **46** x=-2.1 **47** x=4 **48** x=1 **49** h=3 **50** y=5 **51** w=1 **52** t=6 **53** m=1 **54** x=1 **55** x=-18 **56** a=30 **57** a=-8 **58** y=6 **59** x=28 **60** x=60 **61** x=3 **62** x=1 **63** x=0 **64** x=-1 **65** x=-12 **66** x=0 **67** x=-4.5 **68** a=20/3 **69** x=7 **70** b=-4 **71** x=14/3 **72** b=1 **73** x=4/5 **74** x=27/4 **75** x=18 **76** x=12/7 **77** x=24 **78** x=12 **79** x=-18 **80** x=-6/11 **81** x=4 **82** x=6 **83** x=10/13 **84** x=42 **85** x=-15/22 **86** x=24/11

Exercise 7.2 **1** radius=11.94m **2** breadth=26m **3** distance=2640m **4** 5 sides **5** height=6cm **6** height=11.20cm **7** height=5.37m

Exercise 7.3 **1** x=7 **2** x=40 **3** 14,15,16 **4** 1,2,3 **5** w=8m **6** w=30cm **7** 20°,80°,80° **8** 4.8m **9** 112cm **10** -12 and -41

Exercise 7.4 **1** x=-1 **2** x=-2 **3** x=-4 **4** x=-5 **5** x=-7 **6** x=1 **7** x=3 **8** x=4 **9** x=5 **10** x=9 **11** x=-3/2 **12** x=-1/3 **13** x=-1 **14** x=1/2 **15** x=-4/3 **16** x=2/3 **17** x=3/5 **18** x=-4 or x=-1 **19** x=-2 or x=-1 **20** x=-3 or x=-2 **21** x=-6 or x=-1 **22** x=-6 or x=-2 **23** x=-5 or x=-3 **24** x=-4 or x=-2 **25** x=-7 or x=-2 **26** x=-3 or x=2 **27** x=-6 or x=1 **28** x=4 or x=-2 **29** x=6 or x=-1 **30** x=6 or x=-2 **31** x=3 **32** x=1 or x=9 **33** x=2 or x=6

Exercise 7.5 **1** x=6 or x=-1 **2** x=1 or x=-3 **3** x=0.83 or x=-4.83 **4** x=1.16 or x=-5.16 **5** x=0.32 or x=-6.32 **6** x=0.48 or x=-10.48 **7** x=-1 **8** x=-0.76 or x=-5.24 **9** x=-0.39 or x=-7.61 **10** x=-0.17 or x=-11.83 **11** x=3 or x=-1 **12** x=4.24 or x=-0.24 **13** x=6.74 or x=-0.74 **14** x=8.80 or x=-0.80 **15** x=10.20 or x=-0.20 **16** x=12.32 or x=-0.32 **17** x=13.71 or x=0.29 **18** x=5.65 or x=0.35 **19** x=3.73 or x=0.27 **20** x=7.61 or x=0.39

21 x=3.30 or x=-0.30 **22** x=0.79 or x=-3.79 **23** x=0.85 or x=-5.85 **24** x=5.37 or x=-0.37 **25** x=0.53 or x=-7.53
26 x=7.41 or x=-0.41 **27** x=-0.11 or x=-8.89 **28** x=8.77 or x=0.23 **29** x=-0.48 or x=-10.52
30 x=10.82 or x=0.18
Exercise 7.6 **1** x=2 or x=-1 **2** x=1 or x=-2 **3** x=-1 or x=3 **4** x=-1 **5** x=-2 or x=-1 **6** x=-2 or x=-3 **7** x=-1 or x=-3
8 x=1 **9** x=1 or x=2 **10** x=2 or x=3
Exercise 7.7 **1** x=-0.5 or x=-2 **2** x=1.45 or x=-3.45 **3** x=0.69 or x=-0.29 **4** x=1.32 or x=-0.57
5 x=0.78 or x=-1.28 **6** x=4.19 or x=-1.19 **7** x=-0.21 or x=-4.79 **8** x=1 or x=-0.71 **9** x=-0.5 or x=-1.5
10 x=11.09 or x=-0.09 **11** x=0.39 or x=-1.72 **12** x=1.25 or x=0 **13** x=-1 or x=-2 **14** x=1.41 or x=-1.41
15 x=0 or x=-0.38 **16** x=0.8 or x=0 **17** x=0.5 or x=-0.5 **18** x=0.6 or x=-0.6 **19** x=0.44 or x=4.56
20 x=-1 or x=0.25 **21** x=-1 or x=2.5 **22** x=-0.26 or x=1.26 **23** x=-3.73 or x=-0.27 **24** x=-1.58 or x=-0.42
25 0.76s **26a** x=2.39 (x cannot be negative) **26b** x=2.32 (x cannot be negative) **26c** x=5 (x cannot be negative)
Exercise 7.8 **2** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **3** a=3,b=2,c=-2 **4** x^2-4x+4 **5** (x+3)(x+1) **6** 2 **7** 7 and 3 **8** 3 **9** x^{-12} **10** $7.80
Exercise 7.9 **2** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **3** a=5,b=-1,c=-3 **4** x^2+4x+4 **5** (x+5)(x+2) **6** 3 **7** 6 and 5 **8** -5 **9** x^{-8} **10** $9.10
Exercise 7.10 **2** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **3** a=1,b=-2,c=4 **4** x^2-6x+9 **5** (x+4)(x+1) **6** 4 **7** 4 and 2 **8** -14 **9** x^{-10} **10** $10.40
Exercise 7.11 **1** a=7 **2** a=6 **3** a=7 **4** x=-5/4 **5** x=-35/6 **6** x=-13/4 **7** x=1 or x=-1 **8** x=2 or x=1/2 **9** x=-1/3 or x=2
10 x=1 or x=-4 **11a** $y=x^2+x-2$ **11b** $y=x^2-2x-3$
Exercise 7.12 **1** 32cm **2** 
Exercise 7.13 **1** x=9 **2** x=-1 **3** x=3/2 **4** x=17/5 **5** x=9 **6** x=3 **7** x=2 **8** x=-18 **9** x=7/2 **10** x=6 **11** 5sides
12 length=38m, breadth=17m **13** x=-2 **14** x=-5 **15** x=1 **16** x=4 **17** x=-1 or x=-4 **18** x=-2 or x=-6
19 x=1 or x=-6 **20** x=4 or x=-2 **21** x=6 or x=-1 **22** x=6 or x=-2 **23** x=9 or x=1 **24** x=6 or x=2
25 x=-0.44 or x=-4.56 **26** x=0.37 or x=-2.70 **27** x=10.72 or x=0.28 **28** x=-1 or x=0.25 **29** t=0.26s **30** x=4.47
Exercise 7.14 **1** x=8 **2** x=2 **3** x=2 **4** x=3 **5** x=9/2 **6** x=2 **7** x=11/4 **8** x=-15 **9** x=4 **10** x=12 **11** height=4.5
12 36°,72°,72° **13** x=-3 **14** x=-4 **15** x=2 **16** x=5 **17** x=-1 or x=-3 **18** x=-2 or x=-4 **19** x=-3 or x=2
20 x=5 or x=-2 **21** x=6 or x=-1 **22** x=4 or x=-3 **23** x=8 or x=1 **24** x=12 or x=1 **25** x=-0.38 or x=-2.62
26 x=0.33 or x=-2 **27** x=6.19 or x=0.81 **28** x=-1 or x=1/2 **29** t=0.88s **30** x=2.37

Exercise 8.1 **1a** 1/4=0.75 **1b** 2/4=0.5 **1c** 1/4=0.25 **2a** 1/16=0.0625 **2b** 4/16=0.25 **2c** 6/16=0.375 **2d** 4/16=0.25
2e 1/16=0.0625 **3a** 1/8=0.125 **3b** 3/8=0.375 **3c** 1/8=0.125 **4a** 1/16=0.0625 **4b** 4/16=0.25 **4c** 6/16=0.375
4d 1/16=0.0625
Exercise 8.2 **Exercise 14.7** **1a** 18/36=0.5 **1b** 6/36=0.17 **1c** 18/36=0.5 **1d** 5/36=0.14 **1e** 19/36=0.53
1f 1/36=0.03 **1g** 24/36=0.67 **1h** 6/36=0.17 **1i** 24/36=0.67 **1j** 1 **1k** 0 **2a** 3/25=0.12 **2b** 4/25=0.16
2c 12/25=0.48 **2d** 13/25=0.52 **2e** 2/25=0.08 **2f** 12/25=0.48 **2g** 1/25=0.04 **2h** 13/25=0.52 **3a** 1/9
3b 6/9=2/3 **3c** 2/9 **3d** 7/9
Exercise 8.3 **1** independent **2** dependent **3** dependent **4** independent **5** dependent **6** independent **7** independent
Exercise 8.4 **1b(i)** 1/6 **1b(ii)** 4/6=2/3 **1b(iii)** 2/6=1/3 **2b(i)** 1/12 **2b(ii)** 6/12=1/2 **2b(iii)** 7/12 **3a** 2/12=1/6
3b 2/12=1/6 **3c** 4/12=1/3 **4a** 2/20=0.1 **4b** 6/20=0.3 **4c** 12/20=0.6 **5a** 30/90=1/3 **5b** 12/90=2/15 **5c** 48/90=8/15
Exercise 8.5 **1a** 4/25=0.16 **1b** 25/25=1 **1c** 13/25=0.52 **1d** 4/12=0.33 **2a** 4/23=0.17 **2b** 23/23=1 **2c** 11/23=0.48
2d 4/12=0.33 **3a** 21/29=0.72 **3b** 29/29=1 **3c** 2/29=0.07 **3d** 21/27=0.78
Exercise 8.6 **1** 0.17/0.68=0.25 **2** 0.13/0.32=0.41 **3** 0.53/0.81=0.65 **4** 8/25=0.32 **5** 0.05/0.2=0.25
Exercise 8.7 **1a** a collection of data from the whole population **1b** a collection of data from part of the popuation **1c** the sample doesn't represent the population **1d** must be small enough to be economical but large enough to represent the population **1e** a random sample helps avoid bias because each member of the population has an equal chance of being selected in the sample. **1f** a stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample
2a 17 400 000 people **2b(i)** if only households that agree are sampled then the sample is not random which casts considerable doubt on the validity of the ratings **2b(ii)** a sample size of 25 000 from a population of 120 000 000 is a very small sample size - approximately 0.02% **2b(iii)** the sample is unlikely to refect the viewing habits of the whole population as significant strata, such as cable TV, Internet TV, public buildings etc, are not sampled **2c** because the sample isn't random, is small, and concentrates on only one strata then it is unlikely that the sample is a true representation of the population. Conclusions about the population are likely to have considerable error.
Exercise 8.8 **1** product A = 46% to 52%, product B = 44% to 50% **2** the stratified sample doesn't properly reflect population proportions in each state (eg., Tasmania has a higher proportion of the stratified sample than its population). Some states/territories aren't represented at all in the sample. **3a** assuming 3% error: support 53% to 59%, opposition/don't know 41% to 47% **3b(i)** it is possible that people who don't answer could prefer the NBN but are at work and unable to answer calls to their home **3b(ii)** people with mobile phones are likely to be more interested in broadbad but are not sampled because they don't have a landline phone **3b(iii)** the opinion poll questions are not shown - we don't know how the questions were phrased or the technique used by the pollster. It is also well known that some people don't express their true beliefs **3c** The statement could well be true. With a modest sample error of 3% there can thus be a 6% difference error between A and B. It is very likely that the error could be more that 3%.
Exercise 8.9 **2** HHH,HHT,HTH,THH,HTT,THT,TTH,TTT **3** 3/8 **4** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **5** a=5,b=-3,c=-1 **6** (x+3)(x+2)

7 3 **8** 9 and 2 **9** 5 **10** 32

Exercise 8.10 **2** GGG,GGB,GBG,BGG,GBB,BGB,GBB,BBB **3** 3/8 **4** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **5** a=1,b=-3,c=4 **6** (x-5)(x-1) **7** 3 **8** 7 and 4 **9** -2 **10** 243

Exercise 8.11 **2** HHH,HHT,HTH,THH,HTT,THT,TTH,TTT **3** 1/8 **4** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **5** a=25,b=-1,c=3 **6** (x+5)(x-2) **7** 2 **8** 7 and 2 **9** -6 **10** 256

Exercise 8.12 **1** 0.67 **2** 0.67 **3** 0.5 **4** 0.33 **5a** 0.25 **5b** 0.75 **5c** 0.5 **5d** 0.25 **5e** 0.25 **5f** 0.5 **6a** 48/120 or 0.4 **6b** 12/120 or 0.1 **6c** 72/120 or 0.6 **7a** 24 **7b** 120 **7c** how many ways can four people be arranged in a line **7d** how many ways can 5 people be arranged in a line **7e** Q6b

Exercise 8.13 **1** 18/90=0.2 **2** 3 **3** (8-5)×7=21

Exercise 8.14 **1a** 4/9 **1b** 2/9 **1c** 2/9 **1d** 3/9=1/3 **2b(i)** 1/6 **2b(ii)** 6/6=1 **2b(iii)** 2/6=1/3 **3a** 34/90=17/45=0.38 **3b** 42/90=7/15=0.47 **3c** 25/34=0.74 **4** 0.17/0.35=0.49=49% **5a** the sample proportions do not match the population proportions. For example, WA has twice the sample size of Tasmania yet has almost five times the population. **5b(i)** it is possible that people who don't answer could prefer the NBN but are at work and unable to answer calls to their home **5b(ii)** people with mobile phones are likely to be more interested in broadbad but are not sampled because they don't have a landline phone

Exercise 8.15 **1a** 4/9 **1b** 2/9 **1c** 2/9 **1d** 5/9 **2b(i)** 1/6 **2b(ii)** 2/6=1/3 **2b(iii)** 2/6=1/3 **3a** 19/75=0.25 **3b** 35/75=7/15=0.47 **3c** 19/27=0.70 **4** 0.13/.24==0.54=54% **5a** the sample proportions do not match the population proportions. For example, WA has twice the sample size of Tasmania yet has almost five times the population. **5b(i)** it is possible that people who don't answer could prefer the NBN but are at work and unable to answer calls to their home **5b(ii)** people with mobile phones are likely to be more interested in broadbad but are not sampled because they don't have a landline phone

Exercise 9.1 **1a** variable=x, degree=3, leading coefficient=3, constant term=3 **1b** variable=a, degree=4, leading coefficient=5, constant term=-1 **1c** variable=x, degree=2, leading coefficient=-3.5, constant term=2.7 **1d** variable=y, degree=5, leading coefficient=1, constant term=-8 **1e** variable=x, degree=2, leading coefficient=-2, constant term=4 **1f** variable=x, degree=5, leading coefficient=7, constant term=-5 **2a** 8 **2b** 17 **2c** 1 **2d** -7 **2e** 4 **2f** 6 **2g** -1 **2h** 7 **2i** -23 **2j** $-3b^2-3b+4$ **3a** index not positive whole number **3b** index not positive whole number **3c** index not positive whole number, a coefficient not a real number **3d** **3e** index not positive whole number, a coefficient not a real number **3f** index not positive whole number

Exercise 9.2 **1a** $7x^2+x+3$ **1b** $6x^2-x+7$ **1c** $9x^2-3x$ **1d** $4x^2-3x+4$ **1e** x^3+10x^2+x+3 **1f** $11x^3-4x+5$ **1g** $x^4+4x^3-11x^2-x+3$ **1h** $-3x^5+x^4-4x^3+3x^2-3x-4$ **1i** $4x^2-4x-3$ **1j** $2x^2-x+6$ **2a** $5x^2+8x+3$ **2b** $4x^2+5x+6$ **2c** $3x^2-7x+4$ **2d** $-x^2-x+2$ **3** $11x-4$ **4** $3x^4-6x^3+9x^2+15x-9$ **5** $3x^2+7x+3$ **6** 13 **7** $12x^2+10x-10$

Exercise 9.3 **1a** $-3x^2-7x+7$ **1b** $5x-3$ **1c** $3x^2-7x+4$ **1d** $-2x^2+9x+10$ **1e** $-9x^3-7x+7$ **1f** $3x^3-4x^2+6x-1$ **1g** $7x^4+4x^3-x^2-9x+1$ **1h** $-3x^5-3x^4+4x^3-3x^2+9x-10$ **1i** $-x^4-5x^3+8x^2-3x+1$ **1j** $4x^2-3x+2$ **2a** $-x^2-2x+1$ **2b** $2x^2-x-4$ **2c** $-5x^2+3x+2$ **2d** $3x^2-9x+8$ **3** $2x-8$ **4** 32 **5** $0.5x^2-5x-2.5$

Exercise 9.4 **1a** $10x^4-7x^3-16x^2+6x$ **1b** $9x^4-3x^3-9x^2+10x$ **1c** $15x^3+16x^2-6x-4$ **1d** $9x^3+3x^2-51x-21$ **1e** $-20x^6-20x^5-16x^4+13x^3+5x^2+4x-2$ **1f** $28x^6-8x^5-35x^4+10x^3$ **1g** $-12x^8+31x^5+4x^4-20x^2-5x$ **1h** $-6x^9+12x^8-9x^5-2x^4-4x^3+3$ **1i** $3x^3-17x^2+6x+20$ **1j** $-2x^4+2x^3+10x^2-6x-12$ **2a** $9x^3+21x^2+13+2$ **2b** $2x^3+7x^2+13x+5$ **2c** $-4x^4-3x^3+22x^2-15x$ **2d** $-2x^4+14x^3-23x^2+15x$ **3** $8x^5-4x^3+16x^2-8$ **4** $6x^5+21x^3-4x^2+15x-10$

Exercise 9.5 **1a** 46 **1b** 15 **1c** 31 **1d** 23 **1e** 52 **1f** 34 **2a** 30 r 13 **2b** 29 r 5 **2c** 25 r 7 **2d** 53 r 31 **2e** 105 r 22 **2f** 127 r 0 **3a** no **3b** yes

Exercise 9.6 **1a** x+3 **1b** x+2 **1c** x-3 **1d** x-1 **1e** 2x+1 **1f** x-2 **1g** x^2+4x+3 **1h** x^2-x-2 **2a** x-3 **2b** x+5 **2c** $x^2-7x+16$ **2d** x **3** x-2 **4** x+4 **5** x^2-2x+5 **6a** no **6b** yes **6c** yes **6d** no

Exercise 9.7 **1a** x-2 **1b** x-1 **1c** x+2 **1d** P(3)=3 **1e** P(-1)=0 **1f** P(4)=0 **2** yes **3** yes **4** yes **5** no **6** yes **7** yes **8** yes **9** no **10** yes **11** yes **12** no **13** yes **14** yes **15** yes **16** yes **17** yes **18** no **19** yes **20** no **21** yes **22** yes **23** yes

Exercise 9.8 **1** x=1,x=-1,x=2 **2** x=1,x=2,x=-5 **3** x=1,x=3,x=-3 **4** x=-1,x=2,x=3 **5** x=-1,x=2,x=-2 **6** x=1,x=-2,x=5

Exercise 9.9 **1** 7 **2** -3 **3** 15 **4** -18 **5** -10

Exercise 9.10 **2** no **3** $3x^2-x+1$ **4** $6x^4-6x^3-3x^2$ **5** HHH,HHT,HTH,THH,HTT,THT,TTH,TTT **6** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **7** (x+3)(x+2) **8** 4 **9** 3 **10** 81

Exercise 9.11 **2** yes **3** x^2-5x+3 **4** $6x^3+8x^2-4x$ **5** GGG,GGB,GBG,BGG,GBB,BGB,BBG,BBB **6** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **7** (x+5)(x+1) **8** 5 **9** -10 **10** 256

Exercise 9.12 **2** no **3** $-2x^3+4$ **4** $-6x^4+4x^2+6x$ **5** HHH,HHT,HTH,THH,HTT,THT,TTH,TTT **6** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **7** (x+4)(x+1) **8** 6 **9** -8 **10** 625

Exercise 9.13 **1a** a=21 **1b** 4 **2a** a=4b+1 **2b** b=8 **3** a=4, b=3, c=1 **3b** a=5, b=2, c=6 **4** 23 **5** 34 **6** 3 **7** -1 **8** a=2, b=-3

Exercise 9.14 **1** 9+99÷9 **2** 30

Exercise 9.15 **1a** variable=x, degree=3, leading coefficient=2, constant term=1 **1b** variable=a, degree=4, leading coefficient=3, constant term=-5 **2a** 10 **2b** 3 **3a** $7x^2-5x$ **3b** $8x^4+3x^3+x^2-9x+1$ **3c** $-2x^3-7x^2+8x-2$ **3d** x-3 **5a** x=1, x=-2, x=3 **5b** x=1, x=2, x=3 **6** 3

Exercise 9.16 **1a** variable=x, degree=3, leading coefficient=5, constant term=-1 **1b** variable=b, degree=5, leading coefficient=2, constant term=-7 **2a** 19 **2b** -1 **3a** $4x^2-3x+1$ **3b** x^2-5x+1 **3c** $2x^3+7x^2+4x-4$ **3d** x+3 **5a** x=1, x=-2, x=-1 **5b** x=2, x=-3, x=3 **6** -31

Exercise 10.1 **2** yes **3** $3x^2+x-2$ **4** $10x^4-5x^3-15x^2$ **5** HHH,HHT,HTH,THH,HTT,THT,TTH,TTT **6** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **7** (x+4)(x+1) **8** 2 **9** 5 **10** 16

Exercise 10.2 **1a** x^2+3x+2 **1b** x^2+6x+8 **1c** x^2-4x-5 **1d** $2x^2-7x+3$ **2a** (x+3)(x+2) **2b** (x+5)(x-2) **2c** (x-5)(x+2) **2d** (x-3)(x-2) **3a** x=-1 **3b** x=-2 **3c** x=3 **3d** x=4 **3e** x=5 **3f** x=-1/3 **3g** x=-3, x=-1 **3h** x=-3, x=-2 **3i** x=1, x=4 **3j** x=5, x=-3 **3k** x=-4, x=3 **3l** x=7, x=2 **4a** x=-0.59 or x=-3.41 **4b** x=0.32 or x=-6.32 **4c** x=0.10 or x=-10.10 **4d** x=2.73 or x=-0.73 **4e** x=3 or x=-1 **4f** x=5.19 or x=-0.19 **4g** x=6.16 or x=-0.16 **4h** x=0.41 or x=-7.41 **4i** x=3.24 or x=-1.24 **4j** x=0.65 or x=-7.65 **5a** x=5 **5b** x=-1 **5c** x=3 **5d** x=2 **5e** x=5 **5f** x=3 **5g** x=3 **5h** x=8 **5i** x=2 **5j** x=6/5 **6a** x=-1 **6b** x=2 **6c** x=2 **6d** x=5 **6e** x=-1 or x=-3 **6f** x=-8 or x=-1 **6g** x=-5 or x=1 **6h** x=5 or x=-2 **6i** x=6 or x=-1 **6j** x=-4 or x=3 **6k** x=9 or x=1 **6l** x=12 or x=1 **7a** x=-0.38 or x=-2.62 **7b** x=0.37 or x=-5.37 **7c** x=3.89 or x=-0.39 **7d** x=-0.77 or x=0.43 **8a** 4/9 **8b** 2/9 **8c** 2/9 **8d** 5/9 **9b(i)** 1/6 **9b(ii)** 4/6=2/3 **9b(iii)** 2/6=1/3 **10a** 9/70 **10b** 32/70=16/35 **10c** 9/18=1/2 **11a** the sample proportions do not match the population proportions. For example, WA has the same sample size of Tasmania yet has almost five times the population. **11b(i)** it is possible that people who don't answer could prefer the NBN but are at work and unable to answer calls to their home **11b(ii)** people with mobile phones are likely to be more interested in broadbad but are not sampled because they don't have a landline phone **12a** variable=x, degree=3, leading coefficient=2, constant term=-1 **12b** variable=a, degree=5, leading coefficient=4, constant term=-5 **13a** 6 **13b** -3 **14a** $4x^2+x+6$ **14b** $3x^2-3x+1$ **14c** $5x^3+17x^2+5x-3$ **14d** x-2 **15** $2^2+7\times2-18=0$ **16a** x=1, x=-3, x=-5 **16a** x=1, x=-2, x=-4 **17** 5

Exercise 10.3 **2** no **3** $4x^2-4x+3$ **4** $6x^3+15x^2-3x^x$ **5** GGG,GGB,GBG,BGG,GBB,BGB,BBG,BBB **6** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **7** (x+7)(x+1) **8** 3 **9** -6 **10** 64

Exercise 10.4 **1a** x^2+3x+2 **1b** $x^2+8x+15$ **1c** x^2-3x-4 **1d** $2x^2-5x+2$ **2a** (x+3)(x+2) **2b** (x+4)(x-2) **2c** (x-6)(x+2) **2d** (x-3)(x-2) **3a** x=-2 **3b** x=-1 **3c** x=2 **3d** x=4 **3e** x=6 **3f** x=-1/2 **3g** x=-3, x=-1 **3h** x=-5, x=-1 **3i** x=9, x=-1 **3j** x=5, x=-3 **3k** x=4, x=-3 **3l** x=3, x=5 **4a** x=0.65 or x=-4.65 **4b** x=0.73 or x=-2.73 **4c** x=0.32 or x=-6.32 **4d** x=2.41 or x=-0.41 **4e** x=3.41 or x=0.59 **4f** x=7.61 or x=0.39 **4g** x=4.79 or x=0.21 **4h** x=0.79 or x=-3.79 **4i** x=2.30 or x=-1.30 **4j** x=-0.81 or x=-6.19 **5a** x=5 **5b** x=1 **5c** x=6/5 **5d** x=2 **5e** x=4 **5f** x=60/11 **6a** x=-3 **6b** x=-4 **6c** x=2 **6d** x=5 **6e** x=-1 or x=-5 **6f** x=-4 or x=-2 **6g** x=-3 or x=2 **6h** x=5 or x=-2 **6i** x=-6 or x=1 **6j** x=4 or x=-3 **6k** x=1 or x=8 **6l** x=12 or x=1 **7a** x=-0.21 or x=-4.79 **7b** x=1.19 or x=-4.19 **7c** x=2 or x=0.33 **7d** x=-1 or x=0.5 **8a** 4/9 **8b** 1/9 **8c** 2/9 **8d** 5/9 **9b(i)** 1/12 **9b(ii)** 12/12=1 **9b(iii)** 4/12=1/3 **10a** 18/48=3/8 **10b** 11/48 **10c** 18/25 **11a** the sample proportions do not match the population proportions. For example, QLd has four times the same sample size of NT yet has almost twenty-three times the population. **11b(i)** it is possible that people who don't answer could prefer the NBN but are at work and unable to answer calls to their home **11b(ii)** people with mobile phones are likely to be more interested in broadbad but are not sampled because they don't have a landline phone **12a** variable=x, degree=3, leading coefficient=5, constant term=-1 **12b** variable=b, degree=5, leading coefficient=2, constant term=-7 **13a** 19 **13b** -1 **14a** $4x^2-3x+1$ **14b** x^2-5x+1 **14c** $2x^3+7x^2+4x-4$ **14d** x+4 **15** $3^2+2\times3-15=0$ **16a** x=1, x=-1, x=-2 **16a** x=1, x=2, x=3 **17** -31

Exercise 11.1 **1** \$121.83 **2** \$750.00 **3** \$2500.00 **4** \$5000.00 **5** \$382.50 **6** \$960.63 **7** \$2522.00

Exercise 11.2 **1** \$6075 **2** \$9700 **3** \$4050 **4** \$4690 **5** \$11 918 **6** \$2337 **7** \$648

Exercise 11.3 **1** 4years **2** 2years **3** 16months **4** 90days **5** \$13 158 **6** \$9434 **7** 1.07% per month **8** 0.016% per day

Exercise 11.4 **1** \$11 910 **2** \$12 597 **3** \$13 310 **4** \$638 141 **5** \$669 113 **6** \$701 276 **7** \$84 695 **8** \$5755 **9** \$5498 **10** \$5501

Exercise 11.5 **1** \$161 051 **2** \$176 234 **3** 10 612 people **4** about 7 years **5** about 7 years **6** about 6 years

Exercise 11.6 **1** \$4228 **2** \$90 194 **3** \$977 337 **4** \$1 180 291 **5** \$75 764 **6** \$81 589 **7** 28 million **8** 192 million

Exercise 11.7 **1a** \$1080.00 **1b** \$1083.00 **1c** \$1083.28 **2a** \$146 933 **2b** \$148 985 **2c** \$149 176 **3** \$9000 **4** \$16 000 **5** \$68 000

Exercise 11.8 **2** I=Prt **3** n=6 **4** 1.08 **5** yes **6** $2x^2-3x+2$ **7** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **8** (x-4)(x-1) **9** 2 **10** 18

Exercise 11.9 **2** $A=P(1+\frac{r}{100})^n$ **3** n=12 **4** 1.09 **5** no **6** x^2+5x+6 **7** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **8** (x+6)(x+1) **9** 3 **10** 32

Exercise 11.10 **2** I=Prt **3** n=18 **4** 1.065 **5** no **6** x^2+x-2 **7** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **8** (x-3)(x-2) **9** 3 **10** 28

Exercise 11.11 **1** 25% **2** 6.25% **3** \$297 **4** \$495 **5** 200mL **6** 400mL **7** 22children **8** 43adults

Exercise 11.12 **1** X in any corner **2** 12 edges

Exercise 11.13 **1a** \$6900 **1b** \$12 600 **2** 1.5years **3** \$15 873 **4** 0.97% per month **5** \$9362 **6** \$150 632 **7a** \$140 255 **7b** \$141 763 **7c** \$141 902 **8** \$14 000

Exercise 11.13 **1a** \$4725 **1b** \$4500 **2** 3years **3** \$9390 **4** 9.1% pa **5** \$32 829 **6** \$225 432 **7a** \$168 506 **7b** \$172 892 **7c** \$173 311 **8** \$16 000

Exercise 12.1 **1a** 4.42 **1b** 10.79 **1c** 5.38 **1d** 6.60 **1e** 7.03 **1f** 3.65 **1g** 9.76 **1h** 2.57 **1i** 4.16 **2** 14.14m **3** 7.07

4 125.30km **5** 8.66

Exercise 12.2 **1a** 51.77° **1b** 50.55° **1c** 42.66° **1d** 13.8 **1e** 6.18 **1f** 10.23 **2** 43.26° **3** 4.61m

Exercise 12.3 **1a** 51.99° **1b** 55.36° **1c** 67.16° **1d** 9.76 **1e** 4.93 **1f** 5.72 **2** 21.17° **3** 4.23m

Exercise 12.4 **1a** 38.01° **1b** 34.64° **1c** 22.84° **1d** 9.76 **1e** 6.55 **1f** 12.34 **2** 5.93m **3** 48.19° **4** 15.64°

Exercise 12.5 **1** 160km **2** 161km **3** 653km

Exercise 12.6 **1** 8.58km **2** 94m **3** 107m **4** 521m **5** 15m

Exercise 12.7 **1** 40.08m **2** 56.34m **3** 35.69° **4** 8.34m **5** 10.81m **6** 47.15° **7** 183m/h **8a** 30.16m **8b** 29.13m **9** 17.49m **10a** 93.45 **10b** 54.62 **10c** 53.56 **10d** 121.74 **10e** 220.34 **10f** 284.68 **10g** 8.86 **10h** 58.26

Exercise 12.8 **2** 1/√2 **3** 1/√2 **4** 1 **5** 5 **6** I=Prt **7** yes **8** $2x^2+x-3$ **9** (x+3)(x+2) **10** 3

Exercise 12.9 **2** √3/2 **3** 1/2 **4** √3 **5** √2 **6** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **7** yes **8** $2x^2-x+2$ **9** $(x+3)^2$ **10** 3

Exercise 12.10 **2** 1/2 **3** √3/2 **4** 1/√3 **5** √5 **6** I=Prt **7** no **8** $2x^2-x+3$ **9** (x+4)(x+2) **10** 4

Exercise 12.11 **1** 30 **2** 3 **3** 0.3 **4** 0.03 **5a** 13 **5b** 26 **6a** 1 **6b** 2 **6c** 2.5 **7** a/4 **8** 12 **9** sinα=√3/2, cosα=1/2 **10** cosα=1/√2, tanα=1

Exercise 12.12 **1** $4a^2+2ab$ **2** 133.33km/h or 400/3km/h

Exercise 12.13 **1a** 9.24 **1b** 3.44 **1c** 49.80 **2a** 6.72 **2b** 11.9 **2c** β=36.66° **3** 73km **4** 672m **5a** 85m **5b** 61m **6a** 25.64 **6b** 61.47

Exercise 12.14 **1a** 11.67 **1b** 2.53 **1c** 52.28 **2a** 4.06 **2b** 81.80 **2c** 41.41° **3** 126km **4** 4389m **5a** 78.58m **5b** 37.25m **6a** 279.59 **6b** 190.20

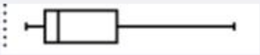
Exercise 13.1 **1** 5 **2** 5 **3** 55 **4** 2.5 **5** 3.45 **6** 23.5 **7** 0 **8** 1

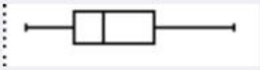
Exercise 13.2 **1** 3 **2** 5 **3** 43 **4** 4 **5** 3 **6** 2 **7** -2 **8** -6

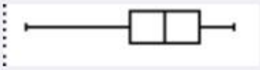
Exercise 13.3 **1** 8.5 **2** 3 **3** 78.5 **4** 5 **5** 5 **6** 7 **7** 7.5 **8** 4.3

Exercise 13.4 **1** (22, 24, 28) **2** (12, 14, 18) **3** (53, 56, 61)
4 (36.5, 44, 48.5) **5** (54.5, 75, 88.5) **6** (142, 152, 162.5)

Exercise 13.5 **1** (220, 290, 340, 560, 990)
2 (0, 1, 3, 15, 33)
3 (1.63, 2.07, 2.38, 2.93, 3.62)
4 (65, 80, 85, 90, 95)

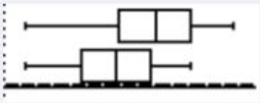




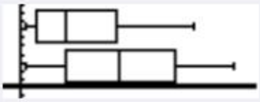


Exercise 13.6

1 10A: (31, 61, 72.5, 83, 97)
10B: (31, 49, 59.9, 71, 83)
10A performed significantly better on the test.
Three-quarters of 10A performed better than half of 10B



2 Ind (2, 6.5, 17, 36.5, 67)
Non (2, 17.5, 38, 59, 82)
Non-ind age distribution significantly larger
Half of the non-ind older than three-quarters of the ind population



Exercise 13.8 **1** 3 **2** 15.67 **3** 2.9 **4** 4.5 **5** 33.33 **6** 83.33 **7** 5.33 **8** 7.77 **9** -3.33 **10** -13.33

Exercise 13.9 **1** 1.15 **2** 28.79 **3** 1.97 **4** 2.69 **5** 1.97 **6** 1.97 **7** 1.97 **8** 1.97 **9** 0.20 **10** 0.20 **11** 1.97 **12** 1.97

Exercise 13.10

1 1st set mean=3.17, sd=0.69 2nd set mean=2.67, sd=1.11 1st set higer mean and is less spread out

2 10A mean=70, sd=17 10B mean=60, sd=14 10A has a higer average and 10A results are more spread out

3 Ind mean=23.76, sd=19.10 Non-ind mean=39.16, sd=23.33 Ind has lower average and less varied

4 Above mean=6.3, sd=0.5 Below mean=5.3, sd=0.7 Higher average oxygen above and less varied

Exercise 13.11 **2** 5 **3** 3 **4** 6 **5** 1/√2 **6** 1/√2 **7** 1 **8** 5 **9** I=Prt **10** no

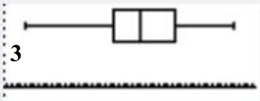
Exercise 13.12 **2** 3 **3** 1.5 **4** 4.5 **5** √3/2 **6** 1/2 **7** √3 **8** √3 **9** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **10** yes

Exercise 13.13 **2** 5 **3** 4 **4** 6.5 **5** 1/√2 **6** 1/√2 **7** 1 **8** √3 **9** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **10** yes

Exercise 13.14 **1a** mode=2 and 4, median=2.5, mean=2.67 **1b** mode=5, median=5, mean=4.57 **2a** 0.37 **2b** 4.107 2c 0.107 **2d** 1/6 **2e** 3/8 **2f** 7/20 **3** 15 **4** 45 **5** 7.8 **6** 8.3 **7** 9km/h **8** 9.8km/h **9** 10,12,12,16,15 is one set of possible scores

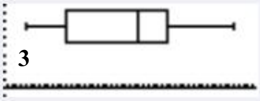
Exercise 13.15 **1** 6 **2** 30 - bottom times difference of top numbers **3a** 1 **3b** 5 **3c** 14 **3d** 30

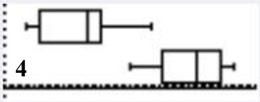
Exercise 13.16 **1a** mode=2, median=2, mean=2.57
1b mode=5, median=5, mean=4.57
2 mean=8.39, Q1=8.2, median=8.3, Q3=8.6,
Range=0.7, sd=0.28



5 DisplayA: mean=9.83, sd=2.74 DisplayB: mean=12.88, sd=2.87 Display A has the lower average although both displays have a similar spread **6** 25.14km/h

Exercise 13.17 **1a** mode=2, median=2, mean=2.57
1b mode=3, median=3, mean=2.56
2 mean=3.62, Q1=3.5, median=3.7, Q3=3.8,





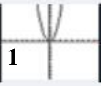
Range=0.7, sd=0.19

5 Ind: mean=6.05, sd=1.56 Non-ind: mean=6.64, sd=1.82 Ind has the lower average although both have a similar spread

6 35.25km/h

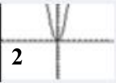
Exercise 14.1

1 x	-3	-2	-1	0	1	2	3
$y = 2x^2$	18	8	2	0	2	8	18



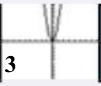
1

2 x	-3	-2	-1	0	1	2	3
$y = 3x^2$	27	12	3	0	3	12	27



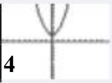
2

3 x	-3	-2	-1	0	1	2	3
$y = 5x^2$	45	20	5	0	5	20	45



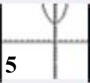
3

4 x	-3	-2	-1	0	1	2	3
$y = x^2 + 2$	11	6	3	2	3	6	11



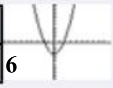
4

5 x	-3	-2	-1	0	1	2	3
$y = x^2 + 5$	14	9	6	5	6	9	14



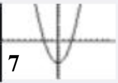
5

6 x	-3	-2	-1	0	1	2	3
$y = x^2 - 3$	6	1	-2	-3	-2	1	6



6

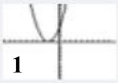
7 x	-3	-2	-1	0	1	2	3
$y = x^2 - 6$	3	-2	-5	-6	-5	-2	3



7

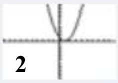
Exercise 14.2

1 x	-3	-2	-1	0	1	2	3
$y = (x + 2)^2$	1	0	1	4	9	16	25



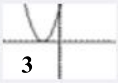
1

2 x	-3	-2	-1	0	1	2	3
$y = (x - 1)^2$	16	9	4	1	0	1	4



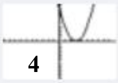
2

3 x	-3	-2	-1	0	1	2	3
$y = (x + 3)^2$	0	1	4	9	16	25	36



3

4 x	-3	-2	-1	0	1	2	3
$y = (x - 3)^2$	36	25	16	9	4	1	0

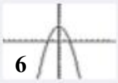


4

5 x	-3	-2	-1	0	1	2	3
$y = -x^2$	-9	-4	-1	0	-1	-4	-9

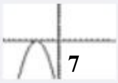
5

6 x	-3	-2	-1	0	1	2	3
$y = -x^2 + 4$	-5	0	3	4	3	0	-5



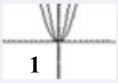
6

7 x	-3	-2	-1	0	1	2	3
$y = -(x + 2)^2$	-1	0	-1	-4	-9	-16	-25

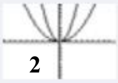


7

Exercise 14.3



1



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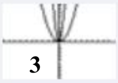
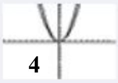


3



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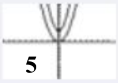

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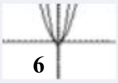

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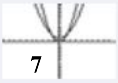

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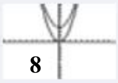

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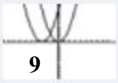

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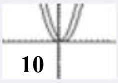

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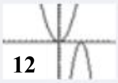

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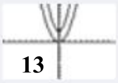

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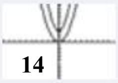

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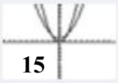

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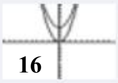
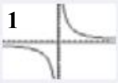


16

Exercise 14.4 **1a** centre=(0,0) radius=3 **1b** centre=(0,0) radius=4 **1c** centre=(3,-1) radius=2 **1d** centre=(-1,5) radius=6 **1e** centre=(4,0) radius=$\sqrt{3}$ **1f** centre=(0,-2) radius=$\sqrt{5}$ **2a** $x^2+y^2=4$ **2b** $x^2+y^2=5$ **2c** $(x-1)^2+(y-3)^2=9$ **2d** $(x+4)^2+(y-1)^2=49$ **2e** $(x-6)^2+(y+4)^2=3$ **2f** $(x+2)^2+(y+3)^2=2$

Exercise 14.5

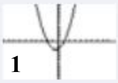
1 x	-3	-2	-1	0	1	2	3
$y=\frac{9}{x}$	-3	-4.5	-9	∞	9	4.5	3


1

x	-3	-2	-1	0	1	2	3
$y=\frac{-4}{x}$	1.33	2	4	∞	-4	-2	-1.3


2

Exercise 14.6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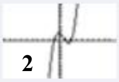
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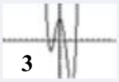
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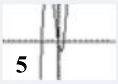
4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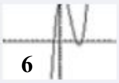
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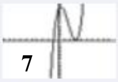
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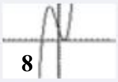
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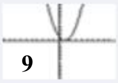
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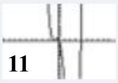
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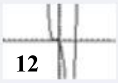
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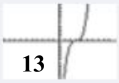
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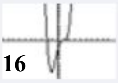
15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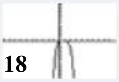
17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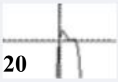
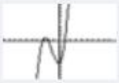
19


20

Exercise 14.7 **1** x=1, x=2, x=-1  **2** x=1, x=-2, x=-3  **3** x=-1, x=-2, x=-4 

4 x=-1, x=3, x=2  **5** x=2, x=3, x=-4  **6** x=2, x=5, x=-3 

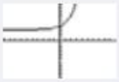
Exercise 14.8

1 x	-3	-2	-1	0	1	2	3
$y = 3^x$	1/27	$\frac{1}{9}$	1/3	1	3	9	27

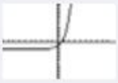
2 x	-3	-2	-1	0	1	2	3
$y = 2\times2^x$	1/4	1/2	1	2	4	8	16

3 x	-3	-2	-1	0	1	2	3
$y = 5\times2^x$	$\frac{5}{8}$	5/4	5/2	5	10	20	40

4 x	-3	-2	-1	0	1	2	3
$y = 2^x + 3$	3.125	3.25	3.5	4	5	7	11



5 x	-3	-2	-1	0	1	2	3
$y = 3^x - 2$	-1.96	-1.89	-1.67	-1	1	7	25



6 x	-3	-2	-1	0	1	2	3
$y = 2^{-x}$	8	4	2	1	1/2	$\frac{1}{4}$	1/8

7 x	-3	-2	-1	0	1	2	3
$y = 3^{-x} - 4$	23	5	-1	-3	-3.67	-3.89	-3.96

Exercise 14.9 **1a** 20000×1.05^t **1b** 25 500 **1c** 14years **2a** $35000\times(1-0.12)^t = 35000\times0.88^t$ **2b** $$18 500 **3a** 250mg **3b** 170mg **3c** almost 2 hours **4a** 27 000 **4b** little over 2 hours

Exercise 14.10 **2** $y=ax^2+bx+c$ **3** y=a/x **4** $x^2+y^2=a^2$ **5** 3 **6** 2 **7** 1/√2 **8** 5 **9** I=Prt **10** no

Exercise 14.11 **2** $y=ax^2+bx+c$ **3** y=a/x **4** $x^2+y^2=a^2$ **5** 3.5 **6** 5.5 **7** 1/2 **8** √5 **9** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **10** yes

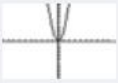
Exercise 14.12 **2** $y=ax^2+bx+c$ **3** y=a/x **4** $x^2+y^2=a^2$ **5** 3 **6** 2.5 **7** 1/2 **8** √8 or 2√2 **9** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **10** no

Exercise 14.13 **1** c **2a** 4 **2b** 8 **2c** 12 **2d** 80 **3a** x^2+x-2 **3b** x^2-2x-3
4 A:y=-x(x+3)(x-2) B:y=x(x+3)(x-2) C:x(x-3)(x-3)(x+2) **5** a **6** (-1,2)
Exercise 14.14 **1** 130 **2** 9,2,7, 4,6,8 5,10,3 is one answer

Exercise 14.15

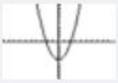
1a) x-intercept is (0,0), y-intercept is (0,0)

1a) x	-3	-2	-1	0	1	2	3
$y = 3x^2$	27	12	3	0	3	12	27



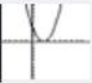
1b) x-intercepts are (-√5,0) and (-√5,0)
y-intercept is (0,-5)

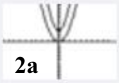
1b) x	-3	-2	-1	0	1	2	3
$y = x^2 - 5$	4	-1	-4	-5	-4	-1	4

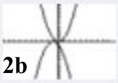


1c) x-intercept is (2,0), y-intercept is (0,4)

1c) x	-3	-2	-1	0	1	2	3
$y = (x-2)^2$	25	14	9	4	1	0	1

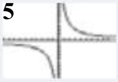


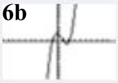
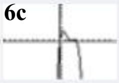
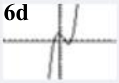

2a


2b

3 centre = (3,-1) radius=2 **4** $(x-3)^2 + (y+2)^2 = 3$

5 x	-3	-2	-1	0	1	2	3
$y=\frac{9}{x}$	-3	-4.5	-9	∞	9	4.5	3

5

6a

6b

6c

6d


7 x	-3	-2	-1	0	1	2	3
$y = 2^x + 3$	3.125	3.25	3.5	4	5	7	11

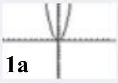

7

8a P=10000×1.05^t **8b** 16300 **8c** 16yrs
9a 250mg **9b** 114mg **9c** 3.5hrs

Exercise 14.16

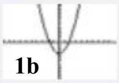
1a) x-intercept is (0,0), y-intercept is (0,0)

1a) x	-3	-2	-1	0	1	2	3
$y = 2x^2$	18	8	2	0	2	8	18


1a

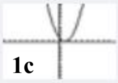
1b) x-intercepts are (-√3,0) and (-√3,0)
y-intercept is (0,-3)

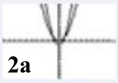
1b) x	-3	-2	-1	0	1	2	3
$y = x^2 - 3$	6	1	-2	-3	-2	1	6

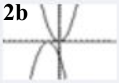

1b

1c) x-intercept is (1,0), y-intercept is (0,1)

1c) x	-3	-2	-1	0	1	2	3
$y = (x-1)^2$	16	9	4	1	0	3	4

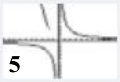

1c

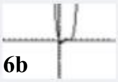
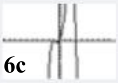

2a

2b


3 centre = (-1,2) radius=3 **4** $(x+2)^2 + (y-3)^2 = 2$

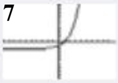
5 x	-3	-2	-1	0	1	2	3
$y=\frac{6}{x}$	-2	-3	-6	∞	6	3	2


5

6a

6b

6c

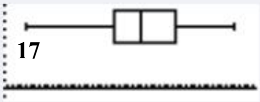
6d

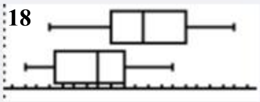
7 x	-3	-2	-1	0	1	2	3
$y = 2^x - 2$	-1.875	-1.75	-1.5	-1	0	2	6

7


8a P=5000×1.02^t **8b** 6100 **8c** 35yrs
9a 500mg **9b** 255mg **9c** 2hrs

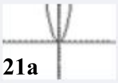
Exercise 15.1 **2** $y=ax^2+bx+c$ **3** y=a/x **4** $x^2+y^2=a^2$ **5** 3 **6** 2 **7** 1/√2 **8** √5 **9** I=Prt **10** yes
Exercise 15.2 **1** $12 750 **2** 1.14yrs **3** $15 385 **4** 1.27% **5** $31 351 **6** $305 485 **7a** $153 862 **7b** $156 568
8 $21 000 **9a** 8.75 **9b** 3.51 **9c** 68.81 **10a** 5.87 **10b** 370.52 **10c** 39.05° **11** 85km **12** 2093m **13a** 83.37m
13b 56.01m **14** 78.74 **15a** mode=2, median=2, mean=2.57 **15b** mode=5, median=5, mean=4.57
16 mean=8.39, Q1=8.2, median=8.3, Q3=8.6, range=0.9, sd=0.28


17

18


19 DisplayA: mean=9.83, sd=2.74 DisplayB: mean=12.88, sd=2.87
Display A has the lower average although both displays
have a similar spread **20** 21.43km/h
21a x-intercept (0,0), y-intercept (0,0)

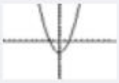
21a) x	-3	-2	-1	0	1	2	3
$y = 2x^2$	18	8	2	0	2	8	18

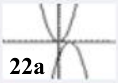

21a

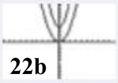
21b

21b x-intercept (-√3,0), (√3,0), y-intercept (0,-3)

21b) x	-3	-2	-1	0	1	2	3
$y = x^2 - 3$	6	1	-2	-3	-2	1	6

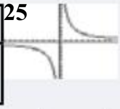



22a


22b

23 centre =(3,-1) radius=2 **24** $(x-3)^2 + (y+2)^2 = 3$

25 x	-3	-2	-1	0	1	2	3
$y=\frac{9}{x}$	-3	-4.5	-9	∞	9	4.5	3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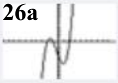
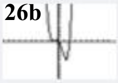
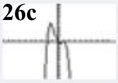
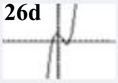
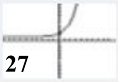

26a


26b


26c

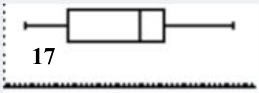

26d

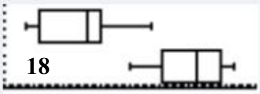
27 x	-3	-2	-1	0	1	2	3
$y = 2^x + 1$	1.125	1.25	1.5	2	3	5	9


27

28a $P=10000\times1.03^t$ **28b** 13400 **28c** 23yrs

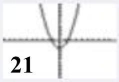
Exercise 15.3 **2** $y=ax^2+bx+c$ **3** y=a/x **4** $x^2+y^2=a^2$ **5** 3.5 **6** 5.5 **7** 1/2 **8** √10 **9** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **10** yes
Exercise 15.4 **1** $17 250 **2** 4yrs **3** $10 929 **4** 0.42% **5** $12 877 **6** $288 144 **7a** $153 862 **7b** $156 568 **8** $13 000 **9a** 7.12 **9b** 5.18 **9c** 32.51 **10a** 6.72 **10b** 8.41 **10c** 54.90° **11** 62km **12** 2629m **13a** 77.59m **13b** 50.22m **14** 25.64 **15a** mode=3, median=3, mean=2.75 **15b** mode=3, median=3, mean=2.56
16 mean=6.62, Q1=6.5, median=6.7, Q3=6.8, range=0.8, sd=0.21


17


18

19 Before: mean=18.41, sd=2.56 After: mean=20.74, sd=3.25
Dexerity after has a higher average and a larger spread **20** 31.67km/h
21 x-intercept (-√2,0), (√2,0), y-intercept (0,-2)

21 x	-3	-2	-1	0	1	2	3
$y = x^2 - 2$	7	2	-1	-2	-1	2	7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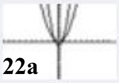
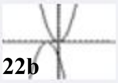
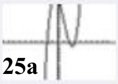

22a

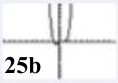

22b

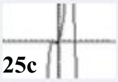
23 $(x-2)^2 + (y-3)^2 = 5$

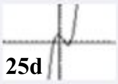
24 x	-3	-2	-1	0	1	2	3
$y=\frac{2}{x}$	-0.67	-1	-2	∞	2	1	0.67


24

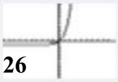

25a


25b


25c


25d

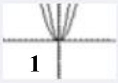
26 x	-3	-2	-1	0	1	2	3
$y = 3^x - 1$	-0.96	-0.89	-0.67	0	2	8	26


26

27a 400mg **27b** 237mg **27c** 5.3hrs

Exercise 16.1 **1** m=1 **2** m=0.2 **3** m=-1 **4** -2/3 **5** m=2.5 **6** m=0.8 **7** m=-2 **8** m=-1 **9** m=1.5 **10** m=-1 **11** m=1.5 **12** m=-1 **13** m=-1/3 **14** m=4/7 **15** m=0.8 **16** m=3 **17** m=0.75
Exercise 16.2 **1** x-axis **2** x-axis **3** x-axis **4** y-axis **5** y-axis **6** y-axis **7** x-axis **8** y-axis **9** x-axis
Exercise 16.3 **1** m_{AB}=-1.5, m_{CD}=2.5, m_{EF}=-1.5, m_{GH}=1.75 AB//EF **2** m_{AB}=3, m_{CD}=-11, m_{EF}=-11, m_{GH}=3, m_{IJ}=0 AB//GH and CD//EF **3a** m_{AB}=4/3, m_{BC}=1/3, m_{CD}=4/3, m_{AD}=1/3 ABCD is a parallelogram **3b** m_{AB}=2, m_{BC}=-3/2, m_{CD}=2, m_{AD}=-3/2 ABCD is a parallelogram **3c** m_{AB}=-1/7, m_{BC}=-1, m_{CD}=-1/7, m_{AD}=7/5 ABCD is not a parallelogram **4** D(1,-3)
Exercise 16.4 **1a** // **1b** ⊥ **1c** ⊥ **1d** neither **1e** neither **1f** ⊥ **1g** ⊥ **1h** neither **2** m_{AB}=2, m_{CD}=-1/2, m_{EF}=-8/3, m_{GH}=2, AB//GH, AB⊥CD, CD⊥GH **3a** m_{AB}=-2, m_{BC}=1/3, m_{AC}=5 NO **3b** m_{AB}=-3, m_{BC}=-1/2, m_{AC}=1/3 YES **3c** m_{AB}=, m_{BC}=, m_{AC}= NO, **4** C(-7,3) is one answer **5** m_{AB}=2, m_{BC}=-1/2, m_{CD}=2, m_{AD}=-1/2, YES
Exercise 16.5 **1a** m=1, c=2 **1b** m=1, c=-3 **1c** m=2, c=1 **1d** m=2, c=-1 **1e** m=-2, c=-5 **1f** m=-1, c=7.5 **1g** m=1.2, c=-2 **1h** m=-0.5, c=-3.6 **2a** y=x+2, m=1, c=2 **2b** y=-x+3, m=-1, c=3 **2c** y=2x-3, m=2, c=-3 **2d** y=-3x-4, m=-3, c=-4 **2e** y=-1.5x+0.5, m=-1.5, c=0.5 **2f** y=2.5x-0.5, m=2.5, c=-0.5 **2g** y=-0.8x+0.2, m=-0.8, c=0.2 **2h** y=2/3x+2, m=2/3, c=2 **3a** m=1, x-int (-2,0) y-int(0,2) **3b** m=1, x-int (3,0) y-int(0,-3) **3c** m=2, x-int (0.5,0) y-int(0,-1) **3d** m=2, x-int (-2,0) y-int(0,4) **3e** m=-1, x-int (-2,0) y-int(0,-2) **3f** m=-1, x-int (5,0) y-int(0,5) **3g** m=1.5, x-int (2,0) y-int(0,-3) **3h** m=-1.2, x-int (-2.5,0) y-int(0,-3) **3i** m=-2, x-int (0.5,0) y-int(0,1) **3j** m=3, x-int (1,0) y-int(0,-3) **3k** m=-0.5, x-int (8/3,0) y-int(0,4/3) **3l** m=2.1, x-int (5/7,0) y-int(0,-1.5)
Exercise 16.6 **1a** $m_1=m_2=3$ **1b** $m_1=m_2=2$ **2a** $m_1(-2)\times m_2(1/2)=-1$ **2b** $m_1(1)\times m_2(1/2) \neq -1$ **2c** $m_1(-2)\times m_2(1/2)=-1$ **3a** m=-2/3 **3b** m=3/2 **4a** m_{AB}=1/3 m_{BC}=1/3 thus collinear **4b** m_{AB}=1/3 m_{BC}=1/3 thus collinear **4b** m_{AB}=1 m_{BC}=1 thus collinear **5a** 45° **5b** 63.43° **5c** -63.43° **5d** 78.69° **5e** -56.31° **6** x=10 **7** x=-11 **8** a=-2 **9** b=2/3 **10** y=-2x+2 **11** y=-0.5x-3 **12** y=0.5x+1

Exercise 16.7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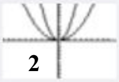
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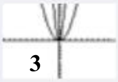
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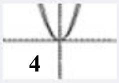
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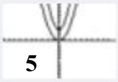
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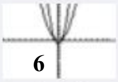
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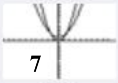
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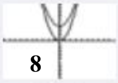
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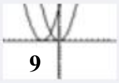
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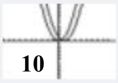
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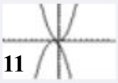
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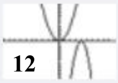
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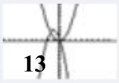
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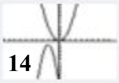
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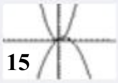
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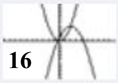
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Exercise 16.8 **1a** centre(0,0), r=3 **1b** centre(0,0), r=4 **1c** centre(0,0), r=2 **2a** centre(3,0), r=2 **2b** centre(1,0), r=3 **2c** centre(4,0), r=1 **2d** centre(-1,0), r=2 **2e** centre(-2,0), r=4 **2f** centre(0,-1), r=2 **2g** centre(0,-3), r=3 **2h** centre(0,-2), r=4 **2i** centre(0,5), r=5 **2j** centre(0,2), r=1 **2k** centre(1,1), r=1 **2l** centre(3,1), r=2 **2m** centre(2,-3), r=3 **2n** centre(-4,2), r=4 **2o** centre(-1,-5), r=5

Exercise 16.9 **2** 3 **3** -1/3 **4** 45° **5** $y=ax^2+bx+x$ **6** 3 **7** no **8** $1/\sqrt{2}$ **9** 5 **10** $3

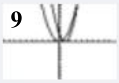
Exercise 16.10 **2** 2 **3** -1/2 **4** -45° **5** y=a/x **6** 3 **7** yes **8** $1/\sqrt{2}$ **9** $\sqrt{5}$ **10** $4

Exercise 16.11 **2** 5 **3** 1/2 **4** 45° **5** $x^2+y^2=a^2$ **6** 6 **7** yes **8** $\sqrt{3}$ **9** $\sqrt{10}$ **10** $4.50

Exercise 16.12 **1** no **2** yes **3** a **4a** 2x+3x=180, x=36° **4b** x+50+x-50=180, x=90° **4c** 4x+6x+5x=180, x=12° **5a** 4.5units^2 **5b** 6.75units^2

Exercise 16.13 **1** 12 edges **2** x=6, or x=-10

Exercise 16.14 **1a** m=1 **1b** m=3/5 **1c** m=-4/3 **2a** $m_{AB}=\infty$ //y-axis **2b** $m_{AB}=0$ //x-axis **2c** $m_{AB}=0$ //x-axis **3** $m_{AB}=3/2$, $m_{CD}=-3/2$, $m_{EF}=-3/2$, $m_{GH}=2/3$ CD//EF, GH $\perp$ CD, GH $\perp$ EF **4a** $m_1=m_2=2$ **4b** $m_1=m_2=3$ **5a** $m_1\times m_2=-1$ **5b** $m_1\times m_2=-1$ **5c** $m_1\times m_2=-1$ **6** x=-22 **7** b=-7 **8** y=-2x+4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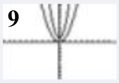
Exercise 16.15 **1a** m=-2/3 **1b** m=1 **1c** m=-4/5 **2a** $m_{AB}=0$ //x-axis **2b** $m_{AB}=\infty$ //y-axis **2c** $m_{AB}=\infty$ //y-axis **3** $m_{AB}=4$, $m_{CD}=-1/4$, $m_{EF}=-1/2$, $m_{GH}=4$ AB//GH, CD $\perp$ AB, CD $\perp$ GH **4a** $m_1=m_2=-1$ **4b** $m_1=m_2=2$ **5a** $m_1\times m_2=-1$ **5b** $m_1\times m_2=-1$ **5c** $m_1\times m_2=-1$ **6** x=-9/2 **7** b=-4/3 **8** y=-3/2x+3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Exercise 17.1 **1** 107° **2** 112° **3** 66° **4** 119° **5** 153° **6** 147° **7** 85° **8** 94° **9** 58° **10** 102° **11** m=137° n=43° **12** a=95° b=46° c=39° d=95°

Exercise 17.2 **1** 43° **2** 78° **3** 90° **4** 60° **5** 90° **6** 63° **7** 119° **8** 82° **9** 46° **10** 90° **11** a=54° b=126° **12** c=52° d=76°

Exercise 17.3 **1** a=59° b=121° **2** x=64° y=116° **3** e=57° f=123° **4** x=37° y=53° z=37° **5** a=42° b=48° c=42° **6** x=58° y=32° z=32°

Exercise 17.4 **1** a=131° b=49° c=49° **2** d=128° e=52° f=52° **3** g=131° h=49° i=131° **4** j=58° k=58° l=122° **5** m=114° n=66° o=114° p=114° **6** q=129° r=51° s=129° t=129° **7** u=57° v=57° w=123° **8** d=107° e=67° f=113° **9** g=54° h=63° i=63° j=63° k=117° **10** x=132° **11** l=51° m=51° **12** n=45° o=83° p=97° q=52°

Exercise 17.5 **1** x=18° y=90° **2** x=15° y=90° **3** x=47° y=43° **4** x=44° y=46° **5** x=44° y=58° **6** x=73° y=67° z=40° **7** a=90° b=35° c=125° **8** x=60° y=60° **9** x=65°

Exercise 17.6 **1** 38.16cm **2** 17.52m **3** 22.36cm **4** x=96°, y=42° **5**

Exercise 17.7 **1** AAA, DC=60 **2** RHS, CD=60

Exercise 17.8 **1** ΔCDE~ΔCAB AAA, CE/CB=1/2, thus CE=EB **2** ΔCDE~ΔCAB SAS, $\angle$CDE=$\angle$CAB thus DE//AB **3** ΔADC~ΔBDC RHS, thus AD=BD **4** ΔADC~ΔBDC SAS, thus AC=BC

Exercise 17.9 **2** all sides equal, all angles equal **3** all sides equal, all angles 90° **4** 360° **5** m=2 **6** m=-1/2 **7** 45° **8** yes **9** $1/\sqrt{2}$ **10** $\sqrt{2}$

Exercise 17.10 **2** two sides equal, angles opposite equal sides are equal **3** opposite sides equal and parallel, equal, all angles 90° **4** 540° **5** m=3 **6** m=1 **7** -45° **8** no **9** $\sqrt{3}/2$ **10** $\sqrt{5}$

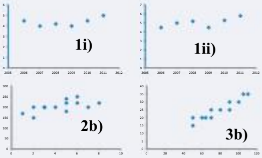
Exercise 17.11 **2** all sides equal, opposite sides parallel **3** opposite sides equal, opposite sides parallel **4** 720° **5** m=5 **6** m=-1/5 **7** 0° **8** yes **9** $\sqrt{3}/2$ **10** $\sqrt{10}$

Exercise 17.12 **1** 31° **2** 77° **3** 60° **4** 50° **5** 36° **6** 22° **7** 60° **8** 76° **9** 51° **10** 67° **11** 23° **12** 71° **13** 39° **14** 120° **15** 147°

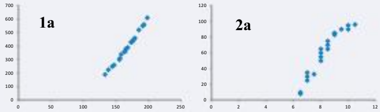
Exercise 17.13 **1** $50 **2** 75° **3** 54° **4a** 93+853+93 = 1039 **4b** 92836 + 12836 = 105672 **4c** 5061 + 5061 + 436 = 10558 **4d** 7514 + 7514 = 15113

Exercise 17.14 **1a** x=121° **1b** y=95° **1c** z=92° **1d** a=91°,b=45°,c=37° **1e** x=47° **1f** y=60° **1g** x=43.5° **1h** x=88°,y=92° **1i** a=57°,b=123° **1j** x=22°,y=68°,z=22° **1k** x=69°,y=21°,z=21° **1l** x=63°,y=63°,z=117° **1m** a=70°,b=55°,c=55°,d=55° **1n** x=135° **1o** x=18°,y=90° **1p** x=48°,y=42° **1q** a=90°,b=32°,c=122° **2** ΔCDE~ΔCAB AAA, DC=60 **3** ΔCDE~ΔCAB AAA, CE/CB=1/2, thus CE=EB **4** x=94°,y=43°

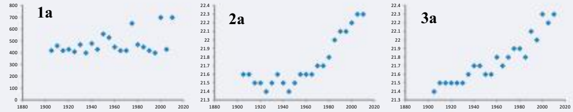
Exercise 17.15 **1a** x=115° **1b** x=140° **1c** z=101° **1d** a=37°,b=42°,c=101° **1e** x=90° **1f** y=118° **1g** z=44.5° **1h** a=46°,b=88°,c=46° **1i** x=60°,y=60° **1j** x=23°,y=67°,z=23° **1k** x=65°,y=25°,z=25° **1l** x=61°,y=61°,z=118° **1m** a=84°,b=48°,c=48°,d=48° **1n** x=58° **1o** x=15°,y=90° **1p** x=46.5°,y=43.5° **1q** x=62°,y=56° **2** ΔCDB~ΔCEA RHS, DE=15 **3** ΔCDE~ΔCAB SAS, $\angle$CDE=$\angle$CAB thus DE//AB **4** 50.92cm



Exercise 18.1 **1i)a)** Year - independent, \$Abillion - dependent **1i)c)** 4 **1i)d)** reasonably confident of 4±2 **1ii)a)** Year - independent, \$Abillion - dependent **1ii)c)** 5 **1ii)d)** reasonably confident of 5±2
2a) Ads-independent, \$Sales-dependent **2c)** It appears that as the number of ads increase the \$Sales also increases - it depends on cost of the ads
3a) km/h-independent, L/100km-dependent **3c)** The slower the speed the more economical
Exercise 18.2 **1** strong positive linear **2** no relationship **3** strong negative linear **4** strong positive non-linear **5** moderate positive linear **6** strong negative linear **7** strong negative non-linear **8** strong positive linear **9** moderate non-linear

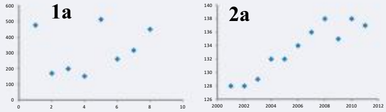


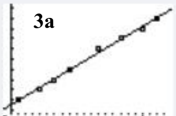
Exercise 18.3 **1b** strong positive linear relationship
1c 161cm is about 350kg ie 5×3.5=17.5g **1c** reasonably confident
2b moderate positive non-linear **2c** age is a third factor that causes both size and test results **2d** a werak relationship as students beging reaching their full shoes size
2e collect data within the class
Exercise 18.4 **1** upward linear trend **2** downward linear trend **3** non-linear downward trend **4** upward linear trend **5** downward linear trend **6** no trend **7** upward linear trend **8** upward linear trend **9** downward linear trend



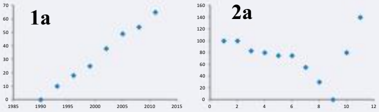
Exercise 18.5 **1b** a diverging trend **1c** between 400 and 700 **1d** difficult to be more accurate than 400 to 700 **2b** upward non-linear trend **2c** 22.4°C
2d maximum error of ±0.1°C
3b upward linear trend **3c** 22.3°C **3d** maximum error of ±0.1°C

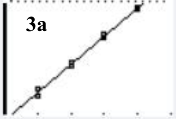
Exercise 18.6 **2b** Freq=0.43×Temp+5.19 **2c** Freq=0.43×23+5.19=15
2d Freq=0.43×18+5.19=13 **3b** GDP=72.47×year-144582
3c GDP=72.47×2002-144582=503 **3d** GDP=72.47×2015-144582=1445
4b GDP=432×year-863000 **4c** GDP=432×2002-863000=1864 **4d** GDP=432×2015-863000=7480
Exercise 18.7 **2** two sides equal, angles opposite equal sides equal **3** all sides equal, all angles 90° **4** 360° **5** m=1 **6** m=-1/2 **7** 45° **8** 1/2 **9** (x+3)(x+1) **10** $\sqrt{5}$
Exercise 18.8 **2** all sides equal, all angles 60° **3** opposite sides equal, all angles 90° **4** 720° **5** m=3 **6** m=3 **7** -45° **8** $1/\sqrt{2}$ **9** (x+5)(x+1) **10** $\sqrt{10}$
Exercise 18.9 **2** all sides equal, opposite sides parallel **3** opposite sides equal, opposite sides parallel **4** 1080° **5** m=2 **6** m=-1/2 **7** 0° **8** $\sqrt{3}/2$ **9** (x+2)(x+1) **10** $\sqrt{13}$
Exercise 18.10 **2** 90° **3** 108° **4** 2008-2011 **5** 1cm/s
Exercise 18.11 **1** 15 years old **2** 14 **3** 21 **4** 130





Exercise 18.12 **1b** no trend **1c** difficult to answer due to lack of a trend **1d** difficult to answer due to lack of a trend **2b** upward non-linear trend
2c 136-138 mm **2d** 90% certain of 137±3 **3b** C=0.43×OD+1.52 **3c** C=0.43×5+1.52=3.7
3d C=0.43×30+1.52=14.4





Exercise 18.13 **1b** strong positive linear
1c 70±5mm **1d** confident if the trend holds
2b downward trend except for last 2 points
2c 200m **2d** very little confidence **3b** Price=82000×Bed+280000
3c Price=82000×3+280000=530000 **3d** Price=82000×5+280000=690000

Exercise 19.1 **1** 15 **2** 2.98 **3** 9 **4** 11.92 **5** 9.79 **6** 43.91
Exercise 19.2 **1** yes **2** yes **3** yes **4** yes **5** yes **6** yes
Exercise 19.3 **1** x=3.75, y=4.16, β=48° **2** a=56.60, b=49.20, θ=41° **3** m=696.32, n=563.87, α=51° **4** x=0.15, y=0.21, β=36° **5** c=8.99, d=21.17, α=23° **6** e=4.82, f=7.72, ϕ=32° **7** x=54.31, y=57.44, β=19° **8** a=452.59, ϕ=51.46°, θ=38.54° **9** x=32.71, y=72.05, α=63°
Exercise 19.4 **1** A=71°, a=27.15m, c=25.35m **2** C=63°, b=122.24m, c=115.19m **3** B=47°, b=22.84m, c=18.36m **4** B=44.2°, C=83.8°, c=8.71m **5** A=27.1°, B=97.9°, b=10.88cm **6** B=76.9°, C=40.1°, c=3.11mm
Exercise 19.5 **1** a=11.64m, B=44.4°, c=56.6° **2** a=6.96m, B=64.2°, c=55.8° **3** a=11.81cm, B=70.1°, c=44.9° **4** a=2.84m, B=57.1°, c=74.9° **5** a=16.46m, B=30.7°, c=31.3° **6** a=74.29cm, B=78.5°, c=44.5°
Exercise 19.6 **1a** 133.99m^2 **1b** 19.87cm^2 **3** 61.17m^2
Exercise 19.7 **1** 1 **2** 0 **3** ∞ **4** 0 **5** -1 **6** 0 **7** -1 **8** 0 **9** ∞ **10** 0 **11** 1 **12** 0 **13** 1/2 **14** -1/2 **15** $-\sqrt{3}$ **16** -1/2 **17** -1/2 **18** $-\sqrt{3}$ **19** -1/2 **20** $\sqrt{3}/2$ **21** $\sqrt{3}$ **22** $1/\sqrt{2}$ **23** $-1/\sqrt{2}$ **24** -1 **25** -1/2 **26** -1/2 **27** -1
Exercise 19.8 **5a** $150\sqrt{3}\text{m}^2$ **5b** $150\sqrt{3}\text{m}^2$ **6a** $10\sqrt{2}$cm **6b** $10\sqrt{3}$cm **6c** 35.26° **6d** $50\sqrt{2}\text{cm}^2$ **7a** $230.4\sqrt{2}$m **7b** 51.82° **7c** m=1.27 **7d** 86000m^2 **8** 50.59m **9** 1cm error **10** 190m

Exercise 19.9 **2** $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ **3** $a^2=b^2+c^2-2bc\cos A$ **4** A=0.5absinC **5** two sides equal, angles opposite equal sides equal **6** m=3 **7** m=-1/3 **8** 1/2 **9** (x-3)(x-2) **10** √2

Exercise 19.10 **2** $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ **3** $a^2=b^2+c^2-2bc\cos A$ **4** A=0.5absinC **5** three sides equal, three angles equal **6** m=2 **7** m=-1/2 **8** 1/√2 **9** (x-3)(x-1) **10** √5

Exercise 19.11 **2** $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ **3** $a^2=b^2+c^2-2bc\cos A$ **4** A=0.5absinC **5** opposite sides equal, opposite sides parallel **6** m=-5 **7** m=5 **8** √3/2 **9** (x-6)(x-1) **10** √10

Exercise 19.12 **1a** (1,1) **1b** (1,√3) **1c** (√3,1) **1d** (3,3) **1e** (6,6√3) **1f** (6√3,6) **2a** $A=x^2/2$, P=2x+x√2 **2b** $A=x^2\sqrt{3}/2$, P=3x+x√3 **2c** $A= x^2\sqrt{3}/2$, P=3x+x√3 **3a** A=4 **3b** A= 3√3 **4a** 7√3 **4b** 7√3

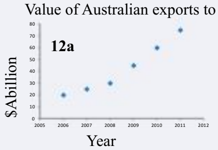
Exercise 19.13 **1** a=70, b=80, c=30 **2** C **3** $1.5a^2 + 6ab + b^2$

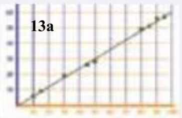
Exercise 19.14 **1a** x=1.62 **1b** x=5.3 **1c** a=30.93, b=29.87, θ=46° **2a** b=95.13m, c=96.62m, C=67° **2b** a=8.31m, B=57.1°, C=51.9° **2c** a=6.68m, B=61.6°, C=76.4° **3a** -1 **3b** -1/2 **3c** -√3 **4** 3130m^2 **8a** 230.4√2m **8b** 51.82° **8c** m=1.27 **8d** 86000m^2

Exercise 19.15 **1a** x=10.63 **1b** b=9.03 **1c** a=66.18, b=63.91, θ=44° **2a** b=97.56m, c=86.58m, A=77° **2b** b=12.82m, A=33.4°, B=87.6° **2c** a=6.14m, B=65.5°, C=74.5° **3a** 0 **3b** 1/2 **3c** -1/√3 **4** 20.63m^2 **8a** 10√2cm **8b** 10√3cm **8c** 35.26° **8d** 50√2cm^2

Exercise 20.1 **2** $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ **3** $a^2=b^2+c^2-2bc\cos A$ **4** A=0.5absinC **5** two sides equal, angles opposite equal sides equal **6** m=2 **7** m=-1/2 **8** 1/2 **9** (x-3)(x-2) **10** √5

Exercise 20.2 **1a** m=-1 **1b** m=0.6 **1c** m=-0.75 **2a** m=0 // x-axis **2b** m=∞ // y-axis **3** m_{AB}=2, m_{CD}=-1/2, m_{EF}=-8/3, m_{GH}=2, AB//GH, AB⊥CD, CD⊥GH **4a** // **4b** // **4c** ⊥ **4d** ⊥ **5** a=-3 **6** b=3 **7** y=-1.5x+2 **8a** 107° **8b** a=43°, b=37°, c=100°, d=43° **8c** x=119° **8d** a=50°, b=80°, c=50° **8e** x=61°, y=61°, z=119° **8f** a=84°, b=48°, c=48°, d=48°, e=132° **8g** x=25°, y=65°, z=25° **8h** x=10°, y=90° **8i** x=44°, y=46° **8j** x=65°, y=50° **9** ΔCDB~ΔCEA SAS, CA/CB=3/2 thus AC=30 **10** ΔCDE~ΔCAB SAS, ∠CDE=∠CAB thus DE//AB

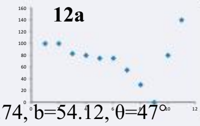


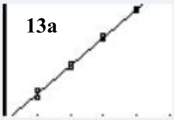


11 MO=34.70cm **12b** strong positive almost linear **12c** $A135billion **12d** Not a lot - many things can happen to the economies of Australia and China **13b** V=0.61×i-0.5 **13c** V=12milliamps **13d** V=73milliamps **14a** x=9.08 **14b** b=8.69 **14c** a=96.84, b=103.85, θ=47° **15a** A=80°, a=110.31, c=97.01 **15b** A=30.7°, B=91.3°, b=17.68 **15c** C=60.1°, B=75.9°, a=1.22 **16a** -1 **16b** 1/2 **16c** -1/√3 **17** 10.95m^2 **21a** 20√2cm **21b** 20√3cm **21c** 35.3° **21d** 200√2cm^2

Exercise 20.3 **2** $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ **3** $a^2=b^2+c^2-2bc\cos A$ **4** A=0.5absinC **5** three sides equal, three angles equal 60° **6** m=5 **7** m=-1/3 **8** √3/2 **9** (x-3)(x-1) **10** √17

Exercise 20.4 **1a** m=-2/3 **1b** m=1 **1c** m=-0.8 **2a** m=0 // x-axis **2b** m=∞ // y-axis **3** m_{AB}=-1.5, m_{CD}=2.5, m_{EF}=-1.5, m_{GH}=1.75 AB//EF **4a** // **4b** // **4c** ⊥ **4d** ⊥ **5** x=-4.5 **6** b=-4/3 **7** y=-1.5x+3 **8a** 110° **8b** a=115°, b=40°, c=25°, d=115° **8c** y=120° **8d** a=60°, b=60°, c=60° **8e** x=59°, y=59°, z=121° **8f** a=82°, b=49°, c=49°, d=49°, e=131° **8g** x=23°, y=67°, z=23° **8h** x=15°, y=90° **8i** x=46°, y=44° **8j** x=60°, y=60° **9** ΔCDE~ΔCAB AAA, DC=60 **10** ΔCDE~ΔCAB AAA, CE/CB=1/2, thus CE=EB





11 MO=40.80cm **12b** downward trend except for last 2 points **12c** 200m **12d** very little confidence **13b** Price=82000×Bed+280000 **13c** Price=82000×3+280000=530000 **13d** Price=82000×5+280000=690000 **14a** x=10.59 **14b** b=6.12 **14c** a=50.74, b=54.12, θ=47° **15a** A=67°, a=92.70, c=90.51 **15b** A=42.1°, B=82.9°, b=13.33 **15c** C=74.8°, B=64.2°, a=5.59 **16a** 0 **16b** √3/2 **16c** 1 **17** 15.47m^2 **21a** 215√2m **21b** 53.07° **21c** m=1.33 **21d** 77000m^2

Index

This text has been written for Year 10 students. The aim of the text is to assist students in investigating and understanding the exciting and very important world of Mathematics and to implement the intent of the Australian Mathematics Curriculum.

A literature review of learning from school textbooks was used to enhance the format of this textbook.

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The author

The author is a mathematics head with 30 years experience, an honours masters in IT education, a PhD in mathematics education, a state award for mathematics education, and extensive publishing experience.

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